Essays on Collusive Agreements and Monitoring

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Abstract

The thesis deals with monitoring strategies designed to implement self-enforcing collusive agreements when individual choices remain hidden and firms use public information stemming from different sources. With imperfect public monitoring, market linkage allows exploiting slack enforcement power but also affects the precision of monitoring and the incidence of equilibrium punishment. The first two chapters focus on multimarket contact and information generated independently in distinct markets. Linkage becomes relevant even if the two markets are identical and may have pernicious effects. In heterogeneous markets, production quotas in each market can be preferable to market sharing agreements even if they imply productive efficiency losses. In the third chapter, firms interact in a single market and get information of varying degrees of precision at different frequencies. The combination of information enhances monitoring and makes collusion immune to a resolution of uncertainty and to aggregate fluctuations.

Keywords: Collusion, imperfect monitoring, multimarket contact, market sharing, trade costs, intra-industry trade.

Resum

La tesi analitza estratègies de supervisió que permeten mantenir acords col·lusoris de consentiment tàcit quan les decisions individuals no són observables i les empreses recorren a informació pública de diverses fonts. Amb supervisió imperfecta, la vinculació de mercats permet relaxar restriccions d'incentius, però també afecta la precisió de la supervisió i la penalització necessària a l'equilibri. Els dos primers capítols es centren en contactes multi-mercat i en informació generada independentment en mercats diferents. La vinculació esdevé rellevant també en el cas de mercats idèntics i pot tenir efectes perniciosos. Amb mercats heterogenis, quotes de producció en cada mercat poden ser preferibles a acords que assignen mercats sencers encara que les quotes impliquin pèrdues d'eficiència productiva. En el tercer capítol, les empreses interactuen en un mercat únic i disposen, de manera irregular, d'informació de graus de precisió diferent. La combinació d'informació millora la supervisió i immunitza la col·lusió davant d'una possible resolució d'incertesa i de fluctuacions agregades.

Conceptes clau: Col·lusió, supervisió imperfecta, contacte multi-mercat, assignació de mercats, costs de comerç, comerç intra-industrial.

Foreword

The theory of repeated games has disclosed the potential impact of iterated interpersonal interaction on individual behaviour. It has become manifest that incentives in a one-shot interaction may strongly differ from the ones arising when an otherwise identical transaction is repeated over time among an at least partially constant set of players. The retention and possibly perpetuation of social or commercial contacts can generate incentive structures that embrace a rich variety of behavioural responses ranging from fierce rivalry to trustful cooperation. In particular, repeated play may allow self-enforcement of promises that cannot otherwise be assumed to be binding. This enables a whole host of interpersonal agreements and social conventions that, because of being very complex or leading to highly uncertain outcomes in the future, are not codifiable as a legally enforceable contract. Incompleteness may be one reason for why formal judicial procedures and legal enforcement fail, a second one are the legal standards themselves. Agreements that are of doubtful legality or univocally illegal require self-enforcement. Collusive side contracts are a typical example of such promises or agreements that are not exogenously enforceable.

The mechanism generating incentives in a context of repeated interaction is remarkably autonomous; it does not fundamentally hinge on the external environment, on physical links or the existence of stock magnitudes that may serve as commitment devices. The requirements for expected future interaction affecting current incentives are (i) effective strategic interaction in the form of behavioural externalities, i.e., the ability of individuals to affect other players' payoffs in the stage game, (ii) the observation and public verifiability of signals about private choices made by other players, or at least compatible beliefs about these choices, and (iii) sufficiently patient players. If these preconditions are met, backward contingency of strategies, i.e., the contingency of current play on the history of accumulated signals, makes future play depend on current choices and can thereby alter the incentives that shape current decisions. Repeated interaction constitutes a form of social cohesion, a social embeddedness of transactions along the

time dimension that allows coordinating equilibrium play.

Unfortunately, the prospects of self-enforcing cooperation are not limited to Pareto-improving individual transactions and to the implementation of broadly targeted, general interests. The same mechanism may be exploited to implement redistribution to particular interest groups in detriment of overall welfare. Since Friedman (1971), the application of the theory of repeated games has contributed a lot to the understanding of the viability and scope of self-enforcing collusive agreements in oligopolistically competitive environments. Beyond the game-theoretic foundation it allowed significant progress in identifying factors and mechanisms that favour or hinder collusive behaviour. This is an area of industrial organisation that transcends the border of academic speculation and systematic theoretical analysis; its progress is followed by practitioners in the field searching for systematic theoretical guidance in their professional decision making. The findings may allow defining adequate screens for detecting and investigating cartels. Perhaps more importantly, they open the possibility of active "ex ante", preventive competition policies that may gainfully complement the traditional approaches that focus on detecting and fighting existing collusion.

A set of results emerged that has become conventional wisdom in the field. It has also become evident, however, that many of these results do not appear to be particularly clear-cut or robust. The range of factors and arguments considered is broad, and many parameters turn out to have non-monotonic and highly context-dependent effects on the scope of collusion. In part, this is due to the structural idiosyncrasies of markets and the variety of strategic interactions and multiplicity of incentives that govern the behaviour of agents trading on these markets. Markets are indeed multifaceted institutions. On the other hand, the evidence that the object of study is a complex phenomenon cannot belie the fact that results are equally driven by methodological concerns related to modelling choice, abstraction and simplification. There has been consciousness of the arbitrariness of the grim trigger strategies used in Friedman (1971). The design of alternative and optimal intertemporal linkages of current and past actions and their effects on incentives has attracted great attention.

The three chapters of my thesis analyse the robustness of three well-known results with respect to a different assumption made in the canonical setting, namely perfect monitoring. The argument relates to point (ii) stated above. Self-enforcement devices are weakened if the

possibility of observing and publicly verifying private choices made by cooperating partners is deferred or complicated. Also this assumption has been scrutinised in the literature, but while the definition of trigger events and the design of punishment strategies are undoubtedly perceived as strategic choices, the information available for monitoring and the use made of it is largely treated as a structural element exogenous to strategic considerations. If hidden information about individual choices is a temporary phenomenon, this assumption is natural. If this information becomes available only very seldomly, however, this treatment is ill-founded. If the coordination of equilibrium play relies on noisy information about past play, uses observations on indirect environmental effects of these actions and possibly complements this knowledge with aggregate information that includes statistical summaries of the hidden individual actions, the availability of different sources of information automatically transforms the collection, selection and use of information into a strategic decision problem.

The thesis deals with monitoring processes designed to implement collusive agreements among oligopolistic firms when strategic choices are generally private information and firms have related information stemming from different sources at their disposal. It is analysed which imperfections result in the monitoring process and how these affect the capacity of coordinating equilibrium play.

In the first two chapters, the information is generated independently in two distinct markets in which rival firms simultaneously interact. This work deals exclusively with imperfect public monitoring and builds on the case of price-setting oligopolists that try sustaining a collusive agreement as in Tirole's (1988) reformulation of the Green and Porter (1984) model. The two sources of information provide publicly verifiable signals that are stochastic and ultimately determined by independent random shocks on demand in two markets, but whose probabilistic distribution depends on the profile of actions played in the stage game. In this context, exogenous realisations of low demand are indistinguishable from uncooperative price undercutting. The two chapters qualify two salient results about multimarket contact and collusion established in the seminal paper by Bernheim and Whinston (1990). Under perfect monitoring, market linkage is irrelevant when the two markets are replications of one another (Bernheim and Whinston, section 3). In chapter 1 it is argued that under imperfect public monitoring, due to informational concerns affecting the monitoring precision and due to the value burning effect of costly punishment

along the equilibrium path, multimarket contact and market linkage are relevant also in symmetric, strictly homogeneous settings. The optimal pattern of punishment and the effects of linkage in the spatial dimension are intimately related to the frequency with which the monitoring process is hampered by demand uncertainty. Interestingly, the optimal spatial linkage may strongly differ from the optimal time contingency of punishment strategies. While the latter implies immediate punishment that is as tough as individual rationality constraints allow and no longer than strictly required ("bang-bang" behaviour), relentless spatial linkage can be very pernicious in highly uncertain environments. Under imperfect monitoring, firms generally prefer more selective, lighter punishment schemes that are "tailored to the crime", a finding that is broadly compatible with evidence from cartel investigations. In reality, defection tends to be followed by communication and additional rule setting rather than price wars and a breakdown of the agreement.

The second chapter revises the result that, in the case of market heterogeneity with symmetric, reciprocal advantages it is always optimal to implement "spheres of influence" (Bernheim and Whinston, section 5). The force driving this result, namely the distribution of slack enforcement power across marketplaces, remains a forceful device in the case of imperfect monitoring. There is a trade-off, however. The spatial linkage that allows pooling incentive constraints reduces the informational efficiency of the monitoring process and causes the occurrence of punishment along the equilibrium path disproportionally often. This trade-off is analysed in a functionally specified model of mutual trade with linear trade costs. It is shown that the pro-collusive effects of a trade liberalisation identified by Lommerud and Sørgard (2001) in a model that uses the same functional specification remains unaffected under imperfect public monitoring. Contrary to what happens in models with perfect monitoring, however, closing intra-industry trade to avoid the waste of trade costs may not always be beneficial. If it implies a reduction of the amount of publicly available information, collusion may become altogether unsustainable; if firms continue observing its potential residual demand even without being actively trading in the foreign market, the enforcement power may nevertheless be reduced due to a more inefficient monitoring and costly linkage. It is shown that with high trade cost the effect of pooling slack enforcement power dominates. Collusive agreements that provide for market retraction and forbearance are then easier to sus-

tain. If on the contrary trade costs are moderate, agreements that provide for intra-industry trade may be easier to sustain and more beneficial especially in more uncertain environments.

In the third chapter, firms interact in a single market but use information of different nature and quality coming from two sources. By assumption, firms are always able to rely on imperfect public monitoring. In some occasions, they may be able to infer the prices set by the rival with certainty. This may be the case when the realisation of aggregate demand is publicly revealed during the period. It is shown that the two kinds of information are typically complementary, but that the way in which they are optimally used for monitoring differs in a substantial way depending on the timing of information revelation, i.e., depending on whether the state of aggregate demand becomes public knowledge before or after firms announce their prices to prospective consumers. The results shed new light on the relation between uncertainty and the sustainability of collusion. Compared to models of pure perfect or pure imperfect monitoring, these effects are reduced to a minimum. The optimal combination of different kinds of information not only mitigates the imprecision of monitoring but makes collusion immune to a possible resolution of uncertainty and the timing of such a resolution. In particular, this implies that the incentives for defection are unaffected by demand fluctuations; the scope of collusion remains constant over the business cycle. The traditional model with perfect monitoring does not consider that in periods of high demand, when the short term gains from deviation are highest, so is the quality of the information available for monitoring. It is shown that information of different quality effectively allows the firms balancing the incentives in periods with revealed high demand and periods in which the additional information is not available.

Contents

1 Multimarket collusive behaviour under imperfect public monitoring

1.1 Introduction

Micro datasets that record production and transactions at the firm level have shed new light on how strategic decisions affect trade flows. A large number of empirical studies document that firms are able to enhance their productivity and competitiveness upon entry by technology adoption and active innovation, but also by improving how production is organised and by strategically aligning the range of products and brands.

Two of the most salient features of firms engaging in international trade are striking from a competition policy perspective. Bernard et al. (2007, p. 116) report that export activity is extremely concentrated. "In 2000, the top 1 percent of trading firms by value (that is, by the sum of imports plus exports) accounted for over 80 percent of the value of total trade, while the top 10 percent of trading firms accounted for over 95 percent of the value of total trade [...]."

A second fact concerns the predominance of multiproduct firms. The strategic role of the product mix has long been recognised. On the production side, specialisation facilitates learning processes and the application of best practices. It allows creating uniqueness and be most responsive to customer needs and wishes without the distractions of selecting, tuning and balancing a product portfolio. It also allows a sharp focus in the marketing tactics, directly targeting and quickly approaching specific customer groups by choosing very precise sources and channels for pre- and post-sales services. It permits creating a niche; instead of being oriented at the average preference, it allows identifying a specific underprovision and serve this demand particularly well. On the other hand, a narrow range does not allow boosting

sales to an already existing customer base by bundling complementary products. A broader range of products generates economies of scope also in organisation, logistics and marketing. Especially in the case of durable or fashionable articles, a broad range may facilitate repeated sales via branding and the creation of corporate identity even with moderate innovation and product updates.

The data confirm that on world markets complex product ranges and multimarket contact play an extraordinary role, quantitatively but even more so qualitatively. Bernard et al. (2007, p.119) state that "[...] in the year 2000, firms that export more than one ten-digit Harmonized System product comprise 57.8 percent of exporting firms and account for more than 99.6 percent of export value." Multiproduct firms are not only more engaged in export trade; they are larger, more productive and make more sales per product than single-product firms.[1] The relevance and ubiquity of diversified product assortments is forcefully documented also in a related study on product switching.[2] The authors report that about one third of a firm's output are recently added or about-to-be dropped products, and one-half of firms alter their mix of five-digit SIC products every five years.

Both high concentration and multimarket contact are generally believed to be of some relevance for the viability and scope of collusive behaviour. The sustainability of collusion based on mechanisms of supervisory deterrence in repeated games rests on the accumulation of market experiences over time. Since the number of market experiences is multiplied if the same set of firms interacts on several markets, it is reasonable to think that multimarket contact might enhance the scope of collusive action. But the mere number of market experiences does not automatically imply a strategic advantage. If interaction happens on all markets simultaneously, more information does not lead to faster detection of deviant behaviour. Even in this case, however, multimarket contact may have an impact on the scope of collusion. In analogy to the past dependency of pricing strategies, which enables pooling and redistributing enforcement power over time, the linkage of pricing strategies across markets allows firms to profitably exploit the second dimension defined by the separate marketplaces.[3]

In a series of models of simultaneous multimarket interaction, Bern-

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[1]See Bernard et al. (2011) and Schoar (2002).

[2]Bernard et al. (2010).

[3]This second dimension may be spatial or more generally related to alternative dimensions of product heterogeneity.

heim and Whinston (1990) show that, under perfect monitoring, the strategic relevance of market linkage hinges on the existence of inter-market differences. In the absence of such heterogeneity, market linkage is shown to be irrelevant.[4] This paper translates part of the analysis conducted by Bernheim and Whinston into a setting characterised by unobservable actions and imperfect public monitoring. It uses a two-market model that builds on Tirole's (1988) reformulation of Green and Porter's (1984) model of collusion with unobservable actions[5]. The analysis is confined to symmetric collusive schemes and therefore implicitly to the generation of incentives by means of value burning. Firms rely on publicly inferable experiences of zero demand to define past-dependent pricing strategies, and these strategies imply that play can be either in a "cooperative phase", where all firms choose collusive actions, or in a "punishment phase" where firms temporarily retaliate against deviating behaviour. The implementation device is pure-strategy temporary Bertrand-Nash reversion.

Monitoring precision and implementation costs.   Adding uncertainty and the issues of observability and verifiability to the analysis of collusive behaviour with multimarket contact implies richer strategic interaction in the inter-market dimension and accentuates the relevance of the patterns of punishment. The pooling of slack enforcement power across marketplaces is no longer the only relevant effect of market linkage. Since strategies are not contingent on actual behaviour but on publicly observed stochastic outcomes, under imperfect monitoring the strategic use of information becomes a relevant aspect of the intertemporal implementation of collusive agreements. Moreover, since punishment becomes an equilibrium phenomenon, enforcement directly affects the collusive expected present discounted value. A trade-off arises between the maximum attainable gains from cooperation and the generation of punishment power. Multimarket contact turns out to be relevant in both regards. Strategic linkage changes the incentive structure by affecting the use of the scarce available information in the monitoring process and the equilibrium cost of implementing the agreement.

With multimarket contact, firms do not only decide whether to deviate but also in how many (and, with inter-market differences, in

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[4]Bernheim and Whinston (1990), sections 2 and 3; in particular proposition 1 on p. 5 and appendix A.

[5]See Tirole (1988), pp. 262 ff.

which) markets to deviate. It enlarges the set of potentially beneficial deviations from a collusive agreement, but also, since pricing strategies may be linked both intertemporally and spatially, the bandwidth of strategies that can be used to sustain such an agreement. Under perfect monitoring, this amplification of the decision margin through multimarket contact is irrelevant[6]. The present analysis shows that under imperfect monitoring firms make extensive use of the additional margin. Firms may prefer single-market deviations over simultaneous deviation in several markets and also may find it beneficial to punish selectively in only a subset of markets. In the following, the phenomenon of partial or selective punishment refers to the same set of firms playing collusively in some markets while simultaneously punishing each other in other markets[7].

The main findings are two. In contrast to Bernheim and Whinston (1990), unobservable actions cause multimarket contact to be relevant even if identical firms with constant returns to scale technologies interact in markets that are identical in expected terms, meaning that they are identical at the moment in which they decide on cooperation or defection. Irrelevance is limited to the case of purely aggregate shocks, i.e., demand shocks that are perfectly positively correlated across marketplaces. The second finding refers to optimal punishments and the trigger event. Abreu, Pierce and Stacchetti (1986) show that the optimal punishment scheme within the set of symmetric trigger strategies consists of responding to a low market price with an immediate punishment that is as tough as individual rationality constraints allow and no longer than strictly required ("bang-bang" behaviour). When firms

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[6]The uselessness of single-market deviations lays at the core of the informal argument behind Bernheim and Whinston's irrelevance result. They challenge the traditional view according to which multimarket contact facilitates collusion because "there is more scope for punishing deviations in any one market" arguing that "once a firm knows that it will be punished in every market, if it decides to cheat, it will do so in every market." See Bernheim and Whinston (1990), p. 3.

[7]Be aware of the different meanings of "partial" collusion that are used in the literature, mainly referring to situations in which (i) not all firms active in the marketplace participate in the (multilateral) collusive agreement, and (ii) the joint monopolistic outcome cannot be credibly sustained, but some other outcome implying payoffs strictly higher than competitive payoffs. Also the behaviour of simultaneously colluding in some entrepreneurial decisions (selling prices, output quantities) while competing in others (capacities, product qualities, R&D activities of advertising efforts) may be mentioned. This, however, is commonly referred to as "semicollusion". In this paper, the designation "partial" strictly refers to multimarket contact.

interact on several markets simultaneously and the process of monitoring is imperfect, such a punishment rule may be counterproductive. Proposition 2 below states that firms generally prefer a more selective punishment, and proposition 3 asserts that in situations of substantial uncertainty firms may even want to deliberately "look the other way" and maintain a cooperative attitude if zero demand is observed in a single market only.

This latter result provides an explanation that is, in some respect, capable of bridging the "somewhat troubling [...] disconnect between optimal collusion theory, simple (grim-strategy) collusion models, and real world firm behavior" bewailed in the literature.[8] We know little about the operating modes of tacit collusive agreements. But for explicit cartel agreements, the information collected in cartel investigations largely disqualifies the theoretical prediction of bang-bang behaviour. Incidents of cheating typically appear to be followed by extensive communication, ingenious problem-solving and constructive behavioural rule setting. Punishment behaviour appears to be partial and hesitant, possibly inert. Even in cases of open violation of the agreement over a continued period of time, firms seem prone to allow matching the deviant practice rather than punishing. Overall, punishment tends to be very much tailored to the crime. This evidence is compatible with findings on the advantages of selective or even more placable forms of punishment with multimarket contact under imperfect monitoring.

Related literature. Linkage has an impact on incentives not only because it enables the exploitation of slack enforcement power but also by affecting how scarce public information is used in the monitoring process and how severely and frequently punishment is executed in equilibrium. All these factors and their relevance for collusion have been studied previously in models with multimarket contact. Perfect monitoring models naturally focus on punishment power and the reallocation of slack enforcement power across markets. The cost of equilibrium punishment is the factor around which Thomas and Willig (2006) develop the idea that multimarket contact may actually reduce the scope of collusion. They study a highly asymmetric environment. In one market, behaviour is periodically publicly revealed and implementation relies on perfect monitoring; in the other, private actions are

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[8]See Cabral (2005).

never observed and collusion is sustained by imperfect public monitoring. In this setting linkage causes "contagion" and provokes substantial value burning. The present analysis eliminates the asymmetry characterising the two markets in Thomas and Willig, which effectively allows them abstracting from the informational aspect characterising the multimarket incentive problem. When behaviour in one market can be perfectly monitored, the strategic value of information is reduced to the single-market case.

The informational aspect of linkage is neatly laid open in Matsushima (2001). He shows that strategically linking more and more imperfectly monitored markets that are inherently unlinked and that generate independent signals effectively allows eliminating uncertainty and approaching perfect monitoring in the limit. The trigger event proposed relies on the ratio of the number of markets in which negative observations are made to the total number of markets, and it implies that a firm has neither an incentive to deviate in all markets simultaneously (by using the Law of Large Numbers, such a deviation can be detected almost certainly) nor to deviate in a single market only (the deviation gains are low). Also in Lee (2012) the available information is central for the design the trigger event. Her analysis is restricted to the case when negative demand shocks happen with probability $\frac{1}{2}$. In this case, unlinked strategies do not allow sustaining the joint monopolistic outcome. The analysis relies on stick-and-carrot strategies and the scope of collusion is measured in terms of the maximum stage game profits sustainable using this kind of punishments. Her finding for i.i.d. shocks that happen with probability $\frac{1}{2}$ is fully compatible with the outcome presented here on the sustainability of the joint monopolistic profits under moderate levels of uncertainty: linkage shows an inferior performance as compared to unlinked strategies. Lee's main concern, however, is not the i.i.d. case. She shows that, when shocks are correlated across markets, a pricing strategy that takes into account these correlations allows enhancing the scope of collusion.[9] Her argument is cognate with Spagnolo (1999) who proves that the irrelevance re-

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[9]This statement does not refer to the first part of her paper concerned with the role of linkage in models where uncertainty is resolved before players decide to cooperate or defect. In this case, the relevance of linkage is related to the law of large numbers and the fact that i.i.d. shocks are averaged out through linkage. In the second part of the paper dealing with imperfect monitoring this effect plays no role since, under relentless linkage, a single (low) observation suffices to trigger punishment. The relevance of linkage in the two sections of Lee (2012) rests on two completely different mechanisms.

sult in Bernheim and Whinston is very sensitive to the assumption of markets being inherently unlinked.[10]

The present paper assumes independent shocks as in Matsushima (2001) but confines the analysis to two markets as in Lee (2012). Contrary to both, it analyses how the scope of collusion and its dependence on different types of cross-market linkage varies with the level of uncertainty, i.e., the degree of imprecision in the monitoring process. This focus on the monitoring friction makes the analysis closely related to the literature on imperfect monitoring in single-market environments in which firms observe outcomes continuously and can react to information quickly. Abreu, Milgrom and Pearce (1991) first showed that collusion may become unsustainable if the frequency of interactions is increased. Sannikov and Skrzypacz (2007, p. 1796) emphasise that the effect is channelled through a deterioration of monitoring:

> With shorter time periods between actions, firms must decide whether to trigger a price war by looking at noisier incremental information. [...] this causes firms to make type I errors by triggering price wars on the equilibrium path disproportionately often, erasing all benefits from collusion.

The insight that symmetric punishment schemes always imply value burning goes back to Radner, Myerson and Maskin (1986). Fudenberg and Levine (2007) and Sannikov and Skrzypacz (2007, 2010) show that this cost crucially depends on the type of information used[11] and that asymmetric collusive schemes may be useless in a context of frequent action because "with a one-dimensional signal and a continuum of quantities to choose from, transfers used to provide incentives for one player interfere with the incentives of the other player."[12] In the present context the trigger event depends on the realisation of a binary random variable and a second, spatial dimension is added to the time

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[10]In Spagnolo's paper the link is due to the strict concavity of the firms' objective function. Independently of the nature of the intrinsic link, an appropriate strategic linkage allows enhancing the scope of collusion with respect to unlinked implementation.

[11]Information processes with increments that are i.i.d. conditional on current actions can be decomposed into a continuous Brownian component and a discontinuous Poisson component. The continuous Brownian signals are shown to lead to substantially higher costs of type I errors (triggering punishment when no deviation occurred) than the Poisson signals associated to sudden, informative events.

[12]Sannikov and Skrzypacz (2007), p. 1797.

dimension. Shocks are assumed to be i.i.d. in both dimensions and the analysis is confined to symmetric equilibria, such that the questions posed by the frequent action literature cannot be evaluated. The main insights concerning the relevance of the monitoring precision and the cost of implementation, however, reappear in the model of multimarket contact. If the factor limiting collusive behaviour is abundance of information, the agreement needs to put a limit on the flexibility of its members to respond to new information. Multimarket contact provides a natural way of imposing discipline by making behaviour conditional on the second, spatial dimension of observations.

The paper is organised as follows. The second section defines a two market extension of Tirole's (1988) reformulation of Green and Porter's (1984) model of collusion with unobservable actions. Within this framework, two price-competing firms interact repeatedly and simultaneously and try to sustain a collusive production quota agreement on both markets. While in the second section the implementation strategies remain unlinked, section 3 explores two different kinds of strategic market linkage, relentless linkage and placable linkage. The scopes of collusion and the equilibrium collusive expected present discounted values are characterised and compared. Section 4 concludes focusing on the differences that distinguish the settings of perfect and imperfect monitoring.

1.2 The benchmark: unlinked implementation

The analysis is confined to the case of two markets and two firms that produce a homogeneous good at constant and equal per unit cost c and trade this good simultaneously, regularly and endlessly at points $t = 0, 1, 2, ...$ on both markets. They discount future payoffs at factor $\delta \in (0, 1)$ and compare intertemporal streams of payoffs using the respective expected present discounted values. Consumers of the indistinguishable good produced by the two firms split up equally in the two marketplaces, 1 and 2; they are immobile and the total number of consumers remains constant over time.

In both markets i $(i = 1, 2)$, and at each date t, total demand for the good is uncertain but otherwise identical. There are two possible states of nature: with probability $\alpha \in (0, 1)$, in the bad demand state, demand is zero while in the good demand state, occurring with opposite probability $1 - \alpha$, demand is given by a continuous function $D(.)$

that implies $D(c) > 0$ and is nonincreasing at prices with strictly positive demand. By assumption, there is a unique monopoly price p^m and industry profits $(p - c)D(p)$ are monotonically increasing in price for $p \leq p^m$ and monotonically decreasing for $p \geq p^m$. The random variable describing the evolution of demand is i.i.d. over time and shocks are also supposed to be purely idiosyncratic, i.e., i.i.d. across marketplaces. In this symmetric setting a bad demand state at t happens independently in each market with the same probability α.

The two firms that produce the identical good are labelled A and B ($j \in \{A, B\}$). If competition prevails in the stage game, the two firms engage in Bertrand price competition. Simultaneously both firms spread the information about the current selling prices among consumers. Once consumers know the two prices, they purchase the good from the lowest-price supplier. When firms announce the same price it is assumed without loss of generality that the residual demand faced by each firm is half the market demand at the price announced independently by both firms. In each period, firms must meet the entire demand for its product at the announced price. Three decision-relevant pieces of information are never observed nor disclosed, not even at the end of the stage game: the prices announced to consumers by the rival firm, the fact that there have effectively been units sold and the exact quantity sold.

In this competitive setting characterised by private information about pricing decisions and sales, the two firms try to sustain a self-enforcing collusive production quota agreement (PQA). In each market $i \in \{1, 2\}$, the agreement specifies a single focal price p_i^{PQA} to be announced to consumers by both firms and two shares of the aggregate demand, s_i^{PQA} and $1 - s_i^{PQA}$, to be served by firms A and B, respectively. The problem of equilibrium selection generally inherent in repeated games is circumvented by focusing exclusively on the fully collusive outcome that consists of both firms charging the unique monopoly price, i.e., $p_i^{PQA} \equiv p^m$, $i = 1, 2$; the sustainability of partially collusive prices $p_i \in (c, p^m)$ is neglected. For what concerns the sharing rule, the firms are supposed to distribute the resulting aggregate demand in a reciprocally proportional manner such as to eliminate the distribution problem involved in bargaining over $(p_i^{PQA}, s_i^{PQA})_{i=1,2}$: firm A is assigned share s in market 1 and $1 - s$ in market 2 while firm B is assigned share $1 - s$ in market 1 and s in market 2.[13] Among the reciprocally proportional sharing rules, special

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[13]This choice is reminiscent of a reciprocal trade model in which market 1 is the

attention is paid to the rule that maximises the scope of collusion, i.e., the sharing rule that relaxes the more stringent of the two incentive constraints of firms A and B.

Since firms' actions are never publicly observed, and since demand uncertainty is independent of the actions taken on both markets, firms are not able to infer deviant behaviour. A firm that abides by the agreement and faces zero demand in a particular period does not know whether the negative experience is due to the realisation of a bad demand shock or due to the rival undercutting its own price. Common knowledge is limited to the fact that, when at least one firm faces zero demand, both partners know that at least one firm faces zero demand. Past zero demand experiences (ZDEs) form the history of public information. They allow for public monitoring because every action profile induces a particular probability distribution over these publicly known outcomes.

The starting point of the analysis is the assumption that firms try to sustain the PQA by explicitly agreeing or implicitly understanding that actions are strategically unlinked across markets, i.e., that the event triggering punishment in market i exclusively depends on the history of publicly verifiable observations made in market i. More concretely, firms assume that the repeated and simultaneous interaction is ruled by the following intertemporal pricing strategy.

Strategy U The *unlinked intertemporal strategy* does not link the trigger event in one market to past observations made in the other market.

- In both markets $i = 1, 2$ the game starts in a collusive phase. Firms $j = A, B$ announce price $p = p^m$, sell $sD(p^m)$ or $(1-s)D(p^m)$ units according to the agreement and continue doing so until one firm makes a ZDE in market i. ZDEs made in market $-i$ are irrelevant for the intertemporal strategy followed in market i. If one or both firms make zero profits in market i, play switches to a punishment phase in this market irrespective of the behaviour followed in market $-i$.

- From the next period onwards, both firms play the stage game Nash equilibrium strategy (announce $p = c$ and sell

$\frac{1}{2}D(c)$ units to consumers) in market i for T periods, irrespective of the behaviour followed in market $-i$. In period $T + 1$, firms revert to collusive actions in market i, again irrespective of actual play in market $-i$.

If a PQA is implemented in this environment by means of strategy U, equilibrium play can be in one of $(T + 1)^2$ possible states

$$\sigma \in \{CC, CP_1, ..., CP_T, P_1C, P_1P_1, ..., P_1P_T, ..., P_TC, ..., P_TP_T\}.$$

For example, CP_3 denotes the state in which collusive play prevails in market 1 while firms are in the third period of a punishment phase in market 2.

Consider an arbitrary period t in state CC. One-period profits of firm A are $s\pi_1^m + (1-s)\pi_2^m$ if no market suffers a negative demand shock. If both firms abide by strategy U, this happens with probability $(1-\alpha)^2$ and, according to the rules, play remains in state CC in the subsequent period. With probability $(1 - \alpha)\alpha$, a bad demand shock occurs in market 2 but not in market 1. Firm A's stage game profits are then $s\pi_1^m + 0$ and punishment comes into effect in market 2 while collusive behaviour is maintained in market 1. Play switches to state CP_1 in the subsequent period. Analogous reasoning for the remaining two possible realisations of demand shocks reveal that, under stationarity, the dynamic path of expected present and future profits of firm A, starting in an arbitrary period in state CC, satisfies

$$\begin{aligned} V_A^{CC} &= (1 - \alpha)^2 \left(s\pi_1^m + (1 - s)\pi_2^m + \delta V_A^{CC} \right) \\ &\quad + (1 - \alpha)\alpha \left(s\pi_1^m + \delta V_A^{CP_1} \right) \\ &\quad + \alpha(1 - \alpha) \left((1 - s)\pi_2^m + \delta V_A^{P_1C} \right) + \alpha^2 \delta V_A^{P_1P_1} \end{aligned}$$

or equivalently, using the fact that $\pi_1^m = \pi_2^m =: \pi^m$,

$$\begin{aligned} V_A^{CC} &= (1 - \alpha)\pi^m + (1 - \alpha)^2 \delta V_A^{CC} + (1 - \alpha)\alpha \delta V_A^{CP_1} \\ &\quad + \alpha(1 - \alpha)\delta V_A^{P_1C} + \alpha^2 \delta V_A^{P_1P_1}. \end{aligned}$$

If by chance both markets suffered a negative shock, play switches to state P_1P_1 in the following period and both firms know that play will remain under punishment for the next T periods in both markets: under stationarity, the expected present discounted value (EPDV) of

firm j, $j \in \{A, B\}$ in state $P_1 P_1$ must satisfy

$$V_j^{P_1 P_1} = \delta V_j^{P_2 P_2} = \delta^2 V_j^{P_3 P_3} = ... = \delta^T V_j^{CC}.$$

$V_j^{P_1 P_1}$ can be expressed recursively in terms of V_j^{CC} only. Looking at the alternative possible shock realisations reveals, however, that it is generally not possible to reduce the $(T+1)^2$-dimensional system of linear equations characterising the stationary path of EPDVs to a small number of states if parallel interaction in two markets is governed by strategy U. For instance, suppose t is a period in which state CP_1 is realised. Following strategy U, firm A announces $p = p^m$ and sells $sD(p^m)$ in market 1 and announces $p = c$ and sells $\frac{1}{2}D(c)$ in market 2. If demand happens to be positive in both markets, firm A's profits are $s\pi_1^m + 0$ and play switches to state CP_2. But firms cannot infer that play will follow the path CP_2 in $t+1$, CP_3 in $t+2$ and so forth, and return to CC in $t+T$. If a bad demand shock hits market 1 (or both markets), firm A makes zero profits and play switches to state $P_1 P_2$. Under stationarity the EPDV $V_A^{CP_1}$ for firm A satisfies

$$
\begin{aligned}
V_A^{CP_1} &= (1-\alpha)^2 \left(s\pi_1^m + 0 + \delta V_A^{CP_2} \right) + (1-\alpha)\alpha \left(s\pi_1^m + 0 + \delta V_A^{CP_2} \right) \\
&\quad + \alpha(1-\alpha) \left(0 + \delta V_A^{P_1 P_2} \right) + \alpha^2 \left(0 + \delta V_A^{P_1 P_2} \right).
\end{aligned}
$$

More generally, the problem is that in cases of selective, single-market punishments the state of play remains responsive to demand shocks as long as collusion is sustained in at least one market. In multimarket models of imperfect public monitoring, partial or selective punishment is a distinctive equilibrium phenomenon. The dimensionality of the equation system cannot be reduced in analogy to the single market case. The determination of the $(T+1)^2$ stationary EPDVs V_j^σ, $\sigma \in \{CC, CP_1, ..., CP_T, ..., P_T C, ..., P_T P_T\}$, for both firms $j \in \{A, B\}$ conditional on punishment length T typically requires solving the corresponding system of $(T+1)^2$ linear equations.

Absence of intrinsic links across markets. In the particular twin market model of collusion with imperfect monitoring specified above, it is possible to identify the scope of collusion and the collusive EPDV with strategy U by looking exclusively on the incentive conditions resulting in each single market when strategy U is spatially truncated and its relevant part applied separately in markets 1 and 2. This truncated version of strategy U coincides with the following strategy

that Tirole (1988) defines for the case of single market interaction with temporary Bertrand reversion.[14]

Strategy GPT The *Green-Porter-Tirole strategy* defines the following trigger event.

- The game starts in a collusive phase. Firm $j = A, B$ announces price $p = p^m$ and sells the fraction of $D(p^m)$ according to agreement at price p^m. It continues playing this action until it makes a ZDE. If firm $j \in \{A, B\}$ faces zero demand, play switches to a punishment phase.

- From the next period onwards, both firms play the stage game Nash equilibrium strategy and announce $p = c$ to consumers for T periods before reverting and starting a new collusive phase. T can, in principle, be finite of infinite.

Abreu et al. (1986) show that, because of the particular recursive structure characterising the Green and Porter model, the game can be analysed in analogy to a repeated game. The equilibrium behaviour in the first period of any proper subgame coincides with the Nash equilibrium strategy in an intertemporally truncated game in which for each possible state of nature in the second period, the game that starts in this state is replaced by the EPDV of its equilibrium strategies. In order to establish the equivalence of strategies U and GPT it therefore suffices to show that single-market profit maximisation and twin-market profit maximisation are equivalent in an arbitrary single period. Two properties of the model specified above ensure that this equivalence holds: first, strategy U defines a strategic, intertemporal linkage but not a spatial linkage: the trigger event in market i does exclusively depend on past ZDEs made in market i. Second, there exist no intrinsic, technological links across markets. The spatial i.i.d. assumption and the separability of the objective functions guarantee that in the stage game the payoff achievable on both markets coincides with the sum of payoffs achievable independently in each single market.[15]

---

[14]See Tirole (1988), pp. 262 ff.

[15]Without intrinsic and strategic links, the analysis of collusion in a single market implicitly takes into account transitions from and to states of partial or selective punishment that happen along the equilibrium path in the twin-market model. The companion paper Schreibweis (2013) proposes an alternative specification that relies on publicly randomised, behavioural punishment strategies. The approach allows defining a low-dimensional state space (independent of

Exemplarily, look at market 1, i.e., the market in which firm A is assigned share s and firm B share $1 - s$. Play can be in one of $T + 1$ possible states $\sigma \in \{C, P_1, P_2, ..., P_T\}$. The stationary EPDVs of firm A's profits in equilibrium satisfy the following $T + 1$-dimensional system of linear equations:

$$
\begin{aligned}
V_A^C &= (1 - \alpha)\left(s\pi^m + \delta V_A^C\right) + \alpha\left(0 + \delta V_A^{P_1}\right), \\
V_A^{P_\tau} &= \delta V_A^{P_{\tau+1}}, \tau = 1, ..., T - 1, \\
V_A^{P_T} &= \delta V_A^C.
\end{aligned}
$$

In single market interaction, finding the stationary solution is simplified by the fact that, when switching from collusion to punishment, it is public knowledge that play will remain under punishment for the next T periods. T certainly is a decision variable, but no firm can gain by unilaterally shortening or prolonging the duration of punishment, both under high and low demand. The intertemporal structure then allows solving $V_j^{P_\tau}$ recursively in terms of V_j^C for $\tau = 1, ..., T$:

$$
V_j^{P_\tau} = \delta^{T - \tau + 1} V_j^C.
$$

In particular, $V_j^{P_1} = \delta^T V_j^C$. This allows reducing the dimensionality of the stationary system to two equations that determine EPDVs V_j^C and $V_j^{P_1}$:

$$
\begin{aligned}
V_A^C &= (1 - \alpha)\left(s\pi^m + \delta V_A^C\right) + \alpha\left(0 + \delta V_A^{P_1}\right), && (1.2.1) \\
V_A^{P_1} &= \delta^T V_A^C. && (1.2.2)
\end{aligned}
$$

Solving the system 1.2.1 and 1.2.2 yields the following expressions of the expected present discounted values of the streams of present and future equilibrium profits for $j = A, B$:

$$
V_j^C = \frac{(1 - \alpha)}{1 - (1 - \alpha)\delta - \alpha\delta^{T+1}} \, s_j \pi^m \qquad (1.2.3)
$$

---

the length of punishment phases) and modelling explicitly the transitions between the few remaining states of cooperation, CC, full punishment, PP, and selective punishments, CP and PC. By laying open the payoffs of all deviation strategies in all possible states of the original game, it provides additional insight into the incentive structure and how it is affected by linkage. The paper is available upon request.

in an arbitrary period in state C and

$$V_j^{P_1} = \frac{(1-\alpha)\delta^T}{1 - (1-\alpha)\delta - \alpha\delta^{T+1}}\, s_j \pi^m \qquad (1.2.4)$$

in a period in state P_1, where $s_A = s$ and $s_B = 1 - s$. The stationary collusive EPDV is strictly increasing in s for firm A and strictly decreasing in s for firm B, irrespective of the chosen T and parameter values α and δ. The stationary collusive EPDVs of both firms are strictly decreasing in T.

