Essays on International Trade and Firm Dynamics

PhD Thesis in Economics

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Per l'Albert i l'Íkram
Si poguessin posar-lo a la primera
jo no els el posaria a la tercera

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Contents

Introduction

This Thesis is concerned with International Trade and Firm Dynamics, and the connection between both. Why do countries trade? In particular, what do we gain by living in an economy with very little barriers to trade? How does trade affect the life of individuals with differing incomes, different capacities of production, and at different distances among them? The answer to these questions is crucial for understanding the well being of not only those of us who live in rich and open economies, but it can also help to understand the potential benefits of those that are currently suffering in poor and isolated countries. The first chapter of this Thesis tries to understand better this question. But for understanding these issues, it is key that we economists have correct models of trade, in particular of those that try to take into account productivity differences of firms. If the models of trade that we are currently using are misleading in terms of the effects caused to the productivity of firms, can we say much about the implications of international trade? It is precisely the second chapter of this Thesis the one that points toward some mismatches between well established results by the empirical literature and the key model of international trade that aims to explain firm level productivity differences. Pushing further the argument, we can ask, do we have appropriate models that capture the behavior of firms? Are the effects that can be computed on the distribution of plants those that arise in more realistic environments? It is precisely in the third chapter of this Thesis a contribution in this particular direction.

The first chapter of this Thesis is entitled *Trade Patterns, Income Differences and Gains From Trade* and is coauthored with my colleague Wyatt J. Brooks. Quantifying the gains from international trade is an area of research that has been widely studied using a variety of trade models. At the same time, it has been shown that non-homotheticities are useful for matching the systematic patterns of trade present in disaggregated trade data. We bring these two literatures together to ask how non-homotheticicities affect our predictions for gains from trade. To do so, we develop a N-country trade model that exactly matches bilateral trade, population, GDP per capita and within country income inequality for many countries. We include non-homotheticicities to match patterns of trade between rich and poor countries that we observe in highly disaggregated trade data. We then make use of the results from Arkolakis, Costinot, and Rodriguez-Clare (2012), which gives a simple formula for gains from trade in a large class of homothetic models, including a version of our model with the non-homotheticity removed. Our main finding is that homothetic models underestimate gains from trade in countries with small populations and low productivities, and overestimate gains in countries with large populations and high productivities. The homothetic model overestimates the gains from being open to trade in the U.S. and Japan by 14% and 22%, and underestimates them in Spain and Italy by 24% and 14%.

The second chapter of this Thesis is entitled *Measured Productivity and International Trade: An Unresolved Puzzle*. Using correct models of firm dynamics when analyzing the impact of trade is key in order to fully understand what are the effects to the supply side of the economy when it engages into trade. There are several models of trade that try to understand the role of trade and firm dynamics, but there is one that is most used by trade economists: the Melitz model (2003).[1] This model explains several features of the data. In particular, it aims to explain why more productive firms export. It is a common agreement among economists that the model is well suited in order to explain these patterns. In this chapter we ask: is it? In particular, we show that measuring productivity in the model's outcome as it is done in the data may lead to some surprising results regarding what do more productive firms do: they may be the non-exporters.

The third chapter of this Thesis is entitled *Distortions, Productivity, and Idiosyncratic Shocks* and is coauthored with my professor José María Da Rocha. We consider policy distortions in a model where plants face idiosyncratic productivity shocks that evolve following a Brownian motion. Introducing idiosyncratic shocks into the model implies that plants have non-constant operating profits and as a result there is an endogenous exit margin and incumbent plants must decide in each period whether or not to remain in the industry. By using the forward Kolmogorov equation, we analytically characterize the Stationary Equilibrium. Our main contribution is to show

[1] It has more than 5000 citations

that if a model is being calibrated/estimated without idiosyncratic shocks, where plants face constant productivity over time and the exit rate is exogenous to fit data generated from a model with shocks and endogenous entry, TFP distortions will be overestimated.

Chapter 1

Trade Patterns, Income Differences and Gains From Trade

(joint with Wyatt J. Brooks)

1.1 Introduction

An important area of research in the study of international trade the measurement of the welfare gains from trade. This question is particularly important for assessing the gains from potential liberalizations of trade policies in countries around the world, and in predicting the effects of bilateral trade agreements. This issue has been widely studied using a variety of models, each of which emphasizes different margins of adjustment when undergoing trade reform. This question has been approached theoretically using models such as Krugman (1980, 1981), Eaton, and Kortum (2002), and Melitz (2003), and has been studied quantitatively in, for instance, Alvarez and Lucas (2007) and Eaton, Kortum and Kramarz (2011). At the same time, another strand of the international trade literature has demonstrated the usefulness of models with non-homotheticicities for matching patterns of trade between countries. Some recent work in this field is Fieler (2011), Markusen (2011) and Simonovska (2011). Non-homothetic preferences allow models to match a variety of facts observed in disaggregated trade data, such as the systematic difference in the volume and composition of goods traded between rich and poor countries.

The goal of this chapter is to combine the findings of these two literatures, and see if non-homotheticities generate qualitatively new predictions about the gains from trade liberalization. To do this, we analyze highly disaggregated bilateral trade data and construct a model consistent with the patterns of trade that we observe between countries of different populations and income levels. We construct an N-country model, calibrated to match the characteristics of countries in the data. Our model exactly matches pair-wise volumes of bilateral trade, population, GDP per capita, and within-country heterogeneity in the income of individuals.

Similar to Markusen (1986), in both the model and data, rich countries trade very similar goods with one another, but trade different goods with poor countries. Yet poor countries also trade very different goods with one another. Likewise, for poor countries there are many goods that are only imported or only exported from a given partner, while this is less true for rich countries. These aspects of the data motivate the need for non-homotheticities, as absolute income levels seem to be an important determinant of trade.

We then ask if the non-homothetic nature of this model generates qualitatively new predictions for the gains from international trade compared to existing homothetic models. In order to make this comparison, we make use of the main result in Arkolakis, Costinot and Rodriguez-Clare (2012), referred to hereafter as ACR. Their result is that, under some specific conditions (balanced trade, constant elasticity of substitution demand structure and balanced trade), the predicted gains from trade can be computed by a simple formula that is only a function of readily observable statistics from the data. Hence, any model satisfying these general assumptions that matches those statistics predicts gains from trade given by their simple formula. Their main result is that most of the widely used models of international trade satisfy their conditions and, therefore, have the same predicted gains from trade whenever they match those statistics.

However, the class of models considered in ACR does not include models with non-homotheticities. Therefore, the gains predicted by our model do not, in general, coincide with those predicted by homothetic models. Our main question, then, is quantitative: how large is the difference between the two? Our strategy for answering this question is to apply the ACR calculation to the output from our model and compare that to the exact gains that we can get by changing trade costs in the model[1]. In this way, the ACR calculation is useful because it stands in for any of

[1]We model all trade costs as iceberg transportation costs rather than tariffs. Throughout the

a large class of homothetic models, including versions of our model that remove the non-homotheticity.

Our main finding is that the difference between these two measures is large for some countries and that the relationship between them is systematic. Homothetic models overestimate gains for some countries and underestimates them for others. Countries with larger populations and higher productivities tend to have overestimated welfare gains. The United States and Japan respectively have a 1.9% and 1.6% gain in the non-homothetic model, but are predicted to have gains of 2.2% and 2.0% in the homothetic model. Meanwhile, smaller and less productive countries tend to be underpredicted. Spain has a 4.3% gain in the non-homothetic model but is predicted to have a 3.4% gain by the homothetic model. However, not all countries exhibit large differences. For instance, India and China, which have very large populations and low productivity, have gains of 2.8% and 1.5% in both the non-homothetic model, and in the ACR calculation. Our interpretation is that the effects of their large population and low productivity offset one another.

To build intuition for these results, we provide a simple, two country non-homothetic model of trade that can be solved analytically. In this environment we are able to prove this relationship analytically. When one country is larger or more productive than the other, its real income is relatively higher, which generates a systematic bias in the ACR calculation due to the non-homotheticity. We are able to sign this bias based on the relative sizes and productivities of the countries. Moreover, we show that, when the countries are equal, this bias disappears and the gains from trade in the non-homothetic model and the ACR equation (and, therefore, the homothetic model) exactly coincide. In this model, we show that the bias is exactly equal to the elasticity of relative income with respect to changes in trade costs.

This highlights our main conclusion, which is that non-homotheticities are important when studying countries that vary substantially in size and income. The difference is more quantitatively important the larger is the difference between the countries. Moreover, our results suggest that further study of the nature of non-homotheticities may, in fact, be informative for predicting gains from trade.

chapter, when we say "gains from trade" we mean comparing observed levels of trade with autarky.

1.1.1 Literature Review

This chapter draws upon the large literature on models of trade with non- homotheticities. A recent study by Fieler (2011) uses a model in which goods vary in their income elasticity of demand to match the patterns of trade across countries of different level of output and population, and studies its relationship to a standard gravity model. In the model, poor countries concentrate consumption on goods with low income elasticity, while rich countries consume high elasticity goods. This model is then used to assess the effects of productivity shocks to countries of different incomes. Our model shares many of the properties of this model, such as the pattern of consumption by income level, so that we are also able to match these facts about the pattern of international trade. We then ask, in our model, which is consistent with these empirical observations, are the gains from trade different than in a model with homothetic preferences?

Markusen (2011) provides a detailed analysis of results from models of non-homothetic international trade. His emphasis is on the role that per capita income plays in determining international trade flows. These roles are highlighted in papers that use models with non-homothetic preferences to match facts about the price and quality of goods traded between countries of differing income levels. Simonovska (2011) uses non-homotheticities to match the observed relationship between the income levels of different countries and the prices of their tradable goods. Fajgelbaum *et al.* (2011) use a model of vertical product differentiation to match facts about the quality differences of products exported from rich and poor countries, and find that the gains from trade liberalization vary across the profile of income levels within each country due to non-homotheticities. Choi *et al.* (2009) use a model of within-country income differences and non-homotheticities to match patterns of trade between countries with differing income distributions.

Matsuyama (2000) shares some of the structural features of the model we develop, such as a positive relationship between the income of an individual and the number of varieties consumed. This feature again allows us to match some of the patterns of bilateral trade between countries of different income levels.

Our model has some similarities with Markusen (1986) and Markusen and Wigle (1990) regarding trade flows. In their models, world trade is divided between a pair of rich, northern countries that takes the usual New Trade form and trade between these northern countries and a poorer partner in the south, which takes a Ricardian-type of trade. The model we develop is Section 4 could easily boil down to a similar trade strucutre, if we were to avoid the production of the luxury good in the poorer countries. Instead, we allow these countries to produce some of the more luxurious goods. This makes a great difference between the strucute of production in our model and theirs.

The chapter is organized as follows: Section 2 describes the data and the facts. Section 3 develops a simple model of trade and non-homotheticities and derives some analytical results. Section 4 develops an enriched model of trade matching facts from Section 2. Section 5 has the quantitative exercise comparing gains in the non-homothetic model and the class of homothetic models considered by ACR. Section 6 concludes.

1.2 Data Analysis

To motivate the usefulness of non-homothetic models of trade, we first demonstrate some facts that homothetic models of trade cannot replicate. We analyze disaggregated trade data from Comtrade, using the 5,227 6 digit Harmonized System (HS6) categories from 2005.[2] Our goal in using HS6 is to have very narrowly defined categories.

We are interested on the raw correlation between imports and exports of goods. If countries tend to buy and sell the same type of goods, then this correlation turns to be positive. If, on the other hand, they buy and sell different types of goods, the correlation will tend to be negative.

Qualitatively, there exist two types of goods in every trade relationship: those that are imported and exported in the two directions (we call them AND goods) and goods that are either imported or exported, but not both (we call them OR goods). We are interested of what fraction of goods is of the OR type versus the AND type in each trade relationship.

[2]We chose 2005 because it does not overlap with the financial crisis that many countries experienced in the latter part of that decade.

1.2.1 Three Facts and Three Examples

We want to first establish three facts about bilateral trade:

1) Trade among pairs of high income countries (G7 and Spain) is characterized by a "hump shaped" relationship between their relative income and how similar their imports and exports are. That is, countries with similar income levels trade more similar goods with one another than they do with countries that are either richer or poorer.

2) This relationship disappears for low income countries (BRICS, Mexico and Turkey). The similarity of their imports from and exporters to their trading partners do not depend on their relative income levels.

3) The fraction of OR goods is large for low income countries trading with low income countries, more moderated for low income countries trading with high income countries, and very small for high income countries trading with high income countries.

It is important here to notice that homothetic models of trade will not be able to match any of these three facts. Homothetic utility and production functions eliminate any role for income differences in consumption and production patterns. That is, by their nature, wealthier consumers (or countries, in this context) consume proportionally more goods than poorer consumers.

These results are similar to those in Markusen (1986): trade between rich countries is characterized by high levels of intra-industry trade, while trade among poor countries is not. To this we add that within the set of rich countries, the degree of similarity of imports and exports depends on the trading partners' relative incomes. Furthermore, we add an element related to the extensive margin: countries at low income levels have zero trade in many categories that they import, while they do export that category.

Before formally establishing the 3 facts that we are interested in, we depict three examples of bilateral trade between trading partners to graphically illustrate these facts. The three relationships are 1) Germany and France (two high income countries); 2) France and Russia (one high income and one middle income); and 3) Russia and Turkey (two middle income countries). Figures 1, 2 and 3 respectively show each of these bilateral relations. Each figure contains a scatter plot in which each point is an HS6 category. The units are the logarithm of the value plus one, in order to display the categories with zero trade volume.

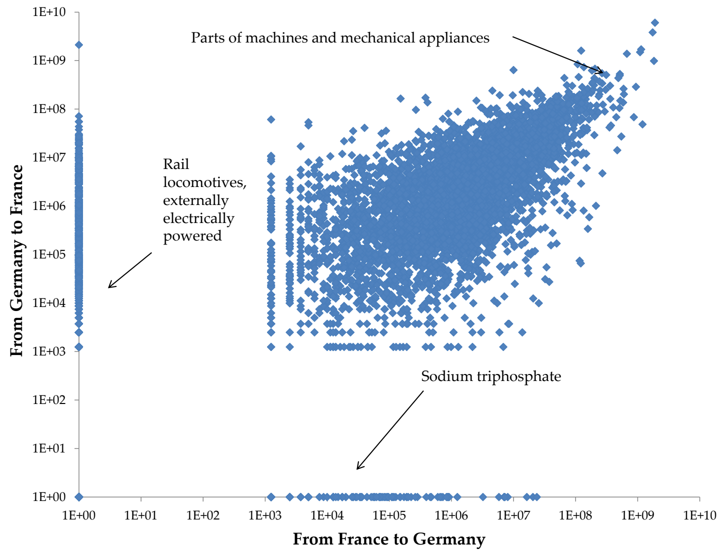

Figure 1: France-Germany bilateral trade

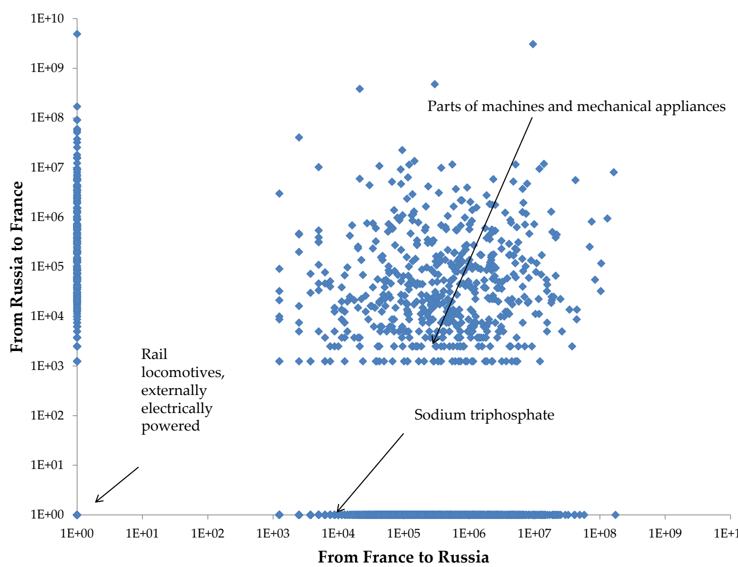

Figure 2: France-Russia bilateral trade

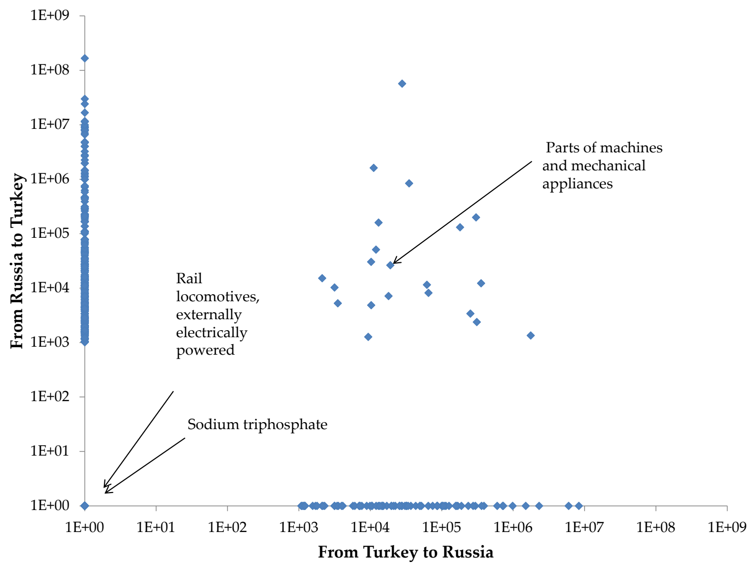

Figure 3: Russia-Turkey bilateral trade

a) In Figure 1, we depict trade volume on all the HS6 categories between France and Germany. We observe the following: most categories are both imported and exported, and being intensively exported is highly correlated with being intensively imported.

b) In Figure 2, we show trade between France and Russia. There are many more trade categories with zero trade volumes, and there are many categories that one country exports but does not import. Furthermore, the relationship between how intensively goods are imported and exported has disappeared.

c) Finally, Figure 3 shows trade between Russia and Turkey. Again, there is very little relationship between the import and export intensity of different product categories. Also, trade is dominated by categories that one country exports, but does not import.

1.2.2 Establishing the Facts

To demonstrate the first fact, we use trade data from the BRICS, Mexico, Turkey (low income countries), the G7 and Spain (high income countries). First, for each country pair we compute the correlation of imports and exports in the 5,227 HS6 categories.

We then take the set of these correlations and run the following regression:

$$corr(I_{x\to y}, X_{x\to y}) = \alpha + \beta_1 \frac{GDPc_x}{GDPc_y} + \beta_2 \left(\frac{GDPc_x}{GDPc_y}\right)^2 + \varepsilon_{x,y}$$

The results of this regression are in the first panel of Table 1. The estimated coefficients imply that $\beta_1 > 0, \beta_2 < 0$ (which indicates a hump shape) and they are both significant at 1 percent level. Furthermore, the implied maximum, $\frac{\beta_1^*}{2\beta_2^*} = 1.07$. Hence, we interpret this to mean that, within the set of high income countries, the further a country pair's ratio of incomes is from 1, the less similar are their imports and exports. This is related to the work by Balassa (1986), which shows a positive relationship between per capita income and an aggregated measure of the extent of intra-industry trade. In contrast, we use bilateral trade pairs and demonstrate a hump-shaped relationship in relative bilateral income.

For the second fact, we show that low income countries exhibit no such relationship between the correlation of imports and exports and the relative income of the trading partner. In order to analyze this, we run the same regression again. The results of this regression are in the second panel of Table 2. We find that neither of the two coefficients is significant at the 5% level.

TABLE 1: REGRESSION

	Rich Countries		
	β_0	β_1	β_2
coefficient	0.06	0.46	−0.21
lower bound	−0.01	0.27	−0.33
upper bound	0.13	0.65	−0.10
fitted maximum			1.07
	Poor countries		
	β_0	β_1	β_2
coefficient	0.12	−0.01	0.00
lower bound	0.08	−0.01	−0.00
upper bound	0.17	0.00	0.00
fitted maximum	does not apply		

For the third fact, Figure 4 shows, for each bilateral pair of countries, the number of goods either imported or exported (but not both) divided by the total number of traded goods in that bilateral relationship plotted against the income of the per capita GDP of one of the trading partners. The point is colored red if the other partner is in the OECD (except Mexico and Turkey), and is labeled blue otherwise. We see that the trade of poor countries is dominated by categories that only have positive trade flows in one direction. For rich countries, this is true to a lesser extent.

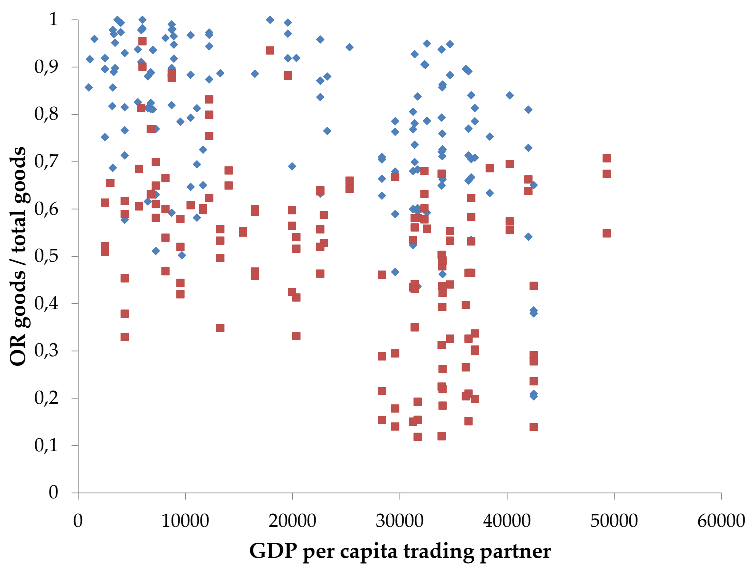

Figure 4: Fraction of OR goods over total goods in the data

We use these three facts to motivate the model developed in later sections. The first fact shows that countries tend to export and import more similar goods the more similar they are. In the next section we develop a very stylized model of bilateral trade with non-homotheticities where we capture this simple idea, and we analytically compare the gains from trade implied by this model to those that would be implied by standard homothetic models.

In the following sections we expand this model in order to account for the second and third facts, and analyze the gains from trade of a richer, calibrated model.

1.3 Welfare Gains and Non-Homothetic Preferences

In this section we develop a simple model to think about the role that non-homothetic preferences play in the analysis of gains from trade. The section is divided into three parts. We first develop a simple, stylized, two country model consistent with the first fact discussed previously. Importantly, in this model the two countries are asymmetric in their productivity levels. Second, we analytically compare the results with the non-homotheticity to homothetic models (described by the ACR calculation). Third, we prove two theorems about the relationship between the gains from trade predicted by this model and those predicted by the ACR calculation. We find that the two coincide for countries with the same income level, but disagree when the countries have different income levels. In particular, the ACR calculation overestimates gains from trade for the poor country, and underestimates the gains for the rich country.

1.3.1 A Simple Model with Non-Homothetic Preferences

We develop a static, 2 country (with populations L_1 and L_2) model with a continuum of goods, each of them indexed by j. This index captures the degree of luxury of the good. The larger the index, the less likely the individual will be willing to consume it, since it already provides enough welfare. The representative household in country l chooses patterns of consumption $c_l(j)$ for each variety j according to

$$\begin{aligned} &\max \left(\int_0^M (c_l(j) + j)^\rho dj \right)^{\frac{1}{\rho}} \\ &st : \int_0^M p_l(j) c_l(j) dj \leq w_l \end{aligned} \tag{1.1}$$

where M is a very large constant.

There is a competitive producer of each good i in each country. In country 1, that firm chooses how many inputs from the domestic market to purchase ($x_{1,1}$) and how many foreign ones, ($x_{1,2}$), in order to maximize profits. In order to get one unit of the country 2 good, the country 1 producer needs to purchase $(1+\tau)$ units.

$$\begin{aligned} &\max p_1(i)c_1(i) - q_1(i)x_{1,1}(i) - q_2(i)x_{1,2}(i)\,(1+\tau) \\ &st : c_1(i) = (\alpha_{1,1}x_{1,1}(i)^\mu + \alpha_{1,2}x_{1,2}(i)^\mu)^{\frac{1}{\mu}} \end{aligned} \tag{1.2}$$

where $\alpha_{1,1}$ and $\alpha_{1,2}$ determine the shares of inputs from each country, and $\mu \in (0,1)$ governs the elasticity of substitution. The problem for the competitive producer in country 2 is symmetric.

We will assume throughout that $\alpha_{1,1} = \alpha_{2,1} = (L_1 z_1)^{1-\mu}$ and $\alpha_{2,2} = \alpha_{1,2} = (L_2 z_2)^{1-\mu}$ so that we maintain the following two properties: 1) if a country splits in two, but there are no trade costs between them, the consumption from the other country is still the sum of the two countries separately and 2) if the other country doubles size but halves productivity, consumption stays the same. It is easily to prove that the stated conditions imply these two properties.

