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In Defense of Implicit Times

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Abstract

The present dissertation explores the hypothesis that English is a language with time-shifting grammatical devices. According to this hypothesis, the linguistic objects of English grammar which serve as the inputs to semantic interpretation have time-sensitive extensions and some of these objects play their roles in the process of interpretation by shifting the times relative to which other objects are interpreted. In a nutshell, English has intensional devices that manipulate times.

The view that English tenses and temporal adverbs like *now* and *then* have time-shifting meanings was endorsed by some of the founders of modern formal semantics during the seventies and early eighties. These theorists studied certain fragments of English and proposed formalizations for them using regimented languages equipped with temporal operators. Subsequent research shed doubt on the operator-based approach of the early formal semanticists. Their intensional accounts of English temporal discourse were abandoned in favor of referential, quantificational, and dynamic theories. As a result, the hypothesis that English is a time-shifting language (in the sense suggested above) is no longer viewed as a tenable option in mainstream formal semantics. This dissertation examines the main lines of argument that have motivated this theoretical move away from the project of intensional semantics. I argue that the prospects for developing a plausible intensional account of English temporal discourse are not as gloomy as it has been assumed in the literature.

I examine four influential lines of argument for the view that English lacks temporal operators. The advocates of this view have argued that operator-based formalizations of English sentences are inadequate for the purposes of natural language semantics because they (i) have expressive limitations that can only be overcome by positing more and more temporal indices in the intensional system, (ii) are ad hoc and inelegant, (iii) fail to explain the pronominal uses of tenses, and (iv) fail to account for the behavior of embedded tenses. Although it is true that the lines of argument (i)-(iv) reveal the explanatory problems of traditional operator-based theories, I suggest that a sophisticated intensional account of English tenses can overcome those problems. If this suggestion is on the right track, the case against the hypothesis that English is a time-shifting language is far less compelling than it appears at first glance.

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Introduction

This dissertation discusses the idea that the tenses of English can be viewed as time-shifting grammatical devices. There are different ways of fleshing out this basic idea. One way consists in using the operators of some system of temporal logic to analyze tenses. Arthur Prior adopted this approach. Although Prior was more a logician than a natural language theorist, he analyzed the logical role of the English tenses using the operators of classical temporal logic.[1] Prior's work influenced early theorizing on tenses in the field of formal semantics. Some of the founders of this field endorsed the view that tenses can be formalized with the aid of temporal operators.[2] By the end of the eighties, however, this view had become marginal in mainstream formal semantics. It was clear by then that a classical Priorean theory of tense (and of temporal discourse in general) does not account for the truth-conditions of various types of sentences. A variable-based paradigm emerged, especially among theorists associated with the tradition of generative grammar.[3] On the accounts proposed by these theorists, LF-structures

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[1] For some considerations regarding Prior's views on this matter, see my footnotes 4 and 6 below. The main contributions of Prior to the development of temporal logic can be found in Prior 1957, 1967, 1968b, and 2003. Some of his early insights in this area were inspired by his research on ancient and medieval logic. Indeed, the earliest system of temporal logic proposed by Prior appeared in a paper devoted to the Master Argument of Diodorus Cronus (Prior 1955). For discussion of the impact of ancient and medieval views on Prior's thought and other issues related to the origins of modern temporal logic, see Prior 1967, Øhrstrøm & Hasle 1993, 1995, Braüner *et al*. 2000, Copeland 2008, and Uckelman 2012a-b.

[2] Among these theorists were Montague (1970, esp. p. 125, 1973), Kamp (1971, pp. 230-232), Lewis (1980, sections 5 and 8), and Dowty (see Dowty 1982 and Dowty *et al*. 1981, chapter 5 and section VIII of chapter 7). The view is also endorsed in Dummett 1973, pp. 382-400, and Salmon 1989. In "Demonstratives", Kaplan toys with the idea that natural languages have temporal operators (see his 1977, sections VI. (i) and XVIII), but he appears to be officially agnostic about it (see pp. 502-504, esp. fn. 28). The assumption that tenses can be plausibly treated as operators is at the heart of Kaplan and Lewis's operator argument for temporalism. For some critical discussion of this argument, see King 2003, Cappelen & Hawthorne 2009, chapter 3, and Glanzberg 2011.

[3] Classical papers by these generative theorists include Enç 1986, 1987, Zagona 1990, Heim 1994, von Stechow 1995, Stowell 1995, Abusch 1997, and Kratzer 1998. Ogihara (1996) symbolizes natural language sentences employing a quantificational formal language. But, as I understand Ogihara's theory, his LFs lack time-denoting constituents (see esp. sections 2.2 and 2.6 of Ogihara 1996 and the appendix to the book).

contain variable-like constituents which denote times. The behavior of tenses is explained in terms of anaphoric and binding principles that apply to such covert time-denoting constituents. In this dissertation I examine the critical literature that motivated this paradigm shift in formal semantics. In my opinion, a striking aspect of this literature is that the operator paradigm has often been discarded in the light of some piece (or pieces) of evidence without careful examination of the ways in which the operator-based theory that is the target of the criticism can be amended. I argue that it is possible to overcome some important explanatory problems of the classical Priorean theories of tense without postulating explicit temporal variables. More specifically, I argue that if we abandon some syntactic and semantic assumptions that the early advocates of temporal operators made, there is room for developing more sophisticated intensional theories that treat tenses as time-shifting elements and can account for the linguistic data that undermined the traditional Priorean approach.

In order to spell out the syntactic and semantic assumptions in which I will be interested, let me briefly characterize Montague's account of English tenses in Montague 1973 (see esp. pp. 252-253, 257-259). This sort of account is a typical target of the critical literature on operator-based theories that I mentioned in the previous paragraph. Like Prior, Montague analyzed the present perfect and the simple future of English using temporal operators. But, unlike Prior, Montague adopted a model-theoretic framework.[4] The PTQ system of Montague translates sentences (1) and (2) into formulas (3) and (4), respectively.

(1) John has run

(2) Mary will talk

(3) H **run** (*j*)

(4) W **talk** (*m*)

[4] Prior did not favor the model-theoretic approach to temporal-logic systems that Montague and other theorists adopted (see Prior 1958b, 1968b, paper XI, 1968a, and Prior & Fine 1977). Prior always preferred a proof-theoretic approach, even after the development of possible world semantics in the early sixties. He was willing to accept temporal instants as individual entities only if they could be constructed out of tensed facts (see Prior & Fine 1977, p. 37). For a nice discussion of Prior's methodological commitments, see Cresswell 2016.

The interpretations of the operator H (informally read 'it has been the case that') and the operator W (informally read 'it will be the case that') are given by rules that determine the truth-values of formulas of the form Hϕ and Wϕ relative to a PTQ intensional model (let us call it $\mathfrak{R}$) and elements i, j, and g that intuitively represent (respectively) a possible world, a moment of time, and an assignment of values to the variables of PTQ's formal language.[5] If $[\![\ ]\!]^{\mathfrak{R}, i, j, g}$ is the function that assigns extensions to the expressions of this formal language, the semantic rules that interpret H and W are as follows:

(H) $[\![\mathsf{H}\phi]\!]^{\mathfrak{R}, i, j, g} = 1$ iff for some moment j' such that $j' \neq j$ and $j' \leq j$,
$[\![\phi]\!]^{\mathfrak{R}, i, j', g} = 1$

(W) $[\![\mathsf{W}\phi]\!]^{\mathfrak{R}, i, j, g} = 1$ iff for some moment j' such that $j' \neq j$ and $j \leq j'$,
$[\![\phi]\!]^{\mathfrak{R}, i, j', g} = 1$

The extension of a sentential expression of the language is a truth-value, which can be either 1 (truth) or 0 (falsity). The intension of an expression (with respect to a model and a variable assignment) is a function from world/moment pairs to extensions. According to the rules (H) and (W), the truth-values of the formulas Hϕ and Wϕ are determined by the truth-value that the intension of the formula ϕ assigns to that formula with respect to some moment of time different from j but temporally related to j. In other words, H and W are intensional operators which *shift* the current moment of evaluation to another moment.

I could say more about the semantic framework of Montague's PTQ system. But let me stop at this point.

Let us call SOAT (for *standard operator-based account of tenses*) the view that the tenses and time adverbs of English (and other natural languages) can be formally represented by means of sentential operators interpreted by rules like (H) and (W). More specifically, let us assume that according to SOAT the formal representations of English sentences are formulas of an intensional first-order language with temporal operators à la Montague. An advocate of SOAT may

[5] A model of PTQ is a quintuple $\langle A, I, J, \leq, F\rangle$, where A, I, and J are non-empty sets (thought of as a set of individuals, a set of worlds, and a set of times, respectively), $\leq$ is a linear order on J, and F is a function that assigns appropriate intensions to the constants of the formal language (see Montague 1973, pp. 257-260).

want to think of these formulas as specifications of the logical forms of English sentences.[6]

There is a significant body of linguistic data that speaks against SOAT. Below I briefly summarize the data by mentioning some examples of the different types of English sentences that have been shown to be problematic for SOAT. All the examples are borrowed from the literature.

(To use a more standard notation, I will call the past and future operators of SOAT P and F instead of H and W. Let me also call $<$ the relation of temporal precedence between moments)

'now'/ 'then' sentences

[Kamp 1971, Vlach 1973, van Benthem 1977]

(5) One day, all persons now alive will be dead

(6) Jones was once going to cite everyone then driving too fast

Sentences (5) and (6) cannot be expressed in the sort of formal language that SOAT uses. (7) and (8) are two attempts to express the truth-condition of (5) with SOAT formulas.

(I assume that the variable *x* ranges over persons)

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[6] Prior 1962 appears to be the earliest publication in which Prior endorsed this view that there are temporal operators in English at the level of logical form. His earlier publications on temporal logic (such as Prior 1955, 1957, and 1958a-c) are focused on the construction of various operator-based logical systems. Although it is true that in those works Prior sought to regiment some natural language statements –e.g. the premises and conclusion of the Master Argument–, in my opinion he was not sufficiently explicit about the nature of the relation between such statements and their tense-logical regimentations. Logical-form-specification is one possible relation, but there are weaker relations –e.g. truth-conditional equivalence– that arguably suffice for the theoretical purposes of those early works. By contrast, Prior's analysis of the tenses of English in Prior 1962 (pp. 6-9) can be reasonably read as one that posits temporal operators in the logical forms of tensed English sentences. Although the expression 'logical form' is not employed in that paper, Prior draws a distinction between surface and logical structure very much in the spirit of Russell's distinction between surface structure and logical form. Moreover, he argues that his logical analysis of tense allows us to solve certain metaphysical puzzles about the nature of time and change (see also Prior 1958c and 1967, chapter I, §8). This argumentative strategy makes more sense if Prior is giving an account of logical forms and not merely a translation into a regimented tense-logical language.

(7) $\mathsf{F}\ \forall x$ (x **is alive** $\rightarrow$ x **is dead**)

(8) $\forall x$ (x **is alive** $\rightarrow$ F x **is dead**)

But (7) and (8) have the truth-conditions given in (9) and (10), whereas the truth-condition of (5) is (11) –u stands for the time of utterance of (5).

(9) There is a time t such that $u < t$ and, for every person x, if x is alive at t, then x is dead at t.

(10) For every person x, if x is alive at u, then there is a time t such that $u < t$ d and x is dead at t.

(11) There is a time t such that $u < t$ and, for every person x, if x is alive at u, then x is dead at t.

(11) is not equivalent to (9)-(10).

Sentence (6) poses a similar expressibility problem.[7]

Pronominal uses of tenses

[Partee 1973]

(12) I didn't turn off the stove

(13) Sheila had a party last Friday and Sam got drunk

Partee considered a scenario in which a speaker utters (12) to communicate that she did not turn off the stove during a particular time interval –for example, during the morning of the day of utterance. (14) and (15) fail to capture the intuitive content of (12) because they quantify unrestrictedly over moments.

(14) $\neg$ P ***I* turn off the stove**

[7] I discuss sentence (6) in chapter 1.

(15) P ¬ ***I* turn off the stove**

(14) says that the speaker has never turned off the stove and (15) says that there is some past time at which the speaker did not turn it off. This is not what (12) intuitively says.

(13) is naturally interpreted as saying that Sam got drunk at the time of the party. There seems to be a relation of temporal anaphora between the times of the two events reported in (13). SOAT does not have a way of capturing this sort of anaphoric relation.[8]

Enç sentences

[Enç 1986]

(16) All rich men were obnoxious children

Enç discussed sentences in which nouns receive temporal interpretations that are independent of the tenses. She considered a reading of (16) in which (16) is used to communicate that all present and past rich men were obnoxious children. SOAT formalizations like (17) and (18) fail to capture this reading.

(17) ∀*x* (*x* **is a rich man** → P *x* **is an obnoxious child**)

(18) P ∀*x* (*x* **is a rich man** → *x* **is an obnoxious child**)

(17) talks only about present rich men. (18) says that there was a time *t* such that all men who were rich at *t* were obnoxious children at *t*.

Embedded tenses

[Enç 1987, Kusumoto 2005]

(19) John heard that Mary was pregnant

(20) Hilary Clinton married a man who became president of the U. S.

[8] Partee also discusses the bound uses of tenses. See Partee 1973, pp. 605-607.

(19) has a simultaneous reading in which Mary's pregnancy and John's hearing overlap. Note that the tense of the complement clause of (19) in a past tense. If the operator P is the formal correlate of the English past, (21) seems to be the natural way of representing (19) according to SOAT.

(The verb *hear* can be represented in SOAT as a 2-place operator **Hear** that makes a formula out of a singular term and another formula)

(21) P ***John*** **Hear** P ***Mary*** **is pregnant**

However, (21) does not express the simultaneous reading of (19). It expresses the shifted reading of (19) in which Mary's pregnancy precedes John's hearing.

Sentence (20) has also a past tense in its embedded clause. If P is the formal correlate of (20)'s embedded past, one would expect that a SOAT formalization of (20) is a formula with a P-operator under the scope of another P-operator. This is what we see in (22).

(22) ∃x P (***Hilary Clinton*** **marry** x ∧ P x **become president of the U. S.**)

But (20) and (22) have different truth-conditions. (22) locates the marriage event after the becoming-president event. (20) can be true if the two events occurred in the opposite order.

Although these examples reveal the empirical inadequacy of SOAT, we can find valuable suggestions as to how to amend SOAT in the writings of Prior and in the writings of several theorists who have been inspired by Prior's work. In this 'Priorean literature', some assumptions of SOAT have been abandoned.

Let me mention three specific assumptions of SOAT that will be relevant for the purposes of this dissertation.

(I) *Single-indexing:* SOAT is a single-index theory. It assigns truth-values to sentential formulas with respect to one time and one world.

In the seventies, Kamp (1971) and Vlach (1973) proposed multiply indexed logical systems with 'now'/'then' operators. With the aid of these operators, it is possible to express the truth-conditions of (5)-(6). To my knowledge, the most sophisticated operator-based theory of tense that incorporates the idea of double-indexing is the account developed in Dowty 1982. Dowty defined past, present, and future operators that relate two temporal points: the time of utterance and a time playing a role analogous to Reichenbach's (1947, §51) point of reference. By relying on operators of this kind, Dowty proposed specific formalizations for the constructions illustrated by the sentences (12), (13), and (19).

(II) *Times as instants:* PTQ is a moment-based system. Its formulas are not assessed with respect to intervals of time.

Some interval-based system have been proposed in the Priorean literature. One logical system that incorporates intervals is Prior's metric tense-logic.[9] On this system, the past and future operators are treated as dyadic operators with an argument place for an interval measure and an argument place for a sentential formula. These operators give rise to formulas of the form $Pn\phi$ and $Fn\phi$, which may be read, respectively, as 'it was the case n-much ago that ϕ' and 'it will be the case n-much hence that ϕ'.[10] Cresswell (2013) shows that sentences like (5) and (6) can also be symbolized in a metric tense-logical language. There are other ways of introducing interval-sensitivity into a temporal logic framework. For example, Blackburn has proposed some extensions of Priorean temporal logic in which new sorts of atomic propositional symbols are introduced.[11] These symbols are true at specific time instants or time intervals. They are the temporal logic analogues of temporal variables, temporal indexicals, and calendar terms.

(III) *Sententiality:* SOAT's temporal operators are sentential. They operate on sentential formulas.

One important claim that I want to defend in this dissertation is that assumption (III) is not essential to the project of intensional semantics. We can postulate time-shifting operators at the sub-sentential level. Such operators might act on

[9] Metric tense-logic is discussed in Prior 1957, chapter 2, pp. 11-15, 1967, chapter 6, and 1968, paper IX.

[10] The interval-measure term appearing in such formulas can be quantified. This operation produces formulas of the form $QnPn\phi$ and $QnFn\phi$, where Q is a monadic quantifier.

[11] See e.g. Blackburn 1993, 1994, and 2006.

predicates, or maybe on sub-predicative syntactic constituents. The possibility of treating tenses as non-sentential operators has been envisaged by a few authors. But it has not been properly explored.[12]

This dissertation is divided into two parts. The first part discusses intensional approaches that abandon assumption (I) but preserve the other two assumptions mentioned above. The second part explores two intensional analyses of English tenses that dispense with assumptions (I)-(III). Each chapter challenges a line of argument against temporal operators. Chapters 1 and 2 consider sentences like (5) and (6). In chapter 1 I criticize van Benthem and Cresswell's suggestion that the problem that these sentences pose for single-index approaches generalizes leading to a semantics of infinite temporal indices. In chapter 2 I criticize King's suggestion that formalizations with multiply indexed operators are inelegant and ad hoc in comparison to formalizations with explicit quantification over times. Chapters 3 and 4 challenge the idea that intensional theories cannot account for sentences like (12)-(13) and (19)-(20). Different intensional tools for modeling tenses will be considered in this dissertation. I start with 'now'/'then'-operators in chapter 1. In chapter 2 I describe some types of operators that make classical multiple-index systems more flexible. In chapter 3 I propose a way of analyzing tenses and modals as sub-sentential operators. The most sophisticated proposal that I discuss is the account of embedded tenses proposed in chapter 4, which represent tenses as LF-constituents with a time-shifting semantics.

[12] See e.g. Enç 1986, pp. 421-422, and Salmon (1989, fn 31). The question of whether there are sub-sentential intensional operators in natural language has been discussed in the philosophical literature on semantic relativism (see Cappelen and Hawthorne 2009, pp. 74-76, and Kölbel 2011, pp. 144-145).

Part 1: Expressibility and Multiple Indexing

1. Multiple Temporal Indexing in Early Formal Semantics

The idea of double indexing is a familiar one in the philosophy of language and in formal semantics. Arguably, Kaplan was the author who contributed the most to its refinement and popularization. But by the time Kaplan was developing his theory of indexicals, double indexing had already been in the air for a few years and double-index logical systems were being studied by theorists such as Kamp (1967, 1971), Vlach (1973), Segerberg (1973), and Åqvist (1973). Interestingly, the motivations that these theorists had for adopting double indexing were subtly different from Kaplan's. One important motivation concerned the possibility of expressing the truth-conditions of certain fragments of English using intensional operators. In this chapter I discuss a family of arguments that rely on considerations about the expressibility of the truth-conditions of English sentences. I will call arguments of this family *expressibility arguments*. Kamp, Vlach, and Segerberg motivated their double-index systems by appeal to arguments of this sort.[1]

Expressibility arguments, as I will characterize them, may be invoked for many different purposes. But I will focus my attention here on some influential expressibility arguments from the literature on multiple indexing. Sections 1.1 and 1.2 provide a formal framework for my discussion. In section 1.3 I analyze the basic structure of expressibility arguments by considering Kamp and Vlach's expressibility arguments for double indexing. In sections 1.4 and 1.5 I examine the expressibility arguments that van Benthem (1977) and Cresswell (1990) gave in order to show that the case for double temporal indexing generalizes and leads to the postulation of infinite temporal indices. I argue that van Benthem and Cresswell's arguments are rather modest in terms of dialectical force. I also

[1] See Kamp 1971, pp. 230-232, Vlach 1973, pp. 2-5, 39-41, and Segerberg 1973, pp. 77-79. Åqvist's motivation for adopting double indexing was to avoid theorem (9c) of his system Å, which he interpreted as precluding the possibility of non-contingent conditionals (see Åqvist 1973, pp. 29, 58-59). As it is well known, Kaplan (1977, section VII) argued for double indexing, and against the view that he called *index-theory*, by reflecting upon the modal and logical properties of sentences like *I am here now*. His argument did not rely on expressibility considerations. When Kaplan considered sentences which may give rise to an expressibility argument for double indexing (see Kaplan 1977, section V), he was presupposing the context/circumstance distinction and was arguing for something different. Expressibility arguments are also absent in Lewis' (1980) defense of the context/index distinction.

briefly discuss an expressibility argument based on an example of Saarinen (1978) that is dialectically more relevant for my purposes in this dissertation.

1.1 Basic syntax and semantics of $\mathscr{L}$-languages

In this section I describe the basic syntax and semantics of a family of formal languages. I will call the languages of this family $\mathscr{L}$-languages. I will reconstruct the expressibility arguments discussed in this chapter as arguments that compare the expressive potential of different $\mathscr{L}$-languages that use temporal operators.

From a syntactic perspective, $\mathscr{L}$-languages are first-order languages equipped with standard quantifiers, standard connectives, and sentence-forming operators. I will assume that every $\mathscr{L}$-language has a vocabulary that contains infinitely many variables, one or more first-order predicates, one or more monadic operators, and the symbols $\neg$, $\wedge$, $\vee$, $\rightarrow$, **(**, **)**, $\exists$, and $\forall$. The vocabulary of the language may also include individual constants and dyadic operators which combine with a singular term and a sentential formula to produce another sentential formula.

Let $\boldsymbol{l}$ be an $\mathscr{L}$-language. Let $\boldsymbol{V}$ be the set of variables of $\boldsymbol{l}$, $\boldsymbol{P}$ be the set of predicates of $\boldsymbol{l}$, $\boldsymbol{I}$ be the (possibly empty) set of individual constants of $\boldsymbol{l}$, $\boldsymbol{M}$ be the set of monadic operators of $\boldsymbol{l}$, and $\boldsymbol{D}$ be the (possibly empty) set of dyadic operators of $\boldsymbol{l}$. Let $\boldsymbol{S}$ be the set $\boldsymbol{V} \cup \boldsymbol{I}$ (the set of singular terms of $\boldsymbol{l}$).

The following rules define the class of well-formed formulas (wffs) of $\boldsymbol{l}$:

(R_1) If Π is an n-place predicate (for some positive integer n), $\Pi \in \boldsymbol{P}$, and $\alpha_1 \ldots \alpha_n$ are singular terms such that $\{\alpha_1, \ldots, \alpha_n\} \subseteq \boldsymbol{S}$, then $\ulcorner \Pi\alpha_1 \ldots \alpha_n \urcorner$ is a wff of $\boldsymbol{l}$

(R_2) If $\mathrm{O} \in \boldsymbol{M}$ and ϕ is a wff of $\boldsymbol{l}$, then $\ulcorner \mathrm{O}\phi \urcorner$ is a wff of $\boldsymbol{l}$

(R_3) If $\Gamma \in \boldsymbol{D}$, $\alpha \in \boldsymbol{S}$, and ϕ is a wff of $\boldsymbol{l}$, then $\ulcorner {}_{\alpha}\Gamma\phi \urcorner$ is a wff of $\boldsymbol{l}$

(R_4) If ϕ, ψ are wffs of $\boldsymbol{l}$ and $\nu \in \boldsymbol{V}$, then $\ulcorner \neg\phi \urcorner$, $\ulcorner (\phi \wedge \psi) \urcorner$, $\ulcorner (\phi \vee \psi) \urcorner$, $\ulcorner (\phi \rightarrow \psi) \urcorner$, $\ulcorner \exists\nu\phi \urcorner$, and $\ulcorner \forall\nu\phi \urcorner$ are wffs of $\boldsymbol{l}$

(R_5) Nothing else is a wff of $\boldsymbol{l}$

Let us turn now to the semantics of $\mathscr{L}$-languages.

Def. 1 Let us say that a quintuple $\langle D, W, T, <, F\rangle$ is an *interpretation of* $\boldsymbol{l}$ if the following conditions hold:

(C_1) D, W, and T are non-empty sets

(C_2) $<$ is a strict linear order on T

(C_3) F is a function with domain $\boldsymbol{I} \cup (\boldsymbol{P} \cup \boldsymbol{D})$ such that for every $\gamma \in \boldsymbol{I} \cup (\boldsymbol{P} \cup \boldsymbol{D})$,

- if $\gamma \in \boldsymbol{I}$, then $F(\gamma) \in D$
- if γ is an n-place predicate and $\gamma \in \boldsymbol{P}$, then $F(\gamma)$ is a function from $W \times T$ to $\wp(D^n)$
- if $\gamma \in \boldsymbol{D}$, then $F(\gamma)$ is a function from $W \times T \times D$ to $\wp(W)$

If we think of D, W, and T (respectively) as a set of individuals, a set of possible worlds, and a set of moments, we can think of F as a function whose job is to assign denotations to the individual constants, predicates, and dyadic operators of $\boldsymbol{l}$. Intuitively, F interprets these symbols in such a way that each individual constant denotes an individual, each n-place predicate denotes a function from pairs of worlds and moments to sets of n-tuples of individuals, and each dyadic operator denotes a function from triples of worlds, moments, and individuals to sets of worlds. Since W and T will be thought of here as a set of worlds and a set of moments, respectively, I will often call the elements of W *world indices* and the elements of T *temporal indices* (or simply *times*).

Let $\mathfrak{I} = \langle D, W, T, <, F\rangle$ be an interpretation of $\boldsymbol{l}$. Let us call any function from $\boldsymbol{V}$ to D a *variable assignment* (with respect to $\mathfrak{I}$). I will adopt a couple of notational conventions concerning variable assignments. First, if g is a variable assignment, $d \in D$, and $\nu \in \boldsymbol{V}$, $g[\nu/d]$ is the variable assignment which maps ν to d and is otherwise identical to g. Second, if g is a variable assignment, F_g is the unique function from $\boldsymbol{S}$ to D such that for any $\alpha \in \boldsymbol{S}$, $F_g(\alpha) = g(\alpha)$ if $\alpha \in \boldsymbol{V}$, and $F_g(\alpha) = F(\alpha)$ if $\alpha \in \boldsymbol{I}$.[2]

I will assume that the monadic operators of an $\mathscr{L}$-language are interpreted by means of semantic clauses that appear in the recursive definition of the notion of truth-value assignment for that language. In order to illustrate how this sort of recursive definition looks like, let us suppose that the monadic operators of $\boldsymbol{l}$ are the Priorean operators P and F.

Let A be the set of all variable assignments (with respect to $\mathfrak{I}$). Let ***Wff*** be the set of all wffs of $\boldsymbol{l}$.

Def. 2 The *truth-value assignment of* ***l*** *with respect to* $\mathfrak{I}$ is the unique (four-place) function $[\![\ ]\!]^{g,\,w,\,t}$ from ***Wff***, A, W, and T to the set $\{1, 0\}$ such that for any $g \in A$, $w \in W$, $t \in T$, n-place predicate $\Pi \in \boldsymbol{P}$, dyadic operator $\Gamma \in \boldsymbol{D}$, variable $\nu \in \boldsymbol{V}$, singular terms α, $\alpha_1 \ldots \alpha_n$ included in $\boldsymbol{S}$, and wffs ϕ and ψ of $\boldsymbol{l}$,

(S1) $[\![\Pi\alpha_1 \ldots \alpha_n]\!]^{g,\,w,\,t} = 1$ iff $\langle F_g(\alpha_1), \ldots F_g(\alpha_n)\rangle \in F(\Pi)(\langle w, t\rangle)$

(S2) $[\![{}_{\alpha}\Gamma\phi]\!]^{g,\,w,\,t} = 1$ iff for any $w' \in W$ such that $w' \in F(\Gamma)(\langle w, t, F_g(\alpha)\rangle)$, $[\![\phi]\!]^{g,\,w',\,t} = 1$

(S3) $[\![\mathsf{P}\phi]\!]^{g,\,w,\,t} = 1$ iff for some $t' \in T$ such that $t' < t$, $[\![\phi]\!]^{g,\,w,\,t'} = 1$

(S4) $[\![\mathsf{F}\phi]\!]^{g,\,w,\,t} = 1$ iff for some $t' \in T$ such that $t < t'$, $[\![\phi]\!]^{g,\,w,\,t'} = 1$

(S5) $[\![\neg\phi]\!]^{g,\,w,\,t} = 1$ iff $[\![\phi]\!]^{g,\,w,\,t} = 0$

(S6) $[\![(\phi \wedge \psi)]\!]^{g,\,w,\,t} = 1$ iff $[\![\phi]\!]^{g,\,w,\,t} = 1$ and $[\![\psi]\!]^{g,\,w,\,t} = 1$

(S7) $[\![(\phi \vee \psi)]\!]^{g,\,w,\,t} = 1$ iff $[\![\phi]\!]^{g,\,w,\,t} = 1$ or $[\![\psi]\!]^{g,\,w,\,t} = 1$

(S8) $[\![(\phi \rightarrow \psi)]\!]^{g,\,w,\,t} = 1$ iff $[\![\phi]\!]^{g,\,w,\,t} = 0$ or $[\![\psi]\!]^{g,\,w,\,t} = 1$

(S9) $[\![\exists\nu\phi]\!]^{g,\,w,\,t} = 1$ iff for some $d \in D$, $[\![\phi]\!]^{g[\nu/d],\,w,\,t} = 1$

[2] In other words, F_g is the unique function that maps each variable of $\boldsymbol{l}$ to the object that that g assigns to that variable and maps each individual constant of $\boldsymbol{l}$ to the object that F assigns to that constant.

(S10) $[\![(\forall \nu\phi)]\!]^{g, w, t} = 1$ iff for every $d \in D$, $[\![\phi]\!]^{g[\nu/d], w, t} = 1$

As usual, 1 and 0 are the truth-values truth and falsity. Clauses (S1)-(S10) tell us how the truth-values of ***l***'s wffs are determined relative to $\Im$. (S3) and (S4) are the semantic clauses of P and F. According to these clauses, the operators P and F shift the temporal index t of $[\![\ ]\!]^{g, w, t}$ to another temporal index that precedes or succeeds t in the linear order $<$. Later in this chapter we will be interested in the expressive power of $\mathscr{L}$-languages equipped with 'now'/'then'-operators. These operators are introduced in section 1.2.

The truth-predicate of ***l*** can be defined in the following manner:

Def. 3 If ϕ is a closed wff of ***l***, $\Im = \langle D, W, T, <, F\rangle$ is an interpretation of ***l***, $w \in W$, and $t \in T$,

ϕ IS TRUE IN ***l*** AT $\langle \Im, w, t\rangle$ iff for every variable assignment g,

$$[\![\phi]\!]^{g, w, t} = 1$$

This truth-predicate applies to closed wffs of ***l*** (wffs of ***l*** without free variables) with respect to triples of interpretations of ***l***, world indices, and temporal indices.

A few comments are in order regarding the basic semantics for $\mathscr{L}$-languages given in this section.

First, our semantics for $\mathscr{L}$-languages is moment-based (or point-based) rather than interval-based. The drawbacks of moment-based approaches to tense and aspect have been known since the early seventies (see Bennett & Partee 1972). During that decade several theorists developed intensional semantic frameworks that incorporated intervals.[3] Nonetheless, interval-based systems are hardly ever mentioned in the literature on temporal expressive power. The contributions to this literature that I am going to discuss in this chapter and the next one rely on moment-based semantic frameworks. This is my main excuse for adopting the same kind of framework here, in spite of the fact that interval-based frameworks

[3] See Kuhn 1976, Åqvist 1978, 1979, Guenthner 1978, Åqvist & Guenthner 1978, Åqvist, Guenthner & Rohrer 1978, Humberstone 1979, Dowty 1979, and chapters 2-6 of Cresswell 1985.

are better suited for the study of natural language.[4] Intervals will play a central role in the accounts of tense that I will propose in Part 2.

Second, although I have stipulated that the relation $<$ is a strict linear order, this stipulation will not play a crucial role in my discussion.[5] If our aim was to investigate the logical properties of $\mathscr{L}$-languages, it might have been desirable to impose less constraints on this relation. What matters for our purposes is that $<$ can be intuitively regarded as an earlier-later relation between the elements of T. I take it that this desideratum is satisfied if $<$ is a strict linear order. But some relation weaker than a linear order may also do the job.[6] As long as $<$ imposes a structure on T that can be reasonably viewed as a temporal structure, we can be open-minded with respect to the properties of $<$.