Since temporary reversion to the one-period Bertrand-Nash equilibrium is clearly a credible punishment, the analysis of best responses is limited to deviations from the collusive action in state C. In models with positive future discounting, immediate deviations are known to be more beneficial under stationarity than deviations at any later date. In an arbitrary period t in state C, the analysis of beneficial unilateral deviations can therefore without loss of generality be confined to one-period best responses in the stage game. Finally, the analysis is facilitated by the fact that there is a single best deviation from the PQA that exploits both decision margins fixed by the agreement, the selling price announced to consumers and the sharing rule. While maintaining collusion requires announcing the collusive focal price p^m and serving a share s (or $1 - s$ in the case of firm B) of the total demand resulting at that price, the one-period best deviation is to slightly undercut p^m and to serve the entire demand at the announced lower price (i.e., set $s = 1$ in the case of firm A or $1 - s = 1$ in the case of firm B). This optimal deviation yields per-period profits of approximately π_m with probability $(1 - \alpha)$ and implies a ZDE on the part of the rival. Since such behaviour triggers a punishment phase with probability one, the best deviation implies a dynamic path of expected present and future profits that under stationarity satisfies

$$V_j^D = (1-\alpha)\left(\pi^m + \delta V_j^{P_1}\right) + \alpha\left(0 + \delta V_j^{P_1}\right) \qquad (1.2.5)$$

for both $j \in \{A, B\}$. The incentive conditions $V_j^C \geq V_j^D$ are then equivalent to

$$(1-\alpha)\delta + \alpha\delta^{T+1} + s_j\left(1 - \delta^{T+1}\right) \geq 1$$

for $j = A, B$. $V_j^C - V_j^D$ is strictly increasing in s for firm A and strictly decreasing in s for firm B, irrespective of the chosen T and parameter
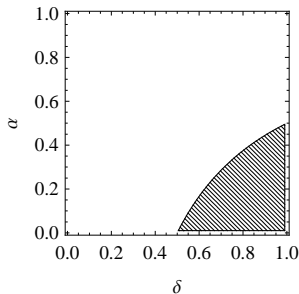
Figure 1.2.1: Scope with strategy U

values α and δ. The agreement that maximises the scope of collusion with respect to s is consequently the one that provides for an equal sharing rule. Setting $s = \frac{1}{2}$, the left hand side of the identical incentive condition of both firms,

$$2(1-\alpha)\delta + (2\alpha - 1)\delta^{T+1} \geq 1, \qquad (1.2.6)$$

is strictly increasing in T for $\alpha < \frac{1}{2}$, independent of T for $\alpha = \frac{1}{2}$ and strictly decreasing in T for $\alpha > \frac{1}{2}$. In the latter case, $V_j^C - V_j^D$ is maximised for $T \to 0$, implying an infinitesimal length of the punishment period. It is immediate that, in the absence of effective punishment, collusion is not sustainable for any $\delta < 1$ ($T = 0$ violates the incentive constraint). The same result is found in the case $\alpha = \frac{1}{2}$, when the sustainability condition is equivalent to $\delta \geq 1$ for any positive integer T. If $\alpha < \frac{1}{2}$, on the other hand, the left hand side is maximised for $T \to +\infty$, implying that collusion is sustainable if $\delta(1-\alpha) \geq \frac{1}{2}$.

The scope of collusion is defined to be the set of parameter constellations (α, δ) under which a PQA is sustainable if s and T are chosen solving

$$\max_{s,T} \left\{ \min_{j \in \{A,B\}} \left\{ V_j^C - V_j^D \right\} \right\},$$

i.e., setting $s_{\text{scopemax}} = \frac{1}{2}$ and the harshest punishment $T_{\text{scopemax}} \to +\infty$. Figure 1.2.1 depicts the scope of collusion in the case of single-market interaction graphically in the unit square of the bidimensional coordinate plane representing the parameter space $\{(\alpha, \delta) | \alpha, \delta \in [0,1]\}$.

Since under imperfect monitoring punishment occurs along the equilibrium path, V_j^C is strictly decreasing in T for both $j = A, B$. Given any parameter constellation (α, δ) at which the PQA $\left(p^m, s = \frac{1}{2} \right)$ is

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sustainable, the firms can't therefore be expected to implement it by playing the harshest possible punishment. Instead, firms prefer to set the shortest punishment length permitting sustainability. Given agreement $\left(p^m, s = \frac{1}{2}\right)$, firm j's choice of T is the smallest integer number that satisfies $V_j^C \geq V_j^D$, $j = A, B$. Up to the integer problem, firms operate on a binding incentive constraint in equilibrium. If $\left(p^m, s = \frac{1}{2}\right)$ is sustainable with $T \to +\infty$, $\frac{\partial V_j^C}{\partial T} < 0$ for $j = A, B$ and the fact that $T = 0$ violates the incentive constraint implies that there exists a strictly positive $\widetilde{T} \in \mathbb{R}$ for which $V_j^C = V_j^D$. The equilibrium punishment length is then the smallest integer larger or equal $\widetilde{T}$.[16]

Absent inter-market differences both in the market structure and firm characteristics, the agreement $\left(p^m, s = \frac{1}{2}\right)$ can be sustained in market 2 under identical conditions. Since the absence of intrinsic technological and strategic links across markets secures independence of the incentive constraints on both markets, the scope of multimarket collusion with strategy U coincides with the scope of single-market collusion in each parallel market, and the equilibrium collusive EPDV V_j^{CC} coincides with the sum $V_j^C(\text{market } 1) + V_j^C(\text{market } 2)$. Proposition 1 summarises the findings characterising the scope of collusion and the maximum achievable collusive EPDV with strategy U. The equilibrium punishment length and the collusive EPDV are derived in appendix A.1.

Proposition 1 With strategy U, the scope of collusion is maximised for $s = \frac{1}{2}$. In this case, collusion can be credibly sustained if $\delta(1 - \alpha) > \frac{1}{2}$. If $\delta \to 1$, collusion remains sustainable for levels of uncertainty $\alpha < \frac{1}{2}$. Given δ and α, the maximum achievable collusive expected present discounted value is, for both firms $j = A, B$,

$$V_j^{CC}(T^*) = \frac{(1 - 2\alpha)\,\pi^m}{1 - \delta}. \qquad (1.2.7)$$

Figure 1.2.2 shows the collusive expected present discounted value in equilibrium, when the punishment length T is chosen such as to maximise V_j^{CC}. It is graphed as a function of α for $\delta = 0.9$ (and

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[16]In what follows, when comparing the maximum achievable collusive EPDVs under different implementation strategies, the integer problem is ignored. It is assumed that the punishment length can take any positive real value, and the binding incentive constraint is used to determine the equilibrium punishment length.
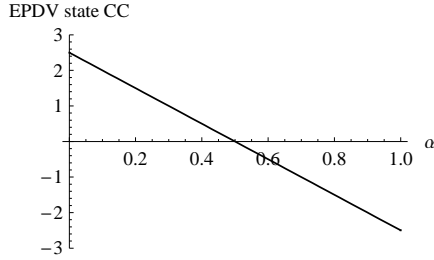
Figure 1.2.2: $V_j^{CC}(T^*)$ with strategy U

$\pi_i^m = \frac{1}{4}$ in markets $i = 1, 2$). The intersection with the abscissa indicates the maximum level of uncertainty compatible with collusion. The expression of V_j^{CC} reveals that a variation of δ affects the scaling of the ordinate but not this intersection.

1.3 Cross-market linkage and collusion

With unlinked dynamic strategies, a ZDE in a collusive phase in market i triggers punishment in market i. ZDEs made in market $-i$ are irrelevant for the implementation of collusion in market i. Under multimarket contact, however, a firm is able to link the implementation strategies of collusive agreements not only intertemporally but also across marketplaces. It can make the trigger event in market i conditional on the past experience made in both markets. In this sense, multimarket contact offers additional degrees of freedom when designing dynamic strategies. Clearly, the additional public information can be used in various ways. A systematic account of the different available options for strategically linking behaviour in market i to the publicly available information in market $-i$ must distinguish two basic types: either a single observation in any market triggers a regime switch in both markets (relentless linkage) or a single observation in any market does not trigger a behavioural reaction at all (placable linkage). Since collusion is known to be unsustainable without punishment, placable linkage necessarily requires that punishment is induced by a twofold observation in both markets.

Relentless linkage. If the classification of possible linkages in two-market interaction is confined to pure strategy trigger events, the two

basic types of relentless linkage and placable linkage are the only types of linkage.[17] Consider first the following spatially linked strategy.

Strategy R The *relentless intertemporal strategy* defines the following trigger event.

- In both markets $i = 1, 2$ the game starts in a collusive phase. Firms charge price $p_i = p^m$ and sell $sD(p^m)$ or $(1-s)D(p^m)$ units according to the agreement and continue doing so until one firm makes a zero demand experience in any market, i.e., in market i or in market $-i$ or in both markets. A ZDE made in market $-i$ has the same relevance for the pricing behaviour in market i as it has a ZDE in market i. If one or both firms face no demand in any market, play switches to a punishment phase in both markets.

- From the next period onwards, both firms play the stage game Nash equilibrium strategy, announce $p_i = c$ and sell $\frac{1}{2}D(c)$ units to consumers for T periods in both markets. In period $T + 1$, firms revert to collusive actions in both markets.

This strategy represents the harshest possible punishment implementable by strategically linking the actions to observations in both markets. A single observation (ZDE) in any market triggers punishment in both markets. It is the strategy that, within the given framework of imperfect monitoring, corresponds to the only type of cross-market linkage that allows sustaining collusive agreements in multimarket models with perfect monitoring.

Before solving the model in the case of strategy R it is worth noting that, by definition, linkage abolishes selective punishment and thereby resolves the technical difficulty associated with the large state space in case of strategy U. With linkage, play can only switch from state CC, in which the two firms are abiding by the collusive agreement in both markets, to state P_1P_1, in which firms punish each other in both markets. A switch to states CP_1 or P_1C is barred. If play enters state P_1P_1, as set out in the previous section, both firms know that play will follow its course or curse for the next T periods before switching back

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[17]The analysis does not consider cross-market linkages that induce topsy-turvy punishment behaviour, i.e., responding to a single ZDE made in market 1 by punishing exclusively in market 2 (or vice versa).

to state CC. The property that allows reducing the dimensionality of the stationary system in the single-market case applies in the twin market model with R-linkage (and P-linkage).

Consider first an arbitrary period in state CC. One-period profits of firm A are $s\pi_1^m + (1-s)\pi_2^m$ if no market suffers a negative demand shock. This case happens with probability $(1-\alpha)^2$ and, according to strategy R, play remains in state CC in the subsequent period. With probability $(1-\alpha)\alpha$, a bad demand shock occurs in market 2 but not in market 1. Firm A's stage game profits are then $s\pi_1^m$ and play switches to state P_1P_1 in the subsequent period. A switch to P_1P_1 also happens with probability $\alpha(1-\alpha)$, when firm A's stage game profits are $(1-s)\pi_2^m$, and probability α^2, with which low demand is realised in both markets and both firms' profits are zero. Under stationarity the dynamic path of expected present and future profits of firm A satisfies

$$
\begin{aligned}
V_A^{CC} &= (1-\alpha)^2 \left(s\pi_1^m + (1-s)\pi_2^m + \delta V_A^{CC} \right) \\
&\quad + (1-\alpha)\alpha \left(s\pi_1^m + \delta V_A^{P_1P_1} \right) \\
&\quad + \alpha(1-\alpha) \left((1-s)\pi_2^m + \delta V_A^{P_1P_1} \right) + \alpha^2 \delta V_A^{P_1P_1}
\end{aligned}
$$

or equivalently, using $\pi_1^m = \pi_2^m$,

$$
V_A^{CC} = (1-\alpha)\pi^m + (1-\alpha)^2 \delta V_A^{CC} + \left(1 - (1-\alpha)^2 \right) \delta V_A^{P_1P_1}. \quad (1.3.1)
$$

Analogous reasoning reveals that firm B's stationary EPDV in state CC satisfies an identical equation.

Cross-market linkage does not affect the EPDVs of the streams of profits in the successive periods of a punishment phase. According to strategy R both firms announce $p = c$ to consumers in both markets. Every punishment phase lasts T subsequent periods. During this time, firms make zero profits in both markets irrespective of the realisation of demand; in period $T+1$ after a punishment phase has been initiated, play returns to state CC. Under stationarity, the EPDV of the present and future profits of firm $j \in \{A, B\}$ in the very first period of a punishment phase therefore satisfies

$$
V_j^{P_1P_1} = \delta^T V_j^{CC}, \quad (1.3.2)
$$

same as in the case of strategy U.

Solving the system 1.3.1 and 1.3.2 in the stationary EPDVs V_j^{CC} and $V_j^{P_1 P_1}$ yields the following expressions of the expected present discounted values. Due to the reciprocally proportional sharing rule, in the symmetric setting they are identical for both firms $j = A, B$ and independent of s_j. With strategy R, the value of colluding in both markets is

$$V_j^{CC} = \frac{(1 - \alpha)}{1 - (1 - \alpha)^2 \delta - (1 - (1 - \alpha)^2) \delta^{T+1}} \pi^m \qquad (1.3.3)$$

while play in state $P_1 P_1$ implies an EPDV

$$V_j^{P_1 P_1} = \frac{(1 - \alpha) \delta^T}{1 - (1 - \alpha)^2 \delta - (1 - (1 - \alpha)^2) \delta^{T+1}} \pi^m. \qquad (1.3.4)$$

Note that $V_j^{CC} > V_j^{P_1 P_1} > 0$ and that both V_j^{CC} and $V_j^{P_1 P_1}$ are strictly decreasing in T.

Since temporary reversion to the one-period Bertrand-Nash equilibrium is a credible punishment, in order for the collusive PQA to be credibly sustainable with strategy R firms must find it optimal to abide by the prescribed strategy also in state CC. Note that the elementary remarks concerning possibly beneficial deviations made in the single-market case remain valid. In each market, there is a single best deviation from the PQA that exploits both decision margins fixed by the agreement. With cross-market linkage, however, the incentive condition for a simultaneous deviation in both markets does not anymore replicate the incentive conditions of independent deviations in markets 1 and 2. For arbitrary reciprocally proportional sharing rules in the two markets, there are three different kinds of possible deviation in state CC: a simultaneous deviation in both markets ($D12$), a deviation in market 1 while maintaining the agreement in market 2 ($D1$), and the opposite, a deviation in market 2 while abiding by the agreement in market 1 ($D2$).

Proposition 2 In comparison with strategy U, R-linkage implies a scope of collusion and a maximum level of uncertainty compatible with collusion that are strictly smaller, and the maximum achievable collusive expected present discounted value is strictly lower for all possible combinations of $\alpha > 0$ and δ.

More concretely, collusion can be credibly sustained if

$$\delta(1-\alpha) > \frac{1}{2(1-\alpha)}. \tag{1.3.5}$$

If $\delta \to 1$, collusion remains sustainable for levels of uncertainty $\alpha < 1 - \frac{1}{\sqrt{2}}$. Given δ and α, the maximum achievable collusive expected present discounted value is

$$V_j^{CC}(T^*) = \frac{\left(2(1-\alpha)^2 - 1\right)\pi^m}{(1-\alpha)(1-\delta)}. \tag{1.3.6}$$

The determination of the optimal deviation in case of R-linkage and the derivation of the incentive condition are relegated to appendix B.1. The relevant incentive condition with strategy R, $V_j^{CC} \geq V_j^{D12}$, is

$$2(1-\alpha)^2\delta + \left(1 - 2(1-\alpha)^2\right)\delta^{T+1} \geq 1 \tag{1.3.7}$$

for both firms independently of the sharing rule s chosen by the firms. The parameter range $\alpha < 1 - \frac{1}{\sqrt{2}}$, in which the PQA is sustainable with strategy R for appropriate δ, is a strict subset of the corresponding α-range with strategy U. For levels of α within this range, $V_j^{CC} - V_j^{D12}$ is strictly smaller than $V_j^{C} - V_j^{D}$ with strategy U, conditional on an equal punishment length T. This implies that the respective scopes of collusion exhibit a strict inclusion relation. Figure 1.3.1 depicts the scopes of collusion graphically. The scope of collusion with strategy U is represented in diagonal hatching, the strictly smaller scope with strategy R in lighter vertical hatching.
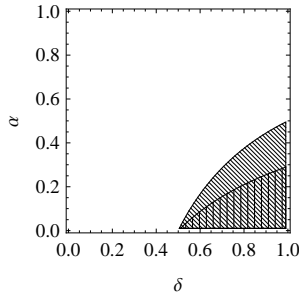

Figure 1.3.1: Scopes with strategies R (vertical) and U

Conditional on a given punishment length T, also V_j^{CC} with strategy R is strictly lower than the corresponding value $2V_j^C$ with strategy U. This comparison ignores the fact that the minimum punishment length T necessary to sustain collusion may be shorter in the case of R-linkage. If punishment is overall less costly along the equilibrium path, the EPDV attainable in equilibrium could be effectively higher with strategy R. The fact that incentives are weaker excludes this possibility, however. In appendix B.1 it is confirmed that the equilibrium length of punishment is strictly shorter with strategy U. Under imperfect monitoring, relentless linkage not only implies an enforcement power that falls short of the one characterising unlinked implementation, but also a strictly larger punishment length and a lower collusive EPDV in equilibrium.

The scopes of collusion and collusive EPDVs with unlinked and R-linked implementation strategies do not coincide for $\alpha > 0$. This finding stands in contrast to the irrelevance result in Bernheim and Whinston (1990). Intrinsic links and inter-market differences absent, multimarket contact and cross-market linkage do not affect the scope of collusive agreements if monitoring is perfect. Under imperfect monitoring, however, the possibility of strategically linking behavioural rules in some markets to observations made in other markets matters even when the shocks are i.i.d. across identical marketplaces and no inter-market differences exist in expected terms. Except in the limit for $\alpha \to 0$, relentless market linkage is decidedly counterproductive from the point of view of collusion-prone competing firms for two reasons. First, R-linkage hampers the monitoring process by making a rough usage of the scarce available information. In addition, it proves to be a rather expensive way of implementing collusive agreements in an environment where firms periodically experience punishment phases along the equilibrium path. Before commenting more thoroughly on these two reasons, the case of P-linkage is presented.

Placable linkage.   The differences between perfect and imperfect monitoring concerning the relevance of multimarket contact for the implementation of collusive agreements do not end here. The superiority of unlinked implementation over R-linkage, i.e., the advantage of selective, partial forms of punishment that are intentionally targeted at the particular type of deviating behaviour that is most likely to have caused the publicly observed outcome, is not the only respect in with the irrelevance result fails to hold under imperfect monitoring.

If self-enforcement relies on retrospective verification of actual defection, R-linkage is the only kind of cross-market linkage that allows credibly sustaining collusion. When implementation is based on the observation of outcomes that are only vaguely correlated with actions, however, an exclusive focus on R-linkage does not allow drawing a complete picture of the relevance of strategic linkage. Imperfect monitoring expands the scope of cross-market linkage in the sense that now there exist additional ways of linking punishment behaviour to observations that enable firms sustaining collusive agreements. Consider the following spatially linked strategy.

Strategy P The *placable intertemporal strategy* defines the following trigger event.

- In both markets $i = 1, 2$ the game starts in a collusive phase. Firms charge price $p = p^m$, sell $sD(p^m)$ or $(1 - s)D(p^m)$ units according to the agreement and continue doing so until one firm makes a ZDE in both markets simultaneously, i.e., in market i and in market $-i$. A single ZDE in either one of the two markets has no effect and play remains in the current collusive phase. If one or both firms face zero demand in both markets, play switches to a punishment phase in both markets.

- From the next period onwards, both firms play the stage game Nash equilibrium strategy, announce $p = c$ and sell $\frac{1}{2}D(c)$ units to consumers for T periods in both markets. In period $T + 1$, firms revert to collusive actions in both markets.

Clearly such a strategy does not allow sustaining collusive agreements under perfect monitoring, when trigger events are responsive to deviations themselves. In such a context, placable linkage "exempts" single-market deviations from punishment altogether by making the trigger event unresponsive to this kind of deviation. The incentive to deviate in a single market is then just too strong. Under imperfect monitoring, however, the trigger event is responsive not to actual deviations in the past but to ZDEs made in the past. With placable linkage, single-market deviations do not trigger selective punishment in the respective market, but they do increase the probability of occurrence of a phase with "full" punishment, i.e., of punishment in both markets in the future. The trigger event thus remains responsive to single-market deviations and thereby reduces their attractiveness

with respect to the case of perfect monitoring. Placable linkage makes single-market deviations more attractive both under perfect and imperfect monitoring, but under imperfect monitoring only moderately so. In comparison with relentless linkage, placable linkage effectively inverts the deviation incentives of colluding firms: they prefer single-market deviation to a deviation in both markets. This inversion turns out to be a forceful device that allows sustaining collusive agreements in situations with substantial levels of uncertainty, in which sustainability becomes impossible with unlinked strategies (and, all the more so, with R-linked strategies).

Consider play in state CC with strategy P. With probability $(1 - \alpha)^2$, demand is high in both markets and firms get the respective collusive profits. With probability $2(1 - \alpha)\alpha$, both firms make zero profits in one market and collusive profits in the other. In all these cases, play continues in state CC at least one more period. With probability α^2, however, when firms make zero demand experiences in both markets, play switches to state $P_1 P_1$. Under stationarity the dynamic path of expected present and future profits of firm A, starting in an arbitrary period in state CC, satisfies

$$
\begin{aligned}
V_A^{CC} &= (1 - \alpha)^2 \left(s\pi_1^m + (1 - s)\pi_2^m + \delta V_A^{CC} \right) \\
&\quad + (1 - \alpha)\alpha \left(s\pi_1^m + \delta V_A^{CC} \right) \\
&\quad + \alpha(1 - \alpha) \left((1 - s)\pi_2^m + \delta V_A^{CC} \right) + \alpha^2 \delta V_A^{P_1 P_1}
\end{aligned}
$$

or equivalently,

$$
V_A^{CC} = (1 - \alpha)\pi^m + \left(1 - \alpha^2 \right) \delta V_A^{CC} + \alpha^2 \delta V_A^{P_1 P_1}. \qquad (1.3.8)
$$

In the absence of inter-market differences, firm B's EPDV satisfies an identical equation.

Recall from the previous discussion that cross-market linkage does not affect the behaviour prescribed by strategy P in a punishment phase. The recursive structure of the EPDVs of expected present and future profits of firm $j \in \{A, B\}$ in the very first period of a punishment phase therefore remains unchanged. It satisfies

$$
V_j^{P_1 P_1} = \delta^T V_j^{CC}, \qquad (1.3.9)
$$

same as in the cases of strategies U and R.

Solving the system of equations 1.3.8 and 1.3.9 yields the following

expressions of the expected present discounted values of the two states under stationarity for both firms $j = A, B$: with strategy P, the value of colluding in both markets is

$$V_j^{CC} = \frac{(1 - \alpha)}{1 - (1 - \alpha^2)\delta - \alpha^2 \delta^{T+1}}\, \pi^m \qquad (1.3.10)$$

while play in state $P_1 P_1$ implies an expected present discounted value

$$V_j^{P_1 P_1} = \frac{(1 - \alpha)\delta^T}{1 - (1 - \alpha^2)\delta - \alpha^2 \delta^{T+1}}\, \pi^m. \qquad (1.3.11)$$

Note that $V_j^{CC} > V_j^{P_1 P_1} > 0$. Both V_j^{CC} and $V_j^{P_1 P_1}$ do not depend on s and are strictly decreasing in T.

The observations concerning potentially beneficial deviations made in the previous cases apply. There is a single best deviation from the PQA in each market; this deviation can be played in each of the two single markets or in both markets simultaneously, giving rise to three different incentive conditions in state CC for a simultaneous deviation in both markets ($D12$), a deviation in market 1 but not in market 2 ($D1$), and the opposite, a deviation solely in market 2 ($D2$). With R-linkage, deviation $D12$ resulted being the unique best deviation for all possible parameter constellations (α, δ), $\alpha, \delta \in (0, 1)$. This shows that Bernheim and Whinston's line of argumentation[18] essentially applies under imperfect monitoring if a strategically equivalent linkage is used. What causes the irrelevance result to fail with strategy R are the effects of linkage on the monitoring precision and the cost of implementation. The first of these effects is irrelevant and the second one absent under perfect monitoring. Strategy P causes a more fundamental change in the incentive structure. As a consequence, the analysis of one-period best responses in a collusive phase is more involved.

Recall the incentives with strategy R. A double-market deviation implies a strictly higher expected one-period payoff than single-market deviations; the continuation value, however, is the same for $D12$, $D1$ and $D2$. With strategy P, a double-market deviation also implies a strictly higher expected one-period payoff than single-market deviations; but single- and double-market deviations do not anymore yield

---

[18]Even if there is "more scope for punishing deviations in any one market", multi-market contact may fail to facilitate collusion because "once a firm knows that it will be punished in every market, if it decides to cheat, it will do so in every market." Bernheim and Whinston (1990), p. 3.

the same continuation value. Strategy $D12$ implies stage game profits of $2\pi^m$ in the favourable state of aggregate demand but provokes the occurrence of a double ZDE with probability one, the only event that triggers a punishment phase. Strategy $D1$ instead yields a strictly lower one-period profit $(2 - s_j)\pi^m$ if demand is positive, but it allows to avoid triggering a punishment phase with probability $1 - \alpha$. Strategy $D2$ implies stage game profits of $(1 + s_j)\pi^m$ if demand is positive in both markets and also allows avoiding a switch to punishment with probability $1 - \alpha$. Appendix B.2 states the induced EPDVs of the three candidates for best deviation and shows that colluding firms (i) always prefer to deviate in a single market only and (ii) are indifferent in which single market to deviate if the sharing rule is set equal to $s = \frac{1}{2}$ such as to maximise the overall scope of collusion. The results are summarised in the following proposition.

Proposition 3 In comparison with strategy U, P-linkage implies a scope of collusion that is strictly smaller if $\alpha \in \left[0, \frac{1}{3}\right)$ and strictly larger if $\alpha \in \left(\frac{1}{3}, \frac{2}{3}\right)$. The scopes coincide if $\alpha = \frac{1}{3}$. For $\alpha \to 0$, when the scope with strategy U is largest, collusion becomes impossible to sustain with strategy P. At the other extreme, the maximum level of uncertainty compatible with collusion is strictly higher with strategy P.

For parameter constellations (α, δ) that allow sustaining the agreement both with strategies P and U, the equilibrium punishment length is strictly higher with P-linkage if $\alpha \in \left[0, \frac{1}{3}\right)$ and strictly higher with strategy U if $\alpha \in \left(\frac{1}{3}, \frac{1}{2}\right)$. Irrespective of the punishment length, the collusive EPDV achievable in equilibrium is always strictly higher with strategy P.

With P-linkage collusion can be credibly sustained if

$$\delta(1 - \alpha) > \frac{1}{1 + 3\alpha}. \tag{1.3.12}$$

If $\delta \to 1$, collusion remains sustainable for levels of uncertainty $\alpha < \frac{2}{3}$. Given δ and α, the maximum achievable collusive expected present discounted value is

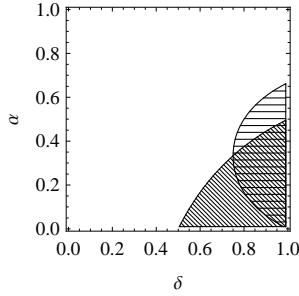$$V_j^{CC}(T^*) = \frac{(2 - 3\alpha)\pi^m}{2(1 - \delta)}. \tag{1.3.13}$$

Figure 1.3.2: Scopes with strategies P (horizontal) and U

How does strategy P perform in comparison with strategy U? The finding that for high enough discount factors P-linkage allows sustaining collusion for levels of uncertainty up to $\alpha = \frac{2}{3}$ while with unlinked strategies collusion is not possible if the negative demand shocks hampering the monitoring process happen with probability higher or equal $\alpha = \frac{1}{2}$ conveys the first central message: placable linkage allows sustaining collusion under levels of uncertainty that are substantially higher than unlinked implementation strategies or R-linkage. Under P-linkage, the impact of the cost of equilibrium punishment on incentives is mitigated.

The relation between the scopes of collusion with strategies P and U, however, is more complicated than the simple inclusion relation characterising the comparison of strategies R and U. In the parameter range $\alpha \in \left(0, \frac{1}{2}\right)$, when collusion is sustainable at least for some parameter constellations with both strategies P and U, $V_j^C - V_j^D$ is strictly decreasing in α in the case of strategy U while the relevant $V_j^{CC} - V_j^{D\text{single}}$ with strategy P is non-monotonic, hump-shaped in α. Incentives become stronger and hence the scope of collusion larger with strategy U in the range $\alpha \in \left(0, \frac{1}{3}\right)$. When the level of uncertainty is moderate, selective punishment with strategy U proves to be the most powerful device for sustaining collusion. In the range $\alpha \in \left(\frac{1}{3}, \frac{1}{2}\right)$, collusion remains sustainable with unlinked strategies but the scope of collusion is larger with P-linkage. For even higher levels $\alpha \in \left[\frac{1}{2}, \frac{2}{3}\right)$, collusion is sustainable only with placable linkage. Since the scope with placable linkage is overall hump-shaped in α, collusion eventually becomes unsustainable for $\alpha \geq \frac{2}{3}$, when demand shocks obfuscating the identification of deviant behaviour happen too frequently. Figure 1.3.2 depicts the two scopes of collusion graphically. The scope of col-

lusion with strategy U is represented in diagonal hatching, the scope with strategy P in lighter horizontal hatching.

In appendix B.2 it is shown how the non-monotonic relation of strategies U and P characterising the scopes of collusion echoes in the respective equilibrium lengths of punishment. The enforcement power is stronger with strategy U if $\alpha < \frac{1}{3}$; if the monitoring process is less precise, strategy P turns out to facilitate enforcement.

Perhaps surprisingly, this dependence of the scope and equilibrium length of punishment on the level of uncertainty α does not translate in the maximum achievable collusive EPDV. In equilibrium, abiding by the agreement is worth strictly more with strategy P than with strategy U, independently of the level of uncertainty. It is shown that the dynamic implementation strategies can be strictly ranked in terms of the collusive EPDVs independently of α and δ. Figure 1.3.3 plots the collusive EPDVs in equilibrium, when the punishment length T is chosen such as to maximise V_j^{CC}, as a function of α in the case of unlinked strategies (solid), relentless linkage (dashed) and placable linkage (dotted) for $\delta = 0.9$ (and $\pi_i^m = \frac{1}{4}$ in markets $i = 1, 2$). Expressions 1.2.7, 1.3.6 and 1.3.13 reveal that the discount factor and the collusive stage-game profits affect the absolute value, but not the relative performance of the three implementation strategies.
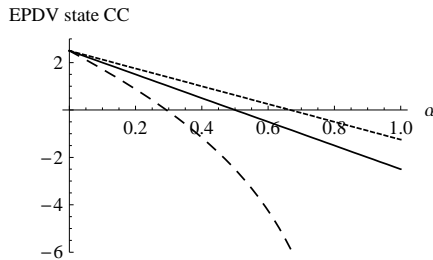
Figure 1.3.3: $V_j^{CC}(T^*)$ with strategies R, U and P

Information, incentives and the cost of retaliation. With respect to unlinked implementation, both types of linkage have the disadvantage of sacrificing publicly verifiable information and therefore suffering from informational imprecision.[19] For positive but moderate

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[19]With linkage, the respective trigger events and return rules are not responsive to all three possible observations (ZDEs in zero, one or two markets) but to

levels of uncertainty, unlinked strategies are therefore strictly superior to any type of linkage. But the informational aspect is only one of three factors affecting incentives under multimarket contact. The second one, the possibility of pooling slack enforcement power across markets, is absent when identical firms with constant returns to scale technologies interact in identical markets. As stressed by the pertinent perfect monitoring literature, relentless linkage is strictly necessary for this factor to be exploited. The model framework analysed is insofar slanted towards the relative advantages of strategies U and P. Also the third factor, the cost of implementation, plays decidedly against relentless linkage. Same as the informational factor, it is relevant only under imperfect monitoring, when punishment phases become an equilibrium phenomenon. In addition to the inefficient use of information, linkages that increase the frequency or the severity of punishment do not only hamper the enforcement power but also imply stronger value burning.

The comparison of strategy U with P-linkage and R-linkage reveals that the opportunistic behaviour optimally responding to collusive play is decidedly different. In the case of strategy U, the scope-maximising sharing rule $s = \frac{1}{2}$ and the value-maximising punishment length imply an equalisation of the incentives to deviate in both markets ($D12$), only in market 1 ($D1$) or only in market 2 ($D2$). As long as firms agree on the equal sharing rule, deviations $D1$ and $D2$ are equally attractive for both firms. In this case, let $V_j^{D\text{single}} := V_j^{D1} = V_j^{D2}$. While strategy U implies $V_j^{D12} = V_j^{D\text{single}}$ in equilibrium, with strategy R it can be shown that $V_j^{D12} > V_j^{D\text{single}}$ independently of T. If firms ever deviate from collusion, they optimally do so in both markets simultaneously. In analogy to Bernheim and Whinston's analysis under perfect monitoring, collusion is in this case effectively sustained against the threat of a double-market deviation, and the optimal deviation is independent of the chosen sharing rule as long as it is reciprocally proportional. On the contrary, strategy P implies $V_j^{D\text{single}} > V_j^{D12}$ in equilibrium and the optimal response to collusive play is deviating in a single market only. In this case, the incentives to deviate crucially hinge on the sharing rule; a firm wants to deviate in the market in which it has the smaller market share.

---

only two of them. P-linkage treats the events of zero and one observation of zero demand the same while R-linkage does not distinguish between the events of one and two such observations.

How do these changes in the incentive structure affect the sustainability of PQAs and the collusive EPDV attainable in equilibrium?

Perfect monitoring. Irrelevance comes about under perfect monitoring because in the absence of inter-market heterogeneity the deviation decision is de facto unswayed by R-linkage. Notice that, relative to U, R-linkage affects neither V_j^{CC} (under perfect monitoring there is no value burning) nor the immediate one-period gains from deviation. What changes is the continuation value in case of deviation. Conditional on T, R-linkage strictly reduces the continuation value of a single market deviation while leaving the continuation value of a double-market deviation unchanged. Since strategy U implies $V_j^{D12} = V_j^{D\text{single}}$, strategy R necessarily implies $V_j^{D12} > V_j^{D\text{single}}$. But if collusion is effectively sustained against V_j^{D12}, the reduction of $V_j^{D\text{single}}$ is irrelevant and the incentive condition effectively remains the same. Obviously, this line of argumentation is valid only if there is no slack enforcement power implying a shorter equilibrium length T with strategy R.

Imperfect monitoring. In the case of imperfect monitoring, the equilibrium adjustment of T causes this irrelevance to break down independently of inter-market heterogeneity. Start noticing that also under imperfect monitoring R-linkage

- does not affect the expected one-period gains from deviation;
- replaces single-market punishment by double-market punishment and thereby makes some instances of punishment (those induced by a single ZDE) more severe.

Conditional on T, the effect of R-linkage on the EPDVs of possible deviations is the same as under perfect monitoring. But now linkage also affects the EPDV of cooperation. The collusive value satisfies

$$
\begin{aligned}
V_j^{CC} &= (1-\alpha)\,\pi^m + (1-\alpha)^2 \delta V_j^{CC} + (1-\alpha)\alpha\delta V_j^{CP_1} \\
&\quad + \alpha(1-\alpha)\delta V_j^{P_1 C} + \alpha^2 \delta V_j^{P_1 P_1}
\end{aligned}
$$

in case of strategy U and

$$
V_j^{CC} = (1-\alpha)\,\pi^m + (1-\alpha)^2 \delta V_j^{CC} + \left(1 - (1-\alpha)^2\right)\delta V_j^{P_1 P_1}
$$

in case of strategy R. These expressions reflect the rougher use of information and the value burning effect due to R-linkage. If T is not otherwise affected, $(1-\alpha)\alpha\delta V_j^{CP_1} + \alpha(1-\alpha)\delta V_j^{P_1 C} > 2\alpha(1-$

$\alpha)\delta V_j^{P_1 P_1}$ and punishment in additional markets implies a decrease of V_j^{CC}. In equilibrium, the lower V_j^{CC} coincides with the EPDV of the best available deviation, which in case of R-linkage is $D12$. A lower V_j^{D12} corresponds to a higher T, such that the enforcement power has effectively been reduced by R-linkage.

An analogous argument applies to strategy P. With respect to strategy U, P-linkage leaves the continuation value of double-market deviations unchanged and strictly raises the continuation value of a single market deviation. With P-linkage single-market deviations become the unique best response to cooperating behaviour. Under perfect monitoring, $V_j^{D\text{single}}$ raises a lot (so much that the PQA becomes unsustainable with strategy P) while V_j^{CC} remains the same. Under imperfect monitoring, $V_j^{D\text{single}}$ raises less and V_j^{CC} also raises. P-linkage reduces the value burning effect relative to strategy U and a PQA remains sustainable with strategy U at least for some parametric constellations (α, δ). The equilibrium effect on T is in this case a priori undetermined. Proposition 3 states that the potential enforcement power of P-linkage relative to strategy U depends on the level of uncertainty α. If the level of α is substantial, the equalisation of V_j^{CC} and $V_j^{D\text{single}}$ implies a lower T, such that the enforcement power is increased by placable linkage. For more moderate levels of α, the enforcement power is larger with strategy U. In terms of the collusive EPDV resulting in equilibrium, it turns out that strategies U and P can be ranked independently of the level of uncertainty α. For any parameter constellation (α, δ) with $\alpha > 0$ that implies sustainability both with strategies U and P, strategy P allows realising strictly higher collusive gains.[20]

1.4 Conclusion

Under imperfect monitoring, the credible sustainability of collusion crucially depends on the use of the scarce available information in the monitoring process and the frequency and intensity with which

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[20]A more forceful result is found when more flexible, random trigger events are taken into account that allow avoiding the informational deficiency characterising strategy P. It can then be shown that for any parameter constellation (α, δ) with $\alpha > 0$ that implies sustainability with strategy U (and not necessarily with strategy P), there exists a more placable form of linkage P_+ that equally allows sustaining collusion and yields a strictly higher collusive EPDV. For a more detailed discussion of the informational aspect of cross-market linkage, see section 4 in the companion paper Schreibweis (2013).

punishment is exerted in equilibrium. The analysis sets out the role of these two elements for sustaining collusion in a multimarket environment. The possibility of linking behaviour across markets adds a degree of freedom to the design of implementation strategies. Both under perfect and imperfect monitoring, it allows pooling enforcement power not only intertemporally but also across markets. When non-cooperative action cannot be verified it moreover allows modulating the cost of enforcement. It has to be taken into account, however, that linkage is never informationally efficient.

In comparison with the case of perfect monitoring, three assertions can be made. First, linkage can generally be expected to be relevant also in the case of identical firms with constant returns to scale technologies that interact in identical markets. Second, in a context in which slack enforcement power is negligible, the particularly harsh R-linkage always exhibits a strictly inferior performance with respect to the alternative, less severe unlinked implementation that uses partial, selective punishment schemes. This inferior performance encompasses both the scope of collusion and the intertemporal gains that result from cooperation. Third, in environments in which monitoring is strongly handicapped by exogenous demand shocks unlinked implementation (and even more so R-linkage) may be dominated by a placable form of linkage. Even though P-linkage, same as R-linkage, is less efficient from the point of view of information processing in the monitoring process, it represents a cheap form of implementation that remains feasible when punishment phases happen frequently in equilibrium. Overall, the following picture emerges. In terms of the enforcement power and scope, the relevance of informational efficiency and the cost of implementation strongly depend on the level of uncertainty, i.e., on the degree of imprecision in the monitoring process. In terms of the maximum collusive EPDV attainable in equilibrium, the implementation cost effect can be shown to always dominate independently of the level of uncertainty. Given a parametric constellation in which collusion is credibly sustainable under various forms of implementation, firms prefer schemes characterised by partial and infrequent punishment. This finding stands in contrast to the "bang-bang" property characterising optimal symmetric intertemporal linkage in the case of single-market interaction, and is consistent with the evidence on real world firm behaviour as documented in cartel investigations.

The results need to be evaluated in their context. The literature on multimarket contact and collusion under perfect monitoring is con-

cerned with the exploitation of slack enforcement power due to differences in market structures and firm characteristics. This mechanism, first set forth by Bernheim and Whinston (1990), is expected to remain fully intact under imperfect monitoring. By assuming the absence of inter-market differences the present analysis deliberately abstracts from the collusion-enhancing effect of pooling incentive constraints. The abstraction allows conceptually separating and focusing on the implementation cost and the information processing effects of linkage, which are in turn absent under perfect monitoring.

A second remark concerns the superiority of placable forms of punishment and the seeming contrast to "bang-bang" punishment behaviour. Also in this respect, the findings do not contradict but qualify previous results. What the superiority of placable linkage in highly uncertain environments documents is a contrast between optimal punishment behaviours in the intertemporal and spatial dimensions. The recommendation of responding to a trigger event with an immediate, tough punishment that lasts no longer than strictly required remains valid. But the definition of the most adequate trigger event is more complex under multimarket contact. When the frequency of demand shocks interfering with the monitoring process is considerable, the likelihood of a ZDE in a single market being effectively caused by a deviation is small. A relentless response in all markets is then informationally inefficient and unnecessarily costly.

Appendices to Chapter One

Appendix A: Section 1.2

Appendix A.1: Proof of proposition 1

Equilibrium length of punishment and collusive EPDV. For sharing rule $s = \frac{1}{2}$, the rule that maximises the joint scope of collusion by relaxing to the largest possible extent the more stringent of the two incentive constraints, the binding sustainability condition $V_j^C = V_j^D$ characterising the equilibrium punishment length T^* in each market is equivalent to

$$2(1-\alpha)\delta - (1-2\alpha)\delta^{T^*+1} = 1$$

or

$$\delta^{T^*} = \frac{2(1-\alpha)\delta}{(1-2\alpha)\delta}$$

for both firms $j = A, B$. Observe that, if $\alpha < \frac{1}{2}$ and $\delta(1-\alpha) \geq \frac{1}{2}$, i.e., when collusion is sustainable with strategy U, both the numerator and denominator are strictly positive and $\delta^{T^*} \in (0, \delta)$. Substituting this expression into the stationary EPDV in an arbitrary period in state C yields

$$V_j^C(T_U^*) = \frac{(1-2\alpha)\pi^m}{2(1-\delta)},$$

which is the value of the stream of present and future equilibrium profits in each of the two markets in which the two firms are operating.

