Theoretical Implications of the Higgs Discovery

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The image of the front page is an event display of a $h \to \gamma + \gamma$ candidate event. Both photon candidates are unconverted. The measured diphoton mass is 126.6 GeV. Only reconstructed tracks with $p_T > 1$ GeV are shown.

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Acknowledgements

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Preface

The discovery of the Higgs particle at the Large Hadron Collider (LHC), presented on July 4th, 2012, is a milestone in the history of particle physics. The Higgs particle was the last particle predicted by the Standard Model (SM) of particle physics to be discovered. The Higgs particle then completes a theoretical puzzle that can be traced back to at least the formulation of the theory of beta decay (E. Fermi, 1934).

The Higgs particle is a keystone in the SM and its discovery is very singular in the following sense. Without it, as far as one can calculate, the Higgsless-SM would contradict the basic rule of Quantum Mechanics that the sum of probabilities adds to one. That is why a central part of the LHC programme was to look for the Higgs particle, or some other more exotic mechanism, that would cure the Higgsless-SM. In this respect, it is very important to measure the Higgs couplings with as much precision as possible, specially if no Beyond the SM (BSM) particle is found, since any deviations from the SM prediction would point to the presence of BSM physics not far from the current energy scale.

So far, the measurement of the Higgs mass and its interaction properties with the other particles is teaching us a big amount of physics. The mass value serves as a discriminator for BSMs and determines wether or not the SM electroweak vacuum develops instabilities at high field values. The Higgs couplings to different SM particles, measured by ATLAS and CMS, are so far compatible with the SM expectations within a $\sim 20\%$ accuracy. However, as already emphasised, a precise determination of the Higgs couplings is very important.

The discovery of the Higgs marks the beginning of a new era in Particle Physics. The field of Particle Physics is facing a situation never encountered before. We have a theoretical framework that is consistent up to very high energies (an hence without any need of modifications of the theory). There are a number of conundrums that certainly require the extension of the SM to include Beyond the SM (BSM) physics, for instance: the strong indirect evidence of large halos of dark matter surrounding galaxies; the strong CP problem; the value of the Higgs mass appears to be finely tuned in any

conceived BSM theory whose lowest mass scale is much bigger than the electroweak scale (Higgs mass hierarchy problem); the striking hierarchy of the SM Yukawa couplings; the matter-antimatter asymmetry, among other theoretical puzzles. Apart from the Higgs hierarchy problem, any of the aforementioned conundrums can be solved at scales much higher than the SM or with very weakly coupled physics. In this respect, if the Higgs mass hierarchy problem and its associated fine-tuning is a good guide, BSM physics is expected at an energy scale reachable by the LHC. However, so far there is no pressure from any collider experiment to accommodate data not being properly fitted by the SM expectations. Time will tell if this is the calm preceding an upcoming storm.

In any case, in order to push forward the boundaries of the field, two very important lines of research consist in

- inspecting any possible sign within the SM theory that may indicate or suggest new physics beyond the SM (BSM),

- the investigation of possible way outs to the constraints that experimental data places on possible BSM.

In fact, the work presented in this thesis can be classified into the two aforementioned broad paths of research. The lines of research followed in the present thesis are mostly motivated by the Higgs discovery. Therefore, a big effort is devoted to the investigation of the implications of the Higgs discovery.

The present thesis is organized in three parts. In the first part we study various aspects related with the stability of the electroweak (EW) vacuum. The topic is introduced in Chapter 2. Then, in Chapter 3 we show that the Higgs mass value is very intriguing from the point of view of the stability of the EW vacuum. If the Higgs mass would had been few GeV's heavier, then the EW vacuum would be absolutely stable. On the other hand, if the Higgs mass would had been lighter by approximately ten GeV's, then the EW vacuum would be too short lived as compared with the age of the universe. We review the results published in Ref. [1] that consist in a NNLO analysis of the EW vacuum stability. In Chapter 4 we discuss a mechanism to stabilise the EW vacuum. This is based on the publication of Ref. [2]. We also discuss various examples of BSM physics that can naturally accommodate such mechanism.

It is very interesting to find concrete theoretical predictions linking particle physics and cosmology. Also, cosmological observables, like imprints of the period of inflation in the Cosmic Microwave Background, can provide experimental information of the highest accessible energy scales that are otherwise not accessible by collider experiments. In Ref. [15] the authors proposed to use the SM Higgs as an inflaton by introducing a large coupling between the Higgs and the Ricci scalar, $\delta\mathcal{L} = -\xi|H|^2 R$. In my opinion,

even-though the proposal is attractive, there are a number of unsettling features. First, the set-up is not radiatively stable. In fact, from the top-bottom point of view, one has to impose a shift symmetry at trans-planckian field values that is broken in a very special way. It is unclear that this can be done without imposing a *functional tuning* (i.e. tuning an infinite number of parameters) on the model. Then, there is a field range of values where the Higgs stops "Higgsing" because it is inflating. This causes problems with perturbative unitary. Lastly, my personal bias, is that I do not see a fundamental reason for paying this prize for the sake of being minimalistic. It seems that nature has cared more on symmetry principles than in being specially minimalistic. In any case, the proposal is an interesting possibility and in Ref. [4] we proposed an embedding of the original theory of Ref. [15]. In the bigger theory we can address some of the aforementioned problems of the original proposal of the Higgs as an inflaton. For example, in the bigger theory, one can compute the coupling of the Higgs to the Ricci scalar. As we commented, this thesis is about the implications of the Higgs discovery. However, due to the lack of space for a proper treatment we will not discuss any further the cosmological implications.

In the second part of this thesis we study possible BSM-induced deviations in the SM Higgs sector. A convenient way to do so is by means of the SM effective field theory (EFT). The topic is introduced in Chapter 5. Subsequently, in Chapter 6 we present a global analysis of the SM EFT. We focus on those observables (with emphasis on Higgs physics) that could present big deviations from the SM expectations and would not be in contradiction with any previous experiment done so far. In the following Chapters 7-9 we study quantum effects in the SM EFT (the anomalous dimension matrix of dimension-six SM operators). This is very interesting for a number of reasons that we review. For instance, physical observables that are unrelated at tree-level, are in fact correlated through perturbative quantum corrections. In Chapter 7 we study, among other things, the interplay between the S-parameter, the triple gauge vertices and the decays $h \to \gamma + \gamma/Z$. In Chapter 8 we study the most relevant quantum effects for dimension-six Higgs operators and we match the results in the SM EFT with various BSMs. In Chapter 9 we present a study of the interplay between EW observables, measured precisely at the Large Electron-Positron Collider (LEP), and Higgs physics. The second part is based on the publications of Refs. [5–7].

One of the results found in the second part of the thesis is that the anomalous dimension matrix of the dimension-six SM operators has a very peculiar structure. It has a lot of vanishing entries. This is surprising because those vanishing entries are allowed by all symmetries in the theory and therefore are not expected to vanish. In the third part of the thesis we present an argument, based on the use of Supersymmetry as a spurious symmetry, that provides a rationale for the structure observed by brute force

calculation in the first place. However, the argument clarifies that the structure seen in the SM is generic (not due to the SM internal symmetries or accidental symmetries) and has applicability to other EFTs. The results of Part III are based on Ref. [8].

The first run of the LHC ended beautifully with the discovery of the Higgs boson and initiated an era of measurements in the EWSB sector that remained only indirectly constrained for several decades. With the next run of the LHC and the high-luminosity program will start an era of precision that will lead certainly to a better understanding of what physics breaks the electroweak symmetry and, hopefully, to the first glimpse of the new physics beyond the Standard Model. We hope that the results we presented in this thesis will be a powerful contribution to that quest.

1. Particle Physics

It is difficult to over-state how marvelous the theory of particles is. Its structure is based on two theories: quantum mechanics and special relativity. Basically this means that if you accept these two theories, on which there is the greatest experimental evidence, the structure of the theory follows by demanding logical consistency. This fact is very satisfactory and gives a sense of inevitability of the physical laws. Therefore, with some exaggeration, the only freedom left for a particle theorist is on the discrete choice of the number of particles and the adjustable parameters that control the interaction strength together with possible masses.

The Standard Model (SM) of particle physics plus the theory of General Relativity (through the Standard Model of Cosmology) can in principle explain observed phenomena ranging from $\sim 10^{-18}$ meters to the largest observable scales $\sim 10^{25}$ meters. Both theories are defined trough a Lagrangian, which very schematically reads

$$
\begin{aligned}
\mathcal{L}_{SM} &= \frac{1}{2}(\partial H)^2 - V(H) + \psi_i^\dagger \sigma \cdot \partial \psi_i + [y_{ij} H \psi_i \psi_j + h.c.] + \mathcal{L}_{s=1} \quad , \text{ where} \\
\mathcal{L}_{s=1} &= \frac{1}{2}(\partial A)^2 + g(\partial A)A^2 + gJ \cdot A + \mathcal{O}(g^2) \quad \text{and} \tag{1.1} \\
\mathcal{L}_{GR} &= \frac{1}{2}(\partial \gamma)^2 + \sqrt{g_N}(\partial \gamma)^2 \gamma + \sqrt{g_N} T \cdot \gamma + \mathcal{O}(g_N) \ , \tag{1.2}
\end{aligned}
$$

where we have expanded the Hilbert-Einstein action $S_{GR} = (16\pi G_N)^{-1} \int d^4 x \sqrt{-g} R$ around a background metric $g = \eta + \sqrt{g_N} \gamma$ to expose its similarity with the $\mathcal{L}_{s=1}$ Lagrangian (we have defined $g_N = 8\pi G_N = M_{Pl}^{-1} \approx 10^{-18}$ GeV). With the recent discovery of the Higgs boson H [9, 10] at the Large Hadron Collider (LHC), so far we have seen *nature making use* of all the particles *she can use* to mediate long range interactions except for the helicity $h = \pm 3/2$.

A sense of unicity of the physical Laws

Let me review some of the concrete evidence of the inevitability of the physical Laws and hence of the underlying structure of the SM of particle physics and General Relativity

of gravitation. Long range interactions are mediated by massless particles of helicity $|h| = \{0, \frac{1}{2}, 1, \frac{3}{2}, 2\}$. This can be seen in a number of ways. One of the historic ways is by considering infrared properties of the S-matrix. The idea is to consider the emission of a soft particle of momentum q from an $(n+1)$-particle amplitude. At leading order in q, the amplitude factorizes into a rational factor times the scattering amplitude of the remaining n particles: $\mathcal{M}_{n+1}(k_1, k_2, \cdots, k_n, q) = F \times \mathcal{M}_n(k_1, k_2, \cdots, k_n) + \mathcal{O}(q^0)$. Then, for the rational factor to be invariant under Lorentz transformations, conservation Laws must be satisfied. For instance, for a helicity $|h| = 1$ soft particle the factor is $F = \sum_{i=1}^{n} e_i \epsilon(q) \cdot k_i / k_i \cdot q$, where $\epsilon(q)$ is the polarisation and e_i the coupling constant between the soft and the ith particle. Under a Lorentz boost $\delta\epsilon \propto q$ and one finds that charge must be conserved $\sum_{i=1}^{n} e_i = 0$, see Ref. [11]. Instead, for helicity $|h| = 2$ the rational factor is $F = \sum_{i=1}^{n} e_i \epsilon_{\mu\nu} k_i^\mu k_i^\nu / q \cdot k_i$. Then, under a Lorentz transformation $\delta_\alpha \epsilon_{\mu\nu} = q_\mu \alpha_\nu + q_\nu \alpha_\mu$ and one discovers that, if no constraints beyond momentum conservation are imposed, the soft $|h| = 2$ massless particle is coupled universally to the remaining species. This is tantamount to the equivalence principle that leads to General Relativity [12]. Now, if the exercise is done for particles of helicity $|h| > 2$ one finds that, for the corresponding factor F to be Lorentz invariant, one needs to impose constrains on the momenta beyond momentum conservation and this constrains the angles on which the particles are emitted after the collision. This is unphysical and therefore helicity $|h| > 2$ particles cannot mediate long range forces. [1]

Another complementary way to derive this result that allows to go further in constraining the possible consistent interactions is the following [13]. The idea is that the 3-particle amplitude [2] is completely fixed by momentum conservation and Lorentz symmetry. In particular, by requiring that the amplitude transforms homogeneously, with weight $-2h_i$, under the little-group scaling of the ith particle of helicity h_i. Next one considers the 4-particle amplitude and requires that it factorizes properly into the lower 3-particle amplitudes as any of the sums of the external momenta go on-shell $(\sum p_i)^2 = 0$. This turns out to impose non-trivial constraints on the couplings of the 3-particle amplitudes. And, for instance, one finds that for a collection of interacting helicity $|h| = 1$ particles their couplings obey the Jacobi identity (and hence satisfy a Lie algebra). The same exercise done for a set of helicity $|h| = 2$ particles implies the equivalence principle together with the fact tat there is no analog of Yang-Mills for helicity $|h| = 2$ particles (instead of the Jacobi identity constraint one gets a commutative algebra that can be diagonalized). Then, again the same exercise shows that the helicity $\pm 3/2$ state necessarily couples to the $|h| = 2$ graviton as linearized N=1 supergravity,

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[1]Soft theorems do not terminate here: sub-leading factors as $q \to 0$ give further conservation laws.
[2]For real momenta, the 3-particle amplitude is only non-vanishing as two of the particles go collinear. However in the following argument all momenta is complexified and we go back to the real line only at the end of the calculation.

and that the constraints cannot be fulfilled for a particle of spin $s > 2$. Hence, Quantum Electrodynamics, Yang-Mills, General Relativity and Supersymmetry do not admit *small* consistent deformations. This is also known through field theoretic methods, but it is particularly nice to see the emergence of the known theories in such a sharp way.

After this short digression on some of the generic structure underlying particle physics, in the rest of the Chapter we present a brief discussion of the SM of particle physics. The presentation does not show the big amount of fun subtleties that surround the SM, most of them present in any QFT, but serves to set the notation and the physical motivations for the upcoming Chapters, which are the main topic of this thesis. The discussion consists in actually defining $\mathcal{L}_{SM}$ in Eq. (1.1), an explanation of its most prominent phenomenological consequences together with its experimental verifications, and a discussion of the theoretical and experimental problems that it faces.

1.1   The Standard Model Lagrangian

The SM is a relativistic Quantum Effective Field Theory (EFT). As such, it can be defined by specifying the gauge group

$$U(1)_Y \otimes SU(2)_L \otimes SU(3)_c \ , \tag{1.3}$$

the field content and its representations under the gauge group

Fields	Names	spin	$U(1)_Y$	$SU(2)_L$	$SU(3)_C$	$SL(2,\mathbb{C})$
q_L^i		$\frac{1}{2}$	$\frac{1}{6}$	**2**	**3**	$(\frac{1}{2}, 0)$
u_R^i	quarks	$\frac{1}{2}$	$\frac{2}{3}$	**1**	**3**	$(0, \frac{1}{2})$
d_R^i		$\frac{1}{2}$	$-\frac{1}{3}$	**1**	**3**	$(0, \frac{1}{2})$
l_L^i	leptons	$\frac{1}{2}$	$-\frac{1}{2}$	**2**	**1**	$(\frac{1}{2}, 0)$
e_R^i		$\frac{1}{2}$	-1	**1**	**1**	$(0, \frac{1}{2})$
H	Higgs	0	$\frac{1}{2}$	**2**	**1**	$(0, 0)$

TABLE 1.1: *The field content of the Standard Model. The index $i = 1, 2, 3$ is called the generation index.*

and, as any physical theory, the energy scales E that it is supposed to describe,

$$E \in [0, \Lambda] \ , \tag{1.4}$$

where $\Lambda \gtrsim 1$ TeV. Given these three pieces of information, we write down the most generic local Lagrangian compatible with the symmetries

$$\mathcal{L}_{SM}^{EFT} = \mathcal{L}_{SM} + \mathcal{L}_{BSM} \ , \tag{1.5}$$

where $\mathcal{L}_{SM} = \mathcal{L}_{Gauge} + \mathcal{L}_{FG} + \mathcal{L}_{Higgs}$ are operators of dimension $d \leq 4$ and $\mathcal{L}_{BSM}$ is comprised of higher dimensional operators $d > 4$. They are defined as follows.

Pure gauge sector:

$$\begin{aligned}
\mathcal{L}_{Gauge} &= -\frac{1}{4} B_{\mu\nu} \left(B^{\mu\nu} - \theta_1 \frac{\alpha_1}{4\pi} \widetilde{B}^{\mu\nu} \right) - \frac{1}{4} W_{\mu\nu}^a \left(W^{a\mu\nu} - \theta_2 \frac{\alpha_2}{4\pi} \widetilde{W}^{a\mu\nu} \right) \\
&\quad - \frac{1}{4} G_{\mu\nu}^A \left(G^{A\mu\nu} - \theta_s \frac{\alpha_s}{4\pi} \widetilde{G}^{A\mu\nu} \right) \ ,
\end{aligned} \tag{1.6}$$

where $\alpha_j = g_j^2/(4\pi)$ and the field strengths are defined as usual $A_{\mu\nu}^a = \partial_\mu A_\nu^a - \partial_\nu A_\mu^a - g_j \epsilon^{abc} A_\mu^b A_\nu^c$. The gauge fields B, W and G correspond to the gauge groups $U(1)_Y$, $SU(2)_L$ and $SU(3)_c$, respectively; the tensor-constants ϵ^{ijk} satisfy the Jacobi identity for the non-abelian groups $SU(2)_L$ and $SU(3)_c$ while they are zero for the $U(1)_Y$. In the path integral quantization we need to add a gauge fixing function and ghosts fields. The six terms in Eq. (1.6) introduce a coupling constant g', g, g_s and an angle θ_1, θ_2, θ_s. The first three have the following approximate values $\{g', g, g_s\} = \{0.4, 0.6, 1.2\}$, at the electroweak scale ~ 100 GeV. See Sec. 1.1.2 for a discussion on the θ_i parameters.

Fermion-Gauge sector: The fermion content and its interaction with the gauge bosons is given by

$$\mathcal{L}_{FG} = \bar{\Psi} i \gamma^\mu D_\mu \Psi \ , \tag{1.7}$$

where we have grouped the fermions into the vector $\Psi^T = (\ q_L^i \quad u_R^i \quad d_R^i \quad l_L^i \quad e_R^i\ )$. The covariant derivative is defined as $D_\mu = \partial_\mu - ig'Y B_\mu - g W_\mu^a T_R^a - g_s G_\mu^A t_R^A$. The matrices T_R^a and t_R^A depend on the fermion's representation: they are given by the three Pauli matrices $\sigma^a/2$ and by the eight Gell-Mann matrices $\lambda^A/2$ for the **2** of $SU(2)_L$ and a **3** $SU(3)_C$; while they are 0 for the singlet representation. We have denoted by Y the $U(1)_Y$ charge $Q_{U(1)}$ of each fermion, for instance $Q_{U(1)_Y} l_L^i = -\frac{1}{2} l_L^i$.

Higgs sector:

$$\begin{aligned}
\mathcal{L}_{Higgs} &= |D_\mu H|^2 + m^2 |H|^2 - \lambda |H|^4 \\
&\quad - \bar{l}_L^i y_{ij}^e e_R^j H - \bar{q}_L^i y_{ij}^d d_R^j H - \bar{q}_L^i y_{ij}^u u_R^j \widetilde{H} + h.c. \ ,
\end{aligned} \tag{1.8}$$

where $D_\mu H = (\partial_\mu - ig'/2 B_\mu - g W_\mu^a \sigma^a/2) H$, $\widetilde{H} = i\sigma_2 H^*$ and y_{ij}^f are 3×3 complex Yukawa matrices, discussed in Sec. 1.1.2.

Higher dimensional operators. Their name is due to the fact that we need to introduce a dimensionful coupling c_i/Λ^{n_i} to match the right dimensions of the Lagrangian:

$$\mathcal{L}_{BSM} \equiv \sum_i \frac{c_i}{\Lambda^{n_i}} \mathcal{O}_i = -\frac{y_\nu^{*ij}}{\Lambda} \epsilon_{\alpha\beta} \epsilon_{\gamma\rho} (l_L^{iT\alpha} C l_L^{i\gamma}) H^\beta H^\rho + h.c. + \frac{c'_{ij}}{\Lambda^2} D^\nu W_{\mu\nu}^a (\bar{l}_i \gamma^\mu l_j)$$

$$+ \frac{c_{ijkm}}{\Lambda^2} \epsilon_{abc} (d_a^i C u_b^j)(q_{c\beta}^k \epsilon^{\alpha\beta} C l_\alpha^m) + h.c. + \mathcal{O}(\Lambda^{-2}) , \qquad (1.9)$$

where $\epsilon_{\alpha\beta}$, ϵ_{abc} and $C = i\gamma_2\gamma_0$ are $SU(2)_L$, $SU(3)_c$ and Lorentz antisymmetric tensor invariants and we are only showing some terms of the series to make a couple of points in the following discussion.

This part of the Lagrangian reflects the *known unknown* physics, in the following sense. It parametrizes possible new physics at a high energy scale Λ. The effects of unknown higher energy physics decouple as

$$(E/\Lambda)^n , \qquad (1.10)$$

where E is the energy of the process under consideration, Λ is the characteristic energy scale of the unknown higher energy physics that decouples, and n a positive number. The effect of the unknown higher energy phenomena can be parametrized in a power expansion of E/Λ. In certain particular cases one can explicitly show the power-law decoupling. For instance, in weakly coupled models $\Lambda \propto M/e$, where M is the mass of the heavy particle and e its coupling to the low energy physics. The decoupling of high energy degrees of freedom is ubiquitous in physics and it takes a precise form in quantum effective field theory (QEFT). Indeed, in QEFT one can compute corrections to the energy scaling of the different operators in the Lagrangian, and they can be resumed using the Renormalization Group Equations (RGE). The combination of the RGE and the EFT rationale makes QEFT very powerful by allowing to focus on the energy scales of interest. Now, energies (or momentum) in a quantum process can come from either a field or a derivative in the Lagrangian. Therefore, the energy expansion translates into an expansion in powers of

$$(\phi/\Lambda)^n , \; (\partial/\Lambda)^n , \qquad (1.11)$$

in the Lagrangian. This explains why, if we are interested in certain energy resolution, we can neglect higher order interactions of the sum $\sum_i c_i \mathcal{O}_i/\Lambda^{n_i}$ that produces corrections beyond our experimental reach or interest.

Coming back to the SM EFT, the measurement of higher dimension operators is crucial because it points to a new energy scale. Presumably new physics, possibly in the form of new degrees of freedom, appear at that energy scale. Two important concrete remarks regarding Eq. (1.9) are that the dimension-five Weinberg operator
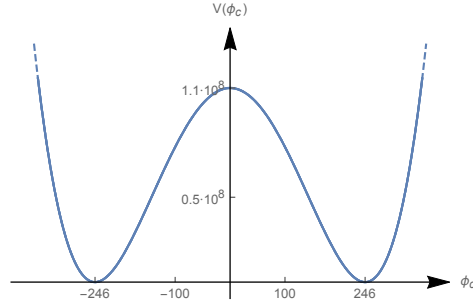
FIGURE 1.1: *Plot of the SM potential $V(\phi_c)$ for the realistic values of $\lambda = 1/8$ and $v = 246$ GeV, for the range of values indicated in the figure.*

$\sim l_i l_j HH$ gives a contribution to the neutrino masses of order $y_\nu v^2/\Lambda$ [14], see Sec. 1.1.2. All operators of the dimension-four SM Lagrangian are invariant under $\{q, u, d\} \to e^{i\alpha Q_B}\{q, u, d\}$. The charge under this symmetry is called baryon number and it is violated by the dimension-six operator $\sim qudl/\Lambda^2$ of Eq. (1.9). This operator has not been directly measured but its coupling strength is tightly constrained by proton decay. Just for completion on the discussion of Eq. (1.9), let us mention that in the limit $\Lambda \to \infty$, the W bosons couple equally to the different lepton families. However, the operator $\sim \partial^2 W \bar{l}_i l_j$ of Eq. (1.9) might lead to a breaking of the universal W-leptons interaction. See Sec. 1.1.3 for a further discussion on accidental symmetries.

Due to historical reasons, is is customary to call *the Standard Model* to the Lagrangian $\mathcal{L}_{SM}$ of Eq. (1.5). The reason is that fixing a finite number of couplings is sufficient for the theory to be predictive. Instead, the Lagrangian $\mathcal{L}_{SM}^{EFT}$ of Eq. (1.5) is predictive for a finite number of couplings at a fixed order in the $(E/\Lambda)^n$ expansion, which is perfectly fine for a physical theory. The point of view taken in the present discussion is to write the generic local Lagrangian containing all the degrees of freedom observed in nature, as dictated by symmetries. Neutrino masses and perturbative quantum gravity require the inclusion of higher order interactions. Thus there is no reason for being minimalistic in the number of particles and there is no beauty or simplicity principle in the field equations themselves. For instance, notice the complexity of the equations of motions derived from $\mathcal{L}_{SM}^{EFT}$. Instead, the simplicity is on the symmetry principle governing the Lagrangian $\mathcal{L}_{SM}^{EFT}$.

1.1.1   Electroweak Symmetry Breaking

The Standard Model tree-level Higgs effective potential in the Lagrangian Eq. (1.8) is given by

$$V(H) = -m^2|H|^2 + \lambda|H|^4 \ , \tag{1.12}$$

Since the SM couplings are small at the electroweak scale ~ 100 GeV the tree-level potential suffices for a qualitative discussion (the couplings satisfy $\alpha_i/(4\pi) \ll 1$, where $\alpha_i = \{\lambda, (y^f)^2, g'^2, g^2, g_s^2\}/(4\pi)$). See Fig. 1.1 for a plot of the potential in Eq. (1.12). The vacuum of the theory is the state that minimizes the Hamiltonian density expectation. For the Higgs sector it is given by

$$\langle 0|\mathcal{H}|0\rangle = V(\phi_c)|_{min} \;, \tag{1.13}$$

subject to $\langle 0|h|0\rangle = \phi_c$, where $H^T = (G^+, h + iG_0)$. Thus, from Eq. (1.12), the minimum of the energy density is given by

$$\langle 0|h|0\rangle = v \;, \tag{1.14}$$

and we say that the Higgs *acquires* a vacuum expectation value. In the $\langle h \rangle = v$ background, the particles of the SM have the masses summarized in Tab. 1.2. The value of v is measured through the muon decay to be $v = 246$ GeV. The masses of the quarks span five orders of magnitude $m_t/m_u \approx 10^5$, while the masses of the neutrinos are much smaller than the electroweak scale $m_\nu \approx 10^{-1}$ eV $\approx 10^{-12}v$. There is no explanation for these hierarchies within the SM, see Sec. 1.1.2 for a further discussion.

The name *electroweak symmetry breaking* originates from the fact that the vacuum state $|0\rangle$ of the SM is not invariant under $SU(2)_L \otimes U(1)_Y$ because $T^i \cdot \langle 0|H^T|0\rangle = T^i \cdot (0, v/\sqrt{2})^T \neq 0$, where T^i is any of the $SU(2)_L$ or $U(1)_Y$ generators. However $(T^3 + Q_Y)|0\rangle = Q_{QED}|0\rangle = 0$ is a symmetry of the vacuum and we say that $SU(2)_L \otimes U(1)_Y$ is broken to $U(1)_{QED}$ electromagnetism.

		Gauge bosons					Leptons		
	m_γ^2	m_Z^2	m_W^2	m_G^2	m_e	m_μ	m_τ	m_ν^i	
SM	0	$\frac{g^2+g'^2}{4}v^2$	$\frac{g^2}{4}$	0	$y_e v/\sqrt{2}$	$y_\mu v/\sqrt{2}$	$y_\tau v/\sqrt{2}$	v^2/Λ	
Value	0	$(91)^2$	$(80)^2$	0	5×10^{-4}	0.1	1.7	3×10^{-10}	

		Quarks						Higgs
	m_u	m_d	m_c	m_s	m_t	m_b		m_h^2
SM	$y_u v/\sqrt{2}$	$y_d v/\sqrt{2}$	$y_c v/\sqrt{2}$	$y_s v/\sqrt{2}$	$y_t v/\sqrt{2}$	$y_b v/\sqrt{2}$		$2\lambda v^2$
Value	2×10^{-3}	4×10^{-3}	1	0.1	173	4		$(125)^2$

TABLE 1.2: *Tree-level masses of the SM particles in terms of the parameters of the Lagrangian in Eq. (1.5) and their experimental values in GeV. The experimental uncertainty on the mass values is below the quoted significant digits. Regarding the neutrino masses, we have quoted their average value as inferred from the cosmic microwave background, assuming the SM.*

The Higgs mechanism

The Higgs boson plays a central role in the SM, and in the present thesis. Let us then review what makes the SM Higgs *a Higgs boson* as compared to other scalars. To make the argument transparent, let us focus on a Higgsless cousin of the SM electroweak interactions by turning the g' coupling to zero; also for the sake of clarity, we omit the SM fermions in the following discussion. The Lagrangian describes a collection of spin $s = 1$ massive gauge bosons, invariant under a global $SU(2)$ transformation $W_\mu \to U W_\mu U^\dagger$ where W's are in the adjoint

$$
\begin{aligned}
\mathcal{L} &= -\frac{1}{4} W^a_{\mu\nu} W^{a\mu\nu} + m_W^2 \mathrm{Tr}\, W_\mu W^\mu + \mathcal{O}(\Lambda^{-1}) \\
&\sim \frac{1}{2}(\partial W)^2 - \frac{1}{2} m_W W^2 - g W^2 \partial W + g^2 W^4 + \mathcal{O}(\Lambda^{-1})
\end{aligned}
\tag{1.15}
$$

where $W^a_{\mu\nu} = \partial_\mu W^a_\nu - \partial_\nu W^a_\mu - g \epsilon^{abc} W^b_\mu W^c_\nu$, $W_\mu = \sigma^a W^a_\mu / 2$ and $\mathcal{O}_i$ are higher dimension operators made of W_μ fields. The theory of Eq. (1.15) is sick in the sense that the cut-off Λ cannot be arbitrarily large, for if we take $\Lambda \gg m_W / g$ perturbative unitarity is lost. There is a number of equivalent ways to see this. Firstly, at high energies the longitudinal gauge bosons's polarization is given by $\epsilon_L^\mu(p) = p^\mu / m_W + \mathcal{O}(m_W / E)$ and therefore the probability amplitudes grow unboundedly with the energy, up to order $\mathcal{O}(\Lambda^{-1})$ corrections. Therefore the validity of Eq. (1.15) is for energies $E \lesssim m_W / g \sim \Lambda$, which can be made more precise by actually computing amplitudes involving external longitudinal bosons. This is closely related to the fact that $-\frac{1}{4} W_{\mu\nu} W^{\mu\nu} + m_W^2 W_\mu W^\mu$ is not renormalizable, and the way to see this is by making the high energy behaviour of the theory manifest at the level of the Lagrangian. This is achieved by making the field redefinition $W_\mu \to U W_\mu U^\dagger + (i/g) U \partial_\mu U^\dagger = (i/g) U (D_\mu U)^\dagger$ on Eq. (1.15), where $U(x) = e^{ig T^a \pi^a / m_W}$, T^a being the $SU(2)$ generators,

$$
\begin{aligned}
\mathcal{L} &= -\frac{1}{4} W_{\mu\nu} W^{\mu\nu} + \frac{m_W^2}{g^2} \mathrm{Tr}\, (D_\mu U)^\dagger D^\mu U + \mathcal{O}(\Lambda^{-1}) \\
&= -\frac{1}{4} W_{\mu\nu} W^{\mu\nu} + \left[D_\mu U \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \right]^\dagger \left[D_\mu U \begin{pmatrix} 0 \\ v/\sqrt{2} \end{pmatrix} \right] + \mathcal{O}(\Lambda^{-1}) ,
\end{aligned}
\tag{1.16}
$$

where $D_\mu = \partial_\mu - i \sigma^a / 2 W^a_\mu$ and $v = 2 m_W / g$. In Eq. (1.16) we have introduced a new field $\pi(x)$, but the Lagrangian is equivalent to the original one of Eq. (1.15), by setting $\pi = 0$ (called the unitarity gauge) they take the same form. In the second equality we have written Eq. (1.16) in a form that makes it clear that global $SU(2)$ is completely broken in the vacuum: $T^a \langle U(0, v/\sqrt{2})^T \rangle = T^a \cdot (0, v)^T \neq 0$.

The advantage of introducing the $\pi(x)$ fields is that the high energy behaviour of the theory is now manifest, at high energies the gauge boson's longitudinal component is

well-described by the $\pi(x)$ fields. This is easily seen by taking the simultaneous limits $m_W \to 0$ and $g \to 0$ while keeping m_W/g fixed. Under this limit (called *decoupling limit*) the gauge bosons decouple from the π fields and Eq. (1.16) reduces to

$$\mathcal{L} = \frac{m_W^2}{g^2} \text{Tr } \partial_\mu U^\dagger \partial^\mu U \; , \tag{1.17}$$

up to terms of order $\mathcal{O}(\Lambda^{-1})$. Corrections to Eq. (1.17) are perturbative in powers of m_W/E and g^2, where E is the energy of the gauge bosons. The most important lesson that we learn from this analysis is that the theory of Eq. (1.17) becomes non-perturbative at energies around m_W/g. At such energies or below we expect new physics beyond the description of Eq. (1.17). More accurately, we can define the strong coupling scale as the loss of perturbative unitarity (in the s-wave scattering it occurs for energies $E \sim 4\pi m_W/g$), or as the scale where the loop expansion breaks down

$$E \sim 4\pi \frac{m_W}{g} \; . \tag{1.18}$$

Again, in the decoupling limit this pathological behaviour is identified as due to the (old sense) non-renormalizability of the theory of pions of Eq. (1.17) or, equivalently, the longitudinal gauge bosons of Eq. (1.15).

Now, the Higgs mechanism is perhaps the simplest way to UV-complete the Lagrangian of Eq. (1.16) into a theory that makes sense at energies much higher than $4\pi\frac{m_W}{g}$. The *mechanism* consists in linearizing the field $U(x) = e^{igT^a\pi^a/m_W}$ by introducing a new field h

$$U \longrightarrow H \equiv U \begin{pmatrix} 0 \\ (v+h)/\sqrt{2} \end{pmatrix} \; , \tag{1.19}$$

and promoting equation Eq. (1.16) into

$$\mathcal{L} = -\frac{1}{4}W_{\mu\nu}W^{\mu\nu} + (D_\mu H)^\dagger D^\mu H + \sum_{i=5} \frac{c_i}{\Lambda^{i-4}}\mathcal{O}_i \; , \tag{1.20}$$

which is the SM electroweak-higgs sector introduced in Eqs. (1.6) and (1.8). [3] Thus, the role of the Higgs in the SM is very particular. Any deviation from the SM higgs predictions requires new physics. With the addition of the Higgs particle h the theory can be used to describe both the low energy physics of massive gauge bosons, Eq. (1.15), and the high energy limit where masses can be ignored and the spin 1 particles interact à la Yang-Mills, as described at the begining of this Chapter.

Notice that there is not a perturbative unitarity problem (and no need for a Higgs mechanism) with a theory of a *photon* and a *Z-boson* that presents the $U(1) \otimes U(1) \to$

---

[3]In all the previous formulas one can restore $g' \neq 0$ straightforwardly.

$U(1)$ breaking pattern as opposed to, for instance, the $U(1)_Y \otimes SU(2)_L \to U(1)_{QED}$ breaking of the SM. The easiest way to see this is from Eq. (1.17), which becomes simply a kinetic term for $\pi(x)$ for $U(x) = e^{ig\pi(x)/m}$; alternatively, start from a Higgs mechanism and decouple the Higgs particle or notice that all amplitudes that lead to violations of unitarity involve triple gauge vertices. A theory of photons and Z's does not need a UV completion. A famous consequence of this is that the Electroweak Precision Tests (EWPT) observables *do* depend on the logarithm of the Higgs mass, preventing us from taking the limit $m_h \to \infty$; and, those terms are proportional to g since we should be able to decouple the Higgs in the $g \to 0$ limit.

1.1.2   Yukawas

As introduced previously in Eqs. (1.7), (1.8) and (1.9), the fermion content of the SM is described by

$$\mathcal{L}_{FG} = \bar{q}_L^i i\slashed{D} q_L^i + \bar{u}_R^i i\slashed{D} u_R^i + \bar{d}_R^i i\slashed{D} d_R^i + \bar{l}_L^i i\slashed{D} l_L^i + \bar{e}_R^i i\slashed{D} e_R^i \tag{1.21}$$

$$\mathcal{L}_{Yuk} = -\bar{q}_L^i y_{ij}^d d_R^j H - \bar{q}_L^i y_{ij}^u u_R^j \widetilde{H} - \bar{l}_L^i y_{ij}^e e_R^j H - \frac{y_\nu^{ij}}{\Lambda}(\bar{l}_L^i \widetilde{H})(H^\dagger i\sigma^2 l_L^{cj}), \tag{1.22}$$

where $q_L = (u_L, d_L)$, $l_L = (\nu_L, e_L)$, $l_L^c \equiv C\bar{l}_L^T$ and in the following discussion we neglect $\mathcal{O}(\Lambda^{-2})$ operators. The Yukawa matrices $y^{d,u,e}$ are generic 3×3 complex matrices and can be written as $L_f y_D^f R_f^\dagger$, where y_D^f is diagonal and R_f, L_f are unitary. Regarding y^ν, it is a symmetric complex 3×3 matrix and can be written as $U_\nu y_D^\nu U_\nu^T$, where y_D^ν is diagonal. Then, upon the unitary transformation $\{q_L, u_R, d_R, l_L, e_R\} \longrightarrow \{L_d q_L, R_u u_R, R_d d_R, L_e l_L, R_e e_R\}$ and setting $\langle H \rangle = (0, v)^T/\sqrt{2}$, Eq. (1.22) reads

$$\begin{aligned}
\mathcal{L}_{Yuk} &= -\left(\bar{d}_L y_D^d d_R + \bar{u}_L V_{CKM}^\dagger y_D^u u_R + \bar{e}_L y_D^e e_R + \bar{\nu}_L U_{PMNS} \frac{y_D^\nu v}{\Lambda} U_{PMNS}^T \nu_L^C\right)\frac{v}{\sqrt{2}} \\
&= -\bar{d}_L \cdot \mathrm{diag}(m_d, m_u, m_b) \cdot d_R - \bar{u}_L V_{CKM}^\dagger \cdot \mathrm{diag}(m_u, m_c, m_t) \cdot u_R \\
&\quad - \bar{e}_L \cdot \mathrm{diag}(m_e, m_\mu, m_\tau) \cdot e_R - \bar{\nu}_L U_{PMNS} \cdot \mathrm{diag}(m_{\nu_1}, m_{\nu_2}, m_{\nu_3}) \cdot U_{PMNS}^T \nu_L^c
\end{aligned} \tag{1.23}$$

where $V_{CKM}^\dagger = L_d^\dagger L_u$ and $U_{PMNS} = L_e^\dagger U_\nu$ while the kinetic and gauge-fermion interactions of Eq. (1.21) remain the same.

It is customary to define the unitary Cabibbo-Kobayashi-Maskawa (CKM) matrix $L_d^\dagger L_u = V_{CKM}^\dagger$. To diagonalize the mass matrices of the quarks in Eq. (1.23), we rotate

u_L^i independently from d_L^i

$$\left(\bar{d}_L y_D^d d_R + \bar{u}_L V_{CKM}^\dagger y_D^u u_R\right)\frac{v}{\sqrt{2}} \xrightarrow{u_L \to V_{CKM}^\dagger u_L} \left(\bar{d}_L y_D^d d_R + \bar{u}_L y_D^u u_R\right)\frac{v}{\sqrt{2}}$$

$$\bar{q}_L^i i\slashed{D} q_L^i \supset \frac{g}{\sqrt{2}}\bar{u}_L^i W_\mu^+ \gamma^\mu d_L^i + h.c. \xrightarrow{u_L \to V_{CKM}^\dagger u_L} \frac{g}{\sqrt{2}}\bar{u}_L^i V_{ij} W_\mu^+ \gamma^\mu d_L^j + h.c. \,,$$

where $W_\mu^\pm = \frac{1}{\sqrt{2}}(W_1^\mu \mp iW_2^\mu)$. The V_{CKM} matrix can be parametrized with three rotation matrices and a phase

$$V_{CKM} = R_x(\theta_{23}) \cdot R_y(\theta_{13}) \cdot \text{diag}(1, e^{i\delta}, 1) \cdot R_z(\theta_{12}) \,, \tag{1.24}$$

where $R_x(\theta_{23})$ is the matrix of a θ_{23} degree rotation in the y-z plane, and analogously for R_y and R_z. The interaction $\frac{g}{\sqrt{2}}\bar{u}_L^i (V_{CKM})_{ij} W_\mu^+ \gamma^\mu d_L^j + h.c.$ is of high phenomenological interest. It induces processes that mix the SM quark families (flavour changing processes) as well as Charge and Parity violation (CPV). Under a Charge and Parity (CP) transformation the above interaction transforms as

$$\frac{g}{\sqrt{2}}\bar{u}_L^i (V_{CKM})_{ij} W_\mu^+ \gamma^\mu d_L^j + h.c. \xrightarrow{CP} \frac{g}{\sqrt{2}}\bar{u}_L^i (V_{CKM})_{ij}^* W_\mu^+ \gamma^\mu d_L^j + h.c. \,,$$

and since V_{CKM} is not real, CP is not preserved in physical processes involving this interaction. [4] These processes are small but measurable in the SM. Extensions of the SM easily contradict the amount of flavour and CPV measured in the SM. The CKM matrix has a striking, hierarchical and regular pattern. The numerical values of the mixing angles of the CKM matrix are

$$|V_{CKM}| = \begin{pmatrix} |V_{ud}| & |V_{us}| & |V_{ub}| \\ |V_{cd}| & |V_{cs}| & |V_{cb}| \\ |V_{td}| & |V_{ts}| & |V_{tb}| \end{pmatrix} = \begin{pmatrix} .97 & .23 & .0035 \\ .23 & .97 & .041 \\ .0087 & .040 & 1.0 \end{pmatrix} = \mathcal{O}(1) \times \begin{pmatrix} 1 & \epsilon & \epsilon^3 \\ \epsilon & 1 & \epsilon^2 \\ \epsilon^3 & \epsilon^2 & 1 \end{pmatrix} \,,$$

where $\epsilon = 0.2$ and the experimental uncertainties are below the quoted significant digits. See, for instance, Ref. [16] for a review of the status of the CKM picture and BSM flavour.

Regarding the leptons, the 3×3 Pontecorvo-Maki-Nakagawa-Sakata matrix U_{PMNS} is commonly parametrized as a sequence of rotations and diagonal phases

$$U_{PMNS} = R_x(\theta_{23}) \cdot R_y(\theta_{13}) \cdot \text{diag}(1, e^{-i\delta}, 1) \cdot R_z(\theta_{12})\text{diag}(e^{i\alpha}, e^{i\beta}, 1) \,. \tag{1.25}$$

---

[4] The imaginary part of V_{CKM} is only physical if it can not be removed by performing redefinitions of the complex fermion's spinor fields. This is indeed the case and it is easy to check that V_{CKM} can be parametrized by three real parameters and one complex phase.

Eq. (1.25) contains three mixing angles θ_{23}, θ_{13}, θ_{12} and three phases ϕ, α, β. Phenomenological aspects of neutrino mixing and mass measurements are not further discussed in this thesis, see Ref. [17] for a review and Ref. [18] for the experimental status.

Finally, we comment on the θ_i parameters of Eq. (1.6). Under a chiral transformation $\psi \to e^{i\frac{\phi_\psi}{2}}\psi$ on the fields $\psi = \{q_L, \bar{u}_R, \bar{d}_R, l_L, \bar{e}_R\}$ apart from the corresponding transformation of Eq. (1.23), the theta parameters shift because of the non-trivial Jacobian of the path integral measure: $\delta\theta_s = -n_f\left(2\phi_{q_L} + \phi_{\bar{u}_R} + \phi_{\bar{d}_R}\right)$, $\delta\theta_2 = -n_f\left(3\phi_{q_L} + \phi_{l_L}\right)$, $\delta\theta_1 = -n_f\left(1/6\phi_{q_L} + 4/3\phi_{\bar{u}_R} + 1/3\phi_{\bar{d}_R} + 1/2\phi_{l_L} + \phi_{\bar{e}_R}\right)$, where $n_f = 3$ is the number of families. Physical observables do not depend on field redefinitions. We can then define a combination of parameters that is invariant under the phase redefinitions of the chiral fermions

$$
\bar{\theta}_{QCD} = \theta_s + \arg\det y^u + \arg\det y^d \tag{1.26}
$$

$$
\bar{\theta}_{EW} = \theta_2 + 2\theta_1 + \frac{8}{3}\arg\det y^u + \frac{2}{3}\arg\det y^d + 2\arg\det y^e. \tag{1.27}
$$

Thus we see that in the SM neither $\theta_2 W_{\mu\nu}\widetilde{W}^{\mu\nu}$ nor $\theta_1 B_{\mu\nu}\widetilde{B}^{\mu\nu}$ are physical by themselves. Below the EWSB scale θ_{EW} is identified with θ_{QED}, i.e. the θ-angle of the photon. Being shift invariant, the parameters of Eqs. (1.26) and (1.27) have a chance of being physical. The terms $A^i_{\mu\nu}\widetilde{A}^{i\mu\nu}$ are total derivatives and only make a contribution when evaluated on certain field configurations, e.g. QCD instantons. Experimentally $\bar{\theta}_{QCD} \leq 10^{-10}$, which is ridiculously small. This is a big puzzle named *strong CP problem*.

1.1.3   Accidental and spurious symmetries

In the preceding subsections we have introduced the SM Lagrangian and commented on all the free parameters of the model. We have now all the information to compute physical processes to measure the parameters and compare the SM predictions with experiment. However, it turns out to be very informative to first recognise various accidental symmetries of the SM. This will point out which are the most critical tests of the SM.

Flavor. In the absence of Yukawa couplings, i.e. $y^f \to 0$, and as $\Lambda \to \infty$ the SM has a global $U(3)^5$ symmetry

$$
G^{SM}_{global}\Big|_{y^f=0} = U(3)_{q_L} \otimes U(3)_{u_R} \otimes U(3)_{d_R} \otimes U(3)_{l_L} \otimes U(3)_{e_R}, \tag{1.28}
$$

that acts on flavour space. This symmetry is explicitly broken by the Yukawas down to a $U(1)^5$ subgroup

$$G_{global}^{SM}\Big|_{y^f \neq 0} = U(1)_B \otimes U(1)_e \otimes U(1)_\tau \otimes U(1)_\mu \otimes U(1)_Y \,, \qquad (1.29)$$

that includes the global part of the $U(1)_Y$ gauge group. [5]

Up to date, all experimental measurements of flavour violation and CP violation in flavour changing processes can be explained by the CKM matrix. New physics can not therefore introduce CP and flavour much beyond V_{CKM}. It is therefore reasonable to promote the quark Yukawas to spurious fields charged under $SU(3)_Q^3 = SU(3)_{q_L} \otimes SU(3)_{u_R} \otimes SU(3)_{d_R}$ as

$$y^u \sim (\mathbf{3}, \bar{\mathbf{3}}, \mathbf{1})\,, \quad y^d \sim (\mathbf{3}, \mathbf{1}, \bar{\mathbf{3}})\,, \qquad (1.30)$$

and require that all perturbations to the dimension-4 Lagrangian are invariant under the $SU(3)_Q^3$. If new physics, or the higher dimensional operators, are invariant under $SU(3)_Q^3$ then all the flavour and CP violation of the SM (beyond $F\tilde{F}$ terms) comes from V_{CKM}. Regarding fermions, similar promotions can be done

$$y^e \sim (\mathbf{3}, \bar{\mathbf{3}})\,, \qquad (1.31)$$

under $SU(3)_{l_L} \otimes SU(3)_{e_R}$. The assumption of Eqs. (1.30) and (1.31) goes under the name of Minimal Flavor Violation (MFV) and it is largely taken in this thesis since we do not deal with the problem of explaining the physical origin of V_{CKM} and m_f, called *the flavour problem*. See Sec. 6.1 of the present thesis, and Ref. [19] and references therein.

Custodial symmetry. In the limit $g', y^f \to 0$, the Higgs sector of the SM, Eq. (1.8), has an enhanced $SU(2)^2$ symmetry,

$$\mathcal{L}_{Higgs}\Big|_{g', y^f, v=0} = |D_\mu H|^2 - V(H) = (D_\mu \Sigma)(D^\mu \Sigma) + m^2 \text{tr}[\Sigma^\dagger \Sigma] - \lambda \text{tr}[\Sigma^\dagger \Sigma]^2 \,, \quad (1.32)$$

where $\Sigma = \frac{1}{\sqrt{2}}(\widetilde{H}, H)$ and it transforms under the global symmetry group $SU(2)_L \otimes SU(2)_R$ as $H \sim (\mathbf{2_L}, \mathbf{2_R})$. The $SU(2)_L$ is *gauged* since $g \neq 0$. The Higgs takes a vev because the vacuum of the theory is at

$$\langle \Sigma^\dagger \Sigma \rangle = v^2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \,, \qquad (1.33)$$

---

[5]It is interesting to note that, apart from $U(1)_Y$, the following $U(1)$'s are (non-simultaneously) non-anomalous: $U(1)_{B-L}$, $U(1)_{L_\mu - L_\tau}$, $U(1)_{L_e - L_\mu}$, $U(1)_{L_e - L_\tau}$.

and the global symmetry group $SU(2)_L \otimes SU(2)_R$ is broken down to the $SU(2)_{L+R}$ by the vacuum, called the custodial symmetry. [6]

Now, consider the quadratic terms in the gauge bosons of the effective Lagrangian

$$\delta\mathcal{L}_{eff} = \Pi_{+-}W^+W^- + \frac{1}{2}\Pi_{33}W^3W^3\,, \tag{1.34}$$

where we have suppressed the Lorentz indices. The two-point functions Π_{ab} receive contributions from SM loops, from higher dimensional operators (upon the Higgs taking a vev) and possibly from new BSM degrees of freedom coupled to the gauge bosons $\Pi_{ab} = \langle J_a J_b \rangle$. Next, consider the parameter $T = [\Pi_{33}(0) - \Pi_{+-}(0)]/m_W$. The T-parameter is in the traceless and symmetric representation of $(\mathbf{3_L} \otimes \mathbf{3_L})$, so it transforms in the $\mathbf{5}$ irrep. Now, if $SU(2)_{L+R}$ is a symmetry of the vacuum, then the spurious symmetry of the parameter T forbids it from having a vev. The recognition of this spurious symmetry that protects the parameter T is very important because it turns out that the relation between the W and Z bosons mass is a prediction of the SM and experiments show that

$$T = \frac{m_W}{m_Z \cos\theta_W} - 1 = \mathcal{O}\left(\frac{\alpha_i}{4\pi}\right)\,, \tag{1.35}$$

is fulfilled to a very good degree. In Eq. (1.35) we have defined $\cos\theta_W = g^2/(g^2 + g'^2)$ and we have set again a finite $g' \neq 0$. The $SU(2)_{L+R}$ custodial symmetry *custodies* this relation and the T-parameter encapsulates the leading violations of the $SU(2)_{L+R}$ custodial symmetry for all BSM theories that couple universally to the SM. The SM parameters g' and y^f can be promoted to spurions such that the SM is formally invariant under $SU(2)_{L+R}$. Imposing this symmetry on the higher dimensional operators and BSMs relaxes the experimental constrains from the measurement of Eq. (1.35). See Sec. 6.1 for further discussion.

1.2   Shadows over the SM

There are various theoretical hints within the SM and GR that point to the presence of new physics. These are the shadows of new physics:

a) The three gauge couplings of the SM approximately coincide at an energy scale around $\sim 10^{16}$ GeV. This feature of the coupling's running might be signalling that, around that scale, the SM gauge groups are embedded into a bigger group with a single gauge coupling, Grand Unified Theories (GUT). In the supersymmetric version of the SM the gauge couplings unify very precisely, see Fig. 1.2.

---

[6]In fact, in the limit $y^u = y^d \neq 0$ there is the enhanced custodial symmetry where (u_R, d_R) are a $\mathbf{2}$ of $SU(2)_R$.

b) The massive nature of the neutrinos is explained via the dimension-five higher dimensional operator of Eq. (1.9). If y_ν is taken of order $\mathcal{O}(1)$, in order to explain the neutrino mass values $m_\nu \lesssim$ eV, then the cut-off scale of the Weinberg operator is around the GUT scale.

c) Gravity is very weak at the electroweak scale because it couples to the SM degrees of freedom via the coupling $\sqrt{g_N}E = E/M_{Pl}$. However, gravitational interactions becomes non-perturbative at energies $E \approx M_{Pl} \approx 10^{18}$ GeV, called the (reduced) Planck mass, not far from the GUT scale.

It is impressive that these three hints point to similar energies. However, these are energy scales much higher than the electroweak scale and it is inconceivable an experiment here on Earth that would directly probe such humongous scales.

In fact, there are various circumstantial evidences that support the picture that there is no new physics directly coupled to the SM until vastly higher scales:

a) The accuracy of the SM predictions are an unprecedented success in science, and none of the predictions is contradicted by any experiment. Flavor and CP tests of the SM suggest that BSM flavour is either not generic or appears at much higher scales.

b) None of the SM couplings hits a Landau pole before the Planck scale. Furthermore, the Higgs quartic coupling is negative at high energies but small enough in absolute value to ensure the stability of the electroweak scale under quantum tunnelling.

c) All possible gauge and gravitational anomalies cancel.
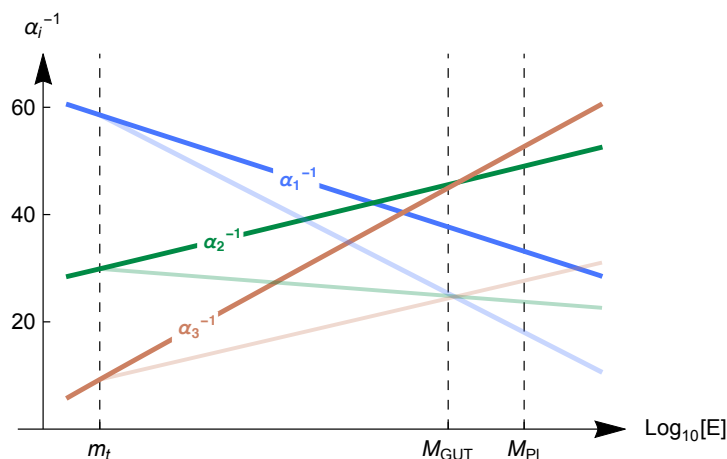
FIGURE 1.2: *Plot of the variation of the SM gauge couplings with the energy, where we have defined $\alpha_i = \{\frac{5}{3}\frac{g'^2}{4\pi}, \frac{g^2}{4\pi}, \frac{g_s^2}{4\pi}\}$. In lighter colors we show the analogous plot in the minimal supersymmetric version of the SM.*

In other words, it is theoretically and experimentally consistent to set $\Lambda \approx M_{Pl}$. Closely related to the theoretical hints, there are theoretical or experimental puzzles (it is not a sharp dichotomy). These are unsettling features of the SM that one wants to explain and gain experimental insight on them. For instance, we would like a deeper understanding of the origin of: the hierarchy of the fermion masses, the origin of the gauge groups, the Higgs potential, why are there three families, why electric charge is quantised, why $\bar{\theta}_{QCD} \ll \mathcal{O}(1)$, dark matter, the period of inflation preceding the Big Bang, etc. However, any of these theoretical conundrums mentioned so far can in principle be explained by new physics at energies far from the EW scale or with very weakly coupled physics.

