EXCHANGE RATE AND WELFARE IN SMALL OPEN ECONOMIES

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PhD Thesis

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Introduction

The birth of a new generation of models in the 1990s was an important development of macroeconomics. The "New Neoclassical Synthesis", as Goodfriend and King (1997) called it, integrated imperfect competition and nominal rigidities into the stochastic dynamic general equilibrium models of the real business cycle literature. A similar development in the subfield of the international macroeconomics is labelled as new open economy macroeconomics (NOEM) and was initiated by the seminal paper of Obstfeld and Rogoff (1995). These models are used to analyze the connection between nominal and real variables and to assess the welfare implications of different monetary and exchange rate policies. The micro-foundations of the models enable them to provide more rigorous results than in the case of the previous generation of policy models.

NOEM models have been designed and used to study a variety of problems in developed economies. However, with appropriate modifications these models can be suitable to answer questions related to emerging market economies. The goal of this dissertation is to adapt NOEM models to the special problems of a certain group of emerging market economies, the economies of European post-communist countries.

For NOEM models to be useful in the applications pursued in this thesis they have to comply with several important empirical findings. The first one is the insight that the external real exchange rate, that is, the relative price of home and foreign tradables, is a key determinant of the real exchange rate. The second one is the connection between nominal and real exchange rate movements. NOEM models are built to explain and analyze the above empirical regularities. However, in European post-communist countries one can also observe long run appreciation of real exchange rates induced by dual inflation, that is, diverging inflation rates of tradables and non-tradables. In other words, in these countries the internal exchange rate has also an important role in real exchange rate determination.

The first chapter of this thesis seeks the answers for two related questions: In NOEM models, how can dual inflation result in real appreciation, and what can cause dual inflation? The answer for the first question is not as trivial as it may seem to be. Most of the empirical literature on this topic explains the coexistence of dual inflation and real appreciation by technological factors, that is, by the faster productivity growth of the tradable sector. But large tradable productivity growth depreciates the external real exchange which offsets the effect of dual inflation unless domestic and foreign tradables are close substitutes. On the other hand, if these goods are close substitutes, then the strong correlation of the nominal and real exchange rates, emphasized by the NOEM literature, is impossible. I demonstrate that the assumption of an internal degree of substitution between domestic and foreign goods does not provide a remedy for this problem. Instead, the solution is the assumption of third degree international price discrimination, or pricing to market. I show that if pricing to market is assumed, then dual inflation can cause real appreciation in NOEM models.

In a traditional model with flexible prices, homogenous goods and financial markets the answer is easy for the second question: only asymmetric sectoral productivity factors cause dual inflation. The empirical literature partly supports this view. The important role of productivity factors is documented by several papers, but the strength of their effects are weaker than traditional models predict. I show that nominal and real rigidities of NOEM models can explain this observation.

The second chapter considers a forthcoming economic policy problem of European transition countries. At the beginning of 2004 ten countries joined the European Union. The new members must join the Monetary Union as well, although the accession date is not specified. This raises several economic policy problems. One of them is the determination of the Euro conversion rate. The, non-academic, economic policy literature emphasizes the importance of estimated misalignment indices of the real exchange rate in the determination of the conversion rate. But other factors are neglected and the connection between the conversion rate and the misalignment indices is not precisely specified.

Since this approach is unsatisfactory from an academic point of view, I study this problem within a NOEM model. Its general equilibrium framework permits a rigorous utility-based welfare analysis of the optimal Euro conversion rate. I show that although an appropriate real exchange rate misalignment have a primary role in the determination of the optimal conversion rate, other factors are also important, like the past inflation rate as well as demand and productivity shocks. Furthermore, I demonstrate that a simple intuitive treatment of the problem provides misleading results.

The third chapter considers the welfare implications of unemployment in open economies. The NOEM literature, following New Keynesian models of closed economies, focuses on two distortionary factors: monopolistic competition and nominal rigidities. Labor market frictions, which may result in unemployment are neglected. This is in sharp contrast with the older Keynesian tradition which considers unemployment as the main source of social welfare costs. The approach of the NOEM literature can be a good modelling strategy if one considers countries like the USA where the labor market is quite flexible. However, to better understand the operation of European economies with rigid labor markets it may be better to introduce unemployment in our models.

The importance of this issue is demonstrated in the third chapter. I show that in open economy models welfare implications can significantly change if unemployment and heterogenous consumption is possible. For example, in existing NOEM models an unexpected devaluation of the nominal exchange rate may have harmful effects on the home country. This is a consequence of the deterioration of the terms of trade: aggregate consumption does not increase as much as in a closed economy environment, hence the marginal disutility of higher labor effort may be greater than the utility gain of consumption. However, if there exists unemployment which is not perfectly insured, then there exists a third effect as well: A devaluation results in lower unemployment and a more even distribution of consumption, which yields higher social welfare. Thus, even if the increase of aggregate consumption is negligible a nominal exchange rate devaluation can improve social welfare.

Chapter 1

Dual inflation and the real exchange rate

1.1 Introduction

The traditional approach in international macroeconomics has attempted to explain real exchange rate behavior by the movements of domestic relative prices, that is, by the *internal real exchange rate*. This was a consequence of the assumptions they employed: strong homogeneity in international goods markets, where *purchasing power parity* (PPP) is dominant and the only source of heterogeneity is the distinction between *tradables* and *non-tradables*. In recent years, however, the literature has switched sides. According to the recent approach consumer markets are segmented, PPP has little explanatory power, and the main determinant of real exchange rate movements is the *external real exchange rate*, which is the relative price of domestic and foreign tradables. This new focus of research was initiated on the basis of empirical findings, see, e.g., the papers of Engel (1999) and Rogoff (1996). It appeared that, as Obstfeld (2001) put it "apparently, consumer markets for tradables are just about as segmented internationally as consumer markets for non-tradables."

After the collapse of the Bretton Woods system, floating exchange rate regimes became widespread. This enabled scrutiny of the relationship between nominal and real exchange rate behavior: It turned out, as first forcefully documented by Mussa (1986), that nominal and real exchange rates were strongly correlated, and moving from fixed to floating exchange rate regimes resulted in a dramatic rise in the variability of the real exchange rate. The need for a comprehensive explanation for the aforementioned empirical findings stimulated the birth of *new open economy macroeconomics*

(NOEM), initiated by the seminal paper of Obstfeld and Rogoff (1995), which combines the heterogeneity of goods with *nominal rigidities* in models with micro-foundations.

Although the empirical literature related to NOEM revealed the importance of the external real exchange rate, in fast-growing and emerging market countries there are considerable movements of the internal real exchange rate. Permanent *dual inflation*, namely a significant divergence of inflation rates for tradable and non-tradable goods, is a frequent phenomenon of such markets: the inflation rate of non-tradables is permanently higher than that of tradables, which results in long-run real appreciation. This phenomenon was documented by Ito et al. (1997) for the case of Japan and some Southeast Asian countries, as well as by Coricelli and Jazbec (2001), Halpern and Wyplosz (2001), Égert (2002), Égert et al. (2002) and Kovács (2002) for European post-communist countries. Of course, this does not mean that in these countries the empirical phenomena emphasized in the NOEM literature are not present. For example, the required disinflation efforts, related to future EMU accession, have revealed that the connection between the consumer price index and the nominal exchange rate is weak, which, of course, violates the PPP and implies a strong co-movement of nominal and real exchange rates.

The objective of this chapter is to build a NOEM model which is able to replicate both sets of empirical facts observable in emerging markets: the strong correlation of the nominal and real exchange rate, and dual inflation accompanied by real appreciation.

The problem is the following: The majority of empirical studies explain emerging markets' dual inflation by the *Balassa - Samuelson* (BS) effect, i.e. the relatively rapid productivity growth in the tradable sector. However, dual inflation accompanies real appreciation only if growth in tradable productivity does not result in a significant depreciation of the external real exchange rate. The external real exchange rate does not depreciate considerably if the common currency prices of domestically produced and foreign tradables cannot strongly deviate from each other, i.e. if domestically produced and foreign tradables are close substitutes. On the other hand, the strong co-movement of the nominal and real exchange rates stressed by the NOEM literature requires considerable deviations in the short run between domestic and foreign tradable prices (denominated in the same currency). Yet this requirement can be fulfilled only if the products of the aforementioned sectors are distant substitutes and/or *pricing to market* (PTM) is possible.

The chapter demonstrates that no intermediate degree of international substitution exists that simultaneously guarantees the operation of the BS effect and strong co-movement of the nominal and real exchange rate. One possible remedy is an assumption of PTM. In this case it is possible that domestically produced export goods are close substitutes of foreign tradables, which ensures the existence of the BS effect. On the other hand, with PTM the common currency price of the exported and locally sold domestically produced goods can be substantially different over the short run. Hence, nominal-exchange-rate movements can influence the behavior of the real exchange rate.

The chapter also shows that a certain combination of real and nominal rigidities has significant impact on the magnitude of the difference between sectoral inflation rates. As a consequence, the size of the effect of asymmetric sectoral productivity growth, in line with empirical observations, becomes smaller than predicted by the models of the traditional approach.

This chapter is structured as follows. *Section 1.2* reviews the main issues in a non-technical manner. *Section 1.3* surveys the empirical literature which initiated the research of this chapter. *Section 1.4* presents the model and the solution technique employed. In *section 1.5* the Balassa – Samuelson hypothesis is examined; under study is how the model can reproduce the coexistence of dual inflation and real appreciation, and the relationship between asymmetric productivity growth and the magnitude of sectoral inflation differentials is examined. *Section 1.6* presents the conclusions.

1.2 Review of studied problems

Before setting up the formal model it is worthwhile reviewing the problems being analyzed by this chapter in a non-technical way.

The first important problem is how a NOEM model can generate the Balassa - Samuelson effect, the widespread explanation for the coexistence of dual inflation and real appreciation.

Let $\mathcal{Q}_t$ denote the natural logarithm of the real exchange rate. By definition $\mathcal{Q}_t = \mathcal{E}_t + \mathcal{P}_t^{F*} - \mathcal{P}_t$, where $\mathcal{P}_t$ is the logarithm of the domestic consumer price index in domestic currency terms, $\mathcal{P}_t^{F*}$ is the logarithm of the foreign consumer price index in foreign currency terms, $\mathcal{E}_t$ is the logarithm of the nominal exchange rate, and t is the time index.[1] Let us assume that the price indices can be decomposed as

$$\mathcal{P}_t = a\mathcal{P}_t^T + (1-a)\mathcal{P}_t^N, \quad \mathcal{P}_t^{F*} = b\mathcal{P}_t^{FT*} + (1-b)\mathcal{P}_t^{FN*},$$

where $\mathcal{P}_t^T$ and $\mathcal{P}_t^{FT*}$ are the logarithms of the domestic and foreign price indices of tradables, $\mathcal{P}_t^N$ and $\mathcal{P}_t^{FN*}$ are the same indices of non-tradables and

[1] Throughout this thesis prices indicated by $*$ are measured in foreign currency.

a and b are parameters. Then the real exchange rate can be expressed as

$$\mathcal{Q}_t = \mathcal{Q}_t^T + \mathcal{Q}_t^R,$$

where $\mathcal{Q}_t^T = \mathcal{E}_t + \mathcal{P}_t^{FT*} - \mathcal{P}_t^T$, i.e. the logarithm of the external real exchange rate, and $\mathcal{Q}_t^R$ is the logarithm of the internal real exchange rate, which is related to sectoral relative prices, i.e. $\mathcal{Q}_t^R = (1-b)\mathcal{P}_t^{FR} - (1-a)\mathcal{P}_t^R$, where $\mathcal{P}_t^R = \mathcal{P}_t^N - \mathcal{P}_t^T$ and $\mathcal{P}_t^{FR} = \mathcal{P}_t^{FN*} - \mathcal{P}_t^{FT*}$. The BS effect is based on two assumptions:

- First, the two sectors use the same production inputs, but the *total factor productivity* (TFP) of the sectors can be different.
- Second, PPP is fulfilled, that is $\mathcal{P}_t^T = \mathcal{E}_t + \mathcal{P}_t^{FT*}$.

The first assumption implies that $\mathcal{P}_t^R = \mathcal{A}_t^T - \mathcal{A}_t^N$ and $\mathcal{P}_t^{FR} = \mathcal{A}_t^{FT} - \mathcal{A}_t^{FN}$ if the sectors have the same constant-returns-to-scale technologies. $\mathcal{A}_t^T$, $\mathcal{A}_t^N$, $\mathcal{A}_t^{FT}$ and $\mathcal{A}_t^{FN}$ denote the logarithms of the sectoral TFP measures. The second assumption implies that the external real exchange rate is constant if the foreign price index is fixed. Hence, if it is assumed that the foreign productivity differential is zero, then

$$d\mathcal{Q}_t = \pi_t^N - \pi_t^T = d\mathcal{A}_t^T - d\mathcal{A}_t^N, \tag{1.1}$$

where d is the difference operator and π_t^s ($s = T,\ N$) are the sectoral inflation rates. That is, if the productivity growth of tradables is higher than that of non-tradables, then the inflation rate of the non-tradables will be higher, and the real exchange rate will appreciate.

Obviously, if PPP is fulfilled and the external real exchange rate is constant, then the main propositions of the NOEM cannot be valid. That is, real exchange rate behavior cannot essentially be determined by movements of the external real exchange rate, which correlates with the nominal exchange rate. Illustrating this contradiction, let us sketch how a typical NOEM model explains the co-movement of the nominal and real exchange rate. Since usually in these models the distinction between tradables and non-tradables is missing, I set $\mathcal{P}_t^R = \mathcal{P}_t^{FR} = 0$. The correlation of the nominal and real exchange rates is guaranteed by the following two conditions:

- It is allowed that $\mathcal{P}_t^T \neq \mathcal{E}_t + \mathcal{P}_t^{FT*}$. This can occur only if the markets of the domestic and foreign tradables are segmented, that is, PPP is not guaranteed since international goods arbitrage is impossible.
- Prices are sticky.

For the sake of clarity, the simplest form of nominal rigidity is used in this example: prices are set one period in advance. Let us assume that at date t an unexpected nominal-exchange-rate shock occurs, which was not accommodated at date $t-1$ when the prices were set. Then the real exchange rate is given by

$$\mathcal{Q}_t = \mathcal{E}_t + \mathcal{P}_{t-1}^{FT*} - \mathcal{P}_{t-1}^{T}.$$

This expression is not necessarily constant according to the first assumption, and the preset prices imply that nominal and real exchange rates are perfectly correlated. Thus, the essential distinction between the traditional and the NOEM approach is not that the latter has usually one sector. One can build two-sector NOEM models as well. Rather it is that they describe differently the behavior of the external real exchange rate $\mathcal{Q}_t^T$.[2]

This chapter studies how the contradiction described above can be resolved. Namely, how it is possible to build a NOEM model in which asymmetric sectoral productivity growth results in dual inflation and real appreciation since the external real exchange rate does not depreciate so much as to neutralize or suppress the appreciation of the internal real exchange rate.[3]

NOEM models guarantee the $\mathcal{P}_t^T \neq \mathcal{E}_t + \mathcal{P}_t^{TF*}$ requirement in two ways. The first way is that they assume that domestic export goods and their foreign rivals are not perfect substitutes. Then, the price of these export goods and their foreign rivals do not need to coincide when expressed in the same currency. The other way involves the assumption of *pricing to market* (PTM), which is often the consequence of third degree international price discrimination. Then it is possible that in the short run the same good have diverging prices in common currency terms at home and abroad.[4]

According to the imperfect substitutability approach, external demand for domestically produced goods is expressed by a formula similar to the following:

$$\mathcal{X}_t = \eta^* \left(\mathcal{E}_t + \mathcal{P}_t^{TF*} - \mathcal{P}_t^{T} \right) + \mathcal{X}_t^*, \tag{1.2}$$

where $\mathcal{X}_t$ is the logarithm of exports, $\mathcal{X}_t^*$ is a variable related to the volume of

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[2] In a short review like this, of course, it is impossible to provide an exact classification of pre-NOEM models. But it is important to note that the external real exchange rate is not fixed in all models in the traditional approach. But this does not influence the validity of my argument since external-real-exchange rate movements are independent from the nominal exchange rate even in these models.

[3] Fagan et al. (2003) study problems related to the BS effect with a two sector NOEM-like model. Although they assume price stickiness in the non-tradable sector, the markets of tradables are internationally homogenous and competitive. In my opinion this is not a solution, but a bypass of the problem.

[4] If PTM occurs, then the assumption of the imperfect substitutability of domestic and foreign tradables is not necessary but possible.

external demand, and η^* is an exogenous parameter. The models of Obstfeld and Rogoff (2000), Galí and Monacelli (2002), and Monacelli (2004) represent this approach.

The parameter η^* measures the substitutability between domestic exports and their rival goods. If $\eta^* = +\infty$, they are perfect substitutes as the traditional approach assumes. Then expression (1.2) takes the simpler form $\mathcal{P}_t^T = \mathcal{E}_t + \mathcal{P}_t^{TF*}$. However, the strong correlation between the nominal and real exchange rate requires that the goods are far substitutes, i.e. η^* is small. In this case if the TFP of domestic tradables increases, then $\mathcal{P}_t^T - \mathcal{E}_t$ will decrease, resulting in depreciation of the external real exchange rate in a small open economy, since foreign prices are not influenced by domestic factors. The problem is whether there exists an intermediate value of η^*, which guarantees a rather strong correlation between nominal and real exchange rates, but where the BS effect still remains valid, as increasing productivity does not cause such a large decrease of $\mathcal{P}_t^T - \mathcal{E}_t$, which neutralizes the appreciation of the internal real exchange rate.

In NOEM models with PTM it is usually assumed that the prices of domestic export goods are sticky in the currency of the destination country. This price setting practice is called *local currency pricing* (LCP). Betts and Devereux (1998), Chari et al. (2002), Devereux and Engel (1999), and Laxton and Pesenti (2003), for example, apply this price setting strategy.[5] If PTM is valid, one can imagine that export prices are sticky in the domestic currency, i.e. *producer currency pricing* (PCP) is performed. Bergin (2004) also considers this case. But usually the PCP assumption is applied without PTM, which is nothing but the imperfect substitutability approach represented by formula (1.2).

In NOEM models with PTM it is less problematic to reconcile the comovement of nominal and real exchange rate and the BS effect than in models with imperfect substitutability of domestic and foreign tradables. Let us briefly illustrate why. For the sake of simplicity let us assume that domestic export goods and their foreign rivals are perfect substitutes. Furthermore, assume that domestic firms are price takers abroad (in this case the LCP versus PCP distinction becomes meaningless). Let us denote by $\mathcal{P}_t^{T*}$ the logarithm of the foreign currency price of the exported domestic goods. The assumption of price taking guarantees that $\mathcal{P}_t^{T*} = \mathcal{P}_t^{FT*}$. Furthermore, assume that the economy is in its long-run equilibrium, when $\mathcal{P}_t^T - \mathcal{E}_t = \mathcal{P}_t^{T*}$. Assume again that $\mathcal{P}_t^T$ and $\mathcal{P}_t^{FT*}$ are set one period in advance. If an un-

[5] A variant of this approach assumes a transportation sector to guarantee the existence of PTM. See, e.g., Benigno and Thoenissen (2002), Monacelli (2003, 2004), and Smets and Wouters (2002).

expected nominal-exchange-rate shock hits the economy, then PTM implies, at least in the short run, that

$$\mathcal{P}_{t-1}^{T} - \mathcal{E}_t = \mathcal{P}_t^{T} - \mathcal{E}_t \neq \mathcal{P}_t^{T*} = \mathcal{P}_{t-1}^{T*}.$$

Thus, as previously, the nominal and the external real exchange rate correlates. On the other hand, in models with PTM the BS effect remains valid, since in the longer run, which is relevant for the BS effect, $\mathcal{P}_t^{T} - \mathcal{E}_t = \mathcal{P}_t^{T*} = \mathcal{P}_t^{FT*}$. That is, the external real exchange rate is fixed. This implies that higher productivity growth results in real appreciation.

The second problem investigated in the chapter is how asymmetric productivity growth influences the magnitude of sectoral inflation differentials. In models of the classical approach if there is no strong sectoral asymmetry, the size of inflation and productivity growth differentials are the same, as in formula (1.1). However, according to empirical studies, e.g. Coricelli and Jazbec (2001), Halpern and Wyplosz (2001), Égert (2002) and Égert et al. (2002) the magnitude of the sectoral price differential is smaller than that of the sectoral productivity differential.

If prices are sticky, the adjustment of the difference between sectoral prices becomes slow, and equation (1.1) correctly describes the behavior of sectoral inflation rates only in the long run. As Woodford (2005) shows, the presence of heterogeneity and real rigidities in capital accumulation makes the above adjustment process even more inert.

My numerical simulations demonstrate that the aforementioned nominal and real rigidities significantly weaken the impact of asymmetric sectoral productivity growth on the difference between sectoral inflation rates, and help to explain the empirically observable magnitude of price and inflation differentials.

1.3 Previous empirical results

This section briefly reviews the empirical literature which initiated the research of this chapter. First, findings related to the internal real exchange rate are surveyed. On this issue the evidence is ambiguous. In developed economies, internal-real-exchange-rate movements are negligible, while in several emerging economies dual inflation is an important phenomenon. Second, findings on the strong relationship between the nominal and real exchange rates are considered, which are relevant in both developed and emerging economies.

1.3.1 Dual inflation and real appreciation

As mentioned in the *Introduction* and discussed in *section 1.2*, the NOEM literature focuses on the behavior of the external real exchange rate, instead of the internal one, which was mainly studied by the previous traditional literature. This switch of interest was partly initiated by the findings of Engel (1999), who, using US data, showed that the volatility of the real exchange rate can be explained nearly perfectly by the movements of the external real exchange rate.

However, the validity of this finding is not general. Even in developed countries one can observe significant movements of the internal real exchange rate, as De Gregorio and Wolf (1994), or more recently López-Salido et al. (2005) have documented, but the real importance of this phenomenon is manifested in high growth and emerging market countries. Several empirical studies demonstrate that the Balassa - Samuelson (BS) effect plays a significant role in these countries.

Balassa (1964) and Samuelson (1964) formulated the hypothesis that the difference in productivity growth rates in tradable and non-tradable sectors results in dual inflation, and as a consequence real appreciation.[6] Ito et al. (1997) showed that mainly in Japan, Korea, and Taiwan, but to some extent in other Southeast Asian countries as well, the BS effect was determinant at particular stages of their development process. It also plays an important role in the transition of European post-communist countries, as the empirical studies of Coricelli and Jazbec (2001), Halpern and Wyplosz (2001), Égert (2002) Égert et al. (2002), and Kovács (2002) have documented.

Coricelli and Jazbec (2001) examined the determinants of the real exchange rate in nineteen transition economies between 1991 and 1998.[7] Halpern and Wyplosz (2001) studied the relevance of the BS effect in nine European post-communist countries by estimating a panel regression for the period 1991-98.[8] Égert (2002) used time series and panel cointegration techniques to study the BS effect in five east European accession countries between 1991 and 2001.[9] Égert et al. (2002) examined the BS effect in nine European accession countries by panel cointegration techniques on a data set covering the

[6] On the Balassa - Samuelson effect see Obstfeld and Rogoff (1996, chapter 4).

[7] The examined countries were Armenia, Azerbaijan, Belarus, Bulgaria, Croatia, Czech republic, Estonia, Hungary, Kazakhstan, Kyrgyzstan, Latvia, Lithuania, Poland, Romania, Russia, Slovakia, Slovenia, Ukraine and Uzbekistan.

[8] The countries in the sample were the Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Russia, Romania and Slovenia.

[9] The examined countries are the Czech Republic, Hungary, Poland, Slovakia and Slovenia.

period from 1995 to 2000.[10] The paper edited by Kovács (2002) summarizes the results of research on the BS effect conducted by the central banks of central European accession countries.[11]

The above studies demonstrate that in most European post-communist countries the coexistence of dual inflation and real appreciation can be observed in their transition period. In addition, dual inflation is related to sectoral productivity growth differentials, and real appreciation is due to the appreciation of both the external and internal real exchange rates.

Coricelli and Jazbec (2001), Halpern and Wyplosz (2001), Égert (2002), and Égert et al. (2002) estimated the relationship between the relative price of non-traded to traded goods and the sectoral productivity differential.[12] Their findings are summarized in *Table 1.1.*

Table 1.1

Empirical long-run relationship between sectoral prices and productivity measures

	Type of regression	Estimated coefficient
Coricelli – Jazbec (2001)	price differential on productivity differential	0.87
Égert (2002)	panel, price differential on productivity differential	0.72
Égert (2002)	individual, price differential on productivity differential	0.49-0.95
Égert et al. (2002)	price differential on productivity differential	0.73-1
Halpern – Wyplosz (2001)	tradable price on tradable productivity	0.43
Halpern – Wyplosz (2001)	non-tradable price on non-tradable productivity	0.32

[10] The studied countries are Croatia, the Czech Republic, Estonia, Hungary, Latvia, Lithuania, Poland, Slovakia and Slovenia.

[11] The examined countries and the length of the data set: the Czech Republic (1994-2001), Hungary (1992-2001), Poland, (1990-2001), Slovakia (1995-2000) and Slovenia (1992-2001).

[12] Since reliable estimates of total factor productivity were not available, due to the lack of capital stock data, they used labor productivity measures.

According to Coricelli and Jazbec (2001, equation 19), if the productivity differential rises by 1 per cent, the relative price rises by 0.87 per cent. Égert (2002, Table 1-7) found significant cointegration relationship between the relative price and productivity differential. The cointegration coefficient measuring the long-run relationship between the relative prices and productivity factors varies from 0.49 to 0.95 in individual country estimates, and 0.72 is the common estimate for the coefficient provided by the panel cointegration analysis. In Égert et al. (2002, Table 5) the same cointegration coefficient ranges from 0.73 to 1, depending on the applied definition of tradable and non-tradable sectors. Unlike the previous studies, Halpern and Wyplosz (2001, Table 7) estimated the effects of tradable and non-tradable productivity developments separately. They found significant coefficients with correct signs, although the estimated coefficients are quite small. If tradable productivity rises by 1 per cent, the sectoral relative price rises by 0.24 per cent in the short run and by 0.43 per cent in the long run. A 1 per cent rise of non-tradable productivity results in a 0.18 per cent decrease of the relative price in the short run and a 0.32 per cent decrease in the long run.

In summary: All papers found a significant relationship between sectoral prices and productivity measures. Magnitudes of estimated coefficients locate in quite a wide range. However, according to all but one estimates, productivity differentials are greater than the accompanying price differentials.

According to the original BS hypothesis, productivity induced real appreciation of the internal real exchange rate results in CPI-based real appreciation, since the external real exchange rate is fixed due to the assumed validity of PPP.

Kovács (2002, Table 1-1) documented that between 1993 and 2002 the annual average real appreciation of the examined countries varied from 2.2 to 5.8 per cent. However, the BS effect does not fully explain the observed CPI-based appreciations. Only 33-72 per cent of it can be attributed to productivity growth induced internal real exchange rate movements, the rest can be assigned to the external real exchange rate. Égert (2002, Table 9) also reveals that productivity induced appreciation of the internal real exchange rate cannot completely explain real appreciation. According to his panel analysis, it is responsible for 38-60 per cent of CPI-based appreciation. He also stresses the importance of a trend appreciation of the external real exchange rate to explain the observed phenomena. Égert et. al (2002) presented similar findings and reinforced the conclusions of the above papers.

Although in this chapter I study only productivity induced dual inflation, I should mention that studies analyzing the BS effect have often detected other non-productivity factors in the determination of the sectoral relative price. Moreover, Arratibel et al. (2002) do not simply provide alternative explanations for dual inflation, they deny the role of productivity factors in the determination of the examined countries. However, the authors admit that one should interpret this result with caution because of the poor quality of productivity data.[13]

1.3.2 The co-movement of the nominal and real exchange rates

As mentioned in the *Introduction*, the NOEM literature was partly initiated by the empirical findings of Mussa (1986), who first documented the strong connection between the nominal and real exchange rates. Using Monacelli (2004), I summarize some important findings. The post-1971 data from 12 developed countries reveal that the unconditional correlation of real and nominal depreciation rates is 0.98. In flexible exchange rate regimes the unconditional variance of the real depreciation rate is nearly equal to the unconditional variance of the nominal depreciation rate.

Violation of purchasing power parity (PPP) is a necessary condition for the above findings. Moreover, the violation of PPP is not a transitory phenomenon, as several empirical studies have shown. Chari et al. (2002) studied the persistency of the real-exchange-rate shocks using HP-filtered quarterly data for the USA and 11 developed European countries for the period 1973:1-2000:1. Their estimated quarterly autocorrelation is 0.84.[14] Though the above empirical results are all related to developed countries, the violation of PPP can also be detected in European post-communist countries, which are the primary focus of this chapter,[15] although the supporting evidence is mainly only stylized facts.

[13] In their paper they studied the inflation processes in 10 European post-communist countries. Their results support the existence of dual inflation in these countries. However, according to their estimations, a positive productivity shock negatively influences the inflation rate in the non-tradable sector.

[14] Diebold et al. (1991) and Lothian and Taylor (1996) using long annual time series of different currencies found much more persistent real-exchange-rate shocks than Chari et al. (2002). It is difficult to explain their findings purely by nominal rigidities. Rogoff (1996) refers to this phenomenon as the '*PPP puzzle*'. Engel and Morley (2001) built an empirical model, which may help to resolve this puzzle.

[15] Hornok et al. (2002) tried to perform econometric estimations on very short time series and the half-time they found is approximately 2.8 years. On the other hand, Darvas (2001) using the data of the Czech Republic, Hungary, Poland, and Slovenia found very short, less than one year, half-lives. But in the studied time periods narrow-band crawling peg regimes were typical in these countries, which may explain his results.

1.4 The model

One of the main focuses of this chapter concerns how to construct a model which can simultaneously guarantee the empirical regularities characterized in *section 1.3*, i.e. the co-movement of the nominal and real exchange rates and generate the Balassa - Samuelson (BS) effect, i.e. the coexistence of productivity based dual inflation and real appreciation

To guarantee the empirically observable correlation between the nominal and real exchange rates the model needs sticky prices and internationally segmented tradable markets. Obviously, to consider the BS effect it is necessary to have at least two sectors with different total factor productivities (TFP).

International market segmentation can be captured in different ways. I therefore compare whether model versions with different descriptions of market segmentation can generate the BS effect. I consider a version (version A) without pricing to market (PTM) and with the assumption that domestic and foreign tradables are imperfect substitutes. In version B PTM combined with local currency pricing (LCP) is added to the model.[16]

The other main topic of the chapter is the relationship between the magnitude of sectoral relative price and productivity differentials. In frictionless, sectorally symmetric models the two quantities are equal. Yet this is not in line with empirical results, which reveal that the relative price of nontradables to tradables is smaller than the sectoral productivity differential. Nominal rigidities help to explain this phenomenon: if prices are sticky the adjustment of the sectoral relative price is not immediate. In addition, as Woodford (2003, chapter 3) demonstrates, *decreasing returns* amplify the impact of sticky prices, making the adjustment process even slower, which provides a better fit in terms of empirical results.

One way of applying decreasing returns in the model is the assumption of fixed capital stock with a constant-returns-to-scale technology. However, one may criticize this approach in that in the relevant time horizon of the Balassa – Samuelson effect, which is longer than a usual business cycle phenomenon, it can be misleading to neglect capital accumulation.

Hence, I choose another way of generating decreasing returns. As Woodford (2005) shows, even if the technology exhibits constant returns to scale, the *lack of an economywide rental market* for physical capital and *frictions in investments formation* combined with sticky asynchronized price setting result in suboptimal input allocation, and as a consequence, scarcity and

[16] Although it is rarely studied in the literature, there is a third logical possibility, namely PTM with producer currency pricing. For the sake of clear presentation I omit discussion of this case.

decreasing returns to scale in the short run.[17]

1.4.1 Households

The domestic economy is populated by a continuum of infinitely-lived identical households. To simplify the notation household indices are dropped, since this does not cause confusion. The utility accrued to a given household at date t is

$$\mathcal{U}\left(c_t, l_t\right) = \frac{c_t^{1-\sigma}}{1-\sigma} - \frac{l_t^{1+\varphi}}{1+\varphi},$$

where c_t is the consumption and l_t is the labor supply of the representative household at date t. Furthermore, σ, $\varphi > 0$. Households discount the future at the rate $0 < \beta < 1$.

The consumption good c_t is composed of *tradable* and *non-tradable* consumption goods:

$$c_t = \left[a_T^{\frac{1}{\eta}} \left(c_t^T\right)^{\frac{\eta-1}{\eta}} + a_N^{\frac{1}{\eta}} \left(c_t^N\right)^{\frac{\eta-1}{\eta}} \right]^{\frac{\eta}{\eta-1}}, \tag{1.3}$$

where c_t^T is the tradable, c_t^N is the non-tradable consumption good, η and $a_T = 1 - a_N$ are non-negative parameters.

The intertemporal budget constraint of a given household is the following:

$$P_t^T c_t^T + P_t^N c_t^N + P_t^B B_t = \zeta_t B_{t-1} + W_t l_t + T_t,$$

where P_t^T and P_t^N are the price indices of tradables and non-tradables, B_t is the household's nominal portfolio at the beginning of date t, P_t^B is its price, and ζ_t is its stochastic payoff. W_t is the nominal wage, while T_t is a lump-sum tax/transfer variable.

It is well known that the linear homogeneity of function (1.3) implies that the households' problem can be solved in two steps. First they maximize the objective function

$$\sum_{t=1}^{\infty} \beta^{t-1} \mathrm{E}_0 \left[\mathcal{U}\left(c_t, l_t\right)\right],$$

with respect to c_t subject to the following modified budget constraint:

$$P_t c_t + P_t^B B_t = \zeta_t B_{t-1} + W_t l_t + T_t, \tag{1.4}$$

[17]There can be different explanations for the lack of a rental market for physical capital. One is based on the existence of *firm-specific* investments and capital goods. The literature of the theory of firms considers this factor very important: one can explain with this phenomenon the size and integration of firms, as Hart (1995) discusses.

non-negativity constraints on consumption, and no-Ponzi schemes. In the budget constraint (1.4) the consumer price index P_t is defined by the following expression:

$$P_t = \left[a_T \left(P_t^T\right)^{1-\eta} + a_N \left(P_t^N\right)^{1-\eta}\right]^{\frac{1}{1-\eta}}. \tag{1.5}$$

Second, knowing c_t it is possible to determine c_t^T and c_t^N by the demand functions

$$c_t^T = a_T \left(\frac{P_t}{P_t^T}\right)^{\eta} c_t, \quad c_t^N = a_N \left(\frac{P_t}{P_t^N}\right)^{\eta} c_t. \tag{1.6}$$

The assumption of complete asset markets implies that the optimal intertemporal allocation of consumption is determined by the following condition in all states of the world:

$$\beta \frac{\Lambda_{t+1} P_t}{\Lambda_t P_{t+1}} = D_{t,t+1}, \tag{1.7}$$

where $\Lambda_t = c_t^{-\sigma}$ is the marginal utility of consumption and $D_{t,t+1}$ is the stochastic discount factor, which satisfies the condition

$$P_t^B = \mathrm{E}_t \left[D_{t,t+1} \zeta_{t+1}\right].$$

Since in this economy the asset markets are also complete internationally, the foreign equivalent of equation (1.7) is also held:

$$\beta \frac{\Lambda_{t+1}^* e_t P_t^{F*}}{\Lambda_t^* e_{t+1} P_{t+1}^{F*}} = D_{t,t+1}, \tag{1.8}$$

where Λ_t^* is the marginal utility of foreign households, P_t^{F*} is the foreign consumer price index in foreign currency terms, and e_t is the nominal exchange rate. For simplicity P_t^{F*} is assumed to be constant. Combining equations (1.7) and (1.8) and applying recursive substitutions yields formula

$$\frac{\Lambda_t e_t P_t^{F*}}{\Lambda_t^* P_t} = \iota, \tag{1.9}$$

where ι is a constant, which depends on initial conditions.