Finally, there is a continuum of competitive intermediate goods producers in country 1 that chooses to maximize profits according to

$$\begin{aligned} & \max q_1(i)\left(x_{1,1}(i) + x_{2,1}(i)\left(1+\tau\right)\right) - w_1 l_1(i) \\ st \quad : \quad & x_{1,1}(i) + x_{2,1}(i)\left(1+\tau\right) \leq z_1 l_1(i) \end{aligned}$$

We allow the two countries to differ in productivity z_i and population size L_i.

The preference structure implies a cutoff

$$J = c(0)\left(\frac{p(0)}{p(J)}\right)^{\frac{1}{1-\rho}} = \left(c(j)+j\right)\left(\frac{p(j)}{p(J)}\right)^{\frac{1}{1-\rho}} \text{ for all } j < J \tag{1.3}$$

This cutoff demonstrates the role of non-homotheticicities[3]. Figure 6 in the Appendix shows the pattern of consumption between the two countries. A useful property of the model is that $p_1(i) = p_1(j) \equiv p_1$ for all i. This follows from the facts that all goods have the same marginal cost, and all markets are competitive. Figure 7 in the Appendix shows the pattern of trade between the two countries: the rich country enjoys a larger set of goods to be consumed than the poor one, and hence the poor country produces some goods that are not consumed domestically. In Figure 8 in the Appendix we show the correlation between imports and exports that is implied by our model as a function of relative incomes. As it was shown in the data, the maximum is exactly 1 when the two countries have the very same productivity.

[3] We always have M being large enough so $J < M$.

Equilibrium wages are such that the labor market clears

$$\int_0^{J_1} x_{1,1}(i)di + \int_0^{J_2} x_{2,1}(i)di\,(1+\tau) = z_1 \tag{1.4}$$

and trade balances.

$$\int_0^{J_1} q_2(i)x_{1,2}(i)di = \int_0^{J_2} q_1(i)x_{2,1}(i)di \tag{1.5}$$

We now characterize the equilibrium of the model. First, we show how welfare is linked to the non-homotheticity of the demand system by showing that total welfare has a one-to-one mapping to the set of goods consumed.

Lemma 1 *Welfare for country i in the model is given by* $\frac{w_i}{p_i} = \frac{J_i^2}{2}$

Proof. *See Appendix* ■

Second, we show that the country that has larger productivity is also the richer country, as measured by total income. We further show that the ratio of productivities is indeed larger than the ratio of incomes.

Lemma 2 *Suppose* $L_1 = L_2$. *If* $z_1 \geq z_2$ *then* $w_1 \geq w_2$ *and* $\frac{z_1}{z_2} \geq \left(\frac{w_1}{w_2}\right)^{\mu}$. *Suppose* $z_1 = z_2$ *and* $L_1 = 1$. *Then,* $L_1 \geq L_2$ *implies* $w_1 \geq w_2$ *and* $\frac{L_1}{L_2} \geq \left(\frac{w_1}{w_2}\right)^{\frac{\mu}{1-\mu}}$

Proof. *See Appendix* ■

In order to proceed, we assume some parameter restrictions in the model. In particular, we assume that the ratio of productivities is large enough. The exact specification is in Condition 1.

Condition 1 Parameters satisfy $z_1^{1+\mu} l_1^{1-\mu} > (1+\tau)^{2\mu} z_2^{1+\mu} l_2^{1-\mu}$

The previous condition is useful in order to characterize further results regarding the bias of welfare gains.

Finally, we show that following trade liberalization, countries with higher productivity benefit less than low productivity countries.

Lemma 3 *The derivative of the ratio of wages increases with increases in trade costs as long as Condition 1 is satisfied.*

Proof. *See Appendix* ∎

This analysis of the model is very useful in order to be able to properly derive results that compare our model to ACR. We do so in the next subsection.

1.3.2 Welfare Gains from Trade

To understand the role that non-homotheticicities play in the welfare gains from trade in the model, it is useful to compare the gains implied by our model to those that would be computed in a homothetic model of trade. The computation in ACR provides a benchmark for a large class of homothetic models. The basic comparison that we will make throughout is the following: what are the exact computed gain from trade implied by the model compared to the ACR computation applied to the output of our model?

The ACR computation requires two statistics. If X_{ij} is the final use by country j of goods produced in country i, then the two statistics are: 1) the import penetration ratio, $1-\lambda_{ij} = X_{ij}/X_{jj}$, and 2) the trade elasticity, $\varepsilon_j^{ii'} = \partial \ln (1-\lambda_{ij}) / \partial \ln (1+\tau_{i'j})$. The ACR computation states that the gains from being open to trade (that is, the welfare difference between the observed level of trade and autarky) is given by:

$$W^{ACR} = 1 - \lambda_{ij}^{-1/\varepsilon_j^{ii}}$$

The main result of ACR is that this computation coincides with those that can be computed in any trade model whose model output match these two statistics if those models meet the following criteria: trade is balanced, profits are a constant fraction of revenue (perfect competition or constant markups, for instance) and the "import demand system is CES". This last assumption is that $\varepsilon_j^{ii'} = \varepsilon < 0$ if $i = i'$ and is 0 otherwise. That is, if a country opens to trade with one country, the proportion of goods consumed from all other countries relative to one another is unchanged. This is an implication of models with homothetic preferences. In our environment, the first two assumptions are certainly satisfied. The third assumption certainly is not due to the non-homotheticity.

We first compute the ACR measure applied to this model.

Proposition 4 *The ACR formula applied to our model implies welfare gains given by*

$$\log(\hat{W}^{ACR}) = \frac{1-\mu}{\mu} \frac{1}{1 - \frac{w_2}{w_1}(1+\tau)\frac{\partial \frac{w_1}{w_2}}{\partial(1+\tau)}} \log\left(\frac{1 + \frac{L_2 z_2}{L_1 z_1}\left(\frac{w_1'}{w_2'}\frac{1}{1+\tau'}\frac{z_2}{z_1}\right)}{1 + \frac{L_2 z_2}{L_1 z_1}\left(\frac{w_1}{w_2}\frac{1}{1+\tau}\frac{z_2}{z_1}\right)}\right) \tag{1.6}$$

Proof. *See Appendix* ■

We then compare this measure to the actual welfare gains in the model.

Proposition 5 *Welfare gains in our model are given by*

$$\log(\hat{W}) = \frac{1-\mu}{\mu} \log\left(\frac{1 + \frac{L_2 z_2}{L_1 z_1}\left(\frac{w_1'}{w_2'}\frac{1}{1+\tau'}\frac{z_2}{z_1}\right)}{1 + \frac{L_2 z_2}{L_1 z_1}\left(\frac{w_1}{w_2}\frac{1}{1+\tau}\frac{z_2}{z_1}\right)}\right) \tag{1.7}$$

Proof. *See Appendix* ■

From equations (1.6) and (1.7) we find the following

$$\frac{\log(\hat{W}^{ACR}) - \log(\hat{W})}{\log(\hat{W}^{ACR})} = \frac{\partial \frac{w_1}{w_2}}{\partial(1+\tau)} \frac{(1+\tau)}{w_1/w_2}$$

.

That is, the bias in the estimated gains in welfare (approximated by $\log(\hat{W})$ and $\log(\hat{W}^{ACR})$) is exactly equal to the elasticity of relative wages with respect to changes in trade costs. This allows us to prove the following theorems about the sign of the bias.

Theorem 6 $\log(\hat{W}^{ACR}) = \log(\hat{W})$ *if* $L_1 = L_2$ *and* $z_1 = z_2$.

Proof. It follows directly from Lemma 3 ■

Therefore, even though the model does not satisfy the assumptions of the ACR result, welfare computed within the model nonetheless coincides with that implied by the ACR result if the productivity levels of the two countries are the same. Hence, when countries have the same income level, the ACR calculation is correct even if the parameters are non-homothetic. However, when productivity (and therefore, income) levels and population sizes differ between countries the ACR computation is biased, as described in the following theorem.

Theorem 7 *The ACR formula overestimates the welfare of the rich and large country and underestimates the welfare of the poor and small country as long as Condition 1 is satisfied.*

Proof. Using Lemma 3 ■

The intuition behind this theorem is simple. A fall in trade costs makes each individual better off by consuming more goods. However, trade must balance. The poor individual gets more marginal utility for the same increase in consumption, and hence his welfare increases relatively more. This translates to a higher relative increase in her wage (see Lemma 3). This increase in wage implies a larger relative increase for the cost of imported goods for the rich individual, and hence, a relatively smaller increase in his imports. This, in turn, translates into a smaller elasticity (see Proposition 4), which biases the ACR prediction for welfare upward.

This simple model is useful in establishing analytical, qualitative predictions. In the next section, we wish to quantify these differences to determine if using non-homotheticities gives meaningfully different predictions for gains from trade. We will expand our model to include many things: goods that are specific in their country of origin, within-country income inequality, and multiple countries. We will show that the expanded model is consistent with all three of the facts described in Section 2, and will demonstrate for what countries and in what situations the model with non-homotheticities disagrees significantly with the ACR calculation.

1.4 Model of Trade with Non-Homothetic Preferences

1.4.1 Household

There are N different countries, each with different population sizes L_j. We assume there is a continuum of differently endowed households in each country. Each household k in country m has labor endowment $l_m(k)$. We further assume that l_m follows a truncated Pareto distribution.

Households consume two types of goods: country specific goods (denoted with S subscripts) and luxury goods (denoted with L subscripts). The set of country specific goods is partitioned into N sets that are each assigned to a different country. Those goods are only produced in those countries, but are purchased by all other countries.

Household $k \in [0, L_m]$ in country m solves the following problem.

$$\max \left(\int_0^M (c_{L,m}(j,k) + j)^\rho \, dj + \sum_{n=1}^N \int_0^{A_n} (c_{S,n,m}(i_n,k))^\rho di_n \right)^{\frac{1}{\rho}} \tag{1.8}$$

$$st \quad : \quad \int_0^M p_{L,m}(j) c_{L,m}(j,k) dj + \sum_{n=1}^N (1 + \tau_{n,m}) \int_0^{A_n} p_{S,n,m}(i_n) c_{S,n,m}(i_n,k) di_n = w_m l_m(k)$$

where country specific goods $i_n \in (0, A_n)$, are produced in country n, and $[0, M]$ is the set of potential variatiesof luxury goods produced in the world.[4] The iceberg transportation cost between countries n and m is $\tau_{n,m}$.

1.4.2 Country Specific Good Firms

Country specific goods are produced by competitive firms in each country. Such a firm in country m producing good i_n, solves the problem

$$\begin{aligned} \pi_{S,m}(i_n) &= \max \sum_{m=1}^N (1 + \tau_{m,n}) \, p_{S,m,n}(i_n) \int_0^{L_n} c_{S,m,n}(i_n,k) dk - w_m l_{S,m}(i_n) \quad (1.9) \\ st &: \sum_{m=1}^N (1 + \tau_{m,n}) \int_0^{L_n} c_{S,m,n}(i_n,k) dk = z_{S,m}(i_n) l_{S,m}(i_n) \end{aligned}$$

where $z_{S,m}(i_n)$ is the country-specific efficiency of the variety. We assume throughout that $\forall i, j \in (0, A_m), z_{S,m}(i) = z_{S,m}(j)$. Hence, although all such firms have the same marginal cost, the number of firms that operate in each country (and in the world) is determined in equilibrium.

1.4.3 Luxury Goods' Production - Final Producer

There is a competitive final firm j for each luxury variety in each country that produces using domestic and foreign goods according to a standard Dixit-Stiglitz aggre-

[4]As in the previous model, M is a very large, exogenous constant.

gator.

$$\pi_{L,m}(j) = \max p_{L,m}(j) \int_0^{L_m} c_{L,m}(j,k)dk - \sum_{n=1}^{N} \int_{s\in\Omega_{n,j}} q_{n,m}(j,s)x_{n,m}(j,s)\left(1+\tau_{n,m}\right)ds \tag{1.10}$$
$$st : \int_0^{L_m} c_{L,m}(j,k)dk = \left(\sum_{n=1}^{N} \int_{s\in\Omega_n} x_{n,m}(j,s)^{\rho}ds\right)^{\frac{1}{\rho}}$$

where Ω_n is the varieties in country n producing the intermediate good used in the production of good j, and ρ governs the elasticity of substitution among varieties.

1.4.4 Luxury Goods' Production

We assume each variety of intermediate good is operated by a perfectly competitive firm that solves the following problem.

$$\max \pi_{X,m}(j,s) = \max \sum_{n=1}^{N} q_{m,n}(j,s)x_{m,n}(j,s)\left(1+\tau_{m,n}\right) - l_{X,m}(j,s)w_m \tag{1.11}$$
$$st : \sum_{n=1}^{N} x_{m,n}(j,s)\left(1+\tau_{m,n}\right) = z_{X,m}l_{X,m}(j,s)$$

1.4.5 Market Clearing Conditions and Trade Balance

Finally, the market for labor clears

$$\int_0^{A_m} l_{S,m}(i)di + \int_0^{M}\int_{s\in\Omega_n} l_{X,m}(j,s)dsdj = \int_0^{L_m} l_m(k)dk \tag{1.12}$$

and trade balances

$$\sum_{n=1}^{N} x_{m,n}(j,s)\left(1+\tau_{m,n}\right) + \sum_{n=1}^{N}\left(1+\tau_{m,n}\right)\int_0^{L_n} c_{S,m,n}(i_n,k)dk \tag{1.13}$$
$$= \sum_{m=1}^{N} x_{n,m}(j,s)\left(1+\tau_{n,m}\right) + \sum_{m=1}^{N}\left(1+\tau_{n,m}\right)\int_0^{L_n} c_{S,n,m}(i_n,k)dk$$

1.4.6 Definition of Equilibrium

In this section we define what an equilibrium is in this economy.

Definition 8 *Given the distribution of skills, the set of iceberg costs, $\tau_{n,m}$, the sets of varieties operated in each country, Ω_m and the mass of country specific goods, A_m, an equilibrium is a vector of functions of prices, $(q_{n,m}(j,s), w_m, p_{E,m}(j), p_{S,m,n}(i_n))$ and a vector of functions of allocations $(c_{S,n,m}(i_n,k), c_{L,m}(j,k), x_{m,n}(j,s), l_{X,m}(j,s), l_{S,m}(i_n), W_m(k))$ such that*

a) Households solve problem (1.8)

b) Country-specific competitive final firms solve problem (1.9)

c) Final prodcers of luxury goods solve problem (1.10)

d) Intermediate luxury goods' producers solve problem (1.11)

e) No firm makes profits

f) Markets of labor clear (equation 1.12)

g) Trade balances (equation 1.13)

The following proposition characterizes the equilibrium:

Proposition 9 *The equilibrium of the model is given by*

$$
\begin{aligned}
p_{L,m} &= \left(\sum_{n=1}^{N} \Omega_n \left(\left(1+\tau_{n,m}\right)\frac{w_n}{z_{X,n}}\right)^{\frac{-\rho}{1-\rho}}\right)^{\frac{\rho-1}{\rho}} \\
W_m(k) &= w_m l_m(k); p_{S,n,m}(i_n) = \frac{w_n}{z_{S,n}(i_n)}; q_{n,m} = \frac{w_n}{z_{X,n}} \\
c_{L,m}(j,k) &= \left(J_m(k)-j\right); c_{S,n,m}(i_n,k) = J_m(k)\left(\frac{p_{L,m}}{p_{S,n,m}(i_n)\left(1+\tau_{n,m}\right)}\right)^{\frac{1}{1-\rho}} \\
\int_{s\in\Omega_s}\int_0^{+\infty} x_{n,m}(j,s)\,dj\,ds &= \frac{\Omega_n \frac{1}{2}\int J_m^2(k)dk}{\left(\left(1+\tau_{n,m}\right)q_{n,m}\right)^{\frac{1}{1-\rho}}\left(\sum_{r=1}^{N}\Omega_r\left(\left(1+\tau_{r,m}\right)\frac{w_r}{z_{X,r}}\right)^{\frac{-\rho}{1-\rho}}\right)^{\frac{1}{\rho}}} \\
l_{X,m}(j,s) &= \frac{\sum_{n=1}^{N} x_{m,n}(j,s)\left(1+\tau_{m,n}\right)}{z_{X,m}} \\
l_{S,m}(i_n) &= \frac{\sum_{m=1}^{N}\left(1+\tau_{m,n}\right)\int_0^{L_n} c_{S,m,n}(i_n,k)dk}{z_{S,m}(i_n)} \\
\textit{where } J_m(k) &= \sqrt{\begin{array}{c}\left(p_{L,m}^{\frac{\rho}{1-\rho}}\sum_{n=1}^{N}\int_0^{A_n}\left(p_{S,n,m}(i_n)\left(1+\tau_{n,m}\right)\right)^{\frac{-\rho}{1-\rho}}di_n\right)^2 + 2\frac{W_m(k)}{p_{L,m}(j)} \\ -p_{L,m}^{\frac{\rho}{1-\rho}}\sum_{n=1}^{N}\int_0^{A_n}\left(p_{S,n,m}(i_n)\left(1+\tau_{n,m}\right)\right)^{\frac{-\rho}{1-\rho}}di_n\end{array}}
\end{aligned}
$$

and the wage satisfies equation 1.12

Proof. *See Appendix* ■

In the next section, we calibrate this economy to match salient features of bilateral trade data, and perform a quantitaive exercise showing the similarities and differences between this type of models and standard ACR predictions.

1.5 Quantitative Exercise

In this section we calibrate the economy of the previous section and perform a quantitative exercise assessing the importance of non-homothetic preferences for welfare gains from trade.

1.5.1 Calibration

Given a list of countries, our economy is governed by the following parameters: $\left\{\alpha_n, b_{0,n}, z_{X,n}, \quad z_{S,n}(i), \Omega_n, A_n, \{\tau_{n,m}\}_{m=1}^{N}, L_n\right\}_{n=1}^{N}$ and ρ.[5] Next we explain how we calibrate the parameters. Table 2 summarizes the calibration.

TABLE 2: CALIBRATED PARAMETERS AND TARGETS

Parameters	Values	Target	Data	Av. |Dev.|
$\tau_{m,n}$	$0 - 3.54$	$\frac{M_{m,n}}{GDP_m}$	$10^{-11} - 0.3$	$< 10^{-7}$
ρ	0.9	*Trade Elasticity*	$(-5, -10)^*$	$(-5.03, -10.63)$
$z_{X,n}$	$0.01 - 1.20$	$\frac{GDPpc_{US}}{GDPpc_m}$	$0.058 - 1$	$< 10^{-5}$
α_m	$1.05 - 2.51$	Gini	$24.9 - 67.4$	0
L_m	$0.11 - 4.63$	$\frac{Population_x}{Population_{USA}}$	$0.11 - 4.63$	0
Ω_n	$0.59 - 1.23$	$\left(L_n z_n \int z^{-\alpha_n} dz\right)^{1-\rho}$		
A_n	$0.0012 - 0.0025$	$\frac{Trade_{US}OR}{Trade_{US}AND}$	0.002	0

*Anderson and Van Wincoop (2003)

We use a truncated Pareto distribution and calibrate the α_n to match the Gini index. The data for the Gini index is taken from the World Bank.[6]

We set $z_{x,n} = z_{s,n}$ for all n and calibrate it to match GDP per capita in each country. The data for GDP per capita in each country is taken from the Penn World Tables. We calibrate L_n to match the population of each country, relative to the US. The calibration of Ω_n follows the properties of the shares of goods that are bought from each country in the Simple Model. This implies that $\Omega_n = (L_n z_n)^{1-\rho}$. We set the country-specific shares of each goods, A_n to be a fraction of Ω_n. The fraction is constant across countries and matches the fraction of goods that the US exports, but doesn't import (approximately 0.2%).

[5]In this exercise, we calibrate the model to the following countries: United States, United Kingdom, Germany, France, Canada, Japan, Italy, Spain, Turkey, India, China, Brazil, South Africa, Russia and Mexico. These countries have been chosen since they are the G8+5, plus Spain and Turkey. We added these last two because we wanted more countries that are middle income, but geographically close to Europe.

[6]Unfortunately, we do not have the Gini coefficients for all the countries for 2005. For those that this coefficient is not available, we use the Gini index for the closest year in which it is available.

We calibrate the $\tau_{n,m}$ matrix in order to match the entire import matrix among all countries in the model. Since our world is only a fraction of the total world, we make total trade flows to actually match each country's imports over GDP in the data. This data comes form the World Bank, and the ratios come from Comtrade.

We set $\rho = 0.90$ in order to match the trade elasticity that one would get from a gravity regression. Anderson and Van Wincoop (2004) show that this number is between -5 and -10. The computed elasticities in our model are in the range of -5.03 to -10.63.

1.5.2 Model Performance

We now compare our model's results to the empirical relationships discussed in Section 2. First, in order to have comparable magnitudes, we discretize the space of luxury goods and account how much each country consumes from the inputs of the others. We use the luxury good final producer's problem and solve for $j = 0$:

$$\int x_{m,m}(0,k)dk = \frac{\int J_m(k)dk}{\left(\sum \Omega_n \left(\frac{q_m}{q_n}\frac{1}{1+\tau_{n,m}}\right)^{\frac{\rho}{1-\rho}}\right)^{\frac{1}{\rho}}}$$

Then, using the first order condition of $x_{m,m}(j)$ we get that:

$$x_{m,m}(j,k) = \begin{cases} x_{m,m}(0)\left(\frac{J_m(k)-j}{J_m(k)}\right)^{\frac{1}{\rho}} & \text{if } j \leq J_m(k) \\ 0 & \text{if } j > J_m(k) \end{cases}$$

Then, bilateral trade flows are given by

$$x_{m,n}(j,k) = x_{m,m}(j,k)\left(\frac{q_m}{q_n\left(1+\tau_{m,n}\right)}\right)^{\frac{1}{\rho}}$$

We then compute the total trade of each category j. We set a grid point of 4500 points that covers up to the highest J_m in the world economy.

Then, we compute the correlation between any pair of two countries' trade flows. Following the exercise from Section 2, we regress this correlation measure against relative GDP per capita and relative GDP per capita squared, in which one of the trading partners is always a rich country.

Table 3 shows the comparison between the model and the data:

Table 3: Regression Comparison (data vs. model)

	Regression Rich Countries					
	Data			Model		
	β_0	β_1	β_2	β_0	β_1	β_2
Coefficient	0.06	0.46	-0.21	-0.04	1.25	-0.48
95% Confidence Interval	-0.01	0.27	-0.33	-0.12	1.00	-0.63
	0.13	0.65	-0.10	0.04	1.50	-0.3
Implied Maximum			1.07			1.29
	Regression Poor Countries					
	Data			Model		
	β_0	β_1	β_2	β_0	β_1	β_2
Coefficient	0.12	-0.01	0.00	-0.00	-0.00	0.00
95% Confidence Interval	0.08	-0.01	-0.00	-0.00	-0.00	-0.00
	0.17	0.00	0.00	0.00	0.00	0.00
Implied Maximum						does not apply

Given that nothing in the parameterization of the model is targetted, the model qualitatively replicates the regression coefficients from the data quite well. For the regression with only rich countries, it does match the hump-shape relationship in the data. The coefficient on the linear term is positive, and on the quadratic term is negative, and they are both significant. The implied maximum of the hump is higher than the data at 1.29. This is mostly driven by the fact that the coefficient on the linear term is much higher than in the data. Similarly, the same regression for the poor countries delivers no relationship between relative income and the correlation measure, as in the data.

Finally, the model performs well on the third fact discussed in Section 2. Due to the presence of country-specific goods, there are many good categories that are only imported or only exported. Because of the non-homotheticity, this is particularly true for the poor countries. In the data, we say that there were a large number of goods categories that only had trade flows in one direction, and that this was particularly true for low income countries. Figure 5 shows this relationship for both groups in the model:

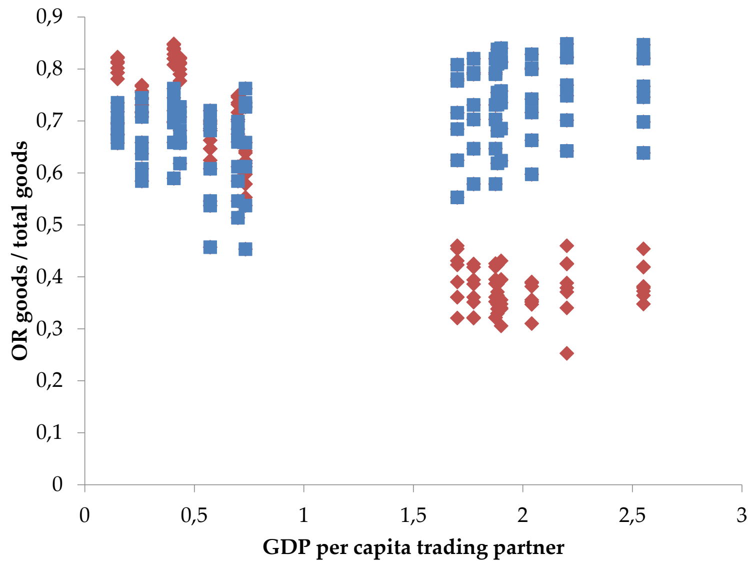

Figure 5: Fraction of OR goods over total goods in the model

The model does well in matching the magnitude of the relationship for both groups. Comparing Figure 5 to Figure 4, we see that there is less dispersion in the model than in the data, but the patterns are quite clear: low income countries have a very large number of goods with trade in only one direction, and this relationship is roughly constant across the income levels of their partners. For high income countries, then have a somewhat smaller number of categories with trade in one direction, and the number of such goods is decreasing in the income of their partner. These relationships are maintained in the output from the model. Again, the calibration does not target these facts.

