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# Business Cycle Fluctuations and Granular Behavior: an Empirical Analysis vs. the Agent-Based Approach


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# Business Cycle Fluctuations and Granular Behavior: an Empirical Analysis vs. the Agent-Based Approach

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*A mis padres*

*A mis iaios*

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*The value of a college education is not the learning of many facts, but the training of the mind to think.*
— Albert Einstein (1921)

*Few if any economists seem to have realized the possibilities that such invariants hold for the future of our science. In particular, nobody seems to have realized that the hunt for, and the interpretation of, invariants of this type might lay the foundations for an entirely novel type of theory.*
— Schumpeter (1949, p. 155), about the Pareto law

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# Abstract

The study of the business cycle has a long tradition in macroeconomics. The proposed theories have typically resorted to exogenous shocks, despite their difficulty in explaining most aggregate fluctuations. On the other hand, microeconomic shocks have been downplayed based on a diversification argument. Recently, an emerging strand of the literature challenges this argument and proposes that the origin of the business cycle can be traced back to idiosyncratic shocks to granular firms, i.e., those large firms whose shocks have an impact at the aggregate level because they do not cancel out with shocks to smaller firms. Despite the fundamental role played by the granular firms in shaping aggregate fluctuations, we lack a method to identify them.

This thesis aims to quantify the granular size of an economy, i.e., the number of granular firms. Empirically, we find that the contribution of idiosyncratic shocks to gross domestic product (GDP) fluctuations is attributed to a very small number of large firms. From a certain number of firms onwards the additional contribution plateaus. Theoretically, we find that this behavior can be explained by the share of economic activity commanded by the largest firm, the volatility of the largest firm with respect to GDP volatility and two summary statistics for large firm dynamics: the tail index of firm size distribution and the size-volatility elasticity. Finally, we use an agent-based model to study in detail how this behavior emerges.

# Resumen

El estudio del ciclo económico tiene una larga tradición en macroeconomía. Las teorías propuestas han recurrido típicamente a perturbaciones exógenas, pese a su dificultad para explicar la mayor parte de las fluctuaciones agregadas. Por otro lado, se ha restado importancia a las perturbaciones de carácter microeconómico basándose en un argumento de diversificación. Recientemente, una vertiente emergente de la literatura desafía este argumento y propone que el origen del ciclo económico puede remontarse a perturbaciones idiosincrásicas a las empresas granulares, es decir, aquellas grandes empresas cuyas perturbaciones tienen impacto a nivel agregado porque no pueden ser compensadas por perturbaciones a otras empresas de menor tamaño. Pese al papel fundamental que juegan las empresas granulares en la configuración de las fluctuaciones agregadas, carecemos de un método que permita su identificación.

Esta tesis tiene como objetivo cuantificar el tamaño granular de una economía, es decir, el número de empresas granulares. Empíricamente, encontramos que la contribución de las perturbaciones idiosincrásicas a las fluctuaciones del producto interior bruto (PIB) se atribuye a un número muy reducido de grandes empresas. A partir de un determinado número de empresas la contribución adicional se estabiliza. Teóricamente, encontramos que este comportamiento puede ser explicado por la cuota de actividad económica de la empresa más grande, la volatilidad de la mayor empresa con respecto a la volatilidad del PIB y dos estadísticos que sintetizan la dinámica de las grandes empresas: el índice de la cola de la distribución del tamaño de las empresas y la elasticidad de la volatilidad al tamaño. Por último, usamos un modelo basado en agentes para estudiar en detalle como emerge este comportamiento.

# Chapter 1

# Introduction: Granular Behavior and Aggregate Fluctuations

Business cycle theories have typically resorted to aggregate exogenous shocks in order to explain aggregate fluctuations in spite of having serious difficulties (Cochrane, 1994).[1,2] On the other hand, the possibility that the origins of business cycles may be traced back to *microeconomic* shocks has long been downplayed by the literature. This dismissal was based on a "diversification" argument, first postulated by Lucas (1977, p. 20):

> A new technology, reducing costs of producing an old good or making possible the production of a new one, will draw resources into the good which benefits, and away from the production of other goods. [...] in a complex modern economy, there will be a large number of such shifts in any given period, each small in importance relative to the total output. There will be much "averaging out" of such effects across markets.

That is, in an economy populated by a large number $n$ of firms hit by independent shocks, the law of large numbers applies and, hence, aggregate volatility—measured by the standard deviation of gross domestic product (GDP)—would be roughly proportional to $1/\sqrt{n}$—a negligible affect.[3]

The diversification argument implicitly assumes a certain degree of homogeneity in the size of firms that is in stark contrast with the observed heterogeneity (e.g., Gibrat (1931), Ijiri and Simon (1977)). Aware of this ill-grounded assumption, Gabaix's (2011) seminal work challenges the convention by introducing the "granular" hypothesis:[4] in the presence of significant heterogeneity at the micro level, the behavior of macroeconomic aggregates is attributable to the incompressible

[1] The candidate shocks traditionally proposed in the literature are: technology (Kydland and Prescott, 1982, Prescott, 1986), monetary (Friedman, 1968), oil (Hamilton, 1983) and credit (Bernanke, 1983).

[2] An exception is the literature on the role of sectoral shocks in generating aggregate fluctuations (see, e.g., Long and Plosser (1983), Horvath (1998, 2000), Dupor (1999), Shea (2002), Conley and Dupor (2003)). See also Jovanovic (1987), Durlauf (1993), Bak et al. (1993).

[3] Throughout this chapter we use the terms "aggregate fluctuations", "business cycle" and "aggregate volatility" interchangeably.

[4] Gabaix (2011) coins the term "granular" to reflect the fact that firms are not atomistic in size.

"grains" of economic activity, the large firms. Importantly, Gabaix (2011) demonstrates that whenever the firm size distribution is sufficiently fat-tailed, idiosyncratic shocks to large (*granular*) firms do not cancel out with shocks to smaller firms and may translate into aggregate fluctuations.[5] In the particular case of power-law distributed sizes,[6] aggregate volatility decays at a rate $1/n^{1-1/\zeta}$ and $1/\ln n$ when, respectively, the shape parameter $\zeta \in (1, 2)$ and $\zeta = 1$—also known as Zipf's law (Zipf, 1949). According to this result, diversification effects due to country size are rather small and thus idiosyncratic shocks do not die out in the aggregate.

The granular view of the economy suggests that the origins of business cycle fluctuations can be traced back to the dynamics of the granular firms. Gabaix (2011), building on the work of Hulten (1978),[7] constructs the so-called "granular residual"—a parsimonious measure of the shock to the largest firms—to investigate the proportion of aggregate volatility that can be accounted for by idiosyncratic fluctuations. He finds that idiosyncratic productivity shocks to the top 100 firms in the US explain an important fraction (one-third) of the fluctuations of GDP and total factor productivity (TFP). Following Gabaix (2011), empirical evidence supporting the granular hypothesis has been found for multiple economic aggregates: GDP (see, e.g., Friberg and Sanctuary (2016), Blanco-Arroyo et al. (2018), Fornaro and Luomaranta (2018), Miranda-Pinto and Shen (2019), Silva and Da Silva (2020), among many others), TFP (Hogen et al., 2017, Gnocato and Rondinelli, 2018, Gutiérrez and Philippon, 2019, Dacic and Melolinna, 2019), investment (Grullon et al., 2013, Karasik et al., 2016), exports (discussed in detail below) and sales (di Giovanni et al., 2014, Yeh, 2017). Dosi et al. (2018) advocate against the "supply granularity" proposed by Gabaix and provide empirical evidence of a "demand granularity", based on investment growth shocks instead. They conclude that demand-driven shocks to the largest 100 US firms account for almost one-fourth of GDP volatility.

The granular nature of some sectors and its crucial role in shaping aggregate fluctuations has also been studied. That is the case of the banking sector. Buch and Neugebauer (2011) construct the "banking granular residual" (Blank et al., 2009) to evaluate how changes in lending by large banks impact on GDP growth.[8] Using a panel data set for 35 European countries, they estimate that idiosyncratic shocks to loan growth at large banks explain about 16% of the short-run, cyclical

[5] The seminal work of Acemoglu et al. (2012) shows that, in the presence of strong interconnections between different firms or sectors, idiosyncratic shocks can propagate throughout the economy, amplifying the initial impact of small shocks. See Carvalho and Tahbaz-Salehi (2019) for a review on production networks in macroeconomics.

[6] A great deal of empirical evidence indicates that the empirical firm size distribution is well approximated by a power law (see, among many others, Okuyama et al. (1999), Axtell (2001), Gaffeo et al. (2003), Fujiwara et al. (2004), Luttmer (2007), di Giovanni and Levchenko (2013)). See Gabaix (2009a) for a review of power laws in economics and finance.

[7] Hulten (1978)'s first-order approximation for frictionless, efficient economies has recently been extended by Baqaee and Farhi (2019, 2020) to study second-order terms—structural microeconomic elasticities of substitution, network linkages, structural microeconomic returns to scale, and the extent of factor reallocation—effects and inefficient economies.

[8] The banking granular residual is first introduced by Blank et al. (2009) in order to explore whether shocks originating at large banks affect the probability of distress of smaller banks and thus the stability of the banking system. The difference with respect to the granular residual is twofold. First, the Domar weights (Domar, 1961, Hulten, 1978) are replaced by the total operating income because bank's sales is not a good proxy of output. Second, bank's productivity is measured by the cost-to-income ratio instead of the sales per employee ratio because it is a better proxy for the efficiency of a bank.

variation in per capita GDP growth within a given country.[9] Bremus et al. (2018) extend this result by providing a theoretical framework that links bank size and aggregate outcomes. They also provide empirical evidence from more than 83 countries supporting the theoretical prediction: individual lending shocks to large banks can explain 11% of aggregate credit growth and 8% of per capita GDP growth. Relatedly, Amiti and Weinstein (2018) identify idiosyncratic bank supply shocks using detailed matched lender-borrower and show empirically that 30-40% of aggregate lending and investment fluctuations in Japan arise from the idiosyncratic supply shocks of granular lenders.

The granular hypothesis has sparked further theoretical developments, such as the heterogeneous firm dynamics setup proposed by Carvalho and Grassi (2019) to evaluate the impact of large firm dynamics on the business cycle. In their framework, fluctuations in the upper tail of the firm size distribution, induced by idiosyncratic shocks to very large firms alone, generates sizeable fluctuations in aggregates. Particularly, the calibration exercise for the US economy shows that aggregate output and productivity fluctuations amount, respectively, to 30% and 24% of that observed in the data, in line with Gabaix (2011)'s empirical estimation.[10] In this context, it is also worth noting the early work of Delli Gatti et al. (2005), which shows that a financial fragility agent-based model (henceforth ABM), based on complex interactions of heterogeneous agents—multiple heterogenous firms and a single bank, can replicate empirical regularities in industrial dynamics and generate aggregate fluctuations of the order of those observed empirically.[11] The ABM approach is particularly useful for studying the impact of firms dynamics on the aggregates due to three main components: *bottom-up perspective* (Tesfatsion, 2002), *heterogeneity* and *interaction* (Kirman, 1992).[12] ABMs are built based on individual behavior and interaction that is rooted in empirical and experimental microeconomic evidence, allowing to evaluate how collective behavior emerges from the interaction of autonomous and heterogeneous agents. Additionally, the ABM approach aims to isolate critical behavior in order to identify those agents that drive the collective result of the system (Pyka and Fagiolo, 2007). Thus, it seems the most suitable approach to evaluate the implications of granular behavior.

The closed-economy framework proposed by Gabaix (2011) has been extended to address the aggregate volatility consequences of granularity in open economies. di Giovanni and Levchenko (2012a) develop a multi-country framework with heterogeneous firms to investigate the role of large firms in explaining cross-country differences in macroeconomic volatility (see, e.g., Koren and Tenreyro (2007), di Giovanni and Levchenko (2009, 2012b)). They argue that trade openness magnifies the granular effect because only the largest and most productive firms export which,

[9] Bremus and Buch (2017) find a similar figure when analyzing how financial openness may affect GDP growth.

[10] Carvalho and Grassi's (2019) setup has been calibrated to study the secular stagnation phenomenon in Japan (Hogen et al., 2017) and the link between concentration and volatility in a spatial approach (Daniele and Stüber, 2020). See also Grassi (2018), who builds a general equilibrium model able to characterize how the structural importance of a firm depends on the interaction between competition intensity, input-output linkages, and firm size.

[11] As a consequence of the global financial crisis of 2008 and the subsequent Great Depression, the model has been extended to multiple heterogenous banks in order to evaluate bank connectivity, financial contagion and aggregate fluctuations (see, e.g., Grilli et al. (2014, 2015, 2020)).

[12] Pyka and Fagiolo (2007) enumerate the main components that tend to characterize economics ABMs.

in turn, allows them to become larger and contribute more to aggregate output fluctuations.[13,14] Therefore, shocks to the largest firms will matter more for aggregate volatility in smaller countries with lower diversification. di Giovanni and Levchenko (2012a) estimate that international trade can increase aggregate volatility by 15–20% in some small open economies. It also increases business cycle comovement between trading countries significantly (di Giovanni and Levchenko, 2010). di Giovanni et al. (2017, 2018) examine the trade-comovement relationship at the firm level to capture the aggregate comovement implications of heterogeneity across firms in both size and the extent of international linkages. They find that internationally connected firms, which tend to be the largest firms, account for over one-half of French aggregate value added and are more correlated with the countries to which they are directly connected through trade and ownership links. Furthermore, they quantify that aggregate correlations would fall by about one-third of the observed aggregate correlations if direct linkages were severed. Multinational companies are a first-order feature of the world economy, accounting for about one-third of gross output in many developed countries (Alviarez, 2019). Kleinert et al. (2015) and Cravino and Levchenko (2017) emphasize the key role played by foreign affiliates in the international business cycle transmission: large foreign affiliates are responsible for 10-16% of comovement between countries. Recently, di Giovanni et al. (2020) set up multi-country model to simulate the propagation of foreign shocks to the French economy and estimate that 40-85% of the impact of foreign fluctuations on French GDP is accounted for by the "foreign granular residual"—the term capturing the fact that larger firms are more affected by the foreign shocks.

Granular forces also shape international trade patterns. Using firm level data for 32 countries, Freund and Pierola (2015) quantify that the average top firm alone accounts for almost 15% of exports and one-third of the variation in the ratio of exports to GDP, whereas the top 5 firms account for 30% and 50%, respectively. Freund and Pierola (2020) extend these results by showing that over one-fourth of aggregate export growth is attributable to the top 5 firms. According to Freund and Pierola (2015), in nearly half of the countries of their sample, the largest 5 firms are also responsible for a revealed comparative advantage in at least one sector which otherwise would not exist. Gaubert and Itskhoki (2021) study more systematically the role of large individual firms in determining the comparative advantage of countries by setting up a granular multi-sector model of trade. The model suggests that granularity accounts for about 20% of the variation in realized export intensity across sectors and that idiosyncratic firm dynamics account for a large share (one-half) of the evolution of a country's comparative advantage over time.[15] Using the model developed

[13]Evidence on the granular nature of international trade is also documented for European countries (Mayer and Ottaviano, 2008, Marin et al., 2015, de Lucio et al., 2017), the US (Bernard et al., 2009) and Japan (Canals et al., 2007). del Rosal (2013) focus on exports by product and finds that the volatility at the product level can affect the growth of aggregate exports in multiple European countries.

[14]Kramarz et al. (2020) show that trade flows are highly concentrated, which makes individual exporters to be strongly exposed to microeconomic supply and demand shocks, and hence bring a large amount of granular risk to the overall economy.

[15]In this vein, de Lucio et al. (2020) find that the granular comparative advantage component explains export specialization in 29% of industries, which account for 47% of the bilateral trade among EU countries, and explains 60% of the variation in export specialization across countries and industries.

by Gaubert and Itskhoki (2021), Gaubert et al. (2021) assess the normative policy implications of granularity in a global economy and argue that, in granular economies, governments have powerful incentives to adopt policies targeted at individual firms due to their substantial market power, and that they tend to create negative international spillovers.

Assessing the macroeconomic implications of rising market power,[16] De Loecker et al. (2020) find that the increase in market power is driven by a few firms that have much higher markups than in the past. Burstein et al. (2020) study the cyclical behavior of markups in a granular setting—heterogeneity in the firm-size distribution enables large firm dynamics to drive the aggregate business cycle—and conclude that sectoral output and markups comove positively in response to shocks to large firms in the sector, whereas they comove negatively in response to shocks to small firms. In turn, the effect of such shocks on the aggregate markup depends on the distribution of sector-level markups and sectoral expenditure shares.

Taken together, the growing theoretical and empirical evidence on the role of individual firms in business cycle fluctuations, gathered by the literature outlined above, points in the same direction: a small number of granular firms may have a non-negligible impact on the aggregate outcomes.[17] And yet, we lack a method that identifies these granular firms. The granular hypothesis is usually tested by constructing the granular residual for an arbitrary number of top firms (e.g., Gabaix (2011)) or for the universe of firms (e.g., di Giovanni et al. (2014)). Note that such "pointwise" estimation may underestimate or overestimate the contribution of the granular term to business cycle fluctuations and thus may mislead the researcher. In addition, the identification of the granular firms may be useful for policy makers, as suggested by the evidence put forth by Gaubert et al. (2021). The main research question that motivates the present thesis is: How many granular firms populate a granular economy?

In Chapter 2 of this thesis, we construct the granular residual using the top 100 Spanish firms, as in Gabaix (2011), and find that idiosyncratic productivity shocks can rationalize about half of variations in GDP growth. Once we have shown that the Spanish economy is granular for this particular number of firms, we then explore how the explanatory power (measured by the $R^2$ statistic) of the granular residual behaves as the number of firms increases gradually. The evolution of $R^2$ is characterized by a sharp increase when a reduced number of large firms is included and by an almost steady value when including additional firms.[18] We check that this behavior (which we refer to as "granular curve") is at odds with that predicted by the representative firm framework by introducing the "equal-weight benchmark", in which all firm are instead symmetric in size. In addition, we note that the granular curve converges to the equal-weight benchmark as the large

[16]See Van Reenen (2018) and Syverson (2019) for a detailed discussion. De Loecker et al. (2021) provide a quantitative framework to assess the causes of market power and its impact on welfare and business dynamism.

[17]Stella (2015) and Wagner and Weche (2020) are the exception. The former tests the granular hypothesis by estimating a dynamic factor model, in the spirit of Foerster et al. (2011), with firm-level data and finds that idiosyncratic shocks have little role in explaining US business cycle fluctuations. Employing the granular residual, the latter conclude that the idiosyncratic movements of the largest 100 firms seem not to be important for an understanding of the aggregate volatility of the German economy.

[18]This type of behavior has also been documented in Brazil (Silva and Da Silva, 2020) and Kazakhstan (Konings et al., 2021).

firms are replaced by small firms. In light of this result, we propose a simple method to calibrate the granular size of the economy—those firms whose idiosyncratic shocks may translate into aggregate fluctuations—based on identifying the point of converge. We estimate the granular size of the Spanish economy to be approximately 450 firms. In other words, if the largest 450 firms did not exist, the Spanish economy would not be granular.

In light of the empirical results obtained in Chapter 2, Chapter 3 raises the following question: What drives the behavior of the granular curve? Building on the models develop by Gabaix (2009a) and Carvalho and Gabaix (2013), we setup a conceptual framework that traces back the volatility of GDP growth to large firms' idiosyncratic shocks. We show that, when the distribution of firm size and the relationship between firm' size and volatility is characterized by a power law,[19] the contribution of idiosyncratic shocks to GDP fluctuations is characterized by the following firm dynamics parameters: productivity multiplier, "Domar" weight (Domar, 1961) of the largest firm, volatility of the largest firm, tail index of firm size distribution, and size-volatility elasticity. Our framework emphasizes the key role played by the largest in shaping aggregate fluctuations, the effect of the size-volatility relationship and the fact that the granular contribution to aggregate fluctuations is bounded. The first result is in line with Carvalho and Grassi (2019), who explore the impact of a negative shock to the largest firm in the economy using an industry dynamics framework that is able to endogenously generate a power law firm size distribution and conclude that business cycles have a "small sample" origin. The second and third results are in line with Yeh (2021), who explores the implications of the size-volatility relationship for origins of business cycles. In contrast to these works, we characterize the maximum contribution of idiosyncratic shocks to aggregate fluctuations and show that changes in the size-volatility elasticity have a larger impact on aggregate volatility than changes in the tail index.

Seeking to answer the question that motivates the present thesis, we employ our framework to quantify the granular size of the economy. We propose three definitions of granular firms that allow us to better quantify granular size because the calibrated size is closer to the point that visually represents the change from the granular to the atomistic regime than the empirical method initially proposed. In particularly, we now estimate that the granular size of the Spanish economy is approximately 50 firms. Thus, this result indicates that we should not consider the transition phase between regimes, as initially proposed in Chapter 2. Finally, we find that the granular size exhibits a cyclical behavior: the number of firms whose idiosyncratic shocks have an impact on the aggregate grows in expansion phases and shrinks in recession phases.

In Chapter 4, we use Delli Gatti et al.'s (2005) ABM to study the emergence of the granular curve behavior, which builds on the *levered aggregate supply* class of models first developed by Greenwald and Stiglitz (1990, 1993). We choose this model for two reasons. First, it is able to generate sizeable aggregate fluctuations from purely idiosyncratic shocks and interaction among the agents (multiple heterogenous firms and a single bank) and to reproduce empirical regularities

[19] Empirical evidence on the power law behavior of the size-volatility relationship is found by Stanley et al. (1996), Lee et al. (1998), Sutton (2002), Koren and Tenreyro (2013), Calvino et al. (2018), Yeh (2021).

such as a power-law firm size distribution and a Laplace distribution of growth rates (Stanley et al., 1996, Amaral et al., 1997). Second, it is analytically tractable, which allows us to precisely identify the drivers of aggregate fluctuations. We show that aggregate volatility is chiefly driven by the direct impact of firm-level specific shocks. The effect of propagation of these shocks due to interactions among the agents plays a minor role. The direct impact of firm-level specific shocks is, in turn, determined by the cross-sectional dispersion of firm sizes. The fact that the model generates a distribution of firm size that is close to a Zipf allows us to attribute the aggregate fluctuations of the economy the dynamics of the largest firm and a the summary statistic that is the tail index. Then, we study the contribution of the large firms' dynamics to aggregate fluctuations and show that it displays the granular curve behavior observed in Chapter 2 and Chapter 3. The framework introduced in Chapter 3 provides a good characterization of such behavior and sheds light on its determinants, which are the following: size of the largest firm, ratio of the representative firm-specific shocks volatility to aggregate volatility and tail index. Finally, we use the granular size measures introduced in Chapter 3 and find that the granular region of this simulated economy is approximately 20 firms.

# Chapter 2

# On the Determination of the Granular Size of the Economy

## 2.1 Introduction

In mainstream macroeconomics, firm-level idiosyncratic shocks are assumed to average out in aggregate (Lucas, 1977), contributing just marginally to macroeconomic fluctuations. This idea has been challenged by the empirical work of Gabaix (2011), who explicitly tests on what extent those shocks account for aggregate fluctuations. He has shown that the idiosyncratic shocks to the largest 100 firms have a significant impact on the business cycle fluctuations of United States, accounting approximately for one-third of GDP variations. Aggregate fluctuations, therefore, can be partially attributed to the destinies of well identified "grains", which are few very large firms. If an economy is characterized by such behavior, it is defined as a "granular economy".

After the seminal work of Gabaix (2011), other studies have found that several macroeconomic variables exhibit granular fluctuations, such as exports (del Rosal, 2013, di Giovanni et al., 2017) or investments (Grullon et al., 2013). The granular behavior can also be observed at sectoral level, for example in the banking (Blank et al., 2009) or manufacturing sector (Wagner, 2012). In those empirical contributions, however, the number of what are considered granular firms is exogenously given.

Based on the methodology proposed by Gabaix (2011), in this paper, we aim at calibrating how many are the granular firms, i.e. to determine the granular size of the economy. To the best of our knowledge, we are the first to address this issue in the literature.

The chapter is organized as follows. The description of the data and the empirical methodology is presented in Section 2.2. Section 2.3 shows the main results. Section 4.5 concludes.

## 2.2 Data and Methodology

In order to perform our empirical analysis, we use the SABI (Sistema de Análisis de Balances Ibéricos) database, which collects accounting data from Spanish firms. For our purpose, we are interested in the annual volume of sales and the corresponding number of employees as well as the activity carried out by each individual firm, which is coded in the SIC code. The initial sample obtained from SABI is made up of the 10000 largest Spanish firms in the period ranging from 1995 to 2016, ranked by their volume of sales.[1] Following Gabaix (2011), firms whose SIC codes are among the following numbers have been filtered out: 1311, 1389, 2911, 2999, between 4900 and 4940, 5052, 5172 and between 6000 and 6999. These firms are, in fact, engaged in activities whose impact on their sale fluctuations are directly related to changes in world commodity prices (e.g., oil companies), which cannot be considered idiosyncratic shocks, or are financial companies, whose sales do not stem from manufactured goods (e.g., banks). After the filtering procedure, the number of remaining firms is 9072. Macroeconomic data (GDP, GDP per capita and GDP deflator) are taken from the World Bank's Development Indicators database.

As proposed by Gabaix (2011), we construct the measure of the idiosyncratic labour productivity shocks to the top $K$ firms, which is called "granular residual":

$$\Gamma_t = \sum_{i=1}^{K} \frac{S_{i,t-1}}{Y_{t-1}} \left( g_{i,t} - \bar{g}_t^Q \right) \ , \tag{2.1}$$

where $S_{i,t-1}$ is the deflated volume of sales of firm $i$ in year $t-1$, $Y_{t-1}$ is the real GDP in year $t-1$ and $g_{i,t} - \bar{g}_t^Q$ is the demeaned labour productivity growth rate, considered as a proxy for the idiosyncratic shock to firm $i$ in year $t$. The term $\bar{g}_t^Q$ is the cross-sectional median of $g_{i,t}$ computed among the top $Q$ firms, with $Q \geq K$. Labour productivity growth of firm $i$ in year $t$ is defined as:

$$g_{i,t} := \Delta \ln \left( \frac{S_{i,t}}{E_{i,t}} \right) = \ln \left( \frac{S_{i,t}}{E_{i,t}} \right) - \ln \left( \frac{S_{i,t-1}}{E_{i,t-1}} \right) \ , \tag{2.2}$$

where $E_{i,t}$ is the number of employees of firm $i$ in year $t$. In order to avoid the effect of outliers, the demeaned productivity growth rates have been winsorized at 90% level.

Following Gabaix, we employ the explanatory power ($R^2$) of the following regression to assess to which extent idiosyncratic shocks account for aggregate fluctuations:

$$g_t^Y = \alpha + \sum_{i=0}^{2} \beta_i \Gamma_{t-i} + \varepsilon_t \ , \tag{2.3}$$

where $g_t^Y$ is per capita real GDP growth rate. Based on the Hulten's theorem (Hulten, 1978), Gabaix (2011) illustrates how the coefficients $\beta_i$s provide an estimation of the factor usage.[2] In

[1] The sum of their sales accounts for approximately 70% of GDP of the Spanish economy.

[2] The estimation of the factor usage provided by $\beta_i$ is a combination of the elasticity of substitution of labor and output elasticities with respect to production inputs.

**Table 2.1.** Explanatory power of the granular residual.

| | GDP Growth$_t$ | | |
|---|---|---|---|
| | (1) | (2) | (3) |
| $\Gamma_t$ | 2.52** | 1.84* | 2.14** |
| | (1.24) | (1.02) | (0.94) |
| $\Gamma_{t-1}$ | | 3.06*** | 2.45** |
| | | (0.95) | (0.93) |
| $\Gamma_{t-2}$ | | | 2.19*** |
| | | | (0.61) |
| Intercept | 0.0187*** | 0.0233*** | 0.0270*** |
| | (0.0053) | (0.0051) | (0.0048) |
| N | 22 | 21 | 20 |
| $R^2$ | 0.185 | 0.421 | 0.537 |
| Adj. $R^2$ | 0.144 | 0.357 | 0.451 |

**Notes**: Results of the regression (2.3) when $Q = K = 100$. Per capita GDP growth $g_t^Y$ is regressed on the granular residual $\Gamma_t$ in column (1), adding one lag in column (2) and adding two lags in column (3). Robust standard errors to autocorrelation are given in parentheses. ***, ** and * indicate significance at 1%, 5% and 10%, respectively.

order to have an intuition for the value of the factor usage of the Spanish economy, we apply the approximate calculation proposed by Gabaix (2011):

$$\sigma_{GDP} = \mu \cdot \sigma_\pi \cdot h \ , \tag{2.4}$$

where $h = 0.048$ is the square root of the Herfindahl index for sales of the 100 largest firms, $\sigma_\pi = 0.13$ is their cross-sectional standard deviation of the productivity growth rate, averaged across the entire period, and $\sigma_{GDP} = 0.024$ is the estimated GDP standard deviation in the considered period. From equation (2.4), the calibrated value of the factor usage is $\mu = 3.8$. The estimated coefficients from equations (2.3) and (2.4) exhibit, indeed, similar values (see Table 2.1).