The solution of the households' problem implies that the real wage is equal to the marginal rate of substitution between consumption and labor, i.e.

$$w_t = c_t^{\sigma} l_t^{\varphi}, \tag{1.10}$$

which determines the labor supply decision.

1.4.2 Production

Final and intermediate goods production

There are two stages of production in the model: in the first step import goods and labor are transformed into differentiated intermediate goods in each sector,[18] while in the second step a homogenous final good is produced in each sector by intermediate products.

As mentioned above, one objective of this chapter is to study how the different descriptions of international goods markets segmentation influence the operation of the BS effect. Therefore, two different model versions are considered and compared. In version A it is assumed that there is no PTM. That is, the domestically produced export goods and the domestically consumed tradable goods have the same prices, if they are measured in the same currency. In version B there is pricing to the market, i.e. the price of the domestically produced export goods and the domestically consumed tradable goods can be different, even if they are measured in the same currency.

To capture these characteristics in version A the assumption is made that the domestically produced export goods and the locally traded tradable goods are the same and produced by the same sector. Hence, two sectors are distinguished in version A: a tradable and a non-tradable one.

In version B there are two types of tradable goods: goods which are traditionally classified as tradable, but in practice they are *local goods*, and another type of tradables that are produced for export. As a consequence, prices of local tradables and export goods denominated in the same currency can be different. Local tradables and the export goods are produced by different sectors.[19]

Let us denote by y_t^s the production of a given sector, where $s = T$, x, N, with T referring to the tradable sector in version A and to the sector of local tradables in version B, x to the exports sector in version B, and N to non-tradables. The final goods are produced in competitive markets by constant-returns-to-scale technologies from a continuum of differentiated inputs, $y_t^s(i)$, $i \in [0, 1]$. The technology is represented by the following CES production

[18]Thus, I apply the approach of McCallum and Nelson (2001), Smets and Wouters (2002) and Laxton and Pesenti (2003), who consider imports as a production input.

[19]To guarantee PTM, of course, the distinction of local tradables and export goods is not necessary. I applied this assumption due to technical reasons. Otherwise in the presence of heterogeneous capital the price setting problem of firms would be intractable. On the other hand, this approach is not unique in the literature. For example, Burnstein et al. (2002) also assumed the existence of local and real tradables. But unlike me, they assumed a quality difference between the two groups: local goods are inferior.

function:

$$y_t^s = \left(\int_0^1 y_t^s(i)^{\frac{\theta-1}{\theta}} \, di \right)^{\frac{\theta}{\theta-1}},$$

where $\theta > 1$. As a consequence, the output price P_t^s is given by

$$P_t^s = \left(\int_0^1 P_t^s(i)^{1-\theta} \, di \right)^{\frac{1}{1-\theta}}, \tag{1.11}$$

where $P_t^s(i)$ denotes the prices of differentiated goods. The demand for good $y_t^s(i)$ is determined by

$$y_t^s(i) = \left(\frac{P_t^s}{P_t^s(i)} \right)^{\theta} y_t^s. \tag{1.12}$$

In each sector the continuum of good $y_t^s(i)$ is produced in a monopolistically competitive market. Each $y_t^s(i)$ is made by an individual firm using the following uniform technology:

$$y_t^s(i) = A_t^s k_t^s(i)^{\alpha} z_t^s(i)^{1-\alpha}, \tag{1.13}$$

where $0 < \alpha < 1$, A_t^s is total factor productivity of sector s, $k_t^s(i)$ is the stock of physical capital available for firm i at date t (it was produced in the previous period), and $z_t^s(i)$ denotes an individual firm's utilization of the composite input z_t^s defined in the following way:

$$z_t^s(i) = N_s l_t^s(i)^{n_s} m_t^s(i)^{1-n_s}, \tag{1.14}$$

where $l_t^s(i)$ is an individual firms' utilization of labor l_t, and $m_t^s(i)$ is the utilization of imported good m_t, n_s is a given non-negative parameters, and $N_s = n_s^{-n_s}(1-n_s)^{n_s-1}$. The price of z_t^s is given by

$$W_t^{z,s} = W_t^{n_s} \left(e_t P_t^{m*} \right)^{1-n_s}, \tag{1.15}$$

where P_t^{m*} is the foreign currency price of the imported good.

Cost minimization and input demand

It is assumed that there is no rental market for physical capital. The necessary capital goods are produced by the firms themselves. As a consequence, firms' optimal input allocation problem cannot be separated from the problem of capital accumulation and cannot be derived from a sequence of static cost minimization problems.

Instead they solve the following dynamic cost minimization problem. Suppose the trajectories of $y_t(i)$, P_t, $W_t^{z,s}$ and $D_{T,t}$ are given. Then a firm should minimize the objective function

$$\sum_{t=T}^{\infty} \mathrm{E}_T \left[D_{T,t} \left(W_t^{z,s} z_t^s(i) + P_t I_t^s(i) \right) \right],$$

with respect to $z_t^s(i)$, $I_t^s(i)$, $k_{t+1}^s(i)$, subject to the technological constraint (1.13) and the investment constraint

$$k_{t+1}^s(i) = (1-\delta) k_t^s(i) + \Phi_s \left(\frac{I_t^s(i)}{k_t^s(i)} \right) k_t^s(i), \tag{1.16}$$

where $I_t^s(i)$ is the investment of firm i at date t. The function Φ_s represents the adjustment costs for investments, and δ is the depreciation rate. As is common in the literature, it is assumed that $\Phi_s' > 0$, $\Phi_s'' < 0$, and that in the steady-state adjustment costs do not exist, i.e. $\Phi_s(I^s/k^s) = I^s/k^s$ and $\Phi_s'(I^s/k^s) = 1$, where variables without time indices refer to the steady-state values.

The first-order conditions of the cost minimization problem are

$$\frac{D_{T,t} P_t}{\nu_t^s(i)} = \Phi_s' \left(\frac{I_t^s(i)}{k_t^s(i)} \right), \tag{1.17}$$

where $\nu_t^s(i)$ is the Lagrange multiplier of the investment equation,[20] and

$$\nu_t^s(i) = \mathrm{E}_T \left[\nu_{t+1}^s(i) \left\{ (1-\delta) + \phi_s \left(\frac{I_{t+1}^s(i)}{k_{t+1}^s(i)} \right) \right\} + D_{T,t+1} P_{t+1} r_{t+1}^s(i) \right], \tag{1.18}$$

where $\phi_s(y) = \Phi_s(y) - y\Phi_s'(y)$, and

$$r_{t+1}^s(i) = \frac{\alpha}{1-\alpha} w_{t+1}^{z,s} \frac{z_{t+1}^s(i)}{k_{t+1}^s(i)}. \tag{1.19}$$

In models with a rental market for physical capital $r_{t+1}^s(i)$ in equation (1.18) represents the rental rate of capital.[21]

Equations (1.13), (1.16), (1.17) (1.18) and (1.19) provide the solution of the cost minimization problem, which determine the paths of $z_t^s(i)$, $k_t^s(i)$,

[20] That is, it is the shadow price of investment. $\nu_t^s(i) \left(D_{T,t} P_t \right)^{-1}$ is the equivalent of Tobin's q in this model.

[21] If there is no adjustment costs for investments, then condition (1.18) becomes $P_t = \mathrm{E}_t \left[D_{t,t+1} P_{t+1} \left((1-\delta) + r_t^s(i) \right) \right]$. As a consequence, $r_t^s(i) = r_t^s = r_t$. In a deterministic setting the previous equation takes the form $1/\beta = r + 1 - \delta$, which is a simple arbitrage condition.

$I_t^s(i)$, $r_t^s(i)$, and $\nu_t^s(i)$ given the paths for $y_t^s(i)$, P_t, $w_t^{z,s}$ and $D_{T,t}$. Knowing $z_t^s(i)$ one can determine the labor and import demand of a particular firm by

$$l_t^s(i) = n_s \frac{W_t^{z,s}}{W_t} z_t^s(i), \tag{1.20}$$

$$m_t^s(i) = (1-n_s) \frac{W_t^{z,s}}{e_t P_t^{m*}} z_t^s(i). \tag{1.21}$$

Firms' investment good is a composition of (local) tradables and non-tradables. The investment good and aggregate consumption good c_t are defined by the same function:

$$I_t^s(i) = \left(a_T^{\frac{1}{\eta}} \mathcal{I}_t^{Ts}(i)^{\frac{\eta-1}{\eta}} + a_N^{\frac{1}{\eta}} \mathcal{I}_t^{Ns}(i)^{\frac{\eta-1}{\eta}} \right)^{\frac{\eta}{\eta-1}}, \tag{1.22}$$

where $\mathcal{I}^{Ts}$ is the demand for (local) tradables of firm i in sector s, and $\mathcal{I}^{Ns}$ is the demand for non-tradables. The particular form of function (1.22) implies that

$$\mathcal{I}_t^{Ts}(i) = a_T \left(\frac{P_t}{P_t^T} \right)^{\eta} I_t^s(i), \quad \mathcal{I}_t^{Ns}(i) = a_N \left(\frac{P_t}{P_t^N} \right)^{\eta} I_t^s(i). \tag{1.23}$$

Price setting

So far, it has been shown how to find the optimal paths of $z_t^s(i)$, $k_t^s(i)$, $l_t^s(i)$, $m_t^s(i)$ conditional on the trajectories of $y_t^s(i)$ and $P_t^s(i)$. Now the optimal paths of the latter two variables will be determined.

Intermediate goods producers follow a sticky price setting practice. As in the model of Calvo (1983) each individual firm in a given time period changes its price in a rational, optimizing, forward looking manner with probability $1-\gamma_s$. Those firms which do not optimize at a given date follow a rule of thumb, as in Christiano et al. (2001) and Smets and Wouters (2003), and update their prices according to the past sectoral inflation rate.

All firms in sector $s = T$, N which follow the simple indexation rule at date T update their prices according to formula

$$P_t^s(i) = P_T^s(i) \left(\frac{P_{t-1}^s}{P_{T-1}^s} \right)^{\vartheta_s}.$$

Those which set their prices rationally take into account that $P_T^s(i)$ (the price they set at date T) will exist with probability γ_s^{t-T} at date t. Thus, they maximize the expected profit function

$$\sum_{t=T}^{\infty} \mathrm{E}_T \left[\gamma_s^{t-T} D_{T,t} \left\{ (1-\tau_s) P_T^s(i) \left(\frac{P_{t-1}^s}{P_{T-1}^s} \right)^{\vartheta_s} - MC_t^s(i) \right\} \right]$$

with respect to $P^s_T(i)$ and $y^s_t(i)$ subject to constraint (1.12), where τ_s is a tax/transfer variable which modifies firms' markup, and $MC^s_t(i)$ is the marginal cost of firm i. In version B of the model the output price of the exports sector in foreign currency terms $P^{x*}_T(i)$ is sticky. Thus, the problem of the firms in the sector is

$$\max_{P^{x*}_T(i), y^x_t(i)} \sum_{t=T}^{\infty} \mathrm{E}_T \left[\gamma_x^{t-T} D_{T,t} \left\{ (1-\tau_x)\, e_t P^{x*}_T(i) \left(\frac{P^{x*}_{t-1}}{P^{x*}_{T-1}} \right)^{\vartheta_x} - MC^x_t(i) \right\} \right],$$

subject to constraint (1.12), where τ_x is also a tax/transfer variable.[22] The log-linear approximations of the solutions of the above price setting problems can be found in *Appendix A.2*.

Since the capital stock available at a given date is predetermined, the variable cost of a firm is $W^{z,s}_t z^s_t(i) + P_t I^s_t(i)$. Thus, its marginal cost is

$$MC^s_t(i) = W^{z,s}_t \frac{\partial z^s_t(i)}{\partial y^s_t(i)}.$$

Expressing $z^s_t(i)$ by the technological constraint (1.13), and differentiating it with respect to $y^s_t(i)$ yields

$$MC^s_t(i) = W^{z,s}_t \left(\frac{y^s_t(i)}{k^s_t(i)} \right)^{\frac{\alpha}{1-\alpha}} (A^s_t)^{\frac{-1}{1-\alpha}}. \tag{1.24}$$

1.4.3 Exports demand

Foreign behavior is not modelled explicitly. It is assumed that the following *ad hoc* equation determines demand for exports:

$$x_t = \left(\frac{P^{FT*}_t}{P^{x*}_t} \right)^{\eta^*} x^*_t, \tag{1.25}$$

where x_t, P^{x*}_t is the foreign currency price of the export goods, P^{FT*} is the foreign currency price of the rival goods (which is constant by assumption), x^*_t is an exogenous parameter representing the volume of demand, and $\eta^* > 0$ is an exogenous parameter.

In version A of the model, exported goods are produced by the tradable sector, and $P^{x*}_t = P^T/e_t$. While in version B local tradables and export goods are different, hence their prices denominated in the same currency can be different, i.e. it is possible that $P^{x*}_t \neq P^T/e_t$.

[22]It is assumed that the government's budget is balanced. Hence, the tax/transfer represented by τ_s ($s = T,\ x,\ N$) is compensated by T_t lump-sum tax/transfer variable in equation (1.4). In the present model the only role of τ_s is to simplify steady-state calculations, see *Appendix A.1*.

1.4.4 Equilibrium conditions

In version A the equilibrium of the tradable sector is given by

$$y_t^T = c_t^T + \sum_{s=T,N} \mathcal{I}_t^{Ts} + x_t. \tag{1.26}$$

In version B the equilibrium conditions of the sector of local tradables and of the exports sector is given by

$$y_t^T = c_t^T + \sum_{s=T,x,N} \mathcal{I}_t^{Ts}, \quad y_t^x = x_t, \tag{1.27}$$

where $\mathcal{I}_t^{Ts} = \int_0^1 \mathcal{I}_t^{Ts}(i)\, di$. The equilibrium condition of the non-tradable sector is

$$y_t^N = c_t^N + \sum_s \mathcal{I}_t^{Ns}, \tag{1.28}$$

where $\mathcal{I}_t^{Ns} = \int_0^1 \mathcal{I}_t^{Ns}(i)\, di$. Finally, the labor market equilibrium condition is

$$l_t = \sum_s \int_0^1 l_t^s(i)\, di. \tag{1.29}$$

1.4.5 Real exchange rate indices

In this chapter the following real exchange indices will be considered:

$$q_t = \frac{e_t P_t^{F*}}{P_t}, \quad q_t^T = \frac{e_t P_t^{FT*}}{P_t^T}, \quad P_t^R = \frac{P_t^N}{P_t^T}, \tag{1.30}$$

where q_t is the CPI-based real exchange rate and q_t^T is the external real exchange rate. The movements of P_t^R, the domestic relative price of non-tradables to tradables, unambiguously determine the fluctuation of the internal real exchange rate, since it is assumed that P^{FT*} and P^{FN*} are constant.

1.4.6 The log-linearized model

To solve the model its log-linear approximation around the steady state is taken. The complete description of the log-linearized model and the derivation of its equations can be found in *Appendix A.3*. In this section, the most important equations of the system are reviewed. Variables without time indices refer to their steady-state values, and the tilde denotes the log-deviation of a variable from its steady-state value.

Aggregate demand

The path of the aggregate consumption is described by

$$\sigma \tilde{c}_t = \tilde{q}_t. \tag{1.31}$$

In version A exports demand is represented by

$$\tilde{x}_t = \eta^* \tilde{q}_t^T, \tag{1.32}$$

since in this version $\tilde{q}_t^T = \widetilde{P}_t^{x*}$. In version B the log-linearized exports demand becomes

$$\tilde{x}_t = -\eta^* \widetilde{P}_t^{x*}. \tag{1.33}$$

Demand for tradable goods depends on exports demand, aggregate consumption and investments, and the sectoral relative price. In version A it takes the form

$$\tilde{y}_t^T = \frac{x x_t + c\tilde{c}_t + I\tilde{I}_t + (c+I)\eta a_N \widetilde{P}_t^R}{c+x+I}, \tag{1.34}$$

where I_t denotes aggregate investments, and $\widetilde{\chi}_t^N$ is and exogenous shift of relative sectoral demand. In version B the demand for tradables is given by

$$\tilde{y}_t^T = \frac{c}{c+I}\tilde{c}_t + \frac{I}{c+I}\tilde{I}_t + \eta a_N \widetilde{P}_t^R. \tag{1.35}$$

Demand for non-tradables depends on the same factors:

$$\tilde{y}_t^N = \frac{c}{c+I}\tilde{c}_t + \frac{I}{c+I}\tilde{I}_t - \eta a_T \widetilde{P}_t^R. \tag{1.36}$$

Price setting

Following Woodford (2005), *Appendix A.2* presents the solution of the price setting problem of *section 1.4.2*. The path of the inflation rate in sector $s = T$, N is given by

$$\pi_t^d - \vartheta_s \pi_{t-1}^d = \beta \mathrm{E}_t \left[\pi_{t+1}^d - \vartheta_s \pi_t^d\right] + \xi_s \widetilde{mc}_t^s, \tag{1.37}$$

where $s = T$, x, N, and $d = x^*$, if $s = x$, otherwise $d = s$. Furthermore, $\pi_t^d = \widetilde{P}_t^d - \widetilde{P}_{t-1}^d$ is the sectoral inflation rate, and $\widetilde{mc}_t^s$ is the average real marginal cost of sector s and

$$\xi_s = \frac{(1-\gamma_s)(1-\beta\gamma_s)}{\gamma_s\left(1 + \frac{\alpha}{1-\alpha}\theta - \psi_s\right)}, \tag{1.38}$$

where parameter ψ_s is defined in *Appendix A.2*. It is assumed that the technology and the price setting parameters of the tradable and the exports sector are the same.

Marginal costs

The previous equations reveal that sectoral real marginal costs play a key role in the price setting process. I therefore summarize the determinants of such costs. The average real marginal cost in sector $s = T$, N is given by

$$\widetilde{mc}_t^s = \frac{\alpha}{1-\alpha}\left(\tilde{y}_t^s - \tilde{k}_t^s\right) - \frac{1}{1-\alpha}\widetilde{A}_t^s + n_s\widetilde{w}_t + (1-n_s)\tilde{q}_t + \chi_s\widetilde{P}_t^R, \quad (1.39)$$

where $\chi_T = a_N$ and $\chi_N = -a_T$. The real marginal cost in the exports sector is

$$\widetilde{mc}_t^x = \frac{\alpha}{1-\alpha}\left(x_t - \tilde{k}_t^x\right) - \frac{1}{1-\alpha}\widetilde{A}_t^T + n_T\left(\widetilde{w}_t - \tilde{q}_t\right) - \widetilde{P}_t^{x*}. \quad (1.40)$$

Policy rule

In this model monetary policy is represented by the following simple log-linear nominal exchange rate rule:

$$d\tilde{e}_t = -\omega\left(a_T\pi_{t-1}^T + a_N\pi_{t-1}^N\right) + \mathcal{S}_t^{de}, \quad (1.41)$$

where $d\tilde{e}_t = \tilde{e}_t - \tilde{e}_{t-1}$ is the nominal depreciation rate, and $\mathcal{S}_t^{de}$ is an exogenous nominal depreciation shock.

1.4.7 Model solution and parameterization

To solve the model Uhlig's (1999) implementation of the *undetermined coefficients* method is used, the numerical results being generated by the aforementioned author's MATLAB algorithm.

Benchmark values of the basic parameters are found in *Table 1.2*. The value of β is taken from King and Rebello (1999). The value α is chosen in such a way that capital's share in GDP is 0.4.[23] The values of σ, φ, a_T, η and δ are widely accepted in the literature. The value of θ was chosen in such a way as to obtain the same degree of strategic complementarity of price setting as in Woodford (2003, 2005). Parameters ε_s measure the degree of investments adjustment costs in sector $s = T$, x, N, their values are taken from Woodford (2005). I take the values of γ_s and ϑ_s from the study of Galí et al. (2001), which also contains Euro area estimates.[24] The value of parameter η^* is not fixed: in the simulation exercises of *section 1.5* several

[23] In this model α is not equal to capital's share in GDP since one has to subtract the value of imports from the value of total output to obtain GDP.

[24] In that study they interpret inflation persistency differently from the approach I use. They use the model of Galí and Gertler (2000) and assume that each firm updates its price

different values are considered. Finally, ω was chosen in such a way that the model fits the empirical findings of *section 1.3*.

Table 1.2
Parameter values of
the benchmark economy

Parameter	
Name	Value
β	0.984
σ	1.000
φ	3.000
a_T	0.500
η	1.000
α	0.250
δ	0.025
ε_s	3.000
θ	10.80
γ_s	0.817
ϑ_s	0.365
ω	1.000

Note: $s = T$, x, N.

1.5 Examination of the Balassa – Samuelson effect

It was discussed in *section 1.3* that there is a strong relationship between the nominal and real exchange rates, and that asymmetric sectoral productivity growth results in dual inflation and real appreciation in developing countries. Under study in this section is how it is possible to reproduce both sets of evidence in a NOEM model.

First, it will be demonstrated that, unlike in the models of the traditional approach, in NOEM models productivity induced dual inflation is not necessarily accompanied by real appreciation, which contradicts the empirical findings discussed previously. It will be shown that the international

in a given period by probability $1-\gamma$. Hence, according to the law of large numbers in a given period $1-\gamma$ fraction of the firms change their prices. But only $1-\vartheta$ fraction of the price setters choose their prices in an optimal forward-looking manner, the rest update their prices according to the past inflation rate. If $\beta = 1$, then the approach I use and the one used by Galí and Gertler coincides, if $\vartheta_s = \vartheta/\gamma$ and $(1-\gamma_s)^2\gamma_s^{-1} = (1-\vartheta)(1-\gamma)^2\gamma^{-1}$, $s = T$, x, N. Although in our case $\beta \neq 1$, as an approximation I used the above mentioned formula to determine the values of γ_s and ϑ_s.

substitution parameter η^* in equations (1.32) and (1.33) has a key role in generating real appreciation. On the other hand, η^* also influences the degree of co-movement of the nominal and real exchange rates. According to my numerical simulations, the assumption of pricing to market (PTM) is necessary to find such a value of η^* which ensures both the strong co-movement of the nominal and real exchange rates and the CPI-based real appreciation related to asymmetric productivity growth.

Second, it will be shown that it is difficult to reproduce the observable slow adjustment of the sectoral relative price to the sectoral productivity differential by frictionless models. However, the coexistence of heterogeneity in capital accumulation and sticky prices help to explain this phenomenon.

1.5.1 Productivity induced dual inflation and real appreciation

As discussed in *section 1.3.1*, in European post-communist countries in the 1990s the fast productivity growth of the tradable sector resulted in dual inflation, i.e. appreciation of the internal real exchange rate, which accompanied the appreciation of the external and the CPI-based exchange rate.

Usually productivity induced coexistence of dual inflation and real appreciation, i.e. the BS effect is analyzed with models of the traditional approach. These models can successfully explain the coexistence of dual inflation and real appreciation, since in these models PPP is assumed, which prevents external real exchange rate movements. On the other hand, due to PPP they cannot reproduce the observable appreciation of the real exchange rate.

It seems that with NOEM models it is even more problematic to explain the discussed empirical phenomena. It is typical in NOEM models that although a positive productivity shock in the tradable sector results in real appreciation of the internal real exchange rate, at the same time, due to increasing productivity, domestic tradables become cheaper, i.e. the external real exchange rate also depreciates. As Beningno and Thoenissen (2002) demonstrated, the latter effect suppresses internal appreciation, hence the CPI-based real exchange rate also depreciates.

This possibility is especially important in version A. Consider the exports demand equation (1.32). If the international substitution parameter $\eta^* = +\infty$ then $\tilde{q}_t^T = 0$, i.e. the external real exchange rate becomes constant, and there will not be any relationship between the nominal and the real exchange rate, which contradicts empirical results. On the other hand, if η^* is low, and $\widetilde{P}_t^T$ is sticky, i.e. it responds to shocks slowly, then $\tilde{q}_t^T = \tilde{e}_t - \widetilde{P}_t^T$ will move together with the nominal exchange rate. However, in this case high tradable-productivity growth may cause strong external-real-exchange depreciation. The question is whether there is an intermediate value of η^* which can replicate both sets of empirical findings in version A of the model.

In version B even a high value of η^* can guarantee a strong co-movement of the nominal and real exchange rates. On the other hand, in this case the foreign currency price of domestically produced export goods $\widetilde{P}_t^{x*}$ does not deviate much from the prices of their foreign rivals. As a consequence, if other factors are kept fixed, the marginal costs of the domestic exports and the tradable sectors are similar, hence $\widetilde{P}_t^T - \tilde{e}_t$ remains relatively stable. Thus, the conjecture is that in version B it is possible to find appropriate values for the substitution parameter, which guarantee that asymmetric sectoral productivity growth results in real appreciation.

First, it is studied which value of the substitution parameter η^* is consistent with the strong co-movement of the nominal and real exchange rates discussed in *section 1.3.2*. In the simulation exercises the depreciation shock $\mathcal{S}_t^{de}$ is the only source of nominal-exchange-rate movements. This approach is supported by several empirical studies. In a closed economy context Smets and Wouters (2003) and Ireland (2004) demonstrated by their estimated models that nominal shocks have a primary role while technological shocks have only an auxiliary role in explaining business cycles. Clarida and Galí (1994) showed that in open economies 35-41 per cent of real exchange rate movements can be attributed to nominal shocks. The prominent importance of the nominal-exchange-rate shocks in emerging markets is documented by Calvo and Reinhart (2002).

In the following simulations all parameters, except η^*, are set to their benchmark values (see *Table 1.2*). *Table 1.3* displays the results. Empirical values of the statistics in the table are taken from *section 1.3.2*.

The time pattern of the reaction of the real exchange rate to the nominal-exchange-rate shock can be captured by the autocorrelation function of the real exchange rate. If $\eta^* = 1$ both versions of the model reproduce the 1-quarter and 1-year value of empirical autocorrelations quite well. However the simulated 2-year autocorrelation coefficients are higher than the observed one.[25]

[25] This contradicts the simulation results of Chari et al. (2002), who found weaker simulated autocorrelations. However, Benigno (2004) demonstrated that if monetary policy is described by a rule with inertia, and the foreign and home country are asymmetric in such a way that monetary shocks result in terms of trade changes, then the required persistence can be attained by the model. These conditions are fulfilled in my model.

Table 1.3
The relationship between nominal and
real exchange rates in the model economy

Version A

Statistics	Data	Parameter values of η^* 1	5	10	15
Autocorrelation of the real exchange rate					
1 quarter	0.84	0.81	0.72	0.66	0.61
1 year	0.50	0.50	0.34	0.23	0.18
2 years	0.25	0.37	0.23	0.16	0.12
The relative variance of the real and nominal depreciations	1	0.88	0.85	0.83	0.81

Version B

Statistics	Data	Parameter values of η^* 1	5	10	15
Autocorrelation of the real exchange rate					
1 quarter	0.84	0.83	0.81	0.80	0.80
1 year	0.50	0.53	0.49	0.47	0.46
2 years	0.25	0.40	0.34	0.32	0.30
The relative variance of the real and nominal depreciations	1	0.94	0.94	0.94	0.94

In version A all the autocorrelation coefficients significantly diminish as η^* increases. In particular, the 1-year coefficient becomes very small compared to the data. On the other hand, in version B the autocorrelation coefficients are much less sensitive to the substitution parameter.

Another measure indicating the strength of the co-movement of nominal and real exchange rates is the relative variance of nominal and real depreciations. In version A this statistic decreases as η^* increases, and becomes significantly smaller than the empirical value. On the other hand, in version B the relative variance does not react to the change of the substitution parameter.

In summary: while model version B is quite insensitive to the change of η^*, version A is sensitive to the variation of the substitution parameter. It can reproduce the empirical results only if η^* has low values, i.e. domestically produced export goods and their foreign rivals are far substitutes.

The next issue is whether dual inflation induced by asymmetric sectoral productivity growth is accompanied by real appreciation. The role of the international substitution parameter η^* in equations (1.32) and (1.33) will be studied by numerical simulations.

In the simulation exercises I imitate some characteristics of productivity developments of transition countries. The model's steady state represents the state of the economy at the beginning of its transition process. Foreign productivity growth is normalized to zero, hence the productivity variables $\widetilde{A}_t^T$ and $\widetilde{A}_t^N$ represent relative productivity of the examined small open economy. In the model transition is driven by increasing productivity. The start of the process is captured by an unexpected productivity shock. It is assumed that during transition the growth rate of productivity is constant. After the transition process the growth rate of productivity in the small open economy will be equal to zero as well. The steady state belonging to the new level of productivity represents the after-transition state of the economy. However, this new state of the economy is beyond my focus. I assume that the transition process is mainly driven by tradable productivity, hence I assume that in the examined transition period the growth rate of non-tradable productivity is equal to zero. In the simulation exercises I set the growth rate of the tradable TFP $d\widetilde{A}_t^T = \widetilde{A}_t^T - \widetilde{A}_{t-1}^T = 1$.

Figures 1.1 and *1.2* display the simulation results for the benchmark economy with $\eta^* = 1$ in version A and B. The first panels of the figures plot the difference between the growth rates of sectoral productivity factors $d\widetilde{A}_t^T - d\widetilde{A}_t^N$, and the inflation differential $\pi_t^R = \pi_t^N - \pi_t^T$. The latter determines the movements of the internal real exchange rate. If π_t^R is positive, then the internal real exchange rate appreciates. The second panels plot the depreciation of the real exchange rate $d\tilde{q}_t$, and the CPI-based external real exchange rate $d\tilde{q}_t^T$. Positive values of $d\tilde{q}_t$ and $d\tilde{q}_t^T$ mean deprecation. The third panels display $\tilde{y}_t^T - \tilde{k}_t^T$ and $\tilde{y}_t^N - \tilde{k}_t^N$. As equations (1.37) and (1.39) reveal, beyond productivity factors these quantities also influence sectoral inflation rates. Finally, the fourth panels plot the growth rates of the real wage and exports. All growth rates are expressed in annualized terms.

Simulation results reveal that although the internal real exchange rate appreciates, the real exchange rate depreciates since the effect of the depreciating external rate is stronger then that of the internal rate. The reason is that the productivity growth of the tradable sector is higher than those of the non-tradable sector and foreign tradable sectors. As a consequence, the relative price of domestically produced tradables to foreign tradables decreases. That is, the external real exchange rate depreciates. If domestically produced and foreign tradables were perfect substitutes, then the reduced relative price would induce a large instant increase of demand for domestic tradables. Hence, domestic real wages and tradable prices would increase and the prices of domestic and foreign tradables denominated in the same currency would equalize immediately. But in the studied case domestic and foreign tradables are far substitutes, hence increasing demand does not result in equalized prices.

Figures 1.3 and *1.4* plot simulation results belonging to an intermediate value of η^* in both versions. The figures reveal that if domestic and foreign tradables are closer substitutes than in the previous case, then the depreciation of the external real exchange rate becomes more moderate. Moreover, in the initial periods it appreciates. However, in the long run even these moderate levels of depreciation prevent appreciation of the CPI-based real exchange rate. As a consequence, even these values of the international substitution parameter η^* are insufficient to reproduce empirical findings.

Figures 1.5 and *1.6* display the results belonging to a relatively high value of η^*. Since in this case export goods are relatively close substitutes of their foreign rivals their prices cannot deviate much, hence the depreciation of the internal real exchange rate is moderate. As a consequence, the CPI-based real exchange rate appreciates in the long run.

Again, initial appreciation of the external exchange rate can be observed. One may ask whether this phenomenon is induced by movements of the nominal exchange rate. Hence I repeat the same exercises with fixed nominal exchange rates (parameter $\omega = 0$ in equation (1.41)). The results are displayed in *Figures 1.7* and *1.8.* In these cases the external real exchange rate still appreciates initially, although the appreciation is weaker.

Since the initial appreciation of the external real exchange rate cannot simply be explained by the policy rule, it is important to discuss what causes this phenomenon. Due to the lack of a rental market for physical capital, as equations (1.37) and (1.39) reveal, the term $\tilde{y}_t^T - \tilde{k}_t^T$ influences price setting in the tradable sector. Relatively slow adjustment of capital and increasing relative demand for local tradables imply that the quantity $\tilde{y}_t^T - \tilde{k}_t^T$ strongly increases in initial periods, and this suppresses the price-reducing effect of tradable productivity growth. As a consequence, the external real exchange rate appreciates. However, in the long run capital accumulation is sufficient and the productivity effect becomes dominant.

In summary: It was demonstrated that the international substitution parameter η^* had a key role in reproducing empirical facts related to the BS effect. If η^* is low, i.e. domestic and foreign tradables are far substitutes, then the external real exchange rate depreciates too much, and prevents the appreciation of the CPI-based real exchange rate. Hence, relatively high values of parameter η^* are the only possible candidates to generate results consistent with empirical findings.[26] However, in version A, when PTM is not allowed, sufficiently high values of η^* result in insufficient and weak relationship between the nominal and real exchange rates. In version A to generate real appreciation at least $\eta^* = 15$ is necessary, but this parameter value induces small autocorrelation coefficients and relative variance of the real exchange rate (recall *Table 1.3*). Hence, PTM seems necessary to appropriately describe the BS effect in NOEM models.

As was discussed in *section 1.3.1*, in European post-communist countries the observed long-run appreciation of the real exchange rate is only partly caused by dual inflation, the long-run appreciation of the external real exchange rate also lies behind this phenomenon. The presented model is not able to reproduce the long-run appreciation of the external real exchange rate. However, due to the assumed frictions in capital formation and the related decreasing-returns-to-scale features of real marginal cost the model can explain initial appreciation of the external real exchange rate. To explain this phenomenon sufficiently it seems necessary to relax the assumption of fixed structure of goods in the model. As Ito et al. (1997) discussed, the export structure of fast developing countries changes, and higher value-added goods gain importance. If the process of improving quality and increasing variety is not properly captured by the statistical system tradable prices may dramatically rise, as Broda and Weinstein (2004) demonstrated.

One more remark. To simplify the exposition I did not discuss the possibility of PTM with producer currency pricing (PCP), but it is possible to show that in the present framework it provides practically the same results as version B. As a consequence, I would rather not take sides in the LCP vs. PCP debate since both approaches can be consistent with the BS effect.[27] PCP can be applied without the assumption of price discrimination. Moreover, in most cases PCP is applied without PTM, which is equivalent to

[26]One may criticize the choice of the applied substitution parameters which are different from the ones used in other open economy models. For example, Backus et al. (1994) use much lower substitution parameter to replicate the empirically observable responses of the trade balance to productivity shocks. My conjecture is that if the inertia of the exports demand is increased, as in Laxton and Pesenti (2003), or the import requirement of exports production is increased, then my model would also be able to reproduce the short run behavior of the trade balance.

[27]LCP vs. PCP is one of the most important undecided debates in the NOEM literature, since the choice of the optimal exchange rate is not independent of this problem. One can read pro LCP arguments in Engel (2002a, 2002b). Obstfeld (2001, 2002) and Obstfeld and Rogoff (2000) presents arguments supporting the PCP approach. Two recent studies on this topic are Bergin (2004), which provides evidence supporting LCP, and Koren et al. (2004) with findings reinforcing PCP.

applying version A. The reason for this is that the arguments of the supporters of PCP remain valid without PTM. However, my results point out that if one wants to capture the particularities of emerging markets, then the PCP approach cannot be applied without the assumption of international price discrimination.

1.5.2 The adjustment of the sectoral relative price

As discussed in *section 1.3.1* and displayed in *Table 1.1*, according to most of the estimations of Coricelli and Jazbec (2001), Halpern and Wyplosz (2001), Égert (2002) and Égert et al. (2002) in the long-run the magnitude of the relative price of non-tradables to tradables P_t^R is significantly smaller than that of the sectoral productivity differential $\widetilde{A}_t^T - \widetilde{A}_t^N$. In addition, Halpern and Wyplosz found that the short-run adjustment of the relative price was very slow.

