Essays on Firm Dynamics and Financial Markets

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To my grandparents.

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Abstract

This thesis aims to better understand the interplay between financial markets and firm dynamics. In the first chapter I study the firm-level implications of asset bubbles. I relax the no-Ponzi-game condition in a model with firm heterogeneity and firm entry and exit. In equilibrium, the price of a firm may contain a bubble component in addition to the fundamental component, i.e. the net present value of profits. I show that bubbles act as subsidies to firm entry because they raise the return on the establishment and investment of new firms. The second chapter investigates different types of exiters by embedding different endogenous firm exit into a model with heterogeneous firms. The main finding is that in a recession, highly leveraged firms with high productivity but few assets become very likely to exit. However, due to the low wage rate in a recession, low leverage firms with low productivity become even more likely to survive. The prediction is consistent with our firm-level evidence about leverage.

Resumen

Esta tesis pretende comprender mejor la interacción entre los mercados financieros y la dinámica de la empresa. En el primer capítulo estudio las implicaciones a nivel de empresa de las burbujas de activos. Relajo la condición de no-Ponzi-game en un modelo con heterogeneidad y entrada y salida de empresas. En equilibrio, el precio de una empresa puede contener un componente de burbuja además del componente fundamental, es decir, el valor actual neto de los beneficios. Muestro que las burbujas actúan como subsidios para la entrada de empresas porque aumentan el retorno sobre el establecimiento y la inversión de nuevas empresas. El segundo capítulo investiga diferentes tipos de salidas mediante la incorporación de salida endógena de empresas en un modelo con empresas heterogéneas. El principal hallazgo es que en una recesión, las empresas altamente apalancadas con alta productividad, pero con pocos activos, tienen muy probabilidades de salir. Sin embargo, debido a la baja tasa de salarios en una recesión, las empresas con poco apalancamiento y baja productividad son aún más propensas a sobrevivir. La predicción es consistente con nuestra evidencia a nivel de empresa sobre apalancamiento.

Preface

The main goal of this thesis is to help understand the interplay between financial markets and firm dynamics. The thesis is composed of two chapters, respectively studying two topics: asset bubbles and firm exit. Firstly, despite the fast growth in the past years, the literature of firm dynamics and the literature of asset bubbles have neglected each other for a long time. Secondly, although firm exit has been an important subject economic research, little attention has been given on the difference among exiters, and the dynamic composition of exiters. This thesis aims to fill these gaps in the literatures.

The first chapter of the thesis studies the macroeconomic effects of asset bubbles from the perspective of firms. I introduce bubbles into a model with firm heterogeneity and firm entry and exit: in a bubbly equilibrium, the price of a firm contains a fundamental component, which represents the net present value of profits, and a bubble component. I show that bubbles act as subsidies to new firms and have the following implications: i) bubbles lower the average productivity and profitability of new firms; ii) bubbles increase the number of firms, wages, and aggregate output; iii) along transition dynamics, bubbles subsidize new firms rather than incumbents, aggravating misallocation and therefore depressing aggregate productivity. The model can be used to discriminate the alternative explanations of business cycles, like shocks to productivity, and shocks to financial frictions. I argue that the Spanish economic expansion before the global financial crisis can be well interpreted as a consequence of a bubble boom, and the recession as an outcome of a bubble crash.

The second chapter is a joint work with Roberto Ramos (Bank of Spain). We investigate the features of different types of exiters, and how

the composition of exiters change over time. We embed different endogenous firm exit into a model with heterogeneous firms. We incorporate different financial frictions. Firms are subject to not only collateral constraints but also non-negative dividend constraints. Firm exit can be either defaulting or non-defaulting, solvent or insolvent. We argue that defaulting exiters are highly leveraged productive firms while non-defaulting exiters are low leverage obese firms. Our model predicts that negative shocks increase defaulting exit rate, but decrease non-defaulting exit rate through lower wages. The results are in line with the documented increase of relative leverage ratio of exiters during the Great Recession, and the recent evidence about the sullying effects of recessions. Moreover, we also show that, during a productivity-driven recession, the increase of defaulting exit stems from the increase of insolvency. However, if a recession is triggered instead by a negative financial shock, the soar of defaulting exit is mainly accounted for by the increasing defaulting exit of solvent firms.

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Chapter 1

ASSET PRICE BUBBLES AND THE DISTRIBUTION OF FIRMS

1.1 Introduction

In the past two decades, most developed economies witnessed spectacular oscillations of asset prices. Notably, the asset price booms and busts generally coincided with fluctuations in credit, investment, and output, and they usually prove difficult to be explained by changes of fundamentals.[1] The speculative and procyclical nature of asset prices has renewed the interest in understanding the macroeconomic implications of asset bubbles. Indeed, it has been argued that asset bubbles may have expansionary effects on aggregate economic activity as they can relax financial con-

[1] See LeRoy (2004), Shiller (2005, 2014), Fernandez-Villaverde and Ohanian (2009) and Fernandez-Villaverde, Garicano, and Santos (2013).

straints.[2] However, it remains unclear what implications bubbles may have at the firm-level. This paper is an attempt to fill this gap in the literature.

I develop a model with bubbles, firm heterogeneity, and firm entry and exit. In equilibrium, the price of a firm may contain a bubble component in addition to the fundamental component, i.e. the net present value of profits. I show that bubbles act as subsidies to firm entry because they raise the return on the establishment and investment of new firms. Within this framework, bubbles have the following implications: i) by subsidizing entry, bubbles lower the average productivity and profitability of new firms; ii) at the aggregate-level, the subsidy on firm entry leads to an expansion in the number of firms, wages, and aggregate output, even in the absence of financial frictions; iii) along transition dynamics, since they subsidize new firms but not incumbent firms, bubbles aggravate misallocation and therefore depress aggregate productivity.

These theoretical insights are important for us to differentiate among the various explanations of business cycles. Besides characterizing the effects of bubbles, I use the model to study shocks to productivity and financial intermediation. I find that, at the aggregate-level, the effects of bubbles are analogous to the effects of positive shocks to productivity and financial intermediation, given that these shocks may also boost the number of firms, wages, credit, and aggregate output. Nonetheless, one distinction stands out when we consider the distribution of firms. Expansionary shocks to productivity and financial intermediation increase the average productivity or profitability of new firms, whereas bubbles lower the average productivity and profitability of new firms by subsidizing the

[2]See Asriyan et al. (2016), Farhi and Tirole (2011), Martin and Ventura (2011,2012), and Miao and Wang (2015).

entry of the less productive firms.

The model can shed light on the recent Spanish experience. Spanish GDP grew significantly between early 1990s and the Great Recession, and this expansion was accompanied by a dramatic boom in asset prices.[3] In principle, the output expansion can be interpreted as an outcome of high productivity and an improvement in financial intermediation. However, using firm-level AMADEUS data for Spain, I find that the new firms entering the market at the peak of the economic expansion were less productive and profitable than the entrants after the outbreak of the financial crisis. It is difficult for fundamental factors such as productivity or financial frictions to reconcile these facts, but they arise naturally as an outcome of bubbles. When an economic expansion is fueled by a bubble, less productive firms find it optimal to enter the market even if the net present value (NPV) of their profits is lower than the entry cost, and the entry of these firms lowers the average productivity and profit of entrants. When a bubble crashes, however, less productive firms no longer find it optimal to enter the market; consequently, the average productivity and profit of new firms increase during the recession.

The model is developed in the spirit of Lucas (1978) and Hopenhayn (1992): firms are heterogeneous in productivity, and the production function exhibits decreasing returns to scale (DRS); incumbents are price takers, and prospective entrants make entry decisions. The novel feature of this model is that the prices of firms contain two components: a fundamental component, which equals the NPV of profits, and a bubble component. Bubbles can subsidize firms along both the extensive and the intensive margins. Along the extensive margin, bubbles subsidize firm

[3] See Fernandez-Villaverde and Ohanian (2009) and Fernandez-Villaverde, Garicano, and Santos (2013).

entry and increase the number of firms. Because of decreasing returns to scale, the subsidy along the extensive margin leads to an expansion of aggregate output, even in the absence of financial frictions. Along the intensive margin, bubbles subsidize the size of a given firm if the size of the bubble component is increasing in the firm size. The subsidy along the intensive margin encourages entrepreneurs to start their firms with more capital.

To help build intuition, I study stationary equilibria in the absence of aggregate uncertainty. I characterize the existence conditions for bubbly equilibria, in which the aggregate bubble is stable over time, even though the bubble component of an individual firm is always explosive. These stationary equlibria feature a closed-form solution. I perform comparative statics analysis on these equilibria and show that the effects of bubbles are very distinct from those of increasing productivity and alleviating financial frictions. However, interestingly, the effects of bubbles are similar to those of an interest rate reduction. A decrease in interest rates has two effects. Firstly, it lowers the cost of capital. Secondly, it increases discount factor and therefore lowers the break-even profit required to compensate the entry cost. I show that the two subsidy effects of bubbles are respectively analogous to the two effects of a declining interest rate.

Moreover, I analyze net output in a stationary bubbly equilibrium. Net output is the amount of consumption good produced in every period, which equals the difference between aggregate output and investment. It can be shown that bubbles have ambiguous effects on net output. On the one hand, because of the subsidy effects, a bubble always increases investment and therefore increases aggregate output. On the other hand, since capital is subject to diminishing returns, the increase in investment may exceed the increase in output. Therefore a bubble does not necessarily improve net output. In fact, a bubble is more likely to boost net output when capital is scarce, or when the size of the bubble is small. Similarly, a financial reform does not necessarily increase net output, especially when the size of the bubble (subsidy) is large. When the size of the bubble is large, investment is heavily subsidized, and financial frictions can restrain entrepreneurs from excessive investment. Hence alleviating financial frictions may reduce net output.

Furthermore, I illustrate how bubbles can give rise to misallocation. I study stochastic bubbles using a dynamic example with parsimonious parameters: in response to a bubble shock, the simulated aggregate output growth increases, wheras the simulated aggregate productivity growth decreases. Intuitively, along transition dynamics, stochastic bubbles generate dispersion in capital intensity across firms, as some cohorts of firms get bubble subsidy but other cohorts do not; thereby bubble shocks aggravate misallocation and dampen aggregate productivity. I also show that bubble shocks can significantly contribute to the magnitude of output fluctuations.

The recent literature on rational bubbles focuses mainly on the macroeconomic implications. Bubbles can serve as either collateral or liquidity, in the presence of financial constraints. Martin and Ventura (2011, 2012, 2016) study an economy with collateral constraints. Bubbles can supplement collateral and thus relax credit constraints and boost investment and output. They also point out that sentiment shocks, the shocks reflected by the stochastic size of bubbles, are important sources of aggregate fluctuations. Miao and Wang (2015) analyze real effect of bubbly collateral in an infinite-horizon framework. They (2014) also investigate how bubbles reallocate resources between productive and non-productive sectors, and how the reallocation affect economic growth. Farhi and Tirole (2011) study how bubbles relax liquidity constraints: if fundamental liquidity is scarce, bubbles can be used as a substitute to be devoted to projects. Ventura (2012) explores the interaction between bubbles and capital cost: bubbles crowd out (inefficient) investment in capital-abundant economies and increase (efficient) investment in capital-scarce economies. My paper is perhaps closest to the one by Olivier (2000), which is the first paper, as far as I know, to study subsidy effect of bubbles. Olivier (2000) extends the horizontal innovation model by assuming that bubbles are attached to different "blue prints", so that bubbles serve as a subsidy to R&D. A common feature of the paper by Olivier (2000) and this paper is that in both papers bubbles can only be achieved after making specific sorts of investment: R&D in the paper by Olivier (2000), and firm entry in the current paper. However, my paper focuses mainly on the implications on the distribution of firms, and studies the aggregate impact of bubbles through the lens of firms.

This paper is also closely related to the wide body of research on firm and industry dynamics. My work is to some extent inspired by Lee and Mukoyama (2015), and Clementi and Palazzo (2015). Lee and Mukoyama (2015) focus on the cyclical pattern of firm entry and exit. Clementi and Palazzo (2015) emphasize the amplifying mechanism of procyclical firm entry and exit. Both works introduce aggregate productivity shocks as the source of fluctuation. My paper is also related to the vast literature incorporating credit constraints to heterogeneous-firm framework. Midrigan and Xu (2014) build up a quantitative Hopenhayn model where new entrepreneurs have no initial assets, nor external financing but can overcome the financial constraint by accumulating funds over time. Calibrating the model with Korean data, they argue that the productivity loss from a low level of firm entry can be very large. Jermann and Quadrini (2007) explain the comovement of stock prices, credit and output growth rate in 1990s as a result of expected increase of growth rate. They argue that the stock market boom in 1990s was induced by the expectation of higher future growth rate; the increased asset prices relaxed the credit constraint and increased the initial investment and capital intensity, and thus increased output and labor productivity.

The recent debt crisis in Eurozone has motivated a literature exploring the credit boom and misallocation in southern Europe. Gopinath et al. (2015) document a significant increase in misallocation of capital in Spain. They argue that adjustment costs and financial constraints play critical roles in generating the dispersion of the marginal product of capital. Garcia-Santana et al. (2016) study misallocation across firms in Spain. They find that deterioration of allocative efficiency in Spain is mainly driven by misallocation across firms rather than across industries. They also find that the measure of misallocation significantly exacerbated using an unbalanced panel with the full sample of firms, compared with the same measure using a balanced panel with permanent sample of firms: the result seems to support that the entry of firms is at the root of the TFP decline in Spain. In my model, bubbles relax financial constraints and subsidize capital inputs for new entrants, so that fluctuations in bubbles generate dispersion in the marginal product of capital across different cohorts of firms, therefore deteriorate allocative efficiency and lower aggregate productivity.

The paper proceeds as follows. Section 2 presents the facts which motivate this paper. Section 3 develops the baseline model. Section 4 delivers the stationary equilibria, as in Hopenhayn (1992) and Melitz (2003), without aggregate shocks. The comparative statics is studied in Section 5. Section 6 studies the dynamic model with aggregate uncertainties. Section 7 concludes with a discussion of future research.

1.2 Motivating Facts

1.2.1 Firm Entry

Figure 1 plots the number of newly registered companies and the total number of companies in Spain since 1999. The data is from the DIRCE, a database provided by Spanish National Statistics Institute and containing statistical information for the population of Spanish companies. The number of entrants experienced an expansion, reaching the height in year 2007, before a rapid downturn during the financial crisis. The accelerating firm entry increased the total number of companies by about one half from 1999 to 2008. However, following the financial crisis and the housing bubble crash, the number of entrants experienced a drastic decline and thereafter stagnated during the prolonged recession. The slowdown of firm entry coincided with a contraction in the number of companies: by 2014 the total number of companies dropped by 10 percent.

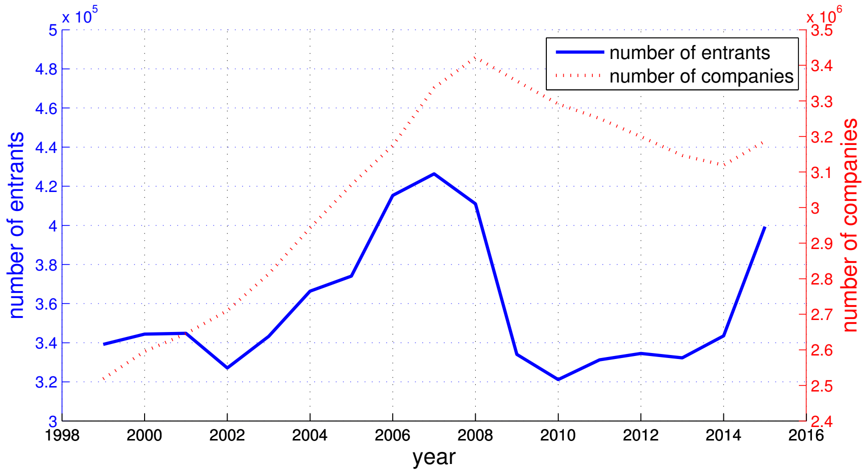

Figure 1: The Number of Newly Registered Companies and the Total number of Companies in Spain, 1999 to 2015

The procyclicality of firm entry shown in Figure 1 is in line with various explanations of business cycles. Lee and Mukoyama (2015) find that the entry rate of the U.S. manufacturing plants is procyclical, and they develop a model with productivity shocks to explain this pattern. Clementi and Palazzo (2015) reproduce the same cyclical pattern of firm entry, using a different quantitative model with productivity shocks. Intuitively, an increase in productivity improves profitability and thus incentivizes firms to enter the market. Gopinath et al. (2015) argue that the decrease in interest rate in early 2000s can potentially explain the increasing number of firms in Spain before the Great Recession, as the low interest rate could enhance the profitability of firms. I show in Section 5 that a financial reform has an ambiguous effect on the equilibrium firm entry, and that investor optimism increases the number of firm entry by subsidizing entrants with bubbles.

1.2.2 Aggregate Productivity and Misallocation

Probably the most salient feature of the Spanish economy is the protracted decline of aggregate productivity. Figure 2 plots the Total Factor Productivity (TFP) in Spain, using Spanish TFP data from Penn World Table 9.0. The TFP declined not only in the recession (after 2008), but also at the time of output expansion (1994-2007).

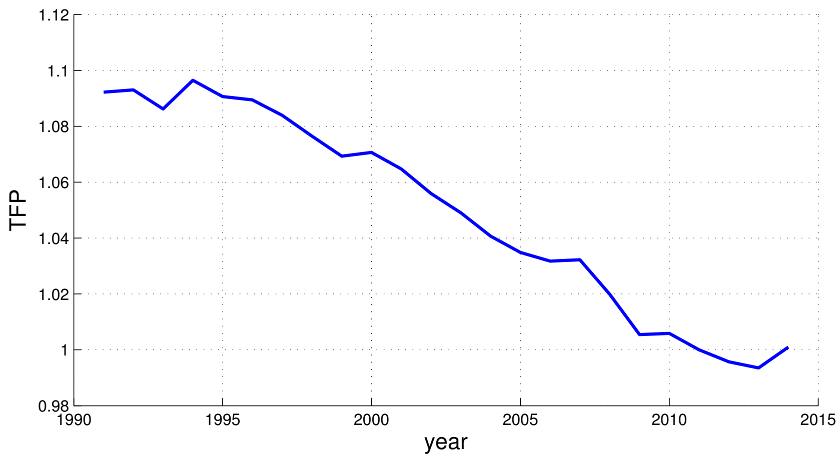

Figure 2: Total Factor Productivity in Spain, 1991-2014

The declining TFP is associated with exacerbating allocative efficiency. The recent literature has documented aggravating misallocation underlying the Spanish boom. Garcia-Santana et al. (2016) find that the TFP in Spain would otherwise grow 0.8% per year on average between 1994 and 2007, if there was no deterioration in allocative efficiency. Gopinath et al. (2015) propose the decrease in interest rate as a source of exacerbating misallocation. While Gopinath et al. (2015) focus on the misallocation among incumbent firms, in my model misallocation arises as bubbles subsidize the capital inputs of new firms and increase their capital intensity: I show in Section 6 that bubble shocks can give rise to the decline of aggregate productivity.

1.2.3 Firm-level Productivity and Profit

I estimate the firm-level revenue-based total factor productivity (TFPR) using the AMADEUS data for Spain during 2003-2009. Assume that firms in a given industry produce identical product according to a Cobb-Douglas production function. For a given industry j, the output of firm i is:

$$y_{it} = \beta_0^j + \beta_l^j l_{it} + \beta_k^j k_{it} + \beta_\iota^j \iota_{it} + \epsilon_{it} \tag{1.1}$$

Where y_{it} is the log of output, for firm i at time t; l_{it}, k_{it}, ι_{it} are respectively the logs of labor input, capital input, and intermediate inputs. The firm-level productivity is measured by Solow residual: $\beta_0^j + \epsilon_{it}$. However, it is well known that OLS estimation of equation (1) yields biased estimation because of the simultaneity problem.[4] I address the estimation problem by implementing the program developed by Levinsohn, Petrin, and Poi (2003), which is based on the approach proposed by Levinsohn and Petrin (2003).[5] In my estimation, y_{it} is measured by annual operation revenue; l_{it} input is measured by the number of employees; k_{it} is measured by total asset; ι_{it} is measured by materials expenditure. The estimation is run at industry-level: the estimation relies on the assumption that the firms in a given industry produce according to homogeneous production

[4]See Olley and Pakes (1996) and Levinsohn and Petrin (2003) for more details.

[5]Levinsohn and Petrin (2003) use intermediate input as a proxy for unobserved productivity in a two-stage estimation.

function and the function does not vary in the sample period.[6] Since the output is measured by operation revenue, the estimated log productivity $\beta_0^j + \epsilon_{it}$ is the log of revenue productivity, or TFPR, so that changes in the estimated productivity also capture changes in prices.[7] I also calculate the firm-level net profit using the data of profit margin and operating revenue.