Appendix B: Section 1.3

Appendix B.1: Proof of proposition 2

Best deviation and scope of collusion. With R-linkage, all three alternative deviation strategies $D12$, $D1$ and $D2$ trigger state P_1P_1 in

the subsequent period with probability one and consequently imply the same continuation value $\delta V_j^{P_1 P_1}$. In the deviation period, conditional on positive demand in both markets the one-period payoff of firm A is $\pi_1^m + \pi_2^m$ in the case of deviation $D12$, $\pi_1^m + (1-s)\pi_2^m$ in the case of deviation $D1$ and $s\pi_1^m + \pi_2^m$ if deviation occurs only in market 2. Since in any PQA by definition $s \in (0,1)$, the deviation payoff is highest in the case of deviation $D12$ for all possible parameter constellations (α, δ). This holds true also in expected terms. The argument replicates in the case of firm B. The EPDV of the stream of present and future profits in the case of the only relevant, best deviation $D12$ satisfies

$$V_j^{D12} = (1-\alpha)2\pi^m + \delta V_j^{P_1 P_1}$$

for $j = A, B$. Under R-linkage, the sustainability condition $V_j^{CC} \geq V_j^{D12}$ is therefore for both $j = A, B$ equivalent to

$$2(1-\alpha)^2\delta + \left(1 - 2(1-\alpha)^2\right)\delta^{T+1} \geq 1.$$

For arbitrary $s \in (0,1)$, $V_j^{CC} - V_j^{D12}$ is strictly increasing in T for $(1-\alpha)^2 > \frac{1}{2}$, independent of T for $(1-\alpha)^2 = \frac{1}{2}$ and strictly decreasing in T for $(1-\alpha)^2 < \frac{1}{2}$. In the latter case, $V_j^{CC} - V_j^{D12}$ is maximised for $T \to 0$, ruling out the sustainability of collusion for any $\delta < 1$. The same happens in the case $(1-\alpha)^2 = \frac{1}{2}$. The only $\alpha \in (0,1)$ satisfying this equation is $\alpha = 1 - \frac{1}{\sqrt{2}}$. In this case, the sustainability condition is equivalent to $\delta \geq 1$ for any positive integer T. If $(1-\alpha)^2 < \frac{1}{2}$, or equivalently $\alpha < 1 - \frac{1}{\sqrt{2}}$, $V_j^{CC} - V_j^{D12}$ is maximised for $T \to +\infty$, implying that collusion is sustainable if $\delta(1-\alpha)^2 \geq \frac{1}{2}$.

In analogy to the case of unlinked implementation, the scope of collusion is defined to be the set of parameter constellations (α, δ) under which a PQA is sustainable if s and T are chosen such as to solve the problem

$$\max_{s,T} \left\{ \min_{j \in \{A,B\}} \left\{ V_j^C - V_j^{D12} \right\} \right\}.$$

With strategy R, the two incentive constraints are independent of s. The incentive constraints are therefore relaxed to the largest possible extent by choosing an arbitrary sharing rule $s \in (0,1)$ and setting the punishment length at its largest possible value, $T \to +\infty$. The scope

of collusion is then characterised by the condition

$$\delta(1-\alpha)^2 \geq \frac{1}{2}.$$

Equilibrium length of punishment and collusive EPDV. Independently of the chosen sharing rule s, the binding sustainability condition $V_j^{CC} = V_j^{D12}$ characterising the equilibrium punishment length T^* is equivalent to

$$2(1-\alpha)^2\delta + \left(1-2(1-\alpha)^2\right)\delta^{T^*+1} = 1$$

or

$$\delta^{T^*} = \frac{2(1-\alpha)^2\delta - 1}{\left(2(1-\alpha)^2 - 1\right)\delta}$$

for both firms $j = A, B$. If $\alpha < 1 - \frac{1}{\sqrt{2}}$ and $\delta(1-\alpha)^2 \geq \frac{1}{2}$, i.e., when collusion is sustainable with strategy R, both the numerator and denominator are strictly positive and $\delta^{T^*} \in (0, \delta)$. It can be verified that in case $\alpha < 1 - \frac{1}{\sqrt{2}}$, i.e., when collusion is sustainable both with R and U, $\delta^{T_R^*} < \delta^{T_U^*}$ and hence $T_R^* > T_U^*$. Substituting the expression into the stationary EPDV in an arbitrary period in state CC yields

$$V_j^{CC}(T_R^*) = \frac{\left(2(1-\alpha)^2 - 1\right)\pi^m}{(1-\alpha)(1-\delta)},$$

which is the value of the stream of joint present and future equilibrium profits of firm $j = A, B$ in both markets in which it is operating. If $\alpha < 1 - \frac{1}{\sqrt{2}}$, $V_j^{CC}(T_R^*) < 2V_j^{C}(T_U^*)$ for all constellations (α, δ) that satisfy the incentive constraint.

Appendix B.2: Proof of proposition 3

Best deviation and scope of collusion. The expected present discounted values of the relevant deviations are

$$V_j^{D12} = (1-\alpha)2\pi^m + \delta V_j^{P_1P_1}$$

for $j = A, B$ in the case of a double-market deviation and, for firm A,

$$
\begin{aligned}
V_A^{D1} &= (1-\alpha)(2-s)\pi^m + (1-\alpha)\delta V_A^{CC} + \alpha\delta V_A^{P_1P_1}, \\
V_A^{D2} &= (1-\alpha)(1+s)\pi^m + (1-\alpha)\delta V_A^{CC} + \alpha\delta V_A^{P_1P_1}
\end{aligned}
$$

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in the case of single-market deviations. Firm B gets the complementary share in in market in which it does not deviate from collusive play. The EPDVs of actions $D1$ and $D2$ chosen by firm B therefore satisfy

$$
\begin{aligned}
V_B^{D1} &= (1-\alpha)(1+s)\pi^m + (1-\alpha)\delta V_B^{CC} + \alpha\delta V_B^{P_1P_1}, \\
V_B^{D2} &= (1-\alpha)(2-s)\pi^m + (1-\alpha)\delta V_B^{CC} + \alpha\delta V_B^{P_1P_1}.
\end{aligned}
$$

Sustainability against deviation $D12$. Firm A prefers $D12$ to $D1$ if

$$
\frac{1 - (1-\alpha^2)\delta - \alpha^2\delta^{T+1}}{\delta - \delta^{T+1}} \geq \frac{1-\alpha}{s}
$$

and $D12$ to $D2$ if

$$
\frac{1 - (1-\alpha^2)\delta - \alpha^2\delta^{T+1}}{\delta - \delta^{T+1}} \geq \frac{1-\alpha}{1-s}.
$$

The set of parameter constellations in which $D12$ is preferred to $D1$ is consequently increasing in s while the set of parameter constellations in which $D12$ is preferred to $D2$ is decreasing in s. The set of (α, δ) in which $D12$ is the optimal deviation is maximised for $s = \frac{1}{2}$. An analogous argument for firm B implies that $D12$ is the best deviation for both firms $j = A, B$ if

$$
\frac{1 - (1-\alpha^2)\delta - \alpha^2\delta^{T+1}}{\delta - \delta^{T+1}} \geq 2(1-\alpha).
$$

Under P-linkage the collusive agreement is sustainable against deviation $D12$ if $V_j^{CC} \geq V_j^{D12}$ for both firms $j = A, B$. This incentive condition, which is independent of s and identical for firms $j = A, B$, is equivalent to

$$
2(1-\alpha)^2\delta + \left(2\alpha^2 - 1\right)\delta^{T+1} \geq 1
$$

or

$$
\frac{1 - (1-\alpha^2)\delta - \alpha^2\delta^{T+1}}{\delta - \delta^{T+1}} \leq (1-\alpha^2).
$$

Since $1-\alpha^2 < 2(1-\alpha) \Leftrightarrow \alpha < 1$, the collusive PQA is not sustainable in all parameter constellations in which $D12$ is the best deviation.

Sustainability against single-market deviations. $D1$ is the best devia-

tion for firm A if

$$s \leq \min\left\{\frac{(1-\alpha)\left(\delta-\delta^{T+1}\right)}{1-(1-\alpha^2)\delta-\alpha^2\delta^{T+1}}, \frac{1}{2}\right\}$$

while $D2$ is the best deviation for firm A if

$$s \geq \max\left\{\frac{1-\delta-(1-\alpha-\alpha^2))\left(\delta-\delta^{T+1}\right)}{1-(1-\alpha^2)\delta-\alpha^2\delta^{T+1}}, \frac{1}{2}\right\}.$$

Under P-linkage firm A prefers colluding rather than playing action $D1$ if

$$\frac{1-(1-\alpha^2)\delta-\alpha^2\delta^{T+1}}{\delta-\delta^{T+1}} \leq \frac{\alpha(1-\alpha)}{1-s}$$

and prefers not to play deviation $D2$ if

$$\frac{1-(1-\alpha^2)\delta-\alpha^2\delta^{T+1}}{\delta-\delta^{T+1}} \leq \frac{\alpha(1-\alpha)}{s}.$$

The set of parameter constellations in which, from the perspective of firm A, abiding by the agreement is preferred to a single-market deviation in market 1 is strictly increasing in s while the set in which abiding by the agreement is preferred to $D2$ is strictly decreasing in s. The set of parameter constellations in which firm A is not tempted by single-market deviations is consequently maximised if $s = \frac{1}{2}$. An analogous line of argumentation yields the same result for firm B. By reciprocal proportionality, in any parameter constellation in which firm A prefers $D1$ to $D2$, firm B prefers $D2$ to $D1$. Taking into account both kinds of single-market deviation, the underlying incentive structures coincide. Note that, for each firm, s determines the relative attractiveness of the two possible deviations in market 1 or market 2 for the same familiar reason that is striking in deterministic single-market models with asymmetric market shares: the gain from deviation is larger for the firm that has a smaller market share. In otherwise identical markets, the maximum incentive to deviate is therefore minimised by agreeing on symmetric market shares. The optimal choice $s = \frac{1}{2}$ equalises the incentives across both kinds of single-market deviation and it also equalises the incentives for single-market deviation of both firms. Given the reciprocally proportional sharing rule, these are necessary conditions for maximising the scope of collusion.[21]

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[21]The possibility to attenuate the situation of conflict by choosing an adequate

If $s = \frac{1}{2}$,

$$V_j^{D1} = V_j^{D2} = V_j^{D\text{single}} := (1 - \alpha)\frac{3}{2}\pi^m + (1 - \alpha)\delta V_j^{CC} + \alpha\delta V_j^{P_1 P_1}$$

for both firms $j = A, B$ and the sustainability condition $V_j^{CC} \geq V_j^{D\text{single}}$ writes

$$\frac{1 - (1 - \alpha^2)\delta - \alpha^2 \delta^{T+1}}{\delta - \delta^{T+1}} \leq 2\alpha(1 - \alpha).$$

In this case both firms $j = A, B$ prefer a single-market deviation over a double-market deviation iff $V_j^{D12} \geq V_j^{D\text{single}}$ or

$$\frac{1 - (1 - \alpha^2)\delta - \alpha^2 \delta^{T+1}}{\delta - \delta^{T+1}} \leq 2(1 - \alpha).$$

Conditional on the length of punishment T, the set of parameter constellations in which the agreement is sustainable against single-market deviations forms a strict subset of the set of parameter constellations for which single-market deviations are preferred to double-market deviations. The incentive condition can equivalently be written in the form

$$(1 + \alpha(2 - 3\alpha))\,\delta + \alpha(3\alpha - 2)\delta^{T+1} \geq 1.$$

This formulation reveals that $V_j^{CC} - V_j^{D\text{single}}$ is strictly increasing in T for $\alpha < \frac{2}{3}$, independent of T for $\alpha = \frac{2}{3}$ and strictly decreasing in T for $\alpha > \frac{2}{3}$. In the latter case, $V_j^{CC} - V_j^{D\text{single}}$ is maximised for $T \to 0$, ruling out the sustainability of collusion for any $\delta < 1$. The same happens in case $\alpha = \frac{2}{3}$, where the sustainability condition is equivalent to $\delta \geq 1$ for any positive integer T. If $\alpha < \frac{2}{3}$, on the other hand, $V_j^{CC} - V_j^{D\text{single}}$ is maximised for $T \to +\infty$, implying that collusion is sustainable if $\delta(1-\alpha) \geq \frac{1}{1+3\alpha}$. Since $2 > 1+3\alpha \Leftrightarrow \alpha < \frac{1}{3}$, the scope of collusion with strategy U is larger than the scope with P-linkage if $\alpha < \frac{1}{3}$.

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sharing rule may in practice well be the most important channel through which multimarket contact enhances the sustainability of collusive agreements. This choice, which belongs to the realm of equilibrium selection, is not made explicit in models of collusion based on the far-sightedness of rational agents involved in repeated interaction.

Equilibrium length of punishment and collusive EPDV. Conditional on a given punishment length T, the collusive EPDV V_j^{CC} in the case of strategy P is strictly higher than the corresponding value with strategy U. This comparison ignores, however, that the optimally chosen minimum punishment length T necessary to sustain the PQA will differ. With strategy P, the binding sustainability condition $V_j^{CC} = V_j^{D\text{single}}$ characterising the equilibrium punishment length T^* is equivalent to

$$\left(1 + \alpha(2 - 3\alpha)\right)\delta + \alpha(3\alpha - 2)\delta^{T^*+1} = 1$$

or

$$\delta^{T^*} = \frac{\alpha(2 - 3\alpha)\delta - (1 - \delta)}{\alpha(2 - 3\alpha)\delta}$$

for both firms $j = A, B$. If $\alpha < \frac{2}{3}$ and $\delta(1 - \alpha) \geq \frac{1}{1+3\alpha}$, i.e., when collusion is sustainable with strategy P, both the numerator and denominator are strictly positive and $\delta^{T^*} \in (0, \delta)$. In the case of strategy P one has to account for the possibility that the optimal choice T^* does imply a change of the optimal deviation. Solving the condition $V_j^{D12} \geq V_j^{D\text{single}}$ for δ^T reveals that both firms prefer a double-market deviation over a single-market deviation iff

$$\begin{cases} \alpha \in \left[0, -1 + \sqrt{3}\right) & \text{and} \quad \delta^T \geq \frac{(2 - \alpha(2+\alpha))\delta - (1-\delta)}{(2 - \alpha(2+\alpha))\delta}, \\ \alpha = -1 + \sqrt{3}, \\ \alpha \in \left[-1 + \sqrt{3}, 1\right) & \text{and} \quad \delta^T \leq \frac{(2 - \alpha(2+\alpha))\delta - (1-\delta)}{(2 - \alpha(2+\alpha))\delta}. \end{cases}$$

Since $\frac{2}{3} < -1 + \sqrt{3}$, the first one is the relevant case. Since

$$\delta^{T^*} < \frac{(2 - \alpha(2+\alpha))\delta - (1 - \delta)}{(2 - \alpha(2+\alpha))\delta}$$

for all constellations (α, δ) that allow for sustainability with strategy P, the analysis of best deviations is consistent.

In case $\alpha < \frac{1}{2}$, i.e., when collusion is sustainable both with R and with U, $\delta^{T_{\mathrm{P}}^*} < \delta^{T_{\mathrm{U}}^*}$ and hence $T_{\mathrm{P}}^* > T_{\mathrm{U}}^*$ iff $\alpha < \frac{1}{3}$. In the range $\alpha \in \left(\frac{1}{3}, \frac{1}{2}\right)$, the equilibrium punishment length is shorter with strategy P. Substituting $\delta^{T_{\mathrm{P}}^*}$ into the stationary EPDV in an arbitrary period in state CC yields

$$V_j^{CC}(T_{\mathrm{P}}^*) = \frac{(2 - 3\alpha)\pi^m}{2(1 - \delta)},$$

which is the value of the stream of joint present and future equilibrium profits of firm $j = A, B$ in both markets in which it is operating. If $\alpha < \frac{1}{2}$, $V_j^{CC}(T_{\mathrm{P}}^*) > 2V_j^C(T_{\mathrm{U}}^*)$ for all constellations (α, δ) that satisfy the incentive constraint.

2 Collusive intra-industry trade under imperfect public monitoring

2.1 Introduction

The hypothesis that exposing home industries to foreign trade fosters competition can safely be considered conventional wisdom. The competitive effects and welfare gains from intra-industry trade were examined more carefully when new trade theory took up forces. Markusen (1981) was probably the first to analyse the question in a two-country model with mutual trade. He showed that in the symmetric case there are bilateral gains from trade "due to the reduction of domestic monopoly power".[1] While the idea that a reduction of trade and transaction costs intensifies the competitive forces is unquestioned, the assertion that trade liberalisation or market integration lead to a superior market performance measured in terms of consumer surplus or overall welfare is less clear.

Allocative gains from potential or actual competition induced by trade liberalisation play a prominent role in models with mutual trade costs like the reciprocal dumping model in Brander (1981) and Brander and Krugman (1983). In these models, trade has a second effect that is not strategic: it induces productive efficiency losses due to the waste in transport. Liberalisation reduces welfare when the loss in productive efficiency dominates the gain in allocative efficiency. Productive efficiency losses, however, are not necessary to overturn Markusen's bilateral gains. Haubrich and Lambson (1986) gave an example of a trade liberalisation that is welfare-reducing for purely strategic reasons. Firms exposed to harsh competition have strong incentives and come up with collusive strategies to circumvent these pressures.[2] In a

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[1]Markusen (1981), p.532.

[2]Shapiro (1989), p. 357, termed the phenomenon "topsy-turvy principle" of tacit

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context of repeated interaction, trade liberalisation has two opposed effects on incentives: larger capture due to the market size effect and stronger retaliation in a more competitive environment.

Later contributions have integrated these ideas using models that explicitly account for the relevance of trade costs and the possibility of self-enforcing collusive behaviour. In this literature two findings are recurrent. Since only domestic trade ensures the avoidance of waste in transport, market sharing appears to be necessary for joint profit maximisation. Moreover, since trade costs imply slack enforcement power, a strategic linkage that allows pooling this slack is the only dynamic strategy that ensures a maximum scope of collusion. The present article argues that both results are the a direct consequence of the exclusive focus on perfect monitoring. It presents a functionally specified example that illustrates when and why these results cannot be expected to hold when firms are forced to rely on imperfect public monitoring for sustaining collusive agreements. The analysis emphasises the relevance of cross-border trade for the obtainment and efficient use of the scarcely available information when actions cannot be directly observed.

Trade cost reductions and the prosecution of international cartels.
Two pieces of empirical evidence have repeatedly sparked interest on the topic. First, the continuing substantial reductions of trade costs over the last decades due to technological, organisational and also regulatory innovation despite the difficult and slow progress within the successive rounds of multilateral trade negotiations. A second centre of attention has been the drastic expansion of antitrust enforcement activities targeted at international cartels starting in the early 1990s.

Some components of trade costs exhibit enormous reductions over the last decades.[3] The average import tariff has fallen from an estimated 20–30% before the Geneva Round in 1947 to approximately 14% in 1952 and steadily thereafter to 3.9% in 2005. In transport costs, the most notable reductions have been in air transport. The revenue per ton-kilometre experienced a reduction of 92% from 1955 to 2004 due

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collusion in infinitely repeated oligopoly games and alerted of its ambivalent nature: "This relationship tends to create some peculiar results: anything (such as unlimited capacities) that makes more competitive behavior feasible or credible actually promotes collusion."

[3] A short account of the empirical evidence on the fall of international trade costs since WWII can be found in the World Trade Report 2008.

to broader-based implementation of technological advances like the jet engine (in the 1960s) and organisational improvements like the Open Skies Agreements implemented in the 1990s. Similar developments in technology and regulation have also profoundly reduced the costs of communication and connectivity. Reported data for Germany show a 60% reduction in prices for domestic phone calls and a reduction of over 90% in international calls from 1947 to 2007.

These numbers need to be put into perspective in two respects. First, there are other cost components that present less pronounced reductions. Glaeser and Kohlhase (2003) calculate that rising fuel prices and regulations caused the price of road transport to be more or less stable over the period 1947 to 1985 in the U.S..[4] In ocean shipping, it seems puzzling that the cost saving effect of technological and institutional innovations like containerisation and open registry are hardly detectable in the data.[5] Problems of measurement and lack of data also complicate the evaluation of cost reductions induced by the elimination of non-tariff barriers like quantitative restrictions, subsidies or standard and technical regulations. The World Trade Report concludes that "A higher number of NTBs over time is more likely to be the result of a better recording of NTBs than an increase in the number."[6]

Second, drastic reductions in several dimensions should not detract from the fact that trade costs are or remain very substantial in many industries. "The death of distance is exaggerated." This is how Anderson and van Wincoop (2004) summarise their empirical findings about the magnitude and relevance of trade costs. They estimate the average trade costs equivalent to a 170% ad valorem tax, breaking down "into 55% local distribution costs and 74% international trade costs

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[4]They report that the prices for trucking services have fallen since the Motor Carrier Act in 1980. For railroads, data show a reduction from 18 cents per ton-mile in 1890 to 2.3 cents in 2000 (measured in 2001 US dollars).

[5]Hummels (2007) describes this puzzle in detail. His first explanation is the fuel prices, ship prices and port costs which rose an annual 14–18% during the 1970s following the oil price shocks. A second explanation appears to be relevant beyond the ocean shipping cost puzzle: the time cost of transport. Containerisation allows for a much faster loading and unloading, an improvement that does not show up in the price indices. Hummels (2001) estimates the time cost of a day's delay in transport and concludes that each day in transit equals 0.8% of the value of the manufactured good, implying that a delay of three days only approximately equals the average worldwide tariff.

[6]Word Trade Organization (2008), page 82.

$(1.7 = 1.55 * 1.74 - 1)$."[7] The disregard of trade costs can certainly generate seriously distorted results in many respects.

Being the object of active and publicly debated policies pursued by national and supranational antitrust authorities, the second obstacle to welfare-enhancing trade liberalisation has become prominent beyond academia. There are early instances of prosecution of international cartels,[8] but the enforcement activity against cartels operating in global markets, the number of prosecutions and the dimension of fines imposed until the 1980s are not comparable to the attention paid to international cartels nowadays. In 2005 Scott Hammond, Deputy Assistant Attorney General for Criminal Enforcement, declared that "Of the nearly $3 billion in criminal fines imposed in Division cases since FY 1997, well over 90% were obtained in connection with the prosecution of international cartel activity."[9]

Why has there been such an expansion of disclosed and prosecuted international collusive agreements? Is this observation indicative of an upsurge in international cartel activity? Is it related to trade cost reductions and recent processes of market integration? These questions remain open. In its 1997 Annual Report, the WTO pointed at "some indications that a growing proportion of cartel agreements are international in scope."[10] In his survey article, Bond (2004) admits a

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[7]See Anderson and van Wincoop (2004), pages 691 and 692. The cost components they consider and try to measure are "transportation costs (both freight costs and time costs), policy barriers (tariffs and non-tariff barriers), information costs, contract enforcement costs, costs associated with the use of different currencies, legal and regulatory costs, and local distribution costs (wholesale and retail)."

[8]U.S. v American Tobacco (1911) involved a number of U.S. and two British firms; at around the same time there was an ongoing potash controversy between the United States and Germany, where potash cartels were instituted and reinstituted repeatedly by the government to control price and production. Ongoing cartelisation in the sector eventually led to the prosecution of the international potash cartel by the U.S. Department of Justice in 1927. Levenstein and Suslow (2008) remark that chemical firms were "particularly active" in international cartels in the interwar period; they mention a 1942 radio address in which Thurman Arnold reported that at the time there were 162 cartel agreements between the German company I.G. Farben and U.S. firms.

[9]Cited in Levenstein and Suslow (2008). Scott Hammond, An Update of the Antitrust Division's Criminal Enforcement Program, Address Before the ABA Section of Antitrust Law Cartel Enforcement Roundtable 2005 Fall Forum 2 (Nov. 16, 2005), available at http://www.usdoj.gov/atr/public/speeches/213247.htm.

[10]WTO (1997), chapter 4.

possible relation to trade liberalisation and in particular to the reduction in communication costs, but also considers alternative explanations. The reform of amnesty or leniency programs and international collaboration in antitrust, for example, have increased the ability to detect and fight cartels. Also Levenstein and Suslow (2008) question the existence of an upsurge in collusive activity. They recall that in the first half of the twentieth century "cartels were tolerated or even encouraged" and that later, "For several decades, international cartels were nominally illegal [...] but enforcement remained limited."[11] They attribute the evidence to a change in attitudes in the early 1990s away from the prosecution of "domestic cartels with limited local effects" to today's aggressive policies fighting collusion at a global level.

Related literature. Davidson (1984) on tariffs, Rotemberg and Saloner (1986) on quotas and Staiger and Wolak (1989) on anti-dumping petitions focus on the impact of trade policies on collusion in single-country, homogeneous good models where domestic firms are exposed to competition with foreign firms. A conceptual issue affecting these models is that the collusive agreements analysed assign positive market shares to all firms whereas joint profit maximisation in the presence of trade costs would require the foreign firms to exit the market. Two alternative formulations tackle this issue. Fung (1992) studies a single-country model in which the competing home and foreign firms produce differentiated goods. Other contributions analyse mutual trade and collusive agreements among firms established in two different countries. Bernheim and Whinston (1990) argue that such a context is particularly adequate for the purpose of analysing collusive behaviour because the possibility of strategically linking actions on several markets allows pooling slack enforcement power and thereby relaxing incentive constraints in critical markets. They also show that in such a context the retraction from markets in which firms have a relative disadvantage is a necessary condition for maximising the scope of collusion.[12] While in Bernheim and Whinston's model the retaliation device is a reversion to Bertrand competition, Fung (1991) shows an analogous result under Cournot reversion. With homogeneous goods, the joint profit maximising agreement requires each firm to sell exclu-

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[11]Levenstein and Suslow (2008), p. 1108.
[12]See Bernheim and Whinston (1990), section 5.

sively in its home market and no intra-industry trade occurs.[13]

Many studies have developed the idea that when mutual trade costs play a role collusive agreements take the form of market retraction and forbearance (or home market monopolisation in the case of a duopoly).[14] Lommerud and Sørgard (2001) show that the effects of trade cost reductions on incentives for home market monopolisation depend on the degree of strategic complementarity. The "harsher punishment" effect of trade cost reductions outreaches the "eased attack" effect in case of Bertrand reversion but not under Cournot reversion.[15] Schröder (2007) points out that Lommerud and Sørgard's result critically depends on the unit cost specification. Both with ad valorem or fixed trade costs the anti-competitive effect very likely disappears. All these studies confine attention to collusive market sharing agreements. Leaving apart models with trade in differentiated goods,[16] there are three analyses of international collusive agreements in which two-way trade plays an active role. Colombo and Labrecciosa (2007) study the impact of variable returns to scale under both Bertrand and Cournot reversion. They show that firms may prefer an agreement providing for intra-industry trade if returns are sufficiently decreasing. Bond and Syropoulos (2008) and Belleflamme and Bloch (2008) identify a strategic rationale for collusive intra-industry trade in the case of quantity-setting firms. In their models, the curvature of revenues or average costs causes the deviation profits to be convex in the assigned shares home and abroad. Deviation incentives are in this case lower for similar market shares, when all firms are active in all markets. When trade costs are low, such that both kinds of agreement yield similar gains, an agreement with intra-industry trade is easier to

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[13]See Fung (1991), proposition 1. He also shows that the result is not robust to the differentiation of the traded products and argues that the evidence of intra-industry trade cannot be taken as an indication for the absence of collusion.

[14]The designation of this kind of collusive agreement is not consistent in the literature. Following Bernheim and Whinston (1990), it is often called the creation of "spheres of influence" (SOI). In what follows, it is referred to as a "market sharing agreement".

[15]The latter result had been shown previously by Pinto (1986). The result in a setting with Bertrand-competing firms was developed simultaneously by Gross and Holahan (2003). While all these models use particular functional specifications, Bhattacharjea and Bhanu Sinha (2012) derive the result under Bertrand reversion in a model with a general demand specification and constant per unit costs.

[16]Colonescu and Schmitt (2003) and Akinbosoye, Bond and Syropoulos (2012) analyse multimarket collusion with differentiated goods. In these models, intra-industry trade occurs naturally.

sustain.

The present work argues that the bias towards collusive agreements that exclude intra-industry trade is a consequence not only of the homogeneous good assumption but also the exclusive focus on environments in which perfect monitoring is possible. With imperfect monitoring, the strategic use of information becomes a relevant aspect of the intertemporal implementation of collusive agreements. In such a context market retraction may suffer a fundamental flaw: if relevant signals are only observed in markets in which a firm actively trades, it substantially limits the access to the scarce, publicly verifiable information. It is argued that in this case a monitoring based on public signals does not allow sustaining market sharing agreements. But even if market retraction does not prevent a firm from getting data on the potential residual demand for its product in the market in which it does not actively trade, market sharing may not be the most advantageous collusive strategy. It is shown that the specific requirements of market sharing agreements make this type of collusive agreement informationally inefficient and rather costly to implement.

The analysis takes into account both advantages of market sharing. Productive efficiency can only be achieved by eliminating expensive export sales. But under imperfect monitoring, a collusive strategy needs to trade productive efficiency off against a lower informational precision in the monitoring process. Second, in the presence of trade costs, slack enforcement power in the foreign market can only be exploited through market linkage. Under imperfect monitoring, however, when punishment is an equilibrium phenomenon, a linked dynamic strategy turns out to be a rather expensive implementation device. It is shown that market sharing agreements require (i) access to public information in the market in which a firm does not actively trade and (ii) market linkage. In contrast, collusive agreements with intra-industry trade are more flexibly implementable with alternative dynamic strategies and in particular with unlinked, independent strategies in each market.

The relative performance of market sharing and agreements with intra-industry trade strongly depends on the dynamic strategy used to implement the agreements. If firms use market linkage in order to pool enforcement power, market sharing implies a larger scope of collusion and a higher equilibrium expected present discounted value than collusive intra-industry trade. Conditional on market linkage, the results are analogous to the ones under perfect monitoring in the case

of Bertrand reversion.

The more interesting results refer to the use of alternative dynamic pricing strategies for sustaining collusion with intra-industry trade. When unlinked strategies are used, deviation incentives of domestic and foreign firms are affected by trade costs in an asymmetric way. It is therefore necessary to determine the assignment of market shares that minimises the maximal incentive to deviate. Given this sharing rule, the relative performance of collusive intra-industry trade and market sharing is more complex. If trade costs are negligible, an agreement with intra-industry trade outperforms a market sharing agreement both in terms of the scope of collusion and in the collusive value achievable in equilibrium. If on the contrary trade costs are very substantial (though not prohibitive), the opposite occurs and the results again match the perfect monitoring case. In an intermediate range with moderate trade costs, the relative performance is context-dependent. If uncertainty is high, informational concerns dominate and an agreement with intra-industry trade requires a lower discount factor and allows implementing a higher value in equilibrium; if uncertainty is moderate, productive efficiency and the advantages of market linkage prevail and market sharing is easier and cheaper to implement in equilibrium.

In general terms, it is found that market sharing agreements are comparatively easier to sustain if uncertainty is moderate while an agreement with intra-industry trade generates stronger enforcement power in environments with substantial uncertainty. As trade costs increase, the threshold level of uncertainty characterising the relative performance is increased. This reflects the underlying forces at work: linkage allows making the fullest use of the retaliatory mechanism; but when the frequency of shocks perturbing the monitoring process becomes substantial, this force reverses. Intransigent punishment turns out to be self-defeating when it has to be made effective in equilibrium too often. The effect can be appreciated not only in terms of the scope but also in terms of the expected present discounted value attainable in equilibrium, which is gradually depressed as demand uncertainty rises.

The paper is organised as follows. The second section describes a two-country model of mutual trade in which two price-competing firms, each one established in one country, interact repeatedly and simultaneously and try to sustain a collusive agreement. The model adds exoge-

nous demand uncertainty and unobservable actions to the description of multimarket collusion with symmetric, mutual advantages in Bernheim and Whinston (1990)[17]. As in Tirole's (1988) reformulation[18] of the model developed by Green and Porter (1984), the prices firms announce to consumers in each period do not become public knowledge at the end of the stage game. Section 3 explores the sustainability of market sharing agreements and analyses the relative performance of market sharing and collusive intra-industry trade conditional on market linkage. Section 4 studies the sustainability of collusive intra-industry trade with unlinked pricing strategies and re-examines the relative performance of the two types of collusive agreement and its dependence on the level of trade costs. Section 5 concludes.

2.2 The model

The analysis is confined to a linear, reciprocally symmetric, two-country, price-setting duopoly model of intra-industry trade in a single, homogeneous good. Consumers of the indistinguishable good split up equally in the two countries $i \in \{1,2\}$; they are immobile and the total number of consumers remains constant over time. There is one firm located in each country. Let firm A be located in country 1 and firm B be located in country 2. Countries 1 and 2 are then referred to as the home and foreign country of firm A, respectively. The firms produce and trade the good simultaneously, regularly and endlessly at points $\tau = 0, 1, 2, ...$ They discount future payoffs at factor $\delta \in (0, 1)$ and compare intertemporal streams of payoffs using the respective expected present discounted values.

The analysis abstracts away from non-convexities and centres on the case of symmetric reciprocal unit cost advantages.[19] The operating expenses caused by the production of one physical unit of the good, which do not affect the relative profitability of cross-border trade, are normalised to zero. The emphasis is on the differential costs of distribution and commercialisation home and abroad. If the unit is sold in the home market, these expenses are assumed to be negligible in comparative terms. Selling the same unit abroad induces constant

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[17]See Bernheim and Whinston (1990), section 5.
[18]See Tirole (1988), pp. 262 ff.
[19]Also the case of absolute cost advantage, in which one firm is is more efficient than the other in both markets, is not analysed.

trading expenses $t > 0$. This asymmetry in the trade costs is the first of two elements determining the profitability of engaging in intra-industry trade. In such a context, productive efficiency requires selling exclusively in the home market. The second element is of informational nature.

In both countries $i \in \{1, 2\}$, and at each date t, total demand for the good is uncertain but otherwise identical. There are two possible states of nature: with probability $\alpha \in (0, 1)$, in the bad demand state, demand is zero while in the good demand state, occurring with opposite probability $1 - \alpha$, demand is defined by $D(p_i) = 1 - p_i$. Home market monopolisation implies a market price $p^m = \frac{1}{2}$. This price defines the prohibitive level of trade costs: if $t > \frac{1}{2}$, a firm cannot competitively sell units in the foreign market. The random variable describing the evolution of demand is i.i.d. over time and shocks are also supposed to be purely idiosyncratic, i.e., i.i.d. across countries.

If competition prevails in the stage game, the two firms engage in Bertrand price competition. Simultaneously both firms spread the information about the current selling prices among consumers. Once consumers know the two prices, they purchase the good from the low-price supplier. When firms announce the same price, the residual demand faced by each firm is assumed to be half the market demand at the price announced by both firms. In each period, firms must meet the entire demand for its product at the announced price. Three decision-relevant pieces of information are never observed nor disclosed, not even at the end of the stage game: the prices announced to consumers by the rival firm, the fact that there have effectively been units sold and the exact quantity sold.

In this competitive setting characterised by private information about pricing decisions and sales, the two firms try to sustain a self-enforcing collusive agreement. The analysis is limited to symmetric dynamic strategies and stationary equilibria, such that the generation of incentives necessarily requires value burning. The enforcement device is pure-strategy temporary Bertrand-Nash reversion. Since under punishment the stage game coincides with the reciprocal dumping model with price-setting firms, the one-period profits in both markets are the following:

- Under punishment a firm makes profits $t(1 - t)$ in the home market and zero profits in the foreign market.

In this strategic context, two kinds of collusive agreements are consid-

ered.

Production quota agreement. When firms interact only in a single market, collusive agreements are typically supposed to take the form of production quota agreements (PQA). Instead of competing, the firms agree on restricted, individual production quotas and corresponding price offers that jointly maximise or at least increase the joint profits made in the industry above the competitive level.[20]

In the present context of price-setting firms and multimarket contact in two countries, a production quota agreement specifies, in each country-specific market $i \in \{1, 2\}$, a single focal price p_i^{PQA} to be announced to consumers by both firms and a sharing rule defining two shares of the aggregate demand s_i^{PQA} and $1 - s_i^{PQA}$, to be served, respectively, by the firm located in country i and the firm not located in market i. The limitation to symmetric strategies manifests itself in the assumption of price uniformity across countries, such that $p_1^{PQA} = p_2^{PQA} =: p^{PQA}$, and in the fact that firms are supposed to distribute the resulting country-specific aggregate demands in a reciprocally proportional manner: if firm A is assigned share s in its home country 1 and $1 - s$ in its foreign country 2, firm B is assigned share $1 - s$ in country 1 and s in country 2.[21] Among the reciprocally proportional rules, a special attention is paid to the rule that maximises the scope of a PQA, i.e., the sharing rule that relaxes the more stringent of the two incentive constraints of firms A and B.[22]

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[20]The standard formalisation ignores that firms may collude not only in prices charged and quantities sold, but also in the level of product quality, in capacities and stocks, means of conveyance, distribution and commercialisation, pre- and post-sales servicing and even in dimensions not directly related to the product market. They may cooperate in R&D or agree to coordinate organisational facilitating practices encompassing, for instance, mutual disclosure of strategic information, joint implementation if best-price policies, vertical restraints or even intra-firm structural adjustments related to the capital structure or managerial compensation plans. It is known from the literature on semi-collusion that linking these dimensions of entrepreneurial choice might affect the scope and incentive compatibility of coordinated action.

[21]Reciprocal symmetry requires the shares to coincide across countries in terms of firm location, i.e. $s_1^{PQA} = s_2^{PQA} =: s^{PQA}$, but not across firms within each country-specific market; it does not imply $s_i^{PQA} = 1 - s_i^{PQA} = \frac{1}{2}$, $i = 1, 2$.

[22]There is no intrinsic property of the model ensuring that firms will choose the sharing rule strictly such as to maximise the scope of collusion. In general, an agreement will depend on the bargaining positions and abilities of the two partners. An agreement could clearly fail for reasons unrelated to its profitability,

In order to make the PQAs comparable with the alternative market sharing agreements also in terms of the collusive expected present discounted values achievable in equilibrium, the analysis focuses on PQAs that implement the fully collusive outcome. This consists of both firms charging the price that maximises the joint global profits attainable on both markets given share s^{PQA} in the home market and $1 - s^{PQA}$ in the foreign market:[23]

$$p^{PQA}\left(s^{PQA}\right) = \arg\max 2\left(s^{PQA}pD(p) + \left(1 - s^{PQA}\right)(p - t)D(p)\right).$$

The price announced to consumers in both market according to the PQA is then $p^{PQA} = \frac{1}{2}(1 + (1 - s^{PQA})t)$ and the total quantity sold in each market $q^{PQA} = \frac{1}{2}(1 - (1 - s^{PQA})t)$. The collusive focal price p^{PQA} lies strictly in between the respective monopolistic prices of the domestic and foreign firms. The single optimal deviation for the price-setting domestic firm is therefore to announce its monopoly price to prospective consumers in the country, while the foreign firm optimally deviates by slightly undercutting the prevailing collusive price. The resulting relevant one-period profits in both markets are the following:[24]

- abiding by the PQA yields $\frac{s}{4}\left(1 - (1 - s)^2 t^2\right)$ in the home market and $\frac{1-s}{4}((1 - t)^2 - s^2 t^2)$ in the foreign market,

- optimally deviating from the PQA yields $\frac{1}{4}$ in the home market and $\frac{1}{4}((1 - t)^2 - s^2 t^2)$ in the foreign market.

Market sharing agreement. In situations of reciprocal advantage arguably a different type of collusive agreement is more likely to occur. Market sharing agreements (MSA) apply the principle of collusive action "divide et impera" not within the market, but across markets.

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i.e., to the problem of equilibrium selection. Under multimarket contact with mutual advantages, however, reciprocal proportionality has a harmonising effect on the overall bargaining positions. This aspect, which is not investigated here, may in practice be an important collusion-enhancing effect of multimarket contact.

[23] The focus on fully collusive outcomes implicitly solves the problem of equilibrium selection inherent in repeated games. The sustainability of partially collusive prices $p_i \in \left(c, p^{PQA}(s^{PQA})\right)$ is neglected. Also note that $p^{PQA}(s^{PQA}) < p^m$ for $s^{PQA} < 1$ and $\lim_{s^{PQA} \to 1} p^{PQA}(s^{PQA}) = p^m$. The productive inefficiency implied by cross-border trade is a key element driving the results.

[24] To save on notation, the superscripts identifying the type of collusive agreement are suppressed where no confusion is possible.

Instead of serving only a restricted share of a market, firms assign whole markets in a cooperative manner and refrain from entering the markets assigned to collaborating firms. The division can be geographical as in the model of intra-industry trade or can more generally refer to alternative dimensions of product differentiation. A MSA is apparently a much simpler private treaty than a PQA. It does not specify prices nor sharing rules. Its characteristic feature is the promise of mutual market retraction and forbearance. In the model with mutual trade costs, joint profit maximisation requires assigning each firm its home market. With only a single domestic firm in each market, abiding by the agreement then implies the monopolisation of the home market. In this case, market prices are $p^{MSA} = \frac{1}{2}$ and in both markets $q^{MSA} = \frac{1}{2}$ many units are sold domestically. No intra-industry trade occurs.

The optimal deviation is, in principle, a significantly more complex operation in the case of a MSA than in the case of a PQA. It requires to secretly entering the foreign market before slightly undercutting the monopolistic price prevailing in that market.[25] Same as PQAs, MSAs are sustained by a threat of temporary Bertrand-Nash reversion to the stage-game equilibrium actions of reciprocal dumping. The resulting relevant one-period profits in both markets are the following:

- abiding by the MSA yields $\frac{1}{4}$ in the home market and zero in the foreign market,

- optimally deviating from the MSA yields $\frac{1}{4}$ in the home market and $\frac{1}{4}(1 - 2t)$ in the foreign market.