1.5.3 Welfare Gains Compared to ACR

In this section we compute the welfare gains from trade in our non-homothetic model, and compare them to the results from a class of homothetic models represented by the ACR computation applied to our model's output. Table 4 summarizes the quantiative results of the measured welfare gains and of ACR methodology.

Table 4: Welfare gains

	Gains relative to autarky				
Country	Model	Homothetic	Difference	z	L
US	1.88%	2.17%	−13.2%	1	1
Japan	1.58%	2.02%	−21.7%	1.03	0.42
South Africa	3.72%	3.62%	2.7%	0.01	0.15
Brazil	1.45%	1.44%	0.9%	0.02	0.61
Russia	2.81%	2.84%	−1.1%	0.32	0.51
Spain	4.25%	3.43%	23.9%	0.70	0.15
France	5.02%	5.04%	−0.5%	0.80	0.21
Germany	5.03%	4.72%	6.7%	1.08	0.28
Canada	14.84%	23.90%	−37.9%	1.21	0.11
China	1.45%	1.46%	−0.8%	0.14	4.63
India	2.76%	2.85%	−3.2%	0.08	3.60
Mexico	5.02%	5.42%	−7.4%	0.13	0.33
Turkey	5.35%	5.85%	−8.6%	0.20	0.22
UK	4.19%	3.92%	6.9%	0.91	0.20
Italy	3.76%	3.30%	14.0%	0.80	0.20

In the first column, we have the welfare gains that our model delivers. The second column is the ACR formula applied to the model-generated data for that country. The third gives the percentage difference. The fourth and fifth column give the productivity and population levels of each country relative to the US.

We see wide variations in the disagreement between the two measures. The main pattern is that identified in the simple model: countries with high productivities and large populations are typically underestimated by the homothetic model, and the opposite is true for small and low productivity countries. The simple model further explains us how to aggregate the components- From Condition 1, we can see that productivity and population are aggregated as $\left(\frac{z_i}{z_j}\right)^{1+\rho}\left(\frac{l_i}{l_j}\right)^{1-\rho}$. However, actual productivity levels have two problems. First, they are model specific, and hence this exercise could not be replicated with actual data. The second problem is that countries do not trade equally with the remainig countries.

In our model, productivity of each country is highly correlated with GDP per capita (0.96). Hence, instead of using productivity, we use GDP per capita. In order to address the second problem, we construct an "ideal GDP per capita of the trading partner". Given that the simple model is silent about how to aggregate the different countries, we weight by imports. In particular, for each country i, we use

$$\frac{z_i}{z_{j(i)}} \simeq \frac{GDPpc_i}{\sum_{j \neq i} GDPpc_j \frac{I_j(1+\tau_{i,j})}{\sum_{k \neq m} I_k(1+\tau_{i,k})}}$$

For the measure of population, we use a similar strategy. In particular, for each country i, we use

$$\frac{l_i}{l_{j(i)}} \simeq \frac{L_i}{\sum_{j \neq i} L_j \frac{I_j(1+\tau_{i,j})}{\sum_{k \neq m} I_k(1+\tau_{i,k})}}$$

Finally, the simple model also highlights the role of iceberg costs.

This pattern is made clear by the following simple regression over each country j:

$$\% \text{ Difference}_i = \alpha + \beta_1 \left(\frac{z_i}{z_{j(i)}}\right)^{1+\rho} \left(\frac{l_i}{l_{j(i)}}\right)^{1-\rho} + \varepsilon_j$$

The simple model predicts that the difference between the models is negatively related to the measure of productivity and population. Table 8 shows the coefficient and significance of the regression.

Table 5: Determinants of Bias

	Coefficient	Confidence Interval (95%)
Constant	4.00**	(-5.72,13.72)
$\left(\frac{z_i}{z_{j(i)}}\right)^{1+\rho} \left(\frac{l_i}{l_{j(i)}}\right)^{1-\rho}$	-7.67**	(-15.22,-0.13)
	$R^2 = 0.27$	(** is significant at 95%)

This shows that the results of the analytical model carry through to the quantitative model. This is why we conclude that the degree of overestimation of homothetic models is increasing in the population and productivity of the country.

Decomposition

In the analytical section, we showed that both population size and productivity can bias the ACR calculation in the model with non-homotheticities. Now we show the magnitudes of each in two different ways. First we remove population differences from the model. We recalibrate[7] the model with population constant across countries and get the results summarized in Table 6:

Table 6: Welfare gains
Constant population

	Gains relative to autarky			z	L
Country	Model	ACR	Difference		
US	2.07%	2.12%	−2.3%	1.00	1
Japan	1.37%	1.38%	−1.2%	1.35	1
South Africa	3.11%	2.88%	8.0%	0.02	1
Brazil	1.44%	1.36%	5.7%	0.02	1
Russia	2.23%	2.07%	7.3%	0.50	1
Spain	3.55%	3.59%	−0.9%	1.59	1
France	4.04%	4.83%	−16.3%	1.52	1
Germany	3.72%	3.54%	5.1%	1.56	1
Canada	1.29%	1.52%	−14.9%	2.83	1
China	4.30%	3.95%	9.0%	0.07	1
India	2.85%	2.68%	6.7%	0.06	1
Mexico	4.02%	4.13%	−2.6%	0.22	1
Turkey	3.35%	2.39%	−1.8%	0.29	1
UK	2.40%	2.23%	7.8%	1.88	1
Italy	2.42%	2.27%	6.3%	1.69	1

We broadly see that the countries whose populations *increased* to U.S. levels had a reduction in the amount that the homothetic model underestimates their gains, while India and China had an increase. In fact, for both of them the homothetic model underestimates their gains when population differences are included, but overestimates them when population differences are removed.

Next, we consider the opposite case: hold productivity fixed across countries and allow population to vary. Recalibrating[8] the model in that case gives use the results in Table 7:

[7] See Table A1 in the Appendix for the parameter values used here.
[8] See Table A2 in the Appendix for the parameter values for this case.

Table 7: Welfare gains

Constant productivity

	Gains relative to autarky			z	L
Country	Model	ACR	Difference		
US	2.08%	2.28%	−8.6%	1	1.00
Japan	1.61%	1.48%	8.8%	1	0.42
South Africa	1.84%	1.82%	1.1%	1	0.15
Brazil	0.34%	0.35%	−1.4%	1	0.61
Russia	2.72%	2.60%	4.7%	1	0.51
Spain	4.28%	4.20%	2.0%	1	0.15
France	17.20%	17.38%	−1.0%	1	0.21
Germany	5.11%	4.94%	3.5%	1	0.28
Canada	38.79%	36.85%	5.3%	1	0.11
China	0.82%	0.85%	−3.4%	1	4.63
India	0.79%	0.81%	−2.6%	1	3.60
Mexico	3.53%	4.16%	−15.0%	1	0.33
Turkey	3.14%	3.01%	4.2%	1	0.22
UK	4.20%	4.04%	3.9%	1	0.20
Italy	4.85%	4.67%	3.9%	1	0.20

These results are driven, to a large extent, by the effect of India and China both becoming as productive as the U.S. Essentially, this has very large effects on some countries that trade a large amount with those two countries (particularly the U.S. and Canada). First, notice that, like with holding population constant, when giving all countries U.S. productivity levels, those that increased their productivity had a reduction in the amount homothetic models underestimate their gains, and the opposite for those countries whose productivity decreased. Second, since there is less variation across countries the magnitude of the differences in general declined. Again, where this is not true is, for example, in Mexico, which has a large degree of trade with large countries, such as the U.S. Again, this demonstrates the effect of population differences on how homothetic models underestimate gains from trade.

Our last exercise is a decomposition of the effects of all the different parameters that vary across countries. Here we keep all parameters at their levels from Table 2, then, in each case, make one of those parameters constant across countries (without otherwise changing the calibration).

Table 8: Decomposition of Welfare gains.

Country	Baseline	Constant Population	Constant Productivity	$\tau = 0$	$A_n = 0$	Constant Inequality
US	−13.2%	−8%	−5%	−19%	−7%	−3%
Japan	−21.7%	9%	−1%	−2%	−16%	−23%
South Africa	2.7%	−1%	1%	4%	4%	3%
Brazil	0.9%	−1%	−2%	3%	3%	3%
Russia	−1.1%	−1%	0%	1%	1%	−1%
Spain	23.9%	4%	21%	3%	11%	15%
France	−0.5%	−20%	4%	1%	4%	7%
Germany	6.7%	0%	−2%	−1%	6%	7%
Canada	−37.9%	−31%	4%	1%	−17%	−32%
China	−0.8%	3%	−9%	−11%	0%	1%
India	−3.2%	2%	−3%	−1%	−2%	−5%
Mexico	−7.4%	−14%	−5%	3%	−4%	5%
Turkey	−8.6%	−16%	−1%	3%	2%	−5%
UK	6.9%	−1%	1%	1%	6%	7%
Italy	14.0%	5%	2%	1%	9%	10%

In the case of constant population, there are two notable features. First, the amount that the homothetic model underestimates gains goes down for European countries. This is consistent with the fact that all their populations increase (which is an increase in real income), and they mostly trade with one another. Second, like in Table 6, removing the large populations from India and China has a large effect on the countries they trade with.

The case of constant productivity is similar to the constant population case. Constant productivity has a large effect on Canada, partly because of its large volume of trade with China and Mexico. Notice that the magnitudes of differences between the baseline case and the constant population case is very similar to that of the constant productivity case.

When all trade costs are removed, countries that were closed become relatively more richer, and those that were very open become relatively poorer. For example, Canada, a relatively open country, has a large increase in the amount that homothetic models underestimate its gains, while the U.S. has a decline. This pattern explains most of the changes in that case. The other two cases do not have a very large effect on the estimation of gains in the non-homothetic model.

1.6 Conclusion

This chapter contributes to the literature on computing the gains from international trade by showing that models with non-homotheticities exhibit markedly different gains from trade than models without. We demonstrated this by developing a model with non-homotheticities, matching patterns of trade between countries, and comparing the gains from trade in that model to a measure that summarizes the gains in a large class of homothetic trade models. Our results demonstrate that homothetic models overstate the gains from trade for high income and large countries, and understate them for low income and small countries. For some countries, though not all, these differences are large.

We interpret our results as demonstrating that homothetic models (and the ACR calculation) are useful when comparing countries of similar income levels, but are not when comparing countries with very different income levels (such as the U.S. and China). Notice that the results for many of the European countries indicate that there is little difference between the predictions of our non-homothetic model and the class of homothetic models. Our theoretical results suggest that this is due to the fact that European countries mostly trade with one another, and they mostly have similar income levels. On the other hand, for the U.S., the ACR calculation and our model give quite different predictions. We interpret this as being due to the U.S.'s high level of trade with countries of lower income levels like Mexico and China.

In future work we hope to explore the role of micro level details about the export activities of firms in non-homothetic models. The ACR result suggests that, in homothetic models, more information about how firms make export decisions, and how their decisions respond to trade costs is irrelevant for computing gains from trade. In future work we wish to explore the extent to which this is true in non-homothetic models. In particular, what aspects of firm-level decision making is important in the class of non-homothetic models? We consider this an important avenue of future research.

Appendix

This Appendix covers the proofs for all the Lemmas and Propositions of this chapter. In most cases, due to symmetry, we only show the result for country 1, and that for country 2 follows by doing the appropriate changes.

Proof of Lemma 1

It follow from (1.1) and the cutoff rule (1.3), since $\int_0^{+\infty} p(j)c(j)dj = p\int_0^J (J-j)\,dj = p\frac{J^2}{2} \leq W$ and $U = \int_0^{+\infty} \log(c(j)+j)dj = J\log(J)$. Walras law implies the last holds with strict equality and total wealth of the household is given by the wage, $W = w$.■

Proof of Lemma 2

We start from

$$c_1(i) = \left(z_1^{1-\mu}x_{1,1}(i)^{\mu} + z_2^{1-\mu}x_{1,2}(i)^{\mu}\right)^{\frac{1}{\mu}}$$

and the first order condition for $x_{1,i}$,

$$x_{1,1}(i) = x_{1,2}(i)\frac{L_1 z_1}{L_2 z_2}\left(\frac{w_2}{w_1}\frac{z_1}{z_2}(1+\tau)\right)^{\frac{1}{1-\mu}} \tag{1.14}$$

and integrating over all the set of goods, we get that

$$\int_0^{J_1} x_{1,2}(i)di = \frac{w_1}{p_1}\frac{1}{(L_2 z_2)^{\frac{1-\mu}{\mu}}}\frac{1}{\left(\frac{L_1 z_1}{L_2 z_2}\left(\frac{w_2}{w_1}\frac{z_1}{z_2}(1+\tau)\right)^{\frac{\mu}{1-\mu}}+1\right)^{\frac{1}{\mu}}} \tag{1.15}$$

using the expression for p_1 that arises from substituting the free entry condition and the first order conditions into (1.2)

$$p_1 = \frac{w_2(1+\tau)}{L_2^{\frac{1-\mu}{\mu}} z_2^{\frac{1}{\mu}}}\left(\frac{L_1 z_1}{L_2 z_2}\left(\frac{w_2}{w_1}\frac{z_1}{z_2}(1+\tau)\right)^{\frac{\mu}{1-\mu}}+1\right)^{\frac{\mu-1}{\mu}} \tag{1.16}$$

which, given symmetry, the expressions for $\int_0^{J_2} x_{2,1}(i)di$ and p_2 are very similar.

Finally, we make use of the labor market clearing equation. Using the expressions for $\int_0^{J_1} x_{1,1}(i)di$ and $\int_0^{J_2} x_{2,1}(i)di$ (similar to equation (1.15)) and the market clearing condition for labor (1.4) we find that

$$
\begin{aligned}
\int_0^{J_1} x_{1,1}(i)di &= \frac{z_1}{\left(\frac{L_2 z_2}{L_1 z_1}\left(\frac{w_1}{w_2}\frac{z_2}{z_1}\left(\frac{1}{1+\tau}\right)\right)^{\frac{\mu}{1-\mu}}+1\right)} \\
\int_0^{J_2} x_{2,1}(i)di &= \frac{w_2 z_1}{(1+\tau)w_1}\frac{1}{\left(\frac{L_2 z_2}{L_1 z_1}\left(\frac{w_1}{w_2}\frac{z_2}{z_1}(1+\tau)\right)^{\frac{\mu}{1-\mu}}+1\right)} \\
L_1 &= \frac{1}{\left(\frac{L_2 z_2}{L_1 z_1}\left(\frac{w_1}{w_2}\frac{z_2}{z_1}\left(\frac{1}{1+\tau}\right)\right)^{\frac{\mu}{1-\mu}}+1\right)} + \frac{w_2}{w_1}\frac{1}{\left(\frac{L_2 z_2}{L_1 z_1}\left(\frac{w_1}{w_2}\frac{z_2}{z_1}(1+\tau)\right)^{\frac{\mu}{1-\mu}}+1\right)}
\end{aligned}
$$

We write $L = \frac{L_1}{L_2}, W = \frac{w_1}{w_2}$ and $Z = \frac{z_1}{z_2}$.

$$
L_1 = \frac{1}{\frac{1}{LZ}\left(\frac{W}{Z}\left(\frac{1}{1+\tau}\right)\right)^{\frac{\mu}{1-\mu}}+1} + \frac{1}{W}\frac{1}{\frac{1}{LZ}\left(\frac{W}{Z}(1+\tau)\right)^{\frac{\mu}{1-\mu}}+1} \tag{1.17}
$$

We want to prove that if $L = L_1 = 1$, then if $Z > 1 \to W > 1$ as well. We prove it by contradiction, Suppose $W < 1$.

$$
\frac{\frac{1}{Z}\left(\frac{W}{Z}\left(\frac{1}{1+\tau}\right)\right)^{\frac{\mu}{1-\mu}} + \left(\left(\frac{1}{Z}\right)^{\frac{1}{1-\mu}} W^{\frac{\mu}{1-\mu}}\right)^2}{\frac{1}{Z}\left(\frac{W}{Z}\left(\frac{1}{1+\tau}\right)\right)^{\frac{\mu}{1-\mu}}+1} = \frac{1}{W}
$$

Then, the term in the left hand side is smaller than 1, and the term in the right hand side is larger than 1, which cannot be. Hence, the result is proven.

The second result is that $Z > 1 \to W^\mu < Z$.. Suppose that $\frac{Z}{W^\mu} < 1$. Then, the left hand side of the previous equation is larger than one, and this would imply that $W < 1$. Hence, $Z > W^\mu$.

For the second part of the Lemma, it is useful to rewrite equation (1.17).

$$
\frac{\left(\frac{1}{L}\left(W\left(\frac{1}{1+\tau}\right)\right)^{\frac{\mu}{1-\mu}}\right) + \left(\frac{1}{L}W^{\frac{\mu}{1-\mu}}\right)^2}{\left(\frac{1}{L}\left(W\left(\frac{1}{1+\tau}\right)\right)^{\frac{\mu}{1-\mu}}+1\right)} = \frac{1}{W}
$$

Notice that in the previous equation, is equivalent to the one used for the first part of the Lemma, up to the reescaling of L for $Z^{\frac{1}{1-\mu}}$

And this concludes the proof.■

Proof of Lemma 3

Using equation (1.17), by the implicit function theorem we get that the derivative is positive. In order to proceed, for simplicity, we keep using the same notation with Z and W.

$$\frac{\partial W}{\partial (1+\tau)} = \frac{1}{1+\tau}\left(\frac{(W-1)\,S\left(2+S\left(T-\frac{1}{T}\right)\right)+(1-S^2)\left(\frac{W}{T}-T\right)}{(S+T)\left(\frac{S}{T}+1\right)(ST+1)\left(S+\frac{1}{T}\right)W}\right)$$
$$\times\frac{1}{\frac{1}{TW^{\frac{\mu}{1-\mu}}\left(\frac{S}{T}+1\right)^2}+\frac{T}{W^{\frac{1}{1-\mu}}(ST+1)^2}+\frac{1}{(ST+1)WS^{\frac{\mu}{1-\mu}}}}$$

where

$T=(1+\tau)^{\frac{\mu}{1-\mu}}>1$

and $S=\frac{1}{LZ}\left(\frac{W}{Z}\right)^{\frac{\mu}{1-\mu}}<1$

Notice that among all the parts in the equation, the only one that determines whether or not the derivative is positive is

$$A=(W-1)\,S\left(2+S\left(T-\frac{1}{T}\right)\right)+(1-S^2)\left(\frac{W}{T}-T\right)$$

since all the other terms are positive.

We make use of equation (1.17), which implies that

$$(W-1)\,S=T\left(1-\frac{1}{LZ}\frac{W}{Z^{\frac{2\mu}{1-\mu}}}\right)$$

In order to find that

$$A=T\left(1-\frac{1}{LZ}\frac{W}{Z^{\frac{2}{1-\mu}}}\right)\left(2+S\left(T-\frac{1}{T}\right)\right)+(1-S^2)\left(\frac{W}{T}-T\right)$$

Suppose that $W>T^2$. Then, A would trivially be positive. So, suppose it is not. In particular, assume that $W=KT^2$, where $K<1$ is the exact value of their ratio.

Then, we can rewrite it as

$$A = T\left(\left(1 - K\frac{1}{LZ}\frac{T^2}{Z^{\frac{2\mu}{1-\mu}}}\right)\left(2 + S\left(T - \frac{1}{T}\right)\right) - \left(1 - S^2\right)\left(1 - K\right)\right)$$

Notice that $2 + S\left(T - \frac{1}{T}\right) > 1 - S^2$, which implies that as long as $1 - K\frac{1}{LZ}\frac{T^2}{Z^{\frac{2\mu}{1-\mu}}} > 1 - K$, the term A is positive. In turn, this is holds when $T < Z^{\frac{\mu}{1-\mu}}\left(LZ\right)^{\frac{1}{2}}$. Getting back to the original parameters, this implies that $z_1^{1+\mu}l_1^{1-\mu} > \left(1+\tau\right)^{2\mu} z_2^{1+\mu}l_2^{1-\mu}$.

Hence, we have proven that under the condition that $z_1^{1+\mu}l_1^{1-\mu} > \left(1+\tau\right)^{2\mu} z_2^{1+\mu}l_2^{1-\mu}$, the derivative of the ratio of wages to an increase in τ, is positive.

This concludes the proof.■

Proof of Proposition 4

Following ACR (footnote 1),[9] we proceed to compute the import penetration ratio.

$$\lambda = 1 - \frac{\frac{w_2}{z_2}x_{1,2}(1+\tau)}{\frac{w_1}{z_1}x_{1,1} + \frac{w_2}{z_2}x_{1,2}(1+\tau)} = \frac{\frac{w_1}{z_1}x_{1,1}}{\frac{w_1}{z_1}x_{1,1} + \frac{w_2}{z_2}x_{1,2}(1+\tau)} = \frac{1}{1 + \frac{L_2 z_2}{L_1 z_1}\left(\frac{w_1}{w_2}\frac{z_2}{z_1}\frac{1}{1+\tau}\right)^{\frac{\mu}{1-\mu}}}$$

And then we compute the trade elasticity, $\epsilon = \frac{\partial \ln\left(\frac{x_{ij}}{x_{jj}}\right)}{\partial \ln(1+\tau)}$. In our model, this ratio is given from equation (1.14):

$$\begin{aligned}
\frac{\partial \log\left(\frac{x_{1,2}(i)}{x_{1,1}(i)}\right)}{\partial \log\left(1+\tau\right)} &= -\frac{1}{1-\mu}\left(1 + \frac{\partial \log\left(\frac{w_2}{w_1}\right)}{\partial \log\left(1+\tau\right)}\right) \\
&= -\frac{1}{1-\mu}\left(1 - \left(1+\tau\right)\left(\frac{w_2}{w_1}\right)\frac{\partial\left(\frac{w_1}{w_2}\right)}{\partial\left(1+\tau\right)}\right) \\
\frac{1}{\epsilon} &= -\frac{1-\mu}{1 - \left(1+\tau\right)\left(\frac{w_2}{w_1}\right)\frac{\partial\left(\frac{w_1}{w_2}\right)}{\partial(1+\tau)}}
\end{aligned}$$

We need to make a correction by intermediate goods (see ACR section 5.2). Hence,

[9] ACR, foornote 1, page 95: "Import penetration [...] can be interpreted as a share of (gross) total expenditures allocated to imports (see Norihiko and Ahmad (2006))".

a change in welfare, according to ACR, is given by

$$\hat{W}^{ACR} = \left(\frac{1 + \frac{L_2 z_2}{L_1 z_1} \left(\frac{w_1'}{w_2'} \frac{1}{1+\tau'} \frac{z_2}{z_1} \right)}{1 + \frac{L_2 z_2}{L_1 z_1} \left(\frac{w_1}{w_2} \frac{1}{1+\tau} \frac{z_2}{z_1} \right)} \right)^{\frac{1-\mu}{\mu} \frac{1}{1 - \frac{w_2}{w_1}(1+\tau) \frac{\partial \frac{w_1}{w_2}}{\partial(1+\tau)}}}$$

and this concludes the proof. ■

Proof of Proposition 5

In order to proceed, recall that the manner in which ACR computes their formula is by making use of real income, w/p. We proceed from equation (1.16)

$$\frac{w_1}{p_1} = K \left(\frac{L_2 z_2}{L_1 z_1} \left(\frac{w_1}{w_2} \frac{1}{1+\tau} \frac{z_2}{z_1} \right)^{\frac{\mu}{1-\mu}} + 1 \right)^{\frac{1-\mu}{\mu}}$$

Hence a change in real income due to a change in τ to τ' implies

$$\hat{W} = \left(\frac{1 + \frac{L_2 z_2}{L_1 z_1} \left(\frac{w_1'}{w_2'} \frac{1}{1+\tau'} \frac{z_2}{z_1} \right)}{1 + \frac{L_2 z_2}{L_1 z_1} \left(\frac{w_1}{w_2} \frac{1}{1+\tau} \frac{z_2}{z_1} \right)} \right)^{\frac{1-\mu}{\mu}}$$

And this concludes the proof.■

Proof of Proposition 9

From the first order conditions that arise from (1.10) and (1.11), we get that

$$\frac{1}{p_{L,m}(j)} = \left(\sum_{n=1}^{N} \Omega_n \left(1 + \tau_{n,m}\right)^{\frac{-\rho}{1-\rho}} \left(\frac{w_n}{z_{X,n}} \right)^{\frac{-\rho}{1-\rho}} \right)^{\frac{1-\rho}{\rho}}$$

which is the usual Dixit-Stiglitz price aggregator that does not depend on the variety, j. Hence, $p_{E,m}(j) = p_{E,m}(k) = p_{E,m}$.