Permanent Sample	2006	2009	Boom	Recesssion
Average log TFPR	5.41	5.30	5.38	5.32
Average normalized log TFPR	-8.00	-8.19	-7.92	-8.17
Average net profit (unit: euros)	6870225	3792534	6450195	4507961
Average profit margin	4.03	-0.81	3.91	0.40

Table 1: Average log Productivity and Profit of Permanent Sample

Table 1 lists the averages of: i) log TFPR $\beta_0^j + \epsilon_{it}$, ii) log TFPR normalized by capital stock $\beta_0^j + \epsilon_{it} - k_{it}$,[8] iii) net profit, and iv) profit margin[9] for the permanent sample which consists of the firms existing throughout the sample period. The two columns in the middle compare year 2006, when the economy was at the peak of economic expansion, and the year 2009, when the economy was deeply trapped in the recession. The firms from the permanent sample had on average higher productivity and net profit in 2006 than in 2009. Besides, the normalized log TFPR, and profit margin, which measure the productivity and profitability in relative terms, were

[6]The industries are identified by a 3-digit SIC code.

[7]Foster, Haltiwanger, and Syverson (2008) study the difference between revenue productivity (TFPR) and quantity productivity (TFPQ).

[8]The normalized log TFPR equals toln $\left(\frac{\exp(\beta_0^j + \epsilon_{it})}{k_{it}}\right)$.

[9]The profit margin can be interpreted as the net profit normalized by revenue.

also lower on average in 2009 than in 2006. The last two coulumns represent a comparison between the boom years 2003-2007, and the recession years 2008-2009. Again, the average productivity and profitability were lower after the outbreak of the Great Recession. The results suggest that the revenue productivity and profitability of existing firms were dampened in the recession, which is consistent with the conventional wisdom that there was a negative productivity shock in the recession.

Entrants	2006	2009	Boom	Recesssion
Average log TFPR	4.99	5.01	4.96	5.00
Average normalized log TFPR	-6.53	-6.50	-6.56	-6.47
Average net profit (unit: euros)	223225	247293	244636	247901
Average profit margin	-2.55	-2.33	-2.49	-2.18

Table 2: Average log Productivity and Profit of Entrants[10]

In contrast, Table 2 reports the opposite pattern for entering firms. The two columns in the middle show that the entrants in 2006 had on average lower productivity and profitability, in both absolute and relative terms, than the entrants in 2009. The comparison suggests, perhaps surprisingly, that the average productivity and profitability of entrants were lower in the boom than in the recession. The pattern still prevails even if we consider more sample years. The last two columns show that the new firms entering the market in 2003-2007 were on average less productive and profitable than the new firms in 2008-2009. In Section 5 I show that this

[10]Gopinath et al. (2015) document that before the Great Recession, the average log productivity over their full sample of firms declined significantly relative to the permanent sample. They argue this implies the entry of less productive firms over time.

pattern would be possible only if the Great Recession features a bubble crash or an increase in interest rate. If the crisis is merely a consequence of dropping productivity, or exacerbating financial frictions, the average productivity and profit of entrants would become otherwise lower in the recession compared with that in the boom.

To sum up, in the past two decades, the Spanish economy was characterized by: i) procyclical firm entry, ii) a persistent decline in aggregate productivity, and iii) an increase in the average productivity and profitability of entrants during the Great Recession. In the rest of this paper, I propose a theory that can rationalize these 3 facts.

1.3 Model Setup

In this section, I set up a model in the spirit of Lucas (1978) and Hopenhayn (1992). The model is a simplified version of the standard Hopenhayn model. I assume exogenous firm exit, time-invariant firm-level productivity, and a fixed number of potential new firms.[11] I incorporate credit constraints for new firms when they make intial investment in capital. The model describes a small open economy where the interest rate is determined exogenously.

1.3.1 Production

There is a mass of atomistic firms, heterogeneous in productivity, producing a single product. Time t runs from zero to infinity. I assume that at time t, firms produce good according to:

[11] The assumptions regarding firm entry and labor supply resemble those used by Clementi and Palazzo (2015) and Carvalho and Grassi (2015).

$$y_t = (A\varphi)^{1-\alpha-\gamma} \cdot n_t^{\alpha} k^{\gamma}, \alpha + \gamma < 1 \tag{1.2}$$

The production displays decreasing return to scale (DRS). A is the common productivity component, which is identical across all firms. φ is the idiosyncratic productivity component, which is firm-specific. Capital k is chosen when the firm is created.[12] Capital input k and productivity components A and φ are time-invariant. n_t is the labor input the firm employs at period t.

Firms take wage as given. The wage is determined in a frictionless market, in which the labor supply is given by a monotonically increasing function of wage w_t:

$$L_{s,t} = w_t^{\varepsilon}$$

where $\varepsilon > 0$.

Let λ_t denote the vector of aggregate state variables at time t. Given the aggregate state λ_t, capital k, and productivity $A \cdot \varphi$, incumbent firms maximize their profit by choosing labor input n_t:

$$\pi\left(\lambda_t, \varphi, k\right) = \max_{n_t} \left\{ (A\varphi)^{1-\alpha-\gamma} \cdot n_t^{\alpha} k^{\gamma} - w_t n_t \right\} \tag{1.3}$$

[12]The assumption enables a closed-form solution of stationary equiliria and greatly simplifies the analysis of aggregate dynamics in Section 6. It also captures the fact that the difference between cohorts of firms are persistent and that the performance of a firm depends heavily on when it was created, a fact documented and studied by Sedlacek and Stern (2016).

1.3.2 Firm Exit and the Value of Incumbents

Every period after producing, there are idiosyncratic exit shocks which are i.i.d. to firms with a constant probability $1-p$. If a firm draws the exit shock, it would exit the market permanently. Firms are traded in equity market at the begining of each period by a group of risk-neutral investors, who have free access to international credit market and take the interest rate as given. The investors discount future profit by interest rate, $\frac{1}{R_t}$. A firm is only tradable if it produces: the firm value becomes zero forever if the firm is forced to exit. The value of any incumbent firm thus satisfies:

$$v_t\left(\lambda_t, \varphi, k\right) = \pi\left(\lambda_t, \varphi, k\right) + \frac{p}{R_t}\widehat{E}_t v_{t+1}\left(\lambda_{t+1}, \varphi, k\right) \tag{1.4}$$

where the operator $\widehat{E}_t$ denotes the conditional expectation at t given that the firm would survive at $t+1$. Equation (4) implies that the value equals the sum of current profit and the discounted expected value, which equals the product of discount factor $\frac{1}{R_t}$, survival probability p, and expected continuation value $\widehat{E}_t v_{t+1}\left(\lambda_{t+1}, \varphi, k\right)$.[13] I show in the subsequent section that under certain circumstances, the value of a firm can be decomposed into a "stationary" fundamental component which equals to the NPV of the future profit, and an "explosive" bubble component which is a pyramid scheme that can be rolled over if the firm survives. The solutions nest the standard solution in Hopenhayn (1992) or in Melitz (2003) where bubble components equal zero.

[13]The expected value equals to the product of survival probability and the expected continuation value:

$$E_t v_{t+1}\left(\lambda_{t+1}, \varphi, k\right) = (1-p)\cdot 0 + p\widehat{E}_t v_{t+1}\left(\lambda_{t+1}, \varphi, k\right) = p\widehat{E}_t v_{t+1}\left(\lambda_{t+1}, \varphi, k\right)$$

1.3.3 Firm Entry and the Credit Constraint

There is a constant mass $\bar{M}$ of potential entrants every period. The potential entrants draw their idiosyncratic productivity φ according to the Pareto distribution function $F(\varphi)$[14]:

$$F(\varphi) = 1 - \varphi^{-\zeta}$$

where $\zeta > 1$[15]. Both the common productivity component A and the idiosyncratic productivity component φ are publicly observable: each potential entrant knows her own productivity level $A\varphi$ before deciding whether or not to enter the market and start production in the subsequent period.

At time t, the entrants issue one-period debt to raise the capital stock k in the international credit market, where the interest rate is given by R_t: the firms become tradable in the equity market once they start production at $t+1$. After getting the credit, entrepreneurs can either flee with a fraction $1-\delta$ of the credit or invest the credit into their firms. The credit contract is incentive-compatible if and oly if:

$$(1-\delta)\,k \leq \frac{1}{R_t}\left(E_t v_{t+1}\left(\lambda_{t+1}, \varphi, k\right) - R_t k\right) \tag{1.5}$$

Throughout the paper, I limit the analysis to the incentive-compatible contracts satisfying inequality (5). Inequality (5) ensures that the payoff from defaulting does not exceed the present value of investing the credit into

[14]The assumption is based on the fact that the firm size distribution is well approximated by the Pareto distribution.

[15]If $\zeta \leq 1$, the mean value of φ would be infinite. The literature has found ζ close to but slightly above 1.

the firm. δ can be motivated by the fraction of credit which entrepreneurs have to spend to evade the enforcement. The size of δ is a proxy for institutional quality: better institution (higher δ) would increase the expenditure of defaulting and prohibit entrepreneurs from stealing the credit; in fact, if $\delta = 1$, the credit constraint is never binding, as the entrepreneurs are unable to steal any credit. Thereafter I refer the "financial reform" to an exogenous increase in δ. Furthermore, inequality (5) implicitly implies that $R_t k \leq E_t v_{t+1}(\lambda_{t+1}, \varphi, k)$, so that the default can not take place once the firm is initiated: the entrepreneurs would lose the firms if they default, and the loss would always outweigh the gains from default.

As for the prospective entrants, the value of entering the market, v^e, is:

$$v_t^e(\lambda_t, \varphi) = \frac{1}{R_t} \max_k E_t \left(v_{t+1}(\lambda_{t+1}, \varphi, k) - R_t k\right) \tag{1.6}$$
$$s.t.\ (1-\delta) R_t k \leq E_t v_{t+1}(\lambda_{t+1}, \varphi, k) - R_t k$$

Entering the market incurs two sunk costs: 1) the cost of physical capital $R_t k$, and 2) entry cost c_e, which measures the utility loss from setting up a firm in the market. The prospective entrant will enter the market if and only if the value of entering is no less than the entry cost c_e:

$$v_t^e(\lambda_t, \varphi) \geq c_e \tag{1.7}$$

where $c_e \geq 0$.

1.4 Stationary Equilibria

The model I develop in the previous section is essentially a simplified Hopenhayn model with credit market frictions. In a dynamic equilibrium:

the labor market and the equity market clear over time; incumbents and entrants maximize their payoff. To help build intuition, in this section I investigate the stationary equilibria as studied by Hopenhayn (1992) and Melitz (2003). Throughout this and the next section, aggregate state variables are stable over time: $\lambda_t = \lambda$[16]. In Section 6 I study instead the equilibria with time-varying λ_t.

In the literature of firm dynamics, the value of a firm equals the NPV of profits. At the begining of this section, I show that under certain circumstances, the functional equation (4) features multiple solutions, in which the value of a firm contains a positive bubble component in addition to the NPV of profits, and each solution corresponds to a unique stationary equilibrium.[17]In Section 4.2 I study the input choices, and the subsidy effect of bubbles on capital input (the intensive margin). The equilibria are characterized in Section 4.3, where I investigate the subsidy effect of bubbles on firm entry (the extensive margin). The aggregate variables and other variables of interest are explored in Section 4.4.

1.4.1 Value Function Decomposition

The solution to the functional equation (4) can be decomposed into a fundamental component which equals the NPV of future profits, and a bubble component which is a pyramid scheme:

$$v_t(\lambda, \varphi, k) = b_t(\lambda, \varphi, k) + f(\lambda, \varphi, k)$$

[16]Unless otherwise specified, letters without time subscript denote the variables in stationary equilibria.

[17]The model is similar to the one used by Gali (2014) in the sense that bubble component coexist with positive profit (or rent). Unlike in Gali (2014), bubbles in my model are assumed to be destroyed when firms stop production. This property implies a different necessary condition under which the bubble and fundamental components can coexist.

where $f(\lambda, \varphi, k)$ denotes the fundamental component, which equals the NPV of the firm's future profits, and $b_t(\lambda, \varphi, k)$ denotes the bubble component, which is a pyramid scheme. Notably, I keep the time subscript for the bubble component rather than the fundamental component. The reason is that in a stationary equilibrium, the fundamental component is pinned down by the time-invariant variables λ, φ, and k, which determine the size of profit π. However, the bubble component arises as an outcome of investor sentiments, and can change independently with respect to λ, φ, and k, so that the time subscript is indispensable.[18] We have:

$$f(\lambda, \varphi, k) = \pi(\lambda, \varphi, k) + \frac{p}{R} f(\lambda, \varphi, k) = \frac{R}{R-p} \pi(\lambda, \varphi, k) \qquad (1.8)$$

The necessary and sufficient condition for the fundamental component to exist is that $p < R$, otherwise the NPV of profits is equal to infinity. Meanwhile we have the recursive formula for bubble component $b_t(\lambda, \varphi, k)$:

$$b_t(\lambda, \varphi, k) = \frac{p}{R} \widehat{E}_t b_{t+1}(\lambda, \varphi, k) \qquad (1.9)$$

$\hat{b}_t(\lambda, \varphi, k)$ is a Ponzi game component and has to be rolled over every period. Equation (9) implies that, given that $p < R$, the expected future size (upon continuation) is strictly larger than the current size of bubble component. In other words, the bubble component is expected to grow over time insofar as the firm stays in the market.

The size of the bubble component depends on the investor sentiment, so it has multiple possible processes. Throughout the paper, I limit the

[18]Moreover, as discussed below, the process of bubble component is non-stationary.

analysis to the special case that, for any new firm at time t:

$$b_t^N(\lambda, \varphi, k) = \hat{b}_t f(\lambda, \varphi, k) \tag{1.10}$$

where $b_t^N(\lambda, \varphi, k)$ denotes the size of bubble components for new firms at t. In stationary equilibria: $\hat{b}_t = \hat{b}$. The assumption underlying equation (10) is that every new firm is created with a bubble component proportional to the fundamental component, and the proportion is constant across the entrant cohort.[19]

The standard solution to functional equation (4) features a zero bubble component, and the value of incumbent firms equals their fundamental component. In a bubbly equilibrium, the solution to equation (4) features a positive bubble component. It is of our interest to know whether a positive bubble component is sustainable in a stationary equilibrium. We have the following proposition:[20]

PROPOSITION 1: *There exist multiple bubbly stationary equilibria in which functional equation (4) has multiple solutions with a positive bubble component* $\left(\hat{b} > 0\right)$*, if and only if* $p < R < 1$. *In a bubbly equilibrium, the size of the bubble component is not stationary, while the aggregate value of bubbles is stable over time.*

The intuition is that, even though at the firm-level, the bubble compo-

[19]The main results of this paper rely on the mechanism of subsidy effect, which work in the presence of disperse $\hat{b}$ across an entrant cohort. It becomes clear, however, in the subsequent sub-section that intra-cohort (or cross-section) variation of $\hat{b}$ would give rise to misallocation across firms in a given cohort. Miao and Wang (2014) explore the interplay between cross-sectional variation of bubble and misallocation. I abstract from this type of misallocation and focus instead on the misallocation across different age cohorts, which is a consequence of time-variation of bubble. See more details in Section 6.

[20]The proof can be found in Appendix 1.

nent upon continuation is non-stationary and can grow to infinity, at the aggregate level, $1-p$ fraction of bubbles disappear as a consequence of firm exit, ensuring a stable size of aggregate bubbles.[21] Throughout the rest of Section 4 and 5, I assume that $p < R < 1$, so that there exist sustainable stationary bubbly equilibria.

1.4.2 Choice of Inputs

The demand for labor can be derived from the F.O.C. of incumbents:

$$\alpha \left(A\varphi\right)^{1-\alpha-\gamma} n^{\alpha-1} k^{\gamma} = w \tag{1.11}$$

Since incumbents recruit labor in a frictionless market, they choose the optimal size of labor which equates the marginal product of labor to market wage.

The demand for capital can be derived analogously from the F.O.C. of entrants, which are however exposed to the credit constraint. If the collateral constraint is not binding, the demand for capital is given by:

$$\frac{R\left(1+\hat{b}\right)\gamma}{R-p}\left(1-\alpha\right)\left(A\varphi\right)^{1-\alpha-\gamma} n^{\alpha} k^{\gamma-1} = R \tag{1.12}$$

Entrants equate the marginal incumbent value of a new firm to the interest rate, whenever the credit constraint is not binding. Equation (12) suggests that bubbles serve as a subsidy on capital input: the demand for capital is strictly increasing in the size of bubble creation ratio $\hat{b}$. In fact, the ratio

[21] There is no bubbly equilibrium if the aggregate bubbles also grows to infinity, since there is not sufficient resource to purchase the bubbles. For more discussions of sustainable bubbly equilibrium, see Tirole (1985), Martin and Ventura (2012), and Gali (2014).

$\hat{b}$ is mathematically equivalent to a rate of subsidy on capital. Intuitively, the size of the bubble component is proportional to the fundamental component, which is increasing in the size of capital input, so the entrants in a bubbly economy ($\hat{b} > 0$) are encouraged to invest more in physical capital to create a larger bubble component.

If the collateral constraint is binding, the demand for capital is given by:

$$\frac{R\left(1+\hat{b}\right)}{(R-p)(2-\delta)}(1-\alpha)(A\varphi)^{1-\alpha-\gamma}(n^*)^{\alpha}(k^*)^{\gamma-1} = R \tag{1.13}$$

The size of capital input is bounded by the pledgeable value of firm. An increase in $\hat{b}$ increases the constrained optimal capital input as well as the unconstrained optimal capital input. In a bubbly economy, bubbles serve as a subsidy on capital regardless of whether the financial constraint is binding; moreover, when the collateral constraint is binding, bubbles also serve as additional collateral to entrepreneurs, so the increase in capital demand is an outcome of both a subsidy effect and a collateral effect.

It is crucial for our analysis to verify whether the credit constraint is binding. We have the following proposition:

PROPOSITION 2: *In a stationary equilibrium, the collateral constraint for entrants is binding if and only if*

$$\gamma > \frac{1}{2-\delta}$$

The proposition is immediate given by equations (12) and (13). The underlying intuition is straightforward: larger δ implies higher pledgeability, while higher γ implies a higher need for capital. Firms are more likely to be financially constrained if the need for capital is large or if the pledgeability is small; apparently, the constraint is never binding if $\delta = 1$. Most importantly, the proposition holds irrespective of whether there exist bubbles. Although the existence of bubbles increases the value of pledgeable assets, it propotionally increases the demand of capital, and the two effects cancel out: if the inequality holds, bubbles still increase the size of pledgeable value and thus the constrained optimal capital input, but increase even more the unconstrained optimal size of capital.

1.4.3 Free Entry Condition and Labor Market Clearing Condition

I prove in Appendix 2 that there always exists a cutoff productivity φ^*: a firm enters the market as long as it draws a productivity $\varphi > \varphi^*$, otherwise it does not enter the market since the entry value $v^e(\lambda, \varphi)$ is not high enough to cover the entry cost c_e. An equilibrium can be represented by a pair: (φ^*, π^*), where φ^* denotes the cutoff productivity for prospective entrants, and π^* denotes the cutoff level profit. Notably, φ^* is interchangable with m, the amount of firm entry, since m is a bijective (and decreasing) function of φ^*.[22]

Two conditions are necessary to characterize the equilibrium (φ^*, π^*): free entry condition (henceforth FE) and factor markets clearing condition (henceforth LMC). Free entry condition establishes the relationship between φ^* (or m) and π^* when the entry value for marginal entrants equals

[22] Given the law of large numbers, we have:

$$m(\varphi^*) = (1 - F(\varphi^*)) \cdot \bar{M}$$

to the entry cost c_e. Labor market clearing condition establishes the link between φ^* (or m) and π^* when the labor market clears. Both conditions are contingent on whether credit constraint are binding, and on the investor sentiment, or more specifically, the size of the bubble creation ratio $\hat{b}$:[23]

PROPOSITION 3: *The FE and LMC in a stationary equilibrium are given by (respectively)*

$$FE:\ \pi^* = \frac{R-p}{\left(1+\hat{b}\right)\left(1-\min\left(\frac{1}{2-\delta},\gamma\right)\right)}c_e \tag{1.14}$$

$$LMC:\ \pi^* = \left[(1-\alpha)\left(A\varphi^*\right)^{1-\alpha-\gamma}\left(\frac{\left(1+\hat{b}\right)\min\left(\frac{1}{2-\delta},\gamma\right)}{R-p}\right)^{\gamma} n\left(\lambda,\varphi^*\right)^{\alpha}\right]^{\frac{1}{1-\gamma}} \tag{1.15}$$

where $n\left(\lambda,\varphi*\right)$ *denotes the size of labor input for cutoff entrants:*

$$n\left(\lambda,\varphi^*\right) = \left(\frac{(1-p)\left(\alpha\left(\frac{\left(1+\hat{b}\right)(1-\alpha)\min\left(\frac{1}{2-\delta},\gamma\right)}{\alpha(R-p)}\right)^{\gamma}\left(A\varphi^*\right)^{1-\alpha-\gamma}\right)^{\frac{\varepsilon}{1-\gamma}}}{\bar{M}\int_{\varphi\geq\varphi*}\left(\frac{\varphi}{\varphi^*}\right)dF\left(\varphi\right)}\right)^{\frac{1}{1-\frac{\varepsilon}{1-\gamma}(\alpha+\gamma-1)}} \tag{1.16}$$

The FE and LMC conditions simultaneously pin down the stationary equilibrium, which is illustrated in Figure 3. The horizontal line represents the FE condition. The profit π^* given in equation (14) can be interpreted as the "reservation profit" for prospective entrants: a break-

[23] The proof of Proposition 3 can be found in Appendix 3.

even level of profit for prospective entrants to compensate the entry cost. Free entry condition implies that a prospective entrant would not enter the market if her profit after entry is lower than π^* (lower than the break-even FE line). The "reservation profit" for a prospective entrant is independent from her own productivity, so that the FE line is horizontal.

