SUPERSYMMETRY AND ELECTROWEAK SYMMETRY BREAKING FROM EXTRA DIMENSIONS

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Chapter 1

Introduction

1.1 A Brief Historical Introduction

The birth of supersymmetry [1] is related with the birth of string theory in the late 1960's. It was known that the string (bosonic) action was anomaly free only in 26 spacetime dimensions. Ramond, Neveu and Schwarz proposed to add d fermionic field doublets to the d original bosonic fields of the string. The result was an action anomaly free in 10 dimensions and invariant under a symmetry relating fermionic and bosonic variables.

A few years later Wess and Zumino extended the idea of referring fermions and bosons to four dimensional quantum field theories constructing the first supersymmetric action consisting on a multiplet of one Majorana fermion, one complex scalar and an auxiliary (non-propagating) complex scalar. A bit after they found a supersymmetric action for a multiplet containing a gauge field.

A few years before, in 1967, Coleman and Mandula had proven, under resonably general assumptions, that the most general symmetry of the S-matrix of a relativistic quantum theory of fields has a Lie algebra consisting on the Poincaré generators[1]: P_μ and $J_{\mu\nu}$ and a finite number of operators commuting with all the Poincaré generators and furnishing a Lie algebra (or subalgebra) of a compact Lie group. This seemed to discard the Wess-Zumino model as a good quantum theory of fields since according to the Coleman-Mandula theorem any S-matrix symmetry mixes fermions

[1] In theories with only massless particles, in addition to the Poincaré algebra, the theorem allows the conformal algebra.

with fermions and bosons with bosons, but not fermions with bosons.
Wess and Zumino solved the apparent contradiction pointing out that the Coleman-Mandula theorem works only with commutation relations between the generators while the supersymmetry generators satisfy anticommutation relations. In fact, Gol'fand and Likhtman were those who, in 1971 and independently of Wess and Zumino, extended the Poincaré algebra to a graded Lie algebra with commutation and anticommutation relations and demanding invariance under this symmetry they found a supersymmetric action in four dimensions.

1.2 How many dimensions do we live in?

Along the different attemps to unify fundamental forces (occured from the late of 19th century) the concept of extra dimensions [2] has played an important role:
Once the relativistic invariance of the Maxwell's theory of electrodynamics was recognized it became clear (due to Minkowski's work) that the unification of electricity and magnetism required space and time unification into a four dimensional continuum "spacetime".
Gunnar Nordström was the one who proposed, before the appearance of Einstein's theory of gravity, a five dimensional Maxwell-like theory together with a 5-conserved current. He noticed that in the cylindrical case (none of the fields depends on the fifth dimension) one could identify the fifth component of Maxwell's vector with a scalar gravitational potential, and the fifth component of the conserved current with the trace of the energy-momentum tensor. The rest of the fields and current components were indentified with the usual Maxwell vector and electromagnetic current, respectively. Thus, Nordström found a unification between electromagnetism and scalar (newtonian) gravity. When relativistic theory of gravity appeared, its unification with Maxwell's theory was a question that quickly arose. In 1919, in the wake of Einstein's theory of gravity, the mathematician Theodor Kaluza proposed an Einstein-like theory in five dimensions, more precisely, in $\mathcal{M}^4 \times S^1$, namely, with the fifth dimension (y) compactified on a circle. In order to avoid bivaluations of the fields, those must admit a Fourier expansion with respect to the fifth coordinate. Integrating the action with respect to y Kaluza found an effective 4-d action made

up of an infinite tower of modes. The zero one consisted on one symmetric 4-tensor (graviton), one 4-vector (Maxwell vector) and a non propagating sacalar (the so called: radion), verifying the well-known 4-d equations of Einstein-Maxwell.[2]
Furthermore, there is no consistent way for building a quantum theory of gravity in 4 dimensions. The candidate to such a theory, the string theory, is consistent in 10 dimensions (with the help of supersymmetry).

1.3 Why supersymmetry

The supersymmetry has a list of nice properties that make it a powerful candidate to describe some of the physics beyond the Standard Model (SM): supersymmetric lagrangians are not quadratically corrected (the so called non-renormalization theorem) only logarithmically[3], which means that the masses do not receive any correction [4] (nor infinite neither finite) only wave functions are corrected.
The running of the gauge couplings of the SM do not match exactly at any scale, nevertheless, taking into account the supersymmetric degrees of freedom all of them coincide at a scale $\backsim 10^{16}\ GeV$.
Furthermore, local supersymmetry invariance implies local Poincaré invariance, namely, a theory of gravity, in fact a theory of *supergravity* [4] though it is not a renormalizable theory and it is seen as an effective theory of the string theory.
Obviously, the supersymmetry can not be an exact symmetry since none of the supersymmetric partners of the SM particles predicted by the theory have been found yet. The nature of supersymmetry breaking is still an open question although a commonly accepted and used mechanism is the so called Scherk-Schwarz mechanism to be described later on.

[2]The Kaluza action gives undesirable constraints on the fields if one takes into account its variation with respect to the radion, for this reason Kaluza set it directly to 1. Afterward, Jordan realized that the action presents, in addition to the general change of coordinate invariance, an invariance under global scale transformations. On the other hand, a possible solution to the action is the flat Minkowski spacetime. This solution serves as a natural vacuum which is not invariant under the global scale transformation and thus the scalar serves as a Goldstone boson acquiring a vacuum expectation value.

[3]A supersymmetric theory having a $U(1)$ (local) gauge invariance can present a mass renormalization and even a supersymmetry breaking by radiative corrections due to the generation of a Fayet-Iliopoulos term [3]. Nevertheless, one can avoid that problem by imposing a traceless generator of the $U(1)$ gauge symmetry.

In the following chapters we will review the the construction of supersymmetric theories and the process of supersymmetry breaking.

This thesis consists mainly in the application of supersymmetry from extradimensional models to some phenomenological aspects of Physics beyond the Standard Model such as ElectroWeak Symmetry Breaking (EWSB) and neutrino masses. Actually these are the central questions around which the present work gravitates. The main results of the thesis are summarized in the following:
We propose a supersymmetric model in five dimensions, where the fifth one is a finite interval (at the scale of TeV), with mass-like boundary terms. Supersymmetry is broken by the boundary conditions à la Scherk-Schwarz and the EWSB is induced by radiative corrections with a tiny fine tuning of the parameters.
Within the same class of models we exhaustively investigate the possibility for yielding an ultra light mass for neutrinos by letting the right handed neutrino to propagate in the five dimensional bulk with bulk mass M and arbitrary mass-like boundary terms. We find that in the general case the model yields a sub-eV Majorana mass for the SM left handed neutrinos. There is, however, a particular bulk-boundary configuration where a global $U(1)$ symmetry arises and prevents the lepton number from being violated, yielding thus a Dirac mass connecting left and right handed neutrinos, whose value is exponentially suppressed by the bulk right handed neutrino mass.

Chapter 2

Supersymmetric Theories and The Minimal Supersymmetric Standard Model (MSSM)

In the previous chapter we mentioned the construction of supersymmetric theories. In this chapter we will briefly review how to construct them from the supersymmetric algebra and finally we will present the MSSM as an example of a supersymmetric theory.

2.1 Construction of a supersymmetric theory

2.1.1 Supersymmetric algebras

Our aim is to construct a relativistic theory of fields respecting supersymmetry and, possibly, an internal symmetry. According to the Coleman-Mandula theorem [5], the Lie algebra of the symmetry of our S-matrix must be made up with: The Lorentz-Poincaré generators $J^{\mu\nu}$, P^{μ} and a set of generators T^A furnishing a Lie algebra of a compact Lie group, all of them satisfying [6]:

$$[P^\mu \ , \ P^\nu] \ = \ 0 \tag{2.1}$$
$$i[J^{\mu\nu} \ , \ J^{\rho\sigma}] \ = \ \eta^{\nu\rho} \ J^{\mu\sigma} - \eta^{\mu\rho} \ J^{\nu\sigma} - \eta^{\sigma\mu} \ J^{\rho\nu} + \eta^{\sigma\nu} \ J^{\rho\mu} \tag{2.2}$$
$$i[P^\mu \ , \ J^{\rho\sigma}] \ = \ \eta^{\mu\rho} \ P^\sigma - \eta^{\mu\sigma} \ P^\rho \tag{2.3}$$
$$i[J^{\mu\nu} \ , \ T^A] \ = \ 0 \tag{2.4}$$
$$i[P^\mu \ , \ T^A] \ = \ 0 \tag{2.5}$$
$$-i[T^A \ , \ T^B] \ = \ f_C^{AB} \ T^C \tag{2.6}$$

With $\eta^{\mu\nu}$ the Minkowski flat metric. In addition to (2.1)-(2.6) we have a set of generators (Q^r_α) (supersymmetry generators) verifying anti commutation relations which together should furnish a graded Lie algebra, that is: We can define a parity operator P such that we can always classify the generators as even ones (those satisfying $[P, M] = 0$) and odd ones (those satisfying $\{P, Q\} = 0$). Then is easy to see that the next relations do hold:

$$\left.\begin{array}{rcl} 0 & = & [P \ , \ [even \ , \ even]] \\ 0 & = & \{P \ , \ [even \ , \ odd]\} \\ 0 & = & [P \ , \{odd \ , \ odd\}] \end{array}\right\} \tag{2.7}$$

Let us write the generalized Jacobi indentities [7]:

$$0 \ = \ [[B_1 \ , \ B_2] \ , \ B_3] + [[B_3 \ , \ B_1] \ , \ B_2] + [[B_2 \ , \ B_3] \ , \ B_1] \tag{2.8}$$
$$0 \ = \ [[B_1 \ , \ B_2] \ , \ F_3] + [[F_3 \ , \ B_1] \ , \ B_2] + [[B_2 \ , \ F_3] \ , \ B_1] \tag{2.9}$$
$$0 \ = \ \{[B_1 \ , \ F_2] \ , \ F_3\} + \{[F_3 \ , \ B_1] \ , \ F_2\} + [\{F_2 \ , \ F_3\} \ , \ B_1] \tag{2.10}$$
$$0 \ = \ [\{F_1 \ , \ F_2\} \ , \ F_3] + [\{F_3 \ , \ F_1\} \ , \ F_2] + [\{F_2 \ , \ F_3\} \ , \ F_1] \tag{2.11}$$

Where we have denoted by B the even generators and F the odd ones. Taking into account (2.7), the Poincaré algebra and the Jacobi identities one can see that:

$$[Q^r_\alpha \ , \ J^{\mu\nu}] = A^{\mu\nu}_{\alpha\beta} Q^r_\beta \tag{2.12}$$

With the matrix $A^{\mu\nu}$ verifying:

$$i[A^{\mu\nu} \ , \ A^{\rho\sigma}] = \eta^{\nu\rho} \ A^{\mu\sigma} - \eta^{\mu\rho} \ A^{\nu\sigma} - \eta^{\sigma\mu} \ A^{\rho\nu} + \eta^{\sigma\nu} \ A^{\rho\mu} \tag{2.13}$$

and thus, furnishing a representation of the Lorentz group or, in other words: α is a spinor index [1]. For that reason, we will briefly summarize the properties of irreducible representations of the Lorentz group in spacetime of arbitrary dimension [8]. Consider the Dirac algebra in a spacetime of dimension d [2]

$$\{\Gamma^i\ ,\ \Gamma^j\} = 2\delta^{ij} \tag{2.14}$$

with $i, j = 1, 2, \cdots, d$, Γ^i hermitian matrices and the usual Γ^0 is now defined as $i\Gamma^d$. The dimensionality of the minimal representation of the Lorentz group is, then:

$$\begin{cases} 2^{\frac{d}{2}} & \text{for } d \text{ even} \\ 2^{\frac{d-1}{2}} & \text{for } d \text{ odd} \end{cases}$$

We will distinguish between even and odd dimensions:

- d even
  The minimal representation is reducible because the hermitian operator $\Gamma^{d+1} \equiv \Gamma^0 \cdots \Gamma^{d-1}$ commutes with all the generators of the group. So the irreducible representation of the Lorentz group in $d = 2n$ dimensions has dimensionality 2^{n-1} obtaining thus the representation in Weyl spinors

- d odd
  We can use the Dirac algebra in $d - 1$ dimensions plus Γ^d matrix to generate the Lorentz algebra. Therefore, the minimal representation in odd dimensions is irreducible.

One can prove that it is always possible to define a matrix (charge conjugation matrix) satisfying:

$$C_\pm^T = \pm C \text{ and } \Gamma_\mu^T = \pm C_\pm \Gamma_\mu C_\pm^{-1} \tag{2.15}$$

Nevertheless, not both signs are possible for all dimensions (for odd dimension only one of the signs is possible). The charge conjugation matrix allows to define (not in

[1]Index r cannot mix non trivially under Lorentz transformation unless it lives in the same representation as α. Otherwise, we would violate the Coleman-Mandula theorem.

[2]We will consider only one timelike direction.

all dimensions) a real constraint on spinors: In general we can have Dirac spinors (D) (without any constraint), Majorana spinors (M), Weyl spinors (W) (only on even dimension), Majorana-Weyl spinors (MW) or Simplectic-Majorana-Weyl spinors [3] (SMW). All those properties (except the dimensionality of the representation) depend on the dimension of the spacetime with periodicity $d \to d+8$. The table 2.1 summarizes the different possibilities of spinors. The number in the last column is the number of real degrees of freedom for the irreducible representation.
In order to have irreducible representations of supersymmetry, the generators Q^r should be spanned in the spinor irreducible representation according to the spacetime dimension, thus, in four dimensions Q^r should be a Majorana spinor (or a complex Weyl spinor) while in five dimensions Q^r is a 4-component Dirac spinor. Note that $N = 1$ in five dimensions is equivalent to $N = 2$ in four dimensions. Due to the periodicity of the spinor representation there are only 8 different types of supersymmetric algebras. The reader can find an excellent explanation and a whole derivation of the algebra structure in reference [1]. We will write the supersymmetric

Table 2.1: Spinors allowed in several dimensions

d	spinor allowed	real d.o.f.
1	M	1
2	MW	1
3	M	2
4	M	4
5	D	8
6	SMW	8
7	D	16
8	M	16

algebras for the cases concerning us (with vanishing central charges [1]):

[3]Whenever a consistent Majorana condition $\lambda^* = \mathcal{C}\ \lambda$ is not possible it can be defined a Symplectic-Majorana condition by doubling the number of degrees of freedom: $\lambda_i^* = \Omega_{ij}\mathcal{C}\ \lambda_j$ with Ω an antisymmetric matrix satisfying $\Omega^*\Omega = -\mathbf{1}$.

- for $d = 4$ and $N = 2$

$$\left.\begin{array}{rcl} [J^{\mu\nu} \ , \ Q^s] & = & -\sigma^{\mu\nu} \, Q^s \\ \{Q^r \ , \ \bar{Q}^s\} & = & 2i\delta^{rs}\sigma_\mu P^\mu \\ \{Q^r \ , \ Q^s\} & = & 0 \end{array}\right\} \tag{2.16}$$

- for $d = 4$ and $N = 1$

$$\left.\begin{array}{rcl} [J^{\mu\nu} \ , \ Q] & = & -\sigma^{\mu\nu} \, Q \\ \{Q \ , \ \bar{Q}\} & = & 2i\sigma_\mu P^\mu \\ \{Q \ , \ Q\} & = & 0 \end{array}\right\} \tag{2.17}$$

- $d = 5$ and $N = 1$

$$\{Q^r \ , \ Q^s\} = -i\epsilon^{rs} \, \gamma^M C P_M \tag{2.18}$$

  With ϵ^{ij} the total antisymmetric tensor, $\gamma^M = \{\gamma^\mu \ , \ i\gamma^5\}$, C the antisymmetric charge conjugation matrix and $\sigma^\mu = (\mathbf{1} \ , \ \vec{\sigma})$.

Once we have briefly presented the supersymmetric algebra we will deal with the representations of the supersymmetry.

2.1.2 Supersymmetric theories

In this section we will construct a supersymmetric action for $d = 4$, $N = 1$ supersymmetry and for $d = 5$, $N = 1$ supersymmetry. First of all we will ask ourselves by the field content on each supermultiplet (the irreducible representation of the superalgebra). We will distinguish massless from massive particles [1, 6] because its spin structures are very different:

- 4 dimensional $N = 1$ supermultiplets
  - Massless particle states:

    For massless particles we must define what is called helicity $h = \hat{p} \cdot \vec{J}$ with $\hat{p}$ the normal 3-momentum vector and $\vec{J}$ the 3-angular momentum. Each irreducible representation of the homogeneous Lorentz group with massless particles consists on a single helicity state with eigenvalue $\lambda = \frac{n}{2}$ (for some integer number n). Furthermore, if parity is a conserved quantity then for each λ-helicity state, there is a $-\lambda$-helicity state because $\{P\ ,\ h\} = 0$ [4]. Consider now (2.17) with $P^\mu = E(1,0,0,1)$

$$\{Q_{\frac{1}{2}}\ ,\ Q^*_{\frac{1}{2}}\} = 4iE \quad (2.19)$$

$$\{Q_{\frac{1}{2}}\ ,\ Q_{\frac{1}{2}}\} = 0 \quad (2.20)$$

$$\{Q_{-\frac{1}{2}}\ ,\ Q^*_{-\frac{1}{2}}\} = 0 \quad (2.21)$$

$$\{Q_{-\frac{1}{2}}\ ,\ Q_{-\frac{1}{2}}\} = 0 \quad (2.22)$$

with $[J_3\ ,\ Q_{\pm\frac{1}{2}}] = \mp\frac{1}{2}Q_{\pm\frac{1}{2}}$. This tells us that if we have a state of maximum helicity λ_{max} then we will have another state of helicity $\lambda_{max} - \frac{1}{2}$. Of course, we will have helicities $\pm\lambda_{max}$ and $\pm(\lambda_{max} - \frac{1}{2})$ if parity is conserved. In four dimensions, with $N = 1$ we have, so, massless on-shell supermultiplets of the form:

$$(\psi\ ,\ \phi) \text{ for } \lambda_{max} = 1/2 \quad (2.23)$$

$$(V^\mu\ ,\ \xi) \text{ for } \lambda_{max} = 1 \quad (2.24)$$

Where $\psi\ ,\ \xi$ are Majorana spinors [5] and $V^\mu\ ,\ \phi$ are one vector and one complex scalar, respectively.

– Massive particle states:
For massive particles we can define the spin group (generated by the three spatial rotations) and now the irreducible representations of the homogeneous Lorentz group consist on states $|j\ ,\ \sigma>$ with j some integer or half-integer positive number, and σ running with steps of $+1$ from $-j$ to j. Since $[P\ ,\ \vec{J}] = 0$, $P\ |j\ ,\ \sigma >= \eta\ |j\ ,\ \sigma>$ with $|\eta|^2 = 1$. For any supermultiplet there is always at least one spin irreducible representation $|j\ ,\ \sigma>$ such that $Q_{\pm\frac{1}{2}}\ |j\ ,\ \sigma >= 0$ for $\sigma = -j, \cdots, j$.

[4]The photon is an example of two helicity states because QED preserves parity. On the other hand the SM neutrinos have only one helicity because SM do not have parity as a symmetry

[5]For matter we could use complex Weyl representation but the vector multiplet discribes the gauge sector which is in the adjoint representation, namely: real representation, and hence we use Majorana spinors.

  * For $j = 0$ we find a supermultiplet $(\psi\ ,\ \phi)$ where ψ is a complex Weyl spinor and ϕ is a complex scalar.
  * For $j = 1/2$ we find a vector supermultiplet $(\Psi\ ,\ \phi\ ,\ V^\mu)$ where Ψ is a Dirac fermion, ϕ is a real scalar and V^μ is a (massive) gauge vector.

- 5 dimensional $N = 1$ supermultiplets
  As we saw in the previous section the number of supersymmetric charges in the irreducible representation of $N = 1$ in 5 dimensions doubles the corresponding number of the $N = 1$ 4-dimensional case. So from the 4-d point of view, $N = 1$ in five dimensions is equivalent to $N = 2$ in four dimensions. If we proceed like we did in the case of $N = 1$ in four dimensions we find, for the massless case the following on-shell supermultiplets:

$$(\psi\ ,\ \phi_i) \text{ for matter case} \tag{2.25}$$

$$(V^\mu\ ,\ \xi_i\ ,\ \Sigma\ ,\ \Omega) \text{ for the gauge sector} \tag{2.26}$$

  where ψ is a Dirac fermion, ϕ_i is a doublet under $SU(2)_R$ of complex scalars, V^μ is a 4-vector, ξ_i is an $SU(2)_R$ doublet of Symplectic-Majorana spinors [6] and Σ and Ω are real scalars. In the five dimensional multiplet one of the scalars is the fifth component of the gauge field.

To derive an invariant action we will furnish a representation of the supersymmetry algebra with the field content above found. Nevertheless, we need auxiliary degrees of freedom (obviously non physical) if we want the algebra to close. This is because the supermultiplets do not have the same number of off-shell bosonic and fermionic degrees of freedom. In other words, we would need the equations of motion.
For the case of $N = 1$ in four dimensions we have the off-shell supermultiplet

$$(\psi\ ,\ \phi\ ,\ F) \tag{2.27}$$

With F a complex scalar and we take ψ as a complex Weyl spinor. ψ has dimension of $E^{\frac{3}{2}}$, ϕ has dimension of E and F has dimension of E^2 [7]. From (2.17) we see that Q

[6] See section 3.3.
[7] Since F is a non propagating degree of freedom the action can not have any derivative of that field and the simplest way of entering it is quadratically: $|F|^2$.

has dimension of $E^{\frac{1}{2}}$, therefore, the supersymmetry parameter ϵ (which is a complex Weyl spinor valued Grassmann parameter) should have dimension of $E^{-\frac{1}{2}}$. Taking into account the dimensions of the fields and demanding (2.17) to be satisfied we find the supersymmetric variations [4]:

$$\begin{aligned} \delta_\epsilon\, \psi &= i\sqrt{2}\; \sigma^\mu \bar{\epsilon} \partial_\mu \phi + \sqrt{2}\; \epsilon F \\ \delta_\epsilon\, \phi &= \sqrt{2}\; \epsilon\; \psi \\ \delta_\epsilon\, F &= i\sqrt{2}\; \bar{\epsilon}\bar{\sigma}^\mu \partial_\mu \psi \end{aligned} \tag{2.28}$$

with $\bar{\sigma}_\mu = \sigma_2(\sigma_\mu)^*\sigma_2$. Invariant lagrangians under this transformations are:

$$\mathcal{L}_0 = i\partial_\mu \bar{\psi} \bar{\sigma}^\mu \psi + \phi^* \partial^2 \phi + F^* F \tag{2.29}$$

$$\mathcal{L} = \mathcal{L}_0 + m(\; AF - \frac{1}{2}\psi\psi + h.c.) \tag{2.30}$$

For the vector multiplet we find the off-shell supermultiplet is

$$(\xi \;,\; V_\mu \;,\; D) \tag{2.31}$$

with D a real auxiliary scalar. If we express ξ in terms of its Weyl components

$$\xi = \begin{pmatrix} \lambda \\ \bar{\lambda} \end{pmatrix}$$

and $\bar{\lambda} = i\sigma_2 \lambda^*$, the supersymmetric transformations can be written as follows [4]

$$\delta_\epsilon\, V_{\mu\nu} = i\epsilon\; \sigma_{[\mu}\partial_{\nu]}\bar{\lambda} + i\bar{\epsilon}\; \bar{\sigma}_{[\mu}\partial_{\nu]}\lambda \tag{2.32}$$

$$\delta_\epsilon\, \lambda = \sigma^{\mu\nu}\epsilon V_{\mu\nu} + i\epsilon D \tag{2.33}$$

$$\delta_\epsilon\, D = \bar{\epsilon}\bar{\sigma}^\mu \partial_\mu \lambda - \epsilon \sigma^\mu \partial_\mu \bar{\lambda} \tag{2.34}$$

with $V_{\mu\nu} = \partial_\nu V_\nu - \partial_\nu V_\mu$ and $\sigma^{\mu\nu} = \frac{1}{4}(\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu)$.
A lagrangian invariant under these transformations is:

$$\mathcal{L} = i\lambda\sigma^\mu\partial_\mu\bar{\lambda} + \frac{1}{4}V^{\mu\nu}V_{\mu\nu} - \frac{1}{2}D^2 \tag{2.35}$$

In four dimensions and $N = 1$ it is possible to derive systematically the invariant lagrangian as well as the form of supersymmetry transformations themselves, using the superfield formalism. For $N \geq 2$ and with no central charges, we can use the $R-$ *symmetry* of the superalgebra to construct an invariant lagrangian from the most general invariant one with $N = 1$. The reader can find exhaustive explanations in references [4, 1].
Finally, for the case of $N = 1$ in five dimensions we have the off-shell multiplets [9], and supersymmetric transformations:

$$\begin{aligned} \Phi &= (\phi^i \ , \ \psi \ , \ F^i) \\ \delta_\xi \ \phi^i &= -\sqrt{2}\epsilon^{ij}\bar{\xi}^j\psi \\ \delta_\xi \ \psi &= \sqrt{2}i\gamma^M\partial_M\phi^i\epsilon^{ij}\xi^j + \sqrt{2}F^i\xi^i \\ \delta_\xi \ F^i &= -\sqrt{2}i\bar{\xi}\gamma^M\partial_M\psi \end{aligned} \tag{2.36}$$

for matter multiplet, and

$$\begin{aligned} V &= (V^M \ , \ \lambda^i \ , \ \varphi \ , \ X^a) \\ \delta_\xi \ V^M &= i\bar{\xi}^i\gamma^M\lambda^i \\ \delta_\xi \ \lambda^i &= (\Sigma^{MN}V_{MN} - \gamma^M D_M\varphi)\xi^i - i(X^a\sigma^a)^{ij}\xi^j \\ \delta_\xi \ \varphi &= i\bar{\xi}^i\lambda^i \\ \delta_\xi \ X^a &= \bar{\xi}(\sigma^a)^{ij}\gamma^M D_M\lambda^j - [\varphi \ , \ \bar{\xi}(\sigma^a)^{ij}\lambda^j] \end{aligned} \tag{2.37}$$

for vector multiplet. Where X^a is a triplet, under $SU(2)_R$, of auxiliary real Lorentz sacalars, $V = gV^aT^a$ with T^a the generators of the gauge symmetry group and g the gauge coupling constant, $D_M f = \partial_M f - i[V_M, f]$ and $\Sigma^{MN} = \frac{1}{4}[\gamma^M, \gamma^N]$. The supersymmetric lagrangians are given in the next chapter.
Up to now we have reviewed, from a quite general point of view, the properties of supersymmetry and the construction of an invariant theory. In the next section we will briefly present a supersymmetric version of the standard model.

2.2 Supersymmetric version of the Standard Model (MSSM)

In the previous chapter we presented some reasons to consider supersymmetry as a good candidate to be the symmetry of the theory beyond the standard model. Let us concretize a bit more about what we meant with this. The standard model is a gauge theory describing electroweak and strong interactions with the local symmetry group: $SU(3)\times SU(2)\times U(1)$ and, thus having 12 spin 1 gauge bosons: g^a for strong interactions, W^i for weak interactions and Y for the hypercharge [10]. Its spin 1/2 content is [8]:

$$\begin{aligned} Q_i = \begin{pmatrix} u_i \\ d_i \end{pmatrix} &= (3,2,1/6) \\ \bar{u}_i &= (\bar{3},1,-2/3) \\ \bar{d}_i &= (\bar{3},1,1/3) \end{aligned} \tag{2.38}$$

$$\begin{aligned} L_i = \begin{pmatrix} \nu_i \\ e_i \end{pmatrix} &= (1,2,-1/2) \\ \bar{e}_i &= (1,1,1) \end{aligned} \tag{2.39}$$

with i a family index, and its spin 0 content (Higgs particle) is

$$h = \begin{pmatrix} h^0 \\ h^- \end{pmatrix} = (1,2,-1/2) \tag{2.40}$$

the parenthesis gives the representation of $(SU(3), SU(2), U(1))$ where the corresponding field lies.

In addition to the gauge couplings the standard model has Yukawa couplings between fermions and the Higgs as well as a potential for Higgs of the form: $\mu^2 h^\dagger h + \lambda(h^\dagger h)^2$, with μ^2, λ real parameters. For negative values of μ^2 the potential minimizes at $h \neq 0$ giving a Vacuum Expectation Value (VEV) to the Higgs breaking, thus, spontaneously the electroweak symmetry and giving masses to the rest of the particles including $SU(2)\times U(1)$ gauge bosons [11]. The standard model, nevertheless,

[8] We are considering neither right-handed neutrino multiplets nor neutrino mass Yukawa couplings for they would not affect the conclusions of this work. They could be easily incorporated.

is an effective theory of a more fundamental one including gravity at higher scales. The experimental results show the standard model as a good theory at energies $\lesssim 100\ GeV$, so there should be a cutoff of the theory. The problem with the standard model arises precisely from its renormalization properties. The standard model is a renormalizable theory which means that all the divergences appearing in perturbation theory can be absorbed by the redefinition of the lagrangian parameters and therefore there is no natural scale that serves as a cutoff of the theory. On the other hand, the fermion masses are protected by the gauge symmetry to be zero (if we want left- and right-handed fermions to be differently affected by the symmetry) but there is no reason to set to zero the scalar masses. And that is the reason of the quadratic divergences in the correction of μ^2 parameter. The regularization of such divergences depends on the scheme introducing new arbitrary mass scale. Notice that were the theory not renormalizable we would need a scale under which the theory is valid and this is a natural cutoff of the theory determining the Fermi masses of the standard model particles. The problem now is that the masses of the particles depend on the arbitrary scale introduced by the regularization scheme. In our search of a more fundamental theory containing the standard model as a low energy description we can not accept such an arbitrariness. The only known way to protect scalar masses is supersymmetry. As we have seen before, supersymmetry relates fermions with bosons with the same momentum eigenvalue (in particular: with the same mass). Therefore, if the fermions are massless due to a gauge symmetry so will the bosons be. Furthermore, the nonrenormalization theorems assure us the presence of no quadratic divergences. Let us start, so, with $N = 1$ supersymmetry and two Higgs supermultiplets[9]. The most general superpotential allowed contains the super-Yukawa terms [1]:

$$[(D_i H_1^0 - U_i H_1^-)\bar{D}_j]\ ,\ [(E_i H_1^0 - N_i H_1^-)\bar{E}_j] \tag{2.41}$$

and

$$[(D_i H_2^+ - U_i H_2^0)\bar{U}_j] \tag{2.42}$$

[9]We must use two Higgs supermultiplets in order to cancel the $SU(2) \times U(1)$ anomalies.

where we have the supermultiplets:

$$Q_i = \begin{pmatrix} U_i \\ D_i \end{pmatrix}$$

$$L_i = \begin{pmatrix} N_i \\ E_i \end{pmatrix}$$

$$H_1 = \begin{pmatrix} H_1^0 \\ H_1^- \end{pmatrix} \text{ and } H_2 = \begin{pmatrix} H_2^+ \\ H_2^0 \end{pmatrix}$$

According to (2.41)-(2.42) a VEV acquired by H_1^0 gives masses to down-type quarks and leptons while a VEV acquired by H_2^0 gives masses to up-type quarks. Notice that two Higgses, therefore, are needed to give masses to all quarks and leptons. Nevertheless, the superpotential allows terms which violate lepton and baryon number conservation. In order to avoid this problem one can make use of the so called R-parity, which is a discrete subgroup of a continuous $U(1)_R$ symmetry [10]. This means that the minimal supersymmetric extension of the standard model comes from $N = 2$ supersymmetry. The so called Minimal Supersymmetric Standard Model (MSSM).
We have said nothing about supersymmetry breaking yet and it is obvious from experimental results that it can not be present at energies of the validity region of the standard model. One could think that supersymmetry is broken in a way similar to the breaking of $SU(2) \times U(1)$. Nevertheless, there are some rules not affected by the symmetry breaking (protected by color and electric charge symmetries). For instance, the fermions with electric charge $-e/3$ are: d, s and b, for which: $m_d^2 + m_b^2 + m_s^2 \backsimeq (5 \text{ GeV})^2$, and if there are no other fermions their supersymmetric partners should hold a similar relation: $\sum m^2 \backsimeq (5 \text{ GeV})^2$ which is excluded by experimental data (there is no evidence of scalar matter at energies below 7 GeV). The breaking of supersymmetry, therefore, must come from higher scales. For instance it could be performed in extra dimensions (via the VEV acquired by other fields). For this reason, in the next chapter we will present some breaking mechanisms of supersymmetry in five dimensions.

[10] Coming from the $SU(2)_R$ symmetry that $N = 2$ supersymmetry presents.

Chapter 3

Compactification of Extra Dimensions, Orbifolds and Supersymmetry Breaking

3.1 Motivation

The motivation for the compactification of the extra dimensions is rather simple: Up to quite high energies ($E \sim 100\ GeV$) we do not detect the presence of more than four spacetime dimensions in nature. This means that all of the extra dimensions (if real) must be very small (the smaller distance you want to reach the higher energy you must spend to). The idea is that the known particles live in our four dimensional world, while there are very massive particles living in the $(4+d)$-dimensional bulk, and they are so massive that a large amount of energy must be spent in order to produce them. In addition to this, all the extra dimensions must be space-like, otherwise we would have closed time-like curves (because extra dimensions are compactified) and this violates causality [2]. An elegant procedure to "reduce the size" of the extra dimensions is the compactification [12]. Althogh we have loosely introduced the notion of compactification as the process of reducing the size of a space it has a precise meaning in the context of topological spaces. There is a natural way of compactifying a given (non compact) topological space through the action of a group. In the following we will briefly describe this method starting by giving the basic notions on topological spaces and differentiable manifolds.

3.2 Compactification of spaces

Previously, we have given an intuitive notion of compactification as reducing the size. Nevertheless, this process has a precise definition in the context of topological spaces. In the following two sections we will briefly review the basic notions about topological spaces and differentiable manifolds. The interested reader can find excellent introductory courses on these topics in Ref. [13, 14, 15].

3.2.1 Topological spaces

The notion of topology on a general space is a generalization of the natural notion of closeness between points in $\mathbb{R}^n$, derived from the Euclidean metric, where a neighborhood of a given point x_0 is defined as the open ball $B_\epsilon(x_0) = \{x \in \mathbb{R}^n | \ |x - x_0| < \epsilon, \ \epsilon > 0\}$. For a general set X we define a topology, denoted by $\mathcal{T}(X)$, as a collection of subsets of X verifying the following properties

1. $X \in \mathcal{T}(X)$.
2. $\emptyset \in \mathcal{T}(X)$ where $\emptyset$ denotes the empty set.
3. If $\{U_1, \cdots, U_k\}$ is a (finite) collection of elements from $\mathcal{T}(X)$ then $U_1 \cap \cdots \cap U_k \in \mathcal{T}(X)$.
4. If $\{U_\alpha\}$ is a collection, with an arbitrary number (perhaps infinite), of elements from $\mathcal{T}(X)$ therefore $\cup_\alpha U_\alpha \in \mathcal{T}(X)$.

Thus the open sets of X are the elements of $\mathcal{T}(X)$ while the closed sets are those whose complementary belongs to $\mathcal{T}(X)$. Notice that according to that definition the whole set as well as the empty set are both open and closed at the same time. A set (or space) X with a defined topology is called a topological set (or space). One can define, as a generalization of the corresponding Euclidean notion, the closure of a set as: if $A \subset X$ is a subset of a topological space then the closure of A, denoted by $\bar{A}$, is the set of points $x \in X$ such that for any open set $U \ni x$, $U \cap A \neq \emptyset$. Then an alternative definition of a closed set is that any set A is closed if, and only if, $A = \bar{A}$.

It is clear that given a set one can provide many different topologies for it, for instance, the most naive topology of a given set X is $\mathcal{T}(X) = \{X, \emptyset\}$ from where the only open sets are the whole space X and the empty set. At the other extreme, the finest topology that can be given to X is $\mathcal{T}(X) = \mathcal{P}(X)$, the set of all possible partitions (subsets) of X, where every element of X is an open set. As these extreme cases illustrate, the notion of open set can be totally different from the intuitive open balls in $\mathbb{R}^n$.

Induced topology

Suppose we have provided the set X with a topology $\mathcal{T}(X)$. A subset $Y \subset X$ inherits a topology from X in the following way: $V \subset Y$ belongs to $\mathcal{T}(Y)$ if, and only if, there is $U \in \mathcal{T}(X)$ such that $U \cap Y = V$. It is easy to check that this definition is indeed a topology but the open sets of Y may not be open sets in X. For instance, consider in $\mathbb{R}$, with the usual Euclidean topology, the closed unity interval $Y = [0,\, 1]$. Then the open sets of Y are of the from

$$\begin{cases} (\epsilon,\, \delta) & 0 \leq \epsilon,\, \delta \leq 1 \\ [0,\, \delta) & 0 < \delta \leq 1 \\ (\epsilon,\, 1] & 0 \leq \epsilon < 1 \end{cases}$$

where the last two class of sets are not open in $\mathbb{R}$.

Continuous functions

Let X and Y be two topological spaces and $f : X \to Y$ an application between them. We say that f is a continuous function if $f^{-1}(V) \in \mathcal{T}(X)$, $\forall\, V \in \mathcal{T}(Y)$, that is, the set of points in X whose mapping by f is a given (arbitrary) open set in Y, form an open set in X. The induced topology defined above makes the natural inclusion $i : Y \hookrightarrow X$ to be continuous. Nevertheless, the image by a continuous function of an open set is not necessarily an open set. As an example, consider the function $f : \mathbb{R} \to \mathbb{R}$ given by $f(x) = x^2$. The image of the open interval $(-1,\, 1)$ is $[0,\, 1)$ which is not open although the function is perfectly continuous. This is true, however, for any homeomorphism. A function $h : X \to Y$ is called a homeomorphism if it is continuous, 1 to 1 and exhaustive and if, in addition,

$h^{-1}: Y \to X$ is as well continuous. Notice that in this case the spaces X and Y are topologically equivalent, since to any open set $V \in \mathcal{T}(Y)$ we can assign a unique open set $h^{-1}(V) \in \mathcal{T}(X)$ and vice versa.