A firm that abides by an agreement and faces zero demand in a particular period does not know whether the negative experience is due to the realisation of a bad demand shock or due to the partner undercutting its own price. Since firms are not able to infer deviant behaviour, not even with a time lag, a collusive agreement cannot be supported

---

[25]The necessity of secretly entering the foreign market (possibly by creating own, new distribution channels) induces a cost that deteriorates the potential gains from deviation. It may also imply a higher probability of early detection in the course of proceedings. Both seem important practical aspect making a MSA easier to sustain than a PQA, where a deviation does not require market penetration but simply changing prices and expanding production. But note that this observation is limited to the geographical interpretation of the "spheres of influence". The present analysis follows the literature in assuming that deviations themselves are costless and that detection is equally (un)likely in the cases of PQA and MSA.

by perfect monitoring. Common knowledge is limited to the fact that, when in some country at least one firm faces zero demand, both cartel partners know that at least one firm faces zero demand in this country. These past zero demand experiences (ZDEs) form the history of public information. They allow for public monitoring because every action profile induces a particular probability distribution over these publicly known outcomes.

If ZDEs can be made only in a country in which the firm under consideration is active, i.e., where it makes an offer to consumers and where it plans to sell positive amounts, market retraction and forbearance eliminate the availability of public signals. In such circumstances MSAs cannot be sustained with imperfect public monitoring and PQAs exhibit, in a trivial sense, a higher performance both in terms of the scope of collusion and the expected discounted value that can be achieved in equilibrium. In what follows, the analysis presupposes that in each period a firms gets information not only about the actual individual demand after making a price announcement to consumers but also about the potential individual demand in a market in which it makes no price announcement. ZDEs are then made independently of active trade. It will become apparent that the decision of engaging in intra-industry trade, though not affecting quantitatively the amount of public information available, is nevertheless relevant for the relative performance of PQAs and MSAs by affecting the precision of the monitoring process. This is the second, informational element determining the relevance of intra-industry trade.

Multimarket contact opens the possibility of strategically linking country-specific punishment behaviour to the relevant observations (ZDEs) made in the different countries. In the single market case, there is only a single source of relevant observations and a single target point of retaliatory action. Consequently there is only one way to define the trigger event: a ZDE in the market necessarily triggers punishment in that market. If observations are made in several markets or countries, the optimal definition of the trigger event needs to specify how many ZDEs and possibly which particular origin of a ZDE causes a switch from collusion to punishment in how many and possibly which countries. An event in country i may cause punishment only in market i or in both markets. Similarly, a behavioural switch to punishment in country i can be made conditional on a ZDE made in country i or in both countries. Among the several possibilities for defining trigger events in a context of multimarket contact, the analysis focuses on the

following two:

- Unlinked strategies: A ZDE made in country i, $i \in \{1, 2\}$, triggers punishment in country i and only in country i.

- Relentless linkage: A single ZDE made in country i, $i \in \{1, 2\}$, triggers punishment in both countries 1 and 2. Two simultaneous ZDEs made in both countries also trigger punishment in both countries 1 and 2.

The selection of dynamic strategies is not arbitrary. Unlinked strategies and relentless linkage have been most prominent in the literature because they are the only trigger events that allow sustaining collusive agreements under perfect monitoring.[26] These strategies are therefore most adequate for contrasting the conditions under which the different kinds of collusive agreement are sustainable under perfect and under imperfect public monitoring.

2.3 Collusive agreements sustained with relentless linkage

2.3.1 Market sharing agreement

Multimarket contact has ambivalent effects on the scope of collusion. When incentives for cooperation are induced through a mechanism of supervisory deterrence, the possibility of market linkage adds an element of rigour. As Bernheim and Whinston (1990) point out, the traditional view according to which multimarket contact facilitates collusion because "there is more scope for punishing deviations in any one market" has to be qualified because "once a firm knows that it will be punished in every market, if it decides to cheat, it will do so in every market."[27] In the context of perfect monitoring, the effect of stronger deterrence is exactly offset by the increased scope of beneficial

---

[26]The first chapter of the thesis argues that under imperfect public monitoring there exist more trigger events that allow sustaining PQAs under imperfect monitoring; events that fail to provide adequate incentives under perfect monitoring. In particular, there is a third trigger event, called "placable linkage", that allows supporting collusion under imperfect monitoring: A ZDE only in country i, $i \in \{1, 2\}$, triggers no punishment, neither in country 1 nor 2. A double ZDE made simultaneously in markets 1 and 2 triggers punishment in both markets.

[27]See Bernheim and Whinston (1990), p. 3.

deviation if there is no inter-market heterogeneity causing slack of enforcement power in some markets.

Under imperfect monitoring, two further aspects become relevant. Chilling is a forceful mechanism for aligning incentives, but is has its disadvantages. One inconvenience is the informational requirements. Sanctioning collusive practices is down to previous disclosure of individualised information on infractions. If this information is hidden, the viability of collusion hinges on the availability and use of correlated information. It has been mentioned that sustaining a MSA is impossible when a firm is not able to observe its potential individual demand in a market in which it does not actively trade. There does simply not exist sufficient publicly verifiable information if each firm makes ZDEs in its domestic market only. If a ZDE is due to a deviation, it is public knowledge that at least one firm has made zero profits; a negative demand shock, however, is observed only by the domestic firm. No public knowledge is generated. The following proposition asserts that the relevance of information for collusion is not exclusively a quantitative issue of availability. It matters how the available information is used.

Proposition 1 MSAs can only be supported with relentless linkage. Unlinked strategies do not allow sustaining MSAs.

The second part is a direct consequence of the definitions of the two alternative dynamic pricing strategies. These are stated more carefully in the following.

Strategy U The *unlinked strategy* does not make the trigger event in one country conditional on past observations made in the other country.

- In each country i $(i = 1, 2)$, the game starts in a collusive phase. Firms continue playing the actions prescribed by either the PQA or the MSA until one of the firms actively trading in country i makes a ZDE in country i. If one or both firms make a ZDE in country i, play switches to a punishment phase in this market irrespective of the observations made in country $-i$.

- From the next period onwards, both firms play the stage game Nash equilibrium strategy in country i for T periods, irrespective of the actions taken in country $-i$. In period

$T + 1$, firms revert to collusive actions in country i, again irrespective of actual play in country $-i$.

Abiding by a MSA yields expected profits $(1-\alpha)\frac{1}{4}$ in the home market and zero profits in the foreign market. Suppose firm A deviates, enters the market in country 2 and undercuts firm B's monopoly price. Firm B makes a ZDE in its home country. According to strategy U, play switches to punishment in country 2 in the subsequent period irrespective of actual play in market 1, which remains monopolised by firm 1. In the punishment phase, firm 2 makes zero profits in country 1 and $(1-\alpha)t(1-t)$ in country 2 while firm 1 makes $(1-\alpha)\frac{1}{4}$ in its home country and zero profits in country 2, exactly the amounts it gets when honouring the agreement. Since the one-period deviation payoff is strictly higher, strategy U does not provide adequate incentives for sustaining a MSA.

This is different in the case of relentless linkage which, in comparison to strategy U, replaces all instances of partial punishment with full punishment. To complete the proof of proposition 1, it is now shown that relentless linkage has the potential to sustain a MSA in at least some parametric constellations. In the following, MSA-R identifies a MSA sustained with strategy R.

Strategy R The *relentless strategy* defines the following trigger event.

- In each country i $(i = 1, 2)$, the game starts in a collusive phase. Firms continue playing the actions prescribed by either the PQA or the MSA until one firm makes a zero demand experience in some country, i.e., in country i or in country $-i$ or in both countries. If one or both firms face no demand in some country, play switches to a punishment phase in both countries.

- From the next period onwards, both firms play the stage game Nash equilibrium strategy for T periods in both countries. In period $T + 1$, firms revert to collusive actions in both countries.

Call CC the state of play in which both firms A and B abide by the agreement in both countries 1 and 2; accordingly, CP_t denotes the state in which collusive play prevails in country 1 while firms are in the tth period of a punishment phase in country 2. If the behaviour in both markets is linked, equilibrium play can be in one of $T+1$ possible

states $\sigma \in \{CC, P_1P_1, P_2P_2, ..., P_TP_T\}$.[28] When play switches to state P_1P_1 due to a negative shock or due to a deviation, since Bertrand-Nash reversion is a credible strategy both firms know that play will remain under punishment for the next T periods in both countries. This implies that $V_j^{P_tP_t}$, $t = 1, ..., T$, can be expressed recursively exclusively in terms of V_j^{CC} for $j = A, B$.

Consider first an arbitrary period in state CC. With probability $(1 - \alpha)^2$, if no market suffers a negative demand shock, the sum of one-period profits of firm A in countries 1 and 2 is $\frac{1}{4} + 0$ and according to strategy R play remains in state CC in the subsequent period. The same one-period profits are realised with probability $(1 - \alpha)\alpha$, when a bad demand shock occurs in country 2 but not in country 1. The ZDE experienced by firm B in this case, however, causes a switch to state P_1P_1 in the subsequent period. Punishment is also triggered with probability $\alpha(1-\alpha)$, when a bad shock hits country 1, and probability α^2, with which low demand is realised in both countries. In both events firm A's stage game profits are zero in the two countries. Under stationarity therefore the dynamic path of expected present and future profits of firm A satisfies

$$
\begin{aligned}
V_A^{CC} &= (1 - \alpha)^2 \left(\frac{1}{4} + 0 + \delta V_A^{CC} \right) + (1 - \alpha)\alpha \left(\frac{1}{4} + 0 + \delta V_A^{P_1P_1} \right) \\
&\quad + \left(\alpha(1 - \alpha) + \alpha^2 \right) \left(0 + 0 + \delta V_A^{P_1P_1} \right)
\end{aligned}
$$

or, equivalently,

$$
V_A^{CC} = (1 - \alpha) \frac{1}{4} + (1 - \alpha)^2 \delta V_A^{CC} + \left(1 - (1 - \alpha)^2 \right) \delta V_A^{P_1P_1}. \quad (2.3.1)
$$

Analogous reasoning applies to firm B. Reciprocal dumping in both countries in states $P_1P_1, ..., P_TP_T$ implies that only the domestic firm makes approximately profits $t(1 - t)$ from home-market sales for T periods. In period $T + 1$ after a punishment phase has been initiated, play returns to state CC. Under stationarity, the EPDV of the present and future profits of firm j in the very first period of a punishment

---

[28] With unlinked strategies the state space is significantly larger; equilibrium play can then be in any of the $(T + 1)^2$ states $\sigma \in \{CC, CP_1, ..., CP_T, P_1C, P_1P_1, ..., P_1P_T, ..., P_TC, P_TP_1, ..., P_TP_T\}$.

phase therefore satisfies

$$
\begin{aligned}
V_A^{P_1 P_1} &= \left((1-\alpha)^2 + (1-\alpha)\alpha\right) \left(\frac{1-\delta^T}{1-\delta} t(1-t) + 0 + \delta^T V_A^{CC}\right) \\
&\quad + \left(\alpha(1-\alpha) + \alpha^2\right) \left(0 + 0 + \delta^T V_A^{CC}\right)
\end{aligned}
$$

or

$$
V_A^{P_1 P_1} = (1-\alpha) \frac{1-\delta^T}{1-\delta} t(1-t) + \delta^T V_A^{CC}. \tag{2.3.2}
$$

An analogous equation holds for firm B. Solving the system of equations 2.3.1 and 2.3.2 in the stationary EPDVs V_j^{CC} and $V_j^{P_1 P_1}$ yields the following expressions that are identical for both firms $j = A, B$. The value of sustaining a MSA with strategy R in both markets is

$$
V_j^{CC} = \frac{1}{4} \left(\frac{1-\alpha}{1-\delta}\right) \frac{(1-\delta) + 4t(1-t)\left(1-(1-\alpha)^2\right)\delta(1-\delta^T)}{1-(1-\alpha)^2\delta - \left(1-(1-\alpha)^2\right)\delta^{T+1}}
$$

while play in state $P_1 P_1$ implies an EPDV

$$
\begin{aligned}
V_j^{P_1 P_1} &= \frac{1}{4} \left(\frac{1-\alpha}{1-\delta}\right) \left(\frac{4t(1-t)\left(1-(1-\alpha)^2\delta\right)}{1-(1-\alpha)^2\delta - \left(1-(1-\alpha)^2\right)\delta^{T+1}}\right. \\
&\quad \left. + \delta^T \frac{1-\delta-t(1-t)\left(1-(1-\alpha)^2\delta\right)}{1-(1-\alpha)^2\delta - \left(1-(1-\alpha)^2\right)\delta^{T+1}}\right).
\end{aligned}
$$

It can be shown that $V_j^{CC} > V_j^{P_1 P_1} > 0$ if $0 < t < \frac{1}{2}$ and that both V_j^{CC} and $V_j^{P_1 P_1}$ are strictly decreasing in T.

Temporary reversion to the one-period Bertrand-Nash equilibrium is a credible punishment. In order for the collusive MSA to be credibly sustainable with strategy R, firms must find it optimal to abide by the prescribed implementation strategy also in state CC. Firm A's single best deviation from the MSA consists of secretly entering the market in country 2 and slightly undercutting the monopolistic price set by firm B. These actions yield an intertemporal EPDV

$$
\begin{aligned}
V_A^D &= (1-\alpha)^2 \left(\frac{1}{4} + \frac{1-2t}{4} + \delta V_A^{P_1 P_1}\right) + (1-\alpha)\alpha \left(\frac{1}{4} + 0 + \delta V_A^{P_1 P_1}\right) \\
&\quad + \alpha(1-\alpha) \left(0 + \frac{1-2t}{4} + \delta V_A^{P_1 P_1}\right) + \alpha^2 \left(0 + 0 + \delta V_A^{P_1 P_1}\right).
\end{aligned}
$$

Since markets are identical in expected terms, the discounted stream

of profits following a deviation is

$$V_j^D = (1 - \alpha) \frac{1 - t}{2} + \delta V_j^{P_1 P_1} \qquad (2.3.3)$$

for both $j = A, B$. The incentive condition $V_j^{CC} \geq V_j^D$, $j = A, B$, is then equivalent to

$$2(1 - t)(1 - \alpha)^2 \delta + \left(1 - 2(1 - t)(1 - \alpha)^2\right) \delta^{T+1} \geq 1. \qquad (2.3.4)$$

Let $\Psi_{\text{MSA-R}} := 2(1 - t)$. $V_j^{CC} - V_j^D$ is strictly increasing in T for $(1 - \alpha)^2 > \frac{1}{\Psi_{\text{MSA-R}}}$, independent of T for $(1 - \alpha)^2 = \frac{1}{\Psi_{\text{MSA-R}}}$ and strictly decreasing in T for $(1 - \alpha)^2 < \frac{1}{\Psi_{\text{MSA-R}}}$. In the latter case, $V_j^{CC} - V_j^D$ is maximised for $T \to 0$, ruling out the sustainability of collusion for any $\delta < 1$. The same happens in the case $(1 - \alpha)^2 = \frac{1}{\Psi_{\text{MSA-R}}}$. The only $\alpha \in (0, 1)$ satisfying this equation is $\alpha = 1 - \frac{1}{\sqrt{\Psi_{\text{MSA-R}}}}$. In this case, the sustainability condition is equivalent to $\delta \geq 1$ for any positive integer T. If $(1 - \alpha)^2 > \frac{1}{\Psi_{\text{MSA-R}}}$, or equivalently $\alpha < 1 - \frac{1}{\sqrt{\Psi_{\text{MSA-R}}}}$, $V_j^{CC} - V_j^D$ is maximised for $T \to +\infty$, implying that collusion is sustainable if

$$\Psi_{\text{MSA-R}} (1 - \alpha)^2 \delta \geq 1. \qquad (2.3.5)$$

Scope of collusion. The dependence of the scope of a MSA-R on the level of trade costs t depends exclusively on factor $\Psi_{\text{MSA-R}}$. The following lemma states its crucial property.

Lemma 3.1 $\Psi_{\text{MSA-R}}$ is strictly decreasing in t.

The implicit dependence of the scope of a MSA-R on t is graphically illustrated in Figure 2.3.1. It depicts the scope of a MSA-R in the unit square of the bidimensional coordinate plane representing the parameter space $\{(\alpha, \delta) | \alpha, \delta \in [0, 1]\}$ exemplarily for the three transportation cost levels $t = 0.1$, $t = 0.2$, $t = 0.3$.

The figure suggests that the maximum level of uncertainty compatible with collusion is strictly decreasing in the level of trade costs t while the minimum discount factor compatible with collusion is strictly increasing in t. In the case of a MSA sustained with strategy R, these levels are

$$\alpha_{\text{MSA-R}} := 1 - \frac{1}{\sqrt{\delta \Psi_{\text{MSA-R}}}} \qquad (2.3.6)$$
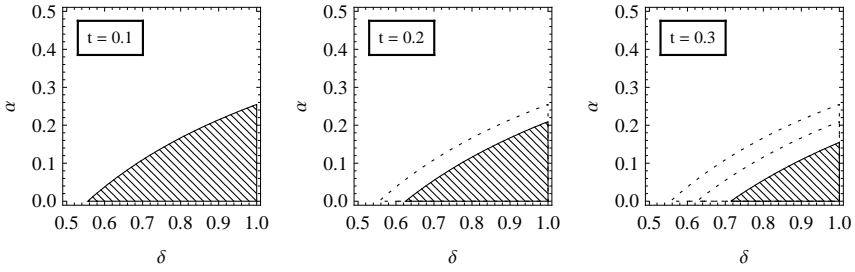
Figure 2.3.1: Scope MSA-R

and

$$\delta_{\text{MSA-R}} := \frac{1}{\Psi_{\text{MSA-R}} \left(1 - \alpha\right)^2}. \tag{2.3.7}$$

The fact that $\Psi_{\text{MSA-R}}$ is strictly decreasing in t confirms the intuition. The two panels of figure 2.3.2 depict $\alpha_{\text{MSA-R}}$ and $\delta_{\text{MSA-R}}$ as a function of t for two exemplifying values of the corresponding parameter.
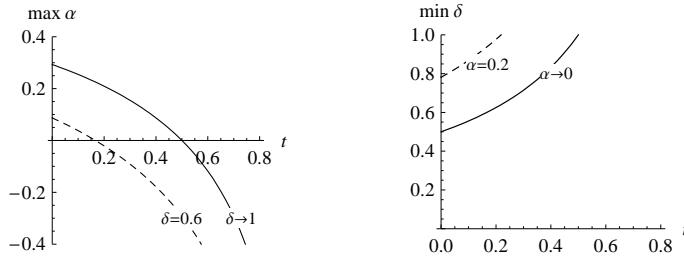

Figure 2.3.2: Maximum uncertainty and minimum patience compatible with MSA-R

For $t \to 0$, $\alpha_{\text{MSA-R}}$ is as high as $1 - \frac{1}{\sqrt{2}} \approx 0.292893$ if $\delta \to 1$ and $\delta_{\text{MSA-R}}$ approaches $\frac{1}{2}$ if $\alpha \to 0$. If the discount factor is high enough, a MSA remains sustainable with strategy R as long as the threat of punishment is effective, i.e. $t < \frac{1}{2}$, when market penetration and intra-industry trade are economically viable. While $\alpha_{\text{MSA-R}}(t \to 0)$ is strictly and continuously decreasing in the discount rate δ and remains positive for $\delta > \frac{1}{2}$, $\delta_{\text{MSA-R}}(t \to 0)$ is strictly increasing in α and remains strictly smaller than one for $\alpha < 1 - \frac{1}{\sqrt{2}}$.

Equilibrium punishment and maximum collusive EPDV. Since under imperfect monitoring punishment occurs along the equilibrium path and V_j^{CC} is strictly decreasing in T, the choice $T = +\infty$ that maximises the scope of collusion cannot be part of an equilibrium strategy of any firm. Given a parameter constellation (α, δ, t) at which the MSA is sustainable with strategy R for some $T \in \mathbb{N}$, firm j's equilibrium choice of T is the smallest integer satisfying $V_j^{CC} \geq V_j^D$. Up to the integer problem, firms operate on a binding incentive constraint in equilibrium. If $T \to +\infty$ allows sustaining the MSA with strategy R, since $\frac{\partial V_j^{CC}}{\partial T} < 0$ for all $T > 0$, and since $T = 0$ violates the incentive constraint, there exists a strictly positive $\widetilde{T} \in \mathbb{R}$ for which $V_j^{CC} = V_j^D$. The equilibrium punishment length T^* is then the smallest integer larger or equal $\widetilde{T}$.[29]

The equilibrium punishment length and the collusive EPDV are derived in appendix A.1. When a MSA is sustained with relentless market linkage, the latter is

$$V_j^{CC}(T^*) = \frac{1 - 2(1-t)\left(1 - (1-\alpha)^2\right)}{4(1-\alpha)(1-\delta)} \tag{2.3.8}$$

for both firms $j = A, B$.

Lemma 3.2 T^* and $V_j^{CC}(T^*)$ are both strictly increasing in t.

The proof is relegated to appendix A.1. The positive dependence of the equilibrium punishment length on t reflects the fact that the enforcement power with strategy R is strictly decreasing in t. Given the previous characterisation of the scope of collusion, this finding is expected.

The negative dependence of the enforcement power on t implies that a seemingly pro-competitive reduction of trade costs effectively enlarges the scope of collusion. The result confirms analogous findings under perfect monitoring in an otherwise corresponding strategic environment.[30] Under perfect monitoring, an increase in the cost of trade

---

[29]In what follows, when comparing the maximum achievable collusive EPDVs under different dynamic strategies, the integer problem is ignored. It is assumed that the punishment length can take any positive real value, and the binding incentive constraint is used to determine the equilibrium punishment length.

[30]In a model that uses the same functional specification of the fundamentals assumed here, Lommerud and Sørgard (2001) show that if the agreement is sustained by a threat of Bertrand-Nash reversion, the latter effect dominates and higher transportation costs have a negative impact on the enforcement power.

is known to have two opposing effects on incentives: it reduces the short-term gains from invading the other firm's home market and, at the same time, it makes the expected, subsequent punishment less severe. Under imperfect monitoring, the analysis of incentives must take into account a third effect. Conditional on T, an increase in t does not only increase the continuation value of a deviation, but also that of maintaining the agreement. Since $\frac{\partial t(1-t)}{\partial t} > 0$ iff $t < \frac{1}{2}$, a higher t raises both $V_j^{P_1 P_1}$ and V_j^{CC}, and the latter effect tends to favour collusion. The equilibrium adjustment of the punishment length reflects the impact of the three forces. Lemma 3.2 asserts that in the linear specification the third effect does not alter the impact t has on the overall enforcement power.

Even though V_j^{CC} is strictly decreasing in T and the equilibrium length of punishment T^* is strictly increasing in t, the expression $V_j^{CC}(T^*)$ reveals that the equilibrium collusive EPDV is strictly increasing in t. Higher trade costs require longer periods of punishment in equilibrium and thereby degrade the collusive EPDV, but they also imply that maintaining market sharing induces more significant gains in productive efficiency. In the linear specification, the latter effect dominates.

2.3.2 Production quota agreement

In this subsection it is shown that with strategy R a MSA is not only effectively sustainable but that, in analogy to what happens in a context of perfect monitoring, it performs better than any PQA both in terms of the scope of collusion and in terms of the collusive expected present discounted value achievable in equilibrium. PQA-R denotes a PQA sustained with strategy R.

The following analysis and the result hinge on a particular implication of relentless linkage. When an arbitrary agreement is sustained with strategy R, a partial deviation in only a single country and a full deviation in both countries imply the same continuation value: both of them trigger punishment in both countries with probability one. Since only a full deviation allows a deviating firm to take full advantage of the potential gains, the optimal deviation consists of simultaneously breaking the agreement in both countries. Let Dhf indicate a simultaneous deviation in the home and foreign countries.

The characterisation of the equation system that determines the EPDVs in periods of cooperation and mutual punishment is relegated to ap-

pendix A.2. It is shown that in case of a PQA-R, $V_j^{CC} > V_j^{P_1 P_1}$ only if

$$s > s_{\min} := 2\sqrt{\frac{1-t}{t}} - \frac{1-t}{t}, \qquad (2.3.9)$$

i.e., if the market share assigned in the domestic market is large enough. This condition sets an effective lower bound on the sharing rules compatible with collusion if $t > \frac{1}{5}$. Apart from this detail, the analysis conditional on the single candidate for a best deviation from collusive play is straightforward. The incentive condition $V_j^{CC} \geq V_j^{Dhf}$ is equivalent to

$$\Psi_{\text{PQA-R}}(1-\alpha)^2 \delta + \left(1 - \Psi_{\text{PQA-R}}(1-\alpha)^2\right)\delta^{T+1} \geq 1, \qquad (2.3.10)$$

for both $j = A, B$, where

$$\Psi_{\text{PQA-R}} := \frac{2 - 6t + 5t^2 - t^2 s^2}{1 - 2t(1-t)s - 2t^2 s^2}. \qquad (2.3.11)$$

$V_j^{CC} - V_j^{Dhf}$ is strictly increasing in T if $(1-\alpha)^2 > \frac{1}{\Psi_{\text{PQA-R}}}$ or, equivalently, $\alpha < 1 - \frac{1}{\sqrt{\Psi_{\text{PQA-R}}}}$. Sustaining a PQA with strategy R and sharing rule s then remains feasible for some parametric constellations as long as $1 - \frac{1}{\sqrt{\Psi_{\text{PQA-R}}}} > 0$ or, equivalently, $\Psi_{\text{PQA-R}} > 1$. It is shown in appendix A.2 that this condition is coincident with condition $s > s_{\min}$ ensuring $V_j^{CC} > V_j^{P_1 P_1}$. If the necessary condition is satisfied, a PQA can be credibly sustained with R-linkage if

$$\Psi_{\text{PQA-R}}(1-\alpha)^2 \delta \geq 1. \qquad (2.3.12)$$

Scope of collusion. Factor $\Psi_{\text{PQA-R}}$ determines the relevance of parameter t and of the sharing rule s for the sustainability of a PQA-R. The following lemma characterises the dependence of $\Psi_{\text{PQA-R}}$ on these two magnitudes.

Lemma 3.3 $\Psi_{\text{PQA-R}}$ is strictly decreasing in t and strictly increasing in s.

The proof is in appendix A.3. Since $\Psi_{\text{PQA-R}}$ is strictly decreasing in t for $0 < s < 1$, the pro-collusive effects of a trade cost reduction found in the case of a MSA-R remain forceful in the case of a PQA-R. The dependence of $\Psi_{\text{PQA-R}}$ on s implies that the maximum level of uncertainty compatible with a PQA-R is increasing in s and the

minimum discount factor required in the case of a PQA-R is decreasing in the sharing rule s. The respective expressions

$$\alpha_{\text{PQA-R}} := 1 - \frac{1}{\sqrt{\delta \Psi_{\text{PQA-R}}}} \qquad (2.3.13)$$

and

$$\delta_{\text{PQA-R}} := \frac{1}{\Psi_{\text{PQA-R}}(1-\alpha)^2} \qquad (2.3.14)$$

are represented in the two panels of figure 2.3.3 as a function of t for exemplifying sharing rules $s \to 1$ (solid line, coincides with $\alpha_{\text{MSA-R}}$ for $\delta \to 1$), $s = 0.9$ (dashed) and $s = 0.5$ (dotted)[31]. The negative dependence of $\Psi_{\text{PQA-R}}$ on t for $t \le \frac{1}{2}$ implies that $\alpha_{\text{PQA-R}}$ is strictly decreasing in t while $\delta_{\text{PQA-R}}$ is strictly increasing in t.
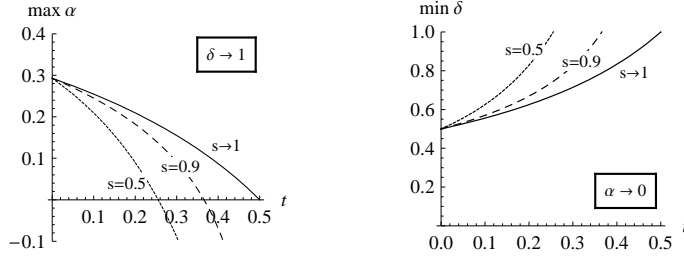

Figure 2.3.3: Maximum uncertainty and minimum patience compatible with PQA-R

Relative performance of MSA-R and PQA-R. More importantly, the dependence of $\Psi_{\text{PQA-R}}$ on s explains to a large extent the relative performance of a MSA-R and a PQA-R. This relation hinges exclusively on the properties of $\Psi_{\text{PQA-R}}$ in comparison with $\Psi_{\text{MSA-R}}$. Direct substitution allows verifying that $\Psi_{\text{PQA-R}}(s = s_{\min}) = 1$ and that $\lim_{s \to 1} \Psi_{\text{PQA-R}} = \Psi_{\text{MSA-R}}$. The fact that $\Psi_{\text{PQA-R}}$ is strictly increasing in the sharing rule s chosen by the firms directly then implies that $\Psi_{\text{PQA-R}} < \Psi_{\text{MSA-R}}$ for all $0 < s < 1$. The scopes of a PQA with alternative s thus exhibit a strict inclusion relation, and the scope of a MSA-R is strictly larger than the scope of any PQA-R. In the limit

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[31]$\alpha_{\text{PQA-R}}$ is evaluated at $\delta \to 1$ and $\delta_{\text{PQA-R}}$ evaluated at $\alpha \to 0$. The expressions reveal that the dependence of $\alpha_{\text{PQA-R}}$ on δ and of $\delta_{\text{PQA-R}}$ on α replicates the ones identified for $\alpha_{\text{MSA-R}}$ and $\delta_{\text{MSA-R}}$.
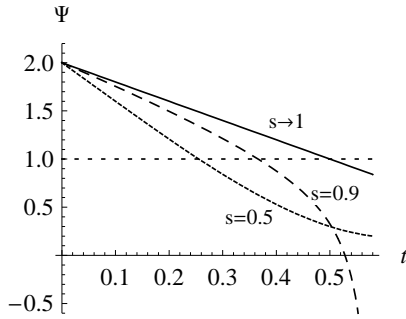
Figure 2.3.4: $\Psi_{\text{MSA-R}}$ and $\Psi_{\text{PQA-R}}$ evaluated at different s

for $s \to 1$, when intra-industry trade in the case of a PQA becomes negligible, the scopes converge. The following proposition states that the advantage of a MSA-R is not limited to the enforcement power it allows to exert; a MSA-R is also superior in terms of the collusive EPDV achievable in equilibrium.

Proposition 2 In comparison with a MSA, any PQA sustained with strategy R implies a strictly smaller scope of collusion in terms of α, δ and t, and a strictly lower equilibrium collusive expected present discounted value for all possible combinations of α, δ and t in which both agreements are sustainable.

The proof is in appendix A.4. Figure 2.3.4 graphically illustrates the relation between $\Psi_{\text{PQA-R}}$ and $\Psi_{\text{MSA-R}}$ that drives the results. The graph $\Psi_{\text{PQA-R}}(t)$ is depicted exemplarily for values $s \to 1$ (solid line, coincides with $\Psi_{\text{MSA-R}}(t)$), $s = 0.9$ (dashed) and $s = 0.5$ (dotted). The horizontal, light dotted line represents $\Psi_{\text{PQA-R}}(t)$ evaluated at $s = s_{\min}$. The intersections of $\Psi_{\text{PQA-R}}(t)$ for alternative s with this horizontal line characterise the maximum trade cost levels compatible with a PQA-R and the corresponding sharing rule.

While for small enough α and large enough δ a MSA-R and a PQA-R with $s \to 1$ are sustainable for all non-prohibitive trade cost levels, more generally a PQA-R with sharing rule $s < 1$ is viable only if

$$t < \frac{3 - s - 2\sqrt{1-s}}{5 - 2s + s^2}. \tag{2.3.15}$$

This limit is strictly increasing in s and for $s \to 1$ it approaches $\frac{1}{2}$.
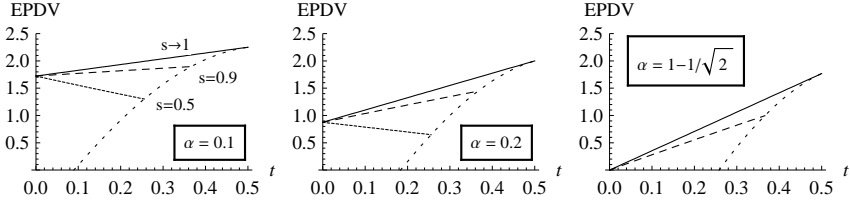
Figure 2.3.5: Collusive EPDV with MSA-R and PQA-R for alternative s

Equilibrium punishment and maximum collusive EPDV. The collusive expected present discounted value achievable in equilibrium with a PQA-R is, for both $j = A, B$,

$$V_j^{CC}(T^*) = \frac{\left(1 + (1-t)^2 - t^2 s^2\right)(1-\alpha)^2 - \left(1 - 2t(1-t)s - 2t^2 s^2\right)}{4(1-\alpha)(1-\delta)}.$$

$$(2.3.16)$$

It is shown in appendix A.4 that the equilibrium punishment length is strictly decreasing in s and $V_j^{CC}(T^*)$ strictly increasing in s for all $0 < s < 1$ and $0 < t < \frac{1}{2}$. Hence, with relentless linkage it is always beneficial to reduce the volume of intra-industry trade. This is the case for two reasons. The extent to which V_j^{CC} is directly affected by the waste of transportation cost depends negatively on s. Second, intra-industry trade puts a strain on $V_j^{CC} - V_j^{Dhf}$, reduces the enforcement power and implies a longer equilibrium punishment length. Generating the necessary incentives is therefore more costly with a lower s.

Also the dependence of $V_j^{CC}(T^*)$ on t depends on the sharing rule s. This is illustrated in figure 2.3.5 which documents, for $\delta = 0.9$ and α-levels 0.1, 0.2 and $\alpha = 1 - \frac{1}{\sqrt{2}}$ (the maximum level compatible with collusion under R-linkage), the EPDVs in state CC achievable with a MSA or a PQA with $s \to 1$ (solid line), with $s = 0.9$ (dashed) or with $s = 0.5$ (dotted). The light dotted line depicts the equilibrium value of a PQA-R for $s = s_{\min}$. A PQA can be sustained only if $s > s_{\min}$, i.e., to the left of the light dotted line.

The panels of the figure reveal that, depending on the chosen s, the collusive EPDV in equilibrium may be increasing or decreasing in t. For sharing rules close enough to $s = 1$, the dependence of the value on t follows the pattern identified in the case of a MSA-R. More im-

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portantly, the figure reflects the superiority of a MSA-R over a PQA-R in terms of the collusive EPDV. This advantage is independent of δ and becomes more pronounced with higher levels of uncertainty.

The superiority of market sharing agreements over production quota agreements is well documented in a variety of different settings under perfect monitoring. In the words of Bernheim and Whinston:[32]

> [...] any solution to this problem must involve $\underline{\lambda} = 1$. That is, the less efficient firm *completely* withdraws from each market.

Conditional on relentless linkage, the previous analysis confirms this relation also in a setting of imperfect public monitoring.[33] The scopes of PQAs for alternative, reciprocally proportional sharing rules exhibit a strict inclusion relation and only in the limit, for $s \to 1$, the scope of a PQA coincides with that of a MSA. In analogy to what happens under perfect monitoring, this result is independent of the trade cost level as long as it allows for economically viable and therefore credible punishment. The only difference between the two modes of monitoring consists in the fact that under perfect monitoring the punishment length does not affect the collusive EPDV achievable in equilibrium. Under imperfect monitoring, the superior enforcement power with a MSA also translates into a shorter punishment length and a higher value in equilibrium.

2.4 Production quotas sustained with unlinked pricing strategies

The previous analysis highlights that, from the viewpoint of two collusion-prone, price-competing oligopolists, a MSA has a couple of promising features as compared to a PQA. Not only that, by avoiding costly intra-industry trade, it allows enhancing the average productive efficiency; also strategically speaking it appears to be a superior device.

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[32]See Bernheim and Whinston (1990), p. 12, referring to the case of reciprocal symmetric cost advantages. $\underline{\lambda}$ is the share of each firm in its home market.

[33]Bernheim and Whinston's analysis of the perfect monitoring case is, in various dimensions, broader in scope than the present study. They assume a general demand specification and distinguish various specifications of the cost function; their analysis is not limited to fully collusive outcomes and neither to reciprocally proportional sharing rules. Finally, they do also consider whether the results extend beyond stationary, symmetric-payoff equilibria.

Only in the limit, when intra-industry trade with a PQA is reduced to insignificant amounts, the two kinds of agreement generate the same incentives and the implementation of the two kinds of agreement is equally costly. But recall the second element affecting the profitability of intra-industry trade mentioned in the model description, the informational argument related to the process of monitoring. This aspect has not played any role in the discussion so far. The reason lies in the exclusive consideration of strategy R and the particular use it makes of the available information.

In a context of perfect monitoring, the focus on relentless linkage is natural. Not only does it enable the firms to avoid the waste of trade costs by implementing a MSA, it also allows exploiting different dimensions of heterogeneity for strengthening incentives. On the other hand, potential disadvantages of relentless linkage are irrelevant under perfect monitoring. These refer to the informational management in the monitoring process and the cost of implementation when punishment becomes an equilibrium phenomenon. Every kind of linkage has the disadvantage of sacrificing publicly verifiable information. Spatially linked strategies suffer from informational imprecision because the respective trigger events are not responsive to the full set of possible observations (in the two-country model, ZDEs in zero, one or two markets) but to only two of them. More specifically, relentless linkage treats the events of one and two observations of zero demand the same. For strictly positive levels of uncertainty, strategy U can therefore be expected to outperform linked strategies in terms of informational efficiency.

Informational efficiency determines the cost of implementing a collusive agreement. Under perfect monitoring, the idea that a more rigorous threat may actually weaken enforcement is certainly counterintuitive. Strategy R is the "harshest" possible punishment implementable by strategically linking the actions to observations in both countries. It does not increase the frequency of punishment compared to strategy U, but it strengthens its severity. As long as punishment is never executed in equilibrium, severity remains hypothetical and leaves the collusive EPDV unchanged. This changes when retaliatory behaviour can be falsely triggered by an unfavourable exogenous demand shock. The execution of punishment then causes an erosion of the intertemporal gains from cooperation and possibly weakens the enforcement power. Under imperfect monitoring, the strength of the retaliatory reactions with relentless linkage has ambivalent effects.

This section tackles the question how strategies R and U perform in terms of (i) productive efficiency, (ii) the capacity of pooling incentive constraints, (iii) information management in the monitoring process and (iv) the resulting cost of implementation. Since relentless linkage is the only dynamic pricing strategy that allows sustaining a MSA, the analysis is confined to the comparison of a PQA sustained with strategy U (henceforth, PQA-U) and a MSA-R.

In the absence of intrinsic links across markets, i.e., given the spatial i.i.d. assumption and the additive separability of the objective functions, the scope of a PQA sustained on both markets with strategy U can be characterised by analysing separately, in each single country i ($i = 1, 2$), the incentives of both firms A and B that are induced by truncating strategy U along its spatial dimension. In equilibrium, play can then be in one of $T + 1$ possible states $\sigma^i \in \left\{ C^i, P_1^i, ..., P_T^i \right\}$ in country i, where C^i means that both firms are abiding by the agreement in country i while P_t^i, $t = 1, ..., T$, indicates the situation firms face when they entered a punishment phase $t - 1$ periods ago in country i. The symmetry resulting from identical, mutual trade costs, the uniform pricing requirement and the reciprocally proportional sharing rule moreover imply that the analysis in country 2 is a perfect replication of the analysis in country 1, with the roles of the domestic and foreign firms inversed. In what follows, subindex h designates the home market firm and f the foreign market firm in country i, $i \in \{1, 2\}$.

Scope of collusion. Consider an arbitrary country $i \in \{1, 2\}$, and assume that the firms try to sustain a PQA by explicitly agreeing or implicitly understanding that actions are strategically unlinked across markets, i.e., that the event triggering punishment in market i exclusively depends on the history of publicly verifiable observations made in market i in the way specified by strategy U.

With strategy U, the optimal deviation for the domestic firm is to announce its monopoly price in the home market to prospective consumers. The foreign firm optimally defects by slightly undercutting the prevailing collusive price.

It is known that when firms interact in a single market, ceteris paribus, it is harder to discipline more efficient firms and also firms that have small market shares. These forces jointly determine the scope-maximising sharing rule also in the present framework. The characterisation

of the equation system that determines the EPDVs in periods of co-operation and mutual punishment is relegated to appendix B.1. It is shown that in case of a PQA-U, $V_h^C > V_h^{P1}$ only if

$$-4(1-t)t + (1-t^2)s + 2t^2s^2 - t^2s^3 > 0. \qquad (2.4.1)$$

Numerical evaluation of the only root with real part within the interval $(0,1)$ allows asserting that this inequality is satisfied if $s_{\min} < s < 1$, where $s_{\min}$ is strictly increasing in t and satisfies $\lim_{t \to 0} s_{\min} = 0$ and $\lim_{t \to 1} s_{\min} = 1$.

