Essays in Experimental Economics:

Some Macroeconomic Issues

Alvaro Andrés Perdomo Strauch

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DIRECTOR DE LA TESI
Prof. Rosemarie Nagel. Departament Economia i Empresa

Universitat Pompeu Fabra Barcelona

To Cristina and Nicolás

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Abstract

This dissertation uses an experimental approach to analyze the behavior of people in controlled environments based in different theoretical issues of relatively recent discussion in macroeconomic theory. The first chapter focuses in a rational bubbles environment. It shows that people behavior is affected by the presence of sunspot messages; however, they do not necessarily react optimally to these messages. In addition, people can adapt their strategies to the optimal equilibrium strategies if these are not too much complex, but not all people have the same ability to implement this adaptation process. The second chapter shows that in an information constraint environment people pay more attention to the most important and variable sources of information. In addition, it reveals that the strategic complementarity between the choices of people is a key ingredient to explain the size of the interaction effects. The third chapter analyzes some predictions attributed to the multiple equilibria that are present in the global game models with endogenous information structure. It shows in a lab experiment that the weakest policy makers have a higher probability that their policies become unsustainable. In addition, it founds that if the uncertainty about the strength of the policy makers increases, then the probability that people attack their policies also increases.

Resumen

Esta tesis utiliza una aproximación experimental para analizar el comportamiento de las personas en ambientes controlados basados en diferentes temas teóricos de discusión relativamente reciente en teoría macroeconómica. El primer capítulo se enfoca en un ambiente de burbujas racionales. Éste muestra que el comportamiento de las personas se ve afectado por la presencia de mensajes "*sunspot*"; sin embargo, ellas no necesariamente reaccionan óptimamente a los mismos. Adicionalmente, las personas pueden adaptar sus estrategias a las estrategias óptimas de equilibrio si estas no son muy complejas; sin embargo, no todas las personas tienen la misma habilidad para adaptarse. El segundo capítulo muestra que en un ambiente con restricciones de información las personas ponen más atención a las fuentes de información más importantes y más variables. Adicionalmente, revela que la complementariedad estratégica entre las decisiones de las personas es un ingrediente esencial para explicar los efectos de interacción. El tercer capítulo analiza algunas predicciones que se atribuyen a los múltiples equilibrios que se presentan en los modelos de juegos globales que tienen una estructura de información endógena. Éste muestra en un experimento de laboratorio que los hacedores de política más débiles tienen una mayor probabilidad de que sus políticas se vuelvan insostenibles. Adicionalmente, éste encuentra que si la incertidumbre acerca de la fortaleza de los hacedores de política se incrementa, entonces la probabilidad de que las personas ataquen sus políticas también se incrementa.

Foreword

This dissertation consists in three self-contained papers. All papers analyze the behavior of people in controlled environments that are based in different theoretical issues of relatively recent discussion in macroeconomic theory; in particular: rational bubbles (Chapter 1), rational inattention (Chapter 2) and global games with an endogenous information structure (Chapter 3).

Almost all the lab experiments that have been done about asset price bubbles are based on theoretical models that do not have bubbling equilibria, therefore these experiments have not considered all possibilities of analysis that the environments with bubbling equilibria provide.

The goal of Chapter 1 ("Bubbles and Crashes: A Laboratory Experiment") is to analyze how people behave in a lab experiment when they face an economic environment in which there exist bubbling equilibria. As far as we know there are only two papers in this area of research[1]: Morgan and Brunnermeier (2010) and Kang, Ray and Camerer (2012)[2]. The experiments of these papers reveal several aspects about the behavior of people: (1) people effectively take advantage of the presence of rational bubbles, but they usually attack the bubbles too early; (2) the early attacks are more usual in people that have higher levels of risk aversion or anxiety; (3) during the experiments people learn to wait until the moment in which is optimal to attack the bubble; and (4) if they can observe the choices of the other people then there is a herding behavior when someone decides to attack the bubble. In Chapter 1 we propose a rational bubbles model[3] with some characteristics that do not appear in the theoretical model used as benchmark in Morgan and Brunnermeier (2010) and Kang, Ray and Camerer (2012); then, we propose a lab experiment based in this theoretical model. We found many of the results obtained in the experiments commented above. However, we also obtained some results that are new: in the bubbling equilibrium environment, people do not necessarily react optimally to sunspot messages even though the presence of these messages affect their behavior; and people can adapt their strategies to the optimal equilibrium strategies if these strategies are not too much complex, but not all people have the same ability to implement this adaptation process.

Mackowiak and Wiederholt (2009) built a rational inattention macroeconomic model that can explain some important stylized facts of the US economy; these explanations are possible due to the following two theoretical results obtained in their model: (1) when idiosyncratic conditions are more variable or more important than aggregate conditions, firms pay more attention to idiosyncratic conditions than to aggregate

[1] Moinas and Pouguet (2013) propose a bubble game that can be adapted to produce rational or irrational bubbles. However, we do not include this paper to the list because the speculative analysis proposed in their paper can be done perfectly using only the irrational bubbles version of the model.

[2] The lab experiments of both papers are based in the same theoretical model which is explained in the first sections of Morgan and Brunnermeier (2010).

[3] Based on the model built by Abreu and Brunnermeier (2003)

conditions; and (2) there are interaction effects because firms track endogenous variables.

The goal of Chapter 2 ("Multiple Sources of Information and Strategic Behavior") is to examine in a lab experiment the two theoretical results obtained by Mackowiak and Wiederholt (2009). We propose a rational inattention model that also obtains these two theoretical results. In addition, to better understand the interaction effects, we also contemplate an extension to the model in which we consider the possibility of strategic behavior. We then propose a lab experiment that follows closely our model. In the experiment, the behavior of the participants is coherent with the theoretical results obtained by Mackowiak and Wiederholt (2009); that is, the agents pay more attention to the sources of information that are more important and variable, and the interaction between agents that have incomplete information affects the choices of the other agents (i.e. there is an interaction effect). Our main finding in this chapter is that the strategic complementarity between the choices of the participants in the experiment is quite important to explain the deviations to the equilibrium strategies.

In Chapter 3 we discuss a new model related to global games. One appealing characteristic of the standard global game models is the equilibrium uniqueness obtained in these models. However, many papers have found that the multiplicity of equilibria may reappear when the endogeneity of the information structure is taken into account. Angeletos and Pavan (2013) propose a general global game model with endogenous information that also obtains multiple equilibria. One of the novelties of their research is that they identify some specific predictions that are inherent to these equilibria.

The goal of Chapter 3 ("Global Games with Endogenous Policy Intervention: An Experiment") is to analyze if the predictions of Angeletos and Pavan (2013) can also be obtained in a lab experiment that is based on an endogenous policy intervention global game model. In the first part of the chapter we explain the model. Then we explain the experimental design and the results obtained in the experiment. We found some discrepancies between the behavior of the participants in the experiment and the equilibrium strategies predicted by the model. However, some of the predictions found by Angeletos and Pavan (2013) are also obtained in the experiment. For instance, the weakest policy makers have a higher probability that their policies become unsustainable. In addition, we found that if the uncertainty about the strength of the policy makers increases, then the probability that the agents attack their policies also increases.

Contents

Chapter 1

BUBBLES AND CRASHES: A LABORATORY EXPERIMENT

1.1 Introduction

One of the most interesting questions about financial markets is how traders behave when there is an asset price bubble[1]. It is complicated to answer this question using exclusively field data because of a lack of control over the environment in which the bubble happens. In particular, with field data, it is difficult to have control over the fundamental value of the assets and over the information that the traders have. On the other hand, the analysis of bubbles using information obtained from lab experiments is more transparent in the sense that the fundamental value and the information that the agents have can be predetermined directly in the experimental design.

Some scholars have traditionally argued that bubbles happen due to the irrationality of traders[2]. However, since the eighties some economists have tried to show that bubbles also begin and are perpetuated in environments in which rational traders play an important role.

In a finite horizon economy in which all traders are rational and there is common knowledge about the price and the fundamental value of the assets[3] the principle of backward induction implies that it is not possible to have a bubbling equilibrium[4].

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[1] An asset price bubble happens when the price of an asset is higher than its fundamental value during several periods.

[2] For instance, when Sir Isaac Newton was asked about the continuance of the rising of South Sea stock? - He answered, "that he could not calculate the madness of people." (Spence, Anecdotes, 1820, p. 368). During the South Sea Bubble Newton initially made substantial profits and then re-invested; he lost ultimately £20,000, a fortune at that time (Carswell, 1960). More recently, in 1996 Alan Greenspan (Chairman of the Federal Reserve of the United States from 1987 to 2006) used the expression irrational exuberance to describe the movements of speculative bubbles in financial markets.

[3] Therefore, when there is a bubble each trader knows the bubble, each trader knows that all other traders know the bubble, each trader knows that all other traders know that all traders know the bubble, and so on.

[4] There is not a bubbling equilibrium because in the last period (e.g. period T) the rational agents are not dispose to face the lost that implies to have overvalued assets. Thus in period T, given the assumption of common knowledge, no one is disposed to buy these assets. Therefore, in period T-1, given the assumption of common knowledge, no one is disposed to have and consequently to buy overvalued assets. In conclusion, by backward induction, in the first period no one is disposed to buy overvalued assets so the bubble is never realized at the equilibrium.

Therefore, to introduce this kind of equilibrium in an environment of rational traders we have to assume that these traders trade according to an unbounded horizon or that there are some asymmetries in beliefs such that the presence of bubbles is not common knowledge.

Respect to the first assumption, for instance, Tirole (1982) in a discrete-time finite-horizon setting showed that price bubbles depend on the myopia of traders and these are ruled out if traders adopt a truly dynamic maximizing behavior; Blanchard and Watson (1982) explained using an infinite horizon model that rational agents only hold a bubble asset if the bubble grows in expectation ad infinitum[5]; and Tirole (1985) found that in overlapping generations economies rational asset price bubbles only happen at the equilibrium when the interest rate is lower than the growth of the economy, that is bubbles are only possible in economies that are dynamically inefficient. Abel, Mankiw, Summers and Zeckhauser (1989) criticize the result obtained by Tirole (1985) because they have found using empirical data that the U.S. economy is dynamically efficient[6]. However, many economists introducing mainly financial frictions into the analysis have shown that bubbles are also possible in dynamically efficient economies (e.g. Farhi and Tirole, 2012; Martin and Ventura, 2012).

Respect to the second point, there are many ways to break the assumption of common knowledge in a finite horizon economy. For instance, Allen and Gorton (1993) show that when there is asymmetric information between investors and portfolio managers, portfolio managers can gain from buying overpriced assets, since trading allows them to fool their clients into believing that they have superior trading information[7]; De Long, Shleifer, Summers and Waldman (1990) show that the presence of behavioral traders imply a noise trader risk that incentives rational traders to perpetuate the bubbles; Abreu and Brunnermeier (2003) show that the asymmetries of information about the existence of the bubbles also motivate rational traders to perpetuate the bubbles; finally, Allen, Morris and Postlewaite (1993) and Scheinkman and Xiong (2003) show that with heterogeneous beliefs and short-sale constraints rational agents pay prices that exceed their own valuation of the fundamental value because they believe that in the future they will find agents that are willing to pay more (i.e. they will find agents that have more optimistic beliefs).

There are many papers that have analyzed asset price bubbles using lab experiments[8]. In the last twenty-five years many of these experiments follow closely the setting proposed by Smith, Suchanek and Williams (1988). This setting is based in an efficient double-auction market model in which bubbles are not possible at the equilibrium. The interesting aspect of their experiment is that they obtain asset price bubbles and crashes in a finite horizon economy in which the fundamental value of the assets (and consequently the bubble) is common knowledge. The most frequent argument respect to this result is that the participants in the experiment speculate; however, Lei, Noussair and Plott (2001) using a similar setting to the one proposed by Smith, Suchanek and

[5] It means that the transversality condition that avoids the presence of overvalued assets at the infinitum does not apply.

[6] That is, in U.S. the interest rate is higher than the growth of the economy.

[7] A portfolio manager that does not trade would reveal that she does not have private information. Consequently, portfolio managers have incentives to churn bubbles at the expense of their uninformed client investors.

[8] For instance, look at the literature referenced in Sunder (1992)

Williams (1988) with the only difference that speculation is not allowed[9] have found that bubbles also appear. On the other hand, Crocket and Duffy (2015) propose an experiment based on a version of the consumption asset pricing model of Lucas (1978) that does not have a bubbling equilibrium, and they find that in identical economies bubbles are low (high) when consumption smoothing is induced via a concave (linear) utility.

The main characteristic of the experiments remarked in the previous paragraph is that they have found bubbles in settings in which according to the economic theory bubbles should not appear. However, Brunnermeier and Morgan (2010) have proposed a clock game experiment based on a clock game model in which there is a bubbling equilibrium. In this game there is a clock that goes backwards when there is a bubble, if the clock arrives to zero the bubble bursts however it also bursts if there is an enough mass of traders that have already sold the only asset that each trader has. The traders do not know the exact moment at which the clock begins, but they are informed in an asynchronized way about this event, therefore the size of the bubble (an also the presence of the bubble) is never common knowledge. At the equilibrium the agents sell their asset at the moment at which the earnings of having the asset are equal to expected earnings of selling it. The main results obtained in the model (which are partially supported by their lab experiment) are: (1) the delay of selling the asset decreases as the agents are more synchronized respect to the moment they are informed about the bubble; and (2) if the bubble becomes common knowledge, due to a signal that is observed by everyone, then traders herd immediately. Kang, Ray and Camerer (2012) also take as setting the model proposed by Brunnermeier and Morgan (2010), the main difference is that in their experiment one human player is trading against some computerized players (in the original experiment there are only human players). Kang, Ray and Camerer (2012) test the first result obtained by Brunnermeier and Morgan (2010) and they also analyze how is the learning process that the human traders have after many times playing the game[10].

Most of the lab experiments observing asset price bubbles are based on theoretical models that do not have a bubbling equilibrium, therefore they do not take into account all the richness that can be found in models that consider this kind of equilibrium. The goal of this chapter is to propose a lab experiment that uses a benchmark model with bubbling equilibrium. More specifically, we propose a modified version of the model built by Abreu and Brunnermeier (2003), we explain the experimental design used in our lab experiment and finally we analyze how human agents behave in this theoretical context. The theoretical model proposed by Brunnermeier and Morgan (2010) have also some similarities with Abreu and Brunnermeier (2003). However, there are also some differences that we have not avoided, for instance we allow traders to buy and sell their

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[9] In particular, in Lei, Noussair and Plott (2001) participants are not allowed to adopt the dual role of buyers and sellers.

[10] One important aspect highlighted by almost all lab experiments about bubbles is that the participants have to play the game many times in order to give them the opportunity to better understand the way in which the experiment works. For instance, the experiments that follow Smith, Suchanek and Williams (1988) regularly find that after many times playing the same game there is a learning process such that the number and the size of bubbles decreases. That is, after many times playing the game the behavior of the participants approaches to the behavior of the rational traders at the theoretical equilibrium.

assets as many times as they want, we inform traders sequentially about the bubble and we propose a sunspot extension which is also proposed in the original model[11].

The rest of the chapter is organized as follows. Section 1.2 explains the theoretical model. Section 1.3 presents the experimental design. Section 1.4 shows an econometric analysis about the results obtained in the experiment. Finally, section 1.5 presents some final comments.

1.2 Model

The model that we propose is a version of the model built by Abreu and Brunnermeier (2003)[12]. The technical details of this version are analyzed in Appendix A1. In the next paragraphs we will explain the most important assumptions and results obtained with the model.

The main differences with the original model is that Abreu and Brunnermeier (2003) assume a continuum of traders and continuous time. In the new version of the model the number of traders is finite and time is discrete[13].

Assume an economy with two kinds of agents which are either rational or irrational. Irrational agents are only useful to explain the emergence and the size of the bubbles[14], but they cannot recognize the future bubble collapse. Therefore, these agents are not going to be analyzed explicitly in our model[15]. On the other hand, there are N rational agents (traders) who understand that an eventual collapse of a bubble is inevitable. The rational agents are risk neutral and they know that a coordinated attack to the bubble can precipitate its collapse, but they cannot communicate to each other to coordinate an attack.

There are two types of assets: safe and risky. The price of each kind of asset has a stock market value and a fundamental value. The stock market value of the safe assets is always equal to its fundamental value and both grow every period at a rate of r. On the other hand, until period $t_0 - 1$ the stock market value of the risky asset is equal to its fundamental value and both grow every period at a rate of $g > r$. But, from period t_0 afterwards the fundamental value of the risky assets grows every period at a rate of r, and the stock market value of the risky assets grows every period at a rate of g, except in the period when the bubble bursts, in this period the stock market value of the risky asset falls until to catch up its fundamental value. Therefore, if a rational agent has a portfolio of only risky assets when the bubble bursts then she loses all capital gains obtained in these assets during bubble's appreciation (Figure 1.1).

[Figure 1.1]

[11] In Brunnermeier and Morgan (2010) a trader finishes to play as soon as she sells the only asset that she has; traders are informed in a uniform and independent way about the bubble; and their model and experiment do not consider the sunspot extension.

[12] We do not use exactly the same version of the model that was proposed by these authors because some assumptions of the original model have problems to be implemented in a lab experiment.

[13] Time is discretized in a way that traders can only make at most one kind of financial transaction per period.

[14] That is, irrational agents sustain the price of the bubble during the bubbling period.

[15] Similarly, the analysis of irrational agents is also not considered in the model of Abreu and Brunnermeier (2003).

We will assume that at time $t = 0$ rational traders are fully investing in the risky asset and they cannot recognize immediately when there is a bubble, but since period t_0 (i.e. since the beginning of the bubbling period) they are sequentially aware, one trader per period, about its existence. Therefore, in period t_0 there is only one trader who knows the existence of the bubble, in period $t_0 + 1$ a new agent is aware about the bubble and this process continues until period $t_0 + N - 1$ in which all agents are aware about the existence of the bubble.

When rational traders are aware about the existence of the bubble, they cannot recognize exactly the number of periods that has elapsed since the emergence of the bubble and how many other rational traders recognize the bubble before or after them. In particular, period t_0 is unknown, however all traders know that variable $t_0 \in [1, 2{,}3 \ldots, \infty)$ and it has a geometric cumulative distribution function $\Phi(t_0) = 1 - (\mathcal{P})^{t_0}$ where $\mathcal{P}$ is the probability that a bubble does not begin in a specific period.

In every period all traders decide the kind of portfolio they want to have, but diversification is not allowed[16]. Therefore, every period a trader who has a portfolio of risky assets has to decide if she wants to sell her portfolio to buy a portfolio of safe assets. Similarly, every period a trader who has a portfolio of safe assets has to decide if she wants to sell her portfolio to buy a portfolio of risky assets. In the model buyback shares (i.e. buyback portfolios) is allowed.

The bubble can burst by exogenous or endogenous reasons. It bursts by endogenous reasons when an enough number of rational traders have a portfolio of safe assets. More specifically, it bursts in a specific period by endogenous reasons if τ_κ traders have portfolios of safe assets[17]. It implies that the value of the portfolio of a specific trader can be affected by the actions of the other traders if these actions cause an endogenous burst of the bubble. On the other hand, the bubble bursts by exogenous reasons when it is old and if an endogenous bursting has not happened previously. More specifically, the exogenous burst happens at time $t_0 + \bar{\tau}$ where $\bar{\tau}$ is the maximum age that a bubble can has.

All agents who have a portfolio of risky assets at the moment of the bubble bursting suffer capital losses because of the fall in the stock market price (as it is shown in the right hand side of Figure 1.1). However, in the model there is a rule in which no more than τ_κ traders can have portfolios of safe assets in the same period. Therefore, if in a specific period (e.g. period t) some traders decide to change their portfolio of risky assets by a portfolio of safe assets, they can do it without any problem except in the case in which our rule is violated. In this specific case, some of the traders who try to get the portfolio of safe assets cannot make the financial transaction. These traders are chosen randomly such that in period $t + 1$ the number of traders with a portfolio of safe assets is equal to τ_κ and consequently there is an endogenous bursting of the bubble.

As we will explain in the next section, in the experiment we are more interested to analyze the endogenous burst of the bubble. Therefore, we will assume (as it is

[16] Abreu and Brunnermeier (2003) allow diversification, however in equilibrium it is not optimal to have a diversified portfolio. Therefore, our assumption is not critical and allows us to have a shortcut to describe faster the solution of the model. Additionally, the assumption of not allowing diversification is directly related to the experiment proposed in the next section of the chapter.

[17] Since only one new trader is informed per period about the existence of the bubble, then τ_κ also represents the length of time until the number of traders who know the existence of the bubble is enough to burst it.

explained in Appendix A1) a combination of parameters such that in equilibrium an endogenous burst is ensured.

Define τ^* as the lowest integer value of τ that solves the following inequality:

$$\left(\frac{g-r}{1+r}\right)-\left\{1-\left[\frac{1+r}{1+g}\right]^{(\tau_\kappa+\tau)}\right\} h(t_i+\tau|t_i)\geq 0 \quad (1.1)$$

where period t_i is the period at which trader t_i is aware about the bubble[18] and $h(t_i+\tau|t_i)=\frac{\frac{[1-\mathcal{P}]}{\mathcal{P}}}{1-(\mathcal{P})^{\tau_\kappa}}$ is the hazard rate that the bubble will burst at period $t_i+\tau$ [19]. Therefore, there exists a unique trading equilibrium[20] in which all traders that become aware about the bubble after or at period τ_κ get the portfolio of safe assets τ^* periods after the period in which they become aware of it. On the other hand, all traders who become aware about the bubble before period τ_κ get the portfolio of safe assets at period $\tau_\kappa+\tau^*$. In this equilibrium the portfolio of safe assets is maintained until the bubble bursts.

In equation (1.1) the first term represents the earnings obtained per asset by continuing one additional period with the portfolio of risky assets and the second term represents the expected potential losses per asset by continuing one period more with the portfolio of risky assets, where $\left[\frac{1+r}{1+g}\right]^{(\tau_\kappa+\tau)}$ represents the decrease in prices a trader with a portfolio of risky assets faces when the bubble burst at period $t_0+\tau_\kappa+\tau$.

Finally, notice that τ^*can also be represented as[21]:

$$\tau^* \lfloor\geq\rfloor \left[\ln\left(1-\frac{\frac{g-r}{1+r}}{\frac{\frac{[1-\mathcal{P}]}{\mathcal{P}}}{1-(\mathcal{P})^{\tau_\kappa}}}\right)\left[\ln\left(\frac{1+r}{1+g}\right)\right]^{-1}\right]-\tau_\kappa \quad (1.1')$$

From expressions (1.1) and (1.1') we can infer that τ^* is the same for all t_i agents and it does not depend on time. This result is mainly due to the assumption that t_0 is randomly determined by a geometric distribution function that is memoryless[22].

Public signals or synchronized events

Until now, we have not assumed any mechanism to coordinate an attack to the bubble. However, in addition to the previous assumptions, assume that the agents know the existence of public signals (or synchronized events) that can induce a synchronized

[18] That is, in the model t_i is a random variable such that $t_i\in[t_0,t_0+1,t_0+2,\dots,t_0+N-1]$

[19] That is, $h(t_i+\tau|t_i)$ is the probability that the bubble that has survived until period $t_i+\tau$ will burst at period $t_i+\tau$

[20] A trading equilibrium (when diversification is not allowed) is defined as a Perfect Bayesian Nash Equilibrium in which every trader who has a portfolio of safe assets (correctly) believes that all traders who became aware of the bubble prior to her also have portfolios of safe assets.

[21] The operator $\lfloor\geq\rfloor$ means the minimum integer value of τ^* that satisfies the inequality.

[22] A similar result is obtained by Abreu and Brunnermeier (2003) who assumed that t_0 is randomly determined by an exponential distribution function (this distribution function is also memoryless)

attack to the bubble. Assume these signals appear randomly at the rate θ and are only observed by traders who already became aware about the bubble.

Define τ^{**} as the lowest integer value of τ that solves the following equation

$$\frac{g-r}{1+r} - \left[1 - \left(\frac{1+r}{1+g}\right)^{(\tau+\tau_\kappa)}\right] h(t_i + \tau | t_i) -$$
$$\theta \sum_{s-\tau_\kappa=0}^{\tau} \sum_{t_0=t_i+\tau-1+s-\tau_\kappa} \left[\frac{s-\tau_\kappa}{\omega}\left(1 - \left(\frac{1+r}{1+g}\right)^{(\tau_\kappa+\tau)}\right)\right] \phi(t_0|t_i) \geq 0 \qquad (1.2)$$

where $\frac{s-\tau_\kappa}{\omega}$ is the probability that a trader, who observe the public signal, has a capital loss when there is a successful synchronized attack to the bubble. ω is the number of traders who pretend to get the portfolio of safe assets in the period immediately after the public signal[23] and $s - \tau_\kappa \geq 0$ is the number of traders who could not get the portfolio of safe assets in the period immediately after the public signal (so this $s - \tau_\kappa$ traders have capital losses when the bubble bursts due to the synchronized attack to the bubble). $h(t_i + \tau|t_i) = \frac{\frac{[1-\mathcal{P}]}{\mathcal{P}}}{1-(\mathcal{P})^{\tau_\kappa}}$ is the hazard rate that the bubble will burst at period $t_i + \tau$; and $\phi(t_0|t_i) = \frac{\mathcal{P}^{-N}[1-\mathcal{P}]}{\mathcal{P}^{-N}-1}$ is the conditional probability mass function of t_0. Notice that if there are not public signals then $\theta = 0$ and we obtained an equation identical to equation (1.1).

In equation (1.2) the first term represents the earnings obtained per asset by continuing one additional period with the portfolio of risky assets. The second term represents the expected potential losses per asset by continuing one period more with the portfolio of risky assets if there is an endogenous burst. Finally, the last term represents the expected potential losses if there is a synchronized attack to the bubble which was motivated by the appearance of a public signal.

Under the new assumptions there are multiple trading equilibria[24] but (as it is explained in Appendix A1) there exists a unique responsive equilibrium[25]. In this equilibrium each trader t_i always sells the portfolio of risky assets at the instances of synchronizing events t_e such that $t_e \geq t_i$ and $t_e \geq \tau_\kappa$ (i.e. in t_e there is a synchronized endogenous attempt to burst the bubble). Furthermore, trader t_i stays with the portfolio of safe assets for all $t \geq t_i + \tau^{**}$ except in the event that the last synchronized endogenous burst attempt failed in which case she re-buys the portfolio of risky assets for the interval $t \in (t_e, t_e + \tau^{**})$, unless a new synchronizing event occurs in the interim.

To summarize, in the model proposed in this section we have found the optimal delays ($\tau^*$and τ^{**}) traders have in two different scenarios. In the first one the individuals are aware in private but sequentially about the existence of a bubble. In the other one,

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[23] More specifically, $\omega = \begin{cases} \tau & if\ s-\tau_\kappa \leq \tau \leq s \\ s & if\ s < \tau \leq N \end{cases}$. From this equation, notice that τ is also the maximum number of traders who are aware about the bubble and has the portfolio of risky assets.

[24] For instance, if all traders do not pay attention to public events then we obtained the same equilibrium analyzed previously.

[25] A responsive equilibrium is a trading equilibrium in which each trader believes that all other traders will synchronize (selling their portfolio of risky assets) at each synchronizing event if it happens at or after period τ_κ.

some agents also observe public signals that can help them to coordinate endogenous attempts to burst the bubble. In the next section we proposed an experiment to analyze if this behavior is coherent when an individual is facing an economy populated by computational traders who behave as rational traders and have beliefs which are consistent with the trading and the responsive equilibria respectively.

1.3 Experimental design

The experiment is based in the theoretical model proposed in the previous section. We prepared two sessions, one session was for the case in which there were no public signals (baseline session) and the other corresponds to the cases in which individuals can observe a public signal without any informational content (sunspot session). 19 subjects were recruited from the UPF Leex Lab to participate in each session, and no subject appeared in more than one session. The software used in the experiment was z-tree. At the beginning of each session, the subjects were seated at computer terminals and given a set of instructions, which were then read aloud by the experimenter. A copy of the instructions appears in Appendix B1. To ensure that subjects understood the game structure, some examples and some questions of understanding were administered at the end of the instructions.

In the instructions of the experiment we avoided to use some words or sentences that could lead to negative (or positive) misunderstandings. For instance, in the experiment instead of denoting the assets as risky or safe (as was done in the theoretical model) we named them as private and public assets respectively. Similarly, we did not use the word bubble, so when there was a bubble we talked about a divergence between the stock market value and the true value of the private assets[26]. In the rest of the chapter we will continue the explanation with the same terminology that we have used in section 1.2.

Each participant played the same game 50 times in succession, all under the same treatment. Each participant played in his own market with 24 computer players (i.e. $N = 25$) programmed to follow the theory predicted equilibrium strategy[27]. Participants were informed that they were playing with computer players, who would receive the same amount of information as they had and who would employ a consistent strategy throughout the experiment. However, the strategy of the computer players remained unknown to the participants. All the time subjects knew the current value of their respective portfolios but during each game they never knew the current value of the portfolio of the computer participants; only at the end of each game the human participants were informed about the highest three portfolio values obtained by the computer players. In addition, at the end of each game the agents were informed about

[26] In addition, we changed the word games by transaction rounds and we did not denominate the participants of the experiment as players.

[27] We also considered the possibility of only having human players within a game. However, this alternative has two inconveniences that were found in a pilot experiment: (1) each game takes too much time and (2) the amount of information that we can obtain per individual is low. These two inconveniencies became critical because most of the experiments that analyze asset price bubbles have found that the learning process is important.

the current and cumulative earnings (in ECUs)[28] they have at that moment in the experiment.

To minimize problems due to the Active Participation Hypothesis we restrict the human trader to interact in the game only when she is aware about the existence of the bubble[29]. More specifically, in each game the first screen that human subjects can see in their computers corresponds to the period in which they are informed about the existence of the bubble (notice that this period changes from game to game because t_0 and consequently t_i are randomly determined). In addition, we will analyze if this restriction helps us also to solve at least in part the time effect problem found in the experiments done by Brunnermeier and Morgan (2010) and Kang, Ray and Camerer (2012). In our experiment the time effect happens if the individuals tend to sell their portfolio of risky assets too early (i.e. if in the experiment the values of $\tau^*$and τ^{**} are lower than the results obtained in the theoretical model).

At the start of each game (i.e. in period 0), the human player and the 24 computer players received a portfolio of risky assets that has the same value. Since the solution of the theoretical model is not affected by the value of the portfolio (or by the level of prices) we decided to normalize this value in 1 ECU at the moment the human players are informed about the existence of the bubble. That is, in the first screen that the agents see in every game the current value of their portfolio is always equal to 1 ECU. With this procedure we are trying to avoid money illusion effects[30] and to simplify the information that individuals can see in the screen of their computers. In addition, the participants in the experiment do not need to know the value of their portfolio without the normalization because this value is not critical to determine the earnings that the agents have in every game.

In each game when a human player begins to participate (i.e. when she becomes aware about the bubble) z-tree randomly has already determined the age of the bubble, and consequently the periods of the endogenous and the exogenous bursts. However, the players do not have this information and consequently they cannot infer exactly how many subjects were aware about the bubble before them and the period in which the bubble bursts. They only know that since the first period there is a probability of 5% per period that the bubble begins (i.e. $1-\mathcal{P}=0.05$).

Individuals are informed that diversification is not allowed and that each game ended once 15 traders have a portfolio of safe assets in the same period (i.e. $\tau_\kappa=15$) or when 100 periods have elapsed since the beginning of the bubble (i.e. $\bar{\tau}=100$). However, they do not know that the experiment is programmed such that the exogenous burst never happens. More specifically, they are not informed that the baseline experiments

[28] ECU (or Experimental Currency Unit) is the monetary denomination used in the experiment.

[29] The Active Participation Hypothesis implies that subjects in experimental markets trade because they feel that they are supposed to trade, even if it does not increase their payoffs (Lei, Noussair and Plott, 2001). Even though our experiment does not follow the structure of Smith, Suchanek and Williams (1988) (which is the structure used by Lei, Noussair and Plott, 2001), the fact that the bubble does not emerge immediately implies that agents can become impatient (remember that in our experiment the bubble emerges in a random period because it is critical that agents cannot identify perfectly the moment at which it begins).

[30] We consider that it is important to eliminate money illusion effects because period t_0 is randomly chosen, then the value of the portfolio at period t_i (i.e. when the individual t_i is informed about the bubble) without normalization should not reveal any information about the size of the bubble or about the period in which it bursts.

always end one period after, one period before or at the period $t_0 + \tau_\kappa + \tau^*$[31]. In addition, they do not know that in the cases in which there exist public signals the bubble does not burst later than $t_0 + \tau_\kappa + \tau^{**}$; and they do not know that each time there is a public signal all computer traders who became aware about the bubble decides to obtain the portfolio of safe assets.

In addition to the 5€ show-up fee, subjects were paid whatever they earned during the experiment. The earnings of each game were determined according to the following equation:

$$Earnings\ per\ game = 0.01\ € \times \left(\frac{Final\ value\ of\ the\ portfolio\ of\ the\ human\ trader}{Final\ fundamental\ value\ of\ the\ portfolio\ of\ private\ assets}\right)$$

$$= 0.01\ € \times \left(\frac{Normalized\ final\ value\ of\ the\ portfolio\ of\ the\ human\ trader}{Normalized\ final\ fundamental\ value\ of\ the\ portfolio\ of\ private\ assets}\right)$$

Earnings averaged 12.2€, and each session lasted around 90 minutes. To summarize, the parameter values used in the experiment and known to the traders were:

- Number of traders: $N = 25$
- Number of traders necessary for bubble bursting: $\tau_\kappa = 15$
- Probability that a bubble begins: $(1 - \mathcal{P}) = 0.05$, then $\mathcal{P} = 0.95$
- $g = 10\%,$
- $r = 1\%,$
- Delay of the exogenous crash: $\bar{\tau} = 100$

Therefore, according to the theoretical model in the baseline session there was always an endogenous crash[32] (with no buyback) and the optimal equilibrium delay was $\tau^* = 13$[33]. Thus, if an individual becomes aware about the bubble after period 15 she should strategically delay for 13 periods until she obtains the portfolio of safe assets and she should maintain this portfolio until the end of the game. On the other hand, according to the theoretical model if an individual is informed about the bubble before or at period 15, then she will obtain the portfolio of safe assets in period 28 ($\tau_\kappa + \tau^* = 15 + 13$). Consequently, the bubble almost always bursts in period $t_0 + \tau_\kappa + \tau^* = t_0 + 28$ (where t_0 is randomly determined in each game)[34].

On the other hand, in the sunspot session we considered also the presence of a random synchronized event that appears every period at a rate of 5% (i.e. $\theta = 0.05$) and this event is only known by the agents who already know about the existence of the

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[31] Notice that the game can finish one period before (after) period $t_0 + \tau_\kappa + \tau^*$ if the human player retains the portfolio of risky assets less (more) time than the theory predicts.

[32] The condition to ensure an endogenous burst is satisfied. From Appendix A1 we know that this condition is $\frac{\frac{g-r}{1+r}}{1-\left(\frac{1+r}{1+g}\right)^{\bar{\tau}}} < \frac{\frac{[1-\mathcal{P}]}{\mathcal{P}}}{1-(\mathcal{P})^{\tau_\kappa}}$ (if we replace the parameters by the respective value that we are considering we get 0.092<0.098)

[33] This value was obtained numerically from equation (1.1) and it is also equal to the analytical value that results from equation (1.1').

[34] It can burst also one period after or one period before depending on the behavior of the human player.

bubble. In this case the optimal delay is $\tau^{**} = 7$[35] and the game always finishes before or at $t_0 + \tau_\kappa + \tau^{**} = t_0 + 22$.

In a clock game experiment the length of each period is a time interval that has been previously predetermined (in this kind of experiments regularly the length of each period is a fraction of a second). On the other hand, in the experiment that we propose the participants have direct control over the length of each period (i.e. each period changes immediately the participant makes a decision). Therefore, we consider that our experiment has the advantage that the human traders can reason more carefully their decisions and they are not mainly acting as a reflex action.

Finally, since in the experiment we are using a human player faced with only computer players, then our experiment has some of the advantages already commented by Kang, Ray and Camerer (2012), in particular: (1) we can control collusive strategies[36]; (2) human players do not face strategic uncertainty about behavior of other people[37]; and (3) using computer opponents also prevents subject from being able to guess when other agents have sold from physical sound such as key pressing[38].

Baseline and Sunspot Sessions.

The main goal of our experiment is to evaluate how a human player behaves when she faces the theoretical setting proposed in the previous section. Remember that the other players (i.e. the computer traders) play an equilibrium strategy that is unknown to the human player. In addition, we want to analyze how the presence of public signals affects the behavior of the human players. To reach these goals we propose the following two sessions[39]:

- **Baseline Treatment Session:** In this session the traders are randomly but sequentially informed by a private message when there is a bubble. The purpose of this session is to replicate the model when there is private and sequential awareness about the existence of the bubble but there is not public information that can be useful to coordinate an attack to the bubble.
- **Sunspot Treatment Session:** In this session traders also receive a public message without any informational content. This message is only observed by all those traders who are already aware about the bubble and it appears with a probability of 5%. The public message that we propose is "*Do not leave for tomorrow what you can do today*" There are other messages that we could have used; for instance, Duffy and Fisher (2005) show that the semantics of the sunspot message matter.

[35]This value was obtained numerically from equation (1.2). Notice that it is not possible to get τ^{**} using an analytical procedure.

[36] That is, subjects cannot coordinate to wait longer than they would otherwise, and split the high payoff outside the lab.

[37] Instead, they face the challenge of learning the computerized agents' strategy

[38] That is, we are avoiding information leakage

[39] Besides these two scenarios, Abreu and Brunnermeier (2003) also proposed a situation in which there are random temporary price drops that are observed by all traders. We did not analyze this extension because its solution is similar in structure to the sunspot extension and it increases in an important amount the complexity of the experimental design. More specifically, this extension requires that we have to consider in the analysis a function τ_p (that depends on the history of price drops) such that given a price drop at any period t_p, then all traders t_i who became aware of the bubble prior to period $t_p - \tau_p$ sells the portfolio of risky assets and all the other traders do not sell.

However, according to our theoretical model a neutral message is enough to get coordination.

In this session the computer traders that are already aware about the bubble always get the portfolio of public (safe) assets immediately after the public message appears. This treatment session is useful to analyze additional questions as: How sensible is the behavior of the human traders respect to a sunspot message? Do the agents buy back the portfolio of safe assets if there is no bubble bursting after a synchronized event?[40]

1.4 Results

Remember from the last section that in the baseline scenario if a trader is aware about the bubble before period 15 (i.e. if $t_i < 15$)[41] then her trading equilibrium strategy is to sell the portfolio of risky assets in period 28 (i.e. $T(t_i) = 28$)[42] to continue with the portfolio of the safe assets until the bursting of the bubble. On the other hand, if a trader is aware about the bubble at or after period 15 (i.e. if $t_i \geq 15$) then her trading equilibrium strategy is to sell the portfolio of risky assets 13 periods after she was aware about the bubble (i.e. $\tau^* = 13$) to continue with the portfolio of the safe assets until the bursting of the bubble.

Similarly, remember that in the sunspot scenario if a trader is aware about the bubble before period 15 then the recursive equilibrium strategy is to sell the portfolio of risky assets in period 22 and if a trader is aware about the bubble at or after period 15 then her trading equilibrium strategy is to sell the portfolio of risky assets 7 periods after this awareness (i.e. $\tau^{**} = 7$). In addition, it is important to remember that the recursive equilibrium strategy implies that if a sunspot appears at or after period 15 then the traders who are already aware about the bubble immediately get the portfolio of safe assets and if the bubble does not burst then they re-buy the portfolio of risky assets for the interval $(t_e, t_e + \tau^{**})$[43], unless a new sunspot occurs in the interim.

In this section we evaluate, in the experiment, how close or far are the strategies followed by the human traders with respect to the strategies followed by the theoretical agents (and consequently, by the computer traders). The section is divided in three parts. In section 1.4.1 we will examine if the strategies followed by the participants in the experiment differ from the equilibrium strategies found in our theoretical model. In section 1.4.2 we will analyze the elements that affect the individual strategies of the individuals in the experiment. Finally, in section 1.4.3 we will analyze if the strategies followed by the human traders converge to the strategies followed by the computer traders.

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[40] Respect to the second question remember that in the model in the equilibrium without public signals there is not buyback but with public signals there is buyback when a synchronized attack could not burst the bubble.

[41] Remember that in the experiment 15 (i.e. τ_κ) is the number of traders who are enough to burst the bubble.

[42] In appendix A1 $T(t_i)$ is defined as the first instant at which trader t_i sells her portfolio of risky assets.

[43] Remember from section 1.2 and Appendix A1 that t_e is the period in which the sunspot (i.e. the synchronizing event) appears.

1.4.1. Equilibrium strategies (Theory vs. Experimental Sessions)

From the experimental design, notice that we did not consider explicitly a training section. In particular, we are assuming that players learnt during the experiment. Since our goal in sections 1.4.1 and 1.4.2 is not to analyze the learning process of the participants during the experiment, then we confine the analysis in these sections only to the last 25 times each human trader plays the game[44].

In Table 1.1 we present t-tests to analyze if the delays found in the theoretical model are statistically the same as the delays obtained in the experiment. As a result, we found that in all cases the theoretical equilibrium values are outside the 95% confidence interval; therefore, in all cases the hypothesis that the results obtained in the experiment are equal to the theoretical values is rejected. In particular notice that the theoretical equilibrium values are always higher than the 95% confidence interval of our estimated values[45]; that is, on average human traders behaved more as risk averse traders.

[Table 1.1]

In each session 950 observations were collected[46]. However, in Table 1.1 we are not using all this information because: (1) 475 observations were omitted since, as we explained before, we are confining the analysis only to the last 25 times each participant plays the game; (2) in the analysis we are only considering the games in which the players can theoretically get bubbling earnings if they apply the trading equilibrium strategy (i.e. in the analysis we are not considering games in which $t_i \geq t_0 + \tau_\kappa$)[47]; and (3) we are not taking into account games in which the bubble bursts before the agents sell the portfolio of risky assets (i.e. in Table 1.1. we are omitting in the analysis the right censored delays).

[Table 1.2]

If we recode the delays that are right censored such that these delays are assigned a value that is 1 unit higher than the maximum value obtained in each session[48], then from the t-tests that appear in Table 1.2 we obtained that in almost all cases the theoretical equilibrium value is higher than the 95% confidence interval. Only in the sunspot session in the case in which $t_i \geq \tau_\kappa$ we cannot reject in a 95% confidence interval the hypothesis that the theoretical value of the optimal delay is equal to the results obtained in the experiment.[49,50]

[44] We also did the analysis using the last 40 times each human trader plays the game (i.e. in this case we considered 10 training periods). In this case there is more volatility; however, the main conclusions of the chapter do not change.

[45] These results were also validated by non-parametric sign tests and by non-parametric Wilcoxon signed-rank tests.

[46] Remember that in each session 19 players were playing the same game 50 times in succession.

[47] That is, we are only taking into account the games in which traders can reveal as risk averse, risk neutral or risk lover traders. We are not taking into account the games if the traders do not have the option to reveal themselves as risk lover traders.

[48] For instance, from figure 1.2 and figure 1.3 we can appreciate that these assigned values are 35 in the baseline session when $t_i \geq 15$ and 26 when $t_i < 15$

[49] These results were also validated by non-parametric sign tests and by non-parametric Wilcoxon signed-rank tests.

In addition, from Figure 1.2 (i.e. in the baseline session when the traders are informed about the bubble before period 15) notice that: (1) on average traders are risk averse; (2) 17.94% of the times traders have an irrational behavior because they decide to sell their portfolio of risky assets before period 15 (i.e. before there were a high enough mass of traders that can burst the bubble); (3) only 25.64% of the times the traders sell the portfolio of risky assets between periods 26 and 30 (i.e. around the theoretical equilibrium value), and (4) there are 5.13% missing values (right censored delays).

[Figure 1.2]

Similarly, from Figure 1.3 (i.e. in the baseline session when traders are informed about the bubble at or after period 15), notice that: (1) we confirm that on average traders are risk averse; (2) only 23.33% of the times the traders sell the portfolio of risky assets between 11 and 15 periods after they were informed about the bubble (i.e. around the theoretical equilibrium value), and (3) there are 9.52% missing values (right censored delays).[51]

[Figure 1.3]

Therefore, from the previous results we conclude that in our experiment the human traders are less patient than the rational traders of the theoretical model[52]. We guess that this conclusion is the result of three assumptions used in the theoretical model that may not be validated by the participants in the experiment. First, in the model the rational agents are risk neutral, but in the experiment human traders may exhibit some amount of risk aversion. Second, the model assumes that the distribution of t_0 is memoryless (i.e. it does not depend on time), however the human traders may not interiorize this characteristic of the distribution into the analysis. Third, some participants in the experiment sometimes play irrationally in the sense that they play strictly dominated strategies; remember that in the baseline scenario 17.94% of the times traders decide to sell the portfolio of risky assets before the period in which they know there is a high enough mass of traders that can burst the bubble.

A characteristic that is part of the equilibrium strategy followed by the rational traders in the theoretical model is that if an agent receives a sunspot message before period 15 then she does not react to this message, but if she receives the message at or after period 15 then she immediately gets or continue with the portfolio of safe assets[53].

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[50] From Table 1.1 (or Table 1.2) notice that the confidence intervals of the delays in the sunspot session are always lower than the confidence intervals of the baseline session (for the same case). This result is in concordance with the theoretical results obtained in section 1.2. However, we consider that this result is mainly influenced by the fact that each human player was playing with many computer players who followed an equilibrium strategy which have a lower delay in the case of the sunspot scenario.

[51] Similar results are obtained in the case of the sunspots experiments.

[52] In the case of the sunspot session, we can argue that in Tables 1.1 and 1.2 agents seem to sell their portfolio of risky assets too early because the theoretical equilibrium values that appear in the tables are overvaluated. In particular, in these tables we do not consider that at the theoretical equilibrium the traders sell their portfolio of risky assets also when they observe sunspot messages. However, as we will see in section 1.4.2, this conclusion does not change if we take into account appropriately the sunspot messages.

[53] It is important to clarify that during the first 25 times the game was played (i.e. during the learning part of the experiment), the participants in the sunspot session experienced situations in which the bubble bursts and situations in which the bubble does not burst immediately after a sunspot message.

From Table 1.3 it seems that in the experiment they do not necessarily apply quite well this strategy because in only 39.02% of the times (i.e. 16 of 41) the agents with a portfolio of risky assets do not react (selling their portfolio) to the sunspot message when it appears before period 15 and only 71.95% of the times (i.e. 118 of 164) they immediately get or continue with the portfolio of safe assets when the message appears at or after period 15. In addition, from the first part of Table 1.3, when the traders received the sunspot message before period 15, notice that 78.08% of the times traders shows an irrational behavior because in 43.84% of the times (i.e. 32 of 73) they already have a portfolio of safe assets when they receive the message and in 34.24% of the times (i.e. 25 of 73) they react immediately to the message; however, in this scenario if we consider all games in which the participants were informed about the bubble before period 15, only in 50.00% of the games the participants in the experiment show an irrational behavior.

[Table 1.3]

Another characteristic of the equilibrium strategies obtained in the theoretical model is that in the baseline scenario the traders followed a trigger strategy in which they maintain the portfolio of risky assets an optimal number of periods, they sell this portfolio and they continue with portfolio of safe assets until the bubble bursting. That is, there is no buyback to get the portfolio the risky assets again. However, in the experiment we observe buybacks as it is shown in Tables 1.4 and 1.5.

[Table 1.4]

[Table 1.5]

Finally, respect to the sunspot scenario, the behavior of the rational traders in the theoretical model is more complex because the buyback of the portfolio of risky assets is the optimal behavior when there is not a bursting of the bubble immediately after a sunspot message. However, as you can appreciate in Table 1.6 in the experiment only 21.7% of the people (i.e. 13 of 60) that have the portfolio of safe assets decide to get again the portfolio of risky assets after a sunspot that does not imply a bubble bursting. This result is not quite unexpected because as we showed previously in Table 1.3 the participants in the experiment do not incorporate quite well the presence of the sunspot to their respective strategies.

[Table 1.6]

1.4.2. What explains the strategies in the experiment sessions?

In the previous section we have shown that the theoretical equilibrium strategies are different from the strategies followed by the human traders in the experiment. Therefore, in this section we are interested to find the elements that affect their strategies. In particular, we are interested to analyze how the following three variables are affected:

- $FirstChoice(g)$: Delay until the first time an agent gets the portfolio of safe assets in the current game g[54] (this variable was represented by τ^{*} in the baseline model and τ^{**} in the sunspot model).
- $FirstChoiceC(g)$: Period in which the agent gets the first time the portfolio of safe assets in the current game g (this variable was represented by $t_i(g)+\tau^{*}$ in the baseline model and by $t_i(g)+\tau^{**}$ in the sunspot model).
- $Numberattacks(g)$: Number of times the agent sells the portfolio of risky assets to get a portfolio of safe assets in the game g.[55]

To analyze the previous variables, we considered four kinds of independent variables[56,57]:

1. Time dependence variables:
   - $Periodti(g)$: Period in which an agent is informed about the bubble in the current game (in the theoretical model this variable was represented by $t_i(g)$).
2. Adaptive behavior variables:
   - $Periodcburst(g-1)$: Period in which the bubble burst in the previous game (i.e. approximately $t_0(g-1)+\tau_\kappa+\tau^{*}$ in the baseline experiment or approximately $t_0(g-1)+\tau_\kappa+\tau^{**}$ in the sunspot experiment)
   - $Goodtiming(g-1)$: Dummy variable equal to 1 (0) if the trader finishes the previous game with a portfolio of safe (risky) assets
   - $Periodtime(g-1)$: Number of periods the agent was aware about the bubble before the bubble bursting in the previous game
   - $Regret(g-1)$: Difference of earnings obtained by the computer player who got the higher earnings in the previous game respect to the earnings obtained by the human player in that game.

   With these variables we analyze how the agents behave depending on the experience obtained in the previous game. According to the theoretical model the adaptive behavior variables do not affect our dependent variables.
3. Trend variables:
   - $Game(g)$: Number of the game that is currently played by the human player.

   With this variable we analyze if the decisions of the human players are following a deterministic trend.
4. Sunspot variables:

   The sunspot variables try to capture the effect of the sunspot messages in the decisions of the players.

[54] Remind that in each game when an agent is informed about the bubble the value of her portfolio is equal to 1 ECU. Therefore, this variable is equivalent to use the portfolio value when the agent decided the first time to sell the portfolio of risky assets to get a portfolio of safe assets.

[55] We also analyzed a variable that represents the number of times the agent sells the portfolio of safe assets to get a portfolio of risky assets in the game g; however, the results did not change.

[56] The main characteristic of these variables is that all are known (or deduced instantly) by traders all the time or at the end of the previous game. In particular, notice that the adaptive behavior variables are the only variables that are known by traders at the end of the previous game.

[57] We also considered persistence variables (represented by the past values of the respective dependent variables) but the dynamic panel model that we regressed using the estimators proposed by Arellano and Bond (1991) and Arellano and Bover (1995) showed that our main conclusions were not affected and most of the time the persistence variables did not have statistical significant effects over the dependent variables.

- $Sunspot(g)$: Dummy variable equal to 1 if there is a sunspot.
- $Sunspotsum(g)$: Sum of sunspots in each game g.
- $Succ\_attack(g-1)$: Dummy variable equal to 1 if in the previous game there were a bubble bursting immediately after the sunspot. Notice that this is also an adaptive behavior variable. According to the theoretical model this variable does not affect our dependent variables.

5. Other variables:
   - $Periodtime(g)$: Number of periods the agent was aware about the bubble before the bubble bursting in the present game. This variable is important to control right side truncation in $FirstChoice(g)$ and $FirstChoiceC(g)$ and to analyze if the number of periods played per game affects $Numberattacks(g)$.

Tables 1.7a and 1.7b summarize the effects deduced from the theoretical model. Tables 1.8a and 1.8b show the results of the econometric estimations. More specifically, we used fixed effects panel models to analyze the variables $FirstChoice(g)$ and $FirstChoiceC(g)$[58], and conditional fixed effects Poisson panel models in the case of the variable $Numberattacks(g)$[59].

[Table 1.7a]

[Table 1.7b]

According to the theoretical model and the parameters used in the experiment if $Periodti(g) \geq 15$ $[Periodti(g) < 15]$ then at the equilibrium $FirstChoice(g)$ $[FirstChoiceC(g)]$ remains constant in the baseline session and only reacts to the presence of sunspot messages in the sunspot session. Therefore, $FirstChoice(g)$ $[FirstChoiceC(g)]$ should not be affected by any variable in the baseline session and can only be affected negatively by the variable $Sunspot(g)$ in the sunspot session (Table 1.7a).

[Table 1.8a]

[Table 1.8b]

From the econometric estimations note that in the sunspot session the variable $Sunspot(g)$ has the predicted negative effect over $FirstChoice(g)$ and $FirstChoiceC(g)$; however, we also recognize three additional effects in our experiment (Table 1.8a):

i. An adaptive behavior effect[60] in the baseline session that affects $FirstChoice(g)$. That is, the results of the previous game affect the behavior of traders that are informed later than period 15 in the baseline session.

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[58] We used the Robust Hausman test explained by Cameron and Trivedi (2009) (based on Wooldridge (2002)) to choose between the fixed and the random effects panel models.

[59] We use a Poisson approach because the variable $Numberattacks(g)$ only has small discrete positive values in the database.

[60] The agents adjust their decisions depending on information obtained in the previous game (even when they know that both games are independent)

ii. A deterministic trend effect that affects positively (negatively) $FirstChoice(g)$ in the baseline (sunspot) session. That is, agents slightly increase (decrease) their delays during the baseline (sunspot) session.

iii. A positive time effect in $FirstChoiceC(g)$. That is, if a human trader is informed near to period 1 about the bubble then she sells the portfolio of risky assets in an earlier period than if she is informed near to period 15. This effect is also appreciated in Figure 1.4

[Figure 1.4]

Finally, according to the theoretical model and the parameters used in the experiment, in the baseline session $Numberattacks(g)$ is equal to 0 if $Periodtime(g) < 13$ and equal to 1 if $Periodtime(g) \geq 13$. On the other hand, in the sunspot experiment $Numberattacks(g)$ depends positively on the number of sunspot messages. Therefore, in Table 1.7b the variable $Numberattacks(g)$ is only affected by the variable $Sunspotsum(g)$. The effect of the variable $Sunspotsum(g)$ is validated by the data; however, there is an impatience effect because the variable $Numberattacks(g)$ is also affected by the number of periods the participants play each game[61] (i.e. by the variable $Periodtime(g)$).

1.4.3. Do the strategies of the human traders converge to the strategies of the theoretical traders?

In section 1.4.1 we have found that the strategies followed by the human traders in the experiment regularly differ from the theoretical equilibrium strategies. Now, we are interested to check if the strategies of the human traders converge to the optimal theoretic strategies followed by risk neutral traders.

In particular, we will analyze if the variable $Convergence(g)$, defined as the distance of the strategies of the human traders[62] respect to the strategies of the theoretical traders, approaches to zero during the experiment (i.e. when the variable $Game(g)$ increases). If this situation happens, it means that human traders adapt their strategies respect to the strategies followed by the traders.

Formally, depending on the scenario, the variable $Convergence(g)$ is defined as[63]:

(a) Baseline scenario:

$$Convergence(g) = \begin{cases} |FirstChoiceC - 28| & if\ t_i < 15 \\ |FirstChoice - 13| & if\ t_i \geq 15 \end{cases}$$

[61] This effect happens when the agents already have the portfolio of safe assets but they are impatient because the game does not finish, so they decide to get the portfolio of risky assets again.

[62] In this definition the strategies of the human traders are determined by the first time these traders decide to sell their portfolio of risky assets to get the portfolio of safe assets (i.e. these strategies are determined by $FirstChoice$ and $FirstChoiceC$)

[63] Remember that t_i is the period at which a human trader is informed about the bubble and t_e is the period at which a sunspot message appears and is observed by the human trader (in this section, if the human trader observes many sunspot messages in the same game, t_e represents the first one depending on the corresponding conditional).

(b) Sunspot scenario:

$$Convergence(g) = \begin{cases} |FirstChoiceC - 22| & if\ t_i < 15\ \ and\ there\ is\ no\ 15 < t_e < 22 \\ |FirstChoiceC - t_e| & if\ t_i < 15\ \ and\ there\ is\ at\ least\ one\ 15 < t_e < 22 \\ |FirstChoice - 7| & if\ t_i \geq 15\ and\ there\ is\ no\ t_e < t_i + 7 \\ |FirstChoiceC - t_e| & if\ t_i \geq 15\ and\ there\ is\ at\ least\ one\ t_e < t_i + 7 \end{cases}$$

[Table 1.9]

In Table 1.9, using fixed effects panel models we get three interesting results: (1) human traders learn to react to the presence of sunspots; (2) in the baseline model human trader strategies converge to the optimal strategies when the human trader is informed about the bubble at or after period 15; and (3) human trader strategies do not converge to the strategies of the computer traders if these strategies are complex. In particular, respect to the last point, notice that: (i) there is not a convergence in the sunspot scenario because the structure of the strategies of the computer traders is more complicated than this structure in the baseline scenario; and (ii) when a human trader is informed about the bubble earlier than period 15 she has less games to learn (e.g. in the baseline model in only 17.05% of the 950 games a human trader was informed about the bubble before period 15), and also the optimal strategy is more complex to learn because in this case you have to follow the optimal strategy of the computer trader who is informed about the bubble in period 15 (remember that this strategy is unknown since the beginning of the experiment).[64]

At this point, it is important to comment that in the experiment we observe some human traders with strategies converging to the strategies of the computer traders and others using strategies that do not converge. Two examples are in Figures 1.5; in particular, notice that the strategies of human trader 15 in the baseline scenario converges to the optimal strategies when she is informed about the bubble before period 15 (red circles) and when she is informed about the bubble at or after period 15 (blue triangles); the opposite happens to the strategies of human trader 5 in the baseline scenario. Finally, in Table 1.10 using conditional fixed effects Poisson panel models we have found that in the baseline session human traders change less times their portfolio during the experiment; that is, their strategies approach to the optimal trigger strategy proposed by the theoretical model. In this table we did not analyze the behavior of the variable $Numberattacks$ during the sunspot session because the strategy of the computer traders when a bubble does not burst after a sunspot was very complex to follow by the human traders.

[Figure 1.5]

[Table 1.10]

[64] On the other hand, notice (using Figure 1.4 and Table 1.9) that when $Periodti < 15$ then an increment of the variable $Periodti$ decreases the value of the variable $Convergence$. This result is not strange because remember from section 1.4.1 that human traders are on average risk averse, then if a human trader is informed about the bubble closer to period 15 then she will attack the bubble in a period nearer to period 22 or 28 (in the sunspot and baseline scenario respectively) compared to a human trader who is informed about the bubble in a period closer to period 1.

1.5 Final comments

In the experiment that we propose in the chapter the optimal delay is not easy to get without the background, the tools and the time necessary to find and solve equations (1.1) and (1.2). Therefore, it is not strange that the participants in the experiment did not follow exactly the same equilibrium strategy proposed in the theoretical model. However, there are some particular characteristics about their strategies that are interesting to highlight.

First, human traders were most of the time risk averse; then, they speculate less than the traders in the theoretical model[65]. Therefore, in the experiment there was a pressure to burst the bubble earlier. This result differs from the conclusion obtained by Smith, Suchanek and Williams (1988) because in their experiment, respect to their theoretical model, there is a pressure to increase the size and the horizon of the bubbles.[66]

Second, human traders show some level of irrational behavior in the sense that their strategies were affected by trend and adaptive variables[67]. It can be argued that even after more than 25 games the participants were learning about the optimal strategy to follow. However, the irrational behavior also happens because 6 participants in the baseline session (i.e. 31.6% of the participants) and 12 in the sunspot session (i.e. 63.2% of the participants) in at least one game sold the portfolio of risky assets in periods lower than 15. Remember that, by construction of the experimental design, before period 15 is common knowledge that the mass of traders informed about the bubble was not enough to burst it.

Third, in around 30% of the games the human traders change their portfolio more than the optimal times predicted by the theoretical model. Therefore, they experienced some level of impatience.

Fourth, almost all participants in the sunspot experiment recognize in a questionnaire, answered at the end of the experiment, that the sunspot message was important to determine the period in which the bubble bursts. In particular, we have found econometrically that the presence of sunspots messages motivates participants to attack the bubble earlier (Tables 1.8a and 1.9). However, the participants did not necessarily react instantly to these messages; more specifically, in periods greater or equal than 15, a little more than 50% of the times a participant with a portfolio of risky assets decides to sell this portfolio immediately after the appearance of the sunspot message (Table 1.3).

Fifth, human traders do not adapt their strategies to converge to the optimal strategies played by the computer traders if the last ones are too much complex. However, they can do it if the optimal strategies are easy to follow.

Finally, as it is shown in Figure 1.5, all human traders do not have the same ability to adapt their strategies such that these converge during the experiment, at least partially, to the optimal strategies played by the computer traders.

[65] Remember that the traders in the theoretical model are risk neutral.

[66] Remember that Smith, Suchanek and Williams (1988) obtain bubbles in experiments based on a theoretical setting in which bubbles do not exist at the equilibrium.

[67] Remember that in every game the beginning of the bubble and the moment at which each agent is informed about the bubble is determined randomly. Therefore, the rational agents do not have a theoretical reason to incorporate the information of adaptive variables to establish their strategies.

To summarize, the main differences of our human traders respect to the theoretical traders is that the first ones experience some level of bounded rationality[68], they are usually not risk neutral (most of the time they are risk averse), sometimes they behave clearly irrationally, they suffer impatience and they do not necessarily react optimally to sunspot messages (even though the presence of these messages affect their behavior). However, human traders can adapt their strategies to the optimal equilibrium strategies if these strategies are not too much complex, but not all human traders have the same ability to implement this adaptation process.

[68] In the sense that the rationality of the participants in the experiment is limited by the information they have, the cognitive limitations of their minds, and the finite amount of time they have to make a decision.

1.6 Appendix A1[69]: The model – Technical details

All the assumptions and part of the notation used in the following propositions were already presented in section 1.2.

Definitions:

$\boldsymbol{N}$: Total number of rational agents (traders)

$\boldsymbol{t_0}$: Period in which the bubble appears and the first trader is aware about the existence of the bubble.

$\boldsymbol{t_i}$: Period in which trader t_i becomes aware of the bubble[70].

$\boldsymbol{s(t, t_0)}$: Given a bubble that emerges in period t_0, $s(t, t_0)$ is the number of traders that have the portfolio of safe assets in period $t \geq t_0$.

$\boldsymbol{\tau_\kappa}$: Number of traders who are enough to burst the bubble. Since only one new trader becomes aware of the bubble in each period, then τ_κ also represents the length of time until the number of traders who know the existence of the bubble is enough (assuming that the rest of traders only have portfolios of risky assets) to burst it.

$\boldsymbol{T^*(t_0)}$: Bursting time of the bubble for a given realization of t_0.

$\boldsymbol{T(t_i)}$: First instant at which trader t_i sells her portfolio of risky assets.

$\boldsymbol{\tau_{t_i}} = T(t_i) - t_i$: Length of time trader t_i chooses to ride the bubble subsequent to becoming aware of the mispricing.

$\boldsymbol{\Phi(t_0)} = 1 - (\mathcal{P})^{t_0}$: Cumulative distribution function of t_0.

$\boldsymbol{\Pi(t|t_i)} = \Phi(T^{*-1}(t)|t_i)$: Trader $t_i's$ beliefs about the bursting date of a bubble. That is, trader $t_i's$ conditional cumulative distribution function of the bursting date at time t. [71]

$\boldsymbol{\pi(t|t_i)} = \Pi(t|t_i) - \Pi(t-1|t_i)$: Associated conditional density of $\Pi(t|t_i)$. [72]

$\boldsymbol{FV_t}$**:** Fundamental value of a portfolio of risky assets that has been maintained until period t

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[69] Given that our model is a version of the model proposed by Abreu and Brunnermeier (2003), then some proofs follow closely the proofs proposed by these authors.

[70] Since only one new trader becomes aware of the bubble in each period, then $t_0 + N - 1 \geq t_i \geq t_0$.

[71] If trader t_i believes that the bubble bursts at $t_0 + \zeta$. Then, $\Pi(t|t_i) = \Pi(t_i + \tau|t_i) = \frac{(\mathcal{P})^{-N} - (\mathcal{P})^{-(\zeta-\tau)}}{(\mathcal{P})^{-N} - 1}$

[72] If trader t_i believes that the bubble bursts at $t_0 + \zeta$. Then, $\pi(t|t_i) = \pi(t_i + \tau|t_i) = \frac{(\mathcal{P})^{-(\zeta-\tau+1)}[1-\mathcal{P}]}{(\mathcal{P})^{-N} - 1}$

$\boldsymbol{P_t}$**:** Stock market price of a portfolio of risky assets that has been sold in period t $\boldsymbol{t_e}$: The date of a synchronizing event $\boldsymbol{\Gamma(t)}$: Cumulative distribution function that the bubble bursts due to the synchronized event prior to t. $\boldsymbol{\gamma(t)}$: Associated density of $\Gamma(t)$

Equilibrium: Definitions.

Trading Equilibrium: A trading equilibrium is defined as a Perfect Bayesian Nash Equilibrium in which whenever a trader's risky asset holding is less than her maximum, then the trader (correctly) believes that the risky asset holding of all traders who became aware of the bubble prior to her are also at less than their respective maximum.

Since in the model we are assuming that diversification is not allowed[73], then in this context we can refine the definition of a trading equilibrium.

Trading Equilibrium (refine definition): When diversification is not allowed, a trading equilibrium is defined as a Perfect Bayesian Nash Equilibrium in which every trader who has a portfolio of safe assets (correctly) believes that all traders who became aware of the bubble prior to her also have portfolios of safe assets.

Responsive Equilibrium: A responsive equilibrium is a trading equilibrium in which each trader believes that all other traders will synchronize (selling their portfolio of risky assets) at each synchronizing event if it happens at or after period τ_κ[74].

The refine definition of trading equilibrium and the fact that in period 0 all traders have portfolios of risky assets implies directly the following corollary.

Corollary 1: *In a trading equilibrium when trader t_i sells her portfolio of risky assets, all traders t_j where $t_0 \leq t_j < t_i$ also have already sold, or will at that moment sell their portfolio of risky assets.*

[73] That is, all traders have portfolios of only risky assets or portfolios of only safe assets.

[74] The last part of the definition of the recursive equilibrium is an extension to the original definition proposed by Abreu and Brunnermeier (2003). This extension is important because in our model (and similarly in Abreu and Brunnermeiers' model) it is never an equilibrium strategy to get the portfolio of risky assets before period τ_κ (i.e. before the number of traders who know about the bubble is enough to burst it).

What happen if there is private and sequential awareness about the bubble?

Proposition 1: *Define τ^* as the minimum integer value of τ that solves the following inequality* $\left[\frac{g-r}{1+r}\right] - \left(1 - \left[\frac{1+r}{1+g}\right]^{(\tau_\kappa+\tau)}\right)\frac{\frac{[1-\mathcal{P}]}{\mathcal{P}}}{1-(\mathcal{P})^{\tau_\kappa}} \geq 0$ *and suppose* $\frac{\frac{g-r}{1+r}}{1-\left(\frac{1+r}{1+g}\right)^{\bar{\tau}}} < \frac{\frac{[1-\mathcal{P}]}{\mathcal{P}}}{1-(\mathcal{P})^{\tau_\kappa}} < \frac{\frac{g-r}{1+r}}{1-\left(\frac{1+r}{1+g}\right)^{\tau_\kappa}}$. *Then there exists a unique trading equilibrium in which all traders t_i such that $t_i \geq \tau_\kappa$ get the portfolio of safe assets τ^*periods after they become aware of the bubble. On the other hand, all traders t_i such that $t_i < \tau_\kappa$ get the portfolio of safe assets at period $\tau_\kappa + \tau^*$.*

Lemma 1 and Proposition 2 are important to prove Proposition 1.

Lemma 1: *In equilibrium, trader t_i believes at time $T(t_i)$ that at most τ_κ traders became aware of the bubble prior to her. In other words, the lower bound of support of trader t_i's posterior beliefs about t_0 is $t_0^{supp}(t_i) \geq t_i - \tau_\kappa$*[75].

Proof of Lemma 1 (by contradiction): Assume in equilibrium, trader t_i believes at time $T(t_i)$ that $\hat{\tau} > \tau_\kappa$ traders became aware of the bubble prior to her. This assumption is a contradiction because all traders $t_j \in [t_0, t_0 + \tau_\kappa)$ have an incentive to participate in an endogenous burst attempt before the possible crash at $T(t_i)$. In particular, Corollary 1 implies that the number of traders that have the portfolio of safe assets at $T(t_i)$ is $s(T(t_i), t_0) = \hat{\tau} > \tau_\kappa$ with strictly positive probability, so the bubble would have already burst before period $T(t_i)$. □

Proposition 2: *In equilibrium, trader t_i maintains the portfolio of safe assets for all $t \geq T(t_i)$, until the bubble bursts. That is, in equilibrium there is no buyback.*

Proof of Proposition 2 (by contradiction): Assume there exists an equilibrium in which trader t_i who gets the portfolio of safe assets at $T(t_i)$ stays with this portfolio at least until period $T(t_i + \vartheta)$, for some $\vartheta \in \{1,2,3, ...\}$[76] independent of t_i. By Corollary 1 trader t_i cannot re-buy the portfolio of risky assets until after trader $t_i + \vartheta$ first re-buys the portfolio of risky assets[77]. The same reasoning applies to trader $t_i + \vartheta$ with respect to $t_i + \vartheta + 1$. Applying the same logic iteratively we conclude that trader t_i stays with the portfolio of safe assets until the bubble bursts exogenously at period $t_0 + \bar{\tau}$ or until the bubble bursts endogenously in an earlier period. □

[75] The upper bound of support of trader t_i's posterior beliefs about t_0 is t_i because traders become aware of the bubble when effectively there is a bubble.

[76] $\vartheta \neq 0$ because traders can do at most one kind of financial transaction per period, so traders cannot buy and sell the same kind of portfolio in the same period. Therefore, since by definition trader t_i buys the portfolio of safe assets at $T(t_i)$, then she can sell this portfolio at least at $T(t_i) + 1$.

[77] That is, if trader $t_i + \vartheta$ has a portfolio of safe assets and trader t_i has a portfolio of risky assets then there is a violation of Corollary 1. More specifically, there is a violation of the definition of trading equilibrium.

Proof of Proposition 1: This proof has been divided in four steps:

First step: (*Circumstances under which any trader t_i prefers to have a portfolio of risky assets or a portfolio of safe assets*). Proposition 2 implies that in equilibrium there is no buyback. Therefore, the expected payoff at period 0 of selling the portfolio of risky assets at period t is[78]:

$$(Exp.P)_t = \frac{FV_{T^{*-1}(\cdot)}(1+r)^{t-T^{*-1}(\cdot)+1}}{(1+r)^{t+1}}\pi(T^{*-1}(\cdot)|t_i) + \cdots + \frac{FV_{t_i}(1+r)^{t-t_i+1}}{(1+r)^{t+1}}\pi(t_i|t_i) + \frac{FV_{t_i+1}(1+r)^{t-t_i}}{(1+r)^{t+1}}\pi(t_i+1|t_i) + \cdots + \frac{FV_{t-2}(1+r)^3}{(1+r)^{t+1}}\pi(t-2|t_i) + \frac{FV_{t-1}(1+r)^2}{(1+r)^{t+1}}\pi(t-1|t_i) + \frac{P_t(1+r)}{(1+r)^{t+1}}[1-\Pi(t-1|t_i)] .$$

That is, $(Exp.P)_t$ is equal to the expected payoff obtained if the bubble bursts for endogenous reasons in any period prior to t $\left(\frac{FV_{T^{*-1}(\cdot)}(1+r)^{t-T^{*-1}(\cdot)+1}}{(1+r)^{t+1}}\pi(T^{*-1}(\cdot)|t_i) + \cdots + \frac{FV_{t-1}(1+r)^2}{(1+r)^{t+1}}\pi(t-1|t_i)\right)$ plus the payoff obtained if there is no bubble burst prior to t and the portfolio of risky assets is sold at period $\left(\frac{P_t(1+r)}{(1+r)^{t+1}}[1-\Pi(t-1|t_i)]\right)$.

The previous equation can also be written as

$$(Exp.P)_t = \frac{FV_{T^{*-1}(\cdot)}}{(1+r)^{T^{*-1}(\cdot)}}\pi(T^{*-1}(\cdot)|t_i) + \cdots + \frac{FV_{t_i}}{(1+r)^{t_i}}\pi(t_i|t_i) + \frac{FV_{t_i+1}}{(1+r)^{t_i+1}}\pi(t_i+1|t_i) + \cdots + \frac{FV_{t-2}}{(1+r)^{t-2}}\pi(t-2|t_i) + \frac{FV_{t-1}}{(1+r)^{t-1}}\pi(t-1|t_i) + \frac{P_t}{(1+r)^t}[1-\Pi(t-1|t_i)]$$

Thus, the expected payoff at period 0 of selling the portfolio of risky assets at period $t+1$ is:

$$(Exp.P)_{t+1} = \left\{(Exp.P)_t - \frac{P_t}{(1+r)^t}[1-\Pi(t-1|t_i)]\right\} + \frac{FV_t}{(1+r)^t}\pi(t|t_i) + \frac{P_{t+1}}{(1+r)^{t+1}}[1-\Pi(t|t_i)]$$ then[79],

$$(Exp.P)_{t+1} = (Exp.P)_t + P_0[1-\Pi(t|t_i)]\left(\frac{1+g}{1+r}\right)^t\left\{\left(\frac{1+g}{1+r}\right) + \left[\frac{1+r}{1+g}\right]^{\left(t-T^{*-1}(\cdot)\right)}\frac{\pi(t|t_i)}{[1-\Pi(t|t_i)]} - \frac{[1-\Pi(t-1|t_i)]}{[1-\Pi(t|t_i)]}\right\}$$

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[78] Remember that $T^*(t_0)$ is the bursting time of the bubble for a given realization of t_0. Therefore, $T^{*-1}(t)$ is the birth time of a bubble (i.e. t_0) given that it bursts at period t.

