Chapter 3

Achieving Efficient Outcomes in Economies with Externalities

3.1 Introduction

A natural field for the emergence of regulatory concerns is the economies with externalities where the agents' actions have significant effects on the other agents. In this paper we try to deal with this kind of economic environments where we design simple mechanisms that help the agents achieve non cooperatively the efficient outcome. To note that, in such economies the realization of efficient outcomes typically fails since the agents are unable to internalize the externalities.

We design a mechanism, a dynamic negotiation protocol that the agents undertake in order to agree on certain production decision, that implements the efficient outcome and ensures the equal sharing of surplus that cooperation generates. The main characteristics of the mechanism are that it is simple and it implements the desired outcome in the absence of asymmetric information among the players. The mechanism is simple in the sense that the participants do not necessarily use a large space of complex strategies or device sophisticated equilibrium strategies. We are interested in mechanisms that are applicable in real-life negotiations. Hence, the simplicity adds further credibility to the results, specially when we try to relate them with actual processes of negotiation. The assumption of complete information deserves certain clarifications. On the one hand, several economic interactions in the presence of externalities are undertaken by agents who have perfect information about others as they repeatedly met before. For example, the negotiations among the members of the European Union, where the countries concerned have been jointly negotiating over several issues. In such an environment, any potential inefficiency that may arise is caused by the presence of conflicting interests, rather than by the existence of private information. On the other hand, as Green and Laffont (1979) showed in the context of provision of a public good, which is a particular case of our problem, there does not exist a mechanism that is first-best Pareto optimal (efficient and budget balanced) and strategy-proof,[1] one has to design a mechanism sacrificing one of the above desirable properties. The nature of the mechanism we design allows us to avoid the problem of strategy proofness, provided there does not exist asymmetric information among the players, since we do not require the agents to send messages to the principal revealing their preferences. This way we are able to maintain the other two properties, namely efficiency and budget balance.

We present two alternative configurations of the mechanism. In the first one,

[1]For example, the pivotal mechanisms suggested by Clarke (1971) and Groves (1973), are efficient and strategy-proof, but are not budget balanced.

production takes place at every period. Hence, the non-cooperative outcome is the reference point of the players at the beginning of the process. The outcome continues to be inefficient till the players reach an agreement. This is the case, for instance, in any pollution abatement negotiation among different countries where the countries keep polluting according to the non-cooperative equilibrium until a consensus is reached. In the second case, the outcome of no-cooperation is only a potential result of the negotiation process. The economy does not produce anything and no profit is realized unless the negotiation has ended. Hence the non-cooperative outcome will prevail only if the negotiation fails. An example of this is a game in which the players have to agree on the implementation of a public project, or in the future exploitation of a common resource, *viz,* a fishing stock, a forest, a mine, etc.

The mechanisms we present have some nice features. First, they are relatively simple. In each, we start by auctioning the right to have the initiative in the negotiation. The winner proposes an allocation that is implemented if the rest of the players unanimously accept it; in case of a rejection the process is started again.[2] It is worth noting that the process is completely autonomous, the players do not need to send messages to the principal for him to decide on the optimal allocation, the regulator is completely passive in all the process of implementation.[3] Another interesting characteristic is that the payoff of the players is not random, hence the distributive objectives are attained deterministically and not in expected terms. The mechanisms are not based on very drastic threats made by the designer. Therefore, they have the attractive feature of being *renegotiation proof*, in

[2]Crawford (1979) was the first to propose methods for allocating resources based on a process of "auctioning the leadership".

[3]This is not the case in most of the existing mechanisms to implement an efficient allocation in a public goods economy. See, for instance, Jackson and Moulin (1992), or Bag (1997), where the players are asked to submit reports to the principal about their valuation for the project.

the sense that all the rules set by the regulator are credible. They do not lead to a Pareto dominated allocation in any subgame. The designer does not threat the players to force them to behave efficiently, either with expelling them from the game or by breaking down the negotiation.

One final comment about the mechanisms concerns their outcome. The sharing of the profits that emerges is based on the principle of equal sharing of the surplus that cooperation generates. We consider this way of allocating profits particularly appealing for environments with transferable utility, as it corresponds to two classical solutions for bargaining problems: the Nash Bargaining Solution and the Kalai-Smorodinski Solution.

The mechanisms we design share similar features with that suggested by Pérez-Castrillo and Wettstein (2001a). They construct a simple (both the process of implementation and the equilibrium strategies, are natural) game form in which the outcome of any Subgame Perfect Equilibrium coincides with the Shapley Value payoffs in zero-monotonic environments with transferable utility.

At this juncture we mention some previous research in the context of implementation of efficient outcomes in the presence of externalities. Varian (1994) presents a simple mechanism based on a compensation scheme chosen independently by each of the players. Eyckmans (1997) uses a variation of Varian's proposal to implement a proportional solution to international pollution control problems. However, these mechanisms consist of an artificial component, a penalty function that punishes the players when their announcements of the proposed compensation scheme are not symmetric. This is a useful analytical tool, but clearly reduces the real applicability of the resulting mechanism.

The provision of public goods has also been the area of active research in implementation theory. Jackson and Moulin (1992) propose an efficient mechanism to implement an indivisible public project and also to share its costs. Bag (1997) modifies the model of Jackson and Moulin allowing for divisible public goods. Pérez-Castrillo, and Wettstein (2001b) design a very simple mechanism that selects an efficient alternative over a set of possible choices.

Our work also shares some features of the literature on repeated bargaining. Nevertheless, in our work the agents share a surplus that is generated by the cooperation, whereas bargaining theory addresses the share of surplus that is exogenously given.

Baron and Ferejohn (1989) develop an interesting model of repeated bargaining in a context of political negotiation. In their approach a member is recognized (randomly) to make a proposal that can be posteriorly amended by other members. Even if our model is different from theirs, their process of random assignment of the initiative plus a potential process of amendments is close in spirit to our auction of the right to become the proposer. Chen and Ordeshook (1998) analyze a voting rule in a dynamic context with spatial preferences based on the requirement of unanimity and show how any policy that is Pareto optimal in the dynamic sense, can be sustained as the status-quo in perpetuity.

Seidmann and Winter (1998) design a model of gradual coalition formation based on a repeated process of negotiation in which proposals are sequentially made. Our negotiation game shares some characteristics with their process of bargaining since they give one player the right to make a proposal that the other players will accept or reject. However, the aim of their work is completely different from ours since, they study when the grand coalition is formed in games that are semi-strict superadditive, distinguishing two scenarios: one with reversible actions and another one in which they are irreversible.

Our paper is organized as follows: Section 3.2 presents the model. Section 3.3 solves a static version of the mechanism. We study the dynamic mechanism in Section 3.4. Section 3.5 presents an example in which we apply our negotiation game to the implementation of a public project. We provide an extension of the mechanism that allows for an exogenous weighting in the distribution of the profits in Section 3.6. Finally, Section 3.7 concludes by presenting some implications of the analysis performed.

3.2 The Model

Our model consists of n players, each one producing an amount $x_i \in \mathbb{R}_+$ of good i. Let $\mathcal{N} = \{1, ..., n\}$ denote the set of players. The profits of each agent depend on his own choice x_i, and also on the production of the other players x_{-i}. The function $\Pi_i(x_i, x_{-i})$, where $\Pi_i : \mathbb{R}^n_+ \to \mathbb{R}$, represents the revenues to player i. We do not restrict ourselves to positive profits, hence our construction allows for a situation in which the players share optimally the costs associated with a certain production decision x.

We assume that the agents are endowed with quasilinear preferences over a numéraire commodity (money, denoted by $M_i \in \mathbb{R}$). Hence, the utility of each agent ($U_i(x, M_i)$) can be described by $U_i(x, M_i) = \Pi_i(x_i, x_{-i}) + M_i$. Therefore, our model is essentially a transferable utility (TU) game.

We will undertake Subgame Perfect implementation, therefore we will assume that there is complete information among the players about their personal characteristics and production decisions.[4]

The vector $x^N = (x_1^N, .., x_i^N, ..., x_n^N)$ is a Nash (non-cooperative) equilibrium if:[5]

$$x_i^N \in \arg\max_{x_i} \Pi_i(x_i, x_{-i}^N), \forall i \in \mathcal{N}. \tag{3.1}$$

Analogously, the vector $x^F = (x_1^F, .., x_i^F, ..., x_n^F)$ is the Pareto Optimal level of production if it is obtained from the maximization of the joint profits:

$$x^F = (x_1^F, .., x_i^F, ..., x_n^F) \in \arg\max_x \sum_{i=1}^{n} \Pi_i(x_i, x_{-i}). \tag{3.2}$$

In the Nash Equilibrium (NE), the players are unable to internalize the externalities generated by the decisions of the other players and hence the NE outcome results in a loss of efficiency. Let S measure the surplus that cooperation generates.[6]

$$S = \sum_{j=1}^{n} \Pi_j(x^F) - \sum_{j=1}^{n} \Pi_j(x^N) > 0. \tag{3.3}$$

Finally, the evolution of profits over time is measured by the common discount factor $\delta \in (0, 1)$.[7]

3.3 The Static Mechanism

In this Section we will construct and solve a simplified mechanism, in which we do not allow for a repetition of the negotiation in case of a rejection of the proposed allocation.

We describe the mechanism Γ^0 as follows:

[4] Actually, the mechanism would maintain its properties (in expected terms) if we assumed that the agents have incomplete but symmetric information.

[5] We assume that the profit functions are such that a Nash equilibrium exists and is unique.

[6] In order to deal with situations in which cooperation yields a positive surplus to share, we take this difference strictly positive.

[7] In the presentation of the results we will provide insights on how the model changes if we allow for heterogeneous discount factors.

1. The players simultaneously make bids, where $b_j^i \in \mathbb{R}\ \forall j \neq i$ denotes the bid of player i to player j. We define the net bid of player i as $B^i = \sum_{j\neq i} b_j^i - \sum_{j\neq i} b_i^j$. Let $\alpha = \arg\max_i(B^i)$, in case of multiplicity α is chosen randomly. Finally α pays b_j^α to every $j \neq \alpha$.

2. Player α makes a proposal $\{T_j, x_j^*\}$ to every $j \neq \alpha$, where x_j^* is the production commitment and $T_j \in \mathbb{R}$ is a transfer. Player α also simultaneously chooses x_α^*.

3. Other players sequentially accept or reject the offer.[8] If nobody rejects the proposal made by α, the allocation is implemented. Otherwise the negotiation is broken, and the players move to the non-cooperative outcome.

Let us denote by $\Pr_i$ the overall discounted utility that player i gets from playing the static mechanism. In the following Proposition we characterize the Subgame Perfect Equilibrium (SPE) of the mechanism Γ^0.

