Anexo 3.5

Deducción de las ecuaciones del movimiento de una viga de Euler-Bernoulli giratoria

DEDUCCIÓN DE LAS ECUACIONES DEL MOVIMIENTO DE UNA VIGA DE EULER-BERNOULLI GIRATORIA

Hipótesis principales: viga Euler-Bernoulli, no se considera fuerza centrífuga, no se considera fuerza de Coriolis, la masa del extremo es despreciable, el momento de inercia de las articulaciones es despreciable, accionada mediante un actuador lineal.

```
> restart;
> with(linalg):
Warning, the protected names norm and trace have been redefined and
unprotected

> m_g:=0;J_g:=0;G:=0;
```

$$m\_g := 0$$

$$J\_g := 0$$

$$G := 0$$

La presencia de un actuador lineal divide a la viga en dos regiones. La primera (1) va desde la articulación al actuador lineal y la segunda (2) del actuador lineal al extremo libre. Determinación de la velocidad absoluta de un punto situado en el eje centroidal de la viga. A continuación se determina el cuadrado de esta velocidad.

```
> VX1:=vector([-
Wln1(x,t)*diff(theta(t),t),0,diff(Wln1(x,t),t)+x*diff(theta(t
),t)]);
```

$$VX1 := \left[-\mathrm{Wln1}(x,t)\left(\frac{\partial}{\partial t}\theta(t)\right), 0, \left(\frac{\partial}{\partial t}\mathrm{Wln1}(x,t)\right)+x\left(\frac{\partial}{\partial t}\theta(t)\right)\right]$$

```
> VX2:=vector([-
Wln2(x,t)*diff(theta(t),t),0,diff(Wln2(x,t),t)+x*diff(theta(t
),t)]);
```

$$VX2 := \left[-\mathrm{Wln2}(x,t)\left(\frac{\partial}{\partial t}\theta(t)\right), 0, \left(\frac{\partial}{\partial t}\mathrm{Wln2}(x,t)\right)+x\left(\frac{\partial}{\partial t}\theta(t)\right)\right]$$

```
> VX1_2:=expand(dotprod(VX1,VX1,'orthogonal'));
```

$$VX1\_2 := \mathrm{Wln1}(x,t)^2\left(\frac{\partial}{\partial t}\theta(t)\right)^2+\left(\frac{\partial}{\partial t}\mathrm{Wln1}(x,t)\right)^2+2\left(\frac{\partial}{\partial t}\mathrm{Wln1}(x,t)\right)x\left(\frac{\partial}{\partial t}\theta(t)\right) + x^2\left(\frac{\partial}{\partial t}\theta(t)\right)^2$$

```
> VX2_2:=expand(dotprod(VX2,VX2,'orthogonal'));
```

$$VX2\_2 := \mathrm{Wln2}(x,t)^2\left(\frac{\partial}{\partial t}\theta(t)\right)^2+\left(\frac{\partial}{\partial t}\mathrm{Wln2}(x,t)\right)^2+2\left(\frac{\partial}{\partial t}\mathrm{Wln2}(x,t)\right)x\left(\frac{\partial}{\partial t}\theta(t)\right) + x^2\left(\frac{\partial}{\partial t}\theta(t)\right)^2$$

Determinación de la energía cinética de una rebanada cualquiera.

```
> dEc1:=(1/2*A*rho*diff(theta(t),t)^2*Wln1(x,t)^2+1/2*A*rho*diff(Wln1(x,t),t)^2+A*rho*diff(theta(t),t)*diff(Wln1(x,t),t)*x+1/2*A*rho*diff(theta(t),t)^2*x^2)*dx;
```

