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Real Options Analysis in Real Estate Investments and Developments

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ABSTRACT

The analysis of real estate projects poses a great number of challenges. The uniqueness of the characteristics of each of type of real estate means that an assessment must be carefully adapted to each case. One of the most determining aspects is the management flexibility of real estate projects. Their long duration, belonging to a strongly cyclical sector, the important level of investment required and other aspects such as the variation of technical solutions link their success to the management skills and capabilities of managers. In practice, the options available to managers constitute one of the main assets of a project. These real options increase the value of projects through two effects. First, as the theory of real options demonstrates, it is possible to assign an objective value to flexibility, which is added to the value of the project estimated strictly by discounting its cash flows. Second, the presence of more or less flexibility in the project makes it more adaptable, reducing its relative risk with respect to less flexible projects, and consequently reducing the cost of capital required by the investor.

The objective of this thesis is to provide a better understanding of both aspects. Regarding the first, the challenge is in adapting the methodology of real options to real cases more common in the daily analysis of a real estate analyst. More than four decades after the first authors described methods for the valuation of real options in real estate, the current reality is that the value of options is almost always still estimated in an intuitive manner. Regarding the second aspect, the effect on the cost of capital, this work focuses on identifying, ordering, quantifying and interrelating the determining factors between management flexibility and the cost of capital required.

From the present analysis, it can be inferred that it is possible to quantify the value of the flexibility of a real estate project. Furthermore, this value is sufficiently significant and reliable, and conceptually it can come from different sources. It can also be concluded that in each project there are decisive factors for the investor to decide whether to invest or not, depending on the relationship between the potential return and the inherent flexibility. In both aspects, the skills and capabilities of the manager are crucial, since he is responsible for enhancing the flexibility of a project before and during its course.

JEL: G12, G31, R32

Keywords: Real options, asset pricing, capital budgeting, real estate investments, cost of capital

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Chapter 1

Introduction to Real Options in Real Estate Sector

1.1. Introduction

Flexibility in the management of real estate assets results in the prospect of various options. These options not only increase the value of the asset or the investment, but also reduce the risk of such investment to the extent that they bring greater adaptability to market uncertainties. Although valued intuitively by practitioners, real options are rarely valued in a systematic manner, despite the existence of option valuation theories that provide tools for this.

The theory of real options, which has been developed over the past 40 years, provides tools that are not used in most cases because of the complexity involved. This doctoral thesis aims to describe the main aspects affecting the application of the theory of real options to the valuation of real estate investment.

Today, more than 30 years after the first attempts to apply real options theory to real estate analysis, still too few practitioners apply the existing theory to the valuation of these types of assets or projects. The main reasons for this reluctance are the complexity of the pricing models that should be applied, together with the difficulty to value the key asset pricing variables involved, undermining the credibility of the results and reducing investors' confidence. All these factors discourage analysts from using real options in their models as a standard valuation and decision-making tool.

This means that, after more than three decades trying to disseminate complex methodologies with little success, a valuable opportunity has been lost to use elementary management principles in practice. Therefore, in an attempt to overcome these obstacles, alternative approaches applicable in the real world are needed. These new approaches should be capable of focusing on valuation methodologies that capture the complexity and the nature of real estate investments in such a way that managers and investors can rely on their implementation.

The challenge is the identification of an adequate methodology able to achieve credibility among investors and practitioners. The methodology should be based on principles grounded in and contrasted by their use in other types of markets. Using reliable values for the key variables is also crucial in this context. This is the perspective this doctoral thesis intends to adopt. However, the use of real options also implies strategic decisions. For this reason it is not possible to delink this research from the strategic aspects.

1.2. The real estate sector

The real estate sector represents the main reference of the largest economies in the world. Its importance in advanced economies in terms of production, employment and added value is crucial. A real estate company is a firm or corporate entity that builds, leases, sells and manages homes. Therefore, there are various activities that can be developed by the same organization in this field, which may cover different or even all phases of the real estate production process. We must understand the sector from the broadest perspective, including all activities related to real estate.

The companies in this sector are often structured vertically, so that they carry out production activities as well as the subsequent tenure and management of the goods generated. The accumulated wealth around urban real estate means, in the main economies of the world, a prevailing weight close to or more than 50% of total cumulated wealth, whose value has doubled in the last century.[1]

In the specific case of Spain, based on 2013, a year at the lowest point of the economic cycle in terms of construction, it is estimated that the gross added value (GAV) of the real estate sector exceeded €135,000 million, which is 13% of the gross domestic product (GDP). The structure of this GAV varies according to the moment in the cycle. Building construction comprises more than 50% of this GAV in bullish periods, while institutional real estate transactions represent more than 60% in bearish periods. In addition, there is a third component in the sector, between 15% and 20% of GDP depending on the time in the cycle, which corresponds to specialized activities within the construction of buildings.[2]

During the period 2007 to 2015, the strong correction of the values of real estate assets, unprecedented in many countries such as Spain, generated special concern for the adequacy of the valuation methods used for projects. This has led to greater professionalization of project analysis and asset valuation. But which methods are traditionally used?

The use of option contracts, or the generation of such options, is a constant that is intrinsic to development in the real estate business. Indeed, real options are also present in a wide variety of other industries: natural resource investments, oil and gas exploration and production, car and manufacturing industries, the airline industry, telecommunications, the motion picture

[1] See data collected by Thomas Piketty in http://piketty.pse.ens.fr/en/capital21c2

[2] Data from the Instituto Nacional de Estadística and the Spanish Ministerio de Fomento.

industry, biotech and pharma industries, and R&D management (Copeland & Antikarov, 2001; Mun, 2005; Kodukula & Papudesu, 2006). The real estate sector is strongly linked to the use of options, sometimes even without the latter being perceived by participants (Baldi, 2013; Geltner & de Neufville, 2018). Besides, real estate is among the sectors where management flexibility is high, which is the main factor in generating options. Moreover, given the nature of real estate projects, those who manage them have considerable influence over their value, which in turn affects the value of the options embedded in the projects.

1.3. Frequently used methodologies in the valuation of projects and real estate assets

Three widely used methodologies used in the valuation of projects and real estate assets can be distinguished: capitalization value, market comparison value and replacement value. In all three cases, an analysis of current market prices and the application of discount rates are required, which implies a certain degree of subjective valuation.

1.3.1. Capitalization value

Capitalization value is the most frequently used methodology for projects, and is used in the following two ways.

It can be calculated by capitalizing perpetual incomes, based on the net rental income from the asset, once the transformation is completed, and subtracting the net present value (NPV) from the required investment. It is also used for existing assets, in which case the lease market prices, duly capitalized, are used to obtain the value of the asset. This requires the valuation of the net rent received from the asset (based on a market study) in addition to considering a rate of return acceptable for the risk assumed.

Alternatively, the value can be calculated by calculating the NPV of the flows that the development of the project will generate, taking into account the final sale of the resulting assets. In order to do so, it is necessary to estimate the sales prices through market research and the discount rate that corresponds to the level of risk assumed by the investors.

1.3.2. Comparison value

Comparison value is the methodology most frequently applied to existing assets. It is based on a market study where the most similar assets are identified and the asset price is assimilated.

1.3.3. Replacement value

Replacement value is calculated by computing the NPV of the costs for rebuilding the asset, to which it is necessary to add the value of a plot of the same characteristics. This form of valuation is frequently used in appraisals and gives little importance to the business cost of an opportunity.

1.4. What is a "real estate project"?

The concept of a real estate investment project is especially broad. The main examples are described below.

1.4.1. Actions on the land (planning and management)

The actions on the land are the first in a chain of subsequent operations. Normally, they are carried out by specialized investors in this typology, since these are projects with very different financial characteristics to other projects. On the one hand, 'planning' refers to the transformation operations of rustic land into urban land. On the other hand, 'management' refers to the operations of transforming urban land into urbanized land and are composed of, first, ordination, and second, subdivision. Management operations also refer to urban modification or improvement plans (modification of the urban plot) and parcelisation with compensation. In this first typology, the frequent lack of a definition of the variable time is crucial to achieving the success of the project. These are very long-term projects (frequently periods between five and seven years), in which administrative times are critical. However, these are also projects where capital returns are particularly high.

Even in the case of the most intensive type of equity, due to the difficulty of leveraging at a low cost, a typology is often used where the returns on equity (ROE) are higher and more variable. This constitutes a type of transformation of the administrative form of the asset.

The initial asset could be rustic land, without possibilities of real estate use, and the result is concrete plots, with a perfectly defined use. The investor does not know what he or she will obtain (what rights on which plots and with what use or value) until the end of the investment. Here the ability of the manager and the presence of options is of great importance, as important as the acting administration allows.

1.4.2. Urban transformation interventions

Urban transformation interventions are what is understood as urbanization projects or urban transformation. The technical definition of these projects is closely linked to the process of land parcelisation, although it is common for their development to be carried out by another investor. This usually happens because the execution periods are again long, linked to inaccurate administrative times, and also very difficult to leverage financially with financial entities at market prices. The main difference, in this case, is that the investor does know what he or she will get, that is, which urbanized land and with what exact characteristics and services.

It is the first intervention that involves a transformation of the asset, which underlines the importance of the technical component and the risks that this implies. In addition, the raw material with which one is working (the land) is especially prone to unforeseen events. In spite of all this, it is a phase where ROEs are not particularly high, therefore these projects are only coveted by investors who are very specialized in this kind of process.

1.4.3. Real estate development of a sector

These are large-scale building operations, which usually include various types of buildings. It is usual for these operations to be carried out by more than one investor group and for the operation to be structured in phases. The establishment of a commercial and technical strategy for the development of these operations is decisive for its results. It is therefore a very important source of options, which makes it difficult to agree on its total value as a project.

These operations are less capital intensive than the operations described under the previous points, since it is frequently possible to leverage them in the market at low prices. This allows ROE values to be achieved that are frequently higher than those of the operations on land. However, their duration can be very long, which also exposes them to high market risk.

1.4.4. Real estate development of a plot

The real estate development of a plot is the real estate project par excellence, with results and times that are much more controlled than those of the projects described above. Contracts are often agreed with construction companies in the "turn-key" format (all inclusive), where the developer is only exposed to market risks. Depending on the representativeness of the cost of the land on the total GAV, returns greater than twice the contributed equity can be obtained, which makes these highly coveted projects.

In some cases, even future purchase contracts are established, which reduces the market risk very significantly. However, the dimensions or characteristics of the plot can allow the development in phases, or also postpone the start of the project at any time, or even pause the project according to its evolution. These are potential option-bearing projects.

1.4.5. Integral transformation of an asset

The integral transformation of an asset is a usual operation in areas of high urban intensity, with historic buildings or with a potential benefit in the change of use of the building. The transformation may be motivated by the need for an update of the building (without producing a change of use) or may be done to obtain greater profitability of the asset, giving it another use. In the first case, the profitability is limited, except in the case of a speculative operation. In the second case, the profitability is usually conditioned by what current regulations determine and some possible changes to them.

Some examples would be the integral refurbishment of a building to be able to increase its rent quickly (e.g. because it is in an area with significant recent rent growth), or the change of use of an office building for residential use. It is common for opportunistic investors to be involved because of the possibility of acquiring an asset below its real market value. In any case, these are plausible operations, although questionable given their low added value.

1.4.6. Projects in profitability

These are projects of one of the last two typologies described above that are developed by the same investor who claims to finally hold the property and its economic exploitation. Its main characteristic is that it involves very long-term investments, looking for a yield on the invested capital, rather than strictly an ROE in the medium term. These are projects with low exposure to the market, because cyclical changes do not affect them. However, they are very capital-intensive projects and only suitable in areas with high rental demand (residential or commercial).

However, they are the standard projects in some building typologies, such as the hotel industry, and the presence of options, both the abandonment of the project and the extension or pause of the activity, are compulsory for an analysis. Another important characteristic of this type of project is that its value is calculated in a substantially different way to that of projects thought of as development and sale. These must be accompanied by an ex-post exploitation account, in addition to having a completely different tax analysis.

1.5. Discounted cash flow is not enough, we need options

The uncertainties facing the investment process in the real estate sector are countless. Despite this, the experienced investor in the sector will be able to distinguish those investments that are of major importance and with regard to which it is crucial to find coverage and preserve the value of the investment. Correct identification of such risk factors is key for business valuation. Likewise, the ability to find coverage for these risks, identifying them using options theory, is particularly important.

The application of the theory of real options to investments in real estate has a double meaning. On the one hand, as the literature of recent decades indicates, a perspective based solely on discounted cash flow (DCF), without the addition of the value of the implicit options, is incomplete and would certainly lead to the underestimation of a large number of projects. In addition, the ability to use options as hedging instruments becomes especially interesting in capital-intensive investment scenarios.

In 1985, 12 years after Black and Scholes presented their theory of option valuation centered on listed assets, Titman for the first time applied these theories academically to the real estate sector. Today, more than 30 years later, very few apply the theory of real options to the valuation of these types of assets or projects (e.g. Titman, 1985). There is complete consensus on the complexity of the models proposed, together with the difficulty to value the key variables, undermine the credibility of the results and reduce the investor's confidence. All these causes also discourage analysts to use them with assiduity in their models (e.g. De Neufville, Scholtes & Wang, 2006).

One frequently finds complaints in the existing literature about the limited diffusion of option-based valuation models. Unfortunately, just as often these same authors present models whose complexity is important enough to arouse suspicion and mistrust among other experts (e.g. Rocha, Salles, Garcia, Sardinha & Teixeira, 2007). Because of this, those who must apply daily financial principles that have to be taken to the street investor reject these models.

However, the use of option contracts, or the generation of such options, is in a way intrinsic to the development of the real estate business. Few economic sectors are so closely linked to the use of options as real estate, sometimes even without the latter being perceived by participants (Baldi, 2013). In addition, there are few sectors where management flexibility is so high, which is the main factor generating options. Moreover, given the nature of real estate projects, those who manage them have a great influence on their value, which actually affects the value of the options embedded in the projects. However, this is unusual when the underlying assets are not real estate.

From these arguments, it can be deduced that there are enough academic foundations to evaluate the important number of options generated in one of the main economic sectors of most economies, although their practical use is reduced to a small number of concrete cases, especially to large-scale projects. The interesting thing about this circumstance is that practically all transactions and projects, representing the bulk of the sector, consider the value of such management flexibility only in an intuitive way. The problem is that the quantification of value in an intuitive way, not rigorous enough or excessively complex in its methodology, renders such valuation completely meaningless when it is transmitted to the potential investor.

1.6. Real options: Literature review

From a financial point of view, the "real option" is the value that is derived from management flexibility. Having different possibilities to modify the course or nature of an investment is a determinant of its market value. Myers (1977) was the first author to use the concept of “real options” to refer to the economic valuation of management flexibility in investment projects of any kind.

Analysis of the theory of real options, its application and the importance of assessing flexibility to manage real options correctly have been addressed by several authors. Since the 1990s, Trigeorgis, Brennan, Schwartz, Smit and Kulatilaka – to quote only a few of the key names – have written significant papers, alone or in partnership, that have guided subsequent analyses.

The increase in the use of real options is justified as a response to the dissatisfaction of corporate practitioners and strategists, as well as some academics, with traditional capital budgeting techniques such as DCF (see e.g., Dean, 1951; Hayes & Abemalhy, 1980). There is extensive concern that this methodology can often undervalue investment opportunities or lead to underinvestment. This undervaluation is a consequence of practitioners not valuing important strategic considerations properly. Another important area that deserves more attention, and where real options have a significant role to play, is that of strategy. This section describes stages in the evolution of the real options literature.

Where there are disagreements, the first aspect to be solved, is the most appropriate methodology to use. Hertz (1964) and Magee (1964) proposed the use of simulation and decision trees to capture the value of future operating flexibility. Baldwin and Trigeorgis (1993) proposed to solve the problem and improve analysis results by developing specific adaptive capabilities as a tool for acquiring, generating and managing real options.

Myers (1987) has acknowledged that traditional DCF methods have inherent limitations for valuing investments with strategic options, suggesting the use of option pricing to better value such investments. Other authors, such as Hodder (1986), have argued that the problem arises from the misuse of DCF in practice. For Kester (1984), the solution is also to adapt the DCF method to include the strategic and competitive aspects of growth options. In any case, all these authors agree that the DCF method is inadequate and that the solution is to use one or the other option pricing methodology.

Other research studies have centered on attempts to value investments using a series of investment outlays that can be strategically switched to different business models, in order to help maximize a project's interdependencies value. For instance, Geske (1979) has suggested valuing a compound option: an option to acquire another option (an opportunity only available if earlier investments are undertaken). Kulatilaka (1988) has described a model to evaluate the option to switch among different business solutions. Margrabe (1978) has valued an option to exchange one risky asset for another. These papers all suggest methods to help analyze the generic option to switch among alternative uses and related options (switch among alternative inputs or outputs).

Cox and Ross (1976) have theorized that an option can be replicated from an "equivalent risk-free portfolio" of traded securities. This portfolio becomes essential to understand the real option valuation in discrete time. Such risk-neutral valuation enables present-value discounting, at a risk-free interest rate, independent of the investor's risk attitude. This allows the use of risk-neutral probabilities in order to achieve arbitrage-free price systems. There are two such methodological trends in general terms: The quantitative analysis of real options derived from Black and Scholes (1973) and Merton (1973) in pricing financial options, and the binomial approach of Cox, Ross, and Rubinstein (1979) which enables a simplified valuation of options in discrete time.

The lack of a traded underlying asset is one of the challenges to overcome. To address this problem, Rubinstein (1976), in a key finding in real options application, proved that standard option pricing formulas can also be derived under risk aversion and that the existence of continuous trading opportunities enabling a riskless hedge or risk neutrality is not strictly necessary. Kasanen and Trigeorgis (1993) have argued that real options may be valued like financial options, even though they may not be traded. In other words, the existence of a traded "twin security" that is perfectly correlated with the nontraded real asset in complete markets is enough for real option valuation.

Another complexity of analysis is the fact that there are very different kinds of options, and each type can be analyzed differently. Several authors have centered their work on studying different kinds of options. Ingersoll and Ross (1992) have reconsidered the option to wait, usual in the real estate market, in the light of the beneficial impact of a potential future interest rate or other market changes. Similar, but not the same, is the option to defer or initiate an investment that has been examined by McDonald and Siegel (1986), Paddock, Siegel and Smith (1998) and Tourinho (1979). They used for their analyses the valuation of offshore petroleum leases or natural reserves valuations. More concretely, in the real estate sector, Majd and Pindyck (1987) have valued the option to delay the sequential construction of projects that take time to build, when there is a maximum rate at which the investment can be carried out. Many authors, such as Carr (1988) and Trigeorgis (1993), have studied specifically sequential or staged investments.

The option to permanently abandon a project for its salvage value, seen as an American put option, is the main focus of the work of Myers and Majd (1990). Trigeorgis and Mason (1987) and Pindyck (1988) examined options to alter the operating scale or capacity choice, which refers to the option to increase or reduce the size of an investment at a certain moment. Corporate growth options, a similar concept to the previous one, were analyzed by Brealey and Myers (1991). Kester (1984) looked at future investment opportunities. McDonald and Siegel (1985) and Brennan and Schwartz (1985) addressed the option to temporarily shut down a project and restart operations, which can also be applied to a staged real estate project. Kulatilaka and Trigeorgis (1993) analyzed the options to switch use (outputs or inputs). Baldwin and Ruback (1986) demonstrated that future price uncertainty creates a valuable switching option that benefits some kinds of projects with a "short life".

However, real projects are never a model with a single option to be analyzed. Projects are often more complex in that they involve a combination of real options whose values may interact. Moreover, some of these options are a creation of their managers. Some studies deal with this point. An example is Brennan and Schwartz (1985), who determined the combined value of the options to shut down (and restart) a mine, and to abandon it for salvage.

Thus, the combined value of a collection of real options may differ from the sum of separate option values. Using a numerical analysis method suitable for valuing complex multi-option investments, Trigeorgis (1991) studied option combinations. He demonstrated that the incremental value of an additional option, in the presence of other options, is generally less than its value in isolation. Moreover, its value is reduced whenever more options are present. Trigeorgis also depicted situations where the value of option interactions can be small or large, and negative as well as positive.

In order to calculate the value of all these options, scientists use two types of numerical techniques: some approximate the underlying stochastic processes directly and are generally more intuitive (e.g., Boyle (1977) using the Monte Carlo simulation or Trigeorgis (1991) using the binomial method), and others approximate the resulting partial differential equations.

More analytic approximations are also available: Geske and Johnson (1984) have proposed a compound-option analytic polynomial approximation approach, while Barone-Adesi and Whaley (1987) have suggested a quadratic approximation. The point is that many of these techniques are too theoretical to become reliable enough for real-world investors.

In terms of strategic management, some authors have obtained many advantages from the use of real options analysis, as will be summarized next.

Roberts and Weitzman (1982) have argued that in sequential decisions it may be optimal to make investments with a negative NPV, provided that they lead to valuable information on potential future investments that are profitable. This possibility is directly proportional to the level of uncertainty. This justifies making investments only because of the options they provide.

By contrast, Baldwin (1982) concluded that in the case of companies with power within oligopolistic markets that face sequential decisions that limit their options (because they are irreversible), a positive NPV is not enough. They will need extra value that compensates the value of these options.

Dixit (1989) analyzed the input and output options. He noted that in the presence of sunk costs or exit costs, the optimal decision to reverse an investment should not always be taken based on the prices observed in the short term. It may be that the costs of this option are not compensated by the increase in value that is favored by the change. To this analysis of Dixit, applied to international trade, Bell (1993) added the effect of the ability to trade options under very volatile exchange rates.

Kester (1984) performed qualitative analyses assessing the strategic aspects of growth options, structured as sequential decisions. Also, Trigeorgis and Kasanen (1991) examined the value of sequential decisions in interrelated projects, where the value would come from the synergies among them.

To this analysis and assessment, Ang and Dukas (1991) added the study of asymmetric information, its value and the possible entry of new competitors. These studies are complemented by Smit and Ankum (1993) who suggested the principles of industrial organization and game theory (studying the reaction of competitors and the effect on options).

Thus, we see that the theory of real options has been applied to a multitude of scientific and analytical fields. Virtually all fields where management offers flexibility margins (exploitation of natural resources, flexible production in factories, decisions between rent or purchase, and R&D) are likely to complement the analysis with the use of real options. This also happens in the real estate sector.

The application of the theory of real options to investments in real estate has a double meaning. On the one hand, as the literature of recent decades indicates, a perspective based solely on DCF, without the addition of the value of the implicit options, is incomplete and would undoubtedly lead to underestimation in a large number of projects. Besides, the ability to use options as hedging instruments becomes especially interesting in capital-intensive investment scenarios.

With regard to soil management, Titman (1985) and Williams (1991), among other authors, have studied how the value of land should not only be based on those uses that are initially assigned to it; the value of its future best alternative potential should be added to them. Quigg (1993) even provided empirical results of land valuations that incorporated real options, specifically the option of waiting to promote, offering better approximations to the value that the market sets for them.

1.7. Positioning of the research and structure of the thesis

This research aims to contribute, through the identification of adequate scientific methodology, to the application of options theory to the most frequent operations in the real estate sector. The objective is therefore to select and apply real options valuation methodologies particularly oriented to real estate management challenges, and illustrate the theoretical proposals with applications to real cases.

With regard to positioning, this research can be classified within the field of analysis of corporate finance, specifically the valuation of projects and assets. In addition, there are two components beyond the process of calculating these options that are equally important.

The first is the strategic component of the analysis. This involves the management of these options, their search, design and the ability to use them correctly. The second is the analysis of the influence that the presence of real options has on the cost of capital required for a project. Within this field of capital budgeting, this research proposal analyzes and applies a valuation methodology that incorporates the value of the options embedded in the nature of the projects under analysis to the traditional DCF methods as the best alternative to price managerial flexibility and add active management to project development. Furthermore, this research is framed within the construction of methodologies aimed at systematizing the valuation of real estate projects.

The structure of this thesis is as follows:

- First, this introductory chapter presents the motivation of this doctoral dissertation as well as its objectives. It presents introductory data on the valuation of projects in the real estate sector, the contribution of real options to such valuations and the methodology of options pricing to be applied.
- Second, this thesis includes two case studies:
  - Chapter 2: Pricing refurbishment and new building construction: This first case study is a clear example of a real option, very common in the real estate sector, and a first pricing proposal. The option in this first case is to change the nature of the investment. The valuation methodology is based on the Black-Scholes-Merton formula.
  - Chapter 3: Pricing real estate multi-stage projects options: This second case presents the second type of most frequent real options in the real estate sector and another methodology for its evaluation. It also introduces an econometric analysis of the

variable volatility. The option in this case is to carry out the project in phases. The evaluation methodology chosen is the binomial tree.

- Chapter 4: Real options in real estate projects: The role of the cost of capital: this chapter analyzes the influence of the presence of real options on the cost of capital required for real estate projects.
- The overall conclusions of the research are presented in Chapter 5.

The study of real cases has three main targets: first, to provide a pricing method that is adequate enough for practitioners to value the options embedded in real estate projects; second, to apply this methodology to frequent business scenarios in the real estate sector; and third, to analyze the effect of the presence of such options on the cost of capital required for real estate developments.

1.8. Key variables in the analysis of real options

The formulation of the most widespread models leaves no doubts about the key variables to be analyzed. In this section, the variables described are those referenced in the work of Copeland and Antikarov (2001), following the basic findings on the variables that affect options prices (Hull, 2018, Chapter 11).

1.8.1. The value of the underlying asset

Of course, it must be a fluctuating value in the market (which involves risk). The increase in the value of the underlying asset is positively correlated with the value of the call option, while in the put option the effect is the reverse. This stems from the nature of both types of options and clearly indicates that the correct identification of the type of option is crucial in any analysis.

The most significant aspect to be highlighted is that, in the case of real estate assets, as well as in a significant number of projects or real assets, the manager can modify the value of the underlying asset. This should not happen in the case of financial assets, if the market functions correctly.

1.8.2. The volatility of the value of the underlying asset

An increase in volatility, insofar as it is an increase in risk, causes the value of a hedging instrument to increase. Therefore, volatility in one of the critical variables such as the value of the real estate asset is decisive for the calculation of the value of any type of option on it. This is much more intuitive if derivatives are conceived as risk-hedging instruments, although they can also be used to speculate, in which case the same effect also occurs.

1.8.3. The exercise price or strike price

As with an option on a financial asset, the exercise price of an option on a real estate asset is negatively related to its value when it is a call option, and positively in the case of a put option. It is easy to understand if we take it to the extreme: for example, the option to buy a small apartment in a suburb for €20 million is worth nothing.

1.8.4. The duration of the option or time to expiration

The longer the term before an option expires, the higher the value of the option, whether it is a purchase or a sale (call or put), since the probability that it will eventually be put into value (in money) is higher. This is valid for any option, whether it is European (with a fixed exercise date) or American (with the flexibility to choose the exercise date). It is true that this has a different influence depending on the type of option, although the sign of the relationship is the same in both cases.

1.8.5. The risk-free interest rate

An option, like any asset with implicit risk, has a relationship to the price of money. An increase in the risk-free interest rate makes money expensive in all its forms, and therefore it increases the value of a call option and reduces the value of a put option (whoever must buy has to pay the money at a more expensive rate). Such interest rate shall be valued as an average estimate for the entire life of the option.

1.8.6. The income generated by the underlying asset

With regard to the income generated by an underlying asset, financially it would be the dividends; in the case of a real option on an industrial production project, it would be the earnings before interest, taxes, depreciation, and amortization (EBITDA) generated; and in the case of a real estate investment, it would be the net income generated. In this case, two aspects converge:

- First, the sign of the correlation between these flows and the value of the option is debatable. That is, it is highly probable that the flows generated have no relation to the future or expected flows, if it is an asset transformation project.
- Second, unlike with a financial asset, it is possible that the flows generated have no relation to the price of the particular option that is valued.

1.9. Which variables deserve more attention in this research study?

The method used by Black and Scholes is defined for European options without dividends. The consideration of the relationship of the flows generated by the asset in the valuation of an option on a property is especially confusing and complex. More importantly, given the asset profile, the required return is relatively low relative to other assets, which usually leads to a high price/earnings ratio (between 16 and 23, for example), and is fairly constant. In addition, its amortization period is especially high, even if part of it (the land) is infinite. Therefore, the income generated during the period that an option can last is not really a determinant. We can consider its value negligible and simplify the whole analysis considerably.

The risk-free interest rate is undoubtedly important for the analysis. In any case, it is an easy value to choose when applying it in a formulation.

The duration of the option and the exercise price are crucial variables, although they are exogenous. They could correspond to the values in the pacts that the parties reach, in some cases, or be determined by regulations, the market or other factors. Their relevance in our analysis will be the option price elasticities with respect to these parameters. In any case, estimating them does not become a problem.

Regarding the other two variables, the value of the underlying asset and its volatility, we are faced with two particularly important variables. In any case, to obtain both we should focus on techniques of market analysis. The first one is determined by market analysis. The second should be modeled in a rigorous, simple and standardized way, in order to limit its complexity. The procedure for the calculation of volatility is determined later, in detail, and is one of the main challenges of the present work.

1.10. Main typologies of real options and their use in real estate assets

This section describes the most common types of options, sorted by their frequency of occurrence.

