Essays on Monetary Economics

Paraskevi Pappa

DEPARTMENT OF ECONOMICS

UNIVERSITAT POMPEU FABRA

Doctoral Thesis

Barcelona, August 2001

ITHAKA

As you set out for Ithaka, hope your road is a long one,
full of adventure, full of discovery.
Laistrygonians, Cyclops, angry Poseidon-don't be afraid of them:
you'll never find things like that on your way
as long as you keep your thoughts raised high,
as long as a rare excitement stirs your spirit and your body.
Laistrygonians, Cyclops, wild Poseidon-you won't encounter them
unless you bring them along inside your soul,
unless your soul sets them up in front of you.
Hope your road is a long one.
May there be many summer mornings when, with what pleasure, what joy,
you enter harbors you're seeing for the first time;
may you stop at Phoenician trading stations
to buy fine things, mother of pearl and coral, amber and ebony,
sensual perfume of every kind - as many sensual perfumes as you can;
and may you visit many Egyptian cities
to learn and go on learning from their scholars.
Keep Ithaka always in your mind.
Arriving there is what you're destined for.
But don't hurry the journey at all.
Better if it lasts for years,
so you are old by the time you reach the island,
wealthy with all you've gained on the way,
not expecting Ithaka to make you rich.
Ithaka gave you the marvellous journey.
Without her you wouldn't have set out.
She has nothing left to give you now.
And if you find her poor, Ithaka won't have fooled you.
Wise as you will have become, so full of experience,
you'll have understood by then what these Ithakas mean.

Constantinos Cavafis

Acknowledgments

If somebody would ask me what I have learned the last five years in Barcelona, I would have answered: *I have understood what these Ithakas mean.*

For that, first, I would like to thank Tryphon Kollintzas for believing in me. If it was not for Tryphon, I would not have set out in the first place.

I have started my trip accompanied by my crew. Some of these crew members have been lost (or forgotten), others have remained and others have been recruited on the way. I would like to thank them all for their presence (short or long) in my trip. Especially, I would like to thank Dora, Ifigeneia and Vanghelis for being always there. In Spain I met Eva, Arpad, Manolo, Sandro and Morten. I would like to thank Eva for being sincere and loyal to me, Arpad for passing me the ball to score when playing football and for rolling me on the dance floor, Manolo for his great "fabadas," and excursions in Raval, Sandro for supplying me with cigarettes when everything around the university was closed, and Morten for encouraging me to continue.

In Sweden I wrote my first paper and I faced the first sea storm when I had to present in public for the first time. I would like to thank Fabrizio Zillibotti for the walk that he offered me before the presentation trying to calm me down and for all his help during my first steps in economics. In England I met a very special and "neiss' person: Katharine Neiss. She, probably without knowing it, taught me to be careful, systematic and patient in my work.

As you already know from Homer, sailing for five years you often face the angry sea, you meet sirens that try to lure you, you get tired and you blame always "*angry*" and "*wild*" Poseidon - or Jordi Gali in my case. With the completion of my thesis I have realized that my trip would not have been as rich without Poseidon. Jordi has been the real captain and I would like to thank him for his guidance and his insistence for perfection.

My sea was not only made by curves of impulse responses or AR(1) processes, sometimes it had very real aspects which were much more scaring than the simulated ones. On those waters Fabio taught me to sail and to survive and I want to thank him for that.

The story finishes where it started: in those who I left and have been waiting for me. I would like to dedicate my thesis to them: to my family and especially to Ageliki and Lefteris, and to Kira Maria.

I would like to thank the Generalitat de Catalunya, Programa de Beques Predoctorales de la Direcció General de Recerca for financial support.

Contents

1. Introduction 1

2. Should the ECB and the FED Cooperate?
Optimal Monetary Policy in a Two-Country World 3

1. Introduction 4

2. The Reduced-Form Model 6

3. Optimal Monetary Policy 9

4. Calibration 12

5. International Monetary Policy Regimes 14

5.1. Cooperative Monetary Policy 14

5.2. Non-Cooperative Monetary Policy 16

5.3. Monetary Union 18

6. Approximating The Optimal Policy 21

6.1. The Optimal Plan 21

6.2. A Simple Feedback Rule 22

6.3. The Taylor Rule 24

7. Conclusions 25

Appendices 30

The Model 30

Deriving the Reduced Form Equations 37

Deriving the Social Objective in an Open Economy 41

Approximating the Optimal Plan 44

3. A Monetary Model of Factor Utilization 47

1. Introduction 47

2. A model of time-varying factor utilization 50

2.1. Households 51

2.2. Firms. 53

2.3. Government 55

2.4. Market Clearing 55

2.5. Equilibrium 56

2.6. Shocks 56

3. Benchmark Model 56

3.1. Calibration 56

3.2. Impulse Responses 59

4. Other Sticky Price Models 62

4.1. Calibration 62

4.2. Comparison of Impulse Responses. 64

5. Model Simulations. 66

5.1. Model Statistics 66

5.2. Output and Real Marginal Cost. 70

5.3. Persistence. 71

6. Robustness. 72

6.1. General Preferences. 72

6.2. Variable Capital Utilization. 73

6.3. Labour Hoarding and Variable Labour Effort 74

6.4. Endogenous Monetary Policy 74

7. Conclusions. 75

Appendices 77

Appendix 1 77

Appendix 2 78

4. Sustaining a Monetary Union 80

1. Introduction 80

2. A Two-Country Model. 82

2.1. Consumers 83

2.2. Producers 85

2.3. Governments 86

2.4. Market Clearing 86

2.5. Exogenous Processes 86

2.6. Characterization of Equilibrium. 87

3. Optimal Independent Monetary Policies 87

4. Optimal Monetary Policy in a Monetary Union:
The Case of No Participation Constraints. 89

5. Optimal Monetary Policy in a Monetary Union:
The Case of Participation Constraints. 92

6. Conclusions. 97

Appendices 99

A.1. The Unconstrained Maximum. 99

A.2. The Constrained Maximum. 101

A3. Determination of the nominal interest
rate in the unconstrained maximum. 102

A4. Determination of the nominal interest rate
in the constrained maximum 104

Bibliography 105

Chapter 1

Introduction

The thesis consists of three chapters:

Chapter 2 investigates, in the context of a two-country model with monopolistic competition and price stickiness, the implications for macroeconomic stability and the welfare properties of three distinct kinds of monetary policy arrangements: (a)cooperative, (b) noncooperative and (c) monetary union. The cooperative first best can be achieved if domestic inflation is set equal to zero in both countries at all times. In the non cooperative equilibrium welfare is not maximized due to the presence of *beggar-thy-neighbor* policies with the costs of non-cooperation increasing with the degree of openness of the economy. A monetary union is welfare-improving relative to non cooperation only for countries that have strong trading links.

In chapter 3 (co-authored with Katharine Neiss), we investigate the propagation mechanism of monetary shocks in an otherwise standard sticky price model, modified to incorporate factor hoarding in the form of variable capital utilisation rates and labour effort. In contrast, to previous studies, we find that real effects of monetary shocks can be generated at relatively low degrees of nominal rigidities. Factor hoarding enriches the propagation mechanism, by flattening the marginal cost responses to monetary shocks. The assumption of labour hoarding is crucial for generating persistence, while the assumption of variable capital utilisation allows us to generate realistic investment volatility without having to introduce capital adjustment costs.

Finally, Chapter 4 investigates whether monetary policy in a currency area should stabilize an aggregate of inflation and output or whether it should take into account the dispersion of these variables across regions when the currency area is characterized by asymmetric shocks across regions and when participation constraints define the set of feasible policies. We find that even if the two regions are identical, asymmetric shocks might create asymmetric responses of regional variables. Participation constraints introduce a trade-off between efficiency and incentives in settings with lack of a strong enforcement technology. In order to deal with incentive problems the central bank has to manipulate the future consumption paths by adopting a more expansionary policy with respect to regional productivity shocks.

Chapter 2

Should the ECB and the FED Cooperate? Optimal Monetary Policy in a Two-Country World[1]

1 Introduction

The implementation of the European Monetary Union has changed the way macroeconomic policy has been conducted within and outside Europe. The establishment of a common currency and a common central bank has in fact created a new major contender to the United States and Japan in the world monetary avenue. One question of crucial importance for developments in the world economy is whether the central banks of these three groups should cooperate or not in pursuing stabilization policies. The main purpose of this paper is to study different types of international monetary policy arrangements and to investigate how the first best solution can be implemented.

I use a two-country model where each country is specialized in the production of a bundle of differentiated goods, the production is monopolistically competitive and prices are staggered. I study the welfare properties and the implications for macroeconomic stability of three types of international monetary policy arrangements: (a) cooperative, (b) non-cooperative and (c) monetary union. Here, monetary policy non-cooperation occurs when each policymaker maximizes the utility of the domestic consumers taking as given the foreign policy variables. Cooperation in the conduct of monetary policy is modelled by assigning the conduct of monetary policy into a "supranational institution" that maximizes the average utility of both countries consumers. Finally, monetary union is modeled in the same way, but with the additional constraint that exchange rates have to be fixed. I simulate the model under

[1] I would like to thank Jordi Gali for invaluable advice and Fabio Canova and Morten Ravn for many useful discussions.

these three different assumptions and perform welfare comparisons. In the discussion I set aside time-consistency issues; i.e., the policies I am analyzing are policies under commitment. The decision maker in each case cannot ignore past commitments and thus the policies analyzed are not in general time consistent in the sense discussed by Kydland and Prescott (1977).

The question of whether central banks should coordinate their monetary policy actions is not new. Many authors in the past have analyzed similar issues. Hamada (1974) studies the independent and strategic nature of monetary policies in an n-country game under fixed exchange rates and where each monetary authority decides on its credit expansion to maximize its objective function. He shows that the non cooperative solution does not generally lie in the Pareto frontier. Under some specific assumptions about the creation of international reserves and preferences of balances of payments, he concludes that the non cooperative solution might lead to inflationary pressures. In another paper on international policy coordination Oudiz and Sachs (1984) study the trade off between coordination and policy competition in a dynamic setting. they study both policies under commitment and discretion. The non cooperative equilibrium is associated with losses due to the presence of beggar-thy-neighbor policies. Cooperation may be useful in moving the countries to the efficient equilibrium. Moreover they show that policies under commitment yield higher level of social welfare than discretionary policies. On the other hand, Rogoff (1985) suggests that cooperation between central banks maybe suboptimal if there is no cooperation between the authorities and private agents.

Recently several authors have devoted time to the development and the evaluation of monetary models that bring imperfect competition and nominal rigidities into basic dynamic general equilibrium models. Since Obstfeld and Rogoff (1995) applications to open economy issues have focused to a large extent to revisiting the economics of international monetary policy interdependence. Corsetti and Pesenti (1997) carry out a welfare analysis of international monetary and fiscal policy interdependence in a model where the terms of trade play a central role in the process of adjustment to external shocks. Benigno (1998) building on the analysis of the latter, considers two countries of unequal size and the inherent conflict between monopolistic distortions and externalities on the terms of trade. He finds that in the cooperative solution, the smaller country is pushed to the competitive output level, while the larger country retains some monopoly power in order to preserve its terms of trade.

This paper is constructed around the, by now, standard stochastic general equilibrium two country model with imperfect competition and price rigidities which allows for the welfare evaluations of different policies. The objective function of the central bank is derived by approximating the expected discounted utility function of the representative consumer with a second order Taylor expansion. Rotemberg and Woodford (1998) have first derived this objective for a closed economy. In the current research, I show that in an open economy this objective includes in addition to inflation and output gap variability, the variability of the terms of trade gap. Relative prices enter the welfare criterion because they play a crucial role in the transmission of foreign shocks when there is no international price discrimination. The concern about optimal reallocation of resources between the two economies translates in an objective of the Central Bank which includes the variability and the expected value of the terms of trade gap (see also Benigno (1999) for the case of no home-bias in consumption).

By construction the cooperative solution is "the best". Non cooperative policy-making is Pareto inefficient because of the presence of beggar-thy-neighbor policies. In the non-cooperative solution the policymakers have incentives to deviate from the flexible price allocation. Each policymaker can use the terms of trade as a tool to export inflation abroad. The equilibrium is characterized by low variability of the nominal exchange rate and inefficient movements of relative prices. When the two countries are almost autarchic foreign prices affect very little domestic inflation and the inefficient movement of relative prices affects very little domestic variables. For that reason, the costs of non-cooperation are small for economies that are nearly closed.

Monetary Union is welfare improving compared to the non cooperative equilibrium, as long as the two economies are highly open to trade and are characterized by low degrees of nominal rigidities. For a small degree of openness of the economy, the distortions associated with the inertia of the terms of trade dominate the gains of coordination inside the union and the countries are better off conducting their monetary policy independently. Also, the costs from fixing the nominal exchange rate are an increasing function of the degree of nominal rigidities.

Optimal monetary policy in a two-country world is achieved by setting domestic inflation equal to zero in both countries at all times. The cooperative first best can be approximated with an interest rate rule that has very strong feedback from domestic inflation only. In practise, such a drastic response of the nominal interest rate to changes in inflation is not reasonable. However, under plausible values for the coefficient in inflation, a Taylor rule can approximate the optimal solution. Moreover, it reduces the welfare losses associated with the non-cooperative and the monetary union solutions for high degrees of openness of the economies.

Finally, given the small degree of openness for the European and the US economies, the model predicts that the gains from cooperation are small and cooperative and non-cooperative optimal policies are almost equivalent.

The paper is organized as follows. The next section describes the reduced form model. Section 3 presents the optimal monetary policy for each of the three policy regimes and offers welfare comparisons between the different regimes. Section 4 considers the form of interest rate feedback rule that can approximate the optimal dynamic responses to shocks. Finally, the last section contains concluding remarks and possible extensions for future research. In the Appendices I collect a brief outline of the model and the technical derivations of various relationships used in the paper.

2 The Reduced-Form Model

The economy consists of two countries. Each country is populated by identical, infinitely lived agents. Each agent produces a single differentiated good and consumes the goods produced in both economies. Agents derive utility from consuming an index of consumption goods and disutility from supplying output. Households maximize the expected discounted value of their utility flow.

I assume that the production units are imperfectly competitive and at each point in time, each domestic producer is allowed to reset its price with a constant probability, independently of the time elapsed since the last adjustment. There are shocks to the production of the differentiated goods at home and abroad. Producers face domestic and foreign demand for their product, but do not engage in international price discrimination.

I assume that capital markets are complete[2]. That is, agents can fully share both

[2] Although there are some reasonable instances where the wealth effects from market incompleteness are very small (Cole and Obstfeld (1991) and Chari et al. (1996)), the assumption of complete

the idiosyncratic risk linked to the randomness in the pricing technology and the risk from country specific productivity shocks.

In this section, I describe the reduced form of the economy in its log-linear form. A brief outline of the structural model is contained in Appendix 1, while details of the derivation of the following conditions are contained in Appendix 2 . Lower case variables denote the percentage deviations from respective steady state values and variables with stars denote foreign variables. c_t denotes home consumption, R_t is the domestic nominal interest rate , π_{Ht} is domestic inflation (where $\pi_{Ht} \equiv p_{Ht} - p_{Ht-1}$) and s_t is the terms of trade defined as the ratio of foreign to domestic prices. Finally, c_t^n and s_t^n denote the flexible price level of consumption and terms of trade respectively. I only present the equations for the home country since the equations for the foreign country are analogous.

The *aggregate demand* of the economy is characterized by the following conditions:

$$c_t = E_t c_{t+1} - \frac{1}{\sigma} \widehat{R}_t - E_t \pi_{Ht+1}) + \frac{\alpha}{\sigma} E_t\{\Delta s_{t+1}\} \tag{1}$$

$$(\widehat{R}_t - \pi_{Ht+1}) - (\widehat{R}_t^* - \pi_{Ht+1}^*) = E_t\{\Delta s_{t+1}\} \tag{2}$$

The *aggregate supply* is characterized by:

$$\pi_{Ht} = \beta E_t \pi_{Ht+1} + \kappa_c (c_t - c_t^n) + k_s (s_t - s_t^n) \tag{3}$$

where $k_c = k(\frac{\sigma+\omega}{1+\theta\omega})$, $k_s = \frac{k\alpha(1+\omega\eta[1+\frac{(1-2\alpha)}{\sigma}])}{1+\theta\omega}$, $k = \frac{1-\gamma}{\gamma}(1-\gamma\beta)$ and σ is the intertemporal elasticity of substitution, η is the elasticity of substitution between foreign and domestic goods, θ is the elasticity of substitution among differentiated goods produced within the country, α is the degree of openness of the economy, ω is the inverse of the elasticity of labor supply, and $1-\gamma$ is the probability that a monopolistic producer faces for resetting her price. The last two terms in (3) are proportional to the real marginal costs of the domestic producers.

Finally the *exogenous stochastic processes* for the productivity shocks are assumed an AR(1) process:

$$\xi_{t+1} = \Gamma \xi_t + \varepsilon_t \tag{4}$$

international financial markets is necessary in my analysis, since the welfare criterion I use cannot account for wealth effects at all.

where $\xi_t' = [z_t, z_t^*]$, $E_t\varepsilon_{t+1} = 0, E_t\varepsilon_t\varepsilon_t' = D$.

Equation (1) is derived from the first order condition for the representative household in the home country. It states that aggregate demand depends on expectations of future monetary policy as well as expectations for domestic inflation. In a closed economy context the last term is absent from the aggregate demand equation. In contrast, in an open economy framework, expectations for changes in the terms of trade affect domestic demand. Equation (2) is the real interest parity condition relating the movements of the real interest rate differential to the expected variations in the relative prices.

Equation (3) is the implied aggregate supply relation. The typical starting point for the derivation of equation (3) is the environment of monopolistically competitive firms that face a constraint on price adjustments. The constraint is that in any given period each firm has a fixed probability $1-\gamma$ of changing its price and, hence, a probability γ of keeping its price unchanged. Using the partial adjustment pricing assumption and the optimal price setting for the firms that receive a signal for changing their price and loglinearizing around steady state yields equation (3). In this equation domestic inflation rate depends on the expectations of future price setting behavior and on the deviations of the terms of trade and domestic consumption from their potential. The latter are defined as:

$$
\begin{aligned}
c_t^n &= \frac{\omega}{\omega+\sigma}\zeta z_t + \frac{\omega\alpha\eta[1+\frac{(1-2\alpha)}{\sigma}]}{\omega+\sigma}\zeta z_t^* \\
s_t^n &= \frac{\omega\alpha\eta[1+\frac{(1-2\alpha)}{\sigma}]}{1+\omega}\zeta(z_t - z_t^*)
\end{aligned}
$$

where c_t^n and s_t^n are the level of consumption and terms of trade that would prevail if prices were worldwide flexible.

If monopolistic distortions are neutralized, the flexible price equilibrium is the efficient outcome. In the absence of nominal distortions, monetary policy has no effects in real variables and these variables are only affected by domestic and foreign real disturbances. Under flexible prices, a positive productivity shock, independently of its origin increases domestic consumption. The terms of trade are affected by the relative size of the two productivity shocks. In the flexible price allocation any changes in productivity should be accommodated by changes in relative prices. For example an increase in productivity in the home country is off set in equilibrium by a depreciation of the terms of trade which works as an insurance for the consumers of the foreign country which did not experience a productivity shock.

Under flexible prices the international transmission of country specific productivity shocks depends on the elasticity of substitution between home and foreign goods, η, the intentemporal elasticity of substitution, σ, and the degree of openness of the economy, α. A positive productivity shock at home decreases the relative price of domestically produced goods and thus increases the demand for domestic goods at home and abroad. Both domestic and foreign consumers substitute foreign for domestic goods, depending on η. At the same time, domestic consumers' income increases and domestic demand for both foreign and domestic goods increase, depending on σ. Abroad, on the one hand, the decrease in demand for foreign goods decreases demand for foreign output , but on the other hand, the domestic income effect increases demand for foreign output. The reaction of foreign output depends on the relative strength of the income and the substitution effects and on the degree of openness of the economy.

According to equation (3), the domestic inflation rates depend on the path of real marginal costs where the real marginal costs are decomposed in two components, the consumption gap and the deviations of the terms of trade from its potential. The terms of trade affect domestic inflation because they affect indirectly the real marginal costs. Although producers set domestic prices when they minimize costs they discount wages with CPI prices. With no international price discrimination CPI prices are directly affected by changes in relative prices. Also, as the economy becomes autarchic relative prices affect less domestic inflation ($k_s \longrightarrow 0$ as $\alpha \longrightarrow 0$)

In summary the economy is characterized by the aggregate supply equations at home and abroad (3), the domestic and foreign aggregate demand equations (1), the real interest parity (2), and the law of motion for the exogenous variables: domestic and foreign productivity shocks (5). Finally, to fully characterize the economy we should characterize the way monetary policy is conducted.

3 Optimal Monetary Policy

In the following analysis I focus on the role of monetary policy as a stabilization policy. The equilibrium of the described economy is suboptimal because of the market power distortion and the presence of nominal rigidities. I assume that employment is subsidized in equilibrium so as to neutralize the monopolistic competition distortion.

As we shall see later this comes with a cost. The complete elimination of the monopolistic distortions may conflict with the strategic use of the terms of trade. In the absence of monopolistic distortions in the non cooperative equilibrium each central bank has an incentive to deviate from the flexible price allocation. Each country may gain by contracting inflation.

Moreover, I assume that the liquidity services of money are very small. By doing so, I eliminate the monetary distortion that would pull optimal policy towards the Friedman rule. In general, when prices are flexible and the monopolistic distortion is neutralized the equilibrium allocation is efficient. Thus, the objective of the (global) social planner should be to fully neutralize the effects of nominal rigidities and restore the flexible price allocation.

Optimal monetary policy entails the optimization of a social objective function, given the aggregate constraints in the economy. A natural welfare criterion that allows an evaluation of the losses associated with the distortions in the economy is the discounted sum of the utility flows of the households. This criterion has been introduced by Rotemberg and Woodford(1997) for a closed economy. The central bank's objective in an open economy is different because variations in the relative prices affect differently production across countries. A central bank which is concerned about the optimal reallocation of resources will tolerate changes in the terms of trade that match changes in the flexible-price rate of relative prices. In appendix 3 I show that for an open economy the average utility of the representative consumer in each country can be approximated by the objective:

$$
\begin{aligned}
W_t^i = E_0 \left\{ \Phi \sum_{t=0}^{\infty} \beta^t L_t^i \right\} \text{ for } i = H, F \\
\text{with } L_t^H = \{\phi_{\overline{s}}(s_t - s_t^n) + \phi_c(c_t - c_t^n)^2 + \pi_{Ht}^2 + \phi_s(s_t - s_t^n)^2\} \\
\text{and } L_t^F = \{-\phi_{\overline{s}}(s_t - s_t^n) + \phi_c(c_t^* - c_t^{*n})^2 + \pi_{Ft}^2 + \phi_s(s_t - s_t^n)^2\}
\end{aligned}
\tag{5}
$$

where Φ, ϕ_c, ϕ_s and $\phi_{\overline{s}}$ depend on the structural parameters of the model[3].

In order for the central bank to replicate the flexible price allocation it should close the gaps of consumption and the terms of trade. That policy implies from equation (3) zero domestic inflation and constant prices for each individual firm. For that reason, the social planner seeks to minimize the weighted average of these

[3] where: $\Phi = -1/2 u_c C \frac{\gamma}{(1-\gamma)(1-\gamma\beta)} \theta(1+\theta\omega), \phi_{\overline{s}} = \frac{(1-\gamma)(1-\gamma\theta)}{\gamma\theta} \frac{2\eta\alpha[1+\frac{(1-2\alpha)}{\sigma}]]}{(1+\theta\omega)}$

$\phi_c = \frac{(1-\gamma)(1-\gamma\theta)}{\gamma\theta} \frac{\sigma+\omega}{1+\theta\omega},$ $\phi_s = \frac{(1-\gamma)(1-\gamma\theta)}{\gamma\theta} \frac{\alpha[1+\omega\eta[1+\frac{(1-2\alpha)}{\sigma}]]}{(1+\theta\omega)}$

gaps and the variability of domestic inflation. The weights depend on structural parameters of the model such as, the degree of nominal rigidity (which determines the size of the sticky price distortion) the intentemporal elasticity of substitution and the elasticity of substitution between home goods (which is associated with the losses from the inefficient movements in consumption) and the elasticity of substitution between home and foreign goods and the degree of openness (which are related with the distortion from the inefficient movements of relative prices).

The welfare criterion in (6) looks very similar to the social objective function used by Rotemberg and Woodford (1998) (and extended by Benigno (1999b)). The major difference lies in the presence of the terms concerning the variability and the expected value of the terms of trade gap[4]. Monetary policy should try to minimize the variability of the deviations of the terms of trade from its natural rate, in order to achieve the optimal allocation of resources when the economy experiences asymmetric shocks. In the limit, when the two countries are economically independent, $\alpha \longrightarrow 0$, $\phi_s \longrightarrow 0$, since there is almost no reallocation of resources across countries. The difference between the criterion offered in (6) and the ones offered by Benigno (1999) and Benigno-Benigno (2001) lies in the assumption on preferences for consumption. The latter authors assume no home bias in consumption and unitary elasticity between home and foreign goods. As it was first highlighted in Corsetti and Pesenti (1997) under such assumptions perfect risk sharing in consumption is guaranteed and purchasing power parity holds. Under the more general assumptions for consumption preferences adopted in the present framework purchasing power parity does not hold and despite the presence of complete financial markets it is not efficient to equalize consumption rates across countries. Since PPP does not hold, the weight of the social objective on the terms of trade deviations from their natural rate is different[5].

Using the national central banks' objective forms I will analyze the properties of the equilibrium under alternative hypotheses regarding the way monetary policy is conducted. I will consider three alternatives: (a) Cooperative, (b) Non-cooperative

[4]Also, in Rotemberg and Woodford (1998) it is the output gap rather than the consumption gap that enters the welfare criterion. This is because in a closed economy, in the absence of capital, output is equal to domestic consumption and they can both be used interchangeably in the welfare criterion. However, in an open economy output is equal to a weighted average of domestic and foreign consumption.

[5]Note that if one sets $\alpha = 0.5$ and $\eta = 1$ the weights of the social welfare function collapse to the ones used in the welfare criterion in Benigno (1999) for a monetary union.

monetary policy and (c) Monetary union.

In the following analysis, I assume that the policymakers can choose the entire future (state contingent) evolutions of the control variables, once and for all, at date zero. In other words, I am only considering optimal monetary policy *under commitment* on the part of the policymaker. The assumption of commitment is important, since the private sector expectations about the evolution of prices affect the forward looking terms in equations (1) - (5). In general, the optimal plan is not time consistent, but it delivers a better outcome than a time-consistent plan that results from optimization under discretion. Woodford (1999) comparing the optimal plan to the time consistent plan under discretion, for a similar framework shows that discretionary optimization is suboptimal for stabilizing the economy (see also Oudiz and Sachs (1984)).

4 Calibration

Because, the problems of the central bankers under the different policy regimes do not have closed form solutions, I have resorted to simulations to compare the welfare outcomes of different policy regimes. To conduct simulations I have calibrated the parameters of the model using the United States and Europe as follows. In the benchmark case, the two countries are assumed to be symmetric and time is taken to be quarters. Table 1 provides a summary of the various parameter values used in the simulations of the benchmark economy[6].

[6]In the literature of new open economy macroeconomics (see, Lane (1998) for a review) the assumed value of $\sigma = 1/6$. Following Backus et al.(1992) I set $\sigma = 1/2$. I do not follow Rotemberg and Woodford (1997) in calibrating the value of the inverse of the labor supply elasticity ($\omega = 0.4633$), since it implies a very elastic labor supply. However, performing simulations using these values do not change the results on the trade off between the three international policy arrangements.

Table 1: Benchmark Parameter values

Parameter	Description	Value
β	Discount factor	$1.03^{-1/4}$
$1/\sigma$	Constant of Relative Risk Aversion	½
η	Elasticity of substitution between home and foreign goods	1.0
θ/θ-1	Gross steady state mark-up	1.2
1-α	Home bias in consumption	0.85
$1/\omega$	Elasticity of labor supply	0.3
γ	Probability that a firm will be unable to change its price	0.75
Technology shocks	$\Gamma = \begin{bmatrix} .906 & .088 \\ .088 & .906 \end{bmatrix}$, and var(z)=var(z)=0.00852	Corr(z,z*)= .258

I set the discount factor $\beta = 1.03^{-1/4}$, so as the annual real interest rate equals 4%. Following Backus, Kehoe and Kydland (1992), I set the intertemporal elasticity of substitution $\sigma = 2$. The elasticity of substitution between home and foreign goods is estimated between [1,2] (Chari et al. (1998)). I set $\eta = 1$.

The parameter θ, the elasticity of substitution among differentiated goods is set equal to 6. Since in the steady state θ equals the mark-up of prices over marginal costs, this value implies a mark-up of 20%.

I set the elasticity of labor supply equal to 0.3. The degree of price stickiness, measured by the parameter γ, is set equal to 0.75, which implies that the average frequency of price adjustments is four quarters.