For $j \in \{h, f\}$, the incentive condition $V_j^C \geq V_j^D$ is

$$\Psi_j(1-\alpha)\delta + (1 - \Psi_j(1-\alpha))\,\delta^{T+1} \geq 1, \qquad (2.4.2)$$

where

$$\Psi_h := \frac{(1-2t)^2}{1 - s + (1-s)^2 st^2} \qquad (2.4.3)$$

for the more efficient domestic firm and

$$\Psi_f := \frac{1}{s} \qquad (2.4.4)$$

for the foreign, less efficient firm. For both $j = h, f$ $V_j^C - V_j^D$ is strictly increasing in T if $\Psi_j(1-\alpha) > 1$ or $\alpha < 1 - \frac{1}{\Psi_j}$.

The scope of collusion is the set of parametric constellations (α, δ, t) that satisfies the incentive conditions of both firm h and firm f. The following lemma characterises the sharing rule s that maximises the scope by balancing the incentives of the more efficient domestic firm and the firm that is assigned a smaller share of the market.

Lemma 4.1 The sharing rule s_{maxscope} that maximises the scope of the PQA-U in country $i = 1, 2$ is the one that satisfies $\Psi_h = \Psi_f$ or, equivalently,

$$1 - (2 - 4t + 3t^2)s - 2t^2s^2 + t^2s^3 = 0. \qquad (2.4.5)$$

The proof is in appendix B.2. It follows directly from the observation that Ψ_h is strictly increasing in s while Ψ_f is strictly decreasing in s for $s \in (0,1)$. Because of the complexity of the expression s_{maxscope}, the characterisation of the scope of PQA-U and its comparison with the case of a MSA-R is realised using numerical approximations of the relevant relations. Numerical evaluation also allows to assert that
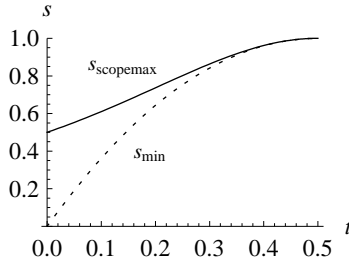
Figure 2.4.1: Scope-maximising sharing rule with PQA-U

s_{maxscope} is strictly increasing in the cost of intra-industry trade. If the trade cost becomes negligible, it converges towards $\frac{1}{2}$ while, when trade costs become prohibitive, it naturally converges towards forbearance, $s_{\mathrm{maxscope}} \to 1$. In the same vein it can be shown that $s_{\mathrm{maxscope}} > s_{\mathrm{min}}$ for all $0 < t < \frac{1}{2}$, ensuring $V_h^C > V_h^{P_1}$. s_{maxscope} is depicted jointly with s_{min} as a function of t in figure 2.4.1.

Let $\Psi_{\mathrm{PQA\text{-}U}} := \Psi_h(s = s_{\mathrm{maxscope}}) = \Psi_f(s = s_{\mathrm{maxscope}})$. If the necessary condition $\Psi_{\mathrm{PQA\text{-}U}}(1 - \alpha) > 1$ is satisfied, a PQA can be credibly sustained with strategy U if

$$\Psi_{\mathrm{PQA\text{-}U}}(1 - \alpha)\delta \geq 1. \tag{2.4.6}$$

Relative performance of MSA-R and PQA-R. Since a MSA-R is sustainable if $\Psi_{\mathrm{MSA\text{-}R}}(1 - \alpha)^2 \delta \geq 1$ while a PQA-U is sustainable if $\Psi_{\mathrm{PQA\text{-}U}}(1 - \alpha)\delta \geq 1$, a direct comparison of $\Psi_{\mathrm{PQA\text{-}R}}$ and $\Psi_{\mathrm{PQA\text{-}U}}$ does now not allow to fully assess the relative performance of the two kinds of agreement. The comparison is made in terms of both the maximum level of uncertainty compatible with collusion and the minimum discount factor required. In the case of a PQA sustained with strategy U, these levels are

$$\alpha_{\mathrm{PQA\text{-}U}} := 1 - \frac{1}{\delta \Psi_{\mathrm{PQA\text{-}R}}}. \tag{2.4.7}$$

and

$$\delta_{\mathrm{PQA\text{-}U}} := \frac{1}{\Psi_{\mathrm{PQA\text{-}U}}(1 - \alpha)}. \tag{2.4.8}$$

Proposition 3 There exists a threshold level $t_{\mathrm{scope}} \approx 0.27155$ approx. such that:

If $t = 0$, the scope of a PQA-U is strictly larger than the scope of a MSA-R; $\delta_{\text{PQA-U}}(t = 0) < \delta_{\text{MSA-R}}(t = 0)$ for all $\alpha < \frac{1}{2}$ and $\alpha_{\text{PQA-U}}(t = 0) > \alpha_{\text{MSA-R}}(t = 0)$ for all $\delta > \frac{1}{2}$.

If $0 < t < t_{\text{scope}}$, the scopes do not exhibit an inclusion relation. There exists a threshold $\delta(t)$ such that $\alpha_{\text{PQA-U}}(t) < \alpha_{\text{MSA-R}}(t)$ if $\frac{1}{2} < \delta < \delta(t)$ while $\alpha_{\text{PQA-U}}(t) > \alpha_{\text{MSA-R}}(t)$ if $\delta(t) < \delta < 1$; and there exists a threshold $\alpha(t)$ such that $\delta_{\text{PQA-U}}(t) > \delta_{\text{MSA-R}}(t)$ if $0 < \alpha < \alpha(t)$ while $\delta_{\text{PQA-U}}(t) > \delta_{\text{MSA-R}}(t)$ if $\alpha(t) < \alpha < \frac{1}{2}$.

If $t_{\text{scope}} < t < \frac{1}{2}$, the scope of a PQA-U is strictly smaller than the scope of a MSA-R; then $\delta_{\text{PQA-U}}(t) > \delta_{\text{MSA-R}}(t)$ for all $\alpha < \frac{1}{2}$ and $\alpha_{\text{PQA-U}}(t) < \alpha_{\text{MSA-R}}(t)$ for all $\delta > \frac{1}{2}$.

The threshold levels t_{scope}, $\delta(t)$ and $\alpha(t)$ are characterised in appendix B.1. The two panels in figure 2.4.2 depict numerical plots of these relations. The left panel shows the relation between t and δ implicitly defined by the equality $\alpha_{\text{PQA-U}} = \alpha_{\text{MSA-R}}$. It allows verifying that $\delta(t)$ is strictly increasing in t and that it satisfies $\lim_{t \to 0} \delta(t) = \frac{1}{2}$ and $\lim_{t \to t_{\text{scope}}} \delta(t) = 1$. The right panel shows the relation between t and α implicitly defined by the equality $\delta_{\text{PQA-U}} = \delta_{\text{MSA-R}}$. It reveals that $\alpha(t)$ is strictly increasing in t and satisfies $\lim_{t \to 0} \alpha(t) = 0$ and $\lim_{t \to t_{\text{scope}}} \alpha(t) = 0.171759$ approx.
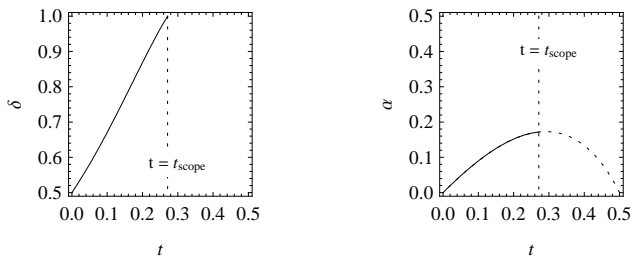
Figure 2.4.2: Threshold levels $\delta(t)$ and $\alpha(t)$

The relation of the scopes of a PQA-U and a MSA-R is graphically illustrated in figure 2.4.3. The figure depicts the scopes of a MSA-R and a PQA-U in the unit square of the bidimensional coordinate plane $\{(\alpha, \delta) | \alpha, \delta \in [0, 1]\}$ for the three transportation cost levels $t = 0.1$, $t = 0.2$, $t = 0.3$. The scope of collusion with a MSA-R is represented in the diagonal hatching; the scope with a PQA-U in the lighter vertical hatching.

The figure reveals that strategy U in combination with a sharing rule that adequately balances the incentives of home and foreign firms allows overruling the dominance of a MSA over a PQA in certain circumstances. Underlying the relative performance of a PQA-U and a MSA-R in terms of the scope are the different factors that affect the enforcement power. Strategy U relies on intra-industry trade and does not allow exploiting the effects of pooling the slack enforcement power across countries. On the plus side, it implements an accurate, selective punishment tailored to retaliate specifically against the particular type of anti-cooperative behaviour that is most likely to have caused the observations made on both markets. The trigger event discriminates between the exact number and the origin of observations. The efficient use of the available information allow a more selective and precise punishment, enhancing the enforcement power in uncertain environments and reducing the overall cost of implementation. When t is large, however, the joint impact of productive inefficiency and the inability to pool slack enforcement power overturn the relative advantages of unlinked implementation. In this case the relative performance of MSA versus PQA mimics the one identified in the previous section.

The analysis reveals that the overall amount of information available is not the only relevant factor. Observing market data in both countries does not automatically prevent firms from triggering punishment too often or too severely. Relentless linkage has two inconveniences. First, it wastes information. The retaliatory response strategy R defines is contingent only on the absence or presence of ZDEs; it does not discriminate between a single ZDE and double ZDEs. Second, it discards the possibility of partial, selective punishment in single markets. It always applies the harshest punishment possible, which under imperfect
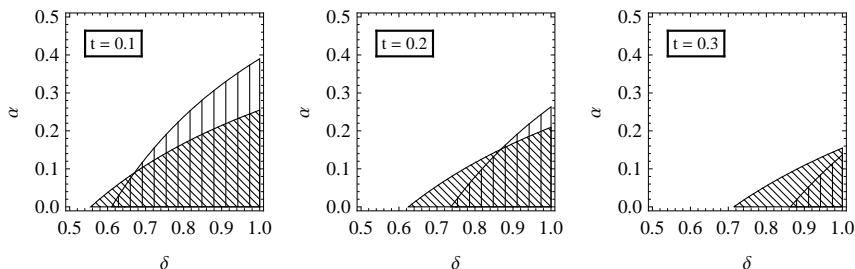

Figure 2.4.3: Scopes with PQA-U (vertical) and MSA-R

monitoring is costly to implement. Strategy U eliminates the first weakness of strategy R and mitigates the second. If the heterogeneity across countries is small, these factors turn out to be decisive and a PQA-U easier to sustain than a MSA-R.

Equilibrium punishment and maximum collusive EPDV. Appendix B.4 characterises the equilibrium punishment length T^* required to generate the necessary incentives for both firms in country i. This length is minimised for $s = s_{\mathrm{maxscope}}$, when $\Psi_h = \Psi_f$ and the incentive conditions of both firms coincide.

In the present symmetric setting, a PQA-U can be sustained in the both countries under identical conditions. Substituting T^* in the expressions of the stationary expected present discounted values in state C for the domestic firm, V_h^C, and the foreign firm, V_f^C, then allows inferring the equilibrium EPDV of a firm in state CC of the original game in which a PQA is sustained on both markets with strategy U,

$$
V_h^C(T^*) + V_f^C(T^*) \tag{2.4.9}
$$
$$
= \frac{s\left(1 - (1-s)^2 t^2\right) - \alpha + ((1-s) - \alpha)\left((1-t)^2 - s^2 t^2\right)}{4(1-\delta)},
$$

where $s = s_{\mathrm{maxscope}}$.

The relative performance of a MSA-R and a PQA-U in terms of the collusive EPDV achievable in equilibrium is characterised and numerically evaluated for $s = s_{\mathrm{maxscope}}$ in appendix B.4. The findings are summarised in the following proposition.

Proposition 4 The relative performance in terms of the collusive EPDV does not depend on δ.

If $\alpha = 0$, the collusive EPDV achievable in equilibrium with a PQA-U is strictly lower than the value achievable with a MSA-R for all t that allow sustaining both agreements.

If $0 < \alpha < 0.174095$ approx., there exists two threshold levels $t^1_{\mathrm{value}}(\alpha) \in (0, 0.29997)$ approx. and $t^2_{\mathrm{value}}(\alpha) \in \left(0.29997, \frac{1}{2}\right)$ such that:

- if $t^1_{\mathrm{value}}(\alpha) < t < t^2_{\mathrm{value}}(\alpha)$, the collusive EPDV with a PQA-U is strictly lower than the collusive EPDV with a MSA-R

- if $0 < t < t^1_{\text{value}}(\alpha)$ or $t^2_{\text{value}}(\alpha) < t < \frac{1}{2}$, the collusive EPDV with a PQA-U is strictly higher than the collusive EPDV with a MSA-R

If $\alpha > 0.174095$ approx., the collusive EPDV achievable in equilibrium with a PQA-U is strictly larger than the value achievable with a MSA-R for all t that allow sustaining both agreements.

The proof is in appendix B.4 and the result is graphically illustrated in figure 2.4.4. The figure depicts, for $\delta = 0.9$ and α-levels 0, 0.1 and 0.2, the overall equilibrium collusive EPDV of an arbitrary firm taking into account that in the case of a multimarket PQA-U it engages in intra-industry trade and obtains gains as a domestic firm in one country and as a foreign firm in the second country. The solid line depicts firm j's ($j = A, B$) collusive EPDV with a MSA-R, the dotted line firm j's ($j = A, B$) collusive EPDV with a PQA-U with $s = s_{\text{maxscope}}$. The vertical lines depict the maximum trade cost level compatible with the respective agreement (for $\delta = 0.9$).
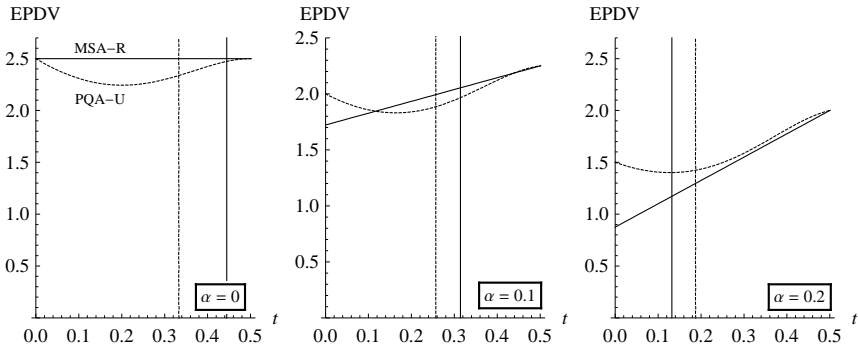
Figure 2.4.4: Collusive EPDV with PQA-U and MSA-R

A caution is indicated when interpreting the figure. The panel in the centre suggests that threshold t^2_{value} and the associated t-range $t^2_{\text{value}}(\alpha) < t < \frac{1}{2}$ are irrelevant since neither of the two agreements are sustainable in this range. This intuition may not necessarily hold true. δ only affects the scaling of the two EPDVs, not its relative performance. But the location of the vertical lines indicating the maximum t-levels compatible with the two agreements crucially depend on δ. If $\delta \to 1$, these lines are shift towards higher t-levels.

What can be stated is that threshold t^1_{value} is increasing in α while

t_{value}^2 is decreasing in α for $0 < \alpha < 0.174095$. The relation between t and α implicitly defined by the equality $V_h^C(T_{\text{PQA-U}}^*) + V_f^C(T_{\text{PQA-U}}^*) = V_j^{CC}(T_{\text{MSA-R}}^*)$ is depicted in figure 2.4.5. It allows verifying that, if $\alpha \to 0$, threshold t_{value}^1 converges to zero, t_{value}^2 to one and a MSA-R yields a higher collusive EPDV for all t for which both types of agreement are sustainable; if $\alpha \to 0.174095$ approx., the threshold is approximately $t_{\text{value}} \approx 0.29997$. No intersection exists when $\alpha > 0.174095$ approx.; then $V_h^C(T_{\text{PQA-U}}^*) + V_f^C(T_{\text{PQA-U}}^*) > V_j^{CC}(T_{\text{MSA-R}}^*)$ for all t that allow sustaining both kinds of agreement.
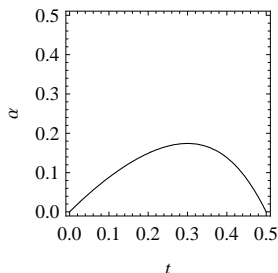
Figure 2.4.5: Relation between t and α characterising equal collusive EPDVs

Under imperfect monitoring, the enforcement power and the associated equilibrium punishment length is one important determinant of the relative performance of agreements in terms of the collusive EPDV. The second one is the productive inefficiency caused by intra-industry trade. In environments in which monitoring is hampered by high demand uncertainty, a PQA-U outperforms a MSA-R for all t that allow sustaining both kinds of agreement. In this case, compromising on the efficiency of monitoring by making a rough use of the available information has a huge cost. When uncertainty is more moderate, the picture is more differentiated. A PQA-U then outperforms a MSA-R only when t is sufficiently small. In such a situation waste due to cross-border trade is low and pooling incentive constraints is less of an issue. If monitoring is precise enough and trade costs are substantial, finally, the advantages of a MSA in terms of productive efficiency and the pooling of slack enforcement power dominate. Also in this case the retaliatory reaction is stronger with a MSA-R than with a PQA-U, but in equilibrium this relentless retaliation will be short due to the efficient pooling of slack enforcement power across countries. A

MSA-R then becomes cheaper to implement than a PQA-U.

2.5 Conclusion

Trade costs generate slack enforcement power in foreign markets that can be exploited in order to strengthen the incentives for multimarket collusion. Since trade costs also render intra-industry trade in homogeneous goods productively inefficient, market sharing agreements can be expected to be the ones that maximise joint profits. These agreements assign positive shares exclusively to the most efficient firms in each market. This argument based on technological idiosyncrasies loses appeal when the firms involved produce substantially differentiated products, but in the context of intra-industry trade of products that exhibit a high degree of homogeneity it is forceful. With few exceptions, the research on collusive behaviour in models of mutual trade has focused on market sharing agreements.

This paper adds a second limitation to the argument beyond the degree of product differentiation. It argues that in market environments in which perfect monitoring of collusive action is not possible the analysis needs to consider not only the productive gains of market retraction but also its informational consequences. The latter always exist if the markets in different countries generate independent, valuable information for monitoring.

The paper explores anti-competitive effects of trade costs and the relative performance of market sharing and agreements providing for intra-industry trade. It stresses the particular requirements of collusive behaviour under imperfect public monitoring. It turns out that in some circumstances the informational requirements of monitoring may break the superiority of market sharing agreements. There exist positive ranges of parameter constellations of trade costs, frequency of demand shocks and the discount rate in which the positive effect of intra-industry trade on the precision of monitoring overcompensates the productive efficiency losses caused by the two-way, cross-border trade.

The informational deficiencies of market sharing are particularly strong if a firm gets public knowledge only in those markets in which it actively trade. But the problem is not limited to the access of information; it also concerns its use. Market sharing necessarily relies on

market linkage, and linkage makes a rather coarse use of the available information. With unlinked strategies, punishment is more tailored to the crime. This dynamic strategy is not able to generate the same punishment force as relentless linkage, but in a highly uncertain environment the more precise monitoring nonetheless allows generating a higher overall enforcement power. Moreover, it implies less value burning in equilibrium.

The relative performance of market sharing versus production quotas in all markets is strongly context-dependent. High uncertainty exacerbates the informational scarcity and the imprecision of monitoring; in this case, the advantages of unlinked implementation become crucial. High trade costs, on the other hand, raise the productive efficiency gains from market retraction and increase slack enforcement power in the home market. Both effects can most efficiently be exploited with market sharing. By assuming perfect monitoring, the traditional analysis blinds out the informational aspect and focuses exclusively on productive efficiency and the balancing of incentives across markets. Using a functionally specified example, this paper shows that informational concerns have the potential to overshadow these traditional arguments. From a competition policy perspective, the absence of market sharing should not be carelessly interpreted as an absence of collusive action.

Appendices to Chapter Two

In order to economise notation, the one-period profits are expressed as fractions of the single-market monopoly profit $\pi^m = \frac{1}{4}$. In the case of a market sharing agreement (MSA),

- abiding by the agreement yields a fraction $\gamma_h^C := 1$ of π^m in the home market and a fraction $\gamma_f^C := 0$ of π^m in the foreign market,

- optimally deviating from the agreement yields $\gamma_h^D := 1$ of π^m in the home market and $\gamma_f^D := 1 - 2t$ of π^m in the foreign market.

In the case of a production quota agreement (PQA),

- abiding by the agreement yields a fraction $\gamma_h^C := s\left(1 - (1-s)^2 t^2\right)$ of π^m in the home market and a fraction $\gamma_f^C := (1-s)\left((1-t)^2 - s^2 t^2\right)$ of π^m in the foreign market,

- optimally deviating from the PQA yields $\gamma_h^D := 1$ of π^m in the home market and $\gamma_f^D := (1-t)^2 - s^2 t^2$ of π^m in the foreign market.

Under punishment, independently of the collusive agreement firms may try to sustain,

- a firm makes a fraction $\gamma_h^P := 4t(1-t)$ of π^m in the home market and a fraction $\gamma_f^P := 0$ of π^m in the foreign market.

Appendix A: Section 2.3

Appendix A.1: Proof of lemma 3.2

Equilibrium length of punishment and collusive EPDV. The binding sustainability condition $V_j^{CC} = V_j^D$ characterising the equilibrium punishment length T^* in each market is equivalent to

$$\Psi_{\text{MSA-R}}(1-\alpha)^2 \delta + \left(1 - \Psi_{\text{MSA-R}}(1-\alpha)^2\right)\delta^{T^*+1} = 1$$

where $\Psi_{\text{MSA-R}} := 2(1-t)$ or

$$\delta^{T^*} = \frac{\Psi_{\text{MSA-R}}(1-\alpha)^2\delta - 1}{\left(\Psi_{\text{MSA-R}}(1-\alpha)^2 - 1\right)\delta} = 1 - \frac{1-\delta}{\left(\Psi_{\text{MSA-R}}(1-\alpha)^2 - 1\right)\delta}$$

for both firms $j = A, B$. If $\alpha < 1 - \frac{1}{\sqrt{\Psi_{\text{MSA-R}}}}$ and $\delta(1-\alpha)^2 \geq \frac{1}{\Psi_{\text{MSA-R}}}$, i.e., when collusion is sustainable under strategy R, both the numerator and denominator are strictly positive and $\delta^{T^*} \in (0, \delta)$. Substituting this expression into the stationary EPDV in an arbitrary period in state CC yields

$$V_j^{CC}(T^*) = \frac{1 - 2(1-t)\left(1 - (1-\alpha)^2\right)}{4(1-\alpha)(1-\delta)},$$

which is, for both firms $j = A, B$, the value of the stream of present and future equilibrium profits if the MSA is sustained. According to the MSA, it corresponds to the stream of present and future equilibrium profits in the home market only.

Since

$$\frac{\partial \delta^{T^*}}{\partial t} = -\frac{2(1-\alpha)^2(1-\delta)}{\left(2(1-t)(1-\alpha)^2 - 1\right)^2 \delta} < 0,$$

the equilibrium punishment length T^* is strictly increasing in t. Conditional on the parameter constellation (α, δ), the partial effect of t on $V_j^{CC}(T^*)$ is

$$\frac{\partial V_j^{CC}(T^*)}{\partial t} = \frac{1 - (1-\alpha)^2}{2(1-\alpha)(1-\delta)} > 0.$$

Appendix A.2: Sustainability of a PQA with strategy R

If both firms implement a PQA with strategy R, under stationarity the dynamic path of expected present and future profits of firm $j \in \{A, B\}$ satisfies

$$V_j^{CC} = (1-\alpha)\left(\gamma_h^C + \gamma_f^C\right)\pi^m + (1-\alpha)^2\delta V_j^{CC} + \left(1 - (1-\alpha)^2\right)\delta V_j^{P_1 P_1}$$

in state CC and

$$V_j^{P_1 P_1} = (1-\alpha)\frac{1 - \delta^T}{1 - \delta}\gamma_h^P \pi^m + \delta^T V_j^{CC}$$

in state $P_1 P_1$. Solving the system of two equations in the stationary EPDVs V_j^{CC} and $V_j^{P_1 P_1}$ yields the following expressions conditional

on choices s, T and parameters α, δ and t:

$$V_j^{CC} = \left(\frac{1-\alpha}{1-\delta}\right) \frac{(1-\delta)\left(\gamma_h^C + \gamma_f^C\right) + \left(1-(1-\alpha)^2\right)\delta(1-\delta^T)\gamma_h^P}{1-(1-\alpha)^2\delta - \left(1-(1-\alpha)^2\right)\delta^{T+1}} \pi^m,$$

$$
\begin{aligned}
V_j^{P_1 P_1} &= \left(\frac{1-\alpha}{1-\delta}\right)\left(\frac{\left(1-(1-\alpha)^2\delta\right)\gamma_h^P}{1-(1-\alpha)^2\delta - \left(1-(1-\alpha)^2\right)\delta^{T+1}}\right. \\
&\quad \left. + \delta^T \frac{(1-\delta)\left(\gamma_h^C + \gamma_f^C\right) - \left(1-(1-\alpha)^2\delta\right)\gamma_h^P}{1-(1-\alpha)^2\delta - \left(1-(1-\alpha)^2\right)\delta^{T+1}}\right)\pi^m.
\end{aligned}
$$

It can be easily verified that $V_j^{CC}, V_j^{P_1 P_1} > 0$. $V_j^{CC} > V_j^{P_1 P_1}$ and V_j^{CC} and $V_j^{P_1 P_1}$ are strictly decreasing in T if and only if $\left(\gamma_h^C + \gamma_f^C\right) > \gamma_h^P$. In terms of the functional specification chosen, this condition is equivalent to $(1 - 6t + 5t^2) + 2t(1-t)s + t^2 s^2 > 0$. For $0 \le t \le 1$ it is satisfied if $s < -\frac{1-t}{t} - 2\sqrt{\frac{1-t}{t}}$ and if $s > -\frac{1-t}{t} + 2\sqrt{\frac{1-t}{t}}$. More concretely, since $-\frac{1-t}{t} + 2\sqrt{\frac{1-t}{t}} < 0 \Leftrightarrow t < \frac{1}{5}$, it is satisfied for all $0 < s < 1$ if $0 \le t < \frac{1}{5}$ and for $2\sqrt{\frac{1-t}{t}} - \frac{1-t}{t} < s < 1$ if $\frac{1}{5} \le t < \frac{1}{2}$. In the latter case,

$$s > 2\sqrt{\frac{1-t}{t}} - \frac{1-t}{t}$$

is a necessary condition for sustainability of a PQA under strategy R. The lower bound for s is strictly increasing in t and approaches $s = 1$ for $t \to \frac{1}{2}$.

Best deviation and incentive condition. The three alternative deviation strategies Dhf, Dh and Df that firm $j \in \{A, B\}$ considers under R-linkage all trigger state $P_1 P_1$ in the subsequent period with probability one and consequently imply the same continuation value $\delta V_j^{P_1 P_1}$. More concretely, in the cases of deviations Dhf, Dh and Df the stationary EPDV of the present and future profits of firm

$j \in \{A, B\}$ satisfies

$$V_j^{Dhf} = (1 - \alpha) \left(\gamma_h^D + \gamma_f^D \right) \pi^m + \delta V_j^{P_1 P_1}$$
$$V_j^{Dh} = (1 - \alpha) \left(\gamma_h^D + \gamma_f^C \right) \pi^m + \delta V_j^{P_1 P_1}$$
$$V_j^{Df} = (1 - \alpha) \left(\gamma_h^C + \gamma_f^D \right) \pi^m + \delta V_j^{P_1 P_1}.$$

Since in any PQA by definition $s \in (0, 1)$, $\gamma_i^C < \gamma_i^D$ for $i = h, f$ and the deviation payoff is highest in the case of deviation Dhf for all possible parameter constellations (α, δ). Under R-linkage, for all (α, δ, t) the incentive condition $V_j^{CC} \geq V_j^{Dhf}$ is equivalent to

$$\Phi_1 (1 - \alpha)^2 \delta + \left(\Phi_2 - \Phi_1 (1 - \alpha)^2 \right) \delta^{T+1} \geq \Phi_2$$

for both $j = A, B$, with $\Phi_1 := 2 - 6t + (5 - s^2)t^2$ and $\Phi_2 := 1 - 2st + 2s(1 - s)t^2$. For arbitrary t, $\Phi_2 > 0$ iff

$$s \in \left(\frac{-(1 - t) - \sqrt{3 - 2t + t^2}}{2t}, \frac{-(1 - t) + \sqrt{3 - 2t + t^2}}{2t} \right).$$

Since $\frac{-(1-t) - \sqrt{3 - 2t + t^2}}{2t} < 0$ if $t > 0$ and $\frac{-(1-t) + \sqrt{3 - 2t + t^2}}{2t} > 1$ if $0 < t < \frac{1}{2}$, $\Phi_2 > 0$ if $0 < s < 1$, $0 < t < \frac{1}{2}$. The incentive condition can be equivalently rewritten

$$\Psi_{\text{PQA-R}} (1 - \alpha)^2 \delta + \left(1 - \Psi_{\text{PQA-R}} (1 - \alpha)^2 \right) \delta^{T+1} \geq 1,$$

where
$$\Psi_{\text{PQA-R}} := \frac{\Phi_1}{\Phi_2} = \frac{2 - 6t + 5t^2 - t^2 s^2}{1 - 2t(1 - t)s - 2t^2 s^2}.$$

$V_j^{CC} - V_j^{Dhf}$ is strictly increasing in T for $(1 - \alpha)^2 > \frac{1}{\Psi_{\text{PQA-R}}}$, independent of T for $(1 - \alpha)^2 = \frac{1}{\Psi_{\text{PQA-R}}}$ and strictly decreasing in T for $(1 - \alpha)^2 < \frac{1}{\Psi_{\text{PQA-R}}}$. In the latter case, $V_j^{CC} - V_j^{Dhf}$ is maximised for $T \to 0$, ruling out the sustainability of collusion for any $\delta < 1$. The same happens if $(1 - \alpha)^2 = \frac{1}{\Psi_{\text{PQA-R}}}$. The only $\alpha \in (0, 1)$ satisfying this equation is $\alpha = 1 - \frac{1}{\sqrt{\Psi_{\text{PQA-R}}}}$, and the incentive condition is equivalent to $\delta \geq 1$ for any non-negative integer T. If $(1 - \alpha)^2 > \frac{1}{\Psi_{\text{PQA-R}}}$, or equivalently $\alpha < 1 - \frac{1}{\sqrt{\Psi_{\text{PQA-R}}}}$, $V_j^{CC} - V_j^{Dhf}$ is maximised for $T \to +\infty$, implying that collusion is sustainable if $\delta(1 - \alpha)^2 \geq \frac{1}{\Psi_{\text{PQA-R}}}$.

It is shown below that $\Psi_{\text{PQA-R}}$ is strictly decreasing in t, implying that $1 - \frac{1}{\sqrt{\Psi_{\text{PQA-R}}}}$ is strictly decreasing in t, for all $s \in (0,1)$ and $t \in \left(0, \frac{1}{2}\right)$. Sustaining a PQA with strategy R fixing sharing rule s remains feasible for small enough levels of uncertainty as long as $1 - \frac{1}{\sqrt{\Psi_{\text{PQA-R}}}} > 0$ or, equivalently, $\Psi_{\text{PQA-R}} > 1$ or $(1 - 6t + 5t^2) + 2t(1-t)s + t^2 s^2 > 0$. The condition is equivalent to $\left(\gamma_h^C + \gamma_f^C\right) > \gamma_h^P$, the condition that ensures $V_j^{CC} > V_j^{P_1 P_1}$ and that V_j^{CC} and $V_j^{P_1 P_1}$ are strictly decreasing in T.

Appendix A.3: Proof of lemma 3.3

$\Psi_{\text{PQA-R}}$ is well defined if $1 - 2t(1-t)s - 2t^2 s^2 \neq 0$. This is satisfied if $t = 0$. If $t \neq 0$, $1 - 2t(1-t)s - 2t^2 s^2 = 0$ is equivalent to $s = \frac{-t + t^2 \pm \sqrt{3t^2 - 2t^3 + t^4}}{2t^2}$. If $t > 0$, $-t + t^2 - \sqrt{3t^2 - 2t^3 + t^4} < 0$ and $-t + t^2 + \sqrt{3t^2 - 2t^3 + t^4} > 1$ is satisfied iff $0 < t < \frac{1}{2}$. $\Psi_{\text{PQA-R}}$ is thus defined for all $0 < s < 1$ and $0 \leq t < \frac{1}{2}$.

The partial effect of s on $\Psi_{\text{PQA-R}}$ is

$$\frac{\partial \Psi_{\text{PQA-R}}}{\partial s} = \frac{2t \left((2 - 8t + 11t^2 - 5t^3) + t(3 - 12t + 10t^2)s + t^2(1-t)s^2\right)}{\left(1 - 2t(1-t)s - 2t^2 s^2\right)^2}.$$

If $\Psi_{\text{PQA-R}}$ is well defined, so is this partial effect. If $t = 0$, it is zero. If $t > 0$, the effect is strictly positive iff $\psi_{\text{PQA-R}}^s := (2 - 8t + 11t^2 - 5t^3) + t(3 - 12t + 10t^2)s + t^2(1-t)s^2$ is strictly positive. $\psi_{\text{PQA-R}}^s$ is strictly increasing in s if

$$\frac{\partial \psi_{\text{PQA-R}}^s}{\partial s} = 2t^2(1-t)s + t(3 - 12t + 10t^2)$$

is strictly positive. If $0 < t < 1$, $2t^2(1-t)s + t(3 - 12t + 10t^2) > 0 \Leftrightarrow s > \frac{3 - 12t + 10t^2}{-2t + 2t^2}$.

Since $\frac{3 - 12t + 10t^2}{-2t + 2t^2} < 0 \Leftrightarrow 0 < t < \frac{6 - \sqrt{6}}{10}$ or $\frac{6 + \sqrt{6}}{10} < t < 1$, $\psi_{\text{PQA-R}}^s$ is strictly increasing in s for all $0 < s < 1$ if $0 < t < \frac{6 - \sqrt{6}}{10}$. Since $\psi_{\text{PQA-R}}^s(s = 0) = 2 - 8t + 11t^2 - 5t^3 > 0$ is satisfied if $t < 1$, $\psi_{\text{PQA-R}}^s > 0$ for all $0 < s < 1$ if $0 < t < \frac{6 - \sqrt{6}}{10}$.

If $\frac{6 - \sqrt{6}}{10} < t < \frac{1}{2}$, $\psi_{\text{PQA-R}}^s$ is strictly increasing in s if $s > \frac{3 - 12t + 10t^2}{-2t + 2t^2}$

and strictly decreasing in s if $s < \frac{3-12t+10t^2}{-2t+2t^2}$. If $0 < t < 1$,

$$\psi_{\text{PQA-R}}^s \left(s = \frac{3 - 12t + 10t^2}{-2t + 2t^2} \right) = -\frac{1}{4(1 - t)} + 4t(2 - 6t + 5t^2) > 0$$

is equivalent to $-1 + 32t - 128t^2 + 176t^3 - 80t^4 > 0$. The polynomial has a real root at $t = \frac{1}{2}$, and numerical approximation reveals that it has complex roots at approximately $t \approx 0.0362481 + 6.93889 \cdot 10^{-18}\,\mathbf{i}$, $t \approx 0.783654 + 1.11022 \cdot 10^{-16}\,\mathbf{i}$ and $t \approx 0.880098 - 1.11022 \cdot 10^{-16}\,\mathbf{i}$. It is strictly positive in the range $0.04 < t < \frac{1}{2}$, an interval that strictly includes the relevant range $\frac{6-\sqrt{6}}{10} < t < \frac{1}{2}$. Positivity of $\psi_{\text{PQA-R}}^s$ at the infimum $s = \frac{3-12t+10t^2}{-2t+2t^2}$ then implies that $\psi_{\text{PQA-R}}^s$ is strictly positive for all $\frac{6-\sqrt{6}}{10} < t < \frac{1}{2}$ and $\frac{3-12t+10t^2}{-2t+2t^2} < s < 1$. If $\frac{6-\sqrt{6}}{10} < t < \frac{1}{2}$ and $s < \frac{3-12t+10t^2}{-2t+2t^2}$, $\psi_{\text{PQA-R}}^s$ is strictly decreasing in s. Positivity of $\psi_{\text{PQA-R}}^s$ at the supremum $s = \frac{3-12t+10t^2}{-2t+2t^2}$ implies that $\psi_{\text{PQA-R}}^s$ is strictly positive for all $\frac{6-\sqrt{6}}{10} < t < \frac{1}{2}$ and $0 < s < \frac{3-12t+10t^2}{-2t+2t^2}$.

Finally note that, if $t = \frac{6-\sqrt{6}}{10}$, $\frac{\partial \psi_{\text{PQA-R}}^s}{\partial s} = 0$. In this case, however, $\psi_{\text{PQA-R}}^s > 0$ for all $s \in \mathbb{R}$. Hence, $\psi_{\text{PQA-R}}^s > 0$ if $0 < s < 1$ and $0 < t < \frac{1}{2}$, implying $\frac{\partial \Psi_{\text{PQA-R}}}{\partial s} > 0$ if $0 < s < 1$ and $0 < t < \frac{1}{2}$.

The partial effect of t on $\Psi_{\text{PQA-R}}$ is

$$\frac{\partial \Psi_{\text{PQA-R}}}{\partial t} = \frac{2\left((-3 + 2s) + (5 - 4s + 3s^2)t + s(1 - 6s + s^2)t^2\right)}{\left(1 - 2t(1 - t)s - 2t^2 s^2\right)^2}.$$

The effect is strictly negative iff $\psi_{\text{PQA-R}}^t := (-3 + 2s) + (5 - 4s + 3s^2)t + s(1 - 6s + s^2)t^2$ is strictly negative. $\psi_{\text{PQA-R}}^t$ is strictly increasing in t if

$$\frac{\partial \psi_{\text{PQA-R}}^t}{\partial t} = 5 - 4s + 3s^2 + 2s(1 - 6s + s^2)t$$

is strictly positive. Since $1 - 6s + s^2 > 0 \Leftrightarrow s < 3 - 2\sqrt{2}$ or $s > 3 + 2\sqrt{2}$, $5 - 4s + 3s^2 + 2s(1 - 6s + s^2)t > 0 \Leftrightarrow t > \frac{-5 + 4s - 3s^2}{2s - 12s^2 + 2s^3}$ if $0 < s < 3 - 2\sqrt{2}$ and $t < \frac{-5 + 4s - 3s^2}{2s - 12s^2 + 2s^3}$ if $3 - 2\sqrt{2} < s < 1$.

Since $\frac{-5 + 4s - 3s^2}{2s - 12s^2 + 2s^3} < 0 \Leftrightarrow s < 3 - 2\sqrt{2}$ or $s > 3 + 2\sqrt{2}$, $\psi_{\text{PQA-R}}^t$ is strictly increasing in t for all $0 < t < \frac{1}{2}$ if $0 < s < 3 - 2\sqrt{2}$. Since $\psi_{\text{PQA-R}}^t\left(t = \frac{1}{2}\right) = -\frac{1}{2} + \frac{1}{4}s + \frac{1}{4}s^3 < 0$ is satisfied if $s < 1$, $\psi_{\text{PQA-R}}^t < 0$ for all $0 < t < \frac{1}{2}$ if $0 < s < 3 - 2\sqrt{2}$.

Since $1 - 6s + s^2 < 0$ for $3 - 2\sqrt{2} < s < 1$, $\frac{-5+4s-3s^2}{2s-12s^2+2s^3} > \frac{1}{2} \Leftrightarrow$ $5 - 3s - 3s^2 + s^3 > 0$ in this s-range. The inequality is satisfied if $1 - \sqrt{6} < s < 1$ or $s > 1 + \sqrt{6}$. Consequently, $\frac{-5+4s-3s^2}{2s-12s^2+2s^3} > \frac{1}{2}$ for $3 - 2\sqrt{2} < s < 1$ and $\psi_{\text{PQA-R}}^t$ is strictly increasing in t for all $0 < t < \frac{1}{2}$ and $3 - 2\sqrt{2} < s < 1$. Since $\psi_{\text{PQA-R}}^t < 0$ is satisfied at the supremum $t = \frac{1}{2}$, $\psi_{\text{PQA-R}}^t < 0$ for all $0 < t < \frac{1}{2}$ if $3 - 2\sqrt{2} < s < 1$.

For $s = 3 - 2\sqrt{2}$, finally, $\psi_{\text{PQA-R}}^t(s = 3 - 2\sqrt{2}) = 3 - 4\sqrt{2} + 4\left(11 - 7\sqrt{2}\right)t$, an expression that is negative for $t < \frac{3-4\sqrt{2}}{-44+28\sqrt{2}} \approx 0.603553$. Hence, $\psi_{\text{PQA-R}}^t < 0$ if $0 < s < 1$ and $0 < t < \frac{1}{2}$, implying $\frac{\partial \Psi_{\text{PQA-R}}}{\partial t} < 0$ if $0 < s < 1$ and $0 < t < \frac{1}{2}$.