Derived topologies

1. The product topology.
   Let X and Y be two topological spaces. The topology of their cartesian product, $X \times Y$, is defined as follows: $\mathcal{U} \subset X \times Y$, i.e. $\mathcal{U} = U_X \times U_Y$, is an open set if, and only if, the canonical projections $\Pi_X(\mathcal{U}) = U_X \subset X$ and $\Pi_Y(\mathcal{U}) = U_Y \subset Y$ are open sets in X and Y, respectively. Again this topology makes the canonical projections to be continuous functions.

2. The quotient topology.
   Let X be a topological space and $Y \subset X$. We can define the quotient space X/Y by the set of equivalence classes defined by the equivalence relation: $x, x' \in X$ are equivalent if, and only if, $x \in Y$ and $x' \in Y$. The topology on the quotient space is then defined through the quotient projection

$$\begin{array}{cccc} P: & X & \to & X/Y \\ & x & \mapsto & [x] \end{array}$$

   as: $U \subset X/Y$ is open if, and only if, there is $\tilde{U} \in \mathcal{T}(X)$ such that $P(\tilde{U}) = U$. As the cases above, with this topology the quotient projection is a continuous map.

Topological properties of spaces

1. Separability (Hausdorff spaces).
   A topological space X is said to be Hausdorff (or separable) if for any pair of points $x, x' \in X$ there are open sets $U_x \ni x$, $U_{x'} \ni x'$ such that $U_x \cap U_{x'} = \emptyset$. The Hausdroff spaces have topological properties which are close to the intuitive notion of Euclidean topology in $\mathbb{R}^n$: if X is a Hausdorff space, any single point $x \in X$ is a closed set, if $\{x_1, \cdots, x_k\} \subset X$ is a finite subset then

each point x_i is an isolated point from the rest, as a consequence of this we have that if $A \subset X$ then $x \in \bar{A}$ if, and only if, any open set $U \ni x$ contains an infinite number of points of A.

2. Connectedness.
   X is connected if only X itself and $\emptyset$ are subsets being open and closed at the same time. An equivalent definition is: X is connected if it can not be expressed as the disjoint union of open sets.

3. Arc connectedness.
   A topological space, X, is arc connected if for any pair of points $x, x' \in X$ there is a continuous function $c : [0,\ 1] \subset \mathbb{R} \to X$ where the closed interval has the topology induced by $\mathbb{R}$, such that $c(0) = x$ and $c(1) = x'$. It turns out that any arc connected space is connected but the reciprocal is not necessarily true.

4. Compactness.
   Let X be a topological space and $\{U_i\}_{i\in I}$, where I is a set of indices, a collection of open sets of X such that $\cup_{i\in I} U_i = X$. We then say that $\{U_i\}_{i\in I}$ is an open covering of X. Let now $I' \subset I$, if $\cup_{j\in I'} U_j = X$ we say that $\{U_j\}_{j\in I'}$ is a subcovering of X.
   A topological space X is said to be compact if for any open covering $\{U_i\}_{i\in I}$ there is a finite subcovering, that is, if from the set $\{U_i\}_{i\in I}$ we can take a finite number, say $\{U_1, \cdots, U_k\}$ such that $U_1 \cup \cdots \cup U_k = X$. For instance consider the set $(0, 1] \in \mathbb{R}$, with the induced topology. An open covering can be $\left\{\left(\frac{1}{n},\ 1\right]\right\}_{n>0}$. Non the less, by taking an arbitrary but finite number out of the covering, say $\{U_{n_1}, \cdots, U_{n_k}\}$ we cover the set $(1/n_{max},\ 1]$ with $n_{max} = \max\{n_1, \cdots, n_k\}$. Thus $(0, 1]$ is not a compact space. In fact, it can be shown that a subset of $\mathbb{R}^n$ (with the usual topology) is compact (according to the previous definition) if, and only if, it is closed and bounded.

All these properties are preserved by homeomorphisms, that is, if X, Y are two equivalent topological spaces (i.e. related by a homeomorphism) then X verifies

some of the above properties if, and only if, Y verifies it/them as well. Moreover, the image by a continuous function of a connected (arc connected) and/or compact space is as well connected (arc connected) and/or compact. The separability, however, is not preserved by continuous functions, in general. Finally, if X and Y are two topological spaces then $X \times Y$ is Hausdorff and/or connected (arc connected) and/or compact if, and only if, each space holds the same(s) property(ies).
Once the notion of continuity is properly defined on a general space, one can add a richer structure by requiring smoothness, that is: differentiability, which generalizes the regular surfaces in $\mathbb{R}^3$. These are the so called differentiable manifolds, which we immediately define. In what follows we will think of $\mathbb{R}^n$ as a topological space provided with the usual Euclidean topology and any subset of $\mathbb{R}^n$ will have the induced topology.

3.2.2 Differentiable manifolds

An intuitive definition of a manifold is a space which locally looks like $\mathbb{R}^n$, although its global structure can be very different. The sphere in $\mathbb{R}^3$, S^2, is an example. An open neighborhood of a given point $p \in S^2$ is an open sheet containing that point, which is homeomorphic to an open ball in $\mathbb{R}^2$. However the whole sphere is very different from $\mathbb{R}^2$, actually S^2 is a compact space (since it is a closed and bounded subset of $\mathbb{R}^3$) while $\mathbb{R}^2$ is not. In fact any regular surface in R^3 is an example of a differentiable manifold since by definition a surface is regular if one can locally and univocally parameterize it with an open set of $\mathbb{R}^2$. A precise definition of a general manifold is given now:
A differentiable manifold of dimension n consists of:

1. A topological space, M [1].

2. An atlas of M, which is a collection of pairs $\{(U_i, \phi_i)\}_{i\in I}$, where $\{U_i\}_{i\in I}$ is an open covering of M and $\phi_i : V_i \subset \mathbb{R}^n \to U_i \subset M$ is a homeomorphism between U_i and an open set, V_i, in $\mathbb{R}^n$, such that whenever $U_i \cap U_j \neq \emptyset$ the

[1] Strictly speaking one should require certain restrictions on the topology of M since we finally want it to locally look like an open set of $\mathbb{R}^n$

homeomorphism

$$\phi_j^{-1} \circ \phi_i : \phi_i^{-1}\left(U_i \cap U_j\right) \subset \mathbb{R}^n \to \phi_j^{-1}\left(U_i \cap U_j\right) \subset \mathbb{R}^n \,,$$

is a differentiable function. Each pair (U_i, ϕ_i) is called a local chart.

3. An equivalence relation between atlas given by: If $\{(U_i, \phi_i)\}_{i\in I}$ and $\{(V_\alpha, \psi_\alpha)\}_{\alpha\in A}$ are two atlas of M we say that they are equivalent if, and only if, $\{(U_i, V_\alpha, \phi_i, \psi_\alpha)\}_{\substack{i\in I\\ \alpha\in A}}$ is again an atlas of M. That is the union of the atlas is an atlas.

Notice that the only non trivial requirement in (3) is that whenever $U_i \cap V_\alpha \neq \emptyset$ the homeomorphisms

$$\phi_i^{-1} \circ \psi_\alpha : \psi_\alpha^{-1}\left(U_i \cap V_\alpha\right) \subset \mathbb{R}^n \to \phi_i^{-1}\left(U_i \cap V_\alpha\right) \subset \mathbb{R}^n \,,$$
$$\psi_\alpha^{-1} \circ \phi_i : \phi_i^{-1}\left(U_i \cap V_\alpha\right) \subset \mathbb{R}^n \to \psi_\alpha^{-1}\left(U_i \cap V_\alpha\right) \subset \mathbb{R}^n \,,$$

are differentiable functions. The rest of requirements for $\{(U_i, V_\alpha, \phi_i, \psi_\alpha)\}_{\substack{i\in I\\ \alpha\in A}}$ to be an atlas are immediately satisfied since each collection is already an atlas.
A differentiable manifold is then a space that locally can be univocally parameterized by n real coordinates such that this parameterization changes smoothly over the whole space. For this reason the open set in a local chart is often referred to as a coordinate neighborhood. In addition, such a neighborhood is topologically equivalent to $\mathbb{R}^n$. Examples of manifolds are: $\mathbb{R}^n$ itself, any smooth surface in $\mathbb{R}^3$ like the sphere and the torus.
A very important structure associated to any differentiable manifold is the tangent bundle, which loosely speaking is the space obtained from the manifold itself by attaching to each point $p \in M$ the set of all tangent vectors to the manifold at that point. For a regular surface $S \subset \mathbb{R}^3$ it is clear what is the tangent space at each point $p \in S$, denoted by T_pS: it is the set of all vectors in $\mathbb{R}^3$ with origin at p and which are tangent to the surface. The tangent bundle of S, denoted by TS, is then the set of pairs

$$TS = \cup_{\substack{p\in S\\ V\in T_pS}} (p, V) \ .$$

For a general manifold, however, we could have no ambient space to which refer a tangent vector. Instead we give a formal definition of tangent vector which depends on the manifold structure only. First of all we have to specify the notion of differentiable functions defined on the manifold: let M be a differentiable manifold with an atlas

$$\{U_\alpha,\, \phi_\alpha : V_\alpha \subset \mathbb{R}^n \to U_\alpha\}_{\alpha\in\mathfrak{A}}\ ,$$

and $f : M \to \mathbb{R}$ be a continuous function. Then f is said to be differentiable if $f \circ \phi_\alpha^{-1} : V_\alpha \subset \mathbb{R}^n \to \mathbb{R}$ is differentiable for any local chart. We denote $\mathcal{D}(M, \mathbb{R})$ the set of real valued differentiable functions defined on M. Let now $c : (-\epsilon, \epsilon) \subset \mathbb{R} \to M$ a (differentiable) curve in M such that $c(0) = p \in U_\alpha \subset M$ and $\epsilon > 0$ is small enough so that $c(-\epsilon, \epsilon) \subset U_\alpha$. We say that c is differentiable if $\phi_\alpha \circ c : (-\epsilon, \epsilon) \to V_\alpha \subset \mathbb{R}^n$ is a differentiable curve. A vector tangent to M at p, V_p, is then defined as an operator acting on any differentiable function $f \in \mathcal{D}(M, \mathbb{R})$ such that $V_p[f] = \frac{\mathrm{d}}{\mathrm{dt}} f(c(t))_{t=0}$. Let $(x^1, \cdots, x^n) \in V_\alpha \subset \mathbb{R}^n$ be local coordinates of the coordinate neighborhood $U_\alpha \subset M$ and $(c^1(t), \cdots, c^n(t))$ the local expression of c in this system of coordinates with $v^k \equiv \left(\frac{\mathrm{d}}{\mathrm{dt}} c^k(t)\right)_{t=0}$. Then, by the chain rule we have

$$V_p[f] = \sum_k v^k \, \frac{\partial}{\partial x^k}\, f_p\, .$$

Since this is for any differentiable function f we can identify the vector with the differential operator

$$V_p \equiv \sum_k v^k \left(\frac{\partial}{\partial x^k}\right)_p\, ,$$

where the subscript indicates the point where the partial derivative is evaluated. This is a linear operator verifying the Leibniz rule, i.e. $V_p[fg] = f\, V_p[g] + g\, V_p[f]$. In addition, for any $v = (v^1, \cdots, v^n) \in \mathbb{R}^n$ the differentiable curve $(v^1 t, \cdots, v^n t)$ has v as tangent vector. Moreover, for any pair of vectors $u, v \in \mathbb{R}^n$ and for any pair of real numbers $\lambda, \mu \in \mathbb{R}$ the differentiable curve $(\lambda u + \mu v)\, t$ has $\lambda u + \mu v$ as tangent vector. Thus the set of tangent vectors to M at p, T_pM, has structure of vector space and it is isomorphic to $\mathbb{R}^n$. The isomorphism is explicitly given by

$$\begin{array}{ccc} \mathbb{R}^n & \longrightarrow & T_pM \\ \left(0,\cdots,\overset{i}{1},\cdots 0\right) & \mapsto & \left(\frac{\partial}{\partial x^i}\right)_p \end{array}$$

Since the coordinates $(x^1,\cdots,x^n)$ are globally defined on the coordinate neighborhood U_α, the partial derivatives $\partial/\partial x^k$ form a basis globally defined on the whole tangent space to U_α, TU_α.

Suppose that $U_\alpha \cap U_\beta = W \neq \emptyset$ with x^k_α and x^k_β being the corresponding local coordinates, i.e., related by the diffeomorphism $x^k_\beta = \left(\phi^{-1}_\beta \circ \phi_\alpha\right)^k (x_\alpha)$. Then if V is a tangent vector to M at $p \in W$, its expressions in the above local coordinate basis,

$$V = \sum_k v^k_\alpha \left(\frac{\partial}{\partial x^k_\alpha}\right)_p = \sum_l v^l_\beta \left(\frac{\partial}{\partial x^l_\beta}\right)_p , \tag{3.1}$$

are related via the Jacobian of the above diffeomorphism as

$$v^k_\beta = \sum_l \frac{\partial}{\partial x^l_a} \left(\phi^{-1}_\beta \circ \phi_\alpha\right)^k v^l_\alpha .$$

This shows that the tangent bundle itself has structure of differentiable manifold with local charts

$$\begin{array}{cccc} \tilde{\phi}_\alpha : & V_\alpha \times \mathbb{R}^n & \longrightarrow & TU_\alpha \cong U_\alpha \times \mathbb{R}^n \\ & (x,v) & \mapsto & \left(\phi_\alpha(x), \sum_k v^k \left(\frac{\partial}{\partial x^k}\right)_{\phi_\alpha(x)}\right) \end{array}$$

However, the global structure of the tangent bundle TM may not be the direct product $M \times \mathbb{R}^n$. This is due to the fact that, unlike the case of $\mathbb{R}^n$, in a general manifold one has no naturally defined global basis for the tangent vectors. Thus the components of a vector determine it univocally on a local chart but one has to specify the change of coordinates with the neighboring charts in order to consistently describe the vector.

Finally, a vector field, is a map that to each point in M (or in an open subset of M) assigns a unique tangent vector. A vector field is said to be differentiable if its local expression in any local chart is a differentiable $\mathbb{R}^n$ vector field.

Once we have a notion of what is a manifold we will describe the process of compactification a given non compact manifold through the action of a group starting by the definition of the action of a group on a manifold.

3.2.3 Compactification by the action of a group

Let M be a manifold and G a group acting on M in the sense that there is a differentiable map

$$\begin{array}{rccl} \Gamma: & M \times G & \rightarrow & M \\ & (p, g) & \mapsto & \Gamma_g(p) \in M \end{array}$$

verifying

1. $\Gamma_e = Id$ where $e \in G$ is the identity element in G and Id is the identity map in M.

2. $\forall g, g' \in G$, $\Gamma_g \circ \Gamma_{g'} = \Gamma_{gg'}$.

Thus it makes sense if we simply write $\Gamma_g(p) = g\,p$.
We will further assume that the action of G on M is *properly discontinuous*, namely: $\forall p \in M$ there is an open neighborhood U of p such that $g(U) \cap U \neq \emptyset$ if, and only if, $g = e$, which is to say that the set of points in M which are connected to each other by a group transformation, known as an orbit of G, is a discrete set in the topology of M. Notice that the properly discontinuous action implies that any element of G (except the identity) leaves no invariant point on M, i.e. $\forall g \neq e$, $g\,p \neq p$, $\forall p \in M$. It is then said that G acts *freely* on M.
The compactification of M follows from the identification of all the points in an orbit, that is: $p, q \in M$ are identified if, and only if, $p = g\,q$ for some $g \in G$. The compactified space is, thus, the set of orbits in M, that is M/G. Recall that M induces a topology on M/G such that the canonical projection

$$\begin{array}{rccc} \Pi: & M & \rightarrow & M/G \\ & p & \mapsto & [p] \end{array}$$

is a continuous function. A sufficient condition for M/G to be a compact space is that there is a compact set $C \in M$, such that $\cup_{g\in G}\, g(C) = M$. Hence $M/G = \Pi(C)$ being the image of a compact space by a continuous function is compact itself. Furthermore, It can be shown that M/G naturally inherits a structure of differentiable manifold from M such that the above projection is a differentiable function (more precisely it is a local diffeomorphism, i.e. a smooth 1 to 1 function with locally smooth inverse [13]). Let us give some examples of what we have stated:

1. The circle S^1
   Consider in $\mathbb{R}$ the action of the group of translations by a multiple of the unity, that is

$$g_n : \begin{array}{ccc} \mathbb{R} & \longrightarrow & \mathbb{R} \\ x & \longmapsto & x+n \end{array}$$

   with $n \in \mathbb{Z}$, hence $G \cong \mathbb{Z}$. The action is, in addition, properly discontinuous. To see this notice that $\forall x \in \mathbb{R}$ the open interval $\mathfrak{I}_x = \{y \in \mathbb{R}|\, |y-x| < 1/2\}$ verifies $g_n\left(\mathfrak{I}_x\right) \cap \mathfrak{I}_x = \emptyset$, $\forall n \neq 0$ since

$$|y+n-x| \geq |\,|n| - |y-x|\,| > |n| - 1/2 > 1/2\,, \qquad n \neq 0\,.$$

   In this case the compact set C can be taken as $C = \{x \in \mathbb{R}|\, |x| \leq 1/2\}$, thus $\mathbb{R}/\mathbb{Z} \cong S^1$ is a compact differentiable manifold.

2. The n dimensional torus $\times^n S^1$
   The case above can be directly generalized to higher dimensions as the action of $\oplus^n\mathbb{Z}$ over $\mathbb{R}^n$.

3. A less trivial example: The Klein Bottle
   Consider the group, say G_K, of homeomorphisms on $\mathbb{R}^2$ generated by $\tau_1\left(x,y\right) = \left(x+1,y\right)$ and $\tau_2\left(x,y\right) = \left(-x,y+1\right)$, that is: the set of all possible compositions of these elements and their inverses. This group is nor abelian neither freely generated since $\tau_1\tau_2\tau_1 = \tau_2$. However, this identity allows us to express any element of G_K as $\left(\tau_2\right)^{k_2}\left(\tau_1\right)^{k_1}$ with $k_1, k_2 \in \mathbb{Z}$, although this correspondence is not a group homomorphism.

Consider now the open set

$$U_{(a,b)} = \left\{ (x,y) \in \mathbb{R}^2 \middle| \, \|(x-a, y-b)\| < 1/2 \right\} ,$$

since $(\tau_2)^{k_2}(\tau_1)^{k_1}(x,y) = \big((-1)^{k_2}(x+k_1), y+k_2\big)$ we thus have a properly discontinuous action. Moreover, $C = \{ (x,y) \in \mathbb{R}^2 | \, ||(x,y)|| \leq 1/2\}$ is a compact subset of $\mathbb{R}^2$ which covers $\mathbb{R}^2$ under the action of G_K, therefore $\mathbb{R}^2/G_K$ is a compact and differentiable manifold (In fact it is homeomorphic to the Klein Bottle).

The last example serves as an illustration of the compactification procedure, non the less it is not suitable for a field theory to be defined on it since the Klein Bottle is a non orientable space and hence non integrable. In the context of a (special) relativistic (quantum) field theory, which will be defined on $\mathcal{M}_4 \times R^k$, $\mathcal{M}_4$ being the 4D Minkowski space, the compactification will concern only the k extra dimensions. Thus the $SO(1,3+k)$ space symmetry group will be spontaneously broken to $SO(1,3) \times SO(k)$.
It is clear that at the level of a (quantum) field theory we must demand certain restrictions on the fields to be well defined on the compactification manifold. This can be done in two different ways:

- The usual compactification (à la Kaluza).
  Here we restrict the fields to be invariant under the group action, namely: If f is a field defined on the manifold M and $g : M \to M$ is an element of the considered group, we require $f \circ g = f$. Thus f will be naturally defined on M/G as $\tilde{f}([p]) \equiv f(p)$. Applied to $\mathcal{M}_4 \times \mathbb{R} \to \mathcal{M}_4 \times S^1$ compactification, we have that any 5D, $\psi(x,y)$ field has the symmetry $\psi(x,y) = \psi(x, y+2\pi R)$ with R the circle radius. Thus they may be written as a Fourier expansion, $\psi(x,y) = \sum_k \psi_n(x) e^{i2k\pi R}$.

- A non trivial compactification.
  Suppose the action for the field theory is

$$S = \int_M \mathcal{L}(f) \, . \tag{3.2}$$

In this case we do not restrict the fields themselves but we require them to transform as a symmetry of (3.2) under the group action. That is, $f \circ g = \mathfrak{t}_g f$ where $\mathfrak{t}$ is some homomorphism of G acting on the space of the fields and leaving the action of the theory invariant. Over M/G thus we would have a multivalued field, this problem can be formally avoided by thinking of f as defining a non trivial fiber bundle on M/G. This kind of compactification was introduced by Joel Scherk and John Schwarz [16] in 1979 as a mechanism to break supersymmetry spontaneously by generating masses to some (but not all) of the fields of the supergravity multiplet. We will describe this symmetry breaking mechanism at the end of this chapter.

3.2.4 Orbifolds

In the process of compactification we start with a smooth manifold to end up with another smooth space. Non the less, if the compactified theory has to reproduce the known low energy physics it has to describe 4D chiral fermions, however, if the compactification is taken to be "smooth", like the cases above, that is not possible. To illustrate this consider the previous 5D compactification. As it was pointed out in the previous chapter, the irreducible minimal representation of the Dirac algebra consists on 4 components Dirac spinors

$$\Psi = \begin{pmatrix} \psi_1 \\ \bar{\psi}_2 \end{pmatrix} , \tag{3.3}$$

thus, after the compactification (take it trivial, for simplicity) we are left with a tower of Dirac spinors Ψ^n, hence no chiral states are present. But with a non smooth compactification we can be left with a theory describing chiral fermions, as we will show in a moment. Before that, let us define what is a non smooth compactification.

Now M will denote a connected (possibly compact) and differentiable manifold while G will be a group acting non freely on M, leaving invariant a discrete set of points, say $\mathfrak{D} \subset M$. Moreover we will assume the action of G on $M \backslash \mathfrak{D}$ (M minus the set $\mathfrak{D}$) to be properly discontinuous. First notice that this action is itself well defined since if there were $p \in M \setminus \mathfrak{D}$ and $g \in G$ such that $g\,p \in \mathfrak{D}$, then by definition there would be $g' \in G$, with $g' \neq e$, such that $g' g\, p = g\, p$ and hence $g^{-1} g' g\, p = p$, which is a contradiction with the fact that $p \notin \mathfrak{D}$ since $g^{-1} g' g \neq e$. By the same argument $G\,(\mathfrak{D}) \subset \mathfrak{D}$.

Furthermore $M \setminus \mathfrak{D}$ is open in M and thus it is a differentiable manifold, therefore by the properly discontinuous action of G, $\tilde{M} \equiv (M \setminus \mathfrak{D})\,/G$ is again a differentiable manifold. In this way we can see the space M/G as the disjoint union $\tilde{M} \cup \tilde{\mathfrak{D}}$, where $\tilde{\mathfrak{D}} \equiv \mathfrak{D}/G$ and clearly belongs to the closure of $\tilde{M}$. Thus the latter discrete set can be thought of as a set of singular points attached to a smooth space. Nevertheless this does not necessarily imply that the space M/G is singular itself in the sense that it has points where the differentiable structure breaks down, as the next two examples illustrate.

1. $M = \{z \in \mathbb{C} |\ |z| = 1\} \cong S^1$ and $G = \{Id,\, \mathfrak{C}\} \cong \mathbb{Z}_2$ with $Id(z) = z$ and $\mathfrak{C}(z) = z^*$, $\forall z \in M$. Clearly, G has $\{-1,\, 1\}$ as invariant points and its action on $S^1 \setminus \{-1,\, 1\}$ is properly discontinuous. Actually $(S^1 \setminus \{-1,\, 1\})\,/G \cong (-1,\, 1) \subset \mathbb{R}^2$, hence $S^1/G \cong [-1,\, 1]$. However, an open set in $[-1,\, 1]$ containing the point $\{1\}$ (alternatively $\{-1\}$) is of the form $(\epsilon,\, 1]$ (alternatively $[-1,\, \epsilon)$) with $|\epsilon| < 1$ which is not homeomorphic to any open interval in $\mathbb{R}$. Thus the differentiable structure is broken at the fixed points.

2. $M = \mathbb{C}$ and G the cyclic group generated by $e^{\mathrm{i}2\pi\frac{1}{n}}$ $n \geq 2$, i.e. $G \cong \mathbb{Z}_n$. The only invariant point in C under G is the origin and one can easily check that the action of G on $M \setminus \{0\}$ is properly discontinuous. Finally, it turns out that $\mathbb{C}/G$ is the upper half of a cone with deficit angle $2\pi/n$. In contrast to the previous example the cone admits a global differentiable structure since it is globally homeomorphic to $\mathbb{R}^2$.

At the level of field theories, we demand the fields to transform under the action of the orbifolding group as a symmetry of the corresponding action. More concretely,

$$f \circ g = T_g\, f\,, \tag{3.4}$$

with T some homomorphic image of G in the group of symmetry transformations. If g_0 is the generator of the orbifolding group verifying $g_0^n = e$, therefore $\tau^n \equiv T_{g_0}^n = \mathbf{1}$.

An interesting consequence gives rise when we consider the action on a fixed point, say $g\,p_0 = p_0$. In this case $f(g\,p_0) = f(p_0) = T_g f(p_0)$, thus we have a constrain on the fields at the fixed points. The operator T_g is called the parity operator. Applied to $\mathcal{M}_4 \times S^1/\mathbb{Z}_2$ orbifold and considering 5D Dirac spinors Ψ, we can take $T \equiv \gamma_5$, thus on the fixed points we have the constrain $\Psi = \gamma_5 \Psi$, i.e. a well defined chirality. In what follows we will review the process of supersymmetry breaking by the Scherk-Schwarz compactification in orbifolds. For simplicity we shall restrict to a five dimensional case.

3.2.5 Scherk-Schwarz compactification in orbifolds

Our starting point was a smooth manifold M modulated by a discrete group $\mathcal{G}$, acting freely on the former (with operators τ and represented in the field space by operators T), defining, thus, a compact smooth manifold $C = M/\mathcal{G}$. Now we introduce a second identification by another discrete group $\mathcal{H}$ acting non freely on C (with operators ζ and represented in the field space with operators Z) defining an orbifold $C/\mathcal{H}$. We can think of ζ_h and τ_g as functions acting on the covering space M, in this way we are taking $\mathcal{G}$ and $\mathcal{H}$ as subgroups of a larger discrete group $\mathcal{J}$ acting on M. In general, $g \cdot h \neq h \cdot g$ which means that $\mathcal{J} \neq \mathcal{G} \times \mathcal{H}$ (it is not the direct product). As before, if we want our theory to be defined in the orbifold $C/\mathcal{H}$ the operators R acting on field space and furnishing a representation of $\mathcal{J}$ must leave the action invariant. There are, nevertheless, constraints on the operators T and Z as we can see by analyzing the simple case of $S^1/\mathbb{Z}_2 \equiv [0, \pi R]$:
The freely acting group $\mathbb{Z}$ is the translation $\tau(y) = y + 2\pi R$ while the orbifolding group $\mathbb{Z}_2$ is the reflection with respect to the origin $\zeta(y) = -y$. It is easy to see that $\zeta \cdot \tau \cdot \zeta = \tau^{-1}$ (the group $\mathcal{J}$ is the semi direct product between $\mathbb{Z}$ and $\mathbb{Z}_2$) or equivalently $\tau \cdot \zeta \cdot \tau = \zeta$ and analogous constraints must satisfy the operators T and Z :

$$Z \cdot T \cdot Z = T^{-1} \iff T \cdot Z \cdot T = Z \tag{3.5}$$

and

$$Z^2 = 1 \tag{3.6}$$

Equality (3.6) implies that Z is diagonalizable with eigenvalues ± 1 . Since T and Z are global symmetries of the theory we can write

$$T = e^{i\vec{\beta}\vec{\lambda}} \tag{3.7}$$

With λ_i hermitian and traceless matrices. With this, the constraints (3.5) translates into:

$$\{\vec{\beta}\vec{\lambda}, Z\} = 0 \tag{3.8}$$

In the special case when $[T, Z] = 0$ the condition (3.5) reads $T^2 = 1$. The operator T is called the twisting operator while Z is called the parity operator.

3.3 Supersymmetry breaking by *Orbifolding*

We will analyze the different supersymmetry breaking mechanisms due to the existence of extra dimensions. In order to simplify the analysis we will consider the case of a five-dimensional $N = 1$ supersymmetric theory compactified in $\mathcal{M}^4 \times S^1/\mathbb{Z}_2$. Basically, we will follow the formalism used in [9]

We start with 5-d spacetime with metric $\eta_{MN} = diag\,(+1, -1, -1, -1, -1)$, $M = \mu, 5$ and Dirac matrices $\gamma^M = \left(\gamma^\mu, \gamma^{\dot{5}}\right)$ [2] with

$$\gamma^\mu = \begin{pmatrix} 0 & \sigma^\mu \\ \bar{\sigma}^\mu & 0 \end{pmatrix}, \quad \gamma^{\dot{5}} = \begin{pmatrix} -i & 0 \\ 0 & i \end{pmatrix} \tag{3.9}$$

where $\sigma^\mu = (1, \vec{\sigma})$ and $\bar{\sigma}^\mu = (1, -\vec{\sigma})$. The off-shell vector multiplet consists on

$$\left(A_M, \Sigma, \lambda^i, \vec{X}\right)^a \tag{3.10}$$

Where A_M is a five dimensional vector, Σ is a real scalar, $\vec{X} = (X_1, X_2, X_3)$ is a real auxiliary field transforming as a vector in the adjoint representation of $SU(2)_R$ and λ^i are Symplectic-Majorana spinors transforming as a doublet under $SU(2)_R$.

[2]The dot is used to distinguish it from the usual $\gamma^5 = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$.

In addition, all the fields carry an index a living in the adjoint representation of a gauge symmetry group. Symplectic-Majorana spinors[3] are defined

$$\lambda^i = \begin{pmatrix} \lambda_L^i \\ \epsilon^{ij}\bar{\lambda}_{jL} \end{pmatrix}, \quad \bar{\lambda}_{jL} \equiv -i\sigma^2 \left(\lambda_L^j\right)^* \tag{3.11}$$

where λ_L are Weyl spinors and ϵ_{ij} is the total antisymmetric 2-tensor with the convention $\epsilon^{12} = +1$

As we have seen in the previous chapter, the off-shell matter multiplet (hypermultiplet) consists on

$$\left(A^i, \psi, F^i\right) \tag{3.12}$$

Where A^i are two complex scalars transforming as a doublet under $SU(2)_R$, ψ is a Dirac fermion and F^i are two auxiliary complex scalars transforming as a doublet under $SU(2)_R$. The supersymmetric transformations in the vector multiplet are given by (2.37):

$$\begin{aligned}
\delta_\xi A^M &= i\bar{\xi}_i\gamma^M\lambda^i \\
\delta_\xi \Sigma &= i\bar{\xi}_i\lambda^i \\
\delta_\xi \lambda^i &= \left(\gamma^{MN}F_{MN} - \gamma^M D_M\Sigma\right)\xi^i - i(\vec{X}\cdot\vec{\sigma})^{ij}\xi^j \\
\delta_\xi X^a &= \bar{\xi}_i(\sigma^a)^{ij}\gamma^M D_M\lambda^j - i\left[\Sigma, \bar{\xi}_i(\sigma^a)^{ij}\lambda^j\right]
\end{aligned} \tag{3.13}$$

where all fields are Lie algebra valued, namely:

$$\mathbb{V} = g\mathbb{V}^a T^a \tag{3.14}$$

with T^a generators of the gauge symmetry group and g the gauge coupling. While the supersymmetric transformations in the matter multiplet are given by (2.36):

$$\begin{aligned}
\delta_\xi A^i &= -\sqrt{2}\epsilon^{ij}\bar{\xi}_j\psi \\
\delta_\xi \psi &= i\sqrt{2}\gamma^M D_M A^i\epsilon^{ij}\xi^j + \sqrt{2}F^i\xi^i \\
\delta_\xi F^i &= -i\sqrt{2}\bar{\xi}_i\gamma^M D_M\psi
\end{aligned} \tag{3.15}$$

[3]In five dimensions there are no dynamical Majorana spinors because the kinetic term $\bar{\psi}\gamma^M\partial_M\psi$ is a total derivative.

Both, supersymmetry parameters and supersymmetry generators are Symplectic-Majorana spinors. This supersymmetry structure is valid in flat 5-d spacetime as well as toroidal 5-d spacetime ($\mathcal{M}^4 \times S^1$) . In the case of orbifolds, $\mathcal{M}^4 \times S^1/\mathbb{Z}_2$, there are four dimensional branes at the fixed points ($y = 0$, $y = \pi R$) where supersymmetry is reduced from $N = 2$ to $N = 1$ as we will show bellow. As discussed above, due to the orbifold group a parity assignment arises on the fields:

$$\phi(x, -y) = Z_\phi \phi(x, y) \tag{3.16}$$

where $Z_\phi = \pm 1$ are the intrinsic parities. Obviously, parities must be assigned such that they leave the bulk lagrangian invariant. Fields with negative parity must vanish at fixed points but have non vanishing derivatives $\partial_5 \phi$ which can couple with fields living in the boundaries, while fields with positive parity are not forced to vanish at the boundaries. We will separately consider the cases of matter and vector multiplets.

3.3.1 Vector multiplet

We will consider here the vector multiplet (3.10) and orbifold conditions that do not break the gauge structure. The parity assignments are chosen to be those in table 3.1, where we have also included the parities of the supersymmetric parameters.

Table 3.1: Parities of the vector multiplet

	$Z = +1$	$Z = -1$
A^M	A^μ	A^5
Σ		Σ
λ^i	λ^1_L	λ^2_L
X^a	X^3	$X^{1,2}$
ξ^i	ξ^1_L	ξ^2_L

Notice that Σ is odd and so it does not couple to the wall, while $D_5\Sigma = \partial_5\Sigma$ is then even and gauge-covariant on the wall.

From table 3.1 we can see that ξ_L^1 is the parameter of the $N = 1$ supersymmetry on the wall. The supersymmetric transformations (3.13) reduce on the walls to the following transformations generated by ξ_L^1 on the even-parity states:

$$\begin{aligned}
\delta_\xi A^\mu &= i\xi_L^{1\dagger}\bar{\sigma}^\mu\lambda_L^1 - i\lambda_L^{1\dagger}\bar{\sigma}^\mu\xi_L^1 \\
\delta_\xi\lambda_L^1 &= \gamma^{\mu\nu}F_{\mu\nu}\xi_L^1 - i(X^3 - \partial_5\Sigma)\xi_L^1 \\
\delta_\xi X^3 &= \xi_L^{1\dagger}\bar{\sigma}^\mu D_\mu\lambda_L^1 - i\xi_L^{1\dagger}D_5\bar{\lambda}_L^2 + h.c. \\
\delta_\xi\partial_5\Sigma &= -i\xi_L^{1\dagger}D_5\bar{\lambda}_L^2 + h.c.
\end{aligned} \tag{3.17}$$

From the last two equations in (3.17) we have:

$$\delta_\xi(X^3 - \partial_5\Sigma) = \xi_L^{1\dagger}\bar{\sigma}^\mu D_\mu\lambda_L^1 \tag{3.18}$$

which shows that the $N = 1$ vector multiplet on the brane in the Wess-Zumino (WZ) gauge is given by (A^μ, λ_L^1, D) where the auxiliary D-field is $D = X^3 - \partial_5\Sigma$ [17, 18].

In this way the five dimensional action can be written as

$$S = \int d^5x \left\{\mathcal{L}_5 + \sum_i \delta(y - y_i)\mathcal{L}_{4i}\right\} \tag{3.19}$$

where $y_i = 0, \pi R$ in the present case. The bulk Lagrangian should be the standard one for a five dimensional super-Yang-Mills theory

$$\mathcal{L}_5 = \text{tr}\left[-\frac{1}{2}F_{MN}^2 + (D_M\Sigma)^2 + \bar{\lambda}i\gamma^M D_M\lambda + \vec{X}^2 - \bar{\lambda}[\Sigma, \lambda]\right] \tag{3.20}$$

with $\text{tr}\, t^A t^B = \delta^{AB}/2$. The boundary Lagrangian should have the standard form corresponding to a four-dimensional chiral multiplet localized on the brane, (ϕ, ψ_L, F) and coupled to the gauge $N = 1$ multiplet $(A^\mu, \lambda_L^1, X^3 - \partial_5\Sigma)$. The chiral multiplet is supposed to transform under the irreducible representation R of the gauge group and we will call t_R^A the generators of the gauge group in the corresponding representation. The brane Lagrangian is then written as

$$\begin{aligned}
\mathcal{L}_4 &= \text{tr}\left[|D_\mu\phi|^2 + \bar{\psi}_L i\bar{\sigma}^\mu D_\mu\psi_L + |F|^2\right] \\
&- ig\sqrt{2}(\lambda_L^{1A}\phi^\dagger t_R^A\psi_L + \bar{\psi}_L t_R^A\phi\bar{\lambda}_L^1) + g\,\phi^\dagger t_R^A\phi(X_3^A - \partial_5\Sigma^A)
\end{aligned} \tag{3.21}$$

The Lagrangian involving the auxiliary field X_3^A and the scalar field ϕ is

$$\int d^5x \left\{ \frac{1}{2}(X_3^A)^2 + g\,\delta(y)\phi^\dagger t_R^A \phi (X_3^A - \partial_5 \Sigma^A) \right\} \tag{3.22}$$

Integrating out the auxiliary fields X_3^A yields the boundary Lagrangian

$$-g\,\phi^\dagger t_R^A \phi \partial_5 \Sigma^A - \frac{1}{2} g^2 (\phi^\dagger t^A \phi)^2 \delta(0) \tag{3.23}$$

As we can see the formalism provides singular terms $\delta(0)$ on the boundary which arise naturally from integration of auxiliary fields. These singular terms are required by supersymmetry and they are necessary for cancellation of divergences in the supersymmetric limit. These terms can be formally understood as

$$\delta(0) = \frac{1}{\pi R} \sum_{n=-\infty}^{\infty} 1 \tag{3.24}$$

Using Eqs. (3.20) and (3.21) we can write the five dimensional Lagrangian for the Σ^A fields as

$$\begin{aligned} \mathcal{L}_5 &= -\frac{1}{2}(\partial_5 \Sigma^A)^2 - \delta(y) g\,\phi^\dagger t_R^A \phi \partial_5 \Sigma^A - \frac{1}{2} g^2 \left(\phi^\dagger t_R^A \phi\right)^2 \delta^2(y) \\ &= -\frac{1}{2}\left[\partial_5 \Sigma^A + \delta(y) g \phi^\dagger t_R^A \phi\right]^2 \end{aligned} \tag{3.25}$$

We can see that the Lagrangian (3.25) is a perfect square and the corresponding potential has a minimum at

$$\Sigma^A = -\frac{1}{2} g \epsilon(y) \phi^\dagger t_R^A \phi \tag{3.26}$$

where $\epsilon(y)$ is the sign function. We can see that if ϕ acquires a VEV, also Σ^A acquires one breaking the gauge group. The function $\Sigma^A(y)$ is an odd function and has jumps at the orbifold fixed points. This behaviour is typical of odd functions in orbifold backgrounds [19, 20, 21, 22].

