Appendix 1

Switching equilibrium errors lead to switching VECM.

Let us assume that x_t is a $(n \times 1)$ vector of nonstationary variables and that β is the $(n \times r)$ cointegrating matrix such that $z_t = \beta' x_t$ is the stationary $(r \times 1)$ vector of equilibrium errors that follows the Markov switching expression (1). We can always choose an $(n \times k)$ matrix $\beta^\dagger$ such that $\beta^{\dagger\prime}\beta = 0$ and $\beta^{\dagger\prime}\beta^\dagger = I_k$. Let w_t be the $(k \times 1)$ vector $\beta^{\dagger\prime} x_t$ such that $\Delta w_t = n_{s_t} + G_{s_t}(L)\Delta w_{t-1} + \eta_t$, where $\Delta = (1 - L)$, $G_{s_t}(L) = (G^1_{s_t} + \cdots + G^p_{s_t} L^{p-1})$, and $\eta_t \sim N(0, I_k)$, is regime-independent.

Using the relation $F_{s_t}(L) = F_{s_t}(1) + F^*_{s_t}(L)(1-L)$,[1] it is easy to see that

$$\beta' \Delta x_t = m_{s_t} + F^*_{s_t}(L)\beta' \Delta x_{t-1} + \left[F_{s_t}(1) - I_r\right] z_{t-1} + e_t. \tag{A1.1}$$

On the other hand, we can establish that

$$\beta^{\dagger\prime} \Delta x_t = n_{s_t} + G_{s_t}(L)\beta^{\dagger\prime}\Delta x_{t-1} + \eta_t. \tag{A1.2}$$

To simplify notation, we use the symbols Θ and Ξ_{s_t} for $\left(\beta, \beta^\dagger\right)'$ and $\left(m'_{s_t}, n'_{s_t}\right)'$ respectively. Thus, expressions (A1) and (A2) immediately lead to (**??**), where $\mu_{s_t} = \Theta^{-1}\Xi_{s_t}$, $\alpha_{s_t} = \Theta^{-1}\left(\left(F_{s_t}(1) - I_n\right)', 0'\right)'$, $\epsilon_t = (e'_t, \eta'_t)'$,and

$$\pi_{s_t}(L) = \Theta^{-1}\begin{pmatrix} F^*_{s_t}(L) & 0 \\ 0 & G_{s_t}(L) \end{pmatrix}\Theta. \tag{A1.3}$$

Appendix 2

Parameters of the moving average representation depend on previous states.

Let w_t be a $(n \times 1)$ vector of stationary variables (minus its conditional mean a_{s_t}) evolving according to a MS-VAR(p), that is:

$$w_t = b^1_{s_t} w_{t-1} + ... + b^p_{s_t} w_{t-p} + g_t. \tag{A2.1}$$

[1] It follows that, after a little of algebra, $F_{s_t}(L) = F^1_{s_t} + ... + F^p_{s_t} L^{p-1}$ can be written as

$$F_{s_t}(L) = F_{s_t}(1) + F^*_{s_t}(L)(1-L),$$

where $F^*_{s_t}(L) = F^{*1}_{s_t} + ... + F^{*p-1}_{s_t} L^{p-2}$ and $F^{*j}_{s_t} = -\sum_{i=j+1}^{p+1} F^i_{s_t}$.

This may be written as a MS-VAR(1) as follows

$$W_t = F_{s_t} W_{t-1} + G_t, \tag{A2.2}$$

where

$$W_t = \begin{bmatrix} w_t \\ \vdots \\ w_{t-p+1} \end{bmatrix}, F_{s_t} = \begin{bmatrix} b^1_{s_t} & b^2_{s_t} & \cdots & b^{p-1}_{s_t} & b^p_{s_t} \\ I_n & 0_n & \cdots & 0_n & 0_n \\ \vdots & \vdots & \cdots & \vdots & \vdots \\ 0_n & 0_n & \cdots & I_n & 0_n \end{bmatrix}, G_t \begin{bmatrix} g_t \\ 0_n \\ \vdots \\ 0_n \end{bmatrix}.$$

Assuming stationarity, recursive substitution in expression (A2.2) leads to

$$W_t = G_t + F_{s_t} G_{t-1} + F_{s_t} F_{s_{t-1}} G_{t-2} + F_{s_t} F_{s_{t-1}} F_{s_{t-2}} G_{t-3} + ..., \tag{A2.3}$$

which implies that parameters in (**??**) are:

$$C^j_{s_t, s_{t-(j-1)}} = \left[F_{s_t} \cdots F_{s_{t-(j-1)}} \right]^{nn}, \tag{A2.4}$$

for $j > 0$, with $C^0 = I_n$, and where $[\Lambda]^{nn}$ denotes the upper left $(n \times n)$ block of the matrix Λ.

Immediately, one can see that, if we were able to know the sequence of states, the switching parameters of the vector moving average representation were the solution of the recursive system

$$C^j_{s_t, s_{t-(j-1)}} = C^{j-1}_{s_t, s_{t-(j-2)}} b^1_{s_{t-(j-1)}} + C^{j-2}_{s_t, s_{t-(j-3)}} b^2_{s_{t-(j-2)}} + ... + C^{j-p}_{s_t, s_{t-(j-p-1)}} b^p_{s_{t-(j-p)}}, \tag{A2.5}$$

with $C^h_{s_t, s_{t-(h-1)}} = 0$ if $h < 0$, and $C^0 = I$.

Appendix 3

Deriving expression (6).

Recall the state-dependent parameters of the moving average expression

$$C_{s_{t\downarrow}}(L) = (I + C^1_{s_t} L + C^2_{s_t, s_{t-1}} L^2 + C^3_{s_t, s_{t-2}} L^3 + ...). \tag{A3.1}$$

Le us define $C(1)$ as follows

$$C(1) = I + C^1 + C^2 + C^3 + ..., \tag{A3.2}$$

where

$$C^{j}=\sum_{i_0=1}^{q}\ldots\sum_{i_{j-1}=1}^{q}P(s_t=i_0,\ldots,s_{t-(j-1)}=i_{j-1})C^{j}_{s_t,s_{t-(j-1)}}. \tag{A3.3}$$

Thus, expression (A3.1) may be rewritten as

$$\begin{aligned} C_{s_{t\downarrow}}(L) \;=\; & C(1)+\left\{\underbrace{(-C(1)+I)}_{C^{*0}}-\underbrace{(-C(1)+I)}_{C^{*0}}L\right\}+ \\ & \left\{\underbrace{\left(-C(1)+I+C^{1}_{s_t}\right)}_{C^{*1}_{s_t}}L-\underbrace{\left(-C(1)+I+C^{1}_{s_{t-1}}\right)}_{C^{*1}_{s_{t-1}}}L^{2}\right\}+ \\ & \left\{\underbrace{\left(-C(1)+I+C^{1}_{s_{t-1}}+C^{2}_{s_t,s_{t-1}}\right)}_{C^{*2}_{s_t,s_{t-1}}}L^{2}-\underbrace{\left(-C(1)+I+C^{1}_{s_{t-2}}+C^{2}_{s_{t-1},s_{t-2}}\right)}_{C^{*2}_{s_{t-1},s_{t-2}}}L^{3}\right\}+ \\ & \vdots \end{aligned} \tag{A3.4}$$

This implies that expression (**??**) holds, with

$$C^{*j}_{s_t,s_{t-(j-1)}}=-C(1)+I+C^{1}_{s_{t-(j-1)}}+C^{2}_{s_{t-(j-2)},s_{t-(j-1)}}+C^{3}_{s_{t-(j-3)},s_{t-(j-1)}}+\ldots+C^{j}_{s_t,s_{t-(j-1)}}. \tag{A3.5}$$

Appendix 4

Estimating the parameters of the moving average expression.