The upward sloping curve represents the LMC condition. The profit π^* given in equation (15) denotes the profit of cutoff entrants when the labor market clears. Mathematically, equations (15) and (16) imply that the cutoff profit π^* is increasing in the cutoff productivity φ^*, or alternatively, decreasing in the number of entrants.[24] Intuitively, moving leftward along the LMC curve (reducing φ^*) corresponds to: 1) a decline in the productivity of cutoff entrants, and 2) an increase in the number of firms which bids up the equilibrium wage.[25] Consequently, the cutoff profit decreases as we move leftward along the LMC curve. Therefore LMC implies an increasing cutoff profit if φ^* increases. In the equilibrium, the labor market clears, and the profit of cutoff entrants equals the "reservation profit".

[24] See the proof in Appendix 4.

[25] See discussions with more details in Section 4.4.

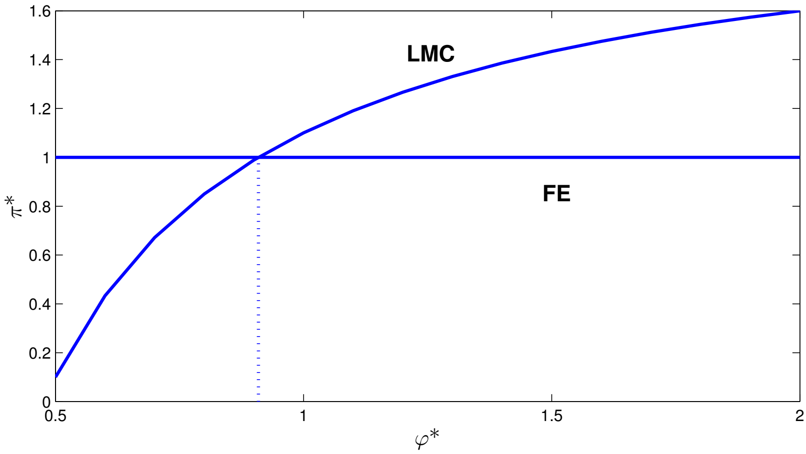

Figure 3: Free Entry Condition (FE) and Labor Market Clearing Condition (LMC)

Notably, equation (14) suggests that the cutoff profit is monotonically decreasing in the size of the bubble creation ratio $\hat{b}$. In a bubbly equilibrium, entrants get a bubble component in addition to the fundamental component, so the existence of bubbles lowers the "reservation profit" requested by entrepreneurs to compensate their entry cost c_e. Again, bubbles serve as a subsidy, but what equation (14) captures is the subsidy effect of bubbles on firm entry, or the subsidy at the extensive margin, rather than the subsidy effect on capital as studied in 4.2.

The subsidy effect on capital, or the subsidy at the intensive margin, is captured by equation (15), the LMC condition. This subsidy effect, on the contrary, tends to increase the cutoff profit: in a bubbly equilibrium, the entrants are encouraged to invest more in capital to increase their profits (and therefore the NPV of profits), so as to raise the size of bubble components. This subsidy effect raises the profitability of entrants in general.

1.4.4 Analysis of the Stationary Equilibrium

Before investigating the comparative statics, I complete characterizing the stationary equilibrium by studying the equilibrium aggregate variables and average productivity and profitability of entrants. The results established here are for the reference of Section 5.

Aggregate Variables

Construct the aggregate production function Y:

$$Y \equiv \int y^j dj = \Phi^{1-\alpha-\gamma} N^\alpha K^\gamma$$

where j denotes the index for firms, $N = \int n^j dj$, $K = \int k^j dj$ denote the aggregate amounts of inputs. The aggregate productivity Φ [26]is given by the following formula:[27]

$$\Phi = A \left(\frac{\bar{M}}{1-p} \right) \left(\int_{\varphi \geq \varphi^*} \varphi dF(\varphi) \right) \tag{1.17}$$

Firm level productivity $A\varphi$ can be viewed as a type of input, intangible asset, and the aggregate productivity Φ equals to the aggregate amount of intangible asset. It is easy to prove that $\frac{d\Phi}{d\varphi^*} < 0$. In this model, an increase of firm entry is equivalent to a decrease in the cutoff φ^*, so equation (17) implies that the stationary equlibria level of aggregate productivity is increasing in the amount of firm entry: an increase in the firm entry raises

[26]The definition of aggregate productivity depends on how we set up the aggregate production function. If we consider the number of firms as an input, as in Hopenhayn (2014), the aggregate productivity would equal the average productivity of all firms.

[27]See the derivation in Appendix 5

the supply of intangible asset, and consequently improves the aggregate productivity Φ.

The aggregate amount of labor and capital are respectively:

$$(labor)\ \alpha\Phi^{1-\alpha-\gamma}N^{\alpha-1}K^{\gamma}=w=N^{\frac{1}{\varepsilon}} \tag{1.18}$$

$$(capital)\ \frac{R\left(1+b\right)}{R-p}\left(1-\alpha\right)\min\left(\frac{1}{2-\delta},\gamma\right)\Phi^{1-\alpha-\gamma}N^{\alpha}K^{\gamma-1}=R \tag{1.19}$$

Equations (18) and (19) are simply aggregate counterparts of Equations (11)-(13). We know from Equation (18) and (19) that, an increase in the firm entry raising the aggregate productivity Φ raises the marginal product of labor and capital, and hence increases the equilibrium level of aggregate inputs. To be more specific, since the production function exhibits DRS, an increase in the number of firms lowers the size of firms, and therefore increases the marginal product of inputs across firms.

Average Productivity

The average productivity of entrants, $\widetilde{\varphi}$, is given by:

$$\widetilde{\varphi}\equiv\frac{\int_{\varphi\geq\varphi*}A\varphi dF\left(\varphi\right)}{\int_{\varphi\geq\varphi*}dF\left(\varphi\right)}=\frac{-\zeta A\varphi^{*}}{1-\zeta} \tag{1.20}$$

where $\zeta>1$. Equation (20) implies that average productivity $\widetilde{\varphi}$ is increasing in the cutoff φ^{*}. Intuitively, a reduction in the cutoff productivity φ^{*}corresponds to the entry of the less productive firms, which lowers the average productivity of entrants.

Average Net Profit

Notably, the profit π in the previous analysis does not take into consideration of capital cost, and is not comparable with the data of net profits. I define net profit in my model as:

$$\pi_{net} \equiv \pi - rk = y - wl - rk$$

where $r \equiv R - 1$[28]. The alternative measure of profits takes into account the capital cost. Furthermore, the average net profit of entrants $\widetilde{\pi}_{net}$ equals

$$\widetilde{\pi}_{net} = \frac{\int_{\varphi \geq \varphi^*} \pi^*_{net} \left(\frac{\varphi}{\varphi^*}\right) dF(\varphi)}{\int_{\varphi \geq \varphi^*} dF(\varphi)} = \frac{-\zeta \pi^*_{net}}{1-\zeta} \tag{1.21}$$

[29]where π^*_{net} denotes the net profit of cutoff firms. Equation (21) implies that the average net profit of entrants (and firms) decreases insofar as the the cutoff net profit π^*_{net} decreases: the entry of the less profitable firms dampens the average net profit of entrants.

Importantly, whenever FE condition is satisfied, the cutoff net profit π^*_{net} takes the following form:

$$\pi^*_{net} = \left(r + \frac{1 - p - r\hat{b}}{\left(1+\hat{b}\right)\left(1 - \min\left(\gamma, \frac{1}{2-\delta}\right)\right)}\right) c_e \tag{1.22}$$

[28]It can be easily proven that the NPV of net profits π_{net} equals the entry value v^e if $p = 1$ and $b = 0$.

[29]Use that:

$$\frac{\pi^i_{net}}{\pi^j_{net}} = \frac{\pi^i}{\pi^j} = \frac{k^i}{k^j} = \frac{\varphi^i}{\varphi^j}$$

Equation (22) is indeed a counterpart of the FE condition given by equation (14) [30]. The cutoff net profit π^*_{net} is not only the profit level of cutoff entrants, but also the "reservation net profit" for prospective entrants: an entrepreneur enters the market so long as the net profit is no less than π^*_{net}. Clearly, the "reservation net profit" in equation (22), akin to the "reservation profit" in the FE condition, is decreasing in the size of the bubble creation ratio $\hat{b}$: the bubble subsidy lowers the break-even net profit for firm entry. Equations (21) and (22) lead us to conclude that bubbles subsidize the entry of the less profitable firms and thereby reduce the average net profit of entrants.

1.5 Comparative Statics

Armed with the results in Section 4, in this section I study the comparative statics. To begin with, I investigate investor optimism (bubbles) in Section 5.1. Then in Section 5.2 and 5.3 I review shocks to productivity and to financial frictions, which have been two orthodox explanations for business cycles. I argue that both explanations cannot fully rationalize the Spanish experience. The implications of a declining interest rate is studied in Section 5.4. Finally, in Section 5.5 I present several results regarding net output in bubbly stationary equilibria.

1.5.1 Bubbles

Figure 4 displays the comparative statics from the bubbleless equilibrium ($\hat{b} = 0$) to a bubbly equilibrium ($\hat{b} > 0$), irrespective of whether credit

[30] The derivation of Equation (22) can be found in Appendix 6.

constraint is binding. The solid lines characterize the bubbleless equilibrium; the dashed lines characterize the bubbly equilibrium. The movement of FE captures the subsidy at the extensive margin: when bubbles emerge, the FE line shifts downward, as the bubble component subsidizes the entry decision, making prospective entrants accept a lower "reservation profit". Bubbles also shift the LMC upwards. The movement of the LMC captures the subsidy at the intensive margin: even in the absence of financial frictions, bubbles subsidize the capital and thus increase the profitability of firms.[31] When the financial constraint is binding, the shift of the LMC is driven by the combination of bubble subsidy and bubbly collateral: the emergence of bubbles (or increase in $\hat{b}$) encourages entrants to increase capital, and the increase in capital is collateralized by the additional bubble component.

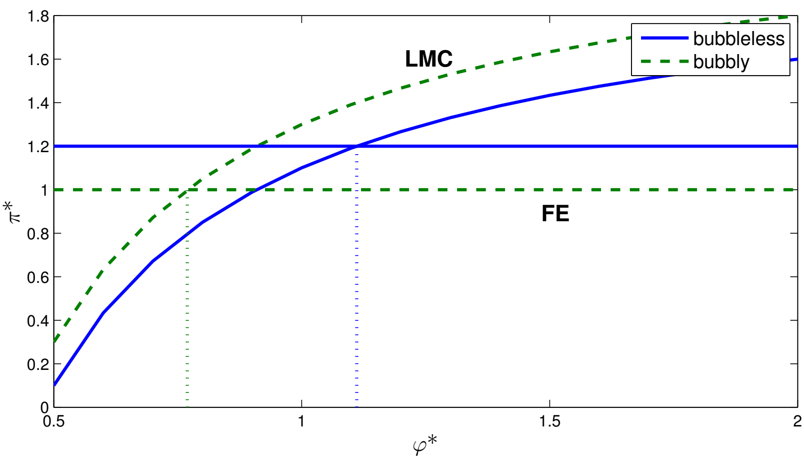

Figure 3: Free Entry Condition (FE) and Labor Market Clearing

[31]Notably, the profit here takes into account only the labor cost, rather than the capital cost.

Condition (LMC)

Figure 4 illustrates how bubbles increase firm entry in equilibrium. Bubbles lower the "reservation profit" required by entrepreneurs. The less productive firms which would enter the market in bubbly episodes would not necessarily enter the market in bubbleless episodes, because the profit is too low compared with the entry cost. In bubbly episodes, however, the entrepreneurs get the bubble component in addition to the fundamental component, they are willing to set up firms even if the profit (or NPV) is very low compared with the entry cost. Besides, the prospective entrants are also encouraged by higher profitability as bubbles subsidize the capital input.

An increase in the bubble creation ratio b increases the stationary equilibrium level of the aggregate output Y, the aggregate productivity Φ, the equilibrium wage w, and the aggregate inputs K and N.[32] The sentiment optimism exerts expansionary effects on the aggregate input and output, even in the absence of financial frictions. Equations (17)-(19) characterize the aggregate demand for inputs, given w and R. The emergence of bubbles lowers the cutoff φ^*, and increases Φ. N and K increase in response to an increase in b, not only because that aggregate productivity Φ increases, or firm entry increases (extensive margin), but also because that bubbles subsidize (and collateralize, if the credit constraint is binding) capital for all entrants (the intensive margin). When markets clear, N and K increase, so does the aggregate output Y.

Mathematically, it is easy to conclude from Equation (20) that the average productivity $\widetilde{\varphi}$ decreases as bubbles lower the cutoff φ^* (thus more firm entry). From equations (21) and (22) we can see that bubbles lower

[32]See the proof in Appendix 7.

the average net profit.[33] Intuitively, if an economy expansion is fueled by bubbles, the less productive and profitable firms enter the market to acquire bubbles, and the entry of these firms lowers the average productivity and net profit. Therefore the documented rise of average revenue productivity and average net profit in the recession is a natural outcome if the recession features a bubble bust, as the less productive and profitable firms would not enter the market after the bubble crash.

1.5.2 Productivity Shocks

In this sub-section, I revisit real business cycles. An increase in the common productivity component A boosts the firm entry. Figure 5 displays the comparative statics following an increase in A. Equations (14)-(16) imply that, regardless of whether collateral constraint is binding, an increase in A shifts the LMC upwards but it has no impact on the FE. The firm entry is higher in the new stationary equilibrium, as φ^* is lower. A universal increase in the productivity does not change the "reservation profit" that entrepreneurs require, but it does increase the profitability of operating firms unanimously. Entrants in the new equilibrium include firms which would not enter without the productivity increase: those firms become entrants in the new equilibrium because of the higher productivity (and profit). Importantly, I have assumed in my setup that the price of product is given; however, an increase in prices is mathematically equiva-

[33] Equation (22) can be rewritten as:

$$\pi^*_{net} = \left(\frac{R - p}{\left(1 + \hat{b}\right)\left(1 - \tilde{\delta}\right)} - \frac{r\tilde{\delta}}{1 - \tilde{\delta}} \right) c_e$$

where $\tilde{\delta} \equiv \min\left(\gamma, \frac{1}{2-\delta}\right)$.

lent to an increase in A. An increase in A therefore can capture not only a universal increase in productivity, but also a general increase in demand.

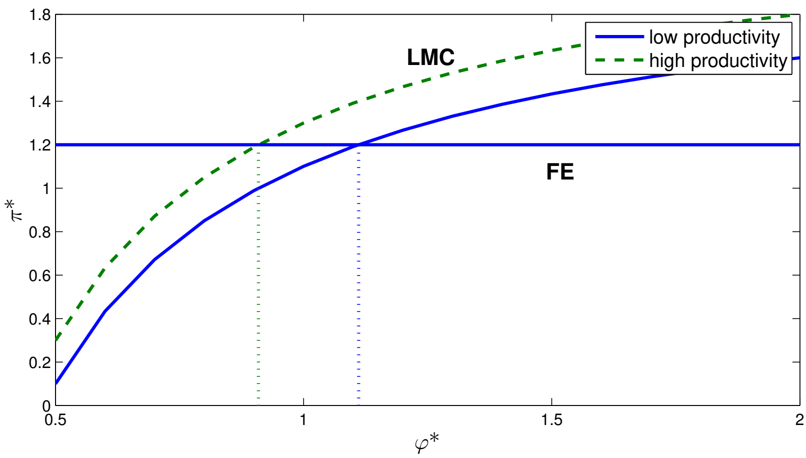

Figure 5: Equlibrium Effects of an Increase in A (Increasing Productivity)

We can further verify that an increase in A undoubtedly raises the aggregate inputs and output. We know from equation (17) that the aggregate productivity Φ raises in response to an improvement in A. In addition, according to equations (18)-(19), the equilibrium level of inputs increases as the marginal products of inputs are boosted by rising Φ. The aggregate output expands, as long as Φ, K, and L all increase.

The change of average productivity $\widetilde{\varphi}$ can be derived using equation (20). The change of $\widetilde{\varphi}$ depends on the change of $A\varphi^*$, which equals the productivity level of cutoff entrants. The increase in A raises the aggregate output Y, and the equilibrium wage w. However, the cutoff profit level π^* remains unchanged in the new equilibrium, so that the productivity of marginal entrants has to increase to maintain the profit level (given the higher w).[34] Therefore, in response to an increasing A, the average productivity $\widetilde{\varphi}$ increases. If the economic downturn in the Great Recession is exclusively a consequence of declining A, we would have observed lower average TFPR of entrants in the recession than in the boom.

Moreover, according to equation (22), changes in A have no impact on the average net profit. The average net profit is solely determined by the level of "reservation net profit". An increase in A enhances the profitability of entrants in general, meanwhile it raises the number of entrants: the increase in the number of entrants lowers the relative fraction of more productive and more profitable entrants. As a result, the average net profit of entrants remains invariant.

1.5.3 Shocks to Financial Frictions

[34] It is easy to check that the capital stock of the cutoff entrant remains invariant.

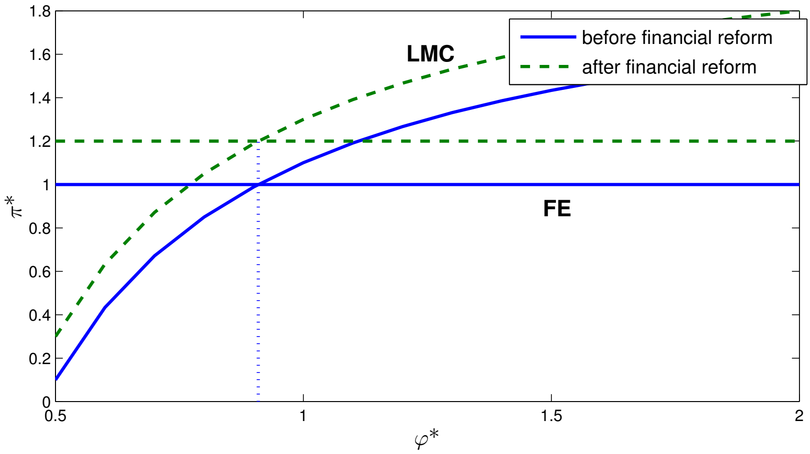

Figure 6: Equlibrium Effects of an Increase in δ (Financial Reform)

Equations (14) to (16) tell us how an increase in δ affects the equilibrium. The financial shock has no impact if the credit constraint is never binding ($\gamma < \frac{1}{2-\delta}$). Figure 6 above graphically illustrates the comparative statics of an increase in δ, if $\gamma > \frac{1}{2-\delta}$. The solid lines denote the equilibrium before the increase in δ; the dashed lines denote the equilibrium after the increase in δ. According to equation (14), an increase in δ increases π^*, shifting the FE line upwards. More specifically, the shift of FE line captures the selection effect: the relaxing of the financial constraint increases the expenditure on capital, hence prohibitting the entry of the less productive firms. Meanwhile, equations (15) and (16) suggest that conditional on φ^*, an increase in δ increases the cutoff profit π^* if the labor market clears. Therefore the LMC curve shifts to the right in reaction to an increasing δ. The shift of LMC curve captures that the less productive firms are more lucrative (i.e. they are encouraged to enter the market) because of higher pledgeability. The overall effect of an increasing δ on the number of firm entry is ambiguous.

Since an increasing δ has ambiguous effect on the equilibrium firm entry, or equivalently, the cutoff productivity φ^*, it is not conclusive how it affects the aggregate productivity Φ and the average productivity $\widetilde{\varphi}$. Moreover, a financial reform lowering the firm entry and thus depressing the aggregate productivity Φ, does not necessarily reduce the aggregate output Y, as it raises pledgeability and possibly increase the aggregate capital K according to equation (19). If a financial reform boosts the firm entry, it would unambiguously expand the output as it does not only increase the aggregate productivity Φ, but also, as implied by equations (18)-(19), increases the aggregate inputs K and L.