3.3.2 Matter multiplet

Hypermultiplets on the walls can be treated in the same way as we have just done with vector multiplets. A hypermultiplet is defined by (A^i, ψ, F^i), where F^i is a doublet of complex auxiliary fields. A consistent set of assignments which yields $N = 1$ supersymmetry on the wall is

Table 3.2: Parities of the hypermultiplet

	$Z = +1$	$Z = -1$
A^i	A^1	A^2
ψ	ψ_L	ψ_R
F^i	F^1	F^2
ξ^i	ξ^1_L	ξ^2_L

Similarly to the vector multiplet case, supersymmetry on the wall is generated by ξ^1_L and it acts on even-parity states as

$$\begin{aligned} \delta_\xi A^1 &= \sqrt{2}\xi^1_L \psi_L \\ \delta_\xi \psi_L &= i\sqrt{2}\sigma^\mu \partial_\mu A^1 \xi^{1*}_L - \sqrt{2}\partial_5 A^2 \xi^1_L + \sqrt{2} F^1 \xi^1_L \\ \delta_\xi F^1 &= i\sqrt{2}\xi^{1\dagger}_L \bar{\sigma}^\mu \partial_\mu \psi_L + \sqrt{2}\xi^{1\dagger}_L \partial_5 \psi_R \\ \delta_\xi \partial_5 A^2 &= \sqrt{2}\xi^{1\dagger}_L \partial_5 \psi_R \end{aligned} \tag{3.27}$$

Putting together the last two equations of (3.27) leads to

$$\delta_\xi (F^1 - \partial_5 A^2) = i\sqrt{2}\xi^{1\dagger}_L \bar{\sigma}^\mu \partial_\mu \psi_L \tag{3.28}$$

which shows that $\mathbb{A} = (A^1, \psi_L, F^1 - \partial_5 A^2)$ transforms as an off-shell chiral multiplet on the boundary. Notice that, as it happened with the case of the vector multiplet, the auxiliary field of a chiral $N = 1$ multiplet on the brane does contain the ∂_5 of an odd field.

We can now write the coupling of the bulk hypermultiplet to chiral superfields $\Phi_0 = (\phi_0, \psi_0, F_0)$ localized on the brane through a superpotential W that depends on ϕ_0 and the boundary value of the scalar field A^1,

$$W = W(\Phi_0, \mathbb{A}) \tag{3.29}$$

The five dimensional action can then be written as in Eq. (3.19) with a bulk Lagrangian

$$\mathcal{L}_5 = |\partial_M A^i|^2 + i\bar{\psi}\gamma^M\partial_M\psi + |F^i|^2 \tag{3.30}$$

and a brane Lagrangian

$$\mathcal{L}_4 = (F^1 - \partial_5 A^2)\frac{dW}{dA^1} + h.c. \tag{3.31}$$

Integrating out the auxiliary field F^1 yields

$$\bar{F}^1 = -\delta(y)\frac{dW}{dA^1} \tag{3.32}$$

and replacing it into the Lagrangian (3.30) and (3.31) gives an action

$$\begin{aligned} S &= \int d^5x \left\{ |\partial_M A^i|^2 + i\bar{\psi}\gamma^M\partial_M\psi \right. \\ &- \left. \delta(y)\left[\left(\partial_5 A^2\frac{dW}{dA^1} + h.c.\right) + \delta(y)\left|\frac{dW}{dA^1}\right|^2\right]\right\} \end{aligned} \tag{3.33}$$

where we again find a singular coupling $\delta(0)$ as required by supersymmetry. Collecting in (3.33) the terms where A^2 appears we get a potential

$$V = \left|\partial_5 A^2 + \delta(y)\frac{dW}{dA^1}\right|^2 \tag{3.34}$$

that is a perfect square and is then minimized for

$$A^2 = -\frac{1}{2}\epsilon(y)\frac{dW}{dA^1} \tag{3.35}$$

Then if supersymmetry is spontaneously broken in the brane, i. e. if

$$\left\langle\frac{dW}{dA^1}\right\rangle \neq 0$$

then A^2 acquires a VEV. This behaviour is reminiscent of a similar one in the Horava-Witten theory [23, 24, 25] in the presence of a gaugino condensation.

3.4 Supersymmetry breaking by Scherk-Schwarz compactification

One could think of breaking supersymmetry by a ”super Higgs” effect via some Goldstone spinor contained in a matter supermultiplet (maximum helicity 1/2). This method, however, can not be general [26] since for extended supersymmetric theories with $N \geq 3$ there is no supermultiplet with helicity less than 1. As an alternative, consider an $N = 1$ five dimensional supersymmetric theory. As shown in the previous chapter, this corresponds to $N = 2$ four dimensional supersymmetry whose filed content consists (for the matter multiplet) in one dirac fermion, Ψ, and two complex scalars, Φ^i, transforming as a doublet under the internal $SU(2)$ group of automorphisms. Now the compactification, $\mathcal{M}_4 \times \mathbb{R} \to \mathcal{M}_4 \times S^1$, is carried out at the level of the fields as

$$\Psi(x, y) = \Psi(x, y + 2\pi R)\,, \tag{3.36}$$

$$\Phi^i(x, y + 2\pi R) = \left\{e^{\mathrm{i}2\pi R\vec{\omega}\cdot\vec{\sigma}}\right\}^{ij} \Phi^j(x, y)\,, \tag{3.37}$$

$$x \in \mathcal{M}_4\,, \qquad y \in \mathbb{R}\,,$$

with $\vec{\sigma} \in \mathfrak{su}(2)$, $\vec{\omega}$ a constant vector and R the S^1 radius. Then by redefining the fields as

$$\Phi^i(x, y) \equiv \left\{e^{\mathrm{i}y\,\vec{\omega}\cdot\vec{\sigma}}\right\}^{ij} \varphi^j(x, y)\,, \tag{3.38}$$

with $\varphi^i(x, y + 2\pi R) = \varphi^i(x, y)$ we can spontaneously generate different masses for scalars and fermions, breaking thus the supersymmetry. This breaking is spontaneous since the supersymmetric algebra

$$\{Q_i, Q_j\} = \epsilon_{ij}\gamma^M C\, P_M + \epsilon_{ij} C\, Z\,, \tag{3.39}$$

survives the dimensional reduction promoting P_5 to a central charge with respect to the 4D Lorentz-Poicaré group. In the original work by Scherk and Schwarz the non

trivial compactification is applied to the $N = 1$ 4D supergravity multiplet to reduce the theory to $N = 2$ 3D. As it is shown, the fourth component of the gravitino, ψ_3, serves as the Goldstone spinor.
Let us explicitly analyze the effect of Scherk-Schwarz compactification in the cases of matter and vector multiplets of an $N = 1$ five dimensional supersymmetric theory. As before we compactify the theory in the orbifold.

3.4.1 Bulk breaking

In the previous section we saw how compactification in the orbifold breaks half of the supersymmetry at the branes. Now we will analyze how Scherk-Schwarz compactification breaks the supersymmetry in the bulk:
Consider the 5-d on-shell (free) lagrangians (3.20) and (3.30)

$$\mathcal{L}_5 = \mathrm{tr}\left[-\frac{1}{2}F^2_{MN} + (D_M\Sigma)^2 + \bar{\lambda}i\gamma^M D_M\lambda - \bar{\lambda}[\Sigma,\lambda]\right] \tag{3.40}$$

$$\mathcal{L}_5 = |\partial_M H^i|^2 + i\bar{\psi}\gamma^M\partial_M\psi \tag{3.41}$$

With

$$\mathbb{H} = \left(H^i, \psi\right) \tag{3.42}$$

the on-shell matter multiplet and

$$\mathbb{V} = \left(A_M, \Sigma, \lambda^i\right) \tag{3.43}$$

the on-shell vector multiplet. Recall that only H^i and λ^i transform as doublets under $SU(2)_R$, the rest of the fields are singlets.
The parity assignments are given by:

$$\left.\begin{array}{ll} H(x,-y) & = P\ H(x,y) \\ \psi(x,-y) & = i\gamma^{\dot{5}}\psi(x,y) \\ A_M(x,-y) & = T_M A_M(x,y) \\ \lambda(x,-y) & = Q\otimes i\gamma^{\dot{5}}\ \lambda(x,y) \end{array}\right\} \tag{3.44}$$

where P and Q act on $SU(2)_R$ indices, $\gamma^{\dot{5}}$ acts on spinor indices and

$$T_M = \begin{cases} +1 & M=\mu \\ -1 & M=5 \end{cases}$$

Notice that (3.44) implies constraints at fixed points: $\left\{ \begin{array}{c} H = P\ H \\ \psi = i\gamma^{\dot{5}}\psi \\ A_M = T_M A_M \\ \lambda = Q \otimes i\gamma^{\dot{5}}\ \lambda \end{array} \right.$ at $y = 0, \pi R$ that work as boundary conditions. The SS-twists are given by:

$$H(x, y + 2\pi R) = e^{i2\pi\vec{\omega}\cdot\vec{\sigma}}\ H(x, y) \tag{3.45}$$

$$\lambda(x, y + 2\pi R) = e^{i2\pi\vec{\eta}\cdot\vec{\sigma}}\ \lambda(x, y) \tag{3.46}$$

where $\vec{\omega}$ and $\vec{\eta}$ are two real valued 3-vectors. As we have discussed in section 3.2.5

$$\{P, \vec{\omega}\cdot\vec{\sigma}\} = 0 \tag{3.47}$$

$$\{Q, \vec{\eta}\cdot\vec{\sigma}\} = 0 \tag{3.48}$$

Trivial solutions to (3.45) are:

$$H(x, y) = e^{i\vec{\omega}\cdot\vec{\sigma}\frac{y}{R}}\ \phi(x, y) \tag{3.49}$$

$$\lambda(x, y) = e^{i\vec{\eta}\cdot\vec{\sigma}\frac{y}{R}}\ \xi(x, y) \tag{3.50}$$

where $\phi(x, y)$ and $\xi(x, y)$ are periodic functions with respect to y. Notice that due to constraints (3.47) and (3.48) ϕ and ξ have the same parity transformations that H and λ , respectively. Therefore, we can expand the fields as

$$\left. \begin{array}{ll} \phi(x, y) & = \sum_n \left(P_+ \cos(n\frac{y}{R}) \ + \ P_- \sin(n\frac{y}{R})\right)\ \phi_n(x) \\ \xi(x, y) & = \sum_n \left(\widetilde{Q}_+ \cos(n\frac{y}{R}) \ + \ \widetilde{Q}_- \sin(n\frac{y}{R})\right)\ \xi_n(x) \\ \psi(x, y) & = \sum_n \left(\Gamma_+ \cos(n\frac{y}{R}) \ + \ \Gamma_- \sin(n\frac{y}{R})\right)\ \psi_n(x) \\ A_M(x, y) & = \sum_n \left(T_{M+} \cos(n\frac{y}{R}) \ + \ T_{M-} \sin(n\frac{y}{R})\right)\ A_M^{(n)}(x) \end{array} \right\} \tag{3.51}$$

Where we have defined

$$\left. \begin{array}{ll} P_\pm & = (1 \pm P)/2 \\ \widetilde{Q}_\pm & = (1 \pm Q \otimes \gamma^5)/2 \\ \Gamma_\pm & = (1 \pm \gamma^5)/2 \\ T_{M\pm} & = (1 \pm T_M)/2 \end{array} \right\} \tag{3.52}$$

In the case of global $SU(2)_R$ symmetry, solutions (3.49) break the symmetry with the fifth component of their kinetic terms. In the case of local $SU(2)_R$ symmetry [4] (in section 3.5 we will see how can this be performed) we have a gauge field $\vec{B}_M$. If $\vec{B}_M$ acquires a non-vanishing VEV such that $\left\langle \vec{B}_5 \right\rangle = \frac{\vec{\omega}}{R}$ all non-singlet fields acquire a mass shift. Consider the actions with the lagrangians (3.40) and (3.41) with the VEV acquired by the gauge field (we consider $SU(2)_R$ doublet fields only)

$$\int_0^{\pi R} dy \int d^4x \; (\mathcal{D}_M\phi)^\dagger \mathcal{D}^M\phi \; + \; i\bar{\xi}\gamma^M\mathcal{D}_M\xi \tag{3.53}$$

with $\mathcal{D}_M = \partial_M + i\frac{\vec{\omega}}{R}\vec{\sigma}\;\delta_{M5}$. If we expand the covariant derivatives and plug (3.51) in, we find

$$\begin{aligned}\int d^4x \sum_n \left\{\partial^\mu\phi_n^\dagger\partial_\mu\phi_n \; - \; \frac{1}{R^2}\phi_n^\dagger\left(n^2+\omega^2+2in(P_+-P_-)\vec{\omega}\vec{\sigma}\right)\phi\right\} \\ + \sum_n \left\{i\bar{\xi}_n\gamma^\mu\partial_\mu\xi_n \; + \; \frac{1}{R}\bar{\xi}_n\left(\widetilde{Q}_- - \widetilde{Q}_+ + i\vec{\omega}\vec{\sigma}\right)\gamma^5\xi_n\right\}\end{aligned} \tag{3.54}$$

While the $SU(2)_R$ singlet fields split into a Kaluza-Klein tower of modes with mass

$$m_n = \frac{n}{R}$$

$SU(2)_R$ doublet fields acquire masses which are the eigenvalues of their corresponding mass matrix breaking, thus, supersymmetry.
The $N = 1$ supersymmetry of the brane lagrangian (3.21) is broken by radiative corrections through the coupling with gauginos [27, 28].

3.5 Hosotani breaking mechanism

In this section we will show the equivalence between Hosotani breaking and Scherk-Schwarz compactification mechanism [5]. In the five dimensional formulation of local

[4] Now the redefinitions (3.49) are absorbed by means of local invariance.

[5] In five dimensions it is always possible to resolve the Scherk-Schwarz compactification as a Hosotani breaking. A general discussion of the issue for arbitrary number of dimensions can be found in [29] .

supersymmetry the $SU(2)_R$ global symmetry is promoted to a local symmetry mediated by an auxiliary gauge field [30, 31, 32, 33, 34, 35, 36, 37]. We will use the formulation of Ref. [30] where two multiplets are necessary to formulate the five dimensional off-shell supergravity: the minimal supergravity multiplet $(40_B + 40_F)$ and the tensor multiplet $(8_B + 8_F)$. Their parities are summarized in Tables 3.3 and 3.4. Only graviton, gravitino and graviphoton are physical fields, the rest, including those in Table 3.4, are auxiliary fields. The $SU(2)_R$ gauge fixing is done by fixing the compensator field [30]

$$\vec{Y} = e^u \begin{pmatrix} 0 \\ 1 \\ 0 \end{pmatrix} \tag{3.55}$$

that breaks $SU(2)_R \to U(1)_R = \{\sigma^2\}$. The invariant Lagrangian $\mathcal{L}_{grav} = \mathcal{L}_{minimal} +$

Table 3.3: Minimal supergravity multiplet

Field		$Z = +1$	$Z = -1$
g_{MN}	graviton	$g_{\mu\nu}$, g_{55}	$g_{\mu 5}$
ψ_M	gravitino	$\psi^1_{\mu L}$, ψ^2_{5L}	$\psi^2_{\mu L}$, ψ^1_{5L}
B_M	graviphoton	B_5	B_μ
$\vec{V}_M$	$SU(2)_R$-gauge	V^3_μ, $V^{1,2}_5$	V^3_5, $V^{1,2}_\mu$
v^{MN}	antisymmetric	$v^{\mu 5}$	$v^{\mu\nu}$
$\vec{t}$	$SU(2)_R$-triplet	$t^{1,2}$	t^3
C	real scalar	C	
ζ	$SU(2)_R$-doublet	ζ^1_L	ζ^2_L

$\mathcal{L}_{tensor}$ contains the term $(1-e^u)C$ and then the equation of motion of C yields $u = 0$. The relevant terms in $\mathcal{L}_{grav}$ containing these fields are [30]

$$\begin{aligned} \mathcal{L}_{grav} &= -\frac{i}{2}\bar{\psi}_P\gamma^{PMN}\mathcal{D}_M\psi_N - \frac{1}{12}\epsilon^{MNPQR}V^2_M\partial_N B_{PQR} \\ &+ (V^1_5)^2 - 12(t^1)^2 - 48(t^2)^2 - 12N\,t^2 - N^2 \end{aligned} \tag{3.56}$$

Table 3.4: Tensor multiplet

Field	$Z=+1$	$Z=-1$
$\vec{Y}$	$Y^{1,2}$	Y^3
B_{MNP}	$B_{\mu\nu\rho}$	$B_{\mu\nu 5}$
N	N	
ρ	ρ_L^1	ρ_L^2

where γ^{PMN} is the normalized antisymmetric product of gamma matrices,

$$\mathcal{D}_M = D_M + i\sigma^2 V_M^2 \tag{3.57}$$

and D_M is the covariant derivative with respect to local Lorentz transformations. The field equations for the auxiliary fields yield

$$V_5^1 = N = t^1 = t^2 = 0 \tag{3.58}$$

while the field equation for the 3-form tensor B_{MNP} gives

$$\partial_{[M} V_{N]}^2 = 0 \Longrightarrow V_M^2 = \partial_M K \Longrightarrow \begin{cases} V_\mu^2 = 0 \text{ (odd field)} \\ V_5^2 = \text{constant (even field)} \end{cases} \tag{3.59}$$

where K is an odd field and the last implication is suggested by the simplest choice

$$K = y\,\frac{\omega}{R} \tag{3.60}$$

which leads to the background

$$V_5^2 = \frac{\omega}{R} \tag{3.61}$$

and makes the connection between Hosotani and Scherk-Schwarz pictures. Notice that by allowing the fields to have non trivial twist conditions under compactification group we recover the Scherk-Schwarz solutions.

Once we have summarized the basic mechanisms of supersymmetry breaking, we will aply them in the following chapters in order to obtain some phenomenological results concerning the Standard Model.

Chapter 4

Supersymmetry in the Orbifold

One of the key points of the Standard Model is the Higgs mechanism of symmetry breaking [11, 38, 39]. As we saw in section 2.2 it consists on the spontaneous breaking of a symmetry via the VEV acquired by a (scalar) field, non-singlet under the symmetry group. This VEV is acquired spontaneously as the configuration that minimizes the effective action [1] [41]. In the framework of the Standard Model it is used to break $SU(2)_L$ gauge symmetry giving masses to both, the SM matter particles (quarks and leptons) and the gauge bosons of weak interaction ($W^{\pm}$ and Z^0). Nevertheless, the nature of the Higgs particle is quite mysterious, since as far as the Standard Model is concerned, it is the only matter field which is a Lorentz scalar. Because of that, it is worth investigating supersymmetry breaking to induce Higgs mechanism; after all, supersymmetry provides the same number of fermionic and bosonic degrees of freedom, and therefore it has scalar matter fields which could play the role of the Higgs particle.

In this chapter we will present some approaches to the electroweak symmetry breaking from a supersymmetric theory in five dimensions.

[1] It could happen that a symmetry is not broken at tree level but breaks spontaneously with quantum contributions à la Coleman-Weinberg [40].

4.1 Approach to the MSSM from supersymmetry in the orbifold

The starting point is an $N = 1$ supersymmetric five dimensional theory built up with on-shell matter and vector multiplets analogous to (3.12) and (3.10), respectively [2]:

$$(V_M, \lambda_i, \Sigma) \tag{4.1}$$

$$(\psi, H_i)^a \tag{4.2}$$

With $\psi = (\psi_L, \psi_R)^T$, $\lambda^i = \left(\lambda_L^i, \epsilon^{ij}\ i\sigma_2 \lambda_L^{*j}\right)^T$ and ψ_L , ψ_R and λ_L Weyl spinors.
a is an index of an extra $SU(2)_H$ group in its fundamental representation. The vector multiplet is Lie algebra valued as in (3.14) and the two scalar doublets of the matter multiplet will be associated to the two Higgs doublets of the MSSM. The whole on-shell lagrangian is given by [42]:

$$\begin{aligned}
\mathcal{L}_5 &= \frac{1}{g^2}\mathrm{tr}\left\{-\frac{1}{2}F_{MN}^2 + (D_M\Sigma)^2 + i\bar{\lambda}_i\gamma^M D_M\lambda^i - \bar{\lambda}_i[\Sigma,\lambda^i]\right\} \\
&+ |D_M H_i^a|^2 + \bar{\psi}_a(i\gamma^M D_M - \Sigma)\psi^a - (i\sqrt{2}H_i^{a\dagger}\bar{\lambda}_i\psi^a + h.c.) \\
&- H_i^{a\dagger}\Sigma^2 H_i^a - \frac{g^2}{2}\left(H_i^{a\dagger}\vec{\sigma}_i^j T^A H_j^a\right)^2 \\
&+ \mathcal{L}_4
\end{aligned} \tag{4.3}$$

Where $D_M = \partial_M - iV_M$ is the covariant derivative with respect to the gauge symmetry group.
$\mathcal{L}_4$ is a brane lagrangian like (3.21) built up with chiral matter and coupled to vector multiplets as explained in section 3.3.1. In this case the vector multiplet lives in the adjoint representation of $SU(3) \times SU(2) \times U(1)$ gauge group while the Higgs multiplet lives in the fundamental representation.

[2]We will follow the formalism introduced in [42] (from now on, let us work in units of R (the radius of compactification), therefore, by y we will understand: yR and any mass m should be understood as: $\frac{m}{R}$).

The parity assignment is analogous to (3.44)

$$\left.\begin{array}{ll} H(x,-y) & = \sigma_3 \otimes \sigma_3 \; H(x,y) \\ \psi(x,-y) & = \sigma_3 \otimes i\gamma^5 \psi(x,y) \\ A_M(x,-y) & = P_M A_M(x,y) \\ \lambda(x,-y) & = \sigma_3 \otimes i\gamma^5 \; \lambda(x,y) \end{array}\right\} \quad (4.4)$$

Where, obviously, each σ acts on either $SU(2)_R$ or $SU(2)_H$ indices, depending on the case. The SS twist is chosen to be

$$\left.\begin{array}{ll} H(x,y+2\pi n) & = e^{i2\pi n\omega\sigma_2} \otimes e^{i2\pi n\tilde{\omega}\sigma_2} \; H(x,y) \\ \lambda(x,y+2\pi n) & = e^{i2\pi n\omega\sigma_2} \; \lambda(x,y) \\ \psi(x,y) & = e^{i2\pi n\tilde{\omega}\sigma_2} \; \psi(x,y) \end{array}\right\} \quad (4.5)$$

Taking into account the redefinitions (3.49) and proceeding like in section 3.4 we find a 4-d mass lagrangian [42]

$$\begin{aligned} M_4 \;=\; & \sum_{n\neq 0} \Bigg\{ \begin{pmatrix} \lambda_L^{1(n)} & \lambda_L^{2(n)} \end{pmatrix} \begin{pmatrix} \omega & n \\ n & \omega \end{pmatrix} \begin{pmatrix} \lambda_L^{1(n)} \\ \lambda_L^{2(n)} \end{pmatrix} \\ & + \begin{pmatrix} \bar{\psi}_L^{1(n)} & \bar{\psi}_L^{2(n)} \end{pmatrix} \begin{pmatrix} n & -\tilde{\omega} \\ \tilde{\omega} & n \end{pmatrix} \begin{pmatrix} \psi_R^{1(n)} \\ \psi_R^{2(n)} \end{pmatrix} \;+\; h.c. \\ & - H^\dagger \begin{pmatrix} n^2+\omega_-^2 & -2in\omega_- & & \\ 2in\omega_- & n^2+\omega_-^2 & & \\ & & n^2+\omega_+^2 & -2n\omega_+ \\ & & 2n\omega_+ & n^2+\omega_+^2 \end{pmatrix} H \Bigg\} \\ & + \omega\lambda_L^{1(0)}\lambda_L^{1(0)} \;+\; \tilde{\omega}\bar{\psi}_L^{2(0)}\psi_R^{1(0)} \;+\; h.c. \\ & - \omega_-^2|H_0^{(0)}|^2 \;+\; \omega_+^2|H_3^{(0)}|^2 \end{aligned} \quad (4.6)$$

with $H = \begin{pmatrix} H_0^{(n)} \\ H_1^{(n)} \\ H_2^{(n)} \\ H_3^{(n)} \end{pmatrix}$, $\omega_\pm = \tilde{\omega} \pm \omega$ and $H_i^a = H_\mu(\sigma^\mu)_i^a$, $\sigma^\mu = (1,\vec{\sigma})$.

If we diagonalize mass matrices from (4.6) for $n \neq 0$, we have:

- Two Majorana fermions: $\lambda_L^{1(n)} \pm \lambda_L^{2(n)}$, with masses: $|n \pm \omega|$, respectively,
- Two Dirac fermions: $\psi^{1(n)} \pm \psi^{2(n)}$, with masses: $|n \pm \tilde{\omega}|$, respectively and
- Four scalars: $H_0^{(n)} \pm iH_2^{(n)}$ and $H_1^{(n)} \pm H_3^{(n)}$ with masses: $|n \pm \omega_-|$ and $|n \pm \omega_+|$, respectively.

While for $n = 0$ we have

- One Majorana fermion: $\lambda_L^{1(0)}$, with mass: $|\omega|$,
- One Dirac fermion: $\begin{pmatrix} \psi_L^{2(0)} \\ \psi_R^{1(0)} \end{pmatrix}$, with mass: $|\tilde{\omega}|$ and
- Two scalars: $H_0^{(0)}$ and $H_3^{(0)}$, with masses: $|\omega_-|$ and $|\omega_+|$, respectively.

The rest of the fields are singlets and thus, acquire the usual KK spectrum.
We could have massless scalars if either $\omega_- = 0$ or $\omega_+ = 0$ is satisfied. Thus, after SS compactification we could reduce the model to SM with one or two massless Higgs doublets, but there is a problem that will be clear with the help of the next particular example:
Consider $\omega_- = 0$ and $\omega_+ \neq 0$ and compute the self interacting quartic term from (4.3)

$$\begin{aligned} \mathcal{L}_D &= 8g^2 \sum_A \Big\{ \Big(Re\{H_0^\dagger T^A H_1\} + Im\{H_0^\dagger T^A H_2\} \Big)^2 \\ &+ \Big(Im\{H_3^\dagger T^A H_1\} - Re\{H_0^\dagger T^A H_2\} \Big)^2 \Big\} \end{aligned} \tag{4.7}$$

All the scalar fields except $H_0^{(0)}$ are massive, which means that $\langle H \rangle = 0$ for them, but as we can see from (4.7) there is no self interacting quartic term for H_0 [3], in fact, there is no potential for $H_0^{(0)}$ because H_1 and H_2 are odd fields, and hence we can not reproduce the Higgs mechanism. Notice that setting $\omega_- = n$ is totally

[3]By radiative corrections we could shift the massless Higgs to a tachyon, which is a necessary condition for the scalar to acquire a VEV if there is a self interacting quartic coupling.

equivalent to the previous case. It is easy to see from (4.5) that a shift in either ω or $\tilde{\omega}$ by an integer quantity leaves the conditions totally invariant. Radiative corrections will provide a small quartic coupling that will yield a Higgs mass below present experimental bounds.

4.2 Supersymmetric bulk mass

A useful technique to localize dynamically the fields on the branes is to introduce a supersymmetric and odd bulk mass [43, 44]. We will deal with a model containing one irreducible matter multiplet. Nevertheless, as we will see, it is not appropriate to describe the MSSM.
Consider the lagrangian (3.30) with an odd bulk mass $\mathcal{M}$

$$\begin{aligned} S &= \int d^5x \ \left\{ |\partial_M A^i|^2 + i\bar{\psi}\gamma^M\partial_M\psi + |G^i|^2 \right. \\ &+ \left. \mathcal{M}\ \bar{\psi}\psi - \mathcal{M}\ \bar{G}^i A_i - \mathcal{M}\ \bar{A}^i G_i \right\} \end{aligned} \tag{4.8}$$

Where we have defined

$$G^i = \epsilon^{ij}F^j$$

Notice that with that redefinition G transfoms under $SU(2)_R$ like A . We think of $\mathcal{M}$ as a periodic step function of the fifth coordinate:

$$\mathcal{M}(y) = M\epsilon(y) \text{ with } \epsilon(y) = \begin{cases} 1 & \pi > y \geq 0 \\ -1 & -\pi \leq y < 0 \end{cases} \text{ with periodicity } 2\pi$$

Then, the supersymmetric variation of (4.8) under the transformations (3.15) gives [44]:

$$\delta_\xi S = \int d^5x \ \left(-i\sqrt{2}M\partial_5\epsilon(y)\ \bar{\psi}\gamma^{\dot{5}}A^i\xi_i + h.c. \right) \tag{4.9}$$

where $\partial_5\epsilon(y) = 2(\delta(y) - \delta(y-\pi))$. In order to cancel the variation (4.9) we add to (4.8)

$$\int d^5x \ - 2M(\delta(y) - \delta(y-\pi))\bar{A}PA \tag{4.10}$$

with P the parity operator. Now, the whole supersymmetry variation gives:

$$\delta_\xi S = \int d^5x \ 2\sqrt{2}M(\delta(y) - \delta(y-\pi)) \ \left\{ -i\bar{\psi}\gamma^{\dot{5}}A^i\xi_i - \bar{\psi}\xi PA + h.c. \right\} \tag{4.11}$$

and taking into account the parity constraints at fixed points: $A = PA$ and $-i\bar{\psi}\gamma^{\dot{5}} = \bar{\psi}$ at $y = 0, \pi$, (4.11) cancels exactly and thus, the on-shell action

$$\begin{aligned} S &= \int d^5x \; \{|\partial_M A^i|^2 + i\bar{\psi}\gamma^M\partial_M\psi - M^2|A^i|^2 \\ &+ M\epsilon(y)\bar{\psi}\psi - 2M(\delta(y) - \delta(y-\pi))\bar{A}PA\} \end{aligned} \tag{4.12}$$

is invariant under (3.15). Consider, so, the Hosotani mechanism with parity operator $P = \sigma_3$. We can then redefine the fields like in (3.49) with a twist operator $T = \sigma_2$ obtaining thus:

$$\begin{aligned} S &= \int d^5x \; \{|\partial_M A^i|^2 + i\bar{\psi}\gamma^M\partial_M\psi - M^2|A^i|^2 \\ &+ M\epsilon(y)\bar{\psi}\psi - 2M\delta(y)\bar{A}P_0A + 2M\delta(y-\pi)\bar{A}P_\pi A\} \end{aligned} \tag{4.13}$$

with $P_0 = P$ and $P_\pi = e^{-i\pi\omega\sigma_2}\; P\; e^{i\pi\omega\sigma_2}$. Computing the variation of (4.13) we find:

$$\partial^M\partial_M A + M\;\left(M + 2\delta(y)P_0 - 2\delta(y-\pi)P_\pi\right)\;A = 0 \tag{4.14}$$

$$i\gamma^M\partial_M\psi - M\epsilon(y)\psi = 0 \tag{4.15}$$

Integrating (4.14) around $y = 0$ and $y = \pi$ and considering the parity constraints, we find the following boundary conditions:

$$(1 - P_f)\; A|_{y=y_f} = 0 \tag{4.16}$$

$$(1 + P_f)\; (A' - MA)|_{y=y_f} = 0 \tag{4.17}$$

with $f = 0, \pi$, $A' = \partial_5 A$ and we have used $(1 + P_f)\; P_f = 1 + P_f$. While for the fermions the boundary conditions read:

$$(1 - i\gamma^{\dot{5}})\;\psi|_{y=0,\pi} = 0 \tag{4.18}$$

It is convenient, for reasons that will become clear below, to obtain an equation for ψ' at the boundaries using the fermionic equation of motion on top of (4.18) :

$$(1 + i\gamma^{\dot{5}})\;\left(\psi' - M\psi\right)|_{y=0,\pi} = 0 \tag{4.19}$$

Consider, now, the stability of the boundary conditions under supersymmetry, obviously when $\omega = 0$:
First of all, let us make the following definitions:

$$A_\pm = \frac{1}{2}(1 \mp \sigma_3)\ A \tag{4.20}$$

$$\psi_\pm = \frac{1}{2}(1 \mp i\gamma^{\dot{5}})\ \psi \tag{4.21}$$

$$\xi_\pm = \frac{1}{2}(1 \mp i\gamma^{\dot{5}}\sigma_3)\ \xi \tag{4.22}$$

$$\tag{4.23}$$

With these definitions, the boundary conditions can be expressed:

$$A_+ = 0 \tag{4.24}$$

$$A'_- - MA'_- = 0 \tag{4.25}$$

$$\psi_+ = 0 \tag{4.26}$$

Where we have eliminated the subscript f for simplicity. Consistency of the boundary conditions with supersymmetry requires:

$$\delta_\xi\ A_+ = 0 \tag{4.27}$$

$$\delta_\xi\ A'_- - M\delta_\xi\ A'_- = 0 \tag{4.28}$$

$$\delta_\xi\ \psi_+ = 0 \tag{4.29}$$

It is easy to see that supersymmetric variations (3.15) of $A_\pm$ and $\psi_\pm$ are given by:

$$\delta_\xi\ A_\pm = -\sqrt{2}\ \left(\bar{\xi}_\pm\psi_+ + \bar{\xi}_\mp\psi_-\right) \tag{4.30}$$

$$\begin{aligned} \delta_\xi\ \psi_\pm &= i\sqrt{2}\left\{\gamma^\mu\ (\xi_\pm\partial_\mu A_+ + \xi_\mp\partial_\mu A_-) + \gamma^{\dot{5}}\ \left(\xi_\mp A'_+ + \xi_\pm A'_-\right)\right\} \\ &+ \sqrt{2}M\ (\xi_\mp A_+ + \xi_\pm A_-) \end{aligned} \tag{4.31}$$

(4.27) implies $\xi_- = 0$. With this constraint (4.28) is satisfied taking into account (4.19) and, finally, (4.29) is as well satisfied with the necessity of no other constraint.

The constraint on ξ tells us nothing but the fact that supersymmetry is broken by the process of orbifolding as we could see in section 3.3.
If we express the solution of the bulk equations of motion as

$$A^i(x,y) = A(x)\phi^i(y)$$

$$\psi(x,y) = \begin{pmatrix} \chi(x)f(y) \\ \bar{\xi}(x)g(y) \end{pmatrix}$$

with $A(x)$ and $\begin{pmatrix} \chi(x) \\ \bar{\xi}(x) \end{pmatrix}$ solutions of 4-d Klein-Gordon and Dirac equations, respectively, with a mass m each 5-d bulk equation reads:

$$\phi'' + (m^2 - M^2)\phi \;=\; 0 \tag{4.32}$$

$$F' - i(m\ \sigma_2 + iM\ \sigma_3)F \;=\; 0 \tag{4.33}$$

with $F = \begin{pmatrix} f(y) \\ g(y) \end{pmatrix}$. The solutions of (4.32) and (4.33) are given by:

$$\phi(y) \;=\; a\cos\Omega y \;+\; b\sin\Omega y \tag{4.34}$$

$$F(y) \;=\; e^{i(iM\ \sigma_3 + m\ \sigma_2)y}\ F_0 \tag{4.35}$$

where $\Omega = \sqrt{m^2 - M^2}$. With this prescription (4.18) reads:

$$(1-\sigma_3)\ F\big|_{y=0,\pi} = 0 \tag{4.36}$$

By enforcing (4.34) to be consistent with boundary conditions (4.16) and (4.17), we find out that for $\Omega \neq 0$

$$a \;=\; P_+\varphi \tag{4.37}$$

$$\Omega b \;=\; MPa + P_-\varphi \tag{4.38}$$

and

$$\begin{aligned} 0 \;=\; & \left\{P_- e^{i\omega\pi\sigma_2}\left(P_+C_\Omega + \left(\frac{1}{\Omega}(P_- + MPP_+)\right)S_\Omega\right)\right. \\ & + \; P_+e^{i\omega\pi\sigma_2}\left[(P_- + MPP_+)\,C_\Omega - \Omega P_+ S_\Omega\right] \\ & - \; \left. MPe^{i\omega\pi\sigma_2}\left[P_+C_\Omega + \left(\frac{1}{\Omega}(P_- + MPP_+)\right)S_\Omega\right]\right\}\ \varphi \end{aligned} \tag{4.39}$$

Where we have defined C_Ω $(S_\Omega) = \cos\Omega\pi$ $(\sin\Omega\pi)$, $P_\pm = (1 \pm P)/2$ and $\varphi \in \mathbb{C}^2$. If we want a non trivial solution the determinant of the matrix in (4.39) must vanish. After a tedious calculation, that requirement leads to:

$$\sin^2\omega\pi = \frac{m^2}{\Omega^2}\sin^2\Omega\pi \tag{4.40}$$

For $\Omega = 0$ the consistency relation is given by the limit of (4.39) when $\Omega \to 0$ and thus a relation between M and ω is required. For that reason there is no consistent solution for that case. Let us, nevertheless, analyze some interesting solutions to (4.40):

- $|M| \to 0$ In this case $\Omega \sim m$ and (4.40) can be written as $\sin^2\omega\pi \simeq \sin^2 m\pi$ and therefore $m \simeq \omega + n$ which is the usual orbifold spectrum.