Luckily, this is not the whole story. There is a theoretical puzzle that seems to require new physics coupled to the SM with $\mathcal{O}(1)$ strength and close to the electroweak scale: the EW hierarchy problem and its associated fine-tuning problem. The EW hierarchy problem consist in giving a satisfactory answer to *why $v \ll M_{Pl}$?*, where satisfactory typically means that the proposed theory has couplings and ratios of energy scales of order $\sim \mathcal{O}(1)$, as dictated by symmetries, but otherwise generic. At this point the problem looks of the same nature as to *why $m_\nu/m_t \ll 1$?*; however, this similarity is not exact due to the closely related fine-tuning problem of the EW scale. In short, the problem can be stated as follows. In any calculable ultra-violet (UV) completion of the SM conceived so far the Higgs mass value turns out to be around the same energy scale of the UV completion scale,

$$m_h^2 = g_{SM}^2 v^2 \pm g_{BSM}^2 \Lambda^2/(16\pi^2) \ , \tag{1.36}$$

where Λ is the UV completion physical scale (e.g. the heaviest mass of the new UV degrees of freedom). [7] Then, given that the SM contribution is dominated by $g_{SM} \sim \mathcal{O}(1)$ couplings, the parameters of the UV completed theory must be of the order $\sum_i g_{BSM,i}^2 \sim \mathcal{O}(16\pi^2 v^2/\Lambda^2)$ to ensure a Higgs mass as light as $m_h \approx v$. Hence, if $\Lambda \gg v$, then $\sum_i g_{BSM}^i$ is *finely-tuned* and we say that the theory does not look *natural*. The need for new physics not far from the EW scale to avoid fine-tuning is not a theorem. However, it is supported by the effective field theory analysis that, in the SM, the Higgs mass term is not forbidden by any symmetry as the mass parameter is taken to zero and therefore can receive additive contributions, as in Eq. (1.36). The fine-tuning problem can be taken as a motivation for physics beyond the SM not far from the EW scale. The argument is that, if the full BSM theory is *natural* then $\Lambda \sim 4\pi v$. However, there are instances in physics that disfavour the naturalness *"principle"*/strategy as a guide. For instance, the cosmological constant problem seems to be totally at odds with the standard QEFT analysis of naturalness: $\langle T_{\mu\nu} \rangle \sim -(10^{-3}\ \mathrm{eV})^4 g_{\mu\nu} \ll -(M_{Pl})^4 g_{\mu\nu}$. Indeed a prototypical

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[7]We have included a $(4\pi)^{-2}$ factor to recall the Higgs mass calculation in perturbation theory and truncated at one-loop.
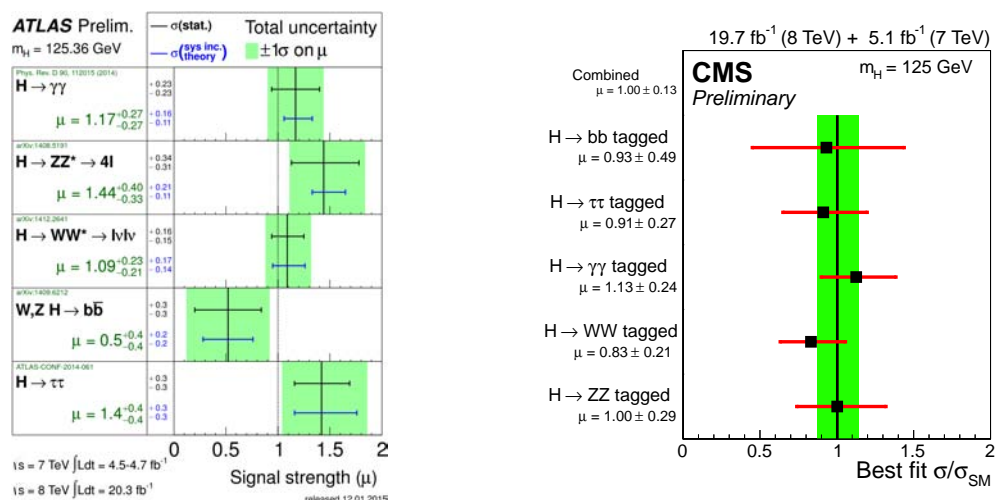
FIGURE 1.3: *The measurment of the signal strength $\mu = \sigma/\sigma_{SM}$ by the experiments ATLAS [22] (left) and CMS [23] (right).*

particle physics solution would be to have SUSY softly broken with $M_{SOFT} \lesssim 10^{-3}$ eV. Thus we see that, even-though it is not guaranteed, it is plausible that the LHC will be able to provide a definite answer on the EW hierarchy problem. See for instance Ref. [20, 21] for further discussion on the fine-tuning problem.

The Higgs

As we have explained, all the collider experiments done so far, at energies $E \lesssim 1$ TeV, have vindicated the Standard Model. The last milestone in the high energy frontier has been the discovery of the Higgs boson [9, 10], see for instance Ref. [24] for a recent theoretical overview. Since the discovery of the Higgs much more data has been analysed and there is no sign of anomalies in the Higgs measurements. More concretely, it has been tested through all its dominant production modes (gluon fusion, vector boson fusion, W or Z Higgstrahlung, and $t\bar{t}H$ production) in each of the most sensitive Higgs boson decays at the Large Hadron Collider (LHC): $\gamma\gamma$, 4-leptons, WW, $\tau\tau$ and $b\bar{b}$. A measure of the agreement with the theoretical expectations is the signal strength μ. It is defined as the observed cross-section times the branching fraction of a process divided by the SM expectation. In Fig. 1.3 we show the combined signal strengths of the different detected channels by both ATLAS and CMS. All in all, the results are consistent with the SM within $\sim 1\sigma$. This is based on an integrated luminosity of ≈ 25 fb^{-1} at an energy of $7 - 8$ TeV, which corresponds to a production of about 10^6 Higgs boson in the dominant production modes. The number of Higgs analysed so far depends on the branching fraction of the different decay channels and its corresponding efficiency and
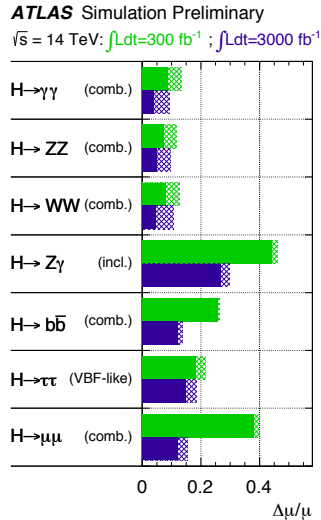
FIGURE 1.4: *The ATLAS prospects for the Higgs signal strength measurements with 300 fb^{-1} and 3000 fb^{-1} [25]. The lower opacity area shows the theoretical uncertainties.*

purity in the reconstruction of the events. For instance, for the decay $h \rightarrow \gamma\gamma$ the number of events produced so far is $N = \mathcal{O}(10^3)$ and it is reduced by a factor ≈ 10 in its reconstruction.

The LHC machine is expected to deliver about 300 fb^{-1} integrated luminosity at an energy of $E = 13-14$ TeV per experiment by the year 2022. There is the possibility that the LHC will be upgraded into the high-luminosity LHC (HL-LHC) and will produce about 3000 fb^{-1} of data at $E = 14$ TeV, per experiment, by the year 2035 [25]. In Fig. 1.4 we show the projected precision signal strength by the ATLAS experiment.

With the overall experimental evidence together with the theoretical expectations (i.e. the essential theoretical bias towards a coherent and satisfying theory) it makes sense to accept that the discovered Higgs boson is indeed the SM Higgs boson. The discovery of a Higgs particle, with mass of approximately 125 GeV, is the most important result from the LHC so far. It is an unprecedented discovery since it is the first observation of fundamental spin zero particle and because of the role of the Higgs in the electroweak symmetry breaking. The measurement of the Higgs couplings is crucial, and part of the main LHC goals, because deviations in the SM expectations are a window to new physics beyond the SM. It is a big issue. We are probing the vacuum structure of the electroweak interactions and it is extremely important that we make sure that we understand it with as much detail as possible. Furthermore, if no new physics beyond the SM is discovered at the TeV scale, the Higgs will remain as one of the few and precious handles for us to understand the governing principles of nature. In the present thesis we take some steps along this line of research.

Part I

Electroweak vacuum stability

2. Introduction

On July 2012 the experiments ATLAS [9] and CMS [10] reported the discovery of a Higgs-like boson. As we have reviewed in Chapter 1, all the measurements done so far are compatible with a SM Higgs and the favoured Higgs mass value is $m_h \approx 125$ GeV. More accurately, the best measurements of the mass value come from the visible decays of the Higgs into photons $h \to \gamma\gamma$ and into four leptons $h \to 4l$. These decays are very clean because the measurements of photons and leptons has much less uncertainties than those processes that hadronize or that have lower branching fractions. Both ATLAS and CMS reported this mass measurement separately in Refs. [23, 27] and recently provided a combination of their measurements in Ref. [28], yielding and average Higgs mass value of

$$m_h = 125.09 \pm 0.21(\text{stat.}) \pm 0.11(\text{syst.}) \text{ GeV} . \tag{2.1}$$

This is a very interesting value for the Higgs mass from the experimental point of view because there are several accessible decay channels of the Higgs boson.

Furthermore, from the theoretical point of view this range of masses is very intriguing. As we show below, the Higgs mass implies that the Higgs quartic coupling λ of Eq. (1.12) is negative at energies $\approx 10^{10}$ and the effective potential has a region in field space of lower energy density than the electroweak vacuum.

In the Standard Model (SM) the Higgs mass is given by

$$m_h^2 = 2\lambda v^2 + \Delta m^2 , \tag{2.2}$$

where Δm^2 denotes small radiative corrections that are numerically dominated by the top Yukawa y_t and strong gauge α_s couplings. Eq. (2.2) is the matching condition that fixes the value of the quartic coupling at the electroweak scale $\sim m_W$. Then, from
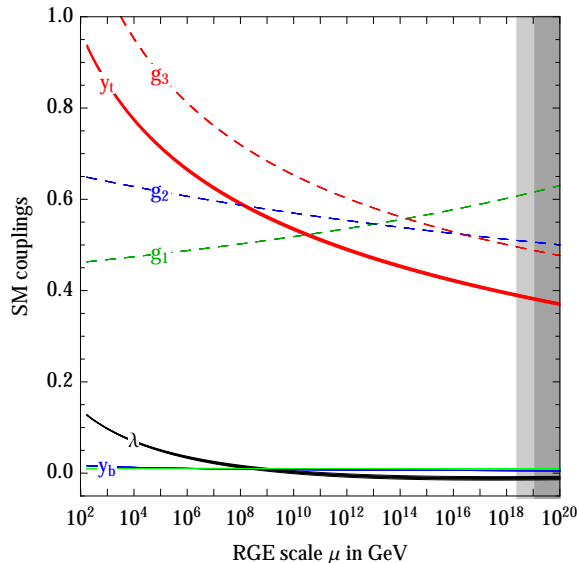
FIGURE 2.1: *SM RG evolution from m_t to M_{Planck} of: the EW gauge couplings $g_1 = \sqrt{5/3}g'$ and g, the strong gauge coupling g_s, the top Yukawa coupling y_t and of the Higgs quartic coupling λ. All the couplings are defined in the $\overline{\text{MS}}$ with two-loop beta functions. Boundary conditions taken at $E = m_t$ with values as in Chapter 3.*

Eq. (2.1) and the measurement of $v \approx 246$ GeV [1] one finds $\lambda_{EW} = \lambda(\mu = m_W) \approx 0.13$. The value of the Higgs quartic coupling λ depends on the energy scale of the scattering process as dictated by the renormalization group (RG) equations. It is given by

$$\lambda(\mu) = \frac{\lambda_{EW}}{1 - \beta_\lambda \log\left(\frac{\mu}{m_W}\right)} + \sum_{n=1} \mathcal{O}\left(\alpha_i^n \log^{n-1}\right), \qquad (2.3)$$

where $\beta_\lambda = 24\lambda^2 + 12\lambda y_t^2 - 6y_t^4 + \ldots$, see e.g. the Appendix of Ref. [29] for the two-loop full expression. Eq. (2.3) is a leading-log resummation of the $\lambda(\mu)$ variation because, using the one-loop beta function, we are resumming the highest (divergent) logarithm power of a fixed loop order calculation. The quartic coupling $\lambda(\mu)$ is negative at high energies. This is due to the negative contribution of the top-quark in the beta function of the quartic coupling. However, the running of $\lambda(\mu)$ is very shallow, see Fig. 2.1. Therefore very small changes in λ_{EW} imply big changes on the energy scale where $\lambda(\mu) = 0$. A priori, the Higgs mass could have been anything from say 115-1000 GeV but it turns out that for $m_h \approx 125$ GeV

$$\lambda\left(\mu = M_{Pl}\right) \approx 0. \qquad (2.4)$$

This is very interesting for various reasons:

- Firstly, it adds one more item to the list of hints (see Sec. 1.2) that point to a special very high energy scale. However, it is fair to say that this hint is less strong

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[1]The parameter v can be measured precisely from the Fermi constant G_F in the process $\mu \to e\nu_e\nu_\mu$.

than the loss of of perturbative unitarity in gravity and the unification or see-saw energy scales. The energy scale where $\lambda = 0$ is not a point of enhanced symmetry [2] and due to the $\lambda(\mu)$ shape and the scalar nature of the Higgs [3] very small new physics thresholds can drastically change the picture of Eq. (2.4). Nevertheless, one might be willing to take Eq. (2.4) as mild evidence that the scale of new physics directly coupled to the SM is very high (e.g. supersymmetry typically requires $\lambda = g_i^2 > 0$).

- Secondly, the effective potential can develop a second minimum at a scale higher than Λ_I, where $\lambda(\Lambda_I) = 0$.

To study the structure of the electroweak vacuum the quantity that matters is not the quartic coupling but the effective potential V_{eff}. Since are interested in the behaviour of V_{eff} for $\phi \gg v$ we can neglect the Higgs mass term. Then, the RG improved effective potential [4] can be written as

$$V_{eff} = \frac{\lambda_{eff}(\phi)}{4}\phi^4 \ , \tag{2.5}$$

where $\phi = \langle 0_J|h|0_J\rangle$, for an external source $\delta\mathcal{L} = \phi \cdot J$. With very good precision (of order $\mathcal{O}(m^2/\phi^2)$) we can study the stability of the electroweak vacuum by analysing the sign of $\lambda_{eff}(\phi)$. At tree level $\lambda_{eff} = \lambda$ and the one-loop RG improved effective potential is given by $\lambda_{eff} = \lambda(\mu = \phi)$, given in Eq. (2.3). In Fig. 2.2 we show a schematic plot of the instability that appears when $\lambda(\phi_*) \lesssim 0$. Notice that, to extrapolate the SM potential up to arbitrary high energy scales in order to study its stability, one needs three pieces of information: the effective potential, the beta functions and the boundary conditions at the EW scale. To be consistent in the precision of the calculation, if the effective potential is computed at n-loops then the beta functions and the matching conditions have to be computed at (n+1)-loops and n-loops, respectively. This resumms up to n-times next to leading order logarithms (N^nLO) of the perturbative calculation of V_{eff}.

The presence of instabilities at high scale are in general problematic because the electroweak vacuum state $|v\rangle$ (that corresponds to $\langle v|\phi|v\rangle \approx 246$ GeV) is no longer a stable minimum at the quantum level. There is a non-zero probability for the electroweak false-vacuum state to decay to the deeper true minimum state $|0\rangle$. The process happens through the nucleation of a bubble of the deeper vacuum, which subsequently expands provided its radius is large enough [33, 34]. Instabilities of this sort are worrisome because we know that the vacuum state $|v\rangle$, upon which the visible matter of the universe

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[2] Notice however that the $\beta_\lambda = 0$ point is remarkably close.

[3] It can couple with any other invariant $\mathcal{L}^{BSM} = |H|^2 \times \mathcal{O}(x)$.

[4] The renormalization group improvement of the effective potential was done in Ref. [30], see also Refs. [31, 32].

rests, is very long lived; at least up to lifetimes much higher than the age of the universe since the Big Bang, $T_U = 10$ Gyr. If the instability of the SM happens to correspond to a very short lifetime (as compared to T_U) it is interpreted as an inconsistency of the theory rather than as meaning that *we have been very lucky until now*. The inconsistency of the theory due to the instability may be cured in a number of ways. For instance, the SM can be deformed by adding bosons coupled to the Higgs. Bosons contribute positively to the running of the quartic coupling and help in ensuring that $\lambda(\phi) > 0$ for scales $M_{Pl} > \phi > 246$ GeV. [5] Or, the SM may be embedded into a BSM with completely different degrees of freedom than the SM. In that hypothetical BSM no longer makes sense to talk about the Higgs potential.

Vacuum decay

In a theory with several local minima, there is the possibility of quantum tunnelling between a local minimum and a point in field space where the value of the potential is lower. The calculation of vacuum tunnelling in field theory is a generalisation of quantum barrier penetration in elementary quantum mechanics to infinite degrees of freedom [33, 34]. One can find excellent reviews of the theory of vacuum decay in Ref. [35–39].

In the SM quantum field tunnelling is relevant in cosmology. An infinitely old universe must be in a true vacuum, no matter how slowly the false vacuum decays. However, the universe is not infinitely old; its is about $T_U \sim 10^{10}$ years old. Then, the relevant
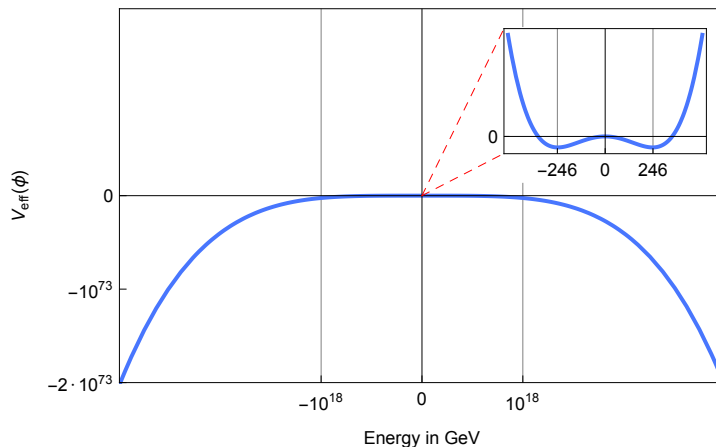
FIGURE 2.2: *Schematic realistic representation of the SM Higgs potential. The region $\phi \in [-500, 500]$ GeV has been zoomed in.*

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[5]Instabilities around or above M_{Pl} are ignored since we expect sizeable quantum gravity effects that we do not want to tackle in a first analysis. The point is that if big instabilities appear at $\phi \ll M_{Pl}$ quantum gravity effects do not have the strength to cure them.

parameter is that cosmic time for which the product of the decay rate per unit time and per unit volume of the false vacuum Γ/V times the volume of the past light-cone [6] is of order unity.

For large field values, Eq. (2.5) is a good approximation of the Standard Model (SM) effective potential and it turns out that the approximation

$$\Gamma/V \approx \max\{\phi^4 \exp(-S_E); \; \lambda_{eff}(\phi) < 0\} \, , \tag{2.6}$$

where $S_E \approx \frac{8\pi^2}{3\lambda_{eff}}$, generally matches the numerical calculation with precision of a few percent [37, 40].

Let us discuss the theoretical uncertainty in Eq. (2.6) for the SM. We do a rough estimate. The uncertainty is dominated by S_E. The quartic coupling at the maximum of Eq. (2.6) is typically $|\lambda_{eff}| \approx (0.01 \pm 0.001)$, see Chapter 3. This implies $B_0 \approx 2600 \pm 100$. An uncertainty of ± 100 in B_0 implies a huge uncertainty in Γ/V. However, we are not so much interested in the precise value of Γ/V but rather on whether or not Γ/V times the past light cone is much smaller than one. The point is that, since tiny changes in λ_{eff} imply big changes in Γ/V, we will always be able to determine if the vacuum is long lived except for a narrow range of values of λ_{eff}. Furthermore, small changes in λ_{eff} can be translated, through the matching condition $\lambda_{eff}(m_{Higgs})$, into an even smaller change in the Higgs mass. For the SM, we can do the following estimate of the width in λ_{eff} where we can not conclude about whether or not the vacuum is long lived in comparison with the age of the universe. From Eq. (2.6), taking $\phi \sim 10^{11}$ GeV [7] and a light-cone volume of $V_u = e^{409}[100 \text{ GeV}]^{-4}$, we see that $\Gamma/V \times V_u \approx 10^{-6}$ and ≈ 100 for $-\lambda_{eff}(\phi) \approx 0.052$ and ≈ 0.054, respectively. This can be traced back to an uncertainty of ~ 1 GeV in a Higgs mass of ~ 120 GeV. Therefore, if the value of the Higgs mass is not close (by ~ 1 GeV) to the boundary where the $\Gamma/V \times V_u \approx 1$, the theoretical uncertainty in Γ/V is small enough; in fact, we find that this is the case for the SM.

In the upcoming Chapter 3 we present a detailed study of the stability of the electroweak vacuum. Then, in Chapter 4 we review a a simple and robust mechanism to stabilise the electroweak vacuum.

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[6]The past light-cone volume can be estimated by multiplying the size of the observable universe by the age of the universe times the speed of light. This is about e^{409}/v^4, where $v \approx 100$ GeV.

[7]This is a typical field value where $V_{eff}(\phi) < 0$, see next Chapter.

3. Stability of the SM

As already emphasised, the Higgs mass value measured by ATLAS and CMS is rather intriguing. Its measured value corresponds to a lifetime of the electroweak vacuum that lies just in between having absolute stability up to the Planck scale M_{Pl} and being unstable, with a lifetime smaller than the age of the universe. In order to conclude about whether or not the SM can be extrapolated up to the Planck scale without any consistency problem a precise calculation is needed. The study of the stability of the SM vacuum has a long history (see also Ref. [39, 41] and references therein). The state-of-the-art analyses of the SM vacuum stability before the Higgs discovery were done at the next-to-leading order (NLO) level [40, 42–49]. This is based on two-loop renormalization-group (RG) equations, one-loop threshold corrections at the electroweak (EW) scale (possibly improved with two-loop terms in the case of pure QCD corrections), and one-loop improved effective potential (see Ref. [3] for a numerically updated analysis). The NNLO calculation was first presented in Ref. [1] and further refined in Ref. [50].

The present Chapter is based on Ref. [1]. We explain the NNLO analysis of the stability of the electroweak vacuum. It is based on the two-loop effective potential, the three-loop beta functions and the matching conditions at two-loops. Subsequently in Sec. 3.5 we summarise the results and discuss the sensitivity of the the study to beyond the SM physics at Planck scale.

3.1 The two-loop threshold correction to $\lambda(\mu)$

As pointed out in Ref. [3], the most important missing NNLO piece for the vacuum stability analysis were the threshold corrections to λ at the weak scale. This is simple to understand from the shape of the function $\lambda(\mu)$, see Fig. 2.1; a small change in the boundary at the EW scale implies a big change in the scale μ where $\lambda(\mu) = 0$.

The most important threshold corrections at the EW scale are corrections due to QCD and top Yukawa interactions, because such couplings are sizeable at low energy,

Fig. 2.1. In this Section we review the calculation of such terms and the associated theoretical uncertainty. As explained below, we will obtain these leading terms in the matching condition, proportional to y_t^6 and $y_t^4 g_s^2$ (where y_t and g_s are the Yukawa of the top quark and the strong coupling) from the calculation of the Higgs mass via the effective potential.

We write the SM potential for the Higgs doublet H in the usual way:

$$V = -m^2 |H|^2 + \lambda |H|^4 , \qquad H = \begin{pmatrix} G^+ \\ (v + h + iG^0)/\sqrt{2} \end{pmatrix} . \qquad (3.1)$$

Up to negligible width effects ($\Gamma_h \approx 4$ MeV), the pole Higgs mass m_h is the solution of the pole equation at the EW minimum

$$M_h^2 = -m^2 + 3\lambda v^2 + \Pi_{hh}(M_h^2) , \qquad (3.2)$$

where m^2, λ and v are $\overline{\text{MS}}$ renormalized quantities and $\Pi_{hh}(p^2)$ is the Higgs self-energy (1PI two-point) function, with external four-momentum p. We rewrite this equation as

$$M_h^2 = \left[-m^2 + 3\lambda v^2 + \Pi_{hh}(0) \right] + \left[\Pi_{hh}(M_h^2) - \Pi_{hh}(0) \right] = \left[M_h^2 \right]_V + \Delta\Pi_{hh}(M_h^2) .$$

This step is convenient because the last term (which is computationally challenging) only gives corrections suppressed by the small Higgs quartic coupling, in view of the smallness of $M_h^2 = 2\lambda v^2$ at tree level. The first piece can be expressed in term of derivatives of the effective potential, V_{eff}. Writing the effective potential as a sum of the tree-level part V_0 plus radiative corrections ΔV

$$V_{eff} = -\frac{m^2}{2} h^2 + \frac{\lambda}{4} h^4 + \Delta V, \qquad (3.3)$$

one finds

$$\left[M_h^2 \right]_V = \left. \frac{\partial^2 V_{eff}}{(\partial h)^2} \right|_{h=v} , \qquad (3.4)$$

where v is the h vev at the minimum of the effective potential, determined by the minimization condition

$$\left. \frac{\partial V_{eff}}{\partial h} \right|_{h=v} = \left[-m^2 h + \lambda h^3 + \frac{\partial \Delta V}{\partial h} \right]_{h=v} . \qquad (3.5)$$

As usual, it is convenient to consider m^2 as a free parameter fixed in terms of v by the above equation, arriving at

$$\left[M_h^2 \right]_V = \left[2\lambda v^2 - \frac{1}{h} \frac{\partial \Delta V}{\partial h} + \frac{\partial^2 \Delta V}{(\partial h)^2} \right]_{h=v} . \qquad (3.6)$$

Defining the operator $\mathcal{D}_m^2$ as [1]

$$\mathcal{D}_m^2 = \left[-\frac{1}{h}\frac{\partial}{\partial h} + \frac{\partial^2}{(\partial h)^2} \right]_{h=v} , \tag{3.7}$$

and noting that $2\lambda v^2 = \mathcal{D}_m^2 V_0$, we can simply write $\left[M_h^2 \right]_V = \mathcal{D}_m^2 V_{eff}$, obtaining the following expression for the Higgs mass:

$$M_h^2 = \mathcal{D}_m^2 V_{eff} + \Delta\Pi_{hh}(M_h^2) . \tag{3.8}$$

Eq. (3.8) gives the Higgs mass squared as the sum of two terms. The first is the Higgs mass obtained from the potential; this is not the complete pole Higgs mass and must be corrected for nonzero external momentum effects, which are taken care of by the last term, $\Delta\Pi_{hh}(M_h^2)$. It is a straightforward exercise to verify that this expression for the pole mass is independent of the renormalization scale μ. In particular, one can prove that

$$\begin{aligned} \frac{d}{d\ln\mu}\left[m_h^2 \right]_V &= -2\gamma\left[m_h^2 \right]_V , \\ \frac{d}{d\ln\mu}\Delta\Pi_{hh}(m_h^2) &= 2\gamma\left[m_h^2 - \Delta\Pi_{hh}(m_h^2) \right] , \end{aligned} \tag{3.9}$$

where γ is the Higgs anomalous dimension, describing its wave-function renormalization, $\gamma \equiv d\ln h/d\ln\mu$.

Using Eq. (3.8) and the one-loop result for V_{eff} one obtains the one-loop Higgs mass correction. The explicit one-loop result for the pole mass is

$$M_h^2 = 2\lambda v^2 + \delta_1 M_h^2 , \tag{3.10}$$

with

$$\begin{aligned} \delta_1 M_h^2 = \frac{1}{(4\pi)^2}\Big\{ &3y_t^2(4m_t^2 - M_h^2)B_0(m_t, m_t, M_h) + 6\lambda^2 v^2(3\ell_h - 6 + \pi\sqrt{3}) \\ &- \frac{v^2}{4}(3g^4 - 8\lambda g^2 + 16\lambda^2)B_0(m_W, m_W, M_h) \\ &- \frac{v^2}{8}(3G^4 - 8\lambda G^2 + 16\lambda^2)B_0(m_Z, m_Z, M_h) + 2m_W^2\left[g^2 - 2\lambda(\ell_W - 1) \right] \\ &+ m_Z^2\left[G^2 - 2\lambda(\ell_Z - 1) \right] \Big\}, \end{aligned} \tag{3.11}$$

where $G^2 = g^2 + g'^2$. All parameters on the right-hand side (including v) are $\overline{\text{MS}}$ running parameters (with the exception of M_h^2, which appears through the external momentum

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[1]Notice that the term in $\mathcal{D}_m^2$ linear in field-derivatives automatically takes into account the cancellation of h-tadpoles (or alternatively, the minimization condition to get the right v).

dependence of the Higgs self-energy). Since we computed Eq. (3.4) in the Landau gauge, v in Eq. (3.11) represents the gauge and scale-dependent vacuum expectation value of the Higgs field as computed in the Landau gauge. Similarly the $\Delta\Pi_{hh}(m_h^2)$ contribution in that equation is computed in the Landau gauge. In Eq. (3.11)

$$B_0(m_a, m_b, m_c) \equiv -\int_0^1 \ln \frac{(1-x)m_a^2 + x m_b^2 - x(1-x)m_c^2 - i\epsilon}{\mu^2} dx \ , \qquad (3.12)$$

and $\ell_x \equiv \ln(m_x^2/\mu^2)$, with m_x the running mass for particle x (e.g. $m_t \equiv y_t v/\sqrt{2}$). One can explicitly check, using the RGEs for these parameters, that this expression for M_h^2 is indeed scale-independent at one-loop order.

Neglecting gauge couplings and setting $M_h^2 = 2\lambda v^2$ in the one-loop terms, one obtains the approximate expression

$$\delta_1 M_h^2 \simeq \frac{2 y_t^2 v^2}{(4\pi)^2} \left[\lambda(2 + 3\ell_t) - 3 y_t^2 \ell_t \right] \ . \qquad (3.13)$$

To compute Eq. (3.8) at the two-loop level one can use the two-loop effective potential [51, 52] to calculate $\left[M_h^2 \right]_V$ and the general results for two-loop scalar self-energies in Ref. [53] (supplemented by the results on two-loop momentum integrals of Ref. [54]) to calculate $\Delta\Pi_{hh}(M_h^2)$. If we only keep the leading two-loop corrections to M_h^2 proportional to y_t^6, $y_t^4 g_s^2$, dropping all sub-leading terms that depend on the EW gauge couplings or λ, our task is simplified dramatically. First, in the two-loop effective potential we only have to consider the diagrams depicted in Fig. 3.1. Their contribution can be extracted from the expressions for V_Y and V_{FV} in the Appendix of Ref. [1]. Second, in the two-loop term $\Delta\Pi_{hh}^{(2)}(M_h^2)$ we can substitute the tree-level value $M_h^2 = 2\lambda v^2$, so that

$$\Delta\Pi_{hh}^{(2)}(M_h^2) \simeq \Pi_{hh}^{(2)}(2\lambda v^2) - \Pi_{hh}^{(2)}(0). \qquad (3.14)$$

It is then clear that the two-loop contributions coming from that term are proportional to λ and are therefore subdominant; we will neglect $\Delta\Pi_{hh}^{(2)}(M_h^2)$ completely. In ref. [1] the $\Delta\Pi_{hh}^{(2)}(M_h^2)$ contribution to the Higgs mass is computed.
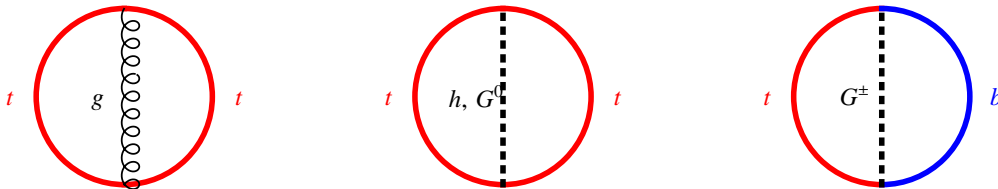
FIGURE 3.1: *Two-loop vacuum diagrams that give the dominant contribution (depending only on g_s and y_t) to the SM two-loop effective potential.*

To find the expression for the Higgs mass at two-loop precision, we must also take into account that M_h^2 has to be evaluated with one-loop precision in the argument of the one-loop term $\Delta \Pi_{hh}^{(1)}(M_h^2)$. Putting together all these pieces, keeping only the y_t^6 and $y_t^4 g_s^2$ terms, we arrive at the following two-loop correction to Eq. (3.10):

$$\delta_2 M_h^2 = \frac{y_t^2 v^2}{(4\pi)^4} \left[16 g_s^2 y_t^2 (3\ell_t^2 + \ell_t) - 3 y_t^4 \left(9\ell_t^2 - 3\ell_t + 2 + \frac{\pi^2}{3} \right) \right] . \tag{3.15}$$

The expression for M_h as a function of λ can be inverted to obtain $\lambda(\mu)$ as a function of the pole Higgs mass M_h. To express $\lambda(\mu)$ in terms of physical quantities (G_μ and the pole masses M_Z, M_W, and M_t) the relations between physical and $\overline{\text{MS}}$ parameters are needed. At the level of accuracy we are working, only the relation between the $y_t(\mu)$ and m_t and the one between $v(\mu)$ and G_μ are required. They are given by:

$$y_t^2(\mu) = 2\sqrt{2} G_\mu M_t^2 \left[1 + \frac{8}{3} \frac{1}{(4\pi)^2} g_s^2 (3 L_T - 4) + \frac{1}{(4\pi)^2} \sqrt{2} G_\mu M_t^2 (-9 L_T + 11) \right], \tag{3.16}$$

$$\begin{aligned}
v^2(\mu) = &\frac{1}{\sqrt{2} G_\mu} + \frac{1}{(4\pi)^2} \left[3 M_t^2 (2 L_T - 1) + M_W^2 (5 - 6 L_W) + \frac{1}{2} M_Z^2 (5 - 6 L_Z) \right. \\
&\left. + \frac{3 M_Z^2 M_W^2}{4(M_Z^2 - M_W^2)} (L_Z - L_W) - \frac{1}{2} M_h^2 - \frac{3 M_W^2 M_h^2}{M_W^2 - M_h^2} (L_W - L_H) \right],
\end{aligned} \tag{3.17}$$

where $L_X = \ln(M_x^2/\mu^2)$, with masses in capital letters denoting pole masses.

We find:
$$\lambda(\mu) = \frac{G_\mu M_h^2}{\sqrt{2}} + \lambda^{(1)}(\mu) + \lambda^{(2)}(\mu) , \tag{3.18}$$

with
$$\lambda^{(2)}(\mu) = \lambda_{\text{QCD,lead.}}^{(2)}(\mu) + \lambda_{\text{Yuk,lead.}}^{(2)}(\mu) + \dots , \tag{3.19}$$

where the ellipsis stands for the sub-leading terms that we are neglecting. The known one-loop term is

$$\begin{aligned}
\lambda^{(1)}(\mu) = \frac{1}{2} G_\mu^2 \frac{1}{(4\pi)^2} \Bigg\{ & \frac{6(L_H - L_W) M_h^6}{M_h^2 - M_W^2} - 8 \left(2 M_W^4 + M_Z^4 \right) - 2(-3 + 6 L_T) M_h^2 M_t^2 \\
& + M_h^4 \left(19 - 15 L_H + 6 L_W - 3\sqrt{3}\pi \right) + 12 (M_h^2 - 4 M_t^2) M_t^2 B_0(M_t, M_t, M_h) \\
& + 2 \left(M_h^4 - 4 M_h^2 M_W^2 + 12 M_W^4 \right) B_0(M_W, M_W, M_h) \\
& + \left(M_h^4 - 4 M_h^2 M_Z^2 + 12 M_Z^4 \right) B_0(M_Z, M_Z, M_h) \\
& + M_h^2 \left[2(8 L_W - 7) M_W^2 + (8 L_Z - 7) M_Z^2 - \frac{6 M_Z^2 M_W^2}{M_Z^2 - M_W^2} (L_Z - L_W) \right] \Bigg\},
\end{aligned} \tag{3.20}$$
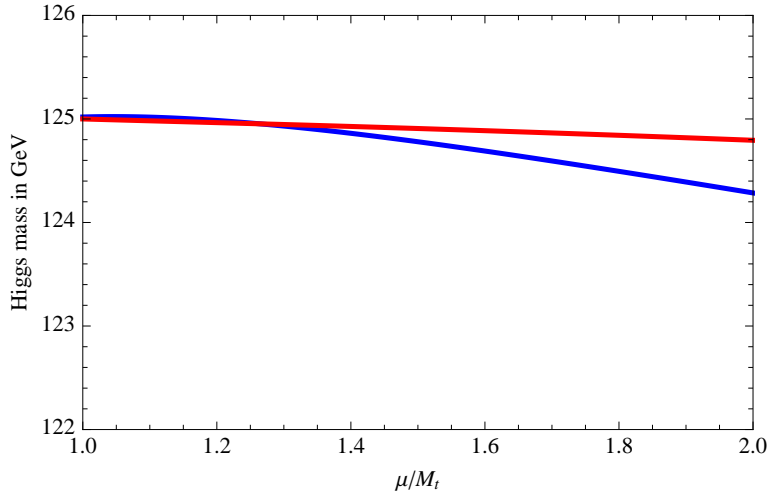
Figure 3.2: *Scale dependence of Eq. (3.8) with one-loop corrections (blue) and with two-loop corrections (red). Both curves are fixed to the value of m_h=125 at $\mu = m_t$.*

and the leading two loop QCD and Yukawa terms are

$$\lambda_{\text{QCD,lead.}}^{(2)}(\mu) = \frac{G_\mu^2 M_t^4}{(4\pi)^4} 64 g_s^2(\mu) \left(-4 - 6L_T + 3L_T^2\right) \ ,$$

$$\lambda_{\text{Yuk,lead.}}^{(2)}(\mu) = \frac{8\sqrt{2} G_\mu^3 M_t^6}{(4\pi)^4} \left(30 + \pi^2 + 36L_T - 45L_T^2\right) \ . \tag{3.21}$$

The above expression for $\lambda(\mu)$ has the correct dependence on the renormalization scale μ, so that both sides of Eq. (3.18) evolve with μ in the same way to the order we work.

In Fig. 3.2 we show the residual scale dependence of M_h in Eq. (3.8) in the $\overline{\text{MS}}$ scheme. We compare the mass calculated using the one-loop and two-loop threshold corrections. As naively expected, the higher loop curve is less sensitive to the choice of renormalizaiton scale. This dependence on the scale will be used later to estimate the theoretical uncertainty in our calculation.

3.2 λ_{eff} and three-loop beta functions.

To study the shape and instability of the effective potential we have to consider Eq. (2.5):

$$V_{eff} \approx \frac{\lambda_{eff}(h)}{4} h^4, \tag{3.22}$$

As we already said, since we want to resum up to next-to-next-to-leading-logarithms of the full loop expansion of V_{eff}, we have to compute $\lambda_{eff}(h)$ at two-loop order and run its couplings with the three-loop order beta functions.

Complete three-loop beta functions for all the SM gauge couplings have been presented in Ref. [55], while the leading three-loop terms in the RG evolution of λ, the top Yukawa coupling (y_t) and the Higgs anomalous dimension have been computed in Ref. [56]. [2]

The explicit two-loop result for $\lambda_{eff}(h)$ can be easily obtained from the two-loop potential, see e.g. the Appendix of Ref. [1]. We report here the simplified expression obtained when, in the two-loop term, we take into account only the contributions from the strong and the top Yukawa couplings [3] [42]:

$$\lambda_{\text{eff}}(h) = e^{4\Gamma(h)} \left\{ \lambda(h) + \frac{1}{(4\pi)^2} \sum_p N_p \kappa_p^2 (r_p - C_p) \right.$$
$$\left. + \frac{1}{(4\pi)^4} y_t^4 \left[8g_s^2(3r_t^2 - 8r_t + 9) - \frac{3}{2} y_t^2 \left(3r_t^2 - 16r_t + 23 + \frac{\pi^2}{3} \right) \right] \right\} . \quad (3.23)$$

Here all couplings are evaluated at the scale determined by the field value ($\mu = h$), the index p runs over $p$article species, N_p counts degrees of freedom (with a minus sign for fermions), the field-dependent mass squared of species p is $m_p^2(h) = \mu_p^2 + \kappa_p h^2$ and C_p is a constant. The values of $\{N_p, C_p, \mu_p^2, \kappa_p\}$ are given in Tab. 3.1. Within the SM they are:

	t	W	Z	h	χ
N_p	-12	6	3	1	3
C_p	$3/2$	$5/6$	$5/6$	$3/2$	$3/2$
μ_p^2	0	0	0	$-m^2$	$-m^2$
κ_p	$y_t^2/2$	$g^2/4$	$(g^2 + g'^2)/4$	3λ	λ

TABLE 3.1:   *The values of $\{N_p, C_p, \mu_p^2, \kappa_p\}$ within the SM.*

The factor

$$\Gamma(h) \equiv \int_{m_t}^{h} \gamma(\mu) \, d\ln\mu , \quad (3.24)$$

where $\gamma \equiv d\ln h / d\ln\mu$ is the Higgs field anomalous dimension, takes into account the wave-function renormalization. We have also defined $r_p \equiv \ln[\kappa_p e^{2\Gamma(h)}]$.

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[2] After the analysis presented in this Chapter was published in Ref. [1], more refined three-loop beta functions for the quartic and Yukawa couplings became available [57, 58].

[3] At high scales, the EW gauge couplings g' and g become comparable in size to y_t and g_s (see Fig. 2.1), but their contribution to $\lambda_{\text{eff}}(h)$ turns out to be numerically small so that Eq. (3.23) is a very good approximation.

3.3   Inputs at the electroweak scale

As far as the SM gauge couplings are concerned, we can directly use results in the literature for the couplings in the $\overline{\text{MS}}$ scheme. In particular, from a global fit of EW precision data, performed with the additional input $M_h \approx 125$ GeV, the following $\overline{\text{MS}}$ values of the electromagnetic coupling and the weak angle renormalized at M_Z are obtained Ref. [59]:

$$\alpha_{\text{em}}^{-1} = 127.937 \pm 0.015 \;, \qquad \sin^2 \theta_{\text{W}} = 0.23126 \pm 0.00005 \;. \tag{3.25}$$

From these we derive

$$\alpha_2^{-1}(m_Z) = \alpha_{\text{em}}^{-1} \sin^2 \theta_{\text{W}} = 29.587 \pm 0.008 \;, \tag{3.26}$$

$$\alpha_Y^{-1}(m_Z) = \alpha_{\text{em}}^{-1} \cos^2 \theta_{\text{W}} = 98.35 \pm 0.013 \;. \tag{3.27}$$

For the strong coupling we adopt

$$\alpha_s(m_Z) = 0.1184 \pm 0.0007 \; [60] \tag{3.28}$$
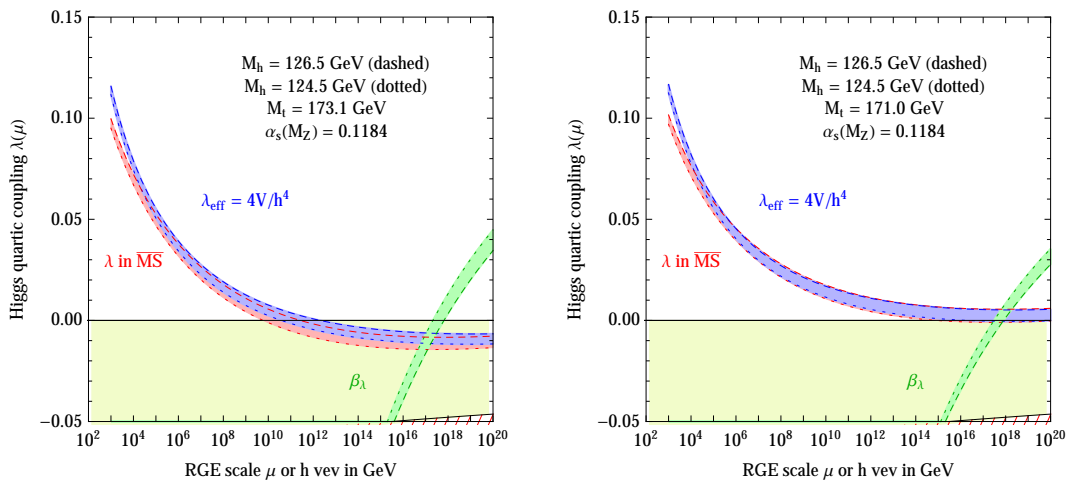
FIGURE 3.3:   *Evolution of the Higgs coupling $\lambda(\mu)$ and its beta function as a function of the renormalization scale, compared to the evolution of the effective coupling $\lambda_{\text{eff}}(h)$, defined in Eq. (3.23), as a function of the field value. Left: curves plotted for the best-fit value of m_t. Right: curves plotted for the lower value of m_t that corresponds to $\lambda(M_{Pl}) = 0$.*

such that, including 3 loop RG running up to M_t and matching to the theory with 6 flavors, we get

$$g_s(m_t) = 1.1645 + 0.0031 \left(\frac{\alpha_s(m_Z) - 0.1184}{0.0007} \right) - 0.00046 \left(\frac{m_t}{\text{GeV}} - 173.15 \right). \quad (3.29)$$

We determine the $\overline{\text{MS}}$ top-quark Yukawa coupling (y_t) starting from the top-quark pole mass (M_t) determined from experiments: $M_t = (173.1 \pm 0.7)$ GeV. This implies the following value for the top Yukawa coupling:

$$y_t(M_t) = 0.93587 + 0.00557 \left(\frac{M_t}{\text{GeV}} - 173.15 \right) - 0.00003 \left(\frac{M_h}{\text{GeV}} - 125 \right)$$
$$- 0.00041 \left(\frac{\alpha_s(M_Z) - 0.1184}{0.0007} \right) \pm 0.00200_{\text{th}}.$$

Next, applying the threshold corrections discussed in Sec. 3.1, we determine the following value for the Higgs self coupling in the $\overline{\text{MS}}$ scheme renormalized at the pole top mass:

$$\lambda(M_t) = 0.12577 + 0.00205 \left(\frac{M_h}{\text{GeV}} - 125 \right) - 0.00004 \left(\frac{M_t}{\text{GeV}} - 173.15 \right) \pm 0.00140_{th}.$$
$$(3.30)$$

The residual theoretical uncertainty, that is equivalent to an error of ± 0.7 GeV in M_h, has been estimated varying the low-energy matching scale for λ between M_Z and $2M_t$. As can be seen in Fig. 3.2, this is a conservative estimate of the theoretical uncertainty.
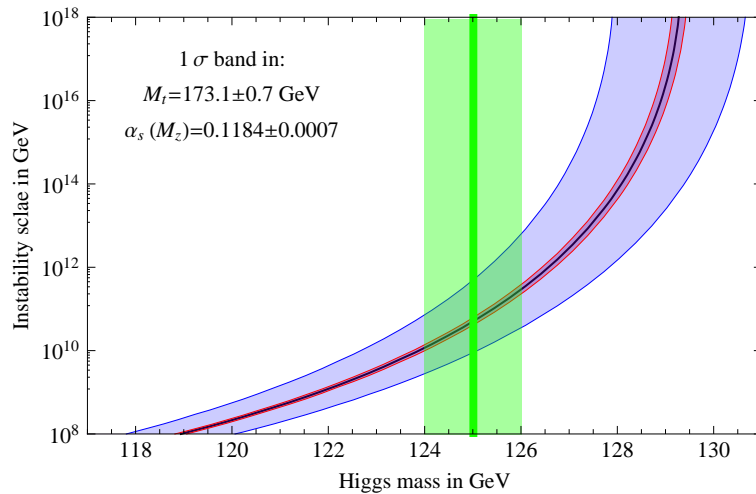
FIGURE 3.4: *The instability scale at which the SM potential becomes negative as a function of the Higgs mass. The blue and the red bands correspond to the experimental uncertainty in m_t and $\alpha_s(M_z)$, respectively. The theoretical error is not shown and corresponds to a ± 1 GeV uncertainty in m_h.*

3.4   Results: is the SM electroweak vacuum stable?

We have now at hand all the pieces needed for the analysis of the EW vacuum stability. In Fig. 3.3 we show the results of the numerical computation of the running of λ and λ_{eff}. We see that for central values of M_t, and taking the experimental value $M_h \approx 125$ GeV, the potential develops an instability around $\sim 10^{11}$ GeV. [4]

Putting all the NNLO ingredients together, we can determine a lower bound for the Higgs mass by requiring that the EW vacuum is absolutely stable up to M_{Pl}, see Fig. 3.4. We obtain the following value:

$$M_h > 129.4 + 1.4\left(\frac{M_t\ [\text{GeV}] - 173.1}{0.7}\right) - 0.5\left(\frac{\alpha_s(m_Z) - 0.1184}{0.0007}\right) \pm 1.0_{\text{th}}\ [\text{GeV}]\ . \quad (3.31)$$

The dominant uncertainties in this evaluation of the minimum M_h value ensuring absolute vacuum stability within the SM are summarized in Tab. 3.2.

The dominant uncertainty is experimental and comes mostly from the measurement of M_t. Although experiments at the LHC are expected to improve the determination of M_t, the error on the top mass will remain as the largest source of uncertainty. The LHC will be able to measure the Higgs mass with an accuracy of about 100–200 MeV, which is far better than the theoretical error with which we are able to determine the condition of absolute stability.

	Estimate of the error	Impact on M_h
M_t	experimental uncertainty in M_t	± 1.4 GeV
α_{s}	experimental uncertainty in α_{s}	± 0.5 GeV
Experiment	**Total combined in quadrature**	$\pm\textbf{1.5}$ **GeV**
λ	scale variation in λ	± 0.7 GeV
y_t	$\mathcal{O}(\Lambda_{\text{QCD}})$ correction to M_t	± 0.6 GeV
y_t	QCD threshold at 4 loops	± 0.3 GeV
RGE	EW at 3 loops + QCD at 4 loops	± 0.2 GeV
Theory	**Total combined in quadrature**	$\pm\textbf{1.0}$ **GeV**

TABLE 3.2:   *Dominant sources of experimental and theoretical errors in the computation of the SM stability bound on the Higgs mass, Eq. (3.31).*

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[4]The value of the instability scale, defined as the field value where the effective potential becomes negative, is not a physical quantity, as it is a gauge dependent quantity. However, whether there is or there is not an instability scale is a physical question, that can be studied with the effective potential. Nevertheless, the instability scale is a useful quantity to use. For instance, if computed in the Landau gauge it is closely related a physical quantity: the mass scale value where new physics that is *integrated in* can stabilise the EW vacuum.

The total theoretical error quoted in Tab. 3.2, is 3 times lower than the uncertainty in the NLO calculation. However, the uncertainty in the threshold corrections to λ at the EW scale still dominates the theoretical error. Another sizable theoretical uncertainty comes from the fact that the pole top mass determined at hadron colliders suffers from $\mathcal{O}(\Lambda_{QCD})$ non-perturbative uncertainties [61]. [5] As far as the RG equations are concerned, the error of 0.2 GeV is a conservative estimate, based on the parametric size of the missing terms. The smallness of this error, compared to the uncertainty due to threshold corrections, can be understood by the smallness of all the couplings at high scales: four-loop terms in the RG equations do not compete with finite three-loop corrections close to the EW scale, where the strong and the top-quark Yukawa couplings are large.

In this Chapter we have presented the most relevant numerical contributions of the NNLO analysis of the EW vacuum stability [1]. This was completed by the full NNLO precision calculation in Ref. [67] where the theoretical uncertainty on the Higgs mass stability bound was further reduced to 0.3 GeV:

$$M_h > 129.4 + 2\left(M_t\ [\text{GeV}] - 173.34\right) - 0.5\left(\frac{\alpha_s(m_Z) - 0.1184}{0.0007}\right) \pm 0.3_{\text{th}}\ \ [\text{GeV}]\ \ .\ [67]$$
(3.32)

Ref. [67] included the full three-loop running of the quartic coupling and the full two-loop calculation of the threshold corrections for both the top Yukawa and the Higgs quartic coupling.

3.5   Summary

We end this Section by showing a plot, Fig. 3.5, that presents the main result. The plot shows that, from the point of view of the stability of the EW vacuum, the Higgs mass is not generic. If we take two random values around the EW scale for the top quark and Higgs mass then, most probably, we will end up with a stable vacuum, or completely unstable, or even not having perturbativity up to M_{Pl}. However, with the preferred experimental values for the top quark and Higgs mass, the SM vacuum is in a metastable situation. This is a perfectly acceptable possibility, however from Fig. 3.5, it is clearly not the most generic point. This fact has motivated various speculations

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[5]The pole mass is defined as the pole of the particle's propagator. It is well defined in perturbation theory for both observable particles, such as the electron, and for quarks [62, 63]. However, the pole mass of the quarks can not be used with arbitrary accuracy because it is affected by non-perturbative QCD effects. In fact, the exact quark propagator does not have a pole because the quarks are confined. Therefore, the pole mass of the top can not be defined outside of perturbation theory. See Refs. [64, 65], and Ref. [66] and references therein.
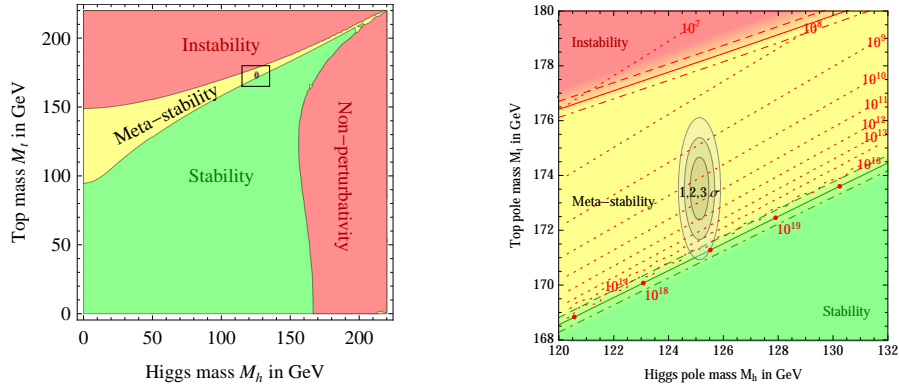
FIGURE 3.5: *Regions of absolute stability, meta-stability and instability of the SM vacuum in the m_t–M_h plane. Right: Zoom in the region of the preferred experimental range of M_h and m_t (the gray areas denote the allowed region at 1, 2, and 3σ). The three boundaries lines correspond to $\alpha_s(m_Z) = 0.1184 \pm 0.0007$, and the grading of the colors indicates the size of the theoretical error. The dotted contour-lines show the instability scale Λ in GeV assuming $\alpha_s(m_Z) = 0.1184$. From Ref. [1].*

like circumstantial evidence for high scale SUSY [68–72] or an IR fixed point of some asymptotically safe gravity [73], among other conjectures.

The answer to the question that gives title to this sub-section should be clear. Assuming there is no new physics up to energy scales of $\sim M_{Pl}$, the SM potential presents a vacuum at large field values that is deeper than the EW vacuum. Therefore, the EW vacuum is unstable. However, the probability of tunnelling out of the EW vacuum is very small, so that its lifetime is much bigger than the age of the universe. Hence, even though the EW vacuum is unstable, the SM can be extrapolated up to M_{Pl} without any consistency problem. In the pure SM, and without input from cosmology, instability arguments can not be invoked to argue in favour of beyond the SM physics below the instability scale.

Sensitivity to Planck scale physics

So far we have assumed the validity of the SM up to the Planck scale. However, it is fair to question the effect of unavoidable gravitational contributions on the stability of the EW vacuum. There are two types of gravitational effects. There are those calculable effects within the EFT of the SM and General Relativity. These include tree level corrections to the bounce action [74]

$$\Delta S_{gravity} = \frac{256\pi^3}{45(RM_{Pl}\lambda)^2}\,, \tag{3.33}$$

and log-divergent loops of gravitons. R is the radius of the bounce solution or, to a good approximation, the value of ϕ that maximises $|\lambda(\phi)|$ for $\lambda < 0$, see Eq. (2.6)

and Ref. [36, 40]. The log-divergent contributions of graviton loops are negligible with respect to the tree-level contribution of Eq. (3.33) to the bounce. In the calculations of this chapter we have neglected Eq. (3.33). We can do an estimate of the relevance of Eq. (3.33) in a similar way as we did at the end of Chapter 2. For instance, taking $R \times M_{Pl} \approx 10$ and $|\lambda(R^{-1})| \approx 0.015$ (corresponding to $M_h \approx 125$ GeV) we find that the bounce action is dominated by the gravitational contribution with a big uncertainty on the bounce action. The contribution of Eq. (3.33) can dominate the bounce action for $R \times M_{Pl} \leq 1$, which it is indeed the case for values of the Higgs and top mass not far from the central experimental values. However, the effect of gravitational contributions on the bounce rapidly decreases for Higgs mass values below $\approx$ 123-125. Its dominance for $R \times M_{Pl} \leq 1$ is due to the fact that the running of λ is very shallow and $|\lambda(M_{Pl})| \ll 1$. A reflection of the this is the narrow strip of metastability for the SM values, see Fig. 3.5.

Secondly there are incalculable gravitational effects due to the presence of quadratic divergences in the theory of SM+GR. These effects are the most sensitive to the unknown Planck scale physics. A convenient way to parametrize them is through higher dimensional operators. For instance, we can include

$$\Delta V = \frac{c}{M_{Pl}^2} \frac{|H|^6}{3} \tag{3.34}$$

to the SM Higgs potential. Clearly the stability of the SM is very sensitive to Planck scale physics in the following sense. We can always destabilise the potential by including the operators like that of Eq. (3.34) with negative sign in the potential. It makes sense then, to take the opposite attitude and ask, for which values of positive $c > 0$ in Eq. (3.34) can the instability of the SM be lifted. This was done in Ref. [75] which found that for $c = \mathcal{O}(1) > 0$ the SM boundary between absolute stability and instability in Fig. 3.5 is mostly unaffected. However, due to the fact that $R \times M_{Pl}$ is not very suppressed, Ref. [75] found that the impact of this operator on the energy density of the true minimum of V_{eff} is huge. This has a direct impact on the lifetime of the vacuum.