To deduce the proof, we consider first the parameter of the vector autoregressive expression as a function of the parameters of the restricted expression. For this purpose, we note that assuming stationarity expression (**??**) can alternatively be seen as

$$y_t=\epsilon^{*}_t+JB_{s_t}J'\epsilon^{*}_{t-1}+JB_{s_t}B_{s_{t-1}}J'\epsilon^{*}_{t-2}+\ldots \tag{A4.1}$$

Premultiplying this expression by $M^{-1}D(L)$ and recalling that $\epsilon^{*}_t=M\epsilon_t$, it is easy to see that

$$\Delta x_t=M^{-1}D(L)M\epsilon_t+M^{-1}D(L)JB_{s_t}J'M\epsilon_{t-1}+M^{-1}D(L)JB_{s_t}B_{s_{t-1}}J'M\epsilon_{t-2}+\ldots \tag{A4.2}$$

Using the relation $D(L)=I_n-DL$, where $D=D_{\perp}(1)$, a close comparison of expressions (5) and (A8) leads to write

$$C^{j+1}_{s_t,s_{t-j}}=M^{-1}JB_{s_t}\ldots B_{s_{t-j}}J'M-M^{-1}DJB_{s_t}\ldots B_{s_{t-(j-1)}}J'M,\ \text{for } j>0 \tag{A4.3}$$

where $C^0 = I_n$, $C^1_{s_t} = M^{-1}JB_{s_t}J'M - M^{-1}DM$.

In the remainder of this proof we assume that a good estimator of $B_{s_t}...B_{s_{t-j}}$ is the following expression

$$\sum_{i_0=1}^{q}\cdots\sum_{i_j=1}^{q} P(s_{t-j}=i_j,...,s_t=i_0|\chi_t)B_{s_t}...B_{s_{t-j}}. \tag{A4.4}$$

To follow with the proof, we need an expression of the joint probabilities. Using the properties of a Markov structure we derive that

$$P(s_{t-j}=i_j,...,s_t=i_0|\chi_t) = \left(e'_{i_0}\xi_{t/t}\right)\frac{\left(e'_{i_0}P'e_{i_1}\right)\left(e'_{i_1}\xi_{t-1/t-1}\right)}{\left(e'_{i_0}\xi_{t/t-1}\right)}\cdots\frac{\left(e'_{i_{j-1}}P'e_{i_j}\right)\left(e'_{i_j}\xi_{t-j/j-1}\right)}{\left(e'_{i_{j-1}}\xi_{t-j+1/t-j}\right)}. \tag{A4.5}$$

where e_j is a $(q \times 1)$ vector with j-th element equals to one and zero elsewhere. We can now apply rational expectations to get the estimates for $(A4.4)$

$$b\left(\widetilde{\xi}^*_{t/t-1}P'^*B\widetilde{\xi}^*_{t-1/t-2}P'^*B...\widetilde{\xi}^*_{t-j+1/t-j}P'^*B\right)\xi^*_{t-j/t-j}. \tag{A4.6}$$

This finishes the intuitive sketch of the proof, the detailed verification of (17) being left to the reader. As an example, and using that $(e_1,...,e_q) = (e_1,...,e_q)' = I_q$, we derive the estimate of

$$C^2_{s_t,s_{t-1}} = M^{-1}JB_{s_t}B_{s_{t-1}}J'M - M^{-1}DJB_{s_t}J'M. \tag{A4.7}$$

For this attempt we need to derive an expression for $P(s_{t-1}=i_1, s_t=i_0|\chi_t)$ and to calculate the estimates of B_{s_t} and $B_{s_t}B_{s_{t-1}}$. Starting from the probability, we have that

$$\begin{aligned} P(s_{t-1}=i_1,s_t=i_0|\chi_t) &= P(s_t=i_0|\chi_t)P(s_{t-1}=i_1|\chi_t,s_t=i_0) \\ &= P(s_t=i_0|\chi_t)P(s_{t-1}=i_1|\chi_{t-1},s_t=i_0) = \left(e'_{i_0}\xi_{t/t}\right)\frac{P(s_{t-1}=i_1|\chi_{t-1},s_t=i_0)}{P(s_t=i_0|\chi_{t-1})} \\ &= \left(e'_{i_0}\xi_{t/t}\right)\frac{\left(e'_{i_0}P'e_{i_1}\right)\left(e'_{i_1}\xi_{t-1/t-1}\right)}{\left(e'_{i_0}\xi_{t/t-1}\right)}. \end{aligned} \tag{A4.8}$$

Hence, the estimate of B_{s_t} is

$$\sum_{i_0} B_{i_0} P(s_t = i_0|\chi_t) = \Big(\underbrace{B_1, ..., B_q}_{b}\Big) \underbrace{\begin{bmatrix} e_1'\xi_{t/t} I_{np} \\ ... \\ e_q'\xi_{t/t} I_{np} \end{bmatrix}}_{\underbrace{\left[\begin{pmatrix} e_1' \\ ... \\ e_q' \end{pmatrix} \xi_{t/t} \otimes I_{np}\right]}_{\left(\xi_{t/t} \otimes I_{np}\right) = \xi^*_{t/t}}} = b\xi^*_{t/t}, \tag{A4.9}$$

and the estimate of $B_{s_t}B_{s_{t-1}}$ becomes

$$\begin{aligned}
\sum_{i_0}\sum_{i_1} B_{i_0}B_{i_1}P(s_{t-1} &= i_1, s_t = i_0|\chi_t) = \\
&= \sum_{i_0} B_{i_0} \frac{\left(e_{i_0}'\xi_{t/t}\right)}{\left(e_{i_0}'\xi_{t/t-1}\right)} \underbrace{\sum_{i_1} \left(e_{i_0}'P'e_{i_1}\right) B_{i_1} \left(e_{i_1}'\xi_{t-1/t-1}\right)} \\
&\qquad \left[\left(e_{i_0}'P'e_1\right) B_1, ..., \left(e_{i_0}'P'e_q\right) B_q\right] \begin{bmatrix} e_1'\xi_{t-1/t-1} I_{np} \\ ... \\ e_q'\xi_{t-1/t-1} I_{np} \end{bmatrix} \\
&\qquad \left[\left(e_{i_0}'P'\left(e_1, ..., e_q\right)\right) \otimes I_{np}\right] B\xi^*_{t-1/t-1} \\
&\qquad \left(e_{i_0}' \otimes I_{np}\right) \left(P' \otimes I_{np}\right) B\xi^*_{t-1/t-1} \\
&= \underbrace{\left(B_1, ..., B_q\right)}_{b} \underbrace{\begin{pmatrix} \left(\frac{e_1'\xi_{t/t}}{e_1'\xi_{t/t-1}}\right) I_{np} & & 0 \\ & ... & \\ 0 & & \left(\frac{e_q'\xi_{t/t}}{e_q'\xi_{t/t-1}}\right) I_{np} \end{pmatrix}}_{\widetilde{\xi}^*_{t/t-1}} \underbrace{\begin{pmatrix} e_1' \otimes I_{np} \\ ... \\ e_q' \otimes I_{np} \end{pmatrix}}_{I_{npq}} P'^* B\xi^*_{t-1/t-1} \\
&= b\widetilde{\xi}^*_{t/t-1} P'^* B\xi^*_{t-1/t-1}.
\end{aligned} \tag{A4.10}$$

Thus, we conclude that the estimate of $C^2_{s_t,s_{t-1}}$ is

$$M^{-1}Jb\widetilde{\xi}^*_{t/t-1}P'^*B\xi^*_{t-1/t-1}J'M - M^{-1}DJb\xi^*_{t/t}J'M \tag{A4.11}$$

Appendix 5

Derivation of forward-looking IRF.