Appendix A.4: Proof of proposition 2

Comparison of the scopes. If $t = 0$, $\Psi_{\text{PQA-R}} = \Psi_{\text{MSA-R}} = 2$. If $t > 0$, the inequality $\Psi_{\text{PQA-R}} < \Psi_{\text{MSA-R}}$ is equivalent to

$$\frac{2 - 6t + 5t^2 - t^2 s^2}{1 - 2t(1-t)s - 2t^2 s^2} < 2(1 - t).$$

If $t \neq 0$, the partial effect of s on the denominator, $\frac{\partial\left(1 - 2t(1-t)s - 2t^2 s^2\right)}{\partial s} = -2t(1 - t + 2st)$, is strictly negative iff $s > -\frac{1-t}{2t}$, i.e., for all $0 < s < 1$. Since at the infimum $s = 0$ the denominator is $1 > 0$, continuity implies that it is strictly positive for all $0 < s < 1$, $0 < t < \frac{1}{2}$. In the relevant parametric range, the inequality is therefore equivalent to $4 - 5t + t(3 - 4t)s > 0$. The partial effect of s on the polynomial on the left hand side, $\frac{\partial(4 - 5t + t(3 - 4t)s)}{\partial s} = t(3 - 4t)$, is strictly positive for $0 < t < \frac{3}{4}$. At $s = 0$, the polynomial writes $4 - 5t$, which is strictly positive if $t < \frac{4}{5}$. In consequence, the polynomial is strictly positive for all $0 < s < 1$ and $0 < t < \frac{1}{2}$.

Comparison of collusive EPDVs. Given sharing rule s, the binding sustainability condition $V_j^{CC} = V_j^{Dhf}$ characterising the equilibrium punishment length T^* is equivalent to

$$\Psi_{\text{PQA-R}}(1 - \alpha)^2 \delta + \left(1 - \Psi_{\text{PQA-R}}(1 - \alpha)^2\right) \delta^{T^*+1} = 1$$

or

$$\delta^{T^*} = \frac{\Psi_{\text{PQA-R}}(1-\alpha)^2\delta - 1}{(\Psi_{\text{PQA-R}}(1-\alpha)^2 - 1)\delta} = 1 - \frac{1-\delta}{(\Psi_{\text{PQA-R}}(1-\alpha)^2 - 1)\delta}$$

for both firms $j = A, B$. If $\alpha < 1 - \frac{1}{\sqrt{\Psi_{\text{PQA-R}}}}$ and $\delta(1-\alpha)^2 \geq \frac{1}{\Psi_{\text{PQA-R}}}$, i.e., when collusion is sustainable under strategy R, both the numerator and denominator are strictly positive and $\delta^{T^*} \in (0, \delta)$. Since $\frac{\partial \delta^{T^*}}{\partial \Psi} > 0$, $\frac{\partial \Psi_{\text{PQA-R}}}{\partial t} < 0$ implies $\frac{\partial \delta^{T^*}}{\partial t} < 0$ or $\frac{\partial T^*}{\partial t} > 0$ and $\frac{\partial \Psi_{\text{PQA-R}}}{\partial s} > 0$ implies $\frac{\partial \delta^{T^*}}{\partial s} > 0$ or $\frac{\partial T^*}{\partial s} < 0$. Substituting the expression into the stationary EPDV in an arbitrary period in state CC yields

$$V_j^{CC}(T^*) = \frac{\left(\gamma_h^D + \gamma_f^D\right)(1-\alpha)^2 - \left(\gamma_h^D - \gamma_h^C\right) - \left(\gamma_f^D - \gamma_f^C\right)}{(1-\alpha)(1-\delta)}\pi^m$$

or

$$V_j^{CC}(T^*) = \frac{\left(1 + (1-t)^2 - t^2s^2\right)(1-\alpha)^2 - \left(1 - 2t(1-t)s - 2t^2s^2\right)}{4(1-\alpha)(1-\delta)},$$

which is the value of the stream of joint present and future equilibrium profits of firm $j \in \{A, B\}$ in both markets in which it is operating. If $\alpha, \delta \in (0, 1)$ and $t > 0$,

$$\frac{\partial V_j^{CC}(T^*)}{\partial s} = \frac{t\left(1 - t + t\left(1 + \alpha(2 - \alpha)\right)s\right)}{2(1-\alpha)(1-\delta)} > 0$$

iff $1 - t + t\left(1 + \alpha(2 - \alpha)\right)s > 0$ or, equivalently, $s > -\frac{1-t}{t(1+\alpha(2-\alpha))}$. Hence, $\frac{\partial V_j^{CC}(T^*)}{\partial s} > 0$ for all $0 < s < 1$ and $0 < t < \frac{1}{2}$.

The collusive EPDV with strategy R is higher in the case of a MSA than in the case of a PQA iff

$$\frac{1 - 2(1-t)\left(1 - (1-\alpha)^2\right)}{4(1-\alpha)(1-\delta)}$$
$$> \frac{\left(1 + (1-t)^2 - t^2s^2\right)(1-\alpha)^2 - \left(1 - 2t(1-t)s - 2t^2s^2\right)}{4(1-\alpha)(1-\delta)}$$

or, equivalently,

$$\frac{(1-s)t\left(2-t\left((1-\alpha)^2-s\left(1+\alpha(2-\alpha)\right)\right)\right)}{4(1-\alpha)(1-\delta)} > 0.$$

If $\alpha, \delta, s \in (0,1)$ and $t > 0$, this condition is equivalent to $2 - t\left((1-\alpha)^2 - s\left(1+\alpha(2-\alpha)\right)\right) > 0$. The partial effect of s on the polynomial on the left hand side,

$$\frac{\partial\left(2-t\left((1-\alpha)^2-s\left(1+\alpha(2-\alpha)\right)\right)\right)}{\partial s} = t\left(1+\alpha(2-\alpha)\right),$$

is strictly positive for $t > 0$ if $1-\sqrt{2} < \alpha < 1+\sqrt{2}$. i.e., for $\alpha \in (0,1)$. At $s = 0$, the polynomial is $2 - t\left(1-\alpha\right)^2$, which is strictly positive for all $0 < t < \frac{1}{2}$. By continuity, the polynomial is then strictly positive for all $0 < s < 1$ and $0 < t < \frac{1}{2}$.

Appendix B: Section 2.4

Appendix B.1: Sustainability of a PQA with strategy U

Incentives home and abroad. Consider country $i \in \{1, 2\}$. Let h denote the domestic firm located in this country and f the firm located in the other country. If firms h and f implement a PQA with a truncated version of strategy U that ignores the rules that refer to the other country, under stationarity the dynamic path of expected present and future profits of the domestic firm satisfies

$$V_h^C = (1-\alpha)\gamma_h^C \pi^m + (1-\alpha)\delta V_h^C + \alpha\delta V_h^{P_1}$$

in state C and

$$V_h^{P_1} = (1-\alpha)\frac{1-\delta^T}{1-\delta}\gamma_h^P \pi^m + \delta^T V_h^C$$

in state P_1, while the corresponding EPDVs of the foreign firm satisfy

$$V_f^C = (1-\alpha)\gamma_f^C \pi^m + (1-\alpha)\delta V_f^C + \alpha\delta V_f^{P_1}$$

in state C and

$$V_f^{P_1} = \delta^T V_f^C$$

in state P_1. Solving the two systems of equations in the stationary EPDVs V_j^C and $V_j^{P_1}$ for $j = h, f$ yields the following expressions conditional on the one-period payoffs that depend on sharing rule s and parameters t and α, δ:

$$V_h^C = \left(\frac{1-\alpha}{1-\delta}\right) \frac{(1-\delta)\gamma_h^C + \alpha\delta(1-\delta^T)\gamma_h^P}{1 - (1-\alpha)\delta - \alpha\delta^{T+1}} \pi^m,$$

$$V_h^{P_1} = \left(\frac{1-\alpha}{1-\delta}\right) \frac{(1-(1-\alpha)\delta)\gamma_h^P + \delta^T\left((1-\delta)\gamma_h^C - (1-(1-\alpha)\delta)\gamma_h^P\right)}{1 - (1-\alpha)\delta - \alpha\delta^{T+1}} \pi^m$$

for the domestic firm and

$$V_f^C = (1-\alpha)\frac{\gamma_f^C}{1 - (1-\alpha)\delta - \alpha\delta^{T+1}} \pi^m,$$

$$V_f^{P_1} = (1-\alpha)\frac{\delta^T\gamma_f^C}{1 - (1-\alpha)\delta - \alpha\delta^{T+1}} \pi^m$$

for the foreign firm. While $V_f^C > V_f^{P_1}$ if $t < \frac{1}{2}$, $V_h^C > V_h^{P_1}$ is only satisfied if $\gamma_h^C > \gamma_h^P$. In terms of the functional specification chosen, this condition is equivalent to $-4(1-t)t + (1-t^2)s + 2t^2s^2 - t^2s^3 > 0$, which is not always satisfied for $0 < s < 1$ and $0 < t < \frac{1}{2}$. The relevant root with real part within the interval $(0,1)$ reveals that a first necessary condition for sustainability under strategy U is

$$s > s_{\min} := \frac{2}{3} - \left(1 - \mathbf{i}\sqrt{3}\right)\frac{3t^2 + t^4}{6t^2\varphi_1} - \left(1 + \mathbf{i}\sqrt{3}\right)\frac{\varphi_1}{6t^2}$$

where

$$\varphi_1 : \quad = \quad \left(9t^4 - 54t^5 + 53t^6\right.$$
$$\left. +3\sqrt{3}\sqrt{t^6\left(-1 + 2t^2 - 36t^3 + 143t^4 - 212t^5 + 104t^6\right)}\right)^{\frac{1}{3}}.$$

A numerical plot of the values $s_{\min}$ resulting for alternative levels of trade costs $t \in \left(0, \frac{1}{2}\right)$ allows stating that $s_{\min}$ is strictly increasing in t and that it converges towards $s = 0$ if $t \to 0$ and towards $s = 1$ if $t \to \frac{1}{2}$.

Best deviation and scope of collusion. With strategy U, the scope of strategic interaction is confined to a single market or country. If the domestic firm optimally deviates, its stationary dynamic path of

expected present and future profits satisfies

$$V_h^D = (1-\alpha)\gamma_h^D \pi^m + \delta V_h^{P_1};$$

the corresponding EPDV of the present and future profits of the foreign firm in case of optimal deviation satisfies

$$V_f^D = (1-\alpha)\gamma_f^D \pi^m + \delta V_f^{P_1}.$$

Given parameter constellation (α, δ, t), a sharing rule s and a punishment length T set by the two firms, the relevant incentive conditions $V_j^C \geq V_j^D$ are equivalent to

$$\Phi_{1,j}(1-\alpha)\delta + (\Phi_{2,j} - \Phi_{1,j}(1-\alpha))\delta^{T+1} \geq \Phi_{2,j}$$

where $\Phi_{1,j} := \gamma_j^D - \gamma_j^P$ and $\Phi_{2,j} := \gamma_j^D - \gamma_j^C$ for $j \in \{h, f\}$. Remind that, for the foreign firm, $\gamma_f^P = 0$. Since $\Phi_{2,h}, \Phi_{2,f} > 0$, these conditions can be expressed equivalently as

$$\Psi_j(1-\alpha)\delta + (1 - \Psi_j(1-\alpha))\delta^{T+1} \geq 1$$

where $\Psi_j := \frac{\Phi_{1,j}}{\Phi_{2,j}}$ for $j \in \{h, f\}$. $V_j^C - V_j^D$ is strictly increasing in T for $(1-\alpha) > \frac{1}{\Psi_j}$, independent of T for $(1-\alpha) = \frac{1}{\Psi_j}$ and strictly decreasing in T for $(1-\alpha) < \frac{1}{\Psi_j}$. In the latter case, $V_j^C - V_j^D$ is maximised for $T \to 0$, ruling out the sustainability of collusion for any $\delta < 1$. The same happens if $(1-\alpha) = \frac{1}{\Psi_j}$. The incentive condition is in this case equivalent to $\delta \geq 1$ for any positive integer T. If $(1-\alpha) > \frac{1}{\Psi_j}$, or equivalently $\alpha < 1 - \frac{1}{\Psi_j}$, $V_j^C - V_j^D$ is maximised for $T \to +\infty$. Setting $T \to +\infty$ such as to maximise the scope of collusion, the incentive condition writes $\delta(1-\alpha) \geq \frac{1}{\Psi_j}$.

Appendix B.2: Proof of lemma 4.1

In terms of the functional specification chosen, $\Psi_h = \frac{(1-2t)^2}{1-s+(1-s)^2 st^2}$ and $\Psi_f = \frac{1}{s}$. If $0 < s < 1$, $\Psi_f > 0$ and $\frac{\partial \Psi_f}{\partial s} = -\frac{1}{s^2} < 0$. Since $1 - s + (1-s)^2 st^2 = (1-s)\left(1 + s(1-s)t^2\right) > 0$ if $0 < s < 1$ and $0 < t < \frac{1}{2}$ and $(1-2t)^2 > 0$ if $t \neq 0$, $\Psi_h > 0$. The partial effect of s on Ψ_h is

$$\frac{\partial \Psi_h}{\partial s} = \frac{(1-2t)^2 \left(1 - (1 - 4s + 3s^2)t^2\right)}{(1-s)^2 \left(1 + s(1-s)t^2\right)^2}.$$

The roots satisfying $(1-s)^2 \left(1+s(1-s)t^2\right)^2 = 0$ are $s = 1$ and $s = \frac{1}{2} \pm \frac{1}{2}\sqrt{\frac{4+t^2}{t^2}}$. Since $\frac{1}{2} - \frac{1}{2}\sqrt{\frac{4+t^2}{t^2}} < 0$ and $\frac{1}{2} + \frac{1}{2}\sqrt{\frac{4+t^2}{t^2}} > 1$ for $0 < t < \frac{1}{2}$, the derivative is well-defined in the relevant parametric range. For $t \neq 0$, $1 - (1 - 4s + 3s^2)t^2 > 0$ iff $\frac{2}{3} - \frac{1}{3}\sqrt{\frac{3+t^2}{t^2}} < s < \frac{2}{3} + \frac{1}{3}\sqrt{\frac{3+t^2}{t^2}}$. Since $\frac{2}{3} + \frac{1}{3}\sqrt{\frac{3+t^2}{t^2}} > 1$ for all $t \neq 0$ and $\frac{2}{3} - \frac{1}{3}\sqrt{\frac{3+t^2}{t^2}} < 0$ if $-1 < t < 1$ and $t \neq 0$, $\frac{\partial \Psi_h}{\partial s} > 0$ if $0 < t < \frac{1}{2}$ and $0 < s < 1$.

If $\alpha < 1 - \frac{1}{\Psi_j}$, i.e., if incentives can be generated for firm j, punishment length $T_{\text{maxscope}} = +\infty$ maximises the scope independently of the chosen sharing rule $0 < s < 1$. For any $T > 0$, $V_j^C - V_j^D$ is strictly increasing in Ψ_j for both $j = h, f$. The s maximising firm j's scope is then the one that maximises Ψ_j. Since Ψ_h is strictly increasing and Ψ_f strictly decreasing in s, the sharing rule that maximises the joint scope of collusion in country i is the one that equalises the incentives to deviate of the two firms. $\Psi_h = \Psi_f$ is equivalent to $1 - (2 - 4t + 3t^2)s - 2t^2s^2 + t^2s^3 = 0$. The only root that has a real part $\in (0,1)$ is

$$s_{\text{maxscope}} = \frac{2}{3} + \left(1 - \mathbf{i}\sqrt{3}\right)\frac{6t^2 - 12t^3 + 13t^4}{32^{\frac{2}{3}}t^2\varphi_2} + \left(1 + \mathbf{i}\sqrt{3}\right)\frac{\varphi_2}{62^{\frac{1}{3}}t^2}$$

with

$$\varphi_2 := \left(-9t^4 + 72t^5 - 70t^6\right.$$
$$\left. +3\sqrt{3}\sqrt{t^6\left(-32 + 192t - 589t^2 + 1040t^3 - 1044t^4 + 528t^5 - 144t^6\right)}\right)^{\frac{1}{3}}.$$

A numerical analysis allows stating that $s_{\text{maxscope}} > s_{\text{min}}$ for all $0 < t < \frac{1}{2}$. The asymptotic behaviour of s_{maxscope} and s_{min} indicates that $s_{\text{maxscope}} > s_{\text{min}}$ for $t \to 0$ while $\lim_{t \to \frac{1}{2}} s_{\text{maxscope}} = \lim_{t \to \frac{1}{2}} s_{\text{min}} = 1$. Figure 2.5.1 depicts the graph of $s_{\text{maxscope}} - s_{\text{min}}$ as a function of t. Numerically it is confirmed that $s_{\text{maxscope}} - s_{\text{min}}$ is strictly positive for $0 < t < \frac{1}{2}$.
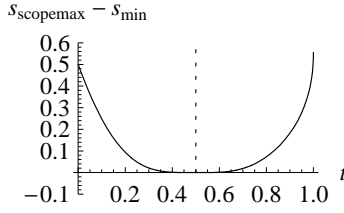
Figure 2.5.1: $s_{\text{maxscope}} - s_{\text{min}}$ as a function of t

A numerical plot of the values s_{maxscope} resulting for alternative levels of trade costs $t \in \left(0, \frac{1}{2}\right)$ also allows stating that s_{maxscope} is strictly increasing in t and that it converges towards $s = \frac{1}{2}$ if $t \to 0$ and towards $s = 1$ if $t \to \frac{1}{2}$.

Conditional on s, the partial effect of t on Ψ_h is

$$\frac{\partial \Psi_h}{\partial t} = -\frac{2(1-2t)(2+s(1-s)t)}{(1-s)\left(1+s(1-s)t^2\right)^2}.$$

The derivative is well-defined in the relevant parametric range. If $s(1-s) > 0 \Leftrightarrow 0 < s < 1$, $2 + s(1-s)t > 0 \Leftrightarrow t > -\frac{2}{s(1-s)}$, which is satisfied for $0 < t < \frac{1}{2}$. Conditional on $s \in (0,1)$, the partial derivative of t on Ψ_h is strictly negative in the relevant parameter range and the joint scope defined by $\Psi_{\text{PQA-U}}(1-\alpha)\delta \geq 1$ therefore exhibits a strict inclusion relation for increasing t. Since $\frac{\partial \Psi_h}{\partial s} > 0$ and s_{maxscope} is strictly increasing in t, the dependence of $\Psi_{\text{PQA-U}} := \Psi_h(s = s_{\text{maxscope}})$ on t seems a priori ambiguous. Since $\Psi_h(s = s_{\text{maxscope}}) = \Psi_f(s = s_{\text{maxscope}})$, $\frac{\partial \Psi_f}{\partial t} = 0$ and $\frac{\partial \Psi_f}{\partial s} < 0$ imply, however, that $\frac{\partial \Psi_{\text{PQA-U}}}{\partial t} < 0$ in the range $0 < t < \frac{1}{2}$. This is confirmed also numerically in figure 2.5.2 plotting the graph of $\Psi_{\text{PQA-U}}$ as a function of t.
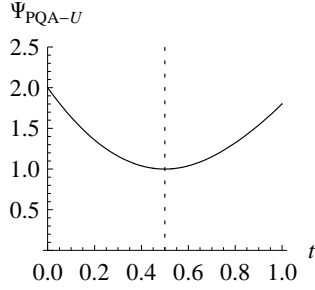
Figure 2.5.2: $\Psi_{\text{PQA-U}}$ as a function of t

Appendix B.3: Proof of proposition 3

Maximum level of uncertainty. For arbitrary $\delta \in (0,1)$, a PQA-U is sustainable if $\Psi_{\text{PQA-U}}(1-\alpha)\delta > 1$ and a MSA-R if $\Psi_{\text{MSA-R}}(1-\alpha)^2\delta > 1$. The maximum α-level that allows sustaining collusion is thus higher for a PQA-U than for a MSA-R iff

$$1 - \frac{1}{\delta\Psi_{\text{PQA-U}}} > 1 - \frac{1}{\sqrt{\delta\Psi_{\text{MSA-R}}}}$$

or, equivalently, $\delta\left(\Psi_{\text{PQA-U}}\right)^2 > \Psi_{\text{MSA-R}}$ or

$$\delta(1-2t)^4 > 2(1-t)\left(1 - s + (1-s)^2 s t^2\right)^2.$$

Figure 2.5.3 depicts the graph of the function satisfying $f(t) = \delta\left(\Psi_{\text{PQA-U}}\right)^2 - \Psi_{\text{MSA-R}}$ evaluated at $s = s_{\text{maxscope}}$ exemplarily for values $\delta = 1$ (solid), $\delta = 0.8$ (dashed), $\delta = 0.6$ (dotted).

Figure 2.5.3: $\delta\left(\Psi_{\text{PQA-U}}\right)^2 - \Psi_{\text{MSA-R}}$ as a function of t

For $\delta \to 1$, the function has a root at approximately $t(\delta \to 1) \approx$

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0.2711184. It takes positive values for $t \in [0, t(\delta \to 1))$ and negative values in the range $t \in \left(t(\delta \to 1), \frac{1}{2}\right)$. Hence, the 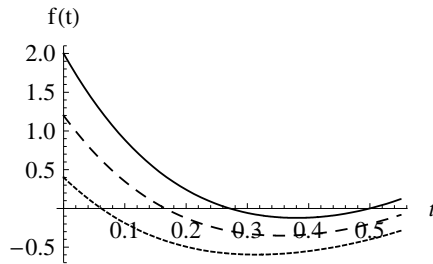maximum level of uncertainty compatible with PQA-U is higher than the level compatible with MSA-R if $t < t(\delta \to 1)$ and lower for higher values of the transportation cost. Similarly, $t(\delta = 0.8) \approx 0.1658178$ and $t(\delta = 0.6) \approx 0.0613861$. The left panel of figure 2.4.2 in the text depicts the graph of the relation $t(\delta)$ implicitly defined by $\delta \left(\Psi_{\text{PQA-U}}\right)^2 - \Psi_{\text{MSA-R}} = 0$, evaluated at $s = s_{\text{maxscope}}$. The relation documents that, in relative terms, the strength of incentives for implementing a PQA-U is increasing in the factor of discounting.

Minimum discount factor. The minimum δ-level that allows sustaining collusion is lower for a PQA-U than for a MSA-R iff

$$\frac{1}{\Psi_{\text{PQA-U}} \left(1 - \alpha\right)} < \frac{1}{\Psi_{\text{MSA-R}} \left(1 - \alpha\right)^2}$$

or, equivalently, $\Psi_{\text{PQA-U}} > \Psi_{\text{MSA-R}} \left(1 - \alpha\right)$ or $(1 - 2t)^4 > 2(1 - t) \left(1 - s + (1 - s)^2 s t^2\right)^2 (1 - \alpha)$. Figure 2.5.4 depicts the graph of the function satisfying $g(t) = \Psi_{\text{PQA-U}} - \Psi_{\text{MSA-R}} \left(1 - \alpha\right)$ evaluated at $s = s_{\text{maxscope}}$ exemplarily for values $\alpha \to 0$ (solid), $\alpha = 0.1$ (dashed), $\alpha = 0.2$ (dotted).
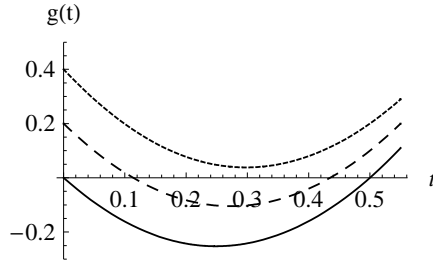
Figure 2.5.4: $\Psi_{\text{PQA-U}} - \Psi_{\text{MSA-R}} \left(1 - \alpha\right)$ as a function of t

For $\alpha = 0$, the function has roots at $t_1(\alpha = 0) = 0$ and $t_2(\alpha = 0) = \frac{1}{2}$ and takes strictly negative values for all $0 < t < \frac{1}{2}$. The minimum discount factor compatible with a PQA-U is higher than the minimum factor compatible with a MSA-R for all $0 < t < \frac{1}{2}$. If $\alpha = 0.1$, the function has roots at approximately $t_1(\alpha = 0.1) \approx 0.1135256$ and $t_2(\alpha = 0.1) \approx 0.4351184$. It takes positive values for $t \in [0, t_1(\alpha = 0.1))$ and $t \in \left(t_2(\alpha = 0.1), \frac{1}{2}\right]$ and negative values in

the range $t \in (t_1(\alpha = 0.1), t_2(\alpha = 0.1))$. The minimum discount factor compatible with PQA-U is lower than the minimum compatible with MSA-R for either very low or very high (though not prohibitive) values t and higher for intermediate values of the transportation cost. If $\alpha = 0.2$, $g(t) > 0$ for all $0 < t < \frac{1}{2}$, implying that the minimum discount factor compatible with PQA-U is always lower than the minimum factor compatible with MSA-R. The right panel of figure 2.4.2 in the text depicts the graph of the relation $t(\alpha)$ implicitly defined by $\Psi_{\text{PQA-U}} - \Psi_{\text{MSA-R}} (1 - \alpha) = 0$, evaluated at $s = s_{\text{maxscope}}$. It allows inferring that the minimum discount factor compatible with PQA-U is lower than the minimum factor compatible with MSA-R for arbitrary t if $\alpha > 0.1728603$. The relation documents that, in relative terms, the strength of incentives for implementing a PQA-U is increasing in the level of uncertainty, and that the strength of incentives for implementing a MSA-R is strongest for a transportation cost level of approximately $t \approx 0.29104207$.

Appendix B.4: Proof of proposition 4

Equilibrium length of punishment and collusive EPDV. In the case of a PQA-U, the basic assumptions on the parameters ensure that V_f^C is strictly decreasing in T independently of the chosen sharing rule; if moreover $\gamma_h^C > \gamma_h^P$, also V_h^C is strictly decreasing in T. Firms then always prefer to set the minimum punishment length required to generate the necessary incentives. Conditional on s, the minimum punishment length T_j^* necessary to discipline firm $j \in \{h, f\}$ from deviating is implicitly defined by $V_j^C = V_j^D$ or

$$\Psi_j(1 - \alpha)\delta + (1 - \Psi_j(1 - \alpha)) \delta^{T_j^* + 1} = 1$$

or

$$\delta^{T_j^*} = 1 - \frac{1 - \delta}{(\Psi_j(1 - \alpha) - 1) \delta}.$$

Providing incentives for both firms requires setting $T^* = \max\left\{T_h^*, T_f^*\right\}$. The strict positive dependence of Ψ_h (strict negative dependence of Ψ_f) on s implies that T_h^* is strictly decreasing (T_f^* strictly increasing) in s in the relevant range $s \in (0, 1)$ for $t \in \left(0, \frac{1}{2}\right)$. Since the value $s = s_{\text{maxscope}}$, for which $\Psi_h = \Psi_f$, satisfies $s_{\text{maxscope}} \in (0, 1)$, there exist sharing rules $s \in (0, 1)$ for which $T_h^* < T_f^*$ and alternative rules for which for which $T_h^* > T_f^*$. The minimisation of the punishment length

T^* thus necessarily requires choosing the scope-maximising sharing rule in equilibrium. In this case, let $T^* := T_h^* = T_f^*$. Since

$$\frac{\partial \delta^{T_j^*}}{\partial \Psi_{\text{PQA-U}}} = \frac{(1-\alpha)(1-\delta)}{\left(\Psi_{\text{PQA-U}}(1-\alpha)-1\right)^2 \delta} > 0$$

and $\frac{\partial \Psi_{\text{PQA-U}}}{\partial t} < 0$ for $s \in (0,1)$ and $t \in \left(0, \frac{1}{2}\right)$, $\frac{\partial \delta^{T^*}}{\partial t} < 0$ and the equilibrium punishment length T^* is strictly increasing in t.

If a PQA-U is sustained with sharing rule $s \in (0,1)$, substituting the resulting punishment lengths T_h^* and T_f^* into the stationary EPDV in an arbitrary period in state C yields

$$V_h^C(T^*) = \frac{\gamma_h^C - \alpha}{1-\delta}\pi^m$$

for the domestic firm, where $\gamma_h^C - \alpha = s\left(1-(1-s)^2 t^2\right) - \alpha$, and

$$V_f^C(T^*) = \frac{\gamma_f^C - \alpha\gamma_f^D}{1-\delta}\pi^m$$

for the foreign firm, where $\gamma_f^C - \alpha\gamma_f^D = ((1-s)-\alpha)\left((1-t)^2 - s^2 t^2\right)$. Taking into account that in the case of a multimarket PQA-U both firms engage in intra-industry trade and obtain gains as a domestic firm in one country and as a foreign firm in the second country, the global EPDV $V_h^C(T_{\text{PQA-U}}^*) + V_f^C(T_{\text{PQA-U}}^*)$ attainable with a PQA-U is larger than the corresponding collusive EPDV $V^{CC}(T_{\text{MSA-R}}^*)$ attainable with a MSA-R iff

$$\frac{\left(\gamma_h^C + \gamma_f^C\right) - \alpha\left(1 + \gamma_f^D\right)}{1-\delta}\pi^m > \frac{1 - \left(1-(1-\alpha)^2\right)\left(1 + \gamma_f^D\right)}{(1-\alpha)(1-\delta)}\pi^m$$

or $\alpha - 2\left(1-(1-\alpha)s\right)t + (1-\alpha)\left((1-\alpha)-2s+(1+\alpha)s^2\right)t^2 > 0$. Note that the discount factor δ is irrelevant when evaluating the relative performance of the agreements in terms of the collusive EPDVs in equilibrium. Same as π^m, it scales both EPDVs proportionally. For $\delta = 0.9$, figure 2.5.5 depicts the graph of the function satisfying $h(t) = V_h^C(T_{\text{PQA-U}}^*) + V_f^C(T_{\text{PQA-U}}^*) - V^{CC}(T_{\text{MSA-R}}^*)$ evaluated at $s = s_{\text{maxscope}}$ exemplarily for values $\alpha \to 0$ (solid), $\alpha = 0.1$ (dashed), $\alpha = 0.2$ (dotted).
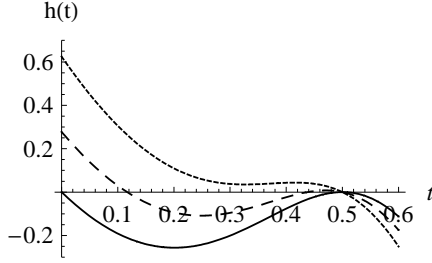
Figure 2.5.5: $\mathrm{EPDV_{PQA\text{-}U}} - \mathrm{EPDV_{MSA\text{-}R}}$ as a function of t

Qualitatively, relation $h(t)$ resembles the comparison of the minimum δ-levels that allow sustaining collusion. For $\alpha = 0$, the function has roots at $t_1(\alpha = 0) = 0$ and $t_2(\alpha = 0) = \frac{1}{2}$ and takes strictly negative values for all $0 < t < \frac{1}{2}$. The collusive EPDV of a PQA-U is lower than the value of a MSA-R for all $0 < t < \frac{1}{2}$. If $\alpha = 0.1$, the function has roots at $t_1(\alpha = 0.1) = 0.116694$ and $t_2(\alpha = 0.1) = 0.4378985$. It takes positive values for $t \in [0, t_1(\alpha = 0.1))$ and $t \in \left(t_2(\alpha = 0.1), \frac{1}{2}\right]$ and negative values in the range $t \in (t_1(\alpha = 0.1), t_2(\alpha = 0.1))$. The maximum collusive EPDV of a PQA-U is higher than the EPDV of a MSA-R for either very low or very high (though not prohibitive) values t and lower for intermediate values of the transportation cost. If $\alpha = 0.2$, $h(t) > 0$ for all $0 < t < \frac{1}{2}$, implying that the maximum collusive EPDV attainable with a PQA-U is always higher than the maximum value of a MSA-R. Figure 2.4.5 in the text depicts the graph of the relation $t(\alpha)$ implicitly defined by $V_h^C(T^*_{\mathrm{PQA\text{-}U}}) + V_f^C(T^*_{\mathrm{PQA\text{-}U}}) - V^{CC}(T^*_{\mathrm{MSA\text{-}R}}) = 0$, evaluated at $s = s_{\mathrm{maxscope}}$. It allows inferring that the maximum collusive EPDV attainable with a PQA-U is higher than the maximum EPDV of a MSA-R for arbitrary t if $\alpha > 0.174095$. The relative performance of a PQA-U in terms of the collusive EPDV attainable in equilibrium is generally improving in the level of uncertainty; the relative advantage of a MSA-R is strongest for a transportation cost level of $t = 0.29997$.

3 Multi-tier information, monitoring and collusion

3.1 Introduction

The theory of collusion based on mechanisms of supervisory deterrence in repeated games asserts that collusive agreements are more difficult to sustain in markets that are subject to demand fluctuations. Two different lines of argumentation provide a theoretical explanation of the phenomenon.

The first argument refers to market transparency and surveillance; it concerns the problem of observability and inferability of actions taken by other market participants. The argument was first formalised by Green and Porter (1984) and Abreu, Pearce and Stacchetti (1986). Effective supervisory deterrence hinges on the detection of non-cooperative behaviour. When conspiring firms cannot directly observe the selling prices announced to potential consumers and the individual quantities sold, or infer this private information from market data, the sustainability of collusion is endangered. In the presence of elements obscuring the process of mutual monitoring it is therefore reasonable to expect collusive agreements being more fragile. If the requirement is only to detect a deviation, not necessarily to identify the deviating firm, lack of observability is necessarily related to some kind of uncertainty. Different dimensions of uncertainty imply that external changes in the environment are confounded with behavioural effects, thereby reducing the precision of the monitoring process.

The first argument assumes that uncertainty is not resolved when firms take their strategic decisions. The second argument assures that uncertainty may become even more dangerous for collusive agreements once the realisation of the random variable has become public knowledge, i.e., after its resolution.[1] If firms already know the outcome of the random event when deciding on defection, their incentives depend

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[1]Strictly speaking, the difference between the two explanations is not so much

strongly on this realisation. Focusing on aggregate demand uncertainty, the one-period gains from deviation are substantially higher in a revealed state of high demand than in a state of low demand. More importantly, they are higher than expected one-period gains, which is the relevant magnitude when firms decide on cooperation or defection without knowing the state of aggregate demand. In this sense, revealed uncertainty is more dangerous than uncertainty itself.

The idea that demand uncertainty impairs collusive agreements is undisputed. The discussion in response to the two alternative explanations has centred on the evolution of incentives over the business cycle and on the question whether price wars are more likely to break out during booms or recessions. It is significant that the question has centred less on the concept of a price war and particular assumptions underlying the formal models than the empirical question whether price wars occur in times of peaking or rather in periods of ailing demand. The empirical research on the topic started with Porter (1983) and an analysis of the Joint Executive Committee, a 19th century railroad cartel. Rotemberg and Saloner (1986), who first formalised the second argument, agree that the question is essentially empirical. They provide several pieces of evidence, including a reassessment of Porter's data in the light of the own theory. The question can hardly be considered settled. What Aiginger (1999) calls the "granddaddy of empirical work on game theoretical models" has been a repeated and arduous discovery of mostly insignificant effects. Available evidence is largely inconclusive.

The discussion suffers from a conceptual shortcoming. It is flawed by the fact that the two explanations talk past each other, that they are not readily comparable. They certainly share common theoretical building blocks. Both model collusive behaviour as a self-enforcing arrangement of otherwise non-binding promises in a repeated game. But the two explanations do not share a common set of premises that allows for mutual qualification or falsification. Within their respective frameworks, the two lines of argumentation invalidate the relevance of the alternative. A change of the timing convention eliminates the confusion about external effects and actions and thus annihilates the

_____

the resolution of uncertainty but predictability. The first argument suggests that price wars should not break out when a decline of demand is foreseen in advance but rather in response to an unexpected downturn. The second argument, on the other hand, asserts that an agreement is in danger when there is the expectation of an imminent increase in demand.

premises of the first argument. On the other hand, if uncertainty remains unresolved, incentives cannot depend on the realisation.

This paper develops a model that allows analysing the impact of the two relevant effects of uncertainty, the deterioration of the monitoring precision and the effect of averaging out deviation gains, in a common framework. Its characteristic feature is the irregular availability of two alternative sources of information for monitoring.

The traditional approach has been to analyse optimal implementation of collusive agreements conditional on a particular mode of monitoring. Excluding private monitoring, which is belief-based and has a different quality, the literature considers two classes of frictions in the monitoring process. Perfect monitoring uses information that allows disclosing the actions of rival firms with certainty but with a time lag. In the following, such information is called "deterministic" information. Imperfect (public) monitoring is based on signals whose distribution depends on the equilibrium profile of actions. In this case, no deterministic link exists between deviations and observable outcomes. Firms try coordinating equilibrium play using shared noisy information. This kind of information is referred to as "correlated" information.

The focus on a single source of information is natural in a context of perfect monitoring; when strategic choices are revealed with certainty, additional information cannot make any difference. If the coordination of equilibrium play relies on noisy information and monitoring remains imperfect, however, the limitation on a single source can only be justified with reference to conceptual simplification. In practice, firms can be expected to collect and even invest in the acquisition of additional information that allows for a more precise monitoring.

The availability of information will often be contingent on circumstantial factors. A customer may claim a low price offered by a competitor; a whistle-blower may approach the firm and disclose or denounce disloyal behaviour. A suspicious firm may itself take more drastic measures and find ways to get information that is difficult to access; by buying it, using bribery or by corporate espionage. But firms can also be expected to have more systematic sources of relevant information. They may observe environmental effects of actions or aggregate information that depends on statistical summaries of actions. Direct communication among market participants can also have a systematic component. After all, the firms' interest in keeping secrecy is dynami-

cally inconsistent, and consumer passiveness in front of disproportionate price increases may be mainly due to a failure of coordination.

Independently of whether alternative sources of information are circumstantial or systematic, the key element that is neglected in the traditional models is that the availability of different sources of information transforms the collection, selection and use of information into a strategic decision problem. The present analysis considers two sources of information, a systematic source consisting of publicly available correlated information and a sporadic source that provides much more precise information and allows resolving uncertainty. More concretely, firms always observe a signal that allows for imperfect public monitoring. With less frequency, they get information of higher "quality" that allows them inferring the actions taken by the rivals with certainty. The analysis determines endogenously the monitoring strategies that maximise (i) the scope of collusion and (ii) the collusive expected present discounted value in equilibrium. These strategies are characterised in terms of the information used or discarded and the retaliatory responses to particular types of information.

The results indicate that there is rich interaction and strong compound effects of different kinds of information. In most parametric constellations, both types of information are actively used and represent complementary enforcement devices. Only if correlated information is very vaguely related with actions and if uncertainty is never resolved before firms decide on defection, the two sources of information may obstruct each other. In this case, the use of correlated information becomes detrimental. More generally, the optimal combination depends on the differential cost of implementation and on the particular functional role played by the two types of information. Different kinds of information are quite versatile; the role they play for enforcement depends strongly on the strategic context.

The findings concerning the optimal use of both sources of information for collusive monitoring with and without prior revelation of uncertainty allow reassessing the relevance of the two seemingly contradicting arguments explaining the effects of uncertainty on collusive action. In line with Rotemberg and Saloner (1986), it is confirmed that one-period gains from deviation are significantly stronger in a state of revealed high demand than in state of revealed low demand. But in the present framework this does not automatically degrade incentives because now two effects of uncertainty coincide: in a state of revealed high demand the gains from deviation are highest, but also the infor-

mation available for monitoring is most precise. On the contrary, in a period without precise information lower expected gains from deviation coincide with an inferior monitoring precision. With unfrequent revelation of high quality information, an optimal use of all available information for monitoring succeeds in balancing the two effects: the deterioration of the monitoring precision and the averaging out of deviation gains. As a result, incentives are the same in periods of high demand and in periods in which uncertainty remains unresolved, and are effectively independent of demand fluctuations. According to the model, incentives remain constant over the cycle. Interestingly, this immunity of incentives from aggregate fluctuations is independent of the frequency with which high quality information becomes available.

More generally, the balancing of the two relevant effects of uncertainty implies that the timing convention and a resolution of uncertainty become irrelevant for the scope of collusion. When correlated information is precise enough, i.e., so precise that it allows generating incentives stand-alone, not only the scopes with and without resolution of uncertainty coincide but it also becomes irrelevant whether uncertainty is resolved before or after firms decide on defection. The point made by Rotemberg and Saloner is not entirely irrelevant, however. An optimal combination of correlated and deterministic information guarantees equal scopes, but it is easy to show that sustaining an agreement will be unambiguously more costly in the case of an early revelation of uncertainty. This latter finding is related to the particular role played by the two types of information for self-enforcement in different strategic environments.

Related literature. Traditional explanations of the likelihood of price wars during booms or recessions were based on technological conditions and argued in favour of defections in periods of slacking demand. In the presence of sunk costs, firms operating under significant increasing returns to scale produce positive amounts even if they are forced to sell these units at prices that do not allow breaking even. Selling these units increases the amount of coverage as long as the selling price is above the (short-run) average variable cost. Firms might then be willing to undergo fierce price wars temporarily when demand is weak. Rotemberg and Saloner (1986) admit the plausibility of this "industrial organization folklore" and even mention additional arguments capable of explaining the occurrence of price wars during recessions. A purely strategic argument is predatory pricing, for example. Un-

der discriminatory access to outside financing, when small firms are credit constrained, "recessions might be the ideal occasions for large established firms to elbow out their smaller competitors." This idea is mentioned not without reminding a counter-argument made by Stiglitz (1984), that "limit pricing may be more salient in booms if the threat of potential entry is also greater [...]."[2]

The available empirical evidence does not discriminate among different arguments predicting pro- or countercyclical movements of prices. The data set used in Porter (1983) and commented in Rotemberg and Saloner (1986) was re-examined by Ellison (1994), who performs the analysis conditional on four alternative definitions of a regime switch between collusive and punishment phases, and Hajivassiliou (1997), who tests the prediction that regime switches follow a Markov process. Ellison finds the unexpected component of demand to be more important than the absolute level of the demand shock and is unable to identify countercyclical price movements. He carefully concludes that "estimates provide some support for the predictions of the first theory", i.e. Green and Porter's argument. Of the four alternative candidates for a regime switch, however, only two are moderately significant.