1.10.1. Postponing option or option to defer

The option to defer consists of the possibility that the manager can delay the start of a project. In the case of real estate assets, it can mean the possibility of deferring the purchase, in other words, to have a purchase option for a certain time period. The possibility of postponing the development of an asset's transformation can also be considered as such an option (construction of a building or reform/change of its use).

Postponing purchase is a frequently used contract. Its use is often merely speculative, so to assess it we should analyze the cost of opportunity that is generated due to a possible arbitrage. If there is no willingness to arbitrate and the asset intended to be financed may have a purely operational use, it would not make sense to assess it. If there is a willingness to arbitrate, the asset's valuation is not reliable; buyer and seller have a very different perception of the value of that option.

The possibility of postponing is an option that depends on the financial structure of the company and the project, as well as on what regulations determine. It is true that this option is presented to many owners in a natural manner, and often it is not valuated. It is closely linked to a key concept whose analysis arouses great interest: determining the right moment to develop a project.

The exercise of this option makes sense when there is a negative change in market conditions that suggests that its future realization, discounting the cost of capital that this may entail, continues to generate a higher profit.

It also makes sense to exercise this option when it is verified that the market information obtained when time evolves generates greater benefit than the cost of capital employed in the waiting time.

1.10.2. The option to change the scale of the project

The option to change the scale of a project is common in large-dimension projects, which may consist of phases, or the execution of part of the project influences the cost/benefit analysis of other subsequent projects. The possibility of dividing the project into phases allows the risk to be scaled to the extent that it can be developed with more information on the development of the market. In this case, the value is derived from the creation of future opportunities from the present development. This is a type of option on which a significant amount of research is being conducted, although it is a rare option.

This option will be exercised when it is inferred from current market information that the optimal project size does not match the maximum possible size.

1.10.3. Option to fragment the investment or create stages of the project

This option consists of generating stages of investment and development, in other words, a growth plan, for the same business. It differs in nuances from the previous two types of real options, but these nuances are very important. This option is frequent at the beginning of many businesses, and is also frequent in real estate investment. The investment in real estate projects or other projects is usually of the "one-shot" type, although stages occur naturally so that at certain moments the value added is clear and measurable (for example, when a license is granted). Once a certain stage of the investment is initiated (construction started or the asset acquired), it is very difficult to play with its timing, since optimum timing is usually determined but the stages of development are still maintained. It does not make special sense to focus attention in this research on this type of option, since it is unusual to have contracts or rights that must be assessed in this respect.

This option could also be called the option to pause the investment. In fact, it will be exercised when market conditions, for some reason, show that the profit obtained from temporarily stopping the project at a certain time exceeds the cost of capital wasted in this waiting period.

1.10.4. Abandonment option

The option to abandon a project entails the option of selling the project at a certain time because of its low market value. This option occurs naturally, especially in phases between stages as described in the previous section. Conceptually, choosing the option to abandon a project implies having overvalued in some previous analysis the value of the project that is being abandoned. It is clear that it is frequently used, although it seems more logical to analyze the appropriateness of its exercise than its value in itself, since it is not possible to buy or sell it, as it is difficult to put a value on it, since its exercise is based on an undesirable assumption.

Therefore, this option would be exercised only when the current value of continuing is negative. In other words, the losses are greater if the project ends than if it is abandoned at that moment.

1.10.5. Option to change the nature of the investment or switch option

This is the option of modifying the nature or model of the business. An example is to conceive the business as an asset to stay in residential rent or as a building to demolish and to develop a project that maximizes its buildability. Both situations will offer a different profitability, a different cost and a different level of risk. Despite this, having this option at the time of purchasing the building adds value. The possibility of changing the nature of the investment may arise in many ways, for example, by maintaining a commercial use or reforming and transforming it into residential or even hotel use.

This type of option is, in my experience as a practitioner in the real estate sector, the most frequent, the most intuitive of those mentioned above, and the one that is most often valued (applying a more or less orthodox methodology) by the office of any real estate asset analyst. This is the reason why this doctoral dissertation focuses, both at the modeling level and at the case study level, on this type of option. This option is, in all its modalities, a call option with an execution cost equal to the cost of transforming one kind of asset into another. It is important that the possibility of transforming is always a right (even for a limited time) and never an obligation. The underlying asset to which the value of the option is linked is the potential transformation project.

Another interesting feature of the option to transform the nature of a real estate investment is that there can be more than one option of this type, and the fact of such multiplicity of options increases the value of the asset. However, in this respect it should be noted that the value of adding all the options together is less than the sum of adding them separately. This is interesting, given that this is frequent in the options, although in this case it is very intuitive. In addition, the fact of exercising one of the options modifies the value of the others, and of the asset itself, but does not make other options disappear and only changes their value.

They are always market needs which will lead to changes in the nature of the project. It is always the needs of the market that will lead to the change in the nature of the project. In this case, the comparison of the NPV of the project with one format or another is definitive. The NPV of that project must be added the value of those exchange options that are not lost by making the format change.

Chapter 2

Pricing Refurbishment and New Building Construction

2.1. Chapter's Introduction

The first part of this chapter considers and justifies the methodology to be applied. This methodology can be divided into two important sections, the choice of method and the choice of the proxies for the key variables. As with everything in this research study, the objective is to be governed by pragmatic and management-orientated approaches.

The second part presents a real case by way of an empirical application. It is a case that is representative of the daily work of any real estate investment analyst. This part ends by sketching the proposals made and discussing the conclusions reached.

2.2. Methodology

2.2.1. General features

On the one hand, most decisions based on the valuation of real estate projects take into account the value of the options of the asset or project, although such valuations in most cases represent a high degree of subjectivity. In this way, the main decision is essentially based on an analysis of the current value of future flows in a single scenario or business model.

On the other hand, the investor's returns required (always depending on the risk of the project) can be influenced by the presence of these options. This is intuitive, given that flexibility reduces the risk exposure. However, this influence is an estimate not systematized nor expressed in a concrete value. It is not usual to value that a decrease in the required profitability implies the presence of one or another concrete option. It is obvious that an increase in the flexibility of the investment should be reflected in a decrease in the required return, because it allows for higher adaptability to the market, and thus reduces the risk.

Most developers will consider the outcome of their business as a perpetual income, which leaves out any implied modification. Baabak (2007), in an exhaustive study carried out for MIT, observed that most promoters performed sensitivity analyses of their projects on different variables. These variables include: rental prices, interest rates, construction costs and the average time it takes to develop (in terms of the time of the sector cycle or as a result of possible bureaucratic obstacles). The influence of the switch option on the required returns is highly improbable.

2.2.2. The main associated risks

Given the previous reference to the discount rate and risk, it seems important to briefly summarize the main risks associated with development or investment in a real estate property. The first risk to be highlighted is the risk of development itself, since management errors (deviations in time or cost) or failures in the initial analysis of the project, as in any other type of investment, can thwart potential gains. The second risk derives from the market, both in terms of the transformation costs as well as the price changes. It may take longer to sell or rent than expected or it may be at a lower price. There are also risks of a technical nature (with the land, for example) or administrative risks (with licenses or the application of regulations). Finally, there is an interest rate risk (as in almost all investments), because the excessive length of the project financing time could lead to the required rates being modified and being insufficient for the market.

2.2.3. Analysis of the volatility of the underlying, a key factor

We have previously pointed out that the two parameters that will really be decisive for the quality of the outcome of our option value estimation are the value of the underlying and the volatility of the underlying. Volatility is, according to some authors (Copeland, 2003), the most difficult parameter to estimate when it comes to the theory of real options. The volatility of the value of the underlying is derived directly from the volatility of the income that the asset provides. Unlike in the case of a highly liquid or quoted asset being valued, real estate assets provide only historical data on other analogous assets that we must filter as proxies of the asset under analysis. There are different methodologies for doing this (Mun, 2005). The positive thing about them is that they are not exclusive, and we can compare the results among them so that their dispersion serves as a measure for their likelihood.

The most commonly used method is comparison with data from a proxy obtained from the market. This methodology has the advantage of being straightforward and easily assimilated by managers, but it is poor in detecting to what extent the proxy is good (has a comparable distribution).

Another manager-friendly methodology is to determine subjective parameters, using them in subsequent distribution simulations (also subjectively selected by managers). This produces a range of values within which this volatility can move. In this case, we would not have a single value, but we would construct several scenarios (for example, better, intermediate and worse scenarios). This gives us a price range for the option. The real validity of this methodology depends on the extent of this range.

It is also possible to perform a Monte Carlo simulation to find the distribution and standard deviation of the asset value. The "logarithm method of the current values of the cash flows" works on this basis, but its mathematical complexity makes it not particularly attractive for managers. In addition, it requires a subjective estimate of such flows and uses a single discount rate.

There are other methods ("logarithmic cash flow returns method" and "generalized autoregressive conditional heteroscedasticity", among others), but the three described above seem the most suitable ones.

It should be noted that, as far as the market is concerned, the location of a valid proxy (historical data of similar projects), from which sufficient values can be derived, would be a good start to substantiate the volatility data. Depending on the type of asset, such assimilation becomes more or less difficult.

2.2.4. The procedure: The Black and Scholes formula

The Black and Scholes method (also referred to as the Black-Scholes-Merton model when American options are also considered) (1973) describes a theoretical way of evaluating European-type options. It is worth noticing that in 1997 Merton and Scholes were awarded the Nobel Prize in Economics precisely for their contribution to derivatives pricing.

In a synthetic way, the formulation is as follows:

$$w(x,t) = xN(d_1) - Ke^{r(t-t^*)}N(d_2) \qquad (1)$$

$$d_1 = \frac{\ln\frac{x}{k} + (r + \frac{1}{2}v^2)(t^* - t)}{v\sqrt{t^* - t}} \qquad (2)$$

$$d_1 = \frac{\ln\frac{x}{k} + (r - \frac{1}{2}v^2)(t^* - t)}{v\sqrt{t^* - t}} \qquad (3)$$

Where:

w(x,t) is the estimated price of the option based on the price assigned to the underlying (*x*) and the year in which it is valued (*t*).

$N(d_1)$ is the cumulative normal density function.

K is the exercise price (strike price) of the option.

t is the year in which it is valued, being 0 the current moment and t * the moment in which it expires (always measured in years).

v is the standard deviation of the income of the asset (or its price).

r is the annualized risk-free interest.

Our objective is, therefore, to find a reasonable valuation of w(*x,t*) with sufficiently simple methodology. As we will see in the application example, with the case study below, of the different variables to be used, the price assigned to the underlying (*x*) and its volatility (*v*) are those of more complexity and controversy in the determination.

2.3. Application: The Case Study description

2.3.1. Main case features

A case is described below, in order to develop a practical application of this theory. The choice of this real case as an example responds to the high frequency with which similar situations occur, especially in central areas of European cities, as is the case of the Barcelona Eixample, where the asset under analysis is located.

Briefly, it can be summarized as follows: it is a 70-year-old, partially rehabilitated housing building. In order not to compromise confidential data, the exact address is not detailed in the present analysis.

At the time of its acquisition, the possibility of carrying out a reform of the building that would imply an increase of the income that it produces is being considered. Alternatively, urban regulations allow the demolition of the building to build a larger new one (larger number of square meters), which means a significant increase in rents. Obviously, both actions have an additional cost and involve the loss of certain revenues that stop being collected by the time the transformation is being undertaken. Therefore, whoever acquires the asset must value it not only for its current incomes, but also must add the potential of future income in case of transforming the asset. This is obvious, and any professional in the sector analyzes it when acquiring the asset. The difference here is the proposed systematization and the use of real options theory. As presented in the following scheme, there is an initial situation at the time of purchase (0) a Transformation Option 1 and a Transformation Option 2. The asset valuation currently (0) implies adding to the NPV of the current rents the value of the transformation options. This implies calculating the value of the combination of them (that is, those that are compatible). The valuation of the asset after undertaking the complete refurbishment assumes the valuation of the new incomes and continues including also the value of the option of constructing a new building. The value of the asset as a new building, in which we assume a final asset (the new building) that optimizes the asset's income, would not include options in its valuation.

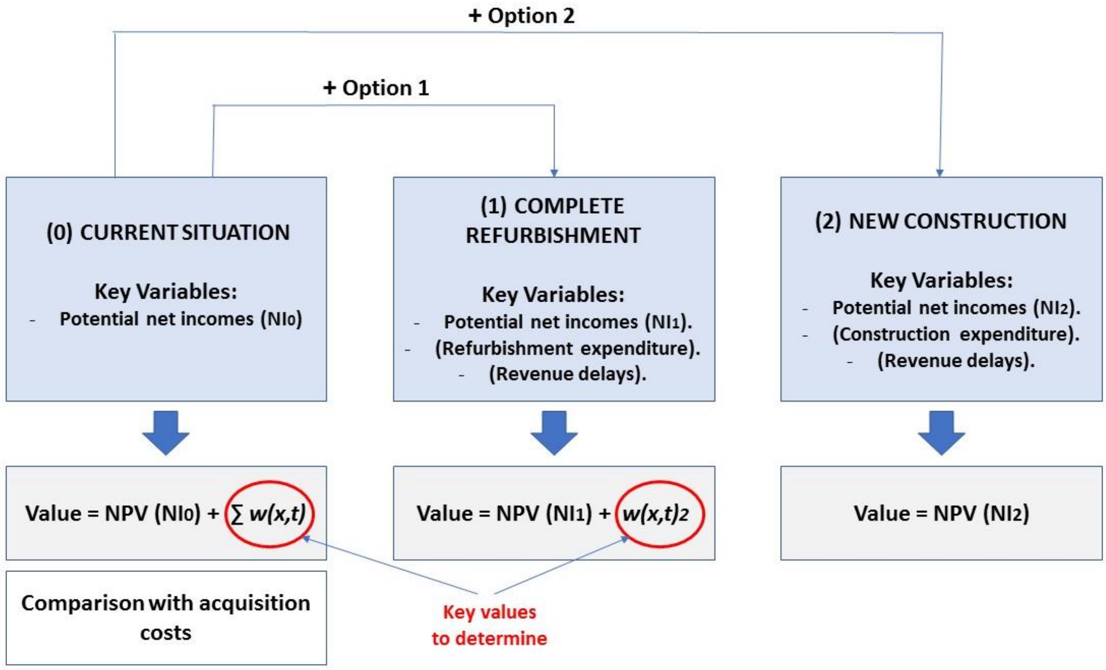


Figure 2.1: Options scheme. Own elaboration

It can be observed that a poorly systematized valuation can induce errors in the valuation, and in fact they this happens frequently.

2.3.2. Case key values

Within the valuation that we want to undertake, there are different key values, some of which must be determined according to the market:

• Revenues that would be obtained in each of the scenarios. This requires a market study of the area. For our analysis we will take that the rents of the concrete building that is analyzed are, previous to the rehabilitation (0), would be of about 16.0 € / sqm / month (Residential); 10.0 € / sqm / month (Retail).

• Cost of transformation (CAPEX of transformation). In this case this cost of transformation is what we will understand as part of the exercise price of the corresponding option. These have been estimated in historical databases and experience of previous projects and valuations of construction companies. The detail is presented in table 2.

• Cost discount rate, which is equivalent to the cost of capital of the transformation. In this case, we can take as a reference in 2017 a 10%.

• Rate discount of the income for the valuation of the asset. In 2017, the return required by an investor in the environment of the building under analysis is a minimum of 4%.

• The risk-free interest rate. For this variable, and to compare with an asset whose relevant time horizon is comparable, the interest rate of a 30-year German government bond is taken, the average yield of which in the first quarter of 2017 is approximately one 1%.3

The value of the asset in either state will be derived from the income. For the analysis of income estimates, there are numerous tools that are based on the market study. For the present analysis, we will take the described ones. This leaves only one variable left to determine: the standard deviation of the market price.

2.3.3. Why Black and Scholes formula?

The method formulated by Black and Scholes is not the only one possible. Indeed, in business practice of applying real options the method of binomial tree analysis (Cox, Ross and Rubinstein, 1979) is applied more frequently. This may be since, at the same time, this methodology has many advantages as a decision tool for a possible scenarios' succession (Smit 1997).

Both models are based on the construction of a risk-free portfolio that replicates the possible returns of such an option. This portfolio's value will be the value of the option. In the case of the Black and Scholes formula, although it counts with variants due to other authors, it was originally designed for the calculation of European options that do not pay dividends.

Despite emphasizing its straightforward implementation compared to other methods, its use will continue to require a "leap of faith" among non-experts. However, the simplicity of its application (once the variables have been determined), combined with the fact that this method is being applied for some decades, facilitates such a "leap". In the world of real estate investment, I doubt that one in a thousand-people using the formula of perpetual income would be able to deduce it. Despite this, the professionals use it daily. I believe the reasons are two: it is not mathematically debatable, and its simplicity of application is extraordinary.

[3] Bloomberg source https://www.bloomberg.com/markets/rates-bonds/government-bonds/germany

We still must consider whether the conditions of its use fit the profile of the option to which we want to use it. On the one hand, the exercise of the option is limited to a specific moment (because it is European) and, on the other hand, it does not have to pay dividends (or, in any case, these should not be significant for the value of the option). Let us focus on the specific case of option that we have described in part 2.3.1.

Existing legislation regarding rental in Spain, and in other countries such as Italy or Belgium, requires a minimum duration of rental contracts. Depending on the country, 3 or 4 years, or even more. In other countries, such as the UK or France, the minimum contracts are for one year. In any case, the option of obtaining an income and, meanwhile, having the option of transforming the asset at any time (which would resemble an American option) is not possible. Usually, when the owner of the property is obtaining rents, and intends to have the transformation option, he or she should devote a period to "empty" the building, that is, to wait for to contracts to expiry progressively.

Two facts are derived from this circumstance: The first one is that the opportunity to transform will show up (or will be achieved) during a very limited time window. The second is that, from the moment that it is decided to try to raise this option (empty or keep empty the building), until that temporary window occurs, the income will be decreasing. In the final stage, income will be very small (or even negative in net terms). All this considering that having an empty building, without any use or activity, not only has little investment sense, but also poses a moral problem, since it would be mere speculation.

Therefore, the most correct way is, in the mid-term, to view the option as a European option whose dividend during the option’s life is negligible (considering, as noted before, that the P/E ratio of real estate assets is especially high).

In other words, in this context the option always exists, but it must be "activated" in advance, giving a specific date (or time) of exercise in which the strike price must be paid. From this perspective, it cannot be an American option.

2.4. How should be interpreted the value of the options we seek?

It is convenient to contextualize the meaning of the value of the options we seek. At the time of acquisition of the asset, in its 0 state, at the same time, we acquire the right to buy the rehabilitated building at a certain time and the new building. We acquire, therefore, two call options.

Although we are considering that we acquire two options when in fact we are acquiring many more. Because, in the course of time, in case of not exercising these options and taking other options of asset management, we would be obtaining others that could be exercised in a longer term. In any case, such longer-term options imply a higher level of uncertainty. This higher uncertainty is due to possible changes in the market and to possible changes in the regulations affecting the projects.

It is convenient to distinguish carefully between cost, value and price in each of the elements that we will find. We need to be careful because these are concepts that interrelate and can be confused.

Indeed, what we are valuing is not the potential profit (expected EBITDA) that one or another transformation entails. This profit is not an option inherent to the purchase, but a business margin. This business margin should be valued by those who must consider the possible sale / rental price of the transformed asset, the costs and the expectation of profit that the business has. Neither confuse the value of the option with the difference between EBITDA's or NPV of different ways you can take the project, or the differences between them.

The value of the option, as we shall see, derives in this case from the potential change in the value of the underlying asset in the market. The NPV difference between one form of project or another (refurbish or build again, for example), is not related to the value of the option, in any case it could be the cause of the option being exercised.

The option to start a business is actually being bought, but at the moment of acquiring the building in state 0, it is not known whether it should be done, whether the optimal financing or market conditions will exist, whether the buyer will be found, etc. All these aspects are part of much more complex considerations in business.

What is certain is that, for instance, at the time of purchaising the building in its state 0, the buyer is also acquiring the option to have the finished building for a price that consists of:

The cost of the building in its state 0 + the transformation costs + the cost of capital of both investments until the new building was 'acquired'. These three components, that will be calculated in the following sections, build up the exercise price of the Call Option adjacent to the purchase of the asset in its 0 state. The value of that option is what we intend to determine, and this depends, as we shall see, largely reflecting the market's expectation of the option's lifetime. In the next section, we will quantify the case under analysis and try to clarify the key aspects of the calculation.

2.5. Option valuation process

In this section we define the steps to determine the value of the different options presented above.

2.5.1. Computing the cost of the asset in every stage

In the case of the asset in its 0 state, we know its cost because it corresponds to its acquisition price plus acquisition costs:

TOTAL EXISTING BUILDING ACQUISITION COSTS	**4.475.100**
Building Price	4.250.000
Notary and Registry	5.600
Taxes and stamps	76.500
Commercial Fees	120.000
Due Diligence	23.000

Table 2.1: Existing building acquisition cost detail

In order to know the cost of the asset in each of the possible subsequent states (1 or 2), we must add the transformation costs of all types and the capital cost of investing the total amount of the investment until the building is transformed (see Table 2).

We already have, therefore, what our total costs of the asset are in the three states in which we have the option to have/acquire it.

Regarding the price of the building, corresponding to the price paid in this real case, note that, as detailed in Table 3, if we demand a 4% gross return to the asset this is not achieved (especially given that part of the current use of the building is office).

This indicates that, logically, the market price has been intuitively increased on the basis of the value that has been intuitively given, among other factors, to these options.

It should be specified that the price finally paid differed by less than 2.5% from the other offers received by the seller. Therefore, it could be expected that the capitalization value of the income that the asset can offer today plus the value of the options we are estimating should closely resemble the price paid.

	1 (Refurbish)	2 (New Constr.)
TOTAL CAPEX & OPEX + COST OF CAPITAL	**1.252.725**	**4.396.237**
Construction costs	***338.640***	***2.246.121***
Construction license	12.640	61.740
Demolition	0	48.231
Urban development	0	10.000
Construction costs	316.000	2.106.150
Service connections (water, medium voltage networks,…)	10.000	20.000
Management, fees and stamps	***53.815***	***422.528***
Demolition's management fee	0	4.823
Project Management	7.584	121.923
Construction Management	13.272	88.458
Architect's fee (Project and construction surveillance)	22.120	103.201
Building Engineer's fee (construction surveillance)	0	44.229
Health and safety coordinator's fee	6.794	33.185
Taxes and stamps	1.264	6.174
Administrative authorizations (facilities-certification body)	2.781	18.534
Topography	0	2.000
Insurances	***0***	***39.345***
Decennial insurance	0	26.708
Technical Control Office	0	12.637
Procedures and charges	***113.163***	***209.687***
Occupancy Certificate	1.200	15.435
Notary - New construction + property division and deed of sale	13.410	17.726
Registry - New construction + property division	10.728	14.180
Other taxes (Public thoroughfares occupation permissions…)	3.160	15.435
Real property tax (landplot)	11.295	24.601
New construction taxes	4.740	23.153
Property division taxes	67.050	88.628
First Occupancy License	1.580	10.531
Cost of capital	***747.108***	***1.478.556***
Considering 10% over the whole investing	747.108	1.478.556

Table 2.2: Transformation / investing cost details

Situation	**Tipology**	**Sqm to rent**	**€/Sqm/ Month**	**Yearly Rent**	**Yield**	**Asset Valuation**
0	Residential	500	14,00 €	84.000,00 €		
	Retail / Office	700	10,00 €	84.000,00 €		
	TOTAL			**168.000,00 €**	**4,00%**	**4.200.000,00 €**

Table 2.3: Asset valuation considering a 4% yield

2.5.2. Analysis of timing: The execution window

As we have indicated, we must consider that it will only be possible to acquire the building in each of its states at a certain moment. Within this period we will consider approximately the term to empty the building and transform it (since obtaining the licenses can be simultaneous to the emptying of tenants).

For the present case, the leases in force have a duration of one year (enough time to obtain a license of any kind). The estimated times of transformation are 6 months for the rehabilitation and fourteen for the transformation into a new building. In the case of new lease agreements we consider 3 years of duration. In case of refurbishment, it would be considered that after rehabilitating the option to acquire the building of new construction is extended 4.15 years after the completion of the rehabilitation (3 years of rental contract plus fourteen months for the transformation).

Combining these times with the contents of Figure 2.1, we have the time summary shown in Table 4.

Option expiration times	
a) Refurbishment	0,50
b) New building transformation	1,15
c) Period until the expiration of the lease agreements	1,00
d) Duration of a new lease	3,00
Time until refurbishment (a+c)	***1,50***
Time until transformation in a new building (b+c)	***2,15***
Time in case of rehabilitating and then transforming into a new building (a+b+c+d)	***5,65***

Table 2.4: Option expiration times considered

2.5.3. Identification of each variable

At this point we are in a position to identify each of the variables that we must use to calculate the different options.

To calculate the exercise price of the option (K), this is the cost at the time of exercising the option, it will be the sum of the cost of acquisition plus the transformation cost plus financial cost.

Calculating the current cost (x), presents an added difficulty: it is impossible to acquire that asset today as it does not exist. What seems most logical is to consider that, since if it existed we would require at least 4% of the cost of our investment, we will use the discount rate to update the cost. In order to accept this proposition, it is necessary to understand that we are not talking about investing today (at risk) and receiving that building (new or rehabilitated) tomorrow, we are actually talking about acquiring today exactly what we can only acquire within a term " *(t*-t)* ", without any type of uncertainty that increases the discount rate.

This consideration, which may be debatable or arbitrary, is in any case a reduction in the value of the options, so that its application is prudent if we are to estimate an option value that is as credible as possible.

In the case of exercising option 1 first and then option 2, for the calculation of *k* we must deduct the income of the 3 years in which they remain rented. These will be counted as 4% of the cost of exercising option 1. The risk-free interest rate estimate is considered at 1% for the entire period. We will have to determine the standard deviation of the cost of the captive (in this case our price), an issue that will be examined below. Table 5 shows already calculated this summary of the variables to be used in the formula, determined so far:

Option 1 Variables		*Option 2 Variables*		*Option 2 (after executing 1) Variables*	
x =	5.361.664,52 €	**x =**	8.103.367,50 €	**x =**	7.186.961,83 €
k =	5.686.560,26 €	**k =**	8.816.317,35 €	**k =**	8.969.819,54 €
τ = (t*-t) =	1,50	τ = (t*-t) =	2,15	τ = (t*-t) =	5,65
r =	1%	**r =**	1%	**r =**	1%

Table 2.5: Option variables summary

2.5.4. Sensitivity analysis as a function of the standard deviation

Before determining the value of the standard deviation, it is worth pausing to analyze what is the sensitivity of the results we will get to the value of that parameter. Considering values between 0% and 10% of annual price variation. As the reader can see, we only see positive values. Obviously, if the investor expects that the cost will decline in the future (bullish market expectations) the decision of the investor would be expected to acquire in the future. Therefore, it does not make sense to perform the analysis considering negative values of *v*.[4]

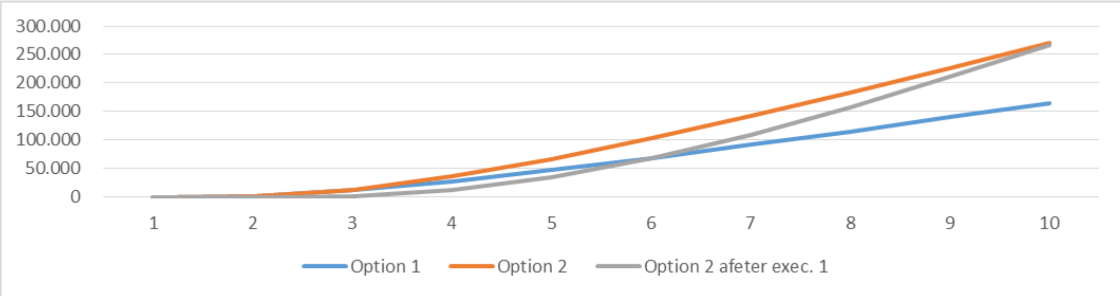
[4] In any case, if we performed the analysis for negative values of *v* we would find only negative values of w (x, t).

With these considerations, we obtain the following table of possible values:

v	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
Option 1	3	1.968	11.461	27.222	46.672	68.296	91.271	115.131	139.597	164.493
Option 2	0	1.410	12.669	35.608	66.557	102.567	141.842	183.302	226.280	270.345
Option 2 after 1	0	24	1.948	12.402	34.794	67.965	109.572	157.520	210.210	266.472

Values in €

Table 2.6: Options values regarding different values of v.



Graph 2.1: Options values regarding different values of v.

2.5.5. Determination of the standard deviation

Table 6 shows the importance of the correct determination of the annual standard deviation that the costs can suffer (also the price). How is the determination of *v* more coherent?

Part of the cost of each of the transformed assets (approx. 70%) will vary with the rental market prices of the area, let us call that year-on-year variation *v1*.

Part of the cost (approx. 20%) will vary with the variation of construction/promotion costs, we will call *v2* to such variation.

Finally part of the costs (approx. 10%) are due to the cost of capital in an investment of this level of risk, to this third component we will call *v3*.