The derivation of the estimates of the parameters of the switching IRF is virtually identical to the derivation of the estimates of the moving average representation analyzed in Appendix 4. During this proof, we will use the same notation we have stated in that appendix.

First, solving recursively, assuming stationarity, premultiplying by $M^{-1}D(L)$ and recalling that $\epsilon_t = \Gamma^{-1}u_t$ (with u_t being the vector of the n fundamental shocks), expression $(A4.2)$ evaluated at $t+j$ becomes

$$\Delta x_{t+h} = u_{t+h} + ... + \left[M^{-1}JB_{s_{t+h}}^{s_{t+1}}J'M - M^{-1}DJB_{s_{t+h}}^{s_{t+2}}J'M\right]\Gamma^{-1}u_t + ... \qquad \text{(A5.1)}$$

where $B_{s_a}^{s_b}$ is the product $B_{s_a}B_{s_{a-1}}\cdots B_{s_b}$. Multiplying expression in brackets by Γ^{-1}, we obtain $\overline{R}^h$, whose row i, column j element is the response at $t+h$ of a unit increase in the jth structural innovation at time t for the value of the first difference of the ith variable.

On the other hand, using again the properties of the Markov structure that the probabilities is assumed to follow, one can express

$$P(s_{t+h} = i_h, ..., s_{t+1} = i_1|\chi_t) = \left(e'_{i_{t+h}}Pe_{i_{t+h-1}}\right)\cdots\left(e_{i_2}P'e_{i_1}\right)\left(e'_{i_1}\xi_{t+1/t}\right). \qquad \text{(A5.2)}$$

Rational expectation hypotheses can be used to propose the following estimation of $B_{s_{t+h}}\cdots B_{s_{t+1}}$

$$\sum_{i_1=1}^{q}\cdots\sum_{i_h=1}^{q}P(s_{t+1} = i_1, ..., s_{t+h} = i_j|\chi_t)B_{s_{t+h}}\cdots B_{s_{t+1}}, \qquad \text{(A5.3)}$$

and the remaining steps to get the general expression (18) are again left to the reader. We instead propose the estimate of the much simpler case of $\overline{R}^2$. Thus, we need first an analytical expression for the estimates of $P(s_{t+2} = i_2, s_{t+1} = i_1|\chi_t)$, $B_{s_{t+2}}$, and $B_{s_{t+2}}B_{s_{t+1}}$.

$$\begin{aligned} P(s_{t+2} &= i_2, s_{t+1} = i_1|\chi_t) = \\ &= P(s_{t+2} = i_2|\chi_t, s_{t+1} = i_1)P(s_{t+1} = i_1|\chi_t) = \left(e'_{i_2}Pe_1\right)\left(e'_1\xi_{t+1/t}\right) \end{aligned} \qquad \text{(A5.4)}$$

Following $(A4.9)$ it is easy to see that the estimate of $B_{s_{t+2}}$ is

$$B_{s_{t+2}}P(s_{t+2} = i_2|\chi_t) = b\xi^*_{t+2/t}. \qquad \text{(A5.5)}$$

Finally, using $(A5.4)$ we propose the following estimate of $B_{s_{t+2}}B_{s_{t+1}}$

$$\begin{aligned}\sum_{i_2=1}^{q}\sum_{i_1=1}^{q}P(s_{t+1} &= i_1, s_{t+2}=i_2|\chi_t)B_{i_2}B_{i_1} \\ &= \sum_{i_2=1}^{q}B_{i_2}\underbrace{\left[\left(e'_{i_2}Pe_1\right)B_1,...,\left(e'_{i_2}Pe_q\right)B_q\right]\begin{bmatrix}\left(e'_1\xi_{t+1/t}\right)I_{np}\\ ...\\ \left(e'_q\xi_{t+1/t}\right)I_{np}\end{bmatrix}}_{\left(e'_{i_2}\otimes I_{np}\right)P^*B\underbrace{\left(\xi_{t+1/t}\otimes I_{np}\right)}_{\xi^*_{t+1/t}}} \\ &= [B_1,...,B_q]\underbrace{\begin{bmatrix}\left(e'_1\otimes I_{np}\right)\\ ...\\ \left(e'_q\otimes I_{np}\right)\end{bmatrix}}_{I_{npq}}P^*B\xi^*_{t+1/t}=bP^*B\xi^*_{t+1/t} \qquad \text{(A5.6)}\end{aligned}$$

It is now straightforward to show that the estimate of $\overline{R}^2$ is

$$\left[M^{-1}JbP^*B\xi^*_{t+1/t}J'M-M^{-1}DJb\xi^*_{t+2/t}J'M\right]\Gamma^{-1}. \qquad \text{(A5.7)}$$

Table 1. Linear common trend model estimates.

a. RVAR model.

$$\begin{pmatrix} \widehat{y}_t \\ \widehat{c}_t \\ \widehat{i}_t \end{pmatrix} = \begin{pmatrix} \underset{(0.129)}{-0.248} \\ \underset{(0.037)}{-0.072} \\ \underset{(0.062)}{-0.254} \end{pmatrix} + \begin{pmatrix} \underset{(0.064)}{0.558} & \underset{(0.125)}{0.335} & \underset{(0.073)}{-0.211} \\ \underset{(0.018)}{-0.095} & \underset{(0.036)}{0.853} & \underset{(0.021)}{-0.025} \\ \underset{(0.031)}{0.224} & \underset{(0.060)}{0.037} & \underset{(0.035)}{0.834} \end{pmatrix} \begin{pmatrix} y_{t-1} \\ c_{t-1} \\ i_{t-1} \end{pmatrix}$$

b. Common trend model.

$$\begin{pmatrix} \Delta y_t \\ \Delta c_t \\ \Delta i_t \end{pmatrix} = \begin{pmatrix} \underset{(0.001)}{0.007} \\ \underset{(0.001)}{0.007} \\ \underset{(0.001)}{0.007} \end{pmatrix} \tau_t + C^*(L)\widehat{\epsilon}_t,$$

where

$$\widehat{\tau}_t = 0.63 + \tau_{t-1}$$

c. Other estimates.

$$M = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}, C(1) = \begin{pmatrix} 0.230 & 0.991 & -0.195 \\ 0.230 & 0.991 & -0.195 \\ 0.230 & 0.991 & -0.195 \end{pmatrix},$$

$$\Gamma^{-1} = \begin{pmatrix} 0.006 & 0.010 & 0.001 \\ 0.007 & 0.001 & 0.003 \\ 0.004 & 0.012 & 0.018 \end{pmatrix}, \alpha = \begin{pmatrix} -0.148 & 0.007 \\ -0.002 & 0.032 \\ -0.186 & 0.172 \end{pmatrix}$$

Table 2. Forecast-error variance decompositions (FEVD) for the linear model.