To set α, note that in the symmetric steady state $\alpha = C_F/C_H$, the share of imported to domestic goods. The value of this parameter does not differ significantly for Europe and the US. According to Chari et al. (1998) imports from Europe to US are roughly 2.0% of GDP, while for Europe this parameter is around 2-4% according to data available at the Eurostat. Thus, the assumption of symmetry is reasonable on these grounds. In the benchmark case, I set the index of openness equal to 0.15, and perform a variety of sensitivity experiments.

Finally, in order to estimate a stochastic process with symmetric Γ and D matrices, consistent with the symmetric characterization of the model, I use the procedure and estimations of Backus, Kehoe and Kydland (1992).

5 International Monetary Policy Regimes

5.1 Cooperative Monetary Policy (CO)

Monetary policy cooperation is modeled as a case in which monetary policy decisions are delegated to a supranational monetary institution which has the objective of maximizing the weighted average of the welfare of the representative consumers in each country. Since I assume that the two countries are symmetric, I constraint the weights to be equal. Intuitively, consumers should be at least as well off when the central banks cooperate as when they do not. Cooperating policymakers can always implement the non-cooperative outcome by simply choosing their non-cooperative strategies. Since that outcome is feasible under cooperation, rational policymakers will never choose something worse.

Following Woodford (1999), the problem of the central authority is to choose stochastic processes c_t, π_{Ht}, $\widehat{R}_t$, c_t^*, π_{Ht}^*, $\widehat{R}_t^*$ and s_t– as a function of an information set I_t, that includes all the history and the information at date t about the future evolution of the exogenous disturbances z_t and z_t^* - to maximize the weighted average of the welfare in the two countries, subject to the constraints given by (1)-(4) (and the respective conditions abroad) at all dates $t \geq 0$.

$$\{c_t, \pi_{Ht}, \widehat{R}_t, c_t^*, \pi_{Ht}^*, \widehat{R}_t^* s_t\} \max E_0\{\Phi \sum_{t=0}^{\infty} \beta^t \{(L_t + L_t^*)\},$$

The solution to the problem is[7]:

$$(1 - \Gamma L) q_{t+1} = (1 - ZL) \xi_{t+1}$$

where $q_t = \{c_t, \pi_{Ht}, \widehat{R}_t, c_t^*, \pi_{Ht}^*, \widehat{R}_t^*, s_t, \phi_t\}$, $\phi_t = \{\phi_{1t}, \phi_{2t}, \phi_{1t}^*, \phi_{2t}^*, \}$ is the vector of deviations of the Lagrange multipliers and $\xi_t = \{z_t, z_t^*\}$[8]. The multipliers $\phi_{10}, \phi_{20}, \phi_{10}^*, \phi_{20}^*$ give the marginal or shadow value of relaxing the aggregate demand and the aggregate supply constraints home and abroad. Since the social planner does not inherit any initial values for his choice variables and the aggregate constraints depend on expectations of the choice variables, a necessary condition for the optimization under commitment is that at time zero $\phi_{10} = \phi_{20} = \phi_{10}^* = \phi_{20}^* = s_0 = 0$[9].

[7] Notice that the linear term in relative prices cancels out for the maximization problem of the global social planner.

[8] The problem of the planner is to solve a quadratic problem subject to linear constraints.

[9] The optimal plan is not time consistent. The time consistency problem arises because along

By definition, the cooperative equilibrium delivers the best outcome. Under complete financial markets, risk sharing in consumption is guaranteed. The only distortion in the economy is the stickiness of prices and the resulting inertia in relative prices. Since the social planner has two instruments available to correct for these distortions she can implement the Pareto optimum allocation which is the flexible price allocation. Optimal monetary policy, closes substantially the gaps in consumption and in the terms of trade and allows for very little variation in domestic inflation and in exchange rates (see Table 2). Since the first best is attained in the cooperative solution, the welfare of the consumers when the two policymakers cooperate can be used as a benchmark for comparing the outcomes of the other policy regimes. In what follows I compare the results of suboptimal solutions in terms of consumption paths. The last column of Table 2, gives the index (OCU, percentage optimal consumption units) of the utility losses in terms of the equivalent optimal consumption decreases associated with the suboptimal equilibria. In Figures 1 and 2 I compare the responses of the domestic macroeconomic variables to a domestic and a foreign productivity shock respectively (the responses of the foreign variables follow a similar pattern).

A positive productivity shock reduces inflation at home. However, the induced depreciation of the nominal exchange rate and consequently of the terms of trade increases demand for domestic goods and thus expectations for future inflation. The movements in the nominal interest rates is such that the effect of the terms of trade depreciation balances the effect of the productivity increase in inflation. As a result, inflation variability is reduced (Figure 1).

Recent empirical work for a closed economy has shown that in response to a positive productivity shock, labor productivity rises more than output while employment decreases. Gali (1998) showed that this stylized fact can be explained in the context of a closed economy general equilibrium model with monopolistic competition and nominal rigidities. However, in an open economy the above stylized fact can be replicated without the need of nominal rigidities. In Figure 1 output reacts less than the change in technology implying a negative movement in employment for our bench-

---

the optimal sequence $\{c_t, \pi_{Ht}, \widehat{R}_t, c_t^*, \pi_{Ht}^*, \widehat{R}_t^* s_t\}_0^\infty$ the lagrange multipliers will not always be zero. Since the Lagrange multipliers are different from zero reoptimization at any point in time when $\phi_{it} \neq 0$ would lead to a new sequence of policies such that ϕ_{it} would again start at zero. Hence, time consistency implies that the lagrange multipliers are zero at all times. Nevertheless, this solution is not feasible.

mark specification. Since cooperation closes the gaps between flexible and sticky price movements of the macroeconomic variables, the reaction of employment under flexible prices should deliver exactly the same pattern of responses (See, also Figure 3, where the responses of the terms of trade and consumption under the different international policy regimes are compared with the natural level responses)[10].

However this is not true for high values of the labor supply elasticity and for $\sigma = 1$. In the theoretical model the demand for labor depends on the domestic price index, while the supply of labor depends on the aggregate price index. Under flexible prices, a positive domestic shock decreases the domestic price index, real wages decrease and demand for labor increases. On the other hand due to the income effect the supply of labor decreases. In the benchmark preferences with a steep labor supply curve these movements in the labor markets lead to a reduction in domestic employment after a positive productivity shock. With a flatter labor supply locus ($\omega = 0.4633$) the reduction in the supply of labor does not exceed the increase in demand and employment increases in equilibrium. Of course this result can be reversed if one changes the values of η and α.

5.2 Non-Cooperative Monetary Policy (NC)

In this section I analyze the interaction between the two countries in a non-cooperative Nash equilibrium. In this case, each monetary authority maximizes the expected utility of its own representative consumer subject to the domestic economy constraints, taking as given the policy of the foreign policymaker. The maximization problem that the domestic policymaker faces is given by:

$$\{c_t, \pi_{Ht}, s_t\} \max E_0 \Phi \sum_{t=0}^{\infty} \beta^t \{\phi_{\bar{s}} s_t + \phi_c (c_t - c_t^n)^2 + \pi_{Ht}^2 + \phi_s (s_t - s_t^n)^2\}$$

subject to (1) and (3), the exogenous process for the productivity shocks and taking as given the foreign variables, c_t^*, π_{Ht}^*,and $\widehat{R}_t^*$. The foreign policymaker behaves

[10] Collard and Dellas (2001) suggest that the above fact can be replicated in an RBC model as long as trade elasticities fall short of unity and the degree of openness is sufficiently high. In the present framework, though, I can generate negative conditional correlation between productivity and employment without having to assume low elasticities of substitution between home and foreign goods ($\eta = 1$ in the benchmark specification).

symmetrically. The objective of the foreign national bank is given by:

$$\{c_t, \pi_{Ht}, s_t\}\max E_0 \Phi \sum_{t=0}^{\infty} \beta^t \{-\phi_{\overline{s}} s_t + \phi_c (c_t^* - c_t^{*n})^2 + \pi_{Ft}^2 + \phi_s (s_t - s_t^n)^2\}$$

The equilibrium is given by the following system:

$$\begin{bmatrix} (1-AL) & (1-BL) \\ (1-FL) & (1-DL) \end{bmatrix} \begin{bmatrix} x_{t+1} \\ x_{t+1}^* \end{bmatrix} = \begin{bmatrix} (1-CL) \\ (1-GL) \end{bmatrix} \begin{bmatrix} z_{t+1} \\ z_{t+1}^* \end{bmatrix}$$

where $x_t = \{c_t, \pi_{Ht}, R_t, s_t\}$, and $x_t^* = \{c_t^*, \pi_{Ht}^*, R_t^*, s_t^*\}$.

The presence of the linear terms in the national banks' objectives causes the inefficiencies associated with the non-cooperative equilibrium. In this framework the flexible price allocation cannot be implemented as a Nash equilibrium, since each national authority, unless $E_0(s_t - s_t^n) = 0$, has an incentive to surprise the foreign policymaker. Notice that the domestic inflation at home is determined by: $\pi_{Ht} = \beta E_t \pi_{Ht+1} + \kappa_c(c_t - c_t^n) + k_s(s_t - s_t^n)$. In the absence of real disturbances (i.e., $s_t^n = c_t^n = 0$), an increase in the terms of trade causes domestic inflation to accelerate, basically because of an increase in demand for domestic goods. In the foreign country it holds that $\pi_{Ht}^* = \beta E_t \pi_{Ht+1}^* + \kappa_c(c_t^* - c_t^{*s}) - k_s(s_t - s_t^n)$. Note that a real depreciation at home causes inflation to fall abroad, while an appreciation causes foreign inflation to rise. This is the heart of the inefficiency of the non cooperative equilibrium: each policymaker may have an incentive to contract the economy in order to appreciate the currency and thereby export inflation abroad at the expense of the other country.

In the presence of productivity shocks (i.e., $s_t^n \neq 0$), in the non cooperative solution, each policymaker has an incentive to deviate from the flexible price allocation. A positive domestic productivity shock decreases real marginal costs and thus domestic inflation. In the flexible price allocation, after such a shock the terms of trade depreciate and as a result demand for domestic goods increases and domestic inflation increases. The movements in the terms of trade are such that the two effects cancel out and inflation remains almost unchanged. In the foreign country, the terms of trade deterioration decreases demand for foreign goods and thus foreign inflation. In the non cooperative equilibrium the domestic policymaker has an incentive to depreciate the domestic currency less than in the flexible price case so as to export inflation abroad. For the foreign policymaker the opposite is true.

In equilibrium the effects of the beggar-thy-neighbor policies cancel out since the two policymakers act in a symmetric way. The exchange is less variable than in the flexible price allocation. In Figure 3, the terms of trade gap is much bigger under non-cooperation than under cooperation. As a result, domestic consumption increases less on impact in the non cooperative equilibrium after a domestic productivity shock[11]. Conversely in the second row of Figure 3, consumption in the foreign country increases more than in the flexible price equilibrium. Finally the competitive policies of the two policymakers do not allow for the minimization of inflation variability in the two countries. In Table 2 it is apparent that a cooperating policy can avoid the 'contractionary bias' in the non cooperative solution to the mutual benefit of both countries.

The "contractionary bias" in the non-cooperative equilibrium could be eliminated for some positive degree of monopolistic distortions. In a recent paper, Benigno-Benigno (2001) show that by appropriately choosing the overall degrees of monopolistic competition, the strategy of zero domestic inflation is a Nash equilibrium. Moreover, this equilibrium implements the flexible price allocation. However, the cooperative equilibrium in the absence of monopolistic distortions gives higher levels of production and consumption and can replicate the flexible price allocation, since in the cooperative solution the strategic role of the terms of trade is completely internalized by the social planner.

The costs of non-cooperation are significant and change with the degree of openness of the economies. For an index of openness equal to 0.25, they account for -0.58% of optimal consumption units and they decrease as the degree of openness decreases. The importance of the exchange rate transmission channel decreases as the economies become autarchic. For that reason, for almost closed economies the costs of non-cooperation are small. According to the data the index of openness of Europe and the States is around 2%. Thus, the model implies that the ECB and the FED do not gain a lot from cooperation and conducting monetary policy independently, in this case, is nearly optimal.

5.3 Monetary Union (MU)

So far, we have seen that policy cooperation is preferable to policy competition. One way to accomplish some of the benefits of policy cooperation is through the

[11]The same is true for output. As it is apparent from Figure 9, both non-cooperation and monetary union imply, also, negative correlations between employment and labor productivity.

establishment of a monetary union. In some cases monetary union is associated with losses, albeit the fact that in a monetary union coordination can be reached. In this case, the problem of the central bank is similar to the problem of the supranational institution of the previous section with the only difference being that in the currency area the nominal exchange rate is fixed. Thus, the problem that the central bank is solving is the same as the problem of section 3.1, with the additional constraint that $\Delta e_t = 0, \forall t$[12].

In a monetary union the fixity of the nominal exchange rate coupled with the rigidity in prices introduces another distortion in the economy: the inertia in relative prices. Notice that there is a trade-off between price stickiness and inertia of relative price distortions. Attempts to neutralize price stickiness,by setting domestic inflation equal to zero, increase the distortions due to the inertia of the terms of trade. Optimality implies a zero inflation rate and both the consumption gap and the terms of trade gap to be zero. In a monetary union, these gaps cannot be closed simultaneously. Low inflation variability implies sluggish relative prices which in turn result to an inefficient reaction of output in response to foreign disturbances and thus higher consumption gap variability. In Table 2 the loss of the exchange rate as an instrument is reflected in the increase in the consumption gap variability, relative to the case of cooperation. The nominal interest rate is also more variable, since it is the only instrument the Central Bank can use for accommodating productivity shocks.

[12]For an analysis of monetary unions in a different modelling framework see Cooley and Quadrini (2000).

Table 2: Welfare Statistics for alternative Regimes

		inflation	Consumption gap	Exchange rate	Terms-of-trade gap	Interest rate	OCU(%)
	α=0.25	0.012	0.63	0.54	0.47	0.70	0.0
	α=0.15	0.007	0.37	0.41	0.30	0.88	0.0
CO	α=0.05	0.002	0.13	0.20	0.14	0.95	0.0
	α=0.02	0.000	0.05	0.12	0.06	0.99	0.0
	α=0.25	0.063	0.68	0.73	1.23	0.77	-0.58%
	α=0.15	0.023	0.48	0.68	1.40	0.49	-0.41%
NC	α=0.05	0.011	0.24	0.12	2.01	0.65	-0.30%
	α=0.02	0.005	0.09	0.11	2.44	0.86	-0.08%
	α=0.25	0.032	0.73	0.0	0.89	1.97	-0.46%
	α=0.15	0.034	0.62	0.0	0.61	1.88	-0.47%
MU	α=0.05	0.039	0.58	0.0	0.50	1.33	-0.48%
	α=0.02	0.042	0.59	0.0	0.53	1.05	-0.62%

The question is whether in the monetary union welfare is improved relative to the non-cooperative case. The answer is not clear cut and it depends on characteristics of the economies such as the degree of openness and the degree of nominal rigidities. In relatively open economies the consumers are better off in a monetary union, while in almost autarchic economies the opposite is true. This is not surprising, since the closer is an economy, the less affected is by foreign variables and thus stabilization of its output and inflation depends mainly on domestic variables.

These results are similar to those in the optimum currency area literature but for entirely different reasons. Mundell (1961) argued that the benefit of a common currency area was its role in minimizing transaction costs and facilitating the flow of information about relative prices. The offsetting force was that fixed exchange rates entailed the loss of independent monetary policies. For a relatively closed economy the loss of the possibility of tailoring monetary policy to the needs of different areas were bigger than the benefits from allocative efficiency. While in the present framework, on the one hand, a common central bank and currency is beneficial because it enables coordination, but on the other hand, it is unfavorable because it distorts the reallocation of resources and imposes excess consumption gap variability.

6 Approximating the Optimal Policy

6.1 The optimal plan

In this section I study what kind of policy rule would approximate the optimal pattern of responses to shocks in a decentralized setting. According to the second welfare theorem any Pareto optimal allocation can be implemented as a competitive equilibrium with transfers. In this section, I study what kind of decentralized monetary policy can guarantee these transfers. I assume that the policy is a feedback rule where the nominal interest rate is a function of endogenous and exogenous variables. I, also, assume that the central bank commits itself to systematically conduct of monetary policy in this particular way and that commitment is understood by the private sector and is credible.

As I show in appendix IV the optimal cooperative plan implies that the nominal interest rate in each country evolves according to:

$$\begin{aligned} \widehat{R}_t &= \Theta(L)\xi_t \\ \widehat{R}_t^* &= \Pi(L)\xi_t \\ \xi_t' &= [z_t, z_t^*] \end{aligned}$$

This implies processes for the other endogenous variables of the form:

$$\begin{aligned} Q(L)s_t &= V(L)\xi_t \\ G(L)\pi_{Ht} &= O(L)\xi_t \\ H(L)y_t &= N(L)\xi_t \end{aligned} \tag{6}$$

where $\Theta(L), \Pi(L), V(L); G(L), O(L), H(L), N(L)$ are first order polynomials and $Q(L)$ is second order in the lag operator.

The optimal cooperative plan is therefore a function of the current and previous states of the economy. This is partly due to the stickiness of prices and partly due to the gains in credibility that the bank achieves by regarding itself as constrained to fulfill previous commitments (Woodford, 1999). Since prices are staggered, the current level of inflation depends not only on the current state of the economy, but also in the past states. Thus, the optimal plan under commitment implies that the changes in the endogenous variables are smooth. Moreover, under the optimal policy all the variations in the variables of interest are proportional to a weighted average of domestic and foreign productivity shocks.

Equation (7) describes how the interest rate should vary as a function of the history of shocks and it is a unique relation that holds in equilibrium. However, (7) cannot be implemented as a monetary policy rule in a decentralized setting. As it is shown in appendix IV, (7) if it is adopted as a feedback rule leads to local indeterminacy of equilibrium. This is because the structural model possesses forward-looking elements and with rational expectations a policy rule may easily be associated with more than one rational expectations equilibria (see, Bernanke and Woodford(1997), Rotemberg and Woodford(1999), and Clarida et al.(2000) for more details on this issue).

6.2 A Simple Feedback Rule

The problem of indeterminacy of rational expectations equilibria can be resolved through sufficiently strong feedback from endogenous variables, such as inflation or output, or the terms of trade. Among the different sets of coefficients for the policy rule that replicate the optimal policy allocation, I consider policy rules within the family (9), in order to be consistent with the symmetric steady state.

$$\widehat{R}_t = v_\pi \pi_{Ht} + v_z z_t + v_{z^*} z_t^* \tag{7}$$

where I allow for the possibility of direct feedback from domestic and foreign productivity shocks. The feedback rule together with the aggregate demand and supply equations complete the system that characterizes the solution of the model. The complete set of structural equations consists of the aggregate demand equation, the aggregate supply equation, the real interest parity condition and (9). This system can be written in the form:

$$KE_t q_{t+1} = \Lambda q_t + M \begin{bmatrix} z_t \\ z_t^* \end{bmatrix} \tag{8}$$

where $q_t' = [c_t, \pi_{Ht}]$. Since there is no predetermined variable in the system, a unique bounded solution exists if and only if the two eigenvalues of the matrix $[K^{-1}\Lambda]$ lie inside the unit circle. Furthermore, a necessary condition for determinacy is that the reaction of the central bank towards inflation is aggressive (Taylor's principle):

$$v_\pi > 1 \tag{9}$$

If (11) is satisfied the equilibrium is determined (see also Benigno-Benigno (2000)). Thus, we turn to the question of whether a feedback rule of the form (9)can replicate the optimal pattern of responses. In order to find the values of the coefficients on the feedback rule that approximate the impulse responses under the optimal cooperative plan I simulate the model described by (1)-(4) and the rule in (9) for both countries, and for various values of the parameters v_{π}, v_z and v_{z^*} and I compare the impulse responses of the simulated economy with the impulse responses that the theoretical model delivers in the optimal solution. The rule that best approximates the optimal policy is the rule with coefficients such that the difference between the impulse responses of the variables of interest for both types of productivity shocks under the two specifications is minimized.

Unfortunately, the elements of the impulse responses functions are not independent. For that reason, minimizing the distance of the impulse response functions under the optimal policy and under (9) by simply summing up the square distance of each step is not appropriate. One way of solving this problem is to change the coordinates of the space where impulse responses are represented. This can be easily done by taking a frequency domain approach, computing the periodogram associated with the cooperative solution and with the solution under (9) and try to find the combination of parameters that minimizes the distance between the periodograms of the simulated series under the two specifications. The comparison is meaningful, since a crucial property of the periodogram is the independence of the different ordinates.[13]

Optimal responses to fluctuations in the productivity shocks are approximated, by a rule which has extremely strong feedback only from domestic inflation ($v_{\pi} \longrightarrow \infty$, $v_z = v_{z^*} = 0$). That is, when the two countries cooperate they should try to stabilize domestic inflation without reacting directly to foreign variables. This result is surprising, because someone would expect that cooperation involves coordination from the part of the policymakers. The numerical results confirm that this is not the case. When each central bank is trying to stabilize domestic inflation it acts in the interest of both countries. However, the variability of the nominal exchange rate under the optimal plan is low. This is because, although both instrument rules react explicitly to domestic conditions only, there is an implicit reaction towards the exchange rate depreciation that deters the exchange rate from moving substantially.

[13]Details of the calculations of the periodograms are given in Appendix 4.

6.3 The Taylor Rule

I now turn to the question of designing a realistic regime that could approximate the optimal patterns of responses to shocks. There are many feedback rules that can approximate the optimal solution. Here the objective is to find rules that involve feedback from variables that are observable and rules that can be adopted in practice by the central banks.

Many authors have shown that a Taylor rule can approximate well the behavior of the interest rates in practise[14]. In this section I analyze how the Taylor rule compares to the optimal policy. In the original paper of Taylor (1993) this rule was defined as an interest rate rule where the interest rate had feedback from domestic inflation and output. Given the results in the previous section, I will consider Taylor-type rules where the nominal interest rate has feedback only from domestic inflation:

$$\widehat{R}_t = v_\pi \pi_{Ht} \tag{10}$$

I analyze the welfare properties of the equilibrium of the two economies when the optimal rule is replaced by (10). In Table 3 I compare the welfare properties of the equilibria when the two economies follow a Taylor rule with the welfare properties of the noncooperative and the monetary union solutions[15]. In the benchmark case, $v_\pi = 1.5$, assumes the value used in the original paper of Taylor (1993). If the two economies were to follow this rule, the consumers would be better off in any of the other policy arrangements[16]. However, for higher values of the coefficient in inflation ($v_\pi = 5$ and $v_\pi = 7$) and for high degrees of openness, a Taylor rule does better than noncooperation. Furthermore, it is preferable to monetary union even for low degrees of openness.

[14]See Clarida-Gali and Gertler (1998, 2000) for the empirical analysis of Taylor rules.

[15]The welfare measure when the two economies follow the optimal plan is zero and for that reason is excluded from the table.

[16]Except for when $\alpha = 0.02$. In this case the consumers are almost indifferent between participating in a monetary union and conducting policy using a Taylor rule.

Table 3: The Taylor Rule

Regimes	α=0.20	α=0.15	α=0.02
	Optimal consumption units (%)		
NC	0.51%	0.41%	0.08%
MU	0.46%	0.47%	0.62%
Taylor v_p=1.5	1.2%	0.69%	0.59%
Taylor v_p=5.0	0.47%	0.47%	0.39%
Taylor v_p=7.0	0.46%	0.35%	0.27%

For $v_\pi = 7$, the Taylor rule is associated with the smaller welfare losses (except for the case that $\alpha = 0.02$). Thus, under plausible values for the coefficient in inflation a Taylor rule can improve upon the equilibria delivered by noncooperation and monetary union.

7 Concluding Remarks

In this paper I evaluate the welfare consequences and the implications for macroeconomic stability of three different kinds of conduct of monetary policy in a two-country world and study the implementation of the welfare maximizing regime.

By definition, international monetary policy cooperation delivers the best outcome. When the two countries cooperate inflation and output variability is reduced and the first best can be achieved. The cooperative solution can be approximated by an interest rate feedback rule that depends strongly only on domestic inflation. Such a rule cannot be adopted in practise as a monetary policy rule. However, under plausible values for the coefficient in inflation a Taylor rule can improve upon the noncooperative and monetary union equilibria.

Non-cooperation implies welfare losses because in the Nash game the policymakers pursue beggar-thy-neighbor policies. In the non cooperative solution the movements of the nominal exchange rate result high variability of the terms of trade gap, in the expense of increased variability in inflation. The welfare costs from non-cooperation increase with the degree of openness of the economy. This is because as the economy becomes autarchic the short run adjustment role of the nominal exchange rate is weakened to the extent that consumer prices are almost unresponsive to exchange rate changes.

Fixing the exchange rate introduces a distortion in the economy, the inertia of the terms of trade, that does not allow the optimal reallocation of resources. Nevertheless, the adoption of a common central bank and currency has the potential of reducing the welfare costs by eliminating the 'contractionary bias' induced by monetary policy competition when the economies are open to trade and are characterized by low degrees of price stickiness. When the economies have weak trade links, there is no need for international monetary policy coordination and the two countries are better off conducting monetary policy independently.

The model indicates that as long as trade interdependencies between Europe and the US are as small as those experienced in the last 50 years, cooperation between the ECB and the FED will produce very little welfare gains. On the other hand, if trade links are asymmetric, non-cooperation implies significant losses for the more open economy (Europe).

Finally, this paper has only focused in the design of optimal monetary policy under commitment. All the papers in the field analyzing issues of monetary policy in the one or in the other way refer to commitment. Commitment is crucial for determining the results and it should be endogenously determined. I would like to focus my future research on enriching the current framework to analyze more deeply the issues of commitment of policy. Learning more about commitment might improve our understanding of the real world.