It is difficult to explain these fact by models of the traditional approach. Applying classical assumptions to the present model,[28] it is easy to show that the relative price is determined by

$$\widetilde{P}_t^R = \frac{n_N}{n_T}\widetilde{A}_t^T - \widetilde{A}_t^N, \tag{1.42}$$

where n_T and n_N are the labor utilization parameters in the technological equation (1.14). If the tradable productivity process $\widetilde{A}_t^T$ is dominant, then the only way to reproduce the aforementioned empirical long-run relationship is to assume that the tradable sector is more labor intensive than the non-tradable one. But this is counterfactual. Beyond this, the above formula implies instant adjustment of the relative price to the productivity differential.

In this section I show that the presence of nominal and real rigidities helps to explain the above empirical findings, even if $n_N \geq n_T$. For expositional simplicity, I assume that $n_N = n_T$. Combine the sticky price equations (1.37) and real marginal cost formulas (1.39), and for expositional simplicity assume that $\xi_T = \xi_N = \xi$ and $\vartheta_T = \vartheta_N = \vartheta$. Then the inflation differential $\pi_t^R = \pi_t^T - \pi_T^N$ is determined by

$$\begin{aligned}\pi_t^R - \vartheta\pi_{t-1}^R &= \beta \mathrm{E}_t\left[\pi_{t+1}^R - \vartheta\pi_t^R\right] + \frac{\xi}{1-\alpha}\left(\widetilde{A}_t^T - \widetilde{A}_t^N\right) \\ &+ \frac{\xi\alpha}{1-\alpha}\left(\tilde{y}_t^N - \tilde{k}_t^N - \tilde{y}_t^T + \tilde{k}_t^T\right) - \xi\widetilde{P}_t^R.\end{aligned} \tag{1.43}$$

[28] Flexible price setting, internationally homogeneous goods and capital markets.

The terms $\tilde{y}_t^N - \tilde{k}_t^N$ and $\tilde{y}_t^T - \tilde{k}_t^T$ appear in the above equation, since due to imperfections in capital accumulation real marginal cost functions have decreasing-returns-to-scale features. In the constant-returns-to-scale version of the present model only the productivity factors $\widetilde{A}_t^T$, $\widetilde{A}_t^N$ and the relative price $\widetilde{P}_t^R$ would influence the evolution of the inflation differential.

Obviously, the speed of the adjustment of $\widetilde{P}_t^R$ depends on the magnitude of the parameter ξ. The smaller ξ is, the slower the adjustment process. The presence of the terms $\tilde{y}_t^N - \tilde{k}_t^N$ and $\tilde{y}_t^T - \tilde{k}_t^T$ also influences the adjustment process. Suppose $\widetilde{A}_t^T$ increases, then formula (1.43) implies that π_t^R and $\widetilde{P}_t^R$ increase as well. As a consequence, the demand for $\tilde{y}_t^T$ will rise and for $\tilde{y}_t^N$ will decrease. But according to the above formula this change of demand will diminish the rise of π_t^R and $\widetilde{P}_t^R$, hence the adjustment process will be slower.

Relative price adjustment in the presence of sticky prices is definitely slower than in the flexible price models of the traditional approach represented by formula (1.42). However, nominal rigidities without frictions in capital accumulation are not sufficient to reproduce the empirical estimates, as the simulation exercise belonging the upper panel of *Figure 1.9* demonstrates. The figure plots the adjustment process of the relative price to the sectoral productivity differential: it displays the fraction of the relative price to the productivity differential, i.e. $\widetilde{P}_t^R/(\widetilde{A}_t^T - \widetilde{A}_t^N)$. In the simulation exercise I apply the same productivity process as previously, and use version B with $\eta^* = 15$, but I assume that capital accumulation is frictionless, i.e. real marginal cost functions exhibit constant-returns-to-scale features, hence the terms $\tilde{y}_t^T - \tilde{k}_t^T$ and $\tilde{y}_t^N - \tilde{k}_t^N$ are missing from formula (1.43). To compare simulation results with empirical estimates I calculated the OLS regression

$$\widetilde{P}_t^R = \rho\left(\widetilde{A}_t^T - \widetilde{A}_t^N\right) + u_t$$

using the simulated ten-year-long time series. The obtained OLS coefficient ρ represents the empirical 'long-run' estimates of the studied relationships. The magnitude of the OLS coefficient ρ is also displayed on the figure. *Figure 1.9* reveals that although the adjustment of $\widetilde{P}_t^R$ is not instant, ρ is nearly equal to 1. However, with one exception the empirical estimates are significantly smaller than this number.

If there are frictions in capital accumulation, and real marginal cost functions have decreasing-returns-to-scale features, the adjustment process becomes slower, since in the constant-returns-to-scale case the adjustment parameter ξ is greater, and terms $\tilde{y}_t^T - \tilde{k}_t^T$ and $\tilde{y}_t^N - \tilde{k}_t^N$ are not present.[29]

[29] In the constant-returns-to-scale case term $1 + \theta\alpha/(1-\alpha) - \psi_s$ is missing from the denominator of formula (1.38).

The lower panel of *Figure 1.9* illustrates this. In this simulation exercise I used the original form of version B with heterogeneous capital ($\eta^* = 15$). The figure reveals that now the adjustment is slower and ρ becomes smaller. However, the coefficient is 0.967, which is still quite far from the majority of the empirical estimates.

One possible way of reducing the speed of the adjustment process is assuming that price setting is more rigid in the non-tradable sector. However, according to my numerical simulations one would have to assume an unrealistically high price setting parameter γ_N to reproduce empirical results. That is why I choose another possibility. Using equations (1.35) and (1.36) one can express term $\tilde{y}_t^N - \tilde{y}_t^T$ in formula (1.43) as

$$\tilde{y}_t^N - \tilde{y}_t^T = -\eta \widetilde{P}_t^R.$$

The parameter η measures the elasticity of substitution between local tradables and non-tradables. The above expression reveals its importance in the adjustment process of the sectoral relative price. If η is high, i.e. tradables and non-tradables are close substitutes, the adjustment becomes slow, since it is more difficult to deviate the prices of close substitutes. It is important to note that this mechanism does not work if the terms $\tilde{y}_t^T$ and $\tilde{y}_t^N$ are not present in real marginal cost functions, i.e. in the constant-returns-to-scale case. The following simulation exercise demonstrates the importance of this mechanism. I use version B with $\eta^* = 15$, but instead of the benchmark value of $\eta = 1$, I use $\eta = 15$. The upper panel of *Figure 1.10* displays the results. Now $\rho = 0.865$, which approximates the empirical estimates quite well.[30] Exceptions are the findings of Halpern and Wyplosz (2001), but their results are rather different from the estimations of others. It is important to note that the increase of the value of η does not alter the results of the previous section. Both the relationship between the nominal and real exchange rates and the behavior of the external and CPI-based real exchange rates remain the same.

One can further reduce the speed of adjustment if asymmetry of sectoral investments adjustment costs is introduced in the model economy of the previous simulation exercise. Assume that $\varepsilon_N = 3$ as in the benchmark economy, but $\varepsilon_T = \varepsilon_x = 10$. The lower panel of *Figure 1.10* plots the results: ρ becomes 0.782.

In summary: Although both flexible price models and sticky price models with flexible capital accumulation can roughly capture the relationship

[30] Altig et al. (2005) emphasize the magnitude of the adjustment parameter ξ in firm-specific-capital models to reconcile micro and macro evidence. However, in the present model instead of the magnitude of ξ, the presence of output terms in real marginal cost functions is the key factor in slackening the adjustment process.

between sectoral price and productivity differentials, they fail to reproduce the exact empirical magnitudes. Frictions in capital accumulation and the accompanying decreasing-returns-to-scale features of real marginal cost help to explain the observed phenomena. However, to reproduce the estimated regularities one has to assume that tradables and non-tradables are not far substitutes. Asymmetry of sectoral investments adjustment costs can also improve the ability of the model to replicate empirical findings.

1.6 Conclusions

This chapter has reviewed how the models of the new open economy macroeconomics (NOEM) can explain the permanent dual inflation and the accompanying real appreciation often observed in emerging markets.

The coexistence of dual inflation and real appreciation is usually explained by the Balassa - Samuelson (BS) effect, i.e. by the faster productivity growth in the tradable sector. Traditionally, the BS effect is derived from models with flexible prices and internationally homogenous tradable goods markets. On the other hand, NOEM models assume sticky prices and/or wages and heterogeneous goods markets. The traditional approach focuses on the determinants of the internal real exchange rate, while NOEM models emphasize the importance of the external real exchange rate.

It was shown that a NOEM model can simultaneously guarantee the strong correlation of nominal and real exchange rates and generate the BS effect only if there is pricing to market in the model.

The chapter also looks at how the presence of nominal rigidities and frictions in capital accumulation modify the effects of asymmetric productivity growth on dual inflation and the external real exchange rate. The chapter demonstrated that in the presence of the aforementioned nominal and real rigidities sectoral real marginal cost functions have decreasing-returns-to-scale features, which help to explain the appreciation of the external real exchange rate, and the slow adjustment of the relative price of non-tradables to tradables observable in post-communist European countries.

Although it was not studied in this chapter, it is worth mentioning here that decreasing-returns-to-scale features can also explain the role of demand factors in generating dual inflation documented in Arratibel et al. (2002) and López-Salido et al. (2005).

Figure 1.1
Balassa-Samuelson effect
No PTM – version *A*
η*=1

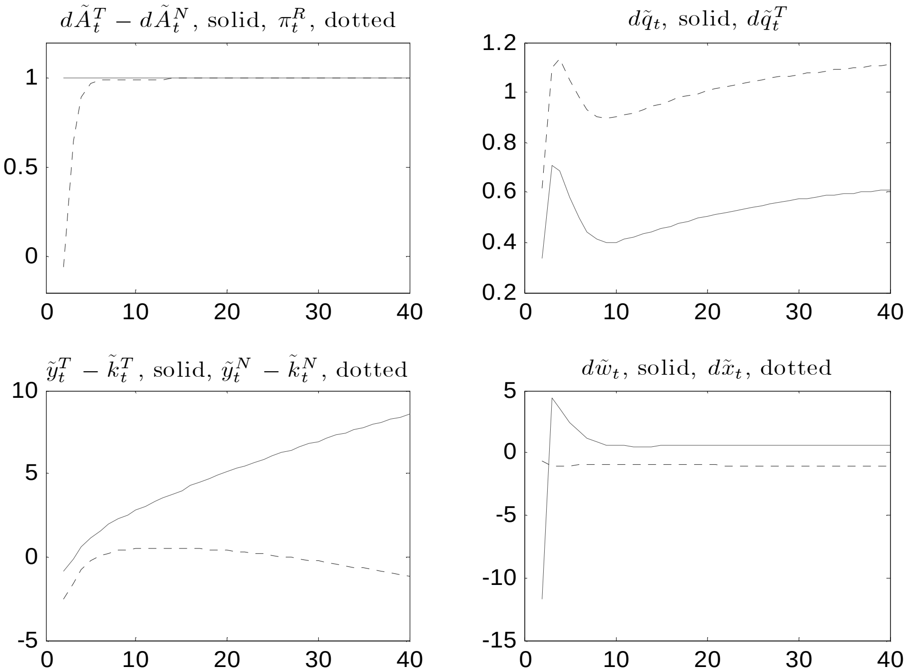
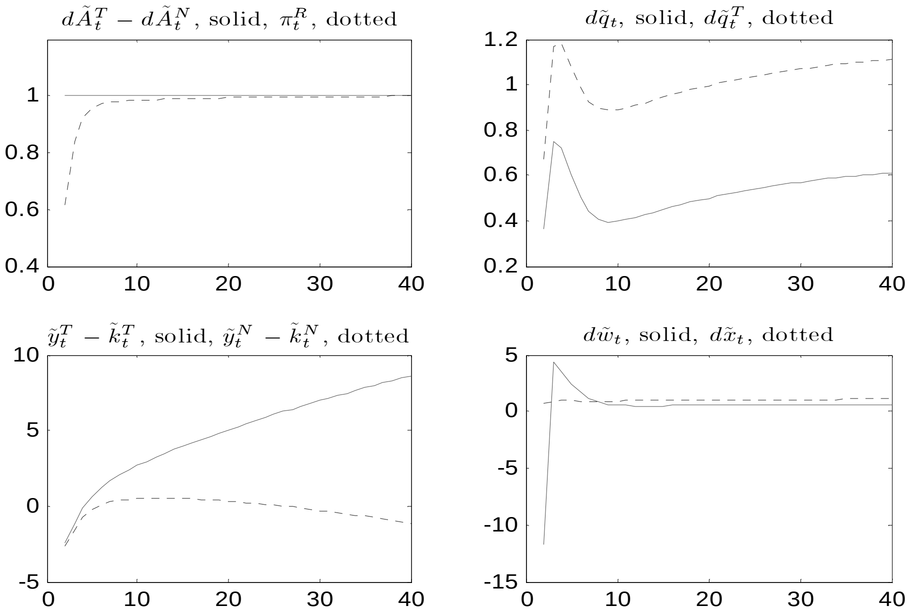


Figure 1.2
Balassa-Samuelson effect
PTM with LCP – version *B*
η*=1



Units on a horizontal axis represent quarters, on a vertical axis percentage points.
Growth rates are displayed in annualized terms.

Figure 1.3
Balassa-Samuelson effect
No PTM – version *A*
η*=5

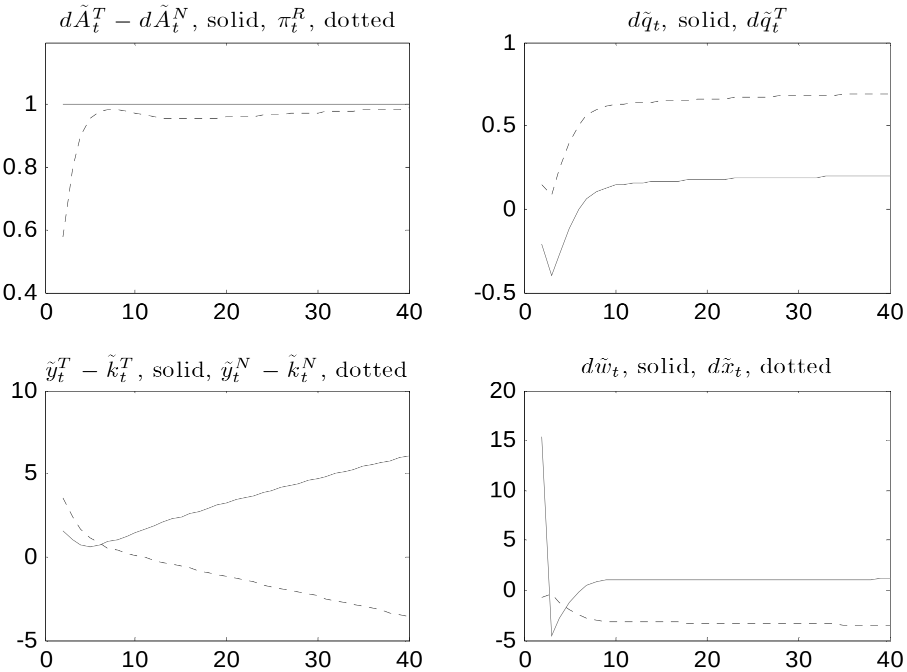
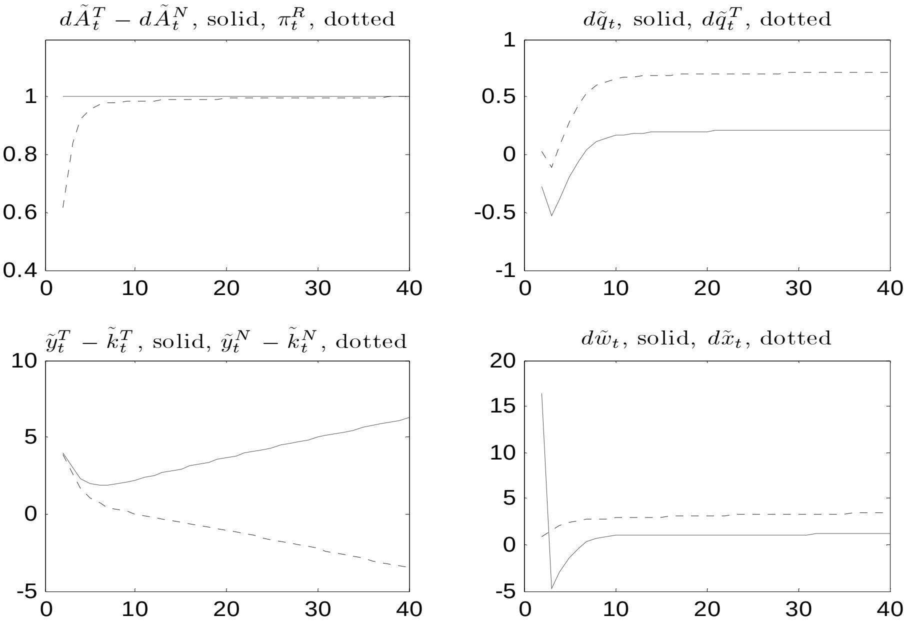


Figure 1.4
Balassa-Samuelson effect
PTM with LCP – version *B*
η*=5



Units on a horizontal axis represent quarters, on a vertical axis percentage points.
Growth rates are displayed in annualized terms.

Figure 1.5
Balassa-Samuelson effect
No PTM – version *A*
η*=15

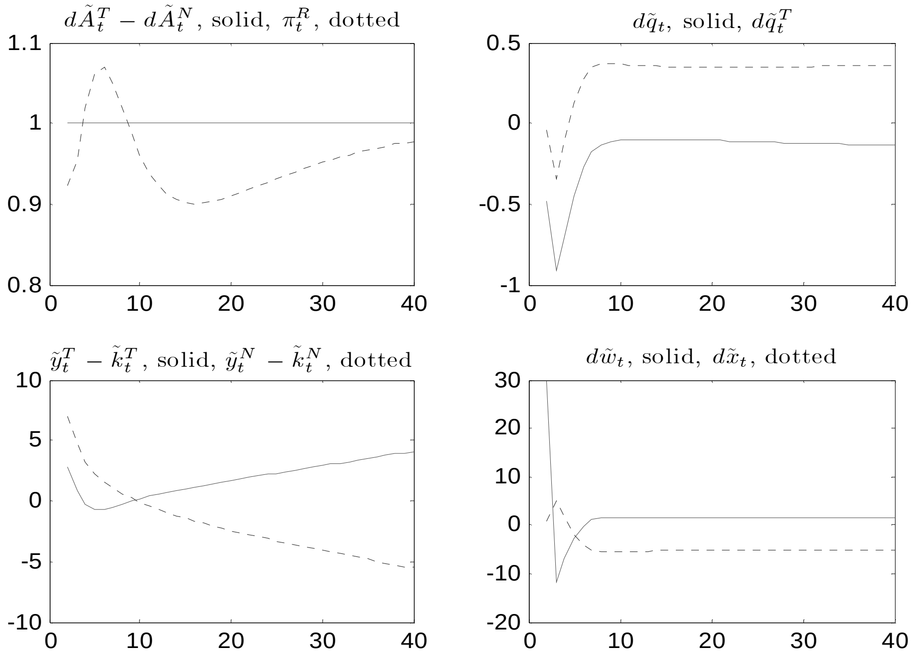
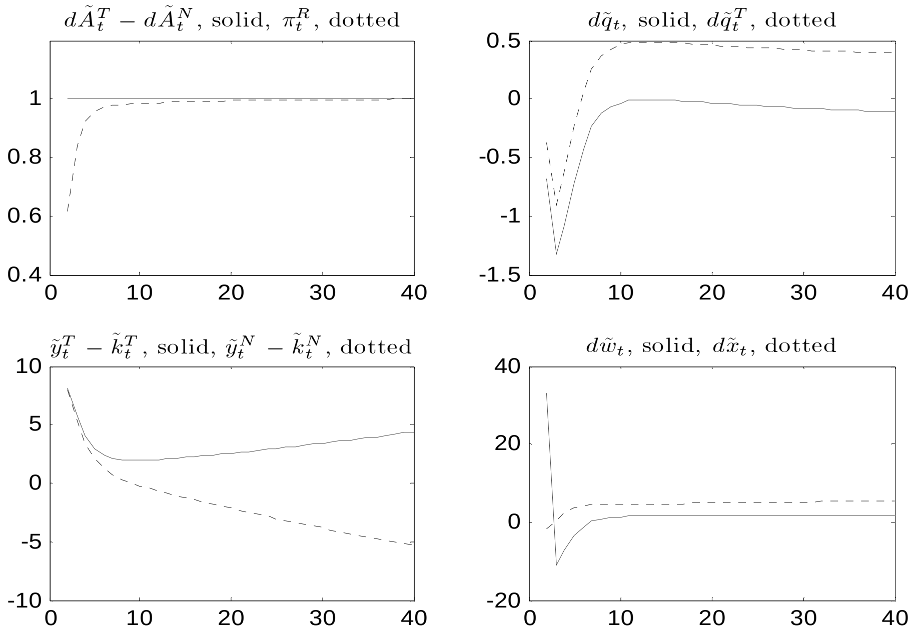


Figure 1.6
Balassa-Samuelson effect
PTM with LCP – version *B*
η*=15



Units on a horizontal axis represent quarters, on a vertical axis percentage points.
Growth rates are displayed in annualized terms.

Figure 1.7
Balassa-Samuelson effect
No PTM – version *A*
η*=15, fixed nominal exchange rate

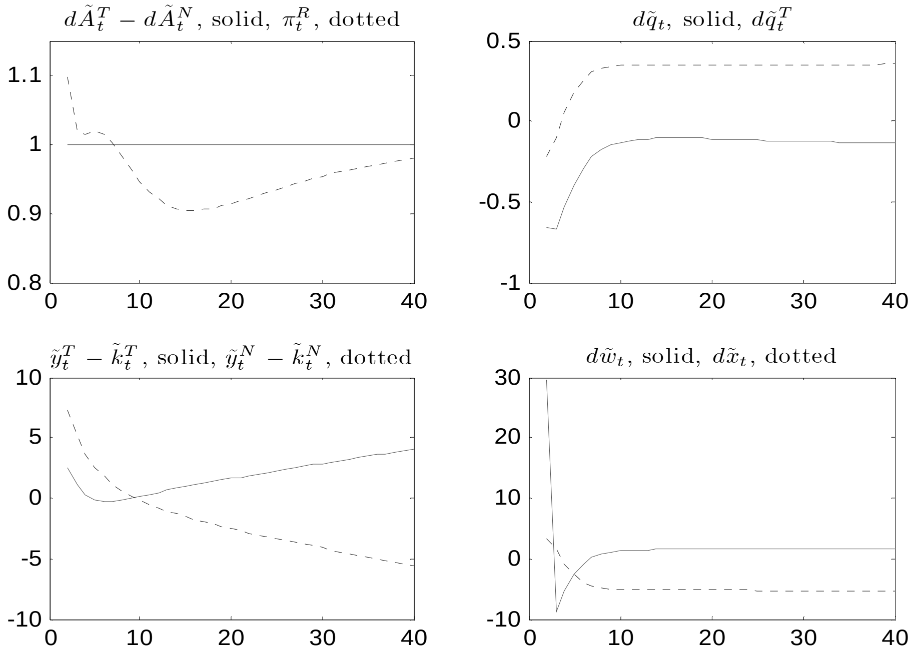
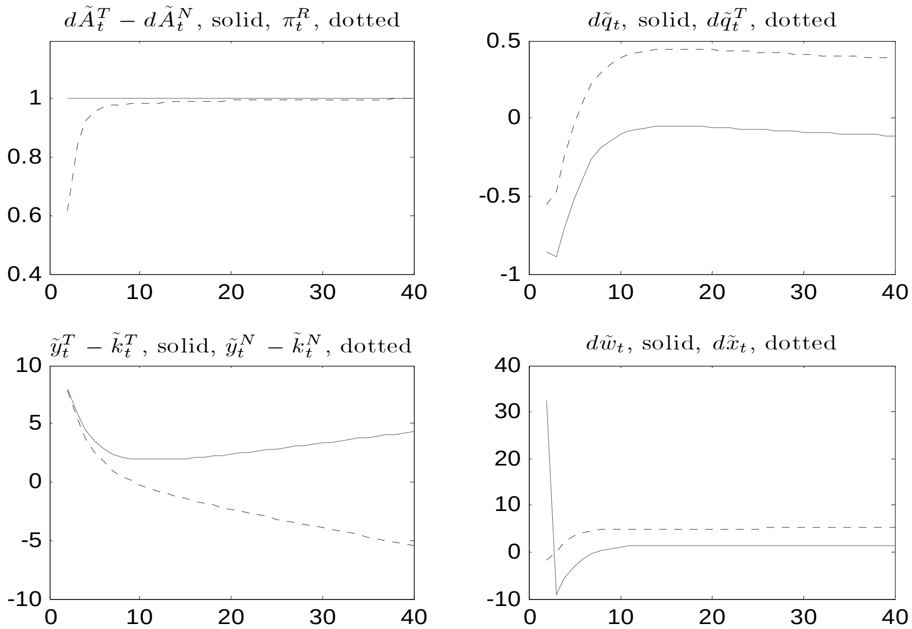
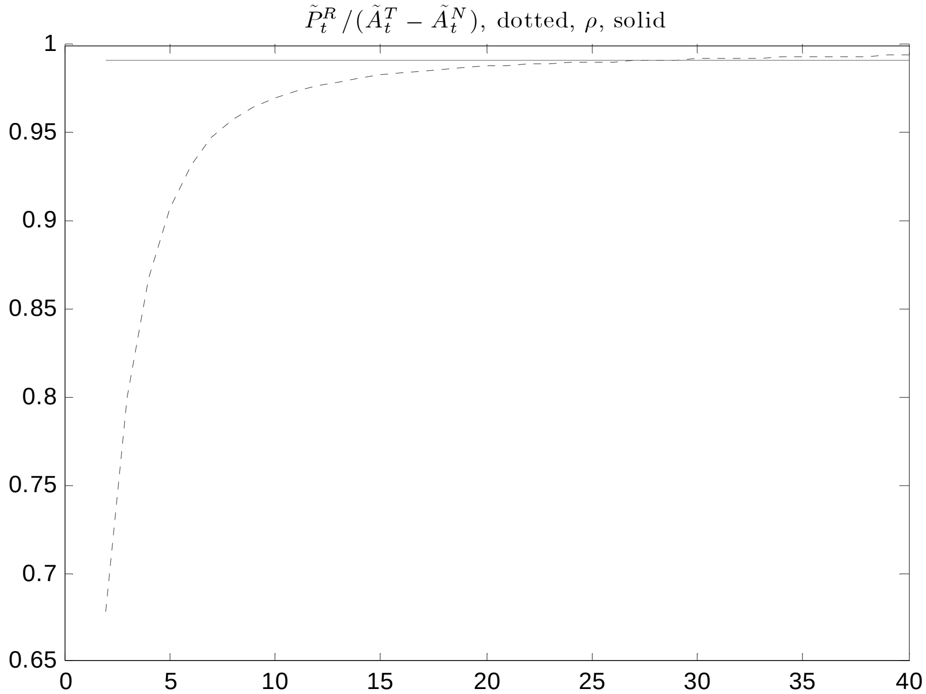


Figure 1.8
Balassa-Samuelson effect
PTM with LCP – version *B*
η*=15, fixed nominal exchange rate

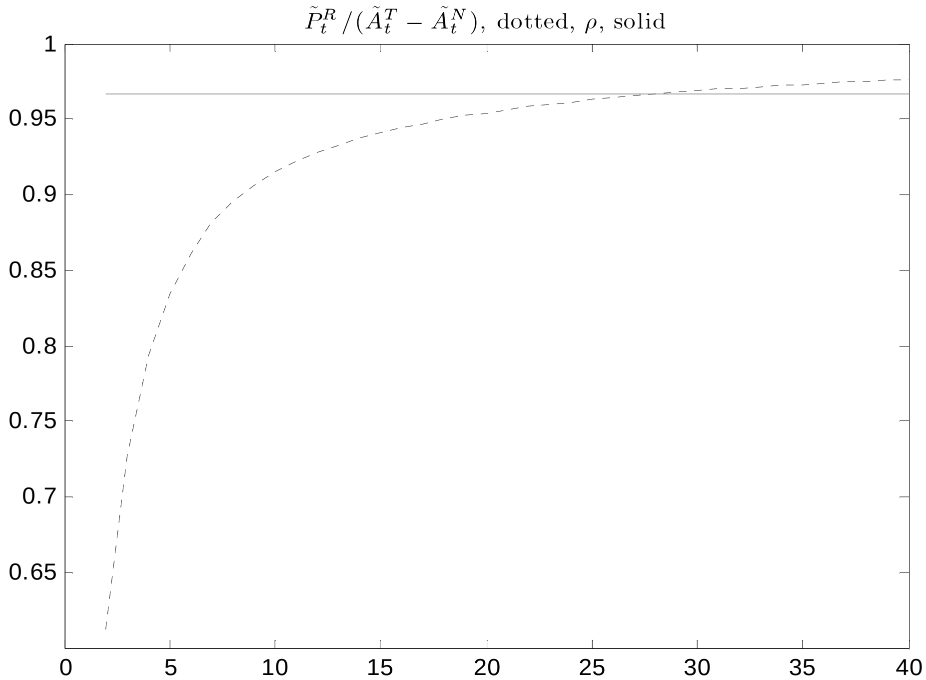


Units on a horizontal axis represent quarters, on a vertical axis percentage points.
Growth rates are displayed in annualized terms.

Figure 1.9
Adjustment of the relative price of non-tradables to tradables P_t^R
Frictionless capital accumulation

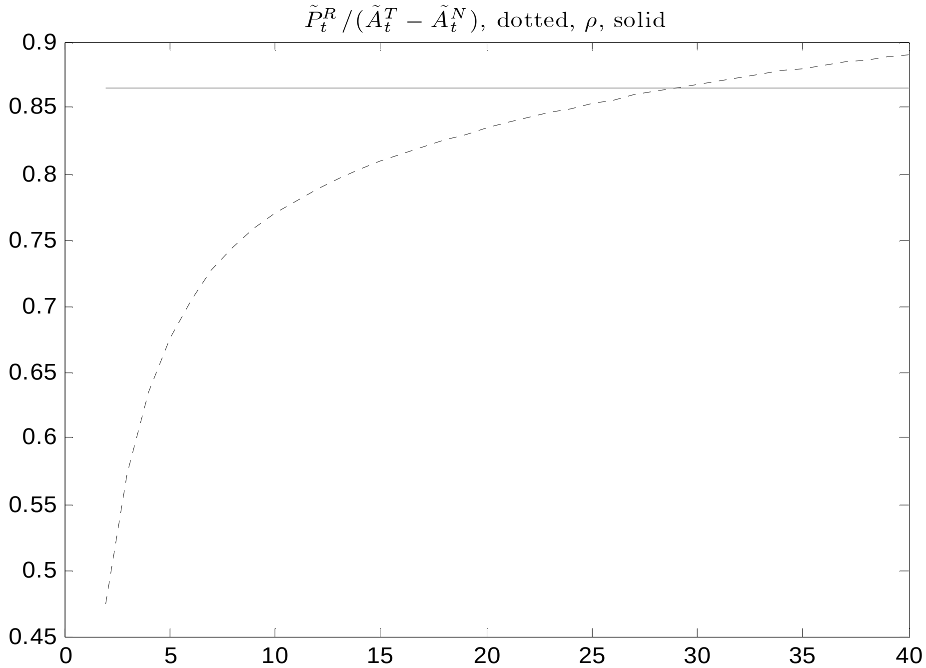


version B, benchmark economy, η*=15

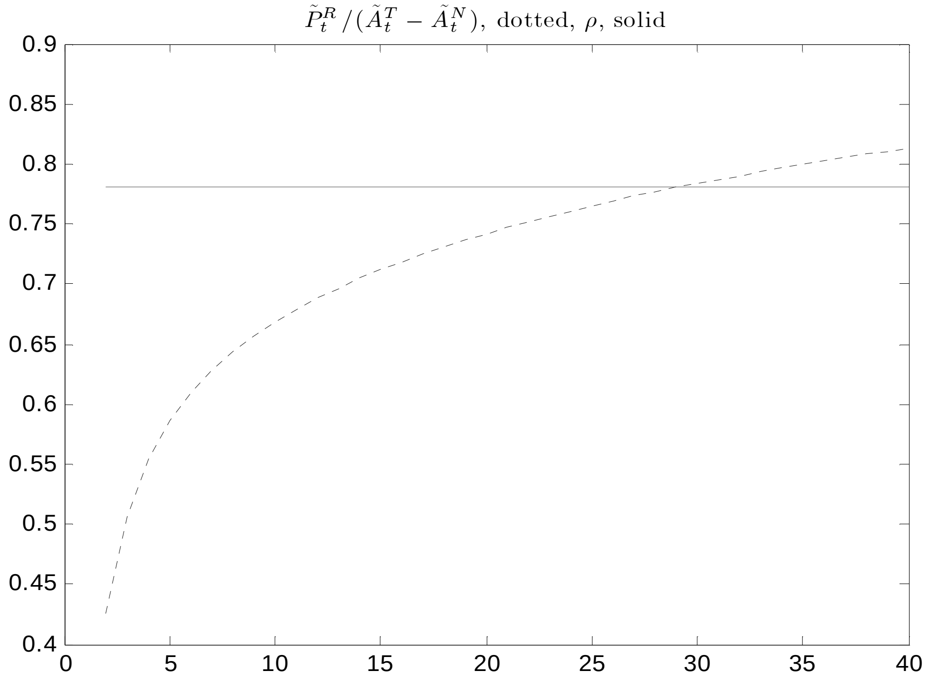


Units on a horizontal axis represent quarters, on a vertical axis percentage points.

Figure 1.10
Adjustment of the relative price of non-tradables to tradables P_t^R
version B, η*=15, η=15



version B, η*=15, η=15, high investments adjustment costs in sector *T* and *x*



Units on a horizontal axis represent quarters, on a vertical axis percentage points.

Chapter 2

The optimal Euro conversion rate

2.1 Introduction

At the beginning of May, 2004, ten countries joined the European Union. It is obligatory for the new member countries to join the Monetary Union, although the deadline for this is not yet specified. Joining the Monetary Union raises several complicated questions of economic policy: policy makers have to decide how and when to meet the Maastricht criteria, they should decide about the date of entering the ERM II exchange rate arrangement, as well as about the corresponding central parity, about the date of joining the Monetary Union, and last but not least, about the Euro conversion rate. This chapter aims to contribute to the solution of this last-mentioned problem.

So far, the academic literature has not paid attention to this problem. The non-academic economic policy literature focuses almost exclusively on one factor, the misalignment of the real exchange rate, namely its deviation from an estimated *equilibrium real exchange rate*.[1] Moreover, it has not provided enough guidelines about how to use the misalignment indices for the determination of the optimal conversion rate. It is not sufficient to base such an important decision on the intuitive wisdom that an overvalued (undervalued) real exchange rate should imply a devaluation (revaluation) of the nominal exchange rate.

This chapter performs a welfare analysis of the problem. The Monetary Union is modelled as an infinitely long, perfectly credible exchange rate peg, and a *new open economy macroeconomics* model is used to provide an algo-

[1] On different equilibrium real exchange rate concepts see the survey of MacDonald (2000) and Driver and Westaway (2003).

rithm to determine how to peg the nominal exchange rate optimally if the accession date values of state and exogenous variables are known.

It is shown that beyond an appropriately defined misalignment index the *past inflation rate* and the level of *real wages* are important state variables worthwhile taking into consideration for the settlement of the conversion rate. Furthermore, the *foreign-business-cycle*, *exports-demand* and *productivity* shocks are the most important exogenous factors necessary for a proper policy decision.

