Universitat Autònoma de Barcelona

Essays on Trade Unions and Collective Bargaining

Rubén PÉREZ-SANZ

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Essays on Trade Unions and Collective Bargaining

Author:
Rubén PÉREZ-SANZ

Supervisor:
Dr. Joan LLULL

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Introduction

Trade unions have been in decline for the last 50 years whereas inequality has risen steadily over the same period. The erosion of bargaining power of individual workers and unions is a key factor that drives wages downwards; simultaneously firms and management have enjoyed the lion's share of the growing income. Among the several causes behind loss of bargaining power on behalf of unions there are two that stand out: the process of globalization and the stringent regulation on collective representation. Regarding the former, unions have lagged behind in the process of globalization as multinational enterprises have expanded to other countries. In the latter, the public good facet of collective agreements is still a hurdle that discourages workers from joining a union.

Understanding the forces behind lack of union coordination in the international arena and behind the diminishing union density around the world is key to recognise how these eventualities affect workers, firms and ultimately society as a whole. The understanding of such mechanisms and their assessment is the chief aim of the present thesis. First of all, I study the conditions of union coordination in an international monopoly market with differentiated goods under the presence of externalities and costs, chapter 1. Secondly, I develop a model to analyse how regulation on minimum wages set in collective agreements affects labour market outcomes such as unemployment, inequality and labour mobility, chapter 2. In the last chapter 3, I use the model developed in chapter 2 to test it with Spanish data.

In chapter 1, International Monopoly Union Coordination Under the Presence of Externalities and Costs, I investigate the reasons behind union inability to coordinate internationally. In the model, I consider an internationally monopoly that holds two differentiated products in two countries. In the two countries there are monopolistic unions that set the wage that fit them best. If a domestic union raised the wage above the optimal level, the firm would reduced the amount of domestic product and would fill the market with one from abroad. It would be detrimental for the domestic union because the loss on employment. However, the foreign union would experience a jump on its demand for labour and could set a higher wage, leaving them undoubtedly better off. Because the problem is symmetric no union is willing to take an unilateral action. On the contrary, if they were to coordinate they could set the wage so that they could internalise the positive externalities that they pose on each other. Whenever these externalities are high enough, which means products are very substitutable, they would be interested in coordinating their wage demands. As an extension, if unions do not hold all the bargaining power, there is a region in which they are never interested in coordinating and in the range where they do so it would be more difficult due to the small internalisation. Also, if unions have different costs of coordinating, both costs need to be relatively low to each other for the unions to coordinate.

In Chapter 2, A Search and Matching Model of Firm Heterogeneity, Minimum

Wages and Collective Agreements, I develop a model with two sided heterogeneity, on-the-job search with sequential auctions, a matching function, minimum wages and collective agreements. The richness of the model allows to be flexible enough to mimic the reality closely. Upon introducing a minimum wage low productive firms suffer the consequences, start laying workers and open less vacancies; in turn large, high productive firms not only have a new stock of unemployed at their disposal, find it easier to fill vacancies and become even larger, but also they face less competition from small firms to rise wages, this stagnates the careers of workers and large firms gain in this margin too. Now, the minimum wage level is not set exogenously but is chosen by collective agents that will set the level that fit them best. Only workers and employers in large firms are able to decide the level of the minimum wage to their advantage, not internalising the effects that they pose on other less skilled workers and low productive firms. If a social planner could let everyone decide the actual level of minimum wages, firms and workers would be willing to reduce the level of minimum wages so that there would be more employment and more dynamic careers.

In Chapter 3, A Structural Model of Minimum Wages in Spain, I test the model described in chapter 2 with Spanish data. For this purpose I use the continuous sample of employment histories (MCVL) provided by the Social Security and the collective agreements registry (REGCON) provided by the Ministry of Labour. Estimating the model through classical minimum distance I discern what are the forces that act on wages and vacancies. The results are in line with other studies that have studied the reallocation of minimum wages in Germany, so the model is a promising venue to analyse the reallocation effects of minimum wages form a behavioural point of view. In particular, I effectively estimate that workers reallocate from small to large firms. Size in my model is in one to one correspondence with productivity, so workers reallocate from low to high productive firms. Furthermore, inequality decreases as the minimum wage rises, not only for those close to the minimum wage level but it also slows wage dynamics, meaning that workers enjoy lower wages through the end of their careers due to curtailed competition. In addition, looking into the political economy side of the model, employers associations hold most of the bargaining power since they are able to set the minimum wage closer to their interest, also they are the most interested in rising the minimum wage as they avoid competition. A social planner would allow everyone to decide on the level of minimum wage so as to maximise social welfare.

Chapter 1

International Monopoly Union Coordination Under the Presence of Externalities and Costs

1.1 Introduction

Despite the increasing globalization at a corporate and political spheres, coordination of labour unions across countries seems to be still in its infancy. A few salient examples of union coordination are in regions where economic integration is sufficiently deep, such as the European Union. The managerial decisions of multinational enterprises (MNE) have consequences across the European plants over the the well-being of workers. As a result, unions are interested in coordinating their actions, or at least to agree to some minimum standards and policies whenever managerial practices are hurtful for workers. Some examples of international unions at European level are IndustriAll for manufacturing, ETF for transport and the EFJ for journalists among others. These organizations have the capacity to negotiate with their correspondent employer's associations and influence the policy making in the European Union.

Nonetheless, the same could be lay down with the trade vis-a-vis with other countries such as US, China, etc. However, in these cases there are not trade union organizations that agree on minimum standards or wage demands. So, the question still remains, why do unions fail to coordinate across countries, in spite of advantages of holding monopoly power? This paper delivers a rationale of why unions are interested in synchronising their actions or why they fail to coordinate internationally based on costs of coordination and the internalization of externalities. The discussion in this paper accommodates realities in which union coordination is observed and realities where is non-existent.

The main point is that MNEs with production facilities in different countries dilute the strength of unions to set a higher wage by flooding domestic markets with close substitutable products from other countries. If a union demands a higher wage, its firm reduces production of that product and floods the market from abroad, increasing the production, and demand for labour in a foreign country, which ultimately facilitates unions in other countries to demand a wage rise.

The domestic union is worse off because it loses employment, whereas foreign unions will be undoubtedly better off as they enjoy higher levels of employment and wages. Because this is the case in every country where the firm operates, no union demands a wage rise in the first place. Thus, unions pose on each other a positive externality that if it were internalised would grant them the opportunity to raise wages and share the employment loss in such a way that both would be better off.

So, what if unions coordinate? The firm cannot threat to fill the void with other products because costs have risen in all markets, the firm reduces production in every market and unions enjoy higher wages that compensate the loss of employment; in this way, unions internalise the positive externality posed on each other. Of course, coordination is costly, unions collude only if products are very substitutable, or in other words if the benefits of internalisation are sufficiently high, so as to compensate for the costs of coordination.

In the context of European Union, multinational firms have taken advantage of the elimination of tariffs, quotas and regulations. Specifically, MNEs have been able to set operations abroad in search for cheaper labour and for weakening the bargaining power of unions, Naylor (1999). The incentives of unions might seem to coordinate their actions across countries in order to curtail the advantage gained by firms when trade is liberalised, Straume (2002). Yet, examples of international coordination among unions is scarce, Schmidt and Keune (2009) survey some examples of transnational bargaing and describe mechanisms that prevent this cooperation.

According to a survey carried out in 2010 in fourteen European countries, Larsson (2012), the most important 'hard' factors in preventing international unions from coordinating are: differences in financial resources, different legal frameworks and policies, employer's associations, similarities of occupational interests and priorities among the leaders and members of the unions. Other factors called 'soft' are cultural, ideological, religious and linguistic dissimilarities, which are regarded by unions as much less important for international cooperation than the 'hard' ones.

Among the 'hard' factors, differences in financial resources among unions and diversity of labour market policies and regulations are the most important hurdles that avoid coordination. Not surprisingly, these hard factors are on the top list of hurdles for sectors that are heavily exposed to international competition like manufacturing. These factors can be traced back to costs that unions have to bear when coordinating across countries.

So, in the model presented here I introduce costs of coordination among unions in different countries that negotiate with a firm. This firm operates two plants in two countries that sells differentiated products in an integrated product market, think of the European Common Market. The labour market is not integrated, maybe because the costs of moving to a different country are too high for example. when product markets are very integrated, the demand of one product affects the demand of the other product. In the case that unions do not coordinate, there is a positive externality of one union on the other when products are substitutes. The more integrated the economy the more substitutable the products are and the stronger the incentives of unions to coordinate. However, upon coordination unions face a cost. Unions will coordinate whenever the internalisation of the positive externalities of coordination are higher than the costs.

Similar to this paper Straume (2002), analyses the incentives of firms and unions to collude when trade is liberalised. In that paper unions collude to set a common wage above the equilibrium level in the one-shot game. Whenever countries being compared have the same level of productivity it is reasonable that they will set a common wage so as not to fall in reciprocal dumping. In this paper, I consider that countries do not necessarily share the same level of productivity and a unions will negotiate the wage in each country taken into account the positive externalities that they pose on each other. Other works following the same vein as that of Straume (2002) are Borghijs and Du Caju (1999) and Buccella (2013).

In a somewhat related paper Eckel and Egger (2017), explore dissimilarities in unions utilities to give a rational of why they do not coordinate. In their paper, monopoly unions in each country negotiate with a firm in autarky, then after trade is liberalised unions are better or worse off depending on their wage orientation. In addition, when coordinating unions form a supranational entity with different objectives than individual partners, thus unions that improved their welfare under liberalisation do not engage in coordination if the supranational union has different preferences to them. Even though differences in priorities have been proven to be an obstacle for union collusion, here I explore the incentives of unions to coordinate under transactional costs and internalisation of externalities. Furthermore, I do not assume a supranational union, but I consider the case of collaboration which seems more natural step towards a deeper international cooperation. So, unions in this setting will preserve their national preferences and objectives; and they coordinate their wages so as to internalise the effects of their strategies.

There is also a close branch of literature based on foreign direct investment (FDI) and multinational enterprises (MNEs) related to the present paper. Pioneering works of Mezzetti and Dinopoulos (1991) and Zhao (1995) find that unions might be welfare improving depending if they are wage or employed oriented. Also, they find the bargain wage under FDI is usually lower and profits of firms and welfare higher. The mechanism that they consider is the threat of a firm to a union to move production abroad if wages are too high. Here I present a different scenario in which markets are already integrated and there are no tariffs, goods can be traded in any country at the same operating cost as the domestic firm. As said previously, firms dilute the bargaining power of unions by flooding the market with close substitutes that prevents unions from coordinating in the first place.

The rest of the paper is organised as follows. Section 2 develops the general model and proposes simplifying assumptions to gain more insight. In section 3 the right-to-manage model and its consequence over coordination is considered. Section 4 analyses the coordination outcome under different transactional costs. Section 5 models explicitly the coordination decision. Section 6 concludes.

1.2 Model

1.2.1 Setting

There is one monopolistic firm operating two plants in two countries, A and B. The two plants produce differentiated products, $I = 1, 2$, that are sold in an integrated product market. In the product market, plants compete a là Cournot and the firm maximises total profits of both plants internalising the negative or positive externalities that each plan could pose on each other through prices, also plants producing these goods can be seen as brands. Since, in the present model goods are substitutes there is no point at considering Bertrand competition, as the profits of each plant would be lower than those if they played Cournot, Singh and Vives (1984). Also in each plant there is a monopoly union that supplies n_i unit of labour at a rate w_i. Following these authors I consider an economy with a continuum of workers with a quadratic utility function that is separable and linear in money, which prevent income effects on the monopolistic sector where the firm operates. More precisely:

$$U(\mathbf{q}) = \sum_{i}^{2} \alpha_i q_i - \frac{1}{2}\left(\sum_{i}^{2}\sum_{j}^{2} \beta_{ij} q_i q_j\right).$$

Which gives rise the to the following inverse demand functions:

$$p_1(q_1, q_2) = \alpha_1 - \beta_1 q_1 - \gamma q_2 \tag{1.1}$$

$$p_2(q_1, q_2) = \alpha_2 - \beta_2 q_2 - \gamma q_1, \tag{1.2}$$

where q_i is the good supplied by the plant i, α_i is a taste parameter for the good i, $\beta_i = \beta_{ii}$ is the sensitivity of price of good i when quantity varies and $\gamma = \beta_{ij} = \beta_{ji} >= 0$ is the sensitivity of the price with respect to the good sold by the other brand, which in this case they will be imperfect substitutes [1].

The production of each brand is given by a production technology that uses one unit of labour to produce one unit of the product, i.e. $q_i = n_i$, so the plant producing the good i maximises

$$\Pi_1(q_1, q_2) = (p_1(q_1, q_2) - w_1) \cdot q_1$$

$$\Pi_2(q_1, q_2) = (p_2(q_2, q_1) - w_2) \cdot q_2.$$

[1] Look at Singh and Vives (1984) second and third notes for conditions of imperfect and perfect substitutability and the range of gamma

The firm, that has its headquarters in country A, operates both plants internalising the negative externalities that both plants could pose in each other, then the firm maximises

$$\Pi_A(q_1, q_2) = \Pi_1(q_1, q_2) + \Pi_2(q_1, q_2) \tag{1.3}$$

with respect to both goods.

Finally, workers do not move across borders and their wages are set by monopoly unions that maximise the wage bill accrue to the workers. Unions can take the decision of coordinating or not, $d_i \in \{0, 1\}$. If they decide to synchronise their actions they bear cost c per unit of labour. Then their respective objective functions are:

$$V_1(\mathbf{w}, \mathbf{d}) = (w_1 - d_1 c) \cdot n_1(\mathbf{w})$$
$$V_2(\mathbf{w}, \mathbf{d}) = (w_2 - d_2 c) \cdot n_2(\mathbf{w}),$$

where $\mathbf{w} = (w_1\ w_2)$ is a vector of prices and $\mathbf{d} = (d_1\ d_2)$ is the vector of decisions of coordination, unions will coordinate if both decide to do so, i.e. $d_1 = d_2 = 1$. In case of coordination they maximise their objective functions as if they were perfectly synchronised, or in other words:

$$V(\mathbf{w}, \mathbf{d}) = V_1(\mathbf{w}, \mathbf{d}) + V_2(\mathbf{w}, \mathbf{d}),$$

where the decision variables of unions are wages and coordination.

The bargain between the union and the firm is that monopoly union model, which is a special case of the right-to-manage model when the union has all the bargaining power, A. L. Booth (1995). The right-to-manage model is widely used in the literature of international unionised oligopolies as opposed to efficient bargaining, even though there should be efficiency gains of doing so and parties should choose this model instead. However, one possible rational of why not to use efficient bargaining, i.e. negotiate over wages and employment, is that related to Charles R. (1984), in which he drops the assumption of certainty. Under uncertain conditions of firm revenues, firms might be unwilling to lock themselves in a contract that they are not going to be able to fulfil. The case of a monopoly union model is more debatable, but it highlights the point made in this paper, reducing the mathematical burden considerably, section 1.3 deals with the case where unions do not hold all the bargaining power, generalising the results to the Right-to-Manage model and its consequences. As is common in the literature the game is built in two stages, first unions set wages, coordinating their actions or not, and then after the firm knows the costs of labour, it chooses the level of employment according to its demand schedule. Then the model is solved by backwards induction.

1.2.2 Competition in the product market

In the last stage the firm chooses output and employment to maximise its profits. Substituting (1.1) and (1.2) into (1.3) and taking into account that the technology is $n_i = q_i$, the profit function of the firm is:

$$\Pi_A(n_1, n_2) = (\alpha_1 - \beta_1 n_1 - \gamma n_2 - w_1)\, n_1 + (\alpha_2 - \beta_2 n_2 - \gamma n_1 - w_2)\, n_2. \tag{1.4}$$

Applying the first order conditions for an optimum on (1.4) :

$$[n_1] : \alpha_1 - w_1 - 2\beta_1 n_1 - 2\gamma n_2 = 0 \Rightarrow n_1 = \frac{\alpha_1 - w_1 - 2\gamma n_2}{2\beta_1}$$
$$[n_2] : \alpha_2 - w_2 - 2\beta_2 n_2 - 2\gamma n_1 = 0 \Rightarrow n_2 = \frac{\alpha_2 - w_2 - 2\gamma n_1}{2\beta_2}.$$

Which are the reaction functions of one plant as a function of the employment of the other. Solving the system of equations, we have the optimal decisions of the firm as a function of wages set by the unions:

$$n_1(w_1, w_2) = \frac{\beta_2\,(\alpha_1 - w_1) - \gamma\,(\alpha_2 - w_2)}{2\,(\beta_1\beta_2 - \gamma^2)} \tag{1.5}$$

$$n_2(w_1, w_2) = \frac{\beta_1\,(\alpha_2 - w_2) - \gamma\,(\alpha_1 - w_1)}{2\,(\beta_1\beta_2 - \gamma^2)}. \tag{1.6}$$

The denominator indicates the strength of the relative responsiveness of prices with respect to quantities of products in both plants. Naturally, The higher the influence of products in its own price, i.e. indicated by β_1, β_2, with respect to the cross influence of one product into the price of the other, i.e. γ, the more freedom has the firm to set the quantities in each plant that maximise profits without the need to consider the externalities that plants pose on each other. As a consequence, inverse demands become less elastic, being relatively less responsive to wages. In the extreme case when $\gamma = 0$ the firm operates two separate monopolies, one in each country.

In the numerator, $(\alpha_i - w_i)$ is the excess of willingness to buy the product by consumers with respect to the cost of the product, in this case the wage. These terms are multiplied by the sensibilities of the other product, since what it matters at this point is how this excess is siphoned off to the demand of the other brand. Clearly, the demand of labour is negative in its own wage and positive in the wage of the other plant. Putting it more formally

$$\frac{\partial n_1(\mathbf{w})}{\partial w_1} = -\frac{\beta_2}{2(\beta_1\beta_2 - \gamma^2)} < 0$$
$$\frac{\partial n_1(\mathbf{w})}{\partial w_2} = \frac{\gamma}{2(\beta_1\beta_2 - \gamma^2)} > 0,$$

and seemingly for the labour demand of union 2. The second result is due to the fact that goods are substitutes.

Note that for the problem to be well defined and labour demands to be positive we need the following conditions to be satisfied. (1) $(\alpha_i - w_i) > 0$, meaning that the cost of producing the good i cannot be higher than the maximum price that the consumers are willing to pay for it. (2) $\beta_2(\alpha_1 - w_1) > \gamma(\alpha_2 - w_2)$, the strength of the siphoning has to be greater for the own product than the product in the other brand. And (3) $\sqrt{\beta_1\beta_2} > \gamma$, the combined influence of inverse demands with respect their quantities should be greater than how they cross-affect each other. We do not need considering the negative part as products are substitutes.

In this case the firm takes wages as given in its labour demands of each product. It is straight forward to see that the labour demand to produce each product is decreasing in its own wage and it is increasing on the wage of the other plant. In this way, the firm is able to replace the losses of one good due to an increase in costs with product of the other plant, cushioning the bargaining power of unions, see Lommerud, Straume, and Sørgard (2006) for a thorough explanation about this mechanism.

1.2.3 No union coordination

In this section unions decide wages to maximise the wage bill without coordinating their actions, so they will decide simultaneously the level of wages that will maximise the wage bill taken the wages of the other union as given. In this case, unions do not bear costs of coordination as their decision is $\mathbf{d} = (0\ 0)$ and consequently the objective function of each union is given by

$$V_1^{N_M}(\mathbf{w}) = w_1 n_1(\mathbf{w}) \tag{1.7}$$

$$V_2^{N_M}(\mathbf{w}) = w_2 n_2(\mathbf{w}). \tag{1.8}$$

Plugging (1.5) and (1.6) into (1.7) and (1.8) respectively, the objective functions of the unions are

$$V_1^{N_M}(\mathbf{w}) = w_1 \frac{\beta_2(\alpha_1 - w_1) - \gamma(\alpha_2 - w_2)}{2(\beta_1\beta_2 - \gamma^2)}$$

$$V_2^{N_M}(\mathbf{w}) = w_2 \frac{\beta_1(\alpha_2 - w_2) - \gamma(\alpha_1 - w_1)}{2(\beta_1\beta_2 - \gamma^2)}.$$

The objective functions are quadratic equations in their own wage, the quadratic term has negative sign and the optimum is a maximum. This reflects the fact that union's objective functions are increasing in wages and employment, but employment is a decreasing function of wages. In addition, these functions are increasing in the wage set by the other union. Clearly, if one union rises wages the production of its plant will be undercut, because both goods are substitutes the firm will fill this void with products of the other brand, demanding more labour and increasing the wages of the union in the other country.

Applying the first order conditions for an optimum we can derive the wage of set by the union in one plant as a function of the wage set in the other plant:

$$w_1^{N_M}(w_2) = \frac{\alpha_1\beta_2 - \gamma(\alpha_2 - w_2)}{2\beta_2}$$
$$w_2^{N_M}(w_1) = \frac{\alpha_2\beta_1 - \gamma(\alpha_1 - w_1)}{2\beta_1}.$$

These are nothing else than the reaction functions of each union with respect to the wages of the other. It is worth noticing that because products are substitutes, wages are strategic complements in the labour market, so an increase in the wage of union 2 will make union 1 to set a higher wage, or in other words $\frac{\partial w_1}{\partial w_2} > 0$. A Nash equilibrium in this setting is a pair of wages such that $w_1^* = w_1(w_2^*)$ and $w_2^* = w_2(w_1^*)$, the solution to this system of equations is

$$w_1^{N_M^*} = \frac{\alpha_1\left(\beta_1\beta_2 - \gamma^2\right) - \beta_1\left(\alpha_2\gamma - \alpha_1\beta_2\right)}{4\beta_1\beta_2 - \gamma^2} \tag{1.9}$$
$$w_2^{N_M^*} = \frac{\alpha_2\left(\beta_1\beta_2 - \gamma^2\right) - \beta_2\left(\alpha_1\gamma - \alpha_2\beta_1\right)}{4\beta_1\beta_2 - \gamma^2}. \tag{1.10}$$

Here the denominator indicates how wages are affected by the externalities that products pose on each other because of quantities. Obviously, they depend positively in the taste parameter of their own product and negatively with the quantity produced by the other product, which is denoted here by $\frac{\alpha_2\gamma - \alpha_1\beta_2}{\beta_1\beta_2 - \gamma}$. Plugging 1.9 and 1.10 into 1.7 and 1.8, the union's objective functions are

$$V_1^{N_M^*} = \frac{\beta_2\left(\alpha_2\beta_1\gamma - \alpha_1\left(2\beta_1\beta_2 - \gamma^2\right)\right)^2}{2\left(4\beta_1\beta_2 - \gamma^2\right)^2\left(\beta_1\beta_2 - \gamma^2\right)} \tag{1.11}$$
$$V_2^{N_M^*} = \frac{\beta_1\left(\alpha_1\beta_2\gamma - \alpha_2\left(2\beta_1\beta_2 - \gamma^2\right)\right)^2}{2\left(4\beta_1\beta_2 - \gamma^2\right)^2\left(\beta_1\beta_2 - \gamma^2\right)}, \tag{1.12}$$

which should be compared to those when unions coordinate their actions.

1.2.4 Union Coordination

Following the outline of the previous point, the first order conditions, wages, and objective functions of unions are derived when they decide to coordinate their actions. In this case, they will maximised their utilities as if they were colluding, nonetheless each union sets its own wage according to its demand schedule. The key factor to bear in mind is that unions internalised the possible negative externalities that they might pose on each other upon setting the wage. The objective function of each union is

$$V_1^{C_M}(\mathbf{w}) = (w_1 - c)\, n_1(\mathbf{w}) \tag{1.13}$$
$$V_2^{C_M}(\mathbf{w}) = (w_2 - c)\, n_2(\mathbf{w}) \tag{1.14}$$

As mention before they maximised their objective function as if they were one, the objective function that both unions maximise is

$$V_A^{C_M}(\mathbf{w}) = V_1^C(\mathbf{w}) + V_2^C(\mathbf{w}) \tag{1.15}$$

Applying the first order conditions over (1.15)

$$w_1^C(w_2) = \frac{\beta_2\,(c + \alpha_1) - \gamma\,(c + \alpha_2 - 2w_2)}{2\beta_2}$$
$$w_2^C(w_1) = \frac{\beta_1\,(c + \alpha_2) - \gamma\,(c + \alpha_1 - 2w_1)}{2\beta_1}$$

Again these are the reaction functions of one union with respect to the wages of the other union. Comparing the reaction function with and without coordination we see that in the former unions weight wages differently, actually they double the weight given to wages of the other union so as to internalise any negative externalities. Of course, the other difference is the coordination cost.

Now, in this case the change in wage provokes a stronger reaction in the other union. Solving the system of equations

$$w_1^{C_M^*} = \frac{c + \alpha_1}{2} \tag{1.16}$$
$$w_2^{C_M^*} = \frac{c + \alpha_2}{2} \tag{1.17}$$

Notice that in this case there are no cross effects of one wage into the other, clearly unions take into account the externalities that they exert into each other and minimise them when coordinating, or in other words they behave so as to remove the externalities. This leaves the final result as a average between the costs of coordination and the willingness to pay for the product, intuitively unions demand higher wage to compensate for the increases in costs. As shown by the objective function of the unions, higher wages come with the downside of depressing demand for labour. The corresponding objective functions at the optimal wages are

$$V_1^{C_M^*} = \frac{(\alpha_1 - c)\,(\beta_2\,(\alpha_1 - c) - \gamma\,(\alpha_2 - c))}{8\,(\beta_1\beta_2 - \gamma^2)} \tag{1.18}$$
$$V_2^{C_M^*} = \frac{(c - \alpha_2)\,(\beta_1\,(c - \alpha_2) - \gamma\,(c - \alpha_1))}{8\,(\beta_1\beta_2 - \gamma^2)}, \tag{1.19}$$

Before we jump to the direct comparison of functions under both situations, it will be illustrative to work out the change in objective functions when the costs of coordination change at the time of coordinating. Looking at the expressions 1.13 and

1.14 there is one direct effect of how costs affect the value of the union and another indirect effect via labour demand. Without loss of generality we can Differentiate the objective function of union 1 with respect to the costs of coordination

$$\frac{\partial V_1^{C_M}}{\partial c} = -n_1(\mathbf{w}) + (w_1 - c)\left\{ \underbrace{\frac{\partial n_1(\mathbf{w})}{\partial w_1}}_{<0} \underbrace{\frac{\partial w_1}{\partial c}}_{>0} + \underbrace{\frac{\partial n_1(\mathbf{w})}{\partial w_2}}_{>0} \underbrace{\frac{\partial w_2}{\partial c}}_{>0} \right\}$$

The final effect will depend on the net forces acting on the labour demands, at this point I shall assume that the wage bill, net of costs, becomes smaller with costs of coordination.