[79] The share of the price of the risky assets that is explained by its fundamental value is:

$$\frac{FV_t}{P_t} = \begin{cases} \frac{(1+g)^{t_0-1}(1+r)^{t-t_0+1}}{(1+g)^t} = \left(\frac{1+r}{1+g}\right)^{(t-t_0+1)} & if\ t = t_0, \dots, T^* \\ 1 & otherwise \end{cases}$$. Therefore,

$$\frac{FV_t}{(1+r)^t} = \left[\frac{1+r}{1+g}\right]^{(t-t_0+1)} \frac{P_t}{(1+r)^t} = P_0\left(\frac{1+g}{1+r}\right)^t\left[\frac{1+r}{1+g}\right]^{(t-t_0+1)}$$

Since $h(t_i+\tau|t_i)=h(t|t_i)=\frac{\pi(t|t_i)}{1-\Pi(t|t_i)}$ is the hazard rate that the bubble will burst at period $t=t_i+\tau$ (i.e. the probability that a bubble that has survived until period t will burst at this period), we obtain[80]

$$(Exp.P)_{t+1}= (Exp.P)_t + P_0[1-\Pi(t|t_i)]\left(\frac{1+g}{1+r}\right)^t\left\{\left(\frac{g-r}{1+r}\right)-\left[1-\left(\frac{1+r}{1+g}\right)^{\left(t-T^{*-1}(\cdot)\right)}\right]h(t|t_i)\right\}$$

Notice that the term $P_0[1-\Pi(t|t_i)]\left(\frac{1+g}{1+r}\right)^{t+1}$ is always positive, therefore if

$$h(t|t_i)=\frac{\frac{[1-\mathcal{P}]}{\mathcal{P}}}{1-(\mathcal{P})^{\left(t-T^{*-1}(t)-\tau\right)}}>\frac{\frac{g-r}{1+r}}{1-\left[\frac{1+r}{1+g}\right]^{\left(t-T^{*-1}(\cdot)\right)}}=Benefit\ cost\ ratio \qquad \textbf{(Condition A1)}$$

the risk neutral trader t_i does not have incentives to have the portfolio of risky assets.

Similarly, if

$$h(t|t_i)=\frac{\frac{[1-\mathcal{P}]}{\mathcal{P}}}{1-(\mathcal{P})^{\left(t-T^{*-1}(t)-\tau\right)}}<\frac{\frac{g-r}{1+r}}{1-\left[\frac{1+r}{1+g}\right]^{\left(t-T^{*-1}(\cdot)\right)}}=Benefit\ cost\ ratio \qquad \textbf{(Condition A2)}$$

the risk neutral trader t_i has incentives to have the portfolio of risky assets.

<u>Second step:</u> (*τ^* defines a symmetric equilibrium).* Suppose that all traders with $t_i \geq \tau_\kappa$ sell the portfolio of risky assets at $t_i+\tau$ and all traders with $t_i<\tau_\kappa$ sell the portfolio of risky assets at $\tau_\kappa+\tau$ for some $\tau\in(0,1,\dots,\bar{t}-\tau_\kappa)$. Then all traders believe that the bubble will bursts, because of endogenous reasons, at $t=t_0+\tau_\kappa+\tau$.

Therefore, τ^* defines an equilibrium if it is equal to the lowest integer value of τ that satisfies the inequality $(Exp.P)_{t_0+\tau_\kappa+\tau}\geq(Exp.P)_{t_0+\tau_\kappa+\tau-1}$ where $t_0+\tau_\kappa$ is the period at which the number of traders informed about the bubble is enough to burst it. So, in equilibrium τ^* is the lowest integer value of τ that satisfies the inequality

$$\left(\left[\frac{1+r}{1+g}\right]^{(\tau_\kappa+\tau)}-1\right)\frac{\frac{[1-\mathcal{P}]}{\mathcal{P}}}{1-(\mathcal{P})^{\tau_\kappa}}+\left[\frac{g-r}{1+r}\right]\ \geq\ 0$$

Equivalently, we can say that in equilibrium τ^* is the lowest integer value of τ that satisfies the inequality $\frac{\frac{g-r}{1+r}}{1-\left(\frac{1+r}{1+g}\right)^{(\tau_\kappa+\tau)}}\geq\frac{\frac{[1-\mathcal{P}]}{\mathcal{P}}}{1-(\mathcal{P})^{\tau_\kappa}}$ where the LHS expression (i.e. the Benefit-Cost Ratio) is decreasing in τ and the RHS (i.e. the Hazard Rate) is not affected by τ[81]. So[82],

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[80] Since $\Pi(t|t_i)-\Pi(t-1|t_i)=\pi(t|t_i)$, then $\frac{[1-\Pi(t-1|t_i)]}{[1-\Pi(t|t_i)]}=1+h(t|t_i)$

[81] That is, $\left(\frac{\frac{[1-\mathcal{P}]}{\mathcal{P}}}{1-(\mathcal{P})^{\tau_\kappa}}\right)_{\Delta\tau}=0$ and $\left(\frac{\frac{g-r}{1+r}}{1-\left(\frac{1+r}{1+g}\right)^{(\tau_\kappa+\tau^*)}}\right)_{\Delta\tau}<0$. Therefore, once this inequality is not satisfied (for a determined value of τ) then it won't be satisfied thereafter.

[82] The operator $\lfloor\geq\rfloor$ means the minimum integer value of τ^* that satisfies the inequality.

$$\tau^* [\geq] \left[\ln\left(1 - \frac{\frac{g-r}{1+r}}{\frac{\frac{[1-\mathcal{P}]}{\mathcal{P}}}{1-(\mathcal{P})^{\tau_\kappa}}} \right) \left[\ln\left(\frac{1+r}{1+g}\right) \right]^{-1} \right] - \tau_\kappa$$

Finally, notice that the assumptions $\frac{\frac{g-r}{1+r}}{1-\left(\frac{1+r}{1+g}\right)^{\bar{\tau}}} < \frac{\frac{[1-\mathcal{P}]}{\mathcal{P}}}{1-(\mathcal{P})^{\tau_\kappa}} \left(= \frac{\frac{g-r}{1+r}}{1-\left(\frac{1+r}{1+g}\right)^{(\tau_\kappa+\tau^*)}} \right) < \frac{\frac{g-r}{1+r}}{1-\left(\frac{1+r}{1+g}\right)^{\tau_\kappa}}$ in the same order imply $\bar{\tau} > \tau_\kappa + \tau^*$ and $\tau^* > 0$. Therefore, τ^*defines a symmetric equilibrium.

Third step: (*Since* $\frac{\frac{g-r}{1+r}}{1-\left(\frac{1+r}{1+g}\right)^{\bar{\tau}}} < \frac{\frac{[1-\mathcal{P}]}{\mathcal{P}}}{1-(\mathcal{P})^{\tau_\kappa}}$ *then the bubble always bursts for endogenous reasons*). Let τ^{EX} to be minimum integer value of τ that solves $\frac{\frac{g-r}{1+r}}{1-\left(\frac{1+r}{1+g}\right)^{\bar{\tau}}} \geq \frac{\frac{[1-\mathcal{P}]}{\mathcal{P}}}{1-(\mathcal{P})^{(\bar{\tau}-\tau)}}$ [83]. Therefore, if each trader t_i believes that the bubble would burst for exogenous reasons at $T^{*-1}(t) + \bar{\tau}$ then they would buy the portfolio of safe assets at $t_i + \tau^{EX}$. Thus, the bubble can burst for exogenous reasons if $\tau_{t_i} > \tau^{EX}$ for at least some t_i. Consider any trader $\tilde{t}_i$, we will show that $\tau_{\tilde{t}_i} > \tau^{EX}$ leads to a contradiction[84].

According to Lemma 1 the inequality $t_0^{supp}(\tilde{t}_i) \geq \tilde{t}_i - \tau_\kappa$ has to be satisfied by all traders. However,

- If $t_0^{supp}(\tilde{t}_i) > \tilde{t}_i - \tau_\kappa$, then the hazard rate at $T(\tilde{t}_i)$ that the bubble will bursts for exogenous reasons is greater than $\frac{\frac{g-r}{1+r}}{1-\left(\frac{1+r}{1+g}\right)^{\bar{\tau}}}$. Then, she has an incentive to sell the portfolio of risky assets strictly prior to $T(\tilde{t}_i)$.

- If $t_0^{supp}(\tilde{t}_i) = \tilde{t}_i - \tau_\kappa$. Since, we are assuming the bubble burst at $t_0 + \tau_\kappa + \tau_{\tilde{t}_i} > t_0 + \tau_\kappa + \tau^{EX}$ then $\frac{\frac{g-r}{1+r}}{1-\left(\frac{1+r}{1+g}\right)^{\tau_\kappa+\tau_{\tilde{t}_i}}} < \frac{\frac{g-r}{1+r}}{1-\left(\frac{1+r}{1+g}\right)^{\tau_\kappa+\tau^{EX}}}$. But, since $\tau_{\tilde{t}_i} > \tau_{t_i}\ \forall t_i$ then $h(T(\tilde{t}_i)|\tilde{t}_i) \geq \frac{\frac{[1-\mathcal{P}]}{\mathcal{P}}}{1-(\mathcal{P})^{\tau_\kappa}}$. Therefore, trader $\tilde{t}_i$ violates Condition A2 at $T(\tilde{t}_i)$.

In conclusion, the bubble always burst for endogenous reasons (i.e. $T(t_0 + \tau_\kappa) < t_0 + \bar{\tau}$).[85]

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[83] Notice that in this equation $t = T^{*-1}(t) + \bar{\tau}$. That is, t is the period in which the exogenous burst happens.

[84] Remember that $\tau_{\tilde{t}_i}$ is the length of time trader $\tilde{t}_i$ chooses to ride the bubble subsequent to becoming aware of the mispricing.

[85] Similarly, notice that the statement of step 3 implies directly $\tau^* < \tau^{EX}$ because the hazard rate at τ^* under the assumed condition is always greater than the benefit cost ratio at τ^{EX}.

Fourth step: (*Uniqueness*). In equilibrium, for traders aware in period $t_i \geq \tau_\kappa$, we have $T(t_i) = t_0 + t_i + \tau^*$ (i.e. they have the same τ_{t_i}) and for traders aware in period $t_i < \tau_\kappa$, we have $T(t_i) = T(\tau_\kappa) = t_0 + \tau_\kappa + \tau^*$.

(a) Minimum and maximum of τ_{t_i} coincide for the traders aware in period $t_i \geq \tau_\kappa$: Remember, Lemma 1 implies that $t_0^{supp}(t_i) \geq t_i - \tau_\kappa$. However, $t_0^{supp}(t_i) > t_i - \tau_\kappa$ can be excluded since trader t_i would be strictly better off by getting the portfolio of safe assets at $T(t_i) + 1$. Hence, given that $t_0^{supp}(t_i) = t_i - \tau_\kappa$ we get $h\big(T(t_i)|t_i, T^*(t_0) \geq T(t_i)\big) = \frac{\frac{[1-\mathcal{P}]}{\mathcal{P}}}{1-(\mathcal{P})^{\left(T(t_i)-T^{*-1}\left(T(t_i)\right)-\tau\right)}}$ which is increasing in t_i. Let two traders $\underline{t_i} \in \arg\min\{\tau_{t_i}\}$ and $\overline{t_i} \in \arg\max\{\tau_{t_i}\}$ and suppose that $\max \tau_{t_i} > \min \tau_{t_i}$, then $h\left(T\left(\underline{t_i}\right)|t_i, T^*(t_0) \geq T\left(\underline{t_i}\right)\right) < h\left(T(\overline{t_i})|t_i, T^*(t_0) \geq T(\overline{t_i})\right)$[86] . However, $\frac{\frac{g-r}{1+r}}{1-\left(\frac{1+r}{1+g}\right)^{\left(\tau_\kappa+\tau_{\underline{t_i}}\right)}} \geq \frac{\frac{g-r}{1+r}}{1-\left(\frac{1+r}{1+g}\right)^{\left(\tau_\kappa+\tau_{\overline{t_i}}\right)}}$. In conclusion, condition A1 and condition A2 cannot be satisfied for both traders $\underline{t_i}$ and $\overline{t_i}$, a contradiction.

(b) For traders aware in period $t_i < \tau_\kappa$, $T(t_i) = T(\tau_\kappa)$: Since at least τ_κ traders are needed to burst the bubble, then no t_i should sell the portfolio of risky assets prior to $T(\tau_\kappa)$ and by corollary 1 will sell this portfolio at $T(\tau_\kappa)$ □

What happen if we incorporate synchronized events (public signals) to the analysis?

Proposition 3: *Define τ^{**} as the minimum integer value of τ that solves the following inequality:*

$$\left[\frac{g-r}{1+r}\right] - \left(1 - \left[\frac{1+r}{1+g}\right]^{(\tau_\kappa+\tau)}\right)\frac{\frac{[1-\mathcal{P}]}{\mathcal{P}}}{1-(\mathcal{P})^{\tau_\kappa}} - \theta \sum_{s-\tau_\kappa=0}^{\tau} \sum_{t_0=t_i+\tau-1+s-\tau_\kappa} \left[\frac{s-\tau_\kappa}{\omega}\left(1-\left(\frac{1+r}{1+g}\right)^{(\tau_\kappa+\tau)}\right)\right]\frac{\mathcal{P}^{-N}[1-\mathcal{P}]}{\mathcal{P}^{-N}-1} \geq 0,$$

then there exists a unique responsive equilibrium. In this equilibrium, if $t_e \geq \tau_\kappa$ each trader t_i always sells the portfolio of risky assets at the instances of synchronizing events $t_e \geq t_i$. Otherwise (i.e. if $t_e < \tau_\kappa$), traders do not react to the synchronized event. Furthermore, each trader $t_i \geq \tau_\kappa$ ($t_i < \tau_\kappa$) stays with the portfolio of safe assets for all $t \geq t_i + \tau^{**}$ ($t \geq \tau_\kappa + \tau^{**}$) except in the event that the last synchronized endogenous burst attempt failed in which case she re-buys the portfolio of risky assets for the interval $t \in (t_e, t_e + \tau^{**})$, unless a new synchronizing event occurs in the interim.

Before doing the proof we need the following proposition:

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[86] Given that $t_0^{supp}(t_i) = t_i - \tau_\kappa$ we get $\pi\big(t_i - \tau_\kappa|t_i, T^*(t_0) \geq T(t_i)\big) = \frac{(\mathcal{P})^{-\tau_\kappa}[1-\mathcal{P}]}{(\mathcal{P})^{-\tau_\kappa}-1}$ which is independent of t_i, and $\Pi\left(T\left(\underline{t_i}\right)|\underline{t_i}, T^*(t_0) \geq T\left(\underline{t_i}\right)\right) = \frac{\frac{[1-\mathcal{P}]}{\mathcal{P}}}{1-(\mathcal{P})^{\tau_{\underline{t_i}}}} < \frac{\frac{[1-\mathcal{P}]}{\mathcal{P}}}{1-(\mathcal{P})^{\tau_{\overline{t_i}}}} = \Pi\left(T(\overline{t_i})|\overline{t_i}, T^*(t_0) \geq T(\overline{t_i})\right)$

Proposition 4: *In equilibrium, trader t_i follows a trigger strategy between two successive synchronizing events (i.e. agents follow interim-trigger-strategies)*

Proof of Proposition 4 (by contradiction): Suppose not. That is, assume there exists and equilibrium in which trader t_i (who get the portfolio of safe assets at $T(t_i)$) stays with the portfolio of safe assets at least until $T(t_i + \vartheta)$ for some $\vartheta \in \{1,2,\dots,N-1\}$ independent of t_i. Hence, a generalized version of Proposition 2 applies between synchronizing events.

More specifically, by Corollary 1 trader t_i cannot re-buy the portfolio of risky assets until after trader $t_i + \vartheta$ first re-buys the portfolio of risky assets. The same reasoning applies to trader $t_i + \vartheta$ with respect to $t_i + \vartheta + 1$. Applying the same logic iteratively we conclude that trader t_i stays with the portfolio of safe assets until: (1) the bubble bursts exogenously at period $t_0 + \bar{\tau}$, (2) there is an endogenous burst or (3) there is a failed attempt to burst the bubble due to a synchronized event.

Respect to the third point, we have to take into account that in a responsive equilibrium, traders belief that the bubble bursts with strict positive probability at each $t_e \geq \tau_\kappa$ if others attack at t_e, too. Therefore, it is always an equilibrium that each trader who observes the synchronized event attacks the bubble at $t_e \geq \tau_\kappa$. However, if there is a failed attempt to burst the bubble, then all traders who participate in the attack update their beliefs and they will re-buy the portfolio of risky assets and sell this portfolio again exactly: (1) when the "first" trader, who did not participate in this common attempt, sells the portfolio of risky assets or (2) when there is a new synchronized event. That is, traders follow interim-trigger-strategies □

Proof of Proposition 3: This proof has been divided in four steps:

First step: (*Circumstances under which any trader t_i prefers a portfolio of risky assets or a portfolio of safe assets*). Proposition 4 implies that the expected payoff at period 0 of selling out the portfolio of risky assets at period t or until she observes the first synchronized event is:

$$(Exp.P)_t = \frac{P_t(1+r)}{(1+r)^{t+1}}[1-\Pi(t-1|t_i)][1-\Gamma(t-2|t_i)]$$
$$+\sum_{z=T^{*-1}(\cdot)}^{t-1}\frac{FV_z(1+r)^{t-z+1}}{(1+r)^{t+1}}\pi(z|t_i)[1-\Gamma(z-1|t_i)]+\sum_{z=t_i}^{t-1}[1-\Pi(z-1|t_i)]\gamma(z-$$
$$1|t_i)\sum_{s-\tau_\kappa=0}^{\tau}\sum_{z+s-\tau_\kappa=t_0}\left[\frac{\left[\left(1-\frac{s-\tau_\kappa}{\omega}\right)P_z+\left(\frac{s-\tau_\kappa}{\omega}\right)FV_z\right](1+r)^{t+1-z}}{(1+r)^{t+1}}\right]\frac{\phi(t_0|t_i)}{1-\Phi(t_i-\tau_\kappa|t_i)}+V(t_i)$$

The first term is the payoff obtained if there is no bubble burst[87] prior to t and the portfolio of risky assets is sell out at period t. The second term is the expected payoff obtained if the bubble does not burst after a synchronized event but it bursts for endogenous reasons in any period prior to t. The third term is the expected payoff obtained if the bubble bursts after a synchronized event[88,89]. Finally, the last term is the

[87] That is, the bubble does not burst endogenously or due to a synchronized event.

[88] $\frac{s-\tau_\kappa}{\omega}$ is the probability that a trader, who observe the public signal, has a capital lost when there is a successful synchronized attack to the bubble; $\omega = \begin{cases}\tau & if\ s-\tau_\kappa \leq \tau \leq s \\ s & if\ s < \tau \leq N\end{cases}$ is the number of traders who pretend to get the portfolio of safe assets in the period immediately after the public signal; and $s-\tau_\kappa \geq 0$

value of the option to re-buy the risky portfolio and to ride the bubble after the first observed failed endogenous burst attempt at period z. The previous equation can also be written as

$$(Exp.P)_t = \frac{P_t}{(1+r)^t}[1-\Pi(t-1|t_i)][1-\Gamma(t-2|t_i)]$$
$$+ \sum_{z=T^{*-1}(\cdot)}^{t-1} \frac{FV_z}{(1+r)^z}\pi(z|t_i)[1-\Gamma(z-1|t_i)] + \quad \sum_{z=t_i}^{t-1}[1-\Pi(z-1|t_i)]\gamma(z-$$
$$1|t_i)\sum_{s-\tau_\kappa=0}^{\tau}\sum_{z+s-\tau_\kappa=t_0}\left[\frac{\left(1-\frac{s-\tau_\kappa}{\omega}\right)P_z+\left(\frac{s-\tau_\kappa}{\omega}\right)FV_z}{(1+r)^z}\right]\frac{\phi(t_0|t_i)}{1-\Phi(t_i-\tau_\kappa|t_i)} + V(t_i)$$

Therefore, the expected payoff of selling out the portfolio of risky assets at period $t+1$ is:

$$(Exp.P)_{t+1} = \left\{(Exp.P)_t - \frac{P_t}{(1+r)^t}[1-\Pi(t-1|t_i)][1-\Gamma(t-2|t_i)]\right\} +$$
$$\frac{FV_t}{(1+r)^t}\pi(t|t_i)[1-\Gamma(t-1|t_i)] + \frac{P_{t+1}}{(1+r)^{t+1}}[1-\Pi(t|t_i)][1-\Gamma(t-1|t_i)] +$$
$$[1-\Pi(t-1|t_i)]\gamma(t-1|t_i)\sum_{s-\tau_\kappa=0}^{\tau}\sum_{t+s-\tau_\kappa=t_0}\left[\frac{\left(1-\frac{s-\tau_\kappa}{\omega}\right)P_t+\left(\frac{s-\tau_\kappa}{\omega}\right)FV_t}{(1+r)^t}\right]\frac{\phi(t_0|t_i)}{1-\Phi(t_i-\tau_\kappa|t_i)}$$

then,

$$(Exp.P)_{t+1} = (Exp.P)_t +$$
$$P_0[1-\Pi(t|t_i)][1-\Gamma(t-1|t_i)]\left(\frac{1+g}{1+r}\right)^{t+1}\left\{1+\left(\frac{1+r}{1+g}\right)^{\left(t+1-T_e^{*-1}(t)\right)}\frac{\pi(t|t_i)}{[1-\Pi(t|t_i)]} -\right.$$
$$\left(\frac{1+r}{1+g}\right)\frac{[1-\Pi(t-1|t_i)]}{[1-\Pi(t|t_i)]}\frac{[1-\Gamma(t-2|t_i)]}{[1-\Gamma(t-1|t_i)]} +$$
$$\left.\left(\frac{1+r}{1+g}\right)\frac{[1-\Pi(t-1|t_i)]}{[1-\Pi(t|t_i)]}\frac{\gamma(t-1|t_i)}{[1-\Gamma(t-1|t_i)]}\sum_{s-\tau_\kappa=0}^{\tau}\sum_{t+s-\tau_\kappa=t_0}\left[1-\frac{s-\tau_\kappa}{\omega}\left(1-\left(\frac{1+r}{1+g}\right)^{\left(t-T_e^{*-1}(t)\right)}\right)\right]\frac{\phi(t_0|t_i)}{1-\Phi(t_i-\tau_\kappa|t_i)}\right\}$$

Since the term $P_0[1-\Pi(t|t_i)][1-\Gamma(t-1|t_i)]\left(\frac{1+g}{1+r}\right)^{t+1}$ is always positive, if the expression inside curly brackets is negative then trader t_i does not have incentives to continue with the portfolio of risky assets until period $t+1$.

Therefore, given $h(t|t_i) = \frac{\pi(t|t_i)}{1-\Pi(t|t_i)}$, $\tilde{h}(t|t_i) = \frac{\gamma(t|t_i)}{[1-\Gamma(t|t_i)]}$, $\phi(t_0|t_i) = \frac{\mathcal{P}^{-N}[1-\mathcal{P}]}{\mathcal{P}^{-N}-1}$ and $\theta = \frac{(1+h(t|t_i))\tilde{h}(t-1|t_i)}{1-\Phi(t_i-\tau_\kappa|t_i)}$ we obtain after some algebraic operations that if

is the number of traders who could not get the portfolio of safe assets in the period immediately after the public signal. $\frac{\phi(t_0|t_i)}{1-\Phi(t_i-\tau_\kappa|t_i)}$ is the conditional probability mass function of t_0 such that $t_0 \geq t_i - \tau_\kappa$, $\phi(t_0|z)$ is the conditional probability mass function of t_0. $1-\Phi(t_i-\tau_\kappa|t_i)$ is the probability that $t_0 \geq t_i - \tau_\kappa$ (i.e. $1-\Phi(t_i-\tau_\kappa|t_i) = \sum_{s-\tau_\kappa=0}^{\tau}\sum_{z+s-\tau_\kappa=t_0}\phi(t_0|t_i)$).

[89] Notice two things: (1) if $s > \tau_\kappa$ the ω traders who bought the safe asset immediately after the public event only receives a convex combination between the pre-crash and post-crash price $\left[\left(1-\frac{s-\tau_\kappa}{\omega}\right)P_z + \left(\frac{s-\tau_\kappa}{\omega}\right)FV_z\right]$; and (2) the sum operator begins in $z = t_i$ because for $z < t_i$, trader t_i does not observe the synchronized event.

$$h(t|t_i)\left(1-\left(\frac{1+r}{1+g}\right)^{\left(t-T_e^{*-1}(t)\right)}\right)+\theta\sum_{s-\tau_\kappa=0}^{\tau}\sum_{t+s-\tau_\kappa=t_0}\left[\frac{s-\tau_\kappa}{\omega}\left(1-\left(\frac{1+r}{1+g}\right)^{\left(t-T_e^{*-1}(t)\right)}\right)\right]\frac{\mathcal{P}^{-N}[1-\mathcal{P}]}{\mathcal{P}^{-N}-1}>\frac{g-r}{1+r} \qquad \textbf{(Condition A3)}$$

the risk neutral trader t_i does not have incentives to have the portfolio of risky assets.

Following a similar reasoning we obtain that if

$$h(t|t_i)\left(1-\left(\frac{1+r}{1+g}\right)^{\left(t-T_e^{*-1}(t)\right)}\right)+\theta\sum_{s-\tau_\kappa=0}^{\tau}\sum_{t+s-\tau_\kappa=t_0}\left[\frac{s-\tau_\kappa}{\omega}\left(1-\left(\frac{1+r}{1+g}\right)^{\left(t-T_e^{*-1}(t)\right)}\right)\right]\frac{\mathcal{P}^{-N}[1-\mathcal{P}]}{\mathcal{P}^{-N}-1}<\frac{g-r}{1+r} \qquad \textbf{(Condition A4)}$$

the risk neutral trader t_i has incentives to have the portfolio of risky assets.

<u>Second step</u>: (τ^{**}*defines a symmetric equilibrium*). Notice that each trader t_i*'s* posterior about t_0 at $T_e(t_i)$ exactly coincides with the posterior she had at $T(t_i)$ in a setting without synchronizing events[90]. Therefore, the hazard rate is also the same at the time trader t_i sells in either setting $h(t|t_i)=h(t_i+\tau|t_i)=\frac{\frac{[1-\mathcal{P}]}{\mathcal{P}}}{1-(\mathcal{P})^{\tau_\kappa}}$. Now, define

$$\varphi(\tau)=\frac{g-r}{1+r}-\frac{\frac{[1-\mathcal{P}]}{\mathcal{P}}}{1-(\mathcal{P})^{\tau_\kappa}}\left(1-\left(\frac{1+r}{1+g}\right)^{(\tau+\tau_\kappa)}\right)-\theta\sum_{s-\tau_\kappa=0}^{\tau}\sum_{t_0=t_i+\tau-1+s-\tau_\kappa}\left[\frac{s-\tau_\kappa}{\omega}\left(1-\left(\frac{1+r}{1+g}\right)^{(\tau+\tau_\kappa)}\right)\right]\phi(t_0|t_i)$$

For equilibrium τ^{**} it is necessary that[91] $\varphi(\tau^{**})\lfloor\geq\rfloor 0$. We argue that there is a τ^{**} such that $\varphi(\tau^{**})\lfloor\geq\rfloor 0$ is (i) unique and (ii) exists. Notice that $\frac{\frac{[1-\mathcal{P}]}{\mathcal{P}}}{1-(\mathcal{P})^{\tau_\kappa}}$ is constant across equilibrium τ^{**}. However, $\left(1-\left(\frac{1+r}{1+g}\right)^{(\tau^{**}+\tau_\kappa)}\right)$ is strictly increasing in τ^{**}, $\phi(t_0|t_i)=\frac{(\mathcal{P})^{-N}[1-\mathcal{P}]}{(\mathcal{P})^{-N}-1}$ is the same, and the upper bound of the second sum operator is increasing in τ^{**}. Thus, $\varphi(\tau^{**})$ is strictly increasing in equilibrium τ^{**}. Uniqueness follows directly.

In a responsive equilibrium, immediately after a synchronizing event at $t_e\geq\tau_\kappa$ each trader who observes this event is assumed to sell the portfolio of risky assets. Therefore, for each trader's point of view, a bubble bursts with strict positive probability at each $t_e\geq\tau_\kappa$ in the equilibrium. Given this belief, it is always optimal that each trader who observes the synchronized event at $t_e\geq\tau_\kappa$ sells the portfolio of risky assets at this time.

To fully specify all relevant strategies, it only remains to consider continuation strategies after a failed burst attempt due to a synchronized event. More specifically,

[90] That is, in any symmetric equilibrium the support of t_0 at $T_e(t_i)$ is also $[t_i-\tau_\kappa,t_i]$

[91] Remember that the operator $\lfloor\geq\rfloor$ means the minimum integer value of τ^{**} that satisfies the inequality.

after this failed burst attempt, traders learn that fewer than τ_κ traders have observed the synchronizing event (i.e. $t_0 > t_e - \tau_\kappa$). Since all other traders who did not observe the synchronizing event only sell the portfolio of risky assets at $t_i + \tau^{**}$, all traders who observed the synchronizing event at $t_e > \tau_\kappa$ can rule out the possibility that the bubble bursts prior to $t_e - \tau^{**}$ provided that no new synchronizing event occurs[92].

Therefore, in any responsive equilibrium all traders who sold the portfolio of risky assets after the last synchronizing event will re-buy the portfolio of risky assets after a failed burst attempt and sell this portfolio again exactly when the "first" trader, who did not participate in this common attempt, sells the portfolio of risky assets. Notice that after $t = t_e - \tau^{**}$, the analysis coincides with a setting without a synchronizing event at t_e. At this point all traders who had participated in the failed sell and subsequently re-buy the portfolio of risky assets, sell this portfolio again.

<u>Third step</u>: (*Since* $\frac{\frac{g-r}{1+r}}{1-\left(\frac{1+r}{1+g}\right)^{\bar{\tau}}} < \frac{\frac{[1-\mathcal{P}]}{\mathcal{P}}}{1-(\mathcal{P})^{\tau_\kappa}}$ *then the bubble always bursts for endogenous reasons*). Assume traders would buy the portfolio of safe assets at $t_i + \tilde{\tau}^{EX}$ if they believe that the bubble would burst for exogenous reasons. However, this is a contradiction because Condition A4 is violated. In particular, notice that the term $\theta \sum_{s-\tau_\kappa=0}^{\tau} \sum_{t_0=t_i+\tau-1+s-\tau_\kappa} \left[\frac{s-\tau_\kappa}{\omega}\left(1-\left(\frac{1+r}{1+g}\right)^{(\tau+\tau_\kappa)}\right)\right] \frac{\mathcal{P}^{-N}[1-\mathcal{P}]}{\mathcal{P}^{-N}-1}$ is always positive an increasing in τ. Therefore, we can follow a procedure similar to step 3 of proposition 1 to show that condition A4 is violated for each trader who sells the portfolio of risky assets only at $t_i + \tilde{\tau}^{EX}$. In conclusion, the bubble always bursts for endogenous reasons (i.e. $T(t_0, \tau_\kappa) < t_0 + \bar{\tau}$)[93]

<u>Fourth step:</u> (*Uniqueness*). In equilibrium, for traders aware in period $t_i \geq \tau_\kappa$, we have $T(t_i) = t_0 + t_i + \tau^{**}$ (i.e. they have the same τ_{t_i}) and for traders aware in period $t_i < \tau_\kappa$, we have $T(t_i) = T(\tau_\kappa) = t_0 + \tau_\kappa + \tau^{**}$.

(a) <u>Minimum and maximum of τ_{t_i} coincide for traders aware in period $t_i \geq \tau_\kappa$</u>: Remember, Lemma 1 implies that $\underline{t_0^{supp}}(t_i) \geq t_i - \tau_\kappa$. However, $\underline{t_0^{supp}}(t_i) > t_i - \tau_\kappa$ can be excluded since trader t_i would be strictly better off by getting the portfolio of safe assets at $T(t_i) + 1$. Hence, given that $\underline{t_0^{supp}}(t_i) = t_i - \tau_\kappa$ we get $h\big(T(t_i)|t_i, T^*(t_0) \geq T(t_i)\big) = \frac{\frac{[1-\mathcal{P}]}{\mathcal{P}}}{1-(\mathcal{P})^{\left(T(t_i)-T^{*-1}(T(t_i))-\tau\right)}}$ which is increasing in t_i. Let two traders $\underline{t_i} \in \arg\min\{\tau_{t_i}\}$ and $\overline{t_i} \in \arg\max\{\tau_{t_i}\}$ and suppose that $\max \tau_{t_i} > \min \tau_{t_i}$,

92 Note that the bubble will not bursts for exogenous reasons prior to $t_e - \tau^{**}$, since the endogenous bursting time $t_0 + \tau_\kappa + \tau^{**}$ occurs strictly before $t_0 + \tau_\kappa + \tau^* < t_0 + \bar{\tau}$. The bubble might only burst prior to $t_e - \tau^{**}$ if a new synchronizing event occurs.

93 Similarly, notice that the statement of step 3 implies directly $\tau^{**} < \tilde{\tau}^{EX}$ because the hazard rate at τ^{**} under the assumed condition is always greater than the benefit cost ratio at $\tilde{\tau}^{EX}$.

then $h\left(T\left(\underline{t_i}\right)|t_i, T^*(t_0) \geq T\left(\underline{t_i}\right)\right) < h\left(T(\overline{t_i})|t_i, T^*(t_0) \geq T(\overline{t_i})\right)$[94] However, $\frac{\frac{g-r}{1+r}}{1-\left(\frac{1+r}{1+g}\right)^{\left(\tau_\kappa+\tau_{\underline{t_i}}\right)}} \geq \frac{\frac{g-r}{1+r}}{1-\left(\frac{1+r}{1+g}\right)^{\left(\tau_\kappa+\tau_{\overline{t_i}}\right)}}$.

Furthermore,

$$\theta \sum_{s-\tau_\kappa=0}^{\tau} \sum_{T_e\left(\underline{t_i}\right)+s-\tau_\kappa-t_0=0} \left[\frac{s-\tau_\kappa}{\omega}\left(1-\left(\frac{1+r}{1+g}\right)^{\left(T_e\left(\underline{t_i}\right)-t_0\right)}\right)\right]\phi(t_0|t_i)$$
$$< \theta \sum_{s-\tau_\kappa=0}^{\tau} \sum_{T_e\left(\overline{t_i}\right)+s-\tau_\kappa-t_0=0} \left[\frac{s-\tau_\kappa}{\omega}\left(1-\left(\frac{1+r}{1+g}\right)^{\left(T_e\left(\overline{t_i}\right)-t_0\right)}\right)\right]\phi(t_0|t_i)$$

In conclusion, conditions A3 and A4 cannot be satisfied for both traders $\underline{t_i}$ and $\overline{t_i}$, a contradiction.

(b) <u>For traders aware in period $t_i < \tau_\kappa$, $T(t_i) = T(\tau_\kappa)$:</u> Since at least τ_κ traders are needed to burst the bubble, then no t_i should sell the portfolio of risky assets prior to $T(\tau_\kappa)$ and by corollary 1 will sell this portfolio at $T(\tau_\kappa)$. □

[94] Given that $t_0^{supp}(t_i) = t_i - \tau_\kappa$ we get $\pi\left(t_i - \tau_\kappa|t_i, T^*(t_0) \geq T(t_i)\right) = \frac{(\mathcal{P})^{-\tau_\kappa}[1-\mathcal{P}]}{(\mathcal{P})^{-\tau_\kappa}-1}$ which is independent of t_i, and $\Pi\left(T\left(\underline{t_i}\right)|\underline{t_i}, T^*(t_0) \geq T\left(\underline{t_i}\right)\right) = \frac{\frac{[1-\mathcal{P}]}{\mathcal{P}}}{1-(\mathcal{P})^{\tau_{\underline{t_i}}}} < \frac{\frac{[1-\mathcal{P}]}{\mathcal{P}}}{1-(\mathcal{P})^{\tau_{\overline{t_i}}}} = \Pi\left(T(\overline{t_i})|\overline{t_i}, T^*(t_0) \geq T(\overline{t_i})\right)$

1.7 Appendix B1: Instructions[95]

Thank you for participating in this experiment on the economics of investment decision making. If you follow the instructions carefully and make good decisions, you might earn a considerable amount of money. The way your earnings are determined is explained at the end of the instructions. The experiment will take around 2 hours.

You are going to participate in transaction rounds 50 times in succession. In every round you will participate with 24 different **computer** agents, these agents will change from round to round. The computer agents are going to follow a predetermined strategy, which remain consistent throughout the experiment, but unknown to you.

Note that:

- You are **NOT** participating in each round with the same set of computer agents.
- You are **NOT** participating in the same round with other human participants in the room. Therefore, your earnings are **NOT** affected by the decisions they are taking or by the rounds in which they are participating.

All participants in each transaction round (i.e. you and the 24 computer agents) are named traders. In this experiment, traders are not trading to each other. However, the decisions taken by other traders (i.e. the computer agents) can affect your own earnings and your decisions can affect the earnings of them. Your goal in each transaction round is to maximize your own earnings (similarly, the goal of each computer agent is to maximize her respective earnings).

How to participate in each transaction round

Each transaction round is composed by many periods. In every period, depending on your decision in the previous period, you will have a portfolio of private assets or a portfolio of public assets.

The price of each kind of asset has a stock market value and a true value. These values are determined in the following way (Figure B1.1):

- The stock market value of the public assets is always equal to its true value and both grow every period at a rate of 1%.
- From period 0 until period $t_0 - 1$ the stock market value of the private assets is always equal to its true value and both grow every period at a rate of 10%. On the other hand, from period t_0 until the end of each round the stock market value of the private assets is different from its true value, in particular: (1) the true value of the private assets grows every period at a rate of 1%, and (2) the stock market value of the private assets grows every period at a rate of 10%, except in the last period of every transaction round when this value falls until to catch up its true value.

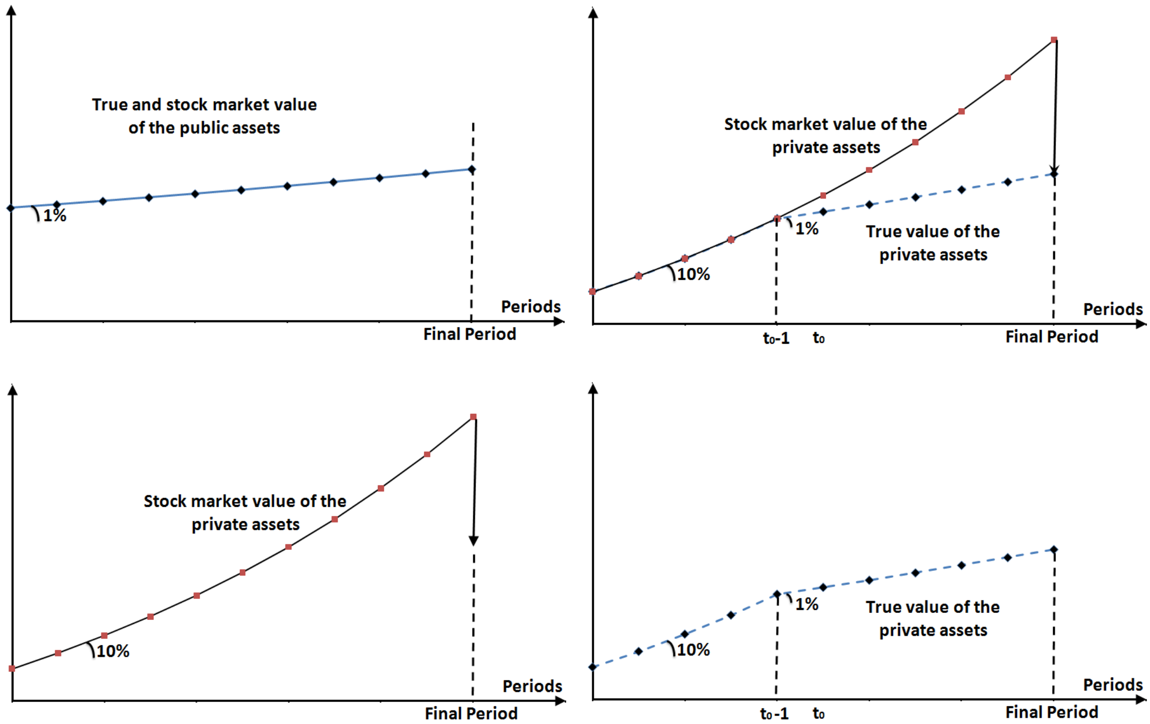
Traders do not know exactly in which period begins the divergence between the true value and the stock market value of the private assets (i.e. traders do not know exactly what period is period t_0). However, they know that since the first period, there is a

[95] Translated from Spanish into English

probability of 5% that in each period the divergence between the true and the stock market value of the private assets begins (i.e. in each period there is a probability of 5% that period t_0 happens).

In addition, they know that the following rule works in every transaction round: **"In period t_0 one of the traders is aware (when she receives a private warning message) that the true value of the private assets is different from its stock market value. In period $t_0 + 1$ a new trader is aware (when she receives a private warning message) that the true value of the private assets is different from its stock market value. This process continues sequentially until period $t_0 + 24$ in which all traders have already been aware that the true value of the private assets is different from its stock market value (i.e. in period $t_0 + 24$ all traders have already received their respective warning message)"**.

Figure B1.1



Note: The number of dots in these figures are only for expositional purposes and are not related to the dynamic of the transaction rounds.

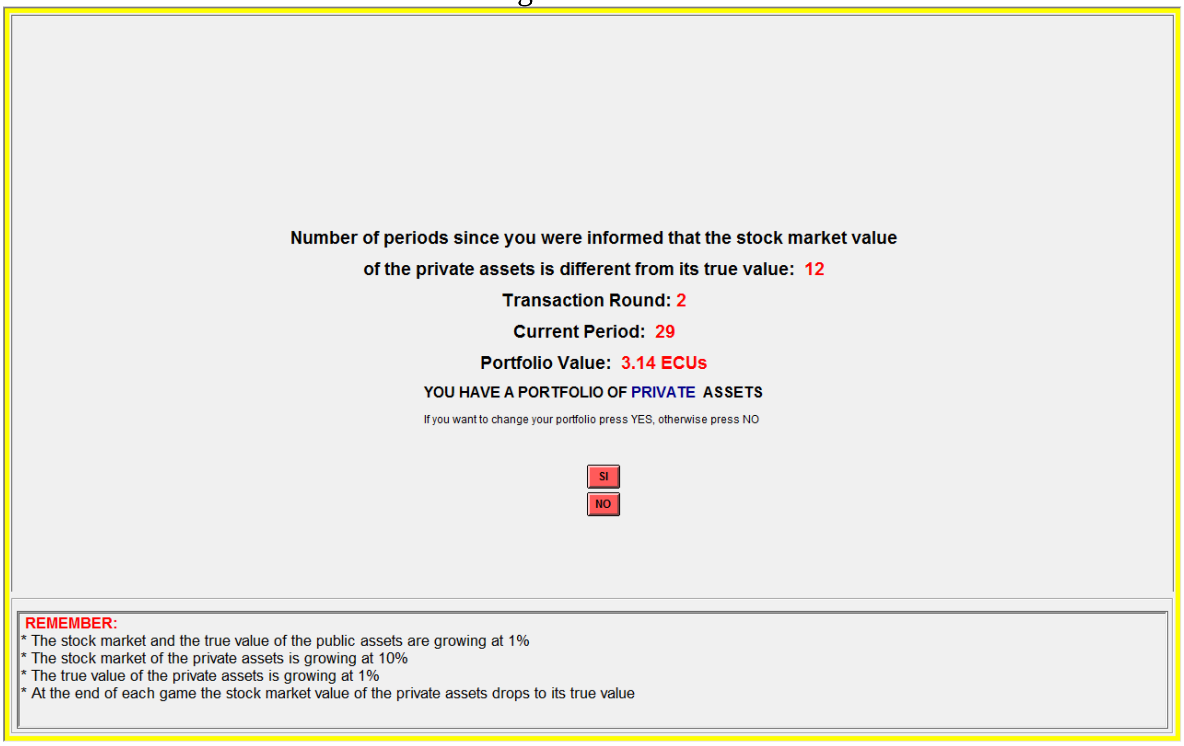
The order in which traders become aware of the bubble is randomly assigned in every round. Therefore, when you receive the warning message, some traders may have already received it, and some might not have received it yet. All the warning messages are private, so you do not know when other traders receive the warning message and they do not know when you receive your warning message (besides, a computer agent does not recognize when another computer agent receive the warning message). However, you are always informed in all periods about the period of the transaction round in which you are participating and about the number of periods that has elapsed since you receive your warning message (Figures B1.2 and B1.3).

You always begin each transaction round with a portfolio of private assets and you will be forced to maintain this portfolio until the period in which you are warned about the divergence between the true and the stock market value of this portfolio (i.e. you are forced to maintain this kind of portfolio until a period that is greater or equal to t_0 and lower or equal to $t_0 + 24$). Since in the first periods of each transaction round you do not take any decision, then the first screen that appears in your computer corresponds to the period in which you are warned about the divergence between the true value and the stock market value of the private assets. From this period and in all the consecutive periods you have to decide if you want to maintain or to change your kind of portfolio. More specifically, during each period if you have a portfolio of private assets you have to decide if you want to change this portfolio to obtain a portfolio of public assets in the next period (Figure B1.2); on the other hand, if you have a portfolio of public assets you have to decide if you want to change this portfolio to obtain a portfolio of private assets in the next period (Figure B1.3).

The above are the only decisions that all traders have to take during the experiment. Every period finishes after you take a decision (i.e. after you click with the mouse the button that has your choice).

> ***Note:*** *In this experiment traders cannot diversify their portfolio (i.e. in each period each trader has a portfolio of only private assets or a portfolio of only public assets). Therefore, the value of your portfolio will grow each period at the same rate that grows the stock market value of the kind of assets that you have in your portfolio. For instance, if in a specific period you have a portfolio of public assets, then the value of your portfolio will grow in this period at 1%.*

Figure B1.2



Each transaction round finishes when the stock market value of the private assets falls until it catches up its true value. This situation happens if one of the following two conditions is satisfied:

1. In the same period at least 15 traders decided to have portfolios of public assets (***endogenous ending***).

> ***Warning:*** *In an endogenous ending <u>no more than</u> 15 traders can have in the same period a portfolio of public assets. Therefore, if more than 15 traders want to have a portfolio of public assets in the same period, then some of the traders who wanted to get the portfolio of public assets in the last period cannot do it, in this case the traders that cannot change the portfolio are randomly chosen.*

Figure B1.3

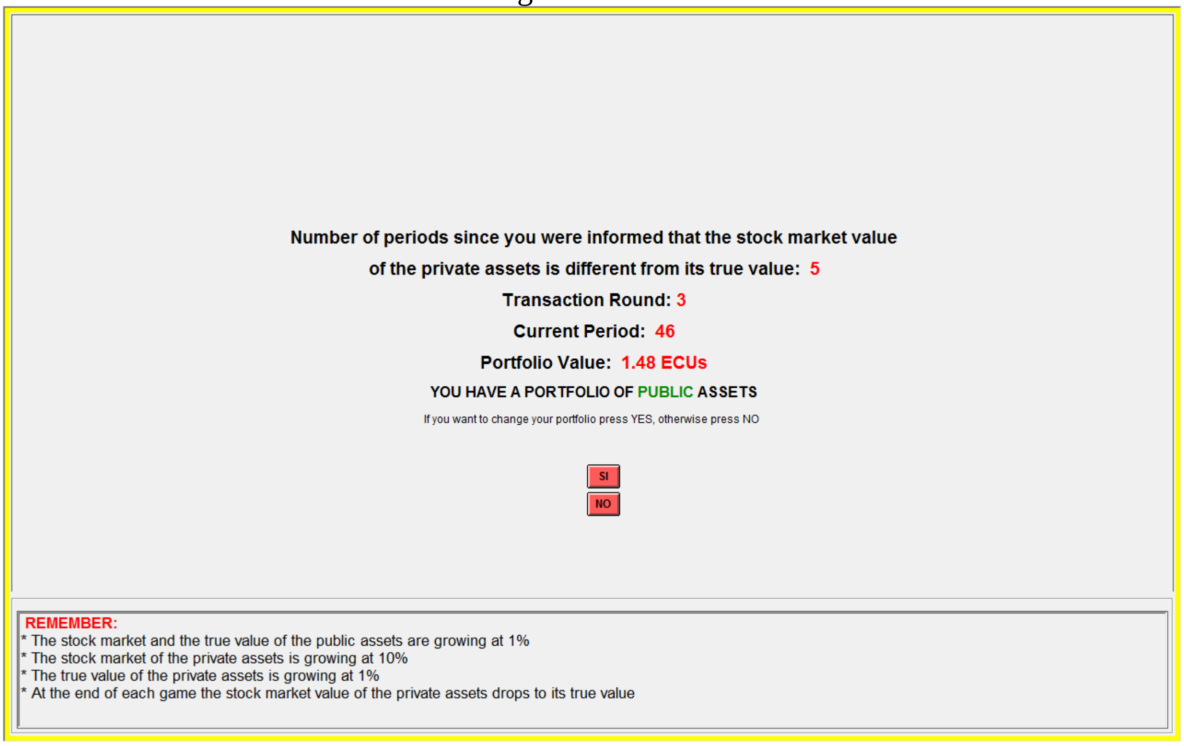


Table B1.1
(Endogenous Ending)

Period	Private Asset: True Value ≠ Stock Market Value	Number of periods since (for the private asset): True Value ≠ Stock Market Value	Number of traders with portfolios of public assets
t0-2	NO	-	
t0-1	NO	-	
t0	YES	0	Less than 15
t0+1	YES	1	Less than 15
t0+2	YES	2	Less than 15
t0+3	YES	3	Less than 15
t0+4	YES	4	Less than 15
…	YES	…	Less than 15
…	YES	…	Less than 15
t	YES	Greater or Equal than 15	15
t+1	**Endogenous Ending**		

2. The period $t_0 + 100$ is reached, even though fewer than 15 traders have portfolios of public assets (***exogenous ending***).

To better understand how both kind of endings work, analyze Tables B1.1 and B1.2.

> ***Note:*** *In the last page of these instructions there are two examples that can help you to understand how evolve the value of your portfolio depending on your decisions.*

Table B1.2
(Exogenous Ending)

Period	Private Asset: True Value ≠ Stock Market Value	Number of periods since (for the private asset): True Value ≠ Stock Market Value	Number of traders with portfolios of public assets
t0-2	NO	-	
t0-1	NO	-	
t0	YES	0	Less than 15
t0+1	YES	1	Less than 15
t0+2	YES	2	Less than 15
t0+3	YES	3	Less than 15
t0+4	YES	4	Less than 15
...	YES	...	Less than 15
t0+98	YES	98	Less than 15
t0+99	YES	99	Less than 15
t0+100	**Exogenous Ending**		

Public Message[96]

All traders know that the following message:

DON'T LEAVE FOR TOMORROW WHAT YOU CAN DO TODAY

appears with a probability of 5% to all traders that already know that there is a divergence between the true value and the stock market value of the private assets.

Final Comments

In period 0 the value of the portfolio of all traders is the same and has been randomly assigned in a value unknown to all traders. However, when you are informed about the divergence between the true value and the stock market value of the private assets (i.e. a period that is greater or equal to t_0 and lower or equal to $t_0 + 24$) the value of your portfolio has been normalized at 1 ECU (experimental currency unit).

In addition, take into account two things: (1) in the screen of your computer you are always notified about the current value of your portfolio and (2) during each transaction round the traders never know the current value of the portfolio of the other traders. However, at the end of each transaction round in the screen of your computer you will be informed about the value of your portfolio, the three highest portfolio values obtained by the computer agents and the earnings that you obtained in the respective transaction round.

On the other hand, at the end of each transaction round one of following three messages will appear in the screen of your computer:

[96] This section only appears in the sunspot sessions

You finishes with a portfolio of public assets.

You finishes with a portfolio of private assets.

I am sorry, you were not allowed to change the portfolio of active assets to get a portfolio of safe assets.

The last message appears if in the last period you try to change the portfolio of private assets to get a portfolio of public assets but unfortunately there is an endogenous ending in which you are not selected to do the change.

Your Earnings

At the end of the experiment you will be paid in cash and in private according to the earnings obtained in all transaction rounds and you will also obtain a show-up fee. The conversion rate that we will use is $1\ ECU = 0.05€$.

By participating in this experiment you have already earned $100\ ECUs$ (i.e. 5€). In addition, in every transaction round your earnings are calculated using the following equation:

$$Earnings\ in\ ECUs = \frac{Final\ value\ of\ your\ portfolio}{Final\ true\ value\ of\ the\ portfolio\ of\ private\ assets} - 1$$

This equation implies that to obtain high earnings is needed that the final value of your portfolio in each round will be high. In addition, note that in every period <u>your earnings in ECUs are always higher or equal to zero</u> because the final value of your portfolio is always higher or equal than the true value of the portfolio of private assets.

Remember that during your participation in each transaction round the final value of your portfolio grows at 10% if you have private assets or at 1% if you have public assets. Therefore, the value of your portfolio at the end of each transaction round is always higher than the true value of the portfolio of private assets (that grows at 1%) and only is equal if the transaction round ends and you already have a portfolio of private assets.

Important: *if a transaction round ends and you already have a portfolio of private assets then your earnings will be zero during this round because at the end of any transaction round the stock market value of the private assets is equal to its true value (remember Figure B1.1). However, you can obtain higher earnings if you maintain during more periods a portfolio of private assets because during the time in which you participate in each transaction round (before its ending) the stock market value of the private assets grows at 10% while the stock market value of the public assets grows at 1% (remember Figure B1.1)*

Before the beginning of the experiment check the test of understanding that is attached to these instructions. Please, focus in your experiment, do not talk to other people in this room and remember that your earnings are **NOT** affected by their decisions or by the rounds in which they are participating.

ARE THERE ANY QUESTIONS?

Test of Understanding

Read the instructions again yourself, try to answer the following questions (the answers are after the last question). If you believe that you already understand how the experiment works, please begin the experiment.

1. *Look at Figure B1.2,*
   a) *What will be the kind of portfolio you have in the next period if you choose YES?*
   b) *What will be the kind of portfolio you have in the next period if you choose NO?*
2. *Look at Figure B1.3,*
   a) *What will be the kind of portfolio you have in the next period if you choose YES?*
   b) *What will be the kind of portfolio you have in the next period if you choose NO?*
3. *You are participating in the same transaction rounds with other people in the room:*
   - *True*
   - *False*
4. *The true value of the private assets is always equal to its stock market value:*
   - *True*
   - *False*
5. *The true value of the public assets is always equal to its stock market value:*
   - *True*
   - *False*
6. *The transaction rounds are designed such that all traders receive at different sequential periods the warning message that the true value of the private assets differs from their stock market value:*
   - *True*
   - *False*
7. *A transaction round end once 15 traders have in the same period a portfolio of public assets:*
   - *True*
   - *False*
8. *How many computer agents participate in each transaction round?*

   ____

   ____
9. *A trader can recognize exactly when another trader has received the warning message that the true value of the private assets is different from is stock market value*
   - *True*
   - *False*
10. *The earnings that you obtain in each transaction round are always greater or equal than zero:*
    - *True*
    - *False*

Answers: (1) a) a portfolio of public assets, b) a portfolio of private assets; (2) a) a portfolio of private assets, b) a portfolio of public assets; (3) false; (4) false; (5) true; (6) true; (7) true; (8) 24; (9) false; (10) true.

Examples

Example: Assume you have in period **t** a portfolio of public assets of 2 ECUs. What is the value of your portfolio in **t+1**?

Answers: There are three cases:

1. If you change to the portfolio of private assets (i.e. if you click YES in period **t**) and the transaction round does not end in period **t** then the value of your portfolio in **t+1** is 2*(1+10%) =2*(1+0.1) =2*1.1= 2.2 ECUs
2. If you change to the portfolio of private assets (i.e. if you click YES in period **t**) and the transaction round ends in period **t** then the value of your portfolio in **t+1** falls to the true value of the portfolio of private assets.
3. If you decide to continue with the portfolio of public assets (i.e. if you click NO in period **t**) the value of your portfolio in period **t+1** is 2*(1+1%) = 2*(1+0.01) = 2*1.01 = 2.02 ECUs

Example: Assume you have in period **t** a portfolio of private assets of 2 ECUs. What is the value of your portfolio in **t+1**?

Answers: There are three cases:

1. If you change to the portfolio of public assets (i.e. if you click YES in period **t**) the value of your portfolio in **t+1** is 2*(1+1%)=2*(1+0.01) =2*1.01=2.02 ECUs
   Note: If in period **t** there is an endogenous ending then there exist the possibility that your change of portfolio is not allowed, in this particular case the value of your portfolio in period **t+1** falls to the true value of the portfolio of private assets
2. If you continue with the portfolio of private assets (i.e. if you click NO in period **t**) and the transaction round does not end in period **t** then the value of your portfolio in period t**+1** is 2*(1+10%) = 2*(1+0.1) =2*1.1 = 2.2 ECUs
3. If you continue with the portfolio of private assets (i.e. if you click NO in period **t**), and the transaction round ends in period **t** then the value of your portfolio in period **t+1** falls to the true value of the portfolio of private assets.

At the end of the experiment the participants also answered a questionnaire in order to get more information about them and about the decisions taken by them in the experiment.

1.8 Tables

Table 1.1
t-tests of the delays (omitting the right censored delays)

Session	Case	Variable	Theoretical Equilibrium Value	Experiment Result					
				Observations	Mean	Standard Error	Standard Deviation	[95% Conf. Interval]	
Baseline	$t_i < \tau_K$	$T(t_i)= \tau_K + \tau^*$	28	74	21.35	0.86	7.39	19.64	23.06
	$t_i \geq \tau_K$	τ^*	13	190	9.07	0.41	5.69	8.25	9.88
Sunspot	$t_i < \tau_K$	$T(t_i)= \tau_K + \tau^{**}$	22	84	14.85	0.62	5.72	13.60	16.09
	$t_i \geq \tau_K$	τ^{**}	7	175	4.59	0.33	4.42	3.93	5.25

Table 1.2
t-tests of the delays (recoding the right censored delays)

Session	Case	Variable	Theoretical Equilibrium Value	Experiment Result					
				Observations	Mean	Standard Error	Standard Deviation	[95% Conf. Interval]	
Baseline	$t_i < \tau_K$	$T(t_i)= \tau_K + \tau^*$	28	78	22.31	0.94	8.30	20.44	24.18
	$t_i \geq \tau_K$	τ^*	13	210	10.68	0.51	7.36	9.68	11.68
Sunspot	$t_i < \tau_K$	$T(t_i)= \tau_K + \tau^{**}$	22	90	15.72	0.68	6.44	14.37	17.07
	$t_i \geq \tau_K$	τ^{**}	7	197	6.54	0.49	6.90	5.57	7.51

Table 1.3
Behavior of the traders in the experiment when there is a sunspot message[97]

Range of periods in which the trader receive the sunspot message	Type of portfolio	Next type of portfolio		
		Safe	Risky	Total
< 15	Safe	31	1	32
	Risky	25	16	41
	Total	**56**	**17**	**73**
≥ 15	Safe	71	5	76
	Risky	47	41	88
	Total	**118**	**46**	**164**

Table 1.4
Number of buybacks in the baseline session[98]

Session	Case	Variable	Theoretical Equilibrium Value	Experiment Result					
				Observations	Mean	Standard Error	Standard Deviation	[95% Conf. Interval]	
Baseline	–	Number of times agents buyback to get the portfolio of risky assets	0	475	0.74	0.07	1.51	0.60	0.87
	$T^*(t_0) < t_i + \tau^*$			187	0.36	0.06	0.82	0.25	0.48
	$T^*(t_0) \geq t_i + \tau^*$			288	0.98	0.10	1.78	0.78	1.19

[97] Notice that given the randomness process of the sunspot message, it does not appear in all games in the sunspot session.

[98] In appendix A1 $T^*(t_0)$ is defined as the bursting time of the bubble for a given realization of t_0. Therefore, the table 1.4 differentiates the cases in which the traders have not or have enough time to apply the optimal delay.

Table 1.5

Number of buybacks to get again the portfolio of risky assets

Session		Number of Buybacks										
		0	1	2	3	4	5	6	7	8	9	10
Baseline*	Number of games	324 68.21%	71 14.95%	35 7.37%	18 3.79%	5 1.05%	10 2.11%	4 0.84%	5 1.05%	1 0.21%	1 0.21%	1 0.21%
	Number of traders	18 94.74%	16 84.21%	10 52.63%	5 26.32%	3 15.79%	3 15.79%	2 10.53%	2 10.53%	1 5.26%	1 5.26%	1 5.26%
Sunspot	Number of games	340 71.58%	78 16.42%	35 7.37%	13 2.74%	3 0.63%	5 1.05%	1 0.21%	0 0.00%	0 0.00%	0 0.00%	0 0.00%
	Number of traders	19 100.00%	14 73.68%	13 68.42%	6 31.58%	3 15.79%	3 15.79%	1 5.26%	0 0.00%	0 0.00%	0 0.00%	0 0.00%

*In the baseline session, the trader that has 8, 9, 10 buybacks are the same (in addition, this is the only trader that always had buybacks). On the other hand, in this session two traders never had buybacks.

Table 1.6

Choices of portfolios in the sunspot session after the appearance of a sunspot message that does not implied a bursting of the bubble

Type of portfolio	Next type of portfolio		
	Safe	Risky	Total
Safe	47	13	60
Risky	9	17	26
Total	**56**	**30**	**86**

Table 1.7a

Dependent Variable	FirstChoice(g)		FirstChoiceC(g)	
Variables /Sessions	Baseline	Sunspot	Baseline	Sunspot
Periodti(g)	0	0	0	0
Periodcburst(g-1)	0	0	0	0
Goodtiming(g-1)	0	0	0	0
Periodtime(g-1)	0	0	0	0
Regret(g-1)	0	0	0	0
Game(g)	0	0	0	0
Sunspot(g)		Negative		Negative
Succ_attack(g-1)		0		0
Database: *Periodti(g)*	≥15		<15	

Table 1.7b

Dependent Variable	Numberattacks(g)		
Variables /Sessions	Baseline		Sunspot
Periodti(g)	0	0	0
Periodtime(g)	0	0	0
Periodcburst(g-1)	0	0	0
Goodtiming(g-1)	0	0	0
Periodtime(g-1)	0	0	0
Regret(g-1)	0	0	0
Game(g)	0	0	0
Sunspotsum(g)			Positive
Succ_attack(g-1)			0
Database: *Periodtime(g)*	<13	≥13	All

Table 1.8a

Fixed effects models

Dependent Variable	FirstChoice(g)		FirstChoiceC(g)	
Variables /Sessions	**Baseline**	**Sunspot**	**Baseline**	**Sunspot**
Periodti(g)	-0.0252	-0.02	**0.731*** **	**0.644*** **
	(0.187)	(0.013)	(0.207)	(0.162)
Periodcburst(g-1)	0.00232	0.00403	0.0137	-0.0019
	(0.014)	(0.009)	(0.022)	(0.022)
Goodtiming(g-1)	-1.02	0.372	1.79	1.83
	(0.941)	(0.532)	(1.678)	(1.158)
Periodtime(g-1)	**0.286*** **	-0.0437	-0.0007	0.0398
	(0.096)	(0.064)	(0.190)	(0.119)
Regret(g-1)	**-0.223** **	0.0636	0.098	0.207
	(0.123)	(0.140)	(0.229)	(0.284)
Game(g)	**0.0836* **	**-0.0433** **	0.0268	0.0166
	(0.042)	(0 .017)	(0.062)	(0.368)
Sunspot(g)		**-1.35** **		**-1.43** **
		(0.423)		(0.616)
Succ_attack(g-1)		0.476		0.459
		(0.449)		(1.034)
Constant	0.108	**4.5*** **	**15.7*** **	**6.41*** **
	(2.844)	(0.986)	(5.082)	(2.592)
R2_adj	0.195	0.193	0.517	0.434
Number of observations	282	255	70	80
Number of groups	19	19	19	18
Database: *Periodti(g)*	**≥15**		**<15**	

Control variable: Periodtime(g). Robust standard errors in parentheses. Significance levels: *p<0.1, **p<0.05, ***p<0.001

Table 1.8b

Conditional fixed effects Poisson panel models

Dependent Variable	Numberattacks(g)		
Variables /Sessions	**Baseline**	**Baseline**	**Sunspot**
Periodti(g)	0.006	0.001	0.000
	(0.006)	(0.002)	(0.002)
Periodcburst(g-1)	-0.004	0.000	0.000
	(0.005)	(0.002)	(0.002)
Goodtiming(g-1)	-0.005	0.093	0.176
	(0.359)	(0.183)	(0.176)
Periodtime(g-1)	-0.016	-0.006	-0.023
	(0.035)	(0.016)	(0.017)
Regret(g-1)	-0.002	0.001	0.045
	(0.051)	(0.023)	(0.035)
Game(g)	-0.006	-0.008	0.001
	(0.012)	(0.007)	(0.006)
Sunspotsum(g)			**0.152** **
			(0.062)
Succ_attack(g-1)			-0.065
			(0.104)
Periodtime(g)	**0.139*** **	**.0486*** **	**0.073*** **
	(0.040)	(0.011)	(0.007)
Log Likelihood	-118.51	-294.41	-440.93
Number of observations	162	276	456
Number of groups	16	19	19
Database: *Periodtime(g)*	**<13**	**≥13**	**All**

Notes: Robust standard errors in parentheses. Significance levels: *p<0.1, **p<0.05, ***p<0.001

Table 1.9

Fixed effects models[+]

Dependent Variable	Convergence (g)							
Variables /Sessions	Baseline				Sunspot			
Game(g)	**-0.0493****	**-0.0502****	0.00055	-0.0067	0.00402	0.00862	0.00712	0.0166
	(0.019)	(0.089)	(0.027)	(0.030)	(0.011)	(0.011)	(0.031)	(0.030)
Periodti(g)		**0.0169***		**-0.481*****		0.00438		**-0.602*****
		(0.010)		(0.141)		(0.008)		(0.100)
Sunspot: t≥15 (g)					**-1.43*****	**-1.46*****	-1.08	**-1.06***
					(0.177)	(0.170)	(0.715)	(0.584)
Constant	**8.45*****	**7.86*****	**8.69*****	**13.3*****	**4.56*****	**4.26*****	**6.98*****	**12.3*****
	(0.491)	(0.584)	(0..689)	(1.559)	(0.293)	(0.385)	(0.758)	(1.231)
R2_adj	0.055	0.0687	-0.007	0.168	0.0818	0.0846	0.0032	0.216
Number of observations	567	567	150	150	535	535	155	155
Number of groups	19	19	19	19	19	19	19	19
Database: *Periodti(g)*	≥15		<15		≥15		<15	

Notes: Robust standard errors in parentheses. Significance levels: *p<0.1, **p<0.05, ***p<0.001

+ Sunspot: t≥15 (g) is a dummy variable equal to one if the human trader receives at least one sunspot message at or after period 15 in game g

Table 1.10

Conditional fixed effects Poisson panel models

Dependent Variable	Numberattacks(g)	
Variables /Sessions	Baseline	
Game(g)	**-0.005*****	**-0.005*****
	(0.002)	(0.0018)
Periodtime(g)		**0.0612*****
		(0.0038)
Log Likelihood	-1267.45	-1130.43
Number of observations	950	950
Number of groups	19	19

Notes: Robust standard errors in parentheses

Significance levels: *p<0.1, **p<0.05, ***p<0.001

1.9 Figures

Figure 1.1

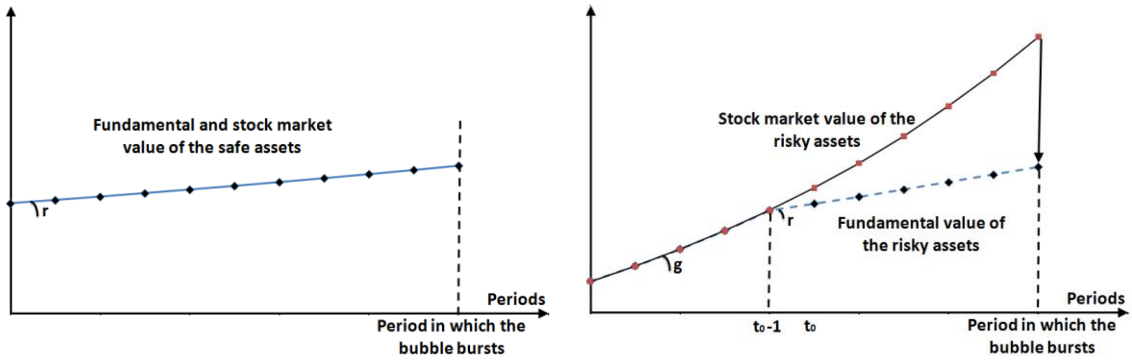
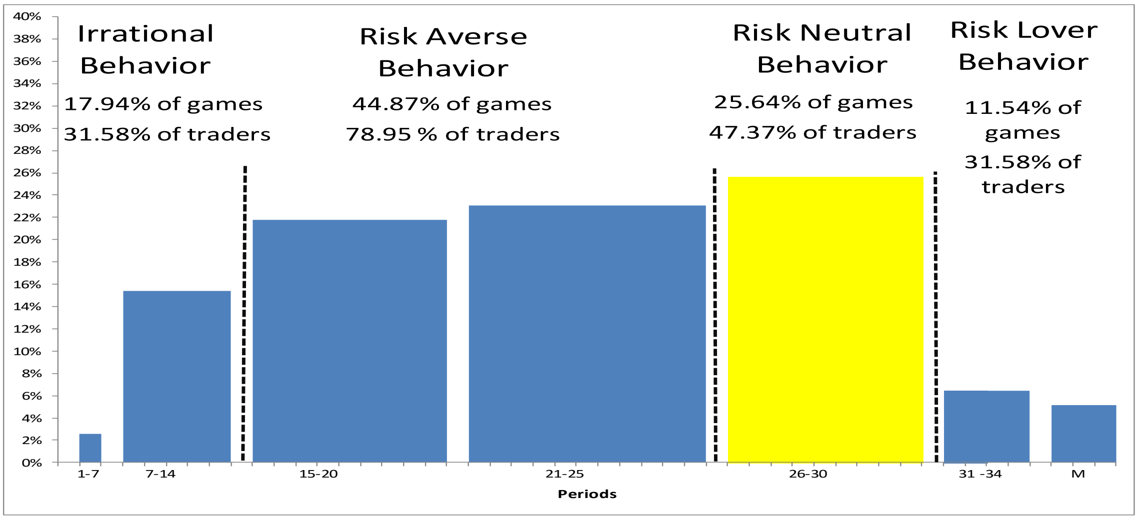


Figure 1.2

Histogram+ (baseline session): Periods in which traders that were informed about the bubble before period 15 sell their portfolio of risky assets[99,100,101]



+ The width of each column depends on the percentage of traders that at least in one game sells in the respective range of periods. The range of the yellow column measures approximately 0.5 standard deviations respect to the mean of the variable used in this figure.

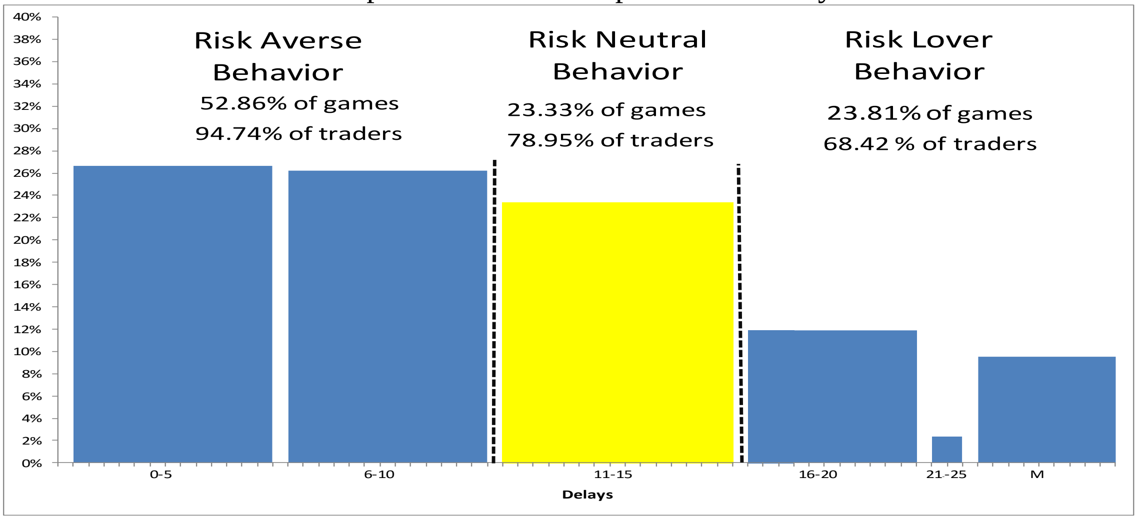
[99] M means missing. That is, these traders have not sold the portfolio of risky assets before the bursting of the bubble.

[100] In this histogram we are only considering the first time the traders decide to sell the portfolio of risky assets.

[101] In each block the first number represents the percentage of games in which all human traders behave as an irrational, risk averse, risk neutral or risk lover trader; and the second number represents the percentage of human traders that behaves at least once as an irrational, risk averse, risk neutral or risk lover trader

Figure 1.3

Histogram+ (baseline session): Delays in which traders that were informed about the bubble at or after period 15 sell their portfolio of risky assets[50,51,52]



\+ The width of each column depends on the percentage of traders that at least in one game sells in the respective range of delays. The range of the yellow column measures approximately 0.8 standard deviations respect to the mean of the variable used in this figure.