- $|M| \to \infty$ In this case $\Omega \sim iM$ and from (4.40) we find $m^2 \simeq \frac{M^2\sin^2\omega\pi}{\sinh^2 M\pi}$ which means that the physical mass is exponentially suppressed. By solving (4.39) and normalizing the solution we find

  $$\phi(y) \sim \mathcal{O}(1)\begin{pmatrix} e^{M(y-\pi)} \\ -\tan\omega\pi\ (e^{M(y-\pi)} - e^{-M(y+\pi)}) \end{pmatrix}$$

  for $M > 0$ and

  $$\phi(y) \sim \mathcal{O}(1)\begin{pmatrix} e^{-|M|(y+\pi)} \\ \tan\omega\pi\ (e^{|M|(y-\pi)} - e^{-|M|(y+\pi)}) \end{pmatrix}$$

  for $M < 0$. As we can see, component 1 of ϕ is exponentially localized at $y = \pi$ or $y = 0$ depending on whether $M > 0$ or $M < 0$, respectively, while the component 2 is always exponentially localized at $y = \pi$.

For the case of fermions (4.36) tells us:

$$P_- \mathcal{V}_\pi P_+\ \Phi = 0 \tag{4.41}$$

where $P_\pm = (1 \pm \sigma_3)/2$, $F_0 = P_+\ \Phi \in \mathbb{C}^2$ and $\mathcal{V}_\pi = e^{(M\sigma_3 - im\sigma_2)\pi}$. (4.41) is satisfied for $\Phi \neq 0$ for every value of Ω. On the other hand, $rank(P_-\mathcal{V}_\pi P_+) \leq 1$

because $rank(P_-) = rank(P_+) = 1$, nevertheless, we want $\Phi \notin Ker(P_+)$ but if $rank(P_-\mathcal{V}_\pi P_+) = 1$ then $Ker(P_-\mathcal{V}_\pi P_+) = Ker(P_+)$ and the solution is trivial. The condition for non trivial solution is, therefore, $P_-\mathcal{V}_\pi P_+ = 0$. This condition gives us the spectrum:

$$m_f^2 = \begin{cases} n^2 + M^2 & n \neq 0 \\ 0 & n = 0 \end{cases} \tag{4.42}$$

which coincides with (4.40) when $\omega = 0$. This fact manifests the compatibility of the boundary conditions with supersymmetry.

In any case, we are left with no massless Higss, except the limiting case above discussed. But in that case we have only one scalar degree of freedom localized at $y = 0$.

Since localizing dynamically the fields on the branes is equivalent to put mass matrices on fixed points, in the following section we will change the point of view and develop a theory in the interval instead of orbifold with masses localized on the branes.

Chapter 5

Supersymmetry in the Interval

5.1 Supersymmetry with boundary terms

Along this chapter we will develop a supersymmetric theory in the interval. First we will use a component field formalism instead of superfields. For that aim we shall not use complex hypermultiplets [45], instead we change to a formalism of "real" hypermultiplets (the formalism used, for instance, in [32]), that is: we make the hypermultiplets to lie in the fundamental representation of an extra $SU(2)_H$ group and impose a reality contidtion respecting the new symmetry. In this formalism, the boundary conditions corresponding to odd bulk masses in the orbifold case, (4.16)-(4.17), are easier to reproduce. Furthermore, within this model we find an interesting source of supersymmetry breaking (mass boundary terms). In the next chapter we shall rewrite the model in terms of superfields. We will consider the hypermultiplet

$$\mathbb{H}^\alpha = (\Psi, \Phi_i, F_i)^\alpha, \tag{5.1}$$

where i is an $SU(2)_R$ index and α is an $SU(2)_H$ index. The reality constraint for the scalar as well as the auxiliary field is:

$$(\bar{\Phi})^i_\alpha \equiv (\Phi^\alpha_i)^* = \epsilon^{ij}\epsilon_{\alpha\beta}\Phi^\beta_j \tag{5.2}$$

While for the fermion the reality constraint is a symplectic Majorana condition:

$$\bar{\Psi}_\alpha \equiv (\Psi_\alpha)^\dagger\gamma^0 = \epsilon_{\alpha\beta}(\Psi^\beta)^T C\,, \tag{5.3}$$

We will consider the total action $\mathcal{S}=\mathcal{S}_{\rm bk}+\mathcal{S}_{\rm bd}$ as the sum of a bulk ($\mathcal{S}_{\rm bk}=\mathcal{S}^0_{\rm bk}+\mathcal{S}^{\rm m}_{\rm bk}$) and a boundary ($\mathcal{S}_{\rm bd}$) term, as

$$\mathcal{S}^0_{\rm bk}=\int_{\mathcal{M}}\left(-\frac{1}{2}\bar{\Phi}\,\partial^2\Phi+\frac{i}{2}\bar{\Psi}\gamma^M\partial_M\Psi+2\bar{F}F\right) \tag{5.4}$$

$$\mathcal{S}^{\rm m}_{\rm bk}=\int_{\mathcal{M}}\left(2i\bar{F}\mathcal{M}\Phi+\frac{1}{2}\bar{\Psi}\mathcal{M}\Psi\right) \tag{5.5}$$

$$\mathcal{S}_{\rm bd}=\int_{\partial\mathcal{M}}\left(\frac{1}{4}\bar{\Psi}S\Psi+\frac{1}{4}(\bar{\Phi}R\Phi)'+\frac{1}{4}\bar{\Phi}N(-1+R)\Phi\right) \tag{5.6}$$

We take $\mathcal{M}$, S and R as hermitian matrices in the $SU(2)_H$ and $SU(2)_R\times SU(2)_H$ indices while N is a real number [1]. We choose $R=T\otimes S$ wtih T a hermitian matrix in $SU(2)_R$ because is a natural choice and it makes sense with supersymmetric transformation laws. The reality of the whole action (5.4)-(5.6) imposes a restriction on the above matrices [44]:

$$\mathcal{M}^\dagger=\mathcal{M},\qquad \mathcal{M}^T=-\sigma^2\mathcal{M}\sigma^2 \tag{5.7}$$

$$S^\dagger=S,\qquad S^T=-\sigma^2S\sigma^2 \tag{5.8}$$

$$T^\dagger=T,\qquad T^T=-\sigma^2T\sigma^2 \tag{5.9}$$

The solution to these constraints are:

$$\mathcal{M}=M\,\vec{p}\cdot\vec{\sigma},\qquad S=\vec{s}\cdot\vec{\sigma},\qquad T=\vec{t}\cdot\vec{\sigma} \tag{5.10}$$

Where $\vec{s}$, $\vec{p}$ and $\vec{t}$ are real and dimensionless vectors ($\vec{p}$ is a unit vector), and M is a mass parameter. First of all, let us check the supersymmetry invariance of the action. The supersymmetric variation of (5.4)-(5.6) can be written as [44]:

$$\delta_\epsilon\mathcal{S}_{\rm bk}=\int_{\partial\mathcal{M}}\left(-\bar{F}\bar{\epsilon}(i\gamma^5)\Psi+\frac{1}{2}\bar{\Phi}\bar{\epsilon}\gamma^\mu\partial_\mu(i\gamma^5)\Psi+\frac{i}{2}\bar{\Phi}\bar{\epsilon}\Psi'-\bar{\epsilon}\bar{\Phi}\mathcal{M}\gamma^5\Psi\right) \tag{5.11}$$

[1] Of course S, $\mathcal{R}$ and N can take different values at the two boundaries. The subindices 0 and π have been omitted for simplicity.

$$\delta_\epsilon \mathcal{S}_{\rm bd} = \int_{\partial\mathcal{M}} \left(-\frac{1}{2}\bar{\epsilon}\gamma^\mu \partial_\mu \bar{\Phi} S \Psi - \frac{1}{2}\bar{\epsilon}\gamma^5 \bar{\Phi}' S \Psi \right.$$
$$\left. + \bar{\epsilon}\bar{F} S\Psi + \frac{i}{2}\bar{\Phi}' R \bar{\epsilon}\Psi + \frac{i}{2}\bar{\Phi} R \bar{\epsilon}\Psi' + \frac{i}{2} N \bar{\Phi}(-1+R)\bar{\epsilon}\Psi \right) \tag{5.12}$$

Discarding a total 4D derivative we can rewrite the sum of (5.11) and (5.12) as

$$\delta_\epsilon \mathcal{S} = \int_{\partial\mathcal{M}} \left(\frac{i}{2}\bar{\epsilon}\bar{\Phi}(1+R)\Psi' + \frac{i}{2}(\bar{\Phi}' + N\bar{\Phi})(-1+R)\bar{\epsilon}\Psi \right.$$
$$\left. -\frac{1}{2}\bar{\epsilon}\gamma^5\bar{\Phi}\mathcal{M}(1+i\gamma^5 S)\Psi - i\bar{\epsilon}\gamma^5 \left(\bar{F} - \frac{i}{2}\bar{\Phi}\mathcal{M} \right)(1 - i\gamma^5 S)\Psi \right) \tag{5.13}$$

Using the BC's, (5.15), (5.16) and (5.17) the first three terms vanish. Finally we use the EOM for F

$$F = -\frac{i}{2}\mathcal{M}\Phi, \qquad \bar{F} = \frac{i}{2}\bar{\Phi}\mathcal{M} \tag{5.14}$$

to deduce that the whole variation is zero.
We can deduce the boundary conditions (BC's) by aplying the variational method to the action (5.4)-(5.6) taking into account the boundary terms. The resulting BC's are:

$$\left(1 + i\gamma^5 S\right)\Psi = 0, \qquad \bar{\Psi}\left(1 - i\gamma^5 S\right) = 0 \tag{5.15}$$

$$(1+R)\,\Phi = 0, \qquad \bar{\Phi}\,(1+R) = 0 \tag{5.16}$$

$$(-1+R)\left[\Phi' + N\Phi\right] = 0, \qquad \left[\bar{\Phi}' + N\bar{\Phi}\right](-1+R) = 0 \tag{5.17}$$

The action is supersymmetric upon the use of BC and the equations of motion for the auxiliary fields, hence the supersymmetric of the action requires the stability of the boundary conditions. To see this consider a second order supersymmetric transformation, that is, if $\Xi' = \Xi + \delta_\epsilon \Xi$ then take $\Xi'' = \Xi + \delta_\eta \Xi'$ and suppose that the boundary conditions are not stable, thus if $BC[\Xi] = 0$ is the functional that gives the boundary conditions then, in general, $BC[\Xi + \delta_\epsilon \Xi] = BC[\delta_\epsilon \Xi] \neq 0$ and since the variation of the action under the second supersymmetry transformation is

$\sim \int_{\partial\mathcal{M}} BC[\Xi']$ it will not cancel. Therefore, proceeding like we did in section 4.2 we find that the BC's (5.15)-(5.17) are compatible with supersymmetry if:

$$T_0 = T_\pi \vec{p}\cdot\vec{s}_f = \frac{N_f}{M} \tag{5.18}$$

Let us now find the general spectrum. First of all notice that (5.15)-(5.17) overdetermine the system unless the 8×8 matrix

$$\left(\begin{array}{cc} 0 & 1+R \\ -1+R & N(-1+R) \end{array}\right). \tag{5.19}$$

has null determinant. This translates into $(1-|\vec{s}_f|\,|\vec{t}_f|)^4 = 0$ which vanishes if $\vec{s}$ and $\vec{t}$ are unit vectors. If we solve the equations of motion imposing the BC's (5.15)-(5.17) (similarly as we proceeded in section 4.2) we find the equations for the mass spectrum of bosons and fermions [44]:

$$A_+ A_- = 0$$

with

$$A_\pm = s_\pm^2 - \frac{N_0 - N_\pi}{\Omega}\tan(\Omega\pi) - \left[c_\pm^2 + \frac{N_0\,N_\pi}{\Omega^2}\right]\tan^2(\Omega\pi) \tag{5.20}$$

for bosons and

$$1 - \tilde{c} - 2\,(c_0 - c_\pi)\frac{M}{\Omega}\tan(\Omega\pi) - \left[1 + \tilde{c} + 2\,c_0\,c_\pi\frac{M^2}{\Omega^2}\right]\tan^2(\Omega\pi) = 0 \tag{5.21}$$

for fermions.
Where we have made use of the following definitions:

$$s_\pm = \sin[\pi(\omega \pm \tilde{\omega})] \qquad c_\pm = \cos[\pi(\omega \pm \tilde{\omega})] \qquad \Omega^2 = m^2 - M^2$$

and

$$\vec{s}_0\cdot\vec{s}_\pi = \cos(2\pi\tilde{\omega}) \qquad \vec{t}_0\cdot\vec{t}_\pi = \cos(2\pi\omega) \qquad \tilde{c} = \cos(2\pi\tilde{\omega}) \qquad c_f \equiv \cos(2\pi\alpha_f) = \vec{p}\cdot\vec{s}_f$$

m being the physical mass. Notice that ω is a Scherk-Schwarz-like parameter[2]. On the other hand, it is straightforward to see that if $\omega = 0$ and $c_f = \frac{N_f}{M}$ (5.20) and (5.21) are the same expression, confirming thus (5.18). Some interesting particular cases are:
$N_f = M = 0$. The bosonic spectrum (5.20) gives $m_n = n \pm \omega \pm \tilde{\omega}$ while the fermionic condition (5.21) yields $m_n = n \pm \tilde{\omega}$ which is in agreement with the model studied in Refs.[27, 42].
For the case $\tilde{\omega} = 0$, $c_0 = c_\pi = 1$ and $N_f = M$ the bosonic spectrum is given by

$$\sin^2(\pi\omega) = \frac{\Omega^2 + M^2}{\Omega^2} \sin^2(\Omega\pi) \tag{5.22}$$

while the fermionic spectrum is: $m_n^2 = n^2 + M^2(1 - \delta_{n0})$ in agreement with the results found in [46].
Another interesting case appears when $\omega = \tilde{\omega} = 1/2$, i.e.,
$c_0 = -c_\pi$ and $N_0 = N_\pi = N$ [3]. The fermionic spectrum (5.21) reduces to:

$$[1 - c_0 \frac{M}{\Omega} \tan(\Omega\pi)]^2 = 0 \tag{5.23}$$

While (5.20) reads:

$$(\Omega^2 + N^2) \frac{\tan^2(\Omega\pi)}{\Omega^2} = 0 \tag{5.24}$$

From (5.23), setting $c_0 = 0$ we find $m_n^2 = M^2 + (n+1/2)^2$. For $c_0 = 1$ there is a light Dirac fermion when $MR \gg 1$: $m \simeq 2Me^{-\pi M}$ (which is exponentially localized on the brane). Notice that the modes n and $-n-1$ may be paired to furnish a Dirac fermion with mass $M^2 + (n+1/2)^2$ which could prevent the $4-D$ effective theory from being anomalous if one would like to identify the zero mode of the hyperscalar field with the Higgs doublet of the Standard Model [4].

[2] In agreement with (5.18), ω does not spoil supersymmetry if it takes integer values.
[3] In that case the supersymmetry is broken by both, SS-like parameter and boundary masses.
[4] We will consider such situation in section 5.2.

On the other hand, (5.24) yields the following bosonic spectrum:

$$\begin{aligned} m_n^2 &= n^2 + M^2 \quad n \neq 0 \\ m_0^2 &= M^2 - N^2 \end{aligned} \tag{5.25}$$

Notice that if $M < N$ the bosonic spectrum presents a tachyon. The last particular case we would like to show explicitly is when $\omega = \tilde{\omega} = 1/2$, $M = 0$, $N_0 = 0$ and $N_\pi = -N \neq 0$. From (5.20)-(5.21) we see that, as before, the Higgsinos are vector-like and their (Dirac) mass spectrum is given by: $m_n = n + 1/2$ $(n \geq 0)$. While the bosonic spectrum is given by: $m_n = n$ $(n > 0)$ and for $N > 0$ we have a tachyonic zero mode $m_0^2 = -m^2$ with m the solution of the equation:

$$m \; \tanh(m\pi) = N \tag{5.26}$$

We would like to finish this section by sheding light on the general conditions that the parameters of our model should satisfy to yield such spectra. First of all notice that the model we are presenting depends on seven parameters: ω, $\tilde{\omega}$, M, N_f and α_f. As we have seen ω acts as a Scherk-Schwarz breaking parameter, $\tilde{\omega}$ accounts for a supersymmetric shift of the mass and the rest of the parameters can break or preserve supersymmetry depending on whether

$$\frac{N_f}{M} - \cos(2\pi\alpha_f) = 0$$

is satisfied or not. These seven parameters are not completely free because the angles between the vectors $\vec{s_0}$, $\vec{s_\pi}$ and $\vec{p}$ are subject to triangular inequalities that lead to the constraint:

$$\frac{(c_0 + c_\pi)^2}{\cos^2(\pi\tilde{\omega})} + \frac{(c_0 - c_\pi)^2}{\sin^2(\pi\tilde{\omega})} \leq 4 \tag{5.27}$$

Which defines an elliptical disk. On the other hand the condition that N_f should satisfy to allow an exacly massless mode in the bosonic spectrum can be easily read off from (5.20):

$$(n_0 - \tau^{-1}) \; (n_\pi + \tau^{-1}) = \cos^2[\pi(\omega \pm \tilde{\omega})] \; (1 - \tau^{-2}) \tag{5.28}$$

where $n_f = \frac{N_f}{M}$ and $\tau = \tanh(M\pi)$. The hyperbola (5.28) divides the (n_0,n_π) plane into diferent regions depending on whether a tachyonic mass mode is allowed or not. In figure 5.1 we show two particular cases in which the elliptic disk of allowed values of (c_0,c_π) is plotted as well. Note that for $\omega = 0$ the elliptic disk can not overlap

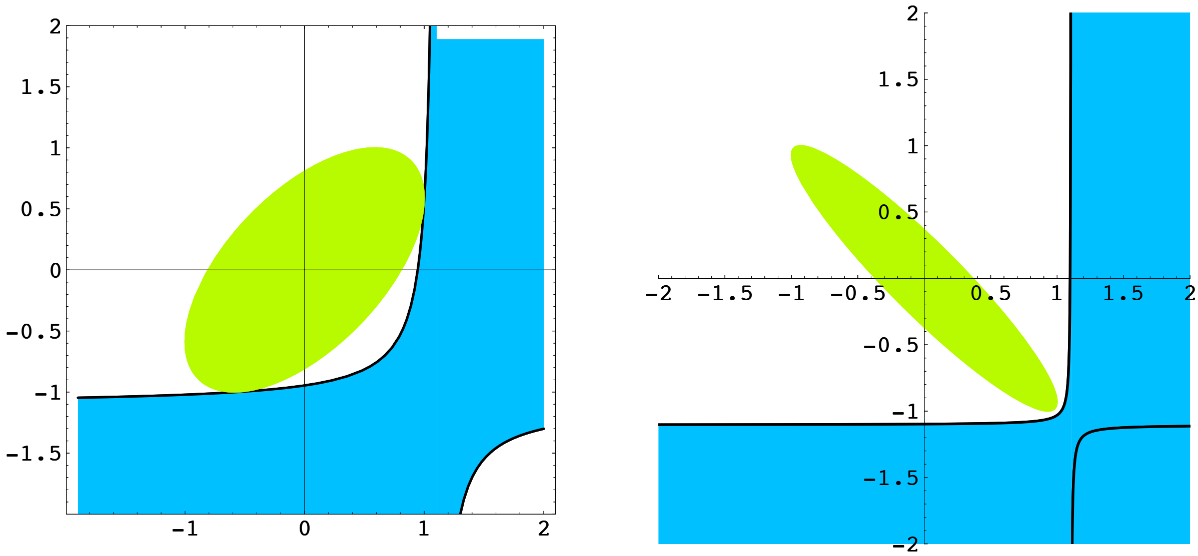

Figure 5.1: *The hyperbola of massless modes for* $\pi MR = 1.5$, $\omega = 0$ *and* $\tilde{\omega} = 0.15$ *(left panel) and* $\tilde{\omega} = 0.44$ *(right panel) in the plane* (n_0, n_π). *The clear region to the upper left has no tachyonic modes, the darkly shaded (blue) region between to the two branches has one tachyonic eigenvalue, and the region to the lower right has two tachyonic modes. The ellipse corresponds to the allowed points in the plane* (c_0, c_π). *The two dots mark the points where the fermions are massless.*

with the shaded region, otherwise the fermionic spectrum will present a tachyon. Nevertheless, for some values of M and $\tilde{\omega}$ there are two points where the ellipse is tangent the hyperbola (5.28) allowing the fermionic spectrum to have massless modes (This is the case in the left panel of fig. 5.1). They are given by [44]:

$$\begin{pmatrix} c_0 \\ c_\pi \end{pmatrix} = \tau^{-1} \begin{pmatrix} \sin^2 \pi\tilde{\omega} \pm \cos \pi\tilde{\omega}\sqrt{\tau^2 - \sin^2 \pi\tilde{\omega}} \\ -\sin^2 \pi\tilde{\omega} \pm \cos \pi\tilde{\omega}\sqrt{\tau^2 - \sin^2 \pi\tilde{\omega}} \end{pmatrix} \tag{5.29}$$

and are obviously constrained to

$$\tau^2 \geq \sin^2 \pi\tilde{\omega} \, . \tag{5.30}$$

The right panel shows a case where hyperfermions do not have massless modes (there is no intersection between the ellipse and the hyperbola), there the supersymmetric spectra ($n_f = c_f$) is not massless anymore.

5.2 Induced ElectroWeak symmetry breaking

In this section we will present a realistic model of electroweak symmetry breaking induced from supersymmetry breaking by boundary terms. The nature of that breaking will be discussed [44].
Of course, if we want to reproduce EW symmetry breaking we should arrange an $SU(2) \times U(1)$ gauge symmetry group to be broken, but here it is a little objection with our formalism. We can not introduce the symmetry by hand because the hypermultiplet can not transform under $SU(2)$ in its fundamental representation due to the reality constraints (5.2)-(5.3) [5]. Instead, we generalize the formalism introduced at the end of the previous section by doubling the number of fields with an index of a new $SU(2)$ group which will be labeled in a suggestive manner: $SU(2)_L$. The reality condition (5.2) is now written as:

$$\bar{\Phi}^i_\alpha = \epsilon^{ij} \rho_{\alpha\beta} \Phi^\beta_j \tag{5.31}$$

where the tensor $\rho_{\alpha\beta}$ can be written in the form [47]

$$\rho = \mathrm{diag}(\epsilon \oplus \epsilon) = \mathbf{1} \otimes \epsilon \quad \text{or} \quad \rho_{\alpha\beta} = \delta_{\alpha_1\beta_1} \epsilon_{\alpha_2\beta_2} \tag{5.32}$$

In particular the reality condition for hyperscalars $\Phi^\alpha_i = \Phi^{\alpha_1,\alpha_2}_i$ is given by

$$\Phi^{\alpha_1,2}_2 = (\Phi^{\alpha_1,1}_1)^* \equiv \bar{\Phi}^1_{\alpha_1,1}, \quad \Phi^{\alpha_1,1}_2 = -(\Phi^{\alpha_1,2}_1)^* \equiv -\bar{\Phi}^1_{\alpha_1,2} \tag{5.33}$$

It is now easy to see that the generators of the symmetry group that preserve the reality constraint must satisfy

$$\rho T^A = -T^{A*} \rho. \tag{5.34}$$

[5] Which is the case of symplectic-Majorana fermions.

The largest possible symmetry group is thus generated by

$$\{\sigma_2 \otimes \mathbf{1},\ \sigma_1 \otimes \sigma_i,\ \sigma_3 \otimes \sigma_i,\ \mathbf{1} \otimes \sigma_i\} \tag{5.35}$$

which is the spinor representation of $SO(5)$. As we will see the BC's will however break this to a subgroup and so does a nonzero mass term in the bulk. The reality constraints on boundary matrices S and $\mathcal{M}$ now read:

$$S^T \rho = -\rho S \qquad \mathcal{M}^T \rho = -\rho \mathcal{M} \tag{5.36}$$

We expect the biggest unbroken subgroup if we choose $S_0 \propto S_\pi \propto \mathcal{M}$, in fact all such choices are equivalent and leave an $SU(2) \times U(1)$ unbroken subgroup. A convenient choice is $S_f \propto \mathbf{1} \otimes \sigma^3$ which leaves the unbroken generators:

$$\{\sigma_2 \otimes \mathbf{1},\ \sigma_1 \otimes \sigma_3,\ \sigma_3 \otimes \sigma_3,\ \mathbf{1} \otimes \sigma_3\} \tag{5.37}$$

The formal proof for the mass eigenstates of bosons and fermions is equivalent to the previous section. As before, the parameters ω, $\tilde{\omega}$ and α_f do make sense now [6]. Let us follow with the case $\omega = \tilde{\omega} = 1/2$ and $N_0 = M = 0$, $N_\pi = -N$. The spectrum for bosons is (5.26), while the fermionic one is given by $m_n = n + 1/2$ $(n \geq 0)$. Solving the boundary conditions for bosons (5.16)-(5.17) we find the eigenstate for the tachyonic mode (5.26) [44]:

$$\begin{pmatrix} \Phi_1^{\alpha_1,1}(x,y) \\ \Phi_2^{\alpha_1,2}(x,y) \end{pmatrix} = \mathcal{N}^{-1} \begin{pmatrix} \cosh(my)\, H^{\alpha_1}(x) \\ \cosh(my)\, [H^{\alpha_1}(x)]^* \end{pmatrix} \tag{5.38}$$

all other components vanishing. Here $H(x)$ is the 4D physical Higgs field and Φ fulfills the BC's with $S_0 = -S_\pi = \mathbf{1} \otimes \sigma_3$ and $T_0 = -T_\pi = -\sigma_3$,

$$\frac{1}{2}(1 + R_f)\Phi(x, y_f) = 0, \qquad R_f = -\sigma^3 \otimes \mathbf{1} \otimes \sigma^3 \tag{5.39}$$

The normalization factor is determined to be

$$\mathcal{N}^2 = \frac{\pi}{2} \left[1 + \frac{\sinh(2\pi m)}{2\pi m}\right]$$

[6] We could have chosen $\mathcal{M} = \mathbf{1} \otimes \vec{p} \cdot \vec{\sigma}$ and $S_f = \mathbf{1} \otimes \vec{s}_f \cdot \vec{\sigma}$, of course, we would not be left with the desired unbroken symmetry, but it is explicitly clear that the parameters $\tilde{\omega}$ and α_f make sense. The case we are considering is obtained when $\tilde{\omega}$,$\alpha_f \in \{0, 1/2\}$.

Notice that $SU(2)_L \otimes U(1)_Y$ acts on the physical Higgs field H in the standard way, i.e. by the generators [7] $\{\frac{1}{2}\sigma^i, \frac{1}{2}\}$. The effective 4D theory is obtained by integrating over the extra dimension. The mass Lagrangian becomes

$$\mathcal{L}_m = m^2 |H|^2 \tag{5.40}$$

The self-coupling quartic term comes out from the integration of $\vec{X}$ auxiliary field form the super-Yang-Mills action (4.3):

$$\mathcal{L}_D = -\frac{1}{8}\, g_A^2\, \left(\bar{\Phi}\; \vec{\sigma}_R \otimes T^A\; \Phi\right)^2 . \tag{5.41}$$

Next we particularize (5.41) to the zero mode Higgs doublet [8] of Eq. (5.38). We get the Lagrangian [9]

$$\mathcal{L}_D = -\frac{1}{8}\left(g_5^2 + g_5'^2\right)|H|^4\, \frac{\cosh^4(my)}{\mathcal{N}^4} \tag{5.42}$$

Putting together Eqs. (5.40) and (5.42), expanding the neutral component of the Higgs doublet as $H^0 = h/\sqrt{2} + i\chi^0$ (where h is the normalized Higgs field with a vacuum expectation value $\langle h \rangle = v = 246$ GeV) and integrating over the fifth dimension we obtain for the Higgs field the tree-level potential

$$V = -\frac{1}{2} m^2\, h^2 + \frac{1}{32}\left(g^2 + g'^2\right)\, \kappa(\pi m R)\, h^4 \tag{5.43}$$

where g and g' are the corresponding 4D gauge couplings [10] and $\kappa(\pi m R)$ defined by

$$\kappa(x) = \frac{12x^2 + 8x\ \sinh(2x) + x\ \sinh(4x)}{2\ [2x + \sinh(2x)]^2} \tag{5.44}$$

[7] We normalize the generators to $\mathrm{tr}\{T^A T^B\} = \frac{1}{2}\delta^{AB}$.

[8] We can assume here that non-zero modes with masses controlled by $1/R \simeq$ few TeV are much larger than the weak scale and they have been integrated out.

[9] For the $SU(2)_L \otimes U(1)_Y$ group with 5D gauge couplings g_5 and g_5'.

[10] 4D and 5D gauge couplings g_4 and g_5 are related to each other as $g_5^2 = \pi R g_4^2$.

Fixing the minimum of the potential to the physical value v one finds the tree-level Higgs mass as a function of the Z-boson mass m_Z

$$m_H^2 = \kappa(\pi m R)\, m_Z^2,$$
$$N^2 - M^2 = \frac{1}{2} m_H^2 \tag{5.45}$$

From the electroweak breaking condition (5.45) and the expansion $\kappa(x) = 1 + 4x^4/45 + \ldots$ we can see that $m \simeq m_H \simeq m_Z$ and for values of the compactification radius $1/R \sim$ few TeV $m\pi R \ll 1$. In this region the mass eigenvalue of Eq. (5.26) can be solved analytically as

$$m^2 \simeq \frac{N}{\pi R}, \tag{5.46}$$

which, along with Eq. (5.45) allows to fix the value of the mass parameter N required by the electroweak breaking condition to

$$N \simeq \frac{\pi}{2} m_Z^2 R \tag{5.47}$$

The electroweak breaking and the Higgs mass we have presented are both at tree level. The electroweak breaking condition (5.47) should be modified by radiative corrections. In particular from those arising from the Yukawa couplings between Higgs and the localized matter sector. On the other hand, the Higgs mass receives one-loop corrections controlled by top-quark mass and (logarithmically) by the mass and mixing angle of the third generation of squarks. Furtheremore, in order to avoid higher sensitivity of the Higgs mass on the cutoff Λ we should demand the condition tr$Y = 0$ to be fullfilled, otherwise a FI term could be generated [3]. The radiative corrections commented above are model-dependent nevertheless we postpone its analysis until chapter 7, where they shall be exhaustively studied within a mildly modified framework. For the moment, however, let us consider the radiative corrections to the Higgs mass comming from the degrees of freedom propagating in the bulk within the present model.

5.3 Stability of the Higgs mass

The Scherk-Schwarz supersymmetry breaking is known to be one-loop finite. Nevertheless here we encounter another source of supersymmetry breaking: the boundary masses N_f. We now want to analyze the stability of the Higgs mass under radiative corrections in the presence of such breaking. For that purpose let us return to the toy model described at the beginning of this chapter. Let ω, $\tilde{\omega}$, N_0, N_π and M take values such that supersymmetry is respected (or at most it is broken by SS parameter only). We will call M_f the departure of N_f from its "supersymmetric" value. We should solve the equations of motion for the "supersymmetric" values of the parameters and treat M_f as a perturbation. Therefore we must calculate one loop diagrams with one or more M_f insertions [11]. For simplicity we can take $\omega = \tilde{\omega} = 1/2$ and $N_f = M = 0$. The spectrum is $m_n = n$ and the eigenstates can be read off from (5.20):

$$
\begin{aligned}
H_1 &= \frac{1}{\sqrt{2\pi}} H_1^{(0)}(x) + \frac{1}{\sqrt{\pi}} \sum_{n=1}^{\infty} \cos ny \; H_1^{(n)}(x) \\
H_2 &= \frac{1}{\sqrt{\pi}} \sum_{n=1}^{\infty} \sin ny \; H_1^{(n)}(x)
\end{aligned}
\tag{5.48}
$$

where we have defined:

$$\Phi_1^1 = H_1 \ , \ \Phi_1^2 = H_2$$

$$\Phi_2^2 = \bar{H}_1 \ , \ \Phi_1^2 = -\bar{H}_2$$

The quartic self-coupling potential is now given by:

$$V_D = \frac{1}{8} g_5^2 (|H_1|^2 + |H_2|^2)^2 \tag{5.49}$$

[11] The zero M_f insertion diagrams might be quadratically divergent due to a generation of a Fayet-Iliopoulus term [48]. Nevertheless it can be seen as a renormalization of the bulk mass, clearly separable from the renormalization of the boundary masses. Of course such divergence could be avoided with a second Higgs which does not interfere in the Electroweak symmetry breaking.

Using (5.48) we can write the lagrangian (5.6) as [44]:

$$\mathcal{L}_{bd} = \frac{1}{\pi} \sum_{m,n=-\infty}^{\infty} \left[M_0 - (-1)^{m+n} M_\pi\right] \, \bar{H}_1^{(m)} H_1^{(n)}] \tag{5.50}$$

The renormalization of the boundary masses is given by loops induced by the quartic potential with M_f insertions. The leading contribution comes from the one M_f insertion diagram and therefore we will concentrate on that kind of loops (see figure 5.2):

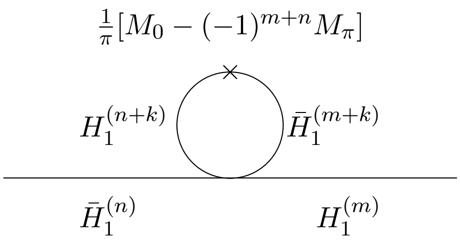

Figure 5.2: One-loop diagram renormalizing M_f.

The contribution from the diagrams in Fig. 5.2 is proportional to the factor

$$I = \frac{1}{\pi} \left[M_0 - (-1)^{m+n} M_\pi\right] g^2 \mathcal{J} \tag{5.51}$$

where $g = g_5/\sqrt{\pi}$ is the 4D gauge coupling, $\mathcal{J}$ is given by the Feynman integral

$$\begin{aligned} \mathcal{J} &= \sum_{k=-\infty}^{\infty} \int \frac{d^4p}{(2\pi)^4} \frac{1}{p^2 + (k+m)^2} \frac{1}{p^2 + (k+n)^2} \\ &= \sum_{\ell=-\infty}^{\infty} \int \frac{d^4p \, dz}{(2\pi)^4} \frac{1}{p^2 + (z+m)^2} \frac{1}{p^2 + (z+n)^2} e^{2i\pi\ell z} \end{aligned} \tag{5.52}$$

and we have made use of Poisson resummation.

The propagators in (5.52) have poles in the complex z-plane at locations $z = -n \pm ip$ and $z = -m \pm ip$. In this way for $\ell \neq 0$ the z-integrations contour can be closed by an infinite semicircle. Picking the residues of the corresponding poles provides the factor

$$e^{-2\pi|\ell|p}$$

that makes the integrand in the remaining integral to exponentially converge in the limit $p \to \infty$ and the corresponding integral to be finite. However for $\ell = 0$ there appears a linear divergence. In fact one can write

$$\begin{aligned} \mathcal{J} &= \int \frac{d^4p\, dz}{(2\pi)^4} \frac{1}{p^2 + (z+m)^2} \frac{1}{p^2 + (z+n)^2} + \text{finite terms} \\ &= \frac{1}{64\pi} \Lambda + \text{finite terms} \end{aligned} \tag{5.53}$$

where Λ is the ultraviolet (UV) cutoff. One can interpret the result in (5.53) as a linear renormalization of the brane mass terms as

$$N_f = M_f\,(1 + \Delta), \quad \Delta = \frac{g^2}{64\pi} \Lambda R + \cdots \tag{5.54}$$

Notice that to leading order the radiative corrections to the boundary mass terms Δ are boundary independent. Therefore the condition $M_0 = M_\pi$ is not spoiled by the (leading) correction in (5.54).

Now that we have the loop-corrected localized soft masses one can go back to (5.20) and recalculate the renormalized bosonic spectrum. In fact for the model under consideration ($N_0 = N_\pi = N$) the bosonic zero mode is a tachyon with a mass [see Eq. (5.24)]

$$m_0^2 = -N^2(1 + \Delta)^2 \; . \tag{5.55}$$

As we can see this breaking is soft from the point of view that it does not induce any cubic counterterm in the 5D theory. However the mass term renormalizes linearly on the boundary, which induces in turn a linear renormalization in the Higgs mass. However this sensitivity does not destabilizes the Higgs mass for values of the cutoff $\Lambda R \lesssim 10^2$: in fact considering for simplicity the weak coupling, $g^2/64\pi \sim 2 \times 10^{-3}$

and $\Delta \lesssim 0.2$. Finally, in models with a single Higgs the quadratically divergent FI term is the dominant effect and we would require a lower cutoff ($\Lambda R \lesssim 10$) to keep this effect small. In fact the quadratically divergent FI term can be avoided if we introduce several (e.g. two) Higgs hypermultiplets with charges satisfying the condition $\mathrm{tr}\mathcal{Q} = 0$, while the linear divergence can be cancelled if the condition $\mathrm{tr}\mathcal{Q}M_f = 0$ is fulfilled for them.

Finally, let us examine the transmission of supersymmetry breaking to the matter sector through loop effects. The sensitivity on the UV cutoff crucially depends on where matter lives.
If it is localized on a brane where supersymmetry breaking occurs we expect to generate local soft mass terms for squarks, and by simple power counting these must scale as $g_5^2\Lambda^2 M_f|\tilde{Q}|^2\delta(y-y_f)$. If matter lives in the bulk a similar dimensional analysis gives $g_5^2\Lambda M_f|\tilde{Q}|^2\delta(y-y_f)$, while if localization of matter and supersymmetry breaking occur at different branes, no local mass terms are generated and we instead obtain finite (nonlocal) soft masses which are insensitive to the cutoff. Again on dimensional grounds, these effective 4D soft terms scale as $g_5^2 R^{-2} M_f|\tilde{Q}|^2$. In the latter case, which we have examined in detail in section 5.2, these are in fact subdominant to the finite contribution from the SS breaking, $g_5^2 R^{-3}|\tilde{Q}|^2$, as $M_f \ll R^{-1}$ in order to decouple the Higgs scale from the compactification radius.

