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Tesi Doctoral

Mathematical learning and language use:

Perspectives from bilingual students in a context of problem solving

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Abstract

The migrant movements throughout the world and the politics around language diversity in many countries have promoted an increasing presence of multilingualism in the mathematics classrooms. There are students who face the challenge of learning mathematics in a language that is not their home language. To further understand the joint learning of mathematics and language, we need to reflect on the processes that the students develop when solving mathematical tasks, and to analyze how they see their involvement in such processes.

The main focus of this PhD study is on the language and mathematical practices by Spanish and English bilingual students in California, United States. In this context, the current policy of “English only” does not facilitate the use of the students' languages in the classroom, except for the case of those who are English dominant. Our assumption is that obstacles to the use of the students' languages may become obstacles to the learning of mathematics.

The PhD study attempts to examine diverse resolutions of mathematical tasks by bilingual students who are in the process of learning the language of instruction. We also consider the perspectives from these bilingual students on the use of their two languages in the resolution of the tasks. To achieve these goals, individual questionnaires with four mathematical activities were issued to students of a middle and a high school. Complementary task-based interviews with students were videotaped and audio recorded.

Several findings have been obtained as regards to connections between the language use and the mathematical practice. It has been documented, for instance, that the students report the use of their two languages for either the oral register or the written one. Moreover, the analysis of the visual mode in the statements of the tasks is particularly interesting as it informs of the attribution of mathematical meanings. On the other hand, the students in the sample do not say to experience the combined use of their languages as a difficulty or an obstacle. Many of them do not even report the use of English when they indeed do it while writing short comments. This leads to one of the major conclusions: the phenomenon of invisibility that frames the students' experience of their language use during their involvement in individual mathematical practices.

Acknowledgments

Love, and do what thou wilt: whether thou hold thy peace, through love hold thy peace; whether thou cry out, through love cry out; whether thou correct, through love correct; whether thou spare, through love do thou spare: let the root of love be within, of this root can nothing spring but what is good (St. Augustine of Hippo ¹).

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¹ ► Extracted from http://en.wikiquote.org/wiki/Augustine_of_Hippo. Consulted September 2012.

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Abbreviations

BICS	Basic Interpersonal Communicative Skills
CALP	Cognitive Academic Language Proficiency
CLT	Concluding Language Themes
CMT	Concluding Mathematical Themes
En-Sp	English-Spanish class. This is the class for the middle school students, where both languages were used during instruction.
HS	High School
L1	First language or home language
L2	Second language or formal language of instruction
LoLT	Language of Learning and Teaching
MS	Middle School
RME	Realistic Mathematics Education

1 Introduction

The researchers and the objects of the research necessarily have a close relationship: the researchers are not considered as strangers in the researched cultures (Barwell & Kaiser, 2005).

In this Chapter we comment on the high level of personal and academic motivation in the initiation and development of the PhD study, “Mathematical learning and language use: Perspectives from bilingual learners in a context of problem solving”. We begin this chapter by mentioning some issues of personal motivation. Later, we academically situate the work in relation to other works in the field of mathematics education and language diversity. After this, we introduce and explain the research question and the two goals that guided the investigation. The last section provides a summary of the structure of the manuscript.

This PhD study is part of a broader study, namely Project EDU2009-07113, “Researching the development of discursive competences in the mathematics classroom”. It is also a product of the Doctoral Program of Science and Mathematics Education Research of the Department of Mathematics and Science Education, at the Universitat Autònoma de Barcelona, Catalonia-Spain. Under the frame of the Project and the Doctoral Program, the ultimate purpose of this research is to better understand the impact of language diversity in the learning of mathematics by students whose dominant language is not the one of the instruction. Throughout the whole investigation, it is assumed that the social and the cultural dimensions play a vital role in the accomplishment of successful mathematical learning trajectories.

1.1 Personal motivation

Catalonia, a region in North Eastern Spain, is officially and de facto a bilingual community. This is why most people in the country (including myself) have been exposed to two languages, Catalan and Spanish, since early childhood. Furthermore, English is a regular school subject in the Catalonian school system. Thus youth people manage at least three languages with more or less fluency, and contributes to the creation of several multilingual scenarios.

When I was a student at school I had the opportunity to take an English summer course in the United Kingdom. It was by that time when I first felt that fluid communication was not

always guaranteed –or at least not in the way that I had wanted– in the interaction with the English family I lived with or with other English people, most of them being monolingual speakers. The summer course had not academic implications for me, and the communication difficulties were not a hard experience in terms of social and school promotion. But that period represents an early contact with the use of a language that was not sufficiently well known by me.

I actually came to experience myself the fact of learning in a language that is not my mother tongue when I took part of an international exchange program and went to Paris, France, to complete my degree in Mathematics. I knew in advance that language would be a difficulty because my proficiency in French was only average. However, I did not encounter so many obstacles as I had expected when I started learning through what was my fourth language. My knowledge of Catalan helped a lot. Catalan and French are linguistically close languages and, besides, I was given most of the information written on the board, which gave me considerable additional time to read and understand the messages. In the lessons, the number of students was small so that the teachers could assist us on a regular basis and attend most language demands. This experience would have not been the same without the enthusiasm I projected from the very beginning of the lessons and which was maintained until the end of the year. The enthusiasm, however, was a resource that would not substitute the need for didactical and pedagogical resources in the different teaching and learning situations.

An important shift in my interpretation of the role of language issues in mathematics education, primarily the relationships with the learning and teaching of mathematics, came when I enrolled the Doctoral Program of Science and Mathematics Education Research at my University. I was positively surprised to discover the existence of investigations informing to what degree mathematics performance and achievement in a school environment are dependent on the use of languages and on the cultures that the different students bring with them. My academic background was in Mathematics and I had not reflected earlier on the idea that each culture and each student could have a particular way to observe and interpret the world, including the world of school mathematics.

My perspective changed again when I more recently went to California to teach mathematics in English. During that academic year I experienced how important it is not only to well manage the languages of the students and the mathematical register through those languages, but also to follow the challenging variations of any language and register being used by the different participants in the mathematics classroom.

Even if I could anticipate some of the difficulties that might arise in the teaching and learning of mathematics from the papers I had read and the seminars in which I had participated in the context of Project EDU2009-07113, I soon was shocked by what I observed in the mathematics lessons. According to my classroom informal observations as a teacher, language was experienced as a barrier by many students. The Mexican and the North-American students had language difficulties in the interaction, and I also had similar difficulties. On the one hand, my Spanish was a different variant of Spanish with respect to the Mexican students' Spanish. On the other, my English was not as good as I had wished.

Moreover, some of the actions and views of the teachers in that school were shocking for me. They recognized the language and cultural challenges that the students of Mexican origin in the North American context should face (inside and outside the classroom) as some of them were of Mexican origin too. Among the teachers that were aware of such difficulties, however, I could see different levels of effort in the search for didactical and pedagogical resources that were oriented to facilitate the learning of mathematics. In informal conversations with them, some said to punctually “allow” the use of Spanish in their classrooms despite the “English only” language policy. That was a learning resource that the teachers were offering to their students and a powerful action in terms of the reaction to the restrictive language policy in the State of California.

Many of the experiences in California have been crucial in my current positioning, along with the knowledge derived from the literature and some preliminary analysis that I conducted with groups of English and Spanish bilingual students. Of course this is not the whole picture of the personal trajectory that has led me to choose mathematics education and language diversity as a research topic for my PhD work. What is actually key in my trajectory is that now I am much more aware of the importance of considering the language of the students from a didactical and a pedagogical point of view instead of taking a deficit perspective.

At present I am a mathematics teacher in Catalonia with a strong interest in prompting a linguistically responsive culture in the mathematics classroom. I have to daily face a quite new situation: the teaching of mathematics to many late arrival newcomers from other parts of the world who have entered the Catalonian school system with a first language that is not of the Romance family. The official language of instruction, Catalan, is quite different from many of my students' languages who are instead highly proficient in Urdu, Punjabi, Arabic, Chinese, etc., and who have very rich funds of knowledge.

1.2 Academic justification

There are many factors which have contributed to the emergence of multilingual education contexts. In nations like the United States or Australia the increase of immigrant people coming from other countries for better opportunities has brought more languages to the classroom. For example, Latino students are a growing population in the United States (Moschkovich 2007; Secada, 1992). Moschkovich (2005, 2007) focuses on students of Mexican origin in the North American school system. The work by this author has contributed to the development of a corpus of research that is oriented to normalize the views on Mexican students in that country. Moschkovich (2011) suggests that bilingualism should not be seen as a problem but rather as a resource for the learning processes.

A similar situation occurs in Catalonia. In addition to the two official languages (Catalan and Spanish) there has been an increase in the number of Spanish dominant students coming from other parts of Spain and countries of South America. Furthermore, there are also immigrant students from Morocco, Eastern Europe or China (Civil & Planas, 2012) arriving to the country. Vietnamese students in Australia (Clarkson, 2006) are another example of the migrant movements and its representation in terms of multilingual schools and classrooms.

Some countries have a multilingual society with many groups speaking different languages so that multilingualism is not directly attached to migration. In South Africa there are up to eleven official languages but as a result of colonization processes, English is de facto 'the' school language (Setati, Molefe & Langa, 2008). English is seen as the language that provides access to social goods and higher education. Papua New offers a similar situation with about 820 living languages, the most languages per head of population in the world (Clarkson & Muke, 2011) and English also the language of instruction.

In the last years there has been a growth of research in the topic of mathematics education and language diversity. Importance and significance attached to multilingualism have been increasing up to the point that the on going 21st ICMI Study is dedicated to this topic (Setati, Nkambule & Goosen, 2011). The choice for the 21st ICMI Study reflects that several groups of researchers are committed to language issues, but also that a lot of work still remains to be done.

On the other hand, in the last years there has been a shift in many practices inside the mathematics classrooms. Many teachers have decided to take the road towards a pedagogical approach that promotes more participationist practices on the side of the

students and less expositions on the side of the teacher. The focus is not so stressed on procedures and concepts only, and work in small groups is considered to be very important. In this context, bilingual learners bring their knowledge through two or more different languages, and shape the mathematical practices in particular ways. Barwell (2005) claims that it might be problematic to teach in a language that is not mastered by the students especially whether the shift towards more participationist practices has not taken place.

Like ours, many studies are centered on the interaction between language diversity and mathematics education from the point of view of the student. Those by Barwell (2005, 2009a), Moschkovich (2007a, 2007b) and Setati (2006), to only say a few, are paradigmatic of this approach. Nevertheless, it has been difficult for us to find investigations that focus on data from the students' individual mathematical work in non interacting situations with peers in the multilingual classroom. This research is expected to contribute to the growth of this research topic.

1.3 Research question and goals

Since the starting of the research design, we wanted to work with bilingual students learning mathematics in classrooms in which there is only an official language of instruction. Bilingual students who do not master the language of instruction may use different strategies in comparison to those of their monolingual peers to overcome some of the difficulties they face in the resolution of mathematical tasks. Many other differences also exist: different levels of English proficiency may be observed by the either implicit or explicit comments of the students themselves. This is why some students with a good management of English were included in the sample. Some other bilingual students in the sample are newcomers to the North American school system. By combining inputs of the new school system with those learned in Mexico, the new insights of their personal experiences can inform the development of mathematical practices.

Taking into account the global sample that is of interest to this investigation and the focus on the students' mathematical learning, two primary research goals were stated. The first goal is the following:

Goal 1. To examine resolutions of mathematical tasks by bilingual students who are in the process of learning the language of instruction.

For this purpose a task-based questionnaire with four mathematical activities was designed according to the school training of the high school students, as reported by some of the teachers in the school. The inclusion of some middle school students was later considered. Such inclusion was appropriate to overcome certain difficulties with broadening the sample. Again, the middle school teacher judged the mathematical activities in the questionnaire as appropriate.

Special attention was paid to the form in which the activities were presented to the students (with a deliberate inclusion of the visual mode in some statements). Furthermore, it was decided that one of the activities in the questionnaire would be similar to what had been done in the high school class. A detailed analysis of the solving process of each student was carried out for this purpose, and attention was paid to the concepts and procedures involved in the different phases of resolution.

After solving the four activities, individual interviews were planned to gain a closer insight to the resolutions and achieve the second goal:

Goal 2. To examine perspectives from these bilingual students on the use of their two languages in the resolution of the tasks.

As commented above, there are many studies that focus on the use of languages when doing mathematics in the classroom: either taken as a whole, when working in small groups or as a combination of both. Nevertheless, we have not found studies in the field that relate the solving of mathematical tasks to the use of language by individuals -on their own and with no external help-. Even if the interviewer was present during the resolution of the mathematical tasks, the students made at least a first attempt to solve the tasks on their own. Some of them asked for concrete aspects but at that point they had already tried to solve the problem and were on the path to doing it.

The views that the students have developed around the use of their two languages should be taken into account as a starting point to construct language resources for the teaching and learning of mathematics. It is therefore very relevant to explore such views and examine whether there are common patterns that favor the mathematical learning or, on the contrary, that make it more difficult. Not only the views but (when possible) also the actions are analyzed in the manuscript.

The two goals that have been already introduced constitute a practical way towards the exploration of the following research question:

Research Question. *Which are some of the relationships emerging from the interaction between the mathematical learning by bilingual students and their use of their two languages?*

The research question attempts to balance the dual focus on the mathematics and the language all throughout the investigation. It is assumed that the examination of *resolutions of mathematical tasks by bilingual students who are in the process of learning the language of instruction* (Goal 1), together with *the examination of perspectives from these bilingual students on the use of their two languages in the resolution of the tasks* (Goal 2), will provide relevant information about relationships emerging from the interaction between the mathematical learning by the students and their language use.

1.4 Structure of the manuscript

So far, I have been using a first person singular to speak about my personal and academic experience. On the other hand, a first person plural has also been used instead of the first person singular to make the reader participant of the narrative, but also as a way to particularly acknowledge the theoretical contributions of my supervisor. For the analysis of data, however, an impersonal form in the writing has been the preferred option in order to let data “speak”, even if I was the interviewer in the conversations with the students.

Each chapter starts with a cite in the field that is illustrative of an important aspect that is later developed in the text. Then there is a description of what can be found, like it has been done in this Introduction. Here we can find a descriptor for each of the sections: personal motivation, academic justification, research question and goals and structure of the manuscript. Chapters are divided in sections which, when needed, contain subsections.

Chapter 2, “Theoretical Framework”, is dedicated to summarize the literature review that served as an inspiration and to situate the research in reference to international works. The first section is an introduction to the research domain with special attention to the definition of language, multilingualism, mathematical register and mathematical discourse. Mathematical concepts and procedures as well as horizontal and vertical mathematizations are also addressed. In the second section there is a review of recent literature referring to mathematics education and language diversity, mainly in relation to multilingual learners. The 21 ICMI Study is a fundamental academic event that denotes the importance of the topic in the domain of Mathematics Education research. It is the

frame that includes some of the referred investigations. In the third section, the focus was to the bilingual learners of school mathematics. Finally language practices in the mathematics classrooms, such as code mixing and code switching, are theoretically explored.

The methodological aspects are commented in Chapter 3, “Methodological Approach”. All throughout the sections, variants of the constant comparative method are described, along with how and when the different parts of the analysis were carried out. This option contributes to ensure credibility and reproducibility. The first section is dedicated to describe the social context of the research and the participants that voluntary took part in it. The information was primarily collected through interviews with students.

The second section presents and discusses the instruments used to collect and organize the data. There is an exhaustive description of the students' interviews and the questionnaire they were asked to answer with the mathematical tasks. The instruments are presented in the same chronological order that resulted from the data collection process. Firstly there is a description of the first semi-structured interview that was used to collect personal data about how the students use their languages inside and outside school. Secondly there is a description of the questionnaire with the four mathematical tasks. Thirdly we show the second semi-structured interview. This is the most important data analyzed in Chapter 4 in relation to the students' use of languages. Both, the questionnaire and the second semi-structured interview are the main data for the analysis in relation to mathematical contents and practices. Finally there is a subsection where we describe in detail some of the mathematical and language aspects of each of the four activities. In this second section of the Chapter, some connections with a previous work (Reverter, 2008) are justified in relation to the design of the questionnaire and the nature of the tasks.

The third section describes in detail the instruments created for the analysis. They are the so called “first reduction” and “second reduction” objects. The process to construct the “third reduction” is also explained. We explain how each component of the instruments is related to the goals of the investigation and their achievement. The chronological itinerary that leads to the fulfillment of each of its parts during the development of the analysis is detailed for purposes of reproducibility. The results coming from the application of these two instruments are presented on the following Chapter.

Chapter 4, “Discussion and Findings”, contains the different rounds in the analysis of data as well as the primary results of the analysis. The mentioned first and second reductions for each of the students in the sample are listed on the first section of the chapter. The first

reduction contains the transcription of the recordings of the interviews with the students, together with the scanned resolutions of the mathematical tasks.

Moreover, there is a mathematical and language analysis of the solving process. The oral and written comments that the students made are also included. The second reduction summarizes the most relevant parts of the first reduction and offers a list of themes that resulted from the analysis of the mathematical and language aspects in conjunction. The first and second reductions become a pair of analytical instruments centered on one student each time. It is the third reduction (i.e., list of emerging themes) that groups the findings that come from more than one student. Such findings are placed in the second section of the chapter and each theme contains a description of an educational phenomenon and a potential explication for it. Extracts of dialogues from different students illustrate the descriptions of the phenomena.

Chapter 5, “Conclusions”, interconnects some of the main findings from Chapter 4 and points to some of the implications for future research. In the first section the conclusions on the design and the methodology are presented. Then the relationships between findings coming from the list of emerging themes are also summarized, with particular attention to the language and mathematical contents in the themes. Finally, some practical implications for the learning and teaching of mathematics are included.

On the final pages, an “Index of objects” can be found. It helps to quickly identify the objects used in the elaboration of the manuscript. Tables, pictures, and graphics have been grouped under the generic name of *objects*. We can also find the list of “References” that have been read and explored for the elaboration of this work. In the case of electronic papers, the latest accessed date is provided. A CD with the electronic version of this work can be found in what has been called “Electronic data”.

2 Theoretical framework

Content learning is inseparably bound up with language learning and vice versa (Barwell, 2005, p. 207).

In this Chapter we present the most relevant investigations which provide a framework for ours. The ideas presented here have guided the analysis of the data, as it is explained in Chapter 3 and reflected in Chapter 4.

In the first section we present a picture of multilingual classrooms, with some of the problematics attached to language diversity. We provide the definitions of language, multilingualism, mathematical register and mathematical discourses. Furthermore, we shift our attention to mathematical procedures and concepts, as well as to the concepts of horizontal and vertical mathematization. These notions will be used in the analysis.

In the second section we describe many researches performed in multilingual mathematics classrooms, mainly in contexts such as South Africa, Papua New Guinea or India. We also include researches centered on the role of the teacher in multilingual classrooms. They illuminate some of the practices that bilingual students have (most of the times bilingual and multilingual learners reflect similar practices).

In the third section the researches that are closer to the focus of this dissertation – centered on bilingual learners of mathematics– are presented.

Finally the fourth section closes the present Chapter by paying attention to practices observed on bilingual students in the mathematics classrooms, such as code mixing and code switching. Researches that directly point to the benefits of such practices are also reported.

2.1 Introducing the research domain

In the last recent years people has been increasingly moving all over the world in search for better opportunities for them and for their families. This has resulted in a raise of the number of multilingual students in classrooms, where this was not common before. In some territories, though, this linguistically diverse situation has been present since many years now. This is the case of historical bilingual regions (Catalonia in Spain, Wales in the United Kingdom) and countries with more than one official language (Belgium, Canada, South Africa).

On the one hand, policies can be used to support minority languages (Peru); on the other hand, the use of some languages is restricted by law (some States in the United States). Furthermore, on some countries there are many official languages and having bilingual students in classrooms is not a new practice. South Africa, for example, has eleven official languages and it is usual to find students with three different home languages in the same classroom. Moreover, sometimes learning occurs through an external imposed language. Thus, in South Africa, English is valued as the high status language to gain access to jobs and to reach the dominant class. In other societies some subjects are taught in a foreign language to promote the acquisition of such language, as it happens with English in the Czech Republic. Still some other countries have language policies for teaching and learning that promote language changes across the different levels of the educational system (primary, secondary, tertiary education) or where most alterations on the language of instruction have taken place during particular historical episodes (Algeria, Pakistan).

So language diversity in the mathematics classroom is primarily caused by historical multilingualism, migration, colonization or globalization, among other factors. Then these languages can be second, third or even additional languages, minority oppressed languages or dominant languages; affected by cultural, politic and social views (Setati, 2006). But what is exactly meant by 'language' in the context of this dissertation?

So far, the term *language* has been used without providing a definition. What does it lay behind such a common term? The online Oxford Dictionary² gives a total of three different meanings for the entry:

1. The method of human communication, either spoken or written, consisting of the use of words in a structured and conventional way:

a study of the way children learn language

[as modifier]:

language development

- a non-verbal method of expression or communication:

body language

2. A system of communication used by a particular country or community:

the book was translated into twenty-five languages

- *Computing* a system of symbols and rules for writing programs or algorithms:

the systems were developed using languages such as Fortran and Basic

3. The style of a piece of writing or speech:

he explained the procedure in simple, everyday language

- the phraseology and vocabulary of a particular profession, domain, or group:

legal language

² ► <http://oxforddictionaries.com/definition/language?q=language>, consulted June 2012. Bold words are kept from the source.

- (usually as **bad/foul/strong language**) coarse or offensive language:
the film contains some violence and bad language

Within the first sense, the definition of language is centered on the verbal form, even if later the gestural language is also considered –it is included within the provided example–. The graphical language, either concrete or abstract (Reverter, 2008), is also included here.

The second meaning for language draws on its use through different countries or particular regions, pointing out the various systems that evolved through the time and that nowadays –even if some of them are not lively used, such as Latin– let communities communicate in a more or less normalized way with its grammatical, syntactical and orthographical rules.

The third sense opens the door to the specific vocabulary used by particular collectives, such as those formed by agriculturists, mathematicians or researchers. It is sometimes limited to the application within the particular field of practices of the community. At the same time this sense relies on the previous one, e.g. Chinese and Tanzanian doctors may use different terms to refer to the same surgical instrument.

The phrase “speak the same language” –exemplified in the Oxford Online Dictionary– makes us aware of the meaning that each individual assigns to the words and sentences. This meaning may vary slightly or be quite different among people and, at the same time, is based on other terms (which theoretically could derive in an infinite iteration of terms' definition to understand the original concept).

As it can be seen with the example of 'language', a word can have different meanings. There is a problem when a word has a sense in one language but does not have it in another language. For example, the consultation of the Spanish word '*lengua*' in the online RAE Dictionary³ returns twelve entries. Six are reproduced here as they are enough for our purpose of showing the differences between the sets of meanings attributed to the words 'language' and '*lengua*'.

1. f. *Órgano muscular situado en la cavidad de la boca de los vertebrados y que sirve para gustación, para deglutir y para modular los sonidos que les son propios.*
2. f. Sistema de comunicación verbal y casi siempre escrito, propio de una comunidad humana.
3. f. Sistema lingüístico cuyos hablantes reconocen modelos de buena expresión. *La lengua de Cervantes es oficial en 21 naciones.*
4. f. Sistema lingüístico considerado en su estructura.

³ ► <http://oxforddictionaries.com/definition/language?q=language>, consulted June 2012. Bold text is kept from the source.

5. f. Vocabulario y gramática propios y característicos de una época, de un escritor o de un grupo social. *La lengua de Góngora. La lengua gauchesca.*
6. f. Badajo de la campana.

Senses 2 to 5 are included in the English meaning commented before. The first one, though, is slightly different, and refers to *tongue* in English. It is not surprising, as the original meaning of both terms, *lengua* and *language*, is precisely the Latin word *lingua*, which designates this part of the body. Sense 6 is definitely not considered among the English definitions. Hence the relationships among languages cannot be unequivocally addressed as some of the meanings are not always easily convertible.

As pointed by Barwell (2012), there is a dominant view of languages as unified and describable, as if there is a standard kind of English or Spanish, for which the rules can be determined. This idea is based on a unifying emphasis; for example when calling for immigrants to learn the national language. When speaking, though, individuals make a particular use of the natural language in each utterance. Of course, for the fluency of the act of communication, there may be some degree of uniformity on the use of the language, but diversity is essential to talk about new concepts and in fact it is what supports the process of developing new vocabulary, which in turn promotes new ways of viewing the world.

Now the concept of multilingualism can be shaped. Depending on the theoretical approaches, different ways arise about how to observe and describe the multiple languages interacting in a particular context. Sometimes multilingualism is defined upon plural monolingualism. That is, multiple languages are used discretely and distinctly even when people may speak several languages. So the limits between one language and another are quite clear. Then languages may be related with specific activities, institutions, contexts. If we take the contexts of school and home, one language may be seen as “for school” and another language may be seen as “for home”.

An alternative approach is to think in terms of hybridized multilingualism – the idea that languages are interwoven within the fabric of social life, to the extent that the boundaries between them can be hard to clearly identify (and, indeed, conceptually problematic). Of course, the nature of the hybridity, the emphasis given to different component languages, the values associated with ‘purity’ or qualifications in one or another language vary from region to region, as well as social context (International Committee of the 21st ICMI study, 2009, p. 4).

Such an insight for hybridized multilingualism is more appropriate for the current study. The hybrid language mix used within mathematics classrooms may be different to the mix used in bars or at home or in the street or... It is not to say that these two views (multilingualism as plural monolingualism or as hybridized multilingualism) are the two

sides of a coin, but still other groundings in between can be found useful to enrich practices of mathematics education.

It is time to see how mathematics and language are intertwined while framing the present investigation. We use the mathematical register to introduce such an interrelationship. *A mathematical register is made up of the semantics and syntax used consistently to describe mathematical ideas* (McDonald, 2011, p. 204). *Formal mathematical language refers to the standard use of terminology (mathematics register) which is usually developed within formal settings like schools* (Setati & Adler, 2000, p. 248).

Mathematical language can be used in both oral or written forms and comprise both formal and informal components. Mathematical discourses are the *ways of talking (about) mathematics, listening to mathematics, acting in a mathematics class or community, interacting mathematically, believing, valuing and using mathematics and /or the mathematics register* (Setati and Adler, 2000, p. 266). So a mathematical register may not exist in a particular language, but mathematical discourses in such a language do exist as the mathematical ideas need to be communicated among individuals as a result of social interaction.

When the language of schooling is English –as it is, at least partially, on the context of the present research– the mathematical register is made of words coming from two different sources: everyday English and Mathematics. Everyday English terms may have the same meaning when used within a mathematical register (e.g. growth). However they may have a subtle different nuance (e.g. between) or a strong different meaning (e.g. table). There are also words specifically belonging to Mathematics that rarely have a meaning outside of it (e.g. pronumeral). The first three examples are retrieved from the activities proposed in the questionnaire (see pages 59 and 60). There is not an appropriate example for the fourth case in our questionnaire. Only percentages or numbers belong specifically to Mathematics, but they have a rather frequent usage on the everyday life.

The difference between both languages that the sample of students participant in our dissertation have (Spanish and English) is not linguistically as big as it may be between other languages, such as Arabic and Italian or Russian and Portuguese. However, it is worthy to consider that in the case of huge differences between the language of instruction and the background language there is a greater chance for error to occur due to language differences rather than to innate ability (McDonald, 2011). Considering this, a statement with a large cognitive demand on the side of language can rest concentration and attention on the side of mathematics.

A major adopted point of view is that rather than consider bilingual learners in a position of deficit, their bilingualism is seen as an opportunity for growth. See, e.g., Civil and Planas, (2012) or Moschkovich (2005, 2011) for details on the idea of language-as-resource). Essien (2011) describes how the approach of language-as-resource dominates the international research paradigm on mathematics education and language diversity: *In recent years, researchers and educationists have paid increased attention to multilingualism as a phenomenon which relates positively to cognitive development, flexibility and the promotion of academic achievement in learners* (p. 99).

Norén's (2011) work shows this point of view (language-as-resource) which is aligned with a view of multilingual speakers in a wider, positive context of teaching and learning. Her research is about how agency influences discourse switches when students solve a statistical problem orally. It is assumed that broader social discourses affect the teaching and learning of mathematics in multilingual classrooms: [...] *discourse and agency [are used] as analytic tools to avoid deterministic deficiency explanations of bilingual students' performance in the mathematics classroom, and to relate to broader social and political issues in Swedish society. I use the concept of discourse according to Foucault as a group of utterances which seem to be regulated in some way, and which seem to have coherence and a supremacy that can be related to broader social contexts. Agency is exercised in discursive practices. Sometimes agency is conscious; however sometimes people act as agents without being aware of the options (Cohen, 1994). This understanding of agency makes it possible to view students and teachers in mathematics classrooms as social actors in practices, in one way or another; agency can be active, but also passive and resistant* (pp. 294-295).

Sweden has about one fifth of students who speak a first language other than Swedish, Arabic being the most extended. The language policy is "Swedish only", though. The case presented by Norén is around a group of ninth grade students solving orally a statistical problem belonging to a national test, drawing on data from a wider study. The focus is a group of four students of a Swedish classroom where Arabic is also used. They are struggling with the meaning of a word, not making sense with the mathematical interpretation of the graphics provided. Norén concludes that discourse switches are affected by agency in the discursive practices of the classroom. When mathematical tasks are developed orally (group cooperation, teacher-student discussions...), students or teachers can take advantage of their bilingual position to overcome language obstacles.

So making students aware of the positive influence of the use of their dominant language has a positive effect when solving mathematical tasks. In the next section, more precise

and extended insights of the benefits of the use of languages in mathematics education are developed. Up to this point, one of the main assumptions of this research has been established: the effective impact of the choice and use of languages on the teaching and learning of mathematics.

Setting aside the focus on the language aspects of the mathematics for a while, now the notions of mathematical concepts and procedures will be discussed, as they are used later on the analysis. Over the last twenty years, mathematics educators have often contrasted conceptual understanding with procedural knowledge. More recently, these notions have been incorporated in the more global terminology of mathematical proficiency (Kilpatrick, Swafford, & Findell, 2001). To be mathematically proficient, a student must have:

- conceptual understanding—comprehension of mathematical concepts, operations, and relations
- procedural fluency—skill in carrying out procedures flexibly, accurately, efficiently, and appropriately
- strategic competence—ability to formulate, represent, and solve mathematical problems
- adaptive reasoning—capacity for logical thought, reflection, explanation, and justification
- productive disposition—habitual inclination to see mathematics as sensible, useful, and worthwhile, coupled with a belief in diligence and one's own efficacy (Kilpatrick et al., 2001, p. 5).

The equilibrium between all five strands is crucial for students to understand and use mathematics. Conceptual understanding allows a student to apply and possibly adapt some acquired mathematical ideas to new situations. Procedural fluency is related not only to the application of algorithms but also to the interrelationship on the steps made to solve the problem. The flexibility on its applications would be included in the strategic competence and the adaptive reasoning. This last mathematical ability is essential to get a global comprehension of the problem and explain it properly. Integrating all these components harmoniously in the different aspects of the school life –and also outside school– will help to promote productive disposition.

Rittle-Johnson & Star (2007) focuses on what they call three critical components of mathematical competence: procedural knowledge, procedural flexibility, and conceptual knowledge in their analysis related with equations, based on the ideas of Kilpatrick and his colleagues above commented.

Procedural knowledge is the ability to execute action sequences to solve problems, including the ability to adapt known procedures to novel problems [...]. Procedural flexibility incorporates knowledge of multiple ways to solve problems and when to use them (Kilpatrick et al., 2001; Star, 2005, 2007) and is an important component of mathematical competence [...]. Finally, conceptual knowledge is “an integrated and functional grasp of mathematical ideas” (Kilpatrick et al., 2001, p. 118). This knowledge is flexible and not tied to specific problem types and is therefore generalizable (although it may not be verbalizable) (p. 562).

Another consideration on the mathematical characterization of the solving process is related with the Realistic Mathematics Education (RME) model. RME was developed by the Freudenthal Institute in Netherlands. Two of its important points of view are that mathematics must be connected to reality and that mathematics should be seen as a human activity. Two types of mathematization help develop this approach.

In the horizontal mathematization, the students come up with mathematical tools which can help them organize and solve a problem located in a real-life situation. Vertical mathematization is the process of reorganization within the mathematical system itself (Zulkardi, 2000). The difference between these two types of mathematization is not always clear, and most of the times both occur simultaneously. Horizontal mathematization involves going from the world of life into the world of symbols. Vertical mathematization means moving within the world of symbols.

We now turn to describe some of the more important researches within the field of Mathematics and Language education that shaped our investigation.

2.2 Mathematics education and language diversity

Bilingual education's long history dates back to biblical times (Secada, 1992). Even this a dilated existence, this topic is of current interest, specially in the field of the Didactics of Mathematics. The International Commission on Mathematical Instruction (ICMI) organizes studies –each one including a conference– about a topic of world-wide relevance in the area. Each study involves an international conference involving from 50 to 100 invited participants. To promote further discussion on the area, a published volume is issued. At the present time –September 2011– ICMI Study 21 is still ongoing. The study conference, entitled “Mathematics education and language diversity” was held on São Paulo, Brazil on September 2011.

Given the growth of research in mathematics education and language diversity and the increasing importance an significance attached to language diversity and multilingualism, this ICMI Study is timely. Across the world, the teaching

and learning of mathematics occurs in context of linguistic and cultural diversity. How do we work with, and work within, this diversity to enhance the learning and teaching of mathematics? In particular, how can the range and complexity of learners' language backgrounds be most effectively used to promote their mathematical learning? These questions are central to this Study and the papers to be presented at the study conference focus on different aspects of these questions (Setati, Nkambule & Goosen, 2011, p. iv).

The ICMI Study 21 was announced in July 2008, as a reaction to the increasing amount of research on language diversity in the field. The International Committee met on February 2009 and issued a Discussion Document, describing what the study should be about, inviting contributions to the study and participants to the conference. Contributions were centered on one or more of the five themes addressed on the Discussion Document: 1. Focus on learning, 2. Focus on teaching, 3. Focus on teacher education, 4. Focus on mathematics education and society, 5. Researching mathematics teaching and learning in multilingual context.

Before commenting some of the more important ICMI Study Conference papers (as well as some other important ideas gathered within them) the focus is turned to a more general observation of the role that language plays on the human intelligence. Cuevas (1984) remarks that *intelligence, achievement, and personality instruments basically test language proficiency* (p. 138). This is important when a person is tested on the language that is not their mother tongue, and raises the question of the reliability of its results in relation to the language proficiency, even when the language is indeed their mother tongue. This observation may have major important consequences on the mathematics classrooms.

When using (or being able to use) more than one language, tensions may arise in relation to such a choice of language(s). Barwell (2012) refers to four kinds of tensions: 1) between school and home languages, 2) between formal and informal language in mathematics, 3) between language policy and mathematics classroom practice and also 4) tensions between a language for learning mathematics and a language for getting on in the world (having access to work, education, ...). Barwell uses the term tension in the *sense that each utterance simultaneously represents both diversity and uniformity. There is a kind of struggle between the uniqueness of each utterance and its conformity to the patterns of language* (p. 320) The referred tensions are not isolated and in fact most of the times more than one are encountered at the same time. Planas (2012) supports the idea of *orchestration* to integrate these tensions in a productive way for the learning of mathematics, as most of the times tensions are inevitable. The fact that both students and teachers are fully aware of them is the first step towards such orchestration and the

resulting benefit for the teaching and learning of mathematics in multilingual classrooms. These and other tensions are presented in this chapter through the work of many authors. Webb & Webb (2008) deepen on the potential of the code switching by training teachers on the exploratory talk in order to apply it to their classrooms. Exploratory talk demands learners to engage critically and constructively with other's ideas and justifications, with alternate hypothesis and joined consensus being eventually reached. Their study supports the benefits of exploratory talk to develop mathematical argumentation, recognizing that this methodology is not easy to implement and teachers need practice and guidance to correctly apply it into their classes. This results in teachers having an important role in the development of student's awareness and use of languages as a tool for reasoning, along with a collaborative and inclusive classroom climate. Webb and Webb note also that the line between the language of mathematics and informal talk should not be stressed, either by teachers or researchers, to allow students develop a more formal mathematical discourse.

Language has also implications from contexts other than education. Sometimes beyond the cognitive aspects there is an issue of power laying down. Setati (2006) shows how this is true in the context of South Africa, where there are eleven official languages but English is seen by parents, teachers, students... as the language that dominates society and which is needed to have access to higher education as well as to the sphere of work.

Whenever one uses a language to speak or write there is not only a transmission of the intended ideas, but a personal identity is created and situated in a particular activity. There are multiple studies that support the use of learners' L1 (Setati & Adler, 2000; Kazima, 2006; Moschkovich, 2005, 2007a). Setati's (2006) analysis, for instance, shows that English is predominant in the mathematics classrooms in South Africa, given its prominent extent in the society. The challenge then is to bring both, English and students's home language(s), together to better develop mathematical understanding. This endeavor is even more challenging in South African classrooms, where students have many different languages at home and where it is usual to find people who speak at least four languages. When students position themselves in relation to an epistemological access to mathematics contradictory discourses arise, as there is a need to have support on languages other than English as the LoLT (Language of Learning and Teaching) but at the same time students are willing to become proficient in the use of English and are aware of its importance to access social goods. In fact, some of the dilemmas of code-switching in multilingual mathematics classrooms do not necessarily need to be thought of as solvable (Setati & Barwell, 2006).

In another research Setati, Molefe and Langa (2008) use the notion of language as a transparent resource to point out that languages can be used together to support at the same time both mathematical and language learning. In general, an access to a practice relates to both, the visibility and invisibility of its resources. While invisibility relates to the integration into activity in a natural way –with no conflicts–, visibility is in the form of extended access to information. The use of language in mathematical classrooms must then be both visible and invisible. Visible in the sense that it must be seen and understood by everybody. Invisible in the sense that when exploring written texts and discussing mathematics it must not shift the attention from the mathematical task towards the use of languages. This distinction is not a dichotomy and using language as a transparent resource (visible and invisible) demands for both notions being played conjointly in a harmonious dance as a continuum. It is like the role played by technology when doing mathematics: technology does not play a central role but it helps to develop the mathematical task with the focus kept on mathematics.

In the research presented by Setati, Molefe and Langa (2008), learners were grouped in the classroom according to their home language, so they could share impressions through it, apart from the usual LoLT (English in the case of South Africa, which is also the case on the research context of this dissertation). Real life tasks, high cognitively demanding, were presented through both English and the corresponding home language (groups were made so that its components shared the same home language). This contributed to a proactive, deliberated and strategic use of their home languages. Results show that home languages (working hand in hand with English) were used as a transparent resource to support mathematical learning.

In the same line and still in the context of South Africa, Vorster (2008) reports the positive inputs students got when the use of Setswana was introduced as an additional language in the teaching of mathematics. Given that teachers and students were used to the employment of English in the mathematics class, most of the students found Setswana vocabulary difficult to understand. But they did not report that the use of both languages was an obstacle. On the contrary, they resorted to the other language when they experienced difficulties with one language. Help on Setswana was available through English/Setswana notes, a glossary and tests being provided in both languages. Even students whose first language was not Setswana experienced it positively. This is another example of how the proactive use of language beyond the formal and institutional practices based on the LoLT promotes language switches which help students achieve a better management of mathematical tasks.

Similar to Setati's (2006) findings, Clarkson and Muke (2011) present a study situated on Papua New Guinea. In this country, since mid 1990s, languages other than English are allowed for instruction in the first three years of schooling, with English being introduced on the third year. Papua New Guinea, with about six and a half million inhabitants and some 820 living languages has the most languages per head of population in the world. Clarkson and Muke's work focuses on eight teachers of this third educational year.

These teachers see English as having a privileged role on the teaching and learning of mathematics. Even if many researches support the use of home languages by multilingual students for cognitive benefits, the teachers of the study do not see it as a the main reason to encourage the learner's first languages use. It seems that the use of the home language is mainly used to promote mathematical English learning and English language learning.

The authors remark that more research would be needed to confirm that Pidgin and Wahgi languages are used to learn the dominant language (English) instead of exploring the nuances among languages. *Although all languages were important in the mathematical discourse in these classrooms, this analysis suggests that the teachers' primary aim when switching between languages was to promote mathematical English through the use of local language* (Clarkson and Muke, 2011, p. 250). Even if other languages are regarded as positive ways to express mathematical ideas, learning English remains crucial for a personal development in the modern society (because the use of English is widely extended). Another finding of the research is that the teachers didn't seem to stick to the use of a particular language (depending on the day they used more one or another language). Moreover, the choice of language did not seem related to the topics of the lesson either.

The findings by Clarkson (2006) seem to be confirmed elsewhere. In their ongoing investigation on a multicultural mathematics classroom in India, where English and Hindi languages are involved, Choudhury and Bose (2011) speculate that the fact of switching from one language to another is not discrete, but it occurs as a continuum. In their tentative conclusion they see language switch as a pedagogical resource for both teachers and students. Teachers can benefit from the metalinguistic and intrinsic characteristics of languages while students can reduce their cognitive load by making use of their language of comfort.

On a research that relates on a considerable number of students, Njuari and Setati (2011) draw on data from a wider study to be conducted in Kenya. In this country the majority of children use multiple languages and risk to fail to develop fluency in English or their home

languages to full proficiency prior to schooling. Data is obtained through four different years from the national exams done in grade 12 (prior to University) in English and Mathematics, from particular multilingual areas of Kenya. Following Cummins (2000), Njuari and Setati (2011) sustain the hypothesis that *the level of L2 competence a bi/multilingual learner attains is partially a function of the level of competence of L1 developed outside school, at the time when intensive exposure to L2 begins. This then implies that the greater the level of language competence in a learner's L1 allows for a stronger transfer of skills across the L2* (p. 281). They speculate that grade 12 learners who switch between L1 and L2 while solving the mathematical tasks count on high cognitive abilities in mathematics. This had helped to overcome the problem of using uniquely one language. If each of the languages is taken individually there is a lack of vocabulary to pursue with the mathematical understanding. Njuari and Setati conjecture that such switches promote a deeper understanding of the mathematical concepts. But this could lead learners to an incomplete development of the language of learning and teaching (English). This may explain theoretically the higher scores in mathematics as compared to the English scores.

Webb, Webb and Foster (2011), through an innovative gathering data method, draw on teachers' experiences. Input comes from teachers who are in rather previously disadvantaged schools in Easter Cape, South Africa, speaking Afrikaans or isiXhosa at home. They were encouraged *to express their emotions serendipitously through poetry* (p. 440) in either language isiXhosa, Afrikaans or English. The interpretation of an isiXhosa speaking person, instead of having the poems translated, helped to have a better insight during the analysis. Here opinions are also reported.

Four dominant constructs were identified and appeared in most of the poems: issues of power, access, identity and language choices. Major importance was given to the learning of English by their students. However some poems showed the importance of the marginalization of their mother tongue and their cultural identities. Writing their own poems as well as reading poems written by others made teachers aware of the importance of their thoughts and feelings. They experienced that it was easier to express their thoughts in their home languages and comprehended students' situation, who have to understand both English and mathematical arguments in such a language while not being proficient in it. At the same time teacher feelings shifted from feeling disempowered and disadvantaged to being able to take control of the situation.

Setati and Adler (2000) present a study where a primary mathematics teacher draws on students' first language (Setswana). This allows her learners to engage in rich

mathematical practices. Thus, code-switching promotes conceptual discourses through the use of the learners' main language, and some of the learners could have not done so without using their main language. Using exclusively the LoLT (English) may risk in discourses remaining formal and procedural.

In the same paper, Setati and Adler study another primary mathematics classroom in a rural area where English is barely used outside school. Then the English discourses of the teacher are of great help for students to gain access to both English a mathematical discourse in English. Nevertheless, the pedagogical approach used by the teacher may be problematic in the pedagogical trends that focus on the students as having a central role as active learners of mathematics –and which discourage *teacher's talk*–.

Secada, Langer-Osuna, Avalos and Zisselsberger (2011) draw an ongoing investigation. The research aims are to help mathematical teachers who work with English Language Learners (ELLs) and to help ELLs engage with mathematically and linguistically challenging problems. The study is situated in Florida, United States, and involves 10 teachers, 2 per grades four to eight, who have a lot of Spanish-speaking English learners in their math classes. Besides, many students in these classes qualify for a free or reduced-cost lunches. Florida has an “English only” language policy. The school district, in order to fulfill the new State Standards has adopted a new textbook, which is not liked by parents or teachers, as basic math problems are turned into complex scenarios. All students have the pressure of high-stakes tests, but specially those who do not master English well enough.

This aforementioned research is still ongoing. In this work the language of mathematical textbooks is being analyzed. Discussion meetings are held in collaboration with teachers and it is expected that students shall be confronted with mathematical texts with different language degrees of difficulty and asked to explain how they understand them. Secada and his colleagues, through the analysis of classroom processes, expect to create a framework for assessing student learning and understanding both in real-time classrooms as well as on mathematical tasks. All of this work is focused on four topics that the district mathematical specialists considered more challenging according to the performance of students in such tests.

Sepeng (2011) uses a socio-cultural perspective, assuming that individual and collective processes are related. Consequently (unrealistic) responses to (real world) problems reflects the students' socio-cultural relationship with school mathematics. The study is conducted in South Africa, with 40 ninth grade learners. Data is gathered through test and video recordings conjointly with field notes, with a neutral participation of the researcher in

the task. Within the case presented, students do not struggle with the comprehension of the wordings' meaning, but with the sense given to *divide the money that it is fair*.

Through the study of the different discourses it became visible students' difficulty to assess the logic of the argumentation and also to provide useful arguments. Sepeng suggests that instead of just applying a well-known mathematical procedure (which are also considered as relevant) real world problems promote student's own making sense methods and that learners should be encouraged to explore the mathematical processes by themselves upon their cultural and personal beliefs. This should occur the other way too: students should be enhanced to use mathematics in real world situations.

Even when teachers are aware of the students' different backgrounds (including language and all their associated implications) and so they use pedagogical strategies to encourage the participation of all students by modifying the traditional way of teaching, it seems that other issues are an obstacle for the full integration of all the different mathematical views that are present in the classroom (Civil & Planas, 2004). In particular dominant students do not easily accept mathematical practices such as bringing personal backgrounds from their private lives into the solving process of a problem, while some minority groups try to do.

Up to this point some researches in relation with multilingual learners and/or which are focused on the role played by the mathematical teachers of bi/multilingual students have been presented. Now we turn specifically to investigations that inform about the interrelationship of mathematics and language of bilingual learners in the mathematics classroom.

2.3 Bilingual learners of school mathematics

Preceding the important corpus of research that has been developed in the last decades, Cuevas (1984) advertised that *an emphasis is placed on using language within a specific academic subject rather than on teaching the language skill needed for effective communication in general. Language is viewed as notions and functions to be mastered. In the case of mathematics, notions refer to the content-specific aspects of the discipline. Functions refer to general language activities (such as giving and receiving factual information or expressing and discovering opinions) and to language functions that are specific to mathematics (such as representation, discussion, translation, and verbalization)* (p. 139). English language learners in the mathematics classrooms have to

cope with the new language of mathematics as well as the new language in which mathematics is taught (English). Furthermore they have to get access to the classroom discourses (Setati & Adler, 2000).

Then special attention should be focused towards the mathematical vocabulary and other language-related skills (grammar, syntax). For example, word problems have sometimes a particular way to be written (Verschaffel, De Corte and Lasure, 1994). At the same time, though, learners should be taught to include real world knowledge when solving word problems in general (Inoue, 2005). Cuevas' warning, does not mean that the focus on the mathematics classroom should become a mere procedural training. Instead, structured activities, investigations and discussions that ensure mathematical understanding should play a central role on the development of both the mathematical content and the mathematical language. To facilitate movement from informal spoken language to formal written mathematical language there are two possible ways: encourage learners to write down their informal utterances and then to work on making the written language more self-sufficient, or else to work on the formality and self-sufficiency of the oral language and then finally write it down. This scheme is more complicated when students do not have the LoLT as their main language. Then this shift from informal oral language to formal written mathematical language is at three levels: from spoken to written language, from main language to the LoLT and from informal to formal mathematical language. Within these pictures many routes are possible to finally acquire written mathematical language.

Even beyond Cuevas' statement, bilingual learners of school mathematics have been described as supporting other practices. Some of the more prominent for this study will be detailed in the next section.

Molina and Ambrose (2010) present a study where first grade bilingual students are faced with problems of similar difficulty both in English and in Spanish separately. Language does not seem to play an important role when solving mathematical problems, even if a small improvement is observed when the problems (and the questions used to guide the solving process when a first attempt is not successful) are carried out through their native language. It looks like that presenting problems close to the students' personal experience is a more significative issue. They conclude, though, that the use of both languages on the mathematical class may be useful.

The word bilingual have already been used many times. We now provide a definition. Bilinguals are

the product of a specific linguistic community that uses one of its languages for certain functions and the other for other functions or situations (Moschkovich, 2005, p. 122).

This definition is based on a sociolinguistic perspective which stresses the social nature of language, considering not only that language is a cognitive product, but also the result of cultural and situated practices. *The proficiency of the languages and the social circumstances determine the contexts in which a bilingual has the opportunity to use one of the languages and how s/he is recognized when doing it* (Planas and Setati, 2009, p. 37). In this thesis the data analyzed is solved mainly individually by the students but there is the collaboration of the interviewer who offers support and who is present in the room during the solving process. So there is indeed some social interaction in some parts of the mathematical solving process. But even considered individually, each student carries on some practices and conceptions of a more social view about how the use of languages within practices need or must be developed –either in general or in particular (for example, when being in a classroom solving mathematical problems)–.

The above concept of 'bilingual' is aligned with the previous definition of multilingualism (see page 14). Definitions of bilingualism range from a fluency in two languages, as if one was a native speaker, to an alternate use of two languages. Nowadays, scholars studying bilingualism do not support the view of bilingualism as a native-like fluency, as the majority of bilinguals have an asymmetric management of both languages, using one language for certain purposes and the other language for other purposes. So it is rare being a bilingual equally fluent in both languages. Barwell (2009b) refers to this broader use of 'bilingualism'. *For that book, and in the context of mathematics classrooms, multilingualism refers to the presence of two or more languages (and so includes bilingualism). Such a presence may be overt or tacit* (p. 2). Planas and Setati (2009) see *bilingualism as a continuum, and assume that even when there is a minimal proficiency in the second language and effective communication result from the ability of the listener, not the speaker, the speaker may be considered bilingual* (p. 37).

Usually bilinguals have been viewed and assessed in comparison to monolingual individuals. A new shift, though, consists of consider bilinguals as a whole, with their particularities and considering the nature of their communicative competence per se. At the same time, bilingualism is an individual, social, cultural, historical and political phenomenon. In some environments being bilingual is recognized and valued, a sign of education. On the contrary, in other areas it may be attached to poverty or cultural deprivation. This is the case of Latino bilinguals in the United States, who have a particular history of a language minority. Some of them arrived to the United States as

immigrants, others are second, third or even a younger immigrant generation and others never immigrated or emigrated, but live in places that belonged in the past to Mexico and which have been recently incorporated to the United States. In the United States, bilingualism is not always considered to be a valuable resource. For example, Spanish is not a high-status second language and Latinos have often been described through deficiency models (Moschkovich, 2005).

Moschkovich (2005) also comments the notion of semilingualism. As it has been left aside and its use confuses rather than sheds lights on the understanding of the use of languages by bilinguals, here just a brief description of this terminology will be provided for informative purposes. The label of semilingual has been used when bilinguals are described as being not true, real or balanced bilinguals. But this aspect has not been proved empirically either, and any normal child living in a particular community has the potential to acquire effortlessly its language with a normal degree of variation and without instruction.

On the one hand, if a student just arrived to a country and s/he is learning the LoLT (Language of Learning and Teaching) there are challenges for her/his participation on the learning of mathematics. On the other hand, if a bilingual student is proficient in the LoLT conflicts about bilingualism are less likely to occur (Barwell 2009b). Between these two situations lie the majority of bilingual students.

Barwell (2009a) makes us aware that solving world problems implies knowing the rules that they have implicitly associated, as well as knowing the language they are written in and applying the mathematics that are needed. His research is a continuation of his previous investigations (see for example, Barwell, 2005). In this case it concerns 9-10 bilingual learners in England, discussing about the process of writing (and solving) mathematical word problems. Three important elements are identified: drawing on personal experience, attention to the word problem genre (its particular form) and the interaction between language learning and mathematical thinking. This last aspects are taken as inseparable interwoven processes. Through many examples it is shown how the discussion about the meaning of particular words at the same time are crucial mathematical aspects of the problem (such as making it solvable or exploring what will be some of its mathematical characteristics). By the use of their personal experience, the students are making meaningful real life problems. An explicit reference to the genre of the problem shows that students are coping with the way word problems are posed on the mathematical classroom and thus preparing themselves to face their understanding on the future. This three aspects that have been highlighted here are, in fact, pointed out by other

researchers (Barwell 2009a). Another benefit for both bilingual students and teachers is that with the word problems task presented the teachers do not need to know in depth what are the particular settings for each of the students of the class, which might be quite demanding and time consuming in some cases.

In a similar analysis, Barwell (2005) evaluates students' solutions and productions as being correct or incorrect. When doing so he is aware that his perspective as a mathematics educator may not necessarily be the same as the one of the students, who may share a sense for the problems that contains some features that are not explicitly reflected within their dialogues. Evaluating the mathematical tasks as wrong does not mean, though, that their work is unproductive. The attention paid to mathematical structure, written form and word problem genre reflects some aspects of the school mathematics that have been internalized by the students, socializing into discursive practices important for school mathematics.

Kazima (2006) explains how language plays a central role in the use of probability vocabulary. Her study is situated in Malawi and involves students whose first language is Chichewa but who have instruction through English (therefore an additional language). Kazima explores the meanings that learners have about four probability words: *certain*, *likely*, *unlikely* and *impossible*. This analysis shows that different meanings are attributed to the same words. She goes further on the investigation to show how this is related to the students' first language, as in Chichewa *likely* is understood as *not unlikely*. If students translate the words from English to Chichewa, then they do the thinking on Chichewa and finally they come back to English to give the answer, problems with words like *very likely*, *equally likely*, *not very likely* and *likely* may be difficult to interpret. As 1) most students learn English just in the school context and 2) other studies suggest that exposure alone is not enough to grasp probability vocabulary, Kazima suggests to complement teacher's explanation of the concepts with facing students to various instances that refine their construction of meanings for the vocabulary. It would be interesting to pay attention to difficulties embedded on students' first languages as those pointed out by Kazima when there is a strong influence of the students first language and it differs from the language of instructions. This is the case, for example, with recent arrival students to a country in which the language of instruction is not mastered by them.

Planas and Setati (2009) examined the use of languages by Catalan-Spanish 12-year-old bilingual students in a Catalan mathematics classroom. The learners were mainly working in small groups according to their main language. Planas and Setati conclude that Spanish dominant bilinguals used Catalan to learn the new vocabulary presented in the

mathematics task –which is the language these tasks were presented on– when working in the small groups. Furthermore, the students chose Spanish as the main language for the discussion in the small groups, although they used Catalan for certain clarifications. A third important conclusion is that in the whole group discussion they contributed only when the teacher directly indicated it, in which case they used Catalan. These findings show how the choice of language is context-sensitive, and that the way in which the teacher structures the lesson influences the use of either one language or the other. It is clear that in the whole group discussions the immigrant Spanish dominant students changed their role to “listeners”; instead of being “questioners”. This had clear consequences in reference to the mathematical content of the whole class discussion as well as for the identities of the individuals as effective achievers of mathematics: a group made by Spanish dominant students was the only team in the class who approached the mathematical task considerably well, but such a work did not become of a public domain. Then Planas and Setati assent that a deeper affirmation of the language identities of the students and a clearer focus on the mathematics might increase the quality of the mathematical talk.

Planas and Civil (2010), using data from the contexts of Catalonia and Arizona, present similar findings. They show how immigrant bilingual students tend to use the language of instruction to address whole group discussions or when speaking to the teacher. They keep their first language when speaking within small linguistically homogeneous groups or when experiencing mathematical difficulties. This sometimes prioritizes language knowledge above mathematical knowledge.

Clarkson (2006) informs that L1 (the first dominant language by the student) facilitates semantic processing. The exclusive use of L2 (the second language or the formal language of instruction) can present problems in relation with long or syntactically complex portions of text as they are harder to hold on the short term memory. It is then when translation becomes efficient to integrate and assimilate the meaning. Usually the learner's L1 better helps them keep the concepts alive, as the network association is richer than in L2. At the same time the use of familiar terms provides security and builds confidence. For example, learners may revert to L1 to verify a verb tense, check for comprehension or clarify syntactic roles. But translation may not always be beneficial or reliable as it might not reflect the exact meaning, for example when a few words in the L2 are replaced by some words from their L1. Furthermore there is a risk of learners focusing firstly on the L1 representations instead of the original L2 forms. Hence the use of code

mixing and switching may help for a better understanding and comprehension of the wordings.

Clarkson (2006) also informs about the use of languages in mathematical classrooms with English students learning mathematics in Spanish in the United States as part of a Spanish immersion program. In the first case the results are that students turned indeed to English when conceptual difficulties arose (comprehension of a problem, understanding of instructions, difficulty of vocabulary, affective responses). For the second research most of the students chose English (the language predominantly used in the classroom) to solve mathematical problems; only a small part of the sample examined reported the use of both languages. So the use of L1 is widely extended when doing mathematics through an additional language.

Another important concept also reported by Clarkson (2006) is metalinguistic awareness: *This concept includes the ability to reflect and manipulate the structural features of language, treating language itself as an object of thought. It is just one aspect of the wider role that metacognition plays in problem-solving behaviour* (p. 195). A child who has no special training in translation is able to produce a meaningful translation when the metalinguistic awareness is developed. This awareness is partially related with finding the key words on a text. Clarkson communicates that a group of 5-9 year-old bilinguals were able to find the meaning of the proposed questions focusing individually on the key words among other non-essential information. Thus it is clear that revealing the meaning of the key words is essential to correctly interpret a statement. When mathematical activities present a task to solve it is essential that the precise meaning of the intended question be understood. It is not the same, for example, to solve an equation than to find an equivalent equation; both tasks having the same symbolic representation (the equation).

Clarkson (2006), in his qualitative analysis, points out how different students switch languages when performing mathematical tasks. He was able to interview the students a couple of years after the collection of the data. This provides a better insight to what might be happening. Among the four case studies presented there are differences on the code switching and the reasons for doing it. Compared to what happens when these students solved mathematical problems again two years later, it results that there is a time when both languages (Vietnamese and English) are smoothly integrated in the mathematical solving process and it becomes difficult to distinguish and identify at which point there is a need to switch between languages. All students participating in the research were highly qualified students –when compared to other students of the same age–, either in relation

with language (both Vietnamese and English) or with Mathematics. They were learning English. Vietnamese had a predominant role at home.

For three of the four students analyzed it was clear when they switched languages on the first data collection. But when data was gathered two years later it appeared that these switches were less evident, and they did not occur (as it had happened in the first interview) for entire processes or subprocesses, but in small portions during the solving process; both languages being integrated naturally. For the other case study there is no evidence to support the assertion, but there is not either information to counterpoint the explanation given above.

Cummins' theoretical framework (2000) states that bilinguals who achieve high levels in both languages tend to achieve high levels in mathematics. Clarkson's (1992, 2006) studies are aligned with this statement. Similarly, in his review of the research on how race, ethnicity, social class and language might affect student achievement in mathematics, Secada (1992) establishes a relationship between the amount of proficiency in a given language and mathematics achievement.

Another reason reported by Clarkson (2006) was the fact that L1 is clearly affected. It could be that using the first language resulted in familiar associations enjoyed by the students, resulting at the same time in a growth of the confidence to process the problems. On the second collection of data, not all the students said they switched languages. All them demonstrated a high level of confidence (maintained even when they ran into difficulties), though. Clarkson attributes this fact to the practice of switching in the past (so such a confidence was built previously).

Meyer and Prediger (2011) qualitatively study how the first language of Turkish-origin students is used for developing conceptual understanding in mathematics while students are dealing with conceptually challenging texts. Students are allowed to make use of their first language to access mathematical knowledge in many countries. However, it is not the case of Germany, where German is the exclusive language of instruction, even if more than one fifth of the students have another mother tongue. As some studies support that being proficient in the language of study is crucial for a success in mathematics, German language learning is the priority in all subjects for the German Department of Education. This situation is similar to the Californian scenario, where the data of this dissertation comes from. In Salinas, California, there is a predominant and enhanced used of English –it is stated by law– over the other languages students may have (Spanish is by far the predominant second language among the bilingual learners population).

Complimentary to that, many other studies point out the importance of the first language to have access to mathematics (Clarkson, 2006). Moschkovich (2007a) and Setati and Duma (2009) consider code switching a social practice of flexible use of languages.

Taking these two views jointly, the use of Turkish may arise as important when doing mathematics. For this purpose, Meyer and Prediger (2011) provided two language versions of the problems. The German version of the problems was given initially, and the Turkish version was provided a moment later (in a preliminary study, the German as well as the Turkish versions of the wording were given at the same time initially, so students – who worked in groups of two or three– had them available during the whole resolution process. Based on the findings of this preliminary study this option was not used in the main study).

Continuing with the research design, initially a German interviewer asked the students to count in Turkish, which (as reported on the conclusions) enabled more students to use Turkish. This event shows appreciation to the Turkish language. It is seen as having enormous effects to allow and promote the use of the students' first language. Once the students had discussed the problem, the interviewer was called by the headmaster and an only-Turkish speaking person, presented as a “caretaker”, pursued with the interview.

Summing up the results, the *case studies allowed to reconstruct moments of interactional cognitive, and/or meta-cognitive benefits of first language use for conceptual understanding. The most important aspect is that the first language enables students to gain access to conceptual understanding, as the linguistic and conceptual development is deeply interwoven* (Meyer and Prediger, 2011, p. 232).

The students that participated in the study seemed to never have been confronted to the use of two languages during a problem-solving process, and often found the metacognitive demands of the researcher during the discussion/interview difficult to respond. Moreover, the language switches were unplanned and unconscious. Meyer and Prediger say that in further research they will consider the use of cameras to make students explain their solution to another Turkish speaking students as a more natural way to enhance Turkish usage.

Civil and Planas (2012) show how language minority students use their first language to gain power in the bilingual mathematics classrooms. They examine this phenomenon in two different contexts, Catalonia and Arizona, where instruction should occur officially through one language only. In the Catalan context, students were observed, on the one hand, to use the instructional language (Catalan) to get familiar with new vocabulary, to situate this new vocabulary in the given task and to begin to organize approaches to solve

the task. On the other hand, students used their first language (Spanish) when discussing mathematics with various degrees of depth –not only repeating the teacher’s ideas–. Even if in the whole class discussions these students did not participate much on the debate, they showed a good extent of agency when using both languages in the group discussions. Looking at the students’ points of view, both Catalan as well as Spanish dominant learners use their languages alternatively in small group discussion to enhance communication among all the members, for practical reasons. The importance of the Catalan language, given that the materials and the whole group discussion are in Catalan, is well recognized, admitting its utility to gain mathematical knowledge. However, as it has been commented, the limited participation during the whole class interaction restricts somehow the access of the Spanish dominant students to the mathematical ideas as active agents.

In the context of Arizona, after Proposition 203 was passed in 2000, English has become the only official language. Some students and teachers were used to use both their languages (English and Spanish) during instruction to better provide access to the mathematical practice. Now some teachers feel that the new English-only policy of does not provide a successful framework to get the most appropriate insight in the mathematical tasks. On the side of the learners, access to whole class discussions is done through English, with the limited participation of those who do not have enough English fluency. In fact, their collaboration to the whole class interaction have been observed through Spanish only after the teacher directly questions them.

The voices of the learners point that they want to be part of the dominant discourse in the school instead of being relegated to a different classroom –where they are assigned while they acquire proficiency in English–. They have the impression of learning less mathematics than their peers who are in a different –mainstream– classroom. At the same time, though, they are aware of the difficulties of having access to mathematics when the official LoLT is not mastered enough. This is shown specially through the experiences of some students after having been in both classrooms. In addition, their teacher also describes it in this way. Their teacher feels that very good students do not have opportunities to show their mathematical expertise when they are requalified as having a certain level of English and so they may enter the mainstream classroom; as once they are there they are not able to respond to the teacher’s questions in English and hence they are relegated from the classroom discourse.

Civil and Planas (2012) remark the importance of the sociopolitical views on the learning of mathematics in multilingual classrooms. Without directly addressing the question

whether languages cause valorizations or valorizations are caused by languages, the important fact is that both elements are intertwined and that learning opportunities are influenced by them.

The acronyms BICS (basic interpersonal communicative skills) and CALP (cognitive academic language proficiency) as used by Cummins (2000) refer to a distinction intended to draw attention to the very different time periods typically required by non native speakers to acquire conversational fluency in their second (and/or third, fourth, etc.) language as compared to grade-appropriate academic proficiency in that language. Although Prediger and Wessel (2011) consider a triad including the (mathematical) technical register, in our investigation the technical register will be included on the CAL register.

Normally two years of initial exposure to the learning language are enough to develop conversational fluency to a functional level whereas at least five years are usually required to attain the level of native speakers in academic aspects of the learning language. When the different paths followed to acquire BICS and CALP have not been recognized, a discriminatory psychological assessment of bilingual students and premature exit from language support programs (e.g. bilingual education in the United States) into mainstream classes has been present.

2.4 Language practices in the mathematics classroom

In this last section of the chapter we describe in detail some practices such as code mixing and code switching that are likely to occur in the bilingual mathematics classroom.

The notion of code switching used in this dissertation draws on that provided by Moschkovich (2005):

Code switching has been used in sociolinguistics to refer to the practice of using more than one language in the course of a single communicative episode (p. 125).

There are some definitions in sociolinguistics about code switching and when it should be applied. It is important to mark that code switching might not be considered *as an individual phenomenon but as a complex and evolving activity and language practice tied to an individual speaker's community or communities* (Moschkovich, 2005, p. 129). It will be distinguished among code mixing and code switching. The first one relates to the use of another language mainly just for a single noun or a noun phrase, while the second one reeffers to a more extensive use of both languages, usually within a single intervention.

Moschkovich points out that some researches do not make this distinction, but accounts for some cases where it should be considered, for example among conversations of children with different ages. To further explore if there is any relevant nuance in such a distinction and given that the students that participate in our research are of different ages, the differentiation between code mix and code switch will be maintained.

According to Moschkovich (2005), code switching has been viewed for many decades embedded on a deficit model, used when the speaker was not able to find the suitable word or phrase in the language employed. Hopefully, thanks to hers and other's studies (see e.g. Barwell, 2009b; Civil & Planas, 2004; Setati, 2006) this deficit model has been almost left aside, at least in the context of most researches. For example, Setati's findings (1998) show how a teacher used code switching with her grade 5 English-Setswana students to promote the understanding of concepts, to encourage participation and to familiarize learners with the language of evaluation. The use of learners' main language in teaching and learning mathematics is encouraged while the learners continue to develop proficiency in the LoLT –at the same time that they are learning mathematics–. Talking is needed to learn; it is a function of fluency and easiness in the language of communication: talk is a social thinking tool (Setati & Adler, 2000). Moschkovich considers that code switching should be regarded as a complex language practice that allows students to make a greater use of the main language while still using the official LoLT. Code switching is not just a random behavior due to some kind of lack of management in one particular language, but should be considered as a communicative strategy related to linguistic and social information.

Moschkovich (2005) distinguishes between code switching and language switching, this last terminology referring to an individual cognitive phenomenon different from the first one. So language switching would refer to *the use of two languages during solitary and/or mental arithmetic computation* (p. 125). The nomenclature of language switching is avoided in the present dissertation. As students work individually they may switch between languages, but this occurrences are just listed and named whether they occur during computations, counting or other processes.

What are sometimes called loans may finally result in the incorporation of a foreign word into a language. *Loans can be loosely defined as single words from one national language used by many members of a bilingual community* (Moschkovich, 2005, p. 130). Using the word “troque” instead of the Spanish word “camión” within a Spanish sentence is an example of a loan. “Chequear” o “checar”, for example, are already accepted words

by the RAE (Real Academia de la Lengua Española) Dictionary⁴. Here loans will be considered instances of code mixing.

Through her review of some works in psycholinguistics, Moschkovich (2005) remarks that it seems that most of the bilinguals prefer to perform operations through the language of instruction. It looks like there is, in general, contradictory information regarding the use of languages when bilinguals perform computations, specially regarding response time and error rates. But one has the impression that allowing them to choose the preferred language as well as giving more time to finish exams that need large quantities of calculations should be fair. On a daily basis, though, these results do not seem to have major consequences. None of the cited studies rely on word problems that hide a rather easy (or not so easy) computational task, nor do they present a more (mathematical) conceptual task involving bilinguals.

Moschkovich (2005) draws on sociolinguistic works which inform that bilingual children under the age of five usually use the language they are spoken to in order to interact with other people. This seems relevant to mathematical classrooms, where the same practice should remain valid. Special caution to the age of the students must be paid, as the results might not be valid in the case of elder siblings. *Sociolinguistic research on code switching among adults shows that this practice can be a reflection of stylistic switches to add color, emphasis, or contrast; random switches of words or phrases that appear in talk with high frequency items; as well as switches related to language dominance, memory, routines, and automatic speech* (p. 131).

Code switching also occurs on mathematics classrooms and it should be regarded according to multiple circumstances and aspects intertwined together. Characteristics such as the particularities of the speakers, the context where the communication occurs (not only physically but also regarding the interpersonal scope) and the semantic objectives of the communication. Bilinguals should be viewed as moving a long a range of modes, with code switching being a normal practice in the bilingual mode. Then bilingual language competence is different from monolingual competence. At first sight, it might seem than a code switch or a code mix would be the result of a lack of vocabulary in one language. But this might not be an appropriate explanation. As bilinguals use the languages depending on the interlocutor, the domain or the topic role, function bilingualism researchers point out that evaluating the levels of language proficiency, the ability to name a concept in a particular language, or using technical nomenclatures in a

⁴► <http://lema.rae.es/drae/?val=chequear>, ► <http://lema.rae.es/drae/?val=checar>. Diccionario de la Lengua Española, 22^a edición, Real Academia Española

particular language may lead to confusion and be, in fact, erroneous. Furthermore, a consistent finding among bilingual population is that code switching is not a reflection of a low level of proficiency in a language or the inability to recall a word.

An important aspect to consider when observing bilingual students is if they are addressing another bilingual or not. A student might be able to make a computation in his/her first language and then decide to translate it to another language depending on the interlocutor. Another issue is whether the setting is private or public (talking in small groups or addressing to the whole class, for example). Furthermore, it is important to take into account the social roles of the participants (teacher, student, aide, elder, ...). Besides, the topic that the conversation is about might be a crucial factor (school topic, conversation about sports, family, ...). One more aspect to consider is the mode used (oral or written).

Particularly related to mathematics, more specific details should be considered. Of course, the mathematical aspects of the situation. *For example, is a student doing computation or engaged in more conceptual activities? What is the mathematical topic (algebra, geometry, etc.)?* (Moschkovich, 2005, p. 132). Another aspect of relevance is how the student has been exposed to each language, either in or out of school; and which topics have been covered in one, the other or both languages. It seems reasonable that if a particular topic has been covered exclusively in a second language, then the student will use the second language to work about it. And it might be that there is special relevance of one language for a particular topic in one language and proficiency in the other language for a different topic. And this is what the analysis of Moschkovich (2005) of a classroom episode between bilingual learners' code switching suggests: using a sociolinguistic perspective is more grounded in what empirical research has demonstrated (enhancing interpretations more complex than a deficiency view of the mathematical or language competence). Two of the more important findings on Moschkovich's (2005) research is the familiarity and the repetition of a point already raised. By using an English word from the statement of the activity during a Spanish utterance, the student is making reference to a well-known terminology, accessible also by her partner through a written form which is present during conversation. By reformulating the ideas already exposed through Spanish, instead of just repeating the exact same English utterances the student benefits of the use of both languages to elaborate and construct mathematical knowledge.

Barwell (2009b) remarks three tensions on the learning and teaching of mathematics. The first one is between mathematics and language. He informs that *in some situations, it is important to pay careful attention to how students express their mathematical ideas, while*

in others, it is important to engage carefully with the mathematics itself (p. 7). That is, a shift from the mathematical content to the mathematical language may make more important the learning of the mathematical vocabulary than the fact of actually doing mathematics. A second tension pointed out by Barwell is between formal and informal language. On the one hand providing a more accurate mathematical register during a mathematical explanation may seem unnatural and result in a series of terms used rather artificially and so being difficult to grasp. On the other hand, providing the right mathematical terminology may empower students to make accurate mathematical descriptions. The third tension is between student's home languages and the official language of schooling. As Setati (2006) and Setati and Adler (2000) point out, teachers as well as students have a conflict regarding the use of the first language (allowing to express fluidly the mathematical ideas and promote a more informal use of mathematical language) and the use of English as LoLT (which is the examining language and is regarded as a language needed to have access to well-paid jobs).

Parvanehnezhad and Clarkson (2008) report two instances of students switching languages in a mathematics classroom. When facing a problem, students must initially read the problem. Then the first difficulties may arise as a result of reading complexities or due to a problematic comprehension of the concepts presented. Also the symbolic mathematical language may result in difficulties to grasp its meaning. Once this first phase is overcome, a second stage regarding which mathematics should be applied or deciding the appropriate mathematical plan to solve the activity may also prompt students to language switching.

Parvanehnezhad and Clarkson also support a constructivist view of teaching and learning. By doing so, they regard students' learning as part of their previous knowledge, so teachers do not have the entire responsibility on the guidance of the learning of mathematics. But when teachers draw on student's previous knowledge they should be aware that for bilingual learners this knowledge is partially rooted in their L1. So when teachers are giving permission for students to use their past experiences and notions they are also allowing them for the use of their L1. This is a major implication derived from their analysis of data.

Through the interviews and other tools of data collection, Parvanehnezhad and Clarkson analyze the way in which sixteen Iranian students use their languages, Persian and English, during the mathematical activity. Three different kinds of problems are posed: open-ended, word problems and symbolic. They speculate that the reasons may be dependent on the item type: for word problems the language context may be more

relevant, while the familiarization with number words may be crucial in the symbolic type. The difficulty of each type seems to play an important role to switching between languages. Motivated students explore different strategies when being stuck attacking a problem, and language switching may well be one of these strategies. Linked with it, Parvanehnezhad and Clarkson report that social and cultural reasons are also important factors that prompt language switching: students bring what they have learned in contexts from outside school (parents, their communities, elders that help them through Persian, friends, playground, etc.). Hence, a mutual development of both their languages help students to further develop and explore the nuances between the mathematical elements.

Van Jaarsveld (2011) proposes a triangular translation to enhance conceptual understanding. The process of learning and acquiring of concepts are seen as constructed through the mastery of vocabulary, as thoughts are indeed articulated through language. Then it is the teacher's responsibility to offer bridges between the students' previous knowledge and vocabulary to the new vocabulary and knowledge that is to be learned. If this is hard to accomplish in a monolingual setting, it is even more difficult in multilingual settings such as those of South Africa, with more than eleven official languages but where exams are provided through English on Afrikaans language (though most of the people has a first language different than English, the LoLT).

In the triangular translation, the teacher exposes the mathematical content to the class in English. Then a proficient isiXhosa learner exposes it through her/his home language. A translation back to English is finally made by a proficient English student. This strategy benefits isiXhosa speakers who listen to the explanation in their mother tongue and also helps the teacher assess how accurate is the English translation when it comes back. The expectation is rather to generate further inquiry instead of producing knowledge.

Furthermore, through the use of isiXhosa and English in the study of the parabola a deeper understanding of "turning point" was developed by the absence of a direct isiXhosa translation and the further discussion of its meaning through both languages. Thus, this exchange benefited both teachers and students and it served to a development of the mathematical register in isiXhosa. As a consequence, the phrase "place of turning" is proposed as a more adequate terminology, which does not have the controversial characteristic of a motion word (turning) and a motionless term (point). It also unveiled the absence of an appropriate vocabulary and deep understanding of graphs' description (interpolating and extrapolating values of the functions or seeing graphs as objects in motion). In these episodes, multilingualism acted as a facilitator that promoted active

discussion around concepts instead of contributing to a merely passive acquisition of English vocabulary.

3 Methodological approach

[...] *credibility occurs when complexities are made visible through the analytic process and are articulated with an openness or «criterion of uncertainty» that acknowledges a certain tentativeness about the final research outcomes* (Thorne, Reimer Kirkham and O'Flynn-Magee, 2004, p. 7).

This Chapter explains which the methodology used in this research is. It also explains how it has been performed, for traceability purposes. It contains three sections which describe both the context and the participants to the project. Furthermore, the instruments used to collect the data, its analysis and the ways and dates in which this gathering process took place are also taken into account. Finally, this chapter includes a mention to the description of the instruments created and used for the analysis.

This research is based on a qualitative analysis: the so-called interpretive description. This methodology, which is aligned with interpretive naturalistic orientations, acknowledges the constructed and contextual nature of human experience. At the same time it allows for shared realities to be considered. The foundation of interpretive description is the qualitative investigation at a smaller scale of a phenomenon of interest to some discipline (in the present case, the Didactics of Mathematics) for the purpose of getting themes and patterns within subjective perceptions and generating an interpretative description leading to an understanding of such a phenomenon (Thorne, Reimer Kirkham and O'Flynn-Magee, 2004).

Using inductive analytic approaches (which are characteristic of interpretative description) researchers look for understandings capable of casting a light upon their characteristics, patterns and structure in some theoretically useful manner. *“Simply stated, interpretive description provides direction in the creation of an interpretive account that is generated on the basis of informed questioning, using techniques of reflective, critical examination, and which will ultimately guide and inform disciplinary thought in some manner”* (Thorne, Reimer Kirkham and O'Flynn-Magee, 2004, p. 3).

Concurrent data collection and analysis, constant comparative analysis and iterative analysis (all of them rigorously developed) give validity to the study and locate the explanatory factors derived from the analysis in the general framework. A further detailed explanation, along with the description of the instruments constructed and used for the analysis is given in the following sections.

The product resulting from an interpretive description can be described as being a coherent conceptual description that points out the commonalities among the data gathered. However, it may also account for the inevitable differences present among the different subjects under investigation. Thus, instead of generating a new whole “truth” its aim is to articulate the descriptive and interpretative insights which may have arisen in the research. This body of knowledge will be incorporated to the theoretical framework so as to make sense to the eccentricities and variations present in each case study or the real life population.

The analytic devices derived from, for example, ethnography, grounded theory or phenomenology, are not always completely satisfying because the object of the analysis derives from that which those processes are able to produce. On the contrary, a rigorous analytic process in interpretive description will involve carefully navigating within and beyond the original theoretical scaffolding (or the analytic framework with which one entered the investigation) in order to fully engage the processes of inductive reasoning. This may include testing and challenging preliminary interpretations as well as conceptualizing an ordered and coherent final product.

On the earlier stages of the analysis the theoretical framework is the path to organize the raw data, but distance has to be taken when alternative conceptual emphases and intrigues arise. Instead of explaining data to the light of a particular theory, or to try and find examples that confirm the initial findings, counterexamples and data with different characteristics must be sought in order to avoid the imposition of a pre-established theory (so all examples would stick to it and almost no new findings will arise). It is also important to avoid a theoretical description. That is, all data needs a rich interpretation to challenge its logic.

On a second phase a more detailed analysis of the data is needed. Among all of qualitative methodological literature there are a lot of descriptions about how to accomplish the investigation. Most techniques can be useful in interpretive description, but within all of them researcher must comprehend the data exhaustively, synthesize meanings, theorize relationships and recontextualize data into findings (Thorne, Reimer Kirkham and O'Flynn-Magee, 2004). It should be noted that the researcher is in charge of organizing the data, selecting part of it according to its relevance or structuring the findings; neither the method nor the data themselves. It is important not to code the data in much detail but rather to have a general picture of what is going. In this sense, the researcher should be asking the question “What is happening here?” repeatedly in order to find the appropriate explanation to the phenomenon being studied.

There may be an infinite number of qualitative interpretations and elements of interest within every data collection. The research question has its inherent limitations and it will be explained and reformulated by taking distance progressively as findings arise. Finally these findings need to be presented through a visible articulated way and contextualized properly on the general set of data.

3.1 Context and participants

This research uses data from two different schools in California, United States of America. One of them is a high school where I taught mathematics as a full-time exchange teacher. The other one is a middle school in the same city and district. Such city, Salinas, is situated approximately in the center of California and its principal economic is agriculture.

Present-day California belonged to Spain for almost 300 years, up to the early XIX century. It was not until 1847 that USA conquered the part of Mexico which was annexed to California. It is not strange, then, to see a lot of Mexican people in many places in California and so it is in the city where the data for this research was gathered.

Now the school system in California will be described. What follows is the normal path which is supposed to be followed by a school student, unless there is a need to retake a school year for any particular reason. After kindergarten, children receive primary education (grades 1 to 6), until 11 years old. Then students go to a middle school for 2 school years (grades 7 and 8). Compulsory education finishes with 4 school years (grades 9 to 12) in a high school, when students are aged 18. At this point a diploma is obtained – if certain requirements have been successfully completed– and the opportunity to go to University arrives.

In middle and high schools, students are organized per subjects according to their level of proficiency within this particular subject. This arrangement is followed more strictly in high schools and in a laxer way in middle schools. According to this system, two students of the same age can be partners in the same English class because they have a similar level of performance in English. Nevertheless, they can be assigned to different Mathematics classes because one of them has not been as successful as the other one when taking mathematical tests.

Making reference to our population of study, most of the students should be enrolled in pre-Algebra in grade 7, Algebra is taken in grade 8 and Geometry in grade 9 –once in high school. But sometimes students prefer to retake Algebra in grade 9. This may also depend on their qualifications. Good students can advance faster, either because they take

summer school classes voluntarily or as a result of brilliant qualifications. Should such a case arise their teacher might propose them to advance to other classes, skipping part of the established itinerary. From a total of 19 students that finally took part in the research, 11 were high school students taking Algebra and 8 were middle school students taking Pre-Algebra (One of the high school students arrived from Mexico on January and was enrolled in Algebra, but switched to Geometry later). *For numerous reasons, the instructional needs of this large population* [English learners on public schools] *warrant serious consideration* (Moschkovich, 2007a, p. 345).

At the High School where I taught, whenever a student came from another country and had Spanish parents s/he was classified as English learner and assigned to a Transitional class, with other peers in a similar situation. If available, a bilingual teacher is in charge of this group. Teaching there should occur in English only, as it is expressed by the policy of the State of California. But in a Transitional class vocabulary and instruction may be provided with a scaffolding to help students gain English confidence and understand assigned tasks. In this sense, a special attention has to be paid in order to help students with language comprehension difficulties to overcome them rather than to help students once they have already fallen behind. A varied set of techniques may be used including: making vocabulary logs, using visual material to support explanations, hands-on methods, etc.

English learners are categorized as “Transitional” until they perform good on a couple of English language state tests, both held once a year: the California English Language Development Test (CELDT) and the English language section of the California Standardized Test (CST). In addition, according to district policies, they need to attain a certain academic performance, so teachers are consulted for this purpose. Then students' progress is monitored for two years. Furthermore parents are consulted and encouraged to participate in the reclassification process of becoming a fluent English proficient. (California Department of Education, 2007).

Students in kindergarten through grade twelve whose home language is not English are required by law to take an English skills test. In California, the test is called the CELDT.

This test helps schools identify students who need to improve their skills in listening, speaking, reading, and writing in English. Schools also give the test each year to students who are still learning English (California Department of Education, 2012a).

CSTs contain four main areas: History-Social Science, English-Language Arts (ELA), Mathematics and Science; all of them are taken on grades two through eleven. The ELA CSTs for grades four through eleven, *consist of 75 multiple-choice questions with an*

additional 6 field-test questions. At grades four and seven, the ELA CSTs also include a writing component, the California Writing Standards Test, which addresses a writing applications standard selected for testing each year (California Department of Education, 2012b).

On the other hand, in the Middle School where I taught there was a special program enhanced to help Spanish students, due to the high number of people with low English abilities in the school. All Middle School people participating in this research belonged to that program. Instruction was conducted mainly through Spanish, as well as informal talk between teacher and students, but materials were in English. On “Object 1: Students' Historical profile”, pages 49 to 53, these middle school students are described as being in an “English-Spanish”, abbreviated “En-Sp” class. Before the brief analysis (see page 47) of this object, the key for its interpretation is given.

“Object 1: Students' Historical profile”, contains English and Spanish cultural references as well as the different students' backgrounds regarding their historical profile as bilinguals. These data was collected for each student following a questionnaire –see “Object 2: Data collection, part 1: Historical bilingual profile” described in the next section, page 57– at the beginning of each interview. All of the student names are fictitious to preserve their anonymity.

In the *Class* column it is marked whether the student belongs to the High School (HS) or the Middle School (MS) and whether s/he belongs to mainstream (M), transitional (Transitional) or English-Spanish (En-Sp) class. Transitional and En-Sp classes have already been described, so the description of this triad will be completed by saying that a mainstream class is one where no specific actions are taken to prevent English understanding difficulties. This is because all of the are expected to be fluent in English.

All comments between brackets do not appear on the recordings. They have been added to facilitate the comprehension to the reader. It is particularly the case of the *USA arrival* column where absolute or relative references to the time or arrival to USA have been added to facilitate the comparison among different students when one of the time allusions is not specified.

In “Object 1: Students' Historical profile” we can appreciate that most of the 19 students are aged 13 to 16, with two of them being 17 years old. All students have Mexican origins, even if some of them are born in California. Except for the first three (Yolanda, Carlos and Coral), the others choose Spanish to interact with me during the interview. There is not any particular criteria to organize the students within this object but to put the three English-interview speakers who were born in USA at the beginning of it.

Miriam, Abel and Damian are also born in USA but preferred to speak in Spanish, who was their first language, except for Miriam, who mastered both languages and was in a mainstream class. Except those born in USA, 11 out of the 13 participants remaining had been in the USA for a period of time ranging from 6 months (the sisters Yael and Julia) to 3 years. Finally, Claudio and Abel had spend 7 years in USA when the interview took place (Abel spent his first 6 years of life in USA, out of this total of 7 years in USA).

Then other items referring to languages use and cultural background related with it are included to get an idea of how each student uses his/her language abilities: if they like living in California; read in Spanish and/or in English; the language they use at home, with friends and at school; and if they have someone helping them with homework. Different degrees of mastering Mathematics and English are represented among all students analyzed.

Object 1: Students' Historical profile
(Beginning)

Name	Class	Interview language	Interview date	Origins	Age	USA arrival	Living in California	Spanish readings	English readings	Home language	Friends language	Main language at school	Homework help
Yolanda HS	Transitional	English	January 2010	Mexico	16	Born in USA. At 9 she leaves to Mexico. 3 years before the interview comes back.	Likes it. It's better. School is better.	No	Books, magazines.	Spanish (parents, uncle, brother of 12), both (sister, cousin).	Both	English. Math, Spanish and Biology in Spanish.	Nobody
Carlos HS	Transitional	English	February 2010	Mexico	14	Born in USA. In March 1995 his parents came back.	Likes it: stores, sports, "maybe school".	Books: sports books. Does not read a lot in Spanish.	Yes (does not remember any title).	Spanish (parents), English (brother), both (cousins)	English	English	Nobody
Coral HS	Mainstream	English	April 2010	Mexico	14	Born in USA. Never lived in Mexico.	Likes it.	No	Books from the library	Spanish (mother – who barely knows English), English (father, brothers)	[not asked]	English. Spanish if her friends speak to her in Spanish.	Sometimes her dad
Miriam HS	Mainstream	Spanish	April 2010	Guanajuato, Mexico	16	Born in USA. When she was a child used to go to Mexico every summer. Now she goes every 2 years.	Likes it	Science or History books instead of books about people. Reads mainly in Spanish to practice and improve reading skills.	Can read perfectly in English but barely does (to improve Spanish reading skills).	Spanish, both (sisters)	Spanish. English just with one.	English. Her Spanish teacher speaks English with the whole class so she (only student who masters Spanish) is not comfortable speaking Spanish.	Father –e.g. in Math. Little help because of his low English fluency. She helps her sisters.

Name	Class	Interview language	Interview date	Origins	Age	USA arrival	Living in California	Spanish readings	English readings	Home language	Friends language	Main language at school	Homework help
Camilo	HS Transitional	Spanish	May 2010	Mexico	16	1 year and 6 months [November 2008]	Did not like it at the beginning (it was too different) but now he is used to.	"La Ganga" newspaper.	Magazines, newspapers, books. Does not like English comics because understands jokes better in Spanish.	Spanish	Spanish. English in a few cases.	Spanish in most of the classes (3 [out of 6])	Nobody
Diandra	HS Transitional	Spanish	April 2010	Oaxaca, Mexico	15	1 year [April 2009]	Does not like it. Very different: weather, always inside houses (do not go out because her parents have to work), a brother still in Mexico.	Books (novels)	Hardly ever because of poor understanding	Spanish	Spanish	Spanish (friends and the 3 teachers that understand Spanish)	Her brother (Camilo) very few times (he does not like to ask her)
Jessica	MS English-Spanish	Spanish	May 2010	Michoacán, Mexico	13	Lived in Mexico until 10 years old. Atlanta and Chicago (1 year). Arrived at California on December 6th 2008.	Does not like living in California: violence... Prefers Chicago (e.g. school).	Magazines	No	Spanish	Spanish	English (Spanish with friends)	Nobody
Ana	MS En-Sp	Spanish	May 2010	Guadalajara, Mexico	12	April 12 2008 [2 years]	Likes it more or less.	Sometimes	[Not asked]	Spanish, English (sister)	Spanish	Spanish (Math, Spanish and Social Studies)	Mother

Name	Class	Interview language	Interview date	Origins	Age	USA arrival	Living in California	Spanish readings	English readings	Home language	Friends language	Main language at school	Homework help
Juan	MS En-Sp	Spanish	May 2010	Guanajuato, Mexico	14	June 2009 [2 years]	Likes it.	"La Ganga" newspaper, soap operas...	Sometimes, but hardly ever	Spanish	Spanish	Spanish	Brother
Angel	MS En-Sp	Spanish	May 2010	Mexico	14	March 2009 [more than a year]	Likes it.	"El Sol" newspaper	"The Californian" newspaper	Spanish	Spanish. English just very few times.	Both. Math is taught in Spanish.	Nobody
Abel	MS En-Sp	Spanish	May 2010	Michoacán, Mexico	13	Born in USA. At 6 went to Mexico. 11 or 12 months before came back to USA.	Likes it	Yes: tales [stories, fairy tales, comic- books],...	Just a little, less than in Spanish	Spanish. A little English (brother, sister).	Spanish. English just sometimes.	Spanish. English only in English class.	Mother and brother
Julia	MS En-Sp	Spanish	May 2010	Mexico	14	Was in USA for a year when she was 1. Came back to USA in October 2009 [6 months ago].	Likes it.	Yes: comic- books ...	Books from the library for English learning purposes	Spanish	Spanish	Spanish	Her sister (Yael)
Ingrid	MS En-Sp	Spanish	May 2010	Mexico	13	9 months [August 2009]	Likes it: school.	Yes	Yes	Spanish. A little English with her brother for learning purposes.	Spanish and English	Spanish	Brother (mainly) and father

Name	Class	Interview language	Interview date	Origins	Age	USA arrival	Living in California	Spanish readings	English readings	Home language	Friends language	Main language at school	Homework help
Yael HS	Mainstream/Transit.. /	Spanish	June 2010	Mexico (American father living in another State)	17	January 2010 [6 months]	Likes it (education).	Novels	No (just compulsory books from the English class)	Spanish (sometimes practices English with her sister – Julia)	Spanish. English with text messages, less frequent.	Spanish when possible (English is compulsory)	Nobody
Julián HS	Transitional	Spanish	June 2010	Mexico	17	October 2007: 2 years and a half	Likes it. Came back to meet his mother, who had not seen since the age of 6. Likes school because there are more opportunities. In Mexico didn't attend school for a certain time.	Novels, tales...	School books, some magazines	Spanish	Both (more Spanish)	[when talking with peers] Spanish (Physical Education, Spanish), English (Health, English), both (Math)	Nobody
Aida HS	Transitional	Spanish	January 2010	Mexicali, Mexico	14	2 years [January 2008]	Likes it.	Books	Books from the school library as part of an after-school English learning program	Spanish	Spanish	[Question not asked]	Nobody

Name	Class	Interview language	Interview date	Origins	Age	USA arrival	Living in California	Spanish readings	English readings	Home language	Friends language	Main language at school	Homework help
Claudio	HS Transitional	Spanish	January 2010	Michoacán, Mexico	16	2003 [7 years]	Likes it a little: free education, less school expenses (material, uniform), no physical punishment at school	Magazines from stores	Books	Spanish, both (sister, brother-in-law, brother)	Both	English	Nobody. Cannot ask his brother-in-law (who knows math) because he is barely at home.
Damian	HS Transitional	Spanish	January 2010	Mexico	14	Born in USA	Prefers Mexico to California: handmade and better food, goes out more frequently. California: less violence.	Sometimes, but can't read very good.	Yes	Spanish (mother), both (sisters, father –to teach him English–)	Mainly English	[With peers] Mainly English. Spanish in English class and with a friend who just arrived to USA.	Sisters (in English)
Zoraida	MS En-Sp	Spanish	May 2010	Michoacán, Mexico	13	May 2009 [1 year before the interview]	Likes it	Yes: newspapers	Sometimes: magazines	Spanish	Spanish and English	Spanish (Math in Spanish)	Cousins

Object 1: Students' Historical profile (End)

3.2 Instruments and data collection

First of all, as I was a Mathematics teacher in a transitional class at a high school, my students were asked to voluntarily participate in the research. They were asked if they wanted to take part in an investigation involving the use of languages while solving mathematical activities (which were not supposed to be very difficult); their mathematical abilities being not an obstacle to take part in the project. Most of them initially showed a real interest in getting involved on the task. So, after getting the principal's approval, a paper asking for permission to participate in the research was made available to all people willing to get involved in the investigation, which had to be signed both by the students and one of their parents.

Thus, my students were invited to complete the task either during lunch time (40 minutes) or once all school classes had finished. As they had to take the school bus to return home most of them finally decided to collaborate at lunch time. In a couple of cases, as we had not enough time during their lunch break, they returned after school to finish the data collection. Another exception was one student interviewed the last day of the school year who came while she should be in class –of course with her teacher's approval.

The initial enthusiasm was not as expected and after a certain amount of time only about 5 students had completed the questionnaire –see pages 57 to 61–. So other bilingual students (who were able to speak Spanish and English) from my other classes as well as from the rest of the school were asked to join the research project. All in all, there were only 7 people who had volunteered. So the offer to participate in the research was also extended to other high school students in the region. Logistic and time issues made impossible to success in collecting data with other volunteers from other high schools. This is why middle school students were finally asked to join the research program too.

Firstly, the research was presented to them by their math teacher, who was also a friend of mine. Later both of us explained what the research was about while the students where in an after school program aimed at learning English. At that moment a form asking for their parents permission was made available to them. Having obtained Middle School's principal's and parents' approval, a total of 8 students were successfully videotaped while completing the questionnaire. These interviews were made in three days (two days with 3 students each and another day with 2 students), once the school day was over, in their mathematics classroom.

I was in charge for gathering all the data and for putting in place all of the logistic issues related with it, like setting up the scenario to make the recordings. For about 6 cases –all of them middle school students– I had the collaboration of another person who was taking care of the camera.

At the beginning of the data collecting process only audio recordings were held in order to make further analysis of the data –it was the case of Aida, Claudio and Damian. These recordings were made using a recording program provided by default on Mac computers (GarageBand).

Later on, video recordings were added to the audio recordings just in case there was a technical failure but also to get more information about the resolution process. For these recordings, a digital video camera using mini DV tapes helped on the task. This recordings were later transferred into an electronic format to ensure due accessibility and transferability among computers.

Almost all of the recordings were made in the students' Math class. Each student completed a questionnaire divided into 3 parts. The first one –see “Object 2: Data collection, part 1: Historical bilingual profile” below–, was a semi-structured interview in order to gather relevant data related to the use of languages. It starts –in English– allowing the student to choose the preferred language to interact with me. In case the language chosen was Spanish, all questions were translated later.

The other questions are intended to retrieve data related to the historical profile as bilinguals and some Spanish and English cultural aspects related with these languages usage. Thus, “Object 1: Students' Historical profile” pages 49-53, which has already been commented on the previous section, collects this data for all 19 students.

Which language do you prefer when being asked questions?

- How old are you?
- Where is your family from?
- When did you arrive at California?
- Do you like living in California?
- Do you read books, comics, magazines, newspapers ... in Spanish?
- Do you read books, comics, magazines, newspapers... in English?
- What language do you speak with your family?
- What language do you speak with your friends?
- What language do you use most at school?
- Who helps you with homework at home?

Object 2: Data collection, part 1: Historical bilingual profile (English version)

Which language do you prefer when being asked questions?

- ¿Cuántos años tiene?
- ¿De dónde es su familia?
- ¿Cuándo llegó a California?
- ¿Le gusta vivir en California?
- ¿Lee libros, cómicos, revistas, periódicos... en Español?
- ¿Lee libros, cómicos, revistas, periódicos... en Inglés?
- ¿Qué lengua habla con su familia?
- ¿Qué lengua habla con sus amigos?
- ¿Qué lengua habla mayoritariamente en la escuela?
- ¿Quién le ayuda con la tarea en casa?

Object 3: Data collection, part 1: Historical bilingual profile (Spanish version)













After this, a questionnaire containing four mathematical activities is given to the participants –see “Object 4: Data collection, part 2: Activities” on pages 59 and 60. Students are then told to solve all mathematical tasks and then fill in the columns on the right with a cross on the appropriate column, depending on the language(s) used. No limitation of time was established to complete the questionnaire. Besides, it is to be taken

into account that the idea of languages being used may be interpreted differently by the different students. For instance, the use can be seen as expressed through thinking, writing and talking or limited to some of them. Consequently, in the analysis there is a necessity to consider the way in which each individual participant is actually deciding where to put the cross. As it will be noted, the interview is expected to inform whether the idea of language use is interpreted as a whole (thinking, writing, etc.) or in a more restricted way.




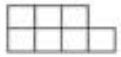
The questionnaire also has a short paragraph at the beginning in which the instructions for its proper completion are stated. All of the documents made available to the students are written exclusively in English to resemble the most to the activities they have to solve at school. The mathematical and language aspects of each activity are discussed in more detail in the following subsections.

For what is worth in reference to the performance of the experience, it is relevant to make clear that I was available during the solving process to help students with particular problems they could find (both, mathematical and language). Actually, they were told they could get help when needed. In some cases I was present during the entire solving process, while some students preferred to work alone and I was doing other tasks in the same class. With the exception of some students at the beginning of the data collection process, the others were told not to delete the mathematical procedure. Even so, some of them did.

QUESTIONNAIRE

Solve the following activities and mark with a cross in the right columns the languages you have used while solving. Write down the solving procedure.	<i>English only</i>	<i>English and Spanish</i>						
<p>ACTIVITY 1</p> <table border="1" data-bbox="392 495 962 813"> <tr> <td data-bbox="392 495 651 636"> <p>JOHN SPORTS</p> </td> <td data-bbox="651 495 962 636"> <p>MIKE SPORTS UNBEATABLE PRICES!!!</p> </td> </tr> <tr> <td data-bbox="392 636 651 719"> <p>40% DISCOUNT</p> </td> <td data-bbox="651 636 962 719"> <p>25% DISCOUNT</p> </td> </tr> <tr> <td data-bbox="392 719 651 813">  </td> <td data-bbox="651 719 962 813">  </td> </tr> </table> <p>In which of these two stores are the shoes cheaper? Why?</p>	<p>JOHN SPORTS</p>	<p>MIKE SPORTS UNBEATABLE PRICES!!!</p>	<p>40% DISCOUNT</p>	<p>25% DISCOUNT</p>				
<p>JOHN SPORTS</p>	<p>MIKE SPORTS UNBEATABLE PRICES!!!</p>							
<p>40% DISCOUNT</p>	<p>25% DISCOUNT</p>							
								
<p>ACTIVITY 2</p> <p>Which of these figures has a greater perimeter? Why?</p> <table data-bbox="392 1312 935 1507"> <tr> <td data-bbox="392 1312 592 1507">  </td> <td data-bbox="592 1312 935 1507">  </td> </tr> </table>								
								

Object 4: Data collection, part 2: Activities 1 and 2

Solve the following activities and mark with a cross in the right columns the languages you have used while solving. Write down the solving procedure.	<i>English only</i>	<i>English and Spanish</i>
<p>ACTIVITY 3 Observe this pattern.</p> <div style="display: flex; justify-content: space-around; align-items: flex-start;"> <div style="text-align: center;">  <p>Figure 1</p> </div> <div style="text-align: center;">  <p>Figure 2</p> </div> <div style="text-align: center;">  <p>Figure 3</p> </div> <div style="text-align: center;">  <p>Figure 4</p> </div> </div> <p>How many tiles does figure 7 have? Why?</p>		
<p>ACTIVITY 4</p> <p>Jamie is shopping in a large department store with many floors. She enters the store at the middle floor, and immediately goes to the credit department. After making sure her credit is good, she goes up one floor to the jewelry department. Then she goes down one floor to the children's department. Then she goes up three floors to the toy department. Finally Jamie goes down ten floors to the main entrance of the store, which is on the first floor, and leaves to go to another store down the street.</p> <p>How many floors does the department store have? Why?</p>		

Object 4: Data collection, part 2: Activities 3 and 4

In the end, another semistructured interview –as it is shown on the object below– put an end to the data gathering process. Students were asked to choose the activity they were willing to start with, in order to make them feel comfortable. They had to choose the order of the remaining activities as well.

Which activity do you prefer to start with?

- How did you solve ACTIVITY 1? What was the first step? And then..?
- What language did you use to start solving the problem?
- Did you switch languages? At what moment did you do so?

- How did you solve ACTIVITY 2? What was the first step? And then..?
- What language did you use to start solving the problem?
- Did you switch languages? At what moment did you do so?

- How did you solve ACTIVITY 3? What was the first step? And then..?
- What language did you use to start solving the problem?
- Did you switch languages? At what moment did you do so?

- How did you solve ACTIVITY 4? What was the first step? And then..?
- What language did you use to start solving the problem?
- Did you switch languages? At what moment did you do so?

- In general, when have you used English? Why do you think you did?
- In general, when have you used Spanish? Why do you think you did?
- Are there any words or phrases that you found difficult in English? If so, which ones?

Object 5: Data collection, part 3: Activities' questionnaire (English version)

For each activity, the same questions were planned to be asked. First of all, students were demanded to explain the solving process, in order to make them recall what they did to solve the problem. Then some questions were asked to see if there had been any switch of language. This control inquiry was followed by a concrete demand about when one particular language usage had occurred.

Once each activity had been discussed, a final set of questions pursued regarding the four activities as a whole. Finally it was demanded if there had been any particular word or phrase on the wordings which had been of special difficulty for them to understand.

¿Con qué actividad quiere empezar?

- ¿Cómo resolvió la ACTIVIDAD 1? ¿Cuál fue el primer paso? ¿Y luego...?
- ¿Qué lengua uso para empezar a resolver el problema?
- ¿Cambió de lengua? ¿En qué momento?

- ¿Cómo resolvió la ACTIVIDAD 2? ¿Cuál fue el primer paso? ¿Y luego...?
- ¿Qué lengua uso para empezar a resolver el problema?
- ¿Cambió de lengua? ¿En qué momento?

- ¿Cómo resolvió la ACTIVIDAD 3? ¿Cuál fue el primer paso? ¿Y luego...?
- ¿Qué lengua uso para empezar a resolver el problema?
- ¿Cambió de lengua? ¿En qué momento?

- ¿Cómo resolvió la ACTIVIDAD 4? ¿Cuál fue el primer paso? ¿Y luego...?
- ¿Qué lengua uso para empezar a resolver el problema?
- ¿Cambió de lengua? ¿En qué momento?

- En general, ¿cuándo ha usado inglés? ¿Por qué cree que lo hizo?
- En general, ¿cuándo ha usado español? ¿Por qué cree que lo hizo?
- ¿Hay alguna palabra o frase que haya encontrado difícil en inglés? Si es así, ¿cuáles?

Object 6: Data collection, part 3: Activities' questionnaire (Spanish version)

The dialogues resulting from this third part of the data collecting process were almost fully transcribed, keeping the original languages (English and Spanish) in which such dialogues occurred. It is to be stated, though, that some modifications were applied on such transcriptions. Thus, for example, those parts containing a paper demand or instructions clarifications were sometimes omitted. References to the solving process – part 2 of the data collection– and scanned solutions were added for clarification purposes.

For each student, all of the information gathered is organized around two objects for further analysis, as it is described in the following section, “Instruments and data analysis”, starting on page 72.

3.2.1 Mathematical and modal characterization of the activities

This subsection aims at explaining the way in which the activities have been designed and which decisions were made while planning the questionnaire. For the purpose of a proper understanding of the terminology used, it is to be noted that within this work the word *statement* has a more general sense than *wording*, as the first one may refer to an activity containing information in the visual mode (as A1, A2 and A3) while the second one is reserved to the solely use of verbal written text (as A4). Furthermore, all throughout this work such terms as 'activity', 'problem' and 'task' are used as synonyms to refer to any one of the four activities of the questionnaire.

Now, some mathematical and modal (Reverter, 2008) aspects of the activities are remarked. Firstly, some general aspects are commented and then the characteristics of each activity are depicted individually. *High or middle school students* or simply *students* refer hereafter to those actual students who participated in the present study.

Mathematical characterization

One of the aims of this characterization is to state the kind of mathematical context in which each activity is embedded. There are 2 problems with an academic context (A2, A3) while two of them count on a realistic context (A1, A4).

Later, references to some solving paths and possible tricks encountered during resolution are also made.

Finally reference to the curriculum for middle and high school students is made. Note that all activities end with the question “Why?”. This is to force students to explain how they get their solutions. The indication “Write down the solving procedure” is also present at the beginning of the questionnaire –see “Object 4: Data collection, part 2: Activities 1 and 2” on page 59– for the same purpose.

All four activities can be solved using arithmetical procedures. Nevertheless, visualization can be important on A2 and, specially, on A4. On the other hand, A3 has a clear geometrical scenario.

It is important to point out that *No question has a single meaning. One way to uncover the multiple meanings of a question is to explore how different participants respond to the*

question (Moschkovich 2007b). Following this assertion, it is true that some of the insights within the several problems were provided by the students, which contributed to provide a new perspective for the activity.


Furthermore, *students' "unrealistic" answers may not be unrealistic at all, but may stem from a "realistic" effort to adapt to the socio-cultural norm of schooling* (Inoue, 2005, p. 71). For instance, the well-known answer of adding the number of sheep and goats to get the age of the captain of a boat may be the result of the captain having one animal as a present for his birthday each year. Inoue's work is centered on word problems, but it can also be extended to other type of problems. For example, $1+1=2$ may not be true if there is one bird and another comes but the latter is a predator of the former. This is the reason why knowing how students have approached the problem and which kind of considerations have they taken into account would lead to a closer perspective of the validity of their solution in regard to the school mathematics. That is, some students may well consider many real world aspects while other may consider more realistic to stick to the culture of the schooling and give a calculational answer even if unrealistic in case they are based on their own understanding of the situation. Verschaffel, De Corte and Lasure (1994) also report that students do not always include real world knowledge when they solve mathematical activities.

Modal characterization

For each activity a modal skeleton is made containing the disposition of the different modes (figurative visual, abstract visual and verbal) present in the statement (Reverter, 2008). When referring to the verbal characterization, a problem is classified as linguistically simple or linguistically dense. This last description occurs when there are a lot of sentences together. In this sense, it is necessary not only to understand their meaning individually, but to connect it all through the text. This has to be done in any particular text, but with mathematical texts being linguistically dense, this procedure takes even more importance.

For the purpose of this work, statements have been considered to be linguistically simple when the verbal mode of it is made by just a few sentences. This is the case of Activities 1, 2 and 3, which contains 3 sentences or less. On the contrary, Activity 4 is linguistically dense.

Activity 1

ACTIVITY 1	
JOHN SPORTS	MIKE SPORTS <small>UNBEATABLE PRICES!!!</small>
40 % <small>DISCOUNT</small>	25 % <small>DISCOUNT</small>
	
In which of these two stores are the shoes cheaper? Why?	

Figurative visual mode	Figurative visual mode
Verbal mode	

Object 7: Activity 1 and modal skeleton of Activity 1

This activity has been adapted from Heuvel-Panhuizen (1996) –see the original version on the next page. The picture of the shoes has been modified to offer an actualized image of the current dressing trends. By doing this, students should get deeper into the situation of buying a shoe. In Salinas most of the people tend to go the Mall where big brand stores are also present. This is the reason why by changing the name of the shops it was expected to develop a view of a familiar store with a familiar treatment.

“Our prices are the cheapest!!!” has been reformulated into “Unbeatable prices”. As it will be seen through the solving process of many students this caused a lot of trouble due to the initial misunderstanding of its meaning. The text was just shortened and it was not expected this would run students into difficulties. However, it is true that the original formulation offers a more common vocabulary.



Finally, it should be pointed out that option b) of the original activity has not been included because it has been considered to be too much directed straightforward towards the solution.

Mathematical characterization

In the statement of this problem there are two advertisements representing two different stores. It goes without saying that this is of course a real-life and everyday situation which

is perfectly known by students. So this is indeed a realistic context. This would be an arithmetical activity, but in reality no calculation can be done due to the lack of initial prices. Even so, correct exemplifications with different initial prices could shed light upon it

Best Buy

EVER SPORTS	World Sports
discount 40%	OUR LIST PRICES ARE THE CHEAPEST !!! now 25% off our list price
	

a In which of the two shops do you think the sale price of the tennis shoes is the lowest?
Explain why you think so.

b Is it also possible that the sale price of the shoes in the other shop is the lowest?
Explain your answer

in order to find the right solution to the situation. Students should be aware of it and consequently say that either more data is needed or that there is no solution, as the initial prices are not given. The same picture of the same shoe in both stores could lead to assume that the initial price is the same for both stores, but this is not stated anywhere on the actual statement. Anyway, this could be an assumption that the students made in an attempt to solve the problem in a school context (Inoue, 2005). The “UNBEATABLE PRICES” sign could also lead to confusion if, as all advertisements intend, it is given a total confidence.

Object 8: Activity extracted from Heuvel-Panhuizen (1996). Activity 1 was adapted from it.

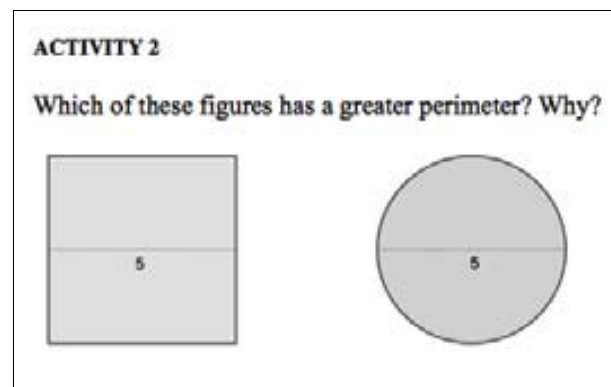
That said, it is to be taken into account that Middle school students had covered the topic of percentages at the beginning of the course, but this kind of activity –where percentages

cannot be applied to a quantity– has not been specifically taught. On the contrary, high school students must know about percentages. For them it is not a specific topic which they are taught the very year when the data collection took place.

Modal characterization

In the wording of the problem, the interrogative pronoun “which” refers to “stores”, which appear later in the sentence. Moreover, which, at the same time, are depicted above the verbal mode, in the visual mode. “UNBEATABLE” (on the visual mode part) and “cheaper” (on the verbal mode part) are likely to pose semantic problems, rather than the rest of the words, which are quite common.

Activity 2



Verbal mode	
Abstract	Abstract
visual	visual
mode	mode

Object 9: Activity 2 and modal skeleton of Activity 2

This activity has not been gathered from any existent document. It has been indeed invented for the specific purpose of the research. This was the last activity included and it was suggested to be included in the experience in order to cover the topic of geometry. It has to be noted that the use of the word “greater” does not refer uniquely to the 1 dimensional notion of perimeter, but it could refer to the 2 dimensional notion of area.

Maybe the fact that the geometrical figures are shaded may lead to confusion on the students that do not have a clear notion of perimeter, because perimeter and area concepts are often studied together. But perimeter is in fact the limit of a planar figure. Formulas may then be interchanged if there is not a solid understanding of concepts.

Mathematical characterization

This activity has an academic context: there are two abstract pictures –a square and a circle– with a dotted line in each one giving a measurement of the figure: the square's side and the circle's diameter. Both distances are five units long, so the circle can be inscribed on the square. If this is done, then both perimeters can be compared visually by dividing the figures in halves or in four parts (thanks to the symmetry that the figures have) and then compare the resulting perimeters in a visual way. A numerical way of solving the activity is calculating both perimeters according to the data provided. If the notion of perimeter is clear, at least both lengths should be compared *grosso modo* visually, saying that one is longer than the other one.




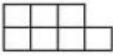
Middle school students covered the topic of perimeter and area formulas just before they solved the questionnaire, while high school students had not seen this topic that year. It is important to note that applying the notion of perimeter goes beyond knowing its meaning or being able to give a definition of it (Kazima, 2006).

Modal characterization

As said for Activity 1, the interrogative pronoun “which” could cause some problems here. In this case, it refers to “figures”, which, at the same time, are depicted below the verbal mode, in the abstract visual mode. Along with “which”, “perimeter” is more likely to pose semantic problems, rather than the rest of the words, which are rather common. Furthermore, as it has already been said, it has not only to be with the conceptual understanding of the word “perimeter” but also its confusion with “area”.

Activity 3

ACTIVITY 3
Observe this pattern.

			
Figure 1	Figure 2	Figure 3	Figure 4

How many tiles does figure 7 have? Why?

Verbal mode			
Abstract visual mode	Abstract visual mode	Abstract visual mode	Abstract visual mode
Verbal mode			

Object 10: Activity 3 and modal skeleton of Activity 3

This activity is similar to the ones presented in the High School textbook (Leslie, Kysh, Sallee, & Hoey, 2008). The figures represent a pattern that might be really easy to follow in the case of the high school students and which should not present major difficulties for middle school students. It was chosen for students to feel comfortable with the mathematical difficulty –at least with this activity– and to see if the high school students resorted on the use of the Language of Learning and Teaching (LoLT).

Mathematical characterization

The activity has an academic context: the four initial figures of a sequence are presented, and it is demanded to find the number of tiles of figure 7. No mention to figures 5 and 6 is made on the statement. The number of tiles per figure forms an arithmetical sequence of growth 2, starting at 1. So the solution can be found by following this sequence. Another way to get the solution is noting that figure n has $2n-1$ tiles (e.g. a row of n tiles and another one of $n-1$ tiles; or 2 rows of $n-1$ tiles with the addition of another tile). Both paths would involve an arithmetical (and also maybe a visual) reasoning. But the solution can be obtained also by drawing the missing figures of the sequence (5, 6 and 7) or by drawing figure 7 directly. In such cases there would be a combination of visual and arithmetical reasoning.

This activity is well known by the high school students participating in this research, who used manipulative tiles in these and other types of exercises. Many figure pattern exercises were done during the course. This exercises usually demanded $x-y$ tables, $x-y$ formulas –where x is the figure number and y the number of tiles–, drawing of figures following the sequence presented according to the pattern and also drawing figures and calculating its number of tiles with figure numbers much larger than the ones initially presented.

Modal characterization

Here the abstract visual mode is in between two pieces of verbal mode. Each piece of visual mode is attached with a piece of verbal mode too –Figure n – as a legend that gives

meaning to the pictures. From a semantic point of view, “pattern” and “tiles” are to be the more difficult words. Both are related with the abstract visual mode: the first one relates to the whole visual mode, while the second one relates to the compounding parts of each figure. The auxiliary “does” may lead to confusion to Spanish learners of English who recently entered in contact with this new language, as Spanish does not use auxiliaries in such a way.

Activity 4

ACTIVITY 4

Jamie is shopping in a large department store with many floors. She enters the store at the middle floor, and immediately goes to the credit department. After making sure her credit is good, she goes up one floor to the jewelry department. Then she goes down one floor to the children’s department. Then she goes up three floors to the toy department. Finally Jamie goes down ten floors to the main entrance of the store, which is on the first floor, and leaves to go to another store down the street.
How many floors does the department store have? Why?

Verbal mode

Object 11: Activity 4 and modal skeleton of Activity 4

This activity was provided by a colleague, with no reference to its original source. It was considered worthy as he commented that it was proposed to his summer school students, resulting to be an engaging activity with detailed argumentation of the solving process provided. Minor changes have been made from the form supplied. A similar statement can be found in Johnson (2001).

Being the most difficult activity on the questionnaire, it is situated at the end, as some students solve the activities in the order they are presented and it might be frustrating for them to be in an impasse situation, not being able to solve the activity. Furthermore, it is the most demanding from a language point of view as it composed exclusively of text, with a lot of sentences. Of course it has been regarded as a difficult activity and its appropriate inclusion into the questionnaire has been discussed, but finally prevailed the fact that it is also important to present challenging activities. Furthermore, the use of a sketch should effectively decrease its difficulty.

Another issue considered was the reference of tall buildings like the one described in the statement (with more than 10 floors) given that in Salinas department stores do not surpass a height of three floors.

Mathematical characterization

The presented situation is embedded in a real-life and everyday activity: a person is shopping in a department store and moving through some of the floors of the building. Even if this statement is classified as having a realistic context, the fact that so tall buildings do not exist in Salinas should be kept in mind.

Two are the key points that help find the answer because they are used as a reference: 1) Jamie enters the building at the *middle floor* and 2) by the end she leaves the store on the *first floor*. The other movements she does meanwhile (she goes up and down through the building) have to be situated relatively to those 2 points, which serve as main absolute references. In this activity making a sketch to represent the situation is of great help. Then visualization is crucial to understand Jamie's movements. If this done, easy arithmetical thinking helps to complete the task.

Note that as there is a middle floor, the total number of floors must be an odd number, and the middle floor must be counted when calculating the total number of floors (eventually making a symmetry through the middle floor: doubling the number of floors through the middle one and adding then consequently the above-mentioned middle floor).

As numbers are not that big, guessing and checking can be another way to solve the problem. Thus, different numbers can be proposed (guessed) as a solution and then check if the conditions presented in the problem are satisfied (following Jamie's movements through the building). Anyway, checking the solution by following again Jaime's movements clearly determines if the obtained solution is right or not.

A third way to solve the problem would imply the use of an equation, with no need of a sketch to represent the situation.

On this activity there isn't any particularly difficult curricular content for both, middle and high school students, as the topic turns around the number line. Rather than conceptual, the problem is clearly more procedural.

Modal characterization

This activity is formed by a verbal mode only. It is linguistically dense and there could be a lot of problems arising from the semantical point of view, although the meaning of some words (e.g.: jewelry, toy) is not crucial to find a solution to the problem. Particularly

confusing can be the expression “making sure her credit is good” if the translation is made in a word to word basis. However, omitting or making a deviated translation of this part should not be relevant for the mathematical procedure.

First of all, the situation is presented and then, on a separate paragraph, the objective of the problem is presented with a question (in fact there are two questions, considering the justification demand that closes all statements –Why?–).

Just by interchanging two letters of Jamie, one gets Jaime. So a female English name is transformed in a male Spanish name. Although this is not consistent with the use of pronouns in the wording –she– and it is not relevant from the mathematical point of view, it can shed light on the use of languages issue. This particular phenomenon was not advised on the design of the questionnaire, but emerged from the analysis of its solutions.

Particularly important are the words “department” and “floor”. Department is close to the Mexican Spanish word “departamento”, which is an apartment. As a result, the term “large department store” can be misinterpreted –as a result of a word-to-word translation, with no attention paid to the adjective function of “large” and considering separately the meaning of “department” and “store”–. “Floor” can be translated to Spanish as “piso”, which is a polysemic word also meaning, among others, “the kind of material the floor is made of”.

At the end of the situation's description it is found “which is on the first floor”, referring to the “main entrance” –situated at the beginning of this long sentence–. A good understanding of this part is a crucial point in the horizontal mathematization. “Goes up” and “goes down” are also essential to correctly situate Jamie's movements through the building. As in Activity 3, the auxiliary “does” in the last line of the text, may lead to confusion to beginner Spanish learners of English, as Spanish does not use auxiliaries in this way.

3.3 Instruments and data analysis

This section reflects how the answers to the activities and the interviews around them are organized to show the mathematical solving process and its relationships with language for each student. For this purpose data is arranged around two objects for each *student*: *Student-First* reduction and *Student-Second* reduction. The first one analyzes the mathematical and language aspects mainly separately, while the second one relates both in a more direct way. Findings combining language and mathematical aspects are organized on Themes–Third reduction regarding all students together. Each of these three steps of the analysis are described in more detail in the corresponding subsections below.

The constant comparative method is at the core of these instruments. In Boeije's (2002) words:

The constant comparative method (CCM) together with theoretical sampling constitute the core of qualitative analysis in the grounded theory approach developed by Glaser and Strauss [...]. Comparison is also the dominant principle of the analysis process in other tradition of qualitative research. All kinds of aids, such as memo writing, close reading and rereading, coding, displays, data matrices and diagrams support the principle of comparison (p. 391).

Issues such as what is being compared, the phase of the research where it (mainly) takes place and the reasons for the comparison will be detailed in order to explicit traceability and to highlight the way in which the work has been systematized. This shall contribute to opening the door of verification and credibility.

Making comparisons, the research findings arise more or less inductively; by categorizing, coding, delineating categories and connecting them. Constant comparison goes hand in hand with *theoretical sampling*. That is, data will be initially arranged according to ideas from previous researches or theoretical ideas. As more and more data are analyzed, categories are modified accordingly to the new findings (creating new categories, modifying or deleting some of them). “*This cycle of comparison and reflection on 'old' and 'new' material can be repeated several times. It is only when new cases do not bring any new information to light that categories can be described as saturated*” (p. 393). When this situation is attained any case can be assigned to one or more of the defined categories with no difficulties. This process is repeated several times and it is not done exclusively at the same time during the main process of the creation of an object. For example, the language items on the Student-First reduction are not only pointed out when the First reduction is mainly created, but some items can be added later, during the constant process of data review. The same practice occurs with the items of the Second and Third reductions.

3.3.1 Student-First reduction

Initially, the interviews were heard several times and just some passages involving mainly both mathematics and language were transcribed. Paragraphs were selected and classified as related to mathematics, language or both. One or more codes were assigned to them. Also the solutions produced by the students for each of the activities were scanned and placed before the transcriptions. The problem there was that it was not easy for a person who had not previously heard the interview to follow the resolution

process. Another issue was that the codes were not explicit enough and reference to their definition was required too often.

Regarding this problematic, a slightly different approach has been taken. Interviews are now (almost) fully transcribed and mathematical and language aspects are described in a few words instead of the compact codification used previously. This first step on the analysis of data is organized around each *Student-First* reduction as described below.

This First reduction of data contains four sections –see “Object 12: First reduction layout” on page 76. This terminology of sections is just used here to help describe how the data is organized and it is not reflected on the analysis. The **first section** is a brief summary explaining actions and abilities of the student in the mathematics classroom, made by the teacher. The remaining sections correspond to the three parts of the data collection process, as described in “Instruments and data collection”, page 55 and following.

The **second section** has the extract of “Object 1: Students’ Historical profile” (pages 49-53) for that particular student. This section was not considered at the beginning of the data analysis but included later, mainly to bring more details and to help to have a deeper explanation on the “Tentative” column (see its characteristics below) of the third section.

The core of the analysis was initially what is now the **third section**. It contains the solution of the activities (see their statements on pages 59 and 60) and the pursuing student-interviewer interaction that resulted from the semi structured interview (see “Object 5: Data collection, part 3: Activities’ questionnaire” on page 61). It is described in more detail below.

The **fourth and last section** is a short summary pointing out the more important math and language aspects of the previous section. This part is the last one to be created and it is not until the First and Second reductions were already shaped that the need for a summary of the solving process was considered necessary for a quick understanding of the solving process, which in some cases took a considerable amount of time and imagination to be decoded. Some extracts of the interview are not easy to understand and the whole resolution process demands occasionally to be read again and again with an open mind to clarify what is happening. Other times it is the resolution of other students what sheds light and permits to clarify the mathematical aspects involved. This summary also facilitates the organization of the elements of the Second reduction.

The section also contains an overall description of the Spanish and English language use made by the students only in relation with the written answers of the four mathematical activities of the activities and the interview around them, using Cummins’ terminology

<p>[description of the level of management of Spanish and English languages; individual summary of each activity]</p>
<p>Activities' (Key ideas) summary [4th SECTION]</p>
<p>Object n: <i>Student-First reduction (End)</i></p>

Object 12: First reduction layout

For each activity there is a heading –*first row*. Column headings are always the same except for the second column, which is the only one varying. The first column heading is “Math”, the third one “Language” and the fourth one “Tentative”. The second column's heading details the activity, the student's name and which one is the column chosen by the student in the questionnaire to point out the language(s) used (“English and Spanish” or “English only”). Thus, for example, *A4. Juan marked “English only”* is the heading of Juan's solution of Activity 4 and posterior interview around it, where he marked the column “English only”. The fact that the language column picked by the student is highlighted here serves the purpose of data reduction and at the same time gives a general picture of the language(s) used by the student. For the fifth part the heading is “General Language Questions”. In this last part there is no “Math” column. In case some mathematical topics arise in relation to the language topics they are commented within such language elements.

A *second row* contains the information that each of the four headings describe. A larger 2nd column contains interviewer's and student's interventions numbered sequentially (starting over in each activity), along with scanned written answers and notes from the video and voice recordings. The content of this column is used to extract the mathematical and language information reflected on columns 1 and 3.

On the left of this larger column, the 1st column extracts some math aspects involved in such activity. Strategic competence and adaptive reasoning (Kilpatrick et al., 2001) will be reflected on this section. Sometimes several math characteristics, though, are embedded on the language items (3rd column) and are not explicitly listed on the first column. As described on the following subsections, the second and third reductions point out when that occurs.

The solving process is divided using *tries* (or approaches to the solutions). Between a try and the following one there is interviewer interaction to clarify, demand, etc. The *0 try* – when it exists– collects features produced during the first try, but which have been

changed by the student with no external help (this is frequent when solving a mathematical exercise). Initially only a dichotomical classification of the answer through right or wrong and a general description of the main mathematical features was provided. Then need of a closer look at the mathematical particularities emerged during the analysis. Even so, this initial kind of “summary” of the activity has been kept. It is marked as **0 try**, **1st try**, **2nd try**,... and is situated after the mathematical items that also refer to such a try have been listed.

On the right of the larger column, the 3rd and 4th columns select the *Language* features (3rd column) from the referred activity and what could be the *Tentative(s)* (4th column) that promoted this language aspects. Reasons do not pretend to be listed exhaustively, but to give a general idea of what is(are) the reason(s) that influenced the language item.

Of course, math and language aspects (along with their “tentative” items) are mainly developed on the first stages of the analysis. Anyway, all of them have been constantly reviewed and modified as new insights emerged, also during the construction of the Second reduction and in relation to the cases of other students.

Finally, for all five parts, there is a *third row* containing “MEMOS”. These are mainly those thoughts that combine language and mathematical aspects arising from the solving process. They are not stated in an exhaustive way. They just have been written down while transcribing the audio or analyzing the math and language aspects separately, in order to be of help later on the Second and Third reduction of data. The “MEMOS” row also contains some items from the math or language column that have called the attention for being rather rare among all the learners under study or for other characteristics. Or even items that are not in the language or math columns but whose inclusion in such columns is left for later consideration. Parts that have not a clear interpretation are also listed here. When the first students were analyzed these memos were written down in an informal way. Later the decision of the usefulness of these notes for the reader become clearer and then were recorded systematically for all students and the initial notes were rewritten in a more understandable way.

The math and language items highlighted on their corresponding columns were obtained at the beginning mainly by fragmenting the solving process. So there was an emphasis on the separate themes that emerged and the focus was on the findings of particular math and language aspects which could be relevant to the research questions. Lately these aspects were integrated on the resolution process, looking at the activity as a whole.

Some questions that guided the construction of the *Student-First* reduction are as follows: Which language aspects are found on the written solutions? Which language aspects are

found during the interview? Which language aspects do the students point out when talking about the mathematical solving process? What reasons do they give for a particular use of the language(s)? What other reasons could be considered? What are the steps followed on the mathematical solving process?

To put an end to this subsection some notation used on the First reduction is described. Dialogue is in chronological order just within each activity. It does not appear the same way it is recorded, but references to the chronological sequence as a whole are added throughout the transcript, indicating when the conversation moves from one activity to another. This is done for clarification purposes, as sometimes it looks like part of the transcribed dialogue in a particular activity does not make much sense for itself. The notation $A_{n,m}$ serves to follow the chronological order. More notation (emphasized here in ***italics and bold***) is described below.

- ***A1*** means Activity 1, and analogously for ***A2***, ***A3*** and ***A4***.
- ***A_{n,m}*** stands for intervention *m* in Activity *n*. So $A_{1,14}$ would mean intervention 14 in Activity 1.
- ***F*** stands for Francesc, the interviewer. We shorten the names of the students by using their initial letter. For example, Ana is shorten by ***A***.
- “***Double quotation marks***” are used to refer to written text, either from the statement or from the student written answer.
- ‘***Single quotation marks***’ are reserved to refer to a part of speech.
- ... at the end and again at the beginning of one person's intervention indicates that the intervention of the other person in between the two sets of 3 points was done interrupting the first person, as in the following example:
 - 23. F: Did you thought...
 - 24. [Interrupting] J: In English.
 - 25. F: ... in English or in Spanish?
- ***[brackets]*** are used in general to add information that is not said to the interview, e.g., the order in which parts of the written solution were created.
- ***[...]*** or ***[pause]*** means that a certain amount of time was spend between utterances. For example, when the interviewer makes a question and –after a while– it is not answered by the student. ***[...]*** also appears when part of the dialogue is omitted; such as when the student asks for an eraser, when the conversation clarifies how to mark the cross in the corresponding column of the questionnaire, when the student is asked which activity s/he prefers to start with or continue with, etcetera.

- **[unintelligible]** appears when a word is not understandable. It is not transcribed.
- **[unintelligible sequence]** appears when more than just a word is not understandable from the recording tape or audio. It is not transcribed.
- A word followed by **[?]**: this mark is used when, according to my understanding, the word before the question mark does seem to be the word said **[?]**, but it is not completely clear. For example, in the sentence 'He did it fast[?]' we are not sure that in fact the word *fast* was, said, even though it appears to be the case.
- **[Dubitative transcription]** follows a sentence with an unclear recording, transcribed anyway.
- **[25%]** is used to refer to Mike store, which has a 25% of discount and **[40%]** alludes to John store, which has a 40% of discount.
- Sometimes the phonetical transcription of a word is added in between slashes, as for example: pi [*/pai/*], for the π number.
- In the math column, brackets [] are used to refer to (part of) the answer as stated by the student, combined with quotation marks in the sense described above in this list. If no quotation marks are used, the answer is either summarized or translated from Spanish.
- 1st, 3rd and sometimes 4th column's items start with numbers. Such numbers refer to the numbered sequence on the 2nd column where such items are found. These numbers pointing to dialogue interventions appear also within parenthesis or dashes in those language and math columns. Sometimes, in the *Tentative* column, when more than one reason is given, such numbers (followed by a point or a colon, indistinctly) precede, if applicable, the reasons. For example:

Language	Tentative
30-40. Use of Spanish to recall mathematical concepts.	Mathematical difficulties, 16: concepts learned in Spanish

One of the reasons for the “Use of Spanish to recall mathematical concepts” (language aspect observed on interventions 30 to 40) is “concepts learned in Spanish” (which is found on intervention 16).

Or in the math columns it could be found, for example:

61. 2nd try. Operational definition of perimeter (validated by interviewer – 62–)

Here the operational definition of the notion of perimeter is found in the intervention 61, belonging to the 2nd try or approach to the answer. The interviewer validates this definition in the intervention 62.

One more thing to consider is that by “Strong bilingual profile” we refer to the good management of both languages, which allows, for example, to switch between languages in a quick and natural way.

3.3.2 **Student-Second reduction**

This second step of the analysis is carried out once the First reduction is already shaped. It is still within each student scope and serves to have a general idea of what happened in each activity and among all of them as a whole in relation with language and mathematics, and so checking if there are any inconsistencies. This part of the analysis accentuates the context and richness of the data as the different math and language items are interpreted as a whole so all the pieces are connected: it represents an attempt to integrate both math and language aspects as they have emerged during the interview as told by the interviewee and along with a consistent interpretation of it.

In this part of the analysis, Language and Mathematics items from each “*Student-First reduction*” object are reported on the corresponding “*Student-Second reduction*” object, which is organized around four profiles –see its layout “Object 13: Second reduction layout”, on page 83. At the same time the math items on the Second reduction are described under its conceptual and procedural characteristics. This helps to have a better insight of the mathematical aspects. Now each profile is commented, followed by a list of the notation used within each one.

The **Historical profile** sums up all the information gathered from “Object 2: Data collection, part 1: Historical bilingual profile”, which is also reflected on the First reduction as “Historical bilingual profile”. To sum up the information in a visual way, when there are two languages and one is underlined, it means that such a language is more used than the other one. E.g., in “English (brother) and Spanish at home” (which might be an item of the Historical profile), Spanish is more used than English at home, English being spoken only with their brother.

The **Bilingual profile** contains five columns. In the first one, the most important language items already listed in the “Language” column of the “First reduction” object, are registered in the order in which they appear within each activity in the referred object. This order when listing the items is established just for practical purposes. It is followed unless a

particular item has already been listed. In such a case it is marked in the corresponding column of the corresponding item's row. The other four columns correspond, in this order, to activities 1 to 4 and are used to see which is/are the activity/es where such item appears. The numbers in each of these columns refer to the tries within each activity. E.g., “2-4” means the item refers to 2nd, 3rd and 4th tries. When the item does not refer to a particular try but to the entire activity, it is marked with an “x” in the corresponding column. The language used by the student to interact with the interviewer is detailed in the heading.

For example:

Bilingual profile (Spanish dominant)	Activity			
Spanish used as thinking language	x		2,4	

Here *student* chose to use Spanish during the interview as it is stated into the heading, between parenthesis. Spanish is used as thinking language in Activity 1 and on the 2nd and 4th tries of Activity 3.

Finally, those items found out in the General Language Questions (GLQ) –asked at the end of the interview– are also listed. For space economy only, sometimes the General Language Questions are placed under the Historical profile. The presence of the four profiles in just one page helps to have an overall overview for that particular student.

- In between the parenthesis, “**English dominant**” or “**Spanish dominant**”, refers to the language chosen by the student at the very beginning of her/his interaction with the interviewer, as commented in the example above.
- Code switching instances are grouped within each n-Activity as **An(i+j+k)**, the numbers within the parenthesis indicating the number of switches done by the student while interacting with the interviewer around such activity. These switches are grouped depending on the math and language involved in the code switch: *i* refers to a code switch involving mathematics, *j* when math is not directly involved and the student is not reading and finally *k* is the number of switches in the n-Activity produced as a result of reading the wording or the solution produced by the student.

The **Procedural and Conceptual profiles** are related with the mathematical items specified in the “Math” column of the “First reduction”. The Procedural profile describes what the student did while the Conceptual profile points out which are the conceptual aspects, as the names of the profiles suggest. These are associated, respectively, with the notions of conceptual understanding and procedural fluency (Kilpatrick, Swafford &

Findell, 2001) commented on Chapter 2. Strategic competence and adaptive reasoning (Kilpatrick et al., 2001) have been incorporated on the First reduction.

- Right (✓) and wrong (✗) symbols are used in each item to quickly visually interpret its correctness. The use of both together (✗/✓) means that positive and a negative aspects are intertwined in the item.
- (✗) –parenthesis included– is used when the item is wrong according to the intended understanding of the activity, but it is right from a mathematical point of view.

For example:

Procedural profile	Conceptual profile
1.1 ✗ Direct comparison of percentages	1.1 ✗ Percentages as absolute value instead of relative

Here *student* gives a wrong answer (✗) on the 1st try (.1) to solve Activity 1 (1.). Percentages are compared directly, so percentages are used as having an absolute value instead of relative.

At the very beginning of the object the facts that relate mathematical and language aspects are listed. These items can be found either at the top (when the 2nd reduction is in just one page) or in a different page (when there is not enough space in only one page these descriptions are moved in block to another page). Here there is a process of analysis, comparison and creativity to organize and present these aspects.

Some questions that guided this second step of the analysis are as follows: What are the conceptual and procedural characteristics of each try? How are mathematical and language aspects related to each other during the solving process? How are codes used with this particular student? (As more students are incorporated to the analysis –mainly on the next step– the consistency of the codification is also checked) Is the storyline consistent? Are there any expressions that are contradictory? How are all the fragments related?

-
- [List of aspects that relate mathematical and language items]
-

Historical profile	Bilingual profile (Spanish dominant)	Activity			
<ul style="list-style-type: none"> • [Information summarized from Historical bilingual profile] • • [Sometimes, if the Activities-Bilingual profile is too large, it is placed partially here, for space organization only] 		x	2		
			x		
	[Language information about each activity]			1	
				x	x
			GLQ		
			GLQ		
Procedural profile	Conceptual profile				
1 X Comparison of percentages X 2.1 √ [Mathematical information √ about each activity: 2.2 ? parts of the solving process] 3 √ 4 √	1 X X { 2.1 √ √ [Mathematical information about each activity: 2.2 ? conceptualization of the procedural profile] 3 √ 4 √				

Object n : Student-Second reduction

Object 13: Second reduction layout

All the first and second reductions are listed in the next Chapter.

3.3.3 Themes–Third reduction

The third step of the analysis is carried out by comparing the findings among all students. Regularities are found on the basis of the constant comparative method as it has been described all throughout this section. The different cases are incorporated progressively to the emergence and definition of themes, with indicators and characteristics of each concept constantly reformulated. So the Third reduction or list of themes emerges through the constant comparison of First and Second reductions among different students, which are progressively incorporated to the comparison. After all the work done during the First and Second reductions, there is already an intuitive idea of what themes can be obtained. Now, minutely, all findings are considered and incorporated to the list of emerging themes. Extracts of the dialogue are quoted to illustrate where these themes are found. As more examples are considered, themes are enlarged, reformulated or merged before the final shape is given. “Contradictory declarations on language use” is an example of a topic detected among many students.

For this purpose, questions such as the following served as a guide: Is the category present in student A also present in student B? What do both students inform about the category? What are the similarities and differences between students A, B, C, ...? What are the criteria underlying this comparison? What combination of codes/concepts occur? What interpretations exist for this? Is there any other particularity in this excerpt? (Boeije, 2002).

All these themes are listed and explained in section 4.2: Findings related to commonalities, page 360.

4 Discussion and findings

I do not wish to simply show, however, that the word problem task provides opportunities for language learning. (...) language-focused exchanges are not separate from the students' mathematical thinking (Barwell, 2009a, p. 74).

This Chapter consists of two large sections that comprise the analysis of the data. The first Section contains the first and second reductions of each student. A detailed description of how its elements are presented and what they contain has been presented in the previous Chapter. A summary is provided below.

For each student both reductions are presented consecutively. The purpose of this organization is to facilitate the analysis within each case study. The order of the students and the corresponding first and second reductions are listed on the next page. To facilitate the distribution of the information on the page both landscape and portrait orientations are used. First reductions are on landscape orientation to facilitate the mathematics and language references to the transcribed data (which is at the central fragment of the page).

The first reduction starts with a short description of the student made by his/her teacher. Later the information provided through the first semi-structured interview is summarized. It contains the student's use of both languages in diverse contexts, other than school. After it, the second semi-structured interview around the four activities of the questionnaire is transcribed (at this point the student had already solved the questionnaire). An important part of the analysis turns around describing important mathematical and language elements of these transcriptions. In the end, a summary is considered to be useful to better understand the solving process. It contains the overview of the solving process for each activity. For information purposes, it also includes a categorization of the students competences for English/Spanish BICS and CALP.

While the first reduction lasts several pages –mainly because of the length of the dialogues–, the second reduction is distributed within one or two pages only. Initially the second reduction contains a list of the most important characterizations that relate some of the mathematical and language characteristics of the solving process. Secondly, it provides a good picture of what happened on the first reduction, summarized around four “profiles”. All four profiles are always on a solely face of the sheet to facilitate the relationships among its elements. When the narratives that accompany them have enough space on the same page, all the information is under just one page. Otherwise, two pages are used.

Section two contains the third reduction of data, list of emerging themes. Within each theme there is an initial description of the theme's characteristics. Possible explanations of the phenomenon described are also provided. In the end, a list of extracts from the dialogues that illustrate the theme is provided and commented individually. Note that parts of the dialogues may belong to more than one theme. In addition, note that the themes on the third reduction are not the narratives of the second reduction. The list of the second reductions served as inspiration for the list of themes presented in the third reduction.

4.1 Findings related to particularities

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Object 14: Yolanda-First reduction
(Beginning)

Yolanda does not perform very good in assignments or exams but she normally completes the assignments. She is organized and has a polite notebook. She enjoys socializing during class. She talks fluently in English.

Math teacher description

Class	Interview language	Interview date	Origins	Age	USA arrival	Living in California	Spanish readings	English readings	Home language	Friends language	Main language at school	Homework help
HS Transitional	English	January 2010	Mexico	16	Born in USA. At 9 she leaves to Mexico. 3 years before the interview comes back.	Likes it. It's better. School is better.	No	Books, magazines.	Spanish (parents, uncle, brother of 12), both (sister, cousin).	Both	English. Maths, Spanish and Biology in Spanish.	Nobody

Historical bilingual profile

Math	A1. Yolanda marked "English only".	Language	Tentative
1-7. Right use of percentages as relative value.	[Dialogue about activities starts at A1,2 after Yolanda solves all activities. At the beginning of the interview, during the data collection process about "Historical bilingual profile", Yolanda gives the same importance to both languages.]	1. Grammatical deviance (its)	Equally spoken words (its, it's)
1-7. Assumption of initial prices based on the discount rate (from personal experience).	<p>1 <i>The cheaper one is the 25% because like the shoes are cheaper they dont discount it a lot, and on the 40% it looks like its more expensive at how much discount they use</i></p> <p>2 F: Can you explain me how have you solved it?</p> <p>3 Y: Well, because on this store is twenty five percent, on this one is forty. So like maybe because this one is forty</p>	8-10. English used for everything	English wording, good English management

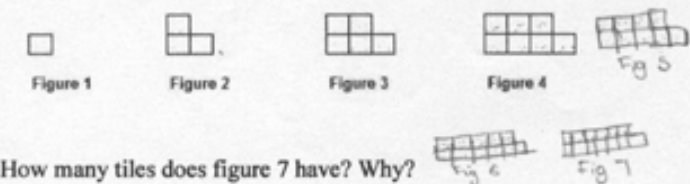
1-7. 1st try. Deviated answer due to assumption of different initial prices.	<p>they have to discount it a lot, because it's more expensive. And this one is twenty five so they don't like ... they only discount a little so maybe because eh... there's... like the store right there [25%] is cheaper. So that's is how I got that this one was the cheaper one.</p> <p>4 F: This one was cheaper?</p> <p>5 Y: Yeah.</p> <p>6 F: So it was cheaper to buy the... the shoes right here [25%] than here?</p> <p>7 Y: Yeah. Because maybe right here [25%] the shoes like they are not that much of expensive than right here [40%].</p> <p>8 F: And you have just crossed here "Only English". You haven't used Spanish at all?</p> <p>9 Y: No.</p> <p>10 F: For any reason? [Pause] No? [Pause] Okay. [Continues in A2,3]</p>		
MEMO	Good English management allows Yolanda to solve the activity entirely in English.		

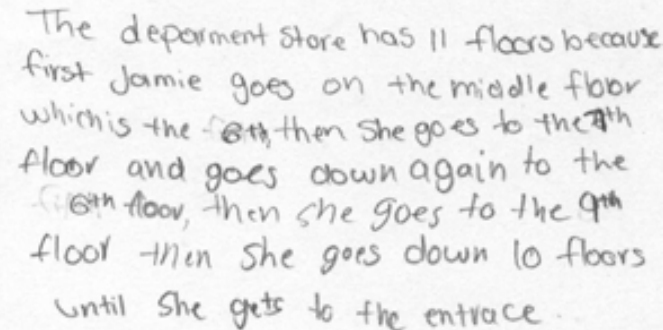
Math	A2. Yolanda marked "English only".		Language	Tentative
1-14. 1st try. Perimeter-area confusion.	1 [Yolanda makes an imaginary segment through the center of the square, from top to the bottom of the square, perpendicular to the dotted line. Later she draws and erases a segment perpendicular to the dotted line from top to the bottom of the square, at approximately one fifth of the length of the square.]	2. Wrong grammar (its)	Colloquial spoken English	
1-14. 1st try. Visual comparison of areas through an imaginary division of the figures into area units.	<p>2 <i>The square because its bigger and because if you start putting squares on the inside you get more than on the circle</i></p> <p>3 [Comes from A1,10] F: How have you solved this activity?</p> <p>4 Y: Well, the perimeter like... I started to [imaginarily] draw like the squares right here [square] inside and I got more right here [square] than here [circle]. So it's this one [square].</p> <p>5 F: You started to draw the squares?</p>	60-68. English used for everything	English wording, good English management	
1-14. 1st try. Deviated answer due to perimeter-area confusion.	6 Y: The squares like do like [she draws a vertical line at one fifth of the width of the square, like the one she did and erased before; see A2,1] right here do one, right here another [draws a 2 nd , 3 rd , and a 4 th vertical line; see A2,25].			
16-17. 2nd try. Right perimeter definition but not right perimeter concept understanding (22-25) even if it looks like it is	<p>7 F: Why that?</p> <p>8 Y: Because that's how I know how to do this.</p> <p>9 F: To... But what's the perimeter?</p> <p>10 Y: Right here... [Makes 2 horizontal lines in the square, see A2,25] Isn't the perimeter the one we use like see how many [unintelligible] there's inside? How many [unintelligible] do we have?</p> <p>11 F: But that's maybe for the area. You were confused with the area.</p> <p>12 Y: Oh, yeah, yeah.</p> <p>13 F: Did you?</p> <p>14 Y: Yeah.</p>			

<p>understood (20-21).</p> <p>22-25. 2nd try. Division of the square as when finding the area.</p> <p>26-37. 2nd try. Perimeter identification on a sheet of paper as the number of sides instead of its length.</p> <p>38-40. 2nd try. Existence of perimeter just with straight lines.</p> <p>40-48. 2nd try. Right perimeter concept.</p> <p>43-51. 2nd try. Right square's perimeter calculation.</p> <p>51. 2nd try. No comparison of perimeters.</p> <p>20-51 2nd try. Incomplete answer due to no comparison of perimeters, with right calculation</p>	<p>15 F: So what is the perimeter?</p> <p>16 Y: The sides? [Makes (imaginarily) 2 semi circumferences to point out 2 sides of the little squares she did (such sides are also part of the top side of the square)]</p> <p>17 F: Mm [agreeing]. Just the sides. You wanna to rethink this activity right now, in a couple of minutes?</p> <p>18 Y: Yeah.</p> <p>19 F: Okay.</p> <p>20 Y: [Makes like 2 semi-circumferences to point out like 2 sides of the little squares she did, which have as top side the top side of the square, as in A2, 25] So the sides are the ones you go like this: one, two, three, four, like this? [Makes a total of 6 imaginary semicircles as she refers to the length of the sides of the little squares, starting from the top left vertex and following the clock sense. She counts up to 4 when she reaches the other vertex.]</p> <p>21 F: Mm [agreeing].</p> <p>22 Y: Okay. [Draws 5 semi-circumferences on the top square's side and 6 in the right square's side. Makes the square divisions bold]</p> <p>23 F: But are you doing squares again?</p> <p>24 Y: Yeah. Because I know how to do it like this.</p> <div data-bbox="456 676 981 1007" data-label="Image"> </div> <p>25 [Final written answer]</p> <p>[Yolanda continues and finishes making up to 5 vertical divisions. Draws 3 horizontal divisions, erasing the 3rd one, which was in the 4th division marked with semi-circumferences. The dotted line can be considered as aligned with the 3rd division.]</p> <p>26 F: Sorry, just a question. I don't know if you have understood what I wanted to tell you. What's the perimeter for example of the [following with the finger the perimeter of the paper which she is using to solve the activity]... of this paper?</p> <p>27 Y: What?</p> <p>28 F: The paper... Just... Eh... Just mark what's the perimeter of the paper.</p> <p>29 Y: All the sides?</p> <p>30 F: All the sides together, right? The outside of the shape we can say. Okay? Okay.</p> <p>31 Y: Yes.</p> <p>32 F: So now, go ahead.</p> <p>33 Y: So it's a four right here or what?</p> <p>34 F: What do you mean? Four? We have four sides, right?</p> <p>35 Y: Yeah.</p> <p>36 F: But how long is gonna be the perimeter?</p>		
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<p>of square's perimeter.</p> <p>52-53. 3rd try. Wrong dotted line interpretation ; or notion of perimeter associated only with straight lines.</p> <p>54. 3rd try. Unknown circle's perimeter formula.</p> <p>54-59. 3rd try. Right visual comparison of perimeters without concise reasons ('is a different shape and it's bigger'). No more detailed explanation demanded.</p> <p>20-59. 3rd try. Answer with visual comparison of perimeters but explanation not detailed enough.</p>	<p>37 Y: Ookay.</p> <p>38 F: In fact what we want is... we wanna compare the outsides of , both figures, right? Of both shapes.</p> <p>39 Y: It's like does it [following the perimeter of the circle] have a side or what?</p> <p>40 F: Well, it's not a straight line, of course, right?, but the outside of this shape [Following the perimeter of the circle with the finger]... we want to compare with the outside of this shape [Following the perimeter of the square with the finger]. Is that?</p> <p>41 Y: Yes.</p> <p>42 F: Okay. Just to make sure you have understood that. [Pause]</p> <p>43 Y: So is it like how much is this side [the top side of the square]?</p> <p>44 F: You know what... how long is that side?</p> <p>45 Y: Five?</p> <p>46 F: Mm [validating].</p> <p>47 Y: So is it twenty?</p> <p>48 F: Mm [validating].</p> <p>49 Y: On all this? [points to the overall shape of the square]</p> <p>50 F: This is twenty, right?</p> <p>51 Y: So do I put it right here [space reserved for the answer] or what?</p> <p>52 F: If you wanna compare, do you know how long is that side [follows circle's perimeter]?</p> <p>53 Y: Like... So it is gonna be like... It's a five[?] right here or no?</p> <p>54 F: Do you know how to calculate the perimeter of a circle? [Pause] Remember the formula? [Pause] Okay. So... But... If you don't know the formula, you can't compare the numbers, right? But do you know another method? Or can you compare both sides, both outsides of both figures without calculating the perimeter? [Pause] Which one is bigger this [interviewer follows the perimeter of the square with the finger] or this [Interviewer follows the perimeter of the circle with the finger]?</p> <p>55 Y: This one [square].</p> <p>56 F: Why?</p> <p>57 Y: Because well... this [square] is a different shape and it's bigger and this one [circle] is like... smaller.</p> <p>58 F: So this is another way. We don't even need to calculate, we can just compare both. Right?</p> <p>59 Y: Yeah.</p> <p>60 F: And here you have used also only English? [As marked with a cross on the "English only" column]</p> <p>61 Y: Do you mean like... the way I think?</p> <p>62 F: Yeah.</p> <p>63 Y: On these two [A1 and A2] I only thought English. On this one [A4] I thought Spanish.</p> <p>64 F: You only thought English here?</p> <p>65 Y: Yeah.</p> <p>66 F: You only thought... all the activity, all the process of solving... only in English.</p> <p>67 Y: Yeah. In this one [A4] I thought Spanish and English.</p> <p>68 F: So can you change the cross right here?</p> <p>69 [In A4 Yolanda changes of column the cross] [Continues in A3,2]</p>		
MEMO	Perimeter-area confusion due to 'large'; "greater" in the wording.		

54-59. Visual comparison not detailed enough (no more detail demanded anyway).
 68-69. Cross on the language columns interpreted initially as language used on the final written answer. Marked below on A4's section.

Math	A3. Yolanda marked "English only".	Language	Tentative
1-3. 1st try . Right answer with arithmetical reasoning through the drawings of Figures 5, 6, 7.	 <p>1 How many tiles does figure 7 have? Why?</p> <p>it has 13 because it adds in each figure 2 tiles so on the 7th figure it goes to 13</p> <p>2 [Comes from A2,69] F: How have you solved that activity?</p> <p>3 Y: Well, first I saw the number of figures and how this counts to add to the new one. I started adding two to each one and it came to thirteen on the last one.</p> <p>4 F: Aha. And... you said that you have used only English here?</p> <p>5 Y: Yeah.</p> <p>6 F: Only English. You have never thought any number, any thing in Spanish?</p> <p>7 Y: That's why I did it fast because... well, I already know this.</p> <p>8 F: What do you mean you already know?</p> <p>9 Y: We are doing it in class, so I already know how to do this, I just put the numbers and ...</p> <p>10 F: The kind of exercise, you mean...</p> <p>11 Y: Yeah.</p> <p>12 F: ... it was easy. Okay. Good. [Continues in A4, 2]</p>	4-6. English used for everything	6-12. Similar exercises done in class
MEMO	7. I did it fast		

Math	A4. Yolanda marked "English and Spanish".	Language	Tentative
5. 0 try. [Ten floors] Ten as the 10 floors Jamie goes down as first thought.	<p>Jamie is shopping in a large department store with many floors. She enters the store at the middle floor, and immediately goes to the credit department. After making sure her credit is good, she goes up one floor to the jewelry department. Then she goes down one floor to the children's department. Then she goes up three floors to the toy department. Finally Jamie goes down ten floors to the main entrance of the store, which is on the first floor, and leaves to go to another store down the street.</p> <p>How many floors does the department store have? Why?</p>	A2, 62-69. Recognition of English and Spanish language use in A4	Bilingual background
1-5. 1 st try. Building with 10 floors as initial answer changed to 11 when writing the answer and Jamie had to go down 10 floors (still wrong answer).	<p>1 </p>	8-9. Translation of some words to Spanish	15. Difficulty in the wording understanding (not the words)
3-5. 1 st try. Dubious recall of middle floor position (during interviewer interaction).	<p>2 [Comes from A3,12] F: How have you solved this activity? 3 Y: This one is difficult because I thought like found/four[?] right here [middle floor]. I thought it was five then she went one up that was six. No this [middle floor] was six. Then she went one up and I think it was seven there. Then she went down it was six again.</p>	8-17. Spanish as counting language	Home language
1-5. 1st try. Wrong solution due to no mathematization of middle floor.	<p>4 F: But what have you done to solve it? Sorry. 5 Y: To solve it, well, I started to read and then like I started counting like the numbers of how many she got, because it was ten floors [pointing to the 'ten' in the wording]. And the first one she went to the middle [pointing to "middle" on the wording] so it was the fifth one. And then she went like down again and it was the forth one. 6 F: And you have just keep adding the numbers? 7 Y: Yeah. 8 F: And why have you used Spanish in this activity [She said it on A2,62-69]? 9 Y: Because I started counting and like [pause] putting this like some words in Spanish so I can like [pause] know what they mean. 10 F: And you counted in Spanish? 11 Y: Yeah. 12 F: Have you counted in Spanish here [A4]? But not in the previous activity, for example. 13 Y: No, this one I did it... 14 F: You counted here [A3] in English but here [A4] you have counted...</p>	17. Spanish and English as thinking languages	17-21. Good knowledge of both languages prompts unconscious switches

	<p>15 Y: Because like... I was confused because they went up and down, up and down...</p> <p>16 F: Aha. And using Spanish was easier for you in that activity?</p> <p>17 Y: Mm [agreeing]. Because I just like don't think, and I just mix them, so...</p> <p>18 F: What do you mean?</p> <p>19 Y: In this one [A4] I just like started counting and then I said some in Spanish and then I... in English again...</p> <p>20 F: And do you know why, when you have switched, when have you decided to switch to Spanish? Do you remember that?</p> <p>21 Y: No, I just keep counting. [Continues in GLQ]</p>		
MEMO	<p>A4, 7. Even if Yolanda agrees with the previous interviewer's utterance, she did NOT <u>ONLY</u> add the number to get the solution. Her reasoning is more elaborated. Probably the interviewer thought Yolanda added the floors and rephrased the 'counting' of Yolanda (A4, 5) into 'adding' (A4, 6). Tendency to agree with interviewer statements.</p> <p>Maybe she is not sure about the middle floor position when she explains the answer to the interviewer because the answer is not correct and there is something going on in her mind, but she does not say anything showing hesitation.</p>		

General Language Questions		Language	Tentative
1	[Comes from A4,21] F: And... So, this [A4] is the only exercise you have been eh... you have used Spanish, right? In general, when solving math problems, when do you think that you use Spanish?	1-6. Spanish with systems of equations	6. Difficulty with numerical mathematics
2	Y: On the work we do right now, to solve systems of equations.		
3	F: To solve systems of equations, that we are doing during class, you use Spanish?		
4	Y: Yeah. I use Spanish in these ones.		
5	F: Why?		
6	Y: Because there are like so much numbers and also because when I talk to Diandra... Well she doesn't know a lot English so I talk to her in Spanish.	3-6. Spanish in peer instruction	6. Collaboration with Spanish dominant students, 6: numbers' presence
7	F: Okay. And in these four activities, in general, when have you used Spanish?		
8	Y: In this one [A4].		
9	F: But why?		
10	Y: And I remember in this one too [A2].		
11	F: You have used also English in this one [A2] you said? I mean Spanish.	10-14, 26, 30-36. Use of Spanish in A2 related with the concept of perimeter (When talking about A2 Spanish is not mentioned!!)	New thoughts come to mind, mathematical difficulties, 16: concepts learned in Spanish
12	Y: I cant' remember.		
13	F: You don't remember. So this [A4] is the only one, but maybe a little bit here [A2].		
14	Y: Yeah.		
15	F: But why do you think you have used Spanish?		
16	Y: Maybe because I am used to, like... on what I know, I think, I use English ... on what I know in Spanish or when it makes it difficult.		
17	F: When it becomes more difficult you use Spanish?		
18	Y: Yeah.		
19	F: You feel more comfortable with Spanish. And when have you used English, here, in general?	15-19. Spanish	16:

<p>20 Y: Right here...</p> <p>21 F: When it was easier...?</p> <p>22 Y: Yeah, like this one it was easier [A3], this one is fast .This one too [A1].</p> <p>23 F: When, when more?</p> <p>24 Y: When I used English in these ones?</p> <p>25 F: Aha.</p> <p>26 Y: Just in this two [A1 and A3]. And I think more in this one [A2]. I used less Spanish in this one [A2].</p> <p>27 F: And why do you think you did it?</p> <p>28 Y: Maybe because it makes more difficult in things that I understand in English.</p> <p>29 F: Is there any word, are there any words or phrases that you found difficult in English?</p> <p>30 Y: This one [A4] got me confused. That's why I start using Spanish. And this one [A2] because I forgot what perimeter was.</p> <p>31 F: But, if it had been <i>perimetro</i> in Spanish, had you understood it better?</p> <p>32 Y: Yeah, because like perimeter, that makes me think of <i>perimetro</i>, so that's what I used to know what it was.</p> <p>33 F: And what do you mean?</p> <p>34 Y: Like it is under[?] in Spanish so that's how I know what it was.</p> <p>35 F: This is why you knew that it was <i>perimetro</i>?</p> <p>36 Y: Yeah.</p> <p>37 F: But you got confused with area at the beginning.</p> <p>38 Y: Yeah, because I confused area with perimeter, <i>pero</i>, I know what perimeter is.</p> <p>39 F: But it was not because of the English?</p> <p>40 Y: No.</p> <p>41 F: And here... Is there any word in English that you haven't understood? Or any phrase, any sentence?</p> <p>42 Y: This one [A3] no and this one [A4] ... Well, this one [A4] I didn't get confused it's just like I got confused in all the things, that was up and down... so I started using Spanish too.</p> <p>43 F: But not because of the words...</p> <p>44 Y: No.</p> <p>45 F: ...itself or the way the sentences were written?</p> <p>46 Y: Yeah. The middle floor, then on the... goes down, then three floors...</p> <p>47 F: But you have understood, you understood the sentences, right?</p> <p>48 Y: Yeah.</p> <p>49 F: And here, in this [A1] you said no... And is... Well, there's not a lot of words here, but you have understood all the words, right?</p> <p>50 Y: Yes.</p>	for particular tasks	Mathematical difficulty, 16: content learned in Spanish
	28. English as initial thinking language	Sufficient English proficiency and no need for additional language complexities
	29-30, 41-48. [A4] Spanish as thinking language	15. Difficulty in the wording understanding (not the words)
	41-48. [A4] Right wording understanding from the language point of view (but mathematical understanding difficulties)	Good English management
MEMO	<p>1-6. Numbers in Spanish due to difficulties.</p> <p>10-14, 26, 30-36. Use of Spanish in A2 (When talking about A2 Spanish is not mentioned!!)</p> <p>15-19. Spanish for particular tasks: 16: Mathematical difficulty, 16: content learned in Spanish</p> <p>Yolanda has a good management of both languages. She has English as initial option, but when she has either mathematical difficulties or understanding difficulties (not just because of the words or meaning) she uses Spanish to try to overcome these issues.</p>	

Yolanda has an excellent English BICS as she chooses to speak in English, thinks mainly in English throughout all four activities and has no problems understanding all words and sentences presented in the statements. Her English CALP is good as she writes fluently in English (despite some grammar errors). Yolanda never speaks Spanish during the interview, but as she spent 4 years in Mexico and speaks Spanish at home with her parents her Spanish BICS must be excellent. There is not much information about her Spanish CALP, which might be at least fair, given her Mexican stage.