Proposition 8 *The SPE outcome of* Γ^0 *is:*

1. $x_i^* = x_i^F$

2. $\Pr_i = \Pi_i(x^N) + \frac{1}{n}S$, *for every* $i \in \mathcal{N}$.

Proof. In order to characterize the Subgame Perfect Equilibrium (SPE), we solve the mechanism by backwards induction, starting from the third stage.

Any player $j \neq \alpha$, will accept the offer if and only if:

$$\Pi_j(x^*) + T_j \geq \Pi_j(x^N). \tag{3.4}$$

[8] The exact order is irrelevant. The only restriction is that the answers are not simultaneous.

Player α will choose x^* and T_j in order to maximize his profits. Hence, $T_j = \Pi_j(x^N) - \Pi_j(x^*)$ and x^* will be such that:

$$x^* = \arg\max_x \Pi_\alpha(x) - \sum_{j\neq\alpha} T_j \equiv \arg\max_x \sum_{j=1}^{n} \Pi_j(x) - \sum_{j\neq\alpha} \Pi_j(x^N).$$

The above implies that $x^* = x^F$.

We have to check whether the proposer will always be interested in his offer being accepted i.e.,

$$\begin{aligned} \Pi_\alpha(x^*) - \sum_{j\neq\alpha} T_j \quad &\geq \quad \Pi_\alpha(x^N) \Longleftrightarrow \sum_{j=1}^{n} \Pi_j(x^*) - \sum_{j\neq\alpha} \Pi_j(x^N) \geq \Pi_\alpha(x^N) \qquad (3.5) \\ &\Longleftrightarrow \quad \sum_{j=1}^{n} \Pi_j(x^F) \geq \sum_{j=1}^{n} \Pi_j(x^N) \Longleftrightarrow S \geq 0. \end{aligned}$$

The above condition always holds good, since the first-best allocation always Pareto dominates the non-cooperative outcome. Now we solve the bidding stage. In order to do it, we make use of two claims derived in the proof of Theorem 1 in Pérez-Castrillo and Wettstein (2001a).[9]

Claim 1 *In any SPE,* $B^i = B^j$, $\forall i, j$, *and hence,* $B^i = 0$, $\forall i \in \mathcal{N}$.

Claim 2 *In any SPE, each player's payoff is the same regardless of who is chosen as the proposer.*

Let Pr_i^j denote the overall profits of player i when j is the proposer, we can compute:

$$\begin{aligned} \sum_{j=1}^{n} \mathrm{Pr}_i^j \quad &= \quad \sum_{j=1}^{n} \Pi_j(x^F) - \sum_{j\neq i} \Pi_j(x^N) - \sum_{j\neq i} b_j^i + \sum_{j\neq i} \left[\Pi_i(x^N) + b_i^j\right] \\ &\Longrightarrow \quad \sum_{j=1}^{n} \mathrm{Pr}_i^j = \sum_{j=1}^{n} \Pi_j(x^F) + \sum_{j\neq i} \left[\Pi_i(x^N) - \Pi_j(x^N)\right] - B^i. \end{aligned}$$

[9] The proof of these Claims is given in the Appendix.

Now, by Claim 1, $B^i = 0$, and by Claim 2, $\sum_{j=1}^{n} \mathrm{Pr}_i^j = n\,\mathrm{Pr}_i$, hence:

$$\mathrm{Pr}_i = \frac{1}{n}\sum_{j=1}^{n} \Pi_j(x^F) + \frac{1}{n}\sum_{j\neq i}\left[\Pi_i(x^N) - \Pi_j(x^N)\right].$$

The above expression can be rewritten, by simple algebraic manipulations, as follows,

$$\mathrm{Pr}_i = \Pi_i(x^N) + \frac{1}{n}\left[\sum_{j=1}^{n} \Pi_j(x^F) - \sum_{j=1}^{n} \Pi_j(x^N)\right] \Longrightarrow \mathrm{Pr}_i = \Pi_i(x^N) + \frac{1}{n}S. \tag{3.6}$$

This completes the proof. ■

There are two important features of the mechanism that are worth noting. The first one concerns its outcome. The outcome of the mechanism is efficient and the surplus is shared equally among the players. In an environment with transferable utility, this surplus sharing corresponds to the Nash Bargaining Solution, as well as to the Kalai-Smorodinski Solution. The revenues of each player consist of his disagreement point (in our framework the profits in the absence of cooperation), and an equal split of the extra surplus that cooperation generates. An important advantage of the mechanism is that it does not give the solution in expected terms, like other mechanisms in the literature do.[10] Here, the profits each player gets is deterministic and equal to Pr_i. This further consolidates our mechanism, since the outcome that each agent gets is individually rational, not only in expected terms, but under any circumstance. Figure 3.1 illustrates the distribution of the profits in an economy with two players.

The second comment is associated to the structure of the mechanism. The proof shows that stages 2 and 3 ensure an efficient allocation, while stage 1 generates an egalitarian

[10] For instance, the main body of the literature in Shapley Value implementation, has made use of this implementation in expected terms: Gul (1989), Krhisna and Serrano (1995), Evans (1996), and Hart and Mas-Colell (1996).

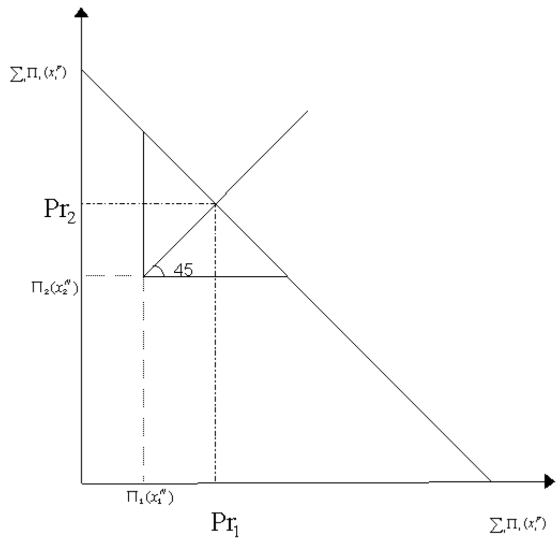

Figure 3.1: Equilibrium Outcome in a Two Players' Economy

split of the surplus. Efficiency is achieved in stages 2 and 3, since the proposer becomes a residual claimer of all the surplus that cooperation generates. His incentives to maximize his earnings are equivalent to the maximization of total surplus, leading therefore to an efficient choice of x. However, the price to pay for the attainment of Pareto optimality is an uneven sharing of the profits. This bias in favor of the proposer is solved in the bidding stage, where the process of "auctioning" the right to have the initiative in the negotiation, vanishes the posterior advantage of the proposer.

However, this simple mechanism has a weak point as it is sustained by a very drastic threat: if the offer is rejected, the outcome is the non-cooperative one. This threat is non-credible since, once an offer has been rejected, trying to negotiate again seems to naturally dominate the breakdown of cooperation. A way to solve this problem is to slightly modify the static mechanism and allow for a repetition of the negotiation process in case of an unsuccessful previous attempt. This is done in the following Section.

3.4 The Dynamic Mechanisms

In this Section, we modify the static mechanism to allow for a repetition of the negotiation process after an unsuccessful attempt to reach an agreement. The only difference with the static mechanism stays in the third stage, since now in case of a single rejection of the proposed allocation the negotiation is broken only in that iteration.

At this point we need to distinguish between two different frameworks in which the negotiation process can be undertaken. These frameworks differ in the timing of the production process. One possibility is to consider the profits as a flow variable, i.e., the production of x, and hence the realization of the profits is undertaken every period. Therefore, until an agreement is reached, the production decision is taken in a non-cooperative way. This makes the absence of cooperation the status-quo of the game. The other possibility is to consider that the production of the good with external effects, is only undertaken once the process of negotiation has come to an end (either successfully, or unsuccessfully). With this configuration, the absence of cooperation is an outside option for the negotiation.

We need to study independently the two environments since the difference in the timings affects the process of repeated negotiation. When the outcome of no cooperation is the status-quo, a process of negotiation will always be followed by another one, as the worst possible outcome is no agreement. However, when the outcome of no cooperation is the outside option, a new negotiation process will start again at the beginning of the next period only if all the players agree. We do not allow for partial agreements and hence, we cannot preclude that a player decides to break the negotiation process if he gains by defecting.

Notice that by requiring unanimity in the acceptance the proposal, as well as in the decision to continue negotiating we give the players the possibility to exert a veto power. As it is shown later, this veto power will be crucial since it precludes the players from strategically giving up the negotiation in order to free-ride the agreement attained by the others. The withdrawal of one player from the negotiation will yield a full breakdown of the process of trying to reach an agreement, and therefore will not be used for strategic purposes.

In the following subsections, we will compute the outcome of the proposed mechanism under both configurations and also discuss about the appropriate economic situations which our model fits into.

3.4.1 The Dynamic Mechanism for Situations with No Cooperation as the Status-Quo

In this subsection, we present a situation in which the players will remain at the non-cooperative outcome until the negotiation succeeds. This means that we are in a framework in which the source of the externality is present at the beginning of the process. The players will be incurring in the inefficiency caused by the absence of cooperation in all the periods from the starting point of the negotiation, until the agreement is reached.

Since profits are a flow variable, for analytical convenience we will normalize the stage profits. The profit of player i is given by $(1-\delta)\Pi_i(\cdot)$. Hence, the overall discounted sum of the profits of player i becomes:

$$\sum_{t=0}^{\infty} \Pi_{it} = \sum_{t=0}^{\infty} (1-\delta)\delta^t \Pi_i(\cdot) = \Pi_i(\cdot). \tag{3.7}$$

Now we will proceed to state formally the mechanism Γ^1 for economies with no cooperation as the status-quo.

At each iteration (attempt to agree), $t \in \mathbb{N}$,

1. The players simultaneously make bids, where $b^i_{jt} \in \mathbb{R}\ \forall j \neq i$ denotes the bid of player i to player j. We define the net bid of player i as $B^i_t = \sum\limits_{j \neq i} b^i_{j_t} - \sum\limits_{j \neq i} b^j_{i_t}$. Let $\alpha_t = \arg\max\limits_i (B^i_t)$, in case of multiplicity α_t is chosen randomly. Finally α_t pays b^{α}_{jt} to every $j \neq \alpha_t$.

2. Player α_t makes a proposal $\{T_{jt}, x^*_{jt}\}$ to every $j \neq \alpha_t$, where x^*_{jt} is the production commitment and $T_{jt} \in \mathbb{R}$ is a transfer. Player α_t also simultaneously chooses $x^*_{\alpha_t t}$.

3. Other players sequentially accept or reject the offer.[11] If nobody rejects the proposal made by α_t, the allocation is implemented. Otherwise the process of negotiation is started again from step 1, in period $t+1$.