Finally, note that the green region of stability in Fig. 3.5 depends on the possible completion of the SM. For a recent emphasis on this issue see Ref. [76]. It is no surprise that one can add new physics to destabilize the EW vacuum. It is also expected that new physics can render the SM point in the left plot of Fig. 3.5 a generic looking point from the stability of the EW vacuum perspective. As an analogy, consider the gauge coupling unification in the (MS)SM. Obviously, one can add new physics to destroy the unification of the gauge couplings. But the point is to take the unification of the gauge couplings as a low energy hint of what physics might take place at the unification scale. In this respect, it seems that most interesting question is: at what energy scales new

physics can stabilize the EW vacuum? For instance, if new physics are added at the Planck scale, that is a too high scale to stabilize the EW vacuum.

It would be interesting to study more thoroughly the gravitational corrections to the bounce action as well as that of higher dimensional operators. As we have argued, these can have an important effect on the lifetime of the EW vacuum for the central values of the SM masses. However, these effects decouple very fast as we move the Higgs mass by a couple of GeV from its critical value ≈ 125 GeV that gives $\lambda(M_{Pl}) \approx 0$. In this respect, the fact that the gravitational contributions can be important ($R \times M_{Pl}$ is not very supressed) give further support to the point that motivated this Chapter: the precise SM Higgs mass value is very singular from the point of view of the stability of the EW vacuum.

4. Stabilisation by a scalar threshold effect

In Chapter 3, we have seen with great detail that the Higgs mass $m_h \approx 125$ GeV is very interesting from the point of view of the stability of the EW vacuum. For such values the SM effective potential develops an instability at field values $\phi \approx 10^{11}$ GeV.

This is of course not necessarily a problem, because the Standard Model is likely to be embedded in a more fundamental theory which may change the shape of the Higgs potential at high high field values. Moreover the lifetime of the EW vacuum is much bigger than the age of the universe. Nevertheless, the actual fate of the EW vacuum depends on the cosmological history. For instance a period of de Sitter inflation in the very early universe may trigger the decay of the electroweak vacuum. [1] The point is that the expanding metric induces fluctuations, proportional to the Hubble rate, to the Higgs field. However, a sizeable Higgs mass during that epoch, as for instance induced by the Ricci curvature, exponentially penalises the fluctuations of the Higgs field and hence the possible vacuum decay. Another source of potential cosmological problems for the Higgs potential instability is the period of reheating. The energy transfer from the inflaton to the SM can be efficient in proving the regions of field space that led to vacuum decay. These issues highly depend on the details of cosmology, however, to avoid potential cosmological constraints it may be preferable to cure any Higgs instability at large field values.

There are, of course, many ways to modify the Higgs potential and raise the instability scale. In this Section, we review a simple and economical mechanism introduced in Ref. [2]. This mechanism requires the existence of a new heavy scalar singlet that acquires a large vacuum expectation value (vev) and has a quartic interaction with the ordinary Higgs doublet. The crucial point is that the matching condition of the Higgs quartic coupling, at the scale where the singlet is integrated out, corresponds to a

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[1] See Ref. [77] for a detailed study of the interplay between the cosmological history and the Higgs vacuum instability.

positive shift, as we evolve from low to high energies. Although the stability condition is also modified by the presence of the singlet, a careful analysis shows that, under some conditions that we will specify later, the threshold correction helps to stabilise the potential. The effect occurs at tree-level and thus can be sizable and, is in general, dominant over loop contributions. Moreover, the effect does not decouple, in the sense that the size of the shift does not depend on the singlet mass, which could take any value lower than the instability scale (but larger than the EW scale). After reviewing the idea of the instability cure by a tree-level threshold effect, we present various examples of beyond the SM physics where the mechanism can be operative.

4.1   Stabilising the Higgs potential with a scalar singlet

To explore the impact of an additional singlet scalar on the stability of the Higgs potential, we consider a tree-level scalar potential of the form

$$V_0 = \lambda_H \left(H^\dagger H - v^2/2 \right)^2 + \lambda_S \left(S^\dagger S - w^2/2 \right)^2 + 2\lambda_{HS} \left(H^\dagger H - v^2/2 \right) \left(S^\dagger S - w^2/2 \right) .$$
(4.1)

Here H is the Higgs doublet, S is a complex scalar field, and V_0 is the most general renormalizable potential that respects a global Abelian symmetry under which only S is charged. Although we will consider here a single complex scalar, most of our conclusions remain valid also in the case of multi-Higgs doublets or real singlet fields (with a Z_2 parity replacing the Abelian symmetry).

For $\lambda_H, \lambda_S > 0$ and $\lambda_{HS}^2 < \lambda_H \lambda_S$, the minimum of V_0 is at

$$\langle H^\dagger H \rangle = v^2/2 , \quad \langle S^\dagger S \rangle = w^2/2 .$$
(4.2)

A nonzero vev of S, which is crucial for the mechanism to work, spontaneously breaks the global symmetry (or the Z_2 parity, for a real singlet) giving rise to a potentially dangerous Goldstone boson. Gauging the symmetry of S or explicitly breaking it by (possibly small) terms in V_0 can be used to evade these problems, but does not conceptually modify our results. For simplicity, we restrict our considerations to the potential in Eq. (4.1), but generalizations are straightforward.

The presence of the new scalar field S modifies the analysis of the stability conditions of the Higgs potential. One effect is the contribution of the singlet to the renormalization group evolution of the Higgs quartic coupling (for recent analyses, see Ref. [78–82] and references therein). The relevant renormalization RG equations above the scale $M_S =$

$\sqrt{2\lambda_S}w$ are, at one-loop

$$(4\pi)^2 \frac{d\lambda_H}{d\ln\mu} = \left(12y_t^2 - 3g'^2 - 9g^2\right)\lambda_H - 6y_t^4 + \frac{3}{8}\left[2g^4 + (g'^2 + g^2)^2\right] + 24\lambda_H^2 + 4\lambda_{HS}^2 ,$$

$$(4\pi)^2 \frac{d\lambda_{HS}}{d\ln\mu} = \lambda_{HS}\left[\frac{1}{2}\left(12y_t^2 - 3g'^2 - 9g^2\right) + 4\left(3\lambda_H + 2\lambda_S\right) + 8\lambda_{HS}\right] ,$$

$$(4\pi)^2 \frac{d\lambda_S}{d\ln\mu} = 8\lambda_{HS}^2 + 20\lambda_S^2 , \tag{4.3}$$

If the singlet mass M_S is below the SM instability scale [2] Λ_I and $(\lambda_{HS}/4\pi)^2 \ln(\Lambda_I/M_S)$ is large enough, the positive contribution to the RG equation for λ_H can prevent it from becoming negative.

4.2   The threshold effect

Besides the loop contribution in the RG equations discussed above, there is a threshold tree-level effect through which the new singlet can affect the stability bound. Let us consider the limit in which M_S is much larger than the Higgs mass ($w^2 \gg v^2$). At the scale M_S we can integrate out the field S using its equation of motion (neglecting derivatives):

$$S^\dagger S = \frac{w^2}{2} - \frac{\lambda_{HS}}{\lambda_S}\left(H^\dagger H - \frac{v^2}{2}\right). \tag{4.4}$$

Replacing Eq. (4.4) in V_0, we obtain the effective potential below the scale M_S:

$$V_{\text{eff}} = \lambda\left(H^\dagger H - \frac{v^2}{2}\right)^2, \qquad \lambda = \lambda_H - \frac{\lambda_{HS}^2}{\lambda_S} . \tag{4.5}$$

This shows that the matching condition at the scale $\mu = M_S$ of the Higgs quartic coupling gives a tree-level shift, $\delta\lambda \equiv \lambda_{HS}^2/\lambda_S$, as we go from λ_H just above M_S to λ just below M_S.

To better understand the origin of the shift in the matching condition, let us consider the mass matrix of the fields h and s, corresponding to the real parts of the doublet H (in unitary gauge) and the singlet S, such that $H^\dagger H = h^2/2$ and $S^\dagger S = s^2/2$. At the minimum, the mass matrix is

$$\mathcal{M}^2 = 2\begin{pmatrix} \lambda_H v^2 & \lambda_{HS} vw \\ \lambda_{HS} vw & \lambda_S w^2 \end{pmatrix} . \tag{4.6}$$

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[2]Stabilizing the potential with degrees of freedom heavier than Λ_I requires sizable couplings, see Refs. [83, 84].

In the limit $\lambda_S w^2 \gg \lambda_H v^2$, the heaviest eigenstate, which is nearly singlet, can be integrated out, leaving behind a "see-saw"-like correction to the lightest eigenvalue

$$m_h^2 = 2v^2 \left[\lambda_H - \frac{\lambda_{HS}^2}{\lambda_S} + \mathcal{O}\left(\frac{v^2}{w^2}\right) \right] \, , \tag{4.7}$$

while $M_S^2 = 2\lambda_S w^2 + 2(\lambda_{HS}^2/\lambda_S)v^2 + \mathcal{O}(v^4/w^2)$. The light state is almost purely h, as the singlet admixture is suppressed by a small mixing angle of order v/w. However, the Higgs mass correction due to the heavy state persists even in the decoupling limit ($w \to \infty$). The negative sign in the shift of the Higgs mass in Eq. (4.7) can be readily understood as coming from the repulsion of mass eigenvalues after turning on the mixing equal to $2\lambda_{HS}vw$.

Naively, as the tree-level shift $\delta\lambda$ corresponds to a larger Higgs quartic coupling above M_S, the chances of keeping it positive seem improved. However, the tree-level conditions for stability change from $\lambda > 0$ in the effective theory below M_S to $\lambda_H > \delta\lambda$ in the full theory above M_S. Thus, it appears that the threshold correction $\delta\lambda$ does not help stability at all. To understand what happens, one has to reexamine the stability conditions more carefully.

First of all, recall that the tree-level potential V_0 in Eq. (4.1) is a good approximation to the full potential if we evaluate couplings and masses (collectively denoted by λ_i below) at a renormalization scale of the order of the field values of interest. Once we express the scalar potential as $V_0[\lambda_i(\mu = \varphi), \varphi]$, potentially large logarithms of the form $\ln m_i(\varphi)/\mu$ (where $m_i(\varphi) \sim \varphi$ is a typical field-dependent mass) are kept small as compared to the tree-level potential. Roughly speaking, this means that V_0 with a fixed μ_c will be reliable as long as one examines $\varphi \sim \mu_c$ and restricts field excursions to $|\varphi - \mu_c| < \mu_c e^{8\pi^2 \lambda_0/\lambda_1^2}$; where λ_0 denotes a coupling in the tree-level potential and λ_1 a coupling affecting the radiative corrections, e.g. the top Yukawa coupling squared. The inequality comes from demanding that the one-loop correction is smaller than the tree-level potential. By adjusting $\mu \sim \varphi$ one can evaluate reliably the potential at all field values, but the previous estimate tells us when we can use $V_0[\lambda_i(\mu_c), \varphi]$, which has a simpler field dependence.

With the parametrization chosen in Eq. (4.1), the EW vacuum corresponds to $V_0 = 0$. Thus the stability condition is $V_0 > 0$ anywhere in field space, away from the EW vacuum. The first and most obvious stability requirement that we should impose is

$$\lambda_H(\mu) > 0 \, , \quad \lambda_S(\mu) > 0 \, , \tag{4.8}$$

at any renormalization scale μ, or else the potential develops unwanted minima lower than the EW vacuum or is unbounded from below at large field values. Next, in order
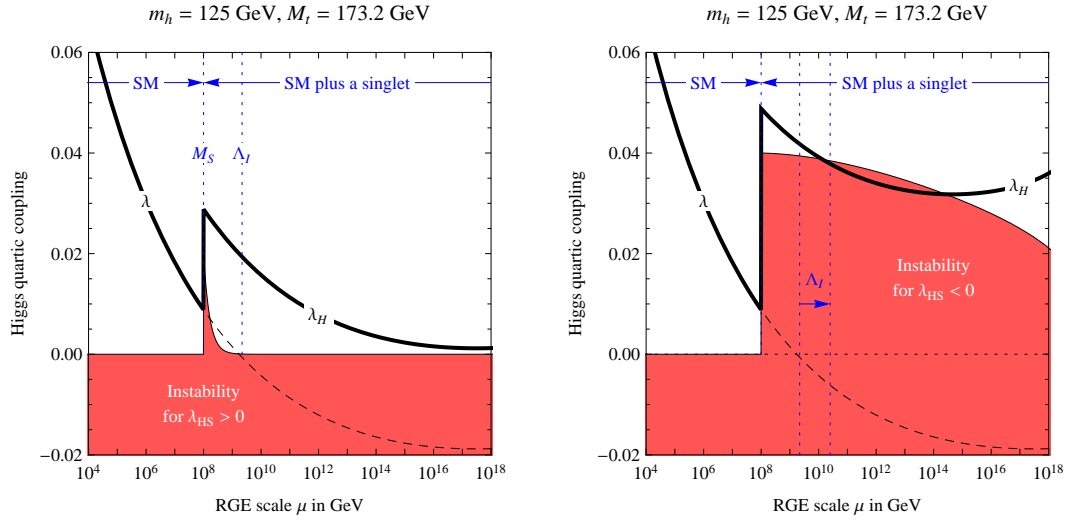
FIGURE 4.1: *Running of the Higgs quartic coupling in the SM and in the model with a scalar singlet, here assumed to have the mass $M_S = 10^8$ GeV. Left: if $\lambda_{HS} > 0$, thanks to the tree level shift at the singlet mass, the coupling never enters into the instability region, even assuming that singlet contributions to the RG equations are negligible. Right: if $\lambda_{HS} < 0$ the instability can be shifted away or avoided only by singlet contributions to the RG equations.*

to discuss the conditions on the coupling constant λ_{HS}, it is convenient to separate the cases in which λ_{HS} is either positive or negative. This separation is meaningful because λ_{HS} renormalizes multiplicatively (as it is the only coupling that connects H and S), see Eq. (4.3), and therefore the RG flow cannot flip its sign.

4.2.1   Case $\lambda_{\mathbf{HS}} > 0$

In this case, V_0 can become negative only when $|S| < w/\sqrt{2}$ (neglecting corrections proportional to v). In this situation, the most dangerous field configuration is well approximated by setting $S = 0$ in Eq. (4.1), such that

$$V_0(H, 0) \approx \lambda_H |H|^4 - \frac{\lambda_{HS}}{2\lambda_S} M_S^2 |H|^2 + \frac{M_S^4}{16\lambda_S} \ . \tag{4.9}$$

The extra stability condition, $V_0 > 0$ at the minimum of Eq. (4.9), is then

$$\lambda_{HS}^2(\mu) < \lambda_H(\mu)\lambda_S(\mu) \ . \tag{4.10}$$

Note that this can be rewritten as $\lambda_H > \delta\lambda = \lambda_{HS}^2/\lambda_S$ and ensures that the light scalar state does not become tachyonic, see Eq. (4.7). If this condition were violated at some

scale μ_*, it would lead to an instability for field configurations with

$$|S| < \frac{M_S}{2\sqrt{\lambda_S}}, \quad \mu_- < |H| < \mu_+, \quad \mu_\pm^2 = \frac{M_S^2 \lambda_{HS}}{4\lambda_H \lambda_S} \left(1 \pm \sqrt{1 - \frac{\lambda_H \lambda_S}{\lambda_{HS}^2}} \right)\Bigg|_{\mu_*}, \qquad (4.11)$$

which could be trusted provided $\mu_- < \mu_* < \mu_+$. Note that, if $\mu_* \gg \mu_\pm$, this would not mean that there is an instability to worry about, as it would be located outside the range of validity of the tree-level approximation $V_0(\lambda_i(\mu_*), \varphi)$. Thus, as long as condition of Eq. (4.10) is satisfied for renormalization scales within a relatively narrow range of energies around M_S (which fixes the mass scale of $\mu_\pm$), there is no instability even if this condition were eventually violated at higher scales. Only if parameters happen to lie near a critical point in which at least one of conditions (4.8) or (4.10) is barely satisfied, radiative corrections can become important and invalidate the stability analysis performed with the tree-level potential. In this case one should resort to the one-loop approximation of the potential; otherwise, our analysis is reliable.

We can now better appreciate how the threshold contribution in Eq. (4.5) can cure the instability of the SM Higgs potential (provided that $M_S < \Lambda_I$). The correction $\delta\lambda$ has the correct sign to shift the Higgs quartic coupling upwards ($\lambda_H = \lambda + \delta\lambda$), although the stability condition is also shifted upwards by the same amount, becoming $\lambda_H > \delta\lambda$. However, for positive λ_{HS}, the condition $\lambda_H > \delta\lambda$ has to be satisfied only at scales of order M_S, while for larger scales it rapidly reduces to the conventional constraint $\lambda_H > 0$. Moreover, one-loop RG effects (although typically less important than the tree-level matching condition) also help to maintain stability. First, λ_S and λ_{HS} will stay positive once they are positive at M_S. Second, $\beta_{\lambda_H} \equiv d\lambda_H/d\ln\mu$ receives extra positive contributions proportional to λ_{HS}^2 and to λ_H^2 (coupling which is numerically larger after the threshold shift). These two RG effects can reduce (or even overcome) the destabilising effect from top loops.

To illustrate the situation, we show in Fig. 4.1, left panel, how the Higgs quartic coupling runs with the renormalization scale. We consider $M_S = 10^8$ GeV $\ll \Lambda_I^{\rm SM} = 2 \times 10^9$ GeV. [3] For simplicity we take the couplings of the singlet to be smaller than the SM top and gauge couplings, in order to better isolate the tree-level threshold effect. The same panel also shows the full stability condition, computed numerically by demanding that $V(H, 0) > 0$: we see that at renormalization scales just above M_S the stability condition of Eq. (4.10) matters, but at larger field values it rapidly becomes irrelevant and only $\lambda_H > 0$ remains.

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[3] Strictly speaking, this is the scale at which $\lambda = 0$, and corresponds to the instability scale of the tree-level RG-improved potential. The Λ_I that we calculate later on is higher and corresponds to the instability of the one-loop RG-improved potential. For simplicity, in Fig. 4.2 we simply plot $\lambda(\mu)$.

To study the efficiency of the stabilisation mechanism, we performed a numerical study using the full one-loop effective potential with SM couplings running at two-loops. We limited the evolution of the unknown singlet couplings to the one-loop level, given that the effect we are considering is at tree level. In order to track accurately the large field behaviour of the one-loop potential one can simply include in the running quartic couplings the finite one-loop contributions not captured by RG evolution and impose the stability conditions on these corrected couplings (the shift in the instability scale can be up to one order of magnitude; see Refs. [3, 42] for further details). The scale at which these one-loop improved running couplings violate the stability condition corresponds then to the field scale at which the potential falls below the EW vacuum. The results are illustrated in Fig. 4.2 which shows the new instability scale Λ_I as a function of the threshold shift $\delta\lambda$ for several singlet scalar masses, $M_S = 10^4, 10^6, 10^8, 10^{10}$ GeV below the SM instability scale, $\Lambda_I^{\mathrm{SM}} \simeq 4 \times 10^{10}$ GeV (for $m_h = 125$ GeV, $m_t = 173.2$ GeV and $\alpha_{\mathrm{s}}(m_Z) = 0.1183$). For each value of M_S, there is a band of values for Λ_I due to the freedom in choosing λ_S, once λ_H and $\delta\lambda = \lambda_{HS}^2/\lambda_S$ are fixed. The lower boundary of each band corresponds to $\lambda_S \ll 1$. This case nearly isolates the impact of the tree-level shift on the instability scale (as the running of λ_H above the singlet threshold is SM-like). The upper boundary of each band corresponds to the largest value of λ_S that we allow by requiring $\lambda_S(\mu) < 4\pi$ up to the Planck scale. Large values of λ_S correspond to large λ_{HS} (for a fixed δ), making the RG effect on λ_H stronger. We conclude that the tree-level shift in λ can have an extremely significant impact in raising the instability

FIGURE 4.2: *For $m_h = 125$ GeV and $\lambda_{HS} > 0$, bands of the modified instability scale Λ_I versus the threshold correction $\delta\lambda$ to the Higgs quartic coupling due to a scalar singlet with mass $M_S = 10^4, 10^6, 10^8, 10^{10}$ GeV (from left to right). For a fixed M_S value the lowest boundary of the band corresponds to small λ_S, λ_{HS} and the highest boundary to $\lambda_S(M_{\mathrm{Pl}}) = 4\pi$.*

scale even for very moderate values of the couplings λ_S and λ_{HS}, and it can easily make the EW vacuum absolutely stable.

4.2.2   Case $\lambda_{\mathbf{HS}} < 0$

In this case V_0 can become negative only for $|S| > w/\sqrt{2}$. In this condition, we can neglect the mass parameters v and w in Eq. (4.1) and approximate the potential by keeping only the quartic terms

$$V_0 \approx \lambda_H |H|^4 + \lambda_S |S|^4 + 2\lambda_{HS}|H|^2|S|^2 \ . \tag{4.12}$$

The stability condition ($V_0 > 0$) is now

$$-\lambda_{HS}(\mu) < \sqrt{\lambda_H(\mu)\lambda_S(\mu)} \ . \tag{4.13}$$

If this condition is violated at some scale μ_* an instability would develop with

$$|S| > \frac{M_S}{2\sqrt{\lambda_S}}, \quad c_- < \frac{|H|}{|S|} < c_+ \quad c_\pm^2 = \left.\frac{-\lambda_{HS}}{\lambda_H}\left(1 \pm \sqrt{1 - \frac{\lambda_H\lambda_S}{\lambda_{HS}^2}}\right)\right|_{\mu_*} . \tag{4.14}$$

As this determines a direction in field space along which the fields H and S slide towards an unbounded instability, condition of Eq. (4.13) has to be satisfied at all renormalization scales larger than M_S. Thus the stability condition for negative λ_{HS} is much more constraining than in the case of positive λ_{HS}.

In the case $\lambda_{HS} < 0$, as the stability condition $\lambda_H > \delta\lambda$ must be satisfied at all scales, the tree-level threshold effect is not sufficient to improve the stability. Then one should resort to RG effects to improve the potential stability, as illustrated in Fig. 4.1, right panel. By using the RG equations of Eq. (4.3), we can derive the evolution of the effective Higgs quartic coupling combination $\lambda \equiv \lambda_H - \lambda_{HS}^2/\lambda_S$ above M_S as

$$\frac{d\lambda}{d\ln\mu} = \beta_\lambda^{\rm SM} + \frac{8}{(4\pi)^2}\left[(\lambda_{HS} - \delta\lambda)^2 + 3\lambda\,\delta\lambda\right] \ , \tag{4.15}$$

where $\beta_\lambda^{\rm SM}$ is the SM beta function for the Higgs quartic coupling and $\delta\lambda = \lambda_{HS}^2/\lambda_S > 0$. We see that the additional term in the beta function of λ is always positive so that RG effects tend to increase the instability scale also in the case $\lambda_{HS} < 0$.

The numerical analysis of the $\lambda_{HS} < 0$ case confirms this expectation. As an illustration, Fig. 4.3 shows the instability scale versus the shift $\delta\lambda$ for the same choice of SM parameters as in Fig. 4.2 and for the particular case $M_S = 10^8$ GeV with three different

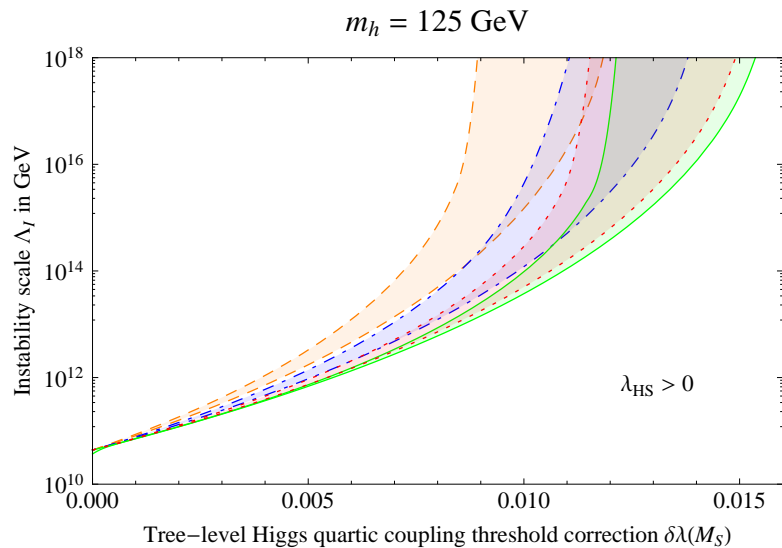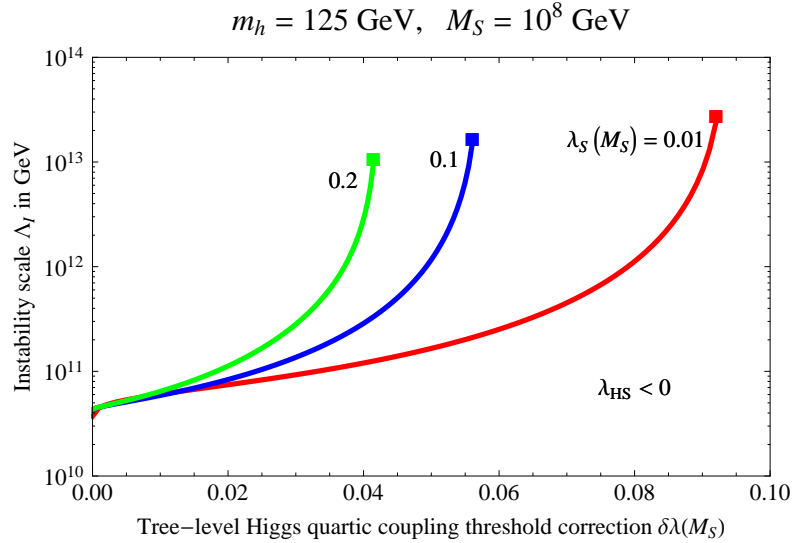$$m_h = 125 \text{ GeV}, \quad M_S = 10^8 \text{ GeV}$$

FIGURE 4.3: *For $m_h = 125$ GeV and $\lambda_{HS} < 0$, the modified instability scale Λ_I versus the threshold correction $\delta\lambda$ to the Higgs quartic coupling due to a scalar singlet with mass $M_S = 10^8$ GeV and $\lambda_S(M_S) = 0.01, 0.1, 0.2$, as indicated.*

values of λ_S as indicated. The end-point of the curves marks the location beyond which (i.e. for larger $\delta\lambda$) the potential becomes completely stable. These end-points occur because λ first decreases as a function of the renormalization scale but, after reaching a minimum, starts increasing at large scales. In comparison with the case $\lambda_{HS} > 0$ (Fig. 4.2) we see that larger values of the shift $\delta\lambda$ are now required to have a significant impact on the instability scale.

The stabilisation mechanism for $\lambda_{HS} < 0$ we have just described is fragile with respect to possible new contributions to the RGEs that can appear if the singlet couples to other sectors of the theory. In contrast, the stabilisation mechanism for $\lambda_{HS} > 0$ is more robust, being based on a tree-level shift. The mechanism is also very effective (because the tree level correction can be easily large) and economical (because it requires only a heavy scalar singlet). The proposed mechanism can be realised in several situations of physical interest [2]. We now discuss two examples were the singlet is required to solve another problem; it gives mass to the right handed See-saw neutrinos but at the same time stabilises the vacuum by the threshold effect. In Re. [2] one more example is given where the singlet is used to unitarize Higgs inflation.

4.3   See-saw

The see-saw is the simplest mechanism to understand the smallness of neutrino masses. It assumes the existence of heavy right-handed neutrino states N (family index suppressed) with

$$\mathcal{L}_N = i\bar{N}\gamma^\mu\partial_\mu N + y_\nu LNH + \frac{M_N}{2}N^2 + h.c. \; . \tag{4.16}$$

After EW symmetry breaking, nonzero neutrino masses are generated

$$m_\nu = \frac{y_\nu^2 v^2}{M_N} \; , \tag{4.17}$$

which are naturally small provided $M_N \gg v$.

The impact of the see-saw mechanism on the stability of the Higgs potential has been discussed in the past [3, 85, 86]. The right-handed neutrino Yukawa couplings can play a x role on β_λ similar to that of the top Yukawa coupling. As they scale like $y_\nu^2 \sim m_\nu M_N$, they become sizable for large M_N and are dangerous for stability only if $M_N \approx 10^{13}$ GeV. For lower M_N the new Yukawas will have a negligible effect on stability.

We do not know what originates the large right-handed Majorana mass, but the simplest idea is to assume that the right-handed neutrinos are coupled to scalar fields carrying two units of lepton number and having a large vev,

$$\frac{\kappa}{2} \, S \, N^2 + \text{h.c.} \tag{4.18}$$

The vev of S, which sets the scale of the Majorana mass, $M_N = \kappa\langle S\rangle$, does not necessarily lead to a Goldstone boson because in unified models $B - L$ is usually a gauge symmetry. In this well-motivated realization of the see-saw the scalar field S could naturally reestablish stability of the electroweak vacuum. In this setting the role of the singlet scalar is therefore double. Upon taking a large vev and decoupling, it leaves behind two effects: a Weinberg dimension-5 operator that gives neutrinos a nonzero mass, and a threshold effect on the Higgs Yukawa coupling that solves the stability problem of the Higgs potential, as long as the mass of S is smaller than Λ_I.

A lower bound on the lightest right-handed neutrino mass M_1 is derived by assuming that the cosmic baryon asymmetry is explained by thermal leptogenesis.[4] In this case, one obtains the bounds [87, 88]:

- $M_1 > 2 \times 10^9$ GeV, if the initial right-handed neutrino density vanishes at high temperature.

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[4]If neutrinos are nearly degenerate in mass, thermal leptogenesis could operate at much smaller values of M_1 and the following lower bounds do not apply.
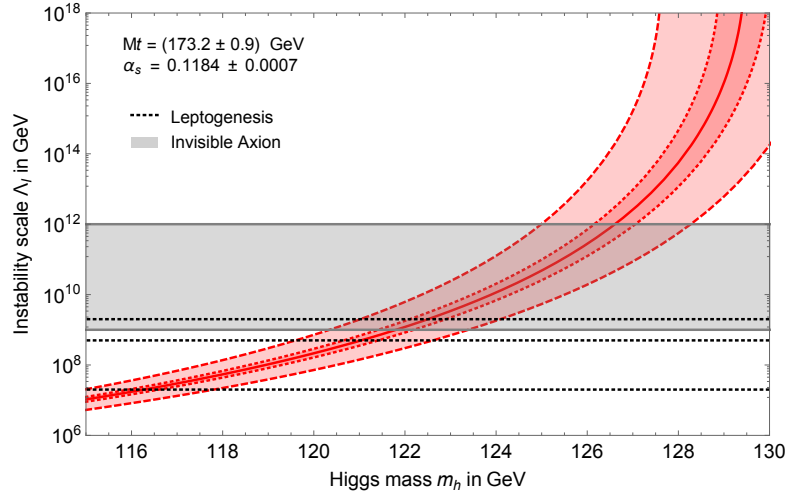
FIGURE 4.4: *The SM instability scale Λ_I increasing as a function of the Higgs mass. The central line corresponds to $m_t = 173.2$ GeV and $\alpha_s(m_Z) = 0.1184$ and the side-bands to 1 sigma deviations as indicated (with the larger deviation for the top mass uncertainty). The horizontal lines mark several values of interest for Λ_I. The three lines are relevant for the see-saw and correspond to lower limits on the mass M_1 of the lightest right-handed neutrino N_1 coming from thermal leptogenesis. The bound depends on the initial density ρ_{N_1}: $M_1 > 2 \times 10^7$ GeV for $\rho_{N_1} \sim 0$; $M_1 > 5 \times 10^8$ GeV for thermal ρ_{N_1} and $M_1 > 2 \times 10^9$ GeV for ρ_{N_1} dominating the universe.*

- $M_1 > 5 \times 10^8$ GeV, if the initial right-handed neutrino density is thermal at high temperature.

- $M_1 > 2 \times 10^7$ GeV, if the initial right-handed neutrino density dominates the universe at high temperature.

Assuming that the mass of S is equal or smaller than its vev, we can infer the range of Higgs masses for which the scalar setting the see-saw scale could cure any instability of the potential. From Fig. 4.4, which gives the SM instability scale as a function of m_h as calculated in Chapter 3, we can easily read off such Higgs masses.[5] At 90% CL in m_t and α_s, we find that the see-saw singlet can potentially eliminate the instability of the EW vacuum, as long as $m_h > 120$ GeV (leptogenesis with vanishing initial right-handed neutrino density), $m_h > 119$ GeV (leptogenesis with thermal initial right-handed neutrino density), or $m_h > 115$ GeV (leptogenesis with dominant initial right-handed neutrino density). These limits are compatible with the Higgs mass of 125 GeV measured by ATLAS and CMS. The instability scale is raised according to the mechanism discussed in the previous Section as long as $M_N \lesssim 10^{13}$ GeV, since the RG effects of y_ν couplings can be neglected.

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[5]Besides the 1-sigma error bands shown, associated with the experimental uncertainties in m_t and α_s, a (conservative) estimate of the higher order radiative corrections not included in the calculation results in a theoretical uncertainty on m_h of ± 1 GeV, see Tab. 3.2.

We conclude that this simple scenario could comfortably account for the cosmological baryon asymmetry through leptogenesis, for the smallness of neutrino masses and cure the Higgs potential instability. The only drawback of this (beautifully simple but depressing) scenario is that it makes plainly explicit the hierarchy problem: a large singlet vev also gives a tree-level contribution to the Higgs mass term in the Lagrangian which requires a large fine-tuning. (This is in contrast with the scenario without the singlet, in which Higgs mass corrections appear at one-loop and are dangerous only when $M_N > 10^7$ GeV [89, 90]).

4.4   Invisible axion

The scalar field S can also be identified with the invisible axion.

DFSZ axion models [91, 92] use the SM fermion content and a two-Higgs doublet structure H_u and H_d augmented by a complex scalar S, neutral under SM gauge interactions, with a coupling $\lambda_{HS} S^2 H_u H_d + \text{h.c.}$, analogous to the one in Eq. (4.1). This interaction is crucial for the axion mechanism, because it transmits the breaking of the global symmetry triggered by the vev of S to the Higgs sector. One or both of the Higgs doublets can remain light, at the electroweak scale. The presence of an instability is subject to the details of the two-Higgs potential [93], but this does not change the essential point. Independently of the model implementation, the field S containing the invisible axion $a = \sqrt{2} \text{Im} \, S$ with large decay constant $f_a \approx \langle S \rangle$ is a perfect candidate to play the role of the field S in our Higgs stabilisation mechanism.

KSVZ axion models [94, 95] use a single Higgs doublet and a complex scalar S coupled to new heavy vector-like fermions Ψ. The Dirac mass term $M \bar{\Psi} \Psi$ is forbidden by imposing the symmetry

$$\Psi_L \to -\Psi_L, \qquad \Psi_R \to \Psi_R, \qquad S \to -S. \tag{4.19}$$

Then the mass of the heavy fermions comes only from the vev of S:

$$\lambda_\Psi \; S \bar{\Psi} \Psi + V(H, S). \tag{4.20}$$

The resulting model has a spontaneously broken U(1) global symmetry

$$\Psi \to e^{i \gamma_5 \alpha} \Psi, \qquad S \to e^{-2i\alpha} S \tag{4.21}$$

which gives rise to a light axion $a = \sqrt{2} \text{Im} \, S$ with large decay constant $f_a \approx \langle S \rangle$. The scalar potential of the theory is precisely of the form in Eq. (4.1), although the coupling

λ_{HS} plays no role in axion phenomenology because both $|S|^2$ and $|H|^2$ are separately invariant under the global symmetry.

The decay constant of the axion is allowed to lie in the range

$$10^9 \,\mathrm{GeV} < f_a < 10^{12} \,\mathrm{GeV} \ . \tag{4.22}$$

The lower bound comes from non-observation of axion emission from stars and supernovæ. The upper bound comes from requiring that the axion dark matter density

$$\Omega_a \approx 0.15 \left(\frac{f_a}{10^{12} \,\mathrm{GeV}} \right)^{7/6} \left(\frac{a_*}{f_a} \right)^2 \tag{4.23}$$

does not exceed the observed value $\Omega_{\mathrm{DM}} \approx 0.23$ under the assumption that the axion vev a_* in the early universe was of the order of f_a [96–98]. The resulting range of singlet mass M_S, which we can roughly take to be the same as the range for f_a in Eq. (4.22), overlaps with the range that can stabilise the SM Higgs potential from $m_h \gtrsim 119$ GeV (for $M_S \approx 10^9$ GeV) to $m_h \gtrsim 124$ GeV (for $M_S \approx 10^{12}$ GeV), as can be inferred from Fig. 4.4.

4.5 Summary

We have presented a stabilisation mechanism of the electroweak vacuum that relies on a threshold correction to the Higgs quartic coupling, whose size is independent of the singlet mass. The necessary ingredients are a singlet self-quartic coupling (λ_S), a mixed quartic coupling with the Higgs (λ_{HS}) and a non-zero vev for the singlet. The mechanism can be operative even for a very heavy singlet, as long as its mass is smaller than the instability scale. Occurring at tree level, the effect is sizeable and robust. The analysis of the effect involves some subtleties because, at the singlet threshold, both the Higgs quartic coupling and the stability conditions are shifted by the same amount. We have shown that, when λ_{HS} is positive, the stability conditions become weaker as the field value is increased above the singlet mass. In this situation, the tree-level contribution is very effective in stabilising the potential. On the other hand, for negative λ_{HS}, the shifts in the Higgs quartic coupling and in the stability condition essentially cancel out, and one has to rely on RG effects. These can help the stabilisation, but larger singlet couplings are needed to obtain the desired effect. The minimal modification of the SM that we have considered, with the addition of one singlet scalar, has motivations that are independent of the stability of the EW vacuum. The new singlet can set the scale of the right-handed neutrino mass in the sea-saw mechanism; or it could play the role of the invisible axion. In both cases, we were able to define the range of Higgs masses for

which the corresponding singlet could also be used to stabilise the SM Higgs potential.
We find that the stabilisation mechanism can be operative in both models for the Higgs
mass value measured by ATLAS and CMS.

Part II

Effective Field Theory of the physics at the electroweak scale

5. Introduction

Quantum Effective Field Theories (EFTs) play a central role in the understanding of physical systems in theoretical physics. Its most prominent applications range from physics of the very early universe to particle physics and condensed matter. The combination of the EFT approach together with the Renormalization Group (RG) permits to focus on the relevant energy scales of the aforementioned physical systems.

Regarding particle physics we are at a very special situation that we have not faced before. The discovery of the Higgs boson at the LHC [9, 10] completes a theoretical puzzle, namely the electroweak interactions. Now we have a theory, the Standard Model (SM), that makes sense up to exponentially higher energies, see Part I. Apart form the fact that in any proposed generalisation of the SM the Higgs mass tends to be of the same order as the physical energy scale of the SM completion, all the other SM conundrums like charge quantisation, unification, flavour or the strong CP problem can be solved at exponentially higher energies or with very weakly coupled physics, see Sec. 1.2.

The null results of the Large Hadron Collider (LHC) in Beyond the SM (BSM) searches together with the fact that the Higgs properties show good agreement with the SM expectations (see Chapter 1) suggest that there is a certain energy gap between the Higgs mass an the new physics scale. In this situation, EFTs provide a practical way to organise the impact of possible new physics. In this respect one can take two extreme point of views in the use of the EFT. On one extreme one can simply take into account all the higher dimensional operators with order $\mathcal{O}(1)$ Wilson coefficients, treat them on equal footing and inspect its consequences. This might not be very informative due to the plethora of higher dimensional operators, even at the dimension-six level. On the other extreme, one can take a singular model and integrate it out, and again inspect its phenomenological consequences. Between these two extremes, there are simple assumptions that can be taken about the BSM theory that allows to weight the importance of the different higher dimension operators and in this way one can study and learn about

the consequences of different classes of models altogether.[1] As we have stressed in the Preface and in Chapter 1, it is very important that, in the upcoming years, we make sure that we understand and measure as much as we can the properties of the Higgs boson. The EFT analysis sheds light to this enterprise as it allows to systematically study possible deviations from the SM expectations by organising the energy scales and possible symmetries or dynamics of the ultraviolet physics.

In Part II of the thesis we embark on the SM EFT study of possible new physics. In Chapter 6 we perform a complete study of the impact of the (dominant) dimension-six operators in the most important Higgs couplings. In particular, we calculate the corrections to single Higgs couplings, relevant for the main Higgs decays and production mechanisms. We will show that, for one family, there are 8 CP-even operators that can only affect Higgs physics and no other SM processes (at tree-level). This corresponds to the number of independent dimension-six operators that can be constructed with $|H|^2$, and implies that Higgs couplings to fermions, photons, gluons, and $Z\gamma$ (for which large corrections are still possible) are characterised by independent Wilson coefficients. The rest of operators that could in principle affect Higgs physics at tree-level also enter in other SM processes and therefore can be constrained by independent (non-Higgs) experiments; we will present the main experimental constraints on these operators.

In the following Chapters 7-9 we go one step further and study several relevant effects that arise due to the operator mixing of the dimenison-six operators through quantum effects. This is very interesting for a number of reasons. In particular we are specially interested in looking for instances of

- possible big deviation from the SM are expected if the RG running of the Wilson coefficients is there (e.g. big contributions to $b \to s\gamma$),

- explicit breaking of assumed symmetries of the BSM degrees of freedom due to the SM running of the dimension-six operators (e.g. contributions to the T-parameter),

- poorly measured or unknown Wilson's coefficients that radiatively generate operators that are precisely measured (e.g. $h \to \gamma Z$ v.s. neutron electric dipole moment or electroweak precision test observables),

- SM processes that are suppressed (they are loop-level interactions or involve small SM couplings) and receive big contributions from higher dimensional operators

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[1]In order to illustrate this point let us mention some examples of assumptions or parametrizations than can be imposed: accidental or global symmetries (e.g. custodial, flavour, R-parity); generic assumptions about the dynamics (operators generator at loop-level v.s. operators generated at tree level); or more refined assumptions about the dynamics and power counting such as in the *SILH* parametrization [99]; one can also asume generic SM↔BSM couplings as (e.g. portals or universally coupled new physics), etc. We will encounter several examples in the next Chapters.

(e.g. possible contributions of tree-level generated operators to loop-level processes like dipoles and certain triple gauge vertices).

In fact, by inspecting RG mixing effects we are further exploiting a key feature of the higher dimensional operators: they connect different kinds of physics that are otherwise not directly connected in the $\Lambda \to \infty$ limit.

We start the study of the RG mixing effects in Chapter 7, where we look for possible connections between the $h \to \gamma + \gamma/Z$ decay and the S-parameter. In addition, in Chapter 7 we compute the contributions of the SM dipoles to the decay $h \to \gamma + \gamma/Z$ and present a digression on the choice of the operator basis. We remark that certain bases facilitate particular physical interpretations, a common fact in physics whenever coordinates are taken.

Then, in Chapter 8 we continue the exploration of the most relevant quantum effects to the relevant Higgs couplings. We apply these results to find the leading-log corrections to the predictions for Higgs-couplings in various BSM scenarios: the Minimal Supersymmetric Standard Model (MSSM), universal theories (such as composite-Higgs models) and models with a non-standard top. We find that the deviations can be as big as 10-20%.

Lastly, in Chapter 9 we focus on universal theories and study the RG mixing and interplay of various Higgs and Electroweak observables. The main result of that Chapter is the anomalous dimension matrix of the 10 bosonic operators (i.e. operators made out of boson fields) related to EW and Higgs observables. Then, we use the RG equations to set bounds on the value of some Wilson coefficients that are otherwise less constrained by direct measurements. We also comment on future prospects, present the anomalous dimension matrix for a set of operators with gluon fields and discuss the available bounds on them.

6. SM Effective Field Theory

6.1 Dimension-six operator basis

Let us consider a BSM sector characterised by a new mass-scale Λ much larger than the electroweak scale M_W. [1] We will assume, among other requirements to be specified later, that this sector preserves lepton and baryon number. By integrating out this sector and performing an expansion of SM fields and their derivatives D_μ over Λ, we obtain an effective Lagrangian made of local operators:

$$\mathcal{L}_{\text{eff}} = \frac{\Lambda^4}{g_*^2} \mathcal{L} \left(\frac{D_\mu}{\Lambda} \ , \ \frac{g_H H}{\Lambda} \ , \ \frac{g_{f_{L,R}} f_{L,R}}{\Lambda^{3/2}} \ , \ \frac{g F_{\mu\nu}}{\Lambda^2} \right) \simeq \mathcal{L}_4 + \mathcal{L}_6 + \cdots , \qquad (6.1)$$

where $\mathcal{L}_d$ denotes the term in the expansion made of operators of dimension d. By g_* we denote a generic coupling, while g_H and $g_{f_{L,R}}$ are respectively accounting for the couplings of the Higgs-doublet H and SM fermions $f_{L,R}$ to the BSM sector, and g and $F_{\mu\nu}$ represent respectively the SM gauge couplings and field-strengths. [2] The Lagrangian in Eq. (6.1) is based on dimensional analysis and the dependence on the couplings is easily obtained when the Planck constant $\hbar$ is put back in place. All couplings introduced in Eq. (6.1) can be useful as bookkeeping parameters. In particular, a term in the Lagrangian that contains n fields, will carry some coupling to the power $n - 2$ (in this counting, λ, the Higgs quartic-coupling, is formally of order g_*^2).

The dominant effects of the BSM sector are encoded in $\mathcal{L}_6$. There are different bases used in the literature for the set of independent $d = 6$ operators in $\mathcal{L}_6$. Although physics is independent of the choice of basis, it is clear that some bases are better suited than others in order to extract the relevant information *e.g.* for Higgs physics. A convenient feature to ask of a good basis is that it captures in few operators the impact of different new-physics scenarios, at least for the most interesting cases. For example, in universal

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[1] In fact $\Lambda \approx 300$ GeV is sufficient in certain cases because it correspond to an expansion parameter of $M_W^2 / \Lambda^2 \approx 0.1$.

[2] With this we are assuming that the SM gauge symmetry is also realized at energies above Λ and therefore the couplings of the gauge bosons to the BSM sector are the SM gauge couplings. We can relax this assumption by replacing g by an arbitrary coupling.

theories, defined as those BSM scenarios whose corrections can be encoded in operators made only of SM bosons, the bases used in Refs. [99, 100] are appropriate since the physics effects can be captured by just 14 CP-even $d = 6$ operators. Therefore, 14 is the number of independent parameters of the new physics effects and this number must be the same in all bases. However, the list of operators required to describe this same physics can contain many more than 14 operators in other bases, as for example in that of Ref. [101]. It follows that if we use such alternative bases to study universal theories there will be correlations among operator coefficients, making the analysis more cumbersome.

Another important consideration for the choice of basis is to separate operators whose coefficients are expected to have different sizes (again, at least in the main theories of interest). For example, it is convenient to keep separated the operators that can be induced at tree-level from integrating weakly-coupled states from those that can only be generated at the one-loop level. This helps in determining the most relevant operators when dealing with a large class of BSM scenarios such as supersymmetric, composite Higgs or little Higgs models among others. As shown in Chapter 7, this criterium is also useful when considering one-loop operator mixing, since one finds that tree-level induced operators often do not contribute to the RGE flow of one-loop induced ones, independently, of course, of the UV origin of the operators. In this particular sense, the basis of Ref. [5] is better suited than that of Ref. [100]. It is obvious that to meet all the criteria given above we do not need to sacrifice generality, as long as one keeps a complete basis of operators, as we do.

The operators of our basis will be broadly classified in three classes [5, 99]. The first two classes will consist of operators that could in principle be generated at tree-level when integrating out heavy states with spin ≤ 1 in renormalizable weakly-interacting theories. As we show in Appendix A, these operators can be written as products of scalar, fermion or vector currents of dimension less than 3. [3] Among these current-current operators we call operators of the first class those that involve extra powers of Higgs fields or SM fermions. They will be proportional to some power of the couplings g_H or $g_{f_{L,R}}$, respectively. The importance of the operators of the first class is that they can be the most sizeable ones when the theory is close to the strong-coupling limit, $g_H, g_{f_{L,R}} \sim 4\pi$. Operators of the second class are instead those that involve extra (covariant) derivatives or gauge-field strengths and, according to Eq. (6.1), are generically suppressed by $1/\Lambda^2$ times a certain power of gauge couplings. Finally, in the third class, we will have operators that cannot be generated from a tree-level exchange of

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[3]This, together with the fact that field-redefinitions through equations of motion do not mix the two types of operators, makes the classification well defined and unambiguous.

heavy fields and can only be induced, in renormalizable weakly-coupled theories, at the one-loop level. In this case, we expect these operators to be suppressed by $g_*^2/(16\pi^2\Lambda^2)$.

We then classify the $d = 6$ operators as

$$\mathcal{L}_6 = \sum_{i_1} g_*^2 \frac{c_{i_1}}{\Lambda^2} \mathcal{O}_{i_1} + \sum_{i_2} \frac{c_{i_2}}{\Lambda^2} \mathcal{O}_{i_2} + \sum_{i_3} \frac{\kappa_{i_3}}{\Lambda^2} \mathcal{O}_{i_3} \, , \tag{6.2}$$

where, for notational convenience, we introduce the one-loop suppressed coefficients

$$\kappa_{i_3} \equiv \frac{g_*^2}{16\pi^2} c_{i_3} \, , \tag{6.3}$$

for the third class of operators. In weakly-coupled theories, $c_i \sim f_i(g/g_*, g_H/g_*, ...)$, where $f_i(g/g_*, g_H/g_*, ...)$ are functions that depend on ratios of couplings. We refer to the operators $\mathcal{O}_{i_1}$ and $\mathcal{O}_{i_2}$ as "current-current" or "tree-level" operators, while we call $\mathcal{O}_{i_3}$ "one-loop" operators.[4]

Let us start defining our basis by considering first operators made of SM bosons only [99]. In the first class of operators, $\mathcal{O}_{i_1}$, we have

$$\mathcal{O}_H = \frac{1}{2}(\partial^\mu|H|^2)^2 \, , \quad \mathcal{O}_T = \frac{1}{2}(H^\dagger \overset{\leftrightarrow}{D}_\mu H)^2 \, , \quad \mathcal{O}_r = |H|^2|D_\mu H|^2 \, , \quad \mathcal{O}_6 = \lambda|H|^6 . \tag{6.4}$$

Here we have defined $H^\dagger \overset{\leftrightarrow}{D}_\mu H \equiv H^\dagger D_\mu H - (D_\mu H)^\dagger H$, with $D_\mu H = \partial_\mu H - ig\sigma^a W_\mu^a H/2 - ig'B_\mu H/2$ (H is taken to have hypercharge $Y_H = 1/2$). For $\mathcal{O}_6$, which involves six Higgs fields, an extra factor g_*^2 could be present. Nevertheless, we have substituted this by λ, the Higgs self-coupling defined as $V = -m^2|H|^2 + \lambda|H|^4$. This is motivated by the fact that the lightness of the Higgs suggests that there is a symmetry protecting the Higgs self-coupling to be of order $\lambda \sim m_h^2/(2v^2) \sim 0.13$. Examples are supersymmetry or global symmetries as in composite Higgs models.

In the second class of operators, $\mathcal{O}_{i_2}$, we have

$$\mathcal{O}_W = \frac{ig}{2}(H^\dagger\sigma^a \overset{\leftrightarrow}{D^\mu} H)D^\nu W_{\mu\nu}^a \, , \quad \mathcal{O}_B = \frac{ig'}{2}(H^\dagger \overset{\leftrightarrow}{D^\mu} H)\partial^\nu B_{\mu\nu} \, ,$$
$$\mathcal{O}_{2W} = -\frac{1}{2}(D^\mu W_{\mu\nu}^a)^2 \, , \quad \mathcal{O}_{2B} = -\frac{1}{2}(\partial^\mu B_{\mu\nu})^2 \, , \quad \mathcal{O}_{2G} = -\frac{1}{2}(D^\mu G_{\mu\nu}^A)^2 . \tag{6.5}$$

Since the last three operators involve two field strengths, we expect $c_{2W} \sim g^2/g_*^2$, $c_{2B} \sim g'^2/g_*^2$, and $c_{2G} \sim g_s^2/g_*^2$.

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[4]For a classification of operators similar in spirit to ours, see Ref. [102].

In the third class of operators, $\mathcal{O}_{i_3}$, we have the CP-even operators

$$\mathcal{O}_{BB} = g'^2|H|^2 B_{\mu\nu}B^{\mu\nu} \quad , \quad \mathcal{O}_{GG} = g_s^2|H|^2 G_{\mu\nu}^A G^{A\mu\nu} \,, \tag{6.6}$$

$$\mathcal{O}_{HW} = ig(D^\mu H)^\dagger \sigma^a (D^\nu H) W_{\mu\nu}^a \quad , \quad \mathcal{O}_{HB} = ig'(D^\mu H)^\dagger (D^\nu H) B_{\mu\nu} \,, \tag{6.7}$$

$$\mathcal{O}_{3W} = \frac{1}{3!} g\epsilon_{abc} W_\mu^{a\,\nu} W_{\nu\rho}^b W^{c\,\rho\mu} \quad , \quad \mathcal{O}_{3G} = \frac{1}{3!} g_s f_{ABC} G_\mu^{A\,\nu} G_{\nu\rho}^B G^{C\,\rho\mu} \,, \tag{6.8}$$

and the CP-odd operators

$$\mathcal{O}_{B\widetilde{B}} = g'^2|H|^2 B_{\mu\nu}\widetilde{B}^{\mu\nu} \quad , \quad \mathcal{O}_{G\widetilde{G}} = g_s^2|H|^2 G_{\mu\nu}^A \widetilde{G}^{A\mu\nu} \,, \tag{6.9}$$

$$\mathcal{O}_{H\widetilde{W}} = ig(D^\mu H)^\dagger \sigma^a (D^\nu H) \widetilde{W}_{\mu\nu}^a \quad , \quad \mathcal{O}_{H\widetilde{B}} = ig'(D^\mu H)^\dagger (D^\nu H) \widetilde{B}_{\mu\nu} \,, \tag{6.10}$$

$$\mathcal{O}_{3\widetilde{W}} = \frac{1}{3!} g\epsilon_{abc} \widetilde{W}_\mu^{a\,\nu} W_{\nu\rho}^b W^{c\,\rho\mu} \quad , \quad \mathcal{O}_{3\widetilde{G}} = \frac{1}{3!} g_s f_{ABC} \widetilde{G}_\mu^{A\,\nu} G_{\nu\rho}^B G^{C\,\rho\mu} \,, \tag{6.11}$$

where $\widetilde{F}^{\mu\nu} = \epsilon^{\mu\nu\rho\sigma} F_{\rho\sigma}/2$. There are two more CP-even operators involving two Higgs fields and gauge bosons, $\mathcal{O}_{WB} = g'g H^\dagger \sigma^a H W_{\mu\nu}^a B^{\mu\nu}$ and $\mathcal{O}_{WW} = g^2|H|^2 W_{\mu\nu}^a W^{\mu\nu\,a}$ (and

SM CP-even operators made of bosons
$\mathcal{O}_H = \frac{1}{2}(\partial^\mu|H|^2)^2$
$\mathcal{O}_T = \frac{1}{2}\left(H^\dagger \overset{\leftrightarrow}{D}_\mu H\right)^2$
$\mathcal{O}_6 = \lambda|H|^6$
$\mathcal{O}_W = \frac{ig}{2}\left(H^\dagger \sigma^a \overset{\leftrightarrow}{D^\mu} H\right) D^\nu W_{\mu\nu}^a$
$\mathcal{O}_B = \frac{ig'}{2}\left(H^\dagger \overset{\leftrightarrow}{D^\mu} H\right) \partial^\nu B_{\mu\nu}$
$\mathcal{O}_{2W} = -\frac{1}{2}(D^\mu W_{\mu\nu}^a)^2$
$\mathcal{O}_{2B} = -\frac{1}{2}(\partial^\mu B_{\mu\nu})^2$
$\mathcal{O}_{2G} = -\frac{1}{2}(D^\mu G_{\mu\nu}^A)^2$
$\mathcal{O}_{BB} = g'^2|H|^2 B_{\mu\nu}B^{\mu\nu}$
$\mathcal{O}_{GG} = g_s^2|H|^2 G_{\mu\nu}^A G^{A\mu\nu}$
$\mathcal{O}_{HW} = ig(D^\mu H)^\dagger \sigma^a (D^\nu H) W_{\mu\nu}^a$
$\mathcal{O}_{HB} = ig'(D^\mu H)^\dagger (D^\nu H) B_{\mu\nu}$
$\mathcal{O}_{3W} = \frac{1}{3!} g\epsilon_{abc} W_\mu^{a\,\nu} W_{\nu\rho}^b W^{c\,\rho\mu}$
$\mathcal{O}_{3G} = \frac{1}{3!} g_s f_{ABC} G_\mu^{A\,\nu} G_{\nu\rho}^B G^{C\,\rho\mu}$

TABLE 6.1: *The operators are grouped in 3 different groups (separated with a solid line) corresponding to the 3 classes of operators defined in Eq. (6.2). Dashed lines separate operators of different structure within a given class. There are, in addition, the 6 CP-odd operators given in Eqs. (6.9)-(6.11).*

the equivalent CP-odd ones), but these can be eliminated using the identities [5]

$$\mathcal{O}_B = \mathcal{O}_{HB} + \frac{1}{4}\mathcal{O}_{BB} + \frac{1}{4}\mathcal{O}_{WB}\,, \qquad (6.12)$$

$$\mathcal{O}_W = \mathcal{O}_{HW} + \frac{1}{4}\mathcal{O}_{WW} + \frac{1}{4}\mathcal{O}_{WB}\,. \qquad (6.13)$$

The operators $\mathcal{O}_{3W}$ and $\mathcal{O}_{3G}$ (and the corresponding CP-odd ones) have three field-strengths and then their corresponding coefficients should scale as $c_{3W} \sim g^2/g_*^2$ and $c_{3G} \sim g_s^2/g_*^2$ respectively.