$$dEc1 := \left(\frac{1}{2} A\, \rho \left(\frac{\partial}{\partial t} \theta(t)\right)^2 \mathrm{Wln1}(x,t)^2 + \frac{1}{2} A\, \rho \left(\frac{\partial}{\partial t} \mathrm{Wln1}(x,t)\right)^2 + A\, \rho \left(\frac{\partial}{\partial t} \theta(t)\right)\left(\frac{\partial}{\partial t} \mathrm{Wln1}(x,t)\right) x + \frac{1}{2} A\, \rho \left(\frac{\partial}{\partial t} \theta(t)\right)^2 x^2\right) dx$$

```
> dEc2:=(1/2*A*rho*diff(theta(t),t)^2*Wln2(x,t)^2+1/2*A*rho*diff(Wln2(x,t),t)^2+A*rho*diff(theta(t),t)*diff(Wln2(x,t),t)*x+1/2*A*rho*diff(theta(t),t)^2*x^2)*dx;
```

$$dEc2 := \left(\frac{1}{2} A\, \rho \left(\frac{\partial}{\partial t} \theta(t)\right)^2 \mathrm{Wln2}(x,t)^2 + \frac{1}{2} A\, \rho \left(\frac{\partial}{\partial t} \mathrm{Wln2}(x,t)\right)^2 + A\, \rho \left(\frac{\partial}{\partial t} \theta(t)\right)\left(\frac{\partial}{\partial t} \mathrm{Wln2}(x,t)\right) x + \frac{1}{2} A\, \rho \left(\frac{\partial}{\partial t} \theta(t)\right)^2 x^2\right) dx$$

Determinación de la energía cinética total.

```
> Ec:= expand(int(dEc1/dx,x=0..lc)+int(dEc2/dx,x=lc..L));
```

$$Ec := \frac{1}{2} \int_0^{lc} A\, \rho \left(\frac{\partial}{\partial t} \theta(t)\right)^2 \mathrm{Wln1}(x,t)^2 + A\, \rho \left(\frac{\partial}{\partial t} \mathrm{Wln1}(x,t)\right)^2 + 2\, A\, \rho \left(\frac{\partial}{\partial t} \theta(t)\right)\left(\frac{\partial}{\partial t} \mathrm{Wln1}(x,t)\right) x + A\, \rho \left(\frac{\partial}{\partial t} \theta(t)\right)^2 x^2\, dx + \frac{1}{2} \int_{lc}^{L} A\, \rho \left(\frac{\partial}{\partial t} \theta(t)\right)^2 \mathrm{Wln2}(x,t)^2 + A\, \rho \left(\frac{\partial}{\partial t} \mathrm{Wln2}(x,t)\right)^2 + 2\, A\, \rho \left(\frac{\partial}{\partial t} \theta(t)\right)\left(\frac{\partial}{\partial t} \mathrm{Wln2}(x,t)\right) x + A\, \rho \left(\frac{\partial}{\partial t} \theta(t)\right)^2 x^2\, dx$$

A continuación se calcula la energía potencial debida a la deformación elástica de una rebanada cualquiera.

```
> dEp1:=(1/2*E*J*diff(psi1(x,t),x)^2)*dx;
> dEp2:=(1/2*E*J*diff(psi2(x,t),x)^2)*dx;
```

$$dEp1 := \frac{1}{2} E\, J \left(\frac{\partial}{\partial x} \psi 1(x,t)\right)^2 dx$$

$$dEp2 := \frac{1}{2} E\, J \left(\frac{\partial}{\partial x} \psi 2(x,t)\right)^2 dx$$

A continuación se calcula la energía potencial **total** debida a la deformación elástica.

```
> Ep:= expand(int(dEp1/dx,x=0..lc)+int(dEp2/dx,x=lc..L));
```

$$Ep := \frac{1}{2} E J \int_0^{lc} \left(\frac{\partial}{\partial x} \psi 1(x, t)\right)^2 dx + \frac{1}{2} E J \int_{lc}^{L} \left(\frac{\partial}{\partial x} \psi 2(x, t)\right)^2 dx$$

ECUACIÓN GLOBAL

La ecuación que se deduce a continuación se aplica a la totalidad de la viga. Su significado físico es: la suma de los momentos de todas las fuerzas en el eje que pasa por la articulación O4 es cero.