Our game is an infinite horizon game, hence the strategy of agent i in the t^{th} iteration ($\sigma_{it} \in \sum_{it}$) will be defined for each possible $t-1$ history $h_{t-1} \in H_{t-1}$.

We define a strategy for i, $\sigma_i \in \sum_i$ by $\sigma_i = (\sigma_{it})_{t=0}^{\infty}$, where $\sigma_{it} \in \sum_{it}$ for all t. Therefore, $\sum = \sum_1 \times \sum_2 \times\ ...\ \times \sum_n$ is the set of strategy profile σ.

We will use the stationary subgame perfect Nash equilibrium (SSPNE) as our solution concept. $E(\sigma)$ is an SSPNE induced by strategy σ if and only if:

[11] The exact order is irrelevant. The only restriction is that the answers are not simultaneous.

i) The strategy of each agent i is optimal after every history, given the strategies of all other agents.

ii) σ is stationary.[12] That is, after two different histories h_{t-1} and $h_{t'-1}$, such that at the beginning of two negotiation rounds, player i faces the same initial situation, then $\sigma_{it}(h_{t-1}) = \sigma_{it'}(h_{t'-1})$. In words, this restriction implies that, in each iteration, the players will only capture from the past the fact that there was no agreement, and will disregard other aspects as who were the previous proposers, which were their proposals, or why they were rejected, as they are payoff irrelevant for the current negotiation process.

With the restriction to stationary strategies, we can write σ_{it} as follows:

$$\sigma_{it}(h_{t-1}) = \left[(b^i_{jt})_{j\neq i}; \left((x_{jt})^n_{j=1}, (T_{jt})_{j\neq i}\right); \tilde{T}_{it}\right]. \tag{3.8}$$

The first term in the strategy profile corresponds to the bids at t. The second one is i's choice in case he is the proposer. The last part defines his behavior when he receives an offer defining $\tilde{T}_{it}$ as the minimum transfer that agent i will accept.

Before analyzing the outcome of the mechanism, we will specify the payoffs the agents will obtain from playing the negotiation game we have presented. Let us denote by $\Pr_{it+1}$ the payoff earned by player i in the negotiation process that starts after $t+1$ previous attempts. The payoff for each player in the t^{th} iteration of the negotiation process is:

1. If all the responders ($\forall j \neq \alpha_t$) accept the offer:

   The proposer (α_t) gets: $(1-\delta)\sum\limits_{\tau=t}^{\infty}\delta^{\tau-t}\Pi_{\alpha_t}(x^*_t) - \sum\limits_{j\neq\alpha_t} T_{jt} - \sum\limits_{j\neq\alpha_t} b^{\alpha_t}_{jt}$.

   The responders ($j \neq \alpha_t$) get: $(1-\delta)\sum\limits_{\tau=t}^{\infty}\delta^{\tau-t}\Pi_j(x^*_t) + T_{jt} + b^{\alpha_t}_{jt}$.

[12] The notion of stationarity we use is a direct application of the concept of stationarity derived by Rubinstein and Wolinsky (1985).

2. If $\exists j \neq \alpha_t$, that rejects the offer:

   The proposer (α_t) gets: $(1-\delta)\Pi_{\alpha_t}(x^N) - \sum_{j \neq \alpha_t} b_{jt}^{\alpha_t} + \delta \Pr_{\alpha_t t+1}$.

   The responders ($j \neq \alpha_t$) get: $(1-\delta)\Pi_j(x^N) + b_{jt}^{\alpha_t} + \delta \Pr_{jt+1}$.

We characterize the outcome of the dynamic negotiation in the following propositions.

Proposition 9 *Let $\sigma^* \in \sum$ be a stationary SPNE strategy profile, and $E(\sigma^*)$ be the SSPNE that it induces. Denote by $\Pr_i$ the discounted payoff of agent i in $E(\sigma^*)$. The outcome of $E(\sigma^*)$ in Γ^1 is, for every $i \in \mathcal{N}$:*

1. $x_i^* = x_i^F$.

2. $\Pr_i = \Pi_i(x^N) + \frac{1}{n}S$.

Proof. In order to characterize the outcome of the SSPNE, we assume that, without loss of generality, we are in the $\hat{t}$-th iteration (attempt to agree). We solve the mechanism by backwards induction starting from the third stage of this iteration.

The construction of the mechanism ensures that all the players other than the proposer ($j \neq \alpha_{\hat{t}}$), will accept the offer if:

$$(1-\delta)\sum_{t=\hat{t}}^{\infty} \delta^{t-\hat{t}} \Pi_j(x^*) + T_{j\hat{t}} \geq (1-\delta)\Pi_j(x^N) + \delta \Pr_{j\hat{t}+1}. \tag{3.9}$$

We find the level of transfers: $T_{j\hat{t}} = (1-\delta)\Pi_j(x^N) + \delta \Pr_{j\hat{t}+1} - \Pi_j(x^*)$. Moving to the second stage, the proposer will choose x^*, according to:

$$\max_x (1-\delta)\sum_{t=\hat{t}}^{\infty} \delta^{t-\hat{t}} \Pi_{\alpha_{\hat{t}}}(x^*) - \sum_{j \neq \alpha_{\hat{t}}} T_{j\hat{t}}$$

$$\Longrightarrow \max_x \left[\sum_{j=1}^{n} \Pi_j(x^*)\right] - \sum_{j \neq \alpha_{\hat{t}}} \left[(1-\delta)\Pi_j(x^N) + \delta \Pr_{j\hat{t}+1}\right].$$

The above gives $x^* = x^F$.

We will have to wait until the equilibrium payoffs are computed, to check whether the proposer will always be interested in his offer being accepted.

Now we have to solve the bidding stage, in order to do it, we make use of two claims similar to the ones derived from the proof of Theorem 1 in Pérez-Castrillo and Wettstein (2001a).[13]

Claim 3 *In any SSPNE,* $B_t^i = B_t^j$, $\forall i, j$, *and hence,* $B_t^i = 0$, $\forall i \in \mathcal{N}$, $\forall t \in \mathbb{N}$

Claim 4 *In any SSPNE, each player's payoff is the same regardless of who is chosen as the proposer.*

Let $\Pr_{i\hat{t}}^j$ denote the overall profits of player i when j is the proposer, and we are in the $\hat{t}$-th attempt to negotiate. We can compute:

$$\sum_{j=1}^{n} \Pr_{i\hat{t}}^j = \left[\sum_{j=1}^{n} \Pi_j(x^F)\right] - \sum_{j \neq i} \left[(1-\delta)\Pi_j(x^N) + \delta \Pr_{j\hat{t}+1}\right] - \sum_{j \neq i} b_{j\hat{t}}^i$$
$$+ \sum_{j \neq i} \left[(1-\delta)\Pi_i(x^N) + \delta \Pr_{i\hat{t}+1} + b_{i\hat{t}}^j\right]$$
$$\Longrightarrow \sum_{j=1}^{n} \Pr_{i\hat{t}}^j = \left[\sum_{j=1}^{n} \Pi_j(x^F)\right] + \delta \sum_{j \neq i} \left[\Pr_{i\hat{t}+1} - \Pr_{j\hat{t}+1}\right] + (1-\delta) \sum_{j \neq i} \left[\Pi_i(x^N) - \Pi_j(x^N)\right] - B_{\hat{t}}^i.$$

Now, by Claim 3, $B_{\hat{t}}^i = 0$, and by Claim 4, $\sum\limits_{j=1}^{n} \Pr_{i\hat{t}}^j = n \Pr_{i\hat{t}}$. These together imply

$$\Pr_{i\hat{t}} = \frac{1}{n} \sum_{j=1}^{n} \Pi_j(x^F) + \frac{(1-\delta)}{n} \sum_{j \neq i} \left[\Pi_i(x^N) - \Pi_j(x^N)\right] + \frac{\delta}{n} \sum_{j \neq i} \left[\Pr_{i\hat{t}+1} - \Pr_{j\hat{t}+1}\right]. \quad (3.10)$$

[13]The proof of these two claims is completely analogous to that of Claims 1 and 2, and is therefore ommited.

The above equation gives a recursive expression of the outcome of cooperation. It allows us to compute $\Pr_{i\hat{t}+1} - \Pr_{j\hat{t}+1}$. After some algebraic manipulations we obtain:

$$\Pr_{i\hat{t}+1} - \Pr_{j\hat{t}+1} = (1-\delta)\left[\Pi_i(x^N) - \Pi_j(x^N)\right] + \delta\left(\Pr_{i\hat{t}+2} - \Pr_{j\hat{t}+2}\right).$$

Introducing this in the expression for $\Pr_{i\hat{t}}$, yields:

$$\Pr_{i\hat{t}} = \frac{1}{n}\sum_{j=1}^{n}\Pi_j(x^F) + \frac{(1-\delta)(1+\delta)}{n}\sum_{j\neq i}\left[\Pi_i(x^N) - \Pi_j(x^N)\right] + \frac{\delta^2}{n}\sum_{j\neq i}\left[\Pr_{i\hat{t}+2} - \Pr_{j\hat{t}+2}\right].$$

The process of recursive substitution can be repeated arbitrarily many times, until we find the following general expression, $\forall m \in \mathbb{N}\backslash\{0\}$:

$$\Pr_{i\hat{t}} = \frac{1}{n}\sum_{j=1}^{n}\Pi_j(x^F) + \frac{(1-\delta)}{n}\left[\sum_{\overline{m}=0}^{m-1}\delta^{\overline{m}}\right]\sum_{j\neq i}\left[\Pi_i(x^N) - \Pi_j(x^N)\right] + \frac{\delta^m}{n}\sum_{j\neq i}\left[\Pr_{i\hat{t}+m} - \Pr_{j\hat{t}+m}\right].$$

At this point we can make us of the fact that as $\delta < 1$, if the negotiation lasts infinitely, the profits to share will lose their present value, hence:

$$\lim_{m\to\infty}\delta^m \Pr_{i\hat{t}+m} = 0,\ \forall i \in \mathcal{N}. \tag{3.11}$$

This allows us to rewrite the payoff function as:

$$\begin{aligned}
\Pr_{i\hat{t}} &= \frac{1}{n}\sum_{j=1}^{n}\Pi_j(x^F) + \frac{(1-\delta)}{n}\left[\sum_{\overline{m}=0}^{\infty}\delta^{\overline{m}}\right]\sum_{j\neq i}\left[\Pi_i(x^N) - \Pi_j(x^N)\right] \\
&\Longrightarrow \Pr_{i\hat{t}} = \Pi_i(x^N) + \frac{1}{n}\left(\sum_{j=1}^{n}\Pi_j(x^F) - \sum_{j=1}^{n}\Pi_j(x^N)\right) \\
&\Longrightarrow \Pr_{i\hat{t}} = \Pi_i(x^N) + \frac{1}{n}S. \qquad (3.12)
\end{aligned}$$

At this point, we can check that the proposer will always be interested in making an "acceptable" offer. In order for this to happen, it has to hold that, $\forall \hat{t} \geq 0$:

$$\sum_{j=1}^{n} \Pi_j(x^*) - \sum_{j \neq \alpha_{\hat{t}}} \left[(1-\delta)\Pi_j(x^N) + \delta \Pr_{j\hat{t}+1} \right] \geq (1-\delta)\Pi_{\alpha_{\hat{t}}}(x^N) + \delta \Pr_{\alpha_{\hat{t}}\hat{t}+1} .$$

Substituting the payoff function, the above condition collapses to the following time-independent inequality:

$$S = \sum_{j=1}^{n} \Pi_j(x^F) - \sum_{j=1}^{n} \Pi_j(x^N) \geq 0,$$

and this condition is always satisfied.