The chapter demonstrates the importance of a *utility-based* approach and that evaluations based on *ad-hoc* welfare criteria may lead to misleading results. It is shown that the persistence of the inflation process implies that the optimal reaction to a positive past inflation rate is the *devaluation* of the nominal exchange rate. This surprising result is the consequence of the form of the exact social welfare function one can derive from the model: what matters is not the inflation rate itself, but its *quasi-difference* if there is inflation *indexation* in the model. Furthermore, it is shown that the optimal solution is sensitive to the *persistency* parameters of the inflation process.

The chapter is structured as follows. *Section 2.2* presents the model. In *section 2.3* the transition dynamics and the impulse responses are analyzed, and the characteristics of the optimal conversion rate are discussed. *Section 2.4* deals with some practical issues. *Section 2.5* presents the conclusions.

2.2 The model

For the study of the determination of the optimal conversion rate one has to take into account two important model building issues. On the one hand, the model should be rich enough to capture important characteristics of real-life economic policy problems. On the other hand, it should be simple enough in order to be suitable for an exact welfare analysis.

Thus, instead of using a highly stylized environment the model is based on a rich theoretical framework. It is a one-sector, fixed-capital variant of version A of the model presented in *chapter 1*. However, it is complemented with sticky wages, and habit formation in consumption.

However, as Woodford (2003, chapter 6) shows, in general equilibrium models some simplifying assumptions are required for the derivation of tractable social welfare functions. Therefore, in this model some restrictions on the relative movements of domestic consumption and exports are imposed, and it is assumed that production inputs, imports and labor, are complements, i.e. firms use a Leontieff technology.

2.2.1 Households

The domestic economy is populated by a continuum of infinitely-lived households. The expected utility function of household j is

$$\sum_{t=1}^{\infty} \beta^{t-1} \mathrm{E}_1 \left[u(H_t(j)) - v(l_t(j)) \right], \tag{2.1}$$

for all $j \in [0,1]$. $H_t(j) = c_t(j) - hc_{t-1}(j)$, where $c_t(j)$, $c_{t-1}(j)$ denote the consumption of household j at date t and $t-1$, parameter $h \in [0,1)$ measures the strength of habit formation, and $l_t(j)$ is the labor supply of household j. Furthermore $u(H) = H^{1-\sigma}/(1-\sigma)$, and $v(l) = l^{1+\varphi}/(1+\varphi)$, σ, $\varphi > 0$, and $0 < \beta < 1$.

The intertemporal budget constraint of a given household can be written in the form

$$P_t c_t(j) + P_t^B B_t(j) = \zeta_t(j) B_{t-1}(j) + W_t(j) l_t(j) + T_t(j), \tag{2.2}$$

where P_t is the consumer price index, $B_t(j)$ is the household's nominal portfolio at the beginning of time t, P_t^B is its price, $\zeta_t(j)$ is its stochastic payoff, $W_t(j)$ is the nominal wage paid to household j and $T_t(j)$ is a lump-sum tax/transfer levied/paid by the government. Households supply differentiated labor, hence the wage paid to individual households can be different. On the other hand, it is assumed that asset markets are complete, and it is possible to eliminate the risk of heterogeneous labor supply and labor income. As a consequence, households have uniform income and consumption, i.e. $c_t(j)$=c_t, and they have the same portfolio, i.e. $B_t(j) = B_t$, for all j and t.

The optimization problem of the households is the following: they maximize the objective function (2.1) subject to budget constraint (2.2), non-negativity constraints on consumption, and no-Ponzi schemes. The assumption of complete asset markets implies that the intertemporal allocation of consumption is determined by the following condition in all states of the world:

$$\beta \frac{\Lambda_{t+1} P_t}{\Lambda_t P_{t+1}} = D_{t,t+1}, \tag{2.3}$$

where Λ_t is the marginal utility of consumption,

$$\Lambda_t = (c_t - hc_{t-1})^{-\sigma} - \beta h \mathrm{E}_t \left[c_{t+1} - hc_t \right]^{-\sigma}, \tag{2.4}$$

and $D_{t,t+1}$ is the stochastic discount factor, which satisfies the condition

$$P_t^B = \mathrm{E}_t \left[D_{t,t+1} \zeta_t \right].$$

Since it is assumed that markets of international assets are also complete, foreign equivalent of equation (2.3) is also held,

$$\beta \frac{\Lambda^*_{t+1} e_t P^{F*}_t}{\Lambda^*_t e_{t+1} P^{F*}_{t+1}} = D_{t,t+1}, \tag{2.5}$$

where P^{F*}_t is the foreign consumer price index in foreign currency terms and e_t is the nominal exchange rate. Furthermore, Λ^*_t is the marginal utility of foreign households,

$$\Lambda^*_t = (c^*_t - hc^*_{t-1})^{-\sigma} - \beta h \mathrm{E}_t \left[c^*_{t+1} - hc^*_t\right]^{-\sigma}, \tag{2.6}$$

where c^*_t denotes foreign consumption, which is driven by an exogenous shock in the present model. Combining equations (2.3) and (2.5), and applying recursive substitutions gives:

$$\frac{\Lambda_t q^d_t P^{F*}_t}{\Lambda^*_t} = \iota, \tag{2.7}$$

where ι is a constant, which depends on initial conditions, and

$$q^d_t = \frac{e_t}{p_t}.$$

Since $q^d_t P^{F*}_t$ is the real exchange rate, q^d_t is called the *domestic component of the real exchange rate.*

There is monopolistic competition in the labor market: As mentioned, labor is differentiated, hence nominal wages can be different, and it is assumed that $W_t(j)$ is set by household j. This implies that the demand for labor supplied by household j is given by

$$l_t(j) = \left(\frac{W_t}{W_t(j)}\right)^{\theta_w} l_t, \tag{2.8}$$

where the aggregate wage index W_t is defined by

$$W_t = \left(\int_0^1 W_t(j)^{1-\theta_w}\, dj\right)^{\frac{1}{1-\theta_w}}.$$

It is assumed that there is sticky wage setting in the model, as in the paper of Erceg et al. (2000). Similarly to Calvo (1983), every individual household at a given date changes its wage in a rational, optimizing forward-looking manner with probability $1 - \gamma_w$. All those households, which do not behave like this at the given date follow a rule of thumb, as in the models of Christiano et al. (2001) and Smets and Wouters (2003), and they update their wages according to the past inflation rate. Each household which sets its price optimally takes into account the above mentioned characteristics of the wage setting process, and the form of the labor demand function represented by equation (2.8). These conditions imply that wage formation is determined by the following equation:

$$\pi_t^w - \vartheta_w \pi_{t-1} = \beta \mathrm{E}_t \left[\pi_{t+1}^w - \vartheta_w \pi_t\right] + \xi_w \left[\widetilde{mrs}_t - \widetilde{w}_t\right] + \tilde{\upsilon}_t^w, \qquad (2.9)$$

where a tilde denotes the percentage deviation of the variables from their steady-state values, and

$$\xi_w = \frac{(1-\gamma_w)(1-\beta\gamma_w)}{\gamma_w(1+\varphi\theta_w)}, \qquad (2.10)$$

where $\pi_t^w = \widetilde{W}_t - \widetilde{W}_{t-1}$ is nominal wage inflation, $\pi_t = \widetilde{P}_t - \widetilde{P}_{t-1}$, is CPI inflation, $\vartheta_w \in [0,1]$ measures the degree of implicit indexation applied by those who follow the rule of thumb, and the average marginal rate of substitution between consumption and labor is defined by

$$mrs_t = l_t^{\varphi} \Lambda_t^{-1}. \qquad (2.11)$$

The interpretation of the exogenous shock $\tilde{\upsilon}_t^w$ is the same as in footnote 14 of the paper of Clarida et al. (1999): it represents some kind of error in wage formation.

2.2.2 Production

Production has a hierarchical structure: at the first stage, import goods and labor are transformed into differentiated intermediate goods, at the second stage, a homogenous final good is produced by the differentiated goods.

Final good y_t is produced in a competitive market by a constant-returns-to-scale technology from a continuum of differentiated intermediate goods $y_t(i)$, $i \in [0,1]$. The technology is represented by the following CES production function:

$$y_t = \left(\int_0^1 y_t(i)^{\frac{\theta-1}{\theta}}\, di\right)^{\frac{\theta}{\theta-1}},$$

where $\theta > 1$. As a consequence, price P_t is given by

$$P_t = \left(\int_0^1 P_t(i)^{1-\theta}\, di\right)^{\frac{1}{1-\theta}},$$

where $P_t(i)$ denotes the prices of differentiated goods $y_t(i)$, and the demand for $y_t(i)$ is determined by

$$y_t(i) = \left(\frac{P_t}{P_t(i)} \right)^{\theta} y_t. \tag{2.12}$$

The continuum of goods $y_t(i)$ is produced in a monopolistically competitive market. Each $y_t(i)$ is made by an individual firm, and they apply the same technology. Firm i uses a decreasing-returns-to-scale technology, [2] which is given by

$$y_t(i) = A_t z_t(i)^{1-\alpha},$$

where $0 < \alpha < 1$, A_t is a uniform exogenous productivity factor of the industry, and $z_t(i)$ denotes the firm i's utilization of composite good z_t,

$$z_t(i) = \min \left[a_l^{-1} l_t(i), a_m^{-1} m_t(i) \right],$$

where $l_t(i)$ is the firm's utilization of composite labor l_t defined as

$$l_t = \left(\int_0^1 l_t(j)^{\frac{\theta_w - 1}{\theta_w}} \, dj \right)^{\frac{\theta_w}{\theta_w - 1}},$$

where $\theta_w > 1$. $m_t(i)$ is the utilization of imported good m_t, and a_l, a_m are given parameters. The price of z_t is determined by

$$W_t^z = a_l W_t + a_m e_t P_t^{m*},$$

where P_t^{m*} is the foreign currency price of the imported good.

The assumptions on the production process imply that $y_t(i)/A_t = z_t^{1-\alpha} = (l_t(i)/a_l)^{1-\alpha} = (m_t(i)/a_m)^{1-\alpha}$. Thus, the demand for labor and import of firm i is determined by

$$l_t(i)^{1-\alpha} = a \frac{y_t(i)}{A_t}, \qquad m_t(i)^{1-\alpha} = (1-a) \frac{y_t(i)}{A_t}, \tag{2.13}$$

where $a = a_l^{1-\alpha}$ and $(1-a) = a_m^{1-\alpha}$.

[2]Decreasing returns to scale is not a common assumption in macroeconomics. But, as is shown in Woodford (2003, chapter 5), provided the technology of the firms exhibits constant returns to scale, physical capital is firm-specific, and the adjustment cost of investment is high, then the presence of sticky prices implies that the firms' behavior becomes similar to the behavior induced by a decreasing-returns-to-scale technology without capital. In addition, it can be assumed that capital is fixed and normalized 1. This can be justified on the basis that at business cycle frequencies capital is uncorrelated with output.

It is assumed that prices are sticky: as in the model of Calvo (1983), each firm at a given date changes its price in a rational, optimizing, forward-looking way with probability $1-\gamma$. Those firms which do not optimize at the given date follow a rule of thumb, as in the models of Christiano et al. (2001) and Smets and Wouters (2003), and update their prices according to the past inflation rate. The optimizing firms take into account the above described characteristics of the price setting process, and the form of the demand function represented by equation (2.12).

Formally, firm i maximizes the expected profit function

$$\sum_{t=T}^{\infty} \mathrm{E}_T \left[\gamma^{t-T} D_{T,t} \left\{ (1-\tau_t) P_T(i) \left(\frac{P_{t-1}}{P_{T-1}} \right)^{\vartheta} - MC_t(i) \right\} \right]$$

with respect to $P_T(i)$ and $y_t(i)$ subject to constraint (2.12), where τ_t is a tax/transfer variable which modifies firms' markup,[3] $\vartheta \in [0,1]$ is the degree of implicit indexation, and $MC_t(i)$ is the marginal cost of firm i. The log-linearized solution of the above problem is given by

$$\pi_t - \vartheta\pi_{t-1} = \beta \mathrm{E}_t \left[\pi_{t+1} - \vartheta\pi_t \right] + \xi \widetilde{mc}_t + \tilde{\upsilon}_t, \tag{2.14}$$

where

$$\xi = \frac{(1-\gamma)(1-\beta\gamma)}{\gamma \left[1 + \theta\alpha(1-\alpha)^{-1} \right]}, \tag{2.15}$$

and mc_t is the average real marginal cost. The exogenous shock $\tilde{\upsilon}_t$ is given by $\tilde{\upsilon}_t = \xi\tau(1-\tau)^{-1} \tilde{\tau}_t + \tilde{\varepsilon}_t$, where τ is the steady state value of $\tilde{\tau}_t$, and the interpretation of $\tilde{\varepsilon}_t$ is the same as that of $\tilde{\upsilon}^w_t$: it represents some kind of error in price formation.

2.2.3 Equilibrium conditions

The equilibrium conditions of the goods and labor market are

$$y_t = c_t + x_t, \tag{2.16}$$

$$l_t = \int_0^1 l_t(i)\, di. \tag{2.17}$$

[3]It is assumed that the government's budget is balanced. The tax/transfer policy represented by τ_t is compensated by the non-distortive T_t lump-sum tax/transfer in the budget constraint (2.2).

2.2.4 The log-linearized model

This section summarizes the log-linearized equations determining trajectories of the endogenous variables for given initial conditions and paths of the exogenous variables. Variables without time indices refer to their steady-state values, and the tilde denotes the log-deviation of a variable from its steady-state value.

Combination of the log-linearized version of equations (2.4), (2.6) and (2.7) provides the following formula for domestic consumption:

$$\begin{aligned}&\left(1+\beta h^2\right)\tilde{c}_t-\beta h \mathrm{E}_t\left[\tilde{c}_{t+1}\right]-h\tilde{c}_{t-1} \\ &=\left(1+\beta h^2\right)\tilde{c}_t^*-\beta h \mathrm{E}_t\left[\tilde{c}_{t+1}^*\right]-h\tilde{c}_{t-1}^*+\frac{(1-h)(1-\beta h)}{\sigma}\left(\tilde{q}_t^d+\widetilde{P}_t^{F*}\right).\end{aligned} \tag{2.18}$$

Foreign behavior is not modelled explicitly, it is just assumed that the following ad hoc formula, similar to the consumption equation, determines the demand for exports:

$$\left(1+\beta h^2\right)\tilde{x}_t-\beta h \mathrm{E}_t\left[\tilde{x}_{t+1}\right]-h\tilde{x}_{t-1}=\eta\tilde{q}_t^d+\tilde{x}_t^*, \tag{2.19}$$

where $0<\eta$, and x_t^* is a shock representing the exogenous component of exports demand.

Let us log-linearize the demand functions, then

$$(1-\alpha)\tilde{l}_t(i)=(1-\alpha)\widetilde{m}_t(i)=\tilde{y}_t(i)-\widetilde{A}_t.$$

This implies that

$$(1-\alpha)\tilde{l}_t=(1-\alpha)\widetilde{m}_t=\tilde{y}_t-\widetilde{A}_t$$

since log-linearization neglects second and higher order approximation error terms.[4] The demand for labor can be derived from the previous expression,

$$(1-\alpha)\tilde{l}_t=a\tilde{c}_t+(1-a)\tilde{x}_t-\widetilde{A}_t, \tag{2.20}$$

where it is used that $c/(c+x)=a$.

Substituting the log-linearized real marginal cost into equation (2.14) yields the price setting equation

$$\begin{aligned}\pi_t-\vartheta\pi_{t-1} &= \beta \mathrm{E}_t\left[\pi_{t+1}-\vartheta\pi_t\right]+\xi\frac{\alpha}{1-\alpha}\left[a\tilde{c}_t+(1-a)\tilde{x}_t\right] \\ &+ \xi\left[a_l\frac{W}{W^z}\widetilde{w}_t+a_m\frac{eP^{m*}}{W^z}\left(\tilde{q}_t^d+\widetilde{P}_t^{m*}\right)-\frac{\widetilde{A}_t}{1-\alpha}\right]+\tilde{\upsilon}_t.\end{aligned} \tag{2.21}$$

[4] If a variable is defined in the following manner: $\mathfrak{z}=\int_0^1\mathfrak{z}(i)\,di$ then its log-linear approximation yields $\tilde{\mathfrak{z}}=\int_0^1\tilde{\mathfrak{z}}(i)\,di+o^2$, where o^2 denotes those second and higher order errors, which were neglected in the approximation process.

If one combines the log-linearized version of equations (2.9) and (2.11), then one obtains the following wage setting equation:

$$\begin{aligned}\pi_t^w - \vartheta_w \pi_{t-1} &= \beta \mathrm{E}_t \left[\pi_{t+1}^w - \vartheta_w \pi_t\right] \\ &+\xi_w \left\{\varphi \tilde{l}_t + \frac{\sigma\left[\left(1+\beta h^2\right)\tilde{c}_t - \beta h \mathrm{E}_t\left[\tilde{c}_{t+1}\right] - h\tilde{c}_{t-1}\right]}{(1-h)(1-\beta h)} - \widetilde{w}_t\right\} + \tilde{\upsilon}_t^w .\end{aligned} \tag{2.22}$$

Finally, two identities close the system:

$$\pi_t^w = \widetilde{w}_t - \widetilde{w}_{t-1} + \pi_t, \tag{2.23}$$

$$\pi_t = \tilde{q}_{t-1}^d - \tilde{q}_t^d + d\tilde{e}_t, \tag{2.24}$$

where $d\tilde{e}_t = \tilde{e}_t - \tilde{e}_{t-1}$, is nominal-exchange-rate devaluation.

The seven-equation system of formulas (2.18) – (2.24) determines the paths of the following seven endogenous variables: $\tilde{q}_t^d$, π_t, π_t^w, $\tilde{c}_t$, $\tilde{x}_t$, $\tilde{l}_t$, and $\widetilde{w}_t$. To be able to consider a wide set of economic policy problems the model contains different type of stochastic shocks, as domestic demand, supply and foreign shocks. They are the foreign-business cycle ($\tilde{c}_t^*$), the exports-demand ($\tilde{x}_t^*$), foreign-CPI ($\widetilde{P}_t^{F*}$), import-price ($\widetilde{P}_t^{m*}$), domestic-price-setting ($\tilde{\upsilon}_t$), nominal-wage ($\tilde{\upsilon}_t^w$) and productivity ($\widetilde{A}_t$) shocks, which are driven by first-order autoregressive processes.

2.2.5 The social welfare function and the consumption gap

An important advantage of utility-based general equilibrium models is that they do not need to rely on an ad-hoc social welfare criteria. It is possible to derive an exact social welfare function from the model itself, and this makes a rigorous welfare analysis possible. However, this remains only a possibility if a model is technically intractable. That is why the finding of Woodford (2003, chapter 6) is important, in that he derived a tractable approximation of the social welfare function in a closed economy framework under relatively mild assumptions.

In a closed economy model it is enough to assume that the steady-state allocation satisfies a certain social welfare criterion. This assumption eliminates first-order terms of the approximation of the social welfare function, which is necessary since the presence of these terms makes optimization results inaccurate.

In small open economy models one needs further assumptions to ensure the above requirement. In the present model I apply parameter restrictions which are related to the behavior of the international trade balance,[5] which combined with Leontieff form of the technology guarantee the elimination of first-order terms.[6]

The above conditions guarantee, as demonstrated in *Appendix B.2* following Woodford (2003, chapter 6), that the second-order approximation of the model-based social welfare function yields a quadratic formula of the form

$$\begin{aligned} & - \sum_{t=1}^{\infty} \beta^{t-1} \mathrm{E}_1 \left[\lambda_c \left(\hat{c}_t - \delta \hat{c}_{t-1} \right)^2 \right] \qquad (2.25) \\ & - \sum_{t=1}^{\infty} \beta^{t-1} \mathrm{E}_1 \left[\lambda_\pi (\pi_t - \vartheta \pi_{t-1})^2 + \lambda_w (\pi_t^w - \vartheta_w \pi_{t-1})^2 \right], \end{aligned}$$

where coefficients λ_c, λ_π, λ_w are functions of parameters of the model, and $0 < \delta \leq h$ is a function of the parameter h.

I refer to the variable $\hat{c}_t$ as the *consumption gap.* The appearance of the lag of the consumption gap in the objective function (2.25) is due to habit formation. Variable $\hat{c}_t$ is defined as the percentage deviation of actual consumption from an appropriately defined *welfare reference level* of consumption, i.e.

$$\hat{c}_t = \tilde{c}_t - \tilde{c}_t^{wr}, \qquad (2.26)$$

where

$$\begin{aligned} & \left[\left(1 + \beta h^2\right) \bar{\sigma} + \frac{\phi}{1-\alpha} + \frac{\alpha}{1-\alpha} \right] \tilde{c}_t^{wr} - \beta h \bar{\sigma} \mathrm{E}_t \left[\tilde{c}_{t+1}^{wr} \right] - h \bar{\sigma} \tilde{c}_{t-1}^{wr} \\ & = \frac{1+\varphi}{1-\alpha} \left[\widetilde{A}_t + (1-a) \left(\tilde{c}_t - \tilde{x}_t \right) \right], \qquad (2.27) \end{aligned}$$

and $\bar{\sigma} = \sigma[(1-h)(1-\beta h)]^{-1}$.

In most models the welfare reference level corresponds to the flexible price and wage version of the model. In this model the welfare reference level is different, since there is an externality, which is the consequence of openness.[7] In the closed economy version of the present model $\tilde{c}^{wr}$ would depend only

[5]To be more specific, it is assumed that the parameters of equations (2.18) and (2.19) satisfy $\eta = (1-h)(1-\beta h)\sigma^{-1}$. This condition ensures that the difference $\tilde{c}_t - \tilde{x}_t$ depends only on stochastic shocks and initial conditions, i.e. it is independent of endogenous variables and economic policy. Others, e.g., Galí and Monacelli (2002) impose different restrictions, however, they also influence the behavior of the trade balance.

[6]It could be an interesting further research topic to relax the above restrictive assumptions, e.g., by the application of the methods described in Benigno and Woodford (2003) or Schmitt-Grohé and Uribe (2004).

[7]Corsetti and Pesenti (2001a) and (2001b) discuss in detail how different actions of domestic economic policy influence foreign welfare.

on the productivity shock $\widetilde{A}_t$. However, to capture the effects of openness the term $(1-a)(\tilde{c}_t - \tilde{x}_t)$ has to be added to the expression defining $\tilde{c}^{wr}$.

Using definitions (2.26) and (2.27) one can obtain the process determining the consumption gap:

$$\left[\left(1+\beta h^2\right)\bar{\sigma} + \frac{\phi}{1-\alpha} + \frac{\alpha}{1-\alpha}\right]\hat{c}_t - \beta h\bar{\sigma}\mathrm{E}_t\left[\hat{c}_{t+1}\right] - h\bar{\sigma}\hat{c}_{t-1} \quad (2.28)$$
$$= \left[\left(1+\beta h^2\right)\bar{\sigma} + \frac{\phi}{1-\alpha} + \frac{\alpha}{1-\alpha}\right]\tilde{c}_t - \beta h\bar{\sigma}\mathrm{E}_t\left[\tilde{c}_{t+1}\right] - h\bar{\sigma}\tilde{c}_{t-1}$$
$$-\frac{1+\varphi}{1-\alpha}\left[\widetilde{A}_t + (1-a)\left(\tilde{c}_t - \tilde{x}_t\right)\right].$$

In usual ad-hoc objective functions of monetary policy there is a CPI inflation rate term. However, the objective function (2.25) contains the quasi-difference of CPI inflation $\pi_t - \vartheta\pi_{t-1}$, and the difference of wage and lagged CPI inflation $\pi_t^w - \vartheta_w\pi_{t-1}$. Since expression (2.25) is derived from the model, the above terms represent certain welfare-decreasing distortions of the model. It is assumed that price and wage setting is asynchronized, hence CPI and wage inflation result in not only a change of the aggregate price and wage indices, but inefficient relative price and wage movements as well. Since the magnitude of relative price and wage distortions depend on the size of implicit indexation, the terms $\vartheta\pi_{t-1}$ and $\vartheta_w\pi_{t-1}^w$ have to appear in the objective function.

2.2.6 Model solution and parameterization

Uhlig's (1999) implementation of the *undetermined coefficients* method is used to derive the resolution of the log-linear model. The numerical results are generated by the aforementioned author's MATLAB algorithm.

The values of the basic parameters in the benchmark economy are given in *Table 2.1. Section 1.4.7* of *chapter 1* argued about the calibration of the parameters β, γ, ϑ. The values of α and θ are common in the literature. Parameters σ and h are taken from Christiano et al. (2001). The values of φ, θ_w, γ_w and ϑ_w are Euro area estimates, taken from the paper of Smets and Wouters (2003). In order to get an appropriate approximation of the social welfare function, it is necessary to ensure that difference of consumption and exports, i.e. $\tilde{c}_t - \tilde{x}_t$ depends only on stochastic shocks and initial conditions. This requirement is fulfilled if the parameters of equations (2.18) and (2.19) satisfy the condition $\eta = (1-h)(1-\beta h)/\sigma$: details are presented in *Appendix B.2.*

The above parameter values are used in the benchmark simulations. However, an alternative version is considered, where the persistency parameters of the CPI and wage inflation processes are much higher, i.e. $\vartheta = \vartheta_w = 0.9$. This sensitivity analysis is motivated by the study of Hornok and Jakab (2003). They used Hungarian, relatively short, data set and found much more persistent inflation than Galí and Gertler (2000), Galí et al. (2001), and Smets and Wouters (2003) estimated using US and European data. It is not clear whether this discrepancy is due to the confused expectations induced by the 2001 exchange-rate regime switch in Hungary, or to more rigid price setting behavior. If the first hypothesis is true, then some near-rational expectation models should explain the phenomenon, e.g., adaptive learning models, see Evans and Honkapohja (2001). But in this case, in the long run the price setting behavior would converge to the practice of developed countries, and low persistency parameters would appropriately describe the inflation process. On the other hand, if the price setting practice in Hungary is significantly different from the practice of developed countries, then inflation would remain highly persistent even in the long run. To select the right explanation needs further research and more data.

Table 2.1
Parameter values of the benchmark economy

Parameter	
Name	Value
β	0.984
σ	1.000
h	0.630
φ	0.755
η	0.141
α	0.333
θ	6.000
θ_w	3.000
γ	0.817
ϑ	0.365
γ_w	0.763
ϑ_w	0.656

2.3 The optimal conversion rate

As mentioned in the *Introduction*, the new member states do not have the option of remaining outside the Monetary Union. Hence, taking this fact as given, in this section I focus on the optimal determination of the Euro conversion rate, and I do not consider whether in new member countries flexible-exchange-rate regimes are superior to joining the Monetary Union.[8]

In this chapter a currency union is modelled as an infinitely long, perfectly credible exchange rate peg, and there is only one policy variable: the monetary authority decides how to peg the nominal exchange rate at the accession date ($t = 1$). Formally, this means that the depreciation rate $d\tilde{e}_1$ is a decision variable, but $d\tilde{e}_t = 0$, for all $t \geq 2$.

To determine the optimal conversion rate is not a trivial problem. It is well known that in some closed and small-open-economy models it is possible to simultaneously stabilize the relevant welfare measures.[9] However, as it is demonstrated in *Appendix B.3*, in the present model policy makers always face a trade-off: it is impossible to jointly stabilize the consumption-gap and inflation terms in the social welfare function (2.25).

The optimal conversion rate is the result of the following optimization problem: policy makers have to choose $d\tilde{e}_1$ in order to maximize the objective function (2.25), subject to equations (2.18)–(2.24) and (2.28), which determine trajectories of endogenous variables, and given

$$Y_0 = \left[\tilde{c}_0,\, \hat{c}_0\; \pi_0,\, \widetilde{w}_0,\, \tilde{x}_0,\, \tilde{q}_0^d\right]',\; S_1 = [d\tilde{e}_1,\, \tilde{c}_1^*,\, \epsilon_1^{c*},\, \tilde{x}_1^*,\, \widetilde{P}_1^{F*},\, \widetilde{P}_1^{m*},\, \widetilde{A}_1,\, \tilde{v}_1,\, \tilde{v}_1^w]',$$

i.e. the vectors of the state and exogenous variables, respectively.[10]

To solve the above optimization problem one has to maximize a quadratic objective function subject to linear constraints. The solution is given by the linear equation

$$\sum_{j=1}^{6} \mathcal{K}_j^Y Y_0(j) + \sum_{s=2}^{9} \mathcal{K}_s^S S_1(s) + \mathcal{K}_1^S d\tilde{e}_1 = 0, \qquad (2.29)$$

where the formulas defining coefficients $\mathcal{K}_j^Y$ and $\mathcal{K}_s^S$ are derived in *Appendix B.4*.

[8] The majority of the NOEM literature assert that flexible-exchange-rate regimes are optimal, see Obstfeld (2001, 2002), since an appropriate exchange rate policy can facilitate the necessary adjustments of real economic variables.

However, monetary authorities may not perfectly control the nominal exchange rate. As Lyons (2001) discusses, exchange rates are excessively volatile relative to the best measures of fundamentals. This issue is an especially important problem in emerging market countries as documented by Calvo and Reinhart (2002). Thus, perfect exchange rate stability supported institutionally by the Monetary Union can improve social welfare in new accession countries.

[9] See, e.g., Goodfriend and King (1997) and Galí and Monacelli (2002).

[10] For technical reasons $\tilde{c}_0$ is replaced by ϵ_1^{c*} which is the corresponding stochastic innovation variable, see *Appendix B.4*.

Equation (2.29) implies that if, keeping everything else fixed, the jth state variable is above its steady-state value by 1 per cent at date $t = 0$, then decision makers should devaluate the nominal exchange rate by $-\mathcal{K}_j^Y/\mathcal{K}_1^S$ per cent in order to settle the optimal conversion rate. Similarly if, keeping everything else fixed, the sth shock variable is above its steady-state value by 1 per cent at date $t = 1$, then a $-\mathcal{K}_s^S/\mathcal{K}_1^S$ per cent devaluation is the optimal response.

To understand better the effect of the policy variable $d\tilde{e}_1$ see *Figure 2.1*, which displays the impulse responses belonging to a 1-percent devaluation at date $t = 1$. In this simulation exercise all initial state variables and exogenous shocks are kept at their steady-state values. As in all subsequent figures, two set of simulations are performed, one for low persistency of inflation ($\vartheta = \vartheta_w = 0.365$), and one for high persistency ($\vartheta = \vartheta_w = 0.9$). The interpretation of one time period in the model is one quarter. The upper four panels of the figure belong to the low persistency case, the lower panels to the high persistency version. The figure displays the price and wage inflation rate in annualized terms.

Due to nominal rigidities, prices react slowly to a nominal devaluation, hence it results in real depreciation. As equations (2.18), (2.19) and (2.28) reveal, real depreciation implies that consumption ($\tilde{c}_t$), exports ($\tilde{x}_t$) and the consumption gap ($\hat{c}_t$) rise . They return to their steady-state values in nearly five years. Since production increases, labor utilization ($\tilde{l}_t$) and the real wage level ($\widetilde{w}_t$) also rise, see equation (2.20), although due to sticky wage formation, the latter changes moderately. Furthermore, as equations (2.21) and (2.22) show, the increase of economic activity implies that CPI inflation (π_t) and wage inflation (π_t^w) rise as well.

In summary: an initial devaluation (revaluation) increases (decreases) the consumption gap, but at the same time increases (decreases) price and wage inflation as well.

2.3.1 Effects of the state variables

Table 2.2 displays the optimal-policy multipliers corresponding to the state variables ($-\mathcal{K}_j^Y/\mathcal{K}_1^S$). The first row of the table contains the results of the model version with low inflation persistency, while the second row belongs to the version with high persistency. The multipliers of the inflation rate are expressed in annualized terms. A positive number in the table refers to a required devaluation.[11]

[11] Variable $\hat{c}_t$ is absent from the table. Although, formally it is a state variable it does not affect any other variables, since it appears only in equation (2.28). Thus, the corresponding

Table 2.2
Optimal multipliers of the state variables

Degree of inflation persistence	State variables				
	$\tilde{q}_0^d$	π_0	$\tilde{c}_0$	$\tilde{x}_0$	$\widetilde{w}_0$
low	-1	0.159	-0.034	-0.035	0.176
high	-1	0.804	-0.063	-0.074	0.303

Note: A positive entry refers to a required devaluation as an optimal policy response.

The value of the multiplier belonging to the domestic component of the real exchange rate implies that if $\tilde{q}_t^d$, which represents practical misalignment indices in the present model, is appreciated by 1 per cent at date $t = 0$, then the decision makers' optimal response is a 1 per cent devaluation of the nominal exchange rate at date $t = 1$. It is easy to understand this result. If one investigates the system of equations (2.18) – (2.23), and (2.28), then it becomes apparent that neither $\tilde{q}_{t-1}^d$ nor $d\tilde{e}_t$ appear in it. On the other hand, in identity (2.24) both appear, and their coefficients are the same in absolute value. This implies that an appropriate initial nominal-exchange-rate movement can perfectly neutralize the effects of real exchange rate misalignment, and the objective function (2.25) attains its maximum.

To understand the effects of past inflation, see *Figure 2.2*, which displays the transition dynamics induced by 1 per cent annual inflation rate at date 0. Since there is inflation inertia in the model, see equation (2.21), a positive inflation rate at date $t = 0$ implies that the inflation rate (π_t) will also be positive at subsequent dates. Due to the fixed nominal exchange rate, this results in real appreciation, i.e. the decrease of $\tilde{q}_t^d$. Hence, equations (2.18) and (2.19) imply that consumption ($\tilde{c}_t$) and exports ($\tilde{x}_t$) will decline. Furthermore, as equation (2.28) reveals, the consumption gap ($\hat{c}_t$) will decline as well. As a consequence, labor demand ($\tilde{l}_t$), see equation (2.20), and the real wage level ($\widetilde{w}_t$) will also decrease. The problem of the policy makers is the following: to increase the consumption gap needs a devaluation, to reduce inflation needs a revaluation.

According to numerical optimization, policy makers have to respond with a devaluation, see the positive coefficients in the second column of *Table 2.2*. The reason for this is that in the social welfare function (2.25) π_t and π_t^w do not appear themselves, rather their divergence from the past inflation rate. Thus, a too quick disinflation decreases welfare, just like a further increase of inflation. Hence, the optimal solution needs moderate disinflation.[12] It

policy multiplier is equal to zero.

[12] This argument, of course, does not mean that in general disinflation with nominal revaluation is necessarily a faulty policy. For example, it is possible that a given country

is interesting to note that if one eliminates terms $-\vartheta\pi_t$ and $-\vartheta_w\pi_t^w$ from the objective function (2.25), then a revaluation would be the optimal policy according to numerical simulations.

Figure 2.3 displays the transition dynamics belonging to an initial positive deviation of consumption. Due to habit formation, there is consumption inertia in the model, see equation (2.18), hence $\tilde{c}_t$ gradually converges to its steady-state value. On the other hand, exports hardly perform any reaction. As a consequence, the consumption gap $\hat{c}_t$ only moderately increases, see equation (2.28). Due to negligible reaction of exports, the rise of production is weak, hence the price and wage inflation mildly increase. As a consequence, if consumption is above its steady-state value by 1 per cent, then the optimal reaction is a small revaluation.

The effects of a 1 per cent initial deviation of exports are similar to that of consumption, hence its graphical analysis is skipped. The main difference is that now exports inertia, see equation (2.19), results in a rise of exports and a moderate increase of the consumption gap. The optimal policy action is again a small revaluation.