Determinación del Lagrangiano total.

```
> e0:=expand(Ec-Ep);
```

$$e0 := \frac{1}{2} \int_0^{lc} A \rho \left(\frac{\partial}{\partial t} \theta(t)\right)^2 \mathrm{Wln1}(x, t)^2 + A \rho \left(\frac{\partial}{\partial t} \mathrm{Wln1}(x, t)\right)^2$$

$$+ 2 A \rho \left(\frac{\partial}{\partial t} \theta(t)\right)\left(\frac{\partial}{\partial t} \mathrm{Wln1}(x, t)\right) x + A \rho \left(\frac{\partial}{\partial t} \theta(t)\right)^2 x^2 \, dx + \frac{1}{2} \int_{lc}^{L}$$

$$A \rho \left(\frac{\partial}{\partial t} \theta(t)\right)^2 \mathrm{Wln2}(x, t)^2 + A \rho \left(\frac{\partial}{\partial t} \mathrm{Wln2}(x, t)\right)^2$$

$$+ 2 A \rho \left(\frac{\partial}{\partial t} \theta(t)\right)\left(\frac{\partial}{\partial t} \mathrm{Wln2}(x, t)\right) x + A \rho \left(\frac{\partial}{\partial t} \theta(t)\right)^2 x^2 \, dx$$

$$- \frac{1}{2} E J \int_0^{lc} \left(\frac{\partial}{\partial x} \psi 1(x, t)\right)^2 dx - \frac{1}{2} E J \int_{lc}^{L} \left(\frac{\partial}{\partial x} \psi 2(x, t)\right)^2 dx$$

La siguiente sustitución es una estrategia para poder derivar el Lagrangiano con MAPLE. Este programa no soporta derivar respecto de una función por lo que se ha de substituir la función velocidad angular por una variable (thetapunto), derivar respecto thetapunto y deshacer la substitución.

```
> e51:=subs(diff(theta(t),t)=thetapunto,e0);
```

$$e51 := \frac{1}{2} \int_0^{lc} A \rho \, thetapunto^2 \, \mathrm{Wln1}(x, t)^2 + A \rho \left(\frac{\partial}{\partial t} \mathrm{Wln1}(x, t)\right)^2$$

$$+ 2 A \rho \, thetapunto \left(\frac{\partial}{\partial t} \mathrm{Wln1}(x, t)\right) x + A \rho \, thetapunto^2 x^2 \, dx + \frac{1}{2} \int_{lc}^{L}$$

$$A\,\rho\,thetapunto^2\,\mathrm{Wln2}(x,t)^2 + A\,\rho\left(\frac{\partial}{\partial t}\mathrm{Wln2}(x,t)\right)^2$$

$$+ 2\,A\,\rho\,thetapunto\left(\frac{\partial}{\partial t}\mathrm{Wln2}(x,t)\right)x + A\,\rho\,thetapunto^2\,x^2\,dx$$

$$-\frac{1}{2}\,E\,J\int_0^{lc}\left(\frac{\partial}{\partial x}\psi1(x,t)\right)^2 dx - \frac{1}{2}\,E\,J\int_{lc}^{L}\left(\frac{\partial}{\partial x}\psi2(x,t)\right)^2 dx$$

```
> e52:=diff(e51,thetapunto);
```

$$e52 :=$$

$$\frac{1}{2}\int_0^{lc} 2\,A\,\rho\,thetapunto\,\mathrm{Wln1}(x,t)^2 + 2\,A\,\rho\left(\frac{\partial}{\partial t}\mathrm{Wln1}(x,t)\right)x + 2\,A\,\rho\,thetapunto\,x^2\,dx$$

$$+$$

$$\frac{1}{2}\int_{lc}^{L} 2\,A\,\rho\,thetapunto\,\mathrm{Wln2}(x,t)^2 + 2\,A\,\rho\left(\frac{\partial}{\partial t}\mathrm{Wln2}(x,t)\right)x + 2\,A\,\rho\,thetapunto\,x^2\,dx$$

```
> e53:=subs(thetapunto=diff(theta(t),t),e52);
```