A1 is solved exclusively through English. Yolanda uses percentages as having a relative value but assumes that initial prices should be different on each store (she attributes this fact to the different amount of discount offered in each store, according to her personal experience).

On A2 Yolanda says she does not use Spanish, but later on the GLQ he says that Spanish helps her with the concept of perimeter. On the 1st try she mismatches the notion of area with perimeter (making a right visual comparison of the number of square units in each figure). On the 2nd try, following a demand from the interviewer, she defines perimeter as 'The sides?' but has not understood its meaning yet, as she continues to make divisions in the square as if she was looking for its area. Interviewer refers to the perimeter of the actual sheet of paper where she is solving the activities and Yolanda defines perimeter as 'All the sides?'. But it is not until the interviewer says 'But how long is gonna be the perimeter?' that Yolanda assimilates the meaning of perimeter and calculates the perimeter of the square. She does not compare it with the perimeter of the circle (she does not know the formula). On a 3rd try, following a demand for comparing both perimeters she does it with vague reasons (but no more detailed argumentation is demanded anyway).

A3 is solved using English as unique language. Yolanda correctly completes the figure sequence up to Figure 7 and gets the demanded number of tiles, knowing that the number of tiles is growing on two tiles per figure.

Answer on A4 is written in English, but Yolanda uses Spanish to translate some words because the situation presented in the wording is too confusing (to better organize the horizontal mathematization). Spanish is used also for counting. Both languages are used when thinking. The answer is not correct because there is not a mathematization of the middle floor. Furthermore, there are some illogical steps on her explanation (when she describes how Jamie's movements fit to her answer –11 floors–).

In the GLQ section Yolanda says that she uses Spanish with numbers and when mathematics become difficult. Spanish is also used in peer

instruction with Spanish dominant students and when working about a topic well known in Spanish. Even though, English is the initial option to solve math problems. When there are some mathematical difficulties (as in A4) Spanish is used to overcome them.

Yolanda only solves A3 completely correctly but she also shows some good mathematical skills when solving all other activities. The interviewer only offers a little help in A2. Maybe Yolanda would have solved all problems had she been given some more clues. For example, just giving her the formula in A2 it would have probably resulted in Yolanda's solving the problem in a right way.

Activities' (Key ideas) summary

Object 14: Yolanda-First reduction (End)

- Assumption of different initial prices in each of the stores (based on personal experience) with no declared use of Spanish.
- Mainly use of English as a thinking language for the perimeter of the geometrical shapes, but introduction of Spanish to overcome the problem with the perimeter concept definition along with the interviewer's interaction (emerged on GLQ not in A2).
- The figure pattern activity is solved quickly and correctly using exclusively English, as similar problems have been done in class.
- Introduction of Spanish as a thinking language in the case of the activity with a dense wording for better understanding of it due to some mathematization difficulties.
- Use of English in peer instruction during class.
- English used as an initial option to solve the problems, but Spanish used to overcome mathematical difficulties, recall concepts or work about topics that are well-known in Spanish.
- Spanish used in relation with numbers.

Historical profile	Bilingual profile (English dominant)	Activity
<ul style="list-style-type: none"> • 16 years old • Transitional class • Born in USA. 9-13 lived in Mexico • Likes California • No Spanish readings • English readings • Spanish (parents, brother) at home • English and Spanish (sister) at home • English and Spanish (Math, Spanish, Biology) at school • Homework help: nobody 	English for everything	x x x
	Deviated grammar	x x
	English as writing language	x x x x
	No right use of geometry vocabulary (diameter, length,...)	
	Spanish as support to interpret wording	
	Spanish as counting language	
	Spanish and English as thinking languages	
	Spanish with systems of equations	GLQ
	Spanish in peer instruction	GLQ
	Spanish for math difficult tasks	GLQ
	English as initial thinking language	GLQ
	Spanish and English as thinking languages	GLQ
	Spanish to recall the concept of perimeter	GLQ
Procedural profile	Conceptual profile	
<p>1. ✓/X Assumption of different initial prices: 40% store should have a lower initial price, as the percentage of discount is greater</p> <p>2.1 (X) Figures divided into little squares (X) Visual comparison on the number of area units in each figure</p> <p>2.2 ✓ Perimeter definition X Division of the square in little squares as when finding the area x Perimeter as a number of sides on the actual paper instead of its length X No identification of circle's perimeter: perimeter exists just with straight lines ✓ Calculation of the square's perimeter</p> <p>2.3 X No calculation of the circle's perimeter (unknown circle's perimeter formula) X No use of alternative approaches X/✓ Argumentation: '[square] is a different shape and it's bigger'</p> <p>3 ✓ Application of a pattern given by adding 2 from one figure to the next in order to draw the sequence of figures</p> <p>4 X Reasoning without distinguishing the singularity of the middle floor in terms of its symmetry function</p>	<p>1. ✓ Notion of percentages as relative value</p> <p>2.1 X Notions of perimeter and area confused ✓ Notion of area</p> <p>2.2 X Perimeter concept</p> <p>2.3 ✓ Perimeter concept</p> <p>3 ✓ Notion of arithmetical sequence associated to a sequence of figures</p> <p>4 X Notion of number line with confused order positions</p>	

Object 15: Yolanda-Second reduction

Object 16: Carlos-First reduction
(Beginning)

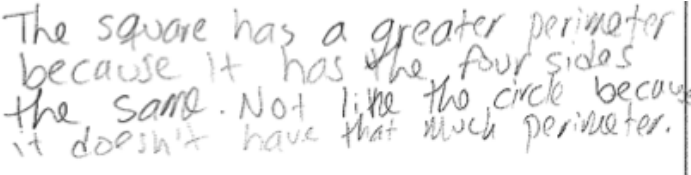
Carlos arrived to the class once the course had started. He is doing the assignments with enthusiasm. As the days go on, he starts to become more and more lazy. He is smart and understands the explanations very well. This allows him to do the tasks quickly. He gets them right most of the times.

Math teacher description

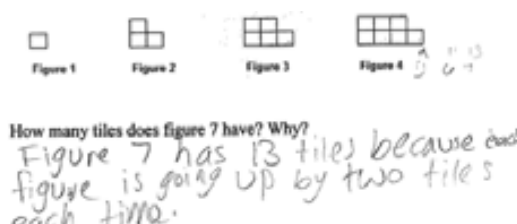
Class	Interview language	Interview date	Origins	Age	USA arrival	Living in California	Spanish readings	English readings	Home language	Friends language	Main language at school	Homework help
HS Transitional	English	February 2010	Mexico	14	Born in USA. In March 1995 his parents came back.	Likes it: stores, sports, "maybe school".	Books: sports books. Does not read a lot in Spanish.	Yes (does not remember any title).	Spanish (parents), English (brother), both languages (cousins)	English	English	Nobody

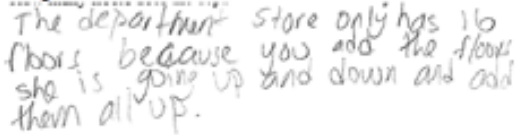
Historical bilingual profile

Math	A1. Carlos marked "English only".		Language	Tentative
2-6. 1st try. Wrong answer due to notion of percentages as absolute value instead of relative (direct comparison of percentages with no initial prices).	1	[Conversation about activities starts at A3,3] [Carlos marked "English only" after reading the questionnaire's instructions]	1. Cross in the "English only" column before solving the exercise	GLQ (5-13, 25-26). Use of Spanish in class not related with mathematics
	2	<i>In John Sport because it has a 40% discount on the shoes so you have to pay less money.</i>	7. English for everything	English dominant
	3	[Comes from A4,9] F: How have you solved this activity?		
	4	C: By looking at the percent off of the shoes. So, I guess John Sports is more... I mean Mike Sports is more expensive than John Sports.		
	5	F: Just looking at the percentage, right?		
	6	C: Yeah.		
	7	F: And you have used here only English too? You never switched to Spanish. [Pause] Ah... Okay, good. [Continues in A2,3]		
MEMO	1. Carlos marked "English only" after reading questionnaire instructions			

Math	A2. Carlos marked "English only".	Language	Tentative
4-10. Unsuccessful naming of a mathematical concept in either English or Spanish to differentiate between the circle and the square.	1 [Carlos marked "English only" once he started writing the answer.] 2 	1. Cross in the "English only" column when writing answer	GLQ (5-13, 25-26). Use of Spanish in class not related with mathematics
13-15. Unknown area concept.	3 [Comes from A1,7] F: Let's go to activity two. 4 C: In this one the square has a greater perimeter because [pause] it has the four sides the same, and the circle has no... something. I forgot how it's called. 5 F: Has no? 6 C: I forgot how it's called.	2. Deviated syntax, but understandable (the four sides the same)	Informal use of English
16. Greater square's perimeter because square has (right) sides and circle has not.	7 F: Do you know the name in Spanish? [Pause] Neither? Can you explain what do you mean? 8 C: Like... it hasn't the like... it has... it's just the figure circle... doesn't have a... Isn't it area or something? 9 F: Area? 10 C: Yeah. Or something. 11 F: Yeah. Has it area or not?	6-9. Unsuccessful naming of a mathematical concept in either English or Spanish	Mathematical vocabulary; mathematical issue rather than linguistic
19-20. Good visual comparison of perimeters, with neither written (2) nor oral (21) explanation.	12 C: I don't know. 13 F: What's the area? 14 C: [Unintelligible] 15 F: The area is the surface, the amount of paper, we can say, we use to... if we would like to make the circle, how many paper we will use. That's the surface. Right? [Pause] So how have you said? Try to explain in other words, maybe.	21-23. English for everything	English dominant
2-21. 1st try . Wrong answer with wrong reasoning (antecedent not conducting to conclusion. On a second sentence, conclusion used also as antecedent) and incorrect perimeter properties application.	16 C: I guess it's like it [pointing to the circle] doesn't have sides, like... maybe in this one [pointing to the square], it has maybe ninety degrees and this one [pointing the circle] doesn't. 17 F: So, just because it [circle] doesn't have sides, is this... ah... the square in fact is greater than the circle? 18 C: Yeah. 19 F: Even if you had a circle like this [Francesc draws a big imaginary circle with the hand], will the square, the perimeter of the square will still be greater than that [the circle]? 20 C: No. 21 F: So, can you try to explain a little bit more? [Pause] Okay. Well. So here again you have only used English, again. 22 C: Yeah. 23 F: Only English. [Continues in GLQ,1]		
MEMO	4-10. Unsuccessful naming of a mathematical concept in either English or Spanish to differentiate between the circle and the square.		

Carlos does not seem much cooperative to describe his mathematical reasoning.

Math	A3. Carlos marked "English only".	Language	Tentative
<p>2-4. 1st try. Right answer with right arithmetical sequence associated to figure pattern.</p>	<p>1 [Carlos marks "English only" after reading wording]</p>  <p>2 [Carlos counts the tiles in each figure by pointing to them. He erases the last word of the 1st line and rewrites it, probably because his handwriting is not clearly intelligible.]</p>	<p>1, 15-17. Cross in the "English only" column before solving the exercise</p>	<p>GLQ (5-13, 25-26). Use of Spanish in class not related with mathematics</p>
	<p>3 F: How did you solve this activity? 4 C: Oh... I was looking at every figure. And each figure is going up by two. So then I keep adding two to each figure so... Then it says [reading] how many tiles does figure seven have. So I keep adding two so it came up to be thirteen figures. Tiles, I mean.</p>	<p>5-14. English for everything</p>	<p>English dominant</p>
	<p>5 F: Okay. What language did you use to start solving the problem? 6 C: English. 7 F: English? 8 C: Yeah. 9 F: And have you switched languages? 10 C: No. 11 F: You have used only English? 12 C: Yeah. 13 F: Eh... So English only, not Spanish for any reason. 14 C: No. 15 F: I have seen, though, that here, for example in activity number four, you have put the cross even before you have finished solving the activity. You know what I mean? Why that? 16 C: I don't know. 17 F: You have put the cross and you were still writing down and then... You were sure you were gonna use only English to solve the activity or...? How have you solved that activity in fact? [Continues in A4,3]</p>		
<p>MEMO</p>	<p>1, 15-17. Cross in the "English only" column before solving the exercise. 17. Carlos does not seem much cooperative to describe his mathematical reasoning.</p>		

Math	A4. Carlos marked "English only".		Language	Tentative
2-6. 1st try . Wrong answer with no relative position of floors (addition of all floors) and no adequate mathematization of middle floor (top of the building considered as highest floor reached).	1	[Carlos marks "English only" before reading the wording. He whispers while he reads the problem in low voice.]	1. Cross in the "English only" column before reading the wording	GLQ (5-13, 25-26). Use of Spanish in class not related with mathematics
	2	 <p>[Carlos erases a couple of words and rewrites them, probably because of his handwriting is not clearly intelligible.]</p>		
	3	[Comes from A3, 17] F: How have you solved that activity in fact? [A3, 17, reproduced also here]	7-9. English for everything	English dominant
	4	C: Jamie started at the middle floor and then she went up one floor, and then she went down one floor. That makes it two floors. And then she goes up three floors to the toy department, so that's five. And then she goes down ten floors to the main entrance. That's fifteen. Plus the one in the middle where she started at, so that was sixteen floors.		
	5	F: So you have add all the numbers?		
	6	C: Yeah.		
	7	F: Mm [continuing conversation]. And here you have used only English?		
	8	C: Yeah.		
	9	F: Only English? You never switched to another language? [Pause] Okay. [Continues in A1,3]		
MEMO	1. Cross in the "English only" column before reading the wording. 2-4. Oral explanation more detailed than written.			

General Language Questions			Language	Tentative
1	F: So while solving these activities, the four of them, you have used only English, in the four of them.		1-4. English for everything	13-14, 27-29. Difficulty of Spanish explanations
2	C: Yeah.			
3	F: You've never used Spanish. Even to think for example [in A3] about this nine, <i>nueve más dos o once más ... trece ...</i> You haven't done that either.		5-13, 25-26. Use of Spanish in class not related with mathematics	27-30, 37-40, 45-49. No academic Spanish, schooling in English
4	C: No.			
5	F: You never use Spanish when you are in class?			
6	C: No.			
7	F: Never. Even to talk with your classmates?			
8	C: Oh, yeah!			
9	F: You use Spanish when you talk with your classmates in... in the class.			
10	C: Yeah, but not about math.			
11	F: You don't talk about math?			
12	C: In Spanish, no.			
13	F: Never? [Pause] Okay. Why do you use only English?			
14	C: I think Spanish is harder to explain stuff.			
15	F: But can you speak Spanish perfectly well?			

16	[Unintelligible sequence: Carlos mumbles something]		
17	F: ¿Podemos hablar un rato en español? ¿Sí? ¿Sí habla usted español bien?		
18	C: Sí.		
19	F: Pero no lo utiliza nunca.		
20	C: No.		
21	F: ¿Toma clases de español, por ejemplo?		
22	C: No.		
23	F: ¿No?		
24	C: No.		
25	F: Y lo utiliza sólo con los amigos o cosas así, pero en clase, para hablar de las materias, nunca lo utiliza.		
26	C: No.		
27	F: ¿Sabe usted por qué? ¿Alguna otra razón me puede dar? ¿Por qué no utiliza el español?		
28	C: Se me hace más difícil.		
29	F: Se le hace más difícil. Porque usted me dijo que no fue a la escuela en Mexico, ¿no?		
30	C: No.		
31	F: Es de una zona... ¿De qué zona es, por cierto? No conozco muy bien Mexico pero tengo algunos amigos por ahí.		
32	C: De Michoacán.		
33	F: ¿Michoacán! Oh, yo tengo un profesor, conozco un profesor que estaba aquí en Salinas que es... era de allí. Hay mucha gente por aquí de Michoacán, ¿no?		
34	C: Sí.		
35	F: ¿Es de un pueblo o de ciudad usted? ¿Dónde vivía?		
36	C: No sé.		
37	F: ¿No recuerda, cuando vivía en Mexico, en dónde estaba?		
38	C: Yo soy nacido aquí.		
39	F: ¡Oh! ¿Nació usted aquí ya?		
40	C: Ajá.		
41	F: Oh, I thought you said that you came here in ... <i>en... como en el noventa y nueve, me dijo, ¿no?</i>		
42	C: I thought you were saying about my parents.		
43	F: ¿Sus padres vinieron aquí?		
44	C: Sí.		
45	[The conversation continues in Spanish, talking about Carlos origins: not transcribed here]		
46	F: ¡Oh! Entonces usted ha vivido toda su vida aquí.		
47	C: Sí.		
48	F: ¡Oh! ¿Y no toma clases de español entonces?		
49	C: No.		
50	F: <i>Nunca ha aprendido el español en la escuela.</i> [Pause] Okay. And why are you in a transitional class?		
51	What's that?		
52	F: ¿ <i>Por qué está en una clase de transicional?</i> T R [/ti/ /ar/]?]		
53	C: Oh!		
54	F: ¿No habla bien el inglés?		

55	C: No.		
56	F: ¿No? Oh, pero se le hace más fácil...		
57	C: [Interrupting] Sí.		
58	F: ...en inglés que en español, por eso.		
59	C: Sí.		
MEMO	35-45. Misunderstanding at the beginning of the interview (Historical bilingual profile) when talking about Carlos' origins.		

It is normal that Carlos has no Spanish CALP because he probably never studied math in Spanish. His parents are Mexican. He says he uses Spanish in class (but not to talk about math) and he talks correctly in Spanish (see GLQ). So he has an excellent Spanish BICS. Carlos thinks of all the problems in English, chooses to use English in the interview too and is using English in class most of the time, so his English BICS is excellent. On the other hand, mainly due to his answer in A2, his English CALP may be categorized as basic. Also his answer in A4 should be more reasoned, but this has to do also with reasoning structure and is not only related to language.

Answer to A1 is not correct due to a direct comparison of percentages.

A2's answer is not logical from a mathematical point of view. While explaining it, Carlos does not name the appropriate mathematical property (not even in Spanish) and does not describe it either. On demand, he does not explain what area is. Finally the reason is attributed to the fact that square's sides have ninety degrees and circle not.

Answer to A3 is correct. Carlos finds that each figure is growing by 2 tiles.

Answer to A4 is not correct because Carlos adds all floors (no relative position, not an adequate mathematization of middle floor).

Carlos does not give much detailed explanations and sometimes he does not even answer some questions (A2,7; A2,21; A3,17).

Activities' (Key ideas) summary

Object 16: Carlos-First reduction (End)

<ul style="list-style-type: none"> - All problems solved entirely in English and with many mathematical errors. - General predisposition to use English language (cross marked before solving each activity) frames opportunities to develop mathematical ideas using Spanish. - Occasional lack of naming in both English and Spanish when searching for a distinctive geometrical property to justify the longest perimeter. 							
Historical profile		Bilingual profile (English dominant)		Activity			
<ul style="list-style-type: none"> • 14 years old • Transitional class • Born in USA • Likes California • Some Spanish readings • English readings • Spanish (parents), English (brother), both (cousins) at home • Mainly English at school • Spanish just when his friends talk to him in Spanish • Homework help: nobody 		"English only" cross marked before finishing problem		x	x	x	x
		English used for everything		x	x	x	x
		Wrong writing language syntax			x		
		Unsuccessful naming of a mathematical concept in either English or Spanish			x		
		English used for everything		GLQ			
		Spanish used in class only around non mathematical contents		GLQ			
Procedural profile		Conceptual profile					
<p>1. X Direct comparison of percentages with no mention of initial prices</p> <p>2 X Counting of the sides of the square to decide the longest perimeter</p> <p>X Incomplete differentiation between the circle and the square</p> <p>X Attempt to explain the notion of area</p> <p>√ Perimeters comparison (between square and bigger imaginary circle drawn by the interviewer), with no justification</p> <p>3 √ Application of a pattern given by adding 2 from one figure to the next</p> <p>4 X No relative ordering of floors</p> <p>X Reasoning without distinguishing the singularity of the middle floor in terms of its symmetry function</p>		<p>1. X Notion of percentage as an absolute value instead of relative</p> <p>2 X Unclear notion of perimeter in relation with areas and polygonal perimeters</p> <p>X Imprecise mathematical vocabulary</p> <p>X Unclear notion of area with references to circles, perimeters, and sides</p> <p>√ Notion of perimeter as a measure</p> <p>X Notion of mathematical validity</p> <p>3 √ Notion of arithmetical sequence associated to a sequence of figures</p> <p>4 X Notion of number line with confused order positions</p>					

Object 17: Carlos-Second reduction

Object 18: Coral-First reduction
(Beginning)

Coral is a not a very smart mathematics student. Her skills are very basic and she does not remember much of the previous days lessons. Her retention skills are poor. Her teacher believes that most of her issues are language based. She speaks A LOT in Spanish during class time and she even asks a couple of questions in Spanish. She does not feel comfortable with English language. She wants to do well but gets frustrated a lot and simply gives up. She is very influenced by her friends and let them affect what she does in class (i.e. they joke around, she jokes around; they work, she works). They also have language issues.

Math teacher description

Class	Interview language	Interview date	Origins	Age	USA arrival	Living in California	Spanish readings	English readings	Home language	Friends language	Main language at school	Homework help
HS Mainstream	English	April 2010	Mexico	14	Born in USA. Never lived in Mexico.	Likes it.	No	Books from the library	Spanish (mother – who barely knows English), English (father, brothers)	[not asked]	English. Spanish if her friends speak to her in Spanish.	Sometimes her dad

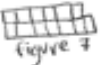
Historical bilingual profile

Math	A1. Coral marked “English only”.	Language	Tentative
1-2. Awareness of the fact that a justification of the answer is demanded.	[Dialogue about activities starts at A3,1] 1 Do I say why right here [empty space for the answer]? 2 F: Yes, please.	3. Deviated grammar (more cheaper)	Written English management, predominant use of in-

<p>7. Influence of visual figurative statement: same brand meaning same price.</p> <p>3,6-9. 1st try. Wrong answer due to assumption of equal initial prices on both stores. Right notion of percentages.</p>	<p>3 <i>The Show that is more cheaper is John Sports because Mike Sports shoes are 25% off and John Sports shoes are 40% off.</i> [She quickly writes down the answer.]</p> <p>4 [Comes from A4,14] F: How did you solve activity one?</p> <p>5 C: Which one do I pick?</p> <p>6 F: Excuse me? Yeah! How have you solved it. What did you think when solving it?</p> <p>7 C: Because like... the more... It's still the same brand, but just a little cheaper in different stores and there is no difference on which store it is, because you are still getting the same thing, but just a little cheaper. And then you just...</p> <p>8 F: So which one is cheaper?</p> <p>9 C: The forty percent.</p> <p>10 F: You write "English only" so you have only used English here?</p> <p>11 C: Yes.</p> <p>12 F: You haven't use Spanish at all. But anyway you have marked "English only" when you were still solving the problem. You haven't even finish. You know what I mean?</p> <p>13 C: Yeah.</p> <p>14 F: You haven't wait till the end to know if you... Maybe you could use Spanish in any part, but you already marked the cross in "English only". Why that?</p> <p>15 C: Because I usually solve math problems or anything in English instead of Spanish.</p> <p>16 F: Only English?</p> <p>17 C: Yeah.</p> <p>18 F: You don't use Spanish for anything.</p> <p>19 C: No.</p> <p>20 F: So, you have done all the problems with... in English. I mean, all the problems you do always, in math, you always use English.</p> <p>21 C: Yes.</p> <p>22 F: Not Spanish at all.</p> <p>23 C: Sometimes a little Spanish.</p> <p>24 F: Sometimes. But... so... if this [happens] sometimes why you where so sure that you were not gonna use Spanish here? And, in fact, haven't you use Spanish at all here in the problem?</p> <p>25 C: No. [Continues in A2,2]</p>	<p>10-25. Use of English for everything</p>	<p>formal/oral English</p> <p>Born in US: non-academic Spanish, mainly use of English in Math problems (A1,13)</p>
	<p>MEMO</p> <p>2. Cross on the "English only" column before writing the answer.</p> <p>7. Oral explanation helps to determine that percentages are treated adequately (as a relative value).</p> <p>23. Sometimes Coral uses Spanish to solve Math problems.</p> <p>Her teacher says she always uses Spanish in class: influence of peer instruction.</p>		

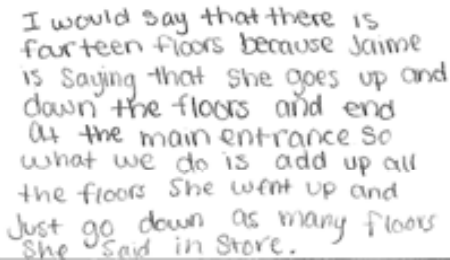
Math	A2. Coral marked "English only".	Language	Tentative
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<p>1. Wrong sentence structure from a logical point of view.</p> <p>1. Deviated use of mathematical vocabulary (length, width, side).</p> <p>3. Deviated visualization of the square's dimensions: sides have different length.</p> <p>1-7. 1st try. Wrong answer with wrong argumentation due to deviated visualization.</p>	<p>1 <i>I would say the circle because each length and width is the same side and its no different than the square.</i></p> <p>2 F: Can you explain me how have you solved it?</p> <p>3 C: Because in the circle the width and length are the same than... are the same size than... are the same size as each... around it. And in the square the width is longer than the length.</p> <p>4 F: In the square the width is longer than the length? You say that?</p> <p>5 C: Yes.</p> <p>6 F: And so the circle has a greater perimeter that the square.</p> <p>7 C: Yes.</p> <p>8 F: And here, again, you have used only English. [Looking at the cross in the "English only" column]</p> <p>9 C: Yes. [Continues in A3,10]</p>	<p>[Coral erases the end of 2nd and 3rd lines and rewrites them.]</p>	1. Deviated spelling, grammar	Written English management, predominant use of informal/oral English
			1. Deviated use of mathematical vocabulary (length, width, side)	Fair English CALP
			8-9. Use of English for everything	Born in US: non-academic Spanish, mainly use of English in Math problems (A1,13)
MEMO				

Math	A3. Coral marked "English only".	Language	Tentative
<p>9-11. 1st try. Right answer with graphical inference reasoning (direct drawing of Figure 7 with 6 tiles on top and 7 at the bottom; without drawing Figures 5 and 6).</p>	<p>1 [Dialogue about activities starts here] C: I don't understand this one. There's four figures and [the wording] says figure seven.</p> <p>2 F: Yes. So what's the problem?</p> <p>3 C: [Reading] How many tiles does figure seven has and why.</p> <p>4 F: So you have to figure it out. Here you have only figures one, two, three, four. And they are asking for figure seven.</p> <p>5 C: Oh!</p> <p>6 F: All right? How many tiles does figure seven have?</p> <p>7 C: Do I draw the figure?</p> <p>8 F: If that helps you, yes, you can do it.</p>	1-8. Wording meaning demand	No (mathematical) solution found
	<p>9  <i>figure 7</i> figure 7 has 13 squares because just by piling figure 6 on top of figure 7 and then you get your solution.</p> <p>[Coral draws directly figure 7 without drawing Figures 5 and 6. Dialogue continues in A4,1]</p>	9. Deviated grammar	Written English management, predominant use of informal/oral English
	<p>10 [Comes from A2,9] F: How have you solved this one?</p> <p>11 C: Because I can see there are just... just adding the two down there, and then they go down two and put three. And then they keep going: the three at the top and the four at the bottom. So I just put the six on top</p>	12-15. Use of English for everything	Born in US – nonacademic Spanish

	with the seven at the bottom. And then I just added all the squares up. 12 F: And again here you have used only English. 13 C: Yes 14 F: Right? Not Spanish at all? 15 C: No. [Continues in A4,15]		
MEMO	A problem with the horizontal mathematization produces the demand for a meaning of the wording.		

Math	A4. Coral marked "English only".	Language	Tentative
14. Wrong explanation from a logic point of view (it is not concisely explained how she finds the 14 floors). 14-16. 1st try. Wrong answer. Nor good written explanation neither oral. Wrong horizontal mathematization without considering the middle floor according to its particular symmetry position and no relative situation of floors.	1 [Comes from A3,9. Coral follows some words with her pencil when reading the wording. She underlines some words.]	1-13. Wording meaning demand	No (mathematical) solution found
	2 C: I kind of don't understand this one.	14. Deviated syntax	Written English management, predominant use of informal/oral English
	3 F: What you don't understand?		
	4 C: Cause it says "Jamie is shopping in a large department, It's in the middle floor", but then it says "down" here. You have to add them all?	14. Deviated grammar	Written English management, predominant use of informal/oral English
	5 F: You will have to figure it out. I cannot tell you how to do that, but if you want I can... You have to... The answer... The question is: you have to say how many floors does the department have, right?		
	6 C: Yes.	17-20. Use of English for everything	Born in US – nonacademic Spanish
	7 F: And here it's explaining what Jamie is doing when she enters the department, she does certain things: go up, go down, right?		
	8 C: Yes.		
	9 F: And at the end she leaves by the... eh... She is in the first floor and she leaves, right? So you will have to figure it out. According to all that information, how many floors does the department store have. Right?		
	10 C: Yes.		
	11 F: What's the total amount of floors. If you think you have to add it, add it. If you think you have to do something else... Just do whatever you think. But do you understand what's the question and what's happening here? What Jamie is doing?		
	12 D: Yes.		
	13 F: Okay. [Coral writes the answer almost immediately]		

	<p>Jamie is stopping in a large department store with many floors. She enters the store at the middle floor, and immediately goes to the credit department. After making sure her credit is good, she goes up one floor to the jewelry department. Then she goes down one floor to the children's department. Then she goes up three floors to the toy department. Finally Jamie goes down ten floors to the main entrance of the store, which is on the first floor, and leaves to go to another store down the street. How many floors does the department store have? Why?</p> <p>14  [Continues in A1,2]</p> <p>15 [Comes from A3,15] F: Can you explain me how have you solved it? 16 C: I added all the floors she said she went up. She went up three floors, plus went up one floor and then she said she first entered the middle floor so there must be a floor on the bottom. And then I just added those up. And then she says she went down one and down ten. So I just added them up. 17 F: Okay. And here you have used again only English? 18 C: Yes. 19 F: Not Spanish at all? 20 C: No. [Continues in GLQ,1]</p>		
MEMO	Coral does not explain the solution properly (A4, 14-16), nor on her written answer neither when explaining the solution to the interviewer. Even though, the oral explanation is more detailed. The interviewer does not ask for a more detailed explanation, given that she used just one language to solve all the problems. Interviewer has not focused on helping Coral with the mathematical issues because she never switched languages. At the beginnig of the data gathereing process these cases were not going to be studied in much detail.		

General Language Questions		Language	Tentative
1	F: But before you have said that you use Spanish sometimes.	3-24. Use of	4-12, 22.
2	C: Yes.	Spanish in	Friends do not
3	F: When do you use Spanish?	discourse	know enough
4	C: When... if the problems that I really do understand I use Spanish.	and thinking	English,
5	F: That you really do understand?	during class	similar
6	C: Yes.	to solve	problems
7	F: And you haven't really understood these ones?	math	done
8	C: Not that much.	problems	previously
9	F: Why not?	23-24. Good	Regular
10	C: They are usually problems that I normally don't do in math.	use of both	management

<p>11 F: You don't do that in a regular class, in the regular class you mean. 12 C: No. 13 F: But in a regular class, you solve the problems in Spanish if you know how to solve them? 14 C: Yes. 15 F: You solve them in Spanish. 16 C: Yes. 17 F: Always? 18 C: Not always, but I use Spanish for some of them. 19 F: To solve the entire problem or to solve part of it or it depends on the day or...? 20 C: Part of it. I solve a part of it. 21 F: So can you tell me, in general, when you use Spanish to solve problems? 22 C: I use them when I guess my friends [use Spanish] [be]cause they don't understand the English, so they say it to me sometimes in Spanish. And then from there I just keep going with the problem. But I understand it in Spanish then I do it in English. 23 F: But you think in Spanish or in English? 24 C: Yes, I think in Spanish, but when I say it, I just do it in English. 25 F: And here, when you were solving these problems, were you thinking in Spanish or in English? 26 C: In English. 27 F: In English in all the problems? 28 C: Yes. 29 F: You haven't thought in Spanish at any time, have you? 30 C: No. 31 F: And is there any word or sentence that you haven't understood here, that you found difficult? 32 C: No.</p>	languages	of English, Spanish friends
MEMO	31-32. No English language difficulties to understand the problems. Coral's teacher (see the beginning of this first reduction) says that Coral normally solves math problems in Spanish. Coral makes some language mistakes and her answers are also difficult to understand from a logic point of view.	

Coral needs to improve her English CALP (A2, 3-5). Even if sometimes it seems more a mathematical issue rather than a language issue, her written answers are not right from a language point of view in none of the four activities. She has an excellent English BICS, as she talks and thinks all the time in English. There is no information about Spanish use.

A1 is solved with no use of Spanish. Coral assumes that both stores have the same initial price because both have the picture of the same shoe and then compares the percentages of discount.

A2 is solved with no use of Spanish. Answer is not correct, with a deviated use of mathematical terms and illogical argumentation. Coral does not

visualizes the lengths correctly (on the square all sides have the same length).

On A3 Coral asks for the meaning of the wording. Her wording understanding does not produce a mathematical procedure to find an answer to the problem. Once the activity's aim is understood she finds the answer by drawing Figure 7 directly (6 tiles on top and 7 at the bottom).

A4 is solved exclusively through English. Coral states that there are fourteen floors, but she does not give a detailed explanation of how she finds this answer (she says she adds the floors Jamie goes up and down) through her written answer. The oral explanation, though, is more detailed, but still does not identify correctly the origin of all the floors added.

Coral uses exclusively English during the entire solving process and interview. Even though, she uses Spanish sometimes in class to talk about math, because her friends don't know enough English.

Activities' (Key ideas) summary

Object 18: Coral-First reduction (End)

<p>- A deviated use of mathematical vocabulary (length, width, side) makes the written answer hard to understand in relation with the perimeter of the geometrical figures. The oral register improves a little the mathematical argumentation.</p> <p>- As the solution is not found directly on the visual mode of the statement (Figure 7 is not present) there is a demand for a wording meaning to clarify the mathematical understanding.</p> <p>- No solution found (wrong mathematization process associated) on a dense wording prompted a demand for a wording meaning.</p> <p>- All exercises. Use of English for everything. Even if Coral mainly uses English at school and also uses English at home, when she did all the exercises in English she could have lost the opportunity to integrate part of the Spanish background, as she does actually do when she works in class with her classmates. In fact, she brought part of this Spanish background in the construction of some of English written answers. As the interviewer never asked anything in Spanish it is not known if that helped her in the resolution.</p>					
Historical profile		Bilingual profile (English dominant)		Activity	
<ul style="list-style-type: none"> • 14 years old • Mainstream class • Born in USA • Likes California • No Spanish readings • English readings • Spanish (mother), English (father) • Mainly English at school • Spanish just when her friends talk to her in Spanish • Homework help: father (sometimes) 		English for everything		x x x x	
		Deviated writing expression		x x x x	
		Deviated use of mathematical vocabulary (length, width, side)		x	
		Demand for a wording meaning			x x
		English for everything		GLQ	
		Spanish in thinking and discourse in group work during class		GLQ	
Procedural profile		Conceptual profile			
<p>1. X Assumption of equal initial prices on both stores: Equal pictures meaning that same thing is bought in both stores</p> <p>2. X Incomplete differentiation between the circle and the square</p> <p>X Visualization of the square's sides: they have different length</p> <p>3. √ Visual abstraction of the figure construction pattern: Direct drawing of Figure 7 (omitting Figures 5 and 6): 6 tiles on top and 7 on the bottom</p> <p>4. X Non relative ordering of floors</p> <p>X Reasoning without distinguishing the singularity of the middle floor in terms of its symmetry function</p>		<p>1. √ Notion of percentage: as a relative value</p> <p>2. X Notion of perimeter is not clearly reflected</p> <p>X Imprecise mathematical vocabulary</p> <p>3. √ Notion of figural sequence associated to a figure pattern</p> <p>4. X Notion of number line with confused order positions</p>			

Object 19: Coral-Second reduction

Object 20: Miriam-First reduction
(Beginning)

Miriam is interested in teacher explanations and tries her best during class, but she says she gets nervous in exams, where she doesn't perform very well.

Math teacher description

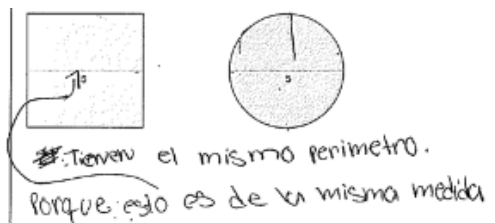
Class	Interview language	Interview date	Origins	Age	USA arrival	Living in California	Spanish readings	English readings	Home language	Friends language	Main language at school	Homework help
HS Mainstream	Spanish	April 2010	Guanajuato, Mexico	16	Born in USA. When she was a child used to go to Mexico every summer. Now she goes every 2 years.	Likes it	Science or History books instead of books about people. Reads mainly in Spanish to practice and improve reading skills.	Can read perfectly in English but barely does (to improve Spanish reading skills).	Spanish, both (sisters)	Spanish. English just with one.	English. Her Spanish teacher speaks English with the whole class so she (only student who masters Spanish) is not comfortable speaking Spanish.	Father –e.g. in Math. Little help because of his low English fluency. She helps her sisters.

Historical bilingual profile

Math	A1. Miriam marked “English and Spanish”.	Language	Tentative
i, 1. Right answer (embedded on a question). Right use of percentages as relative value. vii. 10, 14. Different initial prices (not concreted), to	[Even before the interviewer starts to ask Miriam questions about her use of languages (“Historical bilingual profile”), Miriam looks at A1 and says:] M: Wait! How am I supposed to know which ones are cheaper if they are not telling the price? F: Wait a minute. Before that, I will ask you some questions, right? M: Ah, okay. F: And then we are gonna start with the questionnaire. Which language do you prefer being asked questions? M: Which language? F: English or Spanish. M: Spanish if you want, doesn't matter. F: Whatever you want. M: Hum. F: You choose. M: Any thing. I understand both the same.	i-xii. English utterances	School context, good English management
		2-3. Written answer induced by interviewer (words, not sense)	i, 1. Solution to the problem found
		3, 19-20, 35. Spanish as	2. Previous English

<p>show that each store may be the cheapest.</p> <p>12. Awareness that the answer can be more detailed.</p> <p>33. Different initial prices in both stores (\$10 & \$10, \$20 & \$10, \$10 & \$20) before discount is applied to show that each store may be the cheapest.</p> <p>i, 1-3, 10, 14-16, 26-33. 1st try. Right answer with arithmetical reasoning (general and concrete examples before discount is applied to show that final price can be lower on both stores).</p>	<p>F: But...</p> <p>M: Okay, Spanish.</p> <p>[Once "Historical bilingual profile" questions are asked, Miriam continues:]</p>	unique writing language	interviewer utterance	
	<p>1 M: ¿Y cómo debo de saber cuáles están más baratos si no dice el precio de los zapatos?</p> <p>2 F: Lo puede poner... Si cree... Lo puede poner por escrito. Pues, depende del precio, o... lo que usted crea.</p>	3	3. Spelling variation (presio)	Mexican Spanish does not differentiate pronunciation
	<p>4 [Once all activities are answered, dialogue continues here. This intervention is translated on A4,2 too.] M: I'm done.</p> <p>5 F: We are gonna comment the, the activities now, right? Which one do you prefer?</p>	6 M: [Reading] In which...	4. English utterance	Solving and writing A4 in English
	<p>7 F: Bueno, ¿con cuál quieres empezar?, perdón. Hemos quedado que hablamos en español, ¿no?</p> <p>8 M: [Reading] In which of these two stores are the shoes cheaper? Why?</p>	9 F: ¿Qué has hecho aquí? ¿Cómo ha empezado?	8. Code switching	Reading English wording
	<p>10 M: Yo puse que depende del precio, porque si aquí [25%] están más caros que aquí [40%], entonces sería éste [40%] es más barato que éste [25%]. Y si aquí [40%] están mucho más caros que acá [25%], de todos modos sería mucho más barato éste [25%] que éste [40%].</p>	11 F: ¿Cómo has empezado a resolver el problema?	12, 33. Code switching in discourse (Non mathematical interaction with interviewer)	English dominant
	<p>12 M: Mm... Do you want me like to put an answer?</p> <p>13 F: No. Bueno, con qué te has fijado al principio.</p>	14 M: Pues me fijé en eso. De que si éste [40%] es más bajo el precio va a estar más bajo el zapato en el precio. Y si éste [25%] es más caro, aunque tenga veinticinco por ciento de descuento va a ser más caro. Y si éste [40%] es más caro que éste [25%] de todos modos éste [25%] va a seguir siendo más barato. Y éste [40%] aunque tenga el mayor por ciento de descuento va a seguir siendo más caro.	17-35. English and Spanish as thinking languages	Good management of both languages
	<p>15 F: Depende de los precios dice usted.</p> <p>16 M: Depende de los precios.</p>	17 F: Ajá. Okay. ¿En qué lengua has empezado a resolver el problema?	26-33. Spanish as thinking language	Process of particularization with numbers and money
	<p>18 M: En inglés.</p> <p>19 F: Has empezado en inglés. ¿Y cuándo cambiaste a español?</p>	20 M: Cuando puse depende del precio [answer].	27. Code mixing (yeah)	Hybrid language
	<p>21 F: ¿Sólo al escribir la respuesta?</p> <p>22 F: ¿Estabas pensando en inglés todo el rato?</p>	23 M: ¡¿Mm?! 24 F: All the time in English?	31. Code switching (Mathematical interaction)	English dominant
	<p>25 M: No. Como a medias, pensé en español y después lo pasé a inglés y volví a español.</p> <p>26 F: ¿Y cuándo cambiaste? ¿Te acuerdas de cuándo cambiaste de pensar en inglés a pensar en español?</p> <p>27 M: Yeah. Cuando puse menor precio aquí [40%] y mayor precio acá [25%].</p> <p>28 F: ¿Cuándo pensaste en un precio concreto?</p>			

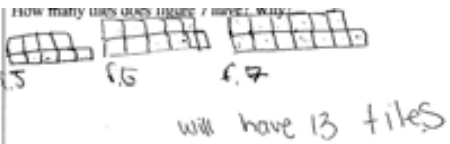
	<p>29 M: Mm [affirming].</p> <p>30 F: ¿En qué precio pensaste?</p> <p>31 M: Like ten dollars.</p> <p>32 F: Ajá. Entonces pensaste diez dólares, no ten dollars. Tu pensaste diez do...</p> <p>33 M: Diez dólares, ajá, aquí [40%] y ponga diez aquí [25%]. Entonces éste [40%] costaría menos que éste [25%]. Y si pusiera veinte acá [25%] y diez acá [40%], éste [40%] sería menos que éste [25%]. Y si los volteara entonces sería menos éste [25%] que éste [40%]... That's what I thought.</p> <p>34 F: ¿Y cuándo cambiaste a inglés luego otra vez?</p> <p>35 M: Eeh... Antes de poner la respuesta. Cuando estaba leyendo esto [percentages and discount in both figures] y luego volví a español cuando lo expliqué [pointing to the answer].</p> <p>36 F: ¿Algún cambio más que te acuerdes?</p> <p>37 M: No.</p> <p>38 F: Vale, otra actividad. [Continues in A2,2]</p>	with inter- viewer)	
		34-35. Eng- lish as think- ing language when reading (no transla- tion)	Good Eng- lish man- agement
MEMO	<p>4. English utterance related with A4.</p> <p>30-43. Good description of mathematical thinking process combining languages.</p> <p>New numerical concrete example when talking about numbers: oral register more detailed than written.</p> <p>Miriam does not use the name of the stores (in English) to refer to them. She always uses Spanish demonstratives (as she talks in Spanish).</p> <p>Use of both languages when thinking. A key point (numerical examples that show that both stores may have a cheaper price) is thought in Spanish.</p>		

Math	A2. Miriam marked "English and Spanish".		Language	Tentative
1, 6, 8, 16. Right dotted line interpretation.	1	 <p>[Circle's line added on A2,32]</p>	1, 2, 16, 22-24. Spanish as writing language (activity thought partially in English, partially in Spanish)	Answers written in English at school
4-5. Right perimeter concept.				
6-8. Symmetry-diameter confusion.	2	[Comes from A1,38] M: [reading] Which of these figures has a greater perimeter and why. Empecé con el inglés y luego en español cuando me fijé en esto [points to the dotted lines]. Y luego al poner la respuesta esta parte ['Tienen el mismo perimetro porque', A2,1] la puse, la quise poner en inglés y luego como que esto ['esto es de la misma medida', A2,1] lo pensé así en español.	2. Code switching	Reading English wording
6-10. symmetry-radius confusion.	3	F: A ver, ¿cómo fue? Perdona, ¿Me lo puedes repetir?	2, 20. Spanish as main thinking language	Interpretation of the dotted lines
	4	M: Yeah. So, dice que que el perimetro [pointing to 'perimetro', A2,1]... Éste es el perímetro, ¿verdad? [follows the perimeter of the circle with the pencil]		
1, 6, 8, 16, 20, 31-33. 1 st try. Wrong answer due to wrong conclusion	5	F: Sí.	4, 14, 34. Code mixing	Hybrid language
	6	M: So, como todo esto de aquí [makes 4 imaginary perpendicular radius in the circle, 2 of them following the dotted line] va a ser igual de longitud, entonces puse que los dos [points to both figures] van a ser igual porque los dos tienen el mismo... [points to the dotted lines] mm... symmetry. Like...		

(equal dotted lines meaning equal perimeter) with deviated visualization.	7	F: ¿Symmetry?	(yeah)	
	8	M: Ajá. Los dos tienen la misma medida de aquí a acá [points to the dotted lines]. So...	4, 6, 8, 16 (x2), 20. Code mixing (so)	Hybrid language
	9	F: Ajá. Esto se llama, por ejemplo, no es la symmetry, ¿no? Esto se llama...	6. Code mixing (symmetry)	Strong bilingual profile, born in USA
	10	M: Radi, radius	6. English utterance	6. Code mixing at the beginning of the utterance
	11	F: Esto es el diámetro y esto es el lado.	6. Code mixing (radius)	Strong bilingual profile, born in USA
	12	M: Okay.	6-8. symmetry-diameter confusion	Explanation not concise enough
	13	F: ¿Sí?	6-10. symmetry-radius confusion	Explanation not concise enough
	14	M: Yeah.	12. Code mixing (okay)	Hybrid language
	15	F: Pero...	25-28. English to interpret 5 (dotted line)	English as thinking language
	16	M: [Interrupting] So pensé que todo esto era igual así [makes 2 segments in the square perpendicular to the dotted line; inscribes a circle in the square], so iban a tener el mismo [follows 3 square sides]. Y aquí ['Tienen el mismo perímetro porque', A2,1] puse, lo puse en español y luego aquí ['esto es de la misma medida', A2,1] empecé a pensar en inglés. [Opposite to what she said in A2,2 !]	28. Code mixing (five)	27. Previous interviewer's English utterance, English thought
	17	F: ¿Y cuándo... ? Empezaste a leer. Bueno, leíste en inglés, ¿no?		
	18	M: Mm [Affirming].		

MEMO | 6, 10, 28. Mathematical academic vocabulary (symmetry, radius, five) in English within Spanish discourse because of English Schooling.

Spanish (A1, 26) and English (A2, 28) to refer to English thoughts.
 25-28. English to interpret mathematical objects (five, dotted line) on pictures.
 Miriam keeps in mind the relation between the solving process and the language usage [even if it is not demanded by the interviewer].
 Strong bilingual profile arises when Miriam is writing the answer (she uses both languages on the Spanish written answer).

Math	A3. Miriam marked “English and Spanish”.	Language	Tentative	
1, 18, 20. 1 st try. Right answer with arithmetical and visual reasoning through the drawings of Figures 5, 6 and 7, being aware that the number of tiles is growing by 2.	 <p>[Miriam counts the tiles by pointing to them with the pen. She also counts the tiles in Figure 7 one by one before writing the answer]</p>	1, 10. English as unique writing language	Similar exercises done in English during class	
	2 [Comes from A2,34] M: Este, eh... [Reading] Observe this pattern y todo esto lo hice en inglés. 3 F: ¿Todo en inglés? 4 M: Mm [Affirming]. 5 F: ¿Sólo inglés? 6 M: Sí. 7 F: ¿Y nunca cambiaste a español para nada más? 8 M: No. Nomás estaba contando los cuadritos en español, pero todo lo demás lo hice en inglés. 9 F: ¿Y nunca cambiaste a español para nada más? 10 M: Mm... No, nomás, em, empecé a leer todo en inglés y luego empecé a hacer las figuras pensando en español y luego volví a inglés acá [points to the answer]. 11 F: ¿Y por qué aquí [A2] escribiste la respuesta en español y aquí [A1] también? 12 M: I don't know. 13 F: Y aquí [A3] en inglés. ¿No sabes? 14 M: No. 15 F: Empezaste aquí [A3] todos los cambios en inglés, todos los pasos del problema en inglés. 16 M: Lo leí en inglés y luego todos esos [points to the figures she drew] los hice pensando en español. 17 F: ¿Al contarlos sólo? 18 M: Ajá. Al contarlos nada más, me fijé que todos llevaban dos más. Como éste, éste ya estaba, y dos más [points to the 2 tiles added to Figures 3 and 4 respect to the previous figures]. Y por cada dos nomás agregaba dos más contando en español. 19 F: Okay. 20 M: Y luego ya al momento de fijarme, conté todo, cuantos tiles había aquí [figure 7] y puse la respuesta en inglés, pero lo había contado en in[glés], en inglés. [Continues in A4,2]	2-8, 16. English as thinking language	Activity solved in English during class, born in USA	
		2. Code switching	Reading English wording	
		8, 10, 16-18. Spanish as counting language	Home language	
		7-10, 16-18. Spanish as thinking language	Home language	
		12. Code switching (non mathematical interaction)	English dominant	
		20. Code mixing (tiles)	Vocabulary used in class	
	MEMO	20. Miriam says she counts in English, but previously repeats several times (8, 10, 16-18) she uses Spanish to count. 20. Miriam uses tiles, but this does not means that she is not able to use the word in Spanish. She just recalls the work done in class. She uses 'cuadritos' in A2,8 and later makes an implicit use of this word (cuadritos) in A2,18.		

Math	A4. Miriam marked "English and Spanish".	Language	Tentative	
1. 1 st try. Two different possibilities for the answer. Awareness of no situation of middle floor (as it is not directly stated on the wording).		1. English as unique writing language	English dominant	
1, 3, 7, 11. 1 st try. Initial sketch with 9 floors [Miriam says 10], adapted (adding 3 floors at the bottom) when Jamie descends 10 floors.		1	1. English spelling variations (flos, midle)	Quick writing
1. 1 st try. Jamie's movements written down on the sketch.			1. English grammatical deviation (were)	Quick writing
1, 3. 1 st try. Good relative situation of floors.		2	3, 7, 21. Code mixing (middle floor)	Drawing (3, 21), mainly English as thinking language (7),
1, 3, 11. 1 st try. ["13 Floors or depends where she first started & were the middle is"] Wrong answer due to no mathematization of middle floor (unknown building's top floor and		3	3. Code mixing (ten)	24-28. Mainly English as thinking language
		4	3, 7. Code mixing (ten floors)	
		5	7. Good Spanish wording summary	Good Spanish management, good internalization
		6	7. Code mixing (so)	Hybrid language
		7	7. Code	Sketch

unknown middle floor position).		luego dice aquí que “she goes up one floor” [points to it in the wording] le puse uno para arriba. Y luego que baja uno, entonces vuelve a quedar a medias. Y luego que sube tres. Y luego que baja diez pisos. Al momento que baja para diez pisos necesitaba agregar tres más. [While explaining Jamie's movements, follows the arrows on her drawing] Así que los agregué y saqué la conclusión de que son trece pisos de...	mixing (five)	'reading'
15. 2 nd try. Wrong Jamie's entrance to the building, arranged with interviewer interaction (16) and wording check (17).	8	F: [Interrupting] ¿Y agregaste tres más dónde?	7. Code switching	Reading English wording
	9	M: Acá abajo.	15. Code mixing (first floor)	?
	10	F: ¿Por qué?	17. Code switching	English on her drawing, English wording imitation
	11	M: Porque dijo que bajó diez pisos y entonces tiene que ir [the long arrow pointing down is stressed: A4, 1] para abajo diez pisos. Entonces del ocho a acá son los diez pisos. Pero como no sé cuántos más están de acá [top of drawn building] para arriba, no sé encones... eh...		
	12	F: ¿Cómo que no lo sabe? ¿No lo puede saber?	21. 2 nd try. English as counting language	pervious English code mixing, English wording
23. 2 nd try. (Deviated) symmetry through middle floor (which is counted twice), with visual reasoning (21) through interviewer interaction (see 14, 16, 18, 20 for indirect hints).	13	M: No, porque...		
	14	F: [Interrupting] Sí lo puede saber, ¿no? ¿Porque por dónde entra?		
	15	M: Por el first floor.		
	16	F: No. ¿Dónde entra ella?		
	17	M: En el middle... [Points to it with a finger on the drawing until she finds the answer on the wording] Wait! [Looks up in the wording] Middle floor.		
	18	F: Yes. ¿Y cuántos tiene abajo?		
	19	M: Diez.		
	20	F: No, de aquí [middle floor] hasta abajo, ¿cuántos hay?		
	21	M: Del middle floor hasta abajo hay ehm... one, two, three, four [counts starting at the middle floor, going down]... one two, three, four, five [starts to count again, now starting at the point she finished, going up]. Five floors. Porque yo lo había hecho diez. Cinco. Pero como tuve que agregar diez [3!] [points to the 3 floors she added later] más [points to the three at the bottom she added later], seis, siete ocho.	21, 33, 35. 2 nd try. Spanish as counting language	Home language
11-23, 30-31. 2 nd try. [8·2=16 floors] Wrong answer due to deviated symmetry through middle floor (which is counted twice).	22	F: ¿Entonces cuántos quedan hasta abajo?	21. Code mixing (five floors)	Pervious English utterance (counting)
	23	M: No, wait. No. Son dieciséis pisos porque serían ocho acá. Tiene que tener ocho encima, ¿no?		
	24	F: Bueno, si acaso luego volvemos sobre cuál es la respuesta correcta. A ver, ¿lo primero que has hecho, qué es? ¿Cómo lo has pensado? ¿En qué momento has cambiado de lengua?		
	25	M: En inglés y namás cambié para poner la, la... mm... [points to the answer] Bueno, todo lo hice en inglés. Lo único que sí puse, fue cuando puse los números, que estaba en español.		
	26	F: Okay... ¿Y cuándo más en español?	21. Code mixing (wait)	?
	27	M: Nada más eso.		
	28	F: Y todo lo otro, ¿todo en inglés?	25-29. Mainly English as thinking language	Large English wording
	29	M: Yeah.		
35. 3 rd try. Right symmetry through middle floor (34: interviewer indicates that middle floor must be counted just once).	30	F: Okay. Volvemos sobre el problema, pues, si lo quieres comentar. Creo que tienes casi la respuesta correcta, pero... A ver, porque el diagrama está bien. ...		
	31	M: Sí, ya entendí. Si tiene entonces ocho pisos abajo, el del medio es el octavo, entonces tienen que ser ocho arriba que son dieciséis pisos.	25-27. 1 st try. Spanish as counting	Numbers learned in home language
11-22, 30-36. 3 rd try. Right answer with arithmetical and	32	F: ¿Cuántos tiene abajo?		
	33	M: Ocho. Uno, dos, tres, cuatro, cinco, seis...		

visual reasoning with interviewer interaction (see 14, 18, 20, 34 for the most relevant hints).	34 F: Pero éste no lo contamos, éste es el del medio.	language	
	35 M: Oh, yeah! Uno, dos tres, cuatro... ¡siete! Entonces serían catorce, quince pisos. [Writes '15' above '13' as final answer: A4,1]	29, 35. Code mixing (yeah)	Hybrid language
MEMO	3, 7, 21. Use of 'middle floor' instead of translating it to Spanish, taken from wording (6) or her own drawing (2, 20). But she also uses the Spanish version 'piso del medio' in line 2. Analogously when 'reading' the 5 from her drawing (A4,6). 7. Good Spanish wording summary denotes good understanding and internalization of the situation presented in the wording, even if later (14) she confused Jamie's entrance floor (first floor instead of middle floor). 11-12, 20-23, 30-36. Miriam benefits of interviewer's hints making steps towards the right solution quickly, probably because she understands the problem very well (she translates it to Spanish: 7) at the beginning. 7, 15, 17, 23, 29, 35. Use of English during Spanish dialogue: Good management of both languages, hybrid language No language difficulties to solve the problem. Use of Spanish (hints, counting) to solve the problem.		

General Language Questions		Language	Tentative
1 F: Así, en general, eh... ¿Cuándo has usado inglés... para resolver estos problemas?		1-2. English as reading language	Good English management with no translation need
2 M: Nada más leyendo la primera parte y poniendo todo en palabras...			
3 F: ¿Qué quieres decir poniendo en palabras?		1-4. English as writing language	Good English management, English wording, schooling in English
4 M: Como las respuestas éstas [A3, A4] en palabras. Y esto [A1, A2] también lo pensaba en inglés pero lo puse en español.			
5 F: Mm [Affirming].			
6 M: Pero todo lo que es de números lo hice como con... eh... en español.		4. Spanish as writing language	Good Spanish management, home language, previous schooling in Spanish
7 F: ¿Y por qué crees que lo has hecho así?			
8 M: Mmm... No sé.			
9 F: ¿Por qué...? ¿Aprendiste a contar en qué idioma en el colegio?			
10 M: Eh... Mi mamá me enseñó cuándo tenía tres años. Aprendí en español.		6-8, 16. Spanish as thinking language with numbers	9-12. First learning of numbers in Spanish (at home)
11 F: Okay. Em... ¿Pero tu en el colegio, en qué idioma aprendiste?			
12 M: Em... en los dos, era bi... [/bi/ : starts to say the word in Spanish] bilingual. So, pero también en matemáticas siempre he tenido muchos problemas y en sexto, desde sexto grado he tenido problemas y mi mamá me ha ayudado, como en dividir, como dividen en México, de donde es ella. So todo eso me ha enseñado. Porque a veces lo que hacen aquí no lo entiendo bien, o sea me lo enseña de otra manera.			
13 F: Alguna frase... Bueno, ¿el español, entonces, en general, cuándo lo has usado?			
14 M: ¿El español?			
15 F: Ajá.		12. Code mixing (bilingual)	Ease of saying
16 M: Namás resolviendo los problemas. Como todo lo que tiene que ver con números y matemáticas. Como esto por ejemplo, como poniendo esto [points to the right side of the building in A4, where the floors are numbered]. Y ya namás lo único que hago en inglés, aquí [A1, A2] como pensé las respuestas en inglés y las puse en español.			
17 F: ¿Y por qué crees que has hecho esto? ¿Alguna idea?		12. Code mixing (so)	Hybrid language

18 M: No. Y aquí [A3, A4] lo pensé en inglés y lo puse en inglés. 19 F: ¿Hay alguna palabra o frase que se te haya hecho difícil en inglés? 20 M: No.	13-16. Spanish as thinking language	Good Spanish management, home language
MEMO	12. Bi[lingüe] / bilingual: Spontaneous use of the other language (English) to maintain fluidity in discourse	

Miriam has an excellent English BICS. She chooses to speak in Spanish (she hesitates on which language to choose at the beginning) but she is in a mainstream class and has no problems understanding words, sentences and meaning from the wordings. Her English CALP is fair, its level is not higher due to the misuse of terms such as 'symmetry' or 'radius' on A2. She has an excellent Spanish BICS, as she talks fluently in Spanish, the language she finally preferred for the dialogue. Her Spanish CALP is good: she does not use a lot the mathematical register in Spanish but when she does it is right (as on A2 answer).

A1's answer is written in English. The solving process is thought in both languages. There is no need for translating the wording as Miriam has a good English management. The arguments for the answer are given orally but not in a written way (the written answer is taken from a previous interviewer's utterance). During dialogue Miriam gives generic and concrete examples (thought in Spanish) to show that the final price cannot be determined with the statement information.

A2's answer is written in Spanish, but a part of it is thought in English (Miriam does not know a justification for this fact). The same measurement of the dotted line leads Miriam to affirm that both figures have the same perimeter. Given that she correctly marks the perimeter of the circle, she does not have a right visualization skill for this task. When she explains the answer she does not manages some mathematical vocabulary (symmetry, diameter, radius) with precision.

A3's answer is written in English. Miriam says she thinks of everything in English, with Spanish reserved for counting (which also has its influence during the drawing of Figures 5, 6 and 7). She finds the right answer by counting the tiles on Figure 7.

A4's answer is written in English. English is as well the main thinking language, with Spanish reserved for counting. During discourse Miriam makes many code mixes. On the first try she makes a sketch and situates the floors in a relative position, but she does not mathematize the middle floor. She is somehow aware of this fact: even if at first sight her written answer might be considered not logical (as she gives two different answers at the same time) it reflects the importance of the middle floor situation. While commenting the activity, on the second try, Miriam does not remember which is the floor which Jamie enters the building. She checks the wording to recall it. With no direct interviewer's hints she is able to mathematize the

middle floor (but counting the middle floor twice), with visual reasoning through the sketch (on the 1st try she says that answer would depend on where the middle floor is situated. The interviewer says it can be found). Finally the interviewer indicates that middle floor must be counted just once. Miriam solves correctly A1 and A3. She also finds the right solution in A4 where she benefits properly from interviewer interaction. She does not always give reasoned written answers (A1, A4) but she is able to explain what she does, orally.

Activities' (Key ideas) summary

Object 20: Miriam-First reduction (End)

- Employment of English (used to think about the wording, with no translation) and Spanish (used to think about numbers and money; which is a key point that leads Miriam to solve the problem correctly). Both languages are used as thinking languages to solve the activity correctly.
- Spanish as a main thinking language leads the mathematical thoughts (the key point is thought in Spanish). English with figures interpretation (dotted line, 5) and mathematical vocabulary in discourse. Good combination of both languages.
- The use of two languages to refer to “tiles” (cuadritos) during dialogue, including a code mixing instance, is a good example of the practice of code mixing as a resource (so it does not have to be included on a deficit model).
- Use of both languages to think about the tiles of the Figures, with predominance of English. Influence of Spanish due to its use as a counting language. Answer is written in English.
- The dense wording activity is solved mainly using English as thinking language, but with Spanish as a counting language.
- English (as expressed loudly during interviewer's interaction) and Spanish (as stated by Miriam and also as expressed loudly during interviewer interaction) as counting languages for the number of floors.
- Use of several code mixing instances to explain the solution, mainly referring to parts of the wording.
- A good Spanish wording summary denotes a good understanding and internalization of the situation presented in the dense wording. Miriam benefits of the interviewer's subtle comments, making steps towards the right solution quickly. Probably because she has understood the problem very well.
- The use of both languages to refer to “middle floor” (piso del medio) during dialogue, including some code mixing instances, is another example of the practice of her using code mixing as a resource.

Historical profile	Bilingual profile (Spanish dominant) [cont]	Activity			
<ul style="list-style-type: none"> • 16 years old • Mainstream class • Born in USA • Likes California • Spanish readings (improve fluency) • Very few English readings (enough fluency) • Spanish at home; both (sisters) • Spanish(English with 1) with friends • English at school (Spanish class too) • Homework help: father (sometimes) 	English utterances (Dialogue previous to the activities)				
	Spanish as unique writing language (A2 answer: partially thought in English)	x	x		
	Written answer induced by interviewer (words, not sense)	x			
	Code switching: A1(x1+2+1), A2(x0+0+1), A3(x0+1+1), A4(x0+1+1)	x	x	x	x
	English and Spanish as thinking languages	x		x	
	Spanish with numbers and money	x			
	Code mix. A1: yeah, A2: yeah(x3), so(x6), symmetry, radius, okay, five, A3: tiles A4: middle floor(x3), ten, ten floors(x2), so, five, first floor, five floors, wait, yeah	x			
	English as reading language (no translation)	x			
	Spanish as main thinking language		x		
	English as reading language	English utterance		x	x
	English as writing language	Vocabulary confusion (symmetry-diameter, symmetry-radius)		x	
	Spanish as writing language	English to interpret "5" (dotted line)		x	
	Spanish as thinking language with numbers	English as unique writing language			x
	Code mix (bilingual, so)	Deviated writing expression (in English)			x
Spanish as thinking language	Good Spanish wording summary			x	
	Spanish as counting language		x	1-2	
	English as counting language			2	
	Mainly English as thinking language			x	
Procedural profile	Conceptual profile				
<p>1. X Argumentation of written answer ✓ Different final prices depending on the initial prices ✓ Exemplification with numerical prices (before discount is applied) that show that each store can be the cheapest</p> <p>2 ✓ Interpretation of the dotted line X Attempt to explain the similarities on both figures X Visualization of the length of both perimeters ✓ Identification of the perimeters of the circle and the square</p> <p>3 ✓ Continuation of a graphical sequence ✓ Counting of the number of tiles in each figure</p> <p>4.1 ✓/X Graphical representation containing all the floors Jamie goes through (but not all the floors of the building) Singularity of the middle floor not materialized in terms of its symmetry function</p> <p>4.2 X Situation of entrance floor (1st floor) ✓/X [8·2=16 floors] (Deviated) symmetry through middle floor</p> <p>4.3 X Middle floor counted once (by</p>	<p>1. X Notion of mathematical validity ✓ Notion of percentages</p> <p>2 X Logic of the reasoning X Imprecise mathematical vocabulary ✓ Perimeter concept</p> <p>3 ✓ Notion of graphical sequence associated to a sequence of figures</p> <p>4.1 ✓ Notion of number line</p> <p>4.2 ✓ Notion of number line</p>				

Object 22: Camilo-First reduction
(Beginning)

Camilo does not like to work during class; he is lazy. But has average –or even better– solving skills.

Math teacher description


Class	Interview language	Interview date	Origins	Age	USA arrival	Living in California	Spanish readings	English readings	Home language	Friends language	Main language at school	Homework help
HS Transitional	Spanish	May 2010	Mexico	16	1 year and 6 months [November 2008]	Did not like it at the beginning (it was too different) but now he has got used to it.	“La Ganga” newspaper.	Magazines, newspapers, books. Does not like English comics because he understands jokes in Spanish Better.	Spanish	Spanish. English in a few cases.	Spanish in most of the classes (3 [out of 6])	Nobody

Historical bilingual profile

Math	A1. Camilo marked “English and Spanish”.	Language	Tentative
1-7. 1st try. Wrong answer due to percentages as absolute value instead of relative (direct comparison of percentages).	<p>[REMARK: Interview occurs during lunch time and at that moment there is a Mexican party outside with music. Maybe Camilo is not completely concentrated on the task as he looks through the window some times. Conversation about activities starts at A1,2 after Camilo solves all activities.]</p> <p>1 <i>In the John's Store por que ahí ay 40% de descuento que en la tienda de Mire</i></p> <p>[Camilo erases and rewrites the first sentence. On the left, the final answer is reproduced.]</p> <p>2 F: ¿Qué ha hecho aquí primero?</p> <p>3 C: Pues la que... Dice cuál... which of these two... In which of these two stores are the shoes cheaper? And why. En la John Store porqué aquí dice de... el discount es cuarenta por ciento y aquí es veinticinco por ciento, es menos. Y aquí descuentan más [40%] y aquí descuentan menos [25%].</p> <p>4 F: ¿Entonces lo primero que ha hecho qué es para resolver esto?</p> <p>5 C: Pues me fijé cuánto es el descuento de cada uno.</p> <p>6 F: Sí.</p> <p>7 C: Y cuál es el... dónde descuentan más. Y dónde descuentan más pues ahí es... queda... es más barato.</p>	1, 30-33. English and Spanish as writing languages (code switching on writing)	34-43 Ease of use in Spanish, school context
		1. Parts of the sentence missing (even though, understandable).	Quick writing

	<p>8 F: ¿Y con qué lengua empezó a resolver el problema? 9 C: Con inglés. 10 F: Mm [validating]. ¿Y cuándo cambió a español? 11 C: Cuando... pues cuando yo quise [laughing]. De todos modos.... 12 F: ¿Pero cuándo? ¿Cuándo quiso cambiar? 13 C: ¿Cuándo? ¿Cómo? 14 F: ¿En qué momento? Porque aquí me puso que usó inglés y español. Entonces me dijo que empezó a resolver la actividad en inglés. 15 C: Ajá. 16 F: ¿Hasta qué momento? ¿Cuándo... cuándo cambió a español? 17 C: Cuando empecé a explicar. 18 F: ¡Oh! ¿Para explicarlo? 19 C: Ajá. 20 F: ¿Pero mientras estaba pensando [pause], por ejemplo en el cuarenta por ciento de descuento? 21 C: En español. 22 F: Esto lo tradujo a español. 23 C: Ajá. [Camilo nods] En español. 24 F: Pensó cuarenta por ciento de descuento, no forty per cent of discount. 25 C: [Camilo nods]. En descuento. 26 F: O sea leyó la actividad en inglés... 27 C: Sí, pero lo pensé en español. 28 F: Luego estuvo pensando en español. 29 C: Sí. 30 F: ¿Y cuando volvió a cambiar a inglés? 31 C: Cuando empecé a explicarlo. 32 F: ¿Empezó con el inglés [pointing to the answer]? 33 C: Ajá. Y terminé con español. 34 F: ¿Por qué? 35 C: Porque... nada más [laughing]. 36 F: ¿Por qué cambió a español... ah... digo a español luego otra vez? 37 C: Así nada más [laughing]. 38 F: Pero algún motivo debe haber. 39 C: No, nada más porque yo quiero. De todos modos pues si lo quisiera poner en inglés también podía pero.... 40 F: ¿Pero entonces por qué lo puso en español? 41 C: Es... no... es igual... no, no... porque es más fácil para mi. 42 F: ¿Es más fácil en español? 43 [Camilo nods] [Continues in A2,2] 44 [Comes from A2,77] Y volviendo a la actividad uno, porque no habíamos dicho esto de las palabras, ¿no? ¿Aquí alguna palabra sí estuvo como manejándola en inglés? 45 C: Discount.</p>	<p>3. Code switching 3. Code mixing (John Store) 3. Code mixing (discount) 20-29. Spanish as main language for thinking 44-49. English words while thinking in Spanish (The shoes, John Store, Mike store, discount) 49. Code switching</p>	<p>Reading English wording Reference to English statement Reference to English statement Home language Reference to English statement Reference to English thoughts</p>
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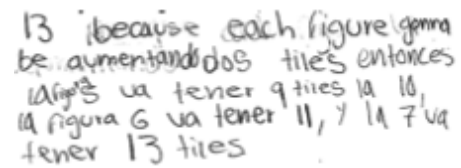
	<p>46 F: ¿El discount?</p> <p>47 C: Sí.</p> <p>48 F: Estuvo pensando en inglés. Discount. ¿Alguna más? ¿Alguna cosa que le venga a la cabeza que estuvo pensando...</p> <p>49 C: The shoes... the shoes... and... The John Store and... y Mike Store. Y eso nada más.</p> <p>50 F: Okay. [Continues in A3,2]</p>		
MEMO	<p>1-3. Oral register more detailed than written.</p> <p>11-12, 35-40. Unknown reasons for the language switching. Finally Camilo points out that writing in Spanish is easier (41-43).</p> <p>14-19. Contradiction: Camilo does not start to write in Spanish. He writes in English at the beginning (30-31).</p> <p>25. Initially Camilo does not say he uses English to think about Discount (25). During dialogue he does use it (3). And later he refers to the English use for 'discount' (45).</p>		

Math	A2. Camilo marked "English and Spanish".		Language	Tentative	
11, 13. 1 st try. Deviated notion of perimeter.	 <p>1 The square because the circle doesn't have perimeter.</p>	[The marks on the circle are added later, see A2,33.]	1, 7, 36-39, 48-51. English as unique writing language	school context, GLQ,14-18: writing in English is shorter	
13. 1 st try. Known procedure to calculate the square's perimeter (addition of its sides).		2 [Comes from A1,43] F: ¿Cómo resolvió ésta?		3. Code switching	Reading English wording
19. 1 st try. Wrong definition of perimeter.		3 C: La pregunta es [reading] Which of these figures has a greater perimeter. Y por qué. Y ya, pues el que tiene más peri...		27. Code mixing (the square)	71. Pronunciation ease (circle)
20. 1 st try. Visual identification of the circle's perimeter and definition of perimeter (by interviewer).		4 F: ¿Cómo empezó a resolver? O sea no me explique la solución, ¿no?, sino cómo usted la pensó.		36-42, 44-65. Mainly Spanish as thinking language	Home language
		5 C: Pues, ¿en qué idioma la pensé o cómo?		60-73. Code mixing (square, circle, perimeter, figure)	English thoughts
		6 F: Sí, bueno, ambas cosas, sí me interesan. Cómo empezó a resolverla...			
		7 C: Empecé leyéndola en inglés y ya la traduje a español y ya lo expliqué en inglés.			
		8 F: Pero, antes de dar la solución, pues usted tiene que pensar, ¿no?, cuál va a ser, cuál de éstas dos va a ser.			
		9 C: Sí.			
		10 F: Entonces mientras está pensando, está pensando pues quizás ésta por esta razón, quizás la otra por otra razón.			
		11 C: ¡Oh! El cuadro porque... Es el cuadro porque el círculo tal vez no tiene perímetro. ¿O sí tiene?			
		12 F: ¿Qué es el perímetro?			
		13 C: Es el... cuándo... dónde... dónde sumas todos lados de un cuadro [follows the perimeter of the square with the pencil]. Luego en el círculo[?] no...			
		14 F: En el cuadrado es sumar todos los...			
		15 C: [interrupting] lados			

21. 1 st try. Visual identification of the square's perimeter.	16 F: ...lados. ¿Y en el círculo? 17 C: Pues en el círculo... Yo no he sacado el del círculo. No sé como se.... 18 F: ¿Qué es el perímetro? Aunque no lo sepa calcular, ¿qué es? 19 C: Lo que... lo ... [makes a circle with the pencil] como explico... es lo que abarca en... adentro. No sé, algo así. 20 F: Es como la parte de afuera de hecho, ¿no? [following the perimeter of the circle with the finger] 21 C: Sí, lo de alrededor del [following the perimeter of the square]	60-77. English as thinking language with isolated words (square, circle, perimeter, figure) but not with entire sentences	Imitation of English statement words, bilingual student
23. 1 st try. Not complete understanding of the notion of perimeter.	22 F: El alrededor de la figura, perfecto. ¿Entonces no puede poner, no, que el... [reading the answer] The circle doesn't have perimter? Porque sí tiene. 23 C: Oh, ¿sí tiene entones? Lo dejo así o... Pues yo no sé de éste [circle] que tenga, pero de éste [square] sí. 24 F: Entonces ahora que ya sabe que es el perímetro, ¿no?, porque aunque esto no sea un lado recto ya sabe que esto también tiene perímetro. Al igual que éste [square] también es la parte de afuera. 25 C: Ajá. 26 F: Como estaba diciendo. ¿Cuál de las dos cree que tiene un perímetro mayor?		
1-3, 10-23. 1st try. ["The square because the circle doesn't have perimeter"]. Wrong answer with wrong reasoning due to conceptual confusion (perimeter exists only with straight sides).	27 C: The square. 28 F: ¿Por qué? 29 C: Porque... porque tal vez dos lados de éste [square], o tres lados de éste, puede abarcar lo que... la línea que hay hasta acá [circle's perimeter]. Sólo viéndolo nada más. 30 F: ¿Sólo viéndolo? 31 C: Sí. 32 F: Así comparando, más o menos. 33 C: Sí, comparando los lados. Como por ejemplo esta línea [one side of the square] puede llegar de aquí tal vez a aquí, y la otra tal vez aquí y la otra tal vez aquí y ya la otra queda libre. [See A2,1 to better understand how Camilo integrates 3 sides of the square in the circle.] 34 F: Sí. Okay. ¿Cómo empezó a plantearse la actividad ahora? 35 C: Pues... 36 F: ¿Qué le pasó por su cabeza? Utilizó... [Pause] Aquí [Language columns of the questionnaire] me puso antes sólo inglés, ¿no? ¿No usó el español para nada?		
24. 2 nd try. Reinforcement of the perimeter definition (by interviewer).	37 C: Pues pensándolo nada más, pero escribiéndolo en puro inglés. 38 F: Entonces escribiéndolo en inglés. 39 C: Sí. ¿Entonces también tengo que poner si lo pensé en español? 40 F: Sí. 41 C: Entonces lo pensé en español. [Camilo changes the cross to the "English and Spanish" column of the questionnaire]		
25-33. 2 nd try. Right perimeter concept.	42 F: Okay. Lo pensó en español. 43 C: Ajá. Y lo expliqué en inglés. 44 F: O sea la primera vez... Hablamos ahora de la primera vez y luego vamos a la segunda vez, cuando yo le he ayudado un poco a entender esto, ¿no? La primera vez empezó, leyó esto [wording] en inglés. 45 C: Sí. Ya. Y lo traducí a español. 46 F: Y luego lo tradujo a español, pensó el problema en español. 47 C: Sí. Sí.		

<p>29-33. 2nd try. Right visual comparison of perimeters' length (by superimposing 3 of the 4 square's sides on the circle).</p> <p>1, 12-33. 2nd try. Right answer with approximate visual comparison (superimposing 3 of the 4 square's sides on the circle).</p>	<p>48 F: ¿Y cuándo cambió a inglés? 49 C: Pues cuando lo expliqué. 50 F: Sí. ¿A la hora de explicarlo? 51 C: Sí. 52 F: ¿No antes? 53 C: No, no antes. 54 F: Vale, ahora la segunda vez, ¿no?, que yo... bueno, que vimos que sí tenía perímetro esto, ¿no? ¿Cómo pensó la actividad? 55 C: ¿En qué idioma o cómo? 56 F: Sí. 57 C: En español. 58 F: ¿En español todo? 59 C: Sí. 60 F: ¿Ninguna cosa en inglés? 61 [Camilo shakes his head saying no.] 62 F: ¿Seguro? 63 C: Sí. 64 F: Porque me dijo por ejemplo the square. 65 C: ¡Oh! Bueno... ¿También eso? ¿También eso incluye? Pues sí, lo pensé en inglés también. Por ejemplo the square, the circle o así. 66 F: ¿Y qué más pensó en inglés? 67 C: Perímetro, perimeter... Luego pues también es uno, dos idiomas. Pienso en uno, como éste es square, luego en círculo. Pensé en inglés y en español también. Por eso puse inglés y español [pointing to the cross he marked on the "English and Spanish" column of the questionnaire; A2,41.] 68 F: ¿Pero también dijo por ejemplo cuadrado aquí? 69 C: Mmm [thinking]... No. Namás square y luego círculo. 70 F: ¿Círculo sí dijo? ¿O circle? 71 C: Yo dije círculo. Aquí dije círculo y aquí dije square. ['cuadro' : A2,11 & A2,13 !!!] Aquí es más fácil de pronunciar círculo. 72 F: Entonces, pues sí, cada vez, aunque sea una palabra, que la piense en inglés, pues sí me interesa que me lo diga, ¿no?, cuándo ha utilizado una palabra en un idioma, otra palabra en otro idioma. Y, por ejemplo, ¿qué más ha pensado aquí en inglés? 73 C: Pues nada más el "perimeter" y luego the "figure" y ya. 74 F: Sólo estas palabras, ningún razonamiento, es decir, "Oh, I'm gonna compare two sides... " 75 C: No, eso no. 76 F: "Or I'm gonna put this side over the other..." 77 C: No, eso no. [Continues in A1,44]</p>		
MEMO	<p>36-41. Camilo understands the language columns of the questionnaire to be related exclusively with the writing language. It is easier to be aware of the language use just on the writing because it is directly observable. 45. C: traduci</p>		

60-65. Unshared meaning of “language use”.
 65. Use of languages not much detailed.
 71,11,13. Different languages used when naming figures: Camilo says he thinks on 'square' (71) but uses 'cuadro' (11,13) and 'square' (65, 67, 69, 71) in discourse. Probably he uses both when thinking.

Math	A3. Camilo marked “English and Spanish”.	Language	Tentative		
1, 8-9. 0 try. Deviated answer due to wrong growth on the number of tiles per figure. 1-3. 1st try. Right answer with arithmetical reasoning.	 <p>[Camilo counts the tiles with the pencil (also during the interview after solving the activity). First Camilo thinks that the increasing number of tiles between figures is 4 tiles. After finishing with A4 he realizes this is wrong and corrects the answer. It was in Spanish but he rewrites part of it in English.]</p> <p>2 [Comes from A1,50] F: ¿Qué hizo aquí? 3 C: Pues nada más en cada figura... Porque la pregunta es [reading] how many ties [tiles!] does figure seven has y por qué. En cada figura va aumentando dos... dos ties [tiles!]. Y pues aquí este empieza con uno, luego va aumentando con dos y ahí tiene tres y luego cinco, luego aquí va a tener siete, luego en la cinco va a tener nueve, en la seis va a tener once y en la siete va a tener trece. 4 F: Entonces, ¿qué es lo primero que pensó aquí? 5 C: Lo pensé primero... Lo leí en inglés y ya después lo pensé en español. Y ya lo escribí en español y en inglés. 6 F: ¿Lo escribió con ambos idiomas? 7 C: Sí. 8 F: ¿Y qué borró ahí? ¿Qué puso por ahí? 9 C: Porque le puse que cada uno iba aumentando cuatro, le puse. Y estaba mal. No. Y era dos pero yo le puse que iba aumentando cuatro. Fue por eso. 10 F: Okay. ¿Qué más cosas pensó en inglés aquí? 11 C: Ties [tiles], la palabra ties [tiles] y luego figure, luego los little square y ya nada más. 12 F: ¿Y entonces qué es lo que pensó concretamente en español? O sea, ¿estaba pensando en español y esas palabras en concreto en inglés? 13 C: No, también en español como por ejemplo cuántos, empecé cuántos ha de tener... cómo... cuánto va a ser esto más grande, cómo dos, aquí dos y aquí otros dos, dos, dos... 14 F: Sí. 15 C: Sumando, iba contando yo en español porque en inglés se me hace... Me enredo mucho. [Continues in A4,2]</p>	1, 5-7, 67-68. Code switching on writing	Home language, GLQ,14-18: writing in English is shorter		
		1. English grammar deviation	Quick writing, lack of organization		
		1. Code mixing on writing (tiles)	Imitating English wording		
		3. Code switching	Reading English wording		
		3. Code mixing (tiles)	Imitating English wording		
		3,11. Misspelling (tiles)	English language in construction		
		5-13. Spanish as main thinking language	Home language		
		11. Code mixing (ti[!]es, figure, little square)	Talking about English thoughts		
		10-11. English as thinking language (ti[!]es, figure, little square)	English wording imitation		
		MEMO	15. Camilo says he counts in Spanish because it is easier than in English.		

Math	A4. Camilo marked “English and Spanish”.	Language	Tentative
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13-28. 1 st try. Right understanding of middle floor and credit department situation.	<p>1 <i>16 por que son los que yo conte cuando subia y bajaba</i> [Camilo counts with his fingers as he reads the wording.]</p> <p>2 [Comes from A3,15] F: Okay. ¿La [actividad] cuatro, pues?</p> <p>3 C: Pues lo leí en inglés, luego cambié a español y lo pensé en español.</p> <p>4 F: ¿Cómo empezó a resolver el problema aquí?, a ver.</p> <p>5 C: Iba leyendo que cuántos pisos iba ella subiendo y bajando y entrando en las tiendas... y así me di cuenta cuánto... Porque aquí dice que ella empezó en el... en el middle floor y luego iba subiendo de piso y bajando, y subiendo y bajando...</p>	3. Spanish linked to thinking	58. Too much text to translate
1, 4-35. 1 st try. [1+1+1+3+10 = 16 floors] Wrong answer due to no relative situation of floors and a wrong mathematization of the middle floor.	<p>6 F: Sí.</p> <p>7 C: Pues por eso.</p> <p>8 F: ¿Y cómo sacó la respuesta?</p> <p>9 C: Pues conté cuántos subió y cuántos bajó y cuántos iba entrando y así.</p> <p>10 F: ¿Y al final qué hizo con todas esas cosas?</p> <p>11 C: Los conté, los pisos que subió y los que bajó.</p> <p>12 F: ¿Puede empezar a resolverlo otra vez, para que lo vea, por ejemplo? [In the discourse below Camilo starts to translate the wording to Spanish and adds up all the numbers with the help of the fingers]</p> <p>13 C: Ella entró a la tienda en el medio... en el piso del medio...</p> <p>14 F: Sí.</p> <p>15 C: Luego inmediatamente fue al departamento de créditos. Ahí es uno. Luego después... se hizo...</p> <p>16 F: ¿Pero es uno qué quiere decir?</p> <p>17 C: Pues en dónde el primer floor, el primer piso. Ya después dice que se hizo... se puso... se hizo... éste... se... ¿cómo se llama?, éste... Se aseguró que su crédito estaba bien.</p>	5. Code mixing (middle floor)	English wording imitation
35-42. 2 nd try. The 10 floors Jamie goes down are given a negative sign, in opposition to the floors Jamie goes up (+). But this is not done with the other floor Jamie goes down.	<p>18 F: Sí.</p> <p>19 C: Ya después subió un piso, a la joyería. [Marks 2 with the fingers]</p> <p>20 F: ¿Por dónde entra, por dónde entra por eso?</p> <p>21 C: En el... en el medio del piso, en el piso del medio.</p> <p>22 F: Sí. Pero, ¿y dónde va para comprobar el crédito?</p> <p>23 C: Pues allí, es el mismo nivel.</p> <p>24 F: ¿Y ha contado esto como uno?</p> <p>25 C: Ajá. Sí.</p> <p>26 F: ¿Por qué?</p> <p>27 C: Porqué aquí no dice que va arriba o va pa abajo, pero no dice que...</p> <p>28 F: Ajá. Sí. Okay. ¿Qué más?</p> <p>29 C: Luego ya después de ir a...</p> <p>30 F: Se va parriba.</p>	13-28. Right understanding of middle floor and credit department situation	Good English management
1, 4-43. 2 nd try. [1+1+1+3-10=4 floors] Wrong answer due to no	<p>31 C: Se va parriba en la joyería, en el departamento de joyería.</p> <p>32 F: Sí.</p> <p>33 C: Ya después ella baja un piso más, dónde el departamento de los niños. [Marks 3 with the fingers]</p> <p>34 F: Sí.</p> <p>35 C: Tal vez ahí cuidan niños, o no sé. Ya son tres. Luego ella va... sube tres pisos más. Luego son seis [marks 6</p>	17. Code mixing (floor)	English wording imitation
		39. Code mixing (four)	Counting in English
		58-69. English linked to thinking	59, 65. Too much text to translate, translation word to word is not suitable
		59. Code switching (after making sure)	Reading English wording
		70-71. Counting in Spanish	Spanish dominant

relative situation of floors and a wrong mathematization of middle floor.	with the fingers, using both hands], tres pisos más ya. Y ya después finalmente baja diez pisos... ¡Oh sí, pero ahí debería de restarlos! Después diez pisos y ahí los cuento y ya son dieciséis pisos. 36 F: ¿Entonces que dijo que tendría que hacer, restarlos? 37 C: Sí, restarlos. 38 F: ¿Lo arreglamos? ¿Arreglamos la respuesta? 39 C: Entonces son... four. 40 F: ¿Cuatro pisos tiene en total? 41 C: Yo pienso que si a... si a diez le quito seis son cuatro, porque iba yo contando diez, [shows 6 fingers] seis, ya después bajó diez, entonces le quito seis y quedan cuatro. 42 F: Entonces el departamento tiene cuatro pisos. 43 C: Ajá, sí.		
47. Awareness of wrong mathematization after inconsistency (44-46: in a building of 4 floors it is not possible to descent 10 floors –shown by interviewer–).	44 F: ¿Pero cuántos me acaba de decir usted que bajó? 45 C: Diez. 46 F: ¿Y cómo puede bajar diez si sólo tiene cuatro? 47 C: Pues es que no lo entiendo bien. Entonces sí son dieciséis. Cuento los seis y los diez que bajó más pabajo. 48 F: ¿Es lo mismo subir un piso que bajarlo? 49 C: Subir un piso que bajarlo así [makes a descending movement with the hand]. 50 F: Es un piso igualmente, ¿no? 51 C: Sí, queda igual. Queda... subiste y luego si... estás en el mismo. 52 F: Pero no, lo que me refiero es que usted ha visto que no es lo mismo subir un piso que bajarlo, a la hora de contarlos, ¿no?		
48-51. 3 rd try. Right relative situation of floors when Jamie goes up 1 floor and down 1 floor.	53 C: Sí. ¿Porque se resta, tal vez, lo que subió, así [makes a descending movement with the hand]? 54 F: Tampoco se restan, pero ¿se suman? 55 C: Sí. 56 F: Si se suman yo creo que esta respuesta tampoco nos funciona. 57 C: No. Entonces seguro no. Esto no... tal vez. Pero yo lo sumé namás los pisos que iba subiendo, el total de pisos que aquí dice.		
52-57. 3 rd try. No further development of a mathematical strategy to solve the problem.	58 F: ¿Qué pensó aquí en inglés? 59 C: Pues aquí sí se me hizo más fácil leerlo en inglés y pensarlo también en inglés. Porque aquí pasarlo a español me cuesta trabajo, ya lo ha visto. Como historias largas, como escrituras largas como ésta. Por ejemplo, como aquí [reading] after making sure, aquí no sé, aquí no sé como en español, como después se puso, se hizo segura, no queda. 60 F: Sí. ¿Pero sí entiende lo que dice? 61 C: Sí. After making sure... 62 F: Porque no tenemos que traducir palabra por palabra. 63 C: No. No.		
1, 4-57. 3 rd try. No answer due to no mathematical approaches developed.	64 F: Después de asegurarse, ¿no? 65 C: Porque no queda bien, no queda bien, como: “Y después haciéndose segura su crédito es bueno...”. Pues ahí... 66 F: Sí. ¿Entonces ahí lo pensó en inglés?		

70-71. All tries. Counting in Spanish.	<p>67 C: Sí.</p> <p>68 F: ¿Y que pensó en español?</p> <p>69 C: No pensé... Namás la respuesta la... la respuesta namás. Namás la escritura que hice aquí [written answer]. No, no pensé nada en español. Lo único fue lo que... en español pensé la respuesta.</p> <p>70 F: ¿Y a la hora de sumar los números, por ejemplo?</p> <p>71 C: ¡Ah! Ahí sí, ahí sí lo sumé en español.</p> <p>72 F: Y quizás un gráfico nos hubiera ayudado en este problema.</p> <p>73 C: Sí.</p> <p>74 F: Porque como sube pisos, entra por ahí... nos hubiera ayudado para hacerlo.</p> <p>75 [Camilo nods]</p> <p>76 F: Pero bueno, está bien. [Continues in GLQ,1]</p>		
MEMO	<p>39, 70-71. Contradiction on language used when counting. Counts in Spanish loudly (15, 33, 35). 47: 'Pues es que no lo entiendo bien' refers to mathematization, not to language comprehension. A4 translated to Spanish, but not read; except 'making sure' (59) but it is translated to Spanish before (17). 3, 58-69. Controversial information about language use: probably Spanish is used on the 1st try and English on the second try.</p>		

General Language Questions		Language	Tentative
1	[Comes from A4,76] F: ¿En general, cuándo ha usado el inglés? Aquí para resolver...	1-6. English as reading language	English statement
2	C: ¿Aquí [pointing to the papers], en general?		
3	F: Sí.	7-8. English as writing language	10: No need of translation, 18, 26: English is shorter than Spanish
4	C: El inglés, al leer, la escritura, al leerlo.		
5	F: ¿Al leerlo?		
6	C: Sí.		
7	F: Y en la escritura.		
8	C: Sí.	13-14, 19-20. Spanish as preferred writing language	26. Better management of Spanish
9	F: ¿Y por qué cree que ha hecho esto?		
10	C: Porque es más fácil para mí porque traducirlo en español, la escritura no queda... no tiene sentido.		
11	F: A la hora de leerlo.		
12	C: Ajá.	21-22. Spanish as thinking language	Spanish dominant
13	F: ¿Y a la hora de escribir?		
14	C: A la hora de escribir porque se me hace más fácil explicarlo en español.		
15	F: Pero me dice que ha usado el inglés también para escribir.		
16	C: Mm [validating]. Pues también porque... pues... no es nada fácil el inglés[?]. No lo sé explicar muy bien.		
17	F: Pero quizás... ¿Por qué? ¿Puede encontrar alguna...? No sé, si lo quiere pensar un poco...		
18	C: En inglés porque no quiero escribir mucho y en inglés hay abreviaturas como aquí namás John store y luego en español el mero escribir la tienda de John y ya es mucho.		
19	F: Okay. ¿Y en general cuando ha usado el español? [The bell rings, announcing the start of the next class.]		
20	C: En general, explicándolo, escribiendo, en la escritura.		

<p>21 F: ¿Y para resolver los problemas? 22 C: Lo he pensado en español. 23 F: Algunos ratos en español y otros ratos en inglés, me dijo, por eso. Un poco de todo. ¿Sabe cuándo ha utilizado una lengua y cuando la otra? 24 C: Sí. 25 F: ¿Cuándo? 26 C: Pues cuando la... el español cuando... El español, pues le digo que lo explico más bien en español y el inglés lo uso porque no quiero escribir mucho. Tiene muchas abreviaturas y se escribe menos.</p>		
MEMO	<p>10. For example, in “making sure” a translation word to word does not make sense (A4,17, A4,59). 11. Interviewer refers to use of English when reading, but Camilo understands it as reading his written production (12). 16. No lo sé explicar muy bien. 23. Interviewer refers to the time when Camilo talks about the activities, but maybe Camilo thinks about his responses on the GLQ. Answers do not stick to the questions. Interviewer refers to the use of languages when thinking, but Camilo answers about the use of languages when writing (26).</p>	

Camilo has an excellent Spanish BICS because he talks fluently in Spanish and his written answers are understandable from a language point of view. His Spanish CALP is good. With no much information about the English use, his English BICS and CALP may be classified as fair or good.

A1 is solved with Spanish as main thinking language, but some words (which are borrowed from the statement: The shoes, John Store, Mike store, discount) are thought in English. Camilo starts to write the answer in English but finishes the sentence in Spanish. The answer is wrong due to a direct comparison of percentages.

A2's answer is written in English. Camilo thinks some words in English –some imitating the wording (figure, perimeter) and some to think about the pictures (circle, square)– but he says that English is not used to articulate sentences when thinking about the mathematical solving process. On the first try he does not have a clear notion of perimeter, but he understands how to calculate the square's perimeter. Interviewer explains what perimeter is. Even if Camilo follows the perimeter of the square with the finger he has not understood completely the notion of perimeter and interviewer clarifies it again. Then Camilo compares both perimeters visually by superimposing 3 of the 4 square's sides on the circle. He finally finds the right answer.

A3's answer is written in English at the beginning and then in Spanish. There is also a code mixing on writing (tiles). Camilo finds the right answer using an arithmetical reasoning, finding that each figure is growing by two tiles (Camilo initially states that the growth is four tiles –0 try– but he

changes it with no external help). He uses Spanish as thinking language but also English (just with some words: tiles, figure, little square).

A4's answer is written in Spanish but the solving process is said to be thought in Spanish (1st try) and in English (probably for the 2nd try only). He admits that he makes additions in Spanish but he says 'four' aloud (so English is used for counting too). All this information together shows how difficult is to speak about the language use. On the 1st try Camilo adds all floors (16). On a 2nd try he realizes that he has not relatively situated the floors, but does not correct this mistake completely ($1+1+1+1-10=4$ floors). The middle floor is not correctly mathematized.

Activities' (Key ideas) summary

Object 22: Camilo-First reduction (End)

<ul style="list-style-type: none"> - Use of both languages for thinking and writing, with predominance of Spanish (dominant language) both on the activities of the percentages and stores and of the perimeter of the geometrical shapes. - Different languages used when naming figures: Camilo says he thinks of 'square' (71) but uses 'cuadro' (11,13) and 'square' (65, 67, 69, 71) in discourse. Probably he uses both when thinking. - Use of both languages for thinking and writing, with predominance of Spanish (dominant language) in relation with the sequence of figures. He switches to English when arranging the written answer. - Use of both languages for thinking in the dense wording. Spanish is used on the first try and (probably) English on the second try. 				
Historical profile	Bilingual profile (Spanish dominant)	Activity		
<ul style="list-style-type: none"> • 16 years old • Transitional class • Born in Mexico. 1.5 years in California • Preferred Mexico to California at the beginning, but now he is used to it • Spanish readings • English readings • Spanish at home • Spanish with friends. • English in a few cases • Spanish (3 out of 6 classes) and English at school • Homework help: nobody 	Code switching in writing	x	x	
	Code switch: A1x(0+1+1), A2x(0+0+1), A3x(0+0+1), A4x(0+0+1)	x	x	x
	Code mix (A1: John Store, discount, A2: the square, square, circle, perimeter, figure, A3: tiles, A4: middle floor, floor, four)	x	x	x
	Spanish as main thinking language (A4: said but not true)	x	x	-
	English as thinking language (A1: The shoes, John Store, Mike store, discount, A2: square, circle, perimeter, figure, A3: ti[l]es, figure, little square)	x	x	
	English as unique writing language	x		
	Code mix on writing (tiles)		x	
	English as main thinking language			x
	Spanish for counting			x
	English as reading language		GLQ	
	English and Spanish (preferred) as writing languages		GLQ	
	Spanish as thinking language		GLQ	
	Procedural profile	Conceptual profile		
1 X Direct comparison of percentages with no mention of initial prices	1 X ✓ Notion of percentages as an absolute value instead of relative			
2.1 ✓ Known procedure to calculate the perimeter of the square	2.1 X Perimeter concept			
2.2 ✓ Visual identification of the perimeters and their comparison by superimposing 3 of the 4 square's sides on the circumference	2.2 ✓ Perimeter concept (thanks to additional interviewer's explanations)			
3.0 X Application of a pattern given by adding 2 from one figure to the next	3.0 ✓ Notion of arithmetical sequence associated to a sequence of figures			
3.1 ✓ Application of a pattern given by adding 2 from one figure to the next	3.1 ✓ Notion of arithmetical sequence associated to a sequence of figures			
4.1, 4.2 X No relative ordering of floors Reasoning without distinguishing the singularity of the middle floor in terms of its symmetry function	4.1, 4.2 Notion of number line with confused order positions			

Object 23: Camilo-Second reduction

Object 24: Diandra-First reduction
(Beginning)

Overall Diandra is a good student. She is applied on the tasks but not very confident on herself. She works hard and likes doing things the right way. She is shy and prefers to talk in Spanish. She is good at Math but not brilliant.

Math teacher description

Class	Interview language	Interview date	Origins	Age	USA arrival	Living in California	Spanish readings	English readings	Home language	Friends language	Main language at school	ingework help
HS Transitional	Spanish	April 2010	Oaxaca, Mexico	15	1 year [April 2009]	Does not like it. Very different: weather, always at home (do not go out because her parents have to work), a brother still in Mexico.	Books (novels)	Hardly ever, because of poor understanding	Spanish	Spanish	Spanish (friends and the 3 teachers who understand Spanish)	Her brother (Camilo) very few times (he does not like to ask her)

Historical bilingual profile

Math	A1. Diandra marked “English and Spanish”.	Language	Tentative
4-12. 1st try. Wrong answer due to notion of percentages as absolute value instead of relative (direct comparison of percentages).	<p>[Activities are solved following the natural order: A1, A2, A3, A4. Activities are commented in this order: A3, A4, A1, A2. Dialogue about activities starts here:]</p> <p>1 D: ¿Las respuestas las tengo que escribir en inglés? 2 F: Eh... como quiera. 3 D: Okay.</p> <p>4 <i>En el numero 1 que es de 40% porque les descuentan mas dinero</i> [Quickly writes down the answer. Continues in A2,1]</p> <p>5 [Comes from A4,69] F: ¿A ver, cómo empezó aquí? 6 D: Pues con la pregunta. Dice que... en cual de las dos tiendas los zapatos son más baratos. ¿Tengo que contárselo? [Rhetoric question: she didn't want to talk anymore, giving the impression of being a little lazy because she asked the interviewer why he let her choose which question she wanted to start with if they were going to comment all of them anyway. Note that the interviewer said after solving the activities they were going to comment all four activities.] Y yo digo que en ésta [40%]. 7 F: ¿En qué se fijó para decir que ésta? 8 D: En el porciento de que le están... quitando, digamos. 9 F: Y... ¿en algo más? ¿Eso fue lo primero que hizo, ya de... 10 D: Sí. 11 F: ...así, de volada? 12 D: Sí. 13 F: Utilizó aquí también inglés y español. ¿Para qué uno y para qué el otro?, a ver. ¿Para qué el inglés y para qué el español? 14 D: Aquí pues para leer la pregunta, pues la traduje al español, cómo para entenderla. Y pues nada más para eso. [Continues in A2,6]</p>	<p>1-3. Norms clarification demand (writing language)</p> <p>3. Code mixing (okay)</p> <p>4. Spanish as writing language</p> <p>4. Spelling variation (mas)</p> <p>13-14. English language linked to reading</p>	<p>Spanish dominant (A4, 66-69)</p> <p>Hybrid language</p> <p>Spanish dominant</p> <p>Quick writing or unknown</p> <p>A4,66-69: Spanish dominant</p>
MEMO	6. Maybe Diandra is tired, she does not want to explain the solving process.		

Math	A2. Diandra marked “English and Spanish”.	Language	Tentative
5-11, 28-43, 67-92. 1st try. Wrong answer with wrong reasoning.	<p>1 [Comes from A1,4] D: ¿Tiene que estar viendo? 2 F: Ah... no... no me estoy fijando, tranquila, tranquila, no pasa nada. Estoy mirando sólo que se quedara grabado pero... no pasa nada. Si está más cómoda, voy haciendo mi trabajo y cuando termine... 3 D: Okay. 4 F: ...me avisa. [Continues in A3,2]</p>	<p>3. Code mixing (okay)</p> <p>5. Spanish as writing language</p>	<p>Hybrid language</p> <p>Spanish dominant</p>



12-17, 22-27. 2 nd try. Unknown circle's perimeter formula.	<p>5 El cuadrado porque tiene mas lados y porque es [Diandra quickly writes the answer]</p> <p>6 [Comes from A1, 14] F: ¿Cómo empezó aquí a resolver esto?</p> <p>7 D: Pues nada más vi el cuadrado y cómo tiene como más lados y cómo esto [circle] casi ... casi no lo entiendo, cómo sacarle el perímetro.</p>	5. Spelling variation (mas)	Quick writing or unknown
18-21. 2 nd try. Right square's perimeter calculation through addition of sides (after interviewer's demand).	<p>8 F: ¿Entonces cómo lo hizo?</p> <p>9 D: Pienso que es esto [square].</p> <p>10 F: ¿Cómo lo hizo?</p> <p>11 D: Porque tiene más... como... éste [follows the sides of the square with the pencil]... por los lados. ¿No?</p> <p>12 F: ¿Pero qué quiere decir que no sabe cómo sacarle el perímetro aquí?</p> <p>13 D: Pues se me hace más difícil en éste [circle]?</p> <p>14 F: ¿De qué?</p> <p>15 D: De sacarle el perímetro.</p> <p>16 F: ¿El calcularlo?</p> <p>17 D: Sí.</p>	93-100. English language linked to reading	Spanish dominant (A4, 66-69)
22-25. 2 nd try. Wrong circle's dotted line interpretation (as the entire perimeter) but right square's dotted line interpretation (21).	<p>18 F: ¿Sabe calcular el perímetro de aquí [square]?</p> <p>19 D: Sí.</p> <p>20 F: ¿Cuánto es?</p> <p>21 D: Si esto es cinco [dotted line], sería cinco, diez, quince, veinte.</p> <p>22 F: Ajá. ¿Y aquí [circle] no se acuerda cómo calcularlo?</p> <p>23 D: ¿Cinco así, o qué? [follows the perimeter of the circle]</p> <p>24 F: ¿Cómo?</p> <p>25 D: ¿Sólo cinco? [following the perimeter of the circle, laughing] No lo entiendo en eso.</p> <p>26 F: Oh, no se acuerda. Es una fórmula. Es el radio, por dos –el diámetro– [showing in the picture what is the diameter] por pi. El número pi, ¿se acuerda del número pi?</p> <p>27 D: Sí. Tres punto catorce dieciséis.</p>		
26. 2 nd try. Circle's perimeter formula introduced by interviewer.	<p>28 F: Ajá. Pues esto sería... por esto. Pero al principio ha dicho: “pues na más lo vi” ¿no?</p> <p>29 D: Sí.</p> <p>30 F: ¿Qué quería decir con esto, nada más lo vi?</p> <p>31 D: Pues porque se me hizo más fácil el cuadrado.</p> <p>32 F: Sí, pero esto [perimeter of the circle] no lo sabe. Entonces como sabe que... Sabe que el cuadrado es veinte, ¿no?, el perímetro.</p> <p>33 D: Sí.</p>		
44-68. 2 nd try. Right comparison of perimeters – between the	<p>34 F: ¿Pero como sabe que es...?</p> <p>35 D: Y éste [circle], tampoco no se la fórmula de éste [circle].</p> <p>36 F: ¿Pero como sabe que éste [square] es mayor que éste [circle]?</p> <p>37 D: No sé, solo lo apunté[?].</p> <p>38 F: ¿Éste [circle] podría ser cuarenta, no? ¿Podría ser éste [circle] mayor que éste [square].</p> <p>39 D: Sí.</p>		

square's picture of the statement and a big imaginary circle drawn by the interviewer—with emphasis on dotted lines but no calculation of perimeters.	<p>40 F: Entonces, ¿por qué puso el cuadrado?, por eso. 41 D: Porque se me hizo más fácil el cuadrado. 42 F: ¿Qué quiere decir más fácil? 43 D: Pues, o sea, no sabía como sacarle la fórmula ni no sabía ni como hacerle y por eso mejor le puse el éste [square]. 44 F: ¿Pero si hubiera sido, por ejemplo, un círculo así de grande? [Interviewer makes a big circle with the finger] 45 D: De todas maneras es lo mismo. 46 F: Hubiera sido también... en vez de ser cinco... hubiera sido esto [diameter]... yo que sé... veinte. [Interviewer makes a big circle with the finger] ¿Hubiera sido éste [square] más grande también [pointing to the square]? 47 D: ¿Cómo, cómo? No lo entiendo. 48 F: El círculo este ahora es cinco y esto también es cinco [dotted lines]. 49 D: Sí. 50 F: ¿Sí? 51 D: Sí.</p>		
86-92. 2 nd try. Possible perimeter-area confusion.	<p>52 F: Pero hubiera podido ser que el círculo, en vez de esto ser cinco, que fuera un círculo así de grande [Interviewer makes a big circle with the finger]. 53 D: ¿Y qué tuviera también cinco [pointing to the dotted line in the circle]? 54 F: No, que tuviera... yo que sé... pues quince aquí [dotted line]. 55 D: ¡Oh!</p>		
12-27, 44-68, 86-92. 2 nd try. No numerical comparison of both perimeters due to unknown circle's perimeter formula. No visual comparison of perimeters neither.	<p>56 F: ¿Cuál sería más grande entonces? 57 D: Pues el círculo. 58 F: ¿El círculo? 59 D: Sí. 60 F: ¿Qué ha hecho para saberlo en éste [circle]? 61 D: Porque el número de esta medida [circle dotted line] es más grande. 62 F: Sí. Porque el número del... el diámetro, a esto lo llamamos el diámetro, es más grande que esto. 63 D: Sí. 64 F: Pero sólo con esto... bueno esto puede ser una... 65 D: Una forma lógica. 66 F: ...un factor, sí. 67 F: ¿Pero se ha fijado también en la forma que tenían ambos, o sea... como... a simple vista, no? ¿Que esto es más largo, recorrer todo esto [Interviewer follows the perimeter of the square with a finger], que recorrer todo esto [Interviewer follows the perimeter of the circle with a finger], o no? 68 D: No. 69 F: Em... Pero es que no... Tiene que haber alguna cosa que la haya hecho decidir por éste [interviewer follows the perimeter of the square with the finger] y no por éste [analogously for the circle's perimeter] [unintelligible]! 70 D: Pues, Mr Reverter, que no ya le dije que nada más porque éste... Pues éste [square] se me hizo más fácil de sacarle y como... 71 F: ¡Sí, pero si saca uno y no saca el otro no los puede comparar! 72 D: Eso sí, pero... [laughing] Es que, Mr Reverter, que no entiende que no sabía como hacerlo en éste... 73 F: [Interrupting] ¡Sí!</p>		

74 D: ...por eso le puse el cuadrado.
 75 F: ¿Sí? ¡Ah! Pero no está segura de esto.
 76 D: O sea, es como si lo ponen a hacer otra... cosas diferentes... pues no va a poder. Si no sabe hacer una cosa... A ver, ponga por ejemplo que usted no sabe sacar esto.
 77 F: Sí.
 78 D: ¿Y si se tiene que decidir cuál es el más grande, cuál escogería?
 79 F: Pues yo diría, por ejemplo... ¡oh!, no me acuerdo de la fórmula, pero... emm... veo que podemos... como esto es cinco [dotted line in the circle] y esto también es cinco [dotted line in the square], por ejemplo, pues podemos meter el círculo aquí dentro del cuadrado y vamos a ver que... recorrer esto así, así, por ejemplo medio círculo, que va a estar así por dentro. El círculo va a ser menor que el cuadrado ¿no?
 80 D: ¡Ah!
 81 F: Porque el círculo iría como por dentro, sería mas corto.
 82 D: Sí.
 83 F: Yo no sé calcularlo, pero sí te sé decir que éste es más pequeño sin calcularlo. Por eso le preguntaba si... si había hecho alguna cosa así parecida.
 84 D: No.
 85 F: Okay.
 86 D: Nomás lo vi como más grande.
 87 F: ¿Lo vio?
 88 D: Sí.
 89 F: Por eso, que me dijo lo vi como más grande, pues no lo está comparando como... parte por parte, pero, pues de alguna forma, la intuición le dice que éste [square] es más grande que éste [circle], ¿no?
 90 D: Sí.
 91 F: Pues es lo que ha hecho, pero sin razonarlo exactamente, ¿no? Como un argumento a simple vista es lo que ha hecho usted.
 92 D: Sí
 93 F: Eh... vale... eh... otra vez aquí, ¿qué lengua?
 94 D: Español.
 95 F: Otra vez, siempre español.
 96 D: Sí.
 97 F: ¿Y nunca cambió a inglés?
 98 D: No. ¿Cómo para...? ¿Cómo para cambiar inglés?
 99 F: No sé, por ejemplo, leer aquí five o alguna cosa así.
 100 D: No. [Continues in GLQ,1]

MEMO

1-4. Diandra prefers to work alone instead of having the interviewer looking at her during the solving process.
 26. The interviewer gives the circle's perimeter formula to Diandra, but does not ask her to calculate the circle's perimeter and compare both results.
 86-92. Possible influence of the possible perimeter-area confusion on the lack of visual comparison of perimeter's length. 'grande' (86) may refer to both the perimeter or the area.

Math	A3. Diandra marked “English and Spanish”.	Language	Tentative
<p>1. 0 try. Drawings of Figure 5, 6.</p> <p>1. 0 try. Visual reasoning, marking the tiles added from one figure to the following one.</p> <p>1. 0 try. Deviated answer with right arithmetical sequence associated to figure pattern, but answer referred to Figure 6 instead of Figure 7 due to confusion on the figures drawn.</p> <p>2. 1st try. Right drawing of Figure 7 according to the pattern.</p> <p>1-8. 1st try. Right answer with visual and arithmetical reasoning (through the drawing of Figure 5, 6, 7).</p>	<p>1 [Diandra marks with a point the tiles added to make a new figure. She follows the figure pattern. She adds two tiles next to figure 4 to make <i>Figure 5</i>. Analogously she makes <i>Figure 6</i>. She writes the answer: “ 11 porque ba creciendo de dos”.]</p>  <p>2 How many tiles does figure 7 have? Why?</p>  <p>3 [Comes from A4,1] F: ¿Cómo resolvió la actividad tres?</p> <p>4 D: Pues nada más le fui aumentando otras dos en cada... [points to the two tiles in figure 2 added from figure 1] hasta llegar a la número siete. Otros dos cuadritos.</p> <p>5 F: Sí. Y... ¿Entonces el primer paso cuál fue, por eso? Eso fue lo que fue al final, ¿no? ¿Pero al principio cómo... cómo lo pensó?</p> <p>6 D: Porque en la primera tiene solamente un cuadro y en la segunda se le aumentan estos dos. Luego en la tercera son estos dos, en la cuarta son estos dos, y ya le fui aumentando nada más otros dos y otros dos [see tiles marked with a point, on A3,2].</p> <p>7 F: Ajá. Okay. En el mismo orden además, ¿no?</p> <p>8 D: Sí.</p> <p>9 F: O sea, siguiendo el patrón. Vale. ¿Qué lenguaje utilizó para empezar a resolver el problema?</p> <p>10 D: ¿Cómo qué lenguaje?</p> <p>11 F: ¿Qué lengua?, perdón.</p> <p>12 D: ¿Cómo?</p> <p>13 F: ¿Qué lengua utilizó: español, inglés,...?</p> <p>14 D: Español.</p> <p>15 F: Para empezar a resolverlo... o sea...usted... porque está en inglés, ¿no?</p> <p>16 D: Sí.</p> <p>17 F: Los enunciados están en inglés... ¿Lo leyó en inglés?</p> <p>18 D: Sí. Y como lo traducí a español.</p> <p>19 F: ¿Y luego pensó en español?</p> <p>20 D: Sí.</p> <p>21 F: ¿Y cuándo volvió a cambiar a inglés?</p> <p>22 D: Nada más cuando leía las preguntas.</p> <p>23 F: ¿Luego la tradujo a español y pensó en español?</p> <p>24 D: Sí. [Continues in A4,2]</p>	<p>2, 9-24. English language linked to reading</p> <p>1, 2. Spanish spelling variation (ba)</p>	<p>Spanish dominant (A4, 66-69)</p> <p>Quick writing or unknown</p>
MEMO	18. 'traduci'		

Math	A4. Diandra marked "English and Spanish".	Language	Tentative
3. 1 st try. Mathematization process has not been easy.	1 <i>18 porque ella visita casi todos los pisos y son en diferentes</i> [Diandra whispers while she is reading the wording. After writing "18 porque" makes a pause and checks the wording. Continues in A3,3.]	1, 8-9, 52-69. 1 st try. English language linked to reading	Spanish dominant (A4, 66-69) 8. English is confusing
12. Mathematical reasoning questioned (indirectly) by interviewer.	2 [Comes from A3,24] F: ¿Cómo resolvió la cuatro? 3 D: Está un poco enredada Mr. Reverter y no la entendí mucho, porque dice que primero sube, luego baja, y luego baja diez pisos, y como que está un poco difícil[?]... Pero yo nada más me puse a contar todos los pisos que ella pasó según. 4 F: ¿Nada más los contó? 5 D: Ajá. 6 F: ¿Cómo empezó por eso? 7 D: ¿Cómo? 8 F: ¿Cómo empezó a pensar la actividad? 9 D: Pues la empecé a leer y le... como la traducí a español para entenderlo más o menos... Más o menos mejor, porque en inglés casi se me hace más confundido.	13-15. 1 st try. Deviated wording question understanding	Spanish dominant (A4, 66-69)
13-15. 1 st try. Deviated wording question understanding (floors Jamie goes through instead of total number of floors on the building). In the mathematization done, both understandings have the same mathematical implications.	10 F: Pero luego, ¿cómo empieza, cómo sabe que tiene que sumar todos? ¿Por qué decide que tiene que sumar? 11 D: [Laughing] Porque lo leo. 12 F: Ajá. Pero podría ser que la respuesta no fuera sumar, ¿no? Podría ser que fuera restarlos todos, o podría ser que fuera... sumar unos y otros no. 13 D: Porque aquí verá [pointing to the wording] de cuántos, éste... departamentos ella ha estado y porqué. Y pues ya, me puse a contarlos, de cuántos ha estado, y cuántos ha subido, cuántos ha bajado, y todo eso. 14 F: Sí. Pero aquí pregunta cuántos departamentos... no en cuántos ha estado, sino cuántos tiene la tienda, ¿no? 15 D: Ajá, por eso. Y como ella sube a diferentes pisos... 16 F: Pero por ejemplo, si sube uno, y baja uno, ¿es esto lo mismo? 17 D: [interrupting] Sí. 18 F: ¿Esto cuenta como dos pisos? 19 D: Sí, y es lo que me confundía, porque nada más decía que subía al piso donde ... de joyas y luego al de juguetes o decía subía uno y bajaba otro, y bajaba pero no decía si... no especificaba si... como... como no le entendía si era el mismo o eran diferentes. 20 F: ¿Quiere que lo revisemos un poco? 21 D: Sí.	9, 20-50. 1 st try. Correct wording understanding with Spanish translation	Good English management
20-49. 1 st try. Right wording understanding.	22 F: A ver, que ponía aquí, la primera fase. ¿Lo quiere irlo leyendo y diciéndome que es lo que entiende? Quizá podemos hacer un... como un pequeño [pause] dibujo para aclararnos un poco, o algo... ¿Por qué no ha intentado hacer un dibujo? 23 D: ¿Para qué?	33. Code switching 73, 87, 179. Code mixing (okay)	Reading English wording Hybrid language

<p>22-25, 74-78, 162-168. 1st, 2nd try. No use of sketches (which are not usually used in class).</p> <p>1-5, 10-19. 1st try. [18 floors] Wrong solution due to no relative positioning of floors (97, 116-119) and highest floor reached considered as the top of the building.</p> <p>74-110, 152. 2nd try. Sketch initiated by Diandra due to interviewer indication. Jamie's movements relatively situated on the sketch while wording is being read.</p> <p>97. 2nd try. Awareness of no relative situation of floors when the credit department is situated on the sketch.</p> <p>70-119. 2nd try. [11 floors] Wrong</p>	<p>24 F: ¿No? 25 D: No. 26 F: Okay. Vamos a ver lo que nos pone. So, Jamie is shopping in a large department store with many floors. Vale, está comprando, Jamie, ¿no? 27 D: Ajá. 28 F: ¿Qué más? 29 D: ¿Qué? ¿Lo leo? 30 F: Sí, bueno, un poco resumiendo que es lo que va haciendo Jamie. 31 D: ¡Ah!, lo leo mejor. 32 F: Mm [agreeing]. 33 D: "She enters the store at the middle floor and immediately goes to the credit department. After making sure her credit is good she goes up one floor to the jewelry department." 34 F: Okay. ¿Qué ha hecho hasta ahora? 35 D: Pues empieza en el... ¿éste es en el miedo del...? 36 F: Ajá. 37 D: ¿Del ese...? 38 F: ¿El medio de qué? 39 D: Del piso, ¿digamos? 40 F: ¿En el medio del? 41 D: Sí, ¿no? De la planta o algo así. 42 F: Entra en el punto medio del edificio, ¿no? 43 D: Ajá, ajá. 44 F: O sea, si el edificio tiene... yo qué sé, veinte plantas o cincuenta y una plantas... 45 D: [Interrupting] Entra en la veinte 46 F: ...pues entra en la veinte... veintiuna. 47 D: Y después se va al departamen... 48 F: [Interrupting] No, perdón. Veintiuna no. Sería veintiséis, ¿sí? 49 D: [Laughing] Y después se va al departamento de crédito. Y después dice que se ... como checa que si su crédito está bien... y después sube un piso al departamento de joyas. Entonces sube uno... son dos entonces aquí. Después dice que va para abajo un piso al departamento de los niños. [Mumbles something not understandable as she continues reading the wording] Y después... después va otra vez arriba tres pisos al... dónde están los juguetes. Y después, finalmente... ella va abajo... ella baja tres [10!] pisos otra vez hasta donde está la entrada principal, que es uno de los primeros pisos y se va a otra, éste... a otra tienda . 50 F: Vale. Sí, está bien. Em... pero de hecho no tenemos que hacer esto, luego si quiere lo podemos comentar. 51 D: Sí. 52 F: ¿Cambió después de traducirlo...? O sea, ¿cuántas veces lo leyó, por ejemplo? ¿Lo entendió con una primera? Lo volvió a leer, ¿no?, supongo luego. 53 D: Sí. 54 F: Para ir contándolos. ¿Cuándo cambió entre inglés y español? 55 D: ¿Cómo?</p>		
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<p>answer due to no mathematization of the middle floor (top of the building as highest floor reached).</p>	<p>56 F: ¿Dice que lo leyó en qué idioma? 57 D: En inglés. 58 F: ¿Luego lo tradujo? 59 D: Cuando estaba leyendo lo traducía. 60 F: ¿Y luego cambió otra vez a inglés en algún momento? 61 D: Para volverlo a leer, pero lo traduzco todo el tiempo cuando lo leo. 62 F: Siempre lo traduce todo el tiempo a, a...</p>		
<p>120-151. 3rd try. After middle floor position is required by the interviewer (120-134), the middle floor is shifted to the 6th floor according to the 11 floors drawn.</p>	<p>63 D: Sí. 64 F: ...a español. Okay. 65 D: Sí. 66 F: ¿Y nada más para eso utiliza el inglés? ¿No piensa en ninguna cosa en inglés? 67 D: ¿Cómo? 68 F: Ningún... no sé, como por ejemplo, pues al añadir los números, al sumarlos, o... 69 D: No, casi no lo utilizo. [Continues in A1,5] 70 F: [Comes from GLQ, 34. From now on, no videotaping, audio recording only.] ¿Quiere que comentemos el problema de las... de las... 71 D: ¿De los pisos? 72 F: Sí.</p>		
<p>152. 3rd try. Interviewer arranges sketch.</p>	<p>73 D: Okay. 74 F: ¿Por qué... por qué no quiere hacer un gráfico? 75 D: ¿Cómo un gráfico? 76 F: Ustedes casi nunca hacen gráficos, como... como un... un sketch, un dibujito que le ayude.</p>		
<p>152-156. Interviewer points out that highest floor reached is not the top of the building. Hint quickly used by Diandra to find the right solution to the problem.</p>	<p>77 D: No, por qué, para qué Mr Reverter dibujitos. 78 F: ¡Pues sí, va bien! Ahora vamos a ver para qué. Si lo quiere poner aquí... 79 D: ¿Aquí? 80 F: Sí. Bueno, como quiera. Em... vamos a dibujar un edificio. 81 D: ¿Qué? ¡Oh! 82 F: Dibuje un edificio, ¿no? 83 D: No, mejor dibuje usted. 84 F: ¡No, usted, usted! 85 D: ¡Ja, ja! ¿Grande o chico? 86 F: Como quiera, no pasa nada. 87 D: Okay. Ya.</p>		
<p>70-161. 3rd try. [7+7+1=15 floors] Right solution with visual (and arithmetical) reasoning.</p>	<p>88 F: Okay. Vamos a ver lo que pasa. Jamie is shopping in a large department with many floors. Okay, está comprando, Jamie. Entra en el edificio, eh... at de middle floor, ¿okay? ¿Por dónde entra? 89 D: ¿Por acá? 90 F: ¡Ajá! Entra por acá. 91 D: Sí. 92 F: Es el piso del medio 93 D: Sí.</p>		

	<p>94 F: Eh. Luego dice: Inmediatamente después va... goes to the crédito department. ¿Okay? ¿Dónde está ahora?</p> <p>95 D: Éste, en el mismo piso.</p> <p>96 F: En el mismo piso, no se mueve, perfecto.</p> <p>97 D: ¡Oooh! [Surprised, low voice]</p> <p>98 F: After making sure her credit is good, she goes up one floor.</p> <p>99 D: Entonces va acá.</p> <p>100 F: Sí. Un piso arriba.</p> <p>101 D: Sí.</p> <p>102 F: Eh.... To the jewelry department. Then she goes down one floor. To the children's department.</p> <p>103 D: ¿Acá?</p> <p>104 F: Ajá, vuelve al punto inicial, ¿sí?</p> <p>105 D: Sí.</p> <p>106 F: Luego dice "then she goes up three floors to the toy department"</p> <p>107 D: Uno, dos, iría acá.</p> <p>108 F: Sí, sí. This is the toy department. Okay. Mmm... Finally, Jamie goes down ten floors.</p> <p>109 D: Dos, tres, hasta acá.</p> <p>110 F: Ten floors. ¿Okay? To the main entrance of the store which is on the first floor and leaves to go to another store, down the street.</p> <p>111 D: ¿Once pisos?</p> <p>112 F: ¿Once pisos? ¿Cuánto le salieron antes? Dieciocho.</p> <p>113 D: Dieciocho.</p> <p>114 F: Porque los añadió todos.</p> <p>115 D: Sí.</p> <p>116 F: ¿Ahora cuántos le salen?</p> <p>117 D: Once.</p> <p>118 F: ¿Por qué?</p> <p>119 D: Porque éste... subía y... dice que iba a diferentes como lugares pero en el mismo piso.</p> <p>120 F: ¿Ve como sube y baja y el dibujo nos ayuda? Más importante aún, si además de sólo hacer el dibujo, le ponemos nombres, le ponemos lo que está pasando, por ejemplo, ¿no? (¡Jewelry department y todo esto que está no!) Pero sí hay una información importante ahí que nos estamos perdiendo. ¿Cuál es? ¿Por dónde entra?</p> <p>121 D: Eh... por... por... ¿por el medio?</p> <p>122 F: Middle floor. ¿Dónde está el middle floor?</p> <p>123 D: Acá.</p> <p>124 F: Éste.</p> <p>125 D: Ajá.</p> <p>126 F: ¿Y cuántas...? ¿Si es el middle floor, qué quiere decir middle floor?</p> <p>127 D: ¿Cómo el del medio?</p> <p>128 F: ¿Cuántos tiene arriba?</p>		
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	<p>129 D: ¿Eh? ¡Oh! Un... pues esto tres.</p> <p>130 F: Tres. ¿Y cuántos tiene abajo?</p> <p>131 D: Siete.</p> <p>132 F: ¿Es éste el middle floor?</p> <p>133 D: No, pero como lo empecé aquí, pues... [laughing]</p> <p>134 F: ¡Pero empezó en el middle floor! ¿Qué quiere decir el piso del medio, que tiene cuántos arriba y cuántos abajo?</p> <p>135 D: Este... cinco acá, cinco acá y el del medio es como el sexto.</p> <p>136 F: ¡No! Pero ha dicho que entra en el del medio, ¿no?</p> <p>137 D: Por eso.</p> <p>138 F: ¡Pues el del medio es éste, es el que entra!</p> <p>139 D: Por eso, ¿y qué es lo que tengo aquí? Empezó acá.</p> <p>140 F: Sí, el del medio. ¿Pero cuántos tiene que tener arriba y abajo si éste es realmente el del medio?</p> <p>141 D: Aquí debería de tener cinco y aquí otros cinco.</p> <p>142 F: Sí. ¿Pero cuántos tiene aquí abajo?</p> <p>143 [Diandra laughs]</p> <p>144 F: ¿Cuántos hay?</p> <p>145 D: Dos, cua.. siete.</p> <p>146 F: Siete me ha dicho antes, por lo tanto...</p> <p>147 D: Sí, pero Reverter, no ve que empecé como lo hice de acá ... Por eso lo cal...</p> <p>148 F: Ajá.</p> <p>149 D: ... el de más acá. Pero luego bajó diez pisos, pues por eso.</p> <p>150 F: Okay, bajó diez. ¿Vale?</p> <p>151 D: Sí.</p> <p>152 F: Déjeme alargar esto. Esto es el... donde entra, middle floor; ésta es la calle. ¿Alguien le ha dicho que esto sea arriba del todo?</p> <p>153 D: No.</p> <p>154 F: Por lo tanto, aquí puede haber más pisos arriba. ¿Sí?</p> <p>155 D: Sí.</p> <p>156 F: ¿Cuántos más hay aquí? Por eso le estaba preguntando, tiene que haber los mismos... si esto es el medio...</p> <p>157 D: ¡Oh, sí, ya! Ya le entiendo, ya le entiendo, ya le entiendo.</p> <p>158 F: ¿Cuántos tiene en total?</p> <p>159 D: Aquí tiene que tener otros cuatro.</p> <p>160 F: Ajá. Por lo tanto, ¿cuántos pisos hay en total?</p> <p>161 D: Cinc... Catorce... quince.</p> <p>162 F: Quince pisos, por eso es importante...</p> <p>163 D: ¡Oh, claro!</p> <p>164 F: ...hacer los dibujos.</p> <p>165 D: Sí, Reverter. Pero usted sabe qué dibujos hacer, o cómo hacerlos.</p>		
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	<p>166 F: ¡No! Lo que pasa es que no están acostumbrados a hacer dibujos en clase. 167 D: ¡Ahí está, Reverter, no nos enseña a hacer dibujos! 168 F: Yo... Bueno los problemas que hacemos tampoco son Word problems, como éstos, ¿no? Son más... 169 D: Pero... pero en el examen venían así, y yo no sabía ni como resolverlos. 170 F: Ah, eso sí es verdad. En el <i>benchmark</i> ahora estuvimos, estos últimos días, estuvimos haciendo un poco más. 171 D: Sí. 172 F: Pues nada, si no quiere nada más, por mi parte ya estoy. 173 D: ¿El qué? 174 F: ¿Sí? 175 D: ¿Eh? 176 F: No, si no me quiere preguntar nada más, digo, yo ya estoy. Ya terminamos. 177 D: Oh, no. No. 178 F: De acuerdo, muchas gracias pues. 179 D: Okay. [End of the dialogue]</p>		
MEMO	<p>31. Diandra starts reading the wording (33) but later she synthesizes most of it in Spanish (49), showing good understanding. 35-48. It is not clear if Diandra understands what "middle floor" means on her 1st try, before interviewer's explanation. She does not use interviewer's hints quickly. 49. Diandra translates ten as 'tres' but neither she nor the interviewer realize. Correct wording understanding not helpful to overcome mathematical problems by herself.</p>		

General Language Questions		Language	Tentative		
1	[Comes from A2,100] F: En general, entonces, ¿cuándo... cuándo ha usado el inglés?	1-2. (In general) Spanish instead of English when possible	Spanish dominant. 22-26. Use of English might result in producing some mistakes and other people laughing for this fact.		
2	D: Sólo cuando estoy hablando con maestros o cuando es necesario usarlo.				
3	F: No, perdón, aquí, en....				
4	D: Ah, ¿en éstos?				
5	F: ...resolviendo éstos... Sí.				
6	D: Pues nada más cuando leo las preguntas porque así como para pensar o para ponerme a resolver no lo uso.				
7	F: Ajá.				
8	D: Sólo uso el español.				
9	F: Okay. ¿Y entonces el español es para todo lo otro?				
10	D: ¿Eh?				
11	F: ¿Para todo lo otro el español?				
12	D: ¿Cómo?				
13	F: Para pensar, para escribir las respuestas,...			3-18. English language linked to reading	15-26. Spanish dominant, Mexico preferred over California
14	D: Sí. ¡Oh sí!				
15	F: ¿Y por qué cree que lo hace así?				
16	D: Porque el español, como es mi primera lengua y es la más, la que mas sé, pues es por eso.				

<p>17 F: ¿Se siente más cómoda... 18 D: Sí. 19 F: ...con el español? Pero... ¿No le gustaría por ejemplo practicar el inglés y intentar aprender un poco de inglés? 20 D: De hecho sí se un poco, pero practicar... 21 F: ¿Un poco más? Como yo... yo practico inglés, aunque no sea... [Diandra laughs] ¡Pues yo practico! 22 D: [laughing] Si yo sólo lo platico con los maestros porque hay veces que como... cuando no saben... se burlan de uno. 23 F: ¿Ah sí? 24 D: En la escuela, cuando estaba en el gimnasio, tenia <i>P.E.</i> [/pi i/ , P. E.: Physical Education] una niña se burló de mí. Por eso como que me da pena, éste, hablar. 25 F: ¿El inglés? 26 D: Sí, sí. 27 F: ¡Guau! Pero... no tiene que tomárselo personal ¿no? Es problema suyo... pero entiendo, entiendo lo que quiere decir. ¿Hay alguna palabra o alguna frase que no haya entendido? 28 D: No. 29 F: ¿Entendió todo bien? 30 D: Sí. 31 F: ¡Bien! Vale, pues... si quiere añadir alguna otra cosa, cuándo usa una lengua o cuándo usa la otra... 32 D: No. 33 F: Sino pues ya estamos, ¿sí? 34 D: Sí. [Continues in A4,70]</p>		(see "Historical bilingual profile"), 22-26. Use of English might result in producing some mistakes and other people laughing for this fact.
	24. Code mixing (<i>PE</i> : Physical Education)	Expression used frequently in English at school
MEMO		

Diandra has an excellent Spanish BICS because she talks fluently and correctly in Spanish. She has a good Spanish CALP, even if she is not always using the most appropriate mathematical terms (A2, 61-62). Diandra has a good English BICS, as she understands all the words and sentences on the four activities. Nothing can be said about her English CALP as she does not use it for writing.

A1 is solved with English language linked to reading. There is a direct comparison of percentages.

A2 is solved with English language linked to reading. The written answer is not mathematically correct ("El cuadrado porque tiene mas lados"). Diandra is able to calculate the square's perimeter through the addition of its four sides. She does not interpret the circle's dotted line correctly (she thinks it is the entire perimeter) and the interviewer introduces the formula to calculate the circle's perimeter (assuming but not demanding that Diandra can calculate the perimeter and find the solution to the problem). Later Diandra refers only to the dotted lines' length when comparing the perimeters of two figures (the square from the statement and a big imaginary circle made by the interviewer) and does not pay attention to its actual

length. There is a possible perimeter-area confusion ('lo vi más grande', where 'grande' might refer to both the area or the perimeter).

A3 is solved with English language linked to reading. Diandra finds that each figure is growing by two tiles and marks which tiles are added from one figure to the next one. She draws figures 5 and 6 just by adding to tiles next to Figure 4 for each one of the new figures. She writes 11 tiles for an answer. But later she reviews the problem (by herself, with no external help), draws figure 7 (again by adding two tiles next to Figure 4) and corrects the written answer. So she combines visual and arithmetical reasoning.

A4 is solved with English language linked to reading. On her 1st try Diandra does not position the floors in a relative way and does not mathematize the middle floor considering its symmetry function. When she starts to talk about the activity she says she has not understood the wording, but this is in fact a deviated mathematization (or maybe she is not convinced about her mathematical procedure) rather than a linguistic issue. She assimilates her deviated understanding of the question (floors Jamie goes through) with the interviewer's translation, finding no difference: according to her mathematization both ways of understanding the wording are the same. On a 2nd try interviewer leads Diandra to make a sketch (previously she refused to do it twice). She correctly situates all of Jamie's movements. When she locates the credit department she is aware of the no relative situation of floors made on her 1st try. Her answer (11 floors) is still wrong because the middle floor is not mathematized according to its symmetry function yet. On the 3rd try, when interviewer asks for the middle floor position Diandra refers to her drawing to state that it is on the 6th floor. After interviewer says that the highest floor reached is not the top of the building Diandra finds the right solution of the problem.

Diandra tries to explain the answers, but the reasons are not always either correct or explicit. Overall it seems that the difficulties Diandra faces are rather a mathematical than a language issue. For example, in A4 she does not seem to have any language comprehension problems and she is able to solve the problem with the interviewer's help. She solves Activities 1, 2 and 3 very quickly but she spends more time on A4.

Activities' (Key ideas) summary

Object 24: Diandra-First reduction (End)

- In all problems English language is linked to reading.
- The notion of percentages (considered as an absolute value) is a mathematical issue.
- A deficient reasoning due to a lack of mathematical arguments related to the geometrical figures is not apparently linked only to language.
- There is a right problem resolution (including self-correction of a mistake) on the figure pattern presented.
- In spite of properly understanding the dense wording of A4, Diandra encounters mathematical difficulties which are not overcome by herself.
- Her deviated understanding of the question in the wording (number of floors Jamie goes through) is seen as similar to the presented meaning (total number of floors on the building) because of Diandra's deviated mathematization (considering the highest floor reached as the top of the building).

Historical profile	Bilingual profile (Spanish dominant)	Activity	
<ul style="list-style-type: none"> • 15 years old • Transitional class • Born in Mexico • 1 year in California • Does not like California • Spanish readings • Almost no English readings • Spanish at home • Spanish with friends • English and Spanish (if possible) at school • Homework help: her brother (sometimes) 	Demand for norms clarification (writing language)	x	
	Code mixing (okay) : A1x1, A2x1, A4x3	x x x	
	English language linked to reading	x x x x	
	Writing deviations (in Spanish)	x x x	
	Deviated understanding of the question in the wording (<i>number of floors Jamie goes through</i> instead of <i>total number of floors on the building</i>)		1
	Correct understanding of the situation described in the wording		1
	Code switching (x1+0+0)		2
	(In general) Spanish instead of English when possible		GLQ
	English language linked to reading		GLQ
	Code mixing ('P E': physical education)		GLQ
Procedural profile	Conceptual profile		
<p>1. X Direct comparison of percentages with no mention of initial prices</p> <p>2.1 X Comparison of the number of sides</p> <p>2.2 X Circle's perimeter not calculated √ Square's sides added to get the perimeter X 5 (Circle's dotted line) seen as circle's perimeter √ Comparison of perimeters –between statement picture of square and big imaginary circle drawn by the interviewer– with emphasis on the dotted lines</p> <p>3.0 √ Application of a pattern given by adding 2 from one figure to the next X Deviation between the figure demanded and figure drawn</p> <p>3.1 √ Application of a pattern given by adding 2 from one figure to the next</p> <p>4.1 Non relative ordering of floors Reasoning without distinguishing the singularity of the middle floor in terms of its symmetry function</p> <p>4.2 Reasoning without distinguishing the singularity of the middle floor in terms of its symmetry function √ Use of sketch (introduced by</p>	<p>1. X Notion of percentages as an absolute value instead of relative</p> <p>2.1 X Notion of perimeter</p> <p>2.2 X Unknown circle's perimeter formula ? Possible confusion with the notions of Perimeter-area (no explicit reference)</p> <p>3.0 √ Notion of arithmetical and graphical sequence associated to a sequence of figures</p> <p>3.1 √ Notion of arithmetical sequence associated to a sequence of figures</p> <p>4.1 Notion of number line with confused order positions</p> <p>4.2 √ Notion of number line</p>		

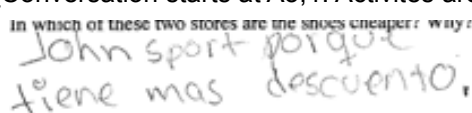
Object 26: Jessica-First reduction
(Beginning)

Jessica has an average mathematical level. She is quite smart, with social abilities. She does not do the homework with regularity.

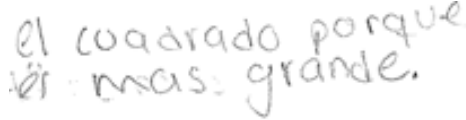
Math teacher description

Class	Interview language	Interview date	Origins	Age	USA arrival	Living in California	Spanish readings	English readings	Home language	Friends language	Main language at school	Homework help
MS English-Spanish	Spanish	May 2010	Michoacán, Mexico	13	Lived in Mexico until 10 years old. Atlanta and Chicago (1 year). Arrived at California on December 6th 2008.	Does not like living in California: violence... Prefers Chicago (e.g. school).	Magazines	No	Spanish	Spanish	English (Spanish with friends)	Nobody

Historical bilingual profile


Math	A1. Jessica marked "English and Spanish".	Language	Tentative
1-15. 1st try. Wrong answer due to notion of percentages as absolute value instead of relative (direct comparison of percentages).	1 [Conversation starts at A3,1. Activites are solved in the natural order: A1, A2, A3, A4.] <i>In which of these two stores are the shoes cheaper: why?</i> 	1. Spanish as writing language	Spanish dominant
	2 [Comes form A3,7, once Jessica writes the answer for all activies] F: ¿Cómo resolvió la actividad uno? ¿Qué fue el primer paso?	1. Deviated Spanish spelling (mas)	Quick writing, unknown
	3 J: Mirar cuál es el descuento de los dos. Y... [pause]	16-35. English language linked to reading	Spanish dominant, reading English wording
	4 F: ¿Y qué? [Pause]		
	5 [Jessica smiles]		
	6 F: ¿Qué hizo luego? Primero miró el descuento... Bueno, se olvidó de poner la crucecita, en dónde... qué lenguas utilizó.		
	7 [Jessica marks the cross in the appropriate column for all four activities]		

<p>8 F: Entonces primero miró el descuento, ¿y luego? 9 J: Escribí. 10 F: ¿Comparó los descuentos? 11 J: Mm [validating]. 12 F: ¿Dónde uno era uno mayor que otro? 13 J: Mm [validating]. 14 F: Y ya escribió la respuesta. 15 J: Sí. 16 F: ¿Con qué lengua empezó a resolver el problema? 17 J: Español. 18 F: ¿Empezó en español? ¿Ya directamente? 19 J: Sí. 20 F: ¿Y cambió al inglés? 21 J: No. 22 F: Nada más, bueno, es que estaba en inglés, ¿no?, el enunciado. 23 J: Mm [validating]. 24 F: ¿Luego empezó en español? 25 J: Mm [validating]. 26 F: ¿Y ya siguió así en español? 27 [Jessica nods] 28 F: Vi que borraba alguna cosa, ¿no? No me fijé muy bien qué era. [Pause] Pero borró algo. ¿Quizás sólo era porque lo escribió mal, la palabra, o algo? 29 [Jessica nods] 30 F: ¿O qué era? 31 J: Sí. 32 F: ¿Entonces para qué usó el inglés? Porque aquí me puso español e inglés, para... 33 J: Porque la pregunta está en inglés. 34 F: Ajá. Y tuvo que leerla en inglés. 35 J: Mm [validating]. [Continues in A2,2]</p>			
MEMO	Jessica's answers are very short.		

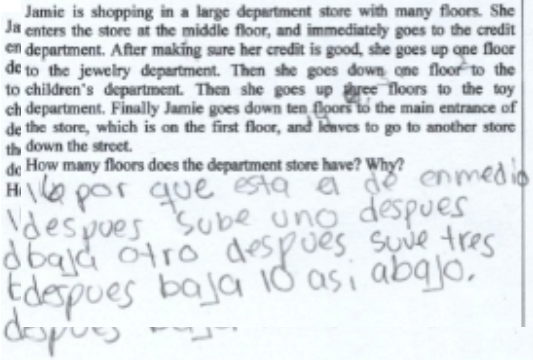
Math	A2. Jessica marked “English and Spanish”.		Language	Tentative
18-22. 1 st try. Unclear perimeter concept.	1	 [Initially she writes: “El cuadrado porque tiene mas”. But immediately erases the second line and states the final answer as displayed.]	1. Spanish as writing language	Spanish dominant
22. 1 st try. Perimeter concept explained by interviewer, both perimeters being visually identified.	2	[Comes from A1,35] F: A ver, ¿cómo empezó ésta?	1. Deviated Spanish spelling (mas)	Quick writing, unknown
23-33. 1 st try. No additional data or arguments added to reasoning.	3	J: Mirando las figuras.	18-22. Unclear meaning of “perímetro”	Hesitation between perimeter and area concepts
1-33. 1 st try. [“el cuadrado porque es mas grande”] Incomplete answer due to lack of detailed reasons. “grande” could be referring to area or length.	4	F: Mm [continuing conversation].	34-56. English language linked to reading	Spanish dominant, reading English wording
	5	J: Y son las mismas. Miden lo mismo. [Points to both dotted lines].		
	6	F: ¿Miden lo mismo? ¿El qué mide lo mismo?		
	7	J: Eeeh... esto [Points to the square's dotted line].		
	8	F: ¿El perímetro? Bueno, esto... la, la... el dato que nos dan sí es lo mismo, ¿no?		
	9	J: Mm [validating].		
	10	F: Sí mide lo mismo, ¿no?		
	11	J: Mm [validating].		
	12	F: ¿Entonces, cómo siguió a partir de ahí?		
	13	J: Mmm... [thinking] [Pause]		
	14	F: ¿Cómo? [Pause] ¿Se acuerda? [Pause] Aquí me puso el cuadrado porque es más grande.		
	15	J: Es que... [Pause]		
	16	F: Ajá. ¿Qué significa más grande?		
	17	J: Que es mayor que éste.		
	18	F: ¿Por lo tanto, qué es...? El perímetro sí sabe qué es, ¿no?		
	19	J: No [low voice].		
	20	F: ¿O no?		
	21	J: No.		
	22	F: ¿No? [Pause] ¿Lo ha estado haciendo en clase o no, el perímetro? ¿Se acuerda? El perímetro es lo de afuera, esto es el perímetro. [Francesc follows both perimeters with the finger.]		
	23	[Jessica nods]		
	24	F: Entonces ahora cambiaría la respuesta, o considera, sigue pensando que el cuadrado tiene más perímetro? ¿O el círculo tiene más perímetro? ¿Cuál de los dos?		
	25	J: El cuadrado.		
	26	F: ¿Por qué?		
	27	J: Está más grande.		
	28	F: Está más grande, entonces esto de afuera es más grande.		
	29	J: Mm [validating].		
	30	F: Vale. ¿Me lo puede justificar de otra manera, también? [Pause] ¿Puede decir el porqué de, con otros argumentos, con otras palabras?		
	31	J: Mmm [low voice, thinking/saying no]		
	32	F: ¿No?		
	33	[Jessica says no with the head.] [Pause]		

34 F: ¿Empezó usando qué lengua?
 35 J: El inglés.
 36 F: ¿Para qué, para qué usó el inglés?
 37 J: Para leer la pregunta.
 38 F: ¿Y luego?
 39 J: Español para escribir.
 40 F: Pero, durante, mientras... , desde que la leyó hasta que escribió, ¿no?, mientras estuvo pensando.
 41 J: Mm [validating].
 42 F: Por ejemplo pues pensó: esta figura es más grande que ésta, o el nombre de la figura, o cualquier cosa. ¿No? ¿Cómo...? Todo este proceso, de los pasos, de, de mirar que esto era lo mismo, eh... ¿Cuándo cambió de inglés a español?
 43 J: Al responder la pregunta.
 44 F: Ajá. ¿Pero desde que la leyó hasta que la respondió, estuvo pensando, ¿no?, como responderla?
 45 J: Mm [validating].
 46 F: Por ejemplo, usted me dijo: Oh, porque ésta es más grande. Por ejemplo. ¿No? ¿En qué momento cambió de inglés a español? [Pause] ¿No? ¿Entiende lo que le digo?
 47 [Jessica says no with the head]
 48 F: Por ejemplo, el nombre de las figuras, ¿en qué los pensó?
 49 J: En español.
 50 F: ¿Los números?
 51 J: En español.
 52 F: En español también. O sea que en inglés, mmm... leyó la pregunta y luego ya lo tradujo, ¿La tradujo cuando ya la leyó? ¿La tradujo a español?
 53 J: Mm [validating].
 54 F: Y luego el razonamiento, de pensar que una era más grande que la otra fue en...
 55 J: Español.
 56 F: En español. Mm [validating]. Vale. [Continues in A3,9]

MEMO A2, 1, 14-17, 27: When Jessica says 'más grande' it is not clear if she is referring to area or perimeter.

Math	A3. Jessica marked "English and Spanish".		Language	Tentative
8. 1 st try. Mathematical reasoning not detailed enough (it is not clear whether there are 25 sides on Figure 4).	1 [Conversion about activities starts here] J: Ésta no la entiendo. 2 F: ¿No la entiende? ¿Por qué no? 3 J: No entiendo la pregunta. 4 F: Dice... ¿Qué es lo que no entiende, la pregunta? How many tiles does the figure have, figure seven have. La figura siete, ¿sí?, cuantos cuadraditos va a tener. Cuántas tiles son, bueno, tejas, cuadraditos. ¿Sí? 5 J: M [surprised]. 6 F: ¿Lo entiende ahora? 7 J: Mm [agreeing]. [Conversation continues at A1,2 after writing the answer for all activities]		1-7. 1st try. Asking initiative	Unknown meaning of wording question, 9-16: unknown meaning (tiles)
8, 17-19. 1 st try. ["25 porque si sumas todos los lados"] Wrong answer with deviated wording question understanding and wrong mathematization.	8  [A3, 1 st try. Written answer has been typed, because Jessica erases it later, on A3,26. On figures 3 and 4 many pencil marks can be appreciated, as if Jessica had been counting.] "25 porque si sumas todos los lados"		8. 1st try. Spanish as writing language	Spanish dominant
19-26. 2 nd try. [7] Wrong answer with no justification.	9 [Comes from A2, 56] F: ¿Qué le ocurrió aquí? ¿Qué es lo que no entendió al principio? 10 J: La pregunta, saber que decía. 11 F: ¿Pero hay alguna palabra que no entendió o por qué se le hizo difícil esto? 12 J: Ésta. [Jessica points to the word tiles] 13 F: F: Tiles. Ah, ¿no usan tiles en clase? 14 J: No. 15 F: Oh, tiles son, ya le he dicho, ¿no?, los cuadraditos éstos. ¿Lo otro sí estaba claro? 16 J: Sí. 17 F: Okay. Y me dice aquí veinticinco. ¿Me puede decir cómo lo hizo, eso? 18 J: Contando las, los lados de un cuadrado.		8. 1st try. Deviated Spanish syntax (si, de)	Quick writing, unknown
28. 3 rd try. Demanding of wording question clarification (by Jessica) due to no mathematical procedure found.	19 F: Mm [validating]. Pero no nos pregunta por los lados, ¿no? Nos pregunta por cuántos cuadraditos. Por ejemplo, ¿cuántos hay aquí? 20 J: Uno. 21 F: Aquí hay... 22 J: Tres. 23 F: Y nos pregunta en la figura siete cuántos va a haber. 24 J: Siete.		42. 4th try. Spanish as writing language	Spanish dominant
27-34. 3 rd try. [15] Wrong answer, but right increasing (of 2 tiles) in an arithmetical sequence associated to the given series of	25 F: ¿Por qué siete? [Pause] 26 J: Mmm... [thinking] 27 F: Esto es figura uno, figura dos, figura tres, cuatro. ¿Sí? Nos pregunta por la número siete, de figura. ¿Lo quiere intentar de [pause] arreglarlo? 28 J: [1 st written answer (A3,8) erased] Figura siete, pues no hay figura siete. 29 F: Sí. ¿Pero se lo puede imaginar, o puede calcularlo de alguna manera? ¿Puede hallar alguna manera de saber cuántas.... cuántos cuadraditos va a haber en la figura siete? 30 J: Mmm... [thinking]. ¿Quince? 31 F: ¿Por qué quince?		42. 4th try. Deviated Spanish spelling variation (bas)	Quick writing, unknown
			51-64. English language linked to reading	Spanish dominant

<p>figures.</p> <p>36. 4th try. Mathematical process reviewed after interviewer indirect question about answer.</p> <p>27-50. 4th try. [13] Right answer with right arithmetical sequence associated to figure pattern.</p>	<p>32 J: Porqué va sumando dos. 33 F: Ajá. 34 J: En cada figura. 35 F: Por lo tanto [pause] a ver. ¿Cree que son quince? 36 J: No, trece. 37 F: Aquí son siete, en la cinco van a ser... 38 J: Nueve. 39 F: En la seis... 40 J: Once. 41 F: Once. Y en la siete trece. Ajá. Perfecto.</p> <div data-bbox="472 475 891 651" style="text-align: center;"> </div> <p>42 [A3, 4th try.]</p> <p>43 F: A ver, ¿aquí cómo empezó? ¿Pensando qué? ¿Cómo hizo para saber que es sumando de dos? 44 J: Mirando éstas [figures from the wording] 45 F: ¿Comparó de una a la siguiente o...? ¿cuáles figuras en concreto comparó, por eso? 46 J: Éstas. 47 F: ¿La uno y la dos? 48 J: Mm [validating]. 49 F: ¿Y luego vio que funcionaba también para las otras? 50 [Jessica nods] 51 F: Aquí empezó también leyendo en inglés, ¿no? El enunciado está en inglés, lo leyó en inglés... 52 J: Mm [validating]. 53 F: Y luego, porque me puso que utilizó las dos lenguas, ¿no?, español y inglés también. ¿En la última vez que lo pensamos también utilizó ambas? 54 J: M [Agreeing/thinking]. 55 F: Sí, porque escribió en español, ¿no? [pointing to the answer]. Entonces lo leyó en inglés, luego, ¿cuándo cambió a español? 56 J: Al escribir. 57 F: Mm [validating]. Y el procedimiento, por ejemplo esto... eh... pensó que son cuadraditos, pensó en las tiles, figure number four... los números los pensó ¿en qué?, ¿en cuál? 58 J: En español. 59 F: ¿En español? ¿Y qué más pensó en español? 60 J: Ir sumando. 61 F: Ajá. Ir sumando dos, ¿es eso? 62 J: Mm [validating]. 63 F: ¿Y en inglés qué pensó? 64 J: Nada. [Continues in A4,2]</p>		
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MEMO		8, 42. Deviated writing expression. // 28. Wording question understanding questioned (by Jessica) due to no mathematical procedure found.			
Math	A4. Jessica marked "English and Spanish".		Tentative		
<p>1-8. 1st try. [middle + 1+1+3+10=16] Wrong answer due to no relative positioning of floors.</p> <p>9. Review of the solving process after the importance of the middle floor is highlighted by the interviewer (8).</p>	<p>1</p> 	<p>[Jessica counts the number of floors while she is reading the wording of the problem. She counts with the fingers.</p> <p>As it can be appreciated on the picture on the left, Jessica writes down the accumulated number of floors as Jamie moves along the building: "2" when Jamie "goes up one floor", "3" when Jamie "goes down one floor", "6" when Jamie "goes up three floors", "14" when "Jamie goes down ten floors".</p> <p>Jessica writes down "14" as initial answer but later changes it to "16".]</p>	<p>1. Spanish as writing language</p> <p>1. Deviated Spanish spelling (por que, esta, despues, suve, asi)</p> <p>12-17. English language linked to reading</p>	<p>Spanish dominant</p> <p>Quick writing, unknown</p> <p>Spanish dominant, reading English wording</p>	
		2	[Comes from A3,64] F: ¿Me puede explicar cómo lo resolvió esto?		
		3	J: Primero leí, y después leí la pregunta y... [pause] fui contando los pisos que iba subiendo. Y conté todo junto. Y después lo escribí.		
4	F: ¿Y cómo lo hizo?, a ver. ¿Cómo lo hizo para obtener la respuesta? ¿Anotó los que iban subiendo, los que iban bajando... y cómo sacó el dieciséis? ¿No? Ahí pone dieciséis. [Pause. Francesc reads Jessica's written answer. Pause.] ¿Y cómo le sale este dieciséis?				
5	J: Sumando todos.				
6	F: ¿Suma todos?				
7	J: Mm [validating].				
8	F: Pero me dice que está en el de en medio, ¿no? ¿Tiene esto alguna importancia?				
9	J: N [thinking]. [Jessica reads de problem again]. N [thinking]. [Pause]				
10	F: ¿Sí, quiere pensarlo un poco más o así lo dejamos?				
11	J: Así.				
12	F: Entonces empezó leyéndolo en inglés. ¿Luego cuándo cambió a español?				
13	J: Para pensar cuántos iba subiendo y bajando.				
14	F: Mm [validating]. ¿Ahí pensó en español?				
15	J: Mm [validating].				
16	F: ¿Volvió luego a pensar alguna otra cosa en inglés?				
17	J: No. [Continues in GLQ,1]				
MEMO		11. Jessica does not wish to get involved on solving the problem again.			

General Language Questions		Language	Tentative
1	[Comes from A3,17] F: Entonces, ¿en general –digamos en todos los problemas, ¿no?– cuándo ha usado el inglés?	1-4, 13-14, 19-20. English language linked to reading	15-18, 21-24. Better management of Spanish
2	J: Al leerlo.		
3	F: Ajá. ¿Sólo al leerlo? ¿Ninguna otra cosa?	5-12. Contradictory affirmations about language use	Automatized use of languages
4	J: No.		
5	F: ¿Y por qué cree que sólo usa el inglés para leerlo?		
6	J: Nnn [thinking].		
7	F: ¿Por qué usted cree que no usa el inglés para resolverlo o para escribirlo?, por ejemplo.		
8	J: N [thinking].		
9	F: ¿Por qué? ¿No se le ocurre alguna explicación de [Jessica moves her shoulders, saying no] por qué sólo usa el inglés para leer?		
10	J: Sí.		
11	F: ¿Por qué?		
12	J: Porque...		
13	F: Vamos con el español. Quizás el español, que lo usa más, es más fácil, ¿no? ¿En general, cuándo cree, cuándo usted usa el español [pause] para resolver aquí las actividades? [pause] Dice que lo ha utilizado, por ejemplo, para escribir, ¿no? Todos los enunciados los ha escrito en español.		
14	J: Mm [validating].		
15	F: ¿Por qué?		
16	J: Porque sé más español que inglés.		
17	F: Sabe más español que inglés entonces se le hace más fácil escribirlo en español.		
18	J: Sí. [Jessica nods]		
19	F: Y para razonar, también utilizó el español, me dijo.		
20	J: Mm [validating].		
21	F: ¿Sí? ¿Por qué? ¿Por qué cree que utiliza más español [Jessica mumbles “no sé”] que inglés?		
22	J: Nn [unclear meaning]		
23	¿Sabe más español?		
24	J: Sí.		
25	F: ¿Alguna palabra o alguna frase que haya encontrado difícil en inglés? [Pause] ¿Hay alguna?		
26	J: No [low voice].		
27	F: En, de todos los enunciados.		
28	J: No.		
29	F: Bueno, ésta sí hemos dicho lo de “tiles”, ¿no? ¿Se acuerda de alguna otra?		
30	J: No.		
31	F: ¿No? ¿Lo otro lo encontró bien?		
32	J: Mm [validating].		
MEMO	5-12. Contradictory affirmations about language use (reasons are said to be known but no reason is provided immediately).		