Third, the treatment of dyadic operators encoded in condition (C_3) and clause (S_2) is based on Hintikka's classical analysis of propositional attitude reports (see Hintikka 1969). I adopt a Hintikka-style approach because it fits nicely with a standard intensional semantics for temporal operators. I am not committed to the view that this kind of approach offers a fully adequate analysis of attitude attributions and speech reports. Theories that rely on unstructured contents have well-known problems. But we do not need to worry about those problems here. Dyadic operators will be used in this chapter to express the truth-conditions of some types of English sentences. Insofar as the truth-conditions of $\mathscr{L}$-language formulas give a reasonable approximation to the intuitive truth-conditions of the relevant English sentences, we are not compelled to look for a more sophisticated analysis of attitude and speech reports.

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[4] Another reason to adopt a moment-based framework is that with it we can provide simple semantic clauses for the logical connectives –such as clauses (S_5)-(S_8) above– without having to deal with the technical problems that arise when we try to interpret those connectives using a notion of truth at an interval. For discussion of these problems, see Cresswell 1985, chapter 3 and chapter 5, section 9, and Landman 1991, chapter 5.

[5] A binary relation on a set is a strict linear order if it is transitive, irreflexive, and connected. The properties of transitivity and irreflexivity entail that strict linear orders are asymmetric. (For accessible definitions of the notions of strict order and linear order, see e.g. Partee, ter Meulen & Wall 1990, chapter 3.)

[6] We could have assumed, for example, that $<$ must be transitive, irreflexive, and backwards linear. (For a definition of backwards linearity, see Øhrstrøm & Hasle 1995, chapter 2.8.) This characterization of $<$ makes forward branching permissible. Since branching time will not be discussed in this dissertation, there is no need to revise (C_2). But it is worth mentioning that my general approach is compatible with a linear or a branching conception of time.

Fourth, note that ∃ and ∀ quantify unrestrictedly over the elements of the set D. There is no requirement that the quantified objects exist at the index points w and t that serve as the current points of evaluation. If we take these indices to be possible worlds and moments, clauses (S_9) and (S_{10}) amount to treating both quantifiers as untensed and possibilist quantifiers. This treatment of quantifiers is standard in the literature on multiple indexing that is the focus of this part of the dissertation.[7]

Later in this chapter we will look at various sentences of English and we will wonder whether their truth-conditions can be expressed with the aid of temporal operators of different kinds. In addressing this question, it will be convenient to have a specific $\mathscr{L}$-language whose non-logical symbols are formal counterparts of some of the words and phrases that appear in the English sentences at stake. Let me call this $\mathscr{L}$-language ***L***. I will assume that P and F are the only monadic operators of ***L***. (However, I will use the symbols H and G as shorthand for $\neg\mathsf{P}\neg$ and $\neg\mathsf{F}\neg$, respectively.) Let me call ***Lex*** (for ***L***'s lexicon) the set of individual constants, predicates, and dyadic operators of ***L***. Below I give an ad hoc list of the primitive symbols of ***L*** that are elements of ***Lex***. This list is based on the English sentences that will be discussed in chapters 1 and 2. ***L*** has no individual constants, predicates or dyadic operators apart from those included in the list.

Individual constants: **Jones**, **Mary**

1-place predicates: **alive**, **barbarian**, **become ruler of the world**, **born**, **child**, **dead**, **drive too fast**, **happy**, **idiot**, **king**, **man**, **miserable**, **rich**, **support the Vietnam War**, **be told**

2-place predicates: **cite**

Dyadic operators: **Admit**, **Believe**

As you can see, the predicates of ***L*** are correlates of certain adjectives, common nouns, and verb phrases of English. These predicates lack extra argument places for times, worlds, situations, or events. The dyadic operators of ***L*** are correlates

[7] For discussion of some formal alternatives to untensed/possibilist quantifiers and constant domains, see Garson 2001 and Braüner & Ghilardi 2007. Notice also that the denotations of individual constants are index-insensitive according to (C_3). By contrast, the denotations of predicates and dyadic operators are sensitive to the indices w and t.

of intensional verbs that take *that*-clauses as complements. The two individual constants of ***L*** are correlates of the proper names *Jones* and *Mary*. (I will assume that these names refer to a specific Jones and a specific Mary.)

Our general definition of an $\mathscr{L}$-language interpretation specifies the class of interpretations of ***L*** (***L***-*interpretations* for short). The truth-value assignment of ***L*** with respect to an ***L***-interpretation is defined as above (see *Def. 2*).

Some ***L***-interpretations assign to the symbols included in ***Lex*** denotations that are inconsistent with the conventional meanings of the English counterparts of those symbols. There are, for example, ***L***-interpretations in which **Jones** denotes Mary, in which the denotation of **famous** assigns to a world/moment pair $\langle w, t\rangle$ a set of individuals that are not famous in w at t, and in which the denotation of **Believe** assigns to a world/moment/individual triple $\langle w, t, d\rangle$ a set of worlds that are not d's doxastic alternatives with respect to w and t. Since we want ***L*** to be a language for specifying truth-conditions, it will be convenient to characterize an ***L***-interpretation that is properly constrained by English meanings.

> *Def. 4* I will say that an ***L***-interpretation $\langle D, W, T, <, F\rangle$ is *the intended interpretation of* ***L*** if (i) W is the set of all possible worlds, (ii) T is the set of all moments of time, (iii) $<$ is the relation of temporal precedence between moments, (iv) D is the set of all individuals that exist in some world at some moment, and (v) F is such that
>
> F (**Jones**) = Jones
>
> F (**Mary**) = Mary
>
> F (**alive**) $= f : W \times T \longrightarrow \wp(D)$
>
> For every $\langle w, t\rangle \in W \times T$,
> $f(\langle w, t\rangle) = \{x \in D$: x is alive in w at $t\}$
>
> F (**barbarian**) $= g : W \times T \longrightarrow \wp(D)$
>
> For every $\langle w, t\rangle \in W \times T$,
> $g(\langle w, t\rangle) = \{x \in D$: x is a barbarian in w at $t\}$
>
> ...

$F\,(\textbf{cite}) = h : W \times T \longrightarrow \wp\,(D \times D)$

For every $\langle w, t\rangle \in W \times T$,
$h\,(\langle w, t\rangle) = \{\langle x, y\rangle \in D \times D\text{: } x \text{ cites } y \text{ in } w \text{ at } t\}$

$F\,(\textbf{Admit}) = f' : W \times T \times D \longrightarrow \wp\,(W)$

For every $\langle w, t, d\rangle \in W \times T \times D$,
$f'\,(\langle w, t, d\rangle) = \{w' \in W\text{: } w' \text{ is a world compatible with what } d \text{ admits in } w \text{ at } t\}$

$F\,(\textbf{Believe}) = g' : W \times T \times D \longrightarrow \wp\,(W)$

For every $\langle w, t, d\rangle \in W \times T \times D$,
$g'\,(\langle w, t, d\rangle) = \{w' \in W\text{: } w' \text{ is a world compatible with what } d \text{ believes in } w \text{ at } t\}$.

I will assume that there is a unique ***L***-interpretation that satisfies this definition. Condition (v) of the definition does not give the value of F for every symbol of ***Lex***. But the attentive reader will have no trouble filling in the blanks.

Admittedly, the denotation-clauses of condition (v) are rather artificial for the case of those ***Lex*** symbols that are correlates of English words and phrases with eventive meanings. It is odd to say, for example, that an individual is born, or cites someone, at a particular moment. Events like being born or citing someone unfold over intervals of time. As I explained above, I will set this problem aside in Part 1. Intervals and events will take up our attention in Part 2.

In using *Def. 4* to specify the intended interpretation of ***L***, I am taking for granted that the denotation function of an ***L***-interpretation can be consistent with English meanings and assign classical set-theoretic denotations to the symbols of ***Lex***. This assumption may seem at odds with the fact that English is a vague language. If the English noun *child* has borderline cases, how can the predicate **child** have a denotation which determines the set of children with respect to every world/moment coordinate? Given that vagueness is not a central topic of this dissertation, I will set aside this problem too. For the purposes of this chapter, we can think of the denotation function of the intended interpretation of ***L*** as a function that assigns denotations to the symbols of ***Lex*** using precisifications of the English counterparts of those symbols.[8]

1.2 $\mathscr{L}$-languages with 'now'/'then'-operators

In the previous section I defined the truth-value assignment of an $\mathscr{L}$-language with operators P and F as a four-place function $[\![\ ]\!]^{g,\,w,\,t}$. A language of this kind is a single-index language. In our present semantic framework, this means that the truth-value assignment of the language has only one argument-place for a world index and one argument-place for a temporal index. In this section I introduce the 'now'/'then'-operators that were studied by Kamp and Vlach in the late sixties and early seventies. I explain how these operators behave when they are used in the context of an $\mathscr{L}$-language equipped with the operators P and F. Since ***L*** is an $\mathscr{L}$-language of this kind, I will characterize the 'now'/'then'-operators of Kamp and Vlach by considering some extensions of ***L*** obtained by adding the relevant 'now'/'then'-operators to ***L***'s vocabulary. The truth-value assignments of $\mathscr{L}$-languages equipped with 'now'/'then'-operators have two or more argument-places for temporal indices. In our semantic framework, this amounts to saying that they are languages with multiple temporal indexing.

In this section and subsequent ones, I will informally describe the semantic effects of some operators in the context of their respective formulas. I will say that those operators "shift", "delete", "store", or "retrieve" a temporal index. One can fully appreciate what an operator does in a formula by writing down a step-by-step derivation of the truth-condition of that formula. In the appendix to this chapter, I provide derivations of this sort for some of the operator-based formulas that we will consider later. My informal remarks about the effects of specific operators may be better understood in the light of the derivations given in the appendix.

---

[8] A precisification can be defined as in Weatherson 2014, section 7.3. The denotation-clauses of condition (v) can be reformulated in a way in which they include explicit reference to precisifications. Given an arbitrary precisification of the noun *child*, F (**child**) could be the function that assigns to every world/moment pair $\langle w, t\rangle$ the set of individuals which, under the chosen precisification, are children in w at t.

Kamp developed the first double-index semantics for an intensional language in a short paper written in the fall of 1967.[9] One of the intensional operators that he characterized in that paper was the so-called 'now'-*operator* N. To see how this operator works, let us consider the $\mathscr{L}$-language that is obtained when we add the 'now'-operator N to the stock of primitive symbols of $\boldsymbol{L}$. I will use subscripts to name the extensions of $\boldsymbol{L}$ in which we will be interested here. Each subscript corresponds to a temporal operator that is added to $\boldsymbol{L}$. Thus, $\boldsymbol{L}_{\mathsf{N}}$ is the language that results from adding N to the list of monadic operators of $\boldsymbol{L}$. The truth-value assignment of every $\mathscr{L}$-language has to be specified by means of an explicit definition analogous to *Def. 2*. Such a definition must provide a semantic clause for each logical symbol of the language. Instead of giving explicit definitions of the truth-value assignments of the different extensions of $\boldsymbol{L}$ that we will consider in this dissertation, I will just give the semantic clauses of the relevant operators.[10] Notice that if two $\mathscr{L}$-languages have the same individual constants, predicates, and dyadic operators, it follows from *Def. 1* that they have the same class of interpretations. The interpretations of $\boldsymbol{L}$ are also the interpretations of all the extensions of $\boldsymbol{L}$ that we will consider in Part 1.

The truth-value assignment of the $\mathscr{L}$-language $\boldsymbol{L}_{\mathsf{N}}$ is a function $[\![\ ]\!]^{g,\, w,\, t,\, t'}$ with two argument-places for temporal indices. I will refer to the first temporal-index argument of a truth-value assignment as *the time of evaluation* and to the (first) world-index argument of a truth-value assignment as *the world of evaluation*. The second temporal-index of the truth-value assignment of $\boldsymbol{L}_{\mathsf{N}}$ can be thought of as the time of utterance of a sentence. I will call it *the time of utterance*.

Here are the semantic clauses of the monadic operators of $\boldsymbol{L}_{\mathsf{N}}$:

Let $\langle D, W, T, <, F\rangle$ be an $\boldsymbol{L}$-interpretation. For any wff ϕ of $\boldsymbol{L}_{\mathsf{N}}$, variable assignment g, $w \in W$, $t \in T$, and $t' \in T$,

(S_3') $[\![\mathsf{P}\phi]\!]^{g,\, w,\, t,\, t'} = 1$ iff for some $t'' \in T$ such that $t'' < t$, $[\![\phi]\!]^{g,\, w,\, t'',\, t'} = 1$

[9] The double-index semantic system outlined in that paper was elaborated on in Kamp 1971. A recent analysis of the influence of Kamp 1967 on Prior 1968a can be found in Blackburn & Jørgensen 2016. For some early references to Kamp's 1967 paper, see Prior 1968a, pp. 104, 110-112, Montague 1968, p. 116, Lewis 1970, pp. 185-186, and Kaplan 1977, p. 510.

[10] See the appendix to this chapter for a definition of the truth-value assignment of a doubly indexed $\mathscr{L}$-language.

(S4’) $[\![\mathsf{F}\phi]\!]^{g, w, t, t'} = 1$ iff for some $t'' \in T$ such that $t < t''$, $[\![\phi]\!]^{g, w, t'', t'} = 1$

(N) $[\![\mathsf{N}\phi]\!]^{g, w, t, t'} = 1$ iff $[\![\phi]\!]^{g, w, t', t'} = 1$

The three operators of $\boldsymbol{L}_{\mathbf{N}}$ shift the time of evaluation of their embedded wffs, but only N does it by reference to the time of utterance. N’s semantic role is to “check” whether its embedded wff gets the truth-value 1 when it is evaluated at the time of utterance. Since $\boldsymbol{L}_{\mathbf{N}}$ does not have operators that can shift the time of utterance, when the truth-value of a wff of $\boldsymbol{L}_{\mathbf{N}}$ is computed, the time of utterance remains available in the “semantic memory” of $\boldsymbol{L}_{\mathbf{N}}$ and can be recovered by N even if N is occurs in a subformula that is embedded under other operators.

According to a standard convention for the interpretation of formal languages with ‘now’/‘then’-operators, the computation of the truth-condition of a given wff must always start at an index coordinate in which the same temporal index occupies all the argument-places for temporal indices. Only at a later stage of the computation this index can be shifted. We can define a truth-predicate for the language $\boldsymbol{L}_{\mathbf{N}}$ that is in line with this convention.[11]

Def. 5 If ϕ is a closed wff of $\boldsymbol{L}_{\mathbf{N}}$, $\Im = \langle D, W, T, <, F\rangle$ is an $\boldsymbol{L}$-interpretation, $w \in W$, and $t \in T$,

ϕ IS TRUE IN $\boldsymbol{L}_{\mathbf{N}}$ AT $\langle \Im, w, t\rangle$ iff for every variable assignment g,

$$[\![\phi]\!]^{g, w, t, t} = 1$$

You can think of the world index and temporal index of a triple $\langle \Im, w, t\rangle$ as the world of utterance and moment of utterance of ϕ. According to *Def. 5*, in order to determine whether the truth-predicate of $\boldsymbol{L}_{\mathbf{N}}$ applies to a closed wff, one has to look first at an index coordinate where t (the moment of utterance) occupies the two argument-places for temporal indices. Other index coordinates may become relevant only when a subformula of the given wff is interpreted.

In his doctoral dissertation, Vlach (1973) developed a logical system in which sentential formulas were semantically assessed with respect to two moments, but each of the two moments was shiftable. His system employed standard Priorean

[11] See e.g. Kamp 1967, p. 4, 1971, pp. 238-239, and Vlach 1967, pp. 5, 39-41.

operators, an operator K which he called *the index operator*, and an operator R called the 'then'-*operator*. Let me call $\boldsymbol{L}_{\mathbf{KR}}$ the $\mathscr{L}$-language obtained by adding Vlach's operators K and R to the language $\boldsymbol{L}$. Here are the semantic clauses of these two operators:

For any wff ϕ of $\boldsymbol{L}_{\mathbf{KR}}$, variable assignment g, $w \in W$, $t \in T$, and $t' \in T$,

(K) $[\![\mathrm{K}\phi]\!]^{g, w, t, t'} = 1$ iff $[\![\phi]\!]^{g, w, t, t} = 1$

(R) $[\![\mathrm{R}\phi]\!]^{g, w, t, t'} = 1$ iff $[\![\phi]\!]^{g, w, t', t'} = 1$

The truth-value assignment of $\boldsymbol{L}_{\mathbf{KR}}$ is a function $[\![\ \ ]\!]^{g, w, t, t'}$ with argument-places for two temporal indices. The first temporal index is shifted by the operators P, F, and R of $\boldsymbol{L}_{\mathbf{KR}}$. The second temporal index is shifted by the operator K. We can view K as an operator that copies the current time of evaluation and stores it in the second-temporal-index slot. At a later stage of a truth-value computation, the operator R may retrieve that time by turning it into the time of evaluation of its embedded formula.[12] R may thus be viewed as a retrieving operator.

Note that there is no difference between the semantic clause of the operator R and the semantic clause of the 'now'-operator N. As Vlach correctly observed (1973, pp. 39-40), however, in a language with the operators K and R there are formulas in which an occurrence of R cannot be read as expressing the meaning of the English word *now*. When R occurs under the scope of an occurrence of K and that occurrence of K is under the scope of another temporal operator, R may shift the time of evaluation of its embedded formula to a time different from the original time of evaluation of the larger formula in which K and R appear. In this sort of sentential environment, R cannot be seen as bringing back the moment of utterance of the larger formula. It simply retrieves a time introduced by another temporal operator.

Of course, I could have avoided having two operators with the same semantic clause by using only the operator N and claiming that this operator can loose its indexical character in some $\mathscr{L}$-languages (e.g. in $\mathscr{L}$-languages that employ N and K). But I think it is better to stick to Vlach's notation and call the operators of

[12] For an illustration of this storing-retrieving process, see the derivation 2.) of the appendix.

his system K and R. This will facilitate my discussion of Vlach's views in the following sections of this chapter.

It is possible to add to $\boldsymbol{L}_{\mathsf{KR}}$ two extra operators K' and R' that act upon a third temporal index. The truth-value assignment of the language $\boldsymbol{L}_{\mathsf{KRK'R'}}$ is a function $[\![\ ]\!]^{g,\, w,\, t,\, t',\, t''}$ with three argument-places for temporal indices. While K' stores a given time of evaluation in the slot of the third temporal index, R' retrieves the time stored in that slot and makes that time the current time of evaluation. The temporal operators of $\boldsymbol{L}_{\mathsf{KRK'R'}}$ have the following semantic clauses:

For any wff ϕ of $\boldsymbol{L}_{\mathsf{KRK'R'}}$, variable assignment g, $w \in W$, $t \in T$, $t' \in T$, and $t'' \in T$,

(S3'') $[\![\mathsf{P}\phi]\!]^{g,\, w,\, t,\, t',\, t''} = 1$ iff for some $t''' \in T$ such that $t''' < t$, $[\![\phi]\!]^{g,\, w,\, t''',\, t',\, t''} = 1$

(S4'') $[\![\mathsf{F}\phi]\!]^{g,\, w,\, t,\, t',\, t''} = 1$ iff for some $t''' \in T$ such that $t < t'''$, $[\![\phi]\!]^{g,\, w,\, t''',\, t',\, t''} = 1$

(K) $[\![\mathsf{K}\phi]\!]^{g,\, w,\, t,\, t',\, t''} = 1$ iff $[\![\phi]\!]^{g,\, w,\, t,\, t,\, t''} = 1$

(R) $[\![\mathsf{R}\phi]\!]^{g,\, w,\, t,\, t',\, t''} = 1$ iff $[\![\phi]\!]^{g,\, w,\, t',\, t',\, t''} = 1$

(K') $[\![\mathsf{K'}\phi]\!]^{g,\, w,\, t,\, t',\, t''} = 1$ iff $[\![\phi]\!]^{g,\, w,\, t,\, t',\, t} = 1$

(R') $[\![\mathsf{R'}\phi]\!]^{g,\, w,\, t,\, t',\, t''} = 1$ iff $[\![\phi]\!]^{g,\, w,\, t'',\, t',\, t''} = 1$

The truth-predicates of $\boldsymbol{L}_{\mathsf{KR}}$ and $\boldsymbol{L}_{\mathsf{KRK'R'}}$ can be defined using a definition along the lines of *Def. 5*.

Def. 6 If ϕ is a closed wff of $\boldsymbol{L}_{\mathsf{KR}}$, ψ is a closed wff of $\boldsymbol{L}_{\mathsf{KRK'R'}}$, $\Im = \langle D, W, T, <, F\rangle$ is an $\boldsymbol{L}$-interpretation, $w \in W$, and $t \in T$,

ϕ IS TRUE IN $\boldsymbol{L}_{\mathsf{KR}}$ AT $\langle\Im, w, t\rangle$ iff for every variable assignment g, $[\![\phi]\!]^{g,\, w,\, t,\, t} = 1$

ψ IS TRUE IN $\boldsymbol{L}_{\mathbf{KRK'R'}}$ AT $\langle \Im, w, t \rangle$ iff for every variable assignment g,

$$[\![\psi]\!]^{g, w, t, t, t} = 1$$

This definition is also in line with the standard convention about truth-condition computations that I mentioned above.

We could add to $\boldsymbol{L}_{\mathbf{KRK'R'}}$ two extra K-like and R-like operators that act upon a fourth temporal index. In the appendix to his dissertation, Vlach showed that it is possible to obtain a hierarchy of K/R systems by systematically increasing the number of temporal indices and introducing suitable 'then'-operators and index operators. Each system of Vlach's hierarchy has more expressive power than the systems located at lower levels. The truth-condition of every formula of each system is computed by looking at index coordinates in which the same moment occupies the different temporal slots that are available in the system at hand. But that moment can be shifted in different ways when a subformula of that formula is interpreted. The strongest system which Vlach outlined was a system that uses infinite K/R operators and whose formulas are assessed with respect to infinite sequences of times.[13] [14]

1.3 The structure of expressibility arguments

At the beginning of this chapter, I mentioned that the study of doubly indexed logical systems was partly motivated by considerations about the possibility of

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[13] Vlach also described an alternative system with just one R operator and points of semantic assessment consisting of finite sequences of moments. In that system, the moments introduced by standard temporal operators are retrieved by iterating R. Vlach claimed that this system has the same strength as the strongest system of his hierarchy –i.e. the one with infinite K/R operators–, but he did not prove this claim.

[14] 'now'/'then'-operators raise interesting logical questions. In general, when you add an intensional operator to a language, you may increase its expressive power, but you may also loose some metalogical properties as a result. Kamp (1971) showed that the 'now'-operator N preserves completeness in languages of propositional and first-order temporal logic. Vlach (1973) proposed an axiomatization for his K/R language and proved that it is complete. For further references to the developments of that period concerning the logic of multiple-index operators, see Blackburn, de Rijke, and Venema 2001, pp. 483-484. For more recent investigations of logical systems with multiple-index operators, see Marx and Venema 1997 and Yanovich 2011, 2015. For a more general discussion of various systems of temporal logic, see Hodkinson and Reynolds 2007.

expressing the truth-conditions of certain kinds of English sentences with the aid of temporal operators. In this section I examine the structure of expressibility arguments by looking at Kamp and Vlach's discussions of sentences (1) and (2).

(1) A child was born who will become ruler of the world

(2) Jones was once going to cite everyone then driving too fast

Kamp (1971, pp. 231-232) argued that a proper symbolization of (1) could be given in a predicate language endowed with the 'now'-operator N, but not in a standard language of temporal logic equipped only with the Priorean operators P and F. He observed that (1) was correctly symbolized by formula (3), but not by the formula obtained if one removes the operator N from (3).

(3) $\mathsf{P}\ \exists x\ (\textbf{child}\ x \wedge (\textbf{born}\ x \wedge \mathsf{N}\ \mathsf{F}\ \textbf{become ruler of the world}\ x))$

Sentence (1) places the world-ruling event at a time that is posterior to the time of utterance of (1). If N is removed from (3), the world-ruling is represented as succeeding the child's birth and no information about whether the world-ruling obtains after the time of utterance is provided. The operator N of (3) retrieves the original time of utterance of (3). Since this is the time which F shifts, (3) places the world-ruling at a time posterior to the time of utterance, which is the desired result.[15]

Vlach (1973, pp. 2-5) argued that the 'now'-operator system of Kamp was not suitable for symbolizing (2). Using the operators K and R of his own system, Vlach symbolized (2) as (4).

(4) $\mathsf{P}\ \mathsf{K}\ \mathsf{F}\ \forall x\ (\mathsf{R}\ \textbf{drive too fast}\ x \rightarrow \textbf{cite Jones}\ x)$

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[15] See the steps 4-6 of derivation 1.) in the appendix. Kamp's symbolization of (1) was slightly different from (3). I have chosen (3) because it is closer to (1) and has all the semantic features that are relevant for the discussion. I take it that the most natural reading of (1) is one in which the future world-ruler is characterized as being a child at the time of her birth. Since in (3) **child** occurs under the scope of P, this is the reading that (3) encodes. But it is not generally the case that nouns are temporally interpreted in accordance with the tenses of their accompanying verbs (see Enç 1986).

Under the reading of (2) in which Vlach was interested, (2) is true just in case there is a past time t with respect to which the following is true: Jones cites at a time posterior to t those who drive too fast at t.[16] The occurrence of the adverb *then* in (2) indicates that the individuals cited by Jones were fast drivers at a past time which is prior to Jones' citing action. (4) captures this aspect of the truth-condition of (2) by means of the operators K and R. The past time which the operator P of (4) introduces is stored by K and later retrieved by R. As a result, the subformula **drive too fast *x*** is evaluated at the past time associated with the operator P, while the subformula **cite Jones *x*** is evaluated at the time posterior to it introduced by the operator F.[17]

Formulas (3) and (4) are, respectively, wffs of the languages $\boldsymbol{L}_{\mathbf{N}}$ and $\boldsymbol{L}_{\mathbf{KR}}$ of section 1.2. The formal apparatus of sections 1.1 and 1.2 can be applied to compute the truth-conditions of the formulas of $\boldsymbol{L}$ and its extensions.

(I assume that truth-conditions are computed using the intended interpretation of $\boldsymbol{L}$ (see *Def. 4*). The possible worlds, moments, and individuals mentioned in our truth-conditions are taken from the intended interpretation. I will refer to the moments of the intended interpretation as *times*.)

Our semantics for $\boldsymbol{L}_{\mathbf{N}}$ and $\boldsymbol{L}_{\mathbf{KR}}$ predicts that (5)-(6) are the truth-conditions of (3)-(4) with respect to a world of utterance w and a time of utterance t (for step-by-step derivations of (5)-(6), see the appendix to this chapter).[18]

(5) There are times t', t'' such that $t' < t < t''$ and there is an individual d such that d is a child in w at t', d is born in w at t', and d becomes ruler of the world in w at t''.

(6) There are times t', t'' such that $t' < t$, $t' < t''$, and, for every individual d, if d drives too fast in w at t', d is cited by Jones in w at t''.

The formal systems of Kamp and Vlach yield truth-conditions along the lines of

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[16] Arguably, gradable adjectives like *fast* require a standard of comparison to be interpreted in context. It seems plausible to think that an utterance of (3) can be assessed as saying something true or false only if context provides a standard of velocity relative to which someone counts as driving too fast. Vlach ignored this context-sensitive aspect of (3). For a recent analysis of the semantics of gradable adjectives, see e.g. Kennedy 2007.

[17] See the steps 2-7 of derivation 2.) in the appendix.

[18] The *if/then*-clauses of our truth-conditions must be understood as material conditionals.

(5)-(6) for (3)-(4).[19] For the purposes of this chapter, it will be useful to reconstruct the arguments of Kamp and Vlach about sentences (1)-(2) as arguments that make claims about the expressive power of $\boldsymbol{L}$, $\boldsymbol{L}_{\mathbf{N}}$, and $\boldsymbol{L}_{\mathbf{KR}}$. These languages can only manipulate times and worlds by means of their temporal operators. For this reason, they are apt for investigating what kinds of truth-conditions can be expressed solely by resort to intensional time-shifts.

We can reconstruct Kamp's analysis of (1) as an argument that relies on three claims. The first claim is that (5) specifies the truth-condition of (1) –assuming that (1) is uttered in world *w* at time *t*. The second claim is that (5) cannot be expressed in $\boldsymbol{L}$. That is to say, no wff of $\boldsymbol{L}$ has (5) as its truth-condition. The third claim is that (5) can be expressed in $\boldsymbol{L}_{\mathbf{N}}$. This amounts to saying that there is some wff of $\boldsymbol{L}$ whose truth-condition is (5). From the three claims it follows that (1)'s truth-condition is expressible in $\boldsymbol{L}_{\mathbf{N}}$ but inexpressible in $\boldsymbol{L}$. Vlach's analysis of (2) can be reconstructed as an argument based on three analogous claims. Just substitute (2) for (1), (6) for (5), $\boldsymbol{L}_{\mathbf{N}}$ for $\boldsymbol{L}$, and $\boldsymbol{L}_{\mathbf{KR}}$ for $\boldsymbol{L}_{\mathbf{N}}$ in my reconstruction of Kamp's argument. The conclusion of the resulting argument is that (2)'s truth-condition can be expressed in $\boldsymbol{L}_{\mathbf{KR}}$ but not in $\boldsymbol{L}_{\mathbf{N}}$.

Let me characterize the two arguments presented in the previous paragraph in slightly more abstract terms. They attempt to establish that a natural language sentence (S) can be symbolized in a formal language $\boldsymbol{l}$ but cannot be symbolized in a different formal language $\boldsymbol{l}'$. To show this, the arguments make use of a regimented truth-condition which is supposed to provide a reading of (S). The truth-condition in question is specified with respect to a world of utterance *w* and a time of utterance *t*. Let us call that truth-condition (R). (R) may be the only reading of (S), its most salient reading, or just one of its readings. Here is a schematic representation of the structure of the two arguments:

Premise 1

Under one reading of (S), (S) is true when uttered in *w* at *t* iff (R)

Premise 2

(R) is inexpressible in $\boldsymbol{l}$

[19] See Kamp 1971, §§ 2, 4, especially pp. 257-259, and Vlach 1973, sections IA, IIA. Since Kamp and Vlach were not studying world-shifting operators, their wffs were not semantically evaluated relative to possible worlds.

Premise 3

(R) is expressible in ***l'***

Conclusion

(S) has a reading that is expressible in ***l*** but inexpressible in ***l'***

I shall call arguments of this form *expressibility arguments*. In order to construct an expressibility argument concerning two formal languages ***l*** and ***l'***, one needs a natural language sentence and a truth-condition. The argument is successful only if its three premises are true. The first premise of the argument fails if the given truth-condition does not really specify a reading of the sentence. To justify the third premise of an expressibility argument, one has to show that there is a formula of the language ***l'*** which has the truth-condition at stake. Typically, a standard derivation suffices to prove that this is so for a particular formula. By contrast, the second premise of the argument can be very difficult to prove. For the second premise requires that there is no formula of ***l*** that has the relevant truth-condition.

The argument of Kamp regarding (1), for example, turned out to be incorrect with respect to *Premise 2*. Kamp (1973, §5) proved that N is not redundant in languages of first-order temporal logic. But (3) is not the kind of formula that illustrates this result. Given that the interpretations of ***L*** have constant domains of individuals –as opposed to domains that vary with times and worlds– we can correctly symbolize (1) by giving the existential quantifier wide scope over P. (7) is a symbolization of this sort.

(7) $\exists x\, (\mathsf{P}\, (\mathbf{child}\ x \wedge \mathbf{born}\ x) \wedge \mathsf{F}\ \mathbf{ruler\text{-}of\text{-}the\text{-}world}\ x)$

The truth-condition of formula (7) is (5). But (7) is a wff of ***L***. Therefore, ***L*** can express truth-condition (5).

Another kind of expressibility argument that has been influential involves a sentence like (8), which is assumed to have the reading given in (9).

(8) One day, everyone now miserable will be happy

(9) There is a time *t'* such that $t < t'$ and, for every individual *d*, if *d* is miserable in *w* at *t*, *d* is happy in *w* at *t'*.

It seems plausible to think that (9) cannot be expressed in ***L***. Consider formulas (10) and (11).

(10) $\forall x$ (**miserable** $x \rightarrow \mathsf{F}$ **happy** x)

(11) $\mathsf{F}\ \forall x$ (**miserable** $x \rightarrow$ **happy** x)

(10) does not express (9) because it does not require that all the persons that are miserable at the time of utterance are happy at the same future time, which is something that (9) requires. (11) talks about people who are miserable at some future time and not about people who are miserable at the time of utterance.

This is just an informal argument which suggests that (9) cannot be expressed in ***L***. In a recent paper, Yanovich (2015, section 5) proves that formulas like (12) are not expressible in standard modal languages without 'now'/'then'-operators.

(12) $\mathsf{F}\ \forall x$ (N **miserable** $x \rightarrow$ **happy** x)

I will not review Yanovich's proof here. But it is worth mentioning that his proof shows that (9) –which is the truth-condition of (12)– cannot be expressed in an $\mathscr{L}$-languages that has only standard Priorean operators. So, if we accept that (9) is the intuitive truth-condition of (8), we can conclude that there is a successful expressibility argument to the effect that (8)'s truth-condition can be expressed in $\boldsymbol{L}_{\mathbf{N}}$ but not in ***L***.