$$e53 := \frac{1}{2}\int_0^{lc} 2\,A\,\rho\left(\frac{\partial}{\partial t}\theta(t)\right)\mathrm{Wln1}(x,t)^2 + 2\,A\,\rho\left(\frac{\partial}{\partial t}\mathrm{Wln1}(x,t)\right)x + 2\,A\,\rho\left(\frac{\partial}{\partial t}\theta(t)\right)x^2\,dx$$

$$+\frac{1}{2}\int_{lc}^{L} 2\,A\,\rho\left(\frac{\partial}{\partial t}\theta(t)\right)\mathrm{Wln2}(x,t)^2 + 2\,A\,\rho\left(\frac{\partial}{\partial t}\mathrm{Wln2}(x,t)\right)x + 2\,A\,\rho\left(\frac{\partial}{\partial t}\theta(t)\right)x^2\,dx$$

Otra vez la misma estrategia para poder derivar respecto del ángulo theta.

```
> e54:=subs(theta(t)=theta,e0);
```

$$e54 := \frac{1}{2}\int_0^{lc} A\,\rho\left(\frac{\partial}{\partial t}\theta\right)^2 \mathrm{Wln1}(x,t)^2 + A\,\rho\left(\frac{\partial}{\partial t}\mathrm{Wln1}(x,t)\right)^2$$

$$+ 2\,A\,\rho\left(\frac{\partial}{\partial t}\theta\right)\left(\frac{\partial}{\partial t}\mathrm{Wln1}(x,t)\right)x + A\,\rho\left(\frac{\partial}{\partial t}\theta\right)^2 x^2\,dx + \frac{1}{2}\int_{lc}^{L} A\,\rho\left(\frac{\partial}{\partial t}\theta\right)^2 \mathrm{Wln2}(x,t)^2$$

$$+ A\,\rho\left(\frac{\partial}{\partial t}\mathrm{Wln2}(x,t)\right)^2 + 2\,A\,\rho\left(\frac{\partial}{\partial t}\theta\right)\left(\frac{\partial}{\partial t}\mathrm{Wln2}(x,t)\right)x + A\,\rho\left(\frac{\partial}{\partial t}\theta\right)^2 x^2\,dx$$

$$-\frac{1}{2} E J \int_0^{lc} \left(\frac{\partial}{\partial x} \psi 1(x,t)\right)^2 dx - \frac{1}{2} E J \int_{lc}^{L} \left(\frac{\partial}{\partial x} \psi 2(x,t)\right)^2 dx$$

```
> e55:=diff(e54,theta);
```

$$e55 := 0$$

```
> e56:=subs(theta=theta(t),e55);
```

$$e56 := 0$$

Construcción de la ecuación de Lagrange para el ángulo theta.

```
> e57:=diff(e53,t)-e56-Mtheta;
```

$$e57 := \frac{1}{2} \int_0^{lc} 2 A \rho \left(\frac{\partial^2}{\partial t^2} \theta(t)\right) \mathrm{Wln1}(x,t)^2 + 4 A \rho \left(\frac{\partial}{\partial t} \theta(t)\right) \mathrm{Wln1}(x,t) \left(\frac{\partial}{\partial t} \mathrm{Wln1}(x,t)\right)$$
$$+ 2 A \rho \left(\frac{\partial^2}{\partial t^2} \mathrm{Wln1}(x,t)\right) x + 2 A \rho \left(\frac{\partial^2}{\partial t^2} \theta(t)\right) x^2 \, dx + \frac{1}{2} \int_{lc}^{L}$$
$$2 A \rho \left(\frac{\partial^2}{\partial t^2} \theta(t)\right) \mathrm{Wln2}(x,t)^2 + 4 A \rho \left(\frac{\partial}{\partial t} \theta(t)\right) \mathrm{Wln2}(x,t) \left(\frac{\partial}{\partial t} \mathrm{Wln2}(x,t)\right)$$
$$+ 2 A \rho \left(\frac{\partial^2}{\partial t^2} \mathrm{Wln2}(x,t)\right) x + 2 A \rho \left(\frac{\partial^2}{\partial t^2} \theta(t)\right) x^2 \, dx - Mtheta$$

ECUACIONES LOCALES

Las ecuaciones que se deducen a continuación se aplican a una rebanada de la viga. Su significado físico es: en cualquier rebanada la suma de fuerzas verticales es nula y la suma de los momentos es cero.