Horizon	y	c	i
1	0.30 (0.20)	0.79 (0.19)	0.10 (0.11)
4	0.41 (0.21)	0.85 (0.18)	0.19 (0.15)
8	0.53 (0.21)	0.90 (0.18)	0.27 (0.16)
12	0.61 (0.19)	0.93 (0.18)	0.30 (0.15)
16	0.66 (0.18)	0.94 (0.17)	0.32 (0.14)
20	0.69 (0.17)	0.95 (0.17)	0.36 (0.14)
24	0.73 (0.17)	0.96 (0.17)	0.39 (0.13)

NOTE: FEVD based on the VECM estimates shown in Table 1. Standard errors (in parentheses) were computed by Monte Carlo simulations (3000 replications).

Table 3. Nonlinear common trend model estimates for state 1.

a. RVAR model.

$$\begin{pmatrix} \widehat{y}_t \\ \widehat{c}_t \\ \widehat{i}_t \end{pmatrix} = \begin{pmatrix} \underset{(0.146)}{-0.427} \\ \underset{(0.041)}{-0.031} \\ \underset{(0.068)}{-0.381} \end{pmatrix} + \begin{pmatrix} \underset{(0.070)}{0.520} & \underset{(0.127)}{0.158} & \underset{(0.086)}{-0.301} \\ \underset{(0.020)}{-0.080} & \underset{(0.036)}{0.831} & \underset{(0.024)}{-0.003} \\ \underset{(0.032)}{0.193} & \underset{(0.059)}{0.003} & \underset{(0.040)}{0.756} \end{pmatrix} \begin{pmatrix} y_{t-1} \\ c_{t-1} \\ i_{t-1} \end{pmatrix}$$

b. Common trend model.

$$\begin{pmatrix} \Delta y_t \\ \Delta c_t \\ \Delta i_t \end{pmatrix} = \begin{pmatrix} 0.0051 \\ 0.0051 \\ 0.0051 \end{pmatrix} \tau_t + C^*(L)\widehat{\epsilon}_t,$$

where

$$\widehat{\tau}_t = 1.25 + \tau_{t-1}$$

c. Other estimates.

$$M = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}, C(1) = \begin{pmatrix} 0.3392 & 0.5704 & -0.1717 \\ 0.3392 & 0.5704 & -0.1717 \\ 0.3392 & 0.5704 & -0.1717 \end{pmatrix},$$

$$\Gamma^{-1} = \begin{pmatrix} 0.0077 & 0.0081 & -0.0012 \\ 0.0052 & 0.0001 & 0.0037 \\ 0.0028 & 0.0161 & 0.0122 \end{pmatrix}, \alpha_1 = \begin{pmatrix} -0.1058 & 0.0205 \\ 0.0610 & 0.0169 \\ -0.1089 & 0.2637 \end{pmatrix}$$

Table 4. Nonlinear common trend model estimates for state 2.

a. RVAR model.

$$\begin{pmatrix} \widehat{y}_t \\ \widehat{c}_t \\ \widehat{i}_t \end{pmatrix} = \begin{pmatrix} \underset{(0.329)}{0.218} \\ \underset{(0.093)}{-0.390} \\ \underset{(0.154)}{-0.364} \end{pmatrix} + \begin{pmatrix} \underset{(0.122)}{0.566} & \underset{(0.353)}{1.322} & \underset{(0.163)}{-0.049} \\ \underset{(0.034)}{-0.188} & \underset{(0.100)}{0.670} & \underset{(0.046)}{-0.192} \\ \underset{(0.057)}{0.185} & \underset{(0.165)}{0.059} & \underset{(0.077)}{0.773} \end{pmatrix} \begin{pmatrix} y_{t-1} \\ c_{t-1} \\ i_{t-1} \end{pmatrix}$$

b. Common trend model.

$$\begin{pmatrix} \Delta y_t \\ \Delta c_t \\ \Delta i_t \end{pmatrix} = \begin{pmatrix} 0.0051 \\ 0.0051 \\ 0.0051 \end{pmatrix} \tau_t + C^*(L)\widehat{\epsilon}_t,$$

where

$$\widehat{\tau}_t = -1.20 + \tau_{t-1}$$

c. Other estimates.

$$M = \begin{pmatrix} 1 & 1 & 1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{pmatrix}, C(1) = \begin{pmatrix} 0.3392 & 0.5704 & -0.1717 \\ 0.3392 & 0.5704 & -0.1717 \\ 0.3392 & 0.5704 & -0.1717 \end{pmatrix},$$

$$\Gamma^{-1} = \begin{pmatrix} 0.0077 & 0.0081 & -0.0012 \\ 0.0052 & 0.0001 & 0.0037 \\ 0.0028 & 0.0161 & 0.0122 \end{pmatrix}, \alpha_2 = \begin{pmatrix} -0.5311 & -0.1228 \\ -0.2014 & 0.0697 \\ -0.5903 & 0.1031 \end{pmatrix}$$

Table 5. Backward-looking Forecast-error variance decompositions (BL-FEVD) for the nonlinear model.

Horizon	y	c	i
1	0.49 (0.06)	0.71 (0.05)	0.15 (0.11)
4	0.61 (0.02)	0.84 (0.03)	0.26 (0.12)
8	0.71 (0.02)	0.90 (0.05)	0.27 (0.11)
12	0.76 (0.02)	0.94 (0.06)	0.35 (0.10)
16	0.80 (0.02)	0.95 (0.06)	0.40 (0.09)
20	0.82 (0.02)	0.96 (0.06)	0.43 (0.08)
24	0.84 (0.02)	0.97 (0.06)	0.50 (0.08)

NOTE: BL-FEVD attributed to the permanent shock based on the nonlinear VECM estimates shown in Tables 3 and 4. Standard errors (in parentheses) were computed by Monte Carlo simulations (3000 replications).

Table 6. Forward-looking Forecast-error variance decompositions (FL-FEVD) for the nonlinear model (shock in 1988.4).

Horizon	y	c	i
1	0.45 (0.11)	0.67 (0.04)	0.11 (0.09)
4	0.55 (0.03)	0.79 (0.02)	0.19 (0.12)
8	0.65 (0.04)	0.79 (0.07)	0.25 (0.14)
12	0.71 (0.04)	0.79 (0.08)	0.28 (0.14)
16	0.75 (0.04)	0.80 (0.09)	0.30 (0.14)
20	0.78 (0.04)	0.80 (0.09)	0.32 (0.14)
24	0.80 (0.04)	0.80 (0.09)	0.34 (0.14)

NOTE: FL-FEVD attributed to a permanent shock hitting the system in 1988.4. These are computed following the nonlinear VECM estimates shown in Tables 3 and 4. Standard errors (in parentheses) were computed by Monte Carlo simulations (3000 replications).

Table 7. Forward-looking Forecast-error variance decompositions (FL-FEVD) for the nonlinear model (shock in 1983.1).

Horizon	y	c	i
1	0.44 (0.15)	0.67 (0.08)	0.10 (0.11)
4	0.56 (0.11)	0.75 (0.05)	0.17 (0.12)
8	0.67 (0.07)	0.65 (0.11)	0.23 (0.12)
12	0.73 (0.05)	0.62 (0.12)	0.26 (0.15)
16	0.76 (0.04)	0.61 (0.17)	0.28 (0.12)
20	0.79 (0.04)	0.60 (0.18)	0.30 (0.12)
24	0.81 (0.04)	0.61 (0.19)	0.31 (0.12)

NOTE: FL-FEVD attributed to a permanent shock hitting the system in 1983.1. These are computed following the nonlinear VECM estimates shown in Tables 3 and 4. Standard errors (in parentheses) were computed by Monte Carlo simulations (3000 replications).