Let us now examine $d = 6$ operators involving SM fermions, considering a single family to begin with. Operators of the first class involving the up-type quark are

$$\begin{aligned}
\mathcal{O}_{y_u} &= y_u|H|^2\bar{Q}_L\widetilde{H}u_R\,, \\
\mathcal{O}_R^u &= (iH^\dagger\overset{\leftrightarrow}{D}_\mu H)(\bar{u}_R\gamma^\mu u_R)\,, \\
\mathcal{O}_L^q &= (iH^\dagger\overset{\leftrightarrow}{D}_\mu H)(\bar{Q}_L\gamma^\mu Q_L)\,, \\
\mathcal{O}_L^{(3)\,q} &= (iH^\dagger\sigma^a\overset{\leftrightarrow}{D}_\mu H)(\bar{Q}_L\gamma^\mu\sigma^a Q_L)\,,
\end{aligned} \qquad (6.14)$$

where $\widetilde{H} = i\sigma_2 H^*$, and in operators $\propto \bar{Q}_L u_R$ we include a Yukawa coupling y_u ($m_u = y_u v/\sqrt{2}$) as an order parameter of the chirality-flip. We also understand, here and in the following, that when needed the Hermitian conjugate of a given operator is included in the analysis. In the first class we have, in addition, the four-fermion operators:

$$\begin{aligned}
\mathcal{O}_{LL}^q &= (\bar{Q}_L\gamma^\mu Q_L)(\bar{Q}_L\gamma^\mu Q_L)\,, & \mathcal{O}_{LL}^{(8)\,q} &= (\bar{Q}_L\gamma^\mu T^A Q_L)(\bar{Q}_L\gamma^\mu T^A Q_L)\,, \\
\mathcal{O}_{LR}^u &= (\bar{Q}_L\gamma^\mu Q_L)(\bar{u}_R\gamma^\mu u_R)\,, & \mathcal{O}_{LR}^{(8)\,u} &= (\bar{Q}_L\gamma^\mu T^A Q_L)(\bar{u}_R\gamma^\mu T^A u_R)\,, \\
\mathcal{O}_{RR}^u &= (\bar{u}_R\gamma^\mu u_R)(\bar{u}_R\gamma^\mu u_R)\,,
\end{aligned} \qquad (6.15)$$

where T^A are the $SU(3)_c$ generators. Other four-fermion operators are linear combinations of the ones appearing in Eq. (6.15); see for example Refs. [101, 103]. Finally, the one-loop (dipole) operators involving the up-type quark are

$$\begin{aligned}
\mathcal{O}_{DB}^u &= y_u\bar{Q}_L\sigma^{\mu\nu}u_R\,\widetilde{H}g'B_{\mu\nu}\,, \\
\mathcal{O}_{DW}^u &= y_u\bar{Q}_L\sigma^{\mu\nu}u_R\,\sigma^a\widetilde{H}gW_{\mu\nu}^a\,, \\
\mathcal{O}_{DG}^u &= y_u\bar{Q}_L\sigma^{\mu\nu}T^A u_R\,\widetilde{H}g_s G_{\mu\nu}^A\,.
\end{aligned} \qquad (6.16)$$

Similar operators to those given above can be written for the down-type quarks and leptons. For one family of fermions these are given in Tab. 6.2. Among them, there is a new type of operators, involving two different types of fermions, which, as we will see,

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[5]For CP-odd operators the identities are $4\mathcal{O}_{H\widetilde{B}} + \mathcal{O}_{B\widetilde{B}} + \mathcal{O}_{W\widetilde{B}} = 0$ and $4\mathcal{O}_{H\widetilde{W}} + \mathcal{O}_{W\widetilde{W}} + \mathcal{O}_{W\widetilde{B}} = 0$.

can have an important impact on Higgs physics at the one-loop level. These are

$$\mathcal{O}_R^{ud} = y_u^\dagger y_d (i\widetilde{H}^\dagger \overset{\leftrightarrow}{D}_\mu H)(\bar{u}_R \gamma^\mu d_R) \,, \tag{6.17}$$

and

$$
\begin{aligned}
\mathcal{O}_{y_u y_d} &= y_u y_d (\bar{Q}_L^r u_R)\epsilon_{rs}(\bar{Q}_L^s d_R) \,, &\quad \mathcal{O}_{y_u y_d}^{(8)} &= y_u y_d (\bar{Q}_L^r T^A u_R)\epsilon_{rs}(\bar{Q}_L^s T^A d_R) \,, \\
\mathcal{O}_{y_u y_e} &= y_u y_e (\bar{Q}_L^r u_R)\epsilon_{rs}(\bar{L}_L^s e_R) \,, &\quad \mathcal{O}'_{y_u y_e} &= y_u y_e (\bar{Q}_L^{r\,\alpha} e_R)\epsilon_{rs}(\bar{L}_L^s u_R^\alpha) \,, \\
\mathcal{O}_{y_e y_d} &= y_e y_d^\dagger (\bar{L}_L e_R)(\bar{d}_R Q_L) \,,
\end{aligned} \tag{6.18}
$$

where $\epsilon = i\sigma_2$ and α labels color (only shown when contracted outside parentheses). These operators are in principle of the first type. Nevertheless in the four-fermion operators of Eq. (6.18) we have incorporated a product of Yukawa couplings since they involve two chirality-flips, while in Eq. (6.17) we have also included Yukawa couplings as it is the case in theories with a flavour symmetry, as discussed below. These operators are then only suppressed by $1/\Lambda^2$ as second-class operators.

There is some redundancy in the operators given above, as it is clear that some of them can be eliminated by field redefinitions (see Appendix A) or using the equations of motion (EoM). For example, the operator $\mathcal{O}_r$ can be eliminated by field redefinitions:

$$c_r \mathcal{O}_r \leftrightarrow c_r \left[\frac{1}{2} \left(\mathcal{O}_{y_u} + \mathcal{O}_{y_d} + \mathcal{O}_{y_e} + \text{h.c.} \right) - \mathcal{O}_H + 2\mathcal{O}_6 \right] \,. \tag{6.19}$$

Also, we could eliminate all 5 operators of Eq. (6.5) by using the EoM for the gauge fields:

$$
\begin{aligned}
D^\nu W_{\mu\nu}^a &= ig H^\dagger \frac{\sigma^a}{2} \overset{\leftrightarrow}{D}_\mu H + g \sum_f \bar{f}_L \frac{\sigma^a}{2} \gamma_\mu f_L \,, \\
\partial^\nu B_{\mu\nu} &= ig' Y_H H^\dagger \overset{\leftrightarrow}{D}_\mu H + g' \sum_f \left[Y_L^f \bar{f}_L \gamma_\mu f_L + Y_R^f \bar{f}_R \gamma_\mu f_R \right] \,, \\
D^\nu G_{\mu\nu}^A &= g_s \sum_q \bar{q} T^A \gamma_\mu q \,,
\end{aligned} \tag{6.20}
$$

where $Y_{L,R}^f$ are the fermion hypercharges and Y_H the Higgs hypercharge. In particular, we could trade $\mathcal{O}_B$ and $\mathcal{O}_W$ with other operators:

$$c_B \mathcal{O}_B \leftrightarrow c_B \frac{g'^2}{g_*^2} \left[-\frac{1}{2}\mathcal{O}_T + \frac{1}{2} \sum_f \left(Y_L^f \mathcal{O}_L^f + Y_R^f \mathcal{O}_R^f \right) \right] \,, \tag{6.21}$$

$$c_W \mathcal{O}_W \leftrightarrow c_W \frac{g^2}{g_*^2} \left[-\frac{3}{2}\mathcal{O}_H + 2\mathcal{O}_6 + \frac{1}{2}\left(\mathcal{O}_{y_u} + \mathcal{O}_{y_d} + \mathcal{O}_{y_e} + \text{h.c.} \right) + \frac{1}{4} \sum_f \mathcal{O}_L^{(3)\,f} \right] \,,$$

where, in the last expression, we have eliminated $\mathcal{O}_r$ using Eq. (6.19).

For one family of fermions the set of operators that we use is collected in Tabs. 6.1 and 6.2. We keep all operators of Eqs. (6.4)-(6.11), since they are the relevant ones for a well-motivated class of BSM scenarios such as universal theories, with the exception of $\mathcal{O}_r$, that we eliminate of our basis using Eq. (6.19). In Tabs. 6.1 and 6.2 there are 58 operators; adding the 6 bosonic CP-odd ones in Eqs. (6.9)-(6.11) leads to a total of 64 operators. We still have 5 redundant operators that once eliminated leave a total of 59 independent operators, in agreement with Ref. [101]. We leave free the choice of which 5 operators to eliminate: *e.g.*, the operators of Eq. (6.5) could be eliminated by using Eq. (6.20) or, alternatively, we could trade 5 operators that contain fermions by the operators in Eq. (6.5). We will use later this freedom in different ways depending on the physics process studied. Other redundant operators are discussed in Appendix A.

Extending the basis to 3 families increases considerably the number of operators. We can reduce it by imposing flavor symmetries, which are also needed to avoid tight constraints on flavor-violating processes. For example, we can require the BSM sector to be invariant under the flavor symmetry $U(3)_{Q_L} \otimes U(3)_{d_R} \otimes U(3)_{u_R} \otimes U(3)_{L_L} \otimes U(3)_{e_R}$, under which the corresponding 3 families transform as triplets, and the Yukawas become 3×3 matrices transforming as $y_d \in (\mathbf{3}, \bar{\mathbf{3}}, \mathbf{0}, \mathbf{0}, \mathbf{0})$, $y_u \in (\mathbf{3}, \mathbf{0}, \bar{\mathbf{3}}, \mathbf{0}, \mathbf{0})$ and $y_e \in (\mathbf{0}, \mathbf{0}, \mathbf{0}, \mathbf{3}, \bar{\mathbf{3}})$ under the non-Abelian part of the flavor group. One can also assume that the Yukawas are the only source of CP violation. This assumption goes under the name of Minimal Flavor Violation (MFV) [19]. In this case the list of operators given in Table 2 can be easily generalized to include 3 families. For example, for operators involving two fermions, we have

$$
\begin{aligned}
(\bar{L}_L \gamma^\mu L_L) &\rightarrow \left[\delta_{ij} + O(y_e y_e^\dagger / g_*^2) \right] (\bar{L}_L^i \gamma^\mu L_L^j) \,, \\
y_e \bar{L}_L e_R &\rightarrow y_e^{ij} \left[1 + O(y_e^\dagger y_e / g_*^2) \right] \bar{L}_L^i e_R^j \,,
\end{aligned}
\tag{6.22}
$$

(i, j are family indices) and similarly for other fermion species. For 4-fermion operators, we have several possibilities to form singlets under the flavor group. For the leptons we find four independent operators:

$$
\begin{aligned}
\mathcal{O}_{LL}^l &= (\bar{L}_L^i \gamma^\mu L_L^i)(\bar{L}_L^j \gamma_\mu L_L^j) \,, \\
\mathcal{O}_{LL}^{(3)\,l} &= (\bar{L}_L^i \gamma^\mu \sigma^a L_L^i)(\bar{L}_L^j \gamma_\mu \sigma^a L_L^j) \,, \\
\mathcal{O}_{LR}^e &= (\bar{L}_L^i \gamma^\mu L_L^i)(\bar{e}_R^j \gamma_\mu e_R^j) \,, \\
\mathcal{O}_{RR}^e &= (\bar{e}_R^i \gamma^\mu e_R^i)(\bar{e}_R^j \gamma_\mu e_R^j) \,,
\end{aligned}
\tag{6.23}
$$

where we are neglecting terms of $O(y_e^2/g_*^2)$, while the independent set of 4-quark operators can be found in the Appendix of Ref. [104]. The MFV assumption that the Yukawas

Up-type quark and other fermions	Down-type quark and leptons	Leptons
$\mathcal{O}_{y_u} = y_u|H|^2 \bar{Q}_L \tilde{H} u_R$	$\mathcal{O}_{y_d} = y_d|H|^2 \bar{Q}_L H d_R$	$\mathcal{O}_{y_e} = y_e|H|^2 \bar{L}_L H e_R$
$\mathcal{O}_R^u = (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{u}_R \gamma^\mu u_R)$	$\mathcal{O}_R^d = (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{d}_R \gamma^\mu d_R)$	$\mathcal{O}_R^e = (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{e}_R \gamma^\mu e_R)$
$\mathcal{O}_L^q = (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{Q}_L \gamma^\mu Q_L)$		$\mathcal{O}_L^l = (iH^\dagger \overleftrightarrow{D}_\mu H)(\bar{L}_L \gamma^\mu L_L)$
$\mathcal{O}_L^{(3)q} = (iH^\dagger \sigma^a \overleftrightarrow{D}_\mu H)(\bar{Q}_L \gamma^\mu \sigma^a Q_L)$		$\mathcal{O}_L^{(3)l} = (iH^\dagger \sigma^a \overleftrightarrow{D}_\mu H)(\bar{L}_L \gamma^\mu \sigma^a L_L)$
$\mathcal{O}_{LR}^u = (\bar{Q}_L \gamma^\mu Q_L)(\bar{u}_R \gamma_\mu u_R)$	$\mathcal{O}_{LR}^d = (\bar{Q}_L \gamma^\mu Q_L)(\bar{d}_R \gamma_\mu d_R)$	$\mathcal{O}_{LR}^e = (\bar{L}_L \gamma^\mu L_L)(\bar{e}_R \gamma_\mu e_R)$
$\mathcal{O}_{LR}^{(8)u} = (\bar{Q}_L \gamma^\mu T^A Q_L)(\bar{u}_R \gamma_\mu T^A u_R)$	$\mathcal{O}_{LR}^{(8)d} = (\bar{Q}_L \gamma^\mu T^A Q_L)(\bar{d}_R \gamma_\mu T^A d_R)$	
$\mathcal{O}_{RR}^u = (\bar{u}_R \gamma^\mu u_R)(\bar{u}_R \gamma_\mu u_R)$	$\mathcal{O}_{RR}^d = (\bar{d}_R \gamma^\mu d_R)(\bar{d}_R \gamma_\mu d_R)$	$\mathcal{O}_{RR}^e = (\bar{e}_R \gamma^\mu e_R)(\bar{e}_R \gamma_\mu e_R)$
$\mathcal{O}_{LL}^q = (\bar{Q}_L \gamma^\mu Q_L)(\bar{Q}_L \gamma_\mu Q_L)$		$\mathcal{O}_{LL}^l = (\bar{L}_L \gamma^\mu L_L)(\bar{L}_L \gamma_\mu L_L)$
$\mathcal{O}_{LL}^{(8)q} = (\bar{Q}_L \gamma^\mu T^A Q_L)(\bar{Q}_L \gamma_\mu T^A Q_L)$		
$\mathcal{O}_{LL}^{ql} = (\bar{Q}_L \gamma^\mu Q_L)(\bar{L}_L \gamma_\mu L_L)$		
$\mathcal{O}_{LL}^{(3)ql} = (\bar{Q}_L \gamma^\mu \sigma^a Q_L)(\bar{L}_L \gamma_\mu \sigma^a L_L)$		
$\mathcal{O}_{LR}^{qe} = (\bar{Q}_L \gamma^\mu Q_L)(\bar{e}_R \gamma_\mu e_R)$		
$\mathcal{O}_{LR}^{lu} = (\bar{L}_L \gamma^\mu L_L)(\bar{u}_R \gamma_\mu u_R)$	$\mathcal{O}_{LR}^{ld} = (\bar{L}_L \gamma^\mu L_L)(\bar{d}_R \gamma_\mu d_R)$	
$\mathcal{O}_{RR}^{ud} = (\bar{u}_R \gamma^\mu u_R)(\bar{d}_R \gamma_\mu d_R)$		
$\mathcal{O}_{RR}^{(8)ud} = (\bar{u}_R \gamma^\mu T^A u_R)(\bar{d}_R \gamma_\mu T^A d_R)$		
$\mathcal{O}_{RR}^{ue} = (\bar{u}_R \gamma^\mu u_R)(\bar{e}_R \gamma_\mu e_R)$	$\mathcal{O}_{RR}^{de} = (\bar{d}_R \gamma^\mu d_R)(\bar{e}_R \gamma_\mu e_R)$	
$\mathcal{O}_R^{ud} = y_u^\dagger y_d (i\tilde{H}^\dagger \overleftrightarrow{D}_\mu H)(\bar{u}_R \gamma^\mu d_R)$		
$\mathcal{O}_{y_u y_d} = y_u y_d (\bar{Q}_L^r u_R)\epsilon_{rs}(\bar{Q}_L^s d_R)$		
$\mathcal{O}_{y_u y_d}^{(8)} = y_u y_d (\bar{Q}_L^r T^A u_R)\epsilon_{rs}(\bar{Q}_L^s T^A d_R)$		
$\mathcal{O}_{y_u y_e} = y_u y_e (\bar{Q}_L^r u_R)\epsilon_{rs}(\bar{L}_L^s e_R)$		
$\mathcal{O}'_{y_u y_e} = y_u y_e (\bar{Q}_L^r e_R)\epsilon_{rs}(\bar{L}_L^s u_R)$		
$\mathcal{O}_{y_e y_d} = y_e y_d^\dagger (\bar{L}_L e_R)(\bar{d}_R Q_L)$		
$\mathcal{O}_{DB}^u = y_u \bar{Q}_L \sigma^{\mu\nu} u_R \tilde{H} g' B_{\mu\nu}$	$\mathcal{O}_{DB}^d = y_d \bar{Q}_L \sigma^{\mu\nu} d_R H g' B_{\mu\nu}$	$\mathcal{O}_{DB}^e = y_e \bar{L}_L \sigma^{\mu\nu} e_R H g' B_{\mu\nu}$
$\mathcal{O}_{DW}^u = y_u \bar{Q}_L \sigma^{\mu\nu} u_R \sigma^a \tilde{H} g W^a_{\mu\nu}$	$\mathcal{O}_{DW}^d = y_d \bar{Q}_L \sigma^{\mu\nu} d_R \sigma^a H g W^a_{\mu\nu}$	$\mathcal{O}_{DW}^e = y_e \bar{L}_L \sigma^{\mu\nu} e_R \sigma^a H g W^a_{\mu\nu}$
$\mathcal{O}_{DG}^u = y_u \bar{Q}_L \sigma^{\mu\nu} T^A u_R \tilde{H} g_s G^A_{\mu\nu}$	$\mathcal{O}_{DG}^d = y_d \bar{Q}_L \sigma^{\mu\nu} T^A d_R H g_s G^A_{\mu\nu}$	

Table 6.2: *44 operators made of one-family of SM fermions. In the first column there are operators made of the up-type quark and other fermions; in the second column there are operators made only of the down-type quark and leptons; the third column lists operators made only of leptons. The operators are grouped in 3 different rows (separated with a solid line) corresponding to the 3 classes of operators defined in Eq. (6.2). Dashed lines separate operators of different structure within a given class.*

are the only source of CP violation implies that the Wilson coefficients are real. For the top quark, having a Yukawa coupling of order one, departures from flavor-universality could be important.

It is useful, in order to understand what operators mix under the RGE, to derive the transformation of the coefficients (or equivalently, of the operators) under the global custodial $SU(2)_L \otimes SU(2)_R$ symmetry and the parity P_{LR} that interchanges $L \leftrightarrow R$. A detailed analysis is given in Appendix C. In Tab. 6.3 we present the quantum numbers of the coefficients of the tree-level operators involving the Higgs.

We emphasize again that the above classification is useful even when one is not working under the minimally-coupled assumption of Ref. [99]. When studying the RGEs of these operators, we will find that, at leading order, current-current operators do not affect the RG running of one-loop suppressed operators (irrespective of their UV origin). Furthermore, the above classification can also be useful to parametrize the effects of strongly-coupled models. In particular, if the Higgs is part of the composite meson states, taking $g_H \sim 4\pi$ gives the correct power counting for strongly-coupled theories with no small parameters. One finds in this case that operators of the first class are the most relevant, while operators of the second and third classes have the same $1/\Lambda^2$ suppression. Also the basis is well suited for characterizing holographic descriptions of strongly-coupled models [99]. In this case $g_H \sim 4\pi/\sqrt{N}$, where N plays the role of the number of colors of the strong-interaction, and then operators of the first and second classes are less suppressed than operators of the third class.

	Spurion	$SU(2)_L \otimes SU(2)_R$	P_{LR}
$[\mathcal{O}] \leq 4$	y_f	$\mathbf{2}_R$	
	g'	$\mathbf{3}_R + \mathbf{1}$	
$[\mathcal{O}] = 6$	c_T	$(\mathbf{3}_R \otimes \mathbf{3}_R)_s$	
	c_H, c_6	$\mathbf{1}$	$+$
	$c_B + c_W$	$\mathbf{1}$	$+$
	$c_B - c_W$	$\mathbf{1}$	$-$
	c_{y_f}	$\mathbf{1}$	
	c_R^f	$\mathbf{3}_R$	
	c_L^f	$\mathbf{3}_R$	
	$c_L^{(3)\,f}$	$\mathbf{1}$	
	c_R^{ud}	$\mathbf{1}$	

TABLE 6.3: *Quantum numbers under the custodial $SU(2)_L \otimes SU(2)_R$ and left-right parity P_{LR} of the SM couplings and coefficients of the tree-level operators involving Higgs fields. We only show the P_{LR}-parities of the coefficients with a well-defined transformation, see Apendix C.*

6.2   Higgs physics

Let us now describe the effects of the $d = 6$ operators on Higgs physics. We will only present the modifications of the Higgs couplings important for single Higgs production and decay, working under the assumption of MFV, allowing however for CP-violating bosonic operators. We split the relevant part of the Lagrangian in two parts,

$$\mathcal{L}_h = \mathcal{L}_h^{(0)} + \mathcal{L}_h^{(1)} \,. \tag{6.24}$$

In $\mathcal{L}_h^{(0)}$ we keep the SM couplings and the effects of the current-current operators of Tabs. 6.1 and 6.2, while $\mathcal{L}_h^{(1)}$ has the effects of the loop operators. We can remove the momentum dependence from the Higgs couplings in $\mathcal{L}_h^{(0)}$ by using the EoM, so that we end up with Higgs couplings at zero momentum. After doing that, we have, in the canonical basis for the Higgs field h,

$$
\begin{aligned}
\mathcal{L}_h^{(0)} \;=\;& g_{hff}\, h(\bar{f}_L f_R + \text{h.c.}) \,+\, g_{hVV}\, h V^\mu V_\mu \,+\, g_{hZf_L f_L}\, h Z_\mu \bar{f}_L \gamma^\mu f_L \\
&+ g_{hZf_R f_R}\, h Z_\mu \bar{f}_R \gamma^\mu f_R \,+\, g_{hW f_L f'_L}\, h W_\mu \bar{f}_L \gamma^\mu f'_L \,,
\end{aligned}
\tag{6.25}
$$

where a sum over fermions is understood and $V = W, Z$. The couplings read [6]

$$
\begin{aligned}
g_{hff} \;&=\; g_{hff}^{\text{SM}} \left[1 - \left(\frac{c_H}{2} + c_{y_f} \right) \xi + \frac{\delta G_F}{2\, G_F} \right] \,, \\
g_{hWW} \;&=\; g_{hWW}^{\text{SM}} \left[1 - \left(c_H - \frac{g^2}{g_*^2} c_W \right) \frac{\xi}{2} + \frac{\delta G_F}{2\, G_F} + 2\, \frac{\delta M_W}{M_W} \right] \,, \\
g_{hZZ} \;&=\; g_{hZZ}^{\text{SM}} \left[1 - \left(c_H - \frac{g^2}{g_*^2} c_Z \right) \frac{\xi}{2} - T + \frac{\delta G_F}{2\, G_F} \right] \,, \\
g_{hW f_L f'_L} \;&=\; \frac{1}{2\sqrt{2}\, v} \frac{g^3}{g_*^2} c_W\, \xi + \frac{2}{v} \delta g_W^{f_L} \,, \\
g_{hZ f_L f_L} \;&=\; \frac{1}{2 v \cos\theta_W} \frac{g^3}{g_*^2} \left(T_L^3 c_Z - Q_f c_B \tan^2\theta_W \right) \xi + \frac{2}{v} \delta g_Z^{f_L} \,, \\
g_{hZ f_R f_R} \;&=\; -\frac{\tan^2\theta_W}{2 v \cos\theta_W} \frac{g^3}{g_*^2} Q_f c_B\, \xi + \frac{2}{v} \delta g_Z^{f_R} \,.
\end{aligned}
\tag{6.26}
$$

Here the SM couplings must be expressed as a function of the input parameters $\alpha = e^2/(4\pi)$, the Fermi constant G_F and the physical m_h, M_Z and fermion masses. In these equations, θ_W is the weak mixing angle, $T_L^3 = \pm 1/2$ stands for the weak isospin values of up and down components of $SU(2)_L$ fermion doublets, Q_f is the fermion electric charge. We have defined

$$\xi \equiv \frac{g_*^2 v^2}{\Lambda^2} \,, \tag{6.27}$$

---

[6] A coupling of $W_\mu^\pm$ to the right-handed current $\bar{f}_R \gamma^\mu f'_R$ is generated from the operator $\mathcal{O}_R^{ud}$ in Eq. (6.17), but we do not include it as it is expected to be suppressed by two Yukawa couplings (due to the MFV assumption) and hence to be small.

with $v \simeq 246$ GeV, and

$$c_Z = c_W + \tan^2 \theta_W \, c_B \ . \tag{6.28}$$

In the couplings of Eq. (6.26), we have introduced

$$\frac{\delta G_F}{G_F} = 2 \left[c_{LL}^{(3)\,l} - c_L^{(3)\,l} \right] \xi \ , \tag{6.29}$$

$$\frac{\delta M_W}{M_W} = \frac{1}{2(1 - 2 \sin^2 \theta_W)} \left[\cos^2 \theta_W \, T - 2 \sin^2 \theta_W \, S + \sin^2 \theta_W \frac{\delta G_F}{G_F} \right] \ , \tag{6.30}$$

and

$$\delta g_W^{f_L} = \frac{g}{\sqrt{2}} c_L^{(3)\,f} \xi \ ,$$

$$\delta g_Z^{f_L} = \frac{g}{2 \cos \theta_W} (2 T_L^3 c_L^{(3)\,f} - c_L^f) \xi,$$

$$\delta g_Z^{f_R} = -\frac{g}{2 \cos \theta_W} c_R^f \xi \ . \tag{6.31}$$

Finally, we have made use of the precision electroweak parameters [105, 106]

$$S = (c_W + c_B) \frac{M_W^2}{\Lambda^2} \ , \quad T = c_T \xi \ . \tag{6.32}$$

As we have stressed in the previous Section, not all the operators appearing in the Higgs couplings of Eq. (6.26) are independent. Once one has decided which are the redundant operators that are not in the basis, one should simply put equal to zero the corresponding operator coefficients.

The second term in the Lagrangian Eq. (6.24) necessarily contains field derivatives. It reads

$$\mathcal{L}_h^{(1)} = g_{\partial hWW} \left(W^{+\mu} W_{\mu\nu}^- \partial^\nu h + \text{h.c.} \right) + g_{\partial hZZ} \, Z^\mu Z_{\mu\nu} \partial^\nu h + g_{hZZ}' \, h Z^{\mu\nu} Z_{\mu\nu} \tag{6.33}$$

$$+ g_{hAA} \, h A^{\mu\nu} A_{\mu\nu} + g_{\partial hAZ} \, Z^\mu A_{\mu\nu} \partial^\nu h + g_{hAZ} \, h A^{\mu\nu} Z_{\mu\nu} + g_{hGG} \, h G^{A\mu\nu} G_{\mu\nu}^A \ ,$$

where we have defined $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$, for $V = W^\pm, Z, A$. The couplings are given by

$$g_{\partial hWW} = -\frac{g^2 v}{2\Lambda^2} \kappa_{HW} \ ,$$

$$g_{\partial hZZ} = -\frac{g^2 v}{2\Lambda^2} (\kappa_{HW} + \kappa_{HB} \tan^2 \theta_W) \ ,$$

$$g_{hAA} = \frac{e^2 v}{\Lambda^2} \kappa_{BB} = \frac{g_{hZZ}'}{\tan^2 \theta_W} = -\frac{g_{hAZ}}{2 \tan \theta_W} \ ,$$

$$g_{\partial hAZ} = -\frac{g^2 v}{2\Lambda^2} \tan \theta_W (\kappa_{HW} - \kappa_{HB}) \ ,$$

$$g_{hGG} = \frac{g_s^2 v}{\Lambda^2} \kappa_{GG} \ . \tag{6.34}$$

The contributions from the CP-violating bosonic operators can be easily obtained from Eq. (6.33) by replacing one of the field strengths $F_{\mu\nu}$ in the operators by $\widetilde{F}_{\mu\nu}$. Only the contributions from the dipole operators (third box of Tab. 6.2) have been neglected since they are assumed to be proportional to Yukawa couplings.

In the list of modified Higgs couplings (6.26), the tree-level operator $\mathcal{O}_6$ does not play any role. The simplest modified coupling containing this operator would be the triple Higgs vertex

$$\delta\mathcal{L}_h^{(0)} = g_{hhh}^{\text{SM}} \left[1 - \left(c_6 + \frac{3c_H}{2} \right) \xi + \frac{\delta G_F}{2\,G_F} \right] h^3 \, , \tag{6.35}$$

where g_{hhh}^{SM} is the SM value for the h^3 coupling. Experimental access to this coupling is not yet possible.

From the couplings in Eqs. (6.25) and (6.33) it is easy to derive the modifications of the main Higgs partial-widths due to $d = 6$ operators [99, 107, 108].[7] The coefficients $c_L^{(3)\,f}, c_L^f, c_R^f$ can also modify the cross-section of $hf\bar{f}$ production, giving contributions that grow with the energy. A particularly interesting case is $pp \to qth$ (q being a light quark) that is dominated by the subprocess $W_L b \to th$. At large energies this grows with the energy as

$$|\mathcal{A}(W_L b \to th)|^2 \simeq \left(\frac{4g_*^2 c_L^{(3)\,q_3}}{\Lambda^2} \right)^2 s(s + t) \, . \tag{6.36}$$

The extraction of new physics through this process has been studied in Ref. [109].

6.3 Experimental constraints on the Wilson coefficients

As we saw in the previous Section, many $d = 6$ operators can directly affect the Higgs couplings. Some of them only affect Higgs physics (at tree-level). Their corresponding coefficients are

$$\{c_H, c_6, c_{y_f}, \kappa_{BB}, \kappa_{GG}, \hat{\kappa}_{WW}, \kappa_{B\widetilde{B}}, \kappa_{G\widetilde{G}}, \hat{\kappa}_{W\widetilde{W}}\} \, . \tag{6.37}$$

The reason for this is clear in the case of c_H and c_6 as these operators contain exclusively Higgs fields; and in the case of c_{y_f}, κ_{BB} and κ_{GG} because, when the Higgs is substituted by its vacuum expectation value (VEV), these operators simply lead to an innocuous renormalisation of SM parameters. The coefficient $\hat{\kappa}_{WW}$ corresponds to the direction in

---

[7]For loop-suppressed partial-widths, such as $h \to \gamma\gamma$, we remind the reader that $d = 6$ operators can have an effect either directly or through modifications of the SM couplings that change the SM loop contribution to that particular decay [99].

parameter space given by [8]

$$\kappa_{HB} = -\kappa_{HW} = 4\kappa_{BB} = c_W = -c_B \equiv 4\hat{\kappa}_{WW} \,, \qquad (6.38)$$

and the reason why this direction is only constrained by Higgs physics is subtle in our basis. The easiest way to see it is to go from our basis, that contains the subset

$$\mathcal{B}_1 = \{\mathcal{O}_W, \mathcal{O}_B, \mathcal{O}_{HW}, \mathcal{O}_{HB}, \mathcal{O}_{BB}\} \,, \qquad (6.39)$$

to the basis containing the subset $\mathcal{B}_3$ defined in Ref. [5]:

$$\mathcal{B}_3 = \{\mathcal{O}_W, \mathcal{O}_B, \mathcal{O}_{WW}, \mathcal{O}_{WB}, \mathcal{O}_{BB}\} \,. \qquad (6.40)$$

One can go from one to another using (7.7). Now, in the basis containing $\mathcal{O}_{WW}$ it is clear that its coefficient cannot be bounded by any non-Higgs SM processes, for exactly the same reasons as κ_{BB}. We can now use Eq. (7.7) to get the expression of $\mathcal{O}_{WW}$ in terms of the operators in $\mathcal{B}_1$,

$$\mathcal{O}_{WW} = 4(\mathcal{O}_W - \mathcal{O}_B) - 4(\mathcal{O}_{HW} - \mathcal{O}_{HB}) + \mathcal{O}_{BB} \,, \qquad (6.41)$$

which leads to the direction given in Eq. (6.38). Similarly, for the CP-odd operators, $\hat{\kappa}_{W\widetilde{W}}$ corresponds to the direction:

$$\kappa_{H\widetilde{B}} = -\kappa_{H\widetilde{W}} = 4\kappa_{B\widetilde{B}} \equiv 4\hat{\kappa}_{W\widetilde{W}} \,. \qquad (6.42)$$

Although the coefficients c_H, c_6 and c_{y_f} have no severe constraints from Higgs physics yet [111], the coefficients κ_{BB} and the difference $\kappa_{HW} - \kappa_{HB}$ are subject to strong constraints from $h \to \gamma\gamma$ and $h \to Z\gamma$ respectively (as these decays are one-loop suppressed in the SM). These give at 95%CL [111]

$$-0.0013 \lesssim \frac{M_W^2}{\Lambda^2}\kappa_{BB} \lesssim 0.0018 \,, \quad -0.016 \lesssim \frac{M_W^2}{\Lambda^2}(\kappa_{HW} - \kappa_{HB}) \lesssim 0.009 \,. \qquad (6.43)$$

Notice that $\kappa_{HW} - \kappa_{HB}$ is odd under P_{LR} [Eq. (C.4)] and could be suppressed with respect to the sum $\kappa_{HW} + \kappa_{HB}$ if the BSM sector respects this parity. Similarly, the coefficient κ_{GG} enters in the production $GG \to h$ and gets the bound [111]:

$$\frac{M_W^2}{\Lambda^2}|\kappa_{GG}| \lesssim 0.004 \,. \qquad (6.44)$$

---

[8]In Ref. [110] these were called blind directions, combinations of operators which a certain group of experiments cannot bound. In the case of $\hat{\kappa}_{WW}$ that group is non-Higgs experiments.

The coefficients of the CP-odd operators enter quadratically in $\Gamma(h \to \gamma\gamma)$ and $\Gamma(h \to Z\gamma)$, and therefore their effects are suppressed with respect to CP-even ones.

Apart from the "Higgs-only" coefficients of Eq. (6.37), the rest of the coefficients of $d = 6$ operators that enter in the Lagrangian of Eq. (6.25) and Eq. (6.33), relevant for single Higgs physics, can in principle be constrained by (non-Higgs) SM processes. In the following we present the main experimental constraints on these Wilson coefficients. We also discuss limits on other Wilson coefficients that, although do not affect Higgs physics at tree-level, could do it at the one-loop level. The details of this study with a full dedicated quantitative analysis is done in Ref. [108]. In what follows we assume MFV (unless explicitly stated) and CP-invariance.

6.3.1   Universal theories

We start considering universal theories, leaving the generalization for later. The new physics effects of these theories are captured by the operators listed in Tab. 6.1. Deviations in the $W^\pm$ and Z^0 propagators can be parametrized by four quantities, S, T, W and Y [106]. The contributions from $d = 6$ operators to S and T have been written in Eq. (6.32); the corresponding equations for W and Y read

$$W = c_{2W} \frac{M_W^2}{\Lambda^2} \ , \quad Y = c_{2B} \frac{M_W^2}{\Lambda^2} \ . \tag{6.45}$$

LEP1, LEP2 ($e^+e^- \to l^+l^-$) and TeVatron allow to constrain independently each of these four quantities, all of them at the per-mille level [106].[9] We saw in (6.32) that S depends only on the combination $c_W + c_B$. The gauge-boson part of the orthogonal combination, $\mathcal{O}_W - \mathcal{O}_B$, contains at least three gauge bosons

$$(\mathcal{O}_W - \mathcal{O}_B)\big|_{\langle H \rangle} = O(V^3) \,, \tag{6.46}$$

and thus it is a blind direction for LEP1 experiments. To constrain this direction, we have to consider the effect of $c_{W,B}$ on triple gauge-boson vertices, which can be cast in the form

$$
\begin{aligned}
\delta\mathcal{L}_{3V} &= ig\cos\theta_W \left[\delta g_1^Z \, Z^\mu \left(W^{-\nu} W_{\mu\nu}^+ - W^{+\nu} W_{\mu\nu}^- \right) + \delta\kappa_Z \, Z^{\mu\nu} W_\mu^- W_\nu^+ \right. \\
&\quad \left. + \frac{\lambda_Z}{M_W^2} Z^{\mu\nu} W_\nu^{-\rho} W_{\rho\mu}^+ \right] + ig\sin\theta_W \left[\delta\kappa_\gamma \, A^{\mu\nu} W_\mu^- W_\nu^+ + \frac{\lambda_\gamma}{M_W^2} A^{\mu\nu} W_\nu^{-\rho} W_{\rho\mu}^+ \right] ,
\end{aligned}
\tag{6.47}
$$

---

[9]LHC data is also useful to constrain W, Y and c_{2G}, which affect quark cross-sections at high energies [104].

where again we have defined $V_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$ for $V = W^\pm, Z, A$. The contributions from $d = 6$ operators to these couplings are given by

$$
\begin{aligned}
\delta g_1^Z &= \frac{M_Z^2}{\Lambda^2}(c_W + \kappa_{HW}) \,, \\
\delta\kappa_\gamma &= \frac{M_W^2}{\Lambda^2}(\kappa_{HW} + \kappa_{HB}) \,, \\
\delta\kappa_Z &= \delta g_1^Z - \tan^2\theta_W \delta\kappa_\gamma \,, \\
\lambda_Z &= \lambda_\gamma = \frac{M_W^2}{\Lambda^2}\kappa_{3W} \,,
\end{aligned}
\tag{6.48}
$$

where we do not include a contribution from c_{2W} since it is constrained to be small, as we have seen before. The third relation, as well as the identity $\lambda_Z = \lambda_\gamma$, are a consequence of limiting the analysis to $d = 6$ operators [100]. The best current limits on triple gauge-boson vertices still come from $e^+e^- \to W^+W^-$ at LEP2 [112], although LHC results are almost as good and will be better in the near future [113–115]. Leaving aside the contributions from κ_{3W}, that we expect to be small in most theories in which the SM gauge bosons are elementary above Λ, we can use the two-parameter fit from LEP2 [112] which at 95%CL reads

$$
\begin{aligned}
-0.046 \leqslant \ \delta g_1^Z \ &\leqslant 0.050 \,, \\
-0.11 \leqslant \ \delta\kappa_\gamma \ &\leqslant 0.084 \,.
\end{aligned}
\tag{6.49}
$$

These are a factor ~ 10 weaker than the constraints on the coefficients S, T, W and Y from LEP1 (for this reason we can neglect their contributions to $e^+e^- \to W^+W^-$). As expected, the two constraints in Eq. (6.49) are orthogonal in parameter space to the direction $\hat{\kappa}_{WW}$ of Eq. (6.38), as can be seen using Eq. (6.48). For this reason, to obtain independent bounds on the 4 parameters c_W, c_B, κ_{HB} and κ_{HW}, we need the constraint Eq. (6.43) combined with Eq. (6.49) and the bound on S. These bounds are at the percent level. In the particular case of $\kappa_i \ll c_i$, as expected in weakly-coupled theories, we obtain the bound

$$
-0.046 \lesssim \frac{M_Z^2}{\Lambda^2}c_W \lesssim 0.050 \,.
\tag{6.50}
$$

As we said, LHC tests of triple gauge-boson vertices are becoming comparable to those from LEP2, and it is foreseen that LHC will surpass LEP2 in these type of measurements [113–115]. It follows that an important implication of our study is that the LHC will have a direct impact on the improvement of the limits on $c_W + \kappa_{HW}$, $\kappa_{HW} + \kappa_{HB}$ and κ_{3W}. We will see in the next Subsection that this conclusion is also valid in non-universal theories.

6.3.2   Non-universal theories

Let us now discuss BSM models without the universal assumption, considering then all operators of the basis. We will follow a different strategy than in the previous Subsection. Let us first look at electroweak leptonic physics for which the experimental constraints are expected to be the strongest ones. Since we assume MFV, dipole operators (third box of Tab. 6.2) give corrections to SM processes proportional to lepton masses and can then be neglected. We use the redundancy in our set of operators to eliminate, by using Eq. (7.42), the 5 operators $\mathcal{O}_{2B,2W,2G}$, $\mathcal{O}_L^{(3)\,l}$ and $\mathcal{O}_L^l$. Taking α, M_Z and G_F as input parameters, the relevant operators for the leptonic data are the 4 operators $\mathcal{O}_T$, $\mathcal{O}_W$, $\mathcal{O}_B$, $\mathcal{O}_R^e$ and the four-lepton operators of Eq. (6.23). LEP1 data and Tevatron afford 4 well-measured experimental quantities: The charged-leptonic width $\Gamma(Z \to l^+l^-)$, the leptonic left-right asymmetry A_{LR}^l, the Z-width into neutrinos $\Gamma(Z \to \nu\bar{\nu}) = \Gamma_Z^{\text{total}} - \Gamma_Z^{\text{visible}}$ and M_W. These allow us to place bounds on the 4 quantities $\{c_T, c_W + c_B, c_R^e, \delta G_F/G_F\}$ [where $\delta G_F/G_F$ is given in Eq. (6.29)] at almost the same level as for universal theories. We again need the LEP2 constraint of Eq. (6.49) from $e^+e^- \to W^+W^-$ to bound the difference $c_W - c_B$ [see Eq. (6.46)]. The only remaining operators are four-lepton interactions but they can also be highly constrained from $e^+e^- \to l^+l^-$ at LEP2.

Having these constraints in mind, we can now move to the quark sector. Higgs-fermion operators, as those in Eq. (7.4), give contributions to the gauge-boson couplings to quarks that make them depart from the leptonic ones by the amounts $\delta g_W^{q_L}, \delta g_Z^{q_L}$ and $\delta g_Z^{q_R}$ given in Eq. (6.31). Experiments put severe bounds on these deviations. For example, we have limits at the per-mille level on deviations from lepton-quark universality from β-decays and semileptonic K-decays [116]. This implies that the coefficient $c_L^{(3)\,q}\xi$ can be constrained at this level.[10] For c_L^q, c_R^u and c_R^d the main constraints come from LEP1 measurements at the Z-pole. These can put bounds on deviations of the Z couplings to quarks, $\delta g_Z^{q_{L,R}}$, and on c_L^q and $c_R^{u,d}$.

As we saw, operators made of top quarks can depart from the MFV assumption due to the large top Yukawa coupling. If this is the case, we can still bound $(c_L^{q3} + c_L^{(3)\,q3})\xi$ from the measurement of the $Zb_L\bar{b}_L$ coupling at LEP1 which also gives a per-mille bound. Interestingly, a P_{LR} symmetry can be imposed in the BSM sector such that $c_L^{q3} = -c_L^{(3)\,q3}$ [see Eq. (C.10)], allowing for large deviations on $c_L^{q3} - c_L^{(3)\,q3}$. Recent LHC measurements of the Wtb coupling [118] put some bounds on $c_L^{(3)\,q3}$ but they are not very strong. Also c_R^t has practically no bound due to the large uncertainty in the determination of the $Zt_R\bar{t}_R$ coupling [119, 120]. Bounds on the Wilson coefficient c_R^{tb},

---

[10]The operator $\mathcal{O}_{LL}^{(3)\,ql} = (\bar{Q}_L\gamma_\mu\sigma^a Q_L)(\bar{L}_L\gamma^\mu\sigma^a L_L)$ also gives contributions to β-decays and K-decays, but this can be independently constrained by recent LHC data [117].

see Eq. (6.17), arise from $b \to s\gamma$ and read $-0.001 \lesssim c_R^{tb} M_W^2/\Lambda^2 \lesssim 0.006$ [121]. These bounds will be improved in the future by the LHC.

Four-fermion operators involving quarks, as those in the first box of Tab. 6.2, can also be constrained by recent LHC data [104], while the coefficients of the operators of the second box of Tab. 6.2 have no severe experimental constraints due to their Yukawa suppression. However, they can affect Higgs physics through operator mixing, as we will see in the next Section. Finally, bounds on dipole operators can be found, for example, in Ref. [107].

We conclude that, concerning the strength of experimental constraints, we can distinguish the following sets of $d = 6$ operators:

1. First, we have those which can only affect Higgs physics. We have 8+3 operators of this type (CP-even plus CP-odd respectively) for one family, with real coefficients given in Eq. (6.37) [11]. As shown in Sec. 6.2, they can independently modify the Higgs decay-width to fermions, photons, gluons and $Z\gamma$, apart from a global rescaling of all Higgs amplitudes due to c_H.

2. A second set of operators are those whose coefficients are severely restricted by electroweak precision data, as explained above. Eliminating, by the EoM of Eq. (7.42), $\mathcal{O}_{2B}, \mathcal{O}_{2W}, \mathcal{O}_{2G}$ and $\mathcal{O}_L^l, \mathcal{O}_L^{(3)\,l}$, these are $c_W + c_B$ and c_T that affect the W/Z propagator, and $c_R^e, c_L^q, c_R^{u,d}, c_L^{(3)\,q}$ that affect $Vf\bar{f}$ vertices.

3. In a third set, we have the operator coefficients that can affect the $ZWW/\gamma WW$ vertices and are, at present, constrained at the few per-cent level. These are the combinations $\kappa_{HB} + \kappa_{HW}$ and $c_W + \kappa_{HW}$ (and also c_{3W} if we include λ_Z in the analysis).

We finally would like to mention that our result is in contradiction with Ref. [122] that obtained a smaller number of parameters to characterize Higgs physics and triple gauge-boson vertices. The origin of this discrepancy is due to the following. In our basis it is clear that physics at LEP1 is not sensitive to the blind direction $c_W = -c_B$, since only the combination $c_W + c_B$ enters in the S parameter. This blind direction, however, becomes more complicated when one goes to other bases, such as that of Ref. [101], in which $\mathcal{O}_W$ and $\mathcal{O}_B$ are eliminated [by using Eq. (7.42)] in favor of operators made of SM fermions. In such bases there is the risk of overestimating the number of independent experimental constraints on the Wilson coefficients.

---

[11]If we relax the MFV assumption that the c_{y_f} are real, in addition to the 3 operators $\mathrm{Re}(c_{y_f})(\mathcal{O}_{y_f} + \mathcal{O}_{y_f}^\dagger)$ we should also consider the 3 CP-odd operators $\mathrm{Im}(c_{y_f})(\mathcal{O}_{y_f} - \mathcal{O}_{y_f}^\dagger)$.

6.4   Summary

As the measurements of the Higgs properties improve, it will be important to understand their implications for BSM models. In this Chapter we have adopted the framework of effective Lagrangians as a tool to study the effects of $d = 6$ operators in Higgs physics. As a first step, we have discussed the choice of operator basis. Our basis has been defined following Ref. [6, 99] that distinguished two classes of operators: tree-level (or current-current) operators, and one-loop operators. This choice can be important when calculating one-loop operator mixing, since most of the tree-level operators do not mix with one-loop operators under RG evolution, see the next Chapters 7-10. Another important property of our basis is that it contains a subset of 5 CP-even operators made of Higgs and gauge field-strengths, that in our case are $\mathcal{O}_{W,B}$ $\mathcal{O}_{HW,HB}$ and $\mathcal{O}_{BB}$, (leaving aside $\mathcal{O}_{GG}$). We have found that it is important to keep these 5 operators to make the connection with experiments more transparent [these subset could also be written with $\mathcal{O}_{WB,WW}$ by using the identities of Eqs. (7.6) and (7.7)]. Bases, such as Ref. [101] and Ref. [122], that eliminate two of these operators in favor of operators made of SM fermions, as it can be done by using the EoM, have dangerous blind directions for LEP1 experiments, which make the contact with experiments more difficult.

We have calculated the modifications that the operators of the effective Lagrangian induce in the Higgs couplings relevant for the main decays and production mechanisms. It has been shown that these operators can be divided in two subsets. There are 11 operators (for one family) with coefficients given in Eq. (6.37), that can only affect Higgs physics and no other SM processes at tree-level. The number 11 can be deduced from counting the number of independent operators one can write as $|H|^2 \mathcal{O}_4$ with $\mathcal{O}_4$ a $d = 4$ operator formed with SM fields. The second subset, formed by the rest of operators, enter in other SM processes and therefore can be constrained by non-Higgs experiments. Among the latter, considering only the CP-even ones, we have found that the least constrained correspond to the two combinations of Wilson coefficients appearing in the measurements of the $ZWW/\gamma WW$ coupling, Eq. (6.48), that LEP2 has only constrained at the few per-cent level. LHC will probe these vertices with better accuracy, so that it will be able to improve these constraints or reveal some BSM deviation.

7. SM Quantum EFT: $h \to \gamma + \gamma/Z$ v.s. S-parameter

Rather than embarking in messy calculations of the one-loop anomalous dimension matrix involving all the SM dimension-six operators, we would like to discuss a simpler calculation that illustrates a number of points. The purpose of this Chapter is to compute the renormalization group equations (RGEs) at the one-loop level of the dimension-six operators responsible for $h \to \gamma + \gamma/Z$. Our main interest is to look for log-enhanced contributions coming from operator mixing. Particularly interesting are those contributions that could arise from mixings with operators induced at tree-level by the theory at high-energies. These can potentially give corrections to the $h\gamma\gamma$ and $h\gamma Z$ couplings of order $\sim g_H^2 v^2 \log(\Lambda/m_h)/(16\pi^2\Lambda^2)$ where g_H is the coupling of the Higgs to the heavy sector and v is the Fermi scale. These contributions would be the leading ones to the decay $h \to \gamma + \gamma/Z$ since this is loop-supressed in the SM.

Ref. [123] argued that this type of contributions could in fact be present for a general class of models as, for example, those in Ref. [99], although the result was based on a calculation that included only a partial list of operators and not the complete basis set. We show however that such corrections are not present. The right choice of operator basis helps in simplifying the calculation of the anomalous dimension matrix as well as its physical interpretation. We work in a basis where the dimension-six operators are classified according to the expected size of their Wilson coefficients. We mainly consider two groups: those operators that can be written as scalar or vector current-current operators (and could therefore arise at the tree-level by the interchange of heavy fields), and the rest, expected to be induced at the one-loop level. By working in this basis, we show that none of the current-current operators affects the running of any loop operator. This in turn implies the absence of a log-enhancement effect. This is already known to happen in other situations. For example, the magnetic moment operator responsible for $b \to s\gamma$ does not receive log-contributions from certain current-current quark operators at the one-loop level [124].

We also show how to reconcile our conclusion with the results of Ref. [123] by completing the calculation done in the basis used in that analysis. Furthermore, we use the results of Ref. [123] to calculate the complete leading-log corrections to the operators responsible for $h \to \gamma\gamma$ and $h \to \gamma Z$. We find that only one-loop operators contribute to these decays, and therefore these effects are not expected to be very large. Finally, we also extend the calculation to include mixing with fermion dipole-moment operators.

Dimension-six operator basis relevant for $h \to \gamma + \gamma/Z$

We start considering only operators made of SM bosons. Let us summarise briefly the basis introduced in Chapter. 6 for the ease of reading the present Chapter. The operators made of bosons can be induced from integrating out heavy states in universal theories, those whose fields only couple to the bosonic sector of the SM (a generalisation including SM fermions will be given later). We can broadly organise the dimension-six operators in three classes of operators. The first two classes consist of operators that can in principle be generated at tree-level when integrating out heavy states, with spin≤ 1, of a weakly-coupled renormalizable BSM theory. The operators of the first class are those that involve extra powers of Higgs fields, and are expected to be suppressed by g_H^2/Λ^2. The operators of the second class involve extra (covariant) derivatives or gauge-field strengths and are generically suppressed by $1/\Lambda^2$. Finally, in the third class we consider operators that, in weakly-coupled renormalizable theories, can only be induced at the loop level. They are summarised in Tab. 6.1. We repeat them here in Tab. 7.1.

$$\mathcal{O}_H = \frac{1}{2}(\partial^\mu |H|^2)^2 \qquad \mathcal{O}_T = \frac{1}{2}(H^\dagger \overset{\leftrightarrow}{D}_\mu H)^2 \qquad \mathcal{O}_6 = \lambda |H|^6$$

$$\mathcal{O}_W = \frac{ig}{2}(H^\dagger \sigma^a \overset{\leftrightarrow}{D^\mu} H)D^\nu W_{\mu\nu}^a \qquad \mathcal{O}_B = \frac{ig'}{2}(H^\dagger \overset{\leftrightarrow}{D^\mu} H)\partial^\nu B_{\mu\nu}$$

$$\mathcal{O}_{2W} = -\frac{1}{2}(D^\mu W_{\mu\nu}^a)^2 \qquad \mathcal{O}_{2B} = -\frac{1}{2}(\partial^\mu B_{\mu\nu})^2 \qquad \mathcal{O}_{2G} = -\frac{1}{2}(D^\mu G_{\mu\nu}^A)^2$$

$$\mathcal{O}_{BB} = g'^2 |H|^2 B_{\mu\nu} B^{\mu\nu} \qquad \mathcal{O}_{GG} = g_s^2 |H|^2 G_{\mu\nu}^A G^{A\mu\nu}$$

$$\mathcal{O}_{HW} = ig(D^\mu H)^\dagger \sigma^a (D^\nu H)W_{\mu\nu}^a \qquad \mathcal{O}_{HB} = ig'(D^\mu H)^\dagger (D^\nu H)B_{\mu\nu}$$

$$\mathcal{O}_{3W} = \frac{1}{3!}g\epsilon_{abc}W_\mu^{a\,\nu}W_{\nu\rho}^b W^{c\,\rho\mu} \qquad \mathcal{O}_{3G} = \frac{1}{3!}g_s f_{ABC}G_\mu^{A\,\nu}G_{\nu\rho}^B G^{C\,\rho\mu}$$

TABLE 7.1: *The operators are grouped in 3 different groups, separated with a solid line. In addition, there are the 6 CP-odd operators given in Eqs. (6.9)-(6.11).*

7.1   Non-renormalization of $h \to \gamma + \gamma/Z$ from JJ-operators

The operator basis introduced in the previous Section is particularly well-suited to describe new-physics contributions to $h \to \gamma\gamma$, which come only from two operators: the CP-even $\mathcal{O}_{BB}$ and the CP-odd $\mathcal{O}_{B\widetilde{B}}$. On the other hand, $h \to \gamma Z$ comes (on-shell) from $\mathcal{O}_{BB}$, $\mathcal{O}_{HB}$, $\mathcal{O}_{HW}$ and their CP-odd counterparts. The relevant Lagrangian terms for such decays are

$$\begin{aligned}
\delta\mathcal{L}_{\gamma\gamma} &= \frac{e^2}{2\Lambda^2}\Big[\kappa_{\gamma\gamma}\, h^2 F_{\mu\nu}F^{\mu\nu} + \kappa_{\gamma\widetilde{\gamma}}\, h^2 F_{\mu\nu}\widetilde{F}^{\mu\nu}\Big], \\
\delta\mathcal{L}_{\gamma Z} &= \frac{e\,G}{2\Lambda^2}\Big[\kappa_{\gamma Z}\, h^2 F_{\mu\nu}Z^{\mu\nu} + \kappa_{\gamma\widetilde{Z}}\, h^2 F_{\mu\nu}\widetilde{Z}^{\mu\nu}\Big],
\end{aligned} \tag{7.1}$$

where $e = gg'/G$ and $G^2 = g^2 + g'^2$. The photon field, $A_\mu = c_w B_\mu + s_w W_\mu^3$, has field-strength $F_{\mu\nu}$, while $Z_\mu = c_w W_\mu^3 - s_w B_\mu$ has field-strength $Z_{\mu\nu}$, where we use $s_w \equiv \sin\theta_w = g'/G$ and $c_w \equiv \cos\theta_w = g/G$. We have

$$\begin{aligned}
\kappa_{\gamma\gamma} = \kappa_{BB}, &\qquad \kappa_{\gamma Z} = \frac{1}{4}(\kappa_{HB} - \kappa_{HW}) - 2s_w^2\kappa_{BB}, \\
\kappa_{\gamma\widetilde{\gamma}} = \kappa_{B\widetilde{B}}, &\qquad \kappa_{\gamma\widetilde{Z}} = \frac{1}{4}(\kappa_{H\widetilde{B}} - \kappa_{H\widetilde{W}}) - 2s_w^2\kappa_{B\widetilde{B}}.
\end{aligned} \tag{7.2}$$

The Wilson coefficients of these dimension-six operators are generated at the scale Λ, at which the heavy new physics is integrated out, and they should be renormalized down to the Higgs mass, at which they are measured in Higgs decays. Let us focus for simplicity on $\kappa_{\gamma\gamma}$, as similar considerations will be applicable to $\kappa_{\gamma\widetilde{\gamma}}, \kappa_{\gamma Z}, \kappa_{\gamma\widetilde{Z}}$. At one-loop leading-log order one has, running from Λ to the Higgs mass m_h:

$$\kappa_{\gamma\gamma}(m_h) = \kappa_{\gamma\gamma}(\Lambda) - \gamma_{\gamma\gamma}\log\frac{\Lambda}{m_h}. \tag{7.3}$$

Here, $\gamma_{\gamma\gamma} = d\kappa_{\gamma\gamma}/d\log\mu$, with μ the energy scale, is the one-loop anomalous dimension for $\kappa_{\gamma\gamma}$. In principle, $\gamma_{\gamma\gamma}$ can depend on the Wilson coefficients of any dimension-six operator in Eq. (6.2). A particularly interesting case would be if the RGEs were to mix the tree-level operators into the RG evolution of one-loop suppressed operators, such as $\mathcal{O}_{BB}$. In that case we would expect $\gamma_{\gamma\gamma} \sim g_H^2/(16\pi^2)$ from mixings with the operators of Eq. (6.4), or $\gamma_{\gamma\gamma} \sim g^2/(16\pi^2)$ from mixings with (6.5). Such loop effect could give a sizeable contribution to $\kappa_{\gamma\gamma}(m_h)$, logarithmically enhanced by a factor $\log\Lambda/m_h$. The initial value $\kappa_{\gamma\gamma}(\Lambda)$, expected to be one-loop suppressed, would then be subleading.