From the general expression of the profits that each player gets from reaching an agreement in the $\hat{t}$-th iteration, it is straightforward to check that all the players are strictly better off from not delaying the cooperation. As $\delta < 1$, then $\forall i \in \mathcal{N}$:

$$\Pr_{i\hat{t}} - \delta \Pr_{i\hat{t}+1} = (1-\delta)\left(\Pi_i(x^N) + \frac{1}{n} S \right) > 0, \ \forall \hat{t} \geq 0.$$

Then the agreement will be reached in the first attempt and therefore:

$$\Pr_i = \Pr_{i0} = \Pi_i(x^N) + \frac{1}{n} S, \forall i \in \mathcal{N}. \tag{3.13}$$

This completes the proof. ■

We have shown that there is a unique candidate for the payoffs in an SSPNE. However, we have left to prove that this equilibrium exists. That is, we need to find an equilibrium strategy profile.

Proposition 10 $\sigma^* \in \sum$, *is an SSPNE strategy profile of the negotiation game with no-cooperation as the status-quo.* σ^* *is such that, for every* $t \in \mathbb{N}$,

$$\sigma^*_{it} = \left[\left(b^{i*}_{jt}\right)_{j \neq i} ; \left(\left(x^*_{jt}\right)_{j=1}^{n}, \left(T^*_{jt}\right)_{j \neq i} \right) ; \tilde{T}^*_{it}, \right],$$

with:

$$\left(b^{i*}_{jt} = \frac{(1-\delta)}{n} S\right)_{j\neq i}$$

$$\left(\left(x^*_{jt} = x^F_j\right)^n_{j=1}, \left(T^*_{jt} = \Pi_j(x^N) + \frac{\delta}{n} S - \Pi_j(x^F)\right)_{j\neq i}\right)$$

$$\tilde{T}^*_{it} = \Pi_i(x^N) + \frac{\delta}{n} S - \Pi_i(x^F).$$

Proof. First, simple algebra shows that the strategy written above yields $\Pr_i$ for every player i (and in general $\Pr_{it}$ $\forall t \in \mathbb{N}$). Second, $B^i_t = 0$ $\forall i$, since $b^j_{it} = b^i_{jt}$ $\forall i, j$ and $\forall t \in \mathbb{N}$. Let us show now that the proposed strategy is best response:

- The third and second stages:

  i)Every respondent j behaves optimally since, $\forall t$,

  $$\Pi_j(x^F) + T^*_{jt} = (1-\delta)\Pi_j(x^N) + \delta \Pr_{jt+1},$$

  and $(1-\delta)\Pi_j(x^N) + \delta \Pr_{jt+1}$ is the maximum j can get by refusing the offer.

  ii) The choice of the proposer is optimal. x^*_t maximizes the revenue to share, and T^*_{jt} is the minimum he has to offer such that the rest of the players accept Moreover, it is easy to check that the proposer always wants to make an acceptable proposal provided $S \geq 0$.

- The first stage:

  There is no profitable deviation in the choice of the bids. If player i increased his total bid, he would be chosen as the proposer with certainty, but his payoff would decrease. If he decreased his bid, he would surely not be the proposer, but his payoff would still be $\Pr_i$. Finally, any other deviation that leaves his total bid unaltered, could have influenced the identity of the proposer, but not player i's payoff.

This completes the proof ■

The above propositions show that this dynamic version of the mechanism induces an optimal provision of the good with externalities, together with an equal share of the surplus that arises from cooperation. This is done in a dynamic setting where the agents take into account the continuation payoffs from mantaining the process of negotiation, in their decision of whether to accept or not the proposal. Note also that in equilibrium, the agreement is reached in the first attempt (with only one round of negotiations).

Moreover, this negotiation protocol has the attractive characteristic of being "renegotiation proof", in the sense that in order to sustain the equilibrium, it does not need any threat that is non-credible. This problem was present in the static mechanism since we linked a rejection of the proposed allocation with a full breakdown of the cooperation. It is worth noting that the mechanism by Pérez-Castrillo and Wettstein (2001a) does not have this property. In their model, if any proposal is rejected, the proposer is taken out from the negotiation and the rest replay the mechanism. This is not "renegotiation proof", as by doing this, the remaining players have a smaller surplus to share.

It can be shown that the mechanism proposed can also deal with the case of heterogeneous discount factors. The efficiency result is unaltered. The sharing of the profits is still based on an equal sharing of the surplus, but the different degrees of "patience" of the players introduces a distortion. In equilibrium, the most patient players gain relatively higher profits, as this gives them a stronger bargaining position.

This mechanism has been designed for markets in which the players are already suffering from the inefficiency that the absence of cooperation generates. This is the case in many economic situations with externalities. The problems of environmental protection are clearly among these. The players (in this context, the countries) will remain polluting at their non-cooperative levels, until they reach an agreement.

However, this is not the only possible framework one may think of. Several economic interactions do not take place until the players, either agree on a cooperative behavior, or decide to act independently. These will be our concern in the next Subsection.

3.4.2 The Dynamic Mechanism for Situations with No Cooperation as the Outside Option

In this subsection we modify the mechanism to encompass a situation where the good is not produced and no payoff is realized until negotiation ends. In this framework the common discount factor plays a crucial role in determining the outcome as it reflects players' willingness to continue the negotiation process in future. A very low value of δ implies that the negotiation is broken early and the non-cooperative payoff accrues to each player

We formally describe the mechanism Γ^2 for situations with no cooperation as the outside option as follows.

At each iteration (attempt to agree), $t \in \mathbb{N}$,

1. The players simultaneously make bids, where $b^i_{jt} \in \mathbb{R}\ \forall j \neq i$ denotes the bid of player i to player j. We define the net bid of player i as $B^i_t = \sum_{j \neq i} b^i_{j_t} - \sum_{j \neq i} b^j_{i_t}$. Let $\alpha_t = \arg\max_i (B^i_t)$, in case of multiplicity α_t is chosen randomly. Finally α_t pays b^{α}_{jt} to every $j \neq \alpha_t$.

2. Player α_t makes a proposal $\{T_{jt}, x^*_{jt}\}$ to every $j \neq \alpha_t$, where x^*_{jt} is the production commitment and $T_{jt} \in \mathbb{R}$ is a transfer. Player α_t also simultaneously chooses $x^*_{\alpha_t t}$.

3. Other players sequentially accept or reject the offer.[14] If nobody rejects the proposal made by α_t, the allocation is implemented and the transfers are realized.

4. If a rejection occurs, the players sequentially choose whether to start a new identical process of negotiation at $t+1$, or break the negotiations where the players move to the non-cooperative outcome.[12]

As in the previous Subsection, our game will have an infinite horizon and the strategy of agent i in the t^{th} iteration ($\sigma_{it} \in \sum_{it}$) will be defined for each possible $t-1$ history $h_{t-1} \in H_{t-1}$. As before we define a strategy for i, $\sigma_i \in \sum_i$ by $\sigma_i = (\sigma_{it})_{t=0}^{\infty}$, where $\sigma_{it} \in \sum_{it}$ for all t. Therefore, $\sum = \sum_1 \times \sum_2 \times \ ... \ \times \sum_n$ is the set of strategy profile σ.

In this subsection, we will enlarge the space of strategies by introducing weakly stationary strategies. This means that we will let the agents make their choices contingent not only on payoff relevant variables, but also on calendar time (even if in our game time is payoff irrelevant). With this space of strategies our solution concept will be Weakly Stationary Subgame Perfect Nash Equilibrium (WSSPNE), which is defined analogously as in the previous subsection, only substituting the requirement of stationarity by weak stationarity. It will be shown later that this enlargement of the strategy space will avoid problems of inexistence of equilibria. Moreover, it will give rise to equilibrium strategies that are "natural" and consistent with the presumed behavior of an individual in real-life negotiation processes. With this restriction on the strategy space, we can write σ_{it} as

[14]The exact order is irrelevant. The only restriction is that the answers are not simultaneous.

follows:

$$\sigma_{it}(h_{t-1}) = \left[(b^i_{jt})_{j\neq i}; \left((x_{jt})^n_{j=1}, (T_{jt})_{j\neq i}\right); \left(\tilde{T}_{it}, a_{it} \in \{\text{retry,quit}\}\right)\right]. \tag{3.14}$$

The first term in the strategy profile corresponds to the bids in the initial stage. The second one is i's choice in case he is the proposer. The last part defines his behavior when he receives an offer; $\tilde{T}_{it}$ is the minimum transfer that agent i will accept, and the other term a_{it} gives i's decision concerning a repetition of the process after a rejection has occurred.

We first state the payoffs the players will obtain from playing the negotiation game, with $\Pr_{it+1}$ denoting the payoff earned by player i in the negotiation process that starts after $t+1$ previous attempts. The payoff for each player in the t^{th} iteration of the negotiation process is:

1. If all the responders ($\forall j \neq \alpha_t$) accept the offer:

   The proposer (α_t) gets: $\Pi_{\alpha_t}(x^*) - \sum_{j\neq\alpha_t} T_{jt} - \sum_{j\neq\alpha_t} b^{\alpha_t}_{jt}$.

   The responders ($j \neq \alpha_t$) get: $\Pi_j(x^*) + T_{jt} + b^{\alpha_t}_{jt}$.

2. If $\exists j \neq \alpha_t$, that rejects the offer:

   (a) If $\forall i \in \mathcal{N}$, $a_{it} = \{\text{retry}\}$:

   The proposer (α_t) gets: $-\sum_{j\neq\alpha_t} b^{\alpha_t}_{jt} + \delta \Pr_{\alpha_t t+1}$.

   The responders ($j \neq \alpha_t$) get: $b^{\alpha_t}_{jt} + \delta \Pr_{jt+1}$.

   (b) If $\exists i \in \mathcal{N}$, $a_{it} = \{\text{quit}\}$:

   The proposer (α_t) gets: $\Pi_{\alpha_t}(x^N) - \sum_{j\neq\alpha} b^{\alpha_t}_{jt}$

   The responders ($j \neq \alpha_t$) get: $\Pi_j(x^N) + b^{\alpha_t}_{jt}$.