1.2.5 Coordination vs non-coordination

At this stage unions decide whether to coordinate or not, they compare the value attained under both situations. Union 1 decides to coordinate whenever:

$$V_1^{C_M^*} \geq V_1^{N_M^*}$$
$$\frac{(c-\alpha_1)\left(\beta_2(c-\alpha_1) - \gamma(c-\alpha_2)\right)}{8\left(\beta_1\beta_2 - \gamma^2\right)} \geq \frac{\beta_2\left(\alpha_1\left(\gamma^2 - 2\beta_1\beta_2\right) + \alpha_2\beta_1\gamma\right)^2}{2\left(4\beta_1\beta_2 - \gamma^2\right)^2\left(\beta_1\beta_2 - \gamma^2\right)}, \tag{1.20}$$

and seemingly for union 2. Solving for the costs of coordination in the quadratic inequality 1.20, the next proposition states that

Proposition 1. *Firms decide to coordinate whenever* $c \leq \hat{c}^M := \alpha_1 - \frac{1}{2(\beta_2-\gamma)}\left(\alpha_2\gamma - \sqrt{\Phi}\right)$ *where the expression of* Φ *is left in the appendix A.1 .*

Proof. Solve for c in (1.20). The positive part can be discarded since the cost of coordination cannot be higher than the taste parameter. ■

The equality derived in the previous proposition is a difficult and hardly explorable expression. In order to illustrate the point made in this paper some further assumptions are to be taken into account. I shall assume that goods have some degree of substitutability, for which $\alpha_1 = \alpha_2 = \alpha$ and $\beta_1 = \beta_2 = \beta$, then $\frac{\gamma}{\beta}$ is a measure of the substitutability of the goods. Following the same steps as in the previous proposition, proposition 1 is rewritten as:

Proposition 2. *Unions decide to coordinate whenever* $c \leq \tilde{c}^M := \alpha - \frac{2\alpha\sqrt{\beta(\beta-\gamma)}}{(2\beta-\gamma)}$.

Clearly, the more substitutable goods are the greater the negative externalities that unions pose on each other. The reason is that consumers can trade off one good for the other without diminishing much their utility, the firm takes advantage of this fact by providing a close substitute good from abroad, reducing the capacity of unions to demand a wage rise. When unions coordinate their action they offset these externalities, but concerting their actions comes at a cost. Then unions will decide to act as a monopoly at an international level, whenever the savings of internalisation are above the costs of coordination. Figure 1.1 Shows the region of coordination in the $c\gamma$-plane. As it is clear from the graph the more substitutability between the goods the larger the minimum cost from which they start to coordinate. For illustration purposes consider the line $\gamma = 0.3$, for which the minimum cost below which they start coordinating is $c = 0.1$, any cost less than that will cause the unions to go together, and to negotiated separately for higher values.

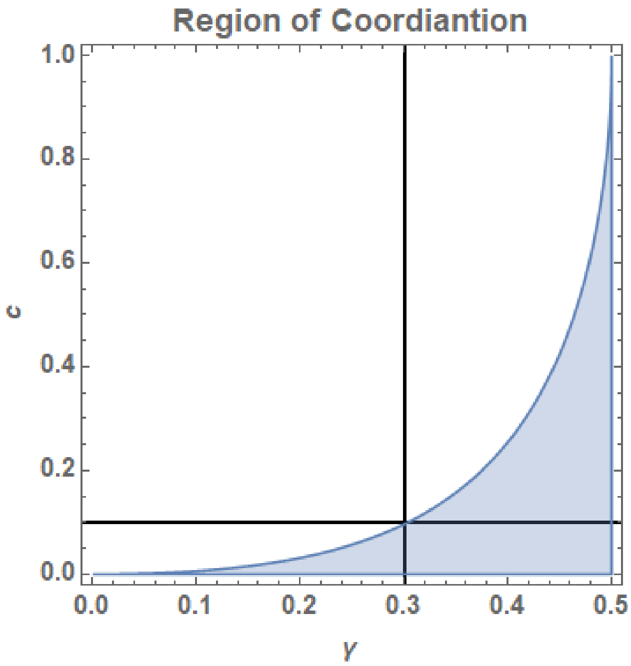


FIGURE 1.1: Coordination region for $\alpha = 1$ and $\beta = \frac{1}{2}$

The point done in the previous paragraph is made clear in figure 1.2. For $c = 0.1$ we see how the objective functions of the unions behave for different levels of substitutability. When $\gamma = 0$, both markets are independent and the firm acts as a monopoly in each market, obviously the value of the unions of not coordinating is higher than when coordinating as they incur in a cost but there is no externalities to internalised. Union values under each situation are decreasing on the substitutability of goods, however the rate at which the value of a union decreases is faster under non-coordination that under coordination, because of the internalisation. As a limiting example, when goods are perfect substitutes, i.e. $\gamma = \beta$, the firm is able to extract all the surplus from the unions. If a union raises its wage marginally more than the other union, the firm will swift all the production to the other plant and will operate from there. This is not so when unions coordinate, in this case unions raise wages in both plants, preventing the firm from flooding domestic markets with goods from other countries and retaining part of the surplus for themselves.

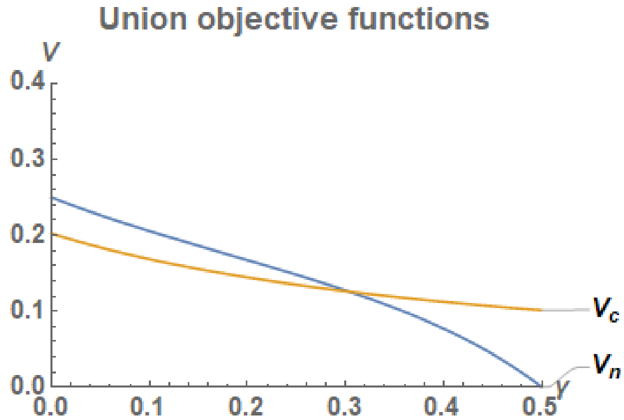


FIGURE 1.2: Value functions under coordination and not coordination as a function of γ. $\alpha = 1$, $\beta = \frac{1}{2}$ and $c = 0.1$

1.2.6 Welfare Analysis and Outcomes

Regarding the social welfare is fair to ask if coordination decisions of unions are socially desirable. Also it is important to acknowledge how the different components of the welfare change under both situations. In this respect, the social welfare function is defined as

$$W\left(\mathbf{q}^k\right) = U\left(\mathbf{q}^k\right) - \sum_{i=1}^{2} p_i^k q_i^k + \Pi_A\left(\mathbf{q}^k\right) + \sum_{i=1}^{2} V_i\left(\mathbf{q}^k\right)$$

where k indicates the coordination structure, either unions are coordinated $k = C_M$ or they are not $k = N_M$. Remember that the production technology is such that one unit of the good is produced with one unit of labour. What is more, because the objective functions of consumers, the firm and the unions are linear in q_i, the revenues of one agent is the cost of the other, simplifying the above expression to:

$$W\left(\mathbf{q}^k\right) = U\left(\mathbf{q}^k\right) - c\sum_{i=1}^{2} q_i^k$$

This expression will allow us to evaluate the possible gains or losses of the decisions of unions, which is stated in this proposition:

Proposition 3. *Social welfare is superior under non-coordination than under coordination, i.e.* $W\left(\mathbf{q}^{N_M}\right) \geq W\left(\mathbf{q}^{C_M}\right)$

Proof. See Appendix A.2 ■

Table 1.1 summarises the outcomes of prices, product/labour demands and wages. Direct comparison of both states shows that prices and wages are higher under coordination, whereas product demand, or equivalently labour demand, is lower.

Clearly, the fact that social welfare is lower in the case of union coordination does not mean that every agent is worse off, indeed unions are better off as a result

	p_1	n_1	w_1
N_M	$\frac{\alpha_2\beta_1\gamma+2\alpha_1\left(5\beta_1\beta_2-\gamma^2\right)}{2(4\beta_1\beta_2-\gamma^2)}$	$\frac{\beta_2\left(\alpha_2\beta_1\gamma-\alpha_1\left(2\beta_1\beta_2-\gamma^2\right)\right)}{2(4\beta_1\beta_2-\gamma^2)(\beta_1\beta_2-\gamma^2)}$	$\frac{\alpha_1\left(\beta_1\beta_2-\gamma^2\right)-\beta_1(\alpha_2\gamma-\alpha_1\beta_2)}{4\beta_1\beta_2-\gamma^2}$
C_M	$\frac{1}{4}\left(5\alpha_1-c\right)$	$\frac{\beta_1(\alpha_1-c)-\gamma(\alpha_1-c)}{4(\gamma^2-\beta_1\beta_2)}$	$\frac{1}{2}\left(\alpha_1+c\right)$
D_{C-N}	>0	<0	>0

TABLE 1.1: Outcome expressions under coordination and non-coordination

whereas the firm and consumers are both of them worse off, below is left a sum-up table with the main idea:

	V_1	Π_1	CS
N_M	$\frac{\beta_2\left(\alpha_2\beta_1\gamma-\alpha_1\left(2\beta_1\beta_2-\gamma^2\right)\right)^2}{2(4\beta_1\beta_2-\gamma^2)^2(\beta_1\beta_2-\gamma^2)}$	$\Pi_1^{N_M^*}$	$CS^{N_M^*}$
C_M	$\frac{(c-\alpha_1)(\beta_2(c-\alpha_1)-\gamma(c-\alpha_2))}{8(\beta_1\beta_2-\gamma^2)}$	$\Pi_1^{C_M^*}$	$CS^{C_M^*}$
D_{C-N}	>0	<0	<0

TABLE 1.2: Union and firm objective functions and consumer surplus

Particular forms of profits and consumer surpluses are left in the appendix A.3 to be compared.

1.3 Right-to-manage Model of Union Coordination

In reality, unions cannot set the wage unilaterally, but have to bargain with the employer to a certain wage level, whereas the employer retains the right-to-manage. Under this scenario union's strategies do not generate as many externalities as in the monopoly case and the profits of internalisation are lower, then unions find it relatively more costly to coordinate and the chances of collusion are lower. So why unions hold less bargaining power? In the negotiating process there are several factors that tilt the balance towards the side of the employers' associations.

First of all is the density, or the share of workers affiliated to a union. As pointed out by Addison (2020), trade unions have had a constant decline of membership since the 1980's across the developed world due to a changes in law and the production technology. So employers can substitute unionised by non-unionised labour more easily and better impose their wage demands.

Secondly the free-rider problem. Collective agreements are a source of public goods from which free-riders can reap the benefits of the union without being affiliated, which ultimately lowers their bargaining position, see A. L. Booth (1985) and A. L. Booth and Bryan (2004).

Apart from that, how easy is for employers to substitute labour for other factors of production affects union's position to negotiate. Even if the union has the monopoly of the labour supply, when labour and capital are highly substitutable, employers can swiftly replace workers with capital to circumvent union's demands.

Without being exhaustive, the bargaining structure also affects the outcomes of the negotiation, either by the level of bargaining or because the intense coordination among different unions, see for example Calmfors and Driffill (1988a). Also Visser et al. (2013), thoroughly examines the impact of the structure of negotiation over bargained outcomes across OCDE countries after the Great Recession. The gist of his paper being that different structures leads to bargaining outcomes closer or farther from unions objectives.

All of these factors erode the union capacity to negotiate, also empirical analysis suggest that still unions bargain wages that are well above the reservation wages of workers, so there must be a point in between where the union can impose at least some of its demands. How close unions are to the competitive outcome or the monopoly union is an empirical matter not analysed here. However, it is illustrating to consider how unions change their decisions as their bargaining position changes.

1.3.1 No Union Coordination

In this section as well as the following ones, I assume that goods have some degree of substitutability, i.e. $\alpha_1 = \alpha_2 = \alpha$ and $\beta_1 = \beta_2 \neq \gamma$. The step that is changing is the bargaining solution between the union and the firm, hence the consumer's utilities and the firm reaction functions with respect to the product of each plant are still the same.

The wage that unions and the firm negotiate, solves the following Nash-Bargaining programme:

$$w_i^{N_R*} = \arg\max_{w_i} \Pi_A(\mathbf{w})^{1-\delta} V_i(\mathbf{w}, \mathbf{d})^{\delta} \quad \forall i = 1, 2. \tag{1.21}$$

Clearly, for the particular case when $\delta = 1$ we are back to the monopoly union model. As in the previous section, the wage of union 1 depends on the one set by union 2, solving the system of equations for each the wage, the result is

$$w_i^{N_R^*} = \frac{\alpha\delta(\beta-\gamma)}{\beta(\delta+1)-\gamma}.$$

In the limiting case in which $\delta = 0$ the wage attained by the union coincides with the reservation wage, which here has been normalised to 0. The corresponding objective function of the union under this regime is

$$V_i^{N_R^*} = \frac{\alpha^2\delta(\beta-\gamma)(\beta-\gamma(1-\delta))}{2(\beta+\gamma)(\beta(1+\delta)-\gamma)^2} \quad \forall i = 1, 2 .$$

Naturally, as unions have less bargaining power as in the monopoly union model the value the objective function attained under the right-to-manage model is lower for values of $\delta \in (0,1)$. This can be proven by direct comparison

$$V_i^{N_R^*} < V_i^{N_M^*}$$
$$\frac{\alpha^2\delta(\beta-\gamma)(\beta-\gamma(1-\delta))}{2(\beta+\gamma)(\beta(1+\delta)-\gamma)^2} < \frac{\alpha^2\beta(\beta-\gamma)}{2(\gamma-2\beta)^2(\beta+\gamma)},$$

which is true for every value of $\delta \in [0,1)$.

1.3.2 Union Coordination

If both unions decide to coordinate the decision vector takes the value $\mathbf{d} = (1,1)$. There is only one problem that needs to be solved with respect to both wages, namely

$$\mathbf{w}^{C_R^*} = \arg\max_{\mathbf{w}} \Pi_A(\mathbf{w})^{1-\delta} V_A(\mathbf{w},\mathbf{d})^{\delta} .$$

Since the problem is symmetric both unions set the same wage, which is

$$w_i^{C_R^*} = \frac{\delta\alpha + (1-\delta)c}{2} .$$

The negotiated wage is the average between what the union can extract from the product and the costs it has to bear, weighted by the bargaining power. It is reasonable that the higher the bargaining power of the union the more can extract from the product and bears less of the costs. Plugging this wage into the objective function of the union when coordinating, we arrive at

$$V_i^{C_R^*} = \frac{\delta(2-\delta)(c-\alpha)^2}{8(\beta+\gamma)} .$$

As in the case of non-coordination the value attained under the right-to-manage model is always lower than under the monopoly union, which can be proven again by direct inspection of the two value functions:

$$V_i^{C_R^*} < V_i^{C_M^*} \iff \frac{\delta(2-\delta)(c-\alpha)^2}{8(\beta+\gamma)} < \frac{(c-\alpha)^2}{8(\beta+\gamma)}$$

for $\delta \in [0,1)$.

1.3.3 Comparison under Right-to-Manage model

In other to know the cost threshold from which unions start coordinating we have to compare the objective functions under both situations:

$$V_i^{N_R^*} < V_i^{C_R^*}$$
$$\frac{\alpha^2\delta(\beta-\gamma)(\beta-\gamma(1-\delta))}{2(\beta+\gamma)(\beta(1+\delta)-\gamma)^2} < \frac{\delta(2-\delta)(c-\alpha)^2}{8(\beta+\gamma)}. \tag{1.22}$$

The next proposition applies for the right-to-manage model

Proposition 4.

1. *Unions decide to coordinate whenever* $c < \tilde{c}^R := \alpha - \frac{2\alpha\sqrt{\beta(\beta-\gamma)}}{(2\beta-\gamma)} \frac{1}{\sqrt{(1-\delta^2)}}$

2. $\tilde{c}^R < \tilde{c}^M, \quad \forall 0 < \gamma < \beta.$

Proof. 1. Solve for c in (1.22). The positive part can be discarded since for the problem to be well defined the cost of coordination cannot be higher than the taste parameter.

2. $\tilde{c}^M - \tilde{c}^R = \frac{1}{\sqrt{1-\delta^2}} - 1 > 0$

■

The second part of the proposition states that: for unions to coordinate abroad they need to be strong at home. As one can see from figure 1.3, the coordination region under the right-to-manage shrinks as the bargaining power of unions decreases. This is because, having less bargaining power in their respective countries the gains of coordination are smaller. In this case, for a given level of substitutability of goods, the threshold at which unions start coordinating is much lower, meaning that they are not willing to assume as many costs as when they are a monopoly. The first point of the proposition goes along the same lines as in the monopoly model, meaning that when costs are sufficiently low unions will decide to coordinate.

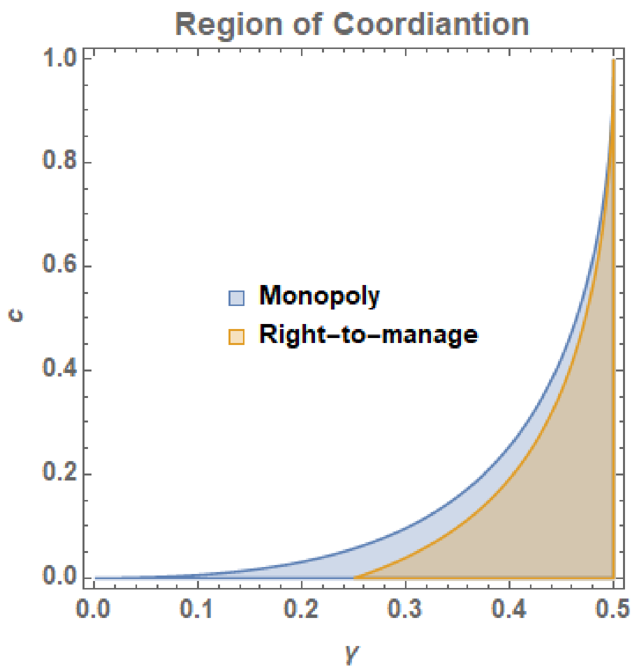


FIGURE 1.3: Value functions under coordination and not coordination as a function of γ. $\alpha = 1, \beta = \frac{1}{2}$ and $c = 0.1$

Figure 1.4 shows a direct comparison of objective functions under both regimes and decisions, from the figure we can distinguish four regions. First, that under sufficiently low costs of coordination, coordination is always a superior outcome for the union than no coordination under both regimes. Second, for high enough costs, unions will always decide to negotiate vis-a-vis with the company, as the gains are not sufficient to cover the costs. Third, if these costs were in an intermediate range, the union would be better off negotiating on its own, provided it had monopoly power. And last, for this intermediate range it could be the case, that the union would decide to coordinate before not coordinating provided it had enough bargaining power (see that no coordination is still superior).

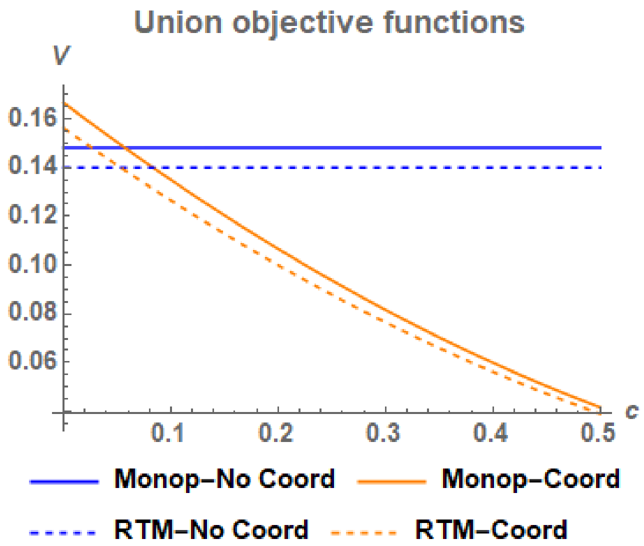


FIGURE 1.4: Value functions under coordination and not coordination as a function of γ. $\alpha = 1$, $\beta = \frac{1}{2}$, $\gamma = \frac{1}{4}$ and $\delta = \frac{1}{2}$

1.4 Different Costs

This section analyses the consequences of dropping the assumption of equal costs of coordination. The difficulty of this analysis resides in accounting from the feedback effect of the costs of one union into the reaction function of the other. The main result is that both costs need to be relatively low in order to induce workers representatives to coordinate. It is not sufficient that one of the unions has low costs since it induces the other union not to engage in coordination.

There are several reasons of why unions might face different costs upon collusion. Maybe some unions are more efficient at organising or some union leaders are more willing to take action towards coordination than others. Another possible rationale could be the disparity of legal frameworks among countries, protecting or making it easier for unions to take actions in this way. Also, different financial strength is a key factor preventing unions from organising abroad. Of course, other reasons make different costs of coordination plausible, like employers' action to prevent union coordination among others.

1.4.1 Comparison under different costs

Here we turn back to the monopoly union model with substitutable goods. The difference with respect to section 1.2 is in the objective functions of unions when they coordinate. The functions to consider now are

$$V_1^{C_D}(\mathbf{w},\mathbf{d}) = (w_1 - d_1 c_1)\, n_1(\mathbf{w}) \tag{1.23}$$
$$V_2^{C_D}(\mathbf{w},\mathbf{d}) = (w_2 - d_2 c_2)\, n_2(\mathbf{w}). \tag{1.24}$$

Since unions have all the bargaining power, they just need to choose the wages that fit them best, under differentiated costs the wages are:

$$[w1] : w_1^{C_D^*} = \frac{1}{2}(\alpha + c_1)$$
$$[w2] : w_2^{C_D^*} = \frac{1}{2}(\alpha + c_2).$$

The interpretation of these expression goes along the same lines exposed in section 1.2. As it is the case in the previous sections, wages should increase linearly with costs of coordination. Notice that unlike Straume (2002), unions do not necessarily agree to a common wage, but they choose the one according to the productivity of their plant. The utilities when coordinating are:

$$V_1^{C_D^*} = \frac{(\alpha - c_1)(\alpha(\beta-\gamma) - \beta c_1 + \gamma c_2)}{8(\beta^2 - \gamma^2)}$$
$$V_2^{C_D^*} = \frac{(\alpha - c_2)(\alpha(\beta-\gamma) - \beta c_2 + \gamma c_1)}{8(\beta^2 - \gamma^2)}$$

Before we jump to the direct comparison of functions under both situations, it will be exemplifying to work out the derivative of the objective function with respect to both coordination costs. Looking at the expressions 1.23 and 1.24 there is one direct effect of how costs affect the value of the union and another indirect effect via labour demand. Differentiating the objective function with respect to both costs of coordination

$$\frac{\partial V_1^{C_D}}{\partial c_1} = -n_1(\mathbf{w}) + (w_1 - c_1)\underbrace{\frac{\partial n_1(\mathbf{w})}{\partial w_1}}_{<0}\underbrace{\frac{\partial w_1}{\partial c_1}}_{>0} < 0$$
$$\frac{\partial V_1^{C_D}}{\partial c_2} = (w_1 - c_1)\underbrace{\frac{\partial n_1(\mathbf{w})}{\partial w_2}}_{>0}\underbrace{\frac{\partial w_2}{\partial c_2}}_{>0} > 0$$

The interesting equation is the second one, it shows that the wage bill net of coordination costs is increasing in the costs of union 2, as explained previously unions raise their demand for higher wages to partially offset costs of coordination, when this is done by union 2 it depresses its demand for labour and products. The firm partially fills the void with the product of the other brand, raising the demand for labour in country 1 and increasing its wage demands as a consequence. conversely a reduction in the costs of union 2 means lower utility for union 1, this reduction in utility might lead to union 1 opting out of an agreement. Regarding the first equation, it states what intuition previously suggested, that the wage bill net of coordination costs is decreasing in these costs.

When comparing utilities we can discern the regions under which both unions coordinate for a given cost of the other union, the utilities to be compared are:

$$V_1^{C_D^*} > V_1^{N_D^*}$$
$$\frac{(\alpha - c_1)\left(\alpha(\beta - \gamma) - \beta c_1 + \gamma c_2\right)}{8\left(\beta^2 - \gamma^2\right)} > \frac{\alpha^2 \beta \left(\gamma - \beta\right)}{2\left(\gamma - 2\beta\right)^2\left(\beta + \gamma\right)} . \tag{1.25}$$

As said before the objective function of the union when it does not coordinate under different costs is the same as in the monopoly union model. Then, with no loss of generality:

Proposition 5. *Union one decides to coordinate whenever*

$$c_1 \leq \hat{c}_1(c_2) = \alpha - \frac{1}{2\beta}\left(\gamma(\alpha + c_2) - \sqrt{\frac{\alpha^2\left(16\beta^4 - 32\beta^3\gamma + 20\beta^2\gamma^2 - 4\beta\gamma^3 + \gamma^4\right)}{(\gamma - 2\beta)^2} - 2\alpha\gamma^2 c_2 + \gamma^2 c_2^2}\right)$$

Proof. Same steps as in proposition 1. ■

The same equation applies for the region of the other union as the problem is symmetric. This equation sets the maximum cost that a union is willing to bear in order to coordinate actions. If union 2 has and increase in costs of coordination that means that union 1 will be more willing to coordinate since will be able to reap more profits out of the relation. The maximum cost that union 1 can bear is α which is the price of the product at which the demand is zero, and this is stated in the following proposition.

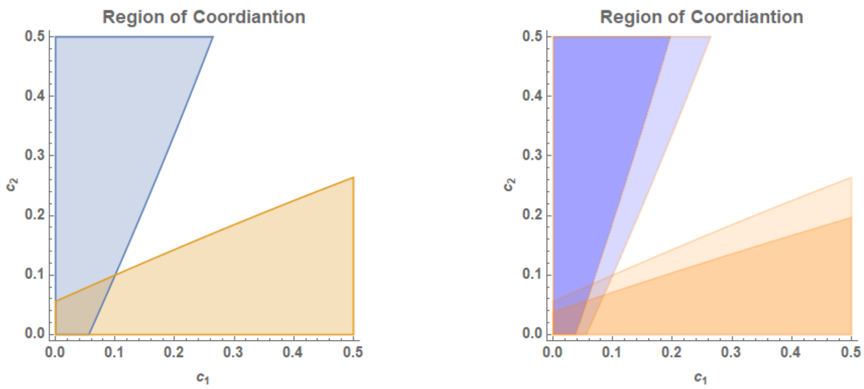
Proposition 6. *The maximum value at which union 1 is willing to coordinate as $c_2 \to \infty$ is $\hat{c}_1(c_2) = \alpha$*

Proof. Complete the square for the terms related to c_2 inside the squared root and discard the rest as $c_2 \to \infty$, since they become arbitrarily relatively small. Then operate and the result follows. ■

Of course, this is from the point of view of union 1, the other union will face the same situation. It is the case that for high costs of coordination for union 2, it will no longer be willing to collude. Since the problem is symmetric both unions will have to have relatively low levels of costs for the coordination to be profitable. However, the relation is not linear, the fact that unions are willing to coordinate for a given level of costs does not imply that for lower cots of one union the relation is still profitable for both of them. Actually, It might be the case that for lower costs of the rival union, it might not be able to extract enough profits out of the relationship.

For example, consider the orange-shaded region in figure 1.5a. This is the region of under which union 2 is willing to coordinate as the costs of union 1 change, and seemingly for the blue-shaded area. The double-shaded area is the region where both of them coordinate. Now take a look at the locus $(.1, .1)$, at this point both are willing to coordinate, however if one of the unions would have lower costs of coordination, e.g. c_2 is lower, it would be able to extract more from the relation leaving the rival union with less to enjoy, making the relation to break apart.