It should be noted that our intention should be to make a conservative estimate of such data. Recall that higher volatilities increase the value of the options.

Regarding the volatility of construction costs, these are similar to CPI's forecast for the coming years. According to the Spanish INE, this is about 3% for the territory of Catalonia.[5] The volatility of the cost of capital for the coming years is linked to the increase in rates, which is especially

[5] http://www.funcas.es/Indicadores/Indicadores_mapa.aspx?Id=2

uncertain. In any case, yearly increases above 1% are not expected according to the majority of analysts.[6]

In the case of the price of the house, its estimated variation for the next years is much more local (due more to the area). To analyze the evolution in the area under analysis we will use a database belonging to a web of marketing and rental of dwellings that allows us to delimit the area.[7] In it we can observe that the year-on-year price increase in the area of the *Ensanche* has been (to March 2017) around 35% for sale and 16% for rent. Obviously, it is a timely data due to the conjuncture and introduces the suspicion that a price correction may occur.

Although this makes it difficult to find an estimate of objective volatility, it is clear that in a more volatile environment such as the current one, the consideration and use of options becomes more relevant.

In order to be prudent, we will take half the volatility of last year's rental price: this is 8%.

Finally, and for simplicity, we consider that the three variables are independent and, for the calculation of the volatility of the set, we use the following formula:

$$Being\ \mathrm{X}_1, \mathrm{X}_2\ and\ \mathrm{X}_3\ the\ components\ of\ the\ variables\ X$$

$$Var(0.7\mathrm{X}_1 + 0.2\mathrm{X}_2 + 0.1\mathrm{X}_3) = Var(0.7\mathrm{X}_1) + Var(0.2\mathrm{X}_2) + Var(0.1\mathrm{X}_3)$$

$$v = \sqrt{0.7^2 \cdot 0.08^2 + 0.2^2 \cdot 0.03^2 + 0.1^2 \cdot 0.01^2} = 0{,}0563\ \ or\ 5.63\% \qquad (4)$$

2.5.6. Pricing the different options

With the values proposed, based on the estimation of a standard deviation of 5.63%, the values of the respective options are:

OPTION 1	60.168,57 €
OPTION 2	88.894,73 €
OPTION 2 AFTER 1	54.683,99 €

[6] See Bankinter's Blog: https://blog.bankinter.com/economia/-/noticia/2016/8/31/prevision-tipos-interes-europa
[7] See Habitaclia's Analysis: http://www.habitaclia.com/informes/precio-medio-viviendas-distrito_eixample-barcelona.htm

In situation 0, that is to say, at the time of the purchase of the building, there are two possibilities that maximize the value of the compatible options: exercise option 1 and then option 2 (€ 60,168.57 + € 54,683.99 = € 114,852.56), or either directly run option 2 (€ 88,894.73). Contrary to what might seem, as far as options are concerned, the first combination outweighs Option 2 alone. We understand, therefore, that this is the value we acquire in terms of options when it comes to acquiring the building and we represent it in Figure 2.2.

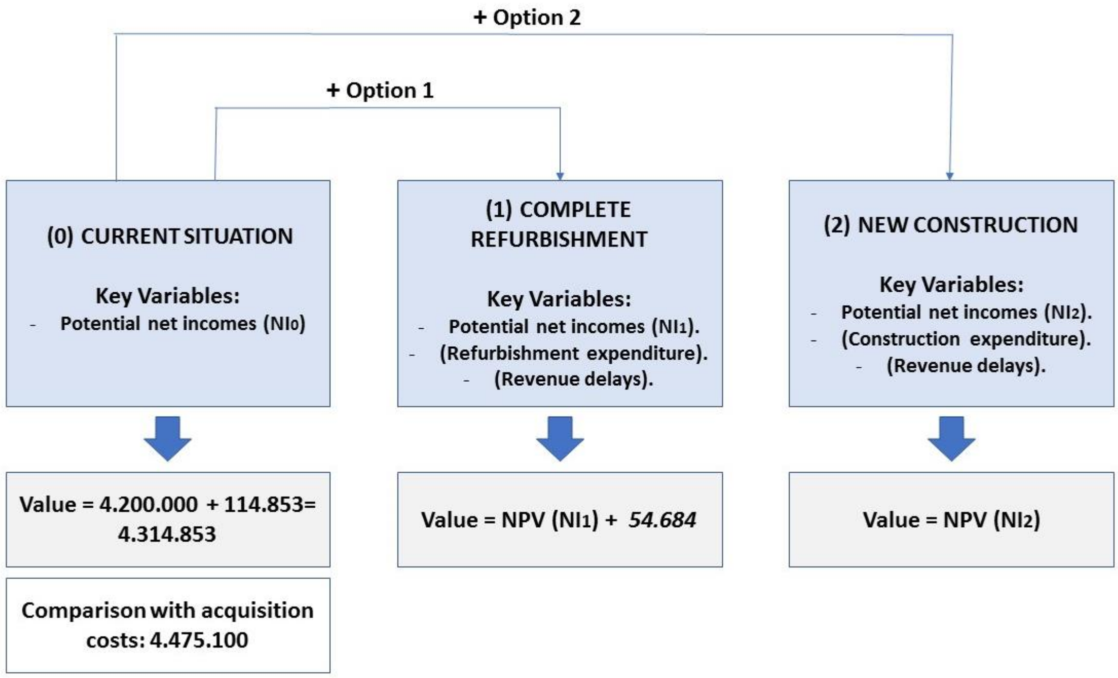


Figure 2.2: Options value scheme. Own elaboration

If we take a look at Table 6, we see that if we consider an 8% as *v* value, the addition of the two options already makes the value of the asset (valued only as an update of flows plus options) almost equal to the acquisition cost.

2.5.7. Price sensitivity regarding the different variables

The results obtained for the value of the indicated sum of options may suffer variations with respect to 4 factors basically: the volatility of the underlying asset (*v*), which has already been mentioned, but also due to changes in the risk-free interest rate (r), the exercise time of the option and (*T*), and finally by variations in the strike price (*K*).

Previously, we have considered volatility as the most determining factor. However, it is interesting to perform a combined sensitivity analysis of this volatility with the other factors. In this, we will see that it is not clear that the main influence on the value of the option is the volatility. In order to deepen into this analysis, we have carried out three additional sensitivity studies, which are shown in the figures below.

The first combined analysis, see Figure 2.3 below, includes the variable r. It assumes that *r* grows up to 900%. In view of the results, each percentage point of changes in the risk-free interest rate generates a change in the value of the option that is much more significant than that caused by the same change in volatility.

In Figure 2.4 we see a combined sensitivity analysis of the volatility with changes of the strike price. In this case, we have made small percentage increases in the cost of exercising the option. We see again that these small changes generate very significant changes. In fact, for small volatilities, by slightly increasing the cost of exercising the option, the option becomes worthless. We also see that, with the increase in volatility, the sensitivity to *K* is being reduced progressively. In the graph, we see a surface with a double slope.

Finally, Figure 2.5 portrays a sensitivity analysis that has been carried out combining volatility with the term of exercising the option. In this case, the same percentage changes that have been applied to the strike price have been applied to the period. Although now the effect of changes in *T* is less significant than the effect of changes in *K* was before (besides being an opposite effect), we continue to observe that as volatility increases in the effect of the change of T is reduced.

Thus, it is obvious that it is not possible to compare which of the factors exerts a more significant influence on the result, basically because they refer to very different concepts and it is not possible to establish clear parameters of comparison among them. However, the conclusion of the three previous analyses shows that changes in volatility not only have a decisive effect on the value of the option we have calculated here, they also influence significantly the ability of the other factors to affect the result.

We see, therefore, that the importance of analyzing volatility does not lie so much in the fact that its variations result in changes in value. It is also important for its effect on the influence on other variables and, above all, it stands out as the most controversial parameter to be determined.

We can say that it would be easy to reach a consensus on the value of the other parameters, although we have already seen what the important effect of small variations of these is. However, in the case of the volatility of the underlying, an uncertainty linked to the evolution of the market and, often, unpredictable, makes it a disputed variable. It is possible that any analysis requires working with a sensitivity study based on this variable.

Sum of option value sensitivity analisys according to r and v

	1.00%	2.00%	3.00%	4.00%	5.00%	6.00%	7.00%	8.00%	9.00%
1.63%	549.20 €	3,862.97 €	27,555.24 €	173,495.28 €	529,997.35 €	963,458.69 €	1,389,483.35 €	1,797,150.05 €	2,185,767.32 €
2.63%	7,716.72 €	25,770.60 €	90,785.79 €	267,356.57 €	582,723.12 €	980,757.82 €	1,395,219.95 €	1,798,997.48 €	2,186,273.06 €
3.63%	28,115.52 €	70,330.92 €	169,899.78 €	361,354.17 €	653,413.19 €	1,017,616.14 €	1,411,826.96 €	1,806,733.12 €	2,189,907.31 €
4.63%	64,429.55 €	131,677.01 €	255,876.92 €	455,371.29 €	732,430.07 €	1,070,152.19 €	1,441,673.46 €	1,822,910.54 €	2,198,924.15 €
5.63%	114,852.56 €	203,763.04 €	345,291.03 €	549,366.54 €	815,742.44 €	1,133,177.13 €	1,483,682.61 €	1,848,905.45 €	2,214,773.30 €
6.63%	176,178.18 €	282,716.92 €	436,627.57 €	643,317.13 €	901,500.85 €	1,203,076.70 €	1,535,348.29 €	1,884,446.90 €	2,238,331.66 €
7.63%	245,585.48 €	366,204.08 €	529,122.05 €	737,206.71 €	988,755.53 €	1,277,583.04 €	1,594,301.95 €	1,928,287.20 €	2,269,585.53 €
8.63%	320,958.88 €	452,788.15 €	622,347.01 €	831,021.68 €	1,076,971.12 €	1,355,269.56 €	1,658,702.01 €	1,978,961.60 €	2,307,886.09 €
9.63%	400,777.63 €	541,551.74 €	716,043.41 €	924,749.74 €	1,165,822.37 €	1,435,212.73 €	1,727,195.98 €	2,035,139.49 €	2,352,325.44 €

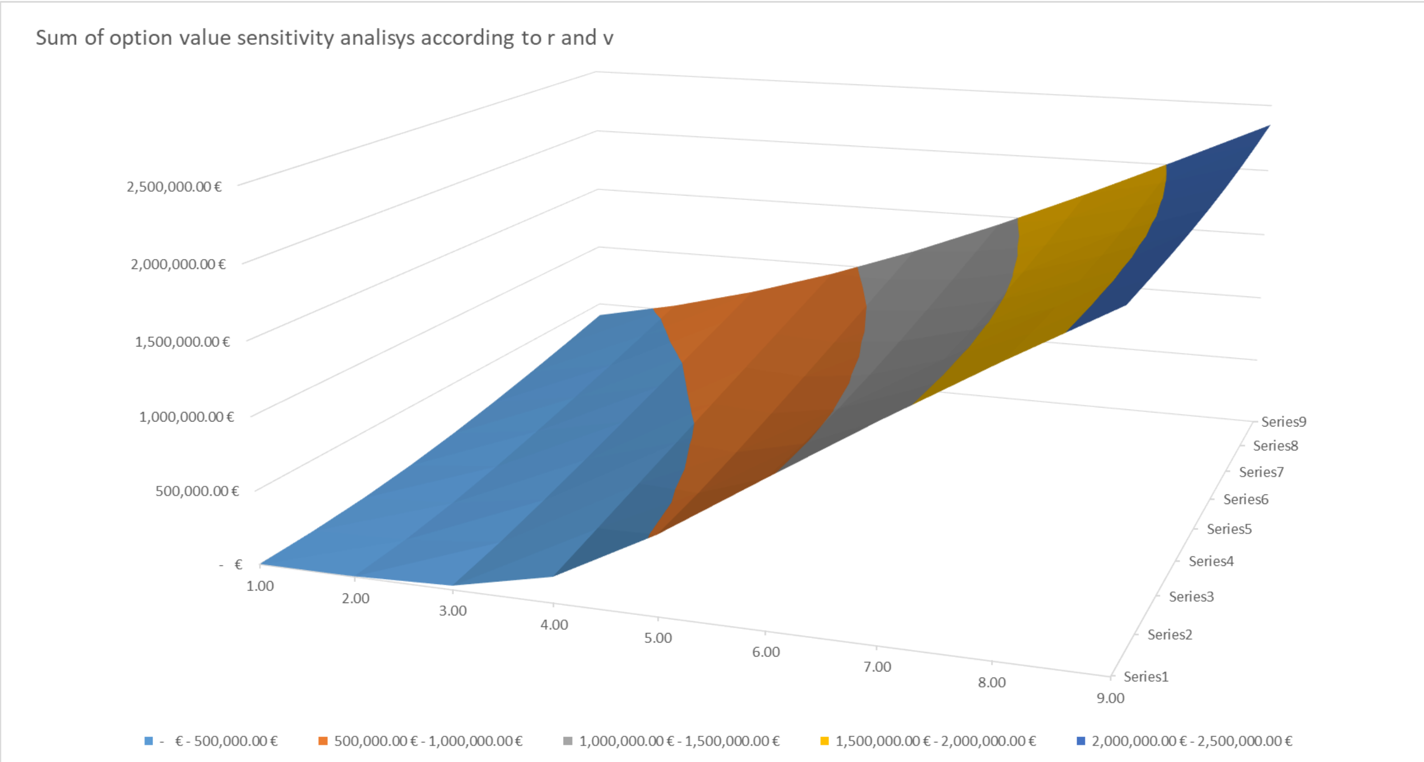


Figure 2.3: Sum of the option value sensitivity analysis according to r and v

Sum of option value sensitivity analisys according to K increase and v

	100.00%	102.00%	104.00%	106.00%	108.00%	110.00%	112.00%	114.00%	116.00%
1.63%	549.20 €	21.99 €	0.41 €	0.00 €	0.00 €	0.00 €	0.00 €	0.00 €	0.00 €
2.63%	7,716.72 €	1,848.25 €	357.56 €	60.47 €	10.39 €	2.09 €	0.48 €	0.11 €	0.03 €
3.63%	28,115.52 €	12,369.88 €	5,065.22 €	1,992.64 €	783.08 €	317.37 €	133.92 €	58.14 €	25.47 €
4.63%	64,429.55 €	37,333.54 €	20,948.53 €	11,530.60 €	6,313.53 €	3,480.14 €	1,944.43 €	1,101.91 €	630.86 €
5.63%	114,852.56 €	77,081.67 €	50,837.23 €	33,152.01 €	21,513.52 €	13,971.89 €	9,118.33 €	5,991.68 €	3,963.90 €
6.63%	176,178.18 €	129,194.10 €	93,738.55 €	67,521.18 €	48,449.80 €	34,740.01 €	24,953.92 €	17,985.91 €	13,017.42 €
7.63%	245,585.48 €	190,919.04 €	147,393.49 €	113,232.17 €	86,737.40 €	66,375.45 €	50,823.94 €	38,986.35 €	29,982.54 €
8.63%	320,958.88 €	259,964.42 €	209,561.07 €	168,348.54 €	134,951.15 €	108,079.51 €	86,571.71 €	69,414.78 €	55,750.06 €
9.63%	400,777.63 €	334,585.00 €	278,385.39 €	231,053.86 €	191,465.96 €	158,541.99 €	131,279.44 €	108,773.70 €	90,228.12 €

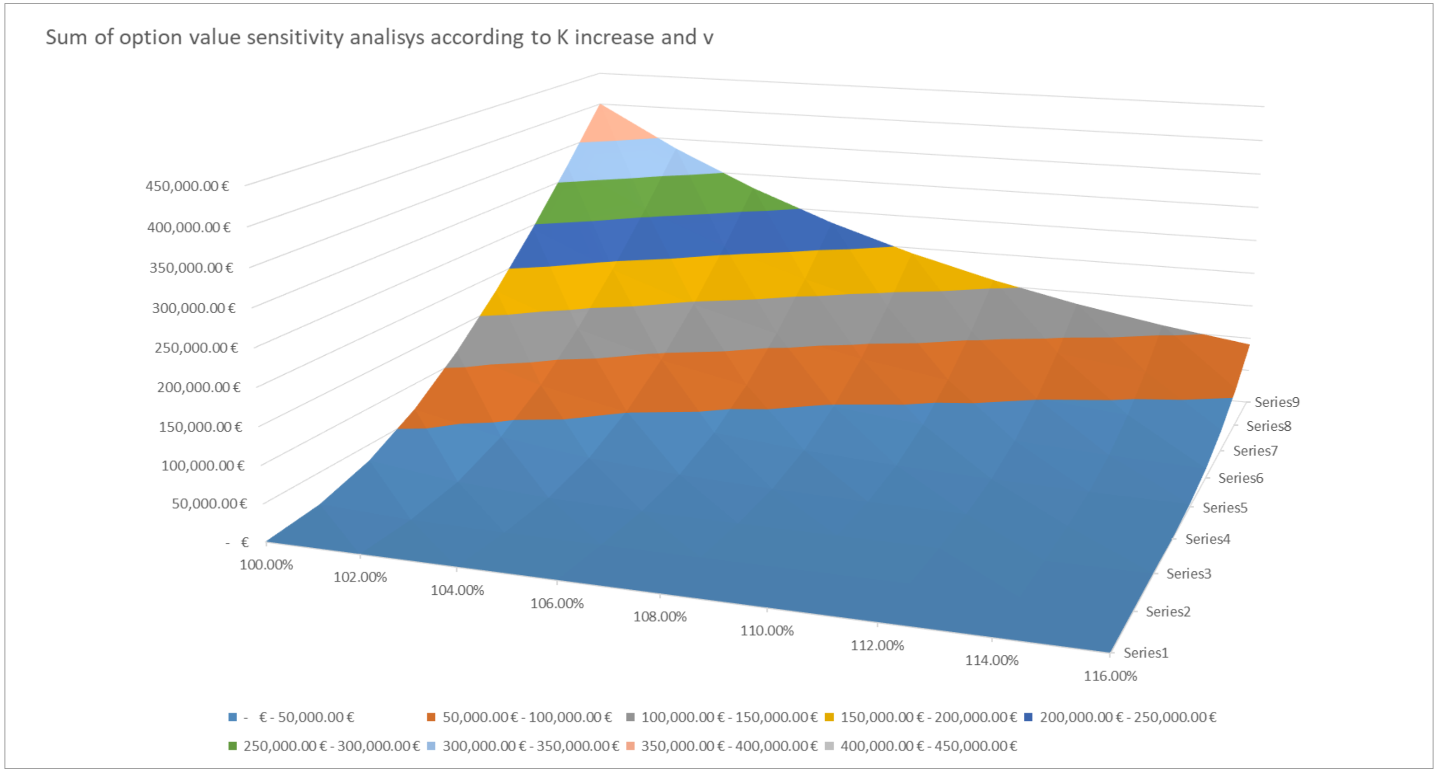


Figure 2.4: Sum of the option value sensitivity analysis according to K increase and v

Sum of option value sensitivity analisys according to T increase and v

	100.00%	102.00%	104.00%	106.00%	108.00%	110.00%	112.00%	114.00%	116.00%
1.63%	549.20 €	614.87 €	685.80 €	762.19 €	844.25 €	932.19 €	1,026.22 €	1,126.53 €	1,233.36 €
2.63%	7,716.72 €	8,246.63 €	8,800.08 €	9,377.77 €	9,980.36 €	10,608.56 €	11,263.06 €	11,944.56 €	12,653.76 €
3.63%	28,115.52 €	29,677.11 €	31,288.97 €	32,951.36 €	34,664.52 €	36,428.65 €	38,243.91 €	40,110.45 €	42,028.35 €
4.63%	64,429.55 €	67,425.46 €	70,484.47 €	73,605.77 €	76,788.53 €	80,031.94 €	83,335.16 €	86,697.37 €	90,117.73 €
5.63%	114,852.56 €	119,410.46 €	124,029.19 €	128,707.21 €	133,443.02 €	138,235.17 €	143,082.24 €	147,982.84 €	152,935.62 €
6.63%	176,178.18 €	182,284.81 €	188,441.21 €	194,645.58 €	200,896.21 €	207,191.42 €	213,529.64 €	219,909.34 €	226,329.06 €
7.63%	245,585.48 €	253,179.07 €	260,807.18 €	268,468.07 €	276,160.10 €	283,881.71 €	291,631.42 €	299,407.81 €	307,209.54 €
8.63%	320,958.88 €	329,968.23 €	338,995.13 €	348,038.10 €	357,095.77 €	366,166.82 €	375,250.04 €	384,344.27 €	393,448.42 €
9.63%	400,777.63 €	411,135.87 €	421,494.21 €	431,851.53 €	442,206.82 €	452,559.11 €	462,907.53 €	473,251.24 €	483,589.50 €

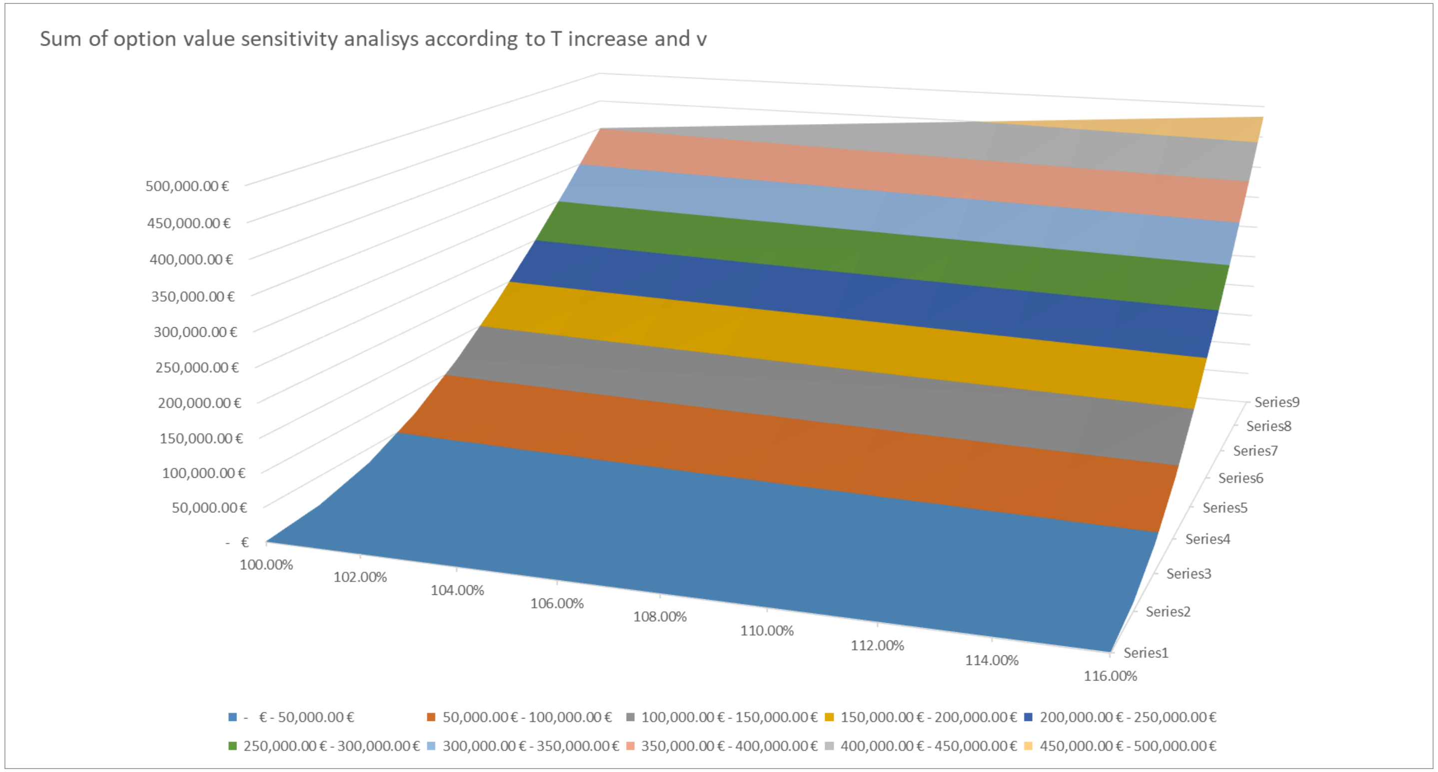


Figure 2.5: Sum of the option value sensitivity analysis according to T increase and v

2.6. Chapter conclusions and final remarks

In this chapter, we have analyzed a frequent case in the real estate market, with real data. The methodology used allows for replication by any analyst and real estate practitioner.

It is worth summarizing and highlighting some aspects:

• In all the considerations that may be arbitrary, we have opted for the alternative that the value of the option would decrease. This has meant that the value of the resulting option can be considered as 'minimum' and, at least, hardly questionable downward.

• This is the part of the value that is not usually considered and, logically, as it always happens in the options market, its value depends on the volatility of the asset, i.e., on market expectations.

This real-world orientated approach can be useful as a basis for the analysis of the presence of options in more complex investment projects. The fact of adapting a standard methodology such as the Black and Scholes equation incorporates important advantages. On the one hand, it facilitates its use for analysts while, on the other hand, it generates trust to the potential investor in the value that is added to the project or at the price that should be paid for the option. As long as the characteristics of the real estate project match with the characteristics of the Black and Scholes setting, this pricing alternative becomes a good choice.

After analyzing the sensitivity of the result to the different variables, we have observed that the volatility of the underlying asset plays an important role in the result. This is not only directly influential on the value of the option, it is also determinant for the capacity of the other variables to influence the result.

Nevertheless, once this approach has been analyzed and applied empirically to a real estate business case, further research must explore other valuation alternatives to evaluate their potential scope and the possible trade-off between reliability, complexity and applicability by managers.

Chapter 2 Annex 1: Table 2.2. Details

	1 (Refurbish)		
TOTAL BUILDING PRICE + CAPEX & OPEX	**4,705,618**		
TOTAL CAPEX & OPEX + COST OF CAPITAL	**1,252,726**		
TOTAL CAPEX & OPEX OF EXECUTION	**505,618**		
Construction costs	***338,640***		
Construction license	12,640	2.5%	Of Construction costs + Townhall managerial tax
Demolition	0	N/A	
Urban development	0	N/A	
Construction costs	316,000	According to builder's budget	
Service connections (water, medium voltage networks,…)	10,000	Estimative according to project's tipology	
Management, fees and stamps	***53,815***		
Demolition's management fee	0	N/A	
Project Management	7,584	2.4%	Of Construction costs per year
Construction Management	13,272	4.2%	Of Construction costs
Architect's fee (Project and construction surveillance)	22,120	7.0%	Of Construction costs
Building Engineer's fee (construction surveillance)	0	N/A	Due to intervention tipology
Health and safety coordinator's fee	6,794	30.0%	Of Architect's fee
Taxes and stamps	1,264	0.4%	Of Construction costs
Administrative authorizations (facilities-certification body)	2,781	0.5%	Of total capex and Opex
Topography	0	N/A	Due to intervention tipology
Insurances	***0***		
Decennial insurance	0	N/A	Due to intervention tipology
Technical Control Office	0	N/A	Due to intervention tipology
Procedures and charges	***113,163***		
Occupancy Certificate	1,200		60 € per Unit
Notary - New construction + property division and deed of sale	13,410	0.2%	Of total building price & capex and Opex + Notary
Registry - New construction + property division	10,728	0.2%	Of total building price & capex and Opex + Notary
Other taxes (Public thoroughfares occupation permissions…)	3,160	Estimative according to project's tipology	
Real property tax (landplot)	11,295	Yearly payment, accodind to latest payments	
New construction taxes	4,740	1.0%	Of construction cost + notary costs
Property division taxes	67,050	1.0%	Of construction cost + notary costs
First Occupancy License	1,580	0.5%	Of Construction costs (Technical fee)
Cost of capital	***747,108***		
Considering 10% over the whole investing	747,108		

	2 (New Constr.)		
TOTAL BUILDING PRICE + CAPEX & OPEX	**7,117,681**		
TOTAL CAPEX & OPEX + COST OF CAPITAL	**4,396,237**		
TOTAL CAPEX & OPEX OF EXECUTION	**2,917,681**		
Construction costs	***2,246,121***		
Construction license	61,740	2.9%	Of Construction costs
Demolition	48,231		According to builder's budget
Urban development	10,000		Estimative according to project's tipology
Construction costs	2,106,150		According to builder's budget
Service connections (water, medium voltage networks,…)	20,000		Estimative according to project's tipology
Management, fees and stamps	***422,528***		
Demolition's management fee	4,823	10.0%	Of Demolition costs
Project Management	121,923	2.4%	Of Construction costs per year
Construction Management	88,458	4.2%	Of Construction costs
Architect's fee (Project and construction surveillance)	103,201	5.0%	Of Construction costs
Building Engineer's fee (construction surveillance)	44,229	2.0%	Due to intervention tipology
Health and safety coordinator's fee	33,185	30.0%	Of Architect's fee
Taxes and stamps	6,174	0.3%	Of Construction costs
Administrative authorizations (facilities-certification body)	18,534	0.6%	Of total capex and Opex
Topography	2,000		Estimative according to project's tipology
Insurances	***39,345***		
Decennial insurance	26,708	1.2%	Of Construction costs
Technical Control Office	12,637	0.6%	Of Construction costs
Procedures and charges	***209,687***		
Occupancy Certificate	15,435	60 €	per Unit
Notary - New construction + property division and deed of sale	17,726	0.2%	Of total building price & capex and Opex + Notary
Registry - New construction + property division	14,180	0.2%	Of total building price & capex and Opex + Notary
Other taxes (Public thoroughfares occupation permissions…)	15,435		Estimative according to project's tipology
Real property tax (landplot)	24,601		Yearly payment, accodind to latest payments
New construction taxes	23,153	1.0%	Of construction cost + notary costs
Property division taxes	88,628	1.0%	Of construction cost + notary costs
First Occupancy License	10,531	0.5%	Of Construction costs (Technical fee)
Cost of capital	***1,478,556***		
Considering 10% over the whole investing	1,478,556		

Chapter 3

Pricing Real Estate Multi-Stage Projects Options

3.1. Chapter's introduction

This chapter pursues three clear objectives:

- Work based on a specific type of option: In this case, the option of carrying out projects in phases will be analyzed.
- Evaluate this option using a very common methodology to evaluate options, analysis by a binomial tree. The spirit of the research, which is to propose a simple method that allows the use and understanding of real options for those who are not experts in finance, is not renounced.
- Propose an econometric model to determine the volatility of the underlying asset in this context.