Using the first order conditions from problem (1.8) and the result that prices are not dependent on j, we get that

$$\frac{W_m(k)}{p_{L,m}} = \left(p_{L,m}^{\frac{\rho}{1-\rho}} \sum_{n=1}^{N} \int_0^{A_n} \left(p_{S,n,m}(i_n)\left(1+\tau_{n,m}\right)\right)^{\frac{-\rho}{1-\rho}} di_n\right) J_M(k) + \frac{1}{2} J_M^2(k)$$

Using again the FOC from (1.10) and (1.11), and combining it with the integral over k for the previous equation, we get that

$$\int_{s\in\Omega_s} \int_0^{+\infty} x_{n,m}(j,s) dj ds = \frac{\Omega_n \frac{1}{2} \int J_m^2(k) dk}{\left(\left(1+\tau_{n,m}\right) q_{n,m}\right)^{\frac{1}{1-\rho}} \left(\sum_{r=1}^{N} \Omega_r \left(\left(1+\tau_{r,m}\right) \frac{w_r}{z_{X,r}}\right)^{\frac{-\rho}{1-\rho}}\right)^{\frac{1}{\rho}}}$$

Combining the first order conditions from the household's problem with those from the country specific good, we get that

$$\begin{aligned} c_{S,n,m}(i_n,k) &= J_m(k) \left(p_{L,m}(j) \frac{z_{S,m}(i_n)}{w_m} \frac{1}{\left(1+\tau_{n,m}\right)}\right)^{\frac{1}{1-\rho}} \\ c_{L,m}(j,k) &= \left(J_m(k) - j\right) \end{aligned}$$

All the other equations arise by definition the equation for $W_m(k)$, for labors or by markets being perfectly competitive, $q_{n,m}$ and $p_{S,n,m}(i_n)$. This concludes the proof.■

Tables and Figures

Table A1: Calibrated Parameters and Targets. Constant Population

Parameters	Values	Target	Data	Av. |Dev.|
$\tau_{m,n}$	$0 - 4.76$	$\frac{M_{m,n}}{GDP_m}$	$10^{-11} - 0.3$	$< 10^{-7}$
ρ	0.9	$Trade\ Elasticity$	$(-5, -10)^*$	$(-6, -9.4)$
$z_{X,n}$	$0.02 - 2.83$	$\frac{GDPpc_{US}}{GDPpc_m}$	$0.058 - 1$	$< 10^{-5}$
α_m	$1.05 - 2.51$	Gini	$24.9 - 67.4$	0
L_m	1	1		0
Ω_n	$0.81 - 1.19$	$\left(L_n z_n \int z^{-\alpha_n} dz\right)^{1-\rho}$		
A_n	$0.0016 - 0.0024$	$\frac{Trade_{US}OMEGA}{Trade_{US}An}$	0.002	0

*Anderson and Van Wincoop (2003)

Table A2: Calibrated Parameters and Targets. Constant Productivity

Parameters	Values	Target	Data	Av. |Dev.|
$\tau_{m,n}$	$0 - 4.16$	$\frac{M_{m,n}}{GDP_m}$	$10^{-11} - 0.3$	$< 10^{-5}$
ρ	0.9	$Trade\ Elasticity$	$(-5, -10)^*$	$(-7.6, -9.2)$
$z_{X,n}$	1	1	$-$	0
α_m	$1.25 - 2.5$	Gini	$24.9 - 57.4$	0
L_m	$0.11 - 4.63$	$\frac{Population_x}{Population_{USA}}$	$0.11 - 4.63$	0
Ω_n	$0.69 - 1.50$	$\left(L_n z_n \int z^{-\alpha_n} dz\right)^{1-\rho}$		
A_n	$0.0014 - 0.003$	$\frac{Trade_{US}OMEGA}{Trade_{US}An}$	0.02	

*Anderson and Van Wincoop (2003)

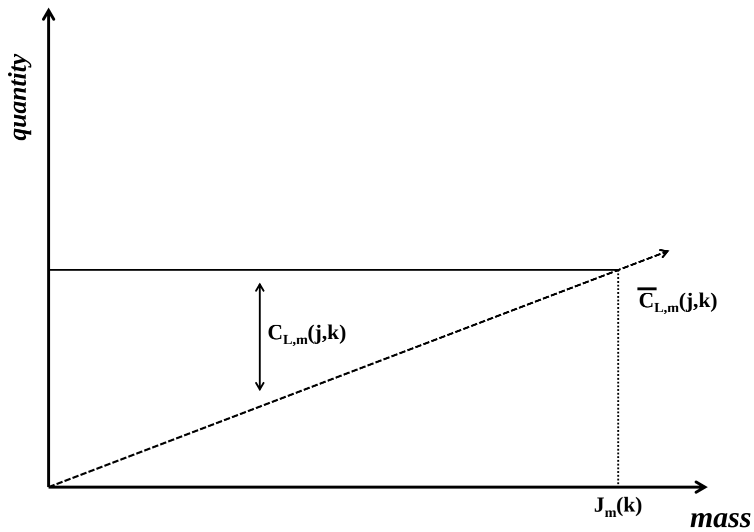

Figure 6: Consumption of a given country (upper triangle)

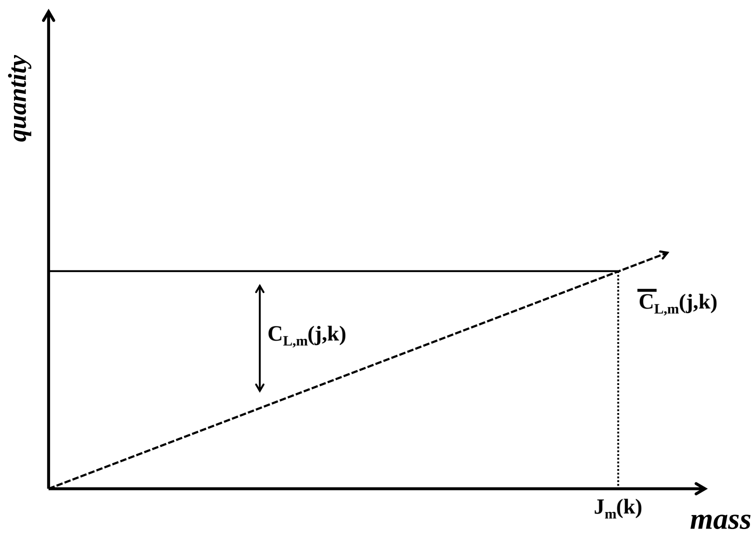

Figure 7: Trade Pattern

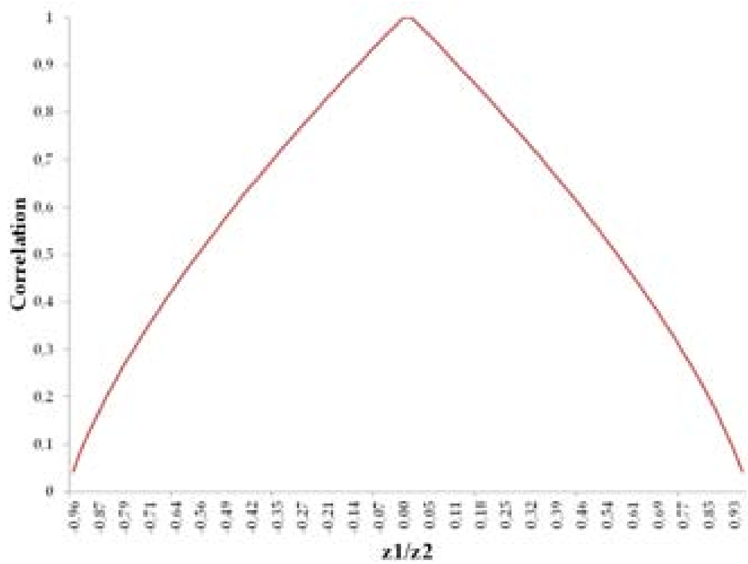

Figure 8: Relationship between correlation and productivity - simple model

Chapter 2

Measured Productivity and International Trade: An Unresolved Puzzle

2.1 Introduction

The Melitz (2003) model has become the workhorse model of International Trade for the last decade. The model rationalizes why more productive firms export, why less productive firms do not, and also why after trade liberalization only the more productive ones remain active, by means of reallocation toward the most productive firms.

Does it really account for these facts?

As Foster, Haltiwanger and Syverson (2008) point out, there are two different manners to account for productivity: revenue based productivity (R-productivity) and quantity based productivity (Q-productivity). The first one uses revenue of firms, while the second uses directly the quantity that is produced. Most of the literature measuring the impact of trade has been using R-productivity since it is hard to find data where price and quantity are separated. In this chapter we study what are the results of performing the analysis done in the data to the output of the Melitz model. Surprisingly enough, it is not necessarily true that better firms export when measured as R-productivity, and it is always true that new exporters' productivity falls after a trade liberalization - contrary to what is found in the empirical literature.

We show that the outcome of the Meltiz model, if measured as R-productivity, may not account for the findings it is thought to account for. This seemingly puzzling result emerges because what the model defines as productivity is very different from the R-productivity that is measured in the data. .

In the model, the term "productivity" refers to the technology that a given firm uses in order to produce some quantity of goods using labor, not accounting for the fixed costs. However, the firm is not getting linear revenues with labor used, and hence the R-productivity, as revenue per worker, is not a linear function of this technology. In order to clarify terms, we use the term *efficiecncy* to refer to the technology that a firm has in order to create goods using variable labor (the model uses φ); and the term *R-productivty* to refer the measurement of productivity that is consistent with what most of the empirical literature uses (we use $v(\varphi)$).

One of the main contributions of this chapter is to precisely show how does efficiency and R-productivity relate to each other in the Meltiz model. Contrary to what one might think, the relationship is not linear, and more important, it is not monotone. How is this possible? The answer has to do with the very foundations of the model, and in particular, to the question of *why do some firms export?*

The answer to this question given by the model is a combination of the demand structure (consumers want goods from different places), and the supply structure (some firms cannot afford selling everywhere).

From the demand point of view, firms are able to sell their goods abroad because individuals have love-for-variety preferences (after Dixit and Stiglitz, 1977) and hence they have downward sloping demand functions for their goods in all the countries. This demand structure imposes a mark-up that firms are able to charge when they sell goods. It turns out that it is precisely this mark-up the maximal R-productivity that a firm can have when it sells domestically ($\frac{\sigma}{\sigma-1}$, where σ is the elasticity of substitution). When a firm also sells abroad, the maximal R-productivity is a corrected term of this, and it has to do with the possibility for some firms to charge prices slightly higher because of the iceberg costs they face. In particular, this maximal R-productivity is given by $\frac{\sigma\left(1+n\tau^{1-\sigma}\right)}{(\sigma-1)(1+n\tau^{-\sigma})}$, where n is the number of countries the firm exports and τ is the iceberg cost of shipping goods to foreign countries. An environment in which all firms would either be exporters or non-exporters with no other mechanism separating them than an exogenous decision (as it is similarly done, for instance, in Brooks and Dovis, 2013), would imply a distribution of R-productivities that would consist of two mass points, instead of a distribution of them, in which exporters would be $\frac{1+n\tau^{1-\sigma}}{1+n\tau^{-\sigma}}$ times more productive that non-exporters. Notice that in a two country world, with standard elasticity of substitution ($\sigma = 2$, and in Ruhl, 2003), the R-productivity of exporters can never be greater than 53% of that of non-exporters, and it would happen with a (maybe too large) iceberg cost of 2.41.

The supply side structure of the model has to do with the reason why some firms choose to export and some firms choose not to do so: the existence of fixed costs of operation. Some firms decide not to export because the fixed cost of exporting (in terms of labor) that these firms should pay is so large, that it does not pay off. It turns out that this cost should be accounted for when accounting for R-productivity differences. Firms that do not export use less labor in order to operate than firms that decide to export. In turn, this affects the ratio of output per worker, since the latter have an extra term in the denominator.

The first result of this chapter is to show that many exporters' R-productivity is smaller than the R-productivity they would have if they decided not to export - although their decision is optimal. This has a first implication regarding the usefulness of R-productivity.[1] If the measured productivity of an exporter is lower than it would be if they chose not to export, how meaningful can productivity of firms be? The reason for some firms to be of this type is that by engaging into the export market, they need to pay a fixed cost of exporting that they would not need to pay otherwise. This extra cost makes measured productivity to be lower than it would be if they decided to export.

The second result of this chapter is to show that it may be that many exporters' productivity is lower than the productivity of some non-exporters. This clearly has an implication with regard to the common understanding of which firms choose to export. The paper argues that "*[R]elatively more productive establishments are much more likely to export*" (p. 1695). That is why, *"[The Melitz model] shows how the exposure to trade induces only the more productive firms to export while simultaneously forcing the least productive firms to exit*" (p.1696). Hence, this second result of the chapter shows that it is not necessarily true that relatively more productive establishments are much more likely to export. In particular, we show that under some parametrization of the model, all exporters are less productive than some non-exporters.

The analysis then focuses on what are the results of a trade liberalization. In the

[1]See Foster *et al.* (2008) for another critique on R-productivity measurements

Melitz model, there are two main barriers to trade: the iceberg cost and the fixed cost of exporting. We first show that all the firms that were not exporting before the trade liberalization suffer a decrease in their measured productivity when the liberalization takes place, irrespectively if they decide to export or they decide not to export.

There are two main causes for this effect: first, the input (labor) becomes more scarce, and hence it lowers the productivity of firms, since they have to pay a higher cost for hiring the same amount of labor. The second effect has to do with the fixed costs that firms that start to export have to pay.

This theoretical finding, in particular for the new exporters, sharply contrasts with the results of the empirical literature measuring R-productivity of new exporters. Empirical trade literature has found a lot of evidence that new exporters' R-productivity increases after trade liberalization. Just to cite a few of the papers that find evidence on this, the seminal contribution of Pavcnik (2002) for Chilean plants, followed by Bernard, Jensen and Schott (2006) for US plants, De Loecker (2007) for Slovenian plants, Amiti and Konings (2007) for Indonesian plants, Park *et al.* (2010) for China during the 1997 financial crisis.[2] In all of them, authors show that exposure to trade makes new exporters to experience increases in their R-productivity.

Repeating the analysis fo what happens to the Melitz model when productivity is measured as Q-productivity is not necessary, because of two effects. First, when it comes to measure what the effect of a trade liberalization is on the productivity of new exporters, the answer between the two types of productivities is exactly the same. The reason for this result is that the manner to compute the change in productivity would include the same two objects as for the analysis of R-productivity, but both terms divided by the price the firm charges. Since the environment is of constant mark-ups, the two prices turn out to be the same, and hence they cancel out. This implies that the same problems the Melitz model has when it comes to R-productivity of new exporters, are also present when the analysis is done with Q-productivity. What does not happen, though, is the static analysis. Since prices are decreasing with efficiency, when the analysis of Q-productivity is done to analyze what firms export, it is true that more productive firms export, and in particular, there is no upper-bound to this measure.

Hence, out of the two stylized facts that we discuss in this chapter, one of them, the larger measured productivity of new exporters after a trade liberalization, is at

[2]See Syverson (2011) for an extensive review of the literature

odds with the results of the Melitz model, irrespectively if the analysis is with R- or with Q-productivity. The second fact, the one in which productivity of exporters is always larger than that of non-exporters, may be at odds with R-productivity measurement, but not with Q-productivity.

This chapter is not the first academic piece to stress the difference between R-productivity and Q-productivity. For instance, Katayama, Lu and Tybout (2009) show that standard measurement of productivity is misleading in environments with heterogeneous mark-ups, variation in input prices or market power.

Some other papers also focus on causes of this growth other than the lowering of tariffs or trade costs, typically mentioning the *learning-by-exporting* strategy, or innovations prior the entrance to the export market. These alternative mechanisms, which would certainly have an effect in the Melitz model, are excluded from this analysis, since this chapter only focuses on the direct effect of international trade on the measured productivity of firms in an environment with static efficiency.

In two very related papers, Gibson (2006) and Bajona, Gibson, Kehoe and Ruhl (2008) already show that, computing aggregate productivity and aggregate real GDP in a Melitz-like model does not deliver aggregate productivity increases. For instance, Kehoe and Ruhl (2010) show that this mechanism can account for the surprisingly low increase in real GDP per capita in Mexico.

This chapter is organized as follows: In the next section we solve the Melitz model for the case in which the efficiencies are Pareto distributed (as it was first done in Chaney, 2008). In section 3 we show what measured productivity is in the model, and how does it relate to the efficiency of firms, and show the properties of measured productivity. Section 4 analyses what is the effect of trade liberalization on the measurement of plants' productivity, and how it sharply contrasts with the common view of it.

2.2 The solution of the Melitz Model

In this section we introduce the main equations of the Melitz model, and the solution of it, for the particular case in which efficiency is Pareto distributed (as it was first done in Chaney, 2008), and the total mass of individuals is $L = 1$. In the model, labor is paid a wage, w, which is later on normalized to 1. We stick to this normalization throughout. We first introduce a definition of equilibrium in the model, and then we characterize it.

Definition: *Given a fixed cost of producing domestically, f, a fixed cost of exporting, $f_x > f$, a number of countries to trade with, $n > 1$, an iceberg cost, $\tau > 1$, an elasticity of substitution, $\sigma > 1$, a Pareto distribution of efficiencies,$f(\varphi)$, characterized by φ^o and κ, a fixed cost of entry, f_e and a destruction rate of firms, δ, an equilibrium in the Melitz model consists of a demand for goods $\{q^d, q^d_x\}$ domestic and foreign prices $\{p, p_x\}$, domestic and foreign quantities $\{q, q_x\}$, profits made by firms, π, aggregate quanty, Q, aggregate price index, P, total mass of firms operating the economy, M, demand for workers l, a decision rule for firms of whether they export or they do not, χ, productivity cutoffs, φ^* and φ^*_x, average productivities, $\tilde{\varphi}, \tilde{\varphi}_x$, a set of firms selling to each country, Ω, such that*

(i) The demand for goods solves the problem of individuals

$$\begin{aligned} & \max_q \left(\int_{i\in\Omega} q^d(i)^{\frac{\sigma-1}{\sigma}} di \right)^{\frac{\sigma}{\sigma-1}} \\ st \quad : \quad & \int_{i\in\Omega} q(i)p(i)di = 1 \end{aligned}$$

(ii) Firms maximize their profits, and choose whether or not to become exporters

$$\begin{aligned} & \max pq^s + p_x q^s_x \chi - l \\ st \quad : \quad & l = \begin{cases} f + \frac{q^s}{\varphi} & \text{if } \chi = 0 \\ f + \frac{q^s}{\varphi} + n\left(f_x + \frac{q^s_{ex}}{\varphi}\right) & \text{if } \chi = 1 \end{cases} \end{aligned}$$

(iii) There is free entry of firms

$$f_e \delta = \left(1 - \int_{\varphi^*}^{\infty} f(\varphi)\, d\varphi\right) \int_{\varphi^*}^{\infty} \pi(i) f(\varphi)\, d\varphi$$

(iv) Markets clear

$$\begin{aligned} q^d &= q^s \\ \int_{i\in\Omega} l(i)di &= 1 \\ PQ &= 1 \end{aligned}$$

Given the previous definition of the Meltiz model, we proceed to characterize it in the following Lemma.

Lemma 1: *The Melitz Model (2003) with Pareto distribution, and mass 1 of workers is characterized by:*

(i) Prices and Quantities at the individual Level

$$
\begin{aligned}
p &= \frac{1}{\rho\varphi} \\
p_{ex} &= \frac{\tau}{\rho\varphi} \\
q &= QP^{\sigma}p^{-\sigma} \\
q_{ex} &= QP^{\sigma}p_x^{-\sigma}
\end{aligned}
$$

(ii) Demand for workers

$$
l = \begin{cases} f + \frac{q}{\varphi} & \text{if non exporter} \\ f + \frac{q}{\varphi} + n\left(f_x + \frac{q_{ex}}{\varphi}\right) & \text{if exporter} \end{cases}
$$

(iii) Cutoff points and export probabilities

$$
\begin{aligned}
\tilde{\varphi}_x^{\sigma-1} &= \tilde{\varphi}^{\sigma-1}\tau^{\sigma-1}\frac{f_x}{f} \\
\tilde{\varphi}^{\sigma-1} &= \varphi^{*\sigma-1}\left(\frac{\kappa}{\kappa-\sigma+1}\right) \\
\tilde{\varphi}_x^{\sigma-1} &= \varphi_x^{*\sigma-1}\left(\frac{\kappa}{\kappa-\sigma+1}\right) \\
p_x &= \frac{\varphi^{*\kappa}}{\varphi_x^{*\kappa}}
\end{aligned}
$$

(iv) Mass of Firms, Aggregate Price Index, Total Quantity, cutoff productivity

$$
\begin{aligned}
M &= \frac{1}{\sigma\left(f+\left(\tau\left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}}\right)^{-\kappa} n f_x\right)\frac{\kappa}{\kappa-\sigma+1}} \\
P &= \frac{(\sigma f)^{\frac{1}{\sigma-1}}}{\rho\varphi^*} \\
Q &= P^{-1} \\
\varphi^* &= \varphi^o\left(\frac{f}{\delta f_e}\frac{\sigma-1}{\kappa-\sigma+1}\left(1+n\tau^{-\kappa}\left(\frac{f_x}{f}\right)^{\frac{-\kappa+\sigma-1}{\sigma-1}}\right)\right)^{\frac{1}{\kappa}}
\end{aligned}
$$

Proof: *See Appendix*

In this section we have named all the equations necessary to compute the Melitz model with the assumption that efficiencies follow a Pareto distribution. In the next section we show how individual measured productivity is defined in this environment.

2.3 Measured Productivity

In the empirical literature, measured productivity is computed as the ability that workers in plant i have at producing output y_i. The empirical literature typically deals with firms using many different inputs (intermediates and capital) and belonging to different industries, with different exposures to international trade. See Pavcnick (2002) for a comprehensive review of the empirical literature.

In the Melitz model, however, we do not need to worry about these issues, since all the firms are thought to belong to the same industry, and the only factor of production is labor. Hence, the correct measure in this framework of productivity of a given firm is output per worker and output per labor cost. Due to the normalization of salaries to 1, the two measures coincide.

In the original Melitz model the fixed cost of exporting and the fixed cost of operating are paid in labor, and so we do in this chapter. However, if these costs were paid in terms of the final good, then this cost should be counted for as investment and enter to some sequence of capital (as it is done in chapter 3 of this Thesis). In this case, though, we would have similar results, since the fixed costs, instead of being an additive term in the denominator, would substract part of the numerator.

In the following Proposition we show what measured productivity of a plant is in the Melitz model.

Proposition 1: *In the Melitz (2003) model, measured individual productivity in the cross section is given by*

$$\nu(\varphi) = \begin{cases} \frac{\sigma}{\left(\frac{\varphi^*}{\varphi}\right)^{\sigma-1}+\sigma-1} & \textit{if non exporter} \\ \frac{\sigma\left(1+n\tau^{1-\sigma}\right)}{\left(\frac{\varphi^*}{\varphi}\right)^{\sigma-1}\left(1+n\frac{f_x}{f}\right)+(\sigma-1)(1+n\tau^{-\sigma})} & \textit{if exporter} \end{cases}$$

where

$$\varphi^{*\kappa} = \varphi^{o\kappa}\left(\frac{f}{\delta f_e}\left(\frac{\sigma-1}{\kappa-\sigma+1}\right)\left(1+n\tau^{-\kappa}\left(\frac{f_x}{f}\right)^{\frac{-k+\sigma+1}{\sigma-1}}\right)\right) \tag{2.1}$$

Proof: *See Appendix*

Once we have measured productivity for each type of firm, we can see how is the behavior of this measurement along the set of efficiencies, $\varphi \in (\varphi^*, \infty)$. The following Lemma shows its basic properties

Lemma 2:

a) *Measured productivity of non exporters is increasing, concave and have a horizontal asymptote to the mark-up.*

b) *Measured productivity of exporters is increasing, concave for all firms* $\varphi > \left(\frac{(\sigma-2)}{\sigma(\sigma-1)}\frac{\left(\frac{f}{f_x}+n\right)}{(\tau+n\tau^{1-\sigma})}\right)^{\frac{1}{\sigma-1}}\varphi_x^*$ *and has an asymptote larger than the mark up.*

Proof: *See Appendix*

Now that we already described how are the measured productivity functions, we proceed to analyse the static picture that emerges in the comparison of exporters and non exporters in this framework.

2.3.1 Static analysis

In order to continue, we define the following concepts.

Definition: *A firm is an unproductive exporter of type I if the measured productivity of the firm would increase by choosing not to export, although she is optimally choosing to export*

Definition: *A firm is an unproductive exporter of type II if the measured productivity of the firm is smaller than the most productive non-exporter*

The concept of unproductive exporter of type I arises from the fact that under some parametrizations of the Melitz model, there are some firms that are optimally choosing to export, although due to the extra cost they face, namely f_x and τq_x, their measured productivity is smaller than it would be if she would not choose to export. The existence of this property triggers the concept of measured productivity. If in their optimal choice, some firms are better off with a smaller-than-maximal productivty, the link between optimality and large levels of productivty is then broken. This would be of no surprise if the technology had decreasing returns to scale, as, for instance, a Cobb-Douglas technology. The problem, however, arises when this is also true with a linear technology, specially since measured productivity would be an increasing function of efficiency if there was no change in the status of the firm.

Next Lemma and Corollaries show conditions that make firms to be in this set.