Figure 1.4

Positive time effect when human traders are informed about the bubble before period 15

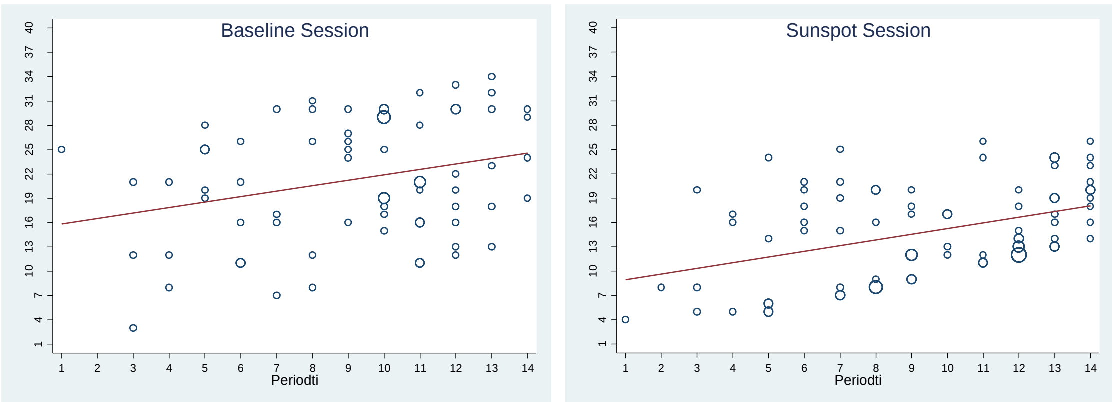
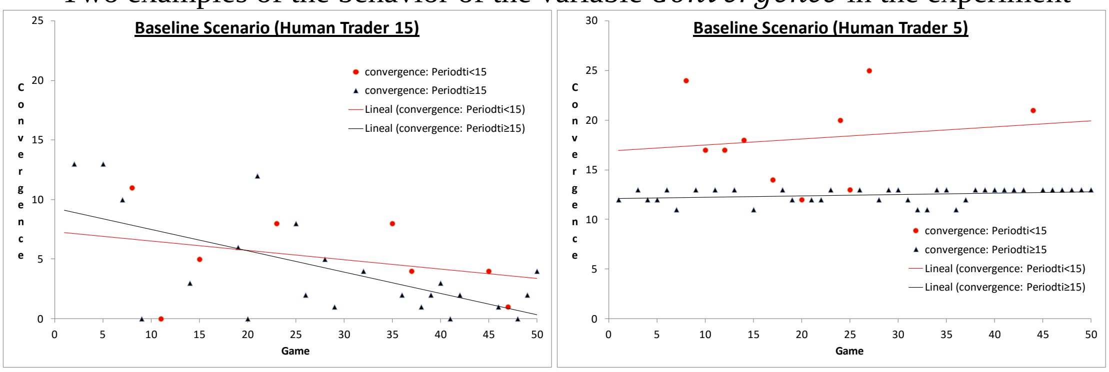


Figure 1.5

Two examples of the behavior of the variable *Convergence* in the experiment



Chapter 2

MULTIPLE SOURCES OF INFORMATION AND STRATEGIC BEHAVIOR

2.1. Introduction

Mackowiak and Wiederholt (2009) built a rational inattention macroeconomic model in which price setting firms face a trade-off between paying attention to aggregate conditions and paying attention to idiosyncratic conditions; the firms have to decide the amount of attention they pay to both kinds of conditions.

This model is useful to explain some characteristics of the US empirical data[1]. These explanations are possible due to the following two theoretical results obtained in the model (Mackowiak and Wiederholt, 2009, p. 770):

1. *When idiosyncratic conditions are more variable or more important than aggregate conditions, firms pay more attention to idiosyncratic conditions than to aggregate conditions*.

In their model firms adjust prices in every period, but nonetheless impulse responses of prices to shocks are sticky and delayed relative to the impulse responses under perfect information. The extent of stickiness in a particular impulse response depends on the amount of attention allocated to that type of shock. If firms pay more attention to idiosyncratic conditions than to aggregate conditions then prices respond strongly and quickly to idiosyncratic shocks, but only weakly and slowly to aggregate shocks.

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[1] More specifically, it explains two empirical findings: (1) why the price level responds slowly to monetary policy shocks, despite the fact that individual prices change fairly frequently and by large amounts, and (2) why sectoral prices respond quickly to sector-specific shocks and slowly to monetary policy shocks (e.g. look at also the empirical research of Boivin, Giannoni and Mihov (2009)).

In addition, Mackowiak, Moench and Wiederholt (2009) using US real data compare the results obtained from the standard Calvo model, the sticky information model built by Mankiw and Reis (2002) and the rational inattention model proposed by Mackowiak and Wiederholt (2009). According to the empirical data in the median sector 100 percent of the long-run response of the sectoral price index to a sector-specific shock occurs in the month of the shock. The standard Calvo model and the standard sticky-information model can match this finding only under extreme assumptions concerning the profit-maximizing price. The model of Mackowiak and Wiederholt (2009) can match this finding without these extreme assumptions. Furthermore, according to the empirical data there is little variation across sectors in the speed of response of sectoral price indexes to sector specific shocks. The rational inattention model matches this finding, while the Calvo model predicts too much cross-sectional variation.

2. *There are interaction effects (feedback effects) because firms track endogenous variables*[2].

The interaction effects happen when the aggregate variables are affected by the choices of agents that have information constraints, and through this way the individual choices of the rest of agents are also affected by these constraints[3]. More specifically, the information constraints imply an overreaction (or underreaction) of the aggregate and individual variables respect to a scenario in which there are not (or less) information constraints. For instance, in Mackowiak and Wiederholt (2009) when other firms pay limited attention to aggregate conditions, the price level responds less to a nominal shock than under perfect information. In addition, if prices are strategic complements then each firm has even less incentive to attend to aggregate conditions. The price level responds even less to a nominal shock, and so on.

The above two theoretical results are not easy to analyze using field data because it is difficult to observe directly the amount of attention that people (or firms) pay to all relevant sources of information that they have; therefore, it is hard to categorize how important and variable are these different sources of information for them. In addition, if at least one of the sources of information is endogenous to the decision process, sometimes using field data you cannot control how much interaction there is between the people (or firms) who are interacting in this context.

On the other hand, in a lab experiment you can: control the number of sources of information, predetermine in certain contexts the quality of the information that these sources provide[4], and manipulate the amount of information that the participants have. In addition, in a lab experiment you can control the level of endogeneity that the different variables have, so you can better analyze how the interaction between the agents is.

We propose a lab experiment to analyze the two theoretical results obtained by Mackowiak and Wiederholt (2009); our analysis is the first one that uses a lab experiment to study this paper. It is important to clarify that we do not propose an experimental design that follows closely the model of Mackowiak and Wiederholt (2009) because they did not build a model that can be tested directly in a lab experiment. We propose a simple theoretical model in which we can reach the same two theoretical results obtained in their paper but that can be easily implementable in a lab experiment.

[2] Mackowiak and Wiederholt (2009) use the expression "feedback effects" instead of "interaction effects"; however, in economics is more common the use of the second expression instead of the use of the first expression when one tries to explain the kind of phenomenon that Mackowiak and Wiederholt describe, so in the rest of the paper we will continue using the last expression instead of the first one.

[3] Mackowiak and Wiederholt (2015) also propose a model that has interaction effects. In this paper they explain the interaction effects of their model using several examples, for instance: "Let us explain the interaction effects by focusing on monetary policy shocks. *To begin, suppose that only a single firm is subject to rational inattention, while all other firms and all households have perfect information* (this is the case with the interaction effects are switched off). *The profit-maximizing price response of this single firm to a monetary policy shock is several times larger in absolute terms than in the baseline economy* (where all other firms and households also have limited attention). *As a result, this single firm chooses to allocate five times more attention to monetary policy than in the baseline economy*" Mackowiak and Wiederholt (2015, p. 1526)

[4] For instance, the information that a particular source of information provides about a random variable has better quality if the information of this source reduces the volatility of the variable more than another source or if it explains a higher part of the value of the variable.

Mackowiak and Wiederholt (2009) assume that the aggregate conditions are taken as given by the firms[5]. This assumption is helpful to find a unique solution in their model; however, it is quite restrictive in terms of the analysis of the interaction effects because it avoids the presence of strategic behavior; then, in the last part of the model and in the biggest part of the experiment we will allow the presence of this kind of behavior to better analyze the implications of the interaction effect.

The rest of the chapter is organized as follows. Section 2.2 explains our theoretical model. Section 2.3 presents the experimental design. Section 2.4 shows the results obtained in the experiment. Finally, section 2.5 presents the conclusions of the chapter.

2.2. Model

In our model the goal of each agent i is to take a choice, about a variable q_i, to maximize her utility function; therefore, in this section we firstly explain the main characteristics of the optimization problem that each agent solves. In particular, we assume that the utility function of each agent i is directly affected by the distance between q_i respect to some aggregate terms and respect to an idiosyncratic term. One of the aggregate terms that affects the utility function of the agents is the average choices of all the agents in the economy; this term is helpful to analyze how the interaction effect works.

Remember that the main goal of the chapter is to analyze the two theoretical results obtained by Mackowiak and Wiederholt (2009). The first theoretical result is clearer and easier to understand if we eliminate the interaction effect from the optimization problem of the agents. Therefore, in section 2.2.1 we use a version of our model which is an individual choice problem that does not include, in the utility function of the agent, the average choices of all the agents in the economy.

The second theoretical result obtained by Mackowiak and Wiederholt (2009) is analyzed in sections 2.2.2 and 2.2.3[6]. Since in both sections we are analyzing the interaction effect, we retake again the most general version of our model in which the utility function is also affected by an aggregate variable that depends on the choices of all agents in the economy. The main difference between the models in section 2.2.2 and 2.2.3 is that in the last one the agents have uncertainty about the choices of the other agents; however, in the first one we assumed a commitment mechanism that force the agents to reveal the choices that they will take.

The agents in Mackowiak and Wiederholt (2009) solve an infinite horizon discrete time optimization problem. However, in their model, the two results that we want to analyze are obtained in each period independently of what happens in other periods. Therefore, we propose a static model in which there is an economy with $N > 1$ agents;

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[5] More specifically, Mackowiak and Wiederholt (2009) assume that there is a continuum of firms that take as given the aggregate and idiosyncratic conditions. In their model the idiosyncratic conditions and almost all the aggregate conditions (except the price level) follow exogenous stochastic processes. The individual prices are optimally chosen by the respective firms, the price level is the weighted sum of all individual prices and it is taken as given by the firms when these decide the individual prices, so their model does not give the opportunity to analyze the strategic behavior in the individual price decision.

[6] In both sections we also explain in which way the first theoretical result of Mackowiak and Wiederholt (2009) is taken into account.

the goal of each agent i is to choose the value of the variable q_i that maximizes the following utility function[7]:

$$\max_{q_i} U_i = X_i - W_i \left[\frac{1}{2} b(q_i - l_i)^2 + \frac{1}{2} c \left(q_i - \frac{\sum_{j=1}^{N} q_j}{N} - v\right)^2\right] \quad \forall\, i = 1,2, \dots, N$$

subject to $b > 0$ and $c > 0$; where X_i and W_i are predetermined positive parameters. Therefore, the utility of each agent i is lower if the distances of the variable q_i respect to her idiosyncratic term l_i and to the aggregate term $\frac{\sum_{j=1}^{N} q_j}{N} + v$ increase, where the ratios $\frac{b}{b+c}$ and $\frac{c}{b+c}$ are the relative weights that each agent i gives to the square of these two distances. The aggregate term in the utility function is composed by two elements: (1) the average choices of all agents in the economy (i.e. $\frac{\sum_{j=1}^{N} q_j}{N}$) and (2) a variable v that affects all agents but that is independent of agent choices. The idiosyncratic term l_i only affects directly the utility of the agent i, is independent of agent choices and is uncorrelated with the idiosyncratic terms of the other agents[8]. Since our model is static, then l_i and v are predetermined real numbers.

The optimization problem that we have proposed above is standard in many areas of the economic analysis. For instance, this kind of problem appears when the firms have to establish the price of their products using as reference information obtained from their respective markets and from the whole market (e.g. the aggregate price index and other macroeconomic conditions). Similarly, this kind of problem is solved by any firm when it has to determine the quality of its product using as reference its niche market, the quality choices of its competitors and additional conditions that can affect the whole economy, In addition, our optimization problem can also represents a government that has to take a choice about any policy action taking into account the characteristics of its own country, the choices of the governments of other countries and additional characteristics that affect the world context. Finally, this problem is also solved by the members of any household or club when their decisions have to take into account not only personal interests, but also the decision of the other members and exogenous elements that are not controlled by any of the members of their group.

Notice that in the optimization problem of agent i, the variables q_i and $q_{j\neq i}$ are strategic complements where the size of this complementarity is decreasing respect to

[7] Similar functions have been used in other papers of experimental economics; for instance, look at the experiment of Cornand and Heinemann (2014). The purpose of this kind of functions is to represent a decision environment that is easier to understand by the participants in an experiment than the more complex decision environment of the original model; however, these functions should incorporate all the elements that are necessary to understand the problem that the experimenter wants to study. In particular, the most important characteristic of our utility function is that it shows an agent i that faces a trade-off between choosing a q_i closer to her idiosyncratic condition or closer to the aggregate conditions.

[8] In our model, the main difference between an idiosyncratic term and an aggregate term is that and idiosyncratic term only directly affects the utility of a particular agent (e.g. l_i affects U_i but not U_j for all $j \neq i$). On the other hand, an aggregate term directly affects the utility of all agents. We are considering two kind of aggregate terms, one that can be directly affected by the choices of any agent (e.g. $\frac{\sum_{j=1}^{N} q_j}{N}$) and one that cannot be directly affected by the choice of a particular agent (e.g. v)

the number of agents and increasing respect to c[9]. This strategic complementarity is crucial to understand the behavior of the agents in the model and to better understand the behavior of the participants in our lab experiment.

Let z_i represents the part of the aggregate term in the utility function that is not directly affected by q_i; that is, in the optimization problem of agent i we have $z_i = \frac{\sum_{j=1}^{N} q_{j \neq i}}{N-1} + \left(\frac{N}{N-1}\right) v$ where $\sum_{j=1}^{N} q_{j \neq i} \equiv \sum_{j=1}^{N} q_j - q_i$. In addition, to simplify notation, assume the constant term $a = c\left(\frac{N-1}{N}\right)^2$, then our optimization problem can be expressed as:

$$\max_{q_i} U_i = X_i - W_i \left[\frac{1}{2} b(q_i - l_i)^2 + \frac{1}{2} a(q_i - z_i)^2\right] \; where \; i = 1,2, \dots, N \qquad (2.1)^{10}$$

subject to $b > 0, \; a > 0$

Equation (2.1) represents the deterministic version of our model. Therefore, the next step is to introduce the rational inattention characteristics to this model. We assume that the agents do not have enough time or capacities to process all relevant information, then each agent i has to optimally choose the amount of information she processes in order to approach to the real value of l_i and z_i.

We will analyze three versions of the model. In the first one there is no interaction between the agents (section 2.2.1); in the second there is interaction between the agents through a commitment mechanism[11] that forces each agent i to choose a freely predetermined value of q_i such that the interaction between the agents does not give the opportunity to strategic behavior[12] (section 2.2.2) ; finally, in the last one there is interaction between the agents and they are not forced to set a predetermined value of q_i, so in this version we consider the presence of strategic behavior (section 2.2.3).

In all versions of the model, each agent i has to solve the following optimization problem that is the stochastic version of the problem presented in equation (2.1),

$$\max_{q_i} \; E_i(U_i| *) = X_i - W_i \left\{\frac{1}{2} b E_i[(q_i - l_i)^2| *] + \frac{1}{2} a E_i[(q_i - z_i)^2| *]\right\}$$

where $i = 1,2, \dots, N$ and $b > 0, \; a > 0$. $E(x| *)$ is the expected value of x given some information constraints represented by the moment by the symbol “$*$”.

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[9] Mathematically the strategic complementarity between these variables in our model is represented by $\frac{\partial U}{\partial q_i \partial q_{j \neq i}} = W_i c \frac{(N-1)}{N^2} > 0$. Therefore, if $N \to \infty$ or $c \to 0$ then the level of complementarity between q_i and $q_{j \neq i}$ approaches to zero.

[10] In this new specification the strategic complementarity is represented by $\frac{\partial U}{\partial q_i \partial q_{j \neq i}} = W_i \frac{a}{N-1} > 0$. Therefore, the strategic complementarity between q_i and $q_{j \neq i}$ increases when a increases.

[11] This mechanism can be a contract, a law, an institution, a technology or any other mechanism that can bind the agents to respect some predetermine agreements.

[12] In this version, q_i is freely determined by each agent i; however, all agents publically know in advance the unmodified values of all q_i. It implies that $Q = \sum q_i$ is taken as given as it happens with the price level in the model of Mackowiak and Wiederholt (2009).

2.2.1. Without interaction. An individual decision making approach

In this version of the model we assume that the optimization problem of agent i is not affected by the average choices of all agents in the economy. That is, the term $\frac{\sum_{j=1}^{N} q_j}{N}$ doesn't affect the utility function U_i, and consequently the aggregate term in the utility function is only represented by v[13]. Remember that z_i represents the part of the aggregate term in the utility function that is not directly affected by q_i; therefore, in this version of the model each agent i has to solve the following optimization problem

$$\max_{q_i} \; E_i(U_i|*) = X_i - W_i \left\{ \frac{1}{2} b E_i[(q_i - l_i)^2|*] + \frac{1}{2} a E_i[(q_i - z_i)^2|*] \right\}$$

where $b > 0, a > 0$ and $z_i = v$. Respect to this optimization problem, we have to clarify one thing; in the previous section we assumed that a is a positive parameter proportional to the positive parameter c. Therefore, to avoid to use to many expressions in our explanation of the model, we assume without loss of generality that $a = c$.

Notice that in the model of this section is not possible to analyze the interaction effect. However, we can analyze directly a version of the first result obtained by Mackowiak and Wiederholt (2009): "When l_i is more variable or more important than z_i, the agent i pays more attention to l_i than to z_i. Similarly, if z_i is more variable or more important than l_i the agent i pays more attention to z_i than to l_i". That is, in this version of the model agent i has to solve two individual decision making problems: (1) she has to choose if she pays more attention to l_i or to z_i, and (2) she has to choose if q_i is closer to l_i or closer to z_i.

Until now, we have not explained in detail the kind of uncertainty that the agents face. We will assume, as it was assumed by Mackowiak and Wiederholt (2009), a rational inattention structure. Therefore, agent i has to solve a rational inattention problem in two stages. In the first stage agent i, given her own information processing capacity constrain κ, must decides the optimal amount of attention she would pay to the unknown expressions l_i and z_i[14]. Finally, in the second stage each agent i has to solve the following optimization problem:

$$\max_{q_i} \; E_i\big(U_i|I_T(z_i); I_V(l_i)\big) = X_i - W_i \left[\tfrac{1}{2} b E_i[(q_i - l_i)^2|I_V(l_i)] + \tfrac{1}{2} a E_i[(q_i - z_i)^2|I_T(z_i)] \right] \quad (2.1')$$

where $b > 0,\ a > 0$. $E_i[(q_i - x)^2|I(x)]$ is the expected value of $(q_i - x)^2$ given $I(x)$, where $I(x)$ is the optimal amount of attention paid by agent i to reduce the entropy (i.e. the uncertainty) of variable x

In section 2.2.1.1 we explain the main considerations that each agent i has to take into account to solve the first stage of the problem. In section 2.2.1.2 we explain the

[13] The problem that that the agents solve is not one in which $N = 1$, it is one in which $\frac{\sum_{j=1}^{N} q_j}{N}$ does not enter in the utility function (i.e. the choices of the agents $j \neq i$ does not affect U_i))

[14] Appendix A2 summarizes the maximum levels of the information processing capacity constraints assumed in different papers

second stage of the problem and we integrate both stages to explain the optimal behavior of agent i in the model. Finally, in section 2.2.1.3 we reveal how our model obtains the first result of Mackowiak and Wiederholt (2009).

2.2.1.1. The first stage.

To solve the first stage of our rational inattention problem we need to explain: (1) the characteristics of the information the agents have to process in order to approach to the exact value of the random variables l_i and z_i; (2) how we measure the amount of uncertainty that each agent i has about the random variables l_i and z_i; and (3) how each agent i decides the optimal amount of attention she spends to process the information about both unknown variables.

Characteristics of the information that the agents do not know about l_i and z_i

The agents do not know the exact value of the variables l_i and z_i. In particular, they have an initial prior about each variable[15], these priors are represented by the variables l_{i0} and z_{i0}. Given the prior, then the agent i has to process many pieces of information in order to approach to the exact values of l_i and z_i. A priori the agents do not know these pieces but they know their statistical distribution. Given these distributions and given their own processing capacity constraints, each agent i has to decide what and how much information she will process[16].

As we do in the lab experiment that we explain in section 2.3, we assume that in order to know the exact values of l_i and z_i, agent i has to process "one" piece of information in each case. This assumption has the empirical advantage that that the first theoretical result of Mackowiak and Wiederholt (2009) can be analyzed directly[17]. Therefore, we are assuming that the exact values of the variables l_i and z_i are represented by:

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[15] For instance, in a dynamic model, the prior of a variable can be represented by the value of the variable in the previous period or by the expected value of the variable given some known information.

[16] Some rational inattention models explain the information constraint in term of signals. For instance, the agents cannot observe the exact value of a determine variable, but they can observe signals about this variable. When the information constraint decreases, it means an improvement in the quality or the quantity of the signals they are processing, where this improvement implies that the agents have a better approach to the real value of the unknown variable. In our model when the information constraint decreases, the agents can get new pieces of information that were not previously available, so they can also have a better approach to the real value of the unknown variable.

[17] In addition, this assumption makes the experiment easier and faster to understand and play, and it also makes the experiment more controllable. However, this assumption has the theoretical weakness that the agent can only pay full attention to the aggregate or to the idiosyncratic conditions, but she cannot pay partial attention to any of them. Therefore, in Appendix B2 we explain a model in which in order to know the exact values of l_i or z_i, agent i has to process "three" pieces of information in each case; this is the simplest version of our model in which we can get a clear example in which an agent optimally (given her processing capacity constraint) decides to: (1) pay full attention only to the aggregate conditions, (2) pay full attention only to the idiosyncratic conditions or (3) pay partial attention to the aggregate and idiosyncratic conditions.

$$l_i = l_{i0} + \varepsilon_i \quad where \;\; \varepsilon_i \in \{-r_{l_i}, 0, r_{l_i}\}, \qquad r_{l_i} > 0, \qquad l_{i0} \; is \; known \tag{2.2}$$

$\text{Prob}(\varepsilon_i = -r_{l_i}) = \text{Prob}(\varepsilon_i = 0) = \text{Prob}(\varepsilon_i = r_{l_i}) = \frac{1}{3}$ and

$$z_i = z_{i0} + \mu_i \;\; where \;\; \mu_i \in \{-r_{z_i}, 0, r_{z_i}\}, \;\; r_{z_i} > 0, \qquad z_{i0} \; is \; known \tag{2.3}$$

$\text{Prob}(\mu_i = -r_{z_i}) = \text{Prob}(\mu_i = 0) = \text{Prob}(\mu_i = r_{z_i}) = \frac{1}{3}$.

The random variables ε_i and μ_i represent the pieces of information that the agent i does not know about l_i and z_i, where the parameters r_{l_i} and r_{z_i} are the dispersion of ε_i and μ_i respectively. Notice that we have assumed that ε_i and μ_i have a discrete uniform independent distribution, then l_i and z_i also have a discrete uniform independent distribution.

Entropy of l_i and z_i

In information theory, entropy is a measure of the uncertainty about a random variable. In particular, depending on the level of uncertainty about l_i we have in our model two levels of entropy:

(1) If ε_i is unknown, then $\varepsilon_i \in \{-r_{l_i}, 0, r_{l_i}\}$ where $Prob_{\varepsilon_i}(-r_{l_i}) = Prob_{\varepsilon_i}(0) = Prob_{\varepsilon_i}(r_{l_i}) = \frac{1}{3}$. Then, in this case the entropy about l_i is: $H_0(l_i) = -3\left(\frac{1}{3}\log_2\frac{1}{3}\right) = 1.585$ bits

(2) If ε_i is known , then the entropy about l_i is: $H_1(l_i) = 1\log_2 1 = \log_2 1 = 0$ bits (i.e. there is no uncertainty about l_i)

On the other hand, given the probabilistic distribution of μ_i, and using the same procedure that we used with l_i, we get: $H_0(z_i) = 1.585$ bits and $H_1(z_i) = 0$ bits.

Solution to the first stage of the rational inattention optimization problem

Let $I_t(f) = H_X(f) - H_{X+t}(f)$ be defined as the amount of attention that an agent has to spend to reduce the entropy of variable f from $H_X(f)$ to $H_{X+t}(f)$ where $H_X(f)$ represents the initial level of entropy of the variable f and t represents the number of pieces of information that are known in $H_{X+t}(f)$ but are unknown in $H_X(f)$. Table 2.1 summarizes the amounts of attention that agent i has to spend if she wants to reduce her entropy from $H_0(z_i)$ and $H_0(l_i)$ to $H_1(z_i)$ and $H_1(l_i)$ respectively. That is, if agent i wants to know both terms μ_i and ε_i (i.e. if she pays full attention to both variables such that at the end these variables are completely known), then she has to spend 3.17 bits of attention (i.e. $I_1(l_i)$+$I_1(z_i)$). If agent i wants to know μ_i or ε_i but not both terms, then she has to spend 1.58 bits of attention (i.e. $I_1(l_i)$ or $I_1(z_i)$). On the other hand, if she does not pay attention to any variable then she spends 0 bits of attention.

[Table 2.1]

In the first stage of our rational inattention problem, agent i chooses the amount of attention she pays to each random variable such that the utility obtained in the second stage is the highest possible. However, her choice is affected by the information processing capacity constrain κ that this agent has[18]. For instance, if $\kappa = 3$, then from Table 2.1 we know that the agent i only can choose the amount of attention she pays from the set: $A(z_i, l_i) = \{(I_0(z_i), I_1(l_i)), (I_1(z_i), I_0(l_i)), (I_0(z_i), I_0(l_i))\}$. That is, the cell of Table 2.1 that belong to the set $A^C(z_i, l_i) = \{(I_1(z_i), I_1(l_i))\}$ are not available to her.

2.2.1.2. The second stage

When agent i minimizes equation (2.1') subject to equations (2.2) and (2.3), then the optimal value q_i^* chosen by agent i is

$$q_i^* = \frac{b}{b+a} E_i(l_i | I_V(l_i)) + \frac{a}{b+a} E_i(z_i | I_T(z_i)) \quad \forall\, T, V \in \{0,1\} \tag{2.4}$$

According to the equation (2.4), q_i^* is the sum of two terms: (1) the relative weight that l_i has in the utility function times the expected value that agent i has about this variable, and (2) the relative weight that z_i has in the utility function times the expected value that agent i has about this variable. Another way to write the equation (2.4) is

$$q_i^* = \frac{b}{b+a}(l_i - \Delta_{l_i}) + \frac{a}{b+a}(z_i - \Delta_{z_i}) = q_i^{*(CI)} - \left[\frac{b}{b+a}\Delta_{l_i} + \frac{a}{b+a}\Delta_{z_i}\right] \tag{2.4'}$$

where Δ_{l_i} and Δ_{z_i} are deviations of l_i and z_i respect to $E_i(l_i | I_V(l_i))$ and $E_i(z_i | I_T(z_i))$ respectively, and $q_i^{*(CI)}$ is the optimal value of q_i when she does not face any information constraint[19]. Therefore, the optimal utility of agent i is[20]:

$$U_i^* = X_i - W_i \left[\frac{ab(z_i - l_i)^2}{2(b+a)} + \frac{\xi^2}{2(b+a)}\right] = U_i^{*(CI)} - \frac{W_i \xi^2}{2(b+a)} \qquad \forall\, i = 1,2, \dots, N\,. \tag{2.5}$$

where $\xi = b\Delta_{l_i} + a\Delta_{z_i} = b(l_i - E_i(l_i | I_V(l_i))) + a(z_i - E_i(z_i | I_T(z_i)))$. In equation (2.5), the expression inside the square brackets is the optimal losses of agent i where the first term shows the optimal losses of agent i given by the distance between z_i and l_i (i.e. if z_i is close to l_i then the optimal losses are low), and the second term shows the amount of the optimal loss of agent i that is due to the lacks of information about l_i and

[18] In Appendix A2 there is a table that summarizes the maximum value of the information processing capacity constrain assumed in different papers that use rational inattention in their models. For instance, Mackowiak and Wiederholt (2009) most of the time assume that $\kappa = 3$ bits; however sometimes they consider values of κ that goes from 1 to 5 bits.

[19] More specifically, in this version of the model $q_i^{*(CI)} = \frac{b}{b+a} E_i(l_i | I_1(l_i)) + \frac{a}{b+a} E_i(z | I_1(z)) = \frac{b}{b+a} l_i + \frac{a}{b+a} z \quad \forall\, i, j = 1,2, \dots, N$.

[20] Equation (2.5) is obtained by replacing equation (2.4) or equation (2.4') into equation (2.1).

z_i. Therefore, equation (2.5) implies that the highest utility of agent i happens when this agent does not have information constraints. Finally, notice that the amount of the optimal loss of agent i that is due to the lacks of information about l_i and z_i depends on: (1) the weights that l_i and z_i have in the function (i.e. b and a respectively), (2) the deviation of $E_i\left(l_i|I_V(l_i)\right)$ respect to l_i (i.e. Δ_{l_i}) and (3) the deviation of $E_i\left(z_i|I_T(z_i)\right)$ respect to z_i (i.e. Δ_{z_i}). Therefore, if there is an unexpected shock, or more precisely, a shock that has not been processed by agent i, then the optimal utility of agent i is lower due to the information constraint that this agent has.

2.2.1.3. How our model gets the first result obtained by Mackowiak and Wiederholt (2009)?

Table 2.2 uses the equations (2.2), (2.3) and (2.5) to get the losses of agent i that are due to the information constraints (i.e. $\left(L_i^{IC}\right)_{[I_T(z);I_T(l_i)]} = \frac{W_i\xi^2}{2(b+a)}$); that is, this table shows the losses of agent i depending on the different levels of attention that she pays to l_i and z_i. The lowest value of L_i^{IC} is $\left(L_i^{IC}\right)_{[I_1(z_i);I_1(l_i)]}$ and the highest value is $\left(L_i^{IC}\right)_{[I_0(z_i);I_0(l_i)]}$; that is, when agent i has full information about both variables there are no losses due to the lacks of information and these losses are the highest when she does not have any information about both variables. Proposition 1 summarizes the main results obtained from Table 2.2.

[Table 2.2]

Proposition 1: *In our model we get the following results:*

1. $(L_i^{IC})_{[I_T(z_i);I_V(l_i)]} < (L_i^{IC})_{[I_{T'}(z_i);I_{V'}(l_i)]}$ *if* $T > T'$ *and* $V \geq V'$. *That is, the losses of agent* i *are lower if she pays more attention to* z_i *and at least the same attention to* l_i.

2. $(L_i^{IC})_{[I_T(z_i);I_V(l_i)]} < (L_i^{IC})_{[I_{T'}(z_i);I_{V'}(l_i)]}$ *if* $V > V'$ *and* $T \geq T'$. *That is, the losses of agent* i *are lower if she pays more attention to* l_i *and at least the same attention to* z_i.

Proof: It is direct from Table 2.2 □

In the rational inattention literature there is not a consensus about the maximum value of the information processing capacity constraint that the agents have. Proposition 2 shows in our model what are the optimal choices of attention depending on the value of κ.

Proposition 2: *In our model, given* κ, *the optimal choices of attention of agent* i *are*

- $[I_1(z_i); I_1(l_i)]$ *if* $\kappa \in [3.17, +\infty)$
- $[I_1(z_i); I_0(l_i)]$ *if* $\kappa \in [1.58,\ 3.17)$ *and* $\left[ar_{z_i}\right]^2 > \left[br_{l_i}\right]^2$
- $[I_0(z_i); I_1(l_i)]$ *if* $\kappa \in [1.58,\ 3.17)$ *and* $\left[ar_{z_i}\right]^2 < \left[br_{l_i}\right]^2$

- $[I_0(z_i); I_0(l_i)]$ $if\, \kappa \in (-\infty, 1.58)$

Proof: It is direct from Tables 2.1 ad 2.2 □

Therefore, in Proposition 2 for each value of κ we have arrived to a conclusion equivalent to the theoretical result obtained by Mackowiak and Wiederholt (2009): "*When l_i is more variable or more important than z_i (i.e. $ar_{z_i} < br_{l_i}$), the agent i pays more attention to l_i than to z_i. Similarly, if z_i is more variable or more important than l_i (i.e. $ar_{z_i} > br_{l_i}$), the agent i pays more attention to z_i than to l_i".*

2.2.2. With interaction through a commitment mechanism

In this version of the model there is interaction because we assume $z_i = \frac{\sum_{j=1}^{N} q_{j \neq i}}{N-1} + \left(\frac{N}{N-1}\right) v$, then the choices of each agent i are affected by the choices of the other $N-1$ agents. In the rest of this section, we will assume without loss of generality that $N = 2$, therefore $z_i = q_j + 2v$.

If we assume that the rationality of all agents, the shape of the utility functions, the set $l = \{l_1, l_2\}$ and the parameter v are common knowledge; then, the Nash equilibrium in this economy is determined by the pair of choices $\left(q_1^{*(CI)}, q_2^{*(CI)}\right)$[21] where[22] $q_i^{*(CI)} = \frac{b+a}{b+2a} l_\text{i} + \frac{a}{b+2a} l_j + \frac{a}{b} 2v \;\forall i$ and it can be reached without any communication between the agents.

However, if it is common knowledge that the only element of the set l that can be known directly by each agent i is her respective l_i, then the Nash equilibrium choices $\left(q_1^{*(CI)}, q_2^{*(CI)}\right)$ can also be obtained using a particular commitment mechanism. The goal of this commitment mechanism is that each agent i reveals through her choice of q_i all information she has about her respective idiosyncratic market l_i. This mechanism has three steps. First, before taking a definitive choice about q_i each agent i is free to choose any preliminary value of q_i (denoted by $\hat{q}_i$)[23], all agents publicly observe the preliminary values chosen by the other agents and they can choose new preliminary values $\hat{q}$. This process continues until no one decides to choose new values of $\hat{q}_i$. Second, each agent i is committed to choose a q_i that is equal to the last preliminary value chosen of this variable (i.e. $q_i = \hat{q}_{i(Last)}$). Third, there is a penalty enough high to the respective agent i, to avoid that her final choice differs from her respective best response function $q_i^* = \frac{b}{b+a} l_i + \frac{a}{b+a}\left(q_j + 2v\right)$[24]. The second and third steps ensures

[21] The superscript CI means "complete information"

[22] Remember from the beginning of section 2.2, that the goal of each agent i is:

$$\max_{q_i}\; U_i = X_i - W_i\left[\frac{1}{2} b(q_i - l_i)^2 + \frac{1}{2} a(q_i - z_i)^2\right]$$

[23] The $\hat{q}_i$ are free of cost.

[24] When the strategy of agent j is to choose her best response function q_j^*, then agent j is revealing l_j to agent i; then, the choice of q_i that maximizes U_i is $q_i^{MAX} = \frac{(b+a)^2}{(b+a)^2+ab} l_i + \frac{ab}{(b+a)^2+ab} l_j + \frac{2a^2}{(b+a)^2+ab} v$

that at the equilibrium $(q_1^*, q_2^*) = \left(q_1^{*(CI)}, q_2^{*(CI)}\right)$, and the communication structure of the first step ensures the convergence to this equilibrium.

On the other hand, if there is a rational inattention problem such that variables l_i and z_i are not perfectly observed and the agents have processing capacity constraints. Then, the problem that each agent solves is:

$$\max_{q_i} \; E_i\big(U_i|I_T(z_i); I_V(l_i)\big) = X_i - W_i\left[\frac{1}{2}bE_i[(q_i - l_i)^2|I_V(l_i)] + \frac{1}{2}aE_i[(q_i - z_i)^2|I_T(z_i)]\right]$$

In this problem, we assume that the random variables $E_i\big(l_i|I_V(l_i)\big) = E_i\big(l_{i0} + \varepsilon_i|I_V(l_i)\big)$ and $E_i\big(z_i|I_T(z_i)\big) = E_i\big(z_{i0} + \mu_i|I_T(z_i)\big)$ have the same characteristics proposed in section 2.2.1.1; in addition, we also assume that the statistical distribution of ε_i and μ_i is common knowledge. Therefore, at the equilibrium the best response function of agent i is[25]:

$$q_i^* = \frac{b}{b+a}E_i\big(l_i|I_V(l_i)\big) + \frac{a}{b+a}E_i\big(z_i|I_T(z_i)\big) \quad \forall\, i,j = 1,2;\; T,V \in \{0,1\}\,. \qquad (2.6)^{26}$$

Notice that q_i^* is the sum of two terms: (1) the relative weight that the idiosyncratic term has in the utility function times the expected value that agent i has about this variable, and (2) the relative weight that the aggregate variable has in the utility function times the expected value that agent i has about this variable. Therefore, you can appreciate that the best response function already includes the main forces that Mackowiak and Wiederholt (2009) found in their model: (a) the importance of the aggregate and the idiosyncratic terms in the function that the agents are optimizing (i.e. a and b respectively), (b) the variability of the aggregate and idiosyncratic terms (this variability appears implicitly in the randomness of l_i and z_i, that is included in $E_i\big(l_i|I_V(l_i)\big)$ and $E_i\big(z_i|I_T(z_i)\big)$) and (3) the interaction effect which is due to the presence of the variable $z_i = f\big(\{q_{j\neq i}\}\big)$ in the equation. Another way to write the best response function is

$$q_i^* = \frac{b}{b+a}\big(l_i - \Delta_{l_i}\big) + \frac{a}{b+a}\big(z_i - \Delta_{z_i}\big) \qquad (2.6')$$

where Δ_{l_i} and Δ_{z_i} are deviations of l_i and z_i respect to $E_i\big(l_i|I_V(l_i)\big)$ and $E_i\big(z_i|I_T(z_i)\big)$.

Therefore, at the equilibrium we have

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which differs from $q_i^{*(CI)}$. That is, the pair of choices (q_1^*, q_2^*) is not a Nash equilibrium unless we impose a penalty enough high to the agents to avoids deviations from this equilibrium.

[25] Notice that the best response function in the case with rational inattention differs from the best response function when there is not a rational inattention problem; then the penalty imposed in the third step of the commitment mechanism has to take into account these differences. More specifically, it has to take into account the amount of information that each agent has about her idiosyncratic and aggregate variables.

[26] From equations (2.2) and (2.3) you can see that the terms $E_i\big(l_i|I_V(l_i)\big)$ and $E_i\big(z_i|I_T(z_i)\big)$ can take many potential values depending on the choices of agent i in the first stage of the problem. That is, depending on the decision of agent i about the amounts of attention $I_V(l_i)$ and $I_T(z_i)$ that she pays given her capacity constraint.

$$U_i^* = X_i - W_i\left[\frac{ab\left(z_i^{*(CI)} - l_i\right)^2}{2(b+a)} + \frac{\xi^2}{2(b+a)}\right] = U_i^{*(CI)} - \frac{W_i\xi^2}{2(b+a)} \quad \forall\, i$$

where $\xi = b\Delta_{l_i} + a\Delta_{z_i} = b\big(l_i - E_i\big(l_i|I_V(l_i)\big)\big) + a\left(z_i - E_i\big(z_i|I_T(z_i)\big)\right)$. Notice that this utility function is similar to the utility function obtained in the previous section. Then, using the same argument we can prove the first result obtained by Mackowiak and Wiederholt (2009). In particular, since l_i represents the idiosyncratic conditions and z_i represents the aggregate conditions, in our model, as it happens in Mackowiak and Wiederholt (2009): *"When idiosyncratic conditions are more variable or more important than aggregate conditions (i.e. $ar_z < br_l$), firms pay more attention to idiosyncratic conditions than to aggregate conditions". Similarly, if aggregate conditions are more variable or more important than idiosyncratic conditions (i.e. $ar_z > br_l$), firms pay more attention to aggregate conditions than to idiosyncratic conditions*.

How our model gets the second result obtained by Mackowiak and Wiederholt (2009)?

Respect to the second theoretical result obtained by Mackowiak and Wiederholt (2009), in our model, the interaction effect can be explained using the following proposition.

Proposition 3: *Assume that all the N agents behave optimally, then if at least one agent i has incomplete information about l_i or z_i, then there is an overreaction or under reaction of the choices of the other agents in the economy respect to the situation in which the information is complete. In addition, there is an overreaction or under reaction of the variable $Q \equiv \sum_{i=1}^{N} q_i$ respect to the case in which all agents have complete information.*

Below I will present the proof for the case in which $N = 2$, the proof for the more general case in which $N \geq 2$ is in Appendix C2.

Proof (when $N = 2$): If there are two agents, then we have the following three cases:

(1) If both agents have complete information, then the solution to the optimization problem proposed in equation (2.1) is:

$q_i^{*(CI)} = \frac{b+a}{b+2a} l_i + \frac{a}{b+2a} l_j + \frac{a}{b} 2v \quad \forall i, j = 1{,}2$ and

$Q^{*(CI)} \equiv \sum_{i=1}^{2} q_i^{*(CI)} = \sum_{i=1}^{2} l_i + \frac{a}{b} 4v.$

Notice that with complete information, the optimal choice of agent i is affected by her own idiosyncratic conditions, the idiosyncratic condition of the other agent and the aggregate conditions.

(2) If one agent has incomplete information (e.g. agent 1) and the other (e.g. agent 2) has complete information we have (the symbol "∼" means that one agent has complete information and the other agent has incomplete information):

$$\tilde{q}_1^* = \frac{b+a}{b+2a} l_1 + \frac{a}{b+2a} l_2 + \frac{a}{b} 2v - \frac{b+a}{b+2a}\left(\Delta_{l_1} - \frac{a}{b}\Delta_{z_1}\right) = q_1^{*(CI)} - \frac{b+a}{b+2a}\left(\Delta_{l_1} + \frac{a}{b}\Delta_{z_1}\right),$$

$$\tilde{q}_2^{*(CI)} = \frac{b+a}{b+2a} l_2 + \frac{a}{b+2a} l_1 + \frac{a}{b} 2v - \frac{a}{b+2a}\left(\Delta_{l_1} + \frac{a}{b}\Delta_{z_1}\right) = q_2^{*(CI)} - \frac{a}{b+2a}\left(\Delta_{l_1} + \frac{a}{b}\Delta_{z_1}\right)$$

and

$$\tilde{Q}^* \equiv \tilde{q}_j^{*(CI)} + \tilde{q}_i^* = \sum_{i=1}^{2} l_i + \frac{a}{b} 4v - \left(\Delta_{l_1} + \frac{a}{b}\Delta_{z_1}\right) = Q^{*(CI)} - \left(\Delta_{l_1} + \frac{a}{b}\Delta_{z_1}\right).$$

Notice that the agent with incomplete information has a higher deviation in $\tilde{q}_i$ respect to $q_i^{*(CI)}$ than the agent with complete information. Finally, notice that the aggregate deviation due to the information constraint is higher than the individual deviations.

(3) If both agents have incomplete information, then:

$$q_i^* = \frac{b+a}{b+2a} l_i + \frac{a}{b+2a} l_j + \frac{a}{b} 2v - \frac{1}{(b+2a)}\left((b+a)\Delta_{l_i} + a\Delta_{l_j}\right) - \frac{a}{b(b+2a)}\left((b+a)\Delta_{z_i} + a\Delta_{z_j}\right).$$

That is, $q_i^* = q_i^{*(CI)} - \left[\frac{b+a}{b+2a}\left(\Delta_{l_i} + \frac{a}{b}\Delta_{z_i}\right) + \frac{a}{b+2a}\left(\Delta_{l_j} + \frac{a}{b}\Delta_{z_j}\right)\right]$, and

$$Q^* \equiv \sum_{i=1}^{2} q_i^* = \sum_{i=1}^{2} l_i + \frac{a}{b} 4v - \left(\sum_{i=1}^{2}\Delta_{l_i} + \frac{a}{b}\sum_{i=1}^{2}\Delta_{z_i}\right) = Q^{*(CI)} - \left(\sum_{i=1}^{2}\Delta_{l_i} + \frac{a}{b}\sum_{i=1}^{2}\Delta_{z_i}\right).$$

Notice that the agents are more affected by her own information constraint than the information constraint of the other agent. We also have again that the aggregate deviation due to the information constraints is higher than the individual deviations.

Since there are differences between $q_i^{*(CI)}$ respect to q_i^*, $\tilde{q}_1^*$ and $\tilde{q}_2^{*(CI)}$, and between $Q^{*(CI)}$ respect to $\tilde{Q}^*$ and Q^* then our proposition has been proved. □

It means that if at least one agent has incomplete information, then there is an overreaction or underreaction of the choices of all agents in the economy respect to the situation in which the information is complete. In addition, there is an overreaction or underreaction of the aggregate variable respect to the case in which all agents have complete information. In other words, there is an interaction effect like the situation found by Mackowiak and Wiederholt (2009) in their model[27].

[27] Another way to explain the interaction effect is: from the proof of Proposition 3, notice that when $\frac{a}{b}$ is higher (i.e. when the strategic complementarity is higher) the losses that each agent faces due to "Δ_{z_i} and Δ_{z_j}" are more important and consequently the agents are more inclined to pay more attention to their

In addition, notice that even with complete information about all the idiosyncratic conditions the agents do not internalize all the interaction that exist in the economy. For instance, assume that the utility of a social planner is equal to the sum of the utilities of all agents and to make the explanation faster assume that $W_i = W\ \ \forall i$, then in the case of two agents we have that the optimal choice of the social planer is $q_i^{SP} = \frac{b+2a}{b+4a} l_i + \frac{2a}{b+4a} l_j$ which is different to the value of $q_i^{*(CI)}$ obtained above.[28]

Finally, notice in the proof of proposition 2 that if $a \to 0$ (i.e. if the strategic complementarity goes to zero) then the interaction effect is lower (i.e. the effect of the information constraints in the individual choices of the agents and in the aggregate Q decreases). Similarly, observe that $\lim_{a\to 0} q_i^{SP} = \lim_{a\to 0} q_i^{*(CI)}$ because the size of the externalities that were not internalized by agents is lower.

2.2.3. With interaction and without any commitment mechanism[29]

In the previous sections, as it was assumed in Mackowiak and Wiederholt (2009), we considered scenarios in which the choices of the agents are not affected by the strategic behavior of the other agents, and we arrived to the same two theoretical results obtained in their paper. However, in many decision-making situations, the agents confront not only uncertainty about future states of nature, but also uncertainty about actions that other agents will take[30]. Then, in these situations the agents have to take into account their beliefs about other agents' behavior; agents' beliefs about other agents' beliefs about other agents' behavior, agents' beliefs about other agents' beliefs about other agents' beliefs about other agents' behavior, and so on. That is, the strategic behavior is an important element that affects the interaction between the agents in the economy; so that, it is an important factor to take into account in the analysis of the interaction effect.

For that reason, we propose a change to the model of the previous section to introduce the strategic behavior into the analysis. In the new model there is not a commitment mechanism that forces the agents to choose any particular value of q. It means that the final value of z_i is never known with certainty before the moment at which all agents decide their final q. To avoid misunderstandings in the explanation of this version of the model, we will assume without loss of generality that $v = 0$, so we will assume during the rest of this section that $z_i = \frac{\sum_{j=1}^{N} q_{j\neq i}}{N-1}$.

Now, let us assume for the moment that each agent i does not have inattention constraints respect to the value of l_i and respect to the preliminary chosen values of all

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respective aggregate conditions. Thanks to Mirko Wiederholt for making me aware about this explanation.

[28] In the more general case in which $N \geq 2$, we have $q_i^{SP} = \frac{b(N-1)+2a}{b(N-1)+2aN} l_i + \frac{2a}{b(N-1)+2aN} \sum_{j=1}^{N} l_{j\neq i} + [2-N]\frac{a}{b}\left(\frac{N}{N-1}\right) v$ which is clearly different to the value of $q_i^{*(CI)}$ obtained in Appendix C2.

[29] This section was written for the case in which $N \geq 2$, then it is directly implementable for the special case in which $N = 2$

[30] For instance, in industries with a small number of dominant firms the price-setting behavior of a single firm can have an important impact on the profits of its competitors. Similarly, the decision of a household member most likely affects the decisions of the other household members.

q_j[31] (i.e. all $\hat{q}_j$ are known). Since in this section we have assumed that the agents are not forced to choose any particular value of q, then the optimization problem that each agent i solves when she chooses the final value of q_i is

$$\max_{q_i} \; E_i(U_i|*) = X_i - W_i\left[\frac{1}{2}b(q_i - l_i)^2 + \frac{1}{2}aE_i[(q_i - z_i)^2|*]\right] \text{ where } i = 1,2,\dots,N,$$

$b > 0$, $a > 0$ and $z_i = \frac{\sum_{j=1}^{N} q_{j\neq i}}{N-1}$. Therefore, to solve this optimization problem each agent i has to build her own beliefs respect to the behavior of the aggregate conditions z_i, where the symbol " $*$ " means any kind of information that can help agent i to build her beliefs about the value of z_i. As soon as this agent establishes her own belief about z_i then her best response function is

$$q_i^* = \frac{b}{b+a}l_i + \frac{a}{b+a}E_i(z_i|*).$$

That is, this equation represents the optimal choice of agent i given what she believes the other $N-1$ agents are doing. Notice that if the agents do not have the option to interact before they take their choice about q_i, then in our model the only information that each agent has to help her to solve the optimization problem is her own idiosyncratic condition, so she has to build her belief about z_i using an uninformative prior.

However, since we are adopting the same assumption of the previous section in which before taking a definitive choice about q_i, the agents can choose many preliminary values of this variable ($\hat{q}$) that do not have any cost and are not binding. Then, the agents can use these preliminary values as messages in a two-way cheap talk communication environment in which all agents are at the same time senders and receivers of information.

The literature about cheap talk usually has used models simpler than our model. However, from this literature we can get some insights to better understand how the interaction of the agents is affected by the introduction of this pre-play non-binding communication mechanism[32]. In the cheap talk models, the equilibria depend on the way the cheap talk affects the beliefs that the agents have about the behavior of the other agents (i.e. the equilibria depend on the kind and the amount of information that the agents can get by using the cheap talk). For instance, in all cheap talk models there is always a babbling equilibrium. In our model this babbling equilibrium happens when all agents believe that all $\hat{q}_i$ are uninformative about q_i, then in this context the optimal choice of each agent i is to use an uninformative prior about z_i.

However, even recognizing that the babbling equilibrium always exists, many models conclude that a cheap talk communication environment can be meaningful and

[31] Remember from the previous section that the presence of a commitment mechanism implies that without rational inattention each agent i always knows l_i and the final value of z_i, then the unique Nash equilibrium value $q_i^{*(CI)}$ can be achieved even when the agents are not informed directly about the idiosyncratic values that the other agents face.

[32] In particular, we are more interested in papers that assume situations in which: (1) the agents are at the same time senders and receivers of information, (2) the agents can interact many times in the cheap talk period and (2) nobody has perfect information (so they probably have something to learn in the cheap talk pre-play period).

informative. For instance, the first paper about cheap talk was written by Crawford and Sobel (1982), in the context of one sender and one receiver they conclude that more communication can occur through cheap talk when the players' preferences are more closely aligned[33]. So in our model, we should expect that the set of $\hat{q}$ values reveals more about the q values when the strategic complementarity of the choices of the agents is high because the agents are more interested to coordinate their choices[34]. Rabin (1994) in a game with perfect information and where all agents are senders and receivers of information shows that when there are enough rounds of talk, in every plausible equilibrium each player gets an expected payoff at least as great as he would get in the other player's favorite Nash equilibrium. That is, in our model by using the cheap talk the agents can expect to get a result better than the result obtained in the babbling equilibrium. Similarly, Aumann and Hart (2003) show that more outcomes preferred by both players are obtained if they have a long conversation than by a single message. In our model, it implies that if the agents interact by choosing many $\hat{q}$ values before they take their choice about q, then the result obtained by them is preferred than the one in which they only have the opportunity to choose only one $\hat{q}$ before they take their final choice about q. Park (2002) found that in his model a sequential cheap talk is better to get coordination than simultaneous cheap talk. In our model, we have not restricted the order and the moment at which the agents choose their $\hat{q}$ values, so we are more open to see in the lab experiment the kind of dynamic that the participants implement. Finally, some papers like Goltsman and Pavlov (2014) have found that in their models is necessary to introduce at least simple mechanisms like a mediator to allow the cheap talk messages to be informative. In our lab experiment we did not introduce any of these mechanisms; however, in the experiment we found that the interaction of the participants during the cheap talk time was not uninformative.[35]

Now, assume that the agents have rational inattention constraints respect to l_i and $\hat{z}_i = \frac{\sum_{j=1}^{N} \hat{q}_{j \neq i}}{N-1}$. In particular, assume that the information problem due to rational inattention is similar to the problem explained in section 2.2.1.1 where the uncertainty about l_i is represented by $l_i = l_{i0} + \varepsilon_i$, the uncertainty about $\hat{z}_i$ is represented by $\hat{z}_i = \hat{z}_{i0} + \mu_i$ and the statistical distribution of ε_i and μ_i is the same explained in section 2.2.1.1 (as it happens in section 2.2.2, we maintain the assumption that the statistical distribution of ε_i and μ_i is common knowledge). The optimization problem that the agents are solving is:

[33] But, perfect communication cannot occur unless the players' preferences are perfectly aligned.

[34] In other words, the agents have incentives to send messages to the other agents that works as coordination devices

[35] On the other hand, there are some papers that have used lab experiments to analyze cheap talk situations, some lessons obtained from these are: Cooper, DeJong, Forsythe and Ross (1989) who analyzed the role of pre-play communication in symmetric battle of the sexes game found that many rounds of talk will yield more coordination than one round. Duffy and Feltovich (2002) in an experiment that uses as reference three simple but famous games (the prisoner's dilemma, stag hunt and chicken games) found that cheap talk and the observation of the behavior of the participants in the experiment in previous games make cooperation and coordination more likely and increase payoffs. If you want to know more about different lab experiments that use communication via cheap talk you can read Crawford (1998).

$$\max_{q_i} \; E_i\left(U_i | *, I_V(l_i), \hat{z}_i\left(I_T(\hat{z}_i)\right)\right)$$
$$= X_i - W_i\left[\frac{1}{2} b E_i[(q_i - l_i)^2 | I_V(l_i)] + \frac{1}{2} a E_i\left[(q_i - z_i)^2 | *, E_i\left(\hat{z}_i | I_T(\hat{z}_i)\right)\right]\right]$$

where $= 1,2, \dots, N$, $b > 0$, $a > 0$, $z_i = \frac{\sum_{j=1}^{N} q_{j \neq i}}{N-1}$ and $\hat{z}_i = \frac{\sum_{j=1}^{N} \hat{q}_{j \neq i}}{N-1}$. Therefore, we always have a babbling equilibrium in which $E_i\left[(q_i - z_i)^2 | *, E_i\left(\hat{z}_i | I_T(\hat{z}_i)\right)\right] = E_i[(q_i - z_i)^2 | *]$ and consequently the agents only pay attention to l_i because $\hat{z}_i$ is uninformative about the value of z_i. Notice that this equilibrium also validates the first theoretical result of Mackowiak and Wiederholt (2009) because in this equilibrium there is only one valid source of information "l_i", the other source does not say anything about z_i, so it must be completely ignored. Respect to the second theoretical result, in the babbling equilibrium there is not an interaction effect due to rational inattention about the aggregate variable because $\hat{z}_i$ and $E_i\left(\hat{z}_i | I_T(\hat{z}_i)\right)$ do not communicate anything about z_i. However, the interaction effect does not disappear because the utility function still depends on z_i.

On the other hand, if we assume that all agents get full information about z_i by observing $\hat{z}_i$. Then our analysis is equivalent to the analysis presented in section 2.2.1[36]. However, the cheap talk models that have assumed agents that are at the same time senders and receivers of information[37] consider that the full information equilibrium is not possible due to the strategic behavior of the agents[38].

Therefore, in our experiment, during the cheap talk time, each participant i has to deal with a dual problem. On the one hand, she has to decide how much truthful information she reveals about q_i such that there is a positive interaction that can help her. But, on the other hand, she has to decide how much false information about q_i she sends in order to take advantage of the strategic behavior.

2.3. Experimental Design

We have prepared an experiment divided in four stages where each stage is divided in many rounds. The stages of the experiment are summarized in Table 2.3. The first two stages (i.e. the decision making stages) are based in the version of the model proposed in section 2.2.1 and the last two stages (i.e. the interaction stages) are based in the version of the model proposed in section 2.2.3.[39]

---

[36] More specifically, in the proof of proposition 3 we showed that with a commitment mechanism (which is equivalent to a cheap talk environment with full information revelation) the equilibrium happens when $q_i = q_i^{*(CI)}$ if there is not the inattention constraint or the equilibrium happens when $q_i = q_i^*$ if there is an inattention constraint.

[37] For instance, Rabin (1994) and Goltsman and Pavlov (2014)

[38] In particular, remember that if one agent gets full information before the other agents during the cheap talk time, then she is not interested to reveal new relevant information about her in order to take advantage of her better strategic situation.

[39] We did not prepare stages to analyze the version of the model presented in section 2.2.2 because of two reasons: (1) the two conclusions of Mackowiak and Wiederholt (2009) can be analyzed using only the versions of the model of sections 2.2.1 and 2.2.3, and (2) as we explained in section 2.2.3, the presence of the strategic behavior enriches the analysis of the interaction effect.

[Table 2.3] [40]

The experiment has been programmed in z-tree, 24 subjects were recruited from the UPF Leex Lab to participate in one of three identical sessions and no subject appeared in more than one session. The subjects were seated at computer terminals and given a set of instructions, which were then read aloud by the experimenter before the beginning of each stage. A copy of the instructions appears in Appendix D2. To ensure that the subjects understood each stage, some questions of understanding were prepared before the beginning of each stage.

To get more information in the experiment and to see clearer the consequences of the strategic behavior in the interaction process, in the interaction stages we only prepared experiment sessions in which $N = 2$. More specifically, in the first two stages the participants know that they are playing in a decision making experiment that is independent on the decisions of the other players. However, in stages 3 and 4, all participants know that in each round they have been randomly matched with another participant; they will not get to know with whom they are matched, each participant knows that her final choices affect the earnings of the person who has been matched with her and she also knows that her earnings are affected by the final choices of the person who has been matched with her. In addition, to make the experiment easier to play and to explain, we have assumed that $a + b = 1$.

In section 2.3.1 we will explain how our model has been parameterized; in section 2.3.2 we will give more details about the experiment and in section 2.3.3 we compare our lab experiment with other lab experiments (mainly with the experiment proposed by Cornand and Heinemann, 2014)

2.3.1. Parameterization

The earnings of the participants in the experiment depend positively on the utility they individually get in each round[41]. In particular, in the rounds of the first two stages the utility function that the participants are solving is:

$$U_i = 2000 - W_i\left[\frac{1}{2}b(q_i - l_i)^2 + \frac{1}{2}a(q_i - z_i)^2\right] \qquad (2.7A)$$

and in the rounds of the last two stages the utility function that the participants are solving is[42]:

[40] It is important to emphasize that in the experiment the agent i does not have complete information about the idiosyncratic conditions or aggregate conditions of the other agents. This characteristic is not relevant in the decision making stages; however, it is quite relevant in the interaction stages.

[41] The participant i knows that her total earnings are the sum of the earnings she gets in the 50 rounds of the experiment. In each round, these earnings depend on the utility they get in the round.

[42] In sections 2.2.2 and 2.2.3 we have shown that the imperfect information about the aggregate terms z_i and z_j is responsible of the presence of the interaction effects. This situation happens even in the case in which there is not uncertainty about v. Therefore, in order to make the experiment simpler to the participants we have decided to take the case in which $v = 0$ (i.e. $z_i = q_j$).

$$U_i = 2000 - W_i\left[\frac{1}{2}b(q_i - l_i)^2 + \frac{1}{2}a(q_i - z_i)^2\right] \text{ where } z_i = q_j \quad (2.7B)$$

The parameters $a \in (0,1)$ and $b \in (0,1)$, in the rounds of stages 3 and 4, are always known by the participants and are randomly chosen for each couple of participants at the beginning of every round such that $a + b = 1$. All couples know that both members have the same parameters a and b. In the rounds of stages 1 and 2, the parameters $a \in (0,1)$ and $b \in (0,1)$ are always known by the participants and are randomly chosen for each participant at the beginning of every round such that $a + b = 1$.

Let the indicator parameters $\mathbb{I}_{2\&4}$, $\mathbb{I}_2$ and $\mathbb{I}_4$ mean: $\mathbb{I}_{2\&4} \in \{0,1\}$ in the rounds of stages 2 and 4 and $\mathbb{I}_{2\&4} = 0$ otherwise, $\mathbb{I}_2 \in \{0,1\}$ in the rounds of stage 2 and $\mathbb{I}_2 = 0$ otherwise, and $\mathbb{I}_4 \in \{0,1\}$ in the rounds of stage 4 and $\mathbb{I}_4 = 0$ otherwise; where we assume $\mathbb{I}_{2\&4} \in \{0,1\}$ necessarily implies $\mathbb{I}_2 = 0$ and $\mathbb{I}_4 = 0$ (and $\mathbb{I}_2 = 0$ or $\mathbb{I}_4 = 0$ necessarily implies $\mathbb{I}_{2\&4} \in \{0,1\}$). Then the problem that the participants solve at the beginning of the incomplete information rounds (i.e. the rounds in stages 2 and 4) can be parameterized in the following way: using the information about a, $l_{i0} - \mathbb{I}_{2\&4}r_l$, l_{i0}, $l_{i0} + \mathbb{I}_{2\&4}r_l$ and $z_{i0} - \mathbb{I}_2 r_z$, z_{i0} ,$z_{i0} + \mathbb{I}_2 r_z$ in stage 2 or $\hat{z}_{i0} - \mathbb{I}_4 r_z$, $\hat{z}_{i0}$, $\hat{z}_{i0} + \mathbb{I}_4 r_z$ in stage 4; each participant has to choose if she wants to know with certainty (i.e. to pay full attention to) l_i or z_i in stage 2, and l_i or $\hat{z}_i$ in stage 4.

In the experiment, the variable l_i follows the same theoretical structure proposed to this variable in section 2.2. That is, in the rounds of stages 1 and 3, $l_i = l_{i0}$ where $l_{i0} \in [-200,200]$ is a parameter randomly chosen for each participant i at the beginning of every round and all participants know their respective l_i. On the other hand, in the rounds of stages 2 and 4 the participants know that $l_i \in \{l_{i0} - \mathbb{I}_{2\&4}r_l, l_{i0}, l_{i0} + \mathbb{I}_{2\&4}r_l\}$ where $Prob(l_i = l_{i0} - \mathbb{I}_{2\&4}r_l) = \text{Prob}(l_i = l_{i0}) = \text{Prob}(l_i = l_{i0} + \mathbb{I}_{2\&4}r_l) = \frac{1}{3}$, the parameters $r_l \in [0,20]$ and $l_{i0} \in [-200,200]$ were randomly chosen and implicitly revealed to each participant i at the beginning of every round.

Similarly, the variable z_i follows the theoretical structure proposed to this variable in the models of section 2.2. That is, in the rounds of stage 1, $z_i = z_{i0}$ where $z_{i0} \in [-200,200]$ is a parameter randomly chosen for each participant i at the beginning of every round and all participants know their respective l_i. On the other hand, in the rounds of stage 2 the participants know that $z_i \in \{z_{i0} - \mathbb{I}_2 r_z, z_{i0}, z_{i0} + \mathbb{I}_2 r_z\}$ where $\text{Prob}(z_i = z_{i0} - \mathbb{I}_2 r_z) = \text{Prob}(z_i = z_{i0}) = \text{Prob}(z_i = z_{i0} + \mathbb{I}_2 r_z) = \frac{1}{3}$; the parameters $r_z \in [0,20]$ and $z_{i0} \in [-200,200]$ were randomly chosen and implicitly revealed to each participant i at the beginning of every round. To summarize, in the stages 1 and 2 there is not a structural difference between l_i and z_i.

In contrast, in the rounds of stage 3, $\hat{z}_i = \hat{q}_j$ where this value is continuously updated to the participant i every time the participant j chooses a new $\hat{q}_j$ (the participant j is the participant who has been matched with participant i in a specific round of the stages 3 and 4). In the rounds of stage 4, $\hat{z}_i = \hat{q}_j \in \{\hat{z}_{i0} - \mathbb{I}_4 r_z, \hat{z}_{i0}, \hat{z}_{i0} + \mathbb{I}_4 r_z\}$ where $\text{Prob}\left(\hat{q}_j = \hat{z}_{i0} + \mathbb{I}_4 r_z\right) = \text{Prob}\left(\hat{q}_j = \hat{z}_{i0}\right) = \text{Prob}\left(\hat{q}_j = \hat{z}_{i0} + \mathbb{I}_4 r_z\right) = \frac{1}{3}$, the parameter $r_z \in [0,20]$ was randomly chosen and implicitly revealed to each couple of participants at the beginning of every round. If the participant i is paying full attention to $\hat{z}_i$ (or similarly, $\hat{q}_j$) then this value is continuously updated to the participant i every time the participant j chooses a new $\hat{q}_j$, but if the participant i is not paying full attention to $\hat{z}_i$ then their three potential values are continuously updated to the participant i every time the participant j chooses a new $\hat{q}_j$.

Finally, the parameter W_i was randomly chosen for each participant at the beginning of every round such that the maximum utility that all participants can get per round is inside the interval $[100,2000]$.

2.3.2. How does the experiment work?

In each round of stages 1 and 2 the participants have 60 seconds (in each round of stages 3 and 4 the participants have 90 seconds) to choose all integer numbers they reach or they want to choose in a line named Real Value, this line is bounded by two integer numbers[43]. The participants know that depending on the "last number" they choose they will obtain a utility (*Note: score is the name given to the utility in the experiment*). Figure 2.1 shows the maximum score (i.e. the maximum utility) they can get per round[44]. To get the maximum score the participant i knows that in every round her last chosen number (i.e. q_i) has to be close to l_i and z_i[45].

[Figure 2.1a]

[Figure 2.1b]

Figures 2.2a to 2.2d show examples of the kind of screens that the participants observe during the rounds of each stage. In the Real Value line, the variable l_i is represented by a purple number and the term z_i is represented by a red number (both numbers are symbolized by squares of the same color); the white squares are the bounds of the line. It implies that in the rounds of Stages 3 and 4 any prior that the participants built about z_i (i.e. q_j) must take into account that it has to be inside the range limited by the white squares. The participants know that in all rounds the range of integer numbers inside the white squares always include the number that has the maximum utility; but, it also includes numbers with negative utilities. In the Real Value line, the last number

[43] We could also use a line with an unbounded range of integer numbers. However, we consider that the dynamic of the experiment is more interesting if we consider a bounded line because in this case is clearer to the agents how to create uninformative priors about z_i using as reference the bounds of the line.

[44] This figure appears in the instructions of the experiment

[45] Notice that they know from Figure 2.1a that the maximum utility they can get per round is a value inside the interval [100, 2000] and that their utility is lower if they finish the round with a number farther away from the number with the highest possible utility. They are not informed about the maximum utility and the number that has this utility; however, they know from Figure 2.1b that at the left hand side (similarly, at the right hand side) of the number that has the maximum utility the difference in utility between two equidistant numbers is lower if these numbers are nearer to the number that has the maximum utility. For instance, in the Figure 2.1b you can appreciate that $A_2 - A_1 = A_3 - A_2 = A_4 - A_3$ but $P_2 - P_1 > P_3 - P_2 > P_4 - P_3$. Therefore, in the experiment the participants are implicitly informed that the utility function is quadratic.

In the experiment we did not want to evaluate explicitly the mathematical abilities that the participants have to solve a particular equation. Therefore, we did not give directly to the participants the mathematical shape of the utility function that they were maximizing (i.e. they did not see directly the equations (2.7a) and (2.7b)) only the graphical representation of Figure 2.1 and the values of a and b (these values change randomly from one round to the next) chosen by the participant during a round (i.e. $\hat{q}_{it}$) is symbolized by a black square as it appears in the Figures 2.2a to 2.2d.

[Figure 2.2a]

[Figure 2.2b]

[Figure 2.2c]

[Figure 2.2d]

Each participant knows that the way to get a high utility is to finish each round with a q_i, that is at the same time very close to the red and the purple numbers (z_i and l_i respectively). In addition, each participant knows that in some rounds they will get a higher utility if q_i is closer to z_i than to l_i and in others the opposite happens. For instance, at the bottom of Figures 2.2a to 2.2d there is a paragraph that says:

> **If you choose a number that is equidistant to the red number and to the purple number, then:**
>
> **37% of your score is due to the closeness of your number respect to the red number and**
>
> **63% of your score is due to the closeness your number has respect to the purple number**

That is, in this particular round $a = 0.37$ and $b = 0.63$[46].

Notice in Figures 2.2b and 2.2d that during the rounds of stages 2 and 4 the participants do not observe in the Real Value line the red or the purple number (and its respective square). Instead, in these rounds the participants observe a line named “Potential Values”. This line tells the participants, using numbers symbolized by circles, the three probable values that the red and purple numbers (i.e. z_i and l_i in Figure 2.2b or $\hat{z}_i$ and l_i in Figure 2.2d) can take in the Real Value line. They know that the numbers of the same color have the same probability and that the distances between these numbers (i.e. r_z and r_l) always remain constant during each round.

On the other hand, at the beginning of each round of the stages 2 and 4 the participants have to choose which of the numbers (red or purple) they want to know exactly in the Real Value line. In order to do a good choice, the participants are informed about: (a) the Potential Values line they will face during the round (i.e. they know $l_{i0} - r_l$, l_{i0} and $l_{i0} + r_l$, and $z_{i0} - r_z$, z_{i0} and $z_{i0} + r_z$ in Stage 2 or $\hat{z}_{i0} - r_z$, $\hat{z}_{i0}$ and $\hat{z}_{i0} + r_z$ in Stage 4) and (b) the values of a and b.

The participants know that in the rounds of stages 1 and 2 all numbers (red and purple) that appear in the Real Value and Potential Values lines remains constant. The same happens with the purple numbers in the rounds of stages 3 and 4. The red numbers do not remain constant in the rounds of stages 3 and 4; each participant knows that in these rounds her red number is equal to the last number chosen by the person who is matched with her (similarly, she knows that the last number chosen by her is symbolized by the red number that appears in the Real Value line of the person who is matched with her).

[46] Remember that in stages 3 and 4, each participant knows that she and the person who is matched with her have the same values of a and b.

In the rounds of stages 1 and 3 each participant knows that every time she chooses a number, according to the information that is available to her, she is informed about the utility that she can get if she finishes the round with this number. In the rounds of stage 3 the utility is automatically updated every time the person who is matched with her chooses a new number. On the other hand, in the rounds of stages 2 and 4 each participant knows that every time she chooses a number, according to the information that is available to her, she is informed about the maximum and minimum utility that she can get if she finishes the round with this number. In the rounds of stage 4 the maximum and minimum utility are automatically updated every time the person who has been matched with her chooses a new number.

In all rounds, each participant has access to a table that record: (i) the numbers chosen by the participant during the round, and (ii) the utility (or the maximum and minimum utilities) that were reported when the participant chooses these numbers (e.g. look at the table that is at the right hand side of Figures 2.2a to 2.2d). This table is not updated in the rounds of stages 3 and 4 as soon as the person who is matched with the participant chooses new numbers.

To summarize, in the rounds of stages 1 and 2 the agents have enough information to know directly the optimal value of q_i (i.e. the number that gives them the maximum utility given the level of information that they have); however, they can use the 60 seconds (mainly in the first rounds of stage 1) to better familiarize with the structure of the experiment by choosing many preliminary values of q_i (remember that the last chosen value is the only value that will determine their payoffs). On the other hand, notice that in the rounds of stages 3 and 4, the participants can use their 90 seconds to interact with the other participant by choosing many $\hat{q}_i$ that are always publically observed. They also know that the last value of $\hat{q}_i$ chosen by her during this 90 seconds is equal to q_i, so this value is her only chosen value that will affect the payoff of both players during the round. In particular, notice that in the rounds of these stages the participants are interacting in a cheap talk environment; therefore, if a participant wants to take strategic advantage with respect to the other participant, then she will be tempted to choose q_i during the last 2 seconds of the round because during this time the other player does not have enough time to react to this choice.

2.3.3. Comparison with other lab experiments

Our lab experiment is the first one that analyzes the main results obtained in the model of Mackowiak and Wiederholt (2009). In the literature, there are other lab experiments that study different rational inattention topics and models[47], or cheap talk environments[48] but none of them looks like the experiment that we propose and none of them analyzes the topics that we are studying in our experiment.

Our experimental design has some similarities with the lab experiment proposed by Cornand and Heinemann (2014); they propose a lab experiment based on a model (or more precisely a game) built by Morris and Shin (2002) which is not a rational inattention or a cheap talk model. If we use the same notation that we used in section

[47] For instance, Pinkovskiy (2009), Martin (2012), Caplin and Martin (2013), Caplin and Dean (2013a, b and 2015) and Cheremukhin, Popova, and Tutino (2015).

[48] For instance, look at the literature referenced in the footnote 35.