3.2. The significance of options valuation in multi-stage projects

Multi-stage projects are understood as those that can be developed fragmented into two or more parts. The execution of the first stage does not presuppose that the second stage has to be executed, although it improves the development conditions of the latter. In the case of a real estate investment, as in many others, the costs and success of marketing are variables that are estimated but that entail risk.

The realization of a project in phases allows obtaining more accurate information about what the results of only one part of the project on the whole investment would be. This allows fragmenting the risk while reducing its size. For investments as capital intensive as real estate, it is of special value to undertake an investment with complementary possibilities if it is observed that the additional information is favorable.

The development or not of the following phases, or the way they are developed, is conditioned by the positive or negative indicators obtained in the first phase. McDonald and Siegel (1986) and Dixit and Pindyck (1994) have pointed out that, in scenarios with uncertainty and with limited liquid and long-term investments, the possibility of modifying this investment has a specific economic value. In this case, the essence of the value acquired with the option is the additional information that makes the potential investment more attractive. It is necessary to differentiate where the value of this type of option concerns the one proposed in the first case study of this thesis. In the first example, the value lies in the right, but not the obligation, to undertake a specific investment, and its value is determined by the expectation of the market, that is, by the volatility of the underlying asset. In this second case, however, the value is generated by having more information, that is, by having a lower level of risk in the investment, as well as by the presence of technical interrelations between both projects.

These types of projects are frequent in the real estate sector, which makes their study more interesting. A project that offers the possibility of being carried out in phases has an added value, and the objective is to quantify this numerically using a well-tested methodology in the pricing of financial assets.

3.3. Objectives of this chapter

3.3.1. Practical application of the options methodology in multi-staged projects

For the potential investor, knowing the value of starting the first phase of the project will be equivalent to assessing its financial result plus the residual value of the project in case the subsequent stages are not carried out. In the case of a real estate project, which concerns us here, the residual value of the project would be the value that could be obtained by selling the remaining phases, that is, to sell the unbuilt land plot. This is an objective accounting valuation of the value of the resulting unbuilt land plot.

For the calculation of this value, it is indifferent whether the land plot is sold to another investor or kept in the portfolio. It is essential to understand that in the valuation of the resulting land plot there is a DCF plus an option to build the other phases. This structure would lead us to an approach like that posed in the first case study of this thesis.

The execution of the first phase may interfere to a greater or lesser extent in subsequent ones. There are different levels of interrelation:

- Knowledge of the market: Through the process of marketing the first phase, we have first-hand, valuable information to sell with greater success in the following phases, for example consumer tastes, market prices and volume of demand.
- Knowledge of costs and specificities of development: From knowledge of the subsoil to the peculiarities of the local climate, this information allows us to understand better the costs and risks of subsequent phases.
- Technical interactions: It is interesting to take into account the technical interrelation that occurs between phases. The execution of the first phase may involve, for example, obtaining joint licenses, the need for collective facilities or the construction of community zones between stages. These facts imply a higher valuation of the following phases of the business.

When pricing the option, we will consider these three aspects. It is true that, in the third case, that of the technical interactions, the phased execution can produce a trade-off between the increase in unit costs in the implementation of the first phase only and the benefits that the following stages receive by "finding part of the work already done." However, it does not always have to be that way. The case that we present contains a clear example: the construction of a garden area with a swimming pool, valid for both: one building or two.

Regarding the first two points, there is a fundamental factor: evaluating the information. Quantifying this information can only be done through its translation into risk reduction and, therefore, in the cost of capital. This translation is the main difficulty that this analysis poses, and for this reason Chapter 4 focuses on the triangle present among the options, the reduction of risk and the reduction of capital cost. In this chapter, therefore, we will have to assume a more subjective estimate of the change in the cost of capital, that is, we will make a hypothesis in which investors demand a specific price if specific information is known, and a higher price if it is unknown.

3.3.2. Raising a methodology that is broadly applicable

For the purposes of the current analysis, the methodology to be used needs to fulfill two characteristics: it should be pragmatic for practitioners in its implementation and understandable and reliable in the eyes of investors. More precisely, the proposal is to use a binomial tree analysis. Indeed, this method is the one we find most frequently in the literature referring to real options, given its discrete time approach and the nature of real assets, but above all, for the purposes of this case study analysis, this method is especially well suited to the characteristics of the options generated in a multiphase project. Other methods (e.g. Shen & Pretorius, 2006; Geltner & de Neufville, 2018) have also been designed to be used in the real estate sector and to take into account institutional interactions, but they focus more specifically on the financial status of a company, which takes us away from the project itself.

3.3.3. An econometric model for calculating the volatility of the local housing market

As stated previously, for the valuation it will be necessary to estimate the residual value of the second phase, which requires an analysis of potential market movements for the assets that do not belong to the first phase. This means that we need to estimate the volatility of the underlying asset, this being, in the case at hand, the real estate. This problem can be solved by analyzing the sensitivity of the option values to volatility in the market. In this chapter, we will estimate the volatility of the real estate market using an econometric model. This model aims at identifying the primary determinants of volatility and the sensitivity of this volatility to each of the determinants.

3.4. Methodology

Section 4 outlines the valuation methodology to be followed for the case study that will be presented. This section aims to clarify what exactly is being assessed, that is, identify the peculiarities of the option to evaluate what differentiates it from other cases and based on which methodology an assessment will be made. A recurring example is presented, followed by the used data. The example of the calculation of the proposed option concerns real data from a project on the island of Mallorca. It is a project whose characteristics induce an approach in two phases.

Once the data of the case to be analyzed is known, the critical variables of the real estate market volatility are identified. The proposed econometric model is defined, and the estimated volatility is calculated using real data. Finally, with these data and based on the proposed framework, the values of the real option are calculated. The aim of this part of the work is to demonstrate objective numerical values that quantify the option's added value. At the end of this chapter the most outstanding aspects are presented, as well as the main conclusions reached.

3.5. Valuation method: Using the binomial tree

A binomial tree is a schematic tool that allows us to illustrate the value of an asset over different periods of time, with a change in the state of its nature. The simplest form represents the current value and two possible values in the next period. It is a simple form of discrete analysis of the value of an asset over time. The number of periods can be extended as far as needed, according to the nature of the underlying asset.

Following the seminal contribution by Cox, Ross and Rubinstein (1979) and the subsequent papers by Harrison and Kreps (1979) and Harrison and Pliska (1981), in this case study we price a call option using the risk-neutral probabilities approach in discrete time. In other words, we determine the value of the call option that we acquired when carrying out the project that gives us the option of a second phase. This value should be added to the assessment we make of the development of the first phase (which we will value considering DCF). This approach allows us to implement a method that is intuitive and understandable enough and has been exhaustively contrasted in multiple areas of option evaluation.

To use this methodology, we first have to determine what is the expected value of the underlying asset after the analysis period, both for a bear market and for a bullish market. For this, an econometric model is applied using historical data for its definition, namely concrete data of both bullish and bearish periods in the sector. Once both volatilities have been determined and the current value of the underlying asset has been determined, we will have all the tools available to calculate an objective value of the option.

3.5.1. Reasons for using the binomial tree

In the case of the real estate market, the peculiarity is that, historically, price increases are smooth, except for specific periods when the decreases are much stronger. The real estate market is very sensitive to credit accessibility, and it is usually a safe haven. Therefore, there are periods in which investments in real estate accrue so that, over very short periods, severe price corrections may occur. This aspect makes this type of option analysis especially interesting: We do not have "symmetric" volatility, because the decreases are more pronounced than the increases. Carr (1988), Majd and Pindyck (1987), and Trigeorgis (1993a), among other authors, have analyzed sequential investments.

At the same time, given that we do not need to know the probability of starting a period of ascent or descent, we can determine separately the volatility in bullish and bearish markets or determine two different values (*u* and *d*). The use of binomial trees in this modeling schematizes especially well a decision in phases: a first decision is made with specific information, among which is the value of the option, and later additional information is available. This allows us to make the next decision. The value of the option seems strictly related only to make the first decision. Therefore, how do we assess the three levels of interrelation described in section 3.3.1.? The answer lies in the fact that in the determination of the disbursement to carry out the second phase, we start from the premise that there are specific knowledge and certain favorable conditions that affect the cost of the second phase positively, reducing it.

However, carrying out the project in two phases also produces an adverse effect: there is an increase in indirect costs (costs of technical management, logistics, financial, etc.). For a correct analysis, the added expenses of management should be compared with the value of the option. If the management costs are lower than those of the option, it makes no sense to consider the option of carrying out the entire project in a single phase and therefore there is no reason to believe that one would incur the risk of doing so.

3.5.2. Peculiarities of using this method for a real estate project in two possible phases

To decide whether to undertake a project of this type, it will be necessary to check whether the current value of the first phase plus the current value of the option is higher than the current value of the costs that will be incurred. This is a traditional DCF analysis, to which we will add the value of the call option that corresponds to the possibility of carrying out the second phase. Panayi and Trigeorgis (1998) have argued that if we add more phases, the effect of composite options would be produced, which would be an important added complexity.

The market price of the finished house (in square meter, for example), is our market variable to be analyzed. In any case, it is also important to know the residual value of the unbuilt land, if the second phase is not carried out. This residual value is added to the value of the first phase of the project, with a peculiarity, as it happens with the value of the second phase, that the value of built square meter can go up or down, which determines the two possible scenarios. The value of the site, directly related to the price of the square meter, can also rise or fall between the time of the first decision and the time of the second; in fact, the value of the site can be inferred from the final sale value of the house. Which of the two estimated values is appropriate to consider then? Does it make sense to add the value of the option to the residual value of the lot? Does it make sense to contemplate what the residual value of the site is, even though it is profitable to carry out the second phase?

These questions make the initial approach difficult, concerning the values to be considered. There is a part of the investment in the second phase that, when acquiring all the land at the beginning, is carried out simultaneously at the beginning of phase 1. The most intuitive way to raise it, is to consider the residual value of the part of the cost of phase 2 in the case of a decrease in market value. When we consider the necessary disbursement to carry out stage 2, we will not consider the value of the land, since it is a disbursement that was made at the time of deciding to undertake phase 1. In this way, it is entirely reasonable to add the residual value of the site to the value of the NPV of phase 1.

In addition to this peculiarity, the case of a real estate project in two phases also raises another aspect that does not always arise in real option analysis. The result of the second phase of the project can be known at any time before the start of the project, so it is likely that there is no decision about carrying out the second phase; rather, it is the market that will tell us whether it will be carried out or not. Let us suppose that, after phase 1 commercialization, phase 2 reserves are taken, and within a short timeframe, the reserves of houses to be built in phase 2 are sold out. This would be a happy circumstance for the investor, who has not even had to wait for the market evolution and has saved the decision of whether to invest or not. He or she had a choice to decide and has allowed the market to determine the decision: he or she bears no risk (commercial at least). This may seem conceptually to distort the idea of the increase in information that is acquired with the option (since phase 1 may be just being implemented on the site), but in fact it is the most significant expression that with the option of carrying out two stages, the investor has acquired a significant extra value: he or she has bought the market response.

3.5.3. Defining key variables and the methodology for their determination

The following list presents, in an orderly manner, the variables to be calculated and how we will determine them:

- The market value of phase 1 of the project: We evaluate it using the DCF method, adding the value of the call option to carry out phase 2 as well as the residual value of the land in case of a bear market. We need to know the estimated sale price of the finished housing in square meter (through market research).
- The disbursement necessary to develop phase 2 (*K*): We determine it by calculating the required Opex. We also consider the cost of the land already acquired and paid at the time of buying the site for phase 1.

- The value of phase 2 of the project: It is the NPV of the project phase if analyzed at current market prices – we named it *S*.
- The volatilities of the underlying asset of our option (*S*): In this case, we determine two possible volatilities, one for an upward cycle and another for a down period, and we represent them as *u* and *d*. To identify these values, we use an econometric model that will use historical values of both bullish and bearish phases of the real estate market cycle. This model is described in greater detail later in this chapter.
- A risk-free interest rate: We assimilate it to a 30-year Spanish debt bond, which is around 2%.

With these variables and the previously defined formulas, we can determine the call value for the case study.

3.5.4. The volatility problem: Importance versus difficulty of determining volatility

When referring to the methodology used, we cannot ignore the critical weight of the value of the volatility for the result. In the previous case, we solved this problem by including a sensitivity analysis of the outcome concerning the volatility. However, this very negatively affects the whole method as far as practical utility is concerned. Volatility becomes the critical factor for the option values, but it remains an exogenous parameter that is very difficult to define. The methodology we use, through an econometric model, results in an analysis parallel to the value of the option itself becoming equally important. A much more complicated methodology is identified in the literature in this regard, although in this case, we focus on the differentiated analysis for a bullish or bearish market (without contemplating which is the probability of one or the other happening). To apply the methodology of calculating options, this approach is appropriate, although discrepancies in the method to determine the values *u* and *d* should not detract from the part of the analysis that refers strictly to the option.

3.6. Case study: Description of the case characteristics

3.6.1. Development in 2 possible phases in Mallorca (one-step tree)

First we will study the simplest version, constructing a one-step binomial tree.

The case (example) for the analysis is a housing promotion in a coastal town on the island of Mallorca (Spain). It is a set of 36 apartments and 59 parking distributed in 2 buildings. The largest of the buildings consists of 20 homes with 5 units per floor and four floors in total. The second of the buildings have the same number of levels and 4 houses per floor. The project includes the construction of a communal garden area with a central pool. The lot belongs to a single owner, due to its size, it could be parceled in two lots, being able to build one of the buildings and ignoring the other. A summary of the project is shown in the drawings below.

The summary of the business plan for both phases, separately, is as follows. The reader will note that a residual solar cost has been considered when performing the first phase as the acquisition cost of solar of the second phase. As indicated, in the business plan of phase 1, it is considered that at the end of this phase the value is retained (whether it is sold).

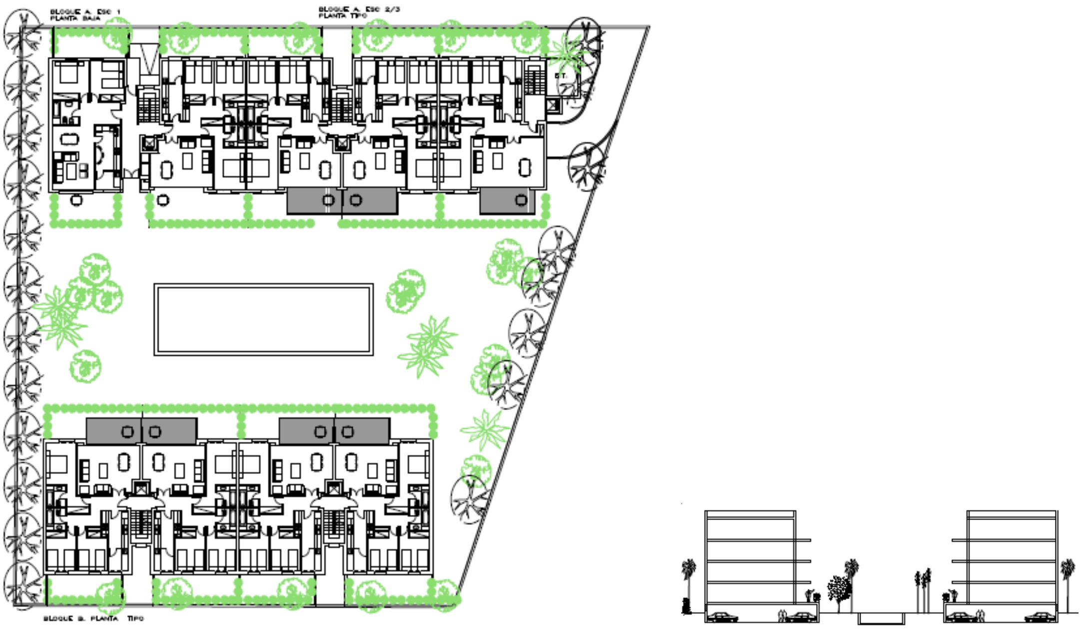


Figure 3.1: Project description.

MALLORCA PHASE 1	Total
Income (Flats)	5.316.000
Income (Parkings)	396.000
Total Income	**5.712.000**
Asset acquisition	1.130.000
Other Costs Relatet to Asset acquisition	58.505
Construction Costs (Flats) + Garden	2.508.826
Construction Costs (Parkings)	356.484
Management, fees and stamps	470.323
Financial Expenditures	46.781
Procedures and charges	267.589
Comercial Expenditures	182.460
Total Costs	**5.020.968**
Developer Margin	**691.032**
% Developer Margin	12,10%

MALLORCA PHASE 2	Total
Income (Flats)	3.864.000
Income (Parkings)	312.000
Total Income	**4.176.000**
Asset acquisition	360.000
Other Costs Relatet to Asset acquisition	0
Construction Costs (Flats)	1.250.331
Construction Costs (Parkings)	157.686
Management, fees and stamps	310.965
Financial Expenditures	35.332
Procedures and charges	128.312
Comercial Expenditures	134.780
Total Costs	**2.377.406**
Developer Margin	**1.798.594**
% Developer Margin	43,07%

Table 3.1: Business Plan Summary

The construction cost of the building of phase 1 is disproportionately higher because it includes the construction of the pool and the conditioning of the garden area. Also, the expenses related to the acquisition of the asset are produced in full at the beginning of phase 1. It is true that part of this cost is used for the second phase, which would contradict a standard accounting analysis. But the truth is that, when calculating the cost of the solar to the residual value in case of selling, it is already considering that the acquisition costs would be passed on. In the case of a bear market, the remaining lot is sold with losses on its reasonable book value. Sales charts would be as follows:

MALLORCA PHASE 1

Development inputs	Flats / Parkings	Total sqm Built	Price (x sqm)	Income	Income x Unit
Parking -1	33	755,00	12.000	396.000	12.000
Ground floor	5	590,00	2.100	1.239.000	247.800
Level 1	5	590,00	2.100	1.239.000	247.800
Level 2	5	590,00	2.100	1.239.000	247.800
Level 3	5	590,00	2.100	1.239.000	247.800
Landplot	1			360.000	360.000
Total (Flats)	**20**	**3.115**	**2.100,00 €**	**5.712.000 €**	

MALLORCA PHASE 2

Development inputs	Flats / Parkings	Total sqm Built	Price (x sqm)	Income	Income x Unit
Parking -1	26	590,00	12.000	312.000	12.000
Ground floor	4	460,00	2.100	966.000	241.500
Level 1	4	460,00	2.100	966.000	241.500
Level 2	4	460,00	2.100	966.000	241.500
Level 3	4	460,00	2.100	966.000	241.500
Total (Viv.)	**16**	**2.430**	**2.100,00 €**	**4.176.000 €**	

Table 3.2: Sales (Income) Summary

To simplify the calculations, we consider that the project has bank financing, so that the partners contribute a certain percentage of the Capex + Opex. This amount, due to the conditioning of the bank, remains invested in the project until its completion. This approach fits well with the reality of a project of this type in the current framework. Besides, this approach allows considering, regarding capital cost, that the partners make this investment at the beginning and receive the principal of their investment plus the profit margin at the end of the operation (that is to the deed of the property to the name of the buyers). In order not to have to analyze detailed cash flow, we will assume that the level of sales required by the financing bank expects that the payments on account of the buyers. These payments are invested in the project in a way monitored by the entity, are sufficient to that the project is developed without any extra contribution from the partners.

The first phase has an estimated duration of 20 months from its inception, that is, from the initial investment, since it is expected that it would take approximately 4 months to obtain the license and 16 months in the construction and sale of the building. The second phase would last 14 months, since the construction is simpler, and the permission would already be granted (given that it would be requested for the whole). We will consider that the necessary disbursement for the second phase is made at the moment of initiating this phase, that is to say that the investment is returned in 14 months.

3.6.2. Identifying the values of the variables involved

The initial payment to undertake the first phase corresponds to the total costs (Gross Asset Value or GAV), minus the construction costs strictly, which can be financed 100% by a bank. This represents an initial outlay of 2,512,142 euros. Since the return will happen after the delivery of the building to the buyers, that amount, along with the benefit will be paid back after 20 months.

The amount received by investors after the end of the first phase is 2,882,690 euros. Given the risk involved in the operation and using the project cost of capital as the discount rate (Modigliani & Miller, 1958), assessing its location and how the investment process should be developed, the cost of capital that we will consider is 10% per year. With this valuation, the NPV that we obtain for the first phase is 152,633 euros.

Note that the DCF is calculated with a cost of capital higher than the risk free interest rate. That is, a priori, contradictory to the principles on which calculating the value of these options is based. It is worth remembering that the calculation is based on the creation of a risk-free portfolio, which means that we should use the risk-free interest rate for all capital updates. However, from a conservative point of view, it could be argued whether those previous calculations, which would be made by the analyst if he were not going to use options, should be calculated with the cost of capital that the investor considers the project (not the acquisition of the option). Valuing both alternatives, we opted to use the cost of capital that the smaller NPV offers. So, using this type of interest we are obtaining the minimum value that would correspond to the option.

3.6.3. Same development in a multi-step binomial tree

In the previous section, we propose the valuation of the option through a binomial tree with one period, which assumes the course of a year between the initial disbursement for phase 1 and phase 2. Thereafter, we will do the same calculation but with a bigger tree that divides the same year into more parts. This results in a tree in several steps instead of one.

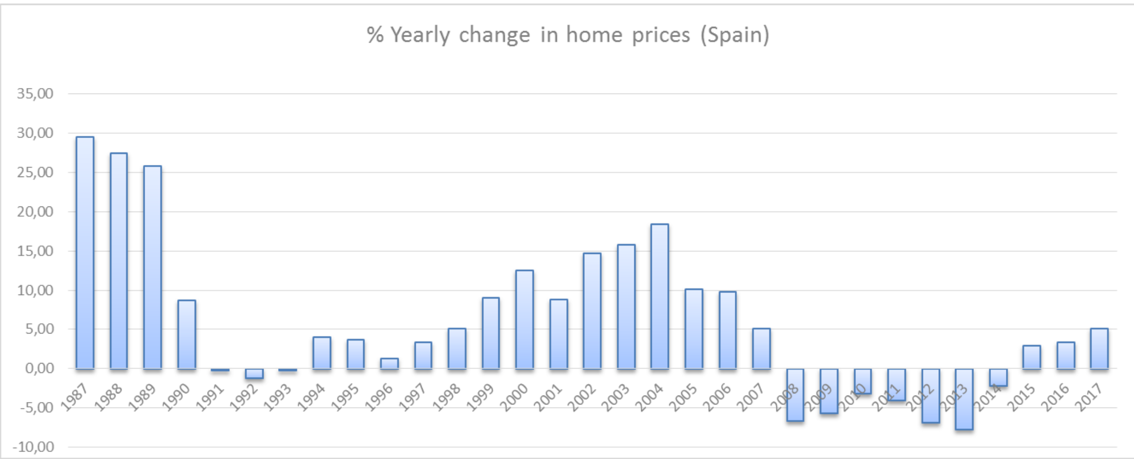
This calculation allows us two things: First of all we can know the option value as the time progresses within the same year and the market is evolving upwards or downwards. Secondly, it allows us to see how the valuation of the option is adjusted significantly as we add more intermediate valuations. As expected, when adding intermediate calculations, the value of this option tends towards what we would obtain with the Black and Scholes formula (Hull & Basu 2016).

3.7. Determination of the volatility of the asset price

3.7.1. Definition of the econometric model to be used

We want to determine what percentage we can expect the value of phase 2 of the project, and subsequent, to increase in case of a favorable evolution of the market. In the same way, we want to determine what percentage we can expect that value to be reduced in the opposite case. Given that this increase or reduction in value depends on the market price, in the first place, we will have to analyze what is the expected potential growth of this price in case of an upward evolution or a decrease in the case of bearish development. Later we will have to determine the relationship between the expected price movements and the value of the second phase (the NPV that we would obtain in both cases).

According to the work carried out by Pagés and Maza for the Bank of Spain in 2.003 (Pagés & Maza 2.003), the determinants of housing prices in the Spanish market are mainly the evolution of the price of money, the growth of disposable income per capita, the growth of the stock market and the demographic development. With the study of these variables, we will try to construct two models, one that helps us to forecast the expected movements in the bullish phase and another in the bearish period. We have taken a sample of such data from the last thirty years (from 1987 to 2.017). Annex 1 shows the used data. On the one hand, it is difficult to find comparable statistical data from previous periods and, on the other hand, the behavior and the conditioning factors of the market have varied too much if we deviate more than three decades. With all this, we have observed several remarkable aspects.



Graph 3.1: Volatility in home prices in Spain 1987-2017, in change %.

3.7.2. Description of the variables and assessment of the results obtained

In the first place, although common sense and the generalized opinion of the market actors make evident that these factors are related, the relation is weak from an analysis strictly based on linear least squares (LLS). Observe first, in Table 3, the result of modeling of this type without segregating the data corresponding to the bullish part of the cycles of the bearish part.

	Coefficient	St. Deviation	t- Value	p-Value	
Constant	-4.99997	3.02981	-1.65000	0.11140	
NNDI_PC_yevar	2.51606	0.47486	5.29800	1.72000E-05	***
y_sp_bond	0.40156	0.34876	1.15100	0.26050	
sp_stock_yevar	0.07856	0.04306	1.82500	0.08000	*
popmin25_yevar	2.13471	1.69717	1.25800	0.22010	
popmore24_yevar	4.19661	1.69054	2.48200	0.02010	**

Table 3.3: LLS for data without segregation between bulls and bears (30 observations).

In this modeling, the dependent variable is the annual percentage variation of the housing price index for the whole of Spain. These data come from the appraiser Sociedad de Tasación SA. The first variable (NNDI_PC_yevar), which shows a lower p-value, is the annual variation of disposable income per capita and comes from the database made by Prados de Escosura. The second (y_sp_yevar) corresponds to the annual yield of the Spanish 3-year bond and comes from data from the Bank of Spain. The third variable (sp_stock_yevar), also considerably correlated, shows the annual percentage variation of the Spanish stock market index, taking as a reference the index of the Madrid Stock Exchange. These have also been obtained from the Bank of Spain. Finally, the last two variables (popmin25_yevar and popmore24_yevar) represent the annual percentage variation of the Spanish population under and over 25 years old, the data are from the Eurostat database. As can be seen, the correlation is much better for those over 25 years of age.

From this perspective, the relationship between the variables is acceptable. The problem is that we need to make a predictive use of this data, so we use the data of the previous year that we want to estimate. In table 4 we see how the quality of modeling decreases when using data from the last year. This sample has 31 observations for each variable.

	Coefficient	St. Deviation	t- Value	p-Value	
Constant	-6.44532	3.55502	-1.81300	0.08240	*
bull-bear	9.40615	4.36786	2.15300	0.04150	**
NNDI_PC_yevar_t_1	0.95557	0.61692	1.54900	0.13450	
y_sp_bond_t_1	0.34008	0.43148	0.78820	0.43830	
sp_stock_yevar_t_1	0.08064	0.04827	1.67100	0.17800	
popmin25_yevar_t_1	0.76284	1.89262	0.40310	0.69050	
popmore24_yevar_t_1	1.51309	1.88578	0.80240	0.43020	

Table 3.4: LLS with the same data as in Table 3 but one year behind the independents.

The independent variables used for this model are the same as for the previous one, but a year has been anticipated in time. Besides, we have added the dichotomous variable that distinguishes between bullish and bearish (bull_bear). In Tables 5 and 6 we will see how this approach evolves when making differentiated models for the data of bullish periods of the real estate market and vice versa.

	Coefficient	St. Deviation	t- Value	p-Value
Constant	-0.74580	6.14946	-0.12130	0.90510
NNDI_PC_yevar_t_1	1.55753	0.90825	1.71500	0.10700
y_sp_bond_t_1	0.30022	0.55826	0.53780	0.59860
sp_stock_yevar_t_1	0.07280	0.06587	1.10500	0.28650
popmin25_yevar_t_1	1.07959	2.63209	0.41020	0.68750
popmore24_yevar_t_1	3.30150	2.74209	1.20400	0.24720

Table 5: LLS with the same data as in Table 4 but only those with a bullish cycle (21 observ.).