## 2.3 Results

### 2.3.1 The Spanish Economy is Granular

We first check whether the Spanish economy is granular by computing the explanatory power of the granular residual for a given number of large firms. Table 2.1 shows the results of the estimation of the coefficients $\beta_i$s, considering different specifications of the OLS in equation (2.3).

With $K = Q = 100$,[3] our results indicate that the Spanish economy is granular since the granular residual accounts approximately for 45% of variations of GDP growth. This value turns out to be higher than the explanatory power reported by Gabaix for the American economy. Our results provide a further empirical support to the granular hypothesis, extending its validity to the Spanish economy.

The identification of the Spanish (or American) economy as a granular economy is based on an exogenous choice for the number of large firms in equation (2.1). Such "pointwise" estimation of the $R^2$ does not provide information on the extent of the granular region since the number of considered firms is arbitrarily chosen. Therefore, we may underestimate the contribution of the granular term to the GDP fluctuations, considering too few granular firms, or overestimate its impact, including too many firms in equation (2.3).

### 2.3.2 The Granular Size of the Spanish Economy

We propose a novel methodology in order to calibrate the granular size of the economy, using the Spanish data as an illustrative example. To be more precise, our aim is to calibrate the number of the granular firms, $K^*$. As a first step, we analyze how the explanatory power of the granular residual behaves when we progressively increase $K$ in equation (2.1), in the range $1 \leq K \leq Q = 1000$.[4] Figure 2.1 shows the evolution of the $R^2$ as a function of $K$, to which we refer as the "granular curve" (the upper curve in Figure 2.1). This curve is characterized by: (i) a sharp increase of the $R^2$ when a reduced number of large firms is gradually included in the calculation of the granular residual (roughly the largest one hundred firms); (ii) an almost steady value of the $R^2$ when including additional firms.

In order to validate our results, let us introduce the "equal-weight" benchmark by replacing the empirical weights in equation (2.1) with constant weights for all firms, i.e. posing $S_{it} = S_t^*$, while keeping unchanged the corresponding idiosyncratic shocks.[5,6] Such benchmark quantifies the contribution of the granular residual to the GDP fluctuations of an economy composed by equal-size firms (representative firm). Within the representative firm framework, the contribution of the firm-level idiosyncratic shocks to aggregate fluctuations is, indeed, marginal. The comparison of the equal-weight benchmark to the granular curve gives a clear indication of the relevant role played by the very large firms in the characterization of business cycle fluctuations. Our results indicate that

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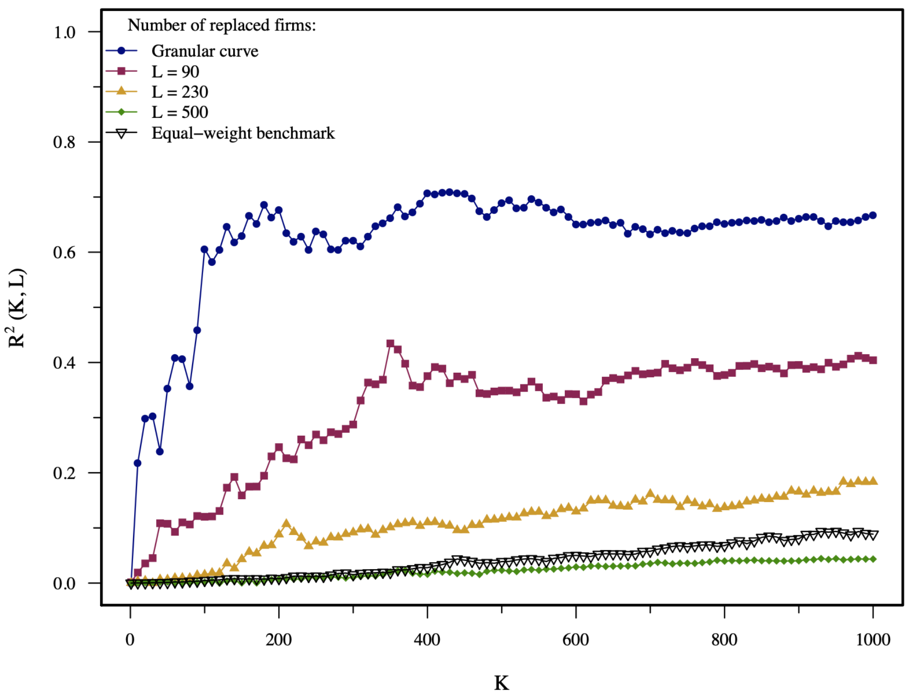
[3] We analyze the specification $K = Q = 100$ to have a direct comparison to the estimates reported by Gabaix (2011).

[4] We now include one lag in the OLS because of the short length of the time series. Our results are robust when including two lags or considering the entire sample of available firms (material upon request).

[5] We consider the volume of sales $S_t^* = S_{1000,t}$ of the largest 1000th firm for each year $t$, and we assign its value to all firms in that year when computing $\Gamma_t$. The choice of the particular value for $S_t^*$ is irrelevant for the behavior of the benchmark, as soon as $S_t^*$ does not coincide with the size of a granular firm.

[6] We limit the variability of $\beta_i$ to the interval $[0, 3.5]$ in order to avoid that the coefficients $\beta_i$ in the regression (2.3) increase artificially their value. The upper bound is chosen as a conservative value, averaging the estimated coefficients from Table 2.1 and the calibrated value of $\mu$ from equation (2.4). Without introducing the bounded interval for $\beta_i$, the coefficients can exhibit values unrealistically high (some time higher than 30), considering that $\beta_i$ are proxies for the factor usage. Interestingly, when computing the granular curve, the coefficients $\beta_i$ never crosses the boundaries.

**Figure 2.1.** Explanatory power as a function of $K$ and $L$.




**Notes**: Explanatory power of the regression in equation (2.3) as a function of an increasing number of firms $K$ and for different values of $L$, $R^2(K, L)$. The incremental step is $\Delta K = 10$.

the heterogeneity of firms cannot be discarded when modeling aggregate fluctuations. As a further evidence of the importance of the heterogeneity of firms, Figure 2.1 shows the transition from the granular curve to the equal-weight benchmark, when we progressively remove the $L$ largest firms in $\Gamma_t$.[7] The curves representing the explanatory power of the OLS regression as a function of $K$ and for given values of $L$, $R^2(K, L)$, exhibit smoother curvatures for large values of $L$, reaching lower explanatory power. In particular, the curve $R^2(K, 500)$ is almost indistinguishable from the equal-weight benchmark, indicating that the remaining heterogeneity among firms has a negligible impact on aggregate fluctuations.

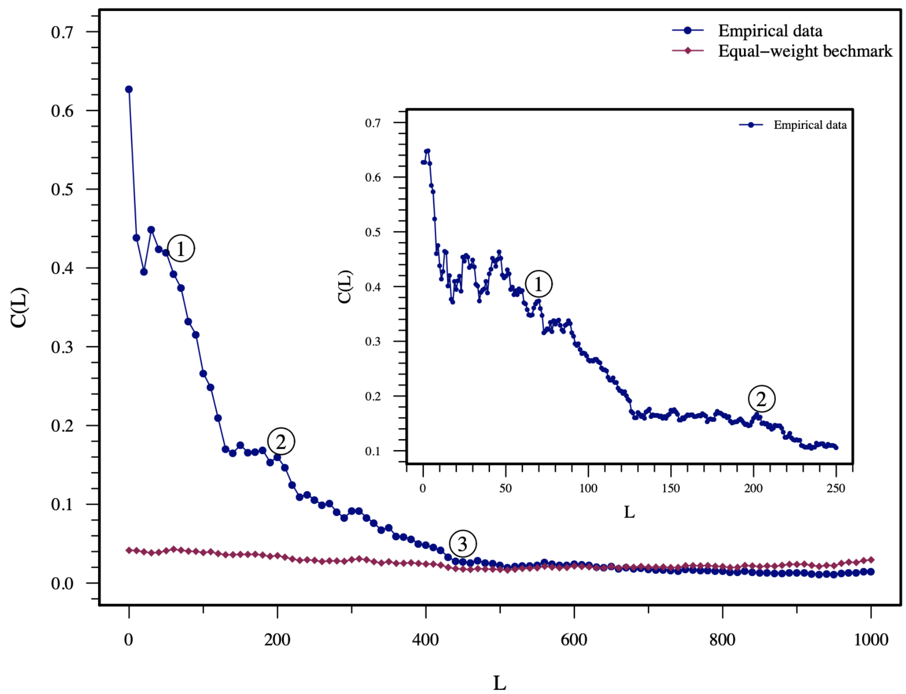
In order to calibrate the granular size of the economy, we empirically analyze the sensitivity of the $R^2(K, L)$ curves to increased values of $L$, i.e. to a gradual elimination of the larger firms. Figure 2.2 plots the average cumulative explanatory power, i.e. the average cumulative $R^2$s, as a function of $L$:

$$C(L) = \frac{1}{Q} \sum_{K=1}^{Q} R^2(K, L) \,. \tag{2.5}$$

A simple method to calibrate $K^*$ is, therefore, to approximatively identify the interval where

[7] We replace the $L$ largest firms with smaller size firms, ranging from the position $Q+1$ to $Q+L$ in the ranked sample. In this way, the considered sample is always composed of $Q$ firms.

**Figure 2.2.** Evolution of the $C(L)$ curve and the equal-weight benchmark.




the $C(L)$ curve intersects the curve of the average cumulative explanatory power of the equal-weight benchmark. Point 3 in Figure 2.2 indicates that the granular size of the Spanish economy is approximately $K^* \approx 450$ firms.

Interestingly, the inset of Figure 2.2 shows that the $C(L)$ curve does not decreases steadily. Instead, it exhibits some well-defined regions where it remains almost unchanged (the plateaus indicated by points 1 and 2 in Figure 2.2). It seems that, within the group of granular firms, we can identify an "inner granular structure", due to different degrees of heterogeneity among the granular firms. In principle, we could introduce alternative criteria to calibrate $K^*$, taking into account the granular inner structure. However, this comes at a cost of introducing a certain arbitrariness into the choice of $K^*$.

## 2.4 Conclusions

In this paper, we have introduced a novel methodology to calibrate the number of granular firms in an economy. We have applied such method to the Spanish economy, calibrating in approximately 450 its number of the granular firms. We plan to apply our methodology to other countries. An international comparison will allow to refine the definition of the granular size of the economy introduced in this paper, by including the empirically identified inner granular structure.

# Chapter 3

# Granular Firms and Aggregate Fluctuations

## 3.1 Introduction

Traditionally, business cycles theories have dismiss the possibility that *microeconomic* shocks may originate aggregate fluctuations due to a "diversification" argument (Lucas, 1977). Gabaix's (2011) seminal work challenges the convention by introducing the "granular" hypothesis: in the presence of significant heterogeneity at the micro level, the behavior of macroeconomic aggregates is attributable to the incompressible "grains" of economic activity, the large firms.[1] In this view, idiosyncratic shocks to the *granular* firms play a crucial role in shaping aggregate fluctuations. And yet, we lack a framework that provides a theoretically founded method for identifying the number of granular firms that populate a granular economy.

The first attempt to quantify the granular size of the economy (i.e., the number of granular firms) is made by Blanco-Arroyo et al.'s (2018) empirical work, who find that the contribution of idiosyncratic shocks to gross domestic product (GDP) fluctuations increases rapidly when the very top firms are taken into account and an almost steady value from a given number of firms onwards. They refer to this behavior as "granular curve".[2] The granular curve clearly shows two well differentiated regimes: the granular regime, which is composed of a small number of large firms whose idiosyncratic perturbations can lead to aggregate fluctuations, and the atomistic regime, which is composed of those firms whose effect on the aggregate is negligible. Blanco-Arroyo et al. (2018) propose an empirical method to estimate the granular size based on replacing large firms by smaller ones and comparing the resulting granular curve with the counterfactual case in which all firms are of equal size. The granular size is then determined by the number of large firms

[1]Recent contributions that also seek to understand the microeconomic underpinnings of aggregate fluctuations are Acemoglu et al. (2012), di Giovanni and Levchenko (2012a), Carvalho and Gabaix (2013), di Giovanni et al. (2014), Grassi (2018), Baqaee (2018), Baqaee and Farhi (2019), Carvalho and Grassi (2019).

[2]This type of behavior has also been documented in Brazil (Silva and Da Silva, 2020) and Kazakhstan (Konings et al., 2021).

that, once removed, cause the empirical curve to converge to the counterfactual case. However, this procedure seems to be too conservative, as the convergence point is much larger than point that visually represents the change of regime in the granular curve.

This paper seeks to shed light on the determinants of the granular curve behavior and to quantify the granular size of a granular economy more precisely. Building on the models developed by Gabaix (2009a) and Carvalho and Gabaix (2013), we setup a conceptual framework that traces back the volatility of GDP growth to large firms' idiosyncratic shocks. We show that, when the distribution of firm size is power law (see, e.g., Axtell (2001), Luttmer (2007), di Giovanni and Levchenko (2013)) and the firms' idiosyncratic volatility depends on size as a power law (see, e.g., Stanley et al. (1996), Koren and Tenreyro (2013), Yeh (2017)), GDP fluctuations are shaped by five parameters that capture the large firms dynamics: (i) productivity multiplier, (ii) Domar weight (Domar, 1961) of the largest firm, (iii) volatility of the largest firm, (iv) tail index of firm size distribution, and (v) size-volatility elasticity.

Theoretically, our framework provides three key results. First, the largest firm contains a great deal of information on the characteristics of the economy and plays a crucial role in driving aggregate fluctuations. This result is in line with Carvalho and Grassi (2019), who develop a heterogeneous firm dynamics setup in which aggregate fluctuations are caused by firm-level disturbances alone and conclude that business cycles have a "small sample" origin.

Second, the granular contribution to aggregate fluctuations is bounded. When the firm size distribution is power law, the contribution of idiosyncratic shocks to aggregate fluctuations exhibits an asymptotic value. This finding is in line with Yeh (2021), who explores the effect the contribution of idiosyncratic shocks to aggregate fluctuations when including the size-volatility relationship. The fact that exists a maximum contribution leaves room to traditional alternative factors, such as oil and monetary policy shocks, and amplification mechanism, such as "cascade effects" propagated throughout the input-output network (Acemoglu et al., 2012).

Third, the effect of the size-volatility relationship in shaping aggregate fluctuations is non-negligible. The literature that studies the granular origins of aggregate fluctuations has typically downplayed the effect of the size-volatility relationship by arguing that the estimates come from biased and non-representative samples (Gabaix, 2011).[3] Recently, Yeh (2021) estimates the relationship using the universe of U.S. firms and concludes that it is statistically different from zero even when taking the large firms only. We show it is incompatible to assume that the weak form of Gibrat's (1931) law for volatilities holds and use the volatility of the largest firms (e.g., Gabaix (2011), Carvalho and Grassi (2019)) and that changes in the size-volatility relationship have greater impact on aggregate fluctuations than tail index changes, which have been the main object of study.

We then employ our setup to study the granular curve behavior observed in the data. As

[3]This critique stems from the fact that the estimation has typically been carried out using firms in *Compustat* database. As argued Gabaix (2011), Compustat only comprises large traded firms that are expected to be more volatile than non-traded firms, as small volatile firms are more prone to seek outside equity financing, while large firms are in any case very likely to be listed in the stock market.

in Blanco-Arroyo et al. (2018), we focus on the top 1000 Spanish firms. The estimation of the parameters support the hypotheses on which it is based: the distribution of firm size and the size-volatility relationship follow a power law behavior.

Empirically, we show that the granular curve is well characterized by our framework and find that the average maximum contribution of top Spanish firms's idiosyncratic shocks to the GDP fluctuations is 23%. This estimate is in line with previous empirical estimations that are purely econometric (see, e.g., Gabaix (2011), Blanco-Arroyo et al. (2018), Fornaro and Luomaranta (2018), Miranda-Pinto and Shen (2019), Silva and Da Silva (2020)). Then, we propose a set of measures that allow to quantify the granular size of the economy more precisely than the empirical method initially proposed by Blanco-Arroyo et al. (2018), as the estimated size is closer to the point that visually represents the change from the granular to the atomistic regime than the empirical method initially proposed. In particularly, we estimate that the granular size of the Spanish economy is approximately 50 firms.

The results are robust to changes in the number of firms and to time-varying parameters. Our baseline estimation focuses on the largest 1000 firms and considers the entire period available. In an alternative approach, we increase the number of firms to 2500 in steps of 500 and find that our framework continues to characterize the empirical granular curve and the calibrated number of firms remains in the region that we visually identify as the change of regime. We also explore the granular curve behavior in smaller time windows. After calibrating the volatility of the largest firm, we show that the framework provides a good characterization of the changes observed in the empirical granular curve through the business cycle. Finally, we find that the granular size exhibits a cyclical behavior: the number of firms whose idiosyncratic shocks have an impact on the aggregate grows in expansion phases and shrinks in recession phases.

**Related Literature** Our paper draws on, and contributes to, two strands of literature: the granular origins of aggregate fluctuations and the empirical industrial dynamics literature. Our conceptual framework sheds light on the components that drive Gabaix's (2011) "granular residual" and, hence, relates to the recent empirical literature that investigates the proportion of aggregate shocks that can be accounted for by idiosyncratic to the large firms (see, e.g., Gabaix (2011), di Giovanni et al. (2014), Stella (2015), Magerman et al. (2016), Yeh (2017)).[4] It also relates to the scarce theoretical literature that studies how large firms dynamics shape aggregate fluctuations (di Giovanni and Levchenko, 2012a, Carvalho and Grassi, 2019, Daniele and Stüber, 2020, Gaubert and Itskhoki, 2021). Although, unlike the literature, our framework takes the firm size distribution and the size-volatility relationship as exogenously given.

This paper is also related to the empirical industrial dynamics literature that studies the firm size distribution (see, e.g., Axtell (2001), Gaffeo et al. (2003), Fujiwara et al. (2004), Luttmer (2007), di Giovanni and Levchenko (2013)) and the size-volatility relationship (e.g., Stanley et al.

[4] Previous literature that seeks to understand the microeconomic underpinnings of aggregate fluctuations includes Jovanovic (1987), Durlauf (1993), Bak et al. (1993), Nirei (2006).

(1996), Lee et al. (1998), Sutton (2002), Koren and Tenreyro (2013), Calvino et al. (2018), Yeh (2021)). In line with the bulk of the recent literature, we find that the upper tail of the firm size distribution follows a power law with exponent larger than one and that the weak form of Gibrat's (1931) law for volatilities, typically assumed in the granular literature, does not hold for the largest firms in the economy.

**Outline** The remainder of this paper is organized as follows. Section 3.2 discusses the conceptual framework that traces back the origins of business cycles fluctuations to large firm's dynamics. Section 3.3 presents the data and estimates the variables that constitute our model. Section 3.4 characterizes the behavior of the empirical contribution of idiosyncratic shocks to large firms to aggregate fluctuations, quantifies the granular size of the economy and explores its cyclical behavior. Section 4.5 concludes. Derivations and robustness checks can be found in the Appendix.

## 3.2 Conceptual Framework and Motivation

This section builds on the models develop by Gabaix (2009a) and Carvalho and Gabaix (2013) to shed light on how idiosyncratic firm shocks shape aggregate fluctuations. We follow the literature and assume that the upper tail of the firm size distribution and the size-volatility relationship follow a power law. Under these assumptions, GDP growth volatility is driven by five components: (i) productivity multiplier, (ii) "Domar" weight of the largest firm, (iii) volatility of the largest firm, (iv) tail index of firm size distribution, and (v) size-volatility elasticity. Furthermore, changes in the size-volatility elasticity have a larger impact on aggregate volatility than changes in the tail index.

### 3.2.1 Conceptual framework

Consider an economy populated by $n$ competitive firms that produce intermediate and final goods using capital, labor and intermediate inputs supplied from one another. According to Hulten (1978), after a Hicks-neutral idiosyncratic productivity shock $\varepsilon_i = dA_i/A_i$ to firm $i$, the shock to aggregate total factor productivity (TFP) $\Lambda$ is

$$\frac{d\Lambda}{\Lambda} = \sum_{i=1}^{n} \frac{S_i}{Y} \varepsilon_i, \tag{3.1}$$

where $S_i$ is firm $i$'s value of sales (gross output) and $Y$ is GDP (aggregate value added). $S_i/Y$ is the so-called "Domar" weight (Domar, 1961).[5] The sum of the Domar weights in (3.1) can be greater than one. This reflects the fact that the change in factor efficiency creates extra output, which serves to increase final demand and intermediate inputs—see Carvalho and Gabaix (2013) for an

[5] See Carvalho and Gabaix (2013) for an intuition on the use of use the concept of gross output, rather than net output (i.e., value added).

intuition.[6] The weighted sum of idiosyncratic productivity shocks is none other than Gabaix's (2011) "granular residual" (see Section 3.4.1).

Gabaix (2009a) and Carvalho and Gabaix (2013) show that, in absence of other disturbances, GDP growth $dY/Y$ is proportional to TFP growth $d\Lambda/\Lambda$: $dY/Y = \mu d\Lambda/\Lambda$, for some productivity multiplier $\mu \geq 1$. Thus, GDP growth is equal to

$$\frac{dY}{Y} = \mu \sum_{i=1}^{n} \frac{S_i}{Y} \varepsilon_i. \tag{3.2}$$

Assume that productivity shocks are uncorrelated across firms (i.e., $\operatorname{cov}(\varepsilon_i, \varepsilon_i) = 0 \ \forall i$) and firm $i$'s has a variance of shocks $\sigma_i = \operatorname{var}(\varepsilon_i)$.[7] Then, we have that the volatility of GDP growth is

$$\sigma_Y = \mu \sqrt{\sum_{i=1}^{n} \left(\frac{S_i}{Y}\right)^2 \sigma_i^2}. \tag{3.3}$$

The square root of the weighted sum is Gabaix's (2011) "granular" volatility, Carvalho and Gabaix's (2013) "fundamental" volatility and di Giovanni et al.'s (2014) "direct effect".

Gabaix's (2011) seminal work introduces the "granular" hypothesis: in the presence of significant heterogeneity at the firm-level, economic fluctuations are attributable to the incompressible "grains" of economic activity, the large firms. The intuition is as follows. When the distribution of firm size in equation (3.3) is sufficiently fat-tailed, idiosyncratic shocks to the granular firms do not die out in the aggregate, because they do not cancel out with shocks to smaller firms. Thus, the origins of aggregate fluctuations can be traced back to the dynamics of the *granular* firms.

Our first goal is to shed light on the industrial dynamics factors that drive the volatility of GDP growth. To this end, we first study the distribution of firm size—measured by the value of sales—and then the volatility of idiosyncratic productivity shocks.

### 3.2.2 Firms size distribution

A plethora of empirical evidence finds that the the entire firm size distribution, or at least its upper tail, is well approximated by a power law (see, e.g., Axtell (2001), Fujiwara et al. (2004), Luttmer (2007), di Giovanni and Levchenko (2013), among many others).[8,9] Given our focus on the large firms, the evidence put forth by the literature in favor of the power law distribution makes this a natural baseline to consider. Therefore, we assume that the counter cumulative distribution

[6] Hulten's (1978) first-order approximation for frictionless, efficient economies has recently been extended by Baqaee and Farhi (2019, 2020) to study the role played by second-order effects, such as complementarity, substitutability, returns to scale, factor reallocation, and network structure.

[7] Throughout this section, we drop the time subscript for the sake of simplicity.

[8] See Gabaix (2009b) for a review of power laws in economics and finance.

[9] Gibrat (1931) and the literature that followed (see Sutton (1997) for a review) describe the firm size distribution by a lognormal. Recent studies using census data conclude that the lognormal behavior emerges in non-representative samples (Axtell, 2001).

function (CCDF) of sales $S$ is characterized by

$$\mathbb{P}\left(\text{firms} > S_i\right) = \left(\frac{S_{\min}}{S_i}\right)^{\zeta}, \tag{3.4}$$

for $S_i > S_{\min}^{1/\zeta}$, with $\zeta \in [1, 2)$. The CCDF (3.4) corresponds to a density $p\left(S_i\right) = \zeta S_{\min}^{\zeta} S_i^{-(\zeta+1)}$. We introduce introduce the cut-off $S_{\min}$ to account for the fact that only the upper tail of the sales distribution could display a power-law behavior (Fujiwara et al., 2004). We bound the tail index $\zeta$ in the range $[1, 2)$ to ensure that the distribution is fat-tailed and, hence, the economy is granular. Traditionally, business cycle theories have discarded the possibility that aggregate fluctuations may originate from microeconomic shocks to firms due to a "diversification argument" (Lucas, 1977). In particular, in an economy populated by a large number $n$ of firms hit by independent shocks, the law of large numbers applies and, hence, GDP volatility would be roughly proportional to $1/\sqrt{n}$—a negligible effect. As shown by Gabaix (2011), this would be the case if $\zeta \geq 2$. However, when $\zeta$ lies in the range $[1, 2)$, as estimated by the literature above, the law of large numbers does not apply and GDP volatility decays much slower. For instance, when $\zeta = 1$, known as Zipf's law (Zipf, 1949), the rate of decay is $1/\ln n$. Thus, shocks to individual large firms may translate into aggregate fluctuations.

### 3.2.3 Size-volatility relationship

The works of Meyer and Kuh (1957) and Hymer and Pashigian (1962) are the first to document the negative relationship between firm's volatility, measured by the standard deviation of firm's sales growth rate, and its size, measured by the average value of sales. Additional contributions find that this relationship is described by a power law (see, e.g., Stanley et al. (1996), Lee et al. (1998), Sutton (2002), Koren and Tenreyro (2013), Calvino et al. (2018), Yeh (2021)). We follow the literature and assume that the power-law behavior also holds for the relationship between size and volatility of shocks. Thus, the relationship between the volatility of the idiosyncratic productivity shock $\sigma_i$ and the value of sales $S$ is described by the law

$$\sigma_i\left(S\right) = \sigma_{\min}\left(\frac{S_{\min}}{S_i}\right)^{\alpha}, \tag{3.5}$$

with $\alpha \in [0, 1/2]$. As in equation (3.4), we introduce the cut-off $S_{\min}$ and its corresponding volatility $\sigma_{\min}$. The intuition typically provided to explain the limiting cases $\alpha = 0$ and $\alpha = 1/2$ is based on a diversification argument. As argued by Amaral et al. (1997), in a firm made up of many units, which are of identical size and grow independently of one another, fluctuations as a function of size decay as a power law with an exponent $\alpha = 1/2$ because the law of large numbers applies. On the contrary, if there are very strong correlations between the units, the growth dynamics are indistinguishable from the dynamics of structureless organizations and, hence, $\alpha = 0$. The latter is the case predicted by Gibrat's (1931) weak law, namely, there is no size dependence of $\sigma$. Thus, the average volatility is well captured the volatility of all firms (i.e., $\sigma_i = \overline{\sigma}\ \forall i$, where $\overline{\sigma}$ is the average volatility).

The literature above estimates $\alpha$ between the two limiting cases even for large firms.[10] The most common mechanisms proposed to explain the size-volatility relationship are based on output (Klette and Kortum, 2004) and establishment (Foster et al., 2001, 2006) diversification. Recently, Yeh (2017) rules out these mechanisms and concludes that large firms face smaller price elasticities and therefore respond less to a given-sized productivity shock than small firms do, as implied by Decker et al. (2020). Despite the lack of consensus, it is important to emphasize that our results do not hinge on a particular microfoundation.

### 3.2.4 Aggregate fluctuations

**Proposition 1** (GDP fluctuations)**.** *If the firm size distribution and the relationship between size and volatility are power-law, then GDP fluctuations have the following form. If $\zeta' \neq 1$,*

$$\sigma_Y = \mu \frac{S_{\max}}{Y} \sigma_{\max} \left\{ \frac{2}{2-\zeta'} \left[ 1 - \Gamma\left(2/\zeta'\right) n^{1-2/\zeta'} \right] \right\}^{1/2}, \tag{3.6}$$

*where $S_{\max}/Y$ and $\sigma_{\max}$ are, respectively, the Domar weight and volatility of the largest firm, $\Gamma(\cdot)$ is the Gamma function, $n$ is the number of firms that populate the economy and the tail index $\zeta' \equiv \zeta/(1-\alpha)$ consists in the tail index of the firm size distribution $\zeta$ and the size-volatility elasticity $\alpha$. If $\zeta' = 1$,*

$$\sigma_Y = \mu \frac{S_{\max}}{Y} \overline{\sigma} \frac{\pi}{\sqrt{6}}, \tag{3.7}$$

*where $\overline{\sigma}$ is a representative volatility.*

**Proof**. See Appendix A.1. ■

According to equation (3.7), when the firm size distribution is Zipf (1949) (namely, the tail index $\zeta$ is equal to 1) and the weak form of Gibrat's (1931) law for variances holds (namely, the elasticity $\alpha$ is equal to 0), the volatility of GDP growth caused by idiosyncratic shocks alone is determined by the following firm dynamics variables: productivity multiplier, Domar weight of the largest firm and representative volatility. On the other hand, when deviations from Zipf law (i.e., $\zeta \in \big(1, 2\big)$) and/or Gibrat law (i.e., $\alpha \in \big(0, 1/2\big]$) exist, equation (3.6) shows that the volatility of GDP growth is driven instead by the following variables: productivity multiplier, Domar weight and volatility of the largest firm, number of firms in the economy, tail index and size-volatility elasticity.