Figure 2.4 plots the transition dynamics belonging to 1 per cent initial deviation of the real wage level. As equations (2.21), (2.22) and (2.23) reveal, this results in a small increase of price inflation and a significant decrease of wage inflation. Since the nominal exchange rate is fixed, the increase of inflation accompanies real appreciation. Hence, equations (2.18), (2.19) and (2.28) imply that consumption, exports and the consumption gap will decrease. As a consequence, the decision makers should devaluate.

Numerical values of the optimal multipliers change significantly if one modifies the persistency parameters. It comes as no surprise that the multiplier of the inflation rate changes the most, in the high persistency version it becomes nearly four times bigger than in the low persistency case. But multipliers of other variables are nearly doubled as well.

2.3.2 Effects of the exogenous shocks

Now the effects of exogenous stochastic shocks will be investigated. Using the estimation results of Ireland (2004) it is assumed that the autoregressive parameters of the shocks are equal to 0.95.

Let us study the optimal-policy multipliers corresponding to the exogenous shocks ($-\mathcal{K}_s^S/\mathcal{K}_1^S$). The results are summarized in *Table 2.3*. Shocks $\tilde{v}_t$ and $\tilde{v}_t^w$ are normalized in such a way that they induce 1 per cent extra price

can only meet the deadline of the Maastricht criteria of the Monetary Union if it performs such a radical disinflation that can be reconciled only with exchange rate revaluation.

or wage inflation in the baseline version of the model.

Table 2.3
Optimal multipliers of the exogenous shocks

Degree of inflation persistence	Exogenous shocks							
	$\tilde{c}_1^*$	$\tilde{c}_0^*$	$\tilde{x}_1^*$	$\widetilde{P}_1^{F*}$	$\widetilde{P}_1^{m*}$	$\tilde{v}_1$	$\tilde{v}_1^w$	$\widetilde{A}_1$
low	-0.596	-0.448	-1.255	-0.479	-0.177	-0.379	-0.418	0.634
high	-0.943	-0.632	-1.646	-0.675	-0.231	0.480	-0.595	0.724

Note: A positive entry refers to a required devaluation as an optimal policy response.

Let us start with the foreign-business-cycle shock $\tilde{c}_t^*$. *Figure 2.5* plots the corresponding impulse responses. The shock has a significant positive impact on consumption, see equation (2.18), and labor demand, see equation (2.20). As a consequence, equations (2.21) and (2.22) imply that CPI inflation and wage inflation rise. Hence, $\tilde{q}_t^d$ appreciates, and this implies that exports will decline, see equation (2.19). Since the consumption gap and the inflation rates increase the optimal response of the policy is a revaluation, as *Table 2.3* displays.

The effects of the exports-demand shock $\tilde{x}_t^*$ and the foreign-CPI shock $\widetilde{P}_t^{F*}$ are similar. Although, the increase in wage inflation is weaker in both cases, and the increase in labor utilization is stronger when $\tilde{x}_t^*$ is considered. Hence, negative optimal-policy multipliers belong to these shocks.

The import-price shock $\widetilde{P}_t^{m*}$ has only a negligible impact on the variables. A positive import-price shock implies that inflation rates rise and the consumption gap declines. Hence, only a numerical simulation can reveal that the optimal policy is a mild initial revaluation.

Figure 2.6 displays the impacts of the price-setting shock $\tilde{v}_t$ in equation (2.21). The size of the shock is set at 0.08 per cent, hence it yields just 1 percentage point extra inflation at date $t = 1$, if the persistence is low. Its positive impact on inflation implies that $\tilde{q}_t^d$ appreciates. As a consequence, equations (2.18), (2.19) and (2.28) imply that consumption, exports and the consumption gap decline. As equation (2.20) reveals, labor demand also declines. Hence, the real wage level decreases as well. Furthermore, equation (2.22) implies that the wage inflation rate will be negative. The change of the inflation persistence parameters modifies the trajectories: the strong undershooting of the inflation rate is especially interesting in the high persistency case.

The consumption gap and price inflation increases, wage inflation decreases. As a consequence, it is again impossible to guess the optimal policy in an intuitive way. According to numerical optimization, as *Table 2.3* displays, the optimal policy is an initial revaluation in the low persistency case, and a devaluation in the high persistency case. The latter result is related to the undershooting of inflation in the high persistency case. A devaluation smoothes better the volatile inflation rate. Furthermore, as was mentioned, this phenomenon is related to the fact that the social welfare function (2.25) contains terms $\pi_t - \vartheta\pi_{t-1}$ and $\pi_t^w - \vartheta_w\pi_{t-1}$, instead of π_t and π_t^w themselves.

Figure 2.7 plots the impulse responses related to the nominal-wage shock $\tilde{v}_t^w$ in equation (2.22). The size of the shock is set in such a way that it induces 1 percentage point extra wage inflation at date $t = 1$, if the persistence is low. The reactions induced by $\tilde{v}_t^w$ are similar to that of $\tilde{v}_t$. However, it induces a greater increase of wage inflation, and a very small rise of inflation. Furthermore, the real wage level increases, and the decline of consumption, exports, and labor utilization are smaller. A positive $\tilde{v}_t^w$ shock requires a revaluation, since the induced rate of wage inflation is much higher than that of CPI inflation.

The impulse responses generated by the productivity shock $\widetilde{A}_t$ can be found in *Figure 2.8.* A rise of productivity influences negatively inflation, see equation (2.21), this results in depreciation of the real exchange rate, since the nominal exchange rate is fixed. Thus, equation (2.18) implies that consumption rises. But this boom is relatively small compared to the size of productivity growth, hence the net effect of these two factors on labor demand is negative, see equation (2.20). Equation (2.27) implies that increasing productivity coincides with increasing $\tilde{c}_t^{wr}$. Hence, although consumption increases, the consumption gap $\hat{c}_t$ declines, since by definition $\hat{c}_t = \tilde{c}_t - \tilde{c}_t^{wr}$.[13] As a consequence, a positive productivity shock $\widetilde{A}_t$ requires significant devaluation, since it reduces the consumption gap and price and wage inflation.

Note that again most of the result are quite sensitive to the persistency parameter of the inflation process.

2.3.3 Summary

The domestic component of the real exchange rate has a key, but not exclusive role in determining the optimal conversion rate. Its role is prominent, since its optimal multiplier is stable, independent of the parameter values of the model, and the value of its multiplier is significantly higher in absolute value than that of other state variables.

[13] As Galí (2002) discussed, it is a general feature of New Keynesian models that a rise of productivity results in decreasing labor demand. Moreover, this can be supported by recent empirical studies, although these findings sharply contradict the predictions of *real business cycle* (RBC) literature.

On the other hand, a decision based exclusively on the initial real exchange rate would be suboptimal, since an undervalued or overvalued real exchange rate always coincides with deviations of other variables, and some of these variables have significant multipliers. The past inflation rate and the past real wage level are the most important state variables, while the shock of the foreign business cycle, exports demand and productivity are the most important exogenous factors.

It is important to note that the optimal-policy multipliers of most variables change significantly if persistency parameters are modified. Thus, for the right decision in a certain economy it would be necessary to thoroughly know the empirical characteristics of price and wage formation processes.

2.4 Practical issues

This section briefly reviews how one should apply the theoretical results of this chapter in practice.

To start with, I clarify the interpretation of the steady state of the present model. In this model, as is usual in business cycle literature, the long-run paths of the variables are filtered out. The object of the analysis is the cyclical behavior of the variables around the long-run trajectories, and not the long-run behavior. Hence, the steady state of the model represents these long-run paths, which are not modelled explicitly.

It is assumed that the long-run growth of the real variables in the model are determined by two factors: long-run technological progress and the convergence of per capita real income to the growth path of developed countries, i.e. the transitional dynamics. These processes can be properly captured by neoclassical open economy models of growth – see, e.g., Barro et al. (1992) – and long-run evolution of nominal variables are determined by long-run money supply.

It is demonstrated that one of the key determinants of the optimal conversion rate is a certain misalignment index, namely, the deviation of the domestic component of the real exchange rate from its steady state value, or in other words, its deviation from its long-run path. That is why it is important to identify empirically the long-run trajectory of this variable. The literature dealing with practical equilibrium real exchange rate estimations give some guidelines. Although the notions of that literature are not perfectly compatible with the categories of general equilibrium neoclassical models of growth, one can try to reconcile them.

In their survey of equilibrium real exchange rate concepts Driver and Westaway (2003) define the long-run equilibrium as the point when net wealth is in full stock-flow equilibrium, so that changes to asset stocks are zero. In a neoclassical model of growth this happens when the transition process is over, i.e. per capita real incomes are equalized. Since the steady state of my model does not represent this state of the economy, it rather corresponds to the medium-run equilibrium concepts of practical misalignment calculations.

The key nominal endogenous variable of this model is the inflation rate. In the model its steady state value is zero. But in reality the long-run inflation rate is usually positive: in the EU it is around two percent. That is, it is useful to add two percentage points to the inflation numbers of this chapter.

One may criticize my result related to the inflation rate as irrelevant, since the Maastricht criteria require that the inflation rate of accession countries cannot be significantly higher than their long-run values. But this chapter demonstrates that if inflation persistency is high, then at the end of a disinflation process there can occur serious undershooting of the long-run value of inflation, see *Figure 2.2*. Due to procedural reasons, at the accession date it is improbable that the conversion rate can be modified as much as optimality would require. Thus, if in a given economy the persistence of inflation is high or uncertain, then it is better to start the disinflation process long before the accession. If there is enough time for disinflation, then the potential undershooting will disappear prior to the accession date, and the inflation rate will be close enough to its long-run value. Hence, a nominal-exchange-rate alignment would not be necessary when the country joins the Monetary Union.

Some limitations of the model for direct policy application have to be mentioned.

First, the conversion rates of accession countries are a result of a multilateral decision, and the welfare of all the concerned countries is taken into account: but in this model the welfare of only one accession country is considered.

Second, it is not taken into account in this model that accession countries can be viewed as competing peripheral countries, which trade with the center of the EU. In this case, as it was shown in the paper of Corsetti et al. (1999), a given nominal exchange rate movement can have different effects depending on the relative weight of trade with the center and with other peripheral countries.

Third, in the model there is no unemployment. However, *chapter 3* will demonstrate that some welfare implications of NOEM models are sensitive to the assumptions about the labor market.

Fourth, in this model the government's budget is always balanced. However, a budget deficit may have inflationary effects, see, e.g., Woodford

(2001). Thus, it has implications for the determination of the conversion rate.

Finally, the Balassa – Samuelson (BS) effect (i.e. the productivity induced divergence of sectoral inflation rates and the accompanying real appreciation) is not considered here. *Chapter 1* reviewed its empirical significance, and it may have implications for the choice of the optimal conversion rate: as Aoki (2001) and Benigno (2001) demonstrated, the optimal policy should put more weight to stabilization of the inflation rate in the sector with stronger nominal rigidities. Thus, in economies with diverging sectoral inflation rates, and with significantly different sectoral rigidities it is not sufficient to consider only the average inflation rate for the determination of the optimal conversion rate.

The reason why the BS effect is neglected is that in order to derive a tractable social welfare function one has to impose restrictions on the co-movement of consumption and export. (See the details in *section 2.2.5* and *Appendix B.2.*) The restrictions applied in this model contradict the pricing to market assumption, which is necessary in a NOEM model to generate the BS effect, as it was demonstrated in *chapter 1.*

2.5 Conclusions

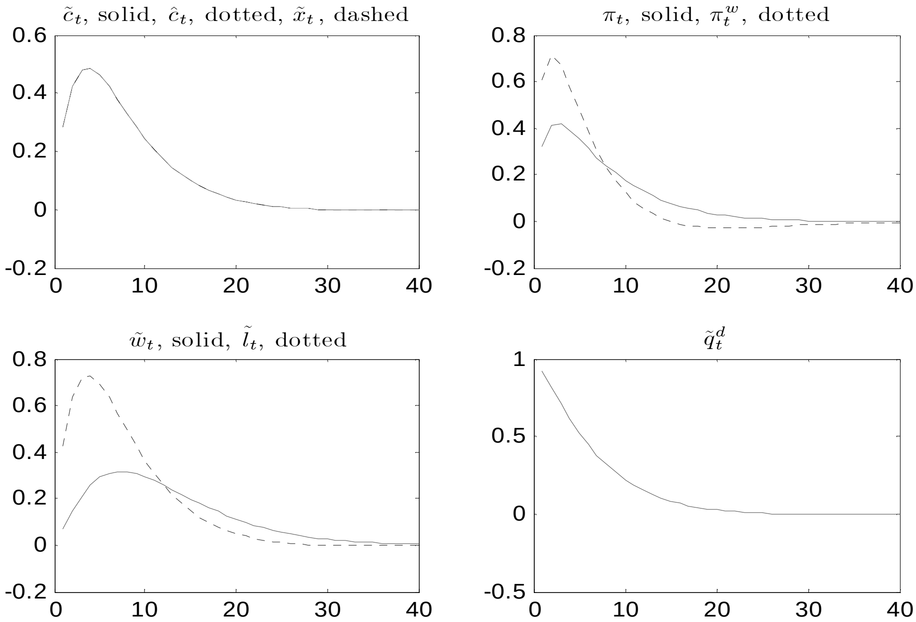
This chapter has examined how a country joining the European Economic and Monetary Union should choose its conversion rate. It was shown that, contrary to the widespread approach of the non-academic economic policy literature, it is not enough to base this decision exclusively on one factor, namely, the real exchange rate. It was demonstrated that although the misalignment of the real exchange rate was a key factor in the determination of the conversion rate, it did not have an exclusive role in determining the optimal conversion rate.

A proper misalignment index is proved to be a robust, parameter independent, and significant factor. On the other hand, the inflation rate and the real wage level are another key state variables which have to be taken into consideration for the determination of the conversion rate. Furthermore, the foreign-business-cycle, exports-demand and productivity shocks are exogenous factors also containing significant information for proper policy decision-making.

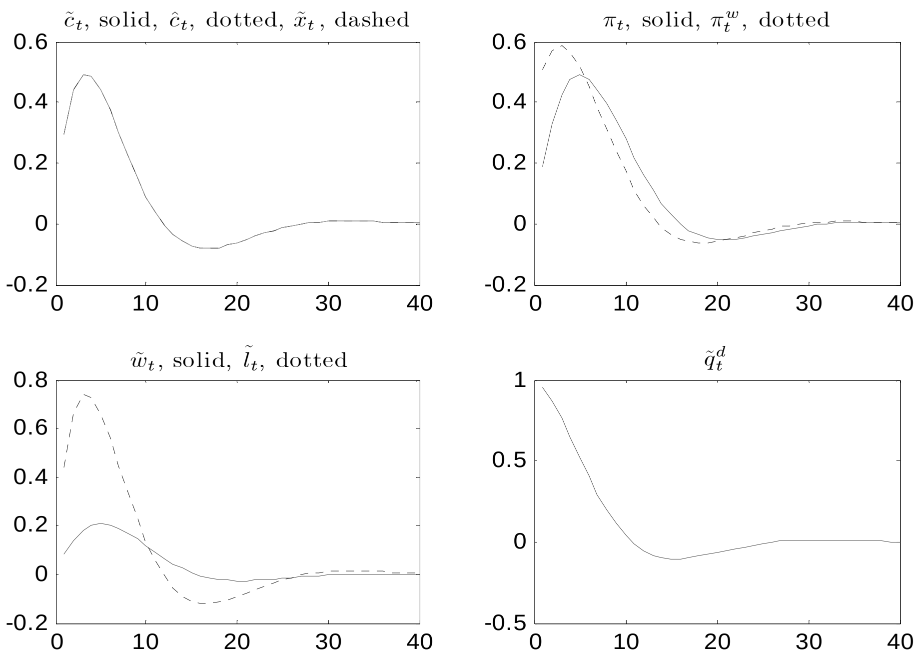
The importance of using a model-based social welfare function instead of ad-hoc welfare criteria was also demonstrated. Due to the persistence of inflation process, the exact social welfare function derived from the model contains both contemporary and past rates of inflation. As a consequence, the optimal policy reaction to some variables substantially differs from that derived from models with ad-hoc policy objective functions.

Furthermore, it was shown that the optimal exchange rate policy changed significantly if persistency of CPI and wage inflation were modified. Thus, for the right decision on the conversion rate in a certain economy it is necessary to have a thorough knowledge of the empirical characteristics of price and wage setting processes.

Figure 2.1
1 per cent initial devaluation (*de*$_1$)
Small persistence of inflation

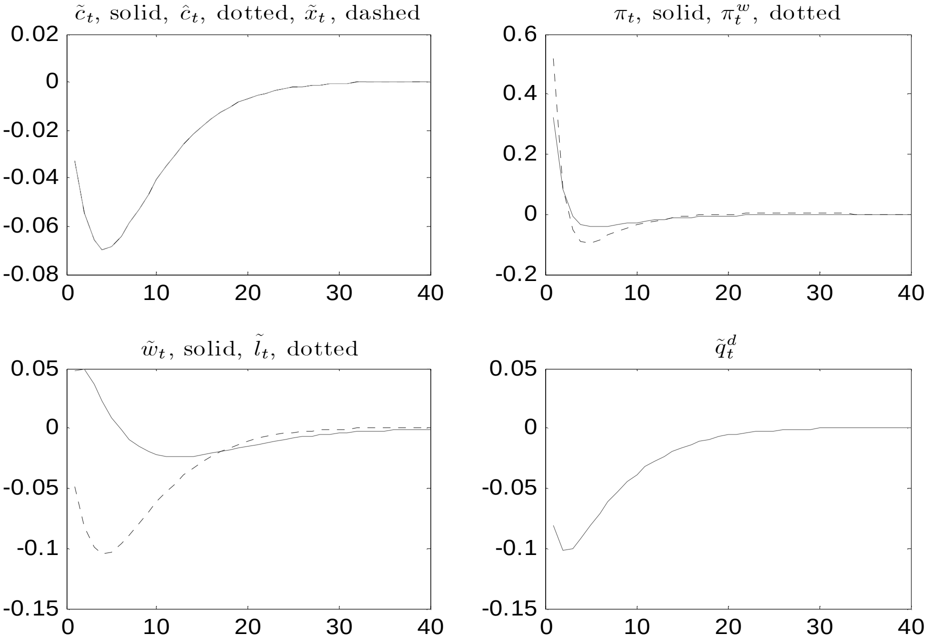


Large persistence of inflation

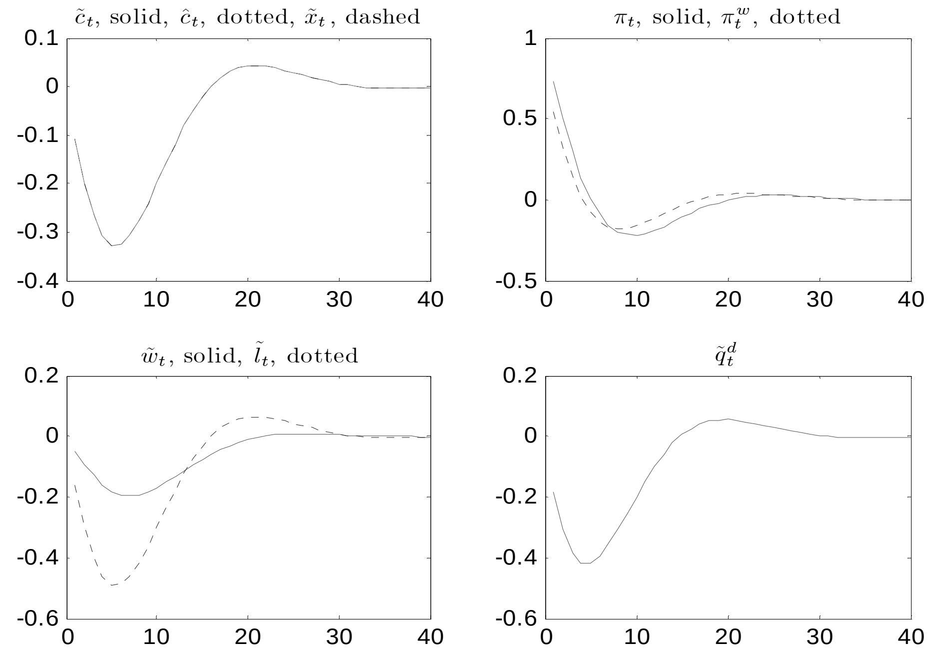


Units on a horizontal axis represent quarters, on a vertical axis percentage points.
Inflationary variables are displayed in annualized terms.

Figure 2.2
1 per cent initial deviation of the rate of inflation (π_0)
Small persistence of inflation

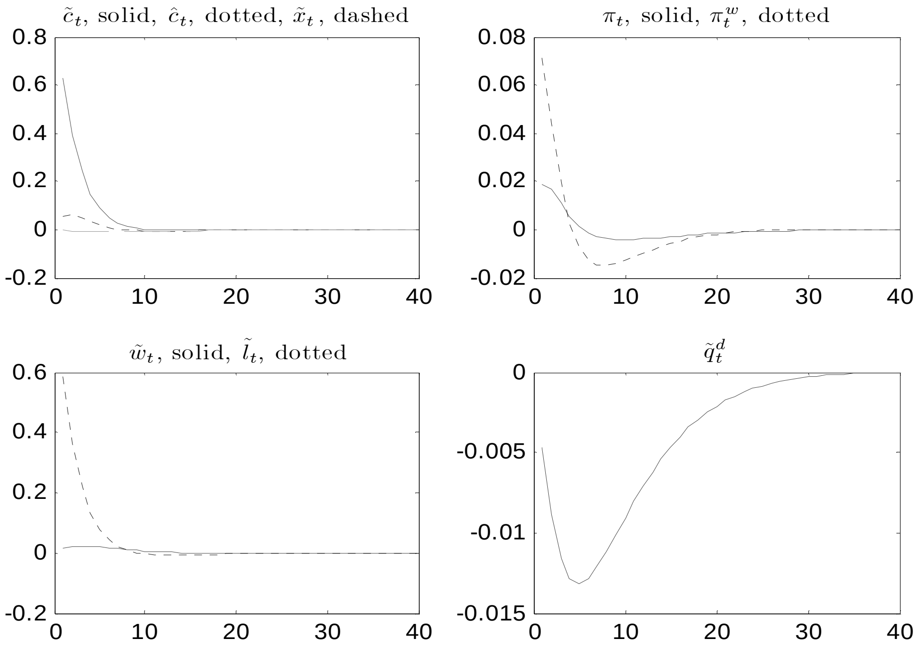


Large persistence of inflation

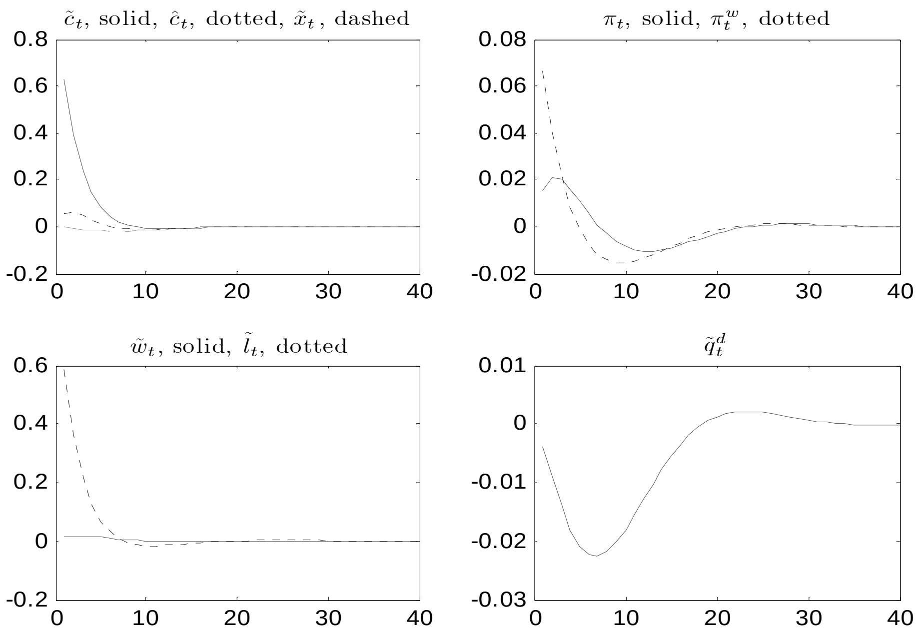


Units on a horizontal axis represent quarters, on a vertical axis percentage points.
Inflationary variables are displayed in annualized terms.

Figure 2.3
1 per cent initial deviation of consumption (c_0)
Small persistence of inflation

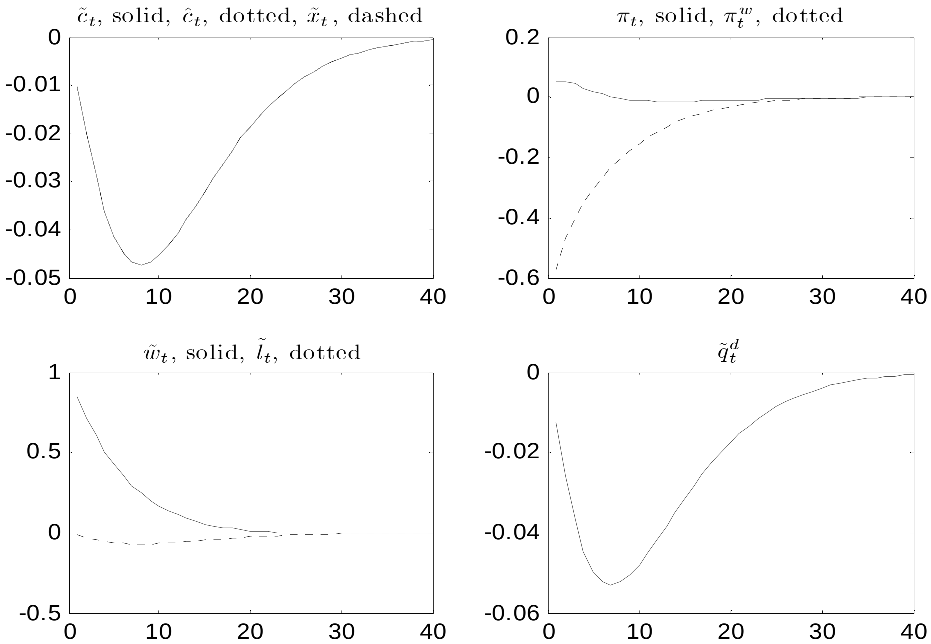


Large persistence of inflation

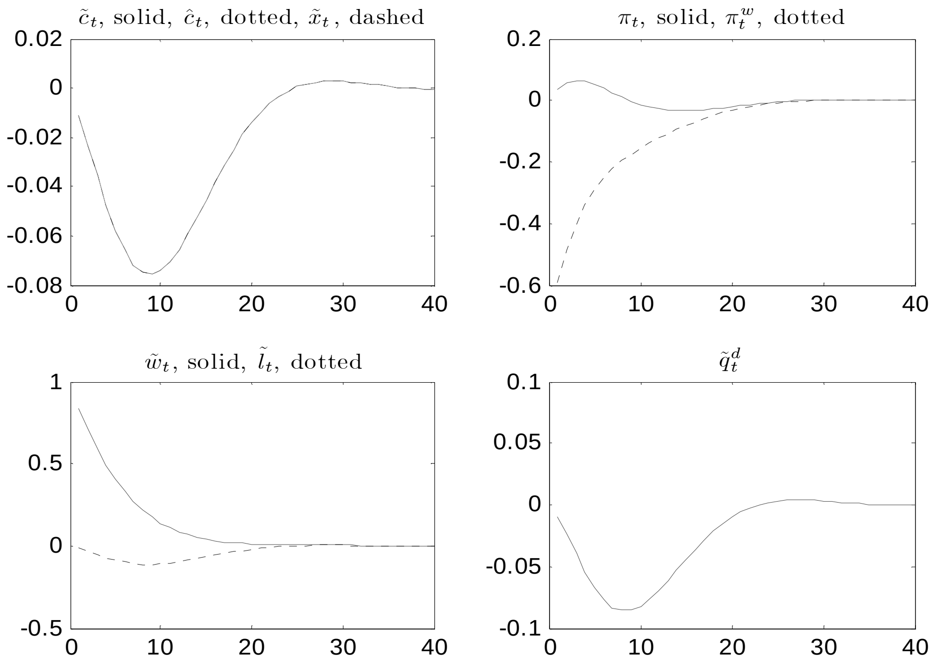


Units on a horizontal axis represent quarters, on a vertical axis percentage points.
Inflationary variables are displayed in annualized terms.

Figure 2.4
1 per cent initial deviation of the real wage (w_0)
Small persistence of inflation

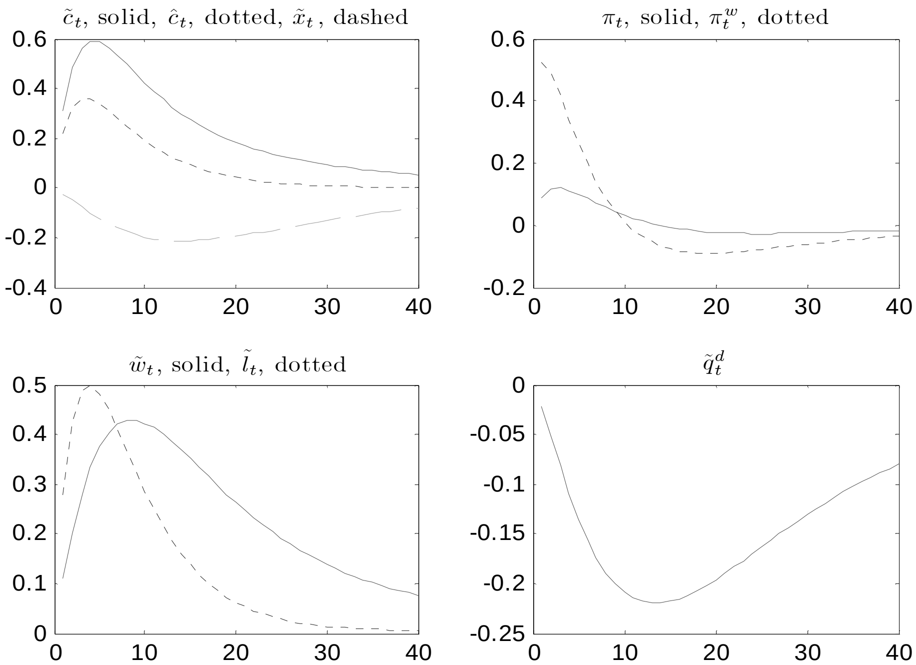


Large persistence of inflation

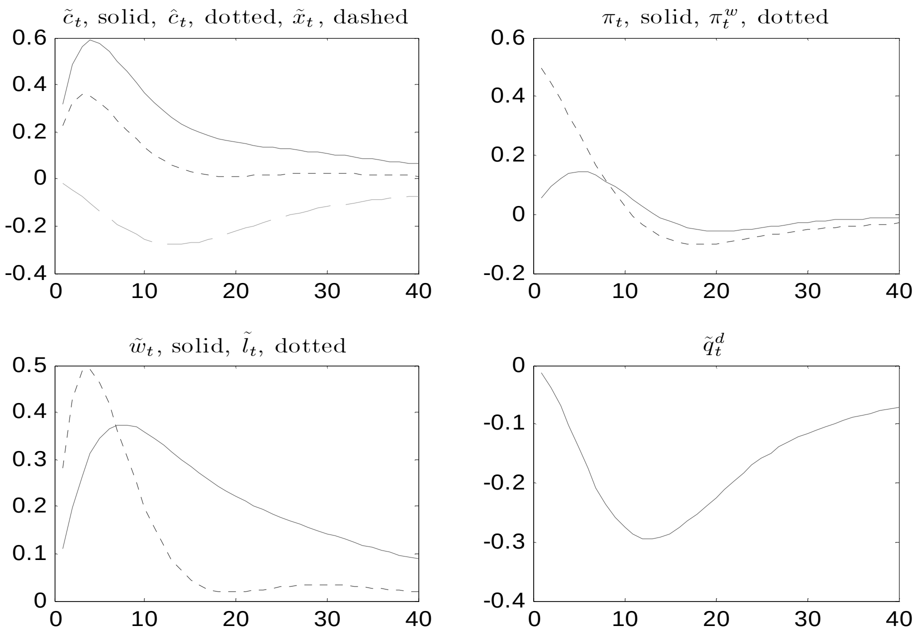


Units on a horizontal axis represent quarters, on a vertical axis percentage points.
Inflationary variables are displayed in annualized terms.

Figure 2.5
1 per cent foreign-business-cycle (c_1*) shock
Small persistence of inflation

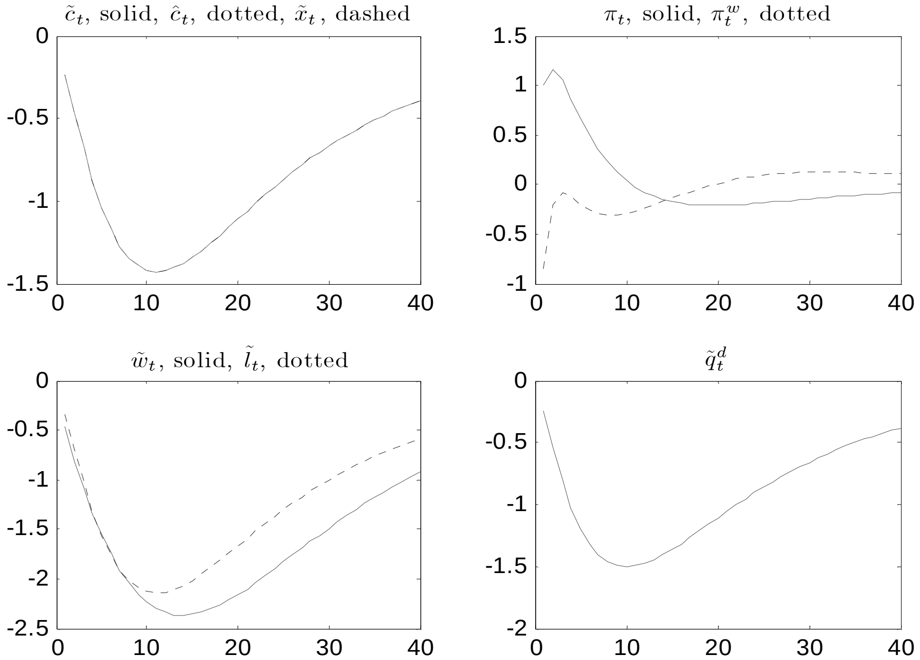


Large persistence of inflation

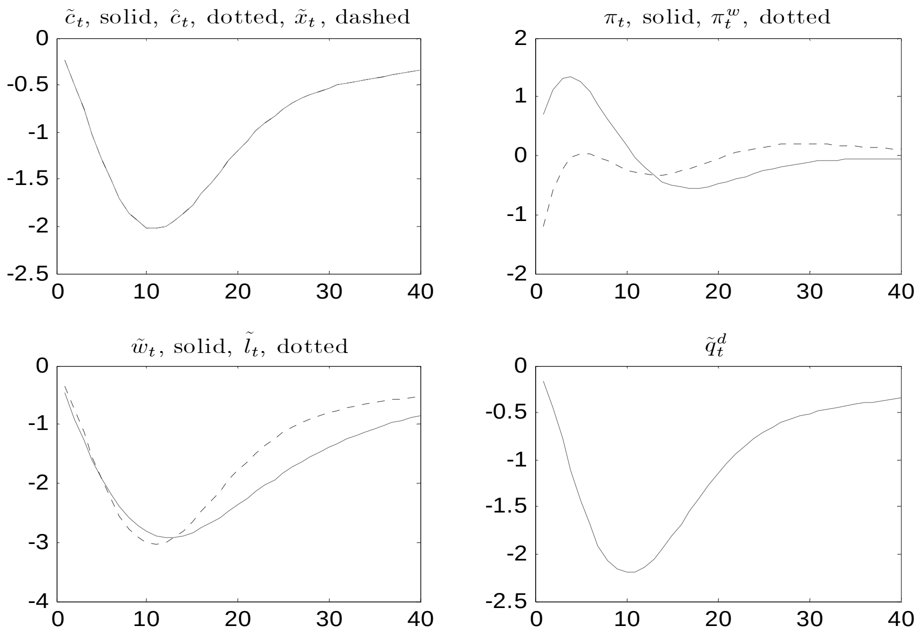


Units on a horizontal axis represent quarters, on a vertical axis percentage points.
Inflationary variables are displayed in annualized terms.