We have not yet considered expressibility arguments involving formulas with dyadic operators. Consider (13)-(17).

(13) Mary once believed that Jones would be rich now

(14) There is a time *t*' such that *t*' $<$ *t* and, for every world *w*' compatible with what Mary believes in *w* at *t*', Jones is rich in *w*' at *t*.

(15) P $_{\mathbf{Mary}}$**Believe** N **rich Jones**

(16) P $_{\mathbf{Mary}}$**Believe rich Jones**

(17) There is a time t' such that $t' < t$ and, for every world w' compatible with what Mary believes in w at t', Jones is rich in w' at t'.

(14) is the truth-condition of the $\boldsymbol{L}_{\mathbf{N}}$ formula (15). Arguably, this truth-condition is not expressible in $\boldsymbol{L}$. For the only reasonable candidate for expressing (14) in $\boldsymbol{L}$ is (16). But the truth-condition of (16) is (17), which is not equivalent to (14). Of course, this is also an informal argument. But it suggests that sentences with attitude verbs may produce sound expressibility arguments involving the formal languages $\boldsymbol{L}$ and $\boldsymbol{L}_{\mathbf{N}}$.

What about Vlach's claim that (2) is expressible in a language with K and R but not in a language that only has standard Priorean operators and the operator N? This claim seems correct. In a proper symbolization of (6), the operators P and F must appear before the quantifier $\forall$ and F must appear under the scope of P. If the formal language does not have a storing operator like K, there is no way to store the past time that P introduces before this time is shifted by the operator F. As a result, the subformula **drive too fast *x*** cannot be evaluated at the past time associated with the operator P. If this is so, (6) cannot be expressed in a language like $\boldsymbol{L}_{\mathbf{N}}$. Thus, if we accept that (6) is a reading of (2), this informal argument suggests that (2)'s truth-condition is expressible in $\boldsymbol{L}_{\mathbf{KR}}$ but not in $\boldsymbol{L}_{\mathbf{N}}$.

I hope my discussion in this section has served to clarify how expressibility arguments work and what kind of considerations are relevant in order to assess them. In the next section I discuss van Benthem's (1977) attempt to show that doubly indexed operator-based languages are insufficiently expressive.

1.4 van Benthem against Vlach

van Benthem's 1977 review of the early development of temporal logic is often approvingly quoted by the critics of operator-based systems.[20] In that review, van Benthem discussed the expressibility arguments of Kamp and Vlach that we examined in section 1.3. He observed that the debates among temporal logicians about 'now'/'then'-operators had led to the postulation of increasingly complex temporal logical systems. One central thesis of van Benthem's review was that

[20] See e.g. Cresswell 1990, chapter 2, King 2003, section 4, and Schaffer 2012, section 2.2. See also Ogihara 1996, p. 28 and Schlenker 2006, p. 514, fn. 11.

this tendency makes temporal logic systems converge towards a logical system with explicit quantification over times. van Benthem argued for this thesis in a section of the review devoted to Vlach's K/R system. The central argument of van Benthem against Vlach was that the case for double indexing generalizes. Just as there are natural language sentences that make the adoption of double indexing necessary for a temporal logician, there are also sentences that force her to adopt triple temporal indexing. van Benthem suggested that this is just the beginning of a slippery slope that ultimately leads to the postulation of infinite temporal indices. In this section I argue that van Benthem's argument against Vlach is not compelling.

van Benthem offers one example to show that there are sentences of English which force a temporal logician like Vlach to adopt triple indexing. According to van Benthem (1977, pp. 417-418), sentence (18) is not expressible in Vlach's K/R system.

(18) There will always jokes be told that were told at one time in the past

Needless to say, (18) is an odd sentence. Its most salient reading seems to be (19). (As in section 1.3, *w* and *t* are the world of utterance and time of utterance of the formulas that we are considering.)

(19) For every time *t*' such that $t < t'$, there is a time *t*'' such that $t'' < t$, and there is a joke *d* such that *d* is told in *w* at *t*', and *d* is told in *w* at *t*''.

Informally, (19) says that for any future time *t*', there was a past time *t*'' such that a joke that will be told at *t*' was told at *t*''. But (19) is not the reading of (18) in which van Benthem is interested. (19) is expressible in the systems of Kamp and Vlach. It is expressed, for example, by formula (20). For simplicity, I assume that the bound variable of (20) ranges over jokes.

(20) G ∃*x* (**be told** *x* ∧ N P **be told** *x*)

van Benthem gives a formal specification of the (alleged) reading of (18) which he has in mind. It is the reading where 'at one time in the past' takes wide scope over 'there will always jokes be told'. In formal terms:

(21) There is a time *t*’ such that *t*’ $<$ *t* and such that for every time *t*” such that *t* $<$ *t*”, there is a joke *d* such that *d* is told in *w* at *t*’ and *d* is told in *w* at *t*”.

Roughly, (21) says that there was a past time *t*’ such that for any future time *t*”, some joke that was told at *t*’ will be told at *t*”. (21) entails (19), but (21) can be false in a scenario in which (19) is true. It is controversial whether (18) has the reading specified in (21). Most English speakers have to make an effort to read (18) as expressing (21). Some speakers report that they do not get such a reading.

van Benthem does not offer a proof to the effect that (21) cannot be expressed in Vlach’s system. He considers and rejects a number of possible symbolizations of (21) in terms of the operators P, G, K, and R.

It is not difficult to find ways of expressing (21) if we switch to a triple-index framework. In section 1.2, I described the formal language $\boldsymbol{L}_{\mathbf{KRK'R'}}$, which is a triply indexed extension of $\boldsymbol{L}$ containing two types of K/R operators that act on the second and third temporal indices of the function $[\![\ ]\!]^{g,\, w,\, t,\, t',\, t''}$. (22) is a wff of $\boldsymbol{L}_{\mathbf{KRK'R'}}$ that expresses (21).[21]

(22) P K’ R G ∃*x* (**be told** *x* ∧ R’ **be told** *x*)

In order to obtain the truth-condition specified in (21), one has to make sure that one of the occurrences of the subformula **be told *x*** is evaluated at the past time associated with P. The other occurrence of **be told *x*** must be evaluated at the time associated with the operator G that is under the scope of P. Let me give an informal description of how (23) does this. A derivation of (22)’s truth-condition starts by looking at an index coordinate in which the time of utterance of (22) occupies the three temporal slots which are available in the semantic memory of $\boldsymbol{L}_{\mathbf{KRK'R'}}$. The operator P of (22) shifts the time of evaluation of its embedded formula to a time prior to the time of utterance. K’ stores this past time in the slot of the third temporal index. Since the time of utterance of (22) is still stored in the second-temporal-index slot, the joint effect of the operators R and G is to introduce a time posterior to the original time of utterance, which becomes the

[21] Although H and G are not in the official list of monadic operators of ***L***, recall that we are treating H and G as abbreviations of ¬P¬ and ¬F¬.

time of evaluation of the first occurrence of the subformula **be told *x***. Finally, the operator R' recovers the past time stored in the third-temporal-index slot and evaluates the second occurrence of **be told *x*** at that time.

In brief, van Benthem's argument involving (18) is an expressibility argument in the sense of section 1.3. The argument seeks to show that (18) has a reading that is expressible in a triply indexed language like $\boldsymbol{L}_{\mathbf{KRK'R'}}$ but not in a doubly indexed language like $\boldsymbol{L}_{\mathbf{KR}}$.

As I mentioned in section 1.2, Vlach had considered the possibility of constructing multiple-index logical systems with three or more temporal indices. However, van Benthem dismisses the strategy of dealing with (18) and similar counterexamples by positing more temporal indices and more operators.

> In an appendix, Vlach mentions a safety valve which blocks this counterexample and similar ones. It consists in adding operators N_1, N_2 … and corresponding K_1, K_2 … in any quantity. This will take care of all cases of cross-reference, but [...] such a move degenerates into using a typographical variant of predicate logic (with subscripts instead of variables), merely without *calling* it predicate logic.[22]
>
> (van Benthem 1977, p. 418)

Here van Benthem is talking about Vlach's hierarchy of K/R systems. Contrary to what van Benthem suggests, the strongest system of Vlach's hierarchy is not a notational variant of a predicate logic system with explicit quantification over times. In two recent papers, Yanovich (2011, 2015) has shown that modal logic first-order languages with infinite temporal indices and equipped with infinite 'now'/'then'-operators have less expressive power than extensional first-order languages with full quantification over times. Yanovich shows that the widespread belief to the contrary among philosophers and linguists is the result of a misinterpretation of some formal results presented in Cresswell 1990. The basic first-oder modal language with 'now'/'then'-operators that Cresswell characterizes in that book has also an operator of universal modality which makes the language much more powerful than a modal first-oder language equipped 'now'/'then'-operators but which lacks the universal modality operator. (For discussion of this point, see esp. Yanovich 2015, sections 1 and 7)

Let us return to van Benthem's expressibility argument regarding (18).

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[22] Cresswell (1990, p. 22) concurs with this remark.

As I explained before, van Benthem's assumption that truth-condition (21) is a reading of (18) is controversial and van Benthem assumes without proof that (21) cannot be expressed in a language with operators K and R such as $\boldsymbol{L}_{\mathsf{KR}}$. Even if we grant him these two assumptions, there are double-index extensions of $\boldsymbol{L}$ that can express (21). One such extension of $\boldsymbol{L}$ is the language $\boldsymbol{L}_{\mathsf{P^+F^+R}}$, which employs the operators P^+ and F^+ that I will characterize in section 2.1. In this language it is possible to define a monster operator G^+ that shifts the second temporal index. But instead of simply copying and pasting the first temporal index –as K does–, G^+ shifts the second temporal index to a time posterior to it. With the aid of G^+, (21) can be expressed as (23).

(23) $\mathsf{P}\ \mathsf{G}^+\ \exists x$ **(be told** $x \wedge$ R **be told** x**)**

By writing down a proper truth-condition derivation, one can show that (21) is the truth-condition of (23) –a step-by-step derivation of (21) from (23) is given in the appendix to this chapter. The fact that (21) can be expressed in $\boldsymbol{L}_{\mathsf{P^+F^+R}}$ is important. It shows that triple indexing is not necessary in order to express (21). Thus, van Benthem's example (18) does not provide a motivation for adopting triple temporal indexing.

Incidentally, notice that an English sentence like (24) seems to come closer to expressing (21) than (18) does.

(24) There was a time in the past such that it will always be the case that some joke that was told then will be told again

(21) seems to be one possible reading of (24). But it is important to stress that if (18) and (24) have a reading that corresponds to (21), this is so because of the role that the expressions 'at one time in the past' and 'there was a time in the past' play in these sentences. It does not seem possible to express (21) in English without explicit quantification over times. However, a theorists who uses 'now'/ 'then'-operators to analyze the behavior of tenses and temporal adverbs like *now* and *then* does not need to be committed to the view that explicit quantification over times in natural language must be semantically represented in terms of temporal operators. When the surface structure of a sentence involves explicit talk about times, such a theorist can –and maybe should– prefer a symbolization that employs singular terms that refer to times and quantifiers that quantify over

times. If this is so, an advocate of ‘now’/‘then’-operators may not need to worry about examples like (18) and (24). I will return to this point in the next section.

1.5 More counterexamples to double temporal indexing

Let us move now to Cresswell’s discussion of multiple temporal indexing in Cresswell 1990. Like van Benthem, Cresswell (1990, chapter 2) argues that the kind of considerations that motivate double temporal indexing ultimately lead to infinite temporal indexing. The key example that he uses to argue for this claim is sentence (25).

(25) There will be times such that all persons now alive will be happy at the first or miserable at the second.

The reading of (25) in which Cresswell is interested is (26).

(26) There are times t', t'' such that $t < t'$, $t < t''$ and such that for any person d, if d is alive in w at t, then d is happy in w at t' or d is miserable in w at t''.

To express (26), the semantics of an operator-based language must keep track of three times: the time of utterance t and two (possibly different) future times. In a proper regimentation of (25), the operators that introduce the two future times should take wide scope over the universal quantifier and the disjunction should appear under the scope of the universal quantifier. Formulas (27)-(28) express truth-conditions different from (26) because they do not meet these conditions. I assume that $\boldsymbol{x}$ ranges over persons in (27)-(28).

(27) $\forall x\ (\textbf{alive}\ x \rightarrow (\mathsf{F}\ \textbf{happy}\ x \vee \mathsf{F}\ \textbf{miserable}\ x))$

(28) $(\mathsf{F}\ \forall x\ (\mathsf{N}\ \textbf{alive}\ x \rightarrow\ \textbf{happy}\ x) \vee \mathsf{F}\ \forall x\ (\mathsf{N}\ \textbf{alive}\ x \rightarrow\ \textbf{miserable}\ x))$

One may try to put the two F operators before the universal quantifier and use the operator K to store the future time introduced by one of these operators. This strategy is illustrated in (29).

(29) F K F ∀*x* **(alive** *x* → **(happy** *x* ∨ R **miserable** *x*))

However, (29) evaluates the subformula **alive *x*** at the future time introduced by the second F operator. If we want to predict truth-condition (26), **alive *x*** should be evaluated at the original time of utterance.

Using the triple-index language $\boldsymbol{L}_{\mathbf{KRK'R'}}$ of section 1.2, we can give a correct symbolization of (25).

(30) F K F ∀*x* (R' **alive** *x* → **(happy** *x* ∨ R **miserable** *x*))

In (30), R' retrieves the original time of utterance, which is stored in the slot of the third temporal index, and turns it into the time of evaluation of **alive *x***. But, as Cresswell notes, the problem posed by (25) generalizes. For the disjunction in (25) can be indefinitely extended and each extended version of (25) introduces a new time in the truth-condition of the resulting sentence. Consider (31).

(31) There will be times such that all persons now alive will be happy at the first, or miserable at the second, or dead at the third.

Sentence (31) poses a problem for $\boldsymbol{L}_{\mathbf{KRK'R'}}$ analogous to the problem that (25) poses for the $\boldsymbol{L}_{\mathbf{KR}}$. Since we can generate more complex variants of (25) without limit, Cresswell concludes that only an operator-based system with infinite times in the semantics can symbolize all the variants of (25).

(25) and its variants may be problematic for a theorist who is interested in the project of capturing the expressive power of a natural language with the aid of 'now'/'then'-operators. The expressive power of natural language is the topic of Cresswell's book. Cresswell is right in pointing out that if we want to express the truth-conditions of (25) and its variants without using explicit quantification over times and by employing only standard Priorean operators and 'now'/'then'-operators, then we have to posit infinite temporal indices in the semantics. But the first logicians who studied 'now'/'then'-operators were not pursuing this project. Prior was interested in the project of expressing overt talk about times in

temporal logic systems. But he invented hybrid logic for that purpose.[23] The ‘now’/‘then’-operator languages of Kamp and Vlach were not designed to formalize overt talk about times in English. Kamp and Vlach introduced their ‘now’/‘then’-operators to study some fragments of English involving tenses and temporal adverbs like *now* and *then*. But they did not hold that every kind of temporal expression that occurs in a sentence must be accounted for in terms of intensional operators. Their views about the formal representation of tenses and temporal adverbs did not entail that every piece of temporal discourse should be regimented using only temporal operators.

At the beginning of the appendix to Vlach 1973, Vlach said quite clearly that sentences like (25) and (31) were not a target of his K/R system.

> There are other ways of doing the sort of thing that the system of this paper is supposed to do and many systems that are stronger than the present one. The present system was chosen for its relative simplicity, and because it seems sufficient to handle most actual English examples that would naturally be expressed without the use of expressions that refer explicitly to times, like ‘the first moment’.
>
> (Vlach 1973, p. 418)

Here Vlach seems to be excluding from the range of application of his system sentences like (25) and (31). He did not think that such sentences were problematic for his double-index theory. Although he described a hierarchy of multiply indexed systems that were stronger than the original double-index K/R-system, he did not attempt to motivate any of those systems on the basis of observations about natural language. He thought that the doubly indexed K/R-system sufficed for symbolizing the fragment of English for which that system was designed.[24]

Kamp made a similar qualification in his 1971 paper on the ‘now’-operator. In a footnote, he rejected the possibility of symbolizing sentences like *A child was*

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[23] For some discussion of Prior’s views on this matter, see Blackburn 2006 and Blackburn & Jørgensen 2016.

[24] It is worth mentioning that Vlach was not solely interested in sentences involving the word *then*. In a section of his dissertation devoted to highlighting further advantages of the K/R-system over the N-system, he suggested that the operators K and R could be used to symbolize English linear narratives exhibiting intersentential anaphora (Vlach 1973, chapter I, section 3). The study of similar linear narratives would later motivate the development of dynamic accounts of temporal discourse (see Hinrichs 1981, 1986, Kamp 1981, Kamp & Rohrer 1983, and Partee 1984).

born who will become ruler of the world using explicit temporal quantifiers. He wrote: "... one can object to symbolizations involving quantification over such abstract objects as moments, if these objects are not explicitly mentioned in the sentences that are to be symbolized" (Kamp 1971, p. 231, fn. 1). The *if*-clause of this passage suggests that he would not have objected to a symbolization of (25) involving explicit quantification over times.

In short, a doubly indexed treatment of tenses and indexical time adverbs is perfectly compatible with the view that English sentences which exhibit overt quantification over times and overt reference to times at the surface level must have formal representations involving explicit temporal quantification and explicit temporal reference. Since (25) and its variants are sentences of this sort, these sentences do not show that infinite temporal indexing is mandatory for a double-index theorist who accepts this view.

Let us leave aside any potential counterexample to double temporal indexing which relies on expressions that quantify overtly over times at the level of surface syntax. Are there English sentences which are inexpressible in a doubly indexed language and do not involve overt talk about times?

Relevant examples are scarce in the literature on multiple-indexing. Saarinen (1978, part I) considers a number of interesting examples that are relevant to our question. (32) is an adaptation of one of his examples.

(32) Every man who ever supported the Vietnam War will have to admit that now he believes that he was an idiot then.

If we want to symbolize (32) with 'now'/'then'-operators, the predicate **idiot** must be evaluated at the past time at which the Vietnam War was supported and the dyadic operator **Believe** must appear under the scope of some operator that retrieves the original time of utterance. The semantics of the formal language has to store these two times and keep them in the semantic memory until the last stages of the computation. But, on the other hand, an F operator must appear in front of the dyadic operator **Admit**. If we are using a double-index framework, it follows that one of the two times stored in the semantic memory will have to be deleted by the operator F.

To illustrate the point with a concrete attempt of formalization, look at (33). (For the sake of simplicity, (33) does not represent the semantic contribution of *have to* in (32).)

(33) $\forall \boldsymbol{x}$ **(man** $x \rightarrow$ **(**H ... **support the Vietnam War** $x \rightarrow$ ... F ${}_x$**Admit** ... ${}_x$**Believe idiot** x**))**

Either the time of utterance or the past time associated with H will be deleted by the operator F. Putting double-index operators in any of the positions marked by the ellipses will not help us to solve this problem. Thus, (32) seems to be a sentence that is inexpressible in a double-index language and does not have the problematic features of the examples of van Benthem and Cresswell.

However, it is not obvious that one can generate variants of (32) that make infinite temporal indexing mandatory for someone who wants to represent the English tenses and the adverbs *now* and *then* in terms of intensional operators. The obvious way of generating a relevant variant of (32) is to put one extra attitude verb in (32). Consider (34).

(34) Every man who ever supported the Vietnam War will have to admit that now he believes that he will know that he was an idiot then.

Regardless of exactly which reading you get in reading this sentence, it is clear that (34) does not require keeping track of the two future times associated with *will have to admit* and *will know*. The semantics of a triple-index language such as $\boldsymbol{L}_{\mathbf{KRK'R'}}$ can delete one of these two times once the future operators that introduce those times have done their jobs. Thus, (34) is not a kind of example that makes quadruple indexing mandatory for an intensional account of tenses and time adverbs like *now* and *then*. As far as I can see, it is unclear whether such an example can be provided.[25]

My provisional conclusion is that the dialectical force of expressibility arguments depends partly on our background theoretical assumptions. Examples like (18) and (25) are not problematic for someone who is interested in modeling the English tenses as time-shifting operators. Example (32) shows that $\mathscr{L}$-languages with double temporal indexing have important expressive limitations. But it is not clear that there are variants of example (32) which can show that intensional

[25] Saarinen considers various English examples of greater complexity than (32). But I think none of his examples requires quadruple indexing in order to be represented in a multiply indexed framework. Saarinen (1978, Part II) proposes an alternative game-theoretical system with backwards-looking operators.

formal languages with three or more temporal indices have equally interesting expressive limitations.

Appendix to chapter 1

Let $\Im = \langle D, W, T, <, F\rangle$ be the intended interpretation of ***L***. The derivations that I give in this appendix yield the truth-conditions of some wffs of the $\mathscr{L}$-language $\boldsymbol{L}_{\mathsf{NKRG}^+}$ with respect to an arbitrary world *w* of *W* and an arbitrary time *t* of *T*, which must be thought of as the world of utterance and the time of utterance of the relevant English sentences. Before each derivation, I mention the steps of it that are of special interest in connection with the discussion of chapter 1. Right after each derivation step, I mention the rule or definition from which that step follows. Most derivation steps are based on (S_1')-(S_{10}') –which are the double-index analogues of the clauses (S_1)-(S_{10}) of section 1.1– and on the clauses (N), (K), (R) and (G^+) (see below). I also simplify the derived truth-conditions in ways that do not affect their contents.

Semantic clauses

The truth-value assignment of $\boldsymbol{L}_{\mathsf{NKRG}^+}$ with respect to $\Im$ is the unique five-place function $[\![\ \ ]\!]^{g,\, w,\, t,\, t'}$ with range $\{1, 0\}$ such that for any variable assignment *g* (with respect to $\Im$), $w \in W$, $t \in T$, $t' \in T$, *n*-place predicate Π of $\boldsymbol{L}_{\mathsf{NKRG}^+}$, dyadic operator Γ of $\boldsymbol{L}_{\mathsf{NKRG}^+}$, singular terms α, $\alpha_1 \ldots \alpha_n$ of $\boldsymbol{L}_{\mathsf{NKRG}^+}$, wffs ϕ, ψ of $\boldsymbol{L}_{\mathsf{NKRG}^+}$, and variable ν of $\boldsymbol{L}_{\mathsf{NKRG}^+}$,

(S_1') $[\![\Pi\alpha_1 \ldots \alpha_n]\!]^{g,\, w,\, t,\, t'} = 1$ iff $\langle F_g(\alpha_1), \ldots F_g(\alpha_n)\rangle \in F(\Pi)(\langle w, t\rangle)$

(S_2') $[\![{}_\alpha\Gamma\phi]\!]^{g,\, w,\, t,\, t'} = 1$ iff for every $w' \in W$ such that $w' \in F(\Gamma)(w, t, F_g(\alpha))$, $[\![\phi]\!]^{g,\, w',\, t,\, t'} = 1$

(S_3') $[\![\mathsf{P}\phi]\!]^{g,\, w,\, t,\, t'} = 1$ iff there is some $t'' \in T$ such that $t'' < t$ and $[\![\phi]\!]^{g,\, w,\, t'',\, t'} = 1$

(S_4') $[\![\mathsf{F}\phi]\!]^{g,\, w,\, t,\, t'} = 1$ iff there is some $t'' \in T$ such that $t < t''$ and $[\![\phi]\!]^{g,\, w,\, t'',\, t'} = 1$

(S5’) $[\![\neg\phi]\!]^{g, w, t, t'} = 1$ iff $[\![\phi]\!]^{g, w, t, t'} = 0$

(S6’) $[\![(\phi \wedge \psi)]\!]^{g, w, t, t'} = 1$ iff $[\![\phi]\!]^{g, w, t, t'} = 1$ and $[\![\psi]\!]^{g, w, t, t'} = 1$

(S7’) $[\![(\phi \vee \psi)]\!]^{g, w, t, t'} = 1$ iff $[\![\phi]\!]^{g, w, t, t'} = 1$ or $[\![\psi]\!]^{g, w, t, t'} = 1$

(S8’) $[\![(\phi \rightarrow \psi)]\!]^{g, w, t, t'} = 1$ iff $[\![\phi]\!]^{g, w, t, t'} = 0$ or $[\![\psi]\!]^{g, w, t, t'} = 1$

(S9’) $[\![\exists\nu\phi]\!]^{g, w, t, t'} = 1$ iff for some $d \in D$, $[\![\phi]\!]^{g[\nu/d], w, t, t'} = 1$

(S10’) $[\![(\forall\nu\phi)]\!]^{g, w, t, t'} = 1$ iff for every $d \in D$, $[\![\phi]\!]^{g[\nu/d], w, t, t'} = 1$

(N) $[\![\mathsf{N}\phi]\!]^{g, w, t, t'} = 1$ iff $[\![\phi]\!]^{g, w, t', t'} = 1$

(K) $[\![\mathsf{K}\phi]\!]^{g, w, t, t'} = 1$ iff $[\![\phi]\!]^{g, w, t, t} = 1$

(R) $[\![\mathsf{R}\phi]\!]^{g, w, t, t'} = 1$ iff $[\![\phi]\!]^{g, w, t', t'} = 1$

(G^+) $[\![\mathsf{G}^+]\!]^{g, w, t, t'} = 1$ iff for every $t'' \in T$ such that $t' < t''$, $[\![\phi]\!]^{g, w, t, t''} = 1$

Truth predicate

If ϕ is a closed wff of $\boldsymbol{L}_{\mathbf{NKRG}^+}$, $w \in W$, and $t \in T$,

ϕ IS TRUE IN $\boldsymbol{L}_{\mathbf{NKRG}^+}$ AT $\langle \Im, w, t\rangle$ iff for every variable assignment g,
$[\![\phi]\!]^{g, w, t, t} = 1$

1.) *Derivation of truth-condition (5) from formula (3)*

(3) $\mathsf{P}\ \exists x$ **(child** $x \wedge$ **(born** $x \wedge \mathsf{N}\ \mathsf{F}$ **become ruler of the world** x**))**

(5) There are times t', t'' such that $t' < t < t''$ and there is an individual d such that d is a child in w at t', d is born in w at t', and d becomes ruler of the world in w at t''

Key steps of the derivation: 5, 6

Derivation: (3) IS TRUE IN $\boldsymbol{L}_{\mathbf{NKRG}^+}$ AT $\langle \mathfrak{I}, w, t\rangle$ iff for every variable assignment g,

1. $[\![\mathsf{P}\ \exists x\ (\mathbf{child}\ x \wedge (\mathbf{born}\ x \wedge \mathsf{N}\ \mathsf{F}\ \mathbf{become\ ruler\ of\ the\ world}\ x))]\!]^{g, w, t, t} = 1$

   by the definition of TRUE IN $\boldsymbol{L}_{\mathbf{NKRG}^+}$

2. There is some $t' \in T$ such that $t' < t$ and $[\![\exists x\ (\mathbf{child}\ x \wedge (\mathbf{born}\ x \wedge \mathsf{N}\ \mathsf{F}\ \mathbf{become\ ruler\ of\ the\ world}\ x))]\!]^{g, w, t', t} = 1$

   by (S_3')

3. There is some $t' \in T$ such that $t' < t$ and there is some $d \in D$ such that $[\![(\mathbf{child}\ x \wedge (\mathbf{born}\ x \wedge \mathsf{N}\ \mathsf{F}\ \mathbf{become\ ruler\ of\ the\ world}\ x))]\!]^{g[x/d], w, t', t} = 1$

   by (S_9')

4. There is some $t' \in T$ such that $t' < t$ and there is some $d \in D$ such that $[\![\mathbf{child}\ x]\!]^{g[x/d], w, t', t} = 1$, $[\![\mathbf{born}\ x]\!]^{g[x/d], w, t', t} = 1$, and $[\![\mathsf{N}\ \mathsf{F}\ \mathbf{become\ ruler\ of\ the\ world}\ x]\!]^{g[x/d], w, t', t} = 1$

   by two applications of (S_6')

5. There is some $t' \in T$ such that $t' < t$ and there is some $d \in D$ such that $[\![\mathbf{child}\ x]\!]^{g[x/d], w, t', t} = 1$, $[\![\mathbf{born}\ x]\!]^{g[x/d], w, t', t} = 1$, and $[\![\mathsf{F}\ \mathbf{become\ ruler\ of\ the\ world}\ x]\!]^{g[x/d], w, t, t} = 1$

   by (N)

6. There is some $t' \in T$ such that $t' < t$, there is some $d \in D$ such that $[\![\textbf{child } x]\!]^{g[x/d], w, t', t} = 1$, $[\![\textbf{born } x]\!]^{g[x/d], w, t', t} = 1$, and there is some $t'' \in T$ such that $t < t''$ and $[\![\textbf{become ruler of the world } x]\!]^{g[x/d], w, t'', t} = 1$

by (S_4')

7. There is some $t' \in T$ such that $t' < t$, there is some $d \in D$ such that $F_{g[x/d]}(x) \in F(\textbf{child})(\langle w, t'\rangle)$, $F_{g[x/d]}(x) \in F(\textbf{born})(\langle w, t'\rangle)$, and there is some $t'' \in T$ such that $t < t''$ and $F_{g[x/d]}(x) \in F(\textbf{become ruler of the world})(\langle w, t''\rangle)$

by (S_1')

8. There is some $t' \in T$ such that $t' < t$, there is some $d \in D$ such that $g[x/d](x) \in F(\textbf{child})(\langle w, t'\rangle)$, $g[x/d](x) \in F(\textbf{born})(\langle w, t'\rangle)$, and there is some $t'' \in T$ such that $t < t''$ and $g[x/d](x) \in F(\textbf{become ruler of the world})(\langle w, t''\rangle)$

by the definition of F_g

9. There is some $t' \in T$ such that $t' < t$, there is some $d \in D$ such that $d \in F(\textbf{child})(\langle w, t'\rangle)$, $d \in F(\textbf{born})(\langle w, t'\rangle)$, and there is some $t'' \in T$ such that $t < t''$ and $d \in F(\textbf{become ruler of the world})(\langle w, t''\rangle)$

by the convention that $g[x/d](x) = d$

10. There is some $t' \in T$ such that $t' < t$, there is some $d \in D$ such that $d \in \{x \in D: x \text{ is a child in } w \text{ at } t'\}$, $d \in \{x \in D: x \text{ is born in } w \text{ at } t'\}$, and there is some $t'' \in T$ such that $t < t''$ and $d \in \{x \in D: x \text{ becomes ruler of the world in } w \text{ at } t''\}$

by function application and the definition of F for the predicates **child**, **born**, and **become ruler of the world**

11. There are times t', t'' such that $t' < t < t''$ and there is an individual d such that d is a child in w at t', d is born in w at t', and d becomes ruler of the world in w at t''

simplification of step 11

2.) *Derivation of truth-condition (6) from formula (4)*

(4) $\mathsf{P\ K\ F}\ \forall x$ **(**R **drive too fast** $x \rightarrow$ **cite Jones** x**)**

(6) There are times t', t'' such that $t' < t$, $t' < t''$, and, for every individual d, if d drives too fast in w at t', d is cited by Jones in w at t''

Key steps of the derivation: 3, 7

Derivation: (4) IS TRUE IN $\boldsymbol{L}_{\mathbf{NKRG}}{}^{+}$ AT $\langle \Im, w, t\rangle$ iff for every variable assignment g,

1. $[\![\mathsf{P\ K\ F}\ \forall x\ (\mathsf{R}\ \textbf{drive too fast}\ x \rightarrow \textbf{cite Jones}\ x)]\!]^{g,\, w,\, t,\, t} = 1$

   by the definition of TRUE IN $\boldsymbol{L}_{\mathbf{NKRG}}{}^{+}$

2. There is some $t' \in T$ such that $t' < t$ and $[\![\mathsf{K\ F}\ \forall x\ (\mathsf{R}\ \textbf{drive too fast}\ x \rightarrow \textbf{cite Jones}\ x)]\!]^{w,\, t',\, t} = 1$

   by (S_3')

3. There is some $t' \in T$ such that $t' < t$ and $[\![\mathsf{F}\ \forall x\ (\mathsf{R}\ \textbf{drive too fast}\ x \rightarrow \textbf{cite Jones}\ x)]\!]^{g,\, w,\, t',\, t'} = 1$

   by (K)

4. There is some $t' \in T$ such that $t' < t$, there is some $t'' \in T$ such that $t' < t''$, and $[\![\forall x\ (\mathsf{R}\ \textbf{drive too fast}\ x \rightarrow \textbf{cite Jones}\ x)]\!]^{g,\, w,\, t'',\, t'} = 1$

   by (S_4')

5. There is some $t' \in T$ such that $t' < t$, there is some $t'' \in T$ such that $t' < t''$, and, for every $d \in D$, $[\![(\mathsf{R}\ \textbf{drive too fast}\ x \rightarrow \textbf{cite Jones}\ x)]\!]^{g[x/d],\, w,\, t'',\, t'} = 1$

   by (S_{10}')