In order to characterize the WSSPNE, we need to define a series of thresholds for δ. Let $\overline{\delta} = \frac{\max_i\{\Pi_i(x^N)\}}{\frac{1}{n}\sum\limits_{j=1}^{n}\Pi_j(x^F)}$, and $\widehat{\delta}_m$ be the unique fixed point of $f_m(\delta)$, where

$$f_m(\delta) = \frac{\max_i\{\Pi_i(x^N)\}}{\frac{1}{n}\sum\limits_{j=1}^{n}\Pi_j(x^F) + \delta^m\left(\max_i\{\Pi_i(x^N)\} - \frac{1}{n}\sum\limits_{j=1}^{n}\Pi_j(x^N)\right)}, \ \forall m \in \mathbb{N}. \tag{3.15}$$

$\overline{\delta}$ and $\widehat{\delta}_m$ are such that; $\forall m \in \mathbb{N}$, $\widehat{\delta}_m < \widehat{\delta}_{m+1} < \overline{\delta}$, with $\lim_{m\to\infty}\widehat{\delta}_m = \min\{\overline{\delta}, 1\}$.[15]

The outcome of this dynamic negotiation mechanism Γ^2 is presented in the following propositions.

Proposition 11 *Let $\sigma^* \in \sum$ be a weakly stationary SPNE strategy profile, and $E(\sigma^*)$ be the WSSPNE that it induces. Denote by* $\Pr_i$ *the discounted payoff of agent i in $E(\sigma^*)$. The outcome of $E(\sigma^*)$ in Γ^2 is, for every $i \in \mathcal{N}$:*

1.- $x_i^* = x_i^F$.

2.- The payoff function is:

$$\Pr_i = \begin{cases} \Pi_i(x^N) + \frac{1}{n}S & \text{If } \delta \leq \widehat{\delta}_0 \\ \frac{1}{n}\sum\limits_{j=1}^{n}\Pi_j(x^F) + \delta^m\left(\Pi_i(x^N) - \frac{1}{n}\sum\limits_{j=1}^{n}\Pi_j(x^N)\right) & \text{If } \delta \in (\widehat{\delta}_{m-1}, \widehat{\delta}_m],\ \forall m \in \mathbb{N}\backslash\{0\} \\ \frac{1}{n}\sum\limits_{j=1}^{n}\Pi_j(x^F) & \text{If } \overline{\delta} < 1,\ \forall \delta > \overline{\delta}. \end{cases}$$

Proof. The proof is similar to that of Proposition 9, although the optimal behavior of the players is more complicated since, now that no-cooperation is the outside option, it

[15] The formal characterization of $\{\widehat{\delta}_m\}_{m=0}^{\infty}$ is provided in the Appendix.

is not always true that after a failed attempt to negotiate, the players will be willing to retry again in the next period. More formally, each player i, when facing the t-th attempt to agree, will compare the proposal with:

$$\max\{\Pi_i(x^N), \delta \operatorname{Pr}_{it+1}\}. \tag{3.16}$$

Therefore, while deciding whether to accept or not the proposed allocation the player will also choose what to do if he rejects it. As the answers are given sequentially and in order to continue negotiating all have to agree in it, the game will always be replayed after an unsuccessful iteration if, $\forall i \in \mathcal{N}$ and $\forall t \in \mathbb{N}$,

$$\Pi_i(x^N) < \delta \operatorname{Pr}_{it+1}. \tag{3.17}$$

We will first solve the mechanism assuming this inequality holds, and once we have computed the equilibrium payoffs, we will check whether it is actually satisfied. If (3.17) holds, then by an analogous process of backwards induction to the one used in Proposition 9, we can write the equilibrium payoffs of player i in the t-th iteration of the game as the following recursive equation,

$$\operatorname{Pr}_{it} = \frac{1}{n}\sum_{j=1}^{n}\Pi_j(x^F) + \frac{1}{n}\sum_{j\neq i}\left[\delta \operatorname{Pr}_{it+1} - \delta \operatorname{Pr}_{jt+1}\right], \ \forall t \in \mathbb{N}$$

Applying again the process of recursive substitution, plus the fact that as $\delta < 1$, if the negotiation lasts infinitely the profits to share will have no present value:

$$\lim_{m\to\infty} \delta^m \operatorname{Pr}_{it+m} = 0, \ \forall i \in \mathcal{N}.$$

We can solve the equation and obtain that:

$$\operatorname{Pr}_{it} = \frac{1}{n}\sum_{j=1}^{n}\Pi_j(x^F). \tag{3.18}$$

From the general expression of the profits that each player gets from reaching an agreement in the t-th iteration, as $\delta < 1$ it is straightforward to check that all the players are strictly better off from not delaying the cooperation, i.e.,

$$\Pr_{it} - \delta \Pr_{it+1} = (1-\delta)\left(\frac{1}{n}\sum_{j=1}^{n}\Pi_j(x^F)\right) > 0,\ \forall t \geq 0,\ \forall i \in \mathcal{N}.$$

Then the agreement will be reached in the first attempt and therefore:

$$\Pr_i = \Pr_{i0} = \frac{1}{n}\sum_{j=1}^{n}\Pi_j(x^F), \forall i \in \mathcal{N}. \tag{3.19}$$

However this equilibrium is sustained under the assumption that (3.17) holds, substituting the payoffs, the condition becomes the following time-independent inequality:

$$\delta \sum_{j=1}^{n}\Pi_j(x^F) > n \max_i\{\Pi_i(x^N)\}.$$

Solving for δ we get that this will be the equilibrium if $\delta > \overline{\delta}$, with

$$\overline{\delta} = \frac{\max_i\{\Pi_i(x^N)\}}{\frac{1}{n}\sum\limits_{j=1}^{n}\Pi_j(x^F)}. \tag{3.20}$$

However, $\delta < 1$ as it is the discount factor. Therefore, the payoffs we computed will be the equilibrium outcome for every $\delta > \overline{\delta}$, if $\overline{\delta} < 1$.

Therefore, if $\delta \leq \overline{\delta}$, the WSSPNE cannot be sustained by an infinite repetition of the negotiation process. This implies that there exists an iteration $\hat{t}$ such that exists a player i with $\Pi_i(x^N) \geq \delta \Pr_{i\hat{t}+1}$. In this iteration, if the negotiation gets to it, the Nash outcome will be the outside option and proceeding as before, we can compute the outcome:

$$\Pr_{i\hat{t}} = \frac{1}{n}\sum_{j=1}^{n}\Pi_j(x^F) + \frac{1}{n}\sum_{j\neq i}\left[\Pi_i(x^N) - \Pi_j(x^N)\right].$$

Now, given this is the payoff for the $\hat{t}$ iteration, we need to know what will happen in $\hat{t}-1$. The outside option in $\hat{t}-1$ will be Nash, if there exists a player i such that $\Pi_i(x^N) \geq \delta \Pr_{i\hat{t}}$. Substituting the value obtained for $\Pr_{i\hat{t}}$ and simplifying this yields a time independent condition for the non-repetition of the negotiation:

$$\delta \leq \widehat{\delta}_0 \equiv \frac{\max_i\{\Pi_i(x^N)\}}{\frac{1}{n}\sum_{j=1}^{n}\Pi_j(x^F) + \left(\max_i\{\Pi_i(x^N)\} - \frac{1}{n}\sum_{j=1}^{n}\Pi_j(x^N)\right)} < \min\{\overline{\delta}, 1\}. \tag{3.21}$$

If $\delta \leq \widehat{\delta}_0$, in any iteration a rejection of a proposal is followed by a breakdown of the negotiations. Therefore, the game is as if it was static, and the equilibrium outcome is:

$$\begin{aligned} \Pr_i &= \Pr_{i0} = \frac{1}{n}\sum_{j=1}^{n}\Pi_j(x^F) + \frac{1}{n}\sum_{j\neq i}\left[\Pi_i(x^N) - \Pi_j(x^N)\right] \Longrightarrow \\ \Pr_i &= \Pi_i(x^N) + \frac{1}{n}\left(\sum_{j=1}^{n}\Pi_j(x^F) - \sum_{j=1}^{n}\Pi_j(x^N)\right) = \Pi_i(x^N) + \frac{1}{n}S. \end{aligned} \tag{3.22}$$

We have now to find the equilibrium outcome for $\delta \in (\widehat{\delta}_0, \min\{\overline{\delta}, 1\}]$. In this range of parameter values neither always breakdown, nor infinite repetition of the negotiation process can be part of an equilibrium strategy. If $\delta > \widehat{\delta}_0$, we know that in $\hat{t}-1$ the outside option will be to negotiate again and hence,

$$\begin{aligned} \Pr_{i\hat{t}-1} &= \frac{1}{n}\sum_{j=1}^{n}\Pi_j(x^F) + \frac{1}{n}\sum_{j\neq i}\left[\delta \Pr_{i\hat{t}} - \delta \Pr_{j\hat{t}}\right] \\ &\Longrightarrow \Pr_{i\hat{t}-1} = \frac{1}{n}\sum_{j=1}^{n}\Pi_j(x^F) + \frac{\delta}{n}\sum_{j\neq i}\left[\Pi_i(x^N) - \Pi_j(x^N)\right]. \end{aligned}$$

But in order to find equilibrium payoffs we need to know which will be the outside option in $\hat{t}-2$. Moving to the non-cooperative outcome will prevail if there exists a player i such that $\Pi_i(x^N) \geq \delta \Pr_{i\hat{t}-1}$. Proceeding as before, this will happen if:

$$\begin{aligned}
\delta &\leq \widehat{\delta}_1, \text{ with } \widehat{\delta}_1 \text{ being the fixed point of } f_1(\delta), \text{ where} \\
f_1(\delta) &= \frac{\max_i\{\Pi_i(x^N)\}}{\frac{1}{n}\sum\limits_{j=1}^{n}\Pi_j(x^F) + \delta\left(\max_i\{\Pi_i(x^N)\} - \frac{1}{n}\sum\limits_{j=1}^{n}\Pi_j(x^N)\right)}. \qquad (3.23)
\end{aligned}$$

By construction $\widehat{\delta}_1 \in (\widehat{\delta}_0, \min\{\overline{\delta}, 1\}]$. Hence, if $\delta \in (\widehat{\delta}_0, \widehat{\delta}_1]$ a rejection of a proposal is followed by no more than one repetition of the negotiation. The equilibrium outcome is given by

$$\begin{aligned}
\Pr_i &= \Pr_{i0} = \frac{1}{n}\sum_{j=1}^{n}\Pi_j(x^F) + \frac{\delta}{n}\sum_{j\neq i}\left[\Pi_i(x^N) - \Pi_j(x^N)\right] \\
&\Longrightarrow \Pr_i = \frac{1}{n}\sum_{j=1}^{n}\Pi_j(x^F) + \delta\left[\Pi_i(x^N) - \frac{1}{n}\sum_{j=1}^{n}\Pi_j(x^N)\right]. \qquad (3.24)
\end{aligned}$$