In summary, in order to unions to be willing to coordinate actions, costs of coordination of each union have to be relatively small for each other. Or in other words when $c_1 \leq \hat{c}_1(c_2)$ and $c_2 \leq \hat{c}_2(c_1)$ ought to be met simultaneously.



(A) Region of coordination under different costs. $\gamma = \frac{1}{4}$

(B) Change in regions as goods become less substitutable. $\gamma_{light} = .3$ and $\gamma_{dark} = \frac{1}{2}$

FIGURE 1.5: Region of coordination. $\alpha = 1$ and $\beta = \frac{1}{2}$

To gain more insight, we can see how these regions change as we consider goods with different grades of substitutability. In figure 1.5b, the light-shaded area represent the regions in which unions are keen to coordinate, this works along the same lines as figure 1.5a. As goods with less grades of substitutability are considered unions are less agreeable, the regions at which they could reach an agreement shrink, dark-shaded area. As in previous sections, the negative externalities that they pose on each other become smaller and the levels of costs that the are keen to support become smaller as well.

1.5 Coordination Offering

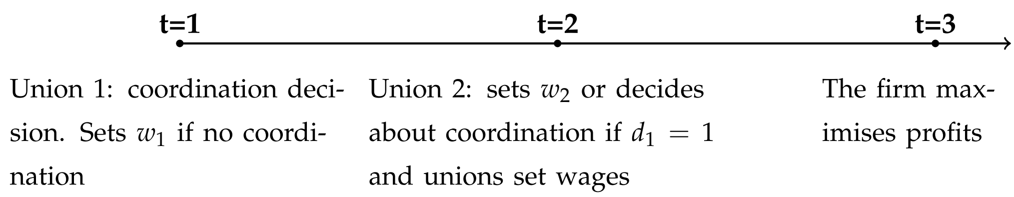
This section analyses the situation in which one of the unions has the possibility to set wages before the union in the other country. The key insight is that the leader is less keen to coordinate as it is able to extract more profits when it goes alone. In the case analysed in this paper the leader enjoys the firs-mover advantage. The downwards-sloping reaction functions that are considered in the product market are translated into a strategic complementarity in the market of the factor, labour, which naturally gives the first mover an advantage, see Gal-Or (1985) for a full description of the mechanics.

There are several reasons that can lead to a union to move first, here it is assumed that union 1 holds some kind of privileged information, it might be because its proximity to the decision making, i.e. headquarters, or because information about the business strategy leaks through and the union is able to use it. This section is related to Dowrick (1986), in which they analyse the case of two firms competing a là Cournot. They argue that it might be in the interest of unions to be assigned the roles of leader and follower if it is in the interest of both of them to do so. For this to happen, both unions need to have very different pay-offs function. In this paper the roles have already been assigned and it assumed that it is in their interest to do so.

1.5.1 Setting

The most important change at this point is the timing of events and the kind of game that is played under each option. Without loss of generality union 1 holds privileged information, due to this fact it has the opportunity to set its wage first if it finds profitable to do so.

In the first period union 1 decides whether to coordinate or not. If it decides not to coordinate, it sets its wage and it becomes a Stackelberg leader. However, If it offers union 2 to coordinate, it is assumed that it discloses information needed to concert action losing its leader condition, so that unions set wages simultaneously. In the second period, if union 1 has decided not to promote an agreement, union 2 sets its wage unilaterally; if union 1 makes an offer to coordinate, union 2 either accepts or rejects the offer and both unions set wages at the same time. In the last period the firm maximises profits in both brands at once and pay-offs are realised.



The game is solved by backwards induction. In the last period the firm sets quantities, and demands labour, maximising profits in both brands and taking the wages of monopoly unions as given. In period two, there are two scenarios for union 2 depending whether union 1 has decided to coordinate or not. If union 1 has decided to coordinate the game boils down to that already shown in section 1.2. In case union 1 decides to set its wage unilaterally, union 2 decides its level of wages taking w_1 as given. Finally, in the first period union 1 sets w_1 knowing the reaction function of union 2. Since the case of coordination has already been shown it is just left to work out the solution under the Stackelberg leader. Once the solution is derived, union 1 to compares its objective function under both regimes and takes a decision.

1.5.2 Reaction function of union 2

In the last step the firm makes its production choices given wages, which sets the labour demand schedule. Then Union 2 solves the following problem

$$\underset{w_2}{max} \quad V_2(w_1^*, w_2) = w_2 \cdot n_2(w_1^*, w_2) \; . \tag{1.26}$$

Where w_1^* indicates the wage set by union 1 optimally. Notice that $c = 0$ as unions do not coordinate. Applying the first order condition we can derive the optimal wage chosen by union 2 as a function of w_1^*, namely the reaction function of union 2, $R_2(w_1^*)$:

$$w_2 = R_2(w_1^*) = \frac{\alpha(\beta - \gamma) + \gamma w_1^*}{2\beta} \; .$$

Clearly the reaction function of union 2 is upwards sloping in w_1^*, i.e. $\frac{\partial R_2(w_1^*)}{\partial w_1^*} > 0$. Because of this fact wages are strategic complements and the leader does not enjoys the first mover advantage as suggested by Gal-Or (1985). The economic intuition is that products are still strategic substitutes, hence if union 1 increases its wage, it also raises the costs of brand 1. Due to this fact, the firm reduces the production in brand 1 and fills this void with products of brand 2, at least partially, which depresses the demand of labour for union 1 and expands it for union 2. Finally, Union 2 faces an increase in demand of which it takes advantage of by increasing its wage. So, even though both unions increase their wage, it is not as profitable for union 1 as it is for union 2 due to changes in the labour demands that they face.

1.5.3 Objective function of Union 1

Union 1 takes into account how union 2 will react to its strategy and will maximise its objective function accordingly, so union 1 solves the following problem

$$\underset{w_1}{max} \quad V_1(w_1, R_2(w_1)) = w_1 \cdot n_1(w_1, R_2(w_1)) \; .$$

After applying the first order condition the wage set by the union is

$$w_1^{N_S^*} = \frac{\alpha(\beta - \gamma)(2\beta + \gamma)}{4\beta^2 - 2\gamma^2}$$

and the respective objective functions after each union has maximised are

$$V_1^{N_S^*} = \frac{\alpha^2(\beta - \gamma)(2\beta + \gamma)^2}{16\beta(\beta + \gamma)\,(2\beta^2 - \gamma^2)}$$
$$V_2^{N_S^*} = \frac{\alpha^2(\beta - \gamma)\left(4\beta^2 + 2\beta\gamma - \gamma^2\right)^2}{32\beta(\beta + \gamma)\,(2\beta^2 - \gamma^2)^2} \; .$$

Even though, the rank of utilities has been pointed out in the previous point, it will be useful for what it comes next to compare these expressions with that of the monopoly union model.

Proposition 7. *For suffiently high costs of coordinaiton, $c > \hat{c}^M$, the objective function of union 1 being the Stakelberg leader is always higher than in the Cournot game. The rank of union utilities is:*

- $V_1^{N_S^*} < V_2^{N_S^*}$
- $V_1^{N_M^*} < V_1^{N_S^*}$

Proof. See appendix ∎

So, the intuition behind this proposition is that unions find more profitable to let one of them act as a leader rather than synchronise their actions due to double marginalisation. Also, because strategic complementarity the follower, union 2, is the one with a second mover advantage.

1.5.4 Coordination decision

In the first step union 1 decides whether to make an offer of coordination with union 2 or move first. Union 1 knows that for sufficiently high coordination costs neither union is interested in coordinating, still union 1 has the possibility of deciding whether to be the leader or to set the wage simultaneously with union 2. The other scenario is that costs are sufficiently low that both of them find profitable to coordinate rather than not when they set wages at the same time. In this case, union 1 will compare the benefits of coordinating against setting its wage first. Below I take each situation in turn.

As seen in section 1.2 for sufficiently high costs of coordination, $c > \hat{c}^M$, union 1 does not coordinate and then makes the comparison between getting ahead and move first or waiting and setting the wage simultaneously with the union 2. As shown in the proposition 7 union 1 will always find profitable to move first, even if it does not enjoy the first mover advantage.

In the case that the costs of coordination are sufficiently low, i.e. $c \leq \hat{c}^M$. Union 1 still has to decide whether to coordinate or to act as Stackelberg leader. For union 1 to have incentives to coordinate, the costs need to be even lower than in the simultaneous game. Union 1 coordinates whenever $V_1^{C_M^*} \geq V_1^{N_S^*}$, which leads to the next proposition

Proposition 8.

1. *Union 1 decides to coordinate whenever* $c \leq \hat{c}^S := \alpha - \alpha(2\beta + \gamma)\frac{\sqrt{(\beta-\gamma)}}{\sqrt{2\beta(4\beta^2-\gamma^2)}}$
2. $\hat{c}^M > \hat{c}^S$

As it has been the case in previous sections, the first point of the proposition just states what is the condition under which union 1 is willing to coordinate. The second point merely states that union 2 always coordinates whenever union 1 decides to do so, as the threshold of coordination under the simultaneous game is higher than that of the sequential.

1.6 Conclusion

Unless unions' strategies are very interrelated, i.e. they cause large externalities on each other, unions do not find profitable to coordinate in the present of transactional costs. I have shown that the more substitutes products were the more interesting is for unions to coordinate actions. Upon coordination unions are usually better off whereas total welfare diminishes, this is the result of labour rationing which decreases production. Also, several extensions were considered, in the first one I have shown the right-to-manage model in which unions hold less bargaining power, as a result unions' actions did not produced large externalities so as to compensate for the costs of coordination. Hence the coordination region shrinks accordingly. In the second extension I considered different costs of coordination for each union, clearly it was not enough that unions had low coordination costs, but they have to be relatively low with respect to each other; too low transactional costs for the foreign union reduces the wage demanded by it, the firm shifts production from the domestic product to the foreign one and the domestic union losses from coordination. In the last extension, I modelled the coordination decision by letting one of the unions move first. Surprisingly, both unions are better-off in the Stackelberg game than under the Cournot game because of the double-marginalisation, consequently cost of coordination need to be lower for unions to be interested in colluding.

Chapter 2

A Search and Matching Model of Firm Heterogeneity, Minimum Wages and Collective Agreements

2.1 Introduction

How do Central and Western European systems of collective bargaining affect labour market outcomes? What is the optimal bargaining protocol? How different sized firms are affected? These are the sort of questions that the present work answers. Arguably, the most important aspect of collective contracts is the minimum wage level. Participants of the political process (insiders) might use minimum wages as a tool to raise wages or reduce competition, not internalising the effects on the outsiders. This work delivers a model to study minimum wage bargaining between firms and workers and study the effects on welfare and labour market outcomes such as unemployment, wage distribution and firm size.

The main contribution of this paper is to bring forth how minimum wages affect firms of different productivities and sizes. Minimum wages are relevant because they redistribute profits, employment and market power along the size distribution. Furthermore, in cases like Spain unions and employers associations in large firms have the power to set the minimum wage. Hence, if unions and employers' associations in large firms can set the minimum wage level, and if the level of minimum wage affects firms along the size distribution differently; then, the type of bargaining protocol that decides who sits in the negotiating table matters.

To analyse this I built a search and matching model with minimum wages and two-sided heterogeneity, on top of this I introduce unions and employer's associations in large firms that bargain for the level of minimum wages. Then, I estimate the model using the Spanish administrative records with complete employment histories of workers. The main result is that unions and employers are interested in keeping minimum wages 11% higher of what would it be desirable. When raising the minimum wage level, low productivity firms become smaller and make about 4% less profits, whereas high productive firms become larger and increase their profits about 7%.

The mechanism that drives these results has two parts. On the one hand, low productive or small firms suffer a cost increase due to the rise in the minimum wage level. The higher costs reduce expected profits which induce small firms to post less vacancies, and then they become smaller. What is more, small firms have to lay off low-skilled workers since they are not productive enough to pay for the minimum wage, adding to a new stock of unemployed. On the other hand, high productive or large firms are not as much affected by the minimum wage rise, since they pay higher wages in the first place. However, now they have a new stock of unemployed at their disposal, making it easier to fill vacancies and becoming larger as a result. On top of that, large firms face less competition from smaller firms when they poach for other workers, these workers cannot use outside offers to demand a wage rise as often, which makes their careers (wages increases) less dynamic. As a consequence, large firms not only become larger but have more profits as they pay workers less.

Turning to the bargaining protocol, the threshold for participation in the political process is exogenously determined according to the firm size. Unions and employers' associations in large firms can elect their representatives to negotiate the level of the minimum wage (insiders). On the other side, employed and employers in small firms, and unemployed are left out of this mechanism to elect representatives (outsiders). Because of this, insiders do not internalise the effects of their decisions on the outsiders and find it profitable to raise the level of minimum wage more than what is desirable. A policy maker would set the cut-off point of participation lower to allow more people to decide as a way to increase social welfare.

A counter factual derived from estimations show that workers represented by the union have a steady decrease in utility as minimum wages grow, this is because the positive effects of having more vacancies at their disposal do not compensate for the increasing competition to fill these vacancies, effectively lowering the probability to match. On the demand side, high productive employers have more unemployed at their disposal, increasing the expected value of opening a vacancy, in turn, they open more vacancies and become larger, increasing the value of their jobs. If the minimum wage is too high, employers will not be able to hire workers and will miss chances of meeting previously profitable workers. Because of this, a hump-shaped curved for the utility of the employers. As a whole, the estimates suggest that employers' associations have a larger say in setting the minimum wage, as the latter is fixed where most convenient for them, indicating that they held most of the bargaining power. Increasing the participation in the political process, negotiators would set a lower the minimum wage at €1150, with large firms reducing their profits by 4.4% and smaller firms reentering the market, and increasing their profits by 6.9%.

This work contributes to the literature in several ways. In the literature of trade unionisation, unions bargain vis-a-vis with the company for better wages, whereas management chooses the level of employment, see A. Booth (1995) for a review. This framework fits better in Anglo-Saxon and Nordic countries. The present work departs from that literature in two ways. First, negotiations are carried out at a sectoral level, not within the firm, consequently collective contracts set the minimum requirements that all agents in the market have to abide by. In a recent paper, Krusell and Rudanko (2016) try to fill this gap assuming the union bargains on behalf of the whole active workforce, employed or unemployed, which make sense since there is no heterogeneity in their model; yet it misses the fact that there is no *a priori* reason why unions should worry about the whole workforce at a sector-wide level or even the unemployed. Then the second departure of the model notices that unions and employers associations do not represent only their affiliates but a wider base of principals who can vote. This feature makes it necessary to account for the political process.

To address this vent, I built on the works of Cahuc, Postel-Vinay, and Robin (2006) (CPR) and Flinn and Mabli (2009) (FM). In the latter, authors presented a unifying framework of search and matching models with firms competing *à la Bertrand*, free-entry and minimum wages; in their model there is match-specific productivity drawn at random, leaving no room to model firm size. In the former, the authors model two-sided heterogeneity to account for worker and firm fix effects, which in turn allowed to introduce a measure of firm size. Both models account for individual wage bargaining, which I do not deem right for this paper, as estimations of CPR show the bargaining power of the worker is close to zero for the lower ranks. The avenue that I take in this work is to account for two-sided heterogeneity in the work of FM to acknowledge that only workers in the largest firms will be able to elect their representatives and in this way participate the political process. On top of that, I carry on introducing unions and employers associations, letting minimum wages be endogenous.

2.2 Base model

I construct a search and matching model with OTJ-search and two-sided heterogeneity where workers do not hold bargaining power and hiring is costly for firms. I built on the work of Postel-Vinay and Robin (2002) in order to draw wage and mobility dynamics and two-sided heterogeneity, this implies that person and firm fix effects can be considered and that seemingly equal pairs, matches with same productivity and ability, pay different wages since workers are subject to different histories of wage offers. From Lise, Meghir, and Robin (2016), endogenous number of jobs is imported, which in their model is a measure of the number of firms. This allows to consider firms of different sizes depending on their productivity; also, notation is taken from here. Furthermore, the model pulls out from Lise and Robin (2017) the condition for the number of firms and its parameterizatiton, although in their paper is related to vacancies.

2.2.1 Setting

The Economy is populated with a continuum of workers indexed by x, representing the ability, which is exogenously given, publicly observable, and distributed over the interval $x \in [\underline{x}, \overline{x}]$ according to a beta distribution $l(x)$ with parameters (a_x, b_x) and which quantity is normalised to L. Workers are either unemployed or employed, in both cases they search for jobs to find better alternatives, the search effort of employed is s and the unemployed effort is normalised to one. Since workers search for other jobs while employed, they have the opportunity to bring other companies into Bertrand competition with their incumbent employers in order to gain a pay rise or change jobs otherwise, the process will be explained in detailed in the following sections. Let $u(x)$ be number of workers of type x among the unemployed and $U = \int u(x')dx'$ total unemployment.

Firms are ranked according to technology y that is uniformingly distributed in $y \in \left[\underline{y}, \overline{y}\right]$. Firms hold a number of jobs $n(y)$ that might be filled or vacant, the total number of jobs by held by all firms is $N = \int n(y')dy'$, which is endogenously determined due to the free-entry condition (FEC). The number of vacancies opened by the firm with productivity y is $v(y)$ and the number of workers of type x employed in this firm is denoted by $h(x, y)$, the size of the firm is then $h_y(y) = \int h(x', y)dx'$, and the distribution of workers across firms is $h_x(x) = \int h(x, y')dy'$. The total number of vacancies opened in the economy is $V = \int v(y')d'$ and seemingly the total number of employed people is $H = \int h_y(y')dy'$.

All agents in the economy discount time at the same factor ρ. The flow income as uneomployed is $f(x, y) = xb$ whereas upon matching the firm and the worker start producing a flow output $f(x, y) = xy$, the chief point is that workers are perfectly substitutable and there are no complementarities among then within the firm. Matches are exogenously terminated by a Poisson process with parameter δ or endogenously when there is a job-to-job transition.

2.2.2 The Matching Process

Search is random and undirected within the matching set, which under the case without minimum wages is $[\underline{x}, \overline{x}] \times [\underline{y}, \overline{y}]$. Notice that neither workers nor firms veto matches when they meet, since matches offer values to the unemployed or vacant jobs at least as high as their outside options. Unemployed and employed workers compete for vacancies with different search costs, the search cost of unemployed is normalised to one and for employed is denoted by s.

Let k be a parameter that characterises all key rates of meeting and which definition is:

$$k = \frac{M\left(U + s(L - U), V\right)}{\left[U + s\left(L - U\right)\right]V}.$$

Where M is a Cobb-Douglas meeting function of the searchers and vacancies with equal weights and a meeting efficiency parameter η. From here we can define the rate at which unemployed workers meet a vacancy as $kV \cdot \frac{v(y)}{V} = kv(y)$, whereas employed workers meet vacancies at a rate $skv(y)$. On the other side of the market, vacancies meet unemployed workers at a Poisson rate $ku(x)$ and meet employed ones at a rate $skh(x, y)$.

2.2.3 Value functions

At this point it is worth remembering that workers do not hold bargaining power vis-à-vis with the employer. In other words, firms take all of the surplus for themselves when a meeting is materialised in a match. The only way a worker can demand a pay rise to its employer is by dragging other firms into Bertrand competition. This setting differs from those of Dey and Flinn (2005) and Cahuc, Postel-Vinay, and Robin (2006) where workers are assumed to hold some bargaining power, at least at a theoretical level. There are three reasons why I chose not to include it. First of all, the bargaining power of workers is close to zero as they show in their empirical extracts. Secondly, the model captures the fact that workers usually do not have bargaining power and let the agent (union) to negotiate on their behalf. Lastly, it reduces the mathematical and computational burden. All derivations of value functions without minimum wages can be found in appendix B.1 together with the corresponding wage offers in appendix C.1.

Unemployed

The value of unemployed worker of type x is denoted by $W_0(x)$ and receives a flow bx for what produces while unemployed, notice that b is common to all workers, i.e. all have the same technology at home but the flow increases with the ability. This captures the fact that unemployed workers with high-skill have better wages and more generous unemployment benefits than those with less ability. Other rationale could be that those unemployed do some informal jobs that are going to be paid according to the ability. In reality, the unemployment insurance payment is usually a function of the wage earned and the time employed, at the same time the wage is a function of time and ability. Hence the mean time is captured by b which is the same for all workers and the ability by x. The wage offered to the unemployed $\phi_0(x, y)$ is such that the firm takes all the surplus of the match for itself, so that the worker is indifferent between taking or rejecting the offer. Notice that the offer depends on both arguments x and y. Then, the starting wage is implicitly defined as

$$W_0(x) = W_1\left(\phi_0(x, y), x, y\right), \quad \forall y \in \left[\underline{y}, \overline{y}\right],$$

with $W_1(w, x, y)$ being the value of an x employed worker earning a wage w at firm of type y. From here it can be drawn the continuation value of unemployment as:

$$(\rho + kV)W_0 = bx + k \int W_1\left(\phi_0(x, y'), x, y'\right) v(y')dy'.$$

Which by the previous definition solves as:

$$\rho W_0 = bx$$

Employed

As mentioned before the value of an employed worker is $W_1(w, x, y)$, nonetheless I will introduce $W_S(w, x, y) = W_1(w, x, y) - W_0(x)$, which is the net surplus accounted to an x-worker earning the wage w in a y-firm, mainly to save notational burden. As workers search on the job they can bring firms into competition in order to be granted a pay rise or switch companies otherwise. In this way, whenever a worker comes across a wage offer from an poaching firm $y' \leq y$, she can use it as outside option to negotiate vis-a-vis with her current employer. Upon meeting a firm, the worker faces three situations: the alternative firm does not have enough productivity to pay the current wage and the current relation does not change; another situation results in a wage increase for the worker and a third one materialises in a new match (a Job-to-Job transition).

The first case might be such that the worker encounters a firm y' that does not even have enough productivity to pay for his current wage and make profits, i.e. $xy' - w \leq 0$. The set of these firms ranges from the firm with least productivity $\underline{y}$ to the threshold $q(w, x, y)$. The productivity threshold $q(w, x, y)$ leaves the worker indifferent between extracting the whole surplus of the poacher and staying in her current firm earning the same wage or in other words

$$W_S\left(w, x, y\right) = S(x, q).$$

In this case the worker does not swap firms nor sees her wage risen, hence the wage offer does not have any effect on her.

In the following scenario the outside firm ranks in $y' \in (q, y]$, i.e. the productivity of the incumbent company is higher that the poaching one, still the latter has enough productivity to oblige the former grant a pay rise to the worker. Let $\phi(x, y', y)$ be the offer done by firm $y' < y$ to an x-type employee working at firm y. The final job offer will leave her indifferent between staying in her current firm with a wage promotion, which is the case, or changing firms and is implicitly defined as

$$W_S(\phi(x, y', y), x, y) = S(x, y').$$

Notice that the poaching firm will never raise its offer above xy' since loses would materialise: $xy' - \phi(x, y', y) < 0$.

Seemingly, when the productivity of the poacher is $y' > y$ the offer granted to the worker $\phi(x, y, y')$ is such that leaves her indifferent between staying with the incumbent with a wage xy or changing jobs to a more productive firm but earning less wage. Again, the wage offer is implicitly defined as

$$W_S\left(\phi(x, y, y'), x, y'\right) = S(x, y).$$

With these expressions at hand the surplus continuation value of an employed worker, net of lay-off shocks and future wage offers, would be

$$\begin{aligned}\left[\rho + \delta + sk\overline{V}\left(q(w, x, y)\right)\right] W_S(w, x, y) = w - \rho W_0(x) & \\ + sk \int_{q(w,x,y)}^{y} W_S(xy', x, y')\, v(y')dy' & \\ + sk \int_{y}^{\overline{y}} W_S(xy, x, y)\, v(y')dy' & \qquad (2.1)\end{aligned}$$

The first term in the right-hand side is wage earn at every point in time. The second is the continuation value of the loss when the match is destroyed. The third one accounts for the increase in value to the worker when promoted to a higher wage and the fourth is the continuation value that comes from a job-to-job transition. For derivations of key equations see Postel-Vinay and Robin (2002).

Vacant Jobs

Following the above discussion vacancies are filled according the search efforts of both sides of the market and the amount of each of them in the economy. Firms create jobs, filled or vacant, until the marginal cost equals the expected revenue of a filled job. Define the continuation value of holding a vacancy $\Pi_0(y)$ as

$$\rho\Pi_0(y) = -c'\left(n(y)\right) + kJ(y),$$

where

$$J(y) = \int_{\underline{x}}^{\overline{x}} S(x', y)u(x')dx' + s\int_{\underline{x}}^{\overline{x}}\int_{\underline{y}}^{y}\left(S(x', y) - S(x', y')\right) h(x', y')dy'dx'.$$

The first term in the right-hand side is the marginal cost of exerting effort $c'\left(n(y)\right) = c_0 n(y)^{c_1}$, to which I impose convexity to ensure an equilibrium exists and the second term is the expected value of filling a vacancy.

Filled Jobs

Firms discount future at the same rate as workers and have a stream flow of profits $xy - w$, when a job is exogenously destroyed, production ceases and a vacancy is immediately opened. As previously highlighted, when vacancies are open, offers accrue to both types of workers, employed and unemployed at a Poisson rates $kv(y)$ and $skv(y)$ respectively. Matches start by firms appropriating the whole surplus. As offers from less productive companies accrue to the worker, the current company has to give up the surplus that the poaching firm grants to the worker. When a worker finds a higher viable alternative the firm has no other option than let her go. Thus, the net continuation surplus $\Pi_S(w,x,y) = \Pi_1(w,x,y) - \Pi_0(y)$ of a filled job is

$$\left(\rho + \delta + sk\overline{V}\left(q(w,x,y)\right)\right)\Pi_S\left(w,x,y\right) = xy - w \\ + sk\int_q^y S(x,y) - S(x,y')v(y')dy'.$$

The first two terms indicate the flow stream of profits made by the firm for a particular match. The second term is the continuation value of the surplus that it make out of the match minus the possible promotions that has to grant the worker when coming across wage offers.

Surplus

From previous sections it is clear that all the continuation values are defined in terms of the match surplus, whilst not being defined until now. Define the surplus in the usual way, i.e. $S(x,y) = \Pi_1(w,x,y) - \Pi_0(y) + W_1(w,x,y) - W_0(x)$. As a result summing over all these expressions in the right-hand side it can easily be proven that the equation for the surplus is

$$(\rho + \delta)\, S(x,y) = yx - bx$$

Which is quite simple expression since we have imposed $\Pi_0(y) = 0,\ \forall y$.

2.2.4 Equilibrium

The exogenous elements of the model are the distribution of workers $l(x)$, the support of this distribution,$[\underline{x}, \overline{x}]$, the support of this distribution,$[\underline{y}, \overline{y}]$, the discount factor ρ, the job destruction δ, the search intensity of employed workers s, the value of leisure b and the production technology $f(x,y) = xy$. Given these parameters, the equilibrium can be characterised by defining the distributions of employees, unemployed, vacancies and wages can be worked out, along with the free entry condition.