Figure 1: Impulse Responses to a Domestic Productivity Shock

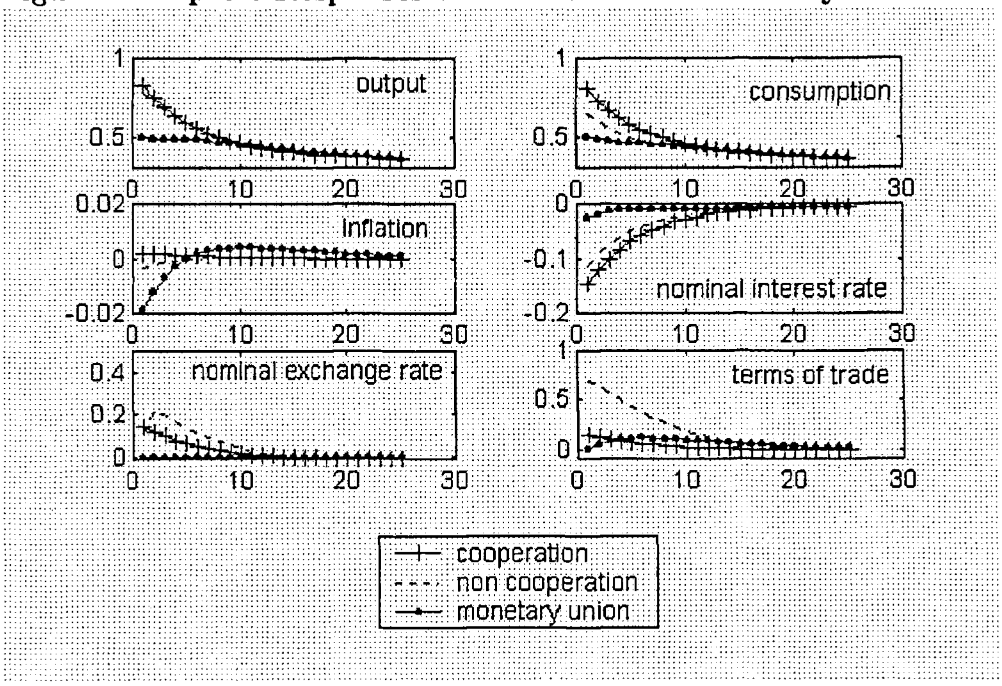


Figure 2: Impulse Responses to a Foreign Productivity Shock

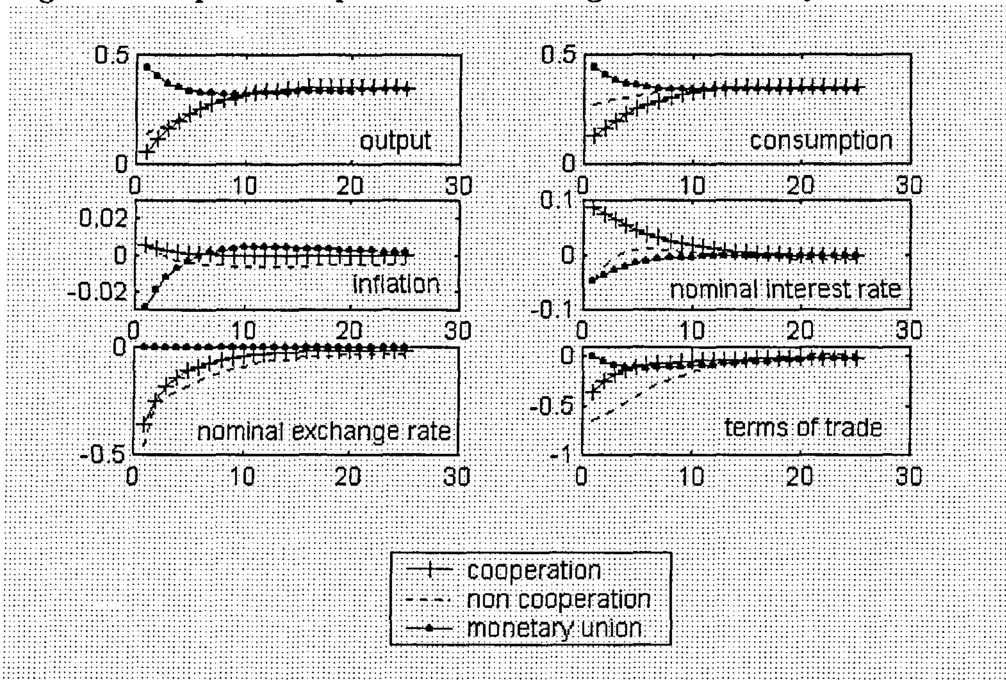
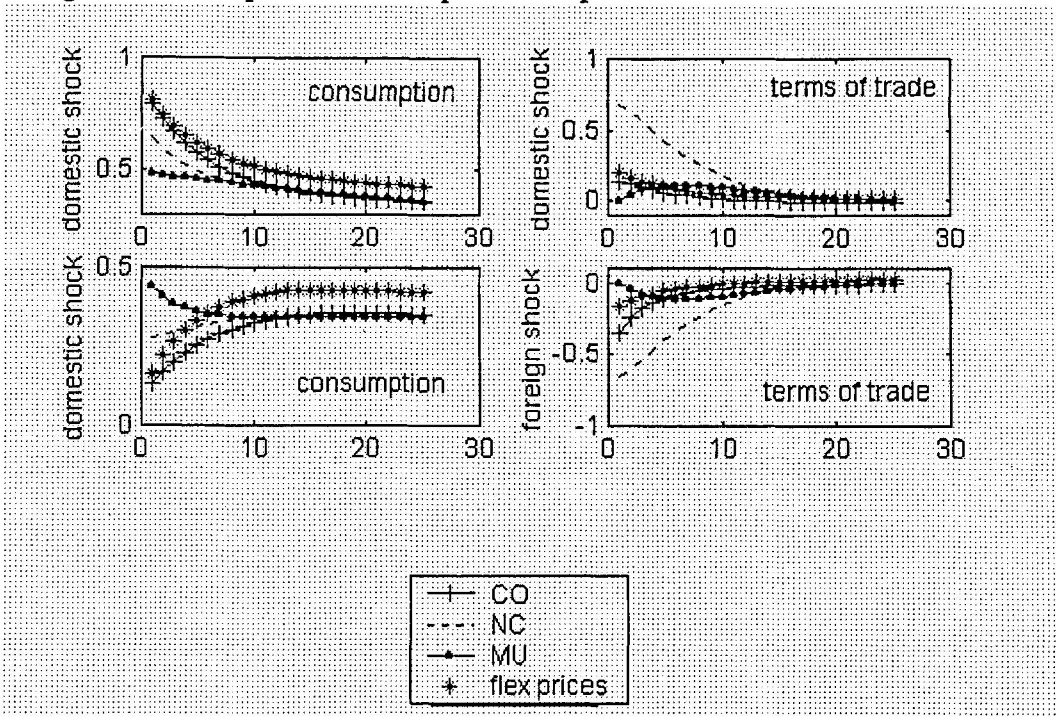


Figure 3: Comparison of Impulse Response Functions



APPENDIX 1

The Model

The problem of the Representative Agent

Each country is inhabited by a continuum of agents on the interval [0,1]. There is no migration across countries. In each period t, the economy experiences one of finitely many events s_t. Let h^t denote the history of realized states from period zero until period t, i.e., $h^t = \{s_0, s_1, ..., s_t\}$. The probability, as of period zero, of any particular history h^t is $\pi(h^t)$. The initial realization s_0 is given. In this section I present the problem of a generic household i, at home (the problem of a foreign household (j) is analogous). I assume that preferences are identical across countries and the foreign variables are denoted with stars throughout the analysis. Agents obtain utility from consumption and real money balances and disutility from producing goods. The objective of the domestic household i is to maximize:

$$EU = t = 0\sum^{\infty}\sum_{h^t}\beta^t\pi(h^t)\left[u(C(i,h^t)) + m(\frac{M(i,h^t)}{P(h^t)}) - v(Y(i,h^t), z(h^t))\right] \tag{A1.1}$$

where the index i denotes a variable that is specific to agent i, and β is a discount factor. u is an increasing concave function and $C(i, h^t)$ is defined as:

$$C(i,h^t) = [(1-\alpha)^{\frac{1}{\eta}}C_H(i,h^t)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}}C_F(i,h^t)^{\frac{\eta-1}{\eta}}]^{\frac{\eta}{\eta-1}} \tag{A1.2}$$

In turn, $C_s(i, h^t)$, $s = H, F$, are given by the following CES aggregators:

$$C_s(i,h^t) = \left(\int_0^1 C_s(\tau,h^t)^{\frac{\theta-1}{\theta}}d\tau\right)^{\frac{\theta}{\theta-1}} \quad \text{for } s = H, F$$

Here, η is the constant elasticity of substitution between foreign and domestic goods and $\theta \geq 1$ is the constant elasticity of substitution across goods produced within the country, $(1-\alpha)$ describes the degree of home bias in the consumption of goods.

$m(.)$ is an increasing concave function of the real money balances, $M(i, h^t)$ is the agents nominal money balances at the end of period t, while $P(h^t)$ is the domestic CPI defined as:

$$P(h^t) = [(1-\alpha)P_H(h^t)^{1-\eta} + \alpha P_F(h^t)^{1-\eta}]^{\frac{1}{1-\eta}} \tag{A1.3}$$

and depends on the price index for domestic goods, $P_H(h^t)$ and the price index for imported goods expressed in home currency, $P_F(h^t)$, where $P_s = [\int_0^1 P_s(\tau)^{1-\theta} d\tau]^{\frac{1}{1-\theta}}$, $s = H, F$.

Finally, $v(.)$ is an increasing convex function of agent i's supply of her product $Y(i, h^t)$. V(Y) can be interpreted as the inverse of the disutility of working (Rotemberg and Woodford (1998)). If we define $Y(i, h^t) = f(N(i, h^t)\ z(h^t))$, and the disutility of working by $g(N(i, h^t))$, then $v(Y(i, h^t), z(h^t))$ can be rewritten as $g(f^{-1}(Y(i, h^t))/z(h^t))$, where $z(h^t)$ is a country-specific productivity shock.

The optimal allocation of any given expenditure within each category of goods yields the demand functions:

$$C_s(i, h^t) = \left(\frac{P_s(i, h^t)}{P_s(h^t)}\right)^{-\theta} C_s(h^t) \qquad ; s = H, F$$

for all $i \in [0, 1]$. Since there is no international price discrimination, $P_F(i, h^t) = e(h^t) P_H^*(i, h^t)$, $\forall i \in [0, 1]$, where $e(h^t)$ is the nominal exchange rate expressed as the price of foreign currency in terms of home currency and $P_H^*(i, h^t)$ is the price of foreign good i denominated in foreign currency.

Moreover, the optimal allocation of expenditures between domestic and foreign goods imply:

$$C_H(h^t) = (1 - \alpha)\left(\frac{P_H(h^t)}{P(h^t)}\right)^{-\eta} C(h^t) \quad ; \quad C_F(h^t) = \alpha\left(\frac{P_F(h^t)}{P(h^t)}\right)^{-\eta} C(h^t)$$

In the symmetric equilibrium, α measures the share of imported to domestic goods and can be used as an index of the openness of the economy. The total demand for good i is then given by:

$$Y^d(i, h^t) = \left(\frac{P_H(i, h^t)}{P_H(h^t)}\right)^{-\theta} [C_H(h^t) + C_F^*(h^t)] \qquad \text{(A1.4)}$$

where C_F^* is the foreign demand for domestically produced goods.

The representative consumer in the home country receives income from selling her product, from asset holdings and from transfers of the domestic government. Households then consume, accumulate money balances, purchase new assets and decide saving allocations between money and assets. The asset structure available to agents is a set of complete contingent one period bonds denominated in the home currency. The consumers of both countries purchase a portfolio of state-contingent home currency denominated nominal bonds at price $Q(h^t, h^{t+1})$.

As it is highlighted by Cole and Obstfeld (1991), the existence of the state contingent home currency denominated bonds is sufficient to ensure complete international financial markets ex ante. They find that fluctuations in the terms-of-trade play an important role in automatically pooling national output risks, since a country's terms of trade are negatively related with growth in its exports sector. Thus, in my framework, ex ante there is no need for international portfolio diversification, since the terms of trade responses alone provide perfect insurance against output risks. For that reason, I do not include trade in state contingent foreign currency denominated nominal bonds. The budget constraints of a domestic producer-consumer is then written as:

$$\begin{gathered} P(h^t)C(i,h^t) + \sum_{h^t} Q(h^t,h^{t+1})b(i,h^t) + M(i,h^t) \leq \\ (1-\tau)P_H(i,h^t)Y(i,h^t) + M(i,h^t) + b(i,h^t) + TR(i,h^t) \end{gathered} \tag{A1.5}$$

$TR(i, h^t)$ denotes the nominal lump sum transfers from the government of the home country to the domestic household i at date t, while τ is a proportional tax on nominal income.

Ruling out Ponzi Games, the optimality conditions are:

$$\beta\pi(h^{t+1})\left(\frac{u_c(h^{t+1})}{u_c(h^t)}\right)\frac{P(h^t)}{P(h^{t+1})} = Q(h^{t+1},h^t) \tag{A1.6}$$

$$\frac{m_M(h^t)}{P(h^t)} - \frac{u_c(h^t)}{P(h^t)} + \beta\sum_{h^{t+1}}\pi(h^{t+1})\left(\frac{u_c(h^{t+1})}{P(h^t)}\right) = 0 \tag{A1.7}$$

When the set of assets is sufficient to completely span all the states of nature, the price of any other asset can be calculated as a linear combination of the underlying Arrow-Debreu prices. For example, internationally traded nominal one-period non-contingent bonds denominated in the home (foreign) currency will cost $\frac{1}{1+R(h^t)}$ $(\frac{1}{1+R^*(h^t)})$ where $\frac{1}{1+R(h^t)} = \sum_{h^{t+1}} Q(h^{t+1}, h^t)$,
$\left(\frac{1}{1+R^*(h^t)} = \sum_{h^{t+1}} Q(h^{t+1}, h^t)\frac{e(h^{t+1})}{e(h^t)}\right)$. Arbitrage between internationally traded bonds implies:

$$\sum_{h^{t+1}} Q(h^{t+1},h^t)[R(h^t) - R^*(h^t)\frac{e(h^{t+1})}{e(h^t)}] = 0 \tag{A1.8}$$

Equation (A1.6) is a standard Euler equation, while (A1.7)[17] is a money demand equation and (A1.8) is the uncovered interest parity condition.

A first order condition analogous to (A1.6) must hold for the consumers that hold the home currency bonds in the foreign country:

$$\beta\pi(h^{t+1})\left(\frac{u_c^*(h^{t+1})}{u_c^*(h^t)}\right)\left(\frac{P^*(h^t)}{P^*(h^{t+1})}\right)\left(\frac{e(h^t)}{e(h^{t+1})}\right)=Q(h^{t+1},h^t) \tag{A1.9}$$

Defining the real exchange rate as: $q(h^t)=e(h^t)P^*(h^t)/P(h^t)$, combining (A1.6) and (A1.9) and iterating we have:

$$q(h^t)=\chi\frac{u_c^*(h^t)}{u_c(h^t)} \tag{A1.10}$$

Equation (A1.10) is a relationship between real exchange rates and marginal rates of substitution,where $\chi = u_c(s_0)P^*(s_0)/u_c^*(s_0)P(s_0)$ is a constant reflecting initial wealth differences. Equation (A1.10) states that if PPP holds, then the marginal utilities of consumption are equated up to a constant, χ, as agents confront identical commodity prices. Movements in the real exchange rate, or departures from PPP, will be reflected in different consumption rates. Despite the presence of complete financial markets, in the presence of home-bias in consumption, it is not efficient to equalize consumption rates across countries because PPP does not hold (unless $\alpha = 1/2$).

Price Setting

Agents are monopolist in selling their product and prices are staggered. In each period an agent faces a fixed probability $1-\gamma$ of adjusting her price. In this event the agent chooses the price, $P_H^n(i,h^t)$ for the produced differentiated good i so as to maximize expected utility resulting from sale revenues minus the disutility of output supply in each of the future states in which the price commitment still applies taking as given P, P_H, and P_F and subject to the demand function (A1.4).

Partial adjustment price setting implies that each period a measure of $1-\gamma$ of producers have the opportunity of changing their prices, then $P_{Ht+k}^n(i) = P_{Ht}^n(i)$ with probability γ^k for $k = 0, 1, 2, ...$ Notice that all the producers that are allowed

[17] In terms of the equilibrium conditions, the assumption that the instrument of the central bank is the nominal interest rate means that the money market equilibrium condition, (7), can be neglected, since it only determines the path of real money balances.

to reset their price in the same country at a certain time face the same discounted future demands and future marginal costs under the assumption that the new price is maintained. Thus they will set the same price.

Therefore, each producer maximizes expected utility resulting from sales revenues minus the disutility of output supply in each of the future states in which the price commitment still applies. Her state contingent profits may be written as:

$$P_H^n \max \sum_{k=0}^{\infty} \sum_{h^{t+k}} (\beta\gamma)^k Q(h^{t+k}, h^t)\{(1-\tau)P_H^n(i,h^t)Y(i,h^{t+k}) - v(Y(i,h^{t+k}); z(h^{t+k}))\} \tag{A1.11}$$

where nominal revenues are evaluated using the market discount factor $Q(h^{t+k}, h^t) = \beta \prod_{j=1}^{k} \pi(h^{t+j}, h^{t+j-1}) \left(\frac{u_c(h^{t+k})}{u_c(h^t)}\right) \frac{P(h^t)}{P(h^{t+k})}$ which is the same for all consumers because of the complete markets assumption. The sellers maximize (AI.1) subject to the sequence of demand constraints:

$$Y^d(i,h^{t+k}) = \left(\frac{P_H(i,h^t)}{P_H(h^{t+k})}\right)^{-\theta} [C_H(h^{t+k}) + C_F^*(h^{t+k})] \tag{A1.12}$$

Then P_H^n satisfies the first order condition:

$$P_H^n(i,h^t) = \frac{\theta \sum_{k=0}^{\infty} \sum_{h^{t+k}} (\gamma\beta)^k Q(h^{t+k}, h^t) v_y(Y^d(i,h^{t+k}), z(h^{t+k}))Y^d(i,h^{t+k}))}{(\theta-1)(1-\tau) \sum_{k=0}^{\infty} \sum_{h^{t+k}} (\gamma\beta)^k Q(h^{t+k}, h^t) Y^d(i,h^{t+k})} \tag{A1.14}$$

where $Q(h^{t+k}, h^t)$ is the delivery price of one unit of domestic currency in h^{t+k} in units of local currency at h^t and $z(h^t)$ is the domestic productivity shock.

According to (A1.14) the optimal relative price varies with current and expected future demands, discount factors and aggregate prices. Intuitively, firms know that the price they set today may also apply in future periods, so the expected state of the economy in those future periods affects the price that they choose today. If demand is expected to be high next period, for example, a producer will set a high price in the current period so as not to sell at loss because of future inflation. Similarly, depending on the degree of openness of the economy, if the terms of trade are expected to depreciate next period, the producer will set a higher price today so that one period of inflation leaves it closer to maximizing static profits next period.

Also, staggered price setting implies the following law of motion for the domestic price index:

$$P_H(h^t) = [\gamma P_H(h^{t-1})^{1-\theta} + (1-\gamma)P_H^n(h^t)^{1-\theta}]^{\frac{1}{1-\theta}} \tag{A1.15}$$

Finally, the terms of trade, i.e., the relative price of domestic to foreign goods, denoted by $S(h^t)$, equals the ratio of foreign to home prices $P_F(h^t)/P_H(h^t)$.

Fiscal Authority

In this paper I abstract from fiscal policy issues. The fiscal authority distributes the revenues from the proportional tax on nominal income and from seignorage to the households as lump sum transfers. The budget constraint at date t of the fiscal authority in the home country is:

$$\tau \int_0^1 P_H(i,h^t)Y(i,h^t)di = \int_0^1 TR(i,h^t)dj + M(h^t) - M(h^{t-1}) \tag{A1.16}$$

I assume that $\tau = -(\theta - 1)^{-1}$, so that in equilibrium the market power distortion created by the monopolistic structure in the market for differentiated goods is neutralized. Hence the steady state level of output is efficient.

Market Clearing Conditions

For each differentiated good, market clearing implies:

$$Y(i,h^t) = C_H(i,h^t) + C_F^*(i,h^t), \quad \forall i \in [0,1] \tag{A1.17}$$

Moreover aggregate domestic output is the sum of individual outputs:

$$Y(h^t) = \int_0^1 Y(i,h^t)di \tag{A1.18}$$

For the two economies it must be the case that:

$$\begin{aligned} Y(h^t) + Y^*(h^t) = {} & C(h^t)\left(\frac{P_H(h^t)}{P(h^t)}\right)^{-\eta}\left[(1-\alpha) + \alpha S(h^t)^{-\eta}\right] \\ & + C^*(h^t)\left(\frac{P_H^*(h^t)}{P^*(h^t)}\right)^{-\eta}\left[(1-\alpha) + \alpha S(h^t)^{\eta}\right] \end{aligned} \tag{A1.19}$$

The market clearing condition for the contingent bonds is

$$b(h^t) + b^*(h^t) = 0 \tag{A1.20}$$

where $b^*(h^t)$ denotes the foreign consumer's holdings of the home country bonds.

In the money market, the monetary authority supplies money to equate the money demand:

$$M(h^t) = \int_0^1 M(i,h^t)di \tag{A1.21a}$$

In a currency area the two countries share the same currency and consequently there is only one central bank that is entitled to issue money and to conduct monetary policy within the area. In the currency area, the level of money supplied by the common central bank is equal to the demand for money in the two economies:

$$M_{union} = \int_0^1 M_t^i di + \int_0^1 M_t^{*j} dj \qquad \text{(A1.21b)}$$

where M^i (M^*) denotes money demand of domestic (foreign) consumers.

The stationary equilibrium of the economy is characterized by:

$$1 + R = \frac{1}{\beta}$$

The monopolistic competition distortion does not allow the marginal utility of consumption to equate the marginal disutility of producing output, unless $\tau = -(\theta-1)^{-1}$. By using the Euler equation in consumption and (A1.16), (A1.10) we find the steady state value of C which we use to calculate the utility losses in terms of equivalent steady state consumption decreases. In the symmetric steady state the terms of trade and the exchange rate and are one. While Y=Y$^* = C = C^*$ and the prices are determined by the initial conditions P_{H-1}, P^*_{H-1}.

Equilibrium

An equilibrium for the economy described in section 2 is a collection of allocations for home consumers $C(h^t), C_H(h^t), C_F(h^t), M(h^t), b(h^{t+1})$; allocations for foreign consumers $C^*(h^t), C^*_H(h^t), C^*_F(h^t), M^*(h^t), b^*(h^{t+1})$; allocations and prices for domestic goods $Y(i, h^t), P_H(i, h^t)$ for $i \in [0, 1]$; allocations and prices for foreign goods $Y^*(i, h^t), P^*_H(i, h^t)$ for $i \in [0, 1]$;, aggregate price levels $P(h^t), P^*(h^t)$, bond prices $Q(h^{t+1}, h^t)$, equilibrium exchange rates $e(h^t)$ and individual transfers and tax rates $TR(i, h^t), \tau, TR^*(i, h^t), \tau^*$ that satisfy the following conditions: (i) taking as given the prices, consumers allocations solve the consumers' problem, (ii) the price set by each differentiated good producer solves his problem (iii) transfers satisfy (13) (iv) the monetary authority solves its maximization problem and (v) markets clear.

APPENDIX 2

Deriving the Reduced Form Equations

In this appendix I describe the derivation of the reduced form equations for aggregate demand presented in section 2. I ignore the state notation here and I substitute summation over probabilities with the expectation operator. Lower case variables denote the percentage deviations from respective steady state values. That is, for a generic variable X_t, $x_t = \log(X_t/X)$, where X is the steady state value of the variable.

The AS equation

I derive the log linear approximation of the AS Equation for the home country. The foreign's country AS equation is similar and is not included here. A1.14 can be written as:

$$E_t \sum_{k=0}^{\infty} (\gamma\beta)^k [(1-\theta)(1-\tau)u_{c,t+k} \frac{P_{Ht}^n}{P_{t+k}} + \theta v_y(Y_{t+k}, z_{t+k})] Y_{t+k} = 0 \tag{A2.1}$$

From the definition of the aggregate price level it follows that:

$$P_t = P_{Ht}[(1-\alpha) + \alpha S_t^{1-\eta}]^{\frac{1}{1-\eta}} \tag{A2.2}$$

Also, A1.12 combined with the demand for home goods at home and abroad as is given in appendix one, imply that the demand for the home good is given by:

$$Y_t(i) = \left(\frac{P_{Ht}(i)}{P_{Ht}}\right)^{-\theta} \left(\frac{P_{Ht}}{P_t}\right)^{-\eta} \left[(1-\alpha) + \alpha \left(\frac{P_{Ht}}{P_t}\right)^{-\frac{\eta}{\sigma}} \left(\frac{P_{Ht}^*}{P_t^*}\right)^{\frac{\eta}{\sigma}}\right] C_t \tag{A2.3}$$

loglinearizing A3.2 yields:

$$y_t = c_t + \alpha\eta[1 + \frac{(1-2\alpha)}{\sigma}] s_t \tag{A2.4}$$

Replacing A2.2 into A2.1,using A2.4 and taking a log linear approximation around the non-stochastic steady state the percentage deviation of newly set domestic prices to the domestic price index is set according to:

$$\begin{aligned} 0 \;=\; & E_t\{\sum_{k=0}^{\infty} (\beta\gamma)^k [(1-\theta)(1-\tau)\widehat{p}_{Ht+k}^n - \alpha(1-\theta)(1-\tau)u_c C s_{t+k} + (1-\theta)(1-\tau)u_c C c_{t+k} \\ & +\theta C v_{yy}[-\theta \widehat{p}_{Ht+k}^n + \alpha\eta[1 + \frac{(1-2\alpha)}{\sigma}] s_{t+k} + c_{t+k}] + \theta v_{yz} z_{t+k}\} \end{aligned} \tag{A2.5}$$

where $\widehat{p}_{Ht}^n \equiv p_{Ht}^n - p_{Ht}$.

A2.3 can be simplified further:

$$0 = E_t\{\sum_{k=0}^{\infty}(\beta\gamma)^k[\widehat{p}^n_{Ht+k} - \alpha s_{t+k} - \sigma c_{t+k} - \omega[-\theta\widehat{p}^n_{Ht+k} + \alpha\eta[1 + \frac{(1-2\alpha)}{\sigma}]s_{t+k} + c_{t+k} - \overline{Y}_t] \tag{A2.6}$$

where $\sigma = -u_{cc}C/u_c$ and $\omega = v_{yy}/v_y$ and $\overline{Y}_t$ is given by: $\overline{Y}_t = -(v_{yz}/Cv_{yy})z_{t+k}$.

Moreover:

$$\widehat{p}^n_{Ht+k} = \widehat{p}_{Ht} - \sum_{j=1}^{k}\pi_{Ht+j} \tag{A2.7}$$

Linearizing (A1.15) we obtain

$$\widehat{p}^n_{Ht} = \frac{\gamma}{1-\gamma}\pi_{Ht} \tag{A2.8}$$

Replacing the above two conditions for the percentage deviation of newly set domestic prices to the domestic price index into A2.6 we obtain:

$$\pi_{Ht} = \frac{(1-\gamma)(1-\gamma\beta)}{\gamma}[\frac{\sigma+\omega}{1+\theta\omega}c_t + \alpha\{1 + \omega\eta[1 + \frac{(1-2\alpha)}{\sigma}]\}s_t + \frac{\sigma}{1+\theta\omega}\overline{Y}_t] + \beta E_t\pi_{Ht+1} \tag{A2.9}$$

If we define the natural rate of consumption as: $c^n_t = \frac{\omega}{\omega+\sigma}\overline{Y}_t$ and using the definition for the natural rate of the terms of trade we end up with:

$$\pi_{Ht} = \beta E_t\pi_{Ht+1} + k_c(c_t - c^n_t) + k_s(s_t - s^n_t) \tag{A2.10}$$

where:$k_c = k(\frac{\sigma+\omega}{1+\theta\omega})$, $k_s = \frac{k\alpha\{1+\omega\eta[1+\frac{(1-2\alpha)}{\sigma}]\}}{1+\theta\omega}$, $k = \frac{1-\gamma}{\gamma}(1-\gamma\beta)$ and σ is the intentemporal elasticity of substitution, η is the elasticity of substitution between foreign and domestic goods, θ is the elasticity of substitution among differentiated goods produced within the country, and ω is the inverse of the elasticity of labor supply.

Notice that as $\alpha \to 0$, $k_s = 0$,which is the standard closed economy aggregate supply curve (see also, Rotemberg and Woodford, 1998).

Aggregate Demand equations

As in the previous subsection I will present the derivation of the conditions for the domestic economy. The conditions of the foreign economy are analogous.

The marginal utility of consumption equals the lagrange multiplier: $u_c(C_t) = \Lambda_t$. The FOC with respect to next period's domestic bonds gives: $\Lambda_t = R_t\beta E_t\Lambda_{t+1}\frac{P_t}{P_{t+1}}$. Loglinearizing these two expressions results in :

$$-\sigma c_t = \lambda_t \tag{A2.11}$$

$$\lambda_t = E_t[\widehat{R}_t - \pi_{t+1} + \lambda_{t+1}] \tag{A2.12}$$

Solving forward A2.11 gives a version of the Fischer equation:

$$\lambda_t = r_t^l = \sum_{k=0}^{\infty} E_t[\widehat{R}_{t+k} - \pi_{t+k+1}] \tag{A2.13}$$

where r_t^l is the long run real rate of return. Taking first differences in equation (A2.13), and using (A2.11 and 12) results into:

$$c_t = c_{t+1} + (-\sigma)^{-1}(\widehat{R}_t - E_t\pi_{t+1}) \tag{A2.14}$$

Equation (A2.13) determines the aggregate demand for the domestic good. Aggregate demand depends not only on the short-term real interest rate but also on the long run real interest rate, r_t^l, on foreign consumption demand and the relative price of foreign to domestic goods. Instead (A2.14) is the standard form of the IS equation.

The definition of the domestic aggregate price index together with the definition for the terms of trade then implies:

$$\begin{aligned} p_{Ht} - p_t &= -\alpha s_t \\ p_{Ht}^* - p_t^* &= -\alpha s_t^* \\ p_{Ft} - p_{Ht} &= s_t \\ s_t &= -s_t^* \end{aligned} \tag{A2.15}$$

Loglinearizing yields:

$$\pi_t = \pi_{Ht} + \alpha \Delta s_t \tag{A2.16}$$

Using (A2.16) to replace for CPI inflation in A3.4 yields the aggregate demand curve in section 2.

The real interest parity condition follows from the definition of the terms of trade, the law of one price and Uncovered Interest Parity. Loglinearizing (A1.8) yields[18]:

$$\widehat{R}_t - \widehat{R}_t^* = E_t \Delta e_{t+1} \tag{A2.17}$$

the law of one price in turn gives:

$$p_{Ft} = p_{Ht}^* - e_t \tag{A2.18}$$

[18]where the hats indicate percentage deviations from steady state values

combining the above with the definition of the terms of trade yields:

$$(\widehat{R}_t - \pi_{Ht+1}) - (\widehat{R}_t^* - \pi_{Ht+1}^*) = E_t\{\Delta s_{t+1}\} \tag{A2.19}$$

APPENDIX 3

Derivation of the Social Objective Function in an Open Economy

Maximizing the utility of the representative agent in the economy is equivalent with maximizing:

$$W = E_t\{u(C_t^i) - \int_0^1 v(Y_t(\tau); Z_t)d\tau\} \tag{A3.1}$$

In (A4.1) real money balances are excluded from the utility function. Here, I assume that the liquidity services of money are arbitrarily small, thus I can neglect the term concerning real money balances from the utility function. Even, if this term was not negligible, Woodford (1996) and Rotemberg and Woodford (1998) show that in the presence of an interest rate rule, money need not be introduced in the model. The equilibrium condition (A1.7), when monetary policy is specified in terms of an interest rate rule, simply determines the nominal level of money balances. Since this condition plays no role in determining interest rates, inflation or output, money can be ignored and utility can be computed only in terms of consumption and output.

