1 Introduction

This thesis has been developed in the scope of fuzzy logic and its application in the field of modeling. Here we will introduce the motivation for this work by detailing the need of working in this field, the objectives we search, the other people's previous work which can be found nowadays and our main contributions.

1.1 General background

For years fuzzy modeling has been one of the most relevant issues in qualitative analysis, benefited from the linguistic capabilities of fuzzy logic. In this sense let us remember the early ideas of Zadeh [127] when in 1973 he stated with the *Principle of Incompatibility* that *"as the complexity of a system increases, our ability to make precise and yet significant statements about its behavior diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics. It is in this sense that precise quantitative analyses of the behavior of humanistic systems are not likely to have much relevance to the real world's societal, political, economic and other types of problems which involve humans either as individuals or in groups"*.

In order to solve this situation, Zadeh proposed a linguistic modeling in the place of quantitative modeling [127] from which he suggested the *"1) use of so-called linguistic variables instead of or in addition to numerical variables; 2) characterization of simple relations between variables by conditional fuzzy statements; and 3) characterization of complex relations by fuzzy algorithms"*.

After reading Zadeh's work one appreciates its valuable formal algebra able to compute with words which has made it suitable for the analysis and synthesis of complex systems where many variables have to be considered.

Remember its multiple applications. This work will point to this field by emphasizing the interests of Zadeh's original work for computing with really intelligible models.

Many authors have concentrated their efforts in achieving models with a high accuracy in terms of error and by doing so they have omitted the linguistic capabilities of the fuzzy systems. In fact, to obtain high degrees of intelligibility and accuracy is a contradictory purpose and it is difficult to find the proper trade-off between both properties because, in general, one of them prevails over the other. For this reason, sometimes fuzzy modeling is divided into precise fuzzy modeling and linguistic fuzzy modeling.

Since then, several methods able to obtain fuzzy models have appeared, most of them based on clustering techniques. Among the most popular ones we can mention Fuzzy C-means [4, 85], where after clustering the transfer function one can approximate the necessary fuzzy sets to model the system. This method or similar ones like Possibilistic C-means [64] have been the core of different alternatives when working with fuzzy models. Some authors combine clustering techniques with genetic algorithms in order to obtain a fine accuracy [20]. In other methods ellipsoidal regions for the clusters are defined from which they obtain a good performance but at the price of increasing the computational cost [1, 22]. Alternative methods search for the model's minimum entropy [57].

Other investigators have directly searched for the model with the lowest possible error. The most popular method for long has been the use of least-squares when placing the fuzzy sets [112, 109, 110]. Another alternative was the use of least-squares together with gradient descent techniques like ANFIS [47, 50]. All these methods have absolutely forgotten the intelligibility of the resulting models in spite of being very accurate.

For this reason most of them have used linear equations in the structure of the model (Takagi-Sugeno's model) instead of fuzzy quantities (Sugeno-Yasukawa's model). In general these methods fulfill the premise of diminishing the errors committed within the resulting model but unfortunately they are often excessively complex to be understood, which is a pity, as it does not take advantage of one of the main virtues of fuzzy systems. In general the more precision, the more complexity, and thus, less intelligibility. Unfortunately, for years we have witnessed the fight to obtain the lowest error while forgetting Zadeh's early ideas.

In fact many years ago Michio Sugeno [111] suggested the need to dissociate the concepts of fuzzy modeling and qualitative modeling, referring to the first just as *a system description with fuzzy quantities* because nowadays several fuzzy models found in literature are not easily comprehensible. Later he successfully solved the problem of converting a fuzzy model into a qualitative model, when the first is based on Zadeh's premises, that is, if it is structurally simple. Nevertheless only a few researchers have worked towards intelligible fuzzy models since then.