Balance Equations

In equilibrium the distribution of the unemployment rate of workers of type x, $u(x)$, and the number of vacancies of type y, $v(y)$, is determined according to the balance conditions

$$\int h(x,y)dy + u(x) = l(x)$$
$$\int h(x,y)dx + v(y) = n(y).$$

The first condition basically states that the number of employed x-type workers plus the number of unemployed of type x has to be equal to the number of people of ability x. Seemingly, the second condition says that the number of workers in firms with productivity y plus the number of vacancies of type y has to be equal to the number of firms with this productivity.

Flow Equations

The joint distribution wages and matches $G(w|x,y) \cdot h(x,y)$ follows a steady state flow equation where inflows balance the outflows. Matches of x-workers in y-firms earning w or less might arise for two reasons, either workers with ability x are hired directly from unemployment by companies with productivity y or they are poached from less productive firms than q. On the other side, matches of (x,y) pairs might be destroyed by exogenous separations that accrue at a rate δ or because outside firms, with higher productivity than q, poach the worker or make a better offer. Netting this two forces the flow equation stays as

$$\left(\delta + sk\int_q^{\overline{y}} v(y')dy'\right) G(w|x,y) \cdot h(x,y) = \left(u(x) + s\int_{\underline{y}}^{q} h(x,y')dy'\right) kv(y).$$

The same can be worked out for the number of unemployed workers and vacancies of any type, i.e.

$$kVu(x) = \delta h(x)$$
$$\left(\delta + sk\int_y^{\overline{y}} v(y')dy'\right) h_y(y) = \left(U + s\int_{\underline{y}}^{y} h_y(y')dy'\right) kv(y).$$

Where $h_x(x) = \int h(x,y)dy$ and $h_y(y) = \int h(x,y)dx$. Once the flow equations and the balance conditions are defined, $h(x,y)$, $u(x)$ and $v(y)$ can be derived as:

$$v(y) = \frac{\delta + sk\overline{V}(y)}{(\delta + sk\overline{V}(y)) + (kU + skH_y(y))} \cdot n(y)$$
$$u(x) = \frac{\delta}{\delta + kV} l(x)$$
$$h(x,y) = \frac{1}{H} h_x(x) h_y(y)$$
$$G(w|y) = \frac{h_y(q)}{v(q)} \cdot \frac{v(y)}{h_y(y)}$$

and $h_y(y)$ depends solely on $n(y)$, which at this point is exogenously determined. Detailed proofs of steady state equations are found in appendix D.1. It is worth noting that the main advantage of having introduced firm heterogeneity is that we have a measure of firm size. And this firms size will be in a one-to-one correspondence with its productivity level. This turns out to be essential in the political economy part of the model. Finally, the number of jobs created by firms of type y, $n(y)$, is set by the free entry condition described below.

Free Entry Condition

Firms of type y exert increasing effort in recruiting candidates until the cost of maintaining a vacancy and retaining talent equals the expected value of filling it for every y, $\Pi_0(y) = 0$, at equilibrium:

$$c'\left(n(y)\right) = kJ(y).$$

Following the parameterization of Lise and Robin (2017), we are able to draw an expression for the effort exerted by firms of type y as

$$c_0 n(y)^{c_1} = kJ(y).$$

Equilibrium effort by firm-type is then written

$$n(y) = \left(k\frac{J(y)}{c_0}\right)^{\frac{1}{c_1}}.$$

Summing over all companies in the economy, the aggregate equilibrium number of jobs in the economy is worked out:

$$N = \int \left(k\frac{J(y')}{c_0}\right)^{\frac{1}{c_1}} dy'.$$

2.3 Introducing Minimum Wages

At this point minimum wages are introduced into the analysis. I add on to the works of Cahuc, Postel-Vinay, and Robin (2006), Flinn and Mabli (2009) and Flinn and Mullins (2019). I consider wage and mobility dynamics based on firm heterogeneity as opposed to match quality. When considering firm heterogeneity, firms with higher productivity win the Bertrand game when competing for workers. They become larger because high productive firms hire workers from less productive ones. On top of that, larger firms will have a say in the negotiating table at a sectoral level whereas smaller firms will be left out. Furthermore, I account for a continuous measure of worker heterogeneity as a way to control for workers fix effects. At this point is important to recall that workers do no hold bargaining power, which makes sense in the present analysis as low categories of workers are considered.

Introducing minimum wages have several implications. First of all, minimum wages affect the distribution of wages beyond those directly affected, compressing the distribution for a given pair (x, y). Intuitively, when a worker is allowed to search on the job, she has to compensate the employer for the expected forgone profits when she changes companies, which might never occur. If the worker does not have bargaining power she is willing to accept less than her wage today for wage rises in the future. Upon setting a wage floor, high productive firms in the range $[t(x, y), \overline{y}]$ drag the wage down to the legal minimum, increasing the value of being employed by keeping the rate of wage offers, fixing equilibrium objects, but earning a higher wage. Low productive firms, in the range $\left[\underline{\hat{y}}(x), t(x, y)\right]$, will have to compensate the worker for this fact, raising effectively the wage earned.

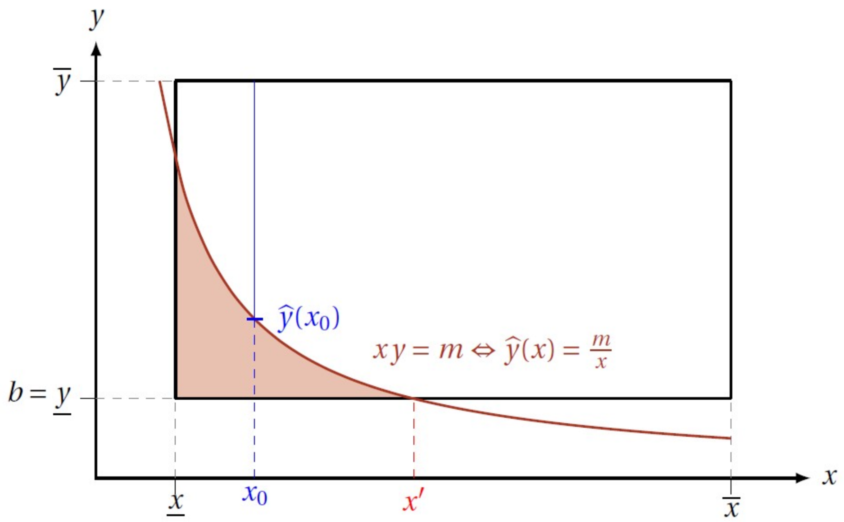


FIGURE 2.1: Matching Space

Another implication is that minimum wages changes the meeting rates at which workers and firms encounter. In figure 2.1 the red shaded area represents the meetings that could have resulted in a match in the absence of a minimum wage but because of it now they are at the disposal of the rest, increasing the chances of meeting for those not directly affected by the minimum wage and potentially decreasing the chances for those affected. In a nutshell, upon introducing minimum wages the number of vacancies and unemployed people increase whereas the tightness, k, decreases monotonically as the minimum wage increases. Coupling both effects, it results in a hump-shaped curve of job offers; at the beginning, job offers accrue at a higher rate for relatively high skill workers and after some threshold these offers start to decrease due to fall in expected revenues of filling a vacancy and firms holding less number of jobs.

As a side effect the value of the match and the outside options of agents change accordingly. The whole surplus is still seized by the employer, workers benefit from the raising wage floor through the higher value of unemployment.

2.3.1 Value functions

In this section the lifetime utility values for the different type of workers are derived when a minimum wage is put in place. For the ease of exposition, workers are grouped in two categories, x^L-type which is in the rank of abilities $[\underline{x}, x']$ and x^H-type employees in $(x', \overline{x}]$. So, employees have different continuation values depending on their abilities. Derivations of value functions with minimum wages can be found in appendix B.2 together with the corresponding wage offers in appendix C.2.

Unemployed

As it is clear from figure 2.1 workers are only hired when contacting a firm with productivity $y' \geq \underline{\widehat{y}}(x) = min\left\{\frac{m}{x}, b\right\}$. Firms with lower productivity than $\underline{\widehat{y}}(x)$ are out of the scope of an x-type unemployed worker and never contacted. The flow value of the unemployed worker $W_0(x;m)$ is increased because firms with high enough productivity cannot trade off less wages today for pay rises tomorrow. I assume that individuals are ex-ante heterogeneous in their valuations before they enter the labour market. Subsequently, they participate of the labour force whenever the value of staying out is strictly lower than the unemployment value $W_0(x;m)$, which under the minimum wage is equivalent to working in $\underline{\hat{y}}(x)$ at the wage $\phi_0\left(x, \underline{\hat{y}}(x)\right)$. Furthermore, notice that the firm with the lowest viable productivity cannot make surplus out of the match, otherwise a firm with marginally less productivity could enter the market. From these considerations the next lemma says,

LEMMA 9. *The value as unemployed is equal to the value of first employment at $\underline{\hat{y}}(x)$. Seemingly, the value of first employment at $\underline{\hat{y}}(x)$ is equal to value product of a match $P(x, \underline{\hat{y}}(x))$. Therefore,*

$$P\left(x, \underline{\hat{y}}(x); m\right) = W_1\left(\phi_0\left(x, \underline{\hat{y}}(x)\right), x, \underline{\hat{y}}(x); m\right) = W_0(x;m)$$

Proof. See appendix E.1.

With these considerations at hand we are ready to calculate what the minimum viable productivity of a firm to hire a worker would be, and hence the lower support of the firm distribution $\underline{\hat{y}}(x)$ for a given ability x. For some ability levels in the range $[\underline{x}, x']$, matches that were profitable without minimum wages they are not anymore and the entry wage is the minimum wage. For those in $x \in (x', \overline{x}]$, the minimum wage only changes the values of being unemployed and employed but not their mobility decisions, therefore the minimum viable productivity of a firm to hire a worker remains unchanged. From these considerations the following lemma states:

LEMMA 10. *Fixing equilibrium objects, the minimum viable productivity of a firm* $\underline{\hat{y}}(x)$ *to hire a worker under the presence of a wage floor is:*

$$\underline{\hat{y}}(x) = \begin{cases} \frac{m}{x} & \text{if } x < x' \\ y_{inf} = b & \text{if } x \geq x' \end{cases}$$

Proof. See appendix E.2.

Employed

As is common in models of OTJ-search with competition a la Bertrand, high productive firms can drag down the wage of the worker in exchange for a more dynamic tenure track, i.e. future offers that will end in wage increases. Upon introducing minimum wages, not every pair (y, y') is able to play a "wage war", more specifically when the poaching firm has very high productivity relative to the incumbent, the former will not be able to lower the wage in its full extend to the worker, because a biding minimum wage is in place. Nonetheless, it does not affect mobility decisions as they are still efficient. Workers see the value of employment risen whenever $t(x, y) < \overline{y}$, even though they might not earn the minimum wage. Intuitively, the worker has the opportunity to work at high productive firms earning no less than the minimum, effectively rising the value of their jobs. The continuation surplus value of an employed worker net of laid-offs would be

$$\begin{aligned}(\rho + \delta)W_S(w, x, y; m) = \\ &w - \rho W_0(x; m) + sk \int_q^y \left[S(x, y'; m) - W_S(w, x, y; m)\right]^+ v(y')dy' + \\ &sk \int_y^{\overline{y}} \left[max\left[S(x, y; m), W_S(m, x, y'; m)\right] - W_S(w, x, y; m)\right]^+ v(y')dy'.\end{aligned}$$

Where $[a]^+ = max[a, 0]$. The first object in the right-hand side is the flow income. The second one is the continuation value of being unemployed. The third term is more interesting, it represents the expected increase in surplus thanks to the fact that the worker can bring two firms into competition staying with the incumbent. The forth term, is the expected increase in surplus derived from switching to more productive firms. $max\left[S(x, y), W_S(m, x, y')\right]$ expresses the possibility that the worker encounters a firm with such productivity that will be able to offer no less than the minimum wage, effectively extracting more surplus for her out of the match. Theoretically, we could find a firm with enough productivity to reduce the wage rate up to the minimum. In practical terms, the firm distribution has its productivity cap at the firm with highest productivity. In turn, it might be the case that $t(x, y) \geq \overline{y}$ and the minimum wage has no direct impact over the worker, although she experiences general equilibrium effects inside the labour market.

As it is obvious, the notation has slightly changed. The reason why is because there is no analytical expression for the threshold $t(x, y)$. This threshold is worked out by iterating the value function to achieve a fix point and finding the cut point where $S(x,y;m) = W_S(m,x,t(x,y);m)$, at which point $t(x,y)$ is implicitly defined.

Employed earning the minimum wage

Because we have introduced the minimum wage we have to consider what is the value for an employed worker earning the minimum wage either because she is been hired directly from unemployment or because she has received and offer for a high productive firm. At the minimum wages the value function is

$$(\rho+\delta)W_S(m,x,y;m) = \\ m - \rho W_0(x;m) + sk\int_{\underline{\hat{y}}(x)}^{y}\left[S(x,y';m) - W_S(m,x,y;m)\right]^{+} v(y')dy' + \\ sk\int_{y}^{\overline{y}}\left[max\left[S(x,y;m), W_S(m,x,y';m)\right] - W_S(m,x,y;m)\right]^{+} v(y')dy'.$$

The only thing that is likely to change is the lower limit of the integral in the third term. I seems that the new limit restricts the matching space of the worker to meet another firm. However, this is not the case, once employed and earning the minimum wage, the worker experience the same restriction as if the minimum were not in place.

Surplus

The new surplus takes into account the increased in value as an unemployed worker, effectively reducing the surplus, together with the increase in the value for an employed worker, leaving the expression

$$(\rho+\delta)\,S(x,y;m) = yx - \rho W_0(x;m) + sk\int\left[W_S(m,x,y';m) - S(x,y;m)\right]^{+} v(y')dy'$$

The change of surplus with respect to the case without minimum wages depends on the interplay of the mentioned objects. Still, the whole surplus is appropriated by the employer and the legal minimum affects the employee positively in two ways. First, it increases the value as unemployed conditioned on participating in the labour market, although participation is not a case of study in the present work, now the worker is indifferent between being unemployed or working at a firm with the least viable productivity higher than before, leaving her better off. The other mechanism at her disposal is again the competition a la Bertrand between firms. Nonetheless, in this case the worker receives a wage offer potentially higher than the one that she would have been given without the presence of a minimum wage, even if she were not earning the statuary minimum. The intuition is that the worker has the opportunity to work at high productive firms earning no less that the minimum, if a poacher did not compensate the employee for this fact, the worker would find it profitable to wait for the next offer come. This is not optimal for the poacher who loses the value of the filled job. Consequently, the poacher offers the employee a wage that leaves her indifferent between working with them at relatively higher wage rate or waiting another period of time.

COROLLARY 11. *The surplus created at the firm with minimum viable productivity is* 0

Proof. Trivial from LEMMA 9.

2.3.2 Equilibrium Under minimum wages

In this section I concentrate in the labour market equilibrium effects of establishing a minimum wage. On the one hand, the set of possible matches is reduced, as $\{x, y\}$ pairs that fall short of m do not form a match anymore. On the other hand, there are more vacancies at the disposal of the rest of workers and more unemployed at the disposal of high productive firms, having ambiguous effects over different firm productivities.

Low productive firms are in general worse off as they find harder to find workers of relatively low ability. However as we move up through the productivity distribution, firms can match with workers with lower abilities. At the same time these firms will have a new stock of unemployed at their disposal, in particular, from those firms with lower productivity that are unable to hire low ability workers. These points are made clear in the following sections.

Balance Equations

These balance conditions have to be rearranged to account for the fact that some meetings do not come true anymore, instead these pairs of workers and vacancies are at the disposal of the rest. More precisely they are

$$l(x) = \underbrace{u(x) + \int_{\underline{y}}^{\hat{y}(x)} h(x,y')dy'}_{\tilde{u}(x)} + \underbrace{\int_{\hat{y}(x)}^{\overline{y}} h(x,y')dy'}_{\tilde{h}_x(x)}$$

$$n(y) = \underbrace{v(y) + \int_{\underline{x}}^{\hat{x}(y)} h(x',y)dx'}_{\tilde{v}(y)} + \underbrace{\int_{\hat{x}(y)}^{\overline{x}} h(x',y)dx'}_{\tilde{h}_y(y)}.$$

Where $\int_{\underline{y}}^{\hat{y}(x)} h(x,y')dy'$ is the new stock of unemployed and $\int_{\hat{y}(x)}^{\overline{y}} h(x,y')dy'$ are are workers that remain employed with ability x that are able to keep their jobs. Exactly the same argument applies for the second and third terms in the firm balance equation. In the first equation, the number of unemployed and employed depend just on x, now affecting the limits of the integral. In the second, the number of vacancies and employed depend just on y.

Flow equations

Remember that no functional forms where assumed on $h(x,y)$, $u(x)$ and $v(y)$; and the new $\tilde{h}(x,y)$, $\tilde{u}(x)$ and $\tilde{v}(y)$ still depend only on their respective variables. Then, the number of vacancies and unemployed people are substituted by their minimum wage counterparts, i.e. $\tilde{v}(y)$ and $\tilde{u}(y)$, which can be readily worked out from the stocks. whereas the flow equation $\tilde{G}_t(w|x,y) \cdot \tilde{h}(x,y)$ is rewritten following exactly the same derivations as the case without minimum wages, showing the same functional form

$$\tilde{h}(x,y) = \begin{cases} \frac{1}{\tilde{H}}\tilde{h}_x(x)\tilde{h}_y(y) & \text{if} \quad xy \geq m \\ 0 & \text{if} \quad xy < m \end{cases}$$

$$\tilde{G}(w|x,y) = \begin{cases} \frac{\tilde{h}_y(q)}{\tilde{v}(q)} \frac{\tilde{v}(y)}{\tilde{h}_y(y)} \cdot \frac{\tilde{H}}{\tilde{h}_x(x)} & \text{if} \quad xy \geq m \\ 0 & \text{if} \quad xy < m \end{cases}$$

Derivations of value functions with minimum wages can be found in appendix D.2. Again, these objects are pinned down by the firm recruiting effort which is determined by the FEC.

Free-Entry Condition

The free-entry condition in which $\Pi_0 = 0$ still holds, as do all the derivations to arrived to the expression for $n(y)$, the only object that changes in this case is the expected value of filling a vacancy which is

$$\begin{aligned} kJ(y;m) = k &\int_{\hat{\underline{x}}(y)}^{\overline{x}} S(x',y;m)\tilde{u}(x')dx' \\ &+ sk \int_{\underline{y}}^{y} \int_{\hat{\underline{x}}(y)}^{\overline{x}} \left(S(x',y;m) - S(x',y';m)\right) \tilde{h}(x',y')dy'dx'. \end{aligned}$$

As it seems clear from the above equation, and figure 2.1, the firm with lowest productivity is undoubtedly worse off because it has less workers to fish from. As we consider higher productivities, firms still lose from those they cannot make profits any more, however they have at their disposal the unemployed not poached by less productive firms, leaving them gradually better off.

2.4 Political Economy

Once the basic framework of the labour market with minimum wages has already been deployed, it is time to consider the bargaining protocol between working unions and employers associations. One of the chief contributions of this work is to endogenise the decision to set minimum wages by reckoning the role of unions and employers. As it is common in collective bargaining systems where the bulk of negotiations are carried out at a sectoral level, what is agreed between unions and employers is usually extended to other participants in the labour market. I focus on the extreme case where collective agreements are applied to the whole labour market, regardless of workers and firms being affiliated to their representative associations or having participated in the political process.

There is much to say about union and employer preferences, what triggers the decision to vote and who can actually vote (there is usually no universal suffrage), representation at the negotiating table, who is affected and accountability. However, I abstract from most of these concerns to keep the model simple and tractable. Nonetheless, two important factors within the political economy sphere are considered. First, who is allowed or able to vote? Either because legal clauses or collective action constraints, participation in the negotiating table is subject to firms reaching a certain size, which in turn means that only the voice of workers in these firms are heard. In terms of my model this requires the following condition

$$h_y(y) \geq \overline{h} \Leftrightarrow y \geq \tilde{y} = h_y\left(\overline{h}\right)^{-1}.$$

Another concern is about union and employers preferences. Traditionally, unions preferences have been model to take into account the fact that they may care in one way or another about wages, unemployment, or income distribution. On the other side of the market, firms have been assumed to maximise profits. At this point is when the complexity of the model starts to pay off. In the present work I deviate from the assumption that unions represent their affiliates and instead they consider the utility of those who actually vote. All the concerns about wages, unemployment and income distribution are directly or indirectly considered through the values that employees assign to them. The functional form reckoned for the union is that of an utilitarian objective function like

$$T(m) = \int_{\tilde{y}}^{\overline{y}} \int_{\underline{x}}^{\overline{x}} \int_{w_{min}}^{\overline{w}} \left(W_1\left(w, x, y; m\right) - W_0\left(x; m\right)\right) G(w|x, y) h(x, y) dw dx dy.$$

And same is applicable in the firm side

$$E(m) = \int_{\tilde{y}}^{\overline{y}} \int_{\underline{x}}^{\overline{x}} \int_{w_{min}}^{\overline{w}} \left(\Pi_1\left(w, x, y; m\right) - \Pi_0\left(x; m\right)\right) G(w|x, y) h(x, y) dw dx dy.$$

Once we know the preferences of unions and employers; and who can vote, we are ready to introduce them into the analysis. In this case, unions and employers do not bargain for wages, employment levels and income distribution directly but they set the level of minimum wages that maximises the Nash-bargaining solution of their respective utilities, or in other words

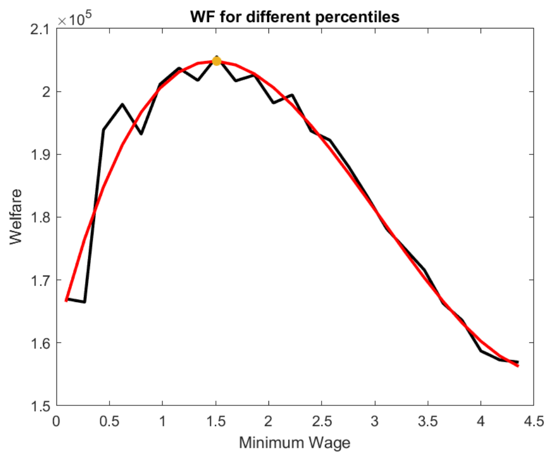
$$m^* = \arg\max_{m} \quad E^{1-\alpha}(m) \cdot T^{\alpha}(m)$$

On the offered side of the market, unions face the typical trade off, higher wages despite higher unemployment for their represented, assuming there is no general equilibrium effects; in the firm side, principals are worse off if just because they have to pay higher wages, in addition low productive firms are not able to hire low productive workers. On the other hand, both coalitions face an additional channel due to the congestion externalities that they exert on each other. Less employment means vacancies are easier to fill, especially for those firms that do not have to lay off workers; on the other side of the market, high skill workers encounter wage offers more frequently. The net effect is ambiguous and structural estimation is carried out to discern what effect is stronger.

2.5 Theoretical Results

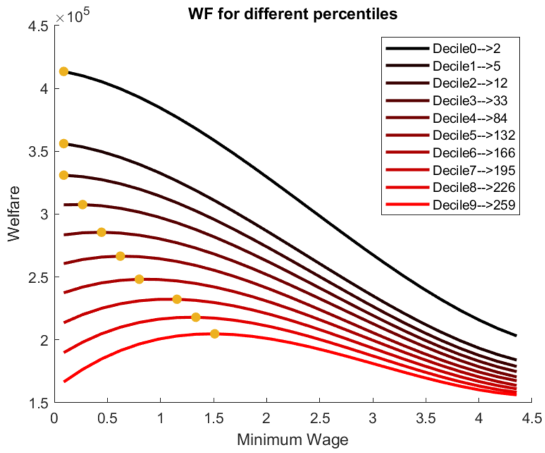
The estimation of the model is a work in progress at the point of writing these lines. therefore I show the results for particular set of parameters. The exercise consist on considering the value of jobs in firms that meet a certain threshold, specifically I consider the matches in the largest firms that add up to the 10% of the working force. Unions and employer's associations bargain on behalf these workers and firms. As pointed out previously, they choose the level of minimum wages that maximise their weighted utilities, or in other words a their social welfare function which is $E^{1-\alpha} \cdot T^{\alpha}$. Then, when pushing up the wage floor this social welfare function displays a hump-shape form as shown in figure 2.2, where the red line is a fit of a third degree polynomial. It seems clear from the picture that the level of minimum wages that these negotiator would choose would be around 1.5

FIGURE 2.2: Top 10% SWF



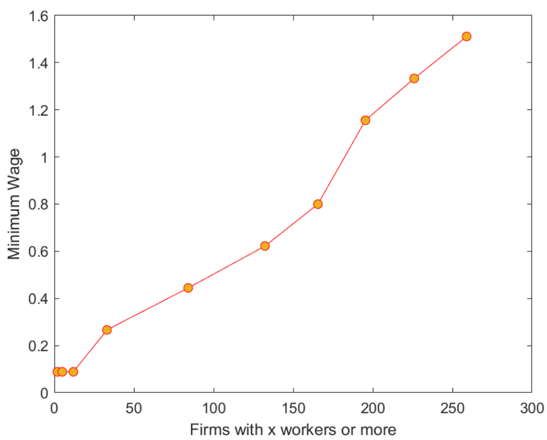
However, other thresholds might be consider as well. It is fair to ask, what would the legal minimum had been if we had increased the threshold to allow the top 20% to participate in negotiations? And what about 30%, 40%, etc. till we allow the who workforce to participate? The next graph in figure 2.3 shows the social welfare functions for the different thresholds and their respective firm size cuts. Higher thresholds means lower skill workers and low productivity firms taking part in negotiations. Since they are the most negatively affected by the increase, they are the most interested in blocking any upward update of the minimum.

FIGURE 2.3: Different Percentiles



As it is clear from the picture, higher thresholds mean less minimum wages being lower. The next graph makes the relation between the participation threshold and minimum wage somewhat more transparent

FIGURE 2.4: Minimum Wage as Function of Threshold



All in all, these results show that rising the legal threshold for participation in the negotiating table or encouraging participation, has positive results in the welfare of participants in the labour market as a whole, for this look at the black curve in figure 2.3 representing the social welfare function when every one is allowed to vote, this curve attains its maximum when there is no minimum wage in place.

2.6 Conclusions

In this chapter I have developed a search and matching model with minimum wages and collective agreements in a one sector economy and two-sided heterogeneity.

The model offers a rationale that links minimum wages, vacancies and collective agents decisions together. Because minimum wages do not affect all firms and workers equally, firms with high productivity are not much affected by the minimum wage but enjoy a new stock of unemployed at their disposal becoming larger and making more profits. Furthermore, small firms struggle with minimum wages and open less vacancies, which means less competition to poach workers. In turn, already employed workers do not come across other firms with which they can demand their current employers a wages rise, so large firms do not grant wage increases as often and gain in this margin too.

In this economy collective agents know everything about the labour market with frictions. Since only trade unions and employers associations in large firms are able to set the minimum wage level, they have the power to set the level of minimum wages that fit them best. Allowing everyone to have a say in the decision making process of minimum wages, i.e. small and large firms, would turn out in a lower level, increasing in this manner social welfare.