Following Rotemberg and Woodford(1997,1998) we compute a second order Taylor series expansion of W around the deterministic steady state where all the shocks are zero. A second order Taylor expansion of the left term gives:

$$\begin{aligned} u &= u(C) + u_c(C_t - C) + \tfrac{1}{2}u_{cc}(C_t - C)^2 \\ &= u(C) + u_c(c_t + \tfrac{1}{2}c_t^2) + \tfrac{1}{2}u_{cc}(c_t + \tfrac{1}{2}c_t^2)^2 + O\left(\|\xi\|^3\right) \\ &= u_cC[c_t + \tfrac{1}{2}(1-\sigma)c_t^2] + t.i.p. + O\left(\|\xi\|^3\right) \end{aligned} \tag{A3.2}$$

where $O(\|\xi\|^3)$ includes terms of order higher than second in the deviations of the variables from their steady state values. For zero steady state inflation and constant tax on output, the variables will deviate from their steady state values only because of variations of the productivity shocks $\xi_t = [z_t, z_t^*]$ around their steady state values. If $\|\xi\|$ is the measure of the size of the shocks, the omitted terms are all of third or higher order in this size. We have expanded C_t with a second order Taylor approximation: $C_t = C(1 + c_t + \frac{1}{2}c_t^2) + O\left(\|\xi\|^3\right)$. $\sigma = -u_{cc}C/u_c$

The second term inside the integral of A3.1 can be written as:

$$v = v(Y,0) + v_y(Y_t(i) - Y) + v_z Z_t + \frac{1}{2}v_{yy}(Y_t(i) - Y)^2 + v_{yz}(Y_t(i) - Y)Z_t + \frac{1}{2}v_{zz}Z_t^2 + O\left(\|\xi\|^3\right) \tag{A3.3}$$

$$\text{or } v = v_y Y[y_t(i) + \frac{\omega}{2}y_t^2(i) - \omega y_t(i)\overline{Y}_t + t.i.p. + O\left(\|\xi\|^3\right)$$

where $\overline{Y}_t : v_{yz}z_t = -v_{yy}Y\overline{Y}_t$ provides a scalar measure of disturbances to the marginal disutility of supply and $\omega = v_{yy}/v_y$. Integrating across all domestic households yields:

$$\int_0^1 v(i)di = v_y Y[E_i y_t(i) + \tfrac{1}{2}[var_i y_t(i) + (E_i y_t(i))^2] + \tfrac{\omega}{2}[var_i y_t(i) + (E_i y_t(i))^2] \\ -\omega E_i y_t(i)\overline{Y}_t] + t.i.p. + O\left(\|\xi\|^3\right) \tag{A3.4}$$

for the aggregate output we have:

$$y_t = E_i y_t(i) + \frac{1}{2}\frac{\theta - 1}{\theta}[(1-\alpha)var_i c_{Ht}(i) + \alpha var_i c^*_{Ht}(i)] \tag{A3.5}$$

Then A3.4 becomes:

$$\int_0^1 v(i)di = v_y Y[y_t + \tfrac{\omega}{2}y_t^2 + \tfrac{1}{2}(\theta^{-1} + \omega)[(1-\alpha)var_i c_{Ht}(i) + \alpha var_i c^*_{Ht}(i)] \\ -\omega y_t \overline{Y}_t] + t.i.p. + O\left(\|\xi\|^3\right) \tag{A3.6}$$

Combining A3.2 and A3.6 together with the fact that $u_c C = v_y Y$, in equilibrium since $(1-\tau) = \frac{\theta}{\theta-1}$, results in:

$$W = v_y Y[c_t + \tfrac{1}{2}(1-\sigma)c_t^2 - y_t - \tfrac{1+\omega}{2}y_t^2 - \tfrac{1}{2}(\theta^{-1} + \omega)[(1-\alpha)var_i c_{Ht}(i) + \alpha var_i c^*_{Ht}(i)] \\ +\omega y_t \overline{Y}_t y_t] + t.i.p. + O\left(\|\xi\|^3\right) \tag{A3.7}$$

Using (A1.10) and (A1.4) the demand for domestic output is given by:

$$Y_t^d = [(1-\alpha) + \alpha s_t^{1-\eta}]^{\frac{\eta}{1-\eta}}[(1-\alpha) + \alpha q_t^{\eta/\sigma}]C_t \tag{A3.8}$$

Loglinearizing (A3.8) and substituting in (A3.7) together with the definitions of the natural rates,results in:

$$W = -\tfrac{1}{2}u_c C\{\eta\alpha\left[1 + \tfrac{1-2\alpha}{\sigma}\right]s_t + (\sigma + \omega)(c_t - c_t^n)^2 + \alpha\{1 + \omega\eta[1 + \tfrac{(1-2\alpha)}{\sigma}]\}(s_t - s_t^n)^2 + \\ +(\theta^{-1} + \omega)[(1-\alpha)var_i c_{Ht}(i) + \alpha var_i c^*_{Ht}(i)]\} + t.i.p. + O\left(\|\xi\|^3\right) \tag{A3.9}$$

Moreover, it also holds that:

$$Evar_i y_t(i) = Evar_i c_{Ht}(i) = Evar_i c^*_{Ht}(i) = \theta^2 Evar_i\{\log p_{Ht}(i)\} \tag{A3.10}$$

Defining $\overline{p}_{Ht} \equiv E_i \log p_{Ht}(i)$, we have:

$$var_i\{\log p_{Ht}(i)\} = var_i\{\log p_{Ht}(i) - \overline{p}_{Ht-1}\} = E_i\{[\log p_{Ht}(i) - \overline{p}_{Ht-1}]^2\} - (\Delta \overline{p}_{Ht})^2 =$$
$$\gamma E_i\{[\log p_{Ht-1}(i) - \overline{p}_{Ht-1}]^2\} + (1-\gamma)\{[\log p^n_{Ht}(i) - \overline{p}_{t-1}]^2\} - (\Delta \overline{p}_{Ht})^2$$
$$= \gamma var_i\{\log p_{Ht-1}(i)\} + (1-\gamma)\{[\log p^n_{Ht}(i) - \overline{p}_{t-1}]^2\} - (\Delta \overline{p}_{Ht})^2 \quad \text{(A3.11)}$$

Given that:$\Delta \overline{p}_{Ht} = (1-\gamma)[\log p^n_{Ht}(i) - \overline{p}_{t-1}]$ implies that:

$$var_i\{\log p_{Ht}(i)\} = \gamma var_i\{\log p_{Ht-1}(i)\} + \frac{\gamma}{1-\gamma}\pi^2_{Ht} + O\left(\|\xi\|^3\right) \quad \text{(A3.12)}$$

Integrating yields:

$$var_i\{\log p_{Ht}(i)\} = \gamma^{t+1} var_i\{\log p_{Ht-1}(i)\} + \sum_{s=0}^{\infty} \gamma^{t-s}\frac{\gamma}{1-\gamma}\pi^2_{Ht} + O\left(\|\xi\|^3\right) \quad \text{(A3.13)}$$

After taking the discounted value, I obtain:

$$\sum_{t=0}^{\infty} \beta^t var_i\{\log p_{Ht}(i)\} = \frac{\gamma}{(1-\gamma)(1-\gamma\beta)} \sum_{t=0}^{\infty} \beta^t \pi^2_{Ht} + t.i.p. + O\left(\|\xi\|^3\right) \quad \text{(A3.13)}$$

Then A3.8 becomes:

$$W = -\tfrac{1}{2}u_c C \tfrac{\gamma}{(1-\gamma)(1-\gamma\beta)}\theta(1+\theta\omega)\{\tfrac{(\sigma+\omega)(1-\gamma)(1-\gamma\beta)}{(1+\theta\omega)\gamma\theta}(c_t - c^n_t)^2 + \tfrac{(1-\gamma)(1-\gamma\beta)}{(1+\theta\omega)\gamma\theta}$$
$$\alpha\{1+\omega\eta[1+\tfrac{(1-2\alpha)}{\sigma}]\}(s_t - s^n_t)^2 + \pi^2_{Ht} + \tfrac{(1-\gamma)(1-\gamma\theta)}{\gamma\theta}\tfrac{2\eta\alpha[1+\frac{(1-2\alpha)}{\sigma}]]}{(1+\theta\omega)}(s_t - s^n_t)\} + t.i.p. + O\left(\|\xi\|^3\right) \quad \text{(A3.14)}$$

Notice that the loss function in (AIII.17) reduces to the loss function used by Rotemberg and Woodford(1997) for a closed economy as the degree of openness of the economy goes to zero ($\alpha \longrightarrow 0$).

APPENDIX 4

Approximating the optimal plan

Deriving the optimal rule

The problem of the central bank in the cooperative solution is to maximize a quadratic function subject to linear constraints. The FOC of the planners problem in the cooperative solution are given by:

$$\delta\lambda_c c_t - k_c\phi_{1t} + \phi_{2t} - \beta^{-1}\phi_{2t-1} = 0 \tag{A4.1}$$

$$\delta^*\lambda_c c_t^* - k_c\phi_{1t}^* + \phi_{2t}^* - \beta^{-1}\phi_{2t-1}^* = 0 \tag{A4.2}$$

$$\delta\pi_{Ht} + \phi_{1t} - \phi_{1t-1} - \beta^{-1}(-\sigma)^{-1}\phi_{2t-1} - \beta^{-1}(-\sigma)^{-1}\phi_{2t-1}^* = 0 \tag{A4.3}$$

$$\delta^*\pi_{Ht}^* + \phi_{1t}^* - \phi_{1t-1}^* - \beta^{-1}(-\sigma)^{-1}\phi_{2t-1}^* - \beta^{-1}(-\sigma)^{-1}\phi_{2t-1} = 0 \tag{A4.4}$$

$$-\phi_{2t}(-\sigma)^{-1}(1+\alpha) + \phi_{2t}^*\alpha(-\sigma)^{-1} = 0 \tag{A4.5}$$

$$-\phi_{2t}^*(-\sigma)^{-1}(1+\alpha) + \phi_{2t}\alpha(-\sigma)^{-1} = 0 \tag{A4.6}$$

$$(\delta+\delta^*)\lambda_s s_t - k_s\phi_{1t} + k_s\phi_{1t}^* = 0 \tag{A4.7}$$

These 7 first order conditions together with the aggregate demand and aggregate supply equations at home and abroad and the UIP condition define a system of 12 equations in 12 unknowns. The system can be written in a vector form as:

$$Q\begin{bmatrix} Eq_{t+1} \\ \phi_t \end{bmatrix} = M\begin{bmatrix} q_t \\ \phi_{t-1} \end{bmatrix} + \Omega\begin{bmatrix} z_t \\ z_t^* \end{bmatrix} \tag{A4.8}$$

where $q_t = [y_t, \pi_{Ht}, \widehat{R}_t, y_t^*, \pi_{Ht}^*, \widehat{R}_t^*, e_t, s_t]'$ and $\phi_t = [\phi_{1t}, \phi_{2t}, \phi_{1t}^*, \phi_{2t}^*,]'$ and Q, M (12x12)and Ω ($12x1$) are matrices. The bottom 7 equations are equations A4.1-A4.7, expressed as the law of motion for the lagrange multipliers, while the other four equations are the aggregate demand and aggregate supply equations for the two countries. Considering a bounded stochastic process for the productivity shocks, a bounded optimal plan exists and it is unique, since the matrix $(Q^{-1})M$ has exactly 5 eigenvalues inside the unit circle and there are 5 predetermined variables in A4.8. The unique bounded solution can be written as:

$$\begin{aligned} \phi_t &= N\phi_{t-1} + G\xi_t \\ x_t &= R\phi_t + S\xi_t \\ \xi_t &= P\xi_{t-1} + u_t \end{aligned} \qquad \text{(A4.9)}$$

From the first equation in A4.9 we can eliminate ϕ_{2t}, and ϕ^*_{2t} and we can express the nominal interest rate as a function of the real shocks in the economy:

$$\begin{aligned} \widehat{R}_t &= R(L)\xi_t \\ \widehat{R}^*_t &= T(L)\xi_t \end{aligned} \qquad \text{(A4.10)}$$

where $\xi_t = [z_t, z^*_t]'$ and $R(L)$ and $T(L)$ are polynomials of first order.

Combining A4.10 with the equilibrium conditions I can reduce the set of equilibrium conditions to the bivariate dynamical set up given by:

$$Dg_t = E_t B g_{t+1} + J\xi_{t+1} + K\xi_t \qquad \text{(A4.11)}$$

where $g_t = [y_t, \pi_{Ht}]'$. The equilibrium will be locally unique if and only if the two eigenvalues of B are inside the unit circle. Yet, this is not the case. The eigenvalues of B are real and positive, but one is bigger than one while the other is less than one. The fact that one eigenvalue has modulus less than one, even though both elements of g_t are not predetermined, implies that for $\{g_t\}$ any bounded stochastic process satisfying A4.11, another bounded solution is given by: $g'_t = g_t + f\zeta_t$ where f is the eigenvector of B associated with the eigenvalue with modulus less than one(μ_1), and the stochastic process ζ_t satisfies: $\zeta_{t+1} = \mu_1\zeta_t + w_{t+1}$, where w_{t+1} is any bounded random variable with zero mean and it is unforcastable at date t. Hence, even if the rule is chosen so as to be consistent with the optimal equilibrium, it does not represent a desirable way of implementing optimal policy. Finally, the problem of indeterminacy of equilibria can be resolved through sufficiently strong feedback from the endogenous variables.

Approximating the Optimal Plan

Here, I present the calculation of the periodograms of the impulse response functions and the criterion I use for approximating the first best.

Let $\Pi(L)$ $(\Pi^*(L))$be the 12×1 vector of moving average polynomials in response to unitary shocks in $z_t(z_t^*)$, for domestic variables. Then,

$$
\begin{aligned}
s^{Op}(\omega) &= \sum_{j=-\infty}^{\infty} e^{-i\omega j} \left|\Pi(j)\right|^2 \\
s^{Op*}(\omega) &= \sum_{j=-\infty}^{\infty} e^{-i\omega j} \left|\Pi^*(j)\right|^2 \text{,where } j = 1, 120
\end{aligned}
$$

I evaluate $s^{Op}(\omega)$ at $\omega = \frac{2\pi}{120}$ (the Fourier frequencies). I do the same for the simulated impulse responses for different values of coefficients in the interest rate rule (27) and I look for values of v_π, v_z, v_{z^*}, that minimize the distance between the two periodograms at medium and low frequencies. The values of the coefficients in the feedback rule that approximate the optimal policy impulse responses are the solutions to the problem:

$$
\min_{v_\pi, v_z, v_{z^*}} \sum_{\omega=0}^{\pi/4} \sum_{i=1}^{12} \phi_i (s_i^{Op}(\omega) - s_i^{rule}(\omega))
$$

$$
\min_{v_\pi, v_z, v_{z^*}} \sum_{\omega=0}^{\pi/4} \sum_{i=1}^{12} \phi_i (s_i^{*Op}(\omega) - s_i^{*rule}(\omega))
$$

where i denotes the number of variables included in the matrix Π. $\phi_i = \phi$ denotes the weights that each variable receive in the minimization problem and I assume equal weights for all the variables. The values of the coefficients that minimize the distance between the periodograms of the impulse responses in a domestic productivity shock and in a foreign productivity shock do not differ substantially.

Chapter 3

A Monetary Model of Factor Utilisation joint with Katharine Neiss (Bank of England)[19]

1 Introduction

The current workhorse for the study of monetary policy is a sticky-price dynamic stochastic general equilibrium model[20]. Calvo (1983) and Rotemberg (1997) provide the theoretical background for introducing nominal price rigidities within a tractable, representative firm framework.This specification, known as the New Phillips curve, has become standard in sticky price models (McCallum, 1997). A central component of these New Keynesian models is that monetary shocks have real effects. Recent work by Kiley (1998) and Chari, Kehoe and McGrattan (2000), however, has shown that the predictions regarding persistence in the Calvo, partial adjustment model do not generally carry over to more realistic models of Taylor - type (1979) staggering[21]. Moreover, in the basic New Keynesian model, the only way to induce protracted effects of monetary policy on real variables is to assume a high degree of price stickiness. This is a rather unsatisfying mechanism given the controversy surrounding the existence of price adjustment costs. But even if there is agreement that these costs exist, there is no agreed framework for modelling the costs that firms face for changing their price[22].

[19]We thank Mark Astley, Larry Ball, Jordi Gali, Jens Larsen, Ed Nelson and Fergal Shortall for helpful discussions. We are particularly indebted to Ed Nelson for his help in coding stochastic simulations.

[20]See for example Rotemberg and Woodford (1997) and McCallum and Nelson (1999).

[21]See for example Jeanne (1997) and Anderson (19xx).

[22]A recent paper by Mankiw (2000) reviews some more general weaknesses associated with the New Keynesian Phillips curve, such as the counter-intuitive predicted relationship between expected inflation and output as noted by Ball (1994).

One way to address these criticisms is to reduce the model's reliance on nominal rigidities for the propagation of shocks, and assume a low degree of nominal rigidity. In doing so, however, the propagation mechanism to monetary shocks in a standard model is essentially eliminated. In a seminal paper, Ball and Romer (1990) suggest that real rigidities have a crucial role in explaining nominal rigidities and the non-neutrality of shocks. Indeed, these arguments are echoed in more recent work by Romer (1996), Christiano, Eichenbaum, and Evans (1997), Kiley (1998), and Chari, Kehoe, and McGrattan (2000). The hope is that real rigidities, coupled with small nominal rigidities, are enough to induce non-neutral effects of monetary policy shocks. In addition, real rigidities have the added benefit of bringing the predictions of a partial adjustment model closer in line with more realistic, but cumbersome, staggered price setting models as noted by Kiley (1998).

A related issue is the importance of fluctuations in investment in the transmission mechanism of monetary policy. Many sticky price models assume an exogenous capital stock (Rotemberg and Woodford, 1997, McCallum and Nelson, 1999). The behaviour of these models with endogenous capital formation has been a key area of recent research (King and Watson 1996, Woodford 2000, Casares and McCallum 2000). A problem with sticky price models with capital is that of too much persistence: output becomes excessively responsive to monetary shocks if capital can be costlessly adjusted. In order to generate realistic dynamics, sticky price models typically introduce a real rigidity in the form of capital adjustment costs– in essence, making these models behave similarly to sticky price models with no capital. But in the case of capital with adjustment costs, Chari, Kehoe, and McGrattan (2000) note that the introduction of capital nevertheless plays a crucial role in reinforcing the lack of persistence in staggered pricing and partial adjustment models.

This paper investigates the persistence properties of a sticky-price variant of Burnside and Eichenbaum's (1996) model with capital and time-varying factor utilisation[23]. Burnside and Eichenbaum's seminal paper showed that i.i.d. shocks to productivity growth could generate persistence in a real business cycle model with time-varying fac-

[23]Fagnart, Licandro, and Portier (1999) investigate the implications of capacity utilisation in a model with explicit micro-foundations, and find that capacity utilisation is an important mechanism for the propagation of technology shocks. Although the depreciation through use assumption in the BE model is a crude way of modelling capacity utilisation, the authors find that it generates similar predictions to those based on a more micro-founded approach.

tor utilisation. We investigate whether the persistence properties of the model carry over to nominal shocks in a sticky-price environment. We find that the introduction of time varying factor utilisation can generate a persistence response to monetary policy shocks, even at relatively low levels of nominal rigidity[24]. Unlike sticky-price models with capital adjustment costs, time-varying factor utilisation elicits an even greater response of investment to policy shocks, and as such allows for a reduction in the assumed degree of price rigidity without sacrificing the persistence properties of the model. In addition, at low levels of nominal rigidity, we are able to generate realistic investment volatility without having to introduce capital adjustment costs.

We compare our results to sticky-price models with capital to explore more generally the relationship between nominal price rigidity and firms' ability to adjust capital services. As described in Chari, Kehoe, McGrattan (2000), in order to generate a persistent output response to monetary shocks in a staggered price setting environment, price inertia must arise endogenously from optimal firm behaviour. One way to achieve this is to reduce the sensitivity of costs to changes in output. In standard models with variable labour input and predetermined capital, firms face sharply rising short-run marginal costs. Firms therefore have relatively limited scope for adjusting their inputs and hence nominal marginal cost. The only option for firms to restore their mark-up in the face of unanticipated shocks is to change their price. In a model where both capital and labour services can respond immediately to unanticipated shocks, firms have an additional margin that they can adjust. Firms are therefore better able to control their marginal costs by varying inputs, and as a result have less of a need to adjust their prices to restore their mark-up. This is the mechanism that a model of time varying factor utilisation exploits to generate persistence at low degrees of nominal rigidity.

In a related paper, Christiano, Eichenbaum and Evans (2001) use staggered wage contracts and variable capital utilisation to generate both output persistence and inflation inertia. Labour market rigidities coupled with variable capital utilisation both in their model and in the one presented in this paper introduce a strong internal propagation mechanism. In our model, labour market 'rigidities' are represented

[24]A related paper by Cook (1999) looks at the propagation mechanism of a real business cycle model with time varying factor utilisation and dynamic complementarities in a limited participation model. He finds that a transitory liquidity shock has a persistent effect on real output, and that time varying factor utilisation plays an important role in augmenting the propagation mechanism.

by labour hoarding (i.e., firms cannot adjust employment instantaneously), whereas Christiano et. al (2001) (in a more complete framework) assume staggered wage contracts. Their model includes various departures from the standard general equilibrium model, such as habit persistence in consumption and investment adjustment costs, in order to account for the response of consumption and investment to a monetary policy shock. For a small degree of price rigidities our model's predictions for the behaviour of output and investment coincide with the ones in their model. In contrast with Christiano et al. (2001) we are not able to generate inertia in inflation since wages are completely flexible in our model.

The paper is organised as follows. In Section 2 we describe the model, Section 3 discusses the calibration and impulse responses of our benchmark model, Section 4 looks at other sticky-price models both with and without capital, Section 5 compares the various model statistics, Section 6 explores the mechanisms that generate persistence in the model and Section 7 concludes.

2 A model of time-varying factor utilisation

This section describes a sticky-price variant of Burnside and Eichenbaum's (1996) model with capital formation and time-varying effort and capital utilisation rates.

The economy consists of infinitely-lived agents, firms, and a government sector. Households and firms optimise intertemporally and have rational expectations. As is usually assumed in the New Keynesian literature, monopolistic firms set their price to maximise profits, but cannot always adjust them instantaneously in response to changing economic conditions. Nominal price stickiness is modelled as in Calvo's (1983) specification of price adjustment. Firms produce a continuum of differentiated goods, which are aggregated to produce a single composite good that can be used for consumption and investment. Households derive utility from the transactions services provided by real balances, and the economy is subject to shocks to real productivity, government spending and the nominal money stock. The key feature of the model is factor hoarding by firms. Following Burnside and Eichenbaum, we assume that the technology for producing differentiated goods depends on capital and labour services. The latter is defined as labour effort times total hours worked. The former is defined as capital utilisation times the existing physical stock of capital. The rate at which capital depreciates is assumed to be a function of the capital utilisation rate. As a result, in equilibrium, firms may over- or under-utilise (e.g. hoard) capital. Moreover, the equilibrium amount of labour units employed (measured in heads) in production is assumed to be chosen prior to the realisation of period shocks.

2.1 Households

Households consume a continuum of differentiated goods indexed by $i \in [0,1]$. The composite consumption good (C_t), which is defined by a Dixit-Stiglitz aggregate over the multiplicity of goods, and price index (P_t) are defined as:

$$C_t = \left[\int_0^1 c_t(i)^{\frac{\rho-1}{\rho}} di \right]^{\frac{\rho}{\rho-1}} \tag{1}$$

and

$$P_t = \left[\int_0^1 p_t(i)^{1-\rho} di \right]^{\frac{1}{1-\rho}} \tag{2}$$

where the elasticity of substitution between differentiated goods, , is assumed to be greater than one.

The economy is inhabited by a large number of households, each of which has preferences defined over the composite consumption good (C_t), real money balances ($\frac{M_t^d}{P_t}$), and leisure. Following Hansen (1985) and Rogerson (1988), we assume that agents face a lottery, which determines whether or not they will be employed[25]. The probability of employment in time t is given by (N_t). Those employed work a fixed shift length h, and incur a fixed cost of commuting out of their total time endowment χ. Whether time spent at work contributes to leisure depends on the level of effort (e_t) expended. Effective hours of leisure for the fraction of the population currently employed are thus given by $\tau - \chi - he_t$.

The proportion $(1 - N_t)$ of the population not currently employed derive leisure from their total time endowment[26]. The proportion of the population currently in

[25]A more general specification of the utility function is adopted in Section 6.

[26]Indivisibility of labour makes aggregation easier, and implies a labour supply elasticity of zero. The steady state elasticity of effort supply is not zero, however, but equal to $\frac{\tau-\chi}{h} - 1$.

employment is assumed to be predetermined. This captures the notion that employment in heads cannot be immediately adjusted in response to unanticipated shocks and that firms must make employment decisions conditional on their view about the future state of demand and technology. The assumption of predetermined employment can be rationalised by the existence of labour market rigidities, such as labour unions, that restrict the ability of firms to instantly adjust employment in response to unanticipated shocks.

Households have access to a complete set of contigent bonds. The representative household chooses a sequence of consumption, effort, nominal money balances and one-period bond holdings (B_{t+1}), capital (K_{t+1}), utilisation (U_t), and employment (N_{t+1}), to maximise lifetime utility:

$$E_t \sum_{j=0}^{\infty} \beta^j \left[\ln C_{t+j} + \theta N_{t+j} \ln\left(\tau - \chi - h e_{t+j}\right) + \theta\left(1 - N_{t+j}\right) \ln(\tau) + \frac{1}{1-\varepsilon} \left(\frac{M_{t+j}^d}{P_{t+j}} \right)^{1-\varepsilon} \right] \tag{3}$$

subject to a series of period budget constraints:

$$P_{t+j} C_{t+j} + P_{t+j} I_{t+j} + M_{t+j}^d + B_{t+j+1} = P_{t+j} w_{t+j} N_{t+j} e_{t+j} + P_{t+j} r_{t+j} U_{t+j} K_{t+j} + M_{t-1+j}^d + \left(1 + R_{t-1+j}\right) B_{t+j} + \Gamma_{t+j} + V_{t+j} \tag{4}$$

$\forall j = 0, 1, ...\infty$, where $\theta > 0$, $\varepsilon > 0$, and $\beta \in (0, 1)$. In the budget constraint, R_{t-1+j}denotes the net nominal interest rate,r_{t+j} denotes the rental rate on capital services, and V_{t+j} and Γ_{t+j} denote lump sum firm profits and government transfers, respectively. Investment (I_{t+j}) is related to the capital stock by:

$$I_{t+j} = K_{t+1+j} - \left(1 - \delta U_{t+j}^{\phi}\right) K_{t+j} \tag{5}$$

Following Greenword, Hercowitz and Huffman (1988) the evolution of capital assumes that using capital more intensively increases the rate at which capital depreciates, where $\delta \in [0, 1)$ and $\phi > 1$. The parameter ϕ is negatively related to the responsiveness of utilisation to shocks, and is interpreted as the elasticity of depreciation with respect to utilisation. For very large values of ϕ, the negative effects of utilisation on depreciation dominate the positive effects of utilisation on output, and firms choose to keep utilisation constant.

The first order conditions for the representative household are given in Appendix 1.

2.2 Firms

There is a continuum of monopolistically competitive firms, indexed by $i \in [0,1]$. Each firm i chooses its factor inputs, labour services ($N_t e_t$) and capital services ($K_t U_t$), in order to minimise costs of producing a given level of output ($\overline{Y}_t$):

$$w_t N_t e_t + r_t K_t U_t \tag{6}$$

subject to its technological constraint on production[27]:

$$\overline{Y}_t \leq (K_t U_t)^{1-\alpha} (N_t h e_t X_t)^{\alpha} \tag{7}$$

where $0 < \alpha < 1$. The process for the level of technology is assumed to follow a logarithmic random walk with drift:

$$X_t = X_{t-1} \exp(\gamma + v_t) \tag{8}$$

where $v_t \tilde{} i.i.d.(0, \sigma_A)$. The firm chooses labour and capital services such that:

$$w_t = \frac{\alpha Y_t mc_t}{N_t e_t} \tag{9}$$

$$r_t = \frac{(1-\alpha) Y_t mc_t}{K_t U_t} \tag{10}$$

[27]An alternative model of factor hoarding, due to Bils and Cho (1994), relates capital utilisation directly to effort in the production function. This specification has perhaps greater intuitive appeal in that increases in total hours worked automatically raises the degree to which the existing physical capital stock is utilised. However, Burnside and Eichenbaum note that the propagation mechanism of this alternative model of factor hoarding is much weaker. Although the specification adopted in the present paper does not mechanically link utilisation to effort, they will nevertheless move together in response to shocks since they are assumed to be complements in production.

where mc_t denotes the unit cost function, or real marginal cost.