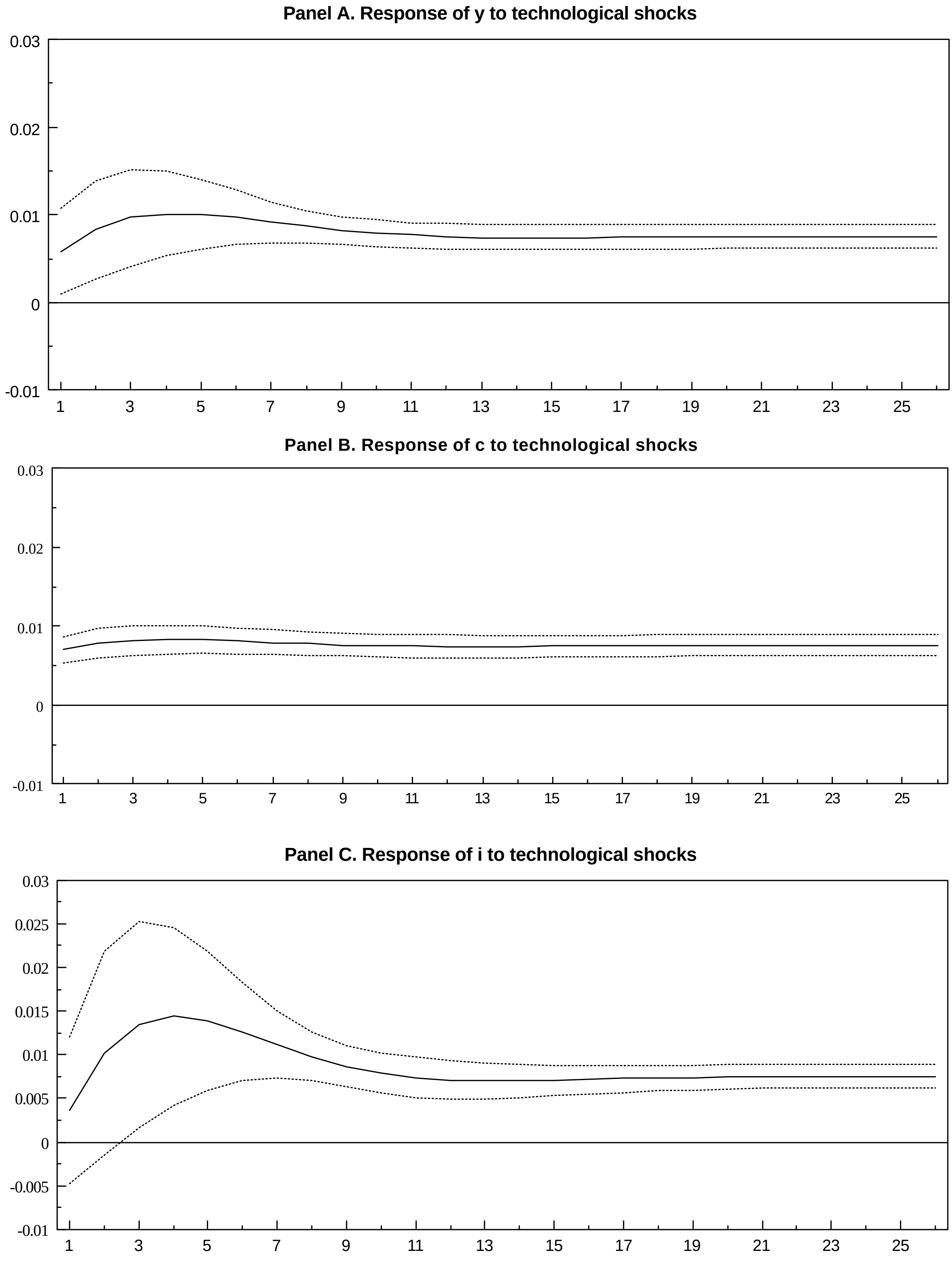

Figure 1. Dynamics responses of output (Panel A), consumption (Panel B) and investment (Panel C) to the technological shock form linear stochastic trend model. Dashed lines are 95% asymptotic confidence intervals.

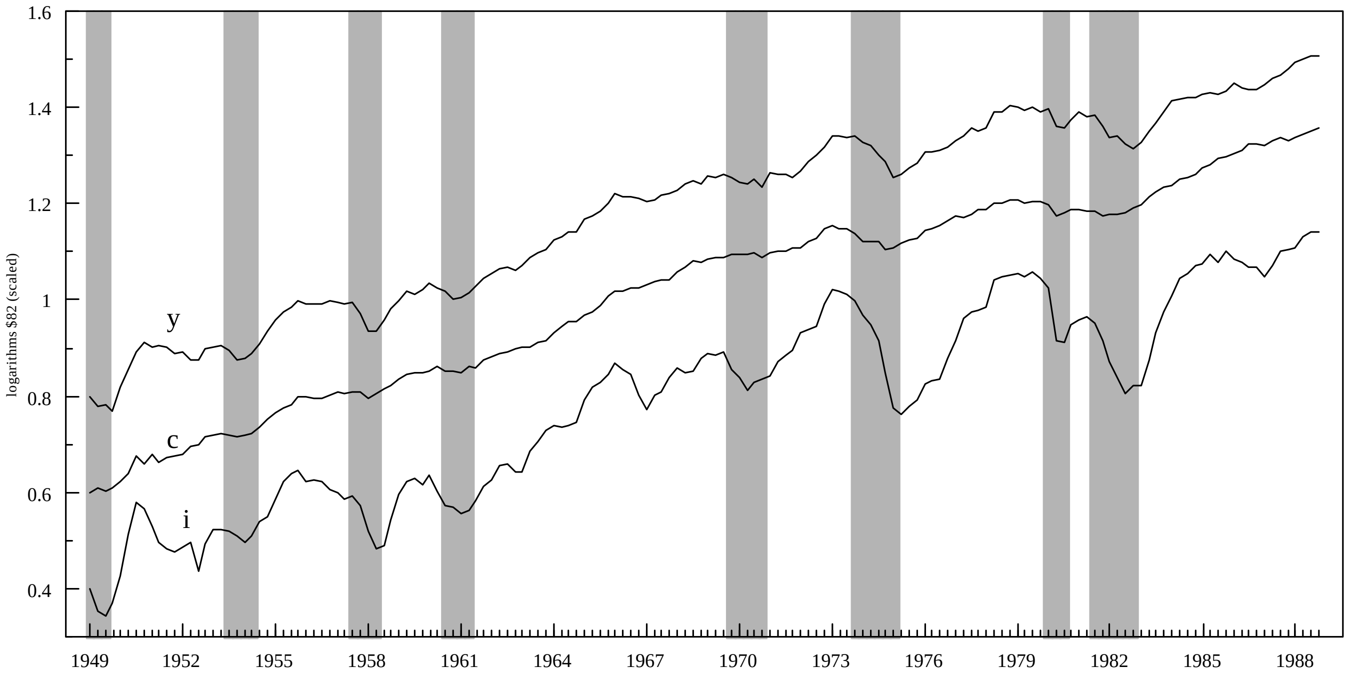

Figure 2. Logarithms of private output (y), consumption (c) and investment (y). To facilitate graphing, constants were added. Shaded areas are NBER recessions.