Chapter 3

A Structural Model of Minimum wages in Spain

3.1 Introduction

The present paper explores the empirical implications of the model devised on chapter 2. The model can be tested on any economy that is subject to minimum wages. Furthermore, the model is specially suitable for the class of labour markets in which a reduced set of agents in the market are able to set the rules, in this case the minimum wage level.

Here, I argue that when a small set of agents have the power to set the level of minimum wages to their advantage, the welfare of the whole economy shrinks. This is so, because the minimum wage level is used by unions and employers associations in large firms to gain labour market power and extract some rents from it. On the one side, union membership with low wages in large firms profits from wage increase, whilst they are not much affected by the negative effects of unemployment, and so they will be interested in moderate increases of the minimum wage. On the other side, employers' associations in large firms face less competition from smaller firms, as a result workers can not find out outside offers as often and experience less wage increases, hence employers gain a wider margin in every worker. In addition, employers have a new stock of unemployed workers at their disposal, they find workers more easily and fill vacancies at higher rate, becoming larger and being more profitable.

So, the present chapter is an application to the Spanish labour market of the aforementioned model. To taste the model, I use two databases that are fit for the two parts of the model: the behaviour of individual agents and the decisions taken by collective ones. To test the implication of the former I use complete employment records from the Spanish Social Security with information about spells, monthly wages and firm size. This allows to compute the labour market equilibrium empirically fitting the most salient characteristics of this market, namely the minimum wage level, wages, finding rates and firm size. For the latter, I use the REGCON database with exhaustive information of collective agreements at both sectoral and firm level, in particular it contains information about the number of representatives for each firm which it is key to assign representation at higher levels of negotiation.

The are three main results. First of all, the most interested in rising the minimum wage level are the employers associations since they gain in two margins, they become larger and more profitable per worker. Secondly, the estimation shows that unions in large firms are willing to rise the level of minimum wages moderately. Reasonable increases of the minimum wage implies higher wages for those workers at the bottom of the wage distribution, which are very scarce. However, all workers in high productive companies find it harder to find alternative jobs, either when unemployed or employed, and their promotions are rarer. Finally, the wage distribution squashes at both ends with the increase in the minimum wage. The minimum wage pushes the bottom of the wage distribution to the right mechanically, together with spill over effects for those workers close to the minimum, but what new here is the fact that the top of the wage distribution is pulled downwards because workers cannot find outside offers as often to demand wage rises.

This chapter is related several parts of the literature that has estimated minimum wages in the context of collective agreements. This strand has analysed collective agreements and how their clauses affect labour market outcomes using quasi-experimental data and a reduce form approach. The model proposed here is deemed to answer questions based on theoretical foundations that are confronted against the data. Some part of this literature began in the late nineties to test predictions laid out by Calmfors and Driffill (1988b), who emphasised the inefficiencies brought about by intermediate levels of negotiations due to market power coupled with lack of internalisation of outcomes. Notable research in this area was done by Hartog, Leuven, and Teulings (2002) who tests this prediction using data on The Netherlands. Yet, they do not inspect how particular provisions affect labour market outcomes. The seminal paper of Cardoso and Portugal (2005) shed light on this by taking minimum wages into the analysis, they found out that adjustments are absorbed reducing the wage cushion and not laying off workers. Another major advancement is Card and Cardoso (2021), in this work they thoroughly analyse collective agreements in Portugal looking at their outcomes such as unemployment, wages or spillovers.

In addition, this paper is related to the strand of the literature that analyses trade unions and its effects on inequality with a reduce form approach (Dinardo, Fortin, and Lemieux (1996) and Faber et al. (2021)). Furthermore, the results of this paper goes along the same lines as Harasztosi and Lindner (2019) and Autor, Manning, and Smith (2016), where they show the spillover effects of minimum wages beyond workers earning the minimum or less. In the model tested in this paper, from a theoretical point of view this spillover effect should affect the whole distribution of wages; in reality this effect is only meaningful for those workers right above the minimum as these authors suggest. A special mention deserves Dustmann et al. (2022) whose work and findings can be rationalised by the present paper; they find that upon introducing minimum wages workers reallocate from small/low productive/ low-pay firms to large/high productive/high-pay firms, which is exactly what the model used here suggest and what the empirical implementation finds.

Last but not least, this paper is related to the literature of structural estimation. Pioneers in this area were Eckstein and Wolpin (1990) and Van Den Berg (1990) in which they estimate models of on the job search focusing on the demand side of the market. Then Ridder and Van Den Berg (1998), Bontemps, Robin, and Van Den Berg (1999) and Christian Bontemps, Jean-Marc Robin and Berg (2000) were the first in estimating the equiliibrium model of Burdett and Mortensen (1998). More recently the work of Postel-Vinay and Robin (2002) adds two-sided heterogeneity and sequential auctions to the estimation; for this empirical strategy to viable they need data with information about the two sides of the market. Ultimately, the approach chosen here is more related to the recent literature (Flinn and Mabli (2009), Lise, Meghir, and Robin (2016), Lise and Robin (2017) and Flinn and Mullins (2019)) that implements either the simulated method of moments or classical minimum distance to fit the data, as there are no close form solutions for the models they propose and neither do mine.

3.2 The Institutional Setting

Most of the theoretical literature about trade unions revolves around their affiliates. However, labour market in Spain is framed within the class of collective negotiating systems where bargaining is a public good, typical of Southern Europe. These systems are characterised by a low density but high coverage of the labour force, regardless the employee is affiliated or not.

Nonetheless the regulatory framework in Spain has some peculiarities worth mentioning. First of all, the important agent in the industrial relations is not the trade union but the work council or committee (*comité*) instead, which is the collegiate organism within companies of more than 50 workers in charge of representing the staff. This council is composed of 5 to 75 members, depending on the size of the firm, and is directly elected among the workforce. Members of the committee, also delegates, can be either union affiliates or independent workers. This 'elections feature' is what justifies the 'public good' facet of the settlement. Then, the number of elected committee members in each firm are recorded by the Ministry of Labour to assign representation at higher levels of negotiations.

Another relevant trait is the way sector-wide negotiations are carried out. The key institution here is the 'Bargaining Commission' which is the body in charge of reaching an understanding. This body is composed of employers associations and trade unions, not independent workers in this case. As noted before, the number of unionised delegates at firm-level elections are recorded and taken into account to assign the representation at higher levels, therefore each union is represented in accordance to their popularity. For example, if there is Union 1 and Union 2 in Firm

1 obtaining 3 and 1 delegates respectively and there is Union 1 and Union 2 in Firm 2 obtaining 1 delegate each, then the bargaining commission will be composed of two thirds of Union 1 and one third of Union 2.

The result of these negotiations is the 'Collective Agreement', whether carried out at firm-level by the committee or at sector-wide by the bargaining commission. This contract rules over any possible matter related to labour, being the most prominent: wage increases, minimum wages, hours and employment. The agreement is published in the Official Bulletin of the State (BOE), has rank of law and Judges might use it to solve potential disputes between workers and employers. Then, this collective contract serves as the minimum standard individual contracts must have. What is more, firm-level agreements override to sector-wide ones, within the latter a narrower scope cannot confront those with a wider one. For example, a province-level agreement for metalurgy cannot set lower standards than a national-level one, I will discuss in more detail in the following section.

3.3 Data

3.3.1 MCVL

To test the model described in previous sections, I make use the *Muestra Continua de Vidas Laborales* (MCVL), a database with complete working histories of employees. The MCVL is a 4% sample of population having a relation with the *Tesorería General de la Seguridad Social* (TGSS) in the year of reference (2015-2017). The TGSS is the institution in charge of the social security finances and releases this database on a yearly basis.

This database has exhaustive information of complete working histories of workers. Relevant for this work are variables relating personal characteristics of the worker such as age, nationality, genre, etc.; Information about firms like id, type of employer, sector and size; large set of features regarding the job relation e.g. type of contract, start and end dates of the relationship, skill, etc. and the database provides information about the monthly base rates of contributions.

Due to information and time constraints explained below the sub-sample of workers selected for the present study are those in the metal sector in the region of Madrid between years 2015 and 2017. As pointed out previously all of them are to be covered by a collective agreement, in particular the agreement *'CONVENIO COLECTIVO DE LA INDUSTRIA, SERVICIOS E INSTALACIONES DEL METAL DE LA COMUNIDAD DE MADRID'* that appeared in the Spanish Official Bulletin on 2 January 2016.

Workers in this subsample are grouped in four categories according to wage floors: unskilled manual workers, administrative stuff, skilled workers, technical supervisors and engineers and graduates. Table 3.1 shows descriptive statistics of the mean wage, wage floor, wage cushion and the percentage of people for which the wage floor binds.

TABLE 3.1: Number of observations and means for selected variables

Categories	Number of observations	Wage, (euros)	Wage floor (euros)	Wage cushion (euros)	Workers earning the wage floor (%)
Unskilled manual workers	1500	1915.1	1273.6	644.1	4.00
Administrative stuff	160	1612.4	1372.7	235.6	16.25
Skilled workers	235	2407.3	1471.6	950.3	4.68
Technical supervisors	137	2776.9	1646.4	1141.0	8.76
Engineers and graduates	67	2816.9	1813.4	1009.7	11.94
Managers	174	3395.2	2161.3	1265.8	0.57

As expected, higher categories earn higher wages and higher wage floors. What is interesting to notice is the fact that higher categories have also higher wage cushions, meaning that the excess of salary above the wage floor is also lager. As one can see, the wage cushion is not monotonically increasing with the category hold, this is because the skill level of the employed in this classification is taken into account two dimensions, responsibility and level of education. In this respect, if we consider two skilled workers, one with a level of education vocational training that is supervisor and the other a graduate but that has just been hired in the company, they will end in two different categories, namely technical supervisors and engineers respectively, the latter will be in a higher category, with a higher wage floor, just because she is a graduate.

It is worth noticing that managers have been left out of the analysis, the reason why they have not been taken into the analysis is twofold. There is no clear sign in the data that this category are affected by its corresponding wage floor. One rationale is that these high level categories have some bargaining power to set their salaries well above and beyond the wage floor. Which leads to the second point, all throughout my analysis I have imposed that wages are not bargained; then had I taken managers, it would have distorted the results, see Cahuc, Postel-Vinay, and Robin (2006) for a seminal paper on this matter.

Apart from that, workers with a salary more than 5000 euros have been removed from the sample for two reasons. First of all, the maximum base of contribution that is taken into account is around 4000 euros for each job relation. However, the social security does not cap the base if the worker happens to have more than one employer or if there has been a job-to-job transition with a substantial wage increase. A second ground is more practical, wages over 5000 euros distorts the wage distribution unreliably. Last, it is unlikely that a worker of a low category earns such a salary. Although considering wages with measurement error is a possibility, it is not within the scope of the current study.

Last but not least workers with a tenure of more than 12 months have been taken into account since lower thresholds apply for those with a lesser duration.

3.3.2 Collective Agreements

As I only focus in one sector and province, the collective agreement that applies is clearly identified. Nonetheless, the skill categories that the social security assigns to workers does not have to coincide with the ones negotiated in collective agreements. In this respect, I have followed the work of Adamopoulou and Villanueva (2020) where they match skill categories for the MCVL. So one challenge is to assign the levels of minimum wages to the correct skill which in the case of the present work has been done by visual exploration, i.e. identifying where is the mass point in the wage level data for each level in the MCVL and assign it to the corresponding level in the collective contract.

Although Card and Cardoso (2021) have pointed the way to go with a vast linkage of collective agreements to worker level data, this process turns out to be a daunting task in the case of Spain since the TGSS does not deliver such information, even though they dispose of it for their inner purposes. Despite of that, the MCVL has two advantages with the respect to *Quadros de Pesoal* which is the Portuguese counterpart. As is common in collective agreements they are usually negotiated for long periods of time that entered the periods over which the clauses are being negotiated, consequently retroactive measures have to be taken. For example, negotiations of collective agreements end up after the period covered. So, a collective agreement signed in 2016, could well rule in 2015. Thus, if wages are updated at a posterior date, one should take into account what part of the increase corresponds to what period. The advantage of MCVL is that it writes a correction in the previous months that are affected and then we do not need to worry of taking backdating into account.

Another concerned could have been the statuary minimum wage which potentially could overlap with wage floors considered. This is certainly not the case of this work, as the sector considered is well above the national minimum wage at the time considered (2017). Even so, minimum wages are unlikely to interfere with wage floors as in the time considered minimum wages were so low that it is unlikely that they overrule sectoral ones.

3.4 Estimation

3.4.1 Method

Using the data described in the previous section, I estimate the model using the classical minimum distance (CMD) estimation. Excellent references for reviewing this area are Newey and McFadden (1994) and Wooldridge (2010), which are followed in this paper. For this purpose moments are taken from data and stored in a vector $\widehat{m}$ of size K. The k^{th}-element is $\widehat{m_k} = \frac{1}{N}\sum_{i=1}^{N} m_{ki}$, where $\widehat{m_k}$ might be any statistic of interest like mean duration of unemployment. Then, the same set of moments is calculated from the model, which is numerically solved, given a set of parameters θ.

Assuming that $\widehat{m} \xrightarrow{p} m_0$, there is a vector of functions such that $m_{k0} = m_k(\theta_0)$, where $m_k(\theta_0)$ is calculated from the model given a parameterization θ_0 which is meant to be unique. The objective is to make these theoretical moments as close as possible as their data counterparts. Then, the problem lays on retrieving the parameters that make the loss function as close to zero as possible, or in other words

$$\widehat{\theta} = \underset{\theta}{\operatorname{argmin}} \Big\{ \big(\widehat{m} - m(\theta)\big)' W \big(\widehat{m} - m\,(\theta)\,\big) \Big\}.$$

Where W is a positive semi-definite weighting matrix, although in the econometric estimation it is just a diagonal matrix with the inverse of variance of their respective moments, which is the proper method to account for different in orders of magnitude.

Once the programme has been set, it is only to choose the right moments to match in order to retrieve the underlying parameters. Since my model introduces free entry of firms to the canonical Postel-Vinay and Robin (2002), it is difficult to argue what moments identify what parameters since moments are affected by all parameters of the model in some way. The approach taken here is the one of Lise and Robin (2017), where they use an heuristic approach to identify all the parameters at once, making sense of the sensitivity of some moments with respect to the parameters that they mean to identify.

Another possible approach, it is the one taken by Flinn and Mullins (2019), where they first solve for a partial equilibrium, i.e. keeping fix the rates at which searchers encounter, which in the present work would be $kv(y), skv(y), u(x)$ and $skh(x, y)$, and take them from data, then they estimate the offer side parameters like the measurement error of wages and worker heterogeneity using a simulated minimum distance estimator. Last, they impose some values on the elasticity of the number of matches with respect to the number of vacancies and the parameter of the cost function, which they use to back out the rest of demand side parameters. So, even though all parameters are identified not all of them are treated as general equilibrium objects, which is the reason why the work of Lise and Robin (2017).

This work shows a heuristic representation of identification as in the works of Lise and Robin (2017) and Lise, Meghir, and Robin (2016) measuring the sensitivity of some moments as parameters change in the SMM estimation. However, in the case of the present paper parameters are estimated using a classical minimum distance estimation. I use the observed distribution of wages, actually 9 deciles, and the mass point at the minimum wage in order to identify parameters related to worker ability $\underline{x}, \overline{x}, a_x, b_x$ and firm productivity $\underline{y}$ and $\overline{y}$. The higher the upper support of worker ability and firm productivity the larger the upper support of the wage distribution, and upper deciles will be more affected as a result. Lower deciles are more sensitive to the lower support of $\underline{x}$ and $\underline{y}$. Depending on how concentrated are wages along deciles, a_x and b_x, skew wages towards the left or right tail depending on the relative strength of the two parameters. Special mention deserves the difference between $\underline{y}$ and $\overline{y}$,

Moments related to durations serve to identify the relative search intensity s, the more effort employed workers exert with respect to those unemployed, the more alternative firms they encounter that allows them to experience a job-to-job transition more often which results in shorter tenures, so job duration will serve as a moment to identify search intensity. Since s is a relative search effort with respect to those unemployed, unemployment durations are taken to identify the parameter. Last but not least, the employed-to-unemployed ratio is used to identify parameters related to the matching function.

Deciles of the size distribution of firms are intimately related to the firm productivity and the costs of opening vacancies c_0 and c_1, which are linked to the free entry condition. The lesser the costs the more vacancies each company opens, there will be more matches and consequently k will rise, in turn durations will be affected accordingly by this.

3.4.2 Estimation of Parameters

Table of the full set of parameters to be estimated is presented in table 3.2.

TABLE 3.2: Parameters

Abilities support	$\overline{x}$	11.67	Productivity support	$\overline{y}$	245.14
	$\underline{x}$	1.33		$\underline{y}$	285.41
Worker heterogeneity	a_x	15.63	Vacancy costs	c_0	1,448.17
	b_x	19.61		c_1	0.09
Search intensity	s	0.18			

Search effort exerted by employed workers is in line with with works of Lise, Meghir, and Robin (2016), Lise and Robin (2017) or Flinn and Mullins (2019), the estimate of $s = 0.18$ is a little bit smaller than in those works, which is coherent with the fact of higher unemployment seen in the Spanish labour market. Parameters that govern the number of jobs, c_0 and c_1, resemble also those seen in the mentioned literature, it actually lies in between of those estimated by Flinn and Mullins (2019). Every opening is more expensive than the previous one, i.e. $1 + c_1 = 1.09$, being the function convex and guaranteeing the equilibrium exists. Also each new opening is going to increase by a factor of $c_0 = 1,448.17$, which seems high but explains the rationale of low firm competition in Spain. Turning to firm related parameters $\underline{y} = 245.14$ and $\overline{y} = 285.41$, it seems that they are relatively close with respect to other estimations carried out, actually they need not to be far apart since it is known that these models give too much market power to large firms, if the difference between these two parameter were too high, wage distributions would be unreasonably skewed towards the lef-tail, which is at odds to a mere visual inspection of the

FIGURE 3.1: Distribution of workers abilities

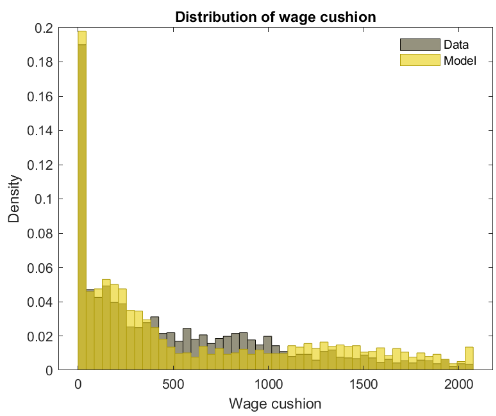
data. Little more can be said about these parameters since they are just a way of ranking firms.

Parameters related to workers do not have a direct economic interpretation, as stated before $\underline{x} = 1.33$ and $\overline{x} = 11.57$ pin down the support of wage distribution together with parameters connected to firm productivity. As seen in figure 3.1 the distribution of abilities in the population is slightly skewed towards the right tail, meaning that there are relatively more workers of low ability.

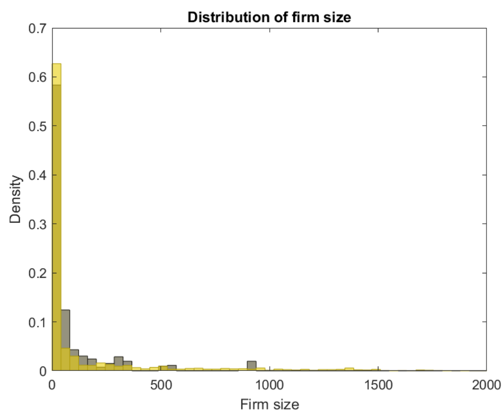
3.4.3 Fit

This section presents empirical and simulated moments in graphs 3.2a and 3.2b as well as table 3.3.

FIGURE 3.2: Fit of data distributions: wage cushion and firms



(A) Distribution of the wage cushion



(B) Distribution of firm size across workers

As it seems clear from figure 3.2a the model fit the distribution of wage cushions closely. Nonetheless, the model tends to over concentrate the distribution over right-end, this is because there are too many large firms in the model. Large firms find it easier to poach workers form low productive ones, because workers are able to bring these same firms into competition, they are granted wage increases more often of what we see in the data, and as a result higher wages. Now, turning to distribution of firms across workers we see that is not perfect, the model underestimates the number of workers in low productive firms, whereas it has a thick long right-tail. High productive firms always win the sequential auction model of offers and counter offers, whenever they come across an unemployed or an employed from a firm with lower productivity they sum one worker to their workforce. As such, in order to generate a wage distribution with a big mass point at the minimum, unreliably large firms are encounter.

Looking at moments associated to durations, we see that the model does a good job fitting the $\frac{H}{U}$-ratio, meaning that there are three times more employed workers than unemployed which results in about 25% of unemployment ratio. As a side effect, unemployment duration will be inevitably high, being the mean duration of unemployment of about 2 years and a half, as unreasonably high as it might, I am focusing at non-employment, and I am not correcting for the fact that people might have leaved the labour force. The largest deviation is from the moments of job duration where my model predicts 7 years of a duration of a job, instead of 5 as we see in the data.

TABLE 3.3: Duration moments

Moments	Data	Model
Unemployment duration	30.1	34.4
Job duration	61.2	84.3
$\frac{H}{U}$-ratio	2.98	2.98

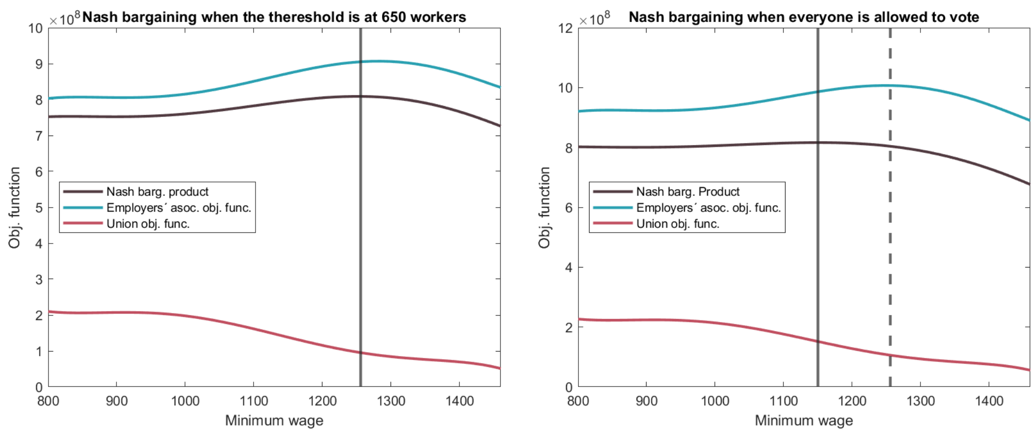
3.5 Welfare and Bargaining

Before looking at how minimum wages affect labour market outcomes, it is important to know how these minimum wages come about. In Spain, only firms with with 50 workers of more are able to set working councils and not all of them actually set one, large firms and their workers might impose the level of minimum wages to their advantage. As pointed out before the cut point to consider a firm large or small is set at 650 workers. Also the level of minimum wages will depend on the bargaining power of both parties. As suggested in the theory section, unions and employers associations negotiate to impose the minimum wage subject to the firms being large enough, the approach taken there is axiomatic and no assumption is done about what the bargaining power α means. Nonetheless, it might be reinterpreted as the relative discounting factors, Ariel Rubinstein (1982).

In order to predict bargaining power from model estimations, I calculate what the results of negotiations on the minimum wage would be for different values of the bargaining power, $\alpha = [0, 1]$. Then I choose the bargaining power of unions and employers' associations in such a way that the observed level of minimum wages would be the result of negotiations. In this case the bargaining power hold by the union would be $\alpha = 0.05$. After this step, I remove the participation threshold and estimate what unions and employers' associations would have chosen in this case.

From figure 3.3a we can see that minimum wages below €1, 000 do not interfere with objective functions of representative agents. Surprisingly, from ranges above €1, 150 the utility of the employers associations starts increasing up to €1, 260, the reason why this behaviour is due to the increment of unemployed stock, who in turn are at the disposal of these firms. Because it is easier to poach an unemployed worker, firms derive more expected value of filling a vacancy. When this level of minimum wages is surpassed, the level of unemployment becomes too high and hiring becomes more and more difficult, reducing effectively the objective function of the employers associations. On the other side of the market, workers are negatively affected by the increase in minimum wages in the whole domain.

FIGURE 3.3: Fit of data distributions: wage cushion and firms



(A) Distribution of the wage cushion (B) Distribution of firm size across workers

If the participation threshold were removed to allow everyone to participate in setting the level of minimum wages, employers' associations and unions would choose a lower level. This is because taking smaller firms into account changes the employers' association objectives to a lower minimum wage. Since this organisation has most of the bargaining power, it will be willing to lower the minimum wage.

3.6 Labour Market Outcomes

Minimum wages affect both sides of the market through different channels. Minimum wages raise the unemployment value of the worker creating incentives to become part of the labour force. Workers that previously earned less that the minimum see their wages rise and simultaneously it is been documented that minimum wages have spill-overs through the wage distribution, so workers that have wages close to the minimum also experience wage increases, Autor, Manning, and Smith (2016). This comes at a cost of reducing employment, however, little evidence has been found in this respect, see Card and Alan B Krueger (2015) book for a review. Nonetheless, Neumark, Schweitzer, and Wascher (2004) have pointed that low skilled workers suffer the most through reduction of hours and employability. On the demand side, firms might adjust through several channels, the most obvious one being employment as commented before. Another possible channel is by hiring higher skill workers, indeed as minimum wage increases, before profitable matches are not anymore and firms have to hire workers with higher ability to cover the same vacancy. Less attention has been drawn to hours effects and the probabilities of part-time or full-time unemployment, for example Katz and Alan B Krueger (1992) find that after a wage increase causes firms to substitute part-time jobs by full-time employment. Out of the labour market firms might be able to pass-through higher costs to prices in their products, which depends on the level of competition in the product market.

In the following sections variables like unemployment, employability, wage dynamics, spill-overs and wage inequality will be considered. Because of how the model has been constructed or because data constraints I will not look at other channels through which minimum wages affect labour market outcomes. Hours will be left out of the analysis since the model is not well suited for this purpose, upon dealing with data wages have been considered in full-time equivalent units (8 hours). Labour market participation is not considered due to the nature of data, as opposed to surveys like the Current Population Survey where people are asked about their searching status, administrative data only records the periods that workers has been on formal employment and because of this limitation, non-employment is regarded instead. Channelling hiking costs via prices is also out the scope of the present work since the data is not well suited for this purpose nor it is the theoretical model.