Jessica has an excellent Spanish BICS because she talks with correctness during the entire interview in Spanish and she does not seem to have any problem when she writes in Spanish. Her Spanish and English CALP need to be improved, as she makes no references to any parts of the figures or specific mathematical vocabulary. Jessica has a fair English BICS, as it seems she only has problems understanding A3. Her answers are generally very short.

Jessica seems not very cooperative when talking about the solving process or the use of languages within it. She does not bring a lot of information by herself to the questions asked. This can be caused by Jessica's use of English exclusively as reading language in all four activities – she never says it is used for any other purpose. Or else, it can also be that she has not developed some of her meta-cognition abilities very much.

A1 is solved incorrectly due to a direct comparison of percentages. English language is linked to reading.

On A2 Jessica slightly modifies her written answer (from “el cuadrado porque tiene mas” to “el cuadrado porque es mas grande”) noticing that maybe she needs to be more specific or give more details when justifying the answer, yet the reasons are not detailed enough. She is not sure about the meaning of “perímetro”. Once this is stated by the interviewer she does not add more details to the explanation. English language is linked to reading.

On the 0 try of A3 Jessica is proactive when asking for the wording meaning, but after the interviewer's explanation the intended meaning is not clear to Jessica yet. On the first try there is a deviated answer as a consequence of the deviated understanding of the wording. From the pencil marks on Figure 3 and Figure 4 it is clear that she counts some square-sides, but it is not clear how Jessica gets her written answer (25 is not the number of square-sides of Figure 4). On her 2nd try Jessica states 7 as answer with no justification, yet without getting the intended meaning to the question. As she does not find the answer directly on the statement pictures, she asks for Figure 7. Once the interviewer explains she has to follow the given series of figures, she states 15 as an answer, with a right increasing (2 tiles per figure) in the arithmetical sequence associated to the figures. With no more clues than a couple of questions about the validity of this answer, Jessica states 13 as a final correct answer on her 4th try. She explains it through interaction with the interviewer.

On A4 Jessica adds all floors [$\text{middle}+1+1+3+10 = 16$]. There is a wrong horizontal mathematization with no relative positioning of floors. English language is linked to reading.

Object 26: Jessica-First reduction (End)

<ul style="list-style-type: none"> - In all activities, English language is linked to reading the statements. - Notion of percentages as an absolute value (direct comparison of percentages) not apparently directly related to the choice of languages. - The geometrical problem is given an intuitive answer not reasoned enough with no use of mathematical specific related vocabulary either in English or Spanish. After interviewer explains what the perimeter is, Jessica does not add more details to her reasoning. - Deviated understanding of the question in the statement (even if it has been translated by the interviewer and the meaning of "tiles" specified and Jessica asserts she understands it). It results in a deviated answer (with an unclear reasoning and not detailed enough). - The statement's question understanding is questioned by Jessica as she does not find Figure 7 in the statement. Its correct understanding prompts a right horizontal mathematization leading to a good approximation to the problem and finally to the right answer. - The floors are not relatively positioned. This does not seem to be related to the use of languages. 						
Historical profile		Bilingual profile (Spanish dominant)		Activity		
<ul style="list-style-type: none"> • 13 years old • English-Spanish class • Mexico until 10. Atlanta. 1 year in Chicago. 1'5 years in California • Does not like California. Prefers Chicago. • Spanish readings • No English readings • Spanish at home • Spanish with friends • English at school, • Spanish with friends • Homework help: nobody 	English language linked to reading		x	x	x	x
	Deviated (Spanish) writing expressions		x	x	x	x
	Unclear meaning of "perímetro"			x		
	Asking initiative (understanding of the question in the wording)					1
	Deviated understanding of the question in the wording					1
	English language linked to reading		GLQ			
	Contradictory affirmations about language use		GLQ			
Procedural profile		Conceptual profile				
1 X Direct comparison of percentages with no mention of initial prices		1 X Notion of percentages as an absolute value instead of relative				
2 X Mathematical reasoning not detailed enough		2 X Unclear concept of 'perímetro'				
3.1 X Focus on the number of sides of a tile		3.1 X Notion of arithmetical sequence associated to a sequence of figures				
3.2 X [7] Reasoning not explained		3.2 X Notion of arithmetical sequence associated to a sequence of figures X Notion of mathematical validity				
3.3 ✓ Finding of a pattern given by adding 2 from one figure to the next X Chosen term of the arithmetical sequence		3.3 ✓ Notion of arithmetical sequence associated to a sequence of figures				
3.4 ✓ Application of a pattern given by adding 2 from one figure to the next		3.4 ✓ Notion of arithmetical sequence associated to a sequence of figures				
4 No relative ordering of floors Reasoning without distinguishing the singularity of the middle floor in terms of its symmetry function		4 Notion of number line with confused order positions				

Object 27: Jessica-Second reduction

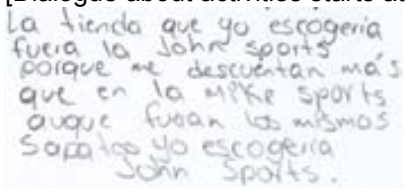
Object 28: Ana-First reduction
(Beginning)

Ana has a good Spanish language management. She is very talkative and quite smart (she sometimes think that she is smarter than she really is.).

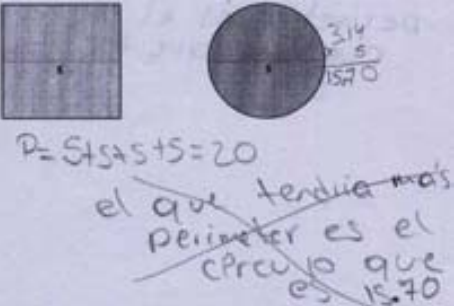
Math teacher description

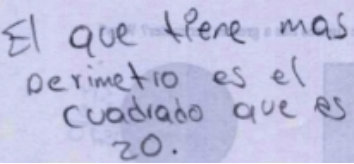
Class	Interview language	Interview date	Origins	Age	USA arrival	Living in California	Spanish readings	English readings	Home language	Friends language	Main language at school	Homework help
MS En-Sp	Spanish	May 2010	Guadalajara, Mexico	12	April 12 2008 [2 years]	Likes it more or less.	Sometimes	[Not asked]	Spanish, English (sister)	Spanish	Spanish (Math, Spanish and Social Studies)	Mother

Historical bilingual profile

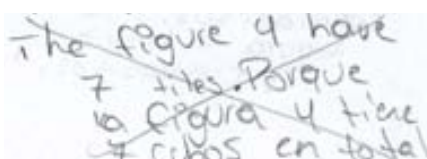
Math	A1. Ana marked "English and Spanish".	Language	Tentative
1-7. 1st try. Wrong answer due to absolute notion of percentages instead of relative (direct comparison of percentages).	1 [Dialogue about activities starts at A3,2, once all answers are written] 	1. Spanish spelling variations (escogeria, sapatos, auque)	Spoken Mexican Spanish has no /z/, /θ/ but /s/ (sapatos), quick writing or unknown (auque, escogeria)
	2 [Comes from A2,47] F: Otra vez, ¿cómo lo hizo eso?		
	3 A: Me fijé, éste, cuál de las dos tiendas tenía ¿más descuento?		
	4 F: Mm [continuing conversation].		
	5 A: Y me fijé que la de John Sport tiene más éste, más descuento que la de Mike Sport.		
	6 F: Y entonces se quedó con la de ...	1, 8-21. English almost exclusively as reading language	Home language, Spanish dominant
	7 A: John Sport.		
	8 F: Okay. ¿Cómo empezó aquí a resolver el problema?		
	9 A: Lo empecé en español, fijándome cuál tenía el descuento más y luego cuándo lo cambié a inglés fue		

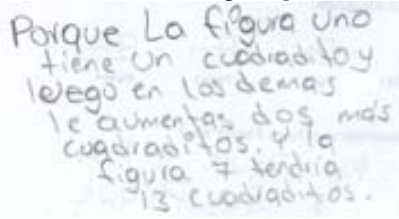
	<p>cuándo ... [pause].</p> <p>10 F: ¿Se acuerda cuándo cambió a inglés?</p> <p>11 A: No. [Pause]</p> <p>12 F: Leyó en inglés, luego lo tradujo a español, ¿estuvo pensando en español?</p> <p>13 A: Sí.</p> <p>14 F: ¿Y ya no se acuerda si cambió otra vez a inglés?</p> <p>15 A: No.</p> <p>16 F: [A4 was going to be commented, but dialogue went back to A1] Por ejemplo, perdone, ¿aquí para comparar los porcentajes, esto lo hizo... lo hizo en inglés?</p> <p>17 A: [Hesitating] Sí.</p> <p>18 F: Perdón, inglés. ¿En inglés o en español?</p> <p>19 A: En español.</p> <p>20 F: Okay. ¿Y al leerlo pues lo tradujo, lo traduzco también, lo tradujo al, al español?</p> <p>21 A: Sí. [Continues with A4, 2]</p>	<p>1, 9-19. Unclear distinction in the use of Spanish and English</p>	<p>Spontaneous use of both languages</p>
		<p>5, 7. Code mixing (John Sports)</p>	<p>English wording imitated</p>
MEMO	<p>1. "au[n]que fueran los mismos zapatos": maybe the visual context (same shoes' pictures) influences the mathematical resolution of the activity (assuming equal initial prices).</p> <p>9-19. She does not remember clearly her using of English during the mathematical solving process of the activity.</p>		

Math	A2. Ana marked "English and Spanish".		Language	Tentative
<p>1. 1st try. Right calculation of perimeters.</p> <p>1-3. 1st try. Deviated final answer due to a mistake on the numerical comparison. Right visual and arithmetical reasoning (square: addition of its sides, circle: application of its perimeter's formula).</p>	<p>1</p>  <p>[Answer crossed out in A2,12]</p>		<p>1, 20-21. 1st try. Spanish as writing language with code mixing (perimeter)</p>	<p>Spanish dominant, wording reproduction</p>
	<p>2 [Comes from A3,41] F: ¿Cómo hizo aquí para resolver esto?</p> <p>3 A: Saqué el perímetro de los, de los, del círculo y del cuadrado. Y del cuadrado sumé cinco más cinco más cinco más cinco y me dio para veinte. Y en el círculo multipliqué trescientos... , tres punto catorce por cinco, que es el perímetro, y me salió quince punto setenta.</p> <p>4 F: Vale, y entonces el más grande pues es el...</p> <p>5 A: Oh, es el... cuadrado.</p> <p>6 F: Mm [validating]. [Pause] ¿Es el cuadrado más grande?</p> <p>7 A: Sí.</p> <p>8 F: Es veinte, ¿no?</p>		<p>1. 1st try. Spanish spelling variation (tendría)</p>	<p>Quick writing, unknown</p>
			<p>13. 2nd try. Spanish as unique writing language</p>	<p>Spanish dominant</p>
			<p>13. 2nd try. Spanish spelling vari-</p>	<p>Unknown, quick writing</p>

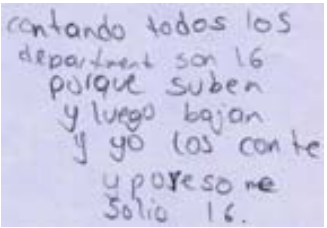
4-5. 2 nd try. Awareness of comparison mistake.	9 A: Sí. 10 F: ¿Se confundió aquí al escribirlo? 11 A: Sí. 12 F: Lo puede cambiar si quiere. [Ana crosses the previous answer out: A1, 1]	ations (mas, perimetro)	
1-13. 2 nd try. Right answer with right visual and arithmetical reasoning (square: addition of its sides, circle: application of its perimeter's formula).	13  14 F: Okay, pues otra vez, a ver, ¿Cómo usamos las lenguas? ¿Cómo usted usó las lenguas para resolver? 15 A: En español. 16 F: ¿Y cambió al inglés [pause] cuándo? 17 A: Cuándo... éste... quise sumar y multiplicar. 18 F: ¿Ahí utilizó el inglés? 19 A: Sí. 20 F: ¿Y luego cuándo volvió al español? 21 A: Al obtener la respuesta, para ver cuál tenía más perímetro. 22 F: Y... pero, por ejemplo, para pensar pues que tenía la fórmula, ¿no?, cómo tenía que sumarlo, ¿cómo lo pensó eso? 23 A: En inglés. 24 F: ¿En inglés que tenía que sumar eso? 25 A: Sí. 26 F: ¿Me lo puede decir, un ejemplo, por ejemplo de cómo lo pensó? Alguna frase... 27 A: Pensé... Pues en la clase de matemáticas Mr Contreras nos dijo que tenemos que sumar todo lo de alrededor y así podemos sacar el perímetro. 28 F: ¿Y pensó en inglés, ahí? 29 A: Ajá. 30 F: You thought like adding the sides of the square, for example. You thought like that? 31 A: Yes. 32 F: ¿Y aquí en el círculo? 33 A: En el círculo que es la forma "A" igual pi por el diámetro. 34 F: Sí. ¿Lo pensó en inglés? 35 A: Sí. 36 F: ¿Cómo se dice en inglés? 37 A: "A" equals pi [/pi/, Spanish pronunciation] times [pause] times perimeter? 38 F: Pi [/pi/] times, five times diameter, right? 39 A: Diameter. 40 F: ¿Sí? ¿Pensó en inglés? Es un poco de todo, veo, ¿no? 41 A: Ajá.	14-15. Spanish as thinking language	Home language
37-42. Incorrect circle's perimeter formula in English: 'perimeter' instead of 'diameter' (correct in Spanish: 33)		16-19. English as adding and multiplying language	27. Math class (partially) in English
		22-41. English as thinking language	27. Math class in English
		31. English utterance (yes)	30. Previous interviewer's English utterance, talking about English thoughts
		37-42. Incorrect utterance of the circle's perimeter formula in English: 'perimeter' instead of 'diameter' (correct in Spanish: 33)	Not enough consolidated Mathematical register in English

	42 F: Porque el diámetro no lo sabía en inglés. Vale. Eh, ¿alguna cosita más en inglés? 43 A: No. 44 F: ¿Ahí está? 45 A: Mm [affirming] 46 F: Y luego lo escribió en español, ¿no? 47 A: Sí. [Continues in A1,2]		
MEMO	1. Code mixing on writing with a mathematical term (perimeter). The notion of perimeter is probably learned during the current school year (she has English materials and receives Spanish explanations too). 27. Influence of the teacher explanations when using English. 37. Ana does not say the formula of the circle's perimeter correctly in English, but is able to say it properly in Spanish and applies it correctly to get the right solution to the problem. As she does it in English during class, she has some thoughts in English. 'A' (37) probably refers to area (instead of perimeter). Such incoherences questions the validity of the operations performed 100% in English (16-19).		

Math	A3. Ana marked "English and Spanish".		Language	Tentative
1-5. 1st try. Wrong answer due to deviated question understanding (figure with seven tiles). Acceptable reasoning according to her question understanding.	1	 <p>[Ana makes some pauses during her writing. There might be a pause (the video is not completely clear) after writing the first sentence –which is in English. The crossing out of the answer takes place in A3,26.]</p>	1. 1 st try. English and Spanish as writing languages	English wording, Spanish dominant
1, 6-8. 2nd try. Cube as synonym of square.			2 F: ¿Cómo resolvió la [actividad] 3? 3 A: Conté cuántos cubos salió, cuántos cuadritos había en cada figura. 4 F: Ajá. ¿Y luego qué? 5 A: Pues la pregunta dice que cuál de las figuras tenía siete cubos. Y yo puse que la figura cuatro. 6 F: ¡Ohhhhh! No, la pregunta dice, eh, eh, [reading] how many tiles does figure seven, es la figura siete, se refiere a la figura siete. ¿Cuántos, tiles sabe qué son? 7 A: Cuadros o cubos. 8 F: Sí, cuadritos. 9 A: Sí.	1. 1 st try. English syntax deviance (the)
6-25. 2nd try. Wrong answer due to calculation of the number of tiles in Figure 6 (instead of 7), but right increasing on the number of	10 F: Cuántos cuadritos tiene la figura siete. [Pause] ¿Sí entiende lo que le pregunta aquí? 11 A: No. 12 F: O sea, la figura siete, aquí no tenemos la figura siete, ¿sí? 13 A: Sí. 14 F: Pero usted puede averiguar, esto es lo que le pregunta, ¿puede decir cuántos cuadritos habría en la figura siete? Y por qué. ¿Sí? ¿Lo quiere pensar otra vez? 15 A: Ajá. 16 F: Okay, pues la figura siete, a ver si me puede decir cuántos cuadritos habría, en la figura siete. [Pause] 17 A: ¿Once?	1. 1 st try. English grammar deviance (have)	English language in construction	
			1, 3.1 st try. "cubos" as synonym of "cuadritos" and "cuadros"	Similar geometrical shapes (but different dimensions)
			5. 1 st try. Deviated wording question	Partial understanding of the question,

<p>tiles per figure.</p> <p>6-27. 3rd try. Right answer with arithmetical reasoning (right arithmetical sequence associated to the figure sequence).</p>	18 F: ¿Por qué once? A ver, ¿cómo lo ha hecho?	<p>understanding (figure with 7 tiles).</p> <p>6-16. 2nd try. Question translated to Spanish by the interviewer.</p> <p>27. 2nd try. Spanish as unique writing language</p> <p>27. 2nd try. spelling variation (tendria).</p> <p>27. 2nd try. Spanish syntax deviance (dos más cuadrillos).</p> <p>28-30, 38-41. 2nd try. Spanish mainly as thinking language</p> <p>30-33. 2nd try. English as counting</p>	<p>finds mathematical solution (1) making sense to her question understanding</p> <p>Deviated wording question understanding</p> <p>Home language, 6-16: translation of the question to Spanish by the interviewer</p> <p>Unknown or quick writing</p> <p>Unknown or quick writing</p> <p>Home language, interviewer's language use</p> <p>34-37. Math class (partially) in</p>
	19 A: Porque en el primer nomás hay uno, y luego en el segundo le aumenta en otros dos, en el tercero le aumenta en otros dos y en el cuarto le aumenta en otros dos.		
	20 F: Sí.		
	21 A: Y luego hay que aumentarle otros dos a los demás.		
	22 F: Sí.		
	23 A: Y sería... Tal vez la figura cinco tuviera [pause] nueve.		
	24 F: Sí.		
	25 A: Oh, aquí me faltaron [dos]. Y luego la figura seis tuviera once y la figura siete tuviera trece.		
	26 F: Mm [validating]. Okay. ¿Lo puede escribir? No hace falta, bueno, puede, puede cruzarlo aquí o puede escribirlo atrás o algo... [crosses out the previous answer: A3,1]		
	27 		
28 F: Okay. Entonces, ahora le voy a preguntar sobre el uso de la lengua y como lo relacionó, o sea, si utilizó inglés, español, ¿no? Aquí me dijo [cross in the 'English and Spanish' column] que usó las dos lenguas, ambas... ¿Cuándo usó una lengua y cuándo usó otra? Vamos a... Bueno, esto lo acaba de hacer ahora, vamos a hacer con la segunda vez, tal como la ha pensado. ¿Empezó pensándola en qué idioma para resolverla?			
29 A: En español.			
30 F: En español. ¿Y cuándo cambió a inglés?			
31 A: Cuándo estaba contándolos.			
32 F: Sí ¿Los contó en inglés?			
33 A: Sí.			
34 F: ¿Por qué cree que los contó en inglés?			
35 A: Em... [pause]			
36 F: ¿Está más acostumbrada a contar en inglés?			
37 A: Sí.			
38 F: ¿Y luego, después de contarlos en inglés, cómo siguió?, a ver.			
39 A: Con en el español.			
40 F: ¿Hasta qué momento? ¿En qué momento volvió a cambiar a inglés? [Pause] ¿Se acuerda de otros momentos, si volvió a pensar alguna otra cosa en inglés?			
41 A: No. [Continues in A2,2]			

		language	English
MEMO	5. Wrong question understanding (as just a few words are correctly understood). The finding of a mathematical solution makes sense to her question understanding. This is why she does not understand the first interviewer's translation of the question (10-11). Quick writing in Spanish (27). Beforehand she uses English and Spanish (1). Writing in English is more demanding, doing it in Spanish is quicker.		

Math	A4. Ana marked "English and Spanish".	Language	Tentative
3-11. 1 st try. Deviated understanding of the situation presented in the wording (3: 'una tienda... ¿vendía piso? Algo así.').	1 	1. Spanish as writing language with code mixing (department)	Home language, incomplete wording understanding
1-45. 1 st try. Wrong answer with deviated horizontal mathematization due to deviated wording understanding. Acceptable mathematical procedure according to deviated interpretation.	2 [Comes from A1,21] F: ¿A ver, cómo resolvió eso? 3 A: Primero leí el problema que decía que, que una tienda... ¿vendía piso? Algo así. 4 F: ¿Una tienda?, perdón. [Pause] Es que no le entendí, sólo al hablar, no le digo que esté mal. Tenía pisos o algo así, me dijo. 5 A: Vendía pisos. 6 F: ¡Oh!, ¿vendía? 7 A: Ajá. 8 F: No, ¿por qué entendió vendía? ¿Vendía pisos? 9 A: [After cheking the wording] ¿O compraba el piso? 10 F: ¿Dónde pone eso? 11 A: Jaime [Spanish male name] is shopping. 12 F: It's Jamie. 13 A: [Reading] Jamie is shopping in a large department store with many floors. 14 F: Mm [okay]. ¿Cómo se dice eso? 15 A: Jamie compró un departamento...	1. Spanish spelling variations (conte, salio)	Unknown or quick writing
8-45. 2 nd try. Right wording understanding (through interviewer's interaction).	16 F: [Interrupting] Está comprando, ¿no?, en un... 17 A: En un departamento. 18 F: Large department, en un eh... centro comercial podríamos decir, ¿no? En un gran centro comercial con varios pisos. 19 A: ¿Y luego él entró al piso que estuvo en medio? 20 F: Sí. Ella, sí, ¿no?, porque es Jamie.	3-11. Incorrect wording translation (vendía pisos / compraba el piso – Jamie is shopping[... with many floors])	Partial sentence understanding, 'piso' is a Spanish polysemic word
44-49. 2 nd try. Negation to carefully reviewing	21 A: Okay. 22 F: Pero bueno, Jaime pues sí es en español así. 23 A: Y luego fue a dónde está la tienda de crédito. 24 F: Mm [validating].	11, 19-20, 25. Jaime-Jamie	Similar writing, no global wording comprehension
		13. Code switching	Reading English wording
		13-18. Incorrect translation (large department store-	Deviated word association

<p>the resolution after correct wording translation (with interviewer interaction) due to similar wording understanding (19-35, 45-46, 48-49).</p> <p>1-49. 2nd try. Wrong answer with deviated horizontal mathematization. No relative positioning of floors, wrong mathematization of the middle floor according to its symmetry particularity.</p>	25 A: Luego él miró que su... o ella miró que su crédito estaba bien.	departamento)	
	26 F: Mm [validating].	13-18. Deviated translation (is shopping in-compró)	13-18. Deviated tense translation, related with 'departamento'
	27 A: Luego ella fue un piso más arriba.		
	28 F: Mm [validating].	21. Code mixing (okay).	English wording
	29 A: Dónde está la, el departa, la tienda de joyas.	35-43. Wrong translation (Main entrance-correo)	Unknown meaning
	30 F: Mm [validating].	50-51. Unshared meaning of the question (50): asked about the first try (50) and answered about the 2 nd try (51).	44-49. Problem solved similarly as she had done earlier, before interviewer's translation
	31 A: Luego ella bajó un, un piso dónde estaba la tienda de niños.	50-57, 64-67. Mainly Spanish as thinking language	Spanish dominant
	32 F: Mm [validating].	58-63. English as thinking language with numbers, to extract key information from the wording	No translation needed (quickest)
	33 A: Y luego ya sube tres pisos más dónde estaba la tienda de juegos.		
	34 F: Ajá.		
35 A: De juguetes. Al final ella bajó diez pisos dónde estaba... main ... dónde estaba la...			
36 F: ¿Qué es lo que encuentra ahí?			
37 A: Main entrance [pointing to the wording].			
38 F: ¿Qué es eso?			
39 A: ¿El éste de correo?			
40 F: Main entrance. [pause] La entrada...			
41 A: De...			
42 F: Principal.			
43 A: Principal.			
44 F: Ajá. ¿Quiere volver a pensar el problema, si no lo había entendido como lo hemos entendido ahora?			
45 A: Dice la pregunta, que es que, éste, cuántos pisos tiene el centro comercial.			
46 F: Mm [validating].			
47 A: Y yo conté todos los pisos y le puse que dieciséis.			
48 F: ¿Quiere volverlo a pensar ahora, si lo entendió de otra manera, el enunciado?			
49 A: [Thinks for a while, looking to the paper] No, nomás lo pensé así, como es.			
50 F: ¿Sí? Entonces, ¿cómo empezó a pensarlo?			
51 A: En inglés porque lo leí y luego, éste, traducí todo esto al español y es cuando entendí que tenía, cuántos éstos, cuántos pisos tenía el centro comercial.			
52 F: Sí. ¿Y siguió pensando en inglés después de leerlo en inglés?			
53 A: No.			
54 F: ¿Algunas cosas las pensó en inglés...			
55 A: [Interrupting] Ajá,			
56 F: ...mientras estaba pensando?			
57 A: Sí. Y luego, y luego lo cambié [traduje] al español.			
58 F: ¿Y qué cosas pensó en inglés, por ejemplo?			
59 A: Los números, los éste, los pisos que bajaba y subía.			
60 F: Sí. ¿Eso lo pensó en inglés?			
61 A: Sí.			
62 F: ¿Qué más? [Pause]			
63 A: Nada más.			

	64 F: ¿Y en español pensó todo lo otro entonces? 65 A: Sí. 66 F: Vale. O sea, después de leerlo, lo traduzco al español y siguió pensando en español pero cuando había los números de los pisos los pensó en inglés? 67 A: Sí. 68 F: Okay. Vamos a hacer tres preguntas más en general y ya estamos, ¿sí? 69 A: Okay. [Continues in GLQ,1]		
MEMO	Despite all translating problems Ana may have on particular parts of the wording (13-18, 35-43), she gets a quite good approximate idea of the wording (19-35, 45-46). This is why she does not review carefully the mathematical procedure once the wording is correctly understood. 50-51. Unshared meaning of the question (50): the interviewer asks about the first try (50) and Ana answers referring to the 2 nd try (51).		

General Language Questions		Language	Tentative
1	F: ¿En general cuándo ha usado el inglés para resolver los problemas?	1-4, 7-14.	English wording
2	A: Leyendo los problemas, de que se trataba cada uno.	English as reading language	
3	F: Mm [continuing conversation].		
4	A: Ahí fue donde tuve que usar el inglés.	5-6. English and Spanish as thinking languages	School and home languages
5	F: Mm [continuing conversation]. ¿Qué más?		
6	A: En algunos problemas lo pensé en inglés y en otros lo pensé en español.		
7	F: ¿Y cuándo usó el inglés?	7-10. English with number names and counting	17-32. English schooling, better-management of numbers in English than in Spanish
8	A: Eh...		
9	F: ¿Para qué cosas?		
10	A: Para los números,...		
11	F: Mm [continuing conversation].		
12	A: ...para leer las preguntas...		
13	F: Mm [continuing conversation].		
14	A: ...y ya.	28, 30. Code mixing	Toponyms
15	F: ¿Y por qué cree que lo hizo así? ¿Por qué usó el inglés para... ? Bueno, para leerlas porque está en inglés, claro, ¿no?		
16	A: Sí.	33-46. Spanish as writing language	39-42. Home language and school in Mexico
17	F: ¿Luego por qué cree que usó el inglés para los números?		
18	A: Porqué me los sé más que en español.	47-62. Difficult	Similar wording
19	F: ¿Se los sabe más en inglés que en español?		
20	A: Sí.		
21	F: ¿Fue a la escuela a México?		
22	A: Sí.		
23	F: Y se los sabe más en inglés que en español.		
24	A: Sí.		
25	F: ¿Cuánto tiempo me ha dicho que llevaba acá?		
26	A: Aquí llevo dos años.		

<p>27 F: ¿En esta escuela? 28 A: Ajá. Aquí llevo uno y en JFK [/dzei ef kei/ 29 F: JFA [/dzei ef ei/], ¿dónde está esto? 30 A: Por la Sandboard [name of the street]. 31 F: ¡Ah! Es otra.. [escuela] 32 A: John Fitzgerald Kennedy. 33 F: Okay. Okay, okay. ¿Y en general cuándo ha usado el español? 34 A: ¿En esto [points to the papers on her desk]? 35 F: Sí. 36 A: Cuando contesté las respuestas. 37 F: ¿Y en qué momento, en qué momento, perdón, de... mientras estaba pensando el proceso de resolución usó el español? 38 A: Cuando estaba pensando cuál iba a ser la respuesta de la pregunta lo pensé en español. 39 F: ¿Por qué? ¿Por qué cree que lo pensó en español? [Pause] ¿Se le ocurre alguna explicación? 40 A: No. 41 F: ¿Qué sabe más, usted diría que sabe más inglés o sabe más español? 42 A: Español. 43 F: ¿Le es más fácil hablar español o inglés? 44 A: Em... español. 45 F: ¿Español más fácil? 46 A: Sí. 47 F: ¿Por último, hay alguna palabra o alguna frase que haya encontrado difícil en inglés? 48 A: No. 49 F: ¿No? Bueno, hemos encontrado las.. 50 A: Ésta. 51 F: Main entrance. ¿Y esto, qué pasó, entonces, aquí al entenderlo? La frase es... hay algunas cosas... bueno, Jaime o Jamie, ¿no? 52 A: Ajá. 53 F: ¿Confundió aquí con Jaime? 54 A: ¡Ah, sí! 55 F: ¿Y que entendió aquí al principio? 56 A: Que él estaba, él estaba, éste, iba a comprar, o fue a comprar a un centro comercial. 57 F: ¿Y que fue a comprar qué, me dijo, pisos? 58 A: Muchas cosas, porque primero fue a, a.. 59 F: [Laughing, both] Okay, 60 A: Fue al es.. 61 F: [Interrupting] ¿Pero sí lo entendió más o menos, cree? 62 A: Sí, sí.</p>	English words forgotten	comprehension in A4
MEMO	GLQ 47-62. She does not remember which words or comprehension problems may have had during the resolution process (mainly in A4). It looks like she hesitates sometimes when giving an answer and sometimes her answers are contradictory. The question about her initial wording interpretation (55) is understood as the situation presented on the wording (56-60) in the beginning by Ana, and not	

as her initial wording interpretation, as it is the intended meaning of the question. Interviewer interrupts Ana's description (61) and he does not understand Ana's interpretation of the question.

Ana has a good Spanish BICS: she speaks Spanish very well. Her Spanish CALP is fair: she sometimes switches to English (on A3 imitates the wording word "tiles") and she sometimes does not use mathematical vocabulary precisely (A3,1: "cubos" instead of "cuadros"). Her English BICS is fair, as she has difficulties understanding and translating A3 and A4 wordings. Her English CALP is fair: Ana uses the English mathematical register sometimes.

On A1 Ana initially says she uses English for reading purposes only. Later she considers English as a thinking language, but she is not able to specify its particular use. She directly compares both percentages.

On A2 Ana uses Spanish as a writing language on her two pieces of answer, with a code mixing (perimeter) on the 1st try. She says operations (addition and multiplication) are performed in English. She does not reproduce the circle's perimeter formula in a completely correct way through English (yet she does it in Spanish previously). So both languages are used when thinking about the problem. Ana calculates both perimeters correctly: the square through the addition of all its sides and the circle through the application of its perimeter's formula. On her written answer she erroneously states that circle has a greater perimeter, but once the interviewer asks for the largest perimeter she quickly corrects it.

On A3 Ana uses English and Spanish on her 1st try written answer, and Spanish on her second written answer (3rd try). She understands the wording question in a deviated way (figure with 7 tiles). The resulting mathematical reasoning is acceptable according to this deviated understanding of the wording. Ana takes 'cuadro' and 'cubo' (square and cube) as synonyms . Once the statement is properly translated by the interviewer, Spanish is the main language for thinking, with English used for counting. She finds the right arithmetical sequence associated to the figure pattern, but she states 11 as a final answer (2nd try). When explaining the mathematical procedure she realizes she has not followed the sequence properly and states 13 tiles as her final answer (3rd try).

A4's answer is written in Spanish with a code mixing instance (department). Ana translates many parts of the wording in a deviated way (so the horizontal mathematization is wrong). Despite all these difficulties, she uses English for counting and Spanish as main thinking language to get a wrong answer (16). She does not change her answer after the wording is correctly interpreted with the interviewer's help,

She solves only A2 correctly on a 1st try. After a right wording interpretation (with the interviewer's interaction) she quickly and correctly solves A3. On A1 she uses percentages as absolute values and on A4 she does not get a more appropriate approximation to the problem. According to her mathematical solving process, maybe a better understanding of the wordings (either by asking someone or by improving her English level) would result in better chances to solve the problem.

Activities' (Key ideas) summary

Object 28: Ana-First reduction (End)

- Notion of percentages as an absolute value instead of relative; not directly related to language use (English restricted almost exclusively to reading language).
 - Code mixing in writing with a mathematical term (perimeter). As the Spanish version of perimeter (perímetro) is used orally and also on a second writing and his mathematical procedure is right, Ana has a good management (linguistical and mathematical) of such a term.
 - English as an adding and multiplying language combined with Spanish as a thinking language in relation with perimeters and figures.
 - Ana does not says the formula of the circle's perimeter correctly in English, but she is able to say it properly in Spanish (she applies it correctly to get the right solution to the problem). The presence of English materials in class could lead Ana to think in English – at least partially– about the circle's perimeter formula (and maybe the square's perimeter formula too) and so to have English as a thinking language.
 - English as a writing language to state the solution (1st part of written answer), switched to Spanish to justify the answer for the sequence of figures. It seems easier to elaborate and organize thinking through Spanish, as Ana is more fluent in this language and after writing the first sentence in English it takes some time for her (also while she is writing the second sentence) to write the second sentence in Spanish. Furthermore she only uses Spanish in her 2nd written solution.
 - Ana gets an operational meaning of “tiles” and uses it properly in the written answer. With the use of 'cubos' the mathematical precision is lost.
 - Partial understanding of a question (some words presented difficulties) leads Ana to a deviated understanding of the question in the wording: which is the figure with seven tiles. The fact that she finds a mathematical solution feeds the (deviated) comprehension and vice versa.
 - English is used as a counting language combined with a main use of Spanish as a thinking language once Ana understands the intended question in relation with the sequence of figures.
 - Despite a partial understanding of the wording (and a wrong horizontal mathematization) Ana is able to focus on the key aspects to make an acceptable vertical mathematization (given the horizontal mathematization done).
 - Ana decides not to change the solution to A4 after the right translation of the wording she obtains with the interviewer's collaboration. Maintenance of vertical mathematization when horizontal mathematization should change (due to different understanding of the wording).
-

Historical profile	Bilingual profile (Spanish dominant)	Activity	
<ul style="list-style-type: none"> • 12 years old • English-Spanish class • 2 years in California • Likes (+/-) California • Spanish readings • Spanish at home, English with sister • Spanish with friends • Mainly Spanish (Math and 2 more) at school • Homework help: mother 	Linguistic deviations (written Spanish)	x x 2 x	
	English language linked to reading	x	
	Unclear distinction in personal language use	x	
	Code mixing (A1: John Sports, A4: okay)	x	x
	Code mixing on writing (A2.1: perimeter, A4: department)		1 x
	Spanish as unique writing language	x 2 2	
	Spanish as thinking language	x x	x
	Deviated translation (vendía pisos / compraba el piso – Jamie is shopping[...] with many floors], large department store-departament, is shopping in-compró, Main entrance- correo)	x	x
	English as adding and multiplying language	x	
	English as thinking language	x	
	English utterance ('yes')	x	
	Incorrect circle's perimeter formula in English (= pi · 'perimeter'), correct in Spanish	x	
	English and Spanish as writing languages		1
	Linguistic deviations (written English)		1
	Cubos = cuadritos, cuadros		1
	Deviated wording question understanding (A3: figure with 7 tiles)		1 x
	Mainly Spanish as thinking language		2 x
	English as counting language		2
	Letters rotation: Jamie - Jaime		x
	Code switching A4(x0+0+1)		x
	English as thinking language with numbers		x
	English as reading language		GLQ
	English and Spanish as thinking languages		GLQ
	English with number names and counting		GLQ
	Code mix (toponyms)		GLQ
	Spanish as writing language		GLQ
Difficult English words forgotten		GLQ	
Procedural profile	Conceptual profile		
<p>1 X Direct comparison of percentages with no mention of initial prices</p> <p>2.1 ✓ Visual and arithmetical reasoning: Calculation of perimeters (square: addition of sides, circle: formula)</p> <p>X Numerical comparison of lengths</p> <p>2.2 ✓ Numerical comparison of lengths</p> <p>3.1 X Search for figure with 7 tiles instead of looking at the tiles of Figure 7</p> <p>3.2 X Application of a pattern given by adding 2 from one figure to the next, due to confusion on the term of the sequence (6th term instead of 7th)</p> <p>3.3 ✓ Application of a pattern given by adding 2 from one figure to the next</p> <p>4.1 No relative ordering of floors Reasoning without distinguishing the singularity of the middle floor in terms of its symmetry function</p> <p>4.2 X No carefully review of the mathematical procedure after the wording is properly translated (in collaboration with the interviewer)</p>	<p>1 X Notion of percentages as an absolute value instead of relative</p> <p>2.1 ✓ Notion of perimeter</p> <p>2.2 ✓ Notion of perimeter</p> <p>3.1 X Confusion between 2D and 3D shapes: 'cubo', 'cuadro'</p> <p>3.2 ✓ Notion of arithmetical sequence associated to a sequence of figures</p> <p>3.3 ✓ Notion of arithmetical sequence associated to a sequence of figures</p> <p>4.1 X Notion of number line with confused order positions</p> <p>4.2 Notion of number line with confused order positions</p>		

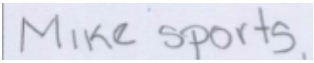
Object 30: Juan-First reduction
(Beginning)

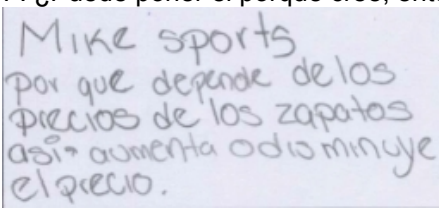
Juan has an interest on the task and he almost always does his homework.

Math teacher description

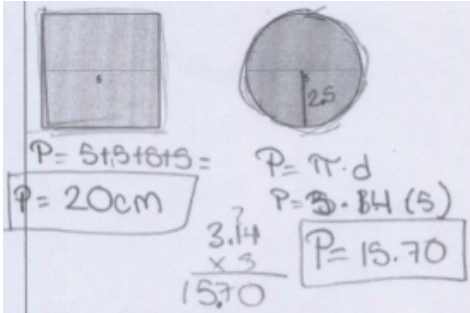
Class	Interview language	Interview date	Origins	Age	USA arrival	Living in California	Spanish readings	English readings	Home language	Friends language	Main language at school	Homework help
MS En-Sp	Spanish	May 2010	Guanajuato, Mexico	14	June 2009 [2 years]	Likes it.	“La Ganga” newspaper, soap operas...	Some times, but hardly ever	Spanish	Spanish	Spanish	Brother

Historical bilingual profile

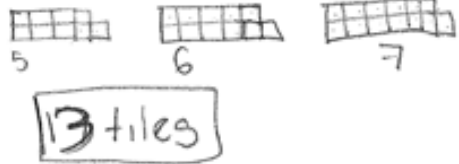
Math	A1. Juan marked “English and Spanish”.		Language	Tentative
1. [“Mike sports”] No written reasoning. 6-25. “Unbeatable prices” influence on answer: 25% store with the	1	 [Juan reviews the problem after reviewing A4 but makes no changes]	1. English as unique writing language	Reproduction of statement pictures text
	2	F: A ver, ¿cómo resolvió esta actividad? [Pause] ¿Cuál fue lo primero que pensó? ¿Qué fue?	13-16. Deviated translation (tabla de precios – unbeatable prices)	Word sliced to relate parts of it with previous knowledge (un/bea/table - table)
	3	J: En el descuento.		
	4	F: ¿Y qué hizo luego?		
	5	J: Me fijé cuál ofrecía más. [Pause] Y... [pause] Mmmm... [pause]		
	6	F: Se fijó en cuál era el que ofrecía más ¿y qué?.		

<p>cheapest initial price.</p> <p>19-21. Right use of percentages as relative value. Abstract examples not detailed enough.</p> <p>25. Right use of percentages as relative value with concrete and detailed examples.</p> <p>21, 27. Written argumentation does not contain all information expressed orally (19-25).</p> <p>1-27. 1st try. [25%] Right answer with assumption of different initial (and final) prices on both stores.</p>	7	J: En cuál... y en esto que decía aquí ["Unbeatable prices"]. Y ofrecía más éste [25%].	<p>27,33. Spanish as unique writing language (1, 27: English too!)</p> <p>27. Deviated spelling (por que)</p> <p>28-33. English as thinking language</p> <p>33-37. English as thinking language to interpret percentages in statement pictures</p>	<p>Interview language, home language</p> <p>Homophony</p> <p>English wording, schooling in English</p> <p>Presence of numbers, English wording</p>
	8	F: ¿Por qué?		
	9	J: Por unbeatable [/un-bi-teibol/] price...		
	10	F: Sí. ¿Qué significa eso?		
	11	J: No...		
	12	F: ¿No decía que se fijó en eso? ¿Con qué, con qué pensó luego? ¿No? ¿Porqué dice "why"? Puede quizás justificar la respuesta.		
	13	J: ¿Porque depende de la tabla de precios [pointing to "Unbeatable prices"]?		
	14	F: ¿Pero qué significa unbeatable prices? ¿Sabe?		
	15	J: No.		
	16	F: Significa precios inmejorables. Que nadie los puede superar, ¿no? Son unos muy buenos precios. ¿Sí entiende ahora?		
	17	J: Sí.		
	18	F: ¿Pero por qué puso Mike Sports?		
	19	J: Porque si tiene un precio muy bajo, de todos modos le descuentan un veinticinco por ciento y costaría menos que éste [40%], porque si ya tiene el precio alto costaría todavía más que éste [25%].		
	20	F: ¿Y por qué tendría éste [25%] el precio alto? ¿Digo el precio más bajo éste [25%]?		
	21	J: Depende del precio de los zapatos.		
22	F: ¿Pero usted cree que éste [25%] tiene un precio más bajo?			
23	J: ¡Sí!			
24	F: ¿Por qué?			
25	J: Porque si le descuentan... y son... Supone que son sesenta, le descuentan veinticinco por ciento. Y si éste [40%] cuesta cien, le descuentan cuarenta. Y tiene precio más bajo éste [pointing to "25%"] que éste [pointing to the shoe picture on the 25% store !!].			
26	F: ¿Puede poner el porqué cree, entonces, que éste es más barato?			
27	 <p>[Entire answer]</p>			
28	F: ¿En qué lengua... con qué lengua empezó a resolver el problema?			
29	J: En inglés y en español.			
30	F: ¿Empezó con las dos?			
31	J: Sí.			
32	F: ¿Para qué utilizó una y para qué la otra?			
33	J: El inglés lo utilicé para leer y entender esto y el español para escribir.			
34	F: Pero desde que leyó la pregunta, ¿no?, hasta que la resolvió, usted me ha dicho que se fijó con los porcentajes, que se fijó también con esto de aquí [Unbeatable prices].			
35	J: Mm [validating].			

	<p>36 F: ¿Mientras estaba pensando todo esto, qué lengua usó? ¿O para qué usó una lengua y para qué usó otra?, si es que usó las dos.</p> <p>37 J: El inglés lo usé para pensar cuánto descuento tenía y el español para, [pause] para anotar la respuesta, porque esto estaba en inglés y me fijé en, en el inglés.</p> <p>38 F: Alguna cosa, alguna otra cosa para la que haya utilizado el inglés?</p> <p>39 J: No. [Continues in A2,2]</p>		
MEMO	<p>1. Answer is written in English. On interviewer's demand, the justification is in Spanish (27). 19-27. Better oral than written argumentation (which is not admissible as a right answer) even with the 2nd try written answer, which is given after oral explanation. 28-33. Juan says he starts to solve the problem using both languages!</p>		

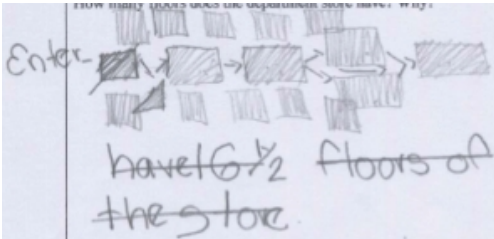
Math	A2. Juan marked "English and Spanish".	Language	Tentative	
1. Addition of units (cm) when calculating the square's perimeter.	 <p>[Juan reviews A2 after finishing all activities. He retraces the perimeter of both figures with the pencil, as it can be appreciated.]</p>	1. No use of natural language	Deviated wording question understanding	
3. Use of 'área' instead of 'perímetro' (no conceptual confusion).		1, 3. Deviated wording question understanding (calculation of perimeters without comparing them)	Activity solved, partial understanding	
1-3. 1st try . No complete written answer due to deviated wording question understanding (3). Arithmetical and visual reasoning (visual identification of square's perimeter and application of circle's perimeter)		<p>2 [Comes from A1,39] F: ¿Cómo resolvió esto? ¡A ver!</p> <p>3 J: Porque dicen que encuentren el perímetro, y el perímetro de un cuadrado es cinco más cinco, [...] lo de... lo de alrededor. Y son cuatro lados, se multiplica por cinco. Suma cinco más cinco, más cinco, más cinco y lo que suma son veinte. Y del círculo es, el área es pi por diámetro y es perímetro igual a tres punto catorce por quince, por cinco, y lo que salió fue quince punto setenta.</p> <p>4 F: ¿Entonces cuál es el que tiene el perímetro más grande?</p> <p>5 J: El cuadrado.</p> <p>6 F: Ajá. ¿Y... por qué me puso aquí este veinticinco?</p> <p>7 J: No, esto... Porque no me acordaba de la fórmula y pensé que era pi por radio al cuadrado y el radio es dos punto cinco.</p> <p>8 F: Sí, bien. ¿Cómo empezó? ¿Con qué lengua empezó a resolver el problema en este caso?</p> <p>9 J: Con el inglés.</p> <p>10 F: ¿Con inglés? ¿Qué hizo con inglés?</p> <p>11 J: Leyendo la pregunta.</p> <p>12 F: ¿Y cómo continua?</p>	3. Use of 'área' instead of 'perímetro' (terminological confusion only, not conceptual)	6-7. Hesitating between area/perimeter formulas during resolution
			8-28. Use of English language linked to	Home language

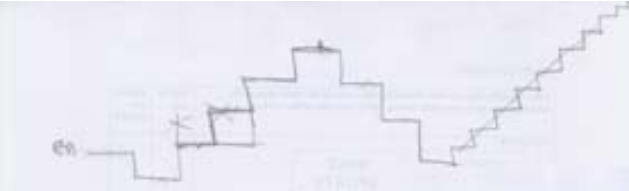
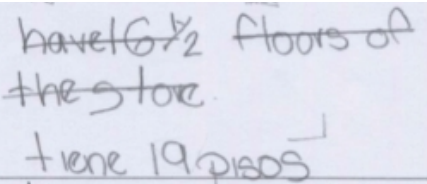
<p>formula).</p> <p>4-5. Comparison of perimeters induced by interviewer.</p> <p>1-5. 2nd try. Right answer with arithmetical and visual reasoning (visual identification of square's perimeter and application of circle's perimeter formula).</p>	<p>13 J: Con el español, haciendo los, [pause] las operaciones.</p> <p>14 F: Y antes de hacer las operaciones, porque tuvo que pensar qué fórmula utilizar, por ejemplo, ¿no?</p> <p>15 J: Porque es la fórmula que... aquí pide el perímetro, es la fórmula de cada figura.</p> <p>16 F: ¿Cómo lo pensó esto que tenía que utilizar la fórmula de cada figura? ¿En qué lengua?</p> <p>17 J: En español.</p> <p>18 F: ¿En español? ¿Alguna cosa pensó en inglés? Por ejemplo este pi... dijo pi o dijo pi [pai/, English version]</p> <p>19 J: Pi.</p> <p>20 F: ¿Lo pensó en español?</p> <p>21 J: Sí.</p> <p>22 F: Okay. ¿Alguna otra cosa que hizo en inglés?</p> <p>23 J: No.</p> <p>24 F: Todo lo pensó en español y el inglés lo utilizó...</p> <p>25 J: [Interrupting:] En la pregunta.</p> <p>26 F: ...sólo dice para...</p> <p>27 J: Leer la pregunta.</p> <p>28 F: Ajá. Okay. [Continues in A3,2]</p>	<p>reading</p>	
<p>MEMO</p>	<p>1. Units (added) in square's perimeter.</p> <p>1. No words used on written answer.</p> <p>1-3. Deviated understanding of the wording, consolidated by a "solution" to the problem found.</p> <p>3. Use of 'área' instead of 'perímetro' (terminological confusion only, not conceptual)</p> <p>All other answers are written initially in English. Use of units, no use of natural language: acceptable skills as an academical problem solver.</p>		

Math	A3. Juan marked "English and Spanish".	Language	Tentative
<p>1-3. 1st try. Right answer with right arithmetical and visual reasoning through the drawings of Figures 5, 6 and 7 according to the given sequence.</p>	<p>How many <u>tiles</u> does figure 7 have? Why?</p>  <p>1 [Juan reviews this activity after reviewing A2: he changes the answer from 11 to 13 and counts the tiles again.]</p> <p>2 [Comes from A2,28] F: ¿Me puede decir aquí también qué es lo que hizo?</p> <p>3 J: Leí la pregunta y que ¿cuántos tendría la figura siete? En la figura uno uno, y para la dos aumentó dos, y para la tres dos y le aumenté dos en cada figura y me salían los trece.</p> <p>4 F: Sí. Otra vez, el uso de las lenguas, ¿no? ¿Cómo lo hizo aquí?</p> <p>5 J: El inglés lo usé para la pregunta y leer esto y el español para contestar.</p>	<p>1, 4-13, 16-23. English as writing language (11: the number "13" is interpreted in Spanish)</p> <p>11. Code switching</p> <p>11. Code mixing (tiles)</p>	<p>5, 11, 13. English wording imitation</p> <p>Reading English wording</p> <p>English wording re-</p>

	<p>6 F: Pero lo contestó en inglés aquí, ¿no? 7 J: Mm [validating]. ¡Sí! 8 F: ¿No lo contestó en español? 9 J: No. 10 F: ¿Cómo es que lo contestó en inglés? 11 J: Porque pregunta [reading] “How many tiles does figure seven have?” [No accurate pronunciation] Y tienes trece tiles. 12 F: ¿Y cómo es que utilizó aquí el inglés para la respuesta? 13 J: Porque salen trece y la palabra de lo que busca es esto [underlines the word “tiles”]. Es lo que buscan y es lo que sale. 14 F: Entonces lo leyó en inglés, ¿no? 15 J: Mm [validating]. 16 F: ¿Y luego cuándo cambió en español? 17 J: Al entender la pregunta. Y para hacer las figuras. 18 F: ¿Luego lo hizo en español ya? 19 J: No, en inglés. Lo terminé en inglés. 20 F: Lo terminó en inglés para escribirlo. 21 J: Ajá, para escribirlo. 22 F: ¿Para qué más utilizó el inglés? 23 J: Para leer la pregunta y el español para entenderla. 24 F: Pero además de escribir esto [answer] en inglés y leer esto [wording] en inglés, utilizó por ejemplo el español para contar las tiles ... 25 J: Sí, para contar. 26 F: ¿Lo contó en qué idioma? 27 J: En español. 28 F: ¿Y en inglés hizo algo? 29 J: No 30 F: ¿Eso es todo? 31 J: Sí. [Continues in A4,2]</p>		production, no adequate translation found
		16-17, 22-31. Spanish as thinking language	Home language
MEMO	1. Answer written in English –as in all of the other answers– but the presence of just one English word is not regarded as important. Borrowing the word “tiles” from the wording, Juan writes a (short) statement in English, a valid answer, even if he thinks of the number (13) in Spanish. 5-7. 'Tiles' is mixed naturally during writing and thinking. No translation is ever used during the dialogue.		

Math	A4. Juan marked “English only”.	Language	Tentative
1. 1 st try. Wrong sketch, but acceptable according to his deviated wording		1. 1 st try. English as unique writing language	Use of English in school Math

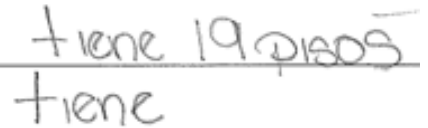
<p>interpretation (3-5).</p> <p>1-13. 1st try. [16'5 floors] Wrong solution due to deviated wording interpretation (tiles instead of floors), but acceptable reasoning according to the interpretation made.</p> <p>22. 2nd try. Sketch with movements inversion (going up instead of going down and vice-versa), addition of 3 extra floors and longitudinal representation of floors (and Jamie's movements) with no complete wording understanding (sentence "which is on the first floor" not properly integrated on the reasoning and on the sketch).</p> <p>6-22, 34, 40-61. 2nd try. [19 floors] Wrong answer due to addition of 3 extra floors, and no relative positioning</p>	 <p>[1st try. Juan reviews the problem after reviewing A3: adds the first and third row of floors. The sentence is crossed out later, when writing 2nd try answer, on A4,22]</p>	1	1. 1 st try. Grammatical and syntax deviances in written English	English language in construction
		2	3, 5, 9, 11, 13. 1 st try. Deviated translation (floor-piso [teja, tile]).	Mexican Spanish, wrong horizontal mathematization
		3	5. 1 st try. Deviated translation (él)	Jamie -Jaime
		4	14-20. 1 st try. Unknown meaning (goes up, goes down)	English language in construction
		5	22, 127-130. 2 nd try. Spanish as unique writing language	Interviewer interaction, Spanish dominant
		6	25-33, 141-142. 1 st try. English for everything	School context
		7	25-26, 33, 131-140. 2 nd -6 th tries. Spanish mainly as thinking	Interviewer interaction, Spanish dominant
		8		
		9		
		10		
11				
12				
13				
14				
15				
16				
17				
18				
19				

of floors.	20 F: Baja un piso. ¿Sí? O sea que está subiendo, bajando, moviéndose por el edificio. ¿Quiere pensar el problema otra vez, quizás?	language	
52. 3 rd try. Sketch with 16 floors. Elimination of the 3 extra floors and positioning partially arranged.	21 J: Sí. [Juan thinks again of the solving process]	40-45. 2 nd try. Incorrect translation (goes up-baja)	English language in construction
40-61. 3 rd try. [16 floors] Wrong answer due to no relative floors positioning. Floors in a horizontal position instead of vertical.	  22	46-51. 2 nd try. Incorrect translation (goes down-sub) self-corrected after interviewer question	English language in construction
62-67. 4 th try. Right situation of middle floor on the picture.	23 F: ¿Ya está? 24 J: Ya. 25 F: Okay. Aquí me dijo... mmm... Perdona, antes de que comentemos cómo lo ha hecho esta vez, ¿sí? Antes puso la cruz solo inglés. ¿Sí? ¿Utilizó solamente el inglés? 26 J: En ésta [1 st try], pero en ésta no [2 nd try].	70-72. 1 st - 3 rd tries. No correct question understanding	English language in construction
68-72. 4 th try. Interviewer introduces that the total number of floors of the building is different than the total number of floors that Jamie goes through.	27 F: ¿Y antes que me puso sólo inglés, no utilizó el español para nada? 28 J: No 29 F: ¿Usted estaba pensando todo el rato en inglés? Aún para dibujar, pues esto [1 st try: A4,1], pensó: with the floor... y todo esto. 30 J: Sí. 31 F: ¿Sí? 32 J: Sí 33 F: Todo en inglés. Y ahora pensó en español, me dijo. A ver, ¿me explica un poco como lo hizo? 34 J: [Looking to the wording:] Que entra y baja [goes up!] uno, luego sube [goes down!] [checks the wording] un pi... uno y aquí... y después tres para llegar al otro departamento [pointing to the 6 th horizontal line, the top, see A4,22] y...	135-140. 2 nd -6 th tries. English to keep track of the floors Jaime goes up and down	References to English wording
70-71. 4 th try. Wording question not understood.	35 F: ¿Department qué significa por eso? 36 J: Departamento. 37 F: ¿Cómo entiende departamento? Es que en español, en España, lo diríamos cómo sección de... no es un departamento, cómo un sitio dónde uno viva, no? 38 J: No, es una... como un sitio especial de algo en una tienda.		
73-79. 4 th try. [19·2 = 38 floors].	39 F: Mm [validating]. Okay. 40 J: Y luego baja... baja... [follows the wording with the pencil] baja tres [points to the 3 floors going down: A4,22].		

<p>19 (from 2nd try answer) is doubled as if where Jamie goes up the same amount of floors can be placed going down and vice-versa. Wrong horizontal mathematization (possible influence of "large").</p> <p>62-79. 4th try. [19·2=38 floors] Wrong answer due to no relative positioning of floors and no right mathematization of the middle floor according to its symmetry particularity.</p> <p>82-85. 5th try. Right situation of the first floor on the sketch.</p> <p>86-89. 5th try. Highest floor reached considered as the top of the building (relying on the sketch).</p> <p>90-93. 5th try. Right situation of the middle floor on</p>	<p>41 F: ¿Baja tres dice?</p> <p>42 J: Sí. Dice [reading] goes up three floors of [to!] the toy departament. Uno, dos... [points to 9th and 10th horizontal lines (within the 3 floors <i>going down</i>), see A4,22].</p> <p>43 F: Pero esto es cuando sube, ¿no? [points to the 3 floors <i>going up</i>: A4,22].</p> <p>44 J: ¡Oh sí!</p> <p>45 F: Goes up es sube.</p> <p>46 J: Aquí llega [pointing to the 6th horizontal line, the <i>top floor</i>: A4,22] [pause] Y final... Sube otros diez [points to the 10 floors going up: A4,22].</p> <p>47 F: ¿Sube?</p> <p>48 J: Sí</p> <p>49 F: ¿Dónde lo dice?</p> <p>50 J: [Checks the wording] No, baja, baja diez para abajo. [Crosses out the 10 floors going up: A4,22]</p> <p>51 F: Ajá, baja diez. Cuando está aquí [6th horizontal line, the <i>top floor</i>: A4,22], ¿no?, porque está aquí, luego baja diez.</p> <div data-bbox="488 612 981 970" style="text-align: center;"> </div> <p>52</p> <p>53 J: Acá baja y llega aquí.</p> <p>54 F: Mm [validating]. ¿Y esto qué es?</p> <p>55 J: Otra entrada de la tienda.</p> <p>56 F: Okay. ¿Y en total cuántos tiene por eso?</p> <p>57 J: Diecinueve.</p> <p>58 F: ¿Había contado antes diecinueve?</p> <p>59 J: Ajá.</p> <p>60 F: ¿Cómo los había sacado estos diecinueve?</p> <p>61 J: Porque es uno, dos, tres, cuatro, cinco, seis... Seis [6th horizontal line, <i>top floor</i>: A4,52] y los diez que baja. Son dieciséis, perdón, con los diez que baja.</p> <p>62 F: ¿Pero por dónde entra?</p> <p>63 J: Por aquí [hesitating, pointing to the 1st horizontal line (entrance floor) on A4,52]</p> <p>64 F: Entra por aquí. ¿Dónde está situado este piso?</p> <p>65 J: ¿En el intermedio?</p>		
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[3rd try. Corrects A4,22 picture, adding 7 floors to the last 3 that were descending. The rightmost vertical segment (at the bottom) was added later: A4,83]

<p>the sketch.</p> <p>94-97. 5th try. Wrong mathematization of the middle floor according to its symmetry particularity (just situated visually on the sketch).</p>	<p>66 F: Ajá. 67 J: Inter... es el intermedio. Y es uno y baja uno. Uno, dos, tres, cuatro, cinco, seis [counting the horizontal lines, until the 6th one, the <i>top</i> floor: A4,52] y diez que baja son dieciséis. 68 F: Dieciséis son los pisos que recorre,... 69 J: [Interrupting:] Ajá. 70 F: ...por lo tanto. ¿Pero cuántos pisos en total tiene el edificio? Es lo que nos pregunta ahí. [Pause] ¿Sí entiende? 71 J: No. 72 F: La pregunta es: el centro comercial, ¿sí?, [reading] How many floors does the department store have? Cuántos pisos, cuántos pisos tiene el centro comercial, no cuántos pisos él recorre, sino cuántos pisos tiene en total el centro comercial. Aunque Jamie no haya pasado por ellos, ¿pero en total cuántos hay en el centro comercial? [Pause]</p>		
<p>82-102. 5th try. [10 floors]. Wrong answer with no explicit reasoning. Maybe the 10 floors come from the 10 floors Jamie goes down.</p>	<p>73 J: ¿Treinta y ocho?. 74 F: ¿Por qué treinta y ocho? 75 J: Porque aquí baja [points to some of the 10 <i>descending</i> floors: A4,52] y puede que también haya para arriba. Y aquí donde subió puede ser que haya abajo. Y multipliqué por dos y salen treinta y ocho. 76 F: ¿Qué es lo que multiplicó por dos? 77 J: Diecinueve. Los diecinueve que recorre, por dos. 78 F: ¿Y va a haber el doble de los que recorre? 79 J: Puede ser. 80 F: ¿Pero lo podemos saber seguro cuantos hay? 81 [Juan sighs/laughs]</p>		
<p>103-108. 6th try. [19·2 = 38 floors]. Inconsistency (19 floors –from 2nd, 4th tries– but refers to drawing with 16 floors).</p>	<p>82 F: ¿Porque dónde esta la calle? ¿No?, para saberlo seguro tendríamos que mirar donde está el piso más alto y dónde está el piso más bajo. 83 J: Aquí está la calle [draws the rightmost vertical segment on A4,52] 84 F: Aquí está el piso más bajo, entonces. 85 J: Sí. 86 F: ¿Y el más alto dónde queda? 87 J: Es éste. 88 F: Éste es el más alto que ha ido ella. ¿Es el más alto del edificio? 89 J: Sí. [pause]</p>		
<p>82-108. 6th try. Wrong answer due to no relative positioning of floors, no right mathematization of the middle floor according to its symmetry</p>	<p>90 F: ¿Sí? ¿Por dónde entra? 91 J: Entró por aquí. 92 F: ¿Y esto qué era? 93 J: El intermedio. 94 F: El piso intermedio. Si entra por aquí y luego sube hasta aquí, ¿es éste el intermedio? ¿Si éste es el más bajo y éste es el más bajo, éste es el intermedio? 95 J: Sí. 96 F: ¿Es éste el intermedio? Éste es el más bajo, éste es el más alto y éste es justo la mitad del edificio. ¿Es así?</p>		

<p>particularity.</p> <p>109-114. Unknown number of floors from the middle to the top of the building.</p> <p>109-121. No relationship between top, middle and bottom floors due to no relative positioning of floors (relying on the deviated sketch).</p> <p>115-122. Wrong number of floors from the middle to the first floor (after the interviewer's demand) due to no relative positioning of floors.</p> <p>118. 7th try. [11 floors]. Wrong answer. Immediate awareness that the answer is wrong. No explicit reasoning. May be related to the 5th try.</p>	<p>97 J: Sí.</p> <p>98 F: Okay. ¿Pues su respuesta [va] a ser, es entonces?</p> <p>99 J: ¿Diez?</p> <p>100 F: ¿Ahora me dice diez?</p> <p>101 [Juan laughs]</p> <p>102 F: ¡Lo que usted crea! [Pause]</p> <p>103  [2nd "tiene" added]</p> <p>104 J: ¿Treinta y ocho?</p> <p>105 F: Multiplicando esto por 2 [points to the top floor: A4,52]</p> <p>106 J: Ajá.</p> <p>107 F: Como decía antes.</p> <p>108 J: Multiplicando por esto... De esto por dos [follows the 6th first horizontal lines with the pencil, starting on the top: A4,52]. ¿Doce? De esto para acá [from entrance floor until top floor: A4,52] son doce y si lo multiplico todo [following the 10 descending floors] serían treinta y ocho [32!, 38 comes from previous answer: A4,22 and A4,57].</p> <p>109 F: Pero hemos dicho que esto... O sea, entonces ¿por aquí arriba [above the top floor: A4,52] cuantos pisos quedan? ¿Hasta arriba del todo del edificio? ¿Porque hemos quedado que éste es el más bajo no?</p> <p>110 J: Mm [validating].</p> <p>111 F: Entonces arriba del todo. [Pause]</p> <p>112 J: No, no sé.</p> <p>113 F: ¿No?</p> <p>114 J: No.</p> <p>115 F: Si [pause] esto es el piso justo del medio [entrance floor], cuántos le quedan por debajo de éste?</p> <p>116 J: Uno.</p> <p>117 F: Uno baja luego [Jamie goes one up and then one down!], pero luego sube tres, luego baja otros diez.</p> <p>118 J: ¿Once? [Pause] No.</p> <p>119 F: ¿Puede contar cuántos quedan hasta abajo?</p> <p>120 J: Uno, dos, tres, cuatro, cinco... quince.</p> <p>121 F: Éstos son todos los que recorre. ¿Se imagina usted en un centro comercial? Sube un piso, baja un piso, sube tres, luego baja diez. ¿Ha llegado usted arriba del todo?</p> <p>122 J: No.</p> <p>123 F: ¿Y puede saber cuántos pisos tiene el edificio en total? Sabe que ha entrado por el medio [pointing to the entrance floor] y se va por el último [pointing to the last floor]. [Pause]</p> <p>124 J: No, no.</p> <p>125 F: Bueno, ¿lo dejamos así si quiere?</p> <p>126 J: Sí.</p>		
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	<p>127 F: Entonces aquí ahora me dijo que ha usado el inglés y el español, ¿no?, esta segunda vez. 128 J: Sí. 129 F: ¿Dónde ha usado el inglés y dónde ha usado el español? 130 J: El inglés en lo de la pregunta y español en la respuesta. 131 F: Ajá. Pero mientras, mientras estaba pensando sube uno, baja tres... lo que sea, ¿qué estaba... con qué idioma estaba pensando todo esto? 132 J: Con el español. 133 F: Con el español. ¿Todo el rato? 134 J: Sí. 135 F: ¿No hay ninguna cosa que haya pensado en inglés? 136 J: No. Nada más cuánto subía y cuánto bajaba, era lo que pensaba. 137 F: ¿En inglés? 138 J: Ajá. 139 F: ¿Goes up or goes down, eso pensó en inglés? 140 J: Ajá. 141 F: Y antes me ha dicho que lo ha pensado todo en... 142 J: En inglés [looking tired]. [Continues in GLQ,1]</p>		
M	6-21. 2nd try. Interviewer's intervention to explain and clarify Jamie's movements along the building.		
E	9, 13. 1st try. 35-39. 2nd try. Right understanding of "department".		
M	22, 34, 117. Initial floors (1 st , 2 nd) not corrected by the interviewer neither on the sketch (22) nor orally (34, 117). Neither the interviewer nor Juan realized		
O	Jamie's movements were inverted on the first 2 floors. Furthermore it was not the sketch of a building. 22. Sketch possibly made longitudinally due to the presence of the word "large" on the wording. 22. No mathematization of the first floor. 80-99, 110-125. Interviewer attempts to clarify the relative floors positioning through the middle floor position. In his 2 nd attempt of solving the problem, he does a better sketch which helps him better understand the situation, but he mismatches "goes up" and "goes down". 2 nd try: maybe "large" influences Juan and he thinks Jamie is on <i>the same floor</i> of a department store but just going up and down some stairs across the building to reach the different departments. Interviewer's unawareness of Juan's particular vision of the building results in hints not being interpreted as pretended by Juan, who continues with his particular vision. He does not mathematize the middle floor, just situates it on his drawing. Maybe the language density of the wording results in multiple wording references when constructing the sketch and produces the addition of 3 extra floors. Juan is tired at the end.		

General Language Questions		Language	Tentative
1	[Comes from A4,142] F: ¿En general, resolviendo estos cuatro problemas, cuándo ha usado el inglés? [Pause] Mientras resolvía los problemas.	1-4. English as reading language	English wording
2	J: En las preguntas.		
3	F: ¿Para qué en las preguntas?	5-6. English as writing	17-24. Easier use
4	J: Para leerlas.		

<p>5 F: Ajá.. ¿Dónde más? 6 J: A veces en las respuestas. 7 F: Mm [continuing conversation]. ¿Y dónde más? 8 J: Nada más. 9 F: Mm [continuing conversation]. ¿Y el español? 10 J: En contar o en pensar la respuesta en español para saber. 11 F: Mm [continuing conversation]. 12 J: O usar las fórmulas. 13 F: Ajá. ¿Qué más? 14 J: Y en escribir las respuestas. 15 F: ¿Y por qué cree que lo ha hecho así –que ha pensado esto en español? 16 J: Porque entiendo más bien el español que el inglés. 17 F: ¿Entonces por qué cree que ha usado... ? Bueno, para leerlo, porque, como el enunciado está en inglés hay que leerlo en inglés, ¿no? ¿Pero por qué lo ha usado también en algunas respuestas como me ha dicho? 18 J: Porque sé algunas palabras y las traduzco al español para entenderlo más. 19 F: Pero ha usado el inglés me dice, en algunas respuestas. 20 J: Sí, en algunas respuestas. 21 F: ¿Por qué? 22 J: Porque se me hace mas fácil en algunas respuestas. 23 F: ¿Se le hace mas fácil en inglés? 24 J: Mm [continuing conversation]. 25 F: ¿Hay alguna palabra o frase que haya encontrado difícil en inglés? 26 J: No. 27 F: Bueno ya hemos comentado algunas aquí, ¿no? ¿Pero aparte de éstas? 28 J: No. 29 F: ¿Las otras las ha entendido bien? 30 J: Sí. 31 F: Bueno, pues esto es todo, si no quiere añadir alguna cosa más que se le ocurre de cuando usa una cosa... 32 J: No. 33 F: Pues ya está todo, muchas gracias.</p>	<p>language</p> <p>9-10. Spanish as thinking language</p> <p>9-10. Spanish as counting language</p> <p>9, 12. Spanish with formulas</p> <p>9, 13-14. Spanish as writing language</p>	<p>of English use in particular cases</p> <p>15-16. Spanish dominant</p> <p>15-16. Spanish dominant</p> <p>15-16. Spanish dominant</p> <p>15-16. Spanish dominant</p>
<p>MEMO</p>	<p>8. Juan does not say he uses English language to think but he does it, for example in A4! 25-30. Juan does not remember that many words caused him trouble during the mathematical solving process. 20. Sometimes it is easier to use (reproduce) words in English.</p>	

Juan has an excellent Spanish BICS and CALP, as he talks fluently in Spanish and uses mathematical terms without problems and accurately. His English BICS is fair and so is his English CALP. His answer on A4 is not well organized from a linguistic point of view and on A1 and A3 answers are too short (all of them written in English). He has major problems understanding A4 and some issues on A2.

A1 is initially written without a justification (“Mike sports”). It is an English text imitating the words from the statement pictures. Later (following interviewer's demand) a justification is added, written in Spanish. Juan confirms that Spanish is used to write the answer. After interviewer's demand, Juan uses percentages in a relative way and supports his answer with a general example, which is not specified enough. Later he gives a specific example which clearly shows that he manages percentages correctly. Due to the influence of “Unbeatable prices” –that Juan translates in a deviated way as 'tabla de precios'– he assumes that 25% store is cheaper. He refers to English as a thinking language to interpret the percentages on the statement's pictures.

On A2 Juan uses English uniquely to read the wording. He understands the wording question in a deviated way: calculate the perimeters of both figures (but with no need for comparison). He calculates the perimeter of the square by adding all its sides and the perimeter of the circle by applying the formula. When talking about it he uses 'área' instead of 'perímetro' but it is a speaking confusion and not a conceptual one. Obviously, when interviewer asks him to compare both perimeters he quickly does it. He adds units to the square's length.

A3's answer is written in English (“13 tiles”), but Juan does not give importance to the use of English in this aspect, as he says he writes in Spanish. He never translates this word, but uses different strategies to refer to it: he points to the wording, uses demonstratives, incurs in some code mixing. Juan draws Figures 5, 6 and 7 to find the right answer (arithmetical and visual reasoning).

A4's answer is written in English and the mathematical procedure is thought in English on the 1st try. Juan shifts to Spanish on the tries to follow (2nd to 7th). On the 1st try Juan does not understand the wording properly but he is able to get a solution. He interprets “floors” as 'tiles' and consequently makes a deviated sketch (according to his deviated understanding). Then interviewer translates the most important information, and Juan makes another sketch, still not correct (Jamie's movements are inverted –going up instead of going down and vice-versa–, 3 extra floors are added and there is a longitudinal representation of floors –and Jamie's movements–). Maybe the word “large” (“large department store”) influences the vertical representation of floors. The wording is not completely understood because the sentence “which is on the first floor” is not properly integrated in the reasoning/sketch. Based on the drawing, Juan states that the building has 19 floors, with no relative positioning of floors. On the 3rd try the sketch is partially arranged (yet the first two of Jamie's movements are not correctly reflected) after the interviewer states the right meaning of “goes up” and “goes down”. The sketch will not vary anymore on the following tries. Floors will be in a horizontal disposition and will not be relatively positioned. On the 4th try Juan correctly situates the middle floor in the drawing, but does not mathematize it properly (no symmetry through the middle floor). Juan recognizes he does not understand the wording question and the interviewer states that the number of floors Jamie goes through are not the total

amount of floors of the building. Then Juan states that the building has 38 floors ($19 \cdot 2$) 'los diecinueve que recorre por dos'. On the 5th try Juan situates the first and middle floors on the drawing, but considers the highest floor reached by Jamie as the top of the building (relying on the sketch). He does not give the appropriate mathematical meaning to the middle floor after interviewer prompts these three main aspects. He says the department store has ten floors with no reasoning (maybe this ten is related with the ten floors Jamie goes down). On the 6th try Juan goes back to the idea of doubling the number of floors ($19 \cdot 2 = 38$), referring to the picture (which has 16 floors instead of 19). On his last try, the 7th, he considers a building with 11 floors but does not reason the answer. Again, Juan does not make the correct relationship between top, middle and bottom floors (reliance on the deviated sketch only). Finally, he does not find the number of floors from the first to the middle floor.

Activities' (Key ideas) summary

Object 30: Juan-First reduction (End)

- English is the initial option to write the answer (“Mike sports”, reproducing the statement picture). Once a justification is demanded by the interviewer, it is written in Spanish.
- A deviated translation of “unbeatable prices” into 'tabla de precios' influences the reasoning, assuming lower initial prices on the 25% store due to its advertisement. Use of both languages for thinking purposes.
- English language is linked to reading in relation with the calculation of perimeters.
- Juan understands the wording in a deviated way (calculate both perimeters). The fact that he finds a mathematical solution reinforces his language comprehension of the problem.
- The sequence of figures and his counting of the number of tiles is carried out through Spanish as thinking language. Even if the answer is written in English (“13 tiles”), it looks like it is thought in Spanish too, with the imitation of the word “tiles” from the wording.
- Use of English language for everything with a dense wording in a first try. Deviated interpretation of the wording results in a deviated answer (with acceptable reasoning according to interpretation).
- Meaning inversion (goes up - goes down) resulted in subsequent deviated sketch using mainly Spanish as a thinking language (keeping English exclusively for Jamie's movements).
- Deviated interpretation of the wording, with the possible influence of “large [department store]”, resulted in floors spread horizontally instead of vertically on the sketch. Such horizontal positioning is not changed later.
- The sequence “which is on the first floor” is not properly contextualized within the problem. Horizontal floors on the sketch help to maintain this idea throughout the different tries and the interviewer's hints are not used as intended.
- Maybe the language density of the wording resulted in multiple wording references when constructing sketch and produced the addition of 3 extra floors (2nd try of A4).
- On the third activity there is just a calculation of perimeter with no natural language usage. All the other answers are written initially in English (even if Juan may interpret the first answer as written in Spanish –as it just refers to the choice of a shop–). So Juan has acceptable skills as a problem solver of school mathematics using the language of instruction.

Historical profile		Bilingual profile (Spanish dominant) [cont]			Activity		
<ul style="list-style-type: none"> • 14 years old • Spanish-English class • 2 years in USA • Likes California • Spanish readings • Very few English readings • Spanish at home • Spanish with friends • Mainly Spanish at school • Homework help: brother 		English as unique writing language			1	x	1
		English and Spanish as writing languages			2		
		Deviated translation (A1: tabla de precios – un/bea/table prices, A4: floor–piso(tile), Jamie-él)			x		1
		English as thinking language (A1: percentages interpretation)			x		1
		English to interpret percentages in wording pictures			x		
		Deviated wording question understanding				x	1-3
		“área” instead of “perímetro” (speaking confusion only)				x	
		English linked to reading language				x	
		Code switching: A3(x0+0+1)					x
		Code mixing (tiles)					x
Bilingual profile (Spanish dominant)							
English as reading language	GLQ	English as reading language					
English as writing language	GLQ	Code switching: A3(x0+0+1)					x
Spanish as thinking language	GLQ	Code mixing (tiles)					x
Spanish as counting language	GLQ	Spanish as thinking language			x	x	x
Spanish with formulas	GLQ	Spanish as unique writing language			x		2
Spanish as writing language	GLQ	Unknown meaning (goes up, goes down)					1
		Meaning inversion (goes up - goes down)					2
		English with Jamie's movements					2-6
Procedural profile		Conceptual profile					
<p>1 ✓ Concrete and abstract examples giving initial prices to the shoes X/✓ Assumption of data not present in the wording (Influence of the advertisement –“unbeatable prices”–on answer)</p> <p>2.1 ✓ Calculation of perimeters: addition of square's sides and application of circle's perimeter formula X Addition of units in the case of the square X No comparison of perimeters</p> <p>2.2 ✓ Comparison of perimeters' length (after interviewer's demand)</p> <p>3 ✓ Drawing of figures 5, 6 and 7 following the figure pattern ✓ Application of a pattern given by adding 2 from one figure to the next</p> <p>4.1 X Horizontal mathematization, with a sketch made with tiles instead of floors due to a deviated wording understanding</p> <p>4.2 X Sketch with movements inversion, addition of 3 extra floors extra, floors positioned from left to right Non relative ordering of floors Reasoning without distinguishing the singularity of the middle floor in terms of its symmetry function</p> <p>4.3 X/✓ Sketch arranged partially, with 10 floors <i>going down</i>, elimination of the 3 extra floors</p> <p>4.4 - 4.7 Non relative ordering of floors Reasoning without distinguishing the singularity of the middle floor in terms of its symmetry function</p>		<p>1 X Notion of mathematical validity ✓ Notion of percentages as relative value</p> <p>2.1 ✓ Notion of perimeter</p> <p>2.2 ✓ Notion of perimeter</p> <p>3 ✓ Notion of arithmetical sequence associated to a sequence of figures</p> <p>4.1 ✓ Notion of addition</p> <p>4.2 Notion of number line with confused order positions</p> <p>4.3 Notion of number line with confused order positions</p> <p>4.4 – 4.7 Notion of number line with confused order positions</p>					

Object 31: Juan-Second reduction

Object 32: Angel-First reduction
(Beginning)

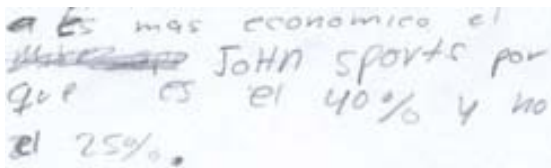
Angel is not a brilliant student (he sometimes does not do his homework) but he does understand the explanations when he pays attention to them.

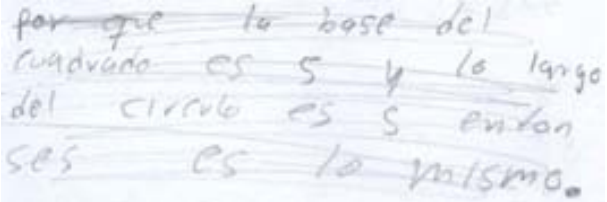
Math teacher description

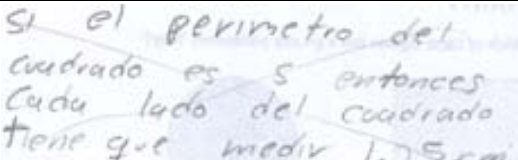
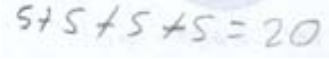
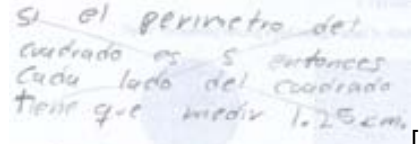

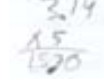
Class	Interview language	Interview date	Origins	Age	USA arrival	Living in California	Spanish readings	English readings	Home language	Friends language	Main language at school	Homework help
MS En-Sp	Spanish	May 2010	Mexico	14	March 2009 [more than a year]	Likes it.	“El Sol” newspaper	“The Californian” newspaper	Spanish	Spanish. English, just very few times.	Both. Math is taught in Spanish.	Nobody

Historical bilingual profile

Math	A1. Angel marked “English and Spanish”.	Language	Tentative
14-18. 1st try. Wrong answer with percentages interpreted as absolute value instead of relative or assumption of equal initial prices (direct percentages comparison).	[Dialogue about activities starts at A4,2] 1 A: ¿Qué significa chapter? 2 F: Excuse me. 3 A: ¿Qué significa chapter? 4 F: Cheaper... Cheaper. ¿Sabe qué significa? 5 A: No. 6 F: Más económicas, más baratas. 7 A: Sí. [Pause] 8 A: ¿Lo puedo poner en español o en inglés? 9 F ¿Cómo? 10 A: ¿Lo puedo poner en español o en inglés? 11 F: ¿La respuesta? 12 A: Sí.	1-4. English reading confusion (cheaper-chapter) 1-7. Unknown vocabulary (cheaper) 8-13. Norms clarification (writing)	Similarly spelled words Unknown word Home language, use of Spanish

	13 F: Como quiera.	language)	allowed when speaking
	14 	14. Deviated grammar (por que)	Homophony, quick writing
	15 [Comes from A4,4] F: ¿Cómo resolvió la actividad uno?	14. spelling variation (economico)	Quick writing, unknown
	16 A: Viendo los porcentajes. 17 F: Ajá. ¿Qué hizo luego? 18 A: Miré cuál es el porcentaje más grande y ya con esos, cuál era más barato. 19 F: Eh... De acuerdo. Me puso aquí inglés y español, ¿sí? ¿Cómo empezó resolviendo la actividad? ¿Qué lengua utilizó para empezar a resolverla? 20 A: El inglés. 21 F: Sí. ¿Para qué utilizó el inglés? 22 A: Para leer los dos [advertisements of both stores]. 23 F: ¿Para leerlos? 24 A: Sí. 25 F: ¿Y para qué más? 26 A: Nada más. 27 F: ¿Luego continuó en español? 28 A: Sí. 29 F: ¿Pensando? 30 [Angel nods] 31 F: ¿Y cuándo volvió a cambiar a inglés? 32 A: Ya no cambié. 33 F: ¿No cambió? ¿Utilizó siempre el español? 34 A: Sí. 35 Vamos con otra actividad, pues. ¿Sí? Es lo que me interesa saber, cuando cambia de un idioma a otro. ¿Sí? 36 [Angels nods] [Continues in A2,2]	14, 19-34. English linked to reading language	Spanish dominant
MEMO	Wording comprehension difficulties overcome by asking the interviewer. Wrong solution due to percentages interpreted as being absolute instead of relative (or assumption of equal initial prices), mainly due to conceptual difficulties.		

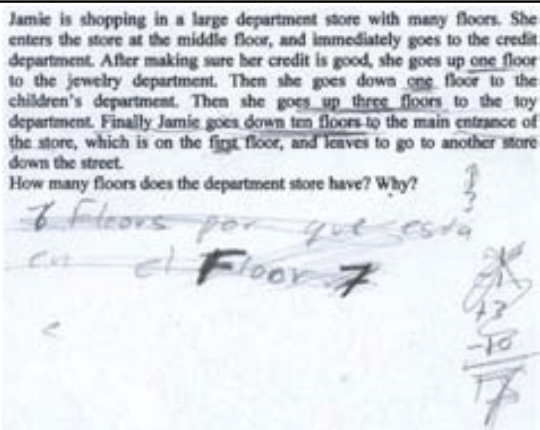
Math	A2. Angel marked "English and Spanish".	Language	Tentative	
1-5, 14-24. 1st try. Wrong answer due to wrong perimeter concept (equal dotted lines meaning equal perimeter). Wrong visualization of figures because circle has central symmetry, but square has not; so maximum distance between two points of the square is not the side's length: it's the diagonal.	 <p>1</p> <p>2 [Comes from A1,36] F: ¿Cómo empezó aquí?, a ver.</p> <p>3 A: Leyendo la pregunta.</p> <p>4 F: Sí. ¿Y luego?</p> <p>5 A: Después en los... en la distancia [the measurements given].</p> <p>6 F: Sí.</p> <p>7 A: Y después ya cambié a español. [the answer].</p> <p>8 F: ¿Esto lo pensó en inglés?</p> <p>9 A: Sí.</p> <p>10 F: ¿La distancia también? ¿En inglés?</p> <p>11 A: Sí.</p> <p>12 F: ¿Y luego qué hizo?</p> <p>13 A: Ya después lo apunté en español.</p> <p>14 F: ¿Qué apuntó?</p> <p>15 A: [reading] Porque la base del cuadrado es cinco y lo largo del círculo es cinco.</p> <p>16 F: [reading] Entonces...</p> <p>17 A: [reading] Entonces es lo mismo.</p> <p>18 F: ¿Tienen los dos el mismo perímetro?</p> <p>19 A: Sí.</p> <p>20 F: ¿Qué es el perímetro?</p> <p>21 A: Lo que mide alrededor del círculo. [points to the diameter]</p> <p>22 F: ¿Qué es alrededor?</p> <p>23 A: Lo que mide la mitad del círculo... así... esto de aquí [diameter]</p> <p>24 F: Esto es el diámetro, se llama, del círculo. ¿No se acuerda qué es el perímetro? ¿Perimeter?</p> <p>25 A: ¡Oh! ¿El perímetro no es... la mitad?</p> <p>26 F: No, esto es el radio. ¿Sí? Esto es el radio, esto es el diámetro, pero ¿el perímetro de un cuadrado, por ejemplo, qué es? O de una figura cualquiera.</p> <p>27 A: Lo que mide alrededor [Angel finally follows the perimeter of the square with the pencil].</p> <p>28 F: Ajá. Esto es el perímetro. ¿Sí? ¿Lo quiere volver a pensar?</p> <p>29 A: Sí. [Angels crosses the answer (A2,1) out]</p>	1. 1 st try. Spanish as writing language	Spanish dominant	
		1. 1 st try. Deviated grammar (por que)	Homophony, quick writing	
		1. spelling variations (por que, círculo, entonses)	Quick writing or unknown	
		2-13. 1 st try. English as thinking language	English wording	
		24-26. Wrong Math vocabulary (radio, diámetro, perímetro)	Guess and check to find the definition of perimeter	
		30. 2 nd try. Spanish as writing language	Spanish dominant	
		30. spelling variation (perimetro)	Quick writing or unknown	
		49, 51-62. 2 nd , 3 rd tries. English linked to reading language	Spanish dialogue, home language	
		23-24. Wrong perimeter concept (as diameter).		
23-26. Guess and check with the notion of radius to find the concept of perimeter.				
27. Visual and gestural identification of square's perimeter.				
20-34. 2nd try. Wrong answer due to dotted line misunderstanding (as				

meaning the entire perimeter). No problem answering: no comparison.	<p>30 </p>		
35-41. Right interpretation of the dotted line with interviewer's help.	<p>31 F: ¿Ya? ¿Cuál tiene mayor perímetro, entonces? 32 A: Si el perímetro de todo es cinco [points to the dotted line with the 5 in the square], entonces cada uno tiene que medir uno veinticinco.</p>		
44-45. Right calculation of square's perimeter through addition of all sides.	<p>33 F: ¿Eso [the dotted line in the square] significa que el perímetro de todo es cinco? 34 A: Sí. 35 F: No significa eso. 36 A: ¿A no? 37 F: Significa que de aquí... esta línea discontinua, del principio al final, esto vale cinco. De la misma forma aquí [in the circle] la línea esta, desde el principio hasta el final, ¿Ve esta línea discontinua? 38 A: Sí.</p>		
46. No comparison of perimeters, even if previously stated (31).	<p>39 F: Esto significa que esta línea [dotted line in the square], igual que ésta [dotted line in the circle], valen las dos cinco. 40 A: ¿Entonces este lado [a square side] es cinco? 41 F: Ajá.</p>		
46-49. Right calculation of circle's perimeter with application of formula after interviewer's demand of comparison between both perimeters.	<p>42 [Angels wants to write the answer again and crosses the previous answer (A2,30) out]. 43 F: ¿Sí? Esto estaba bien como lo ha pensado, pero la línea, pues... 44  45 A: Va a ser veinte. 46 F: Sí. Y entonces, ¿el otro cuánto... ? ¿Cuál es mayor? Porque la pregunta es cuál de los dos es mayor, ¿no? 47 A: ¿Se ha de multiplicar pi por diámetro? 48 F: Sí.</p>		
33-51. 3rd try . Right answer with arithmetical reasoning (Square: 4 sides added, circle: application of formula). Comparison	<p>49  [Entire answer (2nd, 3rd tries)]   50 A: Va a ser mayor el del cuadrado. 51 F: Sí. Vale. ¿Me dice ahora cómo lo ha pensado eso? ¿Cuándo ha utilizado el inglés? ¿Cómo... ? ¿Empezó leyéndolo? ...¿Luego cómo continuó?</p>		


induced by interviewer (46), yet done initially (1).	52 A: Nada más en inglés lo empecé leyendo esta pregunta y nada más. 53 F: ¿En inglés? 54 A: Sí. 55 F: ¿Y a partir de ahí? 56 A: Español. 57 F: ¿Todo? 58 A: Sí. 59 F: ¿Puro español? 60 A: Sí. 61 F: ¿No pensó nada en inglés? 62 A: No. [Continues in A3,2]		
MEMO	Perimeter misconception is not a language issue (24) but a conceptual one. Useful interviewer's hints (26). When the key information is in English, a mathematical thinking in English (1 st try) has more chances to occur. Spanish conversation with the interviewer influences Spanish thinking. Spanish dialogue (23, 25) helps improve Spanish CALP (48).		

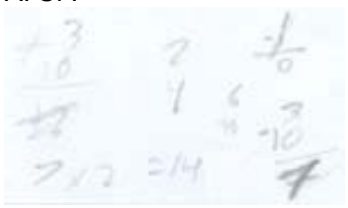
Math	A3. Angel marked "English and Spanish".		Language	Tentative
1-9. 1st try. Right answer with arithmetical sequence associated to figure pattern (number of tiles in figures 5, 6, 7 written down as arithmetical sequence).		[Counts with the fingers while he is writing the answer.]	1. Spanish as writing language	Spanish dominant
1			1. Deviated grammarr ammar (por que)	Homophony, quick writing
2	F: ¿Qué hizo aquí?, a ver. ¿Me explica?		1. spelling variation (ba)	Quick writing
3	A: Dice que... La pregunta es cuántos cuadr... cuadros tiene en la figura número siete.		10-40. English and Spanish as thinking languages	Statement in English, Spanish dominant
4	F: Sí.		24-29. Spanish for answer checking	Spanish dominant
5	A: Y va ser que son trece cuadritos.		9 A: La figura uno es uno, y aquí es tres, aquí son cinco, aquí son siete, en la cinco va a ser nueve, en la seis once y en la siete trece.	
6	F: ¿Por qué?		10 F: Okay. Sí. ¿Cómo usó los lenguajes para pensar todo eso? O sea, ¿empezó en qué idioma?	
7	A: Porque en cada figura van sumando de dos.			
8	F: Sí.			

	<p>11 A: En inglés. 12 F: Sí. ¿Y luego cuándo cambió a español? 13 A: Cuando lo puse a... las... Cuando contesté la pregunta. 14 F: Sí. ¿Al escribirlo? 15 A: Sí. 16 F: Pero la estuvo pensando... 17 A: En inglés. 18 F: En inglés. ¿Qué pensó en inglés? 19 A: Que si en la figura cuatro había sido siete, en la cinco va a ser nueve... 20 F: ¿Esto lo pensó en inglés? 21 A: Sí. 22 F: ¿Todo en inglés? 23 A: Sí, casi todo. 24 F: ¿Qué cositas pensó en español? Que si dice casi todo, algo pensó en español entonces, ¿no? ¿Qué pensó en español? 25 A: La [figura] número siete, si estaba bien. 26 F: ¿La última? 27 A: Sí. 28 F: ¿La última la pensó en español? 29 [Angel nods.] 30 ¿Y qué más en español? 31 A: Lo de abajo, la contesta[ción]... 32 F: La respuesta y este número. Lo demás lo pensó en inglés. El... como... Eh... Saber que se tenía que aumentar de esta a esta dos o de una a la siguiente dos, ¿esto lo pensó en inglés? 33 A: En español. 34 F: Y escribirlo, ¿lo escribió? 35 A: En inglés. 36 F: Lo pensó en inglés para escribir esto. 37 A: Sí. 38 F: Pero pensarlo, como... obse... mirar aquí, saber que se tenía que añadir dos, eso lo pensó... 39 A: En español. 40 F: En español. Okay. [Continues in A4,5]</p>	<p>16-23. English for counting</p>	<p>Schooling in English</p>
MEMO	<p>Right solution using 2 languages. There is some contradictory information about the use of English (16-40). Looks like English has some presence during the thinking process. 25-27. Use of Spanish to check the answer. (Is it really the case?)</p>		

Math	A4. Angel marked "English and Spanish".	Language	Tentative
1-10, 131-150. 1st try. [1 (floor up) - 1 (floor down) + 3 (floors up) - 10 (floors down) = 7] Wrong answer due to no relative situation of all Jamie's movements (though some of them are).	1 	1, 130. 1 st try. Code mixing on writing (floor)	Spanish dominant, statement in English
76-79. Right horizontal mathematization of the first part of the wording.	2 F: ¿Ya está? 3 A: Ya. Pero no está bien la última, no entiendo bien el inglés. 4 F: Okay. Ahora lo vamos a comentar si quiere un poquito más. [Continues in A1,15] 5 [Comes from A3,40] F: Me dijo que había tenido algunos problemas, aquí, ¿no? 6 A: Sí.	1, 27-30. Unclear meaning (floor)	1: code switching on writing, 27-30: no word translation
81-91. Wrong horizontal mathematization of the next part of the wording (floors not rightly positioned in an adequate relative way).	7 F: ¿Que le costó aquí?, a ver. 8 A: Entender el inglés. 9 F: Sí. ¿En dónde? 10 A: En todo. 11 F: Ajá. ¿Vamos paso por paso o cómo quiere hacerlo? 12 A: Pues aquí decía que... en los apartamentos, que tenía tres pisos. 13 F: Sí. No, tenía... Goes up. ¿No? Bueno, ¿cómo nos dice al principio? ¿Cuál es la situación en la... que se describe aquí en esta... en este enunciado? 14 A: Que Jaime [in Spanish] vive en un apartamento. 15 F: Bueno, Jaime [in Spanish] es en inglés, ¿no? Jamie. 16 A: ¡Oh, Jamie! 17 F: Por eso pone luego she, pero bueno. 18 A: Oh, Jamie vive en un apartamen[to]... en un departamento grande. 19 F: ¿Vive?... ¿Shopping, qué es shopping? 20 A: No sé. 21 F: Está [comprando]... va a comprar, ¿no? Dice Jamie se va a comprar en un, no es un departamento sino un centro comercial. Large department store es como un gran centro comercial, ¿sí? 22 A: Sí. 23 F: Con varios pisos. Tenía varios pisos, es muy alto, ¿Sí? 24 [Angel nods]	2-70. 1 st try. Awareness (2-10) of difficulties understanding situation presented in the wording	Spanish dominant, unknown meaning of key points
5-97. 2nd try. [1 (floor up) - 1 (floors down) + 3 (floors up) + 10 (floors down) = 13] Wrong answer due to not an adequate situation of all Jamie's movements, top of the building as the highest floor		12-13. Deviated translation (goes up-tenía)	Pisos [floors] is a polysemic word, general (but deviated) wording understanding
		14, 17-21. Deviated translation (is shopping-vive)	Translation word to word (department store), Mexican Spanish (departamento, apartamento)
		14, 17-21. Deviated translation (department store-departamento)	Translation word to word, omission of the "store", Mexican Spanish

reached.	25 F: Okay. Luego.		(departa- mento -aparta- mento)
98-105. Jamie does not reach the top of the building (introduced by interviewer). Quick use of hints.	26 A: Ella... Y que ella tiene, cómo se llama... 27 F: She enters the store at the middle floor. ¿Qué significa eso? 28 A: Que varios... ¿Store cómo se llama? [whispering] 29 F: ¿Store, qué es? ¿She enters, qué significa she enters? [Pause] Entra, ¿no? No pasa nada si alguna cosa no la sabemos. No tenemos porqué saberlo todo, ¿no? Lo pregunta y ya está. Dice entra en la tienda ¿dónde?, ¿por dónde? The middle floor, at the middle floor.	14-17 Letters rotation (Jamie-Jaime)	Spanish and English names similarity, Spanish dominant, quick reading, no inter-connection of wording sentences ("She" is in the next sentence)
106. Building has 14 floors: 7 floors [half building] times 2 (An even number of floors is never possible with a middle floor).	30 A: Por abajo de, de los departamentos. 31 F: Por el piso del medio, ¿no? Tantos como tenga pues entra justo por el del medio. ¿Sí? 32 A: Sí. 33 F: Y luego se va... inmediatamente 34 A: Inmediatamente... va... hacia... el departamento de crédito. 35 F: El departamento dónde está el... para compr[obar]... chequear ¿sí? Comprobar su crédito. 36 A: Y después... 37 F: ¿Después de? 38 A: Ya su crédito... ¡Aaah...! 39 F: ¿Making sure, qué significa making sure?	26-31. Deviated translation (at the middle floor- por abajo de los departamentos)	Unseen situation (context) in personal experience
106-116. Right symmetry through middle floor (but middle floor not considered).	40 A: No lo sé. 41 F: Después de comprobar, ¿no?, que su crédito está bien... 42 A: Está bien. 43 F: Que tiene buen crédito, ¿sí? 44 A: Ella va un piso arriba. 45 F: Ajá. 46 A: Hacia jewel... 47 F: ¿Jewelry, qué es eso? ¿Se acuerda? La sección de joyas. Donde están las joyas se va. Se va parriba a las joyas.	26-29, 58-61. Unknown meaning (enters, store)	Different previous interpretation of the problem
98-115. 3rd try . [2 · 7 = 14] Wrong answer due to wrong symmetry through middle floor without considering the middle floor.	48 A: Después ella va un piso abajo. 49 F: Ajá. 50 A: Hacia el departamento de los niños. 51 F: Ajá. 52 A: Después ella va tres pisos arriba. 53 F: Sí. 54 A: Para el departamento de los juguetes. 55 F: Sí. Good. 56 A: Y finalmente Jamie va diez pisos hacia abajo. 57 F: Ajá.	35-43. Unknown meaning (making sure)	English language in construction (phrasal verbs are difficult)
116-129. Interviewer introduces that the middle floor must be counted as part of the number of floors of the building.	58 A: Hacia la... la entrada principal. 59 F: Sí. 60 A: ¿De store es de la tienda? 61 F: Sí.	46-47. Unknown meaning (jewelry)	English language in construction
		64-66. Word	English

<p>98-130, 151-154. 4th try. Right answer with middle floor introduced by interviewer (116-129) through gestures (visual and arithmetical reasoning). 7·2+1: 7 floors up and 7 down + middle floor.</p>	<p>62 A: Y ya queda en el primer piso. 63 F: Sí. 64 A: Y después va... ¿Esto qué significa, another? 65 F: Another, hacia otra. 66 A: Hacia otra tienda hacia abajo de la calle. 67 F: Ajá. 68 A: Y dice: ¿Cuántos departa... cuántos pisos... cuántos pisos tiene el de..., el apartamento que va a comprar? 69 F: Sí. Mmm.... Sí. El centro comercial. Cuantos pisos tiene en total el centro comercial. ¿Y por qué? 70 A: Ya me equivoqué. 71 F: Why [pointing to the wording], right? ¿Sí? Lo puede corregir si quiere, ¿sí? 72 [Angel starts to read the problem again.]</p>	<p>meaning demand (another)</p>	<p>language in construction</p>
	<p>73</p>  <p>74 F: ¿Ya está? 75 A: Ya. 76 F: A ver, entonces, ¿me explica cómo lo hizo? 77 A: Dice que subió en el... en el mero medio de su departamento, de la tienda. 78 F: Sí. 79 A: Y después dice que se fue uno para arriba y después bajó uno para abajo y quedó en el mismo. 80 F: Sí. 81 A: Y después de tres para arriba y bajar diez para abajo, entonces si estaba tres para arriba y bajó diez y quedó al principio de la tienda, va a tener trece aparta... trece... pisos. 82 F: Sí. ¿Por qué? 83 A: Porque... 84 F: ¿Cómo le salió ese trece? 85 A: Porque se fue uno para arriba y uno para abajo y quedó otra vez en medio. 86 F: Sí. 87 A: Y después fue tres para arriba. 88 F: Sí. 89 A: Y dice que bajó diez para abajo y quedó en la entrada de la tienda. 90 F: Sí. 91 A: En el primer piso. Y por eso me dio trece. Porque sumé los tres que fue para arriba y los diez que fue para abajo y me dio trece. 92 F: Pero no veo cómo sale el trece. Trece... tres pa arriba y luego diez pabajo. 93 A: Sí. 94 F: ¿Y qué hace? Esto le da... 95 A: Trece.</p>	<p>68-79. Deviated wording question understanding (Jamie wants to buy an apartment)</p>	<p>77, 81. Different previous interpretation of the problem (12, 14, 18), Context</p>
		<p>All tries. 155-179. English linked to reading language</p>	<p>Spanish dominant</p>

- 96 F: ¿Cómo que trece? Ah, ¿trece?... Mmm...
- 97 A: ¿Está mal?
- 98 F: O sea, me dice que los... porque me dijo al principio que entró por el piso del medio, ¿no?
- 99 A: Sí.
- 100 F: Entonces, em... ¿Si se va tres arriba, es esto el... arriba del tope del edificio?
- 101 A: No.
- 102 F: ¿Cómo podemos saberlo, el tope del edificio?
- 103 [Angel whispers]
- 104 F: ¿Sí entiende lo que le digo?
- 105 A: Sí .
- 106 
- 107 A: ¿Va a haber catorce?
- 108 F: ¿Cómo sacó usted catorce? ¿Multiplicó por dos, veo aquí?
- 109 A: Sí. Porqué se fue tres para arriba.
- 110 F: Sí.
- 111 A: Y bajó diez y quedó al principio. Bajó siete para abajo.
- 112 F: Sí.
- 113 A: Y ya está abajo.
- 114 F: Sí.
- 115 A: Por la mitad. Y después este siete lo multiplico por dos porque es la mitad. Y hacen catorce.
- 116 F: Casi. Le queda un pequeño detallito sólo. Sí, es... Sí que es la mitad, pero se ha dejado de contar algo, ¿no? [Francesc makes movements with the hands to show what he is explaining:]
- 117 F: Entra en el piso del medio.
- 118 A: Sí.
- 119 F: Luego dice que sube tres. Luego cuando baja diez, si baja tres se queda en el mismo.
- 120 A: Sí.
- 121 F: ¿Pero luego cuántos le quedan para bajar?
- 122 A: Siete.
- 123 F: Siete. ¿Entonces van a quedar cuántos abajo? ... ¿Abajo, hasta abajo del todo van a quedar?
- 124 A: Siete.
- 125 F: Siete. ¿Arriba del todo van a quedar?
- 126 A: Siete.
- 127 F: ¿Y... cuál nos queda? [Francesc makes a movement with the finger that represented the middle floor]
- 128 A: ¡Oh, el del medio!
- 129 F: Dónde estaba ella, ¿sí?

Jamie is shopping in a large department store with many floors. She enters the store at the middle floor, and immediately goes to the credit department. After making sure her credit is good, she goes up one floor to the jewelry department. Then she goes down one floor to the children's department. Then she goes up three floors to the toy department. Finally Jamie goes down ten floors to the main entrance of the store, which is on the first floor, and leaves to go to another store down the street.

How many floors does the department store have? Why?

6 Floors por que esta en el Floor 7

$$\begin{array}{r} +3 \\ 10 \\ \hline 13 \\ -1 \\ \hline 12 \\ -10 \\ \hline 2 \end{array}$$

$$\begin{array}{r} 2 \\ +6 \\ \hline 8 \\ -10 \\ \hline -2 \end{array}$$

$$\begin{array}{r} 7 \\ \times 2 \\ \hline 14 \\ +1 \\ \hline 15 \end{array}$$

[Entire answer]

131 F: ¿Y qué puso aquí [answer]?, a ver. ¿Al principio qué me puso aquí? ¿Qué es lo que... que luego lo tachó...?

132 A: No me acuerdo.

133 F: Es que no lo entiendo. ¿No se acuerda? Six floors. ¿Eso es un número?

134 A: Sí, que seis para arriba.

135 F: [reading] Six floors...

136 A: Seis pisos.

137 F: ...porque está en el floor siete. ¿Eso lo cambió luego, este siete? ¿O lo dejó al principio?

138 A: Ya ve que... Es que no me acuerdo porque lo puse.

139 F: Okay. Bueno, luego en el video se quedó grabado. Luego miramos como, como lo hizo. Em...

Entonces... ¿Al principio me estuvo haciendo aquí unas cuentas? Bueno, ¿me explica un poco cómo lo hizo? ¿Cuál fue el procedimiento? Aunque luego pues, luego esto no lo contó y hizo algunas cosas más, ¿no? Si se acuerda de todo el procedimiento, cómo lo hizo. Me gustaría que me dijera.

140 A: Primero iba a sumar todo. Y miré que todo salió igual ahorita si lo sumaba todo.

141 F: Pero aquí no... tiene números positivos y números negativos, ¿no?

142 A: Sí. Porque es que decía que iba uno para arriba.

143 F: Ajá.

144 A: Y uno para abajo. Y sumaba uno menos uno y quedaba pues en medio. Y después iba tres para arriba y bajaba diez para abajo.

145 F: Sí.

146 A: Iba a poner que era siete y después tachaba [$1-1 = 0$] y entonces se lo restaba y quedaba siete.

147 F: Sí.

	<p>148 A: Y después ya pensé que era ya dónde quedaba. 149 F: Okay. ¿Eso fue la primera vez que lo hizo? 150 A: Sí. 151 F: ¿Y luego? 152 A: Ya fue cuando vino usted y me explicó lo del inglés. 153 F: Sí. 154 A: Y ya lo hice correcto. 155 F: Sí. Okay. ¿Cómo empezó para..., con qué lengua empezó a resolver el problema? 156 A: Con el inglés, cuando estaba resolviendo las palabras que no sabía en inglés. 157 F: Sí. 158 A: Fue cuando pasaba a español. 159 F: Sí. 160 A: Y cuándo leía, leyéndolo en inglés para ver las palabras que me sabía. 161 F: Sí. 162 A: Después lo traducí a español. 163 F: ¿Y qué hizo con las palabras que se sabía, dijo? Es que no lo entendí lo que me dijo. Con las palabras que se sabía... 164 A: ¿En inglés? 165 F: Sí. 166 A: Lo hacía para traducirlo a español. 167 F: Ajá. Okay. 168 F: ¿Y cuál...? ¿Qué palabras subrayó por aquí? 169 A: Como los pisos que bajó pabajo. 170 F: Sí. 171 A: Y los que iba parriba. 172 F: Okay. Sí. ¿Y luego cómo continuó? 173 A: Haciendo las operaciones. 174 F: ¿Con qué idioma? 175 A: Español. 176 F: Ajá. ¿Y cuándo volvió a cambiar a inglés? 177 A: Ya no. 178 F: ¿Sólo utilizó el inglés para traducirlo? 179 A: Sí. [Continues in GLQ,1]</p>		
MEMO	<p>131-138. Solving process not recalled. Wrong understanding of the situation (1st try) resulting from Spanish and English as writing languages and wrong horizontal mathematization (In the final part of the solving process, considering his situation understanding). +/- for opposite meanings (up/down). Problem solved (incorrectly) despite all the conscious comprehension difficulties mainly through the numbers and “up” and “down”. Use of Spanish to understand the situation with interviewer’s interaction (1st try to 2nd try). Wrong mathematization of middle floor (top of the building as the highest floor reached, middle floor not considered; 3^d and 4th tries) primarily not ascribed to language.</p>		

1st try. Does Angel thinks Jamie lives on the 7th floor and that she has 6 floors at the bottom??

General Language Questions		Language	Tentative
1	F: ¿En general, cuándo ha usado el inglés? ¿Para qué usó el inglés?	1-10. English as reading language	English wording
2	A: ¿Cómo, cómo para qué?		
3	F: Para... como me dijo para leer, por ejemplo.	11-12. (Slight use of) English with numbers	Interest in learning English
4	A: Para leer y para las preguntas.		
5	F: Para leer y para las preguntas. ¿Qué significa para las preguntas?		
6	A: Contestar las preguntas.	17-22, 39-40. Spanish with operations	41-42. Math taught in Spanish
7	F: ¿Contestó en inglés también?		
8	A: No. Pero para leer las preguntas.		
9	F: Ajá. Para leer, como tenía que resolver luego volvió a leer [Angel nods] la pregunta. ¿Es eso?		
10	A: Sí.	17-22, 39-40. Spanish in written answer	41-42. Math taught in Spanish
11	F: ¿Y qué más utilizó en inglés?		
12	A: Y un poquito los números en inglés. Como pensar en inglés.		
13	F: Ajá. ¿Y qué más?		
14	A: Casi nada más.		
15	F: ¿Y por qué cree que lo hizo así, de pensar estas cosas en inglés?	23-34. Spanish and English as thinking languages	35-38. Wish of learning English, keep Spanish
16	A: En verdad no sé.		
17	F: Okay. ¿Y en general cuándo usó el español?		
18	A: En... para contestar las preguntas. Y para las...		
19	F: ¿Para qué más?		
20	A: Para las operaciones.		
21	F: Sí. Operaciones, contestar las preguntas, ¿Qué más?		
22	A: Y nada más.		
23	F: Lo del... el pensar el problema, ¿no? El pensar cómo se tenía que resolver, ¿cómo lo pensó eso?		
24	A: En los dos.		
25	F: Tanto en inglés como en español...		
26	A: Sí.		
27	F: ...el procedimiento.		
28	[Angel nods]		
29	F: ¿Sí?		
30	A: Sí.		
31	F: El pensar pues cómo tenía que usar la información que le daban en el enunciado para sacar la respuesta.		
32	A: A veces en inglés y a veces en español.		
33	F: A veces en un idioma y a veces en otro.		
34	A: Sí.		
35	F: ¿Por qué cree que lo hizo así, a veces en un idioma y a veces en otro?		
36	A: Para que se me pegue más el inglés.		

37	F: Ajá.		
38	A: Y el español para que no se me olvide.		
39	F: Okay. Mmm... Y luego el español me dijo también que lo utilizó para escribir las respuestas...		
40	A: Y hacer las operaciones. Como las sumas o restas.		
41	F: ¿Por qué cree que esto lo hizo así?		
42	A: Porque casi siempre todo lo he hecho, las operaciones las hago en español. Casi nunca he hecho en inglés. Desde que llegué me toca con Mr Conteras y con él puro español y casi no hacemos en inglés las operaciones.		
43	F: Por último, ¿hay alguna palabra o frase que haya... que le haya resultado difícil en inglés a parte de las que ya hemos comentado?		
44	A: No.		
MEMO	1-10. English as a reading language 1-14. Reduced use of English. 17-22, 39-40. Spanish with operations because they are not practiced in English in the Math class.		

Angel has a moderate Spanish CALP (see A2,1) and a good Spanish BICS. He needs to improve his English (CALP and BICS).

A1 is solved with English linked to reading language. Angel asks for the enabled writing languages and he chooses Spanish (in all four activities). Once the meaning of “cheaper” is established (he confuses it with 'chapter' when reading –he does not seem to know the word beforehand–) he directly compares both percentages to state that the store with the 40% discount is cheaper.

A2 is solved on three tries. On the first one English is used as thinking language, along with Spanish. On the others (with interviewer interaction) English is linked to reading language. At the beginning there is a wrong answer due to a wrong conceptualization of the perimeter (equal dotted lines meaning equal perimeter; so perimeter is understood as 'diameter'). Angel does not use the right notion of perimeter. On a guess and check attempt to find the right concept, he postulates the notion of radius as being the perimeter. Later he correctly identifies the square's perimeter and follows it with the finger. On the 2nd try the problem is that he interprets the dotted line as the entire perimeter. Accordingly to this regard he calculates the value of the square's side. He does not say anything about the circle's perimeter (so the wording question is not answered). On a 3rd try, once the dotted line is correctly interpreted –thanks to the interviewer– he quickly calculates the square's perimeter. After interviewer demands for a comparison of perimeters he does it by applying the circle's perimeter formula.

A3 is solved with English and Spanish as thinking languages. Even if there is some contradictory information, this can be due to the fact that English use is not completely defined. It looks like Spanish is used to check the answer. The solution is written in English. Angel writes down the arithmetical

sequence associated to the figure pattern and correctly arguments that there is a growth of two tiles per figure. He says he uses English to count.

Angel says he solves A4 with English linked to reading language. In fact he uses some code mixing on writing, probably due to the uncertain meaning of some statement words, such as floors. He has a lot of English language problems when comprehending the wording: there are many unknown words and he makes many deviated translations. The letters rotation (Jamie-Jaime) reinforces the fact that Spanish is the dominant language and that there is not a global understanding of the wording. Even though, he is able to produce a mathematical solution, giving opposite meaning to the floors where Jamie goes down (negative) and to the ones that she goes up (positive) (1st try). On a 2nd try he correctly situates some of Jamie's movements on a relative way (when Jamie goes up one floor and goes down another one she is on the middle floor) but not all of them (Angel adds 3 and 10, so 3 floors are considered twice). On the 3rd try Angel quickly uses a hint given by the interviewer to say that Jamie does not reach the top of the building, making a symmetry through the middle floor (but not counting the middle floor itself). On the 4th try interviewer shows that the middle floor must be included too.

Angel gives reasoned answers. He shows a good deal of interest on the tasks. Several times he takes the initiative by asking the interviewer (unknown words, norms clarification, general wording comprehension).

Activities' (Key ideas) summary

Object 32: Angel-First reduction (End)

- Asking initiative when encountering an unknown word (cheaper), whose meaning is essential to solve the problem.
- English linked to reading language in relation with percentages and figurative visual mode. Spanish is allowed for writing.
- Incidental use of English as a thinking language stated on 1st try only in relation with geometrical figures (abstract visual mode). Interviewer's interaction and home language influenced the language on the mathematical work on 2nd and 3rd tries.
- Guess and check with related concepts to find the definition of perimeter.
- Right solution with arithmetical progression associated to figure pattern through the use of both languages. Answer is checked through Spanish.
- Use of English for counting in relation with the parts of an abstract figure.
- Despite major English language problems at the time of comprehending a dense wording, Angel is able to organize –somehow– the information mathematically.
- In a dense wording related to the number line, English is used for reading purposes and some code mixing instances (e.g. floors) appear in writing.

Historical profile		Bilingual profile (Spanish dominant)			Activity		
<ul style="list-style-type: none"> • 14 years old • English-Spanish Class • Born in Mexico, 1 year in California • Likes California • Spanish readings • English readings • Spanish at home • <u>Spanish</u> and English with friends • English and Spanish (including Math) at school • Homework help: nobody 		Spanish in written answer			x	x	x
		English reading confusion (cheaper-chapter)			x		
		Unknown vocabulary (A1: cheaper, A4: many)			x		x
		Norms clarification (writing language)			x		
		English linked to reading language (A4.1: code mix on writing)			x	2-3	x
		English and Spanish as thinking languages				1	x
		Spanish for answer checking					x
		English for counting					x
		Code mixing in writing					1
		Situation understanding difficulties					1
English as reading language		GLQ	Deviated translation (goes up-tenía, is shopping-vive, department store-departamento, at the middle floor- por abajo de los departamentos)				1
<u>Spanish</u> and English with numbers		GLQ	Letters rotation: Jaime-Jamie				1
Spanish with operations		GLQ	Unknown meaning (enters, store, making sure, jewelry, another)				1
Spanish in written answer		GLQ	Deviated wording question understanding (Jamie wants to buy an apartment)				1
Spanish and English as thinking languages		GLQ					
Procedural profile		Conceptual profile					
<p>1. X Direct comparison of percentages</p> <p>2.1 X Visualization: circle has central symmetry, square has not</p> <p>2.2 ✓ Visual and gestural identification of square's perimeter</p> <p>X Dotted line interpretation: as the entire perimeter of the square</p> <p>X Unnecessary square's side calculation</p> <p>X Wording question not answered</p> <p>2.3 ✓ Square: 4 sides added</p> <p>X Comparison of perimeters forced by interviewer</p> <p>✓ Circle: application of formula</p> <p>3 ✓ ✓ Application of a pattern given by adding 2 from one figure to the next</p> <p>4.1 ✓/X Relative situation of Jamie's movements</p> <p>4.2 ✓/X Relative situation of (some of) Jamie's movements</p> <p>Reasoning without distinguishing the singularity of the middle floor in terms of its symmetry function</p> <p>4.3 ✓ Jamie does not reach the top of the building (introduced by interviewer)</p> <p>✓/X Symmetry through middle floor</p> <p>✓ Relative situation of Jamie's movements</p> <p>X Middle floor not counted</p> <p>4.4 ✓/X Interviewer introduces the middle floor as part of the number of floors</p>		<p>1. X Notion of percentages</p> <p>2.1 X Notion of perimeter (as diameter)</p> <p>2.2 X Notion of perimeter (as radius)</p> <p>✓/X Guess and check to find perimeter concept</p> <p>✓ Perimeter concept</p> <p>2.3 ✓ Notion of perimeter</p> <p>3 ✓ Notion of arithmetical sequence associated to a sequence of figures</p> <p>4.1 X Notion of number line with confused order positions</p> <p>4.2 X Notion of number line with confused order positions</p> <p>4.3 X Notion of number line with confused order positions</p> <p>4.3 ✓ Notion of number line</p>					

Object 33: Angel-Second reduction

Object 34: Abel-First reduction
(Beginning)

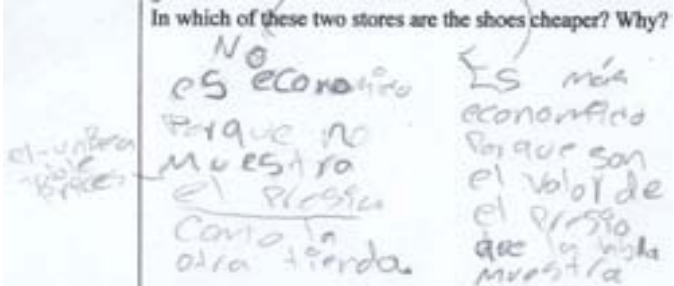
Abel is quite intelligent but does not like to work and consequently most of the days does not do the assigned homework. Sometimes it is hard to understand him when he speaks in Spanish and he needs to have a richer vocabulary.

Math teacher description

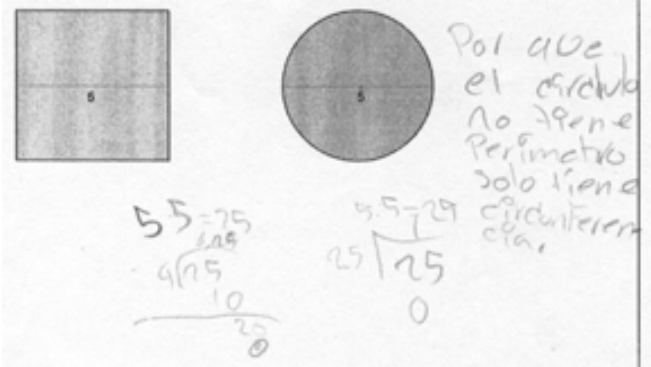
Class	Interview language	Interview date	Origins	Age	USA arrival	Living in California	Spanish readings	English readings	Home language	Friends language	Main language at school	Homework help
MS En-Sp	Spanish	May 2010	Michoacán, Mexico	13	Born in USA. At 6 went to Mexico. 11 or 12 months before came back to USA.	Likes it	Yes: fairy tales, short stories, comic books...	Just a little, less than in Spanish	Spanish. A little English (brother, sister).	Spanish. English just sometimes.	Spanish. English only in English class.	Mother and brother

Historical bilingual profile

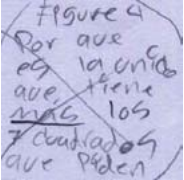
Math	A1. Abel marked "English and Spanish".	Language	Tentative	
12-32. 1st try. [25%] Wrong answer with wrong mathematical argumentation due to the influence of "Unbeatable prices" deviated meaning.	[Starts at A2,1. Abel checks the different wordings several times and in different order. Activities are commented following this order: A3, A4, A2, A1.]	1-10. Wording question not understood	Presence of unknown words	
	1 [Comes from A2,7] F: ¿Está bien? [Interviewer asks Abel how is he feeling because he has the impression that Abel is crying and Abel has not solved any activity after 10 minutes]	12. Deviated Spanish spelling (economico, de el, presio)		Mexican Spanish pronunciation (presio), quick writing (economico, de
	2 A: ¡Oh!, es que no sé yo, aquí no le entiendo lo que hay que hacer.			
	3 F: Ajá. ¿Cuál?			
	4 A: En ésta de los zapatos, cómo, por qué...			
	5 F: ¿Qué es lo que le, qué es lo que le hace falta?			
	6 A: Cómo, aquí, no, no entiendo de todos modos qué debo de hacer, casi.			
	7 F: ¿Qué es lo que no entiende? ¿La pregunta la entiende?			
8 A: No, no casi.				

27-29. Right understanding of "Unbeatable prices".	<p>9 F: En cuál de estos dos... en cuál de estas dos tiendas son los zapatos más baratos, ¿sí?</p> <p>10 [Abel nods]</p> <p>11 F: Más económicos. ¿En cuál de las dos? Y por qué. [Abel writes down the answer for all four activities. Then the dialogue continues in A3,2]</p>		el) or unknown (all 3)
33-36, 40. 25% store with lower prices due to "Unbeatable prices".	<p>12  [Abel writes firstly the right column, then solves other exercises and finally writes the left column]</p>	12. Spanish as writing language	Spanish dominant
38. Right treatment of percentages (as relative value). Assumption of a lower initial price on 25% store (lower enough to have a better final price).	<p>13 F: ¿Cómo empezó aquí a pensar el problema?</p> <p>14 A: Aquí empecé por la Mike [/meik/] Store.</p> <p>15 F: ¿En qué se fijó?</p> <p>16 A: En el porcentaje y en las nis... unbeatable prices [/unbeitebol preis/]. El precio que está en la tabla.</p> <p>17 F: ¿Cómo?</p> <p>18 A: El precio de la... ¿Cómo se llama? Esto de la... Un-beat-</p> <p>19 F: ¿Unbeatable?</p> <p>20 A: Ajá.</p> <p>21 F: ¿Qué significa?</p> <p>22 A: Así...no....</p> <p>23 F: ¿Cómo lo entendió eso?</p> <p>24 A: Esto namás entendí que el, que el precio de la... mesa...</p> <p>25 F: Mm [continuing conversation]</p> <p>26 A: Que está arriba de la mesa. Así lo pensé. Que estaba arriba.</p>	12. Code mixing on writing (unbeatable prices)	Reference to English wording
27-41. 2 nd try. [25%] Wrong answer due to wrong assumption of initial prices in 25% store lower enough to be lower after percentage application (due to "Unbeatable	<p>27 F: F: Esto significa eh... Unbeatable, insuperables. Precios insuperables, ¿no? Que son los mejores precios.</p> <p>28 A: ¡Oh!</p> <p>29 F: Es como un anuncio, ¿no? Tenemos los mejores precios.</p> <p>30 A: ¡Oh! Y puse [reading] es más económico porque son, porque son el valor del precio de, que... que la tabla muestra.</p> <p>31 F: Mm [continuing conversation].</p> <p>32 A: Y en la otra le puse [reading] no es económico porque no muestra el unber... unbeatable [/un-bei-teibol/] prices [/preis/], el de aquí [A1,12, part furthest to the right]. Que si no... Porque no te son los mejores precios, no tiene el anuncio como esa tienda de acá.</p> <p>33 F: ¿Entonces usted dónde compraría los zapatos?</p> <p>34 A: En la Mike [/meik/] Sports.</p> <p>35 F: Le saldrían más económicos, me dice. ¿Por qué?</p>	14, 16, 32, 34. Deviated English pronunciation	Words previously unseen, low English level
		16-29. Deviated translation (unbeatable prices-precio de la tabla, precio que está arriba en la mesa)	Word previously unseen, sliced and translated by fragments

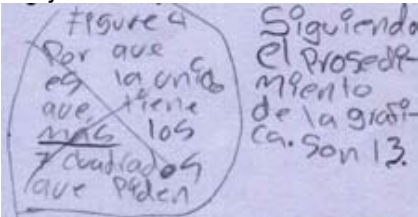
prices”).	36 A: Porque dice que son mejores precios... 37 F: Ajá. 38 A: Y se descuenta menos pero si siguen así pueden comprar más. 39 F: Okay. 40 A: Y acá pues no llama la atención porque no tiene anuncios que digan que son mejores. 41 F: Ajá. [Continues in General Language Questions,1]		
MEMO	(Not transcribed) Abel asks where he has to write the answer (he wants to do it on the languages columns!). No wording question comprehension, but no asking initiative (it is the interviewer who demands if Abel has any problem). 38. Does Abel have a right understanding of percentages? Deviated English pronunciation Questions about language use not asked.		

Math	A2. Abel marked “English and Spanish”.	Language	Tentative
8, 18-20. [circle has no perimeter]. Wrong perimeter concept.	1 [Conversation about activities starts here:] A: ¿En las dos vamos a sacar el perímetro de cuál o cómo? 2 F: Mm [continuing conversation / validating]. ¿Qué es lo que le pregunta? 3 A: Que si cuál es el, la figura, great[er], el perímetro, pero no se qué significa eso. 4 F: ¿Greater? Más grande. 5 A: Ajá. 6 F: Cuál es la que tiene el perímetro más grande. ¿Sí? 7 A: Okay. [Continues in A1,1]	1-7. Checking of the meaning of the wording question	3. Unknown meaning (greater)
15-18. Wrong comparison (it has no logic: circle has no perimeter, different than perimeter equals zero). This assertion is not stated on the written answer.	8 	8. Deviated Spanish spelling (porque, circulo, perimetro)	Quick writing or unknown
19-24. Right perimeter concept. Circle has perimeter if it has no inside (when it is just a	9 F: ¿Qué hizo aquí? 10 A: Aquí dice que... que si cuál era, cuál de las, de las figuras tiene el perímetro ¿más grande? ¿O así? 11 F: Mm [validating]. And why. ¿Qué significa why? 12 A: Por qué. 13 F: Ajá.	8, 18. Deviated Spanish syntax (solo [lo] tiene [la] circunferencia)	Quick writing, Spanish CALP
		8, 27-38. English linked to reading language	Spanish dominant
		9-14. Right	1-7. Inter-

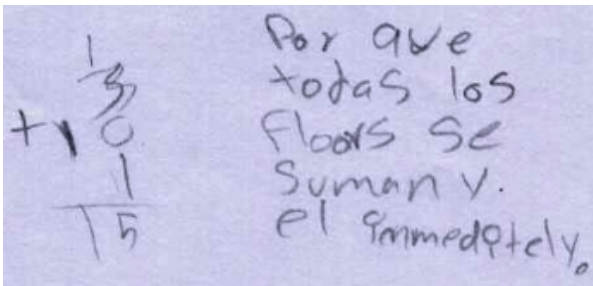
<p>circumference).</p> <p>8. Non-explicit benefit and origin of written operations. Possible area-perimeter confusion.</p> <p>8, 15-24. 1st try. Wrong answer due to wrong perimeter concept, with unclear use of operations.</p>	<p>14 A: Porque...</p> <p>15 F: ¿Y cuál dijo usted que tiene el perímetro más grande?</p> <p>16 A: El cuadrado.</p> <p>17 F: ¿Por qué?</p> <p>18 A: Le puse... Y el círculo y... Puse [reading] porque el círculo no tiene perímetro, sólo tiene ¿circunferencia?</p> <p>19 F: Bueno, pero el perímetro es la circunferencia del círculo, ¿no?, es...</p> <p>20 A: ¡Oooh!</p> <p>21 F: ¿Qué es el perímetro?</p> <p>22 A: ¿Lo que tiene alrededor?</p> <p>23 F: Lo que tiene alrededor. Y esto de hecho se llama circunferencia.</p> <p>24 A: ¡Oh!</p> <p>25 F: Se llama circunferencia... Pero ¿cuánto mide esta circunferencia?, ¿no?, es lo que queremos comparar. Si esta cricu... si el alrededor del círculo es más grande que lo de alrededor del cuadrado. O al revés, o el cuadrado, lo de alrededor del cuadrado es más grande que lo de alrededor del círculo. ¿Cómo lo pensó eso?</p> <p>26 A: En español, so...</p> <p>27 F: Pero ¿cómo lo hizo para comparar, para decir cuál era más grande? ¿Cómo hizo eso, no? ¿En qué lengua empezó a pensar eso?</p> <p>28 A: En español.</p> <p>29 F: ¿Empezó a pensar en español?</p> <p>30 A: Sí.</p> <p>31 F: Y luego continuó....</p> <p>32 A: En español. Escribiendo.</p> <p>33 F: ¿Continuó escribiendo en español?</p> <p>34 A: Sí.</p> <p>35 F: ¿Y cuándo volvió a cambiar a inglés?</p> <p>36 A: No, ya no.</p> <p>37 F: ¿Sólo en español?</p> <p>38 [Abel nods] [Continues in A1,13]</p>	<p>wording question understanding</p>	<p>viewer's previous interpretation</p>
<p>MEMO</p>	<p>8. ["por que el circulo no tiene perimetro solo tiene circunferencia"]. This statement can be related to the fact that circle has area, circumference hasn't.</p> <p>8, 18. More details are needed when explaining the reasoning (written or orally).</p> <p>25-26. Abel himself switches to the use of languages instead of keeping the focus on the mathematical specific content.</p>		

Math	A3. Abel marked "English and Spanish".	Language	Tentative
<p>3, 5-7, 47. 1st try. Deviated use of mathematical specific</p>	<p>1 </p>	<p>1. 1st try. Spanish as writing language with code mixing</p>	<p>English wording reproduction</p>

<p>vocabulary (cuadrado-lado confusion).</p> <p>1-7. 1st try. [Figure 4] Wrong answer due to deviated question understanding (figure with 7 tiles). Right mathematical procedure according to such a deviated understanding.</p> <p>15-21. 2nd try. Figure 7 with 10 tiles due to addition of 1 tile per figure, without considering the pattern.</p> <p>8-21. 2nd try. [10 tiles]. Wrong answer because the arithmetical sequence associated to the figure pattern is not followed.</p> <p>22. There is a pattern that must be followed (indicated by interviewer).</p>	2	[After A1,11, a solution for all activities is written down. Then dialogue continues here:] F: ¿Cómo empezó a resolver esto?	("Figure")	
	3	A: Nada más fue viendo porque, y ya, si cuál de, cuál figura tenía siete lados.	1. Spanish spelling variations (porque, unica)	Quick writing or unknown
	4	F: ¡Oh! No, la pregunta dice... Bueno, dígame lo que ha hecho, luego le, le digo.		
	5	A: Nomás le puse que si cuál figura tenía siete lados.	1-9, 30-32. 1 st try. Deviated question understanding (figure with 7 tiles)	English not enough consolidated, answer to problem found
	6	F: Sí. ¿Siete lados o siete cuadrados?		
	7	A: Siete cuadrados.		
	8	F: Pero lo que decía realmente es ¿qué figura...? mm... ¿Cuántos cuadraditos tiene la figura número siete?		
	9	A: ¡Oh!		
10	F: ¿Lo quiere cambiar? ¿Quiere pensarlo otra vez?			
11	A: ¡Ah, sí! Esta bien.			
12	F: Okay. [Pause]			
13	A: ¡Pero aquí no hay figura siete!			
14	F: Ajá. ¿Pero puede saber cuántos cuadraditos tendría la figura siete? [Pause] No está aquí dibujada, ¿pero puede saber usted cuántos cuadraditos tendría? [Pause]			
15	A: ¿Diez?			
16	F: ¿Cómo lo hizo?	3-5, 7, 47. Deviated use of mathematical specific vocabulary (lados-cuadrados)	Confusion, lack of precision	
17	A: Como haciendo esto así pero poniéndole, incluyéndole un cuadrado.			
18	F: ¿Uno más?			
19	A: Sí.			
20	F: ¿Por qué uno más?			
21	A: Porque va de una figura así, nomás le puede incluir... o puede incluir tres, o los que sean necesarios para que quede lo que tenga que ser.			
22	F: Pero aquí hay... [pointing to the wording] Dice: observa este patrón, ¿no? Aquí hay una serie que sigue una lógica, ¿no? Entonces tiene que mirar cómo funciona esto para ver... tratar de averiguar, pues la cinco, la seis, la siete... Quizás así lo podemos saber, ¿no? Entonces tenemos que mirar cómo cambia de la uno a la dos, de la dos a la tres, de la tres a la cuatro... y ver si lo que usted ha pensado, de añadir una figura [lapse: un cuadrado!], esto funciona. Si es lo que pasa para pasar de una figura a la siguiente.	41. 3 rd try. Spanish as writing language	Interviewer interaction, Spanish dominant	
23	A: ¡Oh!	41. 3 rd try. Spelling deviances on written Spanish (prosedimiento, grafica)	Mexican Spanish, quick writing, unknown	
24	F: ¿Sí entendió? No puede hacerlo usted a su manera, sino que tiene que seguir el modelo que tenemos aquí.			
25	A: Sí. [Pause. As Abel moves the pencil, it looks like he mentally adds tiles on Figure 4, imaging Figure 5, Figure 6 and finally Figure 7 and at the same time he counts the tiles.] ¿Serían trece?			
26	F: ¿Cómo lo hizo?			
27	A: Nada más como siguiendo el procedimiento, de incluyéndole de dos.			
28	F: Sí.			
29	A: Y así le fui incluyendo aquí dos, dos, más dos y así como llegué a la figura siete y me dio trece.	42-57. 1 st try. English linked to reading language	Spanish dominant	
30	F: Ajá. Okay. ¿Lo puede escribir?			
31	A: ¿Esto está mal?			
32	F: ¿Cuántas tiene? Sí, esto es lo que había, habíamos entendido la, la... la respuesta [lapse: pregunta!] mal, ¿no? Puede ponder nada más una línea así y ya está. [Abel circles and crosses out the answer] Si me puede			

<p>27-32. 3rd try. General perspective of the activity –including wording question understanding– not enough consolidated (previous answer –A3,1– not regarded as incorrect).</p>	<p>poner su respuesta y el porqué. 33 [Abels starts to write the answer] 34 F: Mm [validating]. ¿Y cuántas va a tener? No me lo escribió. 35 [Abel Counts again the tiles] 36 F: ¿Qué me dijo antes? ¿Cómo lo encontró? 37 A: Sumándole dos más. 38 F: ¿Entonces aquí cuántas hay? [Pause] 39 A: Trece. 40 F: Ajá. Sí.</p>	<p>54-61. 2nd - 3rd tries. English linked to reading language</p>	<p>Interviewer's interaction, Spanish dominant</p>
<p>22-41. 3rd try. [13 tiles]. Right answer with arithmetical and visual reasoning associated to figure pattern.</p>	<p>41  [A3, entire answer]</p> <p>42 F: ¿Antes [1st try] con qué lengua empezó? 43 A: Nada más empecé a usar el inglés para mi..., lo único que sé más o menos, 44 F: Mm. [continuing conversation] 45 A: Empecé a leer lo que entendía. 46 F: Mm. [continuing conversation] 47 A: Y nada más le puse... me fui por la lógica así de una figura que tiene siete lados. 48 F: Ajá. 49 A: Y nada más le puse la figura 4. 50 F: Okay. ¿Y utilizó el inglés para leerlo? 51 A: Mm. [validating] [Abel nods.] 52 F: ¿Y luego para qué más utilizó el inglés? 53 A: Nomás para eso. 54 F: ¿Luego cambió a español? 55 A: Sí. 56 F: Para pensar cómo lo iba a hacer. ¿Y siguió en español? 57 A: Sí, nomás seguí escribiendo en español. 58 F: Sí. Y la segunda vez, ¿no? Cuándo es que le dije 'Ah, es que habíamos entendido la pregunta mal' ¿Entonces cómo, cómo hizo? ¿Empezó a resolver el problema con qué idioma? 59 A: Con... Ya como me dijo usted aquí que si.. cuál era la figura... No, que cuántos cuadrillos tenía la figura siete... 60 F: Mm. [continuing conversation] 61 A: Y ya lo leí en inglés y claro usted me lo ha dicho en español. 62 F: Mm. [continuing conversation] 63 A: Ya primero lo leí en inglés y después, ya se lo he dicho, en español lo... me... me fui haciendo la idea.</p>		

	64 F: Y el razonamiento, el procedimiento de... de solución, ¿no?, ¿esto lo pensó en qué idioma? 65 A: En español. [Continues in A4,2]		
MEMO	1, 56-57. Code mixing on writing done in a natural way, as he is just writing in Spanish (56-57). 41. Abel does not explain in detail the solving process on his written answer, as he does before in an oral context. 1, 41. Spanish language deviances on writing. 27-32. General perspective of the activity –including wording question understanding– not consolidated enough (previous answer –A3,1– not regarded as incorrect –31–).		

Math	A4. Abel marked “English and Spanish”.	Language	Tentative		
<p>1, 7, 34-39. 1st try. [1+3+10+1=15] Wrong answer due to deviated wording understanding – with many unknown words (40-135). Mathematical argumentation not detailed enough (not even orally).</p>	 <p>Por que todas los floors se suman y. El inmediatamente.</p>	1-3 26-33. Spanish as writing language with code mixing (floors, immediately)	English wording imitation		
		1			
		2	F: ¿Me explica aquí cómo lo hizo eso?	1-25, 138-145. Spanish almost exclusively as thinking language	Spanish dominant
		3	A: Aquí nomás utilicé el inglés para leer esto, ya después el español... lo... el español para escribir. Y aquí nomás me fui por... así leyendo.		
		4	F: Sí.		
		5	A: Y acabando de leer le di una... otra pasada.		
		6	F: Ajá.		
		7	A: Fui leyéndole y ya me di más o menos la idea de que iba a ser sumando.		
		8	F: ¿Y qué pensó en inglés?	14-25. (Minor use of) English as thinking language with additions	Spanish dominant, schooling in English
		9	A: ¿En inglés?		
10	F: Mm [validating].				
11	A: Nada.				
12	F: ¿Sólo al leerlo?				
13	A: Mm [validating].				
14	F: Luego, por ejemplo, a la hora de sumar...				
15	A: Le... cuándo estaba sumando, para estar bien...	23. Code switching (involving addition)	14-25. English thoughts		
16	F: Mm. [continuing conversation]				
17	A: Para saber si está, si iba a estar bien le di otra pasada.				
18	F: ¿Con qué idioma?				
19	A: En inglés, así. Y después...	25. Code mixing (plus)	English thoughts		

	<p>20 F: ¿Sumó one, plus three, plus ten, plus one? 21 A: Sí 22 F: ¿In English? 23 A: Sí, nomás uno más three plus ten plus one y me dió quince, fifteen. 24 F: Pero ¿al pensar eso lo pensó en inglés o en español? 25 A: Ajá... sí... a veces que plus y lo pensé en inglés y ya los números, sumándolo lo pensé en español. 26 F: Sí. ¿Y a la hora de escribir la respuesta? 27 A: En español. 28 F: Pero aquí tiene una palabra en inglés, ¿no? 29 A: ¡Oh, sí! 30 F: ¿Cómo es? 31 A: Floors. [Pause] 32 F: ¿Cómo es que lo puso en inglés? 33 A: ¿Qué? Oh, pues namás que aquí. 34 F: ¿Y por qué los sumó los pisos? 35 A: Los pisos porque... aquí... mmm... [pause] Namás lo fui sumando así como aquí va sumando uno, después ya tres. Los fui poniendo más uno, y después diez y baja... 36 F: ¿De dónde sale el último uno? 37 A: Ajá. 38 F: ¿De dónde sale éste último uno? [poining to the referred 1: A4,1] 39 [Abel does not answer this question] 40 F: ¿Me puede describir quizás un poco que está haciendo aquí? 41 A: Nomás estaba... 42 F: [Interrupting] Lo que sucede, ¿no? Lo que se describe aquí, perdón. 43 A: ¡Oh! Lo que se describe es... 44 F: [Interrupting] ¿Cuál es la situación que se presenta? 45 A: Qué Jaime... [Spanish name]... 46 F: ¿Jaime [Spanish name]? Está en inglés, ¿no? ¿Cómo se dice en inglés? 47 A: Jaime. 48 F: Jaime. Es una chica, ¿no? She. 49 A: ¡Oh sí! [laughing] 50 F: Okay. 51 A: [Reading] Is shopping [in] a large department store with many floors. 52 F: Mm. [continuing conversation] 53 A: Que Jamie, esto, no sé. Department store, sho, sho, shopping. 54 F: Qué va a ser shopping? 55 A: No sé, eso no sé. 56 F: ¿Y eso? 57 A: Un departamento. 58 F: ¿Store?</p>	<p>45-49. Letters rotation (Jamie- Jaime)</p> <p>51, 69, 81, 91, 103, 109-111, 117, 129. Code switching</p> <p>51-68. Unknown meaning (shopping, department store)</p> <p>53, 61, 85, 89, 107, 113, 121, 125, 131, 133. Code mixing: Jamie (53), department store (53), shopping (53, 61), jewelry (85, 89), goes up (93), toy</p>	<p>Spanish and English names simi- larity, Span- ish domi- nant, quick reading, no interconnec- tion of word- ing sen- tences ("She" is in next sen- tence)</p> <p>Reading English wording</p> <p>Unknown meaning</p>
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	<p>59 A: [Looking at A1] No, tampoco.</p> <p>60 F: Es una tienda. En un gran departamento comercial, en un centro comercial grande, ¿qué se puede hacer? [pause] Esto es shopping.</p> <p>61 A: ¿Shopping? No, no, tampoco.</p> <p>62 F: Comprar.</p> <p>63 A: ¡Oh! Jamie fue a comprar en un departamento largo.</p> <p>64 F: Mm. [validating]</p> <p>65 A: En una tienda larga.</p> <p>66 F: En un gran centro comercial. Ajá. Con diversos...</p> <p>67 A: Departamen, diversos departamentos.</p> <p>68 F: Sí. Pisos, más que departamentos. Diversos pisos. También luego vemos que tiene diferentes departamentos, ¿no?, diferentes secciones. Okay.</p> <p>69 A: [Reading] She enters the store at the medium [middle] floor and immediately goes, goes to the credit department.</p> <p>70 F: Mm. [continguing conversation]</p> <p>71 A: Dice que ella entró en el, en el departamento del medio.</p> <p>72 F: ¿Qué significa eso?</p> <p>73 A: En el piso, sino que en el... ¿Cómo se llama?</p> <p>74 F: ¿Qué significa eso, entra en el departamento del medio?</p> <p>75 A: ¿Qué se [pause] subió? Así, se... Entró al segundo departamento o al tercero. En el del medio. [making gestures with the hand meaning a flat surface]</p> <p>76 F: En el de en medio. Ajá. En el piso del medio. Bien.</p> <p>77 A: Y inmedia.. ¿Cómo dice? Inmediata... Así de rápido [pause] inmediato [pause] fue al [pause] del crédito.</p> <p>78 F: Mm. [continuing conversation / validating]</p> <p>79 A: Al crédito. Que fue, de inmediato fue al... ¡Ah! ¿Cómo se dice? ¿El departamento de crédito?</p> <p>80 F: Ajá. Va a mirar su crédito.</p> <p>81 A: Ajá. Después... [Reading] After making sure her credit is good she goes up one floor to the jewelry [/dʒeswerli/] department [/departamen/]. Que después, making... Dice después de que estaba yo creo que segura.</p> <p>82 F: Mm. [validating]</p> <p>83 A: De que su crédito era bueno.</p> <p>84 F: Mm. [validating]</p> <p>85 A: Fue al primer piso del jewelry [/dʒeswerli/] departamento.</p> <p>86 F: ¿Qué dice aquí? Goes up. ¿Qué significa goes up?</p> <p>87 A: ¿Qué up es aba[jo] arriba o ...?</p> <p>88 F: [interrupting] Mm [validating], arriba. Sube un piso.</p> <p>89 A: Ajá. Sube un piso de el jewelry [/dʒeswerli/] departa ..., del apartamento jewelry [/dʒeswerli/] y del ...</p> <p>90 F: ¿Jewelry? ¿What's jewelry? ¿Jewerly department qué significa? La sección de [pause] joyas.</p> <p>91 A: ¡Oh!, joyas. Then she goes down one floor [to] the children department. Que desp[ués]... ella fue al... No sé qué significa down. Esto de aquí [pointing to the wording].</p>	<p>(107), down (113), leaves (121), store (125, 131), have (133)</p> <p>69. English reading deviance (middle-medium)</p> <p>81, 89, 109-111, 121, 129. Reading and pronunciation difficulties</p> <p>85-88. Hesitation about English translation (goes up)</p> <p>92-100. Unknown meaning (goes down)</p> <p>107-109. Meaning not precise enough (toy-juego)</p> <p>109-118. Deviated translation (down ten floors to the main entrance-hacia abajo)</p>	<p>Words with similar meaning and sound</p> <p>Quick translation</p>
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	<p>92 F: ¿Down? She goes down, ¿qué significa goes down? 93 A: Goes up... [gesturing with the hand] 94 F: Goes up, va arriba. 95 A: Arriba. 96 F: Goes down. 97 A: Abajo. 98 F: Hacia abajo. [gesturing with the hand] 99 A: Okay. Se fue hac[ia]... un, un piso hacia abajo. 100 F: Mm [validating]. 101 A: De... ¿Al apartamento de los niños? 102 F: Ajá. 103 A: Ella fue, goes up three floors to the toy department [/departament/]. Ella fue arriba tres apartamentos. 104 F: Mm [validating]. Tres, pisos. 105 A: Ajá. 106 F: ¿Sí? Tres niveles. [gesturing with the hand] 107 A: Sí. Del toy departamento. Del de... ¿Toy? ¿Es juego?. 108 F: Mm [validating]. Juguetes, ¿no? 109 A: ¡Ah, sí, claro! Departamento de juguetes. Final, así final [reading:] Jamie goes down ten floors to the main entrance of to the store which is on the first floor. And leaves, leaves [/lefs/]... 110 F: [Interrupting] Leaves. 111 A: ...leaves to go, go to another store down the street. [reads without a good rhyme] 112 F: Mm [continuing conversation]. 113 A: Que finalmente Jamie fue al, down, hacia abajo diez pisos. 114 F: Mm [validating]. 115 A: Del, de la, ¿de la entrada? 116 F: Hacia la entrada, hasta la entrada principal, ¿no? 117 A: Ajá. De el store which [/wif/]. ¿Which [/wi θ /] no se qué significa? 118 F: De la tienda, ¿no? Y nos dice, la cuál, está hablando de la entrada principal, este "which" se refiere a la entrada principal. La entrada principal, nos dice, está en el... 119 A: Arriba el, del primer piso. 120 F: First floor. Está en el primer piso. 121 A: ¡Oh!, en el primer piso. Y leaves [/leivs/], leaves [/live/], no... 122 F: Eh, se va, leaves es se va. 123 A: Y se va de, aaah, no... ¿cómo se dice? De... ¡Ah! ¿Cómo se...? 124 F: ¿Another? [Pause] Se va para ir a otra... 125 A: Store. 126 F: Tienda, ajá. 127 A: Tienda, abajo de la calle. 128 F: Mm [validating]. 129 A: How many floors does the apartment [/apartament/] store have? Why [/ui/]? Have, dice.</p>	<p>diez pisos de la entrada)</p> <p>117-118. Unknown meaning (which)</p> <p>118-121. Deviated translation (on the first floor-arriba del primer piso)</p> <p>123-127. Unknown meaning (store)</p> <p>131-135. Deviated wording question understanding (cuántos pisos subió o hizo en total)</p>	
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	<p>130 F: Mm [continuing conversation].</p> <p>131 A: Dice, ¿cuántos departa..., éste, pisos del apartamento... [low voice:]¿Store cómo se dice? De la tienda</p> <p>132 F: Mm [validating].</p> <p>133 A: ¿Have, subió o hizo en total?</p> <p>134 F: Hay. El centro comercial, en total, cuántos pisos tiene. ¿Sí?</p> <p>135 A: Ajá.</p> <p>136 F: ¿Quiere pensarlo otra vez, quizás? O...</p> <p>137 A: ¡No! Así ya.</p> <p>138 F: ¿Sí? [Pause] Okay. ¿Qué lenguas utilizó aquí?</p> <p>139 A: El inglés y el español.</p> <p>140 F: ¿El inglés para qué?</p> <p>141 A: Para leer.</p> <p>142 F: Ajá. ¿Y el español?</p> <p>143 A: Para escribir y traducir lo que sé, antes.</p> <p>144 F: ¿Y alguna otra cosa? Por ejemplo, ¿cómo lo pensó, en qué idioma lo pensó eso?</p> <p>145 A: En español. [Continues in A2,9]</p>		
MEMO	<p>1, 26-33. Code mixing made naturally.</p> <p>8-25. Use of English is not clearly determined.</p> <p>Many English language difficulties (goes up/down, Jaime, which,...) to correctly comprehend the situation.</p>		

General Language Questions		Language	Tentative
1	[Comes from A1,41] F: ¿En general, cuándo ha usado el inglés?	1-14. English linked to reading language	12. Spanish dominant
2	A: ¡Oh! ¿Cuándo voy a las tiendas?		
3	F: No, aquí, digo.		
4	A: ¡Oh! Aquí nomás en las preguntas.		
5	F: Entre los cuatro ejercicios.		
6	A: ¿En los cuatro? En éste [no video recording at this point: unknown activity] más.	15-20. Spanish as writing language	25-30. Spanish easier than English
7	F: Entre todo, en general, ¿no?, al resolver éstos, uno, dos, tres y cuatro ejercicios, ¿cuándo ha usado el inglés? ¿Para qué lo ha usado el inglés?		
8	A: Para leer y entender más o menos lo que sé.		
9	F: ¿En qué más?		
10	A: Pues nomás yo creo que para eso.	21-24. Spanish as thinking language	Spanish dominant, 25-30. Spanish easier than English
11	F: ¿Y por qué cree que sólo ha utilizado el inglés para eso?		
12	A: Es que por eso están en inglés y yo me esforcé pues en leerlo en inglés.		
13	F: Sí.		
14	A: Pues nomás para eso y entender.		
15	F: Okay. ¿Y en general cuándo ha usado español?		
16	A: ¡Oh!, en escribir		

<p>17 F: Mm [continuing conversation]. 18 A: Y en las traducciones del inglés. 19 F: Mm [continuing conversation]. 20 A: Para eso, creo que para eso. 21 F: ¿Y para pensar cómo iba a poner la respuesta? 22 A: ¡Oh, sí! 23 F: ¿Cómo lo ha pensado? 24 A: En español. 25 F: ¿Y por qué cree que ha utilizado el español en estos casos? 26 A: Porque yo creo que así es más, para mí es más fácil. 27 F: Ajá. ¿Le resulta más fácil? 28 A: Sí, que en inglés. 29 F: ¿Alguna otra razón? 30 A: No.</p>		
MEMO	1-2. Imagination of real context when solving A1. Use of English language related just to reading. Earlier he says he uses English for other purposes!!	

Abel has a basic Spanish BICS as he is able to communicate in Spanish but sometimes makes some errors that prevent sentences from being fully understandable or make them take a deviated meaning, mainly when it comes to translate English sentences. This seems to be related to the fact that he has a poor Spanish CALP as we can appreciate specially in A2. He needs to improve his English BICS, as he incurs in some deviance when reading English wordings and has difficulties in all statements to understand the meaning of some words. He needs to improve his reading fluency, as observed in A4. Abel has a lot of language difficulties either in understanding English wording or when explaining his written (and sometimes oral) answers. Sometimes he tries to reproduce English wording words within his Spanish explanations, which helps the reader to get an approximate idea of what he does. His English CALP also needs to be improved.

Answer to A1 is written in Spanish with an instance of code mixing (“UnBeatable Prices”). After interviewer translates the wording question, Abel understands “Unbeatable prices” in a deviated way (“tabla de precios”). Such understanding determines that the 25% store is showing the price and consequently is cheaper, whereas 40% is not showing the price (1st try). Once interviewer translates “Unbeatable prices” correctly and asks Abel where he would buy the shoes, Abel adapts his initial answer by emphasizing again “Unbeatable prices” as the main reason to justify his answer (assuming a lower initial price on 25% –as it can happen in a real life situation; lower enough to have a lower final price –after discount is applied).

A2 is solved with English linked to reading language. Abel's understanding of the wording question is not clear, so interviewer clarifies it. The main issue is the wrong perimeter concept, with an unclear answer: circle has no perimeter. This being true, he cannot compare both perimeters as he does orally (it is not the same as the perimeter to be zero). Abel does not specify the use of the operations he makes (which do not seem related to the perimeter but rather to the area).

A3 is written in Spanish with some code mixing ("Figure"). This is the only known use of English, along with the reading of the statement. Firstly Abel interprets the wording question in a deviated way –as which figure has 7 tiles– and accordingly he states that Figure 4 is the solution. Secondly, once the right meaning of the question is established by the interviewer, Abel says that Figure 7 has 10 tiles, adding 1 tile per figure to the figures not present in the statement up to Figure 7. So he does not follow the figure pattern. On the third try, after interviewer points out that there is pattern that needs to be followed, Abel finds out the right solution. But he still hesitates about the correctness of his previous solution. The written answer is not as detailed as the oral explanation and does not contain the number of tiles on Figure 7 (it is written following a second interviewer's demand).

A4 is written in Spanish with some code mixing ("floors", "immediately"). Abel says he uses English language for reading purposes only, but later says that addition was partially performed in English (furthermore there is the mentioned code mixing). Despite a lot of comprehension problems, he states 15 as answer, without a relative position of the floors and without following Jamie's movements properly (skips one movement at the beginning and adds a supplementary floor at the end –because of "immediately"). These problems understanding the wording and the wording question properly refer to the unknown meaning of some words (words so important to solve the problem correctly as "goes up" and "goes down") and to the deviated translation of parts of the statement. When reading, the lack of language fluency along with the fact that some words are changed denote that English language level is a major difficulty in this activity. Maybe it is a so huge an issue that Abel does not want to try to solve the problem again after the statement is correctly translated with interviewer's help.

Abel solves correctly A3 only; and it is after 3 attempts due to a deviated wording understanding (1st try) but also due to the mathematical difficulties associated to the figure pattern (2nd try). The language struggles which he also goes through in all other activities are an obstacle to get to know his mathematical ability. Even so, Abel is able to make an effort and solves all the problems. He is not very enthusiastic in trying to solve the problems once again when he gets the wording meaning as intended with the help of the interviewer.

Activities' (Key ideas) summary

Object 34: Abel-First reduction (End)

- A deviated translation of “Unbeatable prices” determines that the 25% store is showing the price and consequently is cheaper, whereas the one offering 40% is not showing the price.
- When interviewer properly translates “Unbeatable prices” and asks Abel where he would buy the shoes, Abel adapts his initial answer . “Unbeatable prices” is the first argument to justify his answer. Furthermore he refers to a possible real life situation with a lower initial price in the 25% store due to a bigger amount of shoes bought from the supplier, with not enough details.
- Deviated use of mathematical specific vocabulary (cuadrado-lado confusion).
- Asking initiative due to the unknown meaning of a word (greater).
- Activity with geometrical shapes solved restricting English linked to reading language. The main problem is the concept of perimeter.
- English used almost exclusively as a reading language in relation with the figure pattern exercise. Reproducing the word “Figure” from the statement in the written answer is not seen as relevant by Abel.
- A deviated wording question understanding (figure with seven tiles) leads Abel to a deviated mathematical solution. His solution finding feeds his question understanding.
- Deviated mathematical language use (lados-cuadritos) makes explanations not mathematically precise.
- The interviewer's orientation regarding the wording's comprehension is not immediately used to properly follow the arithmetical sequence associated to the figure pattern, and a second interviewer's intervention is needed to pay attention to the mathematical relevance of the figure pattern.
- Abel has many reading difficulties and he encounters many unknown words within the dense wording, evidenced by multiple code switches and mixes. Despite of it, Abel is able to produce a mathematical solution which is explained relying on English words taken from the wording. This helps him to slightly explain the procedure followed.
- Use of English as well as Spanish when performing operations in relation with the number line. it looks like Spanish has a more important role.
- English language linked to reading (no mention of English influence with operations) during the general questions asked at the end of the interview.

Historical profile	Bilingual profile (Spanish dominant)	Activity				
<ul style="list-style-type: none"> • 13 years old • English-Spanish class • USA until 6. 6-12 in Mexico. Back to USA 1 year before interview • Likes California • Spanish readings • English readings (a few) • Spanish at home • A little English with brother and sister • Spanish with friends • English sometimes • Spanish at school • Mother and brother help with homework 	Wording question not understood A2: "greater", A4: many words, syntax	x	x		x	
	Spanish as a writing language with code mixing instances. (A1: unbeatable prices, A3: Figure, A4: floors, immediately)	x		1		x
	Spanish language deviation on writing	x	x	x		
	Deviated translation A1: unbeatable prices-precio de la tabla / precio que está arriba en la mesa A4: down ten floors to the main entrance-hacia abajo diez pisos de la entrada, on the first floor-arriba del primer piso, <i>wording question</i> : cuántos pisos subió o hizo en total	x				x
	English linked to the reading language (A1, A3.1, A4:code mix on writing, A4:operations)		x	x		x
	Deviated mathematical specific language use (lados- cuadrados)				1	
	Deviated question understanding (figure with 7 tiles)				1	
	Spanish as writing language		x		3	
	English and Spanish in operations					x
	Code switch A4(0+2+8)					x
	Letters rotation: Jaime-Jamie					x
	Unknown meaning (shopping, department store, goes up, goes down, toy, store, which)					x
	Code mixing instances: plus, Jamie, department store, shopping (x2), jewelry (x2), goes up, toy, down, leaves, store (x2), have					x
	English linked to reading language					GLQ
	Spanish as a writing language					GLQ
Spanish as a thinking language					GLQ	
Procedural profile	Conceptual profile					
<p>1.1 X Influence of the advertisements on the mathematical reasoning (which is not explained enough according to a standard comprehension)</p> <p>1.2 X Assumption of initial prices in 25% store lower enough to be lower after percentage application (due to "Unbeatable prices")</p> <p>2 X Uncertain use of operations</p> <p>3.1 X/√ Mathematization according to the deviated understanding</p> <p>3.2 X Application of a pattern by adding 1 from one figure to the next</p> <p>3.3 √ Application of a pattern given by adding 2 from one figure to the next</p> <p>4 Nonrelative ordering of floors Reasoning without distinguishing the singularity of the middle floor in terms of its symmetry function</p>	<p>1.1 X Notion of percentage not clearly reflected</p> <p>1.2 X Notion of percentage not clearly reflected</p> <p>2 X Perimeter concept</p> <p>3.1 √ Notion of a whole thing formed by its identical component parts</p> <p>3.2 X Notion of arithmetical sequence associated to a sequence of figures</p> <p>3.3 √ Notion of arithmetical sequence associated to a sequence of figures</p> <p>4 X Notion of number line with confused order position</p>					

Object 35: Abel-Second reduction

Object 36: Julia-First reduction
(Beginning)

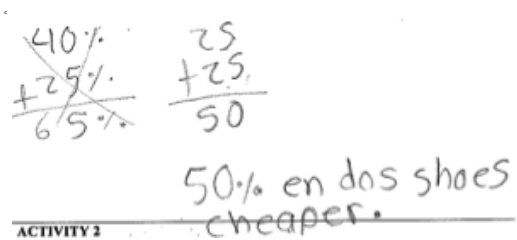
Julia is a hard worker, interested in having good grades and polite in class. She is not very smart, but as a result of her effort she gets pretty good grades.