	Coefficient	St. Deviation	t- Value	p-Value
Constant	-7.27826	3.44959	-2.11000	0.10250
NNDI_PC_yevar_t_1	-0.21288	0.75586	-0.28160	0.79220
y_sp_bond_t_1	0.00058	1.10451	0.00052	0.99960
sp_stock_yevar_t_1	0.02290	0.05968	0.38370	0.72080
popmin25_yevar_t_1	-2.69121	6.07674	-0.44290	0.68080
popmore24_yevar_t_1	1.91870	5.01932	0.38230	0.72170

Table 3.6: LLS with the same data as in Table 4 but only those with a bearish cycle (10 observ.).

According to the p values, the model corresponding to the bullish data is, approximately, comparable regarding reliability to the elaborated without segregating the bullish and bearish values. However, in the bearish period, the estimate has lost part of its reliability.

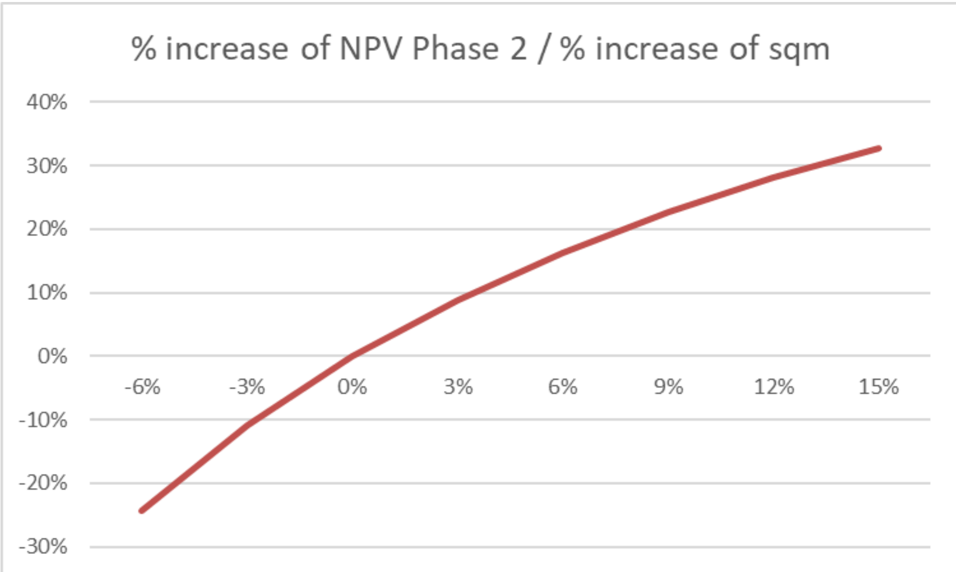
Therefore, for the bullish period we would obtain an equation like the following: $\boldsymbol{HOMEP}_t = -7.75 + 1.55\, \boldsymbol{NNDI}_{\boldsymbol{t-1}} + 0{,}30\, \boldsymbol{YBND}_{\boldsymbol{t-1}} + 1.08\, \boldsymbol{P} < 25_{\boldsymbol{t-1}} + 3.30\, \boldsymbol{P} > 24_{\boldsymbol{t-1}}$

While for the bearish period the estimate should be made with an equation like the one that follows (in which the reference to the variation of the bond has been eliminated):

$$\boldsymbol{HOMEP}_t = -7.27 - 0.21\,\boldsymbol{NNDI}_{\boldsymbol{t-1}} + 0.02\,\boldsymbol{STCK}_{\boldsymbol{t-1}} - 2.69\,\boldsymbol{P} < 25_{\boldsymbol{t-1}} + 1.92\,\boldsymbol{P} > 24_{\boldsymbol{t-1}}$$

3.7.3. Relationship between the variation in market prices and the value of the project (NPV)

In Graph 1, we observe how the variable NPV has a high elasticity with respect to the variable sale price of the sqm. The variables *u* and *d* will be higher than the variation in the market price that we obtain. We observe a logarithmic relation between both variables that we could approximate to this graph 2.



Graph 3.2: Relationship between the increase in the price of the sqm and the NPV increase, in %.

3.8. Option's valuation

In order to find the value of the option we are looking for, we will proceed in 5 steps:

• First, we define what the values for 2017 of the variables we need to find the volatility of the market price sqm of housing for both a bear market and a bear market are.

• Secondly, to determine the values of *u* and *d*, for which we will need to identify the expected upwardgoing and downwardgoing market changes, with the econometric approach defined in the previous section and later transforming them into *u* and *d* by applying this variation to our business plan.

Note that, although all the phases exceed one year and phases 2 and subsequent are moving away from 2017, we maintain the values of *u* and *d* of 2017, since they are the closest in time that we can obtain now.

• Next, we determine the values of *S* and *K* with our business plan.

• Applying the risk-neutral method, we obtain the upwardgoing probability *Pr*.

• Finally, we calculate the option value.

3.8.1. 2017 Market Indicators

Annual variation of Spanish disposable income per capita (in %): 5.06

Annual yield of the Spanish 3-year bond (in %): 0.25

Yearly change of the Spanish stock market index (in %): 7.59

Annual variation of the Spanish population under 25 years old (in %): -0.15

Annual variation of the Spanish population above 25 years old (in %): 0.88

3.8.2. Percentage increase or decrease in prices

To know the market estimates, we substitute the values in the corresponding models.

Bullish period % increase: 2.91%

$$\boldsymbol{HOMEP_t} = -7.75 + 1.55x\mathbf{5.06} + 0{,}30x\mathbf{0.25} + 1.08x - \mathbf{0.15} + 3.30x\ \mathbf{0.88} = \mathbf{2.91}$$

Bearish period % increase: -6,09%

$$\boldsymbol{HOMEP_t} = -7.27 - 0.21x\ \mathbf{5.06} + 0.02x\ \mathbf{7.59} - 2.69x - \mathbf{0.15} + 1.92x\ \mathbf{0.88} = \mathbf{-6.09}$$

3.8.3. Calculations in a two step binomial tree

Values of *u* and *d*

Thus, the value of u and d, applying this increases / decreases in prices to our business plan, would be:

u = 1.0686, and *d*= 0.8564

Value of *S* and *K*

According to our business plan, the NPV of phase 2 amounts to € 1,282,639.

NPV (k=10%)	01/01/2019	01/03/2021
1.282.639	-969.389	2.767.983

We have considered the most unfavorable scenario in which the invested capital is contributed in the first moment of the project. Equity excludes the cost of the constructor, which is financed by credit promoter. The estimated duration is fifteen months, given that licenses are already available and only the construction time is required. The cost of capital is 12% per year, as established above.

K is the disbursement to exercise the option, the initial disbursement minus the value of the land plot, this is 969,389.

Apply (1) to obtain *Pr*

$$Pr\ .\ uS + (1 - Pr)\ .\ dS = S\ .\ (1 + rf) \qquad (1)$$

$$Pr\ .\ 1.0686 \times 1{,}282{,}639 + (1 - Pr)\ .\ 0.8564 \times 1{,}282{,}639 = 1{,}282{,}639\ .\ (1 + 0.02)$$

$$Pr = 0.0942$$

Apply (2) to value the option

$$C= [\ Pr\ .\ Cu + (1 - Pr)\ .\ Cd\ ]\ .(1/(1+rf)) \quad (2)$$

$$C= [\ 0.0942\ .\ 401{,}134 + 0.9057\ .\ 129{,}118\ ]\ .(1/(1.02))= \mathbf{151{,}658}$$

Therefore, we can establish that, according to this calculation, the option to carry out the second phase increases by 151,658 euros the value of the project of the first.

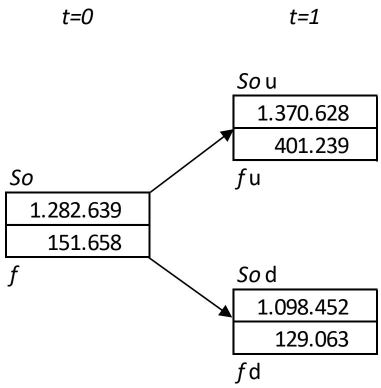


Figure 3.2: One step tree calculation's summary.

3.8.4. Valuation's summary

The NPV of phase 1, considering a cost of capital of 10%, is € 152,633.

The project as a whole, adding to the NPV of phase 1 plus the value of the option is € (152,633+151,658=304,291).

3.9. Calculations in a three, six and twelve-step binomial tree

The calculation using a one-step tree is the most simplified version we can make. Increasing the number of intermediate moments calculated not only obtain data on the evolution of the value of the option over time, we also obtain a better approximation of the value of the option. We will obtain intermediate values of the evolution of the option, discreetly, depending on the evolution of the market in these periods.

In a 3-step tree we divide the year into thirds, in a 6-step in bi-monthly periods, and in a 12-step in monthly periods. The values of *u* and *d* become the cube root of the original values, the values of the underlying values are adapted in each period. However, the value of *p* remains the same as in the one-step tree. Considering these aspects, the calculation is a systematic repetition of the one made in section 3.7.1, starting the calculation at the final moment, since initially we can only calculate the final values of the options. It is interesting to note that in the case of a 3-step tree the result obtained for the value of the option (€ 150,747) implies a reduction of its value by approximately 0.6%. By increasing the number of intermediate calculations, we are adjusting the result we would obtain with a continuous calculation method, such as the calculation with the Black and Scholes formula.

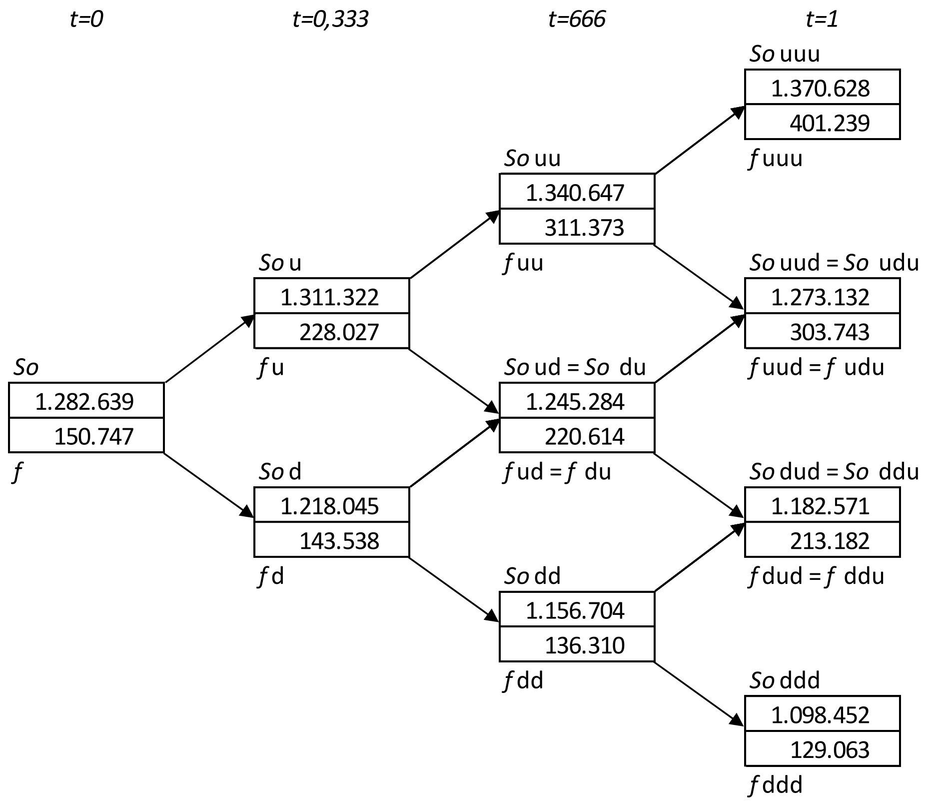


Figure 3.3: Three step tree calculation's summary.

The more we increase the number of steps, the more the result is adjusted to a continuous result. In addition, increasing the tree allows us to observe how the value of the option evolves over time. In the tree of six steps whose result is shown in Figure 3.4, we can observe that the variation of the value of the option between one step and another is greater as we ascend in the tree. That is to say, as time goes by, the value of the option grows much more proportionally as the market advances positively than it decreases when advancing negatively. In other words, waiting for the decision is potentially beneficial for the value of the option. We also see that the value keeps decreasing slightly (adjusting) as we add more precision to the calculation.

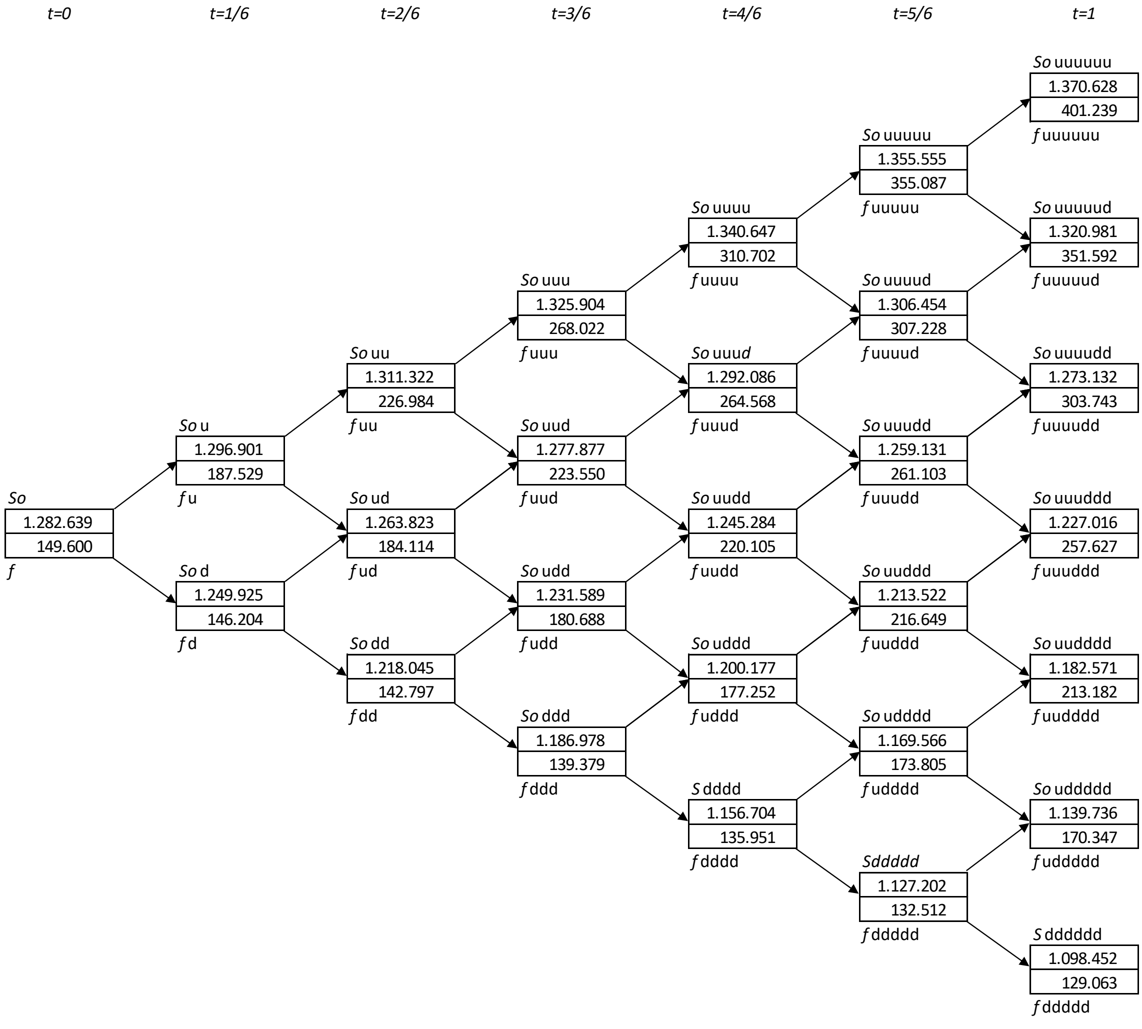


Figure 3.4: Six step tree calculation's summary.

A similar effect, but in more detail we see in the tree of twelve steps (monthly). The small difference in the adjustment and the rest of the information provided, suggests that moving more in detail is unnecessary. Thus, the monthly tree could, in the case of a large project, be sufficient as a detailed tool in decision-making. The next page shows the detail of the monthly tree.

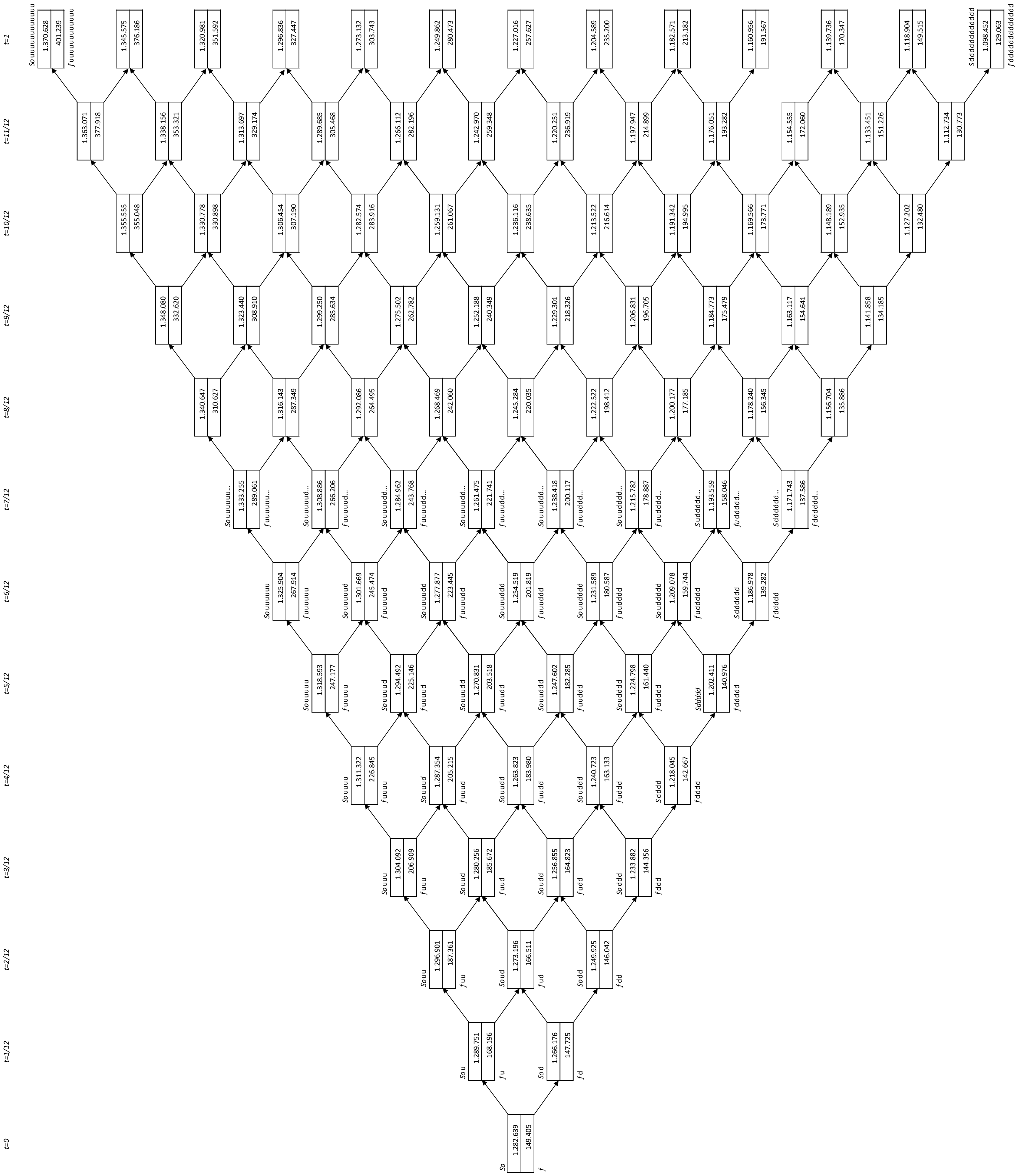


Figure 3.5: Twelve step tree calculation’s summary.

3.10. Variation with a small reduction in volatility

In an option where in both scenarios the project yields benefit, that is, the final NPV of the project is higher than the Price strike, the reductions in volatility affect increasing the value that most resembles the current value of the option, that is the value of the most pessimistic scenario. When the risk-free interest rate is so low in relation to the potential profit (ROE) of the second phase, the value of the option at time zero is very close to the value of the most pessimistic scenario at time 1.

As indicated above, one of the concerns in the calculation is the high sensitivity of the result to volatility. In order to observe how the result evolves as well as its variation in time by slightly modifying the volatility, the calculations made in the previous section have been redone but this time they have been reduced in u and d by 10%. As we see, the option value has increased.

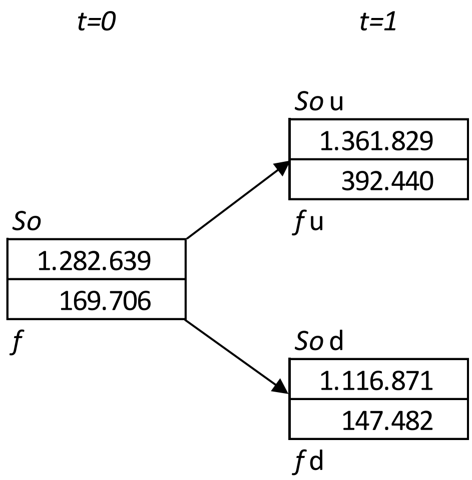


Figure 3.6: One step tree calculation's summary with market volatility reduction.

Figures 3.8 and 3.9 show the diagrams corresponding to the trees of three and six steps respectively. In their distribution of values, we observe the same features found in the previous example.

Figure 3.7 shows the two examples and a third series corresponding to a volatility reduced by 20% compared to the initial one.

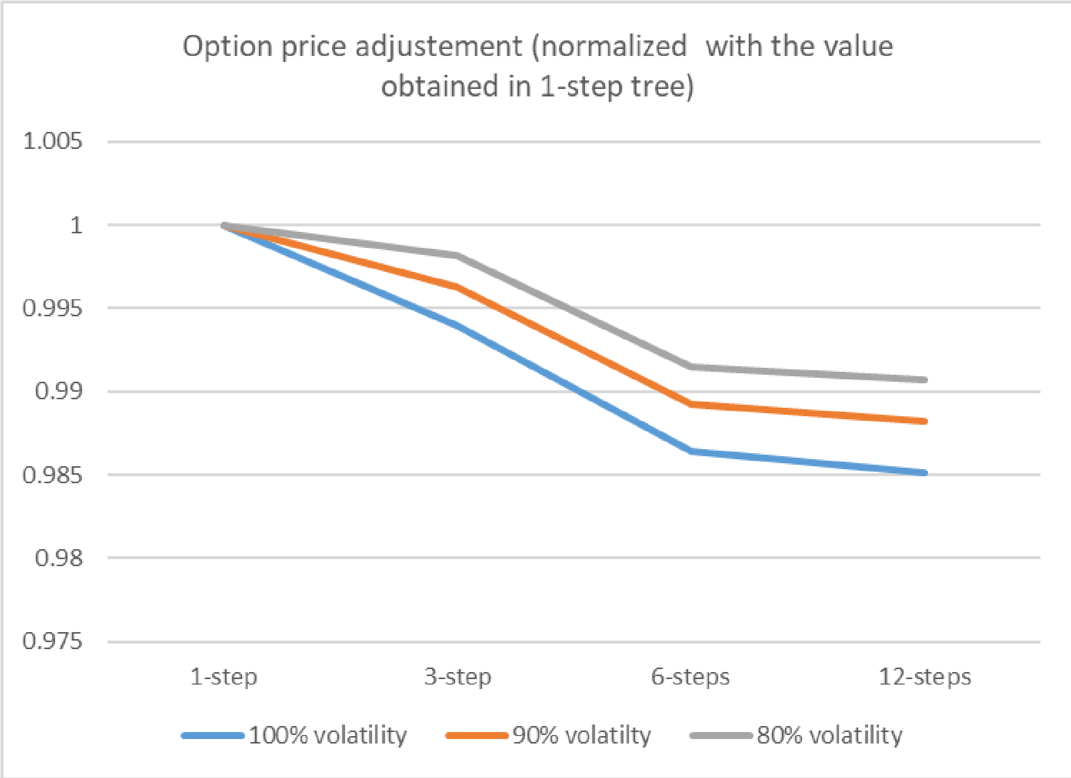


Figure 3.7: Option price adjustment due to tree size increase.

We see that if we normalize the value of the option in each tree, dividing the value between the value obtained with the tree in one step, the adjustment is proportionally greater the greater the volatility. The adjustment curve (the number of steps in which an equivalent adjustment occurs) is maintained with the same shape.

This would lead us to think that:

• The monthly tree already shows an acceptable precision regardless of the option value or the volatility of the underlying.

• The adjustment is not constant with the increment of steps, the adjustment curve takes a certain asymptotic form towards the value of the continuous calculation.

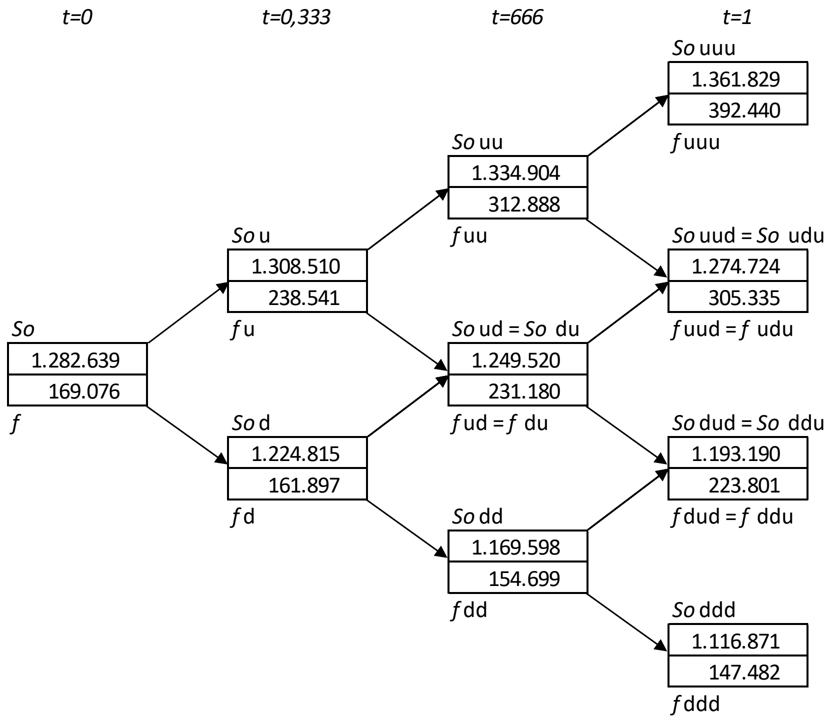


Figure 3.8: Three-step tree calculation's summary with market volatility reduction.

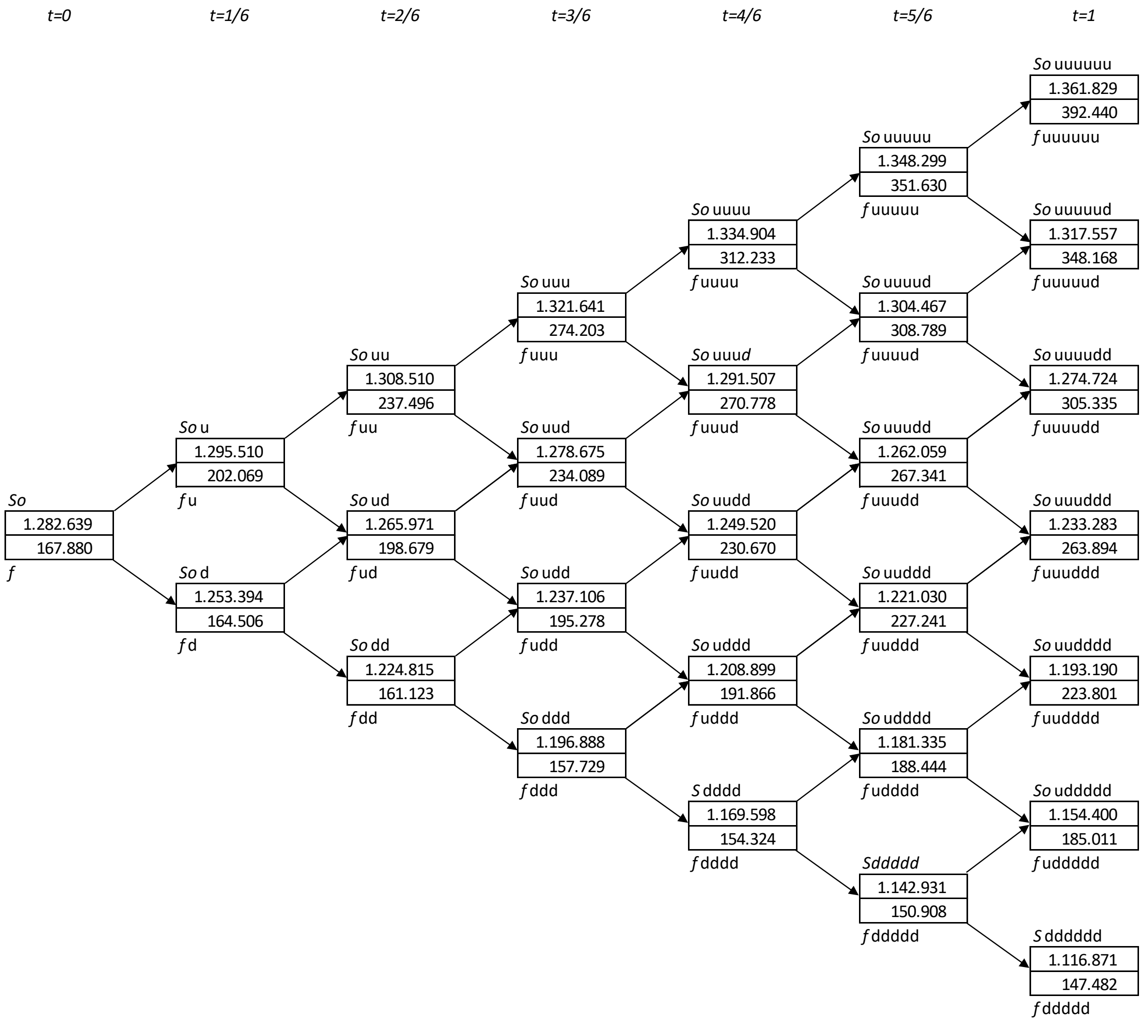


Figure 3.9: Six-step tree calculation’s summary with market volatility reduction.