6. There is some $t' \in T$ such that $t' < t$, there is some $t'' \in T$ such that $t' < t''$, and, for every $d \in D$, if $[\![\mathsf{R}\ \textbf{drive too fast}\ x]\!]^{g[x/d],\, w,\, t'',\, t'} = 1$, then $[\![\textbf{cite Jones}\ x]\!]^{g[x/d],\, w,\, t'',\, t'} = 1$

by (S_8')

7. There is some $t' \in T$ such that $t' < t$, there is some $t'' \in T$ such that $t' < t''$, and, for every $d \in D$, if $[\![\textbf{drive too fast}\ x]\!]^{g[x/d],\, w,\, t',\, t'} = 1$, then $[\![\textbf{cite Jones}\ x]\!]^{g[x/d],\, w,\, t'',\, t'} = 1$

by (R)

8. There is some $t' \in T$ such that $t' < t$, there is some $t'' \in T$ such that $t' < t''$, and, for every $d \in D$, if $F_{g[x/d]}\ (x) \in F\ (\textbf{drive too fast})\ (\langle w, t'\rangle)$, then $\langle F_{g[x/d]}\ (\textbf{Jones}), F_{g[x/d]}\ (x)\rangle \in F\ (\textbf{cite})\ (\langle w, t''\rangle)$

by (S_1')

9. There is some $t' \in T$ such that $t' < t$, there is some $t'' \in T$ such that $t' < t''$, and, for every $d \in D$, if $g[x/d]\ (x) \in F\ (\textbf{drive too fast})\ (\langle w, t'\rangle)$, then $\langle F\ (\textbf{Jones}), g[x/d]\ (x)\rangle \in F\ (\textbf{cite})\ (\langle w, t''\rangle)$

by the definition of F_g

10. There is some $t' \in T$ such that $t' < t$, there is some $t'' \in T$ such that $t' < t''$, and, for every $d \in D$, if $d \in F\ (\textbf{drive too fast})\ (\langle w, t'\rangle)$, then $\langle F\ (\textbf{Jones}), d\rangle \in F\ (\textbf{cite})\ (\langle w, t''\rangle)$

by the convention that $g[x/d]\ (x) = d$

11. There is some $t' \in T$ such that $t' < t$, there is some $t'' \in T$ such that $t' < t''$, and, for every $d \in D$, if $d \in \{x \in D$: x drives too fast in w at $t'\}$, then $d \in \{x \in D$: Jones cites x in w at $t''\}$

by function application and the definition of F for **Jones**, **drive too fast** and **cite**

12. There are times *t*', *t*'' such that *t*' < *t*, *t*' < *t*'', and, for every individual *d*, if *d* drives too fast in *w* at *t*', *d* is cited by Jones in *w* at *t*''

simplification of step 12

3.) *Derivation of truth-condition (21) from formula (23)*

(23) $\mathsf{P}\ \mathsf{G}^{+}\ \exists x\ (\textbf{told}\ x \wedge \mathsf{R}\ \textbf{told}\ x)$

(21) There is a time *t'* such that $t' < t$ and such that for every time *t''* such that $t < t''$, there is a joke *d* such that *d* is told in *w* at *t'* and *d* is told in *w* at *t''*

Key steps of the derivation: 3, 6

Derivation: (23) IS TRUE IN $\boldsymbol{L}_{\mathbf{NKRG}}{}^{+}$ AT $\langle \Im, w, t\rangle$ iff for any variable assignment *g*,

1. $[\![\mathsf{P}\ \mathsf{G}^{+}\ \exists x\ (\textbf{told}\ x \wedge \mathsf{R}\ \textbf{told}\ x)]\!]^{g,\, w,\, t,\, t} = 1$

by the definition of TRUE IN $\boldsymbol{L}_{\mathbf{NKRG}}{}^{+}$

2. There is some $t' \in T$ such that $t' < t$ and $[\![\mathsf{G}^{+}\ \exists x\ (\textbf{told}\ x \wedge \mathsf{R}\ \textbf{told}\ x)]\!]^{w,\, t',\, t} = 1$

by (S_3')

3. There is some $t' \in T$ such that $t' < t$ and for every $t'' \in T$ such that $t < t''$, $[\![\exists x\ (\textbf{told}\ x \wedge \mathsf{R}\ \textbf{told}\ x)]\!]^{g,\, w,\, t',\, t''} = 1$

by (G^{+})

4. There is some $t' \in T$ such that $t' < t$ and for every $t'' \in T$ such that $t < t''$, there is some $d \in D$ such that *d* is a joke and $[\![(\textbf{told}\ x \wedge \mathsf{R}\ \textbf{told}\ x)]\!]^{g[x/d],\, w,\, t',\, t''} = 1$

by (S_9') and the assumption that *x* ranges over jokes

5. There is some $t' \in T$ such that $t' < t$ and for every $t'' \in T$ such that $t < t''$, there is some $d \in D$ such that *d* is a joke, $[\![\textbf{told}\ x]\!]^{g[x/d],\, w,\, t',\, t''} = 1$, and $[\![\mathsf{R}\ \textbf{told}\ x]\!]^{g[x/d],\, w,\, t',\, t''} = 1$

by (S_6’)

6. There is some $t' \in T$ such that $t' < t$ and for every $t'' \in T$ such that $t < t''$, there is some $d \in D$ such that d is a joke, $[\![\textbf{told } x]\!]^{g[x/d], w, t', t''} = 1$, and $[\![\textbf{told } x]\!]^{g[x/d], w, t'', t''} = 1$

by (R)

7. There is some $t' \in T$ such that $t' < t$ and for every $t'' \in T$ such that $t < t''$, there is some $d \in D$ such that d is a joke, $F_{g[x/d]}(x) \in F(\textbf{told})(\langle w, t' \rangle)$ and $F_{g[x/d]}(x) \in F(\textbf{told})(\langle w, t'' \rangle)$

by (S_1’)

8. There is some $t' \in T$ such that $t' < t$ and for every $t'' \in T$ such that $t < t''$, there is some $d \in D$ such that d is a joke, $g[x/d](x) \in F(\textbf{told})(\langle w, t' \rangle)$, and $g[x/d](x) \in F(\textbf{told})(\langle w, t'' \rangle)$

by the definition of F_g

9. There is some $t' \in T$ such that $t' < t$ and for every $t'' \in T$ such that $t < t''$, there is some $d \in D$ such that d is a joke, $d \in F(\textbf{told})(\langle w, t' \rangle)$, and $d \in F(\textbf{told}, w)(\langle w, t'' \rangle)$

by the convention that $g[x/d](x) = d$

10. There is some $t' \in T$ such that $t' < t$ and for every $t'' \in T$ such that $t < t''$, there is some $d \in D$ such that d is a joke, $d \in \{x \in D$: x is told in w at $t'\}$ and $d \in \{x \in D$: x is told in w at $t''\}$

by function application and the definition of F for **told**

11. There is a time t' such that $t' < t$ and such that for every time t'' such that $t < t''$, there is a joke d such that d is told in w at t' and d is told in w at t''

simplification of step 11

2. Rebutting King's argument

In this chapter I criticize King's suggestion that double-index accounts of tenses and time adverbs offer ad hoc and inelegant formalizations of the sort of English sentences that we discussed in section 1.3. In the literature on expressibility, there has been a tendency to focus on 'now'/'then' operators, thereby ignoring other interesting temporal operators that are definable in a double-index framework. My analysis of King's discussion will allow us to appreciate some of the advantages of introducing other kinds of double-index operators.

2.1 Four kinds of temporal operators

A variety of temporal operators can be defined in a multiply indexed framework. In this section I distinguish four categories of double-index temporal operators.

Standard operators

The first category of operators that I would like to mention is the category of standard operators. The operators P, F, H, and G of classical temporal logic belong to this category. Kamp's (1968) operators Since and Until are also standard operators.

Standard operators shift the first temporal index (i.e. *the time of evaluation*) of their embedded formulas. This is one of their defining features. The other characteristic feature of standard operators is that they shift the current time of evaluation to a time that is related to that time. The time that occupies the second-temporal-index slot plays no significant role in the semantic clauses of standard operators. This can be seen by looking back at the double-index clauses for P and F given in section 1.2.

It is awkward that the operators P, F, H, and G of Priorean temporal logic have been traditionally glossed by employing the expressions *it has been the case that*, *it will be the case that*, *it has always been the case that*, and *it will always be the case that*. In ordinary English, these expressions are indexical. They serve to talk about times that lie to the past or future of the time of speech of a sentence. Standard operators only have this kind of effect when the current time of evaluation happens to be the speech time. This is the case when standard operators are not embedded under other operators, or when the closest operators under which they are embedded are operators that retrieve the original time of utterance. But, in general, when a standard temporal operator appears embedded under another standard operator, its shifting action can be independent of the time of speech.

Indexical operators

In a double-index framework, it is possible to define temporal operators that shift the time of evaluation by reference to the second temporal index. The 'now'-operator N is one such operator. Indexical operators shift the time of evaluation of their embedded formulas by using the second temporal index as a point of reference. In a language lacking operators that shift the second temporal index, an indexical operator always shifts the time of evaluation by reference to the original time of utterance.

We can define indexical-operator correlates of the operators P and F. Let us call these indexical operators P* and F*. Using the framework of sections 1.1 and 1.2, they can be defined as follows:

Let $\boldsymbol{L}_{\mathsf{P*F*}}$ be a double-index $\mathscr{L}$-language and $[\![\ ]\!]^{g,\,w,\,t,\,t'}$ be the truth-value assignment of $\boldsymbol{L}_{\mathsf{P*F*}}$ with respect to an interpretation $\langle D, W, T, <, F\rangle$.

For any wff ϕ of $\boldsymbol{L}_{\mathsf{P*F*}}$, variable assignment g, $w \in W$, $t \in T$, and $t' \in T$,

(P*) $[\![\mathsf{P}^*\phi]\!]^{g,\,w,\,t,\,t'} = 1$ iff there is some $t'' \in T$ such that $t'' < t'$ and $[\![\phi]\!]^{g,\,w,\,t'',\,t'} = 1$

(F*) $[\![\mathsf{F}^*\phi]\!]^{g,\,w,\,t,\,t'} = 1$ iff there is some $t'' \in T$ such that $t' < t''$ and $[\![\phi]\!]^{g,\,w,\,t'',\,t'} = 1$

The indexical operators P* and F* shift the current time of evaluation to a time that lies to the past or future of the second temporal index.

We can also define indexical correlates of the standard operators H and G.

(For simplicity, in the rest of this section I will give semantic clauses without making explicit reference to $\mathscr{L}$-languages and their interpretations.)

For any wff ϕ, variable assignment g, $w \in W$, $t \in T$, and $t' \in T$,

(H*) $[\![\mathsf{H}^*\phi]\!]^{g, w, t, t'} = 1$ iff for every $t'' \in T$ such that $t'' < t'$, $[\![\phi]\!]^{g, w, t'', t'} = 1$

(G*) $[\![\mathsf{G}^*\phi]\!]^{g, w, t, t'} = 1$ iff for every $t'' \in T$ such that $t' < t''$, $[\![\phi]\!]^{g, w, t'', t'} = 1$

Just as the semantic clauses of P* and F* involve existential quantification over times that precede or follow the second temporal index, the semantic clauses of H* and G* involve universal quantification over times preceding or following the second temporal index. Of course, we could have introduced H* and G* simply as abbreviations of ¬P*¬ and ¬F*¬, respectively.

In languages like $\boldsymbol{L}_{\mathsf{N}}$ and $\boldsymbol{L}_{\mathsf{P*F*}}$, the indexical operators are anchored to the time of utterance, even when they occur under the scope of other operators. For this reason, a wff of the form FNϕ is equivalent to Nϕ in $\boldsymbol{L}_{\mathsf{N}}$ and a wff of the form FP*ϕ is equivalent to P*ϕ in $\boldsymbol{L}_{\mathsf{P*F*}}$.

Monster operators

It is possible to define operators that shift the second temporal index. Since in some systems this index is the one that plays the role of time of utterance, the operators that shift it can be regarded as monsters in the sense of Kaplan (1977). Monster operators shift a given second-index-time to another time, which can be viewed as a new time of utterance.

As we saw in chapter 1, Vlach's operator K is a monster operator. It is the monster operator that copies the first temporal index and puts it in the position of the second temporal index. In other words, it turns the current time of evaluation into the new time of utterance.

We can introduce monster correlates of P and F. Let us call them P^+ and F^+. Their semantic clauses are as follows:

For any wff ϕ, variable assignment g, $w \in W$, $t \in T$, and $t' \in T$,

(P^+) $[\![\mathsf{P}^+\phi]\!]^{g, w, t, t'} = 1$ iff there is some $t'' \in T$ such that $t'' < t'$ and

$[\![\phi]\!]^{g, w, t, t''} = 1$

(F$^+$) $[\![\mathsf{F}^+]\!]^{g, w, t, t'} = 1$ iff there is some $t'' \in T$ such that $t' < t''$ and $[\![\phi]\!]^{g, w, t, t''} = 1$

These operators shift the current time of utterance –or, if you prefer, the second temporal index– to a time that precedes or follows that time. They leave the time of evaluation unmodified. The monster correlates of H and G, which we may call H$^+$ and G$^+$, can be introduced as abbreviations of $\neg\mathsf{P}^+\neg$ and $\neg\mathsf{F}^+\neg$. Notice that the operators P$^+$ and F$^+$ do not shift the time of utterance of their embedded formulas by relating it to the current time of evaluation. In this respect, they are different from K. But we can define monster analogues of P* and F* that relate the new time of utterance to the current time of evaluation. Let me call them P^{++} and F^{++}.

For any wff ϕ, variable assignment g, $w \in W$, $t \in T$, and $t' \in T$,

(P^{++}) $[\![\mathsf{P}^{++}\phi]\!]^{g, w, t, t'} = 1$ iff there is some $t'' \in T$ such that $t'' < t$ and $[\![\phi]\!]^{g, w, t, t''} = 1$

(F^{++}) $[\![\mathsf{F}^{++}\phi]\!]^{g, w, t, t'} = 1$ iff there is some $t'' \in T$ such that $t < t''$ and $[\![\phi]\!]^{g, w, t, t''} = 1$

P^{++} and F^{++} take you to a new time of utterance that precedes or follows the time of evaluation. The formal properties of these two operators are similar to those of P* and F*. In particular, wffs of the form $\mathsf{F}^+\mathsf{P}^{++}\phi$ and $\mathsf{P}^+\mathsf{F}^{++}\phi$ are equivalent to $\mathsf{P}^{++}\phi$ and $\mathsf{F}^{++}\phi$, respectively. By contrast, a wff of the form $\mathsf{F}^+\mathsf{P}^+\phi$ is not equivalent to $\mathsf{P}^+\phi$ and a wff of the form $\mathsf{P}^+\mathsf{F}^+\phi$ is not equivalent to $\mathsf{F}^+\phi$.

Thus, monster operators can be divided into two subcategories depending on whether they shift the current time of utterance with respect to the current time of evaluation or independently of it. This division is just the monster version of the distinction between standard operators and indexical operators.

Monster/indexical operators

We can also define double-index operators that simultaneously shift the two temporal indices. I will call these operators *monster/indexical* operators. A group of operators of this type that is particularly interesting is the one that includes operators that shift the time of evaluation in the way standard operators do and that also store the new time of evaluation that they introduce. A double-index operator can be a monster/indexical operator. But let me define here a couple of monster/indexical operators of triple-index languages. In the context of a triply indexed language, we can define past and future operators that store their times in different temporal indices. Let us call these operators P^{*+} and F^{*+}. They are defined as follows:

For any wff ϕ, variable assignment g, $w \in W$, $t \in T$, $t' \in T$, and $t'' \in T$

$$(\mathsf{P}^{*+})\ [\![\mathsf{P}^{*+}\phi]\!]^{g,\, w,\, t,\, t',\, t''} = 1 \text{ iff there is some } t''' \in T \text{ such that } t''' < t \text{ and } [\![\phi]\!]^{g,\, w,\, t''',\, t''',\, t''} = 1$$

$$(\mathsf{F}^{*+})\ [\![\mathsf{F}^{*+}\phi]\!]^{g,\, w,\, t,\, t',\, t''} = 1 \text{ iff there is some } t''' \in T \text{ such that } t < t''' \text{ and } [\![\phi]\!]^{g,\, w,\, t''',\, t',\, t'''} = 1$$

While P^{*+} stores the past time that it introduces in the second temporal index, F^{*+} introduces a future time and stores it in the third temporal index. The two 'then'-operators R and R' of section 1.2 can retrieve the times that P^{*+} and F^{*+} store.

The classification of double-index operators presented in this section is not meant to be exhaustive. The operators that we have defined will suffice for the purposes of the next section.

2.2 King's argument

As we saw in chapter 1, there are English sentences whose truth-conditions can be expressed in $\boldsymbol{L}_{\mathbf{N}}$ but not in $\boldsymbol{L}$. (1) is a sentence of this sort. On the other hand, there are English sentences whose truth-conditions are not expressible in $\boldsymbol{L}_{\mathbf{N}}$ but are expressible in $\boldsymbol{L}_{\mathbf{KR}}$. (2) is a sentence of this kind. (3) is a formula of $\boldsymbol{L}_{\mathbf{N}}$ that expresses the truth-condition of (1) and (4) is formula of $\boldsymbol{L}_{\mathbf{KR}}$ that expresses the truth-condition of (2).

(1) One day, all persons alive now will be dead

(2) Once all persons alive then would be dead

(3) $\mathsf{F}\ \forall x\ (\mathsf{N}\ \mathbf{alive}\ x \rightarrow \mathbf{dead}\ x)$

(4) $\mathsf{P}\ \mathsf{K}\ \mathsf{F}\ \forall x\ (\mathsf{R}\ \mathbf{alive}\ x \rightarrow \mathbf{dead}\ x)$

But even if (3)-(4) capture the truth-conditions of (1)-(2), one may think that (3)-(4) do not describe the logical forms of (1)-(2). This is the view that King (2003, pp. 221-222) advocates. He argues that if we hold that (3)-(4) are the logical forms of (1)-(2), we get a messy relation between the surface structures of (1)-(2) and their logical forms. According to King, while there is a prominent similarity between (1) and (2) at the level of surface syntax, (3) and (4) have very different structures. Whereas (3) has two temporal operators, (4) has four operators. King claims that this is ad hoc and points out that quantificational formalizations of (1)-(2) such as (5)-(6) are less ad hoc and allow for a cleaner relation between surface structures and logical forms. In formulas (5)-(6), the predicates **alive** and **dead** contain extra arguments for times. King assumes that the singular term $\boldsymbol{t^*}$ of (5) designates the time of utterance of (1) and the term $\boldsymbol{t''}$ of (6) designates a contextually determined past time.

(5) $\exists \boldsymbol{t}\ (\boldsymbol{t^*} < \boldsymbol{t} \wedge \forall \boldsymbol{x}\ (\mathbf{alive}\ \boldsymbol{x}\ \boldsymbol{t^*} \rightarrow \mathbf{dead}\ \boldsymbol{x}\ \boldsymbol{t}))$

(6) $\exists \boldsymbol{t}\ (\boldsymbol{t''} < \boldsymbol{t} \wedge \forall \boldsymbol{x}\ (\mathbf{alive}\ \boldsymbol{x}\ \boldsymbol{t''} \rightarrow \mathbf{dead}\ \boldsymbol{x}\ \boldsymbol{t}))$

King takes it that his observations about (1)-(6) illustrate the following point:

> ... treating tenses as involving quantification over times (and expressing relations between times) rather than index shifting sentence operators [...] allows for a more plausible account of the relation between the surface structures of English sentences and the syntactic representations of those sentences at the level of syntax that is the input to semantics.
> (King 2003, p. 223)

I do not dispute King's critical remarks regarding (3)-(4). In fact, I think he is right in claiming that (4) contains ad hoc operators that do not correspond to any constituent of the surface structure of (2). Presumably, the only motivation that

Vlach had for positing the operators K and R in the formal representation of a sentence like (2) was that these operators appeared to be required in order to get the truth-condition of the sentence right. In my view, the controversial aspect of King's discussion is his assumption that the point he makes about (3)-(4) generalizes to other operator-based accounts.

> The crucial point is that even though [(1)] and [(2)] appear to have the same number and sort of syntactic constituents combined in the same ways, and differ only in tense and the words 'now' and 'then', they have very different LFs: [(1)]'s LF contains two operators and [(2)]'s contains four! Surely this looks ad hoc, and presupposes a very messy relation between the surface structures of sentences and their LFs. Admittedly, we are looking at only one version of the operator approach, but such ad hocery and messiness in the relation between surface structure and LF is typical of such approaches.[1]
>
> (King 2003, pp. 222-223)

Pace King, an intensional semanticist can offer representations of (2) which do not have the defect that King mentions in the passage. The bound uses of *then* that we encounter in sentences like (2) can be formally represented with the aid of monster/indexical operators. On this approach, (2) can be represented as (7).

(7) $\mathsf{P}^{*+}\ \mathsf{F}\ \forall x\ (\mathsf{R}\ \mathbf{alive}\ x \rightarrow \mathbf{dead}\ x)$

The operator R of (7) retrieves the past time associated with P^{*+} and makes it the time of evaluation of the subformula **alive *x***. The subformula **dead *x*** is evaluated at the future time associated with the operator F.

(8) is the truth-condition of (7) with respect to a world of utterance *w* and a time of utterance *t*.

(8) There are times *t*', *t*'' such that *t*' < *t*, *t*' < *t*'', and, for every individual *d*, if *d* is alive in *w* at *t*', *d* is dead in *w* at *t*''.

I take it that (8) specifies the intuitive reading of (2). King cannot complain that there is no one-to-one correspondence between the operators of (7) and the tem-

[1] I have put my own numbering in this passage. Interestingly, the last sentence of the passage was removed in King 2007, chapter 6 (see pp. 188-189), which is a revised version of King 2003.

poral surface constituents of (2). One can view P^{*+} as representing the semantic contribution of *once*, F as representing the contribution of *would*, and R as representing the contribution of *then*.

King might still complain that (7) has more operators than (3). He might insist that this is bad because (1) and (2) are very similar at the surface level. My reply to this objection is that the truth-condition of (1) involves two different times, whereas the truth-condition of (2) involves three times. For this reason, it is not *prima facie* incorrect to postulate three temporal operators in the formal representation of (2) and just two temporal operators in the representation of (1).

According to the semantics of chapter 1, (9) is the truth-condition of (3).

(9) There is a time *t*' such that $t < t'$ and, for every individual *d*, if *d* is alive in *w* at *t*, *d* is dead in *w* at *t*'.

As you can see, (9) differs from (8) in that (9) distinguishes between three times. Notwithstanding the apparent surface similarity between (1) and (2), the temporal expressions *one day* and *will* do not introduce two different times in the truth-condition of (1). By contrast, the words *once* and *would* introduce two different times in (2). The auxiliary *would* of (2) indicates that the time in which the persons are dead is posterior to the past time in which they were alive.

The formulas (5) and (6) of King have the same number of temporal variables because King assumes that (6) contains the free variable ***t''***. A more orthodox first-order symbolization of (2) would make use of two bound variables ranging over times. Compare (6) and (10).

(10) $\exists \boldsymbol{t}\, \exists \boldsymbol{t'}\, (\boldsymbol{t} < \boldsymbol{t}^* \wedge \boldsymbol{t} < \boldsymbol{t'} \wedge \forall \boldsymbol{x}\, (\textbf{alive}\ \boldsymbol{x}\ \boldsymbol{t} \rightarrow \textbf{dead}\ \boldsymbol{x}\ \boldsymbol{t'}))$

(10) has the same truth-condition as (7). King takes for granted that (6) is a good formalization of (2). But I contend that (2) does not have the free-variable reading which Kings ascribes to it. The presence of *once* in (2) forces a quantified reading along the lines of (8). A contextually given reference time is required in order to interpret the deictic use of *then* in (11).

(11) All persons alive then would be dead

Without a reference time that is implicitly understood, (11) does not express a truth-condition. But (2) does not require a contextually salient reference time to be interpreted. Its intuitive reading is quantificational rather than deictic. Hence, (7) is more accurate as a representation of (2) than (6).

Let me stress that I am not assuming that (3) and (7) accurately represent the contributions of the expressions *one day* and *once* in sentences (1)-(2). An intensional theorist may concede that a more sophisticated formal apparatus is needed in order to account for the semantics of these two expressions. My point is just this: from the mere fact that (3) and (7) differ in the number of operators that they contain one cannot infer that symbolizations with explicit temporal variables are better than (3) and (7) as formalizations of (1)-(2). King's symbolization (6) does not have more quantifiers than (5). But we have just seen that (6) does not predict the right truth-condition for (2). On the other hand, (5) and (10) predict the desired truth-conditions. But these truth-conditions involve a different number of times and the formulas (5) and (10) do not have the same number of quantifiers and bound variables.

In a revised version of his discussion, King (2007, pp. 187-189) tries to illustrate his point using a different group of sentences. Instead of (1)-(4), he considers sentences (12)-(14) and formulas (15)-(17).

(12) A child was born who would be king

(13) A child was born who will be king

(14) A barbarian will be king

(15) $\mathsf{P}\ \exists x\ (\mathbf{child}\ x \wedge (\mathbf{born}\ x \wedge \mathsf{F}\ \mathbf{king}\ x))$

(16) $\mathsf{P}\ \exists x\ (\mathbf{child}\ x \wedge (\mathbf{born}\ x \wedge \mathsf{N}\ \mathsf{F}\ \mathbf{king}\ x))$

(17) $\mathsf{F}\ \exists x\ (\mathbf{barbarian}\ x \wedge \mathbf{king}\ x))$

Here is what King says about (12)-(17):

> ... even though [(12)] and [(13)] appear to have exactly the same number and sort of syntactic constituents combined in the same ways, and differ only in the words 'would' and

> 'will', they have different LFs: [(13)]'s LF contains an additional operator ('N') that corresponds to no operator in [(12)]'s LF. Presumably, this operator is somehow due to the presence of the word 'will' in [(13)]. But then when we consider a sentence containing 'will' alone, such as [(14)] presumably it will be represented as [(17)]. In the case of [(14)], 'will' only contributes one operator to its LF, whereas in [(14)] it contributes two. Surely the fact that the LFs for [(12)] and [(13)] contain different numbers of operators despite the fact that the sentences are exactly similar syntactically and that 'will' contributes different numbers of operators to LFs in different cases looks ad hoc and presupposes a very messy relation between the surface structures of sentences and their LFs.[2]
>
> (King 2007, pp. 188-189)

I do not dispute King's assumption that it is desirable to give a uniform account of the semantic contribution of *will* in sentences (13) and (14). I also agree that (16) and (17) are defective for this reason. However, the intensional semanticist does not need to resort to $\boldsymbol{L}_{\mathsf{N}}$ in order to symbolize (13). The indexical operator F^* of section 2.1 provides an alternative way of symbolizing (13) and (14).

(18) $\mathsf{P}\ \exists x\ (\mathbf{child}\ x \wedge (\mathbf{born}\ x \wedge \mathsf{F}^*\ \mathbf{king}\ x))$

(19) $\exists x\ (\mathbf{barbarian}\ x \wedge \mathsf{F}^*\ \mathbf{king}\ x))$

The crucial thing to note here is that (15) and (18) have the same number of temporal operators. For this reason, King's objection to (16) in the last quoted passage does not apply to (19). Moreover, (18) and (19) are not ad hoc formalizations in the sense of containing operators that are postulated just to obtain the right truth-conditions for (13) and (14). The indexical operator F^* of (18) and (19) seems to capture the contribution of *will* in (13) and (14). Even though in (13) *will* appears in an embedded position, it takes us to a time posterior to the time of utterance. This suggests that F^* is more suitable as an operator correlate of the English word *will* than F. The standard operator F, on the other hand, is more suitable for representing the contribution of *would* in (12). We would get the wrong truth-condition if we replaced F by F^* in (15). We thus observe a nice correlation between the temporal surface constituents of (12)-(15), on the one hand, and the temporal operators of (15), (18), and (19), on the other hand.

The important point, in any case, is that an operator-based representation of (13) does not need to contain more operators than the representations of (12) and

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[2] I have also put my own numbering in this passage.

(14), and we can represent the contribution of *will* in (13) and (14) using the same indexical operator. So, symbolizations (15), (18), and (19) do not have the problems that King mentions with respect to (15)-(17).

I hope my discussion of sentences (1)-(2) and (12)-(14) in this chapter has served to illustrate that 'now'/'then' operators are not the only kind of operators to which a double-index theorist can appeal in order to give the truth-conditions of *prima facie* problematic sentences. Indexical operators, monster operators, and monster/indexical operators can also be used to obtain the truth-conditions of the English sentences discussed in this chapter.

Part 2: Intensionality and Logical Form

3. The Parallel between Pronouns, Tenses, and Modals

Back in the early seventies, Partee (1973) observed some similarities between the linguistic behavior of English personal pronouns and the behavior of the English tense morphemes *Past* and *Present*. She suggested that these behavioral analogies speak for a referential account of English tenses. Partee's discussion of the parallel between tenses and pronouns was the beginning of a trend away from the Priorean conception of tenses as temporal operators. Many semanticists now accept Partee's suggestion that the logical forms of tensed sentences have covert variable-like constituents which denote times. During the late nineties, a trend towards variable-based treatments of modality also began to emerge.[1] The traditional account of modals as intensional operators, which is still accepted by many, has been challenged by several theorists. One crucial line of argument against the operator-based account of modality starts from the observation that modals exhibit all the pronoun-like behaviors of *Past* and *Present* which Partee described in her seminal paper. It is then argued that, just as in the case of tense, this fact provides a motivation for adopting a variable-based approach to modal talk. The first author who suggested that Partee's original analogies extend to the domain of modality was Stone (1997).

This chapter is devoted to examining some of the key examples that Partee and Stone proposed in order to illustrate the parallel between pronouns, tenses, and modals. Partee's examples are presented in section 3.1 and Stone's examples are presented in section 3.2. In section 3.3 I challenge the suggestion that, taken together, the two groups of examples support the view that intensional operators must be dispensed with in favor of variable-based representations.

I present a formal analysis of tenses and modals as context-sensitive operators in section 3.3. I argue that this analysis can predict the right truth-conditions for the English sentences in which Partee and Stone were interested. The formal representations that my analysis posits are similar in various respects to the formulas of the operator-based languages that we discussed in chapters 1 and 2. But

[1] For useful references to this literature on modality, see Schlenker 2006 and Schaffer 2012, section 2.

they have a novel feature: they employ predicate-forming operators that act on sub-predicative syntactic constituents. I will call these constituents *radicals*. In chapter 4 I will propose an account of tenses that is more sophisticated than the analysis of this chapter and which does no model tenses as operators that act on radicals. However, I think that the analysis of tenses outlined in this chapter has some theoretical interest. For this analysis abandons the assumption that the temporal operators that represent tenses are sentential operators. As I mentioned in the introduction, the possibility of dispensing with this assumption while still endorsing an intensional account of English tenses has not been explored in the literature.

3.1 The parallel between pronouns and tenses

Let us take a look at the parallel between pronouns and tenses as introduced in Partee 1973. Sentences (1), (3), and (5) illustrate three prototypical uses of pronouns. Sentences (2), (4), and (6) illustrate what Partee took to be the temporal analogues of such uses.[2]

Deictic reference

(1) She left me

(2) I didn't turn off the stove

Anaphoric reference

(3) Sam took the car yesterday and Sheila took it today

(4a) Sheila had a party and Sam got drunk

(4b) When Susan walked in, Peter left

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[2] Partee offered a more sophisticated taxonomy of the relevant uses of pronouns and tenses in a later paper (Partee 1984, pp. 244-247). In that paper she reappraisses her argument line in Partee 1973 and ends up advocating a dynamic account of tense.

Bound reference

(5) Every woman believes that she is happy

(6a) Whenever Mary smiled, Sam yawned

(6b) When Mary telephoned, Sam was always asleep

Partee relied on examples such as (1)-(6) to advocate a variable-based approach to the semantics of tenses. As she put it: "…there is a considerable and striking parallel in the behavior of tenses and pronouns, at least in English. The corollary seems to be that if pronouns have to be treated as variables and not as sentence operators […] the same must be true for tenses…" (Partee 1973 p. 609). Partee did not provide specific formalizations of her own examples. For purely illustrative purposes, let us consider a simple formalization of (2) containing a temporal variable.

(LF2) ¬ *Turn-off* (*I*, *the stove*, *t*)

In (LF2), *Turn-off* is a triadic predicate, *I* is the first-person individual constant of Kaplan's *LD* formal language (see Kaplan 1989, section XVIII), *the stove* is a definite description, and *t* is a free variable ranging over past intervals of time.