Lemma 3: *Let* $\hat{\varphi}^{\sigma-1} = \frac{1}{\sigma-1}\frac{\frac{f_x}{f}\tau^{\sigma}-\tau}{\tau-1}\varphi^{*\sigma-1}$. *Then, every firm* $\varphi < \hat{\varphi}$ *is an unproductive exporter of type I*

Proof: *Obvious*

Corollary:

a) *If* $\frac{1}{\sigma-1}\frac{\frac{f_x}{f}\tau^{\sigma}-\tau}{\tau-1} > \frac{f_x}{f}\tau^{\sigma-1}$, *the set of unproductive exporters of type I is non-empty*

b) *Under* $\tau = 1$ *and* $f_x > f$ *all exporters are unproductive exporters of type I*

Next we analyze the concept of unproductive exporter of type II, which is a concept that arises from the fact that under some parameterizations of the Melitz model, some exporters seem less productive than some non exporters. This property of the Melitz model is of major importance. It shows that the commonly thought property that exporters are more productive than non-exporters, and in particular, that it is always true that the most productive exporter is more productive than the most productive non-exporter is broken. These findings are important: the mass of firms is Pareto distributed in their efficiency. This implies that when getting the average measured productivity, those firms that are close in efficiency terms to those that do not export account for a much larger share of average than those that have larger efficiency. Thus, it is possible that the average of exporters is lower than that of non exporters.

Lemma 4: *Let* $\check{\varphi}^{\sigma-1} = \left(\frac{1+n\frac{f_x}{f}}{n\tau^{-\sigma}(\tau-1)(\sigma-1)+(1+n\tau^{1-\sigma})\frac{f}{f_x}\tau^{1-\sigma}}\right)\varphi^{*\sigma-1}$. *Then, every firm* $\varphi < \check{\varphi}$ *is an unproductive exporter of type II*

Proof: *Obvious*

Corollary:

a) *If* $\frac{f_x}{f} > \frac{\tau^{2-\sigma}}{\tau(2-\sigma)+(\sigma-1)}$, *the set of unproductive exporters of type II is non-empty*

b) *Under* $\tau = 1$ *and* $f_x > f$ *all exporters are unproductive exporters of type II*

In this section we have developed a static and cross sectional analysis of the Melitz model when firm's productivity is computed as R-productivity. We skip the analysis of Q-productivity in this chapter, because as it could easily be noticed, a crucial point of the results in this section have to do with the asymptote at which productivity measurements always converge when efficiency of firms tends to infinity. Notice that Q-productivty = R-productivty $\rho\varphi$. Hence, the asymptote disappears, and most of the results that are true for R-productivity analysis arenot for Q-productivity.

In the next section, we show what are the effects of changes in this measured productivity when the country performs a trade liberalization.

2.4 The Impact of Trade liberalization

There are two different barriers to trade in the Melitz model: the iceberg cost and the fixed cost of exporting. We analyze the changes in measured productivity for the case in which either of them falls. First, we analyze what happens to firm productivity when the iceberg cost falls from $\tau > 1$ to $\tau' = 1$, and then what is the effect from $f_x > f$ to $f_x' = f$. In order to proceed, we first define how to measure changes in productivty, and then we show how does this productivity measurement change for different types of firms.

Definition: *Change in productivity, Δ, is given by*

$$\Delta = \frac{\nu'(\varphi)}{\nu(\varphi)}$$

where ' stands for after trade liberalization.

Notice that the analysis of this section is equivalent for both, R-productivity and Q-productivity. Because of the definition of Δ and the relationship between R- and Q-productivity the two measures coincide, since the extra term involving Q-productivity cancels with itself.

Recall that after a trade liberalization, there are 4 types of firms: firms that choose not to continue operating, whose measured productivity change cannot be measured, since they disappear, firms that were not exporting and choose to continue without exporting (NN), firms that choose to export now, but were not exporting before (NE) and firms that were exporting and decide to continue exporting (EE). The effect of a trade liberalization, no matter the nature of it, is that measured productivity of all surviving firms that were not exporting initially, drops. Next Theorems explain the result.

Theorem 1: *After a trade liberalization in whcih $\tau > 1$ falls to $\tau' = 1$, measured productivity of all firms that were not exporting falls*
Proof: *See Appendix*

Theorem 2: *After a trade liberalization in which $f_x > f$ falls to $f_x' = f$, measured productivity of firms that were not exporters and continue not to export, falls. Furthermore, measured productivity of firms that start to export for all firms with efficiency $\varphi < \breve{\varphi}$ also falls.*[3]

[3] Where $\breve{\varphi} = \frac{\varphi^{o\sigma-1}\left(\frac{f}{\delta f_e}\frac{\sigma-1}{\kappa-\sigma+1}\right)^{\frac{\sigma-1}{\kappa}}}{(\sigma-1)n\tau^{-\sigma}}\left(\frac{\tau}{\tau-1}\right)\left((1+n)\left(1+n\tau^{-\kappa}\right)^{\frac{\sigma-1}{\kappa}} - \left(1+n\tau^{1-\sigma}\right)\left(1+n\tau^{-\kappa}\left(\frac{f_x}{f}\right)^{\frac{-\kappa+\sigma-1}{\sigma-1}}\right)^{\frac{\sigma-1}{\kappa}}\right)$

Proof: *See Appendix*

Corollary: *After trade liberalization in whcih $f_x > f$ falls to $f_x' = f$ and $\tau > 1$ falls to $\tau' = 1$, all new exporters' productivity drops.*

Theorems 1 and 2 are the main results of this chapter. They show that when there is a trade liberalization - no matter if it is caused by a drop in the iceberg cost or a drop in the fixed cost of exporting - measured productivity of firms that start to export falls. The reason for this result is double.

First, all firms face an incraese in the cost of inputs. In particular, since there is reallocation towards more efficient firms, and those demand more labor, this labor becomes more scarce. This makes all the firms to suffer a loss in their productivity due to this, and in particular, this is the cause of the drop in productivity of firms that never export.

The second argument affects firms that start to export, and has to do with the increase in the labor that they require in order to start this process. It is split in to different arguments. The first argument, which affects Theorem 1, is that these firms require more fixed labor in order to produce the new quantity sold. This extra cost shows up in the denominator, and it is used to sell goods whose price is unchanged from those that are sold domestically. Hence, there is an increase in the input usage, that is relatively more important than the increase in the revenue that it can attract. Hence, the fall in measured productivity.

The other side of this argument, which is the cause for Theorem 2 to hold as it does, is that after a fall of fixed costs to the same level of domestic fixed costs, they amount of revenue per unit of labor that they require is also larger, due to the presence of the iceberg cost. The total effect of these two sides of the argument depends upon the efficiency of each firm. For some parametrizations, measured productivity of some new exporters increases, and for some other parametrizations, the opposite is true.

However, as it is shown in the Corollary that follows both Theorems, measured productivity of all firms that become exporters after a full trade liberalization, $f_x > f$ falls to $f_x' = f$ and $\tau > 1$ falls to $\tau' = 1$, makes all the new exporters' productivity (since are all the surviving firms that were previously not exporting, since no firm would choose not to export in this case) to fall.

2.5 Conclusion and Further Research

In this chapter we have shown a very negative result. When it comes to measured productivity, some of the results that were thought to apply in the workhorse model of international trade are broken. In particular, we show that measuring in the model as productivity is measured in the data has two basic counterfactuals: productivity of new exporters falls, and productivity of exporters may be lower than that of non-exporters.

However, the analysis is limited to the basic model of international trade, and there are many different extensions of this model that could potentially overcome the results. The literature of variable mark-ups, as Holmes, Hsu and Lee (2012) and Edmond, Midrigan and Xu (2012) can shed light on this issue, since the negative relationship between prices and output can be broken.

Appendix

This Appendix covers the proofs for all the Lemmas, Propositions and Theorems of this chapter.

Proof of Lemma 1

Proof. All the steps done in this part use the notation of Melitz (2003). All the equations written below are functions of firm efficiency, φ. We normalize the size of the country to 1, $L = 1$, and use the Pareto distribution with parameters φ^o and κ as the support and the curvature parameters in order to get closed form solutions to the problem. Prices (domestic, p, and foreign, p_{ex}) and quantities (domestic, q, and foreign, q_{ex}) are given by

$$\begin{aligned} p &= \frac{1}{\rho\varphi} p_{ex} = \frac{\tau}{\rho\varphi} \\ q &= QP^\sigma p^{-\sigma} \\ q_{ex} &= QP^\sigma p_x^{-\sigma} \end{aligned}$$

which consist of the solution to the individuals' maximization of utility problem and the firms' maximization of profits problem, where τ is the iceberg cost, σ is the elasticity of substitution, $\rho = \frac{\sigma-1}{\sigma}$, Q is the aggregate consumption bundle, P is the aggregate price index. Since $L = 1$, $PQ = 1$

Next, using the equation of aggregate price index in Melitz (2003), we get that

$$P = \frac{M^{\frac{1}{1-\sigma}}}{\rho} \left(\tilde{\varphi}^{\sigma-1} + np_x \left(\tau^{-1}\tilde{\varphi}_x^{\sigma-1}\right)\right)^{\frac{1}{1-\sigma}}$$

where M is the total mass of domestic producers, $\tilde{\varphi}$ and $\tilde{\varphi}_x$ are the average efficiency of all firms and the average efficiency of exporters respectively and p_x is the probability that a firm exports.

Since

$$\begin{aligned}
\tilde{\varphi}_x^{\sigma-1} &= \tilde{\varphi}^{\sigma-1}\tau^{\sigma-1}\frac{f_x}{f} \\
p_x &= \frac{\varphi^{*\kappa}}{\varphi_x^{*\kappa}} \\
\tilde{\varphi}^{\sigma-1} &= \varphi^{*\sigma-1}\left(\frac{\kappa}{\kappa-\sigma+1}\right) \\
\tilde{\varphi}_x^{\sigma-1} &= \varphi_x^{*\sigma-1}\left(\frac{\kappa}{\kappa-\sigma+1}\right)
\end{aligned}$$

The first equation is directly from Melitz (2003). The second equation follows from the Pareto distribution, where φ^* and φ_x^* are the cutoff efficiency levels for domestic and exports, respectively. The third and fourth equations are from the Pareto distribution and the definition used in Melitz (2003) about average efficiency.

Hence, using all the previous equations into the expression for aggregate price index, we get that

$$P = \frac{M^{\frac{1}{1-\sigma}}}{\rho}\frac{\left(1+n\tau^{-\kappa}\left(\frac{f_x}{f}\right)^{\frac{-\kappa+\sigma-1}{\sigma-1}}\right)^{\frac{1}{1-\sigma}}}{\varphi^*\left(\frac{\kappa}{\kappa-\sigma+1}\right)^{\frac{1}{\sigma-1}}}$$

We can also use the expression for the mass of firms as

$$M = \frac{1}{\sigma\left(\bar{\pi}+f+\left(\tau\left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}}\right)^{-\kappa}nf_x\right)}$$

where $\bar{\pi}$ is average profits.

Next, using the Zero Cutoff Profit condition and the Free Entry condition equations, to get the two equations regarding average profits

$$\bar{\pi} = \delta f_e\left(\frac{\varphi^*}{\varphi^o}\right)^{\kappa} \qquad FE$$

$$\begin{aligned}
\bar{\pi} &= f\left(\left(\frac{\tilde{\varphi}}{\varphi^*}\right)^{\sigma-1}-1\right)+\left(\tau\left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}}\right)^{-\kappa}nf_x\left(\left(\frac{\tilde{\varphi}_x}{\varphi_x^*}\right)^{\sigma-1}-1\right) \\
&= \frac{\sigma-1}{\kappa-\sigma+1}\left(f+\left(\tau\left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}}\right)^{-\kappa}nf_x\right) \qquad ZCP
\end{aligned}$$

Hence, using the ZCP we get that

$$M = \frac{1}{\sigma\left(f + \left(\tau\left(\frac{f_x}{f}\right)^{\frac{1}{\sigma-1}}\right)^{-\kappa} n f_x\right)\frac{\kappa}{\kappa-\sigma+1}}$$

Plugging this expressions into P, the latter simplifies to

$$P = \frac{(\sigma f)^{\frac{1}{\sigma-1}}}{\rho\varphi^*}$$

Finally, combining the FE and the ZCP, we get that

$$\varphi^* = \varphi^o\left(\frac{f}{\delta f_e}\frac{\sigma-1}{\kappa-\sigma+1}\left(1 + n\tau^{-\kappa}\left(\frac{f_x}{f}\right)^{\frac{-\kappa+\sigma-1}{\sigma-1}}\right)\right)^{\frac{1}{\kappa}}$$

and this concludes the characterization. ■

Proof of Proposition 1

Proof. We can compute measured aggregate productivity as revenue per worker (which is also revenue per total use of inputs, that is, TFP) as

$$\nu(\varphi) = \begin{cases} \frac{pq}{l} = \frac{P^{\sigma-1}\left(\frac{1}{\rho\varphi}\right)^{1-\sigma}}{f+\frac{q}{\varphi}} & \text{if non exporter} \\ \frac{pq+np_{ex}q_{ex}}{l} = \frac{P^{\sigma-1}\left(\frac{1}{\rho\varphi}\right)^{1-\sigma}+nP^{\sigma-1}\left(\frac{\tau}{\rho\varphi}\right)^{1-\sigma}}{f+\frac{q}{\varphi}+n\left(f_x+\frac{q_{ex}}{\varphi}\right)} & \text{if exporter} \end{cases}$$

Using the solution of the model from the previous Lemma, it is easy to show that it turns to be

$$\nu(\varphi) = \begin{cases} \frac{\sigma}{\left(\frac{\varphi^*}{\varphi}\right)^{\sigma-1}+\sigma-1} & \text{if non exporter} \\ \frac{\sigma\left(1+n\tau^{1-\sigma}\right)}{\left(1+n\frac{f_x}{f}\right)\left(\frac{\varphi^*}{\varphi}\right)^{\sigma-1}+(\sigma-1)(1+n\tau^{-\sigma})} & \text{if exporter} \end{cases}$$

and this concludes the proof. ■

Proof of Lemma 2

Proof. Although φ^* is not constant, for this type of exercise we can take it as a constant because it does not change, since all policy configurations are constant. Let's define $\nu_{NE}(\varphi)$ for firms that are not exporting and $\nu_E(\varphi)$ for those that export.

Then,

$$\begin{aligned}
\frac{\partial \nu_{NE}(\varphi)}{\partial \varphi} &= \frac{\sigma(\sigma-1)\frac{\varphi^{*\sigma-1}}{\varphi^{\sigma}}}{\left(\left(\frac{\varphi^*}{\varphi}\right)^{\sigma-1}+\sigma-1\right)^2} \\
\frac{\partial^2 \nu_{NE}(\varphi)}{\partial \varphi^2} &= -\frac{\sigma^2(\sigma-1)\varphi^{*\sigma-1}}{\varphi^{\sigma 2}\left(\left(\frac{\varphi^*}{\varphi}\right)^{\sigma-1}+\sigma-1\right)^3}\left(\varphi^{\sigma-1}(\sigma-1)-\varphi^{*\sigma-1}+\frac{2}{\sigma}\varphi^{*\sigma-1}\right)<0
\end{aligned}$$

Where $\varphi^{\sigma-1}(\sigma-1)-\varphi^{*\sigma-1}+\frac{2}{\sigma}\varphi^{*\sigma-1}>0$, since it can be proven in the following two parts.

Suppose $\sigma \geq 2$, then $\varphi^{\sigma-1}(\sigma-1)-\varphi^{*\sigma-1}>0$, because $\varphi>\varphi^*$.

Suppose $2>\sigma>1$, then $-\varphi^{*\sigma-1}+\frac{2}{\sigma}\varphi^{*\sigma-1}>0$, because $\frac{2}{\sigma}-1>0$

Furthermore,

$$\lim_{\varphi\to\infty}\nu_{NE}(\varphi)=\frac{\sigma}{\sigma-1}$$

that is, at the limit the productivity of non exporters is equal to the Dixit-Stiglitz mark-up.

Repeting the previous exercise for the measured productivity of exporters

$$\begin{aligned}
\lim_{\varphi\to\infty}\nu_{NE}(\varphi) &= \frac{\sigma(1+n\tau^{1-\sigma})}{(\sigma-1)(1+n\tau^{-\sigma})}>\frac{\sigma}{\sigma-1} \\
\frac{\partial \nu_E(\varphi)}{\partial \varphi} &= \frac{\sigma(1+n\tau^{1-\sigma})\left(1+n\frac{f_x}{f}\right)(\sigma-1)\frac{\varphi^{*\sigma-1}}{\varphi^{\sigma}}}{\left(\left(1+n\frac{f_x}{f}\right)\left(\frac{\varphi^*}{\varphi}\right)^{\sigma-1}+(\sigma-1)(1+n\tau^{-\sigma})\right)^2} \\
\frac{\partial^2 \nu_E(\varphi)}{\partial \varphi^2} &= \sigma\left(1+n\tau^{1-\sigma}\right)\left(1+n\frac{f_x}{f}\right)(\sigma-1) \\
&\quad\times\frac{\varphi^{*\sigma-1}}{\varphi^{\sigma+1}}\frac{-\sigma(\sigma-1)(1+n\tau^{-\sigma})+\left(1+n\frac{f_x}{f}\right)\left(\frac{\varphi^*}{\varphi}\right)^{\sigma-1}(\sigma-2)}{\left(\left(1+n\frac{f_x}{f}\right)\left(\frac{\varphi^*}{\varphi}\right)^{\sigma-1}+(\sigma-1)(1+n\tau^{-\sigma})\right)^3}
\end{aligned}$$

Combinging the parameters of the last equation that ensure the term being engative with the definition of the cutoff for exporting firms, we get that the term is negative for all firms with efficiency larger than

$$\varphi > \left(\frac{(\sigma-2)}{\sigma(\sigma-1)} \frac{\left(\frac{f}{f_x}+n\right)}{(\tau+n\tau^{1-\sigma})} \right)^{\frac{1}{\sigma-1}} \varphi_x^*$$

■

Proof of Theorem 1

Proof. Change in productivity of firms that were not exporters before and continue without being exporters after the trade liberalization is given by

$$\Delta_{NN} = \frac{\left(\frac{\varphi^o}{\varphi}\left(\frac{f}{\delta f_e}\frac{\sigma-1}{\kappa-\sigma+1}\left(1+n\tau^{-\kappa}\left(\frac{f_x}{f}\right)^{\frac{-\kappa+\sigma-1}{\sigma-1}}\right)\right)^{\frac{1}{\kappa}}\right)^{\sigma-1}+\sigma-1}{\left(\frac{\varphi^o}{\varphi}\left(\frac{f}{\delta f_e}\frac{\sigma-1}{\kappa-\sigma+1}\left(1+n\left(\frac{f_x}{f}\right)^{\frac{-\kappa+\sigma-1}{\sigma-1}}\right)\right)^{\frac{1}{\kappa}}\right)^{\sigma-1}+\sigma-1}$$

Notice that the term in brackets of the numerator differs from that of the denominator only because of the iceberg cost, τ. Since $\tau > 1$, and it is raised to a negative power, the number is trivially smaller than 1.

For firms that become exporters after trade liberalization, the change is given by

$$\Delta_{NE} = \frac{\left(\left(\frac{\varphi^o}{\varphi}\left(\frac{f}{\delta f_e}\frac{\sigma-1}{\kappa-\sigma+1}\left(1+n\tau^{-\kappa}\left(\frac{f_x}{f}\right)^{\frac{-\kappa+\sigma-1}{\sigma-1}}\right)\right)^{\frac{1}{\kappa}}\right)^{\sigma-1}\right)+\sigma-1}{\left(\frac{1+n\frac{f_x}{f}}{1+n}\right)\left(\frac{\varphi^o}{\varphi}\left(\frac{f}{\delta f_e}\frac{\sigma-1}{\kappa-\sigma+1}\left(1+n\left(\frac{f_x}{f}\right)^{\frac{-\kappa+\sigma-1}{\sigma-1}}\right)\right)^{\frac{1}{\kappa}}\right)^{\sigma-1}+\sigma-1}$$

Notice that, as before, the effect on the term in the brackets is the same, but now there is an added term that premultiplies the term in brackets of the denominator by $\frac{1+n\frac{f_x}{f}}{1+n}$ which is trivially larger than one, and hence the measurment is lower than 1. ■

Proof of Theorem 2 and Corollary

Proof. Change in productivity of firms that were not exporters before and continue without being exporters after the trade liberalization is given by

$$\Delta_{NN} = \frac{\left(\frac{\varphi^o}{\varphi}\left(\frac{f}{\delta f_e}\frac{\sigma-1}{\kappa-\sigma+1}\left(1+n\tau^{-\kappa}\left(\frac{f_x}{f}\right)^{\frac{-\kappa+\sigma-1}{\sigma-1}}\right)\right)^{\frac{1}{\kappa}}\right)^{\sigma-1}+\sigma-1}{\left(\frac{\varphi^o}{\varphi}\left(\frac{f}{\delta f_e}\frac{\sigma-1}{\kappa-\sigma+1}\left(1+n\tau^{-\kappa}\right)\right)^{\frac{1}{\kappa}}\right)^{\sigma-1}+\sigma-1}$$

Notice that the term in brackets of the numerator differs from that of the denominator only because of the fixed exporting cost, $f_x > f$. Since the ratio is larger than 1, and it is raised to a negative power, the number is trivially smaller than 1.

For firms that become exporters after trade liberalization, the change is given by

$$\Delta_{NE} = \frac{\left(1+n\tau^{1-\sigma}\right)\left(\left(\frac{\varphi^o}{\varphi}\left(\frac{f}{\delta f_e}\frac{\sigma-1}{\kappa-\sigma+1}\left(1+n\tau^{-\kappa}\left(\frac{f_x}{f}\right)^{\frac{-\kappa+\sigma-1}{\sigma-1}}\right)\right)^{\frac{1}{\kappa}}\right)^{\sigma-1}+\sigma-1\right)}{\left(1+n\right)\left(\frac{\varphi^o}{\varphi}\left(\frac{f}{\delta f_e}\frac{\sigma-1}{\kappa-\sigma+1}\left(1+n\tau^{-\kappa}\right)\right)^{\frac{1}{\kappa}}\right)^{\sigma-1}+\left(\sigma-1\right)\left(1+n\tau^{-\sigma}\right)}$$

This term is not always smaller than 1, in particular, for large enough τ. since it could be that the effect of the iceberg cost raising the revenues in the rest of the world offsets the effect of the increase in the cost that the firm has in order to serve them. However, the previous expression is smaller than one for all the firms with efficiency $\varphi < \breve{\varphi}$

$$\frac{\varphi^{o\sigma-1}\left(\frac{f}{\delta f_e}\frac{\sigma-1}{\kappa-\sigma+1}\right)^{\frac{\sigma-1}{\kappa}}}{\left(\sigma-1\right)n\tau^{-\sigma}}\left(\frac{\tau}{\tau-1}\right)\left(\left(1+n\right)\left(1+n\tau^{-\kappa}\right)^{\frac{\sigma-1}{\kappa}}-\left(1+n\tau^{1-\sigma}\right)\left(1+n\tau^{-\kappa}\left(\frac{f_x}{f}\right)^{\frac{-\kappa+\sigma-1}{\sigma-1}}\right)^{\frac{\sigma-1}{\kappa}}\right) =$$

Finally, in the case of a full liberalization, in which both, $\tau' = 1$ and $f'_x = f$, all non exporters that remain active become exporters. In this case, their measured productivity becomes

$$\Delta_{NE} = \frac{\left(\frac{\varphi^*}{\varphi}\right)^{\sigma-1}+\sigma-1}{\left(\frac{\varphi'^*}{\varphi}\right)^{\sigma-1}+\sigma-1}$$

which is trivially smaller than 1 since $\varphi'^{*} < \varphi^{*}$. ■

Chapter 3

Distortions, Productivity, and Idiosyncratic Shocks

(joint with José María Da Rocha)

3.1 Introduction

Restuccia and Rogerson (2008) show that the policy distortions faced by individual plants can lead to decreases in output and TFP in the range of 30 to 50 percent. Similarly, Guner *et al.* (2008) find that size-dependent policies that reduce average establishment size by 20% generate reductions in output of up to 8.1% and output per establishment of up to 25.6%. In the wake of these findings, an increasing interest in the effects of resource misallocation across heterogeneous plants has emerged. Hsieh and Klenow (2009) find large deviations in resource allocation across firms in India and China that explain differences in TFP relative to the U.S. in the range of 40 to 60 percent. Similar results are obtained by Neuemeyer and Sandleris (2010) for the manufacturing sector in Argentina.

Much of this literature is concerned with understanding the role of distortions in entry decisions for a given distribution of plants. In order to simplify the problem, the literature assumes that productivity for a given plant is constant over time. This assumption has two important implications: The first is that plants have constant operating profits, so once they enter the industry they have no incentive to exit it. The second is that entering plants are the only ones that decide whether to exit the industry or not. That is to say, entering plants decide whether to remain when operating profits are non-negative. If they do then no further exit decisions need to be taken.