    Remarkably, and this is our main result, there is no mixing from tree-level operators (6.4)-(6.5) to one-loop suppressed operators (6.6)-(6.11), at least at the one-loop level. This can be easily shown for the renormalization of $\kappa_{\gamma\gamma}$. The argument goes as follows.

Let us first consider the effects of the first-class operators, Eq. (6.4). Since these operators have four or more H, their contribution to the renormalization of $\kappa_{\gamma\gamma}$ can only arise from a loop of the electrically-charged $G^{\pm}$ with at least one photon attached to the loop. However,

- $\mathcal{O}_6$ has too many Higgs legs to contribute.

- $\mathcal{O}_H$ is simply $\partial_\mu(h^2 + G_0^2 + 2G^+G^-)\partial^\mu(h^2 + G_0^2 + 2G^+G^-)/8$ and this momentum structure implies that a $G^{\pm}$ loop can only give a contribution $\propto \partial_\mu h^2$, which is not the Higgs momentum structure of Eq. (7.1).

- $\mathcal{O}_T$ does not contain a vertex $h^2 G^+G^-$.

- $\mathcal{O}_r$ can be traded with $\mathcal{O}_y$, which clearly can only give one-loop contributions to operators $\propto |H|^2 H$, so it only contributes to the RGE of itself and $\mathcal{O}_6$.

We conclude that there is no contribution from these operators to the RGE of $\kappa_{\gamma\gamma}$. To generalise the proof that no operator in (6.4) contributes to the one-loop anomalous-dimension of any operator in (6.6)-(6.8) [1], we have calculated explicitly the one-loop operator-mixing. We find that the only operators involving two Higgs and gauge bosons that can be affected by (6.4) are the tree-level operators (6.5). The result is given in Sec. 4.

For the operators of Eq. (6.5), proving the absence of one-loop contributions to the anomalous dimension of (6.6)-(6.8) is even simpler. By means of field redefinitions, as those given in the Appendix A, or, equivalently, by using the equations of motion [2], we can trade the operators (6.5) with operators of Eq. (6.4), four-fermion operators and operators of the type

$$
\begin{aligned}
\mathcal{O}_R^f &= (i\, H^\dagger \overset{\leftrightarrow}{D}_\mu H)(\bar{f}_R\gamma^\mu f_R), \\
\mathcal{O}_L^f &= (i\, H^\dagger \overset{\leftrightarrow}{D}_\mu H)(\bar{f}_L\gamma^\mu f_L), \\
\mathcal{O}_L^{f\,(3)} &= (i\, H^\dagger \sigma^a \overset{\leftrightarrow}{D}_\mu H)(\bar{f}_L\gamma^\mu \sigma^a f_L).
\end{aligned}
\tag{7.4}
$$

Now, four-fermion operators contain too many fermion legs to contribute to operators made only of SM bosons. Concerning the operators of Eq. (7.4), after closing the fermion legs in a loop, it is clear that they can only give contributions to operators with the Higgs structure $H^\dagger \overset{\leftrightarrow}{D}_\mu H$ or $H^\dagger \sigma^a \overset{\leftrightarrow}{D}_\mu H$, corresponding to the tree-level operators (6.5). This

---

[1] Obviously, their contribution to the CP-odd operators (6.9)-(6.11) is zero as the SM gauge-boson couplings conserve CP.

[2] That is, $2D^\nu W_{\mu\nu}^a = igH^\dagger\sigma^a\overset{\leftrightarrow}{D}_\mu H + g\bar{f}_L\sigma^a\gamma_\mu f_L$ and $\partial^\nu B_{\mu\nu} = ig'H^\dagger\overset{\leftrightarrow}{D}_\mu H/2 + g'Y_L^f\bar{f}_L\gamma_\mu f_L + g'Y_R^f\bar{f}_R\gamma_\mu f_R$, where $Y_{L,R}^f$ are the fermion hypercharges and a sum over fermions is understood.

completes the proof that no current-current operator contributes to the running of any one-loop suppressed operator.

The calculation above could have also been done in other operator bases. To keep the calculation simple, it is crucial to work in bases that do not mix current-current operators with one-loop suppressed ones. This is guaranteed if we change basis by means of SM-field redefinitions, as shown in Appendix A. We can make use of these field-redefinitions to work in bases that contain only 3 operators made of bosons, the rest consisting of operators involving fermions, such as those in Eq. (6.19), Eq. (7.4) or 4-fermion operators. There are different options in choosing these 3 operators; what is physically relevant are the 3 (shift-invariant) combinations of coefficients in Eq. (D.22). This freedom can be used to select the set of 3 operators most convenient to prove, in the simplest way, that their contribution to the running of $\kappa_{\gamma\gamma}$ and $\kappa_{Z\gamma}$ is zero at the one-loop level. For example, we could have chosen $\mathcal{O}_{2B}$ instead of $\mathcal{O}_T$: since $\mathcal{O}_{2B}$ only affects the propagator of the neutral state B^μ, one can easily see that it cannot contribute to the $h\gamma\gamma$ or $h\gamma Z$ coupling.

7.2 The importance of the choice of basis

The relevance of the possible contributions from tree-level operators to the one-loop RGE of $\kappa_{\gamma\gamma}$ and $\kappa_{\gamma Z}$ has been highlighted recently in Ref. [123]. In fact, that analysis claims that such important effect could actually occur, in contradiction with the results presented in the previous Section. In this Section we show how this contradiction is resolved.

The analysis in Ref. [123], GJMT in what follows, focuses on a subset of dimension-six operators, chosen to be $\mathcal{O}_{BB}$ and the two operators

$$\mathcal{O}_{WB} = gg'(H^\dagger \sigma^a H) W^a_{\mu\nu} B^{\mu\nu} \ , \quad \mathcal{O}_{WW} = g^2 |H|^2 W^a_{\mu\nu} W^{a\mu\nu} \ , \tag{7.5}$$

which are not included in the basis we have used. The relation to our basis follows from the two operator identities:

$$\mathcal{O}_B = \mathcal{O}_{HB} + \frac{1}{4}\mathcal{O}_{WB} + \frac{1}{4}\mathcal{O}_{BB} \ , \tag{7.6}$$

$$\mathcal{O}_W = \mathcal{O}_{HW} + \frac{1}{4}\mathcal{O}_{WW} + \frac{1}{4}\mathcal{O}_{WB} \ , \tag{7.7}$$

which allow us to remove $\mathcal{O}_{WW}$ and $\mathcal{O}_{WB}$ in favor of $\mathcal{O}_B$ and $\mathcal{O}_W$. The two operators $\mathcal{O}_{HW}$ and $\mathcal{O}_{HB}$ were also mentioned in Ref. [123], although their effect was not included in the analysis. To understand the issues involved it will be sufficient to limit the

operator basis to five operators, with the two bases used being

$$B_1 = \{\mathcal{O}_{BB}, \mathcal{O}_B, \mathcal{O}_W, \mathcal{O}_{HW}, \mathcal{O}_{HB}\}, \quad \text{(this work)} \tag{7.8}$$

$$B_2 = \{\mathcal{O}_{BB}, \mathcal{O}_{WW}, \mathcal{O}_{WB}, \mathcal{O}_{HW}, \mathcal{O}_{HB}\}, \quad \text{(GJMT) [123].} \tag{7.9}$$

In relating both bases we will use primed Wilson coefficients for the GJMT basis

$$\mathcal{L}_6 = \sum_i \frac{c'_i}{\Lambda^2} \mathcal{O}_i, \tag{7.10}$$

and the dictionary to translate between B_1 and B_2 is:

$$
\begin{aligned}
\kappa_{HW} &= c'_{HW} - 4c'_{WW}, \\
\kappa_{HB} &= c'_{HB} + 4(c'_{WW} - c'_{WB}), \\
\kappa_{BB} &= c'_{BB} + c'_{WW} - c'_{WB}, \\
c_W &= 4c'_{WW}, \\
c_B &= 4(c'_{WB} - c'_{WW}).
\end{aligned}
\tag{7.11}
$$

From these relations we can directly write the expressions for $\kappa_{\gamma\gamma}$ and $\kappa_{\gamma Z}$ going from (7.2) to the GJMT basis:

$$
\begin{aligned}
\kappa_{\gamma\gamma} &= c'_{BB} + c'_{WW} - c'_{WB}, \\
\kappa_{\gamma Z} &= 2c_w^2 c'_{WW} - 2s_w^2 c'_{BB} - (c_w^2 - s_w^2)c'_{WB} + \frac{1}{4}(c'_{HB} - c'_{HW}).
\end{aligned}
\tag{7.12}
$$

Let us first note that the operator identities (7.6) and (7.7) show that two operators of the GJMT basis, $\mathcal{O}_{WW}$ and $\mathcal{O}_{WB}$, are a mixture of tree-level operators and one-loop suppressed ones of basis B_1. This has the following drawback. Let us suppose that the operator $\mathcal{O}_W$ is generated, for example, by integrating out a heavy SU(2)-triplet gauge boson (see *e.g.* Ref. [125]). This operator can be written in the GJMT basis by using the identity (7.7), but then the coefficients of the operators $\mathcal{O}_{WW}$, $\mathcal{O}_{WB}$ and $\mathcal{O}_{HW}$ generated in this way will all be correlated. In this particular example, we will have $c'_{WW} = c'_{WB} = c'_{HW}/4$. This is telling us that when using the GJMT basis to study the physical impact of this scenario we must include the effects of all operators, and not only a partial list of them, as done in Ref. [123]. Otherwise, one can miss contributions of the same size that could lead to cancellations. The same argument goes through for scenarios generating the tree-level operator $\mathcal{O}_B$. In general, the correlation of the

coefficients in the GJMT basis is explicitly shown in the reversed dictionary:

$$
\begin{aligned}
c'_{WW} &= \frac{1}{4}c_W \,, \\
c'_{WB} &= \frac{1}{4}(c_B + c_W) \,, \\
c'_{BB} &= \frac{1}{4}c_B + \kappa_{BB} \,, \\
c'_{HW} &= c_W + \kappa_{HW} \,, \\
c'_{HB} &= c_B + \kappa_{HB} \,.
\end{aligned}
\tag{7.13}
$$

Obviously, physics does not depend on what basis is used, which is a matter of choice, as long as the full calculation is done in both bases. Reducing, however, the calculations to a few operators in a given basis can be dangerous as this can leave out important effects. This is especially true in bases whose operators are a mixture of operators with Wilson coefficients of different sizes. For this reason the basis B_1 is preferable to B_2.

To explicitly show how this correlation between Wilson coefficients can lead to cancellations in the final result, let us consider a particularly simple example: the calculation of the radiative corrections to the operators $\mathcal{O}_{WW}$, $\mathcal{O}_{BB}$ and $\mathcal{O}_{WB}$ proportional to λ. This is partly given in the analysis of Ref. [123], apparently showing a one-loop mixing from tree-level operators to one-loop suppressed ones. As obtained in Ref. [123], the λ-dependent piece of the anomalous-dimension matrix for $c'_{BB}, c'_{WW}, c'_{WB}$ is given by

$$
\frac{d}{d\log\mu}
\begin{bmatrix} c'_{BB} \\ c'_{WW} \\ c'_{WB} \end{bmatrix}
= \frac{1}{16\pi^2}
\begin{pmatrix} 12\lambda & 0 & 0 \\ 0 & 12\lambda & 0 \\ 0 & 0 & 4\lambda \end{pmatrix}
\begin{bmatrix} c'_{BB} \\ c'_{WW} \\ c'_{WB} \end{bmatrix}
+ \dots .
\tag{7.14}
$$

From (7.12), one obtains the RGE

$$
\gamma_{\gamma\gamma} = \frac{d\kappa_{\gamma\gamma}}{d\log\mu} = \frac{4\lambda}{16\pi^2}(3\kappa_{\gamma\gamma} + 2c'_{WB}) + \dots ,
\tag{7.15}
$$

showing explicitly that the coefficient c'_{WB}, which can be of tree-level size in the GJMT basis [see (7.13)], affects the running of the one-loop suppressed $\kappa_{\gamma\gamma}$. This apparent contradiction with our previous result is, as expected, resolved by adding the effect of the operators $\mathcal{O}_{HW}$ and $\mathcal{O}_{HB}$ in the renormalization of $\kappa_{\gamma\gamma}$. We obtain the (λ-dependent) contributions

$$
\frac{dc'_{BB}}{d\log\mu} = -\frac{3\lambda}{16\pi^2}c'_{HB} \,, \quad
\frac{dc'_{WW}}{d\log\mu} = -\frac{3\lambda}{16\pi^2}c'_{HW} \,, \quad
\frac{dc'_{WB}}{d\log\mu} = -\frac{\lambda}{16\pi^2}(c'_{HB} + c'_{HW}) \,,
\tag{7.16}
$$

which change the RGE (7.15) into

$$\gamma_{\gamma\gamma} = \frac{2\lambda}{16\pi^2}(6\kappa_{\gamma\gamma} + 4c'_{WB} - c'_{HB} - c'_{HW}) \ . \tag{7.17}$$

These additional contributions eliminate the possibly sizeable tree-level correction from c'_{WB}. Indeed, using (7.13), we explicitly see that the contributions proportional to c_W and c_B cancel out, giving

$$\gamma_{\gamma\gamma} = \frac{2\lambda}{16\pi^2}\Big(6\kappa_{\gamma\gamma} - \kappa_{HB} - \kappa_{HW}\Big) \ , \tag{7.18}$$

leaving behind just corrections from one-loop suppressed operators. This is not an accident: this cancellation was expected from our discussion in the previous Section. Beyond the λ-dependent terms we have examined, the same cancellation will necessarily occur for the rest of the potentially sizeable contributions to $\gamma_{\gamma\gamma}$ identified in Ref. [123].

7.3 Renormalization group equation for $\kappa_{\gamma\gamma}$ and $\kappa_{\gamma\widetilde{\gamma}}$

In this Section we use the results of Ref. [123], combined with our results in Sec. 7.1, to obtain $\gamma_{\gamma\gamma}$. Let us write the RGEs for the Wilson coefficients in basis B_2 in a compact way as

$$16\pi^2 \frac{dc'_i}{d\log\mu} = \sum_{j=1}^{5} b'_{i,j} c'_j \ . \tag{7.19}$$

The $b'_{i,j}$ is a 5×5 anomalous-dimension matrix of which the 3×3 submatrix corresponding to $i,j = 1-3$ (that is, c'_{BB}, c'_{WW}, c'_{WB}) was calculated in Ref. [123], while the rest is unknown. From $\kappa_{\gamma\gamma} = \sum_{i=1}^{5} \zeta_i c'_i$ where $\zeta_i = (1,1,-1,0,0)$, we have

$$16\pi^2 \gamma_{\gamma\gamma} = \sum_{i,j=1}^{5} \zeta_i b'_{i,j} c'_j \ . \tag{7.20}$$

Using Eq. (7.13), we can translate this anomalous dimension to our basis. We get

$$
\begin{aligned}
16\pi^2 \gamma_{\gamma\gamma} &= \sum_{i=1}^{5} \zeta_i (b'_{i,BB}\kappa_{BB} + b'_{i,HW}\kappa_{HW} + b'_{i,HB}\kappa_{HB}) \\
&+ \frac{1}{4}c_B \sum_{i=1}^{5} \zeta_i (b'_{i,WB} + b'_{i,BB} + 4b'_{i,HB}) + \frac{1}{4}c_W \sum_{i=1}^{5} \zeta_i (b'_{i,WW} + b'_{i,WB} + 4b'_{i,HW}) \ .
\end{aligned}
\tag{7.21}
$$

From our discussion in Sec. 7.1, we know that the tree-level coefficients c_B and c_W do not appear in this RGE. This means that the two last terms of Eq. (7.21) must be zero, allowing us to extract the sum of the unknown coefficients $b'_{i,HB}$ and $b'_{i,HW}$ in terms of

coefficients calculated in Ref. [123]:

$$\sum_{i=1}^{5} \zeta_i b'_{i,HB} = -\frac{1}{4} \sum_{i=1}^{5} \zeta_i (b'_{i,WB} + b'_{i,BB}) \,, \quad \sum_{i=1}^{5} \zeta_i b'_{i,HW} = -\frac{1}{4} \sum_{i=1}^{5} \zeta_i (b'_{i,WW} + b'_{i,WB}) \,.$$
(7.22)

Notice that $\zeta_4 = \zeta_5 = 0$ is crucial to allow us to restrict the sums in the right-hand-side to terms that were already calculated in Ref. [123]. Plugging the terms (7.22) back in (7.21), one gets

$$16\pi^2 \gamma_{\gamma\gamma} = \sum_{i=1}^{5} \zeta_i \left[b'_{i,BB} \kappa_{BB} - \frac{1}{4}(b'_{i,WB} + b'_{i,WW})\kappa_{HW} - \frac{1}{4}(b'_{i,BB} + b'_{i,WB})\kappa_{HB} \right] \,.$$
(7.23)

Using the coefficients $b'_{i,WW}, b'_{i,WB}$ and $b'_{i,BB}$ from Ref. [123], one arrives at

$$16\pi^2 \gamma_{\gamma\gamma} = \left[6y_t^2 - \frac{3}{2}(3g^2 + g'^2) + 12\lambda \right] \kappa_{BB} + \left[\frac{3}{2}g^2 - 2\lambda \right] (\kappa_{HW} + \kappa_{HB}) \,.$$
(7.24)

This expression gives the one-loop leading-log correction to $\kappa_{\gamma\gamma}(m_h)$. For the resummation of the log terms we would need the full anomalous-dimension matrix. Nevertheless, this is not needed for $\Lambda \sim$ TeV since the log-terms are not very large.

The size of the contributions of Eq. (7.24) to $\kappa_{\gamma\gamma}(m_h)$ is expected to be of two-loop order in minimally-coupled theories. Therefore, we have to keep in mind that the tree-level operators of Eq. (6.4), possibly entering in the RGE of $\kappa_{\gamma\gamma}$ at the two-loop level, could give corrections of the same order. For strongly-coupled theories in which $g_H \sim 4\pi$, we could have $\kappa_i \sim O(1)$, and the corrections from Eq. (7.24) to $h \to \gamma\gamma$ could be of one-loop size. Of course, in principle, the initial values $\kappa_i(\Lambda)$ will give, as Eq. (7.2) shows, the dominant contribution to $h \to \gamma\gamma, \gamma Z$ and not Eq. (7.24). Nevertheless, it could well be the case that $|\kappa_{BB}(\Lambda)| \ll 1$ and $|\kappa_{HB}(\Lambda) - \kappa_{HW}(\Lambda)| \ll 1$ due to symmetries of the new-physics sector. For example, if the Higgs is a pseudo-Goldstone boson arising from a new strong-sector, $\kappa_{BB}(\Lambda)$ is protected by a shift symmetry and can only be generated by loops involving SM couplings, while $\kappa_{HB}(\Lambda) = \kappa_{HW}(\Lambda) \sim g_H^2/(16\pi^2)$ if the strong sector has an accidental custodial O(4) symmetry [3] [99]. In this case Eq. (7.24) could give the main correction to the SM decay $h \to \gamma\gamma$ and could be as large as $\Delta\Gamma_{\gamma\gamma}/\Gamma_{\gamma\gamma}^{SM} \sim g^2 v^2/\Lambda^2 \log(\Lambda/m_h)$ if $g_H \sim 4\pi$. Notice also that there can be finite one-loop corrections to $\kappa_{\gamma\gamma}(m_h)$ from the operators (6.4) and (6.5) which can dominate over those in Eq. (7.24). These were calculated in Ref. [99].

---

[3] We have $O(4) \simeq SU(2)_L \times SU(2)_R \times P_{LR}$ under which P_{LR} interchange $L \leftrightarrow R$. Under this P_{LR} we have $c_{HW} \leftrightarrow c_{HB}$. To make the transformation properties under this symmetry more manifest, it is better to work with $\mathcal{O}_{WB}$, which is even under P_{LR}, instead of $\mathcal{O}_{BB}$.

A similar analysis can be performed for $\kappa_{\gamma\tilde{\gamma}}$, with the simplification that the operator identities corresponding to Eqs. (7.6) and (7.7) are, for the dual field strengths:

$$\mathcal{O}_{H\tilde{B}} + \frac{1}{4}\mathcal{O}_{W\tilde{B}} + \frac{1}{4}\mathcal{O}_{B\tilde{B}} = 0 \,, \tag{7.25}$$

$$\mathcal{O}_{H\widetilde{W}} + \frac{1}{4}\mathcal{O}_{W\widetilde{W}} + \frac{1}{4}\mathcal{O}_{W\tilde{B}} = 0 \,, \tag{7.26}$$

due to the Bianchi identity. The above equations do not mix tree and loop generated operators; hence, from the calculation of Ref. [123] with the set $\{\mathcal{O}_{B\tilde{B}}, \mathcal{O}_{W\widetilde{W}}, \mathcal{O}_{W\tilde{B}}\}$ one can obtain the $\gamma_{\gamma\tilde{\gamma}}$ in terms of the coefficients of the operators $\{\mathcal{O}_{B\tilde{B}}, \mathcal{O}_{H\tilde{B}}, \mathcal{O}_{H\widetilde{W}}\}$ of our basis. One arrives at the expected result: $\gamma_{\gamma\tilde{\gamma}} = d\kappa_{\gamma\tilde{\gamma}}/d\log\mu$ is given by the same expression as $\gamma_{\gamma\gamma}$ but with the corresponding CP-odd coefficients instead of the CP-even ones.

7.4   RGEs for $\kappa_{\gamma Z}$ and $\kappa_{\gamma\tilde{Z}}$ and a new basis

If we try to obtain the RGE for $\kappa_{\gamma Z}$ in the same way as for $\kappa_{\gamma\gamma}$, we face the complication that $\kappa_{\gamma Z}$ depends not only on c'_{BB}, c'_{WW} and c'_{WB}, but also on c'_{HB} and c'_{HW}, and these coefficients were not included in the calculation presented in Ref. [123]. In other words, one would need to calculate the anomalous-dimension matrix elements $b'_{i,j}$ for $i = \{HW, HB\}$ and $j = \{WW, WB, BB\}$, or, in our basis, to complete the 3×3 anomalous-dimension matrix for $\kappa_{BB}, \kappa_{HW}, \kappa_{HB}$.

We can circumvent this difficulty by realizing that the operators $\mathcal{O}_{WW}, \mathcal{O}_{BB}$ and $\mathcal{O}_{WB}$ do not enter in the (one-loop) RGEs for c'_{HW} and c'_{HB}, so that the matrix elements required to get $\gamma_{\gamma Z}$ are in fact zero. In order to see this, notice that both $\mathcal{O}_{HW}$ and $\mathcal{O}_{HB}$ include the trilinear pieces (with two Higgses and one gauge boson):

$$\begin{aligned}
\mathcal{O}_{HW} &= 2ig(\partial^\mu H)^\dagger \sigma^a (\partial^\nu H)\partial_\mu W_\nu^a + \cdots \,, \\
\mathcal{O}_{HB} &= 2ig'(\partial^\mu H)^\dagger (\partial^\nu H)\partial_\mu B_\nu + \cdots \,,
\end{aligned} \tag{7.27}$$
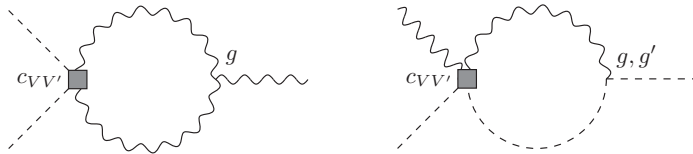
Figure 7.1: *The only two diagrams that could give a contribution (at one loop) from $\mathcal{O}_{WW}$, $\mathcal{O}_{BB}$ and $\mathcal{O}_{WB}$ (with coefficient generically denoted as $c_{VV'}$ in the figure) to the renormalization of $\mathcal{O}_{HW}$ and $\mathcal{O}_{HB}$ (or to $\mathcal{O}_W$ and $\mathcal{O}_B$).*

while $\mathcal{O}_{WW}, \mathcal{O}_{BB}$ and $\mathcal{O}_{WB}$ have two Higgses and at least two gauge bosons. Therefore, in order to generate (at one loop) trilinears like those in (7.27), the only possibility is that one of the two gauge boson legs is attached to the other gauge boson leg or to one of the Higgs legs (see Fig. 7.1). In the first case (Fig. 7.1, left diagram) it is clear that the resulting Higgs structure for the operator generated is either $|H|^2$ or $H^\dagger \sigma^a H$ and not that in (7.27) (in fact, the diagram is zero). In the second case (Fig. 7.1, right diagram) the only structures that result are either $\partial^\mu H^\dagger \partial^\nu (H B_{\mu\nu})$ or $\partial^\mu H^\dagger \sigma^a \partial^\nu (H W^a_{\mu\nu})$, which give zero after integrating by parts.

We can therefore extract $\gamma_{\gamma Z}$ following the same procedure used for $\gamma_{\gamma\gamma}$ in the previous Section, and we obtain

$$16\pi^2 \gamma_{\gamma Z} = \kappa_{\gamma Z} \left[6y_t^2 + 12\lambda - \frac{7}{2}g^2 - \frac{1}{2}g'^2 \right] + (\kappa_{HW} + \kappa_{HB}) \left[2g^2 - 3e^2 - 2\lambda \cos(2\theta_w) \right] ,$$
(7.28)

and a similar expression for $\gamma_{\gamma\widetilde{Z}}$ with the corresponding CP-odd operator coefficients instead of the CP-even ones.

The arguments we have used to prove that $\mathcal{O}_{WW}, \mathcal{O}_{BB}$ and $\mathcal{O}_{WB}$ do not enter into the anomalous dimensions of $\mathcal{O}_{HW}$ and $\mathcal{O}_{HB}$ can be applied in exactly the same way to prove that they do not generate radiatively the operators $\mathcal{O}_W$ and $\mathcal{O}_B$ which have exactly the same trilinear structures displayed in Eq. (7.27) for $\mathcal{O}_{HW}$ and $\mathcal{O}_{HB}$. This immediately implies that the 5×5 matrix of anomalous dimensions will be block diagonal if instead of using the bases in (7.8) and (7.9), we use instead the basis

$$B_3 = \{\mathcal{O}_{BB}, \mathcal{O}_{WW}, \mathcal{O}_{WB}, \mathcal{O}_W, \mathcal{O}_B\} .$$
(7.29)

Calling $\hat{c}_i, \hat{\kappa}_i$ the operator coefficients in this basis, we have

$$\frac{d}{d\log\mu} \begin{pmatrix} \hat{\kappa}_{BB} \\ \hat{\kappa}_{WW} \\ \hat{\kappa}_{WB} \\ \hat{c}_W \\ \hat{c}_B \end{pmatrix} = \begin{pmatrix} \hat{\Gamma} & 0_{3\times 2} \\ 0_{2\times 3} & \hat{X} \end{pmatrix} \begin{pmatrix} \hat{\kappa}_{BB} \\ \hat{\kappa}_{WW} \\ \hat{\kappa}_{WB} \\ \hat{c}_W \\ \hat{c}_B \end{pmatrix} .$$
(7.30)

Taking the anomalous-dimension matrix in the simple form (7.30) as starting point, it is a trivial exercise to transform it to other bases. In the GJMT basis one gets

$$\frac{d}{d\log\mu}\begin{pmatrix} c'_{BB} \\ c'_{WW} \\ c'_{WB} \\ c'_{HW} \\ c'_{HB} \end{pmatrix} = \begin{pmatrix} \hat{\Gamma} & Y' \\ 0_{2\times3} & \hat{X} \end{pmatrix}\begin{pmatrix} c'_{BB} \\ c'_{WW} \\ c'_{WB} \\ c'_{HW} \\ c'_{HB} \end{pmatrix}. \tag{7.31}$$

The 3×3 upper-left block is therefore given by the expression calculated in Ref. [123]:

$$\hat{\Gamma} = \frac{1}{16\pi^2}\begin{pmatrix} 6y_t^2 + 12\lambda - \frac{9}{2}g^2 + \frac{1}{2}g'^2 & 0 & 3g^2 \\ 0 & 6y_t^2 + 12\lambda - \frac{5}{2}g^2 - \frac{3}{2}g'^2 & g'^2 \\ 2g'^2 & 2g^2 & 6y_t^2 + 4\lambda + \frac{9}{2}g^2 - \frac{1}{2}g'^2 \end{pmatrix}, \tag{7.32}$$

while the 2×2 lower-right block $\hat{X}$ has not been fully calculated in the literature. This lack of knowledge affects also the 3×2 block Y', which depends on the entries of $\hat{X}$.

In basis B_1 one gets instead:

$$\frac{d}{d\log\mu}\begin{pmatrix} \kappa_{BB} \\ \kappa_{HW} \\ \kappa_{HB} \\ c_W \\ c_B \end{pmatrix} = \begin{pmatrix} \Gamma & 0_{3\times2} \\ Y & \hat{X} \end{pmatrix}\begin{pmatrix} \kappa_{BB} \\ \kappa_{HW} \\ \kappa_{HB} \\ c_W \\ c_B \end{pmatrix}, \tag{7.33}$$

where now

$$\Gamma = \frac{1}{16\pi^2}\begin{pmatrix} 6y_t^2 + 12\lambda - \frac{9}{2}g^2 - \frac{3}{2}g'^2 & \frac{3}{2}g^2 - 2\lambda & \frac{3}{2}g^2 - 2\lambda \\ 0 & 6y_t^2 + 12\lambda - \frac{5}{2}g^2 - \frac{1}{2}g'^2 & g'^2 \\ -8g'^2 & 9g^2 - 8\lambda & 6y_t^2 + 4\lambda + \frac{9}{2}g^2 + \frac{1}{2}g'^2 \end{pmatrix}, \tag{7.34}$$

while Y is also dependent on the unknown coefficients of $\hat{X}$.[4] We can reexpress Γ in terms of the physically relevant combinations of coefficients $\kappa_{\gamma\gamma}$ and $\kappa_{\gamma Z}$ defined in (7.2) plus the orthogonal combination $\kappa_{ort} \equiv \kappa_{HW} + \kappa_{HB}$. One gets

$$\frac{d}{d\log\mu}\begin{pmatrix} \kappa_{\gamma\gamma} \\ \kappa_{\gamma Z} \\ \kappa_{ort} \end{pmatrix} = \Gamma_o\begin{pmatrix} \kappa_{\gamma\gamma} \\ \kappa_{\gamma Z} \\ \kappa_{ort} \end{pmatrix}, \tag{7.35}$$

where

$$\Gamma_o = \frac{1}{16\pi^2}\begin{pmatrix} 6y_t^2 + 12\lambda - \frac{9}{2}g^2 - \frac{3}{2}g'^2 & 0 & \frac{3}{2}g^2 - 2\lambda \\ 0 & 6y_t^2 + 12\lambda - \frac{7}{2}g^2 - \frac{1}{2}g'^2 & 2g^2 - 3e^2 - 2\lambda\cos(2\theta_w) \\ -16e^2 & -4g^2 + 4g'^2 & 6y_t^2 + 4\lambda + \frac{11}{2}g^2 + \frac{1}{2}g'^2 \end{pmatrix}, \tag{7.36}$$

---

[4]Note that the lower-right block $\hat{X}$ is exactly the same in all the three bases considered.

from which we explicitly see that $\kappa_{\gamma Z}$ does not renormalize $\kappa_{\gamma\gamma}$ and vice versa.

We have seen that the expression for the anomalous-dimension matrix takes the simplest block-diagonal form in basis B_3. This basis has also the virtue of B_1 of keeping separated current-current operators from one-loop suppressed ones. Indeed, using Eqs. (7.6) and (7.7), we can reach B_3 from B_1 by trading two one-loop suppressed operators, $\mathcal{O}_{HW}$ and $\mathcal{O}_{HB}$, by other two one-loop suppressed ones, $\mathcal{O}_{WW}$ and $\mathcal{O}_{WB}$. In spite of the fact that the anomalous-dimension matrix gets its simplest form in basis B_3, there are other advantages in using basis B_1. For example, in B_1 only one operator contributes to $h \to \gamma\gamma$, while there are three in basis B_3. Also B_1 is a more suitable basis to describe the low-energy effective theory expected for a pseudo-Goldstone Higgs boson [99], as it clearly identifies operators invariant under constant shifts $H \to H + c$.

7.5 Dipole operators

The above analysis can be easily extended to include contributions from operators involving SM fermions. We will limit the discussion here to the up-quark sector, having in mind possible large contributions from the top. The extension to other SM fermions is straightforward. We organize again the operators as tree-level and one-loop suppressed ones. Among the first type we have the operators already given in Eq. (6.19), Eq. (7.4), apart from four-fermion operators. In Sec. 7.1, however, we already showed that they cannot contribute to the anomalous dimension of the operators (6.6)-(6.11) at the one-loop level. Among one-loop suppressed operators made with SM fermions, we have the dipole operators

$$
\begin{aligned}
\mathcal{O}_{DB} &= y_u \bar{Q}_L \sigma^{\mu\nu} u_R \, \widetilde{H} g' B_{\mu\nu} \,, \\
\mathcal{O}_{DW} &= y_u \bar{Q}_L \sigma^{\mu\nu} u_R \, \sigma^a \widetilde{H} g W^a_{\mu\nu} \,, \\
\mathcal{O}_{DG} &= y_u \bar{Q}_L \sigma^{\mu\nu} T^a u_R \, \widetilde{H} g_s G^a_{\mu\nu} \,,
\end{aligned}
\tag{7.37}
$$

where T^a are the $SU(3)_C$ generators. These operators can, in principle, give contributions to other one-loop suppressed operators, as those relevant for $h \to \gamma\gamma, \gamma Z$. We have calculated that, indeed, such contributions are nonzero:

$$
\begin{aligned}
16\pi^2 \gamma_{\gamma\gamma} &= 8 y_u^2 N_c Q_u \mathrm{Re}[\kappa_{DB} + \kappa_{DW}] \,, \\
16\pi^2 \gamma_{\gamma\widetilde{\gamma}} &= -8 y_u^2 N_c Q_u \mathrm{Im}[\kappa_{DB} + \kappa_{DW}] \,, \\
16\pi^2 \gamma_{\gamma Z} &= 4 y_u^2 N_c \left\{ \left(\frac{1}{2} - 4 Q_u s_w^2 \right) \mathrm{Re}[\kappa_{DB}] + \left(\frac{1}{2} + 2 Q_u c_{2w} \right) \mathrm{Re}[\kappa_{DW}] \right\} \,, \\
16\pi^2 \gamma_{\gamma\widetilde{Z}} &= -4 y_u^2 N_c \left\{ \left(\frac{1}{2} - 4 Q_u s_w^2 \right) \mathrm{Im}[\kappa_{DB}] + \left(\frac{1}{2} + 2 Q_u c_{2w} \right) \mathrm{Im}[\kappa_{DW}] \right\} \,, \tag{7.38}
\end{aligned}
$$

where $N_c = 3$, $Q_u = 2/3$ is the electric charge of the up-quark, $c_{2w} = \cos(2\theta_w)$, and the κ_i are the one-loop suppressed coefficients of the operators of Eq. (7.37), *i.e.* $\delta\mathcal{L} = \kappa_i \mathcal{O}_i / \Lambda^2 + \text{h.c.}$. In the B_3 basis, Eq. (7.38) arises from

$$\frac{d}{d\log\mu}\begin{pmatrix} \hat\kappa_{BB} \\ \hat\kappa_{WW} \\ \hat\kappa_{WB} \end{pmatrix} = \frac{4N_c y_u^2}{16\pi^2}\begin{pmatrix} 0 & Y_L^u + Y_R^u \\ 1/2 & 0 \\ -(Y_L^u + Y_R^u) & -1/2 \end{pmatrix}\begin{pmatrix} \hat\kappa_{DW} \\ \hat\kappa_{DB} \end{pmatrix}, \qquad (7.39)$$

where $Y_L^u = 1/6$ and $Y_R^u = 2/3$ are the up-quark hypercharges. Similar results follow for the RGE of the Higgs couplings to gluons, κ_{GG} and $\kappa_{G\widetilde{G}}$

$$16\pi^2\gamma_{GG} = 4y_u^2\text{Re}[\kappa_{DG}], \quad 16\pi^2\gamma_{G\widetilde{G}} = -4y_u^2\text{Im}[\kappa_{DG}]. \qquad (7.40)$$

7.6   The S parameter

As we have shown above, the Wilson coefficients of the current-current operators (6.4)-(6.5) do not enter in the one-loop RGEs of the κ_i, but only in their own RGEs. In particular, the only operators with two Higgs bosons and gauge bosons affected by $c_{H,T}$ at one loop are $\mathcal{O}_W$ and $\mathcal{O}_B$ and not those relevant for $h \to \gamma\gamma, \gamma Z$. Indeed, an explicit calculation gives

$$\gamma_W = \frac{dc_W}{d\log\mu} = -\frac{g_H^2}{16\pi^2}\frac{1}{3}(c_H + c_T), \quad \gamma_B = \frac{dc_B}{d\log\mu} = -\frac{g_H^2}{16\pi^2}\frac{1}{3}(c_H + 5c_T). \qquad (7.41)$$

In the basis B_1 of Sec. 6.1, these are the only two Wilson coefficients that enter in the S-parameter [126]. We have $S = 4\pi v^2[c_W(m_Z) + c_B(m_Z)]/\Lambda^2$ where $c_{W,B}(m_Z)$ is the value of the coefficient at the Z mass. The contributions from Eq. (7.41) to $c_{W,B}(m_Z)$ can be sizeable for $g_H \gg 1$ [127], although the value of c_T is highly constrained from the T-parameter [99]. The anomalous dimensions γ_W and γ_B can also receive corrections proportional to $c_{W,B}$, or from one-loop suppressed operators, such as $\mathcal{O}_{BB}$. Nevertheless these contributions are not expected to be sizeable. The coefficients c_W and c_B already contribute at tree-level to S, while the contributions to S from κ_i are expected to be small, $\delta\gamma_W = O(\kappa_i/(16\pi^2))$. Notice that basis B_1 makes very clear the separation between the relevant contributions to S that come from tree-level operators and those to $\kappa_{\gamma\gamma}$, which are from one-loop suppressed operators.

In the GJMT basis the contribution to S arises from the operator $\mathcal{O}_{WB}$ and one has $S = 16\pi v^2 c'_{WB}(m_Z)/\Lambda^2$. In Ref. [123], a partial calculation of the anomalous dimension of $\mathcal{O}_{WB}$ was given. Nevertheless, if the interest is to calculate the running of c'_{WB} in universal theories in which c_W and c_B encode the dominant effects [apart from $c_{H,T}$

whose effects are given in Eq. (7.41)], one also needs, as Eq. (7.13) shows, to include the effects of c'_{HW} and c'_{HB} given in Ref. [100, 128]. This is again due to the fact that the GJMT basis mixes current-current operators with one-loop suppressed ones.

Finally, let us comment on the relation between our basis and one of the most used in the literature, the one originally given in Ref. [129]. After eliminating redundant operators, one ends up with 59 independent operators as listed in Ref. [101]. This basis also keeps separate tree-level operators from one-loop suppressed ones. The set of one-loop suppressed operators is different from ours though: they use $\{\mathcal{O}_{WW}, \mathcal{O}_{WB}, \mathcal{O}_{W\widetilde{W}}, \mathcal{O}_{W\widetilde{B}}\}$ instead of our $\{\mathcal{O}_{HW}, \mathcal{O}_{HB}, \mathcal{O}_{H\widetilde{W}}, \mathcal{O}_{H\widetilde{B}}\}$. The change of basis is given in Eqs. (7.6), (7.7), (7.25) and (7.26). For the tree-level operators they use the minimal set of 3 operators made of SM bosons, in particular $\mathcal{O}_H$, $\mathcal{O}_T$ and $\mathcal{O}_6$, while the rest of operators involves SM fermions: those given in Eq. (6.19), Eq. (7.4) and four-fermion operators. As explained in Appendix A, we can reach this set of operators from our basis by performing field redefinitions. The basis of Refs. [101, 129] is, however, not very convenient for parametrizing the effects of universal theories. Although only a few operators parametrize these theories in our basis (see Sec. 6.1), in the basis of Refs. [101, 129] they require a much larger set of operators. In particular, the two tree-level operators $\mathcal{O}_W$ and $\mathcal{O}_B$ are written in the basis of Refs. [101, 129] as

$$
\begin{aligned}
c_W \mathcal{O}_W &\rightarrow c_W \frac{g^2}{g_H^2} \left[-\frac{3}{2} \mathcal{O}_H + 2\mathcal{O}_6 + \frac{1}{2}\mathcal{O}_y + \frac{1}{4} \sum_f \mathcal{O}_L^{f\,(3)} \right], \\
c_B \mathcal{O}_B &\rightarrow c_B \frac{g'^2}{g_H^2} \left[-\frac{1}{2} \mathcal{O}_T + \frac{1}{2} \sum_f \left(Y_L^f \mathcal{O}_L^f + Y_R^f \mathcal{O}_R^f \right) \right],
\end{aligned}
\tag{7.42}
$$

where Y_L^f and Y_R^f are the hypercharges of the left and right handed fermions, respectively. We can see from (7.42) that the Wilson coefficients in the basis of Refs. [101, 129] are correlated, so that one should include them all in operator analyses of universal theories. As far as the anomalous-dimension matrix is concerned, the basis of Refs. [101, 129] keeps also the same block-diagonal form as the basis of B_3, since loop-suppressed operators $\{\mathcal{O}_{BB}, \mathcal{O}_{WW}, \mathcal{O}_{WB}, \mathcal{O}_{B\widetilde{B}}, \mathcal{O}_{W\widetilde{W}}, \mathcal{O}_{W\widetilde{B}}\}$ do not mix with current-current ones.

7.7   Summary

The $h \to \gamma\gamma$ decay is of special importance because of its clean experimental signature. In this Chapter we have analysed potential effects of new physics in this decay rate (together with the closely related one, $h \to \gamma Z$) following the effective Lagrangian approach, where one enlarges the SM Lagrangian with a set of dimension-six operators.

The choice of the operator basis has been crucial to make the calculations simple and transparent. We have shown the convenience of working in bases that classify operators in two groups. The first is formed by operators which can arise from tree-level exchange of heavy states under the assumption of minimal coupling. This group contains operators that can be written as a product of local currents. A second group contains operators that are generated, from weakly-coupled renormalizable theories, at the loop-level, and thus have suppressed coefficients, see Tab. 7.1

The operators relevant for $h \to \gamma\gamma, \gamma Z$ are, as expected, of the second group, specifically $\mathcal{O}_{BB}$, $\mathcal{O}_{HW}$ and $\mathcal{O}_{HB}$ and their CP-odd counterparts. We have been interested in the anomalous dimensions of these operators that can be generically written as

$$16\pi^2 \frac{d\kappa_{j_3}}{d\log\mu} = \sum_{i_1} b_{j_3,i_1} c_{i_1} + \sum_{i_2} b_{j_3,i_2} c_{i_2} + \sum_{i_3} b_{j_3,i_3} \kappa_{i_3}\,, \qquad (7.43)$$

where $j_3 = BB, HW, HB, B\widetilde{B}, H\widetilde{W}, H\widetilde{B}$. The main purpose of this Chapter has been to calculate b_{j_3,i_1} and b_{j_3,i_2}. Since the corresponding coefficients c_{i_1} and c_{i_2} can be of order one, the RG evolution can enhance the new-physics effect on κ_{i_3} by a factor $\log(\Lambda/m_h)$. Our main result is that such enhancement is not present, because the corresponding elements of the anomalous-dimension matrix vanish

$$b_{j_3,i_1} = b_{j_3,i_2} = 0\,. \qquad (7.44)$$

Therefore, tree-level (current-current) operators do not contribute to the RGEs of the one-loop suppressed operators relevant for the $\gamma\gamma$ and γZ Higgs decay. The result is given in Eq. (7.24) (and its CP-odd analog).

We have also obtained the RGEs for κ_{HW} and κ_{HB}, Eq. (7.33), which affect the decay $h \to \gamma Z$, by realizing that the operators $\mathcal{O}_{BB}$, $\mathcal{O}_{WW}$, $\mathcal{O}_{WB}$ (used in Ref. [123]) do not renormalize (at one-loop) $\mathcal{O}_{HW}$, $\mathcal{O}_{HB}$ (nor $\mathcal{O}_W$, $\mathcal{O}_B$). Exploiting this fact, we have further clarified the structure of the anomalous-dimension matrix for these operators, showing that it takes a particularly simple block-diagonal form in the basis B_3 of Eq. (7.29). The tree-level operators $\mathcal{O}_B$ and $\mathcal{O}_W$ do not mix with the one-loop operators $\mathcal{O}_{WW}$, $\mathcal{O}_{BB}$, $\mathcal{O}_{WB}$ and vice versa, as Eq. (7.30) shows. Enlarging this basis with dipole-moment operators for the SM fermions, we have further computed the effect of such dipoles on $h \to \gamma\gamma, \gamma Z$.

To conclude, we have discussed how the appropriate choice of operator basis can shed light on the physical structure behind the renormalization mixing of operators and reveal hidden simplicities in the structure of the matrix of anomalous dimensions that describes such mixing. This is an early hint of a surprising structure of the one-loop anomalous

dimension matrix. We pursue this hint in Part III of this thesis, were we find that the block diagonal structure presented in this Chapter is rather generic and applies, to a large extent, to the full Standard Model.

8. Anomalous dimensions and Higgs physics

As explained in Chapter 6, out of the 8 CP-even operators that only affect Higgs physics, 5 of them are "tree-level" operators and 3 are "one-loop". The 5 tree-level operators affect directly the Higgs couplings to fermions, the kinetic term of the Higgs and the Higgs self-couplings. In this Chapter calculate the anomalous dimensions of these 5 operators, which allow us to describe the renormalization group (RG) evolution of these Wilson coefficients from the heavy scale Λ, where they are generated, down to the electroweak-scale. [1] We apply these results to find the leading-log corrections to the predictions for Higgs-couplings in several Beyond the Standard Model (BSM) scenarios: the Minimal Supersymmetric Standard Model (MSSM), universal theories (such as composite-Higgs models) and models with a non-standard top. We find that the corrections from this running can be sizable for $\Lambda \sim$ few TeV, and will become more relevant as we have better measurements of the Higgs couplings. We also calculate the anomalous dimensions of the operators contributing to the S and T parameters and to the $Zb\bar{b}$ couplings. The stringent experimental constraints on these quantities can then be translated into indirect bounds on Higgs operators.

8.1   Running effects from Λ to M_W

So far, we have implicitly assumed that the Wilson coefficients were evaluated at the electroweak scale, at which their effects can be eventually measured. However, particular UV completions predict the values of those coefficients at the scale Λ where the heavy BSM is integrated out. The RG evolution from Λ down to the electroweak scale, described by the corresponding anomalous dimensions, can be important in many cases.

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[1] For the other 3 one-loop CP-even operators, as well as for the 3 one-loop CP-odd, the calculation of the main anomalous dimensions has been given in Refs.[5, 123].

Our main interest is to calculate the anomalous dimensions of the Wilson coefficients that can have the largest impact on Higgs physics. As we explained in the previous Section, these are the coefficients listed in Eq. (6.37). In Ref. [5] we already calculated the most relevant anomalous dimensions of the κ_i in Eq. (6.37). We showed that tree-level Wilson coefficients do not enter, at the one-loop level, in the RGEs of the κ_i, a property that allowed us to complete the calculation of Ref. [123] for the anomalous dimensions relevant for $h \to \gamma\gamma, Z\gamma$. In this Section we extend the analysis by calculating the anomalous dimensions for the 5 tree-level Wilson coefficients:

$$\{c_H, c_6, c_{y_t}, c_{y_b}, c_{y_\tau}\} . \tag{8.1}$$

We notice that even in the future, with better measurements of the Higgs couplings, and then better bounds on the Wilson coefficients of (8.1), we still expect these coefficients to give the main BSM contributions to Higgs physics, since other Wilson coefficients, such as c_W, are expected to receive even stronger constraints from LHC (for a given Λ).

Generically, the anomalous dimensions are functions of other Wilson coefficients:

$$\gamma_{c_i} = \frac{dc_i}{d\log \mu} = \gamma_{c_i}(c_j) , \tag{8.2}$$

where μ is the renormalization scale. In the RHS of Eq. (8.2) we keep the c_j coefficients that can potentially give the most significant contributions to the RG running. They are the following. First, those of (8.1) as they have no important experimental constraints and also are the most relevant in BSM scenarios with g_* large. We also keep the Wilson coefficients of operators involving the top quark, departing from the MFV assumption. These are $\mathcal{O}_L^{q3}, \mathcal{O}_R^t, \mathcal{O}_L^{(3)\, q3}$ and $\mathcal{O}_R^{tb}$, in addition to the 4-fermion operators, $\mathcal{O}_{LL}^{q3}, \mathcal{O}_{LL}^{(8)\, q3}$, $\mathcal{O}_{LR}^t, \mathcal{O}_{LR}^{(8)\, t}, \mathcal{O}_{y_t y_b}, \mathcal{O}_{y_t y_b}^{(8)}, \mathcal{O}_{y_t y_\tau}$ and $\mathcal{O}'_{y_t y_\tau}$. We have several motivations to keep them. First, they have no large constraints from experiments. Second, they can induce large effects on the anomalous dimensions, since they are proportional to the top Yukawa coupling. Also their Wilson coefficients can be sizable in many BSM models, such as composite Higgs or supersymmetric theories, as we will discuss. To summarize, we consider in the RHS of Eq. (8.2) the following Wilson coefficients:

$$\{c_j\} = \{c_H, c_6, c_{y_t}, c_{y_b}, c_{y_\tau}, c_L, c_R, c_L^{(3)}, c_R^{tb}, c_{LL}, c_{LL}^{(8)}, c_{LR}, c_{LR}^{(8)}, c_{y_t y_b}, c_{y_t y_b}^{(8)}, c_{y_t y_\tau}, c'_{y_t y_\tau}\} , \tag{8.3}$$

where, from now on, we suppress the q_3 and t superindices in the coefficients for simplicity.

We would like to mention that, even for those Wilson coefficients subjected to experimental constraints, as those discussed in the previous Section, the fact that the constraints apply to the ratios $c_j M_W^2/\Lambda^2$ means that bounds at the percent-level can

allow for $c_j \sim O(1)$ if $\Lambda \sim O(\text{TeV})$. These coefficients could then also give potentially non-negligible effects in the γ_{c_i}. An example of this is c_W. Nevertheless, one can still expect that the dominant effects will be given by the coefficients in Eq. (8.3) since, for a given Λ, they can always be larger than c_W.

In addition, we will also extend our calculation of anomalous dimensions to other Wilson coefficients beyond those in (8.1). These correspond to operators constrained by the present experimental data, and then their anomalous dimensions can also be useful to derive indirect bounds on the coefficients of Eq. (8.3). [2]

The anomalous dimensions presented below correspond to the basis of Tab. 6.1 and Tab. 6.2, after using the five redundancies to eliminate the operators $\{\mathcal{O}_L^l, \mathcal{O}_L^{(3)\,l}, \mathcal{O}_{RR}^e, \mathcal{O}_{LL}^l, \mathcal{O}_{RR}^{(8)\,d}\}$. Nevertheless, removing or not these five operators and keeping the redundancy would not change our results (see Appendix B for more details).

8.1.1   Anomalous dimensions of operators relevant for Higgs physics

We present here the anomalous dimensions for the Wilson coefficients in 8.1, the ones expected to dominate deviations in Higgs physics, including the effects from the Wilson coefficients in Eq. (8.3). They are given by

$$16\pi^2 \gamma_{c_H} = \left[4N_c y_t^2 + 24\lambda - \frac{3}{2}(3g^2 + 2g'^2) \right] c_H + 12 N_c y_t^2 c_L^{(3)} , \tag{8.4}$$

$$16\pi^2 \gamma_{\lambda c_6} = 6 \left[N_c y_t^2 + 18\lambda - \frac{3}{4}(3g^2 + g'^2) \right] \lambda c_6 + 2(40\lambda - 3g^2)\lambda c_H$$
$$- 16 N_c \lambda y_t^2 c_L^{(3)} + 8 N_c y_t^2 (\lambda - y_t^2) c_{y_t} , \tag{8.5}$$

$$16\pi^2 \gamma_{c_{y_t}} = \left[(4N_c + 9)y_t^2 + 24\lambda - \frac{3}{2}(3g^2 + g'^2) \right] c_{y_t} + \left(3y_t^2 + 2\lambda - \frac{3}{2}g^2 \right) c_H$$
$$+ (2y_t^2 + 4\lambda - 3g^2 - g'^2)c_R - 2(y_t^2 + 2\lambda + 2g'^2)c_L$$
$$+ 4(-N_c y_t^2 + 3\lambda + g'^2)c_L^{(3)} + 8(y_t^2 - \lambda)\left[c_{LR} + C_F c_{LR}^{(8)} \right] , \tag{8.6}$$

$$16\pi^2 \gamma_{c_{y_b}} = \left[2(N_c + 1)y_t^2 + 24\lambda - \frac{3}{2}(3g^2 + g'^2) \right] c_{y_b} + \left(2\lambda - \frac{3}{2}g^2 \right) c_H$$
$$+ (2N_c - 1)y_t^2 c_{y_t} + 2(2\lambda + g'^2)c_L + 2 \left[(3 - 2N_c)y_t^2 + 6\lambda + g'^2 \right] c_L^{(3)}$$
$$- 4\frac{y_t^2}{g_*^2} \left(y_t^2 + 2\lambda - \frac{3}{2}g^2 \right) c_R^{tb}$$
$$+ 2\frac{y_t^2}{g_*^2}(\lambda - y_t^2) \left[(2N_c + 1) c_{y_t y_b} + C_F c_{y_t y_b}^{(8)} \right] , \tag{8.7}$$

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[2]Other anomalous dimensions were calculated in Refs. [100, 128].

$$16\pi^2\gamma_{c_{y_\tau}} = \left[2N_c y_t^2 + 24\lambda - \frac{3}{2}(3g^2 + g'^2)\right] c_\tau + \left(2\lambda - \frac{3}{2}g^2\right) c_H + 2N_c y_t^2 [c_{y_t} - 2c_L^{(3)}]$$
$$-2\frac{y_t^2}{g_*^2}N_c(\lambda - y_t^2)\left(2c_{y_t y_\tau} + c'_{y_t y_\tau}\right) , \tag{8.8}$$

where $N_c = 3$ is the number of colors and $C_F = (N_c^2 - 1)/(2N_c)$. Parametrically one has $\gamma_{c_i} \sim g_j^2 c_j/16\pi^2$ and we only keep $g_j^2 = \{y_t^2, g_s^2, g^2, g'^2, \lambda\}$, dropping $g_j^2 = \{y_b^2, y_\tau^2, ...\}$. We remark that, to calculate these anomalous dimensions, one has to take into account that redundant operators removed from our operator basis are nevertheless generated through renormalization at the one-loop level. For details about how to deal with this effect, see Appendices A and B. The need to care about such effect also means that the RGEs depend on the choice of redundant operators (*i.e.* on the basis).