Here is a way of conceiving of the semantics of logical form (LF2). When (2) is felicitously uttered, the context of utterance determines an assignment function g_c that assigns a value to the time variable *t*. Intuitively, $g_c(t)$ is the specific past interval that is relevant for the evaluation of (2) in that context. If one assumes that the propositional contribution of a free variable is the individual that it denotes in a given context, the semantic content of (LF2) may be thought of as a time-specific but world-neutral proposition, namely, a proposition that is true in a world *w* iff a_c (the agent of the context) did not turn off the relevant stove in *w* during the time interval $g_c(t)$.

I must stress that this is just one possible way to go. In what follows I will use formalizations like (LF2) to illustrate how a variable-based account of tenses may work. Needless to say, there are more sophisticated variants of the variable approach (see e.g. Enç 1987 and Ogihara 1996). One does not need to assume, for example, that *t* is a free variable. The logical form of (3) might involve a quantified rather than a free variable. Moreover, one does not need to think that variables are context-sensitive expressions that take different values in different contexts. Some authors rely on this conception of variables (see e. g. Stanley 2007). But Kaplan himself did not treat variables this way in his *LD* formal system.

(LF4b) and (LF6a) are possible formalizations of sentences (4b) and (6a) along the lines of (LF2).

(LF4b) *Walk-in* (*Susan*, *t*) ∧ *Leave* (*Peter*, *t*)

(LF6a) ∀*t* (*Smile* (*Mary*, *t*) → *Yawn* (*Sam*, *t*))

One may think of the content of (LF4b) as a time-specific but world-neutral proposition. Such a proposition would be true in a world *w* iff Susan walks in *w* during $g_c(t)$ and Peter leaves in *w* during $g_c(t)$. Similarly, the semantic content of (LF6a) may be thought of as a time-specific/world-neutral proposition that is true in a world *w* iff, for every past time interval *t* having a contextually constrained size, if Mary smiles in *w* during *t*, then Sam yawns in *w* during *t*. The size of the interval must be constrained because we do not judge (6a) as being true if a very long period of time separates the smiling by Mary and the yawning by John.

A variant of (LF4b) may contain two different variables *t* and *t*' appearing in the predicates *Walk-in* (*Susan*, *t*) and *Leave* (*Peter*, *t*'). In normal contexts of utterance, these variables are interpreted in such a way that $g_c(t) = g_c(t')$. But, in principle, there might be contexts of utterance in which the intervals $g_c(t)$ and $g_c(t')$ are not identical.

3.2 Extending the parallel

As I mentioned above, Stone (1997, section 2) argued that Partee's parallel between pronouns and tenses can be extended to the realm of modality. English modals, he claimed, exhibit the whole range of pronoun-like uses of tenses observed in Partee 1973/1984. Stone proposed examples such as (7) - (9) to illustrate how modals can refer to hypothetical scenarios in a variety of ways.[3] The

[3] Speas (2004) and Schaffer (2012, section 2.1) provide further examples.

interpretation of these examples is not uncontroversial. The contrast between (8) and (9) can be better understood by thinking of the relevant scenarios as abstract situations (see Barwise and Perry 1983).

Imagine a speaker uttering (7) while looking at a guitar amplifier in a store.

(7) My father would kill me

Here the context of utterance makes salient a hypothetical scenario in which the speaker buys the amplifier and plays his guitar with it at home. (7) serves to communicate that if this scenario obtained, the speaker's father would react violently. This, Stone suggests, is comparable to what we observe in (1) and (2). The utterance of each sentence involves an entity –a person, an interval, and a hypothetical scenario, respectively– that becomes salient by virtue of the extra-linguistic context.

(8) If I had a party at my place, my neighbor would call the police

(8) is the modal analogue of (3)-(4). Just as the first conjunct of (4a) introduces a reference time interval relative to which its second conjunct is interpreted, so too the antecedent of (8) serves to specify a concrete scenario against which its consequent must be assessed.

(9) If Mary married one of my brothers, Sarah would envy her

Intuitively, sentence (9) does not concern a single hypothetical scenario. Rather, it specifies a class of such scenarios –at a minimum, one scenario for each brother the speaker has– and states that each scenario belonging to this class is one in which Sarah envies Mary. (9) thus seems to involve bound reference to certain hypothetical scenarios. In this respect, it parallels (5)-(6).

If Stone's argument is correct, (1)-(9) illustrate a structural parallel between pronouns, tenses, and modals in English. If this result is analyzed in the light of the argument discussed in the previous section, the natural conclusion to draw is that English logical forms do not contain modal or temporal operators. Rather, they contain implicit variables that make deictic, anaphoric, and bound reference to times and possibilia. An account of this sort can be implemented in various ways. For illustrative purposes, let us briefly consider a variant of the account in question that formalizes (7) and (9) along the lines of (LF2) and (LF6a).

(LF7) *Kill* (*I's father*, *I*, *s*, *t*)

(LF9) ∀*s* ∀*x* ((*Brother* (*I*, *x*, *s*, *t*) ∧ *Marry* (*Mary*, *x*, *s*, *t*)) → *Envy* (*Sarah*, *Mary*, *s*, *t*))

In (LF7) and (LF9), all predicates contain an argument place for a time and also an argument place for a situation. *s* is a variable ranging over situations that are accessible from the context of utterance (which is also situation) according to a contextual relation of accessibility. *t* is a variable standing for a contextually given present time interval (i.e. an interval that includes the time of utterance).

(LF7) can only be interpreted if the context of utterance provides a specific value for the situation variable *s*. In our example, this contextual value is the hypothetical scenario in which the speaker buys the amplifier and makes noise with it at home. The content of (LF7) is a time/situation-specific proposition that is true iff in that scenario a_c's father kills a_c during the present interval $g_c(t)$.

(LF9) aims to capture the reading of (9) in which the speaker is talking about her actual brothers, as opposed to the non-actual brothers that she may have in a counterfactual scenario. I assume that only scenarios involving actual brothers are contextually accessible. The content of (LF9) is a time/situation-specific proposition that is true iff, for every accessible scenario *s*, if Mary marries one of a_c's actual brothers in *s* during $g_c(t)$, then Sarah envies Mary in *s* during $g_c(t)$.

3.3 Radicals

In this section I want to propose an analysis of the examples that we considered in the previous two sections. The analysis is based on two ideas. The first idea is that intensional operators can be context-sensitive. It is possible to define temporal operators that shift a given time of evaluation to a time that is included in some contextually given interval of time. There can also be modal operators that are sensitive to some situation or set of worlds that is contextually given. The second idea is that tenses and modals can be represented as intensional operators that act on radicals. I argue that the truth-conditions of the examples considered in sections 3.1 and 3.2 can be correctly predicted by an account that combines these two ideas. If I succeed in this task, the moral of the discussion of this chapter is that one can account for the uses of tenses and modals in which Partee and Stone were interested without postulating implicit world/time arguments. We may grant that tenses and modals behave in pronoun-like ways while still insisting that there is a fundamental distinction between reference and predication in natural languages. On the view that I propose in this section, personal pronouns belong to the referential side of this dichotomy, whereas tenses and modals belong to the predicative side.

3.3.1 Deictic uses of tenses and modals

Let us consider once again a deictic use of the English morpheme *Past*.[4]

(10) I frowned

I will assume that the utterance of (10) concerns a contextually provided past time interval. The communicative intention of the speaker is to talk about what she did during that interval of time –as opposed to the whole past.

Let me introduce the account I want to propose in this chapter by interpreting (10). *Prima facie*, in a sentence like (10) *Past* does not modify the whole sentential formula *I frown*, but only its verb. This is what surface syntax suggests. Let PAST be a non-sentential operator that acts on pre-predicative constituents. I will call such constituents *radicals*.[5] Syntactically, radicals are expressions that give rise to predicates when combined with temporal operators such as PAST. Semantically, they get standard predicate extensions relative to worlds and time intervals. The interpretation of an *n*-ary radical $\varphi\ x_1, \ldots x_n$ is a function mapping world/time-interval pairs onto sets of *n*-tuples of individuals. Let *W*, *T*, and *I* be the set of all worlds, the set of all time intervals (I will call them *times*), and the

[4] Following Partee's discussion, I use the labels 'deictic', 'anaphoric', and 'bound' to classify the uses of tenses and modals that we have been considering here. However, these labels are innacurate in the context of the operator-based account that I will put forward in this chapter. On my account, sentences (2), (4), and (6) do not involve reference to times. Therefore, there is no deixis, anaphora or variable-binding taking place in the logical forms of such sentences.

[5] The examples discussed in Enç 1986 –where tense does not seem to affect the interpretation of nominal phrases– provide a motivation for formalizing tenses as non-sentential operators.

set of all individuals. Let $\subseteq$ be the inclusion relation for time intervals. Finally, let $<$ be the earlier-later relation between time intervals.[6]

Let FROWN be the radical associated with the English verb *frown*. Here is the semantic clause that defines the interpretation of FROWN:

$$|\mathbf{FROWN}\ \boldsymbol{x}| = f: W \times T \rightarrow \wp(I), \text{ for any } \langle w, t\rangle \in W \times T,$$
$$f(\langle w, t\rangle) = \{i: i \text{ frowns in } w \text{ at } t\}$$

As you can see, $|\text{FROWN}\ x|$ is the function which maps any world/time pair to the set of individuals that frown in the world of the pair at the time of the pair.

When an n-adic radical $\varphi\ x_1, \ldots x_n$ is modified by the operator PAST, we obtain a predicate of the form PAST$\varphi\ x_1, \ldots x_n$.[7] In classical temporal logic, a formula of the form $\mathsf{P}\phi$ is true at a time t just in case there is a time t' such that $t' < t$ and ϕ is true at t'. We could interpret the predicates of the form PAST$\varphi\ x_1, \ldots x_n$ along similar lines.

$$|\mathbf{PAST}\boldsymbol{\varphi}\ \boldsymbol{x_1, \ldots x_n}| = f: W \times T \rightarrow \wp(I^n), \text{ for any } \langle w, t\rangle \in W \times T,$$
$$f(\langle w, t\rangle) = \{\langle i_1, \ldots i_n\rangle: \text{there is a time } t' < t$$
$$\text{such that } \langle i_1, \ldots i_n\rangle \in |\varphi\ x_1, \ldots x_n|(w, t')\}$$

According to this definition, the extension of PAST$\varphi\ x_1, \ldots x_n$ relative to a given world/time pair $\langle w, t\rangle$ is the set consisting of every n-tuple of individuals such that, for some time t' that precedes t, the n-tuple appears in the extension of $\varphi\ x_1, \ldots x_n$ relative to $\langle w, t'\rangle$. Given this general definition, and given our interpretation of the radical FROWN x, the interpretation of the predicate PASTFROWN x can be easily computed.

$$|\mathbf{PASTFROWN}\ \boldsymbol{x}| = f: W \times T \rightarrow \wp(I), \text{ for any } \langle w, t\rangle \in W \times T,\ f(\langle w, t\rangle) =$$
$$\{i: \text{there is a time } t' < t \text{ such that } i \text{ frowns in } w \text{ during } t'\}$$

[6] Intervals are sets of instants. Given two intervals t and t', $t < t'$ iff every instant of t precedes every instant of t' and $t \subseteq t'$ iff every instant of t is also an instant of t'. For a definition of the notion of interval, see e.g. Ogihara 1996, p. 24.

[7] Here the variables appearing in front of φ are not under the scope of PAST. Radical operators can only take scope over radicals. This is an important feature of the present formalization, since it is what allows us to avoid potential scope conflicts generated by the interaction between nominal phrases and tenses (see Enç 1986).

Let w_c and t_c be the world and time of utterance of (10). If we give the function $|$PASTFROWN $x|$ the pair $\langle w_c, t_c\rangle$ as argument, this function yields the set of those individuals that have frowned in w_c during some time preceding t_c. Thus, if we analyze (10) as being true just in case the speaker of c is a member of this set, we get the wrong truth-condition for (10). As I mentioned before, the utterance of (10) concerns a specific past interval, not the whole past relative to t_c.

There was something right and something wrong about our previous semantic clause for PASTφ $x_1, \ldots x_n$. The right part was the idea that tenses shift the point of evaluation. When (10) is uttered, what matters in order to determine whether it expresses a truth in context is not what is the case at the time of utterance, but rather what was the case at some past time. What was wrong with the clause was the idea of accounting for this assessment shift by unrestrictedly quantifying over past times. In order to fix this problem, we must redefine $|$PASTφ $x_1, \ldots x_n|$.

I will assume that PAST is context-sensitive in two ways. It requires two different times t and t' that should be contextually determined. Think of t as a time of utterance and think of t' as an evaluation time lying to the past of t. These two times are analogues of Reichenbach's (1947, §51) point of speech and point of the event.

The new $|$PASTφ $x_1, \ldots x_n|$ is defined as follows:

$$|\textbf{PAST}\boldsymbol{\varphi}\ \boldsymbol{x_1, \ldots x_n}| = f: W \times T \times T \rightarrow \wp(I^n), \text{ for any } \langle w, t, t'\rangle \in W \times T \times T,$$
$$f(\langle w, t, t'\rangle) = \{\langle i_1, \ldots i_n\rangle: \text{there is a time } t'' \text{ such that } t'' < t,\ t'' \subseteq t',$$
$$\text{and } \langle i_1, \ldots i_n\rangle \in |\varphi\ x_1, \ldots x_n|(w, t'')\}$$

$|$PASTφ $x_1, \ldots x_n|$ maps any given $\langle w, t, t'\rangle$ triple onto the set of those n-tuples $\langle i_1, \ldots i_n\rangle$ that appear in the range of the function $|\varphi\ x_1, \ldots x_n|$ when a time t'' lying to the past of t and mereologically contained in t' is given as input to that function. Let us see what the interpretation of the predicate PASTFROWN x is given our redefinition of $|$PASTφ $x_1, \ldots x_n|$.

$$|\textbf{PASTFROWN}\ \boldsymbol{x}| = f: W \times T \times T \rightarrow \wp(I), \text{ for any } \langle w, t, t'\rangle \in W \times T \times T,$$
$$f(\langle w, t, t'\rangle) = \{i: i \text{ frowns in } w \text{ at some time } t'' \text{ such that}$$
$$t'' < t \text{ and } t'' \subseteq t'\}$$

If we fix the first two argument-positions of $|\textsc{PastFrown}\ x|$ by choosing w_c and t_c, the interpretation of PASTFROWN x relative to w_c and t_c can be seen as a function from evaluation times to sets of individuals.

$$|\textbf{PASTFROWN}\ \boldsymbol{x}|^{w_c, t_c} = f : T \rightarrow \wp(I), \text{ for any } t \in T, f(t) = \{i\text{: } i \text{ frowns in } w_c \text{ at some time } t' \text{ such that } t' < t_c \text{ and } t' \subseteq t\}$$

Let us call $t_{\text{FROWN}(c)}$ the evaluation time that is assigned to FROWN x in the context of utterance c. If, for instance, the speaker of (10) is intuitively talking about what she did on the very day of speech, then $t_{\text{FROWN}(c)}$ is the day of utterance of (10).

$$|\textbf{PASTFROWN}\ \boldsymbol{x}|^{c} = \{i\text{: } i \text{ frowns in } w_c \text{ at some time } t \text{ such that } t < t_c \text{ and } t \subseteq t_{\textbf{FROWN}(c)}\}$$

This is the interpretation of the predicate PASTFROWN x in c. By replacing x with the first-person constant I, we obtain the sentential formula PASTFROWN I, which is the logical form of (10). The truth-conditions of this formula, relative to c, can be easily derived given two standard semantic clauses.

First clause: $|\boldsymbol{I}|^{c} = a_c$

Second clause: If Π is an n-adic predicate and $\alpha_1 \ldots \alpha_n$ are singular terms, $\Pi\alpha_1 \ldots \alpha_n$ is true in c iff $\langle |\alpha_1|^c, \ldots |\alpha_n|^c \rangle \in |\Pi|^c$

From these clauses and our interpretation of PASTFROWN x with respect to c we can infer that PASTFROWN I is true in a context of utterance c iff a_c frowns in w_c during some time t such that $t < t_c$ and $t \subseteq t_{\text{FROWN}(c)}$. In other words, (10) is true in c just in case the speaker of c has frowned in the world of utterance during the contextually determined past interval $t_{\text{FROWN}(c)}$. I take it that this is the right truth-condition for (10).

We can introduce a radical operator for the simple future by slightly modifying our semantic clause for PAST.

$$|\textbf{F}\textbf{UT}\boldsymbol{\varphi}\ \boldsymbol{x_1, \ldots x_n}| = f: W \times T \times T \rightarrow \wp(I^n), \text{ for any } \langle w, t, t' \rangle \in W \times T \times T,$$
$$f(\langle w, t, t' \rangle) = \{\langle i_1, \ldots i_n \rangle: \text{there is a time } t'' \text{ such that } t < t'', t'' \subseteq t', \text{ and } \langle i_1, \ldots i_n \rangle \in |\varphi\ x_1, \ldots x_n|(w, t'')\}$$

The operators PAST and FUT are sensitive to a distinction between the time of utterance and the time of evaluation. In principle, it is possible to define other radical operators that allow for a distinction between the time of utterance, the time of evaluation, and the reference time. As Reichenbach observed, some familiar temporal constructions –e.g. the English past and future perfect– call for such a distinction.

In order to preserve the parallel between tense and modality, let us see how our analysis of (10) can be extended to account for the truth-condition of (7).

(7) My father would kill me

I shall formalize (7) by using sets of possible worlds –as opposed to scenarios/ situations. On the formalization that I want to propose, the modal verb *would* is semantically represented as a radical operator (called WOULD) that modifies the radical KILL x y. To evaluate a sentential formula containing the predicate WOULDKILL x y, a time of evaluation and a set of worlds must be contextually provided. Intuitively, (7) concerns a set of worlds where the speaker buys the guitar amplifier and uses it during a contextually given present time interval. Not every possible world that fits this general description, though, is a member of the set of worlds in question. Only worlds that fit the description and are similar to the world of utterance in certain relevant respects can be included in that set (see Stalnaker 1968 and Lewis 1973). Here are the semantic clauses of WOULD and KILL x y:

$$|\textbf{KILL}\, \boldsymbol{x\ y}| = f: W \times T \rightarrow \wp(I \times I), \text{ for any } \langle w, t \rangle \in W \times T,$$
$$f(\langle w, t \rangle) = \{\langle i, j \rangle: i \text{ kills } j \text{ in } w \text{ during } t\}$$

$$|\textbf{WOULD}\boldsymbol{\varphi}\ \boldsymbol{x_1, \ldots x_n}| = f: \wp(W) \times T \rightarrow \wp(\mathrm{I}^n), \text{ for any } \langle V, t \rangle \in \wp(W) \times T,$$
$$f(\langle V, t \rangle) = \{\langle i_1, \ldots i_n \rangle: \text{for every } w \in V, \text{ there is a time } t' \subseteq t \text{ such that } \langle i_1, \ldots i_n \rangle \in |\varphi\ x_1, \ldots x_n|(w, t')\}$$

$|\text{WOULD}\varphi\ x_1, \ldots x_n|$ is a function mapping set-of-worlds/time pairs to standard predicate extensions –i.e. sets of n-tuples of individuals. $|\text{W}\textsc{ould}\text{K}\textsc{ill}\, x\, y|$ is a function mapping such pairs to sets of pairs of individuals.

$$|\mathbf{W\textsc{ould}K\textsc{ill}}\,\boldsymbol{x}\,\boldsymbol{y}| = f\colon \wp(\mathrm{W}) \times \mathrm{T} \rightarrow \wp(\mathrm{I} \times \mathrm{I}), \text{ for any } \langle V, t\rangle \in \wp(W) \times T,$$
$$f(\langle V, t\rangle) = \{\langle i, j\rangle\text{: for every } w \in V, \text{ there is a time } t' \subseteq t \text{ such that } i \text{ kills } j \text{ in } w \text{ during } t'\}$$

Let $t_{\mathbf{KILL}(c)}$ and $V_{\mathbf{KILL}(c)}$ be the time and set of worlds that serve to interpret (7) in a context of utterance c. $t_{\mathbf{KILL}(c)}$ is a present time interval. $V_{\mathbf{KILL}(c)}$ is a set containing all the worlds relevantly similar to w_c (given a contextual standard of similarity) where a_c plays her guitar with the store amplifier at some time during $t_{\mathbf{KILL}(c)}$. Armed with $t_{\mathbf{KILL}(c)}$ and $V_{\mathbf{KILL}(c)}$, we can compute $|\text{W}\textsc{ould}\text{K}\textsc{ill}\, x\, y|^c$.

$$|\mathbf{W\textsc{ould}K\textsc{ill}}\,\boldsymbol{x}\,\boldsymbol{y}|^c = \{\langle i, j\rangle\text{: for every } w \in V_{\mathbf{KILL}(c)}, \text{ there is a time } t \subseteq t_{\mathbf{KILL}(c)} \text{ such that } i \text{ kills } j \text{ in } w \text{ during } t\}$$

The logical form of (7) is the formula WouldKill $f(I)$ I. I assume that $f(\alpha)$ is a singular-term functor such that $|f(\alpha)|^c$ denotes the father of $|\alpha|^c$. According to our semantics, WouldKill $f(I)$ I is true in a context of utterance c iff, for every $w \in V_{\text{KILL}(c)}$, there is a time $t \subseteq t_{\text{KILL}(c)}$ such that a_c's father kills a_c in w during t. Consequently, (9) is true in c just in case the speaker of c is killed by her father during the present interval $t_{\text{KILL}(c)}$ at any world of $V_{\text{KILL}(c)}$ –which is a set of worlds containing the hypothetical possibilities that are contextually salient in c. This is the right truth-condition for (7).

3.3.2 Anaphoric and bound uses of tenses and modals

In this subsection I am going to formalize examples (4a), (6a), (8) and (9) of sections 3.1 and 3.2. They correspond to the anaphoric and bound uses of *Past* and *would*.

Regarding (4a), I assume that we have two monadic radicals Has-a-party x and Get-drunk x. The logical form of (4a) is (F4a).

(4a) Sheila had a party and Sam got drunk

(F4a) P_{AST} HAS-A-PARTY *Sheila* ∧ PAST GET-DRUNK *Sam*

The analysis of the deictic uses of *Past* proposed in the previous subsection predicts the following truth-conditions for (4a) relative to a context c:

(4a) is true in c iff Sheila has a party in w_c during some time t such that $t < t_c$ and $t \subseteq t_{\text{PARTY}(c)}$, and Sam gets drunk in w_c during some time t' such that $t' < t_c$ and $t' \subseteq t_{\text{DRUNK}(c)}$

The anaphoric reading of (4a) arises when it is assumed that Sam's drunkenness episode occurred during Sheila's party. This amounts to assuming that $t_{\text{DRUNK}(c)} \subseteq t_{\text{PARTY}(c)}$. But this assumption is not encoded in the truth-condition of sentence (4a). Arguably, there are possible contexts of utterance where such assumption should be avoided in order to interpret (4a) correctly. On my account, the choice of a time interval for a radical operator is a matter of pragmatics. The inclusion relations between the past time associated with HAS-A-PARTY x and the past interval associated with GET-DRUNK x may vary depending on the context.

Let us now turn to (6a).

(6a) Whenever Mary smiled, Sam yawned

We have seen how to define the truth-in-context of a sentential formula like PASTFROWN *I*. If we assume that the world and time of utterance are fixed, we can also define a notion of truth with respect to a context and a past time of evaluation. Let us apply this idea to the sentential formulas PASTSMILE *Mary* and PASTYAWN *Sam*.

PASTSMILE *Mary* is true in context c during time t iff Mary smiles in w_c during some time t' such that $t' < t_c$ and $t' \subseteq t$

PASTYAWN *Sam* is true in context c during time t iff Sam yawns in w_c during some time t' such that $t' < t_c$ and $t' \subseteq t$

Let WHENEVER be a dyadic sentential operator defined as follows:

WHENEVER (ϕ, γ) is true in c iff, for every time t that does not exceed a contextual maximal size z_c, if ϕ is true in w_c during t, then γ is true in w_c during t

The truth-in-context for formulas of the form WHENEVER (ϕ, γ) is defined in terms of the notion of truth with respect to a context and a time of evaluation as applied to the embedded sentential formulas ϕ and γ. As in section 3.1, the contextually constrained size is introduced in order guarantee that the event-type described by ϕ and the event-type described by γ are not too distant in time for the standards of the context.

We can now compute the interpretation of (F6a).

(F6a) WHENEVER (PASTSMILE *Mary*, PASTYAWN *Sam*)

WHENEVER (PASTSMILE Mary, PASTYAWN Sam) is true in c iff, for every time t that does not exceed a contextual maximal size z_c and such that $t < t_c$, if Mary smiles in w_c during t, then Sam yawns in w_c during t.

Finally, let us turn our attention to (8) and (9).

(8) If I had a party at my place, my neighbor would call the police

(9) If Mary married one of my brothers, Sarah would envy her

I assume that the function of the antecedents of (8) and (9) is to restrict the set of worlds that is required for the application of the radical operator WOULD. Thus, in (8) the predicate WOULDCALL-THE-POLICE x is evaluated in context c with respect to a set $V_{\mathrm{CALL}(c)}$ containing worlds in which a_c has a party at her place during the present time interval $t_{\mathrm{CALL}(c)}$. Such worlds must also be similar to w_c in the contextually relevant ways. In (9) the predicate WOULDENVY x y is assessed in context c with respect to a set $V_{\mathrm{ENVY}(c)}$ containing worlds in which Mary marries one of a_c's actual brothers. The predicted truth-conditions of (8) and (9), relative to c, appear below:

(8) is true in c iff, for every $w \in V_{\mathrm{CALL}(c)}$, if a_c has a party at a_c's place in w during $t_{\mathrm{HAD\text{-}PARTY}(c)}$, then a_c's neighbor calls the police in w during $t_{\mathrm{CALL}(c)}$.

(9) is true in c iff, for every $w \in V_{\mathrm{ENVY}(c)}$, if Mary marries an actual bother of a_c in w during $t_{\mathrm{MARRY}(c)}$, then Sarah envies Mary in w during $t_{\mathrm{ENVY}(c)}$.

This completes my analysis of the logical forms of examples (4a), (6a), (8) and (9). I conclude that the deictic, anaphoric, and bound uses of English tenses and pronouns can be accounted for by the radical-operator approach to tense and modality that I sketched in this chapter.

4. Embedded Tenses

Embedded tenses pose a challenge to any semantic theory of temporal discourse. In this chapter I propose an intensional account of English embedded tenses. On the account that I will present, the semantic job of a tense is to specify a relation between a perspective time and the time at which an eventuality takes place. By default, the time of utterance is the perspective time that a tense takes as input. But a switch of perspective time can be triggered when a tense appears in certain grammatical environments. I will suggest that intensional verbs and modals are triggers of perspective time shifts.

The account proposed in this chapter can conceivably be implemented using different formal frameworks. I will adopt here an intensional framework. I want to offer a plausible alternative to the various referential theories that have been dominant in the linguistic literature on embedded tenses since the eighties. These theories posit LF-representations containing variable-like constituents that denote time intervals. The formal framework that I am going to adopt also uses syntactic structures that are in line with modern generative syntax. However, while referential theories rely on binding principles to account for the interpretation of embedded tenses, my account relies on intensional time-shifting mechanisms. Although my account is intensional, it dispenses with the assumption that tenses must be represented as sentential operators. I presented an account that that gets rid of this assumption in the previous chapter. The account presented in this chapter is more sophisticated semantically and syntactically.

The behavior of English tenses in embedded positions is quite complex and I will not attempt to account for all the puzzling facts about embedded tenses that have been examined in the linguistic literature. My discussion will be focused on the behavior of the English past tense in the two types of embedded clauses that have received most attention in the literature: complement clauses and relative clauses. I will show that the account proposed in this chapter predicts the different kinds of shifted interpretations that the English past gets in both types of clauses. At the end of the chapter I will suggest that my account is compatible with the postulation of a sequence-of-tense rule for English. This rule accounts for the simultaneous (or overlapping) interpretations of the English past tense in past-under-past environments.

4.1 An intensional semantic framework

This section is devoted to introducing the semantic framework with which I will work in the rest of the chapter. As I said above, I want to propose an account that analyzes English tenses as intensional time-shifting devices. I want to flesh out this idea in such a way that the inputs to semantic interpretation are structures that are well motivated from a syntactic perspective and that can be interpreted with a standard compositional semantics. I will adopt the view that the inputs to semantic interpretation are LF-representations (LFs) of the LF-component of a generative grammar. My assumptions about syntax will be presented in the next section. The semantic framework that I will adopt is a Heim-and-Kratzer-style intensional framework. I am going to assume that the reader is familiar with the λ-notation and the type-driven interpretation rules of Heim and Kratzer.[1] If there are readers who are not quite familiar with the technical semantic machinery employed in this chapter, they are invited to take a look at the appendix to the chapter. There they will find definitions of the key semantic notions that I will be taking for granted as well as formulations of the relevant interpretation rules.

In the semantic framework that I am going to adopt here, LFs are interpreted via lexical entries and interpretation rules which jointly determine intensions. These entries and rules are given in a typed system of semantic domains. In this section I introduce the notions of semantic type and semantic domain, describe the form of the interpretation function of my account, and adopt some assumptions about the denotations of English words of certain categories. I will not add new rules to the stock of interpretation rules of Heim and Kratzer.

Habitual and generic (readings of) English sentences are not analyzed in this chapter. I will focus my attention on episodic sentences. I will adopt the view that eventualities are the truth-makers of episodic sentences.[2] On this view, an episodic sentence is true or false by virtue of the existence of an eventuality with certain characteristics. These characteristics are specified in the truth-condition

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[1] See Heim and Kratzer 1998, esp. pp. 34-39, 43-45, 92-98, 110-115, 184-188, 242-243, and 303-309. The more recent (but yet unpublished) textbook of von Fintel and Heim (2011) discusses in greater detail how the original framework of Heim and Kratzer can be extended to deal with intensional discourse.

[2] The term 'eventuality' was coined by Bach (1981).

of the sentence. I will not make substantive ontological assumptions about the nature of eventualities, but I will assume that events and states are eventualities.

Let me now make some remarks about semantic types and domains. Our basic semantic types will be the types e (the type of individuals), l (the type of eventualities), t (the type of truth-values), and s (the type of world/time coordinates). The building-blocks of semantic domains will be the set of all possible worlds (let us call it W), the set of all moments (let us call it M), the relation of temporal precedence between moments (which I symbolize as $<$), the set of individuals that inhabit worlds/moments (let us call it D), the set of possible eventualities (let us call it E), and the truth-values 1 (truth) and 0 (falsity).[3] Intervals of time are sets of moments with no internal gaps. I will call T the set of all intervals and I will call the elements of T *times*. $<$ is the relation of total temporal precedence between times and $\subseteq$ is the inclusion relation between times.[4] Our basic domains will be the sets D_e (= D), D_t (= $\{1, 0\}$), D_l (= E), and D_s (= $W \times T$). Non-basic types and non-basic domains are recursively specified in the usual way.[5]

I assume that LFs have extensions of type t and singular terms extensions of type e. I also assume that the extensions of verb phrases are of type <l, <e, t>>. In other words, verb phrase extensions are functions which map eventualities to functions that map individuals to truth-values. Intensions will be for us functions mapping pairs of worlds and times to extensions. Thus, the intensions of LFs are of type <s, t>, the intensions of singular terms are of type <s, e>, and the intensions of verb phrases are of type <s, <l, <e, t>>>. I will assume that aspectual features turn verb phrase extensions into extensions of type <e, t>, while tenses and modals turn intensions of type <s, <e, t>> into extensions of type <e, t>.

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[3] In chapter 1 I assumed that the precedence relation of an intended model was a strict linear order. Here I will also assume that $<$ is a strict linear order.

[4] The key semantic notions introduced in this paragraph, including the notion of interval and the relations $<$ and $\subseteq$, are defined in the appendix to this chapter.

[5] For definitions of the notions of time interval, semantic type, and semantic domains, see the appendix to this chapter. Heim and Kratzer (1998, p. 303) treat e and t as the only basic types (see also von Fintel & Heim 2011, p. 10). Although s is not a basic type in their framework, they introduce a rule that generates a type <s, σ> for any given type σ (Eventualities are not mentioned in Heim and Kratzer's definition of a semantic type. But see Kratzer 1998). Even though s and l are basic types in the present framework, e and t have a special status according to the account of tenses proposed in this chapter. Whereas there are English constituents with extensions of type e or of type t, there are no English constituents with extensions or intensions of type s or of type l.