In this chapter we consider policy distortions in a model where plants face idiosyncratic shocks, as in Hopenhayn (1992) and Hopenhayn and Rogerson (1993). As in Restuccia and Rogerson (2008) we also assume that low-productivity plants receive subsidies and high-productivity plants pay taxes. Finally, as in Luttmer (2007)As, we assume that plant productivity follows a Brownian process and we use the forward Kolmogorov equation to analytically characterize the invariant industry distribution of plants.

What do we learn from our model? Introducing idiosyncratic shocks into the model implies that plants have non-constant operating profits and as a result there is an endogenous exit margin. On the one hand, incumbent plants must decide in each period whether or not to remain in the industry. The exit decision is now equivalent to asking whether the plant option value is non-negative. Therefore, plants with non-positive profits may remain in the industry. On the other hand, if there are shocks and endogenous exits, plants are more eager to enter since they are not stuck with a particular productivity level. Hence, unlike a world without shocks, there will be low-productivity plants that are active (waiting for better days). As a result, the productivity of the marginal entering plant and TFP will decrease.

We also analytically characterize cross-sectional dispersion in productivity and show that as the time series volatility of idiosyncratic shocks rises, the option values of plants increase, as does the cross-sectional dispersion of productivity. This is consistent with the findings of Asker *et al.* (2011). Using data from the World Bank's Enterprise Data base on 5,010 establishments in 33 developing countries, they find that countries with greater time-series volatility in productivity are also characterized by greater cross-sectional dispersion in productivity.

What do our findings imply? Assume that a model without idiosyncratic shocks and endogenous exit is calibrated to fit data generated from a model with shocks and endogenous entry. If there are idiosyncratic shocks and endogenous exits, some plants will have negative current-period profits because of the option to exit. Any attempt to fit the model without shocks to data generated from a model with shocks and endogenous entry will underestimate the fixed operating cost to justify the existence of such low-productivity plants. By underestimating the operating cost, the model without shocks overestimates the expected value of entering plants. As a result, in order to keep the same entry level, entry costs must be higher in the model without shocks. Hence, given that distortions on TFP are proportional to the value of plants, a calibrated model without shocks and endogenous entry will overestimate the effect of policy distortions on TFP if the data reflects shocks and endogenous entry decisions.

There is a large, growing body of literature that analyzes the impact that policy distortions have on TFP in models with idiosyncratic shocks, which is carefully reviewed in Hopenhayn (2011). Two papers in that literature that are especially related to ours are Fattal (2011) and Buera *et al.* (2011). Fattal (2011) uses a calibrated model with idiosyncratic shocks to show that misallocation carries big welfare losses when transitional dynamics are taken into account. Buera *et al.* (2011) also use firm dynamics and shocks at plant level. Unlike us, they consider policy configurations that are plant-specific. They show that misallocation in less-developed countries may be due to well-intended policies that were initially chosen to subsidize productive entrepreneurs in order to relax their credit constraints.

The rest of the chapter is organized as follows. We start out by describing the economy in Section 2. In Section 3 we characterize the stationary equilibrium. Section 4 analyzes the effect of policy distortions on TFP with and without shocks and demonstrates that a model without shocks will overestimate the effect of policy distortions on TFP. Section 5 concludes.

3.2 The Economy

We introduce distortions that affect operating profits and the entry and delay-exit decisions of plants in a version of the Hopehayn (1992) and Hopenhayn and Rogerson (1993) stochastic models of plant-level heterogeneity. As in Restuccia and Rogerson (2008), plants have access to a decreasing returns to scale technology, are subject to output taxes and subsidies, and pay a one-time fixed cost of entry and a fixed cost of operation in each period. As in Luttmer (2007), plants experience idiosyncratic productivity shocks that evolve according to a standard Brownian motion. Entry into and exit from the industry are endogenously determined by the combination of fixed costs, taxes, subsidies and idiosyncratic shocks. Finally, there is no capital and households inelastically supply one unit of labor.

3.2.1 Households

There is a continuum of identical households with measure one which consume and receive labor income, plant profits and government transfers. The representative household's utility function is given by

$$\max \int_0^\infty e^{-\rho t} u(c) dt \tag{3.1}$$

where c is consumption. We assume that $0 < \rho < 1$ and that u is continuously differentiable, strictly concave, and monotonically increasing. We assume that the household has an endowment of one unit of time, which is inelastically supplied in the labor market. Therefore, the household's budget constraint can be written as

$$c = w + \pi + T_h,$$

where w, π and T_h are labor income, plant profits and a lump sum transfer from the government, respectively.

3.2.2 Incumbent plants

There is a continuum, with measure M, heterogeneous, infinitely-lived plants that use labor, n, to produce a consumption good according to

$$y(s, n) = s^{1-\gamma} n^{\gamma},$$

where $1 < \gamma < 0$. Cross-Out, the only difference across plants is the level of productivity, s.

We introduce distortions in a very stylized way. We assume that low-productivity plants, which are small, receive subsidies and high-productivity plants, which are big, pay taxes. This kind of policy distortion can be implemented as a combination of a positive output tax rate, τ, and a lump sum subsidy that is equal for all plants, T_f. Hence, a plant producing output y will pay a net tax given by

$$T = \tau y - T_f.$$

We allow s to vary across plants and over time. We assume that the value of $s^{(1-\gamma)}$ for a given plant evolves following a Brownian motion

$$ds^{1-\gamma} = -\mu s^{1-\gamma} dt + \sigma s^{1-\gamma} dz, \tag{3.2}$$

where μ is the drift of the process, σ its standard deviation and dz is the increment of a Wiener process.

Output supply and labor demand decisions are static. An incumbent plant with productivity s hires labor in order to solve the following static profit maximization problem:

$$\pi = \max_{n} (1-\tau) s^{1-\gamma} n^{\gamma} - wn + T_f.$$

A plant's optimal labor demand is given by

$$n(s|w,T) = s\left((1-\tau)\frac{\gamma}{w}\right)^{\frac{1}{1-\gamma}}, \tag{3.3}$$

and its operating profits are equal to

$$\pi(s|w,T) = s\left(\frac{1-\tau}{w^{\gamma}}\left(\gamma^{\gamma}-\gamma\right)\right)^{\frac{1}{1-\gamma}} + T_f. \tag{3.4}$$

We also assume that there is a fixed cost of operation, c_f, and an entry cost, c_{entry}, measured in units of consumption goods. If the plant wants to remain active it must pay the fixed operation cost. Hence, plants may find it optimal to exit the economy and create a new plant in each period. In order to avoid this scheme, we assume that the parameters satisfy $c_{entry}\frac{2\sigma^2\rho}{\mu^2} > c_f$.

Given its current operating profits, an incumbent plant chooses whether or not to remain active by solving the following dynamic problem:

$$\begin{aligned} W(s|w,T) &= \max_{\{stay,exit\}} \left\{\pi(s|w,T) - c_f + (1+\rho dt)^{-1} EW(s+ds|w,T), 0\right\} \\ st &: \; ds^{1-\gamma} = -\mu s^{1-\gamma} dt + \sigma s^{1-\gamma} dz, \end{aligned}$$

where, $W(s|w,T)$ and $EW(s+ds|w,T)$ are the option value and the expected continuation value of an incumbent plant with productivity s respectively. Incumbents choose to remain active if the sum of their current operating profits and their expected continuation value is non negative. The following lemma shows that both the fixed operating cost and the government subsidy imply a minimum plant productivity level if, and only if, the net fixed operating cost is positive, $c_f - T_f > 0$, which we assume throughout.

Lemma 1. *Assume that $c_{entry}\frac{2\sigma^2\rho}{\mu^2} > c_f$ and $c_f - T_f > 0$. Then the minimum plant productivity level, s^*, and the value function of a plant with productivity s, $W(s|w,T)$, are given by*

$$s^*(w,T) = \left(\frac{w^\gamma}{\kappa(\tau)}\right)^{\frac{1}{1-\gamma}} \frac{-\beta}{1-\beta}\frac{\rho+\mu}{\rho}(c_f - T_f) \tag{3.5}$$

and

$$W(s|w,T) = -\frac{(c_f - T_f)}{\rho}\left[\left(1-\frac{s}{s^*}\right)^\beta \frac{1}{1-\beta}\right] + \frac{s}{\rho+\mu}\left(\frac{\kappa(\tau)}{w^\gamma}\right)^{\frac{1}{1-\gamma}} \tag{3.6}$$

where $\kappa(\tau) = (1-\tau)(\gamma^\gamma - \gamma)$ and

$$\beta = \frac{1}{2} + \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{\mu}{\sigma^2}+\frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}} < 0, \tag{3.7}$$

determines the option value size of remaining in the industry, that is, the opportunity cost of exiting.

Proof *See Appendix*

Equation (3.6) highlights the distorting role played by government subsidies in this economy, even when given as a lump sum. An increase in government subsidy increases the option value of remaining in the economy. Therefore, higher subsidies imply a lower minimum productivity level (equation 3.5).

3.2.3 Entering plants

We assume that when potential entrants make their entry decisions they take as given the distribution of productivity, $G(s)$, and the distortions summarized by the tax function T. As in Restuccia and Rogerson (2008), we assume that a potential entrant will optimally decide whether to engage in production after observing its realized draw s. Therefore, the expected present discounted value of entry is

$$\int_{s^*(w,T)}^{\infty} W(s|w,T)dG(s)ds - c_{entry}. \tag{3.8}$$

3.2.4 Government

Finally, the government budget constraint satisfies

$$\int_{s^*(w,T)}^{\infty} \tau y(s)f(s|s^*(w,T))ds - T_f == \frac{T_h}{M} \tag{3.9}$$

where M is the mass of plants in the economy, and $f(s|s^*(w,T))$ is the industry productivity distribution, which is characterized below.

3.3 Stationary Equilibrium

We characterize the stationary equilibrium in a competitive economy where the law of motion of the industry, i.e. the distribution of plants across productivities, must be consistent with the optimal entry and delay-exit policies of plants. By assuming imitation, as in Luttmer (2007), we characterize the invariant industry productivity distribution as a function of the equilibrium wage and the distortions described by the tax function. Competition among potential entering plants and labor market clearing conditions determine wages and the mass of plants in equilibrium.

3.3.1 Industry productivity motion

Given the optimal entry and delay-exit policies of plants the law of motion of industry productivity, i.e. the distribution of plants across productivities, can be characterized as a function of wages and of policy distortions.

Assume that the number of firms is a measure with size one, and let $f(s,t)$ be the number of plants located in the productivity interval $[s, s+ds]$ at time t. The rate of change in the number of plants at time t in the interval of productivity $[s, s+ds]$ is equal to the rate of net departure of incumbents minus the rate of new entry.

Plant productivity s evolves following a Brownian motion with drift μ_s and standard deviation σ_s induced by equation (3.2), where

$$\mu_s = \frac{\mu}{1-\gamma} - \frac{1}{2}\gamma \frac{\sigma^2}{(1-\gamma)^2},$$

and

$$\sigma_s = \frac{\sigma}{1-\gamma}.$$

Therefore, the net departure rate of incumbents when $ds \to 0$ is given by the following Kolmogorov forward equation

$$\frac{\partial f(s,t)}{\partial t} = \mu_s \frac{\partial f(s,t)}{\partial s} + \frac{\sigma_s^2}{2} \frac{\partial^2 f(s,t)}{\partial s^2}, \tag{3.10}$$

with the boundary condition

$$f(s^*(w,T),t) = 0, \quad \forall t \quad \forall s \leq s^*(w,T). \tag{3.11}$$

Equation (3.10) establishes that a fraction μ_s of plants with productivity s, transits to the expected productivity $s - \mu_s$, and then experiences a random motion around $s - \mu_s$ driven by $\frac{\sigma_s^2}{2}\frac{\partial^2 f(s,t)}{\partial s^2}$. The random motion can be rationalized as a *diffusion process* which establishes that the rate of departure is proportional to the negative concentration gradient of density: plants move from productivity levels with a high number of plants to productivity levels with a low number of plants.

The industry law of motion must be consistent with plants' optimal entry and delay-exit policies. Given prices and policy distortions, plants with productivity level $s^*(w,T)$ exit the industry. Therefore, the boundary condition (3.11) indicates that there are no plants with productivity levels lower than $s^*(w,T)$.

Finally, as in Luttmer (2007) we assume that potential entrants imitate incumbent plants. Formally

$$dG(s) = \epsilon f(s|s^*(w,T)), \tag{3.12}$$

where $f(s|s^*(w,T))$ is the invariant distribution of plants in the industry. This assumption is equivalent to assuming that the productivity of entering plants is quite similar to that of incumbents. Luttmer (2007) shows that a model with imitation generates balanced growth and is consistent with the salient features of U.S. firm size distribution.

In stationary equilibrium, at each productivity level, s, the net departure rate of incumbents is equal to the rate of entry and the distribution of plants across productivities is constant over time. Formally,

$$\mu_s \frac{\partial f(s|s^*(w,T))}{\partial s} + \frac{\sigma_s^2}{2}\frac{\partial^2 f(s|s^*(w,T))}{\partial s^2} + \epsilon f(s|s^*(w,T)) = 0 \tag{3.13}$$

determines the mass of new entrants

$$dG\left(s\right) = \frac{1}{2}\frac{\mu_s^2}{\sigma_s^2} f(s|s^*(w,T)), \tag{3.14}$$

and the invariant distribution of plants in the industry[1]

$$f(s|s^*(w,T))) = \left(\mu_s/\sigma_s^2\right)^2 (s - s^*(w,T))e^{-\mu_s/\sigma_s^2(s-s^*(w,T))}. \tag{3.15}$$

3.3.2 Characterization of the Stationary Equilibrium

It is now possible to define the Stationary Equilibrium in this economy. A Stationary Equilibrium for this economy is a wage rate, w, an industry distribution of plants, $f(s|s^*(w,T))$, a mass of firms M, value functions $W(s|w,T)$, $\pi(s|w,T)$, policy functions $n(s|w,T)$, $s^*(w,T)$, aggregate consumption C and policy distortions T and T_h such that:

a) Given wages, w, and distortions, T, $\pi(s|w,T)$ (equation 3.4) solves the problem of incumbent plants and $n(s|w,T)$ (equation 3.3) is the optimal employment level.

b) $W(s|w,T)$ is the value of a plant with productivity s (equation 3.6), and the minimum productivity plant level, $s^*(w,T)$, is given by solving the delay-exit problem (equation 3.5).

c) The productivity distribution solves the forward Kolmogorov equation (equation 3.15).

[1]See Lemma 2 in [**?**].

f) There is free entry of plants.

$$\int_{s^*(w,T)}^{\infty} W(s|w,T)\frac{1}{2}\frac{\mu_s^2}{\tilde{\sigma}^2}f(s|s^*(w,T))ds - c_{entry} = 0 \tag{3.16}$$

d) Government satisfies the budget constraint (equation 3.9).

e) The goods market and the labor market clear

$$\frac{C}{M} + c_{entry} + c_f = \int_{s^*(w,T)}^{\infty} s^{1-\gamma}n^{\gamma}f(s|s^*(w,T))ds \tag{3.17}$$

$$M\int_{s^*(w,T)}^{\infty} nf(s|s^*(w,T))ds = 1 \tag{3.18}$$

To characterize the stationary equilibrium we need to determine the wage, w, the invariant industry distribution, $f(s|s^*(w,T))$, and the mass of plants, M. Note that the for a given wage and distortion, the exit rule s^* determines the invariant distribution of plants in the industry. Therefore, all the information that potential entrants need to make their entry decision is summarized in the wage. Competition among potential entrants determines the wage in equilibrium. Given the wage, the mass of plants is determined by the labor market clearing condition. Formally, using the free entry condition (equation 3.16) and equations (3.6) and (3.5), the cutoff and the wage satisfy

$$s^* = \frac{\left(s^* + \frac{2\sigma_s^2}{\tilde{\mu}}\right)\frac{1}{2}\frac{\mu_s^2}{\sigma_s^2}\left(\frac{-\beta}{1-\beta}\right)}{\frac{\rho c_{entry}}{c_f - T_f} - \int_{s^*}^{\infty}\left(\left(\frac{s}{s^*}\right)^{\beta}\frac{1}{1-\beta} - 1\right)\frac{1}{2}\frac{\mu_s^2}{\sigma_s^2}f(s|s^*)ds}. \tag{3.19}$$

$$w = \left(\left(s^*\frac{1-\beta}{-\beta}\frac{\rho}{\rho+\mu}\frac{1}{c_f - T_f}\right)^{1-\gamma}\kappa(\tau)\right)^{\frac{1}{\gamma}}, \tag{3.20}$$

where $\kappa(\tau) = (1-\tau)(\gamma^{\gamma} - \gamma)$ and M follows directly from the expression of the labor market clearing condition

$$M = \left((1-\tau)s^*\frac{\gamma}{w}\right)^{\frac{1}{1-\gamma}}\left(s^* + \frac{2\sigma_s^2}{\mu_s}\right).$$

Table 3.1: Changes on s^*

	σ^2	μ	ρ	c_{entry}	c_f
s^*	$\downarrow$	$\uparrow$	$\downarrow$	$\downarrow$	$\uparrow$

Proposition 1. *Assume that* $\frac{1}{2}\sigma_s^2 < \mu_s < 2\sigma_s^2$. *Then the steady state equilibrium associated with the policy distortions,* T, *exists and is unique.*

Proof *See Appendix .*

3.3.3 Comparative statics

In a model with idiosyncratic shocks, the productivity of a marginal plant, s^*, depends on c_f and c_{entry} just as it does in a model without shocks: higher operating (entry) costs are positively (negatively) correlated with the minimum plant productivity level.

More interesting is the relationship between s^*, σ^2, μ and ρ. In the appendix we show that if $(\beta + 1) < \frac{\mu_s}{2\sigma_s^2}$ and $\left(\frac{\rho c_{entry}}{c_f - T_f}\right)$ is higher than a lower bound, s^* is decreasing in σ^2 and increasing in μ and ρ. The logic is quite simple. The option value is increasing in σ^2 and decreasing in μ and ρ. If the option value increases there will be low-productivity plants that are active (waiting for better days). Table 3.1 summarizes these findings. Finally, in the Appendix we show that wages and the productivity of the marginal plant are positively correlated if the elasticities of w with respect to μ and ρ are higher than $\frac{\mu}{\mu+\rho}$, and the elasticity of w with respect to σ^2 is higher than a lower bound.

An interesting property of the model is that it rationalizes the empirical findings of Asker *et al.* (2011). They find that the cross-sectional dispersion of productivity and the time-series volatility of productivity are positively correlated. The time-series volatility of productivity is measured in the model by the standard deviation of the Brownian motion. To measure the cross-sectional dispersion of productivity, define the first two moments of the distribution of plant productivity: Mean plant productivity, $E[s]$, and the variance in plant productivity, $Var[s]$, in this economy are given by:

$$E[s] = \int_{s^*}^{\infty} sf(s|s^*)ds = s^* + \frac{2\sigma_s^2}{\mu_s}, \tag{3.21}$$

and

$$Var[s] = \int_{s^*}^{\infty} (s - s^*)^2 f(s|s^*) ds = \frac{2\sigma_s^2}{\mu_s}\left(s^* - 1 + \frac{2\sigma_s^2}{\mu_s}\right) = \frac{2\sigma_s^2}{\mu_s}(E[s] - 1). \quad (3.22)$$

The cross-sectional dispersion of productivity can be measured as

$$\frac{Var[s]}{E[s]} = \frac{2\sigma_s^2}{\mu_s}(1 - E[s]^{-1}) \quad (3.23)$$

Therefore, if a change in the time-series volatility of productivity, σ_s^2, reduces the minimum plant productivity level, s^*, less than proportionally, the time-series volatility of productivity and its cross-sectional dispersion are always positively correlated. The next proposition formalizes this idea.

Proposition 2. *Assume that* $\| \frac{\partial s^*}{\partial \sigma_s^2} \| < 1$. *Then the time-series volatility of productivity and the cross-sectional dispersion of productivity are positively correlated.*

Proof Given that $\frac{1}{2}\sigma_s^2 < \mu_s < 2\sigma_s^2$, if $\| \frac{\partial s^*}{\partial \sigma_s^2} \| < 1$, then $\frac{\partial E[s]}{\partial \sigma_s^2}$ and $\frac{\partial}{\partial \sigma_s^2}\frac{Var[s]}{E[s]} > 0$ ■

3.4 Idiosyncratic Shocks and TFP Distortions

What are the implications of idiosyncratic shocks in terms of TFP distortions? Seeking to answer this question, we first develop a model without shocks, where exit is driven by an exogenous exit rate and potential entrants draw the productivity parameter from a given distribution. Second, we assume that the model is calibrated to match an invariant industry distribution of plants and the entry rate generated in a world with idiosyncratic shocks and endogenous exits. We show that, even though the two models can match the same data, abstracting from idiosyncratic shocks has important implications.

3.4.1 A model without shocks and exogenous exit

Assume that productivity of incumbent plants is constant over time and that potential entrants draw the productivity parameter from an exogenous distribution $G(s)$. To clearly differentiate this scenario from the previous one, we use the notation $\hat{x}$ to describe the endogenous variables.

Because the operational profits of plants, $\pi(s|w,T)$, are constant over time, the value function of a incumbent plant with productivity s is given by,

$$V(s|w,T) = \frac{s\left(\frac{1-\tau}{\hat{w}^{\gamma}}\left(\gamma^{\gamma}-\gamma\right)\right)^{\frac{1}{1-\gamma}} - c_f + T_f}{\rho\lambda} \tag{3.24}$$

where λ is the exogenous exit rate at which firms exit the economy. Therefore, the free entry condition in this setting is given by

$$\int_{\hat{s}}^{\infty} V(s|w,T)dG(s)ds = c_{entry}. \tag{3.25}$$

As above, imitation implies that potential entrants draw the productivity parameter from an exogenous distribution $G(s)$ which is equal to the industry invariant distribution of plants (associated with a mass of one). Assume that the invariant distribution takes the same functional form as $f(s|s^*(w,T))$

$$dG(s) = \left(\left(1-\gamma\right)\mu/\sigma^2\right)^2 (s-\hat{s})e^{-\mu(1-\gamma)/\sigma^2(s-\hat{s})},$$

where the cutoff, $\hat{s}$, is now the entering plant's decision rule for production and assume that the entry rate is the same in both models, i.e.

$$\frac{1}{\lambda} = \frac{1}{2}\frac{\mu_s^2}{\sigma_s^2}.$$

To characterize the stationary equilibrium we need to determine the wage, $\hat{w}$, the entering plant's decision rule for production, $\hat{s}$, and the mass of plants, $\hat{M}$. Note that for a given wage (and distortions), the entry rule is such that

$$\pi(\hat{s}|w,T) = \hat{s}\left(\frac{1-\tau}{w^{\gamma}}\left(\gamma^{\gamma}-\gamma\right)\right)^{\frac{1}{1-\gamma}} + T_f = 0. \tag{3.26}$$

This condition and the free entry condition (equation 3.25) determine the wage in equilibrium and the invariant industry distribution of plants (with a mass of one).

Formally, the marginal plant in the industry and the wage are

$$\hat{s} = \frac{(\hat{s}+\lambda)\frac{1}{\lambda}}{\frac{\rho c_{entry}}{c_f - T_f} + \frac{1}{\lambda}} \tag{3.27}$$

$$\hat{w} = \left(\kappa(\tau) \left(\frac{\hat{s}}{c_f - T_f} \right)^{1-\gamma} \right)^{\frac{1}{\gamma}} \tag{3.28}$$

Finally, $\hat{M}$ follows directly from the expression of the labor market clearing condition

$$\hat{M} \int_{s^*(w,T)}^{\infty} n(s|\hat{w},T) f(s|\hat{s}) ds = 1 \tag{3.29}$$

where $n(s)$ is the labor demand of the plant with productivity s (equation 3.3).

3.4.2 TFP distortions

We are interested in the insights obtained from comparing the two models in terms of distortions. We use two different measures of productivity: i) average plant productivity, $E[s]$, and ii) measured TFP. To compute measured TFP we use labor, $N = 1$, and capital

$$K = M\delta^{-1} \left(\frac{1}{2} \frac{\mu_s^2}{\sigma_s^2} c_{entry} + c_f \right)$$

which is obtained from the value of investment in entry and operating costs with a constant depreciation rate, δ. Therefore, measured TFP is

$$\frac{M \left(\int_{s^*}^{\infty} s^{1-\gamma} n^{\gamma} f(s|s^*) ds \right)}{N^{\gamma} \left(\frac{M}{\delta} \left(\frac{1}{2} \frac{\mu_s^2}{\tilde{\sigma}^2} c_{entry} + c_f \right) \right)^{1-\gamma}} = \left(\frac{s^* + \frac{2\sigma_s^2}{\mu_s}}{\frac{1}{\delta} \left(\frac{1}{2} \frac{\mu_s^2}{\sigma_s^2} c_{entry} + c_f \right)} \right)^{1-\gamma}.$$

The following proposition shows that when idiosyncratic shocks exist at plant level, productivity per firm and TFP are lower than with constant productivity.

Proposition 3. *If there are idiosyncratic shocks, the productivity of the marginal plant is lower than if there are no such shocks. That is $s^* < \hat{s}$.*

Proof *See Appendix.*

Proposition 3 shows a well known property of option theory. Under idiosyncratic shocks, incumbent and entering plants decide to remain in the industry when the option value is non-negative. Therefore, there are plants with negative operating profits that delay their decision to exit the market. This result has a direct implication on the measures of productivity. The corollary below shows that this reduces TFP for any policy distortion.