Determinación del Lagrangiano.

```
> e100:=expand(dEc1-dEp1);
```

$$e100 := \frac{1}{2} dx\, A \rho \left(\frac{\partial}{\partial t} \theta(t)\right)^2 \mathrm{Wln1}(x,t)^2 + \frac{1}{2} dx\, A \rho \left(\frac{\partial}{\partial t} \mathrm{Wln1}(x,t)\right)^2$$
$$+ dx\, A \rho \left(\frac{\partial}{\partial t} \theta(t)\right) \left(\frac{\partial}{\partial t} \mathrm{Wln1}(x,t)\right) x + \frac{1}{2} dx\, A \rho \left(\frac{\partial}{\partial t} \theta(t)\right)^2 x^2$$
$$- \frac{1}{2} E J \left(\frac{\partial}{\partial x} \psi 1(x,t)\right)^2 dx$$

Otra vez la misma estrategia para poder derivar respecto de la velocidad relativa de la rebanada vista desde el sistema de referencia giratorio.

```
> e111:=subs(diff(Wln1(x,t),t)=Wln1punto,e100);
```

$$e111 := \frac{1}{2}\,dx\,A\,\rho\left(\frac{\partial}{\partial t}\theta(t)\right)^2 \mathrm{Wln1}(x,t)^2 + \frac{1}{2}\,dx\,A\,\rho\,Wln1punto^2 + dx\,A\,\rho\left(\frac{\partial}{\partial t}\theta(t)\right) Wln1punto\,x + \frac{1}{2}\,dx\,A\,\rho\left(\frac{\partial}{\partial t}\theta(t)\right)^2 x^2 - \frac{1}{2}\,E\,J\left(\frac{\partial}{\partial x}\psi 1(x,t)\right)^2 dx$$

```
> e112:=diff(e111,Wln1punto);
```

$$e112 := dx\,A\,\rho\,Wln1punto + dx\,A\,\rho\left(\frac{\partial}{\partial t}\theta(t)\right)x$$

```
> e113:=subs(Wln1punto=diff(Wln1(x,t),t),e112);
```

$$e113 := dx\,A\,\rho\left(\frac{\partial}{\partial t}\mathrm{Wln1}(x,t)\right) + dx\,A\,\rho\left(\frac{\partial}{\partial t}\theta(t)\right)x$$

Otra vez la misma estrategia para poder derivar respecto de la posición relativa de la rebanada vista desde el sistema de referencia giratorio.

```
> e114:=subs(Wln1(x,t)=Wln1,e100);
```

$$e114 := \frac{1}{2}\,dx\,A\,\rho\left(\frac{\partial}{\partial t}\theta(t)\right)^2 Wln1^2 + \frac{1}{2}\,dx\,A\,\rho\left(\frac{\partial}{\partial t}Wln1\right)^2 + dx\,A\,\rho\left(\frac{\partial}{\partial t}\theta(t)\right)\left(\frac{\partial}{\partial t}Wln1\right)x + \frac{1}{2}\,dx\,A\,\rho\left(\frac{\partial}{\partial t}\theta(t)\right)^2 x^2 - \frac{1}{2}\,E\,J\left(\frac{\partial}{\partial x}\psi 1(x,t)\right)^2 dx$$

```
> e115:=diff(e114,Wln1);
```

$$e115 := dx\,A\,\rho\left(\frac{\partial}{\partial t}\theta(t)\right)^2 Wln1$$

```
> e116:=subs(Wln1=Wln1(x,t),e115);
```

$$e116 := dx\,A\,\rho\left(\frac{\partial}{\partial t}\theta(t)\right)^2 \mathrm{Wln1}(x,t)$$