We follow Gabaix (2011) and quantify the contribution of idiosyncratic shocks to the volatility

[10] Some exceptions are Hall (1987) and Haltiwanger et al. (2013), who find that Gibrat's law holds for large firms, as deviations observed in the data are attributable to the dynamics of small entrants. Yeh (2021) estimates the relationship using the universe of U.S. firms and finds a strong size-variance relationship even when excluding entrant firms.

of GDP growth using the $R^2$ statistic. If $\zeta' \neq 1$, then

$$R^2 = \mu^2 \left(\frac{S_{\max}}{Y}\right)^2 \left(\frac{\sigma_{\max}}{\sigma_y}\right)^2 \frac{2}{2-\zeta'} \left[1 - \Gamma\left(2/\zeta'\right) n^{1-2/\zeta'}\right]. \tag{3.8}$$

Given the large number of firms that populate an economy, the contribution exhibits an upper bound:

$$\mathcal{A} = \mu^2 \left(\frac{S_{\max}}{Y}\right)^2 \left(\frac{\sigma_{\max}}{\sigma_Y}\right)^2 \frac{2}{2-\zeta'} \tag{3.9}$$

According to equation (4.28), $\mathcal{A}$ increases when any of the following changes take place: the share of economic activity commanded by the largest firm increases, the idiosyncratic volatility of the largest firm with respect to GDP volatility increases, the firm size distribution becomes more homogeneous (i.e., $\zeta$ increases) and the elasticity of volatility to size (i.e., $\alpha$ increases). Note that changes in $\zeta$ impact on $S_{\max}$ and, in turn, on $\sigma_{\max}$. Appendix A.2 discusses how they are related.

If $\zeta' = 1$, then the contribution is

$$R^2 = \mu^2 \left(\frac{S_{\max}}{Y}\right)^2 \left(\frac{\overline{\sigma}}{\sigma_Y}\right)^2 \frac{\pi^2}{6}, \tag{3.10}$$

which coincides with the asymptotic contribution (i.e., $R^2 = \mathcal{A}$). Equation (3.10) represents the maximum contribution of idiosyncratic shocks to GDP volatility because the size of the largest firm in a Zipf distribution is larger than in a power law with tail index greater than 1 (Newman, 2005) and the average volatility is larger than the volatility of the largest firm (see, e.g., Comin and Philippon (2005), Comin and Mulani (2006)).

As mentioned above, the literature (and also our estimates in Section 3.3) shows that there are deviations from Zipf and Gibrat laws in data. Thus, throughout this paper, we characterize GDP volatility using equation (3.6). Interestingly, this characterization suggests to channels through which the contribution decreases: sizes become more homogeneous and volatility is less elastic to changes in size. To which of the two channels the contribution is more sensitive is answered by the following proposition.

**Proposition 2.** *Let $\delta$ be deviations from Zipf's law baseline case, i.e., tail index of the firm size distribution is $\zeta = 1 + \delta$. Then, the excess of sensitivity of the contribution to changes in the size-volatility elasticity with respect to changes in the tail index is*

$$\frac{\partial R^2/\partial \alpha}{\partial R^2/\partial \delta} = \zeta' \geq 1. \tag{3.11}$$

**Proof**. See Appendix A.2. ■

The role of the size-volatility relationship in shaping aggregate fluctuations has been omitted in the granularity literature, as leading modeling assumption is that Gibrat's law holds (Gabaix, 2011, Carvalho and Grassi, 2019). Equation (3.11) extends Yeh's (2021) results by showing that deviations from Gibrat's law, not only attenuate significantly the impact of the granular contribution, but also that thay play an even more important role than changes in the cross-sectional dispersion in firm size, which is the main object of study in the literature.

## 3.3 Data and measurement

In this section, we present the data set on which the results of this work are based and the procedure followed to estimate the parameters that drive aggregate volatility. Our findings are as follows. First, the time average aggregate productivity multiplier is well approximated by its micro-founded version. Second, the Domar weight of the largest has remained rather stable through time. Third, in line with the bulk of the literature presented above, the upper tail of the firm size distribution is well characterized by a power law. Fourth, the size-volatility relationship estimated for the top 1000 firms follows a power law.

### 3.3.1 Data and Summary Statistics

Firm-level data come from SABI (*Sistema de Análisis de Balances Ibéricos*) database. The database is compiled by Bureau Van Dijk Electronic Publishing (BvD). SABI includes information on both listed and unlisted Spanish firms collected from various sources, such as national registers and annual reports. The fact that the data set provides information on unlisted firms is a crucial to avoid strong selection bias, as some of the largest Spanish firms are privately held. The main variables used in the analysis are net sales and number of employees for each firm. The time period is 1994-2018. During this lapse of time, the Spanish economy experienced a rapid economic growth, followed by a double recession (2008:II-2009:IV and 2010IV-2013:II).[11,12]

Given our focus on large firm dynamics, we build our dataset using the largest 200,000 Spanish firms in SABI. We attenuate the impact of exogenous shocks by excluding those firms that are engaged in oil, oil-related and energy activities because their sales come mostly from worldwide commodity prices, rather than real productivity shocks. We also exclude financial and public firms because their sales do not mesh well with the meaning used.[13] Recently, Cravino and Levchenko (2017) and di Giovanni et al. (2018, 2020) find evidence suggesting that foreign shocks are transmitted to the domestic economy through the largest firms and its affiliates. We mitigate the impact of foreign shocks by restricting the sample to those firms whose "global ultimate owner" is based in Spain.[14] We use unconsolidated sales denominated in euros, since sales that are consolidated across the multiple firms that comprise the corporation overestimate the impact of

[11] See Fernandez-Villaverde et al. (2013) and Royo (2013), respectively, for a detailed explanation of the causes and consequences of the economic boom in Spain.

[12] Recession dates are taken from *Asociación Española de Economía* (AEE).

[13] Firms are filtered our using the four-digit SIC primary code. See Appendix C in Gabaix (2011).

[14] A more suitable approach would be to retain those firms whose headquarters are located in Spain. Unfortunately, SABI does not provide this information. We use the global ultimate owner to identify whether the firm is a parent or an affiliate. In the case of individuals and families, the country reported is the country of residence. In the case of firms, it is the country where the firm is based.

**Table 3.1.** Summary statistics.

| | Weight | Sales | Employees | Productivity |
|---|---|---|---|---|
| Average aggregate growth rate | | 0.065 | 0.063 | 0.069 |
| Average individual growth rate | | 0.126 | 0.074 | 0.030 |
| | | Standard deviation of growth rate | | |
| Sample | 0.367 | 0.415 | 0.323 | 0.429 |
| 0 - 20 size percentile | $1.39{\times}10^{-6}$ | 0.694 | 0.427 | 0.681 |
| 21 - 40 size percentile | 0.001 | 0.328 | 0.307 | 0.388 |
| 41 - 60 size percentile | 0.008 | 0.352 | 0.299 | 0.387 |
| 61 - 80 size percentile | 0.040 | 0.306 | 0.252 | 0.321 |
| 81 - 100 size percentile | 0.317 | 0.282 | 0.221 | 0.292 |
| Top 1000 | 0.229 | 0.296 | 0.227 | 0.305 |
| Top 100 | 0.124 | 0.340 | 0.252 | 0.363 |
| Top 10 | 0.057 | 0.276 | 0.223 | 0.316 |
| Average $\sqrt{\mathcal{H}}$ | | 0.065 | 0.068 | |

**Notes**: "Weight" refers to the sum of the Domar weights. "Productivity" refers to labor productivity proxied by the log of the sales per employee ratio, as in Gabaix (2011). "Standard deviation of growth rate" reports the time average standard deviation of growth rates within a percentile category. $\mathcal{H}$ is the Herfindahl index of the total firm shares.

multinational firms and do not provide a reliable picture of the evolution of large firms (Gutiérrez and Philippon, 2019).[15] The resulting sample comprises the top 75,000 Spanish firms.

SABI, as well as other BvD products,[16] has a low coverage for years previous to 1995 and a reporting lag of roughly two years. This particularly affects the years 1994 and 2018 in our sample. We try to overcome these limitations by interpolating missing values with a maximum gap of two consecutive periods. This procedure does not change our conclusions and allows us to increase the representativeness of the sample substantially.[17]

The contant GDP expressed in 2015 euros and GDP deflator come from the OECD's *National Accounts Statistics* (SNA) database (OECD, 2020a). GDP per capita is calculated using total population coming from OECD's SNA database (OECD, 2020b). Total factor productivity (index 100 in 2015) is obtained from the Bank of Spain.[18]

Table 3.1 presents summary statistics for firm-level growth rates for the whole sample. The average growth rate of aggregate sales and employees is lower than the unweighted average of

[15]In particular, we downloaded companies with unconsolidated accounts only (consolidation code U1) and companies that present both consolidated and unconsolidated accounts (consolidation code C2/U2).

[16]Kalemli-Ozcan et al. (2015) discuss in detail how to use ORBIS and AMADEUS (the Global and European supersets of SABI, repectively) to construct representative firm-level datasets.

[17]Figure A.3 shows the number of firms affected by the linear interpolation procedure through time and the number of firms with valid observations.

[18]The time series can be found in the summary indicators table "Structural Indicators of the Spanish economy and of the European Union" (Table 1.4).

firm-level growth rate. The reasoning is because smaller firms tend to grow faster than larger firms, conditional on survival. On the contrary, firm-level productivity, which is defined as the log of the sales per employee ratio (see Section 3.3.2), in smaller firms tend to grow slower than larger firms. This is to be expected, as smaller firms are, on average, less efficient than larger firms (Taymaz, 2005). The table also reports the sum of Domar weights and the averages of firm volatility, measured by the standard deviation, for each size quintile. The results show that exist a high degree of heterogeneity and that smaller firms are more volatile than large firms. Finally, the square root of the Herfindahl index of sales and employees shares have an order of magnitude consistent with that reported by Gabaix (2011) and suggest that the economy is "granular".

### 3.3.2 Idiosyncratic Shocks

Following Gabaix (2011), we focus on the labor productivity shocks.[19] We proxy firm-level labor productivity using the log of its sales per worker ratio: $z_{it} := \text{Sales}_{it}/\text{Employees}_{it}$.[20] The growth rate is then defined simply as $g_{it} = \Delta \ln z_{it}$, where $\Delta$ denotes the difference between years $t$ and $t-1$. Firm-level growth rates are computed using only firms present in the dataset in both years, so that it captures the *intensive margin* growth rates.[21] Suppose that innovations to $g_{it}$ evolve according to the following one-factor model: $g_{it} = \eta_t + \varepsilon_{it}$, where $\eta_t$ is a common shock and $\varepsilon_{it}$ is an idiosyncratic shock. We make the identification assumption that $\mathbb{E}\left[\eta_t \varepsilon_{it}\right] = 0$. Firm $i$'s labor productivity idiosyncratic shock in year $t$ can be estimated as the deviation of its growth rate from the common shock to the top $Q$ firms:

$$\varepsilon_{it}\left(Q\right) = g_{it} - \eta_t\left(Q\right). \tag{3.12}$$

Thus, $\varepsilon_{it}$ captures the residual unexplained by the common shock. This approach to identifying firm-specific shocks is standard in macroeconomics (see, e.g., Koren and Tenreyro (2007), Gabaix (2011) and di Giovanni et al. (2014)).

To estimate equation (3.12), we first need to estimate the common shock $\eta_t\left(Q\right)$. Since our goal is to assess the contribution of idiosyncratic shocks to the largest firms to aggregate fluctuations, we restrict our attention to the top $Q = 1,000$ firms, as in Gabaix's (2011) robustness exercise. This choice is based on the fact that the one-factor model employed to extract $\varepsilon_{it}$ implicitly assumes a certain degree of homogeneity among firms, which is likely to be a less good approximation for a large $Q$. The growth rate of productivity is expected to depend on firm characteristics and factors

[19] Gnocato and Rondinelli (2018) estimate the granular residual with labor productivity shocks and firm-level TFP shocks. They show that both proxies for productivity shocks are highly correlated. See also Syverson (2004).

[20] We use this revenue-based productivity measure because it is not data intensive and is widely used in the literature. An important caveat is that it confounds idiosyncratic demand and factor price affects with efficiency differences (Foster et al., 2008). Therefore, it is not a clear measure of productivity shock, as it would be a measure based on quantities of physical output. Empirically, however, both measures are strongly correlated (see Foster et al. (2008)).

[21] In SABI, the *extensive margin* of entry and exit of firms cannot be calculated because it cannot be distinguished whether the newly observed firms are a genuine entry or an entry into the database. Using the universe of French firms, di Giovanni et al. (2014) show that the extensive margin plays no role in shaping aggregate fluctuations. Osotimehin (2019) finds that it contributes little to the variability of French aggregate productivity.

which, in turn, depend on size. If we consider a large number of firms with very heterogeneous size, firm characteristics can also be very heterogeneous and thus the implicit assumption $\eta_{it} = \eta_t \ \forall i$ may be a poor approximation. With this in mind, Section 3.4.3 shows the robustness of the results to alternative $Q$s. Once the number of potentially granular firms is set, we estimate the common shock to the top $Q$ as the cross-sectional median productivity growth rate, as in Blanco-Arroyo et al. (2018). Given that the time dimension is somewhat limited and that the Great Recession was particularly severe in Spain, the median growth rate seems a more suitable estimate of the common shock that hit the largest firms during these years.[22]

The dataset contains some large outliers, which may be due to mergers, acquisitions or simply measurement errors. We follow the convention in the literature and mitigate their impact by *winsorizing* extreme shocks at 50%.[23] Recently, the winsoring procedure to handle extreme values and outliers has been criticized by Dosi et al. (2018), who argue that it is not necessary when analyzing granularity because large firms have more accurate accounting information and, therefore, do not suffer from large jumps. In addition, they show that Gabaix's (2011) results are heavily influenced by such cleaning procedure. Taking into account Dosi et al.'s (2018) critique, Appendix A, Section A.3, re-estimates the idiosyncratic shocks using the arc-elasticity proposed by Davis et al. (1996). The main advantage of this measure is that it allows us to avoid any winsorizing or trimming procedure. We show that our results do not depend on the definition of the productivity growth rate or the data cleaning strategy.

### 3.3.3 Productivity Multiplier

The frameworks set up by Gabaix (2009a) and Carvalho and Gabaix (2013), among many others, predict that GDP growth volatility is proportional to TFP growth by a factor $\mu$ that represents the productivity multiplier (see equation (3.2)). Therefore, $\mu$ can be directly estimated by the following *relative standard deviations*:

$$\mu = \sigma_Y / \sigma_\Lambda, \tag{3.13}$$

where $\sigma_Y$ and $\sigma_\Lambda$ are the standard deviation of GDP per capita growth and TFP growth, respectively. According to equation (3.13), the estimated multiplier over the period 1994-2018 is 2.92. However, $\mu$ is expected to change through time, that is:

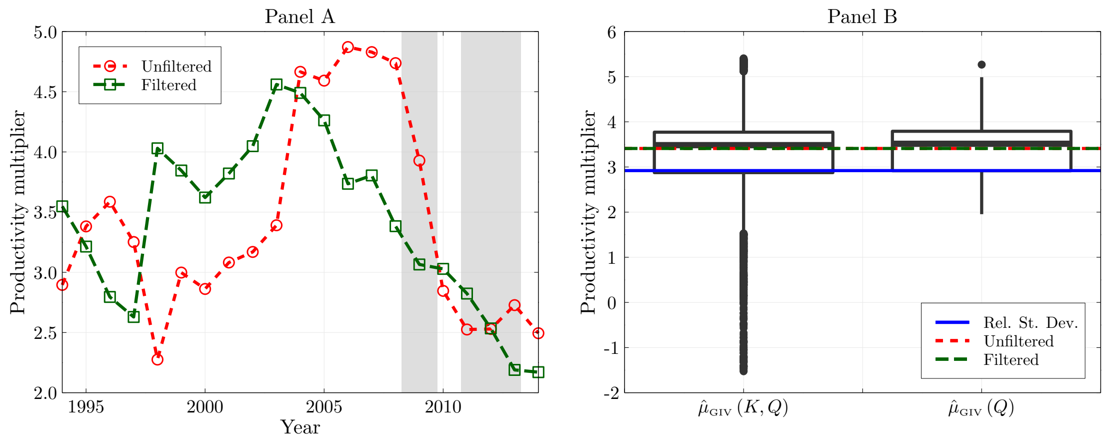
$$\mu_t = \sigma_{Yt} / \sigma_{\Lambda t}. \tag{3.14}$$

We compute the relative standard deviations at year $t$ using a centered rolling window of 10 years. Alternatively, we obtain deviations from the Hodrick-Prescott trend of log GDP per capita and log TFP using a smoothing parameter 6.25 and compute the rolling window. Panel A in Figure 3.1

[22] The conclusions reached in the present paper remain unchanged if we use the mean growth rate to estimate the common shock instead.

[23] More precisely, we set $\hat{\varepsilon}_{it} = \operatorname{sign}(\hat{\varepsilon}_{it})\, 0.5$ if $|\hat{\varepsilon}_{it}| > 0.5$. The winsorizing procedure affects 5% of the top $Q$ Spanish firms in the time period 1995-2018. Results are not materially sensitive to the choice of that threshold.

**Figure 3.1.** Productivity multiplier.




**Notes**: Panel A shows the productivity multiplier $\mu_t = \sigma_{Yt}/\sigma_{\Lambda t}$, computed using a centered rolling window of 10 years. "Unfiltered" computes the standard deviation of growth rates. "Filtered" computes the standard deviation of deviations from the Hodrick-Prescott trend of the time series in logs (the smoothing parameter is 6.25). Shaded lines indicate recession dates, defined using the Spanish Economic Association data. Panel B shows the relative standard deviations calculated using the time period 1994-2018, the average "unfiltered" productivity multiplier and the average "filtered" productivity multiplier. $\hat{\mu}_{\mathrm{GIV}}$ refers to the estimated coefficient in equation (3.15) when the granular residual (3.20) is calculated using the top $K = 1, 2, \ldots, Q$ firms and $K = Q$, where $Q = 500, 1000, \ldots, 10,000$.

shows that the multiplier exhibits a clear cyclical behavior. The time average is equal to 3.41 in the two cases.

Additionally, we use the "granular" instrumental variable (GIV) methodology proposed by Gabaix and Koijen (2020) to estimate the productivity multiplier. Our IV is the granular residual (3.20), which is constructed using the estimated shocks from equation (3.12). The granular residual is a consistent and powerful IV because shocks are idiosyncratic and the firm size distribution presents a high degree of heterogeneity (see Table 3.1 and Section 3.3.4). We run the following ordinary least squares (OLS) regression:

$$g_{Yt} = \text{constant}\,(K) + \mu_{\mathrm{GIV}}\,(K)\,\mathcal{E}_t\,(K) + u_t\,(K)\,, \tag{3.15}$$

for $K = 1, 2, \ldots, Q$, where $Q = 500, 1000, \ldots, 10,000$, $g_{Yt}$ is the growth rate of GDP per capita, $\mathcal{E}_t$ is the granular residual and $u_t$ is the error term. We estimate the productivity multiplier $\mu$ as the coefficient on the GIV $\mathcal{E}_t$. Equation (3.15) is also estimated by Gabaix (2011) to quantify the contribution of the idiosyncratic shocks to the top 100 U.S. firms (i.e., $K = 100$) to GDP growth fluctuations.

Panel B in Figure 3.1 shows the estimated productivity multiplier $\hat{\mu}_{\mathrm{GIV}}$. We find that the median value (3.48) is almost identical to the time average multiplier estimated using (3.14). The multiplier estimated by equation (3.13) can be seen as a lower bound. As the box plot of $\hat{\mu}_{\mathrm{GIV}}\,(Q)$ renders clear, outliers are produced when the granular residual is constructed with a small number of firms, (i.e., small $K$).

In order to simplify the analysis and to be able to clearly identify the contribution of idiosyncratic shocks to aggregate fluctuations, in what follows, we follow the model presented in Section 3.2.1 and assume $\mu$ is constant thought time. Furthermore, we assume that $\mu$ does not depend on the number of large firms $K$. In line with the estimation provided by Blanco-Arroyo et al. (2018), we set $\mu = 3.5\ \forall t, K$.

### 3.3.4 Firm Size Distribution

The CCDF (3.4) implies that the probability of the largest firm $\left(S_{\min}/S_{\max}\right)^{\zeta}$ has a frequency $1/n_{\text{tail}}$, where $n_{\text{tail}}$ is the number of firms whose volume of sales is above the threshold for which the power law behavior holds (i.e., $S_i \geq S_{\min}$). Thus, the size of the largest firm is $S_{\max} = n_{\text{tail}}^{1/\zeta} S_{\min}$. Likewise, the size of the $i$th largest firm is approximately $S_i = \left(n_{\text{tail}}/i\right)^{1/\zeta} S_{\min}$ (see Newman (2005) for a rigorous proof). Taking logs and rearranging, the "Zipf" plot for the power law distribution is characterized by

$$\ln i = c - \zeta \ln S_i, \tag{3.16}$$

where $i$ is the rank of firm $i$ and $c \equiv \ln n_{\text{tail}} + \zeta \ln S_{\min}$. According to equation (3.16), if the upper tail of the firm size distribution is power law, then the log-log plot should display a straight line.

A popular way to estimate the tail index $\zeta$ is to run an OLS using equation (3.16) as the econometric specification. However, Gabaix and Ibragimov (2011) show that this method, known as "log-log rank-size regression", delivers strongly biased estimates in small samples, and suggest the following modification:

$$\ln\left(i - 1/2\right) = c - \hat{\zeta}_{\text{OLS}} \ln S_i + u_i, \tag{3.17}$$

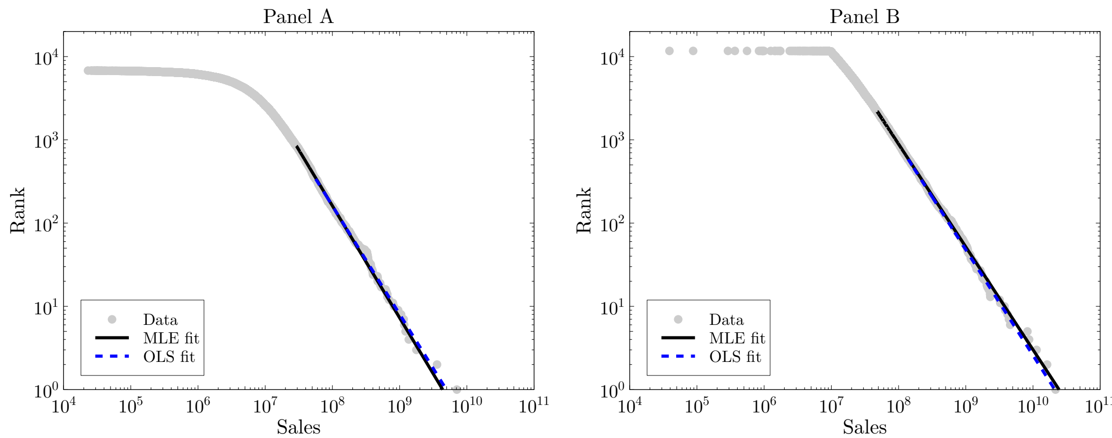
with asymptotic standard error $\hat{\zeta}^{\text{OLS}}\sqrt{2/n_{\text{tail}}}$. We estimate specification (3.17) using two different cut-off points. First, we take the tail that corresponds to 5% of the samples in each year. Note that the size of the tail is arbitrarily chosen following the literature.[24] Second, we take the tail that corresponds to those values of sales above the top $Q$ largest firm. That is, we set $S_{\min} = S_Q$, and, hence, $n_{\text{tail}} = Q$.

Although the log-log rank-size regression method is commonly used in the literature, it has numerous pitfalls (see Clauset et al. (2009) for a detailed explanation). As a cross-check, we also calculate the tail exponent from the density associated to the CCDF (3.4) by using *maximum likelihood estimation* (MLE). The estimator for $\zeta$ is

$$\hat{\zeta}_{\text{MLE}} = \hat{n}_{\text{tail}} \left( \sum_{i=1}^{\hat{n}_{\text{tail}}} \frac{S_i}{S_{\min}} \right)^{-1}, \tag{3.18}$$

[24] It is also a standard approach in the literature to determine the threshold through visual inspection of the empirical distribution. If the distribution has a truncation point, then the threshold is typically set equal to the truncation point. As yet another alternative, we use this approach and find that the estimates are very close to those using the 5% cut-off.

**Figure 3.2.** Firm size distribution.




**Notes**: Panel A and Panel B show the double logarithmic plot of rank against sales in year 1994 and 2018, respectively. "MLE fit" denotes the power-law fit from (3.17) when taking the tail that corresponds to those values above the endogenous threshold determined by Clauset et al.'s (2009) procedure. "OLS fit" denotes the power-law fit from (3.18) when taking the tail that corresponds to 5% of the sample.

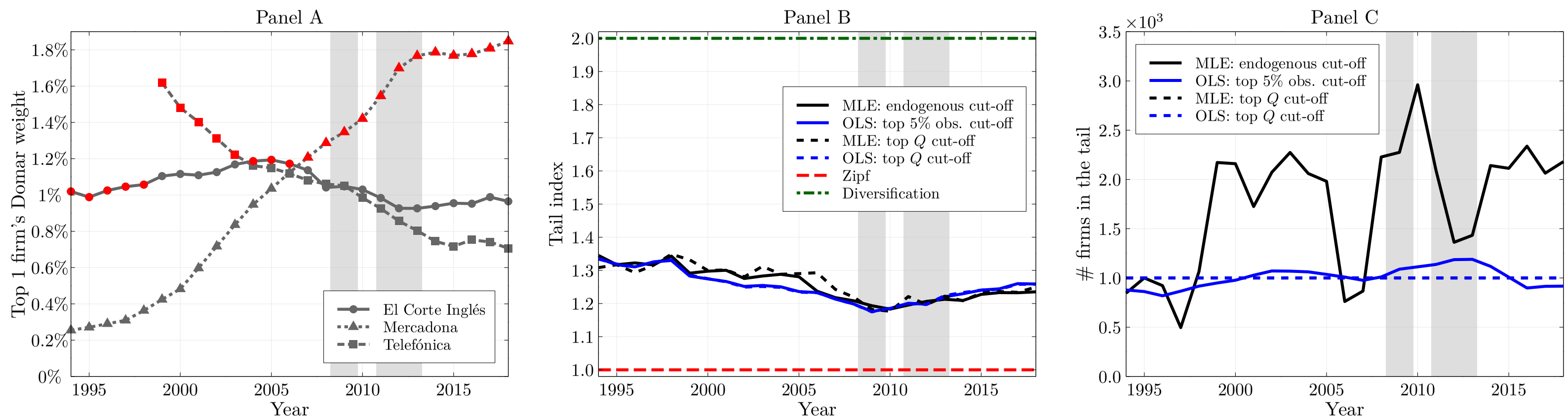
with standard error $\hat{\zeta}_{\text{MLE}}/\sqrt{\hat{n}_{\text{tail}}}$ (see Newman (2005)). We follow Clauset et al. (2009) and estimate $\hat{S}_{\text{min}}$ as the value of sales that minimizes the distance (measured by the Kolmogorov–Smirnov statistic) between the probability distribution of size and the best-fit power-law model above $\hat{S}_{\text{min}}$. Thereofore, $\hat{n}_{\text{tail}}$ is the number of firms whose sales are in the range $\left[\hat{S}_{\text{min}}, S_{\text{max}}\right]$. Additionally, we set $S_{\text{min}} = S_Q$.

Panel A and B in Figure 3.2 show the empirical firm size distribution in year 1994 and 2018, respectively. Particularly, we follow the intuition provided by equation (3.16) and plot the double logarithmic plot of rank vs. sales. The distribution is characterized by a truncation point and an upper tail that displays a straight line characteristic of the power law distribution. This visual identification is confirmed by the fits provided by equations (3.17) and (3.18). Panel B and Panel C in Figure 3.3 present, respectively, the estimates for the tail index $\zeta$ and the cut-offs used in the estimation through time. Despite the fact that the sample coverage grows over time (see Panel B in Figure A.3), the estimates exhibit an almost steady value equal to 1.255 and are not sensitive to the choice of the cut-off. The average tail index is closer to Zipf's (1949) law (i.e., $\zeta = 1$) than the diversification argument (i.e., $\zeta \geq 2$), which implies that the firm size distribution is sufficiently fat-tailed for idiosyncratic shocks to individual firms do not wash out at the aggregate level, because the idiosyncratic shocks to large firms do not cancel out with shocks to smaller firms (Gabaix, 2011).