Math teacher description

Class	Interview language	Interview date	Origins	Age	USA arrival	Living in California	Spanish readings	English readings	Home language	Friends language	Main language at school	Homework help
MS En-Sp	Spanish	May 2010	Mexico	14	Was in USA for a year when she was 1. Came back to USA in October 2009 [6 months ago].	Likes it.	Yes: comicbooks [historietas],...	Books from the library for English learning purposes	Spanish	Spanish	Spanish	Her sister (Yael)

Historical bilingual profile

Math	A1. Julia marked “English and Spanish”.	Language	Tentative
1 (answer crossed out). 0 try. [40+25=65%] Wrong answer due to deviated percentage notion. Addition of percentages applied to different objects (one to each	<p>[Julia works on each activity in the following order: A2, A3, A4, A1, A3, A4, A2, A1. Some parts of her answers are crossed out because she was told not to use an eraser.</p> <p>The following extract –occurred at the beginning of the interview– shows Julia’s opinion about her English language level.]</p> <p>i. F: First of all, I am gonna ask you some questions. Which language do you prefer being asked questions? [Pause] Which language do you prefer to be asked questions? [Pause. Julia shows signs of not understanding through her face] Have you understood my question?</p> <p>ii. [Julia shakes her head saying no]</p> <p>iii. F: ¿Ha entendido mi pregunta?</p> <p>iv. [Julia shakes her head saying no]</p> <p>v. F: ¿Prefiere que hablamos en español?</p> <p>vi. [Julia nods]</p>	i-xvi. Low self-assessment of English language management	Less than a year in USA, English class level
		1. 1 st try. Code mixing on writing	English wording reproduction, no possible translation: “cheaper” is a brand (14-

<p>shoe and store). 5. 1st try. Wrong percentage notion (use of 25 as the price of a shoe). No units used.</p>	<p>vii. F: Hablamos en español. ¿Entiende inglés por eso? viii. J: ¿Eh? ix. F: ¿Entiende inglés? x. J: No. xi. F: ¿Nada? xii. [Julia shakes her head saying no] xiii. ¿Leído tampoco? xiv. [Julia shakes her head saying no] xv. F: ¿Nada? ¿Cuándo llego aquí? xvi. J: Octubre. [Dialogue continues with other questions –see “Historical bilingual profile“ above–. Transcription continues here:]</p>	<p>26) 3. 1st try. Code mixing (store)</p>	<p>English wording imitation</p>
<p>1-29. 1st try. [25+25=50%] Wrong answer due to deviated percentage notion. Addition of percentages influenced by “unbeatable prices” (102-114)).</p>	<p>1  [0 try crossed out: 40% + 25% = 65%]</p>	<p>3-25. 1st try. Deviated wording question understanding</p>	<p>Deviated English sentence construction, unknown meaning of some words (8-10)</p>
<p>47-81. 2nd try. Division of 40% by 2 due to “unbeatable prices” influence (102-114).</p>	<p>2 F: ¿Cómo resolvió la actividad? 3 J: Me preguntan cuánto es en dos store de [pause] de [pause] mmm [pause] de zapatos 4 F: Mm [continuing conversation]. 5 J: Cuánto son en dos. Y si el precio total es de un zapato es veinticinco en dos van a ser cincuenta. 6 F: ¿Cincuenta por ciento? 7 J: Mm. [validating, low voice] 8 F: Pero lo que le pregunta es, a ver, esto cree que es [pause] eh [pause] Bueno, de hecho la pregunta es ¿En cuál de éstas dos tiendas, ¿sí?, son los zapatos...? ¿Cheaper sabe qué quiere decir? 9 J: No.</p>	<p>8-10.1st try. Wording question translated by the interviewer</p>	<p>Deviated meaning given by Julia</p>
<p>56-61. 2nd try. Percentages as absolute value.</p>	<p>10 F: Más baratos. En cuál de las dos tiendas son los zapatos más baratos. Esa es la pregunta, ¿sí? Pero bueno, dígame qué es lo que ha hecho. 11 J: Sumé veinticinco más veinticinco. 12 F: Ajá. 13 J: Y me dio cincuenta. 14 F: Mm [validating]. ¿Y luego qué dice? [Reading] “Cincuenta [por ciento] en dos shoes cheaper”. ¿Qué creía que significaba shoes cheaper?</p>	<p>8-10. 1st try. “cheaper” is given an alternative meaning</p>	<p>Wording makes sense (somehow)</p>
<p>61-77. 2nd try. Deviated expressions in</p>	<p>15 J: Más barata. 16 F: ¿Significaba eso, creía antes? ¿Creía eso antes? 17 J: No. 18 F: ¿Por qué puso entonces... ? ¿Qué significa la frase esa?</p>	<p>14-26. 1st try. “cheaper” as the brand of the shoes</p>	<p>Wording makes sense (somehow)</p>
		<p>30-35. 1st try. English linked to reading language</p>	<p>Spanish dominant</p>
		<p>38-43. 2nd try. Right wording understanding</p>	<p>Interviewer's interaction</p>
		<p>39, 41. Deviated syntax on statement translation</p>	<p>Picture with just one shoe</p>

<p>relation with percentages: unclear notion of percentage.</p> <p>38-83 . 2nd try. [shoes cost 40 on 40% store, 50 on 25% store] Wrong answer due to wrong application of percentages (which are not regarded as the portion to be discounted but as the portion of the price that should be payed: 81-87) and because the percentage in 40% store divided by 2 (due to "unbeatable prices").</p>	<p>19 J: Pues es que no me la sé.</p> <p>20 F: Ajá. ¿Pero por qué lo puso entonces si no sabía, no sabía el significado?</p> <p>21 J: Nomás.</p> <p>22 F: ¿Qué creía que era cheaper?</p> <p>23 J: No sé, no me lo imaginaba que era más barata.</p> <p>24 F: ¿No? ¿Pero qué pensaba que podía significar eso?</p> <p>25 J: Pues la marca de los zapatos.</p> <p>26 F: ¿La marca? [Julia nods] ¡Aah! ¿Y antes por qué me sumó estos dos? ¿No? Porque antes de hacerme esto me sumó estos dos [see A1,1]. ¿Por qué?</p> <p>27 J: Porque pensé que decía que cuántos eran en estos dos, cuál era el porciento.</p> <p>28 F: Ajá.</p> <p>29 J: De esos dos.</p> <p>30 F: ¿En qué idioma empezó a pensar el ejercicio? [Pause] ¿En inglés o en español?</p> <p>31 J: En español.</p> <p>32 F: Ajá. ¿Y cambió luego otra vez a inglés?</p> <p>33 [Julia shakes her head saying no]</p> <p>34 F: ¿Cuándo cambió a inglés? ¿No cambió o sí?</p> <p>35 J: No.</p> <p>36 F: ¿Quiere intentar hacer la actividad otra vez, sabiendo eso?</p> <p>37 J: [Julia nods] Sí.</p> <p>38 F: Okay. ¿Sí entendió, ¿no?, entonces, ahora, qué es lo que significa la pregunta?</p> <p>39 J: Qué en cuál de estas dos tiendas es el más barato.</p> <p>40 F: Mm [validating].</p> <p>41 J: Sale más barato...</p> <p>42 F: Los zapatos.</p> <p>43 J: Mm [confirming]. ¿Lo hago otra vez aquí?</p> <p>44 F: Mm [validating]. O en la otra hoja, si quiere.</p>	<p>('zapato más barato')</p>		
			45. 2 nd try. Spanish as writing language	Spanish dominant, interviewer interaction
			45. Syntax deviance (por que, ha)	Quick writing, unknown
			45. spelling variation (sapatos)	Homophony(specially in Mexican Spanish)
			47, 56-61, 77. 2 nd try. Price and percentage used with similar meanings	Unclear percentage concept
			61-77. 2 nd try. Deviated expressions in relation with percentages ('porcentaje total' , 'porcentaje de la mitad')	Unclear notion of percentage
	81-87. 3 rd try. Awareness of confusion between cuentan-descuentan: percentages regarded as the portion to be discounted, not as the portion of	<p>45 [2nd try written answer]</p> <p>John sports 40% 20 Mike sports 25% 20</p> <p>Es en la John sports por que si compro dos zapatos cuentan 40, y si compro en la Mike sports me va ha costar 50.</p>	84-87 . 3 rd try. Written answer rephrased when reading with "cuentan"- 'descuentan' change and 'por ciento' addition	81. Introduction of specific vocabulary in dicourse (descuentan)

the price that should be payed.	47 J: Dividir cuarenta entre dos porque ese no era el precio total. Y en éste [25% store] sí me lo daban. 48 F: ¿Ese [40%] no era el precio total? 49 J: No.	89. 3 rd try. Introduction of own specific vocabulary in dicourse (rebajan)	Improved wording comprehensi on
88-97. 3 rd try. Percentage of discount (40) divided by 2.	50 F: ¿Qué era esto, el precio de qué? 51 J: De... [pause] No sé. 52 F: Bueno, dígame, dígame cómo lo ha hecho. 53 J: Dividí el cuarenta entre dos. 54 F: Mm [continuing conversation].	102-114. 2 nd -3 rd tries. Deviated translation ("unbeatable"- 'tabla de...')	Meaning given from part of the word (table)
97. 3 rd try. Direct-comparison of percentages.	55 J: Y me dio veinte. 56 F: ¿Y estos veinte qué eran? 57 J: Veinte por ciento. 58 F: ¿De qué? 59 J: De un zapato. 60 F: ¿Y qué significa veinte por ciento de un zapato? [Pause] ¿No? ¿Cómo, cómo, cómo lo mira esto luego para comparar si es más barato esto o esto?	114-119. 2 nd -3 rd tries. English linked to reading language	Spanish dominant
98-101. 3 rd try. Discount as stated on the wording pictures (40 is not divided by 2 on the 40% store).	61 J: Pues es más barato esto [40% store], porque en esto [40% store] no te dan el precio, el porcentaje... [pause] mmm.... [pause] el porcentaje total. 62 F: Mm. [continuing conversation] 63 J: Y en éste [25% store] sí te lo dan. 64 F: ¿Y éste no es el porcentaje total? 65 J: No. 66 F: ¿De qué es el porcentaje esto, entonces? ¿O es el porcentaje de la mitad o cómo funciona? 67 J: Del, que es de la mitad.		
84-101 . 3 rd try. Wrong answer due percentages as absolute value instead of relative (direct percentages comparison).	68 F: ¿Éste es el porcentaje de la mitad? 69 J: Mm [validating]. 70 F: ¿Y éste [25%] es el porcentaje de qué? 71 J: Es el total. 72 F: El total. ¿Entonces esto [40%] luego lo divide entre dos? 73 J: Mm [validating]. 74 F: ¿Por qué? [pause] ¿Por qué lo ha dividido usted entre dos? Simplemente dígame porqué lo ha dividido. Pues yo pensaba dividirlo entre dos porque... 75 J: Mmm... [pause] Porque... [pause] Porque si cuarenta es el... [pause] cuarenta es un entero, quiero ver cuánto, cuánto vale la mitad. Lo dividí entre dos y me salió veinte.		
102-114. 2 nd - 3 rd tries. Influence of "Unbeatable prices" on the treatment of percentages (dividing or	76 F: Mm [continuing conversation]. ¿Y cómo utiliza luego ese veinte? 77 J: Lo utilizo... esto lo utilizo para saber el precio, el porcentaje que... [pause] el porcentaje total. 78 F: Mm [continuing conversation]. 79 J: Y ya. 80 F: Y luego, a ver... ¿Con el Mike, qué pasa con el veinticinco? 81 J: Esto sí son el veinticinco por ciento que te descuentan. 82 F: Mm [continuing conversation].		

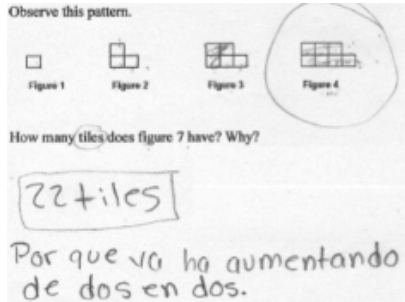
<p>multiplying them by 2).</p> <p>120-121. Solving process quit.</p>	<p>83 J: Y ya.</p> <p>84 F: Entonces, ¿me puede leer la respuesta?</p> <p>85 J: [Reading] Es la John Sport porque si compro zapatos me... [pause] ¡Ah!, aquí no era cuentan.</p> <p>86 F: ¿Qué era? [pause]</p> <p>87 J: Si compro dos zapatos me, me descuentan [pause] el cuarenta por ciento. Y si compro en la Mike Sports me, me descuentan el veinticinco por ciento. [pause]</p> <p>88 F: Entonces, ¿cómo? ¿Lo quiere cambiar eso, o alguna palabra o algo?</p> <p>89 J: Sí. Es en ésta [25% store] en que, en que te rebajan más.</p> <p>90 F: En el veintici... ¿En ésta te rebajan más?</p> <p>91 J: Sí.</p> <p>92 F: ¿Por qué?</p> <p>93 J: Porque te dicen en cuál de las dos tiendas te rebajan, te sale más barato.</p> <p>94 F: Mm [validating].</p> <p>95 J: Y es en ésta [25% store].</p> <p>96 F: ¿Por qué?</p> <p>97 J: Porque aquí [40% store] te rebajan el veinte por ciento y aquí [25% store] el veinticinco por ciento. Te rebajan más aquí [25% store] que aquí [40% store].</p> <p>98 F: ¿Y aquí [40% store] te rebajan la mitad de lo que dice ahí [wording]? ¿En el John Sports [40% store]?</p> <p>99 J: No.</p> <p>100 F: ¿Sí o no? [Pause]</p> <p>101 J: No.</p> <p>102 F: ¿Por qué pensó antes que le rebajaban la mitad?</p> <p>103 J: Porque pensé que ese no era el porcentaje que te rebajaban.</p> <p>104 F: ¿Pero por qué motivo? ¿Qué vio o qué le hizo pensar eso que no... que no era el final sino que tenía que dividirlo entre dos?</p> <p>105 J: En esta palabra, price.</p> <p>106 F: ¿Entonces, esta palabra que le indicó, price?</p> <p>107 J: Que quería saber el precio total.</p> <p>108 F: Esto le indicó que esto era el precio total [25% store]. ¿Y aquí [40% store] como no estaba esta palabra, pensó que éste era sólo la mitad?</p> <p>109 J: Ajá.</p> <p>110 F: Y tenía que dividir esto entre dos. [Announcement interruption] ¿Qué significa esto, unbeatable prices? [Pause] ¿Lo sabe? ¿Por qué no lo ha pregu...?</p> <p>111 J: Es la tabla de... [pause] No lo sé.</p> <p>112 F: ¿Por qué no lo preguntó? [Pause] ¿Lo quiso resolver sin preguntarlo?</p> <p>113 J: Sí.</p> <p>114 F: O sin saberlo. Sí, a veces se pueden resolver los problemas sin saberlo. De hecho significa, unbeatable prices, son como precios insuperables. Esta tienda es como un anuncio publicitario, ¿no? Precios insuperables, que nadie nos puede ganar con nuestros precios. Nadie ofrece precios más baratos que los nuestros. ¿Sí? [Pause] ¿Qué pensó con todo esto en inglés? [Pause] ¿Qué parte del proceso pensó en</p>		
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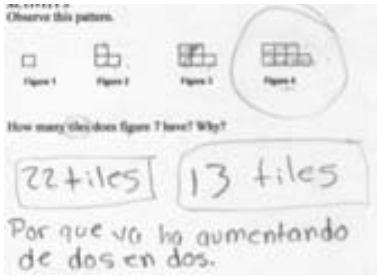
	inglés? ¿Pensó alguna cosita en inglés [pause] mientras estaba resolviendo el problema? 115 J: No. 116 F: ¿Se fijó en esto de prices y lo tradujo al español o pensó en inglés con prices, por ejemplo? 117 J: Lo traducí al español. 118 F: Ajá. ¿Y siguió pensando todo en español? 119 J: Sí. 120 F: ¿Quiere volver a apuntar la respuesta? ¿O lo dejamos así? 121 J: Así. [Continues in A2, 2]		
M	0 try. The initial thought (addition of percentages) not modified (or just slightly) across tries.		
E	i-xvi. Low self-assessment of her English level.		
M	36-37. Effort to get the right solution when she knows it is not correct, but later (121) Julia does not want to think about the solving process anymore.		
O	39, 41. Deviated syntax on statement translation ('zapato más barato'). 38-83. Apparently there is a price-percentage confusion but in fact this is due to a count-discount confusion. I.e., percentages are not regarded as the portion to be discounted but as the portion of the price that should be payed. So the use of the symbol % is omitted on the answer (45). 61-77. Deviated expressions in relation to percentages ('porcentaje total' , 'porcentaje de la mitad') show that the notion of percentage is not clear. 81. Introduction of vocabulary not present in the wording but related with its contents denotes a better insight on the wording interpretation. 98-101. Discount as stated on the wording pictures (no division by 2 in 40% store) after a closed question is asked by the interviewer; idea not retaken later on solving process.		

Math	A2. Julia marked "English and Spanish".	Language	Tentative
1-74. 1 st try. [Por que el perimetro es la suma de todos los lados]. No perimeters comparison.	1 2 [Comes from A1,121] F: ¿Qué hizo aquí, a ver? 3 J: Saqué el, el... Sumé los lados del cuadrado. 4 F: Mm [continuing conversation]. 5 J: Y me dio veinte. Porque el perímetro significa, es la suma de todos los lados. 6 F: Okay. 7 J: De una figura. 8 F: Mm [continuing conversation]. ¿Y luego? 9 J: Sumé los lados, como tiene cuatro, sumé cinco más cinco y me dio veinte. Y ese es el perímetro.	1. 1 st , 2 nd tries. Spanish as unique writing language	Spanish dominant
1-9. 1 st try. Right square's perimeter calculation.	10 F: Okay. ¿Y después de esto qué hizo? 11 J: ¿En éste [circle]? 12 F: Mm [validating]. No sé. ¿Cómo siguió? ¿Después de éste ya pasó aquí? 13 J: Sí. 14 F: ¿O pensó alguna cosa más entremedio? 15 J: No.	1-74. 1 st , 2 nd tries. Not complete understanding of the wording	i-xvi. Low English language management
17, 34-37. 1 st try. [5+10=15 : diameter +	16 F: Okay. ¿Qué hizo aquí pues? 17 J: Aquí... [pause] Para sacar el perímetro es la suma de todos los lados. Pero sumé cinco más la circunferencia, que sería diez. Y cinco más diez son quince.	26-33. 1 st try. English linked to reading language	Spanish dominant
		37. 1 st try.	Good

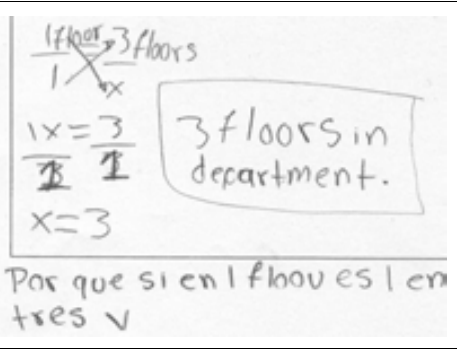
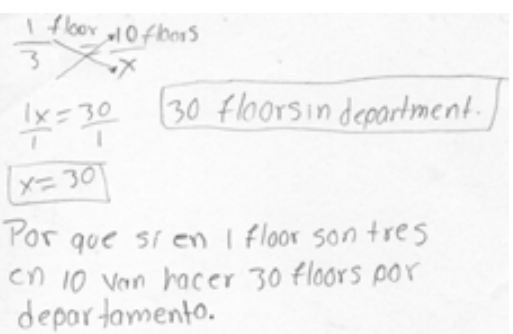
circumference = circle's perimeter]. Inclusion of diameter in circle's perimeter.	18 F: Mm [continuing conversation]. [Pause] 19 J: No le puse qué... qué unidades porque no tiene. 20 F: Ajá. Okay. ¿Y luego qué hizo? 21 J: Ya. 22 F: Dijo que... 23 J: [interrupting] Lo expliqué. 24 F: Anotó la respuesta, ¿no? 25 J: Sí. 26 F: ¿En qué idioma empezó a pensar el, el problema? 27 J: En español. 28 F: ¿En español pensó? ¿Y cuándo cambió a inglés? 29 J: Nunca. 30 F: ¿Nunca cambió? 31 [Julia says no with her head] 32 F: ¿Solamente español? ¿Puro español? 33 J: Sí. 34 F: Okay. ¿Me dijo que, que la circunferencia, esto de fuera es diez, no? 35 J: Sí.	Right use of math vocabulary ('diámetro')	Spanish CALP
18-19. 1 st try. Awareness of no units given on statement. As a result no units on answer.		38-39. 2 nd try. Math vocabulary confusion (perimeter-area)	Perimeter notion and perimeter's formulas not consolidated enough, notion checked with interviewer
1-25, 34-37. 1 st try. Wrong answer due to perimeter as addition of all sides (inclusion of diameter in circle's perimeter) and no perimeters comparison.	36 F: ¿Y luego por qué le añadió este cinco? 37 J: Porque esto es el diámetro del círculo. 38 F: Ajá. ¿Y el perímetro qué es? 39 J: ¿La suma de [pause] lo que le cabe adentro? 40 F: ¿En un cuadrado me ha dicho que el perímetro qué era? 41 J: Es la suma de todos los lados. 42 F: Ajá. ¿Y aquí? 43 J: La suma de es..., de toda la circunf... ¿de la circunferencia? 44 F: ¿La suma de lo de afuera? 45 J: Mm [validating]. 46 F: ¿Y le tenemos que añadir también lo de dentro? [pause] 47 J: No [Julia shakes her head saying no]. 48 F: No. Hay que contar sólo lo de fuera. ¿Sí? Ehm... El perímetro de cualquier figura, ¿no?, por ejemplo el perímetro de la mesa, pues va a ser esto de... [interviewer follows the perimeter of the surface of the table with the finger] es el contorno, ¿no? Entonces aquí [following the circle's perimeter with the finger] también es el contorno de afuera, ¿no? No es, no es lo de adentro. Pero el contorno de fuera, de esto [following the circle's perimeter with the finger], no es diez, ¿no? ¿Se acuerda de cómo lo calculábamos, el... el contorno de la... circunferencia? 49 J: La circunferencia [Pronounced at the same time with interviewer]... es pi por radio. No, pi por diámetro. [Pause]	59-72. 2 nd try. English almost exclusively as reading language	Spanish dominant
38-39. 2 nd try. Math vocabulary confusion (perimeter- area).		63-72. 2 nd try. English with circle's perimeter formula	Topic reviewed in English at School
40-48. 2 nd try. Right notion of perimeter.	50 F: ¿Está segura? [Pause] 51 J: Sí.		

<p>48. 2nd try. Perimeters' identification on physical objects (by interviewer).</p> <p>48 (last 2 sentences). 2nd try. Circle's perimeter is not 10 (by interviewer).</p> <p>49-58. 2nd try. Right calculation of circle's perimeter through application of formula (48: method induced by interviewer).</p> <p>34-58. 2nd try. Incomplete answer with right calculation of perimeters but no comparison.</p>	<p>52 F: ¿Lo pue[de], lo quiere hacer así?</p> <p>53 J: Sí.</p> <p>54</p> <p>55 F: ¿Entonces cómo pensó esta última parte?</p> <p>56 J: Porque la circunferencia es pi por diámetro y el diámetro es de aquí hacia acá [pointing to the circle's diameter on the picture].</p> <p>57 F: Mm [continuing conversation].</p> <p>58 J: Y cómo aquí me dan [pointing to the circle's diameter on the picture], todo lo del diámetro, es cinco. Y tres punto catorce porque eso es lo que equivale pi.</p> <p>59 F: Okay. ¿Y eso lo pensó en español?</p> <p>60 [Julia nods]</p> <p>61 F: ¿Todo o pensó alguna cosita en inglés?</p> <p>62 J: [Julia shakes her head saying no] En español.</p> <p>63 F: ¿Puro español? ¿La fórmula también se la sabe en español?</p> <p>64 J: No.</p> <p>65 F: ¿La fórmula en inglés?</p> <p>66 [Julia nods]</p> <p>67 F: ¿Se la sabía antes cuándo estaba en Mexico en español? ¿O la aprendió aquí?</p> <p>68 J: Me la sabía.</p> <p>69 F: Ajá. ¿Pero la pensó en español ahora? Perdón. La pensó en inglés, me dijo.</p> <p>70 J: Sí.</p> <p>71 F: ¿Como esta cosita que pensó, este pequeño detalle que pensó en inglés, alguna otra cosita aquí pensó? Por ejemplo square en lugar de cuadrado, círculo, circle... ¿Alguna cosa pensó en inglés?</p> <p>72 J: No, nomás esto.</p> <p>73 F: ¿Y aquí [A1] tampoco?</p> <p>74 J: No. [Continues in A3, 2]</p>		
<p>MEMO</p>	<p>17. Maybe 'Pero' indicates there is an error on the mathematical solving process.</p> <p>37. Right use of math vocabulary ('diámetro') leads Julia to remember the right concept of perimeter.</p> <p>38-39. Her math vocabulary confusion (perimeter- area) is the result of a guess and she checks strategy to know what the perimeter is indeed, seems to be related to the confusion when calculating the circle's perimeter.</p> <p>1-74. The interviewer never asks for a comparison of both perimeters, neither Julia compares them. This fact along with the written justification of the answer inform that Julia has not completely understood the question (the comparison).</p>		

Math	A3. Julia marked “English and Spanish”.		Language	Tentative
<p>1, 17-26. 0 try. Wrong answer due to deviated question understanding.</p> <p>1-7. 1st try. Wrong answer due to arithmetical sequence associated to figure pattern with right increasing (2 tiles per figure) but sequence not followed properly.</p> <p>12-14. 2nd try. Awareness of mistake when counting during oral explanation.</p> <p>1-16. 2nd try. Right answer with arithmetical sequence associated to figure pattern.</p>	<p>1</p>  <p>How many tiles does figure 7 have? Why?</p> <p>22 tiles</p> <p>Por que va ha aumentando de dos en dos.</p> <p>2 [Comes from A2,74] F: ¿Qué hizo aquí? 3 J: Aquí me pedía cuántos cuadritos, había, hay en la séptima figura. 4 F: Ajá. 5 J: Y son veintidós. 6 F: Mm [continuing conversation] 7 J: Porque va aumentando de dos en dos. 8 F: ¿Y por qué son veintidós? A ver. ¿Si va aumentando de dos en dos, son veintidós? ¿Me lo puede enseñar esto? 9 [Julia nods] 10 J: Porque aquí son tres cuadritos, acá aumentó dos cuadritos. En esto aumentó dos y entonces son cinco – dos, cuatro, cinco; dos cuatro – cinco y aumentaron dos. En la cuarta figura son siete cuadritos. En la quinta iban a ser nueve cuadritos [counts the tiles on the figures]. En la sexta once cuadros. [Pause] 11 [Interviewer nods] 12 J: Y en la séptima trece cuadritos. 13 F: Ajá. 14 J: Lo conté mal. 15 F: ¿Me lo puede cambiar, pues?</p> <p>[Julia circles figure 4 at the beginning, when she understands the question in another way, see A3,17-18. She circles the word “tiles” from the wording later (A3,38)]</p>	<p>1, 16, 37-38. 1st, 2nd tries. Spanish as writing language with code mixing (tiles)</p> <p>1, 16. 1st try. Deviated Spanish syntax /spelling error (por que)</p> <p>1, 16. 1st try. Deviated Spanish syntax (ha)</p> <p>18. 0 try. Deviated wording question understanding</p>	<p>Spanish dominant, 39-40: English wording question, quick writing (no need of translation) or unknown translation</p> <p>Homophony</p> <p>Quick writing or unknown</p> <p>Understanding of some words from the question, problem solved according to deviated meaning (0)</p>	

	<p>16 </p> <p>[Julia writes down “13 tiles”. She circles the word “tiles” from the wording later (A3,38)]</p> <p>17 F: ¿Por qué me puso este círculo aquí al principio [A3,1 (0 try)]?</p> <p>18 J: Porque pensé que me preguntaban en cuál figura eran, había siete cuadritos.</p> <p>19 F: ¿Y luego cómo supo que no le preguntaban eso?</p> <p>20 J: Porque me le fijé aquí que cuántos habían en la figura siete.</p> <p>21 F: ¿Leyó la pregunta otra vez?</p> <p>22 J: Mm [validating].</p> <p>23 F: ¿Y la entendió de otra manera?</p> <p>24 J: [Julia nods.] Sí.</p> <p>25 F: Nada más entonces al principio pensó que qué figura tenía siete cuadritos entonces pensó que la respuesta era esa, ¿sí?</p> <p>26 J: Sí.</p> <p>27 F: Okay. [Pause. Interviewer explained to Julia that she had to think starting from the beginning, in the first attempt she made to solve the problem (part not transcribed).] ¿Con qué idioma empezó a pensar eso?</p> <p>28 J: Español.</p> <p>29 F: ¿En español?</p> <p>30 J: Sí.</p> <p>31 F: ¿Y luego cuándo cambió a inglés?</p> <p>32 J: Nunca.</p> <p>33 F: ¿Nunca?</p> <p>34 [Julia says no with her head.]</p> <p>35 F: ¿No se acuerda de que utilizó el inglés para nada?</p> <p>36 J: No.</p> <p>37 F: ¿Pero en la respuesta sí la hizo con inglés? [Pause]</p> <p>38 J: Namás en esto [circles the word “tiles” in the wording: A3,16], puse.</p> <p>39 F: Ajá. ¿Por qué?</p> <p>40 J: Porque me pregunta cuántas tiles [tailes/] había en siete figuras. Y porqué.</p> <p>41 F: ¿Y por qué me lo puso esto en inglés?</p> <p>42 J: No sé.</p> <p>43 F: ¿Alguna otra cosita pensó en inglés?</p> <p>44 J: No.</p> <p>45 F: ¿No? ¿Todo en español?</p>		<p>try)</p> <p>27-36. All tries. English almost exclusively as reading language</p> <p>40. Code mixing (tiles)</p> <p>40. Deviated question translation</p> <p>43-48. English as thinking language with “figure”</p> <p>Spanish dominant</p> <p>English wording imitation</p> <p>Understanding of some words from the question, quick translation (2: done correctly before)</p> <p>Wording picture in English</p>
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	<p>46 J: [Julia says no with her head.] Figura.</p> <p>47 F: ¿Figure? ¿Cuándo lo estaba resolviendo estaba pensando en figure?</p> <p>48 J: Sí.</p> <p>49 F: Okay. ¿Alguna otra cosita se acuerda?</p> <p>50 J: No. [Continues in A4, 3]</p>		
MEMO	<p>1, 16. Same structure in answer.</p> <p>27-50. Residual English use.</p> <p>40. Code mixing (tiles)</p> <p>43-48. Does not remember her use of English easily. Linked with blurry awareness of kind of language use.</p> <p>27-43. Transparency.</p>		

Math	A4. Julia marked "English and Spanish".	Language	Tentative
1.0 try. Proportion solved correctly.		1,2. English and Spanish as writing languages	Spanish dominant, 11-14, 33-37: unknown meaning
1. 0 try . Wrong answer due to wrong horizontal mathematization (proportion of floors).	1 	1-2. 54-58. Code mixing on writing (floor)	11-14, 33-37: unknown vocabulary
2. 1 st try. Proportion (not the one on the 0 try) solved correctly.	2 	1,2. Spanish syntax deviance (hacer)	Unknown, homophony
2-9, 37-38. 1st try . Wrong answer due to wrong horizontal mathematization (proportion of floors).	3 [Comes from A3,50] F: ¿Cómo hizo aquí? 4 J: Pues hice una proporción. 5 F: Ajá. 6 J: Que en un floor son tres. 7 F: ¿En un floor son tres qué? 8 J: Mmm... [pause] Floors. 9 F: ¿En un floor son tres floors? 10 J: Sí. 11 F: ¿Qué es un floor? 12 J: No sé. En un floor son tres departamentos y... 13 F: ¿Qué es un floor? [Pause] ¿Lo sabe o no? 14 J: No. 15 F: ¿Departamento, qué es un departamento? 16 J: Es, es un... [pause] ¿Cuartos? 17 F: Sí, puede ser cuartos. ¿Pero en este caso se refiere a cuartos? [Pause] 18 J: No sé. 19 F: Departamentos se refiere, por ejemplo the children's department or the jewelry department, o the toys	1, 2. English deviated syntax (floors in department)	English not enough consolidated
		1-19, 33-37. Deviated wording understanding (deviated meaning of some words: floor, department,...)	Syntax and grammatical (does) deviances, mathematical approximation to the problem found according to deviated understanding
		1-2, 11-14, 33-37. Unknown vocabulary (floor)	i-xvi. Low auto English language management
		6, 8, 38. Code mixing	Quick talk: no need of

<p>department [interviewer points to the wording problem, where these names appear], que está por aquí también, ¿no? ¿Qué significa? Toys department es el departamento de juguetes. Es... ¿Porque dónde está Jamie?</p> <p>20 J: Acá está, la palabra.</p> <p>21 F: Ajá. Pero Jamie digo, ¿no?</p> <p>22 J: ¡Ooooh!</p> <p>23 F: ¿Cómo leyó eso?</p> <p>24 J: En español.</p> <p>25 F: ¿Cómo lo interpretó?</p> <p>26 J: Jaime [in Spanish].</p> <p>27 F: ¡Oh Jaime [in Spanish] lo pensó! Pero luego nos pone she.</p> <p>28 J: Ella.</p> <p>29 F: Ajá. Entonces es Jamie, porque está en inglés.</p> <p>30 J: Mm [validating].</p> <p>31 F: ¿Dónde está Jamie? [Pause]</p> <p>32 J: ¿Se fue de compras a largo departamento?</p> <p>33 F: Sí. A un grande. Un departamento comercial que es muy grande, ¿no? Con varios... [pause] with many...</p> <p>34 J: Con varios...</p> <p>35 F: Floors, ¿Qué son floors?</p> <p>36 [Julia says no with her head.]</p> <p>37 F: Varios pisos, ¿no? Varios pisos. [interviewer makes a movement with the hands, simulating many floors of a building.] Es un departamento muy grande que tiene muchos pisos. ¿Qué más? [Pause] ¿Dónde nos dice qué en cada piso hay tres departamentos?</p> <p>38 J: Aquí [pointing to wording, around “up three floors”]. En un floor hay [pause] yo interpreté uno sobre tres.</p> <p>39 F: ¿Ha entendido lo que está haciendo Jamie en departamen... en el centro comercial? ¿Ha entendido qué es lo que nos describe el enunciado, qué es lo que pasa?</p> <p>40 J: ¿Que no encuentra la salida?</p> <p>41 F: Sí la encuentra, al final. Pero no es lo que está buscando, la salida.</p> <p>42 J: ¿Está buscando cuántos... cuántos niveles hay en... hay en el departamento?</p> <p>43 F: ¿Eso es la pregunta? [Pause] ¿O qué es eso? ¿Entiende la pregunta?</p> <p>44 J: Un poco.</p> <p>45 F: La pregunta del final: How many floors does the department store have?... Cuántos, how many, cuántos [pause] floors son...</p> <p>46 J: Pisos.</p> <p>47 F: Pisos o niveles, como dijo antes, ¿sí?. Does the department store have. And why. ¿Qué significa esta pregunta?</p> <p>48 J: Cuantos pisos... [pause] Ésta [does] no me la sé, el departamento...</p> <p>49 F: Tiene.</p> <p>50 J: Tiene.</p> <p>51 F: Cuántos pisos tiene el departamento. Esto es un verbo auxiliar, ¿no? Cuando... siempre que hacemos preguntas se pone el verbo. Cuántos pisos tiene el centro comercial. Y nos pide luego el porqué.</p>	<p>(floors)</p> <p>15-19. Deviated translation (department)</p> <p>19-30. Letters rotation corresponding to language change (Jamie-Jaime)</p> <p>31. Right understanding of the 1st wording sentence</p> <p>37-38. Deviated wording interpretation ('uno sobre tres')</p> <p>39-51. Deviated wording and wording question understanding</p>	<p>translation, 11-14, 33-37: unknown meaning</p> <p>Translation not accurate, but acceptable enough to solve the problem</p> <p>Spanish dominant, quick reading, low English syntax</p> <p>(At least) partial wording understanding</p> <p>Partial wording understanding (up, three).</p> <p>Multiple unknown words, syntax and grammatical deviances, mathematical approxim-</p>
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	<p>52 F: ¿Quiere que lo revisemos un poco y lo vuelve a hacer o lo dejamos así?</p> <p>53 J: Lo dejamos así.</p> <p>54 F: ¿Sí? Okay.</p> <p>55 F: ¿En general cuándo ha usado el inglés para resolver las preguntas esas?</p> <p>56 J: ¿Aquí en ésta pregunta [pointing to the question in A4]?</p> <p>57 F: Sí. ¿Para qué lo utilizó?</p> <p>58 J: Para poner... para poner los pisos que hay en el departamento. [Continues in GLQ,1]</p>		<p>ation to the problem found</p>
MEMO	<p>1-2. Deviated syntax in English</p> <p>1-2. Deviated orthography, syntax in Spanish</p> <p>1-2. Proportion changed from 0 to 1st try</p> <p>15-19. Deviated translation (department)</p> <p>Partial wording understanding, yet a mathematical solution is found. This, somehow, validates the wording understanding.</p> <p>Wording and wording question understanding, along with tiredness is a too big obstacle to continue with the mathematical solving process.</p>		

General Language Questions		Language	Tentative
1	[Comes from A4,58] F: Sí. ¿Para qué más lo utilizó en general? Por ejemplo, usted dice yo siempre utilizo el inglés para esto. O si he utilizado el inglés cada vez que he encontrado eso.	5-10. English as writing language	8. English learning purposes
2	J: Nada más.		
3	F: ¿Nada más para eso?		
4	[Julia nods.]		
5	F: Bueno, aquí también ha encontrado una... alguna cosita. Pero bueno... ¿No se le ocurre nada más por lo que cree que haya utilizado el inglés?	5-14. English for learning (writing and meaning) purposes	English not consolidated enough.
6	J: Para..		
7	F: [interrupting] ¿Por qué...? ¿Para?, perdón.		
8	J: Para aprender inglés.		
9	F: Ajá.		
10	J: Para poderlo escribirlo.	17-18. Spanish for everything	22-26. Home language, first language
11	F: Mm [validating].		
12	J: Para saber lo que significa.		
13	F: Sí. ¿Por eso utiliza el inglés para escribir?		
14	J: Mm [validating].		
15	F: ¿Qué más?	31-46. Unknown vocabulary (cheaper, which, leaves, which, toy)	English not consolidated enough
16	J: Nada más.		
17	F: ¿Y en general cuando ha utilizado el español?		
18	J: En todo.		
19	F: ¿En todo, lo otro?		
20	[Julia nods.]		

<p>21 F: ¿Y por qué cree que ha utilizado el español en todo lo otro? 22 J: Porque fue mi primer idioma que aprendí. 23 F: Ajá. ¿Alguna otra razón? 24 J: Y porque mis padres lo hablan. 25 F: Ajá. 26 J: Mis hermanos... Toda mi familia lo habla. 27 F: Ajá. Entonces, eh... utiliza el español. 28 J: Sí. 29 F: Eh... ¿Alguna otra cosita? 30 J: No. 31 F: ¿Hay alguna palabra o frase que haya encontrado difícil en inglés, además de las que ya hemos comentado? ¿Alguna otra cosita que le haya resultado difícil por el hecho de estar escrita en inglés ? 32 J: [Julia nods] Sí. 33 F: ¿Como cuál? 34 J: Como ésa. 35 F: Cheaper. Ajá. ¿Qué más? 36 J: Ésta palabra. 37 F: Which. Ajá. En cuál, hemos dicho. [Pause] 38 J: Y... [pause] Esto. 39 F: Leaves. ¿Sabe qué significa? 40 [Julia says no with the head.] 41 F: Deja, ¿no?, o abandona. 42 J: Sí. Toy. 43 F: ¿Un toy no sabe qué es? 44 [Julia says no with the head.] 45 F: Juguete. Un juguete. 46 J: Y ya.</p>		
MEMO	31-46. Unknown vocabulary (cheaper, which, leaves, which, toy)	

Julia has a good Spanish BICS because she communicates orally and written in Spanish. Her Spanish CALP is not that good because she uses Math vocabulary properly in A2 and A4, but not so accurately in A2 and in A1. Julia needs to improve her English BICS and CALP, as she recognizes at the beginning of the interview (A1,i-xvi). All the comprehension difficulties as well as the limited use of correct English can be appreciated through all activities.

Despite all of it, Julia makes mathematical sense of all problems and gets an answer in all of them -even if it was not correct. On the one hand, she

does not solve A1, A2 or A4 correctly, where Julia is able to apply her mathematical knowledge according to her deviated wording understanding. She has major wording comprehension difficulties in A1 and A4 which lead her to a deviated answer. In A2 she gets closer to the solution when she calculates the perimeter as explained in class, after remembering the circle's perimeter formula, and only a comparison of both perimeters is needed to get a completely right answer. On the other hand, A3 is solved correctly once Julia is able to correct her initial deviated question understanding and gets the right mathematical procedure by herself.

A1 is solved with English linked to reading language (besides the code mixing on writing which is not mentioned by her during the interview). Julia does not have a clear notion of percentages as shown during all the different tries made to solve the activity. Initially she writes "50% en dos shoes cheaper". Here there are many problems. Firstly, she thinks that cheaper is the brand of the shoes. Secondly she thinks about the percentage as the part to be paid instead of as the portion to be discounted (this idea is maintained on the the 2nd try). In addition, she does not understand the meaning of "Unbeatable prices" ('tabla de precios'). She thinks this is related with the percentages shown, one being as stated but not the other one: she adds 25% twice on the 25% store (1st try) in the same way she later divides 40% by 2 on the other store (2nd try).

Once the activity's objective is correctly understood, percentages are directly compared. This issue is not abandoned on the whole solving process. The confusion between *cuentan-descuentan* –so percentages are regarded as the portion to be discounted– arises when reading the 2nd try answer. The correct meaning of "Unbeatable prices" is revealed by the interviewer at the end of the A1 dialogue, after many attempts to know why Julia makes that particular treatment of percentages. It seems that Julia understands the discount percentage in the intended way, but the solving process is not continued –maybe she gets tired of repeating the same explanation.

At the very beginning of A1 (0 try), Julia adds the percentages shown on both stores, which supports the idea of her unclear percentages notion.

A2 is solved with a predominant use of Spanish language. English is linked to the interpretation of the statement and to the application of the circle's perimeter formula (2nd try). Julia calculates correctly the perimeter of the square by adding all four sides. In the case of the circle she considers the diameter as part of the perimeter and says that the circumference's length is 10 (so circle's perimeter is 15). She does not use units as they are not present on the statement. She does not compare both perimeters. It is true that interviewer does not ask for this comparison, but this is likely to be due to a partial comprehension of the wording. When the interviewer asks for the notion of perimeter she replies with the notion of area. But when interviewer exposes the case of the square, Julia correctly interprets the perimeter of the circle. To be completely sure of this fact, interviewer shows

the perimeter with physical objects (table, ...) and leads Julia to think about the circle's perimeter formula. As she knows it, she correctly calculates the circle's perimeter. Again she does not compare both perimeters (neither interviewer asks for this comparison).

A3 is solved with English almost exclusively as a reading language. The statement word "picture" is the English influence while thinking. The other instance of English language use is a code mixing ("tiles") on writing. In fact, part of the answer is "22 tiles". Even if there is just an English word, this sentence could be written in English, but there is no reference to the English interpretation of the number 22. The justification is written in Spanish.

Initially Julia understands the wording question as to which figure has seven tiles. But then she corrects this deviated understanding by herself and understands it in the right way. To solve the activity she applies an arithmetical reasoning (arithmetical sequence associated to figure pattern), finding the right growth of 2 tiles per figure. She only makes a mistake when stating the final answer, which is corrected while she shows to the interviewer the steps followed.

A4 is solved with the answer partially written in English and partially written in Spanish (which includes a code mixing: "floors"). This code mixing is due to the unknown meaning of "floors", leading English to be also part of the thinking process. "Floors" plays a crucial role on the mathematical solving process: Julia does not completely understand the situation presented in the wording, neither the wording question. Even so, she produces a mathematical solving process using a well-known procedure, covered in class –she does a proportion on her two attempts to solve the problem (0 try, 1st try, with no interviewer interaction). The finding of a solution validates her deviated wording understanding. A translation of just some words, without giving the correct meaning to the sentence as a whole: 'uno sobre tres' (see A4,37-38) –probably involving the wording words "up" and "three"– is another cause for this deviated wording understanding. The confusion between Jamie and Jaime supports the idea of a partial wording comprehension and highlights the fact that Spanish is the dominant language used when thinking.

Deviated wording and wording question understanding has a central impact on all four mathematical activities. Mathematical production is deviated according to such deviated an understanding.

Activities' (Key ideas) summary

Object 36: Julia-First reduction (End)

- Confusion between “percentages” and “prices”; related with the interpretation of “discount”: percentages regarded as what needs to be paid instead ('cuentan') of what needs to be discounted ('descuentan').
- Deviated wording question understanding (“cheaper” is understood as a brand), with the possible influence of the picture (with just one shoe. This results in the addition of percentages which belong to different objects.
- Deviated translation (unbeatable-tabla de) has a central impact on the mathematical solving process.
- The initial thought of adding percentages is barely changed across the different tries of the solving process.
- Julia's introduction of vocabulary related with the wording (but not present in it) demonstrates a better comprehension of the wording and the situation presented in it. Thus she rephrases her written answer (2nd try) and advances towards an assimilation of the situation presented, overcoming the language and picture comprehension difficulties. However, she considers percentages as having an absolute value instead of relative.
- The deviated question understanding and a focus on the visual mode of the statement resulted in the lack of comparison of both perimeters.
- Perimeter-area confusion as a language issue coming from a struggle with the circle's perimeter. Good management of math vocabulary (circumference, diameter) helped to fix the circle's perimeter concept and to remember the formula to calculate its perimeter.
- Partially correct solution in relation with the notion of perimeters of two geometrical figures using mainly Spanish, with English only with the circle's perimeter formula.
- A deviated wording question understanding (figure with 7 tiles) results in a consequent deviated answer (mutual influence of once into the other); corrected by herself.
- Use of words from the wording (tiles –in the writing–, figure –while thinking–) when solving the problem.
- A deviated wording understanding due to unknown meaning of many words resulted in a mathematical solving process according to a studied procedure (proportions) and based on the deviated interpretation of “department” and “floors”.
- English wording comprehension problems in a dense wording along with a default use of Spanish as a thinking language. Reproduction of some English words in the written answer to support the mathematical explanation.

Historical profile		Bilingual profile (Spanish dominant) [cont]		Activity		
<ul style="list-style-type: none"> • 14 years old • English-Spanish class • USA until 1. Back to USA 8 months before interview • Likes California • Spanish readings • English readings • Spanish at home • Spanish with friends • Spanish at school • Homework help: sister (Yael) 		Code mixing in writing (A1: shoes, cheaper, A3: tiles, A4: floor/s)	1		x	x
		Code mix (A1: store, A4: floor)	1			x
		Deviated wording question understanding (A4: wording not understood neither)	1	x	0	x
		Deviated meaning (A1: cheaper, A4: floor)	1			
		English linked to reading language (A2.2: Circle's perimeter formula in English too, A3: 'figure', "tiles" in English too)	x	x	x	
		Deviated expressions related to percentage	2			
		Deviated writing expressions (in Spanish)	2	x		x
		Specific vocabulary introduced ('descuentan', 'rebajan')	3			
		Deviated translation (unbeatable-tabla de)	1-2			
		English as writing language	GLQ		2	
	English for learning purposes	GLQ		2		
	Spanish for everything	GLQ				x
	Unknown vocabulary (cheaper, leaves, which, toy)	GLQ				x
						x
		Letters rotation: Jamie-Jaime				x
Procedural profile		Conceptual profile				
<p>1.0 X Addition of percentages</p> <p>1.1 X Addition of percentages</p> <p>1.2 X Division of percentages</p> <p>1.3 ✓ Awareness of cuentan-descuentan confusion</p> <p>X Direct percentages comparison</p> <p>✓ Percentages applied to shoes as stated on the wording pictures</p> <p>X 40 divided by 2 on 40% store</p> <p>2.1 ✓ Calculation of the square's perimeter (addition of its sides)</p> <p>X Calculation of the circle's perimeter (inclusion of the diameter and wrong calculation of the circumference's length)</p> <p>X No comparison of perimeters</p> <p>2.2 ✓ Calculation of the circle's perimeter (application of formula)</p> <p>X No comparison of perimeters</p> <p>3.0 X/✓ [figure 4 circled]</p> <p>3.1 X ["23 tiles"] ✓ ["por que va aumentando de dos en dos"]</p> <p>3.2 X ✓ ["13 tiles"]</p> <p>4 X Proportion of floors</p>		<p>1.0 X Notion of percentage</p> <p>1.1 X Notion of percentage</p> <p>1.2 X Notion of percentage as an absolute value</p> <p>1.3 ✓ Percentages as the portion to be discounted</p> <p>X Notion of percentages as an absolute value</p> <p>2.1 ✓ Visual identification of square's perimeter</p> <p>✓ Square's perimeter calculation</p> <p>X Perimeter concept</p> <p>2.2 ✓ Perimeter notion</p> <p>3.0 ✓ Mathematical procedure according to wording understanding</p> <p>3.1 X Arithmetical sequence associated to figure pattern but with ✓ Increase (2 tiles per figure)</p> <p>3.2 ✓ Arithmetical sequence associated to figure pattern</p> <p>4 X Horizontal mathematization</p>				

Object 37: Julia-Second reduction

Object 38: Ingrid-First reduction
(Beginning)

No information on Ingrid's profile coming from her teacher

Math teacher description

Class	Interview language	Interview date	Origins	Age	USA arrival	Living in California	Spanish readings	English readings	Home language	Friends language	Main language at school	Homework help
MS En-Sp	Spanish	May 2010	Mexico	13	9 months [August 2009]	Likes it: school.	Yes	Yes	Spanish. A little English with her brother for learning purposes.	Spanish and English	Spanish	Brother (mainly) and father

Historical bilingual profile

Math	A1. Ingrid marked "English and Spanish".	Language	Tentative
1, 5-11. 1st try. Wrong answer due to percentages as absolute value instead of relative (direct comparison of percentages).	[Starts at A1,2]	1, 26-31. Spanish as writing language	Spanish dominant
	1 <i>el la tienda JOHN SPORTS por que es mayor el descuento.</i> 2 F: ¿Qué hizo, bueno, me puede explicar...? Bueno, lo que vamos a hacer ahora, vamos... Le voy a preguntar que recuerde cómo ha resuelto cada actividad y luego qué lengua ha usado mientras resolvía el problema. ¿Sí? Vamos a hacer eso con todas las actividades. 3 I: Sí. 4 F: ¿Me puede decir cómo empezó a resolver la actividad uno? 5 I: Eh... Viendo en la cuál daban mejor el descuento. 6 F: Sí. ¿Cómo lo vio eso? 7 I: Porque aquí descuentan un cuarenta por ciento y aquí el veinticinco por ciento. 8 F: Mm [continuing conversation]. [Pause] ¿Y así? 9 I: Mm [validating]. 10 F: ¿[En] Nada más se fijó? ¿Sólo eso?	1. Spanish deviated spelling (por que)	Quick writing, unknown
		12-13, 48-55. English as reading language	English wording
		32-47. Spanish as thinking	Spanish dominant


	<p>11 I: Mm [validating].</p> <p>12 F: Okay. ¿Con qué lengua empezó a resolver el problema?</p> <p>13 I: Con el inglés.</p> <p>14 F: Con inglés. ¿Y cuándo cambió a español?</p> <p>15 I: Con algunas palabras [que] no entendía.</p> <p>16 F: ¿Entonces qué hizo, cuándo no entendió algunas palabras?</p> <p>17 I: Pensar qué más me podría decir.</p> <p>18 F: ¿En español o cómo lo pensó?</p> <p>19 I: Mm [validating], en español.</p> <p>20 F: Y entonces, empezó a... con inglés, ¿no?</p> <p>21 I: Sí.</p> <p>22 F: Cuando algunas palabras no entendía las tradujo al inglés, ¿sí?. Perdón, las tradujo al español.</p> <p>23 I: Mm [validating].</p> <p>24 F: ¿Y luego cuándo volvió a usar el español? [Pause] ¿O cuándo empezó a usar el español, no? Porque el enunciado lo leyó en inglés, continuó leyendo en inglés, ¿tradujo algunas palabras en español?</p> <p>25 I: Mm [validating].</p> <p>26 F: ¿Y usó luego el español?</p> <p>27 I: Sí.</p> <p>28 F: ¿Cuándo?</p> <p>29 I: Cuando tuve que responder la pregunta.</p> <p>30 F: ¿A la hora de escribirlo?</p> <p>31 I: Mm [validating].</p> <p>32 F: Okay. ¿Nada más para escribirlo? ¿Estuvo pensando todo el rato en inglés?</p> <p>33 I: No, estuve pensando en español.</p> <p>34 F: ¿En español?</p> <p>35 I: Mm [validating].</p> <p>36 F: ¿Algunas cosas en español y algunas en inglés?</p> <p>37 I: Mm [validating, nodding].</p> <p>38 F: ¿Qué cosas pensó en español? [Pause]</p> <p>39 I: Como qué me estaba pidiendo.</p> <p>40 F: Mm [continuing conversation] [Pause] ¿Qué más? [Pause] Al leer la pregunta, me está diciendo. ¿Es eso: qué le estaba pidiendo [paraphrasing A1,39]? ¿Es eso?</p> <p>41 I: Mm [validating].</p> <p>42 F: Okay. ¿Qué más?</p> <p>43 I: Explicar porqué.</p> <p>44 F: Mm [continuing conversation]. [Pause] ¿Alguna cosita más?</p> <p>45 I: Y ver... cuánto me descontaban en una tienda y cuánto me descontaban en otra.</p> <p>46 F: Mm [validating], muy bien.</p> <p>47 I: Y ya.</p> <p>48 F: ¿Y luego en inglés?</p>	language	
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	<p>49 I: Mm [validating].</p> <p>50 F: ¿Qué? ¿Qué pensó en inglés?</p> <p>51 I: Qué me preguntaba.</p> <p>52 F: ¿Algo más?</p> <p>53 I: No.</p> <p>54 F: ¿Sólo qué le preguntaba?</p> <p>55 I: Mm [validating].</p> <p>56 [Ingrid erases the cross on the “English only” column after interviewer’s explication of the correct use of the columns: she had a cross on both columns] [Continues in A2,2]</p>		
MEMO	English linked to thinking language, remarked many times.		

Math	A2. Ingrid marked “English and Spanish”.	Language	Tentative	
1-6, 13-14. Right square's perimeter calculation.	<p> $P = 5 + 5 + 5 + 5 = 20\text{cm}$ Porque sume todos los lados alrededor de la figura. </p> <p> $P = \pi \cdot d = 3.14 \cdot 5 = 15.70$ por que para sacar el perimetro debo hacer $\pi \cdot d = 3.14 \cdot 5$ </p>	1. Spanish as writing language	Spanish dominant	
1, 15-20. Right circle's perimeter calculation.		1. Orthographical variation (sume, porque, perimetro)	Quick writing, unclear orthography rules	
1-6, 13-14, 15-20. 1st try. Incomplete answer due to right perimeters' calculation but no comparison of lengths.		2 [Comes from A1,56] I: En ésta tuve... Lo hice casi en... Usé los dos también en ésta.	1, 4. Incomplete understanding of the question	Quick reading, 57-60: low English management
		3 F: ¿Podemos ir revisando cómo lo hizo paso a paso y qué lengua utilizó? ¿Cómo empezó?		
		4 I: En ésta empecé leyendo la pregunta, qué me pedía. Y me pedía el perímetro. Y vi las medidas y después sumé los lados para sacar el perímetro.		
		5 F: Sí. Okay. ¿Cómo lo pensó eso?	2, 25-64. Low and non-concrete use of English	Math class in English, influence of the interviewer
		6 I: Pensando que éste [square] era cada lado.		
		7 F: ¿Y qué lengua, qué lenguas utilizó para pensar todo eso?		
		8 I: En español.		
		9 F: ¿Todo en español o alguna cosita en inglés?		
		10 I: Em... Todo en español.		
		11 F: ¿Todo?	7-12, 21-24. Spanish as thinking language	Spanish dominant, 57-60 low English management
		12 I: Sí.		
		13 F: Okay. [Pause] ¿Y luego?		
		14 I: Nada más lo sumé y vi cuánto salía.		
	15 F: Ajá. Sí, lo del círculo, ¿no?, me refería. O si hay alguna otra cosita que tenga del cuadrado, pues sí me lo			

	<p>dice.</p> <p>16 I: No. Con el círculo tenía que sacar el perímetro. Tenía que ser pi que es tres punto catorce multiplicarlo por el diámetro.</p> <p>17 F: Mm [continuing conversation].</p> <p>18 I: Y tenía que multiplicar tres punto catorce por cinco y lo que me salió...</p> <p>19 F: Mm [continuing conversation]</p> <p>20 I: Es eso.</p> <p>21 F: ¿Y cómo lo pensó esto? ¿Qué idiomas utilizó?</p> <p>22 I: El español.</p> <p>23 F: ¿El español?</p> <p>24 I: Mm [validating].</p> <p>25 F: ¿Durante todo el proceso? U otra vez: si se acuerda de alguna cosita que pensó en inglés...</p> <p>26 I: Como perímetro porque el maestro nos ha dicho que siempre tenemos que, que nos tenemos que aprender esas palabras.</p> <p>27 F: ¿Perimeter?</p> <p>28 I: Sí.</p> <p>29 F: ¿Pensó en inglés?</p> <p>30 I: Sí.</p> <p>31 F: ¿Para el círculo?</p> <p>32 I: Sí.</p> <p>33 F: ¿Pero para el cuadrado no?</p> <p>34 I: También, poquito.</p> <p>35 F: Ajá. Okay. Muy bien. ¿Qué más en inglés?</p> <p>36 I: Eh... [pause] Nada más.</p> <p>37 F: ¿La fórmula, por ejemplo?</p> <p>38 I: Mm [continuing conversation]</p> <p>39 F: ¿En qué idioma?</p> <p>40 I: En inglés.</p> <p>41 F: ¿Y el... ? ¿Cómo se lee esto en inglés? [Pause] ¿Cómo lo pensó en inglés?</p> <p>42 I: [Pause] Em...</p> <p>43 F: ¿O lo pensó en español?</p> <p>44 I: Ajá.</p> <p>45 F: ¿En qué idioma lo pensó?</p> <p>46 I: En español.</p> <p>47 F: ¿Sí? ¿Inglés o español?</p> <p>48 I: En español.</p> <p>49 F: ¿Los va manejando los dos, un poco?</p> <p>50 I: Mm [validating, answering before the question is finished, after 'dos']</p> <p>51 F: ¿Y no se acuerda exactamente alguna cosita más que pensó en español, digo en inglés?</p> <p>52 I: Porqué.</p>	5-64. Increase of the visibility of the English use as the interviewer advances	Inter-viewer's focus on the use of English, consciousness and visibility of the use of English

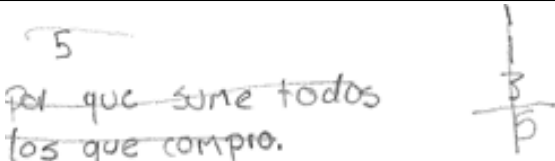
	<p>53 F: ¿El porqué? 54 I: Mm [validating] 55 F: ¿Razonó en, en...? ¿O sea a la hora de empezar a pensar, lo pensó en inglés o sólo leyó el why...? ¿Cómo, cómo pensó el porqué? ¿Por qué dice que el porqué lo pensó en inglés? 56 I: Porque... [pause] 57 F: ¿No me lo puede decir? 58 I: No. 59 F: ¿Más o menos vamos manejando ambos idiomas? 60 I: Mm [validating]. 61 F: ¿O más español o más inglés o cómo cree? 62 I: Un poquito más español, porque mucho inglés no sé. 63 F: Ajá. ¿Pero algo de inglés sí piensa mientras resuelve la...? 64 I: Mm [validating]. 65 F: ¿Y no se le ocurre algún ejemplo, pues por ejemplo... no sé, algún pequeño detalle de qué pensó en inglés? ¿No? [Continues in A3,2]</p>		
MEMO	37-64. Blurry perception of which parts of the solving process are related with each language.		

Math	A3. Ingrid marked “English and Spanish”.	Language	Tentative		
<p>1-13. 1st try. [Figures 1 and 4] Deviated answer due to deviated question understanding: 'With which figures (combined with themselves as many times as desired) 7 tiles can be obtained?'</p> <p>48-62. 2nd try. Right answer with arithmetical sequence associated to figure pattern.</p>	 <p>Figure 1 Figure 2 Figure 3 Figure 4</p>	1. 1 st try. Deviated Spanish syntax (figura)	Quick writing, no review		
	1	<p>How many tiles does figure 7 have? Why?</p> <p><i>figura 1 y 4</i> <i>por que las otras dos saldrian mas que 7.</i></p>	[The lines crossing out the answer are added on the 2 nd try answer (A3,56)]	1. 1 st try. Orthographical variation (saldrian, mas)	Quick writing, unclear orthography rules
	2	[Comes from A2,65] F: ¿Qué hizo?			
	3	I: Ver cuál podría salir siete.			
	4	F: ¿Qué quiere decir cuál podría salir siete?		1, 28-29. 1 st try. Spanish as writing language	Spanish dominant
	5	I: ¿Podrían salir siete cuadros?			
	6	F: ¿Cómo podrían salir?			
	7	I: Poniendo éste [Figure 1] siete veces, y éste [Figure 4] cómo ya tiene siete.			
	8	F: Ajá.			
	9	I: Porque éste [Figure 2] no se podría porque sumando otro serían seis, y si pongo aquí otro, ya serían nueve. Y éste [Figure 3] también ya se pasaría.		1-9. 1 st try. Deviated question understanding (With which	Partial question understanding
	10	F: ¿Entonces cómo empezó aquí a pensar el problema?			
11	I: Em...				

<p>12 F: ¿Cuál fue lo primero que pensó? 13 I: Cuál me podría dar siete cuadros. 14 F: ¿Y qué pensó en inglés aquí? ¿Cómo empezó, perdón, a resolver el problema, con qué idioma? ¿Empezó con español o empezó el inglés? 15 I: En español. 16 F: Ajá. ¿Y cambió a inglés? 17 I: Mm [validating]. 18 F: ¿Luego? ¿En qué momento cambió a inglés, se acuerda? 19 I: Em... Cuando [pause] tenía que ver las figuras. 20 F: ¿Qué quiere decir? ¿Cómo...? ¿Cuándo, cómo las tenía que ver? 21 I: Como la figura uno, la dos, la tres y la cuatro. 22 F: ¿Entonces qué pensó? 23 I: Em... [pause] Cuál de ellas me podría dar siete. 24 F: ¿En inglés? 25 I: Mm [validating]. Más o menos, en inglés y en español. 26 F: ¿Y luego, después de pensar eso, qué hizo? 27 I: Las que me salieron las apunté. 28 F: Sí. ¿Y lo apuntó eso en español? 29 I: Sí. 30 F: ¿Y lo pensó en español también? 31 I: Mm [validating]. 32 F: ¿Antes de apuntarlo pensó eso en español? 33 I: Mm [validating]. 34 F: Okay. ¿Alguna cosita, otra vez, si se acuerda exactamente de qué pensó en inglés? 35 I: Cuántas me podían salir. 36 F: Mm [validating].. ¿Eso lo pensó en inglés? 37 I: Mm [validating]. 38 F: ¿Qué más? 39 I: Nada más. 40 F: ¿Cómo la dijo, esa frase? 41 I: Mmm... 42 F: ¿En inglés dijo, no? 43 I: Mm [validating]. 44 F: ¿Esa frase misma dijo en inglés? 45 I: Sí. 46 F: ¿Cómo la dijo? 47 I: Em... how many. 48 F: Mm [validating]. [Pause] Aquí, las crucecitas, perdón, eran lo mismo todo. O sea que tendría que haber sólo una en todos. [Ingrid erases the crosses in the “English only” column in A2, A3 and A4, as she did before with</p>	<p>figures (combined with them- selves as many times as desired) 7 tiles can be obtained?)</p>	
	<p>14-15, 18-33. 1st try. Spanish as thinking language</p>	<p>Spanish dominant</p>
	<p>18-21. 1st try. English with “Figures”</p>	<p>Reading English wording</p>
	<p>22-25, 34-37, 40-47. 1st try. English as thinking language</p>	<p>English wording</p>
	<p>50. Wording question translated to Spanish (by interviewer)</p>	<p>Wrong understanding</p>
	<p>56. 2nd try. Spanish as writing language</p>	<p>Spanish dominant</p>
	<p>67-68. 2nd try. Spanish as thinking language</p>	<p>Spanish dominant</p>

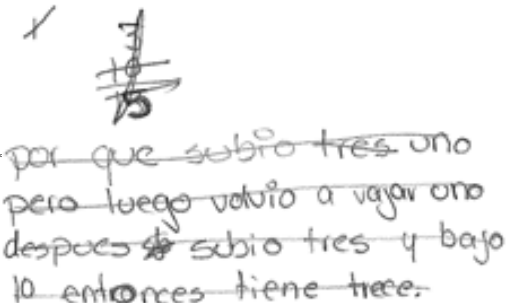
	<p>A1 in A1,56]. Okay. De hecho, lo que nos preguntaba aquí... ¿Ha entendido...?</p> <p>49 I: [Interrupting] Casi no.</p> <p>50 F: ¿... todas las preguntas? ¿No? Nos pregunta cuántas, cuántos cuadraditos tiene la figura siete. Does figure seven have. Cuántos, how many tiles, cuántos cuadraditos does figure seven have. O sea, lo que tendíamos que arreg[lar], que poner aquí es, en la figura siete, cuántos cuadraditos habría.</p> <p>51 I: Mm [continuing conversation].</p> <p>52 F: ¿Lo quiere pensar otra vez o lo dejamos así?</p> <p>53 I: Lo hago otra vez.</p> <p>54 F: ¿Sí?</p> <p>55 I: Mm [validating].</p> <div data-bbox="470 486 1019 582" style="text-align: center;"> </div> <p>How many tiles does figure 7 have? Why?</p> <p>56 <i>figura 1 y 9 por que las otras dos saldrían mas que 7. abrían 13 porque va aumentando dos cuadrados cada vez.</i></p> <p>[Entire answer]</p> <p>57 F: Pensó muy rápido eso.</p> <p>58 I: Sí [smiling].</p> <p>59 F: ¿A ver, qué hizo aquí?</p> <p>60 I: Ver cuántos aumentaba cada vez.</p> <p>61 F: Mm [validating].</p> <p>62 I: Y cuántos saldrían a siete. Cómo aquí es uno, tres, cinco, siete, en la cinco serían nueve, en la seis serían once y en la siete serían trece.</p> <p>63 F: Mm [continuing conversation]. ¿Y con qué lo pensaste? ¿Con qué idiomas pensaste esto? La cruz, si tuvieras que poner la cruz otra vez... Bueno, sería, la respuesta está...</p> <p>64 I: En los dos.</p> <p>65 F: Ajá. ¿En los dos idiomas también?</p> <p>66 I: Sí.</p> <p>67 F: ¿Qué pensaste? ¿Cómo empezaste a pensar eso, en qué idioma?</p> <p>68 I: En español.</p> <p>69 F: ¿Y cuándo cambiaste a inglés?</p> <p>70 I: Cuando tenía que sumarle.</p> <p>71 F: Ajá. ¿Sumaste, hiciste la suma en inglés?</p> <p>72 I: Mm [validating].</p>	69-72. 2 nd try. Additions in English	77-84. Schooling in USA in English
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	<p>73 F: ¿Qué más hiciste en inglés? 74 I: Namás eso. 75 F: ¿Nada más la suma? 76 I: Mm [validating]. 77 F: ¿Aprendiste a sumar en inglés [pause] en la escuela? 78 I: Sí. 79 F: ¿En Mexico estuviste aprendiendo a sumar en inglés? 80 I: No. 81 F: ¿Pero cuando viniste aquí te enseñaron inglés? 82 I: Mm [validating]. 83 F: Ajá. ¿Pero de pequeñita aprendiste en español? 84 I: Sí. [Continues in A4,2]</p>		
MEMO	<p>1-9. 1st try: Written answer with justification. But the oral explanation adds more information as her deviated understanding of the wording question might not be expected. 13-47. English used to read the information associated to the pictures (Figure) but Spanish used as a main thinking language. 48. Ingrid arranges the crosses on the language columns (A2, A3, A4). 62. No use of precise vocabulary and sentences structure in relation with explanations, in Spanish. 22-25, 34-37, 40-47. 1st try. English as a thinking language. Blurry perception of languages use. Contradictory declarations about the language use.</p>		

Math	A4. Ingrid marked "English and Spanish".		Language	Tentative
<p>1-5. 1st try. ["5 por que sume todos los que compro"] Wrong answer due to deviated wording understanding.</p>	<p>1</p>  <p>[The lines crossing out the answer are added later (A4,103)]</p>		<p>1, 12-13. 1st try. Spanish as writing language</p>	<p>Spanish dominant</p>
<p>48. Mathematization of the 10 floors Jamie goes down (by interviewer).</p>	<p>2 [Comes from A3,84] F: ¿Qué hiciste aquí? 3 I: Sumar cuántos pisos había comprado. 4 F: Sí. 5 I: Y [pause] y explicar porqué. 6 F: Mm [continuing conversation]. ¿Y [pause] qué idiomas utilizaste aquí? También pusiste inglés y español, ¿no? 7 I: Mm [validating]. 8 F: ¿Cómo empezaste? ¿Con qué idioma empezaste? 9 I: Con el inglés.</p>		<p>1. 1st try. Deviated Spanish spelling (por que, sume, compro)</p>	<p>Quick writing, unknown</p>
<p>40-116. 2nd try. [13 floors] Wrong answer due to no</p>	<p>10 F: Mm [validating]. ¿Y luego? 11 I: El español. 12 F: ¿Cuándo cambiaste a español? 13 I: Cuando tenía que responder.</p>		<p>2-3, 28-39, 96-101. 1st try. Deviated wording question understanding</p>	<p>Pisos (floors) is a polysemic word in Spanish</p>

<p>mathematization of middle floor (highest floor reached as top of the building), no relative floors positioning.</p> <p>116-121. Right symmetry through middle floor after interviewer controverts final answer.</p> <p>116-129. 3rd try. [14 floors] Wrong answer due to middle floor not counted.</p> <p>130-134. Counting of middle floor introduced by interviewer.</p> <p>116-147. 4th try. [15 floors] Right answer with right arithmetical reasoning.</p>	<p>14 F: Ajá. ¿Y todo el rato éste estuviste pensando en inglés?</p> <p>15 I: La pregunta la estuve pensando en inglés, mm [validating].</p> <p>16 F: ¿Para hacer la suma, por ejemplo?</p> <p>17 I: Mm [validating].</p> <p>18 F: ¿Y para pensar que es... que tenías que hacer esta suma, lo pensaste esto en inglés también?</p> <p>19 I: Sí.</p> <p>20 F: ¿Se te hizo más fácil en inglés?</p> <p>21 I: Poquito.</p> <p>22 F: ¿O por qué lo pensaste en inglés?</p> <p>23 I: Porque [pause] tenía que [jpause] sacar los, las cantidades en inglés.</p> <p>24 F: Mm [continuing conversation]. ¿Y luego, cómo tenías que sacar las cantidades en inglés, también lo otro lo pensaste en inglés?</p> <p>25 I: Ajá.</p> <p>26 F: Okay. Y dices que al final tiene, eh... cinco porque sumaste todos.</p> <p>27 I: Mm [validating].</p> <p>28 F: ¿Qué es lo que nos pregunta? ¿Entendiste la pregunta?</p> <p>29 I: Más o menos.</p> <p>30 F: A ver, ¿qué entendiste?</p> <p>31 I: ¿Qué cuántos pisos había en el departamento? [Pause] Nomás [or '¿No es?'; unclear sound. Ingrid shakes her head saying no].</p> <p>32 F: Department store, se lee esto todo junto.</p> <p>33 I: Mm [validating].</p> <p>34 F: ¿Qué significa department store?</p> <p>35 I: Tienda del[?]/o[?] departamento?</p> <p>36 F: Ajá. Bueno, es como el departamento comercial, o... ¿sí?</p> <p>37 I: Mm [validating].</p> <p>38 F: Es un... como nos dice al principio: large department store. Es un, una gran tienda. ¿Sí?</p> <p>39 I: Mm [validating].</p> <p>40 F: Okay. Mmm... [thinking] ¿Quieres que volvamos a mirar esto o lo dejamos así? Porque es un, es un poco largo esto quizás.</p> <p>41 I: No, lo vamos a ver.</p> <p>42 F: ¿Quiere mirarlo otra vez?</p> <p>43 I: Mm [validating].</p> <p>44 F: Nos pregunta qué cuántas tiendas, cuántos pisos, ¿no?, many floors, cuántas tiendas, cuántos pisos, perdón, cuántos niveles hay en todo el edificio.</p> <p>45 I: Mm [validating].</p> <p>46 F: ¿Sí? ¿Entendió eso cuándo lo volvimos a leer?</p> <p>47 I: Sí.</p> <p>48 F: Okay. ¿Quiere volver a pensar el problema? Porque aquí nos dice en algún momento [pause], em, que baja diez, por ejemplo, ¿no? Goes down ten floors, ¿no? ¿Si baja diez, puede ser que el edificio tenga cinco?</p>	<p>8-9, 14-19. 1st try. English as thinking language</p> <p>16-19. 1st try. Addition performed in English</p> <p>30-39, 68-101. 2nd try. Right understanding of the wording</p> <p>84. 2nd try. Unknown meaning (jewelry)</p> <p>103. 2nd try. Spanish as writing language</p> <p>103. 2nd try. Deviated Spanish spelling (por que, subio, volvio, vajar, despues, bajo) and syntax</p> <p>141, 147. 4th try. Spanish as writing</p>	<p>20-25. Extraction of key information from English wording</p> <p>23-24. English wording</p> <p>Interviewer's explanation</p> <p>English language in construction</p> <p>Spanish dominant</p> <p>Quick writing, unclear orthography rules</p> <p>Spanish dominant</p>
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	<p>49 I: No. 50 F: No puede ser. ¿Lo quiere volver a pensar, un momentito? 51 I: Sí. 52 F: ¿Sí? ¿No se le hará demasiado tarde? ¿Por[que le espera] su mamá? 53 I: Em... No sé. 54 F: Cómo quiera. 55 I: ¿Lo tendría que volver a hacer otra vez todo? 56 F: Bueno, quizás sí. No sé si se puede, si puede aprovechar [algo] que ya haya hecho antes, pero... Como usted quiera. 57 Camera assistant: Si está cansada... 58 F: Sí, si está cansada, pues no pasa nada. O si se tiene que ir. 59 I: Eh, lo puedo volver a hacer. 60 F: ¿Sí? 61 I: Sí. 62 F: Okay. ¿Quiere que le deje pensarlo? O me quedo aquí, como quiera. 63 I: Si me puede ayudar a leerla. 64 F: Ajá. Okay. Sí. ¿Qué quiere que le ayude? ¿Qué?, lo vamos leyendo y traduciendo... 65 I: Mm [validating]. 66 F: ¿O me va preguntando las palabras... ?, cómo quiera. 67 I: Que me lo vaya traduciendo, mejor. 68 F: Ajá. Dice, ¿cómo entendió eso? 69 I: Que Jamie está comprando 70 F: en un 71 I: en una tienda 72 F: comercial 73 I: comercial 74 F: en un gran departamento. En un gran... tienda, en un gran centro comercial. ¿Sí? 75 I: Mm [validating]. 76 F: Con varios pisos, ¿sí? Nos dice: ella entra en la tienda por el piso del medio. 77 I: Mm [validating]. 78 F: ¿Sí? El piso del medio de la tienda. Entra. ¿Sí? ¿Lo ve claro esto? 79 I: Mm [validating]. 80 F: ¿Se lo imagina cómo es? 81 I: Mm [validating]. 82 F: Okay. Luego dice: inmediatamente se va hacia el departamento de crédito. 83 I: Mm [validating]. 84 F: Nada más entrar, por el piso del medio, y se va inmediatamente al departamento de crédito. Después de comprobar que tiene buen crédito, that her credit, after making sure her credit is good. ¿Sí? Comprueba que tiene un crédito bueno. Dice: sube, goes up one floor, sube un piso to the jewelry department. Hacia el departamento de... [pause] ¿jewelry sabe qué es? [Pause] Joyas.</p>	<p>language 141, 147. 4th try. Spanish orthographical variation (por que, harriva, esta) 141, 147. 4th try. Spanish deviated spelling (por que, harriva, esta) and syntax 148-167. 2nd-4th tries. English as thinking language related with numbers: to keep track of the floors and get the answer</p>	<p>Quick writing or unknown Quick writing or unknown Reading English wording</p>
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- 85 I: Mm [continuing conversation].
 86 F: Se va un piso hacia arriba al departamento de joyas después de ver que el, em... que tiene un buen crédito, hemos dicho. ¿Sí?
 87 I: Mm [continuing conversation].
 88 F: Okay. Dice: luego, em... ella baja un piso hacia, to the children's department. Baja, luego de... , se baja un piso hasta el departamento de niños.
 89 I: Mm [continuing conversation].
 90 F: ¿Sí?
 91 I: Mm [validating].
 92 F: Y luego dice: luego, ella va hacia arriba tres pisos hasta el departamento de, toy department, ¿sabe qué es el toy department? Departamento de...
 93 I: ¿Juguetes?
 94 F: Juguetes, bien. Y dice: finalmente Jamie baja diez pisos hasta la entrada principal, main entrance, of the store, de la tienda, la cuál, la entrada principal –which se refiere a la entrada principal–, está en el primer piso. Y desde aquí, and, y, leaves, se va, hacia otra tienda más abajo en la calle.
 95 I: Sí
 96 F: ¿Sí le quedó claro?
 97 I: Sí.
 98 F: ¿Quiere que lo volvamos a ver o alguna cosa?
 99 I: La pregunta.
 100 F: Cuántos pisos tiene, does the department store have. Tiene el departamento comercial. El does es el auxiliar.
 101 I: Mm [continuing conversation].
 102 F: ¿Sí? Siempre me puede preguntar cualquier cosa.
- 103  [2nd try written answer.
Ingrid crosses out the previous answer. “15” instead of “13” in the addition is changed later. The lines crossing out the answer are added later, except for “tres” on the first line.]
- 104 F: A ver, ¿me cuenta cómo lo hizo?
 105 I: Digo cuántos subió.
 106 F: Mm [continuing conversation].
 107 I: Y cuántos bajó.
 108 F: ¿Cuántos subió?
 109 I: Tres.

	<p>110 F: Mm [validating]. 111 I: Y bajó diez. 112 F: Mm [validating]. 113 I: Entonces los sumé porque tenía que ver todos. 114 F: Sumó tres, o sea subió tres... 115 I: Mm [validating]. 116 F: ...y bajó diez. ¿Y esos son todos los del edificio? [Pause] 117 I: No. 118 F: ¿Cómo podemos saber todos los del edificio? 119 I: Eh... Porque ella estaba en medio. 120 F: Mm [continuing conversation]. 121 I: Entonces tiene que tener los dos para el mismo lado. Entonces... [Pause] ¿Tenía siete cada piso? ¿Cada mitad? 122 F: Mm [continuing conversation]. 123 I: Porque de la mitad para abajo son diez. 124 F: ¿Por qué son diez? 125 I: Porque estaba en la mitad y dice que... No, de la mitad para abajo son siete. 126 F: Ajá. 127 I: Porque ella estaba en la mitad, subió tres, pero luego bajó diez hacia abajo, entonces quedaron siete. 128 F: Mm [validating]. Okay. ¿Y por lo tanto cuántos tiene el edificio? 129 I: ¿Catorce? 130 F: ¿Catorce? 131 I: Mm [validating]. 132 F: Siete arriba, me ha dicho antes, siete abajo... 133 I: Mm [validating]. 134 F: Y le falta un pequeño detallito. [Pause] Entra en el piso del medio, tiene siete abajo, siete arriba, ¿cuál le falta por contar? 135 I: Donde ella está. 136 F: ¿Sí? 137 I: Mm [validating]. 138 F: En total son entonces... 139 I: ¿Quince? 140 F: Ajá. ¿Me puede escribir el porqué? 141 <i>Por que son 7 hacia arriba 7 hacia abajo y donde ella esta el 1 los sume y me dio el total.</i> 142 F: ¿Aquí [A4,103: "3+10=13"] cambió eso, no, luego? 143 I: Mm [validating].</p>		
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	<p>144 F: Eso era un trece. Si quiere lo puede tachar, lo de antes, pero ¿cómo, cómo le... ? Porque era tres más diez trece me había puesto, ¿no?</p> <p>145 I: Mm [validating].</p> <p>146 F: ¿Pero me puede decir cómo sacó este siete? Antes me lo explicó, ¿no? ¿Lo puede escribir también?</p> <p>147 <i>Por que son 7 hacia arriba 7 hacia abajo y donde ella esta el 1 los sone y me dio el total.</i></p> <div style="display: flex; align-items: center; justify-content: center;"> <div style="text-align: center;"> $\begin{array}{r} 10 \\ - 3 \\ \hline 7 \end{array}$ </div> <div style="margin: 0 20px;">[Ingrid adds the operations]</div> <div style="text-align: center;"> $\begin{array}{r} 7 \\ + 7 \\ \hline 14 \end{array}$ </div> </div> <div style="display: flex; align-items: center; justify-content: center; margin-top: 10px;"> <div style="text-align: center;"> $\begin{array}{r} 14 \\ + 1 \\ \hline 15 \end{array}$ </div> </div> <p>148 F: Okay. ¿Qué pensó aquí en inglés? [Pause]</p> <p>149 I: La pregunta.</p> <p>150 F: ¿Leer la pregunta?</p> <p>151 I: Mm [validating]. Y el problema.</p> <p>152 F: ¿Y qué más? Mientras luego estaba resolviendo, ¿no? Cuando lo leímos, yo le traduje al español, luego la pregunta también se la traduje, ¿no?, la íbamos leyendo en inglés, y luego empezó a resolver el problema, ¿no?</p> <p>153 I: Mm [validating].</p> <p>154 F: Me hizo todo esto que me dijo siete, y luego lo sumó... Lo sumó y luego finalmente el otro. ¿Qué pensó de estas cosas en inglés?</p> <p>155 I: Cuántos subió y cuántos bajó.</p> <p>156 F: ¿Esto lo pensó en inglés?</p> <p>157 I: Mm [validating].</p> <p>158 F: ¿Qué más?</p> <p>159 I: Cuántos tenía el edificio.</p> <p>160 F: ¿Cuántos tenía en total?</p> <p>161 I: Quince.</p> <p>162 F: “¿Eso pensó en inglés?”, era la pregunta. ¿Eso es lo que pensó en inglés?</p> <p>163 I: Mm [validating].</p> <p>164 F: ¿Cuántos tenía en total?</p> <p>165 I: Mm [validating].</p> <p>166 F: Okay. ¿Qué más?</p> <p>167 I: Nada más. [Continues in GLQ,1]</p>		
MEMO	<p>6-9, 14-19. 1st try. English as a thinking language: 20-25. Extraction of key information from English wording. 48-61. Ingrid wants to solve the problem, even if she knows it is not going to be easy. 63-67. Ingrid asks for the translation of the wording on the second try. 160-165. Different understanding of the question (160).</p>		

General Language Questions		Language	Tentative
1	[Comes from A4,167] F: ¿En general cuándo ha usado el inglés?	1-8. English as reading language	6. Reading English wording
2	I: Cuando tenía que leer la pregunta.		
3	F: Mm [continuing conversation].		
4	I: Y resolver el problema.	4, 7-10. English as thinking language	Schooling in English
5	F: ¿Y por qué lo ha hecho eso con inglés?		
6	I: Porque... [pause] tenía la pregunta en inglés y la tenía que resolver.		
7	F: ¿Y el resolver el problema qué quiere decir [pause] que lo ha hecho con inglés?		
8	I: Mmm... [thinking] [Pause] Nada más la pregunta, no sé.		
9	F: Mm [continuing conversation]. ¿Alguna otra cosita en inglés?	11-12. Operations performed in Spanish [at first she said she did it in English!!]	Spanish dominant
10	I: Los... [pause] nada más.		
11	F: ¿Y en español? ¿En general cuándo ha usado el español?		
12	I: Cuando tenía que hacer las operaciones.		
13	F: ¿Qué más?		
14	I: Cuando tenía que ver el porqué.		
15	F: Mm [continuing conversation]. [Pause] ¿Por qué lo ha hecho esto en español?		
16	I: Porque como yo no hablo mucho inglés se me hace más fácil hacerlo en español.	11-14. Spanish with justifications of written answers	16. More fluent in Spanish than in English
17	F: Mm [validating]. ¿Y qué más cosas ha hecho en español? Me iba a decir algo más, perdone que le he cortado antes.		
18	I: Las medidas.		
19	F: Las medidas...		
20	I: [Interrupting] Las, [pause] las fórmulas.		
21	F: ¿En español también?	20-22. Spanish with formulas	23-24. Known only in Spanish
22	I: Mm [validating].		
23	F: ¿Por qué?		
24	I: Porque en inglés todavía no las he aprendido.		
25	F: Okay. Más cosas en español.	25-28. Additions performed in Spanish	29-30. Easier performance through Spanish
26	I: Las sumas.		
27	F: ¿Las sumas también en español?		
28	I: Mm [validating].		
29	F: ¿Por qué cree que las hace en español las sumas?		
30	I: Porque [pause] eh... es más fácil para mi sacar las cantidades.		
31	F: Okay. ¿Alguna cosita más en español?	35-44. Unknown vocabulary (cheaper, tiles)	English language in construction
32	I: No.		
33	F: ¿Ya?		
34	I: Mm [validating].		
35	F: Por último, ¿hay alguna palabra o frase que no haya entendido en inglés?		
36	I: Sí.		
37	F: ¿Como cuál?		
38	I: Como ésta.		

<p>39 F: Ajá, ¿cheaper? 40 I: Mm [validating]. 41 F: Okay. ¿Qué más? 42 I: Ésta. 43 F: Mm [validating]. Tiles. 44 I: Ajá. Y nada más.</p>		
MEMO	35-44. Unknown vocabulary (cheaper, tiles) 11-12. Operations and other performed in Spanish. Contradiction: in the beginning she says they are performed in English (A3,A4).	

Ingrid has an excellent Spanish BICS because she perfectly and fluently speaks Spanish. Her Spanish CALP is good, as she has no problem with using mathematical vocabulary in Spanish. Her English BICS and CALP are poor. She recognizes it (GLQ,16). She is much more comfortable using Spanish than English.

On A1 English is linked to reading purposes. Ingrid directly compares percentages to say that 40% discount store is cheaper than 25% discount store.

On A2 Ingrid says she uses English but she is not able to specify at which specific time. She calculates both perimeters. In the case of the square she adds all four sides, while in the circle's case she applies the formula. In the justification of the answer she writes down why those are the right perimeters' calculations. Neither Ingrid nor Francesc noticed there is no comparison between both perimeters. But the answer's explanation shows clearly that Ingrid omitted "greater" from the wording question.

On the 1st try of A3 Ingrid says she uses English but she is not able to specify when, as in A2. In relation to English as thinking language she only refers to "Figure" –text present many times associated with the drawing of each figure–. She understands the wording in a deviated way (With which figures (combined with themselves as many times as desired) 7 tiles can be obtained?). Consequently she does a deviated mathematical work, which, on the other hand, seems to be right according to her understanding. On the 2nd try, after interviewer explains the right meaning of the question, she quickly gets the right answer by finding and writing down the arithmetical pattern associated to the figure sequence. She performs all of the operations in English. When asked if she initially learned to add in English she says yes, but when asked again she says she first learned to add in Spanish when she was in Mexico.

On the 1st try of A4 Ingrid uses English in relation to the addition performed. She gets a deviated interpretation of the situation presented on the

wording, which leads her to a deviated answer: “5 por que sumé todos los que compro”. No more details about the solving process are asked, as Ingrid has not a lot of time to stay in the task (her mother is waiting). Even this lack of time, and knowing it is not going to be easy, Ingrid wants to solve the problem again. On the 2nd and following tries, after the interviewer translates the problem into Spanish, Ingrid uses English to keep track of the floors Jamie goes through in order to get a picture of the situation in her mind. She says the department store has 13 floors (3+10), with no mathematization of the middle floor (highest floor reached by Jamie being the top of the building); with floors not situated in a completely correct way: she correctly situates the floors in a relative position when Jamie goes up and down (stating that Jamie ends up where she started) but does not situate properly the 10 floors Jamie goes down. When the interviewer asks her if these (13) are all the floors of the building, she quickly points out the importance of the middle floor and gets 14 floors (7+7) on her 3rd try. Finally the interviewer remarks she does not include the middle floor on the total number of floors of the building, which leads Ingrid to the right answer: 15 floors (7+7+1).

What is surprising in the GLQ is that she says she performs the additions in Spanish, but when she is asked on A3 and A4 she says operations are done in English. She also says that explanations are in Spanish. She informs that she is much more fluent in Spanish, and knows English just a little.

Ingrid justifies all of the answers in a written way, which is not done so systemically by all of the students in the study. When looking to these explanations an idea of the solving process followed by Ingrid can be made, with the exception of A3 (where the oral details complete the written answer).

Activities' (Key ideas) summary

Object 38: Ingrid-First reduction (End)

- Activity involving stores and percentages solved with English used exclusively as a reading language. Mathematical mistake (direct comparison of percentages) not primarily related with language.
- A partial understanding of the statement question leads to a –consequently– deviated solving process (no comparison of perimeters' lengths), with a right mathematical procedure.
- There is a marginal use of the English language to solve the activity not much detailed (rather than pointing to the influence of the word “perimeter”).
- A deviated wording understanding (With which figures (combined with themselves as many times as desired) 7 tiles can be obtained?) leads Ingrid to a deviated mathematical procedure (correct according to her deviance).
- Marginal English use to solve the activity of the sequence of figures.
- Additions performed in English once the statement question is properly understood (in the GLQ she says the opposite)
- A deviated question understanding (due to the density of the wording, among other factors) leads to wrong horizontal mathematization
- Addition of the floors performed in English (though in GLQ she says the opposite!)
- English as a thinking language, mainly to keep track of the floors (numbers) after the wording is properly translated (by the interviewer).
- Within the interview, evolution from Spanish as the unique declared language (apart from reading the wording in English) to the recognition of Spanish as the language for understanding and English as the language for data processing and for operations.

Historical profile	Bilingual profile (Spanish dominant)	Activity			
<ul style="list-style-type: none"> • 13 years old • English-Spanish class • Arrived at USA 9 months before interview • Likes California • Spanish readings • English readings • Spanish at home (A little English with brother) • Spanish and English with friends • Mainly Spanish at school • Homework help: father and mainly brother 	English linked to reading language (A2: low and non concrete English use A3, 1st try: low and non concrete English use A3, 2nd try: additions performed in English)	x	x	x	
	Deviations on Spanish writing	x	x	x	x
	Incomplete understanding of the question (no attention paid to “greater”)		x		
	Low and non-concrete use of English		x		
	Deviated wording question understanding (A3: With which figures (combined with themselves as many times as desired) 7 tiles can be obtained?)			1	1
	English as thinking language (A3: no detailed description of its use)			x	
	Additions performed in English			2	1
	Spanish as a writing and thinking language	x	x	x	x
	Unknown meaning of “jewelry”				2
	Right understanding of the wording				2
	English as a thinking language related to numbers: to keep track of the floors and get the answer			2	-4
	English as a reading language				GLQ
	English as a thinking language (low and non concrete)				GLQ
	Spanish to perform operations				GLQ
	Spanish with justifications of written answers				GLQ
Spanish with formulas				GLQ	
Spanish to perform additions				GLQ	
Procedural profile	Conceptual profile				
1 X Direct comparison of percentages	1 X Notion of percentages as absolute value instead of relative				
2 √ Application of circle's perimeter formula √ Visualization and calculation of square's perimeter X No comparison of perimeters	2 √ Notion of perimeter				
3.1 X/√ Mathematical procedure according to the deviated understanding	3.1 √ Notion of multiples				
3.2 √ Application of a pattern given by adding 2 from one figure to the next	3.2 √ Notion of arithmetical sequence associated to a sequence of figures				
4.1 X [“5 por que sume todos los que compro”]	4.1 √ Notion of number line				
4.2 X Symmetry through middle floor X No relative floors positioning	4.2 Notion of number line with confused order positions				
4.3 √ Symmetry through middle floor X Middle floor not counted	4.3 Notion of number line with confused order positions				
4.4 X Counting of middle floor introduced by the interviewer	4.4 √ Notion of number line				

Object 39: Ingrid-Second reduction

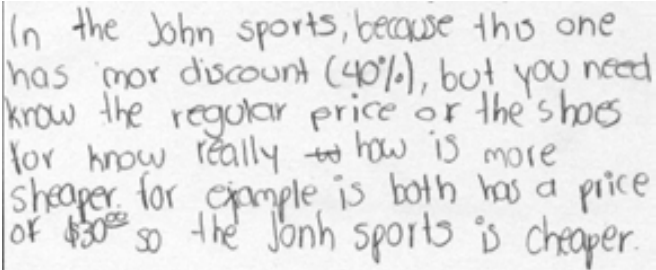
Object 40: Yael-First reduction
(Beginning)

Yael arrived in January to the High School and was assigned to the Algebra 1 Transitional class, as she had Mexican origins. She had a rather poor English comprehension or expression. She is very good at math and demonstrates enthusiasm during explanations and doing the homework. She is very participative and cooperative. Later in the same school year she was assigned to Geometry Mainstream class, as she quickly gained English fluency and the contents of Algebra 1 had already been covered in Mexico.

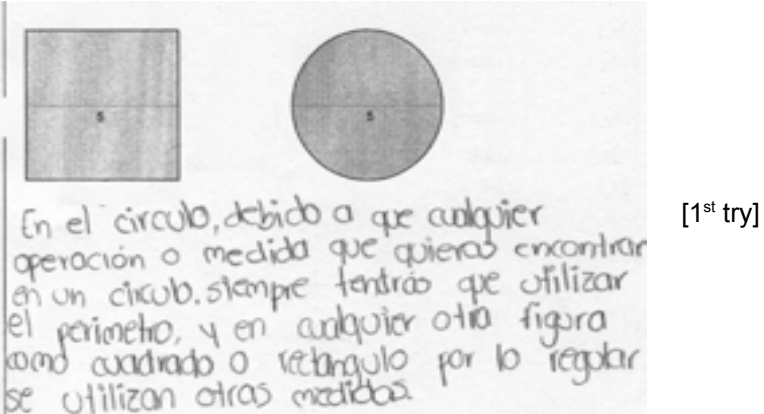
Math teacher description

Class	Interview language	Interview date	Origins	Age	USA arrival	Living in California	Spanish readings	English readings	Home language	Friends language	Main language at school	Homework help
HS Transitional / Mainstream	Spanish	June 2010	Mexico (American father living in another state)	17	6 months before [January 2010]	Likes it (education).	Novels	No (just compulsory books from the English class)	Spanish (just sometimes practices English with her sister – Julia)	Spanish. English with text messages, less frequent.	Spanish when possible (because English is compulsory. In that case she needs to translate all the time.)	Nobody

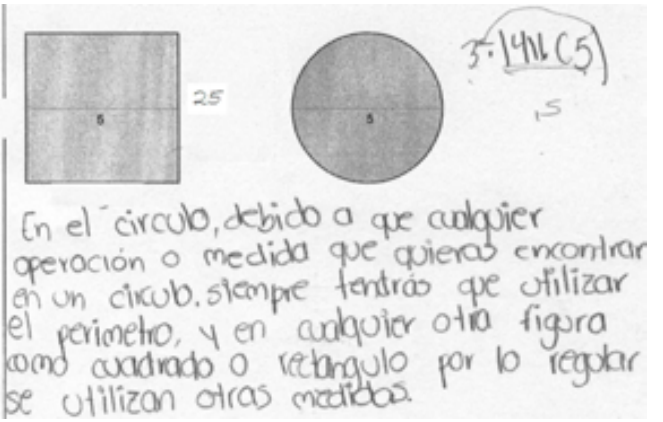
Historical bilingual profile

Math	A1. Yael marked "English and Spanish".	Language	Tentative
1-4. Word meaning demand –the possibly attributed meaning(s) would lead to opposite answers–.	<p>[Activities are solved in the following order: A2, A1, A3, A4. Conversation starts here:]</p> <p>1 Y: [Raising the hand] Tengo una duda. Es que esta palabra me confunde [pointing to "cheaper"].</p> <p>2 F: ¿Cheaper?</p> <p>3 Y: ¿Es barato o caro?</p> <p>4 F: Barato.</p>	1-4. Word meaning demand (cheaper). No distinction between root word and superlative.	Dubitative meaning, important role on the mathe-matization process
5, 20. Awareness of the need of initial prices to get the final price. Assuming equal initial prices (\$30) John sports is cheaper.	<p>5  [Conversation continues on A1,6 once all the activities are solved.]</p>	5, 26, 37-38. Use of English as writing language	English wording
1-5, 20. 1st try. Right answer with awareness of no initial prices. Right treatment of percentages as a relative value.	<p>6 [A4's written answer is finished] Y: Ya está.</p> <p>7 F: ¿Terminó? Okay. Vamos a comentar ahora un poco las cuestiones, ¿sí?</p> <p>8 Y: Sí.</p> <p>9 F: ¿Con cuál quiere empezar?</p> <p>10 Y: Da igual.</p> <p>11 F: Usted escoge. ¿La primera?</p> <p>12 Y: La primera [nodding].</p> <p>13 F: A ver... Bueno, lo que vamos a hacer ahora es que le voy a preguntar... Bueno, ha puesto las crucecitas, ¿no? [On the language(s) columns]</p> <p>14 Y: Sí.</p> <p>15 F: Okay. Le voy a preguntar que me explique los pasos, cómo lo ha resuelto. Y después de hacer esto, qué lengua ha utilizado en cada paso, ¿sí? Si el inglés y el español, mientras estaba pensando el problema.</p> <p>16 Y: Mm [agreeing].</p> <p>17 F: ¿Me puede empezar pues... [pause]?</p> <p>18 Y: Claro que sí.</p> <p>19 F: ...explicando cómo ha hecho esto, ¿cada uno?</p> <p>20 Y: Okay. Aquí aparece la tabla y dice que la primera tienda que es la John Sports los tenis tienen un cuarenta por ciento de descuento y en la Mike tienen un veinticinco por ciento. Yo le puse que era más barato en la primera que en la segunda, porque [tiene] un cuarenta por ciento que es más mucho que un veinticinco. Pero también le puse que necesita saber cuál es el regular precio de los dos, ¿porque si no cómo vas a saber cual! Y puse un ejemplo. Puse por ejemplo si los dos pares, en las dos tiendas, cuestan treinta dólares y con el cuarenta por ciento pues siempre va a ser la John la que va a tener el precio más bajo. Pero si los precios son diferentes pues también va a ser diferente el resultado.</p>	5. Ortho-graphical variations (mor, sheaper)	Quick writing
20, Expanded mathematical talk with arguments		5. Deviated syntax ([to] know, or, how [much], is [4 th sentence], is [5 th sentence], so, the)	English language in construction
		5. Deviated grammar (has)	English language in construction
		24. English as reading language	English wording
		28. Both languages for thinking	34: English wording, 34-36: Spanish as a

	<p>21 F: Ajá. Entonces aquí me puso la cruz. Inglés y español antes de escribir nada. 22 Y: Sí. 23 F: ¿Sí? ¿Se acuerda? ¿Por qué? 24 Y: Sí. Porque viene la respues..., la pregunta en inglés y yo la tengo que traducir a español. 25 F: Okay. 26 Y: Pero la contesté en inglés. 27 F: Okay. 28 Y: Pensando en los dos idiomas [marking two with the fingers]. 29 F: ¿Me puede decir ahora en qué lengua empezó a pensar? 30 Y: En español. 31 F: En español. ¿Qué pensó en español? 32 Y: Todo. 33 F: ¿Todo? ¿Pero no me dijo que pensó en los dos idiomas? 34 Y: Sí. Pensé en español la respuesta y como las gráficas están aquí [statement's visual mode] pues éstas las tenía en mi mente así en inglés, así como vienen aquí. Y ya nada más en español como le complementé para poder saber cuál era el resultado. 35 F: ¿Y para qué más utilizó el inglés? 36 Y: Para traducir mi respuesta. 37 F: Ajá. ¿La escribió en inglés, luego, la respuesta? 38 Y: Mm [validating]. 39 F: Mm [validating]. ¿Alguna cosita más que se acuerda que hizo en inglés? 40 Y: Mmm.... [thinking] No. 41 F: ¿Sí? Okay. [Continues in A2,2]</p>		<p>resource to interpret the English statement and to translate the written answer (Spanish to English)</p>
		<p>29-34. Use of English experienced as transparent</p>	<p>Spanish dominant & motivation in the learning of English</p>
<p>MEMO</p>	<p>1-4. Word meaning demand (cheaper) 20. 'Pero si los precios son diferentes pues también va a ser diferente el resultado'. Adds more information to the written answer, even if it is not yet a complete example, with the other store being cheaper. 20. 'Aquí aparece la tabla'. Possible relationship with 'unbeatable', though it is clearer in other cases. 21-36. More elaborated reasoning through Spanish. Initial English thoughts influenced by English wording. Answer reasoned in Spanish and translated into English (with some writing deviances).</p>		

Math	A2. Yael marked "English and Spanish".	Language	Tentative
1, 20-21, 37. 1 st try. Deviated answer due to deviated wording understanding (2, 38-43, 92-96: In which figure the use of the diameter has a more important role?). Confusion between diameter and perimeter.	 <p>[1st try]</p>	1, 11, 29-31. 1 st try. Spanish as writing language	Spanish dominant, 11, 29-31: Unknown English vocabulary needed to express the answer
21-27. 1 st try. Notion of volume applied to the circle.	2 [Comes from A1,39] F: ¿Hacemos lo mismo con la actividad dos? 3 Y: Sí. Aquí dice que cuál de las figuras necesitas el perímetro. 4 F: O sea, ¿leyó el enunciado primero? 5 Y: Sí. 6 F: ¿Y luego como procedió? 7 Y: Ya luego lo traté de interpretar en inglés...	1. 1 st try. Ortho-graphical variations (circulo, perimetro, rectangulo)	Quick writing or unknown
40-43. Incorrect identification of the procedural topic on comparing measures because "greater" is understood as more useful ('más útil' -41-).	8 F: Mm [agreeing]. 9 Y: Y lo entendí. 10 F: ¿Cómo lo trató de interpretar en inglés? 11 Y: Ehm... Como... Dije, oh no lo voy a traducir al español, y lo entendí así en inglés. Y ya en mi mente como que dije: lo voy a contestar en inglés. Pero había algunas palabras que no sabía cómo decir en inglés y por eso puse mi respuesta en español.	6-18. 1 st try. Use of English as default language	English wording
47-60, 92-94. 2 nd try. Perimeter-diameter confusion.	12 F: Okay. 13 Y: Pero lo estuve pensando en inglés. 14 F: ¿Todo el rato? 15 Y: Mm [agreeing, nodding] 16 F: ¿Hasta cuándo? 17 Y: Pues todo ya solo para... 18 F: Espere, sino no sé [oye] [An announcement interrupts] Entonces leyó esto en inglés me ha dicho, trató de... bueno lo entendió en inglés, sin traducirlo al español, esta vez [Yael nods] E intentó solucionarlo en inglés. 19 Y: Sí, lo solucioné, porque...	6-21, 28-29, 32-35. 1 st try. Use of English as thinking language	English wording, reaction to the interaction with the interviewer
1, 20-27, 36-43, 47-60, 92-94. 1 st	20 F: ¿Y qué pensó solucionándolo en inglés? ¿Cuál fue el procedimiento? ¿Qué es lo que pensó? 21 Y: Que para cualquier operación que tengas que hacer relacionada con un círculo siempre vas a utilizar el perímetro. Y en otra figura como un cuadrado o en rectángulo, ahí casi nunca se utiliza el perímetro. Como si quieres saber... no sé, cuál es el radio, pues tienes que utilizar el perímetro, si quieres saber cuál es el volumen	21-27. 1 st try. Notion of volume applied to the circle	Unknown or confusion (not necessarily related to language)
		37-43, 92-96. 1 st try. Deviated wording	40-41, 92-96. Unknown meaning of

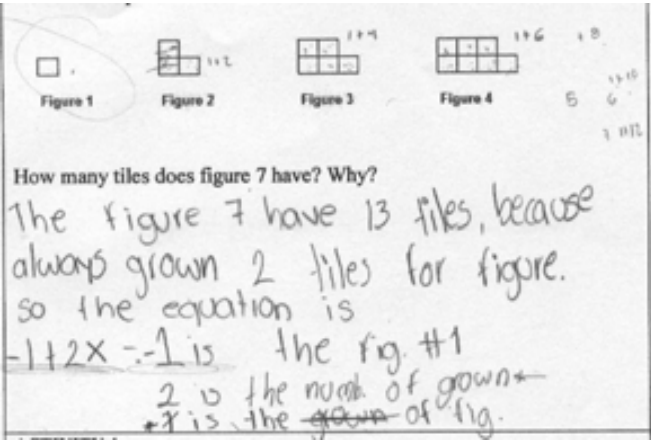
<p>try. Wrong answer due to deviated wording question understanding and perimeter-diameter confusion.</p>	<p>también tienes que utilizar el perímetro del círculo. 22 F: ¿El círculo tiene volumen? 23 Y: El volumen, ah... No, eh ah... Sí tiene volumen, maestro. 24 F: ¿El círculo? ¿Sí? 25 Y: Sí tiene [low voice]. 26 F: ¿Sí? ¿Es una figura plana, no? Que está como en la hoja... 27 Y: Oh, como si es una... ¡Ah! No me acuerdo. [clicks the fingers] 28 F: Bueno, no pasa nada. Luego volvemos a eso. Entonces, ehm... Bueno me puede decir qué es lo que... O sea, todo esto que me ha dicho ahora, lo pensé en inglés, ¿no?</p>	<p>question understanding</p>	<p>some words (greater, perimeter)</p>
<p>61. 2nd try. Operational definition of perimeter (validated by interviewer –62–).</p>	<p>29 Y: Sí. Pero como en mi mente sí sabía, pero ya no sabía cómo escribirlo. 30 F: Entonces lo escribió en español. 31 Y: Lo escribí en español. 32 F: Mm [agreeing]. ¿Y nada más a la hora de escribirlo lo pensé en español? 33 Y: Mm [agreeing, nodding]. 34 F: El resto del problema, ¿lo pensé en inglés? 35 Y: Sí [nodding].</p>	<p>40-41. 1st try. Mismatched meaning (greater -más útil)</p>	<p>English language in construction</p>
<p>62-63. 2nd try. Definition of the circle's perimeter through its (deviated) formula (corrected by interviewer –64–).</p>	<p>36 F: ¿Entonces qué puso, aquí? 37 Y: Que, le puse [reading] en el círculo debido a que cualquier operación o medida que quieres encontrar en un círculo siempre tendrás que utilizar el perímetro y en cualquier otra figura como un cuadrado o rectángulo por lo regular se utilizan otras medidas. 38 F: ¿Y qué le preguntaba el ejercicio aquí? 39 Y: Que cuál de las dos figuras cómo tienes que utilizar el perímetro. Y por qué. Como yo así lo entendí. O como a cuál te es útil. 40 F: Okay. Le pregunta que cuál “has a greater”, que significa “greater”? 41 Y: Pues yo lo entiendo como... como a... Así lo entiendo, como en cuál te es más útil.</p>	<p>47-60, 91-95. 1st - 2nd tries. Perimeter-diameter confusion</p>	<p>Mathematically specific vocabulary not consolidated</p>
<p>69. 2nd try. [square's perimeter = 25] Right mathematical procedure (addition of its sides), with operational error (multiplication of 5 times 5).</p>	<p>42 F: Mayor. Le pregunta en cuál de estas dos figuras el perímetro es mayor. 43 Y: Oh, yo lo entendí como cuál es más útil. 44 F: ¿Lo quiere volver a pensar? 45 Y: ¡Oh, sí! 46 F: Okay. 47 Y: Pero pues en un cuadrado como nunca se utiliza un perímetro. 48 F: Mm [continuing conversation] 49 Y: Y siempre se utiliza en un círculo, pero aquí la medida de los dos es la misma. 50 F: ¿La medida de qué? 51 Y: De los dos perímetros. Porque aquí es cinco y aquí también. 52 F: ¿Qué es el perímetro? 53 Y: El perímetro... pero el perímetro en un círculo, en una... rectángulo o en un cuadrado nunca existe, el perímetro siempre existe como en figuras de círculo.</p>	<p>70-77. 2nd try. Use of Spanish for everything (including calculations)</p>	<p>Spanish dialogue, Spanish dominant, awareness of language difficulties in the 1st try</p>
<p>69. 2nd try. Right comparison of perimeters'</p>	<p>54 F: ¿No existe en un cuadrado el perímetro? 55 Y: No.</p>		

lengths, estimating the circle's perimeter through the application of its formula.	<p>56 F: ¿Qué es el perímetro? 57 Y: El perímetro es como la línea que divide por exactamente la mitad. 58 F: Ohh. Esto es el diámetro [pointing to the diameter of the circle]. ¿Esto? [Marking the diameter of the circle] 59 Y: Mm [agreeing]. 60 F: Es el diámetro. Sí, es verdad que no existe aquí. Pero el perímetro, ¿sabe qué es? ¿Se acuerda? 61 Y: ¿Es la suma de todos los lados? 62 F: Ajá. En un cuadrado, por ejemplo, sí. Aquí no, no hay nada. ¿Pero qué es el perímetro del círculo? 63 Y: ¿El perímetro? ¡Ah! ¿Es la multiplicación del radio por... por "pi"? 64 F: Diámetro por pi, es la fórmula. 65 Y: Ah, ya. 66 F: ¿Puede tratar de decir ahora cuál es el que tiene el perímetro más grande? 67 Y: Es diámetro por pi, ¿no? Entonces...</p>		
61-69. 2nd try . Deviated answer due to error on the addition when calculating the square's perimeter. Right estimation of circle's perimeter through the application of its formula. Right comparison of both lengths.	 <p>68</p>		
79-81. 3rd try . Right calculation of square's perimeter after interviewer's demands for the origin of "25" (78).	<p>69 Y: Va a ser el cuadrado. Porque es la suma de todos los lados. Entonces todos los lados miden cinco, porque es un cuadrado [writing down "5" next to the sides of the square]. Va ser cinco por cinco veinticinco [writes down "25"]. Y aquí [circle] va a ser el diámetro que es cinco por pi. Entonces veinticinco va a ser más, mayor que la suma... que la multiplicación de éstas dos [5 times 3.1416].</p> <p>70 F: Mm [agreeing]. ¿Cómo lo ha pensado esto? ¿Me puede decir otra vez cómo ha utilizado el inglés y cómo ha utilizado el español? ¿En qué lengua empezó a resolver el problema?</p> <p>71 Y: En español. 72 F: ¿Empezó en español ahora? 73 [Yael nods] 74 F: ¿Y cuándo cambió al inglés? 75 Y: No, todo lo hice en español. 76 F: ¿Todo en español ahora? 77 [Yael nods] 78 F: ¿Este veinticinco cómo lo sacó?</p>		
82-85. 3rd try . Right numerical comparison of both perimeters with an estimated value for the circle's perimeter.			
78-86. 3rd try . Right answer			

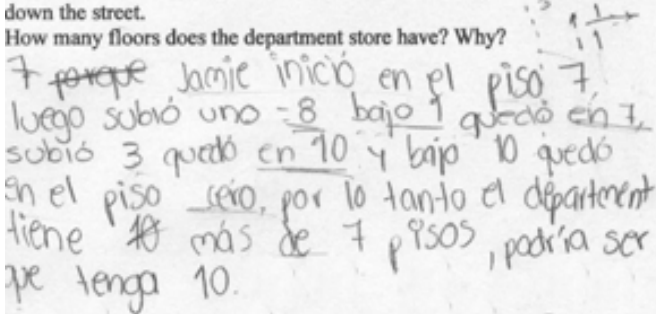
<p>through arithmetical reasoning (calculation of square's perimeter through the addition of all its sides and application of circle's perimeter formula – estimated result, multiplication not performed–).</p>	<p>79 Y: Ehm... si de este lado mide cinco, porque es un cuadrado se supone que todos los lados van a ser iguales. Pues son cinco, cinco, cinco y cinco. Y son veinte.</p> <p>80 F: Ajá.</p> <p>81 Y: Cinco por cuatro, veinte. Y ya me colé.</p> <p>82 F: ¿Entonces continua siendo éste [square] mayor que éste [circle]?</p> <p>83 Y: ¡Sí!</p> <p>84 F: ¿Por qué?</p> <p>85 Y: Porque cinco por tres van a ser quince y si sumas lo de... ¡ay!, si multiplicas los decimales cuándo te... lo complete todo [including the decimals], no se llega a veinte.</p> <div data-bbox="456 456 1099 874" data-label="Image"> <p>5 5 5 5 20 3.14 5 15.7</p> </div> <p>86 En el círculo, debido a que cualquier operación o medida que quieras encontrar en un círculo, siempre tendrás que utilizar el perímetro, y en cualquier otra figura como cuadrado o rectángulo por lo regular se utilizan otras medidas. [3rd try]</p> <p>87 F: Okay. Sin... ¿Sabe alguna otra forma de resolver este problema? ¿Puede resolverlo de otra forma?</p> <p>88 Y: Ah, ah [No].</p> <p>89 F: ¿No?</p> <p>90 [Yael says no with her head]</p> <p>91 F: Okay. Entonces antes lo intentó resolver en inglés y me dijo que utilizó el inglés para todo menos para escribir la respuesta.</p> <p>92 Y: Sí pero me confundí en el diámetro.</p> <p>93 F: Aha. ¿Entendió perímetro como diámetro?</p> <p>94 Y: Mm [agreeing].</p> <p>95 F: Oh y entonces por eso...</p> <p>96 Y: Por eso me confundí en la respuesta.</p> <p>97 F: Mm [continuing conversation]. Vale. ¿Comentamos otra? ¿Cuál?</p> <p>98 Y: La tres. [Continues in A3,2]</p>		
MEMO	<p>1st try oral explanation is more detailed than the written one.</p> <p>7-11. Use of English as the default language.</p> <p>11, 29-31: Unknown English vocabulary is needed to express the answer.</p> <p>38-43. Given that the preceding exercise (A1) was not just a calculation, but more reasoned-oriented, maybe Yael understands this exercise in a deviated</p>		

way.

2nd try: Spanish for calculations and reasoning, due to the mathematical difficulties experienced in the 1st try.

Math	A3. Yael marked "English and Spanish".		Language	Tentative
<p>1, 13. "-1" is the number of tiles of Figure 0 (the series starts with a non geometrical representation; arithmetic is prevalent).</p> <p>13. Move from arithmetical particular statements to algebra with a wrong reference to the notion of equation (though this meaning is suggested in the textbook).</p> <p>1, 13-15, 26-29. 1st try. Right answer with arithmetical reasoning associated to the sequence of tiles. Algebraic check on the number of tiles (formula that relates the figure number and the number of tiles).</p>	 <p>1 The figure 7 have 13 tiles, because always grown 2 tiles for figure. so the equation is $-1+2x = -1$ is the fig #1. 2 is the num of grown + 7 is the num of fig.</p>	<p>[Once the two first lines are written, Yael counts the tiles in each figure several times, this is why most of the tiles are marked with pencil dots. "grown" is crossed out later: A3,15]</p>	<p>1. Use of English as unique writing language</p> <p>1. Grammatical deviation (grown, per, the)</p> <p>2-11, 16-25. Use of English as unique thinking language</p> <p>18-23. No explicit uses of Spanish declared</p>	<p>School context, 8-11, 25: similar problems done in English in the math class</p> <p>English language in construction</p> <p>8-11, 25. Similar problems done in English in the math class</p> <p>Similar problems done in English in the math class, reaction to the emphasis by the interviewer on the use of English</p>
	<p>2 [Comes from A2,98] F: Ajá. A ver, ¿Cómo empezó aquí a resolver el problema?</p> <p>3 Y: Esa, ehm... También empecé en inglés.</p> <p>4 F: Después de leer el enunciado.</p> <p>5 Y: Leí las instrucciones y...</p> <p>6 F: ¿Y después qué hizo? Después de leer las instrucciones.</p> <p>7 Y: Me acordé de lo que habíamos visto en clase, de cómo se resuelven este tipo de problemas.</p> <p>8 F: Ajá. ¿Y cómo se acordó de eso? ¿En qué idioma se acordó?</p> <p>9 Y: En inglés.</p> <p>10 F: ¿Se acordó de lo que hicimos en clase en inglés?</p> <p>11 Y: Sí.</p> <p>12 F: Mm [continuing conversation].</p> <p>13 Y: Y empecé a contar las figuras, el número de tejas de cada figura y... pues hice mi ecuación. Y aquí [A3,1] le puse que menos uno es el número de tejas de la figura uno [Figure 0 is contemplated]. El dos es el número de las figuras que crece, porque en cada una siempre va a crecer dos. En esta son tres, en esta son cinco y en esta son siete. Y el equis es el número como de la figura. Y así es como lo resolví.</p> <p>14 F: ¿El número de la figura?</p> <p>15 Y: El número de la... ¡Sí! Ese es el número de las que crece y este es el número de la... de la figura [she crosses out "grown"].</p> <p>16 F: Okay. Ahora vamos a ver cómo utilizó el inglés, ¿no? Dice que empezó acordándose de lo que hicimos en clase en inglés. ¿Sí?</p> <p>17 Y: Sí.</p> <p>18 F: ¿Cómo continúa después? ¿En qué momento cambia al español?</p> <p>19 Y: No, no cambié al español.</p> <p>20 F: ¿Pensó todo el rato en inglés?</p>			

	<p>21 Y: Sí.</p> <p>22 F: Todas sus palabras en su mente fueron en inglés...</p> <p>23 Y: [Interrupting] Sí [nodding].</p> <p>24 F: ... ¿En este ejercicio?</p> <p>25 Y: Porque había palabras con las que ya me sentía familiarizada porque ya había trabajado mucho con ellas.</p> <p>26 F: Okay. ¿Y utilizó la fórmula?</p> <p>27 Y: Sí.</p> <p>28 F: ¿O primero hizo esto para resolverlo?</p> <p>29 Y: No, primero hice esto [see notes next to the figures on A3,1]. Después hice la fórmula para estar segura de que fuera verdad. Pues ya.</p> <p>30 F: Okay. Creo que quizá lo tendríamos que dejar aquí. Suena la campana ya, ¿no? [It is time to start the next class]. ¿Puede venir a la hora del patio, digo a la hora del lunch y lo terminamos?</p> <p>31 [Yael nods]</p>		
MEMO	<p>Application of two different procedures to find and check the answer.</p> <p>References to the use of English as the unique language in the resolution.</p>		

Math	A4. Yael marked "English and Spanish".	Language	Tentative
<p>1-7, 40-50. No mathematization of the middle floor according to its symmetry condition (Awareness that many possibilities stand for the total number of floors). Right relative situation of floors.</p> <p>1-7, 40-50. 1st try. Incomplete answer due to the non mathematization of the middle floor according to its</p>	<p>Jamie is shopping in a large department store with many floors. She enters the store at the middle floor, and immediately goes to the credit department. After making sure her credit is good, she goes <u>up one</u> floor to the jewelry department. Then she goes <u>down one</u> floor to the children's department. Then she goes <u>up three</u> floors to the toy department. Finally Jamie goes <u>down ten</u> floors to the main entrance of the store, which is on the first floor, and leaves to go to another store down the street.</p> <p>How many floors does the department store have? Why?</p> <p>1</p>  <p>[1st try]</p>	<p>1, 16-19, 30-33, 53-62. 1st try. Use of Spanish as writing language with some code mixing instances (department)</p> <p>8-29, 67-72. 1st try. Use of English linked to thinking</p> <p>53-62. 1st try. Use of Spanish to check the</p>	<p>Spanish dominant</p> <p>English wording</p> <p>Spanish dominant</p>
	<p>2 F: ¿Sí? Vamos a ver si nos da tiempo de hacer esta así rapidito. ¿Me puede explicar cómo empezó aquí?</p> <p>3 Y: ¿En ésta [A4]?</p> <p>4 F: Sí.</p> <p>5 Y: Empecé a leer la actividad y fui marcando como las cosas que yo creí que eran importantes, los números y</p>		

<p>symmetry condition (Awareness that many possibilities stand for the total number of floors). Right relative situation of floors.</p> <p>73. Interviewer indicates that the number of floors can be found.</p> <p>81. Symmetry through middle floor (with no direct clues from the interviewer).</p> <p>81. Sketch of the building with 9 floors instead of 10 when Jamie goes down ten floors.</p> <p>73-88. 2nd try. Deviated answer due to middle floor not included on the total number of floors. Symmetry through middle floor (considering 6 floors instead of 7 from the middle to the first floor because of the</p>	<p>todo eso. Y después leí la pregunta. Que dice que cuántos pisos el departamento de tiendas tiene. Entonces yo como ya tenía mis datos encerrados lo que hice fue como ir haciendo mis gráficas. No son gráficas pero mis números los fui poniendo y ya le puse...</p> <p>6 F: [Interrupting] Pero aquí [drawing on the right, below the wording: A4,1] sí hizo como...</p> <p>7 Y: Ajá, como decía aquí sube uno y aquí baja uno, luego sube tres... Para no confundirme. Y le puse que, ehm, Jamie inicia en el piso siete. Luego subió uno, que dio ocho. Subió uno. Luego bajó uno, bajó uno... y quedó en el siete otra vez, luego subió tres y quedó en el diez, bajó diez y quedó en el piso cero. Por lo tanto puse que el departamento tiene más de siete pisos. Podría ser que tenga diez o más.</p> <p>8 F: Mm [agreeing]. ¿Qué idioma utilizó aquí para empezar?</p> <p>9 Y: Para empezar, empecé en inglés. Empecé leyendo.</p> <p>10 F: ¿Y marcó esto antes de leer la pregunta? ¿Antes de leer la pregunta marcó las palabras clave?</p> <p>11 Y: No, la iba leyendo e iba marcando.</p> <p>12 F: Pero antes de llegar al final ya iba marcando eso.</p> <p>13 Y: Sí.</p> <p>14 F: ¿Antes de saber la pregunta?</p> <p>15 Y: Mm [agreeing].</p> <p>16 F: ¿Cuándo cambió a inglés? Digo, perdone...</p> <p>17 Y: A español.</p> <p>18 F: ... ¿Cuándo cambió a español?</p> <p>19 Y: Ehm, cuándo iba a poner la respuesta.</p> <p>20 F: Pero antes de poner la respuesta pues estuvo pensando todo esto, ¿no? ¿En qué idioma lo hizo?</p> <p>21 Y: Pero esto lo iba pensando en inglés.</p> <p>22 F: ¿Todo esto lo iba pensando en inglés?</p> <p>23 Y: Sí.</p> <p>24 F: ¿Todo? ¿Nunca el español para nada?</p> <p>25 Y: No casi, no.</p> <p>26 F: ¿Alguna cosita sí?</p> <p>27 Y: ¡Ah, ah! [Saying no with her head]</p> <p>28 F: ¿Todo en inglés?</p> <p>29 [Yael nods]</p> <p>30 F: ¿Y a la hora de escribir la respuesta?</p> <p>31 Y: Ya le cambié a español.</p> <p>32 F: Le cambió a español.</p> <p>33 Y: Porque había unas palabras con las que no me sentía segura de que... si significaban eso o no. Ya por eso cambié a español.</p> <p>34 F: Okay. Pues si puede venir a la hora del lunch, un momentito, para terminarlo...</p> <p>35 [Yael nods]</p> <p>36 F: ¿Sí?</p> <p>37 Y: Sí.</p> <p>38 F: Muchas gracias. [Conversation continues 2 hours later]</p>	<p>answer</p> <p>108. Distinction between Spanish as a language for comprehension and English as a language for data processing</p> <p>108-114. 2nd try. Use of English with the wording information</p> <p>114. 2nd try. Use of English with operations</p>	<p>Spanish dominant, language density in the wording</p> <p>110. English wording; no need of translation</p> <p>110. English wording; no need of translation</p>
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<p>deviated sketch; not corrected by the interviewer).</p> <p>95. Addition of the middle floor to the number of floors of the building by the interviewer.</p> <p>78-96. 3rd try. Deviated answer with arithmetical and visual reasoning. Procedural mistake when counting the number of floors from the middle to the bottom (6 instead of 7); validated by the interviewer.</p>	<p>39 F: ¿Me puede explicar otra vez cómo hizo esto?</p> <p>40 Y: Primero lo leí todo e iba subrayando como los números, como las letras pero dónde venían escritos números. Después leí la pregunta. Y fui analizando. Y aquí hice mis gráficas.</p> <p>41 F: ¿Cómo hizo sus gráficas?</p> <p>42 Y: Como decía subió uno y ya ponía uno arriba. Y bajó uno y ya ponía uno abajo. Y pues así lo iba haciendo. Y ya la conclusión que saqué es que inició en el piso siete. Luego subió uno, que es ocho. Bajó uno, está en el siete otra vez. Subió tres, llegó a diez. Y bajó a diez y entonces quedó en el piso cero. Y por lo tanto yo pienso que el departamento tiene más de siete pisos, que podría ser que tenga diez o más.</p> <p>43 F: ¿Y no me puede decir cuántos?</p> <p>44 Y: No sabría exactamente cuántos.</p> <p>45 F: ¿No?</p> <p>46 Y: No. Pero yo digo que son más de siete. Y está entre siete y diez. Entre más de siete y diez.</p> <p>47 F: ¿Más de siete?</p> <p>48 Y: Ajá.</p> <p>49 F: Y más de diez.</p> <p>50 Y: Más de siete. Podría ser diez o más de diez o entre... diez.</p> <p>51 F: Ajá. A ver, ¿me dice cómo utilizó las lenguas aquí? ¿Empezó leyéndolo en inglés?</p> <p>52 Y: Pues sí, porque todo está en inglés.</p> <p>53 F: Ajá. ¿Luego qué hizo? ¿Cuándo cambió a español?</p> <p>54 Y: A español...</p> <p>55 F: ¿Se acuerda?</p> <p>56 Y: Sí. Como cuándo lo estaba haciendo decía: oh, up one y como éstas son palabras que yo ya sé, así lo iba haciendo, ¿no?, todo en inglés. Pero después como para estar segura lo tuve que cambiar a español para saber si... con seguridad si era ésta la respuesta o no.</p> <p>57 F: ¿Cuándo?</p> <p>58 Y: Cuándo ya tenía que, cuándo nada más tenía que... para escribirlo.</p> <p>59 F: Mm [continuing conversation].</p> <p>60 Y: Y para estar segura, para comprobar.</p> <p>61 F: ¿Y cómo lo comprobó eso? ¿Qué quiere decir lo tuve que cambiar? ¿Cómo lo cambió, up one, por ejemplo, a español?</p> <p>62 Y: Pues no sé, le puse como la respuesta. Ya la tenía en mi mente en inglés pero dije no, se me facilita más en español.</p> <p>63 F: ¿Y la respuesta final es siete o... ?</p> <p>64 Y: No, es ehm... . Le puse inició en el piso siete.</p> <p>65 F: ¿La respuesta significa todo esto que escribiste?</p> <p>66 Y: [Nodding] Todo esto, es.</p> <p>67 F: Okay. ¿Y mientras lo estaba pensando lo pensó en inglés?</p> <p>68 Y: En español.</p> <p>69 F: ¿Todo en español?</p> <p>70 Y: Nada más lo que estaba leyendo es como lo estaba pensando así en inglés, así como lo leí.</p>		
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- 71 F: Nada más al leerlo lo estaba pensando en inglés.
 72 Y: No, y también cuando estaba haciendo mis gráficas en lugar de decir, oh subió uno, decía oh, up one o down two, o así.
 73 F: ¿Podemos volver a repasar el enunciado? Hay una pequeña cosita que debería... O sea, sí podemos saber, yo le digo que sí podemos saber...
 74 Y: [Interrupting] Cuántos...
 75 F: ... Cuántos hay. ¿Quiere volverlo a leer? ¿Quiere que la ayude?
 76 Y: Eh, ¡sí!
 77 F: ¿O lo quiere leer por su cuenta?
 78 Y: Dice que está en el piso del medio.... Está en el piso del medio, después sube uno... ¿Puedo usar un lápiz?
 79 F: Sí. [Pause. Goes to get a pencil] Si quiere otra hoja...
 80 Y: No, así está bien.

Jamie is shopping in a large department store with many floors. She enters the store at the middle floor, and immediately goes to the credit department. After making sure her credit is good, she goes up one floor to the jewelry department. Then she goes down one floor to the children's department. Then she goes up three floors to the toy department. Finally Jamie goes down ten floors to the main entrance of the store, which is on the first floor, and leaves to go to another store down the street.

How many floors does the department store have? Why?

81

7 porque Jamie inició en el piso 7
 luego subió uno -8 bajo 1 quedó en 7
 subió 3 quedó en 10 y bajo 10 quedó
 en el piso cero, por lo tanto el department
 tiene 10 más de 7 pisos, podría ser
 que tenga 10.

- 82 Y: Listo, creo que ya la tengo.
 83 F: Okay. A ver, me explica.
 84 Y: Se supone que empieza aquí, en el piso del medio. Luego dice up one, aquí está. Luego down one, aquí se queda en el medio. Luego up three. Es uno, dos, tres. Luego down ten. Uno, dos, tres, cuatro, cinco, seis, siete, ocho, nueve, que son, se hacen diez aquí. [Counts again] Uno, dos, tres, cuatro, cinco, seis, siete, ocho, nueve, diez. Entonces aquí está diez. Y como aquí es la mitad, entonces para completar tengo que poner otros más aquí. Entonces son... Si son seis aquí...

	<p>85 F: ¿Son seis aquí?</p> <p>86 Y: Sí. Porque el... inicia aquí, inicia en el tres. Entonces tuve que contar diez hacia abajo y le completé otros siete para que sí hicieran diez. Entonces, en la mitad, la mitad queda aquí y de la mitad para abajo son seis.</p> <p>87 F: A ver. ¿Me puede decir como son seis de la mitad para abajo?</p> <p>88 Y: Porque cuento de la mitad y es uno, dos, tres, cuatro, cinco, seis. Entonces de la mitad para arriba también tendrían que ser seis, si se supone que está en medio. Y entonces total serían doce. Porque si son seis de la mitad para abajo, aquí también tienen que ser seis. Y seis y seis son doce.</p> <p>89 F: Seis arriba y seis abajo y se deja...</p> <p>90 Y: Y quedan doce.</p> <p>91 F: ¿Y se deja cuál?</p> <p>92 Y: ¿Cómo? ¿En el total?</p> <p>93 F: Mm [validating].</p> <p>94 Y: Doce.</p> <p>95 F: ¿Más...? ¿Ha contado el piso del medio?</p> <p>96 Y: ¡Oh! Con ese serían trece entonces. ¡Oh!</p> <p>97 F: ¿Sí? A ver, ¿cómo lo pensó ahora?</p> <p>98 Y: Pues igual. Leí otra vez esto, los datos y como ya sabía que esto es el medio, pues...</p> <p>99 F: Esto lo sabía ahora, ¿pero antes no se había fijado?</p> <p>100 Y: Antes no me había fijado.</p> <p>101 F: Okay.</p> <p>102 Y: Es cuestión de observarlo bien. Y pues ya hice otra vez una línea, la dividí por dos partes y en la parte... en la que divide es el piso del medio. Y ya fui contando, basándome en los datos que tenía. Y así lo pensé.</p> <p>103 F: Okay. Very good.</p> <p>104 Y: ¿Sí es esto?</p> <p>105 F: Mm [agreeing] ¿Qué idioma utilizó ahora?</p> <p>106 Y: Los dos.</p> <p>107 F: ¿Cómo utilizó cada uno?</p> <p>108 Y: El inglés lo usé como con los datos y ya el español como para entender más.</p> <p>109 F: Pero aquí me puso, por ejemplo, "middle floor", ¿no?</p> <p>110 Y: Sí, por eso, cuándo digo los datos es como toda la información que te dan en el problema. Esa toda la dejo así en inglés, no lo traduzco al español. Y ya nada más para poder poner la respuesta. Y entonces ya me toca pensar en español.</p> <p>111 F: Okay. ¿Lo pensó todo en español también la otra vez?</p> <p>112 Y: No todo.</p> <p>113 F: ¿Qué pensó en inglés?</p> <p>114 Y: ¿En inglés? Los datos que tenía como las operaciones como de sumar y restar, esto lo estaba haciendo todo en inglés.</p> <p>115 F: Okay, ¿qué más?</p> <p>116 Y: Y ya.</p> <p>117 F: ¿Ya? Okay. [Continues in GLQ, 1]</p>		
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MEMO	1. "Middle floor" is not marked, and it is not appropriately mathematized. 56-62, 108. Use of Spanish to understand and check the answer, and differentiated from data processing with English.
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General Language Questions		Language	Tentative
1	[Comes from A4,115] F: Tres preguntitas más, pues, así en general. ¿En general, cuándo ha usado el inglés?	1-10. Use of English as reading language	English wording
2	Y: [Interrupting] ¿Cuándo...?		
3	F: [Continuing] ¿Para qué? Mientras resolvía estas cuatro actividades, ¿no?		
4	Y: Mm [agreeing].		
5	F: ¿Cuándo ha usado el inglés?	1-12. Use of English to extract information of the wording	12: English wording and no need to translate and make additional work, 18-20: to learn English
6	Y: ¿Cuándo lo usé más?		
7	F: No, pues cuándo usted cree que...		
8	Y: [interrupting] En general		
9	F: ... En general al resolver esto usa el inglés.		
10	Y: Lo uso cuando leo el problema, porque todo viene en inglés. Entonces cuando leo lo uso. Y ya también cuando necesito cómo buscar la información. Como los números y eso. También ahí lo uso.		
11	F: ¿Qué quiere decir con los números?		
12	Y: Como en el último problema, como aquí dije: oh, up one... y como vienen los números pero escritos en inglés y entonces así ya no tengo que hacer doble trabajo de traducirlo a español, ya lo dejo así como está.		
13	F: ¿Para hacer las operaciones quiere decir o para qué?		
14	Y: Ajá.		
15	F: Las operaciones entonces las hace en inglés, ¿por no traducirlo a español?	1-19. Use of English to perform operations	12, 16: English wording and no need to translate and make additional work, 18-20: to learn English
16	Y: Sí, para no hacer doble trabajo.		
17	F: Porque... Lo hace así entonces para ahorrarse trabajo. ¿Por alguna otra razón cree que lo hace así? Esto de hacer las operaciones o agarrar la información en inglés?		
18	Y: Sí, porque me va a servir para aprender más inglés.		
19	F: Para aprender más inglés. Muy bien. Algún otro uso del inglés.		
20	Y: Mmm... [thinking]. No. Bueno, sí porque en cada problema como hay como nuevo vocabulario que aprendo. También me sirve porque lo practico.		
21	F: ¿Pero cuándo usa el inglés a la hora de resolver los problemas?		
22	Y: Pues no, sólo ya con lo que dije.		
23	F: ¿Y en general cuándo ha usado el español?		
24	Y: Para estar segura de si es la respuesta correcta o no.		
25	F: ¿Por qué cree que lo hace con español eso?	23-24, 27-28. Use of Spanish to check the answer	25-26. Spanish dominant, more confidence with the home language
26	Y: Porque como es mi idioma que toda la vida he hablado por eso tengo ya más seguridad y con el inglés apenas lo voy aprendiendo.		
27	F: Y para estar segura, ¿qué se refiere con esto, para estar segura de...?		
28	Y: Para comprobar las respuestas, para... Sí, para asegurar-me de que sea la correcta.		

<p>29 F: Mm [agreeing]. Muy bien. ¿Que más cosas [pause] en español ha pensado?</p> <p>30 Y: No. No, es todo también.</p> <p>31 F: ¿Sí? Porque mientras pensó, a veces me dijo... ¿Sí? Se acuerda que mientras estaba pensando los problemas algunas cosas me dijo que las pensó en inglés, algunas cosas en español...</p> <p>32 Y: Algunas cosas que no, como algunas palabras que no me sentía familiarizada como las tenía que traducir. También ahí usé el español, y el inglés. Porque algunas palabras venían en inglés y decía: oh, no sé que es eso, y ya lo traducí.</p> <p>33 F: ¿Lo tradujo? ¿Y cómo... si no sabía qué era, ¿cómo lo tradujo?</p> <p>34 Y: [Laughing] Usted me ayudó.</p> <p>35 F: [Laughing] Okay. Vale, ¿alguna cosita más, del español?</p> <p>36 Y: No.</p> <p>37 F: Y hablando de las frases o palabras que le resultaron difíciles, ¿hay alguna palabra o frase que le haya resultado difícil en inglés?</p> <p>38 Y: Sí, en el problema dos.</p> <p>39 F: ¿Cuál?</p> <p>40 Y: Como me confunden como los términos de cómo llamar a éste, como perímetro y diámetro, eso me confundió.</p> <p>41 F: Okay. Sí me acuerdo. ¿Qué más?</p> <p>42 Y: Y algunas palabras también como que son antónimos, ¿como opuestos? Y yo misma digo, ¡oh!, esto es... Como por ejemplo aquí que decía cheaper yo me confundí y dije: ¿Es esa barato o caro?</p> <p>43 F: Okay.</p> <p>44 Y: Como los antónimos, como los opuestos, eso es lo que me confunde.</p> <p>45 F: ¿Y cómo sabe que significaba una cosa o la otra?</p> <p>46 Y: Porque usted me dijo [laughing].</p> <p>47 F: Sí, no. Usted me pregunto esto es barato o caro, como sabía...</p> <p>48 Y: Porque yo ya sabía esta palabra, pero esto es lo que me confundía.</p> <p>49 F: Okay. Estaba dudando. A veces se confunde si es una cosa o la otra opuesta.</p> <p>50 Y: Sí.</p> <p>51 F: Okay. ¿Alguna otra cosa? ¿Alguna otra palabra o frase?</p> <p>52 Y: Sí, porque como lo de observar los patrones. Como eso ya me sentía bien segura, bien familiarizada como con todo porque esto siempre lo hacíamos en clase. Y en esa no tuve tanta, como no tuve que traducirlo al español mucho. Porque pues ya, como yo decía: oh, esto ya lo sé, o ya lo vi, ya me acordaba.</p> <p>53 F: Mm [agreeing]. Ahí no tuvo problemas.</p> <p>54 Y: No.</p> <p>55 F: ¿Alguna otra cosa con la que tuvo.... ? En la actividad cuatro o no sé, si hay alguna otra cosa...</p> <p>56 Y: No.</p> <p>57 F: ¿No?</p> <p>58 Y: No.</p> <p>59 F: Pues ya está. Si no quiere añadir nada más esto es todo.</p> <p>60 Y: No, ya está.</p> <p>61 F: ¡Pues muchas gracias!</p>	<p>17-21. Use of English to improve the knowledge of this language</p>	<p>18-21. Perspective of herself as English language learner, while being a mathematics learner.</p>
MEMO	31-34. Use of English as default language (12, 16: no need of additional work on translating).	

18-20. Positioning as an English language learner together with the role of mathematics learner. 37-61. Good recall of the English words that presented difficulties while solving the four activities.
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Yael has an excellent Spanish BICS as she chooses to talk in Spanish, which she does with no problems. Her writings show that her Spanish writing skills are very good, also when referring to the informal communication of mathematical talk. Her Spanish CALP is good, as well as her English CALP (Yael has only minor conceptual problems like the difficulty with the notion of perimeter). Her English BICS is basic/good, with minor issues regarding the understanding of some words.

On A1 Yael is aware that no initial prices are given. She uses percentages correctly. English is used on the written answer. Yael also uses English in the thinking process, to interpret the advertisements of the two stores, but she mainly thinks in Spanish.

A2 is thought mainly through English on the 1st try, switching to Spanish when writing the answer because Yael says she does not know some of the words needed to express it. On this first approach, the answer is not correct because she understands the wording question in a deviated way: In which of the figures the use of the perimeter is more useful (“más útil”)? Furthermore, she confuses perimeter with diameter. This two facts could be intertwined (one leading to the other). She applies the notion of volume to the circle. On the 2nd try, once the notion of perimeter is understood, Yael provides operational definitions for the perimeters of the figures (the addition of the sides for the case of the square and the pi times radius for the case of the circle; this last one amended by the interviewer). She says the perimeter of the square is 25 (5·5) and compares it with an estimation of the perimeter of the circle to deduce that the square has a greater perimeter. Yael says this 2nd try is performed through the use of Spanish (so this includes the calculations). On a 3rd try, when asked how she gets the “25”, she realizes she made a mistake and provides the correct answer, again with an estimation for the perimeter of the circle.

A3 is solved correctly with an arithmetical reasoning, adding two tiles from one figure to the next one. A function that relates the figure number and the number of tiles in such figure is also constructed to check the answer (matching the procedure seen in class). The answer is written in English, the language that Yael says she uses to think. She does not mention any use for the Spanish language for this problem.

On the first try of A4, English is used to think about the problem, and there is just a switch to Spanish to check and write the answer (with a code mix on writing –“department”–). There is no mathematization of the middle floor according to its particular symmetry function., Yael is aware that the total

number of floors of the building is uncertain, though. Jamie's movements are correctly situated under a relative interpretation of the floors' positions. Once the interviewer says that the total number of floors can be stated (2nd try), Yael mathematizes the middle floor in relation to its symmetry condition, but making a counting mistake when Jamie goes down ten floors (she draws only nine floors on her sketch). Finally (3rd try) the interviewer states that the middle floor must be included on the total number of floors of the department store. For the second and third tries, English is used to keep track of the wording information and to perform the addition and subtraction operations. However, Spanish is said to be the language for the understanding.

Activities' (Key ideas) summary

Object 40: Yael-First reduction (End)

- Word meaning demand (cheaper) as it is a key word to find the answer (Yael hesitates between the actual meaning of the word or its opposite).
- Thinking process in English in relation with numbers, money and percentages, without translating the wording information (when it is already understood). Spanish support to get the meaning of the wording and to write the answer (which is written in English).
- A deviated understanding of the wording leads to a deviated solution. In particular Yael interprets perimeter as diameter. Thinking process mainly through English.
- Solving process in relation to the square and circle's figures in English, but answer translated to Spanish due to lack of English terminology; on the first approach to the problem.
- Use of Spanish as a thinking language (in fact, there is no mention of English use), including calculations, once the geometrical problem is properly understood (with the help of the interviewer).
- Use of English for everything in relation with a similar problem done in class, by applying similar procedures to those explained in class (formula and arithmetical sequence).
- In a dense wording, use of English as the default language. It helps to identify the essential information (though skipping a key point –the entrance through the middle floor–) and leads Yael to use English as a thinking language in all the approaches to the problem. Also English is used to perform arithmetical operations.
- Use of Spanish to write the answer, despite the thinking process is mainly in English, A4.
- Within the interview, evolution from Spanish as being the unique declared language (apart from reading the wording in English) to the recognition of Spanish as the language for understanding and English as the language for data processing.

Historical profile	Bilingual profile (Spanish dominant)	Activity
<ul style="list-style-type: none"> • 17 years old • Transitional / mainstream class • 6 months in USA • Likes California • Spanish readings • No English readings (just compulsory books from the English class) • Spanish at home (English sometimes with her sister) • Spanish (and some English) with friends • Spanish when possible at school • Homework help: nobody 	Word meaning demand (cheaper)	x
	Use of English as a writing language	x x
	Deviated writing expression	x x x
	Use of English as a reading language	x
	Use of English as a thinking language	x 1 x 1
	Use of Spanish as a thinking language	x
	Use of Spanish as a writing language (A4.1: with code mix –department–)	x 1
	Notion of volume applied to the circle	
	Deviated wording question understanding	1
	Unknown meaning (greater)	1
	Use of Spanish for everything	2
	Use of English for everything	x
	Use of Spanish to check the answer	1
	Use of Spanish for understanding	2-3
	Use of English to process data	2-3
	Use of English with operations	2-3
	Use of English as a reading language	GLQ
	Use of English to extract and use information of the wording	GLQ
	Use of English to perform operations	GLQ
	Use of Spanish to check the answer	GLQ
Use of Spanish to grasp the meaning of unknown words (with interviewer's help)	GLQ	
Procedural profile	Conceptual profile	
<p>1 ✓ Both stores can be cheaper depending on the initial prices ✓ \$30 on both stores results in 40% store cheaper</p> <p>2.1 ✗ Deviated answer, adapted to the deviated wording question understanding (figure where the diameter is more used)</p> <p>2.2 ✗ [square's perimeter: $5 \cdot 5 = 25$] ✓ Comparison of lengths by estimation of the circle's perimeter ($3.1416 \cdot 5$)</p> <p>2.3 ✓ Error detection after review: [square's perimeter = 20] ✓ Comparison of lengths by estimation of the circle's perimeter ($3.1416 \cdot 5$)</p> <p>3 ✓ Application of a pattern given by adding 2 from one figure to the next ✓ Finding and application of the function that relates the number of tiles per figure with the figures' number</p> <p>4.1 ✗ Mathematization of the middle floor according to its symmetry particularity (✓ Awareness that many possibilities stand for the total number of floors) ✓ Relative situation of floors</p> <p>4.2 ✗/✓ Sketch of the building with 9 floors instead of 10 when Jamie goes down ten floors ✗/✓ Symmetry through middle floor to get the top of the building ✗ Middle floor not computed on the total</p> <p>4.3 ✗ Interviewer indicates that middle floor must be included to the total</p>	<p>1 ✓ Notion of percentages as relative value</p> <p>2.1 ✗ Perimeter-diameter confusion ✗ Notion of volume applied to circle</p> <p>2.2 ✗ Perimeter concept ('¿Es la suma de todos los lados?') consolidated by the interviewer ✓ Addition of all four sides for the square's perimeter ✗/✓ Circle's perimeter formula amended by the interviewer</p> <p>2.3 ✓ Perimeter concept</p> <p>3 ✓ Notion of arithmetical sequence associated to a sequence of figures ✓ Notion of function to relate two variables</p> <p>4.1 ✓ Notion of number line</p> <p>4.2 ✓ Notion of number line</p> <p>4.3 ✓ Notion of number line</p>	

Object 41: Yael-Second reduction

Object 42: Julián First Reduction
(Beginning)

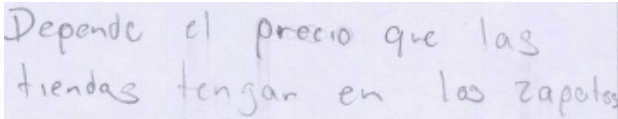
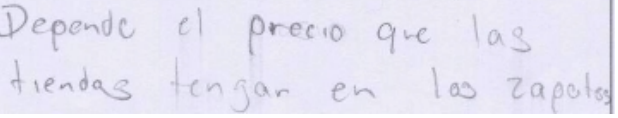
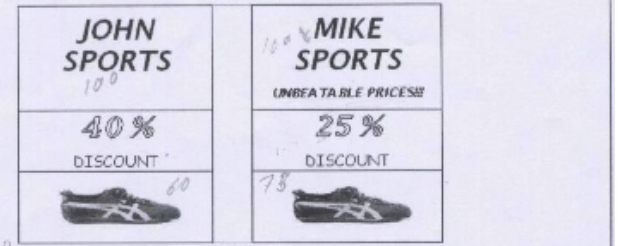
Julián has a good behavior and kind character. He is not always doing the assigned tasks during class and does not do the assignments usually. He has some difficulties understanding the assignments.

Math teacher description

Class	Interview language	Interview date	Origins	Age	USA arrival	Living in California	Spanish readings	English readings	Home language	Friends language	Main language at school	Homework help
HS Transitional	Spanish	June 2010	Mexico	17	October 2007: 2 years and a half	Likes it. Came back to meet his mother, who had not seen since the age of 6. Likes school because there are more opportunities. In Mexico didn't attend school for a certain time.	Novels, tales...	School books, some magazines	Spanish	Both (more Spanish)	[when talking with peers] Spanish (Physical Education, Spanish), English (Health, English), both (Math)	Nobody

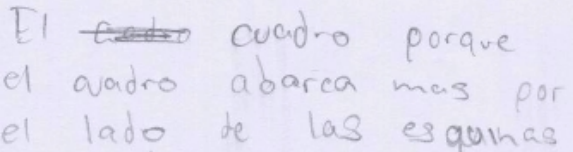
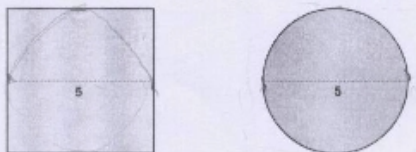
Historical bilingual profile

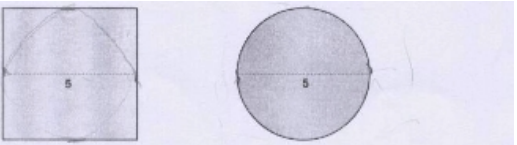
Math	A1. Julián marked "English and Spanish".		Language	Tentative
9-18. 1st try. Right answer with percentages as relative value (different initial prices assigned – without numerical concretion). 20-21. Oral exemplification assigning initial prices to the	[Activities are solved in the following order: A3, A1, A2, A4. Conversation starts at A3,1]		1-9. Translation demand (cheaper)	5. Unknown meaning
	1	[Comes from A3,13] J: Ésta [A1] sí no la entiendo. Esta parte.	4-5. Code switch	Reading English wording
	2	F: ¿No? ¿Qué es lo que no entiende?		
	3	J: Qué tengo que hacer aquí.	18. Spanish as unique writing language	Spanish dominant
	4	F: ¿Qué es lo que le pregunta?		
5	J: Which of these two stores are the shoes cheaper. Cual de estas dos tiendas es la... La cheaper, no entiendo esa palabra.			
	6	F: Más baratas.	18. Syntax deviation (el)	Quick writing, unknown
	7	J: ¡Oh!		
	8	F: ¿No? Más baratas.		
	9	J: Sí, más baratas. Pero no dice la verdad, porque no dice el precio exacto de esto para cuánto es sin el descuento.		
	10	F: Sí. ¿Entonces qué? ¿Qué, le hacen falta datos?	20. Code	Hybrid
	11	J: Sí. Sí, porque imagínese que aquí ofrecen el cuarenta por ciento de descuento pero que tal si lo dan más caro y lo rebajan y ganan igual.		

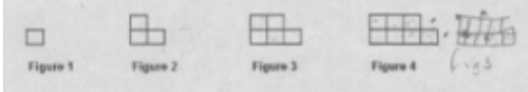
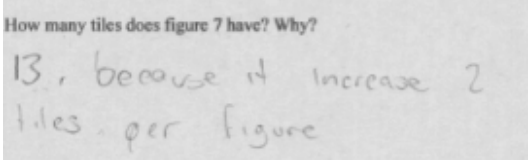
<p>shoes to show that different final prices are possible: \$100 as initial prices and calculation of final prices. No concrete counterexample of the other store being cheaper.</p> <p>53, 56-57. Consideration of 25% store as possible cheapest store after knowing 'unbeatable' meaning (44-52).</p>	12	F: Sí. Pues ponga esto. Lo que me acaba de decir. Lo puede poner como respuesta.			
	13	J: ¿Y qué le pongo aquí entonces, "no sé"?			
	14	F: Pues depende, ¿no? ¿Sí? Lo que... Pues expréselo como quiera.			
	15	J: [Unintelligible sequence]			
	16	F: ¿Sí?, como quiera. Si en lugar de... bueno, si se equivoca ponga una rayita encima y luego ya... Nada más escribe al lado, ¿sí?			
	17	J: Sí.			
	18	 <p>[Continues in A2,1]</p>		<p>mix (okay)</p> <p>20. Code mix (x2) (forty per cent of discount, per cent of discount)</p>	<p>language</p> <p>Reading English statement (advertisements)</p>
	19	[comes from A4,17] F: ¿Qué es lo primero que ha hecho para resolver esta actividad?			
	20	J: Pues una vez, primero la comparación del porcentaje que están de descuento. Okay. Aquí me dice que es forty per cent of discount y aquí es treinta per cent of discount. Pero la diferencia es de que ninguno de estos tiene el precio. Por decir, si aquí [writes down "100" in 40% store, see A1,21] tuviera cien dólares y aquí tuviera cien dólares, así cien dólares [writes down "100 %" in 25% store, and immediately crosses the "%" out, see A1,21]. Sería la diferencia que si son cien dólares y tiene cuarenta por ciento de descuento sería entonces sesenta el precio de esto. Y aquí sería el veinticinco entonces saldría setenta y cinco dólares el par de zapatos. Y entonces aquí me daría cuál es el, uno de los más baratos. Podemos decir esto. Pero como no tiene precio no podemos saber cuál es.			
	21		 <p>[Entire answer]</p>	<p>24-43, 54-55. English almost exclusively as reading language</p> <p>27. Code mix (when)</p> <p>35. Code mix (yeah)</p> <p>44-52. English as thinking language ('Unbeatable')</p> <p>53. Code mix (so)</p> <p>57. Code switch (non mathematically specific)</p>	<p>Spanish dominant</p> <p>Conversation about English thoughts</p> <p>Hybrid language</p> <p>47-49. Unknown meaning, no translation</p> <p>Hybrid language</p> <p>Strong bilingual profile</p>
22	F: Okay. ¿Y cómo lo pensó todo esto [points to the pictures of the stores]?				
23	J: Pues no sé, principalmente...				
24	F: ¿En qué idioma empezó a pensar?				
25	J: ¡Oh! En español.				
26	F: En español, ajá. ¿Y cuándo cambió a inglés?				
27	J: When... Cuando tuve que leer todo esto.				
28	F: O sea empezó...				
29	J: Y en pensar...				

- 30 F: [interrupting] Perdona. Empezó leyendo esto en inglés, ¿no?
 31 J: Sí.
 32 F: ¿Y luego cuándo cambió a español? Permítame que le pregunte así.
 33 J: Cuando... Cuando tuve que ver, o sea, cuál era la diferencia, o sea cuál era la información que necesitaba para resolverlo.
 34 F: ¿Allí empezó a pensar en español?
 35 J: Yeah.
 36 F: ¿Y luego volvió a cambiar a inglés?
 37 J: Oh, no.
 38 F: ¿Siguió en español?
 39 J: Sí.
 40 F: ¿Y continuó en español hasta el final?
 41 J: Sí.
 42 F: ¿No pensó ninguna cosa en inglés?
 43 J: Eh... No. [Continues in A2,17]
 44 [Comes from A2,50] F: Volviendo a éste [A1], perdona, ¿alguna otra cosita –pues ya que hemos visto estos ejemplos, a lo mejor no había pensado antes– que pensó aquí en español? Digo en inglés.
 45 J: ¿Aquí en inglés?
 46 F: Sí.
 47 J: Unbeatable [/anbileitabol/no right pronunciation] price, esto no lo entiendo muy bien, esta palabra. Esa no la utilizo mucho.
 48 F: Unbeatable. ¿Sabe qué significa?
 49 J: No, no estoy muy seguro.
 50 F: Como precios insuperables, sería, ¿no? O precios muy buenos. Unbeatable, que nadie los puede ganar.
 51 J: Oh, sí.
 52 F: Beat es como ganar. Unbeatable, que nadie los puede ganar, ¿no?
 53 J: Oh, sí, sí ya. So, pues no estoy muy seguro porque pues en parte podríamos decir que también puede ser éste [25% store], pero no es por eso lo que dice. Pero no, no. No creo porque tiene aquella más descuento. Yo necesitaría el precio de eso para resolverlos.
 54 F: ¿Pero alguna cosa que pensó aquí en inglés?
 55 J: No.
 56 F: Nada más era por si se le había ocurrido alguna cosa. ¿Entonces dice que podría ser ésta también más barata?
 57 J: Podría ser, pero pues... I'm not sure because... las tiendas utilizan esto para atraer a la gente, y pues... sometimes is not true.
 58 F: Sí. Good. Two more activities.
 59 J: Alright. [Continues in A3,14]

MEMO | 18. Syntax deviation (el)

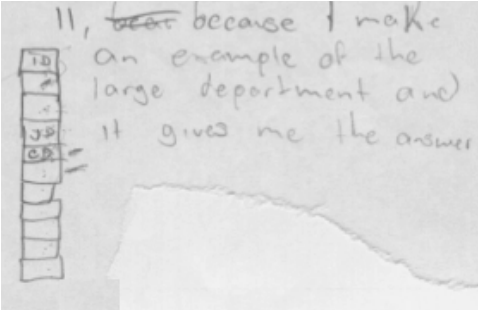
Math	A2. Julián marked "English and Spanish".		Language	Tentative
15-18. Dynamic reasoning based on a translation move.	1	[Comes from A2,18] J: Oh, aquí dice que cuál es el... [Reading] which of these figures has a greater perimeter.	1-9. Translation demand (greater)	Meaning not clear, dotted line interpretation, solution check
	2	F: Sí.		
	3	J: Okay. Pues yo digo que los dos, pues los dos son de cinco aquí en esto [dotted line], ¿no? ¿Pero a qué se refiere con greater... perimeter?		
17. Dotted line matching.	4	F: ¿Qué significa greater?		
	5	J: Éste me hace dudar.		
	6	F: Mayor.		
17. Confusion between diameter and side for the circle's case (terminology issue).	7	J: ¿Mayor? ¿Cómo?	1, 17. Code switch	Reading English wording
	8	F: Más grande que.		
	9	J: Okay. ¿Luego un perímetro más grande? [Makes an imaginary circle with the pencil, below the two figures]		
	10	F: Great, ¿no? Great, greater es el comparativo.		
	11	J: Okay, okay.		
	12	F: Mayor.	3. Code mix (greater perimeter)	Unclear meaning, reading English wording
	13	J: Sí. Entonces lo voy a hacer un poco. Entonces, ¿cuál de estas dos figuras tiene un perímetro más grande?		
	14	F:Cuál de estas dos figuras, ajá.		
	15	J: Oh, pues sería éste, ¿no? El cuadrado. [Moves the pencil like inscribing the circle inside the square]		
	16	 [Continues in A4,1]	3, 9, 11, 17. Code mix (okay)	Hybrid language
19-22. Adequate visual comparison between linear and curvilinear paths with shared extremes.	17	[Comes from A1,43] J: [Reading] Which of these figures has a greater preimeter. Okay. Yo utilicé porque los dos de estos... Aquí el lado de los dos es de cinco [dotted lines]. So si pudiera haber un modo en que moviéramos el círculo para acá [translates the circle dotted line to the square dotted line taking it imaginarily with the fingers], quedaría exactamente esta parte con esta parte [the butts of the dotted lines would coincide]. Y la diferencia de que esto quedaría este lado y este lado [points to the 2 tangent points that would not be in the dotted line: top and bottom]. Pero como es un círculo quedaría esto...	16, 22. Spanish as unique writing language	Spanish dominant, Spanish as thinking language
	18	 [Julián draws the circle inside the square]	17. Confusion between diameter and side for the circle's case	Quick arrangement of arguments or unknown appropriated terminology
1-22. 1 st try. Right answer with visual reasoning, inscribing	19	J: Estas las esquinas quedaría así. Y obviamente lo que son los corners le dan más perimeter a la figura [square].	17. Code mix (so)	Hybrid language
	20	F: Sí. ¿Así lo pensó?		
	21	J: Sí. Y por eso el cuadro, porque [reading] el cuadro abarca más, más por lado de las esquinas que el círculo. [writes down "que el círculo"]	19, 46. Code mix	Strong bilingual

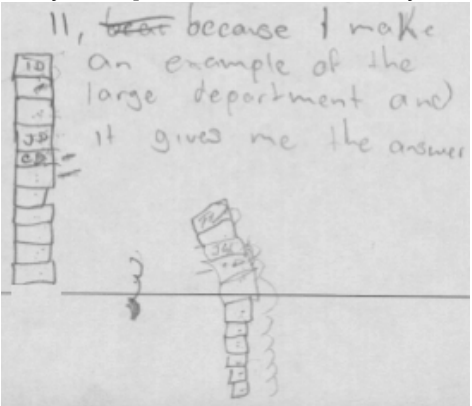
the circle in the square.	 <p>22 El este cuadro porque el cuadro abarca mas por el lado de las esquinas que el circulo.</p> <p>[Entire answer]</p> <p>23 F: A ver, ¿cómo empezó aquí? ¿En qué idioma empezó a pensar el problema? ¿Lo leyó en inglés, no?</p> <p>24 J: Sí, claro, me pregunta [en inglés].</p> <p>25 F: ¿Y luego? ¿Cuándo cambió a español?</p> <p>26 J: Español cuando tuve que pensar, o sea en que... cómo lo podía resolver.</p> <p>27 F: ¿Ahí empezó a pensarlo en español?</p> <p>28 J: Sí.</p> <p>29 F: ¿Cuándo volvió a cambiar a inglés?</p> <p>30 J: Ya no le cambié inglés.</p> <p>31 F: ¿Ya no le cambió a inglés?</p> <p>32 J: No.</p> <p>33 F: ¿Continuó en español?</p> <p>34 J: I only used English at the beginning, so that's why I use more Spanish.</p> <p>35 F: ¿Para leer la pregunta sólo?</p> <p>36 J: Sí.</p> <p>37 F: ¿Pero no pensó ni en el círculo, circle, or square, o five or anything like that in English?</p> <p>38 J: Sí: square, five and the circle.</p> <p>39 F: ¿Eso sí lo pensó en inglés?</p> <p>40 J: Sí. Porque recuerdo las clases, por eso.</p> <p>41 F: ¿Se le hace más fácil esto en inglés?</p> <p>42 J: Cuando recuerdo las cosas de la clase, sí.</p> <p>43 F: ¿Se le hace más fácil hacerlo en inglés?</p> <p>44 J: Porque el procedimiento, recordarlo en español algunas veces no lo recuerdo en español.</p> <p>45 F: Okay. ¿Alguna cosita más que pensó aquí en español ['inglés' !, lapse]?</p> <p>46 J: Pues nada más fue eso: o sea, de la diferencia de los corners, que había esto y...</p> <p>47 F: ¿Pero eso lo pensó en qué idioma?</p> <p>48 J: En español.</p> <p>49 F: Okay. ¿Alguna otra cosita más?</p> <p>50 J: No sé. [Continues in A1,44]</p>	(corners)	profile
		19. Code mix (perimeter)	Strong bilingual profile
		23-39, 45-50. Mainly Spanish as thinking language	Spanish dominant
		34. Code switch	Explanation of his use of English in A2
		37-40. English for picture parts (circle, square, five) when thinking in Spanish	40-42. Mathematical English classes recall
		38. code mix ('square, five and the circle')	Words thought in English, interviewer previous utterance imitation (37)
MEMO	16-22. Oral explanation is richer and more extended than the written one.		