El diferencial de cortante es una fuerza externa que actua en la rebanada. Aunque ha sido incluido el término de energía potencial debida a la deformación por cortante, al derivar respecto de la coordenada Wln no aparece.

```
> dQ1:=E*J*diff(psi1(x,t),`$`(x,3))*dx;
```

$$dQ1 := E\,J\left(\frac{\partial^3}{\partial x^3}\psi 1(x,t)\right)dx$$

Construcción de la ecuación de Lagrange correspondiente a Wln.

```
> e117:=diff(e113,t)-e116-dQ1;
```

$$e117 := dx\,A\,\rho\left(\frac{\partial^2}{\partial t^2}\mathrm{Wln1}(x,t)\right) + dx\,A\,\rho\left(\frac{\partial^2}{\partial t^2}\theta(t)\right)x - dx\,A\,\rho\left(\frac{\partial}{\partial t}\theta(t)\right)^2 \mathrm{Wln1}(x,t) - E\,J\left(\frac{\partial^3}{\partial x^3}\psi 1(x,t)\right)dx$$

No es necesario aplicar Lagrange para obtener la otra ecuación diferencial. Por ser viga Euler-Bernoulli:

```
> psi1(x,t)=diff(Wln1(x,t),x);
```

$$\psi 1(x,t)=\frac{\partial}{\partial x}\,\mathrm{Wln1}(x,t)$$

Ecuación diferencial en Wln.

```
> e118:=subs(psi1(x,t)=diff(Wln1(x,t),x),e117);
```

$$e118 := dx\,A\,\rho\left(\frac{\partial^2}{\partial t^2}\,\mathrm{Wln1}(x,t)\right)+dx\,A\,\rho\left(\frac{\partial^2}{\partial t^2}\,\theta(t)\right)x-dx\,A\,\rho\left(\frac{\partial}{\partial t}\,\theta(t)\right)^2\,\mathrm{Wln1}(x,t)$$
$$-E\,J\left(\frac{\partial^4}{\partial x^4}\,\mathrm{Wln1}(x,t)\right)dx$$

Repetición del proceso para la zona 2.

```
> e200:=expand(dEc2-dEp2);
```

$$e200 := \frac{1}{2}\,dx\,A\,\rho\left(\frac{\partial}{\partial t}\,\theta(t)\right)^2\,\mathrm{Wln2}(x,t)^2+\frac{1}{2}\,dx\,A\,\rho\left(\frac{\partial}{\partial t}\,\mathrm{Wln2}(x,t)\right)^2$$
$$+dx\,A\,\rho\left(\frac{\partial}{\partial t}\,\theta(t)\right)\left(\frac{\partial}{\partial t}\,\mathrm{Wln2}(x,t)\right)x+\frac{1}{2}\,dx\,A\,\rho\left(\frac{\partial}{\partial t}\,\theta(t)\right)^2\,x^2$$
$$-\frac{1}{2}\,E\,J\left(\frac{\partial}{\partial x}\,\psi 2(x,t)\right)^2\,dx$$

```
> e211:=subs(diff(Wln2(x,t),t)=Wln2punto,e200);
```

$$e211 := \frac{1}{2}\,dx\,A\,\rho\left(\frac{\partial}{\partial t}\,\theta(t)\right)^2\,\mathrm{Wln2}(x,t)^2+\frac{1}{2}\,dx\,A\,\rho\,Wln2punto^2$$
$$+dx\,A\,\rho\left(\frac{\partial}{\partial t}\,\theta(t)\right)Wln2punto\,x+\frac{1}{2}\,dx\,A\,\rho\left(\frac{\partial}{\partial t}\,\theta(t)\right)^2\,x^2-\frac{1}{2}\,E\,J\left(\frac{\partial}{\partial x}\,\psi 2(x,t)\right)^2\,dx$$

```
> e212:=diff(e211,Wln2punto);
```