Let us make a quantitative analysis of the size of these radiative effects. Working at one-loop leading log order,

$$c_i(M_t) \simeq c_i(\Lambda) - \gamma_{c_i}\log\frac{\Lambda}{M_t} , \tag{8.9}$$

which is enough if we take $\Lambda \sim 2$ TeV as UV scale and M_t as electroweak scale, we obtain the following radiative modifications of the Wilson coefficients, $\Delta c_i \equiv c_i(M_t) - c_i(2 \text{ TeV})$:

$$\Delta c_H = -0.17\, c_H - 0.49\, c_L^{(3)} , \tag{8.10}$$
$$\Delta\lambda c_6 = -0.36\,\lambda c_6 - 0.015\, c_H + 0.082\, c_L^{(3)} + 0.244\, c_{y_t} ,$$
$$\Delta c_{y_t} = -0.30\, c_{y_t} - 0.035\, c_H - 0.013\, c_R + 0.043\, c_L + 0.13\, c_L^{(3)} - 0.093\, c_{LR} - 0.12\, c_{LR}^{(8)} ,$$
$$\Delta c_{y_b} = -0.12\, c_{y_b} - 0.068\, c_{y_t} + 0.0060\, c_H - 0.012\, c_L + 0.054\, c_L^{(3)} + 0.027\, c_R^{tb}/g_*^2$$
$$+ (0.16\, c_{y_t y_b} + 0.027\, c_{y_t y_b}^{(8)})/g_*^2 ,$$
$$\Delta c_{y_\tau} = -0.096\, c_{y_\tau} - 0.081\, c_{y_t} + 0.0060\, c_H + 0.16\, c_L^{(3)} + (0.012 c_{y_t y_\tau} + 0.061 c'_{y_t y_\tau})/g_*^2.$$

We see that in a few cases, the numerical impact of operator mixing can be significant, like the mixing of $c_L^{(3)}$ into c_H; λc_6 and c_{y_t} into λc_6; and c_{y_t} into itself.

8.1.2   Anomalous dimensions of constrained operators

Other interesting anomalous dimensions to calculate correspond to operators that are at present constrained by experiments. Here we present those of c_T, c_B, c_W, and for the

top quark, c_R, c_L, and $c_L^{(3)}$:

$$16\pi^2 \gamma_{c_T} = \frac{3}{2}g'^2 c_H + 4N_c y_t^2 (c_R - c_L) \, , \tag{8.11}$$

$$16\pi^2 \gamma_{c_R} = \left[2(4 + N_c)y_t^2 - 9g^2 - \frac{7}{3}g'^2 \right] c_R - 4(N_c + 1)\left(y_t^2 - \frac{2}{9}g'^2 \right) c_{RR}$$
$$+ 2N_c \left(y_t^2 + \frac{1}{9}g'^2 \right) c_{LR} + 2y_t^2 \left(\frac{1}{4}c_H - c_L \right) \, , \tag{8.12}$$

$$16\pi^2 \gamma_{c_L} = \left[2(2 + N_c)y_t^2 - 9g^2 - \frac{7}{3}g'^2 \right] c_L + 2\left(y_t^2 + \frac{1}{9}g'^2 \right) \left[(2N_c + 1)c_{LL} + C_F c_{LL}^{(8)} \right]$$
$$- 2N_c \left(y_t^2 - \frac{2}{9}g'^2 \right) c_{LR} - y_t^2 \left(\frac{1}{4}c_H + c_R + 9c_L^{(3)} \right) \, , \tag{8.13}$$

$$16\pi^2 \gamma_{c_L^{(3)}} = \left[2(1 + N_c)y_t^2 - \frac{16}{3}g^2 - 3g'^2 \right] c_L^{(3)} - 2\left(y_t^2 - \frac{1}{3}g^2 \right) \left[c_{LL} + C_F c_{LL}^{(8)} \right]$$
$$+ y_t^2 \left(\frac{1}{4}c_H - 3c_L \right) \, , \tag{8.14}$$

$$16\pi^2 \gamma_{c_W} = \frac{1}{3}g_*^2 \left[16N_c c_L^{(3)} - c_H \right] \, , \tag{8.15}$$

$$16\pi^2 \gamma_{c_B} = \frac{1}{3}g_*^2 \left[\frac{8}{3}N_c (2c_R + c_L) - c_H \right] \, . \tag{8.16}$$

From them we can calculate the leading-log corrections to $c_B + c_W$, c_T and $c_L + c_L^{(3)}$ that are highly constrained by S, T and the Zbb-coupling, as has been discussed in Sec. 6.3. In this way, coefficients that are more loosely constrained by direct processes, such as c_H, c_L or c_R, can get indirect bounds from LEP1 and TeVatron measurements.

Integrating the RGEs of (8.16), at the one-loop leading-log order, between the cutoff scale $\Lambda = 2$ TeV and the electroweak scale, that we take here as M_t, one gets [3]

$$\Delta T = \Delta c_T \xi = [-0.003\, c_H + 0.16\,(c_L - c_R)]\,\xi \, , \tag{8.17}$$
$$\Delta S = \Delta(c_B + c_W)\frac{M_W^2}{\Lambda^2} = \left[0.001\, c_H - 0.01\, c_R - 0.004\, c_L - 0.03\, c_L^{(3)} \right]\xi \, , \tag{8.18}$$
$$\Delta \frac{\delta g_Z^{b_L}}{g_Z^{b_L}} = \frac{\Delta[c_L + c_L^{(3)}]}{1 - (2/3)\sin^2\theta_W}\xi \simeq \Delta[c_L + c_L^{(3)}]\xi \tag{8.19}$$
$$= \left[0.01\, c_R - 0.03\, c_L + 0.06\, c_L^{(3)} - 0.17\, c_{LL} - 0.0064\, c_{LL}^{(8)} + 0.08\, c_{LR} \right]\xi \, ,$$

where $\Delta c_i \equiv c_i(M_t) - c_i(2 \text{ TeV})$ and recall that $\xi \equiv g_*^2 v^2/\Lambda^2$. Notice that even if a P_{LR} symmetry of the BSM sector enforces $c_L + c_L^{(3)} = 0$, we can have a nonzero $c_L + c_L^{(3)}$ from the RG running, since the SM does not respect this parity. The fact that the three quantities above are constrained at the per-mille level implies that the top coefficients,

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[3]The effects of c_H and those of $c_{L,R}$ on T were already calculated in Ref. [127] and Ref. [130] respectively.

$\{c_L, c_R, \dots\} \times \xi$ cannot be of order one. Obviously, we are barring the possibility of cancellations between the initial value of the Wilson coefficients at the scale Λ and the radiative effects $\sim \gamma_{c_i} \log(\Lambda/M_t)$, that could only be possible by accident.

8.2 RGE impact on the predictions of Wilson coefficients

Here we want to study the impact of the evolution of the Wilson coefficients from the UV scale Λ down to the electroweak scale at which they affect Higgs physics. This running can modify the predictions arising from BSM models. We present three examples: two-Higgs doublet models (2HDM), universal theories, and scenarios with sizeable $c_{L,R}$, such as composite-top models.

2HDM and Supersymmetric theories: At tree-level, assuming ordinary R-parity, the only $d = 6$ operators that can be induced in supersymmetric models arise from the exchange of the extra Higgses since these are the only R-even heavy fields. In particular, the MSSM contains an extra heavy Higgs doublet. It is therefore well motivated to look for the impact of an extra heavy Higgs doublet in SM Higgs physics.

Denoting the heavy Higgs by H', defined to have $Y_{H'} = 1/2$, its relevant couplings to the SM fermions and Higgs are given by

$$\mathcal{L}' = -\alpha_u y_u \bar{Q}_L \widetilde{H}' u_R - \alpha_d y_b \bar{Q}_L H' d_R - \alpha_e y_e \bar{l}_L H' e_R - \lambda' H'^\dagger H |H|^2 + h.c. + \cdots \,, \quad (8.20)$$
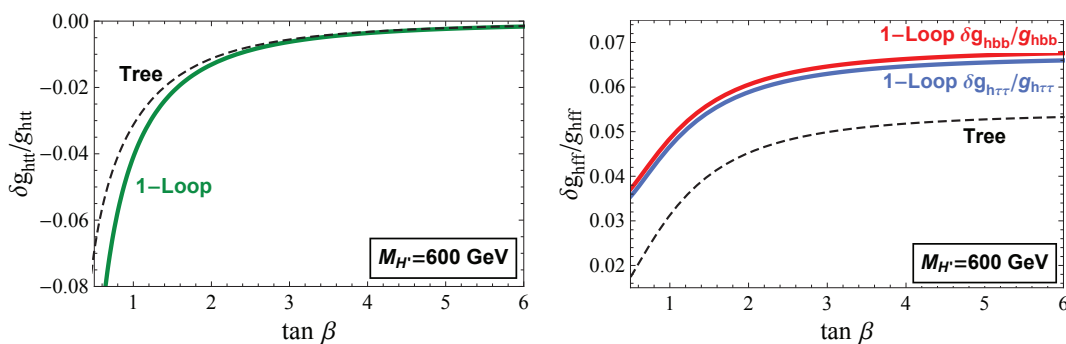
FIGURE 8.1: *Relative modification of the Higgs coupling to fermions, $\delta g_{hff}/g_{hff} = -c_y \xi$, Eq. (6.26), at tree-level (dashed line) and after including RGE effects from Λ to the electroweak scale (solid lines) as a function of $\tan\beta$ in an MSSM scenario with $\Lambda = M_H = 600$ GeV and unmixed stops heavy enough to reproduce $m_h = 125$ GeV. Left plot: top coupling. Right plot: bottom (lower solid line) and tau (upper solid line) couplings.*

where $\alpha_{u,d,e}$ are constants and we assume that λ' is a real number. In particular 2HDMs, these constants are

$$\alpha_u = \alpha_d = \alpha_e = \tan\beta \ , \qquad\qquad \text{for type-I 2HDM} \qquad (8.21)$$

$$\alpha_u = -\cot\beta \ , \quad \alpha_d = \alpha_e = \tan\beta \ , \quad \text{for type-II 2HDM (MSSM)} \qquad (8.22)$$

where $\tan\beta$ defines the rotation from the original basis, in which only one Higgs couples to a given type of fermion, to the mass-eigenstate basis before EWSB. At the order we work ($\sim v^2/\Lambda^2$), $\tan\beta$ coincides with that defined in the MSSM. Integrating out the heavy doublet at tree-level, we obtain the following nonzero coefficients for the third-family $d = 6$ operators:

$$\begin{aligned}
&g_*^2 c_{y_t} = \alpha_t \lambda' \ , \quad g_*^2 c_{y_b} = \alpha_b \lambda' \ , \quad g_*^2 c_{y_\tau} = \alpha_\tau \lambda' \ , \quad g_*^2 \lambda c_6 = \lambda'^2 \ , \\
&g_*^2 c_{LR}^{(8)} = 2N_c g_*^2 c_{LR} = -\alpha_t^2 y_t^2 \ , \quad c_{y_t y_b} = \alpha_t \alpha_b \ , \quad c_{y_t y_\tau} = \alpha_t \alpha_\tau \ .
\end{aligned} \qquad (8.23)$$

We have used $(\bar{Q}_L t_R)(\bar{t}_R Q_L) = -(\bar{Q}_L T^A \gamma^\mu Q_L)(\bar{t}_R T^A \gamma_\mu t_R) - (\bar{Q}_L \gamma^\mu Q_L)(\bar{t}_R \gamma_\mu t_R)/(2N_c)$ and now $\Lambda = M_{H'}$. Under the RGE flow of Eqs. (8.6)-(8.8) the operators $\mathcal{O}_{y_f}$ mix with $\mathcal{O}_{LR}, \mathcal{O}_{LR}^{(8)}, \mathcal{O}_{y_u y_d}$ and $\mathcal{O}_{y_u y_\tau}$. In the type-II 2HDM, we obtain in the one-loop leading-log approximation and neglecting $O(\lambda, g^2, g'^2)$ corrections:

$$\begin{aligned}
g_*^2 c_{y_t}(m_h) &= -\frac{\lambda'}{t_\beta}\left[1 - \frac{21 y_t^2}{16\pi^2}\log\frac{M_{H'}}{m_h}\right] + \frac{3 y_t^4}{4\pi^2 t_\beta^2}\log\frac{M_{H'}}{m_h} \ , \\
g_*^2 c_{y_b}(m_h) &= \lambda' t_\beta\left[1 - \frac{y_t^2}{2\pi^2}\log\frac{M_{H'}}{m_h}\right] + \frac{y_t^2}{16\pi^2}\left[5\frac{\lambda'}{t_\beta} - 14 y_t^2\right]\log\frac{M_{H'}}{m_h} \ , \\
g_*^2 c_{y_\tau}(m_h) &= \lambda' t_\beta\left[1 - \frac{3 y_t^2}{8\pi^2}\log\frac{M_{H'}}{m_h}\right] + \frac{3 y_t^2}{8\pi^2}\left[\frac{\lambda'}{t_\beta} - 2 y_t^2\right]\log\frac{M_{H'}}{m_h} \ ,
\end{aligned} \qquad (8.24)$$

with $t_\beta \equiv \tan\beta$.

To illustrate the impact of these radiative effects, let us consider the MSSM, a model which predicts $\lambda' = (1/8)(g^2 + g'^2)\sin 4\beta$ at tree-level (see for instance Ref. [131]). We take the stop mass scale $M_{\tilde{t}}$ large enough to get $m_h \simeq 125$ GeV through the well-known loop corrections to the Higgs quartic coupling, which at one-loop and zero stop mixing read:

$$\lambda(m_h) = \frac{1}{8}(g^2 + g'^2)\cos^2 2\beta + \frac{3 y_t^4}{16\pi^2}\log\frac{M_{\tilde{t}}^2}{M_t^2} \ , \qquad (8.25)$$

which is precise enough for our illustrative purposes. For consistency we must also include similar radiative corrections to λ', which read at one-loop:

$$\lambda'(M_{H'}) = \frac{1}{8}(g^2 + g'^2)\sin 4\beta - \frac{3 y_t^4}{8\pi^2 t_\beta}\log\frac{M_{\tilde{t}}^2}{M_{H'}^2} \ . \qquad (8.26)$$
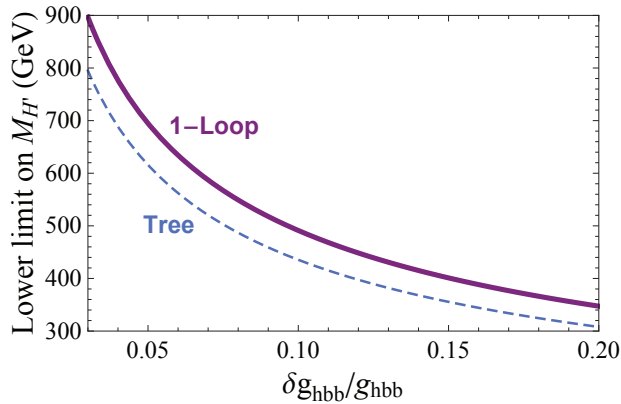
FIGURE 8.2:   *Lower bound on M_H as a function of the upper bound on the relative deviation $\delta g_{hbb}/g_{hbb}$, in an MSSM scenario with $\tan\beta = 5$ and unmixed stops heavy enough to reproduce $m_h = 125$ GeV. The dashed line corresponds to a tree-level analysis (parameters calculated at the scale M_H), while the solid line includes the RG running from M_H down to m_h.*

This gives the value of λ' that we can then plug in Eq. (8.24) to obtain the RG-improved corrections for g_{hff} induced by integrating out the heavy Higgses. The result is shown as a function of t_β in Fig. 8.1, which compares the tree-level result (dashed lines) and the one-loop result (solid lines) which takes into account the running from $\Lambda = M_{H'}$ down to the electroweak scale m_h. One sees that the effect of the running can be quite significant, easily $\sim 50\%$ or more. The importance of this effect can be further appreciated in Fig. 8.2, which shows the lower bound one could set on $M_{H'}$ from an upper bound on $\delta g_{hbb}/g_{hbb}$, the deviation of g_{hbb} from its SM value. By comparing the tree-level bound (dashed line) and the one-loop bound (solid line) one sees that the bound is shifted significantly by the inclusion of the RG corrections from $M_{H'}$ to m_h.

Finally, notice that c_H, which is not generated in the MSSM at tree-level since there are no heavy R-even singlet states, is not generated by the RGE evolution and therefore is also zero in the leading-log approximation.

Universal theories and composite Higgs models: Universal theories predict $c_{y_u} = c_{y_d} = c_{y_e}$. This prediction is modified by the evolution of these coefficients from the scale Λ, where they are generated, down to the electroweak scale. In particular, for $\Lambda = 2$ TeV, we find that the breaking of universality due to the top Yukawa coupling gives

$$
\begin{aligned}
c_{y_t}(m_h) &= c_{y_b}(m_h)\left(1 - \frac{8y_t^2}{16\pi^2}\log\frac{\Lambda}{m_h}\right) - \frac{3y_t^2 c_H}{16\pi^2}\log\frac{\Lambda}{m_h} \simeq 0.88\, c_{y_b}(m_h) - 0.05\, c_H\ , \\
c_{y_b}(m_h) &= c_{y_\tau}(m_h)\left(1 - \frac{y_t^2}{16\pi^2}\log\frac{\Lambda}{m_h}\right) \simeq 0.98\, c_{y_\tau}(m_h)\ .
\end{aligned}
\tag{8.27}
$$

This is a sizeable departure from universality for c_{y_t} that will have to be taken into account when fitting these models to data. Also it is worth noticing that in models in which only c_H is generated (models with only heavy singlets) and $c_{y_f}(\Lambda) = 0$, the value of c_{y_f} is also very small at low-energies, $c_{y_f}(m_h) \simeq 0$. In the minimal composite Higgs model, we also have the prediction $c_H = 1$ [99]. We find that the RG effects give a $\sim 20\%$ reduction of this prediction for $\Lambda \sim 2$ TeV.

Models with a non-SM top: The top is the only quark whose properties are not yet measured with high precision, allowing then sizeable deviations from their SM predicted values. There are also theoretical motivations to expect the top to be the quark with the largest deviations from the SM predictions, as it is the quark with the largest coupling to the Higgs. This is specially true in composite Higgs models where one expects the top to show also certain degree of compositeness. In these examples we can expect sizeable values for c_R, $c_L^{(3)}$, c_L and c_{LR} that can affect, at the one loop-level, the Higgs coefficients c_H and c_{y_f}. As it is clear from Eq. (8.10) the effects of c_R on the RGE evolution of c_H and c_{y_f} are very small. Nevertheless, those from $c_L^{(3)}$ and c_L are quite sizeable, even in the limit $c_L \simeq -c_L^{(3)}$ as required in order to avoid large tree-level contributions to $Zb\bar{b}$. Unfortunately these coefficients also give large one-loop effects to the T and S parameters and Zbb, as Eqs. (8.17)-(8.19) show, and this bounds them to be small (unless ξ is small). Interestingly, the coefficient $c_{LR}^{(8)}$ is not constrained by Eqs. (8.17)-(8.19). Therefore it can give sizeable contributions to the RGE evolution of c_{y_t}:

$$c_{y_t}(m_h) = c_{y_t}(\Lambda) - \frac{2y_t^2}{3\pi^2} c_{LR}^{(8)} \log \frac{\Lambda}{m_h} \, , \tag{8.28}$$

that is of order $\sim 15\%$, assuming $c_{LR}^{(8)} \sim 1$. A nonzero $c_{LR}^{(8)}$ could arise from integrating out a massive gluon coupled to the top.

8.3   Summary

We have calculated the anomalous dimensions of the 5 tree-level operators of the list Eq. (6.37), which allows us to calculate the running of the coefficients from the high-energy scale Λ where they are generated down to the electroweak scale. All technical details of these calculations are discussed in Appendix B. Since the S and T parameters, and the $Zb\bar{b}$ coupling are very well constrained, we have also calculated the anomalous dimension of the operators contributing to these quantities. In this way, we can put indirect bounds on Higgs operators.

We have applied our results to BSM models such as the MSSM, universal theories (as composite Higgs models) and models with non-standard top couplings. In such models we have evaluated the leading-log corrections to the predictions for the Higgs couplings. The corrections from the running can be quite large for $\Lambda \sim$ few TeV, as Fig. 8.2 shows. Our calculation of the anomalous dimensions is an aspect of the physics of the $d = 6$ operators which will become more relevant when we have better measurements of the Higgs couplings.

9. Interplay between Higgs and Electroweak observables

In this Chapter we discuss the anomalous dimension matrix of a set of 13 dimension-six (dim-6) operators composed only of gauge bosons and Higgs fields and estimate the impact of these RG mixing effects on experimental measurements. [1] To be completely general about the possible new physics scenarios one would need to compute the anomalous dimension matrix for all the 59 dimension-six operators (the number of independent operators for one generation of fermions [101, 129]). A given set of experimental observables, however, receives contributions only from a subset of these operators. The dim-6 operators we are focussing our attention on is a particularly interesting subset as they capture most of the possible deformations of the electroweak sector studied at LEP (i.e., electroweak precision tests and triple gauge couplings) and of the Higgs sector being currently studied at the LHC. At the same time, these operators are among the most important ones generated by universal new physics theories. [2]

The Wilson coefficients of the dim-6 operators studied in this Chapter have been constrained at different levels of precision. In particular, the ones contributing to LEP electroweak precision observables have been measured at the per mille level, whereas those parametrizing triple gauge couplings (TGC) and Higgs coupling data have been measured at most at the percent level. This hierarchy in the size of constraints means that, despite the one loop factor, the RG contributions of a weakly constrained coupling to a strongly constrained one can be of the same order as, or even larger than, the bound on the strongly constrained coefficient. This means that the RG-mixing effects of such weakly constrained Wilson coefficients can be measured/constrained by precision measurements of other couplings to which experiments are more sensitive. Indeed, we find interesting instances of coefficients which receive stronger bounds from the RG mixing than from the direct tree-level constraint. For example, we show that the Wilson

---

[1]Some elements of the anomalous dimension matrix have been previously calculated in the literature, see the discussion of the preceding Chapters and Refs. [5, 6, 100, 123, 127, 128, 132–136].

[2]By universal theories we mean theories in which the BSM sector is flavour universal and in addition any new vector state couples to fermions via the SM SU(2)×U(1) currents, see for instance Ref. [106].

coefficients parametrizing deviations in some of the anomalous TGC observables and the correction to the Higgs kinetic term $\hat{c}_H$ receive a stronger bound via their RG-mixing contribution to the electroweak parameters $\hat{S}$, $\hat{T}$, W, Y and $\Gamma_{h \to \gamma\gamma}$ than the direct constraint.

The dimension-six operator basis

We work in the basis defined in Chapter 6, up to a slight rotation (to be defined below) that is motivated by the physics we are interested in: EW observables, Higgs couplings to gauge bosons and QCD observables involving gluons only and the relations among each other as imposed from the running between the scale of new physics to the weak scale. These include the four electroweak oblique pseudo-observables $\hat{S}$, $\hat{T}$, W and Y, the three triple gauge coupling observables g_1^Z, κ_γ and λ_γ, the Higgs couplings to vector bosons, the gluon oblique parameter Z [106] and the anomalous triple gluon coupling parameter $\hat{c}_{3G}$. We describe these observables in more detail in Sec. 9.2.2 and Sec. 9.3.

We have not included the Higgs decays to fermions in our list of observables. The only further dim-6 operators contributing to such observables are the operators $\mathcal{O}_{y_u}, \mathcal{O}_{y_d}$ and $\mathcal{O}_{y_e}$, defined in Chapter 6, whose RG effects have been discussed in Chapter 8. These are weakly constrained operators and new RG-induced constraints can be derived only if they contribute to the running of more strongly constrained operators. In Chapter 8

$$\mathcal{O}_H = \tfrac{1}{2}(\partial^\mu |H|^2)^2 \qquad\qquad \mathcal{O}_T = \tfrac{1}{2}(H^\dagger \overset{\leftrightarrow}{D}_\mu H)^2$$

$$\mathcal{O}_6 = \lambda |H|^6 \qquad\qquad \mathcal{O}_W = \tfrac{ig}{2}(H^\dagger \sigma^a \overset{\leftrightarrow}{D^\mu} H) D^\nu W_{\mu\nu}^a$$

$$\mathcal{O}_B = \tfrac{ig'}{2}(H^\dagger \overset{\leftrightarrow}{D^\mu} H)\partial^\nu B_{\mu\nu} \qquad\qquad \mathcal{O}_{2W} = -\tfrac{1}{2}(D^\mu W_{\mu\nu}^a)^2$$

$$\mathcal{O}_{2B} = -\tfrac{1}{2}(\partial^\mu B_{\mu\nu})^2 \qquad\qquad \mathcal{O}_{2G} = -\tfrac{1}{2}(D^\mu G_{\mu\nu}^A)^2$$

$$\mathcal{O}_{BB} = g'^2 |H|^2 B_{\mu\nu} B^{\mu\nu} \qquad\qquad \mathcal{O}_{WB} = gg' H^\dagger \sigma^a H W_{\mu\nu}^a B^{\mu\nu}$$

$$\mathcal{O}_{WW} = g^2 |H|^2 W_{\mu\nu}^a W^{a\mu\nu} \qquad\qquad \mathcal{O}_{GG} = g_s^2 |H|^2 G_{\mu\nu}^A G^{A\mu\nu}$$

$$\mathcal{O}_{3W} = \tfrac{1}{3!} g \epsilon_{abc} W_\mu^{a\,\nu} W_{\nu\rho}^b W^{c\,\rho\mu} \quad \mathcal{O}_{3G} = \tfrac{1}{3!} g_s f_{ABC} G_\mu^{A\,\nu} G_{\nu\rho}^B G^{C\,\rho\mu}$$

TABLE 9.1: *14 CP-even operators made of SM bosons. The operators have been grouped in two different categories corresponding to operators of the form (SM current) × (SM current) (first group) and operators which are not products of SM currents (bottom group).*

we have shown that there is no such contribution and therefore we do not include these operators in our analysis.

The bosonic operators of the basis used in this Chapter is defined in Tab. 9.1. The only difference with respect to Chapter 6 is the set of operators

$$\{\mathcal{O}_W, \mathcal{O}_B, \mathcal{O}_{WW}, \mathcal{O}_{WB}, \mathcal{O}_{BB}\} \tag{9.1}$$

that is in one-to-one correspondence with

$$\{\mathcal{O}_W, \mathcal{O}_B, \mathcal{O}_{HW}, \mathcal{O}_{HB}, \mathcal{O}_{BB}\} , \tag{9.2}$$

of Chapter 6. The advantage of the choice made, Eq. (9.1), is that the anomalous dimension matrix of the sector $\{\mathcal{O}_B, \mathcal{O}_W\} \times \{\mathcal{O}_{BB}, \mathcal{O}_{WB}, \mathcal{O}_{WW}\}$ is block diagonal, see Chapter 7. Eq. (9.1) is also in one-to-one correspondence with the operators used in Ref. [100]

$$\{\mathcal{O}_{HW}, \mathcal{O}_{HB}, \mathcal{O}_{WW}, \mathcal{O}_{WB}, \mathcal{O}_{BB}\} . \tag{9.3}$$

The four precision parameters $\hat{S}$, $\hat{T}$, W and Y, which in our basis are parametrized by four bosonic dim-6 operators, as we show in Sec. 9.2.2, are sufficient to describe all possible dim-6 contributions to the $e^+e^- \to f^+f^-$ observables at LEP1 and LEP2, only in the limit of universal new physics. As explained in Chapter 6, to be completely general about possible new physics scenarios it would be necessary to include two more operators that contribute to the $e^+e^- \to f^+f^-$ experiment [6, 108]:

$$\mathcal{O}_L = (iH^\dagger \overset{\leftrightarrow}{D}_\mu H)(\bar{L}_L \gamma^\mu L_L) , \quad \mathcal{O}_{LL}^{1,2} = (\bar{L}_L^1 \sigma^a \gamma^\mu L_L^1)(\bar{L}_L^2 \sigma^a \gamma^\mu L_L^2) , \tag{9.4}$$

where the former affects the SM coupling of the Z boson to the left-handed leptons, and the latter affects the measurement of G_F (recall that the super-indices denote the fermion family). There are enough measurements to simultaneously constrain all six operators at the per mille level [137]. The RG contributions of $\{\mathcal{O}_L, \mathcal{O}_{LL}^{1,2}\}$ to the other operators has been discussed in Chapter 8. In this Chapter we do not study the possible RG-contributions of the operators of Tab. 9.1 to $\{\mathcal{O}_L, \mathcal{O}_{LL}^{1,2}\}$. Such RG-contributions could be used to impose some bounds on the weakly constrained operators of Tab. 9.1 since $\{\mathcal{O}_L, \mathcal{O}_{LL}^{1,2}\}$ are constrained at the per-mil level. Such an analysis would require computing many more elements of the full anomalous dimension matrix as well as enlarging the list of observables under consideration; this analysis would be interesting but beyond the scope of the present thesis.

Let us stress that the physics of the operators discussed in this Chapter is self-contained in the following sense. We have identified a set of particularly interesting

observables and its corresponding independent set of operators. We compute the RG equations by working out the (off-shell) effective Lagrangian. Then, operators not included in the basis are radiatively generated and redefined back to our basis at the end of the calculation, see Appendinx D for a more detailed discussion.

9.1   One-loop scaling of EW and Higgs operators

In general, quantum effects mix all the operators among themselves when going from the scale of new physics down to the scale at which the experimental measurements are performed. However, the 3 operators with gluons, $\mathcal{O}_{GG}, \mathcal{O}_{2G}$ and $\mathcal{O}_{3G}$, constitute a separate sector that does not mix with the other 11 bosonic operators at one-loop. [3] So, even if $\mathcal{O}_{GG}$ affects Higgs physics by controlling the dominant production mode of the Higgs boson at the LHC, it can be treated separately from the 3 other Higgs observables we are interested in here. Furthermore since the Higgs self-interactions have not been measured yet, and since $\mathcal{O}_6$ does not enter into the anomalous dimensions of any dim-6 operator other than itself, it can also be omitted from our analysis. For the Higgs- and EW-sector RG study, we can thus restrict the analysis to the following set of 10 dim-6 operators

$$\{\mathcal{O}_H, \mathcal{O}_T, \mathcal{O}_B, \mathcal{O}_W, \mathcal{O}_{2B}, \mathcal{O}_{2W}, \mathcal{O}_{BB}, \mathcal{O}_{WW}, \mathcal{O}_{WB}, \mathcal{O}_{3W}\} \ , \qquad (9.5)$$

and compute the corresponding anomalous dimension matrix. We include all the one-loop contributions proportional to c_i that depend on

$$\{g', g, g_s, \lambda, y_t\} \ , \qquad (9.6)$$

where g', g and g_s are the respective $U(1)_Y$, $SU(2)_L$ and $SU(3)_c$ gauge couplings, λ is the Higgs quartic coupling and y_t is the Yukawa coupling of the top quark. I.e. we neglect the contributions proportional to the Yukawas of the light fermions ($y_b/y_t \sim 0.02$, where y_b is the bottom quark Yukawa).

As in the previous Chapter, we regularized the loop integrals using dimensional regularisation and used $\overline{\text{MS}}$ subtraction scheme. We performed the computation in the unbroken phase of the SM and in the background field gauge, with the gauge fixing term

$$\mathcal{L}^{g.f.} = -\frac{1}{2\xi_A}(D_\mu^{(A)}\delta A^{a\mu})^2 \ , \qquad (9.7)$$

---

[3]The only exception is a contribution from $\mathcal{O}_{2B}$ to the RG of $\mathcal{O}_{2G}$, see Tab. 9.7. This mixing, however, is phenomenologically not very relevant since the Wilson coefficient of $\mathcal{O}_{2B}$ is strongly constrained, as we show in Sec. 9.2.2. In Sec. 9.3 we present the anomalous dimension of the three operators with gluons.

where $\delta A = \{\delta B, \delta W, \delta G\}$ is the quantum field with respect to which the dim ≥ 4 SM action is path-integrated and $D_\mu^{(A)}$ is the covariant derivative with respect to the corresponding background field $A = \{B, W, G\}$. A difference with respect to the previous Chapters is that we performed the calculations with arbitrary gauge fixing parameters ξ_A. In Tab. 9.2-9.3, we give the one-loop anomalous dimensions of the operators of Eq. (9.5), in the basis defined in Sec. 9. We recall that

$$\gamma_{c_i} = 16\pi^2 \frac{dc_i}{d\log\mu} \ . \tag{9.8}$$

As we have found in Chapter 8, upon computing the effective action we find counterterms which correspond to dim-6 operators that are not in our basis (the computation *does not know* our choice of basis). These radiatively-generated redundant operators need to be redefined into operators present in our basis. Upon redefinition, these redundant operators contribute to the anomalous dimensions of the operators in our basis at the same order as other direct contributions coming from one-particle-irreducible graphs. For details on the radiatively generated operators and how we deal with the redundant ones, see Appendix D. The matrices of Tabs. 9.2-9.3 already contain these indirect effects and the physics can be read straightforwardly by inserting those coefficients in tree-level processes. As a consequence of the Nielsen identity, the effective action evaluated on-shell (or equivalently redefining away redundant operators) is gauge invariant and indeed we have checked that the results of Tabs. 9.2-9.3 are independent of the gauge fixing parameters ξ_A of Eq. (9.7).

Apart from gauge invariance, there is another non-trivial consistency check that we have performed. The current-current operators in the left box of Tab. 9.1 can be related to each other and to other current-current operators containing fermions by using the SM EoM, or equivalently by carrying out field redefinitions. In a hypothetical theory

	c_H	c_T
γ_c	$-\frac{9}{2}g^2 - 3g'^2 + 24\lambda + 12y_t^2$	$-9g^2 + \frac{9}{2}g'^2 + 12\lambda$
γ_c	$\frac{3}{2}g'^2$	$\frac{9}{2}g^2 + 12\lambda + 12y_t^2$
γ_c	$-\frac{1}{3}$	$-\frac{5}{3}$
γ_c	$-\frac{1}{3}$	$-\frac{1}{3}$
other γ_c's	0 or $\mathcal{O}(y_l)$	0 or $\mathcal{O}(y_l)$

TABLE 9.2: *Anomalous dimension matrix for the Wilson coefficients of the dim-6 bosonic operators, in the basis defined in Sec. 9, See Tab. 9.3 for the rest of anomalous dimensions.*

without fermions [4], some contributions of the operators in the left box of Tab. 9.1 would vanish using the EoM, i.e. they would form an over-complete set of operators. This would also imply relationships between independently computed entries in the anomalous dimension matrix or, in other words, the anomalous dimensions of this over-complete set are invariant under changes in the field coordinates that respect the SM gauge symmetries. Our matrix passes this consistency check as we shall discuss in detail in Appendix D.3.

	c_B	c_W	c_{2B}	c_{2W}
γ_c	$-\frac{9}{4}g'^2(g'^2-2g^2)-6\lambda g'^2$	$\frac{9}{4}g^2(2g'^2-g^2)-36\lambda g^2$	$-\frac{141}{16}g'^4+3g'^2\lambda$	$\frac{63}{8}g^4+\frac{51}{16}g^2g'^2+18\lambda g^2$
γ_c	$-\frac{9}{4}g'^2g^2-6\lambda g'^2$	$-\frac{9}{4}g'^2g^2$	$3g'^4+\frac{9}{8}g'^2g^2+3\lambda g'^2$	$\frac{9}{8}g'^2g^2$
γ_c	$\frac{g}{6}+6y_t^2$	$\frac{g}{2}$	$\frac{59}{4}g'^2$	$-\frac{g}{4}$
γ_c	$\frac{g}{6}$	$\frac{17}{2}g^2+6y_t^2$	$\left(\frac{29}{8}-\frac{53g}{4g}\right)g'^2$	$\frac{79}{8}g^2+\frac{29}{4}g'^2$
γ_c	$-\frac{2}{3}g'^2$	0	$\frac{94}{3}g'^2$	0
γ_c	0	$-\frac{2}{3}g^2$	$\left(\frac{53}{12}-\frac{53g}{4g}\right)g'^2$	$\frac{331}{12}g^2+\frac{29}{4}g'^2$
γ_c	0	0	0	0
$\gamma_{c_{WW}}$	0	0	0	0
γ_c	0	0	0	0
γ_c	0	0	0	0

	c_{BB}	c_{WW}	c_{WB}	c_{3W}
γ_c	0	0	0	0
γ_c	0	0	0	0
γ_c	0	0	0	0
γ_c	0	0	0	0
γ_c	0	0	0	0
γ_c	0	0	0	0
γ_c	$\frac{g}{2}-\frac{9g}{2}+6y_t^2+12\lambda$	0	$3g^2$	0
$\gamma_{c_{WW}}$	0	$-\frac{3g}{2}-\frac{5g}{2}+6y_t^2+12\lambda$	g'^2	$\frac{5}{2}g^2$
γ_c	$2g'^2$	$2g^2$	$-\frac{g}{2}+\frac{9g}{2}+6y_t^2+4\lambda$	$-\frac{g}{2}$
γ_c	0	0	0	$\frac{53}{3}g^2$

TABLE 9.3: *Anomalous dimension matrix for the Wilson coefficients of the dim-6 bosonic operators, in the basis defined in Sec. 9, see Tab. 9.2 for the rest of anomalous dimensions.*

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[4]The anomalous dimension matrix of this fermionless theory is related, though not equal, to the anomalous dimension matrix we have computed, that is why considering this hypothetical theory provides a non-trivial test of our computation.

9.2   RG-induced contraints on EW and Higgs observables

In this Section we discuss the possibility to use the RGE's to derive constraints on the Wilson coefficients at the weak scale by requiring that none of the RG contributions to these weak-scale Wilson coefficients exceeds the direct bounds [100]. Since the RGE's mix various operators, it becomes possible to put tight constraints on operators loosely constrained by direct measurements via their RG contributions to more severely constrained operators. Then, in Sec. 9.2.2, we apply our method and use EW precision data, triple gauge couplings measurements and Higgs data to derive RG-induced bounds on the set of 10 observables we are interested in.

Renormalizing, order by order, the effective action, the logarithmically divergent terms computed in the previous Section are absorbed in the definition of the renormalized Wilson coefficients. Allowing for arbitrary cancellations in the definition of the renormalized coefficients renders the 1-loop effects small and the indirect bounds which can be obtained in this way are quite weak [138] and not competitive with direct bounds from Higgs physics and anomalous TGC measurements. We follow a different approach, already outlined in Ref. [100]. We are interested in obtaining indirect bounds on the UV value of the Wilson coefficients from low-energy experiments, in this case the 1-loop effect is enhanced by $\sim \log \Lambda/m_H$. Moreover, we assume that no tuned cancellations (or correlations) are present in the definition of the renormalized coefficients and require each log-divergent term not to exceed the direct bounds. In this way, our indirect bounds are much stronger than in Ref. [138] and, more importantly, they are useful in order to obtain insight into the UV physics. In fact, if any of our RG-induced bounds would be violated by a direct measurement this would imply a particular pattern of cancellation (or correlation) in the UV dynamics.

9.2.1   How much fine-tuning is needed to accommodate the data?

The electroweak and Higgs observables we are interested in (specified in Sec. 9.2.2) receive contributions from a particular linear combination of Wilson coefficients, suitably multiplied by the SM couplings:

$$(\text{obs})_i = \kappa_i + \omega_{ij}c_j \equiv \kappa_i + \hat{c}_i \quad \rightarrow \quad \delta(\text{obs})_i = \hat{c}_i \ , \qquad (9.9)$$

where κ_i is the SM contribution, the c_k's are the Wilson coefficients and ω_{ij} is a matrix containing the SM couplings and ratios of scales ($\omega \sim \mathcal{O}(m_W^2/\Lambda^2)$). We defined $\hat{c}_i$ as the linear combinations of the Wilson coefficients which contribute directly to each observable $(\text{obs})_i$ and we shall refer to them in the following as *observable couplings*, with

a slight abuse of language. If the new combinations $\hat{c}_i$ are independent, this corresponds to a change of basis such that to each operator corresponds an observable; we shall call this the *observable basis*.

As an example, consider the process $h \to \gamma Z$ which receives a contribution from the SM (in this case at one loop) as well as a direct contribution from a linear combination of the dim-6 operators. We parametrize this contribution with the *observable coupling* $\hat{c}_{\gamma Z}$, to be defined in Eq. (9.25), which is related to the Wilson coefficients of our basis as ($c_{\theta_W} = \cos\theta_W$ and $s_{\theta_W} = \sin\theta_W$ where θ_W is the mixing angle)

$$\hat{c}_{\gamma Z} = \frac{m_W^2}{\Lambda^2} \left(2c_{\theta_W}^2 c_{WW} - 2s_{\theta_W}^2 c_{BB} - (c_{\theta_W}^2 - s_{\theta_W}^2)c_{WB} \right) \ . \tag{9.10}$$

The above relation defines the coefficients $\omega_{\gamma Z,j}$ for this particular observable.

Now, suppose that this set of observables receives lower and upper bounds from experimental measurements:

$$\delta(\text{obs})_i|_{m_h} = \hat{c}_i(m_h) = \omega_{ij}(m_h)c_j(m_h) \in [\epsilon_i^{low}, \epsilon_i^{up}] \ . \tag{9.11}$$

The observable coupling $\hat{c}_i(m_h)$ (constrained at low energy) is related, through the running, to the high-scale value of the Wilson coefficients $c_j(\Lambda)$, which is not directly known since it is determined by the BSM degrees of freedom that have been integrated out. The matrix $\omega_{ij}(m_h)$ also runs with the scale (in the example of Eq. (9.10) this would be the running of g, g' and v inside m_W and θ_W), however we are not interested in such a running because ω_{ij} is determined by measurements performed at the EW scale and because, for the purpose of this work, we are not interested in the UV value of the SM couplings. This is the reason why we have not taken care of the contributions of the dim-6 operators on the SM couplings, parametrized by κ_i in Eq. (9.9), which would only be necessary if we wanted to relate $\omega_{ij}(m_h)$ to $\omega_{ij}(\Lambda)$ at the order we are working.

This discussion leads us to define the scale-dependent observable couplings as

$$\hat{c}_i(\mu) \equiv \omega_{ij}(m_h)c_j(\mu) \ , \tag{9.12}$$

obtaining

$$\delta(\text{obs})_i|_{m_h} = \hat{c}_i(m_h) = \hat{c}_i(\Lambda) - \frac{1}{16\pi^2}\hat{\gamma}_{ij}\hat{c}_j(\Lambda) \log\left(\frac{\Lambda}{m_h}\right) \ , \tag{9.13}$$

where

$$\hat{\gamma}_{ij} \equiv \omega_{ik}(m_h)\,\gamma_{kl}\,\omega_{lj}^{-1}(m_h) \tag{9.14}$$

and γ_{kl} is the matrix computed in the previous Section. Our interest in Eq. (9.13) is twofold: we want to find instances where a less constrained operator can mix with a

more constrained one by appearing in its RGE's and secondly (but closely related), to learn about the new degrees of freedom at the matching scale. In the following we shall work at leading-log order, which is fine if the hierarchy between the new physics scale Λ and the EW scale is not too big.

The fundamental assumption we make in order to obtain an indirect constrain on the $\hat{c}_j(m_h)$ through the RG is that we require each term in the sum on the r.h.s. of Eq. (9.13), proportional to some coefficient $\hat{c}_j$, to be contained in the experimental intervals associated to the observable $\delta(\text{obs})_i|_{m_h}$:

$$(1 - \delta_i)\hat{c}_i(\Lambda) \in [\epsilon_i^{low}, \epsilon_i^{up}], \tag{9.15}$$

$$-\frac{1}{16\pi^2}\hat{\gamma}_{i\hat{j}}\hat{c}_{\hat{j}}(m_h) \log\left(\frac{\Lambda}{m_h}\right) \in [\epsilon_i^{low}, \epsilon_i^{up}], \tag{9.16}$$

where we defined $\delta_i = \hat{\gamma}_{ii}/(16\pi^2) \log(\Lambda/m_h)$ and in the last line the index $\hat{j}$ is not summed over.[5] We have also used the fact that substituting $\hat{c}_j(\Lambda)$ for $\hat{c}_j(m_h)$ in the $\hat{\gamma}_{ij}\hat{c}_j$ term of Eq. (9.13) amounts to corrections $\mathcal{O}\left((4\pi)^{-4}\log^2(\Lambda/m_h)\right)$ that are beyond our precision (the same is true for the evaluation of γ_{ij}). Notice that this assumption is not only a requirement of the absence of fine-tuning but also an hypothesis on the UV physics, since particular relations, due to symmetry or dynamical accidents, between those combinations could be generically found when considering a BSM theory. From our bottom-up approach we parametrize also this absence of correlations as an absence of tuning. From Eq. (9.15) we can put bounds on the matching-scale Wilson coefficients $c_j(\Lambda)$:

$$c_j(\Lambda) \in \left[\sum_i (1 - \delta_i)^{-1}\omega_{ji}^{-1}\epsilon_i^{low}, \sum_i (1 - \delta_i)^{-1}\omega_{ji}^{-1}\epsilon_i^{up}\right]. \tag{9.17}$$

Notice that, as expected, these bounds grow quadratically weaker with the increase of the UV scale Λ since $\omega^{-1} \sim \Lambda^2/m_W^2$. Using Eq. (9.16), instead, we can put an RG-induced bound on the observable $\delta(\text{obs})_j|_{m_h}$ using the direct constraints on $\delta(\text{obs})_i|_{m_h}$, Eq. (9.11):

$$\begin{aligned}
\text{if } \hat{\gamma}_{\hat{i}j} > 0 : \quad & \delta(\text{obs})_j|_{m_h} \in \frac{16\pi^2}{\log\left(\Lambda/m_h\right)}(\hat{\gamma}_{\hat{i}j})^{-1}[-\epsilon_{\hat{i}}^{up}, -\epsilon_{\hat{i}}^{low}], \\
\text{if } \hat{\gamma}_{\hat{i}j} < 0 : \quad & \delta(\text{obs})_j|_{m_h} \in \frac{16\pi^2}{\log\left(\Lambda/m_h\right)}(\hat{\gamma}_{\hat{i}j})^{-1}[\epsilon_{\hat{i}}^{low}, \epsilon_{\hat{i}}^{up}].
\end{aligned} \tag{9.18}$$

The indirect bounds in Eq. (9.18), grow logarithmically stronger with the increase of the UV scale Λ. However, since the expected effects from new physics decrease quadratically with Λ, assuming order one coefficients c_i, even if the RG-induced bounds on the observables become slightly stronger, their power in investigating the UV degrees of freedom

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[5]In the following we shall denote with a hat all repeated indices not summed over.

becomes much weaker for higher values of Λ, as is clear from Eq. (9.17). It might seem that these bounds are not significant because of the loop factor in the above equation; all the ϵ_i's are, however, not of the same order and if $|\epsilon_i^{low,up}| \ll |\epsilon_j^{low,up}|$, the bound in the above equation can be stronger than the direct bound on $\delta(\text{obs})_j|_{m_h}$, in spite of the loop factor. The RG-induced bounds are, thus, significant only when a weakly constrained coupling appears in the RGE of a strongly coupled one.

Once new physics effects will, hopefully, be observed and the constraints of Eq. (9.11) will not include the zero value in the allowed interval $(0 < \epsilon_i^{low} < |\delta(\text{obs})_i|_{m_h} < \epsilon_i^{up})$, another interesting information that could be extracted from RG effects is a quantification of how much tuned, among themselves, are the electroweak and Higgs observables. First of all, let us define the fine-tuning in an observable as [139]

$$\begin{aligned}
\Delta_i &\equiv \text{Max}_j \left| \frac{\partial \log \delta(\text{obs})_i|_{m_h}}{\partial \log \hat{c}_j(\Lambda)} \right| \\
&\simeq \text{Max} \left\{ \frac{|\hat{c}_i(\Lambda)|}{|\delta(\text{obs})_i|_{m_h}}, \frac{\log(\Lambda/m_h)}{16\pi^2} \frac{\text{Max}_{j\neq i} |\hat{\gamma}_{i\hat{j}}| |\delta(\text{obs})_{\hat{j}}|_{m_h}}{|\delta(\text{obs})_i|_{m_h}} \right\},
\end{aligned} \tag{9.19}$$

where in the second step we separated the diagonal contribution from the off-diagonal ones and, for the diagonal term, we neglected the loop contribution since $\hat{c}_i(\Lambda)$ enters already at tree level and this would be its leading contribution to the tuning. In particular, the fine-tuning Δ_i will satisfy,

$$\Delta_i \geq \frac{\log(\Lambda/m_h)}{16\pi^2} \frac{\text{Max}_{j\neq i} |\hat{\gamma}_{i\hat{j}}| |\delta(\text{obs})_{\hat{j}}|_{m_h}}{|\delta(\text{obs})_i|_{m_h}} > \frac{\log(\Lambda/m_h)}{16\pi^2} \frac{\text{Max}_{j\neq i} |\hat{\gamma}_{i\hat{j}}| \epsilon_{\hat{j}}^{low}}{\epsilon_i^{up}}, \tag{9.20}$$

and one might be able to conclude that a certain degree of fine-tuning among the contributions to the RG flow of some operator is necessary.

9.2.2   EW and Higgs observables

Let us now apply the general formulas of the previous Section to the electroweak and Higgs observables we want to constrain. We have considered the 10 EW and Higgs operators of (9.5) to parametrize BSM corrections to the SM Lagrangian. Let us now describe in detail the set of pseudo-observables, briefly mentioned in Sec. 9, that constrain all these operators and form our basis of observables. These include the four electroweak oblique parameters $\hat{S}$, $\hat{T}$, Y and W; the three anomalous triple gauge coupling (TGC) and three observables related to Higgs physics: the decays to $\gamma\gamma$, γZ and a universal rescaling of all the branching ratios [108]. To derive the RG-induced constraints on these observables we first need to relate them to the operators in (9.5), that is to define the transformation matrix, ω_{ij}, that connect the basis in (9.5) to the observable basis.

We begin with the electroweak precision observables, which are constrained by measurements at LEP1, LEP2 and Tevatron. The first step of the analysis is to fix the SM parameters g, g' and v by the three most precise measurements: the Fermi constant G_F in muon decays, the fine-structure constant α_{em} and the Z-boson mass m_Z. With the input parameters fixed, the SM gives predictions for observables such as Z-pole measurements at LEP 1, the Tevatron measurement of the W-mass and LEP 2 measurements of the $e^+e^- \to f^+f^-$ cross-sections. New physics can affect this analysis by either changing the relationship between the input parameters g, g' and v to the measurement of G_F, α_{em} and m_Z or by directly contributing to the other measurements. All the deviation in the above observables induced by the operators we consider, (9.5), can be parametrized by the $\hat{S}$, $\hat{T}$, W and Y parameters [106] through

$$\Delta\mathcal{L}_{\text{EWPT}} = -\hat{T}\frac{m_Z^2}{2}Z_\mu Z^\mu - \frac{\hat{S}}{4m_W^2}\frac{gg'v^2}{2}(W_{\mu\nu}^3 B^{\mu\nu}) - \frac{W}{2m_W^2}(\partial^\mu W_{\mu\nu}^3)^2 - \frac{Y}{2m_W^2}(\partial^\mu B_{\mu\nu})^2.$$
(9.21)

The contribution of the Wilson coefficients of the operator set in (9.5) to the above observables is given by,

$$
\begin{aligned}
\hat{T} = \hat{c}_T(m_W) &= \frac{v^2}{\Lambda^2}c_T(m_W)\,, \\
\hat{S} = \hat{c}_S(m_W) &= \frac{m_W^2}{\Lambda^2}\left[c_W(m_W) + c_B(m_W) + 4c_{WB}(m_W)\right], \\
Y = \hat{c}_Y(m_W) &= \frac{m_W^2}{\Lambda^2}c_{2B}(m_W)\,, \\
W = \hat{c}_W(m_W) &= \frac{m_W^2}{\Lambda^2}c_{2W}(m_W)\,.
\end{aligned}
$$
(9.22)

The above parameters have been measured very precisely and are constrained at the per mille level. We present the 95 % CL bounds on these parameters, evaluates at the top mass m_t, in Tab. 9.4.

A second set of independent measurements that constrain the operator set in (9.5) are the TGC that were measured in the $e^+e^- \to W^+W^-$ process at LEP2. The phenomenological Lagrangian to describe deviations in the TGC observables from their SM values, is

$$
\begin{aligned}
\Delta\mathcal{L}_{3V} = {}& ig\,g_1^Z c_{\theta_W}Z^\mu\left(W^{+\nu}\hat{W}_{\mu\nu}^- - W^{-\nu}\hat{W}_{\mu\nu}^+\right) + ig\left(\kappa_z c_{\theta_W}\hat{Z}^{\mu\nu} + \kappa_\gamma s_{\theta_W}\hat{A}^{\mu\nu}\right)W_\mu^+W_\nu^- \\
&+ \frac{ig}{m_W^2}\left(\lambda_Z c_{\theta_W}\hat{Z}^{\mu\nu} + \lambda_\gamma s_{\theta_W}\hat{A}^{\mu\nu}\right)\hat{W}_\mu^{-\rho}\hat{W}_{\rho\nu}^+,
\end{aligned}
$$
(9.23)

where $\hat{V}_{\mu\nu} = \partial_\mu V_\nu - \partial_\nu V_\mu$, the photon field $A_\mu = c_{\theta_W}B_\mu + s_{\theta_W}W_\mu^3$ has field-strength $\hat{A}_{\mu\nu}$, while $Z_\mu = c_{\theta_W}W_\mu^3 - s_{\theta_W}B_\mu$ has field-strength $\hat{Z}_{\mu\nu}$ and we use $s_{\theta_W} \equiv \sin\theta_W = $

$g'/\sqrt{g^2 + g'^2}$, $c_{\theta_W} \equiv \cos\theta_W = g/\sqrt{g^2 + g'^2}$ and $e = gs_{\theta_W}$. Note that the above Lagrangian has only three independent parameters at the dim-6 level taken to be g_1^Z, κ_γ and λ_γ here; the other two can be expressed as : $\lambda_Z = \lambda_\gamma$ and $\kappa_Z = g_1^Z - t_{\theta_W}^2 \kappa_\gamma$. These relations are a consequence of the accidental custodial symmetry that is preserved by the dim-6 operators entering in the TGC [140]. The SM contribution is given by $(g_1^Z)_{SM} = (\kappa_\gamma)_{SM} = 1$ and $(\lambda_Z)_{SM} = 0$. The corrections induced by the dim-6 operators in our basis are given by:

$$\delta g_1^Z \equiv \hat{c}_{gZ}(m_W) = -\frac{m_W^2}{\Lambda^2}\frac{1}{c_{\theta_W}^2}c_W(m_W) , \qquad \delta\kappa_\gamma \equiv \hat{c}_{\kappa\gamma}(m_W) = \frac{m_W^2}{\Lambda^2}4c_{WB}(m_W) ,$$

$$\lambda_Z \equiv \hat{c}_{\lambda\gamma}(m_W) = -\frac{m_W^2}{\Lambda^2}c_{3W}(m_W) , \tag{9.24}$$

where $\delta g_1^Z = g_1^Z - (g_1^Z)_{SM}$ and $\delta\kappa_\gamma = \kappa_\gamma - (\kappa_\gamma)_{SM}$. The constraints on these TGC observables are at the percent level (see Tab. 9.4) and thus at least an order of magnitude weaker than the constraints on the electroweak parameters in Eq. (9.22). Note that, for this reason, in Eq. (9.24) we have ignored contributions to the $e^+e^- \to W^+W^-$ process from the couplings in Eq. (9.21).