5.4 Comparison with the Orbifold approach

In order to compare the previous formalism with the more usual orbifold approach, and to also shed light on the nature of the previously considered supersymmetry breaking, we show in this section that the same physical theory can be obtained if one considers the orbifold $S^1/\mathbb{Z}_2$. We assign the following parities to the fields

$$\Psi(-y) = i\gamma^5\sigma_3\Psi(y)\,, \qquad \bar{\Psi}(-y) = -\bar{\Psi}(y)i\gamma^5\sigma_3\,, \tag{5.56}$$

$$\Phi(-y) = \sigma_3\otimes\sigma_3\Phi(y)\,, \qquad \bar{\Phi}(-y) = \bar{\Phi}(y)\sigma_3\otimes\sigma_3\,, \tag{5.57}$$

$$F(-y) = -\sigma_3\otimes\sigma_3 F(y)\,, \qquad \bar{F}(-y) = -\bar{F}(y)\sigma_3\otimes\sigma_3\,. \tag{5.58}$$

We also could introduce Scherk-Schwarz twists for the $SU(2)_R$ and $SU(2)_H$ symmetries. However since the presence of an $\tilde{\omega} \neq 0$ parameter amounts to a supersymmetric mass, while the nature and interpretation of a Scherk-Schwarz twist $\omega \neq 0$ has been widely clarified in the literature [19, 22, 49, 46], we will simplify our disscussion in this section by assuming $\omega = \tilde{\omega} = 0$. Furthermore, we replace the action given in Eqs. (5.4)–(5.6) by

$$\mathcal{S}^0_{\rm bk} = \int \left(-\frac{1}{2}\bar{\Phi}\, \partial^2\Phi + \frac{i}{2}\bar{\Psi}\gamma^M\partial_M\Psi + 2\bar{F}F \right) , \tag{5.59}$$

$$\mathcal{S}^{\rm m}_{\rm bk} = \int \left(2i\bar{F}\mathcal{M}\Phi + \frac{1}{2}\bar{\Psi}\mathcal{M}\Psi \right) , \tag{5.60}$$

$$\mathcal{S}_{\rm bd} = \int \Big(N_0\delta(y) - N_\pi\delta(y-\pi) \Big) \bar{\Phi}\Phi \,. \tag{5.61}$$

In order to have well-defined parity for the mass terms, we take the vector $\vec{p}$ defined in Eq (5.10) to be

$$p = (p_1, p_2, \epsilon(y)p_3) \,, \tag{5.62}$$

where $\epsilon(y)$ is the sign-function. Choosing $p_1 = p_2 = 0$ one reproduces the odd mass terms for hypermultiplets previously considered in the literature [46, 50, 51, 48]. The boundary mass terms involving the N_f parameters are similar to the ones encountered in Eq. (5.6). In fact the boundary conditions (5.57) require $R = -\sigma_3 \otimes \sigma_3$, so that by using this in Eq. (5.6) we find Eq. (5.61). The additional factor of 2 comes from the fact that the support of the delta function on the circle is twice the one on the interval, while the relative sign of the two boundaries reflects our convention of taking the orientation of the boundary at $y = 0$ to be negative. Boundary mass terms –which in the interval give rise to boundary conditions– on the orbifold generate jumps for the profiles of wave functions across the brane. It is easy to calculate these jumps for the special kind of mass terms of Eq. (5.61). All fields are continuous except the ∂_5 derivatives of even bosonic fields, which satisfy

$$(1 + \sigma_3 \otimes \sigma_3)[\Phi'(0^+) + N_0\Phi(0)] = 0 \,, \tag{5.63}$$

$$(1 + \sigma_3 \otimes \sigma_3)[\Phi'(\pi^-) + N_\pi\Phi(\pi)] = 0 \,. \tag{5.64}$$

Here we write the matrix $(1 + \sigma_3 \otimes \sigma_3)$ to project on the even fields only. The spectrum can now be directly inferred from section 5.1. The bosonic one is given by Eq. (5.20) with $\omega = \tilde{\omega} = 0$. For the fermionic one, notice that in order to produce our orbifold boundary conditions, we have to choose $\vec{s}_0 = \vec{s}_\pi = (0, 0, -1)$ and hence must use $c_0 = c_\pi = -p_3$ in Eq. (5.21).

Let us next study supersymmetry of this action. The supersymmetry variation of the bulk action is now given by

$$\delta\mathcal{S}^0_{\rm bk} = 0\,, \qquad \delta\mathcal{S}^m_{\rm bk} = -2\,p_3 M\,[\delta(y) - \delta(y-\pi)]\,\bar{\epsilon}\bar{\Phi}\gamma^5\sigma_3\Psi\,, \tag{5.65}$$

while the boundary piece varies into

$$\delta\mathcal{S}_{\rm bd} = 2i\,[N_0\delta(y) - N_\pi\delta(y-\pi)]\,\bar{\epsilon}\bar{\Phi}\Psi\,. \tag{5.66}$$

Making use of our parity assignments Eq. (5.56) we conclude that for these two pieces to cancel we must have

$$n_0 = n_\pi = -p_3. \tag{5.67}$$

To compare with the interval approach, we note again that there $c_0 = c_\pi = -p_3$ and thus we find that for the action to be supersymmetric, relation (5.18) must hold. Therefore departure from the supersymmetric relation (5.18) implies supersymmetry breaking. Notice that the breaking here is explict and can be viewed as coming from localized soft masses for the even hyperscalars. Splitting the masses N_f into a supersymmetric and a soft piece, $N_f = -p_3 M + M_f$ we can write the localized soft breaking Lagrangian as

$$\mathcal{S}^{\rm hyper}_{\rm soft} = \int \Big(M_0\delta(y) - M_\pi\delta(y-\pi)\Big)\bar{\Phi}\Phi\,. \tag{5.68}$$

Supersymmetry breaking produced by the soft mass terms for even scalars in the action (5.68) bears strong similarities with the usual Scherk-Schwarz supersymmetry breaking by twisted boundary conditions in the gaugino (and gravitino) sector. In fact twisted Scherk-Schwarz boundary conditions for the gauginos λ^i $(i = 1, 2)$ can be produced by localized gaugino soft masses with an action [19, 22, 49, 46]

$$\mathcal{S}_{\rm soft}^{\rm gauge} = \int \bar{\lambda} \left(M_0 \delta(y) - M_\pi \delta(y - \pi) \right) \lambda + \text{h.c.} \tag{5.69}$$

However the nature of supersymmetry breaking by boundary scalar masses is very different from that of the Scherk-Schwarz supersymmetry breaking (which provides a supersoft or finite breaking).

The real formalism has been useful to develop a supersymmetric model in the interval with dynamically obtained boundary terms. However this formalism is not suitable to incorporate coupling terms. For that reason in the next chapter we will show the translation of the real formalism into the superfileld one. The later makes the model closer to what usually appears in the literature and sheds light on the nature of supersymmetry breaking.

Chapter 6

Superfield approach to real formalism

As we mentioned previously, the real formalism is suitable to make contact between an interval approach with boundary matrices and an orbifold model with odd bulk mass. However it is convenient to translate this formalism into superfield language where the coupling terms are easily implemented. This is what we will do in this chapter. The model developed in the previous chapter is defined in a 5D manifold with boundaries $\Sigma = M_4 \times I$ with M_4 the 4D Minkowski space and I the interval $[0, \pi R]$, R being the compactification radius. The field content of the hypermultiplet in 5D is (Φ_i, Ψ, F_i) where Φ_i are complex scalars and F_i are auxiliary fields and both transform as doublets of $SU(2)_R$. Ψ is a Dirac fermion. To have a manifest $SU(2)_R$ covariance in the superalgebra it is used the $N = 2$ 5D structure [32]

$$\{Q_i\,,\,Q_j\} = \epsilon_{ij}\,\gamma^M C\,P_M + \epsilon_{ij} Z\,C\,, \tag{6.1}$$

with a symplectic Majorana (SyM) constrain

$$\bar{Q}^i \equiv Q_i^\dagger \gamma^0 = \epsilon^{ij} Q_j^T C\,, \tag{6.2}$$

where ϵ^{ij} is the total antisymmetric tensor and

$$C = -\mathbf{1} \otimes \mathrm{i}\sigma_2 = \begin{pmatrix} -\mathrm{i}\sigma_2 & 0 \\ 0 & -\mathrm{i}\sigma_2 \end{pmatrix}\,, \tag{6.3}$$

is the 5D charge conjugation matrix verifying $C\,\gamma^M\,C = -\left(\gamma^M\right)^T$. P_M are the spacetime translation generators and Z is a central charge. Consistency with (6.2) imposes Z to be hermitian. The supersymmetric transformations on the fields are given by

$$\begin{aligned} \delta_\chi \Phi_i^\alpha &= \mathrm{i}\bar{\chi}_i \Psi^\alpha\,, \\ \delta_\chi \Psi_\alpha &= -\gamma^M \chi^i \partial_M \Phi_i^\alpha + 2\chi^i F_i^\alpha\,, \\ \delta_\chi F_i^\alpha &= -\tfrac{\mathrm{i}}{2}\bar{\chi}_i \gamma^M \partial_M \Psi^\alpha\,, \end{aligned} \tag{6.4}$$

with

$$\bar{\chi}^i = \epsilon^{ij}\chi_j^T C\,, \tag{6.5}$$

a symplectic Majorana spinor. Under these transformations the variation of the action (5.4)-(5.5)-(5.6) vanishes upon the use of boundary conditions and the equation of motion for the auxiliary field. This is expected since the boundary term is on-shell[1] and as it is shown in Ref. [52], the supersymmetry of the on-shell formulation of the boundary picture requires the boundary conditions to be satisfied. This will be explicitly shown applied to our case in the next section.

6.1 Superfield description

To recast the action in superfields we will take $\vec{t}_0 = \vec{t}_\pi = \vec{t}$ and $N_f = \vec{p}\cdot\vec{s}_f\,M$, according to what we saw in the previous chapter. For simplicity we will take $\vec{t} = \vec{p} = (0,0,1)$. Notice that we can always do so by means of global rotations of $SU(2)_R$ and $S(2)_H$, respectively.

[1] There is no auxiliary field present although it is a mass term.

The reality constrains (5.2)-(5.3) can be solved as

$$
\begin{aligned}
\Phi &= \begin{pmatrix} \Phi_1^1 \\ \Phi_2^1 \\ -\Phi_2^{1*} \\ \Phi_1^{1*} \end{pmatrix} \\
\Psi^1 &= \begin{pmatrix} \psi_L^1 \\ \bar{\psi}_R^1 \end{pmatrix} \\
\Psi^2 &= \begin{pmatrix} -\psi_R^1 \\ \bar{\psi}_L^1 \end{pmatrix}
\end{aligned} \tag{6.6}
$$

and, as it is shown at the end of this chapter, the fields can be split into two chiral multiplets according to

$$H = \Phi + \sqrt{2}\theta\psi + F\theta^2 \,, \tag{6.7}$$

$$H_c = \Phi_c + \sqrt{2}\theta\psi_c + F_c\theta^2 \,, \tag{6.8}$$

just redefining them as

$$
\begin{aligned}
\begin{pmatrix} \Phi \\ \Phi_c \end{pmatrix} &\equiv \begin{pmatrix} \mathrm{i}\Phi_2^{1*} \\ -\mathrm{i}\Phi_1^1 \end{pmatrix} \\
\begin{pmatrix} \psi \\ \psi_c \end{pmatrix} &\equiv \begin{pmatrix} -\mathrm{i}\psi_R^1 \\ -\mathrm{i}\psi_L^1 \end{pmatrix} \\
\begin{pmatrix} F \\ F_c \end{pmatrix} &\equiv \begin{pmatrix} -2F_1^{1*} - \partial_5\Phi_c^* \\ -2F_2^1 + \partial_5\Phi^* \end{pmatrix}
\end{aligned} \tag{6.9}
$$

Therefore, the whole action can be rewritten as[2]

$$\mathcal{S} = \int_\Sigma \left[\mathrm{i}\bar{\psi}_c\bar{\sigma}^\mu\partial_\mu\psi_c + \mathrm{i}\psi\sigma^\mu\partial_\mu\bar{\psi} - \phi_c^*\Box\phi_c - \phi^*\Box\phi + |F_c|^2 + |F|^2\right.$$
$$\left. + F_c\left(-\partial_5 + M\right)\phi + \phi_c\left(-\partial_5 + M\right)F + \psi_c\left(\partial_5 - M\right)\psi + \text{h.c.}\right]$$

$$+ \int_{\partial\Sigma}\left[K - \left(\frac{1}{2}s_3\psi_c\psi - \frac{1}{4}s_+\psi_c\psi_c + \frac{1}{4}s_-\psi\,\psi + \text{h.c.}\right)\right.$$
$$\left. + \frac{1}{2}M\vec{p}\cdot\vec{s}\,\varphi^\dagger(-\mathbf{1} + S)\varphi + \frac{1}{2}(\varphi^\dagger S\varphi)'\right] \qquad (6.10)$$

where K is a Gibbons-Hawking-like term given by

$$K = \frac{1}{2}\partial_5\left(|\phi_c|^2 + |\phi|^2\right) + M\left(|\phi_c|^2 - |\phi|^2\right) - \frac{1}{2}\left(\psi_c\psi + \bar{\psi}_c\bar{\psi}\right) + \phi_c F + \phi_c^* F^*\,,$$

$s_\pm = s_1 \pm \mathrm{i}s_2$, $\vec{s} = (s_1, s_2, s_3)$ and $\varphi = \begin{pmatrix}\phi_c\\ \phi\end{pmatrix}$. Notice that the bulk term of (6.10) is already $N = 1$ invariant without any boundary contribution, which implies that $\mathcal{S}'_{bd} = \mathcal{S}_{bd} + \int_{\partial\Sigma} K$ has to be so. Let us check it explicitly:
The fermionic component of $\mathcal{S}'_{bd}$ is given by

$$\int_{\partial\Sigma}\left[-\frac{1}{2}(1 + s_3)\psi_c\psi + \frac{1}{4}s_+\psi_c\psi_c - \frac{1}{4}s_-\psi\,\psi + \text{h.c.}\right] \qquad (6.11)$$

while for the bosonic sector we have

$$\int_{\partial\Sigma}\left[\frac{1}{2}M\vec{p}\cdot\vec{s}\,\varphi^\dagger(-\mathbf{1} + S)\varphi + \frac{1}{2}\left(\varphi^\dagger\left(\mathbf{1} + S\right)\varphi\right)' + M\varphi^\dagger\sigma_3\varphi + \phi_c F + \phi_c^* F^*\right]$$
$$= \int_{\partial\Sigma}\left\{\frac{1}{2}\varphi^\dagger(-\mathbf{1} + S)\left[\varphi' + M\vec{p}\cdot\vec{s}\,\varphi\right] + \frac{1}{2}\varphi'^\dagger\left(\mathbf{1} + S\right)\varphi\right.$$
$$\left. + \varphi^\dagger\varphi' + M\varphi^\dagger\sigma_3\varphi + \phi_c F + \phi_c^* F^*\right\}\,. \qquad (6.12)$$

[2]For simplicity we omit the subscript f.

Using the boundary conditions

$$(-\mathbf{1}+S)\,[\varphi' + M\vec{p}\cdot\vec{s}\,\varphi] = 0\,, \qquad (\mathbf{1}+S)\,\varphi = 0$$

and the equations of motion for the auxiliary fields

$$F + M\phi_c^* + \partial_5\phi_c^* = 0\,, \qquad F_c + M\phi^* - \partial_5\phi^* = 0\,,$$

(6.12) reduces to

$$\int_{\partial\Sigma} \phi^* F_c^* + \phi_c F \tag{6.13}$$

then it can be easily checked that

$$\begin{aligned}\phi^* F_c^* + \phi_c F &= \frac{1}{2}(1+s_3)\,(F_c\phi + \phi_c F) + \frac{1}{2}s_-\phi\, F - \frac{1}{2}s_+\phi_c F_c + \text{h.c.} \\ &+ \frac{1}{2}\varphi^T \mathrm{i}\sigma_2\,(\mathbf{1}+S)\,\mathcal{F} + \frac{1}{2}\mathcal{F}^\dagger \mathrm{i}\sigma_2\,(\mathbf{1}+S^*)\,\varphi^*\,,\end{aligned} \tag{6.14}$$

where $\mathcal{F} = \begin{pmatrix} F_c \\ F \end{pmatrix}$ and the last two terms separately cancel due to the boundary conditions. To see this, notice that

$$\begin{aligned}&(\mathbf{1}+S)\,\mathcal{F} = (\mathbf{1}+S)\,\mathrm{i}\sigma_2\,[\varphi'^* + M\sigma_3\varphi^*] = \mathrm{i}\sigma_2\,(\mathbf{1}-S^*)\,[\varphi'^* + M\sigma_3\varphi^*] \\ &= \mathrm{i}\sigma_2\,(\mathbf{1}-S^*)\,[\varphi'^* + M\vec{p}\cdot\vec{s}\,\varphi^*] + \mathrm{i}\sigma_2\,(\mathbf{1}+S^*)\,\varphi^* = 0\end{aligned}$$

where we have used the identity $\sigma_2\, S\, \sigma_2 = -S^*$.
This is expected since we are working with supersymmetric boundary conditions and hence they have to be writable in a supersymmetric way.
Thus, as claimed, we can write the whole action in terms of superfields as

$$\begin{aligned}S = &\int_\Sigma \mathrm{d}\theta^4\left[\bar{\mathrm{H}}\mathrm{H} + \bar{\mathrm{H}}_\mathrm{c}\mathrm{H}_\mathrm{c}\right] - \int_\Sigma \mathrm{d}\theta^2 \mathrm{H}_\mathrm{c}(\partial_5 - \mathrm{M})\mathrm{H} + \text{h.c.} \\ &+ \int_{\partial_\Sigma} \mathrm{d}\theta^2\left[\frac{1+\mathrm{s}_3}{2}\mathrm{H}\mathrm{H}_\mathrm{c} + \frac{\mathrm{s}_-}{4}\mathrm{H}\mathrm{H} - \frac{\mathrm{s}_+}{4}\mathrm{H}_\mathrm{c}\mathrm{H}_\mathrm{c}\right] + \text{h.c.}\end{aligned} \tag{6.15}$$

Recall that we have taken $\vec{p} = (0,0,1)$. To have a general mass configuration we simply undo the $SU(2)_H$ rotation. Explicitly, (6.15) can be rewritten in a compact way as

$$S = \int_\Sigma \mathrm{d}\theta^4\, \bar{\mathcal{H}}\mathcal{H} - \frac{1}{2}\int_\Sigma \mathrm{d}\theta^2 \left[\mathcal{H}^{\mathrm{T}}\mathrm{i}\sigma_2\mathcal{H}' - \mathrm{M}\mathcal{H}^{\mathrm{T}}\sigma_1\mathcal{H}\right] + \mathrm{h.c.}$$
$$- \frac{1}{4}\int_{\partial\Sigma} \mathrm{d}\theta^2\, \mathcal{H}^{\mathrm{T}}\mathrm{i}\sigma_2\,(\mathbf{1}+\mathrm{S})\,\mathcal{H} + \mathrm{h.c.} \tag{6.16}$$

where $\mathcal{H} = (H_c,\, H)^T$ (already $SU(2)_H$ covariant[3]) therefore, an arbitrary $SU(2)$ rotation leaves the kinetic term invariant while the mass term is brought into the form

$$M\begin{pmatrix} \beta & \alpha \\ \alpha & -\beta^* \end{pmatrix}, \tag{6.17}$$

with $\alpha \in \mathbb{R}$. In fact, this is, not only, the most general mass term compatible with the $N=2$ structure (see Chapter 8) but the most general compatible with the 5D Lorentz invariance. In terms of a 4 component 5D Dirac spinor the most general mass term can be written as

$$\alpha\bar{\Psi}\,\Psi + \beta\,\Psi^T C\,\Psi + \beta^*\Psi^\dagger C\,\Psi^*, \tag{6.18}$$

with C the 5D charge conjugation matrix. One can easily check that (6.18) expressed in terms of 2 component Weyl spinors yields precisely the mass matrix (6.17).

6.1.1 General boundary term

In this section we will briefly see that the boundary term displayed previously is indeed on-shell equivalent to the most general boundary term that can be written:

$$\tilde{\mathcal{S}}_{\mathrm{bd}} = \int_{\partial\Sigma} d^2\theta\,[\frac{\mu}{2}HH + \frac{\lambda}{2}H_cH_c + \nu HH_c] + h.c. \tag{6.19}$$

where μ, λ and ν are arbitrary complex numbers. The variation of $\mathcal{S}_{\mathrm{bk}} + \tilde{\mathcal{S}}_{\mathrm{bd}}$ yields the boundary term

$$\int_{\partial\Sigma} \mathrm{d}\theta^2\; [\delta\mathrm{H_c}\,(\lambda\mathrm{H_c} + \nu\mathrm{H}) + \delta\mathrm{H}\,(\mu\mathrm{H} + \nu\mathrm{H_c} - \mathrm{H_c})] + \mathrm{h.c.}$$

[3] $\varphi = -\mathrm{i}\begin{pmatrix} \Phi_1^1 \\ \Phi_1^2 \end{pmatrix}$ where the upper indices are those of $SU(2)_H$.

and hence the boundary conditions are

$$\mu H + \nu H_c - H_c = 0 \tag{6.20}$$

$$\lambda H_c + \nu H = 0 \tag{6.21}$$

One easily checks that in order to not overdetermine the system the complex parameters have to satisfy the relation

$$\mu\,\lambda - (\nu - 1)\nu = 0\,, \tag{6.22}$$

and that (6.20)-(6.21) are invariant under the redefinitions

$$H_c \leftrightarrow H,\; \lambda \leftrightarrow \mu,\; \nu \leftrightarrow 1-\nu\,. \tag{6.23}$$

In the special case $\nu = 0$ the boundary conditions reduce to

$$\begin{cases} \lambda = 0,\;\; \mu H - H_c = 0 \\ \text{or} \\ \mu = 0,\;\; H_c = 0 \end{cases} \tag{6.24}$$

while the case $\nu = 1$ is obtained from the previous one by means of the relations (6.23). In the general case $\nu \notin \{0, 1\}$, (6.20)-(6.21) reduce to

$$zH_c + H = 0$$

with $z = \lambda/\nu$. This means that we have a lot of redundancy in the parameters ν, μ, λ since only the complex number z plays a role in solving the boundary conditions. Actually, by letting z to take any complex value we cover the whole set of boundary conditions including $\nu = 0$, which corresponds to $z \to \infty$. As a matter of fact, the parametrization

$$\nu_0 = \frac{1}{2}(1+s_3)\,,\; \mu_0 = \frac{1}{2}s_- = \frac{1}{2}\sqrt{1-s_3^2}\; e^{i\delta}\,,\; \lambda_0 = -\mu_0^*\,,$$

verifies $\mu_0\lambda_0 - (\nu_0 - 1)\nu_0 = \frac{1}{4}\,(1 - \vec{s}^2) = 0$ and in addition $z = \sqrt{\frac{1-s_3}{1+s_3}}\, e^{-i\delta}$, which covers the whole complex plane since $s_3 = \frac{1-|z|^2}{1+|z|^2}$ is well defined and always less (or equal) than one, in absolute value, for all $|z|$.

6.2 Supersymmetry breaking by boundary terms

As we saw in the previous chapter, the supersymmetry is broken by the boundary terms whenever $\vec{t}_0 \neq \vec{t}_\pi$ or $N_f \neq \vec{p}\cdot\vec{s}_f\, M$. The misalignment of the R-matrices is equivalent to have a local R transformation, $e^{\mathrm{i}y\vec{\omega}\cdot\vec{\sigma}}$, such that $T_\pi = e^{\mathrm{i}\pi\vec{\omega}\cdot\vec{\sigma}}\, T_0\, e^{-\mathrm{i}\pi\vec{\omega}\cdot\vec{\sigma}}$ which is a Scherk-Schwarz-like breaking [44] and therefore a soft breaking. This breaking has been widely studied in the literature. A very elegant proposal consists of breaking the supersymmetry at the supergravity level via the expectation value acquired by some auxiliary field of the supergravity multiplet [53, 54], which implies that the breaking pattern should have to be expressable in terms of superfields.

Here we will suggest a breaking mechanism [55] very similar to that in ref. [53] where the case of a warped extra dimension was considered. We will restrict ourselves to the case of a flat space $\mathcal{M}^4 \times I$, where I is the interval $[0, \pi]$, with the metric

$$ds^2 = \eta_{\mu\nu} dx^\mu dx^\nu - R^2 dy^2\,, \tag{6.25}$$

where R is the radion of the compact extra dimension labeled by y, which ranges fron 0 to π. The supersymmetrization of the radion field is given by

$$T = R + \mathrm{i}B_5 + \theta\Psi^5_R + \theta^2 F_T\,, \tag{6.26}$$

where B_5 is the fifth component of the graviphoton, Ψ^5_R is the fifth component of the right-handed gravitino and F_T is a complex auxiliary field. The supersymmetric action will be given by

$$\begin{aligned} S_{\rm bk} = &\int d^5x\, d^4\theta\, \frac{T+\bar{T}}{2}\, \left[\bar{H}H + \bar{H}_c H_c\right] \\ - &\int d^5x\, \left[d^2\theta\, \left(H_c\partial_5 H - M\, T H_c H\right) + \text{h.c.}\right]\,, \end{aligned} \tag{6.27}$$

$$S_{\rm bd} = \int d^4x\, \left[d^2\theta\, \left(\frac{\mu}{2}HH + \frac{\lambda}{2}H_cH_c + \nu H H_c\right) + \text{h.c.}\right]\,, \tag{6.28}$$

Supersymmetry can be spontaneously broken by allowing expectation values for the auxiliary fields

$$\langle T\rangle = R + 2\omega\,\theta^2\,, \tag{6.29}$$

ω being a dimensionless constant. One can easily check that the total action, is on-shell described by the lagrangian

$$L_{\rm bk} = \mathrm{i}\bar{\Psi}\gamma^{\mu}\partial_{\mu}\Psi + \frac{1}{2}\bar{\Psi}\gamma^{5}\partial_{5}\Psi - \frac{1}{2}\partial_{5}\bar{\Psi}\gamma^{5}\Psi$$
$$- \ \Phi^{\dagger}\Box\Phi - |\mathcal{D}_{5}\Phi|^{2} + \frac{1}{2}\left(\Phi^{\dagger}\Phi\right)'' - M^{2}\,\Phi^{\dagger}\Phi + M\,\bar{\Psi}\Psi\,, \tag{6.30}$$

$$L_{\rm bd} = \frac{1}{2}c_{f}M\,\varphi^{\dagger}(-\mathbf{1} - S)\varphi - \frac{1}{2}(\varphi^{\dagger}S\varphi)'$$
$$+\left\{\frac{1}{2}s_{3}\psi_{R}^{1}\psi_{L}^{1} + \frac{1}{4}s_{-}\psi_{R}^{1}\psi_{R}^{1} - s_{+}\psi_{L}^{1}\psi_{L}^{1} + \mathrm{h.c.}\right\}, \tag{6.31}$$

with $\mathcal{D}_5 = \partial_5 + \mathrm{i}\frac{\omega}{R}\sigma_2$. Here we have made the change of variables $y \to Ry$. Finally, by the local ($SU(2)_R$) redefinition: $\Phi \to e^{-\mathrm{i}\frac{\omega}{R}y\sigma_2}\,\Phi$, we can rephrase (6.30)-(6.31) as

$$L_{\rm bk} = -\frac{1}{2}\bar{\Phi}\partial^{2}\Phi + \frac{\mathrm{i}}{2}\bar{\Lambda}\gamma^{M}\partial_{M}\Lambda - \frac{1}{2}M^{2}\bar{\Phi}\Phi\,, \tag{6.32}$$

$$L_{\rm bd} = \frac{1}{4}\bar{\Lambda}S\Lambda + \frac{1}{4}(\bar{\Phi}R\Phi)' + \frac{1}{4}c_{f}M\,\bar{\Phi}(-1+R)\Phi\,, \tag{6.33}$$

where the R matrices are given by

$$R_0 = -\sigma_3 \otimes S_0\,, \qquad R_{\pi} = -T_{\pi}\otimes S_{\pi}\,, \tag{6.34}$$
$$T_{\pi} = e^{\mathrm{i}\omega\pi\sigma_2}\,\sigma_3\,e^{-\mathrm{i}\omega\pi\sigma_2}\,.$$

To study the nature of the breaking due to the departure of n_f from c_f, in the language used in the previous chapter, we shall consider the boundary action (6.15) plus an effective coupling such that the new boundary term now is given by

$$\int_{\partial\Sigma} \mathrm{d}\theta^2 \left[\frac{1+\mathrm{s}_3}{2}\mathrm{H}\mathrm{H}_\mathrm{c} + \frac{\mathrm{s}_-}{4}\mathrm{H}\mathrm{H} - \frac{\mathrm{s}_+}{4}\mathrm{H}_\mathrm{c}\mathrm{H}_\mathrm{c}\right] + \mathrm{h.c.}$$
$$-\frac{1}{\Lambda^3}\int_{\partial\Sigma} \mathrm{d}\theta^2 \bar{\mathcal{N}}\mathcal{N}(\bar{\mathrm{H}}\,\mathrm{H} + \bar{\mathrm{H}}_\mathrm{c}\mathrm{H}_\mathrm{c})\,, \tag{6.35}$$

with Λ the scale of the cutoff and $\mathcal{N}_f$ localized superfields whose auxiliary fields acquire VEVs, say F_f, such that $\frac{|F_f|^2}{\Lambda^3} = N_f$. The new boundary conditions then read

$$\frac{1}{2}(\mathbf{1} - S_f)\begin{pmatrix} H_c \\ H \end{pmatrix} + \frac{\bar{\mathcal{N}}\mathcal{N}}{\Lambda^3}\mathrm{i}\sigma_2\begin{pmatrix} \bar{H}_c \\ \bar{H} \end{pmatrix} = 0\,, \tag{6.36}$$

in (bosonic) components they read

$$\frac{1}{2}(\mathbf{1} - S_f)\begin{pmatrix} \phi_c \\ \phi \end{pmatrix} = 0\,, \tag{6.37}$$

$$\frac{1}{2}(\mathbf{1} - S_f)\left[-M\sigma_1\begin{pmatrix} \bar{\phi}_c \\ \bar{\phi} \end{pmatrix} + \mathrm{i}\sigma_2\begin{pmatrix} \partial_5\bar{\phi}_c \\ \partial_5\bar{\phi} \end{pmatrix}\right] + N_f\mathrm{i}\sigma_2\begin{pmatrix} \bar{\phi}_c \\ \bar{\phi} \end{pmatrix} = 0\,, \tag{6.38}$$

where we have already used the on-shell value of the auxiliary fields. Taking the complex conjugate of the second equation and multiplying on the right by $\mathrm{i}\sigma_2$ and using $\mathrm{i}\sigma_2\, S_f^*\, \mathrm{i}\sigma_2 = S_f$ we find

$$-\frac{1}{2}(\mathbf{1} + S_f)\left[M\sigma_3\begin{pmatrix} \phi_c \\ \phi \end{pmatrix} + \begin{pmatrix} \partial_5\phi_c \\ \partial_5\phi \end{pmatrix}\right] - N_f\begin{pmatrix} \phi_c \\ \phi \end{pmatrix} = 0\,, \tag{6.39}$$

finally, using $\{S_f\,, \sigma_3\} = 2c_f\mathbf{1}$ and (6.37), we are left with

$$\frac{1}{2}(\mathbf{1} - S_f)\begin{pmatrix} \phi_c \\ \phi \end{pmatrix} = 0\,, \tag{6.40}$$

$$\frac{1}{2}(\mathbf{1} + S_f)\left[\partial_5 + c_f M + N_f\right]\begin{pmatrix} \phi_c \\ \phi \end{pmatrix} = 0\,, \tag{6.41}$$

which are the general boundary conditions found in the real formalism. This shows explicitly that this breaking has a soft nature, result that was pointed out in the

calculation of the radiative corrections of such term to the Higgs mass coupling.
In the next chapter we will develop a revisited model for ElectroWeak symmetry breaking from an interval with boundary terms in superfield formalism and the starting point will a generalization of the action (6.15), non the less, to finish this transition chapter we will briefly give the splitting of the 5d hypermultiplet into 4d superfield pieces.

6.3 N=1 splitting

The reality constraint on the supersymmetric parameter (6.5) can be solved as

$$\begin{aligned} \chi^1 &= \begin{pmatrix} \xi \\ \bar{\eta} \end{pmatrix} \\ \chi^2 &= \begin{pmatrix} \eta \\ -\bar{\xi} \end{pmatrix} \end{aligned} \tag{6.42}$$

and thus for the η-transformations (setting $\xi = 0$) [4] (6.4) can be written as

$$\begin{aligned} \delta_\eta(-i\Phi_2^1)^* &= \eta(-i\psi_R^1) \\ \delta_\eta(-i\psi_R^1) &= -i\sigma^\mu\bar{\eta}\partial_\mu(-i\Phi_2^1)^* + \eta(-2F_1^1 + i\partial_5\Phi_1^1)^* \\ \delta_\eta(-2F_1^1 + i\partial_5\Phi_1^1)^* &= -i\bar{\eta}\bar{\sigma}^\mu\partial_\mu(-i\psi_R^1) \end{aligned} \tag{6.43}$$

$$\begin{aligned} \delta_\eta(-i\Phi_1^1) &= \eta(-i\psi_L^1) \\ \delta_\eta(-i\psi_L^1) &= -i\sigma^\mu\bar{\eta}\partial_\mu(-i\Phi_1^1) + \eta(-2F_2^1 - i\partial_5\Phi_2^1) \\ \delta_\eta(-2F_2^1 - i\partial_5\Phi_2^1) &= -i\bar{\eta}\bar{\sigma}^\mu\partial_\mu(-i\psi_L^1) \end{aligned} \tag{6.44}$$

which correspond to a pair of chiral superfields [4]

$$H = \mathrm{i}\Phi_2^{1*} + \sqrt{2}\theta\left(-\mathrm{i}\psi_R^1\right) + \left(-2F_1^{1*} - \mathrm{i}\partial_5\Phi_1^{1*}\right)\theta^2\,, \tag{6.45}$$

$$H_c = \left(-\mathrm{i}\Phi_1^1\right) + \sqrt{2}\theta\left(-\mathrm{i}\psi_L^1\right) + \left(-2F_2^1 - \mathrm{i}\partial_5\Phi_2^1\right)\theta^2\,, \tag{6.46}$$

[4] The $SU(2)_R$ invariance of the supersymmetric algebra allows us to choose any unitary rotation of the symplectic supersymmetric parameters.

in the superspace representation $\bar{D}_{\dot{\alpha}} H = \bar{D}_{\dot{\alpha}} H_c = 0$

$$\bar{D}_{\dot{\alpha}} = -\frac{\partial}{\partial \bar{\theta}^{\dot{\alpha}}} + \mathrm{i}\theta^{\alpha} \sigma^{\mu}_{\alpha\dot{\alpha}} \partial_{\mu} \, . \tag{6.47}$$

Chapter 7

Electroweak symmetry breaking from Scherk-Schwarz supersymmetry breaking.

In chapter 5 it was studied (at tree level) the possibility for iducing the ElectroWeak symmetry breaking from a model in the interval with boundary terms. In particular, we found that for a certain boundary configuration the supersymmetry is softly broken and the lowest mode of the Higgs presents a tachyon at the tree level. This is a very interesting feature since the one-loop positive radiative corrections to the Higgs mass coming from the gauge sector could be partially (or even completely) cancelled by the tree level tachyonic mode and, as a consequence, the negative corrections coming mainly from the top-stop sector (which enter as a two-loop quantum correction) could trigger the ElectroWeak symmetry breaking non-marginally. This is to be compared with the the usual scenario, where the positive gauge correction can not be cancelled by the negative top-stop one and the EkectroWeak symmetry breaking does not take palce [56].

In this chapter we shall study in detail this possibility within the framework of Scherk-Schwarz (SS) supersymmetry breaking rather than the soft source introduced in chapter 5, for this we will show how a tree level tachyon can be equally yielded for the case of SS.

In the next section we will formally present the model.