Following Kamp and Reyle (1993, chapter 5), I would like to distinguish four distinct roles that times play in the interpretation of temporal discourse. A time can play the role of a location time, a reference point, a perspective point, or a time of eventuality. A time of eventuality is the time at which the eventuality described by an episodic sentence occurs. In the semantic framework adopted here, a time of eventuality is a time of evaluation in the traditional sense. Location times are intervals that restrict the temporal location of an eventuality. They can be specified by locating adverbs such as *yesterday* or *on Sunday*. They can also remain implicit. In Partee's scenario involving an utterance of *I didn't turn off the stove*, the location time is salient but implicit. Reference times are times that help to determine temporal motion in discourse. When a sentence is uttered in the context of a discourse, its reference time is typically provided by the nearest eventive sentence which precedes that sentence. The sentence is interpreted as describing an eventuality that stands in a certain temporal relation to the current reference time. This relation is usually a relation of posteriority if the sentence is eventive and a relation of overlap if the sentence is stative. The choice of the relevant relation, however, depends on various semantic and pragmatic factors. A perspective time is the temporal point of view from which an eventuality is characterized. The time of utterance of a sentence often plays the role of perspective time. But other times can play this role too. Later on I will suggest that modals and intensional verbs can take a given time of eventuality and turn it into a perspective time.[6] The notion of perspective time will be central to my account of embedded tenses.

I will assume that English syntactic structures are interpreted by a seven-place interpretation function $[\![\ ]\!]^{g, c, w, Rt, Pt, t}$. This functions takes as inputs a phrase structure subtree, an assignment *g*, a context of utterance *c*, a world of evaluation *w*, a reference time *Rt*, a perspective time *Pt*, and a time of evaluation *t*. Its output is the extension of the given phrase structure subtree with respect to *g*, *c*, *w*, *Rt*, *Pt*, and *t*. I omit the location-time-parameter for the sake of simplicity. The other temporal roles mentioned in the previous paragraph are represented as arguments of the interpretation function. Assignments are by definition partial

[6] Kamp and Reyle (1993, pp. 593-596) argue that the roles of reference time and perspective time have to be distinguished by considering extended flashbacks. For a discussion of the role of perspective times in indirect discourse, see Altshuler 2008.

injective functions from the set of natural numbers to D.[7] I will assume that each context determines the values of various parameters, including the parameters of world of utterance, time of utterance, and assignment of the context. If c is a context, I will call w_c the world of utterance of c, u_c the time of utterance of c, and g_c the assignment of c. The assignment of a context assigns an element of D to every numerical index that occurs in the LF that is being interpreted. If the sentence whose LF we are interpreting is the first sentence of a discourse, the computation of its truth-condition starts at an index coordinate in which u_c is both the perspective time and the time of evaluation. I will presuppose that the interpretation function $[\![\ ]\!]^{g,\, c,\, w,\, Rt,\, Pt,\, t}$ is computed using standard rules such as Functional Application, Predicate Abstraction, and Intensional Functional Application. These rules are formulated in the appendix to this chapter.

Let me now make a few assumptions about the lexical meanings of English words. To begin with, I assume that proper names are context-insensitive rigid designators and that personal pronouns are context-sensitive rigid designators. The extension of a proper name is the individual that the name denotes. Every occurrence of a personal pronoun bears a numerical subscript. The extension of a personal pronoun is the object assigned to its numerical index by the current assignment (see the rule of Terminal Nodes in the appendix). If a pronoun is bound by a proper name, they must bear the same numerical index.

As I mentioned above, I assume that the extensions of verb phrases are of type <l, <e, t>>. The extensions of intransitive verbs are also of type <l, <e, t>>. Transitive verbs have extensions of type <e, <l, <e, t>>>. Below I provide the lexical entries of an activity verb, a stative verb, an accomplishment verb, and an achievement verb. The first verb is intransitive and the other three verbs are transitive.

Activity verb (*run*)

$\lambda e \in D_{\mathsf{l}}\,.\,[\lambda x \in D_{\mathsf{e}}\,.\,e$ is an event of x running$]$

Stative verb (*love*)

$\lambda x \in D_{\mathsf{e}}\,.\,[\lambda e \in D_{\mathsf{l}}\,.\,[\lambda y \in D_{\mathsf{e}}\,.\,e$ is a state of y loving $x]]$

[7] It follows from this definition that the empty set is an assignment. A partial function from the set of natural numbers to D is a function from some subset of the set of natural numbers to D. When $\varnothing$ is the chosen subset of natural numbers, the resulting partial function is a subset of $\varnothing \times D$ and it is easy to show that $\varnothing \times D = \varnothing$.

Accomplishment verb (*build*)

$\lambda e \in D_I$. [$\lambda x \in D_e$. [$\lambda y \in D_e$. e is an event of y building x]]

Achievement verb (*recognize*)

$\lambda e \in D_I$. [$\lambda x \in D_I$. [$\lambda y \in D_e$. e is an event of y recognizing x]]

Notice that these lexical entries do not specify the world and time in which the described eventualities take place. Nor do they give information about whether the eventuality is bounded (i.e. whether it has an initial and a final endpoint) or not. On my account, the job of aspectual and tense features is to specify certain conditions that an eventuality has to satisfy in order to make an uttered sentence true. These conditions concern the location and the internal constitution of the eventuality.

I assume that there is a grammatical feature that distinguishes stative verbs from the other verb groups. While stative verbs bear the feature value [+stative], other verbs bear the value [−stative]. The values [±stative] are associated with different temporal behaviors. Expressions carrying the value [−stative] tend to move the action forward (or backward) in narrative discourse and never give rise to simultaneous readings. The eventualities that they describe are conceptualized as punctual. By contrast, expressions carrying the value [+stative] give rise to simultaneous readings and normally report what goes on at a given reference time without moving the narrative action backward or forward. They describe eventualities that we conceptualize as extended.

In Part 1 I adopted a Hintikka-style treatment of intensional verbs which take *that*-clause complements. Here I preserve the essentials of that treatment. By way of illustration, the lexical entries of the stative verb *believe* and the eventive verb *say* are given below.

Lexical entry of *believe*

$\lambda p \in D_{<s,t>}$ [$\lambda e \in D_I$. [$\lambda x \in D_e$. e is a state of x believing such that $p(\langle w', t' \rangle) = 1$, for every pair $\langle w', t' \rangle \in W \times T$ such that $\langle w', t' \rangle$ is compatible with the content of e]]

Lexical entry of *say*

$\lambda p \in D_{<s,t>}$ [$\lambda e \in D_I$. [$\lambda x \in D_e$. e is an event of x saying such that

p ($\langle w', t' \rangle$) = 1, for every pair $\langle w', t' \rangle \in W \times T$ such that $\langle w', t' \rangle$ is compatible with the content of e]]

As Enç (1986) and other theorists have shown, common nouns that appear in determiner phrases exhibit forms of temporal sensitivity that are independent of the tenses of verbs.[8] I will not investigate here the context sensitivity of nouns. My account of tenses will not make predictions about the temporal interpretation of determiner phrases containing common nouns or adjectives. Nonetheless, I will assume that when a noun or an adjective is the complement of the verb *be*, the corresponding verb phrase has an extension of type <l, <e, t>> and describes a state. So, for example, the extension of the verb phrase *be sick* is the function $\lambda e \in D_l$. [$\lambda x \in D_e$. e is a state of x being sick]] and the extension of the verb phrase *be king* is the function $\lambda e \in D_l$. [$\lambda x \in D_e$. e is a state of x being king]]

4.2 A syntactic framework

Let us turn our attention to syntax. In this chapter I do not discuss sentences containing non-finite clauses. My discussion in the chapter will be focused on the interpretation of declarative finite clauses. The present section is devoted to introducing a framework for the syntactic representation of finite clauses. I will assume that the LFs of English finite clauses are binary and endocentric phrases which are subject to standard movement operations. Since some readers might not be sufficiently familiar with this kind of syntactic structures, in subsection 4.2.1 I offer a brief overview of the syntactic literature on the structure of finite clauses. Those readers who are familiar with this literature can skip subsection 4.2.1. In subsection 4.2.2 I present the main syntactic assumptions that I will make in this chapter.

Roughly speaking, a finite clause is a clause with subject-predicate structure whose syntactically highest verb form is finite. In English, finite verb forms can have nominative subjects and typically exhibit inflections associated with tense and agreement.[9] Whereas non-finite clauses are always embedded under other

[8] For some discussion of the temporal sensitivity of nouns, see Enç 1981, 1986, Musan 1995, and Tonhauser 2002, 2006.

[9] For a more detailed characterization of the distinction between finite and non-finite clauses, see e.g. Radford 1988, section 6.2.

clauses, finite clauses can occur embedded or unembedded. Let us briefly take a look at an unembedded finite clause. (1) is a sentence in the simple present with no embedded tenses.

(1) John loves Mary

This sentence has three surface constituents: the finite verb form *loves* and the names *John* and *Mary*. The verb suffix -*s* is an indicator that the tense associated with (1) is the present tense. In English, this suffix surfaces only when a verb form is in the third person singular and has present tense. In other grammatical environments, the present form of an English verb is simply its base form. The past forms of English regular verbs exhibit an overt indicator of the past tense, namely the suffix -(*e*)*d*. But the past forms of many irregular verbs lack an overt affix associated with the past tense. Linguists commonly assume that any finite verb form of English has a tense, regardless of whether that form exhibits or not an overt tense affix. One way of implementing this idea in a theory of syntax is to hold that at the LF-level the tense of a finite verb form is specified by a tense morpheme which may not be visible at the surface level. Since English employs periphrasis rather than inflection to deal with futurity, it is commonly assumed that English has two tense morphemes. I will call them *past* and *pres*. If *pres* is a functional constituent of (1)'s LF-structure, the question arises as to where *pres* is located at LF and how it is syntactically related to the verb *love*. Different answers to this question have been proposed throughout the history of generative grammar. In subsection 4.2.1 I briefly describe some of the developments which led generative linguists to propose the syntactic treatment of finite clauses that I will adopt here.[10] Due to space limitations, I will not talk about the empirical observations which motivated these theoretical developments. Though I will not reconstruct any syntactic theory in much detail, the interested reader will find references to the relevant literature in my overview.

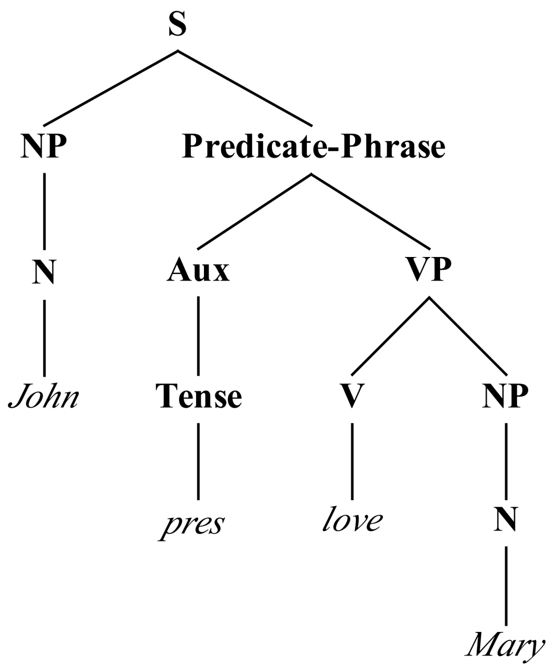
4.2.1 The syntax of tense morphemes in generative grammar

Chomsky's (1957) seminal theory of the auxiliary system of English provided a transformational account of inflectional verb affixation. The theory hypothesized

[10] See Stowell 2012 and Lasnik & Lohndal 2013 for more detailed surveys of these theoretical developments.

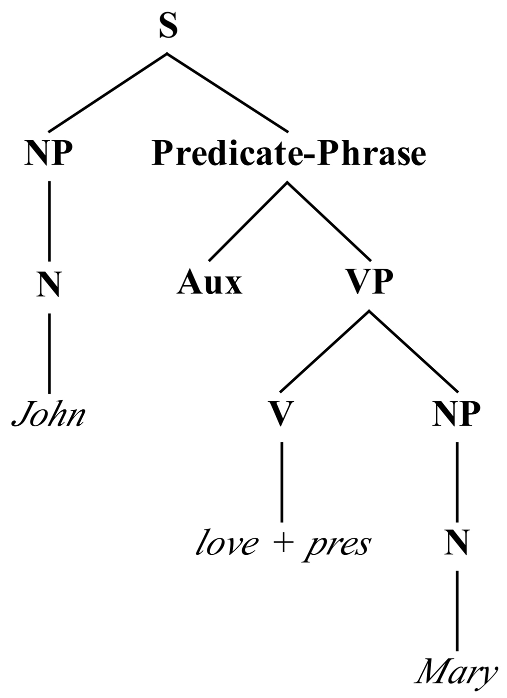
that the inflectional affixes of English verbs originated in pre-verbal syntactic positions and were moved to the post-verbal positions that we observe in surface syntax by means of a transformational rule. The syntactic rule responsible for this transformation came to be known as 'Affix Hopping'. On the so-called 'Standard Theory' of generative grammar (Chomsky 1965), sentence (1) had a deep structure along the lines of (2).

(2)



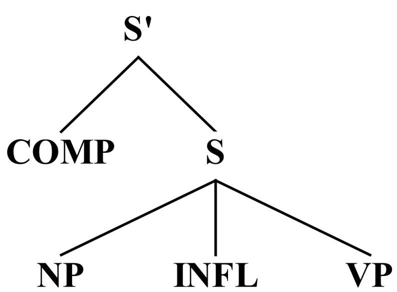
Affix Hopping acted upon deep structures of this sort and transformed them into representations exhibiting post-verbal affixes. The diagram below represents the result of applying Affix Hopping to (2).

(3)



With the advent of Government & Binding Theory (Chomsky 1981), a syntactic element called 'INFL' was postulated at the LF-level. INFL's job was to indicate whether a clause was finite or infinitival and to specify the agreement properties of finite clauses. Chomsky assumed that complement clauses and non-embedded clauses had LF-structures of the following form:

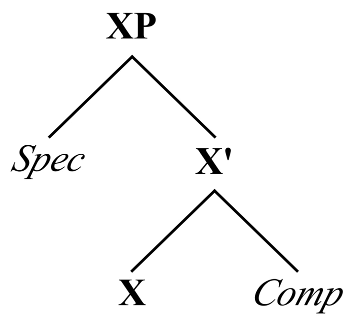
(4)



On this proposal, the two immediate constituent of a complement clause such as *that John loves Mary* (whose syntactic category was S') were a complementizer (e.g. *that*) and a propositional component (i.e. a clause of category S). NP, INFL, and VP were the immediate constituents of the propositional component. INFL had the value [+tense] when it appeared in a finite clause and had the value [−tense] when it appeared in an infinitival clause. In finite clauses, INFL also carried values for the agreement features of person, number, and gender.

The view that complement clauses had the syntactic structure depicted in (4) was quickly abandoned. An important step in the development of Government & Binding Theory was the extension of X-bar Theory to the syntactic categories S and S'. X-bar Theory had been originally invoked to analyze the structure of English phrases headed by nouns, verbs, and adjectives (see Chomsky 1970). During the seventies and early eighties, the theory was extended to phrases of other categories and it eventually became a general theory of phrase structure. The principles of X-bar Theory generate syntactic structures of the following form:

(5)



In this schema, *X* stands for the syntactic category of the head of a phrase. *XP* is the maximal projection of the head and *X'* is its intermediate projection.[11] While the specifier of the head (*Spec*) is a daughter of *XP* and a sister of *X'*, the head's complement (*Comp*) is a daughter *X'*.[12] (For simplicity, non-branching intermediate projections will be sometimes omitted in my phrase structure trees). The

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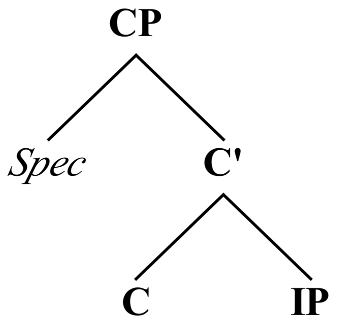
[11] X-bar Theory was so-called because the projections *XP* and *X'* were originally labeled using overbars. I employ here the notation of Chomsky 1986 and later works. Accessible introductions to X-bar Theory can be found in Radford 1988, chapters 4-5, Newson 2006, chapter 3, section 1, and Santorini & Kroch 2007, chapters 4-5.

[12] In English, complements typically follow the head and specifiers typically precede it. Consider, for instance, the noun phrase *the author of that book*. According to X-bar Theory, this phrase has the structure [$_{NP}$ [$_{Det}$ the] [$_{N'}$ [$_{N}$ author] [$_{PP}$ of that book]]]. Here *author* is the head, *the* is the specifier, and *of that book* is the complement. Of course, there are phrases which do not exhibit overt specifiers and complements. Some X-bar theorists assume that every phrase has the form [$_{XP}$ *Spec* [$_{X'}$ [$_{X}$ *head*] *Comp*]] and that the terminal nodes of the specifier and complement positions may be empty. Others assume that phrases can have alternative forms such as [$_{XP}$ [$_{X'}$ [$_{X}$ *head*]]], [$_{XP}$ *Spec* [$_{X'}$ [$_{X}$ *head*]]], or [$_{XP}$ [$_{X'}$ [$_{X}$ *head*] *Comp*]]. I will adopt the latter view in the present chapter.

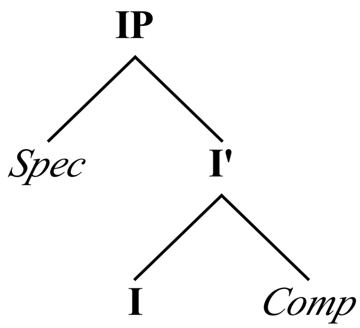
X-bar Theory schema (5) is instantiated by structures that are binary-branching and endocentric (i.e. have a syntactic head). By contrast, some of the structures proposed in Chomsky 1981 and earlier works were exocentric (lacked a head) and had three or more branches.

In order to apply the X-Bar Theory schema to the internal structure of phrases of category S, Stowell (1981, chapter 2) –drawing on Chomsky 1981, chapter 5– proposed an analysis of S-structure where INFL was the head of S, VP was the complement of INFL, and NP was its specifier. He also proposed an analysis of S’ structure where COMP (the complementizer) was the heads of S’ (see Stowell 1981, chapter 6). The old categories S’ and S were then relabeled in conformity with the X-Bar Theory schema. Phrases of category S’ became complementizer phrases (CPs) headed by a complementizer and phrases of category S became inflection phrases (IPs) headed by an inflectional element. (6) and (7) represent the basic structures of CPs and IPs.

(6)



(7)



Pollock (1989) proposed to split IP structures into a tense phrase (TP) headed by a tense morpheme and an agreement phrase (AgrP) carrying agreement features. Although Pollock’s postulation of an AgrP at the LF-level was later rejected, the assumption that the LFs of finite clauses are TPs has been widely adopted in the syntactic literature. In the last two decades, syntacticians have proposed to split CPs, TPs, and VPs in different ways. I will not review these proposals here.[13] For the purposes of this chapter, it suffices to mention that some analyses of the structure of TPs posit an aspect phrase (AspP) between TP and VP. This is an assumption that I adopt in the next subsection.

4.2.2 Syntactic assumptions

I am now in position to introduce the main assumptions that I will make in this chapter regarding the syntax of finite clauses in English.

To begin with, I will assume that the LF-structures of English finite clauses conform to the structural constraints of X-bar Theory. More specifically, I will assume that the LFs of such clauses are either tense phrases (TPs) or modal phrases (MPs) with X-bar style internal structures. If the highest verb of a finite clause is a lexical verb or a non-modal auxiliary, the clause is represented as a TP headed by *pres* or *past*. If the highest verb of the clause is a modal auxiliary, the clause is represented as a MP headed by the corresponding modal.

My account of tenses posits a tense feature with values [±past]. By default, *pres* carries the value [−past] and *past* carries the value [+past].

I will also assume that an aspect phrase (AspP) is always projected above the VP of a finite clause. The head of an AspP carries the features [±perfective] and [±episodic].[14] Auxiliary phrases (AuxPs) and negation phrases (NegPs) can be optionally projected between a VP and a TP/MP. Agreement phrases (AgrPs) will not be represented in our LFs. Finally, I will assume that a determiner phrase (DPs) in object position can be moved out of its associated VP, thereby giving rise to wide scope readings. This movement operation is the result of applying the rule of quantifying raising.

Given the previous syntactic assumptions, (8) is the LF of (1).

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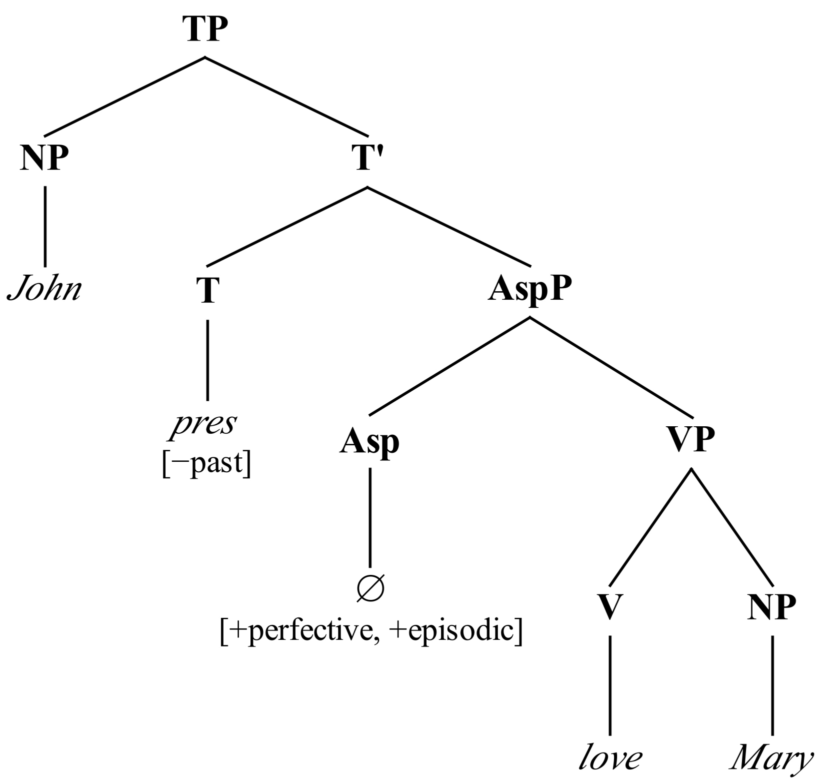
[13] For an overview of split theories, see Radford 2009, chapter 8.

[14] Asp heads may also carry the feature [±perfect]. But I omit this feature because perfects are not discussed in this chapter.

(8)



I will not adopt here the so-called ‘VP-internal subject hypothesis’. According to this hypothesis, subject NPs and subject DPs originate as the specifiers of VPs. They are obligatorily moved out of their original positions to become the specifiers of TPs/MPs. If we adopted the VP-internal subject hypothesis, we could analyze tenses as having denotations of type <<s, t>, t>. Denotations of this type are functions from propositions to truth-values. Instead, we are going to analyze tenses as having denotations of type <<s, <e, t>>, <e, t>>. On this account, the job of tenses is to map properties of individuals to functions from individuals to truth-values.

4.3 Embedded occurrences of *past*

We have now a semantic framework and a stock of syntactic assumptions that will allow us to specify possible interpretations for the English tenses and obtain truth-conditional predictions. As I mentioned above, I am going to assume that the denotations of the English tenses are functions of type <<s, <e, t>>, <e, t>>. Which functions of this semantic type do *pres*, *past*, and *will* denote? First of all, it is important to keep in mind that our intensional framework is quite flexible. There are various possible ways of interpreting the simple tenses of English that are compatible with this framework. In this section I consider three strategies for interpreting the simple tenses of English. The first two strategies analyze these tenses as absolute or relative. The third strategy makes tenses sensitive to the perspective time and assumes that perspective times can be switched in some syntactic environments. I argue that this strategy overcomes the shortcomings of the other two strategies.

I begin by discussing the behavior of *past* in relative and complement clauses. As it is well known, in languages like English embedded stative predicates give rise to simultaneous readings in *past*-under-*past* environments. Simultaneous readings will be considered in section 4.4. In this subsection I focus my attention on cases where embedded *past* carries a meaning of temporal anteriority. The examples that I consider in this section involve eventive embedded predicates.

In the literature on embedded tense, it is usual to distinguish between absolute and relative accounts of tenses.[15] Roughly, an account of *past* as an absolute tense attributes to *past* the lexical meaning TE (time of eventuality) $<$ TU (time of utterance). This type of account assumes that *past* is by default anchored to the time of utterance of a sentence. As we will see shortly, there are types of embedded clauses in which *past* intuitively does not mean TE $<$ TU.[16] An account of *past* as a relative tense attributes to *past* the lexical meaning TE (time of eventuality) $<$ LTE (local time of evaluation). The local time of evaluation of an embedded occurrence of *past* is the time of evaluation that it inherits from the clause in which it is embedded. In the cases that we are going to consider here, a relative account predicts that an embedded *past* is anchored to the time of the matrix eventuality. However, there are types of embedded clauses in which *past* is not intuitively interpreted as meaning TEE (time of eventuality of the embedded clause) $<$ TME (time of the matrix eventuality). The challenge for relative accounts of *past* is to find principles that allow us to predict the right readings in

[15] The distinction is based on Comrie's (1985) analysis of absolute and relative tenses.

[16] For a succinct discussion of the problems of absolute and relative theories of tenses, see Gennari 2003, pp. 35-43. Absolute theories are criticized in von Stechow 1995, pp. 363-366, Ogihara 1996, pp. 181-187, Abusch 1997, and Kusumoto 1999, pp. 20-22 and 30-31.

these problematic cases without abandoning the assumption that the meaning of *past* is TE $<$ LTE. The distinction between absolute and relative accounts can also be drawn with respect to other tenses. There are, for example, absolute and relative accounts of *pres* as well as absolute and relative accounts of *will*. Each family of theories has to deal with problematic uses of the relevant tenses which do not seem to conform to the basic meaning attributed to the tense.

In section 4.1 I characterized the notion of perspective time and I disguised it from other roles which times can play in temporal discourse. Now I want to put forward the hypothesis that the lexical meaning of *past* is TE $<$ PT (perspective time). In other words, *past* imposes the condition that the time of eventuality must precede the temporal point of view from which the eventuality is seen.

For any assignment g, context c, world w, and times Rt, Pt, and t,

(9) **Lexical entry of *past***

$$[\![past_{\ [+\text{past}]}]\!]^{g,\, c,\, w,\, Rt,\, Pt,\, t} = \lambda v \in D_{<\text{s}, <\text{e}, \text{t}>>} .\ [\lambda x \in \text{D} .\ \text{for some } t' \in T \text{ such that } t' < Pt,\ v\,(\langle w, t'\rangle)\,(x) = 1]$$

In order to make predictions about the interpretation of an embedded occurrence of *past*, we need lexical entries for the other tenses. I will provisionally assume that PT $\subseteq$ TE is the lexical meaning of *pres* and that PT $<$ TE is the lexical meaning of *will*. Here are the lexical entries of *pres* and *will*:

For any assignment g, context c, world w, and times Rt, Pt, and t,

(10) **Lexical entry of *pres***

$$[\![pres_{\ [-\text{past}]}]\!]^{g,\, c,\, w,\, Rt,\, Pt,\, t} = \lambda v \in D_{<\text{s}, <\text{e}, \text{t}>>} .\ [\lambda x \in \text{D} .\ \text{for some } t' \in T \text{ such that } Pt \subseteq t',\ v\,(\langle w, t'\rangle)\,(x) = 1]$$

(11) **Lexical entry of *will***

$$[\![will]\!]^{g,\, c,\, w,\, Rt,\, Pt,\, t} = \lambda v \in D_{<\text{s}, <\text{e}, \text{t}>>} .\ [\lambda x \in \text{D} .\ \text{for some } t' \in T \text{ such that } Pt < t',\ v\,(\langle w, t'\rangle)\,(x) = 1]$$

Aspectual features turn functions of semantic type <l, <e, t>> into functions of type <e, t>. AspPs are of type <<l, <e, t>>, <e, t>>. Rules (12)-(14) specify the interpretations of the values [±perfective] and [+episodic]. I will assume that [±perfective] values are computed before [±episodic] values.

For any node α, assignment g, context c, world w, and times Rt, Pt, and t,

(12) **Perfective aspect**

$[\![\alpha_{\text{[+perfective]}}]\!]^{g, c, w, Rt, Pt, t} = \lambda u \in D_{<\mathrm{l}, <\mathrm{e}, \mathrm{t}>>} . [\lambda e \in \mathrm{D_l} . [\lambda x \in D . e$ is bounded and $u(e)(x) = 1]]$

(13) **Imperfective aspect**

$[\![\alpha_{\text{[−perfective]}}]\!]^{g, c, w, Rt, Pt, t} = \lambda u \in D_{<\mathrm{l}, <\mathrm{e}, \mathrm{t}>>} . [\lambda e \in \mathrm{D_l} . [\lambda x \in D . e$ is open and $u(e)(x) = 1]]$

(14) **Episodic aspect**

$[\![\alpha_{\text{[+episodic]}}]\!]^{g, c, w, Rt, Pt, t} = \lambda u \in D_{<\mathrm{l}, <\mathrm{e}, \mathrm{t}>>} . [\lambda x \in D .$ there is an eventuality e such that $u(e)(x) = 1]$

According to rules (12)-(14), the perfective aspect characterizes the eventuality described by a given VP as a bounded (or completed) eventuality –that is to say, as an eventuality that has both an initial and a final endpoint. The imperfective aspect characterizes it as an open eventuality in the sense that it may or may not have endpoints. Rules (12)-(14) are based on Smith's (1997) analysis of English aspect. The episodic aspect introduces an existential quantifier which saturates the eventuality argument, thereby yielding a function of type <e, t>.

Let us apply the rules (9)-(11) to some examples. I will focus my attention on examples involving embedded occurrences of *past*.

According to my account, the time of utterance is, by default, the perspective time with respect to which tenses are interpreted. But the perspective time can be shifted when appropriate triggers are present in the structure of the sentence. Extensional verbs are not perspective shifters. Intensional verbs, by contrast, trigger a perspective shift. This explains the contrast between (15) and (16).[17]

[17] (16) is an example of Ogihara (1996, p. 169).

(I call TEE the time of eventuality of the embedded clause and TME the time of eventuality of the matrix clause. TU is the time of utterance.)

(15) John met a person who saw *Schindler's List*

Unique reading: TME $<$ TU & TEE $<$ TU

(16) John sought a person who saw *Schindler's List*

de re reading: TME $<$ TU & TEE $<$ TU

de dicto reading: TEE $<$ TME $<$ TU

Relative theories of *past* predict that (15) has a reading of type TEE $<$ TME. On this (alleged) reading of (15), the event of seeing *Schindler's List* is anterior to the event of meeting the relevant person. Since it is clear that (15) can be true in a scenario in which John met the person before the time in which the person saw *Schindler's List*, relative theorists must explain how a reading compatible with this scenario is possible.[18] I claim that (15) has only one reading, even though this reading is compatible with different scenarios that make (15) true. The unique reading of (15) is TME $<$ TU & TEE $<$ TU. This is a reading in which the two events are characterized a past events (prior to TU) but no relative order between them is imposed. My account predicts this reading for sentences like (15). Since there are no perspective-shift triggers in (15), the perspective time of both the matrix *past* and the embedded *past* of (16) is the time of utterance. So, when lexical entry (9) is applied, the two occurrences of *past* of (15) take us to a time anterior to TU.