Figure 2.6
0.058 per cent price (υ_I) shock
Small persistence of inflation

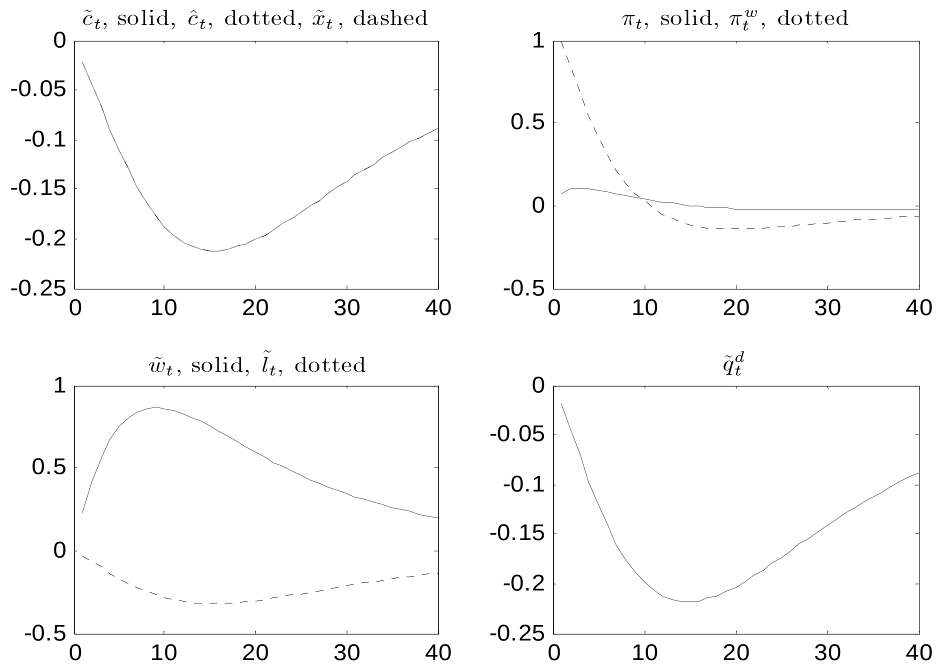


Large persistence of inflation

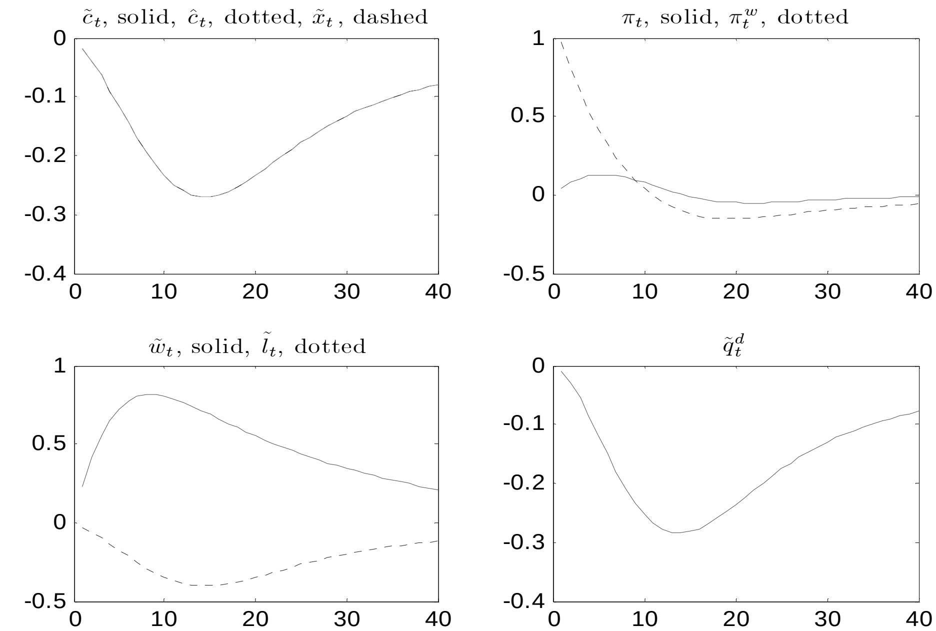


Units on a horizontal axis represent quarters, on a vertical axis percentage points.
Inflationary variables are displayed in annualized terms.

Figure 2.7
0.062 per cent nominal-wage ($\upsilon^{w}{}_{1}$) shock
Small persistence of inflation

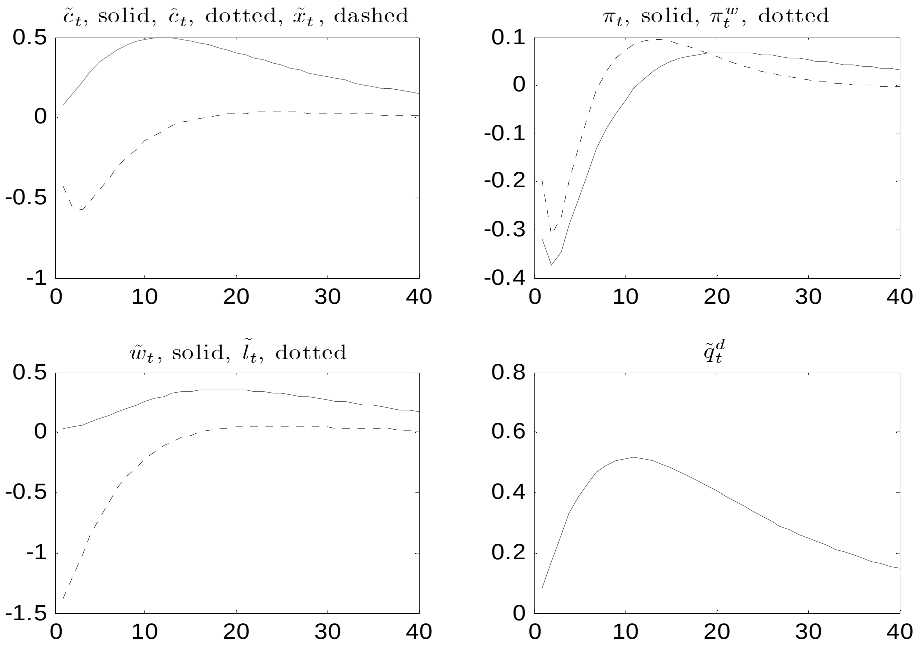


Large persistence of inflation

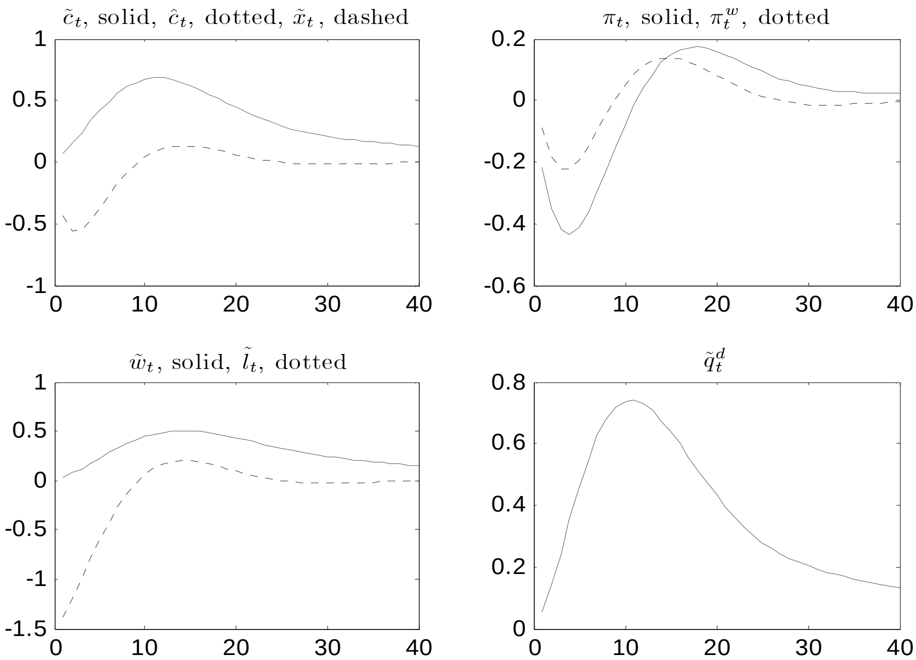


Units on a horizontal axis represent quarters, on a vertical axis percentage points.
Inflationary variables are displayed in annualized terms.

Figure 2.8
1 per cent productivity *(A_1)* shock
Small persistence of inflation



Large persistence of inflation



Units on a horizontal axis represent quarters, on a vertical axis percentage points.
Inflationary variables are displayed in annualized terms.

Chapter 3

A note on the welfare effects of devaluations

3.1 Introduction

Since publication of the seminal paper of Obstfeld and Rogoff (1995), dynamic general equilibrium models with nominal rigidities have become widely used in international macroeconomics. This literature, the so-called new open economy macroeconomics (NOEM), enables researchers to perform exact welfare analysis of monetary and exchange rate policy. Previously this task was much more difficult, due to the lack of micro-foundations in the older generation of open economy monetary models.

The NOEM literature, following the New Keynesian models of closed economies,[1] focuses on two distortionary factors: monopolistic competition and nominal rigidities. Labor market frictions, which may result in unemployment, are neglected. This stands in sharp contrast to the older Keynesian tradition which considers unemployment as the main source of social welfare costs.

The objective of this chapter is to demonstrate that some welfare implications of NOEM models can change dramatically if unemployment and heterogeneous consumption are possible. Consequently, it may be worthwhile resuscitating the older tradition and consider the impacts of unemployment on social welfare.

Corsetti and Pesenti (2001a) show that an unexpected devaluation of the nominal exchange rate may have harmful effects on the home country. This result contradicts traditional wisdom, which considers devaluations as

[1] This approach goes back to Blanchard and Kiyotaki (1987). A thorough review of closed economy New Keynesian literature is presented in Woodford (2003).

welfare-improving acts since they result in an increase in output. This statement is based on an *ad-hoc* welfare criteria: higher output and higher welfare coincide. The crucial insight of Corsetti and Pesenti is that this ad-hoc welfare consideration does not necessarily hold up under a more accurate evaluation: a devaluation results in higher output because home country agents work more. On the other hand, the terms of trade deteriorate. Hence, home consumption does not increase as much as in a closed economy environment. It may occur that the marginal disutility of increasing work effort is higher than the marginal utility of higher consumption.

The terms-of-trade effect remains valid if unemployment exists. However, if there is no full income insurance, then unemployment yields heterogeneous consumption. As a consequence, an exchange-rate devaluation influences both the aggregate level and the distribution of consumption.

In this chapter it is demonstrated that even if a devaluation is harmful for the home country in the case without unemployment, it may improve home welfare if unemployment exists. The reason is simple: higher domestic production requires more labor and decreases unemployment. Lower unemployment yields a more even distribution of consumption, which increases social welfare. If this latter effect is greater than the marginal disutility of higher labor effort, then a devaluation can improve social welfare, even if its positive effect on aggregate consumption is negligible.

The chapter is organized as follows. *Section 3.2* presents two simple stylized small open economy models. In the first one there is no unemployment, while in the second one asymmetric information results in involuntary unemployment. *Section 3.3* analyzes and compares the welfare effects of an unexpected nominal exchange rate devaluation in the two models. Finally, *section 3.4* presents the conclusions.

3.2 Two simple models

This section presents two simple *static* general equilibrium small open economy models to compare the welfare effects of devaluations of the nominal exchange rate in economies with and without unemployment. The results would remain valid in appropriate dynamic models as well, but the purpose here is to present the main idea in the simplest possible setting.

The first model with a Walrasian labor market represents the existing NOEM models. It is designed to replicate the result of Corsetti and Pesenti (2001a), which states that an unexpected nominal-exchange-rate devaluation can harm welfare. The second model drops the assumption of a frictionless labor market, and allows the existence of unemployment. It is used to show that even if a devaluation harms welfare in the presence of a frictionless labor market, it may improve it when there is unemployment in the economy.

3.2.1 Model with Walrasian labor market

Households

Assume the small open economy is inhabited by a representative household which seeks to maximize

$$\ln(c) + x\ln(1-l),$$

where l is labor and c is a composite consumption good defined by

$$c = \min\left[a^{-1}c^h,\, (1-a)^{-1}c^f\right],$$

where c^h and c^f represent consumptions of home and foreign goods, respectively, and $0 < a < 1$. Households maximize their utility function with respect to consumption and labor, subject to the budget constraint

$$P^h c^h + \mathcal{E}P^{f*}c^f = Wl + \Pi,$$

where P^h is the home currency price of a domestically produced good, P^{f*} is the foreign currency price of foreign produced good, $\mathcal{E}$ is nominal exchange rate, W is the nominal wage, and Π is the average profit of home country firms.

The optimization problem can be solved in two steps: First, the utility function is maximized, subject to the following budget constraint:

$$Pc = Pwl + \Pi, \tag{3.1}$$

where w is the real wage and

$$P = aP^h + (1-a)\mathcal{E}P^{f*} \tag{3.2}$$

is the price index of composite consumption. The solution of this problem provides c and l. Second, for a given c one can derive the demand for domestically produced and foreign consumption goods by

$$c^h = ac, \tag{3.3}$$
$$c^f = (1-a)c. \tag{3.4}$$

The optimal solution for c and l is determined by equation (3.1) and the first-order condition

$$\frac{xc}{1-l} = w. \tag{3.5}$$

Production

Production has a hierarchical structure. In the first stage firms use labor to produce a set of intermediate goods. In the second stage, a representative firm transform intermediate goods into a final consumption good on a competitive market. It uses the following constant-returns-to-scale technology:

$$y = \left(\int_0^1 y(i)^{\frac{\theta-1}{\theta}} \, di \right)^{\frac{\theta}{\theta-1}},$$

where y is the production of the final good, $y(i)$ represents intermediate goods, and $\theta > 1$. Since the final good producer is a price taker, the connection between the price of final good and intermediate good prices is given by

$$P^h = \left(\int_0^1 P^h(i)^{1-\theta} \, di \right)^{\frac{1}{1-\theta}},$$

where P^h is the price of y, and $P^h(i)$ is the price of $y(i)$. One can also derive the following demand function for good $y(i)$ from the final good producers' profit-maximization problem:

$$y(i) = \left(\frac{P^h}{P^h(i)} \right)^{\theta} y. \tag{3.6}$$

Each good $y(i)$ is produced by an individual firm, which acts on a *monopolistically competitive* market. The technology of the representative firm is defined by $y(i) = Al(i)$, where A is a common productivity factor, and $l(i)$ is the firm's utilization of labor. Since intermediate goods are not perfect substitutes, each firm can choose different price denoted by $P^h(i)$. It does so by maximizing $\Pi(i) = P^h(i)y(i) - Pwl(i)$ with respect to $P^h(i)$, subject to its technological constraint and the demand function (3.6). The first-order condition provides

$$P^h(i) = \frac{\mu}{A} Pw,$$

where $\mu = \theta/(\theta - 1) > 1$. Since the right hand side of the above formula does not contain any firm-specific variables they choose identical prices, i.e. $P^h(i) = P^h$. As a consequence, equation (3.6) implies $y(i) = y$, and hence $l(i) = l$.

Accordingly, the supply side of home economy can be summarized by the following equations:

$$\begin{aligned} y &= Al, && (3.7) \\ \Pi &= P^h y - Pwl, && (3.8) \\ P^h &= \frac{\mu}{A} Pw. && (3.9) \end{aligned}$$

Exports demand and equilibrium conditions

Foreign behavior is not modelled explicitly. The following ad-hoc exports demand function is used:

$$c^{h*} = \frac{\mathcal{E}P^{f*}}{P^h}c^*, \tag{3.10}$$

where c^{h*} denotes foreign demand for home made goods and c^* is an exogenous variable representing foreign aggregate consumption.

In equilibrium the supply and demand of home made final goods are equal, that is

$$y = c^h + c^{h*}. \tag{3.11}$$

Since this is a one-period model with incomplete international capital markets, international trade is balanced, hence

$$\mathcal{E}P^{f*}c^f = P^h c^{h*}. \tag{3.12}$$

In general equilibrium models, *Walras' law* implies that one condition is redundant: in this model the combination of (3.1), (3.8) and (3.11) provide (3.12). Hence equations (3.1) – (3.5), (3.7) – (3.11) determines the following 10 endogenous variables: P, P^h, w, c, c^h, c^f, c^{h*}, y, l, and Π. The exogenous parameters are a, A, x, and θ. The exogenous variables related to the foreign economy are P^{f*} and c^*. Finally, $\mathcal{E}$ is an exogenous policy variable. It is easy to show that there exists an equilibrium solution of the model.

Sticky prices

So far the *flexible price* version of the model has been described. In the *sticky price* version of the model P^h is *predetermined*. Although it is not modelled explicitly, it is assumed that prices were set in the past. That is, P^h is treated as an exogenous variable. If P^h is exogenously given, then firms' behavior is *demand determined*, and condition (3.9) is dropped. But it is assumed that $\Pi > 0$.

3.2.2 Model with efficiency wages

Labor market data reveal that most fluctuations in aggregate hours are the results of individuals entering and leaving employment rather than continuously employed agents adjusting the number of hours worked. In other words labor fluctuations occur mainly at the *extensive margin* instead of the *intensive margin*.

In the modern business cycle literature Hansen (1985) and Rogerson (1988) first considered the implications of adjustment at the extensive margin. They showed the implications of indivisible labor for aggregate fluctuations. Recent literature is surveyed in Hall (1999), and Mortensen and Pissarides (1999).

In this chapter I do not wish to discuss possible explanations for, and models of unemployment. What is important for my argument is that there is unemployment, and that it causes heterogeneity in households' income if workers are not fully insured against it. The *efficiency-wage* model of Alexopoulos (2004) is appropriate to illustrate my reasoning, and it is simple to incorporate this in my one-period setting.

In efficiency-wage models employers and employees are asymmetrically informed. As a consequence, employers do not choose market clearing wages. Instead they set them in such a way to provide incentive for workers to exercise the required effort. Hence unemployment may occur.

Households

The economy is populated by a continuum of identical households. However, to simplify the notation the household indices are dropped, since this does not cause confusion. Labor is indivisible, hence the worker of a household is either employed or unemployed. However, the worker can choose the level of her labor effort $e \geq 0$ if she is employed. The utility function of a particular consumer is

$$\ln(c) + x \ln\left(1 - e - \chi(e)\zeta\right),$$

where c is the consumption of the composite good defined in the previous model, and x is an exogenous parameter. Furthermore, ζ is the fixed cost of exercising any effort, therefore, $\chi(e) = 0$, if $e = 0$, and $\chi(e) = 1$, if $e > 0$.

The employment contract specifies the nominal wage the worker receives, and a required effort level. Employers cannot directly observe the exercised effort level, but shirkers who provide lower effort level than required are detected by an exogenous monitoring technology with probability $d < 1$. Hence, the contract contains a third element, monetary punishment for shirking. Shirkers obtain only a fraction $s < 1$ of the specified wage.

Let us denote by c^e the consumption of those households who receive their full wage bill, and by c^s the consumption of households belonging to detected shirkers. They are determined by

$$Pc^e = Pw + \Pi - Pf, \tag{3.13}$$

$$Pc^s = sPw + \Pi - Pf, \tag{3.14}$$

where Π is the average profit of home country firms and f is a real lump-sum tax levied by the government.

Since in this model the labor market does not necessarily clear, there can be households whose workers are unemployed. Their consumption is financed by the lump-sum tax f, and by their profit income. Assume that the fraction of employed households is $\lambda \in [0, 1]$. Then the c^u consumption of the unemployed household is defined by,

$$Pc^u = \Pi + \frac{\lambda}{1-\lambda} Pf. \tag{3.15}$$

It is assumed that $c^u < c^e$, in other words, there is only partial unemployment insurance in the model.

Later it will be shown that profit-maximizing employers always design a contract that ensures workers will not shirk, and will voluntarily provide the required effort. The optimal incentive mechanism must satisfy the following constraints:

$$\ln(c^e) + x\ln(1-e-\zeta) \geq \ln(c^u), \tag{PC}$$
$$\ln(c^e) + x\ln(1-e-\zeta) \geq d\ln(c^s) + (1-d)\ln(c^e). \tag{IC}$$

Condition (PC) is the *participation constraint*, which ensures that being employed is better than being unemployed. Condition (IC) is the *incentive compatible* constraint, which ensures that employees do not shirk. In the incentive compatible constraint it is taken into account that a rational shirker always chooses $e = 1$.

Condition (IC) implies that in equilibrium there are no shirkers in the economy. Thus, there is no household which consumes the amount c^s. Hence the average consumption of the composite good is

$$c = \lambda c^e + (1-\lambda)c^u. \tag{3.16}$$

Production

In this model production also has a hierarchical structure. Final and intermediate productions are basically the same as in the previous model. The only difference is that although intermediate good producers have the same constant-returns-to-scale single-input technology as previously, they do not use labor as an input. Instead they need a certain input good, still denoted by l, produced by a third sector. This extra sector is added to the model to simplify the analysis: in this way it is possible to separate the decision of sticky price setting and that of the optimal incentive scheme.

Input good l is produced by a price taking representative firm, which uses labor as input, and has the following technology: $l = B^l e(\lambda - \lambda^s)$, where, as was mentioned, λ is the fraction of employed households, and $\lambda^s \in [0, 1]$ is the fraction of shirkers. The above technology, the previously described specific form of labor contracts, and the fact that $d < 1$ imply that firms design incentive the mechanism to ensure $\lambda^s = 0$. That is, labor contracts satisfy condition (PC) and (IC). It is clear from equations (3.13) and (3.15) that c^e and c^u is a function of w, f, and Π/P. Since the firm cannot manipulate f and the average real profit of the home economy (Π/P), the only way to ensure condition (PC) and (IC) is to set the real wage w appropriately.

It is well known from the literature of mechanism design that (IC) binds. Rearranging it yields a function, which maps the real wage to the required effort level, that is

$$e = e(w) = 1 - \zeta - \left(\frac{c^e}{c^s}\right)^{-\frac{d}{x}}. \tag{3.17}$$

Let us denote the price of l by P^l. Then the firm's problem is to maximize

$$P^l l - Pw\lambda$$

with respect to λ, l and w, subject to the technological constraint

$$l = Be(w)\lambda,$$

and the function (3.17). The first-order conditions are

$$\begin{aligned} P^l &= \frac{Pw}{Be}, \\ e &= e'(w)w, \end{aligned}$$

where the first one is the usual zero-profit condition, and the second one is the *Solow condition* well known in efficiency wage literature.

The behavior of the final and intermediate goods sectors can be described by similar formulas as equations (3.7) – (3.9). Combining them with the above zero-profit condition yields

$$y = ABe\lambda, \tag{3.18}$$

$$\Pi = P^h y - Pw\lambda, \tag{3.19}$$

$$P^h = \frac{\mu}{ABe} Pw. \tag{3.20}$$

Now it will be shown how the optimal incentive mechanism determines the fraction of c^s and c^e. The derivative of function $e(w)$ is

$$e'(w) = \frac{d}{x}\left(\frac{c^e}{c^s}\right)^{-\frac{d}{x}-1} \frac{c^s - sc^e}{(c^s)^2}.$$

Equations (3.13) and (3.14) provide the following formula for the real wage:

$$(1-s)w = c^e - c^s. \tag{3.21}$$

Substituting $e(w)$ and the above two formulas into the Solow condition results in

$$1 + s\left(1 - \frac{c^e}{c^s}\right) - \frac{c^s}{c^e} = (1-s)\frac{x}{d}\left[(1-\zeta)\left(\frac{c^e}{c^s}\right)^{\frac{d}{x}} - 1\right]. \tag{3.22}$$

Equilibrium

To close the model I use the exports demand function (3.10), the equilibrium condition (3.11), and the trade balance equation (3.12).[2]

The system of equations (3.2) – (3.4), (3.10), (3.11), (3.13), (3.15) – (3.22) determines 14 endogenous variables: P, P^h, w, e, c, c^e, c^s, c^u, c^h, c^f, c^{h*}, y, λ, and Π. The solution of the system must satisfy the non-binding inequality (PC) as well. The new exogenous parameters are B, d, s, x, ζ. Furthermore, there is another exogenous policy variable in this model: f.

The above conditions describe the flexible price version of the model. If prices are sticky, then P^h is chosen exogenously, and equation (3.20) is dropped. However, it is required that $\Pi > 0$.

To find the equilibrium solution in the sticky prices version notice that c, c^h, c^f, $\varkappa = c^e/c^s$, and e are independent from the choice of P^h, and from the values of the policy variables. Combining equations (3.4), (3.10), and (3.12) yields

$$c = \frac{c^*}{1-a}.$$

Then equations (3.3) and (3.4) provide c^h and c^f. The variable $\varkappa$ is determined by equation (3.22), which depends only on parameters d, s, x, and ζ. Finally, substitute $\varkappa$ into equation (3.17), so e is given by

$$e = e(w) = 1 - \zeta - \varkappa^{\frac{d}{x}}.$$

.

For given P^h and $\mathcal{E}$ equation (3.2) determines P and equation (3.10) determines c^{h*}. Then it is possible to calculate y by equation (3.11) and λ by (3.18). Combining (3.13), (3.14) and (3.15) yields

$$c^u = c^e\left(1 - \frac{1-\varkappa}{1-s}\right) + \frac{1}{1-\lambda}f.$$

[2] Again, the latter is redundant since the combination of equations (3.13), (3.15), (3.16) and (3.11) provides the same condition.

The above expression and equation (3.16) simultaneously determine c^e and c^u, furthermore $c^s = c^e \varkappa^{-1}$. Equation (3.21) provides w and equation (3.19) provides Π. If f is small enough, then $c^u < c^s < c^e$. Thus the binding inequality (IC) ensures that (PC) is satisfied.

In the flexible price version P^h cannot be set freely. The above discussion reveals that the real wage and the price index of c are functions of P^h. Plugging them into equation (3.20) provides an expression for P^h

$$P^h = \frac{\mu}{ABe} P(P^h) w(P^h).$$

Instead of discussing the conditions for the existence of the flexible price equilibrium, a numerical example is provided, where the solution exists and the result seems reasonable. The values of the exogenous parameters are $a = 0.5$, $AB = 23.72$, $d = 0.25$, $s = 0.81$, $x = 0.03$, $\mu = 1.2$, and $\zeta = 0.7$. The exogenous and policy variables are chosen to be $c^* = 0.5$, $P^{f*} = 1$, $\mathcal{E} = 1$, and $f = 0.04$. Then $c = 1$, $\varkappa = 1.2$, $e = 0.05$. Furthermore, $P^h = 1$, $y = 1$, $\lambda = 0.9$, $c^u = 0.5$, $c^e = 1.06$, and $c^s = c^e \varkappa^{-1}$.

3.3 Welfare analysis of devaluations

3.3.1 The terms-of-trade effect

Now let us consider the effects of an unexpected devaluation of the nominal exchange rate in the sticky price version of the model with a Walrasian labor market.

The preset price of home made goods is $P^h = \bar{P}^h$. Assume that it is expected that the nominal exchange rate would be $\bar{\mathcal{E}}$ with probability 1. The equilibrium values of the variables corresponding to $\bar{P}^h$ are also denoted by the bar.[3]

Suppose that the monetary authority unexpectedly devaluates, that is, the realization of nominal exchange rate is $\hat{\mathcal{E}} > \bar{\mathcal{E}}$. Of course, the value of P^h remains $\bar{P}^h$, but the values of the other endogenous variables will be different from the flexible price solution. Variables with the hat denote their post-devaluation values.

An obvious effect of the devaluation is the rise in exports demand since equation (3.10) implies that

$$\hat{c}^{h*} = \frac{\hat{\mathcal{E}} P^{f*}}{\bar{P}^h} > \frac{\bar{\mathcal{E}} P^{f*}}{\bar{P}^h} = \bar{c}^{h*}.$$

[3] If the realization of the nominal exchange rate is $\bar{\mathcal{E}}$, then the solution of the model is equivalent with that of the flexible price version.

Rising exports may result in rising imports, but not in this particular case. Plug equation (3.10) into the trade balance equation (3.12):

$$\hat{\mathcal{E}}P^{f*}\hat{c}^f = \bar{P}^h \frac{\hat{\mathcal{E}}P^{f*}}{\bar{P}^h} c^*,$$

which implies that $\hat{c}^f = c^*$, that is, domestic consumption of foreign goods is fixed (as already mentioned in the discussion of the efficiency-wage model). Hence, $\hat{c}^f = \bar{c}^f$, although exports rise, the *deterioration* of the terms of trade *offsets* the rise of imports. As a consequence, $\hat{c} = \bar{c} = \mathbf{c}$ and $\hat{c}^h = \bar{c}^h$ by equation (3.3) and (3.4).

Although rising exports do not result in rising domestic consumption, it induces an expansion of home output. Since $\hat{c}^h = \bar{c}$ and $\hat{c}^{h*} > \bar{c}^{h*}$, equation (3.11) implies that $\hat{y} > \bar{y}$. Combining the latter with (3.7) yields $\hat{l} > \bar{l}$.

If the social welfare function is the utility function of the representative consumer, then the welfare effect of the devaluation is obvious:

$$\ln(\mathbf{c}) + x \ln\left(1 - \bar{l}\right) > \ln(\mathbf{c}) + x \ln\left(1 - \hat{l}\right).$$

That is, in this model an unexpected devaluation unambiguously leads to a deterioration in social welfare.

3.3.2 The effect of unemployment reduction

Now let us analyze the effects of an unexpected devaluation in the efficiency wage model. As previously, it induces the expansion of domestic output, hence $\hat{\lambda} > \bar{\lambda}$ by equation (3.18). In other words unemployment falls. Furthermore, the terms-of-trade effect also works since $\hat{c} = \bar{c}$. But in this model there is a third effect as well: Although aggregate consumption does not change, decreasing unemployment results in a *less uneven distribution* of consumption if c^u is kept fixed.

Let us consider the social welfare consequences of the above three effects. The social welfare function is the average of individual households' utility functions:

$$\mathcal{U}(c^e, c^u, \lambda) + \lambda x \ln(1 - e - \zeta),$$

where

$$\mathcal{U}(c^e, c^u, \lambda) = \lambda \ln(c^e) + (1 - \lambda) \ln(c^u).$$

If $\hat{f}$ is chosen in such a way that $\hat{c}^u = \bar{c}^u = \mathbf{c}^u$, then equation (3.16) implies $\hat{c}^e < \bar{c}^e$. Since in the model there is only partial unemployment insurance,

$\mathbf{c}^u < \hat{c}^e$. It is easy to show that

$$\mathcal{U}\left(\hat{c}^e, \mathbf{c}^u, \hat{\lambda}\right) - \mathcal{U}\left(\bar{c}^e, \mathbf{c}^u, \bar{\lambda}\right) =$$
$$\left(\hat{\lambda} - \bar{\lambda}\right)\left[\ln\left(\hat{c}^e\right) - \ln\left(\mathbf{c}^u\right)\right] - \bar{\lambda}\left[\ln\left(\bar{c}^e\right) - \ln\left(\hat{c}^e\right)\right] > 0$$

since the logarithm is a strictly concave function.[4]

Although the social utility of consumption is increased, social welfare is not necessarily greater, since the social disutility of the labor effort is also increased. Formally,

$$\hat{\lambda} x\left(1 - \hat{e} - \zeta\right) > \bar{\lambda} x\left(1 - \bar{e} - \zeta\right),$$

since $\hat{\lambda} > \bar{\lambda}$ and $\hat{e} = \bar{e} = \mathbf{e}$ (recall that the value of e is independent from the policy variables). Devaluation improves social welfare only if

$$\mathcal{U}\left(\hat{c}^e, \mathbf{c}^u, \hat{\lambda}\right) - \mathcal{U}\left(\bar{c}^e, \mathbf{c}^u, \bar{\lambda}\right) > \left(\bar{\lambda} - \hat{\lambda}\right) x \ln(1 - \mathbf{e} - \zeta).$$

Equilibrium values of endogenous variables depend on d/x, but d and x separately do not influence it. As a consequence, for any possible equilibrium allocation one can find such a small value of x that guarantees the above inequality.

Let us illustrate this with an example. Consider the numerical example of *section 3.2.2* as the benchmark equilibrium (the values of the variables corresponding to this equilibrium are denoted by the bar). Suppose that the monetary authority unexpectedly devaluates by 5 percent, that is, $\hat{\mathcal{E}} = 1.05$. $P^h = \bar{P}^h = 1$ remains fixed after the devaluation, and c as well. Furthermore, the government sets f in such a way that c^u does not change. Then $\hat{\lambda} = 0.92$, i.e. the unemployment rate is reduced by 2 percentage points. Recall that $x = 0.03$ ($d = 0.25$). Then one can check that in this case the nominal-exchange-rate devaluation improves social welfare.

One may criticize the discussion of this chapter since the terms-of-trade effect is an implication of balanced international trade, which is an obvious consequence of the static framework with incomplete markets used. However, certain conditions can guarantee balanced trade in dynamic models as well, such as in Corsetti and Pesenti (2001a). Of course, it can be disputed whether

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[4]Strict concavity implies that

$$\left[\ln\left(\hat{c}^e\right) - \ln\left(\mathbf{c}^u\right)\right]\left(\hat{c}^e - \mathbf{c}^u\right)^{-1} > \left[\ln\left(\bar{c}^e\right) - \ln\left(\hat{c}^e\right)\right]\left(\bar{c}^e - \hat{c}^e\right)^{-1}.$$

Furthermore, Since c is constant, equation (3.16) implies that $\bar{\lambda} = \hat{\lambda}\left(\hat{c}^e - \mathbf{c}^u\right)\left(\bar{c}^e - \mathbf{c}^u\right)^{-1}$. The combination of the above two expressions yields the required inequality.

the modelling strategy of neglecting the current account channel is useful or not. But this was not the purpose of this study. The intent was to provide an example to illustrate the significance of the assumptions on the labor market in the welfare analysis of open economies. It was demonstrated that in the presence of unemployment even if the terms-of-trade effect worked, a nominal devaluation might improve social welfare.

3.4 Conclusions

This chapter demonstrates that some welfare implications of new open economy macroeconomics are not robust to the assumptions on the labor market. The effects of an unexpected devaluation of the nominal exchange rate in a model with and without unemployment was compared. In models with Walrasian labor markets a devaluation decreases social welfare if the deterioration of the terms of trade offsets the rise in domestic consumption. However, in the presence of unemployment even such a situation can enhance social welfare since the expansionary effect of the devaluation reduces unemployment, hence the distribution of consumption becomes more even.

Appendix A

Addendum to chapter 1

A.1 The steady state

In this section the non-stochastic steady state of the benchmark model is described. Variables without time indices refer to their steady-state values.

In the steady state there is no difference between the two model versions since $eP^x = P^{x*}$, the technologies of the tradable and the exports sector are the same, and in the steady state nominal rigidities do not exist. Hence, in this section it is sufficient to discuss version A: thus index T will refer both to local and exported tradables. In the steady state there is no intra-household and intra-sector heterogeneity. Therefore the index i of firms are omitted to simplify the notations.