7.1 Free action

The superfield content will be [55]

$$\mathbb{H}^{\alpha,i} = (H,\, \bar{H}_c)^{\alpha,i}\,, \tag{7.1}$$

where α is an $SU(2)_H$ index and i lives in the fundamental representation of some gauge group which will be obviated for the moment. The action is given by $\mathcal{S} = \mathcal{S}_{\rm bk} + \mathcal{S}^{\rm m}_{\rm bk} + \mathcal{S}_{\rm bd}$ where

$$\mathcal{S}_{\rm bk} = \int d^5x\, d^4\theta\, \left(\bar{H}H + H_c\bar{H}_c\right) - \int d^5x\, d^2\theta\, H_c\partial_5 H + \text{h.c.}\,, \tag{7.2}$$

$$\mathcal{S}^{\rm m}_{\rm bk} = \int d^5x \left[d^2\theta\, M\nu H_{c\,1}H_1 + \tilde{\nu}H_{c\,2}H_2 + \mu H_{c\,2}H_1 + \mu_c H_{c\,1}H_2 + \text{h.c.}\right]\,, \tag{7.3}$$

$$\mathcal{S}_{\rm bd} = \int d^4x \left[d^2\theta\, \kappa H_{c\,1}H_1 + \tilde{\kappa}H_{c\,2}H_2 + \lambda H_{c\,2}H_1 + \lambda_c H_{c\,1}H_2 + \text{h.c.}\right]\,. \tag{7.4}$$

The gauge indices have been omited. M is a constant with dimension of mass and the rest of parameters appearing in (7.3)-(7.4) are, a priori, arbitrary complex numbers. It is not very difficult to see that if $\nu\,,\, \tilde{\nu} \in \mathbb{R}$ and $\mu = \mu_c^*$ the bulk action is invariant under $SU(2)_R$ transformations (up to a total derivative). Now (7.3) can be rephrased as

$$\mathcal{S}^{\rm m}_{\rm bk} = \int d^5x\, \left[d^2\theta\, H_c\left(M_0\mathbf{1} + M_1\vec{p}\cdot\vec{\sigma}\right)H + \text{h.c.}\right]\,, \tag{7.5}$$

where

$$M_0 = M\frac{\nu+\tilde{\nu}}{2} \qquad M_1 = M\frac{\sqrt{|\mu|^2 + (\nu-\tilde{\nu})^2}}{2} \tag{7.6}$$

and

$$\vec{p} = (\text{Re}\,\mu,\, \text{Im}\,\mu,\, [\nu-\tilde{\nu}]/2)\,. \tag{7.7}$$

By applying the variation principle to the whole action we find the boundary conditions

$$\begin{pmatrix} \kappa & \lambda_c & & \\ \lambda & \tilde{\kappa} & & \\ & & \tilde{\kappa}-1 & \lambda_c \\ & & \lambda & \kappa-1 \end{pmatrix} \cdot \begin{pmatrix} H_1 \\ H_2 \\ H_{c\,2} \\ H_{c\,1} \end{pmatrix} = 0\,. \tag{7.8}$$

We want to extract two independent degrees of freedom out of (7.8) or, in other words, we want the matrix to have rank 2. This can take place in three different ways:

- The upper matrix has non vanishing determinant while the lower matrix vanishes exactly. This implies $\kappa = \tilde{\kappa} = 1$ and $\lambda = \lambda_c = 0$. In this case the boundary conditions read
$$H = 0\,. \tag{7.9}$$
- The upper matrix is zero while the lower matrix has rank 2. For that case we need $\kappa = \tilde{\kappa} = \lambda = \lambda_c = 0$ and the boundary conditions are
$$H_c = 0\,. \tag{7.10}$$
- Both matrices have rank 1. The solution to this is given by
$$\tilde{\kappa} = 1 - \kappa\,, \qquad \lambda_c \lambda - \kappa(1-\kappa) = 0\,, \tag{7.11}$$
and the boundary conditions become
$$\kappa H_1 + \lambda_c H_2 = 0\,, \qquad \kappa H_{c\,2} - \lambda_c H_{c\,1} = 0\,. \tag{7.12}$$

Of course, we are interested in the third case since the two formers allow the generation of a FI term. A simple inspection of (7.12) shows that only the complex number $z = \lambda_c/\kappa$ does matter for the boundary conditions. In fact, given an arbitrary value of z the conditions (7.11) are satisfied for

$$\kappa = \frac{1}{1+|z|^2}\,, \qquad \lambda_c = \lambda^* = \frac{z}{1+|z|^2}\,, \tag{7.13}$$

and, thus, the boundary action (7.4) yields the same boundary conditions as the action
$$\mathcal{S}_{\rm bd} = \int d^4x \left[d^2\theta \, \frac{1}{2} H_c \left(\mathbf{1} + \vec{s}\cdot\vec{\sigma} \right) H + \text{h.c.} \right]. \tag{7.14}$$

With this, (7.12) can be rewritten as

$$H_c \mathcal{Q}_- = 0\,, \qquad \mathcal{Q}_+ H = 0\,, \tag{7.15}$$

where we have defined the projectors

$$\mathcal{Q}_{\pm} = \frac{1}{2}\left(\mathbf{1} \pm \vec{s}\cdot\vec{\sigma}\right) .$$

The Scherk-Schawarz-like breaking of the supersymmetry will be performed as in chapter 6, thus, the generalized bulk action including the radion superfield is given by

$$S_{\rm bk} = \int d^5x\, d^4\theta\, \frac{T+\bar{T}}{2}\, \left[\bar{H}H + \bar{H}_c H_c\right] \\ - \int d^5x\, \left[d^2\theta\, \left(H_c\partial_5 H - T\, H_c\left[M_0\mathbf{1} + M_1\vec{p}\cdot\vec{\sigma}\right] H\right) + \text{h.c.}\right] , \tag{7.16}$$

The breaking of supersymmetry by boundary masses has been used in the literature to solve the naturalness problem with the μ-term in the MSSM, for instance in Refs. [57, 54], however, as we saw previously, this type of breaking yields linearly divergent corrections to the Higgs mass. In addition, as will be shown in a moment, the Higgs spectrum presents a tachyon at the tree level without the necessity of considering any other breaking than the Scherk-Schwarz-like. For the sake of simplicity, then, we will concentrate in the latter type of supersymmetry breaking. The action is given by

$$S = \int d^5x\, d^4\theta\, \frac{T+\bar{T}}{2}\, \left[\bar{H}H + \bar{H}_c H_c\right] \\ - \int d^5x\, \left[d^2\theta\, \left(H_c\partial_5 H - T H_c\, M\vec{p}\cdot\vec{\sigma}\, H\right) + \text{h.c.}\right] \\ + \int d^4x\, \left[d^2\theta\, \frac{1}{2} H_c\left(\mathbf{1} + \vec{s}\cdot\vec{\sigma}\right) H + \text{h.c.}\right] . \tag{7.17}$$

The traceless election of the bulk mass in (7.17) deserves a further explanation. In the presence of a $U(1)$ gauge sector the traceness part of the bulk mass may be absorbed through a redefinition of the real scalar in the vector supermultiplet yielding a bulk term of the form MV'. Through a partial integration it turns into localized FI terms at the boundaries [51] which suffer from linear divergent

corrections. Therefore, in order to reduce the UV sensitivity of the theory we have discard such mass term.
The boundary conditions derived from (7.17) read[1]

$$(\mathbf{1}+R_f)\ A=0\,,\qquad (\mathbf{1}-R_f)\ [A'+Mc_fA]=0\,,\tag{7.18}$$

for the bosonic sector, while for the fermions we have

$$\left(\mathbf{1}+\gamma^5 S_f\right)\ \Upsilon=0\,,\tag{7.19}$$

where R_f are defined as in (6.34), $c_f=\vec{p}\cdot\vec{s}_f$, $\Omega^2=m^2-M^2$ and m is the physical mass[2]. Furthermore, we have defined

$$A=\begin{pmatrix}\bar{\Phi}_c\\ \Phi\end{pmatrix}\,,\qquad \Upsilon=\begin{pmatrix}\psi\\ \bar{\psi}_c\end{pmatrix}\,.\tag{7.20}$$

The spectrum predicted by (7.18) is given by the zeros of the equation

$$\left(\cos\Omega\pi-\frac{c_0M}{\Omega}\sin\Omega\pi\right)\left(\cos\Omega\pi+\frac{c_\pi M}{\Omega}\sin\Omega\pi\right)=\cos^2(\omega\pm\tilde{\omega})\pi\,,\tag{7.21}$$

where $\cos 2\pi\tilde{\omega}=\vec{s}_0\cdot\vec{s}_\pi$. The spectrum for the fermions [44] is given by (7.21) with $\omega=0$.
To finish this section we just give the solution to the equations of motion for the bosonic sector[3]:

$$A_\pm=\begin{pmatrix}\beta_\pm\sin\Omega y\\ \mp\alpha_\pm\left(\cos\Omega y-\frac{c_0M}{\Omega}\sin\Omega y\right)\\ \alpha_\pm\left(\cos\Omega y-\frac{c_0M}{\Omega}\sin\Omega y\right)\\ \pm\beta_\pm\sin\Omega y\end{pmatrix}=\begin{pmatrix}g_\pm(y)\\ \mp f_\pm(y)\\ f_\pm(y)\\ \pm g_\pm(y)\end{pmatrix}\,,\tag{7.22}$$

where through the last equality we have defined the shorthands $f(y)$ and $g(y)$. The constants $\alpha_\pm$, $\beta_\pm$ verify the relation[4]

$$\alpha_\pm\left(\cos\Omega\pi-\frac{c_0M}{\Omega}\sin\Omega\pi\right)s_\pm(\pi)+\beta_\pm\sin\Omega\pi\,c_\pm(\pi)=0\,,\tag{7.23}$$

[1] Taking into account the local $SU(2)_R$ redefinition.
[2] All the masses are given in units of $\frac{1}{R}$ and the extra coordinate is given in units of R.
[3] We have taken $\vec{s}_0=(0,0,1)$.
[4] It is understood that Ω should be replaced by $\Omega_\pm$.

where $s_\pm(\pi) = \sin(\omega \pm \tilde{\omega})\pi$ and $c_\pm(\pi) = \cos(\omega \pm \tilde{\omega})\pi$. Later on we will indentify this solution with the two MSSM Higgs doublets which couple to the matter sector localized on the brane. Before that, let us incorporate the gauge sector to the whole action.

7.2 Gauge interaction

Consider now an internal symmetry generated by a group $\mathcal{G}$. If we want the symmetry to be local we must add a gauge vector supermultiplet which consists of

$$\left(A_M, \lambda^i, \Sigma, \vec{X}\right) ,$$

where A_M is a five-vector, λ^i are two Majorana spinors which transform as a doublet under $SU(2)_R$, Σ is a real scalar and $\vec{X}$ are three real auxiliary fields which transform as a triplet under $SU(2)_R$. All of these fields live in the adjoint representation of $\mathcal{G}$. We can split the vector supermultiplet in $N = 1$ superfields as

$$V = -\theta\sigma^\mu\bar{\theta}\, A_\mu - \mathrm{i}\bar{\theta}^2\theta\lambda^1 + \mathrm{i}\theta^2\bar{\theta}\bar{\lambda}^1 + \frac{1}{2}\bar{\theta}^2\theta^2\, D\,,$$

$$\chi = \frac{1}{\sqrt{2}}\,(\Sigma + \mathrm{i}A_5) + \sqrt{2}\theta\lambda^2 + \theta^2\, F_\chi\,,$$

where $D = X_3 - \partial_5\Sigma$ and $F_\chi = X_1 + \mathrm{i}X_2$. Under a (super)gauge transformation the variation of the superfields are given by

$$e^{qV} \to U^{-1}\, e^{qV}\, U^{-1\dagger}\,, \qquad e^{-qV} \to U^\dagger\, e^{-qV}\, U\,,$$

$$\chi \to U^{-1}\left(\chi - \frac{\sqrt{2}}{q}\partial_5\right) U\,, \qquad \bar{\chi} \to U^\dagger\left(\bar{\chi} + \frac{\sqrt{2}}{q}\partial_5\right) U^{-1\dagger}\,,$$

and

$$H \to U^{-1}\, H\,, \qquad H_c \to H_c\, U\,,$$

with $U = e^{qK^a\,T_a}$. Here K^a are arbitrary chiral superfields, T_a are the generators of the gauge group and q is a dimensionless charge. The total action for the matter and gauge sectors is given by

$$\begin{aligned}
\mathcal{S} = & \frac{1}{4g_5^2}\int d^5x d^2\theta\, T\, \mathrm{Tr}\,(W^\alpha W_\alpha) + \text{h.c.} \\
& - \frac{2}{g_5^2}\int d^5x d^4\theta \frac{1}{T+\bar{T}}\mathrm{Tr}\left[D_5\, e^{qV}\, \bar{D}_5\, e^{-qV}\right] \\
& + \int d^5x\, d^4\theta\, \frac{T+\bar{T}}{2}\, \left[\bar{H} e^{qV} H + H_c e^{-qV} \bar{H}_c\right] \\
& - \int d^5x d^2\theta\, H_c \left(\partial_5 - T\, M\vec{p}\cdot\vec{\sigma} - \frac{q}{\sqrt{2}}\chi\right) H + \text{h.c.} \\
& + \int d^4x d^2\theta\, \frac{1}{2} H_c\, (\mathbf{1} + \vec{s}\cdot\vec{\sigma})\, H + \text{h.c.}\,,
\end{aligned} \tag{7.24}$$

where g_5 is the gauge coupling in units of $\frac{1}{\sqrt{R}}$ and

$$D_5\, e^{qV} = \frac{1}{q}\partial_5\, e^{qV} - \frac{1}{\sqrt{2}}\chi\, e^{qV} - \frac{1}{\sqrt{2}} e^{qV}\, \bar{\chi}\,,$$

$$\bar{D}_5\, e^{-qV} = \frac{1}{q}\partial_5\, e^{-qV} + \frac{1}{\sqrt{2}}\bar{\chi}\, e^{-qV} + \frac{1}{\sqrt{2}} e^{-qV}\, \chi\,,$$

are the covariant derivatives[5] with respect $\mathcal{G}$. For the Abelian case the bulk action of (7.24) reduces to that in ref. [51], without the radion superfield.
The effect of the Scherk-Schwarz supersymmetry breaking on the gauge sector has been widely studied in the literature from different approaches [53, 42]. From now on we will concentrate on the effective action for the lightest mode of the Higgs field. For that goal we will take into account effective couplings generated by the integration of the massive modes in the gauge sector. In the next section we will show explicitly how the integration of Σ induces an effective quartic coulping on the Higgs field.

[5]Notice that χ is the connection that makes the derivative ∂_5 covariant with respect the gauge transformations [18, 17].

7.3 Effective action for hyperscalars and induced EWSB

In ref. [9] it was shown that the coupling between the gauge sector propagating in the bulk and the mass sector located at the boundaries induced singular terms proportional to $\delta(0)$. In addition, it was proven that taking into account the effective couplings generated by the integration of the gauge scalar Σ one gets rid off the singular terms. In a similar way we will show that the lower mode effective action for the scalar matter sector is the MSSM. For that, as we will see, we need to compute the effective couplings induced by the integration of Σ. On the other hand, the integration of the higher modes in the Kaluza-Klein tower (with masses $\gtrsim 1/R$) makes sense only when we have a light mode (with mass $\ll 1/R$). But, as was shown in [44], the spectrum (7.18) allows massless and very light modes. Actually, for $M \to \infty$ (in practice, $M \gg 1$, in units of $1/R$) (7.18) has the solution

$$\frac{m_{\pm}^2}{M^2} = s_0^2 + 4c_0^2 \left[1 - \frac{2c_{\pm}^2}{1 + c_\pi/c_0}\right] e^{-2M\pi|c_0|}\,, \tag{7.25}$$

where $c_\pm$ has been defined in (7.23).

As pointed out above, the effective couplings for the lightest scalar modes will be given by the integration of the the heavy KK modes. In particular, an effective quartic self interaction is induced by the integragion of Σ. From (7.24) we extract the interaction between Σ and the hyperscalars

$$\mathcal{L}(\Sigma, \Phi, \Phi_c) = \int dy \left[-\frac{1}{2g_5^2}\Sigma^a \Box \Sigma_a - \frac{1}{2g_5^2}\left(\partial_5 \Sigma^a\right)^2 - A^\dagger \left(q\Sigma^a T_a + \mathcal{M}\right)^2 A \right.$$
$$\left. - \frac{g_5^2\, q^2}{2}\left(A^\dagger \vec{\sigma}\, T^a A\right)^2 \right], \tag{7.26}$$

where $\vec{\sigma}$ acts on $SU(2)_R$ space and A is defined as in (7.18). Admiting Σ to be a function of the extra coordinate only[6], the equations of motion read

$$\partial_5^2 \Sigma^a - 2q\, g_5^2\, A^\dagger \mathcal{M} T^a A - 2q^2\, g_5^2\, \Sigma_b\, A^\dagger T^b\, T^a A = 0\,. \tag{7.27}$$

The term liniar in Σ does not yield a quartic coupling in A, instead it induces a very complicated exponential dependence. The solution neglecting such a term is

$$\Sigma^a(y) = 2qg_5^2 \int_0^y d\xi\, F^a(\xi) - y\frac{2qg_5^2}{\pi} \int_0^\pi d\xi\, F^a(\xi)\,, \tag{7.28}$$

where $F^a(y) = \int_0^y d\xi\, A^\dagger \mathcal{M}\, T^a A$ and we have made use of the boundary conditions $\Sigma^a|_{0,\pi} = 0$. Plugging the soultion in (7.26) we find the effective quartic self-interaction[7]

$$\mathcal{L}_{\text{eff}} = 2q^2 g_5^2 \int_0^\pi \left[F^a(y)\right]^2 - \frac{2}{\pi} q^2 g_5^2 \left[\int_0^\pi F^a(y)\right]^2 - \frac{g_5^2\, q^2}{2} \int_0^\pi D(y)\,, \tag{7.29}$$

where we have defined

$$D(y) = \left(A^\dagger \vec{\sigma}\, T^a A\right)^2\,.$$

On the other hand, for the mass eigenvalue (7.25), (7.22) can be rephrased as

$$A_\pm(x,y) = H_\pm(x)\sqrt{Mc_0} \left[\begin{pmatrix} 0 \\ \mp 1 \\ 1 \\ 0 \end{pmatrix} e^{-Mc_0\, y} - \begin{pmatrix} t_\pm \\ \pm\Delta_\pm \\ -\Delta_\pm \\ \pm t_\pm \end{pmatrix} e^{Mc_0\, y} \epsilon^2 \right]\,, \tag{7.30}$$

where we have assumed c_0 to be positive and we have neglected higher order corrections in ϵ. In addition, we have defined $\Delta_\pm = 1 - \frac{2c_\pm^2}{1+c_\pi/c_0}$, $t_\pm = \tan(\omega \pm \tilde{\omega})\pi$

[6]The integration of Σ neglecting the 4D kinetic term is equivalent to sum up all the diagrams with Σ in the propagator for an external 4D momentum, p, such that $p \ll 1/R$. This makes sense since the boundary conditions for Σ derived from (7.24) are $\Sigma|_{0,\pi} = 0$ and therefore, there is no zero mode.

[7]We have a similar coupling between the hyperscalars A and the fifth component of the gauge vector field, A_5, nevertheless such an interaction can not produce any effective quartic coupling since we can always choose a gauge where $A_5 = 0$.

and $\epsilon = e^{-Mc_0\pi}$. $H_\pm(x)$ are 4D complex scalars carrying the gauge indices. From (7.30) and taking into account the definition of A, we can identify the up and down Higgses of the MSSM as

$$H_u \equiv \frac{1}{\sqrt{2}}\left(H_+ + H_-\right), \tag{7.31}$$

$$H_d \equiv \frac{1}{\sqrt{2}}\left(\bar{H}_- - \bar{H}_+\right). \tag{7.32}$$

By doing such identification we are not considering a correction of order ϵ^2 but, for our purposes, this correction is negligible. For $\mathcal{M} = M(s_0, 0, c_0)$ (7.29) takes the form

$$\mathcal{L}_{\text{eff}} = -q^2\frac{g_5^2}{2\pi}\left(\bar{H}_u T^a H_u - H_d T^a \bar{H}_d\right)^2 + \mathcal{O}(\epsilon^2). \tag{7.33}$$

The leading order in (8.52) is precisely the quartic self interaction of the MSSM for $q = 1/2$.

For the mass matrix we have (in the basis $H_{u,d}$)

$$\begin{pmatrix} \mu^2 + m^2 & b \\ b & \mu^2 + m^2 \end{pmatrix}, \tag{7.34}$$

where we have made the definitions

$$m^2 = 8M^2c_0^2\frac{\sin^2\pi\omega\,\cos 2\pi\tilde{\omega}}{1 + c_\pi/c_0}\,\epsilon^2, \qquad b = 4M^2c_0^2\frac{\sin 2\pi\omega\,\sin 2\pi\tilde{\omega}}{1 + c_\pi/c_0}\,\epsilon^2,$$

with

$$\mu^2 = M^2s_0^2 + 4M^2c_0^2\left(1 - 2\frac{\cos^2\pi\tilde{\omega}}{1 + c_\pi/c_0}\right)\epsilon^2,$$

being the supersymmetric mass. Allowing $s_0 \sim \epsilon$ then $c_0 \sim 1$ and $c_\pi \sim \cos 2\pi\tilde{\omega}$. In this case, the tree-level potential for the $H_{u,d}$ basis then reads

$$\begin{aligned} V =& m_u^2|H_u|^2 + m_d^2|H_d|^2 + m_3^2\left(H_uH_d + \text{h.c.}\right) \\ &+ \lambda\left(\bar{H}_u T^a H_u - H_d T^a \bar{H}_d\right)^2, \end{aligned} \tag{7.35}$$

with

$$m_u^2 = m_d^2 = M\, s_0^2 + 4M^2 \sin^2 \pi\omega\, \left[1 - \tan^2 \pi\tilde{\omega}\right] \epsilon^2\,, \tag{7.36}$$

$$m_3^2 = 4M^2 \sin 2\pi\omega \tan \pi\tilde{\omega}\, \epsilon^2 \;\text{ and }\; \lambda = \frac{g_5^2}{8\pi}\,. \tag{7.37}$$

The smallness of s_0 can be understood in terms of spontaneous breaking of a global symmetry by boundary terms. Actually, the geometrical interpretation of the bulk and boundary masses tells us that for $s_0 = s_\pi = 0$ all matrices are aligned along the same axis $\vec{p}$ and hence a $U(1)$ group of rotations around this axis leaves the whole action invariant being then broken by the misalignment of the boundary matrices. The zeroth modes of the Higgs feel this breaking at tree level non the less the breaking at $y = \pi$ is suppressed by the exponential localization of the Higgs while the breaking at $y = 0$ is felt at $\mathcal{O}(1)$. A dinamical solution to this naturalness problem could be the effective coupling of the Higgs with a some (spurion) SM filed, $\mathcal{S}(x)$, localized at $y = 0$

$$\frac{1}{\Lambda}\mathcal{S}(x) H^1(x,0) H_c^2(x,0)\,, \tag{7.38}$$

with Λ the 5D cutoff of the theory. If we suppose the $U(1)$ symmetry to be excat at $y = 0$ at the cutoff scale and this symmetry is only broken by the VEV acquired by $\mathcal{S}(x)$, $\langle\mathcal{S}\rangle$, then the μ-term will be proportional to $\delta = \frac{\langle\mathcal{S}\rangle}{\Lambda}$ being then a small quantity if the breaking take place at a lower scale.

7.4 ElectroWeak symmetry breaking

In this section we will investigate in some detail the possibility of EWSB [55]. The conditions for EWSB and stability of the flat $|H_u| = \pm|H_d|$ directions

$$\begin{aligned} (\mu^2 + m_{H_u}^2)\,(\mu^2 + m_{H_d}^2) &< m_3^4 \\ 2\mu^2 + m_{H_u}^2 + m_{H_d}^2 &> 2|m_3^2| \end{aligned} \tag{7.39}$$

are incompatible with the tree-level induced SS supersymmetry breaking where $m_{H_u}^2 = m_{H_d}^2$. In this way EWSB should proceed radiatively and we must incorporate radiative corrections to the Higgs potential. As matter is strictly localized and Higgses are quasi-localized, SUSY breaking will predominantly be mediated by one-loop gaugino loops that provide a (positive) contribution to the squared masses of squarks, sleptons and Higgses.

In particular the squark masses will be dominated by the contribution from the gluinos which is given by [42, 27, 28]

$$\Delta m^2_{\tilde{t},\tilde{b}} = \frac{2\, g_3^2}{3\pi^4}\, M_c^2\, f(\omega) \tag{7.40}$$

where $M_c = \frac{1}{R}$ and the function $f(\omega)$ is defined by

$$f(\omega) \equiv \sum_{k=1}^{\infty} \frac{\sin(\pi k \omega)^2}{k^3}\ , \tag{7.41}$$

while electroweak gauginos provide a radiative correction to the slepton and Higgs masses as

$$\Delta^{(1)} m^2_{H_u} = \Delta^{(1)} m^2_{H_d} = \frac{3g^2 + g'^2}{8\pi^4}\, M_c^2 f(\omega) \tag{7.42}$$

Furthermore there is a sizable two-loop contribution to the Higgs soft mass terms, as well as to the quartic coupling, coming from top-stop loops with the one-loop generated squark masses given by 7.40. This contribution can be estimated in the large logarithm approximation by just plugging the one-loop squark masses in the one-loop effective potential generated by the top-stop sector [42, 27, 28]. The goodness of this approximation has been shown in Ref. [58, 59, 56] where a rigorous two-loop calculation of the effective potential has been performed. Since in our case the EWSB will not be marginal (as we will see later) it is enough to consider the effective potential in the large logarithm approximation, which yields the two-loop corrections to the Higgs masses

$$\Delta^{(2)} m^2_{H_u} = \frac{3y_t^2}{8\pi^2} \Delta m^2_{\tilde{t}} \log \frac{\Delta m^2_{\tilde{t}}}{\mathcal{Q}^2}\ , \tag{7.43}$$

$$\Delta^{(2)} m^2_{H_d} = \frac{3y_b^2}{8\pi^2} \Delta m^2_{\tilde{b}} \log \frac{\Delta m^2_{\tilde{t}}}{\mathcal{Q}^2}\ , \tag{7.44}$$

where the renormalization scale should be fixed to the scale of SUSY breaking, i.e. the gaugino mass ωM_c [42, 27, 28]. Notice that the corrections from the bottom sector are also considered, which would only be relevant for large values of $\tan\beta$.

A word has to be said about the bulk Higgs-Higgsino one-loop contribution to the soft masses. The reason we did neglect them with respect to the one-loop gauge contribution (and even the leading two-loop one) above is that they are strongly suppressed due to their quasi-localization. The leading $\mathcal{O}(\epsilon)$ corrections come from the Higgs-Higgsino loop contribution to the stop mass. They are proportional to the tree level soft Higgs mass $m_{H_u}^2 \sim M^2\epsilon^2$ and hence suppressed as $\epsilon^2 \log\epsilon$ with respect to the gluon-gluino contribution of (7.40). We will typically find values of $\epsilon \sim 10^{-2}$ and thus these corrections are really subleading. In principle we could easily incorporate in our analysis the radiative corrections to m_3^2 as calculated in Ref. [60]. However for most of the part of parameter space we are interested in, this is only a tiny correction to the tree level value, (7.36), and we will neglect it in our analysis.

Finally, the leading two-loop corrections to the quartic self coupling of H_u and H_d in the potential

$$\Delta V_{\text{quartic}} = \Delta\gamma_u |H_u|^4 + \Delta\gamma_u |H_u|^4 \tag{7.45}$$

are given by

$$\Delta\gamma_u = \frac{3y_t^4}{16\pi^2} \log\frac{\Delta m_{\tilde{t}}^2 + m_t^2}{m_t^2}\,, \tag{7.46}$$

$$\Delta\gamma_d = \frac{3y_b^4}{16\pi^2} \log\frac{\Delta m_{\tilde{b}}^2 + m_b^2}{m_b^2}\,. \tag{7.47}$$

where m_t and m_b are the the top and bottom quark masses respectively.

Electroweak symmetry breaking can now occur in our model in a very peculiar and interesting way. In fact the tree-level squared soft masses m_{H_u,H_d}^2 given in (7.36) are suppressed by the factor ϵ^2 and therefore, for values of $M \sim M_c$ they can be comparable in size to the one-loop gauge corrections $\Delta^{(1)} m_{H_u,H_d}^2$ given by (7.42). Furthermore the tree-level masses m_{H_u,H_d}^2 are negative for values of $\tilde{\omega} > 1/4$ and then there can be a (total or partial) cancellation between the tree-level and one-loop contributions to the Higgs masses. Under extreme conditions they can even cancel, $m^2_{H_u,H_d} + \Delta^{(1)} m^2_{H_u,H_d} \simeq 0$, in which case the negative two-loop corrections $\Delta^{(2)} m^2_{H_u}$ will easily trigger EWSB. On the other hand in the limit of exact localization of the Higgs fields $\epsilon \to 0$ the tree-level masses will vanish and the one-loop gauge and two-loop top-stop corrections have to compete, which will make the EWSB marginal, as pointed out in Refs. [58, 59, 56]. Similarly for $\tilde{\omega} \leq 1/4$ the tree level masses $m^2_{H_u,H_d}$ are positive definite making the EWSB triggering to be difficult, albeit not impossible, for instance, by somehow delocalizing the top-stop right handed (or left-handed) multiplet as it is done in Refs.[58, 59, 56, 60, 61]. These simple arguments prove that there is a wide region in the space of parameters $(\omega, \tilde{\omega}, \epsilon)$ where EWSB easily happens without any fine-tuning of these parameters. Of course EWSB also depends on the Higgsino mass μ and on the compactification scale M_c (or equivalently on the gluino mass as it happens in the MSSM) and we will be concerned about the possible fine-tuning in those mass parameters.

It is easy to check that, due to the smallness of the SUSY breaking scale which will be in the TeV region, as well as the extreme softness of the SS mechanism, the usual fine-tuning problems of the MSSM can almost entirely be avoided. To see this consider the Z mass from the minimization conditions of the potential in the limit $1 \ll \tan^2\beta \ll m_t^2/m_b^2$

$$\frac{m_Z^2}{2} = -(\mu^2 + m^2_{H_u} + \Delta^{(1)} m^2_{H_u} + \Delta^{(2)} m^2_{H_u}) \,. \tag{7.48}$$

As it is intuitively clear, essentially no fine tuning is necessary if we can make EWSB to work with all terms in (7.48) roughly of electroweak size. Let us quantify a little further this statement by considering the sensitivy [62] with respect to the fundamental parameters M_i

$$\Delta_{M_i} = \left| \frac{M_i^2}{m_Z^2} \frac{\partial m_Z^2}{\partial M_i^2} \right| \tag{7.49}$$

where $M_i = \mu, m_{H_u}, M_c$ [8] In terms of these fundamental parameters (7.48) can be

[8] We are defining our fundamental parameters such that the sensitivity on them is really a measure of fine-tuning in the sense of Ref. [63].

rewritten as

$$m_Z^2 = -2\mu^2 - 2m_{H_u}^2 - \kappa M_c^2 \tag{7.50}$$

where typically $\kappa \sim 10^{-3}$, and the corresponding sensitivity parameters are given by

$$\begin{aligned}\Delta_\mu &= \frac{2\mu^2}{m_Z^2} \\ \Delta_{M_c} &= |\kappa|\,\frac{M_c^2}{m_Z^2} \\ \Delta_{M_{H_u}} &= |1 + \Delta_\mu + \text{sign}(\kappa)\Delta_{M_c}|\end{aligned} \tag{7.51}$$

In Fig. 7.1 we plot the three sensitivity parameters in (7.51) for the model, that we will present in section 7.5, corresponding to $\omega = 0.45$, $\tilde{\omega} = 0.35$ and $M = 1.65 M_c$. This model gives a viable spectrum and it is consistent with all electroweak precision observables for $M_c \gtrsim 6.5$ TeV. As one sees from Fig. 7.1 and Eq. (7.51) the largest

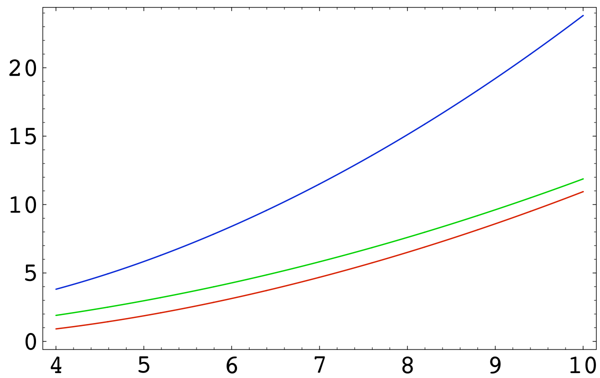

Figure 7.1: *The sensitivity parameters in Eq. (7.51) as functions of M_c in TeV for the case $\omega = 0.45$, $\tilde{\omega} = 0.35$ and $M = 1.65 M_c$. From top to bottom the lines are: $\Delta_{m_{H_u}}$ (blue line), Δ_{M_c} (green line) and Δ_μ (red line).*

sensitivity appears to be with respect to the parameter m_{H_u}. In fact for $M_c = 6.6$ TeV the required amount of fine-tuning is $\sim 10\%$ while for larger values of M_c the fine-tuning naturally increases quadratically. Thus for instance for $M_c = 10$ TeV the fine-tuning is $\sim 4\%$

We can now compare this situation with the one in the MSSM. The gluino mass for a given value of M_c is $M_3 = \omega M_c$ so that in our example, for $M_c \sim 10$ TeV we have $M_3 \sim 5$ TeV. In the MSSM the Z mass squared is proportional to M_3^2 for the same reason as in our model, but with a much larger coefficient $\mathcal{O}(1)$ due to large logarithms $\log m_Z/m_{\text{GUT}}$. A gluino of mass a few TeV in the MSSM will require a ($\tan\beta$ dependent) fine-tuning as large as 0.01%. A careful treatment of the fine tuning issues related to the gluino mass can be found in Ref. [64, 65].

7.5 Supersymmetric spectra and Dark Matter

We will now calculate the Higgs and superpartner spectra for some specific values of the parameters. We would like to plot our predictions as functions of M_c with all other parameters $(\omega, \tilde{\omega}, M)$ fixed. Because of the exponential dependence of the tree level soft masses it will prove convenient to trade M by ϵ (which provides a fixed ratio of M/M_c) when varying over M_c in order to avoid excessively large or small masses.

The parameters ω and $\tilde{\omega}$ give $\mathcal{O}(1)$ coefficients in the soft parameters. Their possible values can be further restricted by demanding that the right-handed slepton mass $m_{\tilde{e}_R}$ be above the mass of the lightest neutralino, as there are strong constraints on charged stable particles [66] and we would like the lightest neutralino to be the lightest supersymmetric particle (LSP) and a Dark Matter candidate. For the nature of the latter notice that gaugino masses are given by ωM_c while Higgsino masses are essentially controlled by the μ-parameter. We thus expect the neutralino to be almost pure Higgsino with a mass basically given by μ. On the other hand the right handed slepton mass is radiatively generated and proportional to $g' M_c$. The size of the μ term is determined by the minimization conditions and will increase $\sim M_c$ for large M_c (as it has to compensate the negative radiative corrections to $m_{H_u}^2$). However the tree level soft mass terms (7.36) increase for smaller $\tilde{\omega}$ which in turn allows for a smaller μ. The requirement that the neutralino be lighter than the charged sleptons thus favours the region $\omega > \tilde{\omega}$.

We then solve the minimization conditions for EWSB which will give us two predictions, $\tan\beta$ and μ as functions of the only left free parameter, M_c. Then all masses will become functions of M_c. In particular in the Higgs sector all masses are obtained from the effective potential where the one-loop corrections to the quartic couplings are included. The mass of the SM-like Higgs is then computed with radiative corrections to the quartic couplings considered at the one-loop level. It is well known that including just the one-loop effective potential overestimates somehow the Higgs masses and improving the effective potential by an RGE resummation of leading logarithms provides more realistic results. In this paper we will nevertheless be content by evaluating masses in the one-loop approximation. The squark and slepton masses are dominated by the gaugino loop contribution and hence grow approximately linearly with M_c. We find [42, 27, 28]

$$(m_{\tilde{q}_L},\, m_{\tilde{u}_R},\, m_{\tilde{d}_R},\, m_{\tilde{\ell}_L},\, m_{\tilde{e}_R}) = (0.110,\, 0.103,\, 0.102,\, 0.042,\, 0.025)\sqrt{f(\omega)}M_c \quad (7.52)$$

where the function $f(\omega)$ is given in Eq. (7.41) [9].

On the other hand the gauginos have a mass given by

$$M_{1/2} = \omega M_c\ , \quad (7.53)$$

and the Higgsinos, charginos and neutralinos, a mass approximately equal to μ, $m_{\tilde{\chi}^\pm} \simeq m_{\tilde{\chi}^0} \simeq \mu$. They are quasi-degenerate in mass and its mass difference can be given to a very good approximation (for $\mu < 0$) by [67]

$$\frac{\Delta m_{\tilde{\chi}}}{m_W} \equiv \frac{m_{\tilde{\chi}^\pm} - m_{\tilde{\chi}^0}}{m_W} \simeq (0.35 + 0.65 \sin 2\beta)\frac{m_W}{M_{1/2}} \quad (7.54)$$

which means that typically e.g. for $M_c \sim 10$ TeV, $\Delta m_{\tilde{\chi}} \sim 1$ GeV. The phenomenology for Tevatron and e^+e^- colliders of models where charginos and neutralinos are quasi-degenerate in mass was worked out in Refs. [68, 69]. The most critical ingredients in the phenomenology of these models are the lifetime and decay modes of $\tilde{\chi}^\pm$ which in turn depend almost entirely on $\Delta m_{\tilde{\chi}}$. Conventional detection of sparticles is difficult since the decay products ($\tilde{\chi}^\pm \to \tilde{\chi}^0\pi^\pm, \tilde{\chi}^0\ell^\pm\nu_\ell, \dots$) are very soft and alternative signals must be considered [68, 69].

[9] Numerically $f(\omega) \lesssim 1$ for the values of ω we will be interested in.

We will now consider in detail a typical example that will be solved numerically and we will plot all the predictions of the model as functions of M_c. We choose $\omega = 0.45$, $\tilde{\omega} = 0.35$ and $M = 1.65 M_c$ as in the previous example of Fig. 7.1 where the fine-tuning in these models is exemplified. The results are shown in Fig. 7.2. The

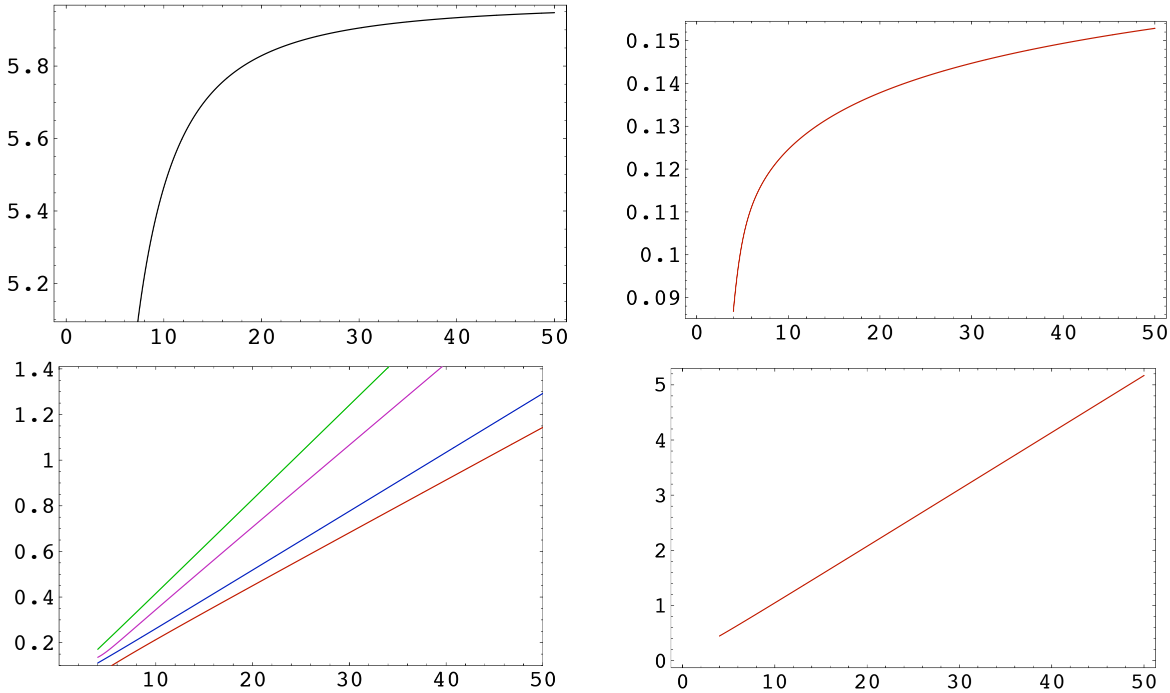

Figure 7.2: *Predictions for the case $\omega = 0.45$, $\tilde{\omega} = 0.35$, $M = 1.65 M_c$ (as in Fig. 7.1) as a function of the compactification scale. Upper left panel:* $\tan\beta$*. Upper right panel: the SM-like Higgs mass m_h. Lower left panel, from top to bottom the lines correspond to the masses of: left-handed sleptons $m_{\tilde{\ell}_L}$ (green line), heavy neutral Higgs (with a mass approximately equal to the pseudoscalar mass) $m_H \simeq m_A$ (magenta line), right-handed sleptons $m_{\tilde{e}_R}$ and neutralinos $m_{\chi^0} \simeq \mu$ (red line). Lower right panel: the squark masses $m_{\tilde{q}}$. All masses are in TeV.*

SM-like Higgs mass easily satisfies the experimental bound $m_{h^0} > 114.5$ GeV for $M_c > 6.5$ TeV. The LSP is the Higgsino-like with mass $\sim \mu$. Electroweak precision observables also put lower bounds on M_c (see e.g. Ref. [28]). For the particularly chosen model the $\chi^2(M_c)$ distribution has a minimum around $M_c \simeq 10.5$ TeV and one deduces $M_c > 4.9$ TeV at 95% c.l.