2.2, then the goal of the participant i in the experiment of Cornand and Heinemann (2014), is to maximize the following utility function

$$U_i = X - \mathbb{1}_{ACD} D(q_i - v)^2 - \mathbb{1}_{BCF} F\left(q_i - q_j\right)^2 \qquad (2.8)$$

where v and q_j are unknown terms (notice that in this utility function there are not idiosyncratic terms and that v is a term common to both participants). The other terms that appear in equation (2.8) are parameters that satisfy the following properties:

$$X = \begin{cases} 100 \ \ in\, Treatments\ A\ and\ B \\ 200\ in\ Treatment\ C \\ 400 \ \ in\ Treatments\ D\ and\ F \end{cases},\ D = \begin{cases} 3\ in\ Treatment\ D \\ 1\ otherwise \end{cases},\ F = \begin{cases} 3\ in\ Treatment\ F \\ 1\ otherwise \end{cases},$$

$$\mathbb{1}_{ACD} = \begin{cases} 1\ in\ Treatments\ A, C\ and\ D \\ 0\ otherwise \end{cases},\ \mathbb{1}_{BCF} = \begin{cases} 1\ in\ Treatments\ B, C\ and\ F \\ 0\ otherwise \end{cases}$$

In this experiment, in the rounds of all treatments, each participant was randomly matched with one of the other participants in the experiment. The participants received a common and a private hint number about v[49]. The common and private hint numbers are randomly selected from the interval $[v - 10, v + 10]$. The utility function is perfectly known by the participants in the experiment (i.e. they observe equation 2.8), then each participant i chooses only one value of q_i and according to this value she gets the utility obtained during the round. In the theory behind this experiment it is not complicated to construct the beliefs about the behavior of the other player because the agents have enough information to do it (the private hint number, the statistical distribution of the two hint numbers and the statistical distribution of v), so they do not need to include additional assumptions. The authors also consider another treatment (Treatment E) in which the utility function is:

$$U_i = 100 - (E_i(v) - v)^2 - \left(E_i\left(E_j(v)\right) - E_j(v)\right)^2$$

In this treatment the participant i chooses $E_i(v)$ and $E_i\left(E_j(v)\right)$ and according to these values she gets the utility obtained during the round. More specifically, Cornand and Heinemann (2014) ran the following five types of sessions:

Type of Sessions	Stage 1	Stage 2	Stage 3	Stage 4	Stage 5
First	Treatment A (5 rounds)	Treatment B (10 rounds)	Treatment C (30 rounds)	Treatment E (5 rounds)	-
Second			Treatment D (30 rounds)		
Third			Treatment F (30 rounds)		

[49] In this experiment, v is always an unknown variable but it has a uniform discrete random distribution in the interval [50,450]

<table>
<tr><th>Type of Sessions</th><th>Stage 1</th><th>Stage 2</th><th>Stage 3</th><th>Stage 4</th><th>Stage 5</th></tr>
<tr><td>Fourth</td><td rowspan="2">Treatment A
(5 rounds)</td><td rowspan="2">Treatment E’
(5 rounds)[50]</td><td>Treatment B’
(10 rounds)[51]</td><td rowspan="2">Treatment C
(20 rounds)</td><td rowspan="2">Treatment B
(10 rounds)</td></tr>
<tr><td>Fifth</td><td>Treatment B’’
(10 rounds)[52]</td></tr>
</table>

They found that the participants attach larger weights to public than to private signals (i.e. the private hint numbers) if they have a motive to coordinate their actions (i.e. if $\mathbb{1}_{BCF}F > \mathbb{1}_{ACD}D$). However, these weights were smaller than in equilibrium and closer to level-2 reasoning of a cognitive-hierarchy model.

The main differences between this experiment and our experiment are:

(1) In the utility function we include an idiosyncratic term that has a different value for each participant and that is never known by the other participant, in Cornand and Heinemann (2014) there are no idiosyncratic terms.

(2) In the utility function of Cornand and Heinemann (2014) there is only one uncertain exogenous and static variable that is common for both players, in our experiment we do not have this kind of variable.

(3) At the beginning of the rounds of stages 2 and 4 we allow the participants to improve the precision of one of our two unknown variables[53]. This option is not present Cornand and Heinemann (2014); however, they send to each participant a signal to help them to improve their choices.

(4) In our experiment, during each round of the stages 3 and 4, there is a continuous interaction between the participants, so they can form endogenously a prior about the behavior of the other participant through a cheap talk environment[54]. In Cornand and Heinemann (2014) the participants do not interact continuously during the round, they make only one decision per round, so they have to use their own signal to form their priors.

2.4. Results

In section 2.4.1 we will analyze the decisions of the participants in the experiment when they had to decide, at the beginning of the rounds of stages 2 and 4, if they wanted to receive during the round perfect information about the purple number (i.e. l_i) or about the red number (i.e. z_i in stage 2 or $\hat{z}_i$ in stage 4). In particular, we will analyze how the results of our experiment approach to the conclusions obtained in the model of

[50] In this treatment the authors use two separate score functions: $U_i = 100 - (E_i(v) - v)^2$ and $U_i = 100 - \left(E_i\left(E_j(v)\right) - E_j(v)\right)^2$ instead of one unified.

[51] This is the same Treatment B except that the participants in the experiment only receive the common signal.

[52] This is the same Treatment B except that the participants in the experiment only receive the private signal.

[53] Remember that, in each round of our experiment, each participant chooses if she wants to know with certainty l_i or z_i in stage 2, and l_i or $\hat{z}_i$ in stage 4.

[54] There are other lab experiments that have used cheap talk environments. However, as far as we know, there is not another lab experiment using this kind of environment to analyze the kind of problem that we propose, so our experiment is a novelty respect to this point.

the previous section and what elements were taking into account by the participants in the experiment when they took their choice.

In section 2.4.2 we will study the behavior of the participants during the rounds of stages 3 and 4. In particular, we are interested to determine if the preliminary choices of the participants during the cheap talk time moments and their final chosen values are in concordance with the results obtained in the theoretical model. We are also interested to see what information of the experiment can help us to better understand the interaction effect. In this section, all $\hat{q}_i$ will be named as $\hat{q}_{it}$ where $t = 1, 2, \dots, last$ represent the order of the choices done by the participant i. Therefore, the last $\hat{q}_i$ will be named as $\hat{q}_{ilast}$ or as q_i because in the experiment $\hat{q}_{ilast} = q_i$.[55]

2.4.1. Choice vs. Optimal Choice

From the results obtained at the beginning of the Stage 2 of the experiment we can analyze how our experiment behaves with respect to the first theoretical result obtained by Mackowiak and Wiederholt (2009). In general terms, we can say that the results obtained in the experiment were close to the results predicted by the theoretical model. In particular, 84% of the times the choices of the participants in this stage were according to the results predicted by our model; that is, most of the time they chose to know perfectly the red number when $[ar_z]^2 > [br_l]^2$ and to know perfectly the purple number when $[ar_z]^2 < [br_l]^2$.

In addition, from the choices compiled at the beginning of the Stage 4, we know that participants in the experiment believed that they could get some useful information from the cheap talk communication interval. For instance, remember that in a babbling equilibrium the optimal choice of the agents always is to know perfectly l_i; however, in our experiment on average in 52% of the rounds of this stage the participants decided to know perfectly l_i and on average in 48% of the rounds they decided to know perfectly $\hat{z}_i$. In addition, we got an interesting heterogeneity between the choices of the participants in the experiment because there were participants who decided to know perfectly l_i in 90% of the rounds and there were similarly other participants who decided to know perfectly $\hat{z}_i$ in 90% of the rounds.

In order to better understand these results, we ran some logit regressions that are reported in Table 2.4. According to these regressions, in Stage 2 when the participants decided between paying attention to l_i or to z_i, they effectively took into account the relative importance in the utility function and variability of both variables; however, they did not use exactly the same specification predicted by the theoretical model (look at the second regression of Table 2.4). On the other hand, in stage 4 when the participants decided between paying attention to l_i or $\hat{z}_i$, they also took into account the differences $a - b$ and $r_z - r_l$, so the participants considered that they could get some useful information about z_i by paying more attention to $\hat{z}_i$ when the variables a and r_z were relatively higher than variables b and r_l respectively.

[55] Notice that in this section we do not pay any special emphasis to understand the behavior of the participants in the stage 1 of the experiment. It happens because the main purpose of this stage was to familiarize the participants with the kind of experiment they were participating.

2.4.2. The interaction between the participants in the experiment

The cheap talk interaction that happened during the rounds of stages 3 and 4 reveals many interesting patterns; a few examples are represented in Figures 2.3a to 2.3f. In these figures, the solid symbols correspond to the choices of two participants that belong to the same group during a specific round (i.e. the solid symbols are the $\hat{q}_i$'s and $\hat{q}_j$'s) where the last solids symbols are their respective final choices (i.e. the last solid symbols are q_i and q_j); in addition, in these figures you can see two hollow symbols near to the q values, each symbol correspond to the best choice of the participant given the last choice of the other participant (i.e. the hollow symbols are q_i^* given q_j and q_j^* given q_i); it means, that the distance between the last solid symbol and the hollow symbol corresponds to losses due to mistakes in the beliefs of the participants given the final choice of the other participant; notice that these distances were very low in some figures, in most of these figures it means that the participants got reliable information during the cheap talk time about the final choice of the other participant.

[Figure 2.3a]

[Figure 2.3b]

[Figure 2.3c]

[Figure 2.3d]

[Figure 2.3e]

[Figure 2.3f]

In Figure 2.3a the optimization problem that both participants face has a high strategic complementarity between q_i and q_j ($a = 0.94$), so the participants had incentives to use the cheap talk time to coordinate such that their q values were close to each other; however, the interaction between both participants favored more the blue participant than the black participant because the q values chosen by both participants were closer to $l_{(blue)}$ than to $l_{(black)}$. In particular, in Figure 2.3a the strategy of the blue participant was to choose her $\hat{q}_{(blue)last}$ near to $l_{(blue)}$ when there were still more than 50 seconds before the end of the round, and she stayed there during the rest of the round, so $q_{(blue)}$ was close to $l_{(blue)}$, and $q_{(red)}$ was attracted to this idiosyncratic value due to the strategic complementarity incentives. However, a high strategic complementarity does not imply necessarily that both agents have to coordinate their q such that these are close to one of the l_i's. For instance, in Figure 2.3b we have that $a = 0.85$ and none of the l_i values seem to be a clear dominant attractor as it happened in Figure 2.3a. In Figure 2.3b, the participants interact repetitively during the cheap talk period and many times they chose $\hat{q}_t$ values that were close to the current best response function value, so the attraction to the idiosyncratic values was not so strong. Finally, notice that in Figures 2.3a and 2.3b both participants could coordinate quite well to choose a q_t near to the q_t of the other participant because at the beginning of these rounds they decided to pay perfect attention to $\hat{q}_t$ and they took advantage of the cheap talk time to implicitly discuss the coordination.

The strategic complementarities in the rounds represented in Figures 2.3c to 2.3e are lower than in the rounds represented in Figures 2.3a and 2.3b. However, the main characteristics of the behavior of the participants during the cheap talk time of these rounds are not the same. In Figure 2.3c during the first 50 seconds the black participant only chose one $\hat{q}_{(black)t}$ and after this period she chose many $\hat{q}_{(black)t}$; on the other hand, the blue participant chose only a few $\hat{q}_{(blue)t}$ at the beginning of the round and stopped when there were still more than 60 seconds before the end of the round; finally, notice that both participants acted strategically choosing the q_i values in the last two seconds of the round not giving the opportunity to the other player to react to these values. In Figures 2.3d and 2.3e all participants were very active during all the cheap talk time; in both figures, the black participants choose the $q_{(black)}$ when there were still some seconds to the end of the round and the blue participants took advantage of this time to choose a better $q_{(blue)}$[56]. The main difference is that in Figure 2.3e both participants always chose values of $\hat{q}_t$ inside a clear interval; on the other hand, in Figure 2.3d the blue participant a few times chose extreme values of $\hat{q}_t$ but it seems the negative potential utilities that were reported to her if she finished the round with these values convinced her to changed immediately to continue choosing $\hat{q}_{(blue)t}$ values in a more normal range.

Finally, in Figure 2.3f, the strategic complementarity is the lowest ($a = 0.02$), so the optimal strategy of both participants was to choose a q_i very close to their own l_i, but during the cheap talk time they had a babbling behavior because sometimes they chose $\hat{q}_i$ values far from l_i. In particular, notice that in the last seconds of the cheap talk time both participants sent non-credible messages where the $\hat{q}_i$ values were far from their respective l_i and suddenly in the last second they chose $q_i = l_i$.

One interesting characteristic in the stages 3 and 4 is that 39% of the times the agents chose q_i in the last two seconds of the rounds. Similarly, in 34% of the times they chose the q_i when there were still 10 or more seconds before the end of the round. So, there were many rounds in which the agents tried to take advantage of the strategic behavior, but there were also others in which it does not happen. Also, respect to this point, there was not a significant difference if the participant was in a round of stage 3 or stage 4.

From section 2.2.3 we know that the best strategy of the agent i in the rounds of stages 3 and 4 is to choose q_i such that it satisfies her best response function given l_i and her beliefs about q_j. Similarly, from sections 2.2.1 and 2.2.2 we know that best choice of the agent in the rounds of stages 1 and 2 is to apply her best response function respect to z_i and l_i. Therefore, in Table 2.5 we ran some regressions to test what elements affect the distance between q_i and her best response value q_i^{BRF} (taken q_j as given in the case of stages 3 and 4). We obtained some interesting results:

1. Before the experimental sessions, a pilot experiment with 20 subjects about stage 1 was done in order to test if the graphical way in which the experiment was proposed works. We obtained, as it happened later in the experimental sessions, that practically since the first rounds, the distance between q_i and q_i^{BRF} was zero or close to zero. It means that stage 1 works quite well as a training stage to face the other

[56] For instance, notice that in Figure 2.3d $q_{(blue)} = q^*_{blue}$ given $q_{(black)}$

stages of the experiment. Therefore, it is not strange that Table 2.5 reports that in stage 1 there is not a particular factor that affect the size of the variable $|q - q^{BRF}|$.

2. In stage 2, before the beginning of the rounds, the participants who took a choice that was according to the predictions of our model (i.e. to pay more attention to l_i if $[ar_{z_i}]^2 < [br_{l_i}]^2$ and to pay more attention to z_i if $[ar_{z_i}]^2 > [br_{l_i}]^2$) obtained a q_i closer to q_i^{BRF}, notice that this choice does not have an statistical effect in the rounds of stage 4 because when there is interaction there are different forces that are working during the cheap talk period as we discussed in section 2.2.3.
3. In stage 2, if the variable that suffers the lack of attention has a high variability (i.e. if $r_{Inattention}$ is high) then the distance between q_i and q_i^{BRF} is high, notice that this variable does not has an important statistical effect in stage 4 because of the same reason commented above.
4. Finally, when there is interaction then the distance between q_i and q_i^{BRF} is higher. In addition, in stages 3 and 4 (i.e. in the interaction stages) the distance between both variables increases if there is a higher strategic complementarity between the choices of the agents (i.e. $|q - q^{BRF}|$ is higher when a is higher).

[Table 2.5]

Remember that when the strategic complementarity is high, the agents have more incentives to coordinate their q_i such that these values can be closed to each other. For instance, in Figure 2.4a and Figure 2.4b you can appreciate that a higher strategic complementarity implies a lower value of $|q_i - q_j|$; similarly, in the regression of Table 2.6 you can see that an increase of a in 0.1 units implies a decrease of the distance $|q_i - q_j|$ on average in 2.7 units.

[Figure 2.4a]

[Figure 2.4b]

[Table 2.6]

In the experiment, as it happens in the theoretical model, we also found that a higher strategic complementarity also implies a higher level of distortions in the economy (i.e. a higher interaction effect). More specifically, we found in stages 3 and 4 that the distance between q_i and $q_i^{*(CI)}$ (i.e. $\left|q_i - q_i^{*(CI)}\right|$) is higher when the strategic complementarity between the choices of the participants is higher as it is reported in the regression in Table 2.7[57]. The same result can also be observed in Figure 2.5 in which we compare the cumulative distribution function of $\left|q_i - q_i^{*(CI)}\right|$ during four different ranges of a in the interaction stages of the experiment. In particular, notice that the C.D.F when $a \geq 0.75$ is always below the other ones (i.e. when $a \geq 0.75$ the distance $\left|q_i - q_i^{*(CI)}\right|$ is higher).

[57] It was not reported in the chapter, but the same situation happens when we compare q_i with the optimal choice of a social planner. That is, a higher strategic complementarity increases the distance between both variables.

[Figure 2.5]

[Table 2.7]

From the regressions in Table 2.7 you can appreciate that there is a positive correlation between q_i and $\hat{q}_{i(Last-1)}$, it implies that the information exchanged during the cheap talk communication time is not irrelevant[58]. In addition, from the Figures 2.3a to 2.3f you can observe that most of the time $\hat{q}_{it}$ and $\hat{q}_{it-1}$ are close to each other. Therefore, it seems interesting to see if there are differences between the elements that determine the distances $|\hat{q}_{it} - \hat{q}_{it-1}|$ where $t \neq last$ respect to the elements that determine the distance $|q_i - \hat{q}_{ilast-1}|$. From the regressions of Table 2.8 we can appreciate that the distances $|\hat{q}_{it} - \hat{q}_{it-1}|$ and $|q_i - \hat{q}_{ilast-1}|$ (or similarly $|\hat{q}_{ilast} - \hat{q}_{ilast-1}|$) exhibit an autoregressive behavior. As we commented above, this autoregressive behavior was already observed in the results of Table 2.7 and in the Figures 2.3a to 2.3f. In addition, there is not any other statistically significant element that affect $|q_i - \hat{q}_{ilast-1}|$. However, it is interesting to appreciate that there are other elements that affect the distance $|\hat{q}_{it} - \hat{q}_{it-1}|$. For instance, the distance $|\hat{q}_{it} - \hat{q}_{it-1}|$ is lower when the range of numbers the agents can choose during the round is narrower; this result is not strange because the wide of this range limits the maximum value that this distance can have. The distance $|\hat{q}_{it} - \hat{q}_{it-1}|$ is lower when t increases (i.e. the volatility of $\hat{q}_{it}$ decreases during the round); in other words, the messages that were sent by the agents stabilized during the round. Finally, notice that a higher strategic complementarity increases de distance $|\hat{q}_{it} - \hat{q}_{it-1}|$, this result is not strange because with a low strategic complementarity the main reference point is l_i which never moves, however when the strategic complementarity is high the main reference point is $\hat{q}_{jt}$ that can move many times during the round.

[Table 2.8]

2.5. Conclusions

We have built model and done a lab experiment to analyze the main theoretical results obtained by Mackowiak and Wiederholt (2009). Their model does not consider the possibility of strategic behavior.

Therefore, we also introduced the possibility of this kind of behavior in our model to better analyze the interaction effect. Our lab experiment validates the results obtained by Mackowiak and Wiederholt (2009); that is, the agents pay more attention to the sources of information that are more important and variable, and the interaction between agents that have incomplete information affects the choices of the other agents (i.e. there is an interaction effect). In addition, we also found that the strategic complementarity between the choices of the participants in the experiment is quite important to explain the main implications of the interaction effect. For instance, when the strategic complementarity is higher, the choices of agents with incomplete

[58] A simple correlation test shows that the correlation between both variables in the interaction stages of the experiment is higher than 80%.

information are farther away from their optimal values with complete information[59] and on average the potential earnings that these agents miss[60] are also higher. These divergences can be explained by the following fact: when the strategic complementarity is low the agents prefer to choose a q_i closer to l_i which is a fix parameter that is at least partially known. However, when the strategic complementarity is high the agents prefer to choose a q_i closer to q_j which is a mobile value, that is potentially unknown and that is subject to strategic behavior.

[59] That is, the choices of the participants are farther away from the unique equilibrium that the model has in the complete information scenario.

[60] Taken as given the choices of the other agents.

2.6. Appendix A2: Highest information processing capacity value (in bits) "κ" used in some papers*:

Paper	κ	Calibration (Methodology)
Sims (2003)	0.641, 0.111, 0.21, 0.269, 0.71, 0.72, 3.56	There is not any methodology. The values are only used as examples.
Peng (2005)	0.1, 2, 10	There is not any methodology. The values are only used as examples.
Peng and Xiong (2006)	Endogenous	$\kappa \in [0.449, 0.857]$. In this paper there is a complex equation according to which κ is affected by: (1) the return variance amplification of the different factors that form the dividends[61], (2) an investor's overconfidence parameter and (3) the discount rate.
Sims (2006b)	Endogenous	He assumes a **shadow price of information** in utility units. $\boldsymbol{\lambda} = 0.5$
Sims (2006a)	Endogenous	He assumes $\lambda = 0.5$ in the case of the log utility function (it implies $\kappa = 0.88$) Later, he assumes $\lambda = 2$ in the case of the CRRA utility function because in this case $\kappa = 0.85$ (i.e. its value is close to the value obtained in the log utility function). In other examples he assumes $\lambda = 0.03$ and 1.
Kasa (2006)	0.073, 0.114	He starts by fixing the detection error bound[62]. Given this, he uses the relationships between detection errors, mutual information and channel capacity to infer the highest capacity constraint under the assumption that the channel is being fully utilized.
Luo (2008)[63]	Many values between 0 and ∞	There is not any methodology. The values are only used as examples.
Tutino (2008)	Endogenous	She assumes $\lambda = 0.02$ in the case of the log utility function (it implies $\kappa = 2.08$). Later, she assumes $\lambda = 0.08$ in the case of the CRRA utility function because in this case $\kappa = 2.13$ (i.e. its value is close to the value obtained in the log utility function). In other examples she assumes $\lambda = 0, 0.2, 2$ and 3. Finally, she also uses many combinations of parameters such that $\kappa = 2.03, 1.99, 1.87, 1.7, 1.41, 1.20, 0.86,$ $0.78, 2.5$ and 0.88.

[61] The dividend for the j^{th} firm in the i^{th} sector is equal to a market factor plus a common factor for sector i plus a firm specific factor for the j^{th} firm in the i^{th} sector.

[62] He establishes priors about what constitutes a reasonable detection error probability. He uses as reference Kailath (1969) and Evans (1974).

[63] This row also applies to Luo and Young (2009, 2010 and 2014)

Paper	κ	Calibration (Methodology)
Mackowiak and Wiederholt (2009)	1, 2, 3, 4, 5	They choose the parameter that bounds the information flow such that the firms set prices that are close to the profit-maximizing prices. Based on this reasoning, they set $\kappa = 3$. The other values are used as examples.
	Endogenous	They consider that the conclusions of their model continue to hold if λ is determined by an increasing, strictly convex cost function. However, in this paper they did not solve the model using this assumption.
Woodford (2009)	Endogenous	He assumes $\lambda = 0, 0.05, 0.5, 5, 50$ and ∞
Lewis (2009)	0.5, 1, 2, 4	The value of $\kappa = 4$ represents in his model an example at which the choices of the agents are very close to the case at which there is not a rational inattention problem. The other values are only used as examples
Van Nieuwerburgh and Veldkamp (2009)	1.64, 11.11 and 30.86	There is not any methodology. The values are only used as examples
Mondria (2010)	$\kappa \in (0.1, 0.5)$	There is not any methodology. The values are only used as examples
Dworczak (2011)	Endogenous	The marginal cost of processing information and the marginal welfare cost of information were chosen such that the technology shock is consistent with the empirical evidence. As a result, the author uses many values of κ in the interval [0,1].
Tutino (2011)	Endogenous	She assumes $\lambda = 0.2$ and 2. κ in the first case is equal to 1.08 and in the second case is equal to 0.73. Other values of λ are 0 and 0.02.
Saint-Paul (2011)	1.2, 1.3, 1.4, 1.5, 1.6	There is not any methodology. The values are only used as examples.
Matějka and McKay (2012)	Endogenous	They assume $\lambda \in [1,3,5]$
Paciello (2012)	Endogenous	κ is chosen to match the speed of inflation adjustment to a technology and monetary policy shocks estimated by Paciello (2011) on U.S. data from 1980 to 2006. He finally uses $\kappa \in [3.5, 3.2, 0.37, 0.22]$
Woodford (2012)	0, 0.01, 0.25,0.5, 0.75, 1, 1.25, 1.5, 2.5, 3.5, 4.5, ∞	There is not any methodology. The values are only used as examples.
Mackowiak and	Not reported	They choose a value of κ such that the posterior variance of the optimal action in normal times

Paper	κ	Calibration (Methodology)
Wiederholt (2014)		equals 0.01[64]
Melosi (2014)	Not reported	There is an algorithm to solve the DSGE model that he proposes. Inside this algorithm κ is chosen such that the average profit losses due to sub-optimal price setting in his imperfect common knowledge model is 60% compared to those in an estimated Calvo model.
Paciello and Wiederholt (2014)	Endogenous	They assume different values for $\frac{\lambda}{\omega}$ (e.g. $10^{-4}, 10^{-5}, 0.9 \times 10^{-4}, 2 \times 10^{-4}, 4 \times 10^{-4}$, 1) where ω is the constant in the price setters' objective function that determines the profit loss in the case of a deviation of the actual price from the profit-maximizing price[65].
Cheremukhin, Restrepo-Echavarria and Tutino (2015)	Endogenous	They assume many values of λ in the interval $[0.84,5]$
Cheremukhin and Tutino (2015)	Endogenous	They assume $\lambda \in [0,0.01,0.05]$
Mackowiak and Wiederholt (2015)	Endogenous	λ_{Firms} is equal to 0.1 percent of the firm's revenue in the non-stochastic steady-state and $\lambda_{Households}$ is equal to 0.1 percent of the household's steady-state consumption
Matějka (2015)	Static Model: 1, 2 Extreme Values: 0, ∞	There is not any methodology. The values are only used as examples.
	Dynamic Model: 1, ∞ $\kappa(t) = \frac{t}{10} \quad \forall t \in \{0 \dots 10\}$ $\kappa(t) = 1 + X(t) \log X(t) + (1 - X(t)) \log(1 - X(t))$ Where $X(t) = 0.5 - 0.05t \;\; \forall t \in \{0 \dots 10\}$	1. There is not any methodology. These values are only used as examples. 2. No sequence of signals across periods is considered, just one signal, which gets tighter in latter periods. 3. $X(t)$ is a noise level decreasing in t, which models knowledge refinement. With increasing time, there is a higher probability that agents receive the correct signal
Matějka and McKay (2015)	Endogenous	They assume $\lambda \in [0,0.4]$
Luo and Young (2016)	0.2, 0.3, 0.5, 0.6, 1	There is not any methodology. The values are only used as examples.

[64] This value means that thinking about the optimal action in normal times reduces the variance of that action by a factor of 100.

[65] They interpret the shadow cost λ as an opportunity cost. If someone is paying more attention to the price setting decision means she is paying less attention to some other activity.

Paper	κ	Calibration (Methodology)
Kacperczyk, Van Nieuwerburgh and Veldkamp (2016)	Unskilled investors: $\kappa = 0$ Skilled Investors: $\kappa = 1$ (or 0.5, or 2 in some examples)	There is not any methodology. The values are only used as examples.
	Endogenous	They propose a numerical exercise using two functional cost functions: $c_1 e^{\lambda}$ and $c_2 \lambda^{\psi}$. However, they choose c_1 and c_2 such that on average $\kappa = 1$, because they want to compare their new results with the case in which κ is exogenous
Matějka (2016)	0.5, 1, 2	There is not any methodology. The values are only used as examples.

*The papers reported in this table are:

CHEREMUKHIN, Anton; RESTREPO-ECHAVARRIA, Paulina and TUTINO, Antonella "A Theory of Targeted Search" Working Paper (February, 2015); 54 p

CHEREMUKHIN, Anton and TUTINO, Antonella "Information Rigidities and Asymmetric Business Cycles" Working Paper (December, 2015); 61 p

DWORCZAK, Piotr. "Fiscal Policy under Rational Inattention" Working Paper (August, 2011); 39 p.

KACPERCZYK, Marcin, VAN NIEUWERBURGH, Stijn and VELDKAMP, Laura. "A Rational Theory of Mutual Funds' Attention Allocation" Econometrica, Vol. 84, No.2 (March, 2016); p. 571-626.

KASA, Kenneth. "Robustness and Information Processing" Review of Economic Dynamics, Vol. 9, Issue 1, (January, 2006); p. 1–33.

LEWIS, Kurt. "The Two-Period Rational Inattention Model: Accelerations and Analyses" Computational Economics, Vol. 33, No. 1 (February, 2009); p. 79-97.

LUO, Yuley. "Consumption dynamics under information processing constraints" Review of Economic Dynamics Vol. 11, No. 2 (April, 2008); p.366–385.

LUO, Yuley and YOUNG, Eric "Rational Inattention and Aggregate Fluctuations" The B.E. Journal of Macroeconomics (Contributions), vol. 9, No.1, (2009); Article 14.

LUO, Yuley and YOUNG, Eric "Risk-Sensitive Consumption and Savings under Rational Inattention" American Economic Journal: Macroeconomics, vol. 2, No.4, (October, 2010); p. 281-325.

LUO, Yuley and YOUNG, Eric "Signal Extraction and Rational Inattention" Economic Inquiry, Vol. 52, No. 2 (April, 2014); p. 811-829.

LUO, Yuley and YOUNG, Eric "Induced Uncertainty, Market Price of Risk, and the Dynamics of Consumption and Wealth" Journal of Economic Theory, Vol.163 (May, 2016); p. 1-41.

MAĆKOWIAK, Bartosz and WIEDERHOLT, Mirko. "Optimal Sticky Prices under Rational Inattention" American Economic Review, Vol. 99, No. 3 (June, 2009); p. 769-803.

MAĆKOWIAK, Bartosz and WIEDERHOLT, Mirko. "Inattention to Rare Events", Working Paper (December, 2014); 48 p.

MAĆKOWIAK, Bartosz and WIEDERHOLT, Mirko. "Business Cycle Dynamics under Rational Inattention", Review of Economic Studies, Vol. 82, No. 4 (October, 2015); p. 1502–1532.

MATĚJKA, Filip. "Rigid Pricing and Rationally Inattentive Consumer" Journal of Economic Theory, Vol. 158, Part B (July, 2015); p. 656-678.

MATĚJKA, Filip. "Rationally Inattentive Seller: Sales and Discrete Pricing" The Review of Economic Studies, Vol. 83, No. 3 (July, 2016); p. 1156-1188.

MATĚJKA, Filip and McKAY, Alisdair. "Simple Market Equilibria with Rationally Inattentive Consumers" American Economic Review, Vol. 102, No. 3 (May, 2012); p. 24-29

MATĚJKA, Filip and McKAY, Alisdair. "Rationally Inattention to Discrete Choices: A New Foundation for the Multinomial Logit Model" American Economic Review, Vol. 105, No. 1 (January, 2015); p. 272-298.

MELOSI, Leonardo. "Estimating Models with Information Frictions" American Economic Journal: Macroeconomics, Vol. 6, No.1 (January, 2014); p. 1-31.

MONDRIA, Jordi. "Portfolio Choice, Attention Allocation, and Price Comovement" Journal of Economic Theory, Vol. 145, No. 5 (September, 2010); p. 1837-1864

PACIELLO, Luigi. “Monetary Policy and Price Responsiveness to Aggregate Shocks under Rational Inattention” Journal of Money, Credit and Banking, Vol. 44, No. 7 (October, 2012); p. 1375–1399.
PACIELLO, Luigi and WIEDERHOLT, Mirko. “Exogenous Information, Endogenous Information and Optimal Monetary Policy” The Review of Economic Studies, Vol. 81, No. 1 (January, 2014); p. 356-388.
PENG, Lin. “Learning with Information Capacity Constraints” Journal of Financial and Quantitative Analysis, Vol. 40, No. 2 (June, 2005); p. 307-329.
PENG, Lin and XIONG, Wei. “Investor attention, overconfidence and category learning” Journal of Financial Economics Vol. 80, No. 3 (June, 2006); p. 563-602.
SAINT-PAUL, Gilles “A Quantized Approach to Rational Inattention” Working Paper (September, 2011); 42 p.
SIMS, Christopher. “Implications of rational inattention” Journal of Monetary Economics, Vol, 50, No.3 (April, 2003) p. 665–690
SIMS, Christopher. “Rational Inattention: A Research Agenda”, Working Paper (March, 2006a), 22 p.
SIMS, Christopher. “Rational Inattention: Beyond the Linear-Quadratic Case”, American Economic Review Vol. 96, No. 2 (May, 2006b), p.158-163.
TUTINO, Antonella. “The Rigidity of Choice. Lifecycle Savings with Information-Processing Limits” Finance and Economics Discussion Series, No. 2008-62 (2008); 65 p.
TUTINO, Antonella. “Rationally Inattentive Macroeconomic Wedges” Journal of Economics Dynamics and Control, Vol. 35, Issue 3 (March, 2011); p. 344–362
VAN NIEUWERBURGH, Stijn and VELDKAMP, Laura. “Information Immobility and the Home Bias Puzzle” The Journal of Finance, Vol. 64, No. 3 (June, 2009); p. 1187-1215
WOODFORD, Michael. “Information-Constrained State-Dependent Pricing” Journal of Monetary Economics, Vol. 56, Supplement (2009); p. S100-S124.
WOODFORD, Michael. “Inattentive Valuation and Reference-Dependent Choice” Working Paper (May, 2012); 85 p.

2.7. Appendix B2: The model without interaction and with three unknown pieces of information.

The agent i does not know the exact value of l_i and z_i. In particular, she has an initial prior about each variable, these priors are represented by l_{i0} and z_{i0}. Given the prior, then the agent i has to process "three" pieces of information in order to approach to the exact values of l_i or z_i[66]. A priori the agent does not know these pieces but she knows their statistical distribution. Given these distributions and given their own processing capacity constraints, agent i has to decide what and how much information she will process.

We assume that the exact values of the variables l_i and z_i are represented by:

$$l_i = l_{i0} + \varepsilon_{i1} + \varepsilon_{i2} + \varepsilon_{i3} \quad \textit{where} \;\; \varepsilon_{ij} \in \{-r_{l_i}, 0, r_{l_i}\}, \;\; r_{l_i} > 0, \;\; l_{i0} \; \textit{is known} \qquad (2.2')$$

$$\text{Prob}(\varepsilon_{ij} = -r_{l_i}) = \text{Prob}(\varepsilon_{ij} = 0) = \text{Prob}(\varepsilon_{ij} = r_{l_i}) = \tfrac{1}{3} \quad \text{and}$$

$$z_i = z_{i0} + \mu_{i1} + \mu_{i2} + \mu_{i3} \;\; \textit{where} \;\; \mu_{is} \in \{-r_{z_i}, 0, r_{z_i}\}, \;\; r_{z_i} > 0, \;\; z_{i0} \; \textit{is known} \qquad (2.3')$$

$$\text{Prob}(\mu_{is} = -r_{z_i}) = \text{Prob}(\mu_{is} = 0) = \text{Prob}(\mu_{is} = r_{z_i}) = \tfrac{1}{3}.$$

The random variables ε_{ij} and μ_{is} represent the pieces of information that the agent i does not know about l_i and z_i[67,68], where the parameters r_{l_i} and r_{z_i} are the dispersion of ε_{ij} and μ_{is} respectively. We have assumed that ε_{ij} and μ_{is} have a discrete uniform

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[66] This assumption has the theoretical advantage that this is the simplest version of our model in which we can get a clear example in which an agent optimally (given her processing capacity constraint) decides to: (1) pay full attention only to the aggregate conditions, (2) pay full attention only to the idiosyncratic conditions or (3) pay partial attention to the aggregate and idiosyncratic conditions.

[67] Therefore, the aggregate terms $\sum_{j=1}^{3} \varepsilon_{\mathrm{ij}}$ and $\sum_{s=1}^{3} \mu_{\mathrm{is}}$ represent the amounts of information that each agent i does not know about l_i and z_i respectively.

[68] You can increase or decrease the number of unknown pieces of information in the model (i.e. the number of the terms represented by ε_{ij} and μ_{is}), but the main results of the model do not change. In this appendix, we are not assuming a more general version of the model in which there is an undetermined number of unknown pieces of information because we could not find a way to generalize the equations that determine the entropy level of the variables l_i and z_i:

$$H_{\mathrm{W}}(l_i) = -\sum_{\mathrm{w}=1}^{\#\,\mathrm{of\,Events}} \left\{ \left[\text{Prob}_{l_{i0}+\sum_{\mathrm{j}=1}^{\mathrm{J}} \varepsilon_{ij}}(\text{Event}_{\mathrm{w}}) \right] \log_2 \left[\text{Prob}_{l_{i0}+\sum_{\mathrm{j}=1}^{\mathrm{J}} \varepsilon_{ij}}(\text{Event}_{\mathrm{w}}) \right] \right\} \quad \text{and}$$
$$H_{\mathrm{D}}(z_i) = -\sum_{d=1}^{\#\,\mathrm{of\,Events}} \left\{ \left[\text{Prob}_{z_{\mathrm{i0}}+\sum_{\mathrm{s}=1}^{\mathrm{S}} \mu_{is}}(\text{Event}_{\mathrm{d}}) \right] \log_2 \left[\text{Prob}_{z_{\mathrm{i0}}\sum_{\mathrm{s}=1}^{\mathrm{S}} \mu_{is}}(\text{Event}_{\mathrm{d}}) \right] \right\})$$

such that these equations can be compared with the equations (2.4) or (2.5). The main problem with the generalization is that in the equation of $H_{\mathrm{W}}(l_i)$ [and something similar happens in the equation of $H_{\mathrm{D}}(z_i)$] there are different combinations of ε_{ij} [or μ_{is} in the case of $H_{\mathrm{D}}(z_i)$] that represent the same $\text{Event}_{\mathrm{w}} = l_{i0} + \sum_{\mathrm{j}=1}^{\mathrm{J}} \varepsilon_{ij}$ where $\varepsilon_{ij} \in \{-r_{l_i}, 0, r_{l_i}\}$ (e.g. the event $l_i = l_{i0}$ can be represented in many different ways like $[l_{i0} + r_{l_i} - r_{l_i} + \sum_{\mathrm{j}=3}^{\mathrm{J}} 0]$ or $[l_{i0} + \sum_{\mathrm{j}=1}^{\mathrm{J}} 0]$ among others).

independent distribution; however, given our priors, notice that the distributions of the variables l_i and z_i are discrete but not uniform[69].

Depending on the level of uncertainty about l_i we have in our model four levels of entropy:

(1) If $\varepsilon_{i1}, \varepsilon_{i2}$ and ε_{i3} are unknown, then
$(\varepsilon_{i1} + \varepsilon_{i2} + \varepsilon_{i3}) \in \{-3r_{l_i}, -2r_{l_i}, -r_{l_i}, 0, r_{l_i}, 2r_{l_i}, 3r_{l_i}\}$; where
$Prob_{\varepsilon_{i1}+\varepsilon_{i2}+\varepsilon_{i3}}(-3r_{l_i}) = Prob_{\varepsilon_{i1}+\varepsilon_{i2}+\varepsilon_{i3}}(3r_{l_i}) = \frac{1}{27};\ Prob_{\varepsilon_{i1}+\varepsilon_{i2}+\varepsilon_{i3}}(-2r_{l_i}) =.$
$Prob_{\varepsilon_{i1}+\varepsilon_{i2}+\varepsilon_{i3}}(2r_{l_i}) = \frac{3}{27}$; $Prob_{\varepsilon_{i1}+\varepsilon_{i2}+\varepsilon_{i3}}(-r_{l_i}) = Prob_{\varepsilon_{i1}+\varepsilon_{i2}+\varepsilon_{i3}}(-r_{l_i}) = \frac{6}{27}$ and
$Prob_{\varepsilon_{i1}+\varepsilon_{i2}+\varepsilon_{i3}}(0) = \frac{7}{27}$.

Therefore, in this case the entropy about l_i is:
$H_0(l_i) = -2\left(\frac{1}{27}\log_2\frac{1}{27} + \frac{3}{27}\log_2\frac{3}{27} + \frac{6}{27}\log_2\frac{6}{27}\right) - \frac{7}{27}\log_2\frac{7}{27} = 2.526$ bits[70]

(2) If ε_{i1} and ε_{i2} are unknown, then $(\varepsilon_{i1} + \varepsilon_{i2}) \in \{-2r_{l_i}, -r_{l_i}, 0, r_{l_i}, 2r_{l_i}\}$; where
$Prob_{\varepsilon_{i1}+\varepsilon_{i2}}(-2r_{l_i}) = Prob_{\varepsilon_{i1}+\varepsilon_{i2}}(2r_{l_i}) = \frac{1}{9}$; $Prob_{\varepsilon_{i1}+\varepsilon_{i2}}(-r_{l_i}) =$
$Prob_{\varepsilon_{i1}+\varepsilon_{i2}}(-r_{l_i}) = \frac{2}{9}$; $Prob_{\varepsilon_{i1}+\varepsilon_{i2}}(0) = \frac{3}{9}$.

Then, in this case the entropy about l_i is: $H_1(l_i) = -2\left(\frac{1}{9}\log_2\frac{1}{9} + \frac{2}{9}\log_2\frac{2}{9}\right) - \frac{3}{9}\log_2\frac{3}{9} = 2.197$ bits

(3) If ε_{i1} is unknown, then $\varepsilon_{i1} \in \{-r_{l_i}, 0, r_{l_i}\}$ where $Prob_{\varepsilon_{i1}}(-r_{l_i}) = Prob_{\varepsilon_{i1}}(0) = Prob_{\varepsilon_{i1}}(r_{l_i}) = \frac{1}{3}$

Then, in this case the entropy about l_i is: $H_2(l_i) = -3\left(\frac{1}{3}\log_2\frac{1}{3}\right) = 1.585$ bits

(4) If all the elements ε_{is} are known , then the entropy about l_i is: $H_3(l_i) = 1\log_2 1 = \log_2 1 = 0$ bits (i.e. there is no uncertainty about l_i)

On the other hand, given the probabilistic distribution of the three unknown pieces of z_i, and using the same procedure that we used with l_i, we get: $H_0(z_i) = 2.526$ bits, $H_1(z_i) = 2.197$ bits, $H_2(z_i) = 1.585$ bits and $H_3(z_i) = 0$ bits.

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[69] More specifically: $Prob_{l_i=l_{i0}-3r_{l_i}} = Prob_{l_i=l_{i0}+3r_{l_i}} = \frac{1}{27}$; $Prob_{l_i=l_{i0}-2r_{l_i}} = Prob_{l_i=l_{i0}+2r_{l_i}} = \frac{3}{27}$; $Prob_{l_i=l_{i0}-r_{l_i}} = Prob_{l_i=l_{i0}+r_{l_i}} = \frac{6}{27}$ and $Prob_{l_i=l_{i0}} = \frac{7}{27}$. Similarly, $Prob_{z_i=z_{i0}-3r_{z_i}} = Prob_{z_i=z_{i0}+3r_{z_i}} = \frac{1}{27}$; $Prob_{z_i=z_{i0}-2r_{z_i}} = Prob_{z_i=z_{i0}+2r_{z_i}} = \frac{3}{27}$; $Prob_{z_i=z_{i0}-r_{z_i}} = Prob_{z_i=z_{i0}+r_{z_i}} = \frac{6}{27}$ and $Prob_{z_i=z_{i0}} = \frac{7}{27}$.

[70] In information theory, the level of entropy is usually measured in terms of bits, nats, or bans. To better understand the meaning of one bit, consider the event in which you throw a fair coin (i.e. the probability of heads is the same as the probability of tails; that is ½); therefore, $H(coin) = -2\log_2\frac{1}{2} = 1$ bit, it means that once you have thrown the coin and you have learnt its outcome, then you have gained one bit of information (i.e. your uncertainty has been reduced by one bit).

Table B2.1 summarizes the amounts of attention that agent i has to spend if she wants to reduce all or part of her entropy from $H_0(z_i)$ and $H_0(l_i)$ to any other level. For instance, if agent i wants to know the terms μ_{i3} and ε_{i3}, then she has to spend 0.66 bits of attention (i.e. $I_1(z_i)$+$I_1(l_i)$). In addition, if she pays full attention to both variables such that at the end these variables are completely known then she needs to spend 5.05 bits of attention; on the other hand, if she does not pay attention to any variable then she spends 0 bits of attention.

Table B2.1

Amounts of attention used by agent i to reduce the entropy of l_i and z from $[H_0(z_i); H_0(l_i)]$ to $[H_{0+S_1}(z); H_{0+S_2}(l_i)]$ where $S_1, S_2 \in \{0,1,2,3\}$

$I(z_i) + I(l_i)$	$I_3(z_i)$	$I_2(z_i)$	$I_1(z_i)$	$I_0(z_i)$
$I_3(l_i)$	5.05 bits	3.47 bits	2.86 bits	2.53 bits
$I_2(l_i)$	3.47 bits	1.88 bits	1.27 bits	0.94 bits
$I_1(l_i)$	2.86 bits	1.27 bits	0.66 bits	0.33 bits
$I_0(l_i)$	2.53 bits	0.94 bits	0.33 bits	0 bits

In the first stage of her rational inattention problem, agent i chooses the amount of attention she pays to each random variable such that the utility obtained in the second stage of the problem is the highest possible, where her choice is affected by the processing capacity constrain κ that this agent has[71].

Table B2.2 uses the equations (2.2'), (2.3') and (2.5) to get the losses of agent i that are due to the information constraints (i.e. $\left(L_i^{IC}\right)_{[I_T(z);I_T(l_i)]} = \frac{W_i\xi^2}{2(b+a)}$); that is, this table shows the losses of agent i depending on the different levels of attention that she pays to l_i and z_i. When agent i has full information about both variables there are no losses due to the lacks of information (i.e. $\left(L_i^{IC}\right)_{[I_3(z_i);I_3(l_i)]} = 0$) and these losses are the highest when she does not have any information about both variables (i.e. $\left(L_i^{IC}\right)_{[I_0(z_i);0(l_i)]} = \frac{W_i}{b+a}\left[a^2 r_{z_i}^2 + \left(\frac{2}{3}\right) b^2 r_{l_i}^2\right]$). Proposition 1' summarizes the main results from Table B2.2.

Table B2.2

Losses of Agent i that are due to the Lacks of Information: $\left(L_i^{IC}\right)_{[I_T(z);I_T(l_i)]}$

	$I_3(z_i)$	$I_2(z_i)$	$I_1(z_i)$	$I_0(z_i)$
$I_3(l_i)$	0	$\frac{W_i}{b+a}\left[\left(\frac{1}{3}\right)a^2 r_{z_i}^2\right]$	$\frac{W_i}{b+a}\left[\left(\frac{2}{3}\right)a^2 r_{z_i}^2\right]$	$\frac{W_i}{b+a}\left[a^2 r_{z_i}^2\right]$
$I_2(l_i)$	$\frac{W_i}{b+a}\left[\left(\frac{1}{3}\right)b^2 r_{l_i}^2\right]$	$\frac{W_i}{b+a}\left[\left(\frac{1}{3}\right)a^2 r_{z_i}^2 + \left(\frac{1}{3}\right)b^2 r_{l_i}^2\right]$	$\frac{W_i}{b+a}\left[\left(\frac{2}{3}\right)a^2 r_{z_i}^2 + \left(\frac{1}{3}\right)b^2 r_{l_i}^2\right]$	$\frac{W_i}{b+a}\left[a^2 r_{z_i}^2 + \left(\frac{1}{3}\right)b^2 r_{l_i}^2\right]$
$I_1(l_i)$	$\frac{W_i}{b+a}\left[\left(\frac{2}{3}\right)b^2 r_{l_i}^2\right]$	$\frac{W_i}{b+a}\left[\left(\frac{1}{3}\right)a^2 r_{z_i}^2 + \left(\frac{2}{3}\right)b^2 r_{l_i}^2\right]$	$\frac{W_i}{b+a}\left[\left(\frac{2}{3}\right)a^2 r_{z_i}^2 + \left(\frac{2}{3}\right)b^2 r_{l_i}^2\right]$	$\frac{W_i}{b+a}\left[a^2 r_{z_i}^2 + \left(\frac{2}{3}\right)b^2 r_{l_i}^2\right]$
$I_0(l_i)$	$\frac{W_i}{b+a}\left[b^2 r_{l_i}^2\right]$	$\frac{W_i}{b+a}\left[\left(\frac{1}{3}\right)a^2 r_{z_i}^2 + b^2 r_{l_i}^2\right]$	$\frac{W_i}{b+a}\left[\left(\frac{2}{3}\right)a^2 r_{z_i}^2 + b^2 r_{l_i}^2\right]$	$\frac{W_i}{b+a}\left[a^2 r_{z_i}^2 + b^2 r_{l_i}^2\right]$

[71] For instance, if $\kappa = 3$, then from Table B2.1 we know that the agent i only can choose the amount of attention she pays from the set: $A(z_i, l_i) = \{(I_0(z_i), I_0(l_i)), (I_0(z_i), I_1(l_i)), (I_1(z_i), I_0(l_i)), (I_1(z_i), I_1(l_i)), (I_0(z_i), I_2(l_i)), (I_2(z_i), I_0(l_i)), (I_2(z_i), I_1(l_i))(I_1(z_i), I_2(l_i)), (I_2(z_i), I_2(l_i))(I_0(z_i), I_3(l_i)), (I_3(z_i), I_0(l_i)), (I_3(z_i), I_1(l_i)), (I_1(z_i), I_3(l_i)), (I_2(z_i), I_2(l_i))\}$. That is, the cells of Table 2.1 that belong to the set $A^C(z_i, l_i) = \{(I_3(z_i), I_3(l_i)), (I_2(z), I_3(l_i)), (I_3(z_i), I_2(l_i))\}$ are not available to her.

Proposition 1': *In our model we get the following results:*

1. $(L_i^{IC})_{[I_T(z_i);I_V(l_i)]} < (L_i^{IC})_{[I_{T'}(z_i);I_{V'}(l_i)]}$ *if* $T > T'$ *and* $V \geq V'$. *That is, the losses of agent* i *are lower if she pays more attention to* z_i *and at least the same attention to* l_i.

2. $(L_i^{IC})_{[I_T(z_i);I_V(l_i)]} < (L_i^{IC})_{[I_{T'}(z_i);I_{V'}(l_i)]}$ *if* $V > V'$ *and* $T \geq T'$. *That is, the losses of agent* i *are lower if she pays more attention to* l_i *and at least the same attention to* z_i.

3. $\left(L_i^{IC}\right)_{[I_T(z_i);I_V(l_i)]} < \left(L_i^{IC}\right)_{[I_{T-M}(z_i);I_{V+N}(l_i)]}$ if $\left[ar_{z_i}\right]^2 > \frac{N}{M}\left[br_{l_i}\right]^2$
   $\left(L_i^{IC}\right)_{[I_T(z_i);I_V(l_i)]} > \left(L_i^{IC}\right)_{[I_{T-M}(z_i);I_{V+N}(l_i)]}$ if $\left[ar_{z_i}\right]^2 < \frac{N}{M}\left[br_{l_i}\right]^2$ *where*
   $T \in \{0,1,2\};\ V \in \{1,2,3\};\ M \in \{1,2,3\};\ N \in \{1,2,3\};\ (V+N) \leq 3\ and\ (T-M) \geq 0$

4. $\left(L_i^{IC}\right)_{[I_T(z_i);I_V(l_i)]} < \left(L_i^{IC}\right)_{[I_{T+M}(z_i);I_{V-N}(l_i)]}$ if $\left[ar_{z_i}\right]^2 < \frac{N}{M}\left[br_{l_i}\right]^2$
   $\left(L_i^{IC}\right)_{[I_T(z_i);I_V(l_i)]} > \left(L_i^{IC}\right)_{[I_{T+M}(z_i);I_{V-N}(l_i)]}$ if $\left[ar_{z_i}\right]^2 > \frac{N}{M}\left[br_{l_i}\right]^2$ *where*
   $T \in \{1,2,3\};\ V \in \{0,1,2\};\ M \in \{1,2,3\};\ N \in \{1,2,3\};\ (V-N) \geq 0$ and $(T+M) \leq 3$.

Proof: It is direct from Table B2.2 □

Proposition 2' shows in our model what are the optimal choices of attention depending on the value of κ.

Proposition 2': *In our model, given* κ*, the optimal choices of attention of agent* i *are*

- $[I_3(z_i); I_3(l_i)]$ *if* $\kappa \in [5.05, +\infty)$
- $[I_3(z_i); I_2(l_i)]$ *if* $\kappa \in [3.47,\ 5.05)$ *and* $\left[ar_{z_i}\right]^2 > \left[br_{l_i}\right]^2$
- $[I_2(z_i); I_3(l_i)]$ *if* $\kappa \in [3.47,\ 5.05)$ *and* $\left[ar_{z_i}\right]^2 < \left[br_{l_i}\right]^2$
- $[I_3(z_i); I_1(l_i)]$ *if* $\kappa \in [2.86,\ 3.47)$ *and* $\left[ar_{z_i}\right]^2 > \left[br_{l_i}\right]^2$
- $[I_1(z_i); I_3(l_i)]$ *if* $\kappa \in [2.86,\ 3.47)$ *and* $\left[ar_{z_i}\right]^2 < \left[br_{l_i}\right]^2$
- $[I_3(z_i); I_0(l_i)]$*if* $\kappa \in [2.53,\ 2.86)$ *and* $\left[ar_{z_i}\right]^2 > 2\left[br_{l_i}\right]^2$
- $[I_2(z_i); I_2(l_i)]$ *if* $\kappa \in [2.53,\ 2.86)$ *and* $\frac{1}{2}\left[br_{l_i}\right]^2 < \left[ar_{z_i}\right]^2 < 2\left[br_{l_i}\right]^2$
- $[I_0(z_i); I_3(l_i)]$ *if* $\kappa \in [2.53,\ 2.86)$ *and* $\left[ar_{z_i}\right]^2 < \frac{1}{2}\left[br_{l_i}\right]^2$
- $[I_2(z_i); I_2(l_i)]$ *if* $\kappa \in [1.88,\ 2.53)$
- $[I_2(z_i); I_1(l_i)]$ *if* $\kappa \in [1.27,\ 1.88)$ *and* $\left[ar_{z_i}\right]^2 > \left[br_{l_i}\right]^2$
- $[I_1(z_i); I_2(l_i)]$ *if* $\kappa \in [1.27,\ 1.88)$ *and* $\left[ar_{z_i}\right]^2 < \left[br_{l_i}\right]^2$
- $[I_2(z_i); I_0(l_i)]$ *if* $\kappa \in [0.94,\ 1.27)$ *and* $\left[ar_{z_i}\right]^2 > \left[br_{l_i}\right]^2$
- $[I_0(z_i); I_2(l_i)]$ *if* $\kappa \in [0.94,\ 1.27)$ *and* $\left[ar_{z_i}\right]^2 < \left[br_{l_i}\right]^2$
- $[I_1(z_i); I_1(l_i)]$ *if* $\kappa \in [0.66,\ 0.94)$
- $[I_1(z_i); I_0(l_i)]$ *if* $\kappa \in [0.33,\ 0.66)$ *and* $\left[ar_{z_i}\right]^2 > \left[br_{l_i}\right]^2$
- $[I_0(z_i); I_1(l_i)]$ *if* $\kappa \in [0.33,\ 0.66)$ *and* $\left[ar_{z_i}\right]^2 < \left[br_{l_i}\right]^2$

- $[I_0(z_i); I_o(l_i)]$ $if \, \kappa \in (-\infty, 0.32)$

Proof: It is direct from Tables 2.1' ad 2.2'[72] □

Therefore, in Proposition 2' for each value of κ we have arrived to a conclusion equivalent to the theoretical result obtained by Mackowiak and Wiederholt (2009): "*When l_i is more variable or more important than z_i (i.e. $ar_{z_i} < br_{l_i}$), the agent i pays more attention to l_i than to z_i. Similarly, if z_i is more variable or more important than l_i (i.e. $ar_{z_i} > br_{l_i}$), the agent i pays more attention to z_i than to l_i".*

For instance, if $\kappa = 3$ then the optimal levels of attention are $[I_3(z_i); I_1(l_i)]$ if $[ar_{z_i}]^2 > [br_{l_i}]^2$ and $[I_1(z_i); I_3(l_i)]$ if $[ar_{z_i}]^2 < [br_{l_i}]^2$. That is, if $[ar_{z_i}]^2 > [br_{l_i}]^2$ then agent i pays full attention to z_i and partial attention to l_i[73]. On the other hand, if $[ar_{z_i}]^2 < [br_{l_i}]^2$ then agent i pays full attention to l_i and partial attention to z_i[74]

Finally, notice in Proposition 2' that when the processing capacity constraint of agent i is $\kappa \in [2.53, 2.86)$, then she can: pay full attention to z_i, pay full attention to l_i or partial attention to z_i and l_i

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[72] In addition, notice from Table B2.2 that $[I_3(z_i); I_0(l_i)]$ is always a better (worse) choice than $[I_0(z_i); I_3(l_i)]$ when $[ar_z]^2 > [br_l]^2$ ($[ar_z]^2 < [br_l]^2$).

[73] In this case agent i only pays attention to $\varepsilon_{i1}, \mu_{i1}, \mu_2$ and μ_{i3} (i.e. ε_{i3} and ε_{i2} remain unknown)

[74] In this case agent i only pays attention to $\mu_{i1}, \varepsilon_{i1}, \varepsilon_{i2}$ and ε_{i3} (i.e. μ_{i3} and μ_{i2} remain unknown).

2.8. Appendix C2: Proof of Proposition 3

We have the following three cases:

1. If all agents have complete information, then the solution to the optimization problem proposed in equation (2.1) is:

   $q_i^{*(CI)} = \frac{b(N-1)+a}{(b+a)(N-1)+a} l_i + \frac{a}{(b+a)(N-1)+a} \sum_{\mathrm{j}=1}^{\mathrm{N}} l_{j\neq i} + \frac{a}{b}\left(\frac{N}{N-1}\right) v \quad \forall i, j = 1,2, \dots N$ and

   $Q^{*(CI)} \equiv \sum_{i=1}^{N} q_i^{*(CI)} = \sum_{i=1}^{N} l_{\mathrm{i}} + \frac{a}{b}\left(\frac{N^2}{N-1}\right) v.$

   Notice that even with complete information, the optimal choice of agent i is affected by her own idiosyncratic conditions, the idiosyncratic conditions of the other agents and the aggregate conditions.

2. If n agents have incomplete information (e.g. the first n agents) and the rest have complete information (e.g. the last $N-n$ agents) we have (the symbol "∼" means that some agents have complete information and the others have incomplete information):

   $\tilde{q}_i^{*} = \frac{b(N-1)+a}{b(N-1)+aN} l_i + \frac{a}{b(N-1)+aN} \sum_{j=1}^{N} l_{j\neq i} + \frac{a}{b}\left(\frac{N}{N-1}\right) v - \frac{b(N-1)+a}{b(N-1)+aN}\left(\Delta_{l_i} + \frac{a}{b}\Delta_{z_i}\right) - \frac{a}{b(N-1)+aN}\left(\sum_{j=1}^{n} \Delta_{l_{j\neq i}} + \frac{a}{b}\sum_{j=1}^{n} \Delta_{z_{j\neq i}}\right)$, then

   $\tilde{q}_i^{*} = \hat{q}_i^{*(CI)} - \frac{b(N-1)+a}{b(N-1)+aN}\left(\Delta_{l_i} + \frac{a}{b}\Delta_{z_i}\right) - \frac{a}{b(N-1)+aN}\left(\sum_{j=1}^{n} \Delta_{l_{j\neq i}} + \frac{a}{b}\sum_{j=1}^{n} \Delta_{z_{j\neq i}}\right)$,

   $\tilde{q}_i^{*(CI)} = \frac{b(N-1)+a}{b(N-1)+aN} l_i + \frac{a}{b(N-1)+aN} \sum_{j=1}^{N} l_{j\neq i} + \frac{a}{b}\left(\frac{N}{N-1}\right) v - \frac{a}{b(N-1)+aN}\left(\sum_{j=1}^{n} \Delta_{l_i} + \frac{a}{b}\sum_{j=1}^{n} \Delta_{z_i}\right)$, then

   $\tilde{q}_i^{*(CI)} = q_i^{*(CI)} - \frac{a}{b(N-1)+aN}\left(\sum_{j=1}^{n} \Delta_{l_i} + \frac{a}{b}\sum_{j=1}^{n} \Delta_{z_i}\right)$ and

   $\tilde{Q}^{*} \equiv \tilde{q}_j^{*(CI)} + \tilde{q}_i^{*} = \sum_{i=1}^{N} l_{\mathrm{i}} + \frac{a}{b}\left(\frac{N^2}{N-1}\right) v - \left(\sum_{j=1}^{n} \Delta_{l_i} + \frac{a}{b}\sum_{j=1}^{n} \Delta_{z_i}\right) = Q^{*(CI)} - \left(\sum_{j=1}^{n} \Delta_{l_i} + \frac{a}{b}\sum_{j=1}^{n} \Delta_{z_i}\right).$

   Notice that the choices of all agents are affected by all information constraints. Finally, notice that the aggregate deviation due to the information constraints is higher than the individual deviations.

3. If all agents have incomplete information, then:

$q_i^* = \frac{b(N-1)+a}{b(N-1)+aN} l_i + \frac{a}{b(N-1)+aN} \sum_{j=1}^{N} l_{j\neq i} + \frac{a}{b}\left(\frac{N}{N-1}\right) v - \frac{1}{b(N-1)+aN}\Big[(b(N-1)+ a)\Delta_{l_i} + a\sum_{j=1}^{N} \Delta_{l_{j\neq i}}\Big] - \frac{a}{b[b(N-1)+aN]}\Big[(b(N-1)+a)\Delta_{z_i} + a\sum_{j=1}^{N} \Delta_{z_{j\neq i}}\Big]$, then

$q_i^* = q_i^{*(CI)} - \left[\frac{b(N-1)+a}{b(N-1)+aN}\left[\sum_{j=1}^{N} \Delta_{l_{j\neq i}} + \frac{a}{b}\Delta_{z_i}\right] + \frac{a}{b(N-1)+aN}\left[\Delta_{l_{j\neq i}} + \frac{a}{b}\sum_{j=1}^{N} \Delta_{z_{j\neq i}}\right]\right]$ and

$Q^* \equiv \sum_{i=1}^{N} q_i^* = \sum_{i=1}^{N} l_{\mathrm{i}} + \frac{a}{b}\left(\frac{N^2}{N-1}\right) v - \left(\sum_{i=1}^{N} \Delta_{l_i} + \frac{a}{b}\sum_{i=1}^{N} \Delta_{z_i}\right) = Q^{*(CI)} - \left(\sum_{i=1}^{N} \Delta_{l_i} + \frac{a}{b}\sum_{i=1}^{N} \Delta_{z_i}\right)$.

Notice that the choice of each agent is affected by her own information constraint and the information constraints of the other agents. However, each agent is more affected by her own information constraint. We also have again that the aggregate deviation due to the information constraints is higher than the individual deviations.

Since there are differences between $q_i^{*(CI)}$ respect to q_i^*, $\tilde{q}_{\mathrm{i}}^*$ and $\tilde{q}_i^{*(CI)}$, and between $Q^{*(CI)}$ respect to $\tilde{Q}^*$ and Q^* then our proposition has been proved □

2.9. Appendix D2: Instructions[75]

Before we begin, there are two important rules. First, please do not speak to any other participant during the experiment. Second, please do not use the computer for any activity other than interacting with the software (e.g. email, or web-surfing). The reason for these two rules is that we are interested in how you make decisions on your own. Talking to other people or using the computer for other activities makes it harder for us to learn about your decision making.

If you follow these instructions and think carefully you can earn a good amount of money. This will be paid to you in cash at the end of the experiment.

How does the experiment work?

The experiment is divided in four stages, the first one includes 5 rounds, the second one 15, the third one 10 and the fourth one 20 (i.e. you will participate in 50 rounds during the experiment). Your main goal in each round is to get the highest possible score. **Your earnings (in €) in each round are equal to the score that you obtain in the round divided by 1000 (i.e. 1000 points =1€).** In all rounds you can get positive or negative scores. Therefore, in each round try to make the best decisions to ensure positive earnings. To avoid big losses, if in a specific round you get a score lower than -2000 points then at the moment of calculating your earnings we will assume that your score has been -2000 points. Therefore

$$Earnings\ in\ the\ experiment = \sum_{j=1}^{40} Earnings\ in\ round\ j = \left[\sum_{j=1}^{40} \frac{max\{-2000, Score\ in\ round\ j\}}{1000}\right] €$$

At the end of each round you will be informed about your score in the round.

Before the beginning of each stage of the experiment, I will read the instructions of the stage aloud in order to ensure you have a good comprehension of the stage. All stages in the experiment are quite similar. Therefore, I will spend a considerable amount of time explaining the Stage 1 such that the other stages will be easier to understand.

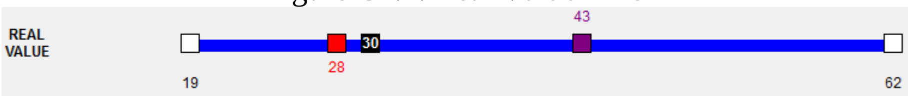
Read this page again, if you have a question of any sort raise your hand and wait until I give you the word.

STAGE 1 (Rounds 1 to 5)

In each round you will see a line named "Real Value" that represents a range of integer numbers (for instance, in Figure C2.1 this line represents the integer numbers that are higher or equal to 19 and lower or equal to 62), the white squares symbolize the numbers that bound this line. Inside the Real Value line there are two numbers, the red one and the purple one, which are symbolized by a red square and a purple square respectively (in Figure C2.1, these two numbers are 28 and 43). The numbers symbolized by the white, red and purple squares are different in each round and are not the same for all the participants in the experiment.

[75] Translated from Spanish into English

Figure C2.1. Real Value Line



In each round you have 60 seconds to choose all integer numbers you reach or you want to choose (later I will give you details about the way you can choose numbers); however, take into account that the score you get in each round “only” depends on the last chosen number during the round. More specifically, **in a specific round you will get a high score if your last chosen number is at the same time a number close to the red and to the purple numbers of the Real Value line**. In some rounds you will get a higher score if you chose a number closer to the red number than to the purple number and in others the opposite happens (later I will give you more details about it). During the course of each round your last chosen number will be symbolized by a black square in the Real Value line (e.g. in Figure 1 the last number chosen by the participant is the number 30).

Figure C2.2 shows the scores you can get in each round depending on your last chosen number. In particular, note the following:

(1) The Maximum Score you can get in each round is generally a score between 100 and 2000 (Figure C2.2a). The Maximum Score and the number with this score won’t be revealed to you, these are different for each participant and change randomly from one round to the other.

(2) Your score will be lower if your last chosen number is further from the number with the Maximum Score. For instance, from the four numbers that are explicitly referenced in Figure C2.2b, the number with the highest score is A_4 and the number with the lowest score is A_1.

(3) At the left hand side (similarly, at the right hand side) of the number that has the Maximum Score the difference in the score between two equidistant numbers is lower if these numbers are nearer to the number that has the Maximum Score (e.g. in Figure 2b you can appreciate that $A_2 - A_1 = A_3 - A_2 = A_4 - A_3$ but $P_2 - P_1 > P_3 - P_2 > P_4 - P_3$)

Figure C2.2a.
Scores you can get in each round

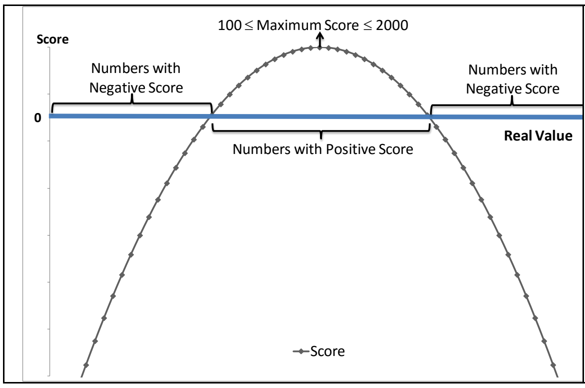
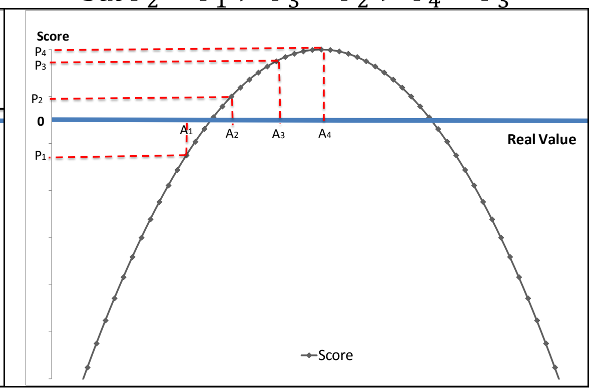


Figure C2.2b.
Example: $A_2 - A_1 = A_3 - A_2 = A_4 - A_3$
but $P_2 - P_1 > P_3 - P_2 > P_4 - P_3$

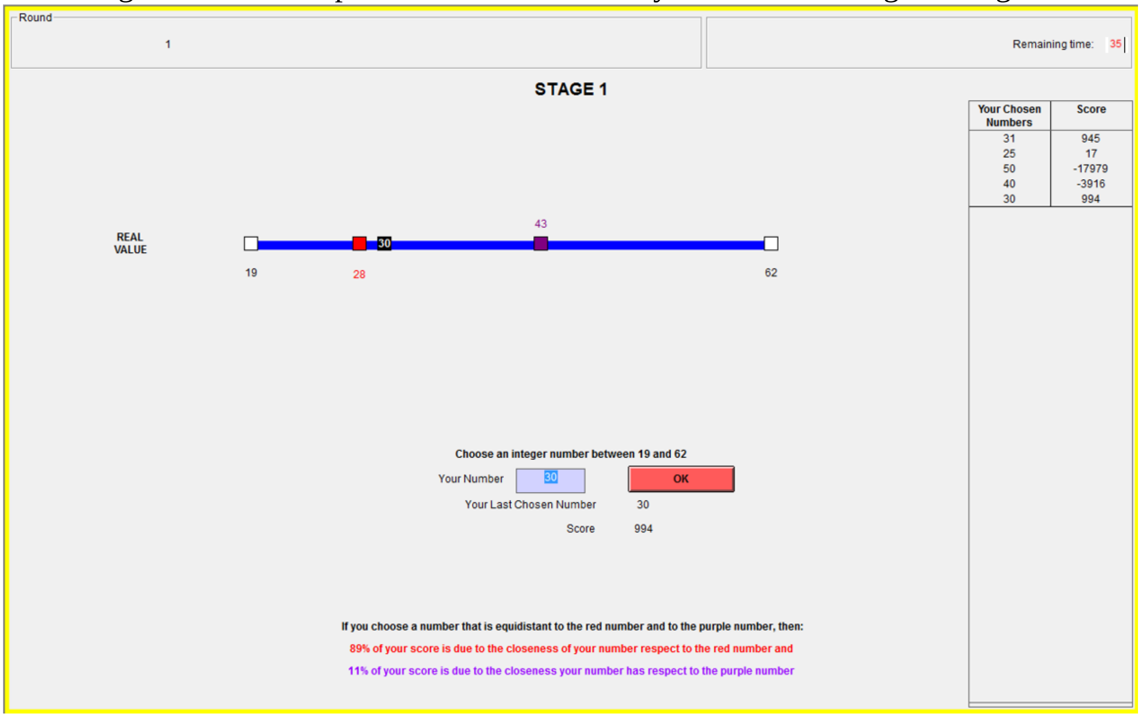


Notice that in Figure C2.2 we do not specify the range in which you can choose numbers (i.e. in the figure you cannot see the numbers symbolized by the white squares that bound the line in Figure 1). However, do not worry because ***in all rounds the range of integer numbers which you can choose always includes the number that has the Maximum Score; but, be careful because this range also includes numbers with negative scores.***

During the rounds of Stage 1 of the experiment you will see a screen similar to the screen that appears in Figure C2.3. Remember that in each round you have 60 seconds to choose all integer numbers you reach or you want to choose (**do not forget that your score in each round depends "only" in your last chosen number**; however, the rest of chosen numbers can be useful to do a better choice); in the top at the right hand side of the screen you are informed about the number of seconds you have before the end of the round (e.g. in Figure C2.3 the participant has 35 seconds before the end of the round).

The way you choose the numbers in each round is the following: Type every chosen number in the cell that appears in the middle of the screen besides the tag "*Your Number*" and every time you type a number press the button OK (if you do not press this button then the typed number won't be taken into account).

Figure C2.3. Example of the kind of screen you will see during the Stage 1



What happens every time you press the button OK?

Three things happen:

1. The last typed number will appear symbolized by a black square in the Real Value line.

2. The last typed number and its corresponding score will be reported below the cell where you typed it (e.g. in Figure C2.3 the last typed number was 30 and the score that corresponds to that number is 994)
3. The last typed number appears at the bottom of the table that is at the right hand side of the screen. This table presents the history of the numbers chosen by you during the round and the score that correspond to each number (e.g. the numbers chosen, in order from the first to the last, by the participant in Figure C2.3 were 31, 25, 50, 40 and 30).

Finally, notice that at the bottom of the screen you will be informed how your score is affected depending on the proximity of your chosen number respect to the red and purple numbers. For instance, at the bottom of Figure C2.3 there is a message that says:

> If you choose a number that is equidistant to the red number and to the purple number, then:
>
> 89% of your score is due to the closeness of your number respect to the red number and
>
> 11% of your score is due to the closeness your number has respect to the purple number

These percentages remain constant during the round, change from one round to the other and are different for each participant.

Please, read the instructions of this stage again. If the instructions are not clear to you, or you have a question of any sort, please raise your hand and sit quietly until I come by to listen to your question. Do not hesitate to ask for help because if you are confused or make a mistake, it could reduce your earnings. The answer to your question might also be helpful for others to hear; if it is, I will repeat your question out loud, and the answer, so everyone can hear them.

STAGE 2 (Rounds 6 to 20)

The instructions of this stage are the same of Stage 1 except that you only will know exactly the red or the purple number of the Real Value line (the other number will be known with some uncertainty). More specifically, at the beginning of each round you will see a screen similar to the screen that appears in Figure C2.4. In this screen you have to choose (pressing with the mouse the corresponding button) which of the two kinds of numbers that appear in the Real Value line you want to know exactly.