The detail of the monthly tree with a 10% reduction in volatility is attached as an annex to this chapter.

3.11. Chapter conclusions and final remarks.

Throughout this chapter, we propose a methodology to assess the flexibility derived from the possibility of carrying out a project in two phases instead of undertaking it in a single stage. The binomial valuation method of real options has been used. This method is particularly well suited to a sequential decision process such as that which occurs when deciding on a future phase once the previous one has begun. Working in a discrete calculation environment allows us to move away from what would be the strict premises of a Brownian movement. This makes us feel more comfortable with the distinction between a volatility value in the case of a bullish and a bearish evolution.

The concept of choice and calculation used is quite different from that seen in the previous chapter. In both cases we deal with a type of investment that does not produce losses, but changes in the final valuation. This is valid given the idiosyncrasy of the sector, since a project where there is a risk of loss would be already directly rejected. In this specific case, since it is a type of option fixed by a decision of sequential realization of projects (phases), the effect of this fact is much clearer.

In a scenario in which the risk-free interest rate is very low in relation to the ROE expected in the sector, the value of the option ends up being very similar to the profit margin obtained in an unfavorable scenario. This, not only should not serve to stop justifying this calculation, on the contrary, it helps to understand the effect of volatility and the cost of capital on the value of the option.

Given the significant specific weight of the volatility in the result of the analysis, a method of prediction using econometric models has been used. Using real data from the last thirty years, two cases have been modeled, to distinguish between bullish and bearish volatility. The bearish fluctuations double the bullish, supporting the need to consider them separately. With this methodology, and always taking the most conservative values possible so as not to overvalue the option, a specific value has been obtained for the option to carry out the second phase.

The calculated value far exceeds the current value of the first phase separately, indicating that it is a valuable decision tool. It should be noted that the value of this option should not be confused with the NPV of the second phase, something that is clear throughout the process. This valuation is intuitive enough for the analyst to apply it and the investor can understand it and, therefore, place their trust in the fixed value.

We have also observed what is happening by making estimates that increase the number of intermediate calculations, introducing values of the option in shorter periods and increasing the number of steps in the tree. On the one hand, it allows us to know how the value of the option evolves throughout the period. On the other hand, we obtain a better approximation of the value of the option.

We observe that the first approximation is significant enough and allows the investor to become aware of the value of the option. However, an increase in steps in the tree, up to the monthly level as what has been done, serves to obtain more adjusted results and, in turn, as a tool for decision making. In a more detailed tree we see how the value of the option grows in the improvement scenario, while the reduction in case of unfavorable evolution is not excessive. In this case, where the distance between the yield of equity is so far from the risk-free interest rate, time plays in favor of the value of the option.

We have carried out an analysis of the same case, introducing a variation in the volatility, obtaining a greater value for the option (logically, since it is a binomial output with both positive scenarios). However, it also helps us to see that the behavior of the adjustment of the value of the option is quite regular in its form, independently of the volatility.

It is important to emphasize that for this valuation it is not necessary to estimate what is the probability that the market will evolve favorably or negatively. As it is not a listed asset, volatility cannot be established objectively, although a good knowledge of the local market can make the analyst justly correct the data obtained with models such as employees. On the other hand, the values used as volatility could be adapted, if necessary, to the subjective estimation that the analyst or investor had of the evolution of the market.

Focusing on the values used for the analysis, we see that a large part of the specific weight of the value of the option falls on the net income available and the demographic variations. It should be noted that obtaining more local historical information, which helps to make the analysis more specific, is very complicated. This has meant the use of more macro data that can give us a rough estimate of local evolution.

Another determining factor in the result, since the method is based on DCF, is the reduction effect that we consider in the cost of capital. Amin & Capozza (1991), Heaney & Jones (1986), Ingersoll & Ross (1992) and Williams (1991), among others, found that positive responses of real investment to interest rate increases can occur in real estate options models. The change in the cost of capital is, together with volatility, one of the two determinants of the value of this type of option. In this case, we have considered a minimum correction, of one percentage point, the differential between both capital costs.

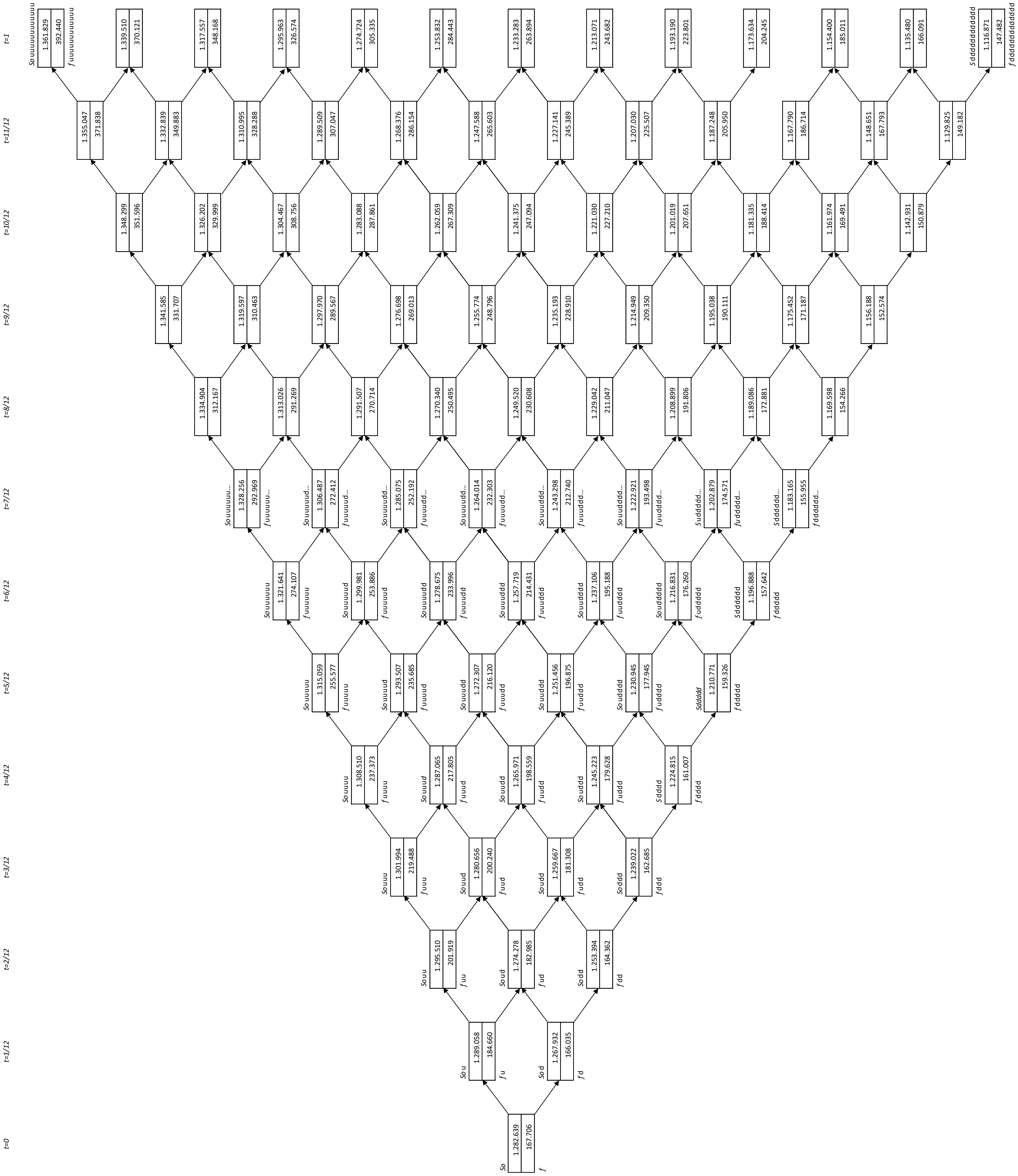
The sure thing is that it would be logical that, having the experience of the first phase, the second one will be financed at a much lower cost. This effect would be even more pronounced in cases such as those described in which the presale could be anticipated. The truth is that, the presence of this option significantly reduces the uncertainty about the outcome of the second phase of the operation, which should have a positive effect on both the capital cost of the second phase and the first phase. In the next chapter, the possibility of quantifying this relationship between the presence of options and the cost of capital will be analyzed in greater depth.

Although we have verified that the methodology provides reliable results, the high dependence of the two mentioned factors (volatility of the underlying and effect of the presence of options on the cost of capital) evidences the need to intensify the research in the determination of these aspects.

Chapter 3 Annex 1: Statistic data employed in volatility calculation

year	new_hom_yevar	bull_bear	NNDI_PC_yevar_t_1	3y_sp_bond _t_1	sp_stock_yevar_t_1	popmin25_yevar_t-1	popmore24_yevar_t-1
1987	29,54	1	3,90	9,12	108,31	-1,13	1,30
1988	27,48	1	7,26	12,71	9,06	-1,23	1,27
1989	25,81	1	6,31	11,32	20,80	-1,38	1,29
1990	8,74	1	6,31	13,15	8,08	-1,58	1,29
1991	-0,21	0	4,14	14,43	-24,73	-1,85	1,34
1992	-1,29	0	2,57	11,58	10,30	-1,88	1,23
1993	-0,22	0	0,11	13,18	-12,99	-1,42	1,50
1994	4,03	1	-2,63	9,00	50,65	-1,40	1,63
1995	3,67	1	1,00	10,49	-11,70	-1,47	1,55
1996	1,31	1	4,53	10,59	12,30	-1,66	1,57
1997	3,39	1	2,32	7,26	38,96	-1,81	1,56
1998	5,12	1	4,21	5,32	42,22	-1,93	1,56
1999	9,00	1	4,71	4,02	37,19	-2,02	1,59
2000	12,47	1	2,06	4,55	16,22	-2,10	1,54
2001	8,84	1	4,90	5,23	-12,68	-2,15	1,54
2002	14,73	1	2,54	4,20	-6,39	-2,13	1,59
2003	15,84	1	0,96	3,97	-23,10	-1,61	1,94
2004	18,38	1	1,06	3,03	27,44	-0,02	2,70
2005	10,06	1	1,22	3,01	18,70	-0,13	2,43
2006	9,82	1	0,78	2,58	20,56	-0,05	2,44
2007	5,14	1	1,67	3,66	34,49	0,14	2,20
2008	-6,64	0	0,80	4,12	5,60	0,66	2,15
2009	-5,68	0	-1,56	3,86	-40,56	1,23	2,24
2010	-3,21	0	-5,12	2,56	27,23	0,63	1,47
2011	-4,04	0	-0,15	3,09	-19,17	-0,31	0,83
2012	-6,90	0	-2,48	4,52	-14,55	-0,36	0,64
2013	-7,82	0	-3,01	4,33	-3,84	-0,25	0,52
2014	-2,21	0	-2,06	2,97	22,71	-0,69	-0,03
2015	2,91	1	2,19	0,97	3,01	-1,00	-0,28
2016	3,31	1	4,08	0,78	-7,42	-0,52	-0,01
2017	5,05	1	5,25	0,21	-2,24	-0,26	0,06

Chapter 3 Annex 2: Monthly binomial tree (with volatility reduction)



Chapter 3 Annex 3: Table 3.1. Detail

MALLORCA PHASE 1 BP	Total		
Uses - Funds use detail	Value	Coments	Amount
Asset purchase			**1,130,000**
Asset Purchase Costs			**58,505**
Due Diligence		Estimative	**15,000**
Acquisition Fee (to commercial)	3.00%	of the purchase value	**33,900**
Structuring Costs		of the purchase value	**0**
Notary + registry	0.45%	of the purchase value	**5,085**
Legal advice	0.20%	of the purchase value	**2,260**
Appraisals	0.20%	of the purchase value	**4,500**
Construction costs			**2,865,310**
Construction license	4.00%	of construction costs	**96,560**
Demolition	0	Per SQM	**0**
Construction costs (Above Ground)	1,023	Per SQM	**2,414,000**
Construction costs (BelowGround)	450	Per SQM	**339,750**
Service connections (water, medium voltage networks,…)	15,000	Estimative	**15,000**
Fees			**470,323**
Architect's fee (Project and construction surveillance)	7.00%	of construction costs	**192,763**
Health and safety coordinator's fee	1.25%	of construction costs	**30,175**
Quality Control	3.43%	of architecture fee	**6,612**
Project Management	1.50%	of Gross Asset Value / year	**86,210**
Construction Management	4.20%	of construction costs	**115,658**
Stamps	0.40%	of construction costs	**9,656**
Administrative	0.88%	of construction costs	**24,233**
Geothecnical Study	0.50%	of construction costs	**12,070**
Topography	2,000	Estimative	**2,000**
Financing costs			**46,781**
Guarantees	20,000	Estimative	**20,000**
Mortgage Tax	1.50%	of the loan	**9,323**
Notary, Registry - Mortgage	0.60%	of the loan	**3,729**
Opening Fee- Acquisition Loan	0.60%	of the loan	**3,729**
Opening Fee- Construction Loan	0.60%	of the loan	**10,000**
Marketing and sales			**182,460**
Marketing	400	per apartment	**8,000**
3D - Renders	1,500	Estimative	**1,500**
Sale Blueprints	600	Estimative	**600**
Sale Material	1,000	Estimative	**1,000**
Sale Fee	3.00%	of the sale value	**171,360**
Insurances, stamps and other taxes			**111,398**
Occupancy Certificate	60	per apartment	**1,200**
Decennial insurance	1.00%	Building costs & fees	**33,356**
Liability insurance (security)	0.36%	of construction costs	**8,690**
Technical Control Office	0.60%	of construction costs	**16,523**
Notary - Newconstruction + property division and deed of sale	0.30%	Acq.+ Capex+ Acq. Costs	**10,808**
Registry - Newconstruction + property division	0.24%	Acq.+ Capex+ Acq. Costs	**8,646**
Project stamps	2,000	Estimative	**2,000**
Garage opening permit	0.8%	of construction costs	**19,312**
IT facilities tax	0.45%	of construction costs	**10,863**
Other development taxes and licenses			**156,191**
Other taxes (Public thoroughfares occupation permissions…)	1%	of construction costs	**24,140**
Real property tax (landplot)	4.5	per SQM (of plot)	**28,035**
Newconstruction taxes	1.50%	of construction costs	**36,210**
Property division taxes	1.50%	Acq.+ Capex+ Acq. Costs	**54,038**
First Occupancy License	0.50%	of construction costs	**13,769**
Total Uses - Investment			**5,020,968**

MALLORCA PHASE 2 BP

Uses - Funds use detail	Total Value	Coments	Amount
Asset purchase			**360,000**
Asset Purchase Costs			**0**
Due Diligence		Estimative	**0**
Acquisition Fee (to commercial)	3.00%	of the purchase value	**0**
Structuring Costs		of the purchase value	**0**
Notary + registry	0.45%	of the purchase value	**0**
Legal advice	0.20%	of the purchase value	**0**
Appraisals	0.20%	of the purchase value	**0**
Construction costs			**1,408,017**
Construction license	4.00%	of construction costs	**44,955**
Demolition	0	Per SQM	**0**
Construction costs (Above Ground)	611	Per SQM	**1,123,863**
Construction costs (BelowGround)	380	Per SQM	**224,200**
Service connections (water, medium voltage networks,…)	15,000	Estimative	**15,000**
Fees			**310,965**
Architect's fee (Project and construction surveillance)	7.00%	of construction costs	**94,364**
Health and safety coordinator's fee	1.25%	of construction costs	**14,048**
Quality Control	3.43%	of architecture fee	**3,237**
Project Management	1.50%	of Gross Asset Value /year	**122,934**
Construction Management	4.20%	of construction costs	**56,619**
Stamps	0.40%	of construction costs	**4,495**
Administrative	0.88%	of construction costs	**11,863**
Geothecnical Study	0.50%	of construction costs	**5,619**
Topography	2,000	Estimative	**2,000**
Financing costs			**35,332**
Guarantees	20,000	Estimative	**20,000**
Mortgage Tax	1.50%	of the loan	**2,970**
Notary, Registry - Mortgage	0.60%	of the loan	**1,181**
Opening Fee- Acquisition Loan	0.60%	of the loan	**1,181**
Opening Fee- Construction Loan	0.60%	of the loan	**10,000**
Marketing and sales			**134,780**
Marketing	400	per apartment	**6,400**
3D - Renders	1,500	Estimative	**1,500**
Sale Blueprints	600	Estimative	**600**
Sale Material	1,000	Estimative	**1,000**
Sale Fee	3.00%	of the sale value	**125,280**
Insurances, stamps and other taxes			**53,784**
Occupancy Certificate	60	per apartment	**960**
Decennial insurance	1.00%	Building costs & fees	**16,629**
Liability insurance (security)	0.36%	of construction costs	**4,046**
Technical Control Office	0.60%	of construction costs	**8,088**
Notary - Newconstruction + property division and deed of sale	0.30%	Acq.+ Capex+ Acq. Costs	**4,452**
Registry - Newconstruction + property division	0.24%	Acq.+ Capex+ Acq. Costs	**3,561**
Project stamps	2,000	Estimative	**2,000**
Garage opening permit	0.8%	of construction costs	**8,991**
IT facilities tax	0.45%	of construction costs	**5,057**
Other development taxes and licenses			**74,528**
Other taxes (Public thoroughfares occupation permissions…)	1%	of construction costs	**11,239**
Real property tax (landplot)	4.5	per SQM (of plot)	**21,870**
Newconstruction taxes	1.50%	of construction costs	**16,858**
Property division taxes	1.50%	Acq.+ Capex+ Acq. Costs	**17,821**
First Occupancy License	0.50%	of construction costs	**6,740**
Total Uses - Investment			**2,377,406**

Chapter 4

Real Options in Real Estate Projects: The Role of Cost of Capital

4.1. Real options and cost of capital links

Up to now, we know that the different alternatives that appear in the development of a real estate project imply the existence of real options embedded in it. These real options, to the extent that they imply management flexibility, confer, per se, an additional value intrinsic to the project. This fact means that the most traditional valuation of the projects, based on the financial update of the cash flows generated (DCF method), is insufficient. To the value obtained through this DCF method, we have to add the value of these real options.

On the other hand, it is obvious that the investor in this project expects to obtain a return on the invested capital. The interest received by the capital is conditioned completely by the level of risk assumed in the investment. In the real estate projects' case, as well as real options in general, we are not dealing with a listed asset where we can apply the traditional methodologies that determine its volatility, and therefore its risk.

Thus, the investors' risk perception of the project determine, to a large extent, the project cost of capital. This perception has, in a significant number of occasions, a predominant subjective component. Depth knowledge of the project is essential to ensure that such investor establishes its criteria on economic compensation expected from it.

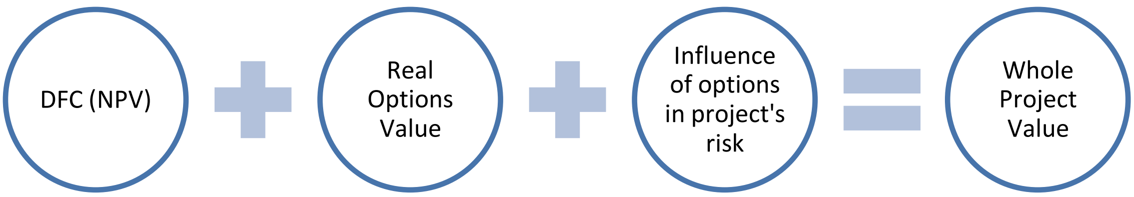


Figure 4.1: Conceptual project's value composition. Own design

Flexibility in management, understood as maneuverability in case of change in the parameters to value or existence of opportunities, reduce the risk of losses on the project and increase their earning potential. Therefore, it is intuitive to think that if the presence of real options reduce the risk that the capital invested is submitted to, the reduction of risk should be reflected in a reduction in the cost of capital required by investors.

This cost of capital reduction could be crucial when determining whether the project is carried out or discarded due to lack of funding. In addition, the cost of capital that is set is an important variable in the process of real option analysis. Therefore, the value of the whole project (DCF plus real options) is clearly conditioned by this cost of capital.

The existence of management flexibility has a double effect: (first) it increases the value of the project by the mere fact of its presence and, at the same time, (second) it reduces the cost of capital which implicitly increases its NPV. It is important to emphasize that these are two different effects, simultaneous but caused by different subjects: market and investor. The first effect is the intrinsic value that this flexibility really has, given the market conditions: the value of the option. The second effect is the subjective effect on the investor of investing in a project that is less exposed to the setbacks of the market: the increase of the whole of the value produced by requiring the investor to have a lower return on its capital in the project. Although both reasons are related to the presence of flexibility, this should not lead us to get confused and think that we are duplicating the same effect.

The determining factors of the first effect are more objective insofar as they are derived from the estimation of the evolution of the market. Those of the second are more subjective and will be different for each investor. In this regard, the investor will not be affected by the value of the option but the perception of risk reduction due to greater flexibility.

As already noted, there is methodology to quantify the first effect, but not so with the second. Assessing their effect on the investor’s confidence in the project is largely subjective and, therefore, there are no methods, ex ante, to quantify the extent of the reduction.

It should be noted that, although the existence of a relationship between real options and the cost of capital seems evident there is still a lot to be said. There is a big subjective component that needs to be under control in order to measure such influence. Actually, recent specific studies in the analysis of real estate market real options ignore this aspect and are based on the valuation of the options themselves (see, for example, Čirjevskis & Tatevosjans 2015).

This chapter discusses under which aspects or considerations the cost of capital required is influenced by the presence of options embedded in the project. Thus, different aspects are presented. First, in this introduction, the concept of capital cost for real estate projects is analyzed. Later, the relationship between the volatility and capital cost is also analyzed. Both, the project and of the options volatility are crucial aspects in the valuation, and both are complex to calculate.

In the second section, we briefly analyze the role of the cost of capital required when evaluating options and projects within the real estate sector. In the third section we study the factors that determine the valuation of the cost of capital required. This is used as a basis for the analysis, later in this chapter, the influence of options on the real estate projects’ cost of capital.

The last section of this chaper collect other aspects to be considered in the analysis and also the final remarks.

4.1.2. About the “cost of capital” concept and the Real Estate sector

One of the characteristics of real estate investments is that they are very capital intensive investments. In addition, these are investments with very long maturing periods. In this context, the cost of capital required by investors is has more influence than in other kind of investments. The capital is usually provided by a company, an investment fund or a private investors. In addition, it is common that part of the construction is leveraged with the capital of a financial institution (a bank).

Thus, in real estate investments, the concept of cost of capital required is not unique, given that we find different costs from different financing sources. Therefore, we would obtain a specific weighted average cost of capital (WACC) for the project. A usual formula, for projects of a certain size, is the constitution of a commercial company specifically for the development of the project, Special (SPV). The different investors, partners or credit entities participating in the project, have different level of warranties in order to recover their investment in case of project failing.

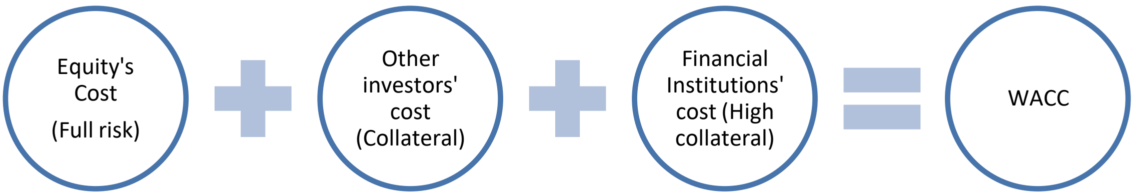


Figure 4.2: Usual cost of capital composition. Own design.

Obviously, there is a relationship between the guarantees offered and the cost of capital required by each funding source. This whole chapter focuses on the cost of capital required by the investment partner, defined as one that, in case of bankruptcy of the project, looses the capital invested completely.

Let's suppose that, before a serious setback a foreclosure is materialized by the financial institution and the remaining assets are not sufficient to cover other debts. We should focus on this "pure" cost of capital required. Conceptually, the investor only value the success of the project as collateral of her investment. If we carry out another type of analysis, we should have to analyze the collateral characteristics.

Usually, the business plan yields a profit margin or an internal rate of return (IRR) which the data that the investor use to decide if it is among the parameters considered appropriate for the acceptance of the development of the project. That is, if it exceeds its required cost of capital for a project with these specific risk characteristics the invested is taken into account.

Often, the profit margin or the IRR is compared with a standardized value for the investor to make the decision. This standardized value usually comes from the previous investments made or from the investment alternatives within the same sector. This decision process is clearly wrong. The specific nature of each investment makes it illogical using standard comparison values: since there are no two real estate projects that carry exactly the same risk, it does not make sense to use the same acceptance criteria.

There may be a project with a higher IRR, but how can this investor that the project has the same level of risk? Unfortunately, the lack of sufficiently detailed criteria, and reliable valuations, makes that a square meter of land in a certain neighborhood is treated as a commodity. All the information helping us to understand how to allocate the required cost of capital alse helps us to make better investment decisions.

4.1.3. The cost of capital of the project and the volatility of the asset value

The main difficulty In determining the cost of capital of the project relies in an area that is not related to the presence of options. It is the volatility of the project value. The volatility concept refers to the change in market price, over a certain period, of the product to sell, (in this case the finished building sqm). Estimates of these variations can be made based on multiple aspects. The basis for this estimates is the observation of past behavior by market research, adjusting the result based on macroeconomic, demographic, or other variables that may affect the price forecasts.

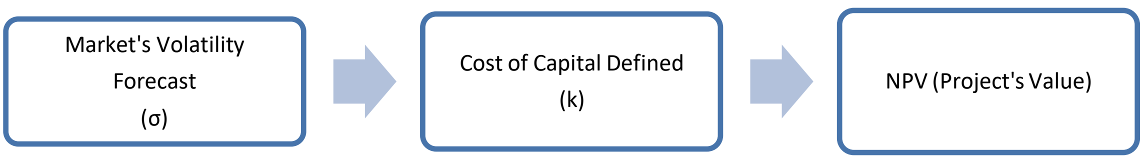


Figure 4.3: Volatility, cost of capital and project’s value. Own design.

The econometric modeling to achieve reliable estimates are faced with important pitfalls such as the lack of sufficiently detailed data series or the multiplicity of factors that affect real estate sector. In any case, we can expect to get “probable” values of volatility.

This indicates that the crucial factor in determining the cost of capital is, in turn, the major problem in the analysis of the value of the project. Both parameters, volatility and the options that may be implicit in the project, are specific to every different project.

4.1.4. The systematic risk

As well known, the cost of capital in an investment of any kind can be divided into two components: the one that refers to systemic risk and the one to the investment specific risk . In the case at hand, the risk in real estate projects is an extreme case. Reasons for that are: a) it is an investment so intensive in capital also for the finalist (for the user), b) it is usually always a highly leveraged investment with long-term credits. In both parts of the economic cycle, bearish and bullish, the price of housing presents a high elasticity with respect to GDP variations.

We can therefore affirm that the main determinant to calculate the cost of capital to be required to the project, the volatility of the price of the sqm, is largely conditioned by macroeconomic factors. As the financial theory indicates, this systemic part is the only one that the investor must pay for, since the specific part is diversifiable. It is clear that, when analyzing projects of this type, the volume of investment makes it practically impossible to diversify the specific risk for a majority of investors.

In turn, it is intuitive that the specific risk part is the one most sensitive to the existence of flexibility, hence to the existance of options. According to this argument, the presence of options can help to compensate the effect of the specific risk of the investment. From the point of view of the investor this helps to compare the analysis of the project to that of any other: basically considering the systemic risk.

This last statement makes even more important that we analyze the effect of options on the cost of capital required. Although there are many difficulties to quantify it, the effect is obvious and deserves a detailed analysis.