It is assumed that $P = P^T = P^N = 1$. Then equations (1.6) and (1.23) imply that

$$c^T = a_T c, \quad c^N = a_N c, \quad \mathcal{I}^{TT} + \mathcal{I}^{TN} = a_T I^T, \quad \mathcal{I}^{NT} + \mathcal{I}^{NN} = a_T I^N. \quad \text{(A.1)}$$

Furthermore, it is assumed that $Px = eP^{m*}m$. Hence,

$$GDP = a^T \left(c^T + \mathcal{I}^{TT} + \mathcal{I}^{TN}\right) + a^N \left(c^N + \mathcal{I}^{NT} + \mathcal{I}^{NN}\right) = c + I,$$

where $I = I^T + I^N$.

Since $\Phi_s(I^s/k^s) = I^s/k^s$ and $\Phi_s'(I^s/k^s) = 1$, in the steady state investments do not have adjustment costs and, as was mentioned, in the steady state nominal rigidities do not exist. Hence, firms' optimization problem will be the same as in the case when there is a rental market for physical capital and the real rental rate of capital is determined by the real interest rate and the depreciation rate. Equation (1.7) implies that the real interest rate is equal to $1/\beta - 1$. If the real rental rate of physical capital, which is uniform in all sectors, is denoted by r, then

$$r = \frac{1}{\beta} - 1 + \delta.$$

This formula represents a special case of equation (1.18). I set the values of τ_T and τ_N in such a way that the markups

$$1 = \tau_s \frac{\theta}{\theta - 1}, \ s = T, \ N.$$

Then it is true for all sectors that the marginal product of capital is equal to r. Thus, equation (1.13) implies that

$$\varkappa = \left(\frac{r}{\alpha}\right)^{\frac{1}{1-\alpha}},$$

where $\varkappa = z^T/k^T = z^N/k^N$. Furthermore, equations (1.13), (1.26), and (1.28) imply that

$$c^T + I^T + x = k^T \varkappa^{1-\alpha}, \quad c^N + I^N = k^N \varkappa^{1-\alpha}. \tag{A.2}$$

Beyond this, in the steady-state equation (1.16) takes the form $I^s = \delta k^s$. Thus, if one defines the $k = k^T + k^N$ aggregate capital stock, then $I = \delta k$.

It is assumed that $w = W = eP^{m*}$, then equation (1.15) implies that $w^z = w$. Since in each sector w^z is equal to the marginal product of z^s

$$w = (1 - \alpha)\varkappa^{-\alpha}.$$

In the benchmark economy $w = 1.367$. Let us denote the exogenous exports/GDP ratio by s_x, and I set $s_x = 0.6$. Since $x = eP^{m*}m$,

$$s_x = \frac{x}{c + I} = \frac{eP^{m*}m}{c + I}. \tag{A.3}$$

It is assumed that in the benchmark economy $n_N = n_T = n$. Then the imports demand equation (1.21) implies that

$$m = (1 - n)\left(z^T + z^N\right).$$

Then one can show that

$$m = (1 - n)\varkappa\left(k^T + k^N\right) = (1 - n)\varkappa k. \tag{A.4}$$

Using the formula $I = \delta k$, the previous expression for m, and equation (A.3) yields

$$c = \mathsf{K}k, \tag{A.5}$$

where

$$\mathsf{K} = eP^{m*}(1-n)\varkappa s_x^{-1} - \delta.$$

By equation (A.2) one can similarly show that

$$k\varkappa^{1-\alpha} = c + \delta k + x = \left(eP^{m*}(1-n)\varkappa s_x^{-1} - \delta\right)k + \delta k + eP^{m*}(1-n)\varkappa k.$$

This implies that

$$n = 1 - \frac{\varkappa^{1-\alpha}}{eP^{m*}\varkappa\left(1+s_x^{-1}\right)}.$$

In the benchmark economy $n = 0.5$.

In the steady state the labor supply function of households (1.10) takes the form

$$w = c^{\sigma} l^{\varphi}. \tag{A.6}$$

As for imports, one can derive a similar expression for labor:

$$l = n\varkappa k. \tag{A.7}$$

Substituting equations (A.5) and (A.7) into equation (A.6) yields an expression for the capital stock:

$$k = \left[w\mathsf{K}^{-\sigma}\left(n\varkappa\right)^{-\varphi}\right]^{\frac{1}{\sigma+\varphi}}.$$

Using this expression one can calculate the steady-state value of the capital stock and investments. In the benchmark economy $k = 20.84$, and $I = \delta k = 0.521$. Then using formula (A.5) yields the value of consumption, $c = 1.629$, and equation (A.7) provides the value of labor, $l = 0.943$.

A.2 Price setting

Following Woodford (2005), one can show that the log-linearized solution of the price setting problem of *section 1.4.2* takes the form

$$\pi_t^d - \vartheta_s \pi_{t-1}^d = \beta \mathrm{E}_t\left[\pi_{t+1}^d - \vartheta_s \pi_t^d\right] + \xi_s \widetilde{mc}_t^s,$$

where $s = T$, x, N, and $d = x^*$, if $s = x$, otherwise $d = s$.

Furthermore, $\widetilde{mc}_t^s$ is the average real marginal cost of sector s, and

$$\xi_s = \frac{(1-\gamma_s)(1-\beta\gamma_s)}{\gamma_s\left(1+\hat{\alpha}\theta-\psi_s\right)},$$

where $\hat{\alpha} = \alpha(1-\alpha)^{-1}$, which is the elasticity of capital in equations (A.25) and (A.27).

Parameter ψ_s can be obtained in the following way. First define λ_s which is the solution of the quartic equation

$$\begin{aligned} 0 &= \left[(1+\hat{\alpha}\theta)(1-\beta\gamma_s\lambda_s)^2 - \gamma_s^2\beta\hat{\alpha}\Xi_s\lambda_s\right] \\ &\times \left\{\beta^2\lambda_s^2 - \left[1+\beta+(1-\beta(1-\delta))\bar{\alpha}\varepsilon_s^{-1}\right]\beta\lambda_s + \beta\right\} \\ &+ \beta(1-\gamma_s)(1-\beta\gamma_s)\hat{\alpha}\Xi_s\lambda_s, \end{aligned}$$

where $\bar{\alpha} = (1-\alpha)^{-1}$, which is the coefficient of capital in equations (A.22) and (A.23), and

$$\Xi_s = \frac{(1-\beta(1-\delta))\,\bar{\alpha}\theta}{\varepsilon_s}.$$

In addition λ_s satisfies a set of three inequalities,

$$\begin{aligned} \lambda_s &< \gamma_s^{-1}, \\ \lambda_s &> \frac{\gamma_s}{\beta(1+\gamma_s)}\left\{\beta^2\lambda_s^2 - \left[1+\beta+(1-\beta(1-\delta))\bar{\alpha}\varepsilon_s^{-1}\right]\beta\lambda_s + \beta\right\} - 1, \\ \lambda_s &< \frac{\gamma_s}{\beta(1-\gamma_s)}\left\{\beta^2\lambda_s^2 - \left[1+\beta+(1-\beta(1-\delta))\bar{\alpha}\varepsilon_s^{-1}\right]\beta\lambda_s + \beta\right\} + 1. \end{aligned}$$

Then

$$\psi_s = \hat{\alpha}\frac{\beta\gamma_s^2\Xi_s\lambda_s}{(1-\beta\gamma_s\lambda_s)^2}.$$

A.3 The complete log-linearized model

To solve the model described in *section 1.4* its log-linear approximation around the steady state is taken. In this section the log-linearized version is described. Variables without time indices refer to their steady-state values, and the tilde denotes the log-deviation of a variable from its steady-state value.

The log-linearization of the price index formula (1.5) yields

$$\widetilde{P}_t = a_T\widetilde{P}_t^T + a_N\widetilde{P}_t^N, \tag{A.8}$$

where I used the assumption that $P = P^T = P^N$.

The log-linearized versions of the real exchange rate indices in equations (1.30), and the assumption that P_t^{F*}, P_t^{FT*} and P_t^{FR} are constant are used for the derivation of the following formulas:

$$\begin{aligned} \pi_t^T &= d\tilde{e}_t - \left(\tilde{q}_t^T - \tilde{q}_{t-1}^T\right), &\text{(A.9)} \\ \pi_t^N &= \pi_t^T + \widetilde{P}_t^R - \widetilde{P}_{t-1}^R, &\text{(A.10)} \\ \pi_t^{x*} &= \widetilde{P}_t^{x*} - \widetilde{P}_{t-1}^{x*}, &\text{(A.11)} \\ \tilde{q}_t &= \tilde{q}_t^T - a_N\widetilde{P}_t^R, &\text{(A.12)} \end{aligned}$$

where $d\tilde{e}_t = \tilde{e}_t - \tilde{e}_{t-1}$ is the depreciation rate of the nominal exchange rate.

Log-linearizing equations (1.10), (1.15), and using the assumption that $W = eP^{m*}$ yields

$$\widetilde{w}_t^{z,s} = n_s\left(\sigma\tilde{c}_t + \varphi\tilde{l}_t\right) + (1 - n_s)\tilde{q}_t, \tag{A.13}$$

for $s = T,\ N$. It is assumed that $n_x = n_T$, hence it is not necessary to have a separate equation for the exports sector.

Using the log-linearized version of equations (1.6), (1.23), (1.30), and using equation (A.8) one can obtain the following expressions:

$$\begin{aligned}\tilde{c}_t^T &= \eta a_N \widetilde{P}_t^R + \tilde{c}_t,\\ \widetilde{\mathcal{I}}_t^{Ts}(i) &= \eta a_N \widetilde{P}_t^R + \tilde{I}_t^s(i).\end{aligned}$$

Let us define $I_t^s = \int_0^1 I_t^s(i)\, di$. Then one can show[1] that

$$\widetilde{\mathcal{I}}_t^{Ts} = \eta a_N \widetilde{P}_t^R + \tilde{I}_t^s.$$

The above formulas imply that the log-linearized version of the equilibrium condition (1.26) takes the form

$$\tilde{y}_t^T = \frac{x\tilde{x}_t + \mathsf{c}\tilde{c}_t + I\tilde{I}_t + (c + I)\,\eta a_N \widetilde{P}_t^R}{c + I + x}, \tag{A.14}$$

where $I_t = \sum_s I_t^s$. Similarly, the log-linearized equilibrium condition (1.27) takes the form

$$\tilde{y}_t^T = \frac{c}{c+I}\tilde{c}_t + \frac{I}{c+I}\tilde{I}_t + \eta a_N \widetilde{P}_t^R. \tag{A.15}$$

Finally, the log-linear approximation of the equilibrium condition (1.28) is

$$\tilde{y}_t^N = \frac{c}{c+I}\tilde{c}_t + \frac{I}{c+I}\tilde{I}_t - \eta a_T \widetilde{P}_t^R. \tag{A.16}$$

The log-linearization of equations (1.9) and (1.30) yields the expression which determines the trajectory of aggregate consumption:

$$\sigma\tilde{c}_t = \tilde{q}_t. \tag{A.17}$$

In version A of the model $\widetilde{P}_t^T - \tilde{e}_t = \widetilde{P}_t^{x*}$, hence the log-linearized version of the exports demand equation (1.25) is

$$\tilde{x}_t = \eta^* \tilde{q}_t^T, \tag{A.18}$$

[1]If a variable is defined in the following manner: $\mathfrak{z} = \int_0^1 \mathfrak{z}(i)\, di$ then its log-linear approximation yields $\tilde{\mathfrak{z}} = \int_0^1 \tilde{\mathfrak{z}}(i)\, di + o^2$, where o^2 denotes those second and higher order errors, which were neglected in the approximation process.

where equation (1.30) was used. In version B the log-linearized exports demand becomes

$$\tilde{x}_t = -\eta^* \widetilde{P}_t^{x*}. \tag{A.19}$$

Define the aggregate stock of physical capital in sector s as $k_t^s = \int_0^1 k_t^s(i)\, di$. Log-linearizing the investment equation (1.16) yields

$$\tilde{k}_{t+1}^s = (1-\delta)\tilde{k}_t^s + \delta \tilde{I}_t^s,$$

where the steady-state properties of Φ_s are used. As a consequence, the log-linearized equation for the aggregate investment is

$$\delta \tilde{I}_t = \sum_s \frac{I^s}{I}\left[\tilde{k}_{t+1}^s - (1-\delta)\tilde{k}_t^s\right], \tag{A.20}$$

where in version A $s = T, N$, in version B $s = T, x, N$.

Let us combine the log-linearized versions of equations (1.10), (1.13), (1.20), (1.29), (1.30), and equation (A.13). Then aggregating the result yields an expression for aggregate labor demand:

$$\tilde{l}_t = \sum_{s=H,x,N} \frac{l^s}{l}\left[(1-n_s)\left(\tilde{q}_t - \sigma\tilde{c}_t - \varphi\tilde{l}_t\right) + \bar{\alpha}\left(\tilde{y}_t^s - \widetilde{A}_t^s\right) - \hat{\alpha}\tilde{k}_t^s\right], \tag{A.21}$$

where, again, in version A $s = T, N$, and in version B $s = T, x, N$, furthermore, $\bar{\alpha} = (1-\alpha)^{-1}$ and $\hat{\alpha} = \alpha\bar{\alpha}$.

Log-linearizing and combining equations (1.7), (1.17) and (1.18) results in

$$\begin{aligned} &\widetilde{\Lambda}_t - \mathrm{E}_t\left[\widetilde{\Lambda}_{t+1}\right] + \varepsilon_s\left(\tilde{k}_{t+1}^s(i) - \tilde{k}_t^s(i)\right) = \\ &\mathrm{E}_t\left[[1-\beta(1-\delta)]\tilde{r}_{t+1}^s + \beta\varepsilon_s\left(\tilde{k}_{t+2}^s(i) - \tilde{k}_{t+1}^s(i)\right)\right], \end{aligned}$$

where $\varepsilon_s = -\Phi_s''(\delta)\delta$. Log-linearizing and combining equations (1.13) and (1.19) yields

$$\tilde{r}_t^s(i) = \widetilde{w}_t^s + \bar{\alpha}\left(\tilde{y}_t^s(i) - \widetilde{A}_t^s - \tilde{k}_t^s(i)\right).$$

Combining the above two equations, aggregating the result, and using the definition of Λ_t results in the equation which determines the evolution of physical capital in sector $s = T$, N:

$$\begin{aligned} &-\sigma\tilde{c}_t + \sigma\mathrm{E}_t\left[\tilde{c}_{t+1}\right] + \varepsilon_s\left(\tilde{k}_{t+1}^s - \tilde{k}_t^s\right) \qquad \text{(A.22)} \\ &= \Delta\mathrm{E}_t\left[\widetilde{w}_{t+1}^{z,s} + \bar{\alpha}\left(\tilde{y}_{t+1}^s - \widetilde{A}_{t+1}^T - \tilde{k}_{t+1}^s\right)\right] + \beta\varepsilon_s\mathrm{E}_t\left[\tilde{k}_{t+2}^s - \tilde{k}_{t+1}^s\right], \end{aligned}$$

where $\Delta = [1-\beta(1-\delta)]$. For the exports sector it is

$$\begin{aligned} &-\sigma\tilde{c}_t + \sigma \mathrm{E}_t\left[\tilde{c}_{t+1}\right] + \varepsilon_T\left(\tilde{k}^x_{t+1} - \tilde{k}^x_t\right) \\ &= \Delta \mathrm{E}_t\left[\widetilde{w}^{z,T}_{t+1} + \bar{\alpha}\left(\tilde{x}_{t+1} - \widetilde{A}^T_{t+1} - \tilde{k}^x_{t+1}\right)\right] + \beta\varepsilon_T \mathrm{E}_t\left[\tilde{k}^x_{t+2} - \tilde{k}^x_{t+1}\right], \end{aligned} \tag{A.23}$$

where the second equilibrium condition of equations (1.27) is used.

In *Appendix A.2* it was shown that the inflation rate in sector $s = T$, N is determined by

$$\pi^s_t - \vartheta_s \pi^s_{t-1} = \beta \mathrm{E}_t\left[\pi^s_{t+1} - \vartheta_s \pi^s_t\right] + \xi_s \widetilde{mc}^s_t. \tag{A.24}$$

The average real marginal cost is defined by

$$mc^s_t = \frac{MC^s_t}{P^s_t},$$

and by using equation (1.24) it can be expressed as

$$\widetilde{mc}^s_t = \hat{\alpha}\left(\tilde{y}^s_t - \tilde{k}^s_t\right) - \bar{\alpha}\widetilde{A}^s_t + \widetilde{w}^{z,s}_t + \chi_s \widetilde{P}^R_t, \tag{A.25}$$

where $\chi_T = a_N$ and $\chi_N = -a_T$.

The equation for the inflation rate in the exports sector in version B can be derived as

$$\pi^{x*}_t - \vartheta_T \pi^{x*}_{t-1} = \beta \mathrm{E}_t\left[\pi^{x*}_{t+1} - \vartheta_T \pi^{x*}_t\right] + \xi_T \widetilde{mc}^x_t. \tag{A.26}$$

To derive the above equation it is assumed that the technology and the price setting parameters of the tradable and the exports sector are the same. Hence, the coefficients of these two equations are the same as in equation (A.24). The average real marginal costs are defined as

$$mc^x_t = \frac{MC^x_t}{e_t P^{x*}_t}.$$

Hence, using equation (1.24) provides the log-linearized real marginal cost formula of the exports sector:

$$\widetilde{mc}^x_t = \hat{\alpha}\left(x_t - \tilde{k}^x_t\right) - \bar{\alpha}\widetilde{A}^T_t + n_T\left(\sigma\tilde{c}_t + \varphi\tilde{l}_t - \tilde{q}_t\right) - \widetilde{P}^{x*}_t. \tag{A.27}$$

As mentioned in *section 1.4.6*, exchange rate policy is represented by the following simple rule:

$$d\tilde{e}_t = -\omega\left(a_T \pi^T_{t-1} + a_N \pi^N_{t-1}\right) + \mathcal{S}^{de}_t, \tag{A.28}$$

where $d\tilde{e}_t = \tilde{e}_t - \tilde{e}_{t-1}$ is the nominal depreciation rate, and $\mathcal{S}_t^{de}$ is the shock of an exogenous nominal depreciation.

Version A of the model (no PTM) contains 18 equations: (A.9), (A.10), (A.12), (A.14), (A.16) – (A.18), (A.20), (A.21), (A.28), and the two-equation systems of formulas (A.13), (A.22), (A.24), and (A.25). This system determines the trajectories of the following 18 endogenous variables: $\tilde{c}_t$, $\tilde{x}_t$, $\tilde{I}_t$, $\tilde{y}_t^T$, $\tilde{y}_t^N$, $\tilde{l}_t$, $\tilde{k}_t^T$, $\tilde{k}^N$, $\widetilde{mc}_t^T$, $\widetilde{mc}_t^N$, $\widetilde{w}_t^{z,T}$, $\widetilde{w}_t^{z,N}$, $\tilde{q}_t$, $\tilde{q}_t^T$, $\widetilde{P}_t^R$, $d\tilde{e}_t$, π_t^T, π_t^N.

To obtain version B (PTM, LCP) replace equations (A.14) and (A.18) by equations (A.15) and (A.19). Furthermore, add equations (A.11), (A.23), (A.26), and (A.27) to the system. This is a system of 22 equations. It determines the paths of the variables belonging to version A, plus the trajectories of $\tilde{k}_t^x$, $\widetilde{mc}_t^x$, $\widetilde{P}_t^{x*}$, π_t^{x*}.

A.4 Second moments of the model

This section provides the formulas for statistics used in *section 1.5.1*. First, let us supplement the log-linearized model of *Appendix A.3* with two new variables, $d\tilde{q}_t$, $d\tilde{q}_t^T$, and the equations defining them:

$$d\tilde{q}_t = \tilde{q}_t - \tilde{q}_{t-1}, \quad d\tilde{q}_t^T = \tilde{q}_t^T - \tilde{q}_{t-1}^T.$$

Let us denote by Y_t the vector of endogenous variables of the augmented system. Since it is assumed that the exogenous shocks of the model are uncorrelated, one can study them separately. Let us denote the nth shock by $\mathcal{S}_t^n$. It is determined by a first-order autoregressive process:

$$\mathcal{S}_t^n = \varrho_n \mathcal{S}_{t-1}^n + \epsilon_t^n, \quad |\varrho_n| < 1, \quad \mathrm{E}\left[\epsilon_t^n\right] = 0, \quad \mathrm{E}\left[(\epsilon_t^n)^2\right] = \varsigma_n^2.$$

The undetermined coefficient method, the solution algorithm used, provides matrix Q and R, and the paths of the endogenous variables are determined by[2]

$$Y_t = QY_{t-1} + R\mathcal{S}_t^n.$$

Define the following variables and matrix:

$$\bar{Y}_t = \begin{bmatrix} Y_t \\ \mathcal{S}_{t+1}^n \end{bmatrix}, \quad \mathcal{E}_t = \begin{bmatrix} 0 \\ \epsilon_{t+1}^n \end{bmatrix}, \quad F = \begin{bmatrix} Q & R \\ \varrho_n & 0 \end{bmatrix}.$$

Then the log-linearized model can be represented by the following *first-order vector autoregressive* process:

$$\bar{Y}_t = F\bar{Y}_{t-1} + \mathcal{E}_t.$$

[2]The eigenvalues of matrix Q are smaller than 1 in absolute value.

Let us denote by g the number of the elements of $\bar{Y}$ and $\mathcal{E}_t$. The variance-covariance matrix of $\mathcal{E}_t$ is Σ, which is a $g \times g$ matrix, with elements equal to zero, except the gth diagonal element, which is equal to ς_n^2.

Let us denote by V_0 the *unconditional variance-covariance* matrix of $\bar{Y}_t$, that is,

$$V_0 = \mathrm{E}\left[\bar{Y}_t \bar{Y}_t'\right],$$

and let us denote by $V_0(ij)$ the element in row i and column j. Apply formula (10.2.16) and (10.2.17) of Hamilton (1994), then

$$\mathrm{vec}(V) = \left(I_{g^2 \times g^2} - \mathcal{A}\right)^{-1} \mathrm{vec}(\Sigma),$$

where $I_{g^2 \times g^2}$ is an appropriate identity matrix, $\mathcal{A} = F \otimes F$. Symbol $\otimes$ represents the *Kronecker product*, and operator *vec* transforms a quadratic matrix into a column vector by stacking the columns of the matrix one below the other, with the columns ordered from left to right.

The *lth autocovariance* matrix is defined by

$$V_l = \mathrm{E}\left[\bar{Y}_t \bar{Y}_{t-l}'\right].$$

Formula (10.2.21) provides an expression for it:

$$V_l = F^l V_0.$$

The *variance* of the ith endogenous variable (that is, the ith element of $\bar{Y}_t$) is $V_0(ii)$. The *covariance* of the ith and jth endogenous variable is $V_0(ij)$. Their *correlation coefficient* is $V_0(ij)\left[V_0(ii)V_0(jj)\right]^{-\frac{1}{2}}$. Finally, the *$l$th autocovariance* of the ith endogenous variable is defined by $V_l(ii)V_0(ii)^{-1}$.

Appendix B

Addendum to chapter 2

B.1 The steady state

In this section the non-stochastic steady state of the model is described. Variables without time indices refer to their steady-state values.

The labor and imports demand functions, i.e. equations (2.13) have the following form in the steady state:

$$l^{1-\alpha} = a\frac{c}{A}, \quad m^{1-\alpha} = (1-a)\frac{c}{A},$$

where it is assumed that in the steady state all firms have the same level of production and input demand. It is assumed that $x/A = m^{1-\alpha}$, as a consequence, $x = c(1-a)/a$, or $l^{1-\alpha} = c/A$. Thus, the labor demand is the same as in a closed economy with similar technology, but $z = l$.

Furthermore, it is assumed that in the steady state international trade is balanced, hence $Px = eP^{m*}m$. Let us take as given the share of export in GDP:

$$s^x = \frac{Px}{Pc + Px - eP^{m*}m} = \frac{x}{c} = \frac{1-a}{a}.$$

According to Hungarian data approximately $s^x = 0.6$, hence $a = 0.625$ and $x = 0.6c$. Coefficients a_l and a_m can be calculated as $a = a_l^{1-\alpha}$ and $(1-a) = a_m^{1-\alpha}$. Thus, $a_l = 0.494$ and $a_m = 0.230$.

It is assumed that in the steady state c is equal to the welfare maximizing consumption level of the above closed economy. The social welfare maximizing allocation can be given by the solution of the following optimization problem:

$$\max_{c_t} \sum_{t=1}^{\infty} \beta^{t-1} \mathrm{E}_1 \left[u(c_t - hc_{t-1}) - v\left(c_t^{\bar{\alpha}} A_t^{-\bar{\alpha}}\right)\right],$$

where $\bar{\alpha} = (1-\alpha)^{-1}$. The corresponding first-order condition in the steady-state is

$$u'(c(1-h))(1-\beta h) = \bar{\alpha} v'\left(c^{\bar{\alpha}} A^{-\bar{\alpha}}\right)) c^{\hat{\alpha}} A^{\bar{\alpha}}, \tag{B.1}$$

where $\hat{\alpha} = \alpha\bar{\alpha}$. This implies that

$$1 = \bar{\alpha}\frac{(1-h)^{\sigma}}{1-\beta h} c^{\sigma+\varphi\bar{\alpha}+\hat{\alpha}} A^{-(1+\varphi)\bar{\alpha}}.$$

For the sake of simplicity, let us choose A such that $1 = (c+x)^{\hat{\alpha}} A^{-\bar{\alpha}}$, i.e. $(c+x)^{\alpha} = A$. $x = (1-a)c/a$, and this implies that $A = (c/a)^{\alpha}$. Substituting this into the previous expression yields

$$1 = \bar{\alpha}\frac{(1-h)^{\sigma}}{1-\beta h} c^{\sigma+\varphi} a^{(1+\varphi)\hat{\alpha}}.$$

Thus, $c = 1.02$ and $A = 1.177$. Using $l^{1-\alpha} = c/A$ one obtains $l = 0.806$.

The steady-state form of the labor supply equation is

$$\frac{W}{P} = \mu^{w}\left((1-h)c\right)^{\sigma} l^{\varphi},$$

where it is assumed that $\mu^{w} = \theta_w/(\theta_w - 1) = 1.5$ and $P = 1$. Hence, $W = 0.481$.

Balanced international trade implies that

$$\frac{P^{m*}}{P} = A x^{-\hat{\alpha}},$$

where I used that $e = 1$ and $m = x^{\bar{\alpha}}/A$. Knowing c one can calculate that $x = 0.612$, hence $P^{m*} = 1.505$.

The steady-state form of the price setting equation is

$$1 = \mu\bar{\alpha}(c+x)^{\hat{\alpha}} A^{-\bar{\alpha}}\left(a_l \frac{W}{P} + a_m \frac{P^{m*}}{P}\right).$$

There is only one value of $\mu = \tau\theta/(\theta-1)$ which satisfies this equation, since $\bar{\alpha}$ is a given parameter, and it is assumed that $(c+x)^{\hat{\alpha}} A^{-\bar{\alpha}} = 1$, and a_l, a_m, W, P^{m*}/P were calculated previously. Let us assume that the government sets the tax/transfer variable τ in such a way that the price setting equation is satisfied, hence $\mu = 1.143$.

B.2 Second-order approximation of the social welfare function

Following Woodford (2003, chapter 6), this section provides a second-order Taylor approximation of the social welfare function, which is the aggregate utility function of households. The social welfare function is the following:

$$\mathcal{U}(Y_0, S) = \sum_{t=1}^{\infty} \beta^{t-1} \left[u(H_t) - \int_0^1 v(l_t(j))\, dj \right], \tag{B.2}$$

where Y_0 is the vector of the date 0 state variables, S=$[S_1, S_2, S_3, \ldots]$, and S_t is the vector of the shock variables at date t. The Taylor approximation of the consumption term around the steady state is

$$\begin{aligned} u(H_t) &= u'(H)\left(\Delta c_t - h\Delta c_{t-1}\right) + \frac{1}{2}u''(H)\left[\left(\Delta c_t\right)^2 + h^2\left(\Delta c_{t-1}\right)^2\right] \\ &- hu''(H)\Delta c_t \Delta c_{t-1} + \text{t.i.p.} + o\left(||S||^3\right), \end{aligned}$$

where $\Delta c_t = c_t - c$, $o\left(||S||^3\right)$ contains the third and higher order error terms, and "t.i.p." means *terms that are independent of policy*, which are constant terms and exogenous variables. Obviously, exchange rate policy does not affect these terms.

Expressions

$$\begin{aligned} \Delta c_t &= c\left(\tilde{c}_t + \frac{1}{2}\tilde{c}_t^2\right) + o\left(||S||^3\right), \\ \left(\Delta c_t\right)^2 &= c\tilde{c}_t^2 + o\left(||S||^3\right) \end{aligned}$$

imply that

$$\begin{aligned} u(H_t) &= u'(H)c\left(\tilde{c}_t + \frac{1}{2}\tilde{c}_t^2\right) - hu'(H)c\left(\tilde{c}_{t-1} + \frac{1}{2}\tilde{c}_{t-1}^2\right) \\ &+ u''(H)c^2\left(\frac{1}{2}\tilde{c}_t^2 + \frac{1}{2}h^2\tilde{c}_{t-1}^2 - h\tilde{c}_t\tilde{c}_{t-1}\right) + \text{t.i.p.} + o\left(||S||^3\right), \end{aligned}$$

where the tilde denotes the log-deviation of a variable from its steady-state value. Rearranging the above expression yields

$$\begin{aligned} u(H_t) &= u'(H)c\left(\tilde{c}_t - h\tilde{c}_{t-1}\right) + \frac{1}{2}u'(H)c\left(\tilde{c}_t^2 - h\tilde{c}_t^2\right) \\ &+ \frac{1}{2}u''(H)c^2\left(\tilde{c}_t - \tilde{c}_{t-1}\right)^2 + \text{t.i.p.} + o\left(||S||^3\right). \end{aligned}$$

Using the fact that

$$\frac{u''(H)c}{u'(H)} = \frac{-\sigma(1-h)^{-\sigma-1}c^{-\sigma}}{(1-h)^{-\sigma}c^{-\sigma}} = \frac{-\sigma}{1-h}$$

one can obtain

$$\begin{aligned} u(H_t) &= u'(H)c\left\{\tilde{c}_t - h\tilde{c}_{t-1} + \frac{1}{2}\left[\tilde{c}_t^2 - h\tilde{c}_{t-1}^2 - \frac{\sigma}{1-h}\left(\tilde{c}_t - h\tilde{c}_{t-1}\right)^2\right]\right\} \\ &+ \text{ t.i.p.} + o\left(||S||^3\right). \end{aligned}$$

The discounted sum of the above expression is

$$\begin{aligned} \sum_{t=1}^{\infty}\beta^{t-1}u(H_t) &= (1-\beta h)u'(H)c\sum_{t=1}^{\infty}\beta^{t-1}\left\{\tilde{c}_t + \frac{1}{2}\left[1-\left(1+\beta h^2\right)\bar{\sigma}\right]\tilde{c}_t^2\right\} \\ &+ (1-\beta h)u'(H)c\sum_{t=1}^{\infty}\beta^{t-1}h\tilde{c}_t\tilde{c}_{t-1} \\ &+ \text{ t.i.p.} + o\left(||S||^3\right), \end{aligned} \tag{B.3}$$

where

$$\bar{\sigma} = \frac{\sigma}{(1-h)(1-\beta h)}.$$

Now let us consider the labor term of the social welfare function. If $l_t(i)$ is the labor input of firm i, then aggregate labor utilization is

$$l_t = \int_0^1 l_t(i)\,di = \mathrm{E}_i\left[l_t(i)\right].$$

This implies that

$$\tilde{l}_t = \mathrm{E}_i\left[\tilde{l}_t(i)\right] + \frac{1}{2}\mathrm{var}_i\left[\tilde{l}_t(i)\right] + o\left(||S||^3\right).$$

Recall equation (2.13),

$$l_t^{1-\alpha}(i) = n_t(i) = a\frac{y_t(i)}{A_t}.$$

Thus, $\tilde{l}_t(i) = \bar{\alpha}\tilde{n}_t(i)$, where $\bar{\alpha} = (1-\alpha)^{-1}$. As a consequence,

$$\tilde{l}_t = \bar{\alpha}\mathrm{E}_i\left[\tilde{n}_t(i)\right] + \frac{1}{2}\bar{\alpha}^2\mathrm{var}_i\left[\tilde{n}_t(i)\right] + o\left(||S||^3\right).$$

As for y_t, it is true for n_t that

$$n_t = \left(\int_0^1 n_t(i)^{\frac{\theta-1}{\theta}} \, di \right)^{\frac{\theta}{\theta-1}},$$

which implies that

$$\mathrm{E}_i \left[\tilde{n}_t(i) \right] = \tilde{n}_t - \frac{1}{2} \frac{\theta - 1}{\theta} \mathrm{var}_i \left[\tilde{n}_t(i) \right] + o\left(||S||^3 \right),$$

thus,

$$\tilde{l}_t = \bar{\alpha} \tilde{n}_t + \frac{1}{2} \bar{\alpha} \theta \left(1 + \theta \hat{\alpha} \right) \mathrm{var}_i \left[\widetilde{P}_t(i) \right] + o\left(||S||^3 \right), \tag{B.4}$$

where $\hat{\alpha} = \alpha \bar{\alpha}$, and it is used that $\mathrm{var}_i \left[\tilde{n}_t(i) \right] = \mathrm{var}_i \left[\tilde{y}_t(i) \right]$, and $\mathrm{var}_i \left[\tilde{y}_t(i) \right] = \theta^2 \mathrm{var}_i \left[\widetilde{P}_t(i) \right]$, which is a consequence of equation (2.12).