38. code mix ('square, five and the circle')				
Math	A3. Julián marked "English and Spanish".		Language	Tentative
8. Graphical representation of a particular case (figure 5).	1	J: ¿Y si no entiendo algo?	3, 6, 13. Code mix (okay)	Hybrid language
	2	F: Sí, me lo puede preguntar. Cualquier duda que tenga...	3, 15(x3), 25. Code mix (so)	Hybrid language
15. Increasing of 2 tiles per figure.	3	J: Okay. Está bien. So... [Pause]	8, 23, 35. English as unique writing language	Similar problems done in English in class
	4	J: ¿Tengo que hacerlo aquí [empty space after the wording], escribir el procedimiento?	8. Grammatical deviance (increase)	Quick writing
15. Visual reasoning on the number of tiles in figure 7 with 6+6+1 tiles.	5	F: Sí.	15 (first line). Code switch	Previous intervention in English (A1,59), English as thinking language
	6	J: Okay.	15 (last lines) (x2). Code switch	Reading English text (wording and answer)
8, 15. 1st try. Right answer with arithmetical and visual abstraction of figure pattern sequence.	7	F: Sí, como si fuera un ejercicio normal... Sí, todo lo que necesite. [Pause]	16-19. English as thinking language	English wording linked to English thoughts, topic explained in English in class (A2,41-44)
			17, 19, 21, 39. Code switch	Conversation about English thoughts
	8		19, 23. Code mix (yeah)	Conversation about English thoughts (19)// Mexican Spanish (23)
	9	J: ¿Puede chequear esto a ver si está bien? El procedimiento que utilicé fue de ver aquí...	20-23, 28-29. Spanish as	Numbers, counting
	10	F: [Interrupting] Si quiere luego al final las comentamos todas.		
	11	J: ¿Pero sí está bien ésta?		
	12	F: Sí, pero si quiere luego al final las comentamos, cuando las termine. Pero sí, creo que sí está bien.		
	13	J: Okay. [Continues in A1,1]		
	14	[Comes from A1,59] [Francesc stands up to fix the camera] F: ¿Con cuál continuamos? [Julián is pointing to A3] ¿Con la tres?		
	15	J: You see that figure one is just one tile. So la figura dos aumenta dos más. Y en la figura tres, al igual que ésta [figure 2], aumentó dos más. So va cambiando así: [figura] cuatro aumenta dos más a la anterior. Y figura cinco, hice un ejemplo aquí [see A3,8] de la figura cinco y le subí dos más y quedó igual. So, [reading] how many tiles does [/dows/] figure seven have. Le puse [reading] "thirteen because it increase two tiles per figure". So lo que utilicé fue que aquí interpreté las tiles que ocupa la figura cuatro y le aumenté dos más y ahí en la figura cinco. Dos más figura seis. Y dos más figura siete. Entonces serían seis acá y seis acá, serían doce, más una, trece.		
16	F: Okay. ¿En qué idioma empezó a pensar esto?			
17	J: In English.			
18	F: ¿Éste lo pensó en inglés?			
19	J: Yeah.			
20	F: ¿Cómo continuó? ¿Cuándo cambió a español?			
21	J: In Spanish, verdad, porque interpretar eso, o sea subirle las... subirle las... Contar cuántas tiles sube.			
22	F: ¿Lo pensó en español eso?			

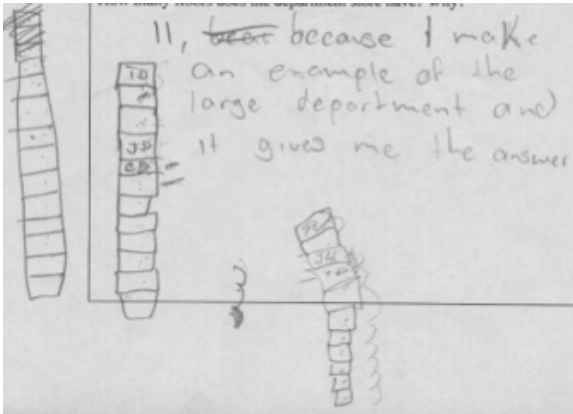
	<p>23 J: [Julián nods] Yeah. Y cambié a inglés cuándo tuve que escribir la answer. 24 F: Sí. 25 J: So eso. 26 F: Luego cambió a inglés otra vez. 27 J: Sí. 28 F: ¿Y el proceso eso de pensar que tenía que añadirle dos, ¿eso lo pensó en español? 29 J: Oh, sí. 30 F: Y iba pensando en los tiles, o iba pensando en alguna palabra en español o cómo lo hacía eso? 31 J: Oh, pues cuadritos. 32 F: ¿Cuadrito pensó? 33 J: Cuadrito, yes. 34 F: Okay. ¿Alguna cosa más aquí del uso de las lenguas? 35 J: Aquí pues no mucho. La verdad es que le puse la [points to the answer] en inglés y no sé. 36 F: ¿Por qué la puso aquí en inglés? 37 J: Pues no estoy muy seguro, la verdad. 38 F: ¿Cómo? 39 J: No estoy muy seguro. I'm not sure because... At the beginning I was thinking in English and just about the process of this pattern. 40 F: Okay. Let's talk about the other one. 41 J: Okay. [Continues in A4,18]</p>	<p>thinking language</p> <p>21. Code mix (tiles)</p> <p>23. Code mix (answer)</p> <p>33. Code mix (yes)</p>	<p>English vocabulary used in class</p> <p>Strong bilingual profile</p> <p>Strong bilingual profile</p>
M	8. Grammatical deviance (increase)		
E	29-32. cuadrito instead of tile. It is good to have a good management of both languages, as when there is a switch need it is easier to continue the thinking		
M	process in such a language. (If Julián did not know 'cuadrito' he must switch to English. This way he continues to think in the language of comfort).		
O	Julián says he starts to think in English (16-19). But maybe it is because the wording is in English and he translates it. Even though, at the end (39) he also says the thinks in English.		

Math	A4. Julián marked "English only".	Language	Tentative
1-4. 1 st try. Credit department situated at the middle floor (checked with interviewer).	<p>1 [Comes from A2,16] J: Una pregunta. 2 F: Sí. 3 J: Aquí dice: [reading] Jamie is shopping in a large department store with many floors. She enters the store at the middle floor and immediately goes to the credit department. ¿Pero en qué piso está el credit department? ¿En el middle floor? 4 F: Sí. 5 J: Okay. Okay. Oh así está... [Draws 3 floors and writes CD in the middle one] [Pause] [Adds 3 floors on top of the drawing] [Pause]</p>	<p>3, 6(x3), 22(x7), 24, 88, 154, 160(x3), 179(x2). Code switch</p>	<p>Reading English wording</p>
5-6, 14. 1 st try. Buildings' sketch containing the floors Jamie goes through. One of	<p>6 J: Aquí me dice que [reading] immediately goes to the credit department. Está aquí en el middle [poiting to the CD floor on his picture], según yo. [Reading] After making sure her credit is good she goes up one floor to the jewelry department. Es aquí, uno arriba. Then she goes up... One, two, three [writes down TD, for toy department]. [1, 2,] tres, cuatro, cinco, seis, siete... [Draws 5 floors to the bottom of his drawing –see A4,14, 1st try– while counting up to the 10 floors Jamie goes down] [Pause] 7 J: [Julian is writing the answer: "11, because", see A4,14, 1st try] ¿Cómo se dice interpreté? ¿Interpreté?</p>	<p>3, 22(x3), 62, 76, 120, 122(x2), 124, 134, 160(x2), 179. Code mix (credit</p>	<p>Strong bi-lingual profile (160: 'crédito... department' and 'credit de-</p>

<p>Jamie's movements is skipped (when she goes down one floor).</p> <p>1-6, 14, 18-23. 1st try. [11 floors] Wrong answer due to no mathematization of middle floor (highest floor reached considered as the building's top).</p> <p>22-25. 1st try (explanation). Error detection of jewelry department situation in 1st drawing (A4, 14), amended on 2nd drawing.</p> <p>22-28, 57-64. 2nd try. Same reasoning as 1st try – [11 floors] Wrong answer due to no mathematization of middle floor (highest floor reached considered as the</p>	<p>8 F: ¿En inglés? 9 J: Ajá. 10 F: No sé... I have interpreted... [Julián writes down "intepretd" (instead of <i>interpreted</i>) in the margin next to A3] Understood. ¿No? Podemos decir, he entendido. 11 J: ¿Interpreted se escribe así [pointing to "intepretd"]? I'm not sure. 12 F: Yo diría I have understood, ¿no? [Pause] 13 F: Yeah, interpret. [checked on the computer] Sí está bien interpret. [Julian has already overcome this issue and is finishing his writing]</p> <p>14  [1st try drawing]</p> <p>15 J: [Unintelligible] That's it. 16 F: ¿Terminó? 17 J: Yeah. [Continues in A1,19] 18 [Comes from A3,41] J: Oh, this part... Ésta está un poco más chungu, un poco más difícil. Todo lo hice en inglés. 19 F: Empezó leyendo el problema. 20 J: Ajá. 21 F: ¿Y luego cómo pensó aquí?, a ver. 22 J: Oh, aquí...[Reading] Jamie is shopping in a large department store. Well, aquí dice... La pregunta es [reading] How many floors does the department store have. Y le puse [1st try: A4,14, reading] eleven, because it make, I make an example of the large department and it gives me the answer. So Jamie shopping is in a large department with many floors. She enters the sotre at the middle floor. So, cuando empecé, puse primero tres cuadros [Starts another drawing, see A4,25]. Para aquí tener un middle. Aquí está el middle de los tres cuadros. So empezó en el credit, credit department. Le puse aquí credit department [A4,25, writes CD]. After making sure her credit is good, she goes up one floor to the jewelry department. So subió uno y éste es el jewelry department [A4,25, writes JW on the floor above CD]. Then she goes down one floor. Oh, wait! Oh! Yeah! Then she goes down one floor to the childs department. So aquí también en el credit department it was the childs department. So regresó otra vez ahí. Then she goes up three floors to the toy department. So subió uno. ¡Oh, <i>pincha madre</i> [or <i>aquí esta mal</i>; or something else: not clear recording]! Me equivoqué entonces. So subió tres. Subió... 23 F: No pasa nada, puede cambiarlo. 24 J: ...toy department [A4,25: writes down TD]. Finally Jam[/dzeim/], Jamie goes down ten floors to the main</p>	<p>department)</p> <p>3, 72, 74, 80(x2), 82, 148, 156, 160(x4), 175(x2). Code mix (middle floor)</p> <p>5(x2), 24, 46, 60, 120, 122, 156, 160, 169, 179. Code mix (okay)</p> <p>6, 22(x2), 68. Code mix (middle)</p> <p>6, 24, 88. Counting in English aloud</p> <p>6, 24, 26, 72, 112, 122, 152, 160, 179. Counting in Spanish aloud</p> <p>7-14. Asking initiative when translating difficulties (intepretd)</p> <p>11. Code switch (non mathemat-</p>	<p>partment' toghether)</p> <p>Strong bilingual profile</p> <p>Hybrid language</p> <p>Strong bilingual profile</p> <p>English wording</p> <p>Spanish dominant</p> <p>Perseverance in English as writing language</p> <p>Writing and thinking in English,</p>
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building's top)-.	entrance. So entonces sería diez. Sería uno, dos, tres, four, five, six, seven, eight, nine, ten. Lo que eh... ohh Okay, wait [adds floors to the 2 nd try drawing, see A4,25].	ically specific)	English word orthographical demand
63. 3 rd try. Solution not valid (by interviewer).	<p>25</p> 	14, 18, 29-38. 1 st try. English for everything. (Except a word demand when writing answer: 6-14)	English wording, good English management
65-87. 3 rd try. Middle floor is assigned to 6 th floor, according to his drawing. Unknown middle floor position.	26 J: Oh my God! [Counts the number of drawn floors] Okay. To the main entrance of the store which is on the first floor and leaves to go to another store down the street. So contando los cuadros que, los floors que hice... So serían tres... y si tu sumas... diez, once. Once éstos. So... Ya.	15, 17. English discourse ('that's it, yeah')	Answer just written and thought in English (A4,14)
88-90. 3 rd try. Drawing started with a <i>random</i> (but greater than 10) number of floors. Initial guess on the numbers of floors as alternative approach (abandoned quickly).	<p>27 F: ¿Once?</p> <p>28 J: Once.</p> <p>29 F: ¿Cómo lo pensó esto? ¿En qué idioma empezó?</p> <p>30 J: En inglés.</p> <p>31 F: ¿Empezó en inglés?</p> <p>32 J: Yeah.</p> <p>33 F: ¿Y luego cuándo cambió a español?</p> <p>34 J: No cambié a español.</p> <p>35 F: ¿No cambió?</p> <p>36 J: Sólo inglés.</p> <p>37 F: ¿Lo pensó todo en inglés?</p> <p>38 J: Sí.</p> <p>39 F: ¿Y esta segunda vez que lo hizo? O sea, esto que hizo ahora mientras me lo explicaba.</p>	18. Code switch (non mathematically specific: 'oh, this part')	Interviewer and Julián previous utterances in English (A3,41)
63-94. 3 rd try. No solution found.	<p>40 J: Eso en español.</p> <p>41 F: ¿Lo hizo en español?</p> <p>42 J: Y un poco de inglés.</p> <p>43 F: ¿Cómo?</p> <p>44 J: Español y inglés.</p> <p>45 F: ¿Cuándo mezcló?, a ver. ¿Cómo, cómo cambió?</p> <p>46 J: Okay. Pensar en inglés, pero interpretando esto [wording], y que traduciendo como al español para así explicar.</p> <p>47 F: ¿Ahora lo tradujo a español?</p>	22, 24, 26. Multiple code switches and mixes.	Strong bilingual profile, reading English text (wording and answer)
92-122. 4 th try. Arrangement of 1 st try drawing. Julian does not remember he		22(x7), 24, 26(x3), 62, 80, 88, 90,	Hybrid language

<p>did a mistake when assigning the jewelry department (102), as he recognized previously (22) and later (118) and states that there are 10 floors on the 1st try drawing and 11 floors on the 2nd try drawing, which is not true (despite interviewer's validation).</p> <p>94-156. 4th try. Interviewer attempts to show where the middle and bottom floors are to find the position of the middle floor. Julián does not respond immediately.</p> <p>125-132. 4th try. Interviewer indicates that top floor reached isn't the building's top floor.</p>	<p>48 J: Yeah.</p> <p>49 F: ¿Pero la primera vez no?</p> <p>50 J: No. I only used the English.</p> <p>51 F: ¿Y ahora porqué sí?</p> <p>52 J: No sé, porque cuándo hablo, usualmente yo utilizo más el español. Por eso.</p> <p>53 F: Okay. Entonces lo estuvo traduciendo a español y siguiendo las instrucciones, ¿cómo? En español, ¿no?</p> <p>54 J: En inglés y español.</p> <p>55 F: Ajá. ¿Y luego cuándo cambió a inglés otra vez?</p> <p>56 J: Yo lo leía... El primero empecé en inglés, después en español. Y pues es todo.</p> <p>57 F: Ajá. ¿Entonces cree que tiene once?</p> <p>58 J: Pues eso es lo que yo pienso.</p> <p>59 F: ¿Por qué tiene once?</p> <p>60 J: Okay. Porque aquí [1st try: A4,14], como empecé aquí pero pues empecé con tres cuadros para obtener la...</p> <p>61 F: Sí, ya me dijo.</p> <p>62 J: ...el medio, so, por eso. Y aquí según si hubiera sido. Como aquí empecé por tres, aquí este credit department...</p> <p>63 F: Le voy a decir yo que la respuesta no es once.</p> <p>64 J: Creo que está doce [or 'trece'] y once, aquí ya [Unclear transcription]. No estoy muy seguro, la verdad.</p> <p>65 F: ¿Pero qué dijo al principio? Empezó con tres, ¿no?</p> <p>66 J: Ajá.</p> <p>67 F: ¿Por qué?</p> <p>68 J: Porque tenía que agarrar el middle.</p> <p>69 F: El middle floor.</p> <p>70 J: Ajá.</p> <p>71 F: ¿Cuál es el middle floor en su dibujo?</p> <p>72 J: Eeeh... sería... doce. Sería... serían doce. Es éste creo. Porque aquí son cinco. Y son once. Serían tres y cuatro, cinco. Y cinco. Pues ahí está el middle floor. [see 1st try: A4,14]</p> <p>73 F: ¿Pero por que piso entró ella?</p> <p>74 J: Por el middle floor.</p> <p>75 F: Ajá. ¿Pero (por) cuál era el middle floor?</p> <p>76 J: Pues el credit department.</p> <p>77 F: Ajá. Pero ahora me lo está cambiando el middle floor, ¿no?</p> <p>78 J: Sí, porque aquí le conté los éstos y pues me cambió [see 2nd try: A4,25].</p> <p>79 F: Pero si entró por éste, no lo puede cambiar el piso de entrada.</p> <p>80 J: Oh, eso sí. Sí. So entonces eso sería el middle floor y pues entonces sí sería aquí [Looks to the wording] [Pause] Pues necesitaría también saber cuál era... cual piso era el middle floor. Porque si era el sexto piso o el... Y si también todo esto creo un poco mejor. Pero está esto un poco a fuerza esto.</p> <p>81 F: ¿Lo puede saber cuál era el piso... en qué número estaba el piso del medio?</p> <p>82 J: No. Porque namás dice the middle floor, no...</p>	<p>122(x2), 126, 152, 156, 160(x3), 175, 179(x2), 181. Code mix (so)</p>	<p>(88: previous utterance in English)</p>	
			<p>22, 122, 160. Code mix (jewelry department)</p>	<p>Strong bilingual profile</p>
			<p>22(x2), 160(x2). Code mix (childs department + 122: child department)</p>	<p>Strong bilingual profile</p>
			<p>24, 88. Code switch when counting</p>	<p>Strong bilingual profile</p>
			<p>24, 98. Code mix (wait)</p>	<p>Previous English utterances (24: counting on previous utterance)</p>
			<p>26. Code mix (floor)</p>	<p>Previous English utterance, English wording imitation</p>
			<p>39-56. 2nd</p>	<p>52. Pre-</p>

<p>143-147, 153-154. 4th try. Interviewer indicates that the number of floors from the middle to the bottom can be found.</p>	<p>83 F: Con esta información sólo no. Pero leyendo más información de por aquí, ¿lo puede saber? 84 J: Sí, se puede saber. 85 F: ¿Lo quiere intentar pensar... 86 J: [Interrupting] ¿Sí? 87 F: ... a ver si lo saca? 88 J: Sí. [Looks to the wording] [Pause] Pues podría interpretarlo así cómo... Podríamos llegarlo a entender como... unos diez o más, porque aquí dice finally Jamie goes down ten floors. So si utilizar este método de... de hacer esto de diez pisos, directamente: uno... six... nine, ten [counts the number of floors as he draws them].</p>	<p>try. English and Spanish as thinking languages</p>	<p>dominant use of Spanish when talking, 46: Spanish dialogue with interviewer</p>
<p>156. 4th try. 10 as possible total number of floors on the building.</p>	<p>89 </p> <p>[3rd try drawing, on the left. The 3 top floors are added and crossed out later, on A4,90]</p>	<p>50. Code switch (non mathematically specific)</p>	<p>Reference to English thoughts</p>
<p>92-162, 174-175, 178-181. 4th try. Right answer with visual and arithmetical reasoning through the drawing of a sketch.</p>	<p>90 J: Utilizar este método, so, ¿podría aquí inventarlo o qué? [Dubitative transcription. Reads the wording. Pause. Unintelligible sequence : sounds like counting up to ten. His finger looks like pointint to the highest floor of the building. Pause.] Entonces eh... Si bajó tres, [adds 3 floors on the top of the 3rd try drawing, see A3,89] aquí, eh... [Pause] Okay, wait. [Julián crosses the top 3 floors out he just added, see A4,89. The finger continues pointing to the same floor. Reads the wording. Pause. Takes out the finger. Pause] Me equivoqué. 91 [Francesc stands up to fix the camera. Pause] 92 J: I'm sorry [when his pencil fell down. Pause. On A4,90-92 Julián checks the wording several times] No sé si... [Pause] Es que es muy confuso, porque... 93 F: ¿Por qué? 94 J: ... no dice cuál es el... cuál es el piso del medio. 95 F: Ajá. De entrada no lo sabe, ¿no? ¿Pero lo puede sacar? O sea, ¿este dibujo que ha hecho [1st try: A4,14] es el mismo que éste [2nd try: A4,25]? 96 J: Sí. 97 F: ¿Es exactamente el mismo? 98 J: Oh, no. Oh, wait. Aquí tengo diez, no sé por qué. 99 F: ¿Cómo? 100 J: No, no es lo mismo.</p>	<p>68. Code mix (middle)</p>	<p>English wording imitation</p>
		<p>90. Code mix ('Okay, wait')</p>	<p>Reading English wording, English thoughts</p>
		<p>92. Code mix (I'm sorry)</p>	<p>Thinking about the problem in English</p>
		<p>160. Code mix (crédito... department)</p>	<p>Translating try, wording imitation</p>
		<p>160. Code mix (toy department)</p>	<p>English wording imitation</p>
		<p>162-173, 166-177. 4th try. Understanding (of the problem in</p>	<p>Spanish dominant, 169: Spanish easier to understand</p>

	<p>101 F: ¿Qué diferencia hay entre éste [1st try: A4,14] y éste [2nd try: A4,25]?</p> <p>102 J: Que aquí [2nd try: A4,25] tiene diez y aquí [1st try: A4,14] tiene once, ¿no? [There are 11 floors on both, but interviewer validates Julian's argumentation]</p> <p>103 F: Ajá. ¿Por qué? ¿Cuál de los dos es el bueno?</p> <p>104 J: Pues yo digo que diez, no estoy muy seguro.</p> <p>105 F: ¿Pero cuál de los dos dibujos es el bueno?</p> <p>106 J: El que yo pensé primero [1st try: A4,14].</p> <p>107 F: ¿Qué diferencia hay entre éste [1st try: A4,14] y éste [2nd try: A4,25]? Aquí cuándo lo leyó por segunda vez, ¿se acuerda?</p> <p>108 J: Sí.</p> <p>109 F: ¿Qué pasó?</p> <p>110 J: [Answering quickly] Pues...</p> <p>111 F: [Interrupting] ¿Qué diferencia hay con éste [1st try: A4,14]?</p> <p>112 J: Pues dice que empezó luego por el tres [A4,25: 3rd floor, starting from the top] y subió una vez, bajó otra vez, y subió tres veces y ya después bajé diez veces. Una, dos, tres, cuatro, cinco, seis, siete, ocho, nueve, diez. [Julián follows the floors with the pencil while explains Jamie's movements throughout the building]</p> <p>113 F: Por lo tanto éste está bien.</p> <p>114 J: Sí.</p> <p>115 F: ¿Sí?</p> <p>116 J: Sí.</p> <p>117 F: ¿Y éste [1st try: A4,14]?</p> <p>118 J: Pues también es lo mismo.</p> <p>119 F: ¿A ver?</p> <p>120 J: Porque aquí empieza desde la credit department. Y después sube uno. Después baja uno. Después sube tres. Uno, dos... [Julián follows the floors with the pencil while he explains the movements throughout the building] Oh, okay. Me equivoqué aquí.</p> <p>121 F: Ajá. Quizás fue cuándo bajó hacia el children department o algo.</p> <p>122 J: Ajá. Luego empiezo aquí eh... Empiezo de arriba uno, dos, tres, cuatro, cinco, aquí. Y aquí empecé. So credit department. Okay. Aquí está el jewelry department. Luego she goes down one floor to the child department. Ya después subió tres pisos: uno, dos, tres. So llegó aquí. Luego aquí debe estar éste, es uno menos. [Makes a movement with the pencil like crossing out the top floor] Y aquí bajó diez: uno, dos tres, cuatro, cinco, seis, nueve, diez. [Adds one floor at the bottom of the building on the 1st try drawing, see A4,157, 2nd drawing on the left] Pero me salió casi casi lo mismo. Pero me salió nomás diferente, porque el credit department es el último... lado más arriba.</p> <p>123 F: Okay. Utilizando cualquiera de estos dos dibujos, ¿dónde empezó? ¿Cuál va a revisar? ¿Dónde empezó ella?</p> <p>124 J: ¿En el credit department?</p> <p>125 F: ¿Y dónde estaba el credit department?</p> <p>126 J: Oh, está hasta arriba, so... Sí está hasta aquí, el cuarto piso de arriba pabajo.</p> <p>127 F: ¿El cuarto piso? ¿Llegó arriba del todo?</p> <p>128 J: ¿Eh?</p>	<p>Spanish (translation)</p> <p>162-173, 166-177. 4th try. Mainly Spanish as thinking language, Spanish while drawing</p> <p>179. Deviated translation (Estar Segura)</p> <p>179. Code mix (department)</p>	<p>Spanish dominant, 169: Spanish easier to understand</p> <p>Quick thinking</p> <p>English wording imitation</p>
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	<p>129 F: ¿Jamie llegó arriba del todo?</p> <p>130 J: Oh, pues también no dice entonces si llegó. Igual le faltaban dos pisos más o dos pisos menos.</p> <p>131 F: No sabemos si llegó arriba del todo, ¿no?</p> <p>132 J: Podía faltar mucho. Ajá.</p> <p>133 F: Pero sí sabemos dónde empezó. ¿Dónde empezó?</p> <p>134 J: En el credit department, en el middle.</p> <p>135 F: Middle floor.</p> <p>136 J: Sí.</p> <p>137 F: Sabemos que empezó en el medio.</p> <p>138 J: Sí.</p> <p>139 F: ¿Sabemos cuántos pisos le quedan hasta arriba?</p> <p>140 J: No.</p> <p>141 F: ¿Y cuántos le quedan hasta abajo?</p> <p>142 J: No.</p> <p>143 F: ¿Cuántos le quedan hasta abajo no lo podemos saber? ¿Lo podemos saber o no [pointing to the paper]?</p> <p>144 J: No.</p> <p>145 F: ¿Por qué no?</p> <p>146 J: Pues...</p> <p>147 F: Sí que podemos saberlo.</p> <p>148 J: Pues aquí cómo dice lo que hizo... O sea, entró por el middle floor, el... el piso del medio.</p> <p>149 F: ¿Qué significa eso?</p> <p>150 J: Que está, que inició desde el medio del edificio.</p> <p>151 F: Sí.</p> <p>152 J: So... Subió un piso más, ya después bajó un piso menos, un piso. Y entonces subió tres veces. Sería una, dos, tres. Y después bajaría diez veces. [Julian makes the movements of going up and down throughout the building with the left hand, representing the middle floor with the right hand –which is not moving] Pero pues...</p> <p>153 F: ¿Y cuándo baja qué pasa? ¿Dónde está cuándo baja?</p> <p>154 J: Llega a la [reading] main entrance of the store. O sea, llegaría a la entrada principal.</p> <p>155 F: Sí.</p> <p>156 J: So sería, aquí me saldría... Lo más rápido es que me saliera diez. Pero hay el problema de que no tengo el middle floor. Okay. O también podría ser... Voy a hacer otro dibujo.</p>		
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157

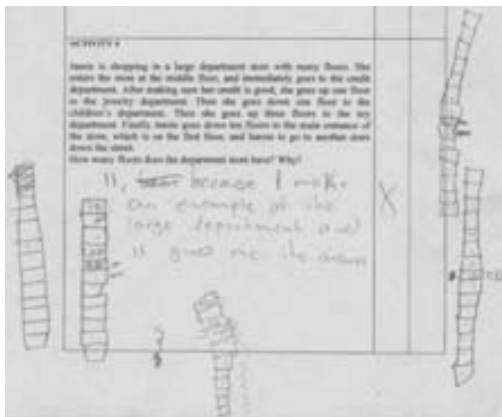
[4th try drawing, on the right]

158 J: ¿Serían quince pisos?

159 F: ¿Cómo sacó este quince?

160 J: Pues por lo que empecé fue por lo mismo. Empecé por los tres cuadros que inicio siempre para sacar el... [Starts another drawing to explain his reasoning (see A4,161, drawing on the right)] Okay. Aquí el middle floor. Aquí está el crédito... [looks to the wording] department, credit department. Pero aquí fue también dónde empezó, so... en el primer, aquí es dónde empezó, dónde empieza esto. [Looks to the wording] Después va un piso arriba, el jewelry department. [Reading] Then she goes down one floor to the childs department. So regresa al credit department dónde está ahí mismo el childs department. [Reading] Then she goes up three floors. So sería uno, dos, tres. Y aquí está...[looks to the wording] el toy department. [Reading] Finally Jamie goes down ten floors. Luego sería uno, dos, tres, cuatro, cinco, seis, siete, ocho, nueve, diez. Ahora está [reading] a la main entrance of the store which is on the first floor and leaves to go to another store. Okay. Si desde aquí inició y éste es el middle floor y de aquí de este lado [down] tenemos son tres, seis, siete. Si de este lado tenemos siete pisos y aquí es el middle floor, so, entonces harían falta cuatro más acá [above] para que sea el middle floor. Calculando así serían siete y siete catorce y éste [middle] serían quince.

161



[Entire answer.
4th try –2nd drawing– on the right. Used to explain the 4th try solving process.]

162 F: Good! ¿Cómo lo pensó esto?

163 J: Oh...

164 F: ¿En qué lengua empezó?

165 J: Pues primero empecé en inglés, o sea...

166 F: Perdón, estamos hablando ahora de la última vez, ¿sí?

167 J: Sí.

168 F: Cuando intentó volver a hacerlo. A ver.

169 J: Okay. Primero intenté comprendiendo esto, comprendiendo toda esta parte [wording]. Comprendiendo cómo era esto. Después lo aprendí a español porque... pues es más fácil comprenderlo en español para mí, pues. Como tuve que hacer eso, o sea, tuve que comprender eso en inglés, cambiarlo al español, y pues también[?] utilicé el español para terminar de hacer.

170 F: Y siguió, desde la última vez que hablamos, que ya hicimos esa cosa [Interviewer points to drawings of 1st, 2nd and 3rd tries], siguió pensando, leyendo esto otra vez, traduciéndolo a español y ¿pensando en qué idioma?

171 J: En... pues en los dos.

172 F: ¿En los dos? ¿Sabe qué cosas pensó en cada idioma?

173 J: Pues nada más esto lo fui comprendiendo así, todo esto [wording] lo fui comprendiendo en inglés. O sea comprendiendo aquí para intentar sacarlo. Y para hacer esta figura [4th try: A4,16] utilicé el español.

174 F: ¿Y cómo se le ocurrió que tenía que añadir?

175 J: Oh, porque como nunca no dice si llegó hasta el último piso, si no llegó, pero lo que dice es que es el middle floor. So para ser el middle floor tiene que tener de los dos el mismo, de los dos lados.

176 F: ¿Y en qué idioma lo pensó eso?

177 J: Eso en español.

178 F: ¿Me puede decir cómo le salieron estos siete de aquí abajo?

179 J: Pues cómo aquí dice que, que... Porque entró a la... [reading] the store at the middle floor. So, como yo hice esto de los tres o empecé acá. Éste es el credit department. [Follows the wording] Después de estar segura que el crédito, su crédito era bueno, se fue para arriba un department. Aquí estaba. Okay. Se fue

	<p>para abajo después otro departamen[to], otro piso. Después se fue tres pisos hacia arriba. Sería uno, dos tres. To the toy department. [Reading] Finally Jamie goes down ten floors. Luego sería uno, dos, tres, cuatro, cinco, seis, siete, ocho, nueve, diez. [NOT reading] Y aquí es la entrada del primer piso, éste. So aquí me salió siete de este lado. [Julián uses the last drawing to exemplify his explanations while he follows the wording, keeping track of it with a finger]</p> <p>180 F: [Counting the floors with the finger] Sí.</p> <p>181 J: Y para... pues porque aquí fue hasta donde llegó lo máximo. Aquí fue, pero también nunca dijo que iba a ser el último piso. So le añadí lo mismo que tenía de este lado [down] para acá [up], para que fuera igual.</p> <p>182 F: Sí. Okay. Muchas gracias, pues es todo. Dos preguntas sólo, en general. [Continues in GLQ, 1]</p>		
<p>M 11-14. Written answer translated to English. Hesitation of a word's translation (with no interviewer immediate help) overcome rephrasing the answer.</p> <p>E 22. 'Jamie shopping is in'. Inverted order of words when reading.</p> <p>M 63-94. 3rd try. Previous interviewer intervention affirming 2nd try was wrong [63] and that it was possible to know where the middle floor was (81-84) helped Julián find an alternative approach (88).</p> <p>O 65-88. Unknown middle floor position because there is no direct situation of it in the wording.</p> <p>102: J: Que aquí [2nd try: A4,25] tiene diez y aquí [1st try: A4,14] tiene once, ¿no? [There are 11 floors on both, but interviewer validates Julian's argumentation]</p> <p>92-122. Arrangement of 1st try drawing. Julian does not remember he did a mistake when assigning the jewelry department (102), as he recognized previously (22) and does later (118) and states that there are 10 floors on the 1st try drawing and 11 floors on the 2nd try drawing, which is not true (despite interviewer's validation).</p> <p>179. Deviated translation (Estar Segura)</p> <p>1st try done individually in English for everything (18, 25, 29-38). Spanish is introduced through interviewer interaction in 2nd try (39-56) and maintained in 3rd and 4th tries.</p> <p>In A4,112 Julián summarizes the movements Jamie did along the building using -for the first time- only Spanish, but that does not help him to find any clue to advance in the solving process. In A4,152 he explains the situation for the second and last time in Spanish. Furthermore he is using his hands to follow Jamie movements throughout the building. After an interviewer's question in Spanish, he answers in English reading from the wording (A4,154) but immediately translates it to Spanish. In A4,157 he finally gets the right answer.</p>			

General language questions		Language	Tentative
1	[Comes from A4,182] F: ¿En general cuándo ha usado inglés para resolver los problemas?	1-6. English as reading language	Wording in English
2	J: Pues principalmente lo utilizo para cuando tengo que comprender el problema. Cuándo es como tipo lectura o cuándo me dan las indicaciones, utilizo más el inglés.	6. English and Spanish as writing languages	Good management of both languages
3	F: ¿Y por qué lo hace -utiliza más el inglés para eso?	6, 10, 23-28. Main use of English in A4	28. Large wording in English, no visual mode
4	J: Porque pues necesito comprender bien la pregunta para poder resolver bien el problema.		
5	F: ¿Qué más piensa en inglés?		
6	J: Pues lo que... pues para un problema lo utilizo más para empezando el problema, comprendiéndolo y... pues ya para resolverlo utilizo más, poco más el español porque estoy más acostumbrado. Luego pues algunas veces finalizo en inglés y otras en español. Pues este tipo aquí [A4], lo primero que utilicé fue... todo lo que utilicé fue puro inglés. Porque tenía que pensar en todo esto de la... de cuál era el último, cuántos pisos tenía el departamento. Eso lo hice más en inglés. Y pues aquí [A3] es un poco más fácil utilizar el español.		
7	F: ¿Aquí es más fácil utilizar el español?	6-20. Main use of Spanish in A3	English more used in A4 than in A3 (on A3 he said he used also English for many things).
8	J: Digo ahí [A3] está... estoy más acostumbrado a utilizar el español.		
9	F: ¿Aquí?		
10	J: Ajá, en esto [A3]. Pues cómo éste [A4] todo era lectura y comprensión entonces utilicé más el inglés.		
11	F: ¿Pero aquí [A3] me escribió la respuesta en inglés?		
12	J: Yes.		
13	F: ¿Utilizó más el español, dice?		
14	J: Pues sí, para contarle los cuadritos y todo eso.		
15	F: ¿Y por qué cree que utilizó más el español?	12. Code mix (yes)	Reference to English thoughts
16	J: Porque pues... ¿Dónde, aquí [A3]?		
17	F: Ajá.		
18	J: Porque tenía que ver la diferencia de los dos. O sea ver la diferencia de cuántos subía y cuánto le aumentaba.	19-22. Mainly use of Spanish as thinking language	32: Spanish dominant, 25-28: information contained in pictures
19	F: ¿Y para qué más utilizó el español?		
20	J: Pues nada más para pensar en cómo iba a tener que hacer esto [A3], cómo...		
21	F: En general, digo, en los cuatro problemas.		
22	J: Pues nada más en cómo... en pensar en cómo lo tenía que hacer... todo. Y en hacerlo. En esta parte, fue ésta de la actividad uno, dos y tres, utilicé más el español para saber cómo hacerlo.		
23	F: ¿Y en la cuatro utilizó más el inglés?	22. Main use of Spanish when reasoning in A1, A2, A3	Larger wording in A4
24	J: El inglés.		
25	F: ¿Por que cree que lo hizo así?		
26	J: Porque una pues el problema está en inglés. La otra...		
27	F: [Interrupting] Éstas [A1, A2, A3] también están en inglés.		
28	J: Oh, sí. Pero de todos modos cómo éste... De todos modos aquí es más... aquí [A2] tiene la figura [points to the square and circle], [unintelligible: circunferencia?/diferencia?]. Aquí [A4] es de más comprensión, de más lectura y pues todavía me hace pensar más en inglés.	33-34. No English difficulties in A4	Good English management
29	F: ¿Y para pensar utilizó más inglés?		
30	J: Sí.		
31	F: ¿Alguna cosa más que quiera decir de cuándo usó el español, cuándo usó el inglés?		
32	J: Pues la diferencia para mí del inglés al español es que utilizo más el español. Pues aquí se puede ver en estos		

	<p>problemas [pointing to the crosses besides each activity], nada más utilicé el inglés acá [A4] porque pues el español estoy más acostumbrado y es un poco más fácil para mí utilizarlo.</p> <p>33 F: ¿Alguna palabra o frase que haya encontrado difícil en inglés?</p> <p>34 J: Oh, sí, ésta: Unbeatable o algo así, greater, que no sabía muy bien, greater. Y aquí [A3] pues... Aquí [A4]... No, pues es que aquí [A4] no estuvo... utilicé más el propio inglés simple. No estuvo tan difícil.</p>		
MEMO	<p>2-20. Focus on Spanish on A3, used for counting. When talking about A3 Julián focused on English. English as an “automatic tool” for dense texts??</p> <p>22-28. Main use of English in A4 because of the larger wording, Main use of Spanish in A1, A2 and A3 due to presence of graphical objects in the statement, which are interpreted in Spanish.</p> <p>As the interview advances, Julián shows an increase in the use of English per activity. Though Julián does not seem to give more examples of the English usage because of the interviewer demands, as the descriptions are rather general.</p>		

Julián has an excellent Spanish BICS because he talks very well in Spanish and his writing answers on A1 and A2 are comprehensible. There are some English utterances and many code mixes and switches, so Julián has a strong bilingual profile, using and mixing both languages indistinctly to communicate his ideas. So his English BICS is good, despite some misspellings during statement reading and writing difficulties. Julián has a good/basic Spanish CALP, as he does not use many mathematical terms in Spanish (sometimes uses English instead: corners, tiles; sometimes uses demonstratives: esto (A2, 3)) but supplies it with a more than acceptable use of Spanish not so specific to mathematics ('lado' meaning 'dotted line') and with direct reference to statement figures. From the multiple code switches and mixes –involving also mathematical terms– his English CALP may be classified as basic.

A1 is solved correctly mainly through Spanish. Julián asks for the meaning of cheaper. He does not know exactly what to write as answer (A1,13). He considers “no sé” as final answer, but gives a rich example during later interaction with interviewer: a particularization with initial prices of what can happen (A1,20-21). As Julián does not know what 'unbeatable prices' means (A1,47) he does not use it in his resolution. But he does consider it later (A1,53,56-58), after knowing its meaning (A1 48-52). Finally Julián decides he still would need to calculate the final prices to decide which store would be cheaper (the interviewer had validated Julián's first answer), confirming his initial answer.

A2 is solved mainly through the use of Spanish. The only known use of English is to interpret parts of the pictures (circle, square, five). Julián initially thinks that both perimeters can have the same length. The solving process is reviewed after his meaning –translation– demand for “greater”. Then he visually compares both perimeters by mentally inscribing the circle on the square: throughout a division of figures into four equal parts he correctly he compares in a visual way linear and curvilinear paths with shared extremes. Moreover, on his oral explanation of the activity, he inscribes the circle into the square (A2,22). There is a confusion between *diameter* and *side* on the case of the circle, but this is a terminological rather than conceptual confusion.

A3 is written in English, and both languages are used during the solving process, specially English. Numbers are linked to Spanish and tiles are thought in Spanish

as 'cuadritos'. Julián draws Figure 5. He finds the number of tiles of Figure 7 in two different ways: as a result of the sequence associated to the figure pattern is growing by 2 and as a result of the figure construction –visual way–, with two rows of 6 tiles plus one tile on the right.

A4 is solved completely in English on the 1st try (although it is not mentioned by Julián, he counts aloud in Spanish and interacts with the interviewer in Spanish), but once interviewer interaction starts –2nd to 4th tries–, Julián uses both languages to think about the problem (the conversation is in Spanish). He reads and checks the wording many times, realizing multiple code mixes and switches. Most of them are due to direct reading of the wording, instead of translating it. He understands it without problems. The only English language related problem encountered is when he asks for the translation of a Spanish word when he is writing the answer on his first try. At the beginning Julián checks with the interviewer if the credit department is in fact at the middle floor, as it is not directly stated on the wording. Then he makes a sketch of the building. He forgets one of Jamie's movements, but this has not a central impact on the answer. The main issue is that he considers the highest floor reached by Jamie as the top of the building. While he explains the answer he realizes he makes a mistake on the drawing and corrects it (2nd try), but he does not change his reasoning, keeping the final answer [11 floors] (as said, the sketch has not a central impact on the answer). Then interviewer states that his solution is not valid. When asked about the position of the middle floor, Julián refers to his sketch to say that it is on the 6th floor. Then he tries an alternative approach (3rd try) initiating a sketch with random (but greater than ten: as Jamie goes down ten is should be greater than ten). This approach is quickly abandoned, as he does not find an answer because he still considers the highest floor reached as the top of the building. Then interviewer tries to show the importance of the middle and bottom floors, but Julián does not take immediate advantage of these hints. After interviewer indicates that top floor reached isn't the building's top floor and that the number of floors from the middle to the bottom can be found Julián makes a right sketch of the situation and finds the right solution.

He solves most of the problems quickly (A1, A2, A3) after knowing what to do. When he has a problem –either knowing the meaning of a word or about norms clarification– he asks the interviewer. He is patient to find the solution in A4. His reasons on A1 and A2 are not very accurately expressed from a mathematical point of view, but they are clearly understandable –no additional explanation is required by the interviewer anyway.

Activities' (Key ideas) summary

Object 42: Julián-First reduction (End)

- Meaning demand –translation– of key word (cheaper) makes Julián find the right solution quickly.
- Problem solved correctly disregarding unknown English word (Unbeatable) but considered as part of the problem solving when its meaning is known.
- Mainly use of Spanish to solve the activity in relation to stores and percentages.
- Code mixing instances involving mathematical words (forty per cent of discount, per cent of discount) during the explanation of the exercise.
- Meaning demand –translation– of key word (greater) makes Julián find the right solution quickly.
- Mainly use of Spanish to get the (right) solution, but inclusion of English terms in discourse (corners, perimeter) as well as in thinking (in this case associated to parts of figure statement: five, circle, square).
- Confusion between diameter and side in the case of the circle (terminological rather than conceptual confusion).
- Oral explanation gives more details than the written one, including a dynamics on the geometry omitted on the written form of the answer.
- Use of both languages (looks like English is more used) for thinking in relation with the sequence of Figures. Spontaneous use of Spanish for counting and 'cuadrito'.
- Code mixing instances involving mathematical-related words (tiles) during the explanation of the exercise.
- English for everything except for one translation demand (interpreté) and for counting aloud on the first try.
- Spanish, along with English, as thinking language with interviewer's interaction, after the first try. Code mixing instances during the solving process, including wording imitation.
- Code switching instances when counting the floors of the sketch.
- Julián expresses that there is a dominant use of English in the dense wording and a more dominant use of Spanish in the other activities because of the visual mode of the statements.

Historical profile		Bilingual profile (Spanish dominant) [cont]	Activity				
<ul style="list-style-type: none"> • 17 years old • Transitional class • Born in Mexico, 2'5 years in USA • Likes California • Spanish readings • English readings • Spanish at home • English and <u>Spanish</u> with friends • English and <u>Spanish</u> at school • Homework help: nobody 		Translation demand (A1: cheaper, A2: greater, A4: interpreté)	x	x		x	
		Spanish as unique writing language	x	x			
		Writing deviances	x	x	x	x	
		Spanish linked to thinking language	x	x	x		
		English as a thinking language [A1: Unbeatable A2: circle, square, five, A3]	x	x	x		
		Code mixing [A1: okay, forty per cent of discount, per cent of discount, when, yeah, so. A2: greater perimter, okay(x4), so, corners(x2), perimeter. A3: okay(x3), so(x5), yeah(x2), tiles, answer, yes. A4: credit department(x14), middle floor (x14), okay(x11), middle(x4), so(x27), jewelry department(x3), childs department(x4), child department, wait(x2), floor, middle, 'okay, wait', 'I'm sorry', department(x2), toy department]	x	x	x	x	
		Code switching A1(x0+1+1), A2(x0+1+2), A3(x0+1+2), A4(x2+3+18)	x	x	x	x	
		English as reading language	GLQ				
		English and Spanish as writing languages	GLQ				
		Mainly use of Spanish as thinking language	GLQ				
English in A4	GLQ						
Main use of Spanish when starting activity (A1, A2, A3)	GLQ						
Code mix (yes)	GLQ						
Procedural profile		Conceptual profile					
<p>1 ✓ Awareness of no initial prices ✓ Assignation of initial prices and final price calculation (orally) ✓ Reconsideration of the solution once the meaning of "Unbeatable" is known</p> <p>2 ✓ Matching of both dotted lines ✓ Translation move (circle inscribed on square) (Orally) ✓ Visual length comparison (between linear and curvilinear paths with shared extremes)</p> <p>3 ✓ Visual and arithmetical abstraction of the figure pattern sequence</p> <p>4.1 ✗ No symmetry through middle floor to get the top of the building ✗ Sketch with jewelry department above middle floor</p> <p>4.2 ✓ Jewelry department arrangement on a new sketch</p> <p>4.3 ✗ 6th floor is middle floor, according to his drawing (after interviewer question) ✓/✗ Drawing started with a <i>random</i> number of floors (but greater than 10). Alternative approach abandoned quickly as no solution is found</p> <p>4.4 ✓/✗ Interviewer affirms: 1) top floor reached isn't the building's top, 2) # floors from the middle to the bottom can be found; which are not immediately used by Julián ✓ Right sketch [15 floors]with symmetry through middle floor due to arithmetical and visual reasoning</p>		<p>1 ✓ Notion of percentages as a relative value</p> <p>2 ✓ Notion of perimeter</p> <p>3 ✓ Notion of arithmetical sequence associated to a sequence of figures</p> <p>4.1 ✗ Notion of number line with confused order positions</p> <p>4.2 ✗ Notion of number line with confused order positions</p> <p>4.3 ✗ Notion of number line with confused order positions</p> <p>4.4 ✓ Notion of number line with confused order positions</p>					

Object 43: Julián-Second reduction

Object 44: Aida-First reduction
(Beginning)

Aida is student with very good behavior in class and interest in the subject. She is shy and started to make classroom contributions little by little. Though she always asks the teacher individually for clarifications. She is a hard worker and always does the homework.

Math teacher description

Class	Interview language	Interview date	Origins	Age	USA arrival	Living in California	Spanish readings	English readings	Home language	Friends language	Main language at school	Homework help
HS Transitional	Spanish	January 2010	Mexicali, Mexico	14	2 years [January 2008]	Likes it.	Books	Books from the school library as part of an after-school English learning program	Spanish	Spanish	[Question not asked]	Nobody

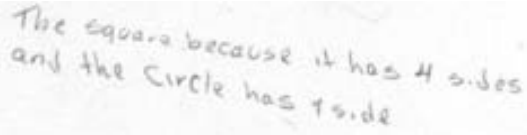
Historical bilingual profile

REMARK: in this transcription there is no video recording (just audio recording). The comments were added in reference to the context.

Math	A1. Aida marked "English and Spanish".	Language	Tentative
3-5, 13-19. 1st try. ["John Sports because they have 40% off"] Wrong answer due to assumption of equal initial prices. 7. 1 st try	<p>[There is no video recording, just audio. Once solved, activities are commented following this order: A3, A4, A1, A2. Conversation starts here:]</p> <p>35 A: ¿Qué quiere decir esto?</p> <p>36 F: Cheaper. Más baratas, más económicas. ¿Sí?</p> <p>37 <i>In which of these two stores are the shoes cheaper? Why?</i> <i>John Sports because they have 40% off.</i></p> <p>38 [Comes from A3,35] F: ¿Cómo resolviste la uno?</p> <p>39 A: Nomás como, porque el, si tiene cuarenta por ciento discount como de, le quitan el cuarenta por ciento de lo que es y a éste le quitan el veinticinco y es... y en éste le quitan más.</p>	<p>1-2. Word meaning demand (cheaper)</p> <p>3, 30-31. Use of English as unique writing language</p> <p>5. Code mix (discount)</p>	<p>23-24. Unknown meaning</p> <p>School context</p> <p>Reading English</p>

(explaining). Awareness of no given initial prices on stores. 1-19. 2nd try. Right answer with awareness of no initial prices. As a consequence both shoes can have the same price.	40 F: Mm [validating]. 41 A: Pero puede ser lo mismo porque no sale el precio de los dos. No sale precio, nomás sale cuánto le quitan. 42 F: No lo has apuntado esto, por eso, ¿no? ¿Por qué no lo has apuntado? [Pause] ¿No?, me acabas de decir pueden ser lo mismo porque no sale el precio. ¿No? 43 A: Sí. 44 F: ¿Y por qué no lo has apuntado esto? [Pause] ¿Sabes por qué no lo has apuntado? ¿Me puedes decir por qué no? ¿O se te acaba de ocurrir ahora esto? 45 A: Porque apenas ahorita. 46 F: ¿Ahorita lo pensaste? ¿Antes no lo habías pensado? 47 A: No. 48 F: ¿Y cómo es que ahora sí? 49 A: Porque no puede, porque como no tienen nada de precio. 50 F: Mm [validating]. ¿Pero antes cómo es que no te fijaste con esto? 51 A: Porque nomás miré como lo que le quitaban. 52 F: Ajá. 53 A: Y por eso le puse. 54 F: Entonces, ¿cómo resolviste esto?, de acuerdo con la lengua, otra vez. Porque aquí pusiste español y inglés, ¿no? ¿Empezaste con qué lengua? 55 A: Con el español. 56 F: ¿Y qué hiciste en español? 57 A: Porque, leí esto como cuarenta por ciento y luego el veinticinco por ciento. Luego le pregunté por esto porque no sabía. 58 F: Ajá, el cheaper. sí. 59 A: Ajá. Y nomás. 60 F: ¿Y el discount sí pensaste en inglés, no? [see 'discount' on A1,5] Tienes cuarenta por ciento... 61 A: [interrupting] Mm [validating]. 62 F: ...discount. No forty percent discount. 63 A: Ajá. 64 F: Y luego volviste a escribir la respuesta... [pause] en inglés. 65 A: En inglés. [Continues in A2,2] 66 [Comes from A4,115] F: La actividad uno, ¿volviste al enunciado? 67 A: Sí. 68 F: ¿Leíste lo que hay dentro de los cuadros? 69 A: Sí. Ajá. 70 F: Y por ejemplo, en los dibujos de aquí, ¿no?, ¿qué es esto? [Pause] El dibujito éste. 71 A: Unos tenis. 72 F: ¿Pensaste en tenis o pensaste en shoes o cómo lo pensaste al ver el dibujito? 73 A: En tenis. [Continues in GLQ,1]		statement
		20-25, 32-39. Use of Spanish as thinking language	Spanish dominant
		32-33. Wording pictures interpreted in Spanish	Spanish dominant
MEMO	4-5. "de lo que es". That informs that Aida (in her comparison of percentages) makes a correctly treatment of percentages, assuming equal initial prices rather that treating percentages as absolute value. Maybe it is the starting point to the right solution (7). Later she says that at the beginning she		

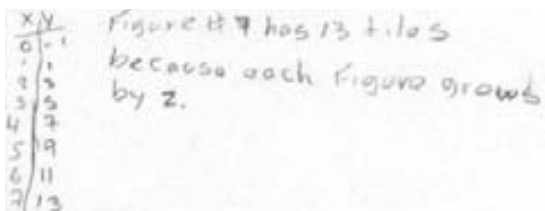
focused on the discount rate only (17). This information is not contained on the written answer.
 17. Information on the visual mode has more influence, distracts (focus on percentages, forgetting initial price).
 32-33. Wording pictures interpreted in Spanish.

Math	A2. Aida marked "English and Spanish".	Language	Tentative
1-5, 8-10. 1 st try. Wrong visualization of perimeter's lengths.	1 	1, 99. Use of English as unique writing language	School context
3, 41-43, 84-85. 1 st try. Reasoning does not rely on the visual mode neither on an arithmetical mode (wrong interpretation of the dotted lines).	2 [Comes from A1,31] F: ¿Me explicas un poco cómo la resolviste? 3 A: Es porque eh... [pause] dice que cuál tiene el perímetro más grande y éste nomás tiene un, como es redondo, nomás es como un lado, porque solamente lo haces. Y el cuadrado tiene cuatro lados. Y puede ser más grande. Este cuadrado es más grande que el círculo. 4 F: ¿Sólo por el número de lados? [Pause] 5 A: Porque aquí está cinco... [Pause] Sí. 6 F: ¿El cinco te ayudó en algo? 7 A: Nnnnoo. 8 F: ¿No? Si aquí hubiera sido, por ejemplo, el cuadrado cinco y el círculo diez, ¿qué hubiera pasado? 9 A: ¿El círculo sería más grande?	34. Unknown circle-related vocabulary (diámetro)	Topic not covered on the present school year
4, 8-10, 84-89. 1 st try. No adequate use of the numbers on the dotted lines.	10 F: Sería más grande. ¿Por lo tanto, de alguna manera, sí que tiene que ver, no el número? 11 A: ¿Los dos son iguales? 12 F: Mm [validating]. ¿Cuándo los dos son iguales qué pasa? 13 A: Los dos es... tienen la misma, el mismo perímetro. 14 F: Ahora, que los dos son iguales, que los dos valen cinco, ¿tienen ambos el mismo perímetro? 15 A: Sssssí. 16 F: Pero aquí me has puesto "The square because it has four sides and the circle has one side" [see A2,1]. ¿Entonces cambias de opinión, ahora?	62-67. English as reading language	Reading English wording
1-10, 40-43. 1 st try. Wrong answer due to wrong dotted line interpretation (resulting in a deviated comparison on the number of	17 A: Porque como si este cinco es cada lado. 18 F: Ajá. 19 A: Y aquí nomás es un lado. 20 F: ¿Pero qué es, qué significa este cinco? 21 A: ¿El total? 22 F: ¿El total de qué? 23 A: Del perímetro. 24 F: Nnnnoo. Este cinco es el diámetro. Es la línea esta discontinua, desde aquí hasta aquí. Aquí [square] también este cinco es desde aquí hasta aquí. Esta línea discontinua. ¿Sí? Esto es lo que es este cinco. 25 A: ¡Oh! [Pause] ¡Oh!, ¿entonces los, tienen la misma, el mismo perímetro?	67-70. Use of English to think about the 'square'	72-81. Familiarity with the English word (inside and outside school)
		70-72. Use of Spanish to think about the circle ('círculo')	Spanish dominant
		90-99. Spanish linked to thinking language	Spanish dominant

sides).	26 F: ¿Los dos tienen lo mismo, siendo ambos cinco?	97. Code mix (square)	Word thought in English
12-13. 2 nd try. Wrong visualization of perimeter's lengths.	27 A: Sí. 28 F: ¿Sí? ¿Por qué? 29 A: Porque... [pause] Porque es la misma cantidad. 30 F: ¿Son igual de largos? Si tu vas, si tu lo midieras esto, el cuadrado y el círculo, con una cuerda, o si vas andando por encima de uno o por encima de otro, son lo mismo, de largos? 31 A: Eh, sí. 32 F: ¿Sí?		
11-15. Dotted line as entire perimeter (on both figures).	33 A: Sí. 34 F: Porque este cinco, o sea, que aquí el diámetro del círculo, ¿sí?, ¿te suena esta palabra, diámetro? ¿No? Okay. Entonces a esto le llamamos diámetro y esto es como el lado, ¿no? Y si esto es cinco, y esto es cinco, tu me dices que ambos son iguales, porque la parte de dentro es igual. Pero tiene esto... ¿Qué la parte de dentro sea igual significa también que la parte de fuera sea igual?		
17-18. 1 st /2 nd try. Right dotted line interpretation on the square.	35 A: Sí. 36 F: ¿Sí? [Pause] Vale, vale. Entonces, a ver, ¿cómo has empezado este problema? ¿Comparando los lados, o cómo? 37 A: Sssí. 38 F: ¿Porque perímetro sí entiendes lo que quiere decir, no?		
19-23. 1 st /2 nd try. Wrong dotted line interpretation on the circle (as the entire perimeter).	39 A: Sí. 40 F: ¿Y cómo has empezado el problema? 41 A: Viendo porque éste nomás tiene un lado y éste tiene cuatro. 42 F: Ajá. Sí. 43 S: Y es más posibilidades que éste sea más grande. 44 F: Si yo te digo: por ejemplo, aquello es un círculo, ¿no?, el ventilador. Pongamos que es un círculo. ¿Cuántos lados tiene?		
24. 2 nd try. Circle's dotted line meaning explained (by interviewer).	45 A: Uno. 46 F: Podemos decir que uno. Bueno, es... no tiene lados rectos, ¿sí?, ¿vale?. 47 A: Sí. 48 F: Dices un lado, como tu dices. Porque no hay, no son rectos, ¿no? 49 A: Sí.		
25-36. 2 nd try. Equal perimeters of both figures. Wrong visual comparison of lengths.	50 F: El perímetro no son lados rectos. ¿Qué es.. qué perímetro es más largo, aquél o el del cuadrado éste [the one in A2 ?] ? 51 A: ¿El cuadrado? 52 F: ¿Éste tiene un perímetro más largo que aquél? 53 A: No, el 54 F: El ventilador es mucho más... 55 A: [interrupting] grande		
38-39. 2 nd try. Right understanding of	56 F: ... largo, por lo tanto no tiene que ver el número de lados, o no es seguro que si tiene más lados rectos, sea el perímetro más grande. ¿Sí? 57 A: Sí.		

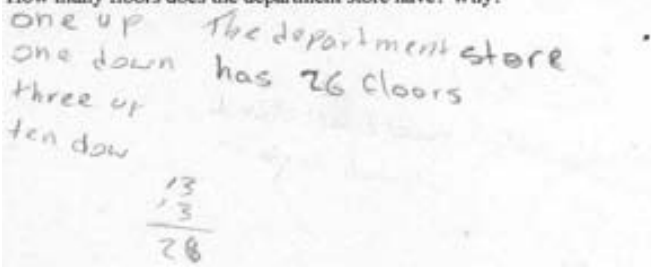
<p>the notion of perimeter affirmed by Aida, but not demonstrated (44-51).</p> <p>44-51. 2nd try. Wrong visual comparison of perimeters lengths (between the square and that of a fan – which is standing on the room). (It is unclear if there is a) right understanding of the notion of perimeter as stated before (38-39)</p> <p>11-39, 44-51. 2nd try. [Figures with equal perimeter] Wrong answer due to wrong visual comparison of perimeter's lengths. It is unclear if Aida has an appropriate insight of the notion of perimeter.</p>	<p>58 F: De hecho aquí éste tiene un perímetro más largo, ¿sí? Pero bueno. Entonces cómo has resuelto la actividad dos? ¿Mirando los lados y luego has vuelto otra vez al inglés?</p> <p>59 A: Ajá.</p> <p>60 F: ¿Has hecho algo más?</p> <p>61 A: No.</p> <p>62 F: ¿No? ¿Entonces, puedes hablarme un poco de las lenguas, por último, resumiendo un poco, aquí?</p> <p>63 A: ¿Cómo?</p> <p>64 F: Sí. Empezaste...</p> <p>65 A: [interrupting] ¡Oh!</p> <p>66 F: ... leyendo el enunciado.</p> <p>67 A: Leí el enunciado en inglés. Luego las figuras las dije en inglés, también.</p> <p>68 F: ¿Las pensaste? ¿El nombre, por ejemplo? ¿Qué nombre le diste a esta figura?</p> <p>69 A: Square.</p> <p>70 F: Square? And this is...</p> <p>71 A: Ssss... [C as in circle] Esa la pensé en español.</p> <p>72 F: En español. ¿Y por qué ésta en inglés?</p> <p>73 A: Porque ésta la oigo más.</p> <p>74 F: Oyes más el square.</p> <p>75 A: Ajá, que el...</p> <p>76 F: Circle.</p> <p>77 A: Ajá.</p> <p>78 F: ¿En clase o dónde la oyes más?</p> <p>79 A: En clase y en... [pause]</p> <p>80 F: ¿Fuera de clase también?</p> <p>81 A: Mm [validating].</p> <p>82 F: Vale, ¿luego que más?</p> <p>83 A: Luego...</p> <p>84 F: Por ejemplo, los números. ¿Qué pasó con los números?</p> <p>85 A: No les di tanta importancia.</p> <p>86 F: ¡Ooooh!, no les diste tanta importancia. Entonces, ¿como los olvidaste un poco?</p> <p>87 A: Ajá.</p> <p>88 F: ¿No pensaste ni siquiera en los números?</p> <p>89 A: Ajá.</p> <p>90 F: Okay. ¿Luego pensaste el problema?</p> <p>91 A: Ajá. Y lo...</p> <p>92 F: ¿En qué idioma?</p> <p>93 A: En español.</p> <p>94 F: En español. ¿Y luego?</p> <p>95 A: Luego como pensé que ese tenía más figuras, en español, y luego que si tenía namás una, en español.</p> <p>96 F: ¿Ésta tenía más? ¿El círculo tenía más?</p>		
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	<p>97 A: Square tenía más que el círculo. 98 F: ¿Más qué? 99 A: Lados. Y luego ya lo pasé al inglés. 100 F: ¿Cómo se llama esto en español? 101 A: Cuadrado. 102 F: Mm [validating]. ¿Utilizas también el español para referirte a esto? ¿O no? ¿No le llamas nunca cuadrado? ¿No utilizas la palabra cuadrado? 103 A: No. 104 F: No. Okay. ¿Y volviste a releer el enunciado? ¿Mientras lo estabas resolviendo, volviste otra vez al enunciado? 105 A: Sí. 106 F: ¿Sí? Y claro, está en inglés, entonces tuviste que pensar en inglés de alguna manera. 107 A: Ajá. [Continues in A3,36]</p>		
MEMO	<p>3, 41-43, 84-85. Reasoning does not rely on the visual mode neither on an arithmetical mode (wrong interpretation of the dotted lines). 11. The interviewer refers to the dotted lines with 'iguales' while Aida refers to the notion of perimeter. 70-72. Spanish with 'círculo'. 95. Lapse: 'figuras' instead of 'lados' (99). 103. Contradiction. Aida uses 'cuadrado' during discourse: A2,3 (where she first introduces the term in the discourse) and A2,51 (where the interviewer says it in the previous intervention) !]</p>		

Math	A3. Aida marked "English and Spanish".	Language	Tentative	
1-3. 1st try . Right answer with arithmetical reasoning (organized on a x-y table as done in class).	<div style="display: flex; align-items: center;"> <div style="margin-right: 10px;">1</div>  </div>	1, 6-16, 19. Use of English as unique writing language	School context, 20-25: English easier than Spanish (in this case, not in general)	
	2 [Comes from A1,3] F: ¿Cómo empezaste entonces a resolver esta actividad?			
	3 A: Me fijé en la figura uno cuánto subió a la figura dos. Y cuánto subía... y subía de dos. Y hice la tabla y cada... subí de dos.			
	4 F: ¿Y con qué lengua has empezado a resolver el problema?			
	5 A: En... español.			
	6 F: ¿Y cuándo has utilizado el inglés? ¿Cuándo has cambiado a inglés?			
	7 A: Cuando escribí la respuesta.		4-5, 18-19. Mainly use of Spanish as thinking language	Spanish dominant
	8 F: Ajá.			
	9 A: Cuando la escribí.			
	10 F: ¿Sólo para escribir la respuesta?		31-32. Use of Spanish as thinking	Home language

	<p>11 A: Sí. 12 F: ¿Y por qué escribiste en inglés la respuesta? 13 A: Nomás así. 14 F: ¿Qué quiere decir nomás? Así, ¿es? Es que es una palabra mexicana que no entiendo mucho, nomás. 15 A: Nomás, porque sí, porque... 16 F: Ajá. ¿Lo hiciste sin pensar?, entonces. El enunciado está en inglés, por eso, ¿sí? Entonces lo leíste en inglés... 17 A: Sí. 18 F: Resolviste el problema... [pause] ¿en qué idioma luego? 19 A: En español, luego la respuesta la escribí en inglés. 20 F: Ajá. ¿Es más fácil escribir la respuesta en inglés que en español? 21 A: Ssssí. 22 F: Más fácil en inglés que en español. 23 A: Es inglés 24 F: ¿Es más fácil? ¿Por qué? 25 A: Porque... [Pause] No sé, se me hace más fácil. 26 F: ¿Escribes en español también, por eso, normalmente? 27 A: Ajá. 28 F: Por eso, por ejemplo, digo, si tienes que hacer una carta a tus padres, perdón una carta a tu amiga o lo que sea, ¿lo haces en español o en inglés? 29 A: En español. 30 F: Ajá. [Continues in A4, 2] 31 [Comes from A4,107] F: Por ejemplo, aquí, en el problema de antes, la actividad tres. ¿Cuándo estabas haciendo la tabla y escribiendo los números, qué pensabas cero, uno, dos, tres o zero, one, two, three? 32 A: Cero, uno... 33 F: ¿Uno, dos, tres? ¿Y equis-ye o ex-wy? 34 A: Exs-wy. 35 F: Exs-wy pero cero, uno, dos, tres. ¡Oh! [Continues in A1,4] 36 [Comes from A2,107] F: ¿Te pasó esto también en alguna de las otras actividades? ¿En la uno o en la tres o en la cuatro? 37 A: En la actividad tres. 38 F: ¿Volviste a leer el enunciado, también? 39 A: Ajá. [Continues in A4,108]</p>	language with numbers	
		33-35. English as thinking language with "x", "y"	School influence
MEMO	<p>4-16. After having been thinking in Spanish, Aida switches to English to write the answer without difficulties. 31-32. Spanish as thinking language with numbers. 36-39. Wording read in English during solving process. 20-25. English easier than Spanish (in this case –for writing the answer–, not in general).</p>		

Math	A4. Aida marked "English and Spanish".		Language	Tentative
10-11. No representation of the situation presented on the statement.	<p>Jamie is shopping in a large department store with many floors. She enters the store at the middle floor, and immediately goes to the credit department. After making sure her credit is good, she goes up one floor to the jewelry department. Then she goes down one floor to the children's department. Then she goes up three floors to the toy department. Finally Jamie goes down ten floors to the main entrance of the store, which is on the first floor, and leaves to go to another store down the street.</p> <p>How many floors does the department store have? Why?</p> <p>one up The department store one down have 15 floors because three up ...[sequence unintelligible]... goes ten down up and down</p>		1, 36. 1 st , 2 nd tries. 93-95. Use of English as unique writing language	School context
1-11, 22-24. 1 st try. [1+1+3+10=15 floors] Wrong answer due to no relative position of floors (wrong horizontal mathematization: 80-88).	1	<p>[A4, 1st try. Part of it (see typed paragraph) has been reconstructed from the writing marks because Aida erased it: A4,34-35]</p>	1, 88-89, 96-99. 1 st try. English as writing language to take notes	100-105. English wording imitation
7-9. 2 nd try. Awareness of comprehension difficulties. In fact these are problems in the horizontal mathematization, but not the understanding of the wording (8-21).	2	<p>[Comes from A3,30] F: ¿Cómo solucionaste la cuatro? A: Sumé los pisos que decía que iba. F: Mm [continuing conversation]. A: Y luego... Nomás los sumé y me salieron quince. F: ¿Los sumaste? A: Sí. [Pause] Es que casi no le entendí muy bien. F: ¿Por qué no lo entendiste muy bien? [Pause] A: Porque dice que empezó como a, en el, en el medio. F: Ajá. ¿Y qué quiere decir que empieza en el medio? ¿Te has imaginado la situación, realmente lo que significa? A: No. F: Pues piensa, a ver. Entramos en un edificio, ¿no? Entramos por el medio. ¿Sí? ¿Luego qué hacemos? A: Sube un piso. F: Ajá, ¿luego? A: Luego lo baja otra vez y está en el medio otra vez. F: Ajá. A: Luego sube tres pisos. F: Mm [validating]. A: Y ya luego baja diez pisos. F: Ajá. Por lo tanto... A: En el medio de la entrada, la, la... la entrada principal. F: Ajá. ¿Por lo tanto ha resuelto bien el problema? [Pause] ¿Es lo que hay que hacer, sumar todos esos pisos? A: ¡Ss! F: ¿Sí? ¿Hay que sumarlos? Pero tu me has dicho que sube uno y luego baja uno, ¿volvemos a estar en el mismo, no? ¿Por lo tanto, por qué los sumas? [Pause] ¿No? Imagínate, por ejemplo, subimos tres y bajamos tres. [Pause] ¿No? Sumamos y nos da otra cosa diferente. Podía ser una situación en el mismo edificio, ¿no?</p>	1. Orthographical variation (dow[n])	Quick writing
9-21. 2 nd try. Right wording comprehension, with movements highlighted: underlined on the wording (1), written down (1), and summarized	10		1. Grammatical deviance (have, corrected on second try: A4,36)	Quick writing, English language learner
	11		9-21. 1 st try. Right understanding of the situation presented on the wording	Good English management
	12		88. Wording translated to Spanish	Spanish dominant
	13		88-93, 106-107. 1 st -4 th tries. Spanish as main	Spanish dominant

during the dialogue (9-21).	25 A: Sí.	thinking language	
15. 2 nd try. Right relative situation of the two initial Jamie's movements.	26 F: Por lo tanto tenemos que cambiar la forma de resolver el problema. ¿No? Tu has entendido la situación. Y aquí has anotado, bueno, primero lo has subrayado, ¿no?, luego lo has anotado aquí, qué es lo que hace. Y ahora piensa cómo lo podemos resolver. ¿Porque por dónde entramos?	108-113. 1 st -4 th tries. Wording read in English during the solving process	English wording
24-26. 2 nd try. Adding all floors is not the right solution (by Interviewer).	27 A: En el medio de... 28 F: [interrupting] Ajá, ¿está información es importante o no? 29 A: Sí. 30 F: Pues a ver cómo podemos utilizar esto. [Pause] 31 A: ¿Podemos poner como cualquier número para hacer como una ecuación? 32 F: Podemos hacer eso, quizás sí. [Pause] 33 A: No sé. 34 F: ¿No? Puede... ¡Intenta hacer una ecuación! ¿Por qué no lo intentas? [Pause] Puedes dejarlo esto [written answer, 1 st try], por si acaso. ¿Puedes continuar escribiendo abajo, la otra respuesta? 35 A: ¡Ah!		
26-30. 2 nd try. Importance of middle floor. Remarked by Interviewer but introduced by Aida (15).	<p>Jamie is shopping in a large department store with many floors. She enters the store <u>at the middle</u> floor, and immediately goes to the credit department. After making sure her credit is good, she goes up <u>one</u> floor to the jewelry department. Then she goes down one floor to the children's department. Then she goes up <u>three</u> floors to the toy department. Finally Jamie goes down <u>ten</u> floors to the main entrance of the store, which is on the first floor, and leaves to go to another store down the street.</p> <p>How many floors does the department store have? Why?</p>		
31-35. 2 nd try. Use of equations as alternative path to solve the problem (idea not developed).	<p>36 </p> <p>[A4, 2nd try. Aida previously erased her first written, see A4,1.]</p>		
36-51. 2 nd try. Right relative situation of the 2 initial Jamie's movements (as in 15), but wrong relative situation of 3-floors-up and 10-floors-down. Right symmetry	37 A: ¿Sí son veintiséis? 38 F: ¿Por qué dices veintiséis ahora? 39 A: Porque ah... dice que está en el medio. 40 F: Ajá, está en el medio. 41 A: En el medio [Aida coughs]. 42 F: Con lo cuál... 43 A: Como dice que subió uno y luego lo bajó, se quedó en el mismo. 44 F: Sí. 45 A: Y luego son, luego bajó diez pisos. 46 F: ¿Antes de bajar diez pisos qué hizo?		

through middle floor (good use of previous hint given by interviewer: 26-30) but middle floor not counted.	<p>47 A: Subió tres. 48 F: Subió tres. 49 A: Y luego de ahí bajó diez pisos que es el primer piso. 50 F: Sí. 51 A: Y como si estaba, estaba en el trece. 52 F: ¿Estaba en el trece? ¿Por qué estaba en el trece? 53 A: Porque subió, cuando subió tres estaba en... [Pause] ¡Oh, no! Cuando subió tres estaba, entonces estaba en el siete y subió tres, sería, estaba en el piso diez porque cuándo bajó diez pabajo era el primer piso.</p>		
8-51. 2 nd try. [13+13=26 floors]. Wrong answer due to wrong relative situation of (some) floors, middle floor not correctly situated and middle floor not counted. (Wrong) mathematization of floors provided by interviewer.	<p>54 F: Ajá. Bien. 55 A: Estaba en el siete. So va a... eh... ¿Eran catorce? 56 F: ¿Catorce? ¿Cómo sacas este catorce?, a ver. 57 A: Porque cuando empezó, empezó en el medio. 58 F: Sí. 59 A: Y la... como la mitad de catorce es siete. 60 F: El doble, quieres decir. La mitad de catorce es siete, sí. Pero si está en el medio, ¿cuántos pisos tiene arriba y cuántos pisos tiene abajo? 61 A: Em... [Pause] Tiene... [Pause] 62 F: ¿En qué piso entra?, entonces. 63 A: En el... [Pause] ¿En el tres? 64 F: ¿En el tres? ¿El tres es el del medio? 65 A: Tres... 66 F: [Interrupting] Más o menos lo tienes, pero hace falta que lo organices, ¿sí? 67 A: Porque... ¿En el ocho? 68 F: En el ocho. ¿Por qué en el ocho? 69 A: Porque abajo hay siete y arriba siete... 70 F: [interrupting] Ahí está. 71 A: ... y ella está al del medio y es el ocho. 72 F: ¿Por lo tanto cuántos pisos hay? 73 A: ¿Catorce? 74 F: Siete abajo, siete arriba. 75 A: Catorce. 76 F: ¡Y el del medio! 77 A: Quince. 78 F: ¿Ves? Por eso, tenías la respuesta bien pero el razonamiento no estaba bien. ¿Ahí ves porque es importante el desarrollo? ¿Sí? 79 A: Sí. 80 F: ¿Qué dificultades has encontrado en este problema? 81 A: Porque no lo leí muy bien. 82 F: No lo leíste muy bien. ¿Pero lo leíste como mínimo dos veces, no? Una vez, luego te vi subrayar... [Pause] subrayar las palabras, ¿no?, los números.</p>		
52-54. 3 rd try. Better relative situation of floors, but not accurate enough (10-3=7, middle floor position).			
68-75. 3 rd try. Right middle floor situation due to interviewer validation, but mathematization not accurate			

<p>enough.</p> <p>40-75. 3rd try. [7+7=14 floors]. Wrong answer due to middle floor not correctly positioned. Deviated use of symmetry (total numbers of floors must be an odd number).</p> <p>56-77. 4th try. [7+7+1=15 floors]. Right answer with arithmetical reasoning. Right middle floor situation (68) but not included on the final answer (interviewer does it, see 76).</p>	<p>83 A: Mm [validating], pero no le puse mucha atención en la, en los des[?] pisos.</p> <p>84 F: En lo de los pisos. ¿Lo hiciste rápido?</p> <p>85 A: Ajá.</p> <p>86 F: ¿Estás un poco cansada?</p> <p>87 A: No.</p> <p>88 F: Querías terminar rápido. Em... ¿Cuándo cambiaste al inglés?</p> <p>89 A: Cuando estaba... [Pause] Cuando andaba haciendo como cuántos subía y bajaba.</p> <p>90 F: O sea, empezaste leyendo el problema en inglés. ¿Luego qué hiciste?</p> <p>91 A: Luego, [Pause] al... como lo quería entender en español.</p> <p>92 F: Ajá.</p> <p>93 A: Luego lo leí, lo que hacía, en... otra vez, cuantos subía y así y luego lo del español otra vez para saberlo, lo, lo, lo... hacía, lo pensaba, en... en lo que subía y eso. Y luego de ahí ya cambié a inglés otra vez.</p> <p>94 F: ¿Otra vez para qué cambiaste a inglés?</p> <p>95 A: Para poner la respuesta.</p> <p>96 F: Pero antes de poner la respuesta, pusiste [reading, 1st try] one up, one down, three up, ten down. Esto lo escribiste en inglés, mientras estabas pensando el problema. ¿Sí, te acuerdas? Escribiste esto para ayudarte a pensar en la solución.</p> <p>97 A: Ajá.</p> <p>98 F: ¿Por lo tanto aquí pensabas un poco en inglés, también?</p> <p>99 A: Sí.</p> <p>100 F: ¿Cómo que lo anotaste en inglés?</p> <p>101 A: Por... [Pause] Porque estaba, cuando lo apunté estaba pensando en estos los que estaban acá [wording(?)].</p> <p>102 F: Que estaban en inglés.</p> <p>103 A: Ajá.</p> <p>104 F: Por eso lo escribiste en inglés.</p> <p>105 A: Sí.</p> <p>106 F: ¿Y no pensaste en... después como... alguna cosa en inglés durante el razonamiento?</p> <p>107 A: No. [Continues in A3,31]</p> <p>108 [Comes from A3,39] F: ¿En la cuatro no?</p> <p>109 A: No, nno.</p> <p>110 F: ¿No volviste a leer el enunciado?</p> <p>111 A: Lo leí tres veces.</p> <p>112 F: Ajá. ¿Y por qué me decías que no? [Pause] ¿No? Te pregunté: “¿Te pasó esto de leer el enunciado otra vez y volver a utilizar el inglés otra vez en alguna de estas otras actividades?”. Y me dijiste: “Sí, en la actividad tres sí me pasó”. Pero en la cuatro me dijiste que no. Y luego me dijiste: “Sí, lo leí tres veces”. ¿Por qué la primera vez me dijiste que no? [Pause] ¿Aida?</p> <p>113 A: No sé.</p> <p>114 F: ¿No sabe? ¿Lo pensaste rápido o por qué?</p> <p>115 A: Ajá. [Continues in A1,32]</p>		
MEMO	9-21. Superfluous information left aside when summarizing the wording.		

53. Middle floor assigned to the 7th floor, validated by interviewer (54) and corrected later (62, 68).
 88. Understanding of the situation in Spanish, but once it is done, use of English extracting parts of the wording (see A4,1).
 106-107. Aida says she does not use English as thinking language. Probably she means she does not articulate large structures in English, because she takes notes in English (1, 88-89, 96-105), so somehow she is already using English.
 108-113. **1-4th tries**. Wording read in English during solving process.

General Language Questions		Language	Tentative
1	[Comes from A1,39] F: En general, ¿cuándo usas el inglés? Al resolver estas cuatro actividades, ¿cuándo has usado el inglés?	1-3. English as writing language	School context
2	A: Cuando pienso más en las respuestas.		
3	F: Al pensar en la respuestas ¿y cuándo más?		
4	A: Y al leerlo, al leerlo.	3-5. English as reading language	Reading English wording
5	F: Al leerlo porque está en inglés, claro. ¿Cuándo más? ¿Y alguna otra ocasión?		
6	A: No.	7-8. Spanish as main thinking language	9-13. Spanish better known than English
7	F: ¿Y cuándo has usado el español, en general?		
8	A: Cuando lo resuelvo lo digo más en español que en inglés.		
9	F: ¿Y por qué crees que haces eso?		
10	A: Porque sé más español que inglés.		
11	F: Entonces te es más fácil pensar en...		
12	A: Español.		
13	F: Mm [validating]. ¿Y hay alguna –a parte de esto que me has preguntado, ¿no?, del cheaper, hay alguna otra palabra o frase que te haya resultado difícil?		
14	A: No.		
15	F: ¿No? Todo lo otro lo entendiste bien.		
16	A: Ajá.		
MEMO	1-3. Effort done when witting in English.		

Aida has a good Spanish BICS as she talks perfectly in Spanish, her home language. Her Spanish CALP is fair. Aida has a good English BICS and CALP as she makes no mistakes on her English writing. She says that her Spanish is better than her English, but she has just a particular problem understanding the different wordings (cheaper, on A1), and no difficulties writing all the answers in English.

Aida solves A1 thinking mainly in Spanish. When she is solving the activity she asks the interviewer for the meaning of “cheaper” (which is essential for an adequate solving process). During discourse she says “discount” in English (imitating the statement). She writes down the answer in English. On her 1st try she compares directly the percentages of discount in both stores (assuming equal initial prices on both stores), saying that store with

40% of discount is cheaper. When she is explaining her solution she realizes that, in fact, it could be that both stores had the same price (2nd try).

In A2 Aida uses English for reading (not only at the beginning but also during the solving process) and writing purposes. She thinks mainly in Spanish but keeps English for 'square'. She does not interpret correctly the dotted line (initially as one side of the figure). As a consequence, she firstly compares the number of lines on both figures (1st try). While interviewer changes several times the dotted line measurements Aida interprets the dotted line in different ways –for example as the whole square's perimeter, changing her initial right interpretation for the case of the square– in an attempt to find the right answer to the problem by guess and check. Then the correct dotted line is interpreted by the interviewer and the word 'diámetro' (diameter) –unknown by Aida– is introduced. Aida does not compare the lengths of both perimeters correctly through the visual way (2nd try). Neither within the statement's square and a round fan that is standing on the class (interviewer proposes this visual comparison as it should be clear which perimeter is larger). As a consequence, likely she does not have a clear the notion of perimeter.

In A3 Aida uses English as reading (not only at the beginning but also during the solving process) and writing language. She thinks mainly in Spanish (also for the numbers from the table and figures) but keeps English for the “x” and “y” variables of the table. She solves this problem quickly with the help of a x-y table relating the figure number and the number of tiles, as done in class.

In A4 Aida uses English as reading language (not only at the beginning but also during the solving process) and also as writing language. She underlines some words in the wording while she is reading it and takes notes in English to summarize the most important information. Even though, Aida says she thinks the entire solving process in Spanish. On the first try she understands the situation presented in the wording, but she is not able to focus on the central aspects to make a good horizontal mathematization: $1+1+3+10=15$ floors is her answer. The interviewer remarks that adding all the floors is not correct and the points Aida needs to focus on. She advances towards the right solution by doing a symmetry through a middle floor: $13+13=26$, which is still not the desired solution. On the third try ($7+7=14$ floors) she only needs to count the middle floor too. This is indicated by the interviewer, after Aida correctly situates the middle floor on the eight floor, but without having understood its situation properly (probably just a guess).

Activities' (Key ideas) summary

Object 44: Aida-First reduction (End)

- Aida asks for the meaning of “cheaper” –which has a major importance for the mathematical solving process–.
- Activity with stores and percentages solved thinking mainly in Spanish, but answer written in English.
- The mathematical error (assumption of equal initial prices) is self-corrected when Aida explains the answer and might be related to the influence of the visual mode on giving information.
- A lack of geometry vocabulary might have influenced the dotted line interpretation.
- The deviated visual comparison of perimeters seems that is not related with languages use but rather to the concept of perimeter, which is not clear.
- Mathematical figure (square) is interpreted through in English while the other (circle) in Spanish.
- Activity of the sequence of figures solved correctly mainly through Spanish, but answer written in English.
- Use of English related with the variables (x,y) when constructing the (x-y) table.
- Activity with a dense wording solved mainly through Spanish, but English is used to write down the key elements of the wording (reproducing exactly part of the wording) and to write down the answer.
- Aida recognizes she has problems understanding the wording, but those are rather issues on the mathematization than on the linguistic understanding of it, as she summarizes it properly.
- Throughout all of the activities English is always used as a unique writing language. Spanish is mainly used as a thinking language during the solving process. English is just used for particular parts of the solving process: square, x-y (on the table), and for writing down and synthesizing the wording information.

Historical profile	Bilingual profile (Spanish dominant)	Activity	
<ul style="list-style-type: none"> • 14 years old • Transitional class • 2 years in USA • Likes California • Spanish readings • English readings • Spanish at home • Spanish with friends • Homework help: herself 	English as writing language	x x x x	
	Unknown meaning (A1: cheaper, A2: diámetro)	1 x	
	Code mix (A1: discount, A2: square)	x x	
	Use of Spanish as main thinking language	x x x x	
	English with 'square'	x	
	English with "x", "y"		x
	English as a writing language to take notes to summarize wording		x
	Right understanding of the wording (linguistically)		1
	English as a writing language		GLQ
	English as a reading language		GLQ
Spanish as a main thinking language		GLQ	
Procedural profile	Conceptual profile		
<p>1.1 X Direct comparison of percentages (assuming equal initial prices on both stores)</p> <p>1.2 ✓ Awareness of no initial prices results in both stores being able to have the same price</p> <p>2.1 X/✓ Multiple dotted line interpretation (as one square's side, entire circle's perimeter, entire square's perimeter)</p> <p>2.2 X [figures with equal perimeter] X Visual comparison of perimeters X Square's perimeter larger than the perimeter of a (real) fan</p> <p>3 ✓ Finding of the growth on the arithmetical sequence associated to the figure pattern</p> <p>4.1 X Relative position of floors</p> <p>4.2 ✓/X Use of equations as alternative approach to solve the problem (idea not developed) X Relative position of floors ✓ Symmetry through a middle floor (Interviewer previously highlighted the importance of the middle floor) X Middle floor situation X Middle floor not counted</p> <p>4.3 X Middle floor position: $10-3=7$ ✓ Situation of the middle floor (8th floor) X Middle floor not included in the total number of floors of the building X [7+7=14 floors]</p> <p>4.4 X Middle floor (added by interviewer)</p>	<p>1.1 ✓ Notion of percentages as a relative value</p> <p>1.2 ✓ Notion of percentages as a relative value</p> <p>2.1 X Notion of perimeter</p> <p>2.2 X Notion of perimeter</p> <p>3 ✓ Notion of arithmetical sequence associated to a sequence of figures</p> <p>4.1 Notion of number line with confused order positions</p> <p>4.2 Notion of number line with confused order positions</p> <p>4.3 Notion of number line with confused order positions</p> <p>4.4 ✓ Notion of number line</p>		

Object 45: Aida-Second reduction

Object 46: Claudio-First reduction
(Beginning)

Claudio is not very confident on himself. Most of the days he does not do his homework and does not like to be on the task. Most of the time is socializing during class. He is not very good at math.

Math teacher description

Class	Interview language	Interview date	Origins	Age	USA arrival	Living in California	Spanish readings	English readings	Home language	Friends language	Main language at school	Homework help
HS Transitional	Spanish	January 2010	Michoacán, Mexico	16	2003 [7 years]	Likes it a little: free education, less school expenses (material, uniform), no physical punishment at school	Magazines from stores	Books	Spanish, both (sister, brother-in-law, brother)	Both	English	Nobody. Cannot ask his brother-in-law (who knows math) because he is barely at home.

Historical bilingual profile

REMARK: in this transcription there is no video recording (just audio recording). The comments were added in reference to the context.

Math	A1. Claudio marked "English and Spanish".	Language	Tentative
3-10. Norms clarification demand (way of answering the problem). 19-23. Comparison of the shoe's brand on both stores.	[Starts here:] 1 C: ¿Y le puedo preguntar preguntas? 2 F: Sí, si hay... Intenta hacerlas solo, pero si tienes alguna duda te intento ayudar. [Pause] 3 C: So. ¿Como aquí, entonces como dice aquí "In which of these two stores are the shoes cheaper? And why." 4 F: Sí. 5 C: So, ¿le tengo que explicar aquí en letra? 6 F: Sí, sí. Por favor. 7 C: ¿Y uso la, la... como John Sports? 8 F: Yeah, whatever you think you have to use. [Pause] 9 C: Y so... aquí namás escribo eso y me voy a la siguiente. 10 F: Sí. Si crees que ya está resuelta puedes pasar a la siguiente, no tienes que hacerlas en el mismo orden, las	3. Code switch	Reading English wording
		3, 5, 9, 11. Code mix (so)	Previous utterance in English (5), Hybrid language (3, 5, 11)
			3-10. Norms clarification

15-23. 1st try. Wrong answer due to percentages as absolute value (direct comparison of percentages) or assumption of equal initial prices on both stores.	11 C: So ¿aquí qué le pongo?	demand (way of answering the problem)	mathematical procedure (justification can be just embedded on the answer)		
	12 F: Qué lengua has utilizado para resolver la actividad. [Claudio was writing 'Espa'] No, sólo pon una cruz dónde, en la columna que toque. [Pause]				
	13 C: Okay. No...				
	14 F: There's no eraser. I don't... Yes, I should have here one. Here.				
	15 <i>John sports por que tiene 40% de descuento.</i> [Continues in A2,1]			7, 19, 23. Code mix (John Sports)	Reference to English name from the wording
	16 [Comes from A4,51] F: ¿Cómo has solucionado la actividad uno?			15. Orthographical variation (por que)	Unknown, quick writing
	17 C: ¿Por qué? Porque allí dice cuarenta por ciento y en la otra dice Jo[hñ], Mike Sports es veinticinco por ciento.			15. Code mix on writing (John Sports)	Reference to English name from the wording
	18 F: ¿Y qué has hecho primero? ¿Qué es lo primero que has hecho?			17. Code mix (Mike Sports)	Reference to English name from the wording
	19 C: Pues miré lo, el porcentaje, lo miré la marca qué es, luego está más barata la John Sports.			15, 24-41. English use linked to reading	Home language
	20 F: ¿Y qué marca es ésta?			33. Code mix ('and Spanish')	Reading English expression
21 C: ¿Eh?					
22 F: ¿Cómo que dices miraste la marca?					
23 C: Porque la marca de John Sports y luego la otra parte, que otra persona está vendiendo.					
24 F: Okay, okay. ¿Qué lengua has utilizado para comenzar a resolver el problema?					
25 C: La o... el español.					
26 F: En español. Aunque está en inglés. Está en inglés por eso, ¿no?					
27 C: Sí.					
28 F: Y has cambiado de lenguas. ¿Has cambiado de lengua?					
29 C: ¿Cómo?					
30 F: De español, has empezado en español pero luego has utilizado el inglés también.					
31 C: ¿Dónde?					
32 F: Para resolver la actividad uno.					
33 C: Sí, por eso es inglés and Spanish. Por eso lo puse.					
34 F: Sí. ¿Y cuándo has utilizado el inglés?					
35 C: Pues para leer.					
36 F: ¿Para leerlo sólo?					
37 C: Pues sí.					
38 F: ¿Nunca pensabas las palabras en inglés? ¿No las pensabas en inglés?					
39 C: No.					
40 F: Y... Pero... Entonces dices que has empezado con español. Aunque lo has leído en inglés, pero has empezado a pensar el problema en español.					
41 C: Sí. [Continues in A4,52]					
MEMO	3-10. Norms clarification demand (way of answering the problem) (No need of writing for mathematical procedure) 15. Orthographical variation (por que) 15. Code mix on writing (John Sports) 15. Spelling deviation (JoHñ)				

19-23. The comparison of the shoe's brand on both stores –both pictures being the same– can led to assume they have the same initial price.
23. Instead of using the name of the store, Claudio personalizes it ('la otra persona está vendiendo').


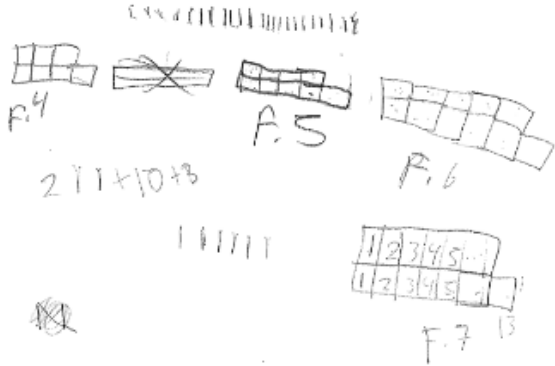
Math	A2. Claudio marked “English and Spanish”.	Language	Tentative
1-10. Interviewer explanation of perimeter concept with physical objects after Claudio's confusion with 'pirámide' (pyramid).	1 [Comes from A1,15] C: ¿Cómo? No lo entiendo. ¿Cómo pirámide? ¿De lo alto? 2 F: El perímetro. 3 C: Ajá. 4 F: ¿Sabes que es el perímetro? 5 C: ¿Que no es lo largo? 6 F: It's like the outside, right? 7 C: Lo... [a school announcement is heard] [Pause] ¿Cómo? No entiendo lo...	1-3. Incorrect translation (perimeter - pirámide)	Poor Spanish CALP
11-13. Activities objective demand (AS no solution or justification is found).	8 F: El perímetro, por ejemplo... 9 C: [Interrupting] Ajá, ¿lo largo o ...? 10 F: ...el perímetro de éste, del ordenador es todo el contorno, ¿sí? Todo esto es el perímetro del ordenador. El perímetro de la hoja, pues es todo esto. [Pause]	11-13. Activity's objective demand	Incomplete statement understanding
1-13. 1st try . No answer due to unclear meaning of perimeter.	11 C: ¿Y también hablo del otro figure [/figura/, with English accent] o...? 12 F:Cuál de los dos, ¿no? What's the question, here? 13 C: Dice “Which of these figures has a greater perimet[er]? And why.” 14 F: Sí, ¿entonces qué tiene...?	11. Code mix (figure)	English wording
15-23. Right final comparison but inexact and not detailed enough explanation.	15 C: Y dice aquí. Yo digo que es el cuadro porque tiene líneas más grandes. 16 F: Ajá. Vale. Intenta justificar la respuesta.	13. Code switch	Reading English wording, previous utterance in English
20. Perimeter size related to area size (no use of appropriate vocabulary).	17  [Continues in A3,1] <i>yo digo que es el cuadro por que tiene lineas grades.</i>	15. Inexact reasoning (grandes)	Poor Spanish CALP
23. Different length of square's sides.	18 [Comes from A4,116] F: Aquí también has puesto la cruz en los idiomas, inglés y español. ¿Qué me puedes decir de esta actividad? ¿Cómo has empezado a resolverla? 19 C: Porque el círculo está más chiquito. 20 F: Ajá. El círculo está más chiquito con lo cual... 21 C: Con lo cual que el cuadro va... es lo más largo. 22 F: Ajá. 23 C: Sí, porque tiene tamaños cortos y tamaños largos. Y esto casi... Yo digo que es lo más grande.	17. Orthographical variation (gra[n]des)	Quick writing
24-27(1 st sentence). Right visual comparison of lengths (perimeter as a whole).		17. Spelling variation (por que)	Quick writing or unknown
27 (2 nd sentence). Right visual comparison of lengths (dividing each		17-23. Lack of details and precision on written and first oral explana-	Poor Spanish CALP

figure in halves across the dotted line) [this assumption is not clearly reflected on the transcript]. 15-27. 2nd try. Right answer with visual comparison of perimeter's lengths (even if the square's sides are given different lengths, 2 by 2).	24	Porque tiene dos tamaños grandes y si lo comparas en una regla a lo largo pues está más grande de los lados. Más los dos del medio. F: ¿Cómo? ¿Más largo de qué lados?		
	25	C: Porque de este lado hazte cuenta de que es una regla.		20. Reference to area instead of length
	26	F: Sí.		21. Adequate use of vocabulary (largo)
	27	C: Y si juntas las dos reglas así a lo largo, entonces lo otro largo, luego más éste largo, pues hacen lo más grande. Y luego si la mitad la comparas es casi la mitad de aquí. [Claudio compared half of each perimeter, dividing them into 2 pieces across the dotted line –this assumption is not clearly reflected on the transcript–]		30-33. English use linked to reading
	28	F: Con el círculo.		
	29	C: Sí.		
	30	F: Aquí también has puesto dos lenguas, inglés y español. ¿Sí? ¿Cuándo has utilizado el español? Perdón, el inglés.		
	31	C: Para leerlo.		
	32	F: ¿Sólo para leerlo, también [as in the previously commented activities, A1 and A4]?		
	33	C: Sí. [Continues in A3,11]		
MEMO	23. Different length of square's sides.			

Math	A3. Claudio marked "English and Spanish".		Language	Tentative
1-4. Awareness of the need of figure 7 to find the answer.	1	[Comes from A2,17] C: Dice "Which figure has... " no, "How many [tiles] does figure seven have? And why. Y a[quí]... pero aquí no es el siete.	1. Code switch	Reading English wording
	2	F: Ajá. So you have to imagine.		
	3	C: Can I just write it here, or...?		
	4	F: Yes, yes, you can write. If you need scratch paper I can give you.	1. Deviated syntax ('aquí no es el siete')	Quick, informal talk
1-4. 1st try. Block of the mathematical procedure as Figure 7 is not represented on the statement.	5	C: Yeah.		
		 <p>yo digo que son 13 cuadros porque cada figure le's aumentados cuadros mas.</p>	2-5. English utterances	1. Previous code switch, 2. interviewer English utterance
6. Construction of Figures –5, 6, 7– following the pattern.	6		12. Code mix (so)	Hybrid language
12. Right	7	C: ¿Sí son trece, no?	17-22. English use linked to reading	Home language dominance

arithmetical reasoning (growth of 2 tiles per figure). 6-16. 2nd try. Right answer with arithmetical and visual reasoning through the drawings of figures 5, 6 and 7 according to the pattern.	8 F: ¿Cómo? 9 C: ¿Son trece? 10 F: Mmmm... Sí, creo que sí. [Continues in A4,1] 11 [Comes from A2,33] F: ¿Qué has hecho aquí? ¿Cómo has empezado? 12 C: Yo empecé porque siempre cada vez que yo miraba algo yo los contaba. Aquí empieza con uno, entonces le sumaron dos más y ya son tres. Son tres más dos. El otro le sumaban dos. Luego otra vez cuenta lo mismo, son cinco. Uno, dos, tres, cuatro, cinco. Cuento los cinco, y le sumaban dos más. So así empecé. 13 F: Pero has hecho los dibujos también, ¿no? 14 C: Sí. 15 F: ¿Por qué has hecho también los dibujos? 16 C: Para poder saber cuántos y sumarle los dos más y así. Hasta cuando llegué a trece. Entonces yo digo que son trece cuadros. 17 F: Aquí también otra vez has puesto la cruz, la cruz, perdón, en inglés y español. 18 C: Sí. 19 F: El inglés, otra vez, ¿cuándo lo has utilizado? 20 C: Para poder, éste, mirar y la pregunta. 21 F: ¿Qué quieres decir para poder mirar? 22 C: Pues leerla. [Continues in GLQ,1]		
MEMO	1-3. No complete math/language understanding of the statement. 20-21. It is not clear if Claudio uses English to interpret the pictures too.		

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Math	A4. Claudio marked “English and Spanish”.	Language	Tentative
1. 1 st try. [10 floors] No structured reasoning (2-9). 5-9. 1 st try. Wording partially read and translated to find a direct answer or a point where a floor can be situated.	1 C: So, aquí que dice: “How many floors does the department store have”. So yo digo que son diez. 2 F: Can you say why? 3 C: Pero porque está... 4 F: [Interrupting] Can you explain that? 5 C: It's because dice aquí que nomás son many floors, pero no te está específicamente diciéndote cuántos. O cuál número, como el primer floor, segundo floor... No está diciendo nada de esto. Nomás dice en medio, éste... te dice abajo, luego dice... Luego dice the other “which is one of the floor [on the 1 st floor!] and leaves to go to another, to another store down the street. How many floors does the department store have.” Luego aquí dice [reading] “which is the, on the first floor”.	1(x2), 31, 41, 45, 87. Code mix (so)	Hybrid language
	6 F: Sí. 7 C: Aquí dice one floor, luego dice goes to the jewelry department. 8 F: ¿Y qué quieres decir? 9 C: Porque se va al departamento de, de joyas. Entonces aquí dice, which is one, which is on the first floor. Pues el departamento de... Luego aquí...	1, 5(x2), 7(x2), 9, 11, 15, 19, 21, 25(x3), 41(x5), 43, 87, 97. Code switch	Reading English wording
13. 1 st try. [10 floors] No reasoning, quick change of opinion (15).	10 F: [Interrupting] Entonces, ¿tenemos suficiente información para resolverlo o no? ¿Sí? Lo puedes ponerlo donde quieras. ¿Sí? Es que no entiendo lo que me estás diciendo.	5. Code switch (It's because) 5. Code mix (many	4. Interviewer previous utterance English wording

<p>1-14. 1st try. [10 floors]. Parts of the wording selected, but no structured, complete or logic reasoning.</p> <p>15(first sentence). 2nd try. [A lot of floors] Answer get directly from the wording.</p> <p>15(2nd, 3rd sentences). 2nd try. Reading wording to find either the answer directly or a point where a floor can be situated.</p> <p>15(last sentence)-21. 2nd try. [20 floors] No reasoning. Reading wording.</p> <p>10-20. 2nd try. [20 floors] Parts of the wording selected, but no structured, complete or logic reasoning.</p> <p>19-46. 3rd try. Reading and translating parts of the wording, but information not organized enough to conclude with a solid argumentation (specially in 41). Several wrong</p>	<p>11 C: [Low voice] How many floors does the department have. Pues aquí namás dice que cuántos de departamentos de...</p> <p>12 F: Ajá, hay.</p> <p>13 C: Hay. Y aquí namás dice que son diez.</p> <p>14 F: ¿Tú crees que son diez?</p> <p>15 C: Yo... ¡no! Pues ahí no te dice específicamente, pero yo digo que son muchos. [Pause] O sea, que aquí dice: "Then she goes she goes up three floors to the toy department. Finally Jamie [/dzeim/] goes to, to, goes down ten floors [pause] to the main entrance of the store which is on the first floor.". Yo digo que nomás hay veinte.</p> <p>16 F: ¿Veinte?</p> <p>17 C: Pues yo digo, no sé.</p> <p>18 F: Pues pon lo... Pon lo que tú creas y luego explica el porqué. [Pause]</p> <p>19 C: Porque aquí dice James [/dzeims/] is stopping, shopping in a large department store with many floors. For "many" means more than one.</p> <p>20 F: Mm [validating].</p> <p>21 C: Enters the store... That's middle. Ya son dos. Porque donde ya estaba ella... Luego dice "Many floors. She enters the store at the middle". [About 5 minutes pause. Whispers something unintelligible several times.] Yo digo que veintidós.</p> <p>22 F: Vale, pues escríbelo, y dime porqué. [Pause] ¿Por qué veintidós?</p> <p>23 C: Porque... Pues si le digo la cuenta, ¿se la digo?</p> <p>24 F: Mm [validating].</p> <p>25 C: Eh, fije. Primero está, Jame [/dzeim/] is shopping in the large department. Entonces el departamento... pues un departamento dónde está ella. Luego dice store which ["with"!]. many floors she enters. Luego dice the sotre at, at the middle, es otro que va, éste, en medio. Son dos... Dos...</p> <p>26 F: [Interrupting] ¿Éste es otro? ¿Esto quiere decir que es otro?</p> <p>27 C: Otro de... Otro abajo. Luego dice...</p> <p>28 F: Mm [understanding]. ¿Pero no será el mismo?</p> <p>29 C: No, porque dice middle, en medio, luego dice [,] arriba, large. ['arriba' as a place in the wording where "large" is situated]</p> <p>30 F: ¿Luego dice arriba dónde? ['arriba' understood in a literal way, as if "above", "top" or a similar word was in the wording]</p> <p>31 C: Arriba, aquí dice. Aquí dice large, luego aquí dice enmedio. So ya está arriba.</p> <p>32 F: Pero, dice que Jamie está, está comprando en un gran departamento comercial con, con diferentes pisos, ¿no?</p> <p>33 C: Mm [understanding].</p> <p>34 F: Dice, entra, en la tienda, en el departamento comercial, por el piso del medio. Pero es la misma tienda.</p> <p>35 C: Mm [validating].</p> <p>36 F: ¿No? Yo creo que es la misma tienda a la que se refiere.</p> <p>37 C: Pues yo digo también la misma. Es lo que digo.</p>	<p>floors)</p> <p>5(x2), 4. Code mix (floors / floor)</p> <p>5, 9, 19, 25. English reading problems</p> <p>15, 19, 25, 41, 87. In-correct 'Jamie' pronunciation</p> <p>19. English utterance (For "many" means more than one)</p> <p>21. English utterance (that's middle)</p> <p>25-39. No global meaning understanding of the first 2 wording's sentences</p> <p>29. Code mix (middle)</p>	<p>English wording</p> <p>Reading difficulties</p> <p>English reading difficulties</p> <p>19. Previous English utterance (reading English wording), explaining English concept</p> <p>21. Previous utterance (reading English wording)</p> <p>40-41. No global comprehension</p> <p>English wording</p>
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<p>assumptions corrected by interviewer.</p> <p>21. 3rd try. When Jamie is at the middle floor Claudio counts 2 floors (supposedly –maybe he has the wording not structured enough and is not thinking on floors–).</p> <p>25-39. 3rd try. Interviewer correction of Claudio’s assumption that when Jamie is in the middle floor Jamie was already into 2 floors (or departments, not clearly stated due to the use of demonstratives).</p> <p>40-41. 3rd try. No sketch of the situation (neither real nor mental).</p> <p>18-47. 3rd try. [22 floors]. Parts of the wording selected, but reasoning not structured, complete or logic.</p> <p>47-49. 4th try. [10 floors] No complete reasoning (only initiated with 3 departments named).</p> <p>47-49. 4th try. [10 floors]. Parts of the wording</p>	<p>38 F: Okay. Con lo cual no podemos contar aquí dos, ¿no?</p> <p>39 C: No, pero entonces ese es uno.</p> <p>40 F: ¿Tú te has imaginado realmente como es eso?</p> <p>41 C: No. Porque ella she goes to the credit department, luego se va al jewelry, luego se va goes up one floor, luego she goes, goes one floor down, luego se va al department children, luego se va al del toy, luego se va three floors up. Luego dice finally James [/dzeims/] goes to down ten floors. So ya está en el floor diez. Luego entra en main entrance of the... at the sotre which is on the first floor and she leaves to go to another sotre down the street.</p> <p>42 F: ¿Pero si dice que baja eh... diez, no significa que está en el diez, no? No es lo mismo.</p> <p>43 C: No, pero dice “down ten floors”.</p> <p>44 F: Sí.</p> <p>45 C: So está arriba y va bajando diez pisos abajo.</p> <p>46 F: Ajá. Okay. Bueno, ¿pones la cruz?</p> <p>47 C: Son, son, son diez. Más el toy, más el jewelry, más el children y luego [d]onde se va para arriba y para abajo.</p> <p>48 F: Bueno, luego si quieres lo...</p> <p>49 C: [Interrupting] Pues yo digo que hay, pues ella dice que son tres. Yo digo que son tres, diez nomás.</p> <p>50 F: Si quiere luego lo revisamos. ¿Sí? ¿Vamos a mirar un poco, comentamos un poco ahora los problemas, cómo los has hecho? ¿Sí?</p> <p>51 <i>yo digo que tiene 15 o 10</i> [It is not clear at which point Claudio wrote the answer, as there is no video recording] [Continues in A1,16]</p> <p>52 [Comes from A1,41] F: ¿Qué otra actividad quieres que miremos ahora?</p> <p>53 C: No sé... Pues yo digo que la de la tienda. No la entendí yo a esa.</p> <p>54 F: ¿La dejamos para el final, quizás? ¿O la quieres mirar ahora?</p> <p>55 C: De una vez, porque ya se le...</p> <p>56 F: Sí. Entones, la de la tienda... Bueno, aquí nos decía que, Jamie en un largo departamento comercial, ¿no?</p> <p>57 C: Mm [continuing conversation].</p> <p>58 F: Que entra en este departamento en el piso del medio. ¿Sí?</p> <p>59 C: Mm [continuing conversation].</p> <p>60 F: Y después dice, eh..., nada más entrar se va al departamento de crédito. ¿Sí?</p> <p>61 C: Mm [continuing conversation].</p> <p>62 F: Entonces quizás imaginarte la situación, cómo sería, en vez de sólo añadir los números que aparecen aquí nos hubiera ayudado a resolver un poco el problema. ¿Quieres imaginarlo e intentarlo resolver o lo dejamos así?</p> <p>63 C: Pues tratarlo, pues también. ¿Entonces para, de qué sirve... para qué estuve aquí?</p> <p>64 F: No, pues ya intentaste los otros, ya resolviste tres, ¿no? Entonces pues esto ya me sirve, también.</p> <p>65 C: ¡Ah, ya!</p>	<p>29, 31. Code mix (large)</p> <p>41. Multiple and complex code switches and mixes</p> <p>41, 47. Code mix (toy)</p> <p>41, 47. Code mix (jewelry)</p> <p>41-45. Deviated rephrasing ('está en el floor diez')</p> <p>47. Code mix (children)</p> <p>71. Code mix (one floor up)</p> <p>89. Code switch</p> <p>93-103. In-correct translation (large department – el departamento más grande)</p>	<p>English wording</p> <p>Reading and summing up English wording without a clear orientation</p> <p>English wording</p> <p>English wording</p> <p>No enough attention paid, no clear expression but right idea</p> <p>English wording</p> <p>English wording (rephrased)</p> <p>Referring to English wording</p> <p>Translation word to word</p>
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selected, but reasoning not structured, complete or logic.	66 F: Pero cómo quieras. Si quieres lo intentamos, sino pues lo dejamos así. 67 C: ¡Pues si quiera! 68 F: ¿Sí? A ver, pues imagínate la situación ésta: estamos en un departamento comercial. Entramos en el piso del medio. ¿Sí?	104-117. English use linked to reading	Home language dominance
52-53. 5 th try. Wording situation not understood (or solution not found).	69 C: Mm [continuing conversation]. 70 F: Y... y luego nos vamos al departamento de crédito. Después, es, ¿cuántos pisos subimos? [Pause] ¿Quizás nos ayudaría ir anotando alguna cosa? ¿O no?		
56-62. 5 th try. Interviewer's translation and explanation of the initial part of the wording.	71 C: Mm [continuing conversation. Pause]. Aquí dice one floor up. 72 F: Mm [validating]. Subimos un piso. 73 C: Ajá. 74 F: Vale. ¿Qué más hacemos?		
62-67. 5 th try. Claudio is willing to solve the problem.	75 C: Luego dice que aquí que bajó tres. 76 F: ¿Baja cuántos? 77 C: Baja uno, abajo. Está en el nueve, entonces. 78 F: O sea, entramos por el del medio. ¿Qué piso es el que entramos?		
68-77. 5 th try. Jamie's initial movements stated by Claudio (71, 75, 77-first sentence) after interviewer's demand.	79 C: El cinco, yo digo. 80 F: ¿Por qué el cinco? 81 C: Porque dice en medio. 82 F: Pero no sabemos dónde está el medio. 83 C: Sí, porque aquí dice diez. La mitad de diez son cinco. 84 F: ¿Porqué sabes... cómo sabes que llega arriba del todo? 85 C: Porque dice aquí. 86 F: Tampoco lo dice que llega hasta arriba del todo, ¿no? 87 C: Pues dice aquí abajo: "She gets, the final James [/dzeims/] get [finally Jamie goes!] down ten floors, enters... main entrance of the store which is on the first floor. Leaves to [go to!, omitted] another store down the street". So baja para abajo.		
77-83. 5 th try. Building with 10 floors. As a consequence, 5 th floor is the middle one and when Jamie goes down 1 floor she is at the 9 th floor.	88 F: Sí, pero no dice que llega arriba del todo, tampoco, ¿no? 89 C: Pues dice pero she starts, isn't it? 90 F: ¿Cómo? 91 C: Empieza ella. 92 F: Mm [validating]. ¿Pero en dónde empieza? 93 C: Pues en el departamento más grande. 94 F: ¿Por qué en el más grande?		
84-93. 5 th try. Jamie starts at the building's top.	95 C: Porque aquí dice: ... 96 F: [Interrupting] ¿Dónde lo dice eso? 97 C: ..."Large department". "Shopping in a large department". 98 F: Large department store, ¿no? 99 C: Mm [continuing conversation].		
56-103. 5 th try. [10 floors]. Assumption of	100 F: Es todo junto. Es un gran centro comercial. 101 C: Mm [continuing conversation].		

<p>building with 10 floors (this 10 is taken from the wording, when Jamie goes down ten floors).</p> <p>104. Solving process quit unilaterally by the interviewer.</p>	<p>102 F: No es que sea un departamento más grande donde ella empieza. Y luego esto sea otro aparte, ¿no? Va todo ligado. ¿Sí?</p> <p>103 C: Mm [continuing conversation].</p> <p>104 F: ¿Qué lengua has utilizado aquí? No has puesto la cruz. [Claudio writes down the cross in the “English and Spanish” column] ¿Ambas también? ¿Qué ha sido la primera cosa que has hecho para resolver el problema?</p> <p>105 C: No, nada más empecé a contar.</p> <p>106 F: Empezaste a contar.</p> <p>107 C: En el departamento.</p> <p>108 F: ¿En qué idioma contabas?</p> <p>109 C: En español.</p> <p>110 F: ¿Y cuándo has utilizado el inglés?</p> <p>111 C: Para leerlo.</p> <p>112 F: ¿Sólo para leerlo?</p> <p>113 C: Mm [validating].</p> <p>114 F: ¿Y en qué momento... ? ¿Cada vez que lo leías, luego lo traducías?</p> <p>115 C: Sí.</p> <p>116 F: Al español. Vale. [Continues in A2,18]</p>		
<p>MEMO</p>	<p>40-41. No sketch of the situation (neither real nor mental). (41. Selection of some parts of the wording with different criteria (mixing movements and department's names with no apparent logic).) 47-49. [10 floors] No complete reasoning (only initiated with 3 departments named) 49. Claudio's interruption: wants to find the answer. 52-68. The interviewer does not want to spend a lot of time in A4, maybe because its getting late (55-56) 56: The interviewer says 'largo' when translating “large”. 58: The interviewer says 'departamento' when translating tradueixo “department store”. 62-67. Claudio is willing to solve the problem. 89-106. Claudio understands the “large department store” as 'the building's top floor'.</p>		

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General Language Questions		Language	Tentative
1	[Comes from A3,22] F: Sí. ¿Y en general cuándo has utilizado el inglés, entonces?	1-16. English use linked to reading	Dominance of home language
2	C: Pues nomás es para... nomás es para leer los problemas y es todo.		
3	F: ¿Por qué crees que sólo has utilizado para leer los problemas?		
4	C: Porque si estuviera... Si no supiera inglés entonces no lo podría leerlo y hacer nada, pues.		
5	F: ¿Pero por qué no lo utilizas también en la resolución el inglés?	17-18. No English understanding difficulties	Average English level in general, no recall of
6	C: No sé.		
7	F: ¿Y cuándo utilizas el español, en general?		
8	C: Pues para hablar, o a veces escribir.		
9	F: Sí, no. Digo para resolver las actividades.		

<p>10 C: ¿Cómo?, no lo entiendo. 11 F: ¿El español cuándo lo has utilizado a la hora de resolver las actividades, no? ¿A la hora de resolver ejercicios de matemáticas, éstos en concreto, cuándo has utilizado el español? 12 C: Pues para responder cosas, nomás. 13 F: Para responder y pensabas también en español, dices. 14 C: Mm [validating]. 15 F: ¿Y por qué crees que pensabas en español? ¿Te sientes más cómodo pensando en español? 16 C: Sí. Mm [validating]. 17 F: ¿Hay alguna palabra o frase que te haya resultado difícil en inglés? 18 C: No.</p>		previous difficulties
MEMO	<p>17-18. No English understanding difficulties. He understands all the words, but neither the correct meaning of all sentences (e.g., due to a translation word to word made sometimes, for example in A4) nor the connection between sentences to get an overall conception of the problem. He has been in USA for many years and is able to understand informal English (good BICS) but lacks of English CALP. From the detailed analysis of the solving process of all activities for the case of Claudio there are some observations and possible implications for the teaching and learning of mathematics. The students need have a good English CALP. Or at least a good Spanish CALP. Without having it it is harder to solve the mathematical activities which have great demands of mathematical register. This is what happens to Claudio on A2, as there are academical procedures implicit on the statement. For those students, maybe it would be useful to solve activities which do not a have high demand of mathematical register.</p>	

Claudio has a fair English BICS, as he has been living in California for 7 years and has no much problem understanding all words and sentences from the wordings (except in A4, where he has major problems due to language issues). There is not much information about his English CALP: he neither writes in English nor talks about the activities (further than reading parts of the wording several times). His Spanish BICS is excellent, as he thinks all the activities in Spanish and choose Spanish as speaking language, making no mistakes. His Spanish CALP is poor, as he barely uses the mathematical register properly –which had been helpful specially in A2– and does not have interiorized the standard written structure well enough in order to answer the problems (“yo digo que”, see A2,17). As stated by himself on GLQ,17-18, Claudio understands almost all the words, but neither the correct meaning of all sentences (e.g., due to a translation word to word made sometimes, for example in A4) nor the connection between sentences to get an overall conception of the problem.

In A1 there is clarification demand about how to justify the answer (5-6). This can be an unexpected situation for Claudio, as there is not a need to show the mathematical procedure (it can be embedded directly in the answer as a justification). The answer is wrong due to percentages as absolute value (direct comparison of percentages) or assumption of equal initial prices on both stores. English use is linked to reading purposes.

During discourse there is a code switch and several code mixing.

In A2 Claudio does not remember what “perimeter” is and translates it as 'pirámide' (pyramid). Once its correct meaning is established and the activity's objective is clarified (or the statement is perfectly understood) he correctly compares both lengths visually: first both perimeters as a whole and then dividing each figure in halves. His written (“el cuadro tiene líneas [más] gra[n]des”) and oral (only initially) arguments are not precise enough, not even correct (area instead of perimeter, square's sides have different lengths). As the conversation advances his reasoning is more understandable and Claudio shows a right visualization of both perimeters lengths.

In A3, as in A2, there is a teacher's clarification demand when a problem is found. The fact that figure 7 is not on the statement, leads Claudio to think that now he has to write some mathematical procedure before writing the answer (earlier he stated the answer with no written mathematical procedure), but wants to check it with the teacher in advance. It is solved correctly with arithmetical and visual reasoning (drawing figures 5, 6 and 7). English use is linked to reading purposes.

In A4 Claudio wants to find the solution directly on the wording and is not able to situate any floor correctly as he does not obtain any direct indication on the wording. He does not do a sketch of the situation (not even after interviewer indication, despite he recognizes it would be useful). All through the dialogue, he mainly maintains the idea of the building having 10 floors. The idea of 10 is extracted directly from the wording, in the point when Jamie goes down 10 floors. Furthermore, he also states 20 and 22 floors as answer. On a second review after commenting other activities, he recognizes he has not understood the problem very well, but he is willing to find the solution of the problem. The initial wrong overall comprehension of the situation presented in the wording is caused, at least partially, by the incorrect translation of some parts of the wording. This fact, along with the prevalent idea of assigning 10 floors to the building is not successfully overcome. The main issue is that his reasoning is not structured and organized enough to make a successful step towards the right solution, but limited to repeat parts of the wording. He says he uses English linked to reading purposes. Claudio makes many codes switches and mixes, especially to refer to parts of the wording (reinforcing the thesis that he understands correctly the words separately but does not give a whole meaning to them appropriately). The code mixes have some influence during the thinking process, but as he has a strong bilingual profile he does not gives much importance to them.

To sum up, Claudio says he uses English for reading purposes only in all four activities. Regarding the mathematical aspects, Claudio solves A2 and A3 correctly. On A1 and A2 major mathematical problems arise.

Activities' (Key ideas) summary

Object 46: Claudio-First reduction (End)

- Demand for the way in which he has to write the answer due to unfamiliar situation in which there is no separated mathematical procedure (embedded on the answer), as the percentages cannot be applied to any price.
- Wrong answer (percentages as an absolute value or assumption of the same initial price on both stores) not related to English or Spanish language usage.
- Wrong perimeter concept (showed in the beginning with an incorrect translation, denoting a poor Spanish CALP).
- Activity's objective is demanded (no complete wording understanding) because no solution is found.
- Lack of mathematical vocabulary results in poor detailed explanation. Understandability is amended after interviewer's demand but more precision should be encouraged.
- No direct continuation of the pattern presented by the figures of the statement. Block derived from the absence of Figure 7 on the statement.
- Right arithmetical and visual reasoning of the pattern associated to the sequence of tiles with English use linked to reading.
- No reasoned mathematical answer along with different language issues (English reading problems, incorrect pronunciation, English utterances during interviewer interaction, complex episode of code switches and mixes) while checking the (dense) wording several times. Language and mathematical problems are intertwined.
- Sometimes the English wording sentences are misunderstood (due to word translation and general misinterpretation of the sentences) which results in a consequent mathematical mistake (wrong horizontal mathematization).
- Important mathematical mistakes (building's top considered as the highest floor reached, no inclusion of the highest floor, wrong symmetry through middle floor) do not seem directly related to the use of language(s).

Historical profile	Bilingual profile (Spanish dominant)	Activity	
<ul style="list-style-type: none"> • 16 years old • Transitional class • 7 years in USA • Likes California a little • Spanish readings • English readings • Spanish at home • English and Spanish at home (sister, brother-in-law, brother) • English and Spanish with friends • Mainly English at school • Homework help: nobody 	Language switching: A1(x0+0+1), A2(x0+0+1), A3(x0+0+1), A4(0+1+21)	x x x x	
	Deviated writing expressions	x x x	
	Code mixing: A1 : so(x4), John Sports(x4), Mike Sports, 'and Spanish'; A2 : figure; A3 : so A4 : so(x5), 'many floors', floors(x2), floor, middle, large(x2), toy(x2), jewelry(x2), children.	x x x x	
	Norms clarification demand (way of answering the problem)	x	
	English use linked to reading	x x x x	
	Incorrect translation (perimeter-pirámide)	x	
	Activity's objective demand (no complete word-ing understanding)	x	
	No detailed explanation (written, oral)	x	
	English utterances		x
	English for reading problems		x
	Incorrect Jamie pronunciation (x5) (mainly as James)		x
	No global meaning understanding		x
	Complex code switching and rephrasing		x
	Incorrect translation (goes down ten floors – está en el floor diez , large department [store] – el departamento más grande)		x
	English use linked to reading		GLQ
	No English understanding difficulties		GLQ
Procedural profile	Conceptual profile		
<p>1 X Norms clarification demand (way of writing the answer) ✓ Shoe's brand comparison on both stores ? Comparison of percentages</p>	<p>1 X Notion of percentages as an absolute value // or ✓ Notion of percentages as a relative value (it is not completely clear)</p>		
<p>2.1 X Interviewer explanation of perimeter with physical objects ✓ Activity's objective demand</p> <p>2.2 X Inexact and not detailed enough explanation ✓ Visual comparison of lengths (figures as a whole and divided in halves across the dotted line)</p> <p>3.1 ✓ Awareness of the need of figure 7 to find the answer</p> <p>3.2 ✓ Drawing of figures 5, 6 and 7 according to the pattern ✓ Application of a pattern given by adding 2 from one figure to the next</p> <p>4 X Different building heights are given with no reasoning (they are just the result of a selection of parts of the wording) X Relative position of the floors X Count of 2 floors when Jamie is at the middle floor X 5th floor as the middle floor X Top of the building as highest floor reached X Symmetry particularity of the middle floor not recognized</p>	<p>2.1 X Perimeter concept</p> <p>2.2 ✓ Concept of perimeter (with interviewer description) X Concept of the square's properties (or wrong visualization of its sides, as they are given different lengths)</p> <p>3.1 X Notion of arithmetical sequence associated to a sequence of figures</p> <p>3.2 ✓ Notion of arithmetical sequence associated to a sequence of figures</p> <p>4 Notion of number line with confused order positions</p> <p>[NOTE: for this activity the different tries are not reflected on the second reduction, as the presentation of the picture altogether offers a better view of the solving process, which is quite confusing for this activity since the different aspects are intertwined]</p>		

Object 47: Claudio-Second reduction

Object 48: Damian-First reduction
(Beginning)

Damian is shy, talks in a low tone of voice, and barely participates during class. He does not try hard to solve math problems or do homework because for him doing the assignments is not so easy and needs more motivation.

Math teacher description

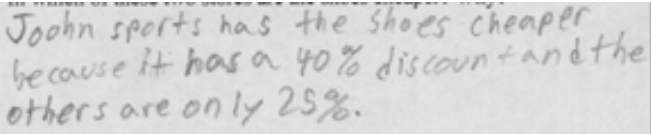
REMARK: in this transcription there is no video recording (just audio recording). The comments were added in reference to the context.

Class	Interview language	Interview date	Origins	Age	USA arrival	Living in California	Spanish readings	English readings	Home language	Friends language	Main language at school	Homework help
HS Transitional	Spanish	January 2010	Mexico	14	Born in USA	Prefers Mexico to California: handmade and better food, goes out more frequently. California: less violence.	Sometimes, but can't read very good.	Yes	Spanish (mother), both (sisters, father – to teach him English–)	Mainly English	[With peers] Mainly English. Spanish in English class and with a friend who just arrived to USA.	Sisters (in English)

Historical bilingual profile

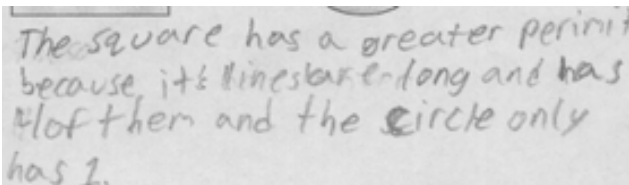
REMARK: The comments on this transcription referring to Damian reading the wording are understood by the context, as there is no video recording (only audio). As usual, when the speech coincides with the wording its is between double commas.

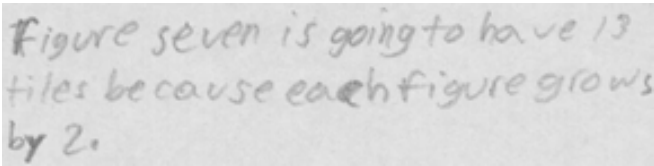
Math	A1. Damian marked "English and Spanish".		Language	Tentative
1-6. Norms clarification (way of writing the answer).	1	D: ¿Lo tengo que escribir así como en senten... seten...? Más o menos es para...	1-6. Norms clarification demand (way of writing the answer)	No need of writing a mathematical procedure (justification can be embedded on answer)
	2	F: What...?		
	3	D: Sentences		
	4	F: Frases.		
	5	D: Eso.		
19-21. 25% as relative value ('Como un cuarto	6	F: Sí, tienes que explicar cómo puedas. Pero al final sí, razonar el porqué y poner una cruz en qué lengua has utilizado, si sólo el inglés o español e inglés. ¿Sí?		
	7	D: Sí.		

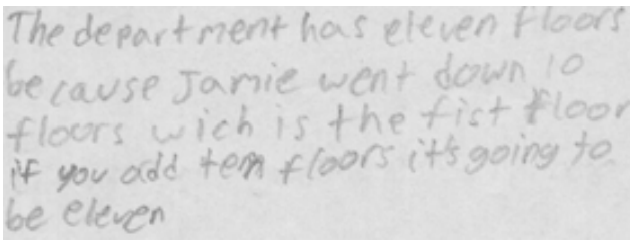
de lo que hay'). 8, 16-21. 1st try. Wrong answer due to assumption of same initial prices on both stores.	<p>8  [Continues in A2,1]</p> <p>9 [Comes from A4,1] D: I'm done. 10 F: ¿Terminaste? 11 D: Sí. 12 F: Vamos a comentar un poco las actividades ahora. ¿Sí? ¿Pusiste las cruces? 13 D: ¿Eh? Sí. 14 F: ¿En el otro lado [the other paper of the questionnaire] también? ¿Con cuál quieres empezar? 15 D: La primera. 16 F: La primera. ¿Cómo la has resuelto? 17 D: Que la de John Sports tiene... están más baratos porque le quitan cuarenta por ciento, le bajan cuarenta por ciento. 18 F: Mm [continuing conversation]. 19 D: Y en el Mike Sports le bajaron veinticinco. Entonces es como una cuarta de lo que... 20 F: ¿Cómo una? 21 D: Como un cuarto de lo que hay. 22 F: Ajá. Okay. ¿Qué lengua has utilizado para empezar a resolver el problema? 23 D: Inglés. 24 F: ¿Inglés? Y dices, de hecho, que has continuado usando el inglés, ¿no? 25 D: Sí. 26 F: ¿Todo en inglés? 27 D: Sí. 28 F: ¿No has utilizado el español para nada? 29 D: Pues sí puedo escribirlo en español, pues. Pero tengo que poner las líneas esas y no sé... 30 F: ¿Cómo? 31 D: Sí puedo escribir en español pero no sé dónde van las líneas porque las pala... las letras de español tienen como líneas y eso, entonces... 32 F: ¿Qué quieres decir líneas? 33 D: Así cómo que esto no tiene nada de líneas. Como la eñe tiene que ponerle la línea esa. Pues la i le puede poner una... de esas líneas arriba y eso. 34 F: Ajá. ¿Los acentos, quieres decir? 35 D: Eso. 36 F: Ajá. Vale. 37 D: Y yo no sé como... 38 F: ¿Pero lo has pensado todo en inglés o has pensado algunas partes en español? 39 D: En español.</p>		<p>only)</p> <p>1. Code mix (senten[ces]) Strong bilingual profile, word in Spanish not found quickly</p> <p>8. English as unique writing language English wording, 28-37: orthography easier in English</p> <p>9. Code switch (non mathematical specific) Previous English thoughts and writing (A4,1)</p> <p>22-27, 38-41, 44-71. English and Spanish as thinking languages; mainly use of Spanish Spanish dominant, school context, English wording</p> <p>42-43. Cross changed in the language columns Deviated understanding of issues languages columns are referring to, 6: interviewer instructions not enough specified</p>
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- 40 F: ¿Lo has pensado en español?
 41 D: Sí, pero lo escribí en inglés.
 42 F: Entonces deberías poner que has usado inglés y español, ¿no?
 43 D: ¡Ah! [Damian changes the cross in the language column]
 44 F: ¿Lo has pensado todo en español, dices?
 45 D: No todo pues, pero una... unas las pensé en inglés.
 46 F: Pero en la actividad uno, digo.
 47 D: ¡Oh!, ésta en español y en inglés.
 48 F: La pensaste tanto en español como en inglés, ¿no?
 49 D: Pues en español y en inglés también.
 50 F: ¿Y te acuerdas cómo empezaste a pensar la actividad?
 51 D: Pues... me fijé en los descuentos primero.
 52 F: Sí. ¿Y cuándo cambiaste de lengua?
 53 D: Cuando estaba poniendo la respuesta.
 54 F: Cuando estabas poniendo la respuesta, cambiaste a inglés.
 55 D: Sí.
 56 F: ¿Pero mientras estabas pensando?
 57 D: Nomás en ésta y en las demás puro inglés.
 58 F: ¿Ésta la pensaste en qué idioma?
 59 D: En español.
 60 F: ¿Y todas las otras en inglés, dices?
 61 D: Sí.
 62 F: ¿Ésta pensaste en inglés el razonamiento? ¿Sí? Dijiste es más barata...
 63 D: Oh, no, ésta la pensé en español y pues.
 64 F: ¿Sí?
 65 D: Sí.
 66 F: ¿Pero algunas cosas en inglés también?
 67 D: Sí.
 68 F: Te veo un poco dudando, por eso digo... ¿Sí? ¿Qué cosas pensaste en inglés?
 69 D: Algo de aquí, sí que puede ser. [Unintelligible]
 70 F: ¿No sabes muy bien?
 71 D: No.
 72 F: Vale, pasemos a otra. Luego, si se te ocurre alguna cosa sobre la actividad uno, pues en qué momento cambiaste o lo que sea, me lo puedes comentar, ¿sí? ¿Qué otra quieres seguir ahora?
 73 D: ¿Vamos a hacer todas las tres, las cuatro, verdad?
 74 F: Sí.
 75 D: Pues las hacemos en orden. [Continues in A2,5]

MEMO 42-43. Cross changed in the language columns due to misunderstanding of its function.
 22-71. It is not clear when he uses English or Spanish as thinking languages, but he says he uses both.

Math	A2. Damian marked "English only".		Language	Tentative
1-4. Perimeter meaning demand (followed by interviewer's explanation). 7. Argumentation not precise but understandable. 5-9. 1st try . No completely right answer due to reasons not always true (less lines meaning shorter perimeter) and not clearly stated.	1	[Comes from A1,8] D: ¿Qué es un perimetre? Perimetre, [both 'perimetre' in Spanish pronunciation] o lo que sea que se dice esta palabra en español. Perimeter [English pronunciation].	1. Unknown mathematical word in Spanish (perimeter)	English schooling for the last years
	2	F: El perimetre, for example, of this table is this. [Interviewer follows the perimeter of the table with the finger] Right?	1. Code mix (perimeter)	Unknown word in Spanish
	3	D: Yeah.		
	4	F: So is the outside.		
	5	 [Dialogue continues in A1,9 after solving all other activities (A3,1 and A4,1)]	1-4. Word meaning demand (perimeter)	Unknown meaning
	6	F: ¿Cómo empezaste para resolver esta actividad?	5. English as unique writing language	English wording, A1,29-37: orthography easier in English
	7	D: [Unintelligible] estaba mirando las líneas de cada figura. Me fijé que esa se miraba más grande y como así hace como uno, hace como en el círculo va a estar como más grande que esto.		
	8	F: ¿Cuál va a estar más grande?		
	9	D: El del cuadrado.	5. Grammatical deviation (it's)	Homophony
	10	F: Ajá. Vale. Aquí, otra vez, ¿Sólo inglés...?	5. Grammatical deviation (long)	Quick writing, no review made
	11	D: Sí.		
	12	F: ¿... o inglés y español?		
	13	D: Nomás en inglés.		
	14	F: Nomás inglés aquí. ¿Pensaste todo en inglés?	10-24. English as unique language to solve the activity	14-19. English with operations
15	D: Nomás para... estos problemas de matemáticas los pienso nomás en inglés.			
16	F: ¿Por qué? ¿Qué quieres decir "como estos problemas"? ¿Qué tipo de problemas?	15-18. English with operations	English dominant	
17	D: Como así como de sumar y multiplicar y eso.			
18	F: Sumar y multiplicar. Pero esto no era de sumar y multiplicar, ¿no?			
19	D: Pues eso sí, pero... un poco sí.			
20	F: Mm [agreeing].			
21	D: Pero todo lo posible[?] está en inglés.			
22	F: Mm [validating]. ¿Y no cambiaste a español?			
23	D: No.			
24	F: Okay. [Continues in A3,2]			
MEMO	1. Unknown mathematical word in Spanish (perimeter). Better English CALP than Spanish CALP.			

Math	A3. Damian marked “English and Spanish”.	Language	Tentative
1. Right arithmetical reasoning (growth of 2 tiles per figure)	1 	1. English as unique writing language	English wording, A1,29-37: orthography easier in English
5. Visual and arithmetical reasoning (direct construction of figure 7, with no need of figures 5 and 6).	2 [Comes from A2,24] F: ¿La tres va a ser también...? ¿La tres sí va a ser inglés y español? 3 D: Sí. [Damian changes the cross in the language column for A3] 4 F: A ver, ¿cómo empezaste esta actividad? 5 D: Me fijé en las figuras éstas. Y las conté que estaban sumando por uno, que estaban subiendo por dos. Y luego estaba pensado que los otros cómo iban a ser por mirando acá . Porque cuatro, cómo aquí es tres y tiene nomás dos y uno de éstos. Dos arriba y dos abajo. Y éste tiene tres arriba y tres abajo. Entonces como menos, es menos uno. Y luego en el siete me fijé. Luego siete menos uno es seis. Y luego conté ésta, estos, estos seis dos veces. Luego conté el último, porque el último nomás tiene uno.	8-25. English and Spanish as thinking languages	Spanish dominant, school context, English wording
6-7. Problem solved in one way (visually) but written justification written in another way (arithmetically).	6 F: Okay. Por esto yo te vi que empezaste y escribiste muy rápido trece y luego dudaste un poco, ¿no?, en escribir “figure grows by two”. Este “by two” te costó un poco escribirlo, ¿no? ¿No? No sé si estuviste pensando otra vez, o estabas repensando algo... ¿No? 7 D: No, pues sí, estaba pensando un poquito de... me fijé... en cuanto estaba creciendo. 8 F: Mm [agreeing]. ¿Empezaste a pensarlo este problema en qué idioma? 9 D: En español. 10 F: ¿Y cómo continuaste? ¿Cuándo cambiaste de idioma? 11 D: Cuando ya empecé a contar. 12 F: ¿Cuándo empezaste a contar...? 13 D: Sí.	10-15. English for counting	English wording
1, 4-7. 1st try . Right answer with arithmetical and visual reasoning (growth of 2 tiles per figure, direct finding of the number of tiles of Figure 7 through a mental representation of it).	14 F: ¿...cambiaste a inglés? 15 D: Sí. 16 F: ¿Y luego cómo seguiste? 17 D: En puro inglés. 18 F: ¿Todo en inglés? 19 D: Sí. 20 F: ¿Y qué es lo que pensaste en español? 21 D: [Short sequence] Como al contarlos, éstos. Iba subiendo por dos, y eso. Así como contando cada vez por dos. 22 F: Mm [continuing conversation]. 23 [Damian mumbles something] 24 F: ¿Cómo? 25 D: Y ya está. [Continues in A4,2]	20-21. Spanish for counting	Home language
MEMO	10-15 / 20-21. English and Spanish for counting. Likely English is used at the beginning to get the number of tiles of Figure 7 and Spanish is used to get the growth of the arithmetical sequence associated to the figure pattern.		

Math	A4. Damian marked "English only".	Language	Tentative
1,3. 1 st try. [10+1=11 floors] No right mathematization of the middle floor (highest floor reached considered as the building's top).	1 	1. English as unique writing language	English wording, A1,29-37: orthography easier in English
3 (from the beginning until 'cuatro', in the 4 th line). 0 try. No relative positioning of floors at the beginning of the solving process.	2 [Comes from A3,25] F: La actividad cuatro, ¿cómo la resolviste ésta? 3 D: Éste, me fijé cuántos pisos subió y cuántos bajó. Entonces ya vi cuántos subió. Pensé que iban a ser trece porque, porque bajó... Digo doce, porque bajó uno y subió uno otra vez y luego subió tres pisos y... No aca..., no aca... Pensé que eran doce. ¿Cómo era? Que iban a... y luego que iban a ser catorce, porque iban los tres, cuatro. Y luego me fijé que bajó diez pisos. Y luego estaba hasta llegar arriba y bajó diez pisos. Y luego de todos los pisos que bajó, llegó hasta el primero y me fijé que estaba en el primero y lo sumé, le sumé diez y ya era el once.	1. Syntax deviations (e.g. "which is", a point is needed after "first floor"; ten floors [plus one])	Quick writing
3 (4 th line: 'Y luego estaba hasta llegar arriba y bajó diez pisos'). 1 st try. Highest floor reached considered as the building's top.	4 F: Ajá. Y aquí pusiste... ¿Está bien esta cruz? 5 D: Sí.	1. Orthographical variation (fi[r]st)	Quick writing
1-3. 1 st try. Wrong answer [10+1=11 floors] due to no mathematization of middle floor (highest floor reached considered as the building's top).	6 F: ¿Sólo inglés? 7 D: Sí. 8 F: Completamente en inglés este problema. 9 D: Sí. 10 F: ¿No pensaste en ningún momento en español para nada? ¿No? 11 D: No. [Continues in GLQ,1]	4-11. English as unique thinking language	English dominant
MEMO	1, 3: Unclear argumentation (it is not specified which one is the floor added to the ten floors) related with syntax deviations. The oral explanation does not completely clarify it. 3 (from the beginning until 'cuatro', in the 4 th line). No relative floors' positioning at the beginning of the solving process.		

General Language Questions		Language	Tentative
1	[Comes from A4,11] F: ¿En general, cuándo usas el inglés? ¿Cuándo has usado, al resolver estos problemas, el inglés?	1-2. English as writing language	A1,29-37: orthography easier in English
2	D: Nomás cuándo... cuándo ya la tuve resuelta. Pero contando, dos y dos, o dos por dos, o tres por tres o algo así, los cuento en español.		
3	F: ¿Y por qué crees que haces esto?	1-2, 5-14. Spanish as language for counting	Home language
4	D: Pues porque a veces se me hace como más fácil en español y a veces más fácil en inglés.		
5	F: ¿Y cuándo utilizas el español?		
6	D: ¿Cómo dice?	13-14. English and Spanish used in relation with numbers	English dominant, home language
7	F: ¿En estas actividades cuándo has utilizado el español?		
8	D: Cuándo conté.		
9	F: ¿Cuándo contaste y cuándo más? ¿Cuándo contaste aquí las figuras [A3], dices?		
10	D: Sí. Y acá en esto, cuándo sumé lo hice porque...	17-22. No English language difficulties	Born in USA, good English level
11	F: ¿Cuándo sumaste?		
12	D: En esa, le digo. Porque... [Unintelligible] pensé en español. Y en ésta porque... lo que le sube y lo que le bajan y eso. Lo que quitan. Y aquí porque le subieron más el descuento y lo que le quitaron.		
13	F: O sea, en la uno, luego, al... ¿en los números pensaste más en español?		
14	D: Más o menos.		
15	F: ¿Y aquí en la dos dices que también pensaste en español?		
16	D: No, en esa no.		
17	F: En ésta no, al final. Vale. ¿Alguna palabra o frase que hayas encontrado difícil en inglés?		
18	D: Nomás las que estoy aprendiendo en inglés.		
19	F: Pero aquí, digo.		
20	D: Ah, aquí, no.		
21	F: ¿No? En estas actividades estamos hablando, ¿no? ¿Lo has entendido todo?		
22	D: Sí.		
MEMO	Looks like he uses more Spanish than English in relation to numbers (1-2), but he uses both languages (4, 13-14). 17-22. No English language difficulties. He does not know what perimeter means in A2 (conceptual difficulty).		

Damian has a good English BICS, as he has no difficulties understanding the activities' statements. The lack of clarity in his written answer on A4 can be due to language issues, mathematical issues or a combination of both. His English CALP is fair (e.g. he does not know what perimeter is in A2). Damian has an excellent Spanish BICS, because he talks in Spanish with no problems. Even though, he recognizes he has some orthographical difficulties when writing in Spanish. His Spanish CALP is worse than his English CALP. His Spanish language level can be related to the fact that he has been in USA and probably schooling occurred in English (at least it is the case during the school year when he was interviewed).

In A1 there is a clarification demand regarding how to justify the answer. This can be an unexpected situation for Damian, as there is not a need to

show the mathematical procedure (it can be embedded directly in the answer as a justification). A1 cannot be considered right due to the assumption of same initial prices on both stores. Percentages are correctly interpreted. The answer is written entirely in English but both languages are used during the solving process, with dominance of Spanish (the use of English is not specified).

A2 is solved entirely with no apparent use of Spanish. After the interviewer's explanation of what perimeter is, it seems that Damian compares correctly both perimeters in a visual way, but his written arguments are not correct (the more number of lines, the larger the perimeter). His oral explanation helps to better understand his reasoning, but it not precise enough.

On A3 Damian infers how a figure is constructed in relation to its figure number. So he has no need of drawing figures 5, 6 or 7, but directly states that figure 7 has 13 tiles (counting mentally the number of tiles in Figure 7 through English). When he has to write the answer he finds the growth per figure (now using the Spanish language), writing this as a justification instead of explaining his first procedure. So he uses both languages for counting. His answer is written in English which is the main language used.

A4 is solved entirely with no apparent use of Spanish. At the beginning Damian does not situate the floors Jamie is moving through in a relative position, but this is later corrected. It is not completely clear the provenance of 1 floor (either top or bottom) on his written answer [$10+1=11$ floors], and a little more clear on his oral explanation. The main issue is that the middle floor is not mathematized (the building's top is considered the highest floor reached by Jamie).

To sum up, there is a predominant use of English, all answers being written in English. The thinking process seems to take place predominantly in English, whereas there are not much details of the use of Spanish (it is related to numbers –see GLQ– , but English is too –see A2). Regarding the mathematical aspects, argumentations are not enough precise. A3 is the only one solved completely correct, but the others are partially correct.

Activities' (Key ideas) summary

Object 48: Damian-First reduction (End)

- Demand for the way in which he has to write the answer due to unfamiliar situation in which there is no separated mathematical procedure but it is embedded on the answer (the percentages cannot be applied to any initial price).
- No apparent language influence on mathematical aspects (right treatment of percentages but wrong assumption of equal initial prices), with predominant use of English but use of both languages (no detailed instance of the use of English).
- English used as unique language to solve the activity of the perimeter of the figures, without an apparent influence of the use of language(s) on the mathematical aspects (right final comparison in a visual way but argumentation not precise enough).
- Both languages are used during the solving process, with a right inference with arithmetical and visual reasoning.
- Both languages are used for counting the tiles of the figures: English in the inference of the number of tiles of Figure 7 and Spanish to get the growth of the number of tiles.
- The mathematical argumentation in relation to the floors of the department store is not clear because the sentences are not constructed with enough precision from a linguistic point of view. English is the unique language used throughout this activity.

Historical profile	Bilingual profile (Spanish dominant)	Activity
<ul style="list-style-type: none"> • 14 years old • Transitional class • Born in USA • Likes more Mexico than California • Some Spanish readings (not fluent) • English readings • English and Spanish at home (English with sisters and to teach father) • English with friends • Mainly English at school (with peers) • Homework help: sisters (in English) 	Norms clarification demand (way of writing the answer)	x
	Code mixing A1: senten[ces], A2: perimeter	x x
	English as unique writing language	x x x x
	Code switch A1(x0+1+0)	x
	English and Spanish as thinking languages (A1: mainly use of Spanish)	x x
	Word meaning demand (perimeter)	x
	English as unique language to solve the activity	x x
	Deviated English writing expressions	x x
	English with operations	x
	English for counting	x
	Spanish for counting	x
	English as writing language	GLQ
	Spanish for counting	GLQ
	English and Spanish with numbers	GLQ
No English language difficulties	GLQ	
Procedural profile	Conceptual profile	
<p>1 X Norms clarification demand (way of writing the answer)</p> <p>X Assumption of same initial prices on both stores</p> <p>2 X Argumentation (perimeter's length related to the number of lines)</p> <p>3 ✓ Application of a pattern given by adding 2 from one figure to the next</p> <p>✓ Mental representation of Figure 7 without drawing the immediate previous figures (5 and 6)</p> <p>4 Non relative ordering of floors Reasoning without distinguishing the singularity of the middle floor in terms of its symmetry function</p>	<p>1 ✓ Notion of percentages as a relative value</p> <p>2 Notion of perimeter not clearly reflected on the argumentation</p> <p>3 ✓ Notion of arithmetical sequence associated to a sequence of figures</p> <p>4 Notion of number line with confused order positions</p>	

Object 49: Damian-Second reduction

Object 50: Zoraida-First reduction
(Beginning)

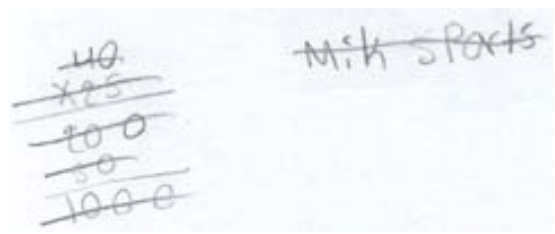
Zoraida has some difficulties to perform well on mathematical activities. She supplies it working hard to get a passing grade.

Math teacher description

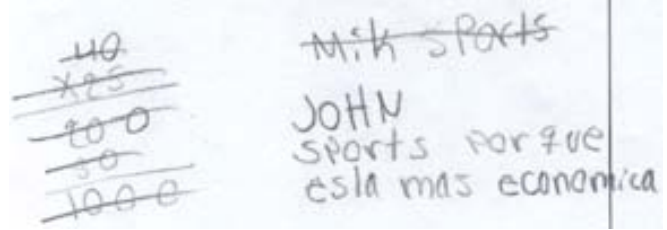
Class	Interview language	Interview date	Origins	Age	USA arrival	Living in California	Spanish readings	English readings	Home language	Friends language	Main language at school	Homework help
MS En-Sp	Spanish	May 2010	Michoacán, Mexico	13	May 2009 [1 year before the interview]	Likes it	Yes: newspapers	Sometimes: magazines	Spanish	Spanish and English	Spanish (Math in Spanish)	Cousins

Historical bilingual profile

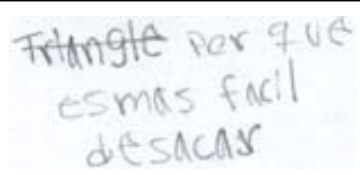
Math	A1. Zoraida marked “English and Spanish”.	Language	Tentative
6, 34-42. 1 st try. No right understanding of the notion of percentage. 6-32. 1 st try. [Mike store] No explicit justification. 45. 2 nd try. Right	[Activities are solved in the natural order: A1, A2, A3, A4. They are commented as follows: A3, A4, A1, A2.] 1 F: Okay, pues aquí tiene estas cuatro actividades, ¿sí? Puede empezar por la que quiera, en el orden que quiera. Nada más le pido que, en vez de utilizar el borrador, por favor, pone una raya por encima si se equivoca o lo que sea y lo escribe al lado, ¿sí? 2 Z: Mm [agreeing]. 3 F: Para saber cómo lo ha hecho. Y al final me pone una crucecita en la columna que sea, ¿sí? Pues le dejo trabajar y luego me avisa cuando termine. 4 Z: Mm [agreeing]. 5 F: ¿De acuerdo? Gracias.	6. English as writing language	English wording
		6. Spelling variation (Mik[e])	Quick writing
		10-14. Right understanding of the wording question	Right translation of the wording question to Spanish
		16. Code	English

interpretation of percentages [by the interviewer].	6	 <p>Handwritten calculations: $40 \times 25 = 100$, $100 - 50 = 50$, $100 - 50 = 50$. The store name 'Mike Sports' is written above the calculations.</p>	[Zoradia's Initial answer. The answer is crossed out later: see A2,94.]	mix (Mike store)	wording
				20. Code mix (John store)	English wording
				22. Code mix (Mike sports)	English wording
				34. Code mix (John sports)	English wording
				79-90. Use of English in relation to the English text of the visual mode	English utterances
				94. Spanish as writing language	Spanish dominant
				94. Spelling variations (mas, economica)	Unknown or quick writing
				60. Code mix (taxes).	
62. 2 nd try. 40% store has greater initial prices than 25% store.	7 F: [Comes from A4,108] La número uno. ¿Cómo empezó aquí? 8 Z: A leer la pregunta.				
63-70. 2 nd try. Right treatment of percentages as relative value.	9 F: Mm [agreeing]. 10 Z: Y ya éste que dice que cuál de las dos tiendas are the shoes cheaper [/caper/]. 11 F: Are the shoes...? 12 Z: The shoes. 13 F: ¿Qué nos pregunta?				
71-76. 2 nd try. Possible reference to the taxes associated to the prices. Right treatment of percentages (taxes) as relative value.	14 Z:Cuál de las dos tiendas tiene [pause] los zapatos más [pause] como más baratos? 15 F: Mm [agreeing]. ¿Y cuál cree que es, usted? 16 Z: La Mike [/meik/] store. 17 F: Mm [agreeing, continuing conversation]. ¿Por qué? 18 Z: Porque tiene un veinticinco de descuento. 19 F: Mm [agreeing]. 20 Z: Y la John store tiene cuarenta por ciento de descuento. 21 F: Mm [agreeing]. 22 Z: Y yo pensé que era la Mike [/meik/] sports. 23 F: ¿Por qué pensó que era esa?				
31-76, 91-94. 2 nd try. Right treatment of percentages. Assumption of equal initial prices [influenced by interviewer].	24 Z: Porque la pregunta dice que cuál de las dos tiendas tiene, eh... 25 F: [Interrupting] O sea, si usted va a comprar los zapatos, ¿cree que aquí serán más baratos? 26 Z: Mm [agreeing]. 27 F: Más baratos. El precio final será más barato aquí que aquí. 28 Z: Mmm... sí. 29 F: ¿Por qué serán más baratos aquí? 30 Z: Porque tiene un veinticinco de descuento. 31 F: ¿Y este número es menor que éste? 32 Z: Sí. 33 F: Y con esto significa que el precio final es más barato. ¿Sí? 34 Z: Eh... no. Pienso que a lo mejor va a ser la John sports porque, éste, si multiplico me va a salir un número más chuiquito que multiplicando por veinticinco. Que multiplicando, sumando veinticinco. 35 F: ¿Cómo tenemos que hacer para calcular esto, que es un veinticinco por ciento de descuento? 36 Z: Dos por punto veinticinco. 37 F: ¿Dos por...? ¿Por qué dos? 38 Z: Porque dice que cuál de las dos tiendas.				