As discussed by Mitzenmacher (2004) and Newman (2005), the log-normal distribution can behave as a power law.[25] As an alternative, we fit a log-normal distribution on the firm size

[25]See Saichev et al. (2009) for a lengthy discussion on the ongoing debate between power law and log-normal in firm size distribution.

**Figure 3.3.** Heterogeneity in firm size.




**Notes**: Panel A shows the Domar weight of those firms that alternate in the top 1. Observations in red denote the weight of the top 1 in year $t$. Panel B shows the estimates for the tail index $\zeta$ using maximum likelihood estimation (MLE) and ordinary least squares (OLS). "Endogenous cut-offs" takes the tail that corresponds to those values above the endogenous threshold determined by Clauset et al.'s (2009) procedure. "Top 5% observations cut-off" takes the tail that corresponds to 5% of the samples in each year. "Top $Q$ cut-off" takes the tail that corresponds to the top 1000 firms. Panel C shows the number of firms in the tail used in each estimation.

distribution using MLE. We impose the same cut-offs for these estimations as in the power law estimations and perform Vuong's (1989) *likelihood ratio test* $\mathcal{R}$ to compare the fits of both models.[26] The sign of $\mathcal{R}$ indicates which model is closer to the true model: if $\mathcal{R}$ is statistically greater than zero, then the test statistic presents evidence in favor of power-law model. Figure A.4 shows that the ratio alternates positive and negative values that are not statistically different from zero. Thus, we cannot conclude which candidate distribution provides a better fit. As argued in Section 3.2.2, we follow the bulk of the literature and assume that the underlying theoretical distribution is power law.

Finally, Panel A in Figure 3.3 shows the Domar weight of the largest firm through time. In our sample, three firms alternate in the top 1: El Corte Íngles (general merchandise store), Telefónica (communications) and Mercadona (food store). The fact that El Corte Íngles is the largest firm in our sample during the period 1994-1998 is consequence of the low coverage in SABI database, as discussed in Section 3.3.1. The reason behind the jump observed between years 1998 and 1999 is that Telefónica enters the sample 1999. In line with Gutiérrez and Philippon (2019), we find that the largest firm's Domar weight has not increased through time.[27] The relative size of Mercadona in 2018 is similar to that of Telefónica in 1999. As baseline, we assume that the average Domar weight of the largest firm (1.4%) captures the evolution in time. In Section 3.4.3, we relax this assumption in order to study how the granular size of the economy changes over the business cycle.

### 3.3.5 Idiosyncratic Shocks Volatility

We estimate the size-shock relationship using the methodology proposed by Koren and Tenreyro (2013), which allows for variation within firms.[28] The volatility of firm-level shocks $\sigma_{i\tau}$ is defined as the standard deviation of idiosyncratic shocks $\varepsilon_{it}$ to firm $i$ over a time block $\tau$. The measure of size $\tilde{S}_{i\tau}$ is the average normalized sales within $\tau$. Normalized sales are defined as $S_{it}/S_{\min,t}$, where $S_{\min,t}$ is the value of sales of the $Q$th firm in year $t$. To use every year in our sample, we calculate $\sigma_{i\tau}$ and $\tilde{S}_{i\tau}$ in a four-year time window. The sample is divided into 6 time blocks (i.e., $\tau = 6$). The econometric specification is

$$\ln \sigma_{i\tau} = \text{constant} + \alpha \ln \tilde{S}_{i\tau} + \varphi_\tau + \varphi_i + u_{i\tau}, \tag{3.19}$$

where $\varphi_\tau$ and $\varphi_i$ control for time blocks and firm fixed effects, respectively. Some firms enter and leave the top $Q$, so they have few observations per block. To reduce the estimated volatility, we only consider those firms that have at least 3 of the 4 years that constitute a block.

[26] We use the normalized log-likelihood ratio: $n_{\text{tail}}^{-1/2}\mathcal{R}/\sigma_{\mathcal{R}}$. The likelihood ratio is $\mathcal{R} = L(\theta_1|x)/L(\theta_2|x)$, where $L$ is the likelihood function and $\theta_1$ and $\theta_2$ are, respectively, a vector of parameters for the power-law model and log-normal model. The standard deviation associated to $\mathcal{R}$ is $\sigma_{\mathcal{R}}$.

[27] Gutiérrez and Philippon (2019) find that the top 20 U.S. firms have not become larger relative to the economy. We also calculate the Domar weight of the largest firm in the U.S. using cite Gabaix's (2011) data set from Compustat North America database and find that General Motors and Walmart alternate in the top 1. During the period 1951-2008, the relative size of the largest has not increased.

[28] See also Yeh (2021), who estimates more systematically the size-variance relationship using the universe of U.S. firms, and quantifies its impact on the explanatory power of the granular residual.

**Table 3.2.** Idiosyncratic shocks volatility and size.

| | $\ln \sigma_{i\tau}$ | |
|---|---|---|
| Constant | $-1.996^{***}$ | $-1.813^{***}$ |
| | (0.029) | (0.064) |
| $\ln \tilde{S}_{i\tau}$ | $-0.062^{**}$ | $-0.150^{***}$ |
| | (0.020) | (0.053) |
| $\varphi_\tau$ | ✓ | ✓ |
| $\varphi_i$ | | ✓ |
| Observations | 5,430 | 5,430 |
| $R^2$ | 0.027 | 0.552 |
| Number of clusters | | 1,861 |

**Notes**: The specifications use the four-year standard deviation of annual productivity shocks to the $Q = 1000$ largest firms in the time period 1995-2018. The size is computed at its mean value over the four-year window. Clustered (by firm) standard errors in parentheses. *, ** and *** indicate significance at 10% , 5% and 1%.

Table 3.2 shows that the estimated size-volatility elasticity is statistically different from zero. Therefore, there are clear deviations from Gibrat's law. When we include firms fixed effects, our estimate is within the range 0.1–0.25, previously estimated in the literature (see, e.g, Stanley et al. (1996), Sutton (2002), Koren and Tenreyro (2013), Yeh (2017), Calvino et al. (2018)).

As a robustness check, Appendix A.3 estimates the elasticity $\alpha$ following Stanley et al.'s (1996) cross-sectional methodology. For a given cross-section of idiosyncratic shocks and size binds, we calculate the standard deviation of shocks and average value of sales within each bin. Then, the elasticity is estimated by running log-log OLS regression of standard deviations on average size. The estimate is very similar to the baseline specification.

Finally, we need to estimate the value for $\sigma_{\max}$. For each firm, we average the standard deviation of shocks $\sigma_{i\tau}$ and normalized sales $\tilde{S}_{i\tau}$ used in the estimation of specification (3.19). As mentioned in Section 3.3.1, SABI has some limitations that makes firm-level volatility estimation quite volatile.[29] We choose $\sigma_{\max}$ to match a standard deviation of 11.6%, corresponding to that of average volatility of the top 30 firms in SABI. This number is comparable to the volatility of the growth rates of sales per employee ratio reported by Gabaix (2011) and is in agreement with previously reported estimates (Comin and Philippon, 2005, Davis et al., 2007, Foster et al., 2008, Haltiwanger, 2011, Bachmann and Bayer, 2014, Castro et al., 2015).

[29] As noted by Gabaix (2011), measuring firm volatility is also difficult because various frictions and identifying assumptions provide conflicting predictions about links between changes in total factor productivity and changes in observable quantities such as sales and employment.

## 3.4 Quantifying the Granular Size of the Economy

In this section, we use the conceptual framework introduced in Section 3.2 and the estimated parameters in Section 3.3 to characterize the behavior of the granular curve first observed by Blanco-Arroyo et al. (2018). In addition, we use our framework to propose a set of measures to quantify the granular size of the economy and find that approximately the top 50 Spanish firms are granular, i.e., idiosyncratic shocks to these firms may translate into aggregate fluctuations. We show that our results are robust to alternative $Q$s and time varying parameters. When we allow the parameters to change through time, we observe that the granular curve and, therefore, the number of granular firms changes with the business cycle. The average contribution of idiosyncratic shocks over the cycle coincides with that observed for the entire time period studied.

### 3.4.1 Granular Curve

Building on Hulten's (1978) result (see equation (3.1)), Gabaix (2011) constructs the "granular residual" $\mathcal{E}_t$, which is a parsimonious measure of the idiosyncratic shocks to the top $K$ firms:

$$\mathcal{E}_t = \sum_{i=1}^{K} \frac{S_{it-1}}{Y_{t-1}} \varepsilon_{it}, \tag{3.20}$$

where firm $i$'s idiosyncratic shocks $\varepsilon_{it}$ in year $t$ are estimated using equation (3.12). Gabaix (2011) and the empirical literature that followed estimate the contribution of the idiosyncratic shocks to the top $K$ firms to GDP fluctuations by regressing the growth rate of GDP $g_{Yt}$ on the granular residual. As noted by Blanco-Arroyo et al. (2018), the estimation is based on an exogenous choice for the number of large firms. Such "pointwise" estimation does not provide information on the extent of the granular size of the economy (i.e., those top firms whose idiosyncratic shocks may translate into aggregate fluctuations), as the number of firms is arbitrarily chosen. Therefore, the contribution of the granular term to the GDP fluctuations may underestimated or overestimated depending on the choice of $K$. Blanco-Arroyo et al. (2018) construct (3.20) for $K = 1, 2, \ldots, Q$ and evaluate the behavior of $R^2$ as $K \to Q$. We follow this approach and estimate the $R^2$ as

$$R^2(K) = \mu^2 \frac{\sigma^2_{\mathcal{E}}(K)}{\sigma^2_Y}, \tag{3.21}$$

where $\sigma^2_{\mathcal{E}}$ is the variance of the granular residual (3.20) and $\sigma^2_Y$ is the variance of the growth rate of GDP per capita. Note that the behavior of $R^2$ will reflect only changes in $\sigma^2_{\mathcal{E}}$, as the productivity multiplier $\mu$ is held constant through $K$ (i.e., $\mu(K) = \mu \ \forall K$). This choice is based on the stability of the multiplier to changes in $K$ (see Panel B in Figure 3.1).

As argued in Section 3.3, our baseline case assumes that firm $i$'s Domar weight and shock

**Table 3.3.** Parameters.

| Parameters | Description | Value |
|---|---|---|
| $\mu$ | Productivity multiplier | 3.5 |
| $S_{\max}/Y$ | Top 1 firm's Domar weight | 0.014 |
| $\sigma_{\max}$ | Top 1 firm's volatility | 0.116 |
| $\sigma_Y$ | GDP growth volatility | 0.023 |
| $\zeta$ | Tail index | 1.255 |
| $\alpha$ | Size-volatility elasticity | 0.150 |

**Notes**: $\mu$ is the average value of the fraction $\sigma_Y/\sigma_\Lambda$. $S_{\max}/Y$ is the average Domar weights of top 1 firm. $\sigma_{\max}$ is the average standard deviation of labor productivity shocks among the top 30 Spanish firms. $\sigma_Y$ is the standard deviation of GDP per capita growth. $\zeta$ is the average tail index.

volatility is constant through time. Hence, the variance of the granular residual is

$$\sigma_{\mathcal{E}}^2 = \sum_{i=1}^{K} \left(\frac{S_i}{Y}\right)^2 \sigma_i^2, \tag{3.22}$$

where $S_i/Y$ is the time average Domar weight and the variance of shock $\sigma_i^2$ is held constant through time. Section 3.3.4 shows that $S_i$ is well described by a power law distribution with exponent 1.255. Thus, we can use equation (3.8) to characterize the contribution of idiosyncratic shocks to aggregate fluctuations as $K \to Q$. Replacing the total number of firms in the economy $n$ by the largest $K$ firms, the explanatory power of the granular residual is

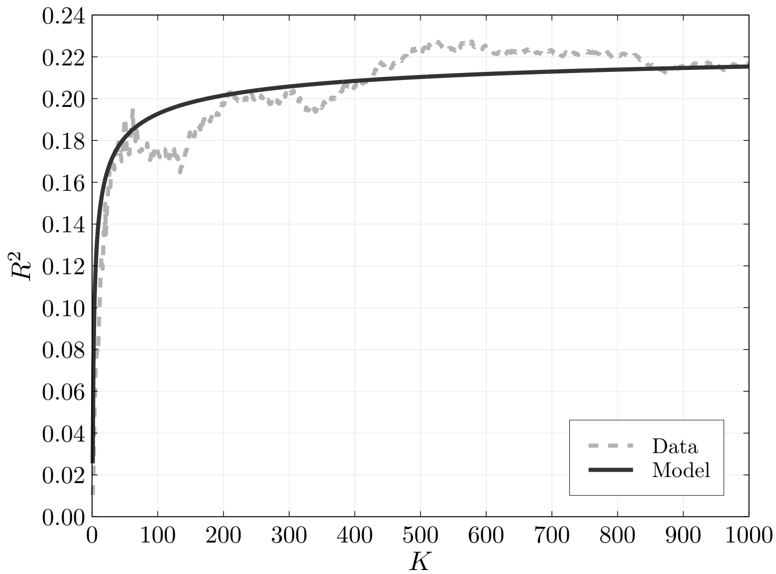
$$R^2(K) = \mu^2 \left(\frac{S_{\max}}{Y}\right)^2 \left(\frac{\sigma_{\max}}{\sigma_Y}\right)^2 \frac{2}{2-\zeta'} \left[1 - \Gamma\left(2/\zeta'\right) K^{1-2/\zeta'}\right]. \tag{3.23}$$

Figure 4.7 shows the behavior of the empirical explanatory power calculated (equation (3.21)). In line with Blanco-Arroyo et al. (2018), we observe the "granular curve" behavior: a rapid increase of $R^2$ when a small number of top firms are included in the granular residual and slow increase after a given number of firms. We also include the predicted behavior by equation (3.23) when we use the parameters estimated in Section 3.3 (see Table 3.3). As the figure renders clear, our framework is able to characterize the dynamics of the empirical explanatory power of the granular residual.

### 3.4.2 Granular Size Measurement

As shown in Figure 4.7, the model developed in Section 3.2 describes well the behavior of the contribution of idiosyncratic shocks to GDP growth fluctuation. Therefore, we can employ our model to provide a set of measures that quantify the granular size of the economy, i.e., how many granular firms populate the economy. We propose three measures based on the following three definitions of granular firms:

**Figure 3.4.** Granular curve.




**Notes**: "Data" refers to $R^2(K)$ calculated using equation (3.21) and "Model" refers to $R^2(K)$ calculated using equation (3.23). The parameters used to calculate (3.23) are presented in Table 3.3.

1. Those firms whose marginal contribution is above a constant contribution.

2. Those firms that account for 75% of the maximum granular contribution.

3. Those firms whose marginal contribution is above the marginal contribution in the equally-weighted firms scenario.

To grasp the intuition of definition 1, let us focus on the top $Q = 1000$ and consider a constant contribution between the largest firm and $Q$. This constant contribution is captured by the secant between firm 1 and $Q$, which is given by

$$\mathcal{M} = \frac{R^2\left(Q\right) - R^2\left(1\right)}{Q-1},$$

where $R^2$ is determined by equation (3.23). Definition 1 seeks to find the number of firms whose marginal contribution to aggregate fluctuations is above the secant, that is: $\partial R^2\left(K\right)/\partial K = \mathcal{M}$. This is none other than the *mean value theorem*, which states that for a given planar arc between two endpoints, there is at least one point at which the tangent to the arc is parallel to the secant through its endpoints. According to Definition 1, the number of granular firms is given by

$$K^*_{\mathcal{M}} = \left[\frac{Q}{1-Q^{1-2/\zeta'}}\left(\frac{2}{\zeta'}-1\right)\right]^{\zeta'/2}. \tag{3.24}$$

Using the estimated parameters presented in Table 3.3, we find that $K^*_{\mathcal{M}} = 82$. The drawback of this measure is its dependence on the number of firms used to compute the granular curve $Q$.

Definition 2 relies on the existence of a maximum granular contribution when the underlying theoretical distribution of firm size is power law (see equation (4.28)). The parameters presented in Table 3.3 give a maximum granular contribution of $\mathcal{A} = 23\%$. We propose an arbitrarily chosen threshold of 75% of this value. Thus, the granular firms are those firms whose accumulated contribution is equal to 17.25%. According to Definition 2 the number of granular firms is given by the following expression

$$K^*_{\mathcal{A}} = \left[\frac{1-\mathcal{T}}{\Gamma\left(2/\zeta'\right)}\right]^{\zeta'/(\zeta'-2)}, \tag{3.25}$$

which is determined by the equation $R^2\left(K\right)/\mathcal{A} = \mathcal{T}$, where $\mathcal{T}$ is set to 0.75. We find that $K^*_{\mathcal{A}} = 36$. The drawback of this measure is the fact that depends on the exogenous threshold $\mathcal{T}$.

In the spirit of Blanco-Arroyo et al. (2018), Definition 3 uses the counterfactual in which all firms are of equal size. Let us assume a representative firm size for all firms ($S_i = \overline{S}\ \forall i$, where $\overline{S}$ is the representative size). According to equation (3.5), the volatility of shocks is identical across firms (i.e., $\sigma_i = \overline{\sigma}\ \forall i$, where $\overline{\sigma}$ is the representative volatility). In this scenario, GDP growth volatility (3.3) becomes

$$\sigma_Y = \mu\frac{\overline{S}}{Y}\overline{\sigma}\sqrt{n}.$$

Assume that the economy is made only up of the top $Q$ (i.e., $n = Q$), the representative size across the top $Q$ firms necessary to match the empirical $\sigma_Y$ is given by

$$\overline{S} = \frac{1}{\mu}\frac{\sigma_Y}{\overline{\sigma}}\frac{Y}{\sqrt{Q}}.$$

Plugging this size into the predicted explanatory power of the equal-weight scenario (i.e., $R^2 = \mu^2\left(\overline{S}/Y\right)^2\left(\overline{\sigma}/\sigma_Y\right)^2 K$) we find that, when all firms are of equal size, the explanatory power is simply the number of top $K$ firms to total number $Q$ of top firms ratio:

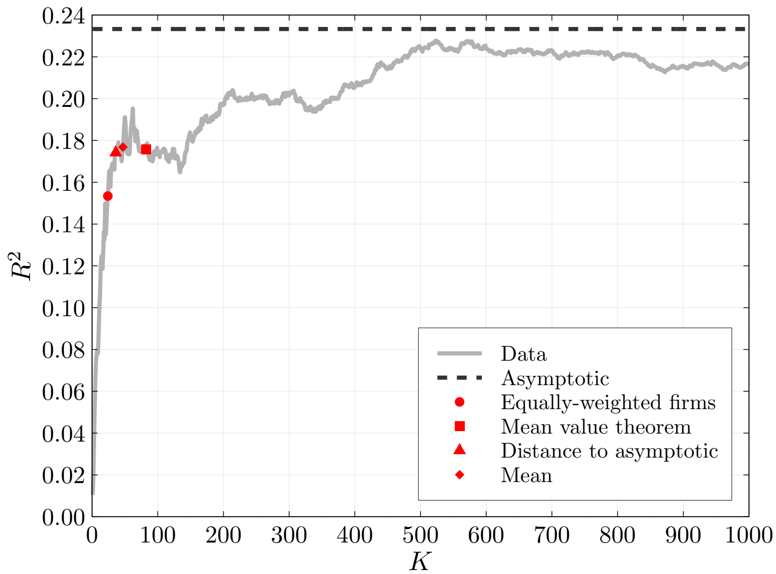
$$R^2_{\overline{S}}\left(K\right) = \frac{K}{Q},$$

where $K = 1, 2, \ldots, Q$. Definition 3 seeks to find the number of firms whose marginal contribution is above the marginal contribution in the equal-weight counterfactual. That is, $\partial R^2\left(K\right)/\partial K = \partial R^2_{\overline{S}}\left(K\right)/\partial K$. According to Definition 3, the number of granular firms is given by

$$K^*_{\overline{S}} = \left(\mu\frac{S_{\max}}{Y}\frac{\sigma_{\max}}{\sigma_Y}\right)^{\zeta'}\left[Q\Gamma\left(\frac{2}{\zeta'}+1\right)\right]^{\zeta'/2}. \tag{3.26}$$

We find that $K^*_{\overline{S}} = 24$. As in Definition 1, the drawback of this measure is its dependence on the number of firms used to compute the granular curve $Q$.

**Figure 3.5.** Granular size of the economy.




**Notes**: "Data" refers to $R^2(K)$ calculated using equation (3.21). "Asymptotic" refers to the maximum contribution of idiosyncratic shocks to the volatility of GDP per capita growth (see equation (4.28)). "Equally-weighted firms" refers to equation (4.29). "Mean value theorem" refers to equation (4.26). "Distance to asymptotic" refers to equation (4.27). "Mean" refers to the value resulting from averaging the three measurements.

Figure 3.5 shows the empirical behavior of the explanatory power of the granular residual, the maximum contribution of idiosyncratic shocks to GDP growth volatility and the calibrated granular size for each definition above. We also include the mean value of the three measures ($K^* = 47$), which appears to be closer to the point that visually represents the change from the granular to the atomistic regime. Therefore, we conclude that the granular size of the Spanish economy is approximately 50 firms. In other words, if the largest 50 firms did not exist, the Spanish economy would not be granular.

A potential concern with our baseline calibration is that two out of the three proposed measures depend on the number of firms $Q$. To address this concern, in Section 3.4.3, we increase $Q$ from 1000 to 2500 in steps of 500 and re-estimate the granular size of the economy. We find that the mean value of the measurements is still within the region that we visually identify as the regime change

Blanco-Arroyo et al. (2018) propose a methodology to calibrate the number of granular firms that consists in gradually replacing the top firms by smaller firms. In each replacement, we compute the empirical explanatory of the granular residual as the number of top firms increases. We observe that the average explanatory power gradually decreases until it converges to a benchmark in which all firms are equally weighted. The number of granular is then the point of convergence, which is approximately 450. However, the decrease in the average explanatory power is not constant.

We identify a "inner granular structure" around firm 50 that is left unexplained. The measures proposed in the present work indicate that the number of granular firms previously estimated by Blanco-Arroyo et al. (2018) is too conservative and it is the inner granular structure which determines the granular size of the economy. Hence, this result indicates that we should not consider the transition phase between granular and atomistic regime when quantifying the number of large firms whose idiosyncratic shocks have a non-negligible impact on the aggregate fluctuations.

### 3.4.3 Extensions

**Alternative $Q$s**

As argued in Section 3.3.2, we focus on the top $Q = 1000$ firms because the homogeneity assumption used to estimate idiosyncratic shocks is likely to be a less good approximation when taking a large $Q$. We now assess the robustness of our results to alternative $Q$s. Specifically, we reestimate equation (3.12) for $Q = 1000, 1500, 2000, 2500$ and the contribution of idiosyncratic shocks to aggregate (3.21) as $K \to Q$. Regarding the approximation (3.23), the only parameters that potentially depend on $Q$ are the volatility of the largest firms $\sigma_{\max}$ and the size-volatility elasticity $\alpha$. We observe that $\sigma_{\max}$, computed as the average standard deviation of the top 30 firms remains unchanged as $Q$ grows large. Therefore, any changes are attributable to the elasticity $\alpha$. The estimation of $\alpha$ using the specification (3.19) is challenging because the introduction of a large number of small firms impacts heavily on the estimate. To attenuate this impact, we average firm $i$'s productivity shock volatility in log $\ln \sigma_{i\tau}$ and normalized sales in log $\ln \tilde{S}_{i\tau}$ over $\tau$ time blocks, and divide them into $\mathcal{B} = 25$ bins using the average normalized sales in log. Then, we compute the average volatility $\sigma_{\mathrm{B}}$ and size $\tilde{S}_{\mathrm{B}}$ within each bin B, with $\mathrm{B} = 1, \ldots, \mathcal{B}$. Finally, we use the following specification to estimate the size-volatility elasticity:

$$\sigma_{\mathrm{B}} = \text{constant} + \alpha \tilde{S}_{\mathrm{B}} + u_{\mathrm{B}}. \tag{3.27}$$

Table 3.4 shows the estimates when $Q$ increases from 1000 to 2500 in steps of 500 firms. The estimates are in line with those previously estimated by specification (3.19). We use these values to calibrate the elasticity that best captures the dynamics of the contribution of idiosyncratic shocks to large firms to aggregate fluctuation. We chose the following values: 0.15 for $Q = 1000$, 0.2 for $Q = 1500$, and 0.18 for $Q = 2000$ and $Q = 2500$. Recall that the rest of the parameters remain as presented in Table 3.3, as they do not depend on $Q$. Figure 3.6 shows that the behavior of $R^2$ is quite stable to changes in the number of large firms taken to estimate the idiosyncratic shocks. We also include the estimated granular size of the economy. In particular, we show the mean value of the three measure proposed in Section 3.4.2. These are $K^* = 47$ for $Q = 1000$, $K^* = 90$ for $Q = 1500$, $K^* = 85$ for $Q = 2000$ and $K^* = 97$ for $Q = 2500$. As expected, the estimated granular size grows as $Q$ grows large, but the estimated values remain within the region that we can visually identify as the change from the granular to the atomistic regime.

**Table 3.4.** Elasticity.

| | $\sigma_{\mathrm{B}}$ | | | |
|---|---|---|---|---|
| | $Q = 1000$ | $Q = 1500$ | $Q = 2000$ | $Q = 2500$ |
| Constant | $-1.933^{***}$ | $-1.882^{***}$ | $-1.885^{***}$ | $-1.889^{***}$ |
| | (0.088) | (0.125) | (0.078) | (0.100) |
| $\tilde{S}_{\mathrm{B}}$ | $-0.169^{***}$ | $-0.193^{***}$ | $-0.165^{***}$ | $-0.150^{***}$ |
| | (0.032) | (0.043) | (0.025) | (0.031) |
| Observations | 25 | 25 | 25 | 25 |
| $R^2$ | 0.585 | 0.506 | 0.677 | 0.535 |

**Notes**: The specifications use the average standard deviation and average normalized sales within each size bin. The number of observations corresponds to the number of bins. *, ** and *** indicate significance at 10% , 5% and 1%

### Time-Varying Parameters

Based on the stability displayed by the share of economic activity commanded by the largest firm and the cross-sectional dispersion of firm sizes through time, our baseline specification characterizes the granular curve behavior and quantifies the granular size of the economy assuming that the determinants remain constant during entire period. We now relax this assumption to study how idiosyncratic shocks to the large firms contribute to GDP growth volatility through time.

The empirical contribution in time window $\tau$ is

$$R_{\tau}^{2}(K) = \mu^{2} \frac{\sigma_{\mathcal{E}\tau}^{2}(K)}{\sigma_{Y\tau}^{2}}, \tag{3.28}$$

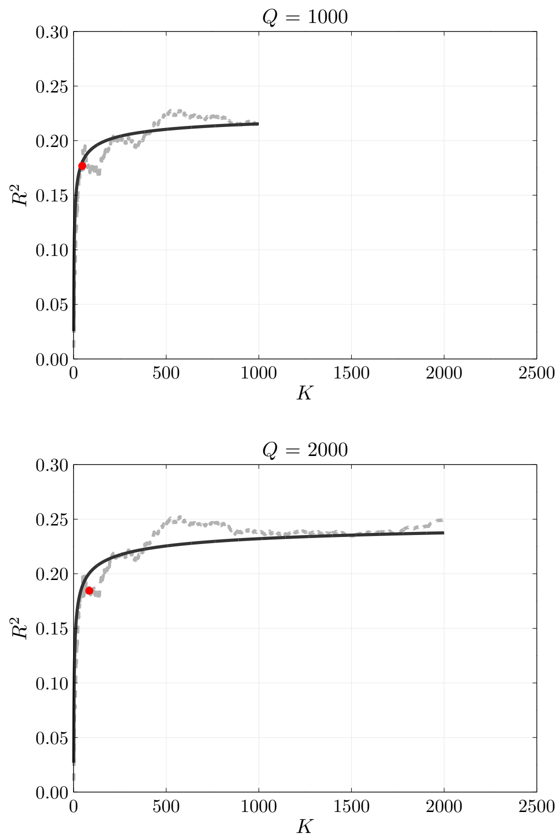
where $\sigma_{\mathcal{E}\tau}^{2}$ is the variance of the granular residual (3.20) in $\tau$ and $\sigma_{Y\tau}^{2}$ is the variance of the growth rate of GDP per capita in $\tau$. We follow Carvalho and Gabaix (2013) and chose $\tau = 10$ years. As in equation (3.21), the evolution of $R_{\tau}^{2}(K)$ will reflect only changes in the relative variance, as the productivity multiplier is held constant at 3.5 through time.