Higgs physics provides the three remaining observables for our basis of observables. We consider the branching ratios $h \to \gamma\gamma/Z\gamma$ and the correction to the Higgs kinetic term,

$$\Delta\mathcal{L}_{Higgs} \supset \frac{\hat{c}_H}{2}(\partial_\mu h)^2 + \frac{\hat{c}_{\gamma\gamma}e^2}{m_W^2}vh\hat{A}_{\mu\nu}\hat{A}^{\mu\nu} + \frac{\hat{c}_{\gamma Z}\, eg}{m_W^2 c_{\theta_W}}vh\hat{A}_{\mu\nu}\hat{Z}^{\mu\nu}. \tag{9.25}$$

The above coefficients, in terms of the Wilson coefficients are given by

$$\hat{c}_H(m_t) = \frac{v^2}{\Lambda^2}c_H(m_t),$$

$$\hat{c}_{\gamma\gamma}(m_t) = \frac{m_W^2}{\Lambda^2}\left(c_{BB}(m_t) + c_{WW}(m_t) - c_{WB}(m_t)\right), \tag{9.26}$$

$$\hat{c}_{\gamma Z}(m_t) = \frac{m_W^2}{\Lambda^2}\left(2c_{\theta_W}^2 c_{WW}(m_t) - 2s_{\theta_W}^2 c_{BB}(m_t) - (c_{\theta_W}^2 - s_{\theta_W}^2)c_{WB}(m_t)\right).$$

We present the constraints on these three observables in Tab. 9.4. The coupling $\hat{c}_{\gamma\gamma}$ is constrained at the per mille level although the constraint on the SM diphoton width has been measured only with $\mathcal{O}(1)$ precision. This is because the SM width is already one-loop suppressed and thus the current $\mathcal{O}(1)$ precision of measurement corresponds to $\hat{c}_{\gamma\gamma} \approx 10^{-3}$. The correction to the Higgs kinetic term $\hat{c}_H$ on the other hand is poorly constrained. This is because $\hat{c}_H$ causes a universal shift in all the Higgs couplings and thus drops out from the branching ratios. Moreover, if only gluon fusion production channels are considered, the coupling c_{GG} mimics the effect of $\hat{c}_H$. Therefore, to disentangle the effect of c_{GG} and constrain $\hat{c}_H$, Higgs production cross-sections in different

channels have to be compared; in particular the weakly sensitive vector-boson fusion (VBF) channels have to be considered.

As we have discussed in the previous chapter, based on their precision of measurement, the observables can be divided into at least two groups. In the first group, containing highly constrained operators, we have the four electroweak parameters and the Higgs diphoton coupling (see Tab. 9.4),

$$\{\hat{c}_S, \ \hat{c}_T, \ \hat{c}_W, \ \hat{c}_Y, \ \hat{c}_{\gamma\gamma}\} \,, \tag{9.27}$$

which have been measured at the per mille level. In the second group we have the $h\gamma Z$ coupling, the couplings related to the three TGC observables $\kappa_\gamma, g_Z^1, \lambda_\gamma$ and $\hat{c}_H$,

$$\{\hat{c}_{\gamma Z}, \ \hat{c}_{\kappa\gamma}, \ \hat{c}_{gz}, \ \hat{c}_{\lambda\gamma}, c_H\} \,, \tag{9.28}$$

which are much more weakly constrained. One can, in fact, further split the above set into c_H which is constrained only at the $\mathcal{O}(1)$ level and the other couplings that are constrained at the few percent level.

We are interested in finding instances where the couplings from the second group in Eq. (9.28) appear in the RGE's of the first group of couplings in Eq. (9.27). To

	Direct Constraint	RG-induced Constraint
$\hat{c}_S(m_t)$	$[-1,2] \times 10^{-3}$ [141]	-
$\hat{c}_T(m_t)$	$[-1,2] \times 10^{-3}$ [141]	-
$\hat{c}_Y(m_t)$	$[-3,3] \times 10^{-3}$ [106]	-
$\hat{c}_W(m_t)$	$[-2,2] \times 10^{-3}$ [106]	-
$\hat{c}_{\gamma\gamma}(m_t)$	$[-1,2] \times 10^{-3}$ [108]	-
$\hat{c}_{\gamma Z}(m_t)$	$[-0.6,1] \times 10^{-2}$ [108]	$[-2,6] \times 10^{-2}$
$\hat{c}_{\kappa\gamma}(m_t)$	$[-10,7] \times 10^{-2}$ [137]	$[-5,2] \times 10^{-2}$
$\hat{c}_{gZ}(m_t)$	$[-4,2] \times 10^{-2}$ [137]	$[-3,1] \times 10^{-2}$
$\hat{c}_{\lambda\gamma}(m_t)$	$[-6,2] \times 10^{-2}$ [137]	$[-2,8] \times 10^{-2}$
$\hat{c}_H(m_t)$	$[-6,5] \times 10^{-1}$ [108]	$[-2,0.5] \times 10^{-1}$

TABLE 9.4: *In this table we present the 95 % CL, direct constraints on the coefficients in the observable basis (second column). The constraints on $\hat{S}$ and $\hat{T}$ presented here the ones obtained after marginalizing on the other parameters in the fit of Ref. [141]. In the analysis we use the $\hat{S}, \hat{T}$-ellipse from Ref. [141] with $U = 0$. Simultaneous constraints on all three of the TGC observables do not exist in the literature, so we have provided the individual constraints on the three couplings without taking into account correlations between them [137]. In the third column we show the RG-induced constraint we are able to obtain under the assumption of no fine-tuning in Eq. (9.29), for $\Lambda = 2$ TeV.*

check this we rotate the anomalous dimension matrix to the observable basis defined by Eq. (9.22), Eq. (9.24), and Eq. (9.26). We present the anomalous dimension matrix in the observable basis in Tab. 9.5. Using this, and fixing $\Lambda = 2$ TeV, we write numerically Eq. (9.13) as

$$(\hat{c}_S, \hat{c}_T, \hat{c}_Y, \hat{c}_W, \hat{c}_{\gamma\gamma}, \hat{c}_{\gamma Z}, \hat{c}_{\kappa\gamma}, \hat{c}_{gz}, \hat{c}_{\lambda\gamma}, \hat{c}_H)^t (m_t) \simeq \qquad (9.29)$$

$$
\begin{pmatrix}
0.9 & 0.003 & -0.03 & -0.08 & -0.02 & -0.02 & -0.04 & 0.05 & -0.01 & 0.001 \\
0.03 & 0.8 & -0.02 & -0.009 & 0 & 0 & -0.03 & 0.01 & 0 & -0.003 \\
0.001 & 0 & 0.9 & 0 & 0 & 0 & -0.001 & 0.001 & 0 & 0 \\
0 & 0 & -0.001 & 0.8 & 0 & 0 & 0 & -0.003 & 0 & 0 \\
0 & 0 & 0 & 0 & 0.9 & 0 & 0.006 & 0 & 0.02 & 0 \\
0 & 0 & 0 & 0 & 0 & 0.9 & 0.007 & 0 & 0.03 & 0 \\
0 & 0 & 0 & 0 & -0.02 & -0.02 & 0.9 & 0 & -0.01 & 0 \\
0.0004 & -0.0007 & -0.0004 & 0.1 & 0 & 0 & -0.0004 & 0.9 & 0 & -0.0007 \\
0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0.9 & 0 \\
-0.02 & 0.03 & 0.01 & -0.4 & 0 & 0 & 0.02 & -0.3 & 0 & 0.8
\end{pmatrix}
\begin{pmatrix}
\hat{c}_S(\Lambda) \\
\hat{c}_T(\Lambda) \\
\hat{c}_Y(\Lambda) \\
\hat{c}_W(\Lambda) \\
\hat{c}_{\gamma\gamma}(\Lambda) \\
\hat{c}_{\gamma Z}(\Lambda) \\
\hat{c}_{\kappa\gamma}(\Lambda) \\
\hat{c}_{gz}(\Lambda) \\
\hat{c}_{\lambda\gamma}(\Lambda) \\
\hat{c}_H(\Lambda)
\end{pmatrix} .
$$

We can now derive the RG-induced constraints by using Eq. (9.18) assuming no fine-tuning among the different terms in the RGE's.

The strongest RG-induced constraints come from the direct bounds on the $\hat{S}, \hat{T}, W$ and Y parameters, i.e. the first four lines in Eq. (9.29). We require that each observable coupling individually satisfies the four RG-induced constraints from these electroweak precision parameters simultaneously. It is very important to take into account the experimental correlations between $\hat{S}, \hat{T}, W$ and Y while imposing these bounds[142–144]. Note that the RG-mixing contributions to $\hat{c}_W$ and $\hat{c}_Y$, from the couplings in the weakly constrained group in Eq. (9.28), is either absent or accidentally much smaller than the ones to $\hat{c}_S$ and $\hat{c}_T$ (see the RG contributions to $\hat{c}_W$ and $\hat{c}_Y$ in the third and fourth row of Eq. (9.29)). We, therefore, look at the constraints on the $\hat{S}-\hat{T}$ plane taking $W = Y = 0$. We use the $\hat{S} - \hat{T}$ ellipse in Ref. [141], which assumes $W = Y = U = 0$, to derive our constraints. We present these RG-induced bounds and compare them with the direct bounds in Tab. 9.4 and in Fig. 9.1. We find that for each of the couplings in the second group we can derive a RG-induced constraint stronger than, or of the same order of, the direct tree-level constraint. We also obtain RG-induced bounds from the direct constraint on $\hat{c}_{\gamma\gamma}$ using the fifth line in Eq. (9.29) and Eq. (9.18),

$$
\begin{aligned}
\hat{c}_{\kappa\gamma} &\in [-0.2, 0.3] \ , \\
\hat{c}_{\lambda\gamma} &\in [-0.05, 0.10] \ ,
\end{aligned}
\qquad (9.30)
$$

but at present these bounds are weaker than those from the direct bounds on electroweak parameters.

Let us briefly comment on alternative choices for our observable basis. [6] For instance, the Higgs decay observables related to $h \to W^+W^-, ZZ$ decays could have been alternatively chosen as part of our observable basis instead of two of the TGC observables (κ_γ and g_Z) but we have kept the TGC in our basis as they are measured more precisely than these Higgs decay observables. This situation is likely to continue in the future. Although, observables like the relative deviation of $h \to W^+W^-, ZZ$ with respect to the SM would be strongly constrained at the 5 %(3 %) level at the LHC with $300\,\mathrm{fb}^{-1}$ ($3000\,\mathrm{fb}^{-1}$) data [145], the bounds on TGC are also expected to become stronger by an order of magnitude at the LHC [145] so that the TGC would still be more precisely measured than these Higgs observables. At linear colliders the decays $h \to W^+W^-, ZZ$ are expected to be measured at the level of 0.5 % [145] and the TGC observables at the 10^{-4} level [146]; again the TGC observables would be more constrained.
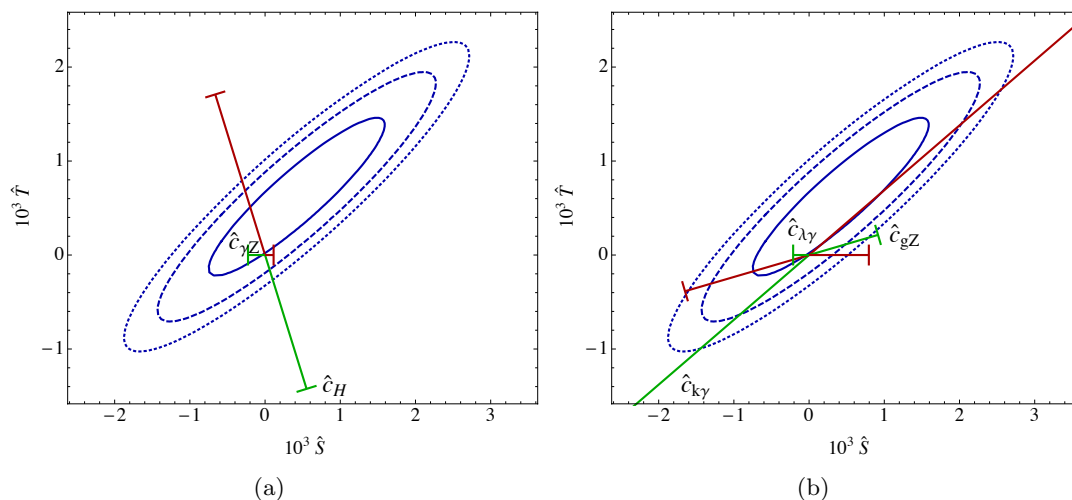
FIGURE 9.1:   *The blue ellipses represent the 68% (solid), 95% (dashed) and 99% (dotted) CL bounds on $\hat{S}$ and $\hat{T}$ as obtained in the fit of Ref. [141] with $U = 0$. The straight lines represent the RG-induced contribution to the oblique parameters from the weakly constrained observable couplings of Eq. (9.28), divided in Higgs couplings (a) and TGC couplings (b), using the first two lines of Eq. (9.29), for $\Lambda = 2$ TeV. The length of the lines corresponds to their present 95% CL direct bounds, see Tab. 9.4; the line is green (red) for positive (negative) values of the parameters.*

---

[6]In general, a change of *observable basis* modifies the anomalous dimension matrix of Tab. 9.5, also for the observables which are maintained in the basis. Thus, the RG-induced constraints we have derived, are applicable only to our particular choice of observables, and for an alternative choice the analysis must be repeated. Note that for our choice of observable basis, $h \to \gamma\gamma$ does not receive a contribution from the $\hat{S}$ parameter even though there is a dependence on c_{WB} in the anomalous dimension. But c_{WB} is actually reconstructing the $\delta\kappa_\gamma$ parameter.

$\gamma_{\hat c}$	$\hat c_S$	$\hat c_T$	$\hat c_Y$	$\hat c_W$	$\hat c_{\gamma\gamma}$
$\gamma_{\hat c}$	$\frac{1}{3}g'^2+6y_t^2$	$-\frac{g}{2}$	$\frac{1}{8}g'^2\left(147-106t_\theta^2\right)$	$\frac{1}{8}\left(77g^2+58g'^2\right)$	$16e^2$
$\gamma_{\hat c}$	$-9g'^2-24t_\theta^2\lambda$	$\frac{9}{2}g^2+12y_t^2+12\lambda$	$\frac{9}{2}g'^2+12t_\theta^2\left(g'^2+\lambda\right)$	$\frac{9}{2}g'^2$	0
$\gamma_{\hat c}$	$-\frac{2}{3}g'^2$	0	$\frac{94}{3}g'^2$	0	0
$\gamma_{\hat c}$	0	0	$\frac{53}{12}g'^2\left(1-3t_\theta^2\right)$	$\frac{331}{12}g^2+\frac{29}{4}g'^2$	0
$\gamma_{\hat c}$	0	0	0	0	$-\frac{9}{2}g^2-\frac{3}{2}g'^2+6y_t^2+12\lambda$
$\gamma_{\hat c}$	$18g'^2-t_\theta^2\left(9g'^2+24\lambda\right)$	$-9g^2+\frac{9}{2}g'^2+12\lambda$	$t_\theta^2\left(-\frac{141}{4}g'^2+12\lambda\right)$	$\frac{63}{2}g^2+\frac{51}{4}g'^2+72\lambda$	0
$\gamma_{\hat c}$	0	0	0	0	0
$\gamma_{\hat c}$	0	0	0	0	$16e^2$
$\gamma_{\hat c}$	$-\frac{g}{6c_\theta}$	$\frac{g}{12c_\theta}$	$\frac{g}{8c_\theta}\left(106t_\theta^2-29\right)$	$-\frac{1}{8c_\theta}\left(79g^2+58g'^2\right)$	0
$\gamma_{\hat c}$	0	0	0	0	0

$\gamma_{\hat c}$	$\hat c_H$	$\hat c_{\gamma Z}$	$\hat c_{\kappa\gamma}$	$\hat c_{gZ}$	$\hat c_{\lambda\gamma}$
$\gamma_{\hat c}$	$-\frac{1}{3}g^2$	$4(g^2-g'^2)$	$\frac{11}{2}g^2+\frac{1}{6}g'^2+4\lambda$	$-c_\theta^2\left(9g^2-\frac{1}{3}g'^2\right)$	$2g^2$
$\gamma_{\hat c}$	$\frac{3}{2}g^2$	0	$9g'^2+24t_\theta^2\lambda$	$-24s_\theta^2\lambda$	0
$\gamma_{\hat c}$	0	0	$\frac{2}{3}g^2$	$-\frac{2}{3}e^2$	0
$\gamma_{\hat c}$	0	0	0	$\frac{2}{3}c_\theta^2 g^2$	0
$\gamma_{\hat c}$	0	0	0	0	$-3g^2$
$\gamma_{\hat c}$	$-\frac{9}{2}g^2-3g'^2+12y_t^2+24\lambda$	0	$-\frac{3}{2}g^2+2\lambda$	0	0
$\gamma_{\hat c}$	0	$-\frac{7}{2}g^2-\frac{1}{2}g'^2+6y_t^2+12\lambda$	$-9g'^2\left(2-t_\theta^2\right)+24t_\theta^2\lambda$	$9\left(g^2c_\theta^2-g'^2s_\theta^2\right)+24\lambda\left(6c_\theta^2-s_\theta^2\right)$	$-\frac{g}{2}\left(11c_\theta^2-s_\theta^2\right)$
$\gamma_{\hat c}$	0	$4(g^2-g'^2)$	$s_\theta^2\left(g^2-2\lambda\right)-c_\theta^2\left(2g^2-2\lambda\right)$	0	0
$\gamma_{\hat c}$	$\frac{g}{12c_\theta}$	0	$\frac{11}{2}g^2+\frac{g}{2}+6y_t^2+4\lambda$	$\frac{17}{2}g^2-\frac{g}{6}+6y_t^2$	$2g^2$
$\gamma_{\hat c}$	0	0	$\frac{g}{6c_\theta}$	0	$\frac{53}{3}g^2$

TABLE 9.5: *Anomalous dimension matrix in the basis of observables.*
We defined $t_\theta = \tan\theta_W$.

Finally, let us discuss the future prospects for these RG-induced effects. As the measurement of the observables we have considered becomes more and more precise, it may be possible to detect signs of new physics. In this case, since some of the observables in Tab. 9.4 will be non-zero one would expect a deviation, via RG-mixing, also in other observables, unrelated at tree level. Note that according to future projections, $\hat{c}_{\gamma\gamma}$, the TGC observables ($\hat{c}_{\kappa\gamma}$, $\hat{c}_{gz}$) and $\hat{c}_{\gamma Z}$ would be measured at the 10^{-4} level [145, 146] at linear colliders and thus all these observables would be sensitive to RG-induced mixing effects of the couplings in Eq. (9.28), if they are above a minimal value.[7] We present these minimum values in Tab. 9.6. If, instead, a deviation is detected in some observable but no such RG-induced deviation in other observables is detected at the level hinted by our analysis, then this would indicate a tuning (or a correlation) among the various RG contributions to the direct measurement, see Eq. (9.19). Take, for example, the first row of Tab. 9.6. Suppose we measure the deviation $\hat{c}_{\lambda\gamma} \sim 1 \times 10^{-2}$, a value larger than the minimum value presented in Tab. 9.6, while instead $h \to \gamma\gamma$ would still remain compatible with zero with the reported sensitivity. From Eq. (9.19) we would then conclude that a fine-tuning of the order $\Delta_{\gamma\gamma} \gtrsim 5$ would be necessary to accommodate the data, or that some particular correlation in the UV physics is needed to induce such cancellation.

	Prospects	$|\hat{c}_{\kappa\gamma}|$	$|\hat{c}_{\gamma Z}|$	$|\hat{c}_{\lambda\gamma}|$	$|\hat{c}_H|$
$\hat{c}_{\gamma\gamma}$	4×10^{-5} [145]	6×10^{-3}	-	2×10^{-3}	-
$\hat{c}_{\gamma Z}$	3×10^{-4} [145]	4×10^{-2}	-	1×10^{-2}	-
$\hat{c}_{\kappa\gamma}$	2×10^{-4} [146]	-	1×10^{-2}	1×10^{-2}	-
$\hat{c}_{gZ}$	2×10^{-4} [146]	0.4	-	-	0.25

TABLE 9.6: *In this table we present the minimum value of the couplings in Eq. (9.28) to which direct measurements of the observables in the first column would be sensitive via the one loop RG-mixing effects computed in this work. The long term projection for the measurement precision for the observables in the first column is given in the second column.*

9.3   Scaling of the gluon operators

In this Section we shall extend the results of the previous Sections and present also the scaling of the bosonic operators that contain gluons, as defined in Tab. 9.1:

$$\{\mathcal{O}_{2G}, \; \mathcal{O}_{GG}, \; \mathcal{O}_{3G}\}. \tag{9.31}$$

The anomalous dimension matrix is shown in Tab. 9.7, where the c_{3G} self-renormalization has been taken from Refs. [132, 133]. This matrix already contains the effect of the redundant operators that are generated radiatively and, upon eliminating them, modify the RG of the operators in Tab. 9.1, see Appendix D for details.

In the same spirit of Sec. 9.2, let us now turn to the observables which are sensitive to these operators and review the present constraints. The Wilson coefficient c_{2G} can be put in one-to-one relation to the parameter Z introduced in Ref. [106] (analogous to the W and Y electroweak parameters):

$$Z = \frac{m_W^2}{\Lambda^2} c_{2G}. \tag{9.32}$$

A bound on this parameter has been obtained by an analysis of dijet events at LHC [104]:

$$-9 \times 10^{-4} \lesssim Z \lesssim 3 \times 10^{-4}. \tag{9.33}$$

	c_{2G}	c_{GG}	c_{3G}	c_{2B}	c_{2W}
γ_c	$\frac{266}{9}g_s^2$	0	0	$g'^2\left(\frac{17}{6}(Y_u^2 + Y_d^2) + 12Y_uY_d\right)$	0
γ_c	0	$-\frac{3}{2}g'^2 - \frac{9}{2}g^2 + 12\lambda + 6y_t^2$	0	0	0
γ_c	0	0	$22g_s^2$	0	0

TABLE 9.7: *Anomalous dimension matrix for the Wilson coefficients of the dim-6 bosonic operators with gluons, in the basis defined in Sec. 9. The contributions to and from the other coefficients of the operators in Eq. (9.5), not reported here, are zero.*

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[7]Future prospects for measurements at the Z-pole predict an enhancement of the precision, with respect to the present one, of about one order of magnitude for ILC [146] and two orders of magnitude for TLEP [147], depending on the observable. Moreover, from runs at energy $\sqrt{s} \sim 2m_W$, the measurement of the W mass is predicted to become more precise by one (ILC) or two (TLEP) orders of magnitude. This will imply an enhancement of the precision in the oblique parameters $\hat{S}$, $\hat{T}$, W and Y. A more detailed study of these future prospects is beyond the scope of this work, since our aim is only to show some examples for future applications of the general idea of RG-induced bounds.

A bound on c_{GG} can be obtained from the analysis of the Higgs production cross section at LHC. The relevant phenomenological Lagrangian is

$$\mathcal{L}_h \supset \hat{c}_{GG} \frac{hv}{m_W^2} g_s^2 G_{\mu\nu}^A G^{\mu\nu\,A}, \tag{9.34}$$

where we defined

$$\hat{c}_{GG} \equiv \frac{m_W^2}{\Lambda^2} c_{GG}. \tag{9.35}$$

The most recent bound, obtained in Ref. [108] after marginalizing over the other deviations from the SM, reads

$$\hat{c}_{GG} \in [-0.8, 0.8] \times 10^{-3}. \tag{9.36}$$

The coefficient c_{3G}, analogous to the $SU(2)_L$ counterpart c_{3W}, would contribute to the anomalous triple gluon couplings. These effects can be measured at LEP, Tevatron and LHC, for example via top-quark pair production, see for example Ref. [148] where it is estimated that LHC should be able to put a bound $|\hat{c}_{3G}| \equiv |c_{3G}|\, m_W^2/\Lambda^2 \lesssim 0.1$.

As can be seen in Tab. 9.7, no mixing to (or from) these gluon operators is present among the operators we considered in Tab. 9.1, the only exception being a contribution from c_{2B} to c_{2G} which, however, is not very interesting since c_{2B} is already very well directly constrained by the oblique Y parameter. For this reason, we are not able to set any indirect constraint using these gluon operators.

9.4 Summary

We computed the scaling and mixing of 13 dim-6 deformations of the SM affecting EW precision observables (4), anomalous EW triple gauge boson couplings (3), QCD observables (2) and Higgs decays (4). This computation has important phenomenological implications. Particularly interesting is the RG-mixing induced among 10 of these observables (the 2 two QCD observables and one Higgs observable, namely $\Gamma(h \to gg)$, constitute a separate sector that does not mix in a relevant way with the severely constrained EW observables.).

These 10 different observables are constrained at very different levels of precision. For example, whereas the electroweak precision observables and the operator coefficient related to the $h \to \gamma\gamma$ partial width are constrained at the per mille level, the TGC and the 2 other Higgs observables are constrained at the percent level at most. As we run down from the new physics scale to the lower scale of experiments, quantum effects mix the observables and the most severely constrained ones receive a contribution from the ones allowed to deviate the most from the SM predictions. These RG-contributions

could in principle be of the same size or even larger than the direct experimental bounds. In other words, the difference in the experimental sensitivities can compensate for the RG-loop factor. Requiring that these RG-contributions do obey individually the direct bounds, i.e. dismissing any possible tuning/correlation among the various RG-terms, we can derive some indirect RG-induced bounds on the weakly constrained observables from the direct measurement of the severely constrained ones. This analysis is particularly relevant for the TGC and the universal shift of the Higgs couplings, as reported in Tab. 9.4.

We also looked at the future prospects for these RG-induced effects. If a deviation from the SM is observed in some of the observables we considered, one would expect a deviation, due to these RG effects, to appear also in other seemingly unrelated observables, in the absence of tuning. If, instead, these RG-induced deviations are not observed, it would mean that some tuning is needed, or it would indicate some correlation among the higher dimensional operators pointing towards a particular structure of the new physics that has been integrated out. We have presented the projected future experimental sensitivity to these RG effects in Tab. 9.6.

Part III

Structure of the anomalous dimension matrix

10. One-loop non-renormalization results in EFTs

As we have discussed at length in Part II of this thesis, following the Higgs discovery, there has been much effort put into the determination of the one-loop anomalous dimensions of the dimension-six operators of the SM EFT [5–7, 123, 135]. This has revealed a rather intriguing structure in the anomalous-dimension matrix, with plenty of vanishing entries that are a priori allowed by all symmetries. Some vanishing entries are trivial since no possible Feynman diagram exist contributing to them. Nevertheless, other zeros result from intricate cancelations without any apparent reason. Similar cancelations had been observed before in other EFTs (see for example [124, 149]).

To make manifest the pattern of zeros in the matrix of anomalous dimensions, it is crucial to work in the proper basis. Refs. [5, 6] pointed out the importance of working in bases with operators classified as "current-current" operators and "loop" operators (as reviewed in Part II). The first ones, which we call from now on JJ-operators, were defined to be those operators that can be generated as a product of spin-zero, spin-1/2 or spin-one currents of renormalizable theories [6, 99, 125], while the rest were called "loop" operators. [1] In this basis it was possible to show [5] that some class of loop-operators were not renormalized by JJ-operators, suggesting a kind of generic non-renormalization rule. The complete pattern of zeros in the SM EFT was recently provided in Ref. [150] in the basis of [101], a basis that also maintains the separation between JJ- and loop-operators. A classification of operators based on holomorphy was suggested to be a key ingredient to understand the structure of zeros of the anomalous-dimension matrix [150].

In the present Chapter we provide an approach to understand in a simple way the vanishing entries of anomalous-dimensions. The reason behind many cancelations is the different Lorentz structure of the operators that makes it impossible to mix them at the one-loop level. Although it is possible to show this in certain cases by simple inspection

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[1] This classification is well-defined regardless of the specific UV-completion. Field redefinitions (or use of the equations of motion) do not mix JJ-operators and loop-operators.

of the one-loop diagrams, we present a more compact and systematic approach based on the superfield formalism. For this reason we embed the EFT into an effective superfield theory (ESFT), and classify the operators depending on their embedding into super-operators. Using the ESFT, we are able to show by a simple spurion analysis (the one used to prove non-renormalization theorems in supersymmetric theories) the absence, in certain cases, of mixing between operators of different classes. We then make the important observation that the superpartner contributions to the one-loop renormalization under consideration trivially vanish in many cases. This allows us to conclude that some of the non-renormalization results of the ESFTs apply to the non-supersymmetric EFTs as well. In other words, we will show that in many cases supersymmetry allows to relate a *non-trivial* calculation to a *trivial* one (that of the superpartner loops). This also provides a way to understand the few exceptions to the ubiquitous rule that JJ-operators do not renormalize loop-operators at the one-loop level.

The Chapter is organized as follows. In Sec. 10.1 we start with a simple theory, the EFT of scalar quantum electrodynamics, to illustrate our approach for obtaining one-loop non-renormalization results. In later subsections, we enlarge the theory including fermions, and present an exceptional type of JJ-operator that renormalizes loop-operators. In Sec. 10.3 we show how to generalize our approach to derive analogous results in the SM EFT and we also discuss the holomorphic properties of the anomalous dimensions. In Sec. 10.4 we show the implications of our approach for the QCD Chiral Lagrangian. We conclude in Sec. 10.5.

10.1   Non-renormalization results in a $U(1)$ EFT

In order to make the logic as transparent as possible, let us start with the simple case of a massless scalar coupled to a $U(1)$-gauge boson with charge Q_ϕ, assuming for simplicity CP-conservation. The corresponding EFT is defined as an expansion in derivatives and fields over a heavy new-physics scale Λ: $\mathcal{L}_{\text{EFT}} = \sum_d \mathcal{L}_d$, where $\mathcal{L}_d$ denotes the terms in the expansion made of local operators of dimension d. The leading terms ($d \leq 6$) in the EFT are given by

$$\mathcal{L}_4 = -|D_\mu \phi|^2 - \lambda_\phi |\phi|^4 - \frac{1}{4g^2} F_{\mu\nu}^2 \,, \qquad \mathcal{L}_6 = \frac{1}{\Lambda^2} \left[c_r \mathcal{O}_r + c_6 \mathcal{O}_6 + c_{FF} \mathcal{O}_{FF} \right] \,, \quad (10.1)$$

where the dimension-six operators are

$$\mathcal{O}_r = |\phi|^2 |D_\mu \phi|^2 \,, \qquad \mathcal{O}_6 = |\phi|^6 \,, \qquad \mathcal{O}_{FF} = |\phi|^2 F_{\mu\nu} F^{\mu\nu} \,. \qquad (10.2)$$

We can use different bases for the dimension-six operators although, when looking at operator mixing, it is convenient to work in a basis that separates JJ-operators from loop-operators, as we defined them in the introduction. Using field redefinitions (or, equivalently, the equation of motion (EOM) of ϕ) we can reduce the number of JJ-operators to only two: for instance, $\mathcal{O}_T = \frac{1}{2}J^\mu J_\mu$ and $\mathcal{O}_6 = J^* J$, where $J_\mu = \phi^* \overset{\leftrightarrow}{D}_\mu \phi$ and $J = |\phi|^2 \phi$. It is convenient, however, to set a one-to-one correspondence between operators and supersymmetric D-terms, as we will show below. For this reason, we choose for our basis $\mathcal{O}_6$ and $\mathcal{O}_r$. [2] The only loop-operator, after requiring CP-invariance, is $\mathcal{O}_{FF}$.

Many of the one-loop non-renormalization results that we discuss can be understood from arguments based on the Lorentz structure of the vertices involved. Take for instance the non-renormalization of $\mathcal{O}_{FF}$ by $\mathcal{O}_r$. Integrating by parts and using the EOM, we can eliminate $\mathcal{O}_r$ in favor of $\mathcal{O}'_r = (\phi D_\mu \phi^*)^2 + h.c.$. Now, it is apparent that $\mathcal{O}'_r$ cannot renormalize $\mathcal{O}_{FF}$ because either $\phi D_\mu \phi^*$ or $\phi^* D_\mu \phi$ is external in all one-loop diagrams, and these Lorentz structures cannot be completed to form $\mathcal{O}_{FF}$. Since, in addition, there are no possible one-loop diagrams involving $\mathcal{O}_6$ that contribute to $\mathcal{O}_{FF}$, we can conclude that in this EFT the loop-operator cannot be renormalized at the one-loop level by the JJ-operators. As we will see, similar Lorentz-based arguments can be used for other non-renormalization results. This approach, however, requires a case by case analysis and it is not always guaranteed that one can find an easy argument to see that the loop is zero without a calculation. In this chapter we present a more systematic and unified understanding of such vanishing anomalous dimensions based on a superfield approach that we explain next.

We first promote the model of Eq. (10.1) to an ESFT and study the renormalization of the dimension-six operators in this supersymmetric theory. The superfield formalism makes it transparent to determine which operators do not mix at the one-loop level. Although in this theory the renormalization of operators involves also loops of superpartners, we will show in a second step that either the ordinary loop (involving ϕ and A_μ) is already trivially zero or it is the superpartner loops which trivially vanish. Therefore, having ensured that there are no cancellations between loops of ordinary matter and supermatter, we are able to extend the supersymmetric non-renormalization results to the non-supersymmetric case. In other words, the advantage of this approach is that we can turn a loop calculation with the ordinary ϕ and A_μ into a calculation with superpartners, where the Lorentz structure of the vertex can make it easier to see that the one-loop contributions are zero.

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[2]In the $U(1)$ case we are considering, $\mathcal{O}_r = \frac{1}{2}(\mathcal{O}_H - \mathcal{O}_T)$ where $\mathcal{O}_H = \frac{1}{2}(\partial_\mu |\phi|^2)^2$.

The dimension-six operators of Eq. (10.2) can be embedded in different types of super-operators. As it will become clear in what follows, it is important for our purposes to embed the dimension-six operators into super-operators with the lowest possible dimension. This corresponds to an embedding into the highest θ-component of the super-operator (notice that we can always lower the θ-component by adding derivatives in superspace). This provides a classification of the dimension-six operators that is extremely useful in analyzing the one-loop mixings. Let us start with the loop-operator $\mathcal{O}_{FF}$. Promoting ϕ to a chiral supermultiplet Φ and the gauge boson A_μ to a vector supermultiplet V, one finds that $\mathcal{O}_{FF}$ can be embedded into the θ^2-component (F-term) of the super-operator

$$\Phi^\dagger e^{V_\Phi} \Phi \, \mathcal{W}^\alpha \mathcal{W}_\alpha = -\frac{1}{2}\theta^2 \mathcal{O}_{FF} + \cdots ,\qquad (10.3)$$

where we have defined $V_\Phi \equiv 2Q_\phi V$, $\mathcal{W}^\alpha$ is the field-strength supermultiplet, and we follow the notation of [151] (using a mostly-plus metric). Since the super-operator in Eq. (10.3) is non-chiral, the $\mathcal{O}_{FF}$ cannot be generated in a supersymmetry-preserving theory at any loop order. For the embedding of the JJ-operators, the situation is different. Some of them can be embedded in a D-term (a $\bar{\theta}^2\theta^2$-component), while for others this is not possible. In the example discussed here, we have

$$\left(\Phi^\dagger e^{V_\Phi} \Phi\right)^2 = -4\theta^2\bar{\theta}^2 \mathcal{O}_r + \cdots ,\qquad (10.4)$$

and therefore $\mathcal{O}_r$ is allowed by supersymmetry to appear in the Kähler potential and is not-protected from one-loop corrections. Nevertheless $\mathcal{O}_6$ must arise from the θ^0-component of the super-operator

$$\left(\Phi^\dagger e^{V_\Phi} \Phi\right)^3 = \mathcal{O}_6 + \cdots ,\qquad (10.5)$$

and then must be zero in a supersymmetry-preserving theory at any loop order.

We can now embed Eq. (10.1) in a ESFT. We use a supersymmetry-breaking (SSB) spurion superfield $\eta \equiv \theta^2$ (of dimension $[\eta] = -1$) to incorporate the couplings of Eq. (10.1) that break supersymmetry. We have [3]

$$\mathcal{L}_4 \;\subset\; \int d^4\theta \left[\Phi^\dagger e^{V_\Phi}\Phi + \lambda_\phi \eta\eta^\dagger (\Phi^\dagger e^{V_\Phi}\Phi)^2\right] + \left[\int d^2\theta\, \mathcal{W}^\alpha \mathcal{W}_\alpha + h.c.\right] ,$$

$$\mathcal{L}_6 \;\subset\; \frac{1}{\Lambda^2}\int d^4\theta \Bigg\{ \tilde{c}_r \left(\Phi^\dagger e^{V_\Phi}\Phi\right)^2 + \tilde{c}_6\, \eta\eta^\dagger(\Phi^\dagger e^{V_\Phi}\Phi)^3 $$

$$+\; \left[\tilde{c}_{FF}\, \eta^\dagger(\Phi^\dagger e^{V_\Phi}\Phi)\mathcal{W}^\alpha\mathcal{W}_\alpha + h.c.\right] \Bigg\}. \qquad (10.6)$$

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[3]Anomaly cancelation requires the inclusion of additional fields that do not play any role in our discussion. We ignore them in what follows.

It is very easy to study the one-loop mixing of the dimension-six operators in the above ESFT using a simple η-spurion analysis. For example, it is clear that there cannot be renormalization from terms with no SSB spurions, such as $\tilde{c}_r$, to terms with SSB spurions, such as $\tilde{c}_{FF}$. Also, corrections from $\tilde{c}_r$ to $\tilde{c}_6$ are only possible through the insertion of λ_ϕ, that carries a $\eta\eta^\dagger$. Similarly, terms with a SSB spurion $\eta^\dagger$ cannot renormalize terms with two SSB spurions $\eta^\dagger\eta$, unless they are proportional to λ_ϕ. This means that $\tilde{c}_{FF}$ can only renormalize $\tilde{c}_6$ with the insertion of a λ_ϕ. The inverse is however not guaranteed: terms with more SSB spurions can in principle renormalize terms with less spurions. For example, $\tilde{c}_{FF}$, that carries a spurion $\eta^\dagger$, could generate at the loop level the operator

$$\int d^4\theta \eta^\dagger \bar{\mathcal{D}}^2 \tilde{\mathcal{O}}_r = \int d^4\theta (\bar{\mathcal{D}}^2 \eta^\dagger)\tilde{\mathcal{O}}_r = \int d^4\theta \tilde{\mathcal{O}}_r \,, \qquad (10.7)$$

where $\tilde{\mathcal{O}}_r = \left(\Phi^\dagger e^{V_\Phi}\Phi\right)^2$ and we have defined $\mathcal{D}^2 \equiv \mathcal{D}_\alpha \mathcal{D}^\alpha$, with $\mathcal{D}_\alpha\Phi = e^{-V_\Phi}D_\alpha(e^{V_\Phi}\Phi)$ being the gauge-covariant derivative in superspace. Therefore one has to check it case by case. For example, $\tilde{c}_6$ could in principle renormalize $\tilde{c}_{FF}$, but it is not possible to write the relevant diagram since it involves a vertex with too many Φ's. This implies that $\tilde{c}_{FF}$ is only renormalized by itself at the one-loop level.

This simple renormalization structure is the starting point from which, by examining more closely the loops involved at the field-component level, we will derive the following non-renormalization results in the non-supersymmetric EFT of Eq. (10.1).

Non-renormalization of $\mathcal{O}_{FF}$ by $\mathcal{O}_r$: The differences between our original EFT in Eq. (10.1) and its supersymmetric version, Eq. (10.6), are the presence of the fermion superpartners for the gauge and scalar: the gaugino, λ, and "Higgsino", ψ. We will show, however, that the contributions from superpartners trivially vanish in the mixing of JJ- and loop-operators. In

$$\int d^4\theta \left(\Phi^\dagger e^{V_\Phi}\Phi\right)^2 = -4\mathcal{O}_r + 2(i\phi^* \overset{\leftrightarrow}{D}_\mu \phi)\psi^\dagger \bar{\sigma}^\mu \psi + 2|\phi|^2(i\psi^\dagger \bar{\sigma}^\mu \overset{\leftrightarrow}{D}_\mu \psi) + \cdots\,, \qquad (10.8)$$

we have only the 3 terms shown that can potentially contribute to $\mathcal{O}_{FF}$ at the one-loop level. These terms can be considered as part of a supersymmetric JJ-operator generated from integrating-out a heavy vector superfield that contains a scalar, a vector and a fermion. Other terms not shown in Eq. (10.8) involve too many fields (see Appendix E) and therefore are only relevant for an analysis beyond one-loop. The first term of Eq. (10.8) can potentially give a contribution to $\mathcal{O}_{FF}$ from a loop of ϕ's, while the second and third term could from a loop of Higgsinos. It is very easy to see that the loop of Higgsinos does not contribute to $\mathcal{O}_{FF}$. Indeed, if in the second term of Eq. (10.8) we close the Higgsinos in a loop, the current $J_\mu = i\phi^* \overset{\leftrightarrow}{D}_\mu \phi$ is left as an external factor,

and it is then clear that we can only generate the JJ-operator $J_\mu J^\mu$. Moreover, the third term of Eq. (10.8) vanishes by using the EOM: $\bar{\sigma}^\mu D_\mu \psi = 0$ (up to gaugino terms that are not relevant here). Therefore, Higgsinos do not contribute at the one-loop level to the renormalization of the loop-operator $\mathcal{O}_{FF}$. We can then extend the non-renormalization result from the ESFT of Eq. (10.6) to the non-supersymmetric EFT of Eq. (10.1) and conclude that *the loop-operator cannot be renormalized at the one-loop level by the JJ-operators.*

Non-renormalization of $\mathcal{O}_r$ by $\mathcal{O}_{FF}$: It remains to study the renormalization from $\mathcal{O}_{FF}$ to $\mathcal{O}_r$. This can arise in principle from a loop of gauge bosons. In the supersymmetric theory, Eq. (10.6), $\tilde{c}_r$ does not carry any SSB spurion and therefore its renormalization by $\tilde{c}_{FF}$ cannot be prevented on general grounds, as we explained before. Nevertheless, we find that operators induced by $\tilde{c}_{FF}$, through a loop of V's, must leave an external factor $\eta^\dagger \Phi^\dagger e^{V_\Phi} \Phi$ from the vertex and then, the only operator that could potentially contribute to $\tilde{c}_r$ must have the form [4]

$$\frac{1}{\Lambda^2} \int d^4\theta \, \eta^\dagger \left(\Phi^\dagger e^{V_\Phi} \Phi \right) \bar{\mathcal{D}}^2 \left(\Phi^\dagger e^{V_\Phi} \Phi \right) + h.c. \,. \tag{10.9}$$

From the EOM for Φ, we have that $\bar{\mathcal{D}}^2 \Phi^\dagger = 0$ up to λ_ϕ terms that bring too many powers of Φ, so that the projection of Eq. (10.9) into $\mathcal{O}_r$ vanishes. Finally, one also has to ensure that redundant JJ-super-operators, that can give $\left(\Phi^\dagger e^{V_\Phi} \Phi \right)^2$ through superfield redefinitions, are not generated at the one-loop level. In particular, the redundant super-operator

$$\frac{1}{\Lambda^2} \int d^4\theta \left(\Phi^\dagger e^{V_\Phi} \Phi \right) \mathcal{D}_\alpha \mathcal{W}^\alpha \,, \tag{10.10}$$

if generated at the loop level, can give a contribution to $\tilde{c}_r$ after superfield redefinitions, or equivalently, after using the EOM of V: $\mathcal{D}_\alpha \mathcal{W}^\alpha + h.c. = -g Q_\phi \Phi^\dagger e^{V_\Phi} \Phi$. We do not find, however, any non-zero contribution from $\eta^\dagger (\Phi^\dagger e^{V_\Phi} \Phi) \mathcal{W}^\alpha \mathcal{W}_\alpha$ to the operator in Eq. (10.10), as such contributions, coming from a V/Φ loop, must be proportional to $\eta^\dagger \mathcal{W}^\alpha \Phi$. [5]

Having shown that supersymmetry guarantees zero contributions to $\tilde{c}_r$ from $\tilde{c}_{FF}$, we must check what are the effects of superpartner loops. From (see Appendix E)

$$\int d^4\theta \eta^\dagger (\Phi^\dagger e^{V_\Phi} \Phi) \mathcal{W}^\alpha \mathcal{W}_\alpha + h.c. = -\mathcal{O}_{FF} \tag{10.11}$$

$$+ \left(2i|\phi|^2 \, \lambda \sigma^\mu \partial_\mu \lambda^\dagger - \frac{1}{\sqrt{2}} \phi^* \lambda \sigma^{\mu\nu} \psi F_{\mu\nu} + h.c. \right) + \dots \,,$$

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[4] Notice that the presence of $\eta^\dagger$, arising from the vertex, requires that the super-operator must have two derivatives $\bar{\mathcal{D}}$ in order to potentially contain $\mathcal{O}_r$.

[5] Of these, the only one that cannot be put to zero by the EOM of Φ is $\int d^4\theta \, \eta^\dagger \mathcal{W}^\alpha \Phi [\bar{\mathcal{D}}_{\dot{\alpha}}, \{\mathcal{D}_\alpha, \bar{\mathcal{D}}^{\dot{\alpha}}\}] e^V \Phi^\dagger$ but, from the identity $[\bar{\mathcal{D}}_{\dot{\alpha}}, \{\mathcal{D}_\alpha, \bar{\mathcal{D}}^{\dot{\alpha}}\}] \sim i \mathcal{W}_\alpha$ [152], one can see that this only contributes to $\tilde{c}_{FF}$.

where $\sigma^{\mu\nu} = \frac{i}{2}(\sigma^\mu\bar{\sigma}^\nu - \sigma^\nu\bar{\sigma}^\mu)$, it is clear that a gaugino/Higgsino loop cannot give a contribution to $\mathcal{O}_r$: the second term of Eq. (10.11), after using the EOM for the gaugino, $\sigma^\mu\partial_\mu\lambda^\dagger = g\phi\psi^\dagger$, can only give a contribution proportional to $|\phi|^2\phi$; while the contribution from the third term must be proportional to $\phi^* F_{\mu\nu}$. None of them have the right Lorentz structure to contribute to $\mathcal{O}_r$. Therefore, we conclude that the *loop-operator $\mathcal{O}_{FF}$ can only renormalize at the one-loop level the JJ-operators that break supersymmetry, like $\mathcal{O}_6$, and not those that can be embedded in a D-term, like $\mathcal{O}_r$.*

10.1.1   Including fermions

Let us extend the previous EFT to include two charged Weyl fermions, q and u, with $U(1)$-charges Q_q and Q_u, such that $Q_\phi + Q_q + Q_u = 0$. We have now extra terms in the Lagrangian (respecting CP-invariance): [6]

$$\Delta\mathcal{L}_4 = iq^\dagger\bar{\sigma}^\mu D_\mu q + iu^\dagger\bar{\sigma}^\mu D_\mu u + y_u\left(\phi qu + h.c.\right),$$

$$\Delta\mathcal{L}_6 = \frac{1}{\Lambda^2}\left[c_{\phi f}\mathcal{O}_{\phi f} + c_{4f}\mathcal{O}_{4f} + c_{y_u}\left(\mathcal{O}_{y_u} + h.c.\right) + c_D\left(\mathcal{O}_D + h.c.\right)\right], \quad (10.12)$$

where $f = q, u$. The JJ-operators are

$$\mathcal{O}_{y_u} = |\phi|^2\phi qu, \quad \mathcal{O}_{\phi f} = i(\phi^* f^\dagger)\bar{\sigma}^\mu D_\mu(f\phi), \quad \mathcal{O}_{4f} = (f^\dagger\bar{\sigma}_\mu f)(f^\dagger\bar{\sigma}^\mu f). \quad (10.13)$$

Instead of $\mathcal{O}_{\phi f}$, we could have chosen the more common JJ-operator $i(\phi^*\overset{\leftrightarrow}{D}_\mu\phi)(f^\dagger\bar{\sigma}^\mu f)$ for our basis. Both are related by

$$\mathcal{O}_{\phi f} = \frac{i}{2}(\phi^*\overset{\leftrightarrow}{D}_\mu\phi)(f^\dagger\bar{\sigma}^\mu f) + \frac{i}{2}|\phi|^2 f^\dagger\bar{\sigma}^\mu\overset{\leftrightarrow}{D}_\mu f, \quad (10.14)$$

where the last term could be eliminated by the use of the EOM. Our motivation for keeping $\mathcal{O}_{\phi f}$ in our basis is that, as we will see later, it is in one-to-one correspondence with a supersymmetric D-term. The only additional loop-operator for a $U(1)$ model with fermions is the dipole operator

$$\mathcal{O}_D = \phi(q\sigma^{\mu\nu}u)F_{\mu\nu}. \quad (10.15)$$

Let us consider the operator mixing in this extended EFT. We will discuss all cases except those for which no diagram exists at the one-loop level. As we said before, in principle, many vanishing entries of the anomalous-dimension matrix can be simply understood from inspection of the Lorentz structure of the different vertices. For example,

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[6] Similar remarks to those made in footnote 3 about anomalies apply to this extended model.

it is relatively simple to check that the JJ-operators $\mathcal{O}_{4f}$ and $\mathcal{O}_{\phi f}$ do not renormalize the loop-operators. For this purpose, it is important to recall that we can write four-fermion operators, such as $(q^\dagger \bar{\sigma}_\mu q)(u^\dagger \bar{\sigma}^\mu u)$, in the equivalent form $q^\dagger u^\dagger q u$. From this, it is obvious that closing a loop of fermions can only give operators containing the Lorentz structure $f^\dagger f$ or qu that cannot be completed to give a dipole operator (nor its equivalent forms, $q\sigma_{\mu\nu}\sigma_\rho D^\rho q^\dagger F^{\mu\nu}$ or $D_\mu \phi q D^\mu u H$). For the case of $\mathcal{O}_{\phi f}$, the absence of renormalization of the dipole operator, as for example from diagrams like the one in Fig. 10.1, can be proved just by realizing that we can always keep the Lorentz structure $\bar{\sigma}^\mu D_\mu(\phi f)$ external to the loop; this Lorentz structure cannot be completed to form a dipole operator. The contribution of $\mathcal{O}_{\phi f}$ to $\mathcal{O}_{FF}$ is also absent, as can be deduced from Eq. (10.14): the first term, after closing the fermion loop, gives the wrong Lorentz structure to generate $\mathcal{O}_{FF}$, while the second term gives an interaction with too many fields if we use the fermion EOM. Finally, $\mathcal{O}_{y_u}$ can only contribute to the Lorentz structure $\phi q u$, not to the dipole one in Eq. (10.15).

We can be more systematic and complete using our ESFT approach. Let us see first how the operators of Eq. (10.12) can be embedded in super-operators. By embedding q and u in the chiral supermultiplets Q and U, we find that the dipole loop-operator must arise from the θ^2-term of a non-chiral superfield:

$$\Phi \left(Q \overset{\leftrightarrow}{\mathcal{D}}_\alpha U \right) \mathcal{W}^\alpha = -\theta^2 \mathcal{O}_D + \cdots . \tag{10.16}$$

Among the JJ-operators of Eq. (10.13), two of them can arise from supersymmetric D-terms and are then supersymmetry-preserving:

$$\left(\Phi^\dagger e^{V_\Phi} \Phi \right) \left(Q^\dagger e^{V_Q} Q \right) = \bar{\theta}^2 \theta^2 \mathcal{O}_{\phi q} + \cdots , \quad \left(Q^\dagger e^{V_Q} Q \right) \left(Q^\dagger e^{V_Q} Q \right) = -\frac{1}{2} \bar{\theta}^2 \theta^2 \mathcal{O}_{4q} + \cdots , \tag{10.17}$$

and similar operators for $Q \to U$, where we again use the short-hand notation $V_Q = 2Q_q V$. Nevertheless, one of the JJ-operators must come from the θ^2-component of a non-chiral superfield that is not invariant under supersymmetry:

$$\left(\Phi^\dagger e^{V_\Phi} \Phi \right) \Phi Q U = \theta^2 \mathcal{O}_{y_u} + \cdots . \tag{10.18}$$
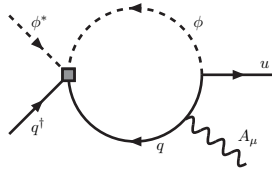

FIGURE 10.1: *A potential contribution from $\mathcal{O}_{\phi q}$ to $\mathcal{O}_D$.*

We can now promote Eq. (10.12) to a ESFT:

$$
\begin{aligned}
\Delta\mathcal{L}_4 &\subset \int d^4\theta \left(Q^\dagger e^{V_Q} Q + U^\dagger e^{V_U} U \right) + \left[\int d^2\theta \, y_u \Phi Q U + h.c. \right] , \\
\Delta\mathcal{L}_6 &\subset \frac{1}{\Lambda^2} \int d^4\theta \Big\{ \tilde{c}_{\phi f}(\Phi^\dagger e^{V_\Phi}\Phi)(F^\dagger e^{V_F} F) + \tilde{c}_{4f}(F^\dagger e^{V_F} F)(F^\dagger e^{V_F} F) \\
&\quad + \left[\eta^\dagger \left(\tilde{c}_{y_u}(\Phi^\dagger e^{V_\Phi}\Phi)\Phi Q U + \tilde{c}_D \Phi \, (Q \overset{\leftrightarrow}{\mathcal{D}}_\alpha U)\,\mathcal{W}^\alpha \right) + h.c. \right] \Big\} , \quad (10.19)
\end{aligned}
$$

where $F = Q, U$.

Non-renormalization of loop-operators from JJ-operators: The embedding of the EFT into the ESFT shows the following rule. Loop-operators ($\mathcal{O}_{FF}$ and $\mathcal{O}_D$) cannot be supersymmetrized, while some JJ-operators can be supersymmetrized ($\mathcal{O}_r$, $\mathcal{O}_{4f}$ and $\mathcal{O}_{\phi f}$) and others cannot ($\mathcal{O}_{y_u}$ and $\mathcal{O}_6$). Supersymmetry then guarantees that loop-operators can at most be generated from the latter ones, $\mathcal{O}_{y_u}$ and $\mathcal{O}_6$, embedded respectively in $\eta^\dagger(\Phi^\dagger e^{V_\Phi}\Phi)\Phi Q U$ and $\eta\eta^\dagger(\Phi^\dagger e^{V_\Phi}\Phi)^3$. By simple inspection of these latter vertices, however, we find that neither of these two operators can possibly renormalize the loop-operators at the one-loop level. Therefore, in the ESFT the loop-operators are not renormalized at one-loop level by the JJ-operators.

To extend the above results to the non-supersymmetric EFT, we must ensure that these non-renormalization results do not arise from cancellations between loops involving "ordinary" fields (A_μ, ϕ, q and u) and loops involving superpartners (λ, ψ, $\tilde{q}$ and $\tilde{u}$). This can be proved by showing that either the former or the latter are zero. In certain cases it is easier to look at the loop of ordinary fields, while in others it is easier to look at the superpartner loops. For example, we have (see Appendix E)

$$
\int d^4\theta \left(Q^\dagger e^{V_Q} Q \right)\left(Q^\dagger e^{V_Q} Q \right) = -\frac{1}{2}\mathcal{O}_{4q} + 2q^\dagger\bar{\sigma}^\mu q(i\tilde{q}^\dagger \overset{\leftrightarrow}{D}_\mu \tilde{q}) + 2(iq^\dagger\bar{\sigma}^\mu \overset{\leftrightarrow}{D}_\mu q)|\tilde{q}|^2 + \cdots ,
$$

(10.20)

from which we see that a renormalization to $\mathcal{O}_D$ can arise either from the first term (by a loop of "quarks" q) or the second and third term by a loop of "squarks" $\tilde{q}$. It is easier to see that the loops of squarks are zero: they can only generate operators containing $q^\dagger\bar{\sigma}^\mu q$ or $q^\dagger\bar{\sigma}^\mu \overset{\leftrightarrow}{D}_\mu q$, that do not have the structure necessary to contribute to the dipole operator $\mathcal{O}_D$ nor to operators related to this one by EOMs, such as $q\sigma_{\mu\nu}\sigma_\rho D^\rho q^\dagger F^{\mu\nu}$. We could proceed similarly for the other operators. For the case of $\mathcal{O}_{\phi f}$, however, the one-loop contribution to $\mathcal{O}_D$ contains scalars and fermions (see Fig. 10.1) and the corresponding graph with superpartners has a similar structure, and therefore is not simpler. Nevertheless, both can be shown to be zero by realizing that $\bar{\sigma}^\mu D_\mu(\phi f)$ can always be kept as external to the loop, and that this Lorentz structure cannot be completed to form a dipole operator. We can conclude that the absence of renormalization of loop-operators by JJ-operators valid in the ESFT also applies to the EFT.