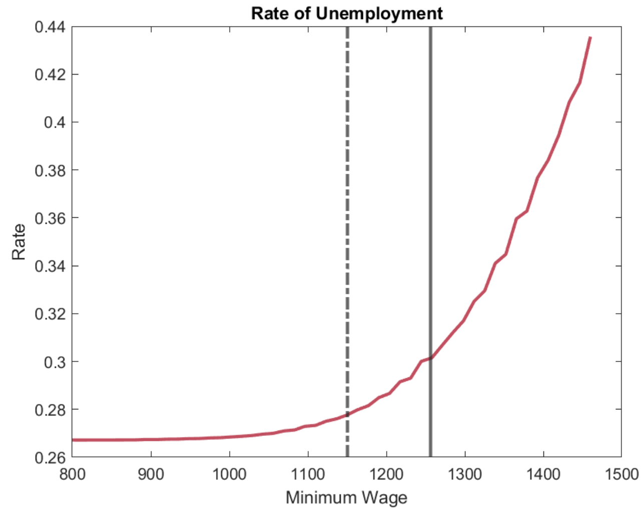
In the following sections I will discus the effect of minimum wages on unemployment, employability, and wage inequality.

3.6.1 Unemployment

Among the different channels that firms use to accommodate an increase in minimum wages, the most controversial one that has been in the limelight since Card and Alan B. Krueger (1994) paper is unemployment. Here, I show a counterfactual analysis for the particular market under study showing that moderate increases of minimum wages do not have an effect on unemployment. However, when minimum wages are hiked too much they start having detrimental effects.

The current level of wage floor in this market, metal industry in Madrid 2015-2017, is negotiated at €1,255.91, with the rest of wage floors normalised to this level. At this level the rate of unemployment is 30.3%, which is the one we actually see in the data. After parameters of the model have been estimated for this level of wage floor, the counterfactual analysis has been carried out taking these parameters as fixed and shifting the wage from the actual level in a range that spans between $m \in [800, 1450]$ euros, as we can see from figure 3.4 moderate increases up to €1,000

FIGURE 3.4: Rate of unemployment as a function of the minimum wage



Note: The solid vertical line represents the current level of wage floor at €1, 255.91 The dash-dotted line indicates the level at €1, 000

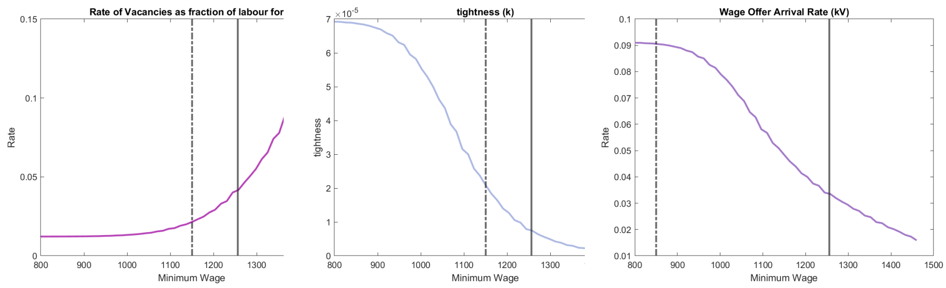
in the minimum wage do not interfere with firm productivity and the level of unemployment rate would be 26.9%, this percentage can be taken as the unemployment rate in the present of frictions. Once this threshold is surpassed, the wage floor adds to these frictions and unemployment increases. In the end the wage floor add 3.49p.p. to unemployment from what it would have been without them at their current level.

3.6.2 Employability

When imposing a wage floor there are two effects that impact in employability. The first one is mechanical, from the supply side, as the wage floor is increased the set of possible firms that are available for pairing is reduced, as shown in the theoretical part, only firms in the range $y \in [\underline{\hat{y}}(x), \overline{y}]$ will be available for workers with ability x, and the same mechanics are present in the other side of the market, where relatively low productive firms are not able to match workers with low capabilities to produce enough to pay for the minimum wage.

The second factor takes into account general equilibrium effects inside the labour market, the discussion goes along the same lines as with the unemployment rate. At the current wage floor of €1, 255.91 the vacancy rate is 5% of the population, had the wage floor been set at €1, 000, the vacancy rate would have been about 1%. At the same time the measure of the tightness falls sharply as the increase in the number of matches does not offset the increasing numbers in the stocks of unemployed and vacancies in the economy. when coupling both effects the outcome is the offer arrival rate to unemployed worker kV, the offer arrival rate is just a vertical stretching of size s. The result could be either increasing or decreasing, but for reasonable estimates of model parameters we see that the offer arrival rate is decreasing in the whole domain of wage floors. The probability of encountering a valid job offer is three times smaller than if there was not a wage floor in place, see figure 3.5c.

FIGURE 3.5: Fit of data distributions: wage cushion and firms



(A) Vacancy rate $\frac{v}{L}$ (B) Tightness: k (C) Offer arrival rate: kV

3.6.3 Wage Inequality

I now turn to the analysis of wage inequality. Dinardo, Fortin, and Lemieux (1996) show how the diminishing of real minimum wage is a major reason for increasing inequality. There, they assume there are no spill overs and no disemployment effects, which is not the case of the present paper and I take both causes, and the direct effect, in turn. As in the cited paper, minimum wages affect directly the distribution of wages by just mechanically cutting off all the wages below it, some workers will go to unemployment and others will experience a wage increase.

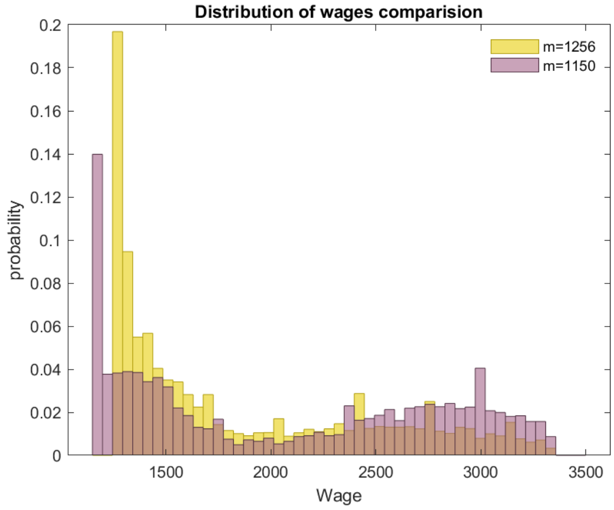
Another channel is the spillover effect. As we have seen in the theoretical part, when minimum wages are increased, workers experience higher values as employed just because they now have the opportunity to work in high productivity firms earning no less than the minimum wage. Higher values of employment result in wages above what they would have had if the minimum wage had not been in place, even if the minimum wage is not biding.

Now, in the previous section we have seen that raising the minimum wage has negative effects on the probability of arrival of wage offers and therefore U-E, E-E transitions and wage increases are reduced accordingly. Looking at the case of an unemployed worker, she receives less offers when employed, meaning that she will lose fewer opportunities in the future and will start with a higher entry wage. So, raising the minimum wage does not only have the direct effect of compressing the wage distribution by rising the lower support or through spillovers but also by increasing entry wages.

Turning to the employed, they encounter less wage offers that end up in a wage increase, also they receive less wage offers from high productive firms, implying in fewer job-to-job transitions. Thus, careers begin with a higher starting wage but at a cost of being stagnated for longer.

In figure 3.6 the effects of previous mechanisms are at play for two levels of the wage floor. As we can see, when the level of wage floor is at the current degree of

FIGURE 3.6: Comparison of Wage distributions under Two Wage Floors



€1, 255.91, the wage distribution has a large mass point in the lower support and wage are concentrated right above this threshold for the reason explained before: Higher lower support, spillovers and higher entry wages. As we move along the distribution to the right tail, wages are being less concentrated because workers do not have as many opportunities. If the minimum wage were lowered to €1, 000 all of this channels would be at work in the reverse order. Lower minimum means the lower support is going to be inferior, also poaching firms will be able to drag wages down further, which results in spillovers being attenuated. Higher rate of wage offer arrivals turns out in reduced entry wages. On the other end of the distribution, these higher rate of wage offers subsequently end in more vibrant careers with more job-to-job transitions and pay rises more often.

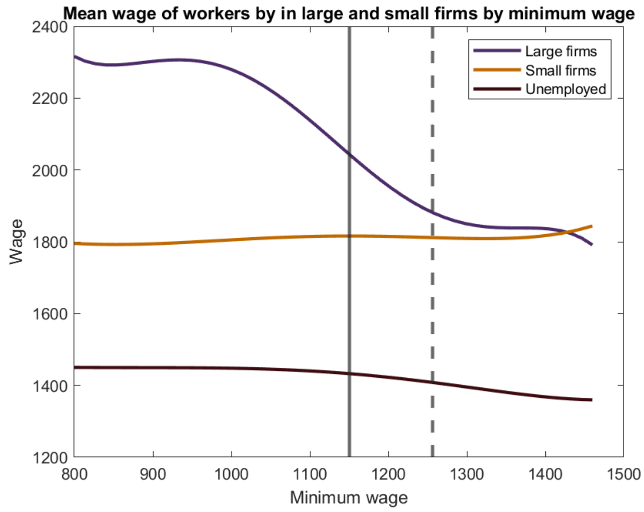
3.6.4 Mean wage

Wage have different evolve differently in large and smaller firms. In large firms the mean wage decreases in most of the domain, this is because wage growth stagnates even thought minimum wages rise and firms pay higher entry wages. Workers in smaller firms see their wages rise as they earn higher entry wages. This takes into account the fact that smaller firms are crowded out of the labour market, posting less vacancies, and holding less filled jobs as a result. Then figure 3.7 depicts the mentioned dynamics.

3.6.5 Mean profit

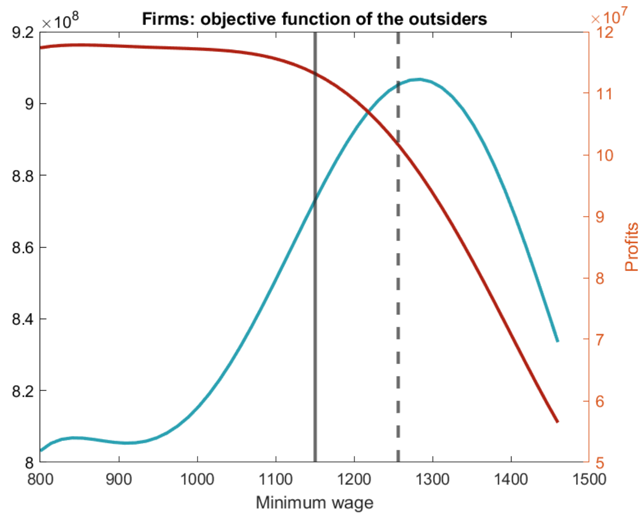
Looking at the evolution of profits on both types of firms we see that profits in large firms are increasing in most of the domain, there are two forces. One is that there are more unemployed workers to poach from. The other is that as large firms open more vacancies, smaller firms post fewer vacancies, the net effect is that there is less

FIGURE 3.7: Mean wage as function of minimum wage



vacancies in the economy and workers do not receive as many offers. Turning to smaller firms, profits are always decreasing since they have to pay higher wages and loose opportunities due to the minimum wage.

FIGURE 3.8: Profit as function of minimum wages



3.7 Discussion and Conclusions

The most important contribution of this paper is a theoretical framework that accommodates the European collective bargaining system, so as to rationalise how negotiating parties use minimum wages to affect labour market outcomes to their advantage. Using this framework, I have carried out the estimation that allows me to measure the different channels through which wage floors spread their impact. Another important trait is that minimum wages are endogeneised, leading to results that partially contradict the view that minimum wages do not have an effect on employment. Nonetheless, there are two features that should be taken into account for future research. First, several wage floors should be considered, so as to capture that different categories are subject to higher wage cushions, research in this area has recently been done and easily to conform. Another feature to be addressed is the large market power of firms, which remains unchallenged by the present work, however this problem could be tackled introducing multiworker firms and cost specific functions. The main advantage of this work is that it can be readily be used in other environments where the access to administrative data and collective agreements is easily accessible.

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Appendix A

Functional Forms

A.1 Form of Φ

The form of Φ in proposition 1 is

$$\Phi = \frac{1}{\left(\gamma^2 - 4\beta_1\beta_2\right)^2}\Big($$
$$- 2\alpha_2\alpha_1\gamma\left(8\beta_1\beta_2\gamma^2\left(\gamma - 2\beta_2\right) - 16\beta_1^2\beta_2^2\left(\gamma - 2\beta_2\right) + \gamma^5\right)$$
$$+ \alpha_2^2\gamma^2\left(-8\beta_1\beta_2\gamma^2 - 16\beta_1^2\beta_2\left(\gamma - 2\beta_2\right) + \gamma^4\right)$$
$$+ \alpha_1^2\left(-8\beta_2\gamma^4\left(\beta_1 + 2\gamma\right) + 16\beta_2^2\gamma^2\left(4\beta_1\gamma + \beta_1^2 + \gamma^2\right) - 64\beta_1\beta_2^3\gamma\left(\beta_1 + \gamma\right) + 64\beta_1^2\beta_2^4 + \gamma^6\right)\Big)$$

A.2 Proof of Proposition 3

We claimed that

$$W\left(\mathbf{q}^{N_M}\right) \geq W\left(\mathbf{q}^{C_M}\right) \Rightarrow U\left(\mathbf{q}^{N_M}\right) \geq U\left(\mathbf{q}^{C_M}\right) - c\sum_{i=1}^{2} q_i^k$$

For our purposes it will suffice to show that $U\left(\mathbf{q}^{N_M}\right) - U\left(\mathbf{q}^{C_M}\right) \geq 0$, since the second term in the right-hand side enters with a negative sign. After substituting the expressions of quantities in the utility functions we arrive at

$$U\left(\mathbf{q}^{N_M}\right) - U\left(\mathbf{q}^{C_M}\right) = \frac{1}{32\left(4\beta_1\beta_2 - \gamma^2\right)^2\left(\beta_1\beta_2 - \gamma^2\right)}\Big($$
$$+ 6c\left(\alpha_1\left(\beta_2 - \gamma\right) + \alpha_2\left(\beta_1 - \gamma\right)\right)\left(\gamma^2 - 4\beta_1\beta_2\right)^2$$
$$+ \left(\alpha_1^2\beta_2 + \alpha_2^2\beta_1\right)\left(28\beta_1\beta_2 - 9\gamma^2\right)\gamma^2$$
$$+ c^2\left(\beta_1 + \beta_2 - 2\gamma\right)\left(4\beta_1\beta_2 - \gamma^2\right)^2$$
$$+ 2\alpha_1\alpha_2\gamma\left(48\beta_1^2\beta_2^2 - 36\beta_1\beta_2\gamma^2 + 7\gamma^4\right)\Big)$$

First let us focus on the first and third terms. Notice that $\beta_1, \beta_2 > \gamma$, then expressions in parenthesis in these terms are positive, the rest are positive parameters or quadratic expressions and as a result they are positive as well. The second term is clearly positive because of the conditions stated in section 1.2. The fourth term requires more work to sign it, the interesting term is the one set in parenthesis which can be written as

$$\left(48\beta_1^2\beta_2^2 - 36\beta_1\beta_2\gamma^2 + 7\gamma^4\right) = \left(\beta_1\beta_2 \ \gamma\right)\underbrace{\begin{pmatrix} 48 & -18 \\ -18 & 7 \end{pmatrix}}_{A}\begin{pmatrix}\beta_1\beta_2 \\ \gamma\end{pmatrix}$$

Notice that the matrix A is positive definite and as such $x^T A x > 0, \quad \forall x \in \mathbb{R}^2$, as a consequence the final term is positive concluding the result.

A.3 Forms of firm profits and consumer surplus

Expressions of firm profits are:

- $\Pi_1^{N_M^*} = \frac{\beta_1\beta_2\left(\alpha_2^2\beta_1\left(3\gamma^2-4\beta_1\beta_2\right)+\alpha_1^2\beta_2\left(3\gamma^2-4\beta_1\beta_2\right)+2\alpha_1\alpha_2\gamma^3\right)}{4(\gamma^2-4\beta_1\beta_2)^2(\gamma^2-\beta_1\beta_2)}$
- $\Pi_1^{C_M^*} = \frac{\alpha_2^2\beta_1+\alpha_1^2\beta_2-c^2(-\beta_1-\beta_2+2\gamma)-2\alpha_2 c(\beta_1-\gamma)-2\alpha_1(\alpha_2\gamma+c(\beta_2-\gamma))}{16(\beta_1\beta_2-\gamma^2)}$

And those of consumer surpluses are:

- $CS^{N_M^*} = \frac{\beta_1\beta_2\left(\alpha_2^2\beta_1\left(3\gamma^2-4\beta_1\beta_2\right)+\alpha_1^2\beta_2\left(3\gamma^2-4\beta_1\beta_2\right)+2\alpha_1\alpha_2\gamma^3\right)}{8(4\beta_1\beta_2-\gamma^2)^2(\beta_1\beta_2-\gamma^2)} + q_0$
- $CS^{C_M^*} = \frac{\alpha_2^2\beta_1+\alpha_1^2\beta_2+\beta_1 c^2+\beta_2 c^2-2\gamma c^2-2\alpha_1(\alpha_2\gamma+c(\beta_2-\gamma))-2\alpha_2\beta_1 c+2\alpha_2\gamma c}{32(\beta_1\beta_2-\gamma^2)} + q_0$

A.4 Proof of Proposition 7

The first part of the proposition states that $V_1^{N_S^*} < V_2^{N_S^*}$, by direct comparison we see that

$$V_2^{N_S^*} - V_1^{N_S^*} = \frac{\alpha^2\gamma^3(\beta-\gamma)(4\beta+3\gamma)}{32\beta(\beta+\gamma)\left(2\beta^2-\gamma^2\right)^2} > 0\,.$$

The second part compares the monopoly model vs the stackelberg one, comparing the utilities of union 1 directly

$$V_1^{N_S^*} - V_1^{N_M^*} = \frac{\alpha^2\gamma^4(\beta-\gamma)}{16\beta(\gamma-2\beta)^2(\beta+\gamma)\left(2\beta^2-\gamma^2\right)} > 0\,.$$

Appendix B

Value Functions

B.1 Derivations of The Value Functions Basic model

Unemployed

In this appendix expressions for the value functions of unemployed, employed, match and surplus are derived for the basic model. I will closely follow the work of PV-R in deriving these analytical forms. First I will start setting the Value function of an unemployed worker in discrete time.

$$\begin{aligned} W_0(x) &= bx\Delta + e^{-r\Delta}\left\{e^{-kV\Delta}W_0(x) + \left(1-e^{-kV\Delta}\right)E\left[W_1\left(\phi_0(x,y),x,y\right)\right]\right\} \\ &= bx\Delta + e^{-r\Delta}\left\{e^{-kV\Delta}W_0(x) + \left(1-e^{-kV\Delta}\right)\int_{\underline{y}}^{\overline{y}} W_1\left(\phi_0(x,y'),x,y'\right)\frac{v(y')}{V}dy'\right\} \end{aligned}$$

rearranging

$$\left(1-e^{-(r+kV)\Delta}\right)W_0(x) = bx\Delta + e^{-r\Delta}\left(1-e^{-kV\Delta}\right)\int_{\underline{y}}^{\overline{y}} W_1\left(\phi_0(x,y'),x,y'\right)\frac{v(y')}{V}dy'$$

Dividing into Δ and taking the limit as Δ tends to zero, i.e. time is continuous, the expressions have the indeterminate form $\frac{0}{0}$. Then applying L'Hospital Rule

$$(r+\kappa V)\,W_0(x) = bx + \kappa\int_{\underline{y}}^{\overline{y}} W_1\left(\phi_0(x,y'),x,y'\right)v(y')dy'.$$

Taking into account that the worker has not bargaining power, in other words $W_0(x) = W_1\left(\phi_0(x,y'),x,y'\right)$, then the above expression is left as

$$rW_0(x) = bx.$$

Employed

Turning to the employed workers of any type, the value function of an x employee working on a firm of type y and earning the wage $w \leq xy$ is derived in the same fashion. Starting from the value of the an employed worker in discrete time

$$
\begin{aligned}
&W_1(w,x,y)\\
&\quad= w\Delta + e^{-r\Delta}\left\{\left(1-e^{-\delta\Delta}\right)W_0(x)\right.\\
&\quad+ e^{-\delta\Delta}\left[e^{-skV\Delta}W_1(w,x,y)+\left(1-e^{-skV\Delta}\right)\left(\int_{\underline{y}}^{q(w,x,y)}W_1(w,x,y)\frac{v(y')}{V}dy'\right.\right.\\
&\quad\left.\left.\left.+\int_{q(w,x,y)}^{y}W_1(xy',x,y)\frac{v(y')}{V}dy'+\int_{y}^{\overline{y}}W_1(xy,x,y')\frac{v(y')}{V}dy'\right)\right]\right\}
\end{aligned}
$$

rearranging

$$
\begin{aligned}
&\left(1-e^{-(r+\delta+skV)\Delta}\right)W_1(w,x,y)\\
&\qquad= w\Delta + e^{-r\Delta}\left(1-e^{-\delta\Delta}\right)W_0(x)\\
&\qquad+ e^{-r\Delta}e^{-\delta\Delta}\left(1-e^{-skV\Delta}\right)\left(\int_{\underline{y}}^{q(w,x,y)}W_1(w,x,y)\frac{v(y')}{V}dy'\right.\\
&\qquad\left.+\int_{q(w,x,y)}^{y}W_1(xy',x,y)\frac{v(y')}{V}dy'+\int_{y}^{\overline{y}}W_1(xy,x,y')\frac{v(y')}{V}dy'\right)
\end{aligned}
$$

Dividing into Δ and taking the limit as Δ tends to zero, i.e. time is continuous, the expressions have the indeterminate form $\frac{0}{0}$. Then applying L'Hospital Rule

$$
\begin{aligned}
&(r+\delta+skV)\,W_1(w,x,y)\\
&\qquad= w+\delta W_0(x)\\
&\qquad+ sk\int_{\underline{y}}^{q(w,x,y)}W_1(w,x,y)v(y')dy'\\
&\qquad+ sk\int_{q(w,x,y)}^{y}W_1(xy',x,y)v(y')dy'\\
&\qquad+ sk\int_{y}^{\overline{y}}W_1(xy,x,y')v(y')dy'
\end{aligned}
$$

rearranging

$$
\begin{aligned}
&\left(r+\delta+sk\overline{V}(q)\right)W_1(w,x,y)\\
&\qquad= w+\delta W_0(x)\\
&\qquad+ sk\int_{q(w,x,y)}^{y}W_1(xy',x,y)v(y')dy'\\
&\qquad+ sk\int_{y}^{\overline{y}}W_1(xy,x,y')v(y')dy'
\end{aligned}
$$

Now we can subtract $\left(r+\delta+sk\overline{V}(q)\right)W_0(x)$ to both sides of the equation and noticing that $W_1(xy,x,y)-W_0(x)=S(x,y)$, we can obtain the continuation value of the surplus accounted to the worker as a function of the whole surplus

$$\begin{aligned}\left(r+\delta+sk\overline{V}(q)\right)W_{10}\left(w,x,y\right)\\&=w-rW_0(x)\\&+sk\int_{q(w,x,y)}^{y}S(x,y')v(y')dy'\\&+sk\int_{y}^{\overline{y}}S(x,y)v(y')dy'\end{aligned}$$

Or rewritting for computational purposes

$$\begin{aligned}(r+\delta)\,W_{10}\left(w,x,y\right)\\&=w-rW_0(x)\\&+sk\int\left[min\left(S(x,y'),S(x,y)\right)-W_{10}\left(w,x,y\right)\right]^{+}v(y')dy'\end{aligned}$$

Where $[a]^+$ is equivalent to $max\,[a,0]$

Value of a Match and Surplus

Define the value of a Match and Surplus as $P(x,y)=\Pi_1(w,x,y)+W_1(w,x,y)$ and $S(x,y)=P(x,y)-W_0(x)$ respectively, again we start setting the value of a match as

$$\begin{aligned}&P(x,y)\\&=yx\Delta+e^{-r\Delta}\left\{\left(1-e^{-\delta\Delta}\right)W_0(x)\right.\\&+e^{-\delta\Delta}\left[e^{-skV\Delta}P(x,y)+\left(1-e^{-skV\Delta}\right)\left(\int_{\underline{y}}^{q}P(x,y)\frac{v(y')}{V}dy'\right.\right.\\&\left.\left.\left.+\int_{q}^{y}P(x,y)\frac{v(y')}{V}dy'+\int_{y}^{\overline{y}}P(x,y)\frac{v(y')}{V}dy'\right)\right]\right\}\end{aligned}$$

rearranging

$$\begin{aligned}\left(1-e^{-(r+\delta+skV)\Delta}\right)P(x,y)\\&=yx\Delta+e^{-r\Delta}\left\{\left(1-e^{-\delta\Delta}\right)W_0(x)\right.\\&\left.+e^{-\delta\Delta}\left[\left(1-e^{-skV\Delta}\right)\int_{\underline{y}}^{\overline{y}}P(x,y)\frac{v(y')}{V}dy'\right]\right\}\end{aligned}$$

Dividing into Δ and taking the limit as Δ tends to zero, i.e. time is continuous, the expressions have the indeterminate form $\frac{0}{0}$. Then applying L'Hospital Rule

$$\begin{aligned}(r+\delta+skV)\,P(x,y) & \\ &= yx+\delta W_0(x) \\ &+ sk\int_{\underline{y}}^{\overline{y}} P(x,y)v(y')dy'\end{aligned}$$

Cancelling terms

$$(r+\delta)\,P(x,y) = yx+\delta W_0(x)$$

We just need to subtract $(r+\delta)W_0(x)$ to both sites to have a close expression for the surplus

$$(r+\delta)\,S(x,y) = yx - rW_0(x)$$

Remember that I am assuming that $\Pi_0(y)=0,\ \forall y$

B.2 Derivations of The Value Functions under Minimum Wages

Unemployed

In this appendix expressions for equilibrium wages are determined for the basic model. I will closely follow the work of PV-R in deriving these analytical forms. First I will start setting the Value function of an unemployed worker in discrete time.

$$\begin{aligned}W_0(x) = bx\Delta + e^{-r\Delta}\Big\{&e^{-kV\Delta}W_0(x) \\ &+\left(1-e^{-kV\Delta}\right)\int max\left[W_1\left(\phi_0(x,y),x,y\right),W_1\left(m,x,y'\right)\right]\frac{v(y')}{V}dy'\Big\}\end{aligned}$$

Since $W_1(w,x,y)$ is monotonically increasing in y there will be a threshold in which $W_1\left(m,x,y'\right)\geq W_1\left(\phi_1(x,y),x,y\right)\ \ \forall y'\geq t_0$ or as implied by the lack of bargaining power of the worker $W_{10}\left(m,x,y'\right)\geq 0$, hence the the threshold is implicitly defined as $W_{10}\left(m,x,t_0(x,y)\right)=0$

$$\begin{aligned}W_0(x) = bx\Delta + e^{-r\Delta}\Big\{&e^{-kV\Delta}W_0(x) \\ &+\left(1-e^{-kV\Delta}\right)\Big(\int_{\underline{y}}^{t_0(x,y)} W_1\left(\phi_0(x,y),x,y\right)\frac{v(y')}{V}dy' \\ &+\int_{t_0(x,y)}^{\overline{y}} W_1\left(m,x,y'\right)\frac{v(y')}{V}dy'\Big)\Big\}\end{aligned}$$