The approximation of contribution (3.28) is

$$R_{\tau}^{2}(K) = \mu^{2} \left(\frac{S_{\max,\tau}}{Y_{\tau}}\right)^{2} \left(\frac{\sigma_{\max,\tau}}{\sigma_{Y\tau}}\right)^{2} \frac{2}{2-\zeta_{\tau}'} \left[1 - \Gamma\left(2/\zeta_{\tau}'\right) K^{1-2/\zeta_{\tau}'}\right], \tag{3.29}$$

where $S_{\max,\tau}/Y_{\tau}$ is the average Domar weight in $\tau$, $\sigma_{\max,\tau}$ and $\sigma_{Y\tau}$ are, respectively, the volatility of shocks to large firms and growth rate of GDP per capita in $\tau$ and $\zeta_{\tau}' \equiv \zeta_{\tau}/(1-\alpha)$. We hold the elasticity $\alpha$ constant at 0.15 because of limited time dimension does not allow us to use Koren and Tenreyro's (2013) methodology. Finally, given the difficulty of measuring the volatility of the largest firms in the data, we choose $\sigma_{\max,\tau}$ that approximates the behavior of $R_{\tau}^{2}(K)$ in the time window $\tau$. The values chosen range between 0.062 and 0.155, which are still within the range of

**Figure 3.6.** Granular curves.

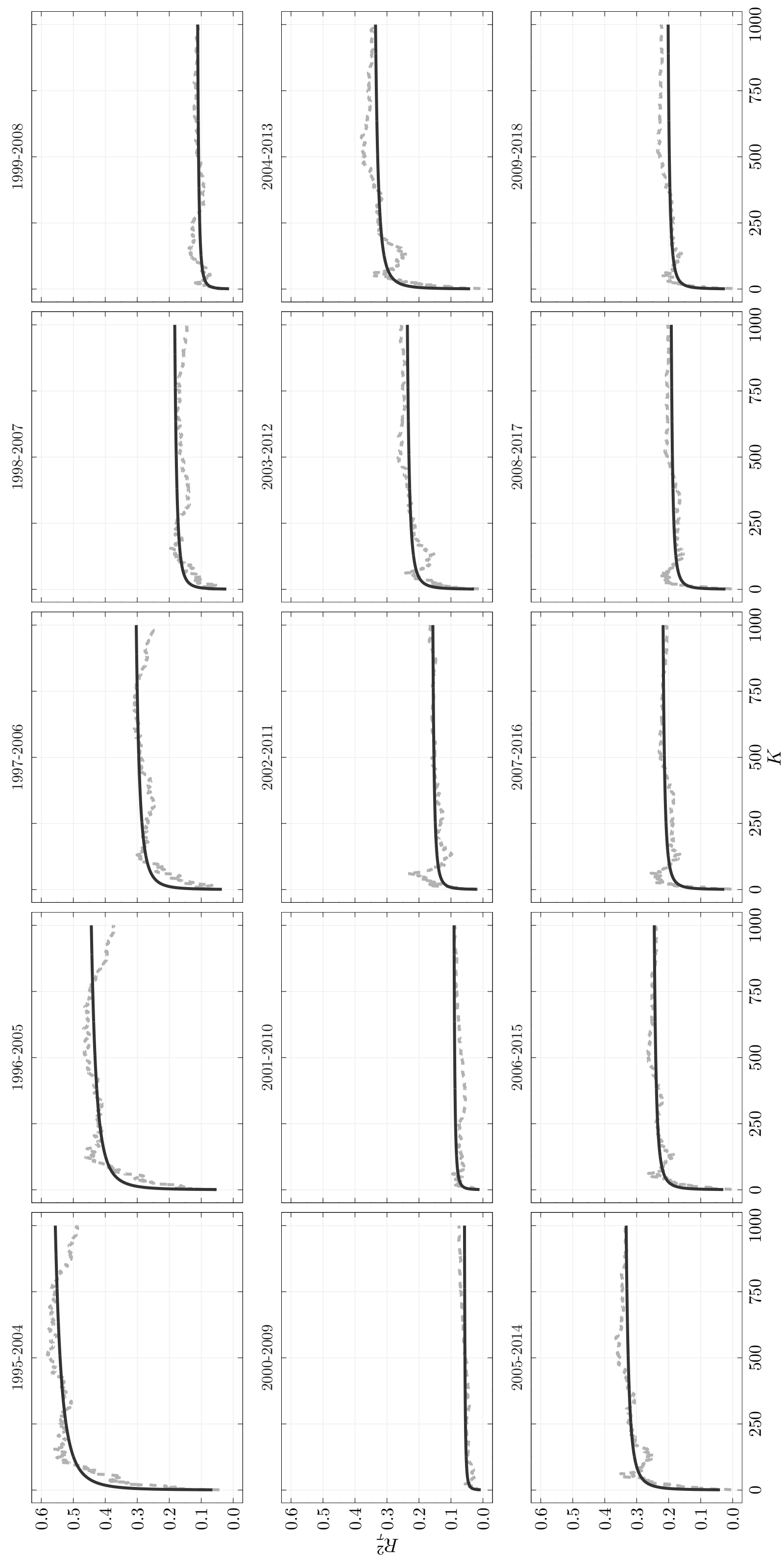



**Notes**: The grey dashed line is $R^2(K)$ calculated using equation (3.21). The solid black line is $R^2(K)$ calculated using equation (3.23). The red dot is the estimated granular size of the economy.

estimates reported by the literature (see Section 3.3.5).

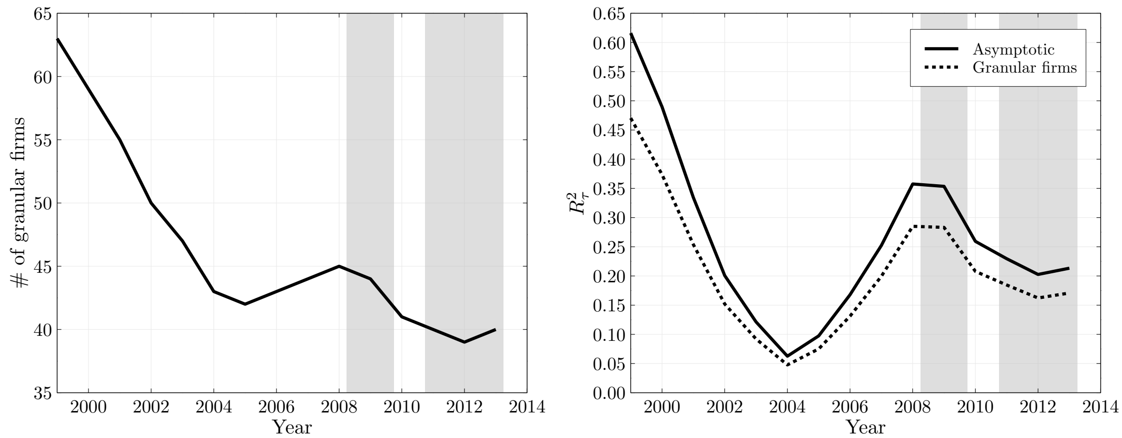
Figure 3.7 plots the empirical contribution of idiosyncratic shocks to GDP fluctuations (equation (3.28)) and its analytical approximation (equation (3.29)). Two results are worth noting. First, the empirical contribution exhibits the granular curve behavior in all the time windows in which the sample is divided. Second, the granular curve exhibits a cyclical behavior: the contribution of idiosyncratic shocks to GDP fluctuations shrinks when taking into account recession years and grows in expansion years. During the period under study, the dynamics of the large firms play a crucial role in shaping aggregate fluctuations in times of relative stability. Nevertheless, when taking into account the years in which the financial crisis (an exogenous shock) hit the Spanish economy, the impact of the dynamics of large firms at the aggregate level becomes almost negligible. In line with this intuition, Figure 3.8 shows that the calibrated number of granular firms (Panel A) and its contribution to aggregate fluctuations (Panel B) increased during the Spanish economic boom and decreased during the burst of the housing bubble.

**Figure 3.7.** Granular curves through time.




**Notes**: The dashed grey line is the empirical $R^2_\tau$ (equation (3.28)). The solid black line is the analytical approximation (equation (3.29)). The estimation of the parameters constituting the approximation is explained in the main text. Each time window $\tau$ consist in 10 years.

**Figure 3.8.** Granular size and its contribution over time.




**Notes**: Panel A shows the average value of the granular sizes resulting from the proposed measures. Panel B shows the maximum contribution of idiosyncratic shocks to volatility of GDP growth (equation (4.28)) and the contribution of granular firms. Year is the centered year in the window $\tau$.

## 3.5 Conclusion

The emergent literature on the granular origins of aggregate fluctuations challenges the tradition in macroeconomics by arguing that, in the presence of significant heterogeneity at the firm level, idiosyncratic shocks to the granular (large) firms do not cancel out with shocks to smaller firms and, thus, translate into aggregate fluctuations. The literature quantifies the contribution of idiosyncratic shocks to aggregate fluctuations using an exogenous given number of large firms, which does not provide information on the granular size of the economy, namely, the number of granular firms. We provide a conceptual to quantify the number granular size of an economy.

The first part of our analysis characterizes, analytically, the contribution of idiosyncratic shocks to the large firms to GDP growth volatility and shows that it is driven by the share of economic activity commanded by the largest firm, the volatility of the largest firm with respect to GDP volatility and two summary statistics for large firm dynamics: the tail index of firm size distribution and the size-volatility elasticity. Additionally, we show that changes size-volatility relationship have greater impact on aggregate fluctuations than tail index changes, which have been the main object of study.

In the second part of the paper, we show that the granular curve is well characterized by our framework and find that the average maximum contribution of top Spanish firms's idiosyncratic shocks to the GDP fluctuations is 23%. We estimate that the granular size of the Spanish economy is approximately 50 firms. Finally, we find that the granular size exhibits a cyclical behavior: the number of firms whose idiosyncratic shocks have an impact on the aggregate grows in expansion phases and shrinks in recession phases.

In future research, we plan to extend our framework to other countries and to include amplification mechanisms of shocks such as the input-output network. These two additional dimensions could provide greater insights on the observed differences in output volatility across countries.

# Chapter 4

# Heterogenous Interacting Agents and Aggregate Fluctuations

## 4.1 Introduction

Blanco-Arroyo et al. (2018) empirical work finds that the contribution of idiosyncratic shocks to gross domestic product fluctuations increases rapidly when the very top firms are taken into account and an almost steady value from a given number of firms onwards. They refer to this behavior as the "granular curve", and it seems to be a characteristic feature of granular economies (see Silva and Da Silva (2020) and Konings et al. (2021). However, there is currently no theoretical framework that studies in detail the determinants of such behavior.

This paper seeks to shed light on the determinants of the granular curve behavior observed by Blanco-Arroyo et al. (2018). Building on Delli Gatti et al.'s (2005) *levered aggregate supply* class model, we characterize analytically the dynamics of the model and show that the origins of aggregate fluctuations can be traced back to the cross-sectional dispersion of firm sizes. In addition, the model generates a granular curve in which the top 2% of firms drive the dynamics of the model and the top 20 are granular firms, i.e., firm-level shocks to these firms may directly translate into aggregate fluctuations.

We decide to use the heterogeneous interacting agent-based model presented by Delli Gatti et al. (2005) because it is able to generate sizeable aggregate fluctuations from purely idiosyncratic shocks and interaction among the agents and to reproduce empirical regularities such as a power-law firm size distribution and a Laplace distribution of growth rates. Moreover, its simplicity allows us to analytically characterize the dynamics and precisely identify the drivers of aggregates fluctuations. This particularly advantageous, as the main drawbacks of the agent-based approach is related to the complexity of the interactions, which typically prevents an analytical solution, leaving only the possibility for Monte Carlo simulations based on a rough calibration of the underlying parameters (see, e.g., LeBaron (2000)).

**Related literature** The paper relates to two distinct literatures: the literature studying the microeconomic origins of aggregate fluctuations and the literature studying role of financial factors in aggregate fluctuations. Gabaix's (2011) seminal work introduces the "granular hypothesis": in the presence of significant heterogeneity at the micro level, the incompressible "grains" of economic activity (large firms) may have a non-negligible impact on the macroeconomic aggregates.[1] Acemoglu et al.'s (2012) seminal work argues that the propagation of idiosyncratic shocks and distortions over input-output linkages can have potentially significant implications for aggregate fluctuations.[2] Delli Gatti et al.'s (2005) model unifies the two arguments by showing that the origin of business cycle fluctuations can be traced back to the ever changing configuration of the network of heterogeneous interacting firms. In line with Carvalho and Grassi's (2019) framework, we show that in Delli Gatti et al.'s (2005) model aggregate volatility dynamics are endogenously driven by the evolution of the cross-sectional dispersion of firm sizes (see Bloom et al. (2018) and Kehrig (2015)).

This paper is also related to the literature studying role of financial factors in aggregate fluctuations. Bernanke and Gertler's (1989) seminal work introduces the "financial accelerator hypothesis": financial factors and monetary shocks may have a non-negligible impact on the aggregate due to the existence of asymmetric information and agency problems which propagate and amplify shocks (see also Bernanke and Gertler (1989, 1990, 1995), Bernanke et al. (1996, 1999), Greenwald and Stiglitz (1988, 1990, 1993), Kiyotaki and Moore (1997, 2002)).[3] Based on Greenwald and Stiglitz's (1990, 1993) framework, Gallegati et al. (2003) and Delli Gatti et al. (2005) use the ABM approach to model an economy in which heterogeneous agents (large number of firms and a bank) interact.[4] This interaction causes several scaling laws observed in the literature to emerge, such as a right-skewed firms' size distribution that is well characterized by a power law (Axtell, 2001, Luttmer, 2007, di Giovanni and Levchenko, 2013) and a firms' size and countries' GDP growth rates distribution that is well characterized by a Laplace (Stanley et al., 1996, Amaral et al., 1997). Building on Delli Gatti et al. (2005), we show that the model is also able generate the granular curve behavior observed empirically by Blanco-Arroyo et al. (2018).

**Outline** The paper is organized as follows. Section 4.2 presents the model setup. Section 4.3 simulates the model and studies the origins of aggregate fluctuations. Section 4.4 quantifies the granular size of the economy. Finally, Section 4.5 concludes. The model without uncertainty and additional content can be found in the Appendix.

[1] Empirical evidence is provided by Friberg and Sanctuary (2016), Blanco-Arroyo et al. (2018), Fornaro and Luomaranta (2018), Miranda-Pinto and Shen (2019), Silva and Da Silva (2020), among many others.

[2] Empirical evidence is provided by Foerster et al. (2011), Carvalho (2014), di Giovanni et al. (2014), Acemoglu et al. (2016), Carvalho et al. (2021).

[3] The alternatives proposed are investment and capital accumulation responses in real business-cycle models (e.g, Kydland and Prescott (1982)), Keynesian multipliers (e.g., Diamond (1982), Kiyotaki (1988), Blanchard and Kiyotaki (1987), Hall (2009), Christiano et al. (2011)), real and nominal rigidities and their interplay (Ball and Romer, 1990), consequences of (potentially inappropriate or constrained) monetary policy (e.g., Friedman and Schwartz (1971), Eggertsson et al. (2003), Farhi and Werning (2016)).

[4] Models related to the one developed by Delli Gatti et al. (2005) have also been used to study bank connectivity, financial contagion and aggregate fluctuations (see, e.g., Grilli et al. (2014, 2015, 2020)).

## 4.2 Model

In this section, we introduce Delli Gatti et al.'s (2005) *levered aggregate supply* class model first develop by Greenwald and Stiglitz (1990, 1993). The model consists in a two-sector economy with goods and credit market. The economy is populated by a large constant number of heterogenous firms and one bank which undertake decisions each discrete time period.[5]

In the goods market, firms produce an homogeneous output using a linear technology with capital as the only input. The model considers an "islands" economy, so that there are no direct linkages between firms. The demand for goods in each island is affected by an *i.i.d.* idiosyncratic real shock. Firms sell all the output they (optimally) decide to produce, hence the model is *supply-driven*.

Due to informational imperfections on the equity market, the capital stock evolves according to investment expenditure, which in turn depend on the firms' ability in raising funds on the credit market. Credit supply is a fraction of the bank's equity base, which is negatively affected as insolvent borrowing firms go bankrupt. Thus, by means of interaction, idiosyncratic shocks propagate throughout the economy, amplifying their impact and translating into aggregate fluctuations.

### 4.2.1 Firms

Consider an islands economy with $N$ firms that produce a homogeneous good using a constant-returns-to-scale technology with capital as the only input. Let firm $i$'s, with $i = 1, 2, \ldots, N$, production function at discrete time $t = 0, 1, 2, \ldots, T$ be

$$Y_{it} = \phi K_{it}, \tag{4.1}$$

where $Y_{it}$ is output, $K_{it}$ is capital and $\phi$ is the productivity of capital, constant and uniform across firms. Capital stock does not depreciate. As in Greenwald and Stiglitz (1990, 1993)'s framework, firms sell all the output produce at an uncertain price. Firm $i$'s selling price $P_{it}$ is assumed to be a random variable with the market price $P_t$ as expected value. As a consequence, the relative price $u_{it} = P_{it}/P_t$ is a random variable with expected value $\mathbb{E}[u_{it}] = 1$ and finite variance.

Following Greenwald and Stiglitz (1990, 1993), firms are assumed to be fully rationed on the equity market, thus the only external source of finance available is credit. In order to increase the level of production, the firm $i$ can finance its capital stock via internal sources, net worth $A_{it}$, or recur to bank loan $L_{it}$. As a result, firms' capital stock motion evolves according to

$$K_{it} = A_{it} + L_{it}. \tag{4.2}$$

We further assume that firms and banks sign long-term contractual relationships. Firm $i$'s debt

[5] Unlike Delli Gatti et al. (2005)'s setup, and for the sake of simplicity, we assume that when a firm goes bankrupt a new one enters the market, so that the number of firms remains constant through time. The entry/exit mechanism is explained in Section 4.2.3.

commitments in real terms at time $t$ are, therefore, $r_{it}L_{it}$, where the real interest rate $r_{it}$ is determined in the credit market (see Section 4.2.2). If, for simplicity, we let debt commitments be equal to the real return on net worth, then financing costs equal to $r_{it}\left(A_{it}+L_{it}\right)=r_{it}K_{it}$.

Firm $i$'s profit in real terms $\pi_{it}$ is given by

$$\pi_{it}^{f}=u_{it}Y_{it}-gr_{it}K_{it}=\left(u_{it}\phi-gr_{it}\right)K_{it}, \tag{4.3}$$

where total variable costs are proportional to financing costs:[6] $gr_{it}K_{it}$, with $g>1$.[7]

Assuming that all profits are retained, the firm accumulates net worth by means of profits. The net worth evolves according to

$$A_{it}=A_{it-1}+\pi_{it} \tag{4.4}$$

Due to the uncertain environment, firms may go bankrupt. Bankruptcy occurs if net worth at time $t$ becomes negative, that is,

$$u_{it}<\frac{1}{\phi}\left(gr_{it}-\frac{A_{it-1}}{K_{it}}\right)\equiv u_{it}^{*}, \tag{4.5}$$

where $A_{it-1}/K_{it}$ is the equity ratio. In words, bankruptcy occurs if the relative price $u_{it}$ is under the threshold $u_{it}^{*}$. The bankrupt firm leaves the market. The exit process depends on the financial fragility: a firm leaves the system if its net worth is so low that an adverse shock makes it become negative, or if the firm suffers a loss so huge as to deplete all the net worth accumulated in the past (see Greenwald and Stiglitz (1993)).

The probability of bankruptcy is $\mathbb{P}\left(u_{it}<u_{it}^{*}\right)$. Therefore, the probability of bankruptcy is an increasing function of the interest rate and the capital stock and a decreasing function of the equity base inherited from the past. For mathematical tractability, $u_{it}$ is assumed is distributed uniformly on the interval $(0,2)$ (i.e., $u_{it}\sim\mathcal{U}(0,2)$), so that the probability of bankruptcy is

$$\mathbb{P}\left(u_{it}<u_{it}^{*}\right)=\frac{1}{2}u_{it}^{*}=\frac{1}{2\phi}\left(gr_{it}-\frac{A_{it-1}}{K_{it}}\right). \tag{4.6}$$

The problem of the firm $i$ consists in maximizing the expected profits $\mathbb{E}\left[\pi_{it}\right]$ minus bankruptcy costs. Bankruptcy costs are due to legal, administrative and reputational costs incurred during the bankruptcy procedure (Greenwald and Stiglitz, 1990). Such costs are expected to increase with firm's output. Following Gallegati et al. (2003) and Delli Gatti et al. (2005), bankruptcy costs are increasing and quadratic in the level of output: $C^{f}=cY_{it}^{2}$, with $c>1$. We can formulate the problem of the firm $i$ as:

$$\max_{K_{it}}\mathbb{E}[\pi_{it}^{f}]-\mathbb{P}\left(u_{it}<\overline{u}_{it}\right)C^{f}=\left(\phi-gr_{it}\right)K_{it}-\frac{\phi c}{2}\left(gr_{it}K_{it}^{2}-A_{it-1}K_{it}\right). \tag{4.7}$$

[6] One can think of retooling and adjustment costs to be sustained each time the production process starts.

[7] Throughout this thesis we use $g$ to denote the growth rate. To keep the notation used by Delli Gatti et al. (2005) and to avoid confusion, in this chapter we add the notation of the variable on which the growth rate is calculated. For example, the capital ($K$) growth rate of firm $i$ in period $t$ is denoted by $g_{K,it}$.

From the first-order condition, the (optimal) capital stock is

$$K_{it}^{*} = \frac{\phi - gr_{it}}{c\phi gr_{it}} + \frac{A_{it-1}}{2gr_{it}}. \tag{4.8}$$

Thus, firm $i$'s capital stock at time $t$ is decreasing with the interest rate and increasing with net worth. Investment is the difference between the optimal capital stock and the capital stock inherited from the previous period, $I_{it} = K_{it}^{*} - K_{it-1}$. Because firms raise funds only on the credit market, due to equity rationing (Greenwald et al., 1984), investment is financed by means of retained profits and new bank loans, $I_{it} = \pi_{it-1} + \Delta L_{it}$, where $\Delta$ denotes the first difference operator. Using (4.8), $i$'s demand for credit is given by

$$L_{it}^{\mathrm{d}} = \frac{(\phi - gr_{it})}{c\phi gr_{it}} - \pi_{it-1} + \left(\frac{1 - 2gr_{it}}{2gr_{it}}\right) A_{it-1}. \tag{4.9}$$

### 4.2.2 Bank

For simplicity, henceforth banks are lumped together in a vertically integrated banking sector (henceforth, the "bank"). Hence, the $N$ heterogeneous firms interact with only one bank on the credit market. The balance sheet of the bank is $L_t^s = E_t + D_t$, where $E_t$ is the bank's equity base and $D_t$ deposits, which are determined as a residual. The regulation of financial intermediaries (Basel I–III) allows banks to lend up to a fraction of their equity, to prevent bankruptcies due to unexpected losses. We assume that the bank is subject to the following prudential rule that prevents it from incurring in excess lending: $L_t^s = E_{t-1}/\nu$, where the risk coefficient $\nu$ is constant.

Credit is allotted to firm $i$ based on the mortgages it offers, which is proportional to its size, and on the amount of cash available to serve debt according to

$$L_{it}^{s} = \lambda L_t^s \kappa_{it-1} + (1 - \lambda) L_t^s a_{it-1} \tag{4.10}$$

where $\kappa_{it-1}$ and $a_{it-1}$ are, respectively, firm $i$'s capital and net worth shares, and $0 < \lambda < 1$. The equilibrium interest for $i$ is determined as credit demand (4.9) and equals credit supply (4.10):

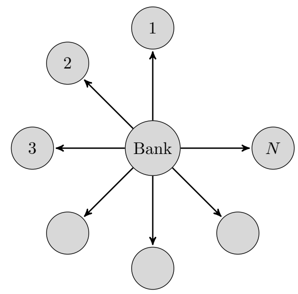
$$r_{it} = \frac{2 + A_{it-1}}{2cg\left[(1/\phi c) + \pi_{it-1} + A_{it-1} + L_{it}^{s}\right]}. \tag{4.11}$$

We assume that the returns on the bank's equity is given by the average of lending interest rates $\overline{r}_t$ (i.e., $\overline{r}_t = N^{-1}\sum_{i=1}^{N} r_{it} L_{it}$) and deposits are remunerated with the borrowing rate $r_t^D = (1 - \omega)\,\overline{r}_t$, where $\omega$ is the profit mark-up for the bank. The bank's profit $\pi_t^b$ is

$$\pi_t^b = \sum_{i=1}^{N} r_{it} L_{it}^{s} - \overline{r}_t\left[(1 - \omega) D_{t-1} + E_{t-1}\right]. \tag{4.12}$$

As assumed above, a firm goes bankrupt when its equity base becomes negative. In such a

**Figure 4.1.** The star interaction network.




*Notes*: The bank is the central node in the interaction network because of its role as credit supplier.

case, $K_{it} < L_{it}$ and the firm cannot refund its own loan. The bank registers a loss or "bad debt" equal to $B_{it} = L_{it} - K_{it}$, which will affect its own equity base negatively. Bad debt for the bank is then

$$B_{it} = \begin{cases} -A_{it} & \text{if } A_{it} < 0, \\ 0 & \text{if } A_{it} \geq 0. \end{cases}$$

The bank's equity base evolves according to the law of motion:

$$E_t = \pi_t^b + E_{t-1} - \sum_{i=1}^{N} B_{it-1}. \tag{4.13}$$

The interaction between firms and the bank in this economy is hence characterized by a "star network" (see Figure 4.1), in which the bank is the "central" node (also known as *hub*) in the interaction network because of its role as credit supplier. This mean field interaction in terms of a *bank effect* (Hubbard et al., 2002) helps to propagate idiosyncratic shocks to firms throughout the economy, amplifying their magnitude and translating them into aggregate fluctuations.[8] The process is as follows. When a firm goes bankrupt as a result of a negative idiosyncratic shock, bank's equity decreases due to the increase in bad debt. In turn, credit supply decreases, raising the interest rate and, therefore, financial costs. This indirect systemic shock increases the risk of bankruptcy for the other firms.

In summary, aggregate fluctuations in this economy are shaped by real idiosyncratic shocks to firms' selling price and a common, systemic shock that is transmitted via the interest rate.

### 4.2.3 Firms' demography

As in Gallegati et al. (2003) and Delli Gatti et al. (2007), when bankrupted firms leave the market, they are replaced by new entrants. This one-to-one replacement keeps the number of firms constant

[8] When the interaction among agents is characterized by an asymmetric structure, such as the one depicted in Figure 4.1, the law of large numbers does not apply and firm-level shocks do not average out (see Acemoglu et al. (2012)).

**Table 4.1.** Parameters and initial conditions.

| Parameter | Value | Description |
|---|---|---|
| $\phi$ | 0.1 | Capital productivity |
| $u$ | $\mathcal{U}(0,2)$ | Relative price |
| $g$ | 1.1 | Total variable cost parameter |
| $c$ | 1 | Bankruptcy cost parameter |
| $\nu$ | 0.08 | Bank's risk coefficient |
| $\lambda$ | 0.3 | Credit allocation parameter |
| $\omega$ | 0.002 | Degree of competition in the banking sector |
| $N$ | 10,000 | Number of firms |
| $T$ | 1,000 | Number of time periods |
| $K_{i0}$ | 100 | Capital stock endowment |
| $A_{i0}$ | 20 | Net worth endowment |
| $\pi_{i0}$ | 0 | Initial profit |
| $B_{i0}$ | 0 | Initial bad debt |
| $A_0^{\text{exit}}$ | 0.0001 | Net worth exit threshold |

**Notes**: $\mathcal{U}(\cdot,\cdot)$ denotes the uniform distribution. The total number of firms is constant due to a one-to-one replacement if a firm goes bankrupt. Bank loan in the initial time period $B_{i0}$ is the difference between capital stock and net worth (see equation (4.2)). Capital stock and net worth growth path is determined, respectively, by $K_{it} = K_{i0}\,(1+\chi)^t$ and $A_{it} = A_{i0}\,(1+\chi)^t$, where $\chi$ is the growth rate of the economy (see equation (B.7)).

through time. As we detail in Appendix B.1, Section B.1.1, the long-run dynamics of the economy described above is well approximated by the growth rate $\chi \equiv \frac{\omega\beta\phi}{g}$. A firm exits the market when its net worth is $A_{it} < A_0^{\text{exit}}\,(1+\chi)^t$, where net worth is in time period $t=0$ is $A_0^{\text{exit}} = 10^{-4}$ for all firms. Entrants at each time period $t$ are endowed with a capital stock and a net worth equal to $K_{it} = K_{i0}\,(1+\chi)^t$ and $A_{it} = A_{i0}\,(1+\chi)^t$, respectively. This entry/exit mechanism aims to keep a certain degree of homogeneity among entrants and incumbents over time in order to prevent that a disproportionately large firm emerges (see Chapter 2 in Pulcini (2017)).