As described in King and Wolman (1996) and Yun (1996), each firm i is allowed to reset its price $(\widetilde{P}_t^i)$ according to a stochastic time-dependent rule that depends on receiving a signal at a constant random rate $(1-\eta)$. The parameter η governs the degree of nominal price rigidity: if η is equal to 0, then prices are perfectly flexible; if η is equal to 1, then firms never have the opportunity to revise their prices. Producers face an idiosyncratic risk due to the uncertainty of price adjustment. The probability that the price set at time t still prevails at $t+j$ is given by η^j. Each firm with an opportunity to change its price will choose it to maximise profits, taking aggregate output (Y_t), the aggregate price level (P_t), nominal marginal cost (MC_t^i), and demand for its good (Y_t^i) as given:

$$E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \eta^j \left[\widetilde{P}_t^i - MC_{t+j}^i\right] Y_{t+j}^i \tag{11}$$

subject to:

$$Y_{t+j}^i = \left[\frac{P_t^i}{P_t}\right]^{-\rho} Y_{t+j}, \forall j = 0, 1, \ldots, \infty \tag{12}$$

The solution to this problem yields the firm's optimal price (P_t^i), which is given by:

$$P_t^i = \frac{\rho}{\rho - 1} \frac{E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \eta^j \left[Y_{t+j}^i mc_{t+j}^i\right]}{E_t \sum_{j=0}^{\infty} \Lambda_{t,t+j} \eta^j Y_{t+j}^i} \tag{13}$$

In the above relationship, $\frac{\rho}{\rho-1}$ is the steady state mark-up, or the inverse of the steady state real marginal cost. Equation (1.13) illustrates that the optimal price depends on current and expected future demand and real marginal cost (mc_t^i). Intuitively, firms know that the price they set today may also apply in future periods, so the expected state of the economy influences the price they choose today.

Given the pricing decisions of each firm i, the aggregate pricing price index evolves according to:

$$P_t = \left[\eta P_{t-1}^{1-\rho} + (1-\eta) \widetilde{P}_t^{1-\rho}\right]^{\frac{1}{1-\rho}} \tag{14}$$

The aggregate price level is therefore a weighted average of prices set in $t-1$, to reflect the fact that some firms cannot change their price in period t, and the optimal price, to reflect the fact that the remaining firms can reset their price to the optimal price at time t.

2.3 Government

Real government purchases of goods and services are modelled as an exogenous stochastic process[28]:

$$G_t = X_t \exp(g_t) \tag{15}$$

where $g_t = \mu(1-\rho_g)+\rho_g g_{t-1}+\varepsilon_{gt}$, with $|\rho_g| < 1$, and $\varepsilon_{gt}\tilde{}\, i.i.d.(0, \sigma_g)$. Government expenditure serves as the second real shock in the model.

The nominal money supply process is assumed to follow[29]:

$$M_t^s = \mu_t M_{t-1}^s \tag{16}$$

where $\mu_t = \rho_\mu \mu_{t-1} + \varepsilon_{\mu t}$, with $|\rho_\mu| < 1$, and $\varepsilon_{\mu t}\tilde{}\, i.i.d.(0, \sigma_\mu)$. Shocks to the growth rate of the nominal money supply introduce a third source of uncertainty in the model.

The government finances its expenditures and lump sum transfers to the representative household through seignorage. It must satisfy its budget constraint, which is given by:

$$P_{t+j}G_{t+j} + \Gamma_{t+j} = M_{t+j}^s - M_{t-1+j}^s \tag{17}$$

for all $j = 0, 1, ..\infty$.

2.4 Market clearing

Finally, the economy is subject to the following resource constraint:

$$Y_{t+j} = C_{t+j} + I_{t+j} + G_{t+j} \tag{18}$$

[28]The process assumed simplifies the analysis of the variables adjusted for growth.

[29]McGrattan (1999) shows that the responses of real variables with respect to productivity and government spending shocks is affected by specific monetary policy rules. In the last section we check for the validity of our results in equilibria where policy is described by a Taylor rule.

In the money market, the equilibrium quantity of nominal money demanded must equal supply:

$$M^d_{t+j} = M^s_{t+j} \tag{19}$$

2.5 Equilibrium

An equilibrium for this economy is a collection of allocations for: consumers $\{C_t, e_t, N_{t+1}, U_t, K_{t+1}, M_t, B_{t+1}\}$; and producers $\{Y_t, K_t, N_t, U_t, e_t\}$; together with prices $\{w_t, r_t, R_{t-1}$ and P^i_t for $i \in [0,1]\}$ that satisfy the following conditions: (a) taking prices as given, consumer allocations solve the consumer's problem, (b) taking all prices but their own as given, producer allocations satisfy the producer's problem, (c) factor markets clear, and (d) the resource constraint holds.

In order to investigate the dynamics of the model, we log-linearise the equilibrium conditions around the steady state. The system of log-linear equations is presented in Appendix 2.

2.6 Shocks

There are three types of shocks in this model: two real shocks (technology and government spending), and a nominal money supply shock. Each shock is assumed to follow an AR(1) process:

Productivity

$$\hat{v}_t = \rho_A \hat{v}_{t-1} + \varepsilon_{At} \tag{20}$$

Government Spending

$$\widehat{g}_t = \rho_g \widehat{g}_{t-1} + \varepsilon_{gt} \tag{21}$$

Money

$$\widehat{\mu}_t = \rho_\mu \widehat{\mu}_{t-1} + \varepsilon_{\mu t} \tag{22}$$

where ε_{At}, ε_{gt}, and $\varepsilon_{\mu t}$ are mutually independent white noise, normally distributed processes.

3 Benchmark model

3.1 Calibration

In this section we describe the parameter values for our benchmark economy.

Table 1: Benchmark calibration

Parameter	Description	Value
β	Discount factor	$1.03^{-1/4}$
α	Elasticity of effective labour	0.674
θ	Preference parameter for leisure	3.89
h	Shift length, in hours	324.8
χ	Fixed cost of travel, in hours	60
τ	Total time endowment in hours	1369.2
γ_x	Gross trend growth rate of technology	1.0034
δ	Steady state rate of depreciation	0.0195
ϕ	Elasticity of depreciation with respect to utilisation	1.56
$\frac{\rho}{\rho-1}$	Gross steady state mark-up	1.145
$1/\varepsilon R^{SS}$	**Interest semielasticity** of money demand	**0.14**
η	Probability that a firm will be unable to change its price	0.25
e, U	Steady state level of effort and capital utilisation	1.0, 1.0
N	Steady state employment rate	0.8
ρ_A	AR(1) parameter on productivity shock	0.0
ρ_μ	AR(1) parameter on money shock	0.603
ρ_g	AR(1) parameter on government spending shock	0.956
σ_A	Standard deviation of technology innovations	0.0072
σ_μ	Standard deviation of money innovations	0.0082
σ_g	Standard deviations of government innovations	0.0146

Most of the parameter values in our benchmark model correspond to those in Burnside and Eichenbaum (1996). $\beta, h, \chi, \tau, \rho$ and ε are not estimated. The discount factor is calibrated such that the steady state annualised real interest rate is equal to 3%. The parameter h, the number of hours worked by an employed person, is calibrated so that the steady state level of effort equals one. The time spent commuting per quarter, χ, is set to 60[30]. This value falls in the middle of the range reported by Burnside and Eichenbaum (1996). The total quarterly time endowment τ is fixed at 1,369.2 hours. The calibrated value for the parameter ρ implies a steady-state markup of 14.5%. The interest semielasticity of money demand[31], $1/eR_{ss}$, is calibrated to take on a value of 0.14, this value is consistent with the values estimated in Stock and Watson (1993)[32].

The parameters $\alpha, \theta, \gamma_x, \delta, \phi, \rho_A, \rho_g, \sigma_A$ and σ_g take the estimated values given in Burnside-Eichenbaum (1996). The persistence ρ_μ and variance σ_μ of the monetary policy shock are taken from the estimation in Yun (1996).

The specification of the utility function in the benchmark model, although consistent with balanced growth (see Cooley and Prescott, 1995), has several unattractive features. The elasticity of labour supply is zero, whereas the elasticity of effective hours of work is related to the elasticity of effort supply which is given by $\frac{\tau-\chi-he_t}{he_t}$. In the steady state this value is equal to approximately 3. Blanchard and Fischer (1988) and Blanchard (1990) show that a very elastic labour supply can generate persistence. This is not the case in our utility specification, since what matters for aggregation is the average labour supply. Mulligan (1999) finds that the assumption of indivisibility of labour does not necessarily have implications for model dynamics. He demonstrates that the models of indivisible and divisible labour are equivalent from a macroeconomic perspective as long as labour supply is identified as average labour supply in the indivisible labour model. In our model, the average labour supply elasticity is equal to 0.33, which falls within the range of empirical estimates [0.1,1][33]. Nevertheless, in Section 6 we consider more general preferences that do not have the above mentioned features.

The key parameter values of the model are ϕ, the parameter governing the degree to which firms will choose to vary capital utilisation in response to shocks, and η the

[30]This amounts to commuting approximately 1 hour per day.

[31]The consumptrion elasticity of money demand equals s/e. In order to maintain comparability with Burnside and Eichenbaum, we set σ=1 in the benchmark specification. However, we check the validity of our results for more plausible values for the consumption elasticity of money demand. More specifically, when we set σ=5 (implying a consumption elasticity of 0.7) our results concerning the behaviour of the variables with respect to monetary disturbances do not change.

[32]See also Mankiw and Summers (1986), Lucas (1988).

[33]The effort elasticity is estmated using simulated data by regressing effort on real wages.

degree of nominal rigidity. As it is illustrated in Burnside and Eichenbaum (1996), the estimate of ϕ depends on the mean of the series for depreciation. The expected value of the rate of depreciation, δ_t, is equal to 0.0195, which implies ϕ=1.56 (The implied two-standard deviation band for quarterly depreciation is 0.0181, 0.0206). In standard sticky-price models, η often takes the value of around 0.75, indicating that firms change their price on average once a year. Estimates of this parameter vary from 0.75 in Gali and Gertler (1999)[34], to 0.5 in See Gali, Gertler, Lopez-Salido (2001)[35]. Our benchmark value of $\eta = 0.25$ assumes a lower degree of nominal rigidity. This is discussed further below.

3.2 Impulse responses

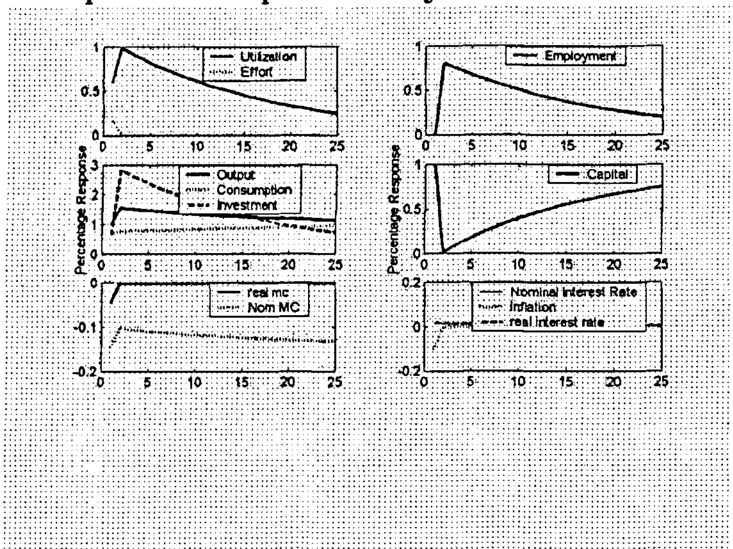
In this section we report the impulse response of the benchmark model to a productivity, government spending, and money shock.

Figure 1 plots the responses of the endogenous variables to a 1% white noise productivity growth shock. Because the assumed degree of nominal rigidity is very low in the benchmark model (the average frequency of price adjustment is a little over once per quarter), the impulse responses reported here essentially replicate those in Burnside and Eichenbaum (1996). The key finding of that paper is that variable factor utilisation magnifies the impact of a real shock. In addition, despite the fact that productivity growth shocks are assumed to be white noise, the real effect of the shock is highly persistent. This is because both labour and capital services can vary in response to shocks. Firms would like to increase their factor inputs to fully exploit the temporarily higher growth rate of productivity. In a standard two-factor input model with predetermined capital, the increased demand for labour services is dampened somewhat by the short-run rigidity of capital, which causes the marginal productivity of labour to decline quite sharply. With variable factor utilisation, firms can increase capital as well as labour services. In doing so, their action magnifies the impact effect of the productivity shock on output.

[34]See Blinder (1994), Sbordone (1998) and Taylor (1999) for empirical evidence of nominal price rigidities in the US.

[35]See also Gali, Gertler and Lopez-Salido (2001).

Figure 1 : Responses to a productivity shock



The effect of the shock is persistent because of the combined effect of utilisation on depreciation and hence the capital stock, and the assumption of labour hoarding. In periods following the shock, the physical capital stock is lower. Output must remain high to finance investment and build up the capital stock. Utilisation and employment remain above steady state to generate the higher output needed to bring the capital stock back to steady state. The transition path of the capital stock back to its steady state is relatively slow, since higher utilisation rates along this path dampen the rate at which capital accumulates. In addition to this mechanism, the assumption that it is costless to adjust hours worked in the period after the shock leads to an immediate response of effort in the impact period. In the second period hours respond by relatively more so that investment in the physical capital stock comes online with increases in employment. These combined effects lead to a hump-shaped response in output.

Figure 2 plots the model impulse responses to a temporary increase in the growth rate of government spending. The effect of a demand shock is also similar to that found in Burnside and Eichenbaum's original model. Utilisation and effort increase on impact to satisfy temporarily higher demand. Higher utilisation in turn temporarily reduces the capital stock. Output must remain higher in subsequent periods, reflected in utilisation and employment rates above steady state, to satisfy the temporarily higher demand and finance investment in order to restore the capital stock to it steady state level.

Figure 2: Responses to a government spending shock

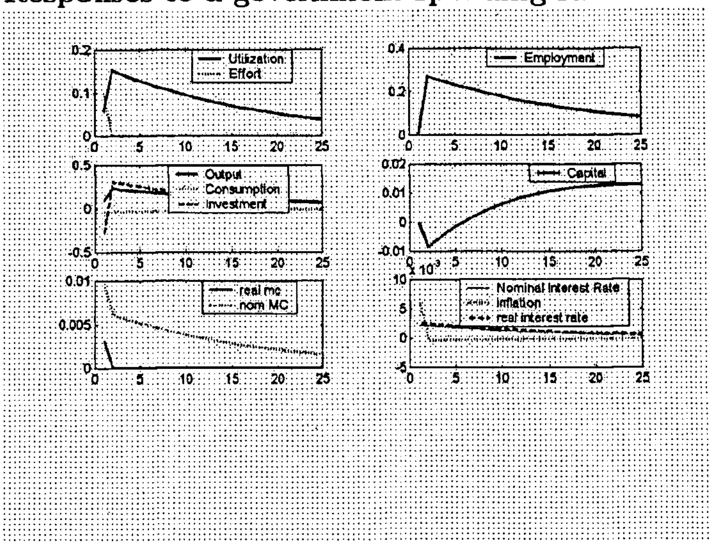


Figure 3: Responses to a money supply shock

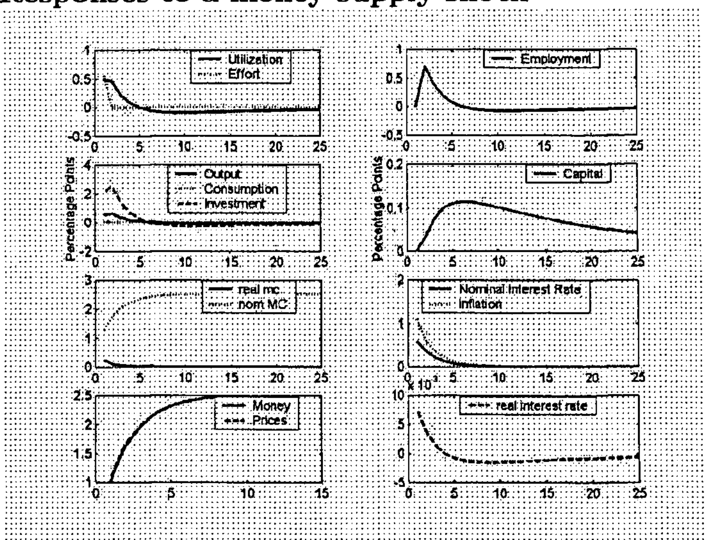


Figure 3 plots the impulse responses to a shock to the money supply rule. Despite the low degree of nominal rigidity assumed, the response of the real variables to the shock is on par with the impact of real shocks in the economy. The effect of the shock is quite persistent in subsequent periods, with variables returning to steady state in approximately five quarters-somewhat longer than the period over which prices are assumed to be fixed at just over one quarter.

Money has real effects because it also moves the aggregate supply curve. A shock in the growth rate of the money supply leads to a proportional increase in the price level, leaving real balances practically unchanged. However, expected inflation increases less than actual inflation, generating a wealth effect that increases the supply of effective labour. Given, the complementarity of effective labour and capital, utilization also increases. As a result, investment increases to replace depreciated capital. This produces persistence in output responses. The hump-shaped response in output arises for the same reason as in the case of a productivity shock: employment in the period after the shock rises by more than the contemporaneous response of effort.

4 Other sticky-price models

In this section we consider alternative sticky-price models, with the aim of comparing the model with time-varying factor utilisation to models without capital, and models with capital and capital adjustment costs.

4.1 Calibration

A model without capital has a similar structure to the one presented above with a fixed level of capital services, and where variations in effort are interpreted as variations in labour supply.

Standard two-factor input models with endogenous capital typically introduce capital adjustment costs in order to reduce the response of real variables to a money shock (see King and Watson, 1996, Chari, Kehoe and McGrattan, 2000, and Casares and McCallum 2000, Woodford 2000). In this case, the capital accumulation equation is assumed to take the form:

$$I_{t+j} = K_{t+1+j} - (1-\delta)K_{t+j} + \frac{b}{2}\left[\frac{K_{t+1+j} - (1-\delta)K_{t+j}}{K_{t+j}} - (\gamma_x - 1 + \delta)\right]^2 K_{t+j} \tag{23}$$

where the parameter b determines the size of the capital adjustment cost.

The calibrated parameter values for a standard one- and two-factor input model both with and without capital adjustment costs are reported in Table 2. Only parameter values that take on a different value from those reported in Table 1 are included below.

Table 2: Models without time-varying factor utilisation and effort		
Parameter	Value	
θ	2.1	
h	1	
χ	0	
τ	1	Steady state time endowment normalised to 1.
ϕ	10000	
e	0.33	Steady state level of employment.
U	1	
N	1	
b	=0 no capital adj costs =19.4 w/ capital adj costs	Capital adjustment cost parameter

In the standard one- or two-factor input model, effort is interpreted as labour supply[36]. Its steady state value is calibrated to . The adjustment cost parameter, b, takes on a value of 19.4, and governs the response of investment to changes in the real return of the asset[37]. When there are no capital adjustment costs ($b = 0$), then the response of investment is unconstrained, and in the case of money shocks leads to unrealistically large responses in investment and output[38]. For this reason, the parameter indexing the size of adjustment costs is a crucial parameter governing the response of sticky-price models with capital. Our benchmark value for b is calibrated in a similar way to that in Casares and McCallum (2000), namely that the semi-elasticity of investment with respect to the return of the real asset is around 3.25%. The resulting value for the adjustment cost parameter falls within the range explored

[36]In this case the elasticity of labour supply is given by $\frac{1-e_t}{e_t}$, which for our calibartion impies a steady state value of around 2, somewhat lower than the typical value assumed in the RBC literature.

[37]If the adjustment costs are described generically by the function $\phi\left(\frac{I_t}{K_t}\right)$, where $1/\phi'\left(\frac{I_t}{K_t}\right)$ is interpreted as Tobin's q, then adjustment costs affect the second derivative $\phi''\left(\frac{I_t}{K_t}\right)$. When adjustment costs are zero, the second derivative is zero. The higher the adjustment costs parameter b the larger the second derivative, and the smaller the response of the investment-to-capital ratio to variations in Tobin's q.

[38]See Casares and McCallum (2000) and Ellison and Scott (2001).

in the above references. These are given in Table 3[39].

Table 3: Capital Adjustment Cost Parameter, b		
Woodford, (2000)	3	Calibrated such that the degree of responsiveness of private expenditure is similar to that estimated in Rotemberg and Woodford (1997)
Casares and McCallum, (2000)	13.4-19.4	Calibrated such that the semi-elasticity of investment with respect to real asset returns is 2.25-3.25%
King and Watson, (1996)	40	Calibrated such that the elasticity of the investment-to-capital ratio with respect to Tobin's q is 1.
Chari, Kehoe, and McGrattan, (2000)	87, 88.5, 110	Calibrated such that $\sigma(I)/\sigma(Y) \cong 3.25$

4.2 Comparison of impulse responses

In this section we compare the impulse responses of our benchmark model of time-varying factor utilisation to those of other sticky-price models[40]. The degree of nominal rigidity assumed implies firms adjust their price on average just over once per quarter ($\eta = 0.25$).

Figure 4 illustrates the basic motivation behind Burnside and Eichenbaum's model, namely that standard models with constant utilisation rates cannot generate a persistent response to a white noise productivity growth shock. The impact effect of the shock, as well as the persistence, is much lower in models without time-varying factor utilisation. In order to generate persistence, a highly autocorrelated productivity shock in levels (eg $\rho_A = 0.95$) is usually assumed in these models.

[39]The estimated parameter assumes quadratic adjustment costs as described by Eq. 1.23.

[40]For brevity we only compare the model responses to productivity and money shocks.

Figure 4: Response to a productivity shock

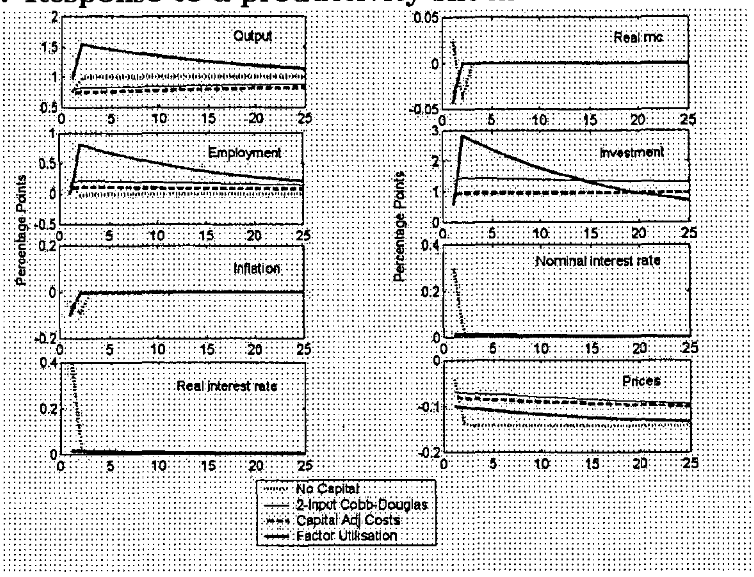
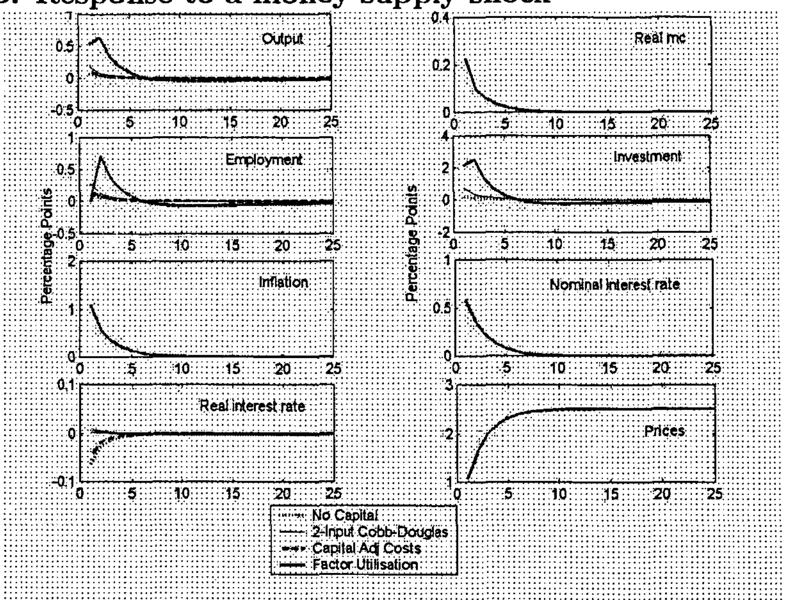


Figure 5: Response to a money supply shock



We now turn to the various model responses to a money shock. These are illustrated in Figure 5.

In models with constant factor utilisation, the low degree of nominal rigidity is reflected in a very small response of real variables. For reasons discussed above, however, our benchmark sticky-price model with factor utilisation does show a relatively large impact and persistent effect of a money shock.

An interesting feature of Figure 5 is the implied relationship between output and real marginal cost across the different models. The response of real marginal cost is similar across models, whereas the response of output is quite different. Why is this the case? Woodford (2000) finds similar cyclical variation in real marginal costs for standard models both with and without capital, which he attributes to relatively small cyclical variation in the capital stock. In models with time-varying factor utilisation, although it is true that for a given level of employment services an increase in capital services reduces marginal cost, the equilibrium response of firms will be to increase employment by more than would otherwise be the case given that they are now able to increase capital services as well. These two competing effects on marginal cost together leave its response unchanged in relation to other model specifications. For a model with time-varying factor utilisation, it seems, real marginal cost is less sensitive to changes in output. This is discussed in more detail in Section 5 below.

5 Model simulations

5.1 Model statistics

In this section we investigate simulated statistics across the different model variants. We allow for both productivity and policy shocks, the standard deviations of which are calibrated as reported in Table 1. In Table 4 we first compare model statistics across the sticky-price model variants assuming our benchmark low degree of nominal rigidity. The relatively low standard deviations for output in the no capital, capital, and capital with adjustment costs cases reflect the limited amount of response in those models to both white noise productivity growth shocks and money shocks in an environment of low nominal rigidity. The statistics are somewhat similar to those reported in King and Watson (1996) for RBC models. The statistics of the factor utilisation model match the data better. Despite the absence of capital adjustment costs, investment does not vary too much, but by more than in a standard sticky-price model with capital. The increased variability of investment is a consequence of the depreciation-through-use assumption and the desire by firms to invest such that the capital stock comes on line with employment.

When we consider conditional moments, we find that all shocks have a substantial role in explaining the variance of the real variables for the factor utilisation model.

In particular, 38% of the variability of output is due to productivity shocks, while 27% is due to monetary shocks. Real supply and demand shocks account for 59% of the variability of investment and the remaining 41% is due to monetary shocks. By comparison, in the no capital model, most of the variability of the real variables is due to real demand shocks and supply shocks have a very small overall effect. This is in contrast to models with capital where fluctuations in real variables are dominated by supply shocks when the assumed degree of nominal rigidity is fairly low. In particular, 56% of the fluctuations in output are due to productivity shocks, while money shocks account for only 17% of these fluctuations. In the model with capital adjustment costs, these values become 65% and 7%, respectively. Again, money has a relatively small role on explaining business cycle fluctuations because the degree of nominal rigidity assumed is very low.

Table 4: Model Statistics, $\eta = 0.25$

		Model Setting			
	Data*	No capital	Capital	Capital with adjustment costs	Time-varying factor utilisation
$\sigma(Y)$	1.72	0.73	0.78	0.71	1.54
$\sigma(C)$	0.86	0.88	0.49	0.58	0.67
$\sigma(I)$	8.24	-	1.50	0.88	4.32
$\sigma(eN)$	1.59	0.73	0.87	0.75	2.10
$\sigma(K)$	0.63**	-	0.92	0.92	0.89
$\sigma(KU)$	-	-	-	-	1.29
$\sigma(R)$	1.29	2.31	2.32	2.20	2.38

*Data for the US are taken from Cooley (1995), Ch. 7, Table 7.1.
**Bils and Cho (1994), HP filtered quarterly US data, 1955:3-1984:1
Model statistics are numerically calculated from model simulations.

We now look at the model statistics in the case where the degree of nominal rigidity is assumed to take on a more standard value of $\eta = 0.75$. These are reported in Table 5. The statistics indicate that the no capital model matches the data fairly well. The main problem is that consumption is too volatile. This could be reduced with the introduction of habit formation in household preferences (see Fuhrer, 2000)[41].

[41] See Christiano, Eichenbaum, and Evans (2001) for an example of habit persistence in a sticky-

The model with capital illustrates the excessively volatile response of investment and hence output when there are no capital adjustment costs. The model with capital and adjustment costs is considered as an intermediate case: the smaller the capital adjustment costs, the closer the model is to a standard two-factor input model; the larger the capital adjustment costs, the closer the model is to behaving as though capital were fixed. The simulations indicate that appropriate calibration of the capital adjustment cost parameters clearly has important implications for the behaviour of the model as noted by Woodford (2000).

Comparing the last two columns illustrates that introducing factor utilisation exacerbates the volatility of investment and hence output. This is due to the additional impact on the capital stock of varying utilisation on depreciation. The introduction of factor utilisation, therefore, seems to worsen the dynamic properties of the model by increasing the sensitivity of investment with respect to monetary shocks. But it is precisely the increased sensitivity of investment that allows for a reduction in the degree of nominal rigidity in a model with time-varying factor utilisation, without sacrificing its response to policy shocks.