Equation (21) implies that the average net profit of entrants $\widetilde{\pi}_{net}$ increases insofar as the cutoff net profit π^*_{net} raises. Equation (22) shows that a financial reform raises the cutoff net profit π^*_{net} (as well as π^* in Figure 6), so it is expected that a financial reform would also raise the average net profit of entrants. If an economic expansion is solely driven by an increasing δ, the average net profit of entrants would be higher in the boom than in the recesssion. However, as shown in Section 2, the opposite is found in the data: compared with the entrants in the recession, the entrants in the boom were on average less profitable.

If the Spanish economic expansion before the financial crisis is jointly explained by high levels of A and δ, we would expect the average net profit of entrants to fall in the recession: a result which is at odds with the facts. To sum up, changes in A and δ can not fully rationalize aforementioned empirical facts.

1.5.4 Interest Rate

In this small open economy, another alternative explanation for economic expansions is a declining interest rate. An important implication of equations (14)-(16) is that: an increase in b is mathematically equivalent to a decrease in the interest rate R. Therefore, like an increase in b, a decrease in interest rate lowers the cutoff profit π^* and increases the firm entry. Intuitively, a decrease in R has two effects: it lowers the cost of capital, encourages capital investment, and therefore raises profitability of firms; meanwhile, it reduces the "reservation profit" as the discount factor $\frac{1}{R}$ increases. The two effects are analogous to the subsidy effects of bubbles on capital input and on firm entry. The comparative statics is shown in Figure 7.

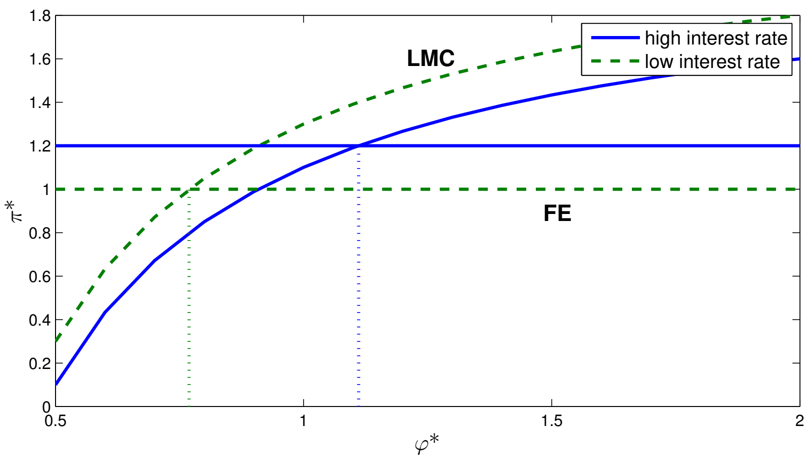

Figure 7: Equlibrium Effects of a Decrease in R

Moreover, lowering the interest rate R improves the aggregate productivity as it expands the firm entry. Besides, lowering R reduces the cost for capital. One can see from equations (18)-(19) that, at the aggregate level, lowering interest rate increases the aggregate output, and boosts the aggregate inputs K and L.

At the firm level, since lowering interest rate decreases the cutoff productivity, Equation (20) implies that it also dampens the average productivity. However, lowering interest rate has ambiguous effect on the average net profit. Recall the equation (22):

$$\pi_{net}^{*} = \left(r + \frac{1 - p - r\hat{b}}{\left(1 + \hat{b}\right)\left(1 - \min\left(\gamma, \frac{1}{2-\delta}\right)\right)} \right) c_e$$

In fact, the equations above suggests that the change in the cutoff (and thus the average) net profit due to a decline in the interest rate depends on the size of $\hat{b}$: if $\hat{b}$ is close to zero, lowering interest rate would lower the cutoff net profit; if $\hat{b}$ is very large, lowering interest rate would improve the cutoff net profit. Intuitively, lowering interest rate, on the one hand, increases the discount factor $\frac{1}{R}$, and hence lowers the "reservation net profit". On the other hand, lowering interest rate unambiguously dampens the cutoff profit π^* and thus dampens the size of the bubble subsidy for cutoff entrants: if the size of bubble creation ratio $\hat{b}$ is large enough, this channel is going to be overwhelming, and thus lowering the interest rate increases the "reservation net profit", as well as the average net profit of entrants.

Notably, the analogy between bubbles and declining interest rates might be an artifact of the assumption that firms only borrow and invest when they enter the market. If incumbent firms can adjust their size of capital, they may respond to an interest rate reduction by increasing their capital stock, in order to take advantage of a lower investment cost. However, a shock to the bubble creation ratio only affects entrants directly, rather than incumbents. Moreover, changes in interest rates are exogenous in this small open economy. It is important to be aware that in a more general model, interest rates can be determined by fundamental factors (like productivity and fiancial frictions) and bubbles. [35]

1.5.5 Net Output

At the end of Section 5, I study the net output in stationary bubbly equilibria. Net output is the amount of consumption good produced in every period:

$$\widehat{Y} \equiv Y - (1-p)\,K$$

$\hat{Y}$ equals to the difference between aggregate output Y and the "depreciation" of physical capital, $(1-p)\,K$, which equals to the amount of capital destroyed with firm exit in every period.[36] As shown in Section 4.3, an increase in $\hat{b}$ decreases φ^*, and increases the aggregate capital stock K in the new stationary equilibrium; consequently, investment cost raise as well in the new stationary equilibrium. An increase in $\hat{b}$ thereby has an ambiguous effect on $\hat{Y}$.

The size of $\hat{b}$ maximizing $\hat{Y}$ in stationary equilibria is analogous to the "Golden Rule Savings rate" in the Solow model. The aggregate capital stock K and productivity Φ are both increasing in $\hat{b}$, and subject to diminishing returns: when $\hat{b}$ raises above a certain level, the increases in the capital depreciation are destined to override the increase in output,

[35]For more discussions about the interplay between interest rates and bubbles, see Farhi and Tirole (2011), and Asriyan et al. (2016).

[36]It has been implicitly assumed that capital cannot be reallocated once the firm exit, otherwise the aggregate amount of capital would grow unboundedly.

and any further increase in $\hat{b}$ deteriorates the net output. Notably, the reasoning above also implies that there always exists an optimal level of $\hat{b}$ (which can equal zero) maximizing the net output $\hat{Y}$.

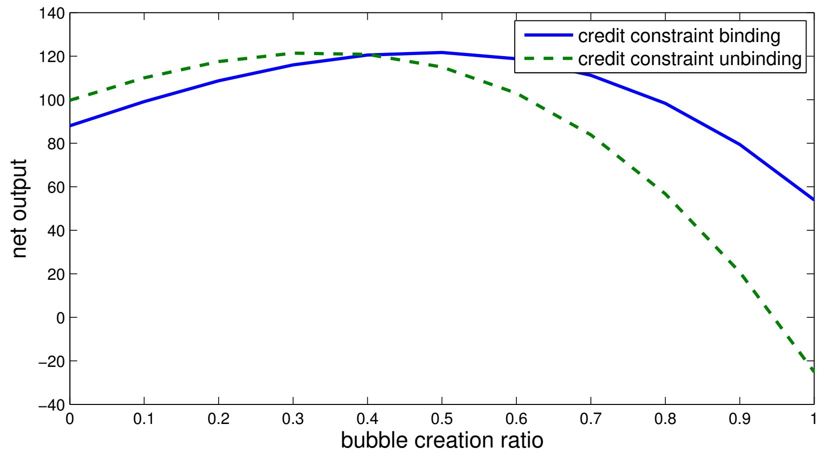

Figure 8: Bubble Creation Ratio and Net Output

Not surprisingly, the size of $\hat{b}$ maximizing $\hat{Y}$ depends on how financially constrained the economy is. Figure 8 shows a comparison between two economies which are only different in pledgeability δ: the solid line denotes an economy with low pledgeability δ and the credit constraint is binding, while the dashed line denotes an economy with high pledgeability δ and the credit constraint is not binding. Despite the fact that in both economies bubbles improve net output when they are small, the size of $\hat{b}$ maximizing $\hat{Y}$ is higher in the economy with lower pledgeability. An alternative interpretation is that bubbles are more likely to improve net output when pledgeability δ is low. Intuitively, bubbles relax the financial constraint and raise the capital stock, and the output expansion from increasing capital stock is more pronounced when there is fewer capital stock: low level of pledgeability depressing the size of capital stock makes bubbles more likely to improve net output.

Perhaps the most astounding feature of Figure 8 is that: when the bubble creation ratio is small, the net output is increasing in pledgeability δ; however, the relationship is reversed when the bubble creation ratio is large. This is again because that the aggregate capital stock K is increasing in $\hat{b}$. When the bubble creation ratio is small, capital is scarce and increasing pledgeability improves the net output as it raises the output greatly. When the size of bubble creation is large, there is abundant capital, and increasing pledgeability deteriorates the net output since the corresponding increase in capital stocks raises depreciation cost but has very limited impact on output.

The mechanism discussed above can be studied mathematically through a special case of economy, where the entry cost $c_e = 0$. When there is no entry cost, an increase in $\hat{b}$ or δ increases the aggregate capital K, rather than the aggregate productivity Φ, which always equals to its maximum level. According to Equation (19), when the financial constraint is binding, the "aggregate" marginal product of capital is given by:

$$\frac{\partial Y}{\partial K} = \gamma \Phi^{1-\alpha-\gamma} N^{\alpha} K^{\gamma-1} = \frac{\gamma (R-p)(2-\delta)}{\left(1+\hat{b}\right)(1-\alpha)}$$

The marginal product $\frac{\partial Y}{\partial K}$ is strictly decreasing in $\hat{b}$ and δ, while a financial reform or an increase in $\hat{b}$ is welfare improving if and only if $\frac{\partial Y}{\partial K} > 1 - p$. In an economy with very high level of $\hat{b}$ (to the extent that $\frac{\partial Y}{\partial K} < 1 - p$), capital is heavily subsidized, and the credit constraint restricts entrepreneurs from excessive investment: a financial reform in such an economy would aggravate the loss in net output.

1.6 Aggregate Fluctuations

In this section I study aggregate fluctuations. Although the model features heterogeneous firms and firm entry and exit, it can be solved without implementing approximation techniques a la Krusell and Smith (1998). In Section 6.1 I lay down the basic idea of my solution method. The stochastic dynamic model can be illustrated through an easy example in Section 6.2.

The main purpose of this section is to study misalloaction. As I show in the previous section, bubbles and an interest rate reduction both increase the aggregate productivity in stationary equilibria. However, misallocation arises in transition dynamics as capital intensity fluctuates across cohorts. Using the example economy I set up in Section 6.2, in Section 6.3 I plot the impulse response functions. I show that the aggregate productivity decreases in response to bubble shocks or negative interest rate shocks, while the aggregate output expands. The results are in line with the stylized fact that the Spanish TFP declined during the output expansion.

I close this section by running counterfactual experiments about bubbles. In Section 6.4 I compare the experiment result with a bubble crash and the result without a bubble crash. The comparison result suggests that bubble shocks can significantly contribute to the magnitude of aggregate fluctuations.

1.6.1 The Model with Aggregate Uncertainties

With aggregate uncertainties included, the model is only solvable with computational tools. A common feature of the stochastic dynamic model with heterogeneous agents is that the aggregate states consist of the entire distribution of heterogeneous agents. A standard approach, developed by Krusell and Smith (1998), is to approximate infinite-dimension states with a finite number of variables. In my model, the assumptions that labor market is frictionless and that there are no idiosyncratic productivity shocks enable me to collapse the information of the entire distribution of firms into one single state variable without approximation.

Recall that the production function is:

$$y_t = (A\varphi)^{1-\alpha-\gamma} \cdot n_t^\alpha k^\gamma, \alpha + \gamma < 1$$

Define the term "efficient capital" as: $\hat{k} :\equiv \left((A\varphi)^{1-\alpha-\gamma}(k)^\gamma\right)^{\frac{1}{1-\alpha}}$, so the production function can be rewritten as:

$$y_t = n_t^\alpha \hat{k}^{1-\alpha}$$

Since the production function is CRS in labor and efficient capital, and the labor market is frictionless, it is immediate that the aggregate output takes the following form:

$$Y_t = N_t^\alpha \hat{K_t}^{1-\alpha}$$

where N_t, $\hat{K}_t$ are respectively the aggregate amount of labor and efficient capital at period t. For any given firm j, the profit π_t^j is pinned down by the wage w_t, the capital input k^j, and the productivity φ^j. The wage w_t is determined by the aggregate efficient capital $\hat{K}_t$, so in order to determine the fundamental component of a firm, we only need to keep track of the process of the aggregate efficient capital $\hat{K}_t$, rather than the evolution of the entire distribution of firms. The aggregate efficient capital $\hat{K}_t$, follows the following process:

$$\hat{K}_{t+1} = p\hat{K}_t + \int_{\varphi \geq \varphi^*_{t+1}(\lambda_t)} \left((A\varphi)^{1-\alpha-\gamma} \left(k\left(\lambda_t, \varphi\right)^{\gamma} \right) \right)^{\frac{1}{1-\alpha}} dF\left(\varphi\right) \quad (1.23)$$

$\hat{K}_{t+1}$ equals the sum of two terms: the remaineder of efficient capital from period t, which equals $p\hat{K}_t$; and the total amount of efficient capital brought by the entrants, which equals the second term on the RHS of equation (23). In our setup, the exit probability of firm j is independent from the level of $\hat{k}^j$, so the LLN imples that $(1-p)\,\hat{K}_t$ is destroyed at the end of the period t and $p\hat{K}_t$ remains. The cutoff level at period $t+1$, φ^*_{t+1}, and the capital input of the entrants at period $t+1$, are both determined at the end of period t, so that they are both functions of aggregate states λ_t, which includes not only $\hat{K}_t$, but also the bubble creation shock $\hat{b}_t$, and the interest rate R_t. The process of $\hat{K}_{t+1}$can therefore be pinned down recursively. I use Value Function Iteration (VFI) to solve for the fundamental component, the algorithm is described in the Appendix 8.

An important difference between the transition dynamics and the stationary equilibria we explore in Section 4 is that, if we write down the aggregate production function as:

$$Y_t = \Phi_t^{1-\alpha-\gamma} N_t^{\alpha} K_t^{\gamma} \quad (1.24)$$

like what I do in Section 4.4, the aggregate productivity Φ_t has no close-form representation in transition dynamics. In order to derive a closed-form Φ_t like in Section 4.4, it is necessary that, for any firm i and j:

$$\frac{n_t^j}{n_t^i} = \frac{k^j}{k^i} = \frac{\varphi^j}{\varphi^i}$$

The equality always holds in a stationary equilibrium; however, with shocks to $\hat{b}_t$ and R_t, the ratio φ^j/k^j fluctuates across different cohorts, entailing misallocation. I show in Section 4.4 that a permanent increase in $\hat{b}_t$ increases aggregate productivity in the long run; however, as shown in Section 6.3, a temporary increase in $\hat{b}_t$ does not necessarily increase aggregate productivity in the short run, because misallocation is exacerbated in bubbly episodes: the capital investment of entrants are subsidized in bubbly episodes, not by the government, but by optimistic investors.

1.6.2 An Example

In this section I set up an example of a stochastic dynamic model. I include only two types of aggregate shocks: sentiment shocks, which are shocks to $\hat{b}_t$, and interest rate shocks, which are shocks to R_t. It is ideal to include as well the shocks pledgeability δ[37], but that would exceedingly prolong the computation time. In any case, as shown in Section 5, the view that the financial crisis in Spain was essentially a sudden drop in A or δ is at odds with the documented increase of the average profit and productivity of entrants. Thus I limit my analysis to interest rate and sentiment shocks, with A and δ fixed.

The baseline parameters are shown in Table 3. I study the benchmark case where $\delta = 0$. The labor and capital intensity α and γ, the

[37] In this model, we can no longer use the law of motion in Equation (23), if we include shocks to A.

Pareto shape parameter ζ, the wage elasticity of labor supply ε, the survival probability p are taken from the literature.[38] A and $\bar{M}$ are irrelevant for our analysis and do not affect our results, so I set them to one.

α	γ	ζ	ε	p	ρ	σ	$\bar{r}$	p_b	p_f	δ	c_e	$\hat{b}$
0.5185	0.3315	1.08	2	0.96	0.7579	0.035	0.0092	0.93	0.89	0	1.5	0.4

Table 3: Parameters

I assume that the net interest rate $r_t :\equiv R_t - 1$ follows:

$$r_t = \bar{r} + \rho r_{t-1} + \epsilon_t \tag{1.25}$$

where $\epsilon_t \sim N\left(0, \left(1-\rho^2\right)\sigma^2\right)$. I estimate ρ, σ, and $\bar{r}$ by regression (25), using Spanish data of annual long-term real interest rate (linearly detrended) from OECD. The choice of ρ, σ, and $\bar{r}$ are based on the estimation results.

The sentiment state $\hat{b}_t$ is assumed to follow a 2-state Markov process $\hat{b}_t \in \left\{0, \hat{b}\right\}$, where $b > 0$: the economy is in a bubbly episode if $\hat{b}_t = \hat{b}$. The transition matrix is given by:

$$\begin{bmatrix} p_f & 1-p_f \\ 1-p_b & p_b \end{bmatrix}$$

p_f denotes the probability of staying in a bubbleless episode; p_b denotes the probability of staying in a bubbly episode. It is important for our

[38]The labor and capital intensity α and γ, and the Pareto shape parameter ζ are taken from Garcia-Santana (2014); the survival probability p is taken from Garcia-Macia (2015); the wage elasticity of labor supply ε is a conventional value in the literature. None of the values is eccentric compared with their counterparts in the literature.

analysis to identify the bubbly episodes among the sample periods, yet it is difficult to do it using macro data. The conventional view is that Spain experienced a bubbly episode coinciding with the rapid economic expansion, before its economy collapsed in early 2008. In media or academia, the bubble in Spain refers mostly to "housing bubble": while the bubble in the context of this paper refers mostly to the stock market bubble, the housing bubble is a sign of investor optimism which would very likely affect the stock market. In fact, Fernandez-Villaverde and Ohanian (2009) show that the Spanish stock market concurrently experienced a dramatic boom, and the size of which was even more exaggerated than those in other developed economies like the U.S. Even if it remains inconclusive whether Spanish economy was bubbly in 1980s, it is plausible to classify the period 1994-2007 as a bubbly episode, and the prolonged recession during 2008-2014 as a bubbleless episode. The calibration of p_f and p_b is based on this classification: p_f and p_b are chosen to maximize the likelihood that the economy was bubbly from 1994 to 2007, and was bubbleless from 2008 to 2014.[39]

The last two parameters: entry cost c_e and the bubble creation ratio, $\hat{b}$, are estimated using Simulated Method of Moments (SMM). Given guesses of c_e, and $\hat{b}$, I simulate the model and calculate the implied target moments: the autocorrelations of GDP growth rate and the TFP growth rate, which is construted according to Equation (24). c_e and $\hat{b}$ are chosen to minimize the difference between the simulated moments and their empirical counterparts calculated using Spanish data.[40] The fittness of the

[39]Year 1993 is assumed to be bubbleless, given the fact that Spain was in recession that year.

[40]GDP data is the Constant GDP per capita for Spain, from World Bank; TFP data is Spanish TFP data from Penn World Table 9.0.

model is reported in Table 4. The autocorrelations of GDP growth and TFP growth in the simulated data are very close to the moments in the real data. I also compare the standard deviations of simulated TFP and GDP growth with the actual data: despite the exclusion of productivity shocks and financial shocks, the model can explain approximately one half of the volatility in TFP growth and one third in GDP growth.

Statistics	Model	Data
Targeted moments		
Autocorr. GDP growth	0.69	0.70
Autocorr. TFP growth	0.60	0.60
Non-targeted moments		
Std. TFP growth	0.0045	0.0098
Std. GDP growth	0.0071	0.0221

Table 4: Moments Fit

1.6.3 Impulse Response Functions

I plot the impulse response using the calibrated model in 6.2. I run simulations with 1010 periods: the shocks are drawn stochastically except that a bubble shock arrives at period $t = 1000$. I repeat the same experiment 1000 times and calculate the average impulse responses following the bubble shock at $t = 1000$. The results are plotted in Figure 9: the starting points at time axis correspond to period $t = 1001$, when the lagged effects of the bubble shock appear. The upper-left panel reports the reaction of GDP growth rate. The GDP growth rate increases in response to a bubble shock. The acceleration in output growth is mainly driven by an acceleration in capital growth, which is reported in the upper-right panel: the growth rate of capital increased by about 1.3 percent points. The lower-left panel suggests that the number of firms also increases in reaction to the temporary bubble shock.

Most notably, we can tell from the last panel that the growth of the aggregate productivity is dampened by a bubble shock. The reason is that the allocative efficiency is deteriorated in a bubbly episode: entrants in a bubbly episode receive bubble subsidy on capital input, and invest more in capital than the cohorts from bubbleless episodes, so that bubble shocks can amplify the dispersion of MPK across different cohorts (and firms). The result is consistent with the stylized fact regarding the Spanish aggregate productivity during 1994-2007.