4.2. The role of cost of capital in the real estate real options calculation

In this section we focus on the calculation of real options. Specifically, in the use we make of the capital cost by applying the most widespread methodology for quantifying the value of such options. We must remember that, in order to calculate the value of an option, we always need the value of its underlying asset.

In our case, these underlying assets are real estate projects. At the same time, in order to calculate the value of these underlying assets, we need to use DCF techniques, that is, we need to calculate, for example, their NPV as a previous step to obtain its market value or gross present value (GPV). Since we cannot calculate its NPV without determining the discount rate of capital, the first step is to know the cost of capital required. As mentioned before, we consider the equity invested, that is, we will not compute the loans linked to the project.

4.2.1. The options' calculation and the risk-free world

The first problem we face in this type of project valuations, and its NPV, is the consideration that any calculation leading to the valuation of an option should be based on a risk-free world. This condition is determined by the very system we use and the theoretical foundations on which it is based. An option value calculation based on a risk-free world would imply that the rate of update of flows should be the risk-free interest rate (only for option values).

What would happen if we used the risk-free interest rate as a discount rate for real estate projects? Keep in mind that a series of very specific factors are combined:

- These are projects with a high level of risk, due to the high elasticity of their financial result with respect to the GDP cycle and the uniqueness of each case.
- These are very capital-intensive projects, that is, considering a small number of partners, the individual effort is usually high and difficult to diversify.[8]

[8] In recent years, investment formulas through crowfunding, including the investment policies of some hedge funds dedicated to real estate, have made the considerations regarding the number of investors, the effort required or the diversification of risk appear nuanced. In any case, we refer here to the most traditional forms of investment in which, driven by a high opportunity cost, the partners are a small number.

If we used the risk-free interest rate when calculating the NPV of the project, compared to market, the value that we would obtain for the projects would be disproportionately high. This would negatively influence the analysis of the result as well as the credibility of the analysis as a whole from the investor's point of view. The abstract concept of the risk-free world, even serving as a basis for the formation of the equivalent portfolio, which serves as the basis for the calculation of the option, would result in a value of the project greater than that which the market intuits, thereby awakening the investor suspicion.

Therefore, if we wanted to be strict in the application of the usual methodology for calculating the value of options, it would not be necessary to value the capital cost required to calculate the NPV (since we would use the risk-free interest rate). However, the use of a market value, with the corresponding risk, makes the value of the project and the option more assimilable by those involved. In addition, given that if we take the opposite criterion we would find ourselves with higher values (of the project and of the option), we are evaluating under a principle of desirable prudence.

To sum up: In the preliminary calculations to be made before calculating the value of the option itself (in the NPV valuations of the projects) the corresponding capital cost sould be used, and risk-free interest sould be used in the application of formula to calculate the value of the option.[9]

4.2.2. The NPV role

The NPV of the underlying is determinant for the value of the option, since it is needed to estimate the expected evolution of the project’s market value. In turn, this is totally conditioned by the discount rate used. Equating the discount rate to the required cost of capital we could say that the value of the option is determined by the cost of capital.

However, although this ia a very common reasoning, this is not always the case. For example, certain types of options, such as the option of postponnig the purchase of a property to maintain it as a rental asset from the first day, obtaining certain profitability. We will understand as the start of the project the acquisition of the asset, that is, the option is a future purchase.

[9] This is applicable whether the Black-Scholes-Merton formulation is applied or if the binomial tree methodology is used.

In that case, we could establish the future acquisition price of the asset through a contract. Therefore, unlike the general case described in the previous section, we would use the risk-free interest rate in the calculation as a whole. This would make sense, since there would be two disbursements: one to pay the option premium and an a secons to pay for the good, at which time the option loses its effectiveness. This is the most simple structure of an option we could find.

But, in that case, the option has a mere speculative purpose. In addition, the fact that it is a real estate property is irrelevant, only to be considered when analyzing the market with which to compare its price.

On the contrary when we use the concept "project" it should require a transformation process with successive disbursements, a future income and probably a limited life. The risk of the investor acquires nuances, as have been described in the preceding sections.

4.2.3. The specific case of the real estate sector

The characteristics of the real estate sector make it specially sensitive to the use of options, however, the same reasons make frequent the use of derivatives (well or wrongly valued), often speculatively. Real estate investors consider, as the basis for their analysis, two references for the cost of capital required:

- The first, which this chapter deals with, is the cost of the money invested in a transformation project. It is in this transformation project, which requires a deep analysis and the necessary know-how, where the risks are greater. This interest rate can be very far (e.i. up to more than 10 percentage points) from the risk-free interest rate.
- The second is the required return to a real estate asset that will not endure a significant transformation when being bought. It is used to estimate its value. Due to the long depreciation period of real estate assets and the fact that they are often used as save haven, this cost of capital is much lower that the previous one. Depending on the asset location, two or three points above the rate of a Government bond could be a reference.

Figure 4.4: Volatility, cost of capital and project's value.

Despite being two distinct concepts, the first is derived from the second. This happens because the value that will be considered to the final production of the project (housing, offices, etc.) will be valued in terms of what is their equivalent income. That is, we must consider what are the net cash flows that produce and apply the required return to the finished building (the second concept) to be able to know its value through the DCF method.

The second value should not be affected by the presence of real options in the project. However, it is this value that reflects the changes in the market. The higher the profitability required to invest in real estate to maintain the portfolio, the lower will be the value considered for the project development, therefore the profitability and the cost of capital will be also lower.[10]

Thus, we face the paradox that the return required by the rental investor, not by the developer, is what reflects the market before and is not affected by the options given. However, since the value of the options is justified only by changes in market prices, only when the second value is likely to vary it makes sense to calculate the value of any option.

[10] Bear in mind that the real estate investor, who invests with the intention of patrimonializing and obtaining a long-term return on his/her investment, usually calculates the acquisition value of the property simply by applying the perpetual income formula. To do this, he/she calculates the estimated income indicated by the market, detracts the expenses and divides the resulting annual amount by the percentage of profitability that he/she intends to obtain.

4.3. The cost of capital drivers in real estate projects

In this third section we analyze which are the main factors that affect the cost of capital required in a real estate project. It is essential that we know these factors before analyzing how they are affected by the presence of options. At this point, no reference is made to the effect of the presence of options, an issue that is the subject of section 4. We will analyze three main factors: the market, the size of the project, and the degree of information available to the investor.

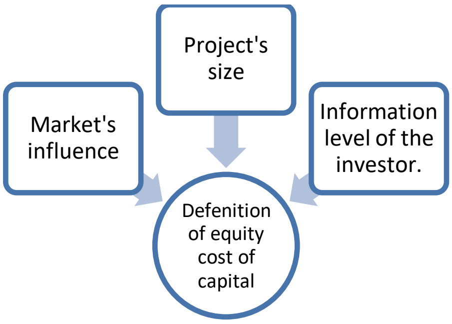


Figure 4.5: Cost of capital in real estate projects, main drivers. Own design.

4.3.1. The market: beta of assets-in-place and its beta of growth opportunities

We can affirm that the trend of the market and the economic cycle is the first important determinant to determine the cost of capital in a real estate investment project. How to approach or refer to this market trend is a priori complex, the first alternative is through the capital structure asset pricing model (CAPM) (Sharpe 1964).

The CAPM method, used in the valuation of listed assets and the cost of capital required, largely serves the literature on determining capital cost as a basis for unlisted assets. In the case of a real estate project, understood as an investment limited in time, this analysis is complicated. Even so, some authors use the sectoral data or the promoter company to determine their covariance correlation with the market and end up defining a fair cost of capital.

In other words, if the group of promoter companies, whether quoted or not, are showing a certain level of volatility in their economic results and returns, it seems reasonable to speak of a beta for real estate projects. Sectoral volatility summarizes us, in a way, the set of risks, that affect projects of these characteristics. Literature is critical with this type of analysis, especially if we analyze the components of this beta as a part corresponding to assets-in-place and another part corresponding to growth opportunities (see, for example, Bernardo, Chowdhry & Goyal 2007).

Assuming an analysis from this base, each new project is an opportunity for growth whose risk has little to do with the risk of projects already carried out in the past. In fact, the same authors indicate that the risk of growth opportunities far exceeds those of assets-in place in practically all sectors. Even so, the CAPM, and the attempt to determine a beta, is still the most used method to calculate the capital cost of an investment project (Graham & Harvey, 2001).

Although risky, this methodology is not so far from an objective assessment. Berk, Green, and Naik (2004) demonstrated in 2004 that many times the decision to undertake a project ends up being reduced to the analysis of the systemic part of the risk. Thus, an analysis of how the business is doing to the listed promoters should be sufficient to gauge the risk that is run when undertaking this type of projects and set the required performance.

Note that the literature on what are the risk factors of a project is very extensive and, sometimes, contradictory. Establishing, from an objective perspective, the set of factors that affect the risk does not seems possible. Not even the methodologies that, through econometric models, try to delimit (ex post) the main factors, achieve results that are conclusive.

Gyourko & Keim (1992) analyzed how the stock market reflects the evolution of the real estate market. For them it is possible to find a portfolio capable of predicting the movements of housing prices. Further analysis contextualized on actual data provides, according to these authors, an insufficient amount of data, thus choose to look at the stock market and infer what will be the estimated return.

In the study by Mao & Wu (2011), focused on the risk factors in the real estate sector. These authors conclude precisely that the methods used to analyze the risk factors in the real estate sector fail in the analysis of the influence on the management development. Mao & Wu pointed out that this can be solved with the use of real options.

Beyond that, the differentiation between assets-in-place and growth opportunities in developing the project has a special meaning in the case of real estate: the key is to buy the asset to be transformed at the right price. Suppose that we have already acquired the asset that we are going to transform (a landplot, for example). If we analyze the project at te purchase moment, we observe that the risk on the investment before starting the transformation is very different from the risk of the whole project.

The risk on the landplot, an asset whose amortization period is infinite, is practically the systemic risk, the danger is reduced to its possible lack of liquidity. Note that, unless the site is acquired by combining bank financing, it is impossible for the value of the investment made to become zero. [11] However, the risk of the capital we use in the transformation is much greater. We could contemplate, conceptually, this part of the risk as the risk of growth opportunities.

It could be reasonable to say that the capital cost estimation starts from a "sectoral" beta, although the singularities of the project should also be a relevant factor. The problem that difficulty in estimating how does the market then pay for the risk of the existing growth opportunities.

The purchase price of the asset to transform should reflect the difference between the cost of capital required "sectoral" and the specific project, i.e. how they are paid in that particular project growth opportunities. The literature shows that even the most faithful followers of the CAPM accept that there are other non-market factors affecting the cost of capital (see e.g. Fama and French, 1992).

In a perfect market, as far as players and information are concerned, the market tells us what exactly the capital cost to be required by the investor is. Imbalances in the market (in the sale price of the asset to be transformed), should lead to the project not being undertaken until these are corrected. That is: two key market inputs, 1) how much I buy and 2) at what price I sell after the transformation, should reflect what is the cost of capital required that the whole market establishes for that project.

There are several problems in this respect as the frequent lack of professionalization of the participants in the market or the distorting effect of the partner's leverage with low-cost financing (obtained from financial institution, for instance).

4.3.2. The project’s size

The second important factor when determining the cost of capital required on a project is size. Usually, the size of the investment required in a project should correlate positively with their

[11] Certainly, there would still be the risk that a change in the urban legislation limited rights to the owner, but even in that case, there would be a grievance that the owner could complain to the legislator.

cost of capital: that is, the larger the project, the higher the required return of the investor. This happens for several reasons:

- The size of the investment acts as a barrier to entry for a significant number of players in the market. Projects of a certain size are bearable only by large companies that may have the initial investment. Even when such initial investment is partially financed by a financial institution, which happens in most cases, the entity will only take as sufficient collateral to certain types of solvents investors.
  There is a situation of demand oligopoly, in the market of assets to be transformed (e.i. landplot), in a natural way, reduces the sale price of the asset to be transformed. For the same reason, a large lot (or building to be transformed) is more illiquid, so the investor will perceive a lower risk. This represents a fundamental factor for the market to adjust the maximum purchase price in relation to the potential sale price transformed.
- The project enters into competition with itself, due to its large size, making the final sale price of the sqm more uncertain. It has already been pointed out that the sale price and market fluctuations are key when determining the cost of capital. A very large project becomes a problem for itself, given that the bargaining power will be influenced by a change in the balance between supply and final demand.
- Economies of scale occur at all levels of the project. All involved in the project (builders, technicians, material suppliers, utility companies, commercial equipment, marketing, financial institutions, etc.) will increase its interest in the project and represents business for them. The first consequence of this is a reduction of costs that will positively affect the outcome of the project. The investor is aware of this fact and will require more profitability from their capital.
- It evolves in the learning curve. In real estate projects, as in any development with unique characteristics, there is an adaptive and learning process that generates the costs of the final sections are significantly lower than those of the initial ones.

It is complex to measure the extent to which the project size affects the required capital cost. Once again, we are faced with the singularity of each project, which makes it difficult to carry out ex post analyzes that help to have an order of magnitude in this respect. This factor is closely related to the previous one because, once again, it will be the market that determines the increase produced.

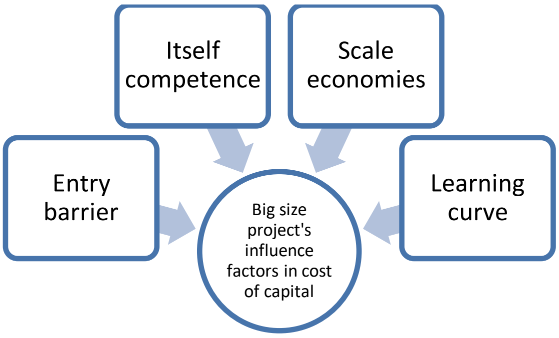


Figure 4.6: Main influence factors due to real estate project's size. Own design.

4.3.3. The information level

As with any other project, the amount of information provided to the investor in the study phase of the project will determine his willingness to invest, his confidence in the project and, therefore, the price of his money. An inverse relationship could then be established between the amount of information and the cost of capital. As pointed out by Christensen, de la Rosa & Feltham (2010), it is not even necessary to have an empirical result for this affirmation, since it is intuitive.

However, some authors (see, for example, Hughes et al., 2007) point out that this effect occurs only if the information provided is related to what systemic risk refers to. We will understand, for the coherence of our analysis, the systemic risk as that which comes to us from the market.

The specific risk is, by definition, diversifiable and, therefore, the investor should not contemplate it in the construction of its portfolio. Certainly, this statement made in the literature on investments in general, is counter-intuitive in the case of projects such as real estate. What type of information is then really relevant in real estate projects for the cost of capital to be affected?

Easley & O'Hara (2004) argue that the quantity and quality of information affects the price of assets. Companies can modify their capital cost by working on their quality of accounting management and their level of analysis of their processes. His model focused on differentiating the influence between public and private information, demonstrating the effect of both. We accept that any information, internal or public, that the company owns is likely to affect its cost of capital. Very specifically, the one that delves into the financial / accounting analysis of the company.

Accepting both trends of opinion, the conclusions of Hughes, Liu & Liu (2007), in their analysis of asymmetric information and its value to influence the cost of capital, indicate that the most valuable information is that which, being private, analyzes the project's sensitivity to systemic shocks. That is, it is only relevant the company specific information that offer informations about the covariance between its financial result and sectorial market variability. Regarding the cost of capital, this covariance information should be a complement of the the market information itself.

4.4. Options' influence on the cost of capital

In this fourth section we focus on the contribution of the options to the three factors described above. We endeavor to deepen in how is the presence of real options in real estate projects related to the determination of the cost of capital by the investor. First consideration is that each type of option is likely to affect the determination of the capital cost of the project in a different way. Therefore, within the three different aspects considered it will be necessary to specify for what type of options is relevant.

Before starting this part of the analysis, we should stop and consider the real estate real option concept, are and how do we perceive their existance. Often the real options appear naturally, most of the time without being wrote in any contract. We perceive that we have a valuable opportunity for flexibility in management and call this a (real) option. Its value will come from offering coverage in the face of market uncertainty, the possibility of resizing the project or the possibility of making decisions in conditions of greater information. These three aspects are those indicated above in section 3.

4.4.1. How can an option offer coverage in the face of market uncertainty?

The answer to this question seems obvious. Actually, options are mainly conceived as hedging or speculative elements in the face of market changes. When it is a real option, the option postpones the decision, taking it to a time when the variation of the market is already known or, in any case, it is easier to guess. As Miller & Park (2002) point out, the real options "comes the rescue" when the investor has to make decisions at the expense of the market.

The financial literature has historically shown us how, under the same conditions, the investor will try to maximize the result of his investment, maximizing his utility function (see, for example, Modigliani & Miller, 1958). It is the market that converts the result of the operation into a random variable and it will be the options that delimit the effect of this variable.

This aspect is, therefore, the highest added value of an option. Take, for example, the option to defer an investment. At a strategic level, being able to decide which the best time to start an investment is allows us to wait for the market trend on our side. The investor's perception is that he not only acquires a profitable business but also that the project will be developed when he considers that market conditions are favorable: the market will wait for him or viceversa.

Something very similar happens with the option to change the nature of the investment. The possibility of adapting our project to fit the characteristics of market demand is equivalent is key to maximize the result of our investment.

The control of the market is, *per se*, a comparative advantage among investments, and therefore an incentive to invest. We have commented previously that the knowledge of the cycle allows us to compare adequately the different results of companies that have operated in the market and give us an idea of the expected results. Then, we should be able to approximate the result to the "sectorial beta" to which we referred. The presence of real options offers us the strategic flexibility to approximate or improve the result to those observed in the market at the same moment of the cycle. The key is knowing how to read correctly the market and the information it offers, since the correct use of the option is decisive to put it in value.[12]

In addition, the presence of options is interpreted by the investor as an opportunity for growth. Previously, we already pointed out the important weight that these "opportunities" have on the perception of our "beta". If our analysis of the project shows that it offers greater management flexibility than the average project, it is reasonable to expect our result to be able to beat the market, since it should be able to adapt in a more optimal way to its evolution.

Regarding this point, a prudent and realistic approach by the analyst is key. Someone can also be fooled into overestimating the growth potential that derives from this flexibility. In any case, any credible approach would have a positive effect on the cost of capital. In other words, the option can also help to mask the weaknesses that the project related with the market evolution.

[12] Along this chapter, reference is made precisely to this aspect: all the value of the option goes through a good management of it, the investor's confidence in the manager and the manager's capacity to know how to transmit to the investor the use that can be given to those options.

4.4.2. Sizing our project through options

The option to carry out the project in phases allows us to define what the real size of our project is. Imagine a case where we have a plot and we can do one or more buildings in a single project and the investor is who decides whether worth developing other phases, depending on the market, the results or her own interests. In the same way, the possibility of developing it in phases allows limiting the negative effects of a large project.

The same applies to the option to expand the project. If we start to develop a project with other plots available in the surroundings, the effect is similar to the option of phasing the project. We could have, at the same time, the advantages of a medium-sized project at the same time that we have the advantages of a large project. Moreover, we could be reducing the disadvantages of having projects of one size or another.

These options are equivalent to having an option to abandon the project with the possibility of being exercised at certain times. That is, the project could be resold somewhere in the middle of its development.

The three types of options are inherent to the characteristics of the project (given by the characteristics of the site, the size or its location) and are hardly reflected contractually, so that there would never be a cost for them or give rise to any speculation. Most likely, it is the evolution of the market that marks whether they are finally exercised or not. In any case, the presence of these options directly affect the dimensioning aspects that are related to the required cost of capital.

Previously, we have indicated that the size of the project usually correlates positively with the cost of capital. The presence of options should then reduce the required capital cost, because it is interpreted as the flexibility that can help limiting the negative effects of having a too large project.

On the one hand, the possibility of positioning itself in a large project, by a medium-sized company, through the acquisition of a strategic phase, supposes the existence of an important opportunity cost. That point will be perceived by the investor with complete certainty. Imagine a building with a strategic location in an entire block that can be transformed. The purchasers of the adjacent buildings have two options to develop smaller projects or acquire the building, improving their project by carrying out a larger project with more optimization possibilities. This building not only has the value that the project that can be made in it, it also has the added value of being able to limit the neighboring projects or to expand if they are acquired.

At the same time, this small "expandable" building does not enter into a commercial competition with itself, but enters into strategic competition with the current owners or neighboring developers. In this case, of course, there is great variability of possibilities. If the neighboring owner has a very large project, he may no longer be interested in expanding it, or on the contrary, the key site that improves his project may be ours. A series of nuances are introduced that will give a subjective value to the option of abandoning (selling the site to the neighbor) or to the option to expand, carrying out more phases or an expanded project. Again, this flexibility reduce the problem of excessive size.

The effect of economies of scale and of the learning curve becomes ambiguous. Investors do not know if they will finally be able to take advantage of it in the development of the first phases, so they would not discount it in their initial cost of capital (maybe in later phases). However, if the manager is skilled enough to take advantage of this, with many of the contractors, the mere fact that future extra hiring can occur will help him in negotiations, probably lowering their prices.

If the subsequent phases are carried out, the effect on the learning curve will be the same, meaning positive effects on the project's revenue. However, the investor should not discount it when forecasting profitability, so the effect is diminished, just as it happens with the economies of scale.

In any case, it is intuitive that the presence of options, let's say "to scale" the project, should never imply an increase in the capital cost of the initial phase, since it is still calculated under the parameters of a smaller project. On the contrary, it can be expected that it will provide an additional incentive for the investor to invest.

It can also happen that, the fact of being able to carry out the project in phases, the payments structure in order to finance the projects are also phased. It could make that the capital requirements are much lower to obtain the same result. Consider, for example, a case where the margin obtained in a first phase could be used to finance the subsequents. This would reduce the barrier effect of entry and would greatly increase the expectation of the operation Return On Equity (ROE). In this case, the increase in the cost of capital would depend on the certainty about carrying out the subsequent phases (the exercise of the option, never the obligation). The greater this certainty, which will depend on the moment of the market, the greater the cost of capital.

4.4.3. Relationship between options and information

We have previously emphasized that the information on project that reveals the strategic tools to overcome, anticipate or help combat the market changes, will reduce our cost of capital. What information should be implicit in an option in order to leads to this effect?

We could say that knowledge of the existence of all options is, to the extent that they allow reacting to an unfavorable market, valuable information to reduce the cost of capital required. Let's see some examples: It is intuitive that, if the investor knows the existence of the option to change the nature of the investment, she will consider it invaluable information and will lead she to interpret the market risk in a different way. Let's suppose, for example, that a building or plot to be transformed into the center of a large city is acquired, with a great shortage of housing and very high prices. The business plan will surely consider the sale of homes to individuals, as it is the most profitable investment.

But suppose that the manager establishes a put option contract with a group of exploitation of assets for rent to students. Although this second option is less profitable in the business plan, it is a guarantee of minimum profitability before a possible change in the market. Even in the most unfavorable case, its effect is equivalent to that of a stop loss. The investor, knowing this information, would show preference for the investment.

However, the greatest effect of the options on the information of the investor is found in the options that allow to expand, carry out phases or even reduce the size of the project. The information that we obtain with the course of the first stages of the project does not always come from the strict evolution of the market. The investor obtains other valuable information with the development of the first phases that provide it, precisely with the information needed when determining the capacity of the project to counteract the market movements. As main examples:

- Information on the adequacy of the product to market demand and the capacity of this product to modify and adapt to it.
- Information on current costs and their suitability or not to the forecasts of the business plan. This shows the adaptation to the part of the market referring to the suppliers.
- Accounting and financial information, real and detailed, of the process and its result.

- Information regarding the quality of management in decision making. This factor is key to investor confidence in the ability to adapt quickly to changes compared to the forecasts.

All this information, well performed, should help in the decision process, and also facilitate to maximize profit. But not only maximize the profit, the course of the project, moreover, its result and the confidence of the investor and the group of participants will be reinforced. In any case, it is internal information, which is part of the project's and the company's know-how, also adding value to it.

In a study conducted by Bulan, Myer & Somerville (2008), in which they analyzed real data on the development of 1214 landplots in Vancouver, they were able to demonstrate, and even quantify, the relationship between the data of the standard deviation of prices (its volatility) and the fact that the promoters chose to delay the start of development over time. They demonstrated how competitiveness affects the value of options (such as postponing development). But in addition, they were offering empirical evidence of the triangle that relates information, the demand for capital and the use of options.

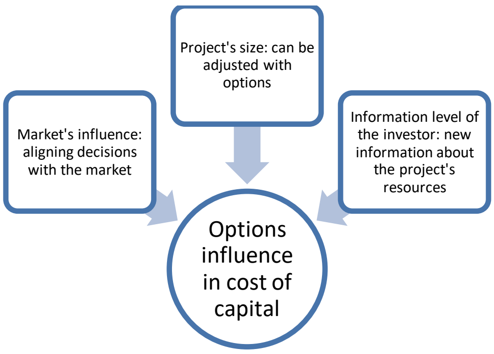


Figure 4.7: Main influence sources of real options in real estate cost of capital. Own design.

We can say that, the presence of certain types of real options, in general all those that allow to postpone or adapt decisions, affects reducing the cost of capital required by the investor. For this reason, it is important that the manager not only report the presence of such options, it is also essential that the person managing the project know how to gather the appropriate information. Manager should know how to communicate it and put it in value before the decision-maker (see, for example , Luehrman 1998). The presence of options, then, is presented as an extra challenge in the project strategic management.

4.5. Other issues to be considered

Up to this point, we have analyzed which are the more relevant aspects when determining the capital cost required by the investor in a real estate project. This si a crucial decision that will make him decide whether to undertake it or not. In addition, we looked how it affects the presence of real options to these decision. Even the mere fact of considering the presence of options has some effects, whose quantification is subjective.

These effects do not have to be always related to the market, the size of the project or the information available when deciding. In this section three of them are pointed out: the effect of trust in management, the comparative effect with the proxies and the sale of the options themselves.

4.5.1. The manager and the investor confidence

The detection, structuring, and analysis of the presence of options supposes, per se, a favorable indication about the capacity of the project manager. It indicates a greater financial knowledge, a greater capacity for adaptation and strategic ability and, probably, a deeper knowledge of the sector as well as the technical possibilities of the project. Therefore, the mere fact of raising the different possibilities of changes in the project as options, valued and well defined, will increase the investor's security when deciding.

In the case of real estate projects, with such long development periods, the ability to "reinvent" the project and keep it flexible for as long as possible adds value and reduces risk to it. As an example, when deciding to acquire a site for a new project in Barcelona, in which this author had the opportunity to participate, a management company valued above other factors (such as the location or even the price), the width of the plot. Precisely the width of the plot allowed, for a small difference of centimeters, that the type floor could be distributed with one or two appartments on each façade (that is, with two or four appartments per floor).

The same manager commissioned a dual result project, with four or two homes per floor. [13] This project maintained the thecnical characteristics allowing both optionsm until the end of contruction process. So, the decision was delayed on how to distribute each plant until reaching the brickwork phase (six months after the start of the works). In the same way,

[13] The project with four houses per floor was used to apply for the license to maximize the number of houses authorized by the municipality, knowing that the regulations always allow modifications that imply a reduction in the total number of homes allowed.

commercialization with these options was prepared until the end of marketing. This prevented the project partners from making the risky decision to build large or small houses in each of the plants.

It would be, therefore, directly the market who decided it, minimizing risk, maximizing revenue and accelerating sales. In this example, the manager created an option, its exercise time and its cost. An option tailored to the project that allowed significant benefits was designed.

In another example, also managed by this author, the management team acquired an small industrial warehouse with the aim of demolishing it and transforming it into homes. This team analyzed the adjoining buildings, also formed by old warehouses to be transformed. In this process they concluded that technically it was not possible to demolish the neighboring warehouse, in a landplot devoted to a much greater project, if simultaneously the small acquired industrial warehouse was not also demolished (given that both buildings shared essential structural elements). So, the new owner of the small warehouse had the power to block the big project permits.

Considering the profile of the investor of the large-scale project, it was understood that the option of acquiring the small lot had a high added value for him. Buying the small allowed him to have control of the development times and to improve a lot the project. It was the investors advised by this management team who made the opportune decisions on whether to exercise the option or not. In this specific case, the additional information made the exercise of the option vary greatly in value over time, which added a crucial strategic component.

These are just two real examples of how the manager can find and define complex and ad-hoc options that, although difficult to quantify a priori, there is no doubt that they reduce the risk of the investment and, therefore, reduce the required cost of capital. But there are many other examples.[14] Something exciting about the real options is that they allow the generation of customized management opportunities. Its conception and exercise are the result of an adequate strategy and a good knowledge of the project. A sample of the manager's ability to add value to the project and reduce its risk.

It is important to clarify that in no case are we suggesting that the capacities or knowledge of the project manager are an implicit value in the option or in the project. The literature points

[14] There are many examples in the literature of real options designed ad-hoc to address a particular business problem. It serves as a curious example how, in the case of international business management, Aabo & Simkins (2003) describe a series of real options aimed at achieving coverage on currency exchange rates.

out that it is the manager who detects the presence of options, analyzes them, and therefore puts them into value.

This is one of the links with the strategic prespective of the options. In the management of options it is crucial to detect the opportunity to exercise them, the best time to do so and the consequences of it. For this the manager's expertise is decisive. This capacity of the manager does not per se affect the characteristics or the value of the option. However, it is essential that the manager knows how to identify the strategic potential of the option and how to exercise it optimally.