## 4.3 Aggregate fluctuations dynamics

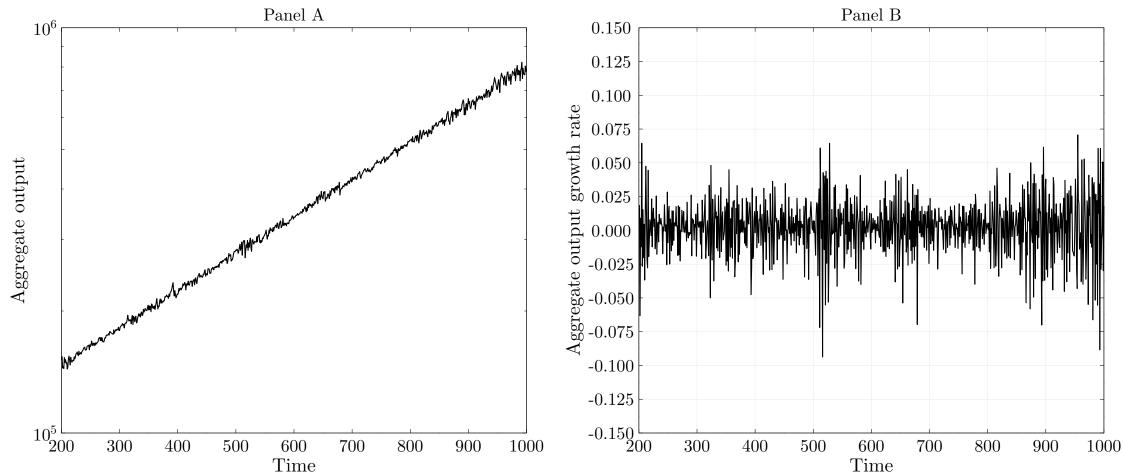
In this section, we study the origins of aggregate fluctuations in the economic system described in Section 4.2 by means of computer simulations and analytically. We consider an economy consisting of $N = 10,000$ firms that lasts $T = 1,000$ time periods. The initial conditions and the values set for each of the parameters are presented in Table 4.1.[9] To exclude the transients, we evaluate only the last 800 simulated periods.[10]

We first show that aggregate output volatility dynamics are chiefly driven by the evolution of the cross-sectional dispersion of firm sizes. The model is able to generate a fat tail distribution in the firm size distribution as a result of the interaction mechanism, without imposing any ad hoc

[9] We use the same parameters as Delli Gatti et al. (2005). Therefore, the model is not calibrated to replicate the behavior observed in the Spanish data presented in Chapter 2 and Chapter 3.

[10] We use Monte Carlo techniques to check the robustness of our qualitative results.

**Figure 4.2.** Logarithm of aggregate output and growth rate of aggregate output.




distribution for the exogenous shock. We show that the upper tail is well described by a power law distribution with exponent close to 1, also known as Zipf law. Under the assumption of Zipf distributed sizes, we find that aggregate output volatility dynamics can be traced back to the dynamics of the largest firm in the economy alone.

### 4.3.1 Aggregate output volatility

The model is able to generate endogenous self-sustained growth characterized by persistent aggregate fluctuations (see Panel A in Figure 4.2). Indeed, aggregate fluctuations, measured by the growth rates of aggregate output (see Panel B in Figure 4.2), are path dependent (i.e., shocks have permanent effects) and exhibit cluster volatility, which is a well-known property in the financial market literature (see, e.g., Cont (2007)). This implies that large changes tend to cluster together, resulting in a persistence in the amplitudes of these changes. The autocorrelation parameter of the aggregate output time series is 0.99 and the standard deviation of growth rates is 0.021, which resembles to the observed standard deviation of GDP growth. Recall that this behavior emerges as a result of *i.i.d.* idiosyncratic shocks affecting individual decision-making processes and interaction between firms and the bank.

To shed light on the origins of aggregate fluctuations, we follow Carvalho and Gabaix (2013) and di Giovanni et al. (2014) and decompose aggregate output volatility

$$\sigma_{Yt}^2 = \sum_{i,j=1,\ldots,N} w_{it-1} w_{jt-1} \sigma_i \sigma_j \rho_{ij}$$

as

$$\sigma_{Yt}^2 = \underbrace{\sum_{i=1}^{N} w_{it-1}^2 \sigma_i^2}_{\mathcal{D}_t} + \underbrace{2 \sum_{1 \le i < j \le N} w_{it-1} w_{jt-1} \sigma_i \sigma_j \rho_{ij}}_{\mathcal{N}_t}, \tag{4.14}$$

where $w_{it-1}$ and $\sigma_i^2$ are, respectively, firm $i$'s output share in total output and volatility of output growth rate (i.e., $\sigma_i^2 = \operatorname{Var}(g_{Y,it})$) and $(\sigma_i \sigma_j \rho_{ij})$ is the variance-covariance matrix (i.e. $\operatorname{Cov}(g_{Y,it}, g_{Y,jt}) = (\sigma_i \sigma_j \rho_{ij})$). Note that aggregate output volatility (4.14) only reflects the changing weights of different firms in the economy, as the variance-covariance matrix is held constant through time. The term $\mathcal{D}_t$ represents the diagonal terms in output growth and the term $\mathcal{N}_t$ represents the non-diagonal terms, i.e., the terms that come from linkages in the economy. Thus, the $\mathcal{N}_t$ term captures the propagation of shocks via the interest rate set by the bank, while the $\mathcal{D}_t$ captures the direct impact of firm-level specific shocks, which are a combination of shocks to price and interest rate.

As discussed below, the entry-exit mechanism introduced in Section 4.2 reduces significantly the share of the economic activity commanded by the largest firm compared to the original mechanism proposed by Delli Gatti et al. (2005), but it still does not prevent the emergence of a disproportionally large firm. Thus, we focus our attention on the span of time during which the size of the largest firm is somewhat contained. In addition, we restrict the analysis to the long-lived firms to attenuate the unrealistic turnover produced by the model.

Figure 4.3 presents the decomposition graphically for the time period 700-800. Since the time series contain low-frequency movements, we filter each series using the Hodrick-Prescott (HP) filter with smoothing parameter 6.25. The $\mathcal{D}$ term explains the majority of aggregate output volatility: $\sqrt{\mathcal{D}_t}/\sigma_t$ is 80% on average. Given that the bulk of aggregate fluctuations come from the diagonal terms in output growth, we assume that the impact of linkages is negligible (i.e., $\sqrt{\mathcal{N}_t} = 0$). Therefore, aggregate volatility is *entirely* driven by variance of individual growth rates:

$$\sigma_{Yt} = \sqrt{\sum_{i=1}^{N} w_{it-1}^2 \sigma_i^2}. \tag{4.15}$$

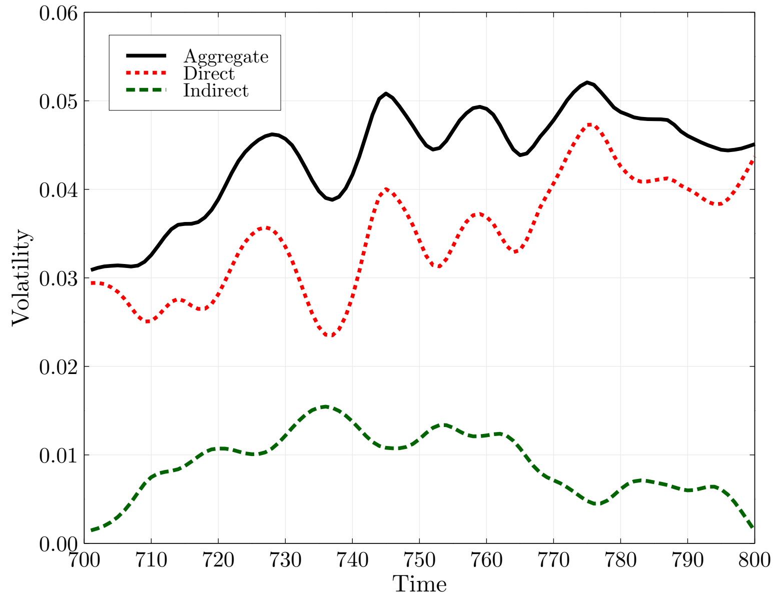
### 4.3.2 Understanding aggregate output volatility

To shed light on the origins of aggregate fluctuations, we start by decomposing the firms' output growth rate. Using equations (4.1), (4.2) and (4.4), the growth rate of capital $g_{Y,it}$ of firm $i$ between time $t-1$ and time $t$ is

$$g_{Y,it} = g_{K,it} \equiv \frac{\Delta K_{it}}{K_{it-1}} = \frac{\Delta L_{it}}{K_{it-1}} + \frac{\Delta A_{it-1}}{K_{it-1}} + \frac{\Delta \pi_{it}^f}{K_{it-1}}.$$

Given that firms accumulate net worth by means of profits, the change in net worth minus profits

**Figure 4.3.** Dynamics of aggregate output volatility.




**Notes**: "Aggregate" is the standard deviation of the growth rate of aggregate output calculated using decomposition (4.14), "Direct" is the square root of diagonal terms in GDP growth (i.e., $\sqrt{\mathcal{D}_t}$) and "Indirect" is the square root of non-diagonal terms (i.e., $\sqrt{\mathcal{N}_t}$). The time series are HP filtered using a smoothing parameter of 6.25.

is zero (i.e., $\Delta A_{it-1} - \pi_{it-1} = 0$).[11] Thus, $g_{Y,it}$ is driven by the new mortgaged debt and the firm's profitability measured by the *return on assets* (ROA):

$$g_{Y,it} = d_{it-1} g_{L,it} + \text{roa}_{it}, \tag{4.16}$$

where $d_{it-1} \equiv L_{it-1}/K_{it-1}$ is the "debt ratio", $g_{L,it} \equiv \Delta L_{it}/L_{it-1}$ is the growth rate of bank loans and $\text{roa}_{it} \equiv \pi^f_{it}/K_{it-1}$ is ROA. In Appendix B.1, Section B.1.1, we show that the expected growth rate of capital and bank loans are equal (i.e., $\mathbb{E}\left[g_{K,it}\right] = \mathbb{E}\left[g_{L,it}\right]$). Additionally, in Section B.1.2 we show that the expected debt ratio is $\mathbb{E}\left[d_{it-1}\right] = 1 - 2\phi$. Then, firm $i$'s expected growth rate of output is determined by its leverage and ROA:

$$\mathbb{E}\left[g_{Y,it}\right] = \ell \mathbb{E}\left[\text{roa}_{it}\right], \tag{4.17}$$

[11] As pointed out by Pulcini (2017), the entry and exit mechanism causes that $\Delta A_{it-1} - \pi_{it-1} \neq 0$ because entrants are endowed with an amount of equity that exceeds that of the failing firms. Thus, the correct approximation is $\Delta A_{it-1} - \pi_{it-1} \approx N^{\text{entry}}_{it-1} < A^{\text{entry}}_{it-1} - A^{\text{exit}}_{it-1} >$, where $N^{\text{entry}}_{it-1}$ is the number of entrants and $< A^{\text{entry}}_{it-1} - A^{\text{exit}}_{it-1} >$ is the average new equity introduced by entrants. We note that the number of entrants relative to the number of incumbents is small enough to assume that $\Delta A_{it-1} - \pi_{it-1} = 0$

where $\ell \equiv 1/(2\phi)$ is the expected leverage. Then, the expected aggregate output growth rate is

$$\mathbb{E}[g_{Yt}] = \ell\mathbb{E}\left[\sum_{i=1}^{N} w_{it-1}\mathrm{roa}_{it}\right]. \tag{4.18}$$

In equation (4.18), we use the fact that $\kappa_{it-1} = w_{it-1}$. Note that firm $i$'s ROA is shaped by the idiosyncratic shock affecting the selling price $u_{it}$ and the common, systemic shock affecting the interest rate $r_{it}$. This equation suggests that aggregate fluctuations dynamics are fully driven by a type of "granular residual" (Gabaix, 2011).

Gabaix (2011) shows that, whenever the firm size distribution is sufficiently fat-tailed, idiosyncratic shocks to large (granular) firms do not cancel out with shocks to smaller firms and may translate into aggregate fluctuations. To grasp the intuition behind Gabaix's hypothesis, let us consider the simplest case in which the variance of ROA is identical across firms and equal to the analytical value, i.e., $\mathrm{Var}(\mathrm{roa}_{it}) = \phi^2/3\ \forall i$ (see Section B.2). Under this assumption, aggregate output volatility is

$$\sigma_{Yt} = \ell\frac{\phi}{\sqrt{3}}\sqrt{h_{t-1}}, \tag{4.19}$$

where $h_{t-1} = \sum_{i=1}^{N} w_{it-1}^2$ denotes the Herfindahl–Hirschman concentration index (HHI) of the economy.[12] The more fat-tailed is the distribution of firm size, the larger will be the HHI and the greater will be the aggregate volatility generated by firm-specific shocks. This argument is in stark contrast with the "diversification argument" (see, e.g., Lucas (1977)): in an economy consisting of a large number $N$ of firms hit by independent shocks, aggregate fluctuations would have a magnitude proportional to $1/\sqrt{N}$ due to the law of large numbers. That is, if all firms are symmetric in size (i.e., $w_{it} = 1/N\ \forall i, t$), then the aggregate volatility would be

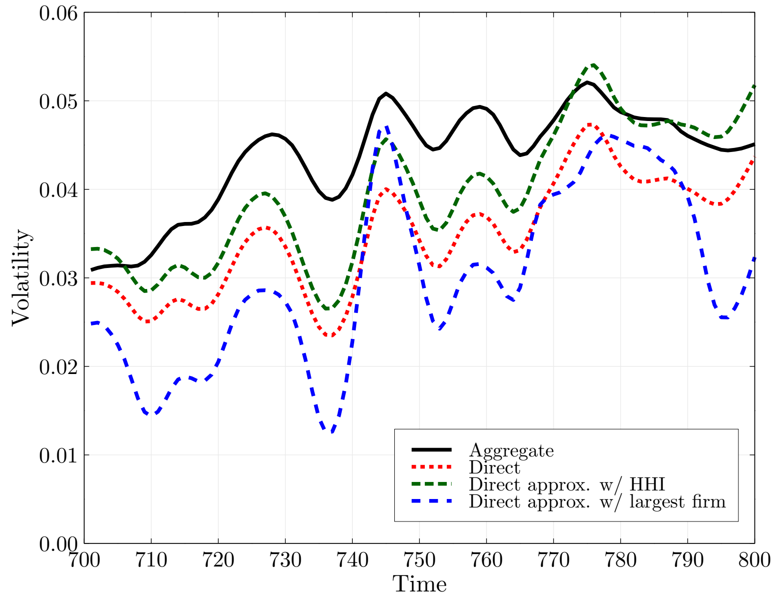
$$\sigma_{Yt} = \frac{\ell\phi}{\sqrt{3N}},$$

and the contribution of firms to aggregate volatility decays rapidly with the number of firms in the economy. In particular, the aggregate volatility implied by equal weights is 0.0028, or 15 times smaller than the average aggregate volatility, which is equal to 0.044.

Figure 4.4 shows that the approximation (4.19) tracks closely the volatility of aggregate output with a highly significant correlation of 0.8 (std. dev. 0.06). We hence conclude that the artificial economy is granular, namely the existence of significant heterogeneity at the firm level matter for the behavior of macroeconomic aggregates.

[12] Equation (4.19) also assumes that the standard deviation of ROA is independent of size, namely "Gibrat's law" for variances holds. We follow Stanley et al. (1996) and assume that, if the relationship between size and volatility exists, then it follows a power law. We test this hypothesis using the methodology proposed by Koren and Tenreyro (2013), which allows for variation within firms. We find that deviation from Gibrat's law are negligible.

**Figure 4.4.** Dynamics of aggregate output volatility using Zipf's law approximation.




**Notes**: "Aggregate" is the standard deviation of the growth rate of aggregate output calculated using decomposition (4.14), "Direct" is the square root of diagonal terms in GDP growth (i.e., $\sqrt{\mathcal{D}_t}$), "Direct approx. w/ HHI" is the approximated $\sqrt{\mathcal{D}}$ term using equation (4.19). "Direct approx. w/ largest firm" is the approximated $\sqrt{\mathcal{D}}$ term using equation (4.22) (discussed below) The time series are HP filtered using a smoothing parameter of 6.25.

### 4.3.3 Firm size dynamics and aggregate output volatility

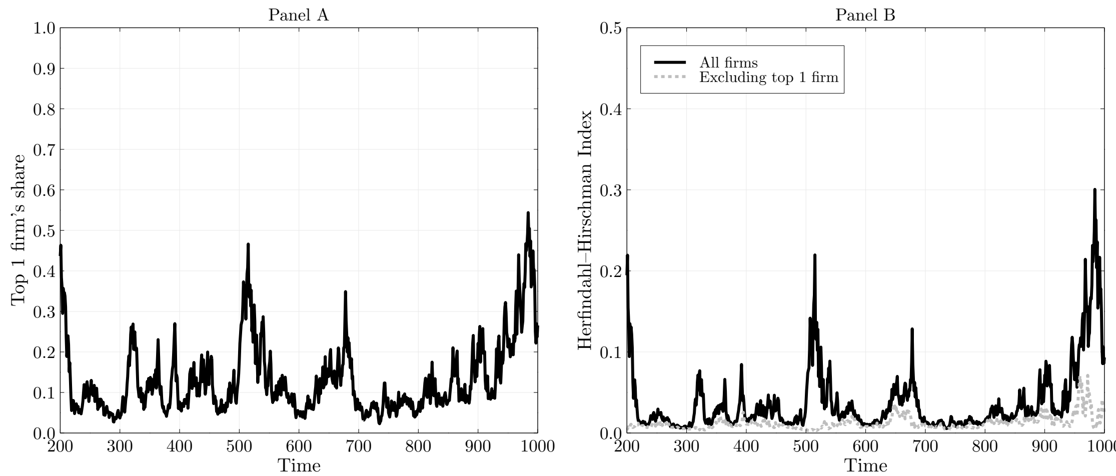
Having established that aggregate volatility dynamics are endogenously driven by the evolution of the cross-sectional dispersion of firm sizes, we next study the firm size distribution to better understand the origins of aggregate fluctuations. As Figure 4.5 renders clear, aggregate fluctuations are mainly driven by the largest firm in the economy. The time periods with high volatility in the growth rate of aggregate output (see Panel B in Figure 4.2) are due to the raise and fall of a disproportionally large firm, which commands more than 1/4 of economic activity in some particular periods.

To further characterize the behavior of the firm size distribution, Panel A in Figure 4.6 plots the counter-cumulative distribution function (CCDF) of firm size in a given time period (we chose $t = 800$). It gives the capital stock $K$ of a given firm in log in the $x$-axis and the probability of finding a firm larger than the corresponding $x$-axis capital stock in log in the $y$-axis. The upper tail of the distribution displays a straight line above a certain threshold. This behavior is characteristic of the Pareto distribution:[13]

$$\mathbb{P}\left(Y > x\right) = \left(\frac{Y}{Y_{\min}}\right)^{-\zeta}, \tag{4.20}$$

[13]The Pareto is type of power law distribution. See Gabaix (2009a) for a review of power laws in economics and finance.

**Figure 4.5.** Heterogeneity in the firm size distribution.




**Notes**: Panel A shows the largest firm's output share in aggregate output through time. Panel B shows the Herfindahl–Hirschman concentration index (i.e., $h_t = \sum_{i=1}^{N} w_{it}^2$) when including all firms and when excluding the largest firm.

where $Y_{\min}$ is the smallest value of $Y$ for which the Pareto behavior holds. The exponent $\zeta$ captures the scaling behavior of the tail of the distribution. For $\zeta > 2$, the first two moments of the distribution are well-defined (i.e., they do not diverge), the conventional law of large numbers applies, and output volatility given by equation (4.19) decays at a rate $1/\sqrt{N}$, as predicted by the diversification argument. Gabaix (2011) and Acemoglu et al. (2012) show that the output volatility decays at a rate $1/N^{1-1/\zeta}$ for $1 < \zeta < 2$ and $1/\ln N$ when $\zeta = 1$. This result means that when $\zeta < 2$, firm-specific shocks do not die out in the aggregate.

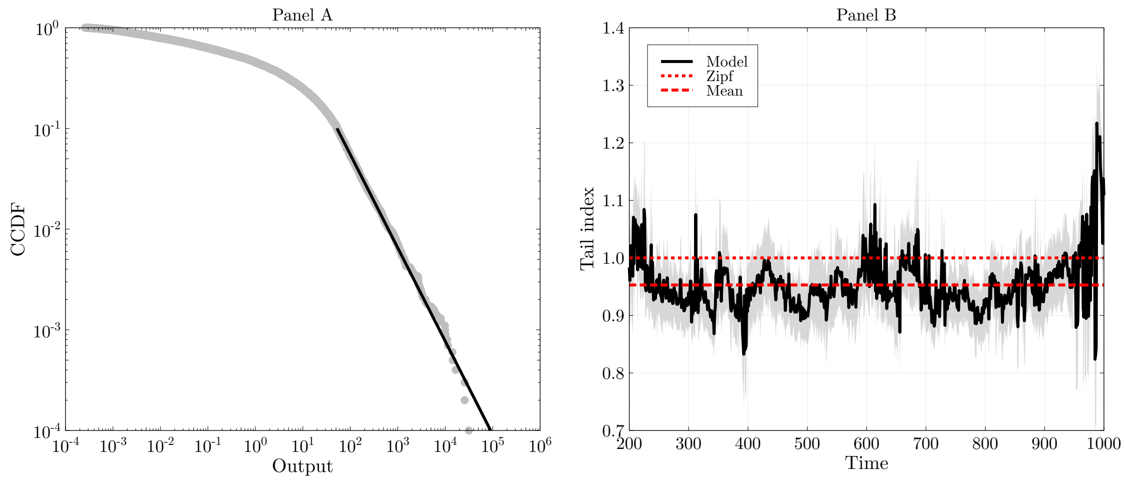
We obtain the tail estimate $\zeta$ by using the maximum likelihood estimation (MLE) method. The estimator for $\zeta$ is

$$\hat{\zeta}_t = \hat{n}_t \left( \sum_{i=1}^{\hat{n}_t} \frac{Y_{it}}{\hat{Y}_{\min,t}} \right)^{-1}, \tag{4.21}$$

with standard error $\hat{\zeta}_t/\sqrt{n_t}$ (see Newman (2005) for a detailed explanation). Following Clauset et al. (2009), we estimate the cut-off $\hat{Y}_{\min}$ as the value of $Y_{\min}$ that minimizes the distance (measured by the Kolmogorov–Smirnov statistic) between the probability distribution of size and the best-fit power-law model above $\hat{Y}_{\min}$. $\hat{n}$ is then the number of firms whose capital stock is in the range $\left[\hat{Y}_{\min,t}, Y_{\max,t}\right]$.

Panel B and C in Figure 4.6 plot the estimated tail index and the estimated number of firms for which the Pareto behavior holds. This region is less than 10% on average (see Figure B.4). The average tail index is close to 1, indicating that the upper tail of the size distribution follows a Zipf's law. Note that the model is able to endogenously generate fat tails, even if uncorrelated shocks are drawn from a uniform distribution. The fat tail distribution emerges as a result of the

**Figure 4.6.** Characterization of the firm size distribution.




**Notes**: Panel A shows the counter-cumulative distribution function (CCDF) of capital stock in time period 800 along with the power law fit obtained by using the maximum likelihood estimation (MLE) method. Panel B shows the dynamics of the tail exponent estimate. Shaded lines correspond to $\pm$ 2 standard errors.

interaction mechanism embodied in the model, without imposing any ad hoc distribution for the exogenous shock (see Mishkin (2011)).

Under the assumption of Zipf distributed sizes, the HHI is $h_{t-1} = \kappa_{\max,t-1} \sum_{i=1}^{N} 1/i^2 = \kappa_{\max,t-1}\pi^2/6$. Therefore, the dynamics of aggregate volatility only depends on the dynamics of the largest firm in the economy:

$$\sigma_{Yt} = \ell \frac{\phi}{\sqrt{3}} \frac{\pi}{\sqrt{6}} w_{\max,t-1} \tag{4.22}$$

where $w_{\max,t-1} \equiv \max\left(w_{i,t-1}\right)$. As previously observed in Figure 4.5, equation (4.22) emphasizes the role played by the largest firm in shaping aggregate fluctuation. Figure 4.4 shows that the evolution of aggregate output volatility can be traced back to the dynamics of the largest firm in the economy. We find a highly significant correlation of 0.8 (std. dev. 0.06). The fact that the aggregate volatility approximated by equation (4.22) is below the $\sqrt{\mathcal{D}}$ term on average is explained by the increase in $\zeta$ when assuming Zipf law with respect to the average. The assumption of Zipf distributed sizes decreases the degree of heterogeneity observed, and thus the approximated volatility.

## 4.4 The granular size of the economy

In this section, we assess whether the heterogeneity in the distribution of firm size that arises from the interaction among the agents that populate the economy produces the granular curve behavior empirically observed by Blanco-Arroyo et al. (2018). We start by constructing a measure of the contribution of large firms volatility to aggregate volatility that attenuates the high turnover in the top firms generated by the model. We find that the model produces granular behavior that is even more pronounced than that observed empirically, as the largest 200 firms alone account for the maximum contribution of firm-level shocks to aggregate volatility. In other words, the aggregate fluctuations produced by the model can be traced back to the dynamics of approximately the largest 2% firms in the economy. Additionally, building on the framework presented in Chapter 3, we find that approximately the top 20 firms are granular, i.e., firm-level shocks to these firms may directly translate into aggregate fluctuations.

### 4.4.1 Granular curve

We study the behavior of the fraction of the aggregate output volatility explained by the top $\mathcal{I}$ firms as $\mathcal{I}$ gradually increases (i.e., $\mathcal{I} = 1, \ldots, N$). The convention in the literature (see, e.g., di Giovanni et al. (2014)) explores the contribution of the top $\mathcal{I}$ firms to aggregate volatility in terms of *relative standard deviations*. That is, the ratio of aggregate volatility accounted for the largest $\mathcal{I}$ firms is

$$R(\mathcal{I}) = \frac{\sigma_Y(\mathcal{I})}{\sigma_Y}, \tag{4.23}$$

where $\sigma_Y(\mathcal{I}) = \sqrt{\operatorname{Var}\left(g_Y(\mathcal{I})\right)} = \sqrt{\operatorname{Var}\left(\sum_{i=1}^{\mathcal{I}} w_{it-1} g_{it}\right)}$ and $\sigma_Y = \sqrt{\operatorname{Var}\left(\sum_{i=1}^{N} w_{it-1} g_{it}\right)}$. Note, however, that the model produces such a high turnover in the top firms that causes $\sigma_Y(\mathcal{I})$ to be extremely volatility for a small $\mathcal{I}$. We attenuate this unrealistic high volatility by measuring the aggregate volatility of the top $\mathcal{I}$ firms as

$$\sigma_Y(\mathcal{I}) = \sqrt{\mathbb{E}\left[g_{Yt}^2\right](\mathcal{I})} = \sqrt{\sum_{i=1}^{\mathcal{I}} \mathbb{E}\left[w_{it}^2\right] \mathbb{E}\left[g_{Y,it}^2\right]}. \tag{4.24}$$

The aggregate volatility of the economy $\sigma_Y$ is the case where $\mathcal{I} = N$. Therefore, by construction, the contribution $R$ is bounded between 0 and 1 (that is, $R(\mathcal{I}) \in (0,1)$).

As in Section 4.3, we focus our analysis on period ranging from 700 to 800, during this span of time the size of the largest firm is somewhat contained, and on the long-lived firms. Figure 4.7 shows that the behavior of the contribution $R$ displays the "granular curve" behavior empirically observed by Blanco-Arroyo et al. (2018): a rapid increase of $R$ when a small number of top firms are taken into account and very slow increase after a given number of firms. In particular, the top 200 long-lived firms in the period 700-800 account for *entire* contribution. Thus, henceforth, we assume that the dynamics of this economy are solely determined by these firms and the rest are negligible.