	<p>39 F: Mm [agreeing].</p> <p>40 Z: Y ya multiplico por una de esas. Ya la de por cuarenta por ciento, por veinticinco por ciento.</p> <p>41 F: ¿Por que me multiplicó aquí cuarenta y veinticinco?</p> <p>42 Z: Porque pensaba que tenía que multiplicar cuarenta por veinticinco.</p> <p>43 F: Mm [agreeing]. ¿Y luego por qué me dice que...? Perdón. ¿Multiplicando el dos por punto veinticinco? Punto veinticinco sí es el porcentaje, ¿pero por qué dos?</p> <p>44 Z: Porque... multiplico por dos porque lo estamos multiplicando por las tiendas.</p> <p>45 F: Pero no tiene nada que ver la tienda ¿no? el precio de una... Bueno puede que sí. Pero los zapatos pues lo que valgan aquí luego le hace el veinticinco por ciento y lo que valgan los zapatos le hace el cuarenta por ciento de descuento. Sí, pero no tiene porque ser doble el precio. [Pause] ¿Sí entiende lo que le digo?</p> <p>46 Z: Mm [agreeing and nodding].</p> <p>47 F: Entonces, ¿cuál diría de las dos?</p> <p>48 Z: La John sports.</p> <p>49 F: Mm [agreeing]. ¿Por qué?</p> <p>50 Z: Porque... [pause] porque si compro dos pares me van a rebajar el cuarenta por ciento.</p> <p>51 F: ¿De uno de los dos o de los dos?</p> <p>52 Z: De uno.</p> <p>53 F: ¿Y del otro no?</p> <p>54 Z: Pues sólo me van a bajar cuarenta de, de los dos. Pero aparte.</p> <p>55 F: ¿Qué quiere decir aparte?</p> <p>56 Z: Que si compro un par me van a rebajar cuarenta y si compro el otro también.</p> <p>57 F: ¿Y esto va a ser más económico que ir aquí?</p> <p>58 Z: Mm [agreeing].</p> <p>59 F: Porque aquí, ¿qué le van a hacer?</p> <p>60 Z: Porque aquí me van a descontar el veinticinco por ciento pero te van a cobrar más por los taxes[?].</p> <p>61 F: ¿Por qué?</p> <p>62 Z: Porque... en lo que... Por que en los precios que se ven aquí en esto [pointing to the statement]. Como de por si en una tienda te dan el veinticinco por ciento y en la otra te dan el cuarenta. Éste, y en la de veinticinco van a dar más caro que en la de cuarenta.</p> <p>63 F: ¿Con el mismo descuento? Si los dos zapatos valen lo mismo ¿no? Digamos que son dos zapatos que valen lo mismo. John sports te hacen el cuarenta por ciento de descuento. Mike sports nos hace un veinticinco por ciento de descuento. ¿Dónde van a ser, dónde te comprarías tú los zapatos, en el que te hace un cuarenta por ciento de descuento o en el que te hace un veinticinco por ciento de descuento?</p> <p>64 Z: En el cuarenta por ciento.</p> <p>65 F: ¿Van a ser éstos mas económicos?</p> <p>66 Z: Ajá.</p> <p>67 F: ¿Por qué?</p> <p>68 Z: Porque es el... te rebajan más que éste.</p> <p>69 F: Mm [agreeing]. ¿Éstos te rebajan más que éstos?</p> <p>70 Z: Ajá.</p>		
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	<p>71 F: ¿Y por qué dijo aquí de los tax? ¿Qué tiene que ver las tasas, –¿no?– los impuestos? ¿Qué tiene que ver los impuestos? Aquí también hay impuestos, ¿no? Y aquí también. Pero si nos rebajan más aquí que aquí...</p> <p>72 Z: Si va a subir más el precio, éste que van a... este... a contar más por el... a contar más por las tasas.</p> <p>73 F: Ah claro. ¿Porque tenemos que pagar más lo dice?</p> <p>74 Z: Ajá.</p> <p>75 F: Okay. Además de que son más caras aquí tenemos que pagar también más impuestos. ¿Es eso lo que quería decir antes?</p> <p>76 Z: Ajá.</p> <p>77 F: Ah, perdón, es que no le entendí bien. Okay. Hemos hablado un poco, me he liado un poco, perdone. ¿Cómo empezó a resolver? ¿Qué es lo primero que hizo entonces para... para resolver la actividad?</p> <p>78 Z: Leer la pregunta y después ver cuál de las preguntas, éste, es la más... la más económica para comprar esos zapatos.</p> <p>79 F: Y... bueno la pregunta está en inglés, ¿no? Leyó en inglés y luego ¿cuándo cambió a español?</p> <p>80 Z: Cuando estuve pensando cuál era la más económica.</p> <p>81 F: Mm [agreeing]. ¿Ahí pensó todo en español?</p> <p>82 Z: Ajá.</p> <p>83 F: ¿O algunas cositas en inglés?</p> <p>84 Z: Algunas cosas en inglés.</p> <p>85 F: ¿Como cuáles?</p> <p>86 Z: Como por decir cuál era la... cómo se hacía, cómo se llama la tienda o para qué eran los zapatos, o si quitando el descuento.</p> <p>87 F: ¿Para qué eran los zapatos? ¿Para qué pensó?</p> <p>88 Z: Como aquí dice sports, como era para hacer deporte...</p> <p>89 F: ¿Pensó en inglés eso?</p> <p>90 Z: Ajá.</p> <p>91 F: ¿Entonces cuál es la buena respuesta?</p> <p>92 Z: La John sports.</p> <p>93 F: Mm [agreeing]. ¿Me escribe también el porqué?</p> <p>94  [2nd try. The previous answer is crossed out.]</p> <p>95 Vale. Vamos finalmente a comentar la dos. Terminamos. ¿Está cansada?</p> <p>96 Z: No.</p> <p>97 F: Okay. Terminamos ya con esa. [Continues in A2,2]</p>		
MEMO	43. The interviewer does not understand the previous utterances and reformulates them. Zoraida elaborates her reasoning () from the interviewer's		

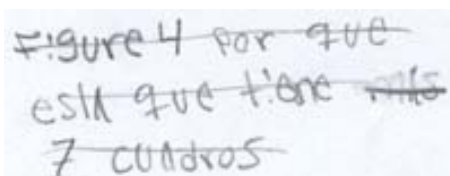
utterance (43) and not from her previous understanding (34-42).
 31-33. These utterances influence Zoraida. She changes the choice she makes in relation to the cheapest store.
 45. Zoraida says 2 (A1,36) referring to 2 stores. The interviewer understands it as twice the price. Maybe the interviewer's understanding influences 'dos pares' (50).

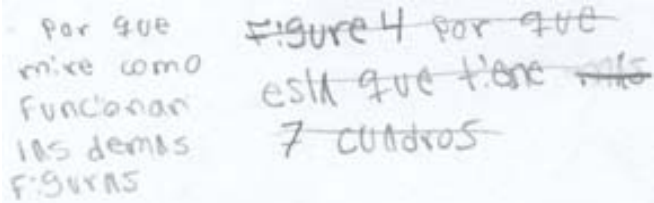
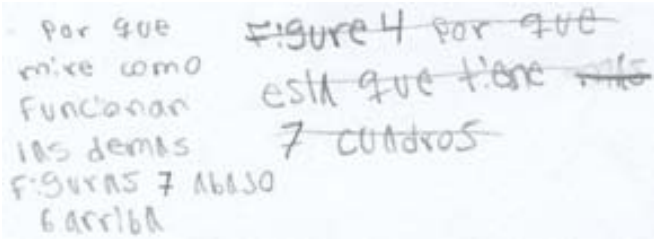
Math	A2. Zoraida marked "English and Spanish".	Language	Tentative	
1-15. 1 st try. Deviated understanding of the wording (which is the figure you can find the perimeter the easiest?). 1, 14-17. 1 st try. Or shift on the understanding of the question (from the initial focus on all the two-dimensional figures to the dichotomy between the figures of the statement – square and circle–).	1  [1st try]	1. Code mix on writing (triangle)	Spanish dominant, English schooling	
	2 [Comes from A1,] F: ¿Cuál de éstas...? Bueno, ¿cómo empezó aquí?	1. Spanish spelling variations (mas, facil)	Quick writing or unknown	
	3 Z: Leyendo la pregunta. Y la pregunta dice que cuál [pause] tiene [pause] el perímetro... Cuál se saca mejor el perímetro.			
	4 F: ¿Cuál se saca, perdón?	1-19. Deviated understanding of the statement's question	Influence of visual mode, 8-12: unknown meaning of greater	
	5 Z: Cuál es el perímetro de éstas o cómo se saca mejor el perímetro.			
	6 F: ¿Qué quiere decir cómo se saca mejor?			
	7 Z: O cómo encuentro el perímetro... o cómo se encuentra mejor el perímetro en estas dos figuras.	8-12. Unknown meaning of greater	English language in construction	
	8 F: Greater, ¿qué significa? [pointing to such a word in the statement]			
	9 Z: Mmm [thinking]	114. Code mix on writing (perimeter)	Spanish dominant, English schooling	
	10 F: ¿Se acuerda?			
	1-23. 1 st try. Wrong answer due to deviated wording question understanding (which is the figure you can find the perimeter the easiest?).	11 Z: No.		
		12 F: Mayor. Cuál de estas dos figuras tiene un perímetro mayor.		
		13 Z: Mm [validating].		
		14 F: ¿Sí?		
		15 [Zoraida nods]		
16 F: ¿Qué había entendido usted, que cómo se es más fácil sacar el perímetro?				
17 Z: Ajá.				
18 F: ¿Y qué contestó?				
19 Z: Que en el triángulo es más fácil de hacer el perímetro.				
20 F: ¿Cuál es el triángulo?				
21 Z: El, el círculo [pointing to the circle].				
22 F: Ajá. ¿Es más fácil?				
23 Z: Ajá.				
24 F: ¿Puede contestar ahora que sabemos que no es la pregunta cuál es más fácil sino cuál tiene el perímetro mayor? ¿Puede, quiere pensarlo un momentito? [Pause]				
25 Z: Ajá. [Pause] ¿El perímetro mayor lo tiene el círculo? [From this point, there is no more video recording, just audio recording.]				

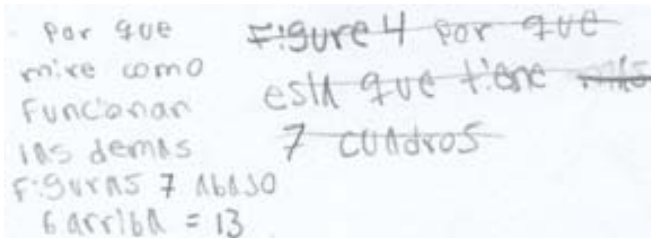
<p>25-35, 56-69. 2nd try. Finding of the perimeter of a figure by manipulative methods. For Zoraida, it is easier to use this methodology in the case of the circle than in the case of the square because of its vertexes.</p>	<p>26 F: ¿El círculo tiene mayor perímetro? ¿Por qué? 27 Z: Porque en éste no se puede sacar bien el perímetro y en éste vas pensando cuál tiene mayor perímetro porque es más fácil de sacar, de ver cuál es el perímetro más... 28 F: ¿Qué es el perímetro? ¿Sabe qué es? 29 Z: Todo lo de alrededor. 30 F: ¿Y es más fácil? 31 Z: Sí. 32 F: ¿Por qué? 33 Z: Porque el perímetro lo puedo sacar con, así como con un hilo y ya, éste [pause], lo mido en la regla y ya así veo cuál es el perímetro. 34 F: Mm [validating]. ¿Y aquí qué pasa con éste? 35 Z: Aquí tengo que éste, tengo esquinas y no casi no puedo... Se me hace más difícil sacar el perímetro en el cuadro que en el círculo. 36 F: ¿Cómo lo sacaría [pause] aquí el perímetro? 37 Z: Pues midiendo cada de ésta, después multiplicarlo por cuatro... 38 F: ¿Y esto le es más difícil? 39 Z: Ajá.</p>		
<p>28-29. 2nd try. Right notion of perimeter.</p>	<p>40 F: ¿Por qué? 41 Z: Porque tengo que multiplicar todo y ya después así veo cómo sale el perímetro. 42 F: ¿Cuánto vale aquí cada lado? 43 Z: Mmm... ¿cinco?</p>		
<p>36-55. 2nd try. Right calculation of the square's perimeter (with interviewer's collaboration).</p>	<p>44 F: Mm [validating] . ¿Entonces sí podemos sacarlo fácil no, todo el perímetro? 45 Z: Pues sí. 46 F: ¿Cómo es, cuánto es el perímetro de aquí? 47 Z: Son veinticinco. 48 F: ¿Cómo lo hace? 49 Z: Porque multiplico cinco por cuatro. 50 F: Mm [validating]. Esto son...</p>		
<p>24-69. 2nd try. Wrong answer due to wrong visualization of the lengths. Different approaches to calculate the perimeter of each figure.</p>	<p>51 Z: Veinticinco. 52 F: Cinco por cuatro son veinte. 53 Z: ¿Veinte? 54 F: ¿Sí? Entonces el perímetro de aquí son veinte. 55 Z: Mm [validating]. 56 F: Y el perímetro de aquí me ha dicho que lo sacaría con un hilo. 57 Z: Ajá, y en una regla. 58 F: ¿Y cómo lo haría? Porque esto es curvo. ¿Cómo pondría el hilo así? 59 Z: Pues que no se mueva el hilo y que se fuera namás por la rayita negra. 60 F: Mm [validating]. 61 Z: Y ya después me fija... después no dejaría que se moviera el hilo y lo pondré en una regla para medirlo.</p>		
<p>70-71. 3rd try.</p>	<p>62 F: ¿Y esto le parecía...? ¿Quería hacer aquí lo mismo del hilo con el cuadrado?</p>		

Indication of use of formulas as alternative way to calculate the perimeter.	63 Z: Pues se me haría más difícil pa poner el... 64 F: ¿Se le haría más difícil para ponerlo por aquí al lado? 65 Z: Por las esquinas. 66 F: ¡Oh! Para que se mantuviera todo. 67 Z: Ajá. 68 F: Pero lo podría hacer como todo trozo por trozo, ¿no?.		
70-76. 3 rd try. Confusion between the circle's area's formula and the square's perimeter's formula.	69 Z: Pues también. 70 F: ¿Sí? [Pause] ¿Alguna otra forma se le ocurre de calcular el perímetro del círculo? 71 Z: Multiplicando [pause] éste [pause] sacando el dos punto veinticinco que es la mitad de cinco. 72 F: Dos punto cinco. 73 Z: Y multiplicándolo por pi radio al cuadrado. 74 F: Radio al cuadrado [pause]. Radio al cuadrado por pi. Esto es... 75 Z: O éste... 76 F: Esto es el área. ¿Sí? 77 Z: O base por altura.		
71. 3 rd try. Wrong division of the diameter by 2.	78 F: Base por altura es el área de... [pause] Del cuadrado también. Pero estamos ahora calculando el perímetro, ¿sí? Se acuerda ahora de cómo era la fórmula del perímetro de [pause] de un círculo? 79 Z: Multiplicando... el...		
73. 3 rd try. Interchange of area and perimeter formulas for the case of the circle.	80 F: [Interrupting] ¿Qué multiplicábamos, se acuerda o no? 81 Z: Dos punto cinco por cinco. Dos punto veinticinco por cinco. 82 F: Eh... 83 Z: O sumando. 84 F: No. Sí teníamos el pi, pero era el diámetro, que es esto el diámetro, por pi. Esto era una forma de sacar el perímetro. Y luego los puedo comparar, ver cuál es mayor. ¿Sí? De la misma forma que ha dicho antes como lo quería hacer al principio con el hilo. ¿Sí? ¿Alguna otra forma tiene de decir... –porque esto– sin calcularlo, cuál de los dos es mayor, si el cuadrado o el círculo?		
77-78. 3 rd try. Formula of the square's area.	85 Z: El cuadrado. 86 F: ¿El cuadrado es mayor, por qué? 87 Z: Porque en éste no sé cómo... cómo conocer el resultado del círculo y en éste sí. 88 F: ¿Pero lo podemos calcular el resultado del círculo? Yo si le doy un hilo, por ejemplo, lo puede calcular eso, ¿no?		
83. 3 rd try. Guess and check to find the formula of the perimeter.	89 Z: Ajá [low voice]. 90 F: Y sin calcularlo... me puede decir, sin saber ningún número... Supongamos que no sabemos que esto es veinte, ¿no? ¿Me puede decir cuál de los dos es más grande? 91 Z: El círculo.		
84. 3 rd try. Circle's perimeter formula introduced by interviewer.	92 F: ¿El círculo es más grande? 93 Z: Ajá. 94 F: ¿Por qué? 95 Z: Porque es como así redondo y, y no te... y el cuadrado, éste, este sitio redondo se va a hacer cómo más chico.		


<p>84-87. 3rd try. No possible comparison of the length, as one of them is unknown.</p> <p>88-97. 3rd try. Deviated visualization of the length of both perimeters.</p> <p>103-108. 3rd try. Right visualization of the length of both shapes.</p> <p>84-97. 3rd try. Wrong answer. No arithmetic reasoning even if the formulas are available. Wrong visualization of the perimeters' lengths.</p> <p>98-114. Right final answer. It is unclear if the lengths are correctly visualized.</p>	<p>96 F: El cuadrado, si lo hace redondo ¿va a quedar más chico que el círculo? 97 Z: Ajá. 98 F: ¿Le puedo dar un... tenemos un hilo por aquí? Podemos hacer como lo dijo usted si acaso. 99 [The interviewer is looking for a lace.] 100 F: ¿No tenemos hilo? ¿Tienes prisa por irte? 101 Z: Ay... no sé. 102 [...] [The interviewer keeps looking for a lace. As he realizes it is getting late, he decides not to use it.] 103 F: Vale, bueno, pues.. ¿Pero así, a simple vista, no? Porque el perímetro, ¿qué es el perímetro? ¿Te están esperando? 104 Z: Ajá. 105 F: Bueno, pues, así a simple vista nada más, ¿cuál es mayor? Tu dices que si esto lo hacemos en forma de círculo, este círculo va a ser mayor que éste? 106 Z: No, va a ser más grande éste que éste. 107 F: Mm [validating]. Por lo tanto, a simple vista, ¿qué, cuál podemos decir que es más grande? 108 Z: El éste, el cuadrado. 109 F: Mm [validating]. Podemos hacerlo sin calcularlo, ¿no? 110 Z: Ajá. 111 F: La forma que tú me has dicho antes, con el hilo, también lo podemos hacer. Y esto sí que es verdad que con todo el hilo entero nos va a costar más, pero una vez llegamos aquí, al vértice podemos parar y luego poner así recto, parar y luego ponerlo así como cachito por cachito una línea recta normalmente es más fácil de hacer que el círculo. 112 Z: Mm [continuing conversation]. 113 F: ¿Me puede apuntar, entonces, pues, la respuesta, en... Also this [pause] and why.</p> <div data-bbox="385 877 1361 1145" data-label="Image"> <p>114 [Final written answer]</p> </div> <p>115 [Continues in GLQ,1]</p>		
MEMO	<p>5. Unclear meaning of the statement's question. 1-12. Activity solved without knowing the meaning of all the words. 33-41. Preference of manipulative methods over arithmetical methods to calculate the perimeter. 87. It is not clear if Zoraida refers to the arithmetic way or to the manipulative (using the lace). 103-108. This sudden change on the comparison of the lengths is not explored in detail (Zoraida is in a hurry).</p>		

Math	A3. Zoraida marked “English and Spanish”.	Language	Tentative
1, 8. 1st try. Wrong answer due to deviated wording understanding (Which figure has 7 tiles?). Right mathematical reasoning according to the deviated understanding.	1  [Zoraida's Initial answer: “Figure 4 por que es la que tiene más 7 cuadros”.]	1. Spanish as writing language with code mix (Figure)	Spanish dominant, English wording
9-13. 2nd try. Translation of the wording question to Spanish (by the interviewer).	2 [Once the activities are solved, the dialogue continues here:] Z: Ya acabé. 3 F: ¿Ya terminó? 4 [Zoraida nods] 5 F: ¿Las cuatro? Okay. A ver... ¿con qué actividad quiere...? Vamos a comentarlas un poco todas. ¿Con cuál quiere empezar? [Zoraida points to the paper] ¿Con la cuatro? 6 Z: No. 7 F: No, con la tres. Okay, cómo resolvió esto? 8 Z: Éste... porque aquí en la pregunta dice que cuál de estas figuras tienen siete... siete cuadros. 9 F: Oh! No dice eso. 10 Z: ¿No? 11 F: Dice ¿Cuántas... cuántos cuadros tiene la figura siete? 12 Z: La figura siete... 13 F: Aquí no está, pero usted... La pregunta es si puede adivinar o puede de alguna manera decir cuántos cuadros va a tener la figura siete. Déjeme cerrar la puerta. ¿Lo quiere pensar otro momentito? 14 Z: Mm [agreeing]. 15 F: Vale. [The interviewer stands up to close the door, as it is too noisy.] ¿Sí puede decir cuántos va a tener la figura siete? 16 Z: Mmm... ¿catorce? 17 F: ¿Porque? 18 Z: Porque multiplico siete por dos. 19 F: ¿Es eso cómo funciona aquí? [Pause] 20 Z: No. 21 F: No funciona multiplicar por dos. Entonces tendríamos que buscar alguna otra cosa. 22 Z: Poner éste siete cuadros abajo y los demás y... Y seis arriba. Y así ya nada más queda un cuadro a derecha o izquierda. 23 F: Mm [agreeing]. ¿Derecha o izquierda? 24 Z: Éste... derecha. 25 F: Ajá. Si quiere hacer lo mismo que así, ¿no? ¿Cómo lo ha hecho esto, cómo lo ha pensado? 26 Z: Haciéndolo cómo son éstos de aquí arriba [pointing to the figures]. 27 F: Y cómo lo... ¿cómo empezó a mirarlo así? 28 Z: Porque aquí en esta figura tiene cuatro, cuatro abajo y tres arriba y además sobra uno a la derecha. 29 F: Y entonces luego lo comparó con... ¿con cuál?	1. Spelling variations (por que) 8. Deviated understanding of the question 9-13. 2nd try. Translation of the wording question to Spanish (by the interviewer)	Quick writing or unknown English language in construction English language in construction
15-20. 2nd try. Deviated reasoning (deviated visualization and consideration of “a missing tile” at the right –23-28–).		49. 3rd try. Spanish as writing language	Spanish dominant
9-20. 2nd try. Wrong answer due to deviated visualization.		49. Spelling variations (por que, mire, demás)	Quick writing or unknown
21-49. 3rd try. Finding of a pattern to construct directly whatever figure,		50-88. Major use of Spanish as thinking language	Spanish dominant
		50-88. Minor use of English as	Spanish dominant, English

with visual and arithmetical reasoning. 9-49. 3rd try. Right answer with visual and arithmetical reasoning, finding a pattern to construct directly whatever figure.	30 Z: Con...	thinking language	statement
	31 F: ¿Miró otra figura?		78-79. Use of Spanish in relation with numbers
	32 Z: Sí.		
	33 F: ¿Después de la cuatro miró otra?		
	34 Z: La... dos y la tres.		
	35 F: ¿Y también funcionaba?		
	36 Z: Ajá.		
	37 F: ¿Sí? Muy bien. Entonces, ¿quiere cambiar la respuesta?		
	38 Z: Ajá.		
	39 F: Me puede escribir aquí al lado...		
	40 		
	41 F: Entonces serían... Me ha dicho ¿cuántas?		
	42 F: Son siete abajo...		
	43 		
	44 F: Por tanto, ¿el total?		
	45 Z: Son... ¿trece?		
	46 F: Mm [agreeing].		
	47 Z: ¿Lo pongo ahí?		
	48 F: Sí, por favor.		

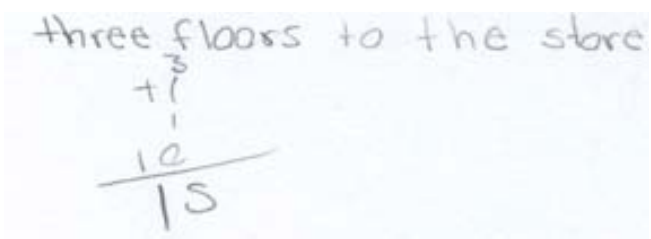
	<p>49  [Zoradia's final answer]</p> <p>50 F: Okay. Vale, ya me ha dicho más o menos cómo lo pensó, ¿sí? Los pasos que ha seguido. Ahora me interesaría que me dijera cómo utilizó el inglés o cómo utilizó el español para resolverlo, ¿sí?. ¿Con qué lengua empezó a resolverlo?</p> <p>51 Z: Con el inglés, porque dice la pregunta que cuántos cuadros tiene la figura siete.</p> <p>52 F: Mm [agreeing].</p> <p>53 Z: Y fue como fui viendo.</p> <p>54 F: ¿Qué pensó en inglés?</p> <p>55 Z: Cuántos cuadros tiene la figura siete.</p> <p>56 F: Pero, por ejemplo, esto que me ha dicho, ¿no? Bueno, al principio lo pensó como... simplemente que... que tenían que ser siete figuras. Perdón, que contase que figura tenía siete, ¿no? Pero luego ya vimos que era la figura siete y adivinar cuántas, cuántos cuadraditos iba a haber en la figura siete.</p> <p>57 Z: Mm [agreeing].</p> <p>58 F: Entonces primero me dijo que sería multiplicando por dos. Vimos que esto no funcionaba para éstas, ¿no?</p> <p>59 Z: No.</p> <p>60 F: Y luego me dijo que esto sería el de abajo, y el de arriba uno menos, ¿sí? El de abajo el mismo número que el de la figura.</p> <p>61 Z: Mm [agreeing].</p> <p>62 F: Entonces... ¿hasta dónde pensó en inglés? ¿Todo esto lo pensó en inglés?</p> <p>63 Z: Nnnn... no.</p> <p>64 F: ¿Qué pensó en inglés?</p> <p>65 Z: Namás la pregunta. La respuesta la pensé en español.</p> <p>66 F: La respuesta la escribió en español.</p> <p>67 Z: Mm [agreeing].</p> <p>68 F: Sí, pero desde que leyó la pregunta –porque está en inglés– luego ¿cuándo empezó a pensar en inglés? ¿O a pensar en español?</p> <p>69 Z: A pensar en español pues... por cómo decían las cosas. Cómo por ejemplo...</p> <p>70 F: [Interrupting] Esto por ejemplo... Dígame, perdone.</p> <p>71 Z: Cómo por así las... Como decía cuántas figuras, la figura tiene un cuadro, la figura tiene dos o así.</p> <p>72 F: Mm [agreeing]. Esto lo pensó en español.</p> <p>73 Z: Mm [agreeing].</p> <p>74 F: ¿Y qué pensó en inglés?</p> <p>75 Z: En inglés pensé en como... qué pasos iba a tener que hacer para hacer ésta la figura siete.</p>		
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	<p>76 F: ¿Como cuál? ¿Me puede decir alguna frase que pensó en inglés?</p> <p>77 Z: Cómo... cómo How many squares [pause] does figure seven.</p> <p>78 F: Y los números, por ejemplo, ¿en qué los pensó?</p> <p>79 Z: En español.</p> <p>80 F: ¿Alguna otra cosa que pensó en inglés?</p> <p>81 Z: No.</p> <p>82 F: Nada más? Lo de... me ha dicho los pasos... ¿cómo serían los pasos? Digo... perdón, los... Por ejemplo el de la figura dos a la figura tres, el comparar las líneas... ¿eso en qué idioma?</p> <p>83 Z: En español.</p> <p>84 F: Básicamente, entonces, en inglés fueron, eh, leerlo...</p> <p>85 Z: Mm [agreeing].</p> <p>86 F: Y luego ¿qué más?</p> <p>87 Z: Luego éste pocas cosas en inglés.</p> <p>88 F: Vale. Vamos a comentar otra, otra actividad. [Continues in A4,2]</p>		
MEMO	9-20. It is possible that Zoraida does not reflect much when she says that the number of figures is twice the figure number. It is also possible that she counts or visualizes the figures in a deviated way.		

Math	A4. Zoraida marked "English and Spanish".	Language	Tentative
5-14. 1 st try. Deviated wording understanding.	1  [Initial answer]	1. English as writing language	English wording
1, 5-14. 1 st try. Wrong answer due to deviated wording understanding.	2 [Comes from A3] Z: ¿Ésta? 3 F: La cuatro. ¿Sí? 4 Z: Mm [agreeing]. 5 F: ¿Cómo, cómo la resolvió ésta? 6 Z: Ésta te dice que how many floors does the department [/departament/] store have. 7 F: Mm [agreeing].	5-14. 1 st try. Deviated wording understanding	English language in construction
15-92. 2 nd try. Wrong answer with no relative situation of floors. No adequate mathematization of middle floor.	8 Z: Y namás tenía que leer esto y ver cuántos tenían, ehm... How many floors does the department [/departament/]. Así llegué [unclear transcript] a encontrar la respuesta. 9 F: ¿Y cómo lo hizo, a ver? Para... porque me dijo... la respuesta es que tiene three, ¿tres? 10 Z: Ajá. 11 F: ¿Cómo lo encontró el tres? 12 Z: Porque leí esto y... [pause] Porque aquí dice que... Nomás tienen que vivir tres personas en un departamento. 13 F: ¿Vivir? 14 Z: Oh éste, dice departamento y dice que, que... que one for to the children... 15 F: ¿Quiere que lo leamos y lo miremos, a ver... cómo... Porque lo pensó en... ¿en qué idioma lo pensó eso, el problema?	15-21. Major use of Spanish as thinking language	Spanish dominant
		15-21. Minor use of English as thinking language	Spanish dominant, English wording
		15-82. Wording translation	15-21. Minor use of English as

	<p>16 Z: En español. 17 F: ¿Todo en español? 18 Z: No, algunas cosas en español y otras en inglés. 19 F: ¿Qué cosas pensó en español? 20 Z: Como por decir como leer esto y ya lo leí en inglés. Y me fijé que tenía un... el número tres y puse tres. Entonces decía que tres [pause], tres floors. 21 F: Mm [agreeing]. 22 Z: To the store. 23 F: Three floors. ¿What happened with these three floors? Then she goes up three floors. ¿Qué significa eso? 24 Z: Goes [pause], ¿arriba tres floors? 25 F: Mm [agreeing]. 26 Z: To the store. 27 F: ¿Qué significa department? 28 Z: Como un departamento o algo así. 29 F: Department store, se lee junto, ¿no? Es como... 30 Z: Como una tienda, un departamento, tienda... 31 F: Es una tienda comercial, ¿no? 32 Z: Ajá. 33 F: No es un departamento donde uno viva. ¿Lo pensó así, como departamento? No, porque si... 34 Z: No, porque dice store. 35 F: Pero departamento... Yo es que no sé muy bien cómo lo dicen en México, pero creo que departamento es como... el lugar de vivir, ¿no?. ¿Es así? 36 Z: Sí. 37 F: Pero aquí no significa eso ¿no? ¿Eso lo tiene ya claro? 38 Z: Mm [agreeing]. [Nodding.] 39 F: Okay. ¿Quiere que lo vayamos... o me lo... lo quiere ir diciendo así como paso por paso, qué es lo que significa cada cosa y lo pensamos otra vez? ¿O...? ¿Tiene ganas de pensarlo otra vez? 40 Z: Mejor lo haga usted. 41 F: A ver. [Pause. The interviewer is waiting for Zoraida to continue, as he has not understood her last utterance.] 42 Z: Dice que... cuántas tiendas caben en un floor. 43 F: ¿Dónde dice eso? 44 Z: Dice cuán... Dice eh.. the department [/departament/] store with [/uitx/] many floors. 45 F: ¿Dónde? Es que no sé dónde está leyendo. ¡Oh! Dice Jamie está de compras o va de compras en un gran centro comercial con varios pisos. ¿No? 46 Z: Mm [agreeing]. 47 F: O sea que va a comprar y el departamento donde va a comprar tiene varios pisos, ¿sí?. Okay, ¿qué más? 48 Z: Dice que entra al, al... aquí dice she enters the store at the middle floor. 49 F: Mm [agreeing]. ¿Qué es eso? 50 Z: Que entró... Está en la tienda de, del medio del departamento, de la tienda.</p>	<p>[with interviewer help]</p> <p>20. Code mix (floors)</p> <p>64-71. Wrong understanding (children's department)</p> <p>72. Deviated understanding (department)</p>	<p>thinking language</p> <p>English wording</p> <p>English language in construction</p> <p>False friend</p>
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	<p>51 F: O sea, entra como por el piso del medio, ¿no? Si tiene varios pisos, pero ella entra pues por una plataforma o lo que sea, por el piso del medio, ¿sí? [The interviewer makes gestures with the hands representing the situation of a middle floor in a building.]</p> <p>52 [Zoraida nods several times.]</p> <p>53 F: ¿Lo ve así, lo entiende? O sea, entra por el piso del medio, ¿sí?</p> <p>54 Z: Mm [agreeing].</p> <p>55 F: ¿Y luego qué hace?</p> <p>56 Z: Dice que inmediato va... Sube... Dice immediately goes to the credit department [/departament/]. Dice que de inmediato va dónde están lo de crédito.</p> <p>57 F: Mm [agreeing]. La sección de crédito, ¿sí?</p> <p>58 Z: Mm [agreeing].</p> <p>59 F: Mm [agreeing]. ¡Muy bien! ¿Qué más?</p> <p>60 Z: Y que después sube otro... éste se va arriba otro piso.</p> <p>61 F: Mm [agreeing]. [Pause.]</p> <p>62 Z: Y que then she goes down one floor to the children's.</p> <p>63 F: Mm [agreeing]. ¿Qué hace entonces?</p> <p>64 Z: Que va dónde namás puede meter[?] un niño.</p> <p>65 F: Goes down significa baja...</p> <p>66 Z: Baja to the...</p> <p>67 F: Un piso.</p> <p>68 Z: Baja un piso to the children's.</p> <p>69 F: Children's department. ¿Qué es el children's department?</p> <p>70 Z: Es como... dónde... dónde están todos los niños, o así.</p> <p>71 F: Ropa de niños, sí, la sección de niños, cosas para niños. [Pause] ¿Y después?</p> <p>72 Z: Después... después goes down ten floors to the main entrance of the store. Después va diez departamentos a... a la entrada</p> <p>73 F: [Interrupting] Diez pisos baja.</p> <p>74 Z: Ajá. Diez departamentos a la entrada de la tienda.</p> <p>75 F: Y nos hemos saltado esto ¿no? Dice que sube...</p> <p>76 Z: Se sube tres departamentos... [Pause]</p> <p>77 F: ¿Departamentos le llaman también a cada piso?</p> <p>78 Z: Ajá. [Zoraida reads the wording.]</p> <p>79 F: A cada planta, ¿sí? ¿Entiende? Floor es como planta, como toda... En una misma planta puede haber pues la sección de niños, la sección de hombres... ¿Sí lo entiende eso?</p> <p>80 Z: Mm [continuing conversation]. Que sube tres departamentos to the toy department [/departament/]?</p> <p>81 F: Mm [agreeing]. Para los juguetes. Sube tres y luego hemos quedado que bajó otros diez, ¿no?</p> <p>82 Z: Ajá.</p> <p>83 F: Y luego, la pregunta ¿qué es?</p> <p>84 Z: How many floors does the department [/departament/] store have? [Pause]</p> <p>85 F: Quizá puede anotar aquí la información o hacerse algún gráfico, porque hay muchas cosas ¿no? Es un</p>		
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	<p>poco...</p>  <p>86</p> <p>87 F: Okay. Entonces, ¿cómo lo arregló eso?</p> <p>88 Z: Sumando todos los pisos que estaban aquí en la pregunta [pointing to the wording]. Y encontré quince, tiene quince pisos en el departamento.</p> <p>89 F: ¿Y hay que sumarlos todos?</p> <p>90 Z: Ajá.</p> <p>91 F: ¿Tanto los que sube como los que baja?</p> <p>92 Z: Mm [agreeing, not sure].</p> <p>93 F: Okay. ¿Qué pensó en inglés?</p> <p>94 Z: Como para decir la respuesta y leer todo esto.</p> <p>95 F: ¿Lo leyó en inglés?</p> <p>96 Z: Mm [agreeing].</p> <p>97 F: Pero luego, por ejemplo, ¿cómo sumó eso?</p> <p>98 Z: Buscando aquí cuántos subía, cuántos departamentos subía y cuántos bajaba.</p> <p>99 F: Y lo sumó en... ¿Qué idioma utilizó para hacer la suma?</p> <p>100 Z: Español.</p> <p>101 F: ¿Qué más pensó? O sea, todo el pensar que se tenía que sumar ¿no? Porque a lo mejor podríamos haber hecho otra cosa en vez de sumar. ¿Cómo, cómo lo pensó eso... en inglés o en español?</p> <p>102 Z: En español.</p> <p>103 F: ¿Qué más pensó en inglés y qué otras cosas pensó en español?</p> <p>104 Z: En español pues pensé que iba a tener que sumar.</p> <p>105 F: Mm [continuing conversation].</p> <p>106 Z: Porque me pregunta en inglés y dice que how many, que cuántos pisos tiene el departamento y ya pensé que tenía que sumar.</p> <p>107 F: Ajá. Okay. Vamos a comentar las otras dos que nos quedan. ¿Cuál quiere?</p> <p>108 Z: Esa. [Continues in A1,7]</p>		
MEMO	Looks like Zoraida does not understand the meaning of "piso". Later she reflects a good understanding.		

General Language Questions		Language	Tentative
1	[Comes from A2,115] F: Vale, tres preguntas más. En general, ¿cuándo usó el inglés?	1-17. Use of	English

2	Z: Pues cuándo los maestros no me hablan en español.	English as reading language	statements, English instructions
3	F: Mm [validating]. [Laughing] Vale.		
4	Z: ¿Aquí, en éstas?	11-12. Use of the English utterances from the wording to write the answer	English language in construction
5	F: Aquí, sí, perdone.		
6	Z: En las preguntas y en los...	21-24. Use of Spanish as writing language	Spanish dominant
7	F: En las preguntas, ¿qué quiere decir? Para...		
8	Z: Para guiarme un poco más.	27-28. Use of Spanish as thinking language	Spanish dominant
9	F: Mm [validating]. ¿Cuándo más?		
10	Z: Y...	29-32. Use of Spanish in relation with numbers	Spanish dominant
11	F: Guiarse un poco más, perdone, ¿qué quiere decir?		
12	Z: Como para decir, como en las preguntas, las leo en inglés y así sé cómo, qué poner en las respuestas.	33-34. Use of Spanish in relation to operations	Spanish dominant
13	F: Mm [validating]. Okay. ¿Cuándo más utilizó el inglés?		
14	Z: Cuándo me están diciendo que cómo resolver las preguntas, así como... como el, cómo aquí. Me dice como que voy a hacer y todo eso.		
15	F: Aquí, antes de empezar las actividades ¿no?		
16	Z: Ajá.		
17	F: Dónde pone solve the following... para leer las instrucciones ¿no? Porque como está en inglés pues hay que leerlo en inglés. Pero durante el procedimiento, mientras va pensando la solución, ¿cuándo utiliza el inglés?		
18	Z: Cuándo pienso algunas cosas en inglés ya me refiero que voy a hacer.		
19	F: ¿Y se acuerda en qué cosas en particular utiliza el inglés?		
20	Z: Como en... en éste, en las preguntas y respuestas cómo las tengo que hacer.		
21	F: Mm [validating]. ¿Y el español, en general, para, cuándo lo usa?		
22	Z: Para responder las esas en español.		
23	F: ¿Para escribir las respuestas?		
24	Z: Para escribir las respuestas en español.		
25	F: ¿Para qué más?		
26	Z: Mm [thinking]		
27	F: Todo esto que ha pensado, por ejemplo ¿no?... Porque desde que lee... desde que lee la pregunta hasta que escribe la respuesta, la lee en inglés y luego escribe la respuesta en español. Pero antes me ha dicho que algunas veces pensaba unas cosas en español, otras veces algunas cosas en inglés, ¿sí? Entonces me gustaría que me dijera pues utilizo el inglés para pensar estas cosas. Utilizo el español para pensar estas cosas.		
28	Z: El español lo utilizo para pensar los resultados y cómo lo voy a hacer. E el inglés lo utilizo para leer las preguntas y los títulos de las hojas.		
29	F: Y los números, por ejemplo, ¿con qué piensa, con qué los piensa los números?		
30	Z: En español.		
31	F: Ajá. ¿Los lee en español?		
32	Z: Sí.		
33	F: Y las operaciones las hace ¿en qué idioma?		
34	Z: Ennnn español.		
35	F: Ajá. Vale, pues si no me quiere decir nada más ya le estoy muy agradecido. Muchas gracias. ¿Alguna cosa que quiera añadir?		

36	Z: No.		
37	F: ¡Gracias! Y perdone que le haya tenido aquí tanto tiempo.		
MEMO	1-3. Zoraida prefers to use Spanish. 1-37. Use of English to read the statements and instructions. No other use of the English language mentioned (in particular, no mention of the use of English as language to think during the mathematical solving process).		

Zoraida has a good Spanish BICS because she communicates orally and written in Spanish. There is not much information about her Spanish CALP because she uses Math vocabulary in English most of the times. Zoraida needs to improve her English BICS and CALP. All the comprehension difficulties as well as the limited use of correct English can be appreciated through all activities.

A1's answer is written initially in English and then in Spanish. On the first try, Zoraida does not justify her reasoning. The interviewer helps her to correctly understand the application of percentages. In the 2nd try, she assumes that both stores may have different prices because the discount rate is different in each store. Assuming equal initial prices on both stores, she correctly gives the final answer.

A2's answer is written initially in Spanish with a code mix. The wording is understood in a deviated way (where is it easier to find the perimeter?). Once the question is properly understood, Zoraida would like to calculate the perimeter of the circle with the help of a lace. She would do it easily in the case of the circle. Zoraida calculates the perimeter of the square. The interviewer provides the formula to calculate the perimeter of the circle. Even though, Zoraida insist on the manipulative method to calculate the perimeter (with the help of a lace). Finally the answer is written in Spanish with a code mix.

A3's answer is initially written in Spanish with a code mix. Zoraida understands the question in a deviated way (which figure has seven tiles?). When the interviewer translates the question she gets a close approach to the solution (he solving process is not detailed). When the interviewer suggest the previous answer is not correct she finds the right answer on the third try.

A4's answer is written in English (both tries). Zoraida does not understand the wording properly. Once the translation is help with the help of the interviewer, she adds up all the floors. She does not mathematizes the middle floor properly. She does not situate the floors in a relative position.

Activities' (Key ideas) summary

Object 50: Zoraida-First reduction (End)

<ul style="list-style-type: none"> - Use of English to interpret part of the visual mode of the statement in a real context - Code mixing when writing the answer in relation to geometrical figures - The deviated wording understanding influences the development of the mathematical activity in relation to the figure pattern. - Use of English when writing in a dense wording. Also after interaction with the interviewer. 			
Historical profile	Bilingual profile (Spanish dominant)	Activity	
<ul style="list-style-type: none"> • 13 years old • English-Spanish class • 1 year in California • Likes more Mexico than California • Spanish readings • Some English readings • Spanish at home • English and Spanish with friends • Mainly Spanish at school • Homework help: cousins 	English as writing language	1	x
	Writing deviations	x	x
	Code mixing (A1: Mike store, John store, Mike sports, John sports, taxes, A4: floor)	x	x
	Spanish as writing language	2	2
	Code mixing on writing (A2: triangle, perimeter, A3: Figure)	x	1
	Deviated statement understanding		1
	Deviated understanding of the question		
	Major use of Spanish as thinking language		x
	Use of Spanish in relation with numbers		x
	Use of English as reading language	GLQ	
	Use of the English utterances from the wording to write the answer	GLQ	
	Use of Spanish as writing language	GLQ	
	Use of Spanish as thinking language	GLQ	
	Use of Spanish in relation with numbers	GLQ	
Use of Spanish in relation to operations	GLQ		
Procedural profile	Conceptual profile		
1.1 X ✓ No detailed reasoning	1.1 X Notion of percentages		
1.2 ✓ Possible different prices on each stores because of different discount rate	1.2 ✓ Notion of percentage as relative value		
2.1 X Deviated mathematization	2.1 ✓ Notion of perimeter		
2.2 X Visual comparison of lengths	2.2 ✓ Notion of perimeter		
2.3 X Visual comparison of lengths	2.3 ✓ Notion of perimeter		
3.1 X Deviated mathematization	3.1 ✓ Mathematical procedure according to wording understanding		
3.2 X Visualization of the figure pattern	3.2 ✓ Notion of funtion		
3.3 ✓ Visualization of the figure pattern	3.3 ✓ Notion of funtion		
4.1 X Deviated mathematization	4.1 Notion of number line with confused order positions		
4.2 Non relative ordering of floors Reasoning without distinguishing the singularity of the middle floor in terms of its symmetry function	4.2 Notion of number line with confused order positions		

Object 51: Zoraida-Second reduction

4.2 Findings related to commonalities

Other practices may arise that are not specific to multilingual classrooms, but which may be used in different ways where multilingualism is present (Setati and Barwell, 2006, p. 29).

As it has been explained on Chapter 3, a list of themes or Third reduction emerged throughout the examination and comparison of the First and Second reductions of all of the students. Now, these themes are described along with an explanation of why this can be happening. Extracts from the dialogues that exemplify its characterization are also provided and accompanied by individual commentaries. Each example, which can be formed by more than one extract, is preceded by a star. Within each theme, the extracts are arranged with the students listed in the same order as they appear in the previous section. The entire list of themes with reference to their pages is the following:

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4.2.1 Contradictory declarations on personal language use

During the dialogue about the activities, when students talk about language use, affirmations are not always consistent from a logical point of view. When this happens it is said that a “contradiction” occurs. The responses given may not always be the true reflect of what indeed occurred when working about a problem; however, there are instances of very good student's recall up to 48 hours after an event (Clarkson 2008). The nature of these contradictions can be classified in four ways:

One way (marked as C1 on the extracts below) is when a student says s/he uses a language for a purpose but later (somewhere on the dialogue or when summarizing the languages' use on the “General Language Questions”) this language purpose is omitted. Maybe they just point out the main use of a language in this last overview. But anyway all uses of that particular language are not mentioned. For example, Ingrid says she uses English to perform additions on A3, but on the GLQ she says calculations are performed in Spanish.

Contradictions that arise from what a student says and what is done are also marked C1. This is the case of Camilo in A4, where he says he counts in Spanish but gives 'four' as an answer, in English.

Another way (C2) is when a student says a language is used for a particular purpose but when later asked, after just some dialogue interventions, this initial idea is not maintained. For example, Angel initially states that A3 is thought mainly through English (16-23) but later he gives more importance to Spanish during the thinking process (32-33, 38-40).

A third way (C3) is when a student says a language is used for a particular purpose but when this is immediately questioned the initial affirmation is changed. This points out that the line that determines when one or the other language is used is not clearly defined. C3 occurs, for example, with Ana in A1.

By looking at the whole interview, or by taking a more general view than just the answer itself, it is clear that sometimes interviewer and interviewee are not talking about the same thing. In some instances this produces a complimentary view of the interviewee's position. For example this happens with Ana in the GLQ, when the interviewer asks her for her initial understanding of the question and she understands it as her actual understanding of the question.

Bi/multilingualism allows for divergent thinking. When words or phrases are used, it is hoped that they will immediately conjure up in the minds of the addressee the same

meaning as the one held by the addresser. However, this does not always happen even when monolinguals communicate with each other (Njurai and Setati, 2011). A sentence can be interpreted in different ways by the two interlocutors, either because of the structure of the sentence itself (as illustrated above) or because each person has her/his own dominant ideas. For example, the instructions of the questionnaire are “[...] mark with a cross in the right columns the languages you have used while solving”. Here some students may interpret “languages” as those used just to write the answer, while others can interpret it as those involved not just in the writing part, but also during thinking and reading (as pretended). These two examples of contradictions are marked as C4.

Specially C2 and C3 are linked to the theme “Blurry perception of language use” (see page 402), as it is not easy to specify when a particular language is used.

C1, C2 and C3 are related with the notion of transparency (Setati, Molefe and Langa, 2008), as language is not seen as an obstacle during the mathematical solving process. But also with the metacognition skills (Clarkson, 2006) which may not be developed enough. It is unknown how aware the students were before the interview of the use of both languages in whatever situation and when solving a mathematical problem in particular, either at the time of doing it or getting a more general scope. But it is true that throughout the interview they developed a new insight on their use of language when solving mathematical problems.

All these contradictions can be deliberated, willing to modify the contents of the interview. Although this can occur in some particular cases, it is not clearly appreciated on the data collected. As students volunteered to participate in the research, no academic benefits might be obtained from such a practice. So probably contradictory declarations are not made on purpose. This is a clear reference to the complexity on the use of languages.

Furthermore, it is a common fact that all of us may well say one thing when in fact we are thinking of another one. For example, the interviewer and Abel do not refer to the same context for the use of English:

★ Abel GLQ,1-4:

1. [Comes from A1,41] F: ¿En general, cuándo ha usado el inglés?
2. A: ¡Oh! ¿Cuándo voy a las tiendas?
3. F: No, aquí, digo.
4. A: ¡Oh! Aquí nomás en las preguntas.

Even the interviewer does not escape from these lapses. He says 'figura' instead of 'cuadrito':

★ Abel A3,22:

22. F: Pero aquí hay... [pointing to the wording] Dice: observa este patrón, ¿no? Aquí hay una

serie que sigue una lógica, ¿no? Entonces tiene que mirar cómo funciona esto para ver... tratar de averiguar, pues la cinco, la seis, la siete... Quizás así lo podemos saber, ¿no? Entonces tenemos que mirar cómo cambia de la uno a la dos, de la dos a la tres, de la tres a la cuatro... y ver si lo que usted ha pensado, de añadir una figura [apase: un cuadrito], esto funciona. Si es lo que pasa para pasar de una figura a la siguiente.

Or maybe sometimes interviewees just answer the first thing that crosses their mind without much reflection. To interpret these contradictions the interview needs to be regarded as a whole, looking for which parts are more repeated (if any) and which ones are complimentary, even if at first sight it looks like they are contradictory.

Now many extracts from the dialogues are presented and commented to exemplify the contradiction on the personal language use.

★ Miriam A1,1-22:



2. [Comes from A1,38] M: [reading] Which of these figures has a greater perimeter and why. Empecé con el inglés y luego en español cuando me fijé en esto [points to the dotted lines]. Y luego al poner la respuesta esta parte ['Tienen el mismo perímetro porque', A2,1] la puse, la quise poner en inglés y luego como que esto ['esto es de la misma medida', A2,1] lo pensé así en español.
3. F: A ver, ¿cómo fue? Perdona, ¿Me lo puedes repetir?
4. M: Yeah. So, dice que que el perimeter [pointing to 'perímetro', A2,1]... Éste es el perímetro, ¿verdad? [follows the perimeter of the circle with the pencil]
5. F: Sí.
6. M: So, como todo esto de aquí [makes 4 imaginary perpendicular radius in the circle, 2 of them following the dotted line] va a ser igual de longitud, entonces puse que los dos [points to both figures] van a ser igual porque los dos tienen el mismo... [points to the dotted lines] mm... symmetry. Like...
7. F: ¿Symmetry?
8. M: Ajá. Los dos tienen la misma medida de aquí a acá [points to the dotted lines]. So...
9. F: Ajá. Esto se llama, por ejemplo, no es la symmetry, ¿no? Esto se llama...
10. M: Radi, radius
11. F: Esto es el diámetro y esto es el lado.
12. M: Okay.
13. F: ¿Sí?
14. M: Yeah.
15. F: Pero...
16. M: [Interrupting] So pensé que todo esto era igual así [makes 2 segments in the square perpendicular to the dotted line; inscribes a circle in the square], so iban a tener el mismo [follows 3 square sides]. Y aquí ['Tienen el mismo perímetro porque', A2,1] puse, lo puse en español y luego aquí ['esto es de la misma medida', A2,1] empecé a pensar en inglés. [Opposite to what she said in A2,2 !]
17. F: ¿Y cuándo... ? Empezaste a leer. Bueno, leíste en inglés, ¿no?
18. M: Mm [Affirming].
19. F: ¿Y luego cuándo cambiaste?, dices, perdona.
20. M: Cambié para cuando éstos [dotted lines] eran los mismos. So pensé que todo era igual [follows the perimeter of the circle] porque estos dos son iguales de [inscribes a circle in the square, as following the perimeter of the square], del perímetro porque de aquí [dotted

lines] son de la misma longitud.

21. F: Ajá. ¿Y lo pensaste esto en español o en inglés?
22. M: En español. Y luego puse esta parte de aquí a acá ['Tienen el mismo perímetro porque', A2,1] en español y esto ['esto es de la misma medida', A2,1] lo estaba pensando en inglés y lo puse en español. [Opposite to what she said in A2,2 !, same way as what she says in A2,16]
23. F: ¿Por qué pensaste esto en inglés?
24. M: No sé.

Miriam says she thinks part of the answer in English and part of it in Spanish (22). Even if there is contradictory information about which part is in English and which part in Spanish (2, 16, 22), the important fact is that both languages are used to think when writing the answer, which is written in Spanish (1). Anyway, she is not aware of the reasons for her switching languages (23-24).

★ Miriam A3, 8-20:

8. M: No. Nomás estaba contando los cuadritos en español, pero todo lo demás lo hice en inglés.
9. F: ¿Y nunca cambiaste a español para nada más?
10. M: Mm... No, nomás, em, empecé a leer todo en inglés y luego empecé a hacer las figuras pensando en español y luego volví a inglés acá [points to the answer].
11. F: ¿Y por qué aquí [A2] escribiste la respuesta en español y aquí [A1] también?
12. M: I don't know.
13. F: Y aquí [A3] en inglés. ¿No sabes?
14. M: No.
15. F: Empezaste aquí [A3] todos los cambios en inglés, todos los pasos del problema en inglés.
16. M: Lo leí en inglés y luego todos esos [points to the figures she drew] los hice pensando en español.
17. F: ¿Al contarlos sólo?
18. M: Ajá. Al contarlos nada más, me fijé que todos llevaban dos más. Como éste, éste ya estaba, y dos más [points to the 2 tiles added to Figures 3 and 4 respect to the previous figures]. Y por cada dos nomás agregaba dos más contando en español.
19. F: Okay.
20. M: Y luego ya al momento de fijarme, conté todo, cuantos tiles había aquí [figure 7] y puse la respuesta en inglés, pero lo había contado en in[glés], en inglés. [Continues in A4, 2]

Miriam says she counts in English (20), but previously she repeats several times that she uses Spanish to count (8, 10, 16-18) (C2).

★ Miriam A4, 7:

7. M: So hice namás ten floors [9!]. Un cuadro con diez pisos así [9!] y ésto no estaba [hides the 3 bottom floors (which are added later: see A4,1)]. [Intervention continues; not transcribed here as this excerpt is illustrative for the comment below]

Miriam says that the sketch has 10 floors but in fact there are only 9 floors. This extract shows that sometimes affirmations are linked to a lack of memory or concentration (C4).

★ Camilo A1,1:

- 1.

Camilo A1,14-31:

14. F: ¿En qué momento? Porque aquí me puso que usó inglés y español. Entonces me dijo

- que empezó a resolver la actividad en inglés.
15. C: Ajá.
 16. F: ¿Hasta qué momento? ¿Cuándo... cuándo cambió a español?
 17. C: Cuando empecé a explicar.
 18. F: ¡Oh! ¿Para explicarlo?
 19. C: Ajá.
 20. F: ¿Pero mientras estaba pensando [pause], por ejemplo en el cuarenta por ciento de descuento?
 21. C: En español.
 22. F: Esto lo tradujo a español.
 23. C: Ajá. [Camilo nods] En español.
 24. F: Pensó cuarenta por ciento de descuento, no forty per cent of discount.
 25. C: [Camilo nods]. En descuento.
 26. F: O sea leyó la actividad en inglés...
 27. C: Sí, pero lo pensé en español.
 28. F: Luego estuvo pensando en español.
 29. C: Sí.
 30. F: ¿Y cuando volvió a cambiar a inglés?
 31. C: Cuando empecé a explicarlo.
- Camilo does not start to write in Spanish actually (4-19), but he writes in English in the beginning (1, 30-31) (C2).

★ Camilo A1,44-49:

44. [Comes from A2,77] Y volviendo a la actividad uno, porque no habíamos dicho esto de las palabras, ¿no? ¿Aquí alguna palabra sí estuvo como manejándola en inglés?
 45. C: Discount.
 46. F: ¿El discount?
 47. C: Sí.
 48. F: Estuvo pensando en inglés. Discount. ¿Alguna más? ¿Alguna cosa que le venga a la cabeza que estuvo pensando...
 49. C: The shoes... the shoes... and... The John Store and... y Mike Store. Y eso nada más.
- Camilo initially says he does not think of discount (see 24-25 on the extract of the above example) but after commenting A2 the dialogue returns to A1 and then Camilo says he does think of 'Discount' (44-48) (C2).

★ Camilo A2,11-13:

11. C: ¡Oh! El cuadro porque... Es el cuadro porque el círculo tal vez no tiene perímetro. ¿O sí tiene?
12. F: ¿Qué es el perímetro?
13. C: Es el... cuándo... dónde... dónde sumas todos lados de un cuadro [follows the perimeter of the square with the pencil]. Luego en el círculo[?] no...

Camilo A2,65-71:

65. C: ¡Oh! Bueno... ¿También eso? ¿También eso incluye? Pues sí, lo pensé en inglés también. Por ejemplo the square, the circle o así.
66. F: ¿Y qué más pensó en inglés?
67. C: Perímetro, perimeter... Luego pues también es uno, dos idiomas. Pienso en uno, como éste es square, luego en círculo. Pensé en inglés y en español también. Por eso puse inglés y español [pointing to the cross he marked on the "English and Spanish" column of the questionnaire; A2,41.]
68. F: ¿Pero también dijo por ejemplo cuadrado aquí?
69. C: Mmm [thinking]... No. Namás square y luego círculo.
70. F: ¿Círculo sí dijo? ¿O circle?
71. C: Yo dije círculo. Aquí dije círculo y aquí dije square. ['cuadro' : A2,11 & A2,13 !!!] Aquí es más fácil de pronunciar círculo.

Camilo says he thinks of 'square' (71). In addition to the use of 'square' (65, 67, 69, 71) he also uses 'cuadro' (11,13) during the discourse. Probably he uses both when thinking.

★ Camilo A4,3:

3. C: Pues lo leí en inglés, luego cambié a español y lo pensé en español.

Camilo A4,35-39:

35. C: Tal vez ahí cuidan niños, o no sé. Ya son tres. Luego ella va... sube tres pisos más. Luego son seis [marks 6 with the fingers, using both hands], tres pisos más ya. Y ya después finalmente baja diez pisos... ¡Oh sí, pero ahí debería de restarlos! Después diez pisos y ahí los cuento y ya son dieciséis pisos.
36. F: ¿Entonces que dijo que tendría que hacer, restarlos?
37. C: Sí, restarlos.
38. F: ¿Lo arreglamos? ¿Arreglamos la respuesta?
39. C: Entonces son... four.

Camilo A4,57-71:

57. C: No. Entonces seguro no. Esto no... tal vez. Pero yo lo sumé namás los pisos que iba subiendo, el total de pisos que aquí dice.
58. F: ¿Qué pensó aquí en inglés?
59. C: Pues aquí sí se me hizo más fácil leerlo en inglés y pensarlo también en inglés. Porque aquí pasarlo a español me cuesta trabajo, ya lo ha visto. Como historias largas, como escrituras largas como ésta. Por ejemplo, como aquí [reading] after making sure, aquí no sé, aquí no sé como en español, como después se puso, se hizo segura, no queda.
60. F: Sí. ¿Pero sí entiende lo que dice?
61. C: Sí. After making sure...
62. F: Porque no tenemos que traducir palabra por palabra.
63. C: No. No.
64. F: Después de asegurarse, ¿no?
65. C: Porque no queda bien, no queda bien, como: "Y después haciéndose segura su crédito es bueno...". Pues ahí...
66. F: Sí. ¿Entonces ahí lo pensó en inglés?
67. C: Sí.
68. F: ¿Y que pensó en español?
69. C: Namás la respuesta, la respuesta nada más. Namás la escritura que dice aquí [written answer]. No pensé nada en español. Lo único fue (lo que pensé...) la respuesta.
70. F: ¿Y a la hora de sumar los números, por ejemplo?
71. C: ¡Ah! Ahí sí, ahí sí lo sumé en español.

Camilo says he thinks of A4 in Spanish at the beginning (3), on the first try. After commenting some mathematical and language aspects of the problem he says the activity is thought of in English (58-69), maybe during the second try. While he explains how he thought of the answer, he talks in Spanish. The expression "making sure her credit is good" is initially translated with some difficulties because Camilo does not find the words in Spanish, but he understands the expression. Later he uses the English expression 'making sure' (59), which is directly taken from the wording. This is an important point that leads Camilo to say that it is easier to think in English, and maybe changing at the same time the initial affirmation of using mainly Spanish as a thinking language.

There is also a contradiction on the language used when counting: Camilo says he adds the numbers in Spanish (70-71) but probably English has some influence too because Camilo says 'four' when giving the answer (39) (C1).

★ Camilo GLQ,1-26:

1. [Comes from A4,76] F: ¿En general, cuándo ha usado el inglés? Aquí para resolver...
2. C: ¿Aquí [pointing to the papers], en general?
3. F: Sí.
4. C: El inglés, al leer, la escritura, al leerlo.
5. F: ¿Al leerlo?
6. C: Sí.
7. F: Y en la escritura.
8. C: Sí.
9. F: ¿Y por qué cree que ha hecho esto?
10. C: Porque es más fácil para mí porque traducirlo en español, la escritura no queda... no tiene sentido.
11. F: A la hora de leerlo.
12. C: Ajá.
13. F: ¿Y a la hora de escribir?
14. C: A la hora de escribir porque se me hace más fácil explicarlo en español.
15. F: Pero me dice que ha usado el inglés también para escribir.
16. C: Mm [validating]. Pues también porque... pues... no es nada fácil el inglés[?]. No lo sé explicar muy bien.
17. F: Pero quizás... ¿Por qué? ¿Puede encontrar alguna...? No sé, si lo quiere pensar un poco...
18. C: En inglés porque no quiero escribir mucho y en inglés hay abreviaturas como aquí namás John store y luego en español el mero escribir la tienda de John y ya es mucho.
19. F: Okay. ¿Y en general cuando ha usado el español? [The bell rings, announcing the start of the next class.]
20. C: En general, explicándolo, escribiendo, en la escritura.
21. F: ¿Y para resolver los problemas?
22. C: Lo he pensado en español.
23. F: Algunos ratos en español y otros ratos en inglés, me dijo, por eso. Un poco de todo. ¿Sabe cuándo ha utilizado una lengua y cuando la otra?
24. C: Sí.
25. F: ¿Cuándo?
26. C: Pues cuando la... el español cuando... El español, pues le digo que lo explico más bien en español y el inglés lo uso porque no quiero escribir mucho. Tiene muchas abreviaturas y se escribe menos.

There are many misunderstandings between the intended meaning of the question given by the interviewer and the way it is understood by Camilo. There is some influence of the fact that they are in a hurry because it is lunch time and soon it will be time to start the class again (19). Firstly the interviewer refers to the use of English when reading (11) on Camilo's previous utterance (10), but Camilo understands it as reading his written production (12). Furthermore, the interviewer refers to the time when Camilo talks about the activities, but maybe Camilo thinks about his responses on the GLQ (23). It is not easy for Camilo to explain it clearly (16) (C4).

★ Jessica A2,18-22.

18. F: ¿Por lo tanto, qué es...? El perímetro sí sabe qué es, ¿no?
19. J: No [low voice].
20. F: ¿O no?
21. J: No.

22. F: ¿No? [pause] ¿Lo ha estado haciendo en clase o no, el perímetro? ¿Se acuerda? El perímetro es lo de afuera, esto es el perímetro. [Francesc follows both perimeters with the finger.]

On A2 Jessica does not know what the perimeter is (neither in Spanish nor in English) even if she has already written an answer.

Jessica A3,9-16:

9. [Comes from A2, 56] F: ¿Qué le ocurrió aquí? ¿Qué es lo que no entendió al principio?
10. J: La pregunta, saber que decía.
11. F: ¿Pero hay alguna palabra que no entendió o por qué se le hizo difícil esto?
12. J: Ésta. [Jessica points to the word tiles]
13. F: F: Tiles. Ah, ¿no usan tiles en clase?
14. J: No.
15. F: Oh, tiles son, ya le he dicho, ¿no?, los cuadraditos éstos. ¿Lo otro sí estaba claro?
16. J: Sí.

On A3 Jessica does not understand the wording question.

Jessica GLQ,25-32:

25. F: ¿Alguna palabra o alguna frase que haya encontrado difícil en inglés? [Pause] ¿Hay alguna?
26. J: No [low voice].
27. F: En, de todos los enunciados.
28. J: No.
29. F: Bueno, ésta sí hemos dicho lo de “tiles”, ¿no? ¿Se acuerda de alguna otra?
30. J: No.
31. F: ¿No? ¿Lo otro lo encontró bien?
32. J: Mm [validating].

On GLQ Jessica does not remember that she did not know the meaning of “tiles” (28) (C1).

★ Ana A1,16-19:

16. F: [A4 was going to be commented, but dialogue went back to A1] Por ejemplo, perdone, ¿aquí para comparar los porcentajes, esto lo hizo... lo hizo en inglés?
17. A: [Hesitating] Sí.
18. F: Perdón, inglés. ¿En inglés o en español?
19. A: En español.

On A1 Anna immediately makes a shift from English as a language for managing percentages (17) to Spanish (19), after the interviewer questioned her answer (C3).

★ Ana GLQ,47-61:

47. F: ¿Por último, hay alguna palabra o alguna frase que haya encontrado difícil en inglés?
48. A: No.
49. F: ¿No? Bueno, hemos encontrado las..
50. A: Ésta.
51. F: Main entrance. ¿Y esto, qué pasó, entonces, aquí al entenderlo? La frase es... hay algunas cosas... bueno, Jaime o Jamie, ¿no?
52. A: Ajá.
53. F: ¿Confundió aquí con Jaime?
54. A: ¡Ah, sí!
55. F: ¿Y que entendió aquí al principio?
56. A: Que él estaba, él estaba, éste, iba a comprar, o fue a comprar a un centro comercial.
57. F: ¿Y que fue a comprar qué, me dijo, pisos?
58. A: Muchas cosas, porque primero fue a, a..
59. F: [Laughing, both] Okay,
60. A: Fue al es..
61. F: [Interrupting] ¿Pero sí lo entendió más o menos, cree?

Ana does not admit she has some problems on the understanding of A4's wording (48), as she later recognizes (50-54) (C1, C3). The question about her initial wording interpretation (55) is understood as the situation presented initially on the wording (56-60). The interviewer interrupts Ana's description (61) without understanding Ana's interpretation of the question (C4).

★ Juan A1,13-17

13. J: ¿Porque depende de la tabla de precios [pointing to "Unbeatable prices"]?
14. F: ¿Pero qué significa unbeatable prices? ¿Sabe?
15. J: No.
16. F: Significa precios inmejorables. Que nadie los puede superar, ¿no? Son unos muy buenos precios. ¿Sí entiende ahora?
17. J: Sí.

On A1 Juan does not translate "Unbeatable prices" properly.

Juan A4,14-16

14. F: Qué significa, a ver... [reading] "goes up three floors". ¿Qué significa "Goes up three floors"? [pause]
15. J: No, no sé.
16. F: Goes up es sube tres pisos.

On A4 Juan does not know the meaning of "goes up".

Juan A4,41-50

41. F: ¿Baja tres dice?
42. J: Sí. Dice [reading:] goes up three floors of [to!] the toy departament. Uno, dos... [points to 9th and 10th horizontal lines (within the 3 floors *going down*): A4,22].
43. F: Pero esto es cuando sube, ¿no? [points to the 3 floors *going up*: A4,22].
44. J: ¡Oh sí!
45. F: Goes up es sube.
46. J: Aquí llega [pointing to the 6th horizontal line, the *top* floor: A4,22] [pause] Y final... Sube otros diez [points to the 10 floors going up: A4,22].
47. F: ¿Sube?
48. J: Sí
49. F: ¿Dónde lo dice?
50. J: [Checks the wording] No, baja, baja diez para abajo. [Crosses out the 10 floors going up: A4,22]

After the previous interviewer's translation of "goes up" and "goes down", Juan interchanges its meaning.

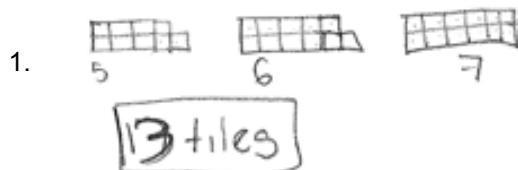
Juan GLQ,25-30

25. F: ¿Hay alguna palabra o frase que haya encontrado difícil en inglés?
26. J: No.
27. F: Bueno ya hemos comentado algunas aquí, ¿no? ¿Pero aparte de éstas?
28. J: No.
29. F: ¿Las otras las ha entendido bien?
30. J: Sí.

Juan says no word or sentence caused trouble to him because of the English language (26) but as he later admits (27) there are many English language problems (C3).

★ Juan A3,1:

How many tiles does figure 7 have? Why?



[Juan reviews this activity after reviewing A2: he changes the answer from 11 to 13 and counts again the tiles.]

2. [Comes from A2,28] F: ¿Me puede decir aquí también qué es lo que hizo?
3. J: Leí la pregunta y que ¿cuántos tendría la figura siete? En la figura uno uno, y para la dos aumentó dos, y para la tres dos y le aumenté dos en cada figura y me salían los trece.
4. F: Sí. Otra vez, el uso de las lenguas, ¿no? ¿Cómo lo hizo aquí?
5. J: El inglés lo usé para la pregunta y leer esto y el español para contestar.
6. F: Pero lo contestó en inglés aquí, ¿no?
7. J: Mm [validating]. ¡Sí!
8. F: ¿No lo contestó en español?
9. J: No.
10. F: ¿Cómo es que lo contestó en inglés?
11. J: Porque pregunta [reading] "How many tiles does figure seven have?" [No accurate pronunciation] Y tienes trece tiles.
12. F: ¿Y cómo es que utilizó aquí el inglés para la respuesta?
13. J: Porque salen trece y la palabra de lo que busca es esto [underlines the word "tiles"]. Es lo que buscan y es lo que sale.

Juan writes the answer in English (1) but he says the answer is in Spanish (5). Probably it is easier for him to reproduce the statement word "tiles" than translating it, which he never does (he makes a code mix –11–, he refers to the wording –13– or uses demonstratives –13–) (C2).

★ Juan A4,25-33:

25. F: Okay. Aquí me dijo... mmm... Perdona, antes de que comentemos cómo lo ha hecho esta vez, ¿sí? Antes puso la cruz solo inglés. ¿Sí? ¿Utilizó solamente el inglés?
26. J: En ésta [1st try], pero en ésta no [2nd try].
27. F: ¿Y antes que me puso sólo inglés, no utilizó el español para nada?
28. J: No
29. F: ¿Usted estaba pensando todo el rato en inglés? Aún para dibujar, pues esto [1st try: A4,1], pensó: with the floor... y todo esto.
30. J: Sí.
31. F: ¿Sí?
32. J: Sí
33. F: Todo en inglés.

Juan GLQ,1-8:

1. [Comes from A4,142] F: ¿En general, resolviendo estos cuatro problemas, cuándo ha usado el inglés? [Pause] Mientras resolvía los problemas.
2. J: En las preguntas.
3. F: ¿Para qué en las preguntas?
4. J: Para leerlas.
5. F: Ajá.. ¿Dónde más?
6. J: A veces en las respuestas.
7. F: Mm [continuing conversation]. ¿Y dónde más?
8. J: Nada más.

Juan says he uses English as a thinking language (A4,25-33). But later he does not refer to the use of English as a thinking language on A4 (GLQ,1-8) (C1).

★ Angel A3,10-40:

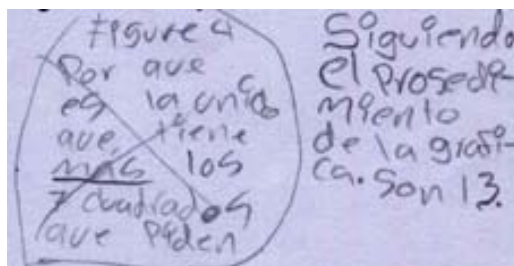
10. F: Okay. Sí. ¿Cómo usó los lenguajes para pensar todo eso? O sea, ¿empezó en qué

- idioma?
11. A: En inglés.
 12. F: Sí. ¿Y luego cuándo cambió a español?
 13. A: Cuando lo puse a... las... Cuando contesté la pregunta.
 14. F: Sí. ¿Al escribirlo?
 15. A: Sí.
 16. F: Pero la estuvo pensando...
 17. A: En inglés.
 18. F: En inglés. ¿Qué pensó en inglés?
 19. A: Que si en la figura cuatro había sido siete, en la cinco va a ser nueve...
 20. F: ¿Esto lo pensó en inglés?
 21. A: Sí.
 22. F: ¿Todo en inglés?
 23. A: Sí, casi todo.
 24. F: ¿Qué cositas pensó en español? Que si dice casi todo, algo pensó en español entonces, ¿no? ¿Qué pensó en español?
 25. A: La [figura] número siete, si estaba bien.
 26. F: ¿La última?
 27. A: Sí.
 28. F: ¿La última la pensó en español?
 29. [Angel nods.]
 30. ¿Y qué más en español?
 31. A: Lo de abajo, la contesta[ción]...
 32. F: La respuesta y este número. Lo demás lo pensó en inglés. El... como... Eh... Saber que se tenía que aumentar de esta a esta dos o de una a la siguiente dos, ¿esto lo pensó en inglés?
 33. A: En español.
 34. F: Y escribirlo, ¿lo escribió?
 35. A: En inglés.
 36. F: Lo pensó en inglés para escribir esto.
 37. A: Sí.
 38. F: Pero pensarlo, como... obse... mirar aquí, saber que se tenía que añadir dos, eso lo pensó...
 39. A: En español.
 40. F: En español. Okay. [Continues in A4, 5]
- Angel initially states that A3 is thought mainly through English (16-23) but later he gives more importance to Spanish during the thinking process (32-33, 38-40). There is even a confusion about the writing language (35) (C2).

★ Abel A3,29-41:

29. A: Y así le fui incluyendo aquí dos, dos, más dos y así como llegué a la figura siete y me dio trece.
30. F: Ajá. Okay. ¿Lo puede escribir?
31. A: ¿Esto está mal?
32. F: ¿Cuántas tiene? Sí, esto es lo que había, habíamos entendido la, la... la respuesta [lapse: pregunta!] mal, ¿no? Puede ponder nada más una línea así y ya está. [Abel circles and crosses out the answer] Si me puede poner su respuesta y el porqué.
33. [Abel starts to write the answer]
34. F: Mm [validating]. ¿Y cuántas va a tener? No me lo escribió.
35. [Abel Counts again the tiles]
36. F: ¿Qué me dijo antes? ¿Cómo lo encontró?
37. A: Sumándole dos más.
38. F: ¿Entonces aquí cuántas hay? [Pause]
39. A: Trece.
40. F: Ajá. Sí.

41.



[A3. Entire answer.]

Abel finds the right answer to the problem (29) after working with it with the support of the interviewer, but he is resistant (31) to believe that his previous answer is not right (see both of them in 41) (C4).

★ Abel, A4,8-25:

8. F: ¿Y qué pensó en inglés?
9. A: ¿En inglés?
10. F: Mm [validating].
11. A: Nada.
12. F: ¿Sólo al leerlo?
13. A: Mm [validating].
14. F: Luego, por ejemplo, a la hora de sumar...
15. A: Le... cuándo estaba sumando, para estar bien...
16. F: Mm. [continuing conversation]
17. A: Para saber si está, si iba a estar bien le di otra pasada.
18. F: ¿Con qué idioma?
19. A: En inglés, así. Y después...
20. F: ¿Sumó one, plus three, plus ten, plus one?
21. A: Sí
22. F: ¿In English?
23. A: Sí, nomás uno más three plus ten plus one y me dió quince, fifteen.
24. F: Pero ¿al pensar eso lo pensó en inglés o en español?
25. A: Ajá... sí... a veces que plus y lo pensé en inglés y ya los números, sumándolo lo pensé en español.

Abel GLQ, 7-30:

7. F: Entre todo, en general, ¿no?, al resolver éstos, uno, dos, tres y cuatro ejercicios, ¿cuándo ha usado el inglés? ¿Para qué lo ha usado el inglés?
8. A: Para leer y entender más o menos lo que sé.
9. F: ¿En qué más?
10. A: Pues nomás yo creo que para eso.
11. F: ¿Y por qué cree que sólo ha utilizado el inglés para eso?
12. A: Es que por eso están en inglés y yo me esforcé pues en leerlo en inglés.
13. F: Sí.
14. A: Pues nomás para eso y entender.
15. F: Okay. ¿Y en general cuándo ha usado español?
16. A: ¡Oh!, en escribir
17. F: Mm [continuing conversation].
18. A: Y en las traducciones del inglés.
19. F: Mm [continuing conversation].
20. A: Para eso, creo que para eso.
21. F: ¿Y para pensar cómo iba a poner la respuesta?
22. A: ¡Oh, sí!
23. F: ¿Cómo lo ha pensado?
24. A: En español.
25. F: ¿Y por qué cree que ha utilizado el español en estos casos?
26. A: Porque yo creo que así es más, para mí es más fácil.
27. F: Ajá. ¿Le resulta más fácil?
28. A: Sí, que en inglés.

29. F: ¿Alguna otra razón?

30. A: No.

Abel uses English on A4 with additions (A4,8-25). He does not refer to the use of English in additions (GLQ,7-10) (C1).

★ Julia A3,43-50:

43. F: ¿Alguna otra cosita pensó en inglés?

44. J: No.

45. F: ¿No? ¿Todo en español?

46. J: [Julia says no with her head.] Figura.

47. F: ¿Figure? ¿Cuándo lo estaba resolviendo estaba pensando en figure?

48. J: Sí.

49. F: Okay. ¿Alguna otra cosita se acuerda?

50. J: No. [Continues in A4, 3]

English use is not quickly recalled by Julia after solving A3, and instances of English use come up *randomly* (C3).

★ Ingrid, A2,35-58:

35. F: Ajá. Okay. Muy bien. ¿Qué más en inglés?

36. I: Eh... [pause] Nada más.

37. F: ¿La fórmula, por ejemplo?

38. I: Mm [continuing conversation]

39. F: ¿En qué idioma?

40. I: En inglés.

41. F: ¿Y el...? ¿Cómo se lee esto en inglés? [Pause] ¿Cómo lo pensó en inglés?

42. I: [Pause] Em...

43. F: ¿O lo pensó en español?

44. I: Ajá.

45. F: ¿En qué idioma lo pensó?

46. I: En español.

47. F: ¿Sí? ¿Inglés o español?

48. I: En español.

49. F: ¿Los va manejando los dos, un poco?

50. I: Mm [validating, answering before the question is finished, after 'dos']

51. F: ¿Y no se acuerda exactamente alguna cosita más que pensó en español, digo en inglés?

52. I: Porqué.

53. F: ¿El porqué?

54. I: Mm [validating]

55. F: ¿Razonó en, en...? ¿O sea a la hora de empezar a pensar, lo pensó en inglés o sólo leyó el why...? ¿Cómo, cómo pensó el porqué? ¿Por qué dice que el porqué lo pensó en inglés?

56. I: Porque... [pause]

57. F: ¿No me lo puede decir?

58. I: No.

Ingrid says she uses English with formulas on A2, but then she changes this affirmation and in the end she does not clearly state which language is used and why (C2).

★ Ingrid A3,69-74:

69. F: ¿Y cuándo cambiaste a inglés?

70. I: Cuando tenía que sumarle.

71. F: Ajá. ¿Sumaste, hiciste la suma en inglés?

72. I: Mm [validating].

73. F: ¿Qué más hiciste en inglés?

74. I: Namás eso.

Ingrid, GLQ,11-12:

11. F: ¿Y en español? ¿En general cuándo ha usado el español?
12. I: Cuando tenía que hacer las operaciones.

Ingrid GLQ,25-28:

20. I: [Interrupting] Las, [pause] las fórmulas.
21. F: ¿En español también?
22. I: Mm [validating].
23. F: ¿Por qué?
24. I: Porque en inglés todavía no las he aprendido.
25. F: Okay. Más cosas en español.
26. I: Las sumas.
27. F: ¿Las sumas también en español?
28. I: Mm [validating].

Ingrid says she uses English with additions (A3,69-72) and also on A2 (where she does not explicitly refer to it, but English has a minor importance). She does not say she uses English with additions on A3 but refers to the opposite in the GLQ (11-12, 25-28) (C1). Ingrid also says she uses English with the formula (even if she is not able to say it properly in English, see the example above) and then refers to the use of Spanish with the formulas (GLQ,20-22) (C1).

★ Ingrid A3,77-84:

77. F: ¿Aprendiste a sumar en inglés [pause] en la escuela?
78. I: Sí.
79. F: ¿En Mexico estuviste aprendiendo a sumar en inglés?
80. I: No.
81. F: ¿Pero cuando viniste aquí te enseñaron inglés?
82. I: Mm [validating].
83. F: Ajá. ¿Pero de pequeñita aprendiste en español?
84. I: Sí. [Continues in A4, 2]

The initial affirmation that Ingrid learned to add in English (77-78) is contradicted when asked again (79-80). Ingrid confirms she learned to add in Spanish when she was in Mexico (83-84) (C3). Her answers are based on monosyllables.

★ Yael, A1,21-38:

21. F: Ajá. Entonces aquí me puso la cruz. Inglés y español antes de escribir nada.
22. Y: Sí.
23. F: ¿Sí? ¿Se acuerda? ¿Por qué?
24. Y: Sí. Porque viene la respues..., la pregunta en inglés y yo la tengo que traducir a español.
25. F: Okay.
26. Y: Pero la contesté en inglés.
27. F: Okay.
28. Y: Pensando en los dos idiomas [marking two with the fingers].
29. F: ¿Me puede decir ahora en qué lengua empezó a pensar?
30. Y: En español.
31. F: En español. ¿Qué pensó en español?
32. Y: Todo.
33. F: ¿Todo? ¿Pero no me dijo que pensó en los dos idiomas?
34. Y: Sí. Pensé en español la respuesta y como las gráficas están aquí [statement's visual mode] pues éstas las tenía en mi mente así en inglés, así como vienen aquí. Y ya nada más en español como le complementé para poder saber cual era el resultado.
35. F: ¿Y para que mas utilizó el inglés?
36. Y: Para traducir mi respuesta.

37. F: Ajá. ¿La escribió en inglés, luego, la respuesta?

38. Y: Mm [validating].

Yael says she uses exclusively Spanish during the entire solving process (32), but when asked again she says she uses English to interpret the visual mode of the statement (34) and to translate the written answer (36) (C3).

★ Aida A2,3:

3. A: Es porque eh... [pause] dice que cuál tiene el perímetro más grande y éste nomás tiene un, como es redondo, nomás es como un lado, porque solamente lo haces. Y el cuadrado tiene cuatro lados. Y puede ser más grande. Este cuadrado es más grande que el círculo.

Aida A2,50-52:

50. F: El perímetro no son lados rectos. ¿Qué es.. qué perímetro es más largo, aquél o el del cuadrado éste [the one in A2 ?] ?

51. A: ¿El cuadrado?

52. F: ¿Éste tiene un perímetro más largo que aquél?

Aida A2,100-104:

122. F: ¿Cómo se llama esto en español?

123. A: Cuadrado.

124. F: Mm [validating]. ¿Utilizas también el español para referirte a esto? ¿O no? ¿No le llamas nunca cuadrado? ¿No utilizas la palabra cuadrado?

125. A: No.

126. F: No. Okay. ¿Y volviste a releer el enunciado? ¿Mientras lo estabas resolviendo, volviste otra vez al enunciado?

Aida says she never uses the word 'cuadrado' in Spanish, but she knows it (101) and in fact she used it before (3, 51) (C2).

★ Aida A2,67-73:

67. A: Leí el enunciado en inglés. Luego las figuras las dije en inglés, también.

68. F: ¿Las pensaste? ¿El nombre, por ejemplo? ¿Qué nombre le diste a esta figura?

69. A: Square.

70. F: Square? And this is...

71. A: Ssss... [C as in circle] Esa la pensé en español.

72. F: En español. ¿Y por qué ésta en inglés?

Aida initially affirms both figures are given English names (67). After the interviewer's demand (68, 70) she says 'square' in English but after a moment of hesitation, she changes to the Spanish denomination for circle (círculo) during A3's thinking process (she starts to say it in English –71–) (C3).

★ Aida A3,6-11:

6. F: ¿Y cuándo has utilizado el inglés? ¿Cuándo has cambiado a inglés?

7. A: Cuando escribí la respuesta.

8. F: Ajá.

9. A: Cuando la escribí.

10. F: ¿Sólo para escribir la respuesta?

11. A: Sí.

Aida A3,31-35:

31. [Comes from A4,107] F: Por ejemplo, aquí, en el problema de antes, la actividad tres.

¿Cuándo estabas haciendo la tabla y escribiendo los números, qué pensabas cero, uno, dos, tres o zero, one, two, three?

32. A: Cero, uno...

33. F: ¿Uno, dos, tres? ¿Y equis-ye o exs-why?

34. A: Exs-why.

35. F: Exs-why pero cero, uno, dos, tres. ¡Oh! [Continues in A1,4]

Aida uses English for the x-y table (33-34) but she did not mention it before (6-11) (C2).

★ Aida A4,108-115:

108. [Comes from A3,39] F: ¿En la cuatro no?

109. A: No, nno.

110. F: ¿No volviste a leer el enunciado?

111. A: Lo leí tres veces.

112. F: Ajá. ¿Y por qué me decías que no? [Pause] ¿No? Te pregunté: “¿Te pasó esto de leer el enunciado otra vez y volver a utilizar el inglés otra vez en alguna de estas otras actividades?”. Y me dijiste: “Sí, en la actividad tres sí me pasó”. Pero en la cuatro me dijiste que no. Y luego me dijiste: “Sí, lo leí tres veces”. ¿Por qué la primera vez me dijiste que no? [Pause] ¿Aida?

113. A: No sé.

114. F: ¿No sabe? ¿Lo pensaste rápido o por qué?

115. A: Ajá. [Continues in A1,32]

Aida initially says she did not read the wording during the solving process (109) but after the interviewer asks it again she says she did read it (111) (C3).

★ Claudio A4,28-32:

28. F: Mm [understanding]. ¿Pero no será el mismo?

29. C: No, porque dice middle, en medio, luego dice [,] arriba, large. ['arriba' as a place in the wording where “large” is situated]

30. F: ¿Luego dice arriba dónde? ['arriba' understood in a literal way, as if “above”, “top” or a similar word was in the wording]

31. C: Arriba, aquí dice. Aquí dice large, luego aquí dice en medio. So ya está arriba.

32. F: Pero, dice que Jamie está, está comprando en un gran departamento comercial con, con diferentes pisos, ¿no?

As Claudio does not make a long enough pause before 'arriba' (29), the interviewer understands that an English translation of the word 'arriba' is present on the wording (30). Then Claudio states that he referred to the word “large” (31) (C4, misunderstanding between the interviewer and the interviewee).

★ Damian, A2,1:

1. D: ¿Qué es un perimetre? Perimetre, [both 'perimetre' in Spanish pronunciation] o lo que sea que se dice esta palabra en español. Perimeter [English pronunciation].

Damian, GLQ,17-22:

17. F: [...] ¿Alguna palabra o frase que hayas encontrado difícil en inglés?

18. D: Nomás las que estoy aprendiendo en inglés.

19. F: Pero aquí, digo.

20. D: Ah, aquí, no.

21. F: ¿No? En estas actividades estamos hablando, ¿no? ¿Lo has entendido todo?

22. D: Sí.

Damian does not mention the unknown word “perímetro” (GLQ,17-22). Maybe because he recognizes the word on A2 but he is not able to recall its meaning (A2,1) (C1).

4.2.2 Languages as a transparent resource in the learning of mathematics

In a few cases, some students show that they use both languages for the same task: counting, writing the answer... In some instances it seems more spontaneous (as it is the case of Yolanda or Miriam). In other cases the shift from one language to the other may be prompted by a cognitive demand: this is the case of Juan, who initially writes down all the answers in English (even if some of them are not regarded in this way by Juan, likely because they are short answers and some key words are taken directly from the wording) but shifts to Spanish (his first language) when he has to give a reasoned answer after the interviewer asks for it.

Even if the interviewer insists on asking what may have prompted the use of languages and when language switches have occurred, it seems that most of the students do not immediately respond to this demand and they do not always give the reasons of why they use one or the other language. So it seems that its use is rather spontaneous. What seems clear is that the alternative use of both languages does not seem problematic when advancing towards the construction of the mathematical solving process.

Making a code switch to demand the meaning of a work using the second language to refer to the unknown word (E.g. '¿Qué significa greater?', Julián A2,4), as well as making a code switch on writing when borrowing English words from the wording (E.g. "[...] yo escogería John Sports.", Ana A1,1), are also examples of using the language as a transparent resource. These are not included here but on the appropriate themes (see the list at the beginning of the section).

Now many extracts from the dialogues are presented and commented to exemplify the use of languages as a transparent resource in the learning of mathematics.

★ Yolanda A2,60-67:

- 60. F: And here you have used also only English? [As marked with a cross on the "English only" column]
- 61. Y: Do you mean like... the way I think?
- 62. F: Yeah.
- 63. Y: On these two [A1 and A2] I only thought English. On this one [A4] I thought Spanish.
- 64. F: You only thought English here?
- 65. Y: Yeah.
- 66. F: You only thought... all the activity, all the process of solving... only in English.
- 67. Y: Yeah. In this one [A4] I thought Spanish and English.

Yolanda GLQ,7-40:

- 7. F: Okay. And in these four activities, in general, when have you used Spanish?
- 8. Y: In this one [A4].
- 9. F: But why?

10. Y: And I remember in this one too [A2].
11. F: You have used also English in this one [A2] you said? I mean Spanish.
12. Y: I cant' remember.
13. F: You don't remember. So this [A4] is the only one, but maybe a little bit here [A2].
14. Y: Yeah.
15. F: But why do you think you have used Spanish?
16. Y: Maybe because I am used to, like... on what I know, I think, I use English ... on what I know in Spanish or when it makes it difficult.
17. F: When it becomes more difficult you use Spanish?
18. Y: Yeah.
19. F: You feel more comfortable with Spanish. And when have you used English, here, in general?
20. Y: Right here...
21. F: When it was easier...?
22. Y: Yeah, like this one it was easier [A3], this one is fast .This one too [A1].
23. F: When, when more?
24. Y: When I used English in these ones?
25. F: Aha.
26. Y: Just in this two [A1 and A3]. And I think more in this one [A2]. I used less Spanish in this one [A2].
27. F: And why do you think you did it?
28. Y: Maybe because it makes more difficult in things that I understand in English.
29. F: Is there any word, are there any words or phrases that you found difficult in English?
30. Y: This one [A4] got me confused. That's why I start using Spanish. And this one [A2] because I forgot what perimeter was.
31. F: But, if it had been *perímetro* in Spanish, had you understood it better?
32. Y: Yeah, because like *perímetro*, that makes me think on *perímetro*, so that's what I used to know what it was.
33. F: And what do you mean?
34. Y: Like it is under[?] in Spanish so that's how I know what it was.
35. F: This is why you knew that it was *perímetro*?
36. Y: Yeah.
37. F: But you where confused with area at the beginning.
38. Y: Yeah, because I confused area with perimeter, pero, I know what perimeter is.
39. F: But it was not because of the English?
40. Y: No.

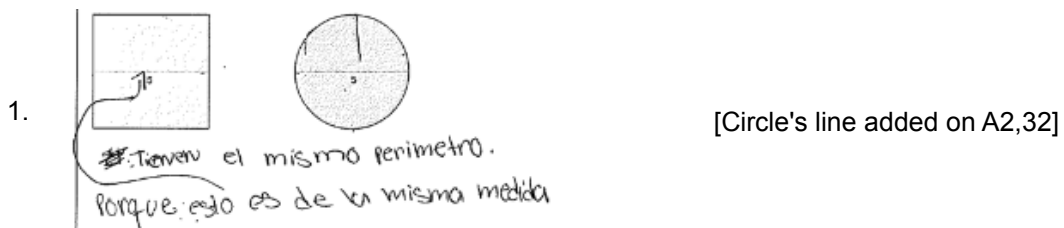
Yolanda says that she does not use Spanish on A2 (60-67). But she says on GLQ that she uses Spanish on A2 (see mainly GLQ,10-14, 26, 30-36), as it helps with the notion of perimeter. The use of Spanish is not perceived as problematic when talking about A2.

★ Yolanda A4,10-21:

10. F: And you counted in Spanish?
11. Y: Yeah.
12. F: Have you counted in Spanish here [A4]? But not in the previous activity, for example.
13. Y: No, this one I did it...
14. F: You counted here [A3] in English but here [A4] you have counted...
15. Y: Because like... I was confused because they went up and down, up and down...
16. F: Aha. And using Spanish was easier for you in that activity?
17. Y: Mm [agreeing]. Because I just like don't think, and I just mix them, so...
18. F: What do you mean?
19. Y: In this one [A4] I just like started counting and then I said some in Spanish and then I... in English again...
20. F: And do you know why, when you have switched, when have you decided to switch to Spanish? Do you remember that?
21. Y: No, I just keep counting. [Continues in GLQ]

Yolanda uses both languages in a flexible way; when she counts she does it in both languages (10-15). She does not force to switch languages voluntarily (17, 19, 21). She is in a transitional class, but she chooses to speak English during the interview. This suggests that she has a good management of English, at least at a practical level. She may switch to Spanish in A4 because of the difficulties in the wording, as the mathematization seems not to be easy. But when saying “I just like don’t think, and I just mix them, so...” (17) she points out how she benefits from the use of both languages.

★ Miriam A2,1-2:



2. [Comes from A1,38] M: [reading] Which of these figures has a greater perimeter and why. Empecé con el inglés y luego en español cuando me fijé en esto [points to the dotted lines]. Y luego al poner la respuesta esta parte ['Tienen el mismo perimetro porque', A2,1] la puse, la quise poner en inglés y luego como que esto ['esto es de la misma medida', A2,1] lo pensé así en español.

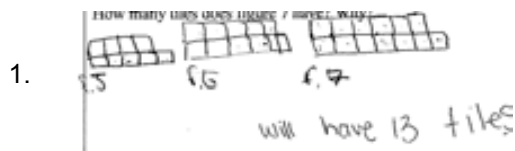
Miriam A2,16-30:

16. M: [Interrupting] So pensé que todo esto era igual así [makes 2 segments in the square perpendicular to the dotted line; inscribes a circle in the square], so iban a tener el mismo [follows 3 square sides]. Y aquí ['Tienen el mismo perimetro porque', A2,1] puse, lo puse en español y luego aquí ['esto es de la misma medida', A2,1] empecé a pensar en inglés. [Opposite to what she said in A2,2 !]
17. F: ¿Y cuándo... ? Empezaste a leer. Bueno, leíste en inglés, ¿no?
18. M: Mm [Affirming].
19. F: ¿Y luego cuándo cambiaste?, dices, perdona.
20. M: Cambié para cuando éstos [dotted lines] eran los mismos. So pensé que todo era igual [follows the perimeter of the circle] porque estos dos son iguales de [inscribes a circle in the square, as following the perimeter of the square], del perímetro porque de aquí [dotted lines] son de la misma longitud.
21. F: Ajá. ¿Y lo pensaste esto en español o en inglés?
22. M: En español. Y luego puse esta parte de aquí a acá ['Tienen el mismo perimetro porque', A2,1] en español y esto ['esto es de la misma medida', A2,1] lo estaba pensando en inglés y lo puse en español. [Opposite to what she said in A2,2 !, same way as what she says in A2,16]
23. F: ¿Por qué pensaste esto en inglés?
24. M: No sé.
25. F: Okay. ¿El cinco [pointing to the '5' in one of the figures], por ejemplo, cómo lo leíste?
26. M: Así en inglés.
27. F: Five?
28. M: Mm [Affirming]. Five.
29. F: Okay. ¿Algún otro cambio que te acuerdes?
30. M: N n [negating]. Eso fue todo.

Miriam says she thinks of a part of the answer in English and of another part of it in Spanish (22); even if there is contradictory information about which part is in English

and which part in Spanish (2, 22), accountability can be given to the use of both languages. She says that she does not know why this happens (24).

★ Miriam A3,1-20:



[Miriam counts the tiles by pointing to them with the pen. She also counts the tiles in Figure 7 one by one before writing the answer]

1. [Comes from A2,34] M: Este, eh... [Reading] Observe this pattern y todo esto lo hice en inglés.
 2. F: ¿Todo en inglés?
 3. M: Mm [Affirming].
 4. F: ¿Sólo inglés?
 5. M: Sí.
 6. F: ¿Y nunca cambiaste a español para nada más?
 7. M: No. Nomás estaba contando los cuadritos en español, pero todo lo demás lo hice en inglés.
 8. F: ¿Y nunca cambiaste a español para nada más?
 9. M: Mm... No, nomás, em, empecé a leer todo en inglés y luego empecé a hacer las figuras pensando en español y luego volví a inglés acá [points to the answer].
 10. F: ¿Y por qué aquí [A2] escribiste la respuesta en español y aquí [A1] también?
 11. M: I don't know.
 12. F: Y aquí [A3] en inglés. ¿No sabes?
 13. M: No.
 14. F: Empezaste aquí [A3] todos los cambios en inglés, todos los pasos del problema en inglés.
 15. M: Lo leí en inglés y luego todos esos [points to the figures she drew] los hice pensando en español.
 16. F: ¿Al contarlos sólo?
 17. M: Ajá. Al contarlos nada más, me fijé que todos llevaban dos más. Como éste, éste ya estaba, y dos más [points to the 2 tiles added to Figures 3 and 4 respect to the previous figures]. Y por cada dos nomás agregaba dos más contando en español.
 18. F: Okay.
 19. M: Y luego ya al momento de fijarme, conté todo, cuantos tiles había aquí [figure 7] y puse la respuesta en inglés, pero lo había contado en inglés, en inglés. [Continues in A4,2]
- Miriam uses English as a unique writing language (1,10) and as a thinking language (2-8, 16). Spanish is used as counting language (8, 10, 16-18) so it is also part of the thinking process (7-10, 16-18). The code mix on 'tiles' (20) along with the code switches (2, 12) form altogether a nice picture of the harmony of both languages constructing the mathematical activity.

★ Miriam A4,7:

7. M: So hice namás ten floors [9!]. Un cuadro con diez pisos así [9!] y ésto no estaba [hides the 3 bottom floors (which are added later: see A4,1)]. Entonces puse que el five [pointing to it in the picture], porque dice que el middle floor [pointing to the wording]. Le puse el cinco [pointing to the middle floor in her drawing]. Y luego dice aquí que "she goes up one floor" [points to it in the wording] le puse uno para arriba. Y luego que baja uno, entonces vuelve a quedar a medias. Y luego que sube tres. Y luego que baja diez pisos. Al momento que baja para diez pisos necesitaba agregar tres más. [While explaining Jamie's movements, follows the arrows on her drawing] Así que los agregué y saqué la conclusión de que son trece pisos de...

Miriam A4,24-29:

24. F: Bueno, si acaso luego volvemos sobre cuál es la respuesta correcta. A ver, ¿lo primero

que has hecho, qué es? ¿Cómo lo has pensado? ¿En qué momento has cambiado de lengua?

25. M: En inglés y namás cambié para poner la, la... mm... [points to the answer] Bueno, todo lo hice en inglés. Lo único que sí puse, fue cuando puse los números, que estaba en español.
 26. F: Okay... ¿Y cuándo más en español?
 27. M: Nada más eso.
 28. F: Y todo lo otro, ¿todo en inglés?
 29. M: Yeah.

Miriam says English is the dominant language used on A4 and the use of Spanish with numbers is rather in the background (25), but there are not language related difficulties. The dialogue is in Spanish and Miriam explains the solution through this language (7) even if she has been thinking in English.

★ Camilo A2,60-65:

60. F: ¿Ninguna cosa en inglés?
 61. [Camilo shakes his head saying no.]
 62. F: ¿Seguro?
 63. C: Sí.
 64. F: Porque me dijo por ejemplo the square.
 65. C: ¡Oh! Bueno... ¿También eso? ¿También eso incluye? Pues sí, lo pensé en inglés también. Por ejemplo the square, the circle o así.

Camilo has a rather general view of the language use, without considering code mixes (65). So his use of both languages is not declared as problematic.

★ Camilo A4,35-39:

40. C: Tal vez ahí cuidan niños, o no sé. Ya son tres. Luego ella va... sube tres pisos más. Luego son seis [marks 6 with the fingers, using both hands], tres pisos más ya. Y ya después finalmente baja diez pisos... ¡Oh sí, pero ahí debería de restarlos! Después diez pisos y ahí los cuento y ya son dieciséis pisos.
 41. F: ¿Entonces que dijo que tendría que hacer, restarlos?
 42. C: Sí, restarlos.
 43. F: ¿Lo arreglamos? ¿Arreglamos la respuesta?
 44. C: Entonces son... four.

Camilo A4,68-71:

68. F: ¿Y que pensó en español?
 69. C: Namás la respuesta, la respuesta nada más. Namás la escritura que dice aquí [written answer]. No pensé nada en español. Lo único fue (lo que pensé...) la respuesta.
 70. F: ¿Y a la hora de sumar los números, por ejemplo?
 71. C: ¡Ah! Ahí sí, ahí sí lo sumé en español.

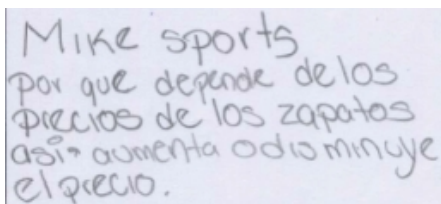
When counting, Camilo says he adds the numbers in Spanish (70-71) but probably English has some influence too because Camilo says 'four' when giving the answer (39), which is not consciously reflected by Camilo.

★ Juan A1,1:

1.  [Juan reviews the problem after reviewing A4 but makes no changes]

Juan A1,27-33:

27. [Entire answer]



28. F: ¿En qué lengua... con qué lengua empezó a resolver el problema?

29. J: En inglés y en español.

30. F: ¿Empezó con las dos?

31. J: Sí.

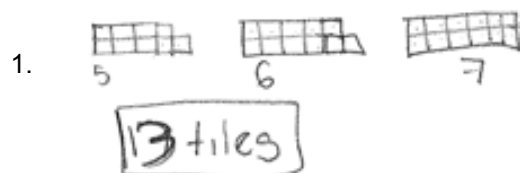
32. F: ¿Para qué utilizó una y para qué la otra?

33. J: El inglés lo utilicé para leer y entender esto y el español para escribir.

It looks like the answer is initially written in English (1) but the justification is in Spanish (27) and Juan says Spanish is used for writing (33). So the use of English is combined with the use of Spanish.

★ Juan A3,1:

How many tiles does figure 7 have? Why?



[Juan reviews this activity after reviewing A2: he changes the answer from 11 to 13 and counts again the tiles.]

2. [Comes from A2,28] F: ¿Me puede decir aquí también qué es lo que hizo?

3. J: Leí la pregunta y que ¿cuántos tendría la figura siete? En la figura uno uno, y para la dos aumentó dos, y para la tres dos y le aumenté dos en cada figura y me salían los trece.

4. F: Sí. Otra vez, el uso de las lenguas, ¿no? ¿Cómo lo hizo aquí?

5. J: El inglés lo usé para la pregunta y leer esto y el español para contestar.

6. F: Pero lo contestó en inglés aquí, ¿no?

7. J: Mm [validating]. ¡Sí!

8. F: ¿No lo contestó en español?

9. J: No.

10. F: ¿Cómo es que lo contestó en inglés?

11. J: Porque pregunta [reading] "How many tiles does figure seven have?" [No accurate pronunciation] Y tienes trece tiles.

12. F: ¿Y cómo es que utilizó aquí el inglés para la respuesta?

13. J: Porque salen trece y la palabra de lo que busca es esto [underlines the word "tiles"]. Es lo que buscan y es lo que sale.

Juan writes the answer in English (1) but he says the answer is in Spanish (5). Probably it is easier for him to imitate the statement word "tiles" than translating it, which he never does (he makes a code mix –11–, refers to the wording –13– or uses demonstratives –13–). So maybe he thinks in Spanish but naturally incorporates the English word "tiles" on the discourse.

★ Juan A4,127-142:

127. F: Entonces aquí ahora me dijo que ha usado el inglés y el español, ¿no?, esta segunda vez.

128. J: Sí.

129. F: ¿Dónde ha usado el inglés y dónde ha usado el español?

130. J: El inglés en lo de la pregunta y español en la respuesta.

131. F: Ajá. Pero mientras, mientras estaba pensando sube uno, baja tres... lo que sea, ¿qué estaba... con qué idioma estaba pensando todo esto?

132. J: Con el español.
 133. F: Con el español. ¿Todo el rato?
 134. J: Sí.
 135. F: ¿No hay ninguna cosa que haya pensado en inglés?
 136. J: No. Nada más cuánto subía y cuánto bajaba, era lo que pensaba.
 137. F: ¿En inglés?
 138. J: Ajá.
 139. F: ¿Goes up or goes down, eso pensó en inglés?
 140. J: Ajá.
 141. F: Y antes me ha dicho que lo ha pensado todo en...
 142. J: En inglés [looking tired]. [Continues in GLQ, 1]

Juan initially explains that he thinks exclusively in Spanish after the first try (131-134). Later he adds that he thinks of the number of floors that Jamie goes up or down (135-140) in English, as if the use of both languages was not interfering his mathematical process.

★ Juan, GLQ, 17-22:

17. F: ¿Entonces por qué cree que ha usado...? Bueno, para leerlo, porque, como el enunciado está en inglés hay que leerlo en inglés, ¿no? ¿Pero por qué lo ha usado también en algunas respuestas como me ha dicho?
 18. J: Porque sé algunas palabras y las traduzco al español para entenderlo más.
 19. F: Pero ha usado el inglés me dice, en algunas respuestas.
 20. J: Sí, en algunas respuestas.
 21. F: ¿Por qué?
 22. J: Porque se me hace más fácil en algunas respuestas.

In some cases it is easier for Juan to use English on the written answer. It may be a result of taking some words directly from the wording. Another influence may be that the questionnaire is being taken within the school environment (with an implicit demand of the use of English). In fact, Juan initially writes all the answers in English. After a justification is demanded, it is written in Spanish. This does not seem to be problematic from the point of view of the student, or at least Juan does not manifest such experience when talking.

★ Yael, A1, 21-38:

21. F: Ajá. Entonces aquí me puso la cruz. Inglés y español antes de escribir nada.
 22. Y: Sí.
 23. F: ¿Sí? ¿Se acuerda? ¿Por qué?
 24. Y: Sí. Porque viene la respues..., la pregunta en inglés y yo la tengo que traducir a español.
 25. F: Okay.
 26. Y: Pero la contesté en inglés.
 27. F: Okay.
 28. Y: Pensando en los dos idiomas [marking two with the fingers].
 29. F: ¿Me puede decir ahora en qué lengua empezó a pensar?
 30. Y: En español.
 31. F: En español. ¿Qué pensó en español?
 32. Y: Todo.
 33. F: ¿Todo? ¿Pero no me dijo que pensó en los dos idiomas?
 34. Y: Sí. Pensé en español la respuesta y como las gráficas están aquí [statement's visual mode] pues éstas las tenía en mi mente así en inglés, así como vienen aquí. Y ya nada más en español como le complementé para poder saber cual era el resultado.
 35. F: ¿Y para que más utilizó el inglés?

36. Y: Para traducir mi respuesta.
 37. F: Ajá. ¿La escribió en inglés, luego, la respuesta?
 38. Y: Mm [validating].

Yael says she uses exclusively Spanish during the entire solving process (32), as if English had not been interfering. It is used to interpret the visual mode of the statement (34) and to translate the written answer (36).

★ Angel A4,1:

Jamie is shopping in a large department store with many floors. She enters the store at the middle floor, and immediately goes to the credit department. After making sure her credit is good, she goes up one floor to the jewelry department. Then she goes down one floor to the children's department. Then she goes up three floors to the toy department. Finally Jamie goes down ten floors to the main entrance of the store, which is on the first floor, and leaves to go to another store down the street.

How many floors does the department store have? Why?

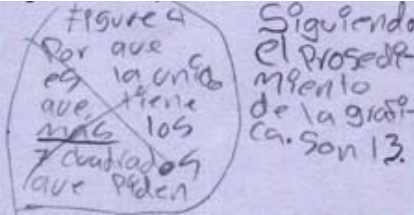
1. ~~6 Floors por que es la~~
~~cu el Floor 7~~

Angel A4,155-179:

155. F: Sí. Okay. ¿Cómo empezó para..., con qué lengua empezó a resolver el problema?
 156. A: Con el inglés, cuando estaba resolviendo las palabras que no sabía en inglés.
 157. F: Sí.
 158. A: Fue cuando pasaba a español.
 159. F: Sí.
 160. A: Y cuándo leía, leyéndolo en inglés para ver las palabras que me sabía.
 161. F: Sí.
 162. A: Después lo traducí a español.
 163. F: ¿Y qué hizo con las palabras que se sabía, dijo? Es que no lo entendí lo que me dijo.
 Con las palabras que se sabía...
 164. A: ¿En inglés?
 165. F: Sí.
 166. A: Lo hacía para traducirlo a español.
 167. F: Ajá. Okay.
 168. F: ¿Y cuál...? ¿Qué palabras subrayó por aquí?
 169. A: Como los pisos que bajó pabajo.
 170. F: Sí.
 171. A: Y los que iba parriba.
 172. F: Okay. Sí. ¿Y luego cómo continuó?
 173. A: Haciendo las operaciones.
 174. F: ¿Con qué idioma?
 175. A: Español.
 176. F: Ajá. ¿Y cuándo volvió a cambiar a inglés?
 177. A: Ya no.
 178. F: ¿Sólo utilizó el inglés para traducirlo?
 179. A: Sí. [Continues in GLQ,1]

Angel uses code mixing in writing (1: "Floors", "Floor") but he does not give any importance to it (155-179).

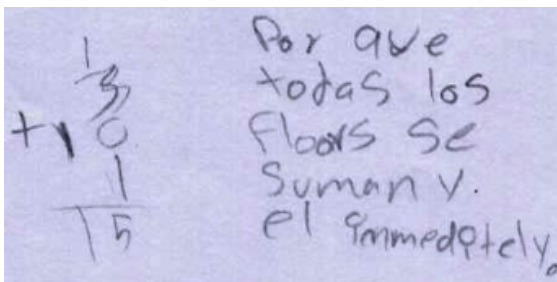
★ Abel A3,41-57:

41.  [A3, entire answer]

42. F: ¿Antes [1st try] con qué lengua empezó?
 43. A: Nada más empecé a usar el inglés para mi..., lo único que sé más o menos,
 44. F: Mm. [continuing conversation]
 45. A: Empecé a leer lo que entendía.
 46. F: Mm. [continuing conversation]
 47. A: Y nada más le puse... me fui por la lógica así de una figura que tiene siete lados.
 48. F: Ajá.
 49. A: Y nada más le puse la figura 4.
 50. F: Okay. ¿Y utilizó el inglés para leerlo?
 51. A: Mm. [validating] [Abel nods.]
 52. F: ¿Y luego para qué más utilizó el inglés?
 53. A: Nomás para eso.
 54. F: ¿Luego cambió a español?
 55. A: Sí.
 56. F: Para pensar cómo lo iba a hacer. ¿Y siguió en español?
 57. A: Sí, nomás seguí escribiendo en español.

Abel does not mention any use of English on his first approach to solve the activity (42-57) but he does use a code mix ("Figure") when writing his answer (41).

★ Abel A4,1:

1. 

Abel A4,8-25:

8. F: ¿Y qué pensó en inglés?
 9. A: ¿En inglés?
 10. F: Mm [validating].
 11. A: Nada.
 12. F: ¿Sólo al leerlo?
 13. A: Mm [validating].
 14. F: Luego, por ejemplo, a la hora de sumar...
 15. A: Le... cuándo estaba sumando, para estar bien...
 16. F: Mm. [continuing conversation]
 17. A: Para saber si está, si iba a estar bien le di otra pasada.
 18. F: ¿Con qué idioma?
 19. A: En inglés, así. Y después...
 20. F: ¿Sumó one, plus three, plus ten, plus one?
 21. A: Sí
 22. F: ¿In English?
 23. A: Sí, nomás uno más three plus ten plus one y me dió quince, fifteen.
 24. F: Pero ¿al pensar eso lo pensó en inglés o en español?
 25. A: Ajá... sí... a veces que plus y lo pensé en inglés y ya los números, sumándolo lo pensé en español.

Abel A4,138-145:

138. F: ¿Sí? [Pause] Okay. ¿Qué lenguas utilizó aquí?

139. A: El inglés y el español.

140. F: ¿El inglés para qué?

141. A: Para leer.

142. F: Ajá. ¿Y el español?

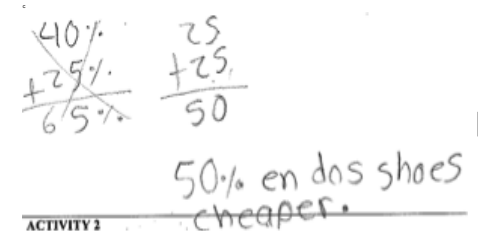
143. A: Para escribir y traducir lo que sé, antes.

144. F: ¿Y alguna otra cosa? Por ejemplo, ¿cómo lo pensó, en qué idioma lo pensó eso?

145. A: En español. [Continues in A2,9]

Abel does not take the initiative to present a use of his English language on the solving process of A4 (8-17). It is not until the interviewer asks it later that Abel admits that English has some influence during the process of addition (18-25). Later, after the meaning of the wording has been discussed, Abel does not makes visible either any use of the English language (138-145). Furthermore, the use of a code mix on writing (1) is never mentioned. So even if English does not play a central role on the solving process, its use is not perceived as problematic by Abel.

★ Julia A1,1:

1.  [0 try crossed out: 40% + 25% = 65%]

Julia A1,30-35:

30. F: ¿En qué idioma empezó a pensar el ejercicio? [Pause] ¿En inglés o en español?

31. J: En español.

32. F: Ajá. ¿Y cambió luego otra vez a inglés?

33. [Julia shakes her head saying no]

34. F: ¿Cuándo cambió a inglés? ¿No cambió o sí?

35. J: No.

Julia says she uses Spanish for everything (30-35) and does not mention the code mix made in the writing (1). She does not seem to give any importance to it.

★ Ingrid A2,5-64:

5. F: Sí. Okay. ¿Cómo lo pensó eso?

6. I: Pensando que éste [square] era cada lado.

7. F: ¿Y qué lengua, qué lenguas utilizó para pensar todo eso?

8. I: En español.

-
9. F: ¿Todo en español o alguna cosita en inglés?
10. I: Em... Todo en español.
11. F: ¿Todo?
12. I: Sí.
13. F: Okay. [Pause] ¿Y luego?
14. I: Nada más lo sumé y vi cuánto salía.
15. F: Ajá. Sí, lo del círculo, ¿no?, me refería. O si hay alguna otra cosita que tenga del cuadrado, pues sí me lo dice.
16. I: No. Con el círculo tenía que sacar el perímetro. Tenía que ser pi que es tres punto catorce multiplicarlo por el diámetro.
17. F: Mm [continuing conversation].
18. I: Y tenía que multiplicar tres punto catorce por cinco y lo que me salió...
19. F: Mm [continuing conversation]
20. I: Es eso.
21. F: ¿Y cómo lo pensó esto? ¿Qué idiomas utilizó?
22. I: El español.
23. F: ¿El español?
24. I: Mm [validating].
25. F: ¿Durante todo el proceso? U otra vez: si se acuerda de alguna cosita que pensó en inglés...
26. I: Como perímetro porque el maestro nos ha dicho que siempre tenemos que, que nos tenemos que aprender esas palabras.
27. F: ¿Perimeter?
28. I: Sí.
29. F: ¿Pensó en inglés?
30. I: Sí.
31. F: ¿Para el círculo?
32. I: Sí.
33. F: ¿Pero para el cuadrado no?
34. I: También, poquito.
35. F: Ajá. Okay. Muy bien. ¿Qué más en inglés?
36. I: Eh... [pause] Nada más.
37. F: ¿La fórmula, por ejemplo?
38. I: Mm [continuing conversation]
39. F: ¿En qué idioma?
40. I: En inglés.
41. F: ¿Y el...? ¿Cómo se lee esto en inglés? [Pause] ¿Cómo lo pensó en inglés?
42. I: [Pause] Em...
43. F: ¿O lo pensó en español?
44. I: Ajá.
45. F: ¿En qué idioma lo pensó?
46. I: En español.
47. F: ¿Sí? ¿Inglés o español?
48. I: En español.
49. F: ¿Los va manejando los dos, un poco?
50. I: Mm [validating, answering before the question is finished, after 'dos']
51. F: ¿Y no se acuerda exactamente alguna cosita más que pensó en español, digo en inglés?
52. I: Porqué.
53. F: ¿El porqué?
54. I: Mm [validating]
55. F: ¿Razonó en, en...? ¿O sea a la hora de empezar a pensar, lo pensó en inglés o sólo leyó el why...? ¿Cómo, cómo pensó el porqué? ¿Por qué dice que el porqué lo pensó en inglés?
56. I: Porque... [pause]
57. F: ¿No me lo puede decir?
58. I: No.
59. F: ¿Más o menos vamos manejando ambos idiomas?
60. I: Mm [validating].
-

61. F: ¿O más español o más inglés o cómo cree?
62. I: Un poquito más español, porque mucho inglés no sé.
63. F: Ajá. ¿Pero algo de inglés sí piensa mientras resuelve la...?
64. I: Mm [validating].

In the beginning Ingrid says she does not use English (5-12, 21-24) but after the insistence of the interviewer and maybe as a result of considering also minor uses of the English language, she admits that “perimeter” has some influence during her thinking process (26-34), because her teacher says that learning vocabulary in English is important (26). It seems that she has detailed all her uses of the English language in the problem, but then it looks like English has some influence on the thinking process. Anyway, these instances are not concreted (37-64).

★ Julia A2,59-70:

59. F: Okay. ¿Y eso lo pensó en español?
60. [Julia nods]
61. F: ¿Todo o pensó alguna cosita en inglés?
62. J: [Julia shakes her head saying no] En español.
63. F: ¿Puro español? ¿La fórmula también se la sabe en español?
64. J: No.
65. F: ¿La fórmula en inglés?
66. [Julia nods]
67. F: ¿Se la sabía antes cuándo estaba en Mexico en español? ¿O la aprendió aquí?
68. J: Me la sabía.
69. F: Ajá. ¿Pero la pensó en español ahora? Perdón. La pensó en inglés, me dijo.
70. J: Sí.

Julia initially says that she uses Spanish during the entire solving process (59-62). She does not give much importance to the use of English in relation to the formula used to calculate the circle's perimeter (63-66, 69-70), where the use of English does not seem to play a central role, as she already knew the formula in Spanish (67-68).

★ Julia A3,27-50:

27. F: Okay. [Pause. Interviewer explained to Julia that she had to think starting from the beginning, in the first attempt she made to solve the problem (part not transcribed).] ¿Con qué idioma empezó a pensar eso?
28. J: Español.
29. F: ¿En español?
30. J: Sí.
31. F: ¿Y luego cuándo cambió a inglés?
32. J: Nunca.
33. F: ¿Nunca?
34. [Julia says no with her head.]
35. F: ¿No se acuerda de que utilizó el inglés para nada?
36. J: No.
37. F: ¿Pero en la respuesta sí la hizo con inglés? [Pause]
38. J: Namás en esto [circles the word “tiles” in the wording: A3,16], puse.
39. F: Ajá. ¿Por qué?
40. J: Porque me pregunta cuántas tiles [tailes/] había en siete figuras. Y porqué.
41. F: ¿Y por qué me lo puso esto en inglés?
42. J: No sé.
43. F: ¿Alguna otra cosita pensó en inglés?
44. J: No.
45. F: ¿No? ¿Todo en español?
46. J: [Julia says no with her head.] Figura.

47. F: ¿Figure? ¿Cuándo lo estaba resolviendo estaba pensando en figure?

48. J: Sí.

49. F: Okay. ¿Alguna otra cosita se acuerda?

50. J: No. [Continues in A4, 3]

Julia initially says Spanish is the only language used to think about the activity (27-36).

Then the interviewer points out that the answer is written in English (37-40). And later she admits that she was thinking about 'figure' in English (45-48). So there is a main use of Spanish but the presence of the English language during the solving process is not seen as problematic by Julia.

★ Julia GLQ,1-20:

1. [Comes from A4,58] F: Sí. ¿Para qué más lo utilizó en general? Por ejemplo, usted dice yo siempre utilizo el inglés para esto. O si he utilizado el inglés cada vez que he encontrado eso.

2. J: Nada más.

3. F: ¿Nada más para eso?

4. [Julia nods.]

5. F: Bueno, aquí también ha encontrado una... alguna cosita. Pero bueno... ¿No se le ocurre nada más por lo que cree que haya utilizado el inglés?

6. J: Para..

7. F: [interrupting] ¿Por qué...? ¿Para?, perdón.

8. J: Para aprender inglés.

9. F: Ajá.

10. J: Para poderlo escribirlo.

11. F: Mm [validating].

12. J: Para saber lo que significa.

13. F: Sí. ¿Por eso utiliza el inglés para escribir?

14. J: Mm [validating].

15. F: ¿Qué más?

16. J: Nada más.

17. F: ¿Y en general cuando ha utilizado el español?

18. J: En todo.

19. F: ¿En todo, lo otro?

20. [Julia nods.]

After commenting the use of English on the mathematical activities (1-16) Julia refers to the use of the Spanish language for everything (18). She decides to ignore the relevance of the English language usage.



★ Yael,A1,29-34:

29. F: ¿Me puede decir ahora en qué lengua empezó a pensar?

30. Y: En español.

31. F: En español. ¿Qué pensó en español?

32. Y: Todo.

33. F: ¿Todo? ¿Pero no me dijo que pensó en los dos idiomas?

34. Y: Sí. Pensé en español la respuesta y como las gráficas están aquí [statement's visual mode] pues éstas las tenía en mi mente así en inglés, así como vienen aquí. Y ya nada más en español como le complementé para poder saber cuál era el resultado.

After saying that she uses both languages to think, Yael does not mention the use of English, as it did not had an important role to her.

★ Julián A2,33-44:

33. F: ¿Continuó en español?

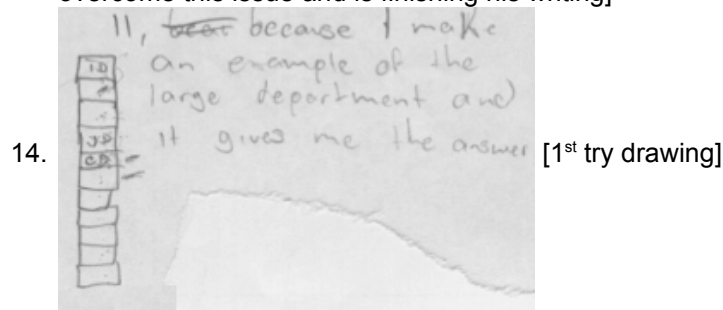
34. J: I only used English at the beginning, so that's why I use more Spanish.
35. F: ¿Para leer la pregunta sólo?
36. J: Sí.
37. F: ¿Pero no pensó ni en el círculo, circle, or square, o five or anything like that in English?
38. J: Sí: square, five and the circle.
39. F: ¿Eso sí lo pensó en inglés?
40. J: Sí. Porque recuerdo las clases, por eso.
41. F: ¿Se le hace más fácil esto en inglés?
42. J: Cuando recuerdo las cosas de la clase, sí.
43. F: ¿Se le hace más fácil hacerlo en inglés?
44. J: Porque el procedimiento, recordarlo en español algunas veces no lo recuerdo en español.

Julián does not mention any use of English during the solving process of A3 (33-36).

Then a specific question is asked and he shows that English is used to name and interpret the visual figurative mode (37-40), influenced by the English classes (40-44).

★ Julián A4,1-18:

1. [Comes from A2,16] J: Una pregunta.
2. F: Sí.
3. J: Aquí dice: [reading] Jamie is shopping in a large department store with many floors. She enters the store at the middle floor and immediately goes to the credit department. ¿Pero en qué piso está el credit department? ¿En el middle floor?
4. F: Sí.
5. J: Okay. Okay. Oh así está... [Draws 3 floors and writes CD in the middle one] [Pause] [Adds 3 floors on top of the drawing] [Pause]
6. J: Aquí me dice que [reading] immediately goes to the credit department. Está aquí en el middle [pointing to the CD floor on his picture], según yo. [Reading] After making sure her credit is good she goes up one floor to the jewelry department. Es aquí, uno arriba. Then she goes up... One, two, three [writes down TD, for toy department]. [1, 2,] tres, cuatro, cinco, seis, siete... [Draws 5 floors to the bottom of his drawing –see A4,14, 1st try– while counting up to the 10 floors Jamie goes down] [Pause]
7. J: [Julian is writing the answer: “11, because”, see A4,14, 1st try] ¿Cómo se dice interpreté? ¿Interpreté?
8. F: ¿En inglés?
9. J: Ajá.
10. F: No sé... I have interpreted... [Julián writes down “intepretd” (instead of *interpreted*) in the margin next to A3] Understood. ¿No? Podemos decir, he entendido.
11. J: ¿Interpreted se escribe así [pointing to “intepretd”]? I'm not sure.
12. F: Yo diría I have understood, ¿no? [Pause]
13. F: Yeah, interpret. [checked on the computer] Sí está bien interpret. [Julian has already overcome this issue and is finishing his writing]



15. J: [Unintelligible] That's it.
16. F: ¿Terminó?
17. J: Yeah. [Continues in A1,19]
18. [Comes from A3,41] J: Oh, this part... Ésta está un poco más chunga, un poco más difícil. Todo lo hice en inglés.

Julián talks with the interviewer in Spanish (1-6), even asking for the translation to English of a word he wants to insert in his answer. He counts aloud in Spanish (also in English) (6), but these uses of the Spanish language are not perceived as important, as he says he only uses English (18).

★ Aida A3,4-16:

4. F: ¿Y con qué lengua has empezado a resolver el problema?
5. A: En... español.
6. F: ¿Y cuándo has utilizado el inglés? ¿Cuándo has cambiado a inglés?
7. A: Cuando escribí la respuesta.
8. F: Ajá.
9. A: Cuando la escribí.
10. F: ¿Sólo para escribir la respuesta?
11. A: Sí.
12. F: ¿Y por qué escribiste en inglés la respuesta?
13. A: Nomás así.
14. F: ¿Qué quiere decir nomás? Así, ¿es? Es que es una palabra mexicana que no entiendo mucho, nomás.
15. A: Nomás, porque sí, porque...
16. F: Ajá. ¿Lo hiciste sin pensar?, entonces. El enunciado está en inglés, por eso, ¿sí? Entonces lo leíste en inglés...

After having been thinking in Spanish, Aida switches to English to write the answer without difficulties in A3.

★ Aida A4,88-107:

88. F: Querías terminar rápido. Em... ¿Cuándo cambiaste al inglés?
89. A: Cuando estaba... [Pause] Cuando andaba haciendo como cuántos subía y bajaba.
90. F: O sea, empezaste leyendo el problema en inglés. ¿Luego qué hiciste?
91. A: Luego, [Pause] al... como lo quería entender en español.
92. F: Ajá.
93. A: Luego lo leí, lo que hacía, en... otra vez, cuantos subía y así y luego lo del español otra vez para saberlo, lo, lo, lo... hacía, lo pensaba, en... en lo que subía y eso. Y luego de ahí ya cambié a inglés otra vez.
94. F: ¿Otra vez para qué cambiaste a inglés?
95. A: Para poner la respuesta.
96. F: Pero antes de poner la respuesta, pusiste [reading, 1st try] one up, one down, three up, ten down. Esto lo escribiste en inglés, mientras estabas pensando el problema. ¿Sí, te acuerdas? Escribiste esto para ayudarte a pensar en la solución.
97. A: Ajá.
98. F: ¿Por lo tanto aquí pensabas un poco en inglés, también?
99. A: Sí.
100. F: ¿Cómo que lo anotaste en inglés?
101. A: Por... [Pause] Porque estaba, cuando lo apunté estaba pensando en estos los que estaban acá [wording(?)].
102. F: Que estaban en inglés.
103. A: Ajá.
104. F: Por eso lo escribiste en inglés.
105. A: Sí.
106. F: ¿Y no pensaste en... después como... alguna cosa en inglés durante el razonamiento?
107. A: No. [Continues in A3,31]

Aida does not give importance to the process of taking notes in English (96-105).

★ Claudio A4,1-5:

1. C: So, aquí que dice: "How many floors does the department store have". So yo digo que son diez.

2. F: Can you say why?
3. C: Pero porque está...
4. F: [Interrupting] Can you explain that?
5. C: It's because dice aquí que nomás son many floors, pero no te está específicamente diciéndote cuántos. O cuál número, como el primer floor, segundo floor... No está diciendo nada de esto. Nomás dice enmedio, éste... te dice abajo, luego dice... Luego dice the other "which is one of the floor [on the 1st floor!] and leaves to go to another, to another store down the street. How many floors does the department store have." Luego aquí dice [reading] "which is the, on the first floor".

Claudio A4,19-21:

19. C: Porque aquí dice James [/dzeims/] is stopping, shopping in a large department store with many floors. For "many" means more than one.
20. F: Mm [validating].
21. C: Enters the store... That's middle. Ya son dos. Porque donde ya estaba ella... Luego dice "Many floors. She enters the store at the middle". [About 5 minutes pause. Whispers something unintelligible several times.] Yo digo que veintidós.

Claudio A4,104-116:

104. F: ¿Qué lengua has utilizado aquí? No has puesto la cruz. [Claudio writes down the cross in the "English and Spanish" column] ¿Ambas también? ¿Qué ha sido la primera cosa que has hecho para resolver el problema?
105. C: No, nada más empecé a contar.
106. F: Empezaste a contar.
107. C: En el departamento.
108. F: ¿En qué idioma contabas?
109. C: En español.
110. F: ¿Y cuándo has utilizado el inglés?
111. C: Para leerlo.
112. F: ¿Sólo para leerlo?
113. C: Mm [validating].
114. F: ¿Y en qué momento... ? ¿Cada vez que lo leías, luego lo traducías?
115. C: Sí.
116. F: Al español. Vale. [Continues in A2,18]

Claudio uses English many times when he is talking with the interviewer and he is solving the problem at the same time (the extracts above are just a small sample of all his instances of code mixes and switches). But this use of English is not reflected by Claudio when asked by the interviewer (104-116) .

4.2.3 Unknown reasons for code mixing, code switching and language choice

Even if in the semi-structured interview it was not planned to ask students about why they switch languages or why they use a particular language for a specific task, when this was demanded some students did not give a reason for it. Sometimes the language used to perform a particular activity can be controlled and modified according to personal preferences or you can adjust it to a level of comfort. But other times its use is more spontaneous, as it happens occasionally when we meet a person who asks us something in a language that is not our first language and we answer and continue the conversation through this same language.

The examples below show that students make code mixes, code switches and use of a particular language to perform a specific mathematical process, but they do not give a justification for these occurrences. This is in consonance with a transparent use of languages (Setati, Molefe & Langa, 2008), as both languages are used without interfering the mathematical solving process. Despite the interviewer's questions to make language visible, the students keep transparency. Planas and Civil (2010) report that *work in sociolinguistics indicates that people who switch languages may not report the reasons for why they switched, and when they do, the given reasons may not be the real ones that explain their switching*. Instead of just focusing on the students' point of view, though, sociolinguistics' analysis considers the talk itself to find patterns. So, even if the label of the theme is "unknown reasons", it could be that the reasons are not made public, because students do not give importance to the question of the interviewer.

★ Miriam A1,2-22:

25. [Comes from A1,38] M: [reading] Which of these figures has a greater perimeter and why. Empecé con el inglés y luego en español cuando me fijé en esto [points to the dotted lines]. Y luego al poner la respuesta esta parte ['Tienen el mismo perímetro porque', A2,1] la puse, la quise poner en inglés y luego como que esto ['esto es de la misma medida', A2,1] lo pensé así en español.
26. F: A ver, ¿cómo fue? Perdona, ¿Me lo puedes repetir?
27. M: Yeah. So, dice que que el perímetro [pointing to 'perímetro', A2,1]... Éste es el perímetro, ¿verdad? [follows the perimeter of the circle with the pencil]
28. F: Sí.
29. M: So, como todo esto de aquí [makes 4 imaginary perpendicular radius in the circle, 2 of them following the dotted line] va a ser igual de longitud, entonces puse que los dos [points to both figures] van a ser igual porque los dos tienen el mismo... [points to the dotted lines] mm... symmetry. Like...
30. F: ¿Symmetry?
31. M: Ajá. Los dos tienen la misma medida de aquí a acá [points to the dotted lines]. So...
32. F: Ajá. Esto se llama, por ejemplo, no es la symmetry, ¿no? Esto se llama...
33. M: Radi, radius
34. F: Esto es el diámetro y esto es el lado.
35. M: Okay.

36. F: ¿Sí?
37. M: Yeah.
38. F: Pero...
39. M: [Interrupting] So pensé que todo esto era igual así [makes 2 segments in the square perpendicular to the dotted line; inscribes a circle in the square], so iban a tener el mismo [follows 3 square sides]. Y aquí ['Tienen el mismo perímetro porque', A2,1] puse, lo puse en español y luego aquí ['esto es de la misma medida', A2,1] empecé a pensar en inglés. [Opposite to what she said in A2,2 !]
40. F: ¿Y cuándo... ? Empezaste a leer. Bueno, leíste en inglés, ¿no?
41. M: Mm [Affirming].
42. F: ¿Y luego cuándo cambiaste?, dices, perdona.
43. M: Cambié para cuando éstos [dotted lines] eran los mismos. So pensé que todo era igual [follows the perimeter of the circle] porque estos dos son iguales de [inscribes a circle in the square, as following the perimeter of the square], del perímetro porque de aquí [dotted lines] son de la misma longitud.
44. F: Ajá. ¿Y lo pensaste esto en español o en inglés?
45. M: En español. Y luego puse esta parte de aquí a acá ['Tienen el mismo perímetro porque', A2,1] en español y esto ['esto es de la misma medida', A2,1] lo estaba pensando en inglés y lo puse en español. [Opposite to what she said in A2,2 !, same way as what she says in A2,16]
46. F: ¿Por qué pensaste esto en inglés?
47. M: No sé.

Miriam gives contradictory information about which part of the answer she thinks of in English and which part she thinks of in Spanish (see interventions 2, 16, 22) (C2). She is not aware of the reasons for this switch (23-24).

★ Miriam A3,10-14:

10. M: Mm... No, nomás, em, empecé a leer todo en inglés y luego empecé a hacer las figuras pensando en español y luego volví a inglés acá [points to the answer].
11. F: ¿Y por qué aquí [A2] escribiste la respuesta en español y aquí [A1] también?
12. M: I don't know.
13. F: Y aquí [A3] en inglés. ¿No sabes?
14. M: No.

Miriam GLQ,13-18:

13. F: Alguna frase... Bueno, ¿el español, entonces, en general, cuándo lo has usado?
14. M: ¿El español?
15. F: Ajá.
16. M: Namás resolviendo los problemas. Como todo lo que tiene que ver con números y matemáticas. Como esto por ejemplo, como poniendo esto [points to the right side of the building in A4, where the floors are numbered]. Y ya namás lo único que hago en inglés, aquí [A1, A2] como pensé las respuestas en inglés y las puse en español.
17. F: ¿Y por qué crees que has hecho esto? ¿Alguna idea?
18. M: No. Y aquí [A3, A4] lo pensé en inglés y lo puse en inglés.

Miriam says she does not know why she sometimes uses English to write or why she uses Spanish (A3,10-14, GLQ,17-18) or why she tends to think in Spanish in relation with numbers (GLQ,17-18).

★ Camilo A1,35-43:

35. C: Porque... nada más [laughing].
36. F: ¿Por qué cambió a español... ah... digo a español luego otra vez?
37. C: Así nada más [laughing].
38. F: Pero algún motivo debe haber.
39. C: No, nada más porque yo quiero. De todos modos pues si lo quisiera poner en inglés también podía pero....
40. F: ¿Pero entonces por qué lo puso en español?

41. C: Es... no... es igual... no, no... porque es más fácil para mi.
 42. F: ¿Es más fácil en español?
 43. [Camilo nods] [Continues in A2,2]
 Initially Camilo does not make clear his reasons for his language switching (35-40).
 Finally, after a yes/no question, Camilo points out that writing in Spanish is easier for him. (41-43).

★ Jessica GLQ,5-12:

5. F: ¿Y por qué cree que sólo usa el inglés para leerlo?
6. J: Nnn [thinking].
7. F: ¿Por qué usted cree que no usa el inglés para resolverlo o para escribirlo?, por ejemplo.
8. J: N [thinking].
9. F: ¿Por qué? ¿No se le ocurre alguna explicación de [Jessica moves her shoulders, saying no] por qué sólo usa el inglés para leer?
10. J: Sí.
11. F: ¿Por qué?
12. J: Porque...

Jessica does not give any reason for her use of English linked to reading.

★ Ana A3,30-37:

30. F: En español. ¿Y cuándo cambió a inglés?
31. A: Cuando estaba contándolos.
32. F: Sí ¿Los contó en inglés?
33. A: Sí.
34. F: ¿Por qué cree que los contó en inglés?
35. A: Em... [pause]
36. F: ¿Está más acostumbrada a contar en inglés?
37. A: Sí.

Ana does not state (35) her reasons for using English as a counting language (30-33) after the interviewer's demand (34). It is not until the interviewer asks a closed question (yes/no) that she responds.

★ Angel GLQ,15-42:

15. F: ¿Y por qué cree que lo hizo así, de pensar estas cosas en inglés?
16. A: En verdad no sé.
17. F: Okay. ¿Y en general cuándo usó el español?
18. A: En... para contestar las preguntas. Y para las...
19. F: ¿Para qué más?
20. A: Para las operaciones.
21. F: Sí. Operaciones, contestar las preguntas, ¿Qué más?
22. A: Y nada más.
23. F: Lo del... el pensar el problema, ¿no? El pensar cómo se tenía que resolver, ¿cómo lo pensó eso?
24. A: En los dos.
25. F: Tanto en inglés como en español...
26. A: Sí.
27. F: ...el procedimiento.
28. [Angel nods]
29. F: ¿Sí?
30. A: Sí.
31. F: El pensar pues cómo tenía que usar la información que le daban en el enunciado para sacar la respuesta.
32. A: A veces en inglés y a veces en español.
33. F: A veces en un idioma y a veces en otro.
34. A: Sí.

35. F: ¿Por qué cree que lo hizo así, a veces en un idioma y a veces en otro?
36. A: Para que se me pegue más el inglés.
37. F: Ajá.
38. A: Y el español para que no se me olvide.
39. F: Okay. Mmm... Y luego el español me dijo también que lo utilizó para escribir las respuestas...
40. A: Y hacer las operaciones. Como las sumas o restas.
41. F: ¿Por qué cree que esto lo hizo así?
42. A: Porque casi siempre todo lo he hecho, las operaciones las hago en español. Casi nunca he hecho en inglés. Desde que llegué me toca con Mr Conteras y con él puro español y casi no hacemos en inglés las operaciones.

Initially Angel does not give any reason (16) for the employment of English during the solution of the mathematical activities. Then he does (36, 38, 42). This also illustrates that the use of languages is somehow unplanned on advance and grounded on the experiences that he had in each language.

★ Julia A3,37-42:

37. F: ¿Pero en la respuesta sí la hizo con inglés? [Pause]
38. J: Namás en esto [circles the word "tiles" in the wording: A3,16], puse.
39. F: Ajá. ¿Por qué?
40. J: Porque me pregunta cuántas tiles [/tailes/] había en siete figuras. Y porqué.
41. F: ¿Y por qué me lo puso esto en inglés?
42. J: No sé.

Julia does not give any reason (42) for writing part of the solution in English (37) (in fact just "22 tiles" and "13 tiles" –37– is written in English; the reasoning is written in Spanish).

★ Julián A3,16-39:

16. F: Okay. ¿En qué idioma empezó a pensar esto?
17. J: In English.
18. F: ¿Éste lo pensó en inglés?
19. J: Yeah.
20. F: ¿Cómo continuó? ¿Cuándo cambió a español?
21. J: In Spanish, verdad, porque interpretar eso, o sea subirle las... subirle las... Contar cuántas tiles sube.
22. F: ¿Lo pensó en español eso?
23. J: [Julián nods] Yeah. Y cambié a inglés cuándo tuve que escribir la answer.
24. F: Sí.
25. J: So eso.
26. F: Luego cambió a inglés otra vez.
27. J: Sí.
28. F: ¿Y el proceso eso de pensar que tenía que añadirle dos, ¿eso lo pensó en español?
29. J: Oh, sí.
30. F: Y iba pensando en los tiles, o iba pensando en alguna palabra en español o cómo lo hacía eso?
31. J: Oh, pues cuadritos.
32. F: ¿Cuadrito pensó?
33. J: Cuadrito, yes.
34. F: Okay. ¿Alguna cosa más aquí del uso de las lenguas?
35. J: Aquí pues no mucho. La verdad es que le puse la [points to the answer] en inglés y no sé.
36. F: ¿Por qué la puso aquí en inglés?
37. J: Pues no estoy muy seguro, la verdad.
38. F: ¿Cómo?
39. J: No estoy muy seguro. I'm not sure because... At the beginning I was thinking in English

and just about the process of this pattern. Julián does not identify any reason for the particular use of both languages on A3. It seems that the use of Spanish while adding the tiles (20-23, 28-29) or thinking of 'cuadritos' (30-33) is rather spontaneous. This is possible as Julián has a good management of both languages at a practical level. He writes the answer in English but cannot justify the decision of choosing this language to write his answer.

★ Aida A3,4-16:

4. F: ¿Y con qué lengua has empezado a resolver el problema?
5. A: En... español.
6. F: ¿Y cuándo has utilizado el inglés? ¿Cuándo has cambiado a inglés?
7. A: Cuando escribí la respuesta.
8. F: Ajá.
9. A: Cuando la escribí.
10. F: ¿Sólo para escribir la respuesta?
11. A: Sí.
12. F: ¿Y por qué escribiste en inglés la respuesta?
13. A: Nomás así.
14. F: ¿Qué quiere decir nomás? Así, ¿es? Es que es una palabra mexicana que no entiendo mucho, nomás.
15. A: Nomás, porque sí, porque...
16. F: Ajá. ¿Lo hiciste sin pensar?, entonces. El enunciado está en inglés, por eso, ¿sí? Entonces lo leíste en inglés...

Aida writes the answer in English (6-9), but does not express any reason towards this fact (13, 15).

4.2.4 Tasks clarifications around language issues

When the work that needs to be done is not clear enough sometimes students ask some questions to the interviewer. This phenomenon occurs due to many reasons. It can be due to the particularities of the Activity 1. As the mathematical procedure does not need to be written aside –it can be embedded on the answer–, students are confused about this rather unusual situation and they ask how they have to write the answer. These are the cases of Claudio and Damian, both on A1.

Another issue, linked with the previous one, arises when the mathematical answer is not found or it is not clear how to justify the reasoning, so a clarification of the statement is demanded –even if it has been correctly comprehended– to help with the mathematical procedure. The cases of Jessica on A3 and Claudio on A2 and A3 illustrate this consideration.

Another reason is present whenever the norms about how to solve the questionnaire are not clear enough, for example when marking with a cross the language(s) used when solving the activity. For instance, Yolanda and Damian did not understand it in the intended way.

One more issue is that students demand if they can write in Spanish. These are the cases of Diandra and Angel, both on A1 (as this is the first activity they solve).

Some explanations to this phenomenon may be that students are greatly involved on the mathematical task and they want to perform well; that there are some aspects that have not been detailed enough; or that students check if (explicit or implicit) classroom norms should be kept when solving the proposed activities so that they can shift to a more comfortable position when using a language. Related to the misunderstanding on the use of the language(s) columns of the questionnaire is the limitation of the solving process to its final product (written answer) or that instructions have not been carefully read and understood, as it has been commented in Chapter 3.

Now extracts to exemplify the described situation are provided and commented.

★ Yolanda A2,63-69:

- 63. Y: On these two [A1 and A2] I only thought English. On this one [A4] I thought Spanish.
- 64. F: You only thought English here?
- 65. Y: Yeah.
- 66. F: You only thought... all the activity, all the process of solving... only in English.
- 67. Y: Yeah. In this one [A4] I thought Spanish and English.
- 68. F: So can you change the cross right here?
- 69. [In A4 Yolanda changes of column the cross] [Continues in A3,2]

Yolanda marked the cross in the “English only” column for all four activities. English was indeed the language chosen to write all the answers.

★ Camilo A2,36-41:

36. F: ¿Qué le pasó por su cabeza? Utilizó... [Pause] Aquí [Language columns of the questionnaire] me puso antes sólo inglés, ¿no? ¿No usó el español para nada?
37. C: Pues pensándolo nada más, pero escribiéndolo en puro inglés.
38. F: Entonces escribiéndolo en inglés.
39. C: Sí. ¿Entonces también tengo que poner si lo pensé en español?
40. F: Sí.
41. C: Entonces lo pensé en español. [Camilo changes the cross to the “English and Spanish” column of the questionnaire]

Camilo A2,60-65:

66. F: ¿Ninguna cosa en inglés?
67. [Camilo shakes his head saying no.]
68. F: ¿Seguro?
69. C: Sí.
70. F: Porque me dijo por ejemplo the square.
71. C: ¡Oh! Bueno... ¿También eso? ¿También eso incluye? Pues sí, lo pensé en inglés también. Por ejemplo the square, the circle o así.

Camilo initially marks the cross on languages columns of the questionnaire depending exclusively on the language used to write the answer (36-41). He has a view of the language use rather general, without considering code mixes (65).

★ Diandra A1,1-3:

1. D: ¿Las respuestas las tengo que escribir en inglés?
2. F: Eh... como quiera.
3. D: Okay.

When Diandra has to write the answer to A1 (the first activity solved) she asks whether answers can be written in Spanish or not. She writes all the answers in Spanish.

★ Angel A1,8-13:

8. A: ¿Lo puedo poner en español o en inglés?
9. F: ¿Cómo?
10. A: ¿Lo puedo poner en español o en inglés?
11. F: ¿La respuesta?
12. A: Sí.
13. F: Como quiera.

Angel asks whether Spanish is allowed as a writing language or not. As he was allowed to choose the language of the interview, this prompts the question of whether to use the school language for mathematics or his preferred language. He does use Spanish for the answer.

★ Ingrid A3,48:

48. F: Mm [validating]. [Pause] Aquí, las crucecitas, perdón, eran lo mismo todo. O sea que tendría que haber sólo una en todos. [Ingrid erases the crosses in the “English only” column in A2, A3 and A4, as she did before with A1 in A1,56]. Okay. De hecho, lo que nos preguntaba aquí... ¿Ha entendido...?

Ingrid did not understand at the beginning that only one column should be selected on the questionnaire and erases the crosses on the “English only” column for all four activities.

★ Claudio A1,3-10:

3. C: So. ¿Como aquí, entonces como dice aquí "In which of these two stores are the shoes cheaper? And why."
4. F: Sí.
5. C: So, ¿le tengo que explicar aquí en letra?
6. F: Sí, sí. Por favor.
7. C: ¿Y uso la, la... como John Sports?
8. F: Yeah, whatever you think you have to use. [Pause]
9. C: Y so... aquí namás escribo eso y me voy a la siguiente.
10. F: Sí. Si crees que ya está resuelta puedes pasar a la siguiente, no tienes que hacerlas en el mismo orden, las puedes revisar cuando quieras... Sí.
11. C: So ¿aquí qué le pongo?
12. F: Qué lengua has utilizado para resolver la actividad. [Claudio was writing 'Espa'] No, sólo pon una cruz dónde, en la columna que toque. [Pause]

Claudio is surprised by A1 having no explicit mathematical procedure and he checks if he just needs to write the answer "en letra" (5). He also demands for the correct way to fill in the language columns of the questionnaire (11-12).

★ Claudio A2,11-15:

11. C: ¿Y también hablo del otro figure [/figura/, with English accent] o...?
12. F:Cuál de los dos, ¿no? What's the question, here?
13. C: Dice "Which of these figures has a greater perimet[er]? And why."
14. F: Sí, ¿entonces qué tiene...?
15. C: Y dice aquí. Yo digo que es el cuadro porque tiene líneas más grandes.

Claudio asks about how to write the answer on A2. He does not have a clear concept of perimeter and during the whole solving process he does not develop a clear written argumentation.

★ Claudio A3,1-3

1. [Comes from A2,17] C: Dice "Which figure has... " no, "How many [tiles] does figure seven have? And why. Y a[quí]... pero aquí no es el siete.
2. F: Ajá. So you have to imagine.
3. C: Can I just write it here, or...?

As Claudio does not find a mathematical procedure to solve A3 he is surprised by his (correct) understanding of the statement question.

★ Julián A3,1-7:

1. J: ¿Y si no entiendo algo?
2. F: Sí, me lo puede preguntar. Cualquier duda que tenga...
3. J: Okay. Está bien. So... [Pause]
4. J: ¿Tengo que hacerlo aquí [empty space after the wording], escribir el procedimiento?
5. F: Sí.
6. J: Okay.
7. F: Sí, como si fuera un ejercicio normal... Sí, todo lo que necesite. [Pause]

Julián starts the questionnaire by solving A3. As he is not sure about how to present the answer, he asks the interviewer about this issue (1, 4).

★ Damian A1,1-7:

1. D: ¿Lo tengo que escribir así como en senten... seten...? Más o menos es para...
2. F: What...?
3. D: Sentences
4. F: Frases.
5. D: Eso.

6. F: Sí, tienes que explicar cómo puedas. Pero al final sí, razonar el porqué y poner una cruz en qué lengua has utilizado, si sólo el inglés o español e inglés. ¿Sí?
7. D: Sí.

Damian is surprised by A1 having no explicit mathematical procedure and he checks if he has to write the answer “en sentences” (1).

★ Damian A1,24-43:

24. F: ¿Inglés? Y dices, de hecho, que has continuado usando el inglés, ¿no?
25. D: Sí.
26. F: ¿Todo en inglés?
27. D: Sí.
28. F: ¿No has utilizado el español para nada?
29. D: Pues sí puedo escribirlo en español, pues. Pero tengo que poner las líneas esas y no sé...
30. F: ¿Cómo?
31. D: Sí puedo escribir en español pero no sé dónde van las líneas porque las pala... las letras de español tienen como líneas y eso, entonces...
32. F: ¿Qué quieres decir líneas?
33. D: Así cómo que esto no tiene nada de líneas. Como la eñe tiene que ponerle la línea esa. Pues la i le puede poner una... de esas líneas arriba y eso.
34. F: Ajá. ¿Los acentos, quieres decir?
35. D: Eso.
36. F: Ajá. Vale.
37. D: Y yo no sé como...
38. F: ¿Pero lo has pensado todo en inglés o has pensado algunas partes en español?
39. D: En español.
40. F: ¿Lo has pensado en español?
41. D: Sí, pero lo escribí en inglés.
42. F: Entonces deberías poner que has usado inglés y español, ¿no?
43. D: ¡Ah! [Damian changes the cross in the language columns]

Probably Damian initially marks in the language column the languages used to write the answer (24-37), not the languages used to think about the problem (38-43).

4.2.5 Blurry perception of language use

The functions of each language when solving a mathematical problem are not clearly stated or remembered. This issue can be given to the fact that metacognition skills have to be involved when talking about the use of languages when solving mathematical activities (Clarkson, 2006). Not all students have the ability of (meta) reflection developed enough. Anyway, awareness about the use of languages is not developed naturally: being able to talk does not mean being able to talk about language. This example shows it is not easy to express one's thoughts, even if they have occurred recently:

★ Aida A1,7-13:

7. A: Pero puede ser lo mismo porque no sale el precio de los dos. No sale precio, nomás sale cuánto le quitan.
8. F: No lo has apuntado esto, por eso, ¿no? ¿Por qué no lo has apuntado? [Pause] ¿No?, me acabas de decir pueden ser lo mismo porque no sale el precio. ¿No?
9. A: Sí.
10. F: ¿Y por qué no lo has apuntado esto? [Pause] ¿Sabes por qué no lo has apuntado? ¿Me puedes decir por qué no? ¿O se te acaba de ocurrir ahora esto?
11. A: Porque apenas ahorita.
12. F: ¿Ahorita lo pensaste? ¿Antes no lo habías pensado?
13. A: No.

Aida gets the correct answer when she is explaining what she did (7) but she does not answer the interviewer's questions (8,10) until there is a yes/no question that fits what she did.

Now more excerpts are provided and commented to exemplify the blurry perception of languages use.

★ Ana A1,9-19:

9. A: Lo empecé en español, fijándome cuál tenía el descuento más y luego cuándo lo cambié a inglés fue cuándo [pause].
10. F: ¿Se acuerda cuándo cambió a inglés?
11. A: No. [Pause]
12. F: Leyó en inglés, luego lo tradujo a español, ¿estuvo pensando en español?
13. A: Sí.
14. F: ¿Y ya no se acuerda si cambió otra vez a inglés?
15. A: No.
16. F: [Ana and Francesc were ready to comment A4, but went back to A1] Por ejemplo, perdone, ¿aquí para comparar los porcentajes, esto lo hizo... lo hizo en inglés?
17. A: [Hesitating] Sí.
18. F: Perdón, inglés. ¿En inglés o en español?
19. A: En español.

Ana initially says she uses English to think (9) but later (9-15) she does not specify its use or when she changes from one language to the other. There is even a rectification from English (17) to Spanish (19) as the language used with percentages (16).

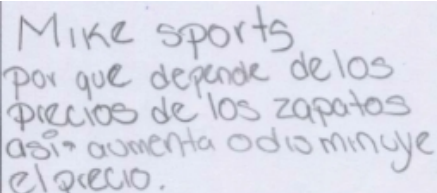
★ Ana A2,22-43:

22. F: Y... pero, por ejemplo, para pensar pues que tenía la fórmula, ¿no?, cómo tenía que

- sumarlo, ¿cómo lo pensó eso?
23. A: En inglés.
24. F: ¿En inglés que tenía que sumar eso?
25. A: Sí.
26. F: ¿Me lo puede decir, un ejemplo, por ejemplo de cómo lo pensó? Alguna frase...
27. A: Pensé... Pues en la clase de matemáticas Mr Contreras nos dijo que tenemos que sumar todo lo de alrededor y así podemos sacar el perímetro.
28. F: ¿Y pensó en inglés, ahí?
29. A: Ajá.
30. F: You thought like adding the sides of the square, for example. You thought like that?
31. A: Yes.
32. F: ¿Y aquí en el círculo?
33. A: En el círculo que es la forma "A" igual pi por el diámetro.
34. F: Sí. ¿Lo pensó en inglés?
35. A: Sí.
36. F: ¿Cómo se dice en inglés?
37. A: "A" equals pi [π], Spanish pronunciation] times [pause] times perimeter?
38. F: Pi [π] times five times diameter, right?
39. A: Diameter.
40. F: ¿Sí? ¿Pensó en inglés? Es un poco de todo, veo, ¿no?
41. A: Ajá.
42. F: Porque el diámetro no lo sabía en inglés. Vale. Eh, ¿alguna cosita más en inglés?
43. A: No.

Ana says she remembers some English explanations from the class when she is solving A3 (27). Even if she says she thinks the formula of the circle's perimeter in English, she does not say it correctly in English (she is able to say it properly in Spanish and applies it correctly to get the right solution in the problem). So English has some influence on the mathematical solving process.

★ Juan, A1, 27-33:

27.  [Entire answer]

28. F: ¿En qué lengua... con qué lengua empezó a resolver el problema?
29. J: En inglés y en español.
30. F: ¿Empezó con las dos?
31. J: Sí.
32. F: ¿Para qué utilizó una y para qué la otra?
33. J: El inglés lo utilicé para leer y entender esto y el español para escribir.

Juan says he uses Spanish as writing language (33), but there is a use of English too (27), even if it refers to a name and it is extracted from the wording. He says he starts to solve the activity through both languages (31).

★ Angel A3,10-40:

41. F: Okay. Sí. ¿Cómo usó los lenguajes para pensar todo eso? O sea, ¿empezó en qué idioma?
42. A: En inglés.
43. F: Sí. ¿Y luego cuándo cambió a español?
44. A: Cuando lo puse a... las... Cuando contesté la pregunta.
45. F: Sí. ¿Al escribirlo?
46. A: Sí.

47. F: Pero la estuvo pensando...
48. A: En inglés.
49. F: En inglés. ¿Qué pensó en inglés?
50. A: Que si en la figura cuatro había sido siete, en la cinco va a ser nueve...
51. F: ¿Esto lo pensó en inglés?
52. A: Sí.
53. F: ¿Todo en inglés?
54. A: Sí, casi todo.
55. F: ¿Qué cositas pensó en español? Que si dice casi todo, algo pensó en español entonces, ¿no? ¿Qué pensó en español?
56. A: La [figura] número siete, si estaba bien.
57. F: ¿La última?
58. A: Sí.
59. F: ¿La última la pensó en español?
60. [Angel nods.]
61. ¿Y qué más en español?
62. A: Lo de abajo, la contesta[ción]...
63. F: La respuesta y este número. Lo demás lo pensó en inglés. El... como... Eh... Saber que se tenía que aumentar de esta a esta dos o de una a la siguiente dos, ¿esto lo pensó en inglés?
64. A: En español.
65. F: Y escribirlo, ¿lo escribió?
66. A: En inglés.
67. F: Lo pensó en inglés para escribir esto.
68. A: Sí.
69. F: Pero pensarlo, como... obse... mirar aquí, saber que se tenía que añadir dos, eso lo pensó...
70. A: En español.
71. F: En español. Okay. [Continues in A4, 5]

Angel initially states that A3 is thought mainly through English (16-23) but later he gives more importance to Spanish during the thinking process (32-33, 38-40). There is even a confusion about the writing language (35). So he is confused and does not precisely defines the uses of each language.

★ Abel A4,8-25:

26. F: ¿Y qué pensó en inglés?
27. A: ¿En inglés?
28. F: Mm [validating].
29. A: Nada.
30. F: ¿Sólo al leerlo?
31. A: Mm [validating].
32. F: Luego, por ejemplo, a la hora de sumar...
33. A: Le... cuándo estaba sumando, para estar bien...
34. F: Mm. [continuing conversation]
35. A: Para saber si está, si iba a estar bien le di otra pasada.
36. F: ¿Con qué idioma?
37. A: En inglés, así. Y después...
38. F: ¿Sumó one, plus three, plus ten, plus one?
39. A: Sí
40. F: ¿In English?
41. A: Sí, nomás uno más three plus ten plus one y me dió quince, fifteen.
42. F: Pero ¿al pensar eso lo pensó en inglés o en español?
43. A: Ajá... sí... a veces que plus y lo pensé en inglés y ya los números, sumándolo lo pensé en español.

There is not a definite use of English when solving A4 and its use does not come to Abel's mind at first sight.

★ Julia A3,43-48:

43. F: ¿Alguna otra cosita pensó en inglés?
 44. J: No.
 45. F: ¿No? ¿Todo en español?
 46. J: [Julia says no with her head.] Figura.
 47. F: ¿Figure? ¿Cuándo lo estaba resolviendo estaba pensando en figure?
 48. J: Sí.

The use of the word “figure” in English is neither quickly nor clearly (as it is said in Spanish –46–) expressed by Julia.

★ Ingrid A2,21-65:

21. F: ¿Y cómo lo pensó esto? ¿Qué idiomas utilizó?
 22. I: El español.
 23. F: ¿El español?
 24. I: Mm [validating].
 25. F: ¿Durante todo el proceso? U otra vez: si se acuerda de alguna cosita que pensó en inglés...
 26. I: Como perímetro porque el maestro nos ha dicho que siempre tenemos que, que nos tenemos que aprender esas palabras.
 27. F: ¿Perimeter?
 28. I: Sí.
 29. F: ¿Pensó en inglés?
 30. I: Sí.
 31. F: ¿Para el círculo?
 32. I: Sí.
 33. F: ¿Pero para el cuadrado no?
 34. I: También, poquito.
 35. F: Ajá. Okay. Muy bien. ¿Qué más en inglés?
 36. I: Eh... [pause] Nada más.
 37. F: ¿La fórmula, por ejemplo?
 38. I: Mm [continuing conversation]
 39. F: ¿En qué idioma?
 40. I: En inglés.
 41. F: ¿Y el...? ¿Cómo se lee esto en inglés? [Pause] ¿Cómo lo pensó en inglés?
 42. I: [Pause] Em...
 43. F: ¿O lo pensó en español?
 44. I: Ajá.
 45. F: ¿En qué idioma lo pensó?
 46. I: En español.
 47. F: ¿Sí? ¿Inglés o español?
 48. I: En español.
 49. F: ¿Los va manejando los dos, un poco?
 50. I: Mm [validating, answering before the question is finished, after 'dos']
 51. F: ¿Y no se acuerda exactamente alguna cosita más que pensó en español, digo en inglés?
 52. I: Porqué.
 53. F: ¿El porqué?
 54. I: Mm [validating]
 55. F: ¿Razonó en, en...? ¿O sea a la hora de empezar a pensar, lo pensó en inglés o sólo leyó el why...? ¿Cómo, cómo pensó el porqué? ¿Por qué dice que el porqué lo pensó en inglés?
 56. I: Porque... [pause]
 57. F: ¿No me lo puede decir?
 58. I: No.

59. F: ¿Más o menos vamos manejando ambos idiomas?
60. I: Mm [validating].
61. F: ¿o más español o más inglés o cómo cree?
62. I: Un poquito más español, porque mucho inglés no sé.
63. F: Ajá. ¿Pero algo de inglés sí piensa mientras resuelve la...?
64. I: Mm [validating].
65. F: ¿Y no se le ocurre algún ejemplo, pues por ejemplo... no sé, algún pequeño detalle de qué pensó en inglés? ¿No? [Continues in A3,2]

In the beginning (21-24) Ingrid does not allude to the use of English when solving the problem, but then she shows signs of using it, even if she cannot specify its use (65). When interviewer asks if she does a particular task in English she agrees most of the times, but then she cannot show clearly what it is done in English. Probably there is a slight English influence in the mathematical solving process of A2.