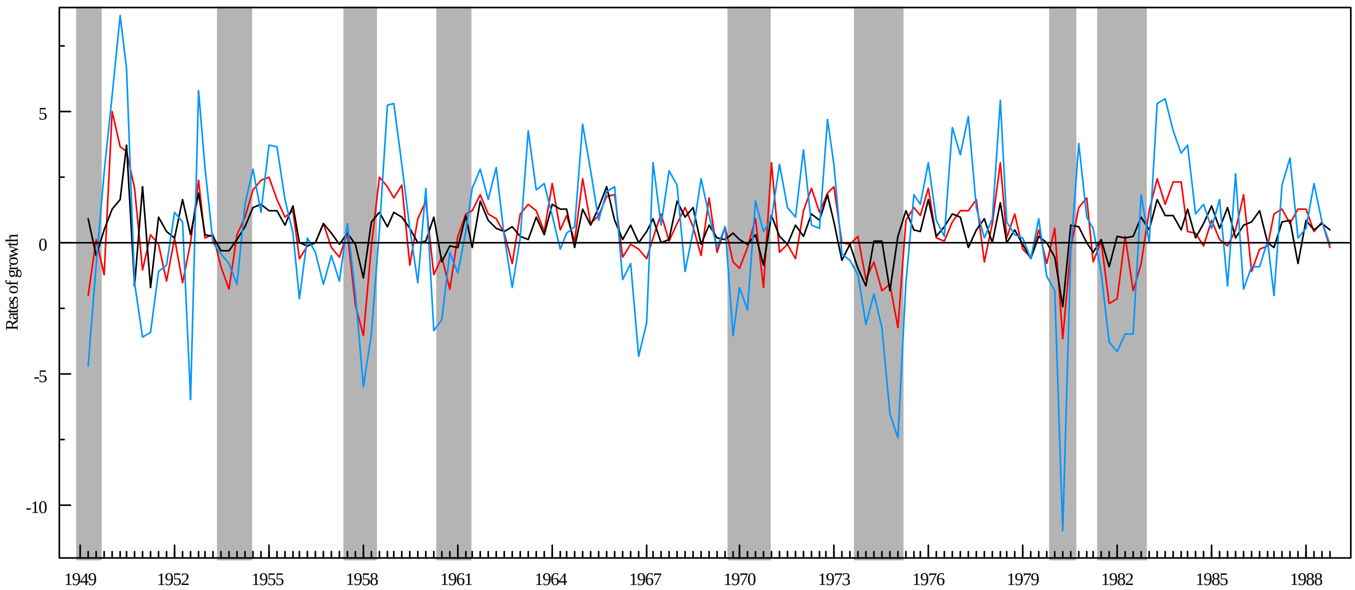

Figure 3. Rates of growth of private output (red), consumption (black) and investment (blue). Shaded areas correspond to the NBER recessions.

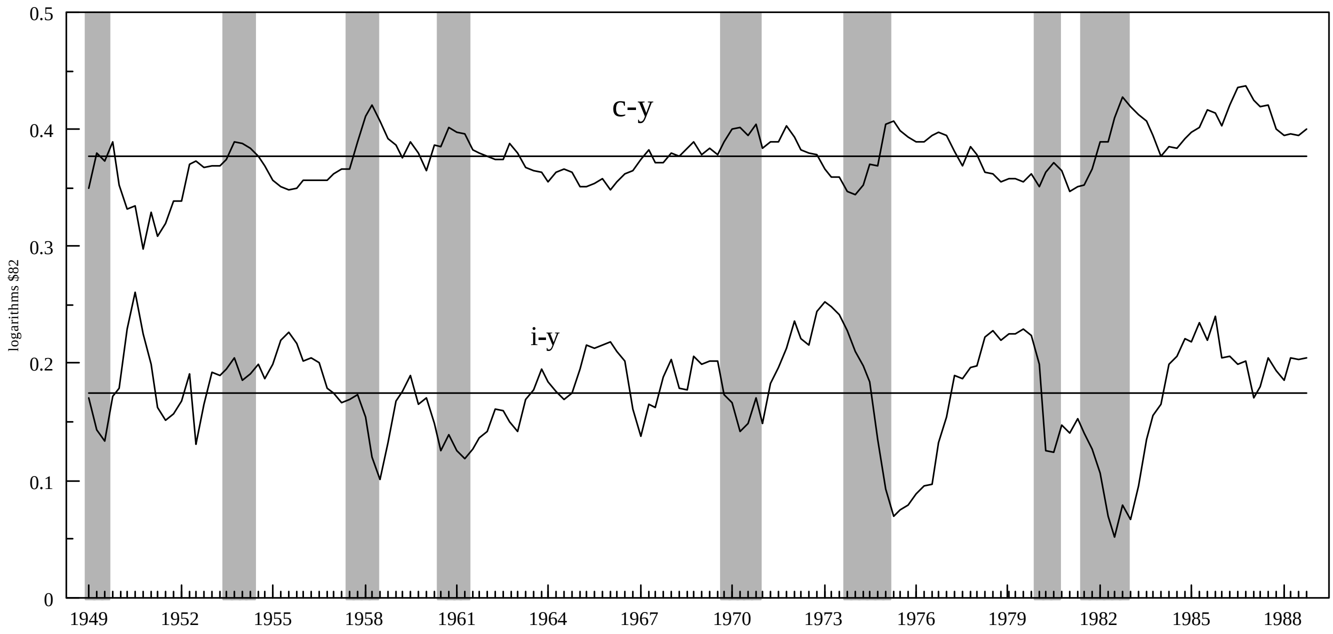

Figure 4: Logarithms of the consumption:output (c-y) and investment:output (i-y) ratios. To facilitate graphing, constants were added. Shaded areas are NBER recessions.