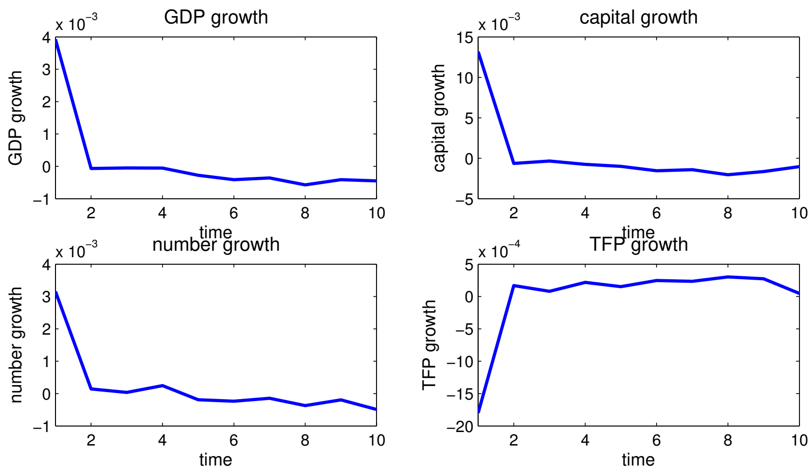

Figure 9: Impulse Response to a Bubble Shock

Like in the stationary equilibria, in the transition dynamics a (temporary) reduction in the interest rate is similar to a bubble shock. Figure 10 plots the impulse responses to a negative shock in interest rate. At time

$t = 1000$, the interest rate falls by one standard deviation: the starting points at time axis correspond to period $t = 1001$. The direction of the impulse responses to a negative interest rate shock are the same as to a bubble shock. In particular, as proposed by Gopinath et al. (2015), a negative interest rate shock can worsen the allocative efficiency and trigger a decline in the aggregate productivity.

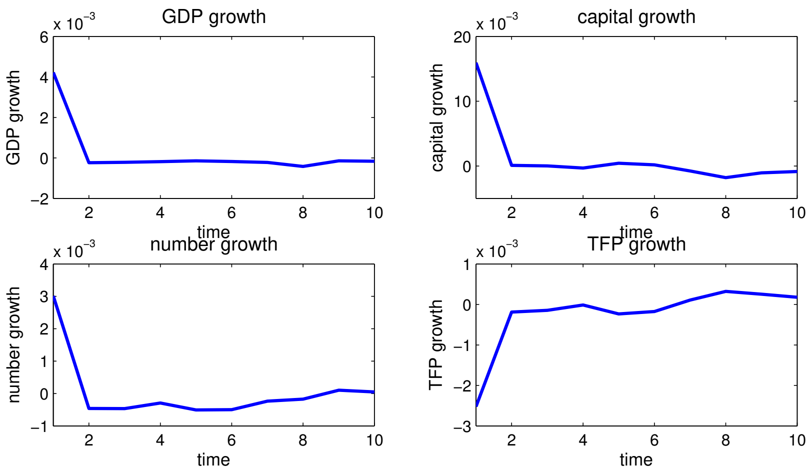

Figure 10: Impulse Response to a Negative Interest Rate Shock

1.6.4 Bubble Crash and the Great Recession

We can evaluate the importance of bubble shocks by performing counterfactual experiments. I run two experiments: in the first experiment I assume that the period 1994-2007 was bubbly, and that the year 1993 and the period 2008-2014 was bubbleless; in the second experiment the sentiment state in the period 1993-2014 is simulated according to the Markov process in our calibration. In both experiments I use actual realization of interest rates for years 1993-2014, and a simulated history of states before year 1993. Each experiment is repeated 1000 times. I plot the average simulated GDP growth rate from both experiments in the Figure 11: the left panel for the first experiment and the right panel for the second experiment. The simulation result are plotted in dashed lines, compared with the real data (growth rate of GDP per capita) in solid lines.

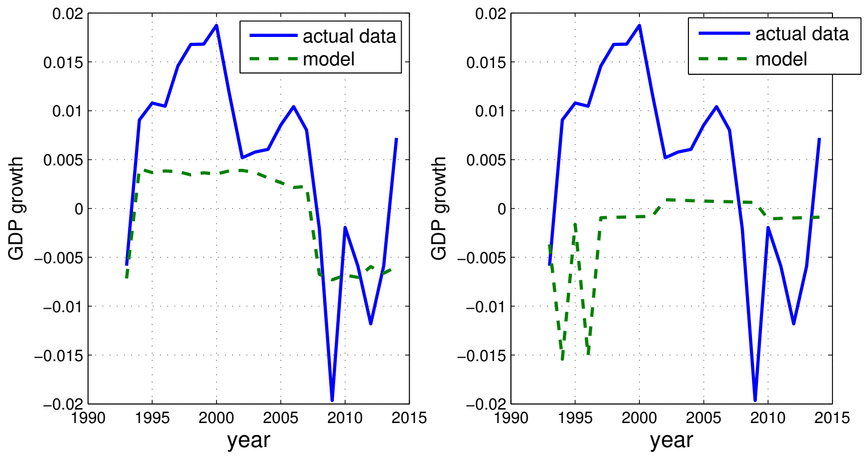

Figure 11: Counter-factual Experiment with Bubble Shocks

In the right panel, our control group, the average effect of bubble shocks is close to zero and the fluctuation of GDP growth is driven solely by the changes in the interest rate. It is clear that the model is unable to reproduce the economic expansion with comparable magnitude. Moreover, without a bubble crash, the model cannot account for the size of the downturn during the Great Recession. In the left panel, our experimental group, bubbles existed during 1993-2007, and did not exist during 2008-2014: with the boom-bust of bubbles, the fluctuations are significantly amplified. It is premature to overstate the importance of bubble shocks in the real world, but the results suggest that bubble shocks are prospective in explaning the dramatic size of aggregate fluctuations.

1.7 Conclusion

To the utmost of my knowledge, this is the first paper studying the implications of asset price bubbles on the distribution of firms, and the first paper investigating the macroeconomic implication of sentiments through firm entry in an economy. In this paper, I construct a model in the spirit of Lucas (1978) and Hopenhayn (1992). I characterize the bubbly equilibria where the price of a firm equals the sum of a fundamental component and a Ponzi game component. The model can shed light on the distinctions between the various explanations of economic expansions, and can rationalize the recent Spanish experience. Although the theory developed in this paper is motivated by and used to interpret the Spanish empirical facts, it is also applicable to study other economies, like other countries in southern Europe, or China, which is experiencing a dramatic boom in asset prices.

The theoretical insights of this paper may be of interest to policy makers. There is a growing literature studying policy implications of bubbles.[41] In practice, policy makers face the challenge of verifying whether an economic expansion is driven by bubbles or other unobservable factors like productivity or demand. The theory I propose here can facilitate this verification.

In order to derive a closed-form solution of stationary equilibrium

[41] See Asriyan et al. (2016), Gali (2014), and Martin and Ventura (2016).

and to simplify the numerical analysis, I abstract from capital adjustment, fixed cost of production, and idiosyncratic productivity shocks. The first extension of the baseline model would be to introduce capital adjustment. It is widely documented that compared with younger firms, older firms are less likely to be financially constrained.[42] The literature argues that this pattern is due to the fact that firms can overcome the financial constraint by accumulating capital stocks.[43] As shown in Section 4, a rational bubble component is expected to grow upon continuation. The growth of bubble components can be an engine for firm growth: as bubble components expand over time, firms gradually grow out of financial constraints since bubbles can act as collateral when fundamental collateral is scarce.[44] In the presence of bubbles, firms can grow even if they do not accumulate capital stock (for instance, capital fully depreciates every period). It has not yet been explored to which extent bubbles contribute to the growth of firms. Introducing capital adjustment of incumbent firms would enable us to study the implications of bubbles on firm growth and moreover, how bubbles affect the evolution of firm size distribution.

The second extension would be to incorporate fixed operating cost and idiosyncratic productivity shocks. A limitation of the baseline model is that there is only exogenous exit rather than endogenous exit: incumbents never choose to exit the market since there is no fixed operating cost and the profit is always non-negative. Firms would possibly choose to exit when facing a positive fixed operating cost and a negative idiosyncratic productivity shock. In the scenario with endogenous exit, bubbles act as a tax on firm exit since exit incurs loss of bubbles. The wave of firm exit in

[42] See Cabral and Mata (2003).
[43] See Midrigan and Xu (2014), and Moll (2014).
[44] See Martin and Ventura (2012, 2016)

the financial crisis could be well explained as a consequence of a bubble crash. Moreover, through the interaction with firm exit, bubbles amplify misallocation given that bubbles sustain the non-productive firms which would otherwise exit the market. Furthermore, bubbles make older firms less likely to exit, since bubble components are expected to grow upon continuation, and older firms are more likely to have larger bubble components: this is consistent with the empirical evidence.[45] In a word, the interaction between bubbles and endogenous exit would be an interesting avenue for future research.

[45] See Dunne, Roberts, and Samuelson (1989).

Appendix 1

This Appendix presents proof for Proposition 1. The aggregate bubbles in stationary equilibria, B, as a result of Law of Large Number, follow the process:

$$B = RB + B(0)$$

or

$$B = \frac{B(0)}{1-R}$$

where $B(0)$ denotes the total amount of bubbles for the entrant. When $B(0) = 0$, the equilibrium is the fundamental equilibrium studied by Hopenhayn (1992) and Melitz (2003), as well as another papers in the literature of firm/industry dynamics. If $B(0) > 0$, the equilibrium is a bubbly equilibrium, and the aggregate bubbles are not explosive if and only if $R < 1$ and $B(0) < \infty$. Given my setup, $B(0)$ is always bounded: for any given entrant, the size of bubbles is proportional to the fundamental component as well as to its profit and output; since the total output for entrants is always bounded, $B(0)$ is bounded as well. Moreover, A bubbly stationary equilibrium is sustainable if B is not larger than the aggregate output, and there always exists a bubble creation ratio b which makes $B(0)$ and B small enough compared with aggregate output.

Q.E.D.

Appendix 2

The fundamental component, $f(\lambda, \varphi, k)$, is increasing in the idiosyncratic productivity component φ, since the profit is increasing in φ, conditional on λ and k. Equation (6) can be rewritten as:

$$v^e(\lambda,\varphi)=\frac{1}{R}\max_k\{v(\lambda,\varphi,k)-Rk\}=\frac{1}{R}\max_k\{(1+b)\,f(\lambda,\varphi,k)-Rk\}$$

The equation above implies that the value function $v^e(\lambda,\varphi)$ is also increasing in φ[46]. Therefore there is a unique cutoff productivity φ^*: an entrant enters the market as long as she draws a productivity $\varphi>\varphi^*$, otherwise she does not enter the market since the entry value $v^e(\lambda,\varphi)$ is not high enough to cover the entry cost c_e[47].

Appendix 3

Without loss of generality, I start with the scenario where $\gamma<\frac{1}{2-\delta}$: the collateral constraint is not binding.

Appendix 3.1: Derive FE

Use that:

$$Rk=\frac{R\left(1+\hat{b}\right)\gamma}{R-p}\pi$$

we can rewrite Equation (6) as:

$$v^e(\lambda,\varphi)=\frac{\left(1+\hat{b}\right)(1-\gamma)}{R-p}\pi\left(\lambda,\varphi,k\left(\lambda,\varphi\right)\right)\qquad(1.26)$$

[46] Use Envelope Theorem.
[47] Use that $v^e(\lambda,\varphi)\to 0$ if $\varphi\to 0$; $v^e(\lambda,\varphi)\to+\infty$ if $\varphi\to+\infty$.

$k(\lambda, \varphi)$ is the capital input chosen by the entrants. FE holds if and only if as for the marginal entrant with $\varphi = \varphi^*$,

$$v^e(\lambda, \varphi^*) = c_e \tag{1.27}$$

Equation (27) implies that at the cut-off productivity level $\varphi = \varphi^*$, entrants are indifferent in whether or not to enter the market. If $\hat{b} = 0$, the stationary equilibrium is a bubbleless, or fundamental equilibrium, as studied by Hopenhayn (1992) and Melitz (2003). We can derive the free entry condition (FE) using Equation (26) and (27):

$$\pi^* = \frac{R - p}{\left(1 + \hat{b}\right)(1 - \gamma)} c_e \tag{1.28}$$

Appendix 3.2: Derive $n(\lambda, \varphi^*)$

We need to derive $n(\lambda, \varphi^*)$ before we derive the LMC. The crucial step is to rewrite the wage w as the marginal product of labor for the marginal entrants. Recall Equation (11) and (12):

$$\alpha (A\varphi)^{1-\alpha-\gamma} n^{\alpha-1} k^{\gamma} = w \tag{1.29}$$

$$\frac{R\left(1 + \hat{b}\right)}{R - p} (1 - \alpha) \gamma (A\varphi)^{1-\alpha-\gamma} n^{\alpha} k^{\gamma-1} = R \tag{1.30}$$

Using Equation (29) and (30), we can derive that

$$k = \frac{w\left(1 + \hat{b}\right)(1 - \alpha)\gamma}{\alpha (R - p)} n \tag{1.31}$$

Plug Equation (31) into Equation (29), we get:

$$w^{1-\gamma} = \alpha \left(A\varphi\right)^{1-\alpha-\gamma} \left(\frac{\left(1+\hat{b}\right)\left(1-\alpha\right)\gamma}{\alpha\left(R-p\right)} \right)^{\gamma} n^{\alpha+\gamma-1}$$

As for the marginal entrant:

$$w^{1-\gamma} = \alpha \left(A\varphi^{*}\right)^{1-\alpha-\gamma} \left(\frac{\left(1+\hat{b}\right)\left(1-\alpha\right)\gamma}{\alpha\left(R-p\right)} \right)^{\gamma} n\left(\lambda,\varphi^{*}\right)^{\alpha+\gamma-1}$$

Using Equation (29) and (30), we have:

$$\frac{n^{i}}{n^{j}} = \frac{\varphi^{i}}{\varphi^{j}} \tag{1.32}$$

where n^{j} denotes the labor demand of firm j. Labor market clearing implies that:

$$\int_{\varphi\geq\varphi*} \left(\frac{\varphi}{\varphi^{*}}\right) n\left(\lambda,\varphi^{*}\right) \frac{\bar{M}}{1-p} dF\left(\varphi\right) = L_{s} = w^{\varepsilon} \tag{1.33}$$

where the left-hand-side is the aggregate labor demand, while the right-hand-side is the aggregate labor supply, conditional on wage; $\frac{M}{1-p}dF\left(\varphi\right)$ measures the amount of firms with productivity φ in the stationary equilibria. Equation (33) can thus be rewritten as:

$$\int_{\varphi\geq\varphi*} \left(\frac{\varphi}{\varphi^{*}}\right) n\left(\lambda,\varphi^{*}\right) \frac{\bar{M}}{1-p} dF\left(\varphi\right) = \left[\alpha \left(A\varphi^{*}\right)^{1-\alpha-\gamma} \left(\frac{\left(1+\hat{b}\right)\left(1-\alpha\right)\gamma}{\alpha\left(R-p\right)} \right)^{\gamma} n\left(\lambda,\varphi^{*}\right)^{\alpha+\gamma-1} \right]^{\frac{\varepsilon}{1-\gamma}} \tag{1.34}$$

Move $n(\lambda, \varphi^*)$ to the left-hand-side:

$$n(\lambda, \varphi^*)^{1-\frac{\varepsilon(\alpha+\gamma-1)}{1-\gamma}} = \frac{(1-p)\left[\alpha (A\varphi^*)^{1-\alpha-\gamma}\left(\frac{\left(1+\hat{b}\right)(1-\alpha)\gamma}{\alpha(R-p)}\right)^{\gamma}\right]^{\frac{\varepsilon}{1-\gamma}}}{\bar{M}\int_{\varphi\geq\varphi*}\left(\frac{\varphi}{\varphi^*}\right)dF(\varphi)} \tag{1.35}$$

or:

$$n(\lambda, \varphi^*) = \left(\frac{(1-p)\left(\alpha\left(\frac{\left(1+\hat{b}\right)(1-\alpha)\gamma}{\alpha(R-p)}\right)^{\gamma}(A\varphi^*)^{1-\alpha-\gamma}\right)^{\frac{\varepsilon}{1-\gamma}}}{\bar{M}\int_{\varphi\geq\varphi*}\left(\frac{\varphi}{\varphi*}\right)dF(\varphi)}\right)^{\frac{1}{1-\frac{\varepsilon}{1-\gamma}(\alpha+\gamma-1)}} \tag{1.36}$$

Appendix 3.3: Derive LMC

The relationship between profit and labor demand is given by[48]:

$$\pi(\lambda, \varphi) = \left[(1-\alpha)(A\varphi)^{1-\alpha-\gamma}\left(\frac{\left(1+\hat{b}\right)\gamma}{R-p}\right)^{\gamma} n(\lambda, \varphi)^{\alpha}\right]^{\frac{1}{1-\gamma}}$$

[48]The equation is immediate if we rewrite capital input as a function of labor input and parameters according to Equation (30), and then use that:

$$\frac{k}{\pi} = \frac{\left(1+\hat{b}\right)\gamma}{R-p}$$

We can thus characterize the labor market clearing condition (LMC):

$$\pi^* = \left[(1-\alpha)\left(A\varphi^*\right)^{1-\alpha-\gamma} \left(\frac{\left(1+\hat{b}\right)\gamma}{R-p} \right)^{\gamma} n\left(\lambda, \varphi^*\right)^{\alpha} \right]^{\frac{1}{1-\gamma}} \tag{1.37}$$

where $n(\lambda, \varphi*)$ is given by Equation (36). The LMC establishes another relationship between cutoff productivity φ^* and profit π^*.

Appendix 3.4: FE and LMC when Credit Constraint is Binding

If $\gamma > \frac{1}{2-\delta}$, we can derive the FE and LMC analogously[49]:

$$FE: \ \pi^* = \frac{R-p}{\left(1+\hat{b}\right)\left(1-\frac{1}{2-\delta}\right)} c_e \tag{1.38}$$

$$LMC: \ \pi^* = \left[(1-\alpha)\left(A\varphi^*\right)^{1-\alpha-\gamma} \left(\frac{\left(1+\hat{b}\right)}{(R-p)(2-\delta)} \right)^{\gamma} n\left(\lambda, \varphi^*\right)^{\alpha} \right]^{\frac{1}{1-\gamma}} \tag{1.39}$$

where $n(\lambda, \varphi*)$ is given by:

$$n\left(\lambda, \varphi^*\right) = \left(\frac{(1-p)\left(\alpha \left(\frac{\left(1+\hat{b}\right)(1-\alpha)}{\alpha(R-p)(2-\delta)} \right)^{\gamma} \left(A\varphi^*\right)^{1-\alpha-\gamma} \right)^{\frac{\varepsilon}{1-\gamma}}}{\bar{M} \int_{\varphi \geq \varphi*} \left(\frac{\varphi}{\varphi*}\right) dF(\varphi)} \right)^{\frac{1}{1-\frac{\varepsilon}{1-\gamma}(\alpha+\gamma-1)}} \tag{1.40}$$

[49]Simply repeat every step but replace γ with δ

Appendix 4

The LMC imples that π^* can be written as a function of φ^*, independent of whether financial constraint is binding:

$$\pi^* = a_0 A^{a_1} \left(\varphi^*\right)^{a_2} \left(1+\hat{b}\right)^{a_3}$$

where:

$$a_1 = \frac{1}{1-\gamma}\left(1-\alpha-\gamma+\frac{\alpha}{1+\frac{\varepsilon}{1-\gamma}\left(1-\alpha-\gamma\right)}\frac{\varepsilon\left(1-\alpha-\gamma\right)}{1-\gamma}\right)>0$$

$$a_2 = \frac{1}{1-\gamma}\left(1-\alpha-\gamma+\frac{1}{1+\frac{\varepsilon}{1-\gamma}\left(1-\alpha-\gamma\right)}\left(\frac{\varepsilon\left(1-\alpha-\gamma\right)}{1-\gamma}+\zeta\right)\right)>0$$

$$a_3 = \frac{1}{1-\gamma}\left(\gamma+\frac{\alpha}{1+\frac{\varepsilon}{1-\gamma}\left(1-\alpha-\gamma\right)}\frac{\varepsilon\left(1-\alpha-\gamma\right)}{1-\gamma}\right)>0$$

and a_0 is a constant independent of φ^*, $\hat{b}$, and A. The LMC condition implies that: 1) π^* is increasing in φ^*, or equivalently, decreaing in the amount of firm entry; 2) if $\hat{b}$ increases, π^* increases for every φ^*.