Corollary 1. *If there are idiosyncratic shocks, TFP is lower than if there are not. That is,* $TFP < T\hat{F}P$.

Proposition 3 and Corollary 1 reveal that, for a given set of parameters, idiosyncratic shocks and endogenous exit reduce productivity per firm and aggregate TFP. However, parameters are obtained by calibrating the model to match the data.

Therefore, assume that a model is calibrated with constant plant productivity and exogenous exit to match TFP, wage and the industry productivity distribution generated by an economy with idiosyncratic shocks and endogenous exit. Note, that we require $TFP = T\hat{F}P$, $w = \hat{w}$ and $s^* = \hat{s}$. The following propositions show that in order to match the same data the fixed operating cost, $\hat{c}_f < c_f$, will be underestimated and the fixed entry cost, $\hat{c}_{entry} > c_{entry}$, will be overestimated.

Proposition 4. *Calibrated operational costs are always lower than actual operational costs, and the gap between the two,* $\mathbf{\Delta}_{,c_f-\hat{c}_f}$*, is given by*

$$\mathbf{\Delta}_{c_f-\hat{c}_f} = \left(\frac{1+\frac{\beta\mu}{\rho}}{1-\beta} \right) (c_f - T_f) > \mathbf{0},$$

which increases with the option value of the marginal plant (measured by the parameter β*)*

Proof *See Appendix.*

If there are idiosyncratic shocks and endogenous exits in the economy, an exit decision is equivalent to asking whether the plant's option value is non-negative.

Therefore, the marginal plant will actually be making negative profits waiting for better days, i.e. $\pi(s^*|w^*, T) - (c_f - T_f) < 0$. However, in a model where plant productivity is constant over time, entering plants decide to remain only when operating profits are non-negative. Therefore, the marginal plant will actually be making zero profits i.e. $\pi(\hat{s}|\hat{w}, T) - (\hat{c}_f - T_f) = 0$. To match the data, $s^* = \hat{s}$, a model without idiosyncratic shocks underestimates the fixed operating cost, i.e. $\hat{c}_f < c_f$. Proposition 4 indicates that the size of the gap between the real and the calibrated operating cost increases when the plant's option value increases. The logic is quite simple: If the option value increases, more plants remain in the economy (waiting for better days) with greater negative profits. Therefore when shocks are very persistent (a low σ^2), the gap should become smaller. Formally,

$$\frac{\partial \mathbf{\Delta}_{c_f - \hat{c}_f}}{\partial \sigma^2} = \frac{\partial \mathbf{\Delta}_{c_f - \hat{c}_f}}{\partial \beta} \frac{\partial \beta}{\partial \sigma^2} > 0.$$

The intuition behind this result is that a small standard deviation implies a lower option value for firms making negative profits that choose to exit sooner, and hence the underestimation becomes smaller.

However when ρ and/or μ increase, the option value decreases and so does the gap. Formally

$$\frac{\partial \mathbf{\Delta}_{c_f - \hat{c}_f}}{\partial \rho} = \underbrace{\frac{\partial \mathbf{\Delta}_{c_f - \hat{c}_f}}{\partial \beta} \frac{\partial \beta}{\partial \rho}}_{Indirect} + \underbrace{\frac{\partial \mathbf{\Delta}_{c_f - \hat{c}_f}}{\partial \rho}}_{Direct} < 0,$$

and

$$\frac{\partial \mathbf{\Delta}_{c_f - \hat{c}_f}}{\partial \mu} = \underbrace{\frac{\partial \mathbf{\Delta}_{c_f - \hat{c}_f}}{\partial \beta} \frac{\partial \beta}{\partial \mu}}_{Indirect} + \underbrace{\frac{\partial \mathbf{\Delta}_{c_f - \hat{c}_f}}{\partial \mu}}_{Direct} = \left(\underbrace{\frac{\frac{\mu}{\sigma^2}}{\sqrt{\left(\frac{\mu}{\sigma^2} + \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}}} - 1}_{<0} \right) \left(\frac{\frac{-\beta}{\rho}}{1-\beta} \right) (c_f - T_f) < 0.$$

Corollary 2. *The gap, $\mathbf{\Delta}$, between c_f and $\hat{c}_f$ is increasing in σ^2 and decreasing in μ and ρ.*

When the operating cost is underestimated the entry cost must also be overestimated. Why? First, because the value of entry is decreasing in the operating cost (see equation 3.25). Second, because the value of entry is higher in a model with shocks (again the option value argument). As a result, in order to keep the same entry level, the entry cost must be higher. Hence, in order to fit the same observed entry level, the calibrated entry cost must be larger than the actual one.

Proposition 5. *Calibrated fixed entry costs are higher than actual costs, and the difference is given by*

$$\Delta_{\hat{c}_{entry}-c_{entry}} = \frac{1}{\rho}\left(\mu c_{entry} + \left(\frac{c_f - T_f}{1-\beta}\frac{\rho+\mu}{\rho\lambda}\right)\left(1 - \int_{s^*(w,T)}^{\infty}\left(\frac{s}{s^*(w,T)}\right)^{\beta} f(s|s^*(w,T))ds\right)\right) > 0.$$

Proof *See Appendix.*

Note that when calibrating lower operating costs and higher fixed costs, the model without idiosyncratic shocks overestimates the value of the plant (see equation 3.24). Equation (3.30) shows that the ratio of TFP distortions is proportional to the ratio of plant values in each model, i.e.

$$\frac{\frac{\partial s^*}{\partial T_f}}{\frac{\partial \hat{s}}{\partial T_f}} = \underbrace{\frac{\frac{c_{entry}}{c_f - T_f}}{\frac{\hat{c}_{entry}}{\hat{c}_f - T_f}}}_{\text{Calibration}} \underbrace{\frac{\rho+\mu}{\rho}\frac{1}{(1-\Lambda)}}_{\text{Dynamics}}, \tag{3.30}$$

where

$$\Lambda = \frac{\mu_s \int_{s^*(w,T)}^{\infty}\left(\left(\frac{s}{s^*(w,T)}\right)^{\beta}\left(\frac{1}{s^*(w,T)} + \frac{1}{-\beta}\left(\frac{\mu_s}{\sigma_s^2} - \frac{1}{s-s^*}\right)\right)\right)\frac{1}{2}\frac{\mu_s^2}{\sigma_s^2} f(s|s^*(w,T))ds}{\left(\frac{\rho\hat{c}_{entry}}{\hat{c}_f - T_f}.\right)^2}.$$

The following proposition shows that the effect of policy distortions on TFP will be overestimated in an economy without shocks.

Proposition 6. *Calibrated models without idiosyncratic shocks and endogenous exit overestimate the effect of policy distortions on TFP, i.e.* $\frac{\partial s^*}{\partial T_f}/\frac{\partial \hat{s}}{\partial T_f} < 1$.

Proof *See Appendix*

Therefore, assuming that incumbent plants always have non-negative profits leads to overestimating plant values. As a result, the relative size of distortions will be overestimated. Hence a calibrated model without idiosyncratic shocks and endogenous exits overestimates the effect of policy distortions on TFP, if the data reflect shocks and endogenous entry decisions.

3.5 Conclusion

We analyze plant-level policy distortions in a tractable stochastic model with analytical solutions. We compare TFP in our model to TFP without idiosyncratic shocks and endogenous exit in two different ways.

First, we demonstrate that models that do not account for firm dynamics always have higher levels of TFP than models with idiosyncratic shocks and endogenous exit. This result is due to the exit option that very inefficient plants have in the stochastic case. This choice variable allows plants to remain in the market longer, becoming more unproductive and therefore decreasing the level of TFP.

Second, we demonstrate that if a model is calibrated without idiosyncratic shocks and endogenous exit to fit data generated by a model with shocks and endogenous entry, the effect of policy distortions on TFP will always be overestimated.

We assume that resources are reallocated through a policy configuration applied at plant level that subsidizes low-productivity plants and taxes high-productivity plants. However, it is well known that idiosyncratic policy distortions affect TFP for all policy configurations since the policy moves plants away from their optimal size.

It is possible to show that similar results can be obtained for more general policy configurations. For example, assume that at the time of entry the tax rate is a lottery that once revealed remains fixed for so long as the plant is in operation. If the future option value of the marginal plants that exit the economy is underestimated, low fixed operating costs are inferred in order to justify the existence of the observed exit rate.

Therefore, our results can be extended to any policy configuration that results from lotteries. For example, in Buera *et al.* (2011) policy initially reallocates capital from unproductive plants towards productive ones. They assume that policies have inertia and are hard to adjust. In that case, over time, as the productivity levels of subsidized plants revert to the mean, subsidized plants are not necessarily the most productive. They show that in the long run this policy configuration is equivalent to a lottery: idiosyncratic taxes and subsidies are uncorrelated with plant productivity levels.

Appendix

Proof of Lemma 1

We follow Dixit and Pindyck (1994) in solving this exit-delay problem. We know that the firm chooses to stay in the market as long as the firm has a positive option value. Hence,

$$\begin{aligned} W(s|w,T) &= \max_{\{stay,exit\}} \{\pi(s|w,T) - c_f + EW(s+ds|w,T), 0\} \\ st &: \; ds^{1-\gamma} = -\mu s^{1-\gamma} dt + \sigma s^{1-\gamma} dz, \end{aligned}$$

Using Ito's calculus, $W(s|w,T)$ satisfies

$$\rho W(s|w,T) = \pi(s|w,T) - c_f - \mu W'(s|w,T)s + \frac{1}{2}\sigma^2 s^2 W''(s|w,T).$$

The value function that solves the above expression is given by

$$W(s|w,T) = Bs^{\beta} + \frac{s}{\rho+\mu}\left(\frac{\kappa(\tau)}{w^{\gamma}}\right)^{\frac{1}{1-\gamma}} - \frac{c_f - T_f}{\rho}$$

where, $\kappa(\tau) = (1-\tau)(\gamma^{\gamma} - \gamma)$ and $\beta = \frac{1}{2} + \frac{\mu}{\sigma^2} - \sqrt{\left(\frac{\mu}{\sigma^2} + \frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}} < 0$. Using the smooth-pasting and value matching conditions for s^*, which ensure continuity and differentiability

$$\begin{aligned} W(s^*|w,T) &= 0 \\ W'(s^*|w,T) &= 0 \end{aligned}$$

gives

$$s^*(s|w,T) = \left(\frac{w^{\gamma}}{\kappa(\tau)}\right)^{\frac{1}{1-\gamma}} \frac{-\beta}{1-\beta}(c_f - T_f)\frac{\rho+\mu}{\rho}$$

and

$$W(s|w,T) = \frac{(c_f - T_f)}{\rho}\left(\left(\frac{s}{s^*(w,T)}\right)^{\beta}\frac{1}{1-\beta} - 1\right) + s\left(\frac{\kappa(\tau)}{w^{\gamma}}\right)^{\frac{1}{1-\gamma}}\frac{1}{\rho+\mu}.$$

Proof of Proposition 1

The aim is to show that this cutoff is unique.

$$\frac{s^*}{\left(s^*+\frac{2\sigma_s^2}{\tilde{\mu}}\right)\frac{1}{2}\frac{\mu_s^2}{\sigma_s^2}\left(\frac{-\beta}{1-\beta}\right)}=\frac{1}{\frac{\rho c_{entry}}{c_f-T_f}-\int_{s^*(w,T)}^{\infty}\left(\left(\frac{s}{s^*(w,T)}\right)^{\beta}\frac{1}{1-\beta}-1\right)\frac{1}{2}\frac{\mu_s^2}{\sigma_s^2}f(s|s^*(w,T))ds} \tag{3.31}$$

The left hand side of this expression is 0 when s^* is 0 and it approaches 1 as $s^* \to \infty$. The same limits for the right hand side are

$$\lim_{s^*\to 0}\frac{1}{\frac{\rho c_{entry}}{c_f-T_f}-\int_{s^*(w,T)}^{\infty}\left(\left(\frac{s}{s^*(w,T)}\right)^{\beta}\frac{1}{1-\beta}-1\right)\frac{1}{2}\frac{\mu_s^2}{\sigma_s^2}f(s|s^*(w,T))ds}\to\frac{1}{\frac{\rho c_{entry}}{c_f-T_f}+\frac{1}{2}\frac{\mu_s^2}{\sigma_s^2}}>0.$$

and

$$\lim_{s^*\to \infty}\frac{1}{\frac{\rho c_{entry}}{c_f-T_f}-\int_{s^*(w,T)}^{\infty}\left(\left(\frac{s}{s^*(w,T)}\right)^{\beta}\frac{1}{1-\beta}-1\right)\frac{1}{2}\frac{\mu_s^2}{\sigma_s^2}f(s|s^*(w,T))ds}\to\frac{1}{\frac{\rho c_{entry}}{c_f-T_f}+\frac{1}{2}\frac{\mu_s^2}{\sigma_s^2}}<1.$$

The fact that the left hand side of expression (3.31) is increasing in s^* is trivial. The right hand side moves according to the term $\int_{s^*(w,T)}^{\infty}\left(\left(\frac{s}{s^*(w,T)}\right)^{\beta}\frac{1}{1-\beta}-1\right)\frac{1}{2}\frac{\mu_s^2}{\sigma_s^2}f(s|s^*(w,T))ds$. If this term increases, the whole right hand side increases as well, and viceversa. In order to proceed with the analysis, we multiply the expression by $1-\beta>0$ and take the derivative with respect to s^*

$$\begin{aligned}&\frac{d}{ds^*}\int_{s^*(w,T)}^{\infty}\left(\left(\frac{s}{s^*}\right)^{\beta}-1+\beta\right)f(s|s^*(w,T))ds\\ =&\int_{s^*(w,T)}^{\infty}\left(\frac{-\beta}{s^*(w,T)}\left(\frac{s}{s^*}\right)^{\beta}+\left(\left(\frac{s}{s^*(w,T)}\right)^{\beta}+1-\beta\right)\left(\frac{-1}{s-s^*}+\frac{\mu_s}{\tilde{\sigma}^2}\right)\right)f(s|s^*(w,T))ds\end{aligned}$$

It is clear that the first term dominates the expression for s^* when it is very close to 0, which makes the sign positive. Since the value has to return to the initial point because the two limits are equal, and the derivative is monotone in s^*, the second term has to be negative. Since the derivative is first increasing and then decreasing, and this path is monotone, the two expressions only intersect once, and hence uniqueness is proved. ■

Derivatives of s^* respect to ρ, μ, σ^2, c_f and c_{entry}

For this section and the following one we assume that the entry cost is large enough,

$$\frac{\rho c_{entry}}{c_f - T_f} > \frac{-\beta\mu_s^2}{2\tilde{\sigma}^2}\left(\int_{s^*(w,T)}^{\infty}\left(\frac{s}{s^*(w,T)}\right)^{\beta}\log\left(\frac{s}{s^*}\right)f(s|s^*)ds + \frac{1}{1-\beta}\right)$$

and

$$\frac{\rho c_{entry}}{c_f - T_f} > \frac{(1-\gamma)\,\mu_s^3}{(1-\beta)\,2s^*}\left(\frac{Es}{\sigma^2(1-\beta)} - \beta(1-\gamma)^{\frac{1}{2}}\right)$$

In the steady state, s^* satisfies

$$\begin{aligned}
H(s^*,\mu,\sigma^2,\rho) &= \frac{s^*}{\left(s^* + \frac{2\sigma_s^2}{\mu_s}\right)\frac{1}{2}\frac{\mu_s^2}{\sigma_s^2}\left(\frac{-\beta}{1-\beta}\right)} - \frac{1}{\frac{\rho c_{entry}}{c_f - T_f} + \frac{1}{2}\frac{\mu_s^2}{\sigma_s^2} - \int_{s^*}^{\infty}\left(\left(\frac{s}{s^*}\right)^{\beta}\frac{1}{1-\beta}\right)g(s|s^*)ds} \\
&= \phi\left[\frac{s^*}{Es\left(\frac{-\beta}{1-\beta}\right)} - \frac{1}{\phi\frac{\rho c_{entry}}{c_f - T_f} + 1 - \int_{s^*}^{\infty}\left(\left(\frac{s}{s^*}\right)^{\beta}\frac{1}{1-\beta}\right)f(\xi)ds}\right] = 0
\end{aligned}$$

where $\phi = \frac{2\sigma_s^2}{\mu_s^2}$, $Es = s^* + \frac{2\sigma_s^2}{\mu_s}$ and $\xi = \frac{\mu_s}{\sigma_s^2}$. From proposition 1, it is known that $\frac{\partial H}{\partial s^*} > 0$, because when the two terms intersect the derivative of the first term is necessarily larger than that of the second term. The aim is to calculate

$$\begin{aligned}
\frac{dH}{d\sigma^2} &= \frac{\partial H}{\partial \beta}\frac{\partial \beta}{\partial \sigma^2} + \frac{\partial H}{\partial Es}\frac{\partial Es}{\partial \sigma^2} + \frac{\partial H}{\partial \phi}\frac{\partial \phi}{\partial \sigma^2} + \frac{\partial H}{\partial \xi}\frac{\partial \xi}{\partial \sigma^2} \\
\frac{dH}{d\mu} &= \frac{\partial H}{\partial \beta}\frac{\partial \beta}{\partial \mu} + \frac{\partial H}{\partial Es}\frac{\partial Es}{\partial \mu} + \frac{\partial H}{\partial \phi}\frac{\partial \phi}{\partial \mu} + \frac{\partial H}{\partial \xi}\frac{\partial \xi}{\partial \mu} \\
\frac{dH}{d\rho} &= \frac{\partial H}{\partial \beta}\frac{\partial \beta}{\partial \rho} - \frac{\phi^2\frac{c_{entry}}{c_f - T_f}}{\left(\phi\frac{\rho c_{entry}}{c_f - T_f} + 1 - \int_{s^*}^{\infty}\left(\left(\frac{s}{s^*}\right)^{\beta}\frac{1}{1-\beta}\right)f(\xi)ds\right)^2}
\end{aligned}$$

Therefore, we compute

$$
\begin{aligned}
\frac{\partial H}{\partial \beta} &= \frac{\phi s^*}{Es\left(\beta\right)^2}\left(\frac{1-\int_{s^*}^{\infty}\left(\frac{s}{s^*}\right)^{\beta} f(s|s^*)ds + \frac{2\sigma_s^2}{s^*\mu_s} - \int_{s^*}^{\infty}\left(\frac{s}{s^*}\right)^{\beta}\log\left(\frac{s}{s^*}\right)\left(1-\beta\right)f(s|s^*)ds}{1+\frac{2\sigma_s^2}{\mu_s s^*}}\right) \\
\frac{\partial H}{\partial Es} &= -\frac{s^*}{Es^2\left(\frac{-\beta}{1-\beta}\right)} \\
\frac{\partial H}{\partial \phi} &= \frac{s^{*2}}{Es^2\left(\frac{-\beta}{1-\beta}\right)^2}\frac{\rho c_{entry}}{c_f - T_f} \\
\frac{\partial H}{\partial \xi} &= -\frac{s^{*2}}{Es^2\left(\frac{-\beta}{1-\beta}\right)^2}\frac{d}{d\xi}\left(\int_{s^*}^{\infty}\left(\frac{s}{s^*}\right)^{\beta}\frac{1}{1-\beta}f(\xi)ds\right)
\end{aligned}
$$

and the partial derivatives of β, which are

$$
\begin{aligned}
\frac{\partial \beta}{\partial \rho} &= -\frac{1}{\sigma^2\sqrt{\left(\frac{\mu}{\sigma^2}+\frac{1}{2}\right)^2+\frac{2\rho}{\sigma^2}}} < 0 \\
\frac{\partial \beta}{\partial \mu} &= \frac{1}{\sigma^2}\left(1-\frac{\left(\frac{\mu}{\sigma^2}+\frac{1}{2}\right)}{\sqrt{\left(\frac{\mu}{\sigma^2}+\frac{1}{2}\right)^2+\frac{2\rho}{\sigma^2}}}\right) > 0 \\
\frac{\partial \beta}{\partial \sigma^2} &= \frac{\mu}{\left(\sigma^2\right)^2}\left(\frac{\sqrt{\left(\frac{\mu}{\sigma^2}+\frac{1}{2}\right)^2+\frac{2\rho}{\sigma^2}+\frac{\rho}{\mu}+\left(\frac{\rho}{\mu}\right)^2}}{\sqrt{\left(\frac{\mu}{\sigma^2}+\frac{1}{2}\right)^2+\frac{2\rho}{\sigma^2}}}-1\right) > 0
\end{aligned}
$$

and

$$
\begin{aligned}
\mu_s &= \frac{\mu}{1-\gamma} - \frac{1}{2}\gamma\frac{\sigma^2}{\left(1-\gamma\right)^2} \\
\sigma_s &= \frac{\sigma}{1-\gamma}
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial Es}{\partial \sigma^2} &= \frac{2\mu}{\left(\sqrt{1-\gamma}\mu - \frac{1}{2}\frac{\gamma}{\sqrt{1-\gamma}}\sigma^2\right)^2} \\
\frac{\partial \phi}{\partial \sigma^2} &= \frac{2\mu + \frac{\gamma}{1-\gamma}\sigma^2}{\left(\sqrt{1-\gamma}\mu - \frac{1}{2}\frac{\gamma}{\sqrt{1-\gamma}}\sigma^2\right)^2} \\
\frac{\partial \xi}{\partial \sigma^2} &= -\frac{\mu}{\left(\sigma^2\right)^2}\left(1-\gamma\right) \\
\frac{\partial Es}{\partial \mu} &= \frac{-2\sigma^2}{\left(\sqrt{1-\gamma}\mu - \frac{1}{2}\frac{\gamma}{\sqrt{1-\gamma}}\sigma^2\right)^2} \\
\frac{\partial \phi}{\partial \mu} &= \frac{-4\sigma^2}{\left(\mu - \frac{1}{2}\gamma\frac{\sigma^2}{(1-\gamma)}\right)^3} \\
\frac{\partial \xi}{\partial \mu} &= \frac{1-\gamma}{\sigma^2}
\end{aligned}
$$

$$
\begin{aligned}
\frac{dH}{d\sigma^2} = {} & \frac{\phi s^*}{Es\left(-\beta\right)}\left(\frac{1-\int_{s^*}^{\infty}\left(\frac{s}{s^*}\right)^{\beta} f(s|s^*)ds + \frac{2\sigma_s^2}{s^*\mu_s} - \int_{s^*}^{\infty}\left(\frac{s}{s^*}\right)^{\beta}\log\left(\frac{s}{s^*}\right)\left(1-\beta\right)f(s|s^*)ds}{1+\frac{2\sigma_s^2}{\mu_s s^*}}\right)\times \\
& \frac{\mu}{\left(\sigma^2\right)^2}\left(\frac{\sqrt{\left(\frac{\mu}{\sigma^2}+\frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2} + \frac{\rho}{\mu} + \left(\frac{\rho}{\mu}\right)^2}}{\sqrt{\left(\frac{\mu}{\sigma^2}+\frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}}} - 1\right) \\
& \underbrace{-\frac{1}{Es\left(\frac{-\beta}{1-\beta}\right)}\frac{2\mu}{\left(\sqrt{1-\gamma}\mu - \frac{1}{2}\frac{\gamma}{\sqrt{1-\gamma}}\sigma^2\right)^2} + \frac{s^{*\frac{1-\beta}{-\beta}}}{Es\left(\frac{-\beta}{1-\beta}\right)}\frac{\rho c_{entry}}{c_f - T_f}\frac{2\mu + \frac{\gamma}{1-\gamma}\sigma^2}{\left(\sqrt{1-\gamma}\mu - \frac{1}{2}\frac{\gamma}{\sqrt{1-\gamma}}\sigma^2\right)^2}}_{>0} \\
& + \frac{s^*}{Es\left(\frac{-\beta}{1-\beta}\right)^2}\frac{d}{d\xi}\left(\int_{s^*}^{\infty}\left(\frac{s}{s^*}\right)^{\beta}\frac{1}{1-\beta}f(\xi)ds\right)\frac{\mu}{\left(\sigma^2\right)^2}\left(1-\gamma\right)
\end{aligned}
$$

$$
\begin{aligned}
\frac{dH}{d\mu} &= \frac{\phi s^*}{Es\left(-\beta\right)}\left(\frac{1-\int_{s^*}^{\infty}\left(\frac{s}{s^*}\right)^{\beta}f(s|s^*)ds+\frac{2\sigma_s^2}{s^*\mu_s}-\int_{s^*}^{\infty}\left(\frac{s}{s^*}\right)^{\beta}\log\left(\frac{s}{s^*}\right)\left(1-\beta\right)f(s|s^*)ds}{1+\frac{2\sigma_s^2}{\mu_s s^*}}\right)\times \\
&\quad \frac{1}{\sigma^2}\left(1-\frac{\left(\frac{\mu}{\sigma^2}+\frac{1}{2}\right)}{\sqrt{\left(\frac{\mu}{\sigma^2}+\frac{1}{2}\right)^2+\frac{2\rho}{\sigma^2}}}\right) \\
&\quad +\frac{s^*}{Es^2\left(\frac{-\beta}{1-\beta}\right)}\frac{2\sigma^2}{\left(\sqrt{1-\gamma}\mu-\frac{1}{2}\frac{\gamma}{\sqrt{1-\gamma}}\sigma^2\right)^2}-\frac{s^{*2}}{Es^2\left(\frac{-\beta}{1-\beta}\right)^2}\frac{\rho c_{entry}}{c_f-T_f}\frac{4\sigma^2}{\left(\mu-\frac{1}{2}\gamma\frac{\sigma^2}{(1-\gamma)}\right)^3} \\
&\quad -\frac{s^{*2}}{Es^2\left(\frac{-\beta}{1-\beta}\right)^2}\frac{d}{d\xi}\left(\int_{s^*}^{\infty}\left(\frac{s}{s^*}\right)^{\beta}\frac{1}{1-\beta}f(\xi)ds\right)\frac{1-\gamma}{\sigma^2}
\end{aligned}
$$

$$
\begin{aligned}
\frac{dH}{d\rho} &= \frac{\phi s^*}{Es\left(-\beta\right)}\left(\frac{1-\int_{s^*}^{\infty}\left(\frac{s}{s^*}\right)^{\beta}f(s|s^*)ds+\frac{2\sigma_s^2}{s^*\mu_s}-\int_{s^*}^{\infty}\left(\frac{s}{s^*}\right)^{\beta}\log\left(\frac{s}{s^*}\right)\left(1-\beta\right)f(s|s^*)ds}{1+\frac{2\sigma_s^2}{\mu_s s^*}}\right)\times \\
&\quad \frac{-1}{\sigma^2\sqrt{\left(\frac{\mu}{\sigma^2}+\frac{1}{2}\right)^2+\frac{2\rho}{\sigma^2}}}-\frac{\phi^2\frac{c_{entry}}{c_f-T_f}}{\left(\phi\frac{\rho c_{entry}}{c_f-T_f}+1-\int_{s^*}^{\infty}\left(\left(\frac{s}{s^*}\right)^{\beta}\frac{1}{1-\beta}\right)f(\xi)ds\right)^2}
\end{aligned}
$$

First, note that except for the term that includes $\frac{d}{d\xi}\left(\int_{s^*}^{\infty}\left(\frac{s}{s^*}\right)^{\beta}\frac{1}{1-\beta}f(\xi)ds\right)$, $\frac{dH}{d\sigma^2}$ is positive and $\frac{dH}{d\rho}$ and $\frac{dH}{d\mu}$are negative.[2]. Hence, if $\frac{d}{d\xi}\left(\int_{s^*}^{\infty}\left(\frac{s}{s^*}\right)^{\beta}\frac{1}{1-\beta}f(\xi)ds\right)>0$, $\frac{dH}{d\sigma^2}>0$, $\frac{dH}{d\rho}<0$, and $\frac{dH}{d\mu}<0$.