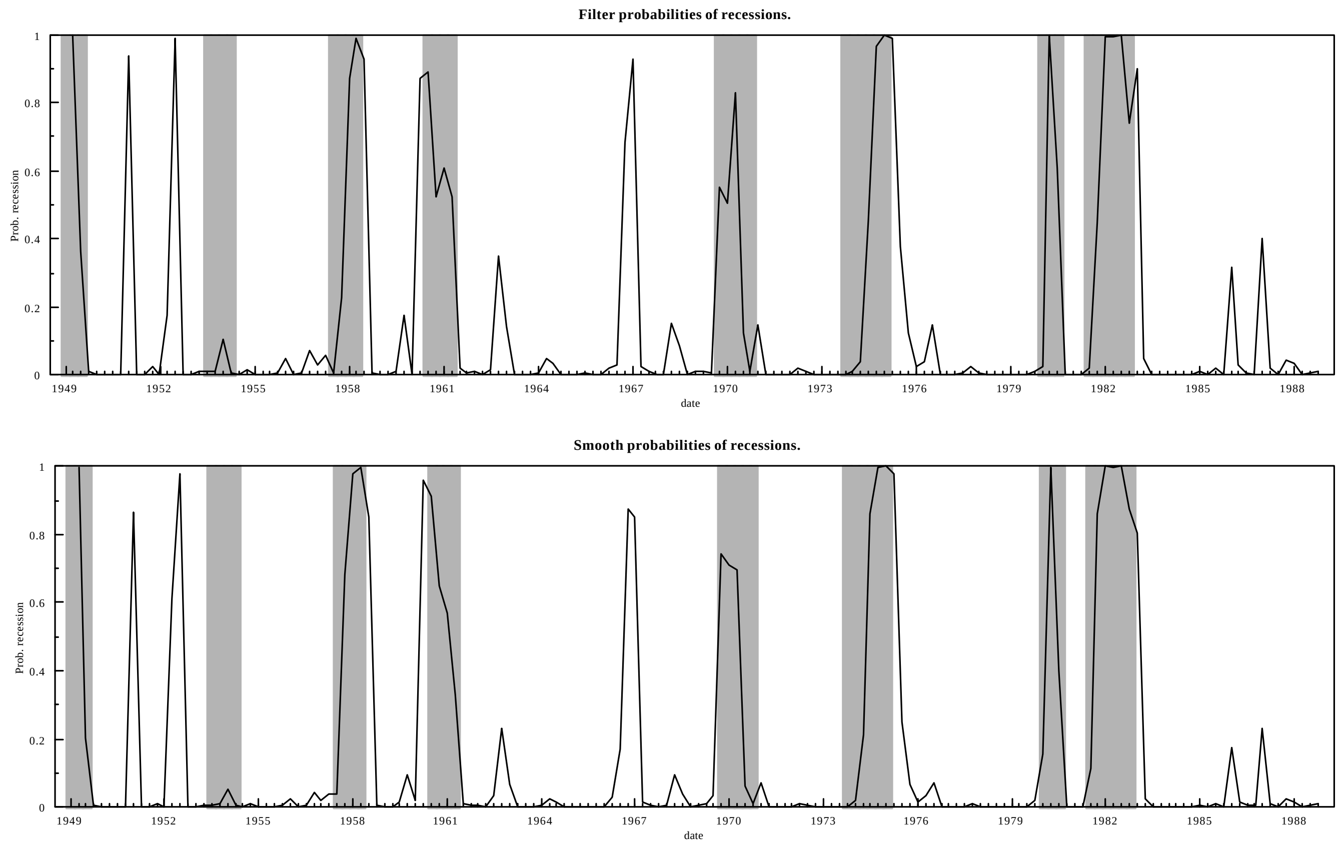

Figure 5: Filter (Panel A) and smooth (Panel B) probabilities of recession from a Markov-switching VAR for logarithms of the consumption:output (c-y) and investment:output (i-y) ratios. Shaded areas are NBER recessions.

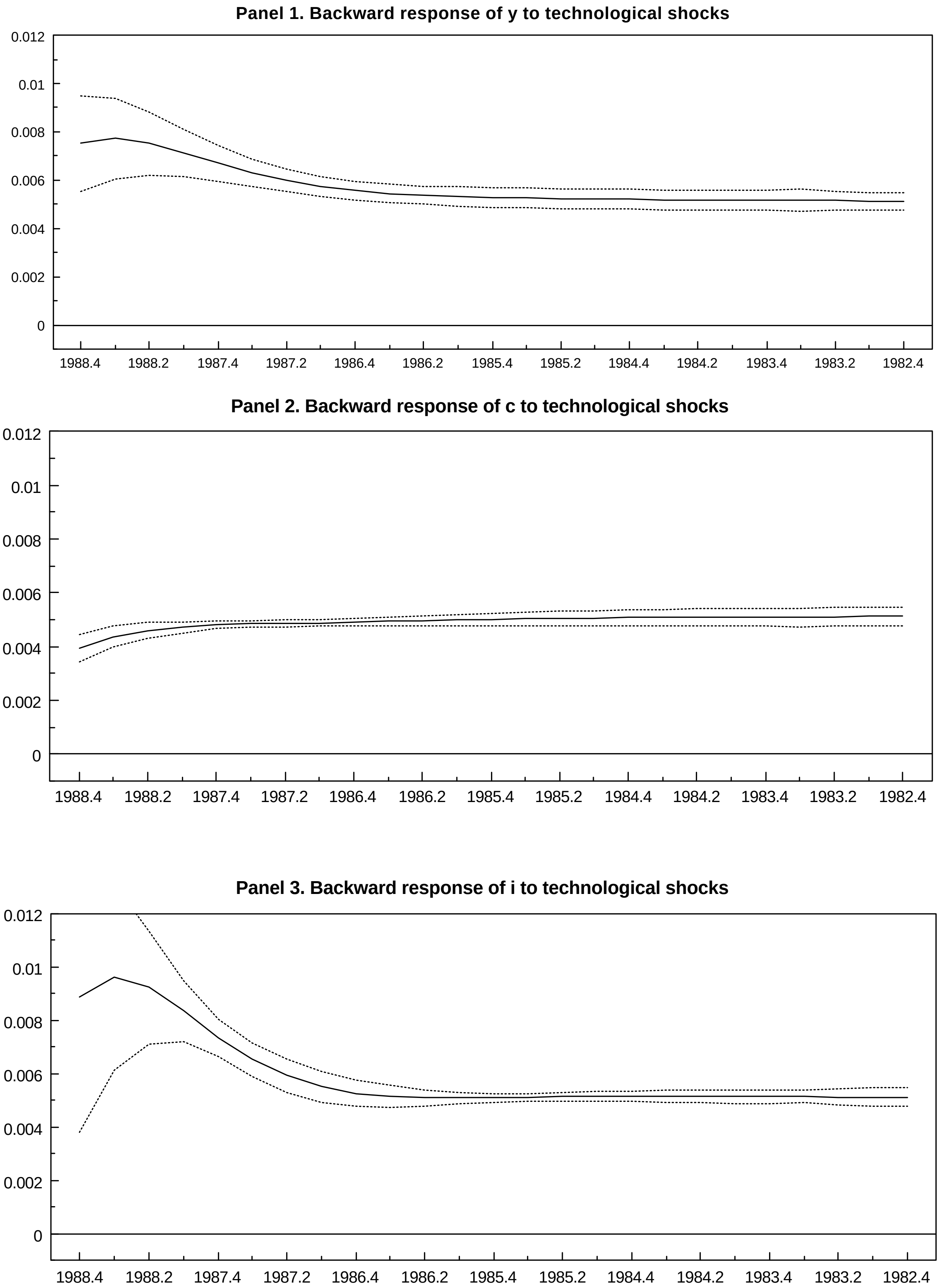

Figure 6: Backward-looking responses of outout (Panel A), consumption (Panel B) and investment (Panel C) in 1988.4 to permanent shocks hitting the system in 1988.4-1982.4. Dashed lines are one-standard-deviation confidence bands.