Because the lack of bargaining power of the worker $W_1(\phi_0(x,y),x,y) = W_0(x)$ we can rewrite

$$W_0(x) = bx\Delta + e^{-r\Delta}\Big\{e^{-kV\Delta}W_0(x) + \left(1-e^{-kV\Delta}\right)\left(\int_{\underline{y}}^{t_0(x,y)} W_0(x)\frac{v(y')}{V}dy' + \int_{t_0(x,y)}^{\overline{y}} W_1(m,x,y')\frac{v(y')}{V}dy'\right)\Big\}$$

rearranging

$$\left(1-e^{-(r+kV)\Delta}\right)W_0(x) = bx\Delta + e^{-r\Delta}\left(1-e^{-kV\Delta}\right)\left(\int_{\underline{y}}^{t_0(x,y)} W_0(x)\frac{v(y')}{V}dy' + \int_{t_0(x,y)}^{\overline{y}} W_1(m,x,y')\frac{v(y')}{V}dy'\right)$$

Dividing into Δ and taking the limit as Δ tends to zero, i.e. time is continuous, the expressions have the indeterminate form $\frac{0}{0}$. Then applying L'Hospital Rule

$$(r+\kappa V)W_0(x) = bx + \kappa\left(\int_{\underline{y}}^{t_0(x,y)} W_0(x)v(y')dy' + \int_{t_0(x,y)}^{\overline{y}} W_1(m,x,y')v(y')dy'\right)$$

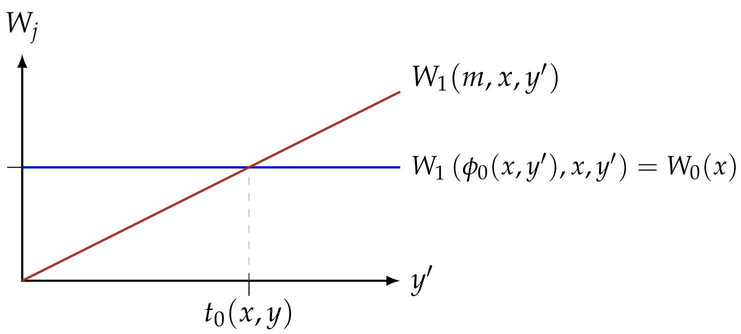
rearranging

$$\left(r+\kappa\overline{V}(t_0(x,y))\right)W_0(x) = bx + \kappa\int_{t_0(x,y)}^{\overline{y}} W_1(m,x,y')v(y')dy'$$

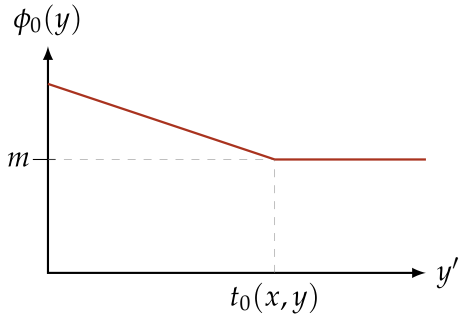
$$rW_0(x) = bx + \kappa\int_{t_0(x,y)}^{\overline{y}} W_{10}(m,x,y')v(y')dy'$$

For computational purposes it is more convenient to write the equation as

$$rW_0(x) = bx + \kappa\int max\left[W_{10}(m,x,y'),0\right]v(y')dy'$$



Threshold $t_0(x,y)$



Wage offer $\phi_0(x,y)$

Employed

Turning to the employed workers of any type, the value function of an x employee working on a firm of type y and earning the wage $w \leq xy$ is derived in the same fashion. Because there is a minimum wage in place, for a particular x, the lower bound of firms might change being $\underline{y}^* = min[\underline{y}, \hat{y}(x)]$, where $\hat{y}(x)$ is minimum viable productivity of a firm to hire a worker of type x when the minimum wage is binding.

$$\begin{aligned}
&W_1\left(w,x,y\right) \\
&\quad = w\Delta + e^{-r\Delta}\left\{\left(1-e^{-\delta\Delta}\right)W_0(x) + e^{-\delta\Delta}\left[e^{-skV\Delta}W_1\left(w,x,y\right)\right.\right. \\
&\quad + \left(1-e^{-skV\Delta}\right)\left(\int_{\underline{y}^*}^{q(w,x,y)} W_1(w,x,y)\frac{v(y')}{V}dy' + \int_{q(w,x,y)}^{y} W_1(xy',x,y)\frac{v(y')}{V}dy'\right. \\
&\quad \left.\left.\left. + + \int_{y}^{t_1(x,y)} W_1(xy,x,y')\frac{v(y')}{V}dy' + \int_{t_1(x,y)}^{\overline{y}} W_1(m,x,y')\frac{v(y')}{V}dy'\right)\right]\right\}
\end{aligned}$$

Rearranging

$$\begin{aligned}
&(1-e^{-(r+\delta+skV)\Delta})W_1\left(w,x,y\right) \\
&\quad = w\Delta + e^{-r\Delta}\left\{\left(1-e^{-\delta\Delta}\right)W_0(x)\right. \\
&\quad + \left(1-e^{-skV\Delta}\right)\left(\int_{\underline{y}^*}^{q(w,x,y)} W_1(w,x,y)\frac{v(y')}{V}dy' + \int_{q(w,x,y)}^{y} W_1(xy',x,y)\frac{v(y')}{V}dy'\right. \\
&\quad \left.\left. + + \int_{y}^{t_1(x,y)} W_1(xy,x,y')\frac{v(y')}{V}dy' + \int_{t_1(x,y)}^{\overline{y}} W_1(m,x,y')\frac{v(y')}{V}dy'\right)\right\}
\end{aligned}$$

Dividing into Δ and taking the limit as Δ tends to zero, i.e. time is continuous, the expressions have the indeterminate form $\frac{0}{0}$. Then applying L'Hospital Rule

$$
\begin{aligned}
(r+\delta+skV)\,W_1\,(w,x,y) \\
&= w+\delta W_0(x) \\
&+ sk\int_{\underline{y}^*}^{q(w,x,y)} W_1(w,x,y)v(y')dy' \\
&+ sk\int_{q(w,x,y)}^{y} W_1(xy',x,y)v(y')dy' \\
&+ sk\int_{y}^{t_1(x,y)} W_1(xy,x,y')v(y')dy' \\
&+ sk\int_{t_1(x,y)}^{\overline{y}} W_1(m,x,y')v(y')dy'
\end{aligned}
$$

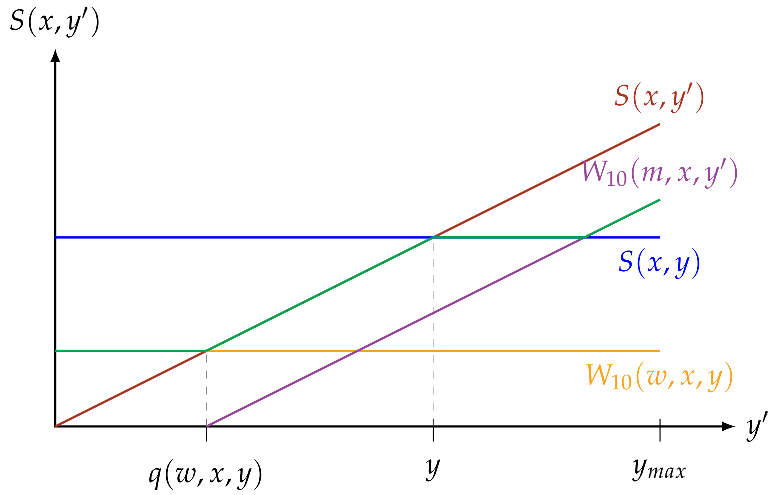
Now, because $\Pi_0(y)=0,\ \forall y$ then $W_1(xy,x,y)=P(x,y)$, substituting this into the previous equation

$$
\begin{aligned}
(r+\delta+skV)\,W_1\,(w,x,y) \\
&= w+\delta W_0(x) \\
&+ sk\int_{\underline{y}^*}^{q(w,x,y)} W_1(w,x,y)v(y')dy' \\
&+ sk\int_{q(w,x,y)}^{y} P(x,y')v(y')dy' \\
&+ sk\int_{y}^{t_1(x,y)} P(x,y)v(y')dy' \\
&+ sk\int_{t_1(x,y)}^{\overline{y}} W_1(m,x,y')v(y')dy'
\end{aligned}
$$

Subtracting $(r+\delta+skV)\,W_0(x)$ from both sites and rearranging

$$
\begin{aligned}
\left(r+\delta+sk\overline{V}(q)\right)W_{10}\,(w,x,y) \\
&= w-rW_0(x) \\
&+ sk\int_{q(w,x,y)}^{y} S(x,y')v(y')dy' \\
&+ sk\int_{y}^{t_1(x,y)} S(x,y)v(y')dy' \\
&+ sk\int_{t_1(x,y)}^{\overline{y}} W_{10}(m,x,y')v(y')dy'
\end{aligned}
$$

Following the picture we can rewrite the expression for computational purposes



Threshold $t_1(x,y)$

and thus avoid calculating $q(w,x,y)$ for every wage

$$\begin{aligned}
&(r+\delta+skV)\, W_1\,(w,x,y) \\
&\quad = w+\delta W_0(x) \\
&\quad + sk\int max\left\{min\left[max\left(W_{10}(w,x,y),S(x,y')\right),S(x,y)\right],W_{10}(m,x,y')\right\}v(y')dy'
\end{aligned}$$

And the threshold $t_1(x,y)$ is defined as

$$\begin{aligned}
W_1\,(xy,x,y) &= W_1\,(m,x,t_1(x,y))\;\; or \\
S\,(x,y) &= W_{10}\,(m,x,t_1(x,y))
\end{aligned}$$

value of a Match and Surplus

Define the value of a Match and Surplus as $P(x,y)=\Pi_1(w,x,y)+W_1(w,x,y)$ and $S(x,y)=P(x,y)-W_0(x)$ respectively. Because there is a minimum wage in place, for a particular x, the lower bound of firms might change being $\underline{y}^*=min[\underline{y},\hat{y}(x)]$, where $\hat{y}(x)$ is minimum viable productivity of a firm to hire a worker of type x when the minimum wage is unemployment binding. again we start setting the value of a match as

$$\begin{aligned}
&P(x,y) \\
&\;= yx\Delta+e^{-r\Delta}\left\{\left(1-e^{-\delta\Delta}\right)W_0(x)\right. \\
&\;+e^{-\delta\Delta}\left[e^{-skV\Delta}P(x,y)+\left(1-e^{-skV\Delta}\right)\left(\int_{\underline{y}^*}^{q}P(x,y)\frac{v(y')}{V}dy'\right.\right. \\
&\;\left.\left.\left.+\int_q^y P(x,y)\frac{v(y')}{V}dy'+\int_y^{t_1(x,y)}P(x,y)\frac{v(y')}{V}dy'+\int_{t_1(x,y)}^{\overline{y}}W_1(m,x,y')\frac{v(y')}{V}dy'\right)\right]\right\}
\end{aligned}$$

rearranging

$$
\begin{aligned}
&\left(1-e^{-(r+\delta+skV)\Delta}\right)P(x,y)\\
&\quad= yx\Delta + e^{-r\Delta}\left\{\left(1-e^{-\delta\Delta}\right)W_0(x)\right.\\
&\quad\left.+\,e^{-\delta\Delta}\left[\left(1-e^{-skV\Delta}\right)\left(\int_{\underline{y}^*}^{t_1(x,y)} P(x,y)\frac{v(y')}{V}dy' + \int_{t_1(x,y)}^{\overline{y}} W_1(m,x,y')\frac{v(y')}{V}dy'\right)\right]\right\}
\end{aligned}
$$

Dividing into Δ and taking the limit as Δ tends to zero, i.e. time is continuous, the expressions have the indeterminate form $\frac{0}{0}$. Then applying L'Hospital Rule

$$
\begin{aligned}
&(r+\delta+skV)\,P(x,y)\\
&\qquad= yx + \delta W_0(x)\\
&\qquad+ sk\int_{\underline{y}^*}^{t_1(x,y)} P(x,y)v(y')dy' + sk\int_{t_1(x,y)}^{\overline{y}} W_1(m,x,y')v(y')dy'
\end{aligned}
$$

Rearranging

$$
\begin{aligned}
&\left(r+\delta+sk\overline{V}(t_1)\right)P(x,y)\\
&\qquad= yx + \delta W_0(x)\\
&\qquad+ sk\int_{t_1(x,y)}^{\overline{y}} W_1(m,x,y')v(y')dy'
\end{aligned}
$$

We just need to subtract $\left(r+\delta+sk\overline{V}(t_1)\right)W_0(x)$ to both sites to have a close expression for the surplus

$$
\left(r+\delta+sk\overline{V}(t_1)\right)S(x,y) = yx - rW_0(x) + sk\int_{t_1(x,y)}^{\overline{y}} W_{10}(m,x,y')v(y')dy'
$$

Rewriting for computational purposes

$$
(r+\delta+skV)\,S(x,y) = yx - rW_0(x) + sk\int max\left[S(x,y), W_{10}(m,x,y')\right]v(y')dy'
$$

Remember that I am assuming that $\Pi_0(y) = 0,\ \forall y$

Appendix C

Wage Offer

C.1 Equilibrium Wage Determination

Employed

In this section I will work out specific expressions for value functions and wages

$$\begin{aligned}\left[r+\delta+s\kappa\overline{V}\left(q(w,x,y)\right)\right]W_1(w,x,y) &= w+\delta W_0(x)\\ &+s\kappa\int_{q(w,x,y)}^{y} W_1(xy',x,y')\ v(y')dy'\\ &+s\kappa\int_{y}^{\overline{y}} W_1(xy,x,y)\ v(y')dy' \end{aligned} \tag{C.1}$$

Now, imposing $w=xy$ implies that $q(xy,x,y)=y$, introducing these concerns into the previous equation:

$$[r+\delta]\,W_1(xy,x,y)=xy+\delta W_0(x)$$

This last expression can be derived with respect to y to have a specific expression of the derivative, then differentiating implicitly and solving for $W_1'(xy,x,y)$

$$W_1'(xy,x,y)=\frac{x}{[r+\delta]}$$

Integrating (2.1) by parts, we have

$$\begin{aligned}\Big[r+\delta\ &+s\kappa\overline{V}\left(q(w,x,y)\right)\Big]W_1(w,x,y)=w+\delta W_0(x)\\ &+s\kappa W_1\left(xy,x,y\right)V(y)-s\kappa W_1\left(w,x,y\right)V\left(q\right)-s\kappa\int_q^y W_1'\left(xy',x,y'\right)V(y')\ dy'\\ &+s\kappa W_1(xy,x,y)\left[V(\overline{y})-V(y)\right]\end{aligned}$$

Noticing that $W_1(w,x,y)=W_1(xq,x,q)$ and after cancelling terms

$$\begin{aligned}[r+\delta]W_1(w,x,y) &= w+\delta W_0(x)\\ &+s\kappa W_1(xy,x,y)V-s\kappa W_1\left(xq,x,q\right)V-s\kappa\int_q^y W_1'\left(xy',x,y'\right)V(y')\ dy'\end{aligned}$$

And by the FTC

$$[r+\delta]\,W_1(w,x,y) = w + \delta W_0(x) \\ + s\kappa \int_q^y W_1'\left(xy',x,y'\right)\overline{V}(y')\,dy'$$

With this general expression at hand we are ready to compute a particular expression of the wages. Start with the fact that an outside offer coming from a firm $\tilde{y} < y$ should comply with the equality $W_1(x\tilde{y},x,x\tilde{y}) = W_1(\phi(x,\tilde{y},y),x,y)$, notice that the threshold $q(x\tilde{y},x,\tilde{y}) = \tilde{y}$

$$x\tilde{y} + \delta W_0(x) + s\kappa \int_{\tilde{y}}^{\tilde{y}} W_1'\left(xy',x,y'\right)\overline{V}(y')\,dy' \\ = \phi(x,\tilde{y},y) + \delta W_0(x) + s\kappa \int_{\tilde{y}}^{y} W_1'\left(xy',x,y'\right)\overline{V}(y')\,dy'$$

Cancelling terms and rearranging

$$\phi(x,\tilde{y},y) = x\tilde{y} - s\kappa \int_{\tilde{y}}^{y} W_1'\left(xy',x,y'\right)\overline{V}(y')\,dy'$$

Unemployed

For the unemployed x workers, entry wages at y firms are $\phi_0(x,y) = \phi(x,y_{inf},y)$.

$$\phi_0(x,y) = xy_{inf} - s\kappa \int_{y_{inf}}^{y} W_1'\left(xy',x,y'\right)\overline{V}(y')dy' \\ = \phi(x,y_{inf},y)$$

Where y_{inf} is the minimum viable productivity of a firm to hire a worker and make no profits or in other words $W_1(xy_{inf},x,y_{inf}) = W_0(x)$, multiplying both sides by $(r+\delta)$ and replacing the LHS by its expression

$$xy_{inf} + \delta W_0(x) + s\kappa \int_{y_{inf}}^{y_{inf}} W_1'\left(xy',x,y'\right)\overline{V}(y')dy' = (r+\delta)W_0(x)$$

Which after cancelling terms becomes

$$y_{inf} = b$$

C.2 Equilibrium Wage Determination under Minimum wages

Employed

Turning to the employed workers of any type, the value function of an x employee working on a firm of type y and earning the wage $w \leq xy$ is set as to equal the wage, plus the value as unemployed with a laid-off rate of δ and offers from outside firms, accruing at a rate $s\kappa V$. When a worker is poached by a firm with productivity $\tilde{y} \leq q$

then $\phi(x, \tilde{y}, y) \leq w$, the poacher does not even reach the necessary productivity to hire the worker and make profits. At this point the minimum viable productivity that the firm has to have in order to provoke a wage increase is defined implicitly in the usual way as $\phi\left(x, q(x, w, y), y\right) = w$. If the offering firm has productivity $[q, y]$, the worker will receive a wage increase due to the Bertrand competition. In the case where the firm would have productivity higher than y, up to a threshold $t_1(x, y)$, the worker will switch jobs, the commonly known interplay between the wage offer and future wages increases plays its role, leaving the discounted future value of wealth fixed in this interval, i.e. the value function reaches a plateau since the firm is able to extract all the value from the match, thus higher productivity (and more likely future wage increases) is offset by a reduction in the wage offered. Now, offers from firms with higher productivity than $t_1(x, y)$ cannot reduce the offer made to the worker to extract all the match value, since the minimum wage acts as a lower bound for wages, in other words the minimum wage is binding. From the onset I will assume that this threshold exists and is unique as will be shown later. The value function of the worker is left as

$$\begin{aligned}\left[r + \delta + s\kappa\overline{V}\left(q(w, x, y)\right)\right] W_1(w, x, y) &= w + \delta W_0(x) \\ &+ s\kappa \int_{q(w,x,y)}^{y} W_1(xy', x, y')\ v(y')dy' \\ &+ s\kappa \int_{y}^{t_1(x,y)} W_1(xy, x, y)\ v(y')dy' \\ &+ s\kappa \int_{t_1(x,y)}^{\overline{y}} W_1(m, x, y')\ v(y')dy' \end{aligned} \tag{C.2}$$

Where the threshold is defined as $W_1(xy, x, y) = W_1\left(m, x, t_1(x, y)\right)$

Adding and subtracting $W_1(xy, x, y)$ in the range $[t_1(x, y), \overline{y}]$ and rearranging terms we have

$$\begin{aligned}\left[r + \delta + s\kappa\overline{V}\left(q(w, x, y)\right)\right] W_1(w, x, y) &= w + \delta W_0(x) \\ &+ s\kappa \int_{q(w,x,y)}^{y} W_1(xy', x, y')\ v(y')dy' \\ &+ s\kappa \int_{y}^{\overline{y}} W_1(xy, x, y)\ v(y')dy' \\ &+ s\kappa \int_{t_1(x,y)}^{\overline{y}} W_1(m, x, y') - W_1(xy, x, y)\ v(y')dy' \end{aligned}$$

The last term in the equation is the increase in utility granted by the minimum wage. Now, imposing $w = xy$ implies that $q(xy, x, y) = y$, introducing these concerns into the previous equation:

$$\begin{aligned}\left[r+\delta+s\kappa\overline{V}\left(y\right)\right]W_1(xy,x,y) &= xy+\delta W_0(x)\\ &\quad+\underbrace{s\kappa\int_{y}^{\overline{y}}W_1(xy,x,y)v(y')dy'}_{s\kappa\overline{V}(y))W_1(xy,x,y)}\\ &\quad+s\kappa\int_{t_1(x,y)}^{\overline{y}}W_1(m,x,y')-\underbrace{W_1(xy,x,y)}_{W_1(xy,x,y)\ \overline{V}(t_1(x,y))}\quad v(y')dy'\end{aligned}$$

Which results in

$$\left[r+\delta+s\kappa\overline{V}\left(t_1(x,y)\right)\right]W_1(xy,x,y) = xy+\delta W_0(x)+s\kappa\int_{t_1(x,y)}^{\overline{y}}W_1(m,x,y')\ v(y')dy'$$

This last expression can be derived with respect to y to have a specific expression of the derivative, then differentiating implicitly

$$\begin{aligned}-s\kappa v\left(t_1(x,y)\right)t_1'(x,y)W_1(xy,x,y)+\left[r+\delta+s\kappa\overline{V}\left(t_1(x,y)\right)\right]W_1'(xy,x,y)\\ = x-s\kappa\underbrace{W_1(m,x,t_1(x,y))}_{W_1(xy,x,y)}v\left(t_1(x,y)\right)t_1'(x,y)\end{aligned}$$

Solve for $W_1'(xy, x, y)$

$$W_1'(xy,x,y) = \frac{x}{\left[r+\delta+s\kappa\overline{V}\left(t_1(x,y)\right)\right]} \tag{C.3}$$

Integrating (C.2) by parts, we have

$$\begin{aligned}\Big[r+\delta\ &+s\kappa\overline{V}\left(q(w,x,y)\right)\Big]W_1(w,x,y) = w+\delta W_0(x)\\ &+s\kappa W_1\left(xy,x,y\right)V(y)-s\kappa W_1\left(w,x,y\right)V\left(q\right)-s\kappa\int_{q}^{y}W_1'\left(xy',x,y'\right)V(y')\,dy'\\ &+s\kappa W_1(xy,x,y)\left[V\left(t_1(x,y)\right)-V(y)\right]\\ &+s\kappa W_1\left(m,x,\overline{y}\right)V-s\kappa\underbrace{W_1\left(m,x,t_1(x,y)\right)}_{W_1(xy,x,y)}V\left(t_1(x,y)\right)-\int_{t_1(x,y)}^{\overline{y}}W_1'\left(m,x,y'\right)\ V(y')dy'\end{aligned}$$

Which after cancelling terms and rearranging, results in

$$\begin{aligned}[r+\delta]W_1(w,x,y) &= w+\delta W_0(x)\\ &\quad+s\kappa\left[W_1(xy,x,y)V-W_1(w,x,y)V-\int_{q}^{y}W_1'(xy',x,y')V(y')\,dy'\right]\\ &\quad+s\kappa\left[W_1(m,x,\overline{y})V-\underbrace{W_1(xy,x,y)V}_{W_1(m,x,t_1(x,y))}-\int_{t_1(x,y)}^{\overline{y}}W_1'(m,x,y')V(y')\,dy'\right]\end{aligned}$$

Now, thanks to the fundamental theorem of calculus we get to the usual expression

$$[r+\delta]W_1(w,x,y) = w + \delta W_0(x) + s\kappa \int_q^y W_1'(xy',x,y')\overline{V}(y')\,dy' + s\kappa \int_{t_1(x,y)}^{\overline{y}} W_1'(m,x,y')\overline{V}(y')\,dy' \tag{C.4}$$

However this expression is not very intuitive, instead it would be better to have the value function defined in the whole support of firms, for which the following change of variables can be performed

$$\left.\begin{aligned} y' &= t_1(x,z) \\ dy' &= t_1'(x,z)dz \end{aligned}\right\} W_1(m,x,y') = W_1(xz,x,z) \Rightarrow W_1(m,x,t_1(x,z)) = W_1(xz,x,z)$$

Deriving with respect to z

$$W_1'\left(m,x,t_1(x,z)\right)t_1'(x,z)dz = W_1'(xz,x,z)$$

Making use of (C.3), (C.4) and the previous expression the final form follows

$$[r+\delta]W_1(w,x,y) = w + \delta W_0(x) + s\kappa x \int_q^y \frac{\overline{V}(y')}{\left[r+\delta+s\kappa\overline{V}\left(t_1(x,y)\right)\right]}\,dy' + s\kappa x \int_y^{\overline{y}} \frac{\overline{V}\left(t_1(x,y')\right)}{\left[r+\delta+s\kappa\overline{V}\left(t_1(x,y)\right)\right]}\,dy' \tag{C.5}$$

With this general expression at hand we are ready to compute a particular expression of the wages. Start with the fact that an outside offer coming from a firm $\tilde{y} < y$ should comply with the equality

$$W_1\left(\phi(x,\tilde{y},y),x,y\right) = max\left[W_1\left(x\tilde{y},x,\tilde{y}\right), W_1\left(m,x,y\right)\right]$$

Notice that because $\tilde{y} < y$, m is not going to be binding and the maximum function will result in $W_1\left(x\tilde{y},x,\tilde{y}\right)$. Then, taking the specific forms of the value functions derived in (C.5) at the particular wages, we can write

$$\phi(x,\tilde{y},y) + \delta W_0(x) + s\kappa x\left\{\int_{\tilde{y}}^{y} \frac{\overline{V}(y')dy'}{\left[r+\delta+s\kappa\overline{V}\left(t_1(x,y)\right)\right]} + \int_y^{\overline{y}} \frac{\overline{V}\left(t_1(x,y')dy'\right)}{\left[r+\delta+s\kappa\overline{V}\left(t_1(x,y)\right)\right]}\right\}$$
$$= x\tilde{y} + \delta W_0(x) + s\kappa x\left\{\int_{\tilde{y}}^{\tilde{y}} \frac{\overline{V}(y')dy'}{\left[r+\delta+s\kappa\overline{V}\left(t_1(x,y)\right)\right]} + \int_{\tilde{y}}^{\overline{y}} \frac{\overline{V}\left(t_1(x,y')dy'\right)}{\left[r+\delta+s\kappa\overline{V}\left(t_1(x,y)\right)\right]}\right\}$$

Which after rearranging becomes

$$\underbrace{\phi(x,\tilde{y},y)}_{\text{wage offer}} = \underbrace{x\tilde{y}}_{\text{max. productivity of the match}} - \underbrace{s\kappa x\int_{\tilde{y}}^{y}\frac{\overline{V}(y')dy'}{\left[r+\delta+s\kappa\overline{V}\left(t_1(x,y)\right)\right]}}_{\text{Trade off of lower wages for future increases}} + \underbrace{s\kappa x\int_{\tilde{y}}^{y}\frac{\overline{V}\left(t_1(x,y')\right)dy'}{\left[r+\delta+s\kappa\overline{V}\left(t_1(x,y)\right)\right]}}_{\text{Extra rent granted by imposing a min. wage}}$$

$$= x\left\{\tilde{y}+s\kappa\int_{\tilde{y}}^{y}\frac{\overline{V}\left(t_1(x,y')\right)-\overline{V}(y')}{\left[r+\delta+s\kappa\overline{V}\left(t_1(x,y)\right)\right]}dy'\right\}$$

Unemployed

As in the case without minimum wages we just need to define the minimum viable productivity of a firm $\hat{y}(x)$ as the value that leaves the worker indifferent between looking for a job and working; and at the same time makes the surplus of the match equal to zero, if the surplus of the match were not zero at the firm with the lowest viable productivity, any other firm with marginally lower productivity could make an offer to the worker and make profits at the same time. Since, there is no restriction on how low the marginal productivity of a firm can be, this is a contradiction, and thus:

$$P(x,\hat{y}(x)) = W_1(m,x,\hat{y}(x)) = W_0(x;m)$$

Where $\hat{y}(x) = \frac{m}{x}$
Proof:

$$P(x,\hat{y}(x)) = x\hat{y}(x) + s\kappa\int_{\hat{y}(x)}^{t_1} P(x,y)v(y')dy' + sk\int_{t_1}^{\overline{y}} W_1(m,x,y')v(y')dy'$$

$$W_1(m,x,\hat{y}(x)) = m + s\kappa\int_{\hat{y}(x)}^{t_1} P(x,y)v(y')dy' + sk\int_{t_1}^{\overline{y}} W_1(m,x,y')v(y')dy'$$

Equating terms we arrive at $x\hat{y}(x) = m$, hence the result $\hat{y}(x) = \frac{m}{x}$.
And the entry wage will be m in any case, $\phi_0(x,\tilde{y}) = m$.