It can be proved that if $(\beta+1)<\xi/2$ then $\Phi=\frac{d}{d\xi}\left(\int_{s^*}^{\infty}\left(\frac{s}{s^*}\right)^{\beta}\frac{1}{1-\beta}f(\xi)ds\right)$ is positive. Applying the Laplace transform and the recurrence relationship $\Gamma(\beta,s^*)=(\beta-1)\Gamma(\beta-1,s^*)+{s^*}^{\beta-1}e^{-s^*}$ the following can be written:

$$
\begin{aligned}
\Phi &= \frac{\xi^2 e^{\xi s^*}}{(1-\beta)\,{s^*}^2}\int_{s^*}^{\infty}\left[-x^{(\beta+2)}+\left(\frac{2}{\xi}+2s^*\right)x^{\beta+1}-s^*(1+s^*)x^{\beta}\right]e^{-\xi x}dx> \\
&\quad \frac{\Gamma(\beta+3,s^*)}{\xi^{(\beta+3)}}+\left(\frac{2}{\xi}+2s^*\right)\frac{\Gamma(\beta+2,s^*)}{\xi^{(\beta+2)}}-s^*(1+s^*)\frac{\Gamma(\beta+1,s^*)}{\xi^{(\beta+1)}} \\
&= \frac{\Gamma(\beta+1,s^*)}{\xi^{(\beta+2)}}\left\{(\beta+1)\left[2\left(\frac{1}{\xi}+s^*\right)-\frac{(\beta+2)}{\xi}\right]-s^*(1+s^*)\xi\right\}
\end{aligned}
$$

where $\Gamma(a,s^*)=\int_{s^*}^{\infty}x^{a-1}e^{-ax}dx$ is the upper incomplete gamma function. Therefore

[2] The first condition in c_{entry} ensures that this last statement is true for the first term, and the second condition does likewise for the case of $\frac{dH}{d\mu}$.

Φ is positive if $(\beta+1)(-\beta) \geq s^*\xi[\xi - 2(\beta+1) - s^*$, which is true if $(1+\beta) < \xi/2$.

Finally,

$$\frac{dH}{dc_{entry}} = \frac{s^{*2}}{Es^2}\frac{(1-\beta)^2}{\beta^2}\left(\frac{\rho\phi}{c_f - T_f}\right)$$

$$\frac{dH}{dc_f} = -\frac{dH}{dc_{entry}}\left(\frac{c_{entry}}{c_f - T_f}\right)$$

Correlation between wages and productivity

Partial derivatives with respect to ρ, μ and σ^2 are:

$$\begin{aligned}
\frac{\partial w}{\partial \rho} &= \frac{\partial w}{\partial \beta}\frac{\partial \beta}{\partial \rho} + \frac{\partial w}{\partial s^*}\frac{\partial s^*}{\partial \rho} + \frac{\partial w}{\partial \rho} \\
&= \underbrace{\frac{1-\gamma}{\gamma}\frac{w}{-\beta\,(1-\beta)}\frac{-1}{\sigma^2\sqrt{\left(\frac{\mu}{\sigma^2}+\frac{1}{2}\right)^2+\frac{2\rho}{\sigma^2}}}}_{<0} \\
&\quad + \underbrace{\frac{\partial s^*}{\partial \rho}\frac{1-\gamma}{\gamma}\frac{w}{s^*}}_{<0} \\
&\quad + \underbrace{\frac{1-\gamma}{\gamma}w\frac{\mu}{(\rho+\mu)\,\rho}}_{>0} \\
&= \frac{1-\gamma}{\gamma}\frac{w}{\rho}\left[\frac{\rho}{\beta\,(1-\beta)}\frac{1}{\sigma^2\sqrt{\left(\frac{\mu}{\sigma^2}+\frac{1}{2}\right)^2+\frac{2\rho}{\sigma^2}}} + \frac{\partial s^*}{\partial \rho}\frac{\rho}{s^*} + \frac{\mu}{\rho+\mu}\right]
\end{aligned}$$

$$
\begin{aligned}
\frac{\partial w}{\partial \mu} &= \frac{\partial w}{\partial \beta}\frac{\partial \beta}{\partial \mu} + \frac{\partial w}{\partial s^*}\frac{\partial s^*}{\partial \mu} + \frac{\partial w}{\partial \mu} \\
&= \underbrace{\frac{1-\gamma}{\gamma}\frac{w}{-\beta(1-\beta)}\frac{1}{\sigma^2}\left(1 - \frac{\left(\frac{\mu}{\sigma^2}+\frac{1}{2}\right)}{\sqrt{\left(\frac{\mu}{\sigma^2}+\frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}}}\right)}_{>0} \\
&\quad \underbrace{+\frac{\partial s^*}{\partial \mu}\frac{1-\gamma}{\gamma}\frac{w}{s^*}}_{>0} \\
&\quad \underbrace{-\frac{1-\gamma}{\gamma}w\frac{1}{\rho+\mu}}_{<0} \\
&= \frac{1-\gamma}{\gamma}\frac{w}{\mu}\left[\frac{\mu}{-\beta(1-\beta)}\frac{1}{\sigma^2}\left(1 - \frac{\left(\frac{\mu}{\sigma^2}+\frac{1}{2}\right)}{\sqrt{\left(\frac{\mu}{\sigma^2}+\frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}}}\right) + \frac{\partial s^*}{\partial \mu}\frac{\mu}{s^*} - \frac{\mu}{\rho+\mu}\right]
\end{aligned}
$$

$$
\begin{aligned}
\frac{\partial w}{\partial \sigma^2} &= \frac{\partial w}{\partial \beta}\frac{\partial \beta}{\partial \sigma^2} + \frac{\partial w}{\partial s^*}\frac{\partial s^*}{\partial \sigma^2} + \underbrace{\frac{\partial w}{\partial \sigma^2}}_{=0} \\
&= \underbrace{\frac{1-\gamma}{\gamma}\frac{w}{-\beta(1-\beta)}\frac{\mu}{(\sigma^2)^2}\left(\frac{\sqrt{\left(\frac{\mu}{\sigma^2}+\frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2} + \frac{\rho}{\mu} + \left(\frac{\rho}{\mu}\right)^2} - \sqrt{\left(\frac{\mu}{\sigma^2}+\frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}}}{\sqrt{\left(\frac{\mu}{\sigma^2}+\frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}}}\right)}_{>0} \\
&\quad \underbrace{+\frac{\partial s^*}{\partial \sigma^2}\frac{1-\gamma}{\gamma}\frac{w}{s^*}}_{<0} \\
&= \frac{1-\gamma}{\gamma}\frac{w}{\sigma^2}\left[\frac{\mu}{-\beta(1-\beta)}\frac{1}{\sigma^2}\left(\frac{\sqrt{\left(\frac{\mu}{\sigma^2}+\frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2} + \frac{\rho}{\mu} + \left(\frac{\rho}{\mu}\right)^2}}{\sqrt{\left(\frac{\mu}{\sigma^2}+\frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}}} - 1\right) + \frac{\partial s^*}{\partial \sigma^2}\frac{\sigma^2}{s^*}\right]
\end{aligned}
$$

Therefore, if the elasticities of w with respect to μ and ρ are higher than $\frac{\mu}{\mu+\rho}$, and the elasticity of w with respect to σ^2 is higher than

$$
\frac{2}{(-\beta)(1-\beta)}\left(\frac{\sqrt{\left(\frac{\mu}{\sigma^2}+\frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2} + \frac{\rho}{\mu} + \left(\frac{\rho}{\mu}\right)^2}}{\sqrt{\left(\frac{\mu}{\sigma^2}+\frac{1}{2}\right)^2 + \frac{2\rho}{\sigma^2}}} - 1\right)
$$

wages and productivity are positively correlated.

Proof of Proposition 3

Equations (3.19) and (3.27) are rewritten so as to have the same term $\frac{s}{\left(s+\frac{2\sigma_s^2}{\mu}\right)\frac{1}{2}\frac{\mu_s^2}{\sigma_s^2}}$ on the one side, and the remaining parts are compared. It is clear that when a term is written in the following manner $\frac{x}{(x+K_1)K_2}$, it is increasing in x for K_1, K_2 positive. Recall that $\frac{1}{2}\frac{\mu_s^2}{\sigma_s^2} = \frac{1}{\lambda}$. Hence,

$$
\begin{aligned}
\frac{s^*}{\left(s^* + \frac{2\sigma_s^2}{\tilde{\mu}}\right)\frac{1}{2}\frac{\mu_s^2}{\sigma_s^2}} &= \frac{\left(\frac{-\beta}{1-\beta}\right)}{\frac{\rho c_{entry}}{c_f - T_f} - \int_{s^*}^{\infty}\left(\left(\frac{s}{s^*}\right)^{\beta}\frac{1}{1-\beta} - 1\right)\frac{1}{2}\frac{\mu_s^2}{\sigma_s^2} f(s|s^*)ds} \\
&= \frac{1}{\underbrace{\frac{\frac{\rho c_{entry}}{c_f - T_f}}{\frac{-\beta}{1-\beta}}}_{> \frac{\rho c_{entry}}{c_f - T_f}} - \int_{s^*}^{\infty}\left(\left(\frac{s}{s^*}\right)^{\beta}\frac{1}{-\beta} - \frac{1}{-\beta} - 1\right)\frac{1}{2}\frac{\mu_s^2}{\sigma_s^2} f(s|s^*)ds} \\
&= \frac{1}{\underbrace{\frac{\frac{\rho c_{entry}}{c_f - T_f}}{\frac{-\beta}{1-\beta}}}_{> \frac{\rho c_{entry}}{c_f - T_f}} + \underbrace{\frac{\frac{1}{\lambda}}{\frac{-\beta}{1-\beta}}}_{> \frac{1}{\lambda}} + \int_{s^*}^{\infty}\left(\frac{\underbrace{1 - \left(\frac{s}{s^*}\right)^{\beta}}_{>0}}{-\beta}\right)\frac{1}{2}\frac{\mu_s^2}{\sigma_s^2} f(s|s^*)ds} \\
&< \frac{1}{\frac{\rho c_{entry}}{c_f - T_f} + \frac{1}{\lambda}} = \frac{\hat{s}}{\left(\hat{s} + \frac{2\sigma_s^2}{\mu_s}\right)\frac{1}{\lambda}}
\end{aligned}
$$

The inequality holds because the denominator of the first expression is larger than that of the second expression ■

Proof of Proposition 4, 5 and 6

This appendix shows that the model without shocks and endogenous exits underestimates the fixed cost of production (Proposition 4) and overestimates the fixed entry cost (Proposition 5). Second, it shows that the distortion is always overestimated (Proposition 6).

The main variables are:

$$
\begin{aligned}
s^* &= \frac{\left(s^* + \frac{2\sigma_s^2}{\mu_s}\right)\frac{1}{2}\frac{\mu_s^2}{\sigma_s^2}\left(\frac{-\beta}{1-\beta}\right)}{\frac{\rho c_{entry}}{c_f - T_f} - \int_{s^*}^{\infty}\left(\left(\frac{s}{s^*}\right)^{\beta}\frac{1}{1-\beta} - 1\right)\frac{1}{2}\frac{\mu_s^2}{\sigma_s^2} f(s|s^*)ds} \\
\hat{s} &= \frac{\left(\hat{s} + \frac{2\sigma_s^2}{\mu_s}\right)\frac{1}{\lambda}}{\frac{\rho \hat{c}_{entry}}{\hat{c}_f - T_f} + \frac{1}{\lambda}} \\
w &= \left(\left(s^* \frac{1-\beta}{-\beta}\frac{\rho}{\rho+\mu}\frac{1}{c_f - T_f}\right)^{1-\gamma}\kappa(\tau)\right)^{\frac{1}{\gamma}} \\
\hat{w} &= \left(\kappa(\tau)\left(\frac{\hat{s}}{\hat{c}_f - T_f}\right)^{1-\gamma}\right)^{\frac{1}{\gamma}}
\end{aligned}
$$

When the parameters are calibrated TFP, the cutoff and wage to be observed (in our case, the actual ones), are targeted. This leads to the following expressions

$$
\begin{aligned}
\frac{1}{\delta}\left(\frac{1}{2}\frac{\mu_s^2}{\sigma_s^2}c_{entry} + c_f\right) &= \frac{1}{\hat{\delta}}\left(\frac{1}{2}\frac{\mu_s^2}{\sigma_s^2}\hat{c}_{entry} + \hat{c}_f\right) \\
\frac{\rho \hat{c}_{entry}}{\hat{c}_f - T_f} - \frac{\rho c_{entry}}{c_f - T_f} &= \left(\frac{1}{-\beta}\right)\left(\frac{\rho c_{entry}}{c_f - T_f} - \int_{s^*}^{\infty}\left(\frac{s}{s^*}\right)^{\beta}\frac{1}{2}\frac{\mu_s^2}{\sigma_s^2} f(s|s^*)ds + \frac{1}{\lambda}\right) \qquad (B.32) \\
\frac{c_f - T_f}{\hat{c}_f - T_f} &= \frac{1-\beta}{-\beta}\frac{\rho}{\rho+\mu}
\end{aligned}
$$

From the calibration it results that $c_f > \hat{c}_f$ (when calibrating, the fixed cost of producing is underestimated) and that $\hat{c}_{entry} > c_{entry}$ (the fixed entry cost is overestimated). The first result is proven in the following Lemma.

Lemma A.1. *The calibrated fixed cost is below the actual fixed cost*

Proof Assume that this is not the case. Then

$$\begin{aligned}
\frac{1-\beta}{-\beta}\frac{\rho}{\rho+\mu} &< 1 \\
(1-\beta)\,\rho &< -\beta\,(\rho+\mu) \\
\frac{\rho}{\mu}+\frac{\mu}{\sigma^2}+\frac{1}{2} &< \sqrt{\left(\frac{\mu}{\sigma^2}+\frac{1}{2}\right)^2+\frac{2\rho}{\sigma^2}} \\
\left(\frac{\rho}{\mu}\right)^2+2\frac{\rho}{\sigma^2}+\frac{\rho}{\mu} &< \frac{2\rho}{\sigma^2}
\end{aligned}$$

which cannot hold because $\frac{\rho}{\mu}$ is positive. ■

Using the last equation that relates the two operational fixed costs, the gap, $\mathbf{\Delta}$, between the two costs is found to be

$$\mathbf{\Delta}_{\hat{c}_f-c_f}=\left(\frac{-1-\frac{\beta\mu}{\rho}}{1-\beta}\right)(c_f-T_f)<0$$

To determine that the calibrated fixed entry cost is higher than the actual cost, equation (3.32) is used and the actual cost is subtracted from the calibrated one

$$\mathbf{\Delta}_{\hat{c}_{entry}-c_{entry}}=\frac{1}{\rho}\left(\mu c_{entry}+\left(\frac{c_f-T_f}{1-\beta}\frac{\rho+\mu}{\rho\lambda}\right)\left(1-\int_{s^*}^{\infty}\left(\frac{s}{s^*}\right)^{\beta}f(s|s^*)ds\right)\right)>0$$

Rewriting the cutoffs

$$\begin{aligned}
s^* &= \frac{\mu_s\left(\frac{-\beta}{1-\beta}\right)}{\frac{\rho c_{entry}}{c_f-T_f}-\int_{s^*}^{\infty}\left(\left(\frac{s}{s^*}\right)^{\beta}\frac{1}{1-\beta}-1\right)\frac{1}{2}\frac{\mu_s^2}{\sigma_s^2}f(s|s^*)ds-\frac{1}{2}\frac{\tilde{\mu}^2}{\sigma_s^2}\left(\frac{-\beta}{1-\beta}\right)} \\
\hat{s} &= \mu_s\frac{\hat{c}_f-T_f}{\rho\hat{c}_{entry}}
\end{aligned}$$

The derivatives of the cutoffs can be taken with respect to the distortion, which only has effects through T_f

$$
\begin{aligned}
\frac{\partial s^*}{\partial T_f} &= \frac{-\mu_s\left(\frac{-\beta}{1-\beta}\right)\frac{\rho c_{entry}}{\left(c_f-T_f\right)^2}}{\begin{array}{c}\left(\frac{\rho c_{entry}}{c_f-T_f}-\int_{s^*}^{\infty}\left(\left(\frac{s}{s^*}\right)^{\beta}\frac{1}{1-\beta}-1\right)\frac{1}{2}\frac{\mu_s^2}{\sigma_s^2}f(s|s^*)ds-\frac{1}{2}\frac{\mu_s^2}{\sigma_s^2}\left(\frac{-\beta}{1-\beta}\right)\right)^2 \\ -\mu_s\left(\frac{-\beta}{1-\beta}\right)\left(\int_{s^*}^{\infty}\left(\left(\frac{s}{s^*}\right)^{\beta}\frac{-\beta}{s^*}\frac{1}{1-\beta}\right)\frac{1}{2}\frac{\mu_s^2}{\tilde{\sigma}^2}f(s|s^*)ds+\int_{s^*}^{\infty}\left(\left(\frac{s}{s^*}\right)^{\beta}\frac{1}{1-\beta}-1\right)\frac{\partial g(s|s^*)}{\partial s^*}ds\right)\end{array}} \\
\frac{\partial \hat{s}}{\partial T_f} &= \frac{-\mu_s}{\rho\hat{c}_{entry}}.
\end{aligned}
$$

And the two expressions can be combined to get

$$
\begin{aligned}
\frac{\frac{\partial s^*}{\partial T_f}}{\frac{\partial \hat{s}}{\partial T_f}} &= \frac{\frac{c_{entry}}{c_f-T_f}}{\frac{\hat{c}_{entry}}{\hat{c}_f-T_f}}\frac{\rho+\mu}{\rho}\frac{1}{1-\frac{\tilde{\mu}\left(\int_{s^*}^{\infty}\left(\left(\frac{s}{s^*}\right)^{\beta}\frac{1}{s^*}\right)\frac{1}{2}\frac{\mu_s^2}{\sigma_s^2}f(s|s^*)ds+\int_{s^*}^{\infty}\left(\left(\frac{s}{s^*}\right)^{\beta}\frac{1}{-\beta}-\frac{1-\beta}{-\beta}\right)\frac{\partial g(s|s^*)}{\partial s^*}ds\right)}{\left(\frac{\rho\hat{c}_{entry}}{\hat{c}_f-T_f}\right)^2}} \\
\frac{\frac{\partial s^*}{\partial T_f}}{\frac{\partial \hat{s}}{\partial T_f}} &= \frac{\frac{c_{entry}}{c_f-T_f}}{\frac{\hat{c}_{entry}}{\hat{c}_f-T_f}}\frac{\rho+\mu}{\rho}\frac{\left(\frac{\rho\hat{c}_{entry}}{\hat{c}_f-T_f}\right)^2}{\left(\frac{\rho\hat{c}_{entry}}{\hat{c}_f-T_f}\right)^2-\mu_s\int_{s^*}^{\infty}\left(\left(\frac{s}{s^*}\right)^{\beta}\left(\frac{1}{s^*}+\frac{1}{-\beta}\left(\frac{\mu_s}{\sigma_s^2}-\frac{1}{s-s^*}\right)\right)\right)\frac{1}{2}\frac{\mu_s^2}{\tilde{\sigma}^2}f(s|s^*)ds}
\end{aligned}
$$

The first term is smaller than one and the rest seem to be larger than one. Yet, the ratio $\frac{\frac{c_{entry}}{c_f-T_f}}{\frac{\hat{c}_{entry}}{\hat{c}_f-T_f}}$ from the calibrated part can be used to get

$$
\frac{\frac{\rho c_{entry}}{c_f-T_f}}{\frac{\rho\hat{c}_{entry}}{\hat{c}_f-T_f}} = \frac{1}{1-\beta}\left(-\beta+\frac{\int_{s^*}^{\infty}\left(\left(\frac{s}{s^*}\right)^{\beta}-1\right)\frac{1}{2}\frac{\mu_s^2}{\sigma_s^2}f(s|s^*)ds}{\frac{\rho\hat{c}_{entry}}{\hat{c}_f-T_f}}\right)
$$

Then the ratio of the effects can be rewritten as

$$
\frac{\frac{\partial s^*}{\partial T_f}}{\frac{\partial \hat{s}}{\partial T_f}} = \underbrace{\frac{-\beta}{1-\beta}\frac{\rho+\mu}{\rho}}_{\text{<1, because of Lemma A.1}}\;\frac{-\beta\left(\frac{\rho\hat{c}_{entry}}{\hat{c}_f-T_f}\right)^2-\frac{\rho\hat{c}_{entry}}{\hat{c}_f-T_f}\int_{s^*}^{\infty}\left(1-\left(\frac{s}{s^*}\right)^{\beta}\right)\frac{1}{2}\frac{\mu_s^2}{\sigma_s^2}f(s|s^*)ds}{-\beta\left(\frac{\rho\hat{c}_{entry}}{\hat{c}_f-T_f}\right)^2-\mu_s\int_{s^*}^{\infty}\left(\left(\frac{s}{s^*}\right)^{\beta}\left(\frac{-\beta}{s^*}+\left(\frac{\mu_s}{\sigma_s^2}-\frac{1}{s-s^*}\right)\right)\right)\frac{1}{2}\frac{\mu_s^2}{\sigma_s^2}f(s|s^*)ds}
$$

The last term is smaller than one as long as

$$
\frac{\rho\hat{c}_{entry}}{\hat{c}_f-T_f}\left(1-\int_{s^*}^{\infty}\left(\frac{s}{s^*}\right)^{\beta}f(s|s^*)ds\right) > \mu_s\int_{s^*}^{\infty}\left(\left(\frac{s}{s^*}\right)^{\beta}\left(\frac{-\beta}{s^*}+\frac{\mu_s}{\sigma_s^2}-\frac{1}{s-s^*}\right)\right)f(s|s^*)ds
$$

Since it is known from Proposition 1 that

$$\int_{s^*}^{\infty} \left(\left(\frac{s}{s^*} \right)^{\beta} \left(\frac{\mu_s}{\tilde{\sigma}^2} - \frac{1}{s - s^*} \right) \right) f(s|s^*) ds < 0,$$

whenever $\left(1 - (1-\beta) \int_{s^*}^{\infty} \left(\frac{s}{s^*}\right)^{\beta} f(s|s^*)ds\right) > 0$,[3] the proof will follow. It turns out that $\int_{s^*}^{\infty} \left(\frac{s}{s^*}\right)^{\beta} f(s|s^*)ds = 0$ ■

[3]To get to this point one should make use of the equality $s^* = \hat{s}$ and use the definition of the latter.

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