Recall that the initial philosophy of fuzzy logic was to be the bridge between human understanding and machine processing. In this challenge, fuzzy model's ability to express the behavior of the real system in a comprehensible manner acquires a great importance. And in this sense, few but some authors have been working for years in this direction in order to recover the premise of giving a good linguistic interpretation in fuzzy modeling or at least a trade-off between accuracy and intelligibility when designing a fuzzy model. Some of these efforts are summarized in [12].

Therefore, in the light of these arguments our efforts have been devoted to the discussion of a new approach to fuzzy modeling able to obtain a good initial intelligible model to undergo further refinements based on qualitative analyses in a simple and intelligible manner. Thus, observe that we will focus this work not only on the final model but also on the method itself.

1.2 Problem statement

The whole procedure for modeling a process or a system has several stages. Nowadays the most accepted procedures consider the following aspects [73] shown in figure 1.1.

- **Data preparation.** We must prepare the data by using filters or transformations in order to find those variables which best explain the process and also its best samples. Despite probably being the most important step, this point will not be considered in this work.

- **Model selection.** We must choose a structure relating the variables. Among the different alternatives in fuzzy modeling, we will work with a Sugeno-Yasukawa's model because of its linguistic capabilities. We will work on this step, by deciding the number of fuzzy sets we need to achieve the model.

- **Parameter estimation.** We must estimate the model's parameters. In the case of fuzzy modeling, these are basically the position of each fuzzy set and the linguistic rules. This work will also deal with this step.

- **Model checking.** We must check the model in order to validate it. If the result is inadequate then we must go back to the previous stages.

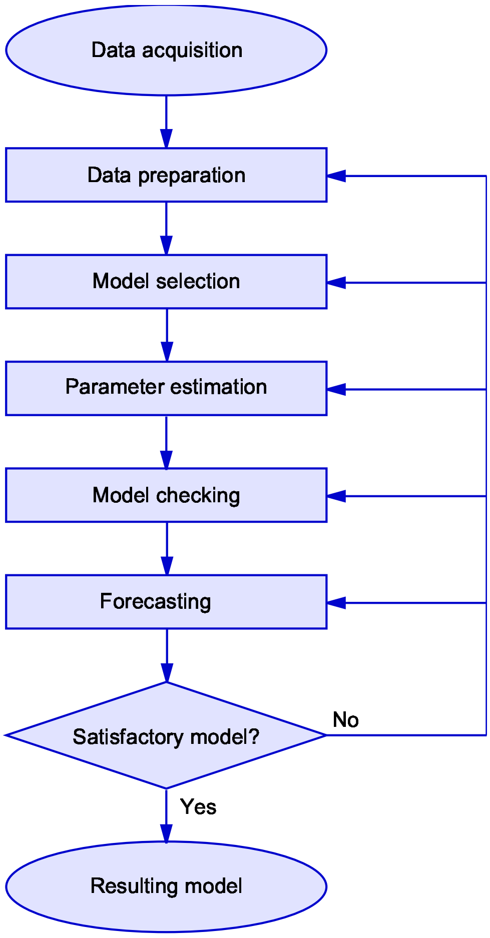

Figure 1.1: Typical modeling procedure.

- **Forecasting.** This is for what the whole procedure is designed for. When we have the model, it is implemented in order to compute forecast.

Thus, in this thesis we will present a set of heuristic criteria devised basically to address the model selection and the parameter estimation. In particular, we are able to determine the number of fuzzy sets, place them in the universe of scope and propose a set of linguistic rules related to them. We will obtain a methodology with a low computational cost but still with a similar performance to other solutions.

We do not seek to conclude that our method is better than others but to obtain an acceptable error in comparison while maintaining the linguistic capabilities of the fuzzy model. In fact, with this methodology we will be able to choose the precision of the model and consequently its degree of intelligibility.