High-skill workers

High-skill workers will show similar expressions as those worked out of low-skill ones. For employed workers the expression for salaries will remain unchanged, since the process for poaching workers is basically the same. Also, since the effective minimum wage $\hat{y}(x)$ is below the minimum viable productivity of a firm, $\hat{y}(x) \leq y_{inf}$, the support of the distribution will remain unchanged. What is likely to change is the support of the distribution of wages since now the entry wage will be the largest between $\phi_0(x,y;m) = [\phi_0(x,y),m]$, depending on the productivity of the initial poacher. Then the expression for entry wages will be a piece-wise function of the form

$$\phi_0(\epsilon, y; m) = \begin{Bmatrix} x\left\{b + s\kappa \int_{y_{inf}}^{y} \frac{\overline{V}(t_1(x,y')) - \overline{V}(y')}{\left[\rho+\delta+\lambda_1\overline{F}(t_1(x,y))\right]} dy'\right\} & \text{if} \quad y' < t_1(x,y)) \\ m & \text{if} \quad y' \geq t_1(x,y)) \end{Bmatrix}$$

Appendix D

Steady State Distributions

D.1 Without Minimum Wages

In this appendix detailed derivations for $v(y)$, $u(x)$ and $h(x, y)$ are worked out. We start with the balance conditions, which are no more that accounting identities, that are met in every point in time

$$\int h_t(x, y')dy' + u_t(x) = l_t(x)$$
$$\int h_t(x', y)dx' + v_t(y) = n_t(y)$$

and flow equations in discrete time.

$$\begin{aligned}
h_{t+1}(x,y) = h_t(x,y) \quad & +kv_t(y)u_t(x)\Delta + sk\int_{\underline{y}}^{y} h_t(x,y')dy'v_t(y)\Delta \quad -\delta h_t(x,y)\Delta - sk\int_{y}^{\overline{y}} v_t(y')dy'h_t(x,y)\Delta \\
u_{t+1}(x) = u_t(x) \quad & -kV_t u_t(x)\Delta \quad +\delta h_{x,t}(x)\Delta \\
v_{t+1}(y) = v_t(y) \quad & -kv(y)U_t\Delta - sk\int_{\underline{y}}^{y} h_{y,t}(y')dy'v_t(y)\Delta \quad +\delta h_{y,t}(y)\Delta + sk\int_{y}^{\overline{y}} v_t(y')dy'h_{y,t}(y)\Delta
\end{aligned}$$

Where $h_{x,t}(x) = \int_{\underline{y}}^{\overline{y}} h_t(x, y')dy'$ and $h_{y,t}(y) = \int_{\underline{x}}^{\overline{x}} h_t(x', y)dx'$. The expressions are easier to work with in continuous time so I rearrange stocks to the LHS and flows to the RHS, Divide by Δ and take the limit as $\Delta \to 0$ to have

$$\begin{aligned}
\dot{h}_t(x,y) = kv_t(y)u_t(x) \quad & +sk\int_{\underline{y}}^{y} h_t(x,y')dy'v_t(y) - \delta h_t(x,y) \quad -sk\int_{y}^{\overline{y}} v_t(y')dy'h_t(x,y) \\
\dot{u}_t(x) = -kV_t u_t(x) \quad & + \delta h_{x,t}(x) \\
\dot{v}_t(y) = -kv_t(y)U_t \quad & -sk\int_{\underline{y}}^{y} h_{y,t}(y')dy'v_t(y) + \delta h_{y,t}(y) \quad +sk\int_{y}^{\overline{y}} v_t(y')dy'h_{y,t}(y)
\end{aligned}$$

Before working out specific expressions for every type of firm and worker, it will be useful to calculate aggregate balance conditions, just aggregate over the set of firm productivities and worker abilities to have

$$H_t + U_t = L_t$$
$$H_t + V_t = N_t$$

Where $H_t = \int\int h_t(x', y')dx'dy'$. L_t is exogenous and given N_t, V_t is pinned down, at this point both conditions are knotted by H_t, so we can write

$$L_t - U_t = N_t - V_t$$

And deriving with respect to time we have that $\dot{U}_t = \dot{V}_t$. Also, we need to have the expressions for the aggregate flows. Integrate $v(y)$, $u(x)$ and $h(x, y)$ over the variables that they depend on. Furthermore, in the aggregate all the workers that quit are the same as those who are poached, i.e. $\int_{\underline{y}}^{y} h_t(x, y')dy'v_t(y) = \int_{y}^{\overline{y}} v_t(y')dy'h_t(x, y)$, hence

$$\left.\begin{array}{ll} \dot{H}_t & = kV_tU_t - \delta H_t \\ \dot{U}_t & = -kV_tU_t + \delta H_t \\ \dot{V}_t & = -kV_tU_t + \delta H_t \end{array}\right\} \overset{SS}{\Longrightarrow} \delta H_t = kV_tU_t$$

Since we are in the S.S., time dependence can be dropped from the notation. Now, Plug the aggregate balance conditions to have H as a function of N, k (endogenous objects) and δ, L (parameters), $\delta H = k(N-H)(L-H)$, this is a quadratic equation on H

$$kH^2 - (\delta + kL + kN)\, H + kLN = 0$$

Which solves as

$$H = \frac{1}{2}\left\{\left(\frac{\delta}{k} + L + N\right) - \sqrt{\left(\frac{\delta}{k} + L + N\right)^2 - 4LN}\right\}$$

Below, it is the proof of why only the negative part is taken. First consider the positive part, i.e.

$$H(N) = \frac{1}{2}\left\{\left(\frac{\delta}{k} + L + N\right) + \sqrt{\left(\frac{\delta}{k} + L + N\right)^2 - 4LN}\right\} > \frac{1}{2}\left\{\left(\frac{\delta}{k} + L + N\right) + \sqrt{\left(\frac{\delta}{k} + L + N\right)^2}\right\} \Leftrightarrow$$
$$\frac{1}{2}\left\{\left(\frac{\delta}{k} + L + N\right) + \sqrt{\left(\frac{\delta}{k} + L + N\right)^2 - 4LN}\right\} > \left(\frac{\delta}{k} + L + N\right)$$

Which means that the number of employed is higher than the number of people in the economy, an absurdity. Turning to the negative part, I would like to work out the maximum and minimum values as a function of N. The minimum value can be easily worked out as

$$H(0) = \frac{1}{2}\left\{\left(\frac{\delta}{k} + L\right) - \sqrt{\left(\frac{\delta}{k} + L\right)^2}\right\} = 0$$

And the maximum

$$\lim_{N\to\infty} H(N) = \lim_{N\to\infty} \frac{1}{2}\left\{\left(\frac{\delta}{k}+L+N\right) - \sqrt{\left(\frac{\delta}{k}+L+N\right)^2 - 4LN}\right\}$$

Which is undetermined, dividing and multiplying by the complement and later on dividing by N in the numerator and denominator we have

$$\begin{aligned}\lim_{N\to\infty} H(N) &= \lim_{N\to\infty} \frac{1}{2}\left\{\frac{4LN}{\left(\frac{\delta}{k}+L+N\right)+\sqrt{\left(\frac{\delta}{k}+L+N\right)^2-4LN}}\right\}\\ &= \lim_{N\to\infty} \frac{1}{2}\left\{\frac{4L}{\left(\frac{\delta}{Nk}+\frac{L}{N}+1\right)+\sqrt{\left(\frac{\delta}{Nk}+\frac{L}{N}+1\right)^2+\frac{4L}{N}}}\right\}\\ &= \frac{1}{2}\left\{\frac{4L}{1+\sqrt{1}}\right\} = L\end{aligned}$$

Which means that as the number of firms tends to infinity the number of employed workers tends to the number of people in the economy. Also, it would be convenient to check if the function is increasing in the whole domain

$$\frac{\partial H}{\partial N} = \frac{1}{2}\left\{1 - \frac{2\left(\frac{\delta}{k}+L+N\right)-4L}{\sqrt{\left(\frac{\delta}{k}+L+N\right)^2-4LN}}\right\} > 0$$

Using the balance conditions U and V can easily be derived. With this expressions at hand we can work out their desegregated counterparts. First, consider the S.S. and drop the time dependence,so

$$\begin{aligned}0 &= kv(y)u(x) \quad +sk\int_{\underline{y}}^{y} h(x,y')dy'v(y) - \delta h(x,y) \quad -sk\int_{y}^{\overline{y}} v(y')dy'h(x,y)\\ 0 &= -kVu(x) \quad + \delta h_x(x)\\ 0 &= -kv(y)U \quad -sk\int_{\underline{y}}^{y} h_y(y')dy'v(y) + \delta h_y(y) \quad +sk\int_{y}^{\overline{y}} v(y')dy'h_y(y)\end{aligned}$$

Now, consider the last expression and rearrange such that U-to-H and H-to-U flows are on the LHS and H-to-H flows are in the RHS. Also, for notational convenience let's write $F_z(z) = \int_{\underline{z}}^{z} f_z(z')dz'$ for any function over the z-characteristic and denote its complement counterpart as $\overline{F}_z(z) = F_z(\overline{z}) - F_z(z) = \int_{z}^{\overline{z}} f_z(z')dz'$

$$kv(y)U - \delta h_y(y) = -skH_y(y)v(y) + sk\overline{V}(y)h_y(y)$$

Integrate both sites from $\underline{y}$ to y to have

$$kV(y)U - \delta H_y(y) = skH_y(y)\overline{V}(y) \tag{D.1}$$

And Integrate the balance condition from $\underline{y}$ to y to have

$$H_y(y) + V(y) = N(y)$$

Then solve last expression for V(y) and plug it into the aggregate flow equation for vacancies to arrive at

$$\begin{aligned} k\left(N(y) - H_y(y)\right)U - \delta H_y(y) &= skH_y(y)\left(V - N(y) + H_y(y)\right) \\ kUN(y) - kUH_y(y) - \delta H_y(y) &= skVH_y(y) - skN(y)H_y(y) + skH^2(y) \\ skH^2(y) + \left(\delta + kU + skV - skN(y)\right)H_y(y) - kUN(y) &= 0 \end{aligned}$$

Again, this is a quadratic equation in $H_y(y)$, which depends solely on k and $N(y)$ and the rest of variables have been previously worked out, as we can see below

$$\underbrace{sk}_{A}H_y^2(y) + \underbrace{(\delta + kU + skV - skN(y))}_{B(N(y))}H_y(y) - \underbrace{kUN(y)}_{C(N(y))} = 0$$

Then

$$H_y(y) = \frac{-B\left(N(y)\right) + \sqrt{B^2\left(N(y)\right) + 4AC\left(N(y)\right)}}{2A}$$

The negative part can safely be discarded as

$$H_y(y) = \frac{-B\left(N(y)\right) - \sqrt{B^2\left(N(y)\right) + 4AC\left(N(y)\right)}}{2A} < \frac{-B\left(N(y)\right) - \sqrt{B^2\left(N(y)\right)}}{2A} = -\frac{2B\left(N(y)\right)}{2A} < 0$$

Whereas the positive part is always greater than zero

$$H_y(y) = \frac{-B\left(N(y)\right) + \sqrt{B^2\left(N(y)\right) + 4AC\left(N(y)\right)}}{2A} > \frac{-B\left(N(y)\right) + \sqrt{B^2\left(N(y)\right)}}{2A} = 0$$

Now, derive the quadratic equation implicitly with respect to to y to find $h_y(y)$

$$2skH_y(y)h_y(y) + (\delta + kU + skV - skN(y))\,h_y(y) - skn(y)H_y(y) - kUn(y) = 0$$

Solving for $h_y(y)$

$$h_y(y) = \frac{kU + skH_y(y)}{\left(\delta + skV + skH_y(y) - skN(y)\right) + \left(kU + skH_y(y)\right)} \cdot n(y)$$

Where

- $\left(\delta + skV + skH_y(y) - skN(y)\right) = \left(\delta + sk\overline{V}(y)\right)$: are flows out of $H_y(y)$ and
- $kU + skH_y(y)$: are flows into $H_y(y)$

It's worth noting that $H_y(y)$ depends on $N(y)$ and so does $h_y(y)$.

At this point we are ready to come with an expression for $v(y)$. Consider again the expression coming from the integrated flow of vacancies in the S.S.

$$kV(y)U - \delta H_y(y) = skH_y(y)\overline{V}(y)$$

It is just left to solve for $V(y)$ and derive to reach the desire result

$$V(y) = \frac{\delta + skV}{kU + skH_y(y)} H_y(y)$$

And deriving

$$v(y) = \frac{\delta + skV}{\left(kU + skH_y(y)\right)^2} kUh_y(y)$$

Which is not very intuitive. In order to have an expression in terms of flows, change $h_y(y)$ by its last derived expression; plug the definition of $V(y)$ and use the integrated flow of vacancies in the S.S. to arrive to the desired result

$$v(y) = \frac{\delta + sk\overline{V}(y)}{\left(\delta + sk\overline{V}(y)\right) + \left(kU + skH_y(y)\right)} \cdot n(y)$$

Once we have worked out a close form expression for the number of vacancies and the number of workers in y-type firms, we can deal with the number of unemployed and the number of workers with x-characteristic. From the differential equation for unemployed, substitute the balance condition for $h_x(x)$, such that

$$kVu(x) = \delta h_x(x) \Leftrightarrow kVu(x) = \delta\left(l(x) - u(x)\right) \Leftrightarrow \left(\delta + kV\right)u(x) = \delta l(x)$$

$$u(x) = \frac{\delta}{(\delta + kV)} l(x)$$

With this expression and basic algebra we work out $h_x(x)$

$$h_x(x) = \frac{kV}{(\delta + kV)} l(x)$$

Finally, we are ready to calculate $h(x, y)$. The steps to arrive at the solution are basically the same as those to compute $h_y(y)$. THen, consider the last expression and rearrange such that U-to-H and H-to-U flows are on the LHS and H-to-H flows are in the RHS.

$$\delta h(x,y) - kv(y)u(x) = sk\int_{\underline{y}}^{y} h(x,y')dy'v(y) - sk\int_{y}^{\overline{y}} v(y')dy'h(x,y)$$

Integrate both sites from $\underline{y}$ to y to have

$$\delta\int_{\underline{y}}^{y} h(x,y')dy' - ku(x)V(y) = -sk\int_{\underline{y}}^{y} h(x,y')dy'\overline{V}(y)$$

rearranging

$$\left(\delta + sk\overline{V}(y)\right) \int_{\underline{y}}^{y} h(x,y')dy' = ku(x)V(y)$$
$$\int_{\underline{y}}^{y} h(x,y')dy' = \frac{ku(x)V(y)}{\left(\delta + sk\overline{V}(y)\right)}$$

And deriving with respect to to y we arrive at the final form

$$h(x,y) = \frac{\delta + skV}{\left(\delta + sk\overline{V}(y)\right)^2} \cdot ku(x)v(y)$$

Which is difficult to interpret. However, we can prove that the following interesting result holds, $h(x,y) = \frac{1}{H}h_x(x)h_y(y)$. Start by plugging in $h(x,y)$ the expressions for $ku(x) = \frac{\delta}{V}h_x(x)$, $v(y)$, and solve for $\left(\delta + sk\overline{V}(y)\right)$ in equation D.1, so that we get

$$h(x,y) = \frac{(\delta + skV)}{\left(\frac{kV(y)U}{H_y(y)}\right)^2} \cdot \frac{(\delta + skV)}{\left(kU + skH_y(y)\right)^2} \cdot kUh_y(y)\frac{\delta}{V}h_x(x)$$

Solve for $\left(kU + skH_y(y)\right)$ in D.1 and use the fact that $kU = \frac{\delta}{V}H$

$$h(x,y) = \frac{(\delta + skV)}{\left(\frac{kV(y)U}{H_y(y)}\right)^2} \cdot \frac{(\delta + skV)}{\left(\frac{(\delta+skV)H_y(y)}{V(y)}\right)^2} \cdot \left(\frac{\delta}{V}\right)^2 Hh_x(x)h_y(y)$$

Cancelling out terms and substituting

$$h(x,y) = \left(\frac{\delta}{kUV}\right)^2 Hh_x(x)h_y(y) = \frac{1}{H^2}Hh_x(x)h_y(y) = \frac{1}{H}h_x(x)h_y(y)$$

Now, we are ready to calculate the distribution of wages from the flow equation

$$\frac{dG_t(w|x,y)\cdot h_t(x,y)}{dt} = kv_t(y)u_t(x) + sk\int_{\underline{y}}^{q} h(x,y')dy' \cdot v(y)$$
$$- \delta G_t(w|x,y)\cdot h_t(x,y) - sk\int_{q}^{\overline{y}} v(y')dy' G_t(w|x,y)\cdot h_t(x,y) = 0$$

rearranging

$$\left(\delta + sk\int_{q}^{\overline{y}} v(y')dy'\right) G(w|x,y)\cdot h(x,y) = kv(y)u(x) + sk\int_{\underline{y}}^{q} h(x,y')dy'v(y)$$

solving for $G_t(w|x,y)$ we have

$$G(w|x,y) = \frac{\left(kv(y)u(x) + sk\int_{\underline{y}}^{q} h(x,y')dy'v(y)\right)}{\left(\delta + sk\int_{q}^{\overline{y}} v(y')dy'\right)} \cdot \frac{1}{h(x,y)}$$

Substitute $h(x,y)$ by the product of the marginals $\frac{1}{H}h_x(x)h_y(y)$. Also use the flow equation for the unemployed $kVu(x) = h_x(x)$ and the aggregate flow equation $kVU = \delta H$ to arrive at $u(x) = \frac{U}{H}h_x(x)$, then after cancelling terms

$$G(w|y) = \frac{\left(kU + sk\int_{\underline{y}}^{q} h_y(y')dy'\right)}{\left(\delta + sk\int_{q}^{\overline{y}} v(y')dy'\right)} \cdot \frac{v(y)}{h_y(y)}$$

Which shows what intuition could have told us in advance, namely that the distribution of wages does not depend on x.

D.2 With Minimum Wages

To find out the distributions of matches, vacancies and unemployed under the minimum wage, $\tilde{h}(x,y)$, $\tilde{v}(y)$ and $\tilde{u}(x)$ respectively, it will just suffice to rewrite them in terms of the old ones. Under the minimum wage some meetings that could have ended in a match are not going to be possible, as the flow revenue of the match it is not enough to pay the minimum wage. Then the balance conditions can be rewritten as

$$\begin{aligned} l(x) &= \underbrace{u(x) + \int_{\underline{y}}^{\hat{y}(x)} h(x,y')dy'}_{\tilde{u}(x)} + \underbrace{\int_{\hat{y}(x)}^{\overline{y}} h(x,y')dy'}_{\tilde{h}_x(x)} \\ n(y) &= \underbrace{v(y) + \int_{\underline{x}}^{\hat{x}(y)} h(x',y)dx'}_{\tilde{v}(y)} + \underbrace{\int_{\hat{x}(y)}^{\overline{x}} h(x',y)dx'}_{\tilde{h}_y(y)}. \end{aligned}$$

Because the previous analysis without minimum wages, we know that under random search there is no sorting under the model assumptions. The implication being to express $h(x,y)$ as the product of two functions that describe abilities and productivities independently, namely $h(x,y) = \frac{1}{H}h_y(y)h_x(x)$, then the balance conditions can be rewritten as

$$\begin{aligned} l(x) &= \underbrace{u(x) + h_x(x)\int_{\underline{y}}^{\hat{y}(x)} \frac{h_y(y')}{H}dy'}_{\tilde{u}(x)} + \underbrace{h_x(x)\int_{\hat{y}(x)}^{\overline{y}} \frac{h_y(y')}{H}dy'}_{\tilde{h}_x(x)} \\ n(y) &= \underbrace{v(y) + h_y(y)\int_{\underline{x}}^{\hat{x}(y)} \frac{h_x(x')}{H}dx'}_{\tilde{v}(y)} + \underbrace{h_y(y)\int_{\hat{x}(y)}^{\overline{x}} \frac{h_x(x')}{H}dx'}_{\tilde{h}_y(y)} \end{aligned}$$

.

And then as

$$l(x) = \underbrace{u(x) + h_x(x)F_y\left(\hat{y}(x)\right)}_{\tilde{u}(x)} + \underbrace{h_x(x)\overline{F}_y\left(\hat{y}(x)\right)}_{\tilde{h}_x(x)}$$
$$n(y) = \underbrace{v(y) + h_y(y)F_x\left(\hat{x}(y)\right)}_{\tilde{v}(y)} + \underbrace{h_y(y)\overline{F}_x\left(\hat{x}(y)\right)}_{\tilde{h}_y(y)}.$$

Integrating over abilities in the first condition and over productivities in the second we work out the aggregate balance conditions

$$L = \underbrace{U + \int_{\underline{x}}^{\overline{x}} h_x(x')F_y\left(\hat{y}(x')\right)dx'}_{\tilde{U}} + \underbrace{\int_{\underline{x}}^{\overline{x}} h_x(x')\overline{F}_y\left(\hat{y}(x')\right)dx'}_{\tilde{H}}$$
$$N = \underbrace{V + \int_{\underline{y}}^{\overline{y}} h_y(y')F_x\left(\hat{x}(y')\right)dy'}_{\tilde{V}} + \underbrace{\int_{\underline{y}}^{\overline{y}} h_y(y')\overline{F}_x\left(\hat{x}(y')\right)dy'}_{\tilde{H}}.$$

For the join distribution of jobs under the minimum wage it will suffice to solve the follow equation for jobs:

$$0 = k\tilde{v}(y)\tilde{u}(x) \qquad +sk\int_{\underline{y}}^{q}\tilde{h}(x,y')dy'\tilde{v}(y)$$
$$-\delta\tilde{G}_t(w|x,y)\cdot\tilde{h}(x,y) \qquad -sk\int_{q}^{\overline{y}}\tilde{v}(y')dy'\tilde{G}_t(w|x,y)\cdot\tilde{h}(x,y).$$

Now the number of vacancies and unemployed people will be substituted by their minimum wage counterparts, i.e. $\tilde{v}(y)$ and $\tilde{u}(y)$, since those vacancies lost by the unemployed or workers with low ability will be at the disposal of the rest, and seemingly the same argument applies for those unemployed that will not be able to cover vacancies in low productive firms. Hence, following the same procedure as before we will arrive at

$$\tilde{h}(x,y) = \frac{\delta + sk\tilde{V}}{\left(\delta + sk\overline{\tilde{V}}(y)\right)^2}\cdot k\tilde{u}(x)\tilde{v}(y)$$
$$\tilde{G}(w|x,y) = \frac{\left(k\tilde{v}(y)\tilde{u}(x) + sk\int_{\underline{y}}^{q}\tilde{h}(x,y')dy'\tilde{v}(y)\right)}{\left(\delta + sk\int_{q}^{\overline{y}}\tilde{v}(y')dy'\right)}\cdot\frac{1}{\tilde{h}(x,y)}$$

Lack of assortative matching will not be the case upon introducing minimum wages, now the minimum viable productivity of a firm (or worker) to form a match will be a function of the worker ability (firm productivity). In this respect minimum wages will introduce negative sorting in our analysis.

Appendix E

Proofs of Lemmas

E.1 Lemma 1

Free-entry implies that the firm with the minimum viable productivity to hire a worker cannot make any surplus out of the match, i.e. $\Pi_1\left(x, \underline{\hat{y}}(x)\right) = 0$, otherwise a firm with marginally less productivity could enter the market, hire a worker and make profits, being a contradiction; then $P\left(x, \underline{\hat{y}}(x)\right) = W_1\left(\phi_0\left(x, \underline{\hat{y}}(x)\right), x, \underline{\hat{y}}(x)\right)$.

With respect to $W_1\left(\phi_0\left(x, \underline{\hat{y}}(x)\right), x, \underline{\hat{y}}(x)\right) = W_0(x, m)$, the same argument along the above lines can be devised. If $W_1\left(\phi_0\left(x, \underline{\hat{y}}(x)\right), x, \underline{\hat{y}}(x)\right) = P\left(x, \underline{\hat{y}}(x)\right) > W_0(x, m)$ then another firm with marginally less productivity could enter and make profits, once again a contradiction.

E.2 Lemma 2

Making use of LEMMA.1 we can equate $P\left(x, \underline{\hat{y}}(x)\right) = W_1\left(\phi_0\left(x, \underline{\hat{y}}(x)\right), x, \underline{\hat{y}}(x)\right)$, which means that

$$\begin{aligned}
&\underline{\hat{y}}(x)x + \delta W_0(x) + sk\int_{\underline{\hat{y}}(x)}^{t(x,y)} P(x,y)v(y')dy' + sk\int_{t(x,y)}^{\overline{y}} W_1(m,x,y')v(y')dy' \\
&\quad = \phi_0\left(x, \underline{\hat{y}}(x)\right) + \delta W_0(x) + sk\int_{\underline{\hat{y}}(x)}^{t(x,y)} P(x,y)v(y')dy' + sk\int_{t(x,y)}^{\overline{y}} W_1(m,x,y')v(y')dy'.
\end{aligned}$$

And after cancelling terms we arrive at:

$$\underline{\hat{y}}(x)x = \phi_0\left(x, \underline{\hat{y}}(x)\right).$$

For convenience define the threshold x' such that $\phi_0\left(x', y_{inf}\right) = m$. There are two cases of interest:

CASE.1: $x < x'$

$$\phi_0\left(x', \underline{\hat{y}}(x)\right) = m \Leftrightarrow \underline{\hat{y}}(x) = \frac{m}{x}.$$

CASE.2: $x \geq x'$

$$\phi_0\left(x', y_{inf}\right) > m \Leftrightarrow \underline{\hat{y}}(x) = y_{inf}.$$