In order to make a good choice notice that in Figure C2.4 you are informed about:

1. The “Potential Values” line: this line tells you using numbers symbolized with circles the three probable values that the red and purple numbers can take in the Real Value line. All numbers of the same color have the same probability (e.g. in Figure 4 the Potential Values line specifies that the purple number in the Real Value line can be 80, 100 or 120 and the probability of each value is 33%).
2. How is affected your score depending on the proximity, in the Real Value line, of your chosen number respect to the red and purple numbers. For instance, at the bottom of Figure C2.4 there is a message that says:

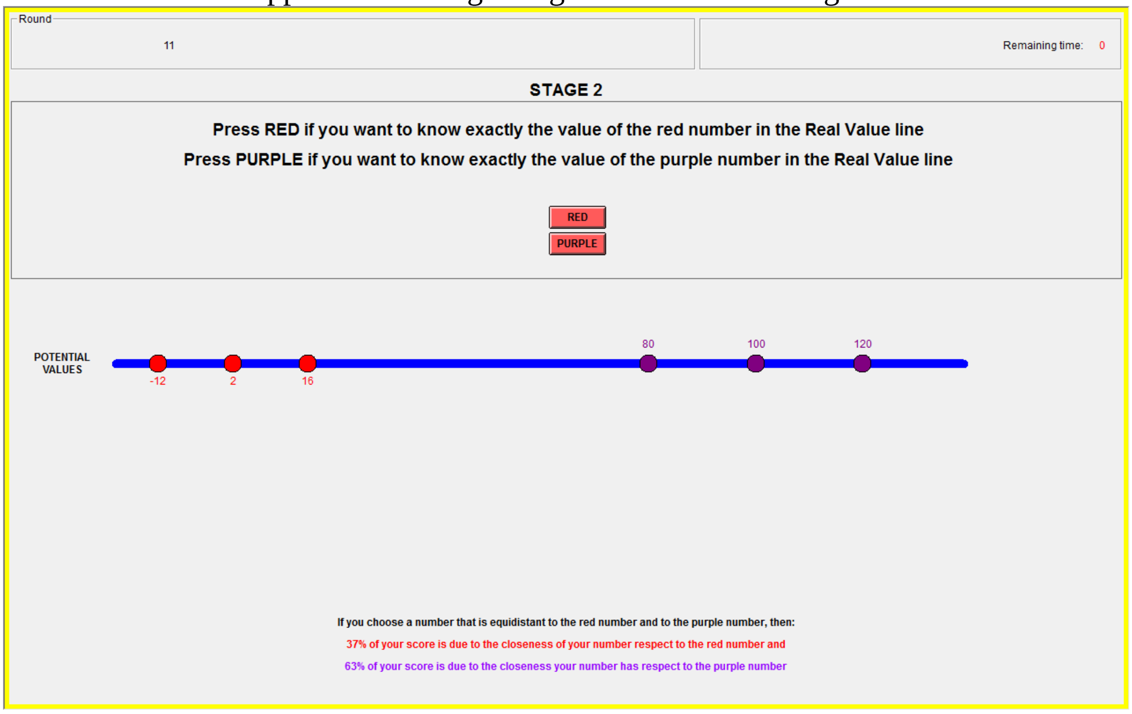
If you choose a number that is equidistant to the red number and to the purple number, then:

37% of your score is due to the closeness of your number respect to the red number and

63% of your score is due to the closeness your number has respect to the purple number

These percentages remain constant during the round, change from one round to the other and are different for each participant.

Figure C2.4. Example of the kind of screen that appears at the beginning of the rounds in Stage 2



If you press the button that says "red" then a screen similar to the screen that appears in Figure C2.5 appears. Notice that this screen is similar to the screens that appeared in Stage 1, except for the following: The Real Value line doesn't show the purple number; instead you can see the Potential Values line reporting the three probable values that the purple number can take in the Real Value line. Remember that these numbers have the same probability (i.e. the probability of each value is 33%) and notice that these numbers are the same numbers that appeared previously in the screen of Figure C2.4.

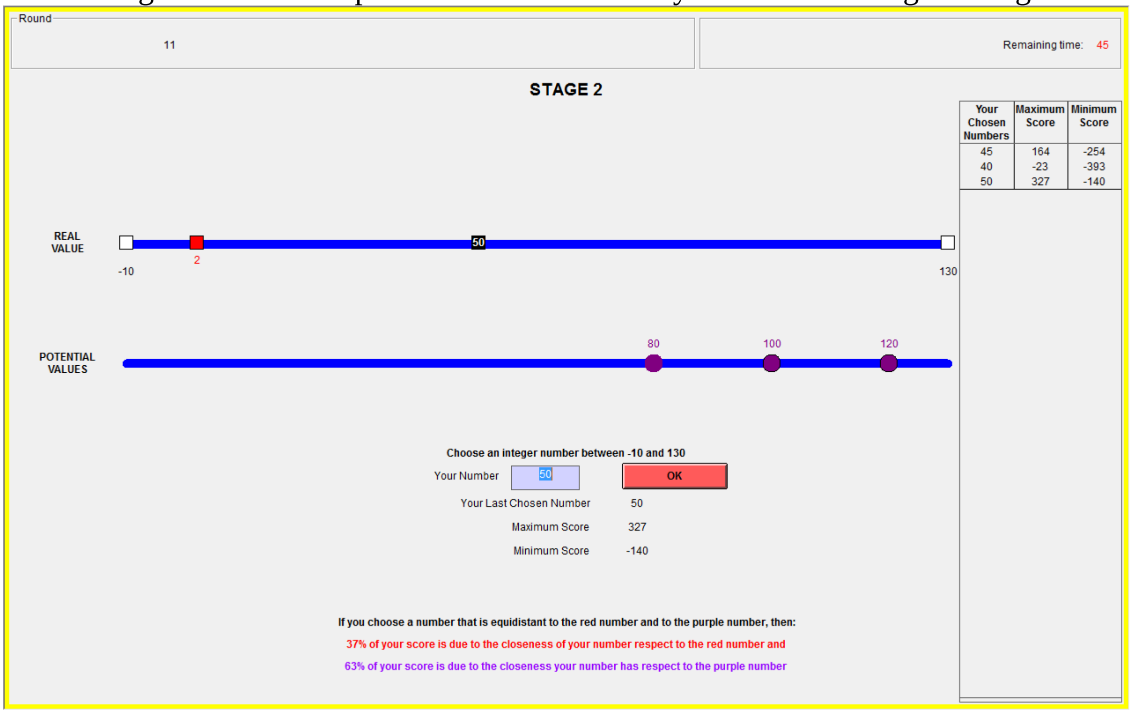
Since you do not know exactly the purple number of the Real Value line then the software does not tell you the exact score you can get per chosen number (as it happened in Stage 1).

However, for each chosen number, the software automatically tells you the maximum and minimum scores that correspond to such number, it uses as sources of information:

1. the Maximum Score you can get in the round,
2. the distance of your chosen number respect to the number that you decided to exactly know in the Real Value line (e.g. in Figure C2.5 the participant chose to know the red number so in this case this distance is $|50 - 2| = 48$) and
3. the distances of your chosen number respect to the three probable values that has the number that you did not decided to exactly know in the Real Value line (e.g. in

Figure C2.5 the participant chose not to know the purple so in this case these distances are $|50-80|=30$, $|50-100|=50$ and $|50-120|=70$).

Figure C2.5. Example of the kind of screen you will see during the Stage 2



Notice that:

- The maximum and minimum scores of your last chosen number are reported below the cell where you typed it, and
- the history of chosen numbers in the current round and their corresponding maximum and minimum scores are reported in the table at the right hand side of the screen.

Finally, I have to clarify that if your chosen number at the beginning of each round was the purple one instead of the red one, the explanation of the previous paragraphs is very similar and practically you only have to change the word purple with red and vice versa.

Please, read the instructions of this stage again. If the instructions are not clear to you, or you have a question of any sort, please raise your hand and sit quietly until the experimenter comes by to listen to your question. Do not hesitate to ask for help because if you are confused or make a mistake, it could reduce your earnings. The answer to your question might also be helpful for others to hear; if it is, I will repeat your question out loud, and the answer, so everyone can hear them.

STAGE 3 (Rounds 21 to 30)

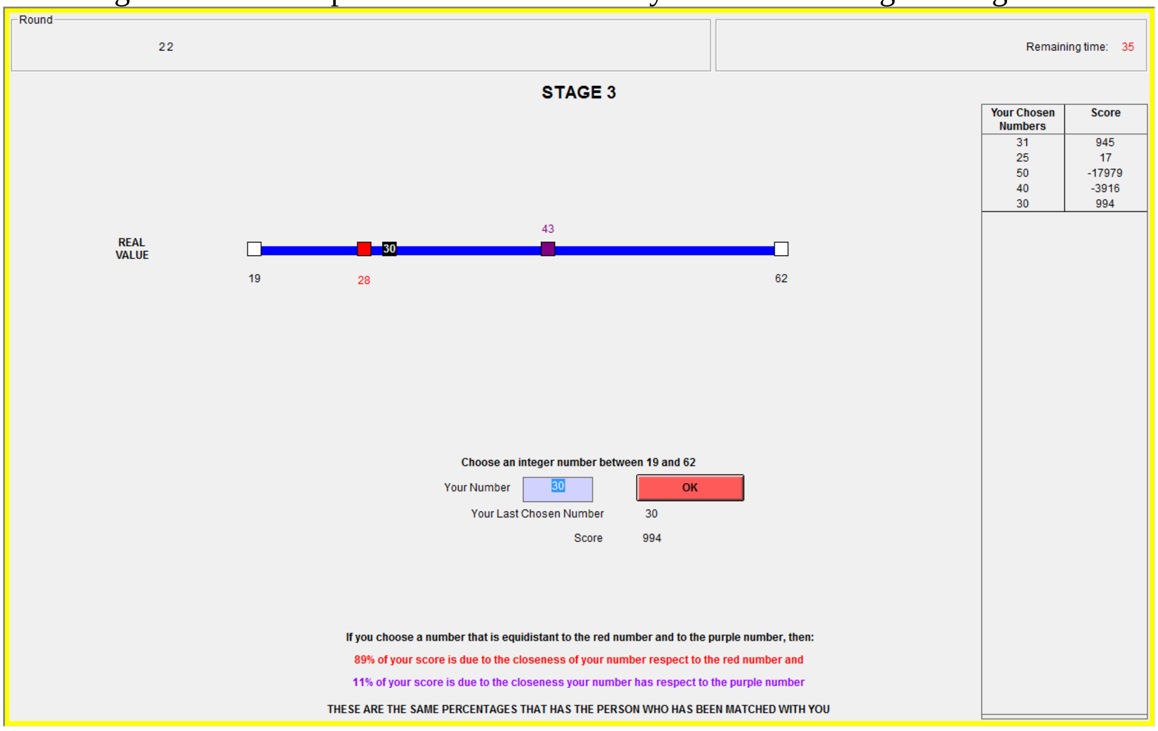
The instructions of this Stage are the same of Stage 1 except for the following:

You are 8 persons participating in this experiment. In each round you will be randomly matched with one of the other 7 participants. You will not get to know with whom you are matched.

In the rounds of Stage 1 the red and the purple numbers that appeared in the Real Value line remained constant. The same happens with the purple number in the rounds of Stage 3. *However, the red number does not remain constant*; in these rounds your red number is equal to the last number chosen by the person who is matched with you as soon as she presses OK. Similarly, the red number that appears in the Real Value line of the person who is matched with you is equal to the last number chosen by you as soon as you press OK. You and the person who is matched with you will have the same Real Value line (i.e. both lines will be bounded by the same white squares); however, the most likely is that the purple numbers are different and the red numbers may differ because they move following the mechanism that was explained above.

During the rounds of this stage you will see a screen similar to the screen that appears in Figure C2.6. Notice that this screen is the same kind of the screen that appeared in the rounds of Stage 1 except for the last line at the bottom of the screen. That is, the last paragraph that appears in the screen is the same paragraph that has the person who is matched with you and it is different for each couple of participants.

Figure C2.6. Example of the kind of screen you will see during the Stage 3



Take into account that your reds number are equal to the numbers chosen by the person who is matched with you, then your score changes as soon as the person who is matched with you chooses new numbers. ***If you want to know all the time how is affected the score of your last chosen number, look at the information that appears below the tag "Your Number" (this information is updated every time the person who is matched with you chooses a new number).*** In addition, <u>be careful because the information that is at the right hand side of the screen NEVER is updated. This table only reports the score of your chosen numbers at the moment you press OK.</u>

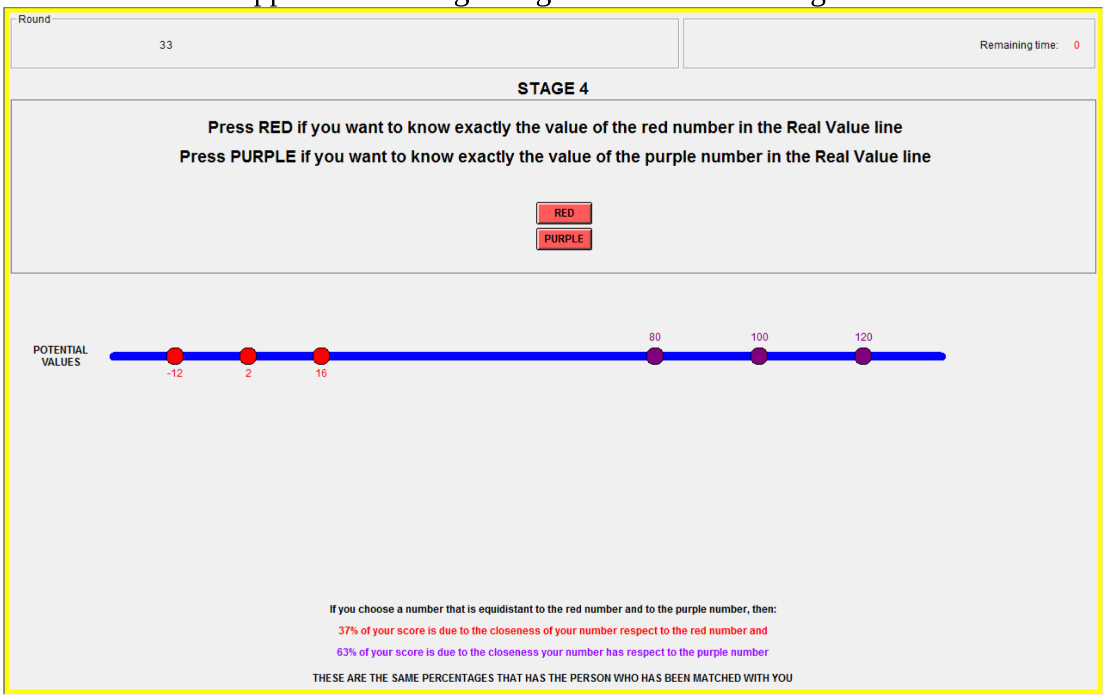
Finally, another difference between the rounds of this Stage respect to the rounds of Stage 1 is that now the rounds last 90 seconds.

Please, read the instructions of this stage again. If the instructions are not clear to you, or you have a question of any sort, please raise your hand and sit quietly until I come by to listen to your question. Do not hesitate to ask for help because if you are confused or make a mistake, it could reduce your earnings. The answer to your question might also be helpful for others to hear; if it is, I will repeat your question out loud, and the answer, so everyone can hear them.

STAGE 4 (Rounds 31 to 50)

The instructions of this stage are the same of Stage 3 except that you only will know exactly the red or the purple number of the Real Value line (the other number will be known with some uncertainty). More specifically, at the beginning of each round you will see a screen similar to the screen that appears in Figure C2.7. In this screen you have to choose (pressing with the mouse the corresponding button) which of the two kinds of numbers that appear in the Real Value line you want to know exactly.

Figure C2.7. Example of the kind of screen that appears at the beginning of the rounds in Stage 4



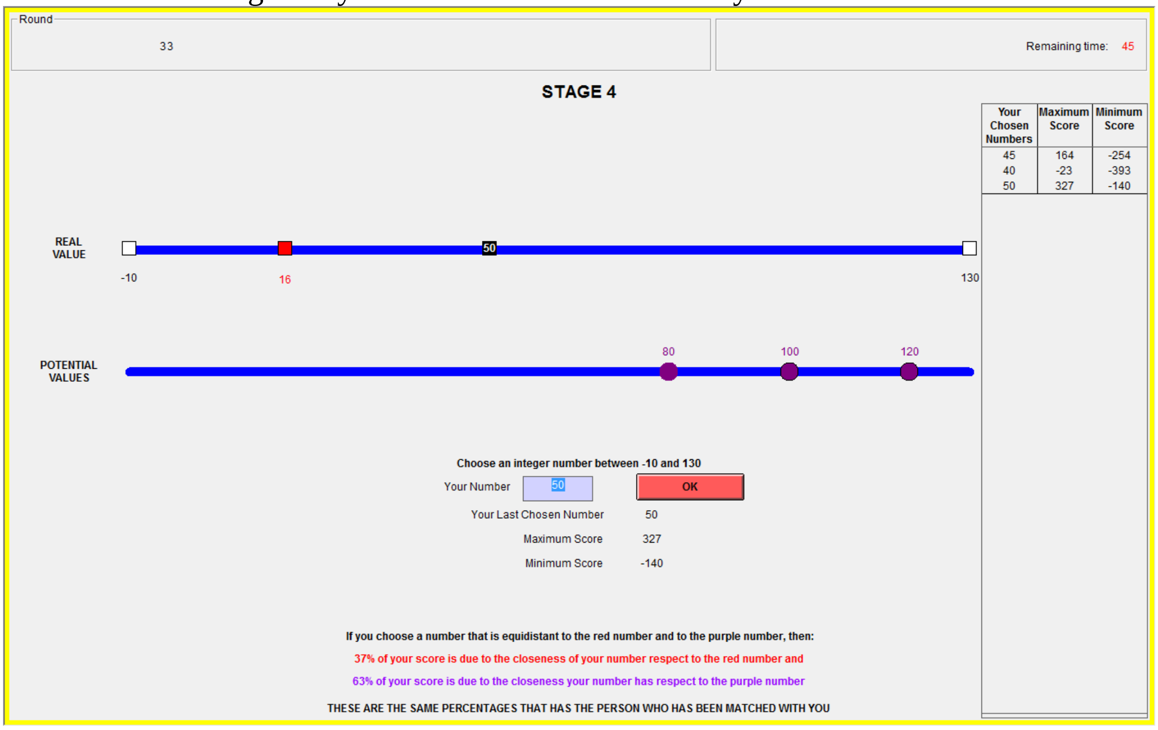
Look at the paragraph that appears at the bottom of Figure C2.7: the percentages of this paragraph remain constant during the round, change from one round to the other and are different for each couple of participant.

If you choose to know exactly the red number, a screen similar to the screen of Figure C2.8 appears. On the other hand, if you choose to know exactly the purple number, a screen similar to the screen of Figure C2.9 appears.

During the experiment, keep in mind the following:

- The red number in your Real Value line is equal to the last number chosen by the person who is matched with you as soon as she presses OK. The same kind of situation happens to the person who is matched with you.
- One of three red numbers that appear in your Potential Values line is always equal to the red number in your Real Value line. In addition, ***every time the person who is matched with you chooses a new number, then the red numbers of the Potential Values line change. However, the distance between these numbers remains constant during the round, changes from one round to the other and is different for each couple of participants***. For instance, the distance between the red numbers of the Potential Values line in Figures C2.7 and C2.9 is the same (i.e. 14).

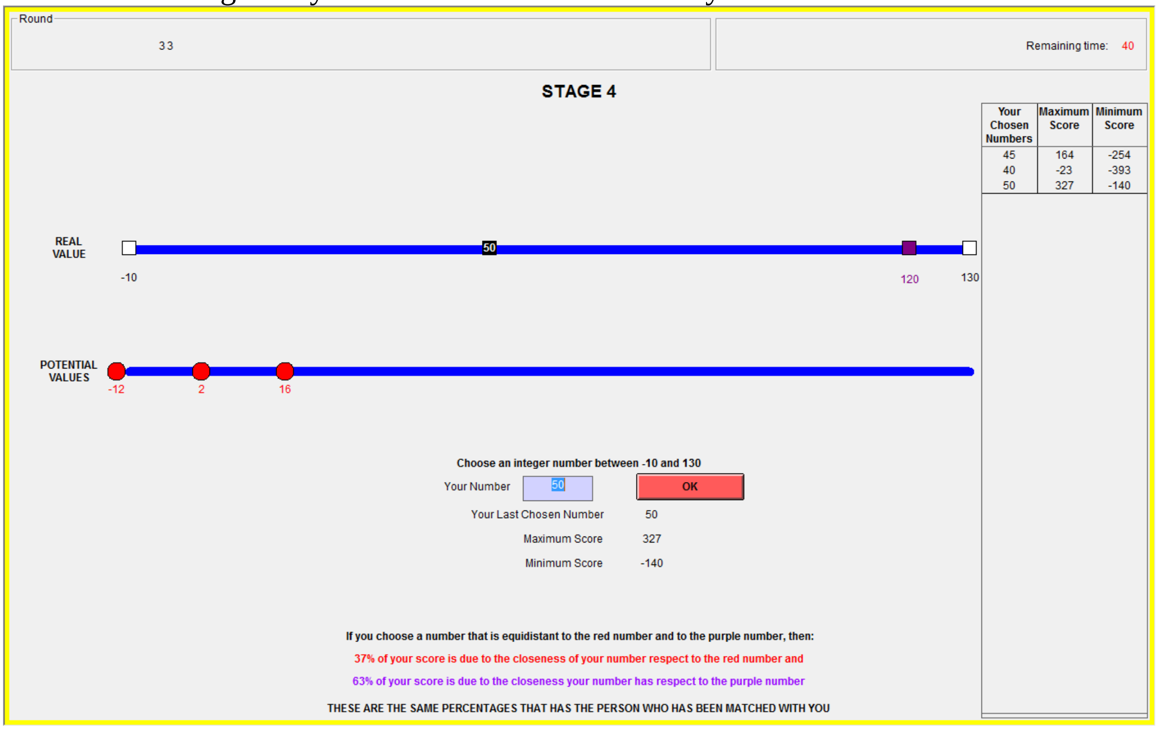
Figure C2.8. Example of the kind of screen you will see during the Stage 4 if you had chosen to know exactly the RED number



- The purple number in your Real Value line remains constant during each round, changes from one round to the other and is different for each participant. Remember that one of the three numbers in the Potential Values line is always equal to purple number in the Real Value line. The purple numbers of the Potential Values line remain constant during each round.

Please, read the instructions of this stage again. If the instructions are not clear to you, or you have a question of any sort, please raise your hand and sit quietly until the experimenter comes by to listen to your question. Do not hesitate to ask for help because if you are confused or make a mistake, it could reduce your earnings. The answer to your question might also be helpful for others to hear; if it is, I will repeat your question out loud, and the answer, so everyone can hear them.

Figure C2.9. Example of the kind of screen you will see during the Stage 4 if you had chosen to know exactly the PURPLE number



2.10. Tables

Table 2.1
Amounts of attention used by agent i to reduce the entropy of l_i and z from $[H_0(z_i); H_0(l_i)]$ to $\left[H_{0+S_1}(z); H_{0+S_2}(l_i)\right]$ where $S_1, S_2 \in \{0,1\}$

$I(z_i) + I(l_i)$	$I_1(z_i)$	$I_0(z_i)$
$I_1(l_i)$	3.17 bits	1.58 bits
$I_0(l_i)$	1.58 bits	0 bits

Table 2.2
Losses of Agent i that are due to the Lacks of Information: $\left(L_i^{IC}\right)_{[I_T(z);I_T(l_i)]}$

	$I_1(z_i)$	$I_0(z_i)$
$I_1(l_i)$	0	$\frac{W_i}{b+a}\left[\left(\frac{1}{3}\right)a^2 r_{z_i}^2\right]$
$I_0(l_i)$	$\frac{W_i}{b+a}\left[\left(\frac{1}{3}\right)b^2 r_{l_i}^2\right]$	$\frac{W_i}{b+a}\left[\left(\frac{1}{3}\right)a^2 r_{z_i}^2 + \left(\frac{1}{3}\right)b^2 r_{l_i}^2\right]$

Table 2.3.
Stages of the experiment

	Decision Making	**Interaction**
Complete Information	Stage 1 (Rounds 1 to 5)	Stage 3[76] (Rounds 21 to 30)
Incomplete Information	Stage 2 (Rounds 6 to 20)	Stage 4 (Rounds 30 to 50)

Notation

In the tables that are below there are only regressions. In these regressions we will use the following notation:

- $\boldsymbol{L.X}$: The value of variable X in the previous round.

Variables in alphabetical order:

- $\boldsymbol{a}$**:** Weight of z_i in the utility function
- $\boldsymbol{a-b}$**:** a minus b
- $\boldsymbol{|a-b|}$**:** Distance between a and b.
- $\boldsymbol{[ar_z]^2 - [br_l]^2}$**:** In stage 2 if this difference is higher than zero the theoretical model says that the agent i prefers to pay more attention to z_i, otherwise she prefers to pay more attention to l_i

[76] Remember that in section 2.2.3 (and also in this stage) each agent i have complete information about $\hat{z}_i$, but not about z_i due to the strategic behavior context.

- $\boldsymbol{Attempts}$: Number of $\hat{q}_t$ values chosen in the round until the participant chooses q_t. This variable does not include $\hat{q}_t$ values that are equal to $\hat{q}_{t-1}$ because these values do not update the available information; in other words, in stages 3 and 4 these repeated choices cannot be observed by the other participant and do not affect the earnings of the participants.
- $\boldsymbol{b}$**:** Weight of l_i in the utility function
- $\boldsymbol{Choice}$: Dummy variable. $Choice = 1$ if a participant chooses to know exactly z_i in Stages 1 and 2 or $\hat{z}_i$ in Stages 3 and 4. $Choice = 0$ if a participant chooses to know exactly l_i.
- $\boldsymbol{Dummy\ Sign\ |\quad |}$**:** Variable equal to 1 if the differences $\hat{q}_{i(Last-1)} - q_i^{(CI)*}$ and $q_i - q_i^{(CI)*}$ have the same sign.
- $\boldsymbol{Experience}$**:** This variable is a dummy equal to 1 if: (1) the player chooses to pay perfect attention to z_i in stage 2 (or $\hat{z}_i$ in Stage 4) and she obtains a high utility relative to the optimal one; or (2) the player chooses to pay perfect attention to l_i and she obtains a low utility relative to the optimal one.
- $\boldsymbol{Inattention}$**:** Dummy variable equal to 1 in stages 2 and 4 and equal to 0 in stages 1 and 3.
- $\boldsymbol{Interaction}$**:** Dummy variable equal to 1 in the interaction stages and equal to 0 in the decision making stages (i.e. this variable is equal to 1 in stages 3 and 4 and equal to 0 in stages 1 and 2).
- $\boldsymbol{Last\ Movement}$**:** Dummy variable. In Stages 3 and 4 $Last\ Movement = 1$ if the participant chose her q_i later than the other participant, otherwise $Last\ Movement = 0$
- $\boldsymbol{|l_i - l_j|}$: In stages 3 and 4 this is the distance between the idiosyncratic values of the two participants.
- $\boldsymbol{\left|\hat{q}_{i(Last-1)} - q_i^{*(CI)}\right|}$**:** In the rounds of stages 3 and 4 this distance refers to the distance of $\hat{q}_{it}$ respect to $q_i^{*(CI)}$ before the participant chooses q_i. Similar definitions (but making reference to a different variables) have the distances $|\hat{q}_{it} - \hat{q}_{it-1}|$, $|\hat{q}_{it-1} - \hat{q}_{it-2}|$, $|q_i - \hat{q}_{ilast-1}|$ and $|\hat{q}_{ilast-1} - \hat{q}_{ilast-2}|$.
- $\boldsymbol{|q - q^{BRF}|}$**:** In the rounds of stages 1 and 2 (i.e. when there is not interaction) this variable measures the distance between the last choice of the participant and her optimal choice. In the stages 3 and 4 (i.e. when there is interaction) this variable measures the distance between the last choice of the participant and her optimal choice given the last choice of the other participant.
- $\boldsymbol{r_{Inattention}}$**:** In stages 2 and 4 this is the dispersion $\boldsymbol{r}$ of the variable that the participant decided not to pay attention at the beginning of the round.
- $\boldsymbol{Range}$: In the real value line, this is the distance between the two white squares.
- $\boldsymbol{Remaining\ Time}$**:** Remaining time before the end of the round.
- $\boldsymbol{Round}$**:** Number of the round
- $\boldsymbol{t}$**:** Subscript of $\hat{q}_{it}$
- $\boldsymbol{Theory\_choice}$**:** In the rounds of stage 2, $Theory_{choice} = 1$ if the choice of attention of the participant at the beginning of the round is the same choice predicted by the theory and $Theory_{choice} = 0$ otherwise. In the rounds stage 4, $Theory_{choice} = 1$ if the choice of attention of the participant follows the same

pattern that the theory says is the optimal in the rounds of stage 2 and $Theory_{choice} = 0$ otherwise.

Table 2.4
Logit: Dependent Variable ($Choice$)

	Stage 2				Stage 4			
	Odd Ratio		Odd Ratio		Odd Ratio		Odd Ratio	
$[ar_z]^2 - [br_l]^2$	1.025	***	0.997		1.020	***	1.004	
	(0.006)		(0.003)		(0.005)		(0.004)	
$a - b$			72.325	***			13.753	***
			(55.765)				(10.244)	
$r_z - r_l$			1.341	***			1.141	***
			(0.048)				(0.038)	
$Constant$	0.643		0.882		4.323		6.278	
	(0.806)		(1.300)		(5.331)		(7.812)	
$Observations$	345		345		440		440	
LL	-146.06		-115.77		-181.60		-168.34	

Notes: Dummies per session and per subject. Robust standard errors in parentheses. Standard errors are clustered at the subject level. Significance levels: * p<0.05; **p<0.01; ***p<0.001. Other control variables included in the regressions are: $Round$, $Range$, $L.Choice$ and $L.Experience$.

Table 2.5
Dependent Variable $|q - q^{BRF}|$

	All		Stage 1		Stage 2		Stage 3		Stage 4	
	Coefficient		Coefficient		Coefficient		Coefficient		Coefficient	
$Interaction$	3.298	**								
	(1.070)									
$Inattention$	1.753	*								
	(0.694)									
a	2.594	***	0.698		-0.855		6.074	***	4.327	**
	(0.572)		(0.586)		(0.881)		(1.535)		(1.161)	
$Theory\_choice$					-2.741	***			-0.152	
					(0.562)				(1.039)	
$r_{Inattention}$					0.187	**			0.124	
					(0.051)				(0.095)	
$Constant$	-0.254		4.632	***	7.333	***	-0.752		1.371	
	(0.910)		(1.530)		(1.862)		(5.412)		(2.841)	
$Observations$	1200		120		360		240		480	
R^2	0.11		0.34		0.17		0.13		0.04	

Notes: Dummies per session and per subject. Robust standard errors in parentheses. Standard errors are clustered at the subject level. Significance levels: *p<0.05; **p<0.01; ***p<0.001. Other control variables included in all or in some regressions are: $Range$, $Round$, $|a - b|$, $Remaining\ Time$, $Last\_movement$, $|l_i - l_j|$, $Attempts$.

Table 2.6

Dependent Variable $|q_i - q_j|$

Stages 3 and 4	Coefficient	
$\boldsymbol{Inattention}$	0.021	
	(1.157)	
$\boldsymbol{a}$	-27.785	***
	(2.623)	
$\boldsymbol{Constant}$	33.573	***
	(1.238)	
$\boldsymbol{Observations}$	720	
$\boldsymbol{R^2}$	0.19	

Notes: Dummies per session and per subject. Robust standard errors in parentheses. Standard errors are clustered at the subject level. Significance levels: *p<0.05; **p<0.01; ***p<0.001.

Table 2.7

Dependent Variable $\left|q_i - q_i^{(CI)*}\right|$

	Stage 3		Stage 4	
	Coefficient		Coefficient	
$\left|\boldsymbol{\hat{q}_{i(Last-1)} - q_i^{(CI)*}}\right|$	0.269	**	0.442	***
	(0.080)		(0.090)	
$\boldsymbol{Dummy\ Sign\ |\quad|}$	4.401	**	3.778	*
	(1.290)		(1.410)	
$\boldsymbol{a}$	14.331	***	11.736	***
	(2.927)		(1.894)	
$\boldsymbol{Choice}$			4.200	**
			(1.439)	
$\boldsymbol{Constant}$	-0.798		-17.187	**
	(5.537)		(5.140)	
$\boldsymbol{Observations}$	239		467	
$\boldsymbol{R^2}$	0.51		0.52	

Notes: Dummies per session and per subject. Robust standard errors in parentheses. Standard errors are clustered at the subject level. Significance levels: *p<0.05; **p<0.01; ***p<0.001. Other control variables included in the regressions are: $Range$, $Round$, $Attempts$, $r_{Inattention}$, $|l_i - l_j|$, $Last\ movement$, $Remaining\ \ Time$

Table 2.8
Regressions in the Interaction stages

Dependent variable:	$\lvert \hat{q}_{it} - \hat{q}_{it-1} \rvert$		$\lvert q_i - \hat{q}_{ilast-1} \rvert$	
	Coefficient		**Coefficient**	
$\lvert \hat{q}_{it-1} - \hat{q}_{it-2} \rvert$	0.395	***		
	(0.026)			
$\lvert \hat{q}_{ilast-1} - \hat{q}_{ilast-2} \rvert$			0.624	***
			(0.048)	
$Range$	0.026	**	0.020	
	(0.009)		(0.014)	
t	-0.104	**		
	(0.029)			
$Attempts$			-0.128	
			(0.088)	
a	2.015	*	-1.688	
	(0.746)		(1.692)	
$Constant$	1.416		0.327	
	(1.648)		(1.937)	
$Observations$	7999		695	
R^2	0.20		0.46	

Notes: Dummies per session and per subject; Robust standard errors in parentheses. Standard errors are clustered at the subject level. Significance levels: *p<0.05; **p<0.01; ***p<0.001. Other control variables included in all or in some the regressions: $Inattention$, $Round$, $Period$, $\lvert l_i - l_j \rvert$, $Remaining\ Time$

2.11. Figures

Figure 2.1a.
Utility the participants can get in each round

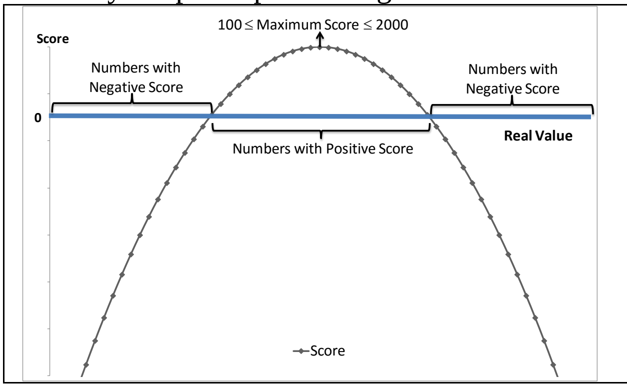


Figure 2.1b.
How the utility is affected by the chosen number?

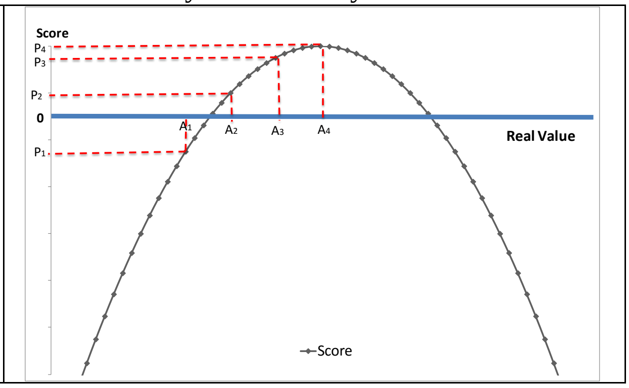


Figure 2.2a
Example of the kind of screen the participants see during the rounds of Stage 1

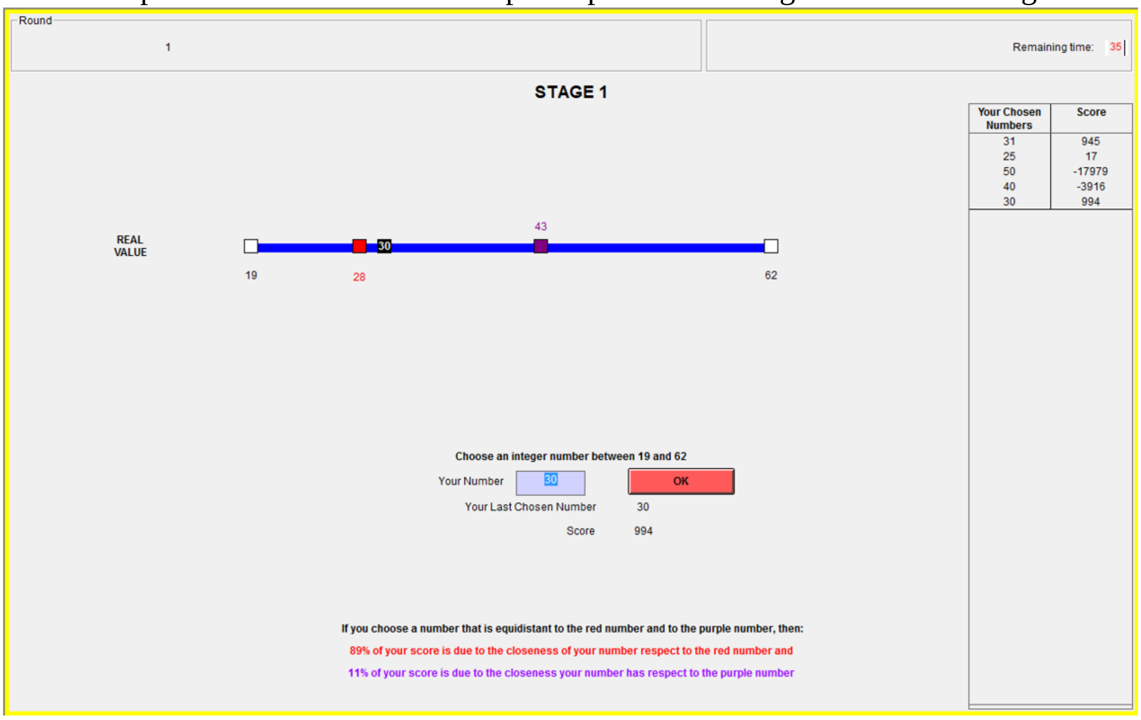


Figure 2.2b

Example of the kind of screen the participants see during the rounds of Stage 2

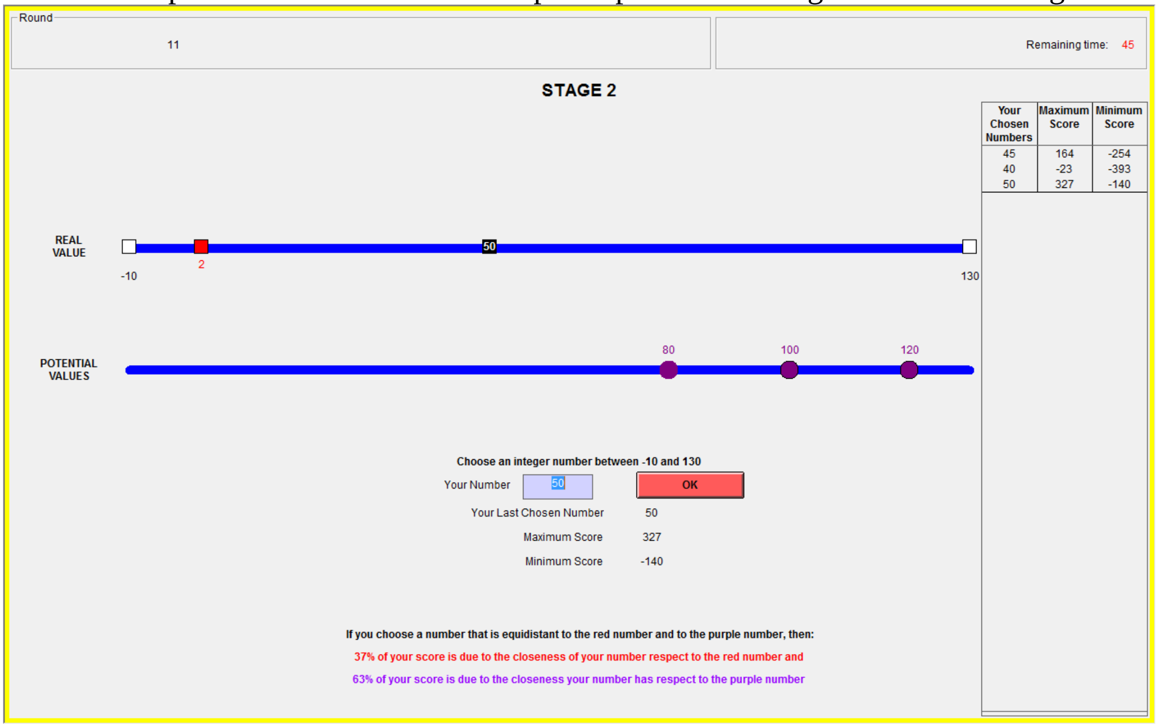


Figure 2.2c

Example of the kind of screen the participants see during the rounds of Stage 3

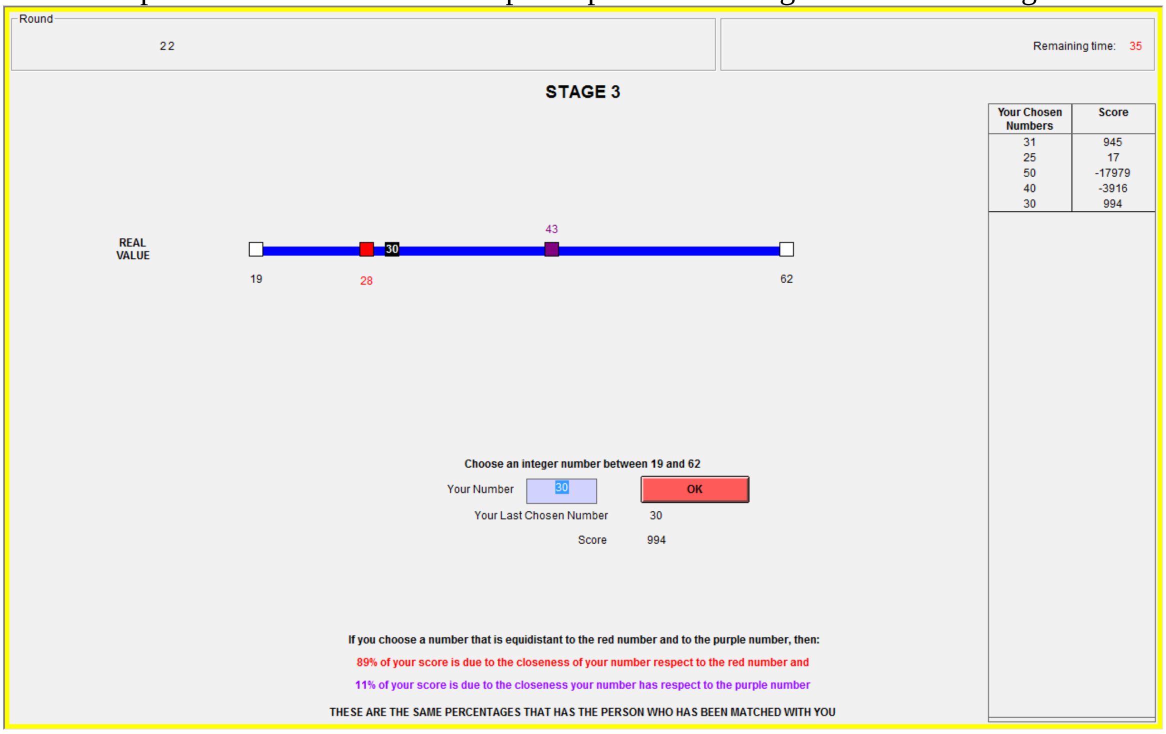


Figure 2.2d

Example of the kind of screen the participants see during the rounds of Stage 4

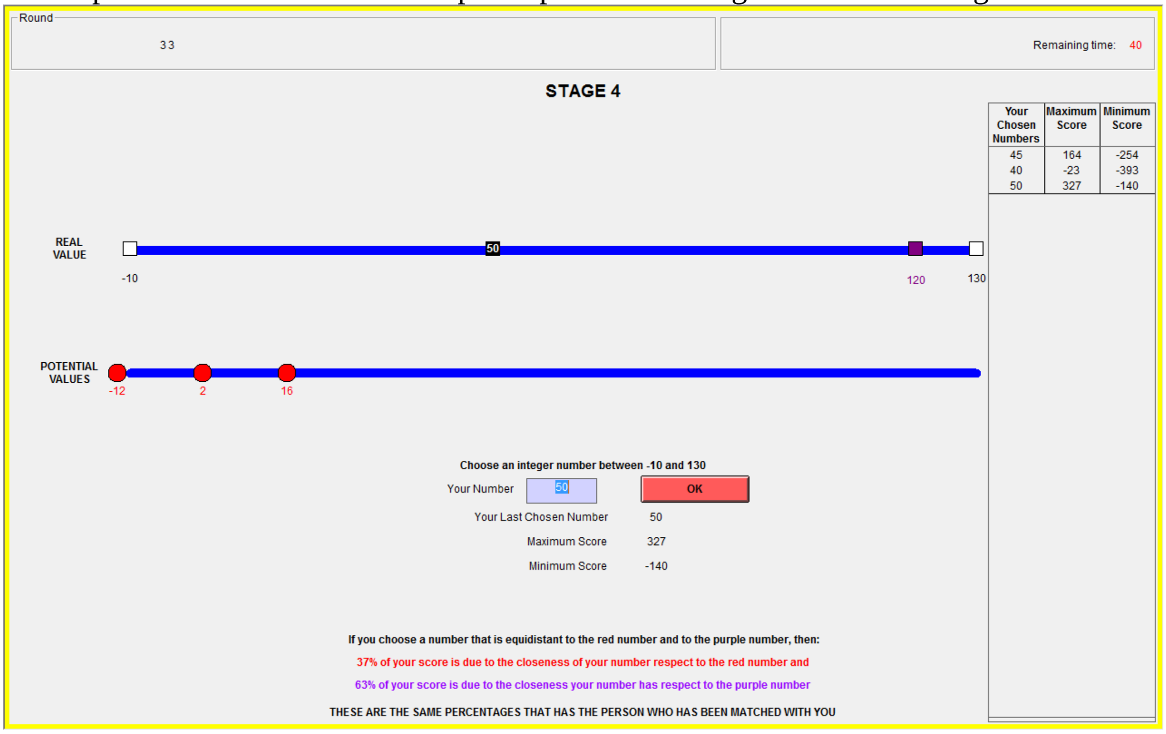
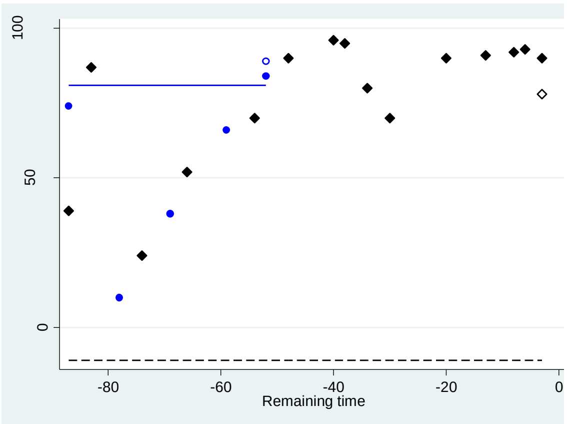
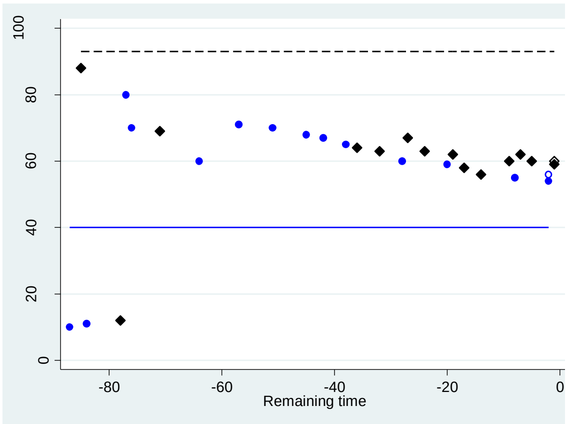


Figure 2.3a



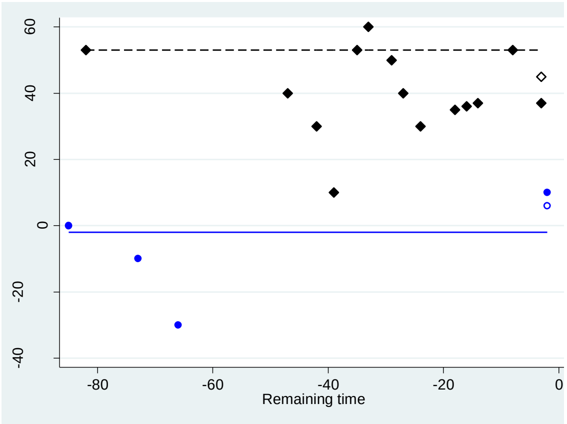
Continuous line: Idiosyncratic value of the blue participant (circles) = 81.
Dashed line: Idiosyncratic number of the black participant (diamonds) = -11.
Round of Treatment 4.
Both Participants know perfectly the red number. $a = 0.94$

Figure 2.3b



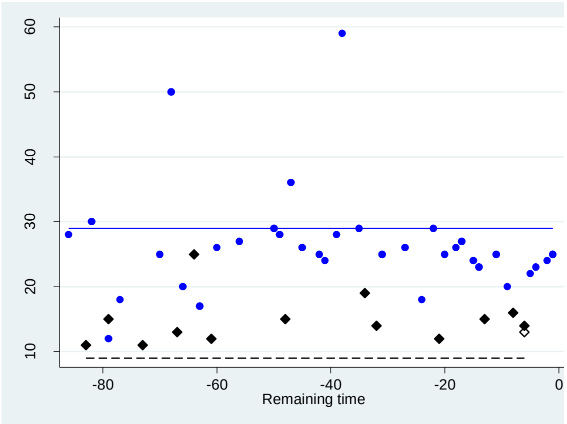
Continuous line: Idiosyncratic value of the blue participant (circles) = 40
Dashed line: Idiosyncratic value of the black participant (diamonds) = 93
Round of Treatment 4.
Both Participants know perfectly the red number. $a = 0.85$

Figure 2.3c



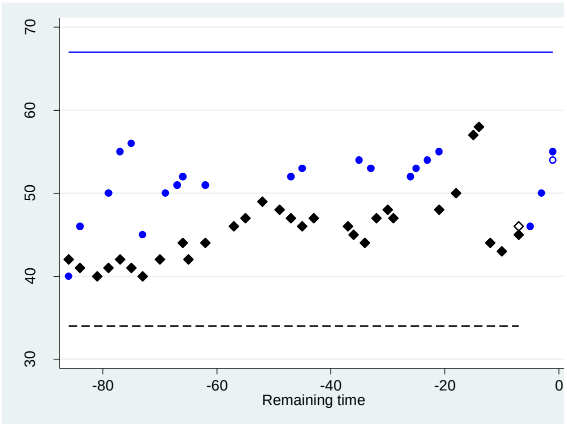
Continuous line: Idiosyncratic value of the blue participant (circles) =-2.
Dashed line: Idiosyncratic value of the black participant (diamonds) = 53.
Round of Treatment 3,
$a = 0.20$

Figure 2.3d



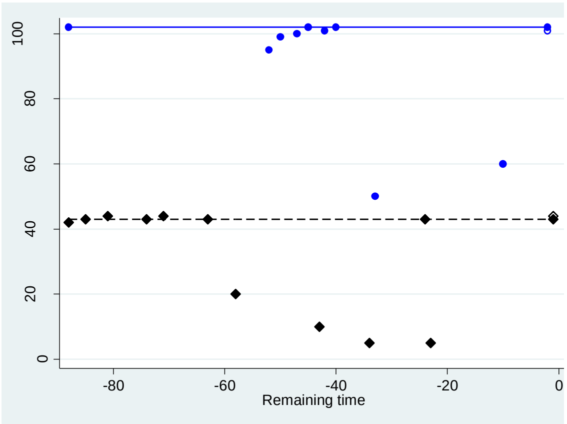
Continuous line: Idiosyncratic value of the blue participant (circles) = 9.
Dashed line: Idiosyncratic value of the black participant (diamonds) = 29.
Round of Treatment 3,
$a = 0.24$

Figure 2.3e.



Continuous line: Idiosyncratic value of the blue participant (circles) = 34
Dashed line: Idiosyncratic value of the black participant (diamonds) = 67
Round of Treatment 3 .
$a = 0.58$

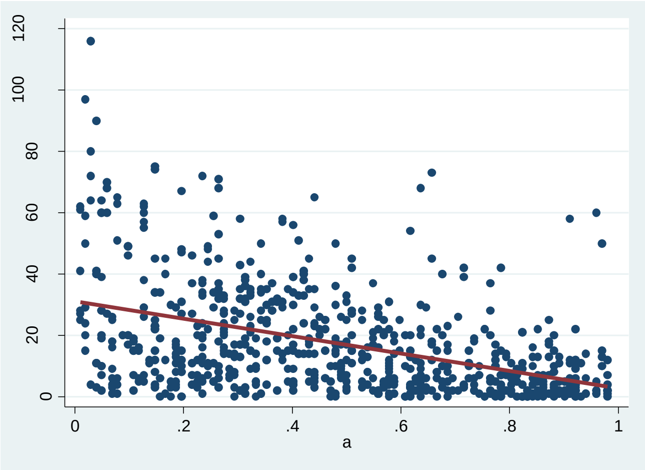
Figure 2.3f.



Continuous line: Idiosyncratic value of the blue participant (circles) = 102
Dashed line: Idiosyncratic value of the black participant (diamonds) = 43
Round of Treatment 4.
Both Participants know perfectly the purple number.
$a = 0.02$

Figure 2.4a

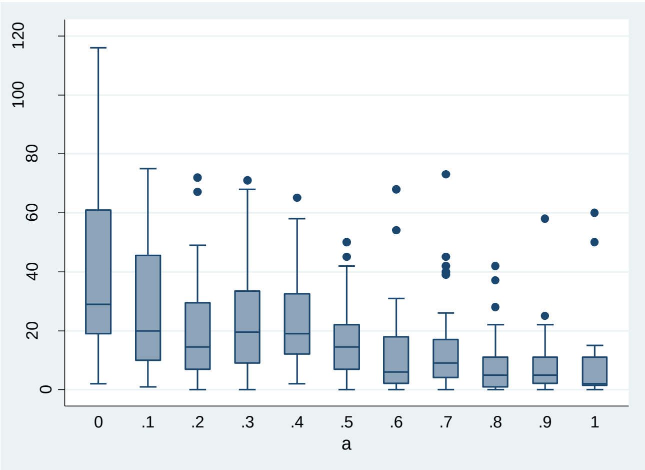
Scatterplot between $|q_i - q_j|$ and a*



*The red line represents the fitted value between $|q_i - q_j|$ and a

Figure 2.4b

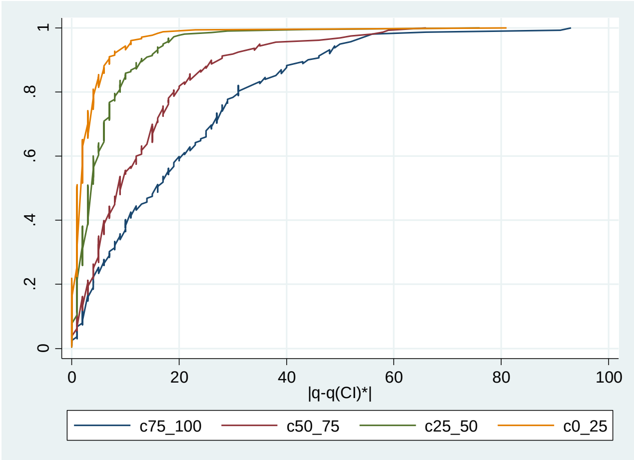
Boxplot between $|q_i - q_j|$ and a*



*To build this figure, the variable a was rounded to its closest one decimal digit number. In each box the top of the box is the 75^{th} percentile, the horizontal line is the median and the bottom of the box is the 25^{th} percentile

Figure 2.5

Cumulative distribution of $\left|q_i - q_i^{(CI)*}\right|$* in the interaction stages



Distance+	**0.00-0.25**	**0.25-0.50**	**0.50-0.75**	**0.75-1.00**	**Distance**+	**0.00-0.25**	**0.25-0.50**	**0.50-0.75**	**0.75-1.00**
≤ 5	85.39%	64.09%	35.00%	25.31%	≤ 60	99.44%	99.55%	99.38	98.15%
≤ 10	94.38%	85.91%	55.63%	40.12%	≤ 70	99.44%	99.55%	100%	98.77%
≤ 20	98.88%	97.73%	81.88%	59.88%	≤ 80	99.44%	100%	100%	98.77%
≤ 30	99.44%	99.09%	91.88%	78.40%	≤ 90	100%	100%	100%	98.77%
≤ 40	99.44%	99.09%	95.63%	88.27%	Maximum Distance	81	76	66	93
≤ 50	99.44%	99.55%	96.88%	95.06%					

*Notation: c75_100 means $a \in [0.75,1]$, c50_75 means $a \in [0.5,0.75]$, c25_50 means $a \in [0.25,0.5]$, c0_25 means $a \in [0,0.25]$, + The distance is in units and the different columns represent the different ranges of a.

Chapter 3

GLOBAL GAMES WITH ENDOGENOUS POLICY INTERVENTION: AN EXPERIMENT

3.1. Introduction

There are many macroeconomic and socioeconomic issues that have been analyzed using global games; for instance, bank runs[1], currency attacks[2], debt crises[3], debt pricing[4], effect of media in political regime change[5], investment dynamics[6], liquidity crashes[7], merger waves[8], party leadership[9], and securitization booms[10].

A global game is an incomplete information game in which a small uncertainty about payoffs implies a significant failure of common knowledge[11]. The global games methodology usually has been used to build strategic environments with incomplete information that are rich enough to consider the important role of higher-order beliefs in economic settings, but simple enough to allow tractable analysis (Morris and Shin 2003; Morris 2008). Another appealing characteristic of global games is that the complete information games that include coordination problems and self-fulfilling beliefs generally arrive to multiple Nash equilibria. However, the standard global games ensure the equilibrium uniqueness by relaxing the assumption of common knowledge[12].

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[1] For instance, the papers written by Rochet and Vives (2004), Goldstein (2005), Goldstein and Pauzner (2005) and Cañon and Margaretic (2014)

[2] For instance, the papers written by Morris and Shin (1998), Corsetti, Dasgupta, Morris and Shin (2004), Cukierman, Goldstein and Spiegel (2004), Goldstein and Pauzner (2005), Guimarães and Morris (2007), Frankel (2012) and Fujimoto (2014)

[3] For instance, the papers written by Corsetti, Guimarães and Roubini (2006) and Zwart (2007)

[4] For instance, the paper written by Morris and Shin (2004b)

[5] For instance, the paper written by Edmond (2013)

[6] For instance, the papers written by Chamley (1999), Heidhues and Melissas (2006), Dasgupta (2007) and Sákovics and Steiner (2012)

[7] For instance, the paper written by Morris and Shin (2004a)

[8] For instance, the paper written by Toxvaerd (2008)

[9] For instance, the paper written by Dewan and Myatt (2007)

[10] For instance, the paper written by Jin (2011)

[11] The global game methodology was firstly proposed by Carlsson and van Damme (1993) in a two-player, two-action game structure. It was extended by Morris and Shin (2003) to games that assume a continuum of players and by Frankel, Morris and Pauzner (2003) to supermodular games.

[12] This equilibrium uniqueness is robust when we assume a two-player, two-action structure or a continuum of players' game. But, in other contexts sometimes we need additional assumptions to get it.

Some recent papers have revealed that multiple equilibria may reemerge once the endogeneity of the information structure is taken into account. This endogeneity can be modeled using different kinds of mechanisms that are commonly observed in real life, such as: the signaling of policy interventions[13], the aggregation of information through prices[14] and learning in dynamic settings[15].

Angeletos and Pavan (2013) propose a general global game model of regime change in which the information is endogenous due to the signaling role of the actions of a policy maker. Using this model, the authors have shown that the global games methodology also brings useful predictions when the endogeneity of information does not lead to a unique equilibrium. In particular, they arrive to the following testable predictions (Angeletos and Pavan, 2013, p. 885): (1) weaker types of policy makers face a higher probability of regime change, (2) the probability of policy intervention is higher when the policy maker is neither too strong nor too weak, (3) the probability of policy intervention is lower when the precision of the agents' information increases, and (4) if the policy maker could commit to a particular policy before observing his type, then he could also guarantee a unique equilibrium[16].

The model proposed by Angeletos and Pavan (2013) has two appealing characteristics. First, in real life there are many situations in which the policy makers cannot isolate themselves from their choices, so it is useful to have models that take into account this kind of conflict. Second, the testable predictions proposed by the model are important to better understand how the policy interventions are affected by the particular interests of the policy maker and how the agents react when this kind of conflict exists. In this chapter we propose a lab experiment to analyze the first three testable predictions proposed by Angeletos and Pavan (2013)[17]. The main problem that we faced in our research is that their model is quite general to be used directly in a lab experiment; therefore, in the next section we explain briefly the version of their model that we used in our experiment, but we left most of the technical details to the Appendixes A3 and B3. In section 3.3 we present the experimental design. In section 3.4 we analyze the results obtained in the experiment; we do not only analyze the three testable predictions of the model, we also examine how was the behavior of the participants in the experiment and how the probability of the agents attacking the status

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For instance, Frankel, Morris and Pauzner (2003) show that in a game with many players and many actions, to get limit uniqueness (i.e. a situation in which as the noise in the global game becomes arbitrarily small, there is a unique strategy profile that survives iterative elimination of strictly dominated strategies) is necessary to assume at least that the strategies of the players are strategic complements and the presence of dominant regions (i.e. regions in which there are strictly dominant actions); on the other hand, to get noise independent selection (i.e. as the noise goes to zero, the equilibrium played, for a given realization of the payoffs, is independent of the distribution of the noise) is necessary to have a local potential game (i.e. a game in which each player's payoffs are quasiconcave in her own action).

[13] A model that introduce this mechanism was built by Angeletos, Hellwig and Pavan (2006).

[14] Some papers that include this mechanism inside their models are Angeletos and Werning (2006), Hellwig, Mukherji and Tsyvinski (2006) and Ozdenoren and Yuan (2008)

[15] Some papers that consider this mechanism are Angeletos, Hellwig and Pavan (2007) and Chassang (2010)

[16] However, the policy maker can prefer such commitment over discretion only insofar as he expects his type to be strong

[17] The fourth testable prediction implies the introduction of additional characteristics to the experimental design that makes each session of the experiment longer and more complex to implement. Therefore, we did not include these characteristics to the experiment and we prefer to focus in the analysis of the other predictions and in the analysis of the behavior of the participants in the experiment.

quo changes when the precision of the agents' information change. Finally, in section 3.5 we present some final comments.

3.2. Model.

This part of the chapter is divided in two sections. In section 3.2.1 we explain general characteristics of the model. In section 3.2.2 we explain the equilibrium strategies followed by the individuals in the model and how these strategies imply the predictions that we want to test.

3.2.1. General characteristics.

Consider a game that represents a society in which there are two possible regimes: the status quo and an alternative to the status quo. At the beginning of the game the prevalent regime is the status quo; however, the kind of regime at the end of the game depends on the choices of the participants in the game. The participants are one policy maker and two agents indexed by i. The payoffs obtained in this game will depend on the choices of the participants and the established regime at the end of the game.

Let $\theta \in \mathbb{R}$ denotes the type of policy maker[18], the policy maker knows his type and establishes a policy $r \in [0, +\infty)$. The only condition that this policy has is that it entails a cost to all individuals in the economy; however, under certain conditions it can discourage the agents to attack the status quo[19]. The agents observe the policy, have incomplete information about the type of policy maker and have to choose individually if they attack the status quo or abstain from attacking. The variable $a_i \in \{0,1\}$ represents the choice of agent i, where $a_i = 1$ if agent i attacks the status quo and $a_i = 0$ if agent i refrains from attacking. The percentage of agents attacking the status quo is denoted by A. In this model, if $\theta > KA$ the policy maker always maintains the status quo and if $\theta \leq KA$ the policy maker always allows the defeat of the status quo (i.e. there is a regime change), where K is a positive finite parameter that regulates how much important is the pressure of the agents when they attack the status quo (i.e. if K is higher, then the probability of regime change increases)[20]. Therefore, KA can be interpreted as the size of the attack.

The goal of the policy maker is to choose the policy r that maximizes her utility $U(\cdot)$ according to the following optimization problem[21]:

$$\max_{r \geq 0} U(\theta, r, KA) = \begin{cases} W(\theta, r, KA) = \theta - KA - cr & if\ \theta > KA \\ L(r) = -cr & if\ \theta \leq KA \end{cases} \tag{3.1}$$

[18] The type of policy maker represents her motivation or ability to maintain the status quo.

[19] Therefore, more than a policy per se, r represents the cost of a policy measure that the policy maker may decide to implement in order to discourage the agents to attack the status quo. However, to make the explanation of the model easier and clearer we will refer to r, in most of the chapter, as the policy implemented by the policy maker.

[20] In the lab experiment, this variable is also useful to calibrate the payoffs of the participants in the experiment.

[21] Therefore, the payoff of the policy maker is directly represented by the utility she obtains in the game.

In equation (3.1) notice that the policy r is not free of cost to the policy maker, the parameter $c \in (0,1]$ regulates the size of this cost. The problem represented in equation (3.1) can be interpreted in the following way: it measures how able (or how willing) is the policy maker of type θ to defend the status quo due to the costs that she incurs by implementing a policy r and by facing an attack to the status quo of size KA. If the status quo survives, then the utility of the policy maker is given by $W(*)$, otherwise it is given by $L(*)$[22].

On the other hand, the goal of each agent i is to choose the action a_i that maximizes her utility $u_i(\cdot)$ according to the following problem[23]:

$$\max_{a_i \in \{0,1\}} u_i(r) = \begin{cases} a_i(y - r - b) \ if \ \theta \leq KA \\ a_i(-r - b) \quad if \ \ \theta > KA \end{cases} \tag{3.2}$$

where $y > b > 0$. In this equation $(y - b)$ represents the maximum utility that each agent i can get when the regime changes and $(r + b)$ is her opportunity cost of attacking the status quo (i.e. her cost of choosing $a_i = 1$). The agents cannot observe θ, but each agent i receives a private signal represented by

$$x_i = \theta + \sigma \vartheta_i \tag{3.3}$$

where $\sigma > 0$ parameterizes the quality of the private signal[24] and $\vartheta_i \sim N(0,1)$ is an idiosyncratic noise which is independently and identically distributed across agents and independent of θ, with absolutely continuous p.d.f ψ and c.d.f Ψ. We will assume that the noise distribution ψ is normal; however, Angeletos and Pavan (2013, p. 910) demonstrate that whatever log-concave distribution works[25]. The problem represented in equation (3.2) can be interpreted in the following way: it measures the utility of each agent given her choice and the choices of the other individuals; if she does not attack the status quo then her utility is equal to zero irrespective of the choices of the other individuals; however, if she attacks the status quo then her utility decreases if the policy chosen by the policy maker increases and if the joint attack of the agents is not high enough to defeat the status quo.

The timing of the game is: (1) the policy maker learns his type θ and sets the policy r; (2) the agents decide simultaneously and individually to attack or abstain from

[22] The equation (3.1) satisfies the conditions that Angeletos and Pavan (2013, p. 889) impose to the policy maker's utility function: a) policy interventions (i.e. when the policy maker chooses a policy $r > 0$) are always costly to her; b) if the status quo is maintained (i.e. if $\theta > KA$), the policy maker prefers a smaller attack (i.e. a smaller KA); c) if setting the policy in $r = 0$ leads to a change of regime (and consequently a policy maker's utility of $L(0) = 0$) while setting the policy to some level $r = s$ where $s > 0$ leads to $KA = 0 < \theta \in (0, +\infty)$ (and consequently a policy maker's utility of $W(\theta, r, 0) = \theta - cr$), then higher types of policy maker have stronger incentives to raise the policy than lower types of policy maker; and d) the policy maker would not prefer to see the regime collapse when it survives (i.e. the policy maker prefers $W(\theta, r, KA)$ than $L(r)$ if $\theta > KA$).

[23] Therefore, the payoff of each agent is directly represented by the utility she obtains in the game.

[24] The precision of the signal that each agent receives is defined by the standard deviation σ, where the precision increases when σ decreases and $\lim_{\sigma \to 0} x_i = \theta$.

[25] Some examples of log-concave distributions are the normal distribution, the multivariate normal distribution, the exponential distribution, the uniform distribution over any convex set, the logistic distribution, the extreme value distribution, the Laplace distribution, the chi distribution and the Subbotin distribution.

attacking the status quo after observing the policy r and the idiosyncratic private signal x_i; and (3) the final regime is determined.

Finally, Angeletos and Pavan (2013; p. 891) assume that an agent who expects a regime change finds it optimal to choose $a = 1$ at least insofar as the policy maker does not play a dominated action; therefore, an agent who expects a regime change, even in the worst case scenario, finds it optimal to choose $a = 1$ when the policy maker does not play a dominated action (i.e. if $y - b - \frac{K}{c} > 0$ then the Angeletos and Pavan's assumption is always satisfied)[26]. In addition, we also assume that $y > 2b$ to ensure that the semiseparating equilibria of the model exist[27].

3.2.2.Equilibrium

The equilibrium characteristics of the model are explained in detail in the Appendixes A3 and B3. We have followed the same line of reasoning proposed by Angeletos and Pavan (2013). In this section we will explain directly the strategies that the individuals follow at the different equilibria of the model. We will also explain how the equilibrium characteristics of the model determine the predictions that will be tested in the lab experiment.

Note that the model combines two kinds of games: (1) a signaling game where the policy maker is the sender and the agents are the receivers; and (2) a global game played

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[26] When an agent expects a regime change, it means that she at least expects $\theta \leq K$ (notice that if $\theta > K$ a regime change cannot occur even if all agents coordinate on the attack). From equation (3.2) notice that when an agent chooses $a = 1$ is because she expects $y - b - r \geq 0$, where the LHS of the inequality represents the utility of an agent when there is a regime change and the agent chooses $a = 1$, and the RHS of the inequality represents the utility of an agent when there is a regime change and the agent chooses $a = 0$.

If $\theta \leq 0$, then a regime change is the unique possible regime outcome, it implies that the utility obtained by the policy maker is necessarily $L(r) = -cr$. Since $L(0) = 0 > -cs = L(s)$ when $s > 0$, then all policies $r = s$ are strictly dominated by the policy $r = 0$ when $\theta \leq 0$. On the other hand, given any policy r, when $\theta \in (0, K]$ the best scenario to the policy maker is when no agents attack (i.e. when the size of the attack is 0, then the status quo survives and consequently her utility is $W(\theta, r, 0) = \theta - cr$) while the worst scenario is when all agents attack (i.e. when the size of the attack is K, then the regime changes and consequently her utility is $L(r) = -cr$). Therefore, the maximal level of r that is not dominated by $r = 0$ is the highest policy r that solves the inequality $W(\theta, r, 0) \geq L(0)$; that is $r = \frac{\theta}{c}$. Consequently, when there is a regime change the worst scenario to the agents, given that the policy maker does not choose dominated strategies, is when the policy maker is the highest that cannot avoid a regime change (i.e. when $\theta = K$) and she chooses $r = \frac{K}{c}$.

[27] In Appendix B3 we explain that in the set of semiseparating equilibria the optimal strategy to the policy maker is to choose the policy $r = s$ in the cases in which the policy $r = 0$ is not the optimal policy. In addition, in the Propositions B4 and B5 of Appendix B, we identify two variables that are important to determine any semiseparating equilibrium in the model, these variables are $x_S^*(\sigma)$ and $\theta_S^{**}(\sigma)$. Both variables depend on the following inverse normal c.d.f $\Psi^{-1}\left(1 - \frac{cs}{K}\left(\frac{b}{y-b}\right)\right)$. It implies that one necessary condition to the existence of $x_S^*(\sigma)$ and $\theta_S^{**}(\sigma)$ is that the inequality $1 > \frac{cs}{K}\left(\frac{b}{y-b}\right)$ is satisfied. In the previous footnote we have seen that in the set of semiseparating equilibria the maximal level of the policy r that is not dominated by the policy $r = 0$ is $r = \frac{K}{c}$. Then the inequality $1 > \frac{cs}{K}\left(\frac{b}{y-b}\right)$ at the equilibrium is always satisfied for all θ if $y > 2b$.

when the agents have to independently decide if they attack or not attack the status quo given the imperfect information that they have. Therefore, the equilibrium concept used in the model is perfect Bayesian equilibrium. It implies that at the equilibrium the individuals are sequentially rational and at the equilibrium path the agent's beliefs are defined by Bayes' rule. In the model, for any σ, we can identify two kinds of equilibria: (1) a unique pooling equilibrium and (2) a set of semiseparating equilibria. In section 3.2.2.1 we will explain the characteristics of the pooling equilibrium and in section 3.2.2.2 we will explain the characteristics of the set of semiseparating equilibria. In both kinds of equilibria, since all positive policies imply a cost to the policy maker, then at the equilibrium the policy maker only chooses a policy $r = s > 0$ when, by implementing this policy, the policy maker can guarantee the survival of the status quo and that the cost of implementing the policy (to the policy maker) is lower than the cost of receiving an attack.

3.2.2.1. The pooling equilibrium.

The propositions about the existence, uniqueness and characteristics of the pooling equilibrium are enunciated and proved in Appendix A3 at the end of the chapter. The main characteristics of this unique pooling equilibrium are: the policy makers always choose the same policy, so this policy is uninformative about θ. Therefore, the agents do not use the information of the policy r to updates their beliefs about the type of policy maker. It implies that in the first stage of the game, the policy maker infers that all policies $r > 0$ are strictly dominated by the policy $r = 0$[28], and consequently she always chooses the policy $r = 0$ at the pooling equilibrium. On the other hand, when the agents observe the policy $r = 0$, then they play a coordination game in which they use a symmetric trigger strategy that has as threshold the unique signal $x^{\#}(\sigma)$. It implies that all individuals may infer that a regime change happens if and only if the type of policy maker is $\theta \leq \theta^{\#}$ where $\theta^{\#}$ is a unique threshold type of policy maker.

In the pooling equilibrium, after observing the policy $r = 0$, the agents follow the trigger strategy: $a = 1$ if $x \leq x^{\#}(\sigma)$ and $a = 0$ if $x > x^{\#}(\sigma)$ where $x^{\#}(\sigma) = \sigma\Psi^{-1}\left(\frac{y-b}{y}\right) + K\left(\frac{y-b}{y}\right)$. On the other hand, the policy maker infers that the threshold type $\theta^{\#}$ is determined by $\theta^{\#} = K\left(\frac{y-b}{y}\right)$[29]. Notice from the equations of $x^{\#}(\sigma)$ and $\theta^{\#}$ that it is not complex to conclude that both values are unique. In addition, notice that the distance between $x^{\#}(\sigma)$ and $\theta^{\#}$ depends on the precision of the signal; that is, if the precision increases (i.e. if σ is lower) then $x^{\#}(\sigma)$ approaches to $\theta^{\#}$.