★ Ingrid A3,22-25:

22. F: ¿Entonces qué pensó?
23. I: Em... [pause]Cuál de ellas me podría dar siete.
24. F: ¿En inglés?
25. I: Mm [validating]. Más o menos, en inglés y en español.

Ingrid A3,34-47:

34. F: Okay. ¿Alguna cosita, otra vez, si se acuerda exactamente de qué pensó en inglés?
35. I: Cuántas me podían salir.
36. F: Mm [validating].. ¿Eso lo pensó en inglés?
37. I: Mm [validating].
38. F: ¿Qué más?
39. I: Nada más.
40. F: ¿Cómo la dijo, esa frase?
41. I: Mmm...
42. F: ¿En inglés dijo, no?
43. I: Mm [validating].
44. F: ¿Esa frase misma dijo en inglés?
45. I: Sí.
46. F: ¿Cómo la dijo?
47. I: Em... how many

Ingrid's use of English is very vague in the beginning (22-25). It gets a little more concise later (34-37). A full sentence in English is not given to exemplify the English influence on the thinking process (40-47). It looks like the use of English helps her to find the answer but her thoughts are not clearly articulated through English.

★ Julián A2,34-40:

34. J: I only used English at the beginning, so that's why I use more Spanish.
35. F: ¿Para leer la pregunta sólo?
36. J: Sí.
37. F: ¿Pero no pensó ni en el círculo, circle, or square, o five or anything like that in English?
38. J: Sí: square, five and the circle.
39. F: ¿Eso sí lo pensó en inglés?
40. J: Sí. Porque recuerdo las clases, por eso.

Julián does not remember the use of English to interpret the abstract figurative mode – the figures– (34-36) until the interviewer points it out (37-40).

★ Damian A1, 22-71:

22. F: Ajá. Okay. ¿Qué lengua has utilizado para empezar a resolver el problema?

23. D: Inglés.
 24. F: ¿Inglés? Y dices, de hecho, que has continuado usando el inglés, ¿no?
 25. D: Sí.
 26. F: ¿Todo en inglés?
 27. D: Sí.
 28. F: ¿No has utilizado el español para nada?
 29. D: Pues sí puedo escribirlo en español, pues. Pero tengo que poner las líneas esas y no sé...
 30. F: ¿Cómo?
 31. D: Sí puedo escribir en español pero no sé dónde van las líneas porque las pala... las letras de español tienen como líneas y eso, entonces...
 32. F: ¿Qué quieres decir líneas?
 33. D: Así cómo que esto no tiene nada de líneas. Como la eñe tiene que ponerle la línea esa. Pues la i le puede poner una... de esas líneas arriba y eso.
 34. F: Ajá. ¿Los acentos, quieres decir?
 35. D: Eso.
 36. F: Ajá. Vale.
 37. D: Y yo no sé como...
 38. F: ¿Pero lo has pensado todo en inglés o has pensado algunas partes en español?
 39. D: En español.
 40. F: ¿Lo has pensado en español?
 41. D: Sí, pero lo escribí en inglés.
 42. F: Entonces deberías poner que has usado inglés y español, ¿no?
 43. D: ¡Ah! [Damian changes the cross in the language column]
 44. F: ¿Lo has pensado todo en español, dices?
 45. D: No todo pues, pero una... unas las pensé en inglés.
 46. F: Pero en la actividad uno, digo.
 47. D: ¡Oh!, ésta en español y en inglés.
 48. F: La pensaste tanto en español como en inglés, ¿no?
 49. D: Pues en español y en inglés también.
 50. F: ¿Y te acuerdas cómo empezaste a pensar la actividad?
 51. D: Pues... me fijé en los descuentos primero.
 52. F: Sí. ¿Y cuándo cambiaste de lengua?
 53. D: Cuando estaba poniendo la respuesta.
 54. F: Cuándo estabas poniendo la respuesta, cambiaste a inglés.
 55. D: Sí.
 56. F: ¿Pero mientras estabas pensando?
 57. D: Nomás en ésta y en las demás puro inglés.
 58. F: ¿Ésta la pensaste en qué idioma?
 59. D: En español.
 60. F: ¿Y todas las otras en inglés, dices?
 61. D: Sí.
 62. F: ¿Ésta pensaste en inglés el razonamiento? ¿Sí? Dijiste es más barata...
 63. D: Oh, no, ésta la pensé en español y pues.
 64. F: ¿Sí?
 65. D: Sí.
 66. F: ¿Pero algunas cosas en inglés también?
 67. D: Sí.
 68. F: Te veo un poco dudando, por eso digo... ¿Sí? ¿Qué cosas pensaste en inglés?
 69. D: Algo de aquí, sí que puede ser. [Unintelligible]
 70. F: ¿No sabes muy bien?
 71. D: No.

In the beginning Damian says he does not use Spanish to solve the problem (22-27). Then he admits he uses both languages to think about the activity (38-47). Later he cannot attribute a particular point where he switches languages.

4.2.6 Mathematical differences between oral and written registers

Written answers do not always contain all the mathematical details and steps towards the solution of the problem. As pointed out by Moschkovich (2007a) *When students are learning a second language, they are able to display content knowledge more easily by showing and telling, rather than through reading text* (p. 346). Some of the students, though, asked to write in Spanish to make this communicative process easier.

Notice that each activity ends up with the question “Why?” in an attempt to force students to justify their answer. Furthermore at the beginning of the questionnaire, it reads “Write down the solving procedure”. A lack of mathematical argumentation is observed when students are demanded to explain the answer during interviewer interaction. Another possibility is that students make another approach to solve the problem during the interaction with the interviewer (2nd try, 3rd try, etc.) and once they are asked to write down the solution, the text does not have as many mathematical details as the discussion provided. Sometimes –as Juan in A1– numerical examples are added to the oral register with respect to the written form.

Summarizing and extracting the more important information cannot be easy in all of the cases and its difficulty increases with reference to complex mathematical thoughts. Here the fact that some answers are written in English, which is not the most comfortable language for most of the students is also important. In some cases, specially on A2, the use of mathematically specific vocabulary would improve the understandability of the written answer (this is the case of Coral).

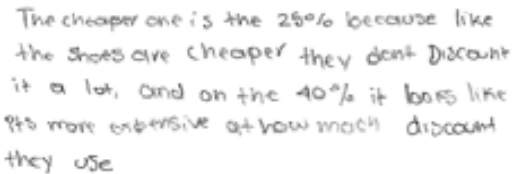
Willing to sum up the answer in a few words (to finish quickly, to write less, to get tired due to laziness, ...) is another reason for this lack of clarity and mathematical precision on the written register, specially for the students that developed an approach to the problem with the interviewer, as in these cases the details are already know (when the process is the opposite way –this is, the written answer is before the oral explanation– it might be that details came to mind after writing the answer). Sometimes, students assume that the answer is clear enough and everybody will understand it the same way they do. For instance, this is what happens to Coral on A4, where just by saying “add the floors up” she assumes that the reader will understand which floors have been added.

In some instances the more detailed explanation is a result of the interactivity provided by the dialogue, while in other instances (as it is the case of Coral in A4) the students directly takes the initiative on giving a more detailed answer.

What all these cases show is that a deficit view of the English language learners should be avoided initially, as they have the potential to explain their reasoning and they may well provide rich new insights on the problem that were not considered initially. Guiding them towards the correct reflection of their mathematical ideas by means of the written text would also be important.

Now excerpts of different case studies are provided to exemplify how the oral register adds mathematical characteristics to the written answer.

★ Yolanda A1,1-7:

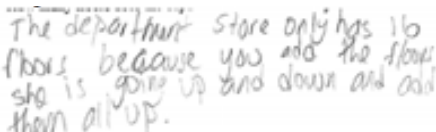
1. 

[Yolanda quickly writes the answer. The questionnaire was in front of her during the previous interview (see its summary above: Historical bilingual profile).]

2. F: Can you explain me how have you solved it?
 3. Y: Well, because on this store is twenty five percent, on this one is forty. So like maybe because this one is forty they have to discount it a lot, because it's more expensive. And this one is twenty five so they don't like ... they only discount a little so maybe because eh... there's... like the store right there [25%] is cheaper. So that's is how I got that this one was the cheaper one.
 4. F: This one was cheaper?
 5. Y: Yeah.
 6. F: So it was cheaper to bought the... the shoes right here [25%] than here?
 7. Y: Yeah. Because maybe right here [25%] the shoes like they are not that much of expensive than right here [40%].

The idea reflected in the written answer (1) is not completely clear without a detailed review of the structure and a close look at the intended meaning. The oral explanation states it clearly (3-7).

★ Carlos A4,2-4:

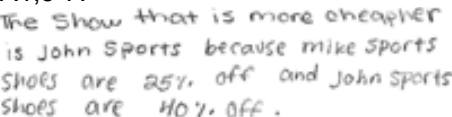
2. 

[Carlos erases a couple of words and rewrites them, probably because of his handwriting is not clearly intelligible]

3. [Comes from A3,17] F: How have you solved that activity in fact? [A3,17, reproduced also here]
 4. C: Jamie started at the middle floor and then she went up one floor, and then she went down one floor. That makes it two floors. And then she goes up three floors to the toy department, so that's five. And then she goes down ten floors to the main entrance. That's fifteen. Plus the one in the middle where she started at, so that was sixteen floors.

In the written answer (2) Carlos says he adds all the floors, but it is on the oral explanation (4) when he details where the sixteen floors come from.

★ Coral A1,3-7:

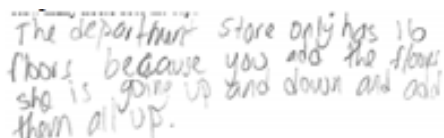
3. 

[She quickly writes down the answer.]

4. [Comes from A4,14] F: How did you solve activity one?
5. C: Which one do I pick?
6. F: Excuse me? Yeah! How have you solved it. What did you think when solving it?
7. C: Because like... the more... It's still the same brand, but just a little cheaper in different stores and there is no difference on which store it is, because you are still getting the same thing, but just a little cheaper. And then you just...

Coral does not reflect that she assumes that both shoes have the same initial prices (as both are the same brand –7–) in her written answer (3).

★ Coral A2,1-7:

1. 

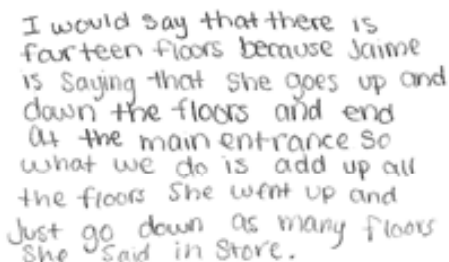
[Carlos erases a couple of words and rewrites them, probably because of his handwriting is not clearly intelligible]

2. F: Can you explain me how have you solved it?
3. C: Because in the circle the width and length are the same than... are the same size than... are the same size as each... around it. And in the square the width is longer than the length.
4. F: In the square the width is longer than the length? You say that?
5. C: Yes.
6. F: And so the circle has a greater perimeter that the square.
7. C: Yes.

A deviated use of mathematical vocabulary (length, width, side) makes the written answer hard to understand. The oral register improves a little the mathematical argumentation.

★ Coral A4,14-17:

Jamie is shopping in a large department store with many floors. She enters the store at the middle floor, and immediately goes to the credit department. After making sure her credit is good, she goes up one floor to the jewelry department. Then she goes down one floor to the children's department. Then she goes up three floors to the toy department. Finally Jamie goes down ten floors to the main entrance of the store, which is on the first floor, and leaves to go to another store down the street.
How many floors does the department store have? Why?

14. 

[Continues in A1,2]

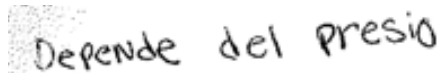
15. [Comes from A3,15] F: Can you explain me how have you solved it?
16. C: I added all the floors she said she went up. She went up three floors, plus went up one floor and then she said she first entered the middle floor so there must be a floor on the bottom. And then I just added those up. And then she says she went down one and down ten. So I just added them up.

In the written answer (14) Coral does not specify how she adds all the floors. They are named on the oral explanation (16) but anyway it is not clear how the 14 floors are figured out.

★ Miriam A1,1-33 :

1. M: ¿Y cómo debo de saber cuáles están más baratos si no dice el precio de los zapatos?
2. F: Lo puede poner... Si cree... Lo puede poner por escrito. Pues, depende del precio, o... lo


que usted crea.

3. 
4. [Once all activities are answered, dialogue continues here. This intervention is translated on A4,2 too.] M: I'm done.
5. F: We are gonna comment the, the activities now, right? Which one do you prefer?
6. M: [Reading] In which...
7. F: Bueno, ¿con cuál quieres empezar?, perdón. Hemos quedado que hablamos en español, ¿no?
8. M: [Reading] In which of these two stores are the shoes cheaper? Why?
9. F: ¿Qué has hecho aquí? ¿Cómo ha empezado?
10. M: Yo puse que depende del precio, porque si aquí [25%] están más caros que aquí [40%], entonces sería éste [40%] es más barato que éste [25%]. Y si aquí [40%] están mucho más caros que acá [25%], de todos modos sería mucho más barato éste [25%] que éste [40%].
11. F: ¿Cómo has empezado a resolver el problema?
12. M: Mm... Do you want me like to put an answer?
13. F: No. Bueno, con qué te has fijado al principio.
14. M: Pues me fijé en eso. De que si éste [40%] es más bajo el precio va a estar más bajo el zapato en el precio. Y si éste [25%] es más caro, aunque tenga veinticinco por ciento de descuento va a ser más caro. Y si éste [40%] es más caro que éste [25%] de todos modos éste [25%] va a seguir siendo más barato. Y éste [40%] aunque tenga el mayor por ciento de descuento va a seguir siendo más caro.
15. F: Depende de los precios dice usted.
16. M: Depende de los precios.
17. F: Ajá. Okay. ¿En qué lengua has empezado a resolver el problema?
18. M: En inglés.
19. F: Has empezado en inglés. ¿Y cuándo cambiaste a español?
20. M: Cuando puse depende del precio [answer].
21. F: ¿Sólo al escribir la respuesta?
22. F: ¿Estabas pensando en inglés todo el rato?
23. M: ¡¿Mm?!
24. F: All the time in English?
25. M: No. Como a medias, pensé en español y después lo pasé a inglés y volví a español.
26. F: ¿Y cuándo cambiaste? ¿Te acuerdas de cuándo cambiaste de pensar en inglés a pensar en español?
27. M: Yeah. Cuando puse menor precio aquí [40%] y mayor precio acá [25%].
28. F: ¿Cuándo pensaste en un precio concreto?
29. M: Mm [affirming].
30. F: ¿En qué precio pensaste?
31. M: Like ten dollars.
32. F: Ajá. Entonces pensaste diez dólares, no ten dollars. Tu pensaste diez do...
33. M: Diez dólares, ajá, aquí [40%] y ponga diez aquí [25%]. Entonces éste [40%] costaría menos que éste [25%]. Y si pusiera veinte acá [25%] y diez acá [40%], éste [40%] sería menos que éste [25%]. Y si los volteara entonces sería menos éste [25%] que éste [40%]... That's what I thought.

Miriam's written answer (3) may be influenced by the interviewer's intervention (2), but Miriam already got the trick of the problem (1). Initially, the oral explanation gives abstract examples (10,14). When talking about the use of languages, numerical instances are also provided (33). So the oral register shows that Miriam has a deeper management of percentages and has the potential to provide a better reasoned answer than what is given by the written response. In fact she is not completely sure about the

correctness of her written answer, as if more details would be needed (12) for a desirable correctness of the solution from a mathematical point of view.

★ Camilo A1,1-3 :

1. 

[Camilo erases and rewrites the first sentence.

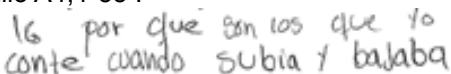
On the left, the final answer is reproduced.]

2. F: ¿Qué ha hecho aquí primero?

3. C: Pues la que... Dice cuál... which of these two... In which of these two stores are the shoes cheaper? And why. En la John Store porqué aquí dice de... el discount es cuarenta por ciento y aquí es veinticinco por ciento, es menos. Y aquí descuentan más [40%] y aquí descuentan menos [25%].

The mathematical reasons stated by Camilo in the written answer (1) can be guessed, but explanation (3) is more detailed and gives no place to doubt that he compares percentages directly giving them an absolute value.

★ Camilo A4,1-35 :

1. 

[Camilo counts with the fingers as he reads the

wording.]

2. [Comes from A3,15] F: Okay. ¿La [actividad] cuatro, pues?

3. C: Pues lo leí en inglés, luego cambié a español y lo pensé en español.

4. F: ¿Cómo empezó a resolver el problema aquí?, a ver.

5. C: Iba leyendo que cuántos pisos iba ella subiendo y bajando y entrando en las tiendas... y así me di cuenta cuánto... Porque aquí dice que ella empezó en el... en el middle floor y luego iba subiendo de piso y bajando, y subiendo y bajando...

6. F: Sí.

7. C: Pues por eso.

8. F: ¿Y cómo sacó la respuesta?

9. C: Pues conté cuántos subió y cuántos bajó y cuántos iba entrando y así.

10. F: ¿Y al final qué hizo con todas esas cosas?

11. C: Los conté, los pisos que subió y los que bajó.

12. F: ¿Puede empezar a resolverlo otra vez, para que lo vea, por ejemplo? [In the discourse below Camilo starts to translate the wording to Spanish and adds up all the numbers with the help of the fingers]

13. C: Ella entró a la tienda en el medio... en el piso del medio...

14. F: Sí.

15. C: Luego inmediatamente fue al departamento de créditos. Ahí es uno. Luego después... se hizo...

16. F: ¿Pero es uno qué quiere decir?

17. C: Pues en dónde el primer floor, el primer piso. Ya después dice que se hizo... se puso... se hizo... éste... se... ¿cómo se llama?, éste... Se aseguró que su crédito estaba bien.

18. F: Sí.

19. C: Ya después subió un piso, a la joyería. [Marks 2 with the fingers]

20. F: ¿Por dónde entra, por dónde entra por eso?

21. C: En el... en el medio del piso, en el piso del medio.

22. F: Sí. Pero, ¿y dónde va para comprobar el crédito?

23. C: Pues allí, es el mismo nivel.

24. F: ¿Y ha contado esto como uno?

25. C: Ajá. Sí.

26. F: ¿Por qué?

27. C: Porqué aquí no dice que va arriba o va pa abajo, pero no dice que...

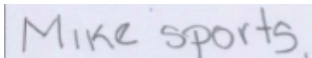
28. F: Ajá. Sí. Okay. ¿Qué más?

29. C: Luego ya después de ir a...

30. F: Se va parriba.
 31. C: Se va parriba en la joyería, en el departamento de joyería.
 32. F: Sí.
 33. C: Ya después ella baja un piso más, dónde el departamento de los niños. [Marks 3 with the fingers]
 34. F: Sí.
 35. C: Tal vez ahí cuidan niños, o no sé. Ya son tres. Luego ella va... sube tres pisos más. Luego son seis [marks 6 with the fingers, using both hands], tres pisos más ya. Y ya después finalmente baja diez pisos... ¡Oh sí, pero ahí debería de restarlos! Después diez pisos y ahí los cuento y ya son dieciséis pisos.

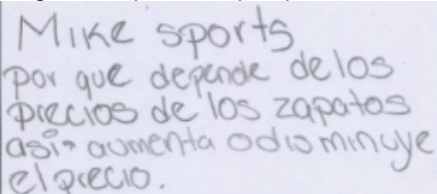
In the written answer (1) it is not explicit which floors Camilo adds to find the answer, neither on the first part of the oral explanation (4-11). After more detail is demanded (12), Camilo details his resolution process (13-35).

★ Juan A1,1:

1.  [Juan reviews the problem after reviewing A4 but makes no changes]

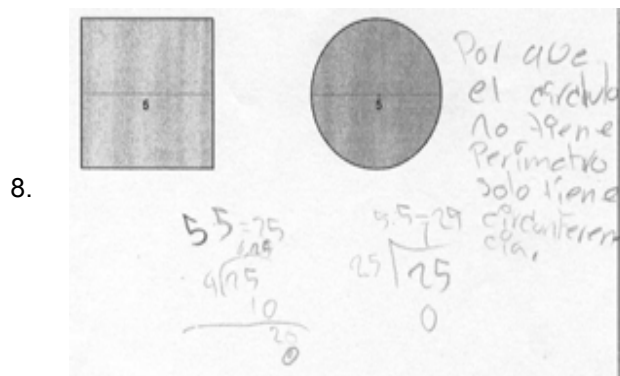
Juan A1,19-27:

19. J: Porque si tiene un precio muy bajo, de todos modos le descuentan un veinticinco por ciento y costaría menos que éste [40%], porque si ya tiene el precio alto costaría todavía más que éste [25%].
 20. F: ¿Y por qué tendría éste [25%] el precio alto? ¿Digo el precio más bajo éste [25%]?
 21. J: Depende del precio de los zapatos.
 22. F: ¿Pero usted cree que éste [25%] tiene un precio más bajo?
 23. J: ¡Sí!
 24. F: ¿Por qué?
 25. J: Porque si le descuentan... y son... Supone que son sesenta, le descuentan veinticinco por ciento. Y si éste [40%] cuesta cien, le descuentan cuarenta. Y tiene precio más bajo éste [pointing to "25%"] que éste [pointing to the shoe picture on the 25% store !!].
 26. F: ¿Puede poner el porqué cree, entonces, que éste es más barato?

27.  [Entire answer]

Initially Juan does not give any reason through the written answer (1). Later he writes down a written argumentation (27) after interviewer's demand (26) but the text is not as rich as his oral explanation (19-25) which contains general and concrete details that show a good management of percentages.

★ Abel A2,8-16:



9. F: ¿Qué hizo aquí?
10. A: Aquí dice que... que si cuál era, cuál de las, de las figuras tiene el perímetro ¿más grande? ¿O así?
11. F: Mm [validating]. And why. ¿Qué significa why?
12. A: Por qué.
13. F: Ajá.
14. A: Porque...
15. F: ¿Y cuál dijo usted que tiene el perímetro más grande?
16. A: El cuadrado.

When asked about which perimeter is greater, Abel adds more information to the written answer: he makes an impossible comparison (15-16); as the circle has no perimeter, it cannot be compared to the perimeter of the square

★ Abel A3, 14-41:

42. F: Ajá. ¿Pero puede saber cuántos cuadraditos tendría la figura siete? [Pause] No está aquí dibujada, ¿pero puede saber usted cuántos cuadraditos tendría? [Pause]
43. A: ¿Diez?
44. F: ¿Cómo lo hizo?
45. A: Como haciendo esto así pero poniéndole, incluyéndole un cuadrado.
46. F: ¿Uno más?
47. A: Sí.
48. F: ¿Por qué uno más?
49. A: Porque va de una figura así, nomás le puede incluir... o puede incluir tres, o los que sean necesarios para que quede lo que tenga que ser.
50. F: Pero aquí hay... [pointing to the wording] Dice: observa este patrón, ¿no? Aquí hay una serie que sigue una lógica, ¿no? Entonces tiene que mirar cómo funciona esto para ver... tratar de averiguar, pues la cinco, la seis, la siete... Quizás así lo podemos saber, ¿no? Entonces tenemos que mirar cómo cambia de la uno a la dos, de la dos a la tres, de la tres a la cuatro... y ver si lo que usted ha pensado, de añadir una figura [apose: un cuadrado!], esto funciona. Si es lo que pasa para pasar de una figura a la siguiente.
51. A: ¡Oh!
52. F: ¿Sí entendió? No puede hacerlo usted a su manera, sino que tiene que seguir el modelo que tenemos aquí.
53. A: Sí. [Pause. As Abel moves the pencil, it looks like he mentally adds tiles on Figure 4, imaging Figure 5, Figure 6 and finally Figure 7 and at the same time he counts the tiles.] ¿Serían trece?
54. F: ¿Cómo lo hizo?
55. A: Nada más como siguiendo el procedimiento, de incluyéndole de dos.
56. F: Sí.
57. A: Y así le fui incluyendo aquí dos, dos, más dos y así como llegué a la figura siete y me dio trece.
58. F: Ajá. Okay. ¿Lo puede escribir?
59. A: ¿Esto está mal?
60. F: ¿Cuántas tiene? Sí, esto es lo que había, habíamos entendido la, la... la respuesta

[lapse: pregunta!] mal, ¿no? Puede poner nada más una línea así y ya está. [Abel circles and crosses out the answer] Si me puede poner su respuesta y el porqué.

61. [Abel starts to write the answer]
 62. F: Mm [validating]. ¿Y cuántas va a tener? No me lo escribió.
 63. [Abel Counts again the tiles]
 64. F: ¿Qué me dijo antes? ¿Cómo lo encontró?
 65. A: Sumándole dos más.
 66. F: ¿Entonces aquí cuántas hay? [Pause]
 67. A: Trece.
 68. F: Ajá. Sí.

69. [A3. Entire answer.]

Abel does not detail how he finds the answer (27-29) in his written expression (41), even if there is an explicit demand by the interviewer to do it (30, 32). Abel even forgets to write how many tiles are in figure 7 and interviewer has to ask it again (34).

★ Ingrid A3,1-9:



1. How many tiles does figure 7 have? Why?

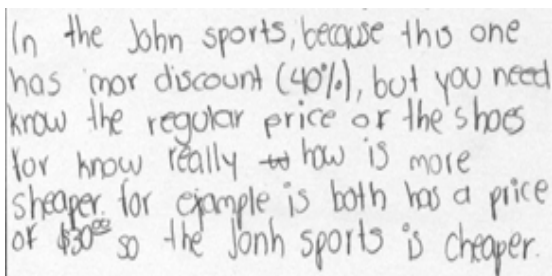
figura 1 49
 por que las otras dos saldrían
 mas que 7.

[The lines crossing out the answer are added on the 2nd try answer (A3,56)]

2. [Comes from A2,65] F: ¿Qué hizo?
 3. I: Ver cuál podría salir siete.
 4. F: ¿Qué quiere decir cuál podría salir siete?
 5. I: ¿Podrían salir siete cuadros?
 6. F: ¿Cómo podrían salir?
 7. I: Poniendo éste [Figure 1] siete veces, y éste [Figure 4] cómo ya tiene siete.
 8. F: Ajá.
 9. I: Porque éste [Figure 2] no se podría porque sumando otro serían seis, y si pongo aquí otro, ya serían nueve. Y éste [Figure 3] también ya se pasaría.

Ingrid explains her reasoning in her written answer (1) but as her comprehension of the wording did not occur in the intended way, the comprehension of the written answer is not immediate. The oral explanation (6-9) clarifies her reasoning.

★ Yael A1,5:

1. 

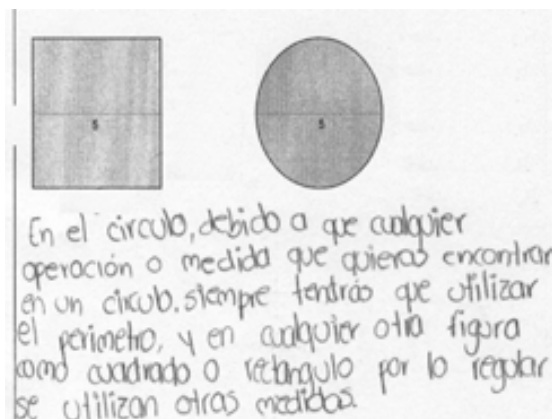
[Conversation continues on A1,6 once all the activities are solved.]

Yael A1,20:

20. Y: Okay. Aquí aparece la tabla y dice que la primera tienda que es la John Sports los tenis tienen un cuarenta por ciento de descuento y en la Mike tienen un veinticinco por ciento. Yo le puse que era más barato en la primera que en la segunda, porque [tiene] un cuarenta por ciento que es más mucho que un veinticinco. Pero también le puse que necesita saber cual es el regular precio de los dos, ¿porque si no como vas a saber cual! Y puse un ejemplo. Puse por ejemplo si los dos pares, en las dos tiendas, cuestan treinta dólares y con el cuarenta por ciento pues siempre va a ser la John la que va a tener el precio más bajo. Pero si los precios son diferentes pues también va a ser diferente el resultado.

'Pero si los precios son diferentes pues también va a ser diferente el resultado' (20) adds more information to the written answer, even if it is not yet a complete example since the other store remains cheaper.

★ Yael A2,1:

1. 

[1st try]

Yael A2, 21:

21. Y: Que para cualquier operación que tengas que hacer relacionada con un círculo siempre vas a utilizar el perímetro. Y en otra figura como un cuadrado o en rectángulo, ahí casi nunca se utiliza el perímetro. Como si quieres saber... no sé, cuál es el radio, pues tienes que utilizar el perímetro, si quieres saber cual es el volumen también tienes que utilizar el perímetro del círculo.

Yael A2,38-52:

38. F: ¿Y qué le preguntaba el ejercicio aquí?

39. Y: Que cuál de las dos figuras cómo tienes que utilizar el perímetro. Y por qué. Como yo así lo entendí. O como a cuál te es útil.

40. F: Okay. Le pregunta que cual "has a greater", que significa "greater"?

41. Y: Pues yo lo entiendo como... como a... Así lo entiendo, como en cuál te es más útil.

42. F: Mayor. Le pregunta en cual de estas dos figuras el perímetro es mayor.

43. Y: Oh, yo lo entendí como cual es más útil.

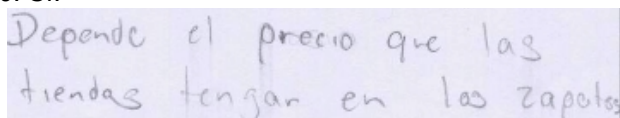
44. F: ¿Lo quiere volver a pensar?

45. Y: ¡Oh, sí!
 46. F: Okay.
 47. Y: Pero pues en un cuadrado como nunca se utiliza un perímetro.
 48. F: Mm [continuing conversation]
 49. Y: Y siempre se utiliza en un círculo, pero aquí la medida de los dos es la misma.
 50. F: ¿La medida de qué?
 51. Y: De los dos perímetros. Porque aquí es cinco y aquí también.
 52. F: ¿Qué es el perímetro?

Yael understands the wording question in a deviated way (39). Just by looking at the written answer (1), the confusion between perimeter and diameter is not apparent, neither on the initial oral explanation (21). It is not until the wording is properly translated by the interviewer (42) that the confusion becomes clearly explicit (49, 51).

★ Julián A1,1-20:

1. [Comes from A3,13] J: Ésta [A1] sí no la entiendo. Esta parte.
2. F: ¿No? ¿Qué es lo que no entiende?
3. J: Qué tengo que hacer aquí.
4. F: ¿Qué es lo que le pregunta?
5. J: Which of these two stores are the shoes cheaper. Cual de estas dos tiendas es la... La cheaper, no entiendo esa palabra.
6. F: Más baratas.
7. J: ¡Oh!
8. F: ¿No? Más baratas.
9. J: Sí, más baratas. Pero no dice la verdad, porque no dice el precio exacto de esto para cuánto es sin el descuento.
10. F: Sí. ¿Entonces qué? ¿Qué, le hacen falta datos?
11. J: Sí. Sí, porque imagínese que aquí ofrecen el cuarenta por ciento de descuento pero que tal si lo dan más caro y lo rebajan y ganan igual.
12. F: Sí. Pues ponga esto. Lo que me acaba de decir. Lo puede poner como respuesta.
13. J: ¿Y qué le pongo aquí entonces, "no sé"?
14. F: Pues depende, ¿no? ¿Sí? Lo que... Pues expréselo como quiera.
15. J: [Unintelligible sequence]
16. F: ¿Sí?, como quiera. Si en lugar de... bueno, si se equivoca ponga una rayita encima y luego ya... Nada más escribe al lado, ¿sí?
17. J: Sí.

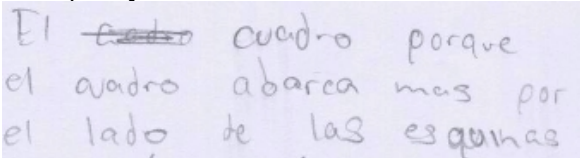
18.  [Continues in A2,1]

19. [comes from A4,17] F: ¿Qué es lo primero que ha hecho para resolver esta actividad?
20. J: Pues una vez, primero la comparación del porcentaje que están de descuento. Okay. Aquí me dice que es forty per cent of discount y aquí es treinta per cent of discount. Pero la diferencia es de que ninguno de estos tiene el precio. Por decir, si aquí [writes down "100" in 40% store, see A1,21] tuviera cien dólares y aquí tuviera cien dólares, así cien dólares [writes down "100 %" in 25% store, and immediately crosses the "%" out, see A1,21]. Sería la diferencia que si son cien dólares y tiene cuarenta por ciento de descuento sería entonces sesenta el precio de esto. Y aquí sería el veinticinco entonces saldría setenta y cinco dólares el par de zapatos. Y entonces aquí me daría cuál es el, uno de los más baratos. Podemos decir esto. Pero como no tiene precio no podemos saber cuál es.

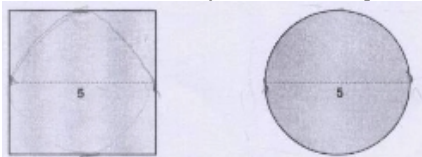
It is not easy (13) for Julián to write down the mathematical argumentation he exposes later (20) and his written answer (18) is influenced by the interviewer's commentary (14) and does not get all the concrete details given orally (20).

★ Julián A2,15-18:

15. J: Oh, pues sería éste, ¿no? El cuadrado. [Moves the pencil like inscribing the circle inside the square]

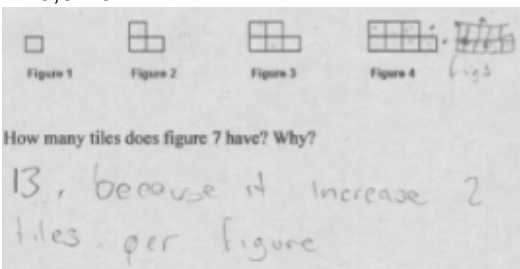
16.  [Continues in A4,1]

17. [Comes from A1,43] J: [Reading] Which of these figures has a greater perimeter. Okay. Yo utilicé porque los dos de estos... Aquí el lado de los dos es de cinco [dotted lines]. So si pudiera haber un modo en que moviéramos el círculo para acá [translates the circle dotted line to the square dotted line taking it imaginarily with the fingers], quedaría exactamente esta parte con esta parte [the butts of the dotted lines would coincide]. Y la diferencia de que esto quedaría este lado y este lado [points to the 2 tangent points that would not be in the dotted line: top and bottom]. Pero como es un círculo quedaría esto...

18.  [Julián draws the circle inside the square]

The written explanation (16) is rather vague and lacks the dynamic reasoning and the inscription (18) resulting from the translation of the circle into the square (17).

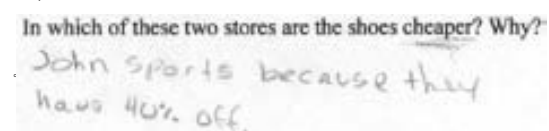
★ Julián A3,8-15:



8. How many tiles does figure 7 have? Why?
13, because it increase 2 tiles per figure
9. J: ¿Puede chequear esto a ver si está bien? El procedimiento que utilicé fue de ver aquí...
10. F: [Interrupting] Si quiere luego al final las comentamos todas.
11. J: ¿Pero sí está bien ésta?
12. F: Sí, pero si quiere luego al final las comentamos, cuando las termine. Pero sí, creo que sí está bien.
13. J: Okay. [Continues in A1,1]
14. [Comes from A1,59] [Francesc stands up to fix the camera] F: ¿Con cuál continuamos? [Julián is pointing to A3] ¿Con la tres?
15. J: You see that figure one is just one tile. So la figura dos aumenta dos más. Y en la figura tres, al igual que ésta [figure 2], aumentó dos más. So va cambiando así: [figura] cuatro aumenta dos más a la anterior. Y figura cinco, hice un ejemplo aquí [see A3,8] de la figura cinco y le subí dos más y quedó igual. So, [reading] how many tiles does [dows/] figure seven have. Le puse [reading] "thirteen because it increase two tiles per figure". So lo que utilicé fue que aquí interpreté las tiles que ocupa la figura cuatro y le aumenté dos más y ahí en la figura cinco. Dos más figura seis. Y dos más figura siete. Entonces serían seis acá y seis acá, serían doce, más una, trece.

Julián does not mention a second way to find the number of tiles of Figure 7 but he does it orally (15).

★ Aida A1,3-7:

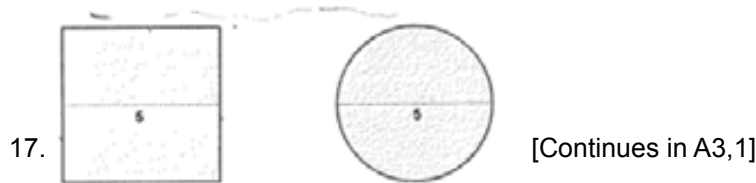
3.  [Continues in A3,2, once all activities are answered]

4. [Comes from A3,35] F: ¿Cómo resolviste la uno?
5. A: Nomás como, porque el, si tiene cuarenta por ciento discount como de, le quitan el cuarenta por ciento de lo que es y a éste le quitan el veinticinco y es... y en éste le quitan más.
6. F: Mm [validating].
7. A: Pero puede ser lo mismo porque no sale el precio de los dos. No sale precio, nomás sale cuánto le quitan.

Aida does not mention that she assumes that the initial prices are equal (3) but in the oral explanation 'de lo que es' (of what it is) she shows it (5).

★ Claudio A2,15-27:

15. C: Y dice aquí. Yo digo que es el cuadro porque tiene líneas más grandes.
16. F: Ajá. Vale. Intenta justificar la respuesta.

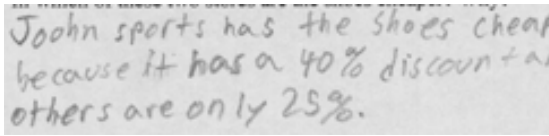


*yo digo que es el cuadro
por que tiene lineas grades.*

18. [Comes from A4,116] F: Aquí también has puesto la cruz en los idiomas, inglés y español. ¿Qué me puedes decir de esta actividad? ¿Cómo has empezado a resolverla?
19. C: Porque el círculo está más chiquito.
20. F: Ajá. El círculo está más chiquito con lo cual...
21. C: Con lo cual que el cuadro va... es lo más largo.
22. F: Ajá.
23. C: Sí, porque tiene tamaños cortos y tamaños largos. Y esto casi... Yo digo que es lo más grande. Porque tiene dos tamaños grandes y si lo comparas en una regla a lo largo pues está más grande de los lados. Más los dos del medio.
24. F: ¿Cómo? ¿Más largo de qué lados?
25. C: Porque de este lado hazte cuenta de que es una regla.
26. F: Sí.
27. C: Y si juntas las dos reglas así a lo largo, entonces lo otro largo, luego más éste largo, pues hacen lo más grande. Y luego si la mitad la comparas es casi la mitad de aquí.
[Claudio compared half of each perimeter, dividing them into 2 pieces across the dotted line –this assumption is not clearly reflected on the transcript–]

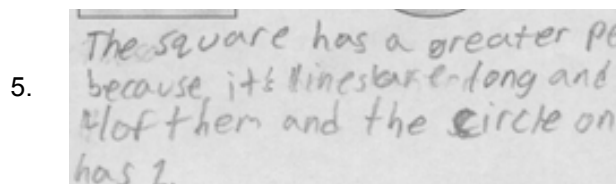
Claudio initially states that the square has a greater perimeter (15) with a rather vague oral argumentation. It is reproduced in the written answer (17), maybe because the interviewer validates it (16), but the demand of a more detailed reasoning (16) is not incorporated. With the explanation given after solving all the activities (19-27) it is clear that Claudio understands the notion of perimeter and correctly visualizes that the square's perimeter is larger (even if the square's sides are given different lengths, 2 by 2, –23–).

★ Damian A1,8-22:

8.  [Continues in A2,1]

9. [Comes from A4,1] D: I'm done.
 10. F: ¿Terminaste?
 11. D: Sí.
 12. F: Vamos a comentar un poco las actividades ahora. ¿Sí? ¿Pusiste las cruces?
 13. D: ¿Eh? Sí.
 14. F: ¿En el otro lado [of the sheet] también? ¿Con cuál quieres empezar?
 15. D: La primera.
 16. F: La primera. ¿Cómo la has resuelto?
 17. D: Que la de John Sports tiene... están más baratos porque le quitan cuarenta por ciento, le bajan cuarenta por ciento.
 18. F: Mm [continuing conversation].
 19. D: Y en el Mike Sports le bajaron veinticinco. Entonces es como una cuarta de lo que...
 20. F: ¿Cómo una?
 21. D: Como un cuarto de lo que hay.
 22. F: Ajá. Okay. ¿Qué lengua has utilizado para empezar a resolver el problema?
- Damian shows a good treatment of percentages (as a relative value, see 19-21) which is not reflected either in his written answer (8) or in his initial oral argumentation (17).

★ Damian A2,5-9:

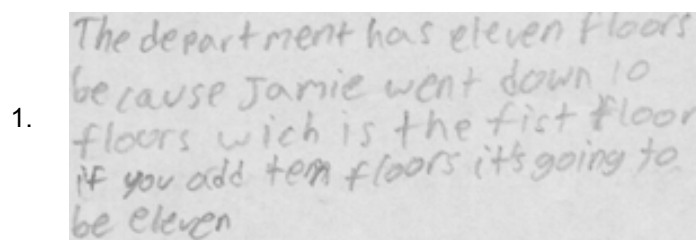


[Dialogue continues in A1,9 after solving all other activities (A3,1 and A4,1)]

6. F: ¿Cómo empezaste para resolver esta actividad?
7. D: [Unintelligible] estaba mirando las líneas de cada figura. Me fijé que esa se miraba más grande y como así hace como uno, hace como en el círculo va a estar como más grande que esto.
8. F: ¿Cuál va a estar más grande?
9. D: El del cuadrado.

Damian's written answer (5) is not clear or even correct (more number of lines does not mean longer). His oral explanation (6-9) shows a slight better treatment of the concept of perimeter.

★ Damian A4,1-3:



2. [Comes from A3,25] F: La actividad cuatro, ¿cómo la resolviste ésta?
3. D: Éste, me fijé cuántos pisos subió y cuántos bajó. Entonces ya vi cuántos subió. Pensé que iban a ser trece porque, porque bajó... Digo doce, porque bajó uno y subió uno otra vez y luego subió tres pisos y... No aca., no aca... Pensé que eran doce. ¿Cómo era? Que iban a... y luego que iban a ser catorce, porque iban los tres, cuatro. Y luego me fijé que bajó diez pisos. Y luego estaba hasta llegar arriba y bajó diez pisos. Y luego de todos los pisos que bajó, llegó hasta el primero y me fijé que estaba en el primero y lo sumé, le sumé diez y ya era el once.

In the written argumentation (1) it is not clearly stated which one is the floor added to get "eleven" (it should be the highest floor, but by the way it is written it could also be

deduced that he might be referring the bottom floor). His oral reasoning (3) sheds a little of light, but it is still a little ambiguous.

4.2.7 L2 for counting and/or operations

Some students use the second language for counting, mainly on A3. As most of the students are Spanish dominant, having the wording in English and the activities being solved in a school environment is a clear influence for using their second language. For the students who are English dominant, it is clear that the home language is one of the reasons for using Spanish.

It happens similarly with the operations. A clear influence contributing to this phenomenon is the English statement, in particular the fact that numbers are written in a graphical mode along with text (e.g. “Figure 1” on A2) and also in verbal mode (e.g. “ten” on A4).

Extracts that reflect such situations are given as follows.

★ Yolanda A4,8-17:

8. F: And why have you used Spanish in this activity [She said it on A2,62-69]?
9. Y: Because I started counting and like [pause] putting this like some words in Spanish so I can like [pause] know what they mean.
10. F: And you counted in Spanish?
11. Y: Yeah.
12. F: Have you counted in Spanish here [A4]? But not in the previous activity, for example.
13. Y: No, this one I did it...
14. F: You counted here [A3] in English but here [A4] you have counted...
15. Y: Because like... I was confused because they went up and down, up and down...
16. F: Aha. And using Spanish was easier for you in that activity?
17. Y: Mm [agreeing]. Because I just like don't think, and I just mix them, so...

Yolanda counts in Spanish on A4 (10-11), while English is the first choice of language for all four activities.

★ Camilo A4,15-41:

15. C: Luego inmediatamente fue al departamento de créditos. Ahí es uno. Luego después... se hizo...
16. F: ¿Pero es uno qué quiere decir?
17. C: Pues en dónde el primer floor, el primer piso. Ya después dice que se hizo... se puso... se hizo... éste... se... ¿cómo se llama?, éste... Se aseguró que su crédito estaba bien.
18. F: Sí.
19. C: Ya después subió un piso, a la joyería. [Marks 2 with the fingers]
20. F: ¿Por dónde entra, por dónde entra por eso?
21. C: En el... en el medio del piso, en el piso del medio.
22. F: Sí. Pero, ¿y dónde va para comprobar el crédito?
23. C: Pues allí, es el mismo nivel.
24. F: ¿Y ha contado esto como uno?
25. C: Ajá. Sí.
26. F: ¿Por qué?
27. C: Porqué aquí no dice que va arriba o va pa abajo, pero no dice que...
28. F: Ajá. Sí. Okay. ¿Qué más?
29. C: Luego ya después de ir a...
30. F: Se va parriba.
31. C: Se va parriba en la joyería, en el departamento de joyería.
32. F: Sí.
33. C: Ya después ella baja un piso más, dónde el departamento de los niños. [Marks 3 with the fingers]
34. F: Sí.

35. C: Tal vez ahí cuidan niños, o no sé. Ya son tres. Luego ella va... sube tres pisos más. Luego son seis [marks 6 with the fingers, using both hands], tres pisos más ya. Y ya después finalmente baja diez pisos... ¡Oh sí, pero ahí debería de restarlos! Después diez pisos y ahí los cuento y ya son dieciséis pisos.
36. F: ¿Entonces que dijo que tendría que hacer, restarlos?
37. C: Sí, restarlos.
38. F: ¿Lo arreglamos? ¿Arreglamos la respuesta?
39. C: Entonces son... four.
40. F: ¿Cuatro pisos tiene en total?
41. C: Yo pienso que si a... si a diez le quito seis son cuatro, porque iba yo contando diez, [shows 6 fingers] seis, ya después bajó diez, entonces le quito seis y quedan cuatro.

Camilo A4,70-71:

70. F: ¿Y a la hora de sumar los números, por ejemplo?
71. C: ¡Ah! Ahí sí, ahí sí lo sumé en español.

Camilo says he uses Spanish to count (70-71) but probably English has some influence too, as he says 'four' (39). The dialogue is in Spanish and he uses this language to count loudly and to name the number of floors Jamie goes up or down (15, 33, 35), with the help of his fingers to keep the count (19, 33, 35, 41).

★ Ana A2,16-42:

16. F: ¿Y cambió al inglés [pause] cuándo?
17. A: Cuando... éste... quise sumar y multiplicar.
18. F: ¿Ahí utilizó el inglés?
19. A: Sí.
20. F: ¿Y luego cuándo volvió al español?
21. A: Al obtener la respuesta, para ver cuál tenía más perímetro.
22. F: Y... pero, por ejemplo, para pensar pues que tenía la fórmula, ¿no?, cómo tenía que sumarlo, ¿cómo lo pensó eso?
23. A: En inglés.
24. F: ¿En inglés que tenía que sumar eso?
25. A: Sí.
26. F: ¿Me lo puede decir, un ejemplo, por ejemplo de cómo lo pensó? Alguna frase...
27. A: Pensé... Pues en la clase de matemáticas Mr Contreras nos dijo que tenemos que sumar todo lo de alrededor y así podemos sacar el perímetro.
28. F: ¿Y pensó en inglés, ahí?
29. A: Ajá.
30. F: You thought like adding the sides of the square, for example. You thought like that?
31. A: Yes.
32. F: ¿Y aquí en el círculo?
33. A: En el círculo que es la forma "A" igual pi por el diámetro.
34. F: Sí. ¿Lo pensó en inglés?
35. A: Sí.
36. F: ¿Cómo se dice en inglés?
37. A: "A" equals pi [/pi/, Spanish pronunciation] times [pause] times perimeter?
38. F: Pi [/pi/] times five times diameter, right?
39. A: Diameter.
40. F: ¿Sí? ¿Pensó en inglés? Es un poco de todo, veo, ¿no?
41. A: Ajá.
42. F: Porque el diámetro no lo sabía en inglés. Vale. Eh, ¿alguna cosita más en inglés?

Ana says she uses English to perform the operations (addition and multiplication) (16-19). She affirms that she remembers some English explanations from the class when she is solving A3 (27). Even if she says she thinks of the formula of the circle's perimeter in English, she does not say it correctly in English (she is able to say it

properly in Spanish; and she applies it correctly to get the right solution to the problem). So English may have some influence when doing the operations but they could not be performed entirely in English.

★ Ana A3,30-37:

- 30. F: En español. ¿Y cuándo cambió a inglés?
- 31. A: Cuándo estaba contándolos.
- 32. F: Sí ¿Los contó en inglés?
- 33. A: Sí.
- 34. F: ¿Por qué cree que los contó en inglés?
- 35. A: Em... [pause]
- 36. F: ¿Está más acostumbrada a contar en inglés?
- 37. A: Sí.

Ana counts the tiles on A3 in English.

★ Ana A4,58-63:

- 58. F: ¿Y qué cosas pensó en inglés, por ejemplo?
- 59. A: Los números, los éste, los pisos que bajaba y subía.
- 60. F: Sí. ¿Eso lo pensó en inglés?
- 61. A: Sí.
- 62. F: ¿Qué más? [Pause]
- 63. A: Nada más.

Ana uses English to add up all the floors Jamie goes through. It is faster than translating them to Spanish.

★ Juan A1,33-37:

- 33. J: El inglés lo utilicé para leer y entender esto y el español para escribir.
- 34. F: Pero desde que leyó la pregunta, ¿no?, hasta que la resolvió, usted me ha dicho que se fijó con los porcentajes, que se fijó también con esto de aquí [Unbeatable prices].
- 35. J: Mm [validating].
- 36. F: ¿Mientras estaba pensando todo esto, qué lengua usó? ¿O para qué usó una lengua y para qué usó otra?, si es que usó las dos.
- 37. J: El inglés lo usé para pensar cuánto descuento tenía y el español para, [pause] para anotar la respuesta, porque esto estaba en inglés y me fijé en, en el inglés.

Juan uses English with the numerical examples treating with percentages on A1.

★ Angel A3,16-23:

- 16. F: Pero la estuvo pensando...
- 17. A: En inglés.
- 18. F: En inglés. ¿Qué pensó en inglés?
- 19. A: Que si en la figura cuatro había sido siete, en la cinco va a ser nueve...
- 20. F: ¿Esto lo pensó en inglés?
- 21. A: Sí.
- 22. F: ¿Todo en inglés?
- 23. A: Sí, casi todo.

Angel A3,38-40:

- 38. F: Pero pensarlo, como... obse... mirar aquí, saber que se tenía que añadir dos, eso lo pensó...
- 39. A: En español.
- 40. F: En español. Okay. [Continues in A4, 5]

Angel GLQ 11-12:

- 11. F: ¿Y qué más utilizó en inglés?
- 12. A: Y un poquito los números en inglés. Como pensar en inglés.

Angel says he uses English to count in the arithmetical sequence associated to the number of tiles per figure (19-21). Even if later he comments that Spanish is also used in this process (38-40). On the GLQ (11-12) he confirms the use of English with numbers.

★ Abel A4,14-25:

14. F: Luego, por ejemplo, a la hora de sumar...
15. A: Le... cuándo estaba sumando, para estar bien...
16. F: Mm. [continuing conversation]
17. A: Para saber si está, si iba a estar bien le di otra pasada.
18. F: ¿Con qué idioma?
19. A: En inglés, así. Y después...
20. F: ¿Sumó one, plus three, plus ten, plus one?
21. A: Sí
22. F: ¿In English?
23. A: Sí, nomás uno más three plus ten plus one y me dió quince, fifteen.
24. F: Pero ¿al pensar eso lo pensó en inglés o en español?
25. A: Ajá... sí... a veces que plus y lo pensé en inglés y ya los números, sumándolo lo pensé en español.

English as well as Spanish have some influence on the addition performed (14-23).

★ Ingrid A3,69-72:

69. F: ¿Y cuándo cambiaste a inglés?
70. I: Cuando tenía que sumarle.
71. F: Ajá. ¿Sumaste, hiciste la suma en inglés?
72. I: Mm [validating].

Ingrid GLQ,1-26:

1. [Comes from A4,167] F: ¿En general cuándo ha usado el inglés?
2. I: Cuando tenía que leer la pregunta.
3. F: Mm [continuing conversation].
4. I: Y resolver el problema.
5. F: ¿Y por qué lo ha hecho eso con inglés?
6. I: Porque... [pause] tenía la pregunta en inglés y la tenía que resolver.
7. F: ¿Y el resolver el problema qué quiere decir [pause] que lo ha hecho con inglés?
8. I: Mmm... [thinking] [Pause] Nada más la pregunta, no sé.
9. F: Mm [continuing conversation]. ¿Alguna otra cosita en inglés?
10. I: Los... [pause] nada más.
11. F: ¿Y en español? ¿En general cuándo ha usado el español?
12. I: Cuando tenía que hacer las operaciones.
13. F: ¿Qué más?
14. I: Cuando tenía que ver el porqué.
15. F: Mm [continuing conversation]. [Pause] ¿Por qué lo ha hecho esto en español?
16. I: Porque como yo no hablo mucho inglés se me hace más fácil hacerlo en español.
17. F: Mm [validating]. ¿Y qué más cosas ha hecho en español? Me iba a decir algo más, perdone que le he cortado antes.
18. I: Las medidas.
19. F: Las medidas...
20. I: [Interrupting] Las, [pause] las fórmulas.
21. F: ¿En español también?
22. I: Mm [validating].
23. F: ¿Por qué?
24. I: Porque en inglés todavía no las he aprendido.
25. F: Okay. Más cosas en español.
26. I: Las sumas.
27. F: ¿Las sumas también en español?

28. I: Mm [validating].
29. F: ¿Por qué cree que las hace en español las sumas?
30. I: Porque [pause] eh... es más fácil para mi sacar las cantidades.

Ingrid uses English in the arithmetical sequence associated to the figure pattern of A3 (69-72). When summarizing the use of languages, Ingrid relates English with reading and also a little with the thinking processes (GLQ,1-10). Later she says operations (GLQ,11-12), justifications (GLQ,13-16), measurements (GLQ,17-19), formulas (GLQ,20-24) and additions (GLQ,25-30) are done through Spanish. So the referred English use with numbers commented on A3 occurs probably mainly in relation to the process of extracting the information from the statement but the core processes might be done through Spanish.

★ Yael A2:

Yael says she uses English to solve A2, with no mention of the use of Spanish for anything, so it should be admitted that English is used also for counting the tiles in each figure.

★ Yael A4,8-29:

8. F: Mm [agreeing]. ¿Qué idioma utilizó aquí para empezar?
9. Y: Para empezar, empecé en inglés. Empecé leyendo.
10. F: ¿Y marcó esto antes de leer la pregunta? ¿Antes de leer la pregunta marcó las palabras clave?
11. Y: No la iba leyendo e iba marcando.
12. F: Pero antes de llegar al final ya iba marcando eso.
13. Y: Sí.
14. F: ¿Antes de saber la pregunta?
15. Y: Mm [agreeing].
16. F: ¿Cuándo cambió a inglés? Digo, perdone...
17. Y: a español.
18. F: ...¿cuándo cambió a español?
19. Y: Ehm, cuándo iba a poner la respuesta.
20. F: Pero antes de poner la respuesta pues estuvo pensando todo esto, ¿no? ¿En qué idioma lo hizo?
21. Y: Pero esto lo iba pensando en inglés.
22. F: ¿Todo esto lo iba pensando en inglés?
23. Y: Sí.
24. F: ¿Todo? ¿Nunca el español para nada?
25. Y: No casi, no.
26. F: ¿Alguna cosita sí?
27. Y: ¡Ah, ah! [Saying no with her head]
28. F: ¿Todo en inglés?
29. [Yael nods]

Yael A4,114:

114. Y: ¿En inglés? Los datos que tenía como las operaciones como de sumar y restar, esto lo estaba haciendo todo en inglés.

Yael says she uses English for everything on the 1st try (8-29), which includes the operations, and also says explicitly that she uses English on the 2nd and 3rd tries for the adding and subtracting operations (114).

★ Julián A4,6:

6. J: Aquí me dice que [reading] immediately goes to the credit department. Está aquí en el middle [pointing to the CD floor on his picture], según yo. [Reading] After making sure her credit is good she goes up one floor to the jewelry department. Es aquí, uno arriba. Then she goes up... One, two, three [writes down TD, for toy department]. [1, 2,] tres, cuatro, cinco, seis, siete... [Draws 5 floors to the bottom of his drawing –see A4,14, 1st try– while counting up to the 10 floors Jamie goes down] [Pause]

Julián A4,24:

24. J: ...toy department [A4,25: writes down TD]. Finally Jam[/dzeim/], Jamie goes down ten floors to the main entrance. So entonces sería diez. Sería uno, dos, tres, four, five, six, seven, eight, nine, ten. Lo que eh... ohh Okay, wait [adds floors to the 2nd try drawing, see A4,25].

Julián uses English to count the floors while he is doing the sketch. He also uses Spanish, and in the dialogue his use of Spanish for counting is, in fact, more extended.

★ Damian A3,4-25:

5. D: Me fijé en las figuras éstas. Y las conté que estaban sumando por uno, que estaban subiendo por dos. Y luego estaba pensando que los otros cómo iban a ser por mirando acá . Porque cuatro, cómo aquí es tres y tiene nomás dos y uno de éstos. Dos arriba y dos abajo. Y éste tiene tres arriba y tres abajo. Entonces como menos, es menos uno. Y luego en el siete me fijé. Luego siete menos uno es seis. Y luego conté ésta, estos, estos seis dos veces. Luego conté el último, porque el último nomás tiene uno.
6. F: Okay. Por esto yo te vi que empezaste y escribiste muy rápido trece y luego dudaste un poco, ¿no?, en escribir “figure grows by two”. Este “by two” te costó un poco escribirlo, ¿no? ¿No? No sé si estuviste pensando otra vez, o estabas repensando algo... ¿No?
7. D: No, pues sí, estaba pensando un poquito de... me fijé... en cuanto estaba creciendo.
8. F: Mm [agreeing]. ¿Empezaste a pensarlo este problema en qué idioma?
9. D: En español.
10. F: ¿Y cómo continuaste? ¿Cuándo cambiaste de idioma?
11. D: Cuando ya empecé a contar.
12. F: ¿Cuándo empezaste a contar...?
13. D: Sí.
14. F: ¿...cambiaste a inglés?
15. D: Sí.
16. F: ¿Y luego cómo seguiste?
17. D: En puro inglés.
18. F: ¿Todo en inglés?
19. D: Sí.
20. F: ¿Y qué es lo que pensaste en español?
21. D: [Short sequence] Como al contarlos, éstos. Iba subiendo por dos, y eso. Así como contando cada vez por dos.
22. F: Mm [continuing conversation].
23. [Damian mumbles something]
24. F: ¿Cómo?
25. D: Y ya está. [Continues in A4,2]

Damian says he uses English to count (10-15) and later she refers to Spanish as the language for counting (20-21). As he explains in the beginning (5), English is likely used at to get the number of tiles of Figure 7 and Spanish is used to get the growth of the arithmetical sequence associated to the figure pattern.

★ Damian A4: Damian says he uses only English in A4, so he uses this language to find the number of floors of the department store.

4.2.8 L1 for counting and/or operations

English dominant students as well as Spanish dominant students use mainly their dominant language to count and/or to perform operations (mainly addition and multiplication, but also division and subtraction). This is consistent with previous findings (see Moschkovich, 2005).

Yolanda and Coral belong to the first group. Yolanda uses English for everything to solve the first three activities of the questionnaire, so he counts the tiles in English on A3. Yolanda uses English to add the four sides of the square in order to get its perimeter's length. She solves A2 uniquely through English. Carlos uses English for everything to solve the four activities of the questionnaire, so he counts the tiles in English on A3. Carlos and Coral use English language to solve all problems, so they also use English on A4 to find the number of floors of the building.

Jessica and Julia are Spanish dominant. Jessica counts the tiles in Spanish on A3 and the floors in Spanish on A4 (in fact English language is linked to reading in all four activities). Julia does not mention the use of English to count the tiles. There is just a slight use of the English language, so she has to count the tiles in Spanish.

Diandra uses Spanish on all of problems: she counts in Spanish on A3, adds in Spanish on A4.

Julia with A1, A2, A4

Ingrid uses mainly Spanish on A2 and she does not relates the use of English with the operations performed to find the perimeters of both figures, so the operations have to be done in Spanish.

Julián says the use of English is linked to read the wording, so he has to perform the percentages in Spanish on A1.

Aida says Spanish is the main language used when solving A4, so the operations she makes to find the number of floors must be done in Spanish.

Claudio uses English just to interpret the statements. On A1 and A2 he does not makes any operation or counting. He counts in Spanish the tiles on A3 and the number of floors in A4.

★ Yolanda GLQ,1-6:

1. [Comes from A4,21] F: And... So, this [A4] is the only exercise you have been eh... you have used Spanish, right? In general, when solving math problems, when do you think that you use Spanish?

2. Y: On the work we do right now, to solve systems of equations.
3. F: To solve systems of equations, that we are doing during class, you use Spanish?
4. Y: Yeah. I use Spanish in these ones.
5. F: Why?
6. Y: Because there are like so much numbers and also because when I talk to Diandra... Well she doesn't know a lot English so I talk to her in Spanish.

Yolanda uses preferentially Spanish in relation with numbers (1-6). But on the questionnaire she uses only English with numbers on all activities. So in class there is a clear influence of the peer interaction when Spanish is used with numbers.

★ Miriam A1,26-33:

26. F: ¿Y cuándo cambiaste? ¿Te acuerdas de cuándo cambiaste de pensar en inglés a pensar en español?
27. M: Yeah. Cuando puse menor precio aquí [40%] y mayor precio acá [25%].
28. F: ¿Cuándo pensaste en un precio concreto?
29. M: Mm [affirming].
30. F: ¿En qué precio pensaste?
31. M: Like ten dollars.
32. F: Ajá. Entonces pensaste diez dólares, no ten dollars. Tu pensaste diez do...
33. M: Diez dólares, ajá, aquí [40%] y ponga diez aquí [25%]. Entonces éste [40%] costaría menos que éste [25%]. Y si pusiera veinte acá [25%] y diez acá [40%], éste [40%] sería menos que éste [25%]. Y si los volteara entonces sería menos éste [25%] que éste [40%]... That's what I thought.

Miriam has a good management of English and she is in a mainstream class. She thinks of part of this problem in English, but when she refers to numbers and money she uses Spanish.

★ Miriam A3,8-18:

8. M: No. Nomás estaba contando los cuadritos en español, pero todo lo demás lo hice en inglés.
9. F: ¿Y nunca cambiaste a español para nada más?
10. M: Mm... No, nomás, em, empecé a leer todo en inglés y luego empecé a hacer las figuras pensando en español y luego volví a inglés acá [points to the answer].
11. F: ¿Y por qué aquí [A2] escribiste la respuesta en español y aquí [A1] también?
12. M: I don't know.
13. F: Y aquí [A3] en inglés. ¿No sabes?
14. M: No.
15. F: Empezaste aquí [A3] todos los cambios en inglés, todos los pasos del problema en inglés.
16. M: Lo leí en inglés y luego todos esos [points to the figures she drew] los hice pensando en español.
17. F: ¿Al contarlos sólo?
18. M: Ajá. Al contarlos nada más, me fijé que todos llevaban dos más. Como éste, éste ya estaba, y dos más [points to the 2 tiles added to figures 3 and 4 respect to the previous figures]. Y por cada dos nomás agregaba dos más contando en español.

Miriam uses Spanish as a counting language on A3 (8, 10, 16-18).

★ Miriam A4,21-35:

21. M: Del middle floor hasta abajo hay ehm... one, two, three, four [counts starting at the middle floor, going down]... one two, three, four, five [starts to count again, now starting at the point she finished, going up]. Five floors. Porque yo lo había hecho diez. Cinco. Pero como tuve que agregar diez [3!] [points to the 3 floors she added later] más [points to the three at the bottom she added later], seis, siete ocho.
22. F: ¿Entonces cuántos quedan hasta abajo?
23. M: No, wait. No. Son dieciséis pisos porque serían ocho acá. Tiene que tener ocho

encima, ¿no?

24. F: Bueno, si acaso luego volvemos sobre cuál es la respuesta correcta. A ver, ¿lo primero que has hecho, qué es? ¿Cómo lo has pensado? ¿En qué momento has cambiado de lengua?
25. M: En inglés y además cambié para poner la, la... mm... [points to the answer] Bueno, todo lo hice en inglés. Lo único que sí puse, fue cuando puse los números, que estaba en español.
26. F: Okay... ¿Y cuándo más en español?
27. M: Nada más eso.
28. F: Y todo lo otro, ¿todo en inglés?
29. M: Yeah.
30. F: Okay. Volvemos sobre el problema, pues, si lo quieres comentar. Creo que tienes casi la respuesta correcta, pero... A ver, porque el diagrama está bien. ...
31. M: Sí, ya entendí. Si tiene entonces ocho pisos abajo, el del medio es el octavo, entonces tienen que ser ocho arriba que son dieciséis pisos.
32. F: ¿Cuántos tiene abajo?
33. M: Ocho. Uno, dos, tres, cuatro, cinco, seis...
34. F: Pero éste no lo contamos, éste es el del medio.
35. M: Oh, yeah! Uno, dos tres, cuatro... ¡siete! Entonces serían catorce, quince pisos. [Writes '15' above '13' as final answer: A4,1]

Miriam uses Spanish as a counting language on the 1st try (25-27), and also she counts in Spanish during the dialogue on the 2nd try (21, 33, 35). But she counts in English on the 2nd try too (21).

★ Camilo A3,13-15:

13. C: No, también en español como por ejemplo cuántos, empecé cuantos ha de tener... cómo... cuánto va a ser esto más grande, cómo dos, aquí dos y aquí otros dos, dos, dos...
14. F: Sí.
15. C: Sumando, iba contando yo en español porque en inglés se me hace... Me enredo mucho. [Continues in A4,2]

Camilo counts the growth of tiles per figure in A3 in Spanish because it is easier than in English (15).

★ Camilo A4,12-39:

12. F: ¿Puede empezar a resolverlo otra vez, para que lo vea, por ejemplo? [In the discourse below Camilo starts to translate the wording to Spanish and adds up all the numbers with the help of the fingers]
13. C: Ella entró a la tienda en el medio... en el piso del medio...
14. F: Sí.
15. C: Luego inmediatamente fue al departamento de créditos. Ahí es uno. Luego después... se hizo...
16. F: ¿Pero es uno qué quiere decir?
17. C: Pues en dónde el primer floor, el primer piso. Ya después dice que se hizo... se puso... se hizo... éste... se... ¿cómo se llama?, éste... Se aseguró que su crédito estaba bien.
18. F: Sí.
19. C: Ya después subió un piso, a la joyería. [Marks 2 with the fingers]
20. F: ¿Por dónde entra, por dónde entra por eso?
21. C: En el... en el medio del piso, en el piso del medio.
22. F: Sí. Pero, ¿y dónde va para comprobar el crédito?
23. C: Pues allí, es el mismo nivel.
24. F: ¿Y ha contado esto como uno?
25. C: Ajá. Sí.
26. F: ¿Por qué?
27. C: Porqué aquí no dice que va arriba o va pa abajo, pero no dice que...
28. F: Ajá. Sí. Okay. ¿Qué más?
29. C: Luego ya después de ir a...

30. F: Se va parriba.
 31. C: Se va parriba en la joyería, en el departamento de joyería.
 32. F: Sí.
 33. C: Ya después ella baja un piso más, dónde el departamento de los niños. [Marks 3 with the fingers]
 34. F: Sí.
 35. C: Tal vez ahí cuidan niños, o no sé. Ya son tres. Luego ella va... sube tres pisos más. Luego son seis [marks 6 with the fingers, using both hands], tres pisos más ya. Y ya después finalmente baja diez pisos... ¡Oh sí, pero ahí debería de restarlos! Después diez pisos y ahí los cuento y ya son dieciséis pisos.
 36. F: ¿Entonces que dijo que tendría que hacer, restarlos?
 37. C: Sí, restarlos.
 38. F: ¿Lo arreglamos? ¿Arreglamos la respuesta?
 39. C: Entonces son... four.

Camilo A4,70-71:

70. F: ¿Y a la hora de sumar los números, por ejemplo?
 71. C: ¡Ah! Ahí sí, ahí sí lo sumé en español.

Camilo says he adds the numbers in Spanish (70-71) and he does count in Spanish loudly (15, 33, 35). But probably English has some influence too because Camilo says 'four' when giving the answer (39).

★ Juan A2,8-13:

8. F: Sí, bien. ¿Cómo empezó? ¿Con qué lengua empezó a resolver el problema en este caso?
 9. J: Con el inglés.
 10. F: ¿Con inglés? ¿Qué hizo con inglés?
 11. J: Leyendo la pregunta.
 12. F: ¿Y cómo continúa?
 13. J: Con el español, haciendo los, [pause] las operaciones.

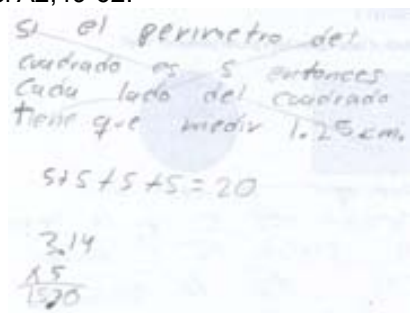
Juan performs an addition and a multiplication in Spanish.

★ Juan A3,24-27:

24. F: Pero además de escribir esto [answer] en inglés y leer esto [wording] en inglés, utilizó por ejemplo el español para contar las tiles ...
 25. J: Sí, para contar.
 26. F: ¿Lo contó en qué idioma?
 27. J: En español.

Juan uses Spanish to count the tiles in each figure.

★ Angel A2,49-62:

49.  [Entire answer (2nd, 3rd tries)]

50. A: Va a ser mayor el del cuadrado.
 51. F: Sí. Vale. ¿Me dice ahora cómo lo ha pensado eso? ¿Cuándo ha utilizado el inglés? ¿Cómo...? ¿Empezó leyéndolo? ...¿Luego cómo continuó?
 52. A: Nada más en inglés lo empecé leyendo esta pregunta y nada más.
 53. F: ¿En inglés?
 54. A: Sí.

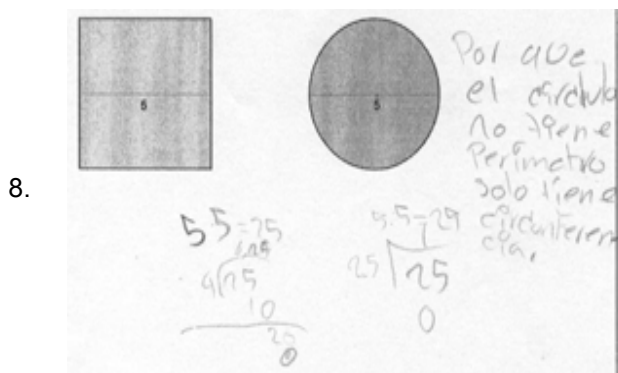
55. F: ¿Y a partir de ahí?
 56. A: Español.
 57. F: ¿Todo?
 58. A: Sí.
 59. F: ¿Puro español?
 60. A: Sí.
 61. F: ¿No pensó nada en inglés?
 62. A: No. [Continues in A3, 2]
 Angel performs the addition and multiplication (49) in Spanish (52-62).

Angel A4, 176-179:

176. F: Ajá. ¿Y cuándo volvió a cambiar a inglés?
 177. A: Ya no.
 178. F: ¿Sólo utilizó el inglés para traducirlo?
 179. A: Sí. [Continues in GLQ,1]

Angel implicitly says he uses Spanish to perform additions and subtractions on A4.

★ Abel A2,8:



Abel A2,25-38:

25. F: Se llama circunferencia... Pero ¿cuánto mide esta circunferencia?, ¿no?, es lo que queremos comparar. Si esta cricu... si el alrededor del círculo es más grande que lo de alrededor del cuadrado. O al revés, o el cuadrado, lo de alrededor del cuadrado es más grande que lo de alrededor del círculo. ¿Cómo lo pensó eso?
 26. A: En español, so...
 27. F: Pero ¿cómo lo hizo para comparar, para decir cuál era más grande? ¿Cómo hizo eso, no? ¿En qué lengua empezó a pensar eso?
 28. A: En español.
 29. F: ¿Empezó a pensar en español?
 30. A: Sí.
 31. F: Y luego continuó....
 32. A: En español. Escribiendo.
 33. F: ¿Continuó escribiendo en español?
 34. A: Sí.
 35. F: ¿Y cuándo volvió a cambiar a inglés?
 36. A: No, ya no.
 37. F: ¿Sólo en español?
 38. [Abel nods] [Continues in A1,13]

Abel does not mention any use of English (25-38) so the operations (8) must be done through Spanish.

★ Abel A3,42-65:

42. F: ¿Antes [1st try] con qué lengua empezó?
 43. A: Nada más empecé a usar el inglés para mi..., lo único que sé más o menos,
 44. F: Mm. [continuing conversation]
 45. A: Empecé a leer lo que entendía.
 46. F: Mm. [continuing conversation]
 47. A: Y nada más le puse... me fui por la lógica así de una figura que tiene siete lados.
 48. F: Ajá.
 49. A: Y nada más le puse la figura 4.
 50. F: Okay. ¿Y utilizó el inglés para leerlo?
 51. A: Mm. [validating] [Abel nods.]
 52. F: ¿Y luego para qué más utilizó el inglés?
 53. A: Nomás para eso.
 54. F: ¿Luego cambió a español?
 55. A: Sí.
 56. F: Para pensar cómo lo iba a hacer. ¿Y siguió en español?
 57. A: Sí, nomás seguí escribiendo en español.
 58. F: Sí. Y la segunda vez, ¿no? Cuándo es que le dije 'Ah, es que habíamos entendido la pregunta mal' ¿Entonces cómo, cómo hizo? ¿Empezó a resolver el problema con qué idioma?
 59. A: Con... Ya como me dijo usted aquí que si.. cuál era la figura... No, que cuántos cuadritos tenía la figura siete...
 60. F: Mm. [continuing conversation]
 61. A: Y ya lo leí en inglés y claro usted me lo ha dicho en español.
 62. F: Mm. [continuing conversation]
 63. A: Ya primero lo leí en inglés y después, ya se lo he dicho, en español lo... me... me fui haciendo la idea.
 64. F: Y el razonamiento, el procedimiento de... de solución, ¿no?, ¿esto lo pensó en qué idioma?
 65. A: En español. [Continues in A4,2]
- Abel does not mention any use of English (42-65), so the tiles of the figure must be counted in Spanish.

★ Abel A4,14-25:

14. F: Luego, por ejemplo, a la hora de sumar...
 15. A: Le... cuándo estaba sumando, para estar bien...
 16. F: Mm. [continuing conversation]
 17. A: Para saber si está, si iba a estar bien le di otra pasada.
 18. F: ¿Con qué idioma?
 19. A: En inglés, así. Y después...
 20. F: ¿Sumó one, plus three, plus ten, plus one?
 21. A: Sí
 22. F: ¿In English?
 23. A: Sí, nomás uno más three plus ten plus one y me dió quince, fifteen.
 24. F: Pero ¿al pensar eso lo pensó en inglés o en español?
 25. A: Ajá... sí... a veces que plus y lo pensé en inglés y ya los números, sumándolo lo pensé en español.

English as well as Spanish have some influence on the addition performed (14-23).

★ Ingrid A4,8-25:

8. F: ¿Cómo empezaste? ¿Con qué idioma empezaste?
 9. I: Con el inglés.
 10. F: Mm [validating]. ¿Y luego?
 11. I: El español.
 12. F: ¿Cuándo cambiaste a español?
 13. I: Cuando tenía que responder.
 14. F: Ajá. ¿Y todo el rato éste estuviste pensando en inglés?
 15. I: La pregunta la estuve pensando en inglés, mm [validating].

16. F: ¿Para hacer la suma, por ejemplo?
17. I: Mm [validating].
18. F: ¿Y para pensar que es... que tenías que hacer esta suma, lo pensaste esto en inglés también?
19. I: Sí.
20. F: ¿Se te hizo más fácil en inglés?
21. I: Poquito.
22. F: ¿O por qué lo pensaste en inglés?
23. I: Porque [pause] tenía que [pause] sacar los, las cantidades en inglés.
24. F: Mm [continuing conversation]. ¿Y luego, cómo tenías que sacar las cantidades en inglés, también lo otro lo pensaste en inglés?
25. I: Ajá.

On the 1st try of A4, Ingrid adds the quantities in English (16-17) because the quantities are given in English (23-35).

Ingrid A4, 148-167:

148. F: Okay. ¿Qué pensó aquí en inglés? [Pause]
149. I: La pregunta.
150. F: ¿Leer la pregunta?
151. I: Mm [validating]. Y el problema.
152. F: ¿Y qué más? Mientras luego estaba resolviendo, ¿no? Cuando lo leímos, yo le traduje al español, luego la pregunta también se la traduje, ¿no?, la íbamos leyendo en inglés, y luego empezó a resolver el problema, ¿no?
153. I: Mm [validating].
154. F: Me hizo todo esto que me dijo siete, y luego lo sumó... Lo sumó y luego finalmente el otro. ¿Qué pensó de estas cosas en inglés?
155. I: Cuántos subió y cuántos bajó.
156. F: ¿Esto lo pensó en inglés?
157. I: Mm [validating].
158. F: ¿Qué más?
159. I: Cuántos tenía el edificio.
160. F: ¿Cuántos tenía en total?
161. I: Quince.
162. F: “¿Eso pensó en inglés?”, era la pregunta. ¿Eso es lo que pensó en inglés?
163. I: Mm [validating].
164. F: ¿Cuántos tenía en total?
165. I: Mm [validating].
166. F: Okay. ¿Qué más?
167. I: Nada más. [Continues in GLQ,1]

During 2nd, 3rd and 4th tries of A4 Ingrid thinks in English in relation with the numbers (154-165).

Ingrid GLQ, 1-26:

1. [Comes from A4,167] F: ¿En general cuándo ha usado el inglés?
2. I: Cuando tenía que leer la pregunta.
3. F: Mm [continuing conversation].
4. I: Y resolver el problema.
5. F: ¿Y por qué lo ha hecho eso con inglés?
6. I: Porque... [pause] tenía la pregunta en inglés y la tenía que resolver.
7. F: ¿Y el resolver el problema qué quiere decir [pause] que lo ha hecho con inglés?
8. I: Mmm... [thinking] [Pause] Nada más la pregunta, no sé.
9. F: Mm [continuing conversation]. ¿Alguna otra cosita en inglés?
10. I: Los... [pause] nada más.
11. F: ¿Y en español? ¿En general cuándo ha usado el español?
12. I: Cuando tenía que hacer las operaciones.
13. F: ¿Qué más?
14. I: Cuando tenía que ver el porqué.

15. F: Mm [continuing conversation]. [Pause] ¿Por qué lo ha hecho esto en español?
 16. I: Porque como yo no hablo mucho inglés se me hace más fácil hacerlo en español.
 17. F: Mm [validating]. ¿Y qué más cosas ha hecho en español? Me iba a decir algo más, perdone que le he cortado antes.
 18. I: Las medidas.
 19. F: Las medidas...
 20. I: [Interrupting] Las, [pause] las fórmulas.
 21. F: ¿En español también?
 22. I: Mm [validating].
 23. F: ¿Por qué?
 24. I: Porque en inglés todavía no las he aprendido.
 25. F: Okay. Más cosas en español.
 26. I: Las sumas.
 27. F: ¿Las sumas también en español?
 28. I: Mm [validating].
 29. F: ¿Por qué cree que las hace en español las sumas?
 30. I: Porque [pause] eh... es más fácil para mi sacar las cantidades.

When summarizing the use of languages, Ingrid relates English with reading and also a little with thinking processes (1-10). Later she says operations (11-12), justifications (13-16), measurements (17-19), formulas (20-24) and additions (25-30) are done through Spanish. So the reference to using English with numbers when A4 is commented occurs probably mainly in relation with the process of extracting the information from the statement but the core processes are done through Spanish.

★ Yael A2,70-77:

70. F: Mm [agreeing]. ¿Cómo lo ha pensado esto? ¿Me puede decir otra vez cómo ha utilizado el inglés y cómo ha utilizado el español? ¿En qué lengua empezó a resolver el problema?
 71. Y: En español.
 72. F: ¿Empezó en español ahora?
 73. [Yael nods]
 74. F: ¿Y cuándo cambió al inglés?
 75. Y: No, todo lo hice en español.
 76. F: ¿Todo en español ahora?
 77. [Yael nods]

Yael says he uses Spanish for everything on the second try, so this includes the calculation (5 times 5).

★ Julián A3-16-39:

16. F: Okay. ¿En qué idioma empezó a pensar esto?
 17. J: In English.
 18. F: ¿Éste lo pensó en inglés?
 19. J: Yeah.
 20. F: ¿Cómo continuó? ¿Cuándo cambió a español?
 21. J: In Spanish, verdad, porque interpretar eso, o sea subirle las... subirle las... Contar cuántas tiles sube.
 22. F: ¿Lo pensó en español eso?
 23. J: [Julián nods] Yeah. Y cambié a inglés cuándo tuve que escribir la answer.
 24. F: Sí.
 25. J: So eso.
 26. F: Luego cambió a inglés otra vez.
 27. J: Sí.
 28. F: ¿Y el proceso eso de pensar que tenía que añadirle dos, ¿eso lo pensó en español?
 29. J: Oh, sí.
 30. F: Y iba pensando en los tiles, o iba pensando en alguna palabra en español o cómo lo

hacía eso?

31. J: Oh, pues cuadritos.
32. F: ¿Cuadrito pensó?
33. J: Cuadrito, yes.
34. F: Okay. ¿Alguna cosa más aquí del uso de las lenguas?
35. J: Aquí pues no mucho. La verdad es que le puse la [points to the answer] en inglés y no sé.
36. F: ¿Por qué la puso aquí en inglés?
37. J: Pues no estoy muy seguro, la verdad.
38. F: ¿Cómo?
39. J: No estoy muy seguro. I'm not sure because... At the beginning I was thinking in English and just about the process of this pattern.

Julián thinks of most of the activity in English (16-19, 23-27, 34-39), but uses Spanish for counting (20-23, 28-29).

★ Julián A4,6:

7. J: Aquí me dice que [reading] immediately goes to the credit department. Está aquí en el middle [pointing to the CD floor on his picture], según yo. [Reading] After making sure her credit is good she goes up one floor to the jewelry department. Es aquí, uno arriba. Then she goes up... One, two, three [writes down TD, for toy department]. [1, 2,] tres, cuatro, cinco, seis, siete... [Draws 5 floors to the bottom of his drawing –see A4,14, 1st try– while counting up to the 10 floors Jamie goes down] [Pause]

Julián A4,24:

25. J: ...toy department [A4,25: writes down TD]. Finally Jam[/dzeim/], Jamie goes down ten floors to the main entrance. So entonces sería diez. Sería uno, dos, tres, four, five, six, seven, eight, nine, ten. Lo que eh... ohh Okay, wait [adds floors to the 2nd try drawing, see A4,25].

Julián A4,26:

26. J: Oh my God! [Counts the number of drawn floors] Okay. To the main entrance of the store which is on the first floor and leaves to go to another store down the street. So contando los cuadros que, los floors que hice... So serían tres... y si tu sumas... diez, once. Once éstos. So... Ya.

Julián uses English to count the floors while he is doing the sketch. He also uses Spanish, and in the dialogue his use of Spanish for counting is, in fact, more extended.

★ Aida A3,4-5:

4. F: ¿Y con qué lengua has empezado a resolver el problema?
5. A: En... español.

Aida A3,31-35:

31. [Comes from A4,107] F: Por ejemplo, aquí, en el problema de antes, la actividad tres. ¿Cuándo estabas haciendo la tabla y escribiendo los números, qué pensabas cero, uno, dos, tres o zero, one, two, three?
32. A: Cero, uno...
33. F: ¿Uno, dos, tres? ¿Y equis-ye o ex-wy?
34. A: Exs-wy.
35. F: Exs-wy pero cero, uno, dos, tres. ¡Oh! [Continues in A1,4]

Aida uses mainly Spanish to solve A3, in particular to think about the number of tiles per figure and its increasing rate (31-33).

★ Claudio A3,11-22:

11. [Comes from A2,33] F: ¿Qué has hecho aquí? ¿Cómo has empezado?
12. C: Yo empecé porque siempre cada vez que yo miraba algo yo los contaba. Aquí empieza con uno, entonces le sumaron dos más y ya son tres. Son tres más dos. El otro le

sumaban dos. Luego otra vez cuenta lo mismo, son cinco. Uno, dos, tres, cuatro, cinco. Cuento los cinco, y le sumaban dos más. So así empecé.

13. F: Pero has hecho los dibujos también, ¿no?

14. C: Sí.

15. F: ¿Por qué has hecho también los dibujos?

16. C: Para poder saber cuántos y sumarle los dos más y así. Hasta cuando llegué a trece. Entonces yo digo que son trece cuadros.

17. F: Aquí también otra vez has puesto la cruz, la cruz, perdón, en inglés y español.

18. C: Sí.

19. F: El inglés, otra vez, ¿cuándo lo has utilizado?

20. C: Para poder, éste, mirar y la pregunta.

21. F: ¿Qué quieres decir para poder mirar?

22. C: Pues leerla. [Continues in GLQ,1]

Claudio says he counts the tiles (12). Later he relates English uniquely with the reading of the statement (17-22) so he must use Spanish to count.

★ Claudio A4,104-109:

104. F: ¿Qué lengua has utilizado aquí? No has puesto la cruz. [Claudio writes down the cross in the "English and Spanish" column] ¿Ambas también? ¿Qué ha sido la primera cosa que has hecho para resolver el problema?

105. C: No, nada más empecé a contar.

106. F: Empezaste a contar.

107. C: En el departamento.

108. F: ¿En qué idioma contabas?

109. C: En español.

Claudio counts the floors on A4 through Spanish.

★ Damian A3,4-25:

26. D: Me fijé en las figuras éstas. Y las conté que estaban sumando por uno, que estaban subiendo por dos. Y luego estaba pensado que los otros cómo iban a ser por mirando acá. Porque cuatro, cómo aquí es tres y tiene nomás dos y uno de éstos. Dos arriba y dos abajo. Y éste tiene tres arriba y tres abajo. Entonces como menos, es menos uno. Y luego en el siete me fijé. Luego siete menos uno es seis. Y luego conté ésta, estos, estos seis dos veces. Luego conté el último, porque el último nomás tiene uno.

27. F: Okay. Por esto yo te vi que empezaste y escribiste muy rápido trece y luego dudaste un poco, ¿no?, en escribir "figure grows by two". Este "by two" te costó un poco escribirlo, ¿no? ¿No? No sé si estuviste pensando otra vez, o estabas repensando algo... ¿No?

28. D: No, pues sí, estaba pensando un poquito de... me fijé... en cuanto estaba creciendo.

29. F: Mm [agreeing]. ¿Empezaste a pensarlo este problema en qué idioma?

30. D: En español.

31. F: ¿Y cómo continuaste? ¿Cuándo cambiaste de idioma?

32. D: Cuando ya empecé a contar.

33. F: ¿Cuándo empezaste a contar...?

34. D: Sí.

35. F: ¿...cambiaste a inglés?

36. D: Sí.

37. F: ¿Y luego cómo seguiste?

38. D: En puro inglés.

39. F: ¿Todo en inglés?

40. D: Sí.

41. F: ¿Y qué es lo que pensaste en español?

42. D: [Short sequence] Como al contarlos, éstos. Iba subiendo por dos, y eso. Así como contando cada vez por dos.

43. F: Mm [continuing conversation].

44. [Damian mumbles something]

45. F: ¿Cómo?

46. D: Y ya está. [Continues in A4,2]

Damian says he uses English to count (10-15) and later she refers to Spanish as the language for counting (20-21). As he explains in the beginning (5), English is likely used to get the number of tiles of Figure 7 and Spanish is used to get the growth of the arithmetical sequence associated to the figure pattern.

4.2.9 L2 for reading and interpreting the statement

There is a list of examples that show several students who declared having used English just to read or interpret the statement, which is in English. The cases where there is just a residual use of English for other reasons (code mixing in writing or interpreting a particular word from the statement) are also included here.

As reported, most of the learners are Spanish dominant so sometimes they may feel more comfortable using their first language. Furthermore, they have been in the United States for a relatively low amount of time. In addition, some students are not pleased with the use of English. So this finding is not surprising and was expected before starting the analysis.

These appreciations can be made throughout the entire solving process (or restricted to the mentioned activities, eventually) and in conjunction to the students Historical profile (see it on the corresponding Second reduction or a more detailed version on the First reduction). So just some extracts have been reported entirely. For the other cases see the texts on First reduction for the referred student.

★ Diandra A1, A2, A3, A4:

Diandra says she uses English to read the statements in all activities and she never mentions any other use of the English language while solving the activities.

In the case of Diandra, she knows English, but she is attached to her Mexican roots and gives a prominent role to the use of Spanish. She is confident with her Spanish language level and has a positive feeling towards it.

★ Jessica A1, A2, A3, A4:

Jessica says she uses English to read the statements on all activities.

★ Juan A2, A3:

Juan says English is uniquely used to read the statement on A2. Almost the same happens on A3, where he uses also the word “tiles” in his written answer, but Juan does not give much importance to this fact.

★ Angel A1,19-34:

19. F: Eh... De acuerdo. Me puso aquí inglés y español, ¿sí? ¿Cómo empezó resolviendo la actividad? ¿Qué lengua utilizó para empezar a resolverla?

20. A: El inglés.

21. F: Sí. ¿Para qué utilizó el inglés?

22. A: Para leer los dos [advertisements of both stores].

23. F: ¿Para leerlos?

24. A: Sí.

25. F: ¿Y para qué más?

26. A: Nada más.

27. F: ¿Luego continuó en español?
28. A: Sí.
29. F: ¿Pensando?
30. [Angel nods]
31. F: ¿Y cuándo volvió a cambiar a inglés?
32. A: Ya no cambié.
33. F: ¿No cambió? ¿Utilizó siempre el español?
34. A: Sí.

Angel says he uses English just at the beginning of the solving process to interpret the visual mode of the statement (19-24) and he does not mention any other use of English (25-33).

★ Angel A2,2-13:

2. [Comes from A1,36] F: ¿Cómo empezó aquí?, a ver.
3. A: Leyendo la pregunta.
4. F: Sí. ¿Y luego?
5. A: Después en los... en la distancia [the measurements given].
6. F: Sí.
7. A: Y después ya cambié a español. [the answer].
8. F: ¿Esto lo pensó en inglés?
9. A: Sí.
10. F: ¿La distancia también? ¿En inglés?
11. A: Sí.
12. F: ¿Y luego qué hizo?
13. A: Ya después lo apunté en español.

Angel A2,51-62:

51. F: Sí. Vale. ¿Me dice ahora cómo lo ha pensado eso? ¿Cuándo ha utilizado el inglés? ¿Cómo...? ¿Empezó leyéndolo? ...¿Luego cómo continuó?
52. A: Nada más en inglés lo empecé leyendo esta pregunta y nada más.
53. F: ¿En inglés?
54. A: Sí.
55. F: ¿Y a partir de ahí?
56. A: Español.
57. F: ¿Todo?
58. A: Sí.
59. F: ¿Puro español?
60. A: Sí.
61. F: ¿No pensó nada en inglés?
62. A: No. [Continues in A3,2]

Angel says that English has some influence while he thinks about the problem on the first try (8-12). During second and third tries, though, and given that the talk with the interviewer is in Spanish, he says that he does not think in English (51-62).

★ Angel A4,155-179:

155. F: Sí. Okay. ¿Cómo empezó para..., con qué lengua empezó a resolver el problema?
156. A: Con el inglés, cuando estaba resolviendo las palabras que no sabía en inglés.
157. F: Sí.
158. A: Fue cuando pasaba a español.
159. F: Sí.
160. A: Y cuándo leía, leyéndolo en inglés para ver las palabras que me sabía.
161. F: Sí.
162. A: Después lo traducí a español.
163. F: ¿Y qué hizo con las palabras que se sabía, dijo? Es que no lo entendí lo que me dijo. Con las palabras que se sabía...
164. A: ¿En inglés?

165. F: Sí.
 166. A: Lo hacía para traducirlo a español.
 167. F: Ajá. Okay.
 168. F: ¿Y cuál...? ¿Qué palabras subrayó por aquí?
 169. A: Como los pisos que bajó pabajo.
 170. F: Sí.
 171. A: Y los que iba parriba.
 172. F: Okay. Sí. ¿Y luego cómo continuó?
 173. A: Haciendo las operaciones.
 174. F: ¿Con qué idioma?
 175. A: Español.
 176. F: Ajá. ¿Y cuándo volvió a cambiar a inglés?
 177. A: Ya no.
 178. F: ¿Sólo utilizó el inglés para traducirlo?
 179. A: Sí. [Continues in GLQ,1]

Angel says he uses English just to read and translate the wording (155-167, 178-179). Then he says the use of Spanish is maintained during the entire solving process (172-177).

★ Abel GLQ,1-30:

1. [Comes from A1,41] F: ¿En general, cuándo ha usado el inglés?
2. A: ¡Oh! ¿Cuándo voy a las tiendas?
3. F: No, aquí, digo.
4. A: ¡Oh! Aquí nomás en las preguntas.
5. F: Entre los cuatro ejercicios.
6. A: ¿En los cuatro? En éste [no video recording at this point: unknown activity] más.
7. F: Entre todo, en general, ¿no?, al resolver éstos, uno, dos, tres y cuatro ejercicios, ¿cuándo ha usado el inglés? ¿Para qué lo ha usado el inglés?
8. A: Para leer y entender más o menos lo que sé.
9. F: ¿En qué más?
10. A: Pues nomás yo creo que para eso.
11. F: ¿Y por qué cree que sólo ha utilizado el inglés para eso?
12. A: Es que por eso están en inglés y yo me esforcé pues en leerlo en inglés.
13. F: Sí.
14. A: Pues nomás para eso y entender.
15. F: Okay. ¿Y en general cuándo ha usado español?
16. A: ¡Oh!, en escribir
17. F: Mm [continuing conversation].
18. A: Y en las traducciones del inglés.
19. F: Mm [continuing conversation].
20. A: Para eso, creo que para eso.
21. F: ¿Y para pensar cómo iba a poner la respuesta?
22. A: ¡Oh, sí!
23. F: ¿Cómo lo ha pensado?
24. A: En español.
25. F: ¿Y por qué cree que ha utilizado el español en estos casos?
26. A: Porque yo creo que así es más, para mí es más fácil.
27. F: Ajá. ¿Le resulta más fácil?
28. A: Sí, que en inglés.
29. F: ¿Alguna otra razón?
30. A: No.

Abel just uses English in a code mixing in writing (A1, A3.1, A4) and in conjunction with Spanish in operations (in A4). Apart from these instances, there is a clear dominance of the use of Spanish when solving all four activities as reflected on the GLQ (1-30)

transcript, where he does not even mention any other use of the English language other than to read the statements (7-10).

★ Julia A1, A2, A3, A4:

Julia says the use of English in A1 is restricted to interpret the statement. Almost the same occurs on A2, where English is used only to think about the formula of the circle's perimeter. And almost the same happens on A3, with English used in a code mixing in writing ("tiles") and in the thinking process in relation with 'figure'. On A4 Spanish is also mainly used while thinking, but both languages are used in the written answer.

Note that in the case of Julia her English language management is rather that of a beginner: she even says that she does not know much English at the beginning of the interview. However, she puts a lot of effort on the tasks and solves all of them.

★ Julia A1:

Julia says she uses English to read the statements in A1.

★ Claudio A1,24-41:

24. F: Okay, okay. ¿Qué lengua has utilizado para comenzar a resolver el problema?

25. C: La o... el español.

26. F: En español. Aunque está en inglés. Está en inglés por eso, ¿no?

27. C: Sí.

28. F: Y has cambiado de lenguas. ¿Has cambiado de lengua?

29. C: ¿Cómo?

30. F: De español, has empezado en español pero luego has utilizado el inglés también.

31. C: ¿Dónde?

32. F: Para resolver la actividad uno.

33. C: Sí, por eso es inglés and Spanish. Por eso lo puse.

34. F: Sí. ¿Y cuándo has utilizado el inglés?

35. C: Pues para leer.

36. F: ¿Para leerlo sólo?

37. C: Pues sí.

38. F: ¿Nunca pensabas las palabras en inglés? ¿No las pensabas en inglés?

39. C: No.

40. F: Y... Pero... Entonces dices que has empezado con español. Aunque lo has leído en inglés, pero has empezado a pensar el problema en español.

41. C: Sí. [Continues in A4,52]

Claudio says he just uses English to read the statement, even if the interviewer insists in other possible uses of the English language.

★ Claudio A2, A3, A4:

Including the extract above about A1, Claudio says he uses English to read the statements in all activities.

4.2.10 Influence of the visual mode on the mathematical procedures

On A1 a majority of the students initially compare percentages directly (as if they had an absolute value instead of the relative value they relate to). This direct comparison is made by Carlos, Coral, Camilo, Diandra, Jessica, Ana, Angel, Ingrid, Aida (later she corrects it), Claudio and Damian. Beyond the underlying mathematical misunderstanding, this can be because both pictures have the same kind of shoes and also due to the prominence of the percentages in the advertisements. On the other hand, the impact of the information given by means of visual modes may be thought of as a result of the weaker role played by the English language. That is, some of the students may be paying special attention to the visual mode as a way of overcoming anticipated difficulties that they might find with the ordinary wording of the problem when it is in English. Although this is a reasonable assumption, the fact is that students who learn in their dominant language also tend to pay special attention to visual data.

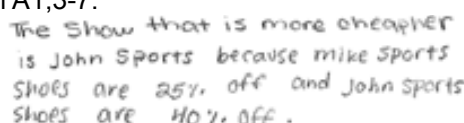
A similar finding is located on A2, where some students do not compare both perimeters as demanded on the statement but just calculate both of them. The presence of the word “perimeter” along with both figures (the square and the circle) prompts this association, omitting the word “greater”, which forces the comparison. As commented on Chapter 3, the middle school students have been calculating the perimeter in the mathematics class.

There is also a related finding on A3, where some students search which of the figures has seven tiles, looking for the answer directly on the visual mode of the statement, instead of projecting the continuation of the series of figures up to Figure 7. It happens with English dominant students (Coral) and also with Spanish dominant students (Ana).

Some of these cases show that a deficit view of the English language learners should be avoided initially, as they have the potential to explain their reasoning and could provide rich new insights on the problem that were not considered initially, even if the understanding of the problems is not always as intended, or else just precisely because of such an understanding.

The extracts below reflect the commented relationship between the visual mode and the mathematical procedures.

★ Coral A1,3-7:

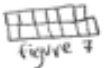
3.  [She quickly writes down the answer.]

4. [Comes from A4,14] F: How did you solve activity one?
5. C: Which one do I pick?
6. F: Excuse me? Yeah! How have you solved it. What did you think when solving it?
7. C: Because like... the more... It's still the same brand, but just a little cheaper in different stores and there is no difference on which store it is, because you are still getting the same thing, but just a little cheaper. And then you just...

Coral reflects the influence of the wording pictures in assuming the same initial price on both stores, as both have the picture of the same shoe (7).

★ Coral A3,1-9 :

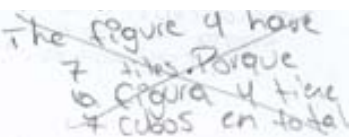
1. [Dialogue about activities starts here] C: I don't understand this one. There's four figures and [the wording] says figure seven.
2. F: Yes. So what's the problem?
3. C: [Reading] How many tiles does figure seven has and why.
4. F: So you have to figure it out. Here you have only figures one, two, three, four. And they are asking for figure seven.
5. C: Oh!
6. F: All right? How many tiles does figure seven have?
7. C: Do I draw the figure?
8. F: If that helps you, yes, you can do it.

9.  figure 7 has 13 squares because just by piling figure 6 on top of figure 7 and then you get your solution.

[Coral draws directly figure 7 without drawing figures 5 and 6. Dialogue continues in A4,1]

Coral does not know how to start the mathematization of the problem and she asks for the meaning of the wording (1-5). Once it is understood she correctly solves the problem (9).

★ Ana A3,1-15:

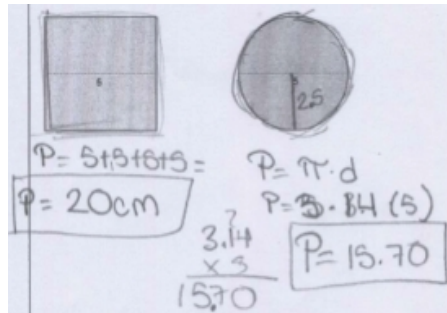
1.  [Ana makes some pauses during her writing. There might be a pause (the video is not completely clear) after writing the first sentence –which is in English. Crossing out the answer takes place in A3,26.]
2. F: ¿Cómo resolvió la [actividad] 3?
3. A: Conté cuántos cubos salió, cuántos cuadrillos había en cada figura.
4. F: Ajá. ¿Y luego qué?
5. A: Pues la pregunta dice que cuál de las figuras tenía siete cubos. Y yo puse que la figura cuatro.
6. F: ¡Ohhhhh! No, la pregunta dice, eh, [reading] how many tiles does figure seven, es la figura siete, se refiere a la figura siete. ¿Cuántos, tiles sabe qué son?
7. A: Cuadros o cubos.
8. F: Sí, cuadrillos.
9. A: Sí.
10. F: Cuántos cuadrillos tiene la figura siete. [Pause] ¿Sí entiende lo que le pregunta aquí?
11. A: No.
12. F: O sea, la figura siete, aquí no tenemos la figura siete, ¿sí?
13. A: Sí.
14. F: Pero usted puede averiguar, esto es lo que le pregunta, ¿puede decir cuántos cuadrillos habría en la figura siete? Y por qué. ¿Sí? ¿Lo quiere pensar otra vez?
15. A: Ajá.

Ana understands the wording question in a deviated way, which, of course, leads her to a different mathematization (1-5). The fact that she finds a mathematical solution could

reinforce her (deviated) understanding of the wording. Many translations and interpretations are given (6, 12, 14) before Ana gets the right interpretation of the wording (14). This is likely because she focus on her own initial understanding.

She has a right operational meaning for “tiles” even if the Spanish translation is not so accurate ('cuadros o cubos') (6-9).

★ Juan A2,1-5:



[Juan reviews A2 after finishing with all activities, remarking the perimeter of both figures with the pencil, as can be appreciated.]

1. [Comes from A1,39] F: ¿Cómo resolvió esto? ¡A ver!
3. J: Porque dicen que encuentren el perímetro, y el perímetro de un cuadrado es cinco más cinco, [...] lo de... lo de alrededor. Y son cuatro lados, se multiplica por cinco. Suma cinco más cinco, más cinco, más cinco y lo que suma son veinte. Y del círculo es, el área es pi por diámetro y es perímetro igual a tres punto catorce por quince, por cinco, y lo que salió fue quince punto setenta.
4. F: ¿Entonces cuál es el que tiene el perímetro más grande?
5. J: El cuadrado.

Juan does not compare both perimeters, either on his written answer (1) or on his oral explanation (3) because he understands the question in a deviated way (3). Probably the influence of the visual mode has a major impact on this deviated comprehension. Of course he quickly makes the comparison after interviewer's inquiry.

★ Julia A1,1-29:

$$\begin{array}{r}
 \cancel{40\%} \\
 + \cancel{25\%} \\
 \hline
 65\%
 \end{array}
 \quad
 \begin{array}{r}
 25 \\
 + 25 \\
 \hline
 50
 \end{array}$$

[0 try crossed out: $40\% + 25\% = 65\%$]

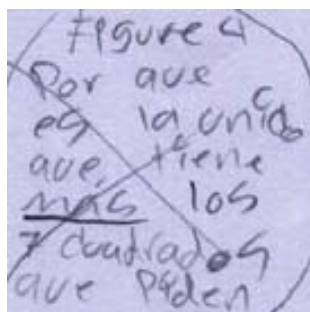
1. $\frac{50\% \text{ en dos shoes}}{\text{cheaper.}}$
2. F: ¿Cómo resolvió la actividad?
3. J: Me preguntan cuánto es en dos store de [pause] de [pause] mmm [pause] de zapatos
4. F: Mm [continuing conversation].
5. J: Cuánto son en dos. Y si el precio total es de un zapato es veinticinco en dos van a ser cincuenta.
6. F: ¿Cincuenta por ciento?
7. J: Mm. [validating, low voice]
8. F: Pero lo que le pregunta es, a ver, esto cree que es [pause] eh [pause] Bueno, de hecho la pregunta es ¿En cuál de éstas dos tiendas, ¿sí?, son los zapatos...? ¿Cheaper sabe qué quiere decir?
9. J: No.
10. F: Más baratos. En cuál de las dos tiendas son los zapatos más baratos. Esa es la pregunta, ¿sí? Pero bueno, dígame qué es lo que ha hecho.

11. J: Sumé veinticinco más veinticinco.
12. F: Ajá.
13. J: Y me dio cincuenta.
14. F: Mm [validating]. ¿Y luego qué dice? [Reading] “Cincuenta [por ciento] en dos shoes cheaper”. ¿Qué creía que significaba shoes cheaper?
15. J: Más barata.
16. F: ¿Significaba eso, creía antes? ¿Creía eso antes?
17. J: No.
18. F: ¿Por qué puso entonces... ? ¿Qué significa la frase esa?
19. J: Pues es que no me la sé.
20. F: Ajá. ¿Pero por qué lo puso entonces si no sabía, no sabía el significado?
21. J: Nomás.
22. F: ¿Qué creía que era cheaper?
23. J: No sé, no me lo imaginaba que era más barata.
24. F: ¿No? ¿Pero qué pensaba que podía significar eso?
25. J: Pues la marca de los zapatos.
26. F: ¿La marca? [Julia nods] ¡Aah! ¿Y antes por qué me sumó estos dos? ¿No? Porque antes de hacerme esto me sumó estos dos [see A1,1]. ¿Por qué?
27. J: Porque pensé que decía que cuántos eran en estos dos, cuál era el porciento.
28. F: Ajá.
29. J: De esos dos.

Maybe the fact that there is one shoe in each store leads Julia to add the percentages (1, 27) and also makes Julia think that “cheaper” is the brand of the shoes (25).

★ Abel A3,1-9:

1.



[The answer is circled and crossed out later, see A3,32]

2. [After A1,11, a solution for all activities is written down. Then dialogue continues here:] F: ¿Cómo empezó a resolver esto?
3. A: Nada más fue viendo porque, y ya, si cuál de, cuál figura tenía siete lados.
4. F: ¡Oh! No, la pregunta dice... Bueno, dígame lo que ha hecho, luego le, le digo.
5. A: Nomás le puse que si cuál figura tenía siete lados.
6. F: Sí. ¿Siete lados o siete cuadrados?
7. A: Siete cuadrados.
8. F: Pero lo que decía realmente es ¿qué figura...? mm... ¿Cuántos cuadraditos tiene la figura número siete?
9. A: ¡Oh!

Abel understands A3 in a deviated way (figure with seven tiles), but he is comfortable with his interpretation as he finds a mathematical procedure to solve the activity that corresponds to his very interpretation.

★ Julia A2,1:

1. [The line crossing out the circle's perimeter was added later: A2,54]

Julia A2,54:

54. [Julia calculates the perimeter of the circle –applying the formula– and crosses out the previous answer.]

Julia does not compare the perimeter's length on any of her two tries (1, 45) to solve A2, neither does the interviewer ask for this comparison. She probably thinks all she needs to do is to calculate both perimeters.

★ Julia A3,1 :

1. [Julia circles figure 4 at the beginning, when she understands the question in another way, see A3,17-18. She circles the word “tiles” from the wording later (A3,38)]

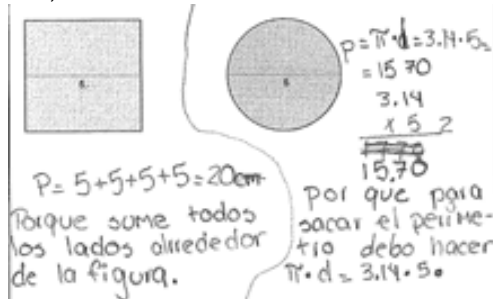
Julia A3,17-26:

17. F: ¿Por qué me puso este círculo aquí al principio [A3,1 (0 try)]?
18. J: Porque pensé que me preguntaban en cuál figura eran, había siete cuadritos.
19. F: ¿Y luego cómo supo que no le preguntaban eso?
20. J: Porque me le fijé aquí que cuántos habían en la figura siete.
21. F: ¿Leyó la pregunta otra vez?
22. J: Mm [validating].
23. F: ¿Y la entendió de otra manera?
24. J: [Julia nods.] Sí.
25. F: Nada más entonces al principio pensó que qué figura tenía siete cuadritos entonces pensó que la respuesta era esa, ¿sí?
26. J: Sí.

Figure 4 is circled on the 0 try, when Julia understands that the aim of the problem is to find a figure with 7 tiles. She gets a deviated wording question understanding (figure

with 7 tiles) with a consequent deviated answer (mutual influence of once into the other) (18); which is corrected by herself before giving the final answer.

★ Ingrid A2,1-4:



- 1.
2. [Comes from A1,56] I: En ésta tuve... Lo hice casi en... Usé los dos también en ésta.
3. F: ¿Podemos ir revisando cómo lo hizo paso a paso y qué lengua utilizó? ¿Cómo empezó?
4. I: En ésta empecé leyendo la pregunta, qué me pedía. Y me pedía el perímetro. Y vi las medidas y después sumé los lados para sacar el perímetro.

Ingrid just calculates both perimeters but does not compare them (1) as it is the way she understands the problem: according to her, what is needed is just to calculate both perimeters (4).

★ Ingrid A3,1-9:



1. How many tiles does figure 7 have? Why?
 figura 1 y 9
 por que las otras dos saldrían mas que 7.

[The lines crossing out the answer are added on the 2nd try answer (A3,56)]

2. [Comes from A2,65] F: ¿Qué hizo?
3. I: Ver cuál podría salir siete.
4. F: ¿Qué quiere decir cuál podría salir siete?
5. I: ¿Podrían salir siete cuadros?
6. F: ¿Cómo podrían salir?
7. I: Poniendo éste [Figure 1] siete veces, y éste [Figure 4] cómo ya tiene siete.
8. F: Ajá.
9. I: Porque éste [Figure 2] no se podría porque sumando otro serían seis, y si pongo aquí otro, ya serían nueve. Y éste [Figure 3] también ya se pasaría.

On the oral explanation (7-9) it is clear how Ingrid understands the statement question (1): 'With which figures (combined with themselves as many times as desired) 7 tiles can be obtained?'. As she finds a solution through a mathematical procedure, the deviated understanding of the statement is reinforced.

★ Aida A1,3-7:

In which of these two stores are the shoes cheaper? Why?

8. John Sports because they have 40% off.

[Continues in A3,2, once all activities are answered]

9. [Comes from A3,35] F: ¿Cómo resolviste la uno?

10. A: Nomás como, porque el, si tiene cuarenta por ciento discount como de, le quitan el cuarenta por ciento de lo que es y a éste le quitan el veinticinco y es... y en éste le quitan más.
11. F: Mm [validating].
12. A: Pero puede ser lo mismo porque no sale el precio de los dos. No sale precio, nomás sale cuánto le quitan.

Aida does not mention that she assumes that the initial prices are equal (3) but on the oral explanation 'de lo que es' (of what it is) she shows it (5). As she is aware that no initial prices are given with no interviewer's help (7) it could be that a focus on the visual mode leads her to make the initial assumption of the prices being the same on both stores.

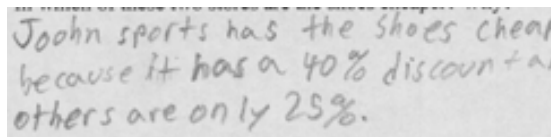
★ Claudio A3,1-5:

1. [Comes from A2,17] C: Dice "Which figure has... " no, "How many [tiles] does figure seven have? And why. Y a[quí]... pero aquí no es el siete.
2. F: Ajá. So you have to imagine.
3. C: Can I just write it here, or...?
4. F: Yes, yes, you can write. If you need scratch paper I can give you.
5. C: Yeah.

Claudio asks for figure 7 (1). As it is not depicted on the statement, he is not sure about the mathematization needed to advance. When the interviewer says he has to pursue the pattern (2) he immediately knows how to continue (3).

★ Damian A1,8-21:

8.



[Continues in A2,1]

9. [Comes from A4,1] D: I'm done.
10. F: ¿Terminaste?
11. D: Sí.
12. F: Vamos a comentar un poco las actividades ahora. ¿Sí? ¿Pusiste las cruces?
13. D: ¿Eh? Sí.
14. F: ¿En el otro lado [of the sheet] también? ¿Con cuál quieres empezar?
15. D: La primera.
16. F: La primera. ¿Cómo la has resuelto?
17. D: Que la de John Sports tiene... están más baratos porque le quitan cuarenta por ciento, le bajan cuarenta por ciento.
18. F: Mm [continuing conversation].
19. D: Y en el Mike Sports le bajaron veinticinco. Entonces es como una cuarta de lo que...
20. F: ¿Cómo una?
21. D: Como un cuarto de lo que hay.

Damian does a good treatment of percentages, as a relative value (19-21). So he assumes equal initial prices on both stores (8, 17). Probably the fact that both stores have the picture of the same shoe influenced this reasoning.

4.2.11 Word mismatching and mathematics-language loop

Sometimes there is a partial or deviated understanding of the wording (or the wording question). But somehow a solution to the activity is found, normally by applying a well-known mathematical procedure. That is, a mathematical meaning is given to the (solving process of the) activity. This (deviated) mathematical comprehension reinforces the (deviated) wording understanding. Brief, the statement comprehension is grounded on the mathematical procedure. Then both, mathematical and language comprehension are validated.

This is related with what has been commented on the previous subsection (“Influence of the visual mode on the mathematical procedures”, page 443), where the loop is between mathematics and its related interpretation of the visual mode of the language. These cases are not listed again here.

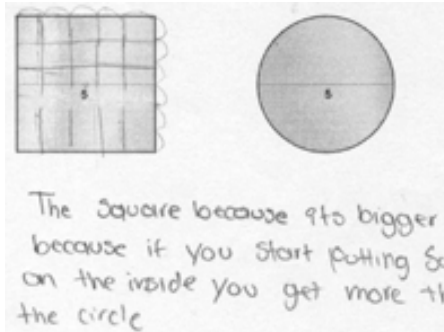
Sometimes the given meaning is quite close to the intended meaning. For example, Ingrid does not compare both perimeters' lengths on A2, but she does calculate them (see the extract on the previous subsection). Other times the interpretation of the wording is quite far from what is established on the statement. For example, Julia makes a proportion on A4.

A reason for this issue to occur can be that the safety on applying mathematical procedures derives on safety towards the language comprehension (less consolidated a priori). Somehow there is a way to go forward. Of course, the misunderstanding of the statement is also an English language issue. What consolidates all this deviances is that the aim of solving the problem –produce a solution (even if it is not known that it is wrong)– is found.

Related with this mathematics-language loop it has been found a particular phenomena with the mismatching of the word “unbeatable” and 'tabla'; the first one present in one of the advertisements for shoes. The decomposition into un-bea-table seems to foster the search for “tablas” in the context of the first problem in the questionnaire. It is clear than a deviated translation of a word can mislead the solving process due to the assumption of incorrect information. It is even more important when this happens with a key word whose understanding is crucial for the comprehension of the problem. Juan, Abel, Angel and Yael's extracts of the dialogues are presented below to exemplify the concrete case of word mismatching between “unbeatable” and “tabla”, along with other examples representing other mathematics-langauge loops.

★ Yolanda A2, 15-37:

15. F: So what is the perimeter?
16. Y: The sides? [Makes (imaginarily) 2 semi circumferences to point out 2 sides of the little squares she did (such sides are also part of the top side of the square)]
17. F: Mm [agreeing]. Just the sides. You wanna to rethink this activity right now, in a couple of minutes?
18. Y: Yeah.
19. F: Okay.
20. Y: [Makes like 2 semi-circumferences to point out like 2 sides of the little squares she did, which have as top side the top side of the square, as in A2, 25] So the sides are the ones you go like this: one, two, three, four, like this? [Makes a total of 6 imaginary semicircles as she refers to the length of the sides of the little squares, starting from the top left vertex and following the clock sense. She counts up to 4 when she reaches the other vertex.]
21. F: Mm [agreeing].
22. Y: Okay. [Draws 5 semi-circumferences on the top square's side and 6 in the right square's side. Makes the square divisions bold]
23. F: But are you doing squares again?
24. Y: Yeah. Because I know how to do it like this.



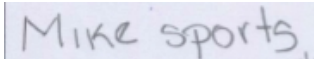
[Final written answer]

25. [Yolanda continues and finishes making up to 5 vertical divisions. Draws 3 horizontal divisions, erasing the 3rd one, which was in the 4th division marked with semi-circumferences. The dotted line can be considered as aligned with the 3rd division.]
26. F: Sorry, just a question. I don't know if you have understood what I wanted to tell you. What's the perimeter for example of the [following with the finger the perimeter of the paper which she is using to solve the activity]... of this paper?
27. Y: What?
28. F: The paper... Just... Eh... Just mark what's the perimeter of the paper.
29. Y: All the sides?
30. F: All the sides together, right? The outside of the shape we can say. Okay? Okay.
31. Y: Yes.
32. F: So now, go ahead.
33. Y: So it's a four right here or what?
34. F: What do you mean? Four? We have four sides, right?
35. Y: Yeah.
36. F: But how long is gonna be the perimeter?
37. Y: Ookay.

After Yolanda states that the procedure she applied is related with the area, the interviewer asks for the meaning of perimeter (15). In her answer –'The sides?' (16)– the notion of perimeter is not consolidated, and the interviewer reaffirms it by using more or less the same words (17). As the procedure employed later by Yolanda (22, 25) does not seem related with the perimeter, the interviewer asks about it (23) and points to the perimeter of an actual sheet of paper as an example (26), but Yolanda gives the same definition of perimeter 'All the sides?' that the one provided at the beginning. It is not until the word 'long' is provided (36) that she understands the notion of perimeter. As commented on Chapter 3, it can be related to the fact that 'long' is

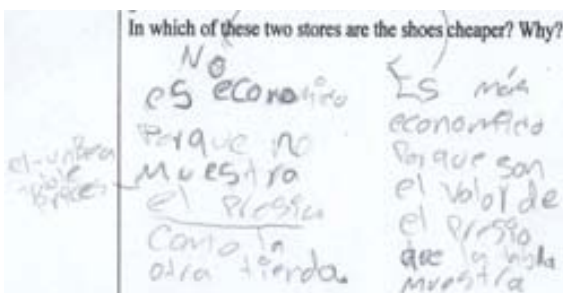
more specific and directly related to the concept of perimeter than the word “greater” present on the statement.

★ Juan A1,1-17:

1.  [Juan reviews the problem after reviewing A4 but makes no changes]
2. F: A ver, ¿cómo resolvió esta actividad? [Pause] ¿Cuál fue lo primero que pensó? ¿Qué fue?
3. J: En el descuento.
4. F: ¿Y qué hizo luego?
5. J: Me fijé cuál ofrecía más. [Pause] Y... [pause] Mmmm... [pause]
6. F: Se fijó en cuál era el que ofrecía más ¿y qué?.
7. J: En cuál... y en esto que decía aquí [“Unbeatable prices”]. Y ofrecía más éste [25%].
8. F: ¿Por qué?
9. J: Por unbeatable [/un-bi-teibol/] price...
10. F: Sí. ¿Qué significa eso?
11. J: No...
12. F: ¿No decía que se fijó en eso? ¿Con qué, con qué pensó luego? ¿No? ¿Porqué dice “why”? Puede quizás justificar la respuesta.
13. J: ¿Porque depende de la tabla de precios [pointing to “Unbeatable prices”]?
14. F: ¿Pero qué significa unbeatable prices? ¿Sabe?
15. J: No.
16. F: Significa precios inmejorables. Que nadie los puede superar, ¿no? Son unos muy buenos precios. ¿Sí entiende ahora?
17. J: Sí.

Juan uses “unbeatable prices” to think about the problem (7). He translates it in a deviated way as ‘tabla de precios’ (13), which leads him to assume that the 25% store has lower initial prices (before the discount is applied). He has a right notion of percentages, as a relative value.

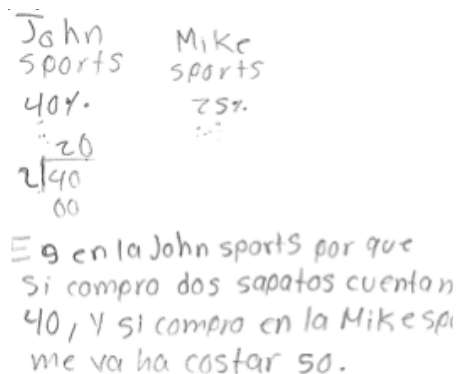
★ Abel A1,12-32:

12.  [Abel writes firstly the right column, then solves other exercises and finally writes the left column]

13. F: ¿Cómo empezó aquí a pensar el problema?
14. A: Aquí empecé por la Mike [/meik/] Store.
15. F: ¿En qué se fijó?
16. A: En el porcentaje y en las nis... unbeatable prices [/unbeitebol preis/]. El precio que está en la tabla.
17. F: ¿Cómo?
18. A: El precio de la... ¿Cómo se llama? Esto de la... Un-beat-
19. F: ¿Unbeatable?
20. A: Ajá.
21. F: ¿Qué significa?
22. A: Así...no....
23. F: ¿Cómo lo entendió eso?
24. A: Esto namás entendí que el, que el precio de la... mesa...
25. F: Mm [continuing conversation]

26. A: Que está arriba de la mesa. Así lo pensé. Que estaba arriba.
 27. F: F: Esto significa eh... Unbeatable, insuperables. Precios insuperables, ¿no? Que son los mejores precios.
 28. A: ¡Oh!
 29. F: Es como un anuncio, ¿no? Tenemos los mejores precios.
 30. A: ¡Oh! Y puse [reading] es más económico porque son, porque son el valor del precio de, que... que la tabla muestra.
 31. F: Mm [continuing conversation].
 32. A: Y en la otra le puse [reading] no es económico porque no muestra el unber... unbeatable [/un-bei-teibol/] prices [/preis/], el de aquí [A1,12, part furthest to the right]. Que si no... Porque no te son los mejores precios, no tiene el anuncio como esa tienda de acá. Abel uses a deviated translation of "Unbeatable prices" (16-26) as 'el precio que está en la tabla' (16) (the price that is on the table), which has a central impact on the answer (12, 32).

★ Julia A1,45:

45. 
[2nd try written answer]

Julia A1,74-87:

74. F: ¿Por qué? [pause] ¿Por qué lo ha dividido usted entre dos? Simplemente dígame porqué lo ha dividido. Pues yo pensaba dividirlo entre dos porque...
 75. J: Mmm... [pause] Porque... [pause] Porque si cuarenta es el... [pause] cuarenta es un entero, quiero ver cuánto, cuánto vale la mitad. Lo dividí entre dos y me salió veinte.
 76. F: Mm [continuing conversation]. ¿Y cómo utiliza luego ese veinte?
 77. J: Lo utilizo... esto lo utilizo para saber el precio, el porcentaje que... [pause] el porcentaje total.
 78. F: Mm [continuing conversation].
 79. J: Y ya.
 80. F: Y luego, a ver... ¿Con el Mike, qué pasa con el veinticinco?
 81. J: Esto sí son el veinticinco por ciento que te descuentan.
 82. F: Mm [continuing conversation].
 83. J: Y ya.
 84. F: Entonces, ¿me puede leer la respuesta?
 85. J: [Reading] Es la John Sport porque si compro zapatos me... [pause] ¡Ah!, aquí no era cuentan.
 86. F: ¿Qué era? [pause]
 87. J: Si compro dos zapatos me, me descuentan [pause] el cuarenta por ciento. Y si compro en la Mike Sports me, me descuentan el veinticinco por ciento. [pause]

Julia A1,102-111:

102. F: ¿Por qué pensó antes que le rebajaban la mitad?
 103. J: Porque pensé que ese no era el porcentaje que te rebajaban.

104. F: ¿Pero por qué motivo? ¿Qué vio o qué le hizo pensar eso que no... que no era el final sino que tenía que dividirlo entre dos?

105. J: En esta palabra, price.

106. F: ¿Entonces, esta palabra que le indicó, price?

107. J: Que quería saber el precio total.

108. F: Esto le indicó que esto era el precio total [25% store]. ¿Y aquí [40% store] como no estaba esta palabra, pensó que éste era sólo la mitad?

109. J: Ajá.

110. F: Y tenía que dividir esto entre dos. [Announcement interruption] ¿Qué significa esto, unbeatable prices? [Pause] ¿Lo sabe? ¿Por qué no lo ha pregu...?

111. J: Es la tabla de... [pause] No lo sé.

Apparently there is a price-percentage confusion (45) and also a count-discount confusion (85-87). I.e., percentages are not regarded as the portion to be discounted but as the portion of the price that should be payed. So the use of the % symbol is omitted on the answer (45).

A deviated meaning on “Unbeatable price” (110-111) leads the mathematical thinking of Julia in relation with the wrong interpretation of the percentages (102-109).

★ Julia A4,1-2:

1.

$\frac{1 \text{ floor}}{1} \rightarrow 3 \text{ floors}$
 $1x = 3$
 $\frac{1x}{1} = \frac{3}{1}$
 $x = 3$

3 floors in department.

Por que si en 1 floor es 1 en tres ✓

2.

$\frac{1 \text{ floor}}{3} \rightarrow 10 \text{ floors}$
 $1x = 30$
 $\frac{1x}{1} = \frac{30}{1}$
 $x = 30$

30 floors in department.

Por que si en 1 floor son tres en 10 van hacer 30 floors por departamento.

Julia A4,35-40:

35. F: Floors, ¿Qué son floors?
 36. [Julia says no with her head.]
 37. F: Varios pisos, ¿no? Varios pisos. [interviewer makes a movement with the hands, simulating many floors of a building.] Es un departamento muy grande que tiene muchos pisos. ¿Qué más? [Pause] ¿Dónde nos dice qué en cada piso hay tres departamentos?
 38. J: Aquí [pointing to wording, around “up three floors”]. En un floor hay [pause] yo interpreté uno sobre tres.
 39. F: ¿Ha entendido lo que está haciendo Jamie en departamen... en el centro comercial? ¿Ha entendido qué es lo que nos describe el enunciado, qué es lo que pasa?
 40. J: ¿Que no encuentra la salida?

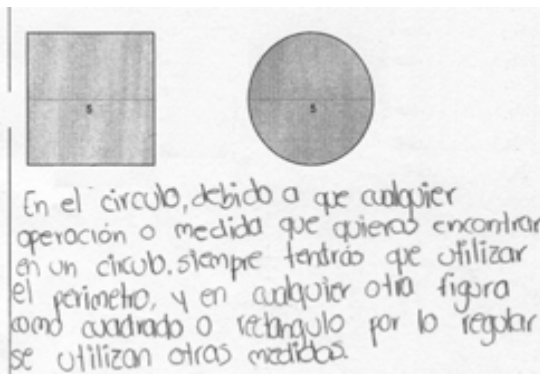
“Floors” plays a crucial role on the mathematical solving process: Julia does not completely understands the situation presented in the wording (40), nor the wording question. However, she produces a mathematical solving process using a well-known procedure –a proportion (38)–, covered in class. She does a proportion on her two attempts to solve the problem 0 try, 1st try –with no interviewer’s interaction (1,2)–. The finding of a solution validates her deviated wording understanding. A translation of just some words, without giving the correct meaning to the sentence as a whole: ‘uno sobre tres’ (one out of three) (see A4,37-38) –probably involving the wording words “up” and “three”– is another cause for this deviated wording understanding.

★ Yael A1,20:

20. Y: Okay. Aquí aparece la tabla y dice que la primera tienda que es la John Sports los tenis tienen un cuarenta por ciento de descuento y en la Mike tienen un veinticinco por ciento. Yo le puse que era más barato en la primera que en la segunda, porque [tiene] un cuarenta por ciento que es más mucho que un veinticinco. Pero también le puse que necesita saber cuál es el regular precio de los dos, ¿porque si no cómo vas a saber cual! Y puse un ejemplo. Puse por ejemplo si los dos pares, en las dos tiendas, cuestan treinta dólares y con el cuarenta por ciento pues siempre va a ser la John la que va a tener el precio más bajo. Pero si los precios son diferentes pues también va a ser diferente el resultado.

Yael says at the beginning ‘aquí aparece la **tabla**’ which might be related to “unbeatable”.

★ Yael A2,1:

1.  [1st try]

En el círculo, debido a que cualquier operación o medida que quieras encontrar en un círculo, siempre tendrás que utilizar el perímetro, y en cualquier otra figura como cuadrado o rectángulo por lo regular se utilizan otras medidas.

Yael A2,38-43:

38. F: ¿Y qué le preguntaba el ejercicio aquí?
 39. Y: Que cuál de las dos figuras cómo tienes que utilizar el perímetro. Y por qué. Como yo así lo entendí. O como a cuál te es útil.
 40. F: Okay. Le pregunta que cual “has a greater”, que significa “greater”?

41. Y: Pues yo lo entiendo como... como a... Así lo entiendo, como en cuál te es más útil.
42. F: Mayor. Le pregunta en cual de estas dos figuras el perímetro es mayor.
43. Y: Oh, yo lo entendí como cual es más útil.
44. F: ¿Lo quiere volver a pensar?
45. Y: ¡Oh, sí!
46. F: Okay.
47. Y: Pero pues en un cuadrado como nunca se utiliza un perímetro.
48. F: Mm [continuing conversation]
49. Y: Y siempre se utiliza en un círculo, pero aquí la medida de los dos es la misma.
50. F: ¿La medida de qué?
51. Y: De los dos perímetros. Porque aquí es cinco y aquí también.
52. F: ¿Qué es el perímetro?
53. Y: el perímetro... pero el perímetro en un círculo, en una... rectángulo o en un cuadrado nunca existe, el perímetro siempre existe como en figuras de círculo.
54. F: ¿No existe en un cuadrado el perímetro?
55. Y: No.
56. F: ¿Qué es el perímetro?
57. Y: El perímetro es como la línea que divide por exactamente la mitad.
58. F: Ohh. Esto es el diámetro [pointing to the diameter of the circle]. ¿Esto? [Marking the diameter of the circle]

Yael understands the wording in a deviated way (38-43). Maybe since the activity she solves before (A1) is not *standard* she gives a different interpretation of this statement too. Or maybe since she understands perimeter as diameter (57) this leads Yael to a different interpretation of the wording. It also could be that the deviated interpretation leads Yael to a misconception on the notion of perimeter.

4.2.12 Obstacles to horizontal mathematization from a dense wording

When the wording of the problem is composed by multiple sentences is it more probable to find words that are not known. More concentration and language skills to connect all the parts of the wording are required in order to get a good mathematization from it. This might be the case of Juan, who understands the situation presented in a very particular way.

Furthermore, having a good memory plays an important role in mathematics and specially in this problem. The capacity of one's working memory has an impact on the ability to solve complex, multi-staged tasks (Tshabalala, 2011). This is even more important when the students need to read, comprehend and communicate in a language that is not their first language, as it happens with the students in this research.

Sometimes it is not crucial to understand the meaning of all the words of the statement to get the right solution, but it is always important to discriminate which are its central aspects in order to initiate the horizontal mathematization. Even if the whole individual meaning of all the sentences presented by the statement is understood, the mathematization of all the statement components needs to be done precisely to pursue with the solving process (Kazima, Pwele & Kasakula, 2011). In a dense wording the information can be spread out among distant parts in the text. This is what happens with Camilo and Diandra. As they have an acceptable English management, they comprehend the meaning of the sentences, but they are not able to mathematize it properly (at least on the initial tries).

★ Yolanda A4,3-21:

3. Y: This one is difficult because I thought like found/four[?] right here [middle floor]. I thought it was five then she went one up that was six. No this [middle floor] was six. Then she went one up and I think it was seven there. Then she went down it was six again.
4. F: But what have you done to solve it? Sorry.
5. Y: To solve it, well, I started to read and then like I started counting like the numbers of how many she got, because it was ten floors [pointing to the 'ten' in the wording]. And the first one she went to the middle [pointing to "middle" on the wording] so it was the fifth one. And then she went like down again and it was the forth one.
6. F: And you have just keep adding the numbers?
7. Y: Yeah.
8. F: And why have you used Spanish in this activity [She said it on A2,62-69]?
9. Y: Because I started counting and like [pause] putting this like some words in Spanish so I can like [pause] know what they mean.
10. F: And you counted in Spanish?
11. Y: Yeah.
12. F: Have you counted in Spanish here [A4]? But not in the previous activity, for example.
13. Y: No, this one I did it...
14. F: You counted here [A3] in English but here [A4] you have counted...

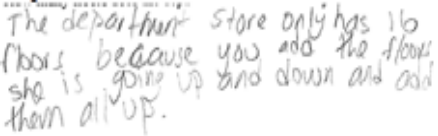
15. Y: Because like... I was confused because they went up and down, up and down...
16. F: Aha. And using Spanish was easier for you in that activity?
17. Y: Mm [agreeing]. Because I just like don't think, and I just mix them, so...
18. F: What do you mean?
19. Y: In this one [A4] I just like started counting and then I said some in Spanish and then I... in English again...
20. F: And do you know why, when you have switched, when have you decided to switch to Spanish? Do you remember that?
21. Y: No, I just keep counting. [Continues in GLQ]

Yolanda GLQ,41-50:

41. F: And here... Is there any word in English that you haven't understood? Or any phrase, any sentence?
42. Y: This one [A3] no and this one [A4] ... Well, this one [A4] I didn't got confused it's just like I got confused in all the things, that was up and down... so I started using Spanish too.
43. F: But not because of the words...
44. Y: No.
45. F: ...itself or the way the sentences were written?
46. Y: Yeah. The middle floor, then on the... goes down, then three floors...
47. F: But you have understood, you understood the sentences, right?
48. Y: Yeah.
49. F: And here, in this [A1] you said no... And is... Well, there's not a lot of words here, but you have understood all the words, right?
50. Y: Yes.

Yolanda finds the problem difficult (3). She starts to think about the problem in English Maybe this difficulty prompts the use of Spanish (15-21), as she understands all the words (GLQ,41-50)

★ Carlos A4, 2-6:

2.  [Carlos erases a couple of words and rewrites them, probably because of his handwriting is not clearly intelligible.]
3. [Comes from A3,17] F: How have you solved that activity in fact? [A3,17, reproduced also here]
4. C: Jamie started at the middle floor and then she went up one floor, and then she went down one floor. That makes it two floors. And then she goes up three floors to the toy department, so that's five. And then she goes down ten floors to the main entrance. That's fifteen. Plus the one in the middle where she started at, so that was sixteen floors.
5. F: So you have add all the numbers?
6. C: Yeah.

Carlos just adds all the floors described on the wording. It is clear that he does not properly mathematize the middle floor and so he does not situate the floors relatively.

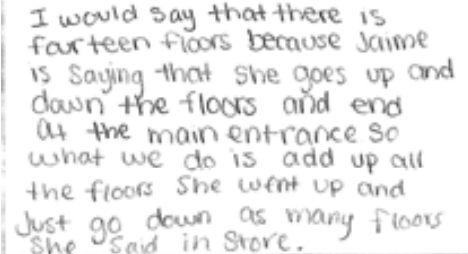
★ Coral A4,1-14:

1. [Comes from A3,9. Coral follows some words with her pencil when reading the wording. She underlines some words.]
2. C: I kind of don't understand this one.
3. F: What you don't understand?
4. C: Cause it says "Jamie is shopping in a large department, It's in the middle floor", but then it says "down" here. You have to add them all?
5. F: You will have to figure it out. I cannot tell you how to do that, but if you want I can... You have to... The answer... The question, is you have to say how many floors does the department have, right?
6. C: Yes.

7. F: And here it's explaining what Jamie is doing when she enters the department, she does certain things: go up, go down, right?
8. C: Yes.
9. F: And at the end she leaves by the... eh... She is in the first floor and she leaves, right? So you will have to figure it out. According to all that information, how many floors does the department store have. Right?
10. C: Yes.
11. F: What's the total amount of floors. If you think you have to add it, add it. If you think you have to do something else... Just do whatever you think. But do you understand what's the question and what's happening here? What Jamie is doing?
12. D: Yes.
13. F: Okay. [Coral writes the answer almost immediately]

Jamie is shopping in a large department store with many floors. She enters the store at the middle floor, and immediately goes to the credit department. After making sure her credit is good, she goes up one floor to the jewelry department. Then she goes down one floor to the children's department. Then she goes up three floors to the toy department. Finally Jamie goes down ten floors to the main entrance of the store, which is on the first floor, and leaves to go to another store down the street.

How many floors does the department store have? Why?

14.  [Continues in A1,2]

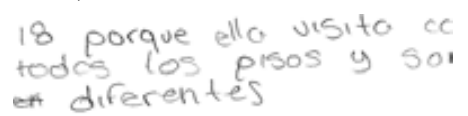
Coral is not sure about the mathematization of the problem she makes and so she asks for the meaning of the wording (2-10), checking her procedure (4). She quickly writes the solution down (13-14) after the dialogue about the wording comprehension (2-13). She has no language difficulties (11-12). She does not solve the problem correctly, anyway (14).

★ Camilo A4,46-47:

46. F: ¿Y cómo puede bajar diez si sólo tiene cuatro?
47. C: Pues es que no lo entiendo bien. Entonces sí son dieciséis. Cuento los seis y los diez que bajó más pabajo.

When saying 'es que no lo entiendo muy bien' (47) Camilo refers not to the meaning of the different wording sentences (from a language point of view), but to the difficulties on the horizontal mathematization of the problem. He previously gives two different answers to the problem, but as both of them are wrong he does not know how to continue.

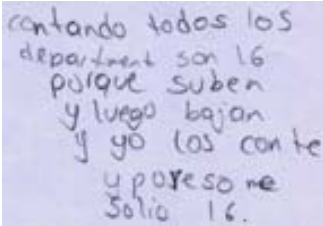
★ Diandra A4,1-15:

1.  [Whispers while is reading the wording. After writing "18 porque" makes a pause and checks the wording. Continues in A3,3]
2. [Comes from A3,24] F: ¿Cómo resolvió la [actividad] cuatro?
3. D: Está un poco enredada Mr. Reverter y no la entendí mucho, porque dice que primero sube, luego baja, y luego baja diez pisos, y como que está un poco difícil[?]... Pero yo nada más me puse a contar todos los pisos que ella pasó según.

4. F: ¿Nada más los contó?
5. D: Ajá.
6. F: ¿Cómo empezó por eso?
7. D: ¿Cómo?
8. F: ¿Cómo empezó a pensar la actividad?
9. D: Pues la empecé a leer y le... como la traducí a español para entenderlo más o menos... Más o menos mejor, porque en inglés casi se me hace más confundido.
10. F: Pero luego, ¿cómo empieza, cómo sabe que tiene que sumar todos? ¿Por qué decide que tiene que sumar?
11. D: [Laughing] Porque lo leo.
12. F: Ajá. Pero podría ser que la respuesta no fuera sumar, ¿no? Podría ser que fuera restarlos todos, o podría ser que fuera... sumar unos y otros no.
13. D: Porque aquí verá [pointing to the wording] de cuántos, éste... departamentos ella ha estado y porqué. Y pues ya, me puse a contarlos, de cuántos ha estado, y cuántos ha subido, cuántos ha bajado, y todo eso.
14. F: Sí. Pero aquí pregunta cuántos departamientos... no en cuántos ha estado, sino cuántos tiene la tienda, ¿no?
15. D: Ajá, por eso. Y como ella sube a diferentes pisos...

Diandra is aware that the mathematization process has not been easy in A4 (3). She gets a deviated question understanding (14: floors Jamie goes through similar to the total number of floors on the building) due to a wrong mathematization with no relative situation of floors (3-5, 10-13). As she considers the highest floor reached as the top of the building (1), her wording understanding is mathematically similar to the one declared in the statement.

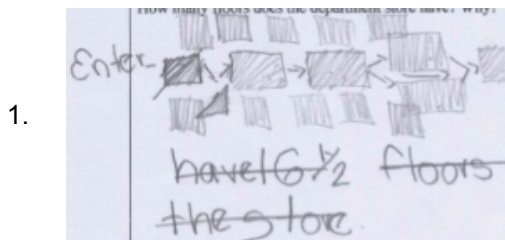
★ Ana A4,1-46:

1. 
2. [Comes from A1,21] F: ¿A ver, cómo resolvió eso?
3. A: Primero leí el problema que decía que, que una tienda... ¿vendía piso? Algo así.
4. F: ¿Una tienda?, perdón. [Pause] Es que no le entendí, sólo al hablar, no le digo que esté mal. Tenía pisos o algo así, me dijo.
5. A: Vendía pisos.
6. F: ¡Oh!, ¿vendía?
7. A: Ajá.
8. F: No, ¿por qué entendió vendía? ¿Vendía pisos?
9. A: [After cheking the wording] ¿O compraba el piso?
10. F: ¿Dónde pone eso?
11. A: Jaime [Spanish male name] is shopping.
12. F: It's Jamie.
13. A: [Reading] Jamie is shopping in a large department store with many floors.
14. F: Mm [okay]. ¿Cómo se dice eso?
15. A: Jamie compró un departamento...
16. F: [Interrupting] Está comprando, ¿no?, en un...
17. A: En un departamento.
18. F: Large department, en un eh... centro comercial podríamos decir, ¿no? En un gran centro comercial con varios pisos.
19. A: ¿Y luego él entró al piso que estuvo en medio?
20. F: Sí. Ella, sí, ¿no?, porque es Jamie.
21. A: Okay.

22. F: Pero bueno, Jaime pues sí es en español así.
 23. A: Y luego fue a dónde está la tienda de crédito.
 24. F: Mm [validating].
 25. A: Luego él miró que su... o ella miró que su crédito estaba bien.
 26. F: Mm [validating].
 27. A: Luego ella fue un piso más arriba.
 28. F: Mm [validating].
 29. A: Dónde está la, el departa, la tienda de joyas.
 30. F: Mm [validating].
 31. A: Luego ella bajó un, un piso dónde estaba la tienda de niños.
 32. F: Mm [validating].
 33. A: Y luego ya sube tres pisos más dónde estaba la tienda de juegos.
 34. F: Ajá.
 35. A: De juguetes. Al final ella bajó diez pisos dónde estaba... main ... dónde estaba la...
 36. F: ¿Qué es lo que encuentra ahí?
 37. A: Main entrance [pointing to the wording].
 38. F: ¿Qué es eso?
 39. A: ¿El éste de correo?
 40. F: Main entrance. [pause] La entrada...
 41. A: De...
 42. F: Principal.
 43. A: Principal.
 44. F: Ajá. ¿Quiere volver a pensar el problema, si no lo había entendido como lo hemos entendido ahora?
 45. A: Dice la pregunta, que es que, éste, cuántos pisos tiene el centro comercial.
 46. F: Mm [validating].

Despite all of the translating problems Ana has on particular parts of the wording (13-18, 35-43), she gets a quite good approximate idea of the wording (19-35, 45-46). Even if she does not connect the meaning of all the sentences, as she confuses Jamie with Jaime (11,19-20, 25). This is why she does not review carefully the mathematical procedure once the wording is correctly understood.

★ Juan A4,1-13:

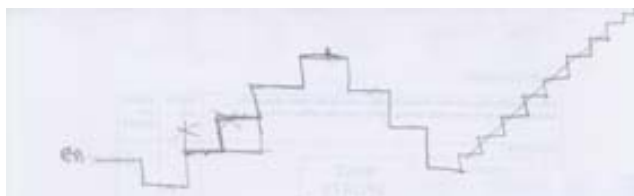


[1st try. Juan reviews the problem after reviewing A3: adds the first and third row of floors. The sentence is crossed out later, when writing 2nd try answer, on A4,22]

2. F: ¿Me puede explicar aquí qué es lo que hizo?
 3. J: Dice que entra y hay un medio piso que es éste [triangle on A4,1]. Y después...
 4. F: ¡Oh! ¿Hay un medio piso qué quiere decir?
 5. J: Medio... Cómo... Él entraba y no está el piso completo. Si es un cuadrado, está, es junto... Se convierte como en un triángulo nomás al entrar.
 6. F: No, esto significa: Imagínese que es un departamento comercial grande. No significa que sólo haya medio piso en el suelo. Sino que significa que entra por el piso del medio, como si hubiera por ahí una escalera o un ascensor o lo que sea... o viene de un edificio de al lado. Y entra, si el edificio es así de alto, por el medio del edificio [Francesc makes gestures with the hands during the explanation to help Juan better understand the situation]. No significa que haya medio piso en el suelo. ¿Sí entiende?
 7. J: Sí.
 8. F: Okay. Quizás entonces convendría cambiar eso, pero bueno, si me quiere terminar de explicar qué es lo que ha hecho.
 9. J: Y luego se encontró otro piso en el departamento de joyería que sería éste.

10. F: Sí. ¿Pero qué hace para ir al departamento de joyería?
11. J: Entra por aquí. Por el medio piso. [refers to the *half tile*, see A4, 1]
12. F: En el piso del medio. ¿Sí? [Juan nods] Okay.
13. J: Se encuentra otro piso del departamento de niños. Que es éste. [referring to 1 *tile*, see A4, 1] Y después se encuentra tres pisos del departamento de juguetes que serían estos tres. [points to 3 *tiles* on A4, 1]

Juan A4,22:



22.

have $6\frac{1}{2}$ floors of
the store.
tiene 19 pisos

[2nd try. The last floor (of the 10 going up) is drawn on the table, as there is no more space on the paper. The line crossing the 10 floors going up is added later; see A4,52]

A deviated translation (3, 5, 9, 11, 13: floor-piso, i.e. teja –tile–) without considering the whole situation resulted in a deviated mathematization (1).

On the second try, Juan does not mathematize correctly that Jamie ends up on the first floor. Maybe the presence of the word “large” on the wording influences Juan's sketch, making him think that Jamie was in *the same floor* of a department store but just going up and down some stairs across the building to reach the different departments. This could be due to a translation of large into “largo”. Interviewer's unawareness of Juan's particular vision of this situation resulted in Juan's not interpreting the hints as pretended by the interviewer, continuing with his particular vision. He does not correctly mathematize the middle floor, he just situates it on his sketch.

★ Angel A4,13-17:

13. F: Sí. No, tenía... Goes up. ¿No? Bueno, ¿cómo nos dice al principio? ¿Cuál es la situación en la... que se describe aquí en esta... en este enunciado?
14. A: Que Jaime [in Spanish] vive en un apartamento.
15. F: Bueno, Jaime [in Spanish] es en inglés, ¿no? Jamie.
16. A: ¡Oh, Jamie!
17. F: Por eso pone luego she, pero bueno.

Angel reads Jaime (Spanish male name) instead of Jamie (English female name) probably because he thinks mainly in Spanish (he says he does it) and most likely because there is a lot of information and he does not pay attention to the word “she” which is not in concordance with the Spanish male name he is interpreting.

★ Abel A4,44-50:

44. F: [Interrupting] ¿Cuál es la situación que se presenta?
45. A: Qué Jaime... [Spanish name]...
46. F: ¿Jaime [Spanish name]? Está en inglés, ¿no? ¿Cómo se dice en inglés?
47. A: Jaime.

48. F: Jaime. Es una chica, ¿no? She.
 49. A: ¡Oh sí! [laughing]
 50. F: Okay.
 Abel reads Jaime (Spanish male name) instead of Jamie (English female name) probably because he thinks mainly in Spanish (he says he does it).

★ Julia A4,3-49:

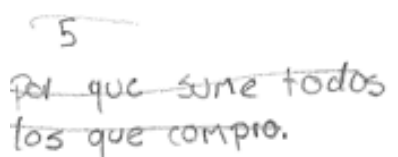

3. [Comes from A3,50] F: ¿Cómo hizo aquí?
4. J: Pues hice una proporción.
5. F: Ajá.
6. J: Que en un floor son tres.
7. F: ¿En un floor son tres qué?
8. J: Mmm... [pause] Floors.
9. F: ¿En un floor son tres floors?
10. J: Sí.
11. F: ¿Qué es un floor?
12. J: No sé. En un floor son tres departamentos y...
13. F: ¿Qué es un floor? [Pause] ¿Lo sabe o no?
14. J: No.
15. F: ¿Departamento, qué es un departamento?
16. J: Es, es un... [pause] ¿Cuartos?
17. F: Sí, puede ser cuartos. ¿Pero en este caso se refiere a cuartos? [Pause]
18. J: No sé.
19. F: Departamentos se refiere, por ejemplo the children's department or the jewelry department, o the toys department [interviewer points to the wording problem, where these names appear], que está por aquí también, ¿no? ¿Qué significa? Toys department es el departamento de juguetes. Es... ¿Porque dónde está Jamie?
20. J: Acá está, la palabra.
21. F: Ajá. Pero Jamie digo, ¿no?
22. J: ¡Ooooh!
23. F: ¿Cómo leyó eso?
24. J: En español.
25. F: ¿Cómo lo interpretó?
26. J: Jaime [in Spanish].
27. F: ¡Oh Jaime [in Spanish] lo pensó! Pero luego nos pone she.
28. J: Ella.
29. F: Ajá. Entonces es Jamie, porque está en inglés.
30. J: Mm [validating].
31. F: ¿Dónde está Jamie? [Pause]
32. J: ¿Se fue de compras a largo departamento?
33. F: Sí. A un grande. Un departamento comercial que es muy grande, ¿no? Con varios... [pause] with many...
34. J: Con varios...
35. F: Floors, ¿Qué son floors?
36. [Julia says no with her head.]
37. F: Varios pisos, ¿no? Varios pisos. [interviewer makes a movement with the hands, simulating many floors of a building.] Es un departamento muy grande que tiene muchos pisos. ¿Qué más? [Pause] ¿Dónde nos dice qué en cada piso hay tres departamentos?
38. J: Aquí [pointing to wording, around "up three floors"]. En un floor hay [pause] yo interpreté uno sobre tres.
39. F: ¿Ha entendido lo que está haciendo Jamie en departamen... en el centro comercial? ¿Ha entendido qué es lo que nos describe el enunciado, qué es lo que pasa?
40. J: ¿Que no encuentra la salida?
41. F: Sí la encuentra, al final. Pero no es lo que está buscando, la salida.
42. J: ¿Está buscando cuántos... cuántos niveles hay en... hay en el departamento?
43. F: ¿Eso es la pregunta? [Pause] ¿O qué es eso? ¿Entiende la pregunta?
44. J: Un poco.
45. F: La pregunta del final: How many floors does the department store have?... Cuántos,

how many, cuántos [pause] floors son...

46. J: Pisos.
 47. F: Pisos o niveles, como dijo antes, ¿sí?. Does the department store have. And why. ¿Qué significa esta pregunta?
 48. J: Cuantos pisos... [pause] Ésta [does] no me la sé, el departamento...
 49. F: Tiene.

Julia has many difficulties to interpret this wording. It is clear that she does not get an idea of the whole picture and that she has difficulties to connect the meaning of the different sentences. For example, she interprets Jamie as Jaime (Spanish male name), ignoring the female pronoun "she". Furthermore, Julia has difficulties with the meaning of some words: floor (11-14, 33-37), department (15-19). She does not interpret the wording question properly (39-51).

★ Ingrid A4,1-3:

1.   [The lines crossing out the answer are added later (A4,103)]
 2. [Comes from A3,84] F: ¿Qué hiciste aquí?
 3. I: Sumar cuántos pisos había comprado.

Ingrid A4,28-39:

28. F: ¿Qué es lo que nos pregunta? ¿Entendiste la pregunta?
 29. I: Más o menos.
 30. F: A ver, ¿qué entendiste?
 31. I: ¿Qué cuántos pisos había en el departamento? [Pause] Nomás [or '¿No es?'; unclear sound. Ingrid shakes her head saying no].
 32. F: Department store, se lee esto todo junto.
 33. I: Mm [validating].
 34. F: ¿Qué significa department store?
 35. I: Tienda del[?]/o[?] departamento?
 36. F: Ajá. Bueno, es como el departamento comercial, o... ¿sí?
 37. I: Mm [validating].
 38. F: Es un... como nos dice al principio: large department store. Es un, una gran tienda. ¿Sí?
 39. I: Mm [validating].

Ingrid 96-101:

96. F: ¿Sí le quedó claro?
 97. I: Sí.
 98. F: ¿Quiere que lo volvamos a ver o alguna cosa?
 99. I: La pregunta.
 100. F: Cuántos pisos tiene, does the department store have. Tiene el departamento comercial. El does es el auxiliar.
 101. I: Mm [continuing conversation].

Ingrid does not consider the 10 floors on her first solution (1). She has problems understanding the wording question (28-39) and its meaning is not interiorized on the first explanation of the interviewer, because there is a second demand for its translation (99) after the dialogue goes on translating the entire wording (46-97, not copied here).

★ Yael A4,73-82:

73. F: ¿Podemos volver a repasar el enunciado? Hay una pequeña cosita que debería... O

sea, sí podemos saber, yo le digo que sí podemos saber...

74. Y: [Interrupting] cuántos

75. F: ... cuántos hay. ¿Quiere volverlo a leer? ¿Quiere que la ayude?

76. Y: Eh, ¡sí!

77. F: ¿O lo quiere leer por su cuenta?

78. Y: Dice que está en el piso del medio.... Está en el piso del medio, después sube uno...

¿Puedo usar un lápiz?

79. F: Sí. [Pause. Goes to get a pencil] Si quiere otra hoja...

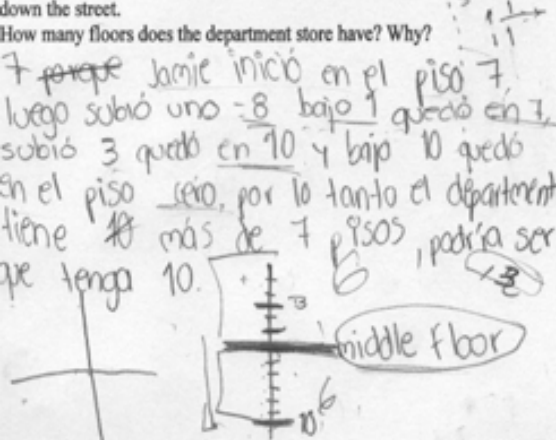
80. Y: No, así está bien.

Jamie is shopping in a large department store with many floors. She enters the store at the middle floor, and immediately goes to the credit department. After making sure her credit is good, she goes up one floor to the jewelry department. Then she goes down one floor to the children's department. Then she goes up three floors to the toy department. Finally Jamie goes down ten floors to the main entrance of the store, which is on the first floor, and leaves to go to another store down the street.

How many floors does the department store have? Why?

81.

7 porque Jamie inició en el piso 7
luego subió uno -8 bajo 1 quedó en 7
subió 3 quedó en 10 y bajo 10 quedó
en el piso seis, por lo tanto el department
tiene 10 más de 7 pisos, podría ser
que tenga 10.



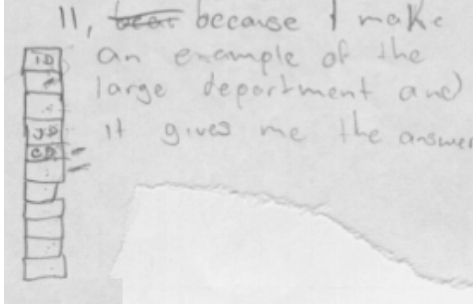
82. Y: Listo, creo que ya la tengo.

Once the interviewer states that the answer can be found, Yael –without any other clue– incorporates the key point of the mathematization –the middle floor– with its symmetry function to her reasoning. By doing this she shows a good comprehension of the problem and a good horizontal mathematization. Maybe the fact that there is a lot of information concentrated on the wording made she unaware of the importance of the middle floor as a key issue in the beginning.

★ Julián A4,14:

14.

11, ~~because~~ because I make
an example of the
large department and
it gives me the answer



Julián A4,123-147:

180. F: Okay. Utilizando cualquiera de estos dos dibujos, ¿dónde empezó? ¿Cuál va a revisar? ¿Dónde empezó ella?

181. J: ¿En el credit department?

182. F: ¿Y dónde estaba el credit department?

183. J: Oh, está hasta arriba, so... Sí está hasta aquí, el cuarto piso de arriba pabajo.
 184. F: ¿El cuarto piso? ¿Llegó arriba del todo?
 185. J: ¿Eh?
 186. F: ¿Jamie llegó arriba del todo?
 187. J: Oh, pues también no dice entonces si llegó. Igual le faltaban dos pisos más o dos pisos menos.
 188. F: No sabemos si llegó arriba del todo, ¿no?
 189. J: Podía faltar mucho. Ajá.
 190. F: Pero sí sabemos dónde empezó. ¿Dónde empezó?
 191. J: En el credit department, en el middle.
 192. F: Middle floor.
 193. J: Sí.
 194. F: Sabemos que empezó en el medio.
 195. J: Sí.
 196. F: ¿Sabemos cuántos pisos le quedan hasta arriba?
 197. J: No.
 198. F: ¿Y cuántos le quedan hasta abajo?
 199. J: No.
 200. F: ¿Cuántos le quedan hasta abajo no lo podemos saber? ¿Lo podemos saber o no [pointing to the paper]?
 201. J: No.
 202. F: ¿Por qué no?
 203. J: Pues...
 204. F: Sí que podemos saberlo.

Julián has a lot of problems with the mathematization of A4. Above there is just an extract to exemplify it. He does not mathematize the middle floor properly (125-130). Furthermore he does not even say how many floors are from the first floor to the bottom floor (141-142), for which he only needs to rely on the sketch he made. All this confusion may well be related with the fact that there is a lot of written information, and it is harder to know which parts need to be mathematized and how. Julián understands the meanings of the English sentences on the wording. Furthermore, he forgets to represent on the sketch one of Jamie's movements on the first try (14). Note that he finally solves the problem (with the help of the interviewer).

★ Aida A4,1-21:

- Jamie is shopping in a large department store with many floors. She enters the store at the middle floor, and immediately goes to the credit department. After making sure her credit is good, she goes up one floor to the jewelry department. Then she goes down one floor to the children's department. Then she goes up three floors to the toy department. Finally Jamie goes down ten floors to the main entrance of the store, which is on the first floor, and leaves to go to another store down the street.
1. How many floors does the department store have? Why?
- one up
one down
three up
ten down
- The department store have 15 floors because ...[sequence unintelligible]... goes up and down
- [A4, 1st try. Part of it (see typed paragraph) has been reconstructed from the writing marks because Aida erased it: A4,34-35]
2. [Comes from A3,30] F: ¿Cómo solucionaste la cuatro?
 3. A: Sumé los pisos que decía que iba.
 4. F: Mm [continuing conversation].
 5. A: Y luego... Nomás los sumé y me salieron quince.
 6. F: ¿Los sumaste?