If $\delta > \widehat{\delta}_1$, the process can be repeated to find $\widehat{\delta}_2$. Defined as the fixed point of $f_2(\delta)$, where

$$f_2(\delta) = \frac{\max_i\{\Pi_i(x^N)\}}{\frac{1}{n}\sum\limits_{j=1}^{n}\Pi_j(x^F) + (\delta)^2\left(\max_i\{\Pi_i(x^N)\} - \frac{1}{n}\sum\limits_{j=1}^{n}\Pi_j(x^N)\right)}. \qquad (3.25)$$

Again by construction $\widehat{\delta}_2 \in (\widehat{\delta}_1, \min\{\overline{\delta}, 1\}]$. Hence, if $\delta \in (\widehat{\delta}_1, \widehat{\delta}_2]$ a rejection of a proposal is followed by at most two more attempts to renegotiate. Therefore, the equilibrium outcome is given by

$$\begin{aligned}
\Pr_i &= \Pr_{i0} = \frac{1}{n}\sum_{j=1}^{n}\Pi_j(x^F) + \frac{\delta^2}{n}\sum_{j\neq i}\left[\Pi_i(x^N) - \Pi_j(x^N)\right] \\
&\Longrightarrow \Pr_i = \frac{1}{n}\sum_{j=1}^{n}\Pi_j(x^F) + \delta^2\left[\Pi_i(x^N) - \frac{1}{n}\sum_{j=1}^{n}\Pi_j(x^N)\right]. \qquad (3.26)
\end{aligned}$$

The same process can be repeated arbitrarily many times. In general, the process of negotiation will be repeated at most m times if $\delta \in (\widehat{\delta}_{m-1}, \widehat{\delta}_m]$, with $\widehat{\delta}_{m-1}$ and $\widehat{\delta}_m$ being the fixed points of $f_{m-1}(\delta)$ and of $f_m(\delta)$ respectively, where:

$$\begin{aligned} f_{m-1}(\delta) &= \frac{\max_i\{\Pi_i(x^N)\}}{\frac{1}{n}\sum_{j=1}^{n}\Pi_j(x^F)+\delta^{m-1}\left(\max_i\{\Pi_i(x^N)\}-\frac{1}{n}\sum_{j=1}^{n}\Pi_j(x^N)\right)} \\ f_m(\delta) &= \frac{\max_i\{\Pi_i(x^N)\}}{\frac{1}{n}\sum_{j=1}^{n}\Pi_j(x^F)+\delta^{m}\left(\max_i\{\Pi_i(x^N)\}-\frac{1}{n}\sum_{j=1}^{n}\Pi_j(x^N)\right)}. \end{aligned} \tag{3.27}$$

Therefore, if $\delta \in (\widehat{\delta}_{m-1}, \widehat{\delta}_m]$, the equilibrium payoff will be

$$\begin{aligned} \Pr_i &= \Pr_{i0} = \frac{1}{n}\sum_{j=1}^{n}\Pi_j(x^F)+\frac{\delta^m}{n}\sum_{j\neq i}\left[\Pi_i(x^N)-\Pi_j(x^N)\right] \\ &\Longrightarrow \Pr_i = \frac{1}{n}\sum_{j=1}^{n}\Pi_j(x^F)+\delta^m\left[\Pi_i(x^N)-\frac{1}{n}\sum_{j=1}^{n}\Pi_j(x^N)\right]. \end{aligned} \tag{3.28}$$

This will be a characterization of the equilibrium outcomes for every possible parameter configuration since

$$\text{If } \overline{\delta} < 1 \text{ then } \quad \lim_{m\to\infty}\widehat{\delta}_m = \overline{\delta}$$

$$\text{If } \overline{\delta} > 1 \text{ then } \quad \lim_{m\to\infty}\widehat{\delta}_m = 1.$$

Hence, we have fully characterized the WSSPN equilibrium payoffs for every possible value of the discount factor δ and the proof is complete ■

This is the unique candidate for the payoffs in a WSSPNE. However, we have left to prove that this equilibrium exists, i.e., we need to find an equilibrium strategy profile (σ^*) defined as:

$$\sigma_{it}^* = \left[\left(b_{jt}^{i*}\right)_{j\neq i}; \left(\left(x_{jt}^*\right)_{j=1}^{n}, \left(T_{jt}^*\right)_{j\neq i}\right); \tilde{T}_{it}^*, a_t^*\right].$$

In order to do it, we present here three alternative constructions for the strategies, that correspond to the different regions obtained in the previous proposition.

Let σ_{it}^0 be such that, for every $t \in \mathbb{N}$

$$\left(b_{jt}^{i*} = \frac{1}{n} S\right)_{j \neq i}$$

$$\left(\left(x_{jt}^* = x_j^F\right)_{j=1}^n, \left(T_{jt}^* = \Pi_j(x^N) - \Pi_j(x^F)\right)_{j \neq i}\right)$$

$$\left(\tilde{T}_{it}^* = \Pi_i(x^N) - \Pi_i(x^F),\ a_t^* = \{\text{break}\}\right)$$

Under this strategy, the players break the negotiations after a failed attempt to agree. Hence, they behave as if the game was static and there were no possibility of renegotiation.

For every $m \in \mathbb{N} \backslash \{0\}$, we define σ_{it}^m such that, for every $t \in \{0, 1, ..., m-1\}$

$$\left(b_{jt}^{i*} = \frac{(1-\delta)}{n} \sum_{j=1}^n \Pi_j(x^F)\right)_{j \neq i}$$

$$\left(\left(x_{jt}^* = x_j^F\right)_{j=1}^n, \left(T_{jt}^* = \frac{\delta}{n} \sum_{j=1}^n \Pi_j(x^F) + \frac{\delta^{m-t}}{n} \left(n\Pi_j(x^N) - \sum_{j=1}^n \Pi_j(x^N)\right) - \Pi_j(x^F)\right)_{j \neq i}\right)$$

$$\left(\tilde{T}_{it}^* = \frac{\delta}{n} \sum_{j=1}^n \Pi_j(x^F) + \frac{(\delta)^{m-t}}{n} \left(n\Pi_i(x^N) - \sum_{j=1}^n \Pi_j(x^N)\right) - \Pi_i(x^F),\ a_t^* = \{\text{retry}\}\right)$$

and for $t = m$

$$\left(b_{jt}^{i*} = \frac{1}{n} S\right)_{j \neq i}$$

$$\left(\left(x_{jt}^* = x_j^F\right)_{j=1}^n, \left(T_{jt}^* = \Pi_j(x^N) - \Pi_j(x^F)\right)_{j \neq i}\right)$$

$$\left(\tilde{T}_{it}^* = \Pi_i(x^N) - \Pi_i(x^F),\ a_t^* = \{\text{break}\}\right)$$

This strategy profile for $m + 1$ stages is repeated for the rest of the periods. This profile corresponds to a negotiation strategy where the value of m determines the potential length of the negotiation, i.e., the maximum number of attempts to agree on a decision.

Finally, we let σ_{it}^{∞} be such that, for every $t \in \mathbb{N}$

$$\left(b_{jt}^{i*} = \frac{(1-\delta)}{n} \sum_{j=1}^{n} \Pi_j(x^F) \right)_{j \neq i}$$

$$\left(\left(x_{jt}^{*} = x_j^F \right)_{j=1}^{n}, \left(T_{jt}^{*} = \frac{\delta}{n} \sum_{j=1}^{n} \Pi_j(x^F) - \Pi_j(x^F) \right)_{j \neq i} \right)$$

$$\left(\tilde{T}_{it}^{*} = \frac{\delta}{n} \sum_{j=1}^{n} \Pi_j(x^F) - \Pi_i(x^F),\ a_t^{*} = \{\text{retry}\} \right)$$

Under this last profile, the players always retry to negotiate after an unsuccessful attempt.

With this, we proceed now to characterize the equilibrium strategies of our negotiation game

Proposition 12 *$\sigma^{*} \in \sum$, is a weakly stationary SPNE strategy profile of the negotiation game with no cooperation as the outside option. σ^{*} is such that, for every $t \in \mathbb{N}$,*

1. *If $\delta \leq \widehat{\delta}_0$, then $\sigma_{it}^{*} = \sigma_{it}^{0}$.*
2. *If $\delta \in (\widehat{\delta}_{m-1}, \widehat{\delta}_m]$, $\forall m \in \mathbb{N} \backslash \{0\}$, then $\sigma_{it}^{*} = \sigma_{it}^{m}$.*
3. *If $\overline{\delta} < 1$, $\forall \delta > \overline{\delta}$, then $\sigma_{it}^{*} = \sigma_{it}^{\infty}$.*

Proof. Simple algebraic calculations show that the stated strategy profile yields the following payoffs, for every $i \in \mathcal{N}$:

1. If $\delta \leq \widehat{\delta}_0$,

$$\begin{aligned} \Pr_i &= \Pr_{i0} = \Pi_i(x^N) + \frac{1}{n} S \\ \Pr_{it} &= \Pr_{i0},\ \forall t \in \mathbb{N}. \end{aligned}$$

2. If $\delta \in (\widehat{\delta}_{m-1}, \widehat{\delta}_m]$, $\forall m \in \mathbb{N}\backslash\{0\}$

$$\Pr_{it} = \tfrac{1}{n}\sum_{j=1}^{n} \Pi_j(x^F) + \delta^{m-t}\left(\Pi_i(x^N) - \tfrac{1}{n}\sum_{j=1}^{n} \Pi_i(x^N)\right) \quad \forall t \in \{0, 1, ..., m\}$$

$$\Pr_{it} = \Pr_{it-(m+1)} \qquad \forall t > m.$$

3. If $\overline{\delta} < 1$, then, $\forall \delta > \overline{\delta}$,

$$\begin{aligned} \Pr_i &= \Pr_{i0} = \frac{1}{n}\sum_{j=1}^{n} \Pi_j(x^F) \\ \Pr_{it} &= \Pr_{i0},\ \forall t \in \mathbb{N}. \end{aligned}$$

Once are computed the payoffs, the rest of the proof is analogous to the one of Proposition 10. ■

The first comment deals with the non-stationarity of the equilibrium strategies. If $\delta \in (\widehat{\delta}_0, \min\{\overline{\delta}, 1\}]$, there is no stationary equilibrium strategy, since as $\delta > \widehat{\delta}_0$ always breaking the negotiation after an unsuccessful attempt is not optimal. On the other hand, retrying always to negotiate is only best response for $\delta > \overline{\delta}$. Therefore, in this parameter region, the equilibrium strategy will only be weakly stationary. In particular, it will be linked to calendar time through the discount factor. The interpretation is the following. There exists a sequence of thresholds for δ, $\{\widehat{\delta}_m\}_{m=0}^{\infty}$ such that if $\delta \in (\widehat{\delta}_{m-1}, \widehat{\delta}_m]$, the players will be willing to repeat (in equilibrium) up to m times the process of negotiation. It is worth noting that we are not imposing this behavior on the players, it endogenously emerges from the mechanism.