In the pooling equilibrium we can check directly how the first testable prediction of Angeletos and Pavan (2013) appears. When θ is lower, the average size of the signals decreases[30], it implies that the probability that the agents receive signals $x \leq x^{\#}(\sigma)$ increases, so the probability that an agent chooses $a = 1$ increases, then the probability that the status quo collapses is higher. The other two testable predictions cannot be

[28] Remember that a higher r implies a higher cost cr to the policymaker

[29] Notice that the assumption $y > 2b$ implies that $x^{\#}(\sigma) > \theta^{\#} > \frac{K}{2}$

[30] Remember that $x_i = \theta + \sigma\vartheta_i$

analyzed in the pooling equilibrium because these predictions require that sometimes at the equilibrium the policies are $r > 0$.

3.2.2.2. The set of semiseparating equilibria.

The propositions about the existence and characteristics of the set of semiseparating equilibria are enunciated and proved in Appendix B3 at the end of the chapter. The main characteristics of the set of semiseparating equilibria are: in any equilibrium, in the first stage of the game, the policy maker chooses the policy $r = 0$ or a policy $r > 0$. Any type of policy maker who chooses a policy $r > 0$ does so by selecting the least costly policy among those policies that are favorable to the survival of the status quo[31]. Let $r = s$ (where $s > 0$) to be the least costly policy that does not imply a regime change. So, it means that at the equilibrium path the policy maker always chooses $r = 0$ or $r = s$. Therefore, in the second stage of the game, if the agents observe a policy $r = s$, then this policy signals them that there won't be a regime change and thus induces them to choose $a = 0$. On the other hand, if the agents observe a policy $r = 0$, then they use a symmetric trigger strategy such that they choose $a = 1$ if $x < x_S^*(\sigma)$ and they choose $a = 0$ if $x > x_S^*(\sigma)$[32] where the threshold signal $x_S^*(\sigma)$ exists and is unique.

Let θ_S^* represents the "lowest" type of policy maker who prefers the policy $r = s$ in which there are no attacks to the status quo to the policy $r = 0$ in which there is a coordinated attack that implies the defeat of the status quo. Therefore, since the implementation of any policy has a cost equal to cr, then any type of policy maker $\theta \geq \theta_S^*$ who wants to implement a policy $r > 0$ will choose the policy $r = s$. It implies that at any equilibrium the policy makers $\theta \geq \theta_S^*$ can always guarantee the survival of the status quo by choosing the policy $r = s$. That is, in the set of semiseparating equilibria the status quo always survives if the type of policy maker is higher than θ_S^*.

Notice that for "sufficiently low" types of policy makers[33] all policies $r > 0$ are strictly dominated by the policy $r = 0$ and a regime change always happens; therefore, the agents iteratively find that $a = 1$ is the dominant strategy for "sufficiently low" signals when they observe $r = 0$. Then the dispersion of information initiates a contagion effect in which, conditional on seeing $r = 0$, the agents find iteratively dominant to choose $a = 1$ for higher and higher signals, then the regime change happens for higher and higher θ. In the limit, this contagion effect guarantees that the

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[31] It happens because according to equation (3.1): [1] the policy $r = 0$ strictly dominates any policy $r > 0$ when the regime changes (so, the policy $r = 0$ is not a useful policy to signal the survival of the status quo), [2] the survival of the status quo is the minimum requirement that a policy maker has to get a positive utility and [3] any policy $r > 0$ always implies a cost cr to the policy maker. Therefore, if two different policies $r > 0$ can ensure the survival of the status quo, then the policy maker prefers the policy that is less costly to implement.

[32] More specifically, the semiseparating equilibria are sustained by the following strategy profile followed by the agents:

- For $r = 0$, each agent implements the trigger strategy $a(x, r) = 1$ if and only if $x < x_S^*(\sigma)$ and $a(x, r) = 0$ if and only if $x > x_S^*(\sigma)$.
- For any $r \in (0, s), a(x, r) = 1$ irrespective of x
- For any $r \geq s,\ a(x, r) = 0$ irrespective of x

However, given θ, remember that any policy $r \in (0, s)$ or $r > s$ is dominated by the policy $r = 0$ or by the policy $r = s$. So, in the equilibrium path we won't see any policy $r \in (0, s)$ or $r > s$.

[33] For example, for types of policy makers $\theta \leq 0$.

regime change happens for all type of policy makers $\theta < min\{\theta_s^*, \theta^\#\}$. However, one of the propositions of Appendix B3 shows that the set of semiseparating equilibria only exist when $\theta_s^* \leq \theta^\#$[34]; it implies that at these equilibria the regime change always happens for all type of policy makers $\theta < \theta_s^*$.

Let $\theta_s^{**}(\sigma)$ denotes the "highest" type of policy maker $\theta \geq \theta_s^*$ who finds it optimal to choose the policy $r = s > 0$ in which there are no attacks to the status quo instead of the policy $r = 0$ in which there is an attack that does not imply the defeat of the status quo[35]. Then, the policy $r = s$ dominates any other policy if the type of policy maker $\theta_s \in (\theta_s^*, \theta_s^{**}(\sigma)]$ and the policy $r = 0$ dominates the policy $r = s$ if the type of policy maker $\theta_s \notin (\theta_s^*, \theta_s^{**}(\sigma)]$. Therefore, in the set of semiseparating equilibria the policy chosen by the policy maker θ_s is $r \in \{0, s\}$, where the policy $r = 0$ is chosen when $\theta_s \notin (\theta_s^*, \theta_s^{**}(\sigma)]$ and the policy $r = s$ is chosen when $\theta_s \in (\theta_s^*, \theta_s^{**}(\sigma)]$. In Appendix B3 we show that the values and the bounds of all variables at the set of semiseparating equilibria are:

Table 3.1

Variable	Value	Bounds
s	s	$\left(\frac{K}{2c}, \frac{\theta^\#}{c}\right]$
θ_s^*	cs	$\left(\frac{K}{2}, \theta^\#\right]$
$\theta_s^{**}(\sigma)$	$\sigma\left[\Psi^{-1}\left(1 - \frac{\theta_s^*}{K}\left(\frac{b}{y-b}\right)\right) - \Psi^{-1}\left(\frac{\theta_s^*}{K}\right)\right] + \theta_s^*$	$\left(\sigma\Psi^{-1}\left(\frac{2y-3b}{2y-2b}\right) + \frac{K}{2}, \theta^\#\right]$
$x_s^*(\sigma)$	$\sigma\Psi^{-1}\left(1 - \frac{\theta_s^*}{K}\left(\frac{b}{y-b}\right)\right) + \theta_s^*$	$\left(\sigma\Psi^{-1}\left(\frac{2y-3b}{2y-2b}\right) + \frac{K}{2}, x^\#(\sigma)\right]$

where $\theta^\#$ and $x^\#(\sigma)$ are the same threshold values commented in the previous section. In Table 3.1, notice that for each type of policy maker θ_s corresponds one value of θ_s^*, $\theta_s^{**}(\sigma)$ and $x_s^*(\sigma)$ in the table. The bounds that appear in the table are useful to compare the behavior of the participants in the experiment with the behavior of the individuals in the theoretical model. Additionally, notice that when s approaches to its upper bound $\frac{\theta^\#}{c}$ then the multiple semiseparating equilibria collapse into the pooling equilibrium[36]. Finally, notice that when the precision of the signal increases then at the limit (i.e. when σ approaches to zero) we get $\lim_{\sigma\to 0}\theta_s^* = \lim_{\sigma\to 0}\theta_s^{**}(\sigma) = \lim_{\sigma\to 0}x_s^*(\sigma) = cs$

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[34] For instance, in the Table 3.1 that is below notice that $\theta_s^* \leq \theta^\#$.

[35] In Appendix B3 we use a contagious argument to get $\theta_s^{**}(\sigma)$. Any policy $r > 0$ is dominated by the policy $r = 0$ if θ is "sufficiently high" (a type of policy maker $\theta \geq \theta_s^*$ is sufficiently high when the expected attack, after observing $r = 0$ and given the signal threshold $x_s^*(\sigma)$, is lower than cs and consequently is lower than θ) then the agents find iteratively dominant to choose $a = 0$ for "sufficiently high" x, conditional on observing $r = 0$. Then the dispersion of information initiates a contagion effect in which, conditional on seeing $r = 0$, the agents find iteratively dominant to choose $a = 0$ for lower and lower signals, then the status quo survives for lower and lower θ. In the limit, this contagion guarantees that all $\theta > \theta_s^{**}(\sigma)$ are able to avoid regime change without choosing $r > 0$, and they obtain a higher utility by choosing the policy $r = 0$ and facing an attack than by choosing the policy $r = s$ and facing no attack.

[36] That is, $\lim_{s\to\theta^\#/c}\theta_s^* = \lim_{s\to\theta^\#/c}\theta_s^{**}(\sigma) = \theta^\#$ and $\lim_{s\to\theta^\#/c}x_s^*(\sigma) = x^\#(\sigma)$

In the set of semiseparating equilibria we can check directly how the first three testable predictions of Angeletos and Pavan (2013) appear: (1) When θ is lower, the average size of the signals decreases, it implies that the probability that the agents receive signals $x \leq x_s^*(\sigma)$ increases, so the probability that an agent chooses $a = 1$ when she observes $r = 0$ increases, then the probability that the status quo collapses is higher; (2) the equilibrium strategy of the policy maker is to set $r = 0$ when $\theta \notin (\theta_s^*, \theta_s^{**}(\sigma)]$ and $r = s > 0$ for all $\theta \in (\theta_s^*, \theta_s^{**}(\sigma)]$, then the second prediction of the model is satisfied; (3) finally, when σ decreases (i.e. when the precision of the agents' information increases) we have that $\theta_s^{**}(\sigma)$ decreases and θ_s^* remains the same, so the range $(\theta_s^*, \theta_s^{**}(\sigma)]$ is smaller and consequently the probability that $r > 0$ (i.e. the probability of policy intervention) decreases.

3.3. Experimental Design

The experiment is based in the theoretical model commented in the previous section. The software used in the experiment was z-tree. 27 subjects were recruited from the UPF Leex Lab to participate in the experiment. There were three sessions and no subject appeared in more than one session. At the beginning of each session the participants were randomly divided in three groups of three people. In each group one member always was the policy maker and the other two always were the agents. The roles were randomly assigned at the beginning of each session and the participants belong to the same group during the 60 rounds of the experiment. Each round replicates the game explained in the previous section[37]. The only difference between the rounds and the sessions was the standard deviations σ as it appears in Table 3.2. That is, the standard deviation of the first half of rounds was different to the standard deviation during the second half of rounds. Additionally, in some sessions the standard deviations were lower at the first half of the experiment than in the second half, and in other sessions the opposite happens.

Table 3.2

Groups	Standard Deviation (σ)	
	Rounds 1 to 30	Rounds 31 to 60
1 to 3	10	15
4 to 6	15	10
7 to 9	15	20

At the beginning of each session, the subjects were seated at computer terminals and given a set of instructions, which were then read aloud by the experimenter. A copy of the instructions appears in Appendix C3. To ensure that subjects understood the game structure, some examples and some questions of understanding were administered at the

[37] That is, the timing of each round of the experiment is: (1) Policy maker stage: The policy maker learns his type θ and sets the policy r; (2) Agents stage: The two agents decide simultaneously and individually whether to attack the status quo after observing the policy r and the idiosyncratic private signals x_i. Consequently, at the end of this stage we can establish if the status quo survives or if there is a regime change; and (3) Payoff stage: The participants get their earnings depending on their particular choices and the kind of regime that stay at the end of the round.

end of the instructions. The subjects received a recruitment fee of 5€; additionally, six rounds were randomly selected at the end of the experiment to pay them the average utility they individually got in these rounds.

The types of policy makers θ and the signals x_i were randomly selected in each round and for each participant (or group) according to the considerations that appear in the theoretical model[38]. The calibration of the other parameters was: $y = 220$, $b = 70$, $K = 100$ and $c = 0.7$[39]. Therefore, the equilibrium thresholds of the sessions were:

- In the pooling equilibrium: $\theta^{\#} = 68.18$, $x^{\#}(10) = 72.91$, $x^{\#}(15) = 75.27$, $x^{\#}(20) = 77.64$.
- In the semiseparating equilibria: $s \in (71.43, 97.40]$, $\theta_s^* \in (50, 68.18]$, and depending on σ and θ_s^* , we have:
  - $\theta_s^{**}(10) \in (57.28, 68.18]$,$\theta_s^{**}(15) \in (60.92, 68.18]$, $\theta_s^{**}(20) \in (64.56, 68.18]$,
  - $x_s^*(10) \in (57.28, 72.91]$, $x_s^*(15) \in (60.92, 75.27]$, $x_s^*(20) \in (64.56, 77.64]$.

3.4. Results

This part of the chapter is divided in five sections in the following way. In section 3.4.1 we compare the strategies followed by the participants in the lab experiment with the equilibrium strategies predicted by the model. Given the differences that there are between the behavior of the subjects in the experiment and in the model, then in sections 3.4.2 to 3.4.4 we analyze if the predictions proposed by Angeletos and Pavan (2013) remain valid; more specifically, in each section using the values of θ and x_i used in the experiment, we first explain how these predictions appear when the participants behave as the theory says, then we analyze if the real behavior of the participants in the experiment also implies the same predictions. Finally, in section 3.4.5 we analyze how the choices of the agents in the experiment are affected but changes in the precision of the private signals.

3.4.1.The strategies taken by the participants in the experiment

According to the theoretical model, in the pooling equilibrium the policy maker always chooses the policy $r = 0$, and in the set of semiseparating equilibria she always chooses the policy $r = s > 0$ if $\theta_s \in (\theta_s^*, \theta_s^{**}(\sigma)]$ and the policy $r = 0$ otherwise. In Figure 3.1a, for all values of σ used in the experiment, we represent the pooling

[38] To be more precise, the idiosyncratic noise ϑ_i was randomly selected to each participant in each round such that this noise satisfies the properties specified in equation (3.3). Then, from the values of θ, σ and ϑ_i, each value of x_i was specified.

[39] Notice that $y - b - \frac{K}{c} > 0$ and $y > 2b$

equilibrium. On the other hand, in Figure 3.1b we represent the semiseparating equilibrium for the case in which $s = 71.44$ and $\sigma = 20$[40].

Figure 3.1a
Theoretical Model:
Pooling Equilibrium

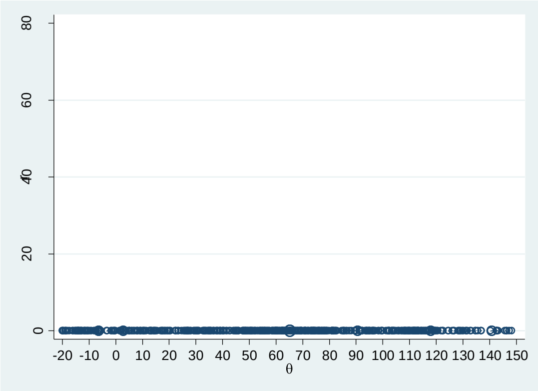
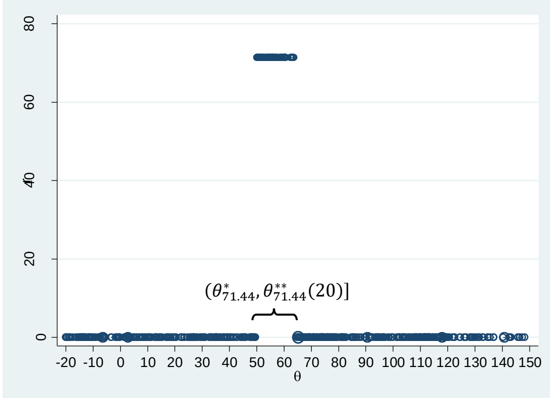


Figure 3.1b.
Theoretical Model:
Semiseparating Equilibrium.
Example: $r \in \{0.00, 71.44\}$ and $\sigma = 20$



In the figures of Appendix E3, given different types of policy makers θ and groups, you can see the policies chosen by the policy makers in the experiment, the size of the attacks and the values of θ in which (during the experiment) the status quo survived or was defeated. In these figures, the two vertical lines delimit the potential range of types of policy makers who at the set of semiseparating equilibria choose a policy $r > 0$[41]. Therefore, according to the model, at the equilibrium at least all policies at the right hand side and at the left hand side of this range must be $r = 0$. On the other hand, in the figures of Appendix E3, the two horizontal lines delimit the potential range that the policy $r = s > 0$ has in the set of semiseparating equilibria[42]. Therefore, according to the model, the equilibrium policies between the two vertical lines must be $r = 0$ (i.e. to be at the bottom of the rectangle B) or $r = s$ (i.e. to be inside the rectangle A)[43].

In Appendix D3 and in the Figures of Appendix E3 you can appreciate that one-third of the policy makers (i.e. the policy makers of the groups 1, 3 and 4) adopted, or almost adopted, a pooling equilibrium strategy in which $r = 0$ for all θ. In contrast, the policy makers of the other groups adopted strategies in which the policy r took many values; in general, these strategies were different to the semiseparating equilibrium strategies. However, in the groups 5, 6 and 9 the policy makers chose the policy $r = 0$ for all types $\theta \leq 50$[44]; that is, the policy makers of these groups, as it happens in the theoretical

[40] Remember, from section 3.3, that in our experiment a policy $r = s > 0$ can take any value in the interval $(71.43, 97.40]$. Therefore, for each value of s we can draw a graph like the one that appears in Figure 3.1b, where the main differences between the different graphs are: the level of $r = s$ and that the interval $(\theta^{*}_{s}, \theta^{**}_{s}(\sigma)]$ shrink and moves to the left until $(68.18, 68.18]$ while s approaches to 97.40.

[41] Using the information of section 3.3 you can see that in the experiment this range is $(50, 68.18]$

[42] Remember that in the experiment this range is $(71.43, 97.40]$

[43] To be more precise, the policies $r = s > 0$ must be located in a triangle inside the rectangle A. From the information of section 3.3 we know that the base of this triangle is $(\theta^{*}_{71.73}, \theta^{**}_{71.43}(\sigma)] = (50, \theta^{**}_{71.43}(\sigma)]$ and the range $(\theta^{*}_{s}, \theta^{**}_{s}(\sigma)]$ shrinks to $(68.18, 68.18]$ while s approaches to 97.40

[44] With a few exceptions in Group 9

model, chose a policy $r = 0$ when the agents did not need to coordinate an attack to defeat the status quo.

In the experiment, we found two interesting patterns of behavior in almost all cases in which the policy makers choose a policy $r > 0$. First, they chose policies $r > 0$ more times than it was predicted by theoretical model[45]. Second, they usually chose policies lower than the theoretical lower bound of s (i.e. policies lower than 71.43); so the policy makers generally did not choose enough credible policies to avoid attacks to the status quo. However, in the experiment there was a policy maker who at least in some rounds, given her chosen policies, could avoid attacks by the agents. More specifically, in the figures of Appendix E3 you can see that the policy maker of group 8 many times adopted policies $r = s$ higher than the policies adopted by the other policy makers; in particular, when $\sigma = 15$ she chose policies $r = s$ close to the policies predicted by the theoretical model (but in a different range of θ)[46]; therefore, these policies worked as policy thresholds to the agents as you can see in Appendix F3[47,48].

In the figures of Appendix F3 you can see how was the behavior of the agents in the experiment given the policy chosen by the policy maker and the private signal that they received (notice that in the figures above the symbols you also have the round in which the decision was taken). In addition, in each figure the shadow area and the dashed line correspond to the signal thresholds $x_s^*(\sigma)$ and $x^{\#}(\sigma)$ respectively, obtained from the theoretical model. From the figures of Appendix F3, we can appreciate that most of the time, the participants in the experiment adopted, or almost adopted, a signal threshold strategy in which they chose $a = 1$ if the signal was lower than a threshold and $a = 0$ otherwise. However, we can also appreciate that most of the time these thresholds seem to be higher than the thresholds obtained in the theoretical model. Therefore, in order to compare the signal thresholds obtained in the experiment with the thresholds of the theoretical model we will use two methodologies.

The first methodology is an adaptation of a methodology proposed by Szkup and Trevino (2015). The first step in this methodology consists in to find the individual signal thresholds per variance. An individual signal threshold is the average, per subject, between the highest value of the signal for which a participant chooses $a = 1$ and the lowest value of the signal for which she chooses $a = 0$[49]. This information is reported in the Figures 3.2a to 3.2c; notice that these figures also show the theoretic threshold $x^{\#}(\sigma)$[50]. The information of Figures 3.2a to 3.2c was classified depending if the policy makers played or not played pooling strategies and depending if the agent

[45] For instance, look at Table 3.5 in section 3.4.3

[46] In the questionnaire at the end of the experiment, this policy maker expressed that she chose $r = 0$ when she considered that θ was too low or too high.

[47] Look at the Figures of the agents 23 and 24 of group 8 when $\sigma = 15$

[48] But, this policy maker when $\sigma = 20$ reduced the size of the positive policies and consequently these stopped being credible.

[49] When we computed the individual signal thresholds we did a wise adjustment in which if the highest value of the signal for which a participant chooses $a = 1$ (or the lowest value of the signal for which she chooses $a = 0$) was an atypical choice taken in the first five periods of the experiment, then to find the individual threshold we took the second highest value of the signal for which a participant chooses $a = 1$ (or the second lowest value of the signal for which she chooses $a = 0$). This adjustment was done in ten cases (when $\sigma = 10$ to the subjects 5, 6, 12, 15 and 17; when $\sigma = 15$ to the subjects 5, 12, 23 and 24; and when $\sigma = 20$ to the subject 27)

[50] Do not forget that $x^{\#}(\sigma)$ is also equal to the upper bound of $x_s^*(\sigma)$

was following (or almost following) a signal threshold[51] (e.g. in Figure 3.2c we compile all cases in which the agents seem not following signal thresholds).

Figure 3.2a*

Individual Signal Thresholds when the Policy Makers Played Pooling Strategies and the Agents Played Signal Threshold Strategies

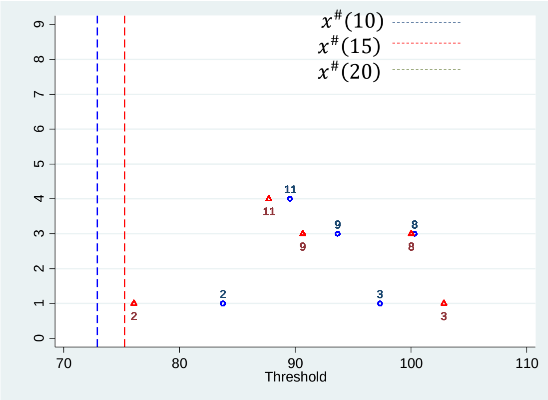


Figure 3.2b*

Individual Signal Thresholds when the Policy Makers DID NOT Played Pooling Strategies and the Agents Played Threshold Strategies

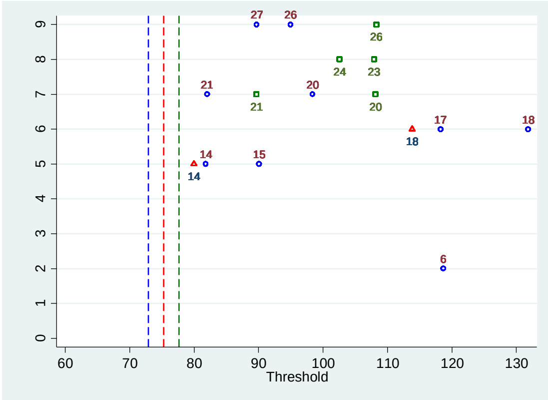
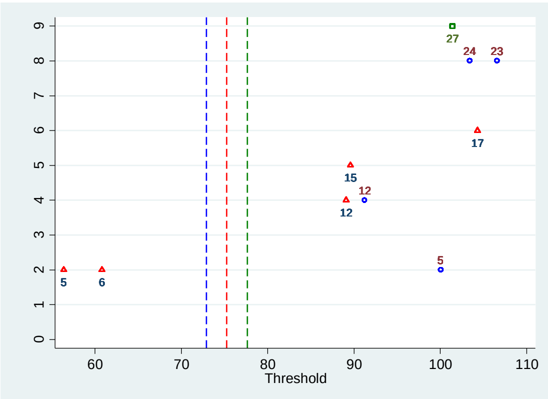


Figure 3.2c*

Individual Signal Thresholds when the Agents DID NOT Played Signal Threshold Strategies



* The labels in the figures correspond to the participants that had the role of agents in the experiment as they are referenced in Appendix F. The red triangles correspond to the individual signal thresholds when $\sigma = 10$, the blue dots correspond to individual signal thresholds when $\sigma = 15$ and the green squares correspond to the individual signal thresholds when $\sigma = 20$.

From Figures 3.2a and 3.2b notice that all individual signal thresholds were higher than the signal thresholds predicted by the theoretical model. In the second step of the first methodology we define the mean individual signal threshold per σ “MEST(σ)” as the average of all the individual signal thresholds per σ; in Table 3.3 you can see the value of this statistic.

The second methodology was proposed by Heinemann, Nagel and Ockenfels (2004), in this methodology we fit a logistic distribution to get the thresholds for the cases

[51] From the figures of Appendix F3 you can differentiate the cases in which the agents followed (or almost followed) signal thresholds strategies from the cases in which the agents did not followed signal thresholds strategies.

reported in the Figures 3.2a and 3.2b[52]. More specifically, Heinemann, Nagel and Ockenfels (2004) says that if the cumulative logistic distribution is given by

$$prob(a = 1) = \frac{1}{1 + e^{(\alpha - \beta x_i(\sigma))}}$$

where a is the action of the agent given the private signal $x_i(\sigma)$, then $-\frac{\alpha}{\beta}$ is the mean threshold and its standard deviation $-\frac{\pi}{\beta\sqrt{3}}$ is a measure of coordination that reflects variation within a group; in Table 3.3 you can see that the signal thresholds using this methodology are not far from the signal thresholds obtained using the other methodology.

Table 3.3[53]

Estimated Signal Thresholds and Equilibrium Signal Thresholds*

	$\boldsymbol{\sigma = 10}$	$\boldsymbol{\sigma = 15}$	$\boldsymbol{\sigma = 20}$	**All $\boldsymbol{\sigma}$**
Figure 3.2a:				
MEST	91.47	92.94	-	92.20
	(9.57)	(5.86)	-	(7.96)
Logit	89.80	91.88	-	90.74
	(9.43)	(6.38)	-	(8.19)
% of observations per σ	41.67%	27.78%	0.00%	27.78%
Figure 3.2b:				
MEST	96.91	100.62	103.30	100.99
	(17.08)	(16.95)	(7.18)	(14.74)
Logit	92.12	101.59	103.67	100.41
	(20.61)	(17.89)	(9.78)	(16.59)
% of observations per σ	16.67%	50.00%	83.33%	44.44%
$x_S^*(\sigma) \in (*, x^{\#}(\sigma)]$	$(57.28, 72.91]$	$(60.92, 75.27]$	$(64.56, 77.64]$	$(57.28, 77.64]$

* The standard deviations are reported in parenthesis

Finally, at the top of the Figures of Appendix E3 and in the Table G3.1 of Appendix G3, notice that most of the time the status quo only survives for values of θ that are higher than the values predicted by the theoretical model[54]. It happens because the agents in the experiment acted more aggressive than the agents in the theoretical model. In part, this aggressive behavior was because, as we saw at the beginning of this section, the positive policies chosen by the policy makers were not enough credible to help to the survival of the status quo.

Until now we have shown that the individual strategies of the participants in the experiment have some differences respect to the equilibrium strategies predicted by the

[52] That is, to fit the logistic distribution we use all the information that appears in Appendix F3 except (1) the cases reported in the Figure 3.2c and (2) the first four periods of each session of the experiment to avoid the atypical values commented previously in the footnote 49.

[53] In this table we did not include the information of Figure 3.2c because in these cases the agents are not playing signal threshold strategies. Therefore, in each column the sum of the percentage of observations per σ is always lower than 100%.

[54] Remember that in the theoretical model the status quo survives, given the equilibrium strategies played by the individuals, if $\theta < \theta_S^*$ (in the figures of Appendix E do not forget that θ_S^* is any of the types of policy makers delimited by the dashed vertical lines)

theoretical model and that these differences affect the kind of regime that remains at the end of each round. However, we have not analyzed if the predictions obtained by Angeletos and Pavan (2013) are enough strong to survive the differences that the strategies of the participants in the experiment have respect to the strategies of the theoretical agents. This will be our purpose in the following three sections[55].

3.4.2. First prediction: *when the type of the policy maker is low there is a higher probability of regime change*

As we explained in the experimental design, the types of policy makers and signals were randomly selected in all rounds and in all groups. Therefore, given the private signals and the policy chosen by the policy maker, the agents decide if they attack or not attack the status quo. In Table 3.4, for the different types of policy makers and signals used in the experiment, we compare the final regimes that we got in the experiment with the final regimes that we would obtain if all individuals behave as the theory says. Since in the theoretical model there are multiple equilibria, then the pair of values that appear in the second and third columns of Table 3.4 correspond to the cases in which θ_S^* is equal to its lower bound and its upper bound values respectively (i.e. when $\theta_{71.43}^* = 50$ and $\theta_{97.40}^* = \theta^{\#} = 68.18$ respectively).

Table 3.4
Percentage of times the status quo survives (and percentage of times there is a regime change) given the values of θ used in the experiment

$\boldsymbol{\theta}$	**Theory***		**Experiment**	
	Status quo survives	**There is a Regime Change**	**Status quo survives**	**There is a Regime Change**
$(-\infty, 0]$	(0.00% - 0.00%)	(100.00% - 100.00%)	0.00%	100.00%
$(0, 20]$	(0.00% - 0.00%)	(100.00% - 100.00%)	1.89%	98.11%
$(20, 40]$	(0.00% - 0.00%)	(100.00% - 100.00%)	1.79%	98.21%
$(40, 60]$	(58.11% - 9.46%)	(41.89% - 90.54%)	0.00%	100.00%
$\mathbf{(60, 80]}$	**(92.92% - 56.64%)**	**(7.08% - 43.36%)**	**17.70%**	**82.30%**
$(80, 100]$	(100.00% - 91.38%)	(0.00% - 8.62%)	65.52%	34.48%
$(100, +\infty)$	(100.00% - 100.00%)	(0.00% - 0.00%)	100.00%	0.00%
$\boldsymbol{Total}$	**(60.93% - 45.74%)**	**(39.07% - 54.23%)**	**33.89%**	**66.11%**

*In these two columns we are taking into account that in the experiment $\theta_S^* \in (50, 68.18]$; therefore, inside the parenthesis we are showing two examples: (1) when $\theta_S^* = 50$ and (2) when $\theta_S^* = \theta^{\#} = 68.18$

For instance, in the cases in which the type of policy maker $\theta \in (60, 80]$[56] the theory predicts that if $\theta_S^* = 50$, then 92.92% of the times the status quo survives (i.e. 7.08% of the times there is a regime change) but if $\theta_S^* = \theta^{\#} = 68.18$, then 56.64% of the times the status quo survives (i.e. 43.36% of the times there is a regime change). Nevertheless, in the experiment (i.e. the fourth and fifth columns of Table 3.4) when the

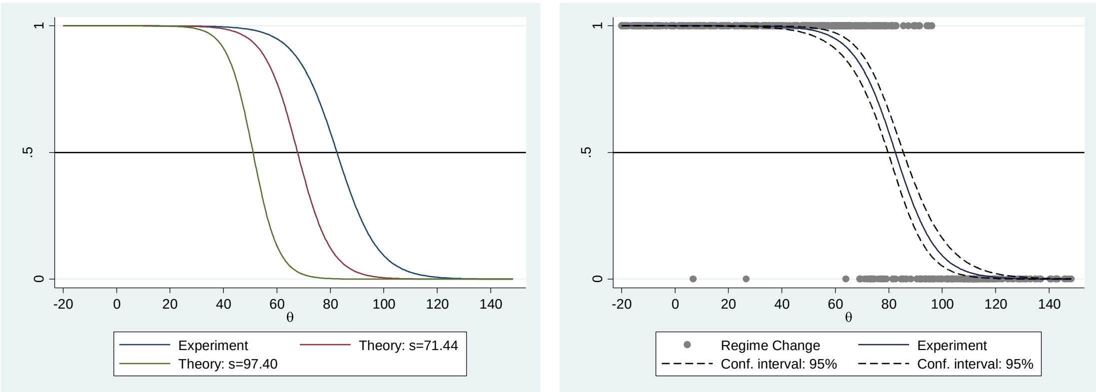
[55] But do not forget that these predictions, as was commented by Angeletos and Pavan (2013), happen with some probability. It means that even when the subjects play equilibrium strategies the predictions may not happen perfectly.

[56] And given the choices of the theoretical agents who are using the private signals of the experiment

type of policy maker $\theta \in (60, 80]$ we got that 82.30% of the times the status quo collapses and in only 17.70% of the times the status quo remains. This result is not strange because, as we explained in section 3.4.1, the participants in the lab experiment behave more aggressive than the agents in the theoretical model.

If we analyze only the punctual values of each row of Table 3.4, then we appreciate significant differences between the results obtained in the experiment and the results when the individuals behave optimally. However, if we analyze the Table 3.4 as a whole, we can appreciate that we got in the experiment almost the same prediction obtained by Angeletos and Pavan (2013): when θ decreases the probability of survival of the status quo also decreases[57]. This conclusion is also in concordance with the predicted logit probabilities represented in Figure 3.3. In this figure notice that the probability of regime change is always lower in the theory than in the experiment, but all curves show the same pattern: a lower θ implies a lower probability of the survival of the status quo.

Figure 3.3[58]
Probabilities of regime change*



*A gray dot equal to 1 means that there is a regime change and a gray dot equal to 0 means that the status quo survives. These dots are the same dots reported at the top of the last figure of Appendix E3.

3.4.3. Second prediction: *the policy chosen by the policy maker is greater than zero when the type of policy maker is neither too high nor too low*

In our experiment, for each value of $r = s \in (71.43, 97.40]$ and $\sigma \in \{10,15,20\}$, we can specify the range at which the policy makers, according to the set of semiseparating equilibria, choose the policy $r = s$ (i.e. we can specify the range $(\theta_S^*, \theta_S^{**}(\sigma)]$. On the other hand, in the pooling equilibrium the policy maker always chooses $r = 0$.

In the second and third columns of Table 3.5 we show, given the values of θ used in the experiment, the percentage of times the theoretical model predicts that the policy

[57] In the Table G3.2 of Appendix G3 you can see that the same conclusion is obtained if we use a more disaggregated classification of θ

[58] The information used to do this figures can be observed at the top of the figures in Appendix E3 (notice that in all these figures, you can also appreciate directly that the probability of the survival of the status quo is lower when the type of policy maker is low)

makers choose a policy $r = 0$ and a policy $r = s \in (71.43, 97.40]$. For instance, if $\theta \in (30\,,40]$, then the theoretical model says that the policy maker always chooses the policy $r = 0$. However, if $\theta \in (50\,,60]$ or if $\theta \in (60\,,70]$, in the set of semiseparating equilibria we expect that some policy makers choose a policy $r \in (71.43, 97.40]$. Therefore, in the second and third columns of Table 3.5 we observe that when $\theta \in (50\,,60]$ or when $\theta \in (60\,,70]$ then on average the percentage of times a theoretical policy maker chooses a policy $r = 0$ is lower or equal than 100% and on average the percentage of times a theoretical policy maker chooses a policy $r \in (71.43, 97.40]$ is higher or equal than 0%. Therefore, in this table (and also in the example used in Figure 1b) you can appreciate that, according to the theoretical model, a positive policy r is chosen when the type of policy maker is neither too high nor too low. That is, if the agents behave as the theory says then the second prediction of Angeletos and Pavan (2013) works.

Table 3.5

Percentage of choices $r > 0$ and $r = 0$ given the θ values used in the experiment

$\boldsymbol{\theta}$	**Theory**[59]		**Experiment**	
	$\boldsymbol{r = 0}$	$\boldsymbol{r \in (71.43, 97.40]}$	$\boldsymbol{r = 0}$	$\boldsymbol{r > 0}$
$(-\infty\,,0]$	100.00%	0.00%	84.13%	15.87%
$(0\,,10]$	100.00%	0.00%	80.00%	20.00%
$(10\,,20]$	100.00%	0.00%	64.29%	35.71%
$(20\,,30]$	100.00%	0.00%	87.50%	12.50%
$(30\,,40]$	100.00%	0.00%	75.00%	25.00%
$(40\,,50]$	100.00%	0.00%	80.00%	20.00%
$\boldsymbol{(50\,,60]}$	**≤100.00%**	**≥0.00%**	75.00%	25.00%
$\boldsymbol{(60\,,70]}$	**≤100.00%**	**≥0.00%**	47.62%	52.38%
$(70\,,80]$	100.00%	0.00%	56.00%	44.00%
$(80\,,90]$	100.00%	0.00%	54.17%	45.83%
$(90\,,100]$	100.00%	0.00%	38.24%	61.76%
$(100, +\infty)$	100.00%	0.00%	60.16%	39.84%
$\boldsymbol{Total}$	**≤100.00%**	**≥0.00%**	**71.85%**	**28.15%**

On the other hand, from the last two columns of Table 3.5 we cannot appreciate clearly the same prediction obtained by Angeletos and Pavan (2013). However, in the Figures of Appendix E3, leaving aside the groups in which the policy makers were following (or almost following) the pooling equilibrium strategy (i.e. the figures of the groups 1, 3 and 4), you can appreciate that 33% of the other policy makers followed the policy $r > 0$ when the type of policy maker was neither too high nor too low. To be more precise, most of the time when the type of policy maker was too low, the participants chose a policy $r = 0$[60]; however, when the type of policy maker was too high we observe that 33% of the policy makers, who did not follow the pooling

[59] Remember that in the experiment the lowest value of θ_s^* is $\theta_{71.43}^*$ =50 and the highest value of $\theta_s^{**}(\sigma)$ is $\theta_{97.40}^{**}(\sigma)$ =68.18

[60] The only policy maker that did not follow this premise was the policy maker of Group 2.

equilibrium strategy, establishes a policy $r = 0$[61]. To summarize, the second theoretical prediction proposed by Angeletos and Pavan (1993) was only partially observed in the experiment.

3.4.4. Third prediction: *The probability of a policy greater than zero decreases when σ is lower.*

In the semiseparating equilibria this prediction happens because, given θ_S^*, then the distance $\theta_S^{**}(\sigma) - \theta_S^* = \sigma\left[\Psi^{-1}\left(1 - \frac{\theta_S^*}{K}\left(\frac{b}{y-b}\right)\right) - \Psi^{-1}\left(\frac{\theta_S^*}{K}\right)\right]$ decreases when σ is lower[62]. On the other hand, if the subjects always play the pooling equilibrium strategies, then the policy makers choose the policy $r = 0$ irrespective of the size of σ.

In the experiment the values of θ in all rounds and in all groups were randomly selected; therefore, using these values and assuming that the policy makers follow the semiseparating equilibrium strategy predicted by the theoretical model, then we may obtain that in some groups the number of rounds in which the policy maker is choosing a positive policy is higher when σ is lower (however, on average we expect the opposite result). For instance, assume that all policy makers in the experiment are following the semiseparating equilibrium at the lower bound of θ_S^* (i.e. $\theta_{71.43}^* = 50$); the red diamonds in Figure 3.4 represent these policy makers. Notice in this figure that the policy makers of the groups 4 and 5 chose more times positive policies in the rounds in which σ was lower and in the other seven groups the opposite happens. That is, if the policy makers behave as the theory says then, on average, we have the third prediction proposed by Angeletos and Pavan (2013).

In the previous sections we have shown that, in our experiment, only in some rounds the policy makers chose policies equal to zero when the type of policy maker was too high[63]; it means, that the variable $\theta_S^{**}(\sigma)$ (and consequently, the range $(\theta_S^*, \theta_S^{**}(\sigma)]$) was not clearly established in the lab sessions. Therefore, the argumentation that we used above, to explain how the size of σ affects the probability of a positive policy, does not work here. However, it does not mean that the precision of the signals does not affect the policies chosen by the policy makers. The blue dots in Figure 3.3 represent the percentage of times the policy makers of each group choose a positive policy for each σ[64]. We ran some regressions to analyze if the size of σ affects the size of the policy r or affects the probability of choosing a policy higher than zero, but we did not find a robust statistical effect (look at for example the Table G3.3 in Appendix G3)[65]. It

[61] Those are the policy makers of the groups 5 and 6. Since the policy makers of the groups 1, 3 and 4 chosen the pooling equilibrium strategy, then the policy makers of the groups 2, 7, 8 and 9 did not choose the policy $r = 0$ when the type θ was too high.

[62] Remember that the equilibrium strategy of the policy maker in the set of semiseparating equilibria is to choose the policy $r = s$ inside the range $(\theta_S^*, \theta_S^{**}(\sigma)]$ and $r = 0$ otherwise

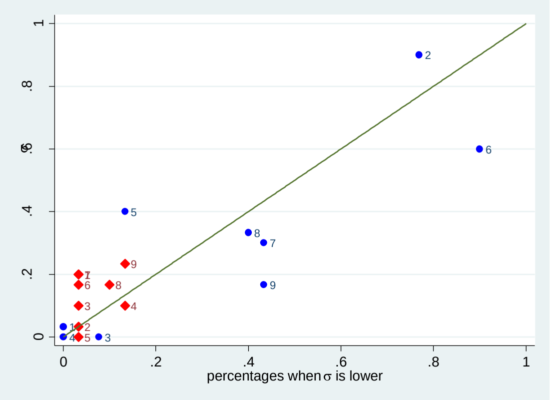
[63] However, do not forget that as it happens in the theoretical model, most of the time the policy makers also chose policies equal to zero when the type of policy maker was too low.

[64] From Table 3.1, remember that in the experiment the first half of the rounds in each session had a different σ respect to the second half of the rounds.

[65] However, we found that a higher type of policy maker has a slightly higher probability of choosing a policy greater than zero.

means that in the experiment we have not validated the prediction formulated by Angeletos and Pavan (2013).

Figure 3.4
Percentage of times the policy makers
of each group choose policies $r > 0$ *[66]



*Theory: Red diamonds (Case: $\theta_S^* = \theta_{71.43}^*$). Experiment: Blue dots. The labels in the figure correspond to the group of the policy maker. The solid line is a 45 degrees line.

3.4.5.A new prediction: *When the agents apply trigger strategies, the probability that an agent attacks the status quo increases when σ is higher.*

In section 3.2 we have seen that an increase in σ affects positively the size of the thresholds $x_S^*(\sigma)$ and $x^{\#}(\sigma)$. When a threshold increase, then the probability of getting a private signal lower than the threshold increases and consequently increases the probability of the agents choosing $a = 1$ when $r = 0$. In section, 3.4.1 we have seen that many agents in our lab experiment played individual thresholds strategies. Then, we have run a logit regression to analyze in our lab experiment how the changes in the size of σ affect the probability of attacking the status quo (look at Table G3.4 in Appendix G3). Remember that in each session of the experiment the first half of rounds has a different value of σ respect to the last half of rounds, then the variable Dummy of Order is equal to 1 in the first half of rounds and zero otherwise. In addition, in the regression we have also included one dummy variable per σ, except when $\sigma = 15$ (we excluded this variable, because from Table 3.2 we know that all individuals in the experiment interacted when $\sigma = 15$ but not when $\sigma = 10$ or $\sigma = 20$).

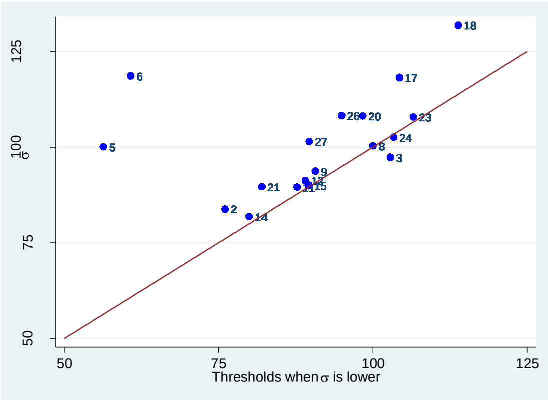
In Table G3.4 we can see that the probability of the agents choosing $a = 1$ decreases when the signal and the policy are higher[67]. On the other hand, the probability

[66] In this figure we are not considering the first four periods of each session to avoid the atypical values commented previously in the footnote 49. However, if we consider these values, the conclusions of this section do not change.

[67] It is clear that a higher signal has a higher probability of been higher than the signal threshold and consequently is higher the probability that the agent who receives the signal chooses $a = 0$. On the other

of choosing $a = 1$ is lower when $\sigma = 10$ than when $\sigma = 15$, and is also lower when $\sigma = 15$ than when $\sigma = 20$. That is, the probability that any agent chooses $a = 1$ increases with σ. This conclusion is in concordance with the information of Figure 3.5 (notice that this figure is another way to represent the information of the Figures 3.2a-3.2c[68] or the information of Table 3.3).

Figure 3.5
Individual signal thresholds
of each agent in the experiment[+]



[+] The label in each dot corresponds to a different agent (look at Appendix F3). The solid line is a 45 degrees line.

3.5. Conclusions

In our research we have found some discrepancies between the behavior of the participants in the lab experiment and the equilibrium strategies predicted by the model. This result is not strange because the model combines a global coordination game with a signaling game. Therefore, the equilibrium strategies of the model require the elimination of dominated strategies (from below and from above), an adequate system of beliefs and that the player's strategies are sequentially rational. Previous papers have shown that these kinds of requirements are not easy to obtain in a lab experiment[69].

Notice that the last requirement was continuously violated by the policy makers in the experiment because their choices were not affected by the precision of the signals[70],

hand, when the policy implemented by the policy maker is higher, then the agents have less incentive to choose $a = 1$ because the choice of this action implies a cost affected positively by r.

[68] Remember that the agents 20 to 27 are the only ones that interacted with $\sigma = 15$ and $\sigma = 20$. The others interacted with $\sigma = 10$ and $\sigma = 15$

[69] For instance, Cabrales, Nagel and Armenter (2007) and Heinemann, Nagel, and Ockenfels (2007) have found that a complete elimination of dominated strategies is not a behavior easy to obtain even in experiments simpler than ours. Similarly, Kübler, Müller and Normann (2008), in a lab experiment that uses as benchmark a job-market signaling model, found some discrepancies in the strategies followed by the participants in the experiment respect to the equilibrium strategies predicted by the model, they argue that it happens because the participants in the experiment did not adopt the same equilibrium system of beliefs obtained in the model.

[70] So, they did not take into account that changes in the precision of the signals affect the equilibrium strategy of the agents. In particular, remember that in the model σ affects positively the value of the signal thresholds of the agents.

and because when they implemented positive policies, these were not high enough to discourage the agents to attack the status quo. In addition, the policy makers did not apply properly the process of elimination of dominated strategies, so they could not correctly identify the range of types of policy makers for which was optimal to apply positive policies. Therefore, it seems that most of the time the policies chosen by the policy makers in the experiment were not revealing reliable information about the variable θ to the agents. Respect to the behavior of the agents, we got that usually they applied signal threshold strategies; however, they attacked the status quo more often than the theory says, even in the groups in which the policy makers followed pooling equilibrium strategies, as you could see in the Figures 3.2a to 3.2c.

These divergences in the behavior of the participants in the experiment respect to the equilibrium strategies gave us the opportunity to test, using the data of the lab experiment, how much strong were the predictions found by Angeletos and Pavan (2013). That is, in the chapter we tested if their predictions remained valid even when the participants in the experiment did not play exactly the same equilibrium strategies predicted by the model. The prediction that better behaves in the experiment was that the low types of policy makers have a higher probability of regime change. This result is in concordance with the facts that (1) any policy (or measure, or rule) has a higher probability of being modified when the policy maker (or any other political or socioeconomic agent in charge of the policy) is less able or less willing to defend the policy, and (2) the speculators (or any other agents that can get benefits if the policy is defeated) have higher incentives of attacking a policy when the policy maker in charge of the policy is less able or less willing to defend the policy.

On the other hand, in the experiment, in concordance with the results obtained in the model, we got that the probability that the agents attack the status quo decreases when the precision of the private signals is higher. In the model this result happens because when the policy maker chooses the policy $r = 0$ then the earnings that the agents can get by choosing $a = 1$ are sufficiently high respect to the losses of choosing $a = 1$[71], such that the agents include a positive risk premium to the signal threshold that increases when the precision of the signal decreases[72]. In the experiment, since the information that the agents could obtained about θ from the policy chosen by the policy makers seems unreliable or uninformative, then agents applied almost always trigger strategies, and we can use the same argument commented previously in the theoretical model to justify the increment on the attacks to the status quo when the precision of the private signals increases.

[71] Remember that we have assumed $y > 2b$.

[72] Remember that in the model the signal thresholds are $x^{\#}(\sigma) = \sigma\Psi^{-1}\left(\frac{y-b}{y}\right) + \theta^{\#}$ and $x_s^*(\sigma) = \sigma\Psi^{-1}\left(1 - \frac{\theta_s^*}{K}\left(\frac{b}{y-b}\right)\right) + \theta_s^*$. Then the risk premium of the signal threshold in the pooling equilibrium is represented by $\sigma\Psi^{-1}\left(\frac{y-b}{y}\right)$ and in the semiseparating equilibria is represented by $\sigma\Psi^{-1}\left(1 - \frac{\theta_s^*}{K}\left(\frac{b}{y-b}\right)\right)$.

Appendix A3: The pooling equilibrium

In this section we will show the propositions and elements that are more relevant to explain the characteristics of the pooling equilibrium of the model. This appendix is based entirely in Angeletos and Pavan (2013). For that reason, if you want to read a more complete explanation of the pooling equilibrium you can read directly their paper. In their paper they use functions that are more general than the functions that we are using; therefore, at some points it can be difficult to visualize their arguments. On the other hand, since we are using specific functions, in our approach we arrive to the close form solutions that we use to analyze the results of the lab experiment.

At the pooling equilibrium the policy makers of all types always choose the same policy, so this policy is uninformative about the type of policy maker θ. Therefore, the agents do not use the information of the policy r to updates their beliefs about the type of policy maker. It implies that in the first stage of the game, the policy maker infers that all positive policies are strictly dominated by the policy $r = 0$, and consequently she always chooses the policy $r = 0$ at the equilibrium. In the second stage the agents play a coordination game in which, when they observe $r = 0$, they use a symmetric trigger strategy that has as threshold the unique signal $x^{\#}(\sigma)$. It implies that all individuals may infer that a regime change happens if and only if the type of policy maker is $\theta \leq \theta^{\#}$ where $\theta^{\#}$ is the unique threshold type of policy maker.

In propositions A1 and A2 we obtain the exact form of the thresholds $x^{\#}(\sigma)$ and $\theta^{\#}$. In proposition A3 we show that both thresholds are unique. In proposition A4 we show that the pooling equilibrium exists and is unique. In particular, we show that that there exists a strategy profile for the agents and the policy maker, and a system of supporting beliefs for the agents that sustain this equilibrium

Before explaining the propositions we have to introduce a few concepts: Let $\mu(\theta^{\#}|x)$ denotes the posterior probability of regime change for an agent who receives a signal x, who considers that the policy r is uninformative about θ, and who believes that there is regime change if and only if $\theta \leq \theta^{\#}$; then $\mu(\theta^{\#}|x) = 1 - \Psi\left(\frac{x-\theta^{\#}}{\sigma}\right)$[73]. In the model, the policy maker is the only individual who knows θ, so she knows that there is a regime change when her type is lower or equal than $\theta^{\#} = K\Psi\left(\frac{x^{\#}(\sigma)-\theta^{\#}}{\sigma}\right)$ where $K\Psi\left(\frac{x^{\#}(\sigma)-\theta}{\sigma}\right)$ is the aggregate size of the attack that happens when the policy maker's type is θ and the agents attack if and only if $x < x^{\#}(\sigma)$[74].

---

[73] From equation (3.3) remember that $x_i = \theta + \sigma\vartheta_i$ where $\vartheta_i \sim N(0,1)$ is an idiosyncratic noise which is independently and identically distributed across agents and independent of θ, with absolutely continuous p.d.f ψ and c.d.f Ψ. It implies that $\theta = x_i - \sigma\vartheta_i$, then:

- $E(\theta|x_i) = E(x_i|x_i) - \sigma E(\vartheta_i|x_i) = x_i$
- $Var(\theta|x_i) = E((x_i - \sigma\vartheta_i)^2|x_i) - [E(\theta|x_i)]^2 = E(x_i^2 - 2x_i\sigma\vartheta_i + \sigma^2\vartheta_i^2|x_i) - x_i^2 =$
  $E(x_i^2|x_i) - 2\sigma E(x_i\vartheta_i|x_i) + \sigma^2 E(\vartheta_i^2|x_i) - x_i^2 = x_i^2 + \sigma^2 - x_i^2 = \sigma^2$

That is, when the agent i receives the signal x_i, she updates her beliefs about θ such that $(\theta|x_i) \sim N(x_i, \sigma^2)$. So, the posterior probability that the agent i, after observing the signal x_i and ignoring the policy r, gives to the type of policy maker $\theta \leq \theta^{\#}$ is: $\mu(\theta^{\#}|x) = Prob(\theta \leq \theta^{\#}|x_i) = \Psi\left(\frac{\theta^{\#}-E(\theta|x_i)}{SD(\theta|x_i)}\right)$ $= \Psi\left(\frac{\theta^{\#}-x_i}{\sigma}\right) = 1 - \Psi\left(\frac{x_i-\theta^{\#}}{\sigma}\right)$. Notice that she uses a Bayesian updating process.

[74] Using the previous footnote, we can easily infer that:

Proposition A1: *Let $x^{\#}(\sigma)$ be any signal x at which, given σ, an agent who believes that a regime change happens if and only if $\theta \leq \theta^{\#}$, is indifferent between choosing $a = 1$ and $a = 0$ when she observes $r = 0$. So, $x^{\#}(\sigma) = \sigma\Psi^{-1}\left(\frac{y-b}{y}\right) + \theta^{\#}$.*

Proof: By definition, $x^{\#}(\sigma)$ is determined from $\left[1 - \Psi\left(\frac{x^{\#}(\sigma)-\theta^{\#}}{\sigma}\right)\right](y-b) - \Psi\left(\frac{x^{\#}(\sigma)-\theta^{\#}}{\sigma}\right)b = 0$[75]. The LHS of this equation is the posterior expected utility of the agent from choosing $a = 1$ and the RHS is the expected utility of the agent from choosing $a = 0$. Then we can obtain directly that $x^{\#}(\sigma) = \sigma\Psi^{-1}\left(\frac{y-b}{y}\right) + \theta^{\#}$.[76] □

Proposition A2: *Let $\theta^{\#}$ denotes a threshold such that, in any such equilibrium, there is regime change if and only $\theta \leq \theta^{\#}$ (i.e. $\theta^{\#}$ is the solution to $\theta^{\#} = K\Psi\left(\frac{x^{\#}(\sigma)-\theta^{\#}}{\sigma}\right)$[77]). Then, $\theta^{\#} = K\left(\frac{y-b}{y}\right)$.*

Proof: If we replace in $\theta^{\#} = K\Psi\left(\frac{x^{\#}(\sigma)-\theta^{\#}}{\sigma}\right)$ the value of $x^{\#}(\sigma)$ obtained in Proposition A1, we get $\theta^{\#} = K\Psi\left(\frac{\sigma\Psi^{-1}\left(\frac{y-b}{y}\right)+\theta^{\#}-\theta^{\#}}{\sigma}\right) = K\left(\frac{y-b}{y}\right)$ □

Proposition A3: *$x^{\#}(\sigma)$ and $\theta^{\#}$ are unique values.*

Proof: Remember that, $\theta^{\#} = K\left(\frac{y-b}{y}\right)$ and $x^{\#}(\sigma) = \sigma\Psi^{-1}\left(\frac{y-b}{y}\right) + \theta^{\#}$. It implies that $x^{\#}(\sigma) = \sigma\Psi^{-1}\left(\frac{y-b}{y}\right) + K\left(\frac{y-b}{y}\right)$. Therefore, the uniqueness of $x^{\#}(\sigma)$ and $\theta^{\#}$ are directly obtained because $\Psi(\cdot)$ is a normal c.d.f and K, b, y, and σ are predetermined parameters □

- $E(x_i|\theta) = E(\theta|\theta) + \sigma E(\vartheta_i|\theta) = \theta$ and
- $Var(x_i|\theta) = E((\theta + \sigma\vartheta_i)^2|\theta) - [E(x_i|\theta)]^2 = E(\theta^2 + 2\theta\sigma\vartheta_i + \sigma^2\vartheta_i^2|\theta) - \theta^2 = E(\theta^2|\theta) + 2\sigma E(\theta\vartheta_i|\theta) + \sigma^2 E(\vartheta_i^2|\theta) - \theta^2 = \theta^2 + \sigma^2 - \theta^2 = \sigma^2$

That is, when a policy maker observes that her type is θ, she updates her beliefs about x_i such that $(x_i|\theta) \sim N(\theta, \sigma^2)$. Since all individuals know that the trigger equilibrium strategy of each agent i is to choose $a_i = 1$ if $x_i < x^{\#}(\sigma)$; thus the posterior probability that the policy maker θ, after observing her type, gives to $a_i = 1$ is: $Prob(x_i < x^{\#}(\sigma)|\theta) = \Psi\left(\frac{x^{\#}(\sigma)-E(x_i|\theta)}{SD(x_i|\theta)}\right) = \Psi\left(\frac{x^{\#}(\sigma)-\theta}{\sigma}\right)$. Therefore, the size of the attack is $K\sum_{i=1}^{2}\frac{a_i Prob\left(x_i<x^{\#}(\sigma)\right)}{2} = K\sum_{i=1}^{2}\frac{\Psi\left(\frac{x^{\#}(\sigma)-\theta}{\sigma}\right)}{2} = K\Psi\left(\frac{x^{\#}(\sigma)-\theta}{\sigma}\right)$. And consequently, she knows that there is a regime change if and only if $\theta \leq \theta^{\#} = K\Psi\left(\frac{x^{\#}(\sigma)-\theta^{\#}}{\sigma}\right)$.

[75] That is, $\mu\left(\theta^{\#}|x^{\#}(\sigma)\right)(y-b) - \left[1 - \mu\left(\theta^{\#}|x^{\#}(\sigma)\right)\right]b = 0$

[76] In the next proposition we will show that $\theta^{\#}$ exists. Therefore, notice that $x^{\#}(\sigma)$ exists because we have assumed $y > b > 0$

[77] Since the expected size of the attack is $K\Psi\left(\frac{x^{\#}(\sigma)-\theta}{\sigma}\right)$. Then the definition of $\theta^{\#}$ implies that $\theta^{\#} = K\Psi\left(\frac{x^{\#}(\sigma)-\theta^{\#}}{\sigma}\right)$.

Proposition A4: *For any σ, our pooling equilibrium exist and is unique.*

Proof: Assume any σ. Then, to get the equilibrium policy of the policy maker we have to establish a strategy profile for the agents and a system of supporting beliefs such that no type of the policy maker finds it optimal to choose a policy $r = s > 0$. Consider the following

$$\text{strategy profile for the agents}: \begin{cases} \text{for } r = 0 \begin{cases} a = 1 \text{ if and only if } x < x^{\#}(\sigma) \\ a = 0 \text{ if and only if } x > x^{\#}(\sigma) \end{cases} \\ \text{for any } r \in (0, \frac{K}{c}], a = 1 \text{ irrespective of } x \\ \text{for any } r > \frac{K}{c}, a = 0 \text{ irrespective of } x \end{cases}.$$

where $x^{\#}(\sigma)$ is the unique threshold obtained in the previous propositions. Now we have to show that, given this profile, for any policy maker of type θ the best policy for her is to choose the policy $r = 0$ (and receive an attack of size $K\Psi\left(\frac{x^{\#}(\sigma)-\theta}{\sigma}\right) \in [0, K]$) than to choose any policy $r \in (0, \frac{K}{c}]$ (and receive an attack of size K) or any policy $r > \frac{K}{c}$ (and receive an attack of size 0)[78]:

a. <u>If $\theta \leq 0$:</u> For these types of policy makers, irrespective of the policy r and irrespective of the size of the attack, there is always a regime change and consequently the utility of the policy maker is always $L(r) = -cr$. Therefore, any policy $r = s > 0$ is strictly dominated by the policy $r = 0$ because we always have $U(\theta, s, *) = L(s) = -cs < 0 = L(0) = U(\theta, 0, *)$.

b. <u>If $\theta \in (0, K]$:</u> For these types of policy makers we will check separately what happen when $r \in (0, \frac{K}{c}]$ and $r > \frac{K}{c}$:
   - Given the strategy profile of the agents, a policy $r \in (0, \frac{K}{c}] > 0$ means that all agents choose $a(x, r) = 1$, the size of the attack is K and there is always a regime change, so the utility of the policy maker is always $U(\theta, r, K) = L(r)$. The policy $r \in (0, \frac{K}{c}] > 0$ is strictly dominated by the policy $r = 0$ because we always have one of the following two situations:
      i. If the policy $r = 0$ implies a regime change (i.e. $\theta \leq K\Psi\left(\frac{x^{\#}(\sigma)-\theta}{\sigma}\right)$), then: $L(r) = -cr < 0 = L(0) = U\left(\theta, 0, K\Psi\left(\frac{x^{\#}(\sigma)-\theta}{\sigma}\right)\right)$.
      ii. If the policy $r = 0$ implies that the status quo survives (i.e. $\theta > K\Psi\left(\frac{x^{\#}(\sigma)-\theta}{\sigma}\right)$), then: $L(r) = -cr < 0 < \theta - K\Psi\left(\frac{x^{\#}(\sigma)-\theta}{\sigma}\right) =$

[78] That is, we will show that for any value of θ we always have $U(\theta, r, K) < U\left(\theta, 0, K\Psi\left(\frac{x^{\#}(\sigma)-\theta}{\sigma}\right)\right)$ for the case in which $r \in (0, \frac{K}{c}]$ and $U(\theta, r, 0) < U\left(\theta, 0, K\Psi\left(\frac{x^{\#}(\sigma)-\theta}{\sigma}\right)\right)$ for the case in which $r > \frac{K}{c}$

$$W\left(\theta,0,K\Psi\left(\frac{x^{\#}(\sigma)-\theta}{\sigma}\right)\right)=U\left(\theta,0,K\Psi\left(\frac{x^{\#}(\sigma)-\theta}{\sigma}\right)\right)$$

To summarize, $U(\theta,r,K)<U\left(\theta,0,K\Psi\left(\frac{x^{\#}(\sigma)-\theta}{\sigma}\right)\right)$ when $r\in(0,\frac{K}{c}]$ and $\theta\in(0,K]$

- Given the strategy profile of the agents, a policy $r>\frac{K}{c}$ means that all agents choose $a=0$, the size of the attack is 0 and the status quo always survives, so the utility of the policy maker is always $U(\theta,r,K)=W(\theta,r,0)$. The policy $r>\frac{K}{c}$ is strictly dominated by the policy $r=0$ because we always have one of the two following situations:
  i. If the policy $r=0$ implies a regime change (i.e. $\theta\le K\Psi\left(\frac{x^{\#}(\sigma)-\theta}{\sigma}\right)$), then: $W(\theta,r,0)=\theta-cr<\theta-K\le 0=L(0)=U\left(\theta,0,K\Psi\left(\frac{x^{\#}(\sigma)-\theta}{\sigma}\right)\right)$[79]
  ii. If the policy $r=0$ implies that the status quo survives (i.e. $\theta>K\Psi\left(\frac{x^{\#}(\sigma)-\theta}{\sigma}\right)$), then: $W(\theta,r,0)=\theta-cr<\theta-K\le\theta-K\Psi\left(\frac{x^{\#}(\sigma)-\theta}{\sigma}\right)=W\left(\theta,0,K\Psi\left(\frac{x^{\#}(\sigma)-\theta}{\sigma}\right)\right)=U\left(\theta,0,K\Psi\left(\frac{x^{\#}(\sigma)-\theta}{\sigma}\right)\right)$[80]

  To summarize, $U(\theta,r,K)<U\left(\theta,0,K\Psi\left(\frac{x^{\#}(\sigma)-\theta}{\sigma}\right)\right)$ when $r\in(0,\frac{K}{c}]$ and $\theta\in(0,K]$

c. <u>If $\theta>K$:</u>. For these types of policy makers, irrespective of the policy r and irrespective of the size of the attack, the status quo always survives and consequently the utility of the policy maker is always $W(\theta,r,*)$. Now, we will check separately what happen in this case when $r\in(0,\frac{K}{c}]$ and $r>\frac{K}{c}$.

- Given the strategy profile of the agents, a policy $r\in(0,\frac{K}{c}]$ means that all agents choose $a=1$, the size of the attack is K, so the utility of the policy maker is always $U(\theta,r,K)=W(\theta,r,K)$. The policy $r\in(0,\frac{K}{c}]$ is strictly dominated by the policy $r=0$ because we always have $W(\theta,r,K)=\theta-cr-K<\theta-K\le\theta-K\Psi\left(\frac{x^{\#}(\sigma)-\theta}{\sigma}\right)=W\left(\theta,0,K\Psi\left(\frac{x^{\#}(\sigma)-\theta}{\sigma}\right)\right)$[81]

  To summarize, $U(\theta,r,K)<U\left(\theta,0,K\Psi\left(\frac{x^{\#}(\sigma)-\theta}{\sigma}\right)\right)$ when $r\in(0,\frac{K}{c}]$ and $\theta>K$

- Given the strategy profile of the agents, a policy $r>\frac{K}{c}$ means that all agents choose $a=0$, the size of the attack is 0, so the utility of the policy maker is always $U(\theta,r,0)=W(\theta,r,0)$. The policy $r>\frac{K}{c}$ is strictly dominated by the

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[79] The inequalities follow from the fact that we are assuming $\theta\in(0,K]$ and $r>\frac{K}{c}$. Therefore, $\theta-cr<\theta-K\le 0$

[80] The inequalities follow from the fact that we are assuming $r>\frac{K}{c}$ and that Ψ is a normal c.d.f (i.e. $\Psi\in[0,1]$)

[81] The first inequality follows from the fact that $r\in(0,\frac{K}{c}]$. The last inequality follows from the fact that we are assuming that Ψ is a normal c.d.f (i.e. $\Psi\in[0,1]$).

policy $r = 0$ because we always have $W(\theta, r, 0) = \theta - cr < \theta - K \leq \theta - K\Psi\left(\frac{x^{\#}(\sigma)-K}{\sigma}\right) < \theta - K\Psi\left(\frac{x^{\#}(\sigma)-\theta}{\sigma}\right) = W\left(\theta, 0, K\Psi\left(\frac{x^{\#}(\sigma)-\theta}{\sigma}\right)\right)$[82]

To summarize, $U(\theta, r, K) < U\left(\theta, 0, K\Psi\left(\frac{x^{\#}(\sigma)-\theta}{\sigma}\right)\right)$ when $r > \frac{K}{c}$ and $\theta > K$

Therefore, at the pooling equilibrium, there exists a unique strategy profile for the policy maker in which she always chooses the policy $r = 0$.

To complete the proof, we have to show that the strategy profile of the agents[83] can be supported by an appropriate system of beliefs:

a. **When $\boldsymbol{r = 0}$:** In this case, for any type of policy maker θ, the posterior beliefs of the agents are represented by $\mu(\theta|\, x, 0) = 1 - \Psi\left(\frac{x-\theta}{\sigma}\right)$[84]. In proposition A1 we showed, given these beliefs, that choosing $a = 1$ if and only if $x < x^{\#}(\sigma)$ is sequentially optimal for the agents when they observe $r = 0$.

b. **When $\boldsymbol{r = s \in (0, \frac{K}{c}]}$:** In proposition B2 we define θ_s^* as the lowest type of policy maker who prefers to implement the policy $r = s$ and face no attack to implement the policy $r = 0$ and suffer a coordinated attack that implies a regime change. In addition, remember from the end of section 3.2.1 that we are assuming that an agent chooses $a = 1$ if she expects $u(*) \geq 0$[85]. Now, let $\mu(\theta|\, x, s)$ be any beliefs that assign probability 1 to the event in which the type of policy maker is $\theta \in [\theta_s^*, K]$[86], irrespective of x. Because for any $\theta \in [\theta_s^*, K]$ there is a regime change, then these beliefs satisfy $y\int_{\theta_s^*}^{K} d\mu(\tilde{\theta}|\, x, s) - b - s \geq 0$ for all x. It implies that any agent i who expects that the other agent chooses $a_j = 1$ when $r = s$ finds it optimal to choose $a_i = 1$ when $r = s$, irrespective of x.

c. **When $\boldsymbol{r = s > \frac{K}{c}}$:** Let $\mu(\theta|\, x, r)$ be any beliefs that assign probability 1 to $\theta > \frac{K}{2}$, irrespective of x_i. Therefore, these beliefs imply that any agent i who expects that the other agent chooses $a_j = 0$ when $r = s$ finds it optimal to choose $a_i = 0$ when $r = s$, irrespective of x_i.

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[82] All the inequalities follow from the fact that we are assuming $r > \frac{K}{c}$, $\theta > K$ and Ψ is a normal c.d.f

[83] Remember that the strategy profile of the agents is $\begin{cases} \text{for } r = 0 \begin{cases} a = 1 \text{ if and only if } x < x^{\#}(\sigma) \\ a = 0 \text{ if and only if } x > x^{\#}(\sigma) \end{cases} \\ \text{for any } r \in (0, \frac{K}{c}], a = 1 \text{ irrespective of } x \\ \text{for any } r > \frac{K}{c}, a = 0 \text{ irrespective of } x \end{cases}$

[84] These beliefs were updated using the Bayes' rule as you can check in the footnote 73.

[85] Remember that this assumption is equivalent to assume that an agent who expects a regime change finds it optimal to choose $a = 1$ at least insofar as the policy maker does not play a dominated action.

[86] Remember that if $\theta > K$, then the status quo always survives, so it is never optimal for the agents to choose $a = 1$ in this situation.

Therefore, in our pooling equilibrium the strategy profile of the agents can be supported by a correct system of beliefs. Finally, notice that at this profile when $r = 0$ there is a unique threshold $x^{\#}(\sigma)$ as it was shown in the proposition A3. □

Appendix B3: The set of semiseparating equilibria

In this section we will show the propositions and elements that are more relevant to explain the characteristics of the set of semiseparating equilibria of our model. This appendix is based entirely in Angeletos and Pavan (2013). For that reason, if you want to read a more complete explanation of this equilibrium you can read directly their paper. In their paper they use functions that are more general than the functions that we are using; therefore, at some points it can be difficult to visualize their arguments. On the other hand, since we are using specific functions, in our approach we arrive to the close form solutions that we use to analyze the results of the lab experiment.

The more relevant characteristics of the propositions that we present below are:

- **Proposition B1:** This proposition argues that, in any equilibrium of the set of semiseparating equilibria, when the policy maker of type θ chooses a policy $r > 0$, she selects the least costly policy among those policies that are favorable to the survival of the status quo[87]. Assume that this policy is $r = s$, then the agents choose $a = 0$ when they observe $r = s$.
- **Proposition B2:** This proposition shows that $\theta_S^* = cs \geq \frac{K}{2}$.
- **Proposition B3:** This proposition shows that the semiseparating equilibria does not exist when $\theta < cs$ for all θ[88]. In addition, it also shows that at the equilibrium no type of policy maker $\theta > \theta_S^*$ experiences a regime change
- **Propositions B4 and B5:** These propositions show that $\theta_S^{**}(\sigma) = \sigma\left[\Psi^{-1}\left(1 - \frac{cs}{K}\left(\frac{b}{y-b}\right)\right) - \Psi^{-1}\left(\frac{cs}{K}\right)\right] + cs$, $\quad x_S^*(\sigma) \equiv X(\theta_S^*, \theta_S^{**}(\sigma); \sigma) = \sigma\Psi^{-1}\left(1 - \frac{cs}{K}\left(\frac{b}{y-b}\right)\right) + cs$[89], and demonstrate that $\lim_{\sigma\to 0}\theta_S^{**}(\sigma) = \lim_{\sigma\to 0}x_S^*(\sigma) = \theta_S^* = cs$[90].
- **Proposition B6:** This proposition uses a contagion argument[91] to delimit from above the range of types $(\theta_S^*, \theta_S^{**}(\sigma)]$ at which the policy maker always chooses $r = s > 0$ in the set of semiseparating equilibria. More specifically, this proposition shows that any policy $r > 0$ is dominated by the policy $r = 0$ if θ is sufficiently high[92], then it shows that the agents find iteratively dominant to choose $a = 0$ for sufficiently high x, conditional on observing $r = 0$. Therefore, it presents a contagion effect argument in which the agents choose $a = 0$ for

[87] From equation (3.1), notice that, when the regime change, all policies $r > 0$ are strictly dominated at the equilibrium by the policy $r = 0$. Therefore, the survival of the status quo is the first requisite that a policy maker needs to satisfy to implement an equilibrium policy $r > 0$ that is not strictly dominated by the policy $r = 0$.

[88] The policy maker of type θ never chooses a policy $r = s$ in which $\theta < cs$ because in this case the policy $r = 0$ strictly dominates the policy $r = s$. Therefore, at the equilibrium the policy makers of type $\theta < \theta_S^*$ never chooses the policy $r = s$.

[89] Notice that one necessary condition to ensure the existence of $\theta_S^{**}(\sigma)$ and $x_S^*(\sigma)$ is that in any equilibrium $1 > \frac{cs}{K}\left(\frac{b}{y-b}\right)$. And we are ensuring that this restriction is always satisfied with the assumption that $y > 2b$ as we explained in the footnote 27

[90] The last part is easily obtained from the equations of θ_S^*, $\theta_S^{**}(\sigma)$ and $x_S^*(\sigma)$.

[91] As the arguments usually used to analyze standard global games

[92] A type of policy maker $\theta \geq \theta_S^*$ is sufficiently high when the expected attack, after observing $r = 0$ and given the signal threshold $x_S^*(\sigma)$, is lower than cs and consequently is lower than θ.

lower and lower x. In the limit, this contagion converges to $\theta_s^{**}(\sigma)$, guaranteeing that all $\theta > \theta_s^{**}(\sigma)$: (i) are able to avoid regime change without intervening, and (ii) obtain a higher utility by choosing the policy $r = 0$ and facing an attack than by choosing the policy $r = s$ and facing no attack.

- **Proposition B7:** In this proposition, the fact that choosing any policy $r > 0$ is dominated for sufficiently low θ, along with the fact that for these types, regime change is inevitable, implies that the agents find it iteratively dominant to choose $a = 1$ for sufficiently low x as long as they do not observe $r > 0$. The dispersion of information then initiates a contagion effect such that, conditional on seeing $r = 0$, agents find it iteratively dominant to choose $a = 1$ for higher and higher x, in which case regime change occurs for higher and higher θ. In the limit, this contagion effect guarantees that regime change occurs for all $\theta < min\{\theta_s^*, \theta^\#\}$in any equilibrium in which the chosen policy is $r = s$[93].