Appendix 5

It is immediate from the definition of aggregate productivity that:

$$\Phi = \frac{\bar{M}}{1-p}\int \left(A\varphi\right)^{1-\alpha-\gamma}\left(\frac{n\left(\lambda,\varphi\right)}{N}\right)^{\alpha}\left(\frac{k\left(\lambda,\varphi\right)}{K}\right)^{\gamma} dF\left(\varphi\right)$$

We can use that:[50]

$$\frac{n(\lambda,\varphi)}{N}=\frac{k(\lambda,\varphi)}{K}$$

and

$$\frac{N}{n(\lambda,\varphi)}=\int_{\varphi'\geq\varphi^*}\left(\frac{\varphi'}{\varphi}\right)dF(\varphi')$$

to derive the expression for aggregate productivity:

$$\Phi=A\left(\frac{\bar{M}}{1-p}\right)^{1-\alpha-\gamma}\left(\int_{\varphi\geq\varphi^*}\varphi dF(\varphi)\right)$$

Appendix 6

Use that:

$$k=\frac{\left(1+\hat{b}\right)\min\left(\frac{1}{2-\delta},\gamma\right)}{R-p}\pi \tag{1.41}$$

We can get, from the identity of net profit, that:

$$\pi_{net}=\frac{R-p-r\min\left(\gamma,\frac{1}{2-\delta}\right)\left(1+\hat{b}\right)}{R-p}\pi \tag{1.42}$$

It is immediate that:

$$\pi^*_{net}=\frac{R-p-r\min\left(\gamma,\frac{1}{2-\delta}\right)\left(1+\hat{b}\right)}{R-p}\pi^* \tag{1.43}$$

[50]The equality holds if firms are universally financially constrained, or universally unconstrained: which is the case in stationary equilibria.

Plug in the FE condition from Equation (14) to Equation (43), we have:

$$\pi_{net}^{*} = \frac{R - p - r\min\left(\gamma, \frac{1}{2-\delta}\right)\left(1+\hat{b}\right)}{\left(1+\hat{b}\right)\left(1-\min\left(\gamma, \frac{1}{2-\delta}\right)\right)} c_e$$

, or:

$$\pi_{net}^{*} = \left(r + \frac{1 - p - r\hat{b}}{\left(1+\hat{b}\right)\left(1-\min\left(\gamma, \frac{1}{2-\delta}\right)\right)}\right) c_e \tag{1.44}$$

Appendix 7

Equations (18) and (19) can be rewritten respectively as:

$$\alpha Y = N^{1+\frac{1}{\varepsilon}} \tag{1.45}$$

$$\frac{R\left(1+\hat{b}\right)}{R-p}(1-\alpha)\min\left(\frac{1}{2-\delta}, \gamma\right) Y = RK \tag{1.46}$$

Using the definition of aggregate production function, and Equation (46), we can derive that:

$$\Phi^{1-\alpha-\gamma}(\alpha Y)^{\frac{\varepsilon\alpha}{\varepsilon+1}}\left(\frac{1+\hat{b}}{R-p}(1-\alpha)\min\left(\frac{1}{2-\delta}, \gamma\right) Y\right)^{\gamma} = Y$$

or

$$Y = \left(\Phi^{1-\alpha-\gamma}\alpha^{\frac{\varepsilon\alpha}{\varepsilon+1}}\left(\frac{1+\hat{b}}{R-p}(1-\alpha)\min\left(\frac{1}{2-\delta}, \gamma\right)\right)^{\gamma}\right)^{\frac{1}{1-\gamma-\frac{\varepsilon\alpha}{\varepsilon+1}}} \tag{1.47}$$

No matter whether financial constraint is binding, we have:

$$\frac{dY}{d\hat{b}} = \frac{\partial Y}{\partial \Phi}\frac{\partial \Phi}{\partial \varphi^*}\frac{\partial \varphi^*}{\partial \hat{b}} + \frac{\partial Y}{\partial \hat{b}} > 0$$

since $\frac{\partial Y}{\partial \Phi} > 0$, $\frac{\partial \Phi}{\partial \varphi^*} < 0$, $\frac{\partial \varphi^*}{\partial \hat{b}} < 0$, and $\frac{\partial Y}{\partial \hat{b}}$. Hence an increase in $\hat{b}$ increases the aggregate output Y in a new stationary equilibrium. According to Equations (45) and (46), aggregate inputs K and N increase if Y increases.

Appendix 8

As for fundamental component, we have the functional equation:

$$f(\lambda_t, \varphi, k) = \pi(\lambda_t, \varphi, k) + \frac{p}{R_t}\widehat{E}_t f(\lambda_{t+1}, \varphi, k) \tag{1.48}$$

It is possible to solve functional equation (48) for fundamental component using value function iteration. The aggregate states consist of the intereset rate R_t, the bubble creation ratio $\hat{b}_t$, and the efficient capital $\widehat{K}_t$[51]. While R_t and $\hat{b}_t$ follow exogenous stochastic process, we need to characterize the law of motion for efficient capital $\widehat{K}_t$. The algorithm include the following steps.

Step 1: Given a set of parameters, guess a function for fundamental component $f_1\left(\varphi, k, \widehat{K}_t, R_t, \hat{b}_t\right)$ and law of motion for $\widehat{K}_t$ conditional on $\widehat{K}_{t-1}$, R_t, and $\hat{b}_t$.

Step 2: Solve the following constrained maximization problem to get

[51]The algorithm I describe here is the one I use to solve the example in 6.2. The algorithm is basically identical if we include a stochastic pledgeability δ_t.

$v^e\left(\varphi,\widehat{K}_t,R_t,\hat{b}_t\right)$ and $k\left(\varphi,\widehat{K}_t,R_t,\hat{b}_t\right)$:

$$v^e\left(\varphi,\widehat{K}_t,R_t,\hat{b}_t\right)=\frac{1}{R_t}\max_k \widehat{E}_t\left(\left(1+\hat{b}_t\right)f_1\left(\varphi,k,\widehat{K}_{t+1},R_{t+1},\hat{b}_{t+1}\right)-R_tk\right) \tag{1.49}$$

$$s.t.\quad \left(1+\hat{b}_t\right)\widehat{E}_tf_1\left(\varphi,k,\widehat{K}_{t+1},R_{t+1},\hat{b}_{t+1}\right)\geq\left(R_t+1-\delta\right)k$$

Step 3: Find the implied law of motion for $\widehat{K}_t$:

$$\widehat{K}_{t+1}=p\widehat{K}_t+\int_{\varphi\geq\varphi^*_{t+1}\left(\widehat{K}_t,R_t,\hat{b}_t\right)}\left((A\varphi)^{1-\alpha-\gamma}\left(k\left(\varphi,\widehat{K}_t,R_t,\hat{b}_t\right)^\gamma\right)\right)^{\frac{1}{1-\alpha}}dF\left(\varphi\right) \tag{1.50}$$

where $\varphi^*_{t+1}\left(\widehat{K}_t,R_t,\hat{b}_t\right)$ is pinned down through the following equation:

$$v^e\left(\varphi^*_{t+1}\left(\widehat{K}_t,R_t,\hat{b}_t\right),\widehat{K}_t,R_t,\hat{b}_t\right)=c_e$$

Step 4: Update our guess for fundamental value function:

$$f_2\left(\varphi,k,\widehat{K}_t,R_t,\hat{b}_t\right)=\pi\left(\varphi,k,\widehat{K}_t,R_t,\hat{b}_t\right)+\frac{p}{R_t}\widehat{E}_tf_1\left(\varphi,k,\widehat{K}_t,R_t,\hat{b}_t\right) \tag{1.51}$$

Step 5: Go back to Step 2, however, use the updated guesses for fundamental value function from Equation (51), and for law of motion from Equation (50).

Iterate Step 2-5 until the fundamental value function converges.

Chapter 2

DIFFERENT TYPES OF FIRM EXIT AND AGGREGATE FLUCTUATIONS

Joint with Roberto Ramos

2.1 Introduction

Firm exit has long been an important subject of economic research. Numerous works have been dedicated to understanding whether recessions have cleansing effects,[1] and how firm exit amplifies and prolongs recessions.[2] To improve the understanding of these issues, we first need to take into account that there exist various types of firm exit. Firm exit may be due to low productivity and profitability, or lack of liquidity. Moreover, exiters may either repay their debt, or default and become bankrupt.

[1] See Caballero and Hammour (1994), and Kehrig (2015).
[2] See Clementi and Palazzo (2016) and Lee and Mukoyama (2013).

Clearly, different types of exiters have different features. However, little attention has been given on the difference among exiters, and the dynamics of exiters composition. Our main goal is to fill this gap in the literature.

Our findings from Spanish firm-level data highlight the quantitative importance of firm exit and the difference between different exiters. We conduct a decomposition of aggregate investment and debt payout, which suggests that a large fraction of disinvestment and deleveraging takes place in the form of firm exit, whereas the existing literature of business cycles focuses mainly on how incumbents behave in recessions. We also show that during the boom of the early 2000s, exiters were on average less leveraged than survivors. However, during the financial crisis, exiters had higher leverage ratio than survivors. We conjecture that the dynamics imply a time-varying composition of exiters.

In this paper we propose a model with different types of firm exit. The model is developed in the spirit of Lucas (1978) and Hopenhayn (1992). We assume price-taking firms, heterogeneous firm-level productivity, and a decreasing-return-to-scale (DRS) production function. We incorporate financial frictions. Firms can use coporate bonds to finance their investment, but bonds have to be backed by collateral. They can also use profits to self-finance, but the dividends cannot be negative.[3]

We investigate different types of exit. Depending on whether an exiter has the ability to avoid negative dividends, it can be either solvent or insolvent. An insolvent firm is unable to raise enough funds to avoid negative dividends if it stays in the market. An insolvent firm is forced to exit the market. An exiter can also be solvent if it is able to avoid negative dividend but finds it optimal to stop producing. Whether insolvency

[3]The assumption implies that firms do not have access to equity markets.

occurs depends on the size of assets, the level of productivity, and the size of outstanding debt. Furthermore, depending on whether an exiter repays its debt, it can be either non-defaulting or defaulting. As to an exiter, whether or not to default relies on the relative size of outstanding debt and collateral assets. If the size of collateral assets is large compared with the size of debt, an exiter chooses to repay its debt to avoid the loss of collateral. In the contrast, if the size of collateral is small relative to the size of debt, an exiter chooses to default. Notably, an insolvent exiter can be either non-defaulting or defaulting. Similarly, a defaulting exiter can be either insolvent or solvent.

The model is calibrated with Spanish firm-level data. In the stationary case, aggregate variables remain time-invariant, while at the firm-level, the average size and the survival probability grow over time. We compare different groups of exiters. Exiters which repay their debt are mostly unproductive firms. Defaulting exiters, on the contrary, are mainly productive firms with little assets. We also find that low profitability, rather than large debt or asset scarcity, are the main cause of insolvency.

We study two different shocks: productivity shocks and financial shocks. Productivity shocks are shocks to the common productivity component. Financial shocks are shocks to the pledgability of collateral assets. Notably, a positive financial shock increases the exit cost of defaulting firms. On the one hand positive financial shocks increase profitability by relaxing financial constraints. On the other hand, prospective entrants take the higher exit cost into consideration and may become less likely to enter the market. Therefore a positive financial shock in our model has an ambiguous effect on output.

Our quantitative results suggest that in aggregate, both shocks are procyclical. More importantly, the model sheds light on the dynamic composition of exiters. Negative shocks increase the defaulting exit rate but lowers the non-defaulting exit rate. In a recession, highly leveraged firms with high productivity but few assets become very likely to exit. However, due to the low wage rate in a recession, low leverage unproductive firms with large assets become even more likely to survive.

We also show that, during a productivity-driven recession, the increase of defaulting exit stems from the increase of insolvency. As aforementioned, defaulting exiters are mostly highly leveraged productive firms with little assets. These firms are very likely to default once they become insolvent, since their debt is large relative to their assets. Therefore the increase of insolvency rate in this group of firms fully transfers into the increase of defaulting exit. However, the insolvency rate does not increase much during a financial crisis. If a recession is triggered by a negative financial shock, the soar of defaulting exit is mainly accounted for by the increasing defaulting exit of solvent firms since the collateral loss from defaulting is low. The results shed light on policy making. We argue that liquidity provision or debt-restructuring policies trying to help insolvent firms may not work well during financial recessions, given that the increasing defaulting exit is mainly a consequence of the default of solvent firms.

The remainder of the paper is organized as follows. In Section 2 we present the motivating facts. The model is introduced in Section 3. We characterize the stationary equilibrium in Section 4. In Section 5 we investigate the equilibrium dynamics in the presence of aggregate uncertainty. Section 6 concludes.

Related Literatures

Our paper is closely related to the wide body of research on firm dynamics and business cycles, especially to the vast literature incorporating credit constraints to a heterogeneous-firm framework. Jermann and Quadrini (2012) propose a model with financial frictions and heterogeneous firms. They argue that financial shocks are important in explaining the magnitude of fluctuations. Buera and Shin (2011, 2013), and Midrigan and Xu (2014) build up quantitative models with firm entry and exit to study the importance of financial frictions in explaining TFP. Lee and Mukoyama (2015), and Clementi and Palazzo (2016) study the cyclical pattern of firm entry and exit. Our paper also contributes to the literature by revisiting the cleansing effects of recessions. Earlier seminal works like Schumpeter (1942) and Caballero and Hammour (1994) suggest that recessions are cleansing. However, using plant-level data, Kehrig (2015) argue that recessions are sullying rather than cleansing. Our finding suggests that not all types of exit are cleansing. Although non-defaulting exit cleans obese firms, defaulting exit destroys firms with good growth potential.

The recent debt crisis in the Eurozone has motivated a literature exploring the credit boom and misallocation in southern Europe. Gopinath et al. (2015) document a significant increase in misallocation of capital in Spain. They argue that adjustment costs and financial constraints play critical roles in generating the dispersion of the marginal product of capital. Garcia-Santana et al. (2016) study misallocation across firms in Spain. They find that deterioration of allocative efficiency in Spain is mainly driven by misallocation across firms rather than across industries. They also find that the measure of misallocation significantly increased using an unbalanced panel with the full sample of firms, compared with the same measure using a balanced panel with a permanent sample of firms. This result seems to support that the entry of firms is at the root of the TFP decline in Spain.

Perhaps the closest paper to ours is the one by Khan and Thomas (2013). They study the cyclical implications of financial frictions in a model with firm entry and exit. Like us, they assume non-negative dividends. However, in their model, firms exit the market only if they are binded by the non-negative dividend constraint to raise enough funds to repay their debt. In other words, the only type of exit in their paper is insolvent exit. In our paper, solvent firms may also exit.

2.2 Motivating Facts

2.2.1 Firm Entry and Exit

Figure 1 plots the number of entrants and exiters of Spain during 1996 to 2012. The data is from the DIRCE, a database provided by the Spanish National Statistics Institute that contains statistical information of the population of Spanish companies. In early 2000s, firm entry experienced a slow expansion. Following the onset of the financial crisis and the housing bubble burst, the number of entrants declined drastically and thereafter stagnated during the prolonged recession. Meanwhile, firm exit increased gradually and soared during the financial crisis.

Figure 1: Firm Entry and Exit

2.2.2 Firm-level Investment

The change of aggregate capital stock can be decomposed into 3 components:

$$K_{t+1} - K_t = K_{t+1}^{en} + \triangle K_{t+1}^{in} - K_t^{ex}$$

where K_t denotes the aggregate capital stock, K_t^{en} denotes the total capital stock of entrants, $\triangle K_t^{in}$ denotes the change in capital, or net investment, of incumbents, K_t^{ex} denotes the capital of exiters.

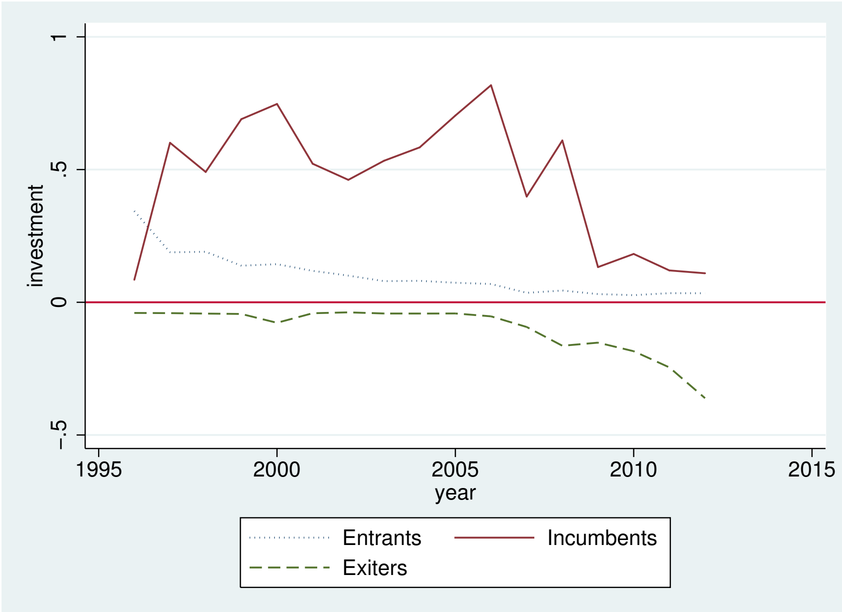

Figure 2: Investment Decomposition

Figure 2 presents the dynamics of the 3 components (normalized by GDP). We use SABI-AMADEUS data of total assets as the measure of the firm-level capital stock. The data is merged with firm entry/exit data from DIRCE, in order to obtain the capital stock of entrants, exiters, and incumbents. We can tell from the graph that after 2008 the loss of capital from firm exit is comparable in magnitude with the investment of incumbents. In fact, firm exit was the main driving force of capital loss during the prolonged recession.

2.2.3 Debt and Leverage of Entrants and Exiters

Similarly, debt payout can be decomposed into:

$$D_{t+1} - D_t = D_{t+1}^{en} + \triangle D_{t+1}^{in} - D_t^{ex}$$

where D_t denotes the aggregate debt, D_t^{en} denotes the debt of entrants, $\triangle D_t^{in}$ denotes the debt payout of incumbents, D_t^{ex} denotes the debt of exiters. The debt is measured by long-term debt, which comes from a firm-level financial dataset of the Bank of Spain. We normalize the three items by GDP and plot the dynamics in Figure 3. It is shown that during the Great Recession, most deleveraging takes place in the form of firm exit.

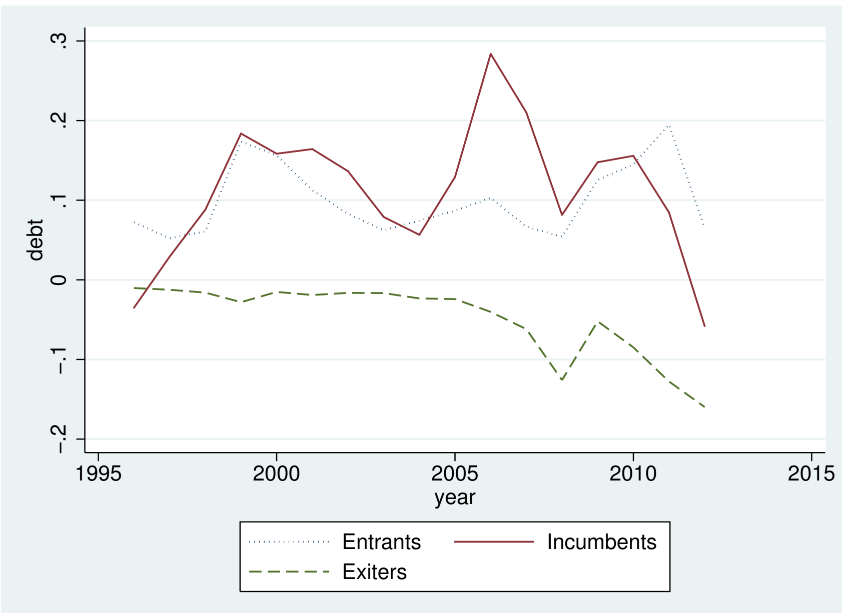

Figure 3: Debt Payout Decomposition

In Figure 4 we plot the average firm-level leverage ratio. The average leverage ratios are calculated using the data of assets and long-term debt from the Bank of Spain, and weighted by size of assets. From the figure we know that during the boom in the early 2000s, exiters tend to be less leveraged firms. However in late 2000s, highly leveraged firms are more likely to exit.