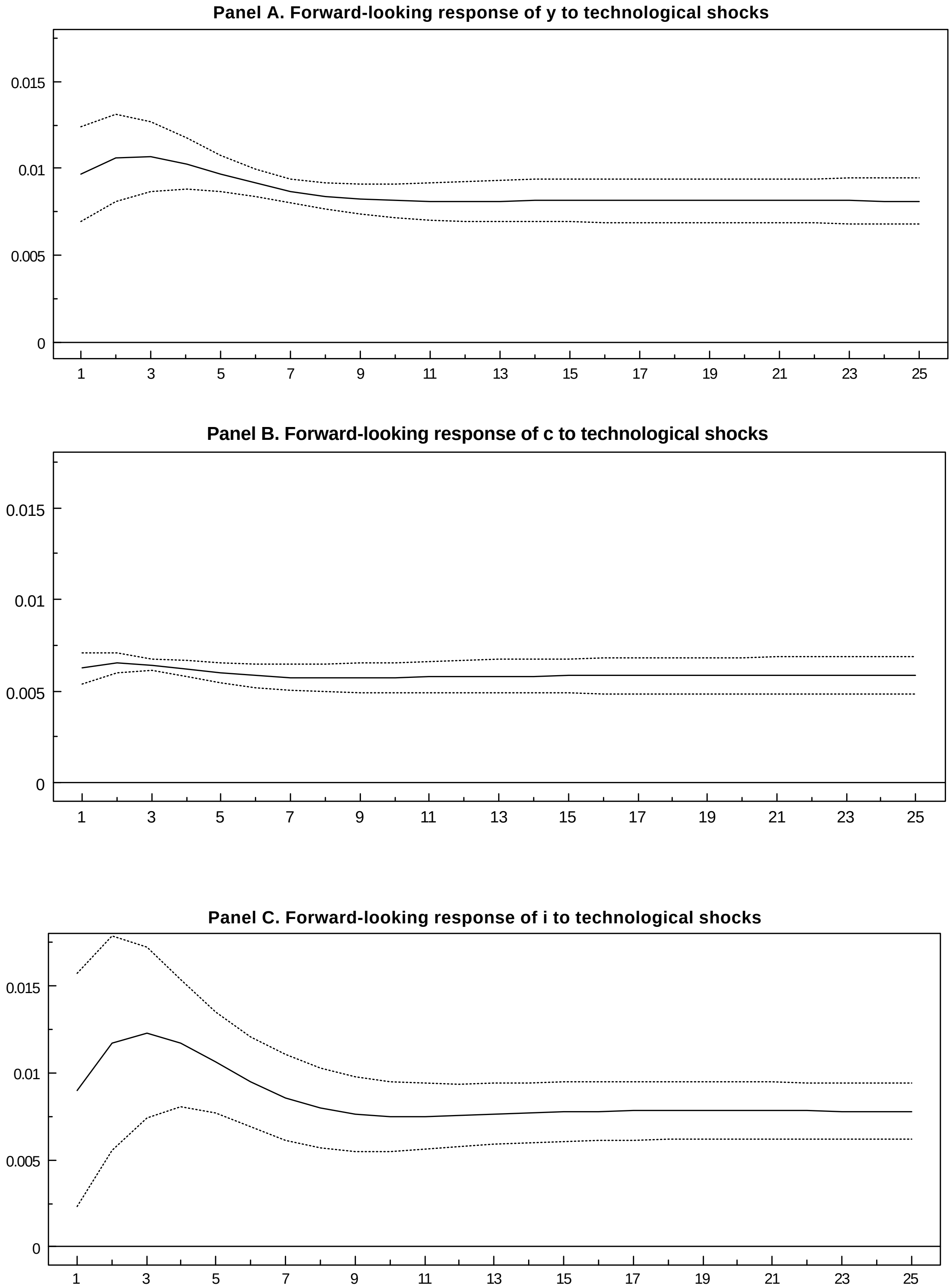

Figure 7: Forward-looking responses of outout (Panel A), consumption (Panel B) and investment (Panel C) to permanent shocks hitting the system in 1988.4. Dashed lines are one-standard-deviation confidence bands computed by Monte Carlo simulation.

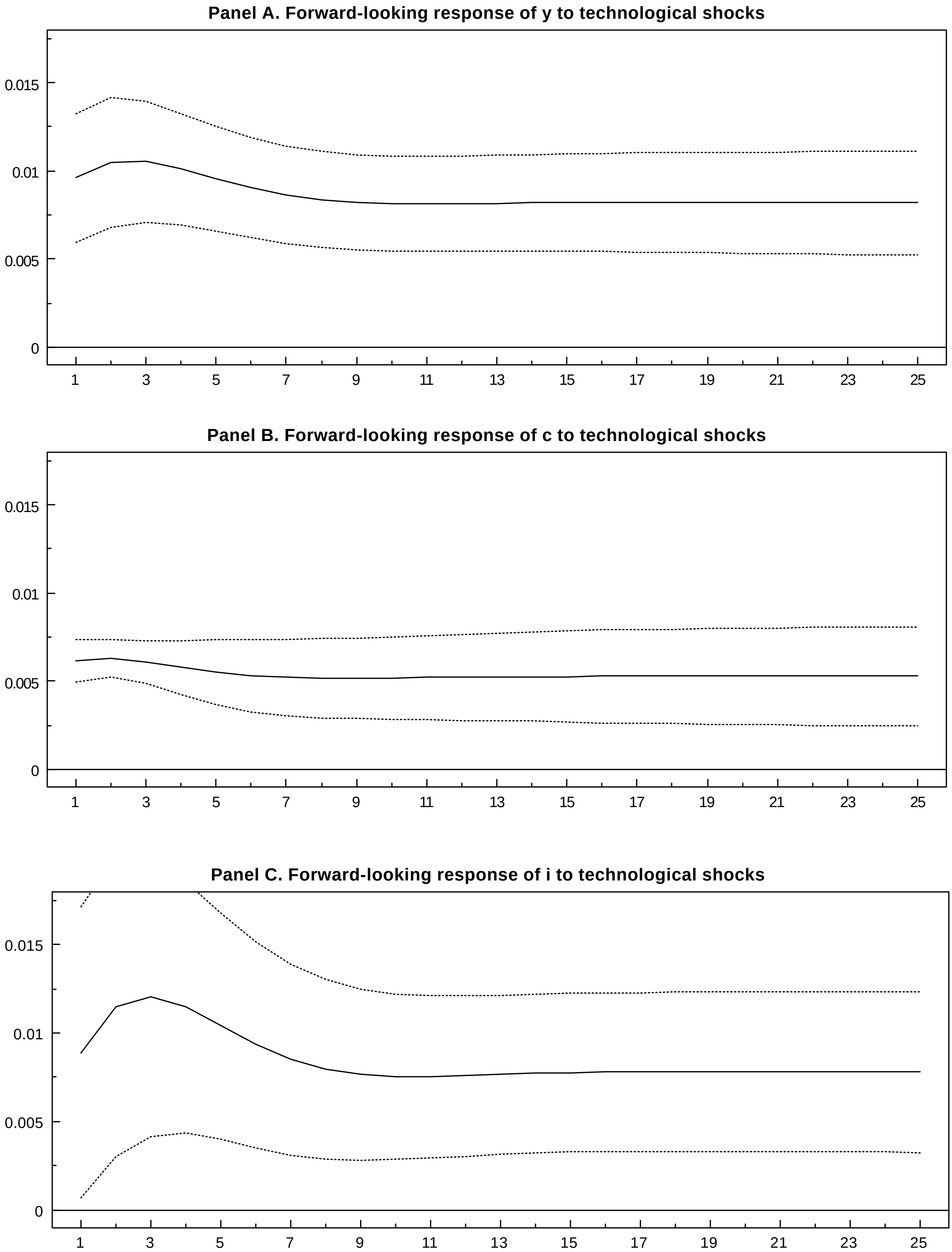

Figure 8: Forward-looking responses of outout (Panel A), consumption (Panel B) and investment (Panel C) to permanent shocks hitting the system in 1983.1. Dashed lines are one-standard-deviation confidence bands computed by Monte Carlo simulations..