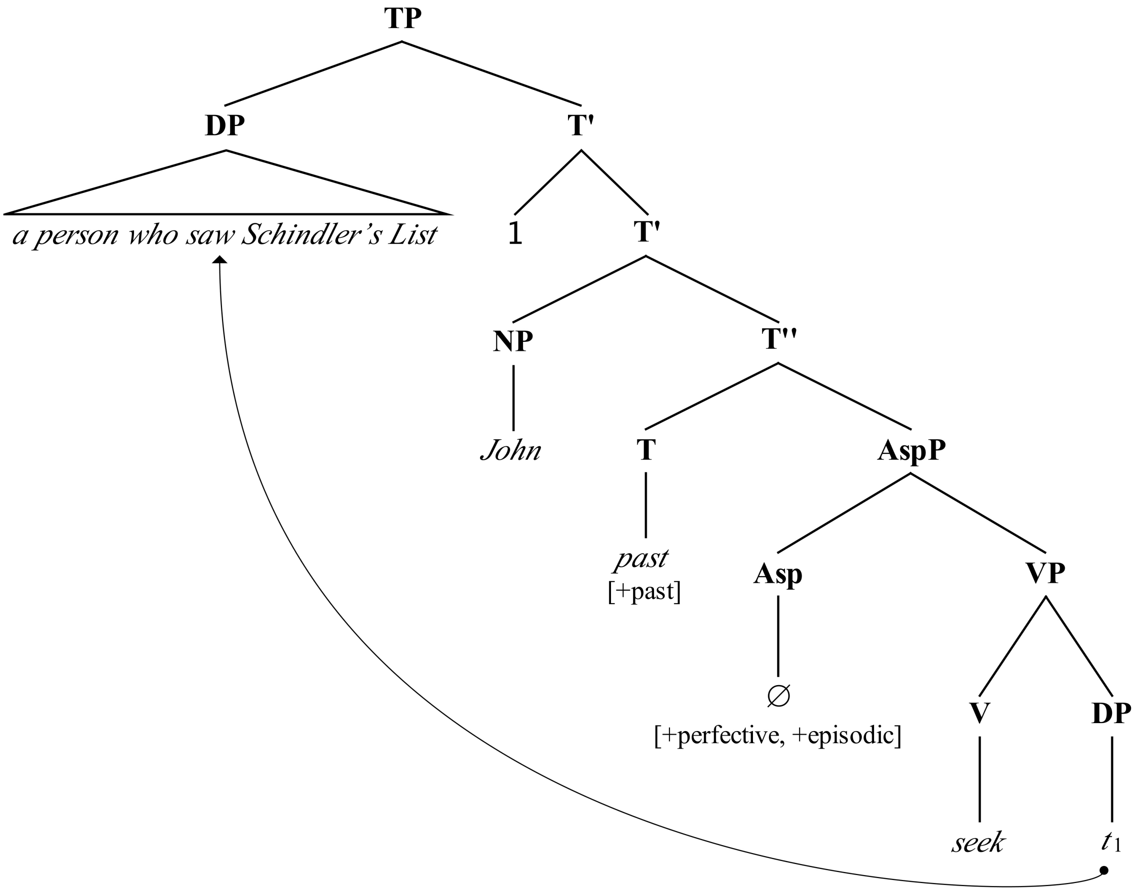
Suppose now that in (16) the intensional verb *seek* switches the perspective time of its grammatical object. Specifically, suppose that *seek* turns TME –the time of seeking in the case of (16)– into the perspective time of the relative clause. When we apply lexical entry (9) to the embedded *past* of (16), TEE –the time of seeing *Schindler's List*– is located before the current perspective time, which by assumption is TME. As a result, we obtain the *de dicto* reading of (16),

[18] In order to get the reading in which the two event times are independent, some relative theorists invoke the rule of quantifier raising (see Ogihara 1996, chapter 5 and Stowell 2007). But Kusumoto (2005, section 2.2) shows that this strategy does not work when more complex relative clauses with *past*-under-*past* structure are considered.

which is the reading on which the predicate *person who saw Schindler's List* characterizes the content of John's seeking action. Under this reading, John was seeking a person who had seen *Schindler's List* before John's present. The worlds compatible with the content of John's seeking action are worlds in which John finds a person who saw *Schindler's List* before John's seeking.

(16) has also a *de re* reading. On this reading, the predicate *person who saw Schindler's List* simply describes the person that John was seeking and it is not assumed that John was thinking about this person under the description given by the predicate. The standard way of accounting for *de re* readings in sentences with relative clauses is to assume that the rule of quantifier raising acts on the LFs of such sentences before the interpretation. I will endorse this explanation of the *de re* reading of (16). By virtue of the quantifier-raising rule, the objet DP *a person who saw Schindler's List* is moved out of the scope of the matrix *past*. The result of this operation is LF (17).

(17)



Since in (17) the object DP has been moved out of the scope of the verb *seek*, this verb does not shift the perspective time of the *past* morpheme of that DP.[19] (17) thus receives an interpretation where TEE is prior to TU but not necessarily precedes TME.

The view that intensional verbs are perspective-time shifters also allows us to explain why sentences like (18) have only a backward-shifted reading.

(18) John said that Bill lost his wallet

backward-shifted reading: TEE $<$ TME $<$ TU

Sentence (18) lacks a reading where TEE is anterior to TU but posterior to TME. Explaining why this reading is absent is a crucial challenge for absolute theories of *past*. The embedded clause of (18) is a complement clause. I assume that in clauses of this kind, the embedded tense is always under scope of the matrix verb. The complement clause of (18) cannot be moved out of the scope of the verb *say*. Consequently, this verb obligatorily triggers a perspective switch that makes TME –the time of saying– the perspective time of the complement clause. When (9) is applied to the *past* of the complement clause, this clause is interpreted as describing an event of loosing a wallet which happens before the time of John's saying (TEE). Thus, (18) only has a backward-shifted reading.

In brief, if we adopt some reasonable syntactic assumptions, the account of *past* that I have proposed here explains the behavior of *past* in past-under-past complement and relative clauses with eventive verbs. In a sentence like (15), the embedded *past* seems to behave as an absolute tense. The eventuality described by the relative clause precedes the time of utterance but it does not necessarily precede the time of the matrix eventuality. In sentences like (16) and (18), the embedded *past* seems to behave as a relative tense. (16) and (18) have readings in which the time of eventuality of the embedded clause is anterior to the time of the matrix eventuality. If we assume that *past* means TE $<$ PT, the hypothesis that intensional verbs switch the perspective time explains this crucial difference between sentences like (15) and sentences like (16) and (18).

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[19] Kusumoto (2005, pp. 330-331) acknowledges that QR-movement can account for *de re* / *de dicto* readings in cases like (16), even though QR-movement fails as an explanation of the truth-conditions of sentences like (15).

One problem for absolute theories of *past* that has been documented in the literature is that the English past can have backwards-shifted interpretations when it is embedded under *will*. Below I provide four examples borrowed from the literature.[20]

(19) Bill will tell you that Mary's exam went well

(20) No matter what you give him to eat, he will eat it and tell you that he liked it

(21) Sue will marry a man she met recently

(22) We will answer every letter that we got

It has been observed that not all English speakers like these sentences. For some English speakers, however, these sentences have a backwards-shifted reading of type TU $<$ TME & TEE $<$ TME and not only a reading of type TU $<$ TME & TEE $<$ TU. It is generally agreed in the literature that the existence of the first type of reading (for some speakers) must be explained somehow. As it turns turns out, my account of *past* predicts the backwards-shifted reading of (19) and (20). The matrix verbs of these two sentences are intensional verbs. Thus, they make TME the time of evaluation of the embedded clause. The application of lexical entry (9) to the embedded clause has the effect of imposing the condition that TEE $<$ TME, but the condition that TEE $<$ TU is not imposed. This is a nice result. However, my account does not predict a reading of type TU $<$ TME & TEE $<$ TU for (19) and (20). Accounting for this kind of reading is important because it is the most salient one (or the only one) for some speakers. In fact, I suspect that the peculiarity of sentences (19)-(22) stems from the fact that the two readings compete with each other to be processed.

Let me propose a modification of my account of *past*. Let us suppose that English has two tense features, which I will call [±past] and [±past i] (the i stands for *indexical*). The feature value [+past] is interpreted as in (9), but we will not think of (9) anymore as the lexical entry of *past*. (9) simply specifies the

[20] See Abusch 1997, p. 37, Kratzer 1998, p. 16, Kusumoto 1999, p. 22, and Gennari 2003, p. 55.

interpretation of [+past]. The feature value [+past i] is interpreted as meaning TE (time of eventuality) $<$ TU. (23) is the interpretation rule of [+past i].

For any assignment g, context c, world w, and times Rt, Pt, and t,

(23) **Rule for [+past i]**

$$[\![past_{\,[+past]}]\!]^{g,\, c,\, w,\, Rt,\, Pt,\, t} = \lambda v \in D_{<s,\, <e,\, t>>} \,.\, [\lambda x \in D \,.\, \text{for some } t' \in T \text{ such that } t' < u_c,\ v(\langle w, t'\rangle)(x) = 1]$$

[+past i] imposes the condition that the time of eventuality must precede the time of utterance. Suppose that *past* carries by default the two feature values [+past] and [+past i]. The intuitive idea is that the semantic contribution of *past* consists in relating the time of eventuality to two possibly different times: the time of utterance and the perspective time. When the perspective time is identical to the time of utterance, *past* simply imposes the condition that TE $<$ TU. But in an intensional environment –where the perspective time is different from the time of utterance– *past* imposes the condition that TE $<$ TU & TE $<$ PT. That is to say, *past* locates the time of eventuality before the time of utterance and before the present of the agent of an attitude report or a speech act report. This hypothesis does not affect my account of past-under-past sentences such as (16) and (18). But it predicts that (19) and (20) have the non-backwards-shifted reading TU $<$ TME & TEE $<$ TU. For the feature value [+past i] of *past* imposes the condition that TEE $<$ TU.

To obtain the backwards-shifted reading of (19)-(20) in our new account, let us assume that *will* optionally deletes the feature value [+past i] of *past*. When *will* deletes the value of [+past i] of a past tense under its scope, *past* $_{[+past]\ [+past\ i]}$ becomes *past* $_{[+past]}$ and the clause embedded under *will* is interpreted using only the condition that [+past] carries, namely the condition that TP $<$ TE. Since in the context of an intensional verb clause TP = TME and TE = TEE, this amounts to saying that *past* $_{[+past]}$ imposes the condition that TEE $<$ TME, which is the condition that generates the backwards-shifted reading of (19)-(20).

We do not have yet an account of the backwards-shifted reading of (21) and (22). Since these sentences do not have an intensional verb, the perspective time of their embedded clauses is the time of utterance. Therefore, *past* $_{[+past]\ [+past\ i]}$

has the same interpretation as *past* $_{[+past]}$ in the relative clauses of (21) and (22). The deletion operation of *will* has no semantic effect. To fix this problem, let us assume that *will* is also a trigger of perspective-time switch. *Will* turns the future time that it introduces into the perspective time of its embedded clause. Thus, the perspective time of the relative clauses of (21) and (22) is TME –the time of marrying and the time of answering the letter. When *will* deletes the feature value [+past i] from *past* $_{[+past]\,[+past\ i]}$ in the embedded clauses of (21) and (22), *past* $_{[+past]}$ imposes the condition that TEE $<$ TME, thereby yielding the backwards-shifted reading of (21) and (22). When *will* does not delete [+past i] from *past* $_{[+past]\,[+past\ i]}$ in the embedded clauses of (21) and (22), *past* $_{[+past]\,[+past\ i]}$ imposes the condition that TEE $<$ TU –and also the condition that TEE $<$ TME, but this condition is entailed by the first one given that in (21) and (22) TU $<$ TME. So, the non-application of [+past i] deletion by *will* gives rise to the non-backwards-shifted reading of (21) and (22).

(The fact that (19) and (20) contain two triggers of perspective-time switches is not problematic. The two switch operations have the result that TME is the perspective time of the complement clauses of (19)-(20).)

The modified account that I have proposed predicts the readings of all the sentences that we have considered so far. The reader might feel that I have made too many assumptions in order to get the right readings. But let me point out that the empirical consequences of the account are quite interesting.

The account can be extended to *pres*. Suppose that *pres* carries by default the two feature values [−past] and [−past i], where [−past] introduces the condition that PT $\subseteq$ TE and [−past i] introduces the condition that TU $\subseteq$ TE.

Consider now sentence (24).

(24) John saw a man who is crying

Since (22) does not have elements that trigger a perspective-time switch, the perspective time of the relative clause of (24) is the utterance time. Thus, (24) has a reading in which the time of utterance must be included in the interval of time in which the man is crying (TU $\subseteq$ TEE), but the time of seeing (TME) does not need to be included in the time of the crying.

By contrast, (25) and (26) have intensional verbs that switch the perspective time.

(25) John looked for a student who understands the incompleteness theorem

(26) John said that Mary is pregnant

The intensional verbs of (25) and (26) make TME the perspective time of the embedded clauses. The feature value [−past] of *pres* $_{[-\text{past}]\ [-\text{past i}]}$ imposes the condition that TME ⊆ TEE, while the feature value [−past i] imposes the condition that TU ⊆ TEE. As a result, the eventuality described by the embedded clause is represented as an eventuality that extends from TME to TU (in the worlds compatible with what John looked for/said). Hence, my account predicts the so-called double-access readings of sentences (25) and (26).[21]

Sentences (27) and (28) are ambiguous.

(27) I will use an iron that is hot

(28) Gianni will say (next week) that Maria is dancing well

[21] I believe that the accounts of double-access sentences that have been proposed in the literature (or at least the ones I am familiar with) are not satisfactory. Abusch (1991, 1994, 1997) and Ogihara (1995a, 1996, 1999) proposed *de re* analyses of double-access readings. Gennari (1999, pp. 95-97, 2003, pp. 42-43) has persuasively argued that such *de re* analyses are inadequate because they impose requirements on the *res* of a double-access sentence that do not need to be satisfied in every scenario in which that sentence is felicitously uttered. The accounts of double-access sentences of authors such as Enç (1987), Stowell (1995, 2007), and Higginbotham (2002) lack an explicit semantics for intensional verbs. For this reason, it is hard to judge whether those accounts predict the right truth-conditions for double-access sentences. The accounts proposed by Gennari (1999, 2003) and by Altshuler and Schwarzschild (2013) characterize the English present as combining a relative requirement –that the described state of affairs overlaps with the local evaluation time– and a deictic requirement –that the described state of affairs is not wholly located before the speech time. These accounts predict that the sentence *John said that Mary is pregnant* is true just in case all the worlds compatible with what John says at some past time are worlds where Mary is pregnant at the time of John's saying and is still pregnant at the time of utterance. This is not the right truth-condition for the sentence. If a week ago John asserted the sentence *Mary is pregnant*, then the sentence *John said that Mary is pregnant* is true today. However, there are worlds compatible with what John said in which Mary's pregnancy is interrupted at some point between the time of John's saying and the present day. Thus, the accounts of double-access sentences of Gennari and Altshuler and Schwarzschild predict that *John said that Mary is pregnant* is false today. My account makes the same erroneous prediction. To avoid this problem, the relevant set of compatible worlds must be restricted in an appropriate manner.

(27) and (28) have a reading of type TU $<$ TME & TME $\subseteq$ TEE (in which the eventuality of the embedded clause occurs in the future). But they also have a reading of type TU $<$ TME & TME $\subseteq$ TEE & TU $\subseteq$ TEE (in which the embedded-clause eventuality extends from the present to the future time of the matrix clause). I suggest that the optional rule of deletion of [+past i] by *will* discussed earlier is a rule that also acts on [−past i]. When this rule is applied to delete [−past i], $pres_{\text{[−past] [−past i]}}$ becomes $pres_{\text{[−past]}}$ and the embedded clause is interpreted as TME $\subseteq$ TEE. When the rule is not applied, the embedded clause is interpreted as TME $\subseteq$ TEE & TU $\subseteq$ TEE.

[−past i] deletion can be found in other present-under-modal constructions.

(29) John should talk to whoever is guarding the entrance

(30) Mary may say that she is in charge

(29) and (30) also have a reading of type TU $<$ TME & TME $\subseteq$ TEE and a reading of type TU $<$ TME & TME $\subseteq$ TEE & TU $\subseteq$ TEE. If we assume that modals like *should* and *may* share with *will* the property of being triggers of perspective-time shifts and of optional [−past i] deletion, we can account for both readings of (29) and (30) using the same basic principles that we applied to account for (27)-(28) and (19)-(22).

We can apply part of the modified account of this section to the analysis of *will*. *will* can be analyzed as a tense that also establishes a relation between the time of evaluation, the time of utterance, and the perspective time. In particular, we can analyze *will* as meaning TU $<$ TE & TP $<$ TE.

(31) **Modified lexical entry of *will***

$$[\![will]\!]^{g,\, c,\, w,\, Rt,\, Pt,\, t} = \lambda v \in D_{<s,\, <e,\, t>>} \,.\, [\lambda x \in D \,.\, \text{for some } t' \in T \text{ such that } Pt < t' \text{ and } u_c < t',\, v(\langle w, t'\rangle)(x) = 1]$$

The principle of feature deletion cannot be applied to embedded occurrences of *will* because *will* always introduces a time of evaluation that is posterior to the time of utterance. But lexical entry (31) is useful for explaining the behavior of *will* in sentences like (32) and (33).

(32) In two days, an official will announce that the president will apologize

(33) Two days ago, an official announced that the president will apologize

The complement clause of (32) is interpreted in such a way that the apology event is posterior to the announcement event. On my account, this reading arises because the matrix *will* switches the perspective time and the embedded *will* takes the new perspective time –the time of the announcement– and shifts it to a time posterior to it. *will* shifts the perspective time in this way because one component of its meaning is TP $<$ TE. In (33), the complement clause describes an event that is posterior to the time of utterance, despite the fact that the embedded *will* is under the scope of a perspective-switching verb. This reading is predicted in my account because the other component of the meaning of *will* is TU $<$ TE. Without one of the two components, we would fail to account of one these two sentences. Therefore, the two components are necessary in order to explain the behavior of *will* in complement clauses.[22]

4.4 Simultaneous readings

Sentence (34), like many other English sentences with stative complements, is ambiguous.

(34) John heard that Mary was pregnant

(34) has a reading in which the content of John's act of hearing represents Mary as being pregnant at the time of the hearing. This is the simultaneous reading of (34), which is the reading in which (34) is true if John heard someone uttering the sentence *Mary is pregnant*. (34) also has a reading in which the content of John's hearing represents Mary as being pregnant before the time of the hearing.

[22] My examples of embedded *pres* and embedded *will* were also borrowed from the literature. See Heim 1994, p. 158, Enç 1987, p. 647, Enç 1997, p. 6, Ogihara 1996, p. 97, Higginbotham 2002, p. 225, and Gennari 2003, p. 37.

This is the so-called shifted reading of (34). On this reading, (34) is true if John heard someone asserting the sentence *Mary was pregnant*.

(35) is an example of a sentence with a relative clause that has also a simultaneous reading.

(35) John saw a man who was crying

There is a reading of (35) in which the time of seeing and the time of crying overlap.

The morpheme *past* can receive a simultaneous interpretation even in cases in which the time of eventuality of the matrix clause lies to the future of the time of utterance.[23] (36) is an example due to Abusch (1988) that has been widely cited in the literature.

(36) John decided a week ago that in ten days at breakfast he will say to his mother that they were having their last meal together

The embedded predicate *were having their last meal together* intuitively refers to an event that is simultaneous with the saying event. The meal event cannot be anterior to the time of utterance. In fact, no event posterior to the meal event is mentioned in (35).

The account of embedded tenses proposed in the previous section predicts the shifted reading of (34). Since *hear* is an intensional verb, the perspective time of the complement clause of (34) is the time of John's hearing. The complement *past* of (34), which carries the feature value [+past], shifts the time of time of John's hearing to a time anterior to it. As a consequence, the worlds compatible with John's hearing are represented as worlds where Mary is pregnant before the time of the hearing. The shifted reading of (34) is thus predicted.

However, the account of section 4.3 does not predict the simultaneous reading of sentences like (34)-(36). I believe that the account of tenses developed in this chapter is compatible with different accounts of simultaneous readings that have been proposed in the literature. A pragmatic-oriented account of these readings along the lines of Gennari's (2003) theory, for example, is compatible with the account of the previous section. The sequence-of-tense rule of Ogihara (1996) is

[23] See e.g. Abusch 1988, pp. 2-3, Heim 1994, pp. 159, and Ogihara 1995, pp. 676-678, 1996, pp. 6-9, 91-93.

compatible with it as well. For concreteness, I adopt here a variant of Ogihara's rule. Here is a formulation of the sequence-of-tense rule for English based on the framework of this chapter.

SOT rule

If an occurrence α of *past* is c-commanded by another occurrence of *past*, the two feature values [+past] and [+past i] of α can be optionally deleted.

This rules accounts for the simultaneous readings of (34)-(35). In the three cases the *past* of the matrix clause c-commands the *past* of the embedded clause. SOT deletes the values [+past] and [+past i] of the embedded *past*. In the type of intensional framework that we have employed in this chapter, a featureless *past* simply transmits to its VP the time of evaluation that it receives from the *past* that c-commands it. So, Mary's pregnancy is interpreted in (34) as obtaining at the time of John's hearing. Similarly, in (35) the crying event interpreted as obtaining at the time of John's seeing and in (35) the meal is interpreted as taking place at the time of the saying event. Thus, SOT yields the right truth-conditions for sentences (34)-(36).

One problem of Ogihara's sequence-of-tense-rule approach is that SOT does not explain why eventive embedded clauses do not have simultaneous readings. SOT is a structural rule that applies to sentences with eventive and stative clauses. But the application of the rule to eventive embedded clauses with past tense has no observable semantic effects. Although this problem suggests that SOT does not tell us the whole story about the genesis of simultaneous readings, I will end my discussion here. In this section I just wanted to suggest that the account of embedded tenses given in 4.3 is compatible with Ogihara's sequence-of-tense approach.

Appendix to chapter 4

Semantic types and domains

The semantic framework of chapter 4 uses a typed system of semantic domains. The basic types of this systems are e (the type of individuals), t (the type of truth-values), l (the type of eventualities), and s (the type of world/time pairs).

Semantic types are recursively defined by the following rules:

- e, t, s, and l are semantic types
- If σ and τ are semantic types, then <σ, τ> is a semantic type
- Nothing else is a semantic type

Let W be the set of all possible worlds. Let M be the set of all moments. Let us call D the set of all individuals that inhabit worlds/moments. If we assume that individuals exist at world/moment coordinates, D can be defined in terms of W and M in the following way:

D := {x: there is some $w \in W$ and there is some $m \in M$ such that x exists in w at m}

Similarly, if we assume that eventualities hold (or 'go on') at worlds/moments, the set of possible eventualities (let us call it E) can be defined in the following way:

E := {e: there is some $w \in W$ and there is some time $m \in M$ such that e holds in w at m}

Let < be the relation of temporal precedence between moments. A subset t of M is a time –i.e. a time interval– just in case, for any moments m, $m' \in t$, if there is some moment m'' such that $m < m'' < m'$, then $m'' \in t$. Let T be the set of all times. Finally, let us assume that 1 and 0 are, respectively, the truth-values truth and falsity.

Semantic types are in one-to-one correspondence with semantic domains. The class of semantic domains is recursively defined as follows:

- $D_e = D$, $D_t = \{1, 0\}$, $D_l = E$, and $D_s = W \times T$
- If σ and τ are semantic types, then $D_{<\sigma, \tau>}$ is the set of all functions from D_σ to D_τ

Relations between times

Different relations between times can be defined in terms of the moments that they contain. For any times t and t',

Temporal precedence
$t < t'$ iff, for every $m \in t$ and every $m' \in t'$, $m < m'$

Temporal inclusion
$t \subseteq t'$ iff, for every $m \in t$, $m \in t'$

Overlap
$t \bigcirc t'$ iff there is some $m \in M$ such that $m \in t$ and $m \in t'$

Interpretation rules

Some interpretation rules are required in order to compute the truth-conditions of the English sentences discussed in chapter 4. Here I state the main rules that were presupposed throughout the chapter.

Recall that an assignment is a partial and injective function from the set of natural numbers to D. If g is an assignment, let us call $dom(g)$ the domain of g.

For any assignment g, context c, world w, times Rt, Pt, and t, and numerical index i,

Functional application (FA)
If α is a branching node with daughters β and γ such that β's

denotation is of type <σ, τ> and γ's denotation is of type σ, then $[\![\alpha]\!]^{g, c, w, Rt, Pt, t} = [\![\beta]\!]^{g, c, w, Rt, Pt, t}$ $([\![\gamma]\!]^{g, c, w, Rt, Pt, t})$

Intensional functional application (IFA)

If α is a branching node with daughters β and γ such that β's denotation is of type <s, σ> and γ's denotation is of type σ, then $[\![\alpha]\!]^{g, c, w, Rt, Pt, t} = [\![\beta]\!]^{g, c, w, Rt, Pt, t}$ $(\lambda\langle w', t'\rangle . [\![\gamma]\!]^{g, c, w', Rt, Pt, t'})$

Predicate abstraction (PA)

If α is a branching node with daughters *i* and β, then $[\![\alpha]\!]^{g, c, w', Rt, Pt, t'} = \lambda x . [\![\beta]\!]^{g[x/i], c, w', Rt, Pt, t'}$

Non-branching nodes (NN)

If α is a non-branching node and β is its daughter, then $[\![\alpha]\!]^{g, c, w, Rt, Pt, t} = [\![\beta]\!]^{g, c, w, Rt, Pt, t}$

Terminal nodes (TN)

- If α is a terminal node occupied by a lexical item, then $[\![\alpha]\!]^{g, c, w, Rt, Pt, t}$ is specified in the lexicon.
- If α_i is a terminal node, α is a pronoun or a trace, and $i \in dom(g)$, then $[\![\alpha_i]\!]^{g, c, w, Rt, Pt, t} = g(i)$.
- If α is a terminal node occupied by a non-indexed pronoun, then $[\![\alpha_i]\!]^{g, c, w, Rt, Pt, t}$ is an element of *D* determined by *c*

Two Derivations

Consider sentence (1) (which is similar to sentence (28) of chapter 4).

(1) Gianni will say that Maria is writing a book

In order to illustrate how the formal apparatus of chapter 4 works, here I will provide truth-condition derivations of the two readings of (1).

Let c be the context in which (1) is uttered. The two readings of (1) can be specified as follows:

First reading of (1)

For some $t \in T$ such that $u_c < t$, there is a bounded eventuality e such that e takes place in w_c at t and e is an event of Gianni saying (something) such that, for any world w' compatible with the content of e, there is some $t' \in T$ such that $t \subseteq t'$, $u_c \subseteq t'$, and there is an open eventuality e' such that e' takes place in w' at t' and e' is an event of Maria writing a book

Schematic representation: TU $<$ TME & TME $\subseteq$ TEE & TU $\subseteq$ TEE

Second reading of (1)

For some $t \in T$ such that $u_c < t$, there is a bounded eventuality e such that e takes place in w_c at t and e is an event of Gianni saying such that, for any world w' compatible with the content of e, there is some $t' \in T$ such that $t \subseteq t'$ and there is an open eventuality e' such that e' takes place in w' at t' and e' is an event of Maria writing a book

Schematic representation: TU $<$ TME & TME $\subseteq$ TEE

To make my derivations more reader-friendly, let me give explicit formulations of the rules and principles of chapter 4 that will be relevant for the derivations. Some of my reformulations of these rules and principles will slightly differ from the formulations given in chapter 4. For the sake of simplicity, I will ignore the assignment parameter and the reference-time parameter of the interpretation function $[\![\ ]\!]^{g, c, w, Rt, Pt, t}$.[1]

Interpretation rules

Let c be a context, w be a world of evaluation, Pt be a perspective time, and t be a time of evaluation.

Functional application (FA)
If α is a branching node with daughters β and γ such that β's denotation is of type $<\sigma, \tau>$ and γ's denotation is of type σ, then $[\![\alpha]\!]^{c, w, Pt, t} = [\![\beta]\!]^{c, w, Pt, t}\ ([\![\gamma]\!]^{c, w, Pt, t})$

Intensional functional application (IFA)
If α is a branching node with daughters β and γ such that β's denotation is of type $<\mathrm{s}, \sigma>$ and γ's denotation is of type σ, then
$[\![\alpha]\!]^{c, w, Pt, t} = [\![\beta]\!]^{c, w, Pt, t}\ (\lambda\langle w', t'\rangle \in D_{\mathrm{s}}\ .\ [\![\gamma]\!]^{c, w', Pt, t'})$

Non-branching nodes (NN)
If α is a non-branching node and β is its daughter, then $[\![\alpha]\!]^{c, w, Pt, t} = [\![\beta]\!]^{c, w, Pt, t}$

Lexical entries

$[\![\textbf{Gianni}]\!]^{c, w, Pt, t} = \text{Gianni}$

[1] The reference-time parameter does not play a significant role in the account of embedded tenses of chapter 4. The default time-shifting principle for reference times must be a principle that turns the time of eventuality introduced by an eventive clause into the new reference time. (Stative clauses normally do not introduce new reference times.) A principle of this sort is required for interpreting non-discourse-initial sentences. The different English sentences that we discussed in chapter 4, however, can be thought of as discourse-initial. For this reason, the reference-time parameter is not essential for the analysis of such sentences, even though it must be central to a general account of temporal discourse.

$[\![\textbf{\textit{will}}]\!]^{c,\, w,\, Pt,\, t} = \lambda v \in D_{<s,\, <e,\, t>>}$. $[\lambda x \in D$. for some $t' \in T$ such that $Pt < t'$ and $u_c < t'$, $v\,(\langle w, t'\rangle)\,(x) = 1]$

$[\![\textbf{\textit{say}}]\!]^{c,\, w,\, Pt,\, t} = \lambda p \in D_{<s,\, t>}$ $[\lambda e \in D_l$. $[\lambda x \in D$. e is an event of x saying such that, for any world w' compatible with the content of e, $p\,(\langle w', t\rangle) = 1]]$

$[\![\textbf{Maria}]\!]^{c,\, w,\, Pt,\, t} = \text{Maria}$

$[\![\textbf{write a book}]\!]^{c,\, w,\, Pt,\, t} = \lambda e \in D_l$. $[\lambda x \in D$. e is an event of x writing a book$]$

I assume that ***that*** and ***be*** are semantically vacuous and leave the interpretation function unchanged.

Interpretations of feature values

If α is a terminal node,

$[\![\alpha_{[+\text{perfective}]\,[+\text{episodic}]}]\!]^{c,\, w,\, Pt,\, t} = \lambda u \in D_{<l,\, <e,\, t>>}$. $[\lambda x \in D$. for some bounded eventuality e such that e LOC $\langle w, t\rangle$, $u\,(e)\,(x) = 1]$

$[\![\alpha_{[-\text{perfective}]\,[+\text{episodic}]}]\!]^{c,\, w,\, Pt,\, t} = \lambda u \in D_{<l,\, <e,\, t>>}$. $[\lambda x \in D$. for some open eventuality e such that e LOC $\langle w, t\rangle$, $u\,(e)\,(x) = 1]$

$[\![\alpha_{[-\text{past}]\,[-\text{past i}]}]\!]^{c,\, w,\, Pt,\, t} = \lambda v \in D_{<s,\, <e,\, t>>}$. $[\lambda x \in D$. for some $t' \in T$ such that $Pt \subseteq t'$ and $u_c \subseteq t'$, $v\,(\langle w, t'\rangle)\,(x) = 1]$

$[\![\alpha_{\,[-\text{past}]}]\!]^{c,\, w,\, Pt,\, t} = \lambda v \in D_{<s,\, <e,\, t>>}$. $[\lambda x \in D$. for some $t' \in T$ such that $Pt \subseteq t'$, $v\,(\langle w, t'\rangle)\,(x) = 1]$

Intuitively, 'e LOC $\langle w, t\rangle$' means that the eventuality e is located at the world/time coordinate $\langle w, t\rangle$. If e is a bounded eventuality (an eventuality with initial and final endpoints), e LOC $\langle w, t\rangle$ just in case e occurs in w during t and t is the smallest time that includes the initial and final endpoints of *e*. If *e* is an open eventuality (an eventuality without endpoints), *e* LOC $\langle w, t\rangle$ just in case *e* occurs in *w* during *t* and some part of (the preparatory phase of) *e* holds at each moment of *t*. I will paraphrase the predicate 'LOC' by saying that *e takes place in w at t*.

Theoretical principles

The account of tenses proposed in chapter 4 relies on the following principles:

Principle 1

If α is the syntactic complement of a modal or an intensional verb, then $[\![\alpha]\!]^{c, w, Pt, t} = [\![\alpha]\!]^{c, w, t, t}$

(Modals and intensional verbs trigger perspective-time shifts)

Principle 2

If α is c-commanded by a modal and α carries the feature value [−past i] or the feature value [+past i], [−past i]/[+past i] can be optionally deleted

(Modals optionally trigger indexical-tense-feature deletion)

Given the syntactic assumptions of chapter 4, (LF 1) is the (default) LF of (1).(Here I will use bracket notation to specify LF-representations.)

(LF 1) $[_{\mathbf{MP}}$ $[_{\mathbf{NP}}$ **Gianni**$]$ $[_{\mathbf{M'}}$ $[_{\mathbf{M}}$ **will**$]$ $[_{\mathbf{AspP}}$ $[_{\mathbf{Asp}}$ $\varnothing_{\text{[+perfective] [+episodic]}}]$ $[_{\mathbf{VP}}$ $[_{\mathbf{V}}$ **say**$]$ $[_{\mathbf{CP}}$ $[_{\mathbf{C}}$ **that**$]$ $[_{\mathbf{TP}}$ $[_{\mathbf{NP}}$ **Maria**$]$ $[_{\mathbf{T'}}$ $[_{\mathbf{T}}$ ***pres***$_{\text{[−past] [−past i]}}]$ $[_{\mathbf{AuxP}}$ $[_{\mathbf{Aux}}$ **be**$]$ $[_{\mathbf{AspP}}$ $[_{\mathbf{Asp}}$ **-ing**$_{\text{[−perfective] [+episodic]}}]$ $[_{\mathbf{VP}}$ **write a book**$]]]]]]]]]]]$

By virtue of Principle 2, (LF 1) can be transformed into (LF 1').