$$e212 := dx\,A\,\rho\,Wln2punto+dx\,A\,\rho\left(\frac{\partial}{\partial t}\,\theta(t)\right)x$$

```
> e213:=subs(Wln2punto=diff(Wln2(x,t),t),e212);
```

$$e213 := dx\,A\,\rho\left(\frac{\partial}{\partial t}\,\mathrm{Wln2}(x,t)\right)+dx\,A\,\rho\left(\frac{\partial}{\partial t}\,\theta(t)\right)x$$

```
> e214:=subs(Wln2(x,t)=Wln2,e200);
```

$$e214 := \frac{1}{2}\,dx\,A\,\rho\left(\frac{\partial}{\partial t}\,\theta(t)\right)^2\,Wln2^2+\frac{1}{2}\,dx\,A\,\rho\left(\frac{\partial}{\partial t}\,Wln2\right)^2+dx\,A\,\rho\left(\frac{\partial}{\partial t}\,\theta(t)\right)\left(\frac{\partial}{\partial t}\,Wln2\right)x$$
$$+\frac{1}{2}\,dx\,A\,\rho\left(\frac{\partial}{\partial t}\,\theta(t)\right)^2\,x^2-\frac{1}{2}\,E\,J\left(\frac{\partial}{\partial x}\,\psi 2(x,t)\right)^2\,dx$$

```
> e215:=diff(e214,Wln2);
```

$$e215 := dx\,A\,\rho\left(\frac{\partial}{\partial t}\,\theta(t)\right)^2\,Wln2$$

```
> e216:=subs(Wln2=Wln2(x,t),e215);
```

$$e216 := dx\, A\, \rho \left(\frac{\partial}{\partial t}\theta(t)\right)^2 \mathrm{Wln2}(x,t)$$

```
> dQ2:=E*J*diff(psi2(x,t),`$`(x,3))*dx;
```

$$dQ2 := E\, J \left(\frac{\partial^3}{\partial x^3}\psi 2(x,t)\right) dx$$

```
> e217:=diff(e213,t)-e216-dQ2;
```

$$e217 := dx\, A\, \rho \left(\frac{\partial^2}{\partial t^2}\mathrm{Wln2}(x,t)\right) + dx\, A\, \rho \left(\frac{\partial^2}{\partial t^2}\theta(t)\right) x - dx\, A\, \rho \left(\frac{\partial}{\partial t}\theta(t)\right)^2 \mathrm{Wln2}(x,t) - E\, J \left(\frac{\partial^3}{\partial x^3}\psi 2(x,t)\right) dx$$

No es necesario aplicar Lagrange para obtener la otra ecuación diferencial. Por ser viga Euler-Bernoulli:

```
> psi2(x,t)=diff(Wln2(x,t),x);
```

$$\psi 2(x,t) = \frac{\partial}{\partial x}\mathrm{Wln2}(x,t)$$

Ecuación diferencial en Wln.

```
> e218:=subs(psi2(x,t)=diff(Wln2(x,t),x),e217);
```

$$e218 := dx\, A\, \rho \left(\frac{\partial^2}{\partial t^2}\mathrm{Wln2}(x,t)\right) + dx\, A\, \rho \left(\frac{\partial^2}{\partial t^2}\theta(t)\right) x - dx\, A\, \rho \left(\frac{\partial}{\partial t}\theta(t)\right)^2 \mathrm{Wln2}(x,t) - E\, J \left(\frac{\partial^4}{\partial x^4}\mathrm{Wln2}(x,t)\right) dx$$

Preparación para intentar resolver las ecuaciones diferenciales obtenidas.

```
> with(ODEtools);
```

[*Solve*, *Xchange*, *Xcommutator*, *Xgauge*, *buildsol*, *buildsym*, *canoni*, *casesplit*, *diff_table*, *equinv*, *eta_k*, *firint*, *firtest*, *gensys*, *hyperode*, *infgen*, *intfactor*, *invariants*, *line_int*, *muchange*, *mutest*, *normalG2*, *odeadvisor*, *odepde*, *odsolve*, *redode*, *reduce_order*, *remove_RootOf*, *solve_group*, *symgen*, *symtest*, *transinv*]

```
> with(PDEtools);
```