This point is better illustrated by considering the conditional moments of the shocks. In the factor hoarding model with a high degree of nominal rigidity (and also in the model with capital and no adjustment costs), the magnitude of the response of real variables with respect to monetary shocks is huge. As a result, monetary shocks are the dominant source of fluctuations in real variables in this case. For example, monetary shocks account for 82% of the fluctuations in output, while productivity disturbances account for just 12% of these fluctuations. Nominal shocks are the main source of fluctuations in the no capital model as well, although the absence of investment dynamics moderates the magnitude of the responses of real variables with respect to monetary disturbances in this case.

---

price model with factor utilisation.

Table 5: Model Statistics, $\eta = 0.75$

		Model Setting			
	Data*	No Capital	Capital	Capital with adjustment costs	Time-varying factor utilisation
$\sigma(Y)$	1.72	1.26	4.03	1.63	16.65
$\sigma(C)$	0.86	1.82	0.51	1.37	1.07
$\sigma(I)$	8.24	-	16.09	3.19	66.63
$\sigma(eN)$	1.59	1.86	6.74	2.76	21.05
$\sigma(K)$	0.63**	-	1.02	0.92	2.94
$\sigma(KU)$	-	-	-	-	13.26
$\sigma(R)$	1.29	1.36	2.49	1.21	2.86

*Data for the US are taken from Cooley (1995), Ch. 7, Table 7.1.
**Bils and Cho (1994), HP filtered quarterly US data, 1955:3-1984:1
Model statistics are numerically calculated from model simulations.

Many authors have tried to evaluate empirically the importance of monetary versus real shocks in explaining business cycle fluctuations. Canova and De Nicolo (2000) find that the relative importance of the two disturbances varies across countries and over time, although their main finding is that demand shocks are the dominant source of fluctuations in real variables for six of the G-7 countries. Roberts (1993) and Faust (1998) also find that monetary shocks have an important role for output variability in the US. On the other hand, Blanchard and Quah (1989), Temin (1998), Stock and Watson (1989), and Astley and Yates (1999) provide empirical evidence that real disturbances are the main source of business cycle fluctuations[42].

Given the mixed evidence on the sources of business cycles fluctuations, it is not possible to evaluate whether the factor hoarding model or the standard sticky price model is more relevant on the basis of conditional moments.

In addition, the factor-hoarding model cannot account for the empirical regularities found in Gali (1999), namely that the estimated conditional correlations of hours and productivity are negative for technology shocks and positive for monetary

[42]Blanchard and Quah (1989) and Gali (1992), (1999) identify supply shocks that have permanent level effects, as assumed in our theoretical framework.

shocks. Although both the standard sticky price models analysed in this section and the benchmark model of factor hoarding can reconcile this feature at higher degrees of nominal rigidity, we have seen that the overall dynamics of the latter model are worse in this case.

5.2 Output and real marginal cost

To investigate further the question of persistence in response to a money shock, we estimate the elasticity of real marginal cost with respect to output. These are reported in Table 6, and reflect the findings of the impulse response functions, namely that although the behaviour of output differs across the model specifications in response to a money shock, the behaviour of real marginal cost is essentially identical. These results are perhaps not surprising, given that the relationship between marginal cost and output is affected by the behaviour of capital. The diverse response of capital across the various model specifications in turn accounts for the range of elasticity estimates.

Table 6: Elasticity of Real Marginal Cost with Respect to Output

	Model Setting			
Models	No capital	Capital	Capital with adjustment costs	Time-varying factor utilisation
$\frac{\partial mc}{\partial y}\frac{y}{mc}$	2.97	1.23	2.08	0.24

The relationship between marginal cost and output is crucial for understanding the model's response to monetary shocks. In principle, money has real effects only if firms react to unanticipated shifts in demand by increasing quantities rather than by increasing prices. Firms might behave so for two reasons: either because they are constrained from changing their price in the short-run (which in turn depends on), or because marginal cost is relatively flat in the short-run. In the latter case, firms have less of an incentive to raise their price in response to a demand shock since they are able to raise their output to satisfy demand without eroding their mark-up. An additional implication of the reduced sensitivity of marginal cost to output in a model of factor hoarding, therefore, is that it provides a potential mechanism by which nominal rigidities arise endogenously.

In a recent paper, Kiley (1998) shows that the assumption of partial price adjustment pricing, such as in the Calvo model we adopt here, imparts in general much more persistence than does Taylor type price setting. However, in the case where the elasticity of marginal cost with respect to output is small (eg less than one), the implications of the two types of price setting are equivalent. Our finding that marginal cost is relatively insensitive to changes in output suggests that the persistent output response to a monetary shock with relatively low price rigidity and time-varying factor utilisation holds more generally in staggered price setting models. This finding differs from that of Chari, Kehoe and McGrattan (2000), who conclude that output persistence cannot be rationalised within a standard business cycle model with staggered price setting without appealing to implausibly large nominal rigidities. In a model with factor utilisation, it seems, small degrees of nominal rigidity can lead to a persistent response to output by flattening the marginal cost curve.

5.3 Persistence

We calculate the conditional autocorrelation coefficient, ξ, of output for money shocks to measure persistence for each of the model variants, assuming their respective baseline degree of nominal rigidity. The values of ξ are reported in Table 7. The greater the greater the autocorrelation coefficient, the more persistent is the response of output[43].

Table 7 shows that the introduction of variable capital utilisation and effort in a sticky-price model increases the persistence of output relative to standard models despite the low degree of nominal rigidity assumed. This is because of the effect of utilisation on depreciation, and of labour hoarding. Labour hoarding prolongs the response of effective labour input, which in turn propagates the response of real variables. On the contrary, models with capital are unable to generate persistence for a low degree of price stickiness. This failure is a reflection of the lack of a strong propagation mechanism in one and two- factor input models. The results in Table 7 also verify the result of Chari, Kehoe and McGrattan (2000) that in standard sticky-price models the introduction of capital reduces persistence by providing agents with

[43] We also calculate a measure of persistence due to Cochrane (1988) for each of the model variants, assuming their respective baseline degree of nominal rigidity. This measure calculates how many periods it takes output to return to of the variance of its first differences. We find that the factor utilisation model is the most persistent based on this measure.

a mechanism for smoothing unanticipated shocks over time.

Table 7: Persistence

Model Setting	Autocorrelation coefficient ξ (η=0.25)	Autocorrelation coefficient ξ (η=0.75)
No capital	0.51 (0.061)	0.74 (0.049)
Capital	0.47 (0.062)	0.47 (0.063)
Capital with Adjustment Costs	0.50 (0.061)	0.72 (0.049)
Time Varying Factor Utilisation	0.76 (0.046)	0.75 (0.047)
Time Varying Factor Utilization+ – standard preferences	0.70 (0.051)	0.68 (0.052)
Variable Capital Utilisation	0.46 (0.063)	0.46 (0.063)
Labour Hoarding and Variable Effort	0.71 (0.05)	0.71 (0.05)

* Numbers in the parenthesis denote standard errors.

+ For $\sigma_n = 4, \sigma_e = 3$, the results are robust to the values of elasticities used as long as $\sigma_e << \sigma_n$.

6 Robustness

In this section we investigate how much each of the various features of the time-varying factor utilisation model contribute to its performance.

6.1 General Preferences

In the benchmark model, preferences are such that the effort supply elasticity is large and time-varying. One could argue that the persistent effects of monetary shocks are a consequence of the particular utility specification adopted. In this section we consider a more general specification of preferences with a smaller and constant elasticity of labour supply. The preferences we consider take the form:

$$u(c_t, N_t, e_t, M_t/P_t) = \log C_t - \lambda_n \frac{1}{1+\sigma_n} N_t^{1+\sigma_n} - \lambda_e \frac{1}{1+\sigma_e} e_t^{1+\sigma_e} + \frac{\gamma}{1-\varepsilon}\left(\frac{M_t}{P_t}\right)^{1-\varepsilon} \tag{24}$$

We find that under factor hoarding monetary shocks can still generate real and persistent effects on output even for this more general specification of preferences as long as the elasticity of the marginal disutility of effort is smaller relatively to that of employment ($\sigma_e << \sigma_n$ see also Gali, 1999)[44]. In Table 7 we report the value of χ for the model of factor utilisation with the more general preference specification. On the basis of this, we conclude that the persistence results in a model of factor hoarding are robust to alternative preference specifications, and are not due to the indivisible labour supply assumption.

The persistence results are mainly drawn from the assumption on labour hoarding. As we have seen in the benchmark case, the combined effect of predetermined employment and the zero elasticity of hours supply were increasing the persistence of the effective labour series and thus of capital and output. An unanticipated shock invokes changes in effort in the impact period, because of labour hoarding. In order to generate a persistent response of effective labour, employment should increase by more than the impact response of effort in the second period. In the specification in Eq (1.24) this happens when the effort supply elasticity is lower than the employment supply elasticity.

6.2 Variable Capital Utilisation

In this section we investigate whether our results on persistence are driven by the assumption of capital utilisation. Thus, we consider a two-factor input model with variable capital utilisation. Such a model has a similar structure to the two-factor input model presented in Section 4 with the additional feature that utilisation is allowed to move. The calibrated values for this model are therefore the same as those reported in Table 2, except that $\phi = 1.56$.

We find that variable capital utilisation alone cannot account for the persistent responses of output to monetary shocks (see Table 7). The introduction of variable capital utilisation increases the magnitude of the response of real variables to unanticipated shocks. However, it is still the case that we can generate realistic investment volatility without having to assume capital adjustment costs at low degrees of nominal rigidities. But this assumption alone cannot account for the increase in persistence of real variables with respect to nominal shocks. In order for this to happen it must be combined with real rigidities such as labour hoarding or real wage rigidities which increase persistence (see also Christiano, Eichenbaum, and Evans, 2001).

[44]We investigate the behaviour of the model over a range of parameter values, $1 < \sigma_e, \sigma_n < 10$.

6.3 Labour Hoarding and Variable Labour Effort

If we assume that there is no capital utilisation and effort is the only variable factor that can vary in response to unanticipated shocks in the impact period, then the impact effect of a monetary shock on real variables is persistent but small at low degrees of nominal rigidities. The model we consider is the same as the one in Section 2 with $\phi = 10000$. An increase in the degree of nominal rigidities raises the impact effect of a nominal shock on real variables, but at a cost of unrealistic investment volatility given that there are no constraints on adjusting capital.

The above analysis suggests that both capital and labour hoarding are important for generating large and persistent responses of real variables to nominal shocks at low degrees of nominal rigidities. For a high degree of nominal rigidity the introduction of capital worsens the dynamic properties of the model by increasing the sensitivity of investment to monetary policy shocks. McCallum and Casares (2000) suggest that introducing capital adjustment costs improve the dynamics of the real variables. Nevertheless, the introduction of capital adjustment costs in the factor hoarding model reduces persistence (the autocorrelation coefficient in output is reduced by 13%)[45].

6.4 Endogenous Monetary Policy

In the preceding discussion we have only analysed equilibria where monetary policy was exogenous. However, the empirical literature has demonstrated that policy movements are largerly due to reactions to the state of the economy. In this section, we analyse whether the persistent properties of our model change when the money supply process in equation (16) is replaced by an interest rate rule. We consider two types of endogenous policy.

A rule that has been frequently used in monetary models of the macroeconomy is the Taylor rule (Taylor (1993)). It has been shown that the Taylor rule can match well the behavior of interest rates in practise.[46] This rule takes the form:

$$R_t = b_\pi \pi_t + b_y y_t + \mu_t$$

[45]From 0.76 in the benchmark model reduces to 0.66 for η=0.75 with capital adjustment costs.

[46]See Calrida, Gali and Gertler (1998, 2000) for an empirical analysis of Taylor rules.

where $b_\pi = 1.5$ and $b_y = 0.5$ as in the original paper of Taylor (1993) and μ_t represents a policy shock.

Following Leeper, Sims and Zha (1996) we, also, analyze equilibria where policy is determined by:

$$R_t = b_m m_t + \mu_t$$

According to this rule the interest rate has feedback only from the money supply contemporaneously, since inflation and output may not be observable when decision about interest rate are taken.

Our findings can be summarized as follows: First, for low degrees of nominal rigidities, after a policy shock, the factor hoarding model generates more persistent responses of output relative to the other sticky price models.

McGrattan (1999) shows that the responses of real variables with respect to productivity and government spending shocks are affected by specific monetary policy rules. However, in our framework, for $\eta = 0.25$, this is not the case. When the monetary authorities follow an endogenous interest rate rule, the responses of the real variables after a productivity or a government spending shock hardly change relative to the case of an exogenous money supply. This is because for low degrees of nominal rigidities the economy behaves as if it had flexible prices. In this framework monetary policy has almost no effects on real variables. However, this is not true when $\eta = 0.75$. When the assumed degree of nominal rigidities is high monetary policy has real effects.

7 Conclusions

Recent debate has focussed on the failure of standard general equilibrium sticky-price models to generate business cycle fluctuations unless an extreme degree of price stickiness is assumed. This paper investigates the propagation mechanism of monetary shocks in an otherwise standard general equilibrium sticky-price model, modified to incorporate factor hoarding in the form of variable capital utilisation rates and labour effort. In contrast to previous studies, we find that real effects of monetary shocks can be generated at a relatively low degree of nominal rigidity.

In addition, we show that our model can generate realistic variances of capital and investment without having to assume capital adjustment costs. Contrary to standard sticky-price models with capital, the introduction of capital in our framework does not affect persistence negatively. Indeed, the increased sensitivity of investment allows for a reduction in the assumed degree of nominal rigidity without sacrificing the model's response to policy shocks.

Finally, we compare the predictions of our model with standard sticky-price models both with and without capital in order to gain insight in the relationship between nominal price rigidity and a firm's ability to adjust capital and labour services. The sensitivity of marginal cost to output is closely related to a firm's ability to adjust its inputs. A model of variable factor utilisation introduces an additional margin by which firms can respond to unanticipated shocks, reducing the effect of output on marginal cost. In other words, variable capital utilisation results in a flattening of the marginal cost schedule thereby introducing the possibility of endogenous price stickiness. On the other hand, standard sticky-price models both with and without capital are subject to increasing short-run marginal costs, which in turn dampens the response of output. The real effect of monetary shocks on output in such models is consequently weak, and can only be accomplished by assuming a relatively high degree of nominal rigidity.

APPENDIX 1

Representative household first order conditions

Allowing to denote the Lagrange multiplier on the household's budget constraint, the first order conditions for the representative agent are given by:

Consumption

$$\frac{1}{C_t} = P_t\lambda_t \tag{A1}$$

Nominal MoneyTAKE GAMA OUT OF THE EQUATION

$$\gamma\left(\frac{M_t^d}{P_t}\right)^{-\varepsilon} = P_t\lambda_t - E_t\beta P_{t+1}\lambda_{t+1} P_t / P_{t+1} \tag{A2}$$

Effort

$$\frac{\theta h N_t}{(\tau - \chi - he_t)} = P_t\lambda_t w_t N_t \tag{A3}$$

Employment

$$E_t\theta \ln\left(\frac{\tau - \chi - he_{t+1}}{\tau}\right) = -E_t P_{t+1}\lambda_{t+1} w_{t+1} e_{t+1} \tag{A4}$$

Utilisation

$$r_t = \delta\phi U_t^{\phi-1} \tag{A5}$$

Capital

$$0 = -P_t\lambda_t + E_t\beta P_{t+1}\lambda_{t+1}\left[1 + r_{t+1}U_{t+1} - \delta U_{t+1}^{\phi}\right] \tag{A6}$$

Bonds

$$0 = -P_t\lambda_t + E_t\beta P_{t+1}\lambda_{t+1}\left[1 + R_t\right] P_t / P_{t+1} \tag{A7}$$

APPENDIX 2

Loglinearized conditions

Note that output, capital, consumption, investment, real money balances, and the Lagrange multiplier are adjusted for growth. The system of equations that solve the model is given by:

Consumption

$$-\hat{c}_t - \hat{\psi}_t = 0 \tag{A8}$$

Effort

$$\hat{\psi}_t + (1-\alpha)\hat{k}_t + (1-\alpha)\hat{u}_t - (1-\alpha)\hat{n}_t - \left[1-\alpha+\frac{eh}{\tau-\chi-eh}\right]\hat{e}_t + \widehat{mc}_t - (1-\alpha)v_t = 0 \tag{A9}$$

Employment

$$0 = E_t\left[(1-\alpha)\hat{k}_{t+1} + (1-\alpha)\hat{u}_{t+1} - (1-\alpha)\hat{n}_{t+1} - (1-\alpha)\hat{e}_{t+1} + \hat{\psi}_{t+1} + \widehat{mc}_{t+1}\right] \tag{A10}$$

Utilisation

$$(1-\alpha-\phi)\hat{u}_t - \alpha\hat{k}_t + \alpha\hat{v}_t + \alpha\hat{n}_t + \alpha\hat{e}_t + \widehat{mc}_t = 0 \tag{A11}$$

Capital

$$\frac{1}{\beta}\hat{\psi}_t = \frac{1}{\beta}E_t\hat{\psi}_{t+1} + E_t\left[\left(\frac{(1-\alpha)y^{ss}}{k^{ss}}(1-\alpha) - \frac{\delta\phi}{\gamma_x}\right)\hat{u}_{t+1} - \frac{(1-\alpha)y^{ss}}{k^{ss}}\left(\alpha\hat{k}_{t+1} - \alpha\hat{n}_{t+1} - \alpha\hat{e}_{t+1} - \widehat{mc}_{t+1}\right)\right] \tag{A12}$$

Euler Equation

$$\hat{\psi}_t - z_t = E_t\hat{\psi}_{t+1} \tag{A13}$$

Fisher Equation

$$R_t - z_t = E_t\pi_{t+1} \tag{A14}$$

Money demand

$$\frac{1}{\varepsilon}\hat{c}_t - \hat{rm}_t - \frac{1}{\varepsilon}\frac{1}{R^{ss}}R_t = 0 \tag{A15}$$

Capital Law of Motion

$$(1-\delta)\hat{k}_t - \delta\phi\hat{u}_t + (\gamma_x - 1 + \delta)\hat{i}_t - (1-\delta)\hat{v}_t = \gamma_x\hat{k}_{t+1} \tag{A16}$$

Resource Constraint

$$\left[1-\alpha+\frac{k^{ss}}{y^{ss}}\frac{(1-\delta)}{\gamma_x}\right]\left(\hat{k}_t-\hat{v}_t\right)+\left[1-\alpha-\frac{k^{ss}}{y^{ss}}\frac{\delta\phi}{\gamma_x}\right]\hat{u}_t+\alpha\hat{n}_t+\alpha\hat{e}_t-\frac{c^{ss}}{y^{ss}}\hat{c}_t-\frac{g^{ss}}{y^{ss}}\hat{g}_t=\frac{k^{ss}}{y^{ss}}\hat{k}_{t+1} \tag{A17}$$

Production Function

$$\hat{y}_t - (1-\alpha)\hat{k}_t - (1-\alpha)\hat{u}_t + (1-\alpha)\hat{v}_t - \alpha\hat{n}_t - \alpha\hat{e}_t = 0 \tag{A18}$$

Calvo Pricing

$$\pi_t - \alpha_\varepsilon\hat{mc}_t = \beta E_t\pi_{t+1} \tag{A19}$$

Money supply process

$$\hat{rm}_{t-1} + \hat{\mu}_t - \frac{1}{\varepsilon}\hat{v}_t - \hat{rm}_t - \pi_t = 0 \tag{A20}$$

Hatted variables denote log-deviations from their steady state values. The variable denotes the nominal Lagrange multiplier , adjusted for growth. is the de-meaned quarterly net inflation rate, and and denote the de-meaned net nominal and real interest rates, respectively.

Chapter 4

Sustaining a Monetary Union[47]

1 Introduction

This paper investigates the optimal conduct of monetary policy in a currency area when the central bank conducts monetary policy taking into account the possibility of defection from the part of the region members. It is constructed around the, by now, standard stochastic general equilibrium two country model with imperfect competition and price rigidities which allows for the welfare evaluations of different policies[48]. Unconstrained optimal policy in a monetary union implies the maximization of the welfare of the consumers of each region subject to the aggregate constraints in the economy. In the constrained optimum we solve the problem of the central bank introducing an additional constraint: the present value of the utility of each region's consumers inside the union should be higher or equal than the present value of the utility of each region's consumers if it were to conduct monetary policy independently. In the discussion below we will aim to answer the following question: Should monetary policy stabilize an aggregate measure of inflation and output or should it take into account the dispersion of inflation and output across regions.

Benigno (1999) studying the unconstrained problem of a central bank in a currency area has given a first answer to this question. He concludes that if the degree of rigidity is the same and if the interest rate reacts to a weighted average of shocks and variables, asymmetric shocks do not create asymmetric responses of inflation, output and nominal interest rates. In the absence of participation constraints, optimal monetary policy enhances the targeting of a weighted average of the harmonized inflation

[47]I would like to thank Albert Marcet for valuable advice and Ramon Marimon for his classes on contract theory. I would also like to thank Richard Clarida and Mark Gertler for useful discussions.

[48]See Woodford(1999), Gali and Monacelli (2000), Devereux and Engel (2000), Benigno (1999), Pappa (2001), Benigno-Benigno (2001))

index of regional consumer prices where the weights are constant and coincide with each country's share of total consumption.

If the participation constraints were never binding the results of the unconstrained with the constrained problem would be identical. This is not the case, since belonging to a currency area implies costs for the region members. In Monacelli (2001) an economy relinquishing its monetary independence may face a potential trade-off between higher instability in real activity and lower instability in inflation. In Benigno (1999) a monetary union is associated with costly inertia in the terms of trade. In Devereux (1999) the dependence of the money shocks in each country lead to increased variability of domestic variables which would be avoided under a flexible exchange rate regime. Also Gali and Monacelli (2000) show that by relinquishing monetary policy a small open economy might increase the volatility of its output, consumption and inflation.

The introduction of participation constraints makes the corresponding maximization problem of the central bank non recursive. However, following Marcet and Marimon (1999), we can obtain a recursive formulation of our problem by expanding the set of state variables to include new variables that summarize the history of the shocks that the central bank should take into account when maximizing welfare with enforceability constraints. The fact that at the initial period our new state variables are well defined allows for a proper recursive formulation. Given the difficulties in constructing a numerical algorithm to compute the equilibria in the presence of participation constraints, we make some specific assumptions on the price setting and on the functional forms to allow for closed form solutions. Prices are fixed one period in advance and the assumed elasticity of substitution between home and foreign goods is unitary. These simplifications allow us to derive explicit expressions for the ratio of consumption between the two regions and the evolution of the nominal interest rate. More general forms can only be solved analytically.

Our results can be summarized as follows: First, we find a trade-off between efficiency and incentives which emerges in settings with lack of a strong enforcement technology. The way to deal with the enforcement problems is to manipulate the future consumption paths of the two countries and sacrifice some risk sharing in consumption. Second, we show that the evolution of the ratio of consumptions between the two countries depends on the frequency with which the participation constraints bind. The individual consumption paths in the presence of participation constraints depend on past temptations to defect. If enforcement constraints are binding for a country member, in the constrained equilibrium, optimal monetary policy implies a persistent increase in consumption for this country in order to prevent defection. An immediate implication of this result is that individual paths of consumption depend on individual histories and not on the aggregate consumption path. Thus, in the presence of binding participation constraints asymmetric shocks do create asymmetric responses of consumption, even if the two countries are identical in all the other respects. Moreover, the asymmetric movements in consumption affect the evolution of inflation and employment and increase the instability of both nominal and real activities.

With intertemporal incentives constraints, the recursive solutions correspond to a central bank's problem where regions' weights vary according to how enforceability constraints have been binding in the past. Optimal monetary policy in this framework, might imply different weights for two otherwise identical countries depending on the history of the shocks that each country has experienced. Consequently, in this case, optimal policy enhances an interest rate rule where the nominal interest rate does not have feedback only from regional inflations, but also from the individual histories of the two regions. When the participation constraints are binding, the central bank pursues a more expansionary policy with respect to productivity shocks than in the unconstrained case.

Finally, the more the participation constraints bind the further the economy moves from the unconstrained first best and the bigger the welfare losses for the members of the union. One conclusion that could be then reached is that unions with more idiosyncratic shocks across regions entail costs in terms of efficiency.

The rest of the paper is structured as follows. Section 2 presents the basic model and Section 3 the problem of the central banks when they conduct policy independently. The unconstrained maximization problem of the central bank in a currency area is presented in Section 4 while Section 5 presents the constrained problem and its equivalent recursive saddle point problem and compares the properties of the constrained and the unconstrained solutions. Some conclusions follow.

2 A Two-Country Model

The world is composed by two economies which feature identical preferences, technology, openness, and distribution of shocks. These two economies are the regions of a currency area. Each region is populated by infinitely lived agents which produce a single differentiated good and consume the goods produced in both economies. The continuum of agents are normalized on the interval [0,1], the population of region H belongs in the [0,n) interval while the population of region F in the [n,1]. Each agent derives utility by consuming an index of consumption goods and disutility from producing output. Households in both countries maximize the expected discounted value of their utility flow. The production units are imperfectly competitive and prices are fixed one period in advance. Producers face domestic and foreign demand for their product but do not engage in international price discrimination.

There are shocks to the production of the differentiated goods at home and abroad. These shocks are asymmetric in the sense that they are not perfectly correlated. At a country level the asset markets are complete while a bond denominated in the union's currency is traded internationally.

2.1 Consumers

There is no migration across countries. In each period t, the economy experiences one of finitely many events s_t. Let h^t denote the history of realized states from period zero until period t, i.e., $h^t = \{s_0, s_1, ..., s_t\}$. The probability, as of period zero, of any particular history h^t is $\pi(h^t)$. The initial realization s_0 is given. Assume that preferences are identical across regions, in the home country consumers have preferences given by:

$$EU = t = 0\sum^{\infty}\sum_{h^t}\beta^t\pi(h^t)\left[u(C^H(i,h^t)) - v(Y_H(i,h^t), z^H(h^t))\right] \tag{1}$$

where the index i denotes a variable that is specific to agent i, and β is a discount factor. u is an increasing concave function:

$$u(C) = \frac{C^{1-\sigma}}{1-\sigma}$$

and v an increasing convex function

$$v(Y,z) = z^{-1}\frac{Y^v}{v}, \qquad v \geq 1$$

z^H is a region-specific productivity shock and $C^H(i,h^t)$ is an index of consumption bundles defined as:

$$C^H(i,h^t) = [(1-\alpha)^{\frac{1}{\eta}} C_H(i,h^t)^{\frac{\eta-1}{\eta}} + \alpha^{\frac{1}{\eta}} C_F(i,h^t)^{\frac{\eta-1}{\eta}}]^{\frac{\eta}{\eta-1}} \tag{2}$$

where C_H and C_F are indexes of consumption across the continuum of differentiated goods produced respectively in region H and F:

$$C_H(h^t) = \left(\left(\frac{1}{n}\right)^{\frac{1}{\theta}} \int_0^n C_H(i,h^t)^{\frac{\theta-1}{\theta}} di\right)^{\frac{\theta}{\theta-1}}, \; C_F(h^t) = \left(\left(\frac{1}{1-n}\right)^{\frac{1}{\theta}} \int_n^1 C_F(j,h^t)^{\frac{\theta-1}{\theta}} dj\right)^{\frac{\theta}{\theta-1}}$$

Here, $\theta > 1$ is the constant elasticity of substitution across goods produced within the region, while the elasticity of substitution between the bundles C_H and C_F is η. Given the indexes of consumption price indexes are defined accordingly:

$$P^H(h^t) = [(1-\alpha)P_H(h^t)^{1-\eta} + \alpha(e(h^t)P_F(h^t))^{1-\eta}]^{\frac{1}{1-\eta}} \tag{3}$$

$$P_H(h^t) = \left(\left(\frac{1}{n}\right) \int_0^n P_H(i,h^t)^{1-\theta} di\right)^{\frac{1}{1-\theta}}, \quad P_F(h^t) = \left(\left(\frac{1}{1-n}\right) \int_n^1 P_F(j,h^t)^{1-\theta} dj\right)^{\frac{1}{1-\theta}} \tag{4}$$

where $e(h^t)$ is the nominal exchange rate, $P_H(i,h^t)$ is the price of goods produced in country H sold in the market of country H and $P_F(j,h^t)$ is the price of goods produced in country F sold in the market of country F. Finally, $P^H(h^t)$ denotes the aggregate price index.

We assume that there is no international price discrimination and the law of one price holds. Note that the foreign consumer price index will be analogously $P^F(h^t) = [(1-\alpha)P_F(h^t)^{1-\eta} + \alpha(P_H(h^t)/e(h^t))^{1-\eta}]^{\frac{1}{1-\eta}}$ and therefore purchasing power parity does not hold in this economy, unless there is no home bias in consumption. The terms of trade are defined as the ratio of the price of the bundle of goods produced in region F relative to the price of the bundle of goods produced in region H, $s(h^t) = P_F(h^t)/P_H(h^t)$. The real exchange rate is accordingly defined as: $q(h^t) = \frac{e(h^t)P^F(h^t)}{P^H(h^t)}$.