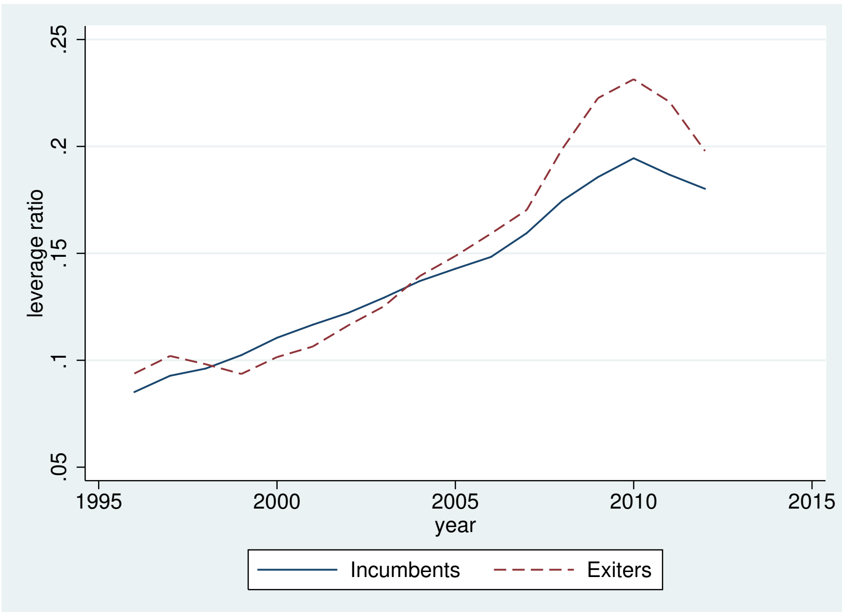

Figure 4: The Leverage Ratio of Incumbents and Exiters

To sum up, we show that during recessions: i) firm exit increases; ii) exiters become more leveraged than incumbents.

2.3 Model Setup

In this section, we set up a model in the spirit of Hopenhayn (1992). We incorporate credit constraints for firms: 1) firms cannot pay negative dividends, and 2) their bonds have to be backed by collateral. Our model describes a small open economy where the interest rate is determined exogenously.

2.3.1 Production

Time is discrete and runs from zero to infinity. A mass of atomistic firms produce a homogeneous product at each time t. The price of the product is normalized to one. The production is subject to a fixed operation cost c_f. Firms take wage w_t as given. For firm j, the output follows:

$$y_t^j = \left[\exp\left(A_t \varphi_t^j\right)\right]^{(1-\gamma)} \left[\left(n_t^j\right)^{1-\alpha} \left(k_t^j\right)^{\alpha}\right]^{\gamma}$$

The production function displays decreasing return to scale ($\gamma < 1$). A_t denotes the common productivity component, which is identical across all firms. φ_t^j denotes the idiosyncratic productivity component, which is specific to firm j. Both A_t and φ_t^j follow an AR1 process:

$$A_{t+1} = \rho_A A_t + \epsilon_{A,t+1}$$

$$\varphi_{t+1}^j = \rho_\varphi \varphi_t + \epsilon_{\varphi,t+1}$$

where $\epsilon_{A,t+1} \sim N\left(0, \sigma_A^2\right)$, $\epsilon_{\varphi,t+1} \sim N\left(0, \sigma_\varphi^2\right)$.

2.3.2 Credit Market and Investment

Firm j can issue one-period bonds to finance its physical capital k_t^j. At time t, firms receive $q_t^j b_{t+1}^j$ by issuing a bond which pays b_{t+1}^j at $t+1$. The bond price q_t^j is given by the no-arbitrage condition:

$$Rq_t^j b_{t+1}^j = \left(1 - E_t p_{t+1}^j\right) b_{t+1}^j + E_t^j p_{t+1}^j \theta_{t+1} k_{t+1}^j \tag{2.1}$$

where p_{t+1}^j denotes the probability of default for firm j at $t+1$, and R denotes the risk-free rate. The bond has to be collateralized by physical capital k_{t+1}^j. θ_{t+1} measures the stochastic pledgability of collateral k_{t+1}^j. The bond holders receive $\theta_{t+1} k_{t+1}^j$ if firm j defaults at $t+1$. We assume θ_t follows an AR1 process:[4]

$$\theta_{t+1} = \rho_\theta \theta_t + \epsilon_{\theta,t+1}$$

where $\epsilon_{\theta,t+1} \sim N\left(0, \sigma_\theta^2\right)$. If firms repays their debt, the remaining resource can be either used for investment i_t^j or dividend d_t^j:

$$d_t^j + i_t^j = \pi_t^j - b_t^j + q_t^j b_{t+1}^j \tag{2.2}$$

where profit π_t^j denotes

$$\pi_t^j = \max_{n_t^j} \left\{ \left[\exp\left(A_t \varphi_t^j\right)\right]^{1-\gamma} \left[\left(n_t^j\right)^{1-\alpha} \left(k_t^j\right)^{\alpha}\right]^{\gamma} - w_t n_t^j - c_f \right\}$$

[4]In principle, continuous AR1 processes can take negative values while pledgability cannot be negative. However, we adopt discrete Tauchen method to approximate the AR1 process. Reasonable values of ρ_θ and σ_θ ensure that θ_t cannot be negative in our computation.

Moreover, we assume that dividends are non-negative:

$$d_t^i \geq 0 \tag{2.3}$$

We also assume that firms cannot hold financial assets, so that they cannot save through purchasing bonds.

$$b_{t+1}^i \geq 0$$

The law of motion of k_{t+1}^j is given by

$$k_{t+1}^j = (1 - \delta)\, k_t^j + i_t^j \tag{2.4}$$

where δ denotes the depreciation rate of physical capital. Notably, firms disinvest if $i_t^j < 0$.

2.3.3 Insolvency

Insolvency is an outcome of the binding non-negative dividend constraint and the binding collateral constraint. In the presence of the non-negative dividend constraint, firms may need to raise funds from the credit market in order to avoid negative dividend. However, as bond price is decreasing in default probability, which is increasing in the size of debt, the income of bond issuance is bounded from above. Firms become insolvent if the limit of fund they can raise through issuing bond is smaller than the fund they need to avoid the negative dividend. Under these circumstances, firms are forced to exit the market.

As shown in 3.2, firms can raise fund through internal finance, external finance or reselling capital stock. After repaying their debt, firms can spend the remainder of funds on either investment or dividend. Equation (2) can be rewritten as:

$$d_t^j = \pi_t^j - b_t^j - \left(k_{t+1}^j - (1-\delta)\, k_t^j\right) + q_t^j b_{t+1}^j \tag{2.5}$$

In order to avoid negative dividend, the external finance $q_t^j b_{t+1}^j$ cannot be lower than the gap g_t^j:

$$q_t^j b_{t+1}^j \geq g_t^j \equiv \left(k_{t+1}^j - (1-\delta)\, k_t^j\right) + b_t^j - \pi_t^j \tag{2.6}$$

Given the collateral constraint (1), the size of external finance is bounded. The limit of external finance, $B_t^j \equiv \max_{b_{t+1}^j} q_t^j b_{t+1}^j$, is a function of capital k_{t+1}^j, which serves as collateral at $t+1$. To be more specific, the size of external finance $q_t^j b_{t+1}^j$ depends on choice variables b_{t+1}^j and k_{t+1}^j, and state variables. The limit of external finance B_t^j is only dependent on state variables. In Figure 5 we plot the gap g_t and external finance limit B_t. Firms choose their optimal future size from the set $\kappa_t = \{k_{t+1} : \; B_t \geq g_t\} \neq \varnothing$. According to the Equation (6), the slope of line g_t is 1, so its location is determined solely by the intercept $b_t - \pi_t - \delta k_t$. On the one hand, if a firm has a high fixed cost c_f, a low productivity $A_t \varphi_t$, or inherits a high debt level b_t, the firm needs more external fund to avoid negative dividend. On the other hand, if a firm has a high level of profit or a high level of capital stock, the firm can use internal finance or resell its capital and rely less on external finance.

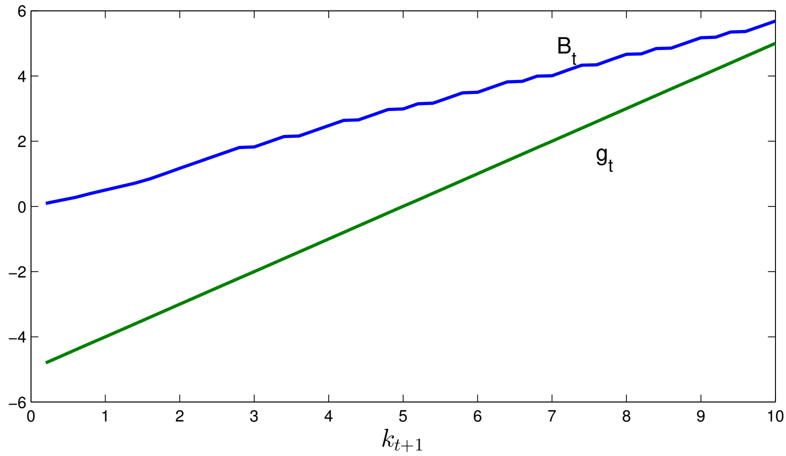

Figure 5: Solvency

Figure 6 illustrates insolvency graphically. If $\kappa_t = \{k_{t+1} : \ B_t \geq g_t\} = \varnothing$, it is impossible to avoid negative dividend, and firm exit becomes inevitable. As aforementioned, the location of gap line depends only on the level of undepreciated capital $(1-\delta)\, k_t$, debt b_t, and profit π_t. The gap line lies above the external finance limit if debt b_t is high, or profit π_t and $(1-\delta)\, k_t$ are low.

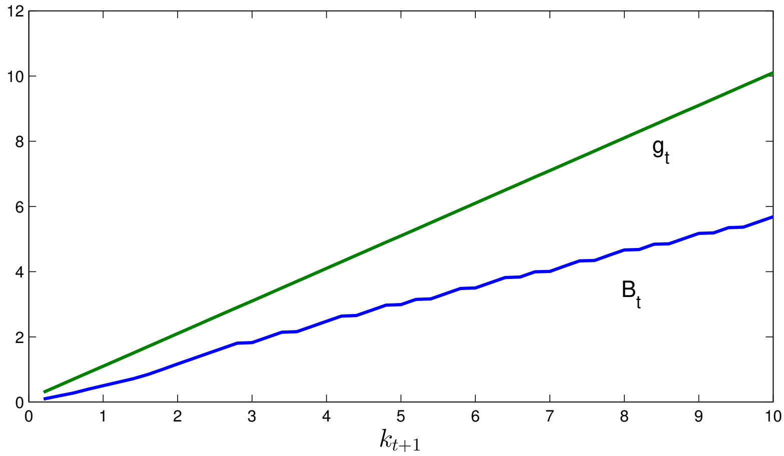

Figure 6: Insolvency

The collateral constraint (1) is critical for insolvency. Without the collateral constraint (1), the bond price is equal to $\frac{1}{R}$. External finance limit $B_t = \max_{b_t} \frac{b_t}{R} = \infty$, so that $B_t \gg g_t$. As a consequence, firms can raise arbitrarily large fund and never become insolvent.

2.3.4 Value Function and Firm Exit

At the begining of time t, firms choose whether or not to stay in the market, and whether or not to repay the debt. We assume that debt restructuring is impossible. Upon defaults, firms are forced to liquidate, and entrepreneurs lose their collateral assets to bond holders. The value function of defaulting is:

$$v^d(\theta_t, k_t) = (1 - \theta_t) k_t$$

where $\theta_t k_t$ denotes the loss of collateral. We assume that there is no dead-weight loss from liquidation. Firms can also exit the market after repaying the debt, under which ciucumstance entrepreneurs obtain physical capital, net of debt repayment. The value function is:

$$v^e(k_t, b_t) = k_t - b_t$$

If a firm is solvent, it has the third option, which is to repay the debt and continue operation. Given wage $w(A_t, \theta_t)$, firms maximize their profit by choosing labor input. The value function of continuing operation solves the following functional equation:

$$\begin{aligned} v^c(A_t, \theta_t, \varphi_t, k_t, b_t) &= \max_{n_t}\left[[\exp(A_t\varphi_t)]^{1-\gamma}\left(n_t^{1-\alpha}k_t^{\alpha}\right)^{\gamma} - w(A_t, \theta_t)\, n_t\right] - c_f - b_t \\ &+ \max_{k_{t+1}, b_{t+1}}\left[q(A_t, \theta_t\varphi_t, k_{t+1}, b_{t+1})\, b_{t+1} - (k_{t+1} - (1-\delta)k_t) + \frac{1}{R}E_t v(A_{t+1}, \theta_{t+1}, \varphi_{t+1}, k_{t+1}, b_{t+1})\right] \end{aligned}$$

$$\begin{aligned} s.t.\ &(\exp(A_t\varphi_t))^{1-\gamma}\left(n_t^{1-\alpha}k_t^{\alpha}\right)^{\gamma} - w(A_t, \theta_t)\, n_t - c_f - b_t \\ &+ q(A_t, \theta_t, \varphi_t, k_{t+1}, b_{t+1})\, b_{t+1} - (k_{t+1} - (1-\delta)k_t) \geq 0 \end{aligned}$$

where $v(A_t, \theta_t, \varphi_t, k_t, b_t)$ denotes the continuation value:

$$v(A_t, \theta_t, \varphi_t, k_t, b_t) = \begin{cases} \max\left\{v^c(A_t, \theta_t, \varphi_t, k_t, b_t), v^e(k_t, b_t), v^d(\theta_t, k_t)\right\}, & \kappa \neq \varnothing \\ \max\left\{v^e(k_t, b_t), v^d(\theta_t, k_t)\right\}, & \kappa = \varnothing \end{cases}$$

Firms become insolvent if $\kappa = \varnothing$. The price schedule for bonds b is determined by Equation (1), and is a function of state variables A_t, θ_t, φ_t, and choice variables k_{t+1}, l_{t+1}.

2.3.5 Firm Entry and Labor Supply

There are infinite potential entrants every period. Potential entrants make the entry decision at the begining of every period. The potential entrants draw their idiosyncratic productivity φ according to normal distribution function $F(\varphi) \sim N(0, \sigma_0^2)$ after paying entry cost entry cost c_e. Entrants start firms with zero debt and initial endowment of capital k_0. The production starts right after paying entry cost and drawing productivity. The entry value of prospective entrants is

$$v^{enter}(A_t, \theta_t) = \int_{\varphi} v(A_t, \theta_t, \varphi_t, k_0, 0) dF(\varphi)$$

Prospective entrants enter the market if and only if the value of entering is no less than the entry cost $c_e w(A_t, \theta_t)$:

$$v^{enter}(A_t, \theta_t) \geq c_e w(A_t, \theta_t) \tag{2.7}$$

where $c_e \geq 0$. c_e is the unit of workers entrepreneurs need to initiate a new firm[5]. As discussed in Section 5, whenever $v^{enter}(A_t, \theta_t) = c_e$, equilibrium wage can be characterized as a function of aggregate states A_t and θ_t. The labor supply is given by a monotonically increasing function of wage:

$$L_{s,t} = [w(A_t, \theta_t)]^{\varepsilon}$$

where $\varepsilon > 0$.

[5]For simplicity, we assume that the workers who initiate new firms are different from the workers who work for firms to produce output.

2.4 Stationary Equilibria

In a dynamic equilibrium: the labor market and the credit market clear over time; incumbents and entrants maximize their payoff. In this section we investigate the stationary equilibrium as studied by Hopenhayn (1992) and Melitz (2003). Throughout this section, aggregate state variables are stable over time: $\sigma_A = 0$, $\sigma_\theta = 0$. The model is calibrated using firm-level data. We characterize the stationary distribution of firms and the model implied composition of exiters. In Section 5 we study the equilibrium in the presence of aggregate shocks.

2.4.1 Calibration

As is common in the business cycle literature, we preset certain parameter values and pick others in order to match our target moments. The preset parameters are listed in Table 1. The capital intensity α, the span of control parameter γ, and the wage elasticity of labor supply ε are conventional values in the literature. We take the values of interest rate R, autocorrelation ρ_φ, standard deviation σ_φ, depreciation rate δ from Gopinath et al. (forthcoming).

Parameter	Concept	Value
α	Capital intensity	0.3
γ	Span of control	0.8
ε	Wage elasticity	2
R	Interest rate	1.15
ρ_φ	Autocorr. idiosyncratic productivity	0.59
σ_φ	Std. idiosyncratic productivity	0.13
δ	Depreciation rate	0.06

Table 1: Preset Parameters

Other parameters are listed in Table 2. We choose these parameters to minimize the difference between a few model-implied moments and their counterparts in data. Our targets include 1) entry/exit rate, which is the average of entry rate (0.11) and exit rate (0.09) during 1996 to 2012[6]; 2) the relative debt size of exiters, which is calculated using our firm-level data of debt; 3) the relative asset size of entrants; 4) the average investment rate of entrants; 5) the standard deviation of investment rate of entrants. 3), 4) and 5) are calculated using SABI-AMADEUS data. 2) and 3) describe features of entrants and exiters relative to average firms. 4) and 5) describe the cross-section of investment among entrants. The moment fitness is presented in Table 3.

[6]In our model implied steady state, entry rate is equal to exit rate.

Parameter	Concept	Value
c_f	Fixed operation cost	0.57
c^{enter}	Entry cost	1.22
θ	Pledgability	0.53
σ_0	Std. initial productivity draw	0.34
k_0	Initial wealth	0.44

Table 2: Parameters

Targeted moments	Model	Data
Entry/exit rate	0.11	0.10
Relative size of debt (exiters)	0.45	0.51
Relative size of assets (entrants)	0.18	0.16
Avg. investment rate (entrants)	0.28	0.28
Std. investment rate (entrants)	1.29	1.29

Table 3: Moment Fitness

Table 4 displays a few other characteristics of average firms. These moments are not comparable with data since their value depends on our choice of grids in numerical solution. However, they serve as a benchmark for us to compare different groups of exiters. We also plot the overall distribution of the productivity component φ in Figure 7. It is shown that the distribution is slightly left-skewed. Intuitively, this is an outcome of selection effect, since firms with low productivity are more likely to exit.

Concept	Value
Average capital k	3.328
Average debt l	0.986
Average productivity φ	0.067
Average marginal product of capital MPK	0.065

Table 4: Average Firm

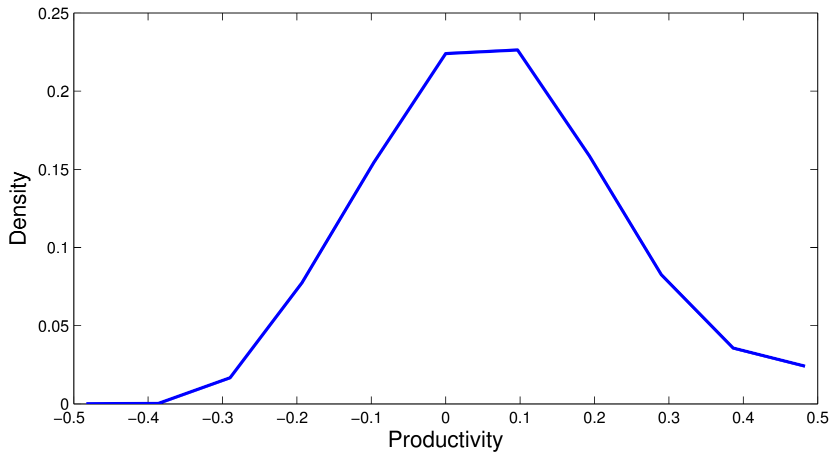

Figure 7: Productivity Distribution

In a stationary equilibrium, firms grow over time while aggregate states remain constant. Figure 8 plots the average size of capital of different age cohorts. The average size is increasing in the age of cohorts. Young firms cannot reach their optimal size because they are subject to financial constraints. However, firms can gradually grow out of financial constraints by accumulating capital stock over time. The increasing firm size also lowers the likelihood of exit. Figure 9 shows that the average probability of exit is decreasing in age. Our results of firm growth are consistent with the finding in seminal works.[7]

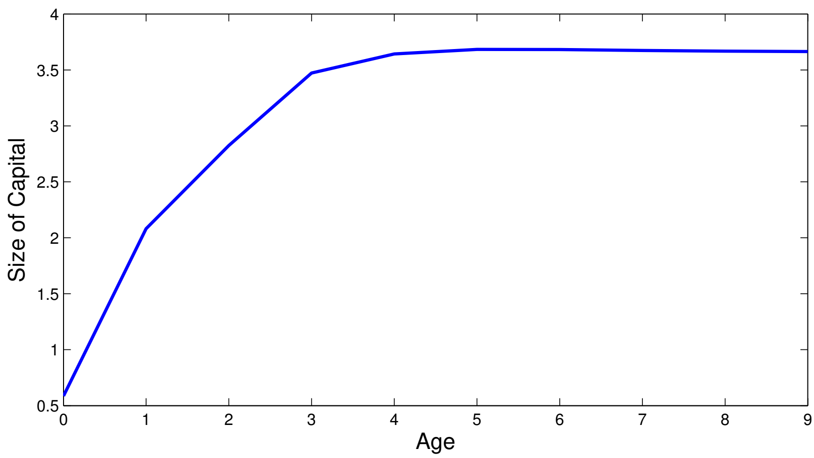

Figure 8: Firm Growth: Size of Capital

[7] See Cabral and Mata (2003), Dunne, Roberts, and Samuelson (1989).