In this respect, there are studies, such as that of Kogut & Kulatilaka (1999) in which they analyze the value of the manager's capabilities as real options per se. These authors explore the connection between strategic theories and their financial value. Investing in these capacities is, therefore, investing in market adaptation, investing in exploring new strategic options. This conception as strategic options is the basis to try to reach a balance between "exploration and exploitation".

For Sudarsanam, Sorwar & Marr (2005) it is possible to measure the value of intellectual capital (IC) through real options. In their study, the growth opportunities derive from this IC, which can be added to the value of the company. This is another example in which a close relationship is established between the value of the company, the quality of its management and the presence of real options. Even, in the case of Lev (2001), a correlation between IC and the return of equity capital is established through econometric analysis.[15] Along the same lines, Tobin's q model could be used to estimate the IC as a difference between the market value and the cost of its tangible assets.

Of course, there is also abundant systematic literature for the correct strategic use of real options available to managers. Part of the capital that is generated in terms of options management is shared and analyzed by the academic community. For instance, the work of Zhou & Wang (2004), specifically focused on the real estate sector, offering an adaptable formulation at different levels of market opening.

Therefore, although it is important to remember that the manager's capacity is not determinant in the value of the option per se, it is undeniable that, for the investor, the knowledge of the

[15] Sudarsanam, Sorwar & Marr (2005) also criticize Lev (2001) for the arbitrariness when choosing the criteria to quantify the IC value.

management capacity has an effect: it reduces the perception of risk in the project and, therefore, it is a potential reducing factor of the required profitability (the cost of capital).

4.5.2. Risk premium and the proxies' projects

Theory indicates that, when choosing between the different investment possibilities, the comparison between the possible investment alternatives and, in turn, the comparison with the systemic risk, guides the investor on her decision. Therefore, the basis of the decision on the return to expect from our capital will be what the similar projects are obtaining. It is commonly accepted that, in the bullish part of the cycle, the market is adequately balancing risk and return within the sector.

But, when we start consider the options that a project entail, and considering that these modify the risk of the investment, it is possible to compare the existing options in the analyzed project with those that are present in the projects that are being used as proxies. In other words, it is convenient to analyze if the options observed in the analyzed project improve those that one would expect to find in the average of projects. Otherwise, if the options we find can be considered "very usual" or "standard" it would not make sense to give a specific weight to this factor to determine the cost of capital. In that case, the cost of capital that the market indicates is the one that will be logic to apply.

The "valuable" options in terms of the cost of money are only those that make a significant difference. As an example, the possibility of establishing phases, the possibility of significantly modifying the nature of the project, those that establish a clear comparative advantage with the average value (high value of the sale of the asset by its characteristics within the market, for example).

Finally, do not forget that all projects are designed departing from a "zero" point. In addition, as we have indicated in the previous section, the most valuable options are both inherent to the project but are a finding or an invention of the manager or the investor. So in the most early stages of the real estate project reside the greatest possibilities of putting value on it without needing to increase the margin: finding ways to conceive it flexible from the very beginnig in order to adapt to the market. The work of a manager, or a fundraising team, shoud include to know how to expose these strategic features as valuable differenciating elements regarding the "proxies".

4.5.3. Options as tradable assets

So far we have referred to the real options in real estate as a value to add to the project's value itself. It is also convenient to analyze the fact that, sometimes, some options can be transformed into a contract between parties that will have a price and that can be sold or purchased. In fact, that is the nature of financial options, on which the valuation principles of real options are based. There are even patented methods that describe how to structure and trade with real options within the real estate sector (see, for example, Ashenmil et al. 2003). How can this affect the estimated cost of capital required?

Depending on the characteristics of the project and considering that the markets were perfect, we should be able to establish real options as hedging instruments. Enough options, bought or sold conveniently, should be able to establish a complete coverage and establish what the benefit of the investment will be, eliminating the random effect, the risk and greatly reducing the cost of capital.

Case, Shiller & Weis (1991) advocate the establishment of derivatives markets that, they argue, well-established markets in this regard could reduce speculation, rationalize rents and is necessary to generate coverage for owners who cannot diversify into other products. Fabozzi, Shiller & Tunaru (2009) argue that the use of derivatives in the real estate sector is desirable, although they point out the extreme difficulty of using such instruments.

When the market starts a trend, it is unique for all players, which makes it very difficult to find a counterpart ready to take a position against that trend. Thus, the use of derivatives as a hedge to cover an owner's risk, eliminating it as it can be done with other assets (such as financial assets), is not easy to establish, although it would follow the same logic as the financial one.

Obviously, this last point described, in which the risk would disappear completely, is paradigmatic, but it is perfectly possible that there are simple cases in which the investor clearly sees his risk reduced. As a counterpart, investor could be limiting it to a lower performance than expected. They do not even have to be options, that they are exercised for a specific reason derived from the project or from the market, they could be, for instance, options that serve to give liquidity to a very long project.

Imagine, for example, a put option with a third investor who might be interested in acquiring the project for a certain period of time. He agrees to contractually commit to it if he receive a certain premium. Imagine also that, even discounting this premium, the margin of the project was enough to generate capital returns in line with what the sector offers. Initially, it may seem complex that an investor is going to continue a project that another discards, or that another is going to get rid of a juicy opportunity cost. But knowledge and positioning in the market of each investor are different, as well as their need to invest or liquidity.

Buying or selling a real option we can limit the uncertainty about the financial outcome of the project. The option can be one that is produced by the special nature of the project or a custom made one. Once again, the strategic ability of the manager, and also the commercial one, will significantly affect the cost of capital.

It will be precisely in the case of tradable options in which it becomes more necessary to be guided by the calculation methodology offered by financial options. However, the parties will set a certain exercise price and a premium. The price can be very far from the prices that would be obtained with this methodology, since in this case it is a non-quoted market with a bilateral oligopoly.

4.6. A model to understand the effect

Next, a model has been designed that aims to represent the set of exposed effects that are produced on the cost of capital derived from the greater or lesser presence of options. To be more illustrative, the model's exposure will be done using an example with data comparable to what we could find in a real case.

Imagine a project with the following distribution:

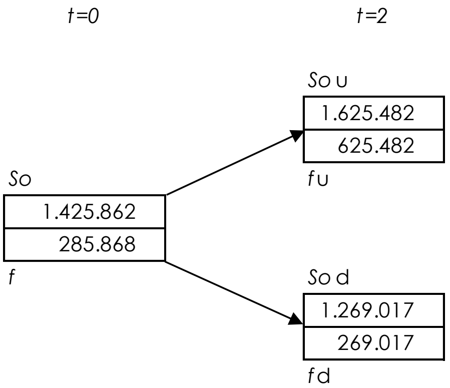
Project description

Phase 1				
Date	01/01/2019	01/06/2019	01/01/2020	01/01/2021
Payment	-5.000.000	-1.000.000	-700.000	9.000.000
K	0,09			
NPV	970.043			

Phase 2				
Date	01/01/2021	01/01/2022	01/03/2023	01/09/2023
Payment	-4.000.000	0	500.000	6.300.000
K	0,09			
NPV	1.425.862			

It is a project that offers the possibility of being carried out in two phases. We could say that it is a project with a scheme like the one exposed in the case of the previous chapter. We have calculated the value of this option following a procedure analogous to the one used in the case of Chapter 3. It is summarized as follows:

Phase 2 Option Value



u=	1,14
d=	0,89
r=	0,02
T=	2
p=	0,0800
k=	1.000.000 *
e^(-r*T)=	0,96078944

* Do not include landplot value.

The total value of the project will be, following the foregoing:

Total Project Value

NPV	Otion Value	Total Value
970.043	285.868	1.255.911

The model to determine the value of k (cost of capital) will be based, on the one hand, on parameters that will depend on the investor. It is an estimate of how the above-mentioned aspects affect the investor. We can consider that these parameters are specific for each investor and will depend on their attitude towards the risk as well as the importance that they give, consciously or not, to the characteristics of the project and the market. These parameters are what we will call the β of the model.

These parameters will be multiplied by the variables shown by the model and the market. The variables and their respective parameters refer to four aspects:

Km	Market sectorial forecast
Ks	Flexibility from 0 to 100% to adapt project to market changes (compared vith the market
Ki	Level of investor's information provided about managerial flexibility (from 0 to 100%)
Kc	Level of managerial complexity regarding options use (from 0 to 3)

These 4 variables represent the previously exposed aspects that relate the presence of options and their influence on the cost of capital. They are derived from the characteristics of the project and the market. For this case, the parameters that we will use as an example are the following:

K determination model			Investor's profile parameters
Market return:	11,00%	Km	βm = 1,000
Size flexibility:	50,00%	Ks	βs = -0,015
Investor info:	35,00%	Ki	βi = -0,008
M. confidence:	2	Kc	βc = -0,005

The interpretation of the profile of this investor, for these specific data, would be the following:

βm equal to 1 indicates that the investor does not underestimate or overestimate the return of the market and its demand for return of capital is aligned with the market.

βs equal to -0,015 indicates that in a maximum flexibility project, as far as size is concerned, the investor would be willing to invest demanding up to 1.5% less return on its capital (up to 150 basis points less).

βi equal to -0.008 means that in a project where the investor had a comparatively maximum level of information regarding management flexibility to be able to adapt to the market, he would be willing to reduce up to 0.8% less return on his capital.

βc equal to -0.055 indicates that in a project of maximum confidence, in terms of management capacity of the flexibility to adapt to changes in the market, the investor would be willing to invest demanding up to 0.5% less than return on capital (up to 50 basis points less).[16]

The value of the variables corresponds to the characteristics of the project. In this regard, the model combines the characteristics of the project with those of the investor. In the case of the confidence level, a discontinuous variable is used, with possible values 0, 1, 2 and 3.

Obviously, it is an idealized model, since in order to quantify both the characteristics of the investor and the project would be necessary to have a database of both that would allow to estimate these parameters and variables. The modeling serves to conceptually capture the interactions and their effects.

The modeling has been created for conceptual purposes, to represent how the cost of capital may be affected as a result of the management of options. However, a sensitivity analysis has also been carried out, based on variations of the variables, of the result of the total valuation of the project. The variation of the cost of capital required generates a double effect: it increases the total value to the extent that it increases both the NPV of the project itself (phase 1) as well as due to the increase that occurs in the value of the option. With this exercise we not only observe the variations that would occur, we can also distribute the origin of the variation between the two parties.

In these analyzes we observe the variation of the two variables that are possible to improve with the addition of options or the improvement thereof, that is, the level of flexibility of the size and the level of information of the investor. We perform first an analysis of the total value (NPV + Option), table 1, and after the parts separately (table 2 and 3).

[16] Note that this parameter is intrinsic to the investor, not the project or the options. It should be noted that the model does not infer any relationship between the capabilities of the manager and the value of the options. In any case, the relationship established is between the manager's capabilities and the possibility of them affecting the risk perception of the investor.

Total value sensibility analisys

Size flexibility

1.255.911	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
10%	1.063.021	1.082.098	1.101.222	1.120.393	1.139.611	1.158.876	1.178.188	1.197.547	1.216.954	1.236.408
20%	1.098.829	1.117.994	1.137.206	1.156.465	1.175.771	1.195.124	1.214.525	1.233.974	1.253.470	1.273.014
30%	1.134.802	1.154.055	1.173.355	1.192.703	1.212.098	1.231.540	1.251.030	1.270.569	1.290.155	1.309.790
40%	1.170.940	1.190.282	1.209.671	1.229.107	1.248.592	1.268.124	1.287.704	1.307.333	1.327.010	1.346.735
50%	1.207.244	1.226.675	1.246.153	1.265.680	1.285.254	1.304.877	1.324.548	1.344.267	1.364.035	1.383.852
60%	1.243.716	1.263.236	1.282.805	1.302.421	1.322.086	1.341.799	1.361.561	1.381.372	1.401.232	1.421.141
70%	1.280.356	1.299.966	1.319.625	1.339.333	1.359.088	1.378.893	1.398.747	1.418.650	1.438.602	1.458.603
80%	1.317.165	1.336.866	1.356.616	1.376.415	1.396.262	1.416.159	1.436.105	1.456.100	1.476.145	1.496.240
90%	1.354.145	1.373.937	1.393.779	1.413.669	1.433.609	1.453.598	1.473.637	1.493.725	1.513.863	1.534.051
100%	1.391.296	1.411.180	1.431.114	1.451.097	1.471.129	1.491.211	1.511.343	1.531.525	1.551.757	1.572.040

Inv. Info

Table 4.1: Total value sensitivity analysis combining the flexibility and the information effects.

Project value sensibility analisys

Size flexibility

970.043	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
10%	867.444	886.505	905.647	924.870	944.175	963.562	983.032	1.002.585	1.022.222	1.041.942
20%	877.600	896.704	915.889	935.156	954.505	973.936	993.450	1.013.047	1.032.729	1.052.495
30%	887.779	906.926	926.155	945.465	964.858	984.333	1.003.892	1.023.534	1.043.260	1.063.071
40%	897.980	917.171	936.443	955.797	975.234	994.754	1.014.357	1.034.044	1.053.815	1.073.672
50%	908.205	927.439	946.755	966.153	985.634	1.005.198	1.024.846	1.044.578	1.064.395	1.084.296
60%	918.453	937.731	957.091	976.533	996.058	1.015.667	1.035.359	1.055.136	1.074.998	1.094.946
70%	928.725	948.046	967.450	986.936	1.006.506	1.026.159	1.045.897	1.065.719	1.085.626	1.105.619
80%	939.019	958.384	977.832	997.363	1.016.977	1.036.675	1.056.458	1.076.325	1.096.278	1.116.317
90%	949.337	968.746	988.238	1.007.813	1.027.472	1.047.215	1.067.043	1.086.956	1.106.955	1.127.040
100%	959.678	979.131	998.668	1.018.288	1.037.991	1.057.780	1.077.653	1.097.612	1.117.656	1.137.787

Inv. Info

Table 4.2: Project (NPV) value sensitivity analysis combining the flexibility and the information effects.

Option value sensibility analisys

Size flexibility

285.868	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
10%	195.577	212.324	229.155	246.070	263.069	280.154	297.324	314.580	331.923	349.354
20%	204.498	221.290	238.166	255.126	272.170	289.301	306.517	323.819	341.209	358.686
30%	213.444	230.280	247.201	264.206	281.296	298.472	315.734	333.083	350.519	368.043
40%	222.413	239.294	256.259	273.310	290.446	307.667	324.976	342.371	359.854	377.425
50%	231.405	248.332	265.342	282.438	299.620	316.888	334.242	351.684	369.214	386.833
60%	240.422	257.393	274.449	291.591	308.819	326.133	343.534	361.023	378.600	396.266
70%	249.463	266.479	283.581	300.768	318.042	335.402	352.850	370.386	388.011	405.724
80%	258.528	275.590	292.737	309.970	327.290	344.697	362.192	379.775	397.447	415.208
90%	267.617	284.724	301.917	319.197	336.563	354.017	371.558	389.189	406.908	424.717
100%	276.730	293.883	311.122	328.448	345.861	363.361	380.950	398.628	416.395	434.252

Inv. Info

Table 4.3: Option value sensitivity analysis combining the flexibility and the information effects.

The analysis of the tables separately does not show relevant information, as it is fictitious data. In any case, it shows us the order of magnitude of the change based on the parameters and the established variables, given that these are values that could perfectly be observed in a real situation. But the most interesting thing is to perform an analysis comparing the tables, which shows how the changes affect one or the other part of the total value. Finally, in table 4, it is analyzed that part of the change of the total value refers to the part of the project itself (without the aggregation of the option).

Two interesting effects are observed. In the first place, the variation of the total referred to the project does not even reach 60%. This means that changes in the value of the option when the capital cost is modified due precisely to the effect of the options, represents a very significant part of the total variation. Secondly, we see that the percentage varies, and always decreases with decreasing the value of k. This can lead us to think that by increasing flexibility or information there is an effect of growth in the total value of the project that will increasingly be due to the increase in the value of the options.

% Variation of total value due to project value (vithout option value variation).

	Size flexibility									
0,0%	10%	20%	30%	40%	50%	60%	70%	80%	90%	100%
10%	53,19%	53,18%	53,17%	53,16%	53,15%	53,14%	53,14%	53,13%	53,12%	53,11%
20%	53,19%	53,18%	53,17%	53,16%	53,15%	53,14%	53,13%	53,12%	53,11%	53,10%
30%	53,18%	53,17%	53,16%	53,15%	53,14%	53,13%	53,13%	53,12%	53,11%	53,10%
40%	53,18%	53,17%	53,16%	53,15%	53,14%	53,13%	53,12%	53,11%	53,10%	53,09%
50%	53,17%	53,16%	53,15%	53,14%	53,13%	53,12%	53,12%	53,11%	53,10%	53,09%
60%	53,17%	53,16%	53,15%	53,14%	53,13%	53,12%	53,11%	53,10%	53,09%	53,08%
70%	53,16%	53,15%	53,14%	53,13%	53,12%	53,11%	53,11%	53,10%	53,09%	53,08%
80%	53,16%	53,15%	53,14%	53,13%	53,12%	53,11%	53,10%	53,09%	53,08%	53,07%
90%	53,15%	53,14%	53,13%	53,12%	53,11%	53,10%	53,10%	53,09%	53,08%	53,07%
100%	53,15%	53,14%	53,13%	53,12%	53,11%	53,10%	53,09%	53,08%	53,07%	53,06%
Inv. Info										

Table 4.4: % Valuation of the total value due to changes in value of the project (not option) part.

4.7. Chapter conclusions and final remarks

Throughout this chapter we intend to analyze the effect of the presence of real options in real estate projects and the cost of capital required by the investor. It is an intuitive but little analyzed effect in a structured way in the literature on real options. For this reason, we have defined a concept of cost of capital referred to pure equity, with full risk on the result of the project. Subsequently have been defined and justified those factors that are the main drivers of the cost of capital in a real estate project.

Specifically, three have been defined: the market as a reference and generator of risk, the size of the project and the level of information available by the investor. After defining the drivers, the ways in which the presence of different types of options can be decisive to modify or affect these factors have been analyzed.

We have seen, with examples from other authors as well as with real cases, that it is logical to expect some effect on the cost of capital required due to the existence of real options. It can be concluded that, although difficult to quantify, the presence of options would be negatively related to the cost of capital required in the three factors.

The options can serve as a hedging instrument against the movements of the market, thereby reducing the risk to this and therefore the cost of capital required. At the same time, certain options serve to resize a real estate project, with which it is possible to modulate the "effect of the project's size" on the cost of capital. In addition, that relevant information that shows the strength of the project with respect to changes in the market, insofar as it affects the part of the systemic risk, is likely to affect the cost of capital.

Finally, three important aspects to be considered when measuring the influence of the presence of options are analyzed. The first is the capacity of the manager and the investor's confidence in it, as a key aspect in the generation and strategic use of the options. The human capital in the management of the project is considered as a strategically key aspect to face the market variations, provide coverage or even liquidity. This capacity of the manager interacts through its effect on the perception of the investor's risk and, therefore, on the reduction of the cost of capital. Management, of course, is not part of the value of the option per se.

The second is the comparative effect of the other projects and their potential in terms of options as a reference. No option implies an added value if it does not represent a differential element between the project and the generic references (proxies) of the market.

Finally, the third is the ability to establish a market for options contracts, quoted or off-market, which gives a new dimension to the strategy with real options. The possibility of establishing the real options with a contract reinforces them and is a much more tangible guarantee for the investor.

It has become evident, therefore, that the presence of options or their creation when conceiving the project can be decisive for it. Management flexibility and good management is a crucial argument to reduce the risk exposure and the perception of risk by the investor. Its quantification, however, is an analysis that would require ex-post or experimental data and is difficult to quantify. It must be borne in mind that in specific cases the weight of the attitude towards the risk of the concrete individual may distort completely the effect that any modification of the project have.

In the last part of this chapter, we have tried to model the effect of the presence of options in real estate projects. To do this, we have created a model that is based on the above mentioned aspects and combines the attitude of the investor with the characteristics of the project. As a result, we have observed an example of how the value of a project might vary as its characteristics vary as far as the interaction of options is concerned.

We have observed that, in a calculation scheme of the total value analogous to that in Chapter 3, the specific weight of the total change in value that corresponds to the value of the option is very significant. This helps to reinforce the idea that the change in the cost of capital due to the presence of options is based on a double effect: part corresponding to the increase in the value of the project itself and part in the increase in the value of the options. In addition, we have seen that the part corresponding to the increase due to the increase in the value of the options is proportionally greater, the greater the effect of these.

It is important that in the field of research in strategic management of real estate projects always take into consideration the real options, those intrinsic to the project and those that could be designed ad hoc. It is important that a deep analysis of the project's potential options be done before determining the cost of capital required by the investor.

No valuation analysis of real options will be complete without a study that attempts to quantify their effect on the cost of capital, adapting by correcting the value of the project as a whole.

Chapter 5

Overall Final Conclusions

5.1. General conclusions

Throughout this doctoral thesis, we have been able to observe the following remarkable aspects:

5.1.1. About the valuations of options in the real estate sector

One of the main conclusions reached is that it is not possible to make a complete assessment of a real estate project without considering the value of the options embedded in the project. Any assessment that ignores the value of management flexibility will be offering a static, inflexible vision that will not adjust to reality. The use of options is the logical consequence of the realization of a long-term project that is subject to risks, and especially of market variations.

On the other hand, often the market does not evaluate these options with rigorous methodology. Daily, options for buying or selling projects are established on. Being the minority those that establish a scientifically justified price to them. Given the importance of the sector, and the consequences recently suffered by the global economy derived from the way in which projects have been valued, the facts pointed above are especially worrying.

We have shown two clear examples from the usual work in the real estate valuation where the value of the management flexibility has significant weight. Throughout this thesis we have seen how the real estate sector has particular characteristics that make it suitable for the design and use of options. Using two widely contrasted methodologies in the financial field, as well as in the valuation of real options, we have assigned a value to this management flexibility.

In both cases it has been done using the most conservative perspective that is, choosing, in case of doubt, the option that gave a lower value to the option. Despite using this conservative methodology, values have been obtained that are very significant in proportion to the value of the project. We have also seen that these securities grow very substantially in scenarios of high market volatility. It must be said that it is a vicious circle, the lack of rigor in the analysis of value is a cause of increased volatility, which, in turn, makes more necessary the application of a more rigorous methodology in valuation.

We can say that applying of real options theory we can obtain more comprehensive projects and real estate asset valuations. In addition, these assessments change substantially and may affect the decision criteria on them. It is possible to find many examples where the addition of the value of flexibility, in the form of options, is decisive and avoids discarding valuable projects. In this thesis, different typologies of options have been pointed out as well as the types of projects likely to contain them.

On the other hand, the methodology used makes the valuation available to investors who are not specialized in the calculation of derivatives. This aspect is key because it allows the result to be valued by the market. In a market such as real estate, which is very open to all types of investors, it is especially important that the valuation methodology used is sufficiently contrasted and transparent. This research contributes to this purpose with some applications.

In the cases analyzed, we have obtained two conservative valuations, which are also supported by a solid methodology for calculating financial options. It is especially important to emphasize the difference of concept between the options analyzed. In the first case studied, as already noted, the value of the option stems from the existence of volatility in the market. To the extent that the market can change, the project's potential to adapt gives it an additional value that has been quantified by the options studied.

In the second case, it is a concept of a different option, since it is the option to carry out a second phase of a project that is being studied. The second, which can only be developed if the first one is developed, has a potential value that can be assessed and added to the value of the project studied. The value of the second stage is conditioned by the evolution of the market.

This also serves to exemplify that the casuistry in terms of the presence of options is, in the real estate sector as in many other economic fields, very broad, as wide as the complex nature of the projects that are being undertaken and the multiple possibilities of maneuver that the manager may identify. The knowledge and assessment of each type of options embedded in the real estate project as well as its optimal combination becomes crucial. This research aims at contributing to confirm with its applications one of the key aspects of real options, already identified in the early 1990s.

Two methodologies have been used, one of continuous-time analysis, based on the formulation by Black and Scholes, and another based on discrete-time binomial trees. In the first case study, sensitivity analyses, including changes in the volatility of the underlying asset, have been carried out.

In the second case study, binomial trees are the methodology chosen to price the option to carry out the project in two phases. This second method, of discrete analysis, has been repeated with different numbers of steps. As expected, increasing the number of steps in the tree contributes to obtain intermediate data that help to understand the evolution of the value of the option in parallel to the evolution of the potential value of the project.

5.1.2. The observed problems

On the other hand, it is important to highlight a significant difficulty in the valuation: its dependence on the correct calculation of the volatility of the underlying asset and its complexity for the calculation. It is very difficult to define the parameters that will determine the volatility of the underlying asset. Even in the case of being able to determine the main influencing factors, the existing data do not allow for a sufficiently reliable assessment.

Being a market with a very important component of localization affecting the price, it is very difficult to establish comparison values with the behavior of other assets. Throughout this work, it has been possible to verify that any valuation is highly dependent on a deep knowledge of the local market, which allows estimating future behavior. We have seen that, of all the variables involved, the volatility of the value of the underlying asset, always the price of the square meter of the asset, is crucial for its effect on value, but also for its effect on the weight of the other variables in the result.

We have observed through an econometric modeling that, even those factors that we would expect to have a more direct relationship, show a low correlation. This happens even separating the data from the different parts of the cycle (bullish and bearish). The scarcity of data currently available on the operations carried out and their characteristics make it very difficult to make reliable projections about the evolution of the market. There is therefore, in the ordering and dissemination of information that the market throws, an expectation of future improvement of the progress in the study of the real causes of market movements.

Given that volatility is a key variable in option pricing, this lack threatens to distort calculations. However, it is possible to adopt “intuitive” values that are valid for analysts and investors in the real estate sector and use them as estimated volatility. A sensitivity analysis, such as the one carried out in this thesis, aims at contributing empirically to overcome this challenge. Otherwise, it makes no sense to pretend to advance in the application of the theory of real options in the real estate sector if one is not able to determine what the margins of market movement are. Obviously, it is not possible to determine in which direction and with which intensity prices will evolve. Nevertheless, one can rely on an adequate knowledge of the volume of existing demand and local supply, combined with the macroeconomic conjuncture.

5.1.3. About the effect of options on the cost of capital

The flexibility increase in the real estate projects management, which we have defined as options, also influences the risk implicit in them and, therefore, in the cost of capital that the investor will require. In Chapter 4, we have analyzed its possible influences.

We have defined three key aspects, such as protection against market variations, the possibilities of resizing the project and the ability to provide valuable information on market variations. The three aspects can come from the presence of certain types of options in the project and their presence is likely to generate a reduction in the risk implicit in it. This is why the presence of real options embedded in the real estate project will reduce the cost of capital required to the project.

In order to outline and better understand these interactions, we have created a model that exemplifies these concepts. The reduction in the cost of capital has a double effect on the value of the project.

5.1.4. Value from the management capacity

Finally, note that the value attributed to the options is value not derived from management but enhanced by from the quality of the decisions along the project's life or the real estate asset. It is not, therefore, value from the strict assessment of the market. We can say that the intrinsic value of the option comes from the variations of the market, although, it becomes crucial that managers have the right skills to identify and price the existing real options.

The value of the option could be wasted if the manager in charge of the project is not able to exercise wisely. Management capacity, both technical and financial, is one of the greatest opportunities that any real estate project can have. The experience and intelligence of the manager is what makes the option valuable. In turn, the investors’ knowledge of the presence of these options and their confidence in the manager can reduce the cost of capital required, since the risk perception of the investor can be influenced by this fact.

5.2. Future research

As observed throughout this research, there are three specific aspects in which it is necessary to move forward if we want to achieve the objective of extending improvements in the valuation methodology of real estate projects by using the theory of real options.

The first aspect that should be improved is the design of valuation models of the volatility of real estate markets at a local level. The analysis of the specific conditions that determine this volatility, linked to its formulation, is crucial. For this, the main obstacle is the access of enough information. A suitable research scenario in this aspect would consist in the use of big data analysis technology as well as the access of information consolidation of real transactions.

This, which seems viable in the long term, is complex in the short and medium term, given the length of the real estate economic cycle, the lack of uniformity of the data from previous decades and the recent use of big data technology.

The second aspect that requires progress is the systematization of the valuation of options. As we have seen, there is a multiplicity of types, as well as a broad methodology. In the same way that there is a standardized methodology for the application of the theory in the case of listed assets, if the market is intended not to generate misunderstandings and distrust of the use of options, it is necessary that the participants in it assimilate homogenized methodologies. This requires the academic dissemination of affordable methodologies among practitioners.

Finally, the third aspect in which to move forward is the study of the effects of management flexibility on the cost of capital required. This happens in the real estate world, very intensive in capital and with very long periods of maturity, but it is also applicable to other investment fields. In this third case, it is an analysis linked to the theories of behavioral psychology. One possible methodology to follow would be the experimental one, although performing experiments on a scale that involves the real estate sector would be very difficult. In any case, it seems possible to design experiments on the cost of capital required by a group of investors based on the options related to the investment.

Progress in these points would be of crucial contribution for the future of real estate valuation.

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