From the previous comments, we summarized our objective as:

> **The search of a methodology which must be simple, intelligible, easily tuned and able to build a comprehensible fuzzy model from which one can easily obtain an understanding of the system and undergo further refinements or analyses.**

1.3 Major lines of work

Despite the existence of a larger number of contributions related to precise fuzzy modeling, in the last years a new tendency is shifting the objectives of fuzzy modeling towards linguistic fuzzy modeling, or at least towards the search of a satisfactory trade-off between intelligibility and accuracy [89, 45, 106, 107, 55, 114, 115, 123, 3, 94, 100, 43, 36, 5, 88]. Here we will cite the major lines in this tendency. Anyway one can find good reviews in [36, 12, 13].

The intelligibility of fuzzy models is achieved by assuring some properties that ensure the good interpretability of results. These properties which will be explained in detail in the following chapter, imply the use of different constraints during the design of the model. In some cases, they are applied only after the model has been obtained with precision fuzzy modeling in order to improve its intelligibility.

Between the different techniques able to deal with these constraints, evolutionary algorithms have been the most applied technique for a long time. Among different evolutionary methods, genetic algorithms have been the most popular because they allow the simultaneous optimizing of the parameters, the rules and also the structure. Nevertheless they are quite complex in terms of computational cost and furthermore they must be tuned in every problem.

In this sense genetic algorithms have been very used in order to obtain a rule reduction [30, 44, 102, 42, 14, 65, 104, 101]. Anyway this has not been the only technique. For example interpolation techniques between neighboring rules have also been considered [60]. In a very similar way some researchers have used alternatives focused on the cooperative coevolution paradigm [96, 91, 92].

Another popular alternative has been the use of the linguistic hedges proposed by Zadeh [126]. For this purpose several techniques have been considered, for example genetic algorithms again during the design process, but in most cases this alternative has been applied after the model has been computed in order to improve its linguistic intelligibility [6, 84, 35, 9, 71, 66, 76, 17, 10].

Other methods which can be found are the merging of membership functions based on a measure of its similitude [105, 27, 53, 28, 37, 108, 38], the dynamic change of the membership functions during the process [41], the use of cooperative rules [11] or relational rules [33], or even a mix of different solutions [34, 15].

1.4 Original contribution

From the former sections one can notice how this work is oriented towards a scientific research, which in fact should have been analyzed in depth since the origins of fuzzy logic. Nevertheless, and as we have already explained, most of the research on fuzzy modeling has not focused on the aspect of intelligibility.

In the previous section we have shown most of the few investigations which have tried to obtain fuzzy models with an adequate trade-off between intelligibility and accuracy. Nevertheless, they are computationally quite complex or focus their efforts on improving the intelligibility of previously computed fuzzy models.

With this work we propose a whole methodology providing intelligible fuzzy models directly from the input-output data and by treating most of the aspects related with the process of modeling. Anyway the method only assures intelligible models in a local manner, that is, rule by rule. Thus, if we wished for a whole intelligible model we should consider some of the other techniques which have been cited.

Furthermore, our methodology will be able to be adapted based on the nature of the problem or the user's preferences.

1.5 Outline of this thesis

Apart from the present introduction, the thesis is divided in three more chapters, the conclusions and the appendices.

The following chapter explains the necessary fuzzy logic concepts to understand the rest of the work. This chapter is explained in the formal manner that one may expect in a thesis but also with examples and didactics that may improve its understanding. The themes considered are not only to do with the working of fuzzy models but also with those aspects which can evaluate their intelligibility and accuracy.

The next chapter explains the methodology we propose in detail. Our solution combines new ways of dealing with data by means of other techniques of a different nature which have been adapted to our method. We explain all the steps and techniques and we also argue every decision and compare them with other alternatives.

The final chapter presents the results obtained with our methodology when applied to many problems that in most cases are typical benchmarks. We take advantage of these examples in order to emphasize several aspects of the methodology and to verify if the objectives have been accomplished. Many of the results are shown directly through the graphics resulting from the implementation of the method with Matlab.

Finally we facilitate the critical conclusions of the thesis in order to open future improvements. Thus, here we do not only explain the advantages of this method but also its problems.

At the end of the thesis one can find the references of the cited works, the list of articles published on this work and also the whole algorithm of our method.