The representative consumer in the home country receives after tax income from selling her product, income from bond holdings and receives transfers, or pay taxes to the regional government. Households then consume, accumulate capital and purchase new assets. The budget constraint of the household i in region H is given by:

$$\begin{gathered} P^H(h^t)C^H(i,h^t) + \textstyle\sum_{h^t} Q(h^t,h^{t+1})b^H(i,h^t) + B(h^{t+1}) \leq \\ (1-\tau)P_H(i,h^t)Y_H(i,h^t) + b^H(i,h^t) + (1+R(h^t))B(h^t) + TR(i,h^t) \end{gathered} \tag{5}$$

The home consumer purchases the set of securities b at price $Q(h^t, h^{t+1})$, which is sufficient to span all the state of nature within region H. She also purchases the nominal non contingent bond denominated in the unions currency B. Note that money does not appear in either the utility function, or the budget constraint. This is because we do not need to introduce money explicitly in the model since monetary policy is specified in terms of an optimal rule.

The domestic consumers' optimal consumption may be described by the following condition (the FOC of the foreign consumers is analogous):

$$\beta\pi(h^{t+1})\left(\frac{u_c^H(h^{t+1})}{u_c^H(h^t)}\right)\frac{P^H(h^t)}{P^H(h^{t+1})}=\frac{1}{1+R(h^t)} \tag{6}$$

2.2 Producers

Agents are monopolists in selling their product and prices are sticky. All sellers fix their price one period in advance and choose the price, $P_H(i,h^t)$ for the produced differentiated good i so as to maximize expected utility resulting from sale revenues minus the disutility of output supply, taking as given P, P_H, and P_F and subject to a downward sloping demand function. Thus the home firm at $t-1$ chooses its price to maximize:

$$\sum_{h^t}\pi(h^t)Q(h^t,h^{t-1})\{(1-\tau)P_H(i,h^t)Y_H(i,h^t)-v(Y_H(i,h^t);z^H(h^t))\} \tag{7}$$

where $z^H(h^t)$ $(z^F(h^t))$is a domestic (foreign) productivity shock.

From the properties of the consumers' preferences it is easy to see that the demand for firm i is given by:

$$Y_H^d(i,h^t)=\left(\frac{P_H(i,h^t)}{P_H(h^t)}\right)^{-\theta}\left[C_H(h^t)+C_F^*(h^t)\right] \tag{8}$$

where $C_F^*(h^t)$ is the foreign demand for domestically produced goods.

The optimal pricing decisions for country H and F satisfy respectively:

$$E_{t-1}\{[U_c(C^H(h^t))\left[(1-\alpha)+\alpha s(h^t)^{1-\eta}\right]^{-\frac{1}{1-\eta}}-V_y(Y^H(h^t),z_t^H)]Y^H(h^t)\}=0 \tag{9}$$

$$E_{t-1}\{[U_c(C^F(h^t))\left[(1-\alpha)+\alpha s(h^t)^{\eta-1}\right]^{-\frac{1}{1-\eta}}-V_y(Y^F(h^t),z_t^F)]Y^F(h^t)\}=0 \tag{10}$$

2.3 Governments

In each country the fiscal authority levies taxes and gives transfers. We assume that governments do not issue bonds. Thus they must always balance their within period budget. For the home country it holds:

$$\tau \int_{i \in H} P_H(i, h^t) Y_H(i, h^t) di = \int_{i \in H} TR(i, h^t) di \tag{11}$$

A similar condition holds for the foreign country.

We also assume that $\tau = -(\theta - 1)^{-1}$, so that in equilibrium the market power distortion created by the monopolistic structure in the market for differentiated goods is neutralized.

2.4 Market Clearing

Total demand for domestic output comes form the demand for consumption of home and foreign consumers. Thus, domestic output is determined by:

$$Y^H(h^t) = \left[(1-\alpha) + \alpha s(h^t)^{1-\eta}\right]^{\frac{1}{1-\eta}} \left[(1-\alpha) C^H(h^t) + \alpha q(h^t)^{\eta} C^F(h^t)\right] \tag{12}$$

and for the foreign country:

$$Y^F(h^t) = \left[(1-\alpha) + \alpha s(h^t)^{\eta-1}\right]^{\frac{1}{1-\eta}} \left[(1-\alpha) C^F(h^t) + \alpha q(h^t)^{-\eta} C^H(h^t)\right] \tag{13}$$

where $q(h^t)$ is the real exchange rate.

2.5 Exogenous Processes

The exogenous stochastic processes for the productivity shocks are assumed to follow a Markov chain and their evolution is approximated by AR(1) processes:

$z_t^i = \exp(v_t^i)$, where $v_t^i = \rho^i v_{t-1}^i + \varepsilon_t^i, i = H, F$ and the cross correlation between the two shocks is ρ^{HF} and ρ^{FH}.

and ε_t is a (1×2) vector of mean zero i.i.d. shocks to technology with a diagonal variance matrix Σ. To keep the model as symmetric as possible we assume that the foreign technology shocks take on identical variances to the home country shocks.

The asymmetry between the two shocks is in the degree that they are correlated. We will allow for positive correlations smaller than one and for negative correlations between the two shocks.

2.6 Characterization of Equilibrium

We now turn to characterize the equilibrium policies and allocations. An equilibrium is a collection of allocations for home consumers $C^H(h^t), M^H(h^t), b(h^{t+1})$; allocations for foreign consumers $C^F(h^t), M^F(h^t), b^*(h^{t+1})$; allocations and prices for domestic goods $Y_H(i,h^t), P_H(i,h^t)$ for $i \in [0,n]$; allocations and prices for foreign goods $Y_F(i,h^t), P_F(i,h^t)$ for $i \in [n,1]$;, aggregate price levels $P^H(h^t), P^F(h^t)$, bond prices $Q^H(h^{t+1},h^t), Q^F(h^{t+1},h^t)$ terms of trade $s(h^t)$ and individual transfers and tax rates $TR(i,h^t), \tau, TR^*(i,h^t), \tau^*$ and policies $R(h^t)$ that satisfy the following conditions: (i) taking as given the prices, consumers allocations solve the consumers' problem, (ii) the price set by each differentiated good producer solves his problem (iii) transfers satisfy (10) (iv) the central bank solves its maximization problem and (v) markets clear. In order to complete the model we need to describe the problem of the central bank in each of the cases we will consider.

3 Optimal Independent Monetary Policies

In this section we analyze the interaction between the two regions when they conduct monetary policy independently. In this case the monetary authorities in each region maximize the expected utility of the domestic consumers subject to the domestic economy constraints and taking as given the policy of the foreign policymaker. The objective of the domestic central bank is defined at each date t as:

$$w^I(h^t) = [u(C^H(h^t) - v(Y^H(h^t), z_t^H)] \tag{14}$$

The welfare criterion is thus defined as:

$$W^I = E_{t-1}\left\{\sum_{h^t} \beta^t \pi(h^t) w^I(h^t)\right\} \tag{15}$$

The domestic policymaker maximizes the welfare of its region's consumers subject to (6) and (9). (The problem of the foreign policymaker is defined in a specular way).

Notice that under flexible prices the optimal pricing decisions for region H and F satisfy:

$$U_c(C^H(h^t))\left[(1-\alpha)+\alpha s(h^t)^{1-\eta}\right]^{-\frac{1}{1-\eta}} = V_y(Y^H(h^t), z_t^H) \tag{16}$$

$$U_c(C^F(h^t))\left[(1-\alpha)+\alpha s(h^t)^{\eta-1}\right]^{-\frac{1}{1-\eta}} = V_y(Y^F(h^t), z_t^F) \tag{17}$$

Under flexible prices a positive productivity shock decreases the marginal disutility of producing domestic output. In equilibrium the terms of trade will deteriorate in order for resources to be allocated optimally.

Now, when the policymakers conduct policy independently welfare might not be maximized due to the presence of beggar-thy-neighbor policies. Notice that from the first order conditions for the domestic firms it holds that: $E_{t-1}\{[U_c(C^H(h^t))$ $[(1-\alpha)+\alpha s(h^t)^{1-\eta}]^{-\frac{1}{1-\eta}} - V_y(Y^H(h^t), z_t^H)]Y^H(h^t)\} = 0$. In the foreign region it also holds that: $E_{t-1}\{[U_c(C^F(h^t))\ [(1-\alpha)+\alpha s(h^t)^{\eta-1}]^{-\frac{1}{1-\eta}}$ $-V_y(Y^F(h^t), z_t^F)]Y^F(h^t)\} = 0$. In a discretionary equilibrium the above conditions are taken as given by the policymaker but need to be satisfied in equilibrium. An unexpected real depreciation at home ($\uparrow s$) causes the foreign marginal disutility of producing to increase, while a real appreciation ($\downarrow s$) causes the foreign marginal disutility of producing to decrease. This is the heart of the inefficiency of the equilibrium when the two regions conduct policy independently: the policymaker in each region may have an incentive to surprise the foreign region by contracting the economy and appreciating the terms of trade. The strength of this incentive depends on the elasticity of substitution between home and foreign goods and the degree of openness of the economies. High substitutability between home and foreign goods and strong trading links increase the incentives for competition between home and foreign policymakers.

The incentives to strategically use the terms of trade for deteriorating the foreign region's conditions could be eliminated if the two policymakers could coordinate in designing policy. One way of coordination is the establishment of a currency area between the two regions. A currency area has the potential of eliminating the policy competition distortion since in a monetary union the central bank internalizes the strategic role of the terms of trade. However, a currency area can partly solve the problem since the fixity of the nominal exchange rate coupled with the fixity of prices leads to the inertia of relative prices in the short-run. As a result, the optimal reallocation of resources cannot be achieved in the impact period of the shock.

Pappa (2001) shows that the regime that eliminates the beggar-thy-neighbor policies, without creating any additional distortions is policy cooperation. Under policy cooperation the nominal exchange rate is free to move and the first best can be achieved. Finally, the "contractionary bias" of the equilibrium when regions conduct policy independently could be eliminated for some positive degree of monopolistic distortions. In a recent paper, Benigno-Benigno (2001) show that by appropriately choosing the overall degrees of monopolistic competition, the flexible price allocation can be implemented as a Nash equilibrium.

4 Optimal Monetary Policy in a Monetary Union: The case of no participation constraints

In general, a currency area might partly solve the problem of the presence of beggar-thy-neighbor policies when the policymakers conduct policy independently. However, when prices are pre-set, the role of the exchange rate of adjusting the economies after a region specific productivity shock is eliminated[49]. This is because when prices are fixed one period in advance productivity shocks have no effect on output, since output is demand determined. The exchange rate would not respond to a region-specific productivity shock even if exchange rates were flexible. Thus in this framework the exchange rate plays no role in the response of the economy to productivity shocks. Nevertheless, the nominal exchange rate can be used as an instrument for implementing the flexible price allocation. The loss of this instrument defines the trade off between entering a monetary union and conducting policy independently for a given region.

Following Benigno (1999) the problem of a central bank in a monetary union is to maximize the weighted average of the utilities of the two regions' consumers subject to the aggregate constraints in the economy and under the assumption that

[49] See Devereux (1999).

the exchange rate is fixed[50]. The average utility flows are defined at each date t as:

$$w^{MU}(h^t) = (1-n)[u(C^H(h^t) - v(Y^H(h^t), z_t^H)] + n[u(C^F(h^t)) - v(Y^F(h^t), z_t^F)] \tag{18}$$

The welfare criterion is thus defined as:

$$W^{MU} = E_0 \left\{ \sum_{t,h^t} \beta^t \pi(h^t) w^{MU}(h^t) \right\} \tag{19}$$

A Ramsey equilibrium is a policy, an allocation for consumption, output and the terms of trade, and price rules, $P^H(.)$ and $P^F(.)$ such that the policy maximizes (19) subject to the constraint of fixed exchange rates, with allocations and prices solving the problem of the consumers and the firms. Any alternative policy should solve the consumer's and the producer's problem at home and abroad[51].

Throughout the paper we suppose there is an institution or commitment technology, through which the central bank can bind itself to a particular set of policies once and for all at date zero. We model that by allowing the central bank choose a policy, at the beginning of time and then having consumers choose their allocations. The Ramsey allocation problem, setting the initial nominal assets of consumers to zero, is

$$\max_{c^H, c^F, s} (1-n) \sum_t \sum_{h^t} \beta^t \pi(h^t) [u(C^H(h^t) - v(Y^H(h^t), z_t^H)] + \tag{A4.10}$$
$$n \sum_t \sum_{h^t} \beta^t \pi(h^t) [u(C^F(h^t)) - v(Y^F(h^t), z_t^F)]\}$$

subject to the choice of intertemporal consumption smoothing at home and abroad (6) and the optimal pricing decisions (9)[52].

---

[50]The only difference with Benigno's framework is that we assume home bias in consumption, prices fixed one period in advance instead of Calvo price setting and we do not use a second order Taylor approximation of the utility of the consumers to derive the social objective.

[51]Notice that we require optimality by consumers and producers for all policies of the central bank. Chari, Cristiano and Kehoe (1996) show that this requirement is a sensible way to solve the central bank's forecasting problem.

[52]Condition (9) is an incentive compatibility contsraint that the central bank has to take into account when designing policy. The inclusion of the expecation operator in the $t-1$ period in (9) implies that the policymaker can gain by surprising price setters. Under commitment the multipliers of equation (9) in the initial period should be set equal to zero.

The first order conditions of the central bank problem in the unconstrained maximum give[53]:

$$\left(\frac{C^H(h^t)}{C^F(h^t)}\right)^{-(1+\sigma)} = \frac{n}{1-n}s(h^t)^{-2} \tag{20}$$

Moreover, in the impact period of the shock $\Delta s(h^t) = 0$,and also for all periods:

$$Y^i(h^t) = C^i(h^t)[(1-\alpha) + \alpha\left(\frac{n}{1-n}\right)^{1/\sigma} s(h^t)^{-\frac{2}{1+\sigma}}], \quad i = H, F \tag{21}$$

The above conditions show that asymmetric shocks do not create asymmetric responses of inflation, consumption, and output. Equation (21) states that in the unconstrained optimum the central bank will set the ratio of consumptions between the two economies equal to the ratio of their weights in the social objective, which coincides with their size. Although international financial markets are incomplete, the central bank's policy guarantees consumption risk sharing between the two regions. In other words, efficiency is assured even without complete financial markets (see also Cole and Obstfeld (1991)). The constancy of the nominal exchange rate combined with the inertial behavior of prices deter the terms of trade from moving in the impact period of the shock. Inflation rates are equalized between the two regions. Equation (22) is derived plugging (21) into the market clearing condition for output (12). It states that output is proportional to consumption and changes in the terms of trade, since the relative prices are inert in the impact period and consumption changes are proportional, real output between the two regions should be proportional as well.

In the impact period of the shock the terms of trade cannot change. However, real output should be proportional in the two regions. This can be achieved by movements in employment. Under flexible prices or exchange rates, the increase in the domestic real GDP is achieved by a rise in production in proportion to the technology shock and a deterioration of the terms of trade, while in the foreign country it is achieved by no change in foreign production and an improvement in the foreign terms of trade. With fixed prices and exchange rates the proportionality of real GDPs in the two regions means that production rises by too little in the domestic region and too much in the foreign region. This is achieved if domestic employment falls and foreign employment increases in response to a domestic technology shock, in place of

[53]The Appendix includes the details of the derivations.

the terms of trade deterioration that would take place under flexible relative prices. In the following period, prices are perfectly flexible and the terms of trade move to restore equilibrium in the labor markets. Thus, unless technology differentials are perfectly correlated, the adjustment to technology shocks under the optimal policy in a currency area requires costly adjustments in labor markets in the impact period of the shock. The more asymmetric are the underlying processes in the two regions, the bigger the inefficiencies in labor markets provoked by the inertia in the terms of trade.

The objective of a central bank in a currency area is to minimize the distortions associated with the fixity of prices (since the monopolistic competition distortions have been neutralized). In other words, the central bank should try to replicate the flexible price allocation. A positive productivity shock in country H should increase consumption in both countries. Since the terms of trade are inert, the change in foreign consumption should occur through monetary policy. The central bank should stimulate demand in both regions. The only instrument available to the policymaker is the nominal interest rate. The equilibrium process of the nominal interest rate (in log-linear form) associated with the optimal policy is given by[54]:

$$\widehat{R}_t = -\frac{\sigma}{(\sigma + v - 1)}(z_t^H + z_t^F) + \frac{1}{2}(\pi_{Ht} + \pi_{Ft}) \qquad (22)$$

Thus, after a regional productivity shock the world nominal interest rate decreases. The decrease in the nominal interest rate stimulates demand at home and abroad (this follows from the Euler equation in the foreign country). The decrease of the nominal interest rate in country H, reinforces the income effect in country H, agents increase consumption of both home and foreign goods and decrease labor. The increased demand for foreign goods is accommodated by an increase in foreign output, through increases in foreign labor supply. Besides the short run inertia of relative prices, the central bank guarantees consumption risk sharing through changes in nominal interest rates, sacrificing efficiency in labor markets. Inflation variability is low in equilibrium since the fall in the nominal interest rate deters inflation in both regions from falling, in H because of the productivity shock and in F because of the decrease in demand of F goods[55].

[54]Details of the derivations are included in the Appendix.

[55]If staggered prices were assumed instead the dynamics would be qualitatively similar, though we would not be able to obtain a complete analytical solution (see Pappa(2001)).

5 Optimal Monetary Policy in a Monetary Union: The Case of Participation Constraints

Although a currency area may have the potential of eliminating the beggar-thy-neighbor policies of the independent-policy equilibrium, it is associated with inefficiencies in the labor markets due to the loss of the nominal exchange rate as an instrument for implementing the flexible price allocation. The costs from the inefficient movements of employment in a currency area might be higher than the benefits from cooperation inside the union for a region member. Since when a region conducts policy independently can use both the nominal interest rate and the exchange rate as instruments for replicating the flexible price allocation. The movements of the nominal exchange rate might reduce the movements of the nominal interest rates and thus the variability of domestic variables when a region conducts policy independently. Of course, this depends on the underlying shocks in the two economies, on their degree of openness and on the substitutability between home and foreign goods. If the shocks in the two regions are very asymmetric the benefits from exchange rate flexibility might outweigh the costs of policy competition when the central bank conducts policy independently. Also, for low degrees of openness the role of the exchange rate as an expenditure switching factor decreases and the strength of the beggar-thy-neighbor policies associated with the non-cooperative equilibrium decreases as well.

For that reason, a central bank when designing policy, should take into account the fact that the country members might be better off outside than inside the union. This consideration from the part of the central bank is translated into the introduction of participation constraints in the maximization problem of the previous section. In other words, a central bank who wants to guarantee participation from the part of its region members should maximize unions utility subject to the additional constraint that the present value of the utility of each region inside the union should be higher or equal with the present value of the utility of each region if it were to conduct monetary policy independently.

The problem of the central bank in this case is given by[56]:

$$\max E_{t-1}\beta^t \left\{(1-n)\{[u(C_t^H) - v(Y_t^H, z_t^H)]\} + n\{[u(C_t^F) - v(Y_t^F, z_t^F)]\}\right\}$$

[56]Hereafter, we omit the dependence of variables from states of nature and we substitute sums over probabilities with the expectation operator for simplicity of notation.

subject to the constraints from the choices of the consumers and the producers and subject to the *intertemporal participation constraints* of the form:

$$\left|\sum_{k=0}^{\infty}\beta^k[u(C_{t+k}^H)-v(Y_{t+k}^H,z_{t+k}^H)]\right|_{MU}\geq\left|\sum_{k=0}^{\infty}\beta^k[u(C_{t+k}^H)-v(Y_{t+k}^H,z_{t+k}^H)]\right|_{independence} \quad (23)$$

$$\left|\sum_{k=0}^{\infty}\beta^k[u(C_{t+k}^H)-v(Y_{t+k}^H,z_{t+k}^H)]\right|_{MU}\geq\left|\sum_{k=0}^{\infty}\beta^k[u(C_{t+k}^H)-v(Y_{t+k}^H,z_{t+k}^H)]\right|_{independence} \quad (24)$$

In what follows we assume that the decision of a region member to leave the union if its participation constraint was violated is irreversible. For any period $t \geq 0$, the constraints restrict the utility of all members in a monetary union to be at least as large as their utility when the monetary authority in each country conducts policy independently. From now on, for ease of notation, we will call the right hand side of (24) and (25) $\phi_H(\omega_t^{IH})$ and $\phi_F(\omega_t^{IF})$ respectively.

The solution to this problem defines a *contract* that takes into account not only feasibility constraints but also incentive or 'existence' constraints. The constrained maximization problem is different from the problem solved in the previous section, since under the new constraints the problem is not recursive and the whole history of the shocks may matter for the optimal decision today (see Kydland and Prescott (1977)). Following Marcet-Marimon (1999) we cast the form of the above problem into an alternative recursive form. If we let $\beta^t\gamma_{it}, i = H, F$, be the multipliers in (24) and (25), the lagrange of the central bank will be given by:

$$\pounds = (1-n)\sum_{t=0}^{\infty}\beta^t\{[u(C_t^H)-v(Y_t^H,z_t^H)]\}+n\sum_{t=0}^{\infty}\beta^t\{[u(C_t^F)-v(Y_t^F,z_t^F)]\}+$$
$$\sum_{k=0}^{\infty}\beta^k\gamma_{Hk}\left[u(C_{t+k}^H)-v(Y_{t+k}^H,z_{t+k}^H)-\phi_H(\omega_t^{IH})\right]+\sum_{k=0}^{\infty}\beta^k\gamma_{Fk}\left[u(C_{t+k}^F)-v(Y_{t+k}^F,z_{t+k}^F)-\phi_F(\omega I_t^F\right.$$

subject to the constraints of the consumers and the producers decisions.

If we define

$$\mu_{im+1}=\sum_{n=0}^{m}\gamma_{in}, \qquad i=H,F \quad (26)$$

and collect all the terms that involve u and v, the Lagrangian becomes:

$$\pounds = \sum_{t=0}^{\infty} \beta^t \mu_{Ht+1} \{[u(C_t^H) - v(Y_t^H, z_t^H)]\} + (\mu_{Ht+1} - \mu_{Ht}) \left[u(C_{t+k}^H) - v(Y_{t+k}^H, z_{t+k}^H) - \phi_H(\omega_t^{IH})\right]$$
$$+ \sum_{t=0}^{\infty} \beta^t \mu_{Ft+1} \{[u(C_t^F) - v(Y_t^F, z_t^F)]\} + (\mu_{Ft+1} - \mu_{Ft}) \left[u(C_{t+k}^F) - v(Y_{t+k}^F, z_{t+k}^F) - \phi_F(\omega_t^{IF})\right] \tag{27}$$

with

$$\begin{aligned} \mu_{Ho} &= (1-n) \text{ and } \mu_{Fo} = n \\ \mu_{it+1} &= \mu_{it} + \gamma_{it} \text{ for } i = H, F \end{aligned} \tag{A4.10}$$

Here the vector $\mu_t = \{\mu_{Ht}, \mu_{Ft}\}$ act as a co-state vector. Notice that the problem defined in (28) does not have future variables in the constraints and all the functions in the constraints are known. The key property of problem (28) is that its solution is recursive, in the sense that only the values of the control variables and the costates μ_t are relevant from past history. In order the solutions in problems (24-25) and (28) to be equivalent in (29) the initial values of the multipliers should coincide with the weights of the central bank's problem, while in future periods $\mu_{it}, i = H, F$ is determined according to the functions related to the participation constraints. This is a special feature of the optimal plan that provides a clear interpretation of the time consistency problem the central bank faces. It is technologically feasible for the central bank to reset $\mu_t = \{1-n, n\}$ at any time t, and this is what it would do if it would ignore past commitments. But such a behavior from the central bank would lead to allocations in which a country might defect from the union. Full commitment from the part of the policymaker means that it commits to the evolution of μ determined by the participation constraints in all periods.

The First Order Conditions of the central bank that solves problem (28)-(29) give:

$$\left(\frac{C_t^H}{C_t^F}\right)^{-\sigma} = \frac{\mu_{Ft+1} + \mu_{Fo}}{\mu_{Ht+1} + \mu_{Ho}} s_t^{-2\eta} \tag{28}$$

Optimal monetary policy amounts to choosing efficiently the time profile of the time dependent weights $\mu_{it+1} + \mu_{io}$, $i = H, F$, in order for the participation constraints to be satisfied in all periods. We will consider two study cases: (i) the participation constraints are never binding, (ii) the participation constraint binds at some period t for one region[57].

If the participation constraints were never binding the results of the unconstrained with the constrained optimum would be equivalent. In (29) $\gamma_{it} = 0$, for $i = H, F$ and $\forall t$. Thus, $\mu_{it+1} = \mu_{it} = \mu_{i0}$ and (30) is identical with (21).

Now, suppose that the constraint is binding for a generic country j. That is suppose for some periods before t the constraint does not bind but at period t, $\gamma_{jt} > 0$, although for $k < t$, $\gamma_{jk} = 0$. Then for the agents of the country j it holds:

$$\mu_{jt+1} = \mu_{j0} + \gamma_{jt} > \mu_{j0} \tag{29}$$

Suppose, for example, that $j = H$, then the denominator in (30) increases. Since s_t is constant for one period, consumption in country H should increase. The monetary union is sustainable if the central bank can prevent its country members from defecting. The monetary authority can do so by increasing the consumption of the region which is likely to defect. The increase in consumption has to be persistent in order for the additional consumption that the specific region receives to be smoothed over time. In the constrained optimum individual paths of consumption depend on individual histories and not on the aggregate consumption path. Moreover, consumption risk sharing does not hold in all periods. The central bank has to sacrifice efficiency in order to guarantee commitment from the part of its region members and the equilibrium allocation is associated with losses.

If a central bank commits itself in the optimal conduct of policy, its objective in the presence of participation constraint is two-fold. The central bank will use the nominal interest rate to stabilize a weighted average of inflation in the two regions, but also to certify participation in the currency area of the two regions. For example, an increase in productivity in region H which creates incentives for region H to default, induces an increase in consumption in that region that lasts much longer after the impact period of the shock. This increase in consumption in region H is induced through a reduction in the nominal interest rate. The nominal interest rate in the constrained optimum is determined according to:

$$\widehat{R}_t = -\frac{\sigma}{(\sigma + v - 1)}(z_t^H + z_t^F) + \frac{1}{2}(\pi_{Ht} + \pi_{Ft}) - \frac{\sigma}{(\sigma + v - 1)}((1-n)\mu_{Ht+1} + n\mu_{Ft+1}) \tag{30}$$

[57] In case that the two regions hit their participation constraints silmuntaneously, the central bank is unable to avoid defection and the currency area disintegrates.

Moreover, the decrease of the nominal interest rate in the constrained optimum is bigger relative to the unconstrained optimum, since $\mu_{Ht+1} > \mu_{H0}$ and $\frac{\sigma}{(\sigma+v-1)} > 0$. The central bank decreases interest rates in order to increase consumption in country *H*. Given that region H is likely to defect the central bank will decrease more the nominal interest rate than in the unconstrained optimum in order to certify that region H will comply with the contract. In the second period prices and terms of trade move to restore equilibrium. The variability of inflation and of nominal exchange rates is increased, since the impact response of the interest rate in the equilibrium with participation constraints is higher. In the period after the shock prices and exchange rates have to move more than in the previous case to restore equilibrium.

If the exchange rate, or alternatively, prices were flexible. The introduction of the participation constraints would not affect at all the behavior of the aggregate variables in the two regions. When countries specialize in production and home and foreign goods are complements, an increase in domestic production is exactly offset by a term of trade deterioration, which favors the foreign country. In equilibrium the income effect dominates the substitution effect and consumption rises in both countries. Thus, although the multipliers would change value in (30) the central bank would be able to deter defection by simply manipulating the terms of trade. The presence of participation constraints in this case would only imply higher variability of the series of the terms of trade and no losses in the efficient reallocation of resources.

The above analysis suggests that when the participation constraints are binding the variability of the output and terms of trade gap and of inflation increases. As it was shown by Pappa (2001)[58] for an open economy the average utility of the representative consumer can be approximated by the objective:

$$\begin{gathered} W_t = E_0\left\{\Phi \sum_{t=0}^{\infty} \beta^t L_t\right\} \\ \text{with } L_t = \{\phi_c(c_t - c_t^n)^2 + \pi_{Ht}^2 + \phi_s(s_t - s_t^n)^2\} \end{gathered} \tag{31}$$

where $(c_t - c_t^n)$ is the gap between actual and flexible price consumption, and $(s_t - s_t^n)$ is the gap between actual and fully flexible terms of trade. It follows from the above discussion and the welfare criterion in (32) that the lack of a strong enforcement mechanism, or the introduction of countries for which the participation constraints

[58] Benigno (1999) introduces a similar criterion for the case of no home bias in consumption.

are often binding decreases the welfare of the consumers of all the region-members in a currency area.

6 Conclusions

We conclude with a brief overview of the paper's primary findings.