Finally in the considered class of models where the neutralino is the LSP and

R-parity is conserved the lightest neutralino is the candidate to Cold Dark Matter. In fact the prediction of $\Omega_{\tilde{\chi}^0}h^2$ can be obtained using the DarkSUSY package [70] and can also be approximated by the expression [71]

$$\Omega_{\tilde{\chi}^0}h^2 \simeq 0.09\,(\mu/TeV)^2 \tag{7.55}$$

In the particular model of Fig. 7.2 the prediction of $\Omega_{\tilde{\chi}^0}h^2$ is given in Fig. 7.3

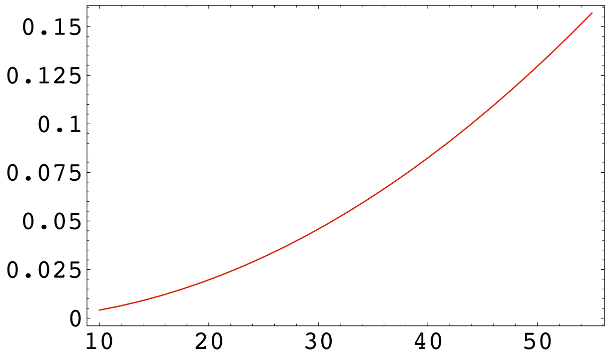

Figure 7.3: $\Omega_{\tilde{\chi}^0}h^2$ *as a function of* M_c *(in TeV) for the model presented in Fig. 7.2.*

Recent WMAP results [72] imply that $0.114 < \Omega_{\tilde{\chi}^0}h^2 < 0.134$. As one can see from Fig. 7.3 this range in $\Omega_{\tilde{\chi}^0}h^2$ points towards the range [10] $49\,\text{TeV} < M_c < 53\,\text{TeV}$. Then for a value of $M_c \sim 50$ TeV the density of Dark Matter agrees with the recent results obtained from WMAP. Notice that for such large values of M_c the neutralinos are almost Dirac particles. However the non-Diracity is spoiled by $\mathcal{O}(m_W/M_{1/2})m_W \sim 300$ MeV which is enough to avoid the strong limits on Dirac fermions that put a lower bound on the non-Diracity around 100 KeV [73, 74]. On the other hand the WMAP range for M_c implies, in the gravitational sector, gravitino masses $m_{3/2} \gtrsim 10$ TeV (depending on the value of the SS parameter ω) are such that gravitinos decay early enough to avoid cosmological troubles and thus solving the longstanding cosmological gravitino problem [75].

[10] Of course, such large values of M_c require a fine tuning $< 1\%$, see section. 7.4.

Chapter 8

Neutrino masses from a flat 5D space

In the previous chapter we presented and developed a model for the electroweak symmetry breaking defined in the interval. Here we continue with the phenomenological insight of this proposal. In particular, we wonder whether this model could predict light mass eigenvalues for standard model neutrinos.
The exponential localization of the Higgs field towards the zero brane induces light effective Yukawa couplings between the Higgs and the matter localized in (or towards to) the π brane and on the other hand, if we allow the RH neutrinos to propagate in the bulk, the eigenvalues of the mass are naturally $m_n \gtrsim R^{-1}$, the invers o the compactifacation radius, therefore, at first glance, it seems to be a chance for a see-saw-like mechanism. In fact, in the last years a great scientific effort has been dedicated to this topic [76, 77, 78, 79, 80, 81]. Non the less, before presenting the model we will briefly review the 4 dimensional (original) see-saw mechanism for neutrino mass generation.

8.1 4D see-saw mechanism

Standard model neutrinos are fermions with well defined chirality, as a consequence their mass couplings should be of Majorana type, otherwise the propagator will mixed the two chiralities, as it is the case of the electron, for example. Thus, the usual Higgs mechanism for mass generation [82, 83] is not suitable. Instead, the see-saw mechanism [84, 85] proposes the existence of heavy right handed neutrinos (RH) which couple with the standard model left-handed ones (LH) through Yukawa couplings. If we call ψ_R and ψ_L the RH and LH neutrinos respectively, M_R the RH mass and λ the Yukawa coupling then we have

$$L = \cdots + M_R \bar{\psi}_R \psi_R + \lambda\, h \bar{\psi}_R \psi_L + \lambda\, h^* \bar{\psi}_L \psi_R \tag{8.1}$$

with h being the Higgs boson. Once it acquires its Vacuum Expectation Value (VEV), say v, the Lagrangian becomes an effective mass matrix given by

$$L = \cdots + \left(\bar{\psi}_L, \bar{\psi}_R\right) \cdot \begin{pmatrix} 0 & \lambda\, v \\ \lambda\, v & M_R \end{pmatrix} \cdot \begin{pmatrix} \psi_L \\ \psi_R \end{pmatrix} . \tag{8.2}$$

If $M_R \gg v\lambda$ then the eigenstates of the mass matrix are

$$\nu_L \equiv \psi_L - \frac{v\lambda}{M_R}\psi_R\,, \qquad \text{with mass} \qquad m_L = \frac{v^2\lambda^2}{M_R}\,,$$

$$\nu_R \equiv \psi_R + \frac{v\lambda}{M_R}\psi_L\,, \qquad \text{with mass} \qquad m_R = M_R\,,$$

thus, if M_R is large enough and λ is small enough we obtain an ultra light Majorana mass. Nevertheless, taking a RH mass $M_R \gtrsim 100\,\text{GeV}$ to yield a Majorana mass of the order of meV we need $\lambda \sim 10^{-7}$, which is a severe suppression.

8.2 5D mechanism for generation of neutrino masses

Many authors have used a 5D generalization of the See-Saw mechanism for neutrino mass generation with a right handed neutrino propagating in the 5D bulk and thus with a mass naturally at the TeV scale. However, as Dienes, Dudas and Gherghetta [77] have shown the effect of the Higher modes in the decomposition of RH neutrino spectrum yields an effective Majorana mass for the SM left handed neutrinos which in general spoils the See-Saw mechanism, as we will see below. Non the less it is still possible to generate ultra light neutrino masses[1] in a way to be

[1]Majorana and Dirac.

discussed in a moment. By now let us introduce the model. For it we will consider an extension of the electroweak symmetry breaking model presented in the previous chapter [86], that is: a supersymmetric ($N = 1$) theory defined in the interval $\Sigma = M^4 \times I$ with a compactification radius $R^{-1} \sim$ few TeV. In what follows we first present the action for the RH neutrinos and then we will apply the general results to the phenomenology of neutrinos and charged leptons. Let us take $N_{(c)} = \phi_{(c)} + \sqrt{2}\,\theta\psi_{(c)} + \theta^2 F_{(c)}$ as the content of the RH hypermultiplet. For them we take the most general supersymmetric action defined in the interval. As we saw in chapter 6 it is given by:

$$\begin{aligned} S = & \int_\Sigma d^4\theta\ \left[\bar{N}N + \bar{N}_c N_c\right] - \int_\Sigma d^2\theta\, N_c \partial_5 N + \text{h.c.} \\ & + \int_\Sigma d^2\theta \left(a N_c N + \frac{b^*}{2} N^2 - \frac{b}{2} N_c^2 \right) + \text{h.c.} \\ & + \int_{\partial\Sigma} d^2\theta \left(\frac{s_-}{4} N^2 - \frac{s_+}{4} N_c^2 + \frac{1+s_3}{2} N N_c \right) + \text{h.c.}\,, \end{aligned} \tag{8.3}$$

where $a \in \mathbb{R}$, $b \in \mathbb{C}$ are constants with dimension of energy and $s_\pm = s_1 \pm \mathrm{i}s_2$,

$$\vec{s} = (s_1, s_2, s_3), \tag{8.4}$$

being a unitary vector. For simplicity we just omitted the subscript indicating the boundary, although, except explicit mention, we take different parameters at $y = 0$ and $y = \pi$. The variational principle on (8.3) yields the boundary conditions

$$\frac{1}{2}\left(\mathbf{1} - \vec{s}\cdot\vec{\sigma}\right) \begin{pmatrix} \phi_c \\ \phi \end{pmatrix} = 0\,,$$

$$\frac{1}{2}\left(\mathbf{1} + \vec{s}\cdot\vec{\sigma}\right)\left[\partial_y + \vec{p}\cdot\vec{s}\,M\,\right] \begin{pmatrix} \phi_c \\ \phi \end{pmatrix} = 0\,, \tag{8.5}$$

$$\frac{1}{2}\left(\mathbf{1} - \vec{s}\cdot\vec{\sigma}\right) \begin{pmatrix} \psi_c \\ \psi \end{pmatrix} = 0\,, \tag{8.6}$$

where we have defined the shorthands

$$\vec{p} = \frac{1}{\sqrt{a^2 + |b|^2}}(b_R, -b_I, a)\,, \qquad M = \sqrt{a^2 + |b|^2}\,. \tag{8.7}$$

The spectrum allowed by these boundary conditions can be read off from Chapter 5, and it is provided be the zeroes of the function [44, 55]

$$\sin^2(\pi\tau) - (c_0 - c_\pi)\frac{M}{\Omega}\tan(\pi\Omega\, R) - \left[\cos^2(\pi\tau) + c_0 c_\pi \frac{M^2}{\Omega^2}\right]\tan^2(\pi\Omega\, R)\,, \qquad (8.8)$$

with $\cos(2\pi\tau) = \vec{s}_0 \cdot \vec{s}_\pi$, $c_f = \vec{p}\cdot\vec{s}_f$ and $\Omega^2 = m^2 - M^2$, m being the physical mass. Notice that the spectrum equation depends only on $SO(3)$ invariants related to the relative configuration of the bulk and boundary matrices. Hence a continuous group of $SU(2)$ transformations acting on the fields lives the spectrum of the theory invariant. To see this explicitly let us rewrite (8.3) in a more compact way as

$$\begin{aligned}\mathcal{S} \;&=\; \int_\Sigma \bar{\mathcal{N}}\mathcal{N}\big|_{\bar\theta^2\theta^2} - \frac{1}{2}M\,\mathcal{N}^T\epsilon\,\vec{p}\cdot\vec{\sigma}\,\mathcal{N}\big|_{\theta^2} - \frac{1}{2}\,\mathcal{N}^T\epsilon\,\mathcal{N}'\big|_{\theta^2} + \text{h.c.} \\ &+\; \frac{1}{4}\int_{\partial\Sigma} \mathcal{N}^T\epsilon\,(\mathbf{1} - \vec{s}\cdot\vec{\sigma})\,\mathcal{N}\big|_{\theta^2} + \text{h.c.}\,, \end{aligned} \qquad (8.9)$$

where

$$\mathcal{N} = \begin{pmatrix} N_c \\ N \end{pmatrix}\,, \qquad (8.10)$$

$\vec{\sigma} \in \mathfrak{su}(2)$ are the Pauli matrices and

$$\epsilon = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\,. \qquad (8.11)$$

Now it is clear that a unitary rotation acting on $(N_c, N)^T$ translates into an $SO(3)$ rotation acting on the vectors defining the matrices, thus the spectrum is left invariant. Geometrically, these (global) unitary transformations change the basis where the bulk and boundary matrices are expressed but the relations between them remain unaltered. Notice that these transformations are not symmetries of the action since albeit they leave the kinetic term invariant, the bulk and boundary mass matrices transform covariantly. Non the less, the solutions connected by them represent, indeed, the same physical 4D state. In the next section we shall use this spectrum invariance to solve the equations of motion.

8.2.1 Spectrum and wave functions

We first note that the general bulk mass configuration displayed above is not really suitable to solve the equations of motion for the fermions. For them we have a mixture of Majorana and Dirac mass term, which add difficulty to the resolution. In particular, as we shall see in a moment, the usual separable solution taken in the orbifold case, i.e. $\Psi(x,y) = \big(f(y)\psi(x), g(y)\bar{\xi}(x)\big)^T$, here does not work in general. However, the set of unitaries

$$\{U(\vec{p})\} \cup \{U(p_1,p_2,0)\} \ , \tag{8.12}$$

where

$$U(\vec{p}) = \frac{1}{2\sqrt{1-s_{\vec{p}}}} \begin{pmatrix} e^{\mathrm{i}\delta_{\vec{p}}} & 0 \\ 0 & e^{-\mathrm{i}\delta_{\vec{p}}} \end{pmatrix} \begin{pmatrix} 1 - s_{\vec{p}} + c_{\vec{p}} & 1 - c_{\vec{p}} - s_{\vec{p}} \\ -1 + c_{\vec{p}} + s_{\vec{p}} & 1 + c_{\vec{p}} - s_{\vec{p}} \end{pmatrix} , \tag{8.13}$$

with

$$c_{\vec{p}} = p_3 \, , \qquad s_{\vec{p}} = \sqrt{1-(p_3)^2} \, ,$$

$$e^{\mathrm{i}\delta_{\vec{p}}} = \frac{1}{\sqrt{2}\,\left[1-(p_3)^2\right]^{1/4}} \left(\sqrt{p_1 + \sqrt{1-(p_3)^2}} - \mathrm{i}\frac{p_2}{|p_2|}\sqrt{-p_1 + \sqrt{1-(p_3)^2}} \right) ,$$

and $U(p_1,p_2,0)$ being the limit of $U(\vec{p})$ when $p_3 \to 0$, i.e.

$$U(p_1,p_2,0) = \frac{1}{\sqrt{2}} \begin{pmatrix} e^{\mathrm{i}\delta_{p_3=0}} & 0 \\ 0 & e^{-\mathrm{i}\delta_{p_3=0}} \end{pmatrix} \begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix} , \tag{8.14}$$

brings $\vec{p}$ to $(0,0,1)$, i.e., a purely Dirac mass coupling.

In this new basis the equations of motion read

$$\left(\partial_5^2 - \Box - M^2\right) \, \phi = 0 \tag{8.15}$$

$$\left(\partial_5^2 - \Box - M^2\right) \, \phi_c = 0 \tag{8.16}$$

$$F + \bar{\phi}_c' + M \, \bar{\phi}_c = 0 \tag{8.17}$$

$$F_c - \bar{\phi}' + M \, \bar{\phi} = 0 \tag{8.18}$$

$$\mathrm{i}\,\bar{\sigma}^\mu \partial_\mu \psi - \partial_5 \bar{\psi}_c - M\,\bar{\psi}_c = 0 \tag{8.19}$$

$$\mathrm{i}\,\bar{\sigma}^\mu \partial_\mu \psi_c + \partial_5 \bar{\psi} - M\,\bar{\psi} = 0\,. \tag{8.20}$$

The spinorial equations can be gather in a single Dirac equation as

$$\left(\mathrm{i}\,\gamma^\mu \partial_\mu - \gamma^5 \partial_5 - M\right)\,\Psi = 0\,, \tag{8.21}$$

with $\Psi = \begin{pmatrix} \psi_c \\ \bar{\psi} \end{pmatrix}$. For simplicity we consider the case $\vec{s}_0 = \vec{s}_\pi = (-s, 0, c)$ and thus the boundary conditions read

$$\begin{pmatrix} 1-c & s \\ s & 1+c \end{pmatrix} \cdot \begin{pmatrix} \phi_c \\ \phi \end{pmatrix}_{0,\pi} = 0 \tag{8.22}$$

$$[\partial_5 + M\,c] \begin{pmatrix} 1+c & -s \\ -s & 1-c \end{pmatrix} \cdot \begin{pmatrix} \phi_c \\ \phi \end{pmatrix}_{0,\pi} = 0 \tag{8.23}$$

$$\begin{pmatrix} 1-c & s \\ s & 1+c \end{pmatrix} \cdot \begin{pmatrix} \psi_c \\ \psi \end{pmatrix}_{0,\pi} = 0 \tag{8.24}$$

hence the spectrum equation reduces to

$$\left[1 + c^2 \frac{M^2}{\Omega^2}\right] \tan^2(\pi \Omega R) = 0\,, \tag{8.25}$$

which has the solution

$$m_0^2 = s^2 M^2\,, \qquad m_n = \frac{1}{R}\sqrt{M^2 R^2 + n^2}\,,\; n = 1, 2, 3, \cdots \tag{8.26}$$

Wave functions

- Bosonic solution

There is no subtlety to deal with the bosonic solution. As usual we suppose the eigenfunctions to verify the 4d klein-Gordon equation $\Box\phi^n_{(c)}(x,y) = -m_n^2\phi^n_{(c)}(x,y)$, hence the general solution to the 5d equations of motion will be given by

$$\Phi(x,y) = A(x)\cos(\Omega y) + B(x)\sin(\Omega y)\,, \tag{8.27}$$

with $\Phi = \begin{pmatrix} \phi_c \\ \phi \end{pmatrix}$. The boundary conditions at $y=0$ impose the restrictions

$$\begin{aligned} A &= \begin{pmatrix} 1+c-s \\ 1-c-s \end{pmatrix} a(x)\,, \\ B &= -\frac{Mc}{\Omega}\begin{pmatrix} 1+c-s \\ 1-c-s \end{pmatrix} a(x) + \begin{pmatrix} -1+c+s \\ 1+c-s \end{pmatrix} b(x)\,, \end{aligned}$$

where $a(x)$, $b(x)$ are independent complex functions verifying the above 4d Klein-Gordon equation. Finally, the boundary conditions at $y=\pi$ impose

$$b(x)\sin(\Omega\pi R) = 0\,, \tag{8.28}$$

$$a(x)\left[\Omega + \frac{c^2M^2}{\Omega}\right]\sin(\Omega\pi R) = 0\,, \tag{8.29}$$

which have two possible solutions

1. $b(x)=0$ and hence $\Omega^2 = (\mathrm{i}\tilde{\Omega})^2 = -c^2M^2$, whose eigenfunction is

$$\begin{aligned} \Phi^0 = \begin{pmatrix} 1+c-s \\ 1-c-s \end{pmatrix}&\left[\cos(\Omega R y) - \frac{c\,M}{\Omega}\sin(\Omega R y)\right]\varphi(x) = \\ &\begin{pmatrix} 1+c-s \\ 1-c-s \end{pmatrix} e^{-McRy}\varphi(x)\,, \end{aligned} \tag{8.30}$$

2. $\Omega R = n \in \mathbb{Z}_+$, with eigenstate

$$\begin{aligned} \Phi^n &= \begin{pmatrix} 1+c-s \\ 1-c-s \end{pmatrix} f^n(y)\,\varphi_1^n(x) \\ &+ \begin{pmatrix} -1+c+s \\ 1+c-s \end{pmatrix} g^n(y)\,\varphi_2^n(x) \end{aligned} \tag{8.31}$$

for $f^n(y) = \cos(\frac{n}{R}y) - \frac{McR}{n}\sin(\frac{n}{R}y)$ and $g^n(y) = \sin(\frac{n}{R}y)$.

- Fermionic solution

The formal solution to (8.21) is given by

$$\Psi = \left[\cos(\sqrt{-\Box - M^2}\, y) + \gamma^5 \frac{\mathrm{i}\,\gamma^\mu \partial_\mu - M}{\sqrt{-\Box - M^2}} \sin(\sqrt{-\Box - M^2}\, y)\right] \Theta(x)\,,$$

where $\Theta(x)$ is the initial value at $y = 0$ and we will assume it to fulfill the 4d Klein-Gordon equation $\Box\,\Theta + m^2\Theta = 0$. Thus the solution for the m-th mode reads

$$\Psi = \left[\cos(\Omega y) + \gamma^5 \frac{\mathrm{i}\,\gamma^\mu \partial_\mu - M}{\Omega} \sin(\Omega y)\right] \Theta(x)\,, \tag{8.32}$$

of course, the initial value will be the solution to the boundary condition, (8.6), at $y = 0$, that is:

$$\Theta = \begin{pmatrix} (1 + c - s)\chi \\ (1 - c - s)\bar{\chi} \end{pmatrix}, \tag{8.33}$$

with χ an arbitrary Weyl spinor. Nevertheless, we are working with a 4d Dirac spinor satisfying a Dirac equation, namely, there has to be another Weyl spinor[2], say ξ, such that, together with χ, both of them verify the equations

$$\mathrm{i}\,\sigma^\mu \partial_\mu \bar{\xi} = m\chi\,, \qquad \mathrm{i}\,\bar{\sigma}^\mu \partial_\mu \chi = m\bar{\xi}\,. \tag{8.34}$$

Plugging (8.34) and (8.33) in (8.32) we obtain

$$\begin{aligned} \Psi &= \begin{pmatrix} (1 + c - s) f_-(y)\,\chi \\ (1 - c - s) f_+(y)\,\bar{\chi} \end{pmatrix} \\ &+ m \begin{pmatrix} (1 - c - s)\,\xi \\ -(1 + c - s)\,\bar{\xi} \end{pmatrix} \frac{\sin(\Omega y)}{\Omega}\,, \end{aligned} \tag{8.35}$$

$$f_\pm(y) = \cos(\Omega y) \pm \frac{M}{\Omega} \sin(\Omega y)\,. \tag{8.36}$$

[2]This is a straight analogy of the orbifold case, where we have a Dirac spinor in the bulk although parity assignment projects out one of the components at the boundary such that there we have a single Weyl (Majorana) spinor.

Finally, the boundary condition at $y = \pi$ imposes the vanishing of

$$\frac{\sin(\Omega\pi R)}{\Omega}\begin{pmatrix} 1-c & s \\ s & 1+c \end{pmatrix}\cdot\begin{pmatrix} -M(1+c-s) & m(1-c-s) \\ M(1-c-s) & -m(1+c-s) \end{pmatrix}\cdot\begin{pmatrix} \chi \\ \xi \end{pmatrix},$$

which is equivalent to

$$\frac{\sin(\Omega\pi R)}{\Omega}\begin{pmatrix} sM & m \\ sM & m \end{pmatrix}\cdot\begin{pmatrix} \chi \\ \xi \end{pmatrix} = 0\,. \tag{8.37}$$

Equation (8.37) has two possible solutions:

1. $\Omega R = n$ with $n = 1, 2, 3, \cdots$
   In this case we have two independent spinorial degrees of freedom, ξ and χ, degenerated in mass.

2. $\Omega R \notin \mathbb{Z}$
   Now the solution should be

   $$m\xi + s\,M\chi = 0\,, \tag{8.38}$$

   however, ξ, χ satisfy the equations (8.34) hence if $\xi = \kappa\chi$ then

   $$\mathrm{i}\,\sigma^\mu\partial_\mu\bar{\xi} = m\frac{1}{\kappa}\xi = \mathrm{i}\,\sigma^\mu\partial_\mu\kappa\bar{\chi} = m\kappa\xi \iff \kappa = 1\,. \tag{8.39}$$

   and therefore

   $$m = -s\,M\,. \tag{8.40}$$

Thus we reencounter the spectrum (8.26). The corresponding wave functions turn out to be

$$\begin{aligned}\Psi^n &= \begin{pmatrix} (1+c-s)f_-^n(y)\,\chi^n \\ (1-c-s)f_+^n(y)\,\bar{\chi}^n \end{pmatrix} \\ &+ \sqrt{n^2+M^2R^2}\begin{pmatrix} (1-c-s)\,\xi^n \\ -(1+c-s)\,\bar{\xi}^n \end{pmatrix}\frac{\sin(\frac{n}{R}y)}{n}\,,\end{aligned} \tag{8.41}$$

$$\Psi^0 = \begin{pmatrix} (1+c-s)\eta \\ (1-c-s)\bar{\eta} \end{pmatrix} e^{-cMy}\,. \tag{8.42}$$

Some comments about these solutions are in order now. First of all the degeneracy of the higher modes is clearly a consequence of the coincidence in the boundary matrices. To understand this, consider the bosonic solution corresponding the the eigenvalue m

$$\left(\partial_5^2 + m^2 - M^2\right) \begin{pmatrix} \phi_c \\ \phi \end{pmatrix} = 0\,, \tag{8.43}$$

$$(\mathbf{1} - S) \begin{pmatrix} \phi_c \\ \phi \end{pmatrix}_{0,\pi} = 0\,, \tag{8.44}$$

$$(\mathbf{1} + S)\,[\partial_5 + c\,M] \begin{pmatrix} \phi_c \\ \phi \end{pmatrix}_{0,\pi} = 0\,, \tag{8.45}$$

both, the boundary conditions and the bulk equations of motion, are invariant under the transformation

$$e^{\mathrm{i}\alpha S}\,, \qquad \alpha \in \mathbb{R}\,, \qquad S = S_0 = S_\pi\,. \tag{8.46}$$

For the sake of simplicity we express the system in the basis were $S = \sigma_3$. Now suppose that the multiplicity of every mass eigenvalue out of (8.25) is one, then we fall in a contradiction, since if

$$\begin{pmatrix} \phi_c \\ \phi \end{pmatrix}, \tag{8.47}$$

is the solution corresponding to the eigenvalue m, then

$$\begin{pmatrix} \phi_c \\ e^{-2\mathrm{i}\alpha}\phi \end{pmatrix}, \tag{8.48}$$

which is a linearly independent $\mathbb{C}^2$ vector, i.e., not proportional to the original state, is a solution to the same equations of motion and boundary conditions, and hence it shares the same mass eigenvalue. It is clear that in a particular case of the form $(\phi_c, 0)$ or $(0, \phi)$ we have not this contradiction. From the supersymmetric structure of boundary conditions it follows that the statement is extendible to the fermionic case. Secondly, notice that the solutions are not, in general, factorizable as $f(y)g(x)$ which reflexes explicitly why the orbifold-like *ansatz* mentioned previously is not suitable. We want to remark in addition that the presence of ξ is not a unitarity

problem at all. From (8.34) it can be expressed in terms of $\bar{\sigma}^\mu \partial_\mu \chi$ which is in concordance with the uniqueness of the solution to (8.21) given a 4D dependence of χ. However they can be thought as off-shell independent degrees of freedom.
Finally, it is clear from (8.32) and (8.15)-(8.16) that given a value of Ω the solution corresponding to $-\Omega$ is exactly the same, since there is no degeneracy associated. Actually the spectrum can be thought as the solution to $\sqrt{m^2 - M^2} = \Omega_0$ ($\sqrt{M^2 - m^2} = \tilde{\Omega}_0$) given a solution Ω_0 of (8.8). The sign of the root is, of course, a matter of convention, but one can not choose both of them at the same time. For instance, by choosing $c\,M$ to be negative one has that the lowest RH mode exponentially localizes towards $y = \pi$ boundary. In fact the sign of Ω ($\tilde{\Omega}$) can be absorbed by the redefinition $y \to \pi R - y$, which shows that both signs cannot coexist simultaneously since they do correspond to the same eigenstate.

8.3 Effective action

In this section we develop the effective action for RH neutrinos coupled with SM matter (the Higgs and the left-handed neutrinos (LH)) and we will obtain the lowest eigenvalue by solving the characteristic polynomial of the effective mass matrix. As we will show this can be alternatively done through the effective mass matrix resulting from the integration of the higher KK modes.
For simplicitywe only consider the action concerning the right handed neutrinos, the left handed neutrinos and the Higgs. For the latter we will consider the action

developed in Ref [55]. The 5D action under study is [3]

$$
\begin{aligned}
\mathcal{S}_{\rm eff} \;=\;& \frac{1}{2}\int_{\Sigma} \mathrm{i}\bar{\psi}_c\bar{\sigma}^\mu\partial_\mu\psi_c + \mathrm{i}\psi_c\sigma^\mu\partial_\mu\bar{\psi}_c + \mathrm{i}\bar{\psi}\bar{\sigma}^\mu\partial_\mu\psi + \mathrm{i}\psi\sigma^\mu\partial_\mu\bar{\psi} \\
-\;& \frac{1}{2}\int_{\Sigma} \psi_c\left(-\partial_5\psi + M\psi\right) + \psi\left(\partial_5\psi_c + M\psi_c\right) + \text{h.c.} \\
-\;& \frac{1}{4}\int_{\partial\Sigma} \psi\left[s_-\psi + (s_3-1)\psi_c\right] + \psi_c\left[(1+s_3)\psi - s_+\psi_c\right] + \text{h.c.} \\
+\;& \int_{y=y_{f_0}} Y_\nu\,\psi_c\nu_L H_c + \text{h.c.}
\end{aligned}
\tag{8.49}
$$

where ν_L denotes the LH neutrino, Y_ν is the 5D Yukawa coupling (with dimension of inverse energy), y_{f_0} stands for the boundary where ν_L is localized, i.e. $f=0$ or $f=\pi$ and H_c is the lowest mode of the Higgs which according to Ref. [55] is given by $H_c \simeq \sqrt{2M_H}\,e^{-M_H y}\,h(x)$. Now we take ψ and ψ_c as

$$
\begin{aligned}
\psi_c =& (1+c-s)e^{-Mcy}\,\eta(x) \\
&+\sum_{n\geq 1}\Big[(1+c-s)f_-^n(y)\,\chi^n(x) + \frac{m_n}{n}(1-c-s)\sin(n\,y/R)\,\xi^n(x)\Big]
\end{aligned}
$$

$$
\begin{aligned}
\psi =& (1-c-s)e^{-Mcy}\,\eta(x) \\
&+\sum_{n\geq 1}\Big[(1-c-s)f_+^n(y)\,\chi^n(x) - \frac{m_n}{n}(1+c-s)\sin(n\,y/R)\,\xi^n(x)\Big]
\end{aligned}
$$

whose components satisfy the (free) equations of motion

$$
\mathrm{i}\bar{\sigma}^\mu\partial_\mu\eta = -s\,M\,\bar{\eta}\,, \tag{8.50}
$$

$$
\mathrm{i}\bar{\sigma}^\mu\partial_\mu\begin{pmatrix}\chi^n\\ \xi^n\end{pmatrix} = \frac{1}{R}m_n\begin{pmatrix}0 & 1\\ 1 & 0\end{pmatrix}\cdot\begin{pmatrix}\bar{\chi}^n\\ \bar{\xi}^n\end{pmatrix}\,, \tag{8.51}
$$

[3] The Yukawa interaction terms are strictly localized on the boundaries since they are not $SU(2)_R$ invariant.

with $m_n = \sqrt{M^2R^2 + n^2}$. By integrating over the fifth coordinate one obtains the following effective 4D action

$$\begin{aligned} S_{\text{eff}} = & \int \mathrm{d}^4\mathrm{x}\ (\mathrm{k}_0)^{-2} \left[\frac{\mathrm{i}}{2} \bar{\eta}\bar{\sigma}^{\mu}\partial_{\mu}\eta + \frac{1}{2} \mathrm{s}\, \mathrm{M}\, \eta^2 \right] \\ & + \sum_{n \geq 1} \int \mathrm{d}^4\mathrm{x} \left[\frac{\mathrm{i}}{2} \bar{\Lambda}_\mathrm{n} \mathrm{k}_\mathrm{n} \bar{\sigma}^{\mu} \partial_{\mu} \Lambda_\mathrm{n} - \frac{1}{2\mathrm{R}} \mathrm{m}_\mathrm{n}\, \Lambda_\mathrm{n}^\mathrm{T} \mathrm{k}_\mathrm{n} \sigma_1 \Lambda_\mathrm{n} \right] \\ & + \int \mathrm{d}^4\mathrm{x}\, \mathrm{Y}_\nu\, \sqrt{2\mathrm{M}_\mathrm{H}} \mathfrak{e}_{\mathrm{f}_\mathrm{h}}\, (1 + \mathrm{c} - \mathrm{s})\, \eta \nu_\mathrm{L} \mathrm{h} \\ & + \sum_{n \geq 1} \int \mathrm{d}^4\mathrm{x}\, \mathrm{Y}_\nu\, \sqrt{2\mathrm{M}_\mathrm{H}} \mathfrak{e}_{\mathrm{f}_\mathrm{h}} (1 + \mathrm{c} - \mathrm{s})\, \chi_\mathrm{n} \nu_\mathrm{L}\, \mathrm{h} + \mathrm{h.c.}\,, \end{aligned} \tag{8.52}$$

where k_0 is the real number

$$(k_0)^{-2} = \frac{2(1-s)}{cM} \left(1 - e^{-2cM\pi R}\right)\,, \tag{8.53}$$

k_n the tower of matrices

$$k_n = 2\pi R \frac{(m_n)^2}{n^2} (1-s) \left[\mathbf{1} + s \frac{MR}{m_n}\, \sigma_1 \right]\,, \tag{8.54}$$

Λ_n stands for

$$\Lambda_n = \begin{pmatrix} \chi_n \\ \xi_n \end{pmatrix}\,,$$

and

$$\mathfrak{e}_{f_h} = e^{-M_H |y_{f_h} - y_{f_0}|}. \tag{8.55}$$

where $f_h = 0$ or $f_h = \pi$ depending on the boundary the H_c zero mode is localized towards. By redefining the modes as

$$\psi_\pm^n = \frac{1}{\sqrt{2}\, k_\pm^{(n)}} \left(\chi^n \pm \xi^n\right)\,, \tag{8.56}$$

$$\zeta = \frac{1}{k_0}\, \eta\,, \tag{8.57}$$

with

$$(k_{\pm}^{(n)})^{-2} = 2\pi R \frac{(m_n)^2}{n^2}(1-s)\left[1 \pm s\frac{MR}{m_n}\right], \tag{8.58}$$

the 4D effective action (8.52) can be rewritten as

$$\begin{aligned} S_{\text{eff}} = & \int \mathrm{d^4x} \left[\frac{\mathrm{i}}{2}\bar{\zeta}\bar{\sigma}^\mu\partial_\mu\zeta + \frac{1}{2}\mathrm{s\,M}\,\zeta^2\right] + \sum_{\mathrm{n}\geq 1}\int \mathrm{d^4x}\,\frac{\mathrm{i}}{2}\left(\bar{\psi}_-^{\mathrm{n}}\bar{\sigma}^\mu\partial_\mu\psi_-^{\mathrm{n}} + \bar{\psi}_+^{\mathrm{n}}\bar{\sigma}^\mu\partial_\mu\psi_+^{\mathrm{n}}\right) \\ & - \sum_{n\geq 1}\int \mathrm{d^4x}\,\frac{1}{2\mathrm{R}}\mathrm{m_n}\left[\left(\psi_+^{\mathrm{n}}\right)^2 - \left(\psi_-^{\mathrm{n}}\right)^2\right] + \int \mathrm{d^4x\,Y^{(0)}}\zeta\nu_{\mathrm{L}}\,\mathrm{h} \\ & + \sum_{n\geq 1}\int \mathrm{d^4x}\left[\mathrm{Y_-^{(n)}}\psi_-^{\mathrm{n}}\nu_{\mathrm{L}}\,\mathrm{h} + \mathrm{Y_+^{(n)}}\psi_+^{\mathrm{n}}\nu_{\mathrm{L}}\,\mathrm{h}\right] + \mathrm{h.c.}\,, \end{aligned} \tag{8.59}$$

where

$$Y^{(0)} = Y_\nu\,(1+c-s)\sqrt{\frac{|cMM_H|}{1-s}}\,\mathfrak{e}_{f_h}\mathfrak{e}_{f_\nu} \tag{8.60}$$

and

$$Y_{\pm}^{(n)} = \frac{1}{\sqrt{2}}Y_\nu\,(1+c-s)\sqrt{|M_H|}\,\mathfrak{e}_{f_h}\,k_{\pm}^{(n)}\,, \tag{8.61}$$

are the 4D effective Yukawa coupling constants. Here we have defined $\mathfrak{e}_{f_\nu}$ as

$$\mathfrak{e}_{f_\nu} = e^{-|cM|\left|y_{f_\nu}-y_{f_h}\right|}\,, \tag{8.62}$$

where $y_{f_\nu} = 0, \pi R$ depending on where the lowest mode of the RH neutrino localizes towards. Once the Higgs gets its vacuum expectation value, $\langle h\rangle = v$, the Yukawa couplings turn into Dirac mass terms and we are thus left with an effective mass matrix connecting LH and RH neutrinos given by

$$\begin{aligned} \mathcal{L}_m = & \frac{1}{2}s\,M\,\zeta^2 + Y^{(0)}v\zeta\nu_L - \sum_{n\geq 1}\frac{1}{2R}m_n\left[\left(\psi_+^n\right)^2 - \left(\psi_-^n\right)^2\right] \\ & + \sum_{n\geq 1} v\left\{Y_+^{(n)}\psi_+^n\nu_L + Y_-^{(n)}\psi_-^n\nu_L\right\}\,. \end{aligned} \tag{8.63}$$

Notice that RH neutrinos appear in (8.59) as Majorana spinors albeit we started with Dirac fermions. However if we redefine the fields as

$$\varphi_+^n = \psi_-^n + \psi_+^n\,, \tag{8.64}$$

$$\varphi_-^n = \psi_-^n - \psi_+^n\,, \tag{8.65}$$

we recover the expected Dirac mass spinors due to the degeneracy in mass of the higher modes.