(LF 1') $[_{\mathbf{MP}}$ $[_{\mathbf{NP}}$ **Gianni**$]$ $[_{\mathbf{M'}}$ $[_{\mathbf{M}}$ **will**$]$ $[_{\mathbf{AspP}}$ $[_{\mathbf{Asp}}$ $\varnothing_{\text{[+perfective] [+episodic]}}]$ $[_{\mathbf{VP}}$ $[_{\mathbf{V}}$ **say**$]$ $[_{\mathbf{CP}}$ $[_{\mathbf{C}}$ **that**$]$ $[_{\mathbf{TP}}$ $[_{\mathbf{NP}}$ **Maria**$]$ $[_{\mathbf{T'}}$ $[_{\mathbf{T}}$ ***pres***$_{\text{[−past]}}]$ $[_{\mathbf{AuxP}}$

[$_{\textbf{Aux}}$ **be**] [$_{\textbf{AspP}}$ [$_{\textbf{Asp}}$ **-ing**$_{[-\text{perfective}]\ [+\text{episodic}]}$] [$_{\textbf{VP}}$ **write a book**]]]]]]]]]]

My account of embedded tenses predicts that (LF 1) and (LF 1') are the two possible LFs of (1). Suppose that (1) is uttered in a context *c*. Suppose also that no discourse occurs before the utterance of (1) and that there is no location time or reference time that is salient in *c*. I will assume that under these conditions an LF ϕ of (1) counts as true in *c* just in case $[\![\ \ ]\!]^{c,\, w_c,\, u_c,\, u_c}$ yields the truth-value 1 when its syntactic input is ϕ.

Truth-condition derivations

Derivation of the first reading of (1)

[$_{\textbf{MP}}$ [$_{\textbf{NP}}$ **Gianni**] [$_{\textbf{M'}}$ [$_{\textbf{M}}$ **will**] [$_{\textbf{AspP}}$ [$_{\textbf{Asp}}$ $\varnothing_{[+\text{perfective}]\ [+\text{episodic}]}$] [$_{\textbf{VP}}$ [$_{\textbf{V}}$ **say**] [$_{\textbf{CP}}$ [$_{\textbf{C}}$ **that**] [$_{\textbf{TP}}$ [$_{\textbf{NP}}$ **Maria**] [$_{\textbf{T'}}$ [$_{\textbf{T}}$ ***pres***$_{[-\text{past}]\ [-\text{past } i]}$] [$_{\textbf{AuxP}}$ [$_{\textbf{Aux}}$ **be**] [$_{\textbf{AspP}}$ [$_{\textbf{Asp}}$ **-ing**$_{[-\text{perfective}]\ [+\text{episodic}]}$] [$_{\textbf{VP}}$ **write a book**]]]]]]]]]] IS TRUE IN *c* iff …

1. $[\![$[$_{\textbf{MP}}$ [$_{\textbf{NP}}$ **Gianni**] [$_{\textbf{M'}}$ [$_{\textbf{M}}$ **will**] [$_{\textbf{AspP}}$ [$_{\textbf{Asp}}$ $\varnothing_{[+\text{perfective}]\ [+\text{episodic}]}$] [$_{\textbf{VP}}$ [$_{\textbf{V}}$ **say**] [$_{\textbf{CP}}$ [$_{\textbf{C}}$ **that**] [$_{\textbf{TP}}$ [$_{\textbf{NP}}$ **Maria**] [$_{\textbf{T'}}$ [$_{\textbf{T}}$ ***pres***$_{[-\text{past}]\ [-\text{past } i]}$] [$_{\textbf{AuxP}}$ [$_{\textbf{Aux}}$ **be**] [$_{\textbf{AspP}}$ [$_{\textbf{Asp}}$ **-ing**$_{[-\text{perfective}]\ [+\text{episodic}]}$] [$_{\textbf{VP}}$ **write a book**]]]]]]]]]]$]\!]^{c,\, w_c,\, u_c,\, u_c}$ = 1

2. $[\![$[$_{\textbf{M'}}$ [$_{\textbf{M}}$ **will**] [$_{\textbf{AspP}}$ [$_{\textbf{Asp}}$ $\varnothing_{[+\text{perfective}]\ [+\text{episodic}]}$] [$_{\textbf{VP}}$ [$_{\textbf{V}}$ **say**] [$_{\textbf{CP}}$ [$_{\textbf{C}}$ **that**] [$_{\textbf{TP}}$ [$_{\textbf{NP}}$ **Maria**] [$_{\textbf{T'}}$ [$_{\textbf{T}}$ ***pres***$_{[-\text{past}]\ [-\text{past } i]}$] [$_{\textbf{AuxP}}$ [$_{\textbf{Aux}}$ **be**] [$_{\textbf{AspP}}$ [$_{\textbf{Asp}}$ **-ing**$_{[-\text{perfective}]\ [+\text{episodic}]}$] [$_{\textbf{VP}}$ **write a book**]]]]]]]]]$]\!]^{c,\, w_c,\, u_c,\, u_c}$ ($[\![$[$_{\textbf{NP}}$ **Gianni**]$]\!]^{c,\, w_c,\, u_c,\, u_c}$) = 1

   by FA

3. $[\![$[$_{\textbf{M'}}$ [$_{\textbf{M}}$ **will**] [$_{\textbf{AspP}}$ [$_{\textbf{Asp}}$ $\varnothing_{[+\text{perfective}]\ [+\text{episodic}]}$] [$_{\textbf{VP}}$ [$_{\textbf{V}}$ **say**] [$_{\textbf{CP}}$ [$_{\textbf{C}}$ **that**] [$_{\textbf{TP}}$ [$_{\textbf{NP}}$ **Maria**] [$_{\textbf{T'}}$ [$_{\textbf{T}}$ ***pres***$_{[-\text{past}]\ [-\text{past } i]}$] [$_{\textbf{AuxP}}$ [$_{\textbf{Aux}}$ **be**] [$_{\textbf{AspP}}$ [$_{\textbf{Asp}}$ **-ing**$_{[-\text{perfective}]\ [+\text{episodic}]}$] [$_{\textbf{VP}}$ **write a book**]]]]]]]]]$]\!]^{c,\, w_c,\, u_c,\, u_c}$ (Gianni) = 1

   by NN and the lexical entry of **Gianni**

4. $[\![[_{\mathrm{M}}\ \textbf{will}]]\!]^{c,\, w_c,\, u_c,\, u_c}\ (\lambda\langle w', t'\rangle \in D_s\ .\ [\![[_{\mathrm{AspP}}\ [_{\mathrm{Asp}}\ \varnothing_{[+\text{perfective}]\ [+\text{episodic}]}]\ [_{\mathrm{VP}}\ [_{\mathrm{V}}\ \textbf{say}]\ [_{\mathrm{CP}}\ [_{\mathrm{C}}\ \textbf{that}]\ [_{\mathrm{TP}}\ [_{\mathrm{NP}}\ \textbf{Maria}]\ [_{\mathrm{T'}}\ [_{\mathrm{T}}\ \textbf{\textit{pres}}_{[-\text{past}]\ [-\text{past i}]}]\ [_{\mathrm{AuxP}}\ [_{\mathrm{Aux}}\ \textbf{be}]\ [_{\mathrm{AspP}}\ [_{\mathrm{Asp}}\ \textbf{-ing}_{[-\text{perfective}]\ [+\text{episodic}]}]\ [_{\mathrm{VP}}\ \textbf{write a book}]]]]]]]]]\!]^{c,\, w',\, u_c,\, t'})\ (\text{Gianni}) = 1$

by IFA

5. $[\lambda v \in D_{<s,\, <e,\, t>>}\ .\ [\lambda x \in D\ .\ \text{for some } t \in T \text{ such that } u_c < t,\ v\,(\langle w_c, t\rangle)\,(x) = 1]]$ $(\lambda\langle w', t'\rangle \in D_s\ .\ [\![[_{\mathrm{AspP}}\ [_{\mathrm{Asp}}\ \varnothing_{[+\text{perfective}]\ [+\text{episodic}]}]\ [_{\mathrm{VP}}\ [_{\mathrm{V}}\ \textbf{say}]\ [_{\mathrm{CP}}\ [_{\mathrm{C}}\ \textbf{that}]\ [_{\mathrm{TP}}\ [_{\mathrm{NP}}\ \textbf{Maria}]\ [_{\mathrm{T'}}\ [_{\mathrm{T}}\ \textbf{\textit{pres}}_{[-\text{past}]\ [-\text{past i}]}]\ [_{\mathrm{AuxP}}\ [_{\mathrm{Aux}}\ \textbf{be}]\ [_{\mathrm{AspP}}\ [_{\mathrm{Asp}}\ \textbf{-ing}_{[-\text{perfective}]\ [+\text{episodic}]}]\ [_{\mathrm{VP}}\ \textbf{write a book}]]]]]]]]]\!]^{c,\, w',\, u_c,\, t'})\ (\text{Gianni}) = 1$

by NN and the lexical entry of **will**

6. $[\lambda x \in D\ .\ \text{for some } t \in T \text{ such that } u_c < t,\ [\![[_{\mathrm{AspP}}\ [_{\mathrm{Asp}}\ \varnothing_{[+\text{perfective}]\ [+\text{episodic}]}]\ [_{\mathrm{VP}}\ [_{\mathrm{V}}\ \textbf{say}]\ [_{\mathrm{CP}}\ [_{\mathrm{C}}\ \textbf{that}]\ [_{\mathrm{TP}}\ [_{\mathrm{NP}}\ \textbf{Maria}]\ [_{\mathrm{T'}}\ [_{\mathrm{T}}\ \textbf{\textit{pres}}_{[-\text{past}]\ [-\text{past i}]}]\ [_{\mathrm{AuxP}}\ [_{\mathrm{Aux}}\ \textbf{be}]\ [_{\mathrm{AspP}}\ [_{\mathrm{Asp}}\ \textbf{-ing}_{[-\text{perfective}]\ [+\text{episodic}]}]\ [_{\mathrm{VP}}\ \textbf{write a book}]]]]]]]]]\!]^{c,\, w_c,\, u_c,\, t}\ (x) = 1]$ $(\text{Gianni}) = 1$

by λ-notation

7. $[\lambda x \in D\ .\ \text{for some } t \in T \text{ such that } u_c < t,\ [\![[_{\mathrm{AspP}}\ [_{\mathrm{Asp}}\ \varnothing_{[+\text{perfective}]\ [+\text{episodic}]}]\ [_{\mathrm{VP}}\ [_{\mathrm{V}}\ \textbf{say}]\ [_{\mathrm{CP}}\ [_{\mathrm{C}}\ \textbf{that}]\ [_{\mathrm{TP}}\ [_{\mathrm{NP}}\ \textbf{Maria}]\ [_{\mathrm{T'}}\ [_{\mathrm{T}}\ \textbf{\textit{pres}}_{[-\text{past}]\ [-\text{past i}]}]\ [_{\mathrm{AuxP}}\ [_{\mathrm{Aux}}\ \textbf{be}]\ [_{\mathrm{AspP}}\ [_{\mathrm{Asp}}\ \textbf{-ing}_{[-\text{perfective}]\ [+\text{episodic}]}]\ [_{\mathrm{VP}}\ \textbf{write a book}]]]]]]]]]\!]^{c,\, w_c,\, t,\, t}\ (x) = 1]$ $(\text{Gianni}) = 1$

by Principle 1

8. $[\lambda x \in D\ .\ \text{for some } t \in T \text{ such that } u_c < t,\ [\![[_{\mathrm{Asp}}\ \varnothing_{[+\text{perfective}]\ [+\text{episodic}]}]]\!]^{c,\, w_c,\, t,\, t}\ ([\![[_{\mathrm{VP}}\ [_{\mathrm{V}}\ \textbf{say}]\ [_{\mathrm{CP}}\ [_{\mathrm{C}}\ \textbf{that}]\ [_{\mathrm{TP}}\ [_{\mathrm{NP}}\ \textbf{Maria}]\ [_{\mathrm{T'}}\ [_{\mathrm{T}}\ \textbf{\textit{pres}}_{[-\text{past}]\ [-\text{past i}]}]\ [_{\mathrm{AuxP}}\ [_{\mathrm{Aux}}\ \textbf{be}]\ [_{\mathrm{AspP}}\ [_{\mathrm{Asp}}\ \textbf{-ing}_{[-\text{perfective}]\ [+\text{episodic}]}]\ [_{\mathrm{VP}}\ \textbf{write a book}]]]]]]]]\!]^{c,\, w_c,\, t,\, t})\ (x) = 1]$ $(\text{Gianni}) = 1$

by FA

9. [$\lambda x \in D$. for some $t \in T$ such that $u_c < t$, [$\lambda u \in D_{<l, <e, t>>}$. [$\lambda x' \in D$. for some bounded eventuality e such that e LOC $\langle w_c, t\rangle$, u (e) (x') = 1]] (⟦[VP [V **say**] [CP [C **that**] [TP [NP **Maria**] [T' [T ***pres***[−past] [−past i]] [AuxP [Aux **be**] [AspP [Asp **-ing**[−perfective] [+episodic]] [VP **write a book**]]]]]]]]⟧$^{c, w_c, t, t}$) (x) = 1] (Gianni) = 1

by the interpretation of [+perfective] [+episodic]

10. [$\lambda x \in D$. for some $t \in T$ such that $u_c < t$, there is a bounded eventuality e such that e LOC $\langle w_c, t\rangle$ and ⟦[VP [V **say**] [CP [C **that**] [TP [NP **Maria**] [T' [T ***pres***[−past] [−past i]] [AuxP [Aux **be**] [AspP [Asp **-ing**[−perfective] [+episodic]] [VP **write a book**]]]]]]]]⟧$^{c, w_c, t, t}$ (e) (x) = 1] (Gianni) = 1

by λ-notation

11. [$\lambda x \in D$. for some $t \in T$ such that $u_c < t$, there is a bounded eventuality e such that e LOC $\langle w_c, t\rangle$ and ⟦[V **say**]⟧$^{c, w_c, t, t}$ ($\lambda\langle w', t'\rangle \in D_s$. ⟦[CP [C **that**] [TP [NP **Maria**] [T' [T ***pres***[−past] [−past i]] [AuxP [Aux **be**] [AspP [Asp **-ing**[−perfective] [+episodic]] [VP **write a book**]]]]]]]⟧$^{c, w', t, t'}$) (e) (x) = 1] (Gianni) = 1

by IFA

12. [$\lambda x \in D$. for some $t \in T$ such that $u_c < t$, there is a bounded eventuality e such that e LOC $\langle w_c, t\rangle$ and [$\lambda p \in D_{<s, t>}$ [$\lambda e' \in D_l$. [$\lambda x' \in D$. e' is an event of x' saying such that, for any world w'' compatible with the content of e', p ($\langle w'', t\rangle$) = 1]]] ($\lambda\langle w', t'\rangle \in D_s$. ⟦[CP [C **that**] [TP [NP **Maria**] [T' [T ***pres***[−past] [−past i]] [AuxP [Aux **be**] [AspP [Asp **-ing**[−perfective] [+episodic]] [VP **write a book**]]]]]]]⟧$^{c, w', t, t'}$) (e) (x) = 1] (Gianni) = 1

by NN and the lexical entry of **say**

13. [$\lambda x \in D$. for some $t \in T$ such that $u_c < t$, there is a bounded eventuality e such that e LOC $\langle w_c, t\rangle$ and e is an event of x saying such that, for any world w' compatible with the content of e, ⟦[CP [C **that**] [TP [NP **Maria**] [T'

$[_{\text{T}}\ \textbf{\textit{pres}}_{[-\text{past}]\ [-\text{past i}]}]\ [_{\text{AuxP}}\ [_{\text{Aux}}\ \textbf{be}]\ [_{\text{AspP}}\ [_{\text{Asp}}\ \textbf{-ing}_{[-\text{perfective}]\ [+\text{episodic}]}]\ [_{\text{VP}}\ \textbf{write a book}]]]]]]\rrbracket^{c,\, w',\, t,\, t} = 1]$ (Gianni) = 1

by λ-notation

14. $[\lambda x \in D$. for some $t \in T$ such that $u_{c} < t$, there is a bounded eventuality e such that e LOC $\langle w_{c}, t\rangle$ and e is an event of x saying such that, for any world w' compatible with the content of e, $\llbracket[_{\text{TP}}\ [_{\text{NP}}\ \textbf{Maria}]\ [_{\text{T'}}\ [_{\text{T}}\ \textbf{\textit{pres}}_{[-\text{past}]\ [-\text{past i}]}]\ [_{\text{AuxP}}\ [_{\text{Aux}}\ \textbf{be}]\ [_{\text{AspP}}\ [_{\text{Asp}}\ \textbf{-ing}_{[-\text{perfective}]\ [+\text{episodic}]}]\ [_{\text{VP}}\ \textbf{write a book}]]]]]\rrbracket^{c,\, w',\, t,\, t} = 1]$ (Gianni) = 1

by the semantic vacuity of **that**

15. $[\lambda x \in D$. for some $t \in T$ such that $u_{c} < t$, there is a bounded eventuality e such that e LOC $\langle w_{c}, t\rangle$ and e is an event of x saying such that, for any world w' compatible with the content of e, $\llbracket[_{\text{T'}}\ [_{\text{T}}\ \textbf{\textit{pres}}_{[-\text{past}]\ [-\text{past i}]}]\ [_{\text{AuxP}}\ [_{\text{Aux}}\ \textbf{be}]\ [_{\text{AspP}}\ [_{\text{Asp}}\ \textbf{-ing}_{[-\text{perfective}]\ [+\text{episodic}]}]\ [_{\text{VP}}\ \textbf{write a book}]]]]\rrbracket^{c,\, w',\, t,\, t}\ (\llbracket[_{\text{NP}}\ \textbf{Maria}]\rrbracket^{c,\, w',\, t,\, t}) = 1]$ (Gianni) = 1

by FA

16. $[\lambda x \in D$. for some $t \in T$ such that $u_{c} < t$, there is a bounded eventuality e such that e LOC $\langle w_{c}, t\rangle$ and e is an event of x saying such that, for any world w' compatible with the content of e, $\llbracket[_{\text{T'}}\ [_{\text{T}}\ \textbf{\textit{pres}}_{[-\text{past}]\ [-\text{past i}]}]\ [_{\text{AuxP}}\ [_{\text{Aux}}\ \textbf{be}]\ [_{\text{AspP}}\ [_{\text{Asp}}\ \textbf{-ing}_{[-\text{perfective}]\ [+\text{episodic}]}]\ [_{\text{VP}}\ \textbf{write a book}]]]]\rrbracket^{c,\, w',\, t,\, t}$ (Maria) = 1] (Gianni) = 1

by NN and the lexical entry of **Maria**

17. $[\lambda x \in D$. for some $t \in T$ such that $u_{c} < t$, there is a bounded eventuality e such that e LOC $\langle w_{c}, t\rangle$ and e is an event of x saying such that, for any world w' compatible with the content of e, $\llbracket[_{\text{T}}\ \textbf{\textit{pres}}_{[-\text{past}]\ [-\text{past i}]}]\rrbracket^{c,\, w',\, t,\, t}\ (\lambda\langle w'', t''\rangle \in D_{s}\ .\ \llbracket[_{\text{AuxP}}\ [_{\text{Aux}}\ \textbf{be}]\ [_{\text{AspP}}\ [_{\text{Asp}}\ \textbf{-ing}_{[-\text{perfective}]\ [+\text{episodic}]}]\ [_{\text{VP}}\ \textbf{write a book}]]]\rrbracket^{c,\, w'',\, t,\, t''})$ (Maria) = 1] (Gianni) = 1

by IFA

18. [$\lambda x \in D$. for some $t \in T$ such that $u_c < t$, there is a bounded eventuality e such that e LOC $\langle w_c, t\rangle$ and e is an event of x saying such that, for any world w' compatible with the content of e, [$\lambda v \in D_{<s, <e, t>>}$. [$\lambda x' \in D$. for some t' $\in T$ such that $t \subseteq t'$ and $u_c \subseteq t'$, v ($\langle w', t'\rangle$) (x') = 1]] ($\lambda\langle w'', t''\rangle \in D_s$. $[\![$[$_{\textbf{AuxP}}$ [$_{\textbf{Aux}}$ **be**] [$_{\textbf{AspP}}$ [$_{\textbf{Asp}}$ **-ing**$_{[-\text{perfective}]\ [+\text{episodic}]}$] [$_{\textbf{VP}}$ **write a book**]]]$]\!]^{c,\ w'',\ t,\ t''}$) (Maria) = 1] (Gianni) = 1

by the interpretation of [−past] [−past i]

19. [$\lambda x \in D$. for some $t \in T$ such that $u_c < t$, there is a bounded eventuality e such that e LOC $\langle w_c, t\rangle$ and e is an event of x saying such that, for any world w' compatible with the content of e, there is some $t' \in T$ such that $t \subseteq t'$, $u_c \subseteq t'$, and $[\![$[$_{\textbf{AuxP}}$ [$_{\textbf{Aux}}$ **be**] [$_{\textbf{AspP}}$ [$_{\textbf{Asp}}$ **-ing**$_{[-\text{perfective}]\ [+\text{episodic}]}$] [$_{\textbf{VP}}$ **write a book**]]]$]\!]^{c,\ w',\ t,\ t'}$ (Maria) = 1] (Gianni) = 1

by λ-notation

20. [$\lambda x \in D$. for some $t \in T$ such that $u_c < t$, there is a bounded eventuality e such that e LOC $\langle w_c, t\rangle$ and e is an event of x saying such that, for any world w' compatible with the content of e, there is some $t' \in T$ such that $t \subseteq t'$, $u_c \subseteq t'$, and $[\![$[$_{\textbf{AspP}}$ [$_{\textbf{Asp}}$ **-ing**$_{[-\text{perfective}]\ [+\text{episodic}]}$] [$_{\textbf{VP}}$ **write a book**]]$]\!]^{c,\ w',\ t,\ t'}$ (Maria) = 1] (Gianni) = 1

by the semantic vacuity of **be**

21. [$\lambda x \in D$. for some $t \in T$ such that $u_c < t$, there is a bounded eventuality e such that e LOC $\langle w_c, t\rangle$ and e is an event of x saying such that, for any world w' compatible with the content of e, there is some $t' \in T$ such that $t \subseteq t'$, $u_c \subseteq t'$, and $[\![$[$_{\textbf{Asp}}$ **-ing**$_{[-\text{perfective}]\ [+\text{episodic}]}$]$]\!]^{c,\ w',\ t,\ t'}$ ($[\![$[$_{\textbf{VP}}$ **write a book**]$]\!]^{c,\ w',\ t,\ t'}$) (Maria) = 1] (Gianni) = 1

by FA

22. [$\lambda x \in D$. for some $t \in T$ such that $u_c < t$, there is a bounded eventuality e such that e LOC $\langle w_c, t\rangle$ and e is an event of x saying such that, for any world w' compatible with the content of e, there is some $t' \in T$ such that $t \subseteq t'$, $u_c \subseteq t'$, and [$\lambda u \in D_{<l, <e, t>>}$. [$\lambda x' \in D$. for some open eventuality e' such that e' LOC $\langle w', t'\rangle$, u (e') (x') = 1]] ($[\![$[VP **write a book**]$]\!]^{c, w', t, t'}$) (Maria) = 1] (Gianni) = 1

by the interpretation of [−perfective] [+episodic]

23. [$\lambda x \in D$. for some $t \in T$ such that $u_c < t$, there is a bounded eventuality e such that e LOC $\langle w_c, t\rangle$ and e is an event of x saying such that, for any world w' compatible with the content of e, there is some $t' \in T$ such that $t \subseteq t'$, $u_c \subseteq t'$, and [$\lambda x' \in D$. for some open eventuality e' such that e' LOC $\langle w', t'\rangle$, $[\![$[VP **write a book**]$]\!]^{c, w', t, t'}$ (e') (x') = 1] (Maria) = 1] (Gianni) = 1

by λ-notation

24. [$\lambda x \in D$. for some $t \in T$ such that $u_c < t$, there is a bounded eventuality e such that e LOC $\langle w_c, t\rangle$ and e is an event of x saying such that, for any world w' compatible with the content of e, there is some $t' \in T$ such that $t \subseteq t'$, $u_c \subseteq t'$, and [$\lambda x' \in D$. for some open eventuality e' such that e' LOC $\langle w', t'\rangle$, [$\lambda e'' \in D_l$. [$\lambda x'' \in D$. e'' is an event of x'' writing a book]] (e') (x') = 1] (Maria) = 1] (Gianni) = 1

by NN and the lexical entry of **write a book**

25. [$\lambda x \in D$. for some $t \in T$ such that $u_c < t$, there is a bounded eventuality e such that e LOC $\langle w_c, t\rangle$ and e is an event of x saying such that, for any world w' compatible with the content of e, there is some $t' \in T$ such that $t \subseteq t'$, $u_c \subseteq t'$, and [$\lambda x' \in D$. for some open eventuality e' such that e' LOC $\langle w', t'\rangle$, e' is an event of x' writing a book] (Maria) = 1] (Gianni) = 1

by λ-notation

26. [$\lambda x \in D$. for some $t \in T$ such that $u_c < t$, there is a bounded eventuality e such that e LOC $\langle w_c, t \rangle$ and e is an event of x saying such that, for any world w' compatible with the content of e, there is some $t' \in T$ such that $t \subseteq t'$, $u_c \subseteq t'$, and there is an open eventuality e' such that e' LOC $\langle w', t' \rangle$ and e' is an event of Maria writing a book] (Gianni) = 1

by λ-notation

27. For some $t \in T$ such that $u_c < t$, there is a bounded eventuality e such that e LOC $\langle w_c, t \rangle$ and e is an event of Gianni saying such that, for any world w' compatible with the content of e, there is some $t' \in T$ such that $t \subseteq t'$, $u_c \subseteq t'$, and there is an open eventuality e' such that e' LOC $\langle w', t' \rangle$ and e' is an event of Maria writing a book

by λ-notation

28. For some $t \in T$ such that $u_c < t$, there is a bounded eventuality e such that e takes place in w_c at t and e is an event of Gianni saying such that, for any world w' compatible with the content of e, there is some $t' \in T$ such that $t \subseteq t'$, $u_c \subseteq t'$, and there is an open eventuality e' such that e' takes place in w' at t' and e' is an event of Maria writing a book

by the meaning of LOC

Derivation of the second reading of (1)

Since (LF 1') only differs from (LF 1) in that $pres_{[-past]}$ is the head of T, we can obtain (1)'s second reading via a step-by-step derivation with twenty-eight steps analogous to the steps 1-28 of the previous derivation. I will only write down the steps of (1)'s-second-reading derivation in which the truth-conditional difference between the two readings of (1) is generated. The other derivation steps simply differ from 1-28 in that $pres_{[-past]\ [-past\ i]}$ is substituted for $pres_{[-past]}$ and the condition that $t \subseteq t'$ and $u_c \subseteq t'$ is substituted for the weaker condition that $t \subseteq t'$.

$[_{\mathbf{MP}}\ [_{\mathbf{NP}}$ **Gianni**$]\ [_{\mathbf{M'}}\ [_{\mathbf{M}}$ **will**$]\ [_{\mathbf{AspP}}\ [_{\mathbf{Asp}}\ \varnothing_{[+perfective]\ [+episodic]}]\ [_{\mathbf{VP}}\ [_{\mathbf{V}}$ **say**$]\ [_{\mathbf{CP}}\ [_{\mathbf{C}}$ **that**$]$

$[_{TP}\ [_{NP}\ \textbf{Maria}]\ [_{T'}\ [_{T}\ \textbf{\textit{pres}}_{[-past]}]\ [_{AuxP}\ [_{Aux}\ \textbf{be}]\ [_{AspP}\ [_{Asp}\ \textbf{-ing}_{[-perfective]\ [+episodic]}]\ [_{VP}$ $\textbf{write a book}]]]]]]]]]$ IS TRUE IN c iff …

17’. [$\lambda x \in D$. for some $t \in T$ such that $u_c < t$, there is a bounded eventuality e such that e LOC $\langle w_c, t\rangle$ and e is an event of x saying such that, for any world w’ compatible with the content of e, $[\![[_{T}\ \textbf{\textit{pres}}_{[-past]}]]\!]^{c, w', t, t}$ $(\lambda\langle w'', t''\rangle \in D_s$. $[\![[_{AuxP}\ [_{Aux}\ \textbf{be}]\ [_{AspP}\ [_{Asp}\ \textbf{-ing}_{[-perfective]\ [+episodic]}]\ [_{VP}\ \textbf{write a book}]]]]]\!]^{c, w'', t, t''}$) (Maria) = 1] (Gianni) = 1

18’. [$\lambda x \in D$. for some $t \in T$ such that $u_c < t$, there is a bounded eventuality e such that e LOC $\langle w_c, t\rangle$ and e is an event of x saying such that, for any world w’ compatible with the content of e, [$\lambda v \in D_{<s, <e, t>>}$. [$\lambda x' \in D$. for some $t' \in T$ such that $t \subseteq t'$, $v(\langle w', t'\rangle)(x') = 1$]] $(\lambda\langle w'', t''\rangle \in D_s$. $[\![[_{AuxP}\ [_{Aux}\ \textbf{be}]\ [_{AspP}\ [_{Asp}\ \textbf{-ing}_{[-perfective]\ [+episodic]}]\ [_{VP}\ \textbf{write a book}]]]]]\!]^{c, w'', t, t''}$) (Maria) = 1] (Gianni) = 1

by the interpretation of [−past]

19’. [$\lambda x \in D$. for some $t \in T$ such that $u_c < t$, there is a bounded eventuality e such that e LOC $\langle w_c, t\rangle$ and e is an event of x saying such that, for any world w’ compatible with the content of e, there is some $t' \in T$ such that $t \subseteq t'$ and $[\![[_{AuxP}\ [_{Aux}\ \textbf{be}]\ [_{AspP}\ [_{Asp}\ \textbf{-ing}_{[-perfective]\ [+episodic]}]\ [_{VP}\ \textbf{write a book}]]]]]\!]^{c, w', t, t'}$ (Maria) = 1] (Gianni) = 1

by λ-notation

…

28’. For some $t \in T$ such that $u_c < t$, there is a bounded eventuality e such that e takes place in w_c at t and e is an event of Gianni saying such that, for any world w’ compatible with the content of e, there is some $t' \in T$ such that $t \subseteq t'$ and there is an open eventuality e’ such that e’ takes place in w’ at t’ and e’ is an event of Maria writing a book

Conclusions

In the first part of this dissertation I examined the literature on multiple temporal indexing with a critical eye. The general conclusion that emerges from Part 1 is that the case against multiply indexed temporal operators is weaker than what the critics of intensional approaches have generally supposed.

In chapter 1 I considered various expressibility arguments for double indexing and triple indexing. I analyzed the basic structure of expressibility arguments by looking at the arguments that Kamp and Vlach gave to motivate their respective double-index systems. I criticized van Benthem and Cresswell's suggestion that the case for double indexing generalizes and leads to infinite temporal indexing. I argued that if we want to represent tenses and indexical time adverbs in terms of intensional operators, we do not need to multiply the number of time indices in order account for the sentences that van Benthem and Cresswell discuss.

In chapter 2 I criticized King's suggestion that operator-based formalizations of English sentences are ad hoc and less elegant than standard quantificational formalizations. King considers English sentences similar to the ones that were discussed in chapter 1. I criticized King's contention that an intensional theorists needs to posit ad hoc and inelegant operators in formalizing these sentence. With the aid of operators of the four categories that I distinguished in the chapter, it is possible to provide representations of those sentences which do not have the problematic features of the particular operator-based formalizations which King considers in his discussion.

In the second part of the dissertation I examined two linguistic phenomena that are *prima facie* problematic for intensional theories of tense: the pronominal uses of tenses and the behavior of embedded tenses.

Chapter 3 was devoted to examining the the so-called deictic, anaphoric and bound uses of tenses and modals. I discussed the suggestion that tenses, modals, and pronouns behave in parallel ways. This parallelism has been an important motivation for the development of variable-based accounts of tense and modality. I proposed an alternative analysis of the classical examples that illustrate the parallelism. According this analysis, tenses and modals are context-sensitive intensional operators that act on a class of sub-sentential syntactic constituents that I called *radicals*. Since radicals are sub-sentential elements, tenses and modals are sub-sentential operators according the analysis given in chapter 3. I argued that this analysis makes the right predictions about the classical examples of Partee and Stone. Although the analysis of chapter 3 treats personal pronouns as referential expressions, it analyzes tenses and modals as non-referential devices.

In chapter 4 I proposed a more sophisticated account of tenses. The account of embedded tenses proposed in chapter 4 was based on the idea that tenses specify a relation between a perspective time and a time of eventuality. By default, the perspective time is simply the time of utterance. But perspective-time shifts are triggered by intensional verbs and modals.

I presented an intensional semantic framework that allowed me to formalize these ideas. I also adopted certain syntactic assumptions that made my account of tenses compatible with modern syntactic theories. I argued that my account explains the shifted interpretations of the embedded English past in complement clauses and relative clauses. I also argued that it is better equipped to account for these interpretations than absolute and relative theories. I briefly discussed the simultaneous readings of *past* in past-under-past environments and suggested that my account of tenses is compatible with an explanation of the simultaneous readings in terms of a sequence-of-tense rule.

The intensional account proposed in chapter 4 vindicates the view that tenses are intensional grammatical devices. I hope that this account can be a promising point of departure for future research.

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