  Later, in proposition B9 we will show that a semiseparating equilibrium exists only if the equilibrium policy $s \in \left(\frac{K}{2c}, \frac{\theta^\#}{c}\right]$, this implies that $min\{\theta_s^*, \theta^\#\} = \theta_s^*$; so in proposition B10 we show that in a semiseparating equilibrium, in which the equilibrium positive policy is established in the range $(\theta_s^*, \theta_s^{**}(\sigma)]$, a regime change happens only if $\theta < \theta_s^*$ and the status quo survives only if $\theta > \theta_s^*$.
- **Propositions B8 to B11:** These propositions show that the set of semiseparating equilibria characterized by the strategies explained in section 3.2.2.2 exist if and only if $s \leq \frac{\theta^\#}{c}$.

Before explaining the propositions, we have to introduce a few concepts and variables: The variable $\Delta U(U(*)_1, U(*)_2) \equiv U(*)_1 - U(*)_2$ represents the utility that the policy maker gets if the situation 1 happens minus the utility that the policy maker gets if the situation 2 happens (these situations are determined by the choices of the policy maker and the regime status at the end of the game)[94]. In the propositions, we will show that at the equilibria when the policy maker chooses a policy $r = 0$ then the optimal trigger strategy of the agents is to choose $a = 1$ if and only if $x < x^*(\sigma)$ and $a = 0$ otherwise; so at the equilibria the size of the attack can be represented by

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[93] The last result is obtained by comparing the agents' incentives to choose $a = 1$ after observing $r = 0$ with the corresponding incentives when they expect at the equilibrium $r = 0$ for all θ. Because the observation of $r = 0$ is most informative of regime change when all types of policy maker who experience regime change set $r = 0$, while some of the types of policy maker who are spared from regime change raise the policy to $r > 0$, the size of the attack when setting $r = 0$ is necessarily larger in any of the equilibria in which some types of policy maker are expected to raise the policy to $r = s$ than in the pooling equilibria where all types of policy maker are expected to set $r = 0$. Hence any type $\theta < \theta^\#$ who does not raise the policy to $r = s$ necessarily experience regime change in equilibrium. Because choosing the policy $r = s$ is dominated for all $\theta < \theta_s^*$, this implies that regime change occurs for any $\theta < min\{\theta_s^*, \theta^\#\}$ in any equilibrium of our set of semiseparating equilibria.

[94] For instance, $\Delta U\big(W(\theta, r', KA), L(r'')\big) \equiv W(\theta, r', KA) - L(r'')$ where in the first situation the policy maker has chosen $r' \geq 0$ and there was no regime change and in the second situation the policy maker has chosen $r'' \geq 0$ and there was a regime change. Similarly, we can also have $\Delta U\big(W(\theta, r', KA), W(\theta, r'', KA)\big) \equiv W(\theta, r', KA) - W(\theta, r'', KA)$ or $\Delta U\big(L(r'), L(r'')\big) \equiv L(r') - L(r'')$ among others.

$K\Psi\left(\frac{x^*(\sigma)-\theta}{\sigma}\right)$ when the type of policy maker is θ[95]. Finally, as was assumed by Angeletos and Pavan (2013, p. 888), we will assume that in the equilibrium, the policy maker does not face uncertainty about regime outcomes.

<u>Proposition B1:</u> *In any equilibrium in which a policy maker of type θ decides a policy $r > 0$, there exists a single policy $r = s > 0$ such that at the equilibrium this policy is chosen when $r = s$ is not dominated by the policy $r = 0$. Furthermore, when $r = s$, all agents decide $a = 0$ for all θ.*

Proof: Since the policy maker does not have uncertainty about the size of the attack; then, she can predict with complete certainty when there is a regime change. Therefore, any type of policy maker who chooses an equilibrium policy $r > 0$ must be waiting that the status quo survives, because otherwise she would be strictly better off by setting $r = 0$.[96] It implies that at the equilibrium if the agents observe the equilibrium policy $r > 0$, then this policy signals them that there won't be a regime change and thus induces each agent to choose $a = 0$ no matter what her signal x is.

For that reason, any type of policy maker θ can always obtains a higher utility by choosing the less costly policy $r > 0$ among those that are played in equilibrium. Therefore, in any equilibrium in which the policy maker of type θ chooses a policy $r > 0$, there exists a single policy $s > 0$ such that this is the policy chosen when the policy $r = 0$ does not dominate all policies $r > 0$. Then, when the chosen policy is $r = s$ all agents decide $a = 0$ for all θ. □

<u>Proposition B2:</u> *Let θ_s^* denotes the lowest type of policy maker who prefers to rise the policy to $r = s$ and face no attack to leave the policy at $r = 0$ and suffer a coordinated attack that implies a regime change. Therefore, $\theta_s^* = cs \geq \frac{K}{2}$.*

Proof: For any $\{\theta, s\}$, $\theta_s^* = inf\left\{\theta \geq \frac{K}{2} \mid \Delta U\big(W(\theta, s, 0), L(0)\big) \geq 0\right\}$. Therefore, given the definition of $U(\cdot)$ in equation (3.1) we get $\theta_s^* = inf\left\{\theta \geq \frac{K}{2} \mid \theta - cs \geq 0\right\} = cs \geq \frac{K}{2}$. The lower bound of θ_s^* is due to the fact that when the type of policy maker is higher than $\frac{K}{2}$, then the agents need a coordinated attack to get a regime change. □

<u>Proposition B3:</u> *For any $s > 0$, if our set of semiseparating equilibria exists, then there exists a type of policy maker $\theta \geq \frac{K}{2}$ such that $\Delta U\big(W(\theta, s, 0), L(0)\big) \geq 0$ (i.e. $\theta \geq$*

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[95] In the first part of the footnote 74 we already explained that when a policy maker observes that her type is θ, she updates her beliefs about x_i such that $(x_i|\theta) \sim N(\theta, \sigma^2)$. Therefore, when the policy maker at the equilibrium chooses a policy $r = 0$, then the posterior probability that the policy maker θ gives to $a_i = 1$ is: $Prob(x_i < x^*(\sigma)|\theta) = \Psi\left(\frac{x^*(\sigma)-E(x_i|\theta)}{SD(x_i|\theta)}\right) = \Psi\left(\frac{x^*(\sigma)-\theta}{\sigma}\right)$. Therefore, the expected size of the attack is $K\sum_{i=1}^{2}\frac{a_i Prob(x_i<x^*(\sigma))}{2} = K\sum_{i=1}^{2}\frac{\Psi\left(\frac{x^*(\sigma)-\theta}{\sigma}\right)}{2} = K\Psi\left(\frac{x^*(\sigma)-\theta}{\sigma}\right)$.

[96] Notice that choosing a policy $r > 0$ when there is a regime change implies a utility to the policy maker that is lower than the utility obtained if the policy is equal to zero. More specifically, $L(r) = -cr < 0 = L(0)$.

$cs \geq \frac{K}{2}$). In addition, any equilibrium in our set of semiseparating equilibria is such that there is no regime change for all $\theta > \theta_S^*$.

Proof: First we will prove that for any $s > 0$ a semiseparating equilibrium does not exist when $\Delta U\big(W(\theta, s, 0), L(0)\big) < 0$ for all θ (i.e. this kind of equilibrium does not exist when $\theta < cs\ \forall\theta$). It happens because the utility that the type of policy maker θ obtains by choosing the policy $r = 0$ is always strictly higher than the utility the same type obtains by choosing the policy $r = s > 0$. More specifically, we always have one of the following situations[97]:

1. $U(\theta, 0{,}0) = W(\theta, 0{,}0) = \theta > \theta - cs = W(\theta, s, 0) = U(\theta, s, 0)$,
2. $U(\theta, 0{,}0) = W(\theta, 0{,}0) = \theta > 0 > -cs = L(s) = U(\theta, s, 0)$ [98],
3. $U\left(\theta, 0, K\Psi\left(\frac{x-\theta}{\sigma}\right)\right) = W\left(\theta, 0, K\Psi\left(\frac{x-\theta}{\sigma}\right)\right) = \theta - K\Psi\left(\frac{x-\theta}{\sigma}\right) > 0 > -cs = L(s) = U(\theta, s, 0)$ [99],
4. $U\left(\theta, 0, K\Psi\left(\frac{x-\theta}{\sigma}\right)\right) = W\left(\theta, 0, K\Psi\left(\frac{x-\theta}{\sigma}\right)\right) = \theta - K\Psi\left(\frac{x-\theta}{\sigma}\right) > 0 > \theta - cs = W(\theta, s, 0) = U(\theta, s, 0)$ [100],
5. $U(\theta, 0,*) = L(0) = 0 > -cs = L(s) = U(\theta, s, 0)$ and
6. $U(\theta, 0,*) = L(0) = 0 > \theta - cs = W(\theta, s, 0) = U(\theta, s, 0)$

To summarize, when $\Delta U\big(W(\theta, s, 0), L(0)\big) < 0$ (i.e. when $\theta < cs$) we got that $U(\theta, 0,*) > U(\theta, s, 0)$ always happens irrespective whether choosing the policy $r = 0$ leads to or not to a regime change and irrespective of the size of the attack when $r = 0$.

Therefore, consider the policies $r = s > 0$ for which there exists a type of policy maker $\theta \geq \frac{K}{2}$ such that $\Delta U\big(W(\theta, s, 0), L(0)\big) \geq 0$ (i.e. consider the policies $r = s$ for which $\theta \geq \theta_S^* = cs \geq \frac{K}{2}$). From proposition B1 we know that at the equilibrium when the established policy is $r = s > 0$, then all agents decide $a = 0$ for any θ. It implies that at the equilibrium any policy maker of type $\theta > \theta_S^*$, by choosing the policy $r = s$, ensures a utility higher than the utility she obtains by setting the policy $r = 0$ and experiencing a regime change (i.e. $U(\theta, s, 0) = W(\theta, s, 0) = \theta - cs > 0 = L(0)$). For that reason, at the equilibrium no type of policy maker $\theta > \theta_S^*$ experiences a regime change. □

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[97] Observe than in all situations we only compare $U(\theta, s, 0)$ with different specifications of $(\theta, 0,*)$, it happens because from Proposition B1 we know that at the equilibrium when the established policy is $r = s > 0$, then all agents decide $a = 0$ for any θ.

[98] The first inequality happens because the utility $W(*)$ implies that there is not regime change, so it means that at least the condition $\theta > 0$ must be satisfied.

[99] The first inequality happens because the utility $W\left(\theta, 0, K\Psi\left(\frac{x-\theta}{\sigma}\right)\right)$ implies that there is not a regime change, so $\theta > K\Psi\left(\frac{x-\theta}{\sigma}\right)$.

[100] The first inequality happens because the utility $W\left(\theta, 0, K\Psi\left(\frac{x-\theta}{\sigma}\right)\right)$ implies that there is not a regime change, so $\theta > K\Psi\left(\frac{x-\theta}{\sigma}\right)$. The second inequality happens because we are assuming $\theta < cs$.

Proposition B4: *Let $\theta_s^{**}(\sigma)$ denotes the highest type of policy maker $\theta \geq \theta_s^*$ who finds it optimal to choose the policy $r = s > 0$ and face no attack when the policy $r = 0$ leads to an attack of size $K\Psi\left(\frac{x_s^*(\sigma)-\theta_s^{**}(\sigma)}{\sigma}\right)$ where $x_s^*(\sigma) \equiv X(\theta_s^*, \theta_s^{**}(\sigma); \sigma) = \theta_s^{**}(\sigma) + \sigma\Psi^{-1}\left(\frac{\theta_s^*}{K}\right) = \theta_s^* + \sigma\Psi^{-1}\left(1 - \frac{\theta_s^*}{K}\left(\frac{b}{y-b}\right)\right)$ is the unique solution to $\left[1 - \Psi\left(\frac{x_s^*(\sigma)-\theta_s^*}{\sigma}\right)\right](y-b) - \left[\Psi\left(\frac{x_s^*(\sigma)-\theta_s^{**}(\sigma)}{\sigma}\right)\right]b = 0$. Then, $\theta_s^{**}(\sigma) = \sigma\left[\Psi^{-1}\left(1 - \frac{cs}{K}\left(\frac{b}{y-b}\right)\right) - \Psi^{-1}\left(\frac{cs}{K}\right)\right] + cs$, and $\lim_{\sigma \to 0} \theta_s^{**}(\sigma) = \lim_{\sigma \to 0} x_s^*(\sigma) = \theta_s^* = cs$.*

Before proving this proposition, we need to explain better the way $x_s^*(\sigma)$ is determined and why $x_s^*(\sigma)$ is unique. Assume an agent who believes that: (1) the regime change happens when $\theta \leq \theta_s^*$ and (2) the policy makers' type is $\theta \notin [\theta_s^*, \theta_s^{**}(\sigma)]$. Therefore, when an agent observes $r = 0$, she is indifferent between choosing $a = 1$ or $a = 0$ if $\left[\frac{1-\Psi\left(\frac{x_s^*(\sigma)-\theta_s^*}{\sigma}\right)}{1-\Psi\left(\frac{x_s^*(\sigma)-\theta_s^*}{\sigma}\right)+\Psi\left(\frac{x_s^*(\sigma)-\theta_s^{**}(\sigma)}{\sigma}\right)}\right](y-b) - \left[\frac{\Psi\left(\frac{x_s^*(\sigma)-\theta_s^{**}(\sigma)}{\sigma}\right)}{1-\Psi\left(\frac{x_s^*(\sigma)-\theta_s^*}{\sigma}\right)+\Psi\left(\frac{x_s^*(\sigma)-\theta_s^{**}(\sigma)}{\sigma}\right)}\right]b = 0$[101]. The LHS of this equation is the expected utility that an agent has from choosing $a = 1$ after observing the policy $r = 0$ and the private signal $x_s^*(\sigma)$, and the RHS of the equation is the expected utility that an agent has from choosing $a = 0$ after observing $r = 0$ (from equation (3.2) remember that $u_i(*)$ is always zero if $a_i = 0$). Therefore, we have $\left[1 - \Psi\left(\frac{x_s^*(\sigma)-\theta_s^*}{\sigma}\right)\right](y-b) - \left[\Psi\left(\frac{x_s^*(\sigma)-\theta_s^{**}(\sigma)}{\sigma}\right)\right]b = 0$.

Proposition B5: Let $KA(\theta_s^*, \theta_s^{**}(\sigma); \sigma) \equiv K\Psi\left(\frac{x_s^*(\sigma)-\theta_s^{**}(\sigma)}{\sigma}\right)$ denotes the aggregate size of the attack when the policy maker's type is $\theta_s^{**}(\sigma)$ and the agents attack if and only if $x < x_s^*(\sigma)$. Therefore, $A(\theta_s^*, \theta_s^{**}(\sigma); \sigma)$ is increasing in σ with $\lim_{\sigma \to 0} KA(\theta_s^*, \theta_s^{**}(\sigma); \sigma) = \begin{cases} 0 \ \ if \ \theta_s^{**}(\sigma) > \theta_s^* \\ \theta^{\#} \ if \ \theta_s^{**}(\sigma) = \theta_s^* \end{cases}$ where $x_s^*(\sigma)$ is the unique x that solves the equality $\left[1 - \Psi\left(\frac{x-\theta_s^*}{\sigma}\right)\right](y-b) - \left[\Psi\left(\frac{x-\theta_s^{**}(\sigma)}{\sigma}\right)\right]b = 0$ and $\theta^{\#} = K\left(\frac{y-b}{y}\right)$ is the same value obtained in Proposition A2.

Proof: From the identity that appears in the proposition we obtain $x_s^*(\sigma) = \sigma\Psi^{-1}(A) + \theta_s^{**}(\sigma)$. Therefore, if we replace this equation in $\left[1 - \Psi\left(\frac{x_s^*(\sigma)-\theta_s^*}{\sigma}\right)\right](y-b) - \left[\Psi\left(\frac{x_s^*(\sigma)-\theta_s^{**}(\sigma)}{\sigma}\right)\right]b = 0$ we get $(y-b)\left[1 - \Psi\left(\Psi^{-1}(A) + \frac{\theta_s^{**}(\sigma)-\theta_s^*}{\sigma}\right)\right] - bA = 0$. Note that the LHS of this equation is: (1) decreasing in A, and (2) increasing in σ.

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[101] From the footnote 73, remember that when the policy is uninformative (as it happens in the semiseparating equilibria inside the range in which all policy makers choose $r = 0$ or in the pooling equilibrium in which $r = 0$ is always chosen) then when the agent i receives the signal x_i, she updates her beliefs about θ such that $(\theta|x_i) \sim N(x_i, \sigma^2)$. Therefore, the posterior probability that the agent i, after observing her private signal, gives to $\theta \leq \theta_s^*$ is: $Prob(\theta \leq \theta_s^*|x_i) = \Psi\left(\frac{\theta_s^*-E(\theta|x_i)}{SD(\theta|x_i)}\right) = \Psi\left(\frac{\theta_s^*-x_i}{\sigma}\right) = 1 - \Psi\left(\frac{x_i-\theta_s^*}{\sigma}\right)$. Similarly, $Prob(\theta > \theta_s^{**}(\sigma)|x_i) = \Psi\left(\frac{x_i-\theta_s^{**}(\sigma)}{\sigma}\right)$ is the posterior probability that the agent i, after observing her private signal, gives to $\theta > \theta_s^{**}(\sigma)$; and consequently $Prob(\theta \notin [\theta_s^*, \theta_s^{**}(\sigma)]|x_i) = 1 - \Psi\left(\frac{x_s^*(\sigma)-\theta_s^*}{\sigma}\right) + \Psi\left(\frac{x_s^*(\sigma)-\theta_s^{**}(\sigma)}{\sigma}\right)$ is the posterior probability that the agent gives to $\theta \notin [\theta_s^*, \theta_s^{**}(\sigma)]$. In the equation linked to this footnote, notice that the posterior probabilities were updated using the Bayes' rule.

Therefore, given θ_s^* and $\theta_s^{**}(\sigma)$, the first property of the equation guarantees that there is a unique solution for A and consequently the equation $\left[1-\Psi\left(\frac{x_s^*(\sigma)-\theta_s^*}{\sigma}\right)\right](y-b)-\left[\Psi\left(\frac{x_s^*(\sigma)-\theta_s^{**}(\sigma)}{\sigma}\right)\right]b=0$ admits a unique solution for $x_s^*(\sigma)$. The second property of the equation and the implicit function theorem imply that $A(\cdot)$ is also increasing in σ. In addition, the results obtained in this proposition imply that[102]

$\lim_{\sigma\to 0} A(\theta_s^*,\theta_s^{**}(\sigma);\sigma)=\begin{cases}0 & if\ \theta_s^{**}(\sigma)>\theta_s^*\\ \frac{y-b}{y} & if\ \theta_s^{**}(\sigma)=\theta_s^*\end{cases}$, and consequently we conclude that

$\lim_{\sigma\to 0} KA(\theta_s^*,\theta_s^{**}(\sigma);\sigma)=\begin{cases}0\ if\ \theta_s^{**}(\sigma)>\theta_s^*\\ \theta^{\#}\ if\ \theta_s^{**}(\sigma)=\theta_s^*\end{cases}$ □

Proof of Proposition B4: The uniqueness of $x_s^*(\sigma)\equiv X(\theta_s^*,\theta_s^{**}(\sigma);\sigma)$ was already proved in proposition B5. Now, by definition of $\theta_s^{**}(\sigma)$ we have $\theta_s^{**}(\sigma)=max\{\dot{\theta}_s,\hat{\theta}_s\}$[103] where $\dot{\theta}_s=sup\left\{\theta\geq\theta_s^*|\ W(\theta,s,0)=W\left(\theta,0,K\Psi\left(\frac{X(\theta_s^*,\theta;\sigma)-\theta}{\sigma}\right)\right) and\ \theta>K\Psi\left(\frac{X(\theta_s^*,\theta;\sigma)-\theta}{\sigma}\right)\right\}$[104] and $\hat{\theta}_s\equiv sup\left\{\theta\geq\theta_s^*|\ \theta=K\Psi\left(\frac{X(\theta_s^*,\theta;\sigma)-\theta}{\sigma}\right)\right\}$[105]. Then,

$\theta_s^{**}(\sigma)=max\left\{\theta\geq\theta_s^*|\theta,\theta=\ X(\theta_s^*,\theta;\sigma)-\sigma\Psi^{-1}\left(\frac{\theta_s^*}{K}\right)\right\}=x_s^*(\sigma)-\sigma\Psi^{-1}\left(\frac{\theta_s^*}{K}\right)\geq\theta_s^*$. Now, if we replace $\theta_s^{**}(\sigma)$ in $\left[1-\Psi\left(\frac{x_s^*(\sigma)-\theta_s^*}{\sigma}\right)\right](y-b)-\left[\Psi\left(\frac{x_s^*(\sigma)-\theta_s^{**}(\sigma)}{\sigma}\right)\right]b=0$ we get

$x_s^*(\sigma)=\sigma\Psi^{-1}\left(1-\frac{\theta_s^*}{K}\left(\frac{b}{y-b}\right)\right)+\theta_s^*$. Then, $\theta_s^{**}(\sigma)=\sigma\left[\Psi^{-1}\left(1-\frac{\theta_s^*}{K}\left(\frac{b}{y-b}\right)\right)-\Psi^{-1}\left(\frac{\theta_s^*}{K}\right)\right]+\theta_s^*$.

Since, $\theta_s^*=cs$, then $\theta_s^{**}(\sigma)=\sigma\left[\Psi^{-1}\left(1-\frac{cs}{K}\left(\frac{b}{y-b}\right)\right)-\Psi^{-1}\left(\frac{cs}{K}\right)\right]+cs$ and

$x_s^*(\sigma)=\sigma\Psi^{-1}\left(1-\frac{cs}{K}\left(\frac{b}{y-b}\right)\right)+cs$[106]

Finally, we will prove that $\lim_{\sigma\to 0}\theta_s^{**}(\sigma)=\lim_{\sigma\to 0}x_s^*(\sigma)=\theta_s^*=cs$. The proof is direct from: [1] $x_s^*(\sigma)=\sigma\Psi^{-1}\left(1-\frac{\theta_s^*}{K}\left(\frac{b}{y-b}\right)\right)+\theta_s^*$, [2] $\theta_s^{**}(\sigma)=x_s^*(\sigma)-\sigma\Psi^{-1}\left(\frac{\theta_s^*}{K}\right)$ and [3] $\theta_s^*=cs$ □

Proposition B6: *For any $s>0$, if our set of semiseparating equilibria exists then there exists a type of policy maker $\theta_s\geq\theta_s^*$ who finds it optimal to chose the policy $r=$*

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[102] Do not forget that $\Psi(+\infty)=1$ because Ψ is a normal c.d.f

[103] $\dot{\theta}_s\geq\theta_s^*$ represents the highest type of policy maker who is indifferent between the policies $r=0$ or $r=s$, where the policy $r=0$ implies an attack that does not imply a regime change. $\hat{\theta}_s\geq\theta_s^*$ represents the highest type of policy maker who experience a regime change higher or equal than cs when she chooses the policy $r=0$.

[104] That is, $\dot{\theta}_s=sup\left\{\theta\geq\theta_s^*|\ cs=K\Psi\left(\frac{X(\theta_s^*,\theta;\sigma)-\theta}{\sigma}\right)\ and\ \theta>K\Psi\left(\frac{X(\theta_s^*,\theta;\sigma)-\theta}{\sigma}\right)\right\}=sup\left\{\theta\geq\theta_s^*|\ \theta_s^*=K\Psi\left(\frac{X(\theta_s^*,\theta;\sigma)-\theta}{\sigma}\right)\ and\ \theta>K\Psi\left(\frac{X(\theta_s^*,\theta;\sigma)-\theta}{\sigma}\right)\right\}$. Then, $\dot{\theta}_s=sup\{\theta\geq\theta_s^*|\ \theta>\theta_s^*\}=\theta\geq\theta_s^*$

[105] That is, $\hat{\theta}_s=sup\left\{\theta|\ K\Psi\left(\frac{X(\theta_s^*,\theta;\sigma)-\theta}{\sigma}\right)\geq\theta_s^*\right\}=sup\left\{\theta|\ X(\theta_s^*,\theta;\sigma)-\sigma\Psi^{-1}\left(\frac{\theta_s^*}{K}\right)\geq\theta\right\}$, then $\hat{\theta}_s=X(\theta_s^*,\hat{\theta}_s;\sigma)-\sigma\Psi^{-1}\left(\frac{\theta_s^*}{K}\right)$

[106] Notice that one necessary condition to ensure the existence of $\theta_s^{**}(\sigma)$ and $x_s^*(\sigma)$ is that in any equilibrium $1>\frac{cs}{K}\left(\frac{b}{y-b}\right)$. And we are ensuring that this restriction is always satisfied with the assumption that $y>2b$ as we explained in the footnote 27.

$s > 0$ and face no attack when the policy $r = 0$ leads to an attack of size $K\Psi\left(\frac{X(\theta_s^*,\theta_s;\sigma)-\theta_s}{\sigma}\right)$[107]. Furthermore, any equilibrium in our set is such that the equilibrium policy $r = s$ is chosen only if $\theta_s \in [\theta_s^*, \theta_s^{**}(\sigma)]$.

Proof: Consider a sequence $\{\theta^n\}_{n=0}^{\infty}$ that that has the following two characteristics:

1) $\theta^0 \equiv inf\{\theta \geq \theta_s^* \mid \Delta U(W(\theta,s,0), W(\theta,0,K)) < 0 \; and \; \theta > K$[108]$\} = inf\{\theta \geq \theta_s^* \mid \theta_s^* > K \; and \; \theta > K\} = \theta_s^* > K$. That is, θ^0 is the lowest type of policy maker higher or equal than θ_s^* who prefers to choose the policy $r = 0$ in which all agents choose $a = 1$ (but that does not imply a regime change) than to choose the policy $r = s$ in which all agents choose $a = 0$ (and consequently the regime survives).
2) For any $n \geq 1$ and $\theta^{n-1} \geq \theta_s^*$, let $\theta^n \equiv inf\left\{\theta \geq \theta_s^* \,\middle|\, U(\theta,s,0) - U\left(\theta,0,K\Psi\left(\frac{X(\theta_s^*,\theta^{n-1};\sigma)-\theta}{\sigma}\right)\right) < 0\right\}$ where $X(\theta_s^*,\theta^{n-1};\sigma)$ is the unique signal x that solves $\left[1-\Psi\left(\frac{x-\theta_s^*}{\sigma}\right)\right](y-b) - \left[\Psi\left(\frac{x-\theta^{n-1}}{\sigma}\right)\right]b = 0$[109]. That is, θ^n is the lowest type of policy maker greater or equal than θ_s^* who prefers a policy $r = 0$ that implies an attack of size $K\Psi\left(\frac{X(\theta_s^*,\theta^{n-1};\sigma)-\theta}{\sigma}\right)$[110] to a policy $r = s$ that implies that no agent attacks. Notice that θ^n requires that $U(\theta,s,0) = W(\theta,s,0) = \theta - cs$ because we are assuming $\theta \geq \theta_s^* = cs > 0$ and consequently the status quo always survives; in addition, θ^n also requires that $U\left(\theta,0,K\Psi\left(\frac{X(\theta_s^*,\theta^{n-1};\sigma)-\theta}{\sigma}\right)\right) = W\left(\theta,0,K\Psi\left(\frac{X(\theta_s^*,\theta^{n-1};\sigma)-\theta}{\sigma}\right)\right)$ because $W(\theta,s,0) - L(0) < 0$ implies $\theta < cs$ which contradicts the fact that θ^n requires $\theta \geq \theta_s^* = cs$. Therefore, the element θ^n requires that in the sequence the status quo always survives, that is; $\theta > K\Psi\left(\frac{X(\theta_s^*,\theta^{n-1};\sigma)-\theta}{\sigma}\right) \geq 0$. Then,
$$\theta^n = inf\left\{\theta \geq \theta_s^* \,\middle|\, W(\theta,s,0) - W\left(\theta,0,K\Psi\left(\frac{X(\theta_s^*,\theta^{n-1};\sigma)-\theta}{\sigma}\right)\right) < 0 \; and \; \theta > K\Psi\left(\frac{X(\theta_s^*,\theta^{n-1};\sigma)-\theta}{\sigma}\right)\right\}.$$
And consequently,
$$\theta^n = inf\left\{\theta \geq cs \,\middle|\, K\Psi\left(\frac{X(cs,\theta^{n-1};\sigma)-\theta}{\sigma}\right) < cs \; and \; \theta > K\Psi\left(\frac{X(cs,\theta^{n-1};\sigma)-\theta}{\sigma}\right)\right\} = \theta \geq \theta_s^* > K\Psi\left(\frac{X(cs,\theta^{n-1};\sigma)-\theta}{\sigma}\right)$$
[111]. That is $\theta^n \in [\theta_s^*, \theta^0]$.

Notice, that the sequence $\{\theta^n\}_{n=0}^{\infty}$ is nonincreasing[112]. Therefore, the way this sequence helps us to delimit from above, in the set of semiseparating equilibria, the

[107] That is, θ_s satisfies (1) $W(\theta_s,s,0) = W\left(\theta_s,0,K\Psi\left(\frac{X(\theta_s^*,\theta_s;\sigma)-\theta_s}{\sigma}\right)\right)$ and $\theta_s > K\Psi\left(\frac{X(\theta_s^*,\theta_s;\sigma)-\theta_s}{\sigma}\right)$ (i.e. $\theta_s > K\Psi\left(\frac{X(\theta_s^*,\theta_s;\sigma)-\theta_s}{\sigma}\right) = cs$) or (2) $\theta_s = K\Psi\left(\frac{X(\theta_s^*,\theta_s;\sigma)-\theta_s}{\sigma}\right)$

[108] That is, the type of policy maker θ^0, irrespective of the size of the attack, never faces a regime change.

[109] Notice that the LHS of this equality is positive if x is sufficiently low and negative if x is sufficiently high.

[110] Notice that this attack resembles the behavior of agents who has the threshold signal in $X(\theta_s^*,\theta^{n-1};\sigma)$.

[111] Notice that $\theta^0 \geq \theta^1$ because Ψ is a c.d.f

range of policy makers that choose the policy $r = s$ is by using the following contagion argument: From the definition of the sequence, we know that in any semiseparating equilibrium in which the policy maker only chooses the policies $r = 0$ or $r = s$, no type of policy maker $\theta \notin [\theta_s^*, \theta^0]$ chooses the policy $r = s$. Subsequently, an agent who expects: (1) that a regime change happens if and only if $\theta < \theta_s^*$ and (2) that the equilibrium policy is $r = 0$ if and only if $\theta \notin [\theta_s^*, \theta^0]$, finds it optimal to choose $a = 1$ if and only if she observes a signal $x < X(\theta_s^*, \theta^0; \sigma)$ when the chosen policy is $r = 0$. It implies that an agent who expects: (1) the survival of the status quo for all $\theta \geq \theta_s^*$ (but possible also for some $\theta < \theta_s^*$) and (2) that the equilibrium policy is $r = 0$ for all $\theta \notin [\theta_s^*, \theta^0]$ (but possible also for some $\theta \in [\theta_s^*, \theta^0]$), never finds it optimal to choose $a = 1$ for a signal $x > X(\theta_s^*, \theta^0; \sigma)$. Knowing this, a policy maker who expects that the other agent chooses $a = 0$ for $x > X(\theta_s^*, \theta^0; \sigma)$ never finds it optimal to choose the policy $r = s$ for any $\theta > \theta^1$. Knowing this, all agents find it optimal to choose $a = 0$ for any $x > X(\theta_s^*, \theta^1; \sigma)$ when they observe the policy $r = 0$, and so on. If we follow this argument iteratively we approach to $\theta_s^{**}(\sigma)$[113].

Therefore, if our set of semiseparating equilibria exists, we have established that any type of policy maker θ_s who belongs to the range $[\theta_s^*, \theta_s^{**}(\sigma)]$ finds it optimal to choose the policy $r = s > 0$ and face no attack when the policy $r = 0$ leads to an attack of size $K\Psi\left(\frac{X(\theta_s^*, \theta_s; \sigma) - \theta_s}{\sigma}\right)$ □

<u>Proposition B7:</u> *For any $s > 0$ if our set of semiseparating equilibria exists, then any equilibrium in this set is such that there is regime change for any $\theta < min\{\theta_s^*, \theta^\#\}$. In addition, $\theta_s^* > \theta^\#$ if and only if $s > \frac{K}{c}\left(\frac{y-b}{y}\right)$*

Proof: Since $\theta_s^* = cs$ and $\theta^\# = K\left(\frac{y-b}{y}\right)$; then, we can prove directly that $\theta_s^* > \theta^\#$ if and only if $s > \frac{K}{c}\left(\frac{y-b}{y}\right)$. On the other hand, to prove take at the equilibrium there is regime change for any $\theta < min\{\theta_s^*, \theta^\#\}$ consider the sequence $\{\theta_n, x_n\}_{n=0}^{\infty}$ that is constructed in the following way:

1) Assume that $\theta_0 \equiv \frac{K}{2}$ and that x_0 is the unique signal x that solves $\left[1 - \Psi\left(\frac{x - \frac{K}{2}}{\sigma}\right)\right](y - b) - \left[\Psi\left(\frac{x - \frac{K}{2}}{\sigma}\right)\right]b = 0$; then, $\left[1 - \Psi\left(\frac{x_0 - \frac{K}{2}}{\sigma}\right)\right]y = b$, so $x_0 = \sigma\Psi^{-1}\left(\frac{y-b}{y}\right) + \frac{K}{2}$. That is,

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[112] The proof is by contradiction; take any $s > 0$ for which there exists a $\theta \geq \theta_s^* = cs$ (i.e. $\Delta U\big(W(\theta, s, 0), L(0)\big) \geq 0$). Now we will use a contradictory argument, assume that there exists an $n \geq 1$ such that $\theta^n > \theta^{n-1}$. Without loss of generality, consider that n is the first step in this sequence. The definition of the sequence implies that for any $\theta > \theta^{n-1}$, we have $\theta \geq cs > K\Psi\left(\frac{X(cs, \theta^{n-2}; \sigma) - \theta}{\sigma}\right)$ (i.e. there is no regime change). Because $\theta^{n-1} \leq \theta^{n-2}$ and because $X(cs, *; \sigma)$ is increasing, this in turn implies that for any $\theta > \theta^{n-1}$, we have $\theta \geq cs > K\Psi\left(\frac{X(cs, \theta^{n-1}; \sigma) - \theta}{\sigma}\right)$ (i.e. $W(\theta, s, 0) - W\left(\theta, 0, K\Psi\left(\frac{X(\theta_s^*, \theta^{n-1}; \sigma) - \theta}{\sigma}\right)\right) < 0$). By the definition of θ^n, this means that $\theta^n \leq \theta^{n-1}$, so we have a contradiction and consequently the sequence $\{\theta^n\}_{n=0}^{\infty}$ is nonincreasing.

[113] Remember that $\theta_s^{**}(\sigma)$ denotes the highest type of policy maker $\theta \geq \theta_s^*$ who finds it optimal to raise the policy to $r = s$ and face no attack when choosing the policy $r = 0$ leads to an attack of size $K\Psi\left(\frac{x_s^*(\sigma) - \theta_s^{**}(\sigma)}{\sigma}\right)$

when the type of policy maker is $\theta_0 \equiv \frac{K}{2}$, and the agent observes the policy $r = 0$ and receives the signal x_0, then her expected utility of choosing $a = 1$ is equal to her expected utility of choosing $a = 0$.

2) For any $n \geq 1$, assume that $\theta_n \equiv min\{\theta_s^*, \theta_n^{'}\}$ where $\theta_n^{'}$ solves $\theta_n^{'} = K\Psi\left(\frac{x_{n-1}-\theta_n^{'}}{\sigma}\right)$, where x_n is the unique signal x that solves $\left[1 - \Psi\left(\frac{x-\theta_n^{'}}{\sigma}\right)\right]y = b$ (i.e. $x_n = \sigma\Psi^{-1}\left(\frac{y-b}{y}\right) + \theta_n^{'}$). Then, $\theta_n^{'} = K\Psi\left(\Psi^{-1}\left(\frac{y-b}{y}\right) + \frac{\theta_{n-1}^{'}-\theta_n^{'}}{\sigma}\right)$ and $x_n = \sigma\Psi^{-1}\left(\frac{y-b}{y}\right) + K\Psi\left(\Psi^{-1}\left(\frac{y-b}{y}\right) + \frac{\theta_{n-1}^{'}-\theta_n^{'}}{\sigma}\right)$).

The way this sequence helps us to define, in the semiseparating equilibria, the lowest type of policy maker who does not face a regime change is by using the following contagion argument[114]: from the sequence, you can see that an agent who (1) observes $r = 0$, (2) believes that at the equilibrium $r = 0$ for all θ and (3) believes that the other agent chooses $a = 0$[115] finds it optimal to choose $a = 1$ if and only if $x \leq x_0$. It implies that an agent who expects that the other agent chooses $a = 0$ and that the equilibrium policy is $r = 0$ for all $\theta < \theta_s^*$ inevitably finds it optimal to choose $a = 1$ for any $x < x_0$. This conclusion is obtained because the observation of the policy $r = 0$ is most informative about regime change when all types of policy makers for whom regime change happens set the policy $r = 0$, while some of the types of policy makers for whom regime change does not happen raise the policy above $r = 0$. But, if both agents choose $a = 1$ to any $x < x_0$, the regime change happens for all $\theta < \theta_1$. Then there exists a signal $x_1 > x_0$ such that an agent who (1) expects that the other agent chooses $a = 1$ if $x < x_0$[116] and (2) believes that at the equilibrium $r = 0$ for all θ, necessarily finds it optimal to choose $a = 1$ for all $x < x_1$. It implies that an agent who expects the equilibrium policy $r = 0$ for all $< \theta_s^*$, but possibly an equilibrium policy $r > 0$ for some $\theta > \theta_s^*$, necessarily finds it optimal to choose $a = 1$ for any $x < x_1$ and so on. From the propositions of appendix A3, we know that $x^{\#} = \sigma\Psi^{-1}\left(\frac{y-b}{y}\right) + \theta^{\#}$ and $\theta^{\#} = K\left(\frac{y-b}{y}\right)$ are the unique solutions to $x' = \sigma\Psi^{-1}\left(\frac{y-b}{y}\right) + \theta'$ and $\theta' = K\Psi\left(\frac{x'-\theta'}{\sigma}\right)$, then our sequence convergence to $min\{\theta_s^*, \theta^{\#}\}$[117]. Then, there is a regime change for all $\theta < min\{\theta_s^*, \theta^{\#}\}$ □

<u>Proposition B8:</u> *For any σ, and any $s \in \left(\frac{K}{2}, \frac{\theta^{\#}}{c}\right]$ there exists at least one type of policy maker θ_s that satisfies $\theta_s = K\Psi\left(\frac{X(cs,\theta_s;\sigma)-\theta_s}{\sigma}\right) \geq \theta_s^*$ where $X(cs, \theta_s; \sigma)$ is the unique x that solves $\left[1 - \Psi\left(\frac{x-cs}{\sigma}\right)\right](y - b) - \left[\Psi\left(\frac{x-\theta_s}{\sigma}\right)\right]b = 0$.*[118]

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[114] Notice that $\{\theta_n\}_{n=0}^{\infty}$ is increasing and bounded from above, therefore it necessarily converges.

[115] In which case regime change happens if and only if $\theta \leq \frac{K}{2}$

[116] So a regime change happens for all $\theta \leq \theta_1$

[117] By definition of θ_s^*, the elements of our sequence cannot be higher than θ_s^*

[118] This threshold identifies the unique signal x at which an agent who believes that $\theta \notin [\theta_s^*, \theta_s]$ and that the regime change happens if and only if $\theta \leq \theta_s^*$ is indifferent between choosing $a = 0$ and $a = 1$ when observing $r = 0$

Proof: Notice that for any policy $s \in \left(\frac{K}{2}, \frac{\theta^{\#}}{c}\right]$, there is at least one policy maker that by choosing the policy $r = s$ in which there is no regime change obtains a utility that is higher than the utility that the same policy maker obtains by choosing the policy $r = 0$ in which there is a regime change[119]. Next, since $X(\theta, \theta; \sigma)$ is the unique signal x that solves $\left[1 - \Psi\left(\frac{x-\theta}{\sigma}\right)\right](y-b) - \left[\Psi\left(\frac{x-\theta}{\sigma}\right)\right] b = 0$ (that is $X(\theta, \theta; \sigma) = \sigma\Psi^{-1}\left(\frac{y-b}{y}\right) + \theta$), then the unique type of policy maker at which $\theta = K\Psi\left(\frac{X(\theta,\theta;\sigma)-\theta}{\sigma}\right)$ is $\theta = K\left(\frac{y-b}{y}\right) = \theta^{\#}$. It implies that there is regime change when $\theta \leq \theta^{\#}$ and there is not regime change when $\theta > \theta^{\#}$.

Because the equation $\theta = K\Psi\left(\frac{X(\theta_s^*,\theta;\sigma)-\theta}{\sigma}\right)$ is continuous in θ and $K > K\Psi\left(\frac{X(\theta_s^*,K;\sigma)-K}{\sigma}\right)$, then there always exists a $\theta_s \geq \theta_s^*$ (with strict inequality when $\theta_s^* < \theta^{\#}$) that satisfies $\theta_s = K\Psi\left(\frac{X(\theta_s^*,\theta_s,\sigma)-\theta_s}{\sigma}\right)$. □

Proposition B9: *The set of semiseparating equilibria exists if and only if* $s \in \left(\frac{K}{2c}, \frac{\theta^{\#}}{c}\right]$.

Proof: The proof consists in to show that the range $(\theta_s^*, \theta_s^{**}(\sigma)]$ of types of policy makers that can sustain the multiple semiseparating equilibria is not well defined when $s \notin \left(\frac{K}{2c}, \frac{\theta^{\#}}{c}\right]$ but is well defined when $s \in \left(\frac{K}{2c}, \frac{\theta^{\#}}{c}\right]$

When $s \leq \frac{K}{2c}$: Then $\theta_s^* = cs \leq \frac{K}{2}$ which contradicts the fact that $\theta_s^* > \frac{K}{2}$ so the range $(\theta_s^*, \theta_s^{**}(\sigma)]$ is not well defined

When $s > \frac{\theta^{\#}}{c}$: Assume that θ_s^* exists, that is $\theta_s^{**}(\sigma) = cs > \theta^{\#} = K\left(\frac{y-b}{y}\right)$. Now we well prove that a type of policy maker $\theta_s^{**}(\sigma) \geq \theta_s^*(\sigma)$ does not exist when $s > \frac{\theta^{\#}}{c}$. Remember from Proposition B4 that $\theta_s^{**}(\sigma) = \sigma\left[\Psi^{-1}\left(1 - \frac{cs}{K}\left(\frac{b}{y-b}\right)\right) - \Psi^{-1}\left(\frac{cs}{K}\right)\right] + cs$; therefore $\theta_s^{**}(\sigma) < \theta_s^*$ when $s > \frac{\theta^{\#}}{c}$[120]. It implies that if $s > \frac{\theta^{\#}}{c}$ then the range $(\theta_s^*, \theta_s^{**}(\sigma)]$ is not well defined.

When $s \in \left(\frac{K}{2c}, \frac{\theta^{\#}}{c}\right]$: Remember that $\theta_s^* = inf\left\{\theta \geq \frac{K}{2} \mid \Delta U\big(W(\theta, s, 0), L(0)\big) \geq 0\right\} = cs$, so θ_s^* is well defined in the range $\left(\frac{K}{2c}, +\infty\right)$. In addition, since $\theta_s^{**}(\sigma) = \sigma\left[\Psi^{-1}\left(1 - \frac{cs}{K}\left(\frac{b}{y-b}\right)\right) - \Psi^{-1}\left(\frac{cs}{K}\right)\right] + cs$, then $\theta_s^{**}(\sigma) > \theta_s^*$ when $s \in \left(\frac{K}{2c}, \frac{\theta^{\#}}{c}\right]$, so θ_s^* is also well defined. Therefore, the range $(\theta_s^*, \theta_s^{**}(\sigma)]$ of types of policy makers that can sustain the multiple semiseparating equilibria is well defined when $s \in \left(\frac{K}{2c}, \frac{\theta^{\#}}{c}\right]$.

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[119] That is, the set $\left\{\theta \geq \frac{K}{2} \mid \Delta U\big(W(\theta, s, 0), L(0)\big) \geq 0\right\} = \left\{\theta \geq \frac{K}{2} \mid \theta \geq cs\right\}$ exists

[120] If $cs > K\left(\frac{y-b}{y}\right)$ (i.e. if $cs > \theta^{\#}$), then $1 < \frac{cs}{K}\left[\left(\frac{y}{y-b}\right)\right] = \frac{cs}{K}\left[\left(\frac{b}{y-b}\right) + 1\right] = \frac{cs}{K}\left(\frac{b}{y-b}\right) + \frac{cs}{K}$, then $1 - \frac{cs}{K}\left(\frac{b}{y-b}\right) < \frac{cs}{K}$. Consequently, since Ψ is a normal c.d.f, we have $\left[\Psi^{-1}\left(1 - \frac{cs}{K}\left(\frac{b}{y-b}\right)\right) - \Psi^{-1}\left(\frac{cs}{K}\right)\right] < 0$, then $\theta_s^{**}(\sigma) < \theta_s^*$.

In addition, notice that when $s \in \left(\frac{K}{2c}, \frac{\theta^{\#}}{c}\right]$ then $\Delta U\left(U(\theta, s, 0), U\left(\theta, 0, K\Psi\left(\frac{x_s^*(\sigma)-\theta}{\sigma}\right)\right)\right) \geq 0$ for all $\theta > \theta_s^*$; more specifically:

I. For any $\theta \in (\theta_s^*, \theta_s^{**}(\sigma)]$, notice that $U(\theta, s, 0) = W(\theta, s, 0)$ because all agents are choosing $a = 0$, then
$\Delta U\left(U(\theta, s, 0), U\left(\theta, 0, K\Psi\left(\frac{x_s^*(\sigma)-\theta}{\sigma}\right)\right)\right) = \Delta U\left(W(\theta, s, 0), U\left(\theta, 0, K\Psi\left(\frac{x_s^*(\sigma)-\theta}{\sigma}\right)\right)\right) \geq 0$[121].

II. For any $\theta > \theta_s^{**}(\sigma)$, notice that $U(\theta, s, 0) = W(\theta, s, 0)$ because all agents are choosing $a = 0$; then
$\Delta U\left(U(\theta, s, 0), U\left(\theta, 0, K\Psi\left(\frac{x_s^*(\sigma)-\theta}{\sigma}\right)\right)\right) = \Delta U\left(W(\theta, s, 0), U\left(\theta, 0, K\Psi\left(\frac{x_s^*(\sigma)-\theta}{\sigma}\right)\right)\right) \leq 0$[122]. □

<u>Proposition B10:</u> *Any equilibrium in our set of semiseparating equilibria is such that there is regime change only if* $\theta < \theta_s^*$ *and the status quo survives if* $\theta > \theta_s^*$

Proof: It is direct from the first part of Proposition B9 and the Propositions B3 and B7 □

<u>Proposition B11:</u> *For any* σ *and any* $s \in \left(\frac{K}{2c}, \frac{\theta^{\#}}{c}\right]$, *there exists an equilibrium in which* $r(\theta) = s$ *for all* $\theta \in (\theta_s^*, \theta_s^{**}(\sigma)]$ and $r = 0$ otherwise.

Proof: We will show that this equilibrium exists at least for the following strategy profile for the agents:

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[121] Notice that:

$$\Delta U\left(W(\theta, s, 0), U\left(\theta, 0, K\Psi\left(\frac{X(\theta_s^*, \theta_s^{**}(\sigma); \sigma)-\theta}{\sigma}\right)\right)\right) = \begin{cases} K\Psi\left(\frac{X(\theta_s^*, \theta_s^{**}(\sigma);\sigma)-\theta}{\sigma}\right) - cs \;\; if\; \theta > K\Psi\left(\frac{X(\theta_s^*, \theta_s^{**}(\sigma);\sigma)-\theta}{\sigma}\right) \\ \theta - cs \quad if\; \theta \leq K\Psi\left(\frac{X(\theta_s^*, \theta_s^{**}(\sigma);\sigma)-\theta}{\sigma}\right) \end{cases}.$$

Therefore,

i. The inequality holds when $\theta \leq K\Psi\left(\frac{X(\theta_s^*, \theta_s^{**}(\sigma);\sigma)-\theta}{\sigma^2}\right)$ because $\theta \in \left(cs, K\Psi\left(\frac{X(\theta_s^*, \theta_s^{**}(\sigma);\sigma)-\theta}{\sigma^2}\right)\right]$ (i.e. $\theta \geq cs$).

ii. The inequality also holds when $\theta > K\Psi\left(\frac{X(\theta_s^*, \theta_s^{**}(\sigma);\sigma)-\theta}{\sigma}\right)$. In particular, remember that $\theta_s^{**}(\sigma) = X(\theta_s^*, \theta_s^{**}(\sigma); \sigma) - \sigma\Psi^{-1}\left(\frac{cs}{K}\right)$. Therefore $K\Psi\left(\frac{X(\theta_s^*, \theta_s^{**}(\sigma);\sigma)-\theta}{\sigma}\right) - cs = K\Psi\left(\Psi^{-1}\left(\frac{cs}{K}\right) + \frac{\theta_s^{**}(\sigma)-\theta}{\sigma}\right) - cs \geq K\Psi\left(\Psi^{-1}\left(\frac{cs}{K}\right) + \frac{\theta_s^{**}(\sigma)-\theta_s^{**}(\sigma)}{\sigma}\right) - cs = 0$, where the inequality happens because we are assuming $\theta \in (\theta_s^*, \theta_s^{**}(\sigma)]$ and remember that Ψ is a c.d.f.

[122] Notice that:

$$\Delta U\left(W(\theta, s, 0), U\left(\theta, 0, K\Psi\left(\frac{X(\theta_s^*, \theta_s^{**}(\sigma); \sigma)-\theta}{\sigma}\right)\right)\right) = \begin{cases} K\Psi\left(\frac{X(\theta_s^*, \theta_s^{**}(\sigma);\sigma)-\theta}{\sigma}\right) - cs \;\; if\; \theta > K\Psi\left(\frac{X(\theta_s^*, \theta_s^{**}(\sigma);\sigma)-\theta}{\sigma}\right) \\ \theta - cs \quad if\; \theta \leq K\Psi\left(\frac{X(\theta_s^*, \theta_s^{**}(\sigma);\sigma)-\theta}{\sigma}\right) \end{cases}.$$

Therefore,

i. The inequality holds when $\theta \leq K\Psi\left(\frac{X(\theta_s^*, \theta_s^{**}(\sigma);\sigma)-\theta}{\sigma^2}\right)$ because $\theta > \theta_s^{**}(\sigma) \geq \theta_s^* = cs$

ii. The inequality also holds when $\theta > K\Psi\left(\frac{X(\theta_s^*, \theta_s^{**}(\sigma);\sigma)-\theta}{\sigma}\right)$. In particular, remember that $\theta_s^{**}(\sigma) = X(\theta_s^*, \theta_s^{**}(\sigma); \sigma) - \sigma\Psi^{-1}\left(\frac{cs}{K}\right)$. Therefore $K\Psi\left(\frac{X(\theta_s^*, \theta_s^{**}(\sigma);\sigma)-\theta}{\sigma}\right) - cs = K\Psi\left(\Psi^{-1}\left(\frac{cs}{K}\right) + \frac{\theta_s^{**}(\sigma)-\theta_s^{**}(\sigma)}{\sigma}\right) - cs \leq K\Psi\left(\Psi^{-1}\left(\frac{cs}{K}\right) + \frac{\theta_s^{**}(\sigma)-\theta}{\sigma}\right) - cs = 0$. where, the inequality happens because we are assuming $\theta > \theta_s^{**}(\sigma)$ and remember that Ψ is a c.d.f.

$$\text{strategy profile for the agents}: \begin{cases} \text{for } r = 0 \begin{cases} a = 1 \text{ if and only if } x < x_s^*(\sigma) \\ a = 0 \text{ if and only if } x > x_s^*(\sigma) \end{cases} \\ \text{for any } r \in (0, s), a = 1 \text{ irrespective of } x \\ \text{for any } r \geq s, a = 0 \text{ irrespective of } x \end{cases} \text{ where } x_s^*(\sigma) \equiv X(\theta_s^*, \theta_s^{**}(\sigma); \sigma)$$

Since all agents chooses $a = 1$ when the policy is $r \in (0, s)$ then the policy $r = 0$ strictly dominates any policy $r \in (0, s)$[123]. Similarly, since all agents chooses $a = 0$ when the policy is $r \geq s$, then the policy $r = s$ dominates any policy $r > s$[124]. In addition, notice that the policy $r = 0$ is dominant for any type of policy maker $\theta \leq \frac{K}{2}$[125]. On the other hand, for the type of policy maker $\theta > \frac{K}{2}$ the utility from choosing the policy$r = s$ is $W(\theta, s, 0) = \theta - cs$ while the utility from choosing the policy $r = 0$ is $U\left(\theta, 0, K\Psi\left(\frac{x_s^*(\sigma) - \theta}{\sigma}\right)\right)$. At the end of Proposition B9 we showed that if $s \in \left(\frac{K}{2c}, \frac{\theta^{\#}}{c}\right]$ then we have $\Delta U\left(U(\theta, s, 0), U\left(\theta, 0, K\Psi\left(\frac{x_s^*(\sigma) - \theta}{\sigma}\right)\right)\right) \geq 0$ for all types of policy maker $\theta > \theta_s^*$; therefore, given the definitions of θ_s^* and $\theta_s^{**}(\sigma)$, we have that choosing the policy $r = s$ is optimal if and only if $\theta \in [\theta_s^*, \theta_s^{**}(\sigma)]$. To summarize, we have established the optimality of the policy maker's strategy.

On the other hand, given the strategy profile of the agents, we have the following situation:

$$\text{When} \begin{cases} r = 0, \begin{cases} \text{there is regime change if and only if } \theta \leq \hat{\theta}_s \\ \text{the status quo survives if and only if } \theta > \hat{\theta}_s \end{cases} \\ r \in (0, s), \begin{cases} \text{there is regime change if and only if } \theta \leq K \\ \text{the status quo survives if and only if } \theta > K \end{cases} \\ r \geq s, \begin{cases} \text{there is regime change if and only if } \theta \leq \frac{K}{2} \\ \text{the status quo survives if and only if } \theta > \frac{K}{2} \end{cases} \end{cases}, \text{ where } \hat{\theta}_s \text{ is the}$$

threshold type of policy maker θ that solves $\theta = K\Psi\left(\frac{x_s^*(\sigma) - \theta}{\sigma}\right)$.

Therefore, for an agent is optimal to follow the equilibrium strategy if and only if her beliefs satisfy the following conditions:

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[123] Notice that in this case for any type of policy maker $\theta \leq K$ there is always a regime change, then choosing the policy $r \in (0, s)$ implies a utility $L(r)$ that is lower than the utility obtained when the policy is $r = 0$ (i.e. $L(r) = -cr < 0 = L(0)$). On the other hand, for any type of policy maker $\theta > K$ the status quo always survives, then choosing the policy $r \in (0, s)$, implies a utility $W(\theta, r, K)$ that is lower than the utility obtained when the policy is $r = 0$ (i.e. $W(\theta, r, K) = \theta - cr - K < \theta - K\Psi\left(\frac{x_s^*(\sigma) - \theta}{\sigma}\right) = W\left(\theta, 0, K\left(\frac{x_s^*(\sigma) - \theta}{\sigma}\right)\right)$

[124] We know that the policy $r \geq s \in \left(\frac{K}{2}, \frac{\theta^{\#}}{c}\right]$ implies that there is not any attack; therefore, choosing the policy to $r > s$ implies a utility that is always lower than the utility when the policy is $r = s$ (i.e. $U(\theta, r, 0) < U(\theta, s, 0)$).

[125] Remember that for any type of policy maker $\theta \leq K$ there is always a regime change, then choosing the policy $r \in (0, s)$ implies a utility $L(r)$ that is lower than the utility obtained when the policy is $r = 0$ (i.e. $L(r) = -cr < 0 = L(0)$). Now, if the policy is $r \geq s$ and the type of policy maker is $\theta \leq \frac{K}{2}$, then there is not a regime change, but we obtain $W(\theta, r, K) = \theta - cr < 0$ because the policy $r \geq s \in \left(\frac{K}{2c}, \frac{\theta^{\#}}{c}\right]$.

when $\begin{cases} r=0, \begin{cases} y\int_{-\infty}^{\hat{\theta}_s} d\mu(\tilde{\theta}|x,0) \geq b \text{ if } x < x_s^*(\sigma) \text{ and} \\ y\int_{-\infty}^{\hat{\theta}_s} d\mu(\tilde{\theta}|x,0) < b \text{ if } x \geq x_s^*(\sigma) \end{cases} \\ r \in (0,s),\ y\int_{-\infty}^{K} d\mu(\tilde{\theta}|x,r) \geq r+b \text{ for all } x \\ r \geq s,\ y\int_{-\infty}^{\frac{K}{2}} d\mu(\tilde{\theta}|x,r) \leq r+b \text{ for all } x \end{cases}$ where the posterior beliefs $\mu(\theta|*)$ are always established using the Baye's rule.

Thus, we have to analyze the following two cases:

1. When $r=0$: the beliefs of the agents are

$$\mu(\theta|x,0) = \begin{cases} \frac{1-\Psi\left(\frac{x-\theta}{\sigma}\right)}{1-\Psi\left(\frac{x-\theta_s^*}{\sigma}\right)+\Psi\left(\frac{x-\theta_s^{**}(\sigma)}{\sigma}\right)} \ for\ any\ \theta \leq \theta_s^* \\ \frac{1-\Psi\left(\frac{x-\theta_s^*}{\sigma}\right)}{1-\Psi\left(\frac{x-\theta_s^*}{\sigma}\right)+\Psi\left(\frac{x-\theta_s^{**}(\sigma)}{\sigma}\right)} for\ any\ \theta \in \left(\theta_s^*, \hat{\theta}_s\right) \end{cases}.$$

   From proposition B5, remember that once the agents observe the policy $r=0$, then $x_s^*(\sigma)$ is the unique signal at which they are indifferent between choosing $a=1$ and $a=0$. Therefore, the beliefs $\mu(\theta|x,0)$ satisfy $\begin{cases} y\int_{-\infty}^{\hat{\theta}_s} d\mu(\tilde{\theta}|x,\underline{r}) \geq b\ \ if\ x < x_s^*(\sigma) \\ y\int_{-\infty}^{\hat{\theta}_s} d\mu(\tilde{\theta}|x,\underline{r}) < b\ \ if\ x \geq x_s^*(\sigma) \end{cases}$.
2. When $r=s$: the beliefs of the agents are $\mu(0|x,s)=0$. Then, the condition $y\int_{-\infty}^{0} d\mu(\tilde{\theta}|x,r) \leq r$ is evidently satisfied.
3. When $r \notin \{0,s\}$, there are many kinds of out-of-equilibrium beliefs that satisfy $y\int_{-\infty}^{K} d\mu(\tilde{\theta}|x,r) \geq r+b$ when $r>s$ and $y\int_{-\infty}^{\frac{K}{2}} d\mu(\tilde{\theta}|x,r) \leq r+b$ when $r \in (0,s)$. For instance,
   - When $r \in (0,s)$: Let $\mu(\theta|\ x,s)$ be any beliefs that assign probability 1 to $\theta \in [\theta_s^*, K]$, irrespective of x. Because for any $\theta \in [\theta_s^*, K]$ where $r \in (0,s)$ there is a regime change, then these beliefs satisfy $y\int_{\theta_s^*}^{K} d\mu(\tilde{\theta}|\ x,r) \geq r+b$ for all x. It implies that any agent i who expects that the other agent chooses $a_j(x,s)=1$ finds it optimal to choose $a_i(x,s)=1$, irrespective of x.
   - When $r>s$: Let $\mu(\theta|\ x,r)$ be any beliefs that assign probability 1 to $\theta > \frac{K}{2}$, irrespective of x_i. Therefore, these beliefs imply that any agent i who expects that the other agent chooses $a_j(x,s)=0$ finds it optimal to choose $a_i(x,s)=0$, irrespective of x_i; that is, for this agent $y\int_{-\infty}^{\frac{K}{2}} d\mu(\tilde{\theta}|x,r) \leq r+b$

Therefore, given the optimality of the agents' strategy and the optimality of the policy maker's strategy we have proved the proposition □

Appendix C3: Instructions[126]

General information

Thank you for your participation in this experiment. If you make good decisions you may be able to earn a considerable amount of money, which will be paid to you privately at the end of the experiment. We ask you not to communicate with each other from now on, as well as to turn off your mobile phones.

Communication between participants is absolutely forbidden during the experiment! Not obeying this rule will lead to immediate exclusion from the experiment and all payments. If you have a question during the experiment, please raise your hand and I will answer your question directly at your desk.

The experiment is divided in two parts. Both parts consist of 30 independent and identical rounds. In this experiment you will be randomly paired with two persons in the room, and you will remain matched with them throughout the 60 rounds of the experiment. That is, you will be in the same group of three people during all the experiment. In each group we have randomly assigned two different kinds of roles ($\boldsymbol{A}$ and $\boldsymbol{B}$), the members of your group will maintain the same role during all the experiment. More specifically, <u>in each group one of the members always has role $\boldsymbol{A}$ ($\boldsymbol{Participant\ A}$) and the other two always have role $\boldsymbol{B}$ ($\boldsymbol{Participants\ B}$)</u>.

In this experiment we will use a virtual currency called *token.* The amount of *tokens* you have at the end of each round (i.e. $\boldsymbol{your\ final\ budget}$ in the round) will depends on your choices and the choices of the other members of your group during the round.

At the end of the experiment, we will randomly select six of the rounds that you played and you will be privately paid an amount that depends directly on the average amount of *tokens* you have at the end of those specific rounds. In the last section of the instructions I will give you more details about how your earnings in Euros will be determined.

Initial budgets assigned to the three members of each group

<u>At the beginning of every round</u>:

- <u>the two $\boldsymbol{Participant\ B}$ will always have 150 *tokens* each one</u>, and
- <u>the $\boldsymbol{Participant\ A}$ will always have a random amount of *tokens*</u> that can be positive or negative. The $\boldsymbol{Initial\ Budget\ of\ A}$ (i.e. the amount of *tokens* that the $\boldsymbol{Participant\ A}$ has at the beginning of every round) is always different from one round to the other.

At the beginning of each round the $\boldsymbol{Participant\ A}$ always will be informed about her initial budget. The $\boldsymbol{Participants\ B}$ won't know the $\boldsymbol{Initial\ Budget\ of\ A}$;

[126] Translated from Spanish into English. Case when $\sigma = 10$ in the first half of rounds and $\sigma = 15$ in the last half of rounds

however, they will receive a private clue about it, this clue is different for each $\boldsymbol{Participant\ B}$.

The clue that the $\boldsymbol{Participants\ B}$ receive about the $\boldsymbol{Initial\ Budget\ of\ A}$.

The clue that each $\boldsymbol{Participant\ B}$ receives about the $\boldsymbol{Initial\ Budget\ of A}$ is a private number "$\boldsymbol{x}$" given in the form of

$$\boldsymbol{x = (Initial\ Butget\ of\ A) + n}$$

where "$\boldsymbol{n}$" is a normally distributed random variable, independently for each $\boldsymbol{Participant\ B}$, with an average value of $\mathbf{0}$ and a standard deviation of $\mathbf{10}$. Therefore, if you are a $\boldsymbol{Participant\ B}$, take into account that **<u>on average,</u> your private number $\boldsymbol{x}$ accurately reflects the value of the $\boldsymbol{Initial\ Budget\ \ of\ A}$, because the value of the random variable $\boldsymbol{n}$ is zero on average.** However, in any given situation, your private number $\boldsymbol{x}$ can differ from the $\boldsymbol{Initial\ Budget\ of\ A}$. **In particular:**

- **There is approximately a probability of $\mathbf{68\%}$ that the $\boldsymbol{Initial\ Budget\ of\ A}$ lies within the interval $\boldsymbol{x - 10}$ and $\boldsymbol{x + 10}$.**
- **There is approximately a probability of $\mathbf{95\%}$ that the $\boldsymbol{Initial\ Budget\ of\ A}$ lies within the interval $\boldsymbol{x - 20}$ and $\boldsymbol{x + 20}$.**
- **There is approximately a probability of $\mathbf{99.7\%}$ that the $\boldsymbol{Initial\ Budget\ of\ A}$ lies within the interval $\boldsymbol{x - 30}$ and $\boldsymbol{x + 30}$.**

Example: You receive a private number of 24.5 (i.e. $x = 24.5$). Then:

- There is approximately a probability of 68% that the $\boldsymbol{Initial\ Budget\ of\ A}$ lies between 14.5 and 34.5.
- There is approximately a probability of 95% that the $\boldsymbol{Initial\ Budget\ of\ A}$ lies between 4.5 and 44.5.
- There is approximately a probability of 99.7% that the $\boldsymbol{Initial\ Budget\ of\ A}$ lies between -5.5 and 54.5.

In the second part of the experiment we will change the standard deviation of the private number. As soon as you finish the last round of the first part we will give you more indications about it.

Decisions in each round

All rounds of the experiment will consist of <u>two sequential stages</u>: Stage 1 and Stage 2. In Stage 1 only participates the $\boldsymbol{Participant\ A}$ and in Stage 2 only participate the two $\boldsymbol{Participants\ B}$.

Stage 1

In this stage the $\boldsymbol{Participant\ A}$ will be informed about her initial budget and she has to choose how many *tokens* must be disposed by **<u>THE OTHER</u>** members of her group (i.e. the two $\boldsymbol{Participants\ B}$). We will use the letter "$\boldsymbol{r}$" to identify the choice of the $\boldsymbol{Participant\ A}$. The value of $\boldsymbol{r}$ must be higher or equal than zero (and it must have two decimals at most). Therefore, as soon as the $\boldsymbol{Participant\ A}$ takes her choice about $\boldsymbol{r}$ then the budget of each of the other two participants in her group decreases the quantity $\boldsymbol{r}$ and the Stage 2 automatically begins.

IMPORTANT RULE: In Stage 1 the $\boldsymbol{Participant\ A}$ also has to dispose $0.7 * \boldsymbol{r}$ *tokens* from her own budget. For instance, if the $\boldsymbol{Participant\ A}$ chooses that $\boldsymbol{r = 10}$, then the budget of each $\boldsymbol{Participant\ B}$ automatically decreases 10 *tokens* and the budget of the $\boldsymbol{Participant\ A}$ decreases 7 *tokens*.

In Stage 1 the two $\boldsymbol{Participants\ B}$ have to wait quiet in front of their computers until Stage 2 begins.

Stage 2

In this stage each $\boldsymbol{Participant\ B}$ is informed about

(1) the choice of the $\boldsymbol{Participant\ A}$ (i.e. $\boldsymbol{r}$),
(2) her $\boldsymbol{updated\ budget}$ (i.e. $\mathbf{150}$ minus $\boldsymbol{r}$), and
(3) her clue about the $\boldsymbol{Initial\ Budget\ of\ A}$ (i.e. her private number $\boldsymbol{x}$)

Each $\boldsymbol{Participant\ B}$ has to choose independently only one of two actions: $\boldsymbol{Action\ z}$ or $\boldsymbol{Action\ w}$:

- If a $\boldsymbol{Participant\ B}$ chooses $\boldsymbol{Action\ w}$ then her $\boldsymbol{final\ budget}$ in this specific round will always be $\mathbf{0}$. This is independent of the choice of the other members of her group (i.e. **the** $\boldsymbol{Participant\ A}$ **and the other** $\boldsymbol{Participant\ B}$).
- If both $\boldsymbol{Participants\ B}$ choose $\boldsymbol{Action\ z}$ then the $\boldsymbol{Participant\ A}$ must dispose 100 additional *tokens*. If only one $\boldsymbol{Participant\ B}$ chooses $\boldsymbol{Action\ z}$ then the $\boldsymbol{Participant\ A}$ must dispose 50 additional *tokens*. If none of the $\boldsymbol{Participants\ B}$ choose $\boldsymbol{Action\ z}$ then $\boldsymbol{Participant\ A}$ must not dispose any additional *token*. We will use letter "$\boldsymbol{Z}$" to denote the aggregate amount of *tokens* that the $\boldsymbol{Participants\ B}$ choose that $\boldsymbol{Participant\ A}$ must dispose. That is, $\boldsymbol{Z}$ can only be equal to: 0, 50 or 100.

IMPORTANT RULE (1): The $\boldsymbol{Final\ Budget\ of\ the\ Participant\ A}$ cannot be lower than $\mathbf{-0.7} * \boldsymbol{r}$ *tokens*. This rule always applies, no matter the value of $\boldsymbol{Z}$ (that depends on the joint choice of both $\boldsymbol{B}$) and the $\boldsymbol{Initial\ Budget\ of\ A}$**.**

IMPORTANT RULE (2): If $\boldsymbol{Z < Initial\ Budget\ of\ A}$ and a $\boldsymbol{Participant\ B}$ has chosen $\boldsymbol{Action\ z,}$ then this $\boldsymbol{Participant\ B}$ has to dispose 220 additional *tokens* (i.e. in this specific case her $\boldsymbol{final\ budget}$ in the round will be necessarily negative. In particular, it will be equal to: $\mathbf{150 - 220} - \boldsymbol{r} = -(\mathbf{70 + r})$ *tokens*).

In Stage 2 the ***Participant A*** has to wait quiet in front of her computer until the round finishes. The round is finished once both ***Participants B*** have chosen between ***Action z*** and ***Action w***.

Final budgets per round

The information of the previous section is summarized below.

Participant A: The ***Final Budget of Participant A*** in each round depends on her initial budget, her choice about $\boldsymbol{r}$ and the amount of *tokens* that the other two participants in her group have chosen she has to dispose (i.e. $\boldsymbol{Z}$). More specifically:

Final Budget of Participant A	
$\boldsymbol{if\ (Initial\ Budget\ of\ A) \leq Z}$	$\boldsymbol{-0.7 * r}$
$\boldsymbol{if\ (Initial\ Budget\ of\ A) > Z}$	$\boldsymbol{(Initial\ Budget\ of\ A) - Z - (0.7 * r)}$

Participants B: The ***Final budget of each Participant B*** will depend on her individual choice (i.e. ***Action z*** or ***Action w***), the amount of *tokens* the ***Participant A*** had chosen that each ***Participant B*** has to dispose (i.e. $\boldsymbol{r}$), and the size of the ***Initial Budget of A*** relative to the aggregate amount $\boldsymbol{Z}$. More specifically, the ***Final Budget of each Participant B*** is determined according to the following combinations:

Final Budget of Participant B		***if she chooses***	
		Action z	***Action w***
and	$\boldsymbol{(Initial\ Budget\ of\ A) \leq Z}$	$\boldsymbol{150 - r}$	**0**
	$\boldsymbol{(Initial\ Budget\ of\ A) > Z}$	$\boldsymbol{-(70 + r)}$	**0**

The screens

Stage 1: During Stage 1, the ***Participant A*** always sees a screen that looks like the screen that appears in Figure C3.1.

If you are a ***Participant A*** you have to type your choice in the cell that is available to do it in the middle of the screen. Notice, that before you take a final choice, you can try (by pressing the button [Press this button if you want to test your choice about r]) different values of $\boldsymbol{r}$ to see how much will be ***your final budget*** in the round depending on your choice and the choices of the other two members of your group (this information will appear in the table that is at the bottom of the screen and will be automatically updated every time you choose a new value of $\boldsymbol{r}$ and press the button [Press this button if you want to test your choice about r]).

When you already have taken a decision about $\boldsymbol{r}$ press the button [Send] .

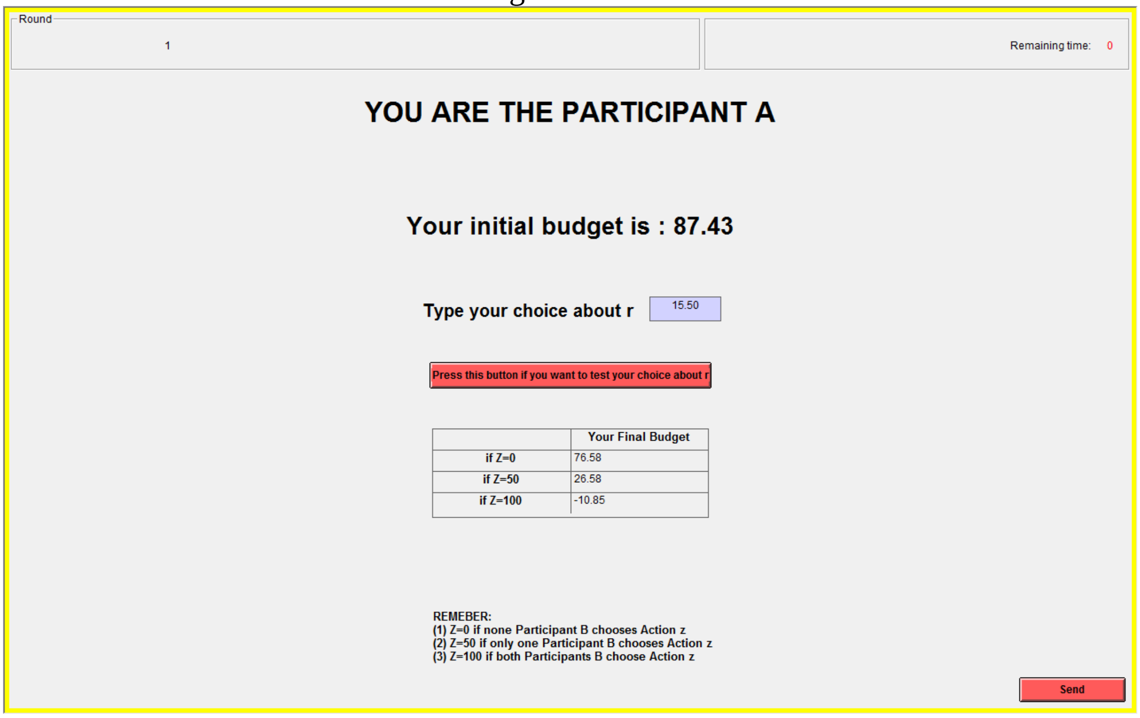
Stage 2: During Stage 2, each ***Participant B*** always sees a screen that looks like the screen of Figure C3.2. If you are a ***Participant B*** you have to take your choice by pressing one of the two buttons that are at the bottom of the screen. Notice that in the screen there is a table that summarizes the $\boldsymbol{potential\ final\ budgets}$ you can get during the round depending on the different combinations.