By construction our mechanism leads to an individually rational outcome for every player. This is given by the fact that the agents can always decide not to negotiate and ensure themselves their non-cooperative payoffs.

Concerning the interpretation of the results we find, first of all, that analogously to the other frameworks we implement the first best level of x. Nevertheless, the split of the profits from cooperation presents some changes that are worth noting. The main difference is that, in this benchmark in which there is no production decision while the negotiation is being undertaken, the degree of "patience" of the players (given by the discount factor δ) defines the intensity of the egalitarianism of the resulting share of the profits.

The share of the profits given by $\Pr_i = \frac{1}{n}\sum_{j=1}^{n}\Pi_j(x^F)$ corresponds to a "fully" egalitarian distribution where the non-cooperative profits of the players do not have influence on the final revenues. A necessary condition for this outcome to be sustainable is that $\overline{\delta} < 1$ i.e. that for every player this "fully" egalitarian outcome is individually rational. In this case equal split will be the solution, provided the loss in utility that postponing the agreement causes is sufficiently low (i.e., if $\delta > \overline{\delta}$). The reason is that, if the value of the profits in the future is sufficiently high, the agents with a good outside option (a high value of $\Pi_i(x^N)$) will lose their bargaining power as breaking the negotiation and moving to the non-cooperative outcome will no longer be a credible threat. Therefore, in this case players' payoff is independent of their disagreement point (the Nash profits). This result can be interpreted, in fact, as an illustration of the "outside option" principle: *only threats which are credible will have effect on outcomes.*[16]

For $\overline{\delta} > 1$, or if not when $\delta \leq \overline{\delta}$, the "fully" egalitarian distribution can no longer be an equilibrium payoff. In this case, the sequence of thresholds for the discount factor $\{\widehat{\delta}_m\}_{m=0}^{\infty}$ determine how influential are the non-cooperative profits on the final sharing. The higher the patience of the players, the smaller will be the importance of the non-cooperative

[16]See, for instance, Sutton (1986) for a clear and intuitive explanation of this principle.

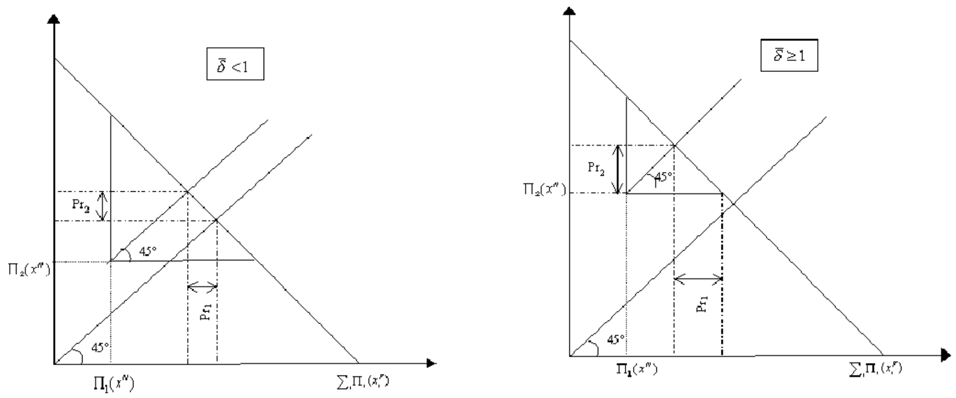

Figure 3.2: Equilibrium Outcome in a Two Players' Economy

payoffs on the outcome of the negotiation process. Finally, if the players' valuation of the future falls below the threshold $\widehat{\delta}_0$, then the equilibrium outcome is the equal split of the gains derived from cooperation plus the profits at the non-cooperative situation. Figure 3.2 illustrates this sharing for the two possible configurations of $\overline{\delta}$ in a two players' economy.

Analogously as in the previous framework, it can be shown that the results are robust to the presence of heterogeneity in the discount factors. The only change is in the distribution of the profits and it goes in the same direction as before, giving a higher share to the relatively more patient players. The new feature is that in this scenario the maximum number of rounds that the negotiation can be repeated (under the equilibrium strategies), is determined by the player that has a high outside option combined with a relatively low degree of "patience".

3.5 An Example: Implementing a Public Project

In this section we provide an example of a specific economic situation where the mechanism can be applied. We will analyze the problem of undertaking an indivisible public project, and distributing the costs derived from its implementation.

Consider an economy with n players. Each player enjoys a private profit γ_i if a given project is undertaken, 0 otherwise. These profits are independent of who undertakes the project. In the construction, player i incurs a cost K_i. We specifically assume that for every $i \in \mathcal{N}$, $K_i > \gamma_i$. Finally, let $K_{\min}$ be the minimum value of K_i across all agents. Let x_i be a binary function that takes value 1 if player i undertakes the project and 0 if he does not. Define $\bar{x}$ as the indicator of whether there exists at least one agent that undertakes the project or not. Formally:

$$\bar{x} = \begin{cases} 1 & \text{if } \exists i \in \mathcal{N}, \text{ s.t. } x_i = 1 \\ 0 & \text{otherwise.} \end{cases}$$

With this we can write the profits of player i as follows:

$$\Pi_i(x_{i,}x_{-i}) = \gamma_i \bar{x} - K_i x_i.$$

This environment can be interpreted as a problem of implementing an infrastructure to be enjoyed by a group of cities or communities. The assumption that for every $i \in \mathcal{N}$, $K_i > \gamma_i$, will represent then that no community is willing to unilaterally undertake the project.

We will be interested in analyzing how the mechanism decides when the project should be undertaken, and also how the costs should be distributed among the players. Therefore, if we denote by Pr_i the revenues that player i obtains when playing the mechanism and by x^* the decision that the mechanism induces concerning the "construction" of the project, we can define the contribution of player i , C_i as follows

$$C_i \equiv \Pi_i(x^*_{i,}x^*_{-i}) - \mathrm{Pr}_i\,.$$

In the following Corollary we will analyze the outcome of the mechanism in this environment.[17]

Corollary 1 *The outcome of the SSPNE of the negotiation game applied to the implementation of a project is:*

1. *The project is implemented* ($\bar{x} = 1$) *iff* $\sum\limits_{j=1}^{n} \gamma_j \geq K_{\min}$.
2. *If agent i undertakes the project, then* $K_i = K_{\min}$.
3. $\mathrm{Pr}_i = \frac{1}{n}(\sum\limits_{j=1}^{n} \gamma_j - K_{\min})$.
4. $C_i = \frac{1}{n}K_{\min} + \left(\gamma_i - \frac{1}{n}\sum\limits_{j=1}^{n} \gamma_j\right)$.

The mechanism results in the implementation of the project if it is socially valuable i.e., if the sum of the benefits across all players outweighs the costs of its construction. Moreover, the negotiation process leads to an efficient choice of the player that will undertake the project, selecting the one with the lowest costs ($K_{\min}$).

Concerning the distributional aspects, the revenues of the agents are fully egalitarian since no player has positive profits in the absence of cooperation. However, the cost contributions of the players are heterogeneous. They have two components, an egalitarian share of the construction costs, $\frac{1}{n}K_{\min}$ and a factor that corrects for the different private

[17] Notice that as in the absence of cooperation the project is not implemented, then $\Pi_i(x^N) = 0$ for every i

profits. If a player values more the project than the average valuation across all agents ($\gamma_i > \frac{1}{n}\sum_{j=1}^{n}\gamma_j$), then his contribution exceeds the n^{th} part of the costs.

3.6 An Extension: Achieving a Weighted Sharing of the Surplus

The mechanism we have constructed can be adapted to allow for exogenous weights that give different "power" to the players involved in the negotiation. These weights ($w_i, \forall i$) have to be exogenous, common knowledge and already given at the start of the negotiation process.

These biases in the distribution can have multiple interpretations. First, they can stand for political pressure, or be a proxy for the property rights over the good with external effects (if it is a natural resource like a fishing stock, these rights are clearly relevant). They can be given another interpretation if we are in a framework of a multilateral negotiation among countries, in this case the weights can be used normalize the profits on the basis of the different population of the countries and achieve a per-capita equal sharing of the surplus.

The way to do incorporate this in the mechanism is by correcting the bidding stage and defining a weighted net bid.[18] The bids made by each player are evaluated according to his relative weight, in order to define the weighted net bid (B_t^{Wi}). Formally:

$$B_t^{Wi} = w_i \sum_{j \neq i} b_{jt}^i - \sum_{j \neq i} w_j b_{it}^j.$$

[18]This is analogous to the variation that Pérez-Castrillo and Wettstein (2001a) do to implement the Weighted Shapley Value.

With this modification the results of the mechanism become as follows.

For the economic situations in which no cooperation is the Status-Quo, the outcome of the Stationary Subgame Perfect Nash Equilibrium of the weighted negotiation game is:[19]

Proposition 13 *The outcome of the SSPNE in the weighted dynamic mechanism with no cooperation as the status-quo is, for every $i \in \mathcal{N}$:*

1. $x_i^* = x_i^F$.

2. $\Pr_i = \Pi_i(x^N) + \frac{w_i}{\sum_{j=1}^n w_j} S$.

Proof. Follows directly from the proof of Proposition 9 ■

The resulting revenues of the players correspond to the weighted equal sharing of the surplus. Each agent instead of getting $\frac{1}{n}$ of the gains from cooperation, gets: $\frac{w_i}{\sum_{j=1}^n w_j}$.

This corrected mechanism can also be applied to the situations in which non cooperation is the outside option. In order to do it we need to define a series of thresholds for the discount factor (δ) analogous to the ones in Subsection 3.4.2.

$\overline{\delta}^w = \max_i \left\{ \frac{\Pi_i(x^N)}{\frac{w_i}{\sum_{j=1}^n w_j} \sum_{j=1}^n \Pi_j(x^F)} \right\}$, and $\widehat{\delta}_m^w$ be the unique fixed point of $f_m^w(\delta)$, where:

$$f_m^w(\delta) = \max_i \left\{ \frac{\Pi_i(x^N)}{\frac{w_i}{\sum_{j=1}^n w_j} \sum_{j=1}^n \Pi_j(x^F) + \delta^m \left(\Pi_i(x^N) - \frac{w_i}{\sum_{j=1}^n w_j} \sum_{j=1}^n \Pi_j(x^N) \right)} \right\}, \ \forall m \in \mathbb{N}. \tag{3.29}$$

$\overline{\delta}^w$ and $\widehat{\delta}_m^w$ are such that; $\forall m \in \mathbb{N}$, $\widehat{\delta}_m^w < \widehat{\delta}_{m+1}^w < \overline{\delta}^w$, with $\lim_{m\to\infty} \widehat{\delta}_m^w = \min\{\overline{\delta}^w, 1\}$.