Domowitz, Hubbard and Petersen (1986, 1987) and Suslow (2005) use cross-sectoral data. The first two contributions analyse US data of 4 digit industries from 1958–81 and report three main findings. The data reveal slight reductions of price levels during recessions; the hypothesis of a regime switch is rejected, however. Price-cost margins are positively correlated with levels of capacity utilisation, but this effect is insignificant in the more concentrated industries that exhibit large price-cost margins. Third, price levels are shown to vary negatively with levels of capacity utilisation, a finding that is interpreted in line with Rotemberg and Saloner. Suslow examines 72 international cartel agreements in 47 industries from the years 1920-39. Her results are more supportive of Green and Porter's argument.

Kandori (1991), Haltiwanger and Harrington (1991) and Bagwell and Staiger (1997) develop Rotemberg and Saloner's model and analyse the effects of serial correlation of shocks, cyclical patterns of demand fluctuations, Markov growth of demand and transitory demand shocks. These structural details allow understanding better the relation between demand fluctuations and incentives for defection. They qualify Rotemberg and Saloner's results in various dimensions, but generally

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[2]Rotemberg and Saloner (1986), footnotes 1 and 2.

without ruling counter-cyclical price-cost margins inoperative.[3] Especially during booms price wars may also be obstructed by strategic or technological elements limiting the exploitation of one-period deviation payoffs. If strategic substitutabilities prevail, or in the case of increasing marginal costs or capacity constraints, incentives for deviation along the cycle may be reversed. Staiger and Wolak (1992) and Fabra (2006) tackle these issues.

Constant most collusive prices over the business cycle have previously been suggested in a model with uncertainty about unit production costs by Athey, Bagwell and Sanchirico (2004). In their model, productive efficiency requires assortative pricing that is strictly increasing in the unit production costs, but such a pricing rule degrades incentives because the resulting sales pattern penalises high-cost firms. For a wide range of parametric constellations, the optimal dynamic strategy is characterised by constant, uniform pricing and the absence of punishment in equilibrium. The model specified in the following shares the first of these characteristics but for a different reason. There are no idiosyncratic shocks and the pricing decision is straightforward. The maintenance of a constant collusive focal price is the result of firms adequately exploiting two alternative sources of information for monitoring.

The paper is organised as follows. The second section defines an extension of Tirole's (1988) reformulation of Green and Porter's (1984) model of collusion with unobservable actions in which firms may rely on imperfect monitoring, but sometimes dispose of additional, more precise information. The analysis is confined to stationary equilibria and strongly symmetric dynamic strategies. The generation of incentives thus necessarily requires value burning. Within this framework, two price-competing firms interact repeatedly and simultaneously on a single market and try to sustain a collusive production quota agree-

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[3]In Bagwell and Staiger (1997), this is true for transitory demand shocks: "a higher transitory demand shock results in a (weakly) lower collusive price, regardless of whether the market is in a boom or a recession phase. In this extended model, therefore, Rotemberg and Saloner's theory of collusive pricing can be interpreted in terms of the response of collusive prices to transitory demand shocks that occur within broader business cycle phases." (p. 83–4) The relation between the cyclicality of prices and the correlation of demand growth, i.e., whether expected growth is positively or negatively correlated with the current rate of growth, is more complex. Prices can move (weakly) pro- and countercyclically depending on the correlation of growth rates over time.

ment. While in the third section the additional information consists of late revelation of the aggregate demand shock, section 4 explores the case of early revelation. Section 5 compares the two settings. Section 6 summarises results and concludes.

3.2 A model of collusion with two-tier information

The analysis is confined to the case of two firms that produce a homogeneous good at constant and equal per unit cost c and trade this good simultaneously, regularly and endlessly at points $t = 0, 1, 2, ...$ in a single marketplace. Future payoffs are discounted at factor $\delta \in (0, 1)$ and intertemporal streams of payoffs are compared in terms of their respective expected present discounted values. The total number of consumers and their willingness to pay for the indistinguishable good produced by the two firms remain constant over time. At each date t, total demand for the good is uncertain but otherwise identical. There are two possible states of nature: with probability $\alpha \in (0, 1)$, in the bad demand state, demand is zero while in the good demand state, occurring with opposite probability $1 - \alpha$, demand is given by a continuous function $D(.)$ that implies $D(c) > 0$ and is nonincreasing at prices with strictly positive demand. By assumption, there is a unique monopoly price p^m and industry profits $(p - c)D(p)$ are monotonically increasing in price for $p \leq p^m$ and monotonically decreasing for $p \geq p^m$. The random variable describing the evolution of demand is i.i.d. over time.

The two firms that produce the identical good are labelled A and B ($j \in \{A, B\}$). If competition prevails in the stage game, the two firms engage in Bertrand price competition. Simultaneously both firms spread the information about their current selling prices among consumers. Once consumers know the two prices, they purchase the good from the lowest-price supplier. When firms announce the same price it is assumed without loss of generality that the residual demand faced by each firm is half the market demand at the price announced independently by both firms. In each period, firms must meet the entire demand for its product at the announced price.

In this competitive setting the two firms try to sustain a self-enforcing collusive production quota agreement (PQA). The agreement specifies

a single focal price p^{PQA} to be announced to consumers by both firms and a sharing rule, i.e., two shares of aggregate demand s^{PQA} and $1 - s^{PQA}$ to be served by firms A and B. The problem of equilibrium selection generally inherent in repeated games is circumvented by focusing exclusively on the fully collusive outcome that consists of both firms charging the unique monopoly price, i.e., $p^{PQA} \equiv p^m$; the sustainability of partially collusive prices $p \in (c, p^m)$ is neglected. To close the model, it is moreover assumed that the two partners fix s such as to maximise the scope of collusion, i.e., they choose the s that relaxes the more stringent of the two incentive constraints of firms A and B.

Tirole's (1988) reformulation of Green and Porter's (1984) model assumes that the following decision-relevant pieces of information are never observed nor disclosed, not even at the end of the stage game: the realisation of the aggregate demand shock, the prices announced to consumers by the rival firm and the number of units the rival firm sells. In this context, the rival's actions remain private information and deviant behaviour cannot be inferred with certainty. A firm that abides by the agreement and faces zero demand in a particular period does not know whether the negative experience is due to the realisation of a bad demand shock or due to the rival undercutting its own price. Common knowledge is limited to the fact that, when at least one firm faces zero individual residual demand, both partners know that at least one firm faces zero demand. These past zero demand experiences (ZDEs) form the history of public information. They allow for imperfect public monitoring because every action profile induces a particular probability distribution over these publicly known outcomes.

The analysis assumes that imperfect public monitoring based on the observation of individual residual demand is feasible in every period $t = 0, 1, 2, ...$. A firm can always rely on publicly verifiable information that is stochastically correlated with the entire profile of price announcements. In some periods, however, there is more precise publicly verifiable information available for monitoring. Let $\gamma \in (0, 1)$ be the probability with which additional information becomes available in period t, $t = 0, 1, 2, ...$ This probability is exogenously given. If firm j makes no ZDE in period t, additional information is useless; it does nothing but confirm what is already public knowledge, namely that both firms abided by the terms of the agreement. If however firm j makes a ZDE in period t, the disclosure of the additional information

may allow for perfect monitoring. In this case the correlated information, insofar as it exclusively serves monitoring purposes, becomes obsolete.

There are many potentially relevant sources of information. Direct disclosure of the rival's strategic decision is one example, but it is not a necessary requirement of perfect monitoring. In some circumstances, price announcements are inferable with certainty from environmental data. This is the case when firms learn about the realisation of the aggregate demand shock in a particular period. Since firms always observe the own residual demand, a revelation of the shock allows contrasting the two pieces of information. If aggregate demand is revealed to be zero, the additional information remains silent about the rival's behaviour since zero aggregate demand and a ZDE are compatible with both cooperative and non-cooperative behaviour. If aggregate demand is revealed to be positive, however, a ZDE uncovers defection from collusive play.

The paper focuses on this particular type of information and a related matter of timing. It analyses the strategic implications of unfrequent revelation of the state of aggregate demand, its relevance for the monitoring process and its implication in terms of the scope of collusion and the expected present discounted value in the following two cases.

- *Deferred revelation of the state of aggregate demand.* With probability $\gamma \in (0,1)$, both firms get reliable and verifiable information about the realisation of the exogenous aggregate demand shock at the end of each period t, after having announced the own prices to prospective consumers.

- *Early revelation of the state of aggregate demand.* With probability $\gamma \in (0,1)$, both firms observe the demand shock realisation before announcing selling prices to prospective consumers.

Note that the two cases do not differ in terms of the time at which all relevant information for effective monitoring is available. Independently of the time of revelation of aggregate demand, individual demand is only observed after prices have been announced and consumers have decided where to purchase. A retaliatory reaction to defection is possible in both cases only with delay. In models of pure perfect monitoring, the timing convention has nevertheless a strong impact on the scope of collusion. The comparison of the two cases reveals that in the present setting with two sources of information,

the timing convention affects the optimal strategy of monitoring, but that the scope of collusion remains unchanged.

The characteristic feature of the model is the unfrequent disclosure of precise, additional information in an environment where monitoring generally relies on correlated signals. In other respects, the analysis maintains common assumptions made in models of oligopolistic collusive behaviour with both perfect and imperfect monitoring. The analysis is limited to stationary equilibria and strongly symmetric dynamic strategies, such that the generation of incentives necessarily requires value burning. The inevitability of periods in which players mutually retaliate against each other makes intertemporally efficient outcomes impossible. The enforcement device is the threat of pure-strategy temporary Bertrand-Nash reversion, and in equilibrium firms implement the strictly necessary punishment such as to maximise the expected present discounted value (EPDV) of the stream of present and future payoffs in an arbitrary collusive period.

Due to the two sources of information, however, there are two well-differentiated histories of public information. The trigger event and the length of punishment phases are contingent on the specific type of information that has triggered the respective punishment. More concretely, the two firms try to sustain the production quota agreement (PQA) by explicitly agreeing or implicitly understanding that repeated interaction is ruled by the following intertemporal pricing strategy.

- The game starts in a collusive phase. Both firms announce price $p = p^m$ and sell $sD(p^m)$ or $(1-s)D(p^m)$ units according to the agreement. Firm $j \in \{A, B\}$ continues playing this action until it makes a ZDE. If firm j faces zero demand in period $t > 0$, it applies the following reasoning.

- If in period t aggregate demand has been revealed to be zero, firm j continues playing cooperatively in period $t + 1$.

  If in period t aggregate demand has been revealed to be high, play switches to a punishment phase of length T_+. From period $t + 1$ onwards, both firms play the stage game Nash equilibrium strategy and announce $p = c$ to consumers for T_+ periods before reverting and starting a new collusive phase.

  If in period t aggregate demand has not been revealed, play switches to a punishment phase of length T_-. From period $t +$

1 onwards, both firms play the stage game Nash equilibrium strategy for T_- periods before starting a new collusive phase.

The length of a punishment phase is T_+ if the ZDE can be publicly traced back to a deviation; it is T_- if no additional information is available; no punishment occurs if the ZDE is known to coincide with zero aggregate demand.

Both T_- and T_+ can, in principle, be zero or positive, finite or infinite. In the following, "hybrid monitoring" (HM) identifies a monitoring strategy with $T_+ > 0$ and $T_- > 0$, indicating that both sources of information are actively used to generate incentives. A hybrid monitoring strategy is conceptually distinguished from its two limiting cases, perfect monitoring (PM) with $T_+ > 0$ and $T_- = 0$ and imperfect public monitoring (IM) with $T_+ = 0$ and $T_- > 0$.

3.3 Deferred revelation of the state of aggregate demand

If the realisation of aggregate demand becomes public knowledge at the end of period t with frequency $\gamma \in (0, 1)$, the trigger event for punishment phases with lengths T_- and T_+ is defined as follows. If firm j makes a ZDE in period t and does not get additional public information about the realisation of aggregate demand before the end of the period, it relies on imperfect public monitoring and starts a punishment phase of length T_- in the subsequent period. If it makes a ZDE and aggregate demand is revealed to be strictly positive, it relies on perfect monitoring and starts a punishment phase of length T_+. In all other cases (i.e., if it does not make a ZDE or if it does make a ZDE and aggregate demand is revealed to be zero), it continues playing cooperatively in the subsequent period. The rival firm relies on the available public information and private knowledge about its own action to correctly infer how the game is continued, including the lengths of possible punishment phases.

Equilibrium play can be in one of $1 + T_- + T_+$ possible states

$$\sigma \in \left\{ C, P_{1_-}, P_{2_-}, ..., P_{T_-}, P_{1_+}, P_{2_+}, ..., P_{T_+} \right\},$$

where C denotes the state in which both firms abide by the agreement, P_{τ_-} for $\tau_- \in \{1, ..., T_-\}$ the τ_-th period of a punishment phase of

length T_- and, accordingly, P_{τ_+} for $\tau_+ \in \{1, ..., T_+\}$ the τ_+th period of a punishment phase of length T_+.

In an arbitrary period t in state C, firm $j \in \{A, B\}$ gets profits $s_j \pi^m$ with probability $1 - \alpha$. A revelation of positive aggregate demand does in this case nothing but confirm the knowledge that firm j already obtained when learning about its individual residual demand: that the rival firm honoured the collusive agreement. According to the rules, play remains in state C in the subsequent period. With probability α, aggregate demand and both firms' one-period profits are zero. The state of aggregate demand is publicly revealed at the end of the period with probability $1 - \gamma$. This additional information is uninformative, however. Both firms' response is then to temper justice with mercy and continue playing cooperatively in the subsequent period. With opposite probability γ, aggregate demand is not revealed. Also in this case price announcements remain private information, but in the absence of additional information both firms rely on imperfect monitoring and play switches to state P_{1-} in the following period.

In an arbitrary period in state C, the stationary path of expected present and future profits of firm $j \in \{A, B\}$ therefore satisfies

$$
\begin{aligned}
V_j^C \;=\; & (1 - \alpha) \left(s_j \pi^m + \delta \left((1 - \gamma) V_j^C + \gamma V_j^C \right) \right) \\
& + \alpha \left(0 + \delta \left((1 - \gamma) V_j^C + \gamma V_j^{P_{1-}} \right) \right)
\end{aligned}
$$

or, equivalently,

$$
V_j^C = (1 - \alpha) s_j \pi^m + (1 - \alpha \gamma) \delta V_j^C + \alpha \gamma \delta V_j^{P_{1-}}. \tag{3.3.1}
$$

Following an unverified ZDE, play switches to state P_{1-} in the following period. Since pure-strategy temporary Bertrand-Nash reversion is a credible punishment, play remains under punishment for the next T_- periods: under stationarity, the EPDV of firm j, $j \in \{A, B\}$ in state P_{1-} satisfies

$$
V_j^{P_{1-}} = \delta^{T_-} V_j^C. \tag{3.3.2}
$$

The resulting stationary EPDVs V_j^C and $V_j^{P_{1-}}$ conditional on the sharing rule $s \in (0, 1)$ and punishment length $T_- \in \mathbb{N}_0$ are stated in appendix A.3.

If firm $-j$ plays cooperatively in the stage game, the most profitable deviation for firm j consists of slightly undercutting the collusive focal

price p^m and serving the entire demand at the announced lower price (setting $s_j = 1$). This deviation yields one-period profits of approximately π_m with probability $(1 - \alpha)$ for firm j while firm $-j$ makes a ZDE. How play proceeds in the subsequent period depends on the informational context. With probability γ, the deviation remains private information and the unverified ZDE triggers a switch to state P_{1_-} in the following period. With probability $1 - \gamma$, it becomes public knowledge that the ZDE has been due to a deviation if aggregate demand is high; play then switches to state P_{1_+} in the next period. If aggregate demand is revealed to be low, the additional information is not revealing and firm $-j$ continues playing cooperatively.

The best response to cooperative play implies a dynamic path of expected present and future profits that under stationarity satisfies

$$
\begin{aligned}
V_j^D &= (1-\alpha)\left(\pi^m + \delta\left((1-\gamma)V_j^{P_{1+}} + \gamma V_j^{P_{1-}}\right)\right) \\
&\quad + \alpha\left(0 + \delta\left((1-\gamma)V_j^C + \gamma V_j^{P_{1-}}\right)\right)
\end{aligned}
$$

or

$$
V_j^D = (1-\alpha)\pi^m + \alpha(1-\gamma)\delta V_j^C + (1-\alpha)(1-\gamma)\delta V_j^{P_{1+}} + \gamma\delta V_j^{P_{1-}} \quad (3.3.3)
$$

for $j \in \{A, B\}$, where the EPDV in state P_{1_+} can be expressed recursively as

$$
V_j^{P_{1+}} = \delta^{T_+} V_j^C \tag{3.3.4}
$$

in analogy to $V_j^{P_{1-}}$.

Scope of collusion. The sustainability conditions $V_j^C \geq V_j^D$ for $j \in \{A, B\}$ are equivalent to

$$
s_j + (1 - \alpha\gamma - s_j\alpha(1-\gamma))\,\delta - (s_j - \alpha)\gamma\delta^{T_-+1} - s_j(1-\alpha)(1-\gamma)\delta^{T_++1} \geq 1.
$$

The scope of collusion is defined to be the set of parameter constellations (α, γ, δ) under which the PQA is sustainable if s and T are chosen solving the problem

$$
\max_{s,T}\left\{\min_{j\in\{A,B\}}\left\{V_j^C - V_j^D\right\}\right\}.
$$

The fact that the choice of the sharing rule is a problem of pure redistribution immediately implies the following result.

Lemma 3.1 Independently of T_- and T_+, the equal sharing rule $s = \frac{1}{2}$ maximises the scope of collusion with respect to s.

The proof is relegated to appendix A.1. With equal sharing, the incentive condition of both firms is

$$\left(2(1 - \alpha\gamma) - \alpha(1 - \gamma)\right)\delta - (1 - 2\alpha)\gamma\delta^{T_-+1} - (1 - \alpha)(1 - \gamma)\delta^{T_++1} \geq 1. \tag{3.3.5}$$

The following proposition characterises the monitoring strategy that maximises the scope of collusion with unfrequent deferred revelation of the state of aggregate demand if firms agree on an equal sharing rule.

Proposition 1 *Pure perfect monitoring* with $\{T_+ = +\infty, T_- = 0\}$ is the scope-maximising monitoring strategy if $\alpha > \frac{1}{2}$ independently of γ; if $\alpha < \frac{1}{2}$, it is outperformed by *hybrid monitoring* with $\{T_+ = +\infty, T_- = +\infty\}$ independently of γ, and by *pure imperfect monitoring* with $\{T_+ = 0, T_- = +\infty\}$ if $\gamma > \frac{\alpha}{1-\alpha}$. Pure imperfect monitoring is never the scope-maximising monitoring strategy; it is strictly dominated by hybrid monitoring for all $\alpha, \gamma, \delta \in (0, 1)$.

The proof is in appendix A.2. The determination of the optimal combination of deterministic and correlated information for maximising the scope of collusion is simplified by the fact that the effects of T_- and T_+ on the incentives of both firms are additively separable. When T_-, the length punishment triggered by an unverified ZDE, has no direct effect on the enforcement power executable with T_+, and vice versa, punishments relying on the two alternative modes of monitoring are independent devices for generating incentives. In order to determine the scope-maximising monitoring strategy it therefore suffices to separately analyse under which parametric constellations the two modes of monitoring add to or derogate the overall enforcement power.

Figure 3.3.1 illustrates the result. It represents, conditional on the three values $\gamma = 0.25$, $\gamma = 0.5$ and $\gamma = 0.75$, the projection of the scopes of collusion with pure imperfect monitoring (dotted), pure perfect monitoring (dashed) and hybrid monitoring (solid) on the parameter space $\{(\alpha, \delta) | \alpha, \delta \in [0, 1]\}$.
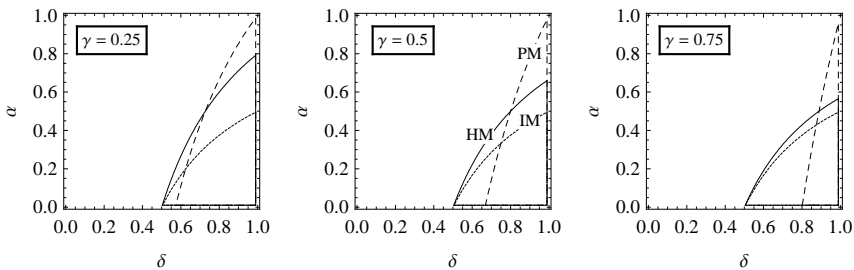
Figure 3.3.1: Scopes with HM (solid), PM (dashed), IM (dotted)

It can be seen that the α-level at which collusion becomes easier to sustain with $\{T_+ = +\infty, T_- = 0\}$ than with $\{T_+ = +\infty, T_- = +\infty\}$, namely $\alpha = \frac{1}{2}$, coincides with the α-level at which pure imperfect monitoring with $\{T_+ = 0, T_- = +\infty\}$ ceases to provide adequate incentives. This threshold is independent of γ because correlated information is available in every period. The relative performance of the two sources of information, however, depends on γ as well as α. Punishment in response to an unverified ZDE generates stronger incentives than punishment based on deterministic information if

$$1 - \gamma < \frac{1 - 2\alpha}{1 - \alpha}, \qquad (3.3.6)$$

i.e., if the relative frequency $1 - \gamma$ of getting deterministic information (which renders the correlated information obsolete) and the frequency α with which imperfect monitoring is handicapped are relatively low.

The relative convenience of the two types of information for monitoring depends on the various factors affecting the enforcement power. Deterministic information discloses a deviation with certainty in the case of high demand and effectively precludes equilibrium punishment in both states of aggregate demand. It is compromised by the periodical demand shocks, but much less so than correlated information. This second source of information never discloses a deviation with certainty and moreover induces equilibrium punishment in the state of low aggregate demand. Its big advantage is its constant availability. These factors are reflected in the inequality above: with high γ and low α imperfect monitoring is most forceful, while in the opposite case perfect monitoring is strongest. This reasoning suggests that the relative advantage of hybrid monitoring with respect to both pure perfect
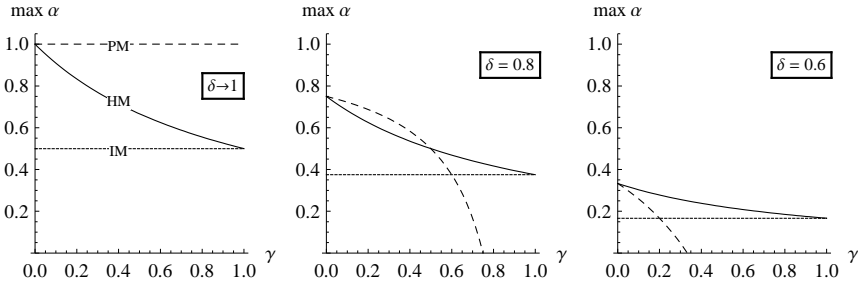
Figure 3.3.2: Maximum uncertainty compatible with collusion

monitoring and pure imperfect monitoring is particularly relevant for intermediate values of γ and α as long as $\alpha \leq \frac{1}{2}$. This fact about the relative performance of the three modes of monitoring is visible in figure 3.3.1. It becomes clearer when plotting the maximum α-level sustainable as a function of γ, conditional on a given discount factor. This is done in the three panels of figure 3.3.2 conditional on values $\delta \to 1$, $\delta = 0.8$ and $\delta = 0.6$. The graphs represent the maximum α-level sustainable with exclusive use of imperfect monitoring (dotted), exclusive use of perfect monitoring (dashed) and under hybrid monitoring (solid).

It can be appreciated that for small γ hybrid monitoring is dominated by pure perfect monitoring while for $\gamma \to 1$ it becomes asymptotically equivalent to pure imperfect monitoring. For intermediate values, the combined use of both sources of information may allow for a significantly more precise monitoring. The comparison of the three panels also reveals that hybrid monitoring is particularly useful when the common discount factor is low.

Equilibrium punishment and maximum collusive EPDV. The analysis of the scope-maximising monitoring strategy ignores that, when correlated information is used, the generation of enforcement power with strictly symmetric punishment strategies necessarily implies value burning. Using imperfect monitoring, it is not possible to provide incentives without accepting a partial deterioration of the intertemporal collusive payoff.

This is different when more precise information is available. It has just been argued that information about the current state of aggregate

demand effectively precludes equilibrium punishment. A first result is therefore immediate.

Lemma 3.2 If perfect monitoring alone allows sustaining collusion, then setting $T_- = 0$ is a necessary condition for maximising the collusive EPDV V_j^C. In this case, firms may always set $T_+ = +\infty$ without prejudice.

In appendix A.3 it is shown that V_j^C is independent of T_+ and that it is strictly decreasing in T_-. The statement cannot immediately be made extensive to situations when $T_+ = +\infty$ alone does not allow sustaining the PQA because in equilibrium T_+ and T_- are not anymore independent devices for generating incentives. The following proposition, however, argues that this equilibrium interdependence of the two sources of information does not alter the relative performance of perfect and imperfect monitoring. If, as in the present model, the attainment of high quality information is costless, both the informational advantage and the cheap implementation makes perfect monitoring a superior practice in all respects.

Proposition 2 In terms of the collusive EPDV attainable in equilibrium, the three types of monitoring are strictly ranked. If *pure perfect monitoring* with $\{T_+ = +\infty, T_- = 0\}$ allows sustaining the collusive agreement, it is the only monitoring strategy that maximises the collusive EPDV in equilibrium; if *hybrid monitoring* with $\{T_+ = +\infty, T_- > 0\}$ and *pure imperfect monitoring* with $\{T_+ = 0, T_- > 0\}$ allow sustaining the collusive agreement but not pure perfect monitoring, imperfect monitoring is strictly dominated by hybrid monitoring.

If there is a trade-off between the maximum attainable gains from cooperation and incentive provision, playing the harshest possible punishment $T_- = +\infty$ typically implies burning value in excess. With hybrid monitoring (or pure imperfect monitoring), firms prefer to set the shortest punishment length permitting sustainability in response to an unverified ZDE. $\frac{\partial V_j^C}{\partial T_-} < 0$ independently of T_+ implies that, given an arbitrary length $T_+ \in \mathbb{N}_0$, the choice T_- that maximises the collusive EPDV of firm j is the smallest non-negative integer number satisfying $V_j^C \geq V_j^D(T_+)$.[4] The question then is which combination

---

[4]In what follows, the integer problem is ignored. It is assumed that a punishment length can take any positive real value, and the binding incentive constraint is used to determine the equilibrium punishment length.

of T_+ and the associated T_- determined by $V_j^C = V_j^D(T_+)$ maximises V_j^C. The proof of the proposition in appendix A.4 answers this question by showing that punishment in response to deterministic information does not hamper, but actually reinforces the incentives that can be independently generated using correlated information. Since in equilibrium the necessary and costly T_- is strictly decreasing in the costless T_+, $T_+ = +\infty$ is a necessary condition for maximising the collusive EPDV V_j^C when pure perfect monitoring alone does not allow implementing the agreement.

Before closing the discussion of late revelation, a final observation may be interesting. It documents the importance even nuances have when defining trigger events and punishment strategies. The postulated trigger event prescribes that a firm refrains from punishing when the aggregate demand has been revealed to be zero. But why should firms use a dynamic strategy that prescribes lenient rather than grim response under ignorance? Results on optimal punishments with both perfect and imperfect monitoring coincide in highlighting the role of most forceful punishments. It may therefore be surprising that in the present context "leniency in response to ignorance" is an essential assumption. The alternative specification "retaliation in response to ignorance" implies a stationary path of expected present and future profits in state C that satisfies

$$
\begin{aligned}
V_j^C &= (1-\alpha)\left(s_j\pi^m + \delta\left((1-\gamma)V_j^C + \gamma V_j^C\right)\right) \\
&\quad + \alpha\left(0 + \delta\left((1-\gamma)V_j^{P_1-} + \gamma V_j^{P_1-}\right)\right)
\end{aligned}
$$

or, equivalently,

$$
V_j^C = (1-\alpha)s_j\pi^m + (1-\alpha)\delta V_j^C + \alpha\delta V_j^{P_1-}.
$$

This expression coincides with the one resulting in the original model when $\gamma = 1$, i.e., when available deterministic information is simply ignored. Proposition 2 asserts that this strategy implies a strictly lower collusive EPDV in equilibrium. The best deviation with "retaliation in response to ignorance" implies

$$
\begin{aligned}
V_j^D &= (1-\alpha)\left(\pi^m + \delta\left((1-\gamma)V_j^{P_1+} + \gamma V_j^{P_1-}\right)\right) \\
&\quad + \alpha\left(0 + \delta\left((1-\gamma)V_j^{P_1-} + \gamma V_j^{P_1-}\right)\right)
\end{aligned}
$$

or

$$V_j^D = (1-\alpha)\pi^m + (1-\alpha)(1-\gamma)\delta V_j^{P_{1+}} + (1 - (1-\alpha)(1-\gamma))\,\delta V_j^{P_{1-}},$$

such that the incentive condition for a firm with share s_j is

$$s_j + (1-\alpha)\,\delta - (s_j - \alpha)\,\delta^{T_-+1} - s_j(1-\alpha)(1-\gamma)\left(\delta^{T_++1} - \delta^{T_-+1}\right) \geq 1.$$

The third term on the left hand side reveals that, contrary to what happens with a lenient response to ignorance, an increase of T_- by one period offsets the enforcement power generated by a one-period length of punishment triggered by a verified ZDE. A grim response to ignorance implicitly downgrades the quality of deterministic information and makes it informationally equivalent to correlated information. Hybrid monitoring does then obviously not allow improving upon pure imperfect monitoring.

3.4 Early revelation of the state of aggregate demand

This section changes the timing convention that governs the strategic interaction of the two price-setting oligopolists that try to sustain a collusive PQA.

It is now assumed that, with frequency $\gamma \in (0,1)$, the realisation of aggregate demand becomes public knowledge at the beginning of each period, i.e., before firms decide on which price they announce to consumers. The trigger event for punishment phases with lengths T_- and T_+ is then defined as follows. If in period t aggregate demand is not revealed, no additional information is available that can be contrasted with own residual demand to make an inference about the price charged by the rival firm. In this case, firm $j \in \{A, B\}$ relies on imperfect public monitoring. If it faces positive individual demand, play continues in state C; if it makes a ZDE, play switches to a punishment phase of length T_- starting in period $t+1$. The strategic context is substantially different when aggregate demand is revealed prior to firm j's decision in period t. If high demand is revealed, a positive residual demand confirms cooperative behaviour while a ZDE discloses defection. In the former case, play continues cooperatively in period $t+1$ while in the latter, it switches to a punishment phase of

length T_+. If the revelation is of low demand, on the other hand, contrasting aggregate and individual demand does not allow inferring the rival's choice. In this case it is assumed that firm j continues playing cooperatively in the subsequent period.

The band width of strategic interaction is now broader; equilibrium play can be in one of $3 + T_- + T_+$ possible states

$$\sigma \in \left\{ C, C_H, C_L, P_{1_-}, P_{2_-}, ..., P_{T_-}, P_{1_+}, P_{2_+}, ..., P_{T_+} \right\}.$$

Suppose that, at the end of period $t - 1$, the intertemporal strategy stipulates that firms continue acting cooperatively. Play in period t can then be in either one of three states. If at the beginning of period t the realisation of aggregate demand is not revealed (probability γ), play enters state C; if the aggregate shock is publicly revealed (probability $1 - \gamma$), states C_H or C_L materialise depending on the level of aggregate demand.

In state C, since no additional information is available, identifying the rival's action with certainty is impossible and monitoring necessarily relies exclusively on correlated information. Firm $j \in \{A, B\}$ makes profits $s_j \pi^m$ with probability $1 - \alpha$ and zero profits otherwise. Positive individual demand discloses both a positive aggregate shock and cooperative behaviour; the rule then prescribes cooperation, such that in the next period again state C is reached with probability γ, state C_H with probability $(1-\gamma)(1-\alpha)$ and state C_L with probability $(1 - \gamma)\alpha$. Since an unverified ZDE in this case triggers a punishment phase oflength T_-, the stationary path of expected present and future profits in state C satisfies

$$
\begin{aligned}
V_j^C \quad = \quad & (1 - \alpha) \left(s_j \pi^m + \delta \gamma V_j^C \right. \\
& \left. + \delta (1 - \gamma) \left((1 - \alpha) V_j^{C_H} + \alpha V_j^{C_L} \right) \right) \\
& + \alpha \left(0 + \delta V_j^{P_{1_-}} \right)
\end{aligned}
\tag{3.4.1}
$$

if firm j abides by the agreement. The best response to state C cooperation in the stage game consists of slightly undercutting the collusive focal price p^m and serving the entire demand at the announced lower price. This action provokes a ZDE of firm $-j$ with probability one, and imperfect monitoring then implies that the continuation game starts with a punishment phase of length T_-. Under stationarity V_j^{DC}

satisfies

$$V_j^{DC} = (1 - \alpha) \left(\pi^m + \delta V_j^{P_{1-}} \right) + \alpha \left(0 + \delta V_j^{P_{1-}} \right). \qquad (3.4.2)$$

In state C_H, abiding by the agreement assures profits $s_j \pi^m$ with certainty. Positive individual profits confirm the public information and the continuation game starts in state C with probability γ, state C_H with probability $(1 - \gamma)(1 - \alpha)$ and state C_L with probability $(1 - \gamma)\alpha$. The EPDV of playing cooperatively satisfies

$$V_j^{C_H} = s_j \pi^m + \delta \gamma V_j^C + \delta (1 - \gamma) \left((1 - \alpha) V_j^{C_H} + \alpha V_j^{C_L} \right). \quad (3.4.3)$$

Also in state C_H the best response consists of slightly undercutting the collusive focal price p^m and selling approximately $D(p^m)$ many units of the good. The rival faces zero residual demand knowing that the aggregate shock is strictly positive; a publicly verified ZDE triggers a punishment phase of length T_+ and $V_j^{DC_H}$ thus satisfies

$$V_j^{DC_H} = \pi^m + \delta V_j^{P_{1+}}. \qquad (3.4.4)$$

In state C_L, a firm makes zero profits and is unable to identify the rival's decision. "Leniency in response to ignorance" implies that the behaviour and, in consequence, the EPDV of the continuation play mimic the situation in state C_H. $V_j^{C_L}$ satisfies

$$V_j^{C_L} = 0 + \delta \gamma V_j^C + \delta (1 - \gamma) \left((1 - \alpha) V_j^{C_H} + \alpha V_j^{C_L} \right). \qquad (3.4.5)$$

In this case no deviation exists that allows increasing the one-period profit in the stage game. If the dynamic strategy provides for a lenient response to ignorance, in state C_L firm j is trivially indifferent between cooperation and defection. In this sense, state C_L does not pose a threat to the sustainability of a collusive PQA under early revelation of the state of demand.

If a punishment phase is initiated, since pure-strategy temporary Bertrand-Nash reversion is a credible punishment, play remains under punishment for the next T_- or, respectively, T_+ periods. The EPDVs in states P_{1-} and P_{1+} can be expressed recursively in terms of the expected continuation value $\gamma V_j^C + (1 - \gamma) \left((1 - \alpha) V_j^{C_H} + \alpha V_j^{C_L} \right)$ that is realised T_- or T_+ periods ahead. In a stationary environment these

values satisfy

$$V_j^{P_{1-}} = \delta^{T_-}\gamma V_j^C + \delta^{T_-}(1-\gamma)\left((1-\alpha)V_j^{C_H} + \alpha V_j^{C_L}\right) \quad (3.4.6)$$

$$V_j^{P_{1+}} = \delta^{T_+}\gamma V_j^C + \delta^{T_+}(1-\gamma)\left((1-\alpha)V_j^{C_H} + \alpha V_j^{C_L}\right) \quad (3.4.7)$$

The values in states C, C_H, C_L, P_{1-} and P_{1+} solving equations 3.4.1, 3.4.3, 3.4.5–3.4.7 conditional on the sharing rule $s \in (0,1)$ and punishment lengths $T_-, T_+ \in \mathbb{N}_0$ are stated in appendix B.1.

Before proceeding with the characterisation of the monitoring strategies that maximise the scope of collusion or minimise the cost of implementation, it is necessary to make a note of caution. V_j^C should not be misinterpreted as an arithmetic mean of $V_j^{C_H}$ and $V_j^{C_L}$. The three values V_j^C, $V_j^{C_H}$ and $V_j^{C_L}$ correspond to states in which different quantity and quality of information is available for monitoring. It is easy to show that $V_j^{C_H} > V_j^{C_L}, V_j^C$, but the order relation between V_j^C and $V_j^{C_L}$ is context-dependent. If imperfect monitoring is imprecise, either due to bad demand shocks occurring too often or due to additional information rendering the observation of correlated outcomes obsolete, the EPDV in state C falls below the EPDV in a periods of revealed low aggregate demand.[5] The inequality that best reflects the informational value of different states is the following: it can be shown that $(1-\alpha)V_j^{C_H} + \alpha V_j^{C_L} > V_j^C$.

Scope of collusion. Credibility of the implementation strategy is guaranteed under punishment and in state C_L. The analysis of best responses focuses on deviations from collusive action in states C and C_H.

Sustaining collusion in state C. It is shown in appendix B.1 that $V_j^C - V_j^{DC}$ in state C is strictly increasing in the share $s_j \in (0,1)$ for both $j = A, B$ also under unfrequent early revelation. With equal

---

[5] If imperfect monitoring becomes very imprecise, there is even the hypothetical possibility that the EPDV in state C falls below the intertemporal values in the initial periods of a punishment phase of length T_+. $V_j^C > V_j^{P_{1+}}$ is not satisfied independently of punishment lenghts T_-, T_+ for all parametric constellations. It is shown in the appendix that this possibility does not affect the monitoring strategies that maximise the scope of collusion or the collusive EPDV in equilibrium.

sharing, the incentive conditions $V_j^C \geq V_j^{DC}$ in state C of both firms $j = A, B$ coincide and are equivalent to

$$(2(1 - \alpha\gamma) - \alpha(1 - \gamma))\,\delta - ((1 - 2\alpha) + \alpha(1 - \gamma))\,\delta^{T_- + 1} \geq 1. \quad (3.4.8)$$

Evidently, deterministic information does not contribute because in state C this sort of information is unavailable. Correlated information allows exerting enforcement power if $\alpha < \frac{1}{1+\gamma}$ (see appendix B.3). The fact that the maximum α-level compatible with collusion depends negatively on γ may seem counterintuitive. Obsolescence of correlated information is decreasing in γ; the sustainability of collusion in state C, however, which relies exclusively on this type of information, appears to be handicapped by high levels of γ. But in state C correlated information is known to be informative. In this situation γ only affects the probability with which the different states will be reached in the next future. A higher γ increases the probability of play continuing in the low information state, it depresses the continuation value and thereby weakens incentives.

Sustaining collusion in state C_H. Also in state C_H, $V_j^{C_H} - V_j^{DC_H}$ is strictly increasing in $s_j \in (0, 1)$ (see appendix B.1). With $s = \frac{1}{2}$, the incentive condition $V_j^{C_H} \geq V_j^{DC_H}$ is identical for both $j = A, B$ and equivalent to

$$(2(1 - \alpha\gamma) - \alpha(1 - \gamma))\,\delta + \alpha\gamma\delta^{T_- + 1} - (1 - \alpha)\delta^{T_+ + 1} \geq 1. \quad (3.4.9)$$

It can be appreciated that sustainability in state C_H is responsive to both correlated and deterministic information. But while deterministic information now allows generating incentives, punishment in response to correlated information hampers the process of monitoring in state C_H. Only deterministic information effectively allows generating incentives. The enforcement power is strictly increasing in T_+ and strictly decreasing in T_- irrespective of parameters α, γ and δ (see appendix B.3).

The scope of collusion under unfrequent early revelation of the state of aggregate demand is the set of parametric constellations (α, γ, δ) that allow sustaining a collusive PQA in both critical states C and C_H, i.e., the intersection of the scopes in states C and C_H. The following lemma relates the scopes in states C and C_H and thereby identifies

the crucial property of the scope of collusion under unfrequent early revelation. The proof is in appendix B.2.

Lemma 4.1 The scope of collusion in state C is a strict subset of the scope of collusion in state C_H if and only if $T_+ > T_-$. When $T_+ = T_-$, the scopes of collusion in states C and C_H coincide.

The lemma reveals a striking particularity of the monitoring process used to sustain collusion under unfrequent early revelation of aggregate shocks: it is independent of parameters α, γ and δ. The independence of γ or $1 - \gamma$, the probability with which deterministic information renders correlated information obsolete, may be particularly surprising. But remember the highly asymmetric effects of T_- and T_+ on the sustainability of a PQA in the different collusive states. If only one source of information is used, collusive agreements are very fragile either in states of revealed high demand or in states in which high quality information is unavailable. The following proposition shows that in such an environment colluding firms may use hybrid monitoring to effectively balance incentives across states.