After observe that the model produces the granular curve, we now try to shed light on the determinants of this behavior. We assume that the upper tail of the distribution of $\mathbb{E}\left[w_{it}^2\right]$ in

**Figure 4.7.** Granular curve.

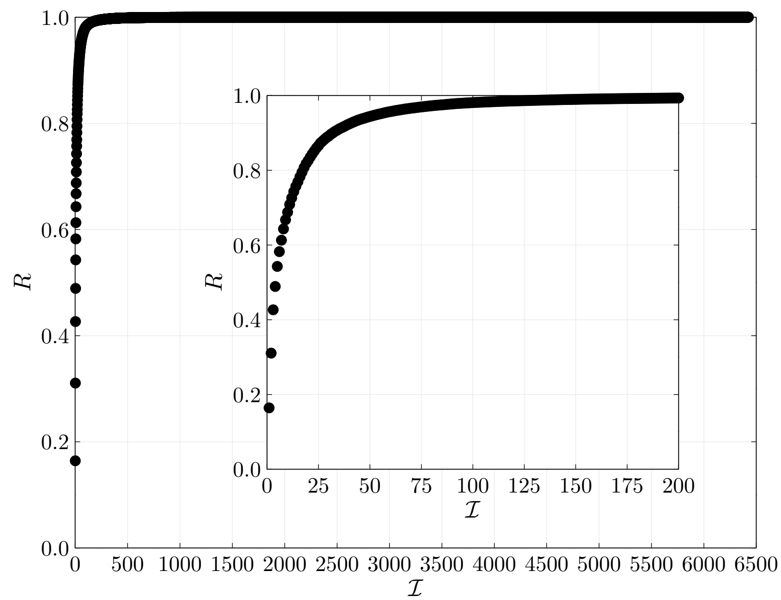



**Table 4.2.** Parameters.

| Parameter | Value | Description |
|---|---|---|
| $w_{\max}$ | 0.0705 | Largest firm share |
| $\phi$ | 0.1 | Capital productivity |
| $\ell$ | 5 | Leverage |
| $\sigma_Y$ | 0.0361 | Aggregate output volatility |
| $\zeta$ | 1.451 | Tail index |

**Notes**: The largest firm's output share in aggregate output is $w_{\max} = \sqrt{\max(\mathbb{E}[w_{it}^2])}$, the expected leverage is $\ell = 1/(2\phi)$, the volatility of aggregate output is $\sigma_Y = \sqrt{\sum_{i=1}^N \mathbb{E}[w_{it}^2]\,\mathbb{E}\left[g_{Y,it}^2\right]}$ and $\zeta$ is the estimated tail index of the distribution of $\mathbb{E}[w_{it-1}^2]$.

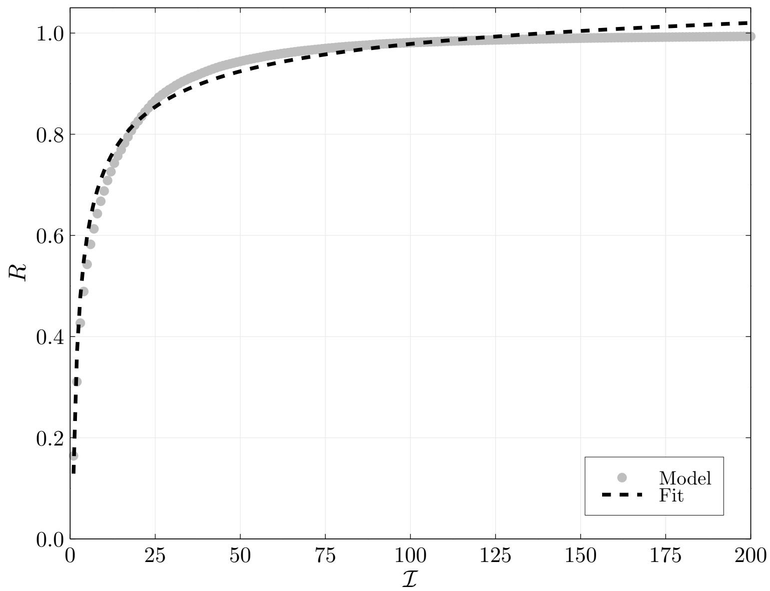
equation (4.24) follows a power law (see Figure B.5). Zaliapin et al. (2005) show that the sum of *i.i.d.* power-law summands can be replaced by the maximum summand. If $\zeta \neq 1$, then

$$\mathbb{E}\left[\sum_{i=1}^N w_i\right] = w_{\max}\frac{1}{1-\zeta}\left[1 - NB\left(N, 1/\zeta\right)\right] \cong w_{\max}\frac{1}{1-\zeta}\left[1 - \Gamma\left(1/\zeta\right)N^{1-1/\zeta}\right],$$

where $B(\cdot,\cdot)$ and $\Gamma(\cdot)$ are, respectively, the Beta and Gamma distributions.[14] In our case, we use

[14] The approximation $B\left(n, 1/\zeta'\right) \sim \Gamma\left(1/\zeta'\right) n^{-1/\zeta'}$ is valid because $n$ is large and $1/\zeta'$ is a constant.

**Figure 4.8.** Granular curve characterization.




**Notes**: "Model" refers to $R(\mathcal{I})$ calculated using equation (4.23) and "Fit" refers to $R(\mathcal{I})$ calculated using equation (4.25). The parameter used to calculate (4.25) are presented in Table 4.2.

the following approximation:

$$\mathbb{E}\left[\sum_{i=1}^{N} w_i^2\right] = w_{\max}^2 \frac{2}{2-\zeta}\left[1-\Gamma\left(2/\zeta\right)N^{1-2/\zeta}\right].$$

Further, we assume that $\mathbb{E}\left[g_{Y,it}^2\right] = \sigma_{Y,i}^2 = \sigma_{\text{roa},i}^2 = \phi^2/3$. With this assumptions in place, we characterize the contribution of the top $\mathcal{I}$ firms to aggregate volatility as

$$R(\mathcal{I}) = w_{\max}\frac{\ell\phi}{\sigma_Y\sqrt{3}}\left\{\frac{2}{2-\zeta}\left[1-\Gamma\left(2/\zeta\right)\mathcal{I}^{1-2/\zeta}\right]\right\}^{1/2}, \tag{4.25}$$

where $w_{\max}^2 \equiv \max\left(\mathbb{E}\left[w_{it}^2\right]\right)$. Table 4.2 presents the estimated parameters use to calculate the predicted contribution. As Figure 4.8 renders clear, our framework is able to characterize the dynamics of the contribution of large firms volatility to aggregate volatility.

### 4.4.2 Granular size measurement

Given the good characterization produced by equation (4.25), we employ our model to provide a set of measures that quantify the granular size of the economy, i.e., how many granular firms populate the economy. We use the three measures proposed in Chapter 3, which are based on the following three definitions of granular firms:

1. Those firms whose marginal contribution is above a constant contribution.

2. Those firms that account for 75% of the maximum granular contribution.

3. Those firms whose marginal contribution is above the marginal contribution in the equally-weighted firms scenario.

As argue above, we focus on the top $Q = 200$ to quantify the granular size of the economy. Definition 1 considers a constant contribution between the largest firm and $Q$ and quantifies the granular size as the number of firms for which the marginal contribution is above the constant contribution. Using the *mean value theorem*, the number of granular firms is given by

$$K^*_{\mathcal{M}} = \left[ \frac{Q}{1 - Q^{1-2/\zeta}} \left( \frac{2}{\zeta} - 1 \right) \right]^{\zeta/2}. \tag{4.26}$$

Using the estimated parameters presented in Table 4.2, we find that $K^*_{\mathcal{M}} = 26$.

Definition 2 relies on the fact that $R$ has an upper bound equal to 1. We propose an arbitrarily chosen threshold of 75% of this value. Thus, the granular firms are those firms whose accumulated contribution is equal to 0.75. According to Definition 2 the number of granular firms is given by

$$K^*_{\mathcal{A}} = \left[ \frac{1 - \mathcal{T}/\mathcal{A}}{\Gamma\left(2/\zeta\right)} \right]^{\zeta/(2-\zeta)} \tag{4.27}$$

where $\mathcal{T}$ is set to 0.75 and the maximum contribution is

$$\mathcal{A} = w_{\max} \frac{\ell\phi}{\sigma_Y \sqrt{3}} \sqrt{\frac{2}{2-\zeta}}. \tag{4.28}$$

We find that $K^*_{\mathcal{A}} = 11$.

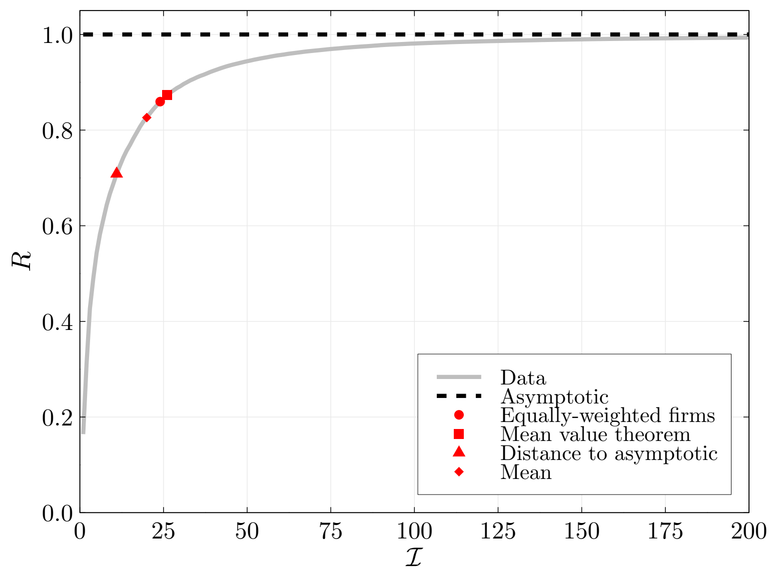
Finally, Definition 3 uses the counterfactual in which all firms are of equal size and defines the number of granular firms as those whose marginal contribution is above the marginal contribution in the equal-weight counterfactual. According to Definition 3, the number of granular firms is given by

$$K^*_{\overline{S}} = \left( w_{\max} \frac{\ell\phi}{\sigma_Y \sqrt{3}} \right)^{\zeta} \left[ Q\Gamma \left( \frac{2}{\zeta} + 1 \right) \right]^{\zeta/2}. \tag{4.29}$$

We find that $K^*_{\overline{S}} = 24$.

Figure 4.9 shows the behavior of the contribution produced by the model and the calibrated granular size for each definition above. We also include the mean value of the three measures ($K^* = 20$), which appears to be closer to the point that visually represents the change from the granular to the atomistic regime. Therefore, we conclude that the granular size of the economy is

**Figure 4.9.** Granular size of the economy.




**Notes**: "Model" refers to $R(\mathcal{I})$ calculated using equation (4.23). "Asymptotic" refers to the maximum contribution of firm-level shocks shocks to the volatility of aggregate output (see equation (4.28)). "Equally-weighted firms" refers to equation (4.29). "Mean value theorem" refers to equation (4.26). "Distance to asymptotic" refers to equation (4.27).

approximately 20 firms. In other words, if the largest 20 firms did not exist, the economy would not be granular.

## 4.5 Conclusion

In this paper we use the heterogeneous interacting agent-based model presented by Delli Gatti et al. (2005), in which the financial fragility of heterogeneous firms and the banking system and their interaction on the credit market play a crucial role in shaping aggregate fluctuations, to study how the granular curve behavior empirically observed by Blanco-Arroyo et al. (2018) emerges. This model is particularly useful to study this type of behavior because it is able to generate sizeable aggregate fluctuations from purely idiosyncratic shocks and interaction among the agents and to reproduce empirical regularities. Moreover, its simple structure allows for analytical derivations, whereby we can identify precisely the main drivers of aggregate volatility.

The first part of our analysis is devoted to shed light on the determinants of the business cycle produce by the model. We show that aggregate volatility is chiefly driven by the direct impact of firm-level specific shocks. The effect of propagation of these shocks due to interactions among the agents plays a minor role. The direct impact of firm-level specific shocks is, in turn, determined by the cross-sectional dispersion of firm sizes. In addition, The fact that the model generates a distribution of firm size that is close to a Zipf allows us to attribute the aggregate fluctuations of the economy the dynamics of the largest firm and a the summary statistic that is the tail index.

In the second part of the paper, we study the contribution of the large firms' dynamics to aggregate fluctuations and show that it displays the granular curve behavior. Building on the framework introduced in Chapter 3, we characterize of such behavior and find that size of the largest firm, ratio of the representative firm-specific shocks volatility to aggregate volatility and tail index are the main drivers of aggregate volatility. Finally, the set of granular size measures introduced in Chapter 3 quantify the granular region, which is approximately 20 firms.

In future research, we plan to apply our framework to extended models such as those that allow for heterogeneity in banks sizes (e.g., Grilli et al. (2014, 2015, 2020)).

# Chapter 5

# General Conclusions and Future Perspectives

As hypothesized in Gabaix's (2011) seminal work, we show that the existence of significant heterogeneity at the firm level causes that the idiosyncratic shocks to the Spanish large firms do not cancel out with shocks to smaller firms and translate into aggregate fluctuations. We follow the literature and test the "granular" hypothesis using an exogenously given number of firms. However, we note that such "pointwise" estimation could be misleading for the researcher because it does not provide information on the extent of the granular size of the economy—those large firms whose idiosyncratic shocks contribute significantly to aggregate fluctuations. We calibrate the granular size by exploring the behavior of the contribution of idiosyncratic shocks to GDP growth volatility as the number of large firms gradually increases and find that the contribution increases rapidly when the very top firms are taken into account and an almost steady value from a given number of firms onwards. We dub this behavior "granular curve". As a first approach to the problem, we propose an empirical method to estimate the granular size based on replacing large firms by smaller ones and comparing the resulting granular curve with the counterfactual case in which all firms are of equal size. The granular size is then determined by the number of large firms that, once removed, cause the empirical curve to converge to the counterfactual case.

In light of these empirical results, we then focus on characterizing the granular curve to better quantify the granular size of the economy. Building on the models develop by Gabaix (2009a) and Carvalho and Gabaix (2013), we setup a conceptual framework in which the volatility of GDP growth is driven by firm-level disturbances alone. Based on the empirical evidence provided by the industrial dynamics literature, we assume that the firm size distribution and the relationship between firm size and volatility is power law. Under these assumptions, we find that the granular curve is driven by the largest firm's share and volatility, the tail index of firm size distribution and the size-volatility elasticity. Our framework emphasizes the key role played by the largest in shaping aggregate fluctuations, the effect of the size-volatility relationship and the fact that the granular contribution to aggregate fluctuations is bounded. In addition, it allows us to better quantify the granular size of the economy because the calibrated size is closer to the point that visually represents the change from the granular to the atomistic regime than the empirical method initially proposed. We also find that the granular size exhibits a cyclical behavior: the number of firms whose idiosyncratic shocks have an impact on the aggregate grows in expansion phases and shrinks in recession phases.

Finally, we study the emergence of the granular curve behavior using Delli Gatti et al.'s (2005) agent-based model, which is able to generate sizeable aggregate fluctuations from purely idiosyncratic shocks and interaction among the agents, as well as to reproduce empirical regularities such as a power-law firm size distribution and a Laplace distribution of growth rates. We show that aggregate fluctuations are mainly driven by the direct impact of firm-specific shocks which, in turn, are determined by the cross-sectional dispersion of firm sizes. The fact that the model is able to generate a power-law distributed sizes allows us to trace back the aggregate output fluctuations to the dynamics of the largest firm and the summary statistics that is the tail index. The model also generates the granular curve behavior documented empirically. We find that this behavior is driven by the size of the largest firm, the ratio of the representative firm-specific shocks volatility to aggregate volatility and the tail index. Finally, we estimate that the granular size of the economy is approximately 20 firms.

The present thesis constitutes the first attempt to quantify the granular size of the economy. In future research, we plan to extend our framework to other countries and to include amplification mechanisms of shocks such as the input-output network. These two additional dimensions could provide greater insights on the observed differences in output volatility across countries.

# Appendices

# Appendix A

# Granular Firms and Aggregate Fluctuations

## A.1 Proof of Proposition 1

We first plug the relationship between size and shock volatility (3.5) into the variance of the granular residual (i.e., the squared granular volatility (3.3)). Rearranging, we have that

$$\sigma_Y^2 = \mu^2 \frac{S_{\min}^{2\alpha}}{Y^2} \sigma_{\min}^2 \sum_{i=1}^{n} \mathcal{S}_i^2,$$

where $\mathcal{S}_i \equiv S_i^{1-\alpha}$. If $S_i$ is drawn from power law distribution (3.4), then $\mathcal{S}_i$ is a power law with exponent $\zeta' \equiv \zeta / (1-\alpha)$.[1] Zaliapin et al. (2005) show that the sum of *i.i.d.* power-law summands can be replaced by the maximum summand. If $\zeta' \neq 1$, then

$$\mathbb{E}\left[\sum_{i=1}^{n} \mathcal{S}_i\right] = \mathcal{S}_{\max} \frac{1}{1-\zeta'} \left[1 - nB\left(n, 1/\zeta'\right)\right] \cong \mathcal{S}_{\max} \frac{1}{1-\zeta'} \left[1 - \Gamma\left(1/\zeta'\right) n^{1-1/\zeta'}\right],$$

where $B(\cdot,\cdot)$ and $\Gamma(\cdot)$ are, respectively, the Beta and Gamma distributions.[2] In our case, we use the following approximation:

$$\mathbb{E}\left[\sum_{i=1}^{n} \mathcal{S}_i^2\right] = \mathcal{S}_{\max}^2 \frac{2}{2-\zeta'} \left[1 - \Gamma\left(2/\zeta'\right) n^{1-2/\zeta'}\right].$$

[1] Recall that the CCDF (3.4) corresponds to a density $p(S) = \zeta S_{\min}^{\zeta} S^{-(\zeta+1)}$, then the density of $\mathcal{S}$ is

$$p(\mathcal{S}) = \frac{p(S)}{d\mathcal{S}/dS} = \frac{\zeta S_{\min}^{\zeta}}{(1-\alpha) S^{1+\zeta-\alpha}} = \frac{\zeta \mathcal{S}_{\min}^{\zeta/(1-\alpha)}}{(1-\alpha) \mathcal{S}^{(1+\zeta-\alpha)/(1-\alpha)}} = \frac{\zeta' \mathcal{S}_{\min}^{\zeta'}}{\mathcal{S}^{1+\zeta'}}.$$

[2] The approximation $B\left(n, 1/\zeta'\right) \sim \Gamma\left(1/\zeta'\right) n^{-1/\zeta'}$ is valid because $n$ is large and $1/\zeta'$ is a constant.

Then, the variance of GDP growth is

$$\sigma_Y^2 = \mu^2 \left(\frac{S_{\max}}{Y}\right)^2 \sigma_{\min}^2 \left(\frac{S_{\min}}{S_{\max}}\right)^{2\alpha} \frac{2}{2-\zeta'} \left[1 - \Gamma\left(2/\zeta'\right) n^{1-2/\zeta'}\right]. \tag{A.1}$$

We use the relationship (3.5) and define $\sigma_{\max} \equiv \sigma_{\min}\left(S_{\min}/S_{\max}\right)^{\alpha}$. Equation (3.6) is the square root of equation (A.1) with $\sigma_{\max}$.

In the case of $\zeta' = 2$, the expression

$$\frac{2}{2-\zeta'} \left[1 - \Gamma\left(2/\zeta'\right) n^{1-2/\zeta'}\right]$$

can be approximated using the Euler–Mascheroni constant $\gamma$, which is defined as:

$$\gamma = \lim_{x \to 0} \left[\frac{1}{x} - \Gamma(x)\right].$$

To see this, let $2/\zeta' = 1 + x$, then

$$\lim_{\zeta' \to 2^-} \frac{2}{2-\zeta'} \left[1 - \Gamma\left(2/\zeta'\right)\right] = \lim_{x \to 0^-} (1+x) \left[\frac{1}{x} - \Gamma(x)\right] = \gamma.$$

And let $2/\zeta' = 1 - x$, then

$$\lim_{\zeta' \to 2^+} \frac{2}{2-\zeta'} \left[1 - \Gamma\left(2/\zeta'\right)\right] = \lim_{x \to 0^+} \left[\frac{1}{x} - \Gamma(x)\right] = \gamma.$$

Alternatively, $\zeta' = 1$ only if $\zeta = 1$ and $\alpha = 0$. Thus, Gibrat's law holds true, and the behavior of the volatility of shocks is well characterized by the average volatility $\overline{\sigma}$. In this case, the variance of GDP growth is

$$\begin{aligned}\sigma_Y^2 &= \mu^2 \frac{1}{Y^2} \overline{\sigma}^2 \sum_{i=1}^{n} S_i^2 \\ &= \mu^2 \left(\frac{S_{\max}}{Y}\right)^2 \overline{\sigma}^2 \sum_{i=1}^{n} \frac{1}{i^2} \\ &= \mu^2 \left(\frac{S_{\max}}{Y}\right)^2 \overline{\sigma}^2 \frac{\pi^2}{6}.\end{aligned}$$

Equation (3.7) is the square root of the above expression.

## A.2 Proof of Proposition 2

We cannot assess de impact of $\delta$ and $\alpha$ on $R^2$ using equation (3.8) because we have to take into their impact on $S_{\max}$ and $\sigma_{\max}$ first. As discussed in Section 3.3.4, $S_{\max} = n^{1/(1+\delta)} S_{\min}$, thus an increase in $\delta$ decreases $S_{\max}$, *ceteris paribus.* Using the definition of $\sigma_{\max}$ and the relationship between $\delta$ and $S_{\max}$, we have that $\sigma_{\max} = \sigma_{\min} n^{-\alpha/(1+\delta)}$. Therefore, *ceteris paribus*, $\sigma_{\max}$ increases when $\delta$ increases and decreases when $\alpha$ increases. Plugging these expressions into equation (A.1) and rearranging, we get that

$$R^2 = \mu^2 \left(\frac{S_{\min}}{Y}\right)^2 \left(\frac{\sigma_{\min}}{\sigma_Y}\right)^2 n^{2/\zeta'} \frac{2}{2-\zeta'} \left[1 - \Gamma\left(2/\zeta'\right) n^{1-2/\zeta'}\right].$$

The impact of $\delta$ and $\alpha$ on $R^2$ is then

$$\begin{aligned}
\frac{\partial R^2}{\partial \delta} &= \frac{\partial \zeta'}{\partial \delta}\frac{\partial R^2}{\partial \zeta'} = \frac{1}{1-\alpha}\frac{\partial R^2}{\partial \zeta'} = \frac{\zeta'}{1+\delta}\frac{\partial R^2}{\partial \zeta'} < 0 \\
\frac{\partial R^2}{\partial \alpha} &= \frac{\partial \zeta'}{\partial \alpha}\frac{\partial R^2}{\partial \zeta'} = \frac{1+\delta}{(1-\alpha)^2}\frac{\partial R^2}{\partial \zeta'} = \frac{\zeta'}{1-\alpha}\frac{\partial R^2}{\partial \zeta'} < 0,
\end{aligned}$$

where

$$\begin{aligned}
\frac{\partial R^2}{\partial \zeta'} = \mu^2 \left(\frac{S_{\min}}{Y}\right)^2 \left(\frac{\sigma_{\min}}{\sigma_Y}\right)^2 \frac{2}{\zeta'^2\left(2-\zeta'\right)^2} \Bigg\{ n^{2/\zeta'} \left[\zeta'^2 + 2\left(2-\zeta'\right)\ln n\right] - \\
n\Gamma\left(2/\zeta'\right)\left[\zeta'^2 + 2\left(2-\zeta'\right)\frac{\Gamma'\left(2/\zeta'\right)}{\Gamma\left(2/\zeta'\right)}\right]\Bigg\} < 0
\end{aligned}$$

As one would expect, the more homogeneity in the firm size distribution (larger $\delta$) and the more inelastic is volatility to size (larger $\alpha$), the lower the contribution of idiosyncratic shocks to GDP fluctuation (smaller $R^2$). The ratio between the two expressions is the elasticity (3.11).

## A.3 Alternative Construction of the Granular Residual

In our baseline specification (see Section 3.3.2), firm-level labor productivity growth rates are defined as yearly natural log differences. Due to the existence of mergers, acquisitions or measurement errors, we observe some large jumps. We follow the convention in the literature and attenuate the impact of these outliers by winsorizing them. This technique has recently been criticized by Dosi et al. (2018), who argue that supply-driven granular shocks play no role when this cleaning procedure is not carried out. In this section, we show that our results are robust to the data cleaning strategy.

### A.3.1 Alternative productivity growth rates

As an alternative approach, we now calculate the firm-level labor productivity growth rates using the arc-elasticity adopted by Davis et al. (1996):

$$g'_{it} \equiv 2\left(\frac{z_{it} - z_{it-1}}{z_{it} + z_{it-1}}\right). \tag{A.2}$$

That is, the denominator is the average of the beginning and end period levels, rather than the beginning period level. This growth rate, which we label DHS, has two main advantages compared with the log difference. First, it ranges from $-2$ to 2 and thus limits the impact of outliers. Second, it avoids pitfalls associated with temporary transitory shocks and measurement errors (Neumark et al., 2011).

The idiosyncratic productivity shocks are estimated as in (3.12), namely the deviation of $g'_{it}$ from the common shock $\eta'_t(Q)$:

$$\varepsilon'_{it} = g'_{it} - \eta'_t(\mathcal{Q}). \tag{A.3}$$

where the common shock to the top $Q = 1000$ firms is estimated as the median productivity growth rate for Spain. The key difference with respect to equation (3.12) is that shocks are already bounded. Thus, these estimated shocks do not present extreme values that need to be winsorized to an exogenously determined threshold, such us 50% in Section 3.3.2.

We now assess the existing differences between specification (3.12) and (A.3) by constructing the empirical probability density of the idiosyncratic shocks. Figure A.1 presents the pooled empirical densities of idiosyncratic productivity shocks to the top $Q$ largest firms. We show the pooled densities rather than the year-by-year densities because they are quite stable through time (see Figure A.5). They exhibit a markedly "tent-shape" form on semi-log scale. This is a well-known behavior of the distribution of firm size growth rates since the seminal work of Stanley et al. (1996). They found that the distribution of the growth rates of sales and employees is well characterized by an *Laplace distribution*. Delli Gatti et al. (2005) show that this behavior is caused by the fact that both variables exhibit a power-law behavior. In particular, they show that when the logarithm of a power-law random variable follows an exponential distribution, the difference of two exponential random variables becomes a Laplace distribution. Therefore, given that our measure of labor productivity is the sales per employee ratio, it is not surprising that the distributions of the productivity growth rate and idiosyncratic shock display also such behavior.[3]

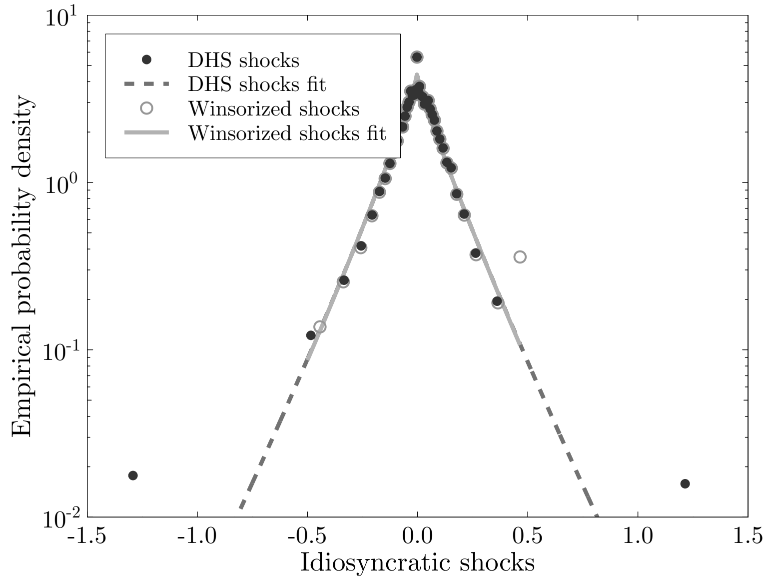
In the literature that studies the empirical distribution of growth rates (see, e.g., Bottazzi et al. (2002), Bottazzi and Secchi (2006), Bottazzi et al. (2011)), it is standard to use the *exponential power distribution*, also known as *Subbotin distribution* (Subbotin, 1923):

$$f_S(x; a, b, m) = \frac{1}{2ab^{1/b}\Gamma\left(1 + 1/b\right)} \exp\left(-\frac{1}{b}\left|\frac{x-m}{a}\right|^b\right), \tag{A.4}$$

where $a$, $b \in \mathbb{R}^+$, $m \in \mathbb{R}$, $\Gamma(\cdot)$ denotes the Gamma function and $x \in \{\varepsilon_{it}, \varepsilon'_{it}\}$. The distribution is characterized by the scale parameter $a$, the shape parameter $b$ and the location parameter $m$. It comprises the Laplace ($b = 1$) and the normal ($b = 2$) distributions as special cases. As expected, the empirical distributions are well approximated by (A.4) (see Figure A.1). The

[3]Recall that firm $i$'s labor productivity growth rate is simply the difference between $i$'s sales and employees growth rates: $g_{it} = \Delta \ln z_{it} = g_{it}^{\text{sales}} - g_{it}^{\text{employees}}$, where $g_{it}^{\text{sales}} = \Delta \ln \text{sales}_{it}$ and $g_{it}^{\text{employees}} = \Delta \ln \text{employees}_{it}$. If both $g_{it}^{\text{sales}}$ and $g_{it}^{\text{employees}}$ are distributed as a Laplace, then so is $g_{it}$. In addition, if $\varepsilon_{it} = g_{it} - \eta_t$, where the shock $\eta_t$ is common to all firms in year $t$, then $\varepsilon_{it}$ is also distributed as a Laplace.