[19] The formal definition of the strategy space for each player, as well as the exact solution concept is ommited, since they are completely analogous to the ones presented in Subsection 3.4.1

With this, we can proceed now to characterize the Weakly Stationary Subgame Perfect Nash Equilibrium of the weighted negotiation game.[20]

Proposition 14 *The outcome of WSSPNE in the dynamic mechanism with no cooperation as the outside option is, for every $i \in \mathcal{N}$:*

1.- $x_i^* = x_i^F$.

2.- The payoff function is:

$$\Pr_i = \begin{cases} \Pi_i(x^N) + \frac{w_i}{\sum_{j=1}^n w_j} S & \textit{If } \delta \leq \widehat{\delta}_0^w \\ \frac{w_i \sum\limits_{j=1}^n \Pi_j(x^F)}{\sum_{j=1}^n w_j} + \delta^m \left(\Pi_i(x^N) - \frac{w_i}{\sum_{j=1}^n w_j} \sum\limits_{j=1}^n \Pi_j(x^N) \right) & \textit{If } \delta \in (\widehat{\delta}_{m-1}^w, \widehat{\delta}_m^w],\ \forall m \in \mathbb{N} \backslash \{0\} \\ \frac{w_i}{\sum_{j=1}^n w_j} \sum\limits_{j=1}^n \Pi_j(x^F) & \textit{If } \overline{\delta}^w < 1,\ \forall \delta > \overline{\delta}^w. \end{cases}$$

Proof. Follows directly from the proof of Proposition 11 ■

In this case, as in the mechanism analyzed in Subsection 3.4.2, the value of the discount factor determines the intensity of the egalitarianism, and hence the impact of the outside option ($\Pi_i(x^N)$) on the equilibrium payoffs. The difference is that now, the distribution rule for the surplus reflects the different weights assigned to each of the players.

[20] We refer the reader to Subsection 3.4.2 for a formal definition of the space of strategies and the solution concept.

3.7 Concluding Remarks

In this paper we have proposed a negotiation process that implements an efficient allocation and ensures equal sharing of the surplus that cooperation generates in economic situations where a group of well informed agents interact in the presence of externalities. Moreover, the mechanism we have constructed has the characteristics of being simple, autonomous, deterministic, and renegotiation proof.

We have presented two alternative versions of the mechanism that can be used in two different types of economies. The first one is designed to deal with environments in which non-cooperation is the status-quo of the game. The fact that the agents earn the non-cooperative profits in all the periods until the agreement is reached, has significant distributional implications for the game. The outcome ensures the non-cooperative profits to each player plus an equal share of the surplus that cooperation generates. The other mechanism is devised for environments in which non-cooperation is only an outside option of the negotiation process. Hence, the non-cooperative outcome can be interpreted as a potential result of the negotiation, the result in case of failure. In this scenario, the discount factor plays a crucial role in the distribution of the profits. The higher the discount factor, the weaker the effect of the Nash (non-cooperative) profits on the final revenues of the players. In particular, if the fully egalitarian solution is individually rational for all the agents, there exists a threshold in the discount factor, beyond which the fully egalitarian distribution is implemented, eliminating all the impact of the non-cooperative profits on the final sharing.

The kind of mechanisms we propose in this paper shares similar features with various practical negotiation protocols that are used to solve collective decision problems. One can put forward the example of negotiation processes among the member states of the European Union while deciding on major issues like union enlargement, redistributive policies, or the assignment of the voting weights to the members. In order to take such decisions, each country is given the presidency of the Union for a period of six months according to a pre-specified order. During this period the country has the right to make proposals that are implemented only if they are unanimously accepted in the half-yearly summit of the EU. The search for efficiency motivates the design of this kind of mechanisms as we do in this paper by giving one agent the initiative and veto power to the remaining players (countries). However, one cannot presume a priori that the members interact in a way that leads to an equilibrium in stationary strategies. There, a country might not fully exploit the advantage of being the proposer during his presidential mandate in order to avoid a tough reaction from the other countries in future. Hence, the equilibrium might be sustained by some sort of trigger strategies ensuring “fairness” in distribution. This process of allocating gains from cooperation clearly differs from our approach. In our mechanisms the profits are distributed in an initial stage where the proposer pays the other players for his right to take the initiative which in turn eliminates his advantage as the proposer.

A final comment to make on the bargaining power of the participant agents in the mechanisms. The two mechanisms we described have different distributional implications. In the first one, when no-cooperation is the status-quo, the situation is similar to a peace process that is being undertaken after the conflict has started. In this case the agent with higher stake gains more since, the equilibrium outcome ensures the status-quo plus an equal share of surplus to each player. Whereas, the second situation where no-cooperation is an outside option is similar to undertaking peace process a priori. In this case the equilibrium might result in a fully egalitarian distribution depending on the value of the discount factor, but not on the relative bargaining position of a particular individual. Higher the value of the discount factor, lower the impact of non-cooperative outcome. This provides our results with the testable implication that the outcome might be more egalitarian in the second situation (pre-conflict negotiation) and that therefore, the "strongest" player in the confrontation would benefit more from his position in in-conflict peace processes.

3.8 Appendix

Proof of Claim 1: (Pérez-Castrillo and Wettstein, 2001a)

We have to show that in any SPE, $B^i = B^j \ \forall i, j$ and hence $B^i = 0 \ \forall i \in \mathcal{N}$.

Denote $\Omega = \{i \in \mathcal{N} \text{ s.t. } B^i = \max_j(B^j)\}$. If $\Omega = n$, then the claim is shown since $\sum_{i=1}^{n} B^i = 0$.

Otherwise, we will show that there exists a profitable deviation for player $i \in \Omega$.

Take some player $j \neq \Omega$. Let player $i \in \Omega$ change his strategy by announcing: $b_k^{i'} = b_k^i + \delta$ for all $k \in \Omega$, $k \neq i$, $b_j^{i'} = b_j^i - |\Omega|\delta$ and $b_l^{i'} = b_l^i$ for all $l \notin \Omega$ and $l \neq j$. The new net bids are: $B^{i'} = B^i - \delta$, $B^{k'} = B^k - \delta$,for all $k \in \Omega$, $k \neq i$, $B^{j'} = B^j + |\Omega|\delta$ and $B^{l'} = B^l$ for all $l \notin \Omega$ and $l \neq j$.

If δ is small enough, so that $B^j + |\Omega|\delta < B^{i'} = B^i - \delta$, then $B^{l'} < B^{i'} = B^{k'}$ for all $l \notin \Omega$ (including j) and for all $k \in \Omega$. Therefore, Ω does not change. However: $\sum\limits_{h \neq i} b_h^i - \delta < \sum\limits_{h \neq i} b_h^i$. ■

Proof of Claim 2: (Pérez-Castrillo and Wettstein, 2001a)

We have to show that in any SPE, each player's payoff is the same regardless of who is chosen as the proposer.

We already know that all the bids B^i are the same. If player i would strictly prefer to be the proposer, he could improve his payoff by slightly increasing one of his bids b_j^i. Similarly, if player i would strictly prefer player j to be the proposer, he could improve his payoff by decreasing b_j^i. The fact that player i does not do so in equilibrium means that he is indifferent to the proposer's identity. ■

Characterization of $\{\hat{\delta}_m\}_{m=0}^{\infty}$.

We will show that $\hat{\delta}_m$ exists for every $m \in \mathbb{N}$ and that $\lim_{m\to\infty} \widehat{\delta}_m = \min\{\overline{\delta}, 1\}$, with $\overline{\delta} = \frac{\max_i\{\Pi_i(x^N)\}}{\frac{1}{n}\sum_{j=1}^{n}\Pi_j(x^F)}$.

We have defined $\hat{\delta}_m$ as the fixed point of $f_m(\delta)$ where:

$$f_m(\delta) = \frac{\max_i\{\Pi_i(x^N)\}}{\frac{1}{n}\sum_{j=1}^{n}\Pi_j(x^F) + \delta^m\left(\max_i\{\Pi_i(x^N)\} - \frac{1}{n}\sum_{j=1}^{n}\Pi_j(x^N)\right)}, \ \forall m \in \mathbb{N}.$$

It is easy to show that: $\frac{\partial f_m(\delta)}{\partial \delta} < 0$ for every $m \in \mathbb{N}$. Moreover:

$$\begin{aligned} f_m(\delta &= 0) = \frac{\max_i\{\Pi_i(x^N)\}}{\frac{1}{n}\sum_{j=1}^{n}\Pi_j(x^F)} > 0 \\ f_m(\delta &= 1) = \frac{\max_i\{\Pi_i(x^N)\}}{\max_i\{\Pi_i(x^N)\} + \frac{1}{n}\sum_{j=1}^{n}\Pi_j(x^F) - \frac{1}{n}\sum_{j=1}^{n}\Pi_j(x^N)} < 1 \end{aligned}$$

Hence, a fixed point always exists and is unique. Moreover, it is straightforward from the construction of $f_m(\delta)$ that $\hat{\delta}_m < \hat{\delta}_{m+1}$ for every $m \in \mathbb{N}$.

The fixed point can be computed as the unique δ_m^* such that:

$$\delta_m^*\left(\frac{1}{n}\sum_{j=1}^{n}\Pi_j(x^F) + (\delta_m^*)^m\left(\max_i\{\Pi_i(x^N)\} - \frac{1}{n}\sum_{j=1}^{n}\Pi_j(x^N)\right)\right) = \max_i\{\Pi_i(x^N)\}$$

We have to distinguish two cases:

1- If $\frac{1}{n}\sum_{j=1}^{n}\Pi_j(x^F) < \max_i\{\Pi_i(x^N)\}$, i.e. if $\bar{\delta} > 1$ then for any $\tilde{\delta} < 1$ if $\lim_{m\to\infty} \hat{\delta}_m^* = \tilde{\delta}$ then the above equality is not fulfilled. Hence as we know that a fixed point always exists and is unique then $\lim_{m\to\infty} \hat{\delta}_m^* = 1$.

2- If $\frac{1}{n}\sum_{j=1}^{n}\Pi_j(x^F) > \max_i\{\Pi_i(x^N)\}$, i.e. if $\bar{\delta} < 1$ then it is straightforward to check that $\lim_{m\to\infty} \hat{\delta}_m = \bar{\delta}$.

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