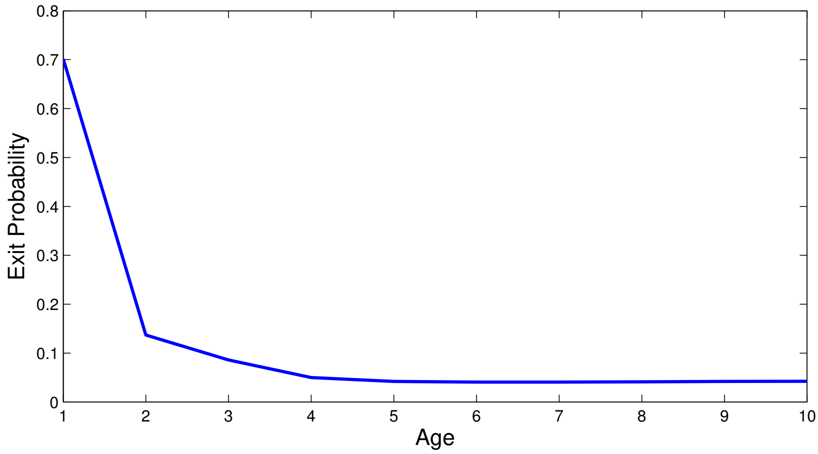

Figure 9: Firm Growth: Exit Probability

2.4.2 Non-defaulting and Defaulting Exiters

Once a firm decides to exit, it still has to choose whether to repay its debt. In this section we compare two groups of exiters: defaulting exiters, the exiters which default on their debt, and non-defaulting exiters, the exiters which repay their debt. Figure 10 displays the distribution of φ across non-defaulting exiters. Different from the population of firms we plot in Figure 7, non-defaulting exiters exhibit right-skewed distribution in productivity. From the graph we also know that these exiters are mostly firms with low level of productivity.

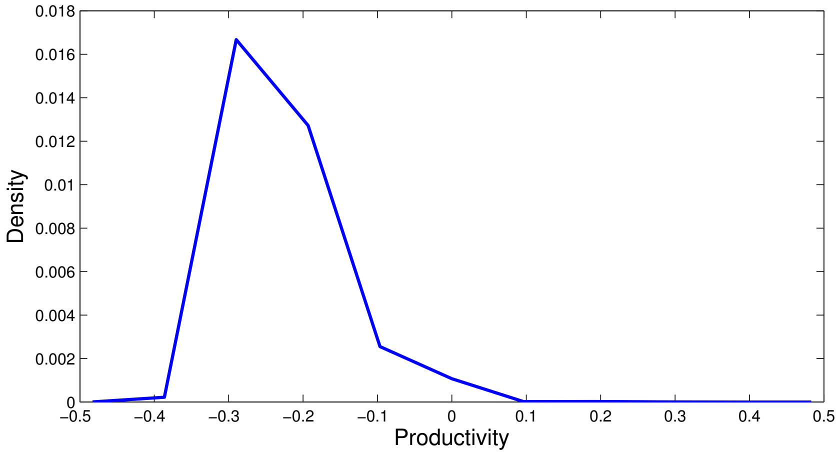

Figure 10: Non-defaulting Exiters

Figure 11 plots the distribution of φ among defaulting exiters. Like the distribution in Figure 10, the distribution of defaulting exiters is right skewed. However, compared with non-defaulting exiters and average firms, defaulting exiter are much more productive.

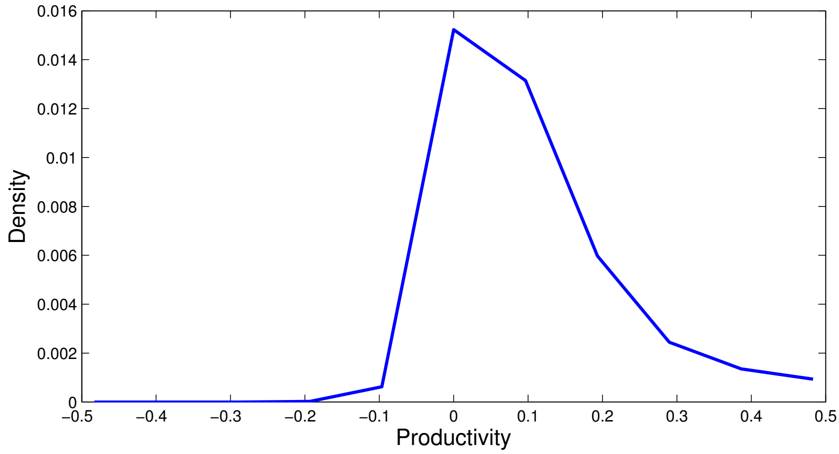

Figure 11: Defaulting Exiters

The comparison between Figure 11 and Figure 7 are surprising at the first glance, as it seems to suggest that productive firms are more likely to default. In Table 5 we list a few summary statistics of the two exiter groups, and we also include the results from Table 4. Although defaulting exiters are more productive than non-defaulting exiters and average firms, they tend to have much less assets. Moreover, defaulting exiters have relatively little debt. It can be infered that defaulting exiters are mostly financially constrained firms. These firms have good growth potential but are more likely to default. As opposed to defaulting exiters, non-defaulting exiters have low productivity but accumlate large capital stock. Not surprisingly, defaulting exiters have on average high marginal product of capital wheras non-defaulting exiters have low marginal product of capital. Table 5 implies that not all types of exit are cleansing. Although non-defaulting exit cleans obese firms, default destroys firms with high growth potential.

Concept	Non-defaulting	Defaulting	Average Firms
Average capital k	3.082	0.910	3.328
Average debt l	0.511	0.348	0.986
Average productivity φ	-0.229	0.102	0.067
Average marginal product of capital MPK	0.041	0.119	0.065

Table 5: Non-defaulting and Defaulting Exiters

Notably, we can use the numbers in Table 5 to calculate the average leverage ratio (weighted by the size of assets). It is easy to prove that the average leverage ratio of every group equals the ratio between average debt and capital. Table 5 imples that defaulting exiters are more leveraged than average firms, while non-defaulting exiters are less leveraged than average firms.

2.4.3 Insolvent and Solvent Exiters

An exiter can be either solvent or insolvent. According to Equation (6), insolvency can be an outcome of low productivity, small size of capital, or high level of debt. In Table 6 we compare solvent exiters, insolvent exiters, and average firms. On average, insolvent exiters have more capital stock but less debt than solvent exiters. In addition, insolvent firms have relatively low productivity. The results suggest that, negative productivity shocks, rather than large debt or asset scarcity, are the main cause of insolvency.

Concept	Insolvent	Solvent	Average Firms
Average capital k	2.014	1.817	3.328
Average debt l	0.268	0.533	0.986
Average productivity φ	-0.161	0.032	0.067
Average marginal product of capital MPK	0.072	0.263	0.065

Table 6: Insolvent Exiters and Solvent Exiters

2.5 Aggregate Fluctuations

In this section we study firm dynamics in the presence of productivity shocks and financial shocks. One common problem in the literature of heterogeneous firms is that the aggregate output depends on time-varying distribution of firms, which has infinite dimensions. Although our model features heterogeneous firms and firm entry and exit, the free entry condition (7) pins down wage as a function of aggregate variables A_t and θ_t, therefore it can be solved without implementing approximation techniques a la Krusell and Smith (1998). We develop a numerical method to solve our model in the presence of aggregate shocks. We assume that prospective entrants are subject to idiosyncratic productivity shocks before entering the market. The assumption ensures that we do not need to know the distribution of firms to characterize wages and aggregate output. Intuitively, equilibrium wage clears the labor market and it depends on the labor demand. However, the supply of prospective entrants is perfectly elastic. Therefore, given aggregate states A_t and θ_t, the labor demand is independent to the distribution of incumbent firms. No matter whatever the current distribution of firms is, if the wage is too low, there will be more entrants, increasing the wage to the equilibrium level; if the wage is too high, there will be less entrants.

The main purpose of this section is to study the impulse response to different shocks. As shown afterwards, even if productivity shocks and financial shocks are analogous in aggregate, they vary in their micro-level implications.

2.5.1 Calibration

In addition to the parameters we use in the stationary equilibrium, we need calibrate the autocorrelation and standard deviation of shocks. The values are chosen to match the standard deviation of 4 statistics: entry rate, exit rate, the growth rate of output, and the debt fraction of exiters. In our scenario, exit takes place at the begining of each period. Therefore in our model exit rate at time t is defined as the ratio of operating firms at $t-1$ which exit at t. The output is measure by Spanish GDP data (PPP, 2011 constant international \$) from World Bank. The results are listed in Table 7.

Parameters	Value	Targeted moments	Model	Data
ρ_A	0.45	Std. entry rate	0.052	0.050
σ_A	0.0053	Std. exit rate	0.023	0.031
ρ_θ	0.11	Std. output growth	0.024	0.024
σ_θ	0.051	Std. debt fraction of exiters	0.027	0.027

Table 7: Parameters and Targets

The model fits target moments well. We compare a few extra moments in Table 8. The model reproduces realistic values of the autocorrelation of output and the correlation between entry rate and output. Even though the model tends to understate the correlation between exit rate and output, it captures the negative relationship between output and exit rate as observed in the data.

Moments	Model	Data
Corr. between entry rate and ouput	0.18	0.19
Corr. between exit rate and ouput	-0.17	-0.42
Autocorr. output[8]	0.40	0.47

Table 8: Moments

2.5.2 Output

In Figure 12 and 13 we plot respectively the impulse responses of output following a negative productivity shock and a negative financial shock. The model is simulated with 120 period, with a negative productivity/financial shock arriving at the 101st period. The size of shocks are equal to 2 standard deviations. The initial distribution of firms is given by the stationary distribution. We repeat the simulations 300 times and take the average of output from period 100 to period 120. The impulse responses in Section 5.3 are computed in an analogous way. In the figures, shocks arrive at time $t = 1$.

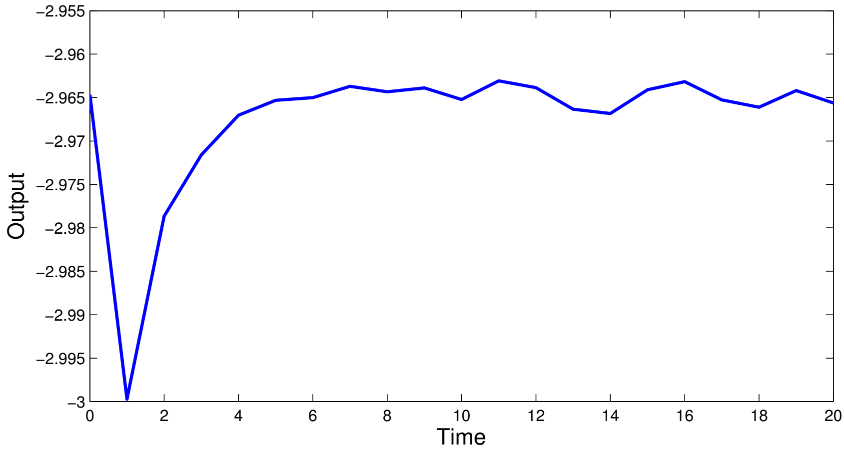

Figure 12: Response to a Negative Productivity Shock

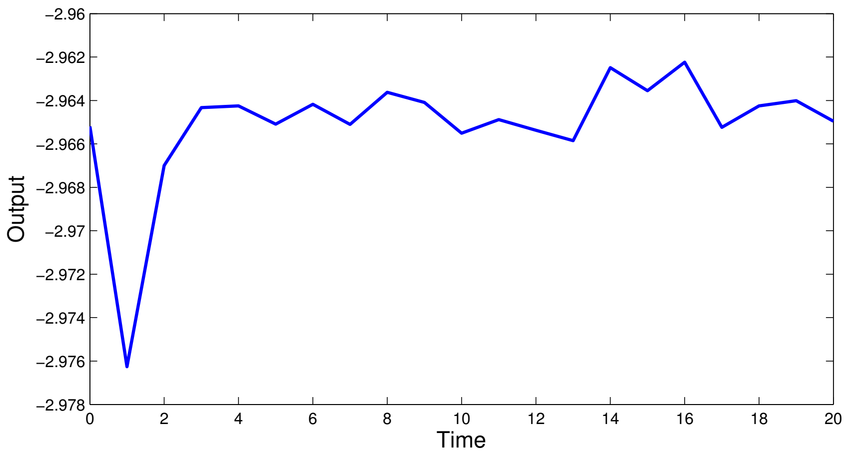

Figure 13: Response to a Negative Financial Shock

Theoretically, negative financial shocks decreases the exit cost of defaulting firms but increase financial frictions. On the one hand negative financial shocks decrease profitability by adding up financial constraints. On the other hand, prospective entrants take the lower cost of exit into consideration and find it easier to enter the market. Therefore a negative financial shock has ambiguous effects on output. Figure 13 suggests the impulse response of a negative financial shock resembles the impulse response of a productivity contraction. Both shocks trigger a recession in output.

2.5.3 Firm Exit

Our main insight is aggregate shocks change the composition of exiters. Figure 14 and 15 illustrate defaulting and non-defaulting exit in response to a negative productivity shock. The dash line denotes the insolvent subgroup of corresponding exiters. Following a negative productivity shock, defaulting exit rate rises, and more surprisingly, non-defaulting exit rate falls. Non-defaulting exit rate decreases because the equilibrium wage falls during a recession. Given our results in Section 4 about non-defaulting exiters, the results here further implies that a negative productivity shock increases the survival probability of obese firms.

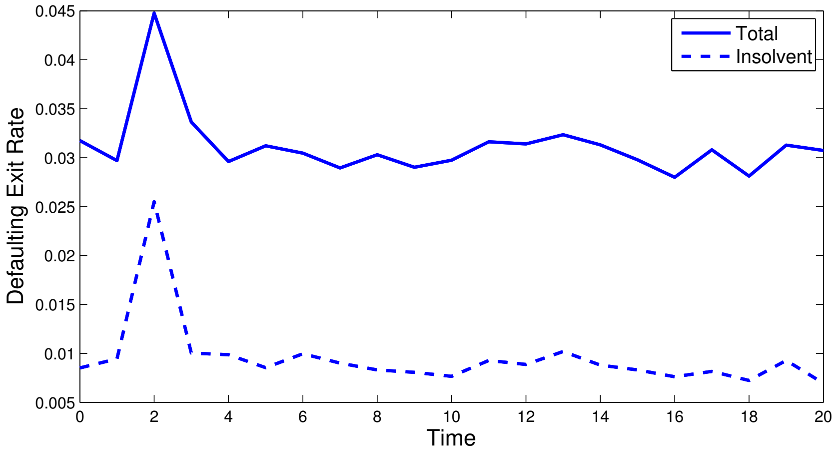

Figure 14: Response to a Negative Productivity Shock

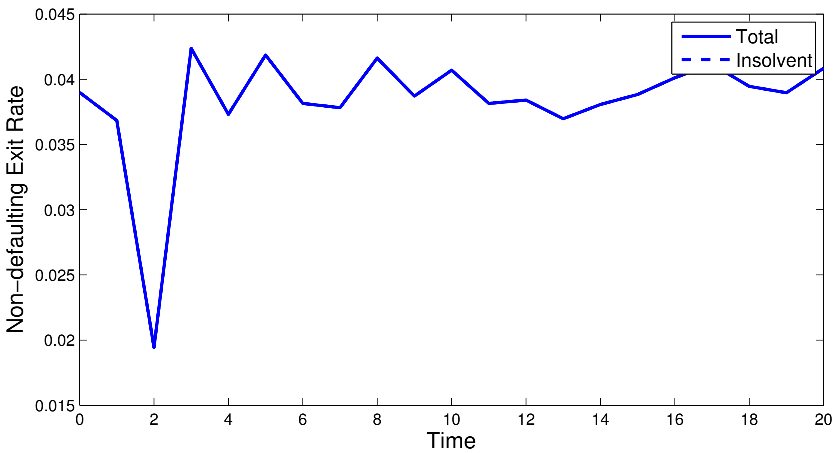

Figure 15: Response to a Negative Productivity Shock

Another notable finding is that the increase in defaulting rate can be accounted for by the increase of the insolvency rate in the group of defaulting exiters. Defaulting exiters, according to Section 4, are in general highly leveraged, in other words, have large debt relative to their assets. Once these firms become insolvent, they are more likely to default rather than to repay since the scrap value they can claim after repaying debt is small relative to their debt. Hence the increase in insolvency fully converts into defaulting exit. Furthermore, we can conclude that basically all the non-defaulting exiters are insolvent exiters.

In Figure 17 and 18 we plot the impulse response to a negative financial shock. A negative financial shock is analogous to a productivity contraction in its implications on the composition of defaulting/non-defaulting exiters. However, a crucial difference is that the change of insolvency rate can explain very little the increase of defaulting rate. Most of the increased defaulting exit are solvent exit. The cost of defaulting is much lower during a financial recession. Therefore a solvent firm also becomes much more likely to default rather than to stay in the market.

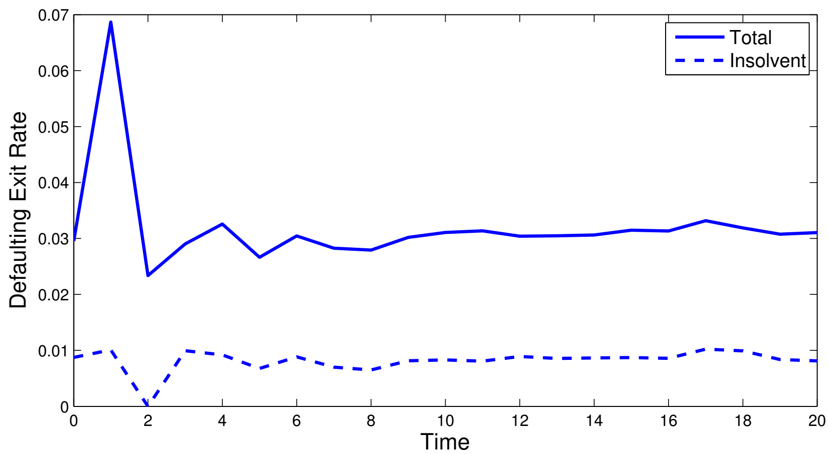

Figure 16: Negative Financial Shock

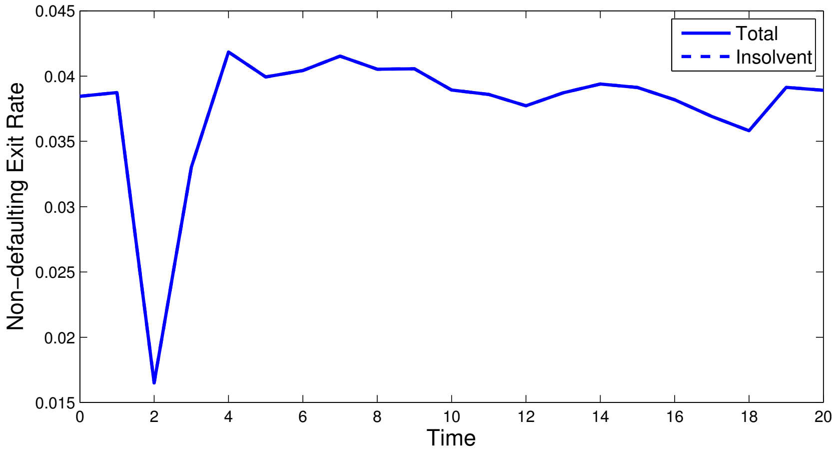

Figure 17: Negative Financial Shock

Our results can rationalize the dynamics of leverage ratio in Figure 4. In our model, defaulting exit rate inceases while non-defaulting exit rate decreases during a recession. As shown in Section 4, compared with non-defaulting exiters, defaulting exiters are more productive and more leveraged. The relative leverage ratio of exiters increase as the fraction of defauting exiters increaes during recessions. Moreover, the results are also in line with the recent evidence about the sullying effects of recessions.[9] Firms with good growth potential find it difficult to withstand negative shocks, wheras obese firms can take advantage of recessions. Last but not least, we speculate that liquidity provision or debt-restructuring policies trying to help insolvent firms may not work well during financial recessions, given that the increasing defaulting exit is mainly a consequence of the default of solvent firms.

[9] See Kehrig (2015).

2.6 Conclusion

The paper sheds light on the features of different types of exiters, and how the composition of exiters change over time. We argue that defaulting exiters are highly leveraged productive firms while non-defaulting exiters are low leveraged obese firms. Our model predicts that negative shocks increase defaulting exit rate, but decrease non-defaulting exit rate through lower wages. The results are in line with the increase of relative leverage ratio of exiters during recession. Moreover, since financial shocks and productivity shocks affect firm exit through different mechanisms, liquidity provision or debt-restructuring policies may not stabilize firm exit well during financial recessions.

An interesting avenue for future research would be to study the interaction between firm exit and credit bubbles. Credit bubbles are roll-over of debt. Bubbles can not only relax collateral constraints, but also relax non-negative dividend constraints since firms can roll over their debt to avoid negative dividend. Therefore we conjecture bubbles lower the likelihood of insolvency. A recent literature has been trying to understand the collateral effects of bubbles.[10] However, the interplay between bubbles and non-negative dividend constraints has yet awaited to be explored.

[10] See Asriyan et al. (2016), Martin and Ventura (2012, 2016).

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