Proposition 3 Hybrid monitoring with $T_- = T_+ = +\infty$ is the only monitoring strategy maximising the scope of collusion under early revelation of the state of aggregate demand irrespective of α, γ and δ.

Also under unfrequent early revelation the two modes of monitoring are complementary devices for implementing a collusive agreement, but compared to unfrequent late revelation in a very different manner. The crucial difference is that both sources of information are now essential. With unfrequent late revelation, the two kinds of information reinforce each other when jointly generating enforcement power in the only existing collusive state (i.e., when $\alpha < \frac{1}{2}$). With unfrequent early revelation, a second source is either useless (state C) or directly counter-productive (state C_H) from an *intra-state* perspective. But the two modes of monitoring are complementary *inter-state*. The best use of both kinds of information is determined by their availability in the two critical states: sustainability in state C_H relies on deterministic information while correlated information is used in state C. The independence of γ is then meaningful: a higher γ implies that deterministic information is available less frequently, but also that it is needed less often.
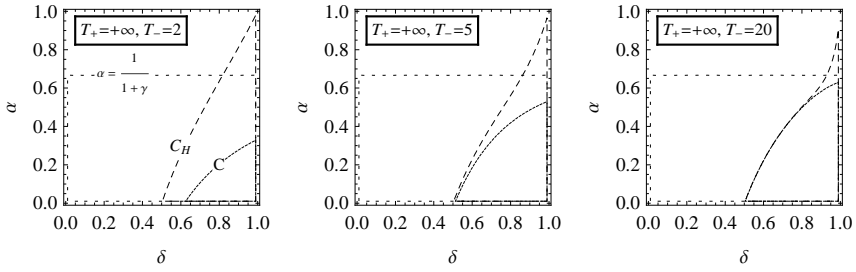
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Figure 3.4.1: Scopes in states C and C_H for $T_+ = +\infty$ and alternative T_-

The proof in appendix B.3 also verifies that, for what concerns the sustainability of collusion, state C is effectively the bottleneck. The relation of the two scopes for varying lengths of punishment T_- is illustrated in figure 3.4.1 conditional on $\gamma = 0.5$.

Equilibrium punishment and maximum collusive EPDV. Due to the particularities of monitoring in the case of an unfrequent early revelation of aggregate demand, the determination of the monitoring strategy that maximises the expected collusive present discounted value in equilibrium $\gamma V_j^C + (1 - \gamma)\left((1 - \alpha)V_j^{C_H} + \alpha V_j^{C_L}\right)$ is straightforward. The result is summarised in proposition 4 and proved in appendix B.4.

Proposition 4 The only monitoring strategy that maximises the collusive EPDV in equilibrium is hybrid monitoring with $T_+ = +\infty$ and $T_- > 0$ defined by

$$\delta^{T_-} = 1 - \frac{1 - \delta}{(1 - \alpha(1 + \gamma))\,\delta}.$$

Also this section is closed with a short remark on the alternative rule "retaliation in response to ignorance". In the case of unfrequent early revelation the benevolent reaction to revealed low aggregate demand is even more essential than in the case of unfrequent late revelation. It is shown in appendix B.5 that a grim response reduces $V_j^{C_L}$ so much that it is not only always strictly lower than V_j^C but also lower than $V_j^{P_{1-}}$ irrespective of punishment lengths T_+ and T_-. It is impossible

to sustain the agreement in state C_L in this case; the joint scope of collusion is empty.

3.5 Demand uncertainty, fluctuations and the sustainability of collusion

A comparison of the scopes of collusive action and the equilibrium expected present discounted values resulting from cooperation allow qualifying and challenging the traditional view on (i) the evolution of incentives over the business cycle and (ii) the robustness of these incentives with respect to the timing convention, the cornerstone differentiating the approaches of Green and Porter (1984) and Rotemberg and Saloner (1986).

Incentives and the business cycle.   In the case of unfrequent late revelation, the state of aggregate demand is unknown when firms decide on cooperation or defection. It cannot affect incentives.

The analysis reveals that the state of aggregate demand may be irrelevant also if it becomes public knowledge before firms take their strategic decisions. It is confirmed that, effectively, the one-period gains from deviation are much stronger in state C_H than in state C_L. But unlike the model by Rotemberg and Saloner (1986), in the present model this disparity has no impact on incentives. The reason being that in a state of revealed high demand, when the gains from deviation are highest (no averaging effect when uncertainty is resolved), so is the precision of the information available for monitoring (no camouflage effect without uncertainty). State C_H is the only state in which perfect monitoring is feasible. Short-term deviation gains are substantially lower in state C and even more so in state C_L, but also the information that is used for monitoring in these states is less precise. In this sense, the result by Rotemberg and Saloner appears to be a direct consequence of their exclusive focus on perfect monitoring. The analysis in section 4 suggests that, contrary to Rotemberg and Saloner's hypothesis, the critical state is generally C, the state in which the information available for monitoring is weakest. The monitoring precision in state C determines the maximal amount of enforcement power that can be generated and the equilibrium cost of punishment affecting the collusive EPDV in equilibrium.

Nonetheless, stating that the results favour Green and Porter's explanation would be equally misleading. Rather, the model asserts that the monitoring strategy that maximises the scope of collusion is precisely the one that balances the two effects of uncertainty on self-enforcement correctly identified by Rotemberg and Saloner (the positive averaging effect of uncertainty) and Green and Porter (the pernicious camouflage effect). According to this finding, the incentives for deviation should be expected to be constant over the cycle, unaffected by eventual revelations that the market is at a peak. The model also predicts that this uniformity of incentives over the business cycle is independent of the frequency with which such revelations are made.

Relevance of the timing convention. The answer given in the unfrequent revelation framework to the second question, concerning the robustness of these uniform incentives with regard to the timing convention, is summarised in proposition 5. It compares the scope of collusion and the collusive EPDV in equilibrium under both early and deferred revelation of the aggregate demand shock.

Proposition 5 If $\alpha < \frac{1}{2}$, the scopes under unfrequent early and late revelation of the shock of aggregate demand coincide. If $\alpha \geq \frac{1}{2}$, the scope under unfrequent late revelation strictly includes the scope with unfrequent early revelation.

For all α, γ and δ that allow sustaining collusion, i.e., irrespective of the monitoring strategy used under unfrequent late revelation, the collusive EPDV in equilibrium is strictly larger under unfrequent late revelation than under unfrequent early revelation of the aggregate demand shock.

The proof is in appendix C.1. The interesting result concerns $\alpha < \frac{1}{2}$. The finding that for $\alpha \geq \frac{1}{2}$ the scope under late revelation strictly includes the scope with early revelation is expected. In this case, correlated information is so imprecise that it becomes useless under unfrequent late revelation. Monitoring can then only rely on deterministic information and the unfrequent revelation framework replicates the traditional finding in perfect monitoring models.

When correlated information allows generating incentives stand-alone and hybrid monitoring expands the scope of collusion under unfrequent late revelation, however, the possibility of optimally combining the two sources of information makes a striking difference. With hybrid

monitoring, the scopes under early and late revelation of the shock of aggregate demand coincide. In terms of the maximum scope of collusion, the timing of the revelation of aggregate shocks becomes irrelevant.

The fact that it makes no difference for the maximum scope of collusion whether uncertainty is resolved before or after firms decide on defection forcefully demonstrates the relevance of both explanations for self-enforcement. If uncertainty has not been resolved, it obfuscates the process of monitoring by making external, environmental changes and behavioural effects indistinguishable (camouflage effect). If it is resolved, the gains from defection are strictly increasing in the realisation of the random event (no averaging of deviation gains). The degree to which uncertainty affects incentives is limited to the net impact of these two effects.

Finally, the relevance of both factors is also reflected in the cost of implementation. If the use of correlated and deterministic information is endogenously determined, hybrid monitoring allows balancing incentives and the maximum scope of collusion is the same with and without resolution of uncertainty. High gains from deviation nevertheless remain a challenge to the self-enforcement of collusive agreements. Under unfrequent early revelation, when the payoff structure in the stage game is less favourable, the implementation of these balanced incentives is more costly. While in models of perfect monitoring the generation of incentives is trivially costless independently of the timing convention, the second part of proposition 5 reveals that, over time, unfrequent early revelation requires more "preventive" punishment in equilibrium than unfrequent low revelation. The maximum amount of enforcement power is the same, but under unfrequent late revelation its implementation is cheaper.

3.6 Conclusion

Collusion theory distinguishes three types of monitoring frictions: perfect monitoring, when the rivals' actions are disclosed with certainty but with a delay; imperfect public monitoring based on the observation of random signals whose distribution depends on the equilibrium profile of actions; and private monitoring that does not rely on common knowledge. The traditional approach has been to analyse optimal implementation of collusive agreements conditional on a particular mode

of monitoring. The present paper explores how monitoring strategies look like if agents exploit several sources of information for monitoring rival behaviour. More concretely, it is assumed that firms generally rely on imperfect monitoring but sporadically get more precise information that allows them identifying the choices made by rival firms. The analysis then endogenously determines the best use of correlated and deterministic information for monitoring.

The results suggest that firms will generally exploit the full range of information available. In most parametric constellations, there are strong compound effects of the two kinds of information. In the different cases analysed, there are actually only two situations in which available information is voluntarily discarded. If correlated information is too imprecise, it does not directly interfere with monitoring based on deterministic information but it degrades the enforcement power by implementing an excessive punishment along the equilibrium path. Just ignoring correlated information then allows enlarging the scope of collusion. The second case refers to situations in which correlated information is precise enough to effectively exert enforcement power, but it is unnecessary because deterministic information alone allows sustaining collusion. Ignoring correlated information does in this case reduce the scope of collusion, but it raises the intertemporal gains from collusion. Both situations can happen only in the case of unfrequent late revelation of the aggregate demand shock; with early revelation, both sources of information are actively used independently of the precision of correlated information and independently of the frequency with which deterministic information becomes available.

Given the quite different modes of operation of the scope-maximising monitoring strategies in the cases of unfrequent early and late revelation of aggregate demand, it may seem surprising that the maximum scope itself remains unaffected as long as imperfect monitoring is sufficiently precise. In the present model, the timing convention does not affect the scope of collusion. This is very different in a model of pure perfect monitoring. It happens because the optimal monitoring strategy balances the positive effect of uncertainty on incentives, namely the averaging of deviation gains, and the negative effect of uncertainty on incentives, the camouflage of behavioural effects. With an adequate monitoring strategy therefore the resolution of uncertainty becomes effectively irrelevant for the sustainability of collusion. The timing convention does affect the intertemporal collusive gains made in equilibrium, however. If uncertainty is unresolved when firms decide

on cooperation or defection, firms have more degrees of freedom when defining their monitoring strategy. In many parametric constellations, pure perfect monitoring as well as hybrid monitoring allow sustaining collusion, possibly even pure imperfect monitoring. Firms are then free to choose the strategy that maximises the collusive expected present discounted value in equilibrium. If uncertainty is resolved, there is no such freedom; state of aggregate demand then directly determines the monitoring strategy required to sustain collusion.

Finally, it has been argued that the results allow reassessing the traditional controversy about the likelihood of price wars during booms and recessions. Conditional on an adequate use of both sources of information, the incentives for cooperation or defection remain constant over the cycle. This is a direct implication of the balancing of the two effects of uncertainty: if aggregate demand has been revealed to be at its peak, the gains from deviation are highest but a deviation is detected with certainty; if the state of aggregate demand has not been revealed, both deviation gains and monitoring precision are low. In empirical analysis, it has been very difficult to identify a higher propensity of defection either in booms or in recessions. Collusion with multi-tier information provides one possible explanation of this inconclusive evidence.

Appendix to Chapter Three

Appendix A: Section 3.3

Appendix A.1: Proof of lemma 3.1 Let $\Delta V_j := V_j^C - V_j^D$. The partial derivative of

$$\Delta V_j = (1 - \alpha\gamma - s_j\alpha(1 - \gamma))\,\delta - (s_j - \alpha)\gamma\delta^{T_-+1}$$
$$-s_j(1 - \alpha)(1 - \gamma)\delta^{T_++1} - (1 - s_j)$$

with respect to s_j is

$$\frac{\partial \Delta V_j}{\partial s_j} = 1 - \alpha(1 - \gamma)\delta - \gamma\delta^{T_-+1} - (1 - \alpha)(1 - \gamma)\delta^{T_++1}.$$

Since $\delta^{T_-}, \delta^{T_+} \in (0, 1]$ for $T_-, T_+ \in \mathbb{N}_0$, $\frac{\partial \Delta V_j}{\partial s_j} > 1 - \alpha(1-\gamma)\delta - \gamma\delta - (1-\alpha)(1 - \gamma)\delta = 1 - \delta > 0$. The scope is thus strictly increasing for firm $j = A$ which is assigned share $s_A = s$ and strictly decreasing for firm $j = B$ with share $s_B = 1 - s$ independently of the punishment lengths T_- and T_+ and independently of parameters α, γ and δ. Consequently the rule $s = \frac{1}{2}$ that assigns equal shares to both firms relaxes the more stringent of the two incentive conditions and thus maximises the joint scope of collusion.

Alternatively, notice that the incentive condition of firm A can equivalently be rewritten as

$$s \geq \underline{s} := \frac{1 - (1 - \alpha\gamma)\delta - \alpha\gamma\delta^{T_-+1}}{1 - \gamma\delta^{T_-+1} - (1 - \gamma)\delta^{T_++1}}$$

while the incentive condition of firm B is equivalent to

$$s \leq \overline{s} := \frac{(1 - \alpha\gamma)\delta - (1 - \alpha)\gamma\delta^{T_-+1} - (1 - \gamma)\delta^{T_++1}}{1 - \gamma\delta^{T_-+1} - (1 - \gamma)\delta^{T_++1}}.$$

There exists a sharing rule allowing for sustainable collusion only if $\underline{s} \leq \overline{s}$ or

$$2(1 - \alpha\gamma)\delta - (1 - 2\alpha)\gamma\delta^{T_- + 1} - (1 - \gamma)\delta^{T_+ + 1} \geq 1,$$

a condition that is equivalent to both firms' incentive constraint in case $s = \frac{1}{2}$.

Appendix A.2: Proof of proposition 1 Let $\Delta V_j := V_j^C - V_j^D$. The partial derivatives w.r.t. the punishment lengths are

$$\frac{\partial \Delta V_j}{\partial T_-} = -(s_j - \alpha)\gamma\, \delta^{T_- + 1} \ln(\delta) \quad \left\{ \begin{array}{ll} > 0 & \text{if } \alpha < s_j \\ = 0 & \text{if } \alpha = s_j \\ < 0 & \text{if } \alpha > s_j \end{array} \right.$$

and

$$\frac{\partial \Delta V_j}{\partial T_+} = -s_j(1 - \alpha)(1 - \gamma)\, \delta^{T_+ + 1} \ln(\delta) > 0.$$

With an equal sharing rule $s_A = s_B = \frac{1}{2}$, the punishment lengths maximising $V_j^C - V_j^D$ are then $\{T_+ = +\infty, T_- = +\infty\}$ if $\alpha < \frac{1}{2}$ and $\{T_+ = +\infty, T_- = 0\}$ if $\alpha \geq \frac{1}{2}$.

With pure imperfect monitoring deterministic information is ignored. In this case, $T_+ = 0$ and $\gamma = 1$ since correlated information is available in every period. A marginal increase of the length of punishment T_- following an unverified ZDE then implies $\frac{\partial \Delta V_j}{\partial T_-} = -(s_j - \alpha)\, \delta^{T_- + 1} \ln(\delta)$. If $\alpha < \frac{1}{2}$, this effect is strictly positive. If an equal length of both punishment phases is taken as a basis for comparison, a marginal increase of the length of punishment T_- following an unverified ZDE under pure imperfect monitoring has a larger positive effect on the overall enforcement power than a marginal increase of the length of punishment T_+ following a verified deviation under pure prefect monitoring if $(s_j - \alpha) > s_j(1 - \alpha)(1 - \gamma)$ or $\gamma > \frac{\alpha}{1 - \alpha}$.

Appendix A.3: Proof of lemma 3.2 The stationary EPDVs V_j^C and $V_j^{P_{1-}}$ of firm j's current and future profits in an arbitrary period in states C and P_{1-} satisfy the following system of linear equations:

$$\begin{aligned} V_j^C &= (1 - \alpha)s_j\pi^m + (1 - \alpha\gamma)\,\delta V_j^C + \alpha\gamma\delta V_j^{P_{1-}}, \\ V_j^{P_{1-}} &= \delta^{T_-} V_j^C. \end{aligned}$$

Solving the equation system yields:

$$V_j^C = \frac{(1-\alpha)s_j\pi^m}{1 - (1-\alpha\gamma)\delta - \alpha\gamma\delta^{T_-+1}}$$

and

$$V_j^{P_{1-}} = \frac{(1-\alpha)\delta^{T_-}s_j\pi^m}{1 - (1-\alpha\gamma)\delta - \alpha\gamma\delta^{T_-+1}}.$$

Since

$$\frac{\partial V_j^C}{\partial T_-} = \frac{\alpha(1-\alpha)\gamma\,\delta^{T_-+1}\ln(\delta)\,s_j\pi^m}{(1 - (1-\alpha\gamma)\delta - \alpha\gamma\delta^{T_-+1})^2} < 0$$

and

$$\frac{\partial V_j^C}{\partial T_+} = 0,$$

In analogy to the standard model with pure imperfect monitoring, V_j^C is strictly decreasing in T_- irrespective of the sharing rule chosen, punishment lengths T_+, T_- and parameter values α, δ and γ. In analogy to the standard model with pure perfect monitoring, punishment phases of length T_+ are never exerted in equilibrium if supervisory deterrence is strong enough to sustain collusion. T_+ can be fixed such as to maximise the scope of collusion without detrimental effects on the overall collusive gains.

Appendix A.4: Proof of proposition 2 Pure perfect monitoring with $\{T_+ = +\infty, T_- = 0\}$ allows sustaining a PQA if $\gamma < \gamma_{\mathrm{PM}} := \frac{(1-\alpha)\delta-(1-\delta)}{(1-\alpha)\delta}$, while with hybrid monitoring and $\{T_+ = +\infty, T = +\infty\}$ it can be sustained if $\gamma < \gamma_{\mathrm{HM}} := \frac{(1-\alpha)\delta-(1-\delta)}{\alpha\delta}$. If $\alpha \geq \frac{1}{2}$, $\gamma_{\mathrm{PM}} \geq \gamma_{\mathrm{HM}}$ and the scope with hybrid monitoring is a subset of the scope with pure perfect monitoring. Lemma 3.2 applies and correlated information is not used in equilibrium. If $\alpha < \frac{1}{2}$, $\gamma_{\mathrm{HM}} > \gamma_{\mathrm{PM}}$ and lemma 3.2 applies only if $\gamma \in (0, \gamma_{\mathrm{PM}}]$. If $\gamma \in (\gamma_{\mathrm{PM}}, \gamma_{\mathrm{HM}})$, a PQA cannot be sustained with $\{T_+ = +\infty, T_- = 0\}$; a combination of both sources of information for monitoring, however, does allow sustaining the PQA with equal sharing.

If $\alpha < \frac{1}{2}$, $V_j^C - V_j^D$ is strictly increasing in both T_+ and T_-. Because of $\frac{\partial V_j^C}{\partial T_-} < 0$, the fact that $T_- = 0$ does not guarantee sustainability if $\gamma \in (\gamma_{\mathrm{PM}}, \gamma_{\mathrm{HM}})$ implies that the optimal monitoring strategy $\{T_+^*, T_-^*\}$ necessarily satisfies $V_j^C = V_j^D$. The binding sustainability condition

is equivalent to

$$(2(1 - \alpha\gamma) - \alpha(1 - \gamma)) \delta - (1 - 2\alpha)\gamma\delta^{T_-+1} - (1 - \alpha)(1 - \gamma)\delta^{T_++1} = 1$$

or

$$\delta^{T_-} = 1 - \frac{1 - (2 - \gamma)\delta}{(1 - 2\alpha)\gamma\delta} - \frac{1 - \gamma}{(1 - 2\alpha)\gamma}\delta^{T_+}$$

for both firms $j = A, B$. Since $\frac{\partial \delta^{T_-}}{\partial \delta^{T_+}} = -\frac{1-\gamma}{(1-2\alpha)\gamma} < 0$, equilibrium punishment length T_- is strictly decreasing in T_+. In equilibrium, punishment lengths T_- and T_+ are complementary in the sense that a higher T_+ strengthens the enforcement power of a given punishment length T_-. Then $\frac{\partial V_j^C}{\partial T_+} = 0$ implies that setting $T_+ = +\infty$ is a necessary condition for payoff-maximisation also if $\gamma \in (\gamma_{\mathrm{PM}}, \gamma_{\mathrm{HM}})$, and the optimal T_- is implicitly determined by

$$\delta^{T_-} = 1 - \frac{1 - (2 - \gamma)\delta}{(1 - 2\alpha)\gamma\delta}.$$

Substituting the equilibrium expression for δ^{T_-} in V_j^C yields the resulting stationary equilibrium EPDVs of a single firm in an arbitrary period in state C under optimal pure perfect and hybrid monitoring in any parameter constellation (α, γ, δ) in which the respective monitoring strategy allows sustaining collusion. If $\gamma \in (0, \gamma_{\mathrm{PM}}]$, the value-maximising monitoring strategy is pure perfect monitoring with $\{T_+ = +\infty, T_- = 0\}$ and the collusive EPDV in equilibrium is

$$V_{\mathrm{PM}}^C = \frac{(1 - \alpha)\pi^m}{2(1 - \delta)}.$$

If $\gamma \in (\gamma_{\mathrm{PM}}, \gamma_{\mathrm{HM}})$, the value-maximising monitoring strategy is hybrid monitoring with $T_+ = +\infty$ and $T_- > 0$ defined by

$$\delta^{T_-^*} = 1 - \frac{1 - \delta(2 - \gamma)}{(1 - 2\alpha)\gamma\delta}$$

and the collusive EPDV in equilibrium is

$$V_{\mathrm{HM}}^C = \frac{(1 - 2\alpha)\pi^m}{2((1 - \delta) - \alpha(1 - \gamma)\delta)}.$$

Appendix B: Section 3.4

Appendix B.1: Preliminaries   The solution of the linearly indepen-
dent system characterising the stationary EPDVs conditional on s_j,
T_-, T_+,

$$
\begin{aligned}
V_j^C &= (1-\alpha)s_j\pi^m + \gamma(1-\alpha)\delta V_j^C \\
&\quad + (1-\gamma)(1-\alpha)\delta\left((1-\alpha)V_j^{C_H} + \alpha V_j^{C_L}\right) + \alpha\delta V_j^{P_{1-}}, \\
V_j^{C_H} &= s_j\pi^m + \gamma\delta V_j^C + (1-\gamma)\delta\left((1-\alpha)V_j^{C_H} + \alpha V_j^{C_L}\right), \\
V_j^{C_L} &= 0 + \gamma\delta V_j^C + (1-\gamma)\delta\left((1-\alpha)V_j^{C_H} + \alpha V_j^{C_L}\right), \\
V_j^{P_{1-}} &= \delta^{T_-}\gamma V_j^C + \delta^{T_-}(1-\gamma)\left((1-\alpha)V_j^{C_H} + \alpha V_j^{C_L}\right), \\
V_j^{P_{1+}} &= \delta^{T_+}\gamma V_j^C + \delta^{T_+}(1-\gamma)\left((1-\alpha)V_j^{C_H} + \alpha V_j^{C_L}\right),
\end{aligned}
$$

is

$$
\begin{aligned}
V_j^C &= (1-\alpha)\left(\frac{1-\alpha(1-\gamma)\delta + \alpha(1-\gamma)\delta^{T_-+1}}{1-(1-\alpha\gamma)\delta - \alpha\gamma\delta^{T_-+1}}\right)s_j\pi^m, \\
V_j^{C_H} &= \left(1 + \frac{(1-\alpha)\delta}{1-(1-\alpha\gamma)\delta - \alpha\gamma\delta^{T_-+1}}\right)s_j\pi^m, \\
V_j^{C_L} &= \left(\frac{(1-\alpha)\delta}{1-(1-\alpha\gamma)\delta - \alpha\gamma\delta^{T_-+1}}\right)s_j\pi^m
\end{aligned}
$$

and

$$
\begin{aligned}
V_j^{P_{1-}} &= \left(\frac{(1-\alpha)\delta^{T_-}}{1-(1-\alpha\gamma)\delta - \alpha\gamma\delta^{T_-+1}}\right)s_j\pi^m, \\
V_j^{P_{1+}} &= \left(\frac{(1-\alpha)\delta^{T_+}}{1-(1-\alpha\gamma)\delta - \alpha\gamma\delta^{T_-+1}}\right)s_j\pi^m.
\end{aligned}
$$

The EPDVs are nonnegative for $\alpha, \gamma, \delta \in (0,1)$, $s_j \in (0,1)$, $T_-, T_+ \in$
$\mathbb{N}_0$ and $\pi^m > 0$. Moreover, $V_j^{C_H} > V_j^{C_L}$ and $V_j^{C_H} > V_j^C$. If $\delta \in$
$\left(0, \frac{1}{1+\alpha(1-\gamma)}\right)$, $V_j^C > V_j^{C_L}$ for all $T_- \geq 0$; if $\delta \in \left(\frac{1}{1+\alpha(1-\gamma)}, 1\right)$, $V_j^C >$
$V_j^{C_L}$ iff $\delta^{T_-} > 1 - \frac{1-\delta}{\alpha\delta(1-\gamma)}$. The parametric range in which $V_j^C < V_j^{C_L}$
for $T_- \to +\infty$ increases in the imprecision of imperfect monitoring and
in the probability $1-\gamma$ with which deterministic information renders
correlated information obsolete. Also observe that $V_j^C > V_j^{P_{1+}}$ iff

$\frac{1-\delta^{T+}}{1-\delta^{T-}} > \alpha(1-\gamma)\delta$. If $T_+ \geq T_-$, this condition is satisfied.

Sustaining collusion in state C. Let

$$
\begin{aligned}
\Delta^C V_j \;&:=\; V_j^C - V_j^{DC} \\
&= \; (1 - \alpha\gamma - s_j\alpha(1-\gamma))\,\delta \\
&\quad - ((s_j - \alpha) + (1 - s_j)\alpha(1-\gamma))\,\delta^{T_-+1} - (1 - s_j).
\end{aligned}
$$

The partial derivative of $\Delta^C V_j$ with respect to s_j is

$$
\begin{aligned}
\frac{\partial \Delta^C V_j}{\partial s_j} \;&=\; 1 - \alpha(1-\gamma)\delta - (1 - \alpha(1-\gamma))\,\delta^{T_-+1} \\
&=\; 1 - \delta + (1 - \alpha(1-\gamma))\,\delta\left(1 - \delta^{T_-}\right) > 0;
\end{aligned}
$$

the scope is strictly increasing for firm $j = A$ and strictly decreasing for firm $j = B$ independently of T_+, T_- and parameters $\alpha, \gamma, \delta \in (0,1)$. The rule $s = \frac{1}{2}$ therefore maximises the joint scope of collusion.

Sustaining collusion in state C_H. Let

$$
\begin{aligned}
\Delta^{C_H} V_j \;&:=\; V_j^{C_H} - V_j^{DC_H} \\
&= \; (1 - \alpha\gamma - s_j\alpha(1-\gamma))\,\delta + (1 - s_j)\alpha\gamma\delta^{T_-+1} \\
&\quad - s_j(1-\alpha)\delta^{T_++1} - (1 - s_j).
\end{aligned}
$$

The partial derivative of $\Delta^{C_H} V_j$ with respect to s_j is

$$
\frac{\partial \Delta^{C_H} V_j}{\partial s_j} = 1 - \alpha(1-\gamma)\delta - \alpha\gamma\delta^{T_-+1} - (1-\alpha)\delta^{T_++1};
$$

it is strictly positive since $\delta^{T_-}, \delta^{T_-} \leq 1$ implies $1 - \alpha(1-\gamma)\delta - \alpha\gamma\delta^{T_-+1} - (1-\alpha)\delta^{T_++1} \geq 1 - \delta > 0$. Also in state C_H the rule $s = \frac{1}{2}$ then maximises the joint scope of collusion.

Appendix B.2: Proof of lemma 4.1 $\Delta^{C_H} V_j \geq \Delta^C V_j$ is equivalent to $\delta^{T_-} \geq \delta^{T_+}$ or $T_+ \geq T_-$ for all $T_-, T_+ \in \mathbb{N}_0$, sharing rules $s_j \in (0,1)$ and parametric constellations $\alpha, \gamma, \delta \in (0,1)$.

Appendix B.3: Proof of proposition 3 *Sustaining collusion in state* C. The partial derivatives w.r.t. the punishment lengths are

$$\frac{\partial \Delta^C V_j}{\partial T_-} = -\left(s_j - \alpha \left(s_j + (1 - s_j)\gamma\right)\right) \delta^{T_- + 1} \ln(\delta)$$

$$\begin{cases} > 0 & \text{if } \alpha < \frac{s_j}{s_j + (1 - s_j)\gamma} \\ = 0 & \text{if } \alpha = \frac{s_j}{s_j + (1 - s_j)\gamma} \\ < 0 & \text{if } \alpha > \frac{s_j}{s_j + (1 - s_j)\gamma} \end{cases}$$

and

$$\frac{\partial \Delta^C V_j}{\partial T_+} = 0.$$

A collusive PQA can be sustained in state C only by using correlated information. If $\alpha > \frac{s_j}{s_j + (1 - s_j)\gamma}$, $T_- = 0$ maximises $\Delta^C V_j$ but does not generate any incentives. The same happens if $\alpha = \frac{s_j}{s_j + (1 - s_j)\gamma}$. If $\alpha < \frac{s_j}{s_j + (1 - s_j)\gamma}$, $T_- = +\infty$ maximises $\Delta^C V_j$ and the incentive condition is satisfied iff $1 - \delta < s_j$ and $\alpha < \frac{s_j - (1 - \delta)}{\delta(s_j + (1 - s_j)\gamma)}$.

With equal sharing, incentives can be generated if $\alpha < \frac{1}{1+\gamma}$ and a collusive PQA can be sustained in state C iff $\delta \geq \frac{1}{2}$ and $\alpha < \frac{2\delta - 1}{\delta(1+\gamma)}$.

Sustaining collusion in state C_H. The partial derivatives w.r.t. the punishment lengths are

$$\frac{\partial \Delta^{C_H} V_j}{\partial T_-} = (1 - s_j)\alpha\gamma\,\delta^{T_- + 1} \ln(\delta) < 0$$

and

$$\frac{\partial \Delta^{C_H} V_j}{\partial T_+} = -s_j(1 - \alpha)\,\delta^{T_+ + 1} \ln(\delta) > 0.$$

Setting $T_- > 0$ degrades the enforcement power for $\alpha, \gamma, \delta \in (0, 1)$, $s_j \in (0, 1)$ independently of T_+. A collusive PQA may nevertheless be sustainable in state C_H if monitoring relies on deterministic information. For all $\alpha, \gamma, \delta \in (0, 1)$ and $s_j \in (0, 1)$, $T_+ = +\infty$ maximises $\Delta^{C_H} V_j$ independently of T_-. Given $T_- \in \mathbb{N}_0$, the incentive condition is satisfied iff $1 - \delta < s_j$ and $\alpha < \frac{s_j - (1 - \delta)}{\delta(s_j + (1 - s_j)\gamma(1 - \delta^{T_-}))}$.

With equal sharing, a collusive PQA can be sustained in state C_H iff $\delta \geq \frac{1}{2}$ and $\alpha < \frac{2\delta - 1}{\delta(1 + \gamma(1 - \delta^{T_-}))}$.

Sustaining collusion in both states C *and* C_H. Since $\Delta^C V_j$ is inde-

pendent of T_+ while $\Delta^{C_H} V_j$ is strictly increasing in T_+, $T_+ = +\infty$ is a necessary condition for maximising the joint scope of collusion. Since $\Delta^{C_H} V_j$ is strictly decreasing in T_-, an analogous statement cannot be made for T_- even though sustaining an agreement in state C essentially hinges upon T_-. But since $\Delta^{C_H} V_j \geq \Delta^C V_j$ if and only if $T_+ \geq T_-$, $T_+ = +\infty$ implies that the scope of collusion in state C is a strict subset of the scope of collusion in state C_H for arbitrary punishment lengths $T_- < +\infty$. If $\alpha < \frac{1}{1+\gamma}$, $\frac{\partial \Delta^C V_j}{\partial T_-} > 0$ then implies that the scope in state C and thus the scope of collusion under unfrequent early revelation of the aggregate shock is maximised with hybrid monitoring and $\{T_+^* = +\infty, T_-^* = +\infty\}$.

Appendix B.4: Proof of proposition 4   The partial derivatives of V_j^C, $V_j^{C_H}$ and $V_j^{C_L}$ with respect to punishment length T_+ are zero, the partial derivatives with respect to T_- are

$$
\frac{\partial V_j^C}{\partial T_-} = \frac{\alpha(1-\alpha)\left(1-(1-\gamma)\delta\right)\delta\, s_j \pi^m}{\left(1-(1-\alpha\gamma)\delta - \alpha\gamma\delta^{T_-+1}\right)^2}\, \delta^{T_-+1} \ln(\delta) < 0,
$$

$$
\frac{\partial V_j^{C_H}}{\partial T_-} = \frac{\alpha(1-\alpha)\gamma\delta^2\, s_j \pi^m}{\left(1-(1-\alpha\gamma)\delta - \alpha\gamma\delta^{T_-+1}\right)^2}\, \delta^{T_-+1} \ln(\delta) < 0,
$$

$$
\frac{\partial V_j^{C_L}}{\partial T_-} = \frac{\partial V_j^{C_H}}{\partial T_-} < 0.
$$

Since T_+ does not affect the collusive EPDV

$$
\gamma V_j^C + (1-\gamma)\left((1-\alpha)V_j^{C_H} + \alpha V_j^{C_L}\right),
$$

$T_+ = +\infty$ can be set without prejudice such as to maximise the scope in state C_H. If $T_+ = +\infty$, state C is the bottleneck for all $T_- > 0$. If the incentive condition in state C is satisfied, so is the incentive condition in state C_H. $\frac{\partial V_j^C}{\partial T_-} < 0$ implies that the only monitoring strategy that is incentive compatible in state C and maximises the collusive EPDV satisfies $V_j^C = V_j^D$. The binding sustainability condition is equivalent to

$$
\left(2(1-\alpha\gamma) - \alpha(1-\gamma)\right)\delta - \left((1-2\alpha) + \alpha(1-\gamma)\right)\delta^{T_-^*+1} = 1
$$

or

$$\delta^{T_-^*} = 1 - \frac{1-\delta}{(1-\alpha(1+\gamma))\,\delta}$$

for both firms $j = A, B$. Substituting the equilibrium expression for δ^{T_-} in V_j^C, $V_j^{C_H}$ and $V_j^{C_L}$ yields the resulting stationary equilibrium EPDV of the collusive agreement under optimal hybrid monitoring

$$
\begin{aligned}
V_{\mathrm{HM}}^{\mathrm{early}} &= \gamma V_j^C(T_+^*, T_-^*) + (1-\gamma)\left((1-\alpha)V_j^{C_H}(T_+^*, T_-^*) + \alpha V_j^{C_L}(T_+^*, T_-^*)\right) \\
&= \frac{((1-2\alpha)+\alpha(1-\gamma)\delta)\,\pi^m}{2(1-\delta)}.
\end{aligned}
$$

Appendix B.5: Comments on "retaliation in response to ignorance"
Assuming the alternative specification "retaliation in response to ignorance", the equations characterising the stationary path of expected present and future profits of cooperative or defective behaviour in states C and C_H don't change. In a stationary solution, $V_j^{C_L}$ satisfies

$$V_j^{C_L} = 0 + \delta V_j^{P_{1-}}.$$

Solving the linearly independent system characterising the stationary EPDVs conditional on s_j, T_-, T_+ yields

$$
\begin{aligned}
V_j^C &= \frac{(1-\alpha)s_j\pi^m}{1-(1-\alpha)\delta - \alpha\delta^{T_-+1}}, \\
V_j^{C_H} &= \frac{\left(1-\alpha\delta^{T_-+1}\right)s_j\pi^m}{1-(1-\alpha)\delta - \alpha\delta^{T_-+1}}, \\
V_j^{C_L} &= \frac{(1-\alpha)\delta^{T_-+1}s_j\pi^m}{1-(1-\alpha)\delta - \alpha\delta^{T_-+1}}
\end{aligned}
$$

and

$$
\begin{aligned}
V_j^{P_{1-}} &= \frac{(1-\alpha)\delta^{T_-}s_j\pi^m}{1-(1-\alpha)\delta - \alpha\delta^{T_-+1}}, \\
V_j^{P_{1+}} &= \frac{(1-\alpha)\delta^{T_+}s_j\pi^m}{1-(1-\alpha)\delta - \alpha\delta^{T_-+1}}.
\end{aligned}
$$

Since $V_j^{C_L} < V_j^{P_{1-}}$, it is impossible to sustain collusion in state C_L.

Appendix C: Section 3.5

Appendix C.1: Proof of proposition 5   Remember the sustainability conditions $V_j^C \geq V_j^D$ for $j \in \{A, B\}$ are

$$(2(1 - \alpha\gamma) - \alpha(1 - \gamma))\,\delta - (1 - 2\alpha)\gamma\delta^{T_- + 1} - (1 - \alpha)(1 - \gamma)\delta^{T_+ + 1} \geq 1.$$

under unfrequent late revelation and

$$(2(1 - \alpha\gamma) - \alpha(1 - \gamma))\,\delta - ((1 - 2\alpha) + \alpha(1 - \gamma))\,\delta^{T_- + 1} \geq 1.$$

under unfrequent early revelation in state C. In the latter case, the scope-maximising monitoring strategy is $T_- = +\infty$, $T_+ = +\infty$ irrespective of α, γ and δ; under unfrequent late revelation, it is $T_- = +\infty$, $T_+ = +\infty$ if $\alpha < \frac{1}{2}$ and $T_- = 0$, $T_+ = +\infty$ if $\alpha \geq \frac{1}{2}$. Using the scope-maximising monitoring strategy, if $\alpha < \frac{1}{2}$ collusion is sustainable iff $(2 - \alpha(1 + \gamma))\,\delta \geq 1$ both under unfrequent early and late revelation; if $\alpha \geq \frac{1}{2}$, collusion is sustainable iff $(2 - \alpha(1 + \gamma))\,\delta \geq 1$ under unfrequent early revelation and iff $(2 - \alpha(1 - \gamma) - \gamma)\,\delta \geq 1$ under unfrequent late revelation. Since $2 - \alpha(1 - \gamma) - \gamma \geq 2 - \alpha(1 + \gamma) \Leftrightarrow \alpha \geq \frac{1}{2}$, the scope of collusion is in this case larger under unfrequent late revelation of aggregate demand.

If $\gamma \in (0, \gamma_{\mathrm{PM}}]$, under unfrequent late revelation the collusive EPDV in equilibrium, optimally sustained with pure perfect monitoring $\{T_+ = +\infty, T_- = 0\}$, is

$$V_{\mathrm{PM}}^C = \frac{(1 - \alpha)\pi^m}{2(1 - \delta)},$$

and under unfrequent early revelation the collusive EPDV in equilibrium, optimally sustained with hybrid monitoring $\{T_+ = +\infty, T_- > 0\}$ where T_- satisfies $\delta^{T_-} = 1 - \frac{1 - \delta}{(1 - \alpha(1 + \gamma))\delta}$, is

$$V_{\mathrm{HM}}^{\mathrm{early}} = \frac{((1 - 2\alpha) + \alpha(1 - \gamma)\delta)\,\pi^m}{2(1 - \delta)}.$$

Direct comparison immediately reveals that $V_{\mathrm{PM}}^C > V_{\mathrm{HM}}^{\mathrm{early}}$ irrespective of α, γ and δ.

If $\gamma \in (\gamma_{\mathrm{PM}}, \gamma_{\mathrm{HM}})$, under unfrequent late revelation the collusive EPDV in equilibrium, optimally sustained with hybrid monitoring $\{T_+ = +\infty, T_- > 0\}$ where T_- satisfies $\delta^{T_-} = 1 - \frac{1 - \delta(2 - \gamma)}{(1 - 2\alpha)\gamma\delta}$, is

$$V_{\text{HM}}^C = \frac{(1 - 2\alpha)\pi^m}{2\left((1 - \delta) - \alpha(1 - \gamma)\delta\right)}.$$

Under unfrequent early revelation the optimal monitoring strategy and the expression of the collusive EPDV in equilibrium remain unaltered. Note that the denominator of V_{HM}^C is positive iff $(1 - \delta) - \alpha(1 - \gamma)\delta > 0 \Leftrightarrow \gamma > \gamma' := 1 - \frac{1-\delta}{\alpha\delta}$. This is satisfied in the range $\gamma \in (\gamma_{\text{PM}}, \gamma_{\text{HM}})$ since $\gamma' < \gamma_{\text{PM}}$. Then $V_{\text{HM}}^C > V_{\text{HM}}^{\text{early}}$ is equivalent to $\gamma < \gamma'' := \frac{(1+\alpha)\delta - 2\alpha}{\alpha\delta}$. Since $\gamma'' > \gamma_{\text{HM}}$, $V_{\text{HM}}^C > V_{\text{HM}}^{\text{early}}$ for all $\gamma \in (\gamma_{\text{PM}}, \gamma_{\text{HM}})$.

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