**Figure A.1.** Empirical probability density of idiosyncratic productivity shocks.




**Notes**: Pooled empirical densities on semi-log scale of idiosyncratic productivity shocks to the top $Q = 1000$ largest firms in Spain during the period 1995-2018. Winsorized shocks and DHS shocks refer to the estimated idiosyncratic shocks according to equation (3.12) (see Section 3.3.2) and (A.3), respectively. The solid and dashed lines show the exponential power distribution fit (A.4) obtained by maximum likelihood estimation of the scale ($a$), shape ($b$) and location ($m$) parameters. The resulting estimates are shown in Table 3.2.

**Table A.1.** Maximum likelihood estimates of the exponential power distribution.

| | $\varepsilon$ | $\varepsilon'$ |
|---|---|---|
| $a$ | $0.122^{***}$ | $0.122^{***}$ |
| | (0.004) | (0.004) |
| $b$ | $0.893^{***}$ | $0.886^{***}$ |
| | (0.069) | (0.067) |
| $m$ | $-0.003^{**}$ | $-0.003^{**}$ |
| | (0.001) | (0.001) |

**Notes**: Maximum likelihood estimates of the exponential power distribution (A.4). Winsorized shocks $\varepsilon$ are estimated using (3.12) and DHS shocks $\varepsilon'$ are estimated using (A.3). Standard errors in parentheses. *, ** and *** indicate significance at 10% , 5% and 1%.

resulting estimates are shown in Table A.1, along with the corresponding standard errors. Notice that all parameters are virtually unaffected by winsorization. Contrary to Dosi et al. (2018), the distribution of productivity shocks remains fat-tailed after the winsorizing procedure.

The fact that the distribution of the idiosyncratic shocks is fat-tailed is in direct contradiction with the prediction of Gibrat's law that the distribution should be normal. Therefore, models that

**Table A.2.** Alternative idiosyncratic shocks volatility and size.

| | $\ln \sigma'_{i\tau}$ | |
|---|---|---|
| Constant | $-2.000^{***}$ | $-1.792^{***}$ |
| | (0.032) | (0.062) |
| $\ln \tilde{S}_{i\tau}$ | $-0.037^{*}$ | $-0.164^{***}$ |
| | 0.022 | 0.062 |
| $\varphi_\tau$ | ✓ | ✓ |
| $\varphi_i$ | | ✓ |
| Observations | 5,435 | 5,435 |
| $R^2$ | 0.028 | 0.557 |
| Number of clusters | | 1,870 |

**Notes**: The specifications use the four-year standard deviation of annual productivity shocks, calculated using DHS growth rates (A.3), to the $Q = 1000$ largest firms in the time period 1995-2018. The size is computed at its mean value over the four-year window. Clustered (by firm) standard errors in parentheses. *, ** and *** indicate significance at 10% , 5% and 1%.

consider Gibrat's law as a baseline (e.g., Gabaix (2011), di Giovanni and Levchenko (2012a) and Carvalho and Grassi (2019)) not only omit the negative relationship between size and volatility but also implicitly impose a degree of homogeneity in the size of idiosyncratic shocks to large firms that is at odds with the piece of evidence presented.

### A.3.2 Alternative estimation of the size-volatility relationship

Before assessing the potential impact of the winsorizing process on the granular curve, we re-estimate the relationship between size and the volatility of DHS shocks using our baseline methodology (see Section 3.3.5). Table A.2 presents the estimates for the elasticity $\alpha$. Compared to our baseline estimation, these results show that the deviation from Gibrat's law is even greater when we do not resort to the winsorization process to handle outliers. Yeh (2019) also finds that the relationship between size and the volatility of growth rates becomes steeper when using DHS growth rates rather than log-difference growth rates.

As an alternative to the Koren and Tenreyro (2013)'s methodology, we also estimate the relationship using the cross-sectional methodology employed by Stanley et al. (1996) and Sutton (2002). Although this methodology captures the degree of dispersion in idiosyncratic shocks rather than firm's shock volatility over time, we decide to take it into account because it has long been used by the literature. The procedure is as follows. First, we pool the normalized sales (i.e., $S_{it}/S_{\min,t}$) and productivity shocks to the top $Q = 1000$ firms over time and divide them into $\mathcal{B} = 16$ bins using normalized sales. Second, we fit the empirical distribution of shocks in each bin B, with $\mathrm{B} = 1, \ldots, \mathcal{B}$, using the distribution (A.4). Thus, we have an estimate for the scale parameter $a_{\mathrm{B}}$, shape parameter $b_{\mathrm{B}}$ and location parameter $m_{\mathrm{B}}$ for each bin B. We compute the cross-sectional

**Table A.3.** Idiosyncratic shocks dispersion and size.

| | $\ln \sigma_{\mathrm{B}}$ | | | |
|---|---|---|---|---|
| | Subbotin fit | | Standard deviation | |
| | Winsorized | DHS | Winsorized | DHS |
| Constant | $-1.633^{***}$ | $-1.550^{***}$ | $-1.418^{***}$ | $-0.983^{***}$ |
| | (0.107) | (0.267) | (0.129) | (0.186) |
| $\ln \tilde{S}_{\mathrm{B}}$ | $-0.102^{***}$ | $-0.174^{*}$ | $-0.187^{***}$ | $-0.247^{***}$ |
| | (0.033) | (0.083) | (0.040) | (0.058) |
| Observations | 16 | 16 | 16 | 16 |
| $\mathrm{R}^2$ | 0.401 | 0.239 | 0.608 | 0.566 |

**Notes**: Winsorized refers to growth rates calculated using (3.12). DHS refers to growth rates calculated using (A.3). Subbotin fit estimates the shocks volatility using (A.4). Standard deviation estimates the shocks volatility using the standard deviation of the shocks within bin B. The number of observations corresponds to number of bins $\mathcal{B}$. Standard errors in parentheses. *, ** and *** indicate significance at 10% , 5% and 1%.

standard deviation[4]

$$\sigma_{\mathrm{B}} = a_{\mathrm{B}} b_{\mathrm{B}}^{1/b_{\mathrm{B}}} \frac{\sqrt{\Gamma\left(3/b_{\mathrm{B}}\right)}}{\sqrt{\Gamma\left(1/b_{\mathrm{B}}\right)}}$$

and the average normalized sales $\tilde{S}_{\mathrm{B}}$ within each bin. Finally, we run the following OLS regression:

$$\ln \sigma_{\mathrm{B}} = \text{constant} + \alpha \ln \tilde{S}_{\mathrm{B}} + u_{\mathrm{B}}. \tag{A.5}$$

The estimated coefficient from (A.5) reflects the relationship between size and dispersion. As noted by Thesmar and Thoenig (2011), the main disadvantage of using cross-sectional dispersion is that it does not remove the average growth rate of the firm and, hence, it does not eliminate the bias in the evolution of firm volatility caused by a change in the distribution of firm's growth potential. However, the results are similar to the baseline specification (see Table A.3).

### A.3.3 Alternative granular residual

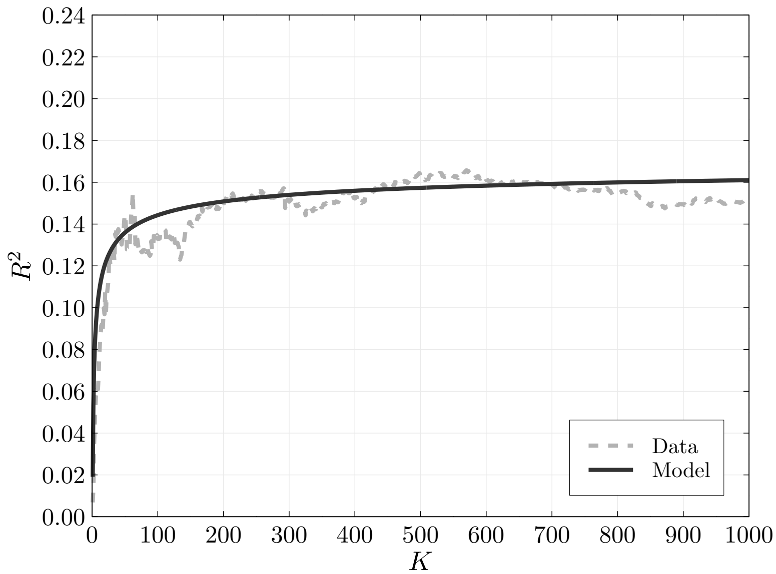
Under DHS definition of growth rates (A.2), the correct weights for aggregation are

$$w'_{it} \equiv \frac{S_{it} + S_{it-1}}{Y_t + Y_{it-1}}, \tag{A.6}$$

[4] Due to its symmetry, the Subbotin density has all central moments of odd order equal to zero. The central moment of order $2l$ reads

$$M_{2l} = \left(ab^{1/b}\right)^{2l} \frac{\Gamma\left(\left(2l+1\right)/b\right)}{\Gamma\left(1/b\right)}.$$

**Figure A.2.** Granular curve with alternative shocks.




**Notes**: The parameters used in the approximation are: $\mu = 3.5$, $S_{\max/Y} = 0.0138$, $\sigma_{\max} = 0.102$, $\sigma_Y = 0.023$, $\zeta = 1.252$ and $\alpha = 0.15$.

the granular residual is

$$\mathcal{E}'_t = \sum_{i=1}^{K} w'_{it}\varepsilon'_{it} \tag{A.7}$$

and GDP growth is

$$g'_Y \equiv 2\left(\frac{Y_t - Y_{t-1}}{Y_t + Y_{t-1}}\right). \tag{A.8}$$

Following the procedure described in Section 3.4, we compute the empirical contribution of idiosyncratic shocks to GDP growth fluctuations, its approximation and calibrate de granular size of the economy. Figure A.2 shows that the granular curve behavior is still present and the approximation characterizes such behavior. The calibrated number of granular firms is 45.

## A.4 Additional Figures

**Figure A.3.** Sample characteristics.

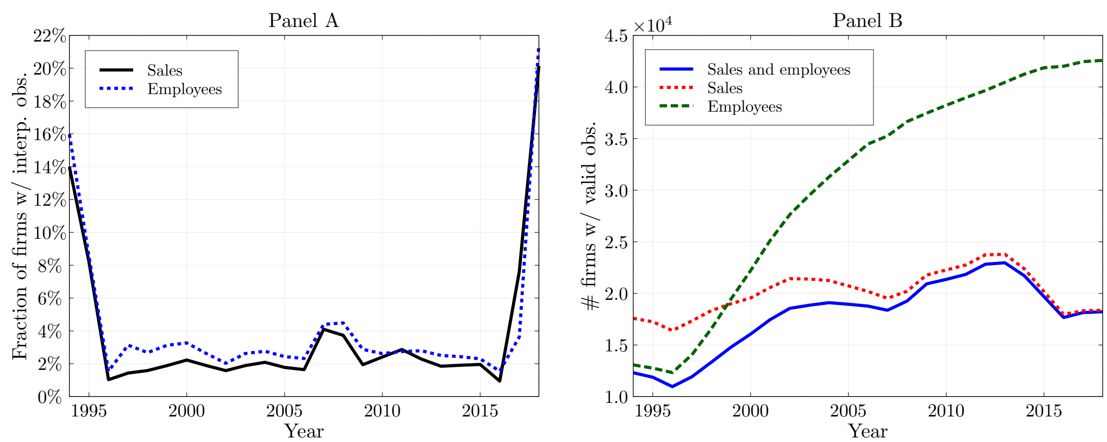
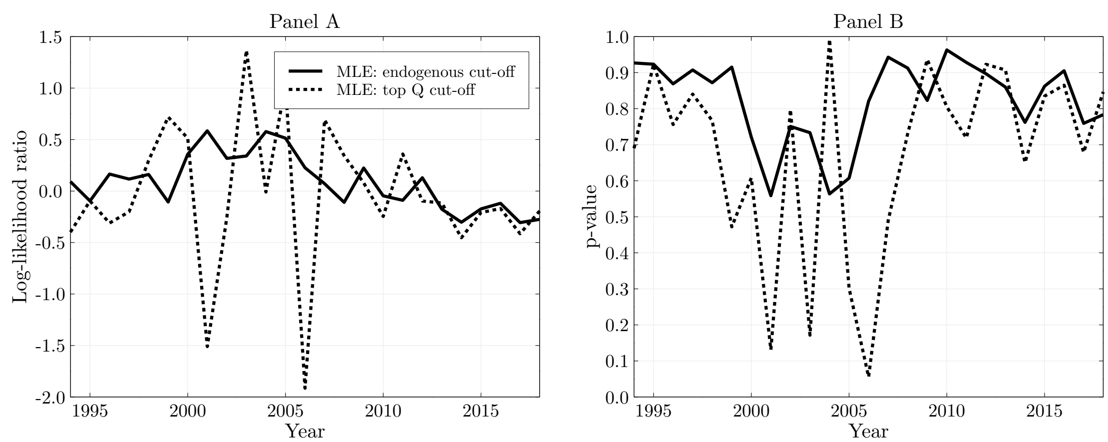
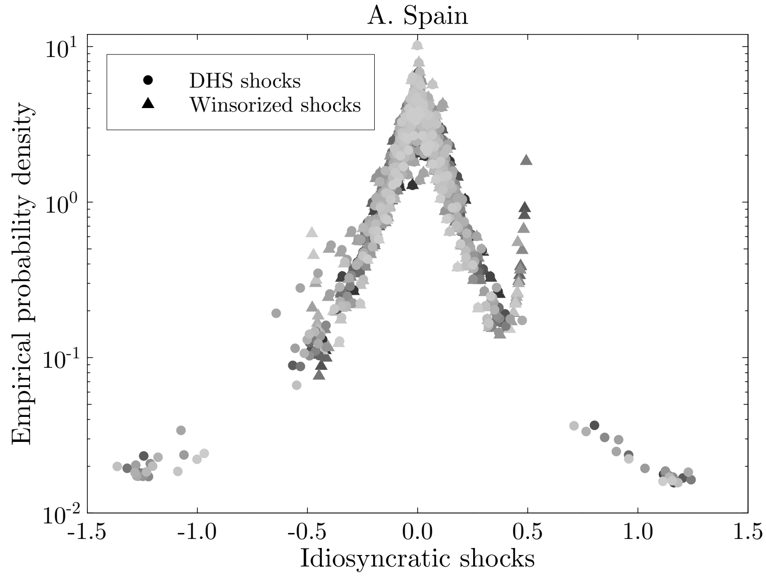



**Figure A.4.** Comparison of candidate distributions.




**Notes**: Panel A compares the power-law and log-normal candidate models using Vuong's (1989) normalized likelihood ratio $\mathcal{R}$. We fit both models on the firm size distribution using MLE. Panel B shows the associated p-value.

**Figure A.5.** Year-by-year empirical probability density of idiosyncratic productivity shocks.




**Notes**: Year-by-year empirical densities on semi-log scale of idiosyncratic productivity shocks to the top $Q = 1000$ largest firms in Spain during the period 1995-2018. Winsorized shocks and DHS shocks refer to the estimated idiosyncratic shocks according to equation (3.12) (see Section 3.3.2) and (A.3), respectively.

# Appendix B

# Heterogenous Interacting Agents and Aggregate Fluctuations

## B.1 The model without uncertainty

### B.1.1 Long-run dynamics

We analyze the long-run dynamics of the economy in absence of uncertainty and hence heterogeneity. We set the relative price equal to its expected value, that is: $u_{it} = 1 \; \forall i, t$. Using equations (4.3), (4.4) and (4.8), and assuming a constant interest rate, the law of motion of motion of net worth is

$$A_t = \left(1 + \frac{\phi - gr_t}{2gr_t}\right) A_{t-1} + \left(\frac{\phi - gr_t}{c\phi gr_t}\right)^2 . \tag{B.1}$$

Our aim is to characterize the dynamics of (B.1) analitically. To do this, we need to determine the growth rate of the economy. Following Pulcini (2017), we start by studying the net worth growth rate of the representative firm. According to the law of motion (4.4), this is

$$g_{At} = \frac{\Delta A_t}{A_{t-1}} = \frac{\pi_t^f}{A_{t-1}}. \tag{B.2}$$

Using the firm's profit (4.3) and the long-run approximation of the optimal capital stock (4.8),[1] the growth rate is then given by

$$g_{At} = \frac{\phi - gr_t}{2gr_t}. \tag{B.3}$$

We now turn to the net worth growth rate of the bank. According to the law of motion (4.13)

[1] In equation (4.8), the term $\frac{A_{t-1}}{2gr_t}$ grows exponentially, so that the term $\frac{\phi - gr_t}{c\phi gr_t}$ is negligible. Then, we can approximate the optimal capital stock in the long run as $K_t \approx \frac{A_{t-1}}{2gr_t}$.

and assuming that the representative firm does not go bankrupt, this is

$$g_{Et} = \frac{\Delta E_t}{E_{t-1}} = \frac{\pi_t^b}{E_{t-1}}. \tag{B.4}$$

Using the bank's profit (4.12), the balance sheet of the bank and the prudential rule, the growth rate is then given by[2]

$$g_{Et} = r_t \omega \beta, \tag{B.5}$$

where $\beta \equiv 1/\nu - 1$.

As in Pulcini (2017), we get the equilibrium interest rate by equating (B.3) to (B.5):

$$r_t = \frac{\sqrt{1 + \frac{8\omega\beta\phi}{g}} - 1}{4\beta\omega} \ \forall t.$$

Given the values of the parameters (see Table 4.1), the interest rate can be approximated by the *Taylor expansion* for $\sqrt{1+x}$, which is $1 + \frac{2}{x} - \frac{1}{8}x^2 + o(x)$:

$$r = \frac{\phi}{g} - \frac{2\omega\beta\phi^2}{g^2}. \tag{B.6}$$

Plugging (B.6) into (B.3) and (B.5), and equating them, we get that the growth rate of the economy is determined by

$$\chi \equiv \frac{\omega\beta\phi}{g}. \tag{B.7}$$

Thus, the law of motion (B.1) is approximated by

$$A_t = A_0 \left(1 + \chi\right)^t, \tag{B.8}$$

where $A_0$ is the net worth in initial time period.

Figure B.1 shows the dynamics of output (Panel A), capital (Panel B) and net worth (Panel C) of the representative firm along with the approximated growth rate resulting from equation (B.7) and a dynamic similar to that presented in equation (B.8). The dynamics of the economy are well characterize by the analytic growth rate.

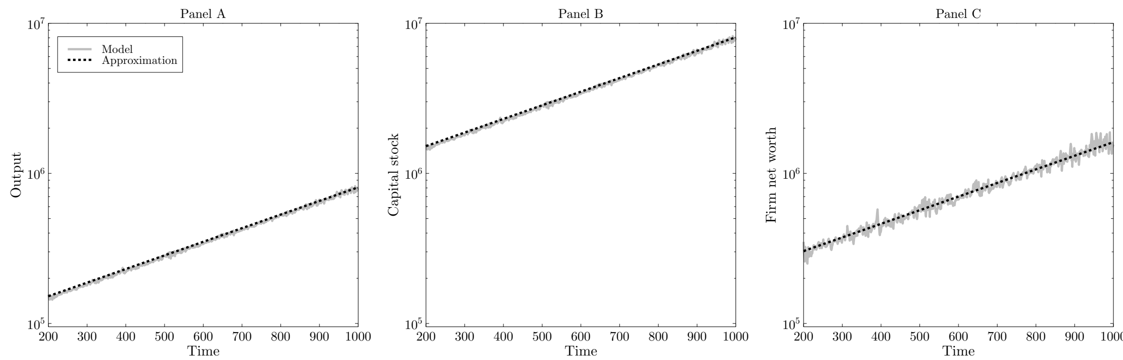
### B.1.2 Relationship between leverage and debt ratios

We approximate the growth rate of net worth (B.2) as

$$g_{At} \approx \mathrm{roa}_t \ell_t, \tag{B.9}$$

[2] We use the approximation $D_t = E_{t-1}/\nu - E_t \approx E_{t-1}/\nu - E_{t-1}$.

**Figure B.1.** Approximation of long-run dynamics.




**Notes**: Panel A approximates aggregate output using $Y_t = Y_0 (1+\chi)^t$, where $\chi$ is the growth rate of the economy (see equation (B.7)). Panel B, approximates aggregate aggregate capital using $K_t = K_0 (1+\chi)^t$. Panel C approximates aggregate firm net worth using $A_t = A_0 (1+\chi)^t$. The values of the parameters to calculate $\chi$ are presented in Table 4.1.

where $\text{roa}_t = \pi_t / K_t$ is the return on assets and $\ell_t = K_t / A_{t-1}$ is the leverage. Using the long-run approximation of optimal capital capital stock (namely, $K_t \approx \frac{A_{t-1}}{2gr_t}$) and the equilibrium interest rate (B.6), we get that $\text{roa}_t = \frac{2\omega\beta\phi^2}{g}$ $\forall t$. Given the fact that $g_{At} = \chi$ $\forall t$, the leverage is

$$\ell_t = \frac{1}{2\phi} \ \forall t. \tag{B.10}$$

Considering the motion of capital stock (4.2), the debt ratio $d_t = L_t / K_t$ satisfies

$$d_t = 1 - 2\phi \ \forall t. \tag{B.11}$$

Figure B.2 presents the leverage (Panel A) and debt (Panel B) ratios along with the approximated values resulting from equations (B.10) and (B.11), respectively. The dynamics of both ratios are well characterized by their respective analytical expressions, as shown by their proximity to the mean value.

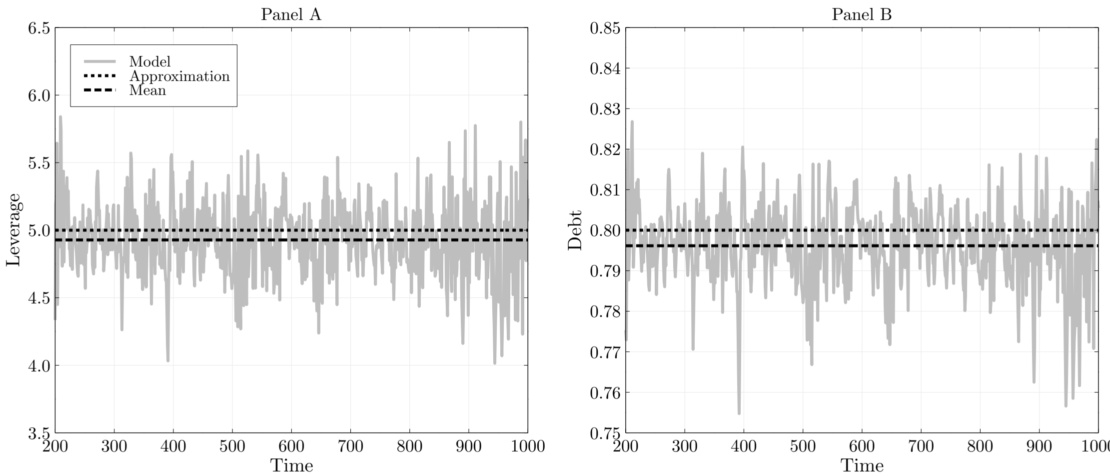
## B.2 Volatility of ROAs

The return on assets (ROA) is defined as the profit to capital ratio:

$$\text{roa}_{it} \equiv \frac{\pi_{it}}{K_{it}} = \phi u_{it} - gr_{it}, \tag{B.12}$$

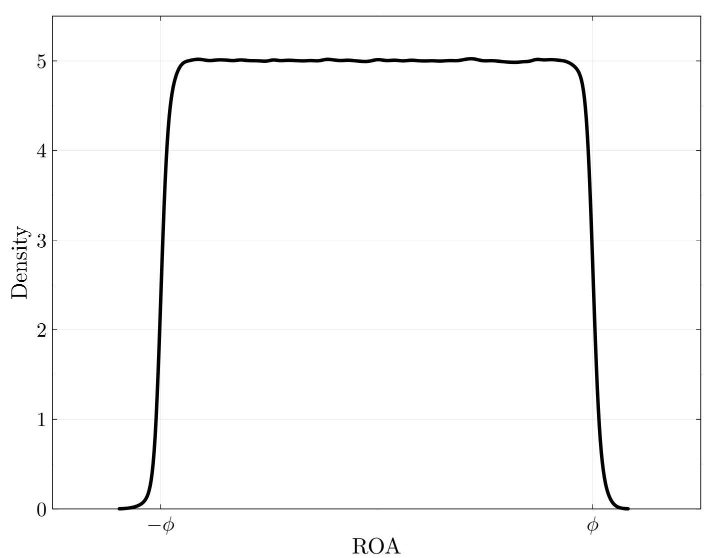
where $u_{it} \sim U(0,2)$. Plugging the deterministic interest rate (B.6) into (B.12) and using the fact that $\phi/g \gg \left(\omega\beta\phi^2\right)/g^2$, firm $i$'s ROA is $\text{roa}_{it} = \phi u_{it} - \phi$. Given the bounds set for $u_{it}$, $roa_{it} \in [-\phi, \phi]$. As expected, Figure B.3 shows that the distribution of ROA is bounded by the

**Figure B.2.** Approximation of leverage and debt ratios.




**Notes**: Panel A approximates the leverage ratio using equation (B.10). Panel B approximates the debt ratio using equation (B.11). Mean is the average of the time series.

**Figure B.3.** Distribution of ROA.




**Notes**: ROAs are pooled across time.

productivity of capital.

The variance of ROA is thus determined by

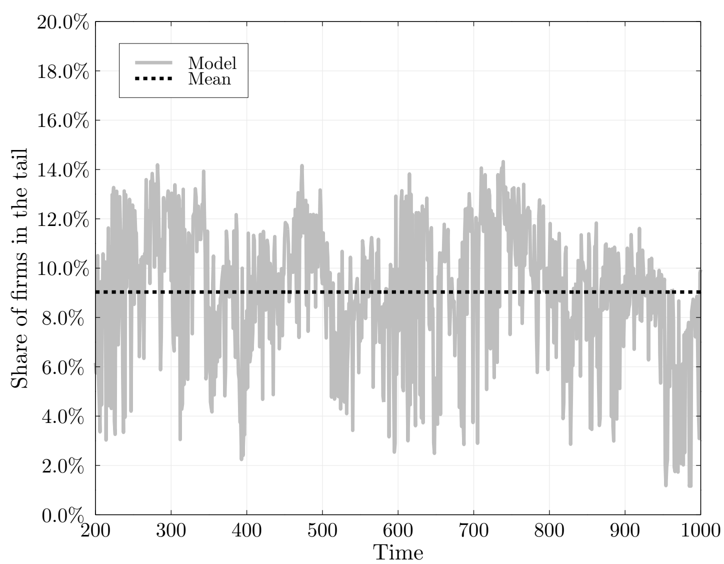
$$\mathrm{Var}\left(\mathrm{roa}_{it}\right) = \int_{-\phi}^{\phi} x^2 p\left(x\right) dx,$$

where $x$ denotes the relative price and the probability density function is $p\left(x\right) = 1/\left(2\phi\right)$. Therefore, the analytical ROA is

$$\mathrm{Var}\left(\mathrm{roa}_{it}\right) = \frac{1}{2\phi}\int_{-\phi}^{\phi} x^2 dx = \frac{\phi^2}{3}.$$

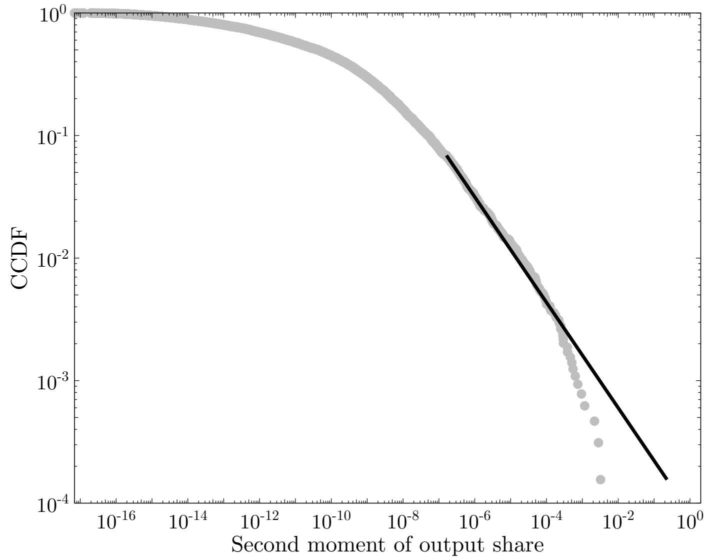
## B.3 Additional figures

**Figure B.4.** Estimated number of firms in the tail.




**Notes**: We presents the ratio $\hat{n}/N$, where $\hat{n}$ is the number of firms whose capital stock provides the best power law fit and determines the tail index in the MLE method (see equation (4.21)).

**Figure B.5.** Size distribution of the second moment of output share.




**Notes**: Second moment of output share refers to $\mathbb{E}\left[w_{it-1}^{2}\right]$ in equation (4.24).

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