- Optimal monetary policy in a currency area should take into account the dispersion of output and inflation across regions when participation constraints define the set of feasible policies. In the constrained maximum, individual paths of consumption depend on individual histories and not on the aggregate consumption path.
- With recursive constraints, optimal monetary policy corresponds to a central bank's problem where regions' weights vary according to how these constraints have been binding in the past. Even if two regions are identical, when the enforceability constraints bind, the weight of the region for which these constraints have been binding more frequently in the past is higher than in the other region.
- In this framework, asymmetric shocks in a currency area do create asymmetric responses of regional variables.
- Participation constraints introduce a trade-off between efficiency and feasibility in settings with lack of commitment. In order to avoid defection from the part of the region-members the central bank has to sacrifice some consumption insurance.
- Frequent violations of the participation constraints in a currency area increases the variability of the macroeconomic variables.
- In the presence of enforceability constraints, the central bank should pursue a more expansionary policy with respect to positive productivity shocks than in their absence.
- The inclusion of countries that seem to do better outside a monetary union might increase instability inside the union.

In the current paper we have limited our analysis in particular utility functions and assumptions for the price setting which allowed us to solve the model analytically. We have only used implicit indicators to support the fact that participation constraints might be binding. In order to address, more specific policy issues such as under which conditions the participation constraints bind, how frequently they bind, or given the stochastic processes for the country members of the EMU when these constraints have been binding, or else when they are going to bind in the future, we have to enrich the dynamics of the model (assume staggered pricing) and solve the model numerically. The numerical solution of the model is out of the scope of this paper. Nevertheless we plan to proceed by solving numerically a more general specification of the presented model in order to be able to address all the above mentioned issues.

APPENDIX 1

The Unconstrained Maximum

In the unconstrained maximum the central bank maximizes the welfare objective subject to the first order conditions of the consumers' problem and taking into account the first order conditions of the firms. The problem of the central bank is summarized as[59]:

$$\max_{R,c^H,c^F,s} E_{t-1}(1-n)\sum_t \beta^t[\frac{C_t^{H\,1-\sigma}}{1-\sigma} - \frac{z_t^{H\,-1}}{v}[(1-\alpha)+\alpha s_t^{1-\eta}]^{\frac{v\eta}{1-\eta}}[(1-\alpha)C_t^H + \alpha q_t^{\eta} C_t^F]^v] +$$
$$n\sum_t \beta^t[\frac{C_t^{F\,1-\sigma}}{1-\sigma} - \frac{z_t^{F\,-1}}{v}[(1-\alpha)+\alpha s_t^{\eta-1}]^{\frac{v\eta}{1-\eta}}[(1-\alpha)C_t^F + \alpha q_t^{-\eta} C_t^H]^v] \quad \text{(A1)}$$

subject to:

$$C_t^{H-\sigma} - E_t\beta\{C_{t+1}^{H\;-\sigma}(1+R_t)\left[\frac{(1-\alpha)+\alpha s_{t+1}^{1-\eta}}{(1-\alpha)+\alpha s_t^{1-\eta}}\right]^{\frac{1}{1-\eta}}\} = 0 \quad \text{(A2)}$$

$$C_t^{F-\sigma} - E_t\beta\{C_{t+1}^{F\;-\sigma}(1+R_t)\left[\frac{(1-\alpha)+\alpha s_{t+1}^{\eta-1}}{(1-\alpha)+\alpha s_t^{\eta-1}}\right]^{\frac{1}{1-\eta}}\} = 0 \quad \text{(A3)}$$

If we set $\eta = 1$, (A1) is simplified:

$$\max_{R,c^H,c^F,s} E_{t-1}(1-n)\sum_t \beta^t\{\frac{C_t^{H\,1-\sigma}}{1-\sigma} - \frac{z_t^{H\,-1}}{v}[(1-\alpha)C_t^H + \alpha s_t C_t^F]^v\} + \quad \text{(A4)}$$
$$n\sum_t \beta^t[\frac{C_t^{F\,1-\sigma}}{1-\sigma} - \frac{z_t^{F\,-1}}{v}[(1-\alpha)C_t^F + \alpha s_t^{-1} C_t^H]^v]$$

subject to:

$$C_t^{H-\sigma} - E_t\beta\{C_{t+1}^{H\;-\sigma}(1+R_t)\} = 0 \quad \text{(A5)}$$

$$C_t^{F-\sigma} - E_t\beta\{C_{t+1}^{F\;-\sigma}(1+R_t)\} = 0 \quad \text{(A6)}$$

The FOC of the central banks problem are given by:

$$(1-n)\{C_t^{H-\sigma} - z_t^{H-1}[(1-\alpha)C_t^H + \alpha s_t C_t^F]^{v-1}(1-\alpha)\} - n z_t^{F-1}\alpha s_t^{-1}[(1-\alpha)C_t^F + \alpha s_t^{-1} C_t^H]^{v-1}$$
$$+ \lambda_t^H(-\sigma)C_t^{H-(\sigma+1)} - \lambda_{t-1}^H(-\sigma)C_t^{H-(\sigma+1)}(1+R_{t-1}) = 0 \quad \text{(A7)}$$

[59]For ease of notation in the appendix we abandon the dependence of the variables in the states of nature and we replace the sum over probabilities with the expectations operation.

$$n\{C_t^{F-\sigma} - z_t^{F-1}[(1-\alpha)C_t^F + \alpha s_t^{-1}C_t^H]^{v-1}(1-\alpha)\} - (1-n)z_t^{H-1}\alpha s_t[(1-\alpha)C_t^H + \alpha s_t C_t^F]^{v-1}$$
$$+ \lambda_t^F(-\sigma)C_t^{F-(\sigma+1)} - \lambda_{t-1}^F(-\sigma)C_t^{F-(\sigma+1)}(1+R_{t-1}) = 0 \quad \text{(A8)}$$

$$-\lambda_t^H \beta C_{t+1}^{H\ -\sigma} - \lambda^F \beta C_{t+1}^{F\ -\sigma} = 0 \quad \text{(A9)}$$

$$E_{t-1}\{(1-n)\frac{z_t^{H\ -1}}{v} v[(1-\alpha)C_t^H + \alpha s_t C_t^F]^{v-1}\alpha C_t^F + n\frac{z_t^{F\ -1}}{v} v[(1-\alpha)C_t^F + \alpha s_t^{-1}C_t^H]^{v-1}\alpha C_t^H s_t^{-2}\} = \quad \text{(A10}$$

Adding and substracting $C_t^{H-\sigma}C_t^F\alpha(1-n)$ and $C_t^{F-\sigma}C_t^H s_t^{-2}\alpha n$ and using the FOC of the firms problem in (A10) we get:

$$\left(\frac{C_t^H}{C_t^F}\right)^{-(\sigma+1)}) = \frac{n}{1-n}s_t^{-2} \quad \text{(A11)}$$

APPENDIX 2

The Constrained Maximum

Using the simplified welfare objective (A4) the Lagrangian for the constrained problem is given by:

$$\begin{aligned}
\pounds \ : \ & E_{t-1}\sum_t \beta^t \mu_{Ht+1}\{[\frac{C_t^{H\,1-\sigma}}{1-\sigma} - \frac{z_t^{H^{-1}}}{v}[(1-\alpha)C_t^H + \alpha s_t C_t^F]^v] + \\
& (\mu_{Ht+1} - \mu_{Ht})[\frac{C_t^{H1-\sigma}}{1-\sigma} - \frac{z_t^{H^{-1}}}{v}[(1-\alpha)C_t^H + \alpha s_t C_t^F]^v - \phi_H(\omega_t^{IH})]\} \\
& + \sum_t \beta^t \mu_{Ft+1}\{[\frac{C_t^{F1-\sigma}}{1-\sigma} - \frac{z_t^{F^{-1}}}{v}[(1-\alpha)C_t^F + \alpha s_t^{-1} C_t^H]^v] + \qquad (A12) \\
& (\mu_{Ft+1} - \mu_{Ft})[\frac{C_t^{F1-\sigma}}{1-\sigma} - \frac{z_t^{H^{-1}}}{v}[(1-\alpha)C_t^F + \alpha s_t^{-1} C_t^H]^v - \phi_H(\omega_t^{IF})]
\end{aligned}$$

subject to A5 and A6 and with

$$\begin{aligned}
\mu_{it+1} &= \mu_{it} + \gamma_{it} \qquad (A13) \\
\text{and } \mu_{im+1} &= \sum_{n=0}^{m} \gamma_{in} \text{ for } i = H, F
\end{aligned}$$

Substituting (A13) into (A12) and taking the derivative with respect to s_t following the same procedure as in the unconstrained maximum results in equation (30) in the text.

APPENDIX 3

Determination of the nominal interest rate in the unconstrained maximum

If we replace the FOC of the central bank with respect to the terms of trade to the FOC of the central bank with respect to domestic and foreign consumption Loglinearizing equation (21) in the text we get:

$$(1-n)\{1 - z_t^{H-1} C_t^{Hv+\sigma-1}[(1-\alpha) + \alpha s_t^{\frac{\sigma-1}{1+\sigma}} \left(\frac{n}{1-n}\right)^{\frac{1}{1+\sigma}}]^{v-1}(1-\alpha)\}$$
$$- n z_t^{F-1} \alpha s_t^{-1}[(1-\alpha) s_t^{-\frac{2}{1+\sigma}} \left(\frac{n}{1-n}\right)^{\frac{1}{1+\sigma}} + \alpha s_t^{-1}]^{v-1} = 0 \quad \text{(A14)}$$

$$n\{1 - z_t^{F-1} C_t^{Fv+\sigma-1}[(1-\alpha) + \alpha s_t^{\frac{1-\sigma}{1+\sigma}} \left(\frac{n}{1-n}\right)^{-\frac{1}{1+\sigma}}](1-\alpha)\}$$
$$- (1-n) z_t^{H-1} \alpha s_t [(1-\alpha) s_t^{\frac{2}{1+\sigma}} \left(\frac{n}{1-n}\right)^{-\frac{1}{1+\sigma}} + \alpha s_t]^{v-1} = 0 \quad \text{(A15)}$$

Loglinearizing the above conditions[60]:

$$z_t^H + z_t^F - (v+\sigma-1) c_t^H +$$
$$\left\{\frac{(v-1)\alpha(\sigma-1)}{1+\sigma}\left(\frac{n}{1-n}\right)^{\frac{1}{1+\sigma}} + 1 - (v-1)\alpha - (v-1)(1-\alpha)\frac{2}{1+\sigma}\left(\frac{n}{1-n}\right)^{\frac{1}{1+\sigma}}\right\} s_t = 0 \quad \text{(A16)}$$

$$z_t^H + z_t^F - (v+\sigma-1) c_t^F -$$
$$\left\{\frac{(v-1)\alpha(\sigma-1)}{1+\sigma}\left(\frac{n}{1-n}\right)^{\frac{1}{1+\sigma}} + 1 - (v-1)\alpha - (v-1)(1-\alpha)\frac{2}{1+\sigma}\left(\frac{n}{1-n}\right)^{\frac{1}{1+\sigma}}\right\} s_t = 0 \quad \text{(A17)}$$

The loglinearized version of the FOC of the consumers gives:

$$c_t^H = E_t c_{t+1}^H - \frac{1}{\sigma}(R_t - \pi_{Ht} - \alpha \Delta s_{t+1}) \quad \text{(A18)}$$

$$c_t^F = E_t c_{t+1}^F - \frac{1}{\sigma}(R_t - \pi_{Ft} + \alpha \Delta s_{t+1}) \quad \text{(A19)}$$

[60]Lower case variables denote the percentage deviations from respective steady state values.

Substitute (A16) to (A18) and (A17) to (A19) to get:

$$\widehat{R}_t = -\frac{\sigma}{\sigma + v - 1}(z_t^H + z_t^F) + \pi_{Ht} - \Omega E_t \Delta s_{t+1} \tag{A20}$$

Repeating the same for procedure for the foreign country:

$$\widehat{R}_t = -\frac{\sigma}{\sigma + v - 1}(z_t^H + z_t^F) + \pi_{Ft} + \Omega E_t \Delta s_{t+1} \tag{A21}$$

where $\Omega = \left\{ \frac{(v-1)\alpha(\sigma-1)}{1+\sigma} \left(\frac{n}{1-n}\right)^{\frac{1}{1+\sigma}} + 1 - (v-1)\alpha - (v-1)(1-\alpha)\frac{2}{1+\sigma}\left(\frac{n}{1-n}\right)^{\frac{1}{1+\sigma}} \right\} - \frac{\alpha}{\sigma}$

Adding (A20) and (A21) results in equation (23) in the text.

APPENDIX 4

A4. Determination of the nominal interest rate in the constrained maximum

The procedure followed for determining the optimal behavior of the nominal interest rate in the constrained maximum is exactly the same as for the unconstrained one. The only difference lies in the presence of the multipliers. Also, notice that we are loglinearizing after we have derived the FOC of the planners problem. Equations (A16) and (A17) in the presence of participation constraints become:

$$z_t^H + z_t^F - (v+\sigma-1)c_t^H + \frac{(v-1)n}{(1+\sigma)}\mu_{Ft} - \frac{(v-1)(1-n)}{(1+\sigma)}\mu_{Ht} - n\mu_{Ft} +$$
$$\left\{\frac{(v-1)\alpha(\sigma-1)}{1+\sigma}\left(\frac{n}{1-n}\right)^{\frac{1}{1+\sigma}} + 1 - (v-1)\alpha - (v-1)(1-\alpha)\frac{2}{1+\sigma}\left(\frac{n}{1-n}\right)^{\frac{1}{1+\sigma}}\right\} s_t = 0 \qquad \text{(A22)}$$

$$z_t^H + z_t^F - (v+\sigma-1)c_t^F - \frac{(v-1)n}{(1+\sigma)}\mu_{Ft} + \frac{(v-1)(1-n)}{(1+\sigma)}\mu_{Ht} - (1-n)\mu_{Ft} -$$
$$\left\{\frac{(v-1)\alpha(\sigma-1)}{1+\sigma}\left(\frac{n}{1-n}\right)^{\frac{1}{1+\sigma}} + 1 - (v-1)\alpha - (v-1)(1-\alpha)\frac{2}{1+\sigma}\left(\frac{n}{1-n}\right)^{\frac{1}{1+\sigma}}\right\} s_t = 0 \qquad \text{(A23)}$$

The rest of the procedure follows as in the case of no participation constraints.

Bibliography

Adao, B., Correia, I. and Teles, P., 2000, "Gaps and Triangles," mimeo UPF.

———, 2000, ''The Monetary Transmission Mechanism: Is it Relevant for Policy?'Unpublished Manuscript, Bank of Portugal.

Astley, M. and Yates, T. (1999), "Inflation and real disequilibria", Bank of England Working Paper Series, no. 103.

Backus, D., Kehoe, P. and Kydland, F., 1992, "International Business Cycles," Journal of Political Economy, 100(4), 745-775.

Ball, Larry. (1994), "Credible disinflation with staggered prices setting", American Economic Review, 84, pp.282-289.

Ball, Larry. (1998), "Another look at long run money demand", NBER Working Paper Series, no. 6597.

Ball, Lawrence and David Romer. (1990), "Real rigidities and the non-neutrality of money", Review of Economic Studies 57, pp. 183-203.

Baxter, M. and M. Crucini, 1994, "Business Cycles and the Asset structure of Foreign Trade," NBER Working Paper Series, wp 4975.

Benigno, Gianluca and Pierpaolo Benigno, 2000, "Monetary Policy Rules and the Exchange Rate," unpublished manuscript, University of California at Berkeley and Princeton University.

———, (2001), "Price Stability as a Nash Equilibrium in Monetary Open Economy Models," mimeo

Benigno, Pierpaolo., 1999a, "A Simple Approach to International Monetary Policy Coordination," International Economic Review, forthcoming.

———, 1999b, "Optimal Monetary Policy in a Currency Area," mimeo, Princeton University

Bernanke, B.,S. and Woodford, M., 1997 "Inflation Forecasts and Monetary Policy," NBER Working Paper Series, 6157, p.60.

Betts, C. and Devereux, M., 1996, "The exchange Rate in a Model of Pricing to Market," European Economic Review, 40, 1007-1021.

———, 1999, "The International Effects of Monetary Policy in a Two-Country Model," Discussion Paper, University British Columbia.

Bils, Mark and Jang-Ok Cho. (1994), "Cyclical factor utilisation", Journal of Monetary Economics, 33, pp. 319-354.

Blanchard, Olivier. (1990), "Why does money affect output? A survey", Handbook of Monetary Economics, vol. 2, Amsterdam: North Holland, pp. 779-835
Blanchard, Olivier and Stanley Fischer. (1988), Lectures in Macroeconomics, Cambridge: MIT Press.
Blanchard, Olivier and Danny Quah. (1989), "The dynamic effects of aggregate demand and supply disturbances", American Economic Review, 79, pp. 654-673.
Blinder, Alan. (1994), "On sticky prices: academic theories meet the real world" in Monetary Policy, Mankiw (ed), University of Chicago Press.
Burnside, Craig and Martin Eichenbaum. (1996), "Factor-hoarding and the propagation of shocks", American Economic Review, 86, no. 5, pp. 1154-74.
Calvo, Guilliermo A., 1983, "Staggered prices in a Utility Maximizing Framework," Journal of Monetary Economics 12, 383-398.
Carraro, C. and Giavazzi, F., 1991, "Can International Co-ordination Really Be Counterproductive?" on International Economic Policy Co-ordination, Blackwell press.
Casares, Miguel and Bennett McCallum. (2000), "An optimizing IS-LM framework with endogenous investment", NBER Working Paper Series, no. 7908.
Canova, Fabio and G. De Nicolo. (2000), "On the sources of business cycles in the G-7", Universitat Pompeu Fabra Working Paper Series.
Chari, V.V., Christiano, L.J., and Kehoe, P.J.1991, "Optimal Fiscal and Monetary Policy: Some Recent Results," Federal Reserve Bank of Minneapolis, Staff Report #147.
Chari, V.V., Kehoe, P., and McGrattan, E. 1998, "Monetary Shocks and Real Exchange Rates in Sticky Price Models of International Business cycles," Federal Reserve Bank of Minneapolis Research Department Staff Report #223.
Christiano, Laurence, Martin Eichenbaum and Charles Evans. (1997), "Sticky price and limited participation models: a comparison", European Economic Review, 41, no. 6, pp. 1173-1200.
Christiano, Laurence, Martin Eichenbaum and Charles Evans. (2001), "Nominal price rigidities and the dynamic effects of a shock to monetary policy", working paper, Northwestern University.

Clarida Richard, and Jordi Gali (1991), "Sources of Real Exchange Rate Movements: How Important are Nominal Shocks?" Carnegie Rochester Conference Series on Public Policy, 41

Clarida Richard, Jordi Gali, and Mark Gertler (1997), "Monetary Policy Rules and Macroeconomic Stability: Evidence and some Theory," Quarterly journal of Economics, forthcoming

________(1998), "Monetary Policy Rules in Practice: Some International Evidence," European Economic Review, 42, 1033-1067.

________(1999), "The Science of Monetary Policy: A New Keynesian Perspective," Journal of Economic Literature, forthcoming

Cochrane, John. (1988), "How big is the random walk in GNP?" Journal of Political Economy 96, pp. 893-920.

Cole, H. and Obstfeld, M., 1991, "Commodity trade and International Risk Sharing," Journal of Monetary Economics, 28, 3-24.

Collard, f. and Dellas, H. (2001) "Technology Shocks and Employment", mimeo.

Cook, David. (1999), "Real propagation of monetary shocks: dynamic complementarities and capital utilisation", Macroeconomic Dynamics, 3, pp. 368-383.

Cooley, Thomas and Gary Hansen. (1995), "Money and the business cycle", in Frontiers of Business Cycle Research, T. Cooley (ed.), Princeton University Press.

Cooley, Thomas and Ed Prescott. (1995), "Economic growth and business cycles", in Frontiers of Business Cycle Research, Princeton University Press, pp. 1-38.

Cooley, F.T. and V.Quadrini (2000), 'Common Currencies vs. Monetary Policy Independence, 'Unpublished Manuscript, New York University

Corsetti, Giancarlo and Paolo Pesenti (1997), 'Welfare and Macroeconomic Interdependence," NBER Working Paper, No. 6307.

Devereux, Michael B. (1999), "A Simple Dynamic General Equilibrium Analysis of the Trade Off between Fixed and Floating Exchange Rates," mimeo, UBC.

Devereux, Michael B. and Charles Engel (1998), "Fixed vs. Floating Exchange Rates: How Price Setting affects the Optimal Choice of Exchange Rate Regime," mimeo.

Devereux, Michael B. and Charles Engel (2000), "Monetary Policy in the Open Economy Revisited: Price Setting and Exchange Rate Flexibility," mimeo.

Duarte, Margarida (2000), "Why Don't Macroeconomic Quantities Respond to Exchange Rate Variability? Comparing Fixed and Flexible Exchange Rates," mimeo University of Rochester.

Ellison, Martin and Andrew Scott. (2001), "Sticky-prices and volatile output", Bank of England Working Paper Series, no. 127.

Fagnart, Jean-Louise, Omar Licandro, and Franck Portier. (1999), "Firm heterogeneity, capacity utilisation, and the business cycle", Review of Economic Dynamics vol. 2, pp. 433-455.

Faust, John. (1998), "On the robustness of the identified VAR conclusions about money", Carnegie-Rochester Conference Series on Public Policy, 49, pp. 207-244.

Friberg, Richard (1998), "In Which Currency Should Exporters set their Price?", Journal of International Economics, 45, 59-76.

Fuhrer, Jeffrey, (2000), "Habit formation in consumption and its implications for monetary policy models", American Economic Review 90, pp. 367-90.

Gali, Jordi. (1992), "Does the IS-LM model fit US postwar data?" Quarterly Journal of Economics, CVII, pp. 708-738.

Gali, Jordi. (1999), "Technology, employment and the business cycle: do technology shocks explain aggregate fluctuations?" American Economic Review, 89, pp. 249-271.

Gali, Jordi and Mark Gertler. (1999), "Inflation dynamics: a structural econometric analysis", Journal of Monetary Economics vol. 44, pp.195-222.

Gali, Jordi, Mark Gertler, and J. David Lopez-Salido. (2001), "European inflation dynamics", forthcoming European Economic Review.

Gali, Jordi and Tommaso Monacelli (1999), "Optimal Monetary Policy and Exchange Rate Volatility in a Small Open Economy," mimeo UPF

Gerlach Stefan and Gert Schnabel, "The Taylor Rule and Interest Rates in the EMU Area," CEPR Discussion Paper Series, No.2271

Greenwood, Jeremy, Zvi Hercowitz, and Gregory Huffman. (1988), "Investment, capacity utilization, and the real business cycle", American Economic Review 78, pp.402-17.

Ireland, Peter, "The Role of Countercyclical Monetary Policy," *Journal of Political Economy,* 104(4), 704-23.

Hamada Koichi (1976), "A Strategic Analysis of Monetary Independence," *Journal of Political Economy,* vol. 84, pp. 677-700.

Hansen, Gary. (1985), "Indivisible labour and the business cycle", Journal of Monetary Economics 16, no. 3, pp. 309-28.

Jeanne, Olivier (1997), "Generating Real Persistent Effects of Monetary Shocks: How much rigidity do we really need?" NBER Working Paper Series , wp. 6258

Kiley, Michael. (1998), "Partial adjustment and staggered price setting", mimeo, Federal Reserve Board.

King, Robert and Mark Watson. (1996), "Money, prices, and interest rates and the business cycle", Review of Economics and Statistics 78, pp. 35-53.

King, Robert and Alex Wolman. (1996), "Inflation targeting in a St. Louis model of the 21st century", Federal Reserve Bank of St. Louis Review.

King, Robert G. and Alexander L. Wolman (1998), "What should Monetary Authority do when Prices are Sticky?" in J.B. Taylor (ed.), Monetary Policy Rules, University of Chicago Press for NBER.

Kollman Robert (1996), "The Exchange Rate in a Dynamic-Optimizing Current Account Model with Nominal Rigidities: A Quantitative Investigation," mimeo.

Kydland, F.E. and Prescott, E.C., 1977, "Rules Rather than Discretion: The Inconsistency of Optimal Plans," *Journal of Political Economy,* 85, 473-491.

Leeper, E., Sims, C. and T. Zha (1996), "What Does Monetary Policy Do?" mimeo Princeton University

Lucas Robert (1988), "Money Demand in the US: A Quantitative Review," Carnegie Rochester Series on Public Policy, 29, 137-167.

Mankiw, Gregory and Lawrence Summers (1986), "Money Demand and the Effects of Fiscal Policy," Journal of Money, Credit and Banking, 18, 415-429.

Mankiw, Gregory, (2000), "The inexorable and mysterious trade-off between inflation and unemployment", NBER Working Paper Series, no. 7884.

Marcet, Albert and Marimon Ramon, (1999), 'Recursive Contracts,' mimeo UPF.

Marimon, R. and A. Scott (eds.) (1998) : *Computational Methods for the Study of Dynamic Economies.* Oxford, UK: Oxford University Press.

McCallum , Bennett. (1997), "Comment", in NBER Macroeconomics Annual, Bernanke and Gertler (eds). Cambridge, MA: MIT Press, pp. 355-359.

McCallum, Ben and Edward Nelson. (1999), "An optimising IS-LM specification for monetary policy and business cycle analysis", Journal of Money, Credit, and Banking, 31, pp. 296-316.

McGrattan, Ellen (1999), "Predicting the Effects of Federal Reserve Policy in a Sticky-Price Model: An Analytical Approach," Federal Reserve Bank of Minneapolis Working Paper Series, wp. 598

Miller, M., Salmon, M. and A. Sutherland, 1991, "Time Consistency, Discounting and the Returns to Co-operation," on International Economic Policy Co-ordination, Blackwell press.

Monacelli, Tommaso (1998), "Into the Mussa Puzzle: Monetary Policy Regimes and the Real Exchange Rate in a Small Open Economy," mimeo, New York University.

Mulligan, B. Casey. (1999), "Microfoundations and macro implications of indivisible labor", NBER Working Paper Series, no. 7116.

Mundell, R. (1961), "A Theory of Optimum Currency Areas," *American Economic Review,*51, 657-675.

Obstfeld, Maurice (1985), "Floating Exchange Rates: Experience and Prospectus," Brooking Papers on Economic Activity, 2, 369-450.

Obstfeld Maurice and Kenneth Rogoff (1995), "Exchange Rate Dynamics Redux," Journal of Political Economy, 103, 624-660.

________(1996), Foundations of International Macroeconomics, Cambridge, MA: MIT Press.

________(1998), "Risk and Exchange Rates," NBER Working Paper No. 6694.

Oudiz, Gilles and Jeffrey Sachs (1984), "International Policy Coordination in Dynamic Macroeconomic Models," NBER Working Paper Series, wp. 1417

Peersman, Gert and Frank Smets (1999), "Uncertainty and the Taylor Rule in a Simple Model of the Euro Area Economy," unpublished.

Roberts, John M. (1993), "The sources of business cycles: a monetarist interpretation", International Economic Review, 34, pp. 923-934.

Rogerson, Richard. (1988), "Indivisible labour, lotteries and equilibrium", Journal of Monetary Economics, vol.21, no. 1, pp. 71-89.

Rogoff Kenneth, 1985, "Can International Monetary Policy be Counterproductive?", *Journal of International Economics,* 18, 199-217.

Romer, David. (1996), Advanced Macroeconomics, New York: McGraw-Hill.

Rotemberg, Julio. (1987), "The New Keynesian microfoundations" in NBER Macroeconomics Annual, S. Fischer (ed.). Cambridge, MA: MIT Press, pp. 69-104.

Rotemberg, Julio and Michael Woodford (1998), "An Optimization Based Framework for the Evaluation of Monetary Policy," NBER Technical Working Paper Series, No.233.

________(1998), "Interest Rate Rules in an Estimated Sticky Price Model," in J.B. Taylor (ed.) Monetary Policy Rules, University of Chicago Press for NBER

Sargent, T.J. and L. Ljungqvist (1998): *Recursive Macroeconomic Theory.* Stanford University Press

Sbordone, Argia. (1998), "Price and unit labor cost: a new test of price rigidity", Institute for International Economic Studies, Stockholm University Seminar Paper, no. 653.

Smets, Frank (1995), "Central Bank Macroeconomic Models and the Monetary policy Transmission Mechanism," Financial structure and the Monetary Policy Transmission Mechanism, Bank of International Settlements, 225-266.

Stock, Jim and Mark Watson. (1989), "New index of coincident and leading economic indicators," NBER Macroeconomic Annual, 4, pp. 351-393.

Stock, John and Mark Watson (1993), " A Simple estimator of Cointegrating Vectors in Higher Order Integrated Systems," Econometrica, Vol.61, No.4, pp. 783-820.

Svensson, Lars, 1998, 'Open Economy Inflation Targeting,' NBER Working Paper Series, WP.6545

Taylor, John. (1979), "Staggered contracts in a macro model", American Economic Review 69, pp.108-13.

Taylor, John (1993), "Discretion versus Policy Rules in Practice," Carnegie - Rochester series on Public Policy, 39, 195-214.

Taylor John and Michael Woodford (1999), Handbook of Macroeconomics, North Holland, Elsevier

Woodford, Michael (1999), "Optimal Monetary Policy Inertia," NBER Working Paper, No. 7261.

Woodford, Michael. (2000), "A neo-Wicksellian framework for the analysis of monetary policy" Chapter 4 of Interest and Prices, manuscript, Princeton University.

Yun, Tack (1996), "Nominal Price Rigidity, Money Supply Endogeneity, and Business Cycles," Journal of Monetary Economics, 37, 345-370.