Since

$$n_t = a \frac{y_t}{A_t} = a \frac{c_t + x_t}{A_t} = \frac{c_t}{A_t} a \left(1 + \frac{x_t}{c_t} \right),$$

variable $\tilde{n}_t$ can be expressed as

$$\tilde{n}_t = \tilde{c}_t - \widetilde{A}_t - \tilde{g}_t,$$

where

$$\tilde{g}_t = \frac{x}{x + c} \left(\tilde{c}_t - \tilde{x}_t \right) = (1 - a) \left(\tilde{c}_t - \tilde{x}_t \right).$$

The inspection of (2.18) and (2.19) reveals that condition

$$\eta = \frac{(1 - h)(1 - \beta h)}{\sigma} \tag{B.5}$$

ensures that the difference between $\tilde{c}_t$ and $\tilde{x}_t$ depends only on stochastic shocks and initial conditions. Furthermore, formula (2.13), a consequence of the Leontieff technology, and condition (B.5) imply that variable $\tilde{r}_t = \widetilde{A}_t + \tilde{g}_t$ also depends only on stochastic shocks and initial conditions. Substituting $\tilde{n}_t = \tilde{c}_t - \tilde{r}_t$ into equation (B.4) yields

$$\tilde{l}_t = \bar{\alpha} \left(\tilde{c}_t - \tilde{r}_t \right) + \frac{1}{2} \bar{\alpha} \theta \left(1 + \theta \hat{\alpha} \right) \mathrm{var}_i \left[\widetilde{P}_t(i) \right] + o\left(||S||^3 \right). \tag{B.6}$$

The approximation of the disutility of labor of household j is given by

$$v(l_t(j)) = v'(l) l \left\{ \tilde{l}_t(j) + \frac{1}{2} (1 + \varphi) \tilde{l}_t^2(j) \right\} + \text{t.i.p.} + o\left(||S||^3 \right).$$

Aggregating this formula one obtains

$$\int_0^1 v(l_t(j))\,dj =$$
$$v'(l)l\left\{\mathrm{E}_j\left[\tilde{l}_t(j)\right]+\frac{1}{2}(1+\varphi)\left[\mathrm{E}_j\left[\tilde{l}_t(j)\right]^2+\mathrm{var}_j\left[\tilde{l}_t(j)\right]\right]\right\}+\text{t.i.p.}+o\left(||S||^3\right).$$

Using equations

$$\begin{aligned}\mathrm{E}_j\left[\tilde{l}_t(j)\right] &= \tilde{l}_t-\frac{1}{2}\frac{\theta_w-1}{\theta_w}\mathrm{var}_j\left[\tilde{l}_t(j)\right]+o\left(||S||^3\right),\\ \mathrm{E}_j\left[\tilde{l}_t(j)\right]^2 &= \tilde{l}_t^2+o\left(||S||^3\right),\end{aligned}$$

and $\mathrm{var}_j\left[\tilde{l}_t(j)\right]=\theta_w^2\mathrm{var}_j\left[\widetilde{w}_t(j)\right]$, an implication of equation (2.8), yields

$$\int_0^1 v(l_t(j))\,dj = \tag{B.7}$$
$$v'(l)l\left\{\tilde{l}_t+\frac{1}{2}(1+\varphi)\tilde{l}_t^2+\frac{1}{2}\theta_w(1+\varphi\theta_w)\mathrm{var}_j\left[\widetilde{W}_t(j)\right]\right\}+\text{t.i.p.}+o\left(||S||^3\right).$$

Substitute equation (B.6) into equation (B.7),

$$\int_0^1 v(l_t(j))\,dj = \tag{B.8}$$
$$\frac{(1-\beta h)u'(H)c}{2}\left\{2\tilde{c}_t+(1+\varphi)\bar{\alpha}\left(\tilde{c}_t^2-2\tilde{r}_t\tilde{c}_t\right)+\theta\left(1+\hat{\alpha}\theta\right)\mathrm{var}_i\left[\widetilde{P}_t(i)\right]\right\}$$
$$+\frac{(1-\beta h)u'(H)c}{2}\left\{\theta_w\bar{\alpha}^{-1}\left(1+\varphi\theta_w\right)\mathrm{var}_j\left[\widetilde{W}_t(j)\right]\right\}+\text{t.i.p.}+o\left(||S||^3\right),$$

where it is used that

$$(1-\beta h)u'(H)c=\bar{\alpha}v'(l)l,$$

since equation (B.1) implies that

$$(1-\beta h)u'(H)c=\bar{\alpha}v'(l)c^{\hat{\alpha}+1}A^{-\bar{\alpha}}=\bar{\alpha}v'(l)\left(\frac{c}{A}\right)^{\bar{\alpha}}.$$

Combining equations (B.3) and (B.8) yields the following expression for the utility function defined by equation (B.2):

$$\begin{aligned}\mathcal{U}(Y_0,S) = &- J\sum_{t=1}^{\infty}\beta^{t-1}\left\{\left[\left(1+\beta h^2\right)\bar{\sigma}+\varphi\bar{\alpha}+\hat{\alpha}\right]\tilde{c}_t^2-2h\bar{\sigma}\tilde{c}_t\tilde{c}_{t-1}\right\}\\ &- J\sum_{t=1}^{\infty}\beta^{t-1}\left\{2(1+\varphi)\bar{\alpha}\tilde{r}_t\tilde{c}_t+\theta\left(1+\theta\hat{\alpha}\right)\mathrm{var}_i\left[\widetilde{P}_t(i)\right]\right\}\\ &- J\sum_{t=1}^{\infty}\beta^{t-1}\theta_w\bar{\alpha}^{-1}\left(1+\theta_w\varphi\right)\mathrm{var}_j\left[\widetilde{W}_t(j)\right]+\text{t.i.p.}+o\left(||S||^3\right),\end{aligned}$$

where $J = (1 - \beta h)u'(H)c/2$. For the calculation of the coefficient of term $\tilde{c}_t^2$ it was taken into account that $(1+\varphi)\bar{\alpha} = 1 + \varphi\bar{\alpha} + \hat{\alpha}$.

As Woodford (2003, chapter 6) shows, the welfare analysis becomes inaccurate, if the approximation of the social welfare function contains first-order terms of endogenous variables. Condition (B.1), formula (2.13), which is an implication of the Leontieff technology, and condition (B.5) ensure that in the objective function there are only second-order terms of endogenous variables.

Define $\tilde{c}_t^{wr}$, the *welfare reference* level of consumption, which is determined by

$$\left[\left(1+\beta h^2\right)\bar{\sigma} + \varphi\bar{\alpha} + \hat{\alpha}\right]\tilde{c}_t^{wr} - \beta h\bar{\sigma}\mathrm{E}_t\left[\tilde{c}_{t+1}^{wr}\right] - h\bar{\sigma}\tilde{c}_{t-1}^{wr} = (1+\varphi)\bar{\alpha}\tilde{r}_t.$$

Using this definition one can express $\mathcal{U}(Y_0, S)$ as

$$\begin{aligned}\mathcal{U}(Y_0,S) = \; & - \; J\sum_{t=1}^{\infty}\beta^{t-1}\left[\left(1+\beta h^2\right)\bar{\sigma} + \varphi\bar{\alpha} + \hat{\alpha}\right]\left\{\tilde{c}_t^2 - 2\tilde{c}_t^{wr}\tilde{c}_t\right\} \\ & - \; J\sum_{t=1}^{\infty}\beta^{t-1}2h\bar{\sigma}\left\{\tilde{c}_t\tilde{c}_{t-1} - \tilde{c}_t\left(\beta\tilde{c}_{t+1}^{wr} + \tilde{c}_{t-1}^{wr}\right)\right\} \\ & - \; J\sum_{t=1}^{\infty}\beta^{t-1}\theta\left(1+\theta\hat{\alpha}\right)\mathrm{var}_i\left[\widetilde{P}_t(i)\right] \\ & - \; J\sum_{t=1}^{\infty}\beta^{t-1}\theta_w\bar{\alpha}^{-1}\left(1+\theta_w\varphi\right)\mathrm{var}_j\left[\widetilde{W}_t(j)\right] + \text{t.i.p.} + o\left(||S||^3\right).\end{aligned} \tag{B.9}$$

The expression

$$-J\sum_{t=1}^{\infty}\beta^{t-1}\left\{\left[\left(1+\beta h^2\right)\bar{\sigma} + \varphi\bar{\alpha} + \hat{\alpha}\right]\left(\tilde{c}_t^{wr}\right)^2 + \tilde{c}_t^{wr}\tilde{c}_{t-1}^{wr}\right\} + 2h\bar{\sigma}\tilde{c}_0\tilde{c}_1^{wr}, \tag{B.10}$$

depends only on the exogenous variable $\tilde{r}_t$ and initial conditions, that is, only on terms that are independent of policy (t.i.p.). Let us define the *consumption gap*,

$$\hat{c}_t = \tilde{c}_t - \tilde{c}_t^{wr}.$$

Using this definition, and adding the discounted sum (B.10) to equation (B.9) yields

$$\begin{aligned}\mathcal{U}(Y_0,S) = \; & - \; J\sum_{t=1}^{\infty}\beta^{t-1}\left\{\mathsf{A}\hat{c}_t^2 - 2\mathsf{B}\hat{c}_t\hat{c}_{t-1}\right\} \\ & - \; J\sum_{t=1}^{\infty}\beta^{t-1}\theta\left(1+\theta\hat{\alpha}\right)\mathrm{var}_i\left[\widetilde{P}_t(i)\right] \\ & - \; J\sum_{t=1}^{\infty}\beta^{t-1}\theta_w\bar{\alpha}^{-1}\left(1+\theta_w\varphi\right)\mathrm{var}_j\left[\widetilde{W}_t(j)\right] + \text{t.i.p.} + o\left(||S||^3\right),\end{aligned} \tag{B.11}$$

where $\mathsf{A} = (1+\beta h^2)\,\bar{\sigma} + \varphi\bar{\alpha} + \hat{\alpha}$ and $\mathsf{B} = h\bar{\sigma}$. Define δ_0 and δ in such a way that

$$\mathsf{A} = (1+\beta\delta^2)\delta_0, \quad \text{and} \quad \mathsf{B} = \delta\delta_0.$$

Then parameter δ is the solution of the following quadratic equation:

$$\mathsf{A}\delta = \mathsf{B}\left(1+\beta\delta^2\right).$$

Let us choose the smaller root of the above equation, which satisfies $0 \le \delta \le h$. Equation (B.11) can be simplified further, since

$$\begin{aligned}
\sum_{t=1}^{\infty}\beta^{t-1}\left(\mathsf{A}\hat{c}_t^2 - 2\mathsf{B}\hat{c}_t\hat{c}_{t-1}\right) &= \delta_0\sum_{t=1}^{\infty}\beta^{t-1}\left[\left(1+\beta\delta^2\right)\hat{c}_t^2 - 2\delta\hat{c}_t\hat{c}_{t-1}\right] \\
&= \delta_0\sum_{t=1}^{\infty}\beta^{t-1}\left(\hat{c}_t^2 - 2\delta\hat{c}_t\hat{c}_{t-1} + \delta^2\hat{c}_{t-1}^2\right) + \delta_0\delta^2\hat{c}_0 \\
&= \delta_0\sum_{t=1}^{\infty}\beta^{t-1}\left(\hat{c}_t - \delta\hat{c}_{t-1}\right)^2 + \text{t.i.p.}
\end{aligned}$$

Substituting the above expression into (B.11) yields

$$\begin{aligned}
\mathcal{U}(Y_0,S) = &- J\sum_{t=1}^{\infty}\beta^{t-1}\delta_0\left(\hat{c}_t - \delta\hat{c}_{t-1}\right) \qquad \text{(B.12)} \\
&- J\sum_{t=1}^{\infty}\beta^{t-1}\theta\left(1+\theta\hat{\alpha}\right)\text{var}_i\left[\widetilde{P}_t(i)\right] \\
&- J\sum_{t=1}^{\infty}\beta^{t-1}\theta_w\bar{\alpha}^{-1}\left(1+\theta_w\varphi\right)\text{var}_j\left[\widetilde{W}_t(j)\right] + \text{t.i.p.} + o\left(||S||^3\right).
\end{aligned}$$

Now it will be shown how it is possible to express the variance of prices and wages by price and wage inflation. First, the assumptions on pricing behavior imply that

$$\text{E}_i\left[\widetilde{P}_t(i)\right] = \gamma\text{E}_i\left[\widetilde{P}_{t-1}(i) + \vartheta\pi_{t-1}\right] + (1-\gamma)\widetilde{P}_t^o, \qquad \text{(B.13)}$$

$$\text{E}_i\left[\widetilde{P}_t^2(i)\right] = \gamma\text{E}_i\left[\left(\widetilde{P}_{t-1}(i) + \vartheta\pi_{t-1}\right)^2\right] + (1-\gamma)\left(\widetilde{P}_t^o\right)^2, \qquad \text{(B.14)}$$

where $\widetilde{P}_t^o$ denotes the price, which is chosen by those firms, which set their price in an optimal forward-looking way in period t.

Let $\bar{P}_t = \text{E}_i\left[\widetilde{P}_t(i)\right]$, then by equation (B.13) it is easy to show that

$$\bar{P}_t - \bar{P}_{t-1} - \vartheta\pi_{t-1} = (1-\gamma)\left(\widetilde{P}_t^o - \bar{P}_{t-1} - \vartheta\pi_{t-1}\right). \qquad \text{(B.15)}$$

Now let us express the variance of prices,

$$\begin{aligned}
\operatorname{var}_i\left[\widetilde{P}_t(i)\right] &= \operatorname{var}_i\left[\widetilde{P}_t(i) - \bar{P}_{t-1} - \vartheta\pi_{t-1}\right] \\
&= \mathrm{E}_i\left[\left(\widetilde{P}_t(i) - \bar{P}_{t-1} - \vartheta\pi_{t-1}\right)^2\right] - \mathrm{E}_i\left[\widetilde{P}_t(i) - \bar{P}_{t-1} - \vartheta\pi_{t-1}\right]^2 \\
&= \mathrm{E}_i\left[\left(\widetilde{P}_t(i) - \bar{P}_{t-1} - \vartheta\pi_{t-1}\right)^2\right] - \left(\bar{P}_t - \bar{P}_{t-1} - \vartheta\pi_{t-1}\right)^2 \\
&= \gamma\mathrm{E}_i\left[\left(\widetilde{P}_{t-1}(i) - \bar{P}_{t-1}\right)^2\right] + (1-\gamma)\left(\widetilde{P}_t^o - \widetilde{P}_{t-1} - \vartheta\pi_{t-1}\right)^2 \\
&- \left(\bar{P}_t - \bar{P}_{t-1} - \vartheta\pi_{t-1}\right)^2,
\end{aligned}$$

where equations (B.13) and (B.14) are used for the derivation of the last equality. Using the above formula and equation (B.15) yields

$$\operatorname{var}_i\left[\widetilde{P}_t(i)\right] = \gamma\operatorname{var}_i\left[\widetilde{P}_{t-1}(i)\right] + \frac{\gamma}{1-\gamma}(\bar{P}_t - \bar{P}_{t-1} - \vartheta\pi_{t-1})^2.$$

Since $\bar{P}_t = \widetilde{P}_t + o\left(||S||^2\right)$, one can obtain

$$\left(\bar{P}_t - \bar{P}_{t-1} - \vartheta\pi_{t-1}\right)^2 = (\pi_t - \vartheta\pi_{t-1})^2 + o\left(||S||^3\right).$$

Combining the previous formulas yields the following expression for the variance of prices:

$$\operatorname{var}_i\left[\widetilde{P}_t(i)\right] = \gamma\operatorname{var}_i\left[\widetilde{P}_{t-1}(i)\right] + \frac{\gamma}{1-\gamma}(\pi_t - \vartheta\pi_{t-1}) + o\left(||S||^3\right). \tag{B.16}$$

One can show that a similar expression is true for the variance of wages, i.e.

$$\operatorname{var}_j\left[\widetilde{W}_t(j)\right] = \gamma_w\operatorname{var}_j\left[\widetilde{W}_{t-1}(j)\right] + \frac{\gamma_w}{1-\gamma_w}(\pi_t^w - \vartheta_w\pi_{t-1}) + o\left(||S||^3\right). \tag{B.17}$$

Using equation (B.16), by recursive substitutions it is possible to show that

$$\begin{aligned}
&\sum_{t=1}^{\infty}\beta^{t-1}\operatorname{var}_i\left[\widetilde{P}_t(i)\right] = \qquad\qquad \text{(B.18)}\\
&\frac{\gamma\sum_{t=1}^{\infty}\beta^{t-1}}{(1-\gamma)(1-\beta\gamma)}(\pi_t - \vartheta\pi_{t-1})^2 + \text{t.i.p.} + o\left(||S||^3\right).
\end{aligned}$$

Similarly equation (B.17) implies that

$$\begin{aligned}
&\sum_{t=1}^{\infty}\beta^{t-1}\operatorname{var}_j\left[\widetilde{W}_t(j)\right] = \qquad\qquad \text{(B.19)}\\
&\frac{\gamma_w\sum_{t=1}^{\infty}\beta^{t-1}}{(1-\gamma_w)(1-\beta\gamma_w)}(\pi_t^w - \vartheta_w\pi_{t-1})^2 + \text{t.i.p.} + o\left(||S||^3\right).
\end{aligned}$$

Substitute equations (B.18) and (B.19) into equation (B.12), and use definitions (2.10) and (2.15), then

$$\begin{aligned}\mathcal{U}(Y_0, S) &= -J\sum_{t=1}^{\infty}\beta^{t-1}\delta_0\left(\hat{c}_t - \delta\hat{c}_{t-1}\right)^2 \\ &-J\sum_{t=1}^{\infty}\beta^{t-1}\left\{\frac{\theta}{\xi}(\pi_t - \vartheta\pi_{t-1}) + \frac{\bar{\alpha}^{-1}\theta_w}{\xi_w}(\pi_t^w - \vartheta_w\pi_{t-1})\right\} + \text{t.i.p.} + o\left(||S||^3\right).\end{aligned}$$

Let $\mathcal{J} = J(\theta/\xi + (1-\alpha)\theta_w/\xi_w)$, then the social welfare function can be expressed as

$$\mathcal{U}(Y_0, S) = -\mathcal{J}\sum_{t=1}^{\infty}\beta^{t-1}L_t + o\left(||S||^3\right) + \text{t.i.p.},$$

where

$$L_t = \lambda_c\left(\hat{c}_t - \delta\hat{c}_{t-1}\right)^2 + \lambda_\pi(\pi_t - \vartheta\pi_{t-1})^2 + \lambda_w(\pi_t^w - \vartheta_w\pi_{t-1})^2,$$

and

$$\begin{aligned}\lambda_c &= \frac{\delta_0}{\theta\xi^{-1} + (1-\alpha)\theta_w\xi_w^{-1}}, \quad \lambda_\pi = \frac{\theta\xi^{-1}}{\theta\xi^{-1} + (1-\alpha)\theta_w\xi_w^{-1}}, \\ \lambda_w &= \frac{(1-\alpha)\theta_w\xi_w^{-1}}{\theta\xi^{-1} + (1-\alpha)\theta_w\xi_w^{-1}}.\end{aligned}$$

This obviously implies that maximization of the expected utility function

$$\mathrm{E}_1\left[\mathcal{U}(Y_0, S)\right]$$

is equivalent to the minimization of the expected loss function

$$\sum_{t=1}^{\infty}\beta^{t-1}\mathrm{E}_1\left[L_t\right].$$

B.3 Stabilization of the consumption gap and inflation

It is a well known feature of closed economy New Keynesian models that if wages are flexible and there are no cost-push shocks, then it is possible for there to be simultaneous stabilization of the relevant welfare measures: the output gap and inflation. See, e.g., Clarida et al. (1999), Goodfriend and King (1997), Galí (2002), and Woodford (2003, chapter 7).

Galí and Monacelli (2002) show that if some certain conditions are satisfied, then even in small open economies the simultaneous stabilization of the output gap and the appropriate measure of inflation is possible.

Let us study whether similar assumptions make simultaneous stabilization possible in my model. Recall that in this model the relevant welfare measure is not the output gap, but a similar concept, the consumption gap. Let us assume that wages are flexible ($1/\xi_w = 0$), and there are no cost push shocks $\tilde{\upsilon}_t = \tilde{\upsilon}_t^w = 0$. Furthermore, for the sake of simpler comparison, suppose $h = 0$, $\alpha = 0$, and $\tilde{x}_t^* = \widetilde{P}_t^{x*} = \widetilde{P}_t^{m*} = 0$.

If wages are flexible, then combining equations (2.20) and (2.22) yields

$$(\sigma + a\varphi)\tilde{c}_t + (1-a)\varphi\tilde{x}_t - \varphi\widetilde{A}_t = \widetilde{w}_t.$$

Substituting this into equation (2.21) yields the following price setting equation:

$$\begin{aligned}
\pi_t - \vartheta\pi_{t-1} &= \beta \mathrm{E}_t\left[\pi_{t+1} - \vartheta\pi_t\right] + \xi a\frac{W}{W^z}(\sigma + a\varphi)\tilde{c}_t + \xi a\frac{W}{W^z}(1-a)\varphi\tilde{x}_t \\
&+ \xi(1-a)\frac{eP^{m*}}{W^z}\tilde{q}_t^d - \left(1 + a\frac{W}{W^z}\varphi\right)\widetilde{A}_t.
\end{aligned}$$

Replace consumption and exports: since $h = 0$ and $\alpha = 0$, equation (2.28) implies that

$$\tilde{c}_t = \nu_c\hat{c}_t + \nu\widetilde{A}_t - (1-a)\nu\tilde{x}_t,$$

where $\nu_c = [\sigma(1-a) + \varphi]/[\sigma(1-a) + \varphi - (1+\varphi)\alpha]$ and $\nu = (1+\varphi)/[\sigma(1-a) + \varphi - (1+\varphi)\alpha]$. Using the above expression and equation (2.19) the price setting equation can be expressed as

$$\begin{aligned}
&\pi_t - \vartheta\pi_{t-1} = \beta \mathrm{E}_t\left[\pi_{t+1} - \vartheta\pi_t\right] + \xi a\frac{W}{W^z}(\sigma + a\varphi)\nu_c\hat{c}_t \\
&+\xi\left\{(1-a)\frac{eP^{m*}}{W^z} + a\frac{W}{W^z}\eta\left[(1-a)\varphi - (\sigma + a\varphi)(1-a)\nu\right]\right\}\tilde{q}_t^d \\
&-\left(1 + a\frac{W}{W^z}\varphi - (\sigma + a\varphi)\nu\right)\widetilde{A}_t.
\end{aligned}$$

Obviously, in this case it is impossible to stabilize simultaneously the consumption gap and inflation, since in the price setting equation, beyond the consumption gap, there is another endogenous variable $\tilde{q}_t^d$, and it is easy to show that the coefficient of the productivity shock is non-zero. Simultaneous stabilization is possible only in the closed economy version of this model, i.e.

when $a = 1$. In this case $W = W^z$, $\nu_c = 1$, and $\nu = (1+\varphi)/(\sigma+\varphi)$, thus the price setting equation becomes the standard New Keynesian closed economy Phillips curve,

$$\pi_t - \vartheta\pi_{t-1} = \beta \mathrm{E}_t\left[\pi_{t+1} - \vartheta\pi_t\right] + \xi(\sigma+\varphi)\hat{c}_t.$$

The model of Monacelli (2003), which can be considered as a generalization of the model of Galí and Monacelli, has the same property, i.e. simultaneous stabilization is impossible. Monacelli in his generalized model relaxes the assumption of perfect import price pass-through. Imperfect pass-through implies his impossibility result. In my model import price pass-through is perfect. But since the imported goods are not used for consumption, the pass-through between the nominal exchange rate and CPI becomes imperfect, as in the model of Monacelli. Thus, it comes as no surprise that in my model simultaneous stabilization is also impossible.

B.4 Calculation of the optimal solution

In this model there is only one policy variable: the monetary authority determines the value of the nominal exchange rate at date $t = 1$, and it remains unchanged later. Formally, this means that the depreciation rate $d\tilde{e}_1$ is a decision variable, but $d\tilde{e}_t = 0$, for all $t \geq 2$. Joining to a currency union is not simply an exchange rate peg, but it means that pegging the exchange rate is perfectly credible, hence rational expectations imply that $\mathrm{E}_1\left[d\tilde{e}_t\right] = 0$, for all $t \geq 2$.

This implies that it is worthwhile treating the depreciation rate formally as a first-order autoregressive process, with autoregressive parameter $\phi_e = 0$, and the realizations of its innovations $\epsilon_t^e = 0$, for all $t \geq 2$. That is,

$$d\tilde{e}_t = \phi_e^{t-1} d\tilde{e}_{t-1} + \epsilon_t^e, \quad \mathrm{E}_1\left[d\tilde{e}_t\right] = \phi_e^{t-1} d\tilde{e}_1.$$

Thus the exogenous variables, namely the policy variable and the stochastic shocks are treated uniformly in the model.

All the stochastic shocks of the model are determined by first-order autoregressive processes, and it is assumed that the shocks are uncorrelated to each other. To simplify the calculations in equation (2.18) $\tilde{c}_{t-1}$ is replaced by

$$\tilde{c}_{t-1} = \frac{\tilde{c}_t - \epsilon_t^{c*}}{\phi_{c*}},$$

where ϕ_{c*} and ϵ_t^{c*} are the corresponding autoregressive parameter and innovation, respectively. The vector of the shocks is

$$S_t = [d\tilde{e}_t,\, \tilde{c}_t^*,\, \epsilon_t^{c*},\, \tilde{x}_t^*,\, \widetilde{P}_t^{F*},\, \widetilde{P}_t^{m*},\, \widetilde{A}_t,\, \tilde{\upsilon}_t,\, \tilde{\upsilon}_t^w]'.$$

The evolution of the exogenous variables is described by process

$$S_t = \Phi S_{t-1} + \mathcal{E}_t, \tag{B.20}$$

where coefficient matrix Φ is diagonal.

Let us supplement the log-linearized model of equations (2.18) – (2.24) with equation equation (2.28). This system of equations is solved by the undetermined coefficients algorithm. The output of the algorithm is the set of Q, $\bar{Q}$, Ω, and $\bar{\Omega}$ matrices, which are used to determine the paths of the endogenous variables,

$$Y_t = QY_{t-1} + \Omega S_t, \qquad \bar{Y}_t = \bar{Q}Y_{t-1} + \bar{\Omega} S_t, \tag{B.21}$$

where

$$Y_t = \left[\tilde{c}_t,\, \hat{c}_t\ \pi_t,\, \widetilde{w}_t,\, \tilde{x}_t,\, \tilde{q}_t^d\right]'$$

is the vector of the state variables, $\bar{Y}_t$ is the vector of other endogenous variables. It is required that the eigenvalues of matrix Q are smaller than 1 in absolute value. Using equations (B.20) and (B.21) one can show by recursive substitutions that

$$\mathrm{E}_1\left[Y_t\right] = K_t Y_0 + G_t S_1, \tag{B.22}$$

where

$$K_t = Q^t, \quad G_t = \sum_{n=1}^{t} Q^{t-n}\Omega\Phi^{n-1}.$$

Let us introduce some new notations:

$$\begin{aligned} K_t^\eta &= K_t - \eta K_{t-1}, \\ G_t^\eta &= G_t - \eta G_{t-1}, \end{aligned}$$

where $\eta = \delta,\ \vartheta,\ \vartheta_w,\ 1$. The row vectors of these matrices are denoted by $K_t^\eta(i\ :)$ and $G_t^\eta(i\ :)$, respectively. Substitute equation (B.22) into the objective function

$$\begin{aligned} &- \sum_{t=1}^{\infty} \beta^{t-1}\mathrm{E}_1\left[\lambda_c\left(\hat{c}_t - \delta\hat{c}_{t-1}\right)^2 + \lambda_\pi(\pi_t - \vartheta\pi_{t-1})^2\right] \\ &- \sum_{t=1}^{\infty} \beta^{t-1}\mathrm{E}_1\left[\lambda_w\left(\pi_t + \widetilde{w}_t - \widetilde{w}_{t-1} - \vartheta_w\pi_{t-1}\right)^2\right], \end{aligned}$$

which was derived in *Appendix B.2*. Using the above notations the objective function takes the form

$$
\begin{aligned}
&- \sum_{t=1}^{\infty} \beta^{t-1} \left\{ \lambda_c \left[K_t^{\delta}(2:) Y_0 + G_t^{\delta}(2:) S_1 \right]^2 + \lambda_\pi \left[K_t^{\vartheta}(3:) Y_0 + G_t^{\vartheta}(3:) S_1 \right]^2 \right\} \\
&- \lambda_w \sum_{t=1}^{\infty} \beta^{t-1} \left[\left(K_t^{\vartheta_w}(3:) + K_t^{1}(4:) \right) Y_0 + \left(G_t^{\vartheta_w}(3:) + G_t^{1}(4:) \right) S_1 \right]^2 .
\end{aligned}
$$

The first-order condition is

$$
-\frac{1}{2} \frac{\partial \left(\sum_{t=1}^{\infty} \beta^{t-1} \mathrm{E}_1 \left[L_t \right] \right)}{\partial \left(d\tilde{e}_1 \right)} = 0,
$$

i.e.

$$
\begin{aligned}
0 &= \lambda_c \sum_{t=1}^{\infty} \beta^{t-1} \left[K_t^{\delta}(2:) Y_0 + G_t^{\delta}(2:) S_1 \right] G_t^{\delta}(21) \\
&+ \lambda_\pi \sum_{t=1}^{\infty} \beta^{t-1} \left[K_t^{\vartheta}(3:) Y_0 + G_t^{\vartheta}(3:) S_1 \right] G_t^{\vartheta}(31) \\
&+ \lambda_w \sum_{t=1}^{\infty} \beta^{t-1} \left[K_t^{\vartheta_w}(3:) + K_t^{1}(4:) \right] Y_0 \left[G_t^{\vartheta_w}(31) + G_t^{1}(41) \right] \\
&+ \lambda_w \sum_{t=1}^{\infty} \beta^{t-1} \left[G_t^{\vartheta_w}(3:) + G_t^{1}(4:) \right] S_1 \left[G_t^{\vartheta_w}(31) + G_t^{1}(41) \right] .
\end{aligned}
$$

The second-order condition is given by the coefficient of $d\tilde{e}_1$, which is evidently positive, since it is the square of an expression. Hence the objective function is concave, thus the first-order condition provides the maximum.

The first-order condition can be expressed alternatively as

$$
0 = \sum_{j=1}^{6} \mathcal{K}_j^{Y} Y_0(j) + \sum_{s=1}^{9} \mathcal{K}_s^{S} S_1(s),
$$

where

$$
\begin{aligned}
\mathcal{K}_j^{Y} &= \sum_{t=1}^{\infty} \beta^{t-1} \left[\lambda_c K_t^{\delta}(2j) G_t^{\delta}(21) + \lambda_\pi K_t^{\vartheta}(3j) G_t^{\vartheta}(31) \right] \qquad \text{(B.23)} \\
&+ \sum_{t=1}^{\infty} \beta^{t-1} \lambda_w \left[K_t^{\vartheta_w}(3j) + K_t^{1}(4j) \right] \left[G_t^{\vartheta_w}(31) + G_t^{1}(41) \right] ,
\end{aligned}
$$

and

$$\mathcal{K}_s^S = \sum_{t=1}^{\infty} \beta^{t-1} \left[\lambda_c G_t^{\delta}(2j) G_t^{\delta}(21) + \lambda_{\pi} G_t^{\vartheta}(3j) G_t^{\vartheta}(31)\right] \tag{B.24}$$
$$+ \sum_{t=1}^{\infty} \beta^{t-1} \lambda_w \left[G_t^{\vartheta_w}(3j) + G_t^{1}(4j)\right] \left[G_t^{\vartheta_w}(31) + G_t^{1}(41)\right].$$

The above coefficients have closed form solutions. Since the eigenvalues of Q are different from each other, there exists a diagonal matrix M and an invertible matrix F, such that $M = FQ\hat{F}$, where $\hat{F} = F^{-1}$ and the diagonal elements of M are the eigenvalues of Q, which are denoted by μ_k. This implies that

$$K_t = FM^{t-1}\hat{F}, \quad G_t = \sum_{n=1}^{t} FM^{t-n}\hat{F}\Omega\Phi^{n-1}.$$

Using this one can show that the element in the ith row and jth column of K_t is given by

$$K_t(ij) = \sum_{k=1}^{6} K(ij,k)\mu_k^t, \tag{B.25}$$

where $K(ij,k) = f(ik)\hat{f}(kj)$, $f(ik)$ and $\hat{f}(kj)$ are the appropriate elements of F and $\hat{F}$, respectively. One can show by some calculations[1] that

$$G_t(is) = \sum_{k=1}^{6} \frac{\mu_k^t - \phi_s^t}{\mu_k - \phi_s} \widehat{\omega}(ik,s), \tag{B.26}$$

where ϕ_s is sth diagonal element of matrix Φ,

$$\widehat{\omega}(ik,s) = \sum_{l=1}^{6} \omega(ls)K(il,k),$$

and $\omega(ls)$ is the element in the lth row and sth column of matrix Ω.

Using equations (B.25) and (B.26) yields

$$K_t^{\eta}(ij) = \sum_{k=1}^{6} K(ij,k)\kappa_{\eta}^{k}\mu_k^{t-1},$$
$$G_t^{\eta}(is) = \sum_{k=1}^{6} \widehat{\omega}(ik,s)\frac{\kappa_{\eta}^{k}\mu_k^{t-1} - \psi_{\eta}^{s}\phi_s^{t-1}}{\mu_k - \phi_s},$$

[1]For the calculations one has to use the fact that in this model $\mu_k \neq \phi_s$, for all k and s.

where $\kappa_\eta^k = \mu_k - \eta$, $\psi_\eta^s = \phi_s - \eta$. Substituting the above expressions into equations (B.23) and (B.24) and using the fact that $\phi_1 = 0$ one can obtain

$$\mathcal{K}_j^Y = \sum_{k=1}^{6}\sum_{l=1}^{6}\widehat{\mathcal{K}}_j^Y(kl),$$

where

$$\begin{aligned}
\widehat{\mathcal{K}}_j^Y(kl) &= \lambda_c K(2j,k)\kappa_\delta^k \frac{\widehat{\omega}(2l,1)}{\mu_l}\left(\frac{\kappa_\delta^l}{1-\beta\mu_k\mu_l}+\delta\right)\\
&+ \lambda_\pi K(3j,k)\kappa_\vartheta^k \frac{\widehat{\omega}(3l,1)}{\mu_l}\left(\frac{\kappa_\vartheta^l}{1-\beta\mu_k\mu_l}+\vartheta\right)\\
&+ \lambda_w \left(K(3j,k)\kappa_{\vartheta_w}^k + K(4j,k)\kappa_1^k\right)\frac{\widehat{\omega}(3l,1)}{\mu_l}\left(\frac{\kappa_{\vartheta_w}^l}{1-\beta\mu_k\mu_l}+\vartheta_w\right)\\
&+ \lambda_w \left(K(3j,k)\kappa_{\vartheta_w}^k + K(4j,k)\kappa_1^k\right)\frac{\widehat{\omega}(4l,1)}{\mu_l}\left(\frac{\kappa_1^l}{1-\beta\mu_k\mu_l}+1\right),
\end{aligned}$$

and

$$\mathcal{K}_s^S = \sum_{k=1}^{6}\sum_{l=1}^{6}\widehat{\mathcal{K}}_s^S(kl),$$

where

$$\begin{aligned}
&\widehat{\mathcal{K}}_s^S(kl) =\\
&\lambda_c\frac{\widehat{\omega}(2l,1)\widehat{\omega}(2k,s)}{(\mu_k-\phi_s)\mu_l}\left[\kappa_\delta^l\left(\frac{\kappa_\delta^l}{1-\beta\mu_k\mu_l}+\delta\right)-\psi_\delta^s\left(\frac{\kappa_\delta^l}{1-\beta\mu_l\phi_s}+\delta\right)\right]\\
&+\lambda_\pi\frac{\widehat{\omega}(3l,1)\widehat{\omega}(3k,s)}{(\mu_k-\phi_s)\mu_l}\left[\kappa_\vartheta^l\left(\frac{\kappa_\vartheta^l}{1-\beta\mu_k\mu_l}+\vartheta\right)-\psi_\vartheta^s\left(\frac{\kappa_\vartheta^l}{1-\beta\mu_l\phi_s}+\vartheta\right)\right]\\
&+\lambda_w\frac{\widehat{\omega}(3l,1)\widehat{\omega}(3k,s)}{(\mu_k-\phi_s)\mu_l}\left(\widehat{\omega}(3k,s)\kappa_{\vartheta_w}^k+\widehat{\omega}(4k,s)\kappa_1^k\right)\left(\frac{\kappa_\vartheta^l}{1-\beta\mu_k\mu_l}+\vartheta_w\right)\\
&-\lambda_w\frac{\widehat{\omega}(3l,1)\widehat{\omega}(3k,s)}{(\mu_k-\phi_s)\mu_l}\left(\widehat{\omega}(3k,s)\psi_{\vartheta_w}^s+\widehat{\omega}(4k,s)\psi_1^s\right)\left(\frac{\kappa_\vartheta^l}{1-\beta\phi_s\mu_l}+\vartheta_w\right)\\
&+\lambda_w\frac{\widehat{\omega}(4l,1)\widehat{\omega}(3k,s)}{(\mu_k-\phi_s)\mu_l}\left(\widehat{\omega}(3k,s)\kappa_{\vartheta_w}^k+\widehat{\omega}(4k,s)\kappa_1^k\right)\left(\frac{\kappa_\vartheta^l}{1-\beta\mu_k\mu_l}+1\right)\\
&-\lambda_w\frac{\widehat{\omega}(4l,1)\widehat{\omega}(3k,s)}{(\mu_k-\phi_s)\mu_l}\left(\widehat{\omega}(3k,s)\psi_{\vartheta_w}^s+\widehat{\omega}(4k,s)\psi_1^s\right)\left(\frac{\kappa_\vartheta^l}{1-\beta\phi_s\mu_l}+1\right).
\end{aligned}$$

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