Information provided at the end of each round

After each round the $\boldsymbol{Participant\ A}$ will be informed about:

- Her initial budget
- Her choice (i.e. $\boldsymbol{r}$)
- The private number that each $\boldsymbol{Participant\ B}$ received (i.e. the $\boldsymbol{x}$ numbers)
- The value of $\boldsymbol{Z}$
- Her $\boldsymbol{final\ budget}$.

Figure C3.1*

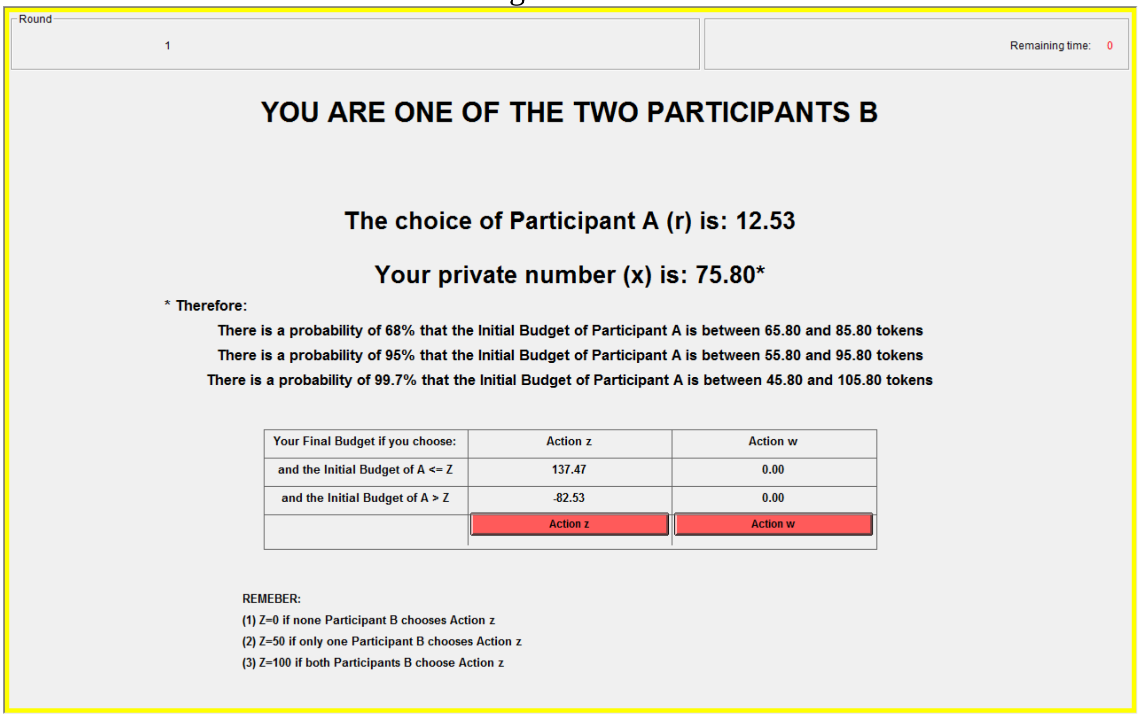


* In this figure notice that $\boldsymbol{Final\ Budget\ of\ Participant\ A}$ is 87.43 and $0.7 * r = 0.7 * 15.50 = 10.85$. Therefore, depending on the value of $\boldsymbol{Z}$, the $\boldsymbol{Final\ Budget\ of\ Participant\ A}$ is: 87.43-0-10.85= 76.58 if $\boldsymbol{Z = 0}$; 87.43-50-10.85= 26.58 if $\boldsymbol{Z = 50}$; or -10.85 if $\boldsymbol{Z = 100}$.

After each round each $\boldsymbol{Participant\ B}$ will be informed about:

- The true value of the $\boldsymbol{Initial\ Budget\ of\ A.}$
- Her clue about the $\boldsymbol{Initial\ Budget\ of\ A}$ (i.e. her private number $\boldsymbol{x}$),
- The choice of $\boldsymbol{Participant\ A}$ (i.e. $\boldsymbol{r}$)
- Her choice (i.e. $\boldsymbol{Action\ z}$ or $\boldsymbol{Action\ w}$)
- The value of $\boldsymbol{Z}$,
- Her $\boldsymbol{final\ budget}$.

Figure C3.2*



* In this figure $r = 12.53$. Then, 150-r=150-12.53=137.37 and –(70+r) = – (70+12.53) = –82.53

After a round is over, you will proceed to the next round and you will face the same decision problem. Remember that the values of the ***Initial Budget of A*** are randomly and independently determined from one round to the other, so a high (or low) ***Initial Budget of A*** in one round does not imply anything about the likely value of the ***Initial Budget of A*** for the next rounds.

Payoffs

As you may notice, the number of *tokens* you get in each round will depend on your choice, on the choice of the people you have been matched with, and on chance.

When you reach the end of the experiment, six of the rounds that you have played will be randomly selected. More specifically, the first chosen round will be randomly selected from the first 10 rounds you played; the second chosen round will be randomly selected from the second 10 rounds you played; the third chosen round will be randomly selected from the third 10 rounds you played; the fourth chosen round will be randomly selected from the fourth 10 rounds you played; the fifth chosen round will be randomly selected from the fifth 10 rounds you played; and the sixth chosen round will be randomly selected from the sixth 10 rounds you played.

Then, we will average the number of *tokens* you obtained in these particular rounds (i.e. we will average ***your final budgets*** during these rounds). This average number will be converted to Euros and will be paid to you in cash. The conversion rate that we will use is 6 *tokens* are equivalent to 1€. You will also receive a show up fee of 5 euros.

Finally, notice that the six rounds that are used to determine your final earnings will be randomly chosen, then every round you participate in the experiment can determine your final earnings.

Please, read the instructions again and pay special attention to the details. If the instructions are not clear to you, or you have a question of any sort, please raise your hand and sit quietly until the experimenter comes by to listen to your question. The answer to your question might also be helpful for others to hear; if it is, I will repeat your question out loud, and the answer, so everyone can hear them.

When the round 30 was over all participants received the following additional instructions that were read loudly:

From this Round Until the Last Round of the Experiment There is a Small Change in the Instructions

As you know, in the experiment the clue that each $\boldsymbol{Participant\ B}$ receives about the $\boldsymbol{Initial\ Budget\ of A}$ is a private number "$\boldsymbol{x}$" given in the form of

$$\boldsymbol{x = (Initial\ Butget\ of\ A) + n}$$

where "$\boldsymbol{n}$" is a random variable, chosen independently for each $\boldsymbol{Participant\ B.}$

Until now we have assumed that $\boldsymbol{n}$ is normally distributed with an average value of $\boldsymbol{0}$ and a standard deviation of $\boldsymbol{10}$.

<u>From now on, and until the end of the experiment we will assume that $\boldsymbol{n}$ is normally distributed with an average value of $\boldsymbol{0}$ and a standard deviation of $\boldsymbol{15}$</u>. Therefore, if you are a $\boldsymbol{Participant\ B}$, take into account that **<u>on average,</u> your private number $\boldsymbol{x}$ will continue accurately reflecting the value of the $\boldsymbol{Initial\ Budget\ of\ A}$, because the value of the random variable $\boldsymbol{n}$ is zero on average.** However, in any given situation, your private number $\boldsymbol{x}$ can differ from the $\boldsymbol{Initial\ Budget\ of\ A}$. **In particular,**

- **There is approximately a probability of 68% that the $\boldsymbol{Initial\ Budget\ of\ A}$ lies within the interval $\boldsymbol{x - 15}$ and $\boldsymbol{x + 15}$**
- **There is approximately a probability of 95% that the $\boldsymbol{Initial\ Budget\ of\ A}$ lies within the interval $\boldsymbol{x - 30}$ and $\boldsymbol{x + 30}$.**
- **There is approximately a probability of 99.7% that the $\boldsymbol{Initial\ Budget\ of\ A}$ lies within the interval $\boldsymbol{x - 45}$ and $\boldsymbol{x + 45}$.**

Example: You receive a private number of 24.5 (i.e. $x = 24.5$). Then,

- There is approximately a probability of 68% that the $\boldsymbol{Initial\ Budget\ of\ A}$ lies between 9.5 and 39.5.
- There is approximately a probability of 95% that the $\boldsymbol{Initial\ Budget\ of\ A}$ lies between -5.5 and 54.5.
- There is approximately a probability of 99.7% that the $\boldsymbol{Initial\ Budget\ of\ A}$ lies between -20.5 and 69.5.

The rest of the instructions of the experiment will remain the same.

Appendix D3: Number of times all policies r were chosen by each group in the experiment

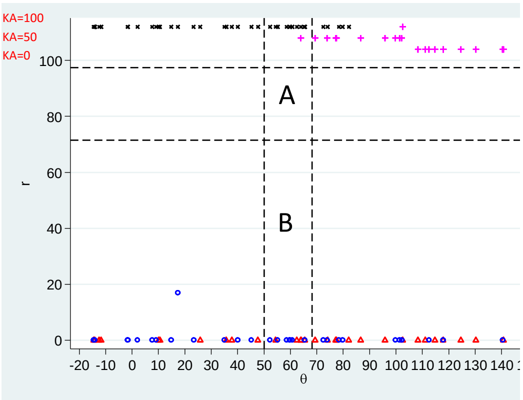
	Groups										Groups								
r	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**	**r**	**1**	**2**	**3**	**4**	**5**	**6**	**7**	**8**	**9**
0.00	59	10	54	60	44	15	35	34	40	**13.00**	0	0	0	0	0	0	1	0	0
0.50	0	0	0	0	2	0	0	0	0	**13.14**	0	0	0	0	0	0	0	0	1
0.60	0	1	0	0	0	0	0	0	0	**13.44**	0	0	0	0	0	0	1	0	0
0.69	0	0	0	0	0	37	0	0	0	**14.00**	0	0	0	0	0	0	1	0	0
1.00	0	4	3	0	0	0	0	0	1	**14.50**	0	0	0	0	0	0	1	0	0
1.50	0	1	0	0	2	0	0	0	0	**15.00**	0	1	0	0	0	1	1	0	0
1.60	0	1	0	0	0	0	0	0	0	**16.00**	0	0	0	0	0	0	0	0	1
2.00	0	4	2	0	1	0	0	0	1	**17.00**	1	0	0	0	0	0	0	0	0
2.30	0	1	0	0	0	0	0	0	0	**18.00**	0	1	0	0	0	0	0	0	0
2.40	0	1	0	0	0	0	0	0	0	**20.00**	0	1	0	0	0	3	0	0	1
2.50	0	3	0	0	0	0	0	0	0	**25.00**	0	1	0	0	0	0	0	0	0
2.80	0	1	0	0	0	0	0	0	0	**30.00**	0	1	0	0	1	1	0	5	0
3.00	0	2	0	0	2	0	0	0	1	**36.00**	0	1	0	0	0	0	0	0	0
4.00	0	1	0	0	0	0	0	0	0	**40.00**	0	0	0	0	0	0	0	0	2
5.00	0	1	1	0	6	0	6	0	0	**46.01**	0	0	0	0	0	0	0	0	1
5.40	0	1	0	0	0	0	0	0	0	**46.19**	0	0	0	0	0	0	0	0	1
6.00	0	1	0	0	0	0	0	0	0	**47.00**	0	0	0	0	0	0	0	0	1
6.50	0	1	0	0	0	0	1	0	0	**50.00**	0	1	0	0	0	1	0	0	1
6.90	0	1	0	0	0	1	0	0	0	**51.00**	0	0	0	0	0	0	0	1	0
7.45	0	1	0	0	0	0	0	0	0	**54.00**	0	0	0	0	0	0	0	0	1
8.59	0	0	0	0	1	0	0	0	0	**59.00**	0	0	0	0	0	0	0	0	1
8.90	0	0	0	0	0	0	1	0	0	**60.00**	0	0	0	0	0	1	0	6	1
9.29	0	1	0	0	0	0	0	0	0	**60.89**	0	0	0	0	0	0	0	0	1
10.00	0	4	0	0	1	0	7	4	1	**69.00**	0	1	0	0	0	0	0	0	0
10.36	0	1	0	0	0	0	0	0	0	**70.00**	0	0	0	0	0	0	0	8	1
10.50	0	1	0	0	0	0	0	0	0	**80.00**	0	0	0	0	0	0	0	2	0
11.00	0	2	0	0	0	0	1	0	0	**85.00**	0	1	0	0	0	0	0	0	0
11.25	0	0	0	0	0	0	1	0	0	**100.00**	0	1	0	0	0	0	0	0	0
11.50	0	1	0	0	0	0	0	0	0	**150.00**	0	0	0	0	0	0	0	0	1
12.00	0	2	0	0	0	0	2	0	0	**151.00**	0	1	0	0	0	0	0	0	0
12.07	0	0	0	0	0	0	1	0	0	**180.00**	0	1	0	0	0	0	0	0	0
12.15	0	0	0	0	0	0	0	0	1	**Total**	**60**	**60**	**60**	**60**	**60**	**60**	**60**	**60**	**60**

APPENDIX E3: Figures (policy maker policies, size of the attacks and final regimes)

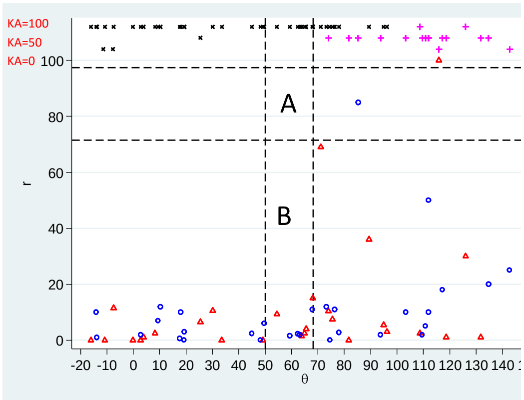
Policies r chosen in the experiment per type of policy maker θ, group and variance σ: In the following figures the vertical lines delimit the range $\theta \in (50.00, 68.18)$ and the horizontal lines delimit the range $r \in (71.43, 97.40]$. In the figures the red triangles (Δ), the blue dots (o), and the green squares (□) correspond to the policies r chosen in the rounds in which $\sigma = 10$, $\sigma = 15$ and $\sigma = 20$ respectively.

Size of the attacks KA and final regime obtained in the experiment, per type of policy maker θ, group and variance σ: The two kind of symbols that are the top of all figures tell us for the different types of policy makers and groups in which cases the status quo survives (+) and in which cases there were a regime change (**x**). In addition, notice that the symbols "+" and "**x**" are divided depending on the size of the attack $KA \in \{0, 50, 100\}$

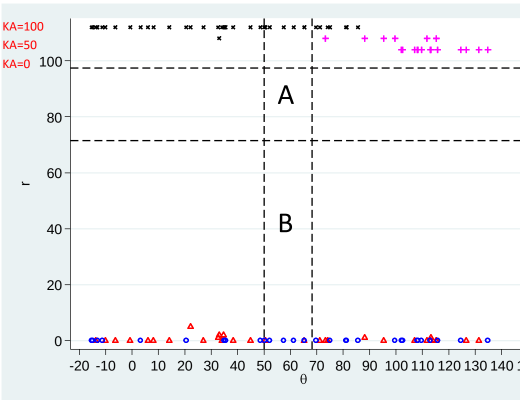
Group 1



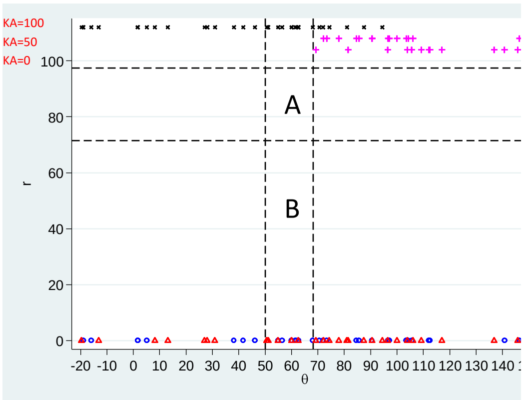
Group 2[127]



Group 3

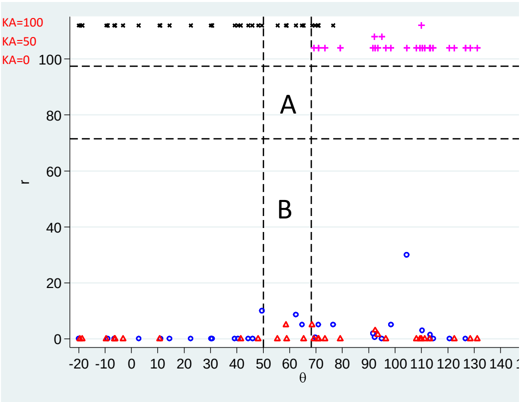


Group 4

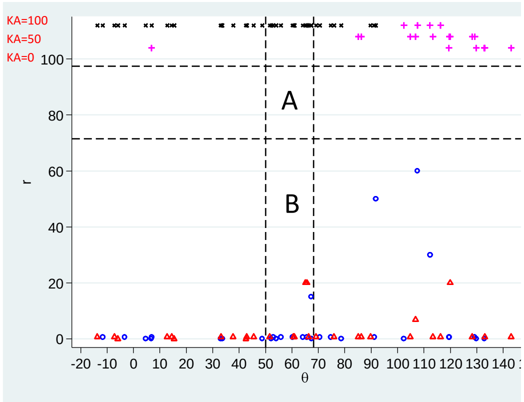


[127] Note: There are two red triangles (θ, r) that were not included in the Figure: (-7.78, 180) and (-11.38,151)

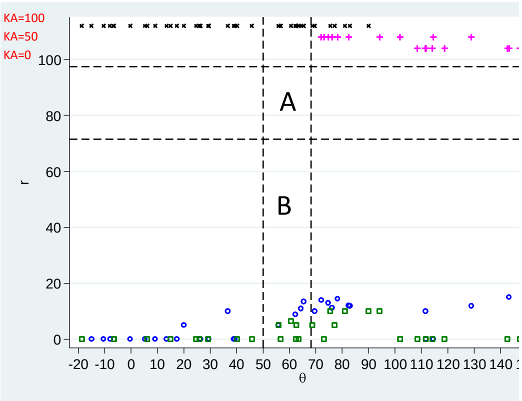
Group 5



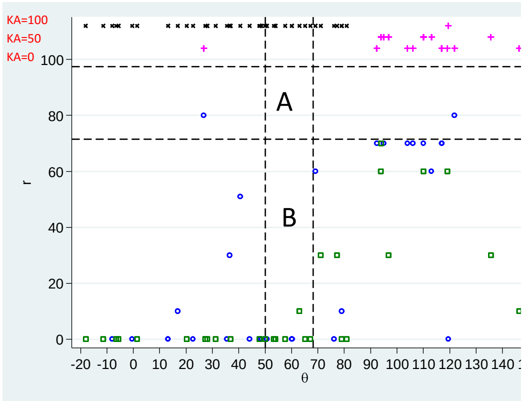
Group 6



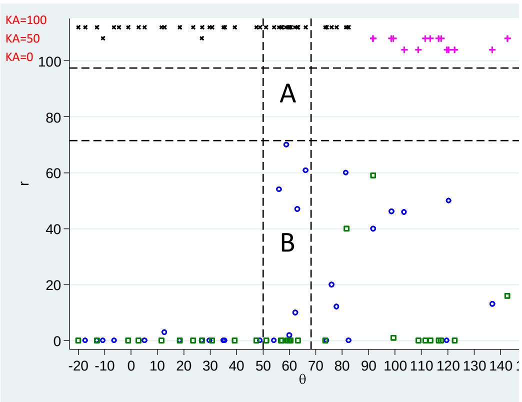
Group 7



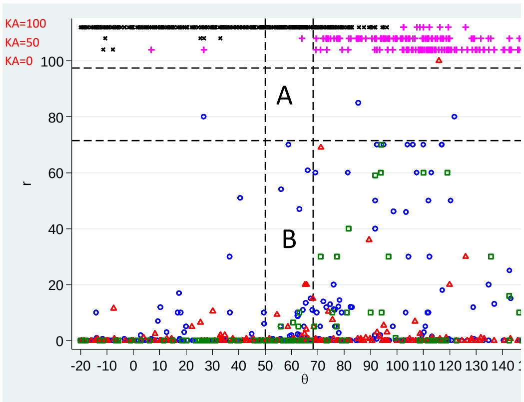
Group 8



Group 9[128]



All Groups[129]



128 Note: There is one green square (θ, r) that were not included in the Figure: (-4.59,150)

129 Note: There are two red triangles (θ, r) that were not included in the Figure: (-7.78,180) and (-11.38, 151). There is one green square (θ, r) that were not included in the Figure: (-4.59, 150)

Appendix F3: Figures (choices of the agents, size of the attacks and final regimes)

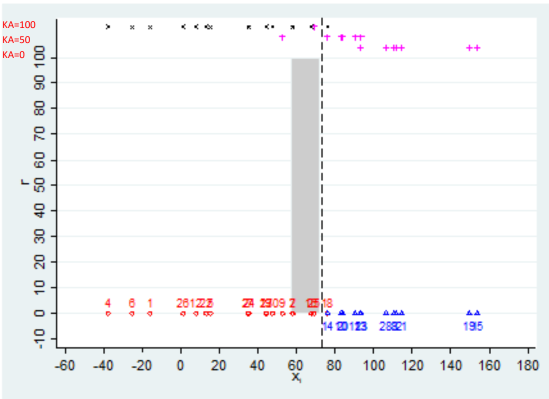
Choices of the agents in the experiment per policy, signal, round and σ: In the figures of this section, given the policy r taken by the policy maker and the private signal x_i the choices of the agents per round are represented by the following symbols:

- A red dot "o" means that the agent has chosen $a = 1$, and
- A blue triangle "Δ" means that the agent has chosen $a = 0$

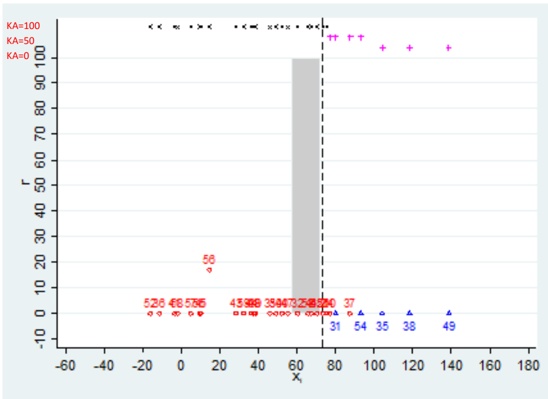
Above each symbol you can see the round in which the choice was taken. In addition, given σ, in all figures you can also see the threshold of the pooling equilibrium (i.e. $x^{\#}(\sigma)$) represented by a dashed line and the threshold of the semiseparating equilibria (i.e. $x_S^*(\sigma)$) represented by the shadow area.

Size of the attacks KA and final regimes in the experiment per signal, agent and variance σ: The two kind of symbols that are the top of all figures tell us for the different types of signals and agents in which cases the status quo survives (+) and in which cases there were a regime change (**x**). Furthermore, notice that the symbols "+" and "**x**" are divided depending on the size of the attack $KA \in \{0, 50, 100\}$

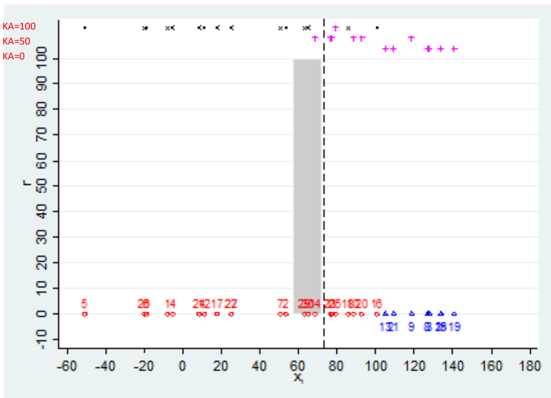
Group 1 - Participant 2 - $\sigma = 10$



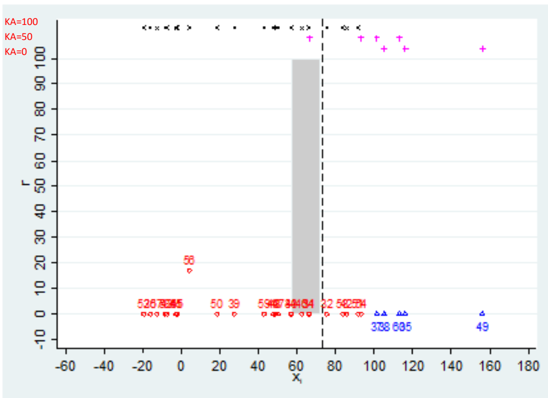
Group 1 - Participant 2 - $\sigma = 15$



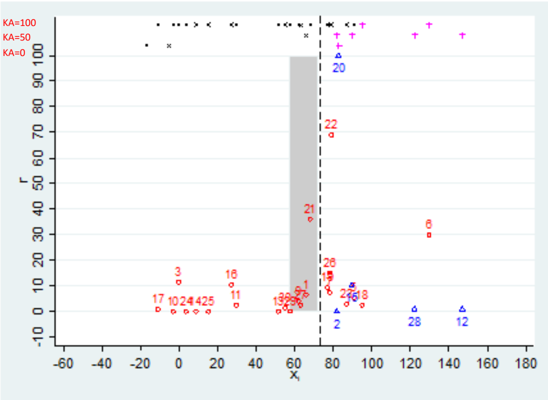
Group 1 - Participant 3 - $\sigma = 10$



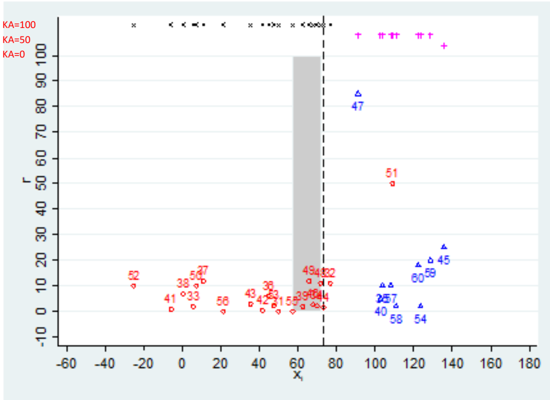
Group 1 - Participant 3 - $\sigma = 15$



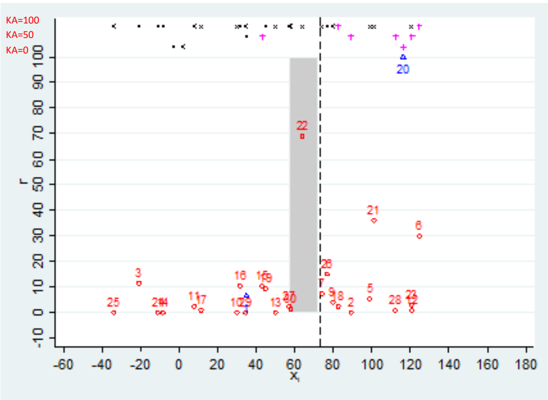
Group 2 - Participant 5 - $\sigma = 10$[130]



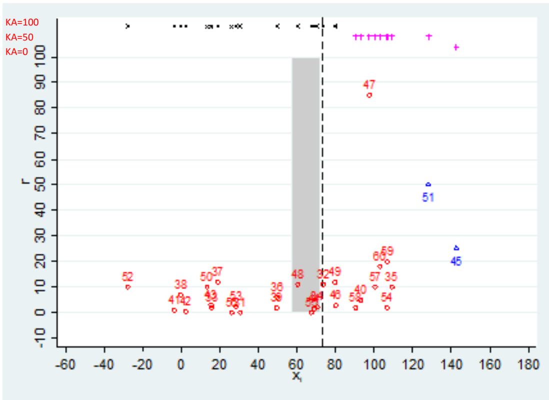
Group 2 - Participant 5 - $\sigma = 15$



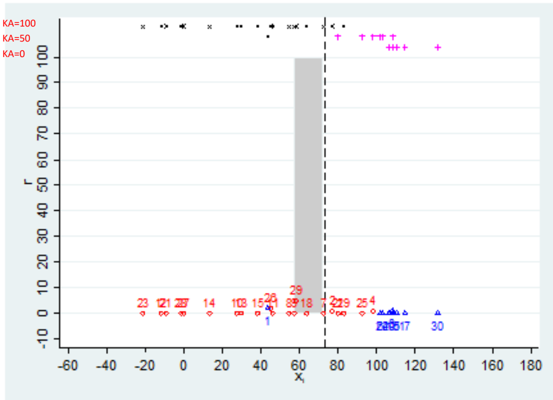
Group 2 - Participant 6 - $\sigma = 10$[131]



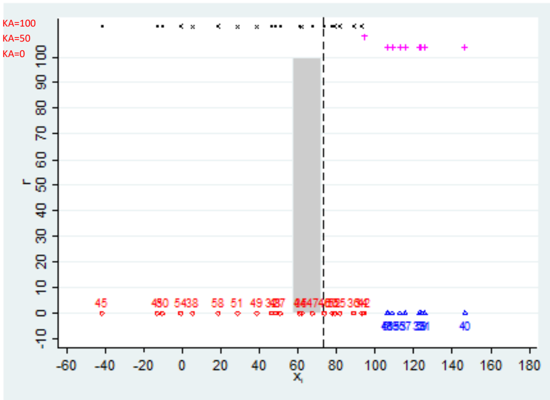
Group 2 - Participant 6 - $\sigma = 15$



Group 3 - Participant 8 - $\sigma = 10$



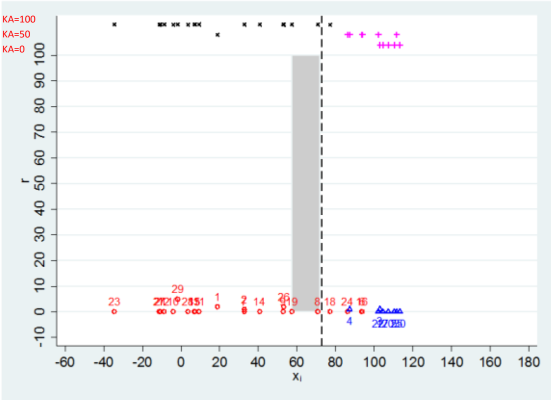
Group 3 - Participant 8 - $\sigma = 15$



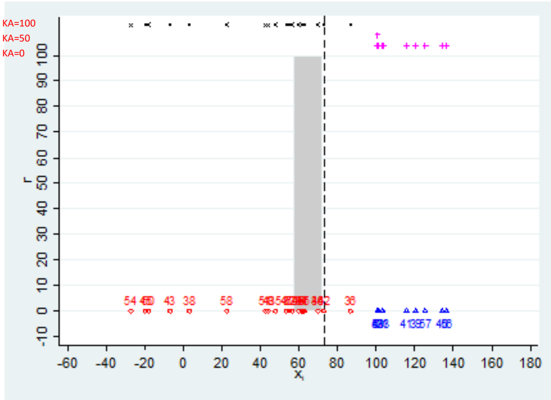
[130] Note: There are two red circles $(x_i, r, period)$ that were not included in the Figure: (-5.47, 180,8) and (-17.000,151,4)

[131] Note: There are two red circles $(x_i, r, period)$ that were not included in the Figure: (1.82, 180,8) and (-3.13,151,4)

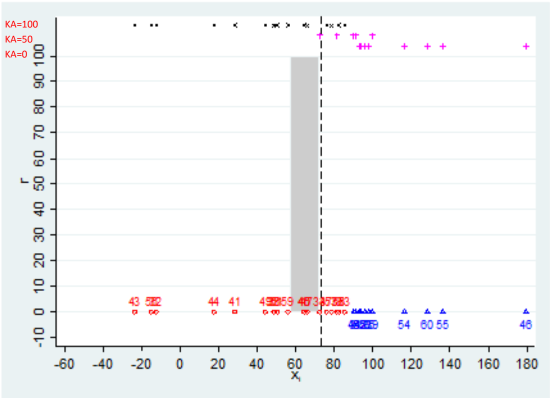
Group 3 - Participant 9 - $\sigma = 10$



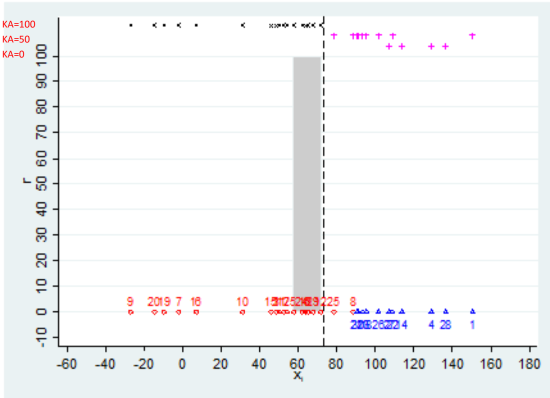
Group 3 - Participant 9 - $\sigma = 15$



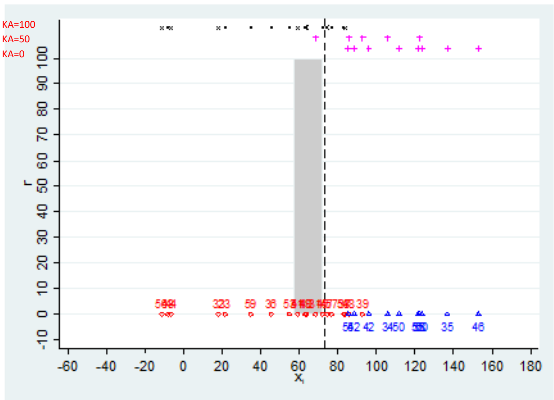
Group 4 - Participant 11 - $\sigma = 10$



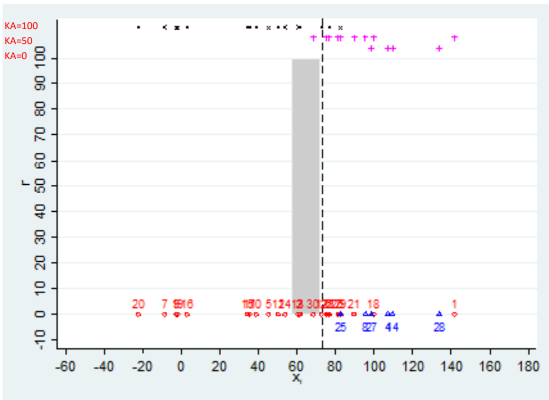
Group 4 - Participant 11 - $\sigma = 15$



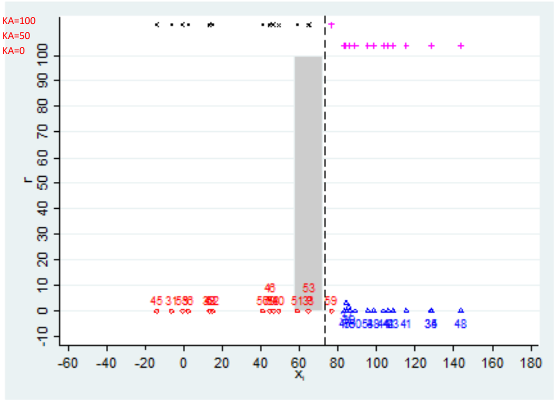
Group 4 - Participant 12 - $\sigma = 10$



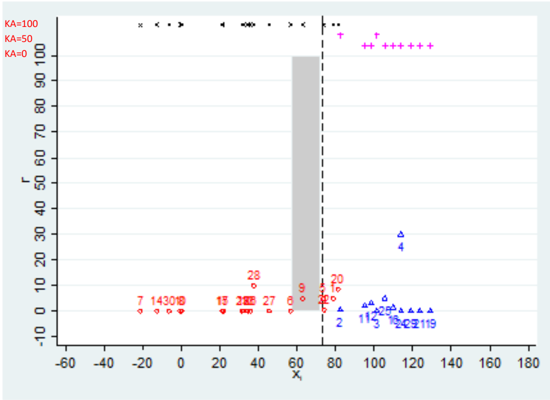
Group 4 - Participant 12 - $\sigma = 15$



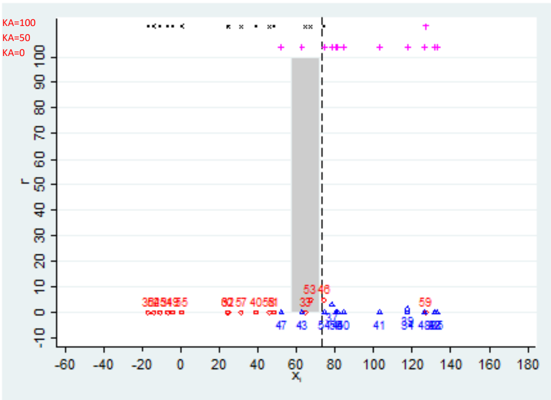
Group 5 - Participant 14 - σ = 10



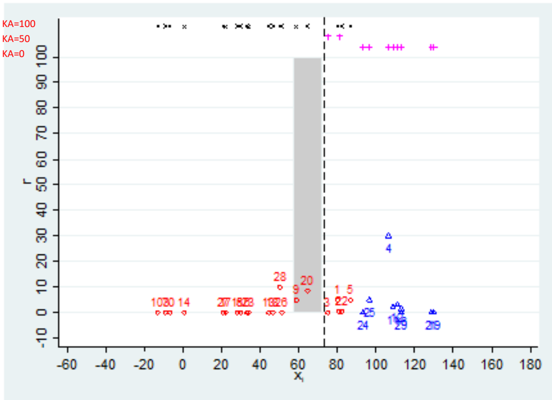
Group 5 - Participant 14 - σ = 15



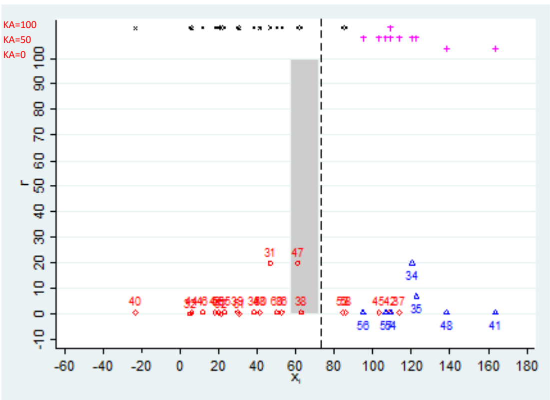
Group 5 - Participant 15 - σ = 10



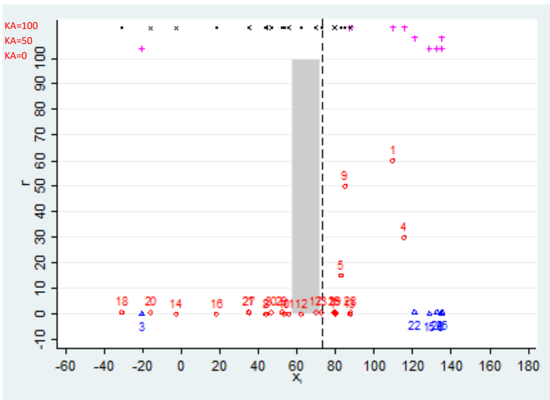
Group 5 - Participant 15 - σ = 15



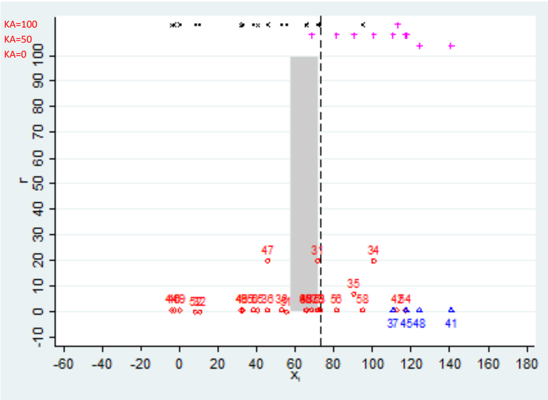
Group 6 - Participant 17 - σ = 10



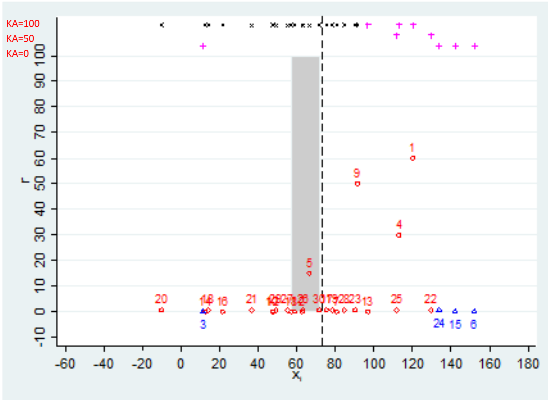
Group 6 - Participant 17 - σ = 15



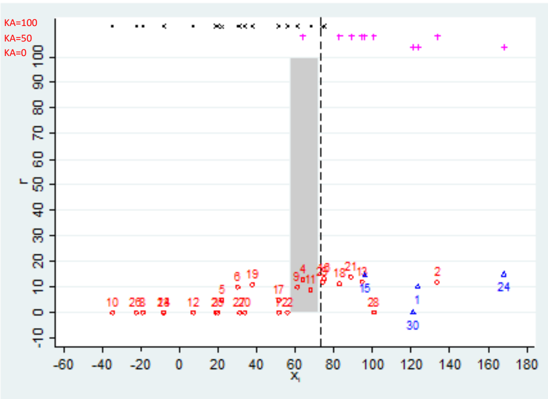
Group 6 - Participant 18 - $\sigma = 10$



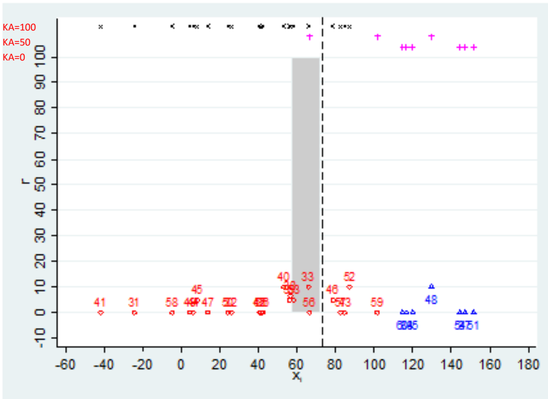
Group 6 - Participant 18 - $\sigma = 15$



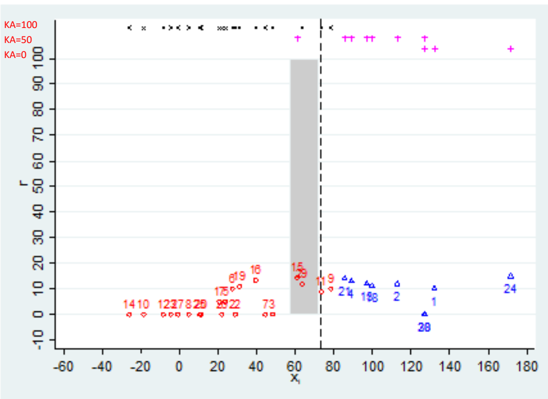
Group 7 - Participant 20 - $\sigma = 15$



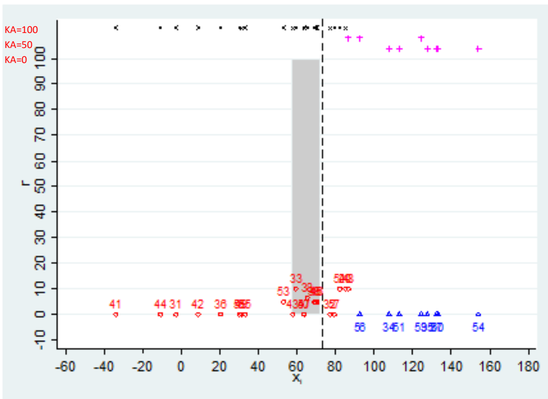
Group 7 - Participant 20 - $\sigma = 20$



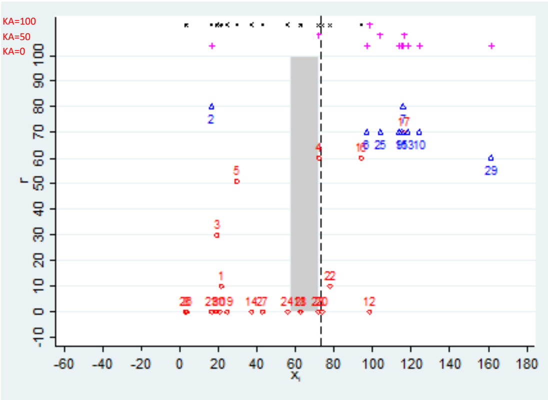
Group 7 - Participant 21 - $\sigma = 15$



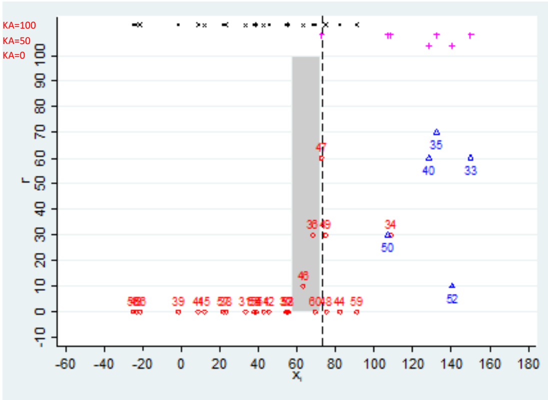
Group 7 - Participant 21 - $\sigma = 20$



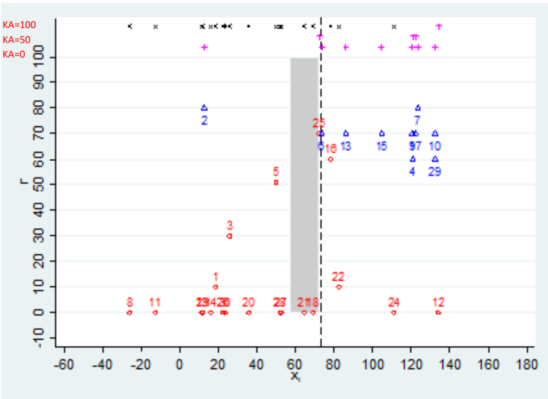
Group 8 - Participant 23 - $\sigma = 15$



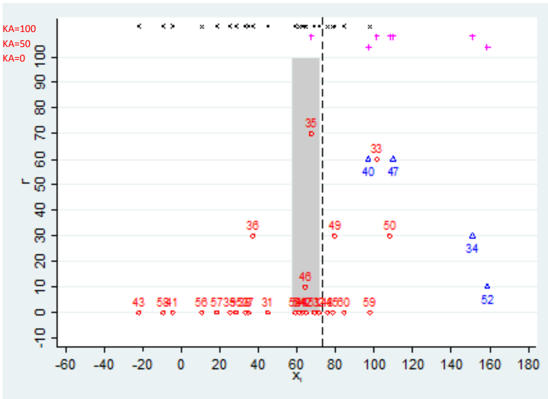
Group 8 - Participant 23 - $\sigma = 20$



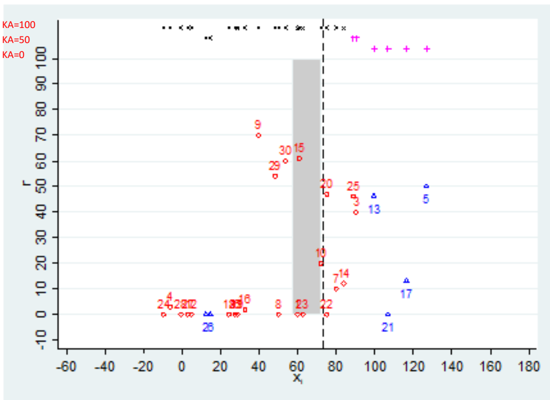
Group 8 - Participant 24 - $\sigma = 15$



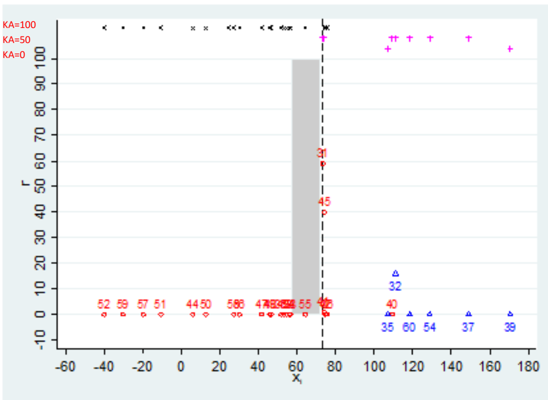
Group 8 - Participant 24 - $\sigma = 20$



Group 9 - Participant 26 - $\sigma = 15$

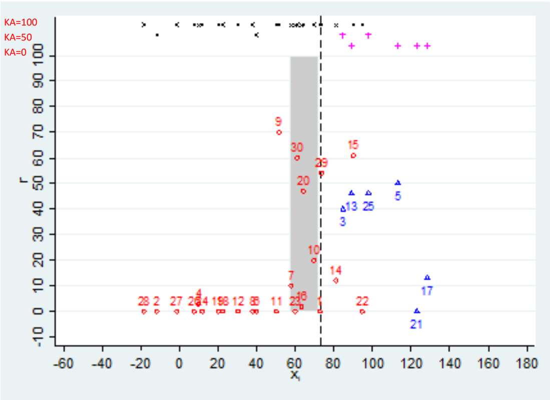


Group 9 - Participant 26 - $\sigma = 20$[132]

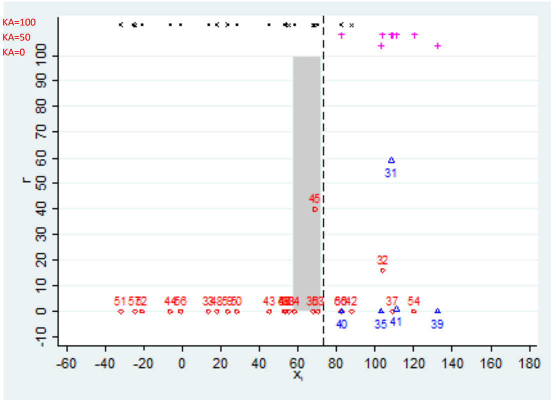


[132] Note: There is one red circle $(x_i, r, period)$ that were not included in the Figure: (24.46,150,46)

Group 9 - Participant 27 - $\sigma = 15$



Group 9 - Participant 27 - $\sigma = 20$[133]



[133] Note: There is one red circle $(x_i, r, period)$ that were not included in the Figure: (-25.47,150,46)

Appendix G3: Other tables

Table G3.1
Policies chosen by the policy makers, choices of the agents
and final regimes per σ and θ

	$r = 0$		$r > 0$		$a = 0$		$a = 1$	
	Regime Change	**Status Quo Survives**	**Regime Change**	**Status Quo Survives**	**Regime Change**	**Status Quo Survives**	**Regime Change**	**Status Quo Survives**
	$\theta \leq 0$							
$\sigma = 10$	80.00%	0.00%	20.00%	0.00%	38.67%	0.00%	61.33%	0.00%
$\sigma = 15$	85.19%	0.00%	14.81%	0.00%	34.57%	0.00%	65.43%	0.00%
$\sigma = 20$	90.91%	0.00%	9.09%	0.00%	33.33%	0.00%	66.67%	0.00%
	$\theta \epsilon (0,50]$							
$\sigma = 10$	65.00%	0.00%	35.00%	0.00%	35.00%	0.00%	65.00%	0.00%
$\sigma = 15$	75.00%	1.32%	22.37%	1.32%	32.89%	2.63%	64.47%	0.00%
$\sigma = 20$	100.00%	0.00%	0.00%	0.00%	33.33%	0.00%	66.67%	0.00%
	$\theta \epsilon (50,70]$							
$\sigma = 10$	45.45%	12.12%	42.42%	0.00%	29.29%	10.10%	58.59%	2.02%
$\sigma = 15$	52.83%	0.00%	47.17%	0.00%	33.33%	0.00%	66.67%	0.00%
$\sigma = 20$	76.19%	0.00%	23.81%	0.00%	33.33%	0.00%	66.67%	0.00%
	$\theta \epsilon (70, 100)$							
$\sigma = 10$	23.08%	43.59%	17.95%	15.38%	13.68%	46.15%	27.35%	12.82%
$\sigma = 15$	26.92%	19.23%	26.92%	26.92%	17.95%	32.69%	35.90%	13.46%
$\sigma = 20$	17.65%	5.88%	41.18%	35.29%	19.61%	27.45%	39.22%	13.73%
	$\theta \geq 100$							
$\sigma = 10$	0.00%	67.44%	0.00%	32.56%	0.00%	83.72%	0.00%	16.28%
$\sigma = 15$	0.00%	51.61%	0.00%	48.39%	0.00%	86.02%	0.00%	13.98%
$\sigma = 20$	0.00%	72.22%	0.00%	27.78%	0.00%	85.19%	0.00%	14.81%

Table G3.2

Percentage of times the status quo survives (and percentage of times there is a regime change) given the values of θ used in the experiment

θ	Theory*		Experiment	
	Status quo survives	**There is a Regime Change**	**Status quo survives**	**There is a Regime Change**
$(-\infty, 0]$	(0.00% - 0.00%)	(100.00% - 100.00%)	0.00%	100.00%
$(0, 10]$	(0.00% - 0.00%)	(100.00% - 100.00%)	4.00%	96.00%
$(10, 20]$	(0.00% - 0.00%)	(100.00% - 100.00%)	0.00%	100.00%
$(20, 30]$	(0.00% - 0.00%)	(100.00% - 100.00%)	0.00%	100.00%
$(30, 40]$	(0.00% - 0.00%)	(100.00% - 100.00%)	4.17%	95.83%
$(40, 50]$	(0.00% - 0.00%)	(100.00% - 100.00%)	0.00%	100.00%
$(50, 60]$	(97.73% - 15.91%)	(2.27% - 84.09%)	0.00%	100.00%
$(60, 70]$	(90.48% - 41.27%)	(9.52% - 58.73%)	6.35%	93.65%
$(70, 80]$	(96.00% - 76.00%)	(4.00% - 24.00%)	32.00%	68.00%
$(80, 90]$	(100.00% - 79.17%)	(0.00% - 20.83%)	41.67%	58.33%
$(90, 100]$	(100.00% - 100.00%)	(0.00% - 0.00%)	82.35%	17.65%
$(100, +\infty)$	(100.00% - 100.00%)	(0.00% - 0.00%)	100.00%	0.00%
Total	**(60.93%-45.74%)**	**(39.07%-54.23%)**	**33.89%**	**66.11%**

*In these two columns we are taking into account that in the experiment $\theta_S^* \in (50, 68.18]$; therefore, inside the parenthesis we are showing two examples: (1) when $\theta_S^* = 50$ and (2) when $\theta_S^* = \theta^{\#} = 68.18$

Table G3.3

Dependent Variable	$Dummy\ (r > 0)$ Odds Ratio		r Coefficient	
$\boldsymbol{\theta}$	1.018	***	0.106	
	(0.006)		(0.069)	
$Period$	0.972		-0.348	
	(0.06)		(0.176)	
$Dummy\ (\sigma = 10)$	0.850		-0.582	
	(0.540)		(2.298)	
$Dummy\ (\sigma = 20)$	0.152	*	3.444	
	(0.119)		(0.315)	
$Dummy\ of\ order$[134]	3.711		1.418	
	(3.157)		(3.262)	
$Constant$	0.646		12.656	*
	(0.286)		(3.966)	
$Observations$	336		336	

Notes: Robust standard errors in parentheses. Standard errors are clustered by group. Significance levels: $^+$p<0.10 *p<0.05; **p<0.01; ***p<0.001. In these regressions we did not use the information of the groups that followed the pooling equilibrium strategies (i.e. the groups 1, 3 and 4).

[134] This dummy is equal to 1 in the first half of the rounds and 0 otherwise.

Table G3.4

Dependent Variable: a

	Odds Ratio		Odds Ratio	
$\boldsymbol{r}$	0.975	**	0.975	**
	(0.009)		(0.009)	
$\boldsymbol{x}$	0.934	***	0.934	***
	(0.009)		(0.009)	
$\boldsymbol{\sigma}$	1.142	**		
	(0.048)			
$\boldsymbol{Dummy\ (\sigma = 10)}$			0.531	***
			(0.098)	
$\boldsymbol{Dummy\ (\sigma = 20)}$			2.052	+
			(0.866)	
$\boldsymbol{Dummy\ of\ order}$	0.853		0.840	
	(0.156)		(0.186)	
$\boldsymbol{Constant}$	165.158		1190.64	
	(118.815)	***	(901.044)	***
$\boldsymbol{Observations}$	1080		1080	
$\boldsymbol{LL}$	-301.94		-301.92	

Notes: Robust standard errors in parentheses. Standard errors are clustered at the subject level. Significance levels: $^{+}p<0.10$ $^{*}p<0.05$; $^{**}p<0.01$; $^{***}p<0.001$.

References

ABEL, Andrew B.; MANKIW, N. Gregory; SUMMERS, Lawrence H. and ZECHHAUSER, Richard J. "Assessing Dynamic Efficiency: Theory and Evidence," Review of Economic Studies, Vol. 56, No. 1 (January, 1989), p. 1-19.

ABREU, Dilip and BRUNNERMEIER, Markus K. "Bubbles and Crashes"; Econometrica, Vol. 71, No. 1 (January, 2003), p.173-204.

ALLEN, Franklin and GORTON Gary. "Churning Bubbles" Review of Economic Studies, Vol. 60, No. 4 (October, 1993), p.813-836.

ALLEN, Franklin; MORRIS, Stephen and POSTLEWAITE, Andrew. "Finite Bubbles with Short Sale Constraints and Asymmetric Information" Journal of Economic Theory, Vol. 61, No. 2 (December, 1993); p.206-229.

ANGELETOS, George-Marios; HELLWIG, Christian and PAVAN, Alessandro. "Signaling in a Global Game: Coordination and Policy Traps" Journal of Political Economy, Vol. 114, No. 3 (June, 2006); p. 452–484.

ANGELETOS, George-Marios; HELLWIG, Christian and PAVAN, Alessandro. "Dynamic Global Games of Regime Change: Learning, Multiplicity, and the Timing of Attacks" Econometrica, Vol. 75, No. 3 (May, 2007); p. 711–756

ANGELETOS, George-Marios and PAVAN; Alessandro. "Selection-Free Predictions in Global Games with Endogenous Information and Multiple Equilibria" Theoretical Economics, Vol. 8, No. 3 (September, 2013); p. 883–938

ANGELETOS, George-Marios and WERNING, Iván. "Crises and Prices: Information Aggregation, Multiplicity and Volatility" American Economic Review, Vol. 96, No. 5 (December, 2006); p. 1720–1736.

ARELLANO, Manuel and BOND, Stephen. "Some Tests of Specification for Panel Data: Monte Carlo Evidence and an Application to Employment Equations". Review of Economic Studies, Vol. 58, No. 2 (April, 1991) p.277-297

ARELLANO, Manuel and BOVER, Olympia. "Another Look at the Instrumental Variable Estimation of Error-Components Models". Journal of Econometrics, Vol. 68, No.1 (July, 1995) p-29-51

BLANCHARD, Olivier J. and WATSON Mark W. "Bubbles, Rational Expectations and Financial Markets" Crises in the Economic and Financial Structure, Paul Wachtel, editor. Lexington, MA: D.C. Heathand Company, (1982); p. 295-316.

BOIVIN, Jean; GIANNONI, Marc and MIHOV, Ilian. "Sticky Prices and Monetary Policy: Evidence from Disaggregated U.S. Data" American Economic Review. Vol. 99, No. 1 (March, 2009); p. 350-384.

BRUNNERMEIER Markus K. and MORGAN, John. “Clock Games: Theory and Experiments” Games and Economic Behavior, Vol. 68, No. 2 (March, 2010) p.532–550

CABRALES, Antonio; NAGEL, Rosemarie and ARMENTER, Roc. “Equilibrium selection through incomplete information in coordination games: an experimental study” Experimental Economics, Vol. 10, No. 3 (September, 2007); p. 221-234

CAMERON, Colin and TRIVEDI, Pravin. “Microeconometrics Using STATA” Stata Press (2009), 692 p.

CAÑON, Carlos and MARGARETIC, Paula “Correlated Bank Runs, Interbank Markets and Reserve Requirements” Journal of Banking & Finance, Vol. 49 (December, 2014); p. 515-533.

CAPLIN, Andrew and DEAN, Mark. “Rational Inattention, Entropy, and Choice: The Posterior-Based Approach” Working Paper (July, 2013a); 51 p.

CAPLIN, Andrew and DEAN, Mark. “The Behavioral Implications of Rational Inattention with Shannon Entropy” Working Paper (August, 2013b); 40 p.

CAPLIN, Andrew and DEAN, Mark. “Revealed Preference, Rational Inattention, and Costly Information Acquisition” American Economic Review, Vol. 105, No. 7 (July, 2015); p. 2183-2203.

CAPLIN, Andrew and MARTIN, Daniel. “Defaults and Attention: The Drop Out Effect” Working Paper (June, 2013); 24 p.

CARLSSON, Hans and VAN DAMME, Eric. “Global Games and Equilibrium Selection” Econometrica, Vol. 61, No. 5 (September, 1993); p. 989–1018

CARSWELL, John. “The South Sea Bubble” London: Cresset Press (1960); Cited by KINDLEBERGER and ALIBER (2005).

CHAMLEY, Christophe. “Coordinating Regime Switches” Quarterly Journal of Economics, Vol. 114, No. 3 (August, 1999); p. 869–905

CHASSANG, Sylvain. “Fear of Miscoordination and the Robustness of Cooperation in Dynamic Global Games with Exit” Econometrica, Vol. 78, No. 3 (May, 2010); p. 973–1006

CHEREMUKHIN, Anton; POPOVA, Anna and TUTINO, Antonella. Journal of Economic Behavior & Organization, Vol. 113, (May, 2015); p. 34-50.

COOPER, Russell; DEJONG, Douglas; FORSYTHE, Robert and ROSS; Thomas. “Communication in the Battle of the Sexes Game: Some Experimental Results” The RAND Journal of Economics, Vol. 20, No. 4 (Winter, 1989); p. 568-587.

CORNAND, Camille and HEINEMANN, Frank "Measuring Agents' Reaction to Private and Public Information in Games with Strategic Complementarities" Experimental Economics, Vol. 17, No. 1 (March, 2014); p. 61-77.

CORSETTI, Giancarlo; DASGUPTA, Amil Stephen; MORRIS, Stephen and SHIN, Hyun Song "Does One Soros Make a Difference? A Theory of Currency Crises with Large and Small Traders" Review of Economic Studies, Vol. 71, No. 1 (January, 2004); p. 87–113

CORSETTI, Giancarlo; GUIMARÃES, Bernardo and ROUBINI, Nouriel "International Lending of Last Resort and Moral Hazard: A Model of IMF's Catalytic Finance" Journal of Monetary Economics, Vol. 53, No. 3 (April, 2006); p. 441–471

CRAWFORD, Vincent. "A Survey of Experiments on Communication via Cheap Talk" Journal of Economic Theory, Vol. 78, No. 2 (February, 1998); p. 286-298

CRAWFORD, Vincent and SOBEL, Joel. "Strategic Information Transmission" Econometrica, Vol. 50, No. 6 (November, 1982); p. 1431-1451

CROCKET, Sean and DUFFY, John. "An Experimental Test of the Lucas Asset Pricing Model", Working Paper, (December, 2015); 76 p.

CUKIERMAN, Alex; GOLDSTEIN, Itay and SPIEGEL, Yossi "The Choice of Exchange-Rate Regime and speculative attacks. Journal of the European Economic Association, Vol. 2, No. 6 (December, 2004); p. 1206–1241.

DASGUPTA, Amil "Coordination and Delay in Global Games" Journal of Economic Theory, Vol. 134, No. 1 (May, 2007); p. 195–225

DE LONG, J. Bradford; SHLEIFER, Andrei; SUMMERS, Lawrence H. and WALDMANN, Robert J. "Noise Trader Risk in Financial Markets" Journal of Political Economy, (August, 1990); p. 703-738.

DEWAN, Torun and MYATT, David "Leading the party: Coordination, Direction, and Communication" American Political Science Review, Vol. 101, No. 4 (November, 2007); p. 827–845

DUFFY, John and FISHER, Eric O'N. "Sunspots in the Laboratory," American Economic Review, Vol.95, No.3 (June, 2005) p.510—529.

EDMOND, Chris. "Information Manipulation, Coordination, and Regime Change" Review of Economic Studies, Vol. 80. No. 4 (October, 2013); p. 1422-1458.

FARHI, Emmanuel and TIROLE, John "Bubbly liquidity", Review of Economic Studies, Vol. 79, No. 2 (April, 2012); p. 678-706.

FRANKEL, David. "Recurrent Crises in Global Games" Journal of Mathematical Economics, Vol. 48, No. 5 (October, 2012); p. 309-321.

FRANKEL, David; MORRIS, Stephen and PAUZNER, Ady. “Equilibrium Selection in Global Games with Strategic Complementarities” Journal of Economic Theory, Vol. 108, No. 1 (January, 2003); p. 1–44.

FUJIMOTO, Junichi. “Speculative Attacks with Multiple Targets” Economic Theory, Vol. 57 (2014); p. 89-132

GOLDSTEIN, Itay and PAUZNER, Ady. “Contagion of Self-Fulfilling Financial Crises due to Diversification of Investment Portfolios” Journal of Economic Theory, Vol. 119, No. 1 (November, 2004); p. 151–183

GOLDSTEIN, Itay and PAUZNER, Ady. “Demand-Deposit Contracts and the Probability of Bank Runs” Journal of Finance, Vol. 60, No. 3 (June, 2005); p. 1293–1327.

GOLDSTEIN, Itay. “Strategic Complementarities and the Twin Crises” Economic Journal, Vol. 115, Issue. 503 (December, 2005); p. 368-390.

GOLTSMAN, Maria and Pavlov, Gregory. “Communication in Cournot Oligopoly” Journal of Economic Theory, Vol. 153 (September, 2014); p. 152-176

GREENSPAN, Alan. “The Challenge of Central Banking in a Democratic Society” Remarks by Chairman Alan Greenspan at the Annual Dinner and Francis Boyer Lecture of the American Enterprise Institute for Public Policy Research, Washington, D.C. (December 5, 1996) http://www.federalreserve.gov/boarddocs/speeches/1996/19961205.htm

GUIMARÃES, Bernardo and MORRIS, Stephen “Risk and Wealth in a Model of Self-Fulfilling currency Attacks” Journal of Monetary Economics, Vol. 54, No. 8 (November, 2007); p. 2205–2230.

HEIDHUES, Paul and MELISSAS, Nicolas “Equilibria in a Dynamic Global Game: The Role of Cohort Effects” Economic Theory, Vol. 28, No. 3 (August, 2006); p. 531–557

HEINEMANN, Frank; NAGEL, Rosemarie and OCKENFELS, Peter. “The Theory of Global Games on Test: Experimental Analysis of Coordination Games with Public and Private Information” Econometrica, Vol. 72, No. 5, (September, 2004); p. 1583-1599.

HELLWIG, Christian; MUKHERJI, Arijit and TSYVINSKI, Aleh “Self-Fulfilling Currency Crises: The Role of Interest Rates” American Economic Review, Vol. 96, No. 5 (December, 2006); p. 1769–1787

JIN, Yu. “Bank Monitoring and Liquidity in the Secondary Market for Loans”. Working Paper. (October, 2011)

KANG, Min Jeong; RAY, Debajyoti and CAMERER Colin. “Measured Anxiety and Choices in Experimental Timing Games” Working Paper (2012); 57 p.

KINDLEBERGER, Charles and ALIBER, Robert. “Manias, Panics, and Crashes: A History of Financial Crises” Fifth Edition, John Wiley & Sons, Inc (2005); 355 p.

KÜBLER, Dorothea; MÜLLER, Wieland and NORMANN, Hans-Theo “Job-Market Signaling and Screening: An Experimental Comparison” Games and Economic Behavior, Vol. 64, No. 1 (September, 2008) p. 219-236

LEI, Vivian; NOUSSAIR, Charles N. and POTT, Charles R. “Nonspeculative Bubbles in Experimental Asset Markets: Lack of Common Knowledge of Rationality vs. Actual Irrationality” Econometrica, Vol. 69, No. 4 (July, 2001), p.831-859.

LUCAS, Robert E. Jr. “Asset prices in an exchange economy,” Econometrica, Vol. 46, No. 6 (November, 1978), p.1429–1445.

MAĆKOWIAK, Bartosz and WIEDERHOLT, Mirko. “Optimal Sticky Prices under Rational Inattention” American Economic Review, Vol. 99, No. 3 (June, 2009); p. 769-803.

MAĆKOWIAK, Bartosz and WIEDERHOLT, Mirko. “Business Cycle Dynamics under Rational Inattention”, Review of Economic Studies, Vol. 82, No. 4 (October, 2015); p. 1502–1532.

MAĆKOWIAK, Bartosz; MOENCH, Emanuel and WIEDERHOLT, Mirko. “Sectoral price data and models of price setting” Journal of Monetary Economics, Vol. 56, Supplement (October, 2009); p. S78-S99.

MANKIW, Gregory and REIS, Ricardo “Sticky Information Versus Sticky Prices: A Proposal to Replace the New Keynesian Phillips Curve” Quarterly Journal of Economics, Vol. 117, No. 4 (November, 2002); p. 1295-1328.

MARTIN, Alberto and VENTURA, Jaume, “Economic Growth with Bubbles” American Economic Review, Vol. 102, No. 6 (October, 2012), p. 3033-3058.

MARTIN, Daniel. “Strategic Pricing and Rational Inattention to Quality” Working Paper (December, 2015); 19 p

MILGROM, Paul and SHANNON, Chris. “Monotone comparative statics” Econometrica, Vol. 62, No. 1 (January, 1994); p. 157-180.

MORRIS, Stephen. “Global Games” in The New Palgrave Dictionary of Economics (DURLAUF, Stephen and BLUME, Lawrence, editors), second edition, Vol. 3 (2008); p. 676-681.

MORRIS, Stephen and SHIN, Hyun Song. “Unique Equilibrium in a Model of Self-Fulfilling Currency Attacks” American Economic Review, Vol. 88, No. 3 (June, 1998); p. 587–597.

MORRIS, Stephen and SHIN, Hyun Song “Social Value of Public Information” American Economic Review, Vol. 92, No. 5 (December, 2002); p. 1522-1534.

MORRIS, Stephen and SHIN, Hyun Song. “Global games: Theory and Applications” in Advances in Economics and Econometrics (DEWATRIPONT, Mathias; HANSEN, Lars Peter and TURNOVSKY, Stephen, editors.), Cambridge University Press (2003); p. 56–114

MORRIS, Stephen and SHIN, Hyun Song. “Liquidity Black Holes” Review of Finance, Vol. 8, No. 1 (2004a); p. 1–18.

MORRIS, Stephen and SHIN, Hyun Song. “Coordination Risk and the Price of Debt” European Economic Review, Vol. 48, No. 1 (February, 2004b); p. 133–153.

OZDENOREN, Emre and YUAN, Kathy “Feedback Effects and Asset Prices” Journal of Finance, Vol. 63, No. 4 (August, 2008); p. 1939–1975

PARK, In-Uck. “Cheap-Talk Coordination of Entry by Privately Informed Firms” The RAND Journal of Economics, Vol. 33, No. 3 (Autumn, 2002); p. 377-393

PINKOVSKIY, Maxim. “Rational Inattention and Choice Under Risk: Explaining Violations of Expected Utility through a Shannon Entropy Formulation of the Costs of Rationality” Atlantic Economic Journal, Vol. 37, No. 1 (March, 2009); p. 99-112.

RABIN, Matthew. “A Model of Pre-Game Communication” Journal of Economic Theory, Vol. 63, No. 2 (August, 1994); p. 370-391.

ROCHET, Jean-Charles and VIVES, Xavier, “Coordination Failures and the Lender of Last Resort: Was Bagehot Right After All?” Journal of the European Economic Association, Vol. 2, No. 6 (December, 2004); p. 1116–1147.

SÁKOVICS, József, and STEINER, Jakub. "Who Matters in Coordination Problems?" American Economic Review, Vol. 102, No. 7 (December, 2012); p. 3439-61.

SCHEINKMAN, José and XIONG, Wei “Overconfidence and Speculative Bubbles” Journal of Political Economy, Vol. 111, No. 6 (December, 2003), p. 1183-1209

SMITH, Vernon L.; SUCHANEK, Gerry L; and WILLIAMS, Arlington W. “Bubbles, Crashes, and Endogenous Expectations in Experimental Spot Asset Markets” Econometrica, Vol. 56, No. 5 (September, 1988), p. 1119-1151

SPENCE, Joseph “Anecdotes, Observations, and Characters of Books and Men” Collected from the Conversation of Mr. Pope, and Another Eminent Persons of his Time, (1820), 501 p.

SUNDER, Shyam. “Experimental Asset Markets: A Survey” The Handbook of Experimental Economics, John H. Kagel and Alvin E. Roth, editors, Princeton University Press (1995), p. 445-500

SZKUP, Michal, and TREVINO, Isabel. “Costly Information Acquisition in a Speculative Attack: Theory and Experiments” Working Paper (2015); 56 p.

TIROLE, Jean. “On the Possibility of Speculation under Rational Expectations” Econometrica, Vol. 50, No. 5 (September, 1982), p. 1163-1181

TIROLE, Jean. “Asset Bubbles and Overlapping Generations” Econometrica, Vol. 53, No. 5 (September, 1985), p. 1071-1100

TOXVAERD, Flavio. “Strategic Merger Waves: A Theory of Musical Chairs” Journal of Economic Theory, Vol. 140, No. 1 (May, 2008); p. 1–26.

WOOLDRIDGE, Jeffrey. “Econometric Analysis of Cross Section and Panel Data” The MIT Press (2002), 1064 p.

ZWART, Sanne. “The Mixed Blessing of IMF Intervention: Signaling Versus Liquidity Support” Journal of Financial Intermediation, Vol. 3, No. 2 (July, 2007); p. 149–174.