7. A: Sí. [Pause] Es que casi no le entendí muy bien.
8. F: ¿Por qué no lo entendiste muy bien? [Pause]
9. A: Porque dice que empezó como a, en el, en el medio.
10. F: Ajá. ¿Y qué quiere decir que empieza en el medio? ¿Te has imaginado la situación, realmente lo que significa?
11. A: No.
12. F: Pues piensa, a ver. Entramos en un edificio, ¿no? Entramos por el medio. ¿Sí? ¿Luego qué hacemos?
13. A: Sube un piso.
14. F: Ajá, ¿luego?
15. A: Luego lo baja otra vez y está en el medio otra vez.
16. F: Ajá.
17. A: Luego sube tres pisos.
18. F: Mm [validating].
19. A: Y ya luego baja diez pisos.
20. F: Ajá. Por lo tanto...
21. A: En el medio de la entrada, la, la... la entrada principal.

Aida A4,80-83:

80. F: ¿Qué dificultades has encontrado en este problema?
81. A: Porque no lo leí muy bien.
82. F: No lo leíste muy bien. ¿Pero lo leíste como mínimo dos veces, no? Una vez, luego te vi subrayar... [Pause] subrayar las palabras, ¿no?, los números.
83. A: Mm [validating], pero no le puse mucha atención en la, en los des[?] pisos.

Aida says she does not understand the wording very well (7-8, 81). But she makes a good summary of it by retrieving the essential information needed for the mathematization (9-21), underlines the main points on the wording (1) and writes down the most important data (1). Even though, the horizontal mathematization is not correct. Aida correctly situates some of the movements that Jamie makes (up and down one floor).

★ Claudio A4,5-17:

5. C: It's because dice aquí que nomás son many floors, pero no te está específicamente diciéndote cuántos. O cuál número, como el primer floor, segundo floor... No está diciendo nada de esto. Nomás dice en medio, éste... te dice abajo, luego dice... Luego dice the other "which is one of the floor [on the 1st floor!] and leaves to go to another, to another store down the street. How many floors does the department store have." Luego aquí dice [reading] "which is the, on the first floor".
6. F: Sí.
7. C: Aquí dice one floor, luego dice goes to the jewelry department.
8. F: ¿Y qué quieres decir?
9. C: Porque se va al departamento de, de joyas. Entonces aquí dice, which is one, which is on the first floor. Pues el departamento de... Luego aquí...
10. F: [Interrupting] Entonces, ¿tenemos suficiente información para resolverlo o no? ¿Sí? Lo puedes ponerlo donde quieras. ¿Sí? Es que no entiendo lo que me estás diciendo.
11. C: [Low voice] How many floors does the department have. Pues aquí nomás dice que cuántos de departamentos de...
12. F: Ajá, hay.
13. C: Hay. Y aquí nomás dice que son diez.
14. F: ¿Tú crees que son diez?
15. C: Yo... ¡no! Pues ahí no te dice específicamente, pero yo digo que son muchos. [Pause] O sea, que aquí dice: "Then she goes she goes up three floors to the toy department. Finally Jamie [/dzeim/] goes to, to, goes down ten floors [pause] to the main entrance of the store which is on the first floor.". Yo digo que nomás hay veinte.
16. F: ¿Veinte?
17. C: Pues yo digo, no sé.

Claudio A4,78-97:

78. F: O sea, entramos por el del medio. ¿Qué piso es el que entramos?
79. C: El cinco, yo digo.
80. F: ¿Por qué el cinco?
81. C: Porque dice en medio.
82. F: Pero no sabemos dónde está el medio.
83. C: Sí, porque aquí dice diez. La mitad de diez son cinco.
84. F: ¿Porqué sabes... cómo sabes que llega arriba del todo?
85. C: Porque dice aquí.
86. F: Tampoco lo dice que llega hasta arriba del todo, ¿no?
87. C: Pues dice aquí abajo: "She gets, the final James [/dzeims/] get [finally Jamie goes!] down ten floors, enters... main entrance of the store which is on the first floor. Leaves to [go to!, omitted] another store down the street". So baja para abajo.
88. F: Sí, pero no dice que llega arriba del todo, tampoco, ¿no?
89. C: Pues dice pero she starts, isn't it?
90. F: ¿Cómo?
91. C: Empieza ella.
92. F: Mm [validating]. ¿Pero en dónde empieza?
93. C: Pues en el departamento más grande.
94. F: ¿Por qué en el más grande?
95. C: Porque aquí dice: ...
96. F: [Interrupting] ¿Dónde lo dice eso?
97. C: ..."Large department". "Shopping in a large department".

Claudio GLQ,17-18:

17. F: ¿Hay alguna palabra o frase que te haya resultado difícil en inglés?
18. C: No.

Claudio just finds some isolated pieces of information in the wording (5-16) but does not put all of them together, and tries with different answers with no justification or looking directly to the statement information (A4,13,16). He does not mathematize the middle floor properly.

Later he continues to point directly to the statement information (a building with ten floors), when he situates the middle floor on the fifth floor (A4,78-83) and gets confused with the statement saying it (A4,84-93), partially influenced by a deviated translation of "large department store" (A4,93-97). Claudio does not experience any difficulty with the understanding of the English wording in relation with its language aspects (GLQ,17-18).

★ Damian A4,3:

3. D: Éste, me fijé cuántos pisos subió y cuántos bajó. Entonces ya vi cuántos subió. Pensé que iban a ser trece porque, porque bajó... Digo doce, porque bajó uno y subió uno otra vez y luego subió tres pisos y... No aca..., no aca... Pensé que eran doce. ¿Cómo era? Que iban a... y luego que iban a ser catorce, porque iban los tres, cuatro. Y luego me fijé que bajó diez pisos. Y luego estaba hasta llegar arriba y bajó diez pisos. Y luego de todos los pisos que bajó, llegó hasta el primero y me fijé que estaba en el primero y lo sumé, le sumé diez y ya era el once.

Damian does not explain his reasoning very well, and gets confused about what he did do. In fact he does not properly mathematize the middle floor and considers the top of

the building as the highest floor reached by Jamie. All this confusions may be due to the fact that there is a lot of information (in a dense wording) that should be organized.

4.2.13 Effect of English language difficulties on the mathematical resolution

Sometimes one or many words from the statement are not understood, but their meaning is not demanded and the mathematical procedure continues anyway. If these words play a central role on the mathematization, the solution might be wrong, as it is the case of Angel or Juan on A4. Angel is the only student who points out the difficulties found on the understanding of the wording (he does it before checking the solutions of the activities with the interviewer), but he solved the problem anyway.

On the contrary, if the word does not play a central role on the mathematization it may not have major consequences on the mathematical solution. It could be that even when the meaning of a word is important for the mathematization process, its omission has not a major impact on the solution. This is what happens to Julián on A1 with “unbeatable”, as this word was included on the statement to make it more realistic, but its intentional omission on the mathematization is acceptable.

Students may not ask for the meaning of some words or passages because they could think their global understanding is already good enough to solve the problem. Another possibility is that they are not confident enough with the interviewer or even just because they do not want to reveal they do not know it.

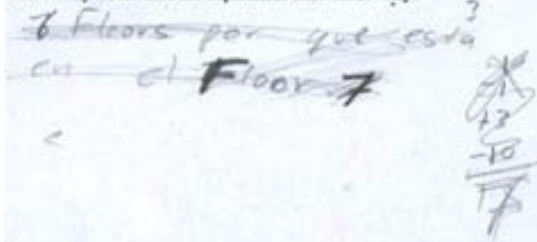
Some of the cases are commented on the “Obstacles to horizontal mathematization from a dense wording” theme, on page 457 and are not reproduced here. Other extracts from the dialogues are presented and commented below.

★ Angel A4, 1-70:

Jamie is shopping in a large department store with many floors. She enters the store at the middle floor, and immediately goes to the credit department. After making sure her credit is good, she goes up one floor to the jewelry department. Then she goes down one floor to the children's department. Then she goes up three floors to the toy department. Finally Jamie goes down ten floors to the main entrance of the store, which is on the first floor, and leaves to go to another store down the street.

How many floors does the department store have? Why?

1.



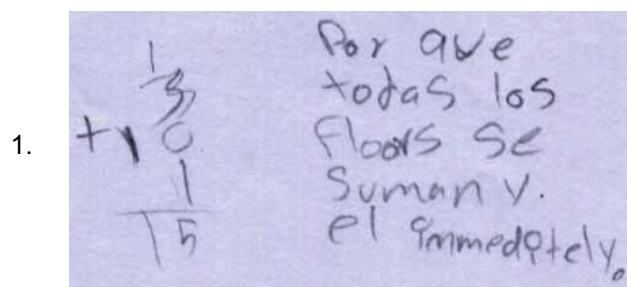
2. F: ¿Ya está?
3. A: Ya. Pero no está bien la última, no entiendo bien el inglés.
4. F: Okay. Ahora lo vamos a comentar si quiere un poquito más. [Continues in A1,15]
5. [Comes from A3,40] F: Me dijo que había tenido algunos problemas, aquí, ¿no?
6. A: Sí.

7. F: ¿Que le costó aquí?, a ver.
8. A: Entender el inglés.
9. F: Sí. ¿En dónde?
10. A: En todo.
11. F: Ajá. ¿Vamos paso por paso o cómo quiere hacerlo?
12. A: Pues aquí decía que... en los apartamentos, que tenía tres pisos.
13. F: Sí. No, tenía... Goes up. ¿No? Bueno, ¿cómo nos dice al principio? ¿Cuál es la situación en la... que se describe aquí en esta... en este enunciado?
14. A: Que Jaime [in Spanish] vive en un apartamento.
15. F: Bueno, Jaime [in Spanish] es en inglés, ¿no? Jamie.
16. A: ¡Oh, Jamie!
17. F: Por eso pone luego she, pero bueno.
18. A: Oh, Jamie vive en un apartamen[to]... en un departamento grande.
19. F: ¿Vive?... ¿Shopping, qué es shopping?
20. A: No sé.
21. F: Está [comprando]... va a comprar, ¿no? Dice Jamie se va a comprar en un, no es un departamento sino un centro comercial. Large department store es como un gran centro comercial, ¿sí?
22. A: Sí.
23. F: Con varios pisos. Tenía varios pisos, es muy alto, ¿Sí?
24. [Angel nods]
25. F: Okay. Luego.
26. A: Ella... Y que ella tiene, cómo se llama...
27. F: She enters the store at the middle floor. ¿Qué significa eso?
28. A: Que varios... ¿Store cómo se llama? [whispering]
29. F: ¿Store, qué es? ¿She enters, qué significa she enters? [Pause] Entra, ¿no? No pasa nada si alguna cosa no la sabemos. No tenemos porqué saberlo todo, ¿no? Lo pregunta y ya está. Dice entra en la tienda ¿dónde?, ¿por dónde? The middle floor, at the middle floor.
30. A: Por abajo de, de los departamentos.
31. F: Por el piso del medio, ¿no? Tantos como tenga pues entra justo por el del medio. ¿Sí?
32. A: Sí.
33. F: Y luego se va... inmediatamente
34. A: Inmediatamente... va... hacia... el departamento de crédito.
35. F: El departamento dónde está el... para compr[obar]... chequear ¿sí? Comprobar su crédito.
36. A: Y después...
37. F: ¿Después de?
38. A: Ya su crédito... ¡Aaah...!
39. F: ¿Making sure, qué significa making sure?
40. A: No lo sé.
41. F: Después de comprobar, ¿no?, que su crédito está bien...
42. A: Está bien.
43. F: Que tiene buen crédito, ¿sí?
44. A: Ella va un piso arriba.
45. F: Ajá.
46. A: Hacia jewel...
47. F: ¿Jewelry, qué es eso? ¿Se acuerda? La sección de joyas. Donde están las joyas se va. Se va parriba a las joyas.
48. A: Después ella va un piso abajo.
49. F: Ajá.
50. A: Hacia el departamento de los niños.
51. F: Ajá.
52. A: Después ella va tres pisos arriba.
53. F: Sí.
54. A: Para el departamento de los juguetes.
55. F: Sí. Good.
56. A: Y finalmente Jamie va diez pisos hacia abajo.
57. F: Ajá.
58. A: Hacia la... la entrada principal.

59. F: Sí.
 60. A: ¿De store es de la tienda?
 61. F: Sí.
 62. A: Y ya queda en el primer piso.
 63. F: Sí.
 64. A: Y después va... ¿Esto qué significa, another?
 65. F: Another, hacia otra.
 66. A: Hacia otra tienda hacia abajo de la calle.
 67. F: Ajá.
 68. A: Y dice: ¿Cuántos departa... cuántos pisos... cuántos pisos tiene el de..., el apartamento que va a comprar?
 69. F: Sí. Mmm.... Sí. El centro comercial. Cuantos pisos tiene en total el centro comercial. ¿Y por qué?
 70. A: Ya me equivoqué.

Angel is aware (2-10) of the English language difficulties he has when solving A4 (11-70) but even so he solves (incorrectly) the problem (1). He makes some good mathematical contributions (giving opposite meaning +/- to opposite movements – up/down–). In fact this is not used on the next try made to solve the problem (the second one) but Angel does use it to get almost the right answer on the third try.

★ Abel A4,1:



Despite all of the comprehension difficulties that Abel encounters on the dense wording of A4 (not copied here) he produces an answer to the problem.

★ Julia A1, 14-26:

14. F: Mm [validating]. ¿Y luego qué dice? [Reading] “Cincuenta [por ciento] en dos shoes cheaper”. ¿Qué creía que significaba shoes cheaper?
 15. J: Más barata.
 16. F: ¿Significaba eso, creía antes? ¿Creía eso antes?
 17. J: No.
 18. F: ¿Por qué puso entonces... ? ¿Qué significa la frase esa?
 19. J: Pues es que no me la sé.
 20. F: Ajá. ¿Pero por qué lo puso entonces si no sabía, no sabía el significado?
 21. J: Nomás.
 22. F: ¿Qué creía que era cheaper?
 23. J: No sé, no me lo imaginaba que era más barata.
 24. F: ¿No? ¿Pero qué pensaba que podía significar eso?
 25. J: Pues la marca de los zapatos.
 26. F: ¿La marca? [Julia nods] ¡Aah! ¿Y antes por qué me sumó estos dos? ¿No? Porque antes de hacerme esto me sumó estos dos [see A1,1]. ¿Por qué?

Julia thinks that “cheaper” is the brand of the shoes.

★ Julián A1,44-57:

44. [Comes from A2,50] F: Volviendo a éste [A1], perdone, ¿alguna otra cosita –pues ya que hemos visto estos ejemplos, a lo mejor no había pensado antes– que pensó aquí en

español? Digo en inglés.

45. J: ¿Aquí en inglés?

46. F: Sí.

47. J: Unbeatable [/anbileitabol/no right pronunciation] price, esto no lo entiendo muy bien, esta palabra. Esa no la utilizo mucho.

48. F: Unbeatable. ¿Sabe qué significa?

49. J: No, no estoy muy seguro.

50. F: Como precios insuperables, sería, ¿no? O precios muy buenos. Unbeatable, que nadie los puede ganar.

51. J: Oh, sí.

52. F: Beat es como ganar. Unbeatable, que nadie los puede ganar, ¿no?

53. J: Oh, sí, sí ya. So, pues no estoy muy seguro porque pues en parte podríamos decir que también puede ser éste [25% store], pero no es por eso lo que dice. Pero no, no. No creo porque tiene aquella más descuento. Yo necesitaría el precio de eso para resolverlos.

54. F: ¿Pero alguna cosa que pensó aquí en inglés?

55. J: No.

56. F: Nada más era por si se le había ocurrido alguna cosa. ¿Entonces dice que podría ser ésta también más barata?

57. J: Podría ser, pero pues... I'm not sure because... las tiendas utilizan esto para atraer a la gente, y pues... sometimes is not true.

The conversation about A1 was over, but it comes back (44). Then Julián informs that he does not know the meaning of “unbeatable” (47-50). However, he found the right solution. When its meaning is known he integrates it to the mathematical reasoning to check if the answer would change and thinks again about the solution (53-57).

4.2.14 Understanding of a key word in the resolution of the problem

It is clear that if the question of the statement is not clear, the mathematical process that the student will apply to solve the problem will hardly be what is demanded. Some students showed a proactive role by asking the interviewer what the meaning of a word was. The same difficulties may arise if there is a word whose meaning is essential in the mathematization. This proactivity may be the result of the students willing to perform well and using all the available resources to do it.

This proactivity happened exclusively on Activities 1, 2 and 3, with the words cheaper, greater and perimeter, respectively. As there is not a lot of text, maybe it is easier to identify the key words needed to solve the problem and consequently the students asked for its meaning. It surprising, thought, that none of them asked for the meaning of a single word in Activity 4.

Mainly the students did not use to ask for the meaning or translation of particular words and advanced on the mathematical solving process without knowing the meaning of some of the words. See, for example, what happens with “Unbeatable prices” on the topic “Word mismatching and mathematics-language loop” on page 450. Some of the reasons for his lack of proactivity may be laziness, shyness, etc.

The extracts below show when students demand for the meaning of a word.

★ Yael A1,1-4:

1. Y: [Raising the hand] Tengo una duda. Es que esta palabra me confunde [pointing to “cheaper”].
2. F: ¿Cheaper?
3. Y: ¿Es barato o caro?
4. F: Barato.

As knowing the meaning of cheaper is a central issue when giving the answer, and Yael is hesitating between opposite meanings (3), she asks the interviewer (1) about it.

★ Angel A1,1-7:

1. A: ¿Qué significa chapter?
2. F: Excuse me.
3. A: ¿Qué significa chapter?
4. F: Cheaper... Cheaper. ¿Sabe qué significa?
5. A: No.
6. F: Más económicas, más baratas.
7. A: Sí. [Pause]

Angel does not know the meaning of cheaper (5) and he asks for its meaning (1,3) before writing down the solution.

★ Julián A1,1-12:

1. [Comes from A3,13] J: Ésta [A1] sí no la entiendo. Esta parte.

2. F: ¿No? ¿Qué es lo que no entiende?
3. J: Qué tengo que hacer aquí.
4. F: ¿Qué es lo que le pregunta?
5. J: Which of these two stores are the shoes cheaper. Cual de estas dos tiendas es la... La cheaper, no entiendo esa palabra.
6. F: Más baratas.
7. J: ¡Oh!
8. F: ¿No? Más baratas.
9. J: Sí, más baratas. Pero no dice la verdad, porque no dice el precio exacto de esto para cuánto es sin el descuento.

Julián does not know the meaning of cheaper (4) and asks for its meaning. In fact, once he grasps its meaning, he immediately gets the answer (9).

★ Julián A2,1-12:

1. [Comes from A2,18] J: Oh, aquí dice que cuál es el... [Reading] which of these figures has a greater perimeter.
2. F: Sí.
3. J: Okay. Pues yo digo que los dos, pues los dos son de cinco aquí en esto [dotted line], ¿no? ¿Pero a qué se refiere con greater... perimeter?
4. F: ¿Qué significa greater?
5. J: Éste me hace dudar.
6. F: Mayor.
7. J: ¿Mayor? ¿Cómo?
8. F: Más grande que.
9. J: Okay. ¿Luego un perímetro más grande? [Makes an imaginary circle with the pencil, below the two figures]
10. F: Great, ¿no? Great, greater es el comparativo.
11. J: Okay, okay.
12. F: Mayor.
13. J: Sí. Entonces lo voy a hacer un poco. Entonces, ¿cuál de estas dos figuras tiene un perímetro más grande?
14. F:Cuál de estas dos figuras, ajá.
15. J: Oh, pues sería éste, ¿no? El cuadrado. [Moves the pencil like inscribing the circle inside the square]

Julián does not know the meaning of greater (4) and asks for its meaning. In fact, once he grasps its meaning, he quickly gets the answer (15).

★ Aida A1,1-3:

1. A: ¿Qué quiere decir esto?
2. F: Cheaper. Más baratas, más económicas. ¿Sí?

In which of these two stores are the shoes cheaper? Why?

3.  [Continues in A3,2, once all activities are answered]

Aida asks for the meaning of “cheaper” –as it is a key word to solve the activity– and then she solves the problem.

★ Claudio A2,1-15:

1. [Comes from A1,15] C: ¿Cómo? No lo entiendo. ¿Cómo pirámide? ¿De lo alto?
2. F: El perímetro.
3. C: Ajá.
4. F: ¿Sabes que es el perímetro?
5. C: ¿Que no es lo largo?
6. F: It's like the outside, right?
7. C: Lo... [a school announcement is heard] [Pause] ¿Cómo? No entiendo lo...

8. F: El perímetro, por ejemplo...
 9. C: [Interrupting] Ajá, ¿lo largo o ...?
 10. F: ...el perímetro de éste, del ordenador es todo el contorno, ¿sí? Todo esto es el perímetro del ordenador. El perímetro de la hoja, pues es todo esto. [Pause]
 11. C: ¿Y también hablo del otro figure [/figura/, with English accent] o...?
 12. F:Cuál de los dos, ¿no? What's the question, here?
 13. C: Dice "Which of these figures has a greater perimet[er]? And why."
 14. F: Sí, ¿entonces qué tiene...?
 15. C: Y dice aquí. Yo digo que es el cuadro porque tiene líneas más grandes.
- Claudio does not know how to pursue with the mathematization (1) and asks for the meaning of 'pirámide' (pyramid), an erroneous translation of "perimeter" (2-3). Maybe he has a vague notion of perimeter (4-5, 7). After the interviewer clarifies the concept (6-10) and remarks the wording's aim (12), Claudio starts the mathematization of the problem (15), now equipped with the necessary tools (furthermore he ends up finding the right answer).

★ Damian A2,1-4:

1. [Comes from A1,8] D: ¿Qué es un perimetre? Perimetre, [both 'perimetre' in Spanish pronunciation] o lo que sea que se dice esta palabra en español. Perimeter [English pronunciation].
2. F: El perimeter, for example, of this table is this. [Interviewer probably –as there is no video– follows the perimeter of the table with the finger] Right?
3. D: Yeah.
4. F: So is the outside.

Damian asks for the meaning of perimeter, as it is essential to solve the problem.

4.2.15 Mathematical and language hints for the facilitation of resolutions

When the wording is not first correctly understood, some difficulties arise regarding the mathematical problem solving. Sometimes and despite an initial deviated understanding, if a good mathematical work has been done, when the intentional meaning of the statement is clarified by the interviewer, the students quickly get a right mathematical solution of the problem.

Also a good treatment of the mathematical aspects in advance leads to a quick benefit of the interviewer's hints, which are used effectively towards the development of a right solution.

Barwell (2009a) shows how some bilingual learners, after constructing the statement of some word problems, develop a good insight of the problem building up a set of interconnected meanings which allows them to solve the activities with ease. In our study and for the present theme, the language and the mathematics support mainly comes from the interviewer.

The examples below show when the focus is on the understanding of the wording (Jessica on A4, Ana on A3, Juan on A2, Ingrid on A3, Julián on A1 and on A2) or on the role played by the interviewer (Miriam on A4, Ingrid on A4, Yael on A4). Its distinction is clear by the comments that accompany the excerpts.

Other examples that point to the theme in this section have been found. However, they have been considered as informative of other themes and this is why they are located elsewhere (for instance, "Understanding of a key word in the resolution of the problem" on page 474. Obvious space restrictions recommend not repeating the different extracts in more than one theme unless the topics of the themes do not have much in common.

★ Miriam A4,1-36:

Jamie is shopping in a large department store with many floors. She enters the store at the middle floor, and immediately goes to the credit department. After making sure her credit is good, she goes up one floor to the jewelry department. Then she goes down one floor to the children's department. Then she goes up three floors to the toy department. Finally Jamie goes down ten floors to the main entrance of the store, which is on the first floor, and leaves to go to another store down the street.

How many floors does the department store have? Why?

[Miriam reads the problem twice. The second time she underlines the key information. Then she makes a sketch of the building, starting to number the floors from the first one until the ninth floor. Miriam writes "middle" in the fifth floor. She follows the key points she underlined. When Jamie has to go down 10 floors she adds 3 more floors at the bottom of the building. Finally she writes the answer: "13 floors or [...]". "13" is turned into "15" at the end of the 3rd try.]

1. [After Miriam finishes to write all the answers] M: I'm done. [Continues in A1,4]
3. [Comes from A3,20] M: [Miriam reads quickly the entire wording. On the seventh line of the wording she says "which is one of the first floor". Immediately she realizes of the mistake and reads it properly.] Aquí le puse que dependía de que, de que, cuál era el middle floor [points to the middle floor in her drawing]. Y como decía ten [points to the wording], yo le puse ten floors primero [9!] y a ver si daba, pero luego, al momento que se fue diez pisos para abajo, me dio la respuesta que eran trece pisos. Pero como no sé cuántos más había acá arriba, entonces puse que podrían ser trece pisos pero depende de dónde haya sido el [pointing to the middle floor on her drawing]... piso del medio.
4. F: A ver, a ver, ¿cómo empezaste?
5. M: Empecé con inglés.
6. F: No, perdona, el primer paso que hiciste.
7. M: So hice namás ten floors [9!]. Un cuadro con diez pisos así [9!] y ésto no estaba [hides the 3 bottom floors (which are added later: see A4,1)]. Entonces puse que el five [pointing to it in the picture], porque dice que el middle floor [pointing to the wording]. Le puse el cinco [pointing to the middle floor in her drawing]. Y luego dice aquí que "she goes up one floor" [points to it in the wording] le puse uno para arriba. Y luego que baja uno, entonces vuelve a quedar a medias. Y luego que sube tres. Y luego que baja diez pisos. Al momento que baja para diez pisos necesitaba agregar tres más. [While explaining Jamie's movements, follows the arrows on her drawing] Así que los agregué y saqué la conclusión de que son trece pisos de...
8. F: [Interrupting] ¿Y agregaste tres más dónde?
9. M: Acá abajo.
10. F: ¿Por qué?
11. M: Porqué dijo que bajó diez pisos y entonces tiene que ir [the long arrow pointing down is stressed: A4,1] para abajo diez pisos. Entonces del ocho a acá son los diez pisos. Pero como no sé cuántos más están de acá [top of drawn building] para arriba, no sé encones... eh...
12. F: ¿Cómo que no lo sabe? ¿No lo puede saber?
13. M: No, porque...
14. F: [Interrupting] Sí lo puede saber, ¿no? ¿Por qué por dónde entra?
15. M: Por el first floor.
16. F: No. ¿Dónde entra ella?
17. M: En el middle... [Points to it with a finger on the drawing until she finds the answer on the wording] Wait! [Looks up in the wording] Middle floor.
18. F: Yes. ¿Y cuántos tiene abajo?
19. M: Diez.

20. F: No, de aquí [middle floor] hasta abajo, ¿cuántos hay?
21. M: Del middle floor hasta abajo hay ehm... one, two, three, four [counts starting at the middle floor, going down]... one two, three, four, five [starts to count again, now starting at the point she finished, going up]. Five floors. Porque yo lo había hecho diez. Cinco. Pero como tuve que agregar diez [3!] [points to the 3 floors she added later] más [points to the three at the bottom she added later], seis, siete ocho.
22. F: ¿Entonces cuántos quedan hasta abajo?
23. M: No, wait. No. Son dieciséis pisos porque serían ocho acá. Tiene que tener ocho encima, ¿no?
24. F: Bueno, si acaso luego volvemos sobre cuál es la respuesta correcta. A ver, ¿lo primero que has hecho, qué es? ¿Cómo lo has pensado? ¿En qué momento has cambiado de lengua?
25. M: En inglés y namás cambié para poner la, la... mm... [points to the answer] Bueno, todo lo hice en inglés. Lo único que sí puse, fue cuando puse los números, que estaba en español.
26. F: Okay... ¿Y cuándo más en español?
27. M: Nada más eso.
28. F: Y todo lo otro, ¿todo en inglés?
29. M: Yeah.
30. F: Okay. Volvemos sobre el problema, pues, si lo quieres comentar. Creo que tienes casi la respuesta correcta, pero... A ver, porque el diagrama está bien. ...
31. M: Sí, ya entendí. Si tiene entonces ocho pisos abajo, el del medio es el octavo, entonces tienen que ser ocho arriba que son dieciséis pisos.
32. F: ¿Cuántos tiene abajo?
33. M: Ocho. Uno, dos, tres, cuatro, cinco, seis...
34. F: Pero éste no lo contamos, éste es el del medio.
35. M: Oh, yeah! Uno, dos tres, cuatro... ¡siete! Entonces serían catorce, quince pisos. [Writes '15' above '13' as final answer: A4,1]
36. F: Mm [Affirming]. Very well. [Continues in GLQ,1]

Miriam makes a good Spanish wording summary (7) which denotes a good understanding and internalization of the situation presented in the wording, even if later (14) she confuses which is the floor which Jamie uses to enter the building (first floor instead of middle floor). Miriam benefits from the interviewer's subtle comments (22) to quickly make steps towards the right solution (11-12, 20-23, 30-36), probably because she has understood the problem very well, but also thanks to her sketch and the mathematization work done through it. In fact, on her initial written answer (1) she is already aware that the middle floor position is a key point to solve the problem and she finally states that the answer will depend on its placement.

★ Jessica A3,1-7:

1. [Conversion about activities starts here] J: Ésta no la entiendo.
2. F: ¿No la entiende? ¿Por qué no?
3. J: No entiendo la pregunta.
4. F: Dice... ¿Qué es lo que no entiende, la pregunta? How many tiles does figure have, figure seven have. La figura siete, ¿sí?, cuantos cuadraditos va a tener. Cuántas tiles son, bueno, tejas, cuadraditos. ¿Sí?
5. J: M [surprised].
6. F: ¿Lo entiende ahora?
7. J: Mm [agreeing]. [Conversation continues at A1,2 after writing the answer for all activities]

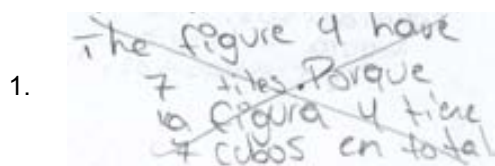
Jessica A3, 27-30:

19. F: Mm [validating]. Pero no nos pregunta por los lados, ¿no? Nos pregunta por cuántos cuadraditos. Por ejemplo, ¿cuántos hay aquí?
20. J: Uno.

21. F: Aquí hay...
22. J: Tres.
23. F: Y nos pregunta en la figura siete cuántos va a haber.
24. J: Siete.
25. F: ¿Por qué siete? [pause]
26. J: Mmm... [thinking]
27. F: Esto es figura uno, figura dos, figura tres, cuatro. ¿Sí? Nos pregunta por la número siete, de figura. ¿Lo quiere intentar de [pause] arreglarlo?
28. J: [1st written answer (A3,8) erased] Figura siete, pues no hay figura siete.
29. F: Sí. ¿Pero se lo puede imaginar, o puede calcularlo de alguna manera? ¿Puede hallar alguna manera de saber cuántas... cuántos cuadraditos va a haber en la figura siete?
30. J: Mmm... [thinking]. ¿Quince?
31. F: ¿Por qué quince?
32. J: Porque va sumando dos.

At the beginning of the solving process Jessica asks for the meaning of the wording (1-7). Without it she cannot solve the problem. Even if she says she understands it (6-7), she produces two wrong answers. With the interviewer's explanation of the statement's question (19-23) it looks like she understands it (20-23) but as she does not find a mathematical procedure to solve the problem, she questions herself about the correct understanding of the statement (28) and then she quickly finds the right mathematical procedure (32).

★ Ana A3,1-25:



[Ana makes some pauses during her writing. There might be a pause (the video is not completely clear) after writing the first sentence – which is in English. Crossing out the answer takes place in A3,26]

- 1.
2. F: ¿Cómo resolvió la [actividad] 3?
3. A: Conté cuántos cubos salió, cuántos cuadritos había en cada figura.
4. F: Ajá. ¿Y luego qué?
5. A: Pues la pregunta dice que cuál de las figuras tenía siete cubos. Y yo puse que la figura cuatro.
6. F: ¡Ohhhhh! No, la pregunta dice, ehhhh, [reading] how many tiles does figure seven, es la figura siete, se refiere a la figura siete. ¿Cuántos, tiles sabe qué son?
7. A: Cuadros o cubos.
8. F: Sí, cuadraditos.
9. A: Sí.
10. F: Cuántos cuadraditos tiene la figura siete. [Pause] ¿Sí entiende lo que le pregunta aquí?
11. A: No.
12. F: O sea, la figura siete, aquí no tenemos la figura siete, ¿sí?
13. A: Sí.
14. F: Pero usted puede averiguar, esto es lo que le pregunta, ¿puede decir cuántos cuadraditos habría en la figura siete? Y por qué. ¿Sí? ¿Lo quiere pensar otra vez?
15. A: Ajá.
16. F: Okay, pues la figura siete, a ver si me puede decir cuántos cuadraditos habría, en la figura siete. [Pause]
17. A: ¿Once?
18. F: ¿Por qué once? A ver, ¿cómo lo ha hecho?
19. A: Porque en el primer nomás hay uno, y luego en el segundo le aumenta en otros dos, en el tercero le aumenta en otros dos y en el cuarto le aumenta en otros dos.
20. F: Sí.
21. A: Y luego hay que aumentarle otros dos a los demás.
22. F: Sí.

23. A: Y sería... Tal vez la figura cinco tuviera [pause] nueve.

24. F: Sí.

25. A: Oh, aquí me faltaron [dos]. Y luego la figura seis tuviera once y la figura siete tuviera trece.

Ana understands the wording question in a deviated way (1-16) and once its meaning is clarified (12-16) she gets the right arithmetical sequence associated to the figure sequence (18-21). As she has already been working with the number of tiles in each figure, this is done quickly. She only gets confused when picking up the right term, but she corrects it while she explains the mathematical procedure (25).

★ Juan A2,3-5:

3. J: Porque dicen que encuentren el perímetro, y el perímetro de un cuadrado es cinco más cinco, [...] lo de... lo de alrededor. Y son cuatro lados, se multiplica por cinco. Suma cinco más cinco, más cinco, más cinco, y lo que suma son veinte. Y del círculo es, el área es pi por diámetro y es perímetro igual a tres punto catorce por quince, por cinco, y lo que salió fue quince punto setenta.

4. F: ¿Entonces cuál es el que tiene el perímetro más grande?

5. J: El cuadrado.

Juan finds the perimeters of both figures (3). So when asked which perimeter is greater he correctly compares both of them(5).

★ Ingrid A3,1:



1. How many tiles does figure 7 have? Why?

~~figura 1 y 4~~
~~por que las otras dos saldrían~~
~~mas que 7.~~

[The lines crossing out the answer are added on the 2nd try answer (A3,56)]

Ingrid A3,48-62:

48. F: Mm [validating]. [Pause] Aquí, las crucecitas, perdón, eran lo mismo todo. O sea que tendría que haber sólo una en todos. [Ingrid erases the crosses in the "English only" column in A2, A3 and A4, as she did before with A1 in A1,56]. Okay. De hecho, lo que nos preguntaba aquí... ¿Ha entendido...?

49. I: [Interrupting] Casi no.

50. F: ¿... todas las preguntas? ¿No? Nos pregunta cuántas, cuántos cuadraditos tiene la figura siete. Does figure seven have. Cuántos, how many tiles, cuántos cuadraditos does figure seven have. O sea, lo que tendíamos que arreg[lar], que poner aquí es, en la figura siete, cuántos cuadraditos habría.

51. I: Mm [continuing conversation].

52. F: ¿Lo quiere pensar otra vez o lo dejamos así?

53. I: Lo hago otra vez.

54. F: ¿Sí?

55. I: Mm [validating].



How many tiles does figure 7 have? Why?

56. ~~figura 1 y 4~~
~~por que las otras dos saldrían~~
~~mas que 7.~~
~~deberían 13 porque va aumentando~~
~~dos cuadrados cada vez.~~

[Entire answer]

57. F: Pensó muy rápido eso.

58. I: Sí [smiling].

59. F: ¿A ver, qué hizo aquí?

60. I: Ver cuántos aumentaba cada vez.

61. F: Mm [validating].

62. I: Y cuántos saldrían a siete. Como aquí es uno, tres, cinco, siete, en la cinco serían nueve, en la seis serían once y en la siete serían trece.

Ingrid understands the wording question in a deviated way (1), but she does a good mathematical work with the figures. When interviewer translates the wording question (50, 62) she quickly (57) finds the answer.

★ Ingrid A4,103-121:



103. ~~por que subió tres uno~~
~~pero luego volvió a bajar uno~~
~~después ~~de~~ subió tres y bajó~~
~~10 entonces tiene trece.~~

[2nd try written answer.

Ingrid crosses out the previous answer.

“15” instead of “13” in the addition is changed later. The lines crossing out the answer are added later, except for “tres” on the first line.]

104. F: A ver, ¿me cuenta cómo lo hizo?

105. I: Digo cuántos subió.

106. F: Mm [continuing conversation].

107. I: Y cuántos bajó.

108. F: ¿Cuántos subió?

109. I: Tres.

110. F: Mm [validating].

111. I: Y bajó diez.

112. F: Mm [validating].

113. I: Entonces los sumé porque tenía que ver todos.

114. F: Sumó tres, o sea subió tres...

115. I: Mm [validating].

116. F: ...y bajó diez. ¿Y esos son todos los del edificio? [Pause]

117. I: No.

118. F: ¿Cómo podemos saber todos los del edificio?

119. I: Eh... Porque ella estaba en medio.

120. F: Mm [continuing conversation].

121. I: Entonces tiene que tener los dos para el mismo lado. Entonces... [Pause]
¿Tenía siete cada piso? ¿Cada mitad?

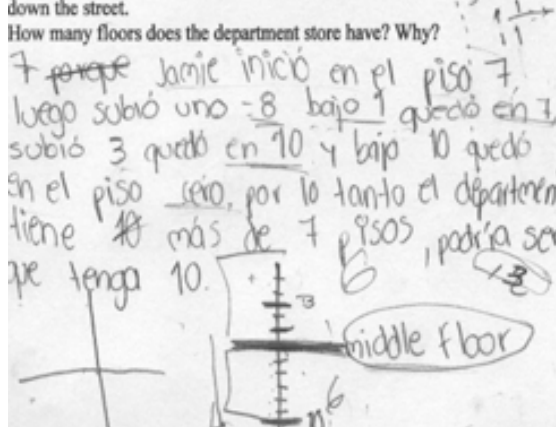
After the translation of the wording by the interviewer, Ingrid produces an answer (103) without considering the singularity of the middle floor. Even if the answer is not found straightforward, the crucial point of incorporating the symmetry particularity of the middle floor is quickly reached by Ingrid after the interviewer questions her answer (116).

★ Yael A4,73-82:

83. F: ¿Podemos volver a repasar el enunciado? Hay una pequeña cosita que debería... O sea, sí podemos saber, yo le digo que sí podemos saber...
84. Y: [Interrupting] cuántos
85. F: ... cuántos hay. ¿Quiere volverlo a leer? ¿Quiere que la ayude?
86. Y: Eh, ¡sí!
87. F: ¿O lo quiere leer por su cuenta?
88. Y: Dice que está en el piso del medio.... Está en el piso del medio, después sube uno... ¿Puedo usar un lápiz?
89. F: Sí. [Pause. Goes to get a pencil] Si quiere otra hoja...
90. Y: No, así está bien.

Jamie is shopping in a large department store with many floors. She enters the store at the middle floor, and immediately goes to the credit department. After making sure her credit is good, she goes up one floor to the jewelry department. Then she goes down one floor to the children's department. Then she goes up three floors to the toy department. Finally Jamie goes down ten floors to the main entrance of the store, which is on the first floor, and leaves to go to another store down the street.

How many floors does the department store have? Why?

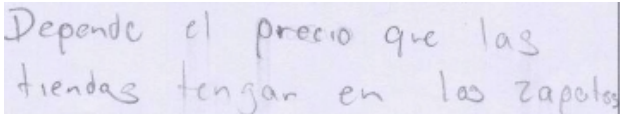
91. 

7 porque Jamie inició en el piso 7
luego subió uno -8 bajo 1 quedó en 7
subió 3 quedó en 10 y bajo 10 quedó
en el piso 1ero, por lo tanto el department
tiene ~~8~~ más de 7 pisos, podría ser
que tenga 10.

92. Y: Listo, creo que ya la tengo.

Once the interviewer states that the answer can be found, Yael –without any other clue– incorporates the key point of the mathematization of the middle floor with its symmetry function to her reasoning. By doing this she shows a good comprehension of the problem and a good horizontal mathematization.

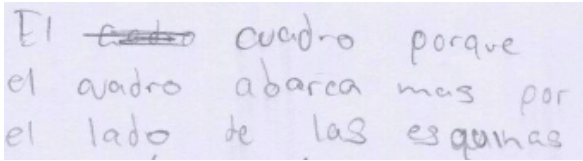
★ Julián A1,1-18:

1. [Comes from A3,13] J: Ésta [A1] sí no la entiendo. Esta parte.
2. F: ¿No? ¿Qué es lo que no entiende?
3. J: Qué tengo que hacer aquí.
4. F: ¿Qué es lo que le pregunta?
5. J: Which of these two stores are the shoes cheaper. Cual de estas dos tiendas es la... La cheaper, no entiendo esa palabra.
6. F: Más baratas.
7. J: ¡Oh!
8. F: ¿No? Más baratas.
9. J: Sí, más baratas. Pero no dice la verdad, porque no dice el precio exacto de esto para cuánto es sin el descuento.
10. F: Sí. ¿Entonces qué? ¿Qué, le hacen falta datos?
11. J: Sí. Sí, porque imagínese que aquí ofrecen el cuarenta por ciento de descuento pero que tal si lo dan más caro y lo rebajan y ganan igual.
12. F: Sí. Pues ponga esto. Lo que me acaba de decir. Lo puede poner como respuesta.
13. J: ¿Y qué le pongo aquí entonces, “no sé”?
14. F: Pues depende, ¿no? ¿Sí? Lo que... Pues expréselo como quiera.
15. J: [Unintelligible sequence]
16. F: ¿Sí?, como quiera. Si en lugar de... bueno, si se equivoca ponga una rayita encima y luego ya... Nada más escribe al lado, ¿sí?
17. J: Sí.
18.  [Continues in A2,1]

Julián is stuck because he does not know the meaning of cheaper. Thanks to his proactive demand (1-5), once he has it he quickly gets the correct answer (9, 18).

★ Julián A2,1-16:

1. [Comes from A2,18] J: Oh, aquí dice que cuál es el... [Reading] which of these figures has a greater perimeter.
2. F: Sí.
3. J: Okay. Pues yo digo que los dos, pues los dos son de cinco aquí en esto [dotted line], ¿no? ¿Pero a qué se refiere con greater... perimeter?
4. F: ¿Qué significa greater?
5. J: Éste me hace dudar.
6. F: Mayor.
7. J: ¿Mayor? ¿Cómo?
8. F: Más grande que.
9. J: Okay. ¿Luego un perímetro más grande? [Makes an imaginary circle with the pencil, below the two figures]
10. F: Great, ¿no? Great, greater es el comparativo.
11. J: Okay, okay.
12. F: Mayor.
13. J: Sí. Entonces lo voy a hacer un poco. Entonces, ¿cuál de estas dos figuras tiene un perímetro más grande?
14. F:Cuál de estas dos figuras, ajá.
15. J: Oh, pues sería éste, ¿no? El cuadrado. [Moves the pencil like inscribing the circle inside the square]

16.  [Continues in A4,1]

Even if Julián has been working on the activity (1-3), in this case the solution is not found as quickly as it happens in the case of A1, once the meaning of greater is known (3-9). Again, Julián's proactivity helps him understand the statement properly and to find the right solution (15-16).

4.2.16 Code mixing and code switching in written and oral registers

Sometimes students use a code mix on writing, mainly borrowing some words from the wording and being included on a Spanish discourse. This is, again, a clear example of the use transparent use of languages (Setati et al, 2008). By using English terms, students achieve at least two objectives. On the one hand, they show they have understood the term—at least they are able to make a practical use of it— and are able to use that particular term. On the other hand, they are able to produce meaningful text through the use of both languages, taking advantage of their bilingualism.

(I)t is important to realise that even though students might use the same words as the teacher, the meanings for students might be very different from that of the teacher (Kazima, 2006, p. 188). Even if this does not appear to be the case in the examples below, by citing this quotation acknowledge is given to this possibility and, at the same time, we show how the combined use of visual and verbal language might help to grasp the meanings of words such as tile (see Juan, A3).

The code switches that the students made during the dialogue were classified on the first reduction . Three categories were created: mathematical content, reading the wording and all other instances. The counting of the number of instances is reflected on the second reduction. For a further perspective considering all the students at a time, see the electronic data attached (“Spreadsheets” folder). Several findings derived from the analysis of this repertory are now commented.

From all 19 students, only 6 of them did use one language uniquely when they wrote the answers. Diandra used Spanish. Yolanda, Carlos, Coral, Aida and Damian wrote all their answers in English. This could be somehow expected: Yolanda, Carlos and Coral preferred to speak in English during the interview and Damian was born in the United States.

A1 promoted more code mixing, as the students referred to the names of the stores. A4's answers also reflected code mixing. In this case the mixing is a direct consequence of the real context of the activity.

Code switching on writing was observed for one student only (Camilo). From the comparison of middle school (MS) students and high school (HS) students, several findings arise. MS students tended to use code mixes when writing, and they used to do it in more than one answer. On the other hand, HS students barely used code mixing on the written answers. The 3 students who did it, did it for only one activity.

Among the HS students the use of English for A3 (similar examples of A3 had been done in class) was greater than for the rest of the activities. But the changes are not significant in comparison with those of A4. English language use was greater on the answer of A3 and A4 than on the answer of A1 and A2. As the sample is small, more investigation would be needed to confirm or challenge such findings.

These findings for the HS students coincide with what the MS students did, even if they were not used to solve activities like A3. However, they were used to find the perimeter of figures.

On the oral register, most of the code switches made by the students were to read information of the wording. Just a small instances of code-switching and code mixing is directly related to mathematical content.

Now many extracts from the dialogues are presented and commented to exemplify the use of both languages when writing.

★ Camilo A1,1:

1. *In the John's store por que
ahi ay 40% de descuento
que en la tienda de Mike*

Camilo A1,35-43:

44. C: Porque... nada más [laughing].
45. F: ¿Por qué cambió a español... ah... digo a español luego otra vez?
46. C: Así nada más [laughing].
47. F: Pero algún motivo debe haber.
48. C: No, nada más porque yo quiero. De todos modos pues si lo quisiera poner en inglés también podía pero....
49. F: ¿Pero entonces por qué lo puso en español?
50. C: Es... no... es igual... no, no... porque es más fácil para mi.
51. F: ¿Es más fácil en español?
52. [Camilo nods] [Continues in A2,2]

Camilo starts to write in English and finishes in Spanish, with a code mix ("Mike", imitating the statement). The reasons of this code switch are not initially stated by Camilo (35-40), but finally he points out that writing in Spanish is easier (41-43).

★ Camilo A3,1:

1. *13 because each figure gonna
be aumentándose tiles entonces
14 figs va tener 9 tiles la 10,
la figura 6 va tener 11, y la 7 va
tener 13 tiles*

[Camilo counts the tiles with the pencil (also during the interview after solving the activity). First Camilo thinks that the increasing number of tiles among figures is 4 tiles. After finishing with A4 he realizes it is wrong and corrects the answer. It was in Spanish but he rewrites part of it in English.]

Camilo initially writes the answer in Spanish but when he corrects it (changing the growth of the number of tiles per figure from four to two) he writes in English. This results in the use of both languages in the same sentence.

★ Camilo GLQ,1-26:

1. [Comes from A4,76] F: ¿En general, cuándo ha usado el inglés? Aquí para resolver...
2. C: ¿Aquí [pointing to the papers], en general?
3. F: Sí.
4. C: El inglés, al leer, la escritura, al leerlo.
5. F: ¿Al leerlo?
6. C: Sí.
7. F: Y en la escritura.
8. C: Sí.
9. F: ¿Y por qué cree que ha hecho esto?
10. C: Porque es más fácil para mí porque traducirlo en español, la escritura no queda... no tiene sentido.
11. F: A la hora de leerlo.
12. C: Ajá.
13. F: ¿Y a la hora de escribir?
14. C: A la hora de escribir porque se me hace más fácil explicarlo en español.
15. F: Pero me dice que ha usado el inglés también para escribir.
16. C: Mm [validating]. Pues también porque... pues... no es nada fácil el inglés[?]. No lo sé explicar muy bien.
17. F: Pero quizás... ¿Por qué? ¿Puede encontrar alguna...? No sé, si lo quiere pensar un poco...
18. C: En inglés porque no quiero escribir mucho y en inglés hay abreviaturas como aquí namás John store y luego en español el mero escribir la tienda de John y ya es mucho.
19. F: Okay. ¿Y en general cuando ha usado el español? [The bell rings, announcing the start of the next class.]
20. C: En general, explicándolo, escribiendo, en la escritura.
21. F: ¿Y para resolver los problemas?
22. C: Lo he pensado en español.
23. F: Algunos ratos en español y otros ratos en inglés, me dijo, por eso. Un poco de todo. ¿Sabe cuándo ha utilizado una lengua y cuando la otra?
24. C: Sí.
25. F: ¿Cuándo?
26. C: Pues cuando la... el español cuando... El español, pues le digo que lo explico más bien en español y el inglés lo uso porque no quiero escribir mucho. Tiene muchas abreviaturas y se escribe menos.

Camilo says that it is shorter to write in English than to write in Spanish (15-18, 26).

★ Jessica A1,1:

- In which of these two stores are the shoes cheaper? why?
1. John sport porque tiene mas descuento.

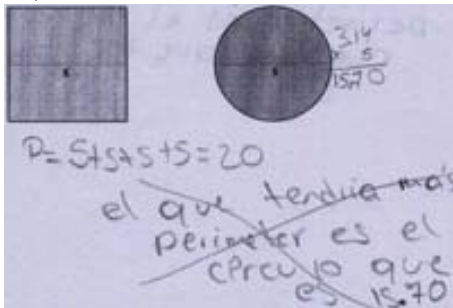
Jessica uses the name of a store when writing in Spanish.

★ Ana A1,1:

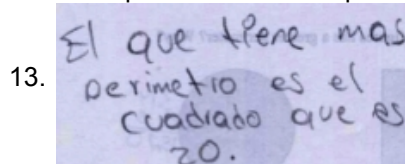
1. La tienda que yo escogería fuera la John sports porque me descuentan más que en la Mike sports aunque fueran los mismos. Si no los yo escogería John Sports.

Ana uses the name of the stores (three times) when writing in Spanish.

★ Ana A2,1:

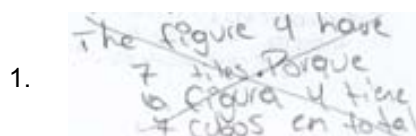


1. $P = 5 + 5 + 5 + 5 = 20$
el que tendría más Perímetro es el círculo que es 15.70
2. [Comes from A3,41] F: ¿Cómo hizo aquí para resolver esto?
3. A: Saqué el perímetro de los, de los, del círculo y del cuadrado. Y del cuadrado sumé cinco más cinco más cinco más cinco y me dio para veinte. Y en el círculo multipliqué trescientos... , tres punto catorce por cinco, que es el perímetro, y me salió quince punto setenta.
4. F: Vale, y entonces el más grande pues es el...
5. A: Oh, es el... cuadrado.
6. F: Mm [validating]. [Pause] ¿Es el cuadrado más grande?
7. A: Sí.
8. F: Es veinte, ¿no?
9. A: Sí.
10. F: ¿Se confundió aquí al escribirlo?
11. A: Sí.
12. F: Lo puede cambiar si quiere. [Ana crosses the previous answer out: A1,1]



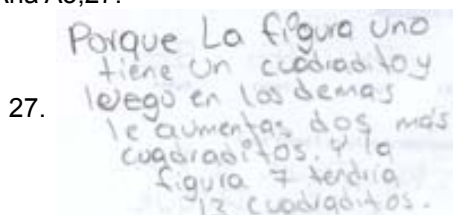
Ana uses the word “perimeter” on his initial written answer (1). She most likely learned this concept during the current school year. It is present on the statement too. This code mixing instance shows how English mathematical vocabulary is being incorporated and integrated on discourse. This does not mean she does not know the equivalent Spanish term, as she uses it orally (2) and when she rewrites the solution (13).

★ Ana A3,1:



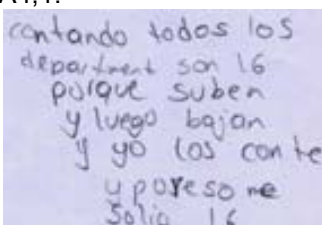
[Ana makes some pauses during her writing. There might be a pause (the video is not completely clear) after writing the first sentence – which is in English. Crossing out the answer takes place in A3,26.]

Ana A3,27:



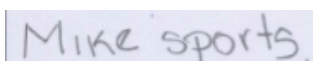
Ana writes the answer quickly through Spanish on her 3rd try (27). On her 1st try (1), at the beginning she started using English on the first sentence but changed to Spanish on the second sentence, to justify the answer. Writing in English may be more cognitively demanding for Ana, and doing it in Spanish is quicker.

★ Ana A4,1:

1. 

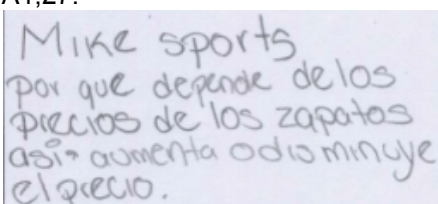
Ana uses the word “department” instead of translating it to Spanish (which is the language the rest of the answer is written with).

★ Juan A1,1:

1. 

[Juan reviews the problem after reviewing A4 but makes no changes]

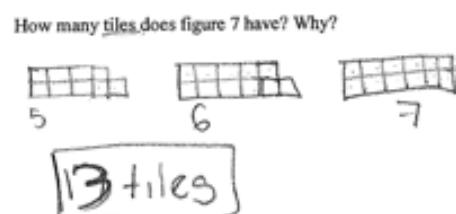
Juan A1,27:

27. 

[Entire answer]

Juan initially writes down the answer in English, but after the dialogue with the interviewer he writes the justification in Spanish.

★ Juan A3,1:

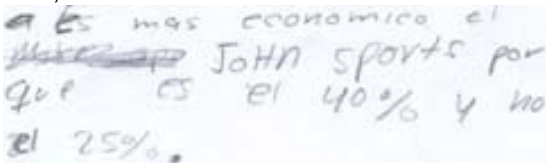
1. 

[Juan reviews this activity after reviewing A2: he changes the answer from 11 to 13 and counts again the tiles.]

2. [Comes from A2,28] F: ¿Me puede decir aquí también qué es lo que hizo?
3. J: Leí la pregunta y que ¿cuántos tendría la figura siete? En la figura uno uno, y para la dos aumentó dos, y para la tres dos y le aumenté dos en cada figura y me salían los trece.
4. F: Sí. Otra vez, el uso de las lenguas, ¿no? ¿Cómo lo hizo aquí?
5. J: El inglés lo usé para la pregunta y leer esto y el español para contestar.
6. F: Pero lo contestó en inglés aquí, ¿no?
7. J: Mm [validating]. ¡Sí!
8. F: ¿No lo contestó en español?
9. J: No.
10. F: ¿Cómo es que lo contestó en inglés?
11. J: Porque pregunta [reading] “How many tiles does figure seven have?” [No accurate pronunciation] Y tienes trece tiles.
12. F: ¿Y cómo es que utilizó aquí el inglés para la respuesta?
13. J: Porque salen trece y la palabra de lo que busca es esto [underlines the word “tiles”]. Es

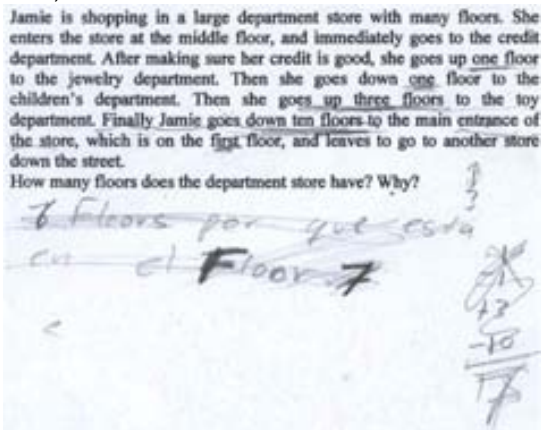
lo que buscan y es lo que sale.
 Juan writes the answer in English (1) but he says the answer is in Spanish (5). Probably it is easier for him to imitate the statement word “tiles” than translating it, which he never does (he makes a code mix –11–, refers to the wording –13– or uses demonstratives –13–). So maybe he thinks in Spanish but naturally incorporates the English word “tiles” on the discourse.

★ Angel A1,14:

14. 

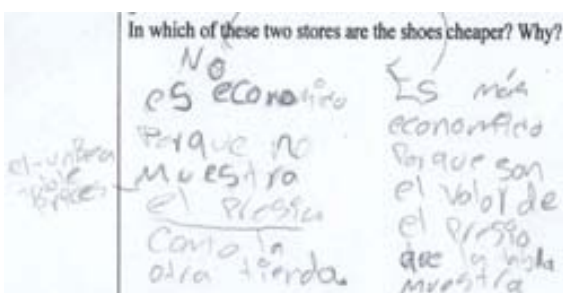
Angel uses the English name of the store (John Sports) in his Spanish written utterance.

★ Angel A4,1:

1. 

Angel uses two code mixes (“Floors”, “Floor”) when he writes the answer, borrowing the English words from the wording.

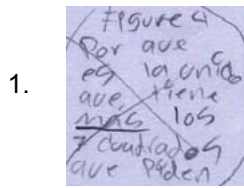
★ Abel A1,12:

1. 

[Abel writes firstly the right column, then solves other exercises and finally writes the left column]

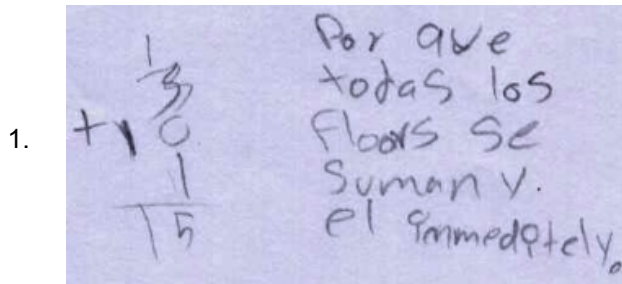
Abel uses “unBeatable Prices” on his written answer, which is taken literally from the statement.

★ Abel A3,1:



Abel uses “Figure 4” on his written answer, which is taken literally from the statement.

★ Abel A4,1:

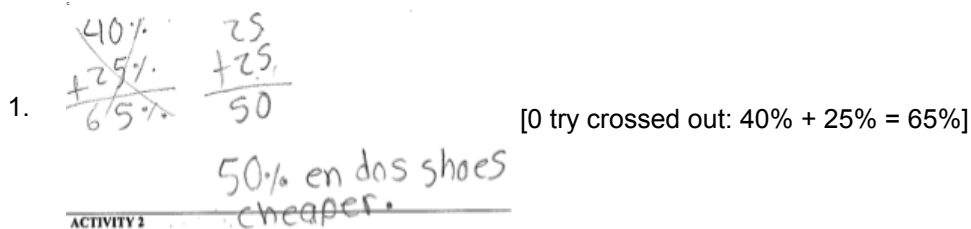


Abel A4,26-33:

- 26. F: Sí. ¿Y a la hora de escribir la respuesta?
- 27. A: En español.
- 28. F: Pero aquí tiene una palabra en inglés, ¿no?
- 29. A: ¡Oh, sí!
- 30. F: ¿Cómo es?
- 31. A: Floors. [Pause]
- 32. F: ¿Cómo es que lo puso en inglés?
- 33. A: ¿Qué? Oh, pues namás que aquí.

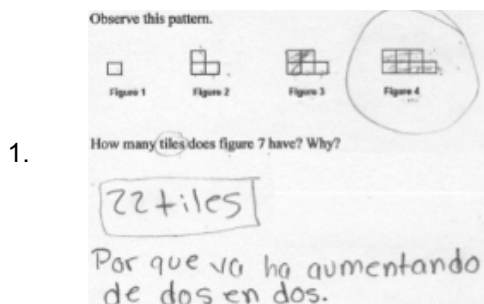
Abel uses the “Floors” and “immediately” (1) borrowing these words from the wording in a transparent way (26-33).

★ Julia A1,1:



Julia uses “shoes cheaper” on her Spanish sentence.

★ Julia A3,1:



[Julia circles figure 4 at the beginning, when she understands the question in another way, see A3,17-18. She circles the word “tiles” from the wording later (A3,38)]

★ Julia A3,16:

16.

22 tiles 13 tiles

Por que va lo aumentando de dos en dos.

[Julia writes down “13 tiles”. She circles the word “tiles” from the wording later (A3,38)]

Julia uses “tiles” in her answer, giving the argumentation in Spanish, both on her 1st try (1) and her 2nd try (16).

★ Julia A4,1-2:

1.

~~1 floor~~ → 3 floors

$1x = 3$

$x = 3$

3 floors in department.

Por que si en 1 floor es 1 en tres ✓

2.

~~1 floor~~ → 10 floors

$1x = 30$

$x = 30$

30 floors in department.

Por que si en 1 floor son tres en 10 van hacer 30 floors por departamento.

Julia uses “floor” within her Spanish argumentation, besides her initial writing of the answer in English.

★ Ingrid A1,1:

1. el la tienda JOHN SPORTS
por que es mayor el descuento.

Ingrid uses the English name of one of the stores (JOHN SPORTS) as a code mix in her Spanish written answer.

★ Yael A4,81:

Jamie is shopping in a large department store with many floors. She enters the store at the middle floor, and immediately goes to the credit department. After making sure her credit is good, she goes up one floor to the jewelry department. Then she goes down one floor to the children's department. Then she goes up three floors to the toy department. Finally Jamie goes down ten floors to the main entrance of the store, which is on the first floor, and leaves to go to another store down the street.

How many floors does the department store have? Why?

81.

7 porque Jamie inició en el piso 7. luego subió uno -8 bajo 7 quedó en 7. subió 3 quedó en 10 y bajo 10 quedó en el piso cero, por lo tanto el department tiene ~~8~~ más de 7 pisos, podría ser que tenga 10.

Yael writes the answer in Spanish, but uses the English word “department”, matching the wording. She also uses English to situate the “middle floor” on his sketch, later on the third try.

★ Claudio A1,15:

15. JoHn sports por que tiene 40% de descuento.

Claudio uses the code mix of “JoHn sports” in his Spanish written answer.

4.2.17 Mathematical procedures in the language of instruction

For the interviewed students, at least part of their school life has been spent in the United States, with English language being more or less present. Then some procedures have been learned through English alone. The cases below of Yolanda, Julián and Aida (A3) reflect this situation, reproducing some mathematical aspects through English and attaching it to the fact that they learned it at school through English.

On the other hand, some students who have arrived recently and who do not master English well enough may tend to stick to the use of procedures and concepts through Spanish. This situation is exemplified with the case of Yael. Also the cases of Ana and Aida (A2) reflect the influence of the school discourses in relation to their use of English. On the contrary, the case of Carlos, who was born in California from Mexican parents, refuses systematically to speak in English about mathematical topics in class.

Similar findings are reported by Vazquez and Powell (2011) who say that students reported it was easier to do mathematics in the same language they learned the materials.

Now extracts from the dialogues are presented and commented to exemplify how the teacher's language influences the choice of language on the students.

★ Carlos, all activities:

As Carlos is born in USA he tends to use English in relation with mathematical problems. Moreover, he says he never uses Spanish in relation with mathematics. He solves the four activities in English. This can be related to issues of power, as Mexican people are sometimes regarded in a pejorative way, in line with the findings of Setati (2006).

★ Yolanda A3,4-11:

4. F: Aha. And... you said that you have used only English here?
5. Y: Yeah.
6. F: Only English. You have never thought any number, any thing in Spanish?
7. Y: That's why I did it fast because... well, I already know this.
8. F: What do you mean you already know?
9. Y: We are doing it in class, so I already know how to do this, I just put the numbers and...
10. F: The kind of exercise you mean...
11. Y: Yeah.

Yolanda solves A3 quickly, entirely through English (4-5), as she knows well the procedure because it has been done in class (7-11).

★ Ana A2,22-29:

22. F: Y... pero, por ejemplo, para pensar pues que tenía la fórmula, ¿no?, cómo tenía que sumarlo, ¿cómo lo pensó eso?
23. A: En inglés.
24. F: ¿En inglés que tenía que sumar eso?

25. A: Sí.
26. F: ¿Me lo puede decir, un ejemplo, por ejemplo de cómo lo pensó? Alguna frase...
27. A: Pensé... Pues en la clase de matemáticas Mr Contreras nos dijo que tenemos que sumar todo lo de alrededor y así podemos sacar el perímetro.
28. F: ¿Y pensó en inglés, ahí?
29. A: Ajá.

Ana refers to her teacher's orientations as a cause for the use of English in relation with the perimeter (27).

★ Yael GLQ,23-28:

23. ¿Y en general cuándo ha usado el español?
24. Y: Para estar segura de si es la respuesta correcta o no.
25. F: ¿Por qué cree que lo hace con español eso?
26. Y: Porque como es mi idioma que toda la vida he hablado por eso tengo ya más seguridad y con el inglés apenas lo voy aprendiendo.
27. F: Y para estar segura, ¿qué se refiere con esto, para estar segura de...?
28. Y: Para comprobar las respuestas, para... Sí, para asegurar-me de que sea la correcta.

Yael refers to the use of Spanish to check the answer (23-28), as she feels more confident with its use given that she has been learning English for a short period of time (26).

★ Julián A2,39-44:

39. F: ¿Eso sí lo pensó en inglés?
40. J: Sí. Porque recuerdo las clases, por eso.
41. F: ¿Se le hace más fácil esto en inglés?
42. J: Cuándo recuerdo las cosas de la clase, sí.
43. F: ¿Se le hace más fácil hacerlo en inglés?
44. J: Porque el procedimiento, recordarlo en español algunas veces no lo recuerdo en español.

Generally, Julián thinks about the mathematical content in the same language he has learned it (107).

★ Aida A2,68-81:

68. F: ¿Las pensaste? ¿El nombre, por ejemplo? ¿Qué nombre le diste a esta figura?
69. A: Square.
70. F: Square? And this is...
71. A: Ssss... [C as in circle] Esa la pensé en español.
72. F: En español. ¿Y por qué ésta en inglés?
73. A: Porque ésta la oigo más.
74. F: Oyes más el square.
75. A: Ajá, que el...
76. F: Circle.
77. A: Ajá.
78. F: ¿En clase o dónde la oyes más?
79. A: En clase y en... [pause]
80. F: ¿Fuera de clase también?
81. A: Mm [validating].

Aida interprets the picture of the square through English (68-69) because she is used to hear it in English either at school or outside the school environment (73-81).

★ Aida A3,31-35:

31. [Comes from A4,107] F: Por ejemplo, aquí, en el problema de antes, la actividad tres. ¿Cuándo estabas haciendo la tabla y escribiendo los números, qué pensabas cero, uno, dos, tres o zero, one, two, three?

32. A: Cero, uno...
33. F: ¿Uno, dos, tres? ¿Y equis-ye o exs-why?
34. A: Exs-why.
35. F: Exs-why pero cero, uno, dos, tres. ¡Oh! [*Continues in A1,4*]
Probably Aida learned to do x-y tables in USA, so she thinks in English for it.

4.2.18 Major use of the wording language during the resolution of the problem

In some cases English is the dominant language during the mathematical solving process. For example Yolanda does it, but when mathematical difficulties are encountered, she shifts to Spanish, as a support to solve the problem. Yolanda chooses to speak in English and she does not show any use of the Spanish language on A1, A2 and A3. Below are the general comments she makes about this topic.

In the case of the English dominant students, this characteristic is less relevant. Carlos and Coral use English to solve all of the activities, with no declared use of Spanish. Also Yolanda uses English in all four activities (combined with Spanish on A4). These extracts are not reproduced, with the exception of Yolanda's for its singularity. Miriam thinks of A1's statement in English and she also uses English to comprehend all of the wordings.

Now extracts from the dialogues are reproduced and commented.

★ Yolanda GLQ,7-50:

7. F: Okay. And in these four activities, in general, when have you used Spanish?
8. Y: In this one [A4].
9. F: But why?
10. Y: And I remember in this one too [A2].
11. F: You have used also English in this one [A2] you said? I mean Spanish.
12. Y: I cant' remember.
13. F: You don't remember. So this [A4] is the only one, but maybe a little bit here [A2].
14. Y: Yeah.
15. F: But why do you think you have used Spanish?
16. Y: Maybe because I am used to, like... on what I know, I think, I use English ... on what I know in Spanish or when it makes it difficult.
17. F: When it becomes more difficult you use Spanish?
18. Y: Yeah.
19. F: You feel more comfortable with Spanish. And when have you used English, here, in general?
20. Y: Right here...
21. F: When it was easier...?
22. Y: Yeah, like this one it was easier [A3], this one is fast .This one too [A1].
23. F: When, when more?
24. Y: When I used English in these ones?
25. F: Aha.
26. Y: Just in this two [A1 and A3]. And I think more in this one [A2]. I used less Spanish in this one [A2].
27. F: And why do you think you did it?
28. Y: Maybe because it makes more difficult in things that I understand in English.
29. F: Is there any word, are there any words or phrases that you found difficult in English?
30. Y: This one [A4] got me confused. That's why I start using Spanish. And this one [A2] because I forgot what perimeter was.
31. F: But, if it had been *perímetro* in Spanish, had you understood it better?
32. Y: Yeah, because like *perímetro*, that makes me think on *perímetro*, so that's what I used to know what it was.
33. F: And what do you mean?
34. Y: Like it is under[?] in Spanish so that's how I know what it was.

35. F: This is why you knew that it was *perímetro*?
36. Y: Yeah.
37. F: But you were confused with area at the beginning.
38. Y: Yeah, because I confused area with perimeter, pero, I know what perimeter is.
39. F: But it was not because of the English?
40. Y: No.
41. F: And here... Is there any word in English that you haven't understood? Or any phrase, any sentence?
42. Y: This one [A3] no and this one [A4] ... Well, this one [A4] I didn't get confused it's just like I got confused in all the things, that was up and down... so I started using Spanish too.
43. F: But not because of the words...
44. Y: No.
45. F: ...itself or the way the sentences were written?
46. Y: Yeah. The middle floor, then on the... goes down, then three floors...
47. F: But you have understood, you understood the sentences, right?
48. Y: Yeah.
49. F: And here, in this [A1] you said no... And is... Well, there's not a lot of words here, but you have understood all the words, right?
50. Y: Yes.

Yolanda says English is the initial option (28) and Spanish is used to recall mathematical concepts (30-40). She also uses it when mathematization becomes difficult (16, 41-48). Spanish is also used on mathematical topics which are well-known in Spanish (16).

★ Miriam A3,2-18:

2. F: ¿Todo en inglés?
3. M: Mm [Affirming].
4. F: ¿Sólo inglés?
5. M: Sí.
6. F: ¿Y nunca cambiaste a español para nada más?
7. M: No. Nomás estaba contando los cuadritos en español, pero todo lo demás lo hice en inglés.
8. F: ¿Y nunca cambiaste a español para nada más?
9. M: Mm... No, nomás, em, empecé a leer todo en inglés y luego empecé a hacer las figuras pensando en español y luego volví a inglés acá [points to the answer].
10. F: ¿Y por qué aquí [A2] escribiste la respuesta en español y aquí [A1] también?
11. M: I don't know.
12. F: Y aquí [A3] en inglés. ¿No sabes?
13. M: No.
14. F: Empezaste aquí [A3] todos los cambios en inglés, todos los pasos del problema en inglés.
15. M: Lo leí en inglés y luego todos esos [points to the figures she drew] los hice pensando en español.
16. F: ¿Al contarlos sólo?
17. M: Ajá. Al contarlos nada más, me fijé que todos llevaban dos más. Como éste, éste ya estaba, y dos más [points to the 2 tiles added to Figures 3 and 4 respect to the previous figures]. Y por cada dos nomás agregaba dos más contando en español.

English is the main thinking language, as expressed initially by Miriam (2-8, 16) even if later the use of Spanish is also recognized (8, 10, 16-18).

★ Miriam A4,24-29:

30. F: Bueno, si acaso luego volvemos sobre cuál es la respuesta correcta. A ver, ¿lo primero que has hecho, qué es? ¿Cómo lo has pensado? ¿En qué momento has cambiado de lengua?
31. M: En inglés y nomás cambié para poner la, la... mm... [points to the answer] Bueno, todo

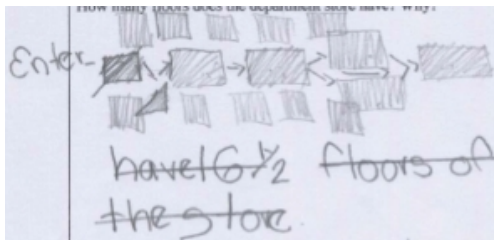
lo hice en inglés. Lo único que sí puse, fue cuando puse los números, que estaba en español.

32. F: Okay... ¿Y cuándo más en español?
33. M: Nada más eso.
34. F: Y todo lo otro, ¿todo en inglés?
35. M: Yeah.

Miriam says English is the dominant language used on A4.

★ Juan A4,1:

1.



[1st try. Juan reviews the problem after reviewing A3: adds the first and third row of floors.

The sentence is crossed out later, when writing 2nd try answer, on A4,22]

Juan A4,25-33:

25. F: Okay. Aquí me dijo... mmm... Perdona, antes de que comentemos cómo lo ha hecho esta vez, ¿sí? Antes puso la cruz solo inglés. ¿Sí? ¿Utilizó solamente el inglés?
26. J: En ésta [1st try], pero en ésta no [2nd try].
27. F: ¿Y antes que me puso sólo inglés, no utilizó el español para nada?
28. J: No
29. F: ¿Usted estaba pensando todo el rato en inglés? Aún para dibujar, pues esto [1st try: A4,1], pensó: with the floor... y todo esto.
30. J: Sí.
31. F: ¿Sí?
32. J: Sí.
33. F: Todo en inglés. Y ahora pensó en español, me dijo. A ver, ¿me explica un poco como lo hizo?

Juan A4,141-142:

141. F: Y antes me ha dicho que lo ha pensado todo en...
142. J: En inglés [looking tired]. [Continues in General Language Questions,1]

Juan takes notes (“Enter”) and writes the answer in English (1). Juan explains that he thinks exclusively in English during the first try (25-33). At the end of the dialogue around A4, Juan also admits that in the beginning he had thought exclusively in English (141-142).

★ Angel A2,2-15:

2. [Comes from A1,36] F: ¿Cómo empezó aquí?, a ver.
3. A: Leyendo la pregunta.
4. F: Sí. ¿Y luego?
5. A: Después en los... en la distancia [the measurements given].
6. F: Sí.
7. A: Y después ya cambié a español. [the answer].
8. F: ¿Esto lo pensó en inglés?
9. A: Sí.
10. F: ¿La distancia también? ¿En inglés?
11. A: Sí.
12. F: ¿Y luego qué hizo?
13. A: Ya después lo apunté en español.
14. F: ¿Qué apuntó?
15. A: [reading] Porque la base del cuadrado es cinco y lo largo del círculo es cinco.

Angel A2, 51-62 :

51. F: Sí. Vale. ¿Me dice ahora cómo lo ha pensado eso? ¿Cuándo ha utilizado el inglés? ¿Cómo... ? ¿Empezó leyéndolo? ...¿Luego cómo continuó?
52. A: Nada más en inglés lo empecé leyendo esta pregunta y nada más.
53. F: ¿En inglés?
54. A: Sí.
55. F: ¿Y a partir de ahí?
56. A: Español.
57. F: ¿Todo?
58. A: Sí.
59. F: ¿Puro español?
60. A: Sí.
61. F: ¿No pensó nada en inglés?
62. A: No. [Continues in A3, 2]

Angel uses English to think about A2 on the first try (8-11), but not on the second try (51-62).

★ Yael A2,6-21:

6. F: ¿Y luego como procedió?
7. Y: Ya luego lo traté de interpretar en inglés...
8. F: Mm [agreeing].
9. Y: Y lo entendí.
10. F: ¿Cómo lo trató de interpretar en inglés?
11. Y: Ehm... Como... Dije, oh no lo voy a traducir al español, y lo entendí así en inglés. Y ya en mi mente cómo que dije: lo voy a contestar en inglés. Pero había algunas palabras que no sabía como decir en inglés y por eso puse mi respuesta en español.
12. F: Okay.
13. Y: Pero lo estuve pensando en inglés.
14. F: ¿Todo el rato?
15. Y: Mm [agreeing, nodding]
16. F: ¿Hasta cuándo?
17. Y: Pues todo ya solo para...
18. F: Espere, sino no se [oye] [An announcement interrupts] Entonces leyó esto en inglés me ha dicho, trató de... bueno lo entendió en inglés, sin traducirlo al español, esta vez [Yael nods] Y intentó solucionarlo en inglés.
19. Y: Sí, lo solucioné, porque...
20. F: ¿Y qué pensó solucionándolo en inglés? ¿Cuál fue el procedimiento? ¿Qué es lo que pensó?
21. Y: Que para cualquier operación que tengas que hacer relacionada con un círculo siempre vas a utilizar el perímetro. Y en otra figura como un cuadrado o en rectángulo, ahí casi nunca se utiliza el perímetro. Como si quieres saber... no sé, cuál es el radio, pues tienes que utilizar el perímetro, si quieres saber cual es el volumen también tienes que utilizar el perímetro del círculo.

Yael A2,28-35:

28. F: Bueno, no pasa nada. Luego volvemos a eso. Entonces, ehm... Bueno me puede decir que es lo que... O sea, todo esto que me ha dicho ahora, lo pensó en inglés, ¿no?
29. Y: Sí. Pero como en mi mente sí sabía, pero ya no sabía como escribirlo.
30. F: Entonces lo escribió en español.
31. Y: Lo escribí en español.
32. F: Mm [agreeing]. ¿Y nada más a la hora de escribirlo lo pensó en español?
33. Y: Mm [agreeing, nodding].
34. F: El resto del problema lo pensó en inglés.
35. Y: Sí [nodding].

Yael says she uses English during the solving process (6-19, 38-35). She switches to Spanish to write the answer (11, 29-33) because she does not know some of the words she would like to use when writing in English.

★ Yael, A3,2-25:

2. [Comes from A2,98] F: Ajá. A ver, ¿Cómo empezó aquí a resolver el problema?
3. Y: Esa, ehm... También empecé en inglés.
4. F: Después de leer el enunciado.
5. Y: Leí las instrucciones y...
6. F: ¿Y después qué hizo? Después de leer las instrucciones.
7. Y: Me acordé de lo que habíamos visto en clase, de cómo se resuelven este tipo de problemas.
8. F: Ajá. ¿Y cómo se acordó de eso? ¿En qué idioma se acordó?
9. Y: En inglés.
10. F: ¿Se acordó de lo que hicimos a clase en inglés?
11. Y: Sí.
12. F: Mm [continuing conversation].
13. Y: Y empecé a contar las figuras, el número de tejas de cada figura y... pues hice mi ecuación. Y aquí [A3,1] le puse que menos uno es el número de tejas de la figura uno [Figure 0!!]. El dos es el número de las figuras que crece, porque en cada una siempre va a crecer dos. En esta son tres, en esta son cinco y en esta son siete. Y el equis es el número como de la figura. Y así es como lo resolví.
14. F: ¿El número de la figura?
15. Y: El número de la... ¡Sí! Ese es el número de las que crece y este es el número de la... de la figura [she crosses out "grown"]].
16. F: Okay. Ahora vamos a ver como utilizó el inglés, ¿no? Dice que empezó acordándose de lo que hicimos en clase en inglés. ¿Sí?
17. Y: Sí.
18. F: ¿Cómo continua después? ¿En qué momento cambia al español?
19. Y: No, no cambié al español.
20. F: ¿Pensó todo el rato en inglés?
21. Y: Sí.
22. F: Todas sus palabras en su mente fueron en inglés...
23. Y: [Interrupting] Sí [nodding].
24. F: ...en este ejercicio?
25. Y: Porque había palabras con las que ya me sentía familiarizada porque ya había trabajado mucho con ellas.

Yael says she uses English to solve the problem, with no mention of Spanish use (2-11, 16-25).

★ Yael A4,8-29:

8. F: Mm [agreeing]. ¿Qué idioma utilizó aquí para empezar?
9. Y: Para empezar, empecé en inglés. Empecé leyendo.
10. F: ¿Y marcó esto antes de leer la pregunta? ¿Antes de leer la pregunta marcó las palabras clave?
11. Y: No la iba leyendo e iba marcando.
12. F: Pero antes de llegar al final ya iba marcando eso.
13. Y: Sí.
14. F: ¿Antes de saber la pregunta?
15. Y: Mm [agreeing].
16. F: ¿Cuándo cambió a inglés? Digo, perdone...
17. Y: a español.
18. F: ...¿cuándo cambió a español?
19. Y: Ehm, cuándo iba a poner la respuesta.
20. F: Pero antes de poner la respuesta pues estuvo pensando todo esto, ¿no? ¿En qué idioma lo hizo?

21. Y: Pero esto lo iba pensando en inglés.
22. F: ¿Todo esto lo iba pensando en inglés?
23. Y: Sí.
24. F: ¿Todo? ¿Nunca el español para nada?
25. Y: No casi, no.
26. F: ¿Alguna cosita sí?
27. Y: ¡Ah, ah! [Saying no with her head]
28. F: ¿Todo en inglés?
29. [Yael nods]

Yael A4,67-72:

67. F: Okay. ¿Y mientras lo estaba pensando lo pensó en inglés?
68. Y: En español.
69. F: ¿Todo en español?
70. Y: Nada más lo que estaba leyendo es como lo estaba pensando así en inglés, así como lo leí.
71. F: Nada más al leerlo lo estaba pensando en inglés.
72. Y: No, y también cuando estaba haciendo mis gráficas en lugar de decir, oh subió uno, decía oh, up one o down two, o así.

Yael A4,108-114:

108. Y: El inglés lo usé como con los datos y ya el español como para entender más.
109. F: Pero aquí me puso, por ejemplo, "middle floor", ¿no?
110. Y: Sí, por eso, cuándo digo los datos es como toda la información que te dan en el problema. Esa toda la dejo así en inglés, no lo traduzco al español. Y ya nada más para poder poner la respuesta. Y entonces ya me toca pensar en español.
111. F: Okay. ¿Lo pensó todo en español también la otra vez?
112. Y: No todo.
113. F: ¿Qué pensó en inglés?
114. Y: ¿En inglés? Los datos que tenía como las operaciones como de sumar y restar, esto lo estaba haciendo todo en inglés.

Yael says she uses English during the thinking process on the 1st try (8-29, 67-72) and English also has an important role on the 2nd and 3rd tries (108-114)

★ Julián A3,16-21:

16. F: Okay. ¿En qué idioma empezó a pensar esto?
17. J: In English.
18. F: ¿Éste lo pensó en inglés?
19. J: Yeah.
20. F: ¿Cómo continuó? ¿Cuándo cambió a español?
21. J: In Spanish, verdad, porque interpretar eso, o sea subirle las... subirle las... Contar cuántas tiles sube.

Julián starts to think about A3 in English (16-19).

★ Julián A4,18:

18. [Comes from A3,41] J: Oh, this part... Ésta está un poco más chungu, un poco más difícil. Todo lo hice en inglés.

Julián GLQ,1-6:

1. [Comes from A4,182] F: ¿En general cuándo ha usado inglés para resolver los problemas?
2. J: Pues principalmente lo utilizo para cuando tengo que comprender el problema. Cuándo es como tipo lectura o cuándo me dan las indicaciones, utilizo más el inglés.
3. F: ¿Y por qué lo hace -utiliza más el inglés para eso?
4. J: Porque pues necesito comprender bien la pregunta para poder resolver bien el problema.
5. F: ¿Qué más piensa en inglés?
6. J: Pues lo que... pues para un problema lo utilizo más para empezando el problema, comprendiéndolo y... pues ya para resolverlo utilizo más, poco más el español porque

estoy más acostumbrado. Luego pues algunas veces finalizo en inglés y otras en español. Pues este tipo aquí [A4], lo primero que utilicé fue... todo lo que utilicé fue puro inglés. Porque tenía que pensar en todo esto de la... de cuál era el último, cuántos pisos tenía el departamento. Eso lo hice más en inglés. Y pues aquí [A3] es un poco más fácil utilizar el español.

Julián GLQ,23-30:

23. F: ¿Y en la cuatro utilizó más el inglés?

24. J: El inglés.

25. F: ¿Por que cree que lo hizo así?

26. J: Porque una pues el problema está en inglés. La otra...

27. F: [Interrupting] Éstas [A1, A2, A3] también están en inglés.

28. J: Oh, sí. Pero de todos modos cómo éste... De todos modos aquí es más... aquí [A2] tiene la figura [points to the square and circle], [unintelligible: circunferencia?/diferencia?]. Aquí [A4] es de más comprensión, de más lectura y pues todavía me hace pensar más en inglés.

29. F: ¿Y para pensar utilizó más inglés?

30. J: Sí.

Julián uses English on his first try to solve A4, being aware of the mathematization difficulties encountered (A4,18). Later on, Julián comments again that he used exclusively English on the first try of A4 (GLQ, 1-6).

Julián says A4 is solved using English alone, while a combination of English and Spanish is used on A1, A2 and A3. He relates this feature with the fact that the statement of A4 does not have pictures, only text (28). Maybe he interprets parts of the pictures in Spanish and, as A4 is formed just by a verbal mode, he does not need to switch to Spanish, as he can understand it in English. It might also be related to the fact that the wording in A4 is much more linguistically dense than the others.

★ Damian A2,10-24:

10. F: Ajá. Vale. Aquí, otra vez, ¿Sólo inglés...?

11. D: Sí.

12. F: ¿... o inglés y español?

13. D: Nomás en inglés.

14. F: Nomás inglés aquí. ¿Pensaste todo en inglés?

15. D: Nomás para... estos problemas de matemáticas los pienso nomás en inglés.

16. F: ¿Por qué? ¿Qué quieres decir "como estos problemas"? ¿Qué tipo de problemas?

17. D: Como así como de sumar y multiplicar y eso.

18. F: Sumar y multiplicar. Pero esto no era de sumar y multiplicar, ¿no?

19. D: Pues eso sí, pero... un poco sí.

20. F: Mm [agreeing].

21. D: Pero todo lo posible[?] está en inglés.

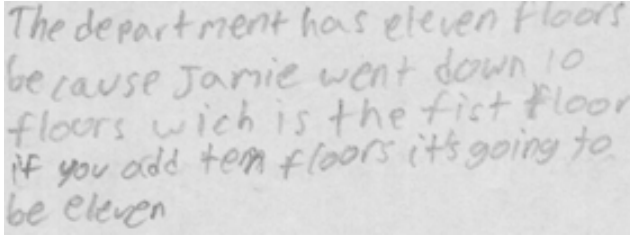
22. F: Mm [validating]. ¿Y no cambiaste a español?

23. D: No.

24. F: Okay. [Continues in A3,2]

Damian says that he only uses English to solve A2. Note that the reasons given for his argumentation (the use of operations –17–) are not reflected on the dialogue about A2.

★ Damian A4,1-11:

1. 

2. [Comes from A3,25] F: La actividad cuatro, ¿cómo la resolviste ésta?
 3. D: Éste, me fijé cuántos pisos subió y cuántos bajó. Entonces ya vi cuántos subió. Pensé que iban a ser trece porque, porque bajó... Digo doce, porque bajó uno y subió uno otra vez y luego subió tres pisos y... No aca..., no aca... Pensé que eran doce. ¿Cómo era? Que iban a... y luego que iban a ser catorce, porque iban los tres, cuatro. Y luego me fijé que bajó diez pisos. Y luego estaba hasta llegar arriba y bajó diez pisos. Y luego de todos los pisos que bajó, llegó hasta el primero y me fijé que estaba en el primero y lo sumé, le sumé diez y ya era el once.
 4. F: Ajá. Y aquí pusiste... ¿Está bien esta cruz?
 5. D: Sí.
 6. F: ¿Sólo inglés?
 7. D: Sí.
 8. F: Completamente en inglés este problema.
 9. D: Sí.
 10. F: ¿No pensaste en ningún momento en español para nada? ¿No?
 11. D: No. [Continues in GLQ,1]

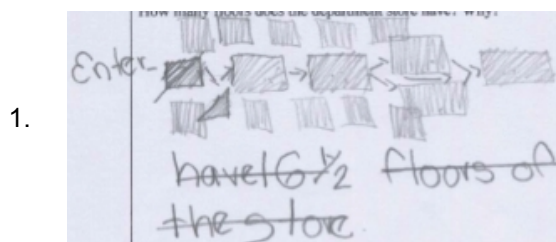
Damian says he uses English alone to solve the activity (4-11) and this is the language used to write the answer down (1).

4.2.19 Experience of mathematical difficulties and language switching

Of course, reading an English utterance could induce to English thoughts a moment after reading the English sequence. It is clear that English has some influence on the resolution of the problems of the questionnaire (at least on the initial phase of reading the problem, as they are written in this language). In some cases, after experiencing some mathematical difficulties on the solving process, when problems are thought again, there is a unique (or predominant) use of the Spanish language. This may be due to a greater confidence of the students on their first language as well as a result of the interaction with the interviewer and an internalization of the objectives of the problem through Spanish. In some other cases this shift may not become visible. It may be produced within the initial phases of the solving process, either consciously or in consciously.

Now the cases of Juan (on A4), Angel (A2), Yael (A2) and Julián (A4) are described when shifting from Spanish to English as main thinking languages.

★ Juan A4,1:



[1st try. Juan reviews the problem after reviewing A3: adds the first and third row of floors.

The sentence is crossed out later, when writing 2nd try answer, on A4,22]

Juan A4,25-33:

25. F: Okay. Aquí me dijo... mmm... Perdona, antes de que comentemos cómo lo ha hecho esta vez, ¿sí? Antes puso la cruz solo inglés. ¿Sí? ¿Utilizó solamente el inglés?

26. J: En ésta [1st try], pero en ésta no [2nd try].

27. F: ¿Y antes que me puso sólo inglés, no utilizó el español para nada?

28. J: No

29. F: ¿Usted estaba pensando todo el rato en inglés? Aún para dibujar, pues esto [1st try: A4,1], pensó: with the floor... y todo esto.

30. J: Sí.

31. F: ¿Sí?

32. J: Sí.

33. F: Todo en inglés. Y ahora pensó en español, me dijo. A ver, ¿me explica un poco como lo hizo?

Juan A4,127-142:

127.F: Entonces aquí ahora me dijo que ha usado el inglés y el español, ¿no?, esta segunda vez.

128. J: Sí.

129. F: ¿Dónde ha usado el inglés y dónde ha usado el español?

130. J: El inglés en lo de la pregunta y español en la respuesta.
131. F: Ajá. Pero mientras, mientras estaba pensando sube uno, baja tres... lo que sea, ¿qué estaba... con qué idioma estaba pensando todo esto?
132. J: Con el español.
133. F: Con el español. ¿Todo el rato?
134. J: Sí.
135. F: ¿No hay ninguna cosa que haya pensado en inglés?
136. J: No. Nada más cuánto subía y cuánto bajaba, era lo que pensaba.
137. F: ¿En inglés?
138. J: Ajá.
139. F: ¿Goes up or goes down, eso pensó en inglés?
140. J: Ajá.
141. F: Y antes me ha dicho que lo ha pensado todo en...
142. J: En inglés [looking tired]. [Continues in GLQ,1]

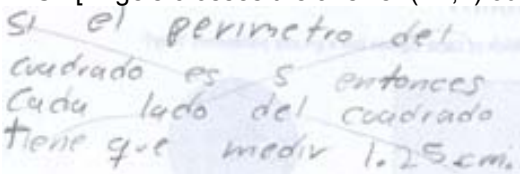
Juan takes notes ("Enter") and writes the answer in English (1). Juan explains that he thinks exclusively in English during the first try (25-33). At the end of the dialogue around A4, Juan also admits that at the beginning he thinks exclusively in English (141-142). But then, with the interviewer interaction he uses mainly Spanish in his other tries to solve the problem (127-140)

★ Angel A2,2-13:

2. [Comes from A1,36] F: ¿Cómo empezó aquí?, a ver.
3. A: Leyendo la pregunta.
4. F: Sí. ¿Y luego?
5. A: Después en los... en la distancia [the measurements given].
6. F: Sí.
7. A: Y después ya cambié a español. [the answer].
8. F: ¿Esto lo pensó en inglés?
9. A: Sí.
10. F: ¿La distancia también? ¿En inglés?
11. A: Sí.
12. F: ¿Y luego qué hizo?
13. A: Ya después lo apunté en español.

Angel A2,20-39:

20. F: ¿Qué es el perímetro?
21. A: Lo que mide alrededor del círculo. [points to the diameter]
22. F: ¿Qué es alrededor?
23. A: Lo que mide la mitad del círculo... así... esto de aquí [diameter]

24. F: Esto es el diámetro, se llama, del círculo. ¿No se acuerda qué es el perímetro?
¿Perimeter?
25. A: ¡Oh! ¿El perímetro no es... la mitad?
26. F: No, esto es el radio. ¿Sí? Esto es el radio, esto es el diámetro, pero ¿el perímetro de un cuadrado, por ejemplo, qué es? O de una figura cualquiera.
27. A: Lo que mide alrededor [Angel finally follows the perimeter of the square with the pencil].
28. F: Ajá. Esto es el perímetro. ¿Sí? ¿Lo quiere volver a pensar?
29. A: Sí. [Angels crosses the answer (A2,1) out]
30. 
31. F: ¿Ya? ¿Cuál tiene mayor perímetro, entonces?
32. A: Si el perímetro de todo es cinco [points to the dotted line with the 5 in the square], entonces cada uno tiene que medir uno veinticinco.
33. F: ¿Eso [the dotted line in the square] significa que el perímetro de todo es cinco?
34. A: Sí.
35. F: No significa eso.
36. A: ¿A no?
37. F: Significa que de aquí... esta línea discontinua, del principio al final, esto vale cinco. De la misma forma aquí [in the circle] la línea esta, desde el principio hasta el final, ¿Ve esta línea discontinua?
38. A: Sí.
39. F: Esto significa que esta línea [dotted line in the square], igual que ésta [dotted line in the circle], valen las dos cinco.

Angel A2,51-62:

51. F: Sí. Vale. ¿Me dice ahora cómo lo ha pensado eso? ¿Cuándo ha utilizado el inglés?
¿Cómo...? ¿Empezó leyéndolo? ...¿Luego cómo continuó?
52. A: Nada más en inglés lo empecé leyendo esta pregunta y nada más.
53. F: ¿En inglés?
54. A: Sí.
55. F: ¿Y a partir de ahí?
56. A: Español.
57. F: ¿Todo?
58. A: Sí.
59. F: ¿Puro español?
60. A: Sí.
61. F: ¿No pensó nada en inglés?
62. A: No. [Continues in A3,2]

Angel uses English as a thinking language on the first try (2-11), but he uses Spanish alone on the 2nd and 3rd tries –he says English is just to read the statement– (51-62), after experiencing major conceptual difficulties (20-39).

★ Yael A2,4-63:

4. F: O sea, ¿leyó el enunciado primero?
5. Y: Sí.
6. F: ¿Y luego como procedió?
7. Y: Ya luego lo traté de interpretar en inglés...
8. F: Mm [agreeing].
9. Y: Y lo entendí.
10. F: ¿Cómo lo trató de interpretar en inglés?
11. Y: Ehm... Como... Dije, oh no lo voy a traducir al español, y lo entendí así en inglés. Y ya en mi mente como que dije: lo voy a contestar en inglés. Pero había algunas palabras que no sabía cómo decir en inglés y por eso puse mi respuesta en español.
12. F: Okay.

13. Y: Pero lo estuve pensando en inglés.
14. F: ¿Todo el rato?
15. Y: Mm [agreeing, nodding]
16. F: ¿Hasta cuándo?
17. Y: Pues todo ya solo para...
18. F: Espere, sino no sé [oye] [An announcement interrupts] Entonces leyó esto en inglés me ha dicho, trató de... bueno lo entendió en inglés, sin traducirlo al español, esta vez [Yael nods] E intentó solucionarlo en inglés.
19. Y: Sí, lo solucioné, porque...
20. F: ¿Y qué pensó solucionándolo en inglés? ¿Cuál fue el procedimiento? ¿Qué es lo que pensó?
21. Y: Que para cualquier operación que tengas que hacer relacionada con un círculo siempre vas a utilizar el perímetro. Y en otra figura como un cuadrado o en rectángulo, ahí casi nunca se utiliza el perímetro. Como si quieres saber... no sé, cuál es el radio, pues tienes que utilizar el perímetro, si quieres saber cuál es el volumen también tienes que utilizar el perímetro del círculo.
22. F: ¿El círculo tiene volumen?
23. Y: El volumen, ah... No, eh ah... Sí tiene volumen, maestro.
24. F: ¿El círculo? ¿Sí?
25. Y: Sí tiene [low voice].
26. F: ¿Sí? ¿Es una figura plana, no? Que está como en la hoja...
27. Y: Oh, como si es una... ¡Ah! No me acuerdo. [he clicks his fingers]
28. F: Bueno, no pasa nada. Luego volvemos a eso. Entonces, ehm... Bueno me puede decir qué es lo que... O sea, todo esto que me ha dicho ahora, lo pensó en inglés, ¿no?
29. Y: Sí. Pero como en mi mente sí sabía, pero ya no sabía cómo escribirlo.
30. F: Entonces lo escribió en español.
31. Y: Lo escribí en español.
32. F: Mm [agreeing]. ¿Y nada más a la hora de escribirlo lo pensó en español?
33. Y: Mm [agreeing, nodding].
34. F: El resto del problema, ¿lo pensó en inglés?
35. Y: Sí [nodding].
36. F: ¿Entonces qué puso, aquí?
37. Y: Que, le puse [reading] en el círculo debido a que cualquier operación o medida que quieres encontrar en un círculo siempre tendrás que utilizar el perímetro y en cualquier otra figura como un cuadrado o rectángulo por lo regular se utilizan otras medidas.
38. F: ¿Y qué le preguntaba el ejercicio aquí?
39. Y: Que cuál de las dos figuras cómo tienes que utilizar el perímetro. Y por qué. Como yo así lo entendí. O como a cuál te es útil.
40. F: Okay. Le pregunta que cuál "has a greater", que significa "greater"?
41. Y: Pues yo lo entiendo como... como a... Así lo entiendo, como en cuál te es más útil.
42. F: Mayor. Le pregunta en cuál de estas dos figuras el perímetro es mayor.
43. Y: Oh, yo lo entendí como cuál es más útil.
44. F: ¿Lo quiere volver a pensar?
45. Y: ¡Oh, sí!
46. F: Okay.
47. Y: Pero pues en un cuadrado como nunca se utiliza un perímetro.
48. F: Mm [continuing conversation]
49. Y: Y siempre se utiliza en un círculo, pero aquí la medida de los dos es la misma.
50. F: ¿La medida de qué?
51. Y: De los dos perímetros. Porque aquí es cinco y aquí también.
52. F: ¿Qué es el perímetro?
53. Y: El perímetro... pero el perímetro en un círculo, en una... rectángulo o en un cuadrado nunca existe, el perímetro siempre existe como en figuras de círculo.
54. F: ¿No existe en un cuadrado el perímetro?
55. Y: No.
56. F: ¿Qué es el perímetro?
57. Y: El perímetro es como la línea que divide por exactamente la mitad.
58. F: Ohh. Esto es el diámetro [pointing to the diameter of the circle]. ¿Esto? [Marking the diameter of the circle]

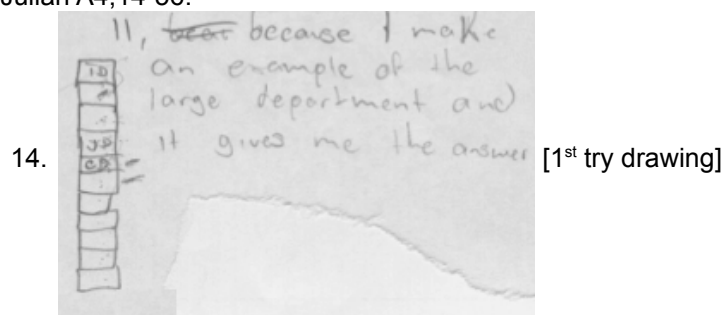
59. Y: Mm [agreeing].
 60. F: Es el diámetro. Sí, es verdad que no existe aquí. Pero el perímetro, ¿sabe qué es? ¿Se acuerda?
 61. Y: ¿Es la suma de todos los lados?
 62. F: Ajá. En un cuadrado, por ejemplo, sí. Aquí no, no hay nada. ¿Pero qué es el perímetro del círculo?
 63. Y: ¿El perímetro? ¡Ah! ¿Es la multiplicación del radio por... por "pi"?

Yael A2,70-77:

70. F: Mm [agreeing]. ¿Cómo lo ha pensado esto? ¿Me puede decir otra vez cómo ha utilizado el inglés y cómo ha utilizado el español? ¿En qué lengua empezó a resolver el problema?
 71. Y: En español.
 72. F: ¿Empezó en español ahora?
 73. [Yael nods]
 74. F: ¿Y cuándo cambió al inglés?
 75. Y: No, todo lo hice en español.
 76. F: ¿Todo en español ahora?
 77. [Yael nods]

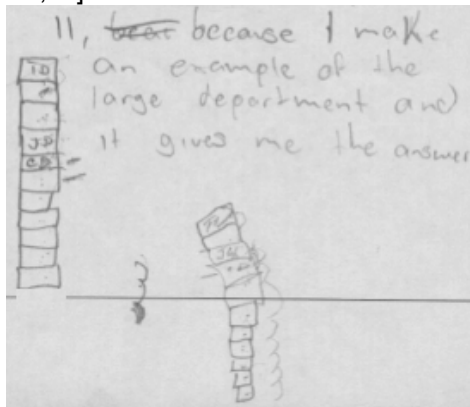
Yael thinks about the solution of A2 in English (4-18, 28-35), but given the mathematical difficulties she encounters on the 1st try (20-27, 36-61) she uses Spanish as the unique thinking language on her 2nd try (70-77).

★ Julián A4,14-56:



15. J: [Unintelligible] That's it.
 16. F: ¿Terminó?
 17. J: Yeah. [Continues in A1,19]
 18. [Comes from A3,41] J: Oh, this part... Ésta está un poco más chunga, un poco más difícil. Todo lo hice en inglés.
 19. F: Empezó leyendo el problema.
 20. J: Ajá.
 21. F: ¿Y luego cómo pensó aquí?, a ver.
 22. J: Oh, aquí...[Reading] Jamie is shopping in a large department store. Well, aquí dice... La pregunta es [reading] How many floors does the department store have. Y le puse [1st try: A4,14, reading] eleven, because it make, I make an example of the large department and it gives me the answer. So Jamie shopping is in a large department with many floors. She enters the sotre at the middle floor. So, cuando empecé, puse primero tres cuadros [Starts another drawing, see A4,25]. Para aquí tener un middle. Aquí está el middle de los tres cuadros. So empezó en el credit, credit department. Le puse aquí credit department [A4,25, writes CD]. After making sure her credit is good, she goes up one floor to the jewelry department. So subió uno y éste es el jewelry department [A4,25, writes JW on the floor above CD]. Then she goes down one floor. Oh, wait! Oh! Yeah! Then she goes down one floor to the childs department. So aquí también en el credit department it was the childs department. So regresó otra vez ahí. Then she goes up three floors to the toy department. So subió uno. ¡Oh, pincha madre [or aquí esta mal; or something else: not clear recording]! Me equivoqué entonces. So subió tres. Subió...
 23. F: No pasa nada, puede cambiarlo.

24. J: ...toy department [A4,25: writes down TD]. Finally Jam[/dzeim/], Jamie goes down ten floors to the main entrance. So entonces sería diez. Sería uno, dos, tres, four, five, six, seven, eight, nine, ten. Lo que eh... ohh Okay, wait [adds floors to the 2nd try drawing, see A4,25].



25. [2nd try drawing, on the right]

26. J: Oh my God! [Counts the number of drawn floors] Okay. To the main entrance of the store which is on the first floor and leaves to go to another store down the street. So contando los cuadros que, los floors que hice... So serían tres... y si tu sumas... diez, once. Once éstos. So... Ya.
27. F: ¿Once?
28. J: Once.
29. F: ¿Cómo lo pensó esto? ¿En qué idioma empezó?
30. J: En inglés.
31. F: ¿Empezó en inglés?
32. J: Yeah.
33. F: ¿Y luego cuándo cambió a español?
34. J: No cambié a español.
35. F: ¿No cambió?
36. J: Sólo inglés.
37. F: ¿Lo pensó todo en inglés?
38. J: Sí.
39. F: ¿Y esta segunda vez que lo hizo? O sea, esto que hizo ahora mientras me lo explicaba.
40. J: Eso en español.
41. F: ¿Lo hizo en español?
42. J: Y un poco de inglés.
43. F: ¿Cómo?
44. J: Español y inglés.
45. F: ¿Cuándo mezcló?, a ver. ¿Cómo, cómo cambió?
46. J: Okay. Pensar en inglés, pero interpretando esto [wording], y que traduciendo como al español para así explicar.
47. F: ¿Ahora lo tradujo a español?
48. J: Yeah.
49. F: ¿Pero la primera vez no?
50. J: No. I only used the English.
51. F: ¿Y ahora por qué sí?
52. J: No sé, porque cuándo hablo, usualmente yo utilizo más el español. Por eso.
53. F: Okay. Entonces lo estuvo traduciendo a español y siguiendo las instrucciones, ¿cómo? En español, ¿no?
54. J: En inglés y español.
55. F: Ajá. ¿Y luego cuándo cambió a inglés otra vez?
56. J: Yo lo leía... El primero empecé en inglés, después en español. Y pues es todo.

Julián A4,162-177:

162. F: Good! ¿Cómo lo pensó esto?
163. J: Oh...
164. F: ¿En qué lengua empezó?
165. J: Pues primero empecé en inglés, o sea...

166. F: Perdón, estamos hablando ahora de la última vez, ¿sí?
167. J: Sí.
168. F: Cuando intentó volver a hacerlo. A ver.
169. J: Okay. Primero intenté comprendiendo esto, comprendiendo toda esta parte [wording]. Comprendiendo cómo era esto. Después lo aprendí a español porque... pues es más fácil comprenderlo en español para mí, pues. Como tuve que hacer eso, o sea, tuve que comprender eso en inglés, cambiarlo al español, y pues también[?] utilicé el español para terminar de hacer.
170. F: Y siguió, desde la última vez que hablamos, que ya hicimos esa cosa [Interviewer points to drawings of 1st, 2nd and 3rd tries], siguió pensando, leyendo esto otra vez, traduciéndolo a español y ¿pensando en qué idioma?
171. J: En... pues en los dos.
172. F: ¿En los dos? ¿Sabe qué cosas pensó en cada idioma?
173. J: Pues nada más esto lo fui comprendiendo así, todo esto [wording] lo fui comprendiendo en inglés. O sea comprendiendo aquí para intentar sacarlo. Y para hacer esta figura [4th try: A4,16] utilicé el español.
174. F: ¿Y cómo se le ocurrió que tenía que añadir?
175. J: Oh, porque como nunca no dice si llegó hasta el último piso, si no llegó, pero lo que dice es que es el middle floor. So para ser el middle floor tiene que tener de los dos el mismo, de los dos lados.
176. F: ¿Y en qué idioma lo pensó eso?
177. J: Eso en español.

Julián uses English for everything during the 1st try (14,18,29-38). While he is explaining the solution to the interviewer, as the talk is in Spanish, both languages are used (39-56) when he makes a different sketch (he does not realize it is different as he follows the wording and redoes the drawing of the building). On his 4th try he mainly uses Spanish to think (162-173, 166-177).

4.2.20 Experience of language learner and use of L2

Most of the students analyzed are Spanish dominant and they are learning a second language (English). When they are in class or in any other ambient, they receive inputs in English and may express themselves in such a language. Becoming aware that oneself is on the way to improve the competences on the mastery of any human activity –and in language in particular– may result in a quicker assimilation of the target.

Some of the students do want to improve their management of the English language. The cases of Angel, Abel, Julia and Yael are now reported, showing how them benefit from the English statements of the mathematical activities.

★ Angel GLQ 35-38:

35. F: ¿Por qué cree que lo hizo así, a veces en un idioma y a veces en otro?

36. A: Para que se me pegue más el inglés.

37. F: Ajá.

38. A: Y el español para que no se me olvide.

Angel says he uses English to learn it (36). At the same time he reaffirms his Spanish dominant position and does not want to forget his first language (38).

★ Abel GLQ,11-14:

11. F: ¿Y por qué cree que sólo ha utilizado el inglés para eso?

12. A: Es que por eso están en inglés y yo me esforcé pues en leerlo en inglés.

13. F: Sí.

14. A: Pues nomás para eso y entender.

Abel recognizes he has to make an effort to understand the statements because they are written in English (12,14). Even if he does not directly recognizes he is benefiting from the use of the English language, Abel is aware that he is an English language learner and he is making an effort to overcome this issue.

★ Julia GLQ,1-11:

1. [Comes from A4,58] F: Sí. ¿Para qué más lo utilizó en general? Por ejemplo, usted dice yo siempre utilizo el inglés para esto. O si he utilizado el inglés cada vez que he encontrado eso.

2. J: Nada más.

3. F: ¿Nada más para eso?

4. [Julia nods.]

5. F: Bueno, aquí también ha encontrado una... alguna cosita. Pero bueno... ¿No se le ocurre nada más por lo que cree que haya utilizado el inglés?

6. J: Para..

7. F: [interrupting] ¿Por qué...? ¿Para?, perdón.

8. J: Para aprender inglés.

9. F: Ajá.

10. J: Para poderlo escribirlo.

11. F: Mm [validating].

12. J: Para saber lo que significa.

Julia says she uses English to learn English (8), to learn the meaning of the words (12) and to improve her writing skills (10).

★ Yael GLQ, 12-20:

12. Y: Como en el último problema, como aquí dije: oh, up one... y como vienen los números pero escritos en inglés y entonces así ya no tengo que hacer doble trabajo de traducirlo a español, ya lo dejo así como está.
13. F: ¿Para hacer las operaciones quiere decir o para qué?
14. Y: Ajá.
15. F: Las operaciones entonces las hace en inglés, ¿por no traducirlo a español?
16. Y: Sí, para no hacer doble trabajo.
17. F: Porque... Lo hace así entonces para ahorrarse trabajo. ¿Por alguna otra razón cree que lo hace así? Esto de hacer las operaciones o agarrar la información en inglés?
18. Y: Sí, porque me va a servir para aprender más inglés.
19. F: Para aprender más inglés. Muy bien. Algún otro uso del inglés.
20. Y: Mmm... [thinking]. No. Bueno, sí porque en cada problema como hay como nuevo vocabulario que aprendo. También me sirve porque lo practico.

Yael says she uses English to learn English (18), incorporating the new vocabulary she finds on the problems (20).

4.2.21 Progressive use of L2 on the dialogue

Some students reflect an adaptation to fulfill the demands of the interviewer. This is clear, for example, in the case of Yael, who progressively incorporates a proactive description of her use of both languages –and an increase of the processes that were accomplished through the English language– to solve the activities as the interview advances and once she notes that the same kind of questions will be asked for every problem. This reflects good metacognitive skills. She might be willing to cope with the demand from the part of the interviewer to focus on the English usage, sharing a fine understanding of the questions. When doing this, Yael –as well all other students– is aware of the importance of both languages when solving mathematical problems and she might benefit from this awareness in the future to learn both, English and Mathematics when using both languages alternatively.

★ Yolanda GLQ,7-40:

7. F: Okay. And in these four activities, in general, when have you used Spanish?
8. Y: In this one [A4].
9. F: But why?
10. Y: And I remember in this one too [A2].
11. F: You have used also English in this one [A2] you said? I mean Spanish.
12. Y: I cant' remember.
13. F: You don't remember. So this [A4] is the only one, but maybe a little bit here [A2].
14. Y: Yeah.
15. F: But why do you think you have used Spanish?
16. Y: Maybe because I am used to, like... on what I know, I think, I use English ... on what I know in Spanish or when it makes it difficult.
17. F: When it becomes more difficult you use Spanish?
18. Y: Yeah.
19. F: You feel more comfortable with Spanish. And when have you used English, here, in general?
20. Y: Right here...
21. F: When it was easier...?
22. Y: Yeah, like this one it was easier [A3], this one is fast .This one too [A1].
23. F: When, when more?
24. Y: When I used English in these ones?
25. F: Aha.
26. Y: Just in this two [A1 and A3]. And I think more in this one [A2]. I used less Spanish in this one [A2].
27. F: And why do you think you did it?
28. Y: Maybe because it makes more difficult in things that I understand in English.
29. F: Is there any word, are there any words or phrases that you found difficult in English?
30. Y: This one [A4] got me confused. That's why I start using Spanish. And this one [A2] because I forgot what perimeter was.
31. F: But, if it had been *perímetro* in Spanish, had you understood it better?
32. Y: Yeah, because like perimeter, that makes me think of *perímetro*, so that's what I used to know what it was.
33. F: And what do you mean?
34. Y: Like it is under[?] in Spanish so that's how I know what it was.
35. F: This is why you knew that it was *perímetro*?
36. Y: Yeah.

37. F: But you got confused with area at the beginning.
38. Y: Yeah, because I confused area with perimeter, *pero*, I know what perimeter is.
39. F: But it was not because of the English?
40. Y: No.

Yolanda says the first time she uses Spanish on A2 (10) even if she is not sure about it (11-14). But later she insists again on the use of Spanish in A2 (26-30). So somehow after having been talking about the use of languages in all activities, at the end of the interview she shows sensibility towards the use of both of them.

★ Angel A2,2-13:

2. [Comes from A1,36] F: ¿Cómo empezó aquí?, a ver.
3. A: Leyendo la pregunta.
4. F: Sí. ¿Y luego?
5. A: Después en los... en la distancia [the measurements given].
6. F: Sí.
7. A: Y después ya cambié a español. [the answer].
8. F: ¿Esto lo pensó en inglés?
9. A: Sí.
10. F: ¿La distancia también? ¿En inglés?
11. A: Sí.
12. F: ¿Y luego qué hizo?
13. A: Ya después lo apunté en español.

Angel A3,10-40:

10. F: Okay. Sí. ¿Cómo usó los lenguajes para pensar todo eso? O sea, ¿empezó en qué idioma?
11. A: En inglés.
12. F: Sí. ¿Y luego cuándo cambió a español?
13. A: Cuando lo puse a... las... Cuando contesté la pregunta.
14. F: Sí. ¿Al escribirlo?
15. A: Sí.
16. F: Pero la estuvo pensando...
17. A: En inglés.
18. F: En inglés. ¿Qué pensó en inglés?
19. A: Que si en la figura cuatro había sido siete, en la cinco va a ser nueve...
20. F: ¿Esto lo pensó en inglés?
21. A: Sí.
22. F: ¿Todo en inglés?
23. A: Sí, casi todo.
24. F: ¿Qué cositas pensó en español? Que si dice casi todo, algo pensó en español entonces, ¿no? ¿Qué pensó en español?
25. A: La [figura] número siete, si estaba bien.
26. F: ¿La última?
27. A: Sí.
28. F: ¿La última la pensó en español?
29. [Angel nods.]
30. ¿Y qué más en español?
31. A: Lo de abajo, la contesta[ción]...
32. F: La respuesta y este número. Lo demás lo pensó en inglés. El... como... Eh... Saber que se tenía que aumentar de esta a esta dos o de una a la siguiente dos, ¿esto lo pensó en inglés?
33. A: En español.
34. F: Y escribirlo, ¿lo escribió?
35. A: En inglés.
36. F: Lo pensó en inglés para escribir esto.
37. A: Sí.
38. F: Pero pensarlo, como... obse... mirar aquí, saber que se tenía que añadir dos, eso lo

pensó...

39. A: En español.

40. F: En español. Okay. [Continues in A4,5]

Angel says he thinks in English on the first try of A2 (8-11). Similarly, at the beginning of the interview he says he uses English exclusively to think about A3 (10-23) but later he says he uses Spanish too (A3,24-40). On A4, though, he says that the use of English is restricted to read the wording.

★ Ingrid:

As the interview advances, it is seen how Ingrid incorporates the uses of English in her discourse. She shifts from the use of Spanish for everything on A1 to the use of English to extract the key information on A4 and in the operations (A3, A4).

★ Yael A2,2-19:

2. [Comes from A1,39] F: ¿Hacemos lo mismo con la actividad dos?

3. Y: Sí. Aquí dice que cuál de las figuras necesitas el perímetro.

4. F: O sea, ¿leyó el enunciado primero?

5. Y: Sí.

6. F: ¿Y luego como procedió?

7. Y: Ya luego lo traté de interpretar en inglés...

8. F: Mm [agreeing].

9. Y: Y lo entendí.

10. F: ¿Cómo lo trató de interpretar en inglés?

11. Y: Ehm... Como... Dije, oh no lo voy a traducir al español, y lo entendí así en inglés. Y ya en mi mente como que dije: lo voy a contestar en inglés. Pero había algunas palabras que no sabía cómo decir en inglés y por eso puse mi respuesta en español.

12. F: Okay.

13. Y: Pero lo estuve pensando en inglés.

14. F: ¿Todo el rato?

15. Y: Mm [agreeing, nodding]

16. F: ¿Hasta cuándo?

17. Y: Pues todo ya solo para...

18. F: Espere, sino no sé [oye] [An announcement interrupts] Entonces leyó esto en inglés me ha dicho, trató de... bueno lo entendió en inglés, sin traducirlo al español, esta vez [Yael nods] E intentó solucionarlo en inglés.

19. Y: Sí, lo solucioné, porque...

Yael A3,2-25 :

2. [Comes from A2,98] F: Ajá. A ver, ¿Cómo empezó aquí a resolver el problema?

3. Y: Esa, ehm... También empecé en inglés.

4. F: Después de leer el enunciado.

5. Y: Leí las instrucciones y...

6. F: ¿Y después qué hizo? Después de leer las instrucciones.

7. Y: Me acordé de lo que habíamos visto en clase, de cómo se resuelven este tipo de problemas.

8. F: Ajá. ¿Y cómo se acordó de eso? ¿En qué idioma se acordó?

9. Y: En inglés.

10. F: ¿Se acordó de lo que hicimos en clase en inglés?

11. Y: Sí.

12. F: Mm [continuing conversation].

13. Y: Y empecé a contar las figuras, el número de tejas de cada figura y... pues hice mi ecuación. Y aquí [A3,1] le puse que menos uno es el número de tejas de la figura uno [Figure 0 is contemplated]. El dos es el número de las figuras que crece, porque en cada una siempre va a crecer dos. En esta son tres, en esta son cinco y en esta son siete. Y el equis es el número como de la figura. Y así es como lo resolví.

14. F: ¿El número de la figura?
15. Y: El número de la... ¡Sí! Ese es el número de las que crece y este es el número de la... de la figura [she crosses out "grown"].
16. F: Okay. Ahora vamos a ver cómo utilizó el inglés, ¿no? Dice que empezó acordándose de lo que hicimos en clase en inglés. ¿Sí?
17. Y: Sí.
18. F: ¿Cómo continúa después? ¿En qué momento cambia al español?
19. Y: No, no cambié al español.
20. F: ¿Pensó todo el rato en inglés?
21. Y: Sí.
22. F: Todas sus palabras en su mente fueron en inglés...
23. Y: [Interrupting] Sí [nodding].
24. F: ... ¿En este ejercicio?
25. Y: Porque había palabras con las que ya me sentía familiarizada porque ya había trabajado mucho con ellas.

Yael A4,8-27:

8. F: Mm [agreeing]. ¿Qué idioma utilizó aquí para empezar?
9. Y: Para empezar, empecé en inglés. Empecé leyendo.
10. F: ¿Y marcó esto antes de leer la pregunta? ¿Antes de leer la pregunta marcó las palabras clave?
11. Y: No, la iba leyendo e iba marcando.
12. F: Pero antes de llegar al final ya iba marcando eso.
13. Y: Sí.
14. F: ¿Antes de saber la pregunta?
15. Y: Mm [agreeing].
16. F: ¿Cuándo cambió a inglés? Digo, perdone...
17. Y: A español.
18. F: ... ¿Cuándo cambió a español?
19. Y: Ehm, cuándo iba a poner la respuesta.
20. F: Pero antes de poner la respuesta pues estuvo pensando todo esto, ¿no? ¿En qué idioma lo hizo?
21. Y: Pero esto lo iba pensando en inglés.
22. F: ¿Todo esto lo iba pensando en inglés?
23. Y: Sí.
24. F: ¿Todo? ¿Nunca el español para nada?
25. Y: No casi, no.
26. F: ¿Alguna cosita sí?
27. Y: ¡Ah, ah! [Saying no with her head]

Yael 51-72:

51. F: Ajá. A ver, ¿me dice cómo utilizó las lenguas aquí? ¿Empezó leyéndolo en inglés?
52. Y: Pues sí, porque todo está en inglés.
53. F: Ajá. ¿Luego qué hizo? ¿Cuándo cambió a español?
54. Y: A español...
55. F: ¿Se acuerda?
56. Y: Sí. Como cuándo lo estaba haciendo decía: oh, up one y como éstas son palabras que yo ya sé, así lo iba haciendo, ¿no?, todo en inglés. Pero después como para estar segura lo tuve que cambiar a español para saber si... con seguridad si era ésta la respuesta o no.
57. F: ¿Cuándo?
58. Y: Cuándo ya tenía que, cuándo nada más tenía que... para escribirlo.
59. F: Mm [continuing conversation].
60. Y: Y para estar segura, para comprobar.
61. F: ¿Y cómo lo comprobó eso? ¿Qué quiere decir lo tuve que cambiar? ¿Cómo lo cambió, up one, por ejemplo, a español?
62. Y: Pues no sé, le puse como la respuesta. Ya la tenía en mi mente en inglés pero dije no, se me facilita más en español.
63. F: ¿Y la respuesta final es siete o... ?
64. Y: No, es ehm... . Le puse inició en el piso siete.

65. F: ¿La respuesta significa todo esto que escribí?

66. Y: [Nodding] Todo esto, es.

67. F: Okay. ¿Y mientras lo estaba pensando lo pensó en inglés?

68. Y: En español.

69. F: ¿Todo en español?

70. Y: Nada más lo que estaba leyendo es como lo estaba pensando así en inglés, así como lo leí.

71. F: Nada más al leerlo lo estaba pensando en inglés.

72. Y: No, y también cuando estaba haciendo mis gráficas en lugar de decir, oh subió uno, decía oh, up one o down two, o así.

After commenting A1, Yael says that English is the main language used when thinking about the problem of A2 (7-19). On A3 (2-11, 16-25) there is no reference to the use of Spanish. Also on A4 (20-27) Yael says she thinks in English alone. The interview is retaken later (as there was not time to finish it when scheduled and it was time to come back to the class) and then the use of the English language when solving A4 is less accentuated by Yael (56).

5 Conclusions

An ecological experiment must allow for reciprocal processes; that is, not only the effect of A on B, but also the effect of B on A. This is the requirement of reciprocity (Bronfenbrenner, 1977).

As stated in Chapter 1, "Introduction", the two main goals of this study are:

Goal 1. To examine resolutions of mathematical tasks by bilingual students who are in the process of learning the language of instruction.

Goal 2. To examine perspectives from these bilingual students on the use of their two languages in the resolution of the tasks.

The successful achievement of the goals has been justified in Chapter 4, "Discussion and Findings". Now we come to the discussion of the main research question, which was also stated in the Introduction as following:

Research Question. Which are some of the relationships emerging from the interaction between the mathematical learning by bilingual students and their use of their two languages?

In this final Chapter, "Conclusions, we draw on the substantial empirical evidence that has been presented in Chapter 4 to summarize two types of conclusions. Firstly, we present conclusions on some of the methodological decisions that have been taken throughout the course of the investigation. Secondly, we introduce conclusions arising from the set of all the emerging themes that constitute the findings. Finally we point to some practical implications for the teaching and learning of mathematics in multilingual settings. Such practical prospective is seen as the 'natural' continuity of a research that was motivated by the phenomenon of school failure in groups of linguistically diverse students. This is a research that was planned to serve for the improvement of practice in the mid term.

5.1 Conclusions on the design and the methods

In this section some possible improvements in the methodology are explored and presented as conclusions of the study. In educational research, there is always a need for a continuous improvement of methods, and one way to approach such need is to work on

revisions of methods that have been already planned, developed and implemented. Methods drawn from this particular research may inform other investigations placed in similar settings and provide a further analysis with similar goals. Moreover, conclusions on the methods may serve for replications using new sets of data.

The research question and the goals of this investigation have been addressed through the organization and development of case studies. Data have been mainly collected by means of individual interviews with 19 Spanish and English bilingual students. These interviews were organized around a task-based questionnaire that was elaborated for the purpose of the research. The compilation of conclusions below have to do with the case studies, the interviews, and the questionnaire. As it will be exposed, there are open questions that require exploration in order to claim that a sufficient critical analysis has been reached.

Concerning the development of the case studies, it is important to think of the practical criteria that were taken to select the students for direct participation in the research. The primary criterion was to have students who were themselves Mexican or whose families were of Mexican origin. A final criterion was to include in the final sample those students who had carefully completed the questionnaire with no more than one missing response. Nevertheless, some of the obstacles in the research process indicate that the exclusive dominance of these two criteria should be revised in the future in that they may be problematic. A few arguments for this assertion are elaborated in the next paragraphs, together with the discussion of some final methodological worries.

According to data from the development of the case studies, was the selection of the students adequate? Having more students volunteering and then making an internal selection within the initial group would have probably resulted into a richer collection of data than just taking directly all 19 students who voluntarily participated in the investigation. However, given that I was in charge of the data collection on my own, I was myself a full time teacher in a high school and the same high schools had approximately my same schedule, it had not been possible to interview students from other high schools in the district, unless they had volunteered to stay after school, which none of them did when they were asked to. Time and space limitations made, therefore, difficult to collect additional data. Luckily some students from a middle school offered their participation after the school day. This fact enlarged the age range of the population under study, and it did not represent any inconvenient as regards to the sustainability of the mathematical tasks in the questionnaire. All students in the middle school, for example, had recently been working on the topic of perimeters and areas in their mathematics

classroom and so they were expected to have the knowledge needed to solve Activity 3 in the questionnaire fresher than their peers from the high school.

Were all the students equally good informants? In a research we not only want informants but rather good informants that actively participate in the collection of data. Some of the students in the sample showed signs of not being completely comfortable when they were interviewed. Sometimes their answers were very short, lacking pro-activity in the explanation of the mathematical solving process or in the use of languages, with plenty of monosyllables like 'yes' and 'no' in response to questions that were asking for qualitative explanations. The major assumption that all students were 'knowers' made it recommendable to insist on direct questions, as it occurred for example with the case of Jessica.

The presence of a peer might have changed the rapport with the interviewer. But we wanted the students to solve the problems individually, as it was what they often were used to when solving written tasks at school. On the other hand, asking the teacher for the selection of a sort of 'communicative' students might have resulted into the compilation of more detailed examples on the use of languages as well as on the mathematical aspects of the solving process. However, such a selection would have been risky in that the teacher might have recommended the most 'academic' students instead of the most participative, or the most co-operating ones.

Was the selection of the physical space for the interviews adequate? For practical reasons, interviewing the students on the same setting and where all students were available at the same time was a reasonable way to get access to data from the learners. For future studies, however, making interviews at the students' homes might be of interest, given that homework sometimes is made individually and confidence may be more easily guaranteed in that context. Issues of privacy, availability and the presence of the families should also be considered to facilitate a more positive rapport with the students in the sample. Although the presence of the parents during the interview at the house would be a significant methodological and theoretical change in the approach to the research, it might be beneficial in many senses as families are a source of (indirect) information concerning their children's perspectives.

Taking into account the comments above, how could future selections gain adequacy? Two main methodological conclusions arise from the analysis. For future studies it would be convenient to choose the final sample from a larger amount of volunteers. Moreover, it would be convenient to ask the ordinary mathematical teacher for the students who tend to be participative and co-operating in the classroom, making sure that a variety of levels

of mathematical achievement is duly represented. Although there are good reasons for recommending these improvements in the methodology, the quantity and the quality of the results that have been obtained indicate that the selection of students was adequate enough.

Concerning the development of the interviews, it is important to reflect on some of the interviewer's actions that may have had an unintended influence on the contents of the students' answers. Some of the inconveniences in relation to the controlling role of the interviewer cannot be ignored, nor can it be ignored the fact that the interviewer inevitably acts as a powerful tool in the representation of data. Like the students, the interviewer is also a 'knower' that informs and frames the collection of data.

One permanent worry has been whether the actions by the interviewer were always adequate. In the first interviews it was assumed that the students preferred to work on their own, as the presence of the interviewer would put pressure on them. In fact one of the students (Diandra) explicitly asked if the interviewer had to physically attend to her resolution of the tasks. Conversely, a few students wanted the interviewer to stay while they were answering the questionnaire. In the subsequent interviews, the students were given the option to choose whether they wanted the interviewer to stay or to leave while they were working on the tasks from the questionnaire. Similarly –and almost for all the cases– the students were informed that they had the option to ask any doubt that they might have. The biggest challenge was, however, to provide a context in which the students freely talked about their language and mathematical experiences when they were asked by an interviewer who was himself a Spanish and English bilingual and a mathematics secondary teacher.

Was the dual role of teacher and interviewer kept balanced during the interview? Knowing some of the students and being their mathematical teacher led sometimes to guide the solving process closely in terms of the students' mathematical learning. It was always intended to take distance from it, that is, to separate the teacher's role from the researcher's. Nevertheless, in punctual situations the "closed-mouth" approach was not accomplished. The students may have grasped by the tone of voice or by gestures whether their answers were mathematically right or wrong. The interviewer presented himself as a researcher and a mathematics teacher, so that it was clear for the students that he knew more than them about mathematical topics. What was actually fundamental was to tell them that they knew much more about the topic of learning mathematics in a language that is not the home language.

Were the questions in the interview always pertinent? The questions were formulated to obtain information about the two research goals, but on some occasions the students did not immediately grasp the intended meanings. The students did not always provide representative examples neither did they give answers as rich as expected. For instance, the resolution of the activity was sometimes not described in much detail and so was the case for the use of the languages. This was what happened with Camilo, a student who did not consider thinking of a particular word in English as a relevant fact to be explained. He actually said that he had thought the solving process in Spanish. When he was directly questioned about the English term 'square', which he had used during the resolution of one of the tasks in the questionnaire, he came to talk about his use of English and Spanish. It can be considered that this case and others inform of how the students perceive their use of their languages. This is why it would be reasonable to keep the design of the questions as it was, though for future studies the inclusion of more directive questions might be included.

Concerning the construction of the questionnaire, it is important to reflect on the selection of the four mathematical tasks and its influence on the emergence of the students' oral actions and written answers. The mathematical contents of the questionnaire are crucial as they were the starting point for conducting the interviews. From the design of the research, it is clear that the characteristics of the mathematical tasks have a determinant role in the detection and characterization of the findings. In other words, the selection of a different set of tasks would have led to the compilation of a different set of findings. *Were the tasks useful enough for the goals of the study?* Four mathematical tasks with different conceptual and procedural topics, communicative modes and wording contexts were proposed. The relevance of each characteristic has been presented earlier in the manuscript, but more research would be necessary to clarify to which extent these variables are interrelated among them and have an impact on the set of findings. What seems clear is that the tasks and the interviews prompted a good amount of themes, as reflected in Chapter 4, Section 2. Two of the tasks (Activity 1 and Activity 4) were not typically posed to the students in their mathematical classrooms, as commented on Chapter 3. For Activity 2 even if the comparison of the perimeters might not be so common, at least its calculation was a common practice for the middle school students. The contents of Activity 3 were familiar for the group of students in the high school. This raises the question of whether the fact that the problems are similar to the ones that the students are used to solve in class has an impact on the results.

Are there changes in the tasks that might reinforce the detection of results? Most of the students were told that they could ask questions about the language and mathematical characteristics of the activities to the interviewer. Even so, indicating such instruction on the sheet of paper (next to the instructions to complete the questionnaire) would help them be more conscious that effective communication was fostered. In relation to this aspect, access to a dual English-Spanish, Spanish-English dictionary might have been also useful to unveil the meaning of concrete words, especially for those students who were more reluctant to speak with the interviewer. Particularly for the students who have arrived more recently to the local school system, a decrease of the complexity of the sentences from the point of view of the language would have kept the focus on the mathematical aspects and consequently the cognitive demands to comprehend the statement would have been lower. The linguistic complexity of the sentences, though, prompted a good amount of findings that show how the mathematics and language aspects are intertwined in the solving process of the tasks.

Had it been more informative to hold the interviews around each activity immediately after its resolution? This question cannot be answered as there is no data to compare both methodological options. However, after the analysis it seems that the two possibilities might be influencing the student's perception of the use of languages. This alternative in the development of the interview would have the potential to make the students focus on meta-cognition skills after each activity had been commented with the interviewer. Maybe it would leave aside the blurry perception in relation to the students' use of their languages (in some cases it was not possible to state if a language was used and to which extent) that on some occasions was manifested during the interview. It would also have the potential disadvantage of making the students more inclined to reflect an increasing use of English as the interview advances.

5.2 Conclusions on the mathematics in the themes

In this section results on the students' mathematical work are commented and presented as conclusions of the study. The main mathematical features in the set of the emerging themes are recapitulated in order to develop "Concluding Mathematical Themes" –CMTs–. If comparing the results from this study with the literature in the field, it can be seen that some of the CMTs challenge a number of existing works, some others confirm themes that have been claimed by other authors, and some others introduce themes that had not been considered before in the seminars of Project EDU2009-07113.

CMT1: Differences in the mathematical performance between the oral and the written registers

The use of oral registers has implicit a greater level of interactivity (that allows for a quick feedback) than the use of written registers, which normally conveys a greater degree of clarity, as the ideas may have been organized in advance.

Many students provided more details of their mathematical reasoning when speaking than in their written answers. On the one hand, this seems to happen when the answer has been written prior to the oral explanation. On some occasions, it is clear that the interactivity with the interviewer facilitated a more detailed expression on the part of the students; in particular, if some of their contributions were not understood, they were asked to refine the explanations. On other occasions, the students directly provided more details on the oral explanation, without a demand for it on the part on the interviewer. Maybe the students had still not reflected much on the way to write the answer when they did it, as they might have wanted to finish it quickly or have just thought that the text would be understandable enough.

On the other hand, when there is an oral approach to the problem (with the help of the interviewer) and then the answer is written down, it might have occurred that the students did not have the ability to reflect on the essential mathematical characteristics or that they had decided not to include all the details in their explanations.

We conclude that the bilingual students in the sample used the oral register to complement the written explanation, either before or after writing it. The quality of the mathematical performance was dependent of the register that was used and the order in which the written effort was placed. There are no evidences that relate such differences to the level of proficiency in the oral and the written registers in the two languages of the students.

CMT2: Connections between the mathematical and the language understanding of the wording

The statements of Activities 1, 2 and 3 have a visual component. This has been a determinant influence for some students during the process of thinking and solving the tasks. I comment on this fact in relation with each activity.

For Activity 1 most of the students directly compared the percentages, without noticing that the initial prices were not given in the statement. To a lesser extent, some of them reflected an assumption of equal initial prices for the two stores. In the first case this association has a major problem concerning the right mathematical treatment of percentages as a relative value. In the second case a major factor may be a particular interpretation of the problem in relation with the students' personal experience.

For Activity 2 some of the students calculated the two perimeters but did not compare them. This close approach to the meaning of the statement was facilitated by the visual mode, with the presence of two geometrical figures (a circle and a square). Some students were familiarized with problems where they had to find out what the perimeter was. They identified the word perimeter in the statement. Maybe they did not pay attention to the rest of the sentence. Maybe they just applied the same procedures they had learned in class. Or maybe they did not know the meaning of some of the words. In all the cases, the students developed a mathematical solving process that was partially correct (this fact represents an improvement in comparison to what had happened in Activity 1, where the mathematical process and conceptualization commented above were not so adequate).

Given that the visual mode played such an important role in Activities 1 and 2, it is reasonable to think that some of the students were perhaps influenced by the visual mode and looked directly for the answer to the problem in the figures which were already on the statement of Activity 3. Indeed, they had not understood the wording properly. Again, the students might have had difficulties in the understanding of some words or the text as a whole. In this case, though, in opposition to what happened with A2, the prominence of the visual mode resulted into a mathematical procedure quite far away from the expected solution.

It seems adequate the metaphor of a loop: the mathematical and language understanding of the wording are intertwined. Initially, the students have a particular focus on the statement. Then the mathematical procedure advances grounded on such understanding and they come to a resolution for the task. Once the students find a solution to the problem –even if they do not know if it is correct or not, they think it is– they automatically think that they have correctly understood the meaning of the statement in an appropriate way. This may be because the objective (to solve the problem) has been accomplished. Then both, mathematics and language comprehension are taken as correct.

Such a mathematics-language loop might be of importance also on the visual mode and the mathematical interpretation of Activity 1, apart from what has been commented above.

A particular phenomenon arises with the mismatching of the word “unbeatable” –a word that is present in one of the advertisements for shoes– and “tabla” (table) illustrates this situation. The potential decomposition into un-bea-table seems to foster the search for “tablas” in the context of the first problem in the questionnaire. Such interpretations lead the students to different assumptions about the meaning of the percentages. It is clear than a deviated translation of a word can mislead the solving process due to the assumption of incorrect information.

Further research would be needed to determine if the natural language used to interpret the visual mode has a substantial influence on the language choice that follows such interpretation of the figures. Specially attention should be paid in relation to the mathematical aspects of the activity and the solving process.

All these cases show that a deficit view of the English language learners in the mathematics classroom should be avoided, as they have the potential to explain their reasoning and provide rich insights to the problem that were not considered initially, even if the understanding of the problems is not always as intended, or just precisely because of such understanding. The influence of a concrete understanding of the statement is stated in the next CMT too.

CMT3: Obstacles to the process mathematization from a dense wording

Like it is confirmed by other studies (e.g., Parvanehnezhad & Clarkson, 2008), students who do not master the language of instruction may encounter problems when interpreting the statements. Activity 4 is the only one of the tasks in the questionnaire that does not have information embedded in a visual mode: all the data is included in the wording. It is a large wording which, due to its extension, may present particular problems in relation to the understanding of some passages from a language point of view.

Once a first attempt to solve the problem takes place, some of the language difficulties are solved with the interviewer's intervention. Nevertheless, for most students, the interviewer has to insist on the mathematical characteristics of the middle floor and the top of the building to guide a right resolution for the problem. Initially the explicit clue is to remark the importance of these floors. If the students do not make progress in the resolution with this clue and references to key points in the wording, the indications gradually become more directed. Despite the hints, some of the students are not able to solve the problem. For those who solve the problem correctly, the solution is not hard to understand, what can be interpreted in terms of the sustainability of the mathematical contents.

As there is a lot of information spread out all throughout the wording, the students were not able to link all of it together. Hence the mathematizations were all of them initially wrong. This is the case specially for those who did not support the mathematization process with a sketch. It had been useful, as the visual mode was absent in the statement. For future cases, recommending the students to use a visual approach –when possible– should be considered as a complimentary support to understand the vocabulary and the meaning of the text as a whole.

An example of this lack of interconnection regarding the meaning of the different pieces of information is that some students interpreted “Jamie” as 'Jaime', so they transformed an English female name into a Spanish male name, omitting the reference to a female name found on the wording (“she”). This interchange of vowels was made by Spanish dominant students. However, also English dominant students encountered similar problems on the mathematization process to those faced by their Spanish dominant peers.

Another consideration is that the meaning of particular words is misunderstood, as what happens with floor (translated as 'piso' which is a polysemic Spanish word). Even if the whole individual meaning of all the sentences presented by the statement is understood, the mathematization of all the statement components needs to be done precisely to pursue with the solving process (Kazima, Pwele & Kasakula, 2011).

In some cases, the meaning of some words was not understood but the students did not ask for its meaning to the interviewer. Furthermore, sometimes a deviated meaning was assigned to particular words. It is also seen in the other exercises. This practice is also observed with monolingual students or in those who has as mother tongue the language of instruction. With bilingual students, and specially those who do not manage the language of learning and teaching, special attention should be paid to this phenomena.

The rest of the activities are less linguistically dense –there are much less words–. Then it is less probable to find words whose meaning is not known. However the students did not use to ask for the meaning of particular words on Activity 4. They did do ask in the rest of the Activities (words such as greater, perimeter, cheaper...). This might be because in the other activities the meaning of the words was judged to be more relevant to understand the activity (and so consequently solve it correctly).

CMT4: *L2 and/or L1 for counting and/or operating*

Most of the students used mainly their first language to count and perform operations, as it is reported in the literature (Moschkovich, 2005). It seems that the initial learning of such a basic activities is deeply rooted on the mother tongue.

But besides this finding, also some of the students resorted on the use of their L2 for counting and/or operating. One of the reasons might be the presence of an English text along with the expression of the numbers in a graphical way, as it is what occurs in Activity 3 (e.g. “Figure 1”). Another reason is that they are learning mathematics through English.

CMT5: Effect of English language difficulties on the mathematical resolution

In a few cases the students did not know the meaning of a word but continued with the resolution of the problem anyway. Once they knew the correct meaning, they incorporated this knowledge to think again around the solving process of the activity. It is important to remark that despite many of the difficulties encountered –either mathematical or not–, all the students wrote a solution to the all the problems. This is, there were no blank answers. Just one of the students pointed proactively that he was aware he had a lot of difficulties because of the language comprehension problems. However, even he solved the activity according to what he said was a limited understanding of the statement.

For future researches it would be interesting to investigate the relationship between the kind of context –specially according to their understanding– and the use of languages. In particular, if realistic approaches to the problems are related to the choice of language and how.

CMT6: Mathematical and language hints for the facilitation of resolutions

Obviously, when the meaning of a word is not known or such a word is given a different meaning, the mathematical process might be incorrect. When trying to solve the problem a mathematization process is developed and hence some mathematical features of the task are studied –coinciding or not with a process leading to the correct answer–. In some cases, the words that were not correctly understood were translated or interpreted by the interviewer during the review of the solving process. Then some students quickly benefited from the correct understanding of the statement. As they had been working on the task and knew some of the mathematical features, they quickly adapted their reasoning to the new understanding, finding the right solution or at least making good progress towards it.

It similarly happened with the mathematical hints. In this case, however, that kind of help resulted less successful. When I considered the students has correctly understood the statement, I resorted on the organization of the mathematical ideas. A considerable amount of time was devoted to this task. The long dialogues around the tasks, specially on Activity 4, reflect this situation.

Further investigation would be useful to determine why some of the students effectively used these hints while others did not. A closer look to the students' languages proficiency might be of interest. We should also pay attention to degree of understanding of the hints given by the interviewer form a language point of view.

CMT7: Performance of *mathematical procedures in the language of instruction*

Some students are aware that they reproduce some procedures or think about specific parts when they solve mathematical problems according to the language of instruction. Some of the students have recently arrived to the United States and some others some time before, but all of them have in common that they have received some instruction about new mathematical topics in California. Then, on the one hand, the use of English is a direct consequence of the imitation of the language of learning and teaching. On the other hand, most students have spent most of their life (including their school life) in Mexico and have a strong feeling of pertinence to Mexico. For them using Spanish language is their priority.

Obviously those students born in the United Stated will hardly ever use Spanish for the School mathematics unless they have attended bilingual educational programs. Another possibility is that their parents taught them some mathematical procedures in Spanish.

CMT8: *Experiences of mathematical difficulties and language switching*

Four students, two for Activity 2 and the other two for Activity 4 switched from English as the main language used to solve the activities to a predominant use of Spanish. One reason may be the interaction in Spanish with the interviewer. From the detailed analysis of the interview and the solving process, we believe that a more important reason is that after having trouble to find the solution of the problem, even if English did not pose major problems, they switched –consciously or not– to their dominant language. This is in consonance with the findings of Clarkson (2006), who points out that bilinguals used their first language to solve problems that are difficult in terms of the mathematical contents involved.

5.3 Conclusions on language use in the themes

In this section results on the students' uses of the two languages are commented and presented as conclusions of the study. The main language features in the set of the emerging themes are recapitulated in order to develop "Concluding Language Themes" – CLTs. If comparing the results from this study with the literature in the field, and like the case with mathematical content, it can be seen that some of the CLTs challenge a number of existing works, some others confirm themes that have been claimed by others authors, and some others introduce themes that had not been considered before in the seminars of Project EDU2009-07113.

CLT1: *Contradictory declarations on personal language use*

The students gave many times contradictory information during the interview. These contradictions were detected either during the interview or once it was over, during the analysis. They have been classified into four typologies, mainly in relation to the moment where they were produced. One case occurred when the student gave information in a particular direction when talking around one of the activities, but then –at the end of the interview, when summarizing the view on the language use– the student said the opposite. Another possibility is that in such a summary, the student did not mention a particular use of the language(s) that was indeed commented before. This is related with CLT2.

A second type of contradiction is when the two contradictory pieces of information appear when the dialogue is centered around one of the four activities. The contradictory information was given some time after the first piece of information (with which there was such a contradiction) was given. Some other aspects of the problem were treated in between. In these cases I was not always able to detect all the contradictions during the dialogue. Sometimes one of the opinions seem stronger than the other when the interview is taken as a whole. Other times both interventions seem to have the same importance.

A third case arises when the student changed of opinion when I asked her or him to repeat the answer or to further explain it. The student's interventions were consecutive on time. In these cases most of the times the contradiction was elucidated and the student selected just one of the views exposed. In a few cases they were not sure if one or the other utterance had actually occurred.

We do not believe that students lied on purpose., The fact that there were some pieces of information that were not matching is somehow expected in any conversation. Words, utterances and ultimately language may have different meanings according to the individuals. We should keep in mind that misunderstandings occur now and then during most of the dialogues. This is the fourth type of contradiction. Sometimes I realized that I was not speaking the same language than the students and redirected the question. In other cases, though, the dialogue was lacking full sense because I was not completely aware of the student's point of view. To be completely sure about the consistency of the interviews, another interview once the data had been analyzed would shed some light on this issue. Unfortunately, the students were not available at that point.

All these cases reveal that talking about the use of language is not a straightforward question. Metacognitive skills should be well developed if we want the students to properly reflect and to inform of how they used their languages when solved mathematical tasks (Clarkson, 2006).

CLT2: *Languages as a transparent resource in the learning of mathematics*. Most of the students did not experience the use of both languages as a problem while they were solving the mathematical tasks. They rather used both languages together as a resource on the construction of the solution. So language was both visible and invisible (Setati, Molefe & Langa, 2008). A clear example of the visibility on the use of English is the practice of code mixing and code switching (just one instance) in the written answer. But some students did refer to the answer as if it had been written in Spanish. This was even more surprising in those cases when the answer was completely written in English.

Furthermore, there are instances of students' code switching and code mixing who do not report the reasons of such a use of both languages. It is as if their use would be rather spontaneous. As reported by Planas and Civil (2010), not always the reasons of the switch are explained by the students.

Of course, some of the students had problems with the understanding of particular words. For instance, as illustrated on the CMT5 (Effect of English language difficulties on the mathematical resolution). Even though, when solving the problems or writing the answer they conjointly used both languages to advance towards the resolution of the activity.

One last relevant aspect is that even if I insisted during the interview on the role that both languages played during the resolution of the activities, most of the students remained reluctant to reflect the use of their second language.

CLT3: Blurry perception of language use and task clarifications

Some students do not clearly distinguish what are the uses of one language and what are the uses of the other language when solving one or some of the activities. This issue can be related to the meta-cognition skills that need to be involved when talking about the use of languages when solving mathematical activities (Clarkson, 2006).

Besides, some students needed clarification around the use of languages. Some of them demand if they can write in Spanish. Furthermore there is a problem with how to interpret the columns of the questionnaire. Some students did not reflected their use of languages in detail, for example considering that it was referring only to the language they used when writing the answer. There is a strong connection these perceptions on the use of languages and the transparency of languages –commented above–.

CLT4: Language for reading and interpreting the statement

Most of the students that participated in the study were Spanish dominant. As expected most of them have used Spanish in most parts of most of the activities. Some of them even declared they used English just to read and interpret the statement.

Oppositely, some of the students reported that they did not use Spanish to solve the problem but used exclusively English. Of course this might be expected for the English dominant students, or even for the students with a good level of English. However a good group of Spanish dominant students informed that they had not used Spanish to solve a particular activity. Some of them did not have a very good management of the English language.

CLT5: Code mixing and code switching in the oral register

People who had been in the United States for a certain amount of time have a certain proficiency of the English language. Most of the students reflected the use of the English language by using code mixes and code switches during the interview, mainly to refer to parts of the wording, but also with some common expressions which are commonly used by the Mexican community on California (such as 'so', 'yeah', etc.). None of the students who chose to speak in English resorted on the use of the other language during the interview. There are just a few examples of code-switching in relation to mathematical

content. The number of code mixes directly referring to mathematical content is a little higher, but very low in comparison to the total number of code mixes.

Other students, on the contrary, stuck to the use of 'Spanish only', as they restricted the use of English to interpret the statement. Halai (2011) reports that students code-switched more during the initial phases of the problem-solving process. Instances of code mixing and code switching on the oral register have been widely reported by many researches (Moschkovich, 2005; Planas & Civil 2010).

Setati and Adler (2010) conclude that code switching *occurred least in the rural primary classrooms, those that we have described as foreign language learning environments* (p. 254). I.e., *in rural schools where there is very limited English infrastructure in the surrounding community for teachers to build on in school* (p. 255). Our results seem to be aligned with these findings, as the MS school students have been less exposed to the English language (see Object 1: Students' Historical profile on page 49).

CLT6: *Code mixing and code switching in the written register*

Some students used English words when they were writing some of the answers in Spanish. Furthermore some of them did not report this code mixing, as if the answer had been written exclusively in Spanish. This fact is also a reflect of the transparency of the use of languages. It reflects how English vocabulary is assimilated and used by English language learners. While they are in the process to proactively integrate the new words on their discourse they reflect –at least– an operational understanding of these words.

In general, when the students where asked for the reasons of such codes mixing or switching, they did not provide a reason. A majority of the written answers reflected code mixing. The middle school students used more code mixes than the high school students. High school students barely used code-mixing on the written answers, in part because they tended to write the answer through English. It seems that they are more used to write the solution of the mathematical activities through English than their colleagues in the middle school. Instances of code mixing and code switching on the written register are not commonly reported on the literature. With code mixing and switching in reference to the written mode, our results seem to contradict those of aforementioned Setati and Adler (2010).

In both registers –written and oral– code mixes and switches are used to share information that is directly accessible, as either the reader or the interlocutor count with the presence of the statement. In Spanish discourses, the students used English to

organize and situate parts of the statement. Similar findings are reported by Civil and Planas (2012). They show that students use the language of instruction to get familiar with new vocabulary, to situate this new vocabulary in the given task and to begin to organize approaches to solve the task.

CLT7: Mathematical issues related with unknown vocabulary

In some cases the students asked for the meaning of particular words. This happened exclusively on Activities 1, 2 and 3, with the words cheaper, greater and perimeter, respectively. As there is not a lot of text, maybe it is easier to identify the key words needed to solve the problem and consequently the students asked for its meaning. It is surprising, though, that none of them asked for the meaning of a single word in Activity 4. But mainly students do not showed a proactive attitude on the demand for the meaning of a word. They had rather pursued with the mathematical solving process without knowing it, or assigning a deviated meaning to them, as the dialogue around the activities reflects.

CLT8: Major use of the wording language during the resolution of the problem

Most of the students said they used mainly English at least on one of the four proposed problems. A closer look to the group of students who did use English during almost all the solving process reveal a disparity depending if they attend middle school or high school, with the second group being more numerous. There are only two of the eight middle school students who did it. This is not surprising, as the middle school students had arrived to the United States relatively recently at the time of the interview. Furthermore they had not been exposed to an “English only” policy in their classrooms (as the explanations of their teacher were sometimes or most of the times in Spanish).

CLT9: Experiences of language learner while using the language of instruction

Some students reported that they were aware of being learning English and as a consequence of not understanding some of the vocabulary they encountered when reading the mathematical tasks. Instead of complaining, most of them took advantage of it. They said that they experienced this situation as a learning opportunity for gaining new vocabulary in the the language of instruction.

The literature in the field has largely discussed the experiences of multilingual learners of mathematics giving value to the learning of the language of instruction, sometimes with

more emphasis in comparison to the value given by the same students to the learning of the mathematics. Setati (1998) and Planas and Setati (2009) are only two examples of the works with this sort of findings.

5.4 Implications for the teaching and learning of mathematics

The prime motivation for undertaking this dissertation was the desire to learn more about the teaching and learning of mathematics in multilingual classrooms. This is why the final part of the manuscript is devoted to comment on practical implications for the mathematics classroom. The discussion below starts with some of the critical actions performed by teachers and students in the multilingual mathematics classroom.

Since the beginning of the interview, we allowed the students to choose the language they preferred to express themselves. Initially they were invited to choose the language they felt more comfortable speaking with. Then if they asked if it was possible to write in Spanish they were allowed to do it. The presence of code switching and code mixing both in the written and oral registers is an example that they actually used both languages. Dominguez (2011) proposes unrestricted access to bilingualism in order to fully participate in the class and communicate mathematically. We agree with Dominguez though certain theoretical orientations are not always possible at the level of the practice. One of the limitations is that teachers do not always share all the students' languages. More practical implications for the support of bilingual students in the mathematical classroom are proposed below.

Through the analysis of all the case studies it is evident that students come to the classroom with a great diversity of language and mathematics backgrounds. The first step towards the correction of a problem is to be aware that the problem exists. When bilingual students enter the classroom we should keep in mind not only the variety of mathematical levels that students normally have in any classroom but also of their different language competence in all of their languages. Even more, and as it has been commented through all the analysis and also on the conclusions above, such languages play a crucial role on the teaching and learning of mathematics, beyond its role as a setting that supports any human communication –and knowledge in particular–.

Special attention should be paid to the way the statements are presented if we want the focus to stay on the mathematical aspects. First of all we have to acknowledge the role

played by the visual mode on the comprehension of the statement. In recent times, there has been and continues to exist, a growth on the spread of the new technologies all through the world and a continuous development of new possibilities for the teaching and learning of mathematics through tablets, cell phones and online applications such as online platforms for group learning. Hence, the visual presentation of the contents is becoming more and more important. Making a closer look to the mathematical activities in relation with its visual mode we have shown how it might determine –changing or omitting– the interpretation of the written information. For the learners who are on their initial phases of developing proficiency on the language of instruction we should take advantage of the visual communication. An adequate combination of both modes –visual and textual– not only to teach procedures but also to promote richer mathematical activities will help the students to expand the development of the language of learning and teaching while, at the same time, they are involved in meaningful mathematical practices.

This same phenomenon sheds some light on what most of the times tends to be a deficit view of the mathematical learners. That is, when the solution provided is not as accurate or does not matches at all what was expected a priori. A closer look at the assumptions that the students made, as well as enhancing them to express in detail their solving process would result in a better understanding of the procedures and concepts applied. The discovering in some cases of rich mathematizations would illuminate new perspectives on similar problems and would show that some of the ideas were in fact well oriented. In some cases the answers did not match the expectations because the students made different assumptions at the beginning of the solving process. It is as the journey of a ship which is deviated just one degree at the departure and ends up many miles away from the supposed arrival point.

Nevertheless, in our analysis, some of the answers that were based on the assumptions from the visual mode seemed to have a rooted mathematical knowledge that was not correct, as the treatment of percentages as absolute value. These understandings were not immediately unveiled. Rather their establishment was approached through the analysis of different case studies. This deep confusion on the mathematical characteristics should be mentioned as a way to encourage the students to write in detail and precision what is their understanding of the task and the solution.

We propose some practical implication derived from the use of the students of the oral register to complement the written register. When the teacher knows the language of instruction, a direct communication is easier to establish with the student that when the teacher does not know it. Even though, when there are a lot of students in the classroom it

might be impossible to speak with a majority of them. When the teacher does not share the dominant language of the students, a triangular translation (Van Jaarsveld, 2011) might be useful. For that purpose there should be at least two students fluent in both the language of instruction and the home language. They would be in charge to translate the message from the teacher to the students and from the students to the teacher, respectively. However, this strategy might be time consuming. Sooner or later we will need to teach the students how to clarify and detail the mathematical aspects. For such a training, the triangular translation might be useful.

It is also important not only to use the visual language as a support for the presentation of the task but also during its resolution. Encouraging students to complement the understanding of the written text and at the same time, starting processes of horizontal mathematization would have been helpful in Activity 4. Hence the presence of figures or sketches might be useful during the entire resolution process and to enhance communication between teachers and students.

We agree with Setati (1998) that students should be allowed to code switch, specially for the understanding of the key concepts. This is in reference to both, language and mathematical related concepts. In particular to the first ones. When the students understood the key words of the statement, specially on the ones that are formed by a few sentences only, they solved the problem straightforward. For those cases, help is useful by means of code switching (Civil & Planas, 2012).

The language hints of the interviewer in relation to the comprehension of the wording had a greater impact on the construction of the solution of the problem than the mathematical hints. As a consequence, it is important than firstly teachers guide their bilingual students on the understanding of the statement and the objective of the problem. They have to ensure that such a correct interpretation of the statement is effectively made, before guiding the mathematical solving process. Furthermore, even during the mathematical process this references to the statement might be important.

Some students reported the use of the language of instruction for (almost) the entire solving process of some activities. Maybe if we ensure understanding of the statement through the language of instruction, the students will tend to maintain such a language for the entire solving process. This would have the benefit of the students' improvement on the LoLT.

Some of the students reported they used their first language in peer instruction, even if the level of the students who reported these practices is good enough to follow the classroom discourse in English. Like other studies suggest (Setati, Molefe & Langa, 2008)) the use of

the first language –for example to group students according to their dominant language– is enhancing.

Most of the students used Spanish as a writing language, as they felt more comfortable with this language. Sometimes, code mixing and switching was a practice on the written answer. Effective practices that promote the progressive use of the language of instruction should be considered, specially if the teacher is not able to understand the student's first language.

Most of the students did not reported particular instances of the use of the English language during the solving process. Even though the use of English was visible, this was partially caused by a different interpretation of the questions addressed by me during the interview. Once their understanding and ours coincided, more details about their particular use of the language of instruction were added. This view of language as a transparent resource should be appreciated by teachers, who in turn should accept this as a common practice. Bilingual learners have their own way to view, interpret and communicate their vision of the world, and this is one of the most important resources in their learning of mathematics.

All these recommendations will need the collaboration of the learners to gain efficiency. An important point will be that they take a proactive disposition to overcome language and mathematical difficulties. Consulting the available fonts (bibliographic references, internet) as well as the teacher along with the help of their classmates should be powerful resources.

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 Cross references.ods	Cross-references used all throughout the text documents
 Languages used in writing.ods	Instances of code-switching and mixing in the written register are classified in relation to the activities and students
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 FRS 00Title with signatures	First page of the thesis (for deposit purposes only)
 FRS 01Abstract_cat	Abstract of the thesis in Catalan
 FRS 01Abstract	Abstract (in English)
 FRS 01bis	Acknowledgments
Acknowledgments	
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 FRS 01bis3 Abbreviations	Abbreviations used
 FRS 1Introduction	1 Introduction
 FRS 2Theoretical	2 Theoretical framework
framework	
 FRS 3Methodological	3 Methodological approach
approach	
 FRS 4_1 Findings related	4 Discussion and Findings. Beginning of the first section
to particularities	of the Chapter
 FRS 4_1 Students-First	Contains 19 files. Each one corresponds to the first
reduction	reduction of each one of the 19 students analyzed.
 FRS 4_2 Students-Second	Contains 19 files. Each one corresponds to the second
reduction	reduction of each one of the 19 students analyzed.
 FRS 4_2 Findings related	4 Discussion and Findings. Second section of the Chapter
to commonalities	
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