Now we can find the eigenvalues of the infinite mass matrix (8.63) by computing its characteristic polynomial [77]

$$P(\lambda) = \det \begin{pmatrix} -\lambda & m_D^{(0)} & \dots & m_{D+}^{(n)} & m_{D-}^{(n)} & \dots \\ m_D^{(0)} & s\,M - \lambda & & & & \\ \vdots & & \ddots & & & \\ m_{D+}^{(n)} & & & -m_n - \lambda & & \\ m_{D-}^{(n)} & & & & m_n - \lambda & \\ \vdots & & & & & \ddots \end{pmatrix}. \tag{8.66}$$

where $m_D^{(0)} = vY^{(0)}$ and $m_{D\pm}^{(n)} = vY_\pm^{(n)}$. The determinant of (8.66) yields

$$P(\lambda) = \tag{8.67}$$

$$\left[(\lambda - s\,M)\prod_{k\geq 1}\left(\lambda^2 - m_k^2\right)\right]\left\{\lambda + \frac{\left(m_D^{(0)}\right)^2}{s\,M - \lambda} + \sum_{l\geq 1}\left[-\frac{\left(m_{D+}^{(l)}\right)^2}{m_l + \lambda} + \frac{\left(m_{D-}^{(l)}\right)^2}{m_l - \lambda}\right]\right\}$$

When $s \neq 0$ the smallest eigenvalue, λ_L, will not be that vanishing either $\lambda - s\,M$ or $\lambda^2 - m_n^2$. Therefore it should verify the equation

$$\lambda_L + \frac{\left(m_D^{(0)}\right)^2}{s\,M - \lambda_L} + \sum_{l\geq 1}\left[\frac{\left(m_{D-}^{(l)}\right)^2}{m_l - \lambda_L} - \frac{\left(m_{D+}^{(l)}\right)^2}{m_l + \lambda_L}\right] = 0\,. \tag{8.68}$$

Since $m_{D\pm}^{(n)}, m_D^{(0)}$ are $\sim vY_\nu/R$, and we will assume it to be much smaller than M, we can expand the solution in powers of $\beta = \frac{vY_\nu}{MR}$ as

$$\frac{\lambda_L}{M} = \sum_{\ell=1}^{\infty} \lambda_{2\ell}\beta^{2\ell}\,. \tag{8.69}$$

which makes sense whenever the lowest order is small. Substituting back in (8.68) we find that at lowest order λ_L is given by

$$\frac{\lambda_L}{M}+\frac{\left(m_D^{(0)}\right)^2}{s\,M^2}=-\sum_{n\geq 1}\frac{R}{Mm_n}\left[\left(m_{D-}^{(n)}\right)^2-\left(m_{D+}^{(n)}\right)^2\right]$$

$$=-\frac{2(1+c)sM_H\,\left(\mathfrak{e}_{f_h}\right)^2Rv^2Y^2}{\pi}\sum_{n\geq 1}\frac{n^2}{(n^2+M^2R^2)(n^2+c^2M^2R^2)} \tag{8.70}$$

Finally the series in (8.70) can be computed analytically by means of a Poisson re-summation giving

$$\lambda_L=-v^2Y_\nu^2\frac{1+c}{s}\,M_H\left(\mathfrak{e}_{f_h}\right)^2\left[2c\left(\mathfrak{e}_{f_\nu}\right)^2+\coth(\pi MR)-c\coth(\pi c\,MR)\right]\,. \tag{8.71}$$

Alternatively this result can be obtained as the diagonalization of the effective action induced after integrating out the higher KK modes $\psi_\pm^n$ in (8.63) for momenta much smaller than their mass (i.e. neglecting their kinetic terms). From (8.63) the equations of motion for $\psi_\pm^n$ are given by

$$\psi_\pm^n=\mp\frac{R\,m_{D\pm}^{(n)}}{m_n}\,\nu_L \tag{8.72}$$

and substituting back in (8.63) we find in matrix form the following effective mass coupling for ν_L and $\zeta\equiv\nu_R$

$$\frac{1}{2}\left(\nu_L,\nu_R\right)\cdot\begin{pmatrix}\mu & m_D^{(0)}\\ m_D^{(0)} & s\,M\end{pmatrix}\cdot\begin{pmatrix}\nu_L\\ \nu_R\end{pmatrix}, \tag{8.73}$$

with

$$\mu=\sum_{n\geq 1}\frac{R}{m_n}\left[\left(m_{D+}^{(n)}\right)^2-\left(m_{D-}^{(n)}\right)^2\right], \tag{8.74}$$

Since μ and $m_D^{(0)}$ are much smaller than M the light and heavy eigenvalues $\lambda_{L,H}$ of the mass matrix in (8.73) are

$$\lambda_L\simeq\mu-\frac{\left(m_D^{(0)}\right)^2}{s\,M}\,,\qquad \lambda_H\simeq s\,M\,. \tag{8.75}$$

where the light eigenvalue λ_L coincides with that found in Eq. (8.71).

A particularly interesting case is found when $s = 0$, that is when the boundary matrices are precisely aligned with the bulk mass matrix. In that case the characteristic polynomial simplifies to

$$P_0(\lambda) = \left[\prod_{k\geq 1}\left(\lambda^2 - m_k^2\right)\right]\left\{\lambda^2\left(1 + 2\sum_{l\geq 1}\frac{\left(m_D^{(l)}\right)^2}{m_l^2 - \lambda^2}\right) - \left(m_D^{(0)}\right)^2\right\} \tag{8.76}$$

where

$$\left(m_D^{(l)}\right)^2 = 2\beta^2 M^2 M_H R \frac{l^2}{\pi(l^2 + M^2R^2)}\,, \tag{8.77}$$

Notice that (8.76) is an equation for λ^2 which means that both $\pm\lambda$ are solutions and thus the set of eigenstates of the whole mass matrix are exactly degenerate by pairs and therefore they can be gathered to yield Dirac fermions. Following the same used above we find that the lowest eigenvalue is given by

$$\lambda_{L\pm} = \pm 2\, Y_\nu v\, \mathfrak{e}_{f_h} \sqrt{M_H M}\, e^{-\pi M y_{f_\nu}}\,, \tag{8.78}$$

The effective mass matrix for ν_L, ν_R will be given in that case by

$$\begin{pmatrix} 0 & m_D \\ m_D & 0 \end{pmatrix}. \tag{8.79}$$

where we have defined $m_D = 2Y_\nu v\, \mathfrak{e}_{f_h}\sqrt{MM_H}\, e^{-\pi M y_{f_\nu}}$. The degeneracy of the spectrum for $s = 0$ can be understood in terms of a symmetry which takes place only within this case. As a matter of fact, $s = 0$ means that the vectors $\vec{p}, \vec{s}_0, \vec{s}_\pi$ are all aligned along the same direction and hence a $U(1)$ subgroup of unitary rotations around this direction axis leaves the action invariant. In terms of the fermion components, these transformations translate into

$$(\eta,\, \chi^n) \to e^{\mathrm{i}\alpha}\,(\eta,\, \chi^n)\,, \qquad \xi^n \to e^{-\mathrm{i}\alpha}\,\xi^n\,, \qquad \nu_L \to e^{-\mathrm{i}\alpha}\,\nu_L \tag{8.80}$$

where α is a real parameter. Notice that this symmetry forbids any Majorana term [4], in particular for ν_L, and hence μ must vanish.

[4] This symmetry plays the role of the lepton number symmetry of the SM.

8.4 Discussion on neutrino masses

In this section we will apply the previous results to discuss the possibility of getting, within this kind of models, an (ultralight) neutrino mass in the sub meV range. The first task will be to set the range of dimensional Yukawa couplings which appear in the 5D action in the leptonic sector

$$\int_{\partial\Sigma} (Y_{\nu_\ell} H_c \nu_L \psi_c + Y_\ell H \ell_L e_R) \tag{8.81}$$

i.e. Y_{ν_ℓ}, with dimension of length, and Y_ℓ, with dimension of square root of length, where $\ell = \{\tau, \mu, e\}$. A naive estimate of wave function renormalization correction to the Yukawa couplings in the 5D theory sets bounds as

$$y_{\nu_\ell} \equiv \frac{Y_{\nu_\ell}}{R} \lesssim \frac{4\pi}{\Lambda R}, \quad y_\ell \equiv \frac{Y_\ell}{\sqrt{R}} \lesssim \frac{4\pi}{\sqrt{\Lambda R}} \tag{8.82}$$

so that taking $\Lambda R \sim 10$ we obtain $\mathcal{O}(1)$ upper bounds on the dimensionless Yukawa couplings $y_{\nu_\ell,\ell}$. We can now distinguish three different scenarios:

8.4.1 Dirac mass

We will assume here that all the SM matter is strictly localized on the $y = 0$ brane and the zero mode of the Higgs is localized towards it as well, thus $\mathfrak{e}_{f_h} = 1$, and the zeroth mode of RH neutrino is exponentially localized towards $y = \pi R$, i.e. $y_{f_\nu} = \pi R$, with $s = 0$ as Fig. 1 shows.

As shown in the previous section we obtain a Dirac mass connecting ν_L and ν_R, which is of order

$$m^D_{\nu_\ell} \sim 2v\, Y_{\nu_\ell} \sqrt{M\, M_H}\, \epsilon_R\,. \tag{8.83}$$

with $\epsilon_R = e^{-M\pi R}$. In Fig. 8.2 we show the Dirac mass as a function of MR for $1/R \sim 5$ TeV, $y_{\nu_\ell} \sim 1$ and $M_H R \sim 1.6$ [55] [5]. We can see from Fig. 8.2 that $m^D_{\nu_\ell} \lesssim 1$

[5] As it is shown there for such value of $M_H R$ the spectrum of the Higgs presents a tachyon at the tree level, which partially cancels the positive one-loop radiative correction to the Higgs mass due to the gauge coupling and allows the EWSB to take place at the two loop level with a modest amount of fine tuning.

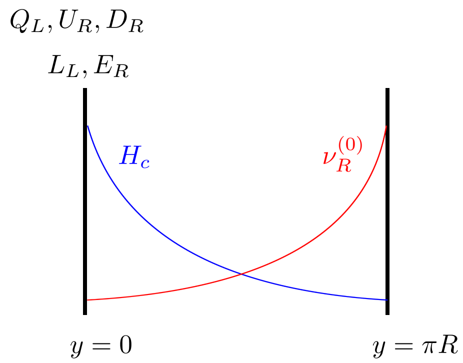

Figure 8.1: Bulk and brane matter distribution for a neutrino Dirac-like mass.

eV for $MR \gtrsim 9$ although $m^D_{\nu_\ell}$ decreases exponentially with MR and thus $m^D_{\nu_\ell} \simeq 1$ meV for $MR \simeq 11$. In this scenario there is no wave function suppression for the charged leptons whose Yukawa couplings should therefore be given by $y_\ell \sim m_\ell/v$.

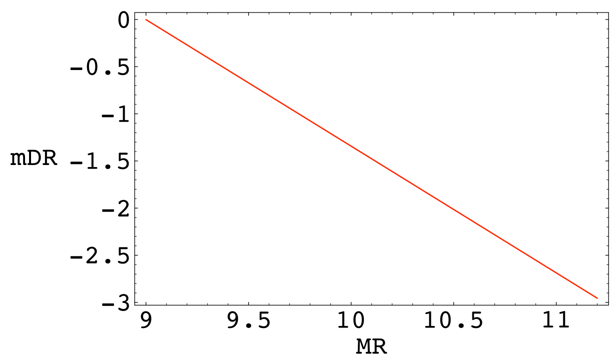

Figure 8.2: $m^D_{\nu_\ell}$ as a function of MR for $M_H R = 1.6$ and $s = 0$.

If $s \neq 0$ in this scenario the lowest eigenvalue is a Majorana mass given by

$$m^M_{\nu_\ell} \sim Y^2_{\nu_\ell} v^2 M_H \left[2c\, e^{-2\pi|cM|R} + \coth(\pi MR) - c\coth(\pi c\, MR) \right] , \qquad (8.84)$$

which in general will be too large, unless a severe, although not extreme, suppression of the 5D Yukawa constants, $y_{\nu_\ell} \sim 10^{-6}$ which is similar to the electron Yukawa coupling in this kind of models y_e.

Yet another possibility could be to localize the lowest RH mode towards $y = 0$, corresponding to $cM > 0$. Considering now $MR \gg 1$ the mass eigenvalue is proportional to $(c + \text{sign}(M) + 2\,e^{-2\pi|M|R} - 2c\,e^{-2\pi c MR})$. Then by choosing $c = -\text{sign}(M)$ we could be left with an exponentially suppressed Majorana mass. However this value of c is not consistent with the initial hypothesis $cM > 0$. In fact the smallness of the Majorana eigenmass is achieved with a different localization of the quark and lepton sector within the SM as we will see in the next section.

8.4.2 Majorana mass

The main obstruction to get a small Majorana mass eigenvalue out of the effective mass matrix (8.73) for the $s \neq 0$ case is that the Yukawa couplings of the higher KK modes are not suppressed if the SM matter is located on the boundary where the Higgs localizes towards. However by allowing the Standard Model matter to be split into different branes , for instance quarks localized in the same boundary (quark brane) where the Higgs localizes towards, while leptons are localized in the opposite boundary (lepton brane) as Fig. 3 shows, then via $\mathfrak{e}_{f_h}$, the whole tower of effective Yukawa couplings will be exponentially suppressed by the Higgs localization and so μ will be.

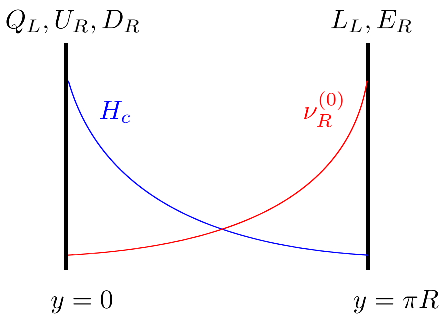

Figure 8.3: Bulk and brane matter distribution for a neutrino Majorana-like mass. The ν_R propagates in the bulk with mass M.

Such a splitting has its justification within the context of intersecting branes in String theory. Briefly, it consists of different Dp-branes wrapping non trivial homology cycles which intersect at M_4 world volumes. The open strings stretching between them reproduce $(p+1)$D gauge multiplets and 4D chiral multiplets. Such Dp-branes intersect (in general) several times, therefore one can replicate 4D chiral matter. The details of this development takes us far from the aim of the present work. Non the less the interested reader can find an excellent review in [87] and references therein.

Now the effective mass matrix is analogous to the previous case except for the global exponential suppression on the Dirac couplings, namely, $\mu \to \epsilon_H^2\,\mu$ with $\epsilon_H = e^{-\pi M_H R}$. In addition, we will assume the lowest mode of the RH neutrino to localize towards the leptonic brane, i.e. $y_{f_\nu} = 0$, corresponding thus to $cM < 0$. We then find that the lowest neutrino Majorana eigenmass is given by

$$m_{\nu_\ell}^M = \epsilon_H^2 v^2 Y_{\nu_\ell}^2 M_H \frac{1+c}{s}\left[2c + \coth(\pi MR) - c\coth(c\,\pi MR)\right] . \tag{8.85}$$

Notice the almost independence on the RH neutrino bulk mass M albeit its presence is absolutely necessary to provide the existence of a lowest Majorana eigenmass [6] it is shielded by the higher RH neutrino modes. In Fig. 8.4 we plot $m_{\nu_\ell}^M$ as a function of $\log_{10} y_{\nu_\ell}$ for fixed values of c and MR. From Fig. 8.4 we can see that generically

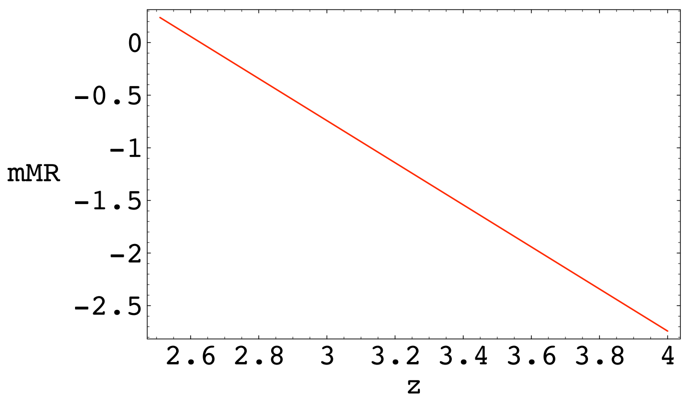

Figure 8.4: Neutrino Majorana mass, $m_{\nu_\ell}^M$, as a function of $-\log_{10} y_{\nu_\ell}$ for $c = -1/2$, $M_H R = 1.6$ and $MR = 5$.

$m_{\nu_\ell}^M \lesssim 1$ eV implies $y_{\nu_\ell} \lesssim 10^{-3}$.

[6] In case of vanishing M we would be left with a lowest Dirac eigenvalue or, at most, with two almost degenerate Majorana eigenstates.

The charged leptons, on the other hand, have masses

$$m_\ell \sim v\, y_\ell \sqrt{M_H R}\, \epsilon_H \,, \tag{8.86}$$

By fixing in this scenario the Higgs localizing mass to its previous value $M_H R = 1.6$ we predict the correct value of the τ mass [66] by means of the 5D Yulkawa coupling $y_\tau \simeq 1$ while $y_\ell \simeq m_\ell / m_\tau$ for the first two generations ($\ell = e, \mu$).

An interesting particular case arises here. Given that cM is negative, in the limit when $|cMR| \gg 1$ Eq. (8.85) reads

$$m^M_{\nu_\ell} \sim Y^2_{\nu_\ell} v^2 \epsilon^2_H M_H \frac{1+c}{s} \left[3c + \text{sign}(M) + 2\text{sign}(M)\, e^{-2\pi|M|R} + 2c\, e^{-2\pi|cM|R}\right] . \tag{8.87}$$

Considering now the value $c = -\frac{1}{3}\text{sign}(M)$ we find a doubly suppressed Majorana eigenmass given. For instance for the case $M > 0$ and $c = -1/3$ one gets

$$m^M_{\nu_\ell} \sim \frac{\sqrt{2}}{3} Y^2_{\nu_\ell} v^2 \epsilon^2_H e^{-\frac{2}{3}\pi M R} . \tag{8.88}$$

which has a doubly suppressed exponential behaviour both from M_H and M. In that case one can get tiny Majorana neutrino masses from the localization of the zero mode of ν_R for $\mathcal{O}(1)$ values of the 5D Yukawa couplings y_{ν_ℓ}. This is shown in Fig. 8.5 where the Majorana mass $m^M_{\nu_\ell}$ is plotted versus $|MR|$ for $y_{\nu_\ell} = 1$ and $c = \pm 1/3$.

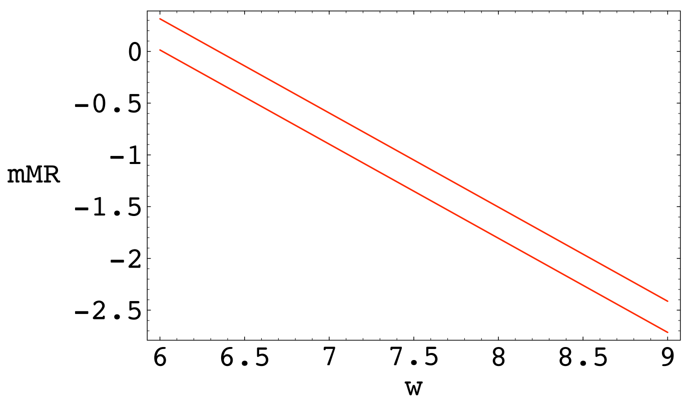

Figure 8.5: Neutrino Majorana mass, $m^M_{\nu_\ell}$, as a function of $|MR|$ for $M_H R = 1.6$ and $c = 1/3$ (upper curve), $c = -1/3$ (lower curve).

Conclusions and Outlook

Along this Thesis we have developed a clear line of research based on models of High Energy Physics in 5 dimensions with the extra dimension defined in the closed interval $(0, \pi R)$ with R at the scale of few TeV^{-1} incorporating global supersymmetry whose aim is to be applied to phenomenology of Physics beyond the SM. In particular we have tried to shed some light on two crucial aspects of the SM Physics: ElectroWeak Symmetry Breaking and the origin of neutrino masses. To this aim we worked out a supersymmetric model in a five dimensional space with boundaries motivated from models defined on the orbifold with odd bulk masses.
As we saw, a geometrical interpretation could be given to supersymmetry breaking by boundary terms, as a mismatch between bulk and boundary mass matrices in two different ways. We identified one of them as a Scherk-Schwarz supersymmetry breaking due to the breaking of the global $SU(2)_R$ symmetry and the other was saw as a soft breaking coming from a possible boundary spurion superfield. While the Scherk-Schwarz breaking is known to be one-loop finte, we checked explicitly that the latter breaking pattern induced linearly divergent corrections to the Higgs mass although its stability is warranted for values of the cut off $\Lambda R \lesssim 10^2$. Finally we used this novel breaking pattern to proposed a model for ElectroWeak Symmetry Breaking since the Higgs spectrum predicted by the boundary conditions presented a tachyon at the tree level.
Non the less, the tachyonic mode is still present in the Higgs spectrum with the Scherk-Schwarz breaking introducing an extra $SU(2)_H$ index in the Higgs multiplet. Therefore, to reduce the UV sensitivity of the model we further investigated the ElectroWeak symmetry breaking process within this context. In particular we exhaustively studied its quantum stability under radiative corrections and we concluded that the tree level tachyon could partially cancel the positive quatum corrections to the Higgs mass coming from the gauge sector and then the EWSB could be triggered with the negative quantum corrections coming from the top-stop sector with a mild fine tuning.

Finally, we continued with the phenomenological insight of this class of models by investigating the possibility for yielding a light neutrino mass. We developed an $N = 1$ supersymmetric action for right handed neutrinos, left handed neutrinos and the Higgs. We found that using the exponential localization towards one of the boundaries of the lowest mode of RH neutrino one can yield an ultra light neutrino Dirac mass while the Higgs exponential localization allows one to get a Majorana like mass at the meV scale for natural values of the 5D Yukawa coupling constants $\sim 10^{-3}R$.
Following the lines of our calculation it should be easy to describe textures of RH neutrino masses describing the different patterns for LH neutrino masses and mixings (see e.g. [88]). It should be enough to introduce the corresponding three-by-three mass matrices and carry on the parallel calculation. In cases where Dirac or Majorana neutrino masses are controlled by the localizing masses, since they depend exponentially on the latter a modest change in the corresponding RH mass eigenvalues should be able to describe realistic neutrino spectra. Of course since the RH neutrino masses are an input in our theory, even if correct spectra do not require to fine-tune any parameters, we cannot call this a "solution to the neutrino mass problem" until some more fundamental theory (e.g. string theory) would give us the correct values for the heavy masses. In fact this was not the aim of our work but a classification of the different solutions of our 5D theory providing realistic spectra for neutrino masses and yielding hints for possible future discoveries at LHC.

Bibliography

[1] Steven Weinberg. The quantum theory of fields. vol. 3: Supersymmetry. Cambridge, UK: Univ. Pr. (2000) 419 p.

[2] T. Appelquist, A. Chodos, and P. G. O. Freund. Introduction to 'modern kaluza klein theories.'. IN *APPELQUIST, T. (ED.) ET AL.: MODERN KALUZA-KLEIN THEORIES* 1-47.

[3] P. Fayet and J. Iliopoulos. Spontaneously broken supergauge symmetries and goldstone spinors. *Phys. Lett.*, B51:461–464, 1974.

[4] J. Wess and J. Bagger. Supersymmetry and supergravity. Princeton, USA: Univ. Pr. (1992) 259 p.

[5] Sidney Coleman and Jeffrey Mandula. All possible symmetries of the s matrix. *Physical Review*, 159:1251, July 1967. Copyright (C) 2008 The American Physical Society; Please report any problems to prola@aps.org.

[6] Steven Weinberg. The quantum theory of fields. vol. 1: Foundations. Cambridge, UK: Univ. Pr. (1995) 609 p.

[7] Peter C. West. Introduction to rigid supersymmetric theories. 1997.

[8] Antoine Van Proeyen. Tools for supersymmetry. 1999.

[9] Eugene A. Mirabelli and Michael Edward Peskin. Transmission of supersymmetry breaking from a 4-dimensional boundary. *Phys. Rev.*, D58:065002, 1998.

[10] Hans Peter Nilles. Phenomenological aspects of supersymmetry. 1995.

[11] Michael Edward Peskin and Daniel V. Schroeder. An introduction to quantum field theory. Reading, USA: Addison-Wesley (1995) 842 p.

[12] M. Quiros. New ideas in symmetry breaking. 2003.

[13] Manfredo Perdigao do Carmo. *Riemannian Geometry*. Birkhäuser Boston, 1 edition, 1992.

[14] M. Nakahara. Geometry, topology and physics. Bristol, UK: Hilger (1990) 505 p. (Graduate student series in physics).

[15] Czes Kosniowski. *A first course in algebraic topology*. Cambridge University Press, 1980.

[16] Joel Scherk and John H. Schwarz. Spontaneous breaking of supersymmetry through dimensional reduction. *Phys. Lett.*, B82:60, 1979.

[17] A. Hebecker. 5d super yang-mills theory in 4-d superspace, superfield brane operators, and applications to orbifold guts. *Nucl. Phys.*, B632:101–113, 2002.

[18] Nima Arkani-Hamed, Thomas Gregoire, and Jay G. Wacker. Higher dimensional supersymmetry in 4d superspace. *JHEP*, 03:055, 2002.

[19] Jonathan Bagger, Ferruccio Feruglio, and Fabio Zwirner. Brane induced supersymmetry breaking. *JHEP*, 02:010, 2002.

[20] Jonathan A. Bagger, Ferruccio Feruglio, and Fabio Zwirner. Generalized symmetry breaking on orbifolds. *Phys. Rev. Lett.*, 88:101601, 2002.

[21] K. A. Meissner, H. P. Nilles, and M. Olechowski. Brane induced supersymmetry breakdown and restoration. *Acta Phys. Polon.*, B33:2435, 2002.

[22] A. Delgado, G. von Gersdorff, and M. Quiros. Brane-assisted scherk-schwarz supersymmetry breaking in orbifolds. *JHEP*, 12:002, 2002.

[23] Petr Horava and Edward Witten. Eleven-dimensional supergravity on a manifold with boundary. *Nucl. Phys.*, B475:94–114, 1996.

[24] Petr Horava. Gluino condensation in strongly coupled heterotic string theory. *Phys. Rev.*, D54:7561–7569, 1996.

[25] Petr Horava and Edward Witten. Heterotic and type i string dynamics from eleven dimensions. *Nucl. Phys.*, B460:506–524, 1996.

[26] P. Fayet. Spontaneous generation of massive multiplets and central charges in extended supersymmetric theories. *Nucl.Phys.* B149:137, 1979.

[27] Ignatios Antoniadis, S. Dimopoulos, A. Pomarol, and M. Quiros. Soft masses in theories with supersymmetry breaking by tev-compactification. *Nucl. Phys.*, B544:503–519, 1999.

[28] A. Delgado, A. Pomarol, and M. Quiros. Supersymmetry and electroweak breaking from extra dimensions at the tev-scale. *Phys. Rev.*, D60:095008, 1999.

[29] Gero von Gersdorff and Mariano Quiros. Localized anomalies in orbifold gauge theories. *Phys. Rev.*, D68:105002, 2003.

[30] Max Zucker. Minimal off-shell supergravity in five dimensions. *Nucl. Phys.*, B570:267–283, 2000.

[31] Max Zucker. Gauged n = 2 off-shell supergravity in five dimensions. *JHEP*, 08:016, 2000.

[32] Max Zucker. Off-shell supergravity in five-dimensions and supersymmetric brane world scenarios. *Fortsch. Phys.*, 51:899–972, 2003.

[33] Tomoyuki Fujita, Taichiro Kugo, and Keisuke Ohashi. Off-shell formulation of supergravity on orbifold. *Prog. Theor. Phys.*, 106:671–690, 2001.

[34] Taichiro Kugo and Keisuke Ohashi. Off-shell d = 5 supergravity coupled to matter-yang-mills system. *Prog. Theor. Phys.*, 105:323–353, 2001.

[35] Taichiro Kugo and Keisuke Ohashi. Superconformal tensor calculus on orbifold in 5d. *Prog. Theor. Phys.*, 108:203–228, 2002.

[36] Taichiro Kugo and Keisuke Ohashi. Supergravity tensor calculus in 5d from 6d. *Prog. Theor. Phys.*, 104:835–865, 2000.

[37] Richard Altendorfer, Jonathan Bagger, and Dennis Nemeschansky. Supersymmetric randall-sundrum scenario. *Phys. Rev.*, D63:125025, 2001.

[38] F. Halzen and Alan D. Martin. Quarks and leptons: An introductory course in modern particle physics. New York, Usa: Wiley (1984) 396p.

[39] J. F. Donoghue, E. Golowich, and Barry R. Holstein. Dynamics of the standard model. *Camb. Monogr. Part. Phys. Nucl. Phys. Cosmol.*, 2:1–540, 1992.

[40] Sidney Coleman and Erick Weinberg. Radiative corrections as the origin of spontaneous symmetry breaking. *Physical Review D*, 7:1888–1910, March 1973.

[41] Mariano Quiros. Finite temperature field theory and phase transitions. 1999.

[42] A. Pomarol and M. Quiros. The standard model from extra dimensions. *Phys. Lett.*, B438:255–260, 1998.

[43] Riccardo Barbieri et al. Radiative electroweak symmetry breaking from a quasi-localized top quark. *Nucl. Phys.*, B663:141–162, 2003.

[44] David Diego, Gero von Gersdorff, and Mariano Quiros. Supersymmetry and electroweak breaking in the interval. *JHEP*, 11:008, 2005.

[45] Peter Breitenlohner and Martin F. Sohnius. Matter coupling and nonlinear sigma models in n=2 supergravity. *Nucl. Phys.*, B187:409, 1981.

[46] G. von Gersdorff, L. Pilo, M. Quiros, D. A. J. Rayner, and A. Riotto. Supersymmetry breaking with quasi-localized fields in orbifold field theories. *Phys. Lett.*, B580:93–101, 2004.

[47] B. de Wit, P. G. Lauwers, and Antoine Van Proeyen. Lagrangians of n=2 supergravity - matter systems. *Nucl. Phys.*, B255:569, 1985.

[48] D. M. Ghilencea, S. Groot Nibbelink, and Hans Peter Nilles. Gauge corrections and fi-term in 5d kk theories. *Nucl. Phys.*, B619:385–395, 2001.

[49] G. von Gersdorff, L. Pilo, M. Quiros, A. Riotto, and V. Sanz. Fermions and supersymmetry breaking in the interval. *Phys. Lett.*, B598:106–112, 2004.

[50] Daniel Marti and Alex Pomarol. Fayet-iliopoulos terms in 5d theories and their phenomenological implications. *hep-ph/0205034*, May 2002. Phys.Rev. D66 (2002) 125005.

[51] R. Barbieri, R. Contino, P. Creminelli, R. Rattazzi, and C. A Scrucca. Anomalies, fayet-iliopoulos terms and the consistency of orbifold field theories. *hep-th/0203039*, March 2002. Phys.Rev. D66 (2002) 024025.

[52] Dmitry V Belyaev. Boundary conditions in the mirabelli and peskin model. *hep-th/0509171*, September 2005. JHEP 0601 (2006) 046.

[53] Daniel Marti and Alex Pomarol. Supersymmetric theories with compact extra dimensions in n=1 superfields. *hep-th/0106256*, June 2001. Phys.Rev. D64 (2001) 105025.

[54] G. F. Giudice and A. Masiero. A natural solution to the mu-problem in supergravity theories. *Physics Letters B*, 206:480–484, May 1988.

[55] D. Diego, G. von Gersdorff, and M. Quiros. The mssm from scherk-schwarz supersymmetry breaking. *hep-ph/0605024*, May 2006. Phys.Rev. D74 (2006) 055004.

[56] Riccardo Barbieri, Guido Marandella, and Michele Papucci. The higgs mass as a function of the compactification scale. *hep-ph/0305044*, May 2003. Nucl.Phys. B668 (2003) 273-292.

[57] Z. Chacko, Markus A Luty, Ann E Nelson, and Eduardo Ponton. Gaugino mediated supersymmetry breaking. *hep-ph/9911323*, November 1999. JHEP 0001 (2000) 003.

[58] Riccardo Barbieri, Guido Marandella, and Michele Papucci. Breaking the electroweak symmetry and supersymmetry by a compact extra dimension. *hep-ph/0205280*, May 2002. Phys.Rev. D66 (2002) 095003.

[59] Riccardo Barbieri, Lawrence J Hall, Guido Marandella, Yasunori Nomura, Takemichi Okui, Steven J Oliver, and Michele Papucci. Radiative electroweak symmetry breaking from a quasi-localized top quark. *hep-ph/0208153*, August 2002. Nucl.Phys. B663 (2003) 141-162.

[60] A. Delgado and M. Quirós. Supersymmetry and finite radiative electroweak breaking from an extra dimension. *hep-ph/0103058*, March 2001. Nucl.Phys. B607 (2001) 99-116.

[61] Riccardo Barbieri, Lawrence J Hall, and Yasunori Nomura. A constrained standard model from a compact extra dimension. *hep-ph/0011311*, November 2000. Phys.Rev. D63 (2001) 105007.

[62] S. Dimopoulos and G. F Giudice. Naturalness constraints in supersymmetric theories with non-universal soft terms. *hep-ph/9507282*, July 1995. Phys.Lett. B357 (1995) 573-578.

[63] Greg Anderson and Diego Castano. Measures of fine tuning. *hep-ph/9409419*, September 1994. Phys.Lett. B347 (1995) 300-308.

[64] David Wright. Naturally nonminimal supersymmetry. *hep-ph/9801449*, 1998.

[65] G. L Kane and S. F King. Naturalness implications of lep results. *hep-ph/9810374*, October 1998. Phys.Lett. B451 (1999) 113-122.

[66] ... S. Eidelman. Review of particle physics. *Physics Letters B*, 592:1–5, July 2004.

[67] Gian F Giudice and Alex Pomarol. Mass degeneracy of the higgsinos. *hep-ph/9512337*, December 1995. Phys.Lett. B372 (1996) 253-258.

[68] John F Gunion and Stephen Mrenna. Probing models with near degeneracy of the chargino and lsp at a linear e+e- collider. *hep-ph/0103167*, March 2001. Phys.Rev. D64 (2001) 075002.

[69] J. F Gunion and S. Mrenna. A study of susy signatures at the tevatron in models with near mass degeneracy of the lightest chargino and neutralino. *hep-ph/9906270*, June 1999. Phys.Rev. D62 (2000) 015002.

[70] Paolo Gondolo, Joakim Edsjo, Piero Ullio, Lars Bergstrom, Mia Schelke, and Edward A Baltz. Darksusy - a numerical package for supersymmetric dark matter calculations. *astro-ph/0211238*, November 2002.

[71] G. F Giudice and A. Romanino. Split supersymmetry. *hep-ph/0406088*, June 2004. Nucl.Phys. B699 (2004) 65-89; Erratum-ibid. B706 (2005) 65-89.

[72] D. N Spergel, R. Bean, O. Doré, M. R Nolta, C. L Bennett, J. Dunkley, G. Hinshaw, N. Jarosik, E. Komatsu, L. Page, H. V Peiris, L. Verde, M. Halpern, R. S Hill, A. Kogut, M. Limon, S. S Meyer, N. Odegard, G. S Tucker, J. L Weiland, E. Wollack, and E. L Wright. Wilkinson microwave anisotropy probe (wmap) three year results: Implications for cosmology. *astro-ph/0603449*, March 2006. ApJS, 170, 377 (2007).

[73] David Smith and Neal Weiner. Inelastic dark matter. *hep-ph/0101138*, 2001. Phys.Rev. D64 (2001) 043502.

[74] Ignatios Antoniadis, Karim Benakli, Antonio Delgado, Mariano Quiros, and Marc Tuckmantel. Splitting extended supersymmetry. *hep-ph/0507192*, July 2005. Phys.Lett. B634 (2006) 302-306.

[75] S. Weinberg. Cosmological constraints on the scale of supersymmetry breaking. *Physical Review Letters*, 48:1303–1306, May 1982.

[76] Nima Arkani-Hamed, Savas Dimopoulos, Gia Dvali, and John March-Russell. Neutrino masses from large extra dimensions. *hep-ph/9811448*, November 1998. Phys.Rev. D65 (2002) 024032.

[77] Keith R Dienes, Emilian Dudas, and Tony Gherghetta. Light neutrinos without heavy mass scales: A higher-dimensional seesaw mechanism. *hep-ph/9811428*, November 1998. Nucl.Phys. B557 (1999) 25.

[78] Yuval Grossman and Matthias Neubert. Neutrino masses and mixings in non-factorizable geometry. *hep-ph/9912408*, December 1999. Phys.Lett. B474 (2000) 361-371.

[79] Tony Gherghetta, Kenji Kadota, and Masahide Yamaguchi. Warped leptogenesis with dirac neutrino masses. *0705.1749*, May 2007. Phys. Rev. D 76, 023516 (2007).

[80] Gia Dvali and Alexei Yu Smirnov. Probing large extra dimensions with neutrinos. *hep-ph/9904211*, April 1999. Nucl.Phys. B563 (1999) 63-81.

[81] Alon E Faraggi and Maxim Pospelov. Phenomenological issues in tev scale gravity with light neutrino masses. *hep-ph/9901299*, 1999. Phys.Lett. B458 (1999) 237-244.

[82] P.W. Higgs. Broken symmetries, massless particlees and gauge fields. *Physics Letters*, 12:132–133, September 1964.

[83] Peter W. Higgs. Broken symmetries and the masses of gauge bosons. *Physical Review Letters*, 13:508–509, October 1964.

[84] Rabindra N. Mohapatra and Goran Senjanovic. Neutrino mass and spontaneous parity nonconservation. *Physical Review Letters*, 44:912–915, April 1980.

[85] Peter Minkowski. mu–¿egamma at a rate of one out of 109 muon decays? *Physics Letters B*, 67:421–428, April 1977.

[86] D. Diego and M. Quiros. Dirac vs. majorana neutrino masses from a tev interval. *0804.2838*, April 2008.

[87] Angel M. Uranga. Intersecting brane worlds. *Classical and Quantum Gravity*, 22:S41–S76, 2005.

[88] M. C Gonzalez-Garcia and Y. Nir. Neutrino masses and mixing: Evidence and implications. *hep-ph/0202058*, February 2002. Rev.Mod.Phys. 75 (2003) 345-402.