[*PDEplot*, *build*, *casesplit*, *charstrip*, *dchange*, *dcoeffs*, *declare*, *difforder*, *dpolyform*, *dsubs*, *mapde*, *separability*, *splitstrip*, *splitsys*, *undeclare*]

```
> infolevel[dsolve]:= 5;
```

$$infolevel_{dsolve} := 5$$

```
> infolevel[pdsolve]:= 5;
```

$$infolevel_{pdsolve} := 5$$

>
Ecuaciones locales.

```
> pde1:=expand((e218/dx))=0;
```

$$pde1 := A\,\rho\left(\frac{\partial^2}{\partial t^2}\mathrm{Wln2}(x,t)\right)+A\,\rho\left(\frac{\partial^2}{\partial t^2}\theta(t)\right)x-A\,\rho\left(\frac{\partial}{\partial t}\theta(t)\right)^2\mathrm{Wln2}(x,t)-E\,J\left(\frac{\partial^4}{\partial x^4}\mathrm{Wln2}(x,t)\right)=0$$

>

```
> pde3:=expand(e118/dx)=0;
```

$$pde3 := A\,\rho\left(\frac{\partial^2}{\partial t^2}\mathrm{Wln1}(x,t)\right)+A\,\rho\left(\frac{\partial^2}{\partial t^2}\theta(t)\right)x-A\,\rho\left(\frac{\partial}{\partial t}\theta(t)\right)^2\mathrm{Wln1}(x,t)-E\,J\left(\frac{\partial^4}{\partial x^4}\mathrm{Wln1}(x,t)\right)=0$$

Ecuación global.

```
> pde6:=expand(e57)=0;
```

$$pde6 := \int_0^{lc} A\,\rho\left(\frac{\partial^2}{\partial t^2}\theta(t)\right)\mathrm{Wln1}(x,t)^2+2\,A\,\rho\left(\frac{\partial}{\partial t}\theta(t)\right)\mathrm{Wln1}(x,t)\left(\frac{\partial}{\partial t}\mathrm{Wln1}(x,t)\right)+A\,\rho\left(\frac{\partial^2}{\partial t^2}\mathrm{Wln1}(x,t)\right)x+A\,\rho\left(\frac{\partial^2}{\partial t^2}\theta(t)\right)x^2\,dx+\int_{lc}^{L} A\,\rho\left(\frac{\partial^2}{\partial t^2}\theta(t)\right)\mathrm{Wln2}(x,t)^2+2\,A\,\rho\left(\frac{\partial}{\partial t}\theta(t)\right)\mathrm{Wln2}(x,t)\left(\frac{\partial}{\partial t}\mathrm{Wln2}(x,t)\right)+A\,\rho\left(\frac{\partial^2}{\partial t^2}\mathrm{Wln2}(x,t)\right)x+A\,\rho\left(\frac{\partial^2}{\partial t^2}\theta(t)\right)x^2\,dx-Mtheta=0$$

Intento de separar las variables.

```
>
answ:=pdsolve([pde3,pde1,pde6],[Wln1(x,t),Wln2(x,t),theta(t)]
,singsol=false);
Error, (in pdsolve/sys) not implemented for composite functions of the
unknowns of the system as in
int(A*rho*diff(diff(theta(t),t),t)*Wln1(x,t)^2+2*A*rho*diff(theta(t),t)*W
ln1(x,t)*diff(Wln1(x,t),t)+A*rho*diff(diff(Wln1(x,t),t),t)*x+A*rho*diff(d
iff(theta(t),t),t)*x^2,x = 0 .. lc)
```

En el mensaje anterior queda claro que el algoritmo actual no soporta todas las posibles ecuaciones integrodiferenciales, en particular las que aparecen aquí.

>