Class of JJ-operators not renormalized by loop-operators: Following the same approach, we can also check whether loop-operators can generate JJ-operators. Let us first work within the ESFT. We have shown already that the loop-super-operator $\eta^\dagger(\Phi^\dagger e^{V_\Phi}\Phi)\mathcal{W}^\alpha\mathcal{W}_\alpha$ cannot generate the JJ-super-operator $(\Phi^\dagger e^{V_\Phi}\Phi)^2$. The same arguments apply straightforwardly to $(F^\dagger e^{V_F}F)(\Phi^\dagger e^{V_\Phi}\Phi)$. For the case of the dipole super-operator, $\eta^\dagger\Phi(Q\overset{\leftrightarrow}{\mathcal{D}}_\alpha U)\mathcal{W}^\alpha$, we have a potential contribution to $(Q^\dagger e^{V_Q}Q)(U^\dagger e^{V_U}U)$ coming from a Φ/V loop. Nevertheless, as the factor $\eta^\dagger Q\overset{\leftrightarrow}{\mathcal{D}}_\alpha U$ remains in the external legs, it is clear that such contribution can only lead to operators containing $\eta^\dagger\mathcal{D}^2$, which are not JJ-super-operators. Similarly, contributions to $(\Phi^\dagger e^{V_\Phi}\Phi)(Q^\dagger e^{V_Q}Q)$ could arise from a U/V loop, but one can always arrange it to leave either $\eta^\dagger\mathcal{D}_\alpha\Phi$ or $\eta^\dagger\mathcal{D}_\alpha Q$ in the external legs [7], which again does not have the structure of a JJ-super-operator (the same applies for $Q\leftrightarrow U$). Finally we must check whether redundant JJ-super-operators, as the one in Eq. (10.10), can be generated by the dipole. Similar arguments as those below Eq. (10.10) can be used to prove that this is not the case. Notice, however, that we cannot guarantee the absence of renormalization by loop-super-operators neither of $\eta^\dagger(\Phi^\dagger e^{V_\Phi}\Phi)\Phi QU$ nor of $\eta\eta^\dagger(\Phi^\dagger e^{V_\Phi}\Phi)^3$. We then conclude that only the JJ-super-operators that preserve supersymmetry (with no SSB-spurions) are safe at the one-loop level from the renormalization by loop-super-operators.

It remains to show that this result extends also to the non-supersymmetric EFT. From Eq. (E.3) of the Appendix E, we have, after using the gaugino EOM and eliminating the auxiliary fields F_i, that loops from superpartners can only give contributions proportional to ϕff, $|\phi|^2 f$, ff or $F_{\mu\nu}f$ (for $f=q,u$). None of these terms can lead to the Lorentz structure of $\mathcal{O}_r$, $\mathcal{O}_{4f}$ nor $\mathcal{O}_{\phi f}$. These are exactly the same JJ-operators that could not be generated (at one loop) from loop-operators in the ESFT.
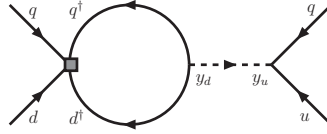
10.1.2   An exceptional JJ-operator

Let us finally extend the EFT to include an extra fermion, a "down-quark" d of charge Q_d, such that $Q_\phi = Q_q + Q_d$. The following extra terms are allowed in the Lagrangian:

$$\Delta\mathcal{L}_4 = id^\dagger\bar\sigma^\mu D_\mu d + y_d\left(\phi^* qd + h.c.\right),$$

$$\Delta\mathcal{L}_6 = \frac{1}{\Lambda^2}\left[c_{y_d}\mathcal{O}_{y_d} + c_{y_uy_d}\mathcal{O}_{y_uy_d} + h.c.\right], \tag{10.21}$$

where we have the additional JJ-operators

$$\mathcal{O}_{y_d} = |\phi|^2\phi^* qd, \quad \mathcal{O}_{y_uy_d} = quqd, \tag{10.22}$$

---

[7] Using integration by parts and the EOM of V, we can write the dipole super-operator as $\int d^4\theta\,\eta^\dagger\Phi(Q\overset{\leftrightarrow}{\mathcal{D}}_\alpha U)\mathcal{W}^\alpha = -\int d^4\theta\,\eta^\dagger[(\mathcal{D}_\alpha\Phi)QU\mathcal{W}^\alpha + 2\Phi(\mathcal{D}_\alpha Q)U\mathcal{W}^\alpha + O(\Phi_i^5)]$ where $\Phi_i = \Phi, Q, U$.

FIGURE 10.2: *Contribution to c_{yy} proportional to $y_d y_u$.*

apart from operators similar to the ones in Eq. (10.12) with f including also the d.

Following the ESFT approach, we embed the d-quark in a chiral supermultiplet D and the operators of Eq. (10.21) into the super-operators:

$$
\begin{aligned}
\Phi^\dagger e^{V_\Phi} Q D &= \theta^2 \phi^* q d + \cdots , \\
\left(\Phi^\dagger e^{V_\Phi} \Phi \right) \Phi^\dagger e^{V_\Phi} Q D &= \theta^2 \mathcal{O}_{y_d} + \cdots , \\
(QU) \mathcal{D}^2 (QD) &= -4\theta^2 \mathcal{O}_{y_u y_d} + \cdots .
\end{aligned}
\tag{10.23}
$$

As all of these operators come from a θ^2-term of non-chiral super-operators, we learn that they can only be generated from supersymmetry-breaking. We can promote Eq. (10.21) into a ESFT in the following way:

$$
\begin{aligned}
\Delta\mathcal{L}_4 &\subset \int d^4\theta \left[D^\dagger e^{V_D} D + \left(\eta^\dagger y_d \Phi^\dagger e^{V_\Phi} Q D + h.c. \right) \right] , \\
\Delta\mathcal{L}_6 &\subset \frac{1}{\Lambda^2} \int d^4\theta\, \eta^\dagger \left[\tilde{c}_{y_d} \left(\Phi^\dagger e^{V_\Phi} \Phi \right) \Phi^\dagger e^{V_\Phi} Q D + \tilde{c}_{y_u y_d} (QU) \mathcal{D}^2 (QD) \right] + h.c.
\end{aligned}
\tag{10.24}
$$

Now, and this is very important, when considering only d, q, ϕ in isolation (without the u fermion), we can always change the supersymmetric embedding of ϕ by considering $\phi^* \in \bar{\Phi}$, where $\bar{\Phi}$ is a chiral supermultiplet of charge $-1/2$. By doing this, we can write the Yukawa-term for the d in a supersymmetric way, $\int d^2\theta\, y_d \bar{\Phi} Q D$, and guarantee that the renormalization of operators involving only ϕ, q, d is identical to the one of ϕ, q, u explained in the previous Section.

It is then clear that supersymmetry breaking from Yukawas can only arise through the combination $y_u y_d$. This allows to explain why contributions to $\mathcal{O}_{y_u y_d}$ from $(q^\dagger \bar{\sigma}_\mu q)(d^\dagger \bar{\sigma}^\mu d)$ must be proportional to $y_u y_d$, as explicit calculations have shown in the SM context [150]. In the ESFT, the operator $(q^\dagger \bar{\sigma}_\mu q)(d^\dagger \bar{\sigma}^\mu d)$ is embedded in a supersymmetry-preserving super-operator and therefore can only generate supersymmetry-breaking interactions, such as $\mathcal{O}_{y_u y_d}$, via the SSB couplings $y_u y_d$ (see Fig. 10.2). The one-loop contributions from superpartners do not affect this result, as Eq. (10.20) shows that they are trivially zero.

The operators $\mathcal{O}_{y_u y_d}$ and $\mathcal{O}_{y_{u,d}}$ are the only JJ-operators that are embedded in the ESFT with the same SSB-spurion dependence as the loop-operators – see Eq. (10.24). Therefore, they can potentially renormalize $\mathcal{O}_D$. Although this was not the case for $\mathcal{O}_{y_{u,d}}$ due to its Lorentz structure, as we explained above, we have confirmed by explicit calculation that $\mathcal{O}_{y_u y_d}$ indeed renormalizes $\mathcal{O}_D$. This is then an exception to the ubiquitous rule that JJ-operators do not renormalize loop-operators.

10.2   A closely related analogy

There is an analogy that helps in clarifying the role that supersymmetry has played in explaining the non-renormalization result in the non-supersymmetric EFT. In QCD, tree-level amplitudes with all plus helicity gluons or all plus helicity except one negative helicity gluon vanish

$$A_n^{tree}\left[g^- g^+ g^+ \cdots g^+\right] = A_n^{tree}\left[g^+ g^+ \cdots g^+\right] = 0\,, \tag{10.25}$$

where all gluons are taken as outgoing. To prove Eq. (10.25) by brute force is hard. The original derivation was done through a smart choice of the polarisation vectors of the gluons, see for instance Ref. [153]. However, the easiest way to prove it is to consider super-QCD and a Ward identity associated to the SUSY generators [154, 155]. Recall that a symmetric vacuum is annihilated by the symmetry generators Q, then the Ward identity for an n-point reads

$$
\begin{aligned}
0 &= \langle 0|\left[Q^\dagger, \mathcal{O}_1(x_1)\mathcal{O}_2(x_2)\cdots\mathcal{O}_n(x_n)\right]|0\rangle \\
&= \sum_i^n (-1)^{\sum_{i<j}|\mathcal{O}_i|}\langle 0|\mathcal{O}_1(x_1)\cdots\left[Q^\dagger, \mathcal{O}_i(x_i)\right]\cdots\mathcal{O}_n(x_n)|0\rangle\,, \tag{10.26}
\end{aligned}
$$

where $\mathcal{O}_i$ are field-operators and we pick a sign every time $Q^\dagger$ is commuted with a fermionic operator. Next, let us take $Q^\dagger$ to be the supersymmetry generator. Recall that it acts on the positive helicity gluon and positive helicity gluino as $[Q^\dagger, a_g^+] \sim 0$, $[Q^\dagger, a_\lambda^+] \sim a_g^+$, respectively. Then, applying the Ward identity to

$$
\begin{aligned}
0 &= \langle 0|\left[Q^\dagger, a_\lambda^+(p_1)a_g^+(p_2)a_g^+(p_3)\cdots a_g^+(p_n)\right]|0\rangle \\
&\propto \langle 0|a_g^+(p_1)a_g^+(p_2)\cdots a_g^+(p_n)|0\rangle\,, \tag{10.27}
\end{aligned}
$$

and a similar one for $\mathcal{O} = a_g^- a_\lambda^+ a_g^+ a_g^+ \cdots a_g^+$, we find that

$$A_{SQCD,n}^{L-loop}\left[g^- g^+ g^+ \cdots g^+\right] = A_{SQCD,n}^{L-loop}\left[g^+ g^+ \cdots g^+\right] = 0\,, \tag{10.28}$$

at all orders in perturbation theory, without making any actual diagrammatic calculation or clever choice of variables. Finally, one notices that in the tree-level scattering amplitude of Eq. (10.25) gluinos are absent. For an amplitude with external gluons only the gluinos are necessary closed in loops and therefore the SQCD result of Eq. (10.28) is inherited by tree-level QCD.

Now the analogy is clear: in both the present example and in the non-renormalization results of the previous Section, we found an exact result in SUSY valid at all loop orders. Then, in both cases we notice the absence of the superpartners at a fixed order in perturbation theory and therefore the supersymmetric result is inherited at that same order by the non-supersymmetric theory. Superpartners were absent at tree-level in the scattering-amplitudes of the present example while they where absent in the one-loop effective action of the previous Section.

10.3 Generalization to the Standard Model EFT

We can generalize the analysis of Sec. 10.1 to dimension-six operators in the SM EFT. We begin by constructing an operator basis that separates JJ-operators from loop-operators. We then classify them according to their embedding into a supersymmetric model, depending on whether they can arise from a super-operator with no SSB spurion (η^0), which therefore preserves supersymmetry, or whether they need SSB spurions, either $\bar{\mathcal{D}}_{\dot{\alpha}}\eta^\dagger$, $\eta^\dagger$, $|\bar{\mathcal{D}}_{\dot{\alpha}}\eta^\dagger|^2$ or $\eta\eta^\dagger$ (that selects the $\bar{\theta}\theta^2$, θ^2, $\bar{\theta}\theta$ and $\bar{\theta}^0\theta^0$ component of the super-operator, respectively), or their Hermitian-conjugates. The supersymmetric embedding naturally selects a SM basis that we present in Tab. 10.1. In this basis, the non-renormalization results between the different classes of operators discussed in the previous Section will also hold.

The operator basis of Tab. 10.1 is close to the basis defined in Ref. [101]. One significant difference is our choice of the only-Higgs JJ-operators, that we take to be $\mathcal{O}_\pm$ and $\mathcal{O}_6$, and of the Higgs-fermion JJ-operator $\mathcal{O}_{Hf}$. As in the $U(1)$ case, this choice is motivated by the embedding of operators into super-field operators, as we have just mentioned (see more details below). Concerning the classification of 4-fermion operators, our $\mathcal{O}_{4f}$ operators correspond not only to types $(\bar{L}L)(\bar{L}L)$, $(\bar{R}R)(\bar{R}R)$ and $(\bar{L}L)(\bar{R}R)$ of Ref. [101], but also to the operator $Q_{ledq} = (\bar{L}_L e_R)(\bar{d}_R Q_L)$ classified as $(\bar{L}R)(\bar{R}L)$ in [101], since this latter operator can be written as a $\mathcal{O}_{4f}$ by Fierz rearrangement. Finally, our $\mathcal{O}_{yy}$ operators correspond to the four operators of type $(\bar{L}R)(\bar{L}R)$ in [101].

To embed the SM fields in supermultiplets we follow the common practice of working with left-handed fermion fields so that Q_L, u_R^c and d_R^c are embedded into the chiral

supermultiplets Q, U and D (generically denoted by F). With an abuse of notation, we use H for the SM Higgs doublet as well as for the chiral supermultiplet into which it is embedded. Finally, gauge bosons are embedded in vector superfields, V^a, and we use the notation $V_\Phi \equiv 2t^a V^a$ where t^a include the generators of the SM gauge-group in the representation of the chiral-superfield Φ.

Concerning the embedding of operators into super-operators, there are a few differences with respect to the $U(1)$ model discussed in the previous Section, as we discuss below. Starting with the JJ-operators, we have a new type of operator not present in the $U(1)$ case, $\mathcal{O}_R^{ud} = (iH^\dagger \overset{\leftrightarrow}{D}_\mu \tilde{H})(\bar{d}_R \gamma^\mu u_R)$, where $\tilde{H} \equiv i\sigma_2 H^*$. This operator cannot be embedded as the others in a D-term due to $\tilde{H}^\dagger H = 0$ and must be embedded as a $\theta^2 \bar{\theta}$ term of a spinor super-operator:

$$\int d^4\theta \; \bar{\mathcal{D}}_{\dot{\alpha}} \eta^\dagger (H^\dagger \bar{\mathcal{D}}^{\dot{\alpha}} \tilde{H}) U^\dagger e^{V_D} D = \mathcal{O}_R^{ud} + \cdots . \tag{10.29}$$

For the JJ-operators involving only the Higgs field, there is also an important difference

	Operators	Super-operators	Spurion
	$\mathcal{O}_+ = D_\mu(H_i^\dagger H_j^\dagger) D^\mu(H^i H^j)$	$(H^\dagger e^V H)^2$	
	$\mathcal{O}_{4f} = (\bar{f}\gamma^\mu t^a f)(\bar{f}\gamma_\mu t^a f)$	$(F^\dagger t^a e^V F)(F^\dagger t^a e^V F)$	η^0
JJ-operators	$\mathcal{O}_{Hf} = i(H^\dagger t^a)_i(\bar{f}t^a)_j \gamma^\mu D_\mu (H^i f^j)$	$(H^\dagger t^a e^V H)(F^\dagger t^a e^V F)$	
	$\mathcal{O}_R^{ud} = (iH^\dagger \overset{\leftrightarrow}{D}_\mu \tilde{H})(\bar{d}_R \gamma^\mu u_R)$	$H^\dagger \bar{\mathcal{D}}^{\dot{\alpha}} \tilde{H} U^\dagger e^V D$	$\bar{\mathcal{D}}_{\dot{\alpha}} \eta^\dagger$
	$\mathcal{O}_- = |H^\dagger D_\mu H|^2$	$|H^\dagger e^V \mathcal{D}_\alpha H|^2$	$|\bar{\mathcal{D}}_{\dot{\alpha}} \eta^\dagger|^2$
	$\mathcal{O}_6 = |H|^6$	$(H^\dagger e^V H)^3$	$|\eta|^2$
	$\mathcal{O}_y = |H|^2 H \bar{f}_R f_L$	$(H^\dagger e^V H) H F F$	
	$\mathcal{O}_{yy} = (\bar{f}_R t^a f_L)(\bar{f}_R t^a f_L)$	$(F t^a F) \mathcal{D}^2 (F t^a F)$	
Loop-operators	$\mathcal{O}_D = H^\dagger \bar{f}_R \sigma^{\mu\nu} t^a f_L F_{\mu\nu}^a$	$H(F t^a \overset{\leftrightarrow}{\mathcal{D}}_\alpha F) \mathcal{W}^{a\,\alpha}$	$\eta^\dagger$
	$\mathcal{O}_{FF} = H^\dagger t^a t^b H F_{\mu\nu}^a (F^{b\,\mu\nu} - i\tilde{F}^{b\,\mu\nu})$	$(H^\dagger t^a t^b e^V H) \mathcal{W}^{a\,\alpha} \mathcal{W}_\alpha^b$	
	$\mathcal{O}_{3F} = f^{abc} F_\mu^{a\,\nu} F_\nu^{b\,\rho} (F_\rho^{c\,\mu} - i\tilde{F}_\rho^{c\,\mu})$	$f^{abc} \mathcal{D}^\beta \mathcal{W}^{a\,\alpha} \mathcal{W}_\beta^b \mathcal{W}_\alpha^c$	

TABLE 10.1: *Left: Basis of dimension-six SM operators classified as JJ-operators and loop-operators. We also distinguish those that can arise from a supersymmetric D-term (η^0) from those that break supersymmetry either by an spurion $\bar{\mathcal{D}}_{\dot{\alpha}}\eta^\dagger$, $\eta^\dagger$, $|\bar{\mathcal{D}}_{\dot{\alpha}}\eta^\dagger|^2$ or $|\eta|^2$. We denote by $F_{\mu\nu}^a$ ($\tilde{F}_{\mu\nu}^a$) any SM gauge (dual) field-strength. The t^a matrices include the $U(1)_Y$, $SU(2)_L$ and $SU(3)_c$ generators, depending on the quantum numbers of the fields involved. Fermion operators are written schematically with $f = \{Q_L, u_R, d_R, L_L, e_R\}$. Right: For each operator in the left column, we provide the super-operator at which it is embedded.*

with respect to the $U(1)$ case. We have now two independent operators, [8] but only one can arise from a supersymmetric D-term: [9]

$$(H^\dagger e^{V_H} H)^2 = -\bar{\theta}^2 \theta^2 \mathcal{O}_+ + \cdots ,\qquad(10.30)$$

where

$$\mathcal{O}_+ = [2\mathcal{O}_r + \mathcal{O}_H - \mathcal{O}_T] = D_\mu(H_i^\dagger H_j^\dagger) D^\mu(H^i H^j) ,\qquad(10.31)$$

with $\mathcal{O}_r$, $\mathcal{O}_H$ and $\mathcal{O}_T$ being the SM analogues of the $U(1)$ operators, obtained simply by replacing ϕ by H. The other independent only-Higgs operator must arise from a SSB term. We find that this can be the $\theta\bar{\theta}$-component of the superfield

$$\bar{\mathcal{D}}^{\dot\alpha}(H^\dagger e^{V_H} H)\mathcal{D}_\alpha(H^\dagger e^{V_H} H) = -4(\bar{\sigma}^\mu\theta)^{\dot\alpha}(\sigma^\nu\bar{\theta})_\alpha \left(D_\mu H^\dagger H \right) \left(H^\dagger D_\nu H \right) + \cdots .\quad(10.32)$$

We can write this operator in a superfield Lagrangian by using the spurion $|\bar{\mathcal{D}}_{\dot\alpha}\eta^\dagger|^2$:

$$\int d^4\theta \ \bar{\mathcal{D}}_{\dot\alpha}\eta^\dagger \mathcal{D}^\alpha\eta \ \bar{\mathcal{D}}^{\dot\alpha}(H^\dagger e^{V_H} H)\mathcal{D}_\alpha(H^\dagger e^{V_H} H) = -16 \ \mathcal{O}_- + \cdots ,\quad(10.33)$$

where

$$\mathcal{O}_- = \frac{1}{2}[\mathcal{O}_H - \mathcal{O}_T] = |H^\dagger D_\mu H|^2 .\qquad(10.34)$$

Concerning loop-operators, we have the new operators $\mathcal{O}_{3F} = f^{abc} F_\mu^{a\,\nu} F_\nu^{b\,\rho} F_\rho^{c\,\mu}$ and $\mathcal{O}_{3\tilde{F}} = f^{abc} F_\mu^{a\,\nu} F_\nu^{b\,\rho} \tilde{F}_\rho^{c\,\mu}$, possible now for the non-Abelian groups $SU(2)_L$ and $SU(3)_c$, which again can only arise from a θ^2-term:

$$f^{abc}\mathcal{D}^\beta \mathcal{W}^{a\,\alpha}\mathcal{W}_\beta^b \mathcal{W}_\alpha^c = i\theta^2 \mathcal{O}_{3F_+} + \cdots ,\qquad(10.35)$$

where we have defined $\mathcal{O}_{3F_\pm} = \mathcal{O}_{3F} \mp i\mathcal{O}_{3\tilde{F}}$. To contain $\mathcal{O}_{3F_+}$, Eq. (10.35) must then appear in the ESFT multiplying the SSB-spurion $\eta^\dagger$, as the rest of loop-operators.

For the loop-operators $\mathcal{O}_{FF} = H^\dagger t^a t^b H F_{\mu\nu}^a F^{b\,\mu\nu}$ and their CP-violating counterparts, $\mathcal{O}_{F\tilde{F}} = H^\dagger t^a t^b H F_{\mu\nu}^a \tilde{F}^{b\,\mu\nu}$, we can proceed as above and embed them together in the super-operators

$$(H^\dagger t^a t^b e^{V_H} H)\mathcal{W}^{a\,\alpha}\mathcal{W}_\alpha^b = -\frac{1}{2}\theta^2 \mathcal{O}_{FF_+} + \cdots .\qquad(10.36)$$

where $\mathcal{O}_{FF_\pm} = \mathcal{O}_{FF} \mp i\mathcal{O}_{F\tilde{F}}$.

---

[8]The $U(1)$-case identity $\mathcal{O}_r = (\mathcal{O}_H - \mathcal{O}_T)/2$ does not hold in the SM due to the fact that H is a doublet.

[9]The operator $(H^\dagger \sigma^a e^V H)^2$ can be reduced to $(H^\dagger e^V H)^2$ by using $\sigma_{ij}^a \sigma_{kl}^a = 2\delta_{il}\delta_{kj} - \delta_{ij}\delta_{kl}$.

10.3.1   One-loop operator mixing

It is straightforward to extend the $U(1)$ analysis of Sec. 10.1 to the operators of Tab. 10.1 to show that, with the exception of $\mathcal{O}_{yy}$, the JJ-operators do not renormalize the loop-operators. The only important novelties arise from the new type of JJ-operators, $\mathcal{O}_R^{ud}$ and $\mathcal{O}_-$. Concerning $\mathcal{O}_R^{ud}$, it is very simple to see that this operator cannot renormalize loop-operators (from a loop of quarks one obtains operators with the Lorentz structure $(i\tilde{H}^\dagger D_\mu H)$; while the Higgs-loop gives operators containing $\bar{d}_R \gamma_\mu u_R$, and none of them can be loop-operators). Concerning $\mathcal{O}_-$, we only need to worry about the renormalization of $\mathcal{O}_{FF}$. This can be studied directly in the ESFT, as superpartner contributions from JJ-operator to loop-operators can be shown to trivially vanish. In the ESFT, the operator $\mathcal{O}_-$ is embedded in a super-operator containing the SSB-spurion $|\mathcal{D}_\alpha \eta|^2$. This guarantees the absence of renormalization of loop-super-operators as these contain the SSB-spurion $\eta^\dagger$. Besides this direct contribution, there is an indirect route by which $\mathcal{O}_-$ could renormalize $\mathcal{O}_{FF}$: by generating $\mathcal{O}_{HF} = i(D^\mu H)^\dagger t^a (D^\nu H) F_{\mu\nu}^a$ which, via integration by parts, can give $\mathcal{O}_{FF}$. The operator $\mathcal{O}_{HF}$ can come from the super-operator $\tilde{\mathcal{O}}_{HF} = \bar{\mathcal{D}}_{\dot{\alpha}} \eta^\dagger \bar{\mathcal{D}}^{\dot{\alpha}} H^\dagger e^{V_H} \mathcal{D}_\alpha H \, \mathcal{W}^\alpha$ that in principle is not protected by a simple SSB-spurion analysis from being generated by super-operators $\propto |\mathcal{D}_\alpha \eta|^2$. Nevertheless, contributions to $\tilde{\mathcal{O}}_{HF}$ must come from Eq. (10.33) with derivatives acting on the two Higgs superfields external to the loop, and due to the derivative contractions, this can only give $\bar{\mathcal{D}}_{\dot{\alpha}} \eta^\dagger \mathcal{D}^\alpha \eta \bar{\mathcal{D}}^{\dot{\alpha}} H^\dagger \mathcal{D}_\alpha H \mathcal{D}_\beta \mathcal{W}^\beta$; by the use of the EOM of V, however, this gives a JJ-super-operator and not $\tilde{\mathcal{O}}_{HF}$.

In the SM case, the exceptional $\mathcal{O}_{yy}$ operators (than can in principle renormalize the dipole operators) are (following the notation in [6])

$$
\begin{aligned}
\mathcal{O}_{y_u y_d} &= (\bar{Q}_L^r u_R)\epsilon_{rs}(\bar{Q}_L^s d_R)\,, \\
\mathcal{O}_{y_u y_d}^{(8)} &= (\bar{Q}_L^r T^A u_R)\epsilon_{rs}(\bar{Q}_L^s T^A d_R)\,, \\
\mathcal{O}_{y_u y_e} &= (\bar{Q}_L^r u_R)\epsilon_{rs}(\bar{L}_L^s e_R)\,, \\
\mathcal{O}'_{y_u y_e} &= (\bar{Q}_L^{r\,\alpha} e_R)\epsilon_{rs}(\bar{L}_L^s u_R^\alpha)\,,
\end{aligned}
\tag{10.37}
$$

where r, s are $SU(2)_L$ indices and T^A are $SU(3)_c$ generators. Although in principle all of these four operators could renormalize the SM dipoles, it is easy to realize that $\mathcal{O}_{y_u y_e}$ will not: the only possible way of closing a loop ($\bar{Q}_L u_R$ or $\bar{L}_L e_R$) does not reproduce the dipole Lorentz structure for the external fermion legs. One concludes that only the three remaining operators in Eq. (10.37) renormalize the SM dipole operators and we have verified this by explicit calculation. These are the only dimension-six JJ-operator of the SM that renormalize loop-operators. Some of these exceptions were also pointed out in [135]. Our analysis completes the list of these exceptions and helps to understand

the reason behind them. From the analysis of the $U(1)$ case, we can also explain the presence of $y_u y_d$ in the renormalization of $\mathcal{O}_{yy}$ from $\mathcal{O}_{4f}$ [150].

It is obvious that no operator other than itself renormalizes $\mathcal{O}_{3F_+}$: no adequate one-loop 1PI diagram can be constructed from other dimension-six operators, since they have too many fermion and/or scalar fields. Nevertheless $\mathcal{O}_{3F_+}$ can in principle renormalize JJ-operators. Let us consider, for concreteness, the case of $\mathcal{O}_{3F_+}$ made of $SU(2)_L$ field-strengths. SM-loop contributions from $\mathcal{O}_{3F_+}$ can generate the JJ-operators $(D_\nu F^{a\,\mu\nu})^2$ and $J^a_\mu D_\nu F^{a\,\mu\nu}$ (where J^a_μ is the weak current), and indeed these contributions have been found to be nonzero by explicit calculation [7]. By using the EOM, $D_\nu F^{a\,\mu\nu} = g J^{a\,\mu}$, we can reduce these two operators to $(J^a_\mu)^2$. Surprisingly, one finds that the total contribution from $\mathcal{O}_{3F_+}$ to $(J^a_\mu)^2$ adds up to zero [7, 150]. We can derive this result as follows. From inspection of Eq. (E.4), one can see that the superpartners cannot give any one-loop contribution to these JJ-operators. Therefore the result must be the same in the SM EFT as in the corresponding ESFT. Looking at the Higgs component of $(J^a_\mu)^2 = (H^\dagger \sigma^a \overleftrightarrow{D}_\mu H)^2 + \cdots$, we see that this operator must arise from the ESFT term $\int (\mathcal{D}^\alpha \eta \mathcal{J}^a_\alpha + h.c.)^2$ where $\mathcal{J}^a_\alpha = H^\dagger \sigma^a \mathcal{D}_\alpha H$. This super-operator, however, cannot be generated from the super-operator in Eq. (10.35), as this operator appears in the ESFT with a different number of SSB-spurions, $\eta^\dagger$. This proves that $\mathcal{O}_{3F_+}$ cannot generate JJ-operators with Higgs. Now, if current-current super-operators with H are not generated, those with Q_L cannot be generated either, since in the ESFT the $SU(2)_L$ vector does not distinguish between different $SU(2)_L$-doublet chiral superfields. This completes the proof that $\mathcal{O}_{3F_+}$ does not renormalize any JJ-operator in the basis of Tab. 10.1.

Concerning the non-renormalization of JJ-operators by loop-operators, the last new case left to discuss is that of $\mathcal{O}_-$ by $\mathcal{O}_{FF}$. The SSB-spurion analysis forbids such renormalization in the ESFT and the result can be extended to the SM EFT as no superpartner-loop contributes either (see Eq. (E.2) in the Appendix E).

At energies below the electroweak scale, we can integrate out W, Z, Higgs and top, and write an EFT with only light quarks and leptons, photon and gluons. This EFT contains four-fermion operators of type $\mathcal{O}_{4f}$, generated at tree-level, that are JJ-operators, and other operators of dipole-type that are loop-operators. Following the above approach we can prove that these four-fermion operators cannot renormalize the dipole-type operators, and this is exactly what is found in explicit calculations [124].

10.3.2 Holomorphy of the anomalous dimensions

It has been recently shown in Ref. [150], based on explicit calculations, that the anomalous dimension matrix respects, to a large extent, holomorphy. Here we would like to
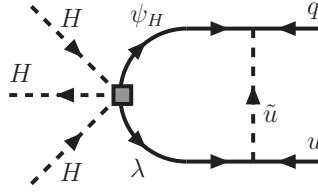
Figure 10.3: *Non-holomorphic contribution to $\mathcal{O}_y$.*

show how to derive some of these properties using our ESFT approach. In particular, we will derive that, with the exception of one case, the one-loop anomalous dimensions of the complex Wilson-coefficients $c_i = \{c_{3F_+}, c_{FF_+}, c_D, c_y, c_{yy}, c_R^{ud}\}$ do not depend on their complex-conjugates c_j^*:

$$\frac{\partial \gamma_{c_i}}{\partial c_j^*} = 0 \,. \tag{10.38}$$

We start by showing when Eq. (10.38) is satisfied just by simple inspection of the SM diagrams. For example, it is easy to realize that holomorphy must be respected in contributions from dimension-six operators in which fermions with a given chirality, e.g., f_α or $f_\alpha f'_\beta$, are kept as external legs; indeed, the corresponding Hermitian-conjugate operator can only contribute to operators with fermions in the opposite chirality. Interestingly, we can extend the same argument to operators with field-strengths if we write the loop-operators as

$$\mathcal{O}_{3F_+} = -\frac{1}{4}\text{tr}\,\mathcal{F}_\alpha{}^\beta \mathcal{F}_\beta{}^\lambda \mathcal{F}_\lambda{}^\alpha \,, \quad \mathcal{O}_{FF_+} = \frac{1}{4}H^\dagger t^a t^b H (\mathcal{F}^a)_{\alpha\beta}(\mathcal{F}^b)^{\beta\alpha} \,, \quad \mathcal{O}_D = H^\dagger f_\alpha (\mathcal{F}^a)^{\alpha\beta} t^a f'_\beta \,, \tag{10.39}$$

where we have defined $\mathcal{F}^{\alpha\beta} \equiv (F^a_{\mu\nu} t^a \sigma^{\mu\nu})^{\alpha\beta}$ that transforms as a $(\mathbf{1}, \mathbf{0})$ under the Lorentz group, and write the Hermitian-conjugate of Eq. (10.39) with $\mathcal{F}^{\dot\alpha\dot\beta}$, a $(\mathbf{0}, \mathbf{1})$ under the Lorentz group, as for example, $\mathcal{O}_{3F+}^\dagger = \mathcal{O}_{3F-} = -\frac{1}{4}\text{tr}\,\mathcal{F}_{\dot\alpha}{}^{\dot\beta} \mathcal{F}_{\dot\beta}{}^{\dot\lambda} \mathcal{F}_{\dot\lambda}{}^{\dot\alpha}$. From Eq. (10.39) it is clear that any diagram with an external $\mathcal{F}_{\alpha\beta}$ respects holomorphy, as it can only generate the operators of Eq. (10.39) and not their Hermitian conjugates. One-loop contributions from $\mathcal{O}_{FF_+}$ in which $H^\dagger t^a t^b H$ is kept among the external fields, however, do not necessarily respect holomorphy. An explicit calculation is needed, and while contributions to $\mathcal{O}_{FF_+}$ vanish by the reasoning given in [123], contributions to $\mathcal{O}_y$ are found not to be holomorphic.

Following our previous supersymmetric approach, it is quite simple to check whether or not loop contributions are holomorphic. In the ESFT, holomorphy is trivially respected as super-operators with an $\eta^\dagger$-spurion renormalize among themselves and cannot induce the Hermitian-conjugate super-operators since those contain an η, and vice versa. This means that possible breakings of holomorphy, at the field-component level,

must be the same in the ordinary SM loop and in its corresponding superpartner loop, as the total breaking must cancel in their sum. Therefore we can look at either one or the other type of loop to check holomorphy. In this way, we can always relate holomorphy to fermion chirality. For example, the breaking of holomorphy in the renormalization of $\mathcal{O}_y$ from $\mathcal{O}_{FF+}^\dagger$ [150], mentioned before, can be easily seen to arise from the diagram of Fig. 10.3. It corresponds to the superpartner one-loop contribution to $\mathcal{O}_y$ arising from the vertex $|H|^2 \lambda^\dagger \bar{\sigma}^\mu \partial_\mu \lambda \sim |H|^2 H \lambda^\dagger \psi_H^\dagger$ of Eq. (10.11), where we have used the EOM of λ (and replaced the $U(1)$ ϕ and ψ by the SM Higgs and Higgsino).

10.4 Implications for the QCD Chiral Lagrangian

We can extend the above analysis also to the QCD Chiral Lagrangian [149]. At $O(p^2)$, we have

$$\mathcal{L}_2 = \frac{f_\pi^2}{4} \langle D_\mu U^\dagger D^\mu U \rangle . \tag{10.40}$$

This is an operator that can be embedded in a D-term as $\int d^4\theta \, \langle \mathcal{U}^\dagger \mathcal{U} \rangle$, where U and its superpartners are contained in $\mathcal{U} \equiv e^{i\Phi}$, with Φ a chiral superfield. At $O(p^4)$, the QCD Chiral Lagrangian is usually parametrized by the L_i coefficients [149] in a basis with operators that are linear combinations of JJ-operators and loop-operators. These are

$$\mathcal{L}_4 = -iL_9 \langle F_R^{\mu\nu} D_\mu U D_\nu U^\dagger + F_L^{\mu\nu} D_\mu U^\dagger D_\nu U \rangle + L_{10} \langle U^\dagger F_R^{\mu\nu} U F_{L\mu\nu} \rangle + \cdots . \tag{10.41}$$

A more convenient basis is

$$\mathcal{L}_4 = iL_{JJ} \langle D_\mu F_L^{\mu\nu}(U^\dagger \overset{\leftrightarrow}{D}_\nu U) + (U \overset{\leftrightarrow}{D}_\nu U^\dagger) D_\mu F_R^{\mu\nu} \rangle + L_{loop} \langle U^\dagger F_R^{\mu\nu} U F_{L\mu\nu} \rangle + \cdots , \tag{10.42}$$

where $L_{JJ} = L_9/2$ and $L_{loop} = L_9 + L_{10}$. It is easy to see that the first operator of Eq. (10.42) is a JJ-operator, while the second is a loop-operator. This latter can only be embedded in a θ^2-term of a super-operator (i.e., $\langle \mathcal{U}^\dagger \mathcal{W}_R^\alpha \mathcal{U} \mathcal{W}_{\alpha L} \rangle$), and therefore it cannot be renormalized by the operator in Eq. (10.40) in the supersymmetric limit. As contributions from superpartner loops can be easily shown to vanish, we can deduce that Eq. (10.40) cannot renormalize L_{loop} at the one-loop level. This is indeed what one finds from the explicit calculation [149]: $\gamma_{L_{loop}} = \gamma_{L_9} + \gamma_{L_{10}} = 1/4 - 1/4 = 0$.

10.5 Summary

In EFTs with higher-dimensional operators the one-loop anomalous dimension matrix has plenty of vanishing entries apparently not forbidden by the symmetries of the theory.
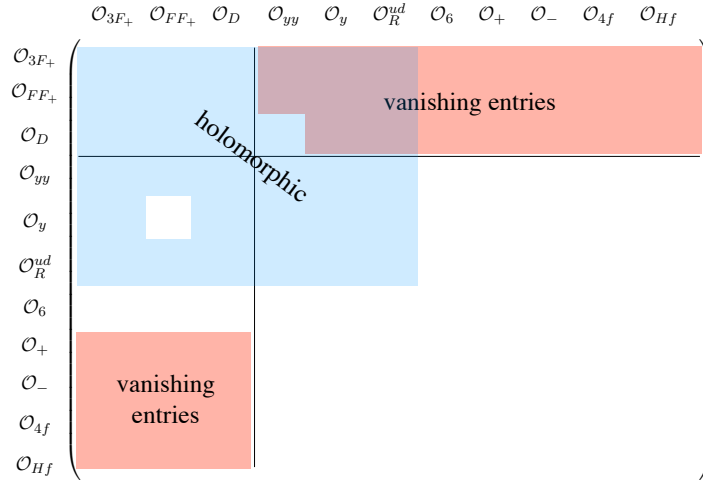
FIGURE 10.4: *Anomalous-dimension matrix of the dimension-six SM operators showing which entries (red-shaded) vanish following the present analysis. We also show the entries (light blue-shaded) that respect the holomorphic condition Eq. (10.38). Solid lines separate loop-operators from JJ-operators.*

In this chapter we have shown that the reason behind these zeros is the different Lorentz structure of the operators that does not allow them to mix at the one-loop level. We have proposed a way to understand the pattern underlying these zeros based on classifying the dimension-six operators in JJ- and loop-operators and also according to their embedding in super-operators (see Tab. 10.1 for the SM EFT). We have seen that all loop-operators break supersymmetry, [10] while we have two classes of JJ-operators, those that can be supersymetrized and those that cannot. This classification is very useful to obtain non-renormalization results based in a pure SSB-spurion analysis in superfields, that can be extended to non-supersymmetric EFTs. In terms of component fields, the crucial point is that the vanishing of the anomalous-dimensions does not arise from cancellations between bosons and fermions but from the underlying Lorentz structure of the operators.

We have explained how this approach works in a simple $U(1)$ model with a scalar and fermions, and have shown how to extend this to SM EFTs and the QCD Chiral Langrangian. The main results are summarized in Fig. 10.4 that shows which entries of the anomalous-dimension matrix for the SM EFTs operators we have proved to vanish. We have also explained how to check if holomorphy is respected by the complex Wilson-coefficients, a property that is fulfilled in most cases, as Fig. 10.4 shows.

Recently, Ref. [156] presented an alternative way to derive similar results to ours. Ref. [156] makes use of the spinor helicity formalism and the generalised unitarity cuts

---

[10]This is not true in general. For instance, in models with two Higgses of opposite hypercharge, H and $\bar{H}$, one can have the supersymmetric loop-operator $\int d^2\theta H\bar{H}\mathcal{W}^\alpha\mathcal{W}_\alpha$. Notice that in such a case supersymmetry also protects that operator from being renormalized in the ESFT.

to analyse, in a very efficient way, all possible Lorentz vertices that can contribute to the renormalization of a given operator. This allows Ref. [156] to organise the non-renormalization results presented in this Chapter in a rather simple way based solely on the weighs w of the amplitudes generated by the operators [11]. In contrast with our analysis based on the supersymmetric spurion η power and the *easy* superpartner analysis presented in this chapter. It would be very interesting to make a connection between Ref. [156] and the spurious supersymmetric analysis of this Chapter, and to clarify if it is possible to extend it to multi-loop renormalization.

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[11]The weigh of an amplitude A is defined as $w[A] = n[A] - h[A]$ where $n[A]$ is the number of particles and $h[A]$ is the sum of the helicites of the particles of the amplitude.

A. Currents, redundant operators and field shifts

In this Appendix we first list, in Sec. A.1, the different currents (of dimension ≤ 3) built from SM fields that enter into the $d = 6$ current-current operators. We examine in Sec. A.2 how these operators can be generated from integrating out heavy particles discussing what type of operators appear depending on the quantum numbers of the heavy fields. Some of these operators are redundant and can be eliminated from the Lagrangian by using the field equations of motion or, equivalently, by field redefinitions. We discuss this point in Sec. A.3, where we give a possible set of field redefinitions that can be used to get rid of the redundant operators.

A.1   Currents of SM fields

For simplicity we limit our examples of currents to the SM with a single family of fermions, the generalization to 3 families being straightforward. The scalar currents are:

$$
\begin{aligned}
J_H &= |H|^2 \ , & J_H^{(2)} &= H|H|^2 \ , & J_{\Box H} &= D_\mu^2 H \ , \\
J_H^a &= H^\dagger \sigma^a H \ , & J_{y_f H} &= y_f \bar{F}_L f_R \ , & J_{y_f H}^A &= y_f \bar{Q}_L T^A f_R \ ,
\end{aligned}
\tag{A.1}
$$

where T^A are the $SU(3)_c$ generators and from now on we use the notation $F_L = \{Q_L, L_L\}$ and $f_R = \{u_R, d_R, e_R\}$ for fields, while $F = \{q, l\}$ and $f = \{u, d, e\}$ are used for the corresponding operator indices. Obviously, one can also have the conjugate currents: $\tilde{J}_H^{(2)} = \tilde{H}|H|^2$, $\tilde{J}_{\Box H} = D_\mu^2 \tilde{H}$, etc.

There are also vector currents made of SM bosons, like:

$$
\begin{aligned}
J_H^\mu &= iH^\dagger \overset{\leftrightarrow}{D}{}^\mu H \ , & J_{W_R}^\mu &= i\widetilde{H}^\dagger \overset{\leftrightarrow}{D}{}^\mu H \ , & J_H^{a\,\mu} &= iH^\dagger \sigma^a \overset{\leftrightarrow}{D}{}^\mu H \ , \\
J_B^\mu &= \partial_\nu B^{\mu\nu} \ , & J_W^{a\,\mu} &= D_\nu W^{a\,\mu\nu} \ , & J_G^{A\,\mu} &= D_\nu G^{A\,\mu\nu} \ ,
\end{aligned}
\tag{A.2}
$$

and made of SM fermions, like:

$$J^\mu_{ff'} = \bar{f}_R \gamma^\mu f'_R \ , \qquad J^\mu_F = \bar{F}_L \gamma^\mu F_L \ , \qquad J^{a\,\mu}_F = \bar{F}_L \sigma^a \gamma^\mu F_L \ , \atop J^{A\,\mu}_f = \bar{f}_R T^A \gamma^\mu f_R \ , \quad J^{A\,\mu}_Q = \bar{Q}_L T^A \gamma^\mu Q_L \ , \tag{A.3}$$

as well as the lepto-quark currents:

$$J^\alpha_{Qe} = y_e \bar{Q}^\alpha_L e_R \ , \quad J^\alpha_{Lu} = y_u \bar{L}_L u^\alpha_R \ , \quad J^\alpha_{Ld} = y_d \bar{L}_L d^\alpha_R \ , \tag{A.4}$$

where we write explicitly the color index α. Finally, we list fermionic currents made of SM fields. They can be $SU(2)_L$ singlets:

$$J_{Df} = i\slashed{D} f_R \ , \quad J_{y_f \bar{f}_R} = \{y^\dagger_u \widetilde{H}^\dagger Q_L \ , y^\dagger_d H^\dagger Q_L \ , y^\dagger_e H^\dagger L_L\} \ , \tag{A.5}$$

doublets:

$$J_{DF} = i\slashed{D} F_L \ , \quad J_{y_f \bar{F}_L} = \{y_u \widetilde{H} u_R \ , y_d H d_R \ , y_e H e_R\} \ , \tag{A.6}$$

or triplets:

$$J^a_{\widetilde{H}F} = \{\widetilde{H}^\dagger \sigma^a Q_L, \widetilde{H}^\dagger \sigma^a L_L\} \ , \quad J^a_{HF} = \{H^\dagger \sigma^a Q_L, H^\dagger \sigma^a L_L\} \ . \tag{A.7}$$

The previous list of SM currents is not complete but contains all the currents necessary to build the current-current operators of our basis (defined in the main body of the paper), as well as many of the redundant operators.

A.2   Current-current operators

The $d = 6$ current-current operators can in principle be generated from the tree-level exchange of heavy fields. We can then classify such operators by the quantum numbers of the exchanged heavy fields. We present such classification below (giving explicit expressions for those redundant operators that appear here for the first time.). Finding possible deformations in SM couplings that can be assigned to particular current-current operators can offer crucial information in identifying the heavy physics responsible for such effects.

○ **Scalar × scalar**

The exchange of a heavy scalar singlet can lead (after integration by parts) to:

$$-J_H \Box J_H = 2\mathcal{O}_H \ . \tag{A.8}$$

From a heavy scalar $SU(2)_L$-doublet we get:

$$\lambda J_H^{(2)\,\dagger} J_H^{(2)} = \mathcal{O}_6 \, , \qquad\qquad J_H^{(2)\,\dagger} J_{\Box H} + \text{h.c.} = -2(\mathcal{O}_H + \mathcal{O}_r) \, ,$$
$$J_{y_u H} \tilde{J}_H^{(2)} = \mathcal{O}_{y_u} \, , \qquad\qquad J_{y_d H}^{\dagger} J_{y_e H} = \mathcal{O}_{y_d y_e} \, , \qquad\qquad (\text{A.9})$$
$$(J_{y_u H})^r \epsilon_{rs} (J_{y_e H})^s = \mathcal{O}_{y_u y_e} \, , \quad (J_{y_u H})^r \epsilon_{rs} (J_{y_d H})^s = \mathcal{O}_{y_u y_d} \, ,$$

and also:

$$J_{\Box H}^{\dagger} J_{\Box H} = \left| D_\mu^2 H \right|^2 \equiv \mathcal{O}_{K4} \, ,$$
$$J_{y_u H} \tilde{J}_{\Box H} = -y_u D_\mu \left(\bar{Q}_L u_R \right) D^\mu \widetilde{H} \equiv -\mathcal{O}_{yH}^u \, . \qquad\qquad (\text{A.10})$$

If the heavy doublet is also charged under $SU(3)_c$ we can get:

$$(J_{Qe}^\alpha)^r \epsilon_{rs} (J_{Lu}^\alpha)^s = \mathcal{O}'_{y_u y_e} \, , \quad (J_{y_u H}^A)^r \epsilon_{rs} (J_{y_d H}^A)^s = \mathcal{O}_{y_u y_d}^{(8)} \, , \qquad\qquad (\text{A.11})$$

while, from a heavy scalar $SU(2)_L$-triplet we would obtain:

$$J_H^a D^2 J_H^a = -2\mathcal{O}_T - 4\mathcal{O}_r \, . \qquad\qquad (\text{A.12})$$

○ **Vector × vector**

From the exchange of a heavy singlet vector one can get:

$$J_H^\mu J_{H\,\mu} = -2\mathcal{O}_T \, , \quad g' J_B^\mu J_{H\,\mu} = 2\mathcal{O}_B \, , \quad J_{uu}^\mu J_{H\,\mu} = \mathcal{O}_R^u \, ,$$
$$J_H^\mu J_{F\,\mu} = \mathcal{O}_L^F \, , \quad J_B^\mu J_{B\,\mu} = -2\mathcal{O}_{2B} \, , \quad J_{uu}^\mu J_{uu\,\mu} = \mathcal{O}_{RR}^u \, , \qquad\qquad (\text{A.13})$$
$$J_{uu}^\mu J_{F\,\mu} = \mathcal{O}_{LR}^u \, , \quad J_F^\mu J_{F\,\mu} = \mathcal{O}_{LL}^F \, , \quad y_u^\dagger y_d J_{W_R\,\mu} J_{ud}^\mu = \mathcal{O}_R^{ud} \, ,$$

as well as

$$g' J_B^\mu J_{uu\,\mu} = g' (\bar{u}_R \gamma^\mu u_R)(\partial^\nu B_{\mu\nu}) \equiv \mathcal{O}_{BR}^u \, ,$$
$$g' J_B^\mu J_{F\,\mu} = g' (\bar{F}_L \gamma^\mu F_L)(\partial^\nu B_{\mu\nu}) \equiv \mathcal{O}_{BL}^F \, . \qquad\qquad (\text{A.14})$$

The exchange of a heavy $SU(2)_L$-triplet vector can produce:

$$J_H^{a\,\mu} J_{H\,\mu}^a = -2\mathcal{O}_H + 4\mathcal{O}_r \, , \quad g J_{W\,\mu}^a J_H^{a\,\mu} = 2\mathcal{O}_W \, , \quad J_F^{a\,\mu} J_{H\,\mu}^a = \mathcal{O}_L^{(3)F} \, ,$$
$$J_W^{a\,\mu} J_{W\,\mu}^a = -2\mathcal{O}_{2W} \, , \quad J_{L\,\mu}^a J_L^{a\,\mu} = \mathcal{O}_{LL}^l \, , \quad J_{Q\,\mu}^a J_Q^{a\,\mu} = 4\mathcal{O}_{LL}^{(8)q} + \frac{2 - N_c}{N_c} \mathcal{O}_{LL}^q \, ,$$
$$\qquad\qquad (\text{A.15})$$

and

$$g J_W^{a\mu} J_{F\,\mu}^a = g(\bar{F}_L \gamma^\mu \sigma^a F_L)(D^\nu W_{\mu\nu}^a) \equiv \mathcal{O}_{WL}^F \, , \qquad\qquad (\text{A.16})$$

while a heavy $SU(3)_c$-octet vector could give:

$$J^A_{u\,\mu}J^{A\,\mu}_u = (1/3)\,\mathcal{O}^u_{RR}\ , \quad J^A_{Q\,\mu}J^{A\,\mu}_Q = \mathcal{O}^{(8)\,q}_{LL}\ , \quad J^A_{Q\,\mu}J^{A\,\mu}_u = \mathcal{O}^{(8)\,u}_{LR}\ . \tag{A.17}$$

○ **Fermion × fermion**

Finally, we list operators that can arise from integrating a heavy fermion. If the fermion is a singlet:

$$\bar{J}_{y_u\bar{u}_R}i\slashed{D}J_{Du} = y_u D_\mu(\bar{Q}_L\widetilde{H})\gamma^\mu\gamma^\nu D_\nu u_R \equiv \mathcal{O}^u_{yR}\ ,$$
$$\bar{J}_{y_u\bar{u}_R}i\slashed{D}J_{y_u\bar{u}_R} + \text{h.c.} = \frac{1}{2}|y_u|^2\left[-\widetilde{\mathcal{O}}^{(3)q}_L + \widetilde{\mathcal{O}}^q_L - \mathcal{O}^{(3)q}_L + \mathcal{O}^q_L\right]\ , \tag{A.18}$$

where

$$\widetilde{\mathcal{O}}^{(3)q}_L = i(\bar{Q}_L\sigma^a\overset{\leftrightarrow}{\slashed{D}}Q_L)(H^\dagger\sigma^a H)\ ,$$
$$\widetilde{\mathcal{O}}^q_L = i(\bar{Q}_L\overset{\leftrightarrow}{\slashed{D}}Q_L)|H|^2\ , \tag{A.19}$$

are redundant operators.

If the fermion integrated-out is a doublet, one can get:

$$\bar{J}_{DQ}i\slashed{D}J_{y_u\bar{Q}_L} = y_u D_\mu\bar{Q}_L\gamma^\mu\gamma^\nu D_\nu(\widetilde{H}u_R) \equiv \mathcal{O}^u_{yL}\ ,$$
$$\bar{J}_{y_u\bar{Q}_L}i\slashed{D}J_{y_u\bar{Q}_L} + \text{h.c.} = |y_u|^2\left[-\mathcal{O}^u_R + \widetilde{\mathcal{O}}^u_R\right]\ , \tag{A.20}$$

with the redundant operator:

$$\widetilde{\mathcal{O}}^u_R = i(\bar{u}_R\overset{\leftrightarrow}{\slashed{D}}u_R)|H|^2\ . \tag{A.21}$$

Finally, from integrating out a heavy fermion triplet, we can get:

$$\bar{J}^a_{\widetilde{H}F}i\slashed{D}J^a_{\widetilde{H}F} + \text{h.c.} = \frac{1}{2}\left[\widetilde{\mathcal{O}}^{(3)F}_L + 3\widetilde{\mathcal{O}}^F_L + \mathcal{O}^{(3)F}_L + 3\mathcal{O}^F_L\right]\ ,$$
$$\bar{J}^a_{HF}i\slashed{D}J^a_{HF} + \text{h.c.} = \frac{1}{2}\left[-\widetilde{\mathcal{O}}^{(3)F}_L + 3\widetilde{\mathcal{O}}^F_L + \mathcal{O}^{(3)F}_L - 3\mathcal{O}^F_L\right]\ . \tag{A.22}$$

To describe the effect of a heavy fermion that is a color octet, one would need to generalize the quark currents of Sec. A.1 by inserting $SU(3)_c$ generators. However, the dimension-6 operators that result have been already found in Eqs. (A.18) and (A.20).

A.3 Field redefinitions and redundant operators

Many $d = 6$ current-current operators are redundant: they can be removed from the Lagrangian by field redefinitions. We will show how field redefinitions can be used for that purpose, focusing here on current-current operators not of the 4-fermion type.

Let us start first with bosonic operators. Consider the following transformations that shift fields by some of the bosonic currents listed in Sec. A.1 (with the same quantum numbers of the shifted fields):

$$H \rightarrow H + \alpha_1 J_H^{(2)}/\Lambda^2 \ , \qquad H \rightarrow H \left(1 - \alpha_2 m^2/\Lambda^2\right) + \alpha_2 J_{\Box H}/\Lambda^2 \ ,$$
$$B_\mu \rightarrow B_\mu + [g'\alpha_B J_{H\,\mu} + \alpha_{2B} J_{B\,\mu}]/\Lambda^2 \ , \qquad W_\mu^a \rightarrow W_\mu^a + [g\alpha_W J_{H\,\mu}^a + \alpha_{2W} J_{W\,\mu}^a]/\Lambda^2 \ ,$$
$$G_\mu^A \rightarrow G_\mu^A + \alpha_{2G} J_{G\,\mu}^A/\Lambda^2 \ , \tag{A.23}$$

with α_i arbitrary parameters (taken real). These transformations induce shifts in the $d = 6$ Wilson coefficients [1] of Eqs. (6.4) and (6.5) plus the redundant operator $\mathcal{O}_{K4} = |D_\mu^2 H|^2$:

$$c_H \rightarrow c_H + 2\alpha_1 + (4\lambda\alpha_2 - \alpha_W g^2)/g_*^2 \ ,$$
$$c_r \rightarrow c_r + 2\alpha_1 + (4\lambda\alpha_2 + 2\alpha_W g^2)/g_*^2 \ ,$$
$$c_6 \rightarrow c_6 - 4\alpha_1 \ ,$$
$$c_T \rightarrow c_T - \alpha_B g'^2/g_*^2 \ ,$$
$$c_B \rightarrow c_B - 2\alpha_B + \alpha_{2B} \ ,$$
$$c_W \rightarrow c_W - 2\alpha_W + \alpha_{2W} \ ,$$
$$c_{2W} \rightarrow c_{2W} + 2\alpha_{2W} \ ,$$
$$c_{2B} \rightarrow c_{2B} + 2\alpha_{2B} \ ,$$
$$c_{2G} \rightarrow c_{2G} + 2\alpha_{2G} \ ,$$
$$c_{K4} \rightarrow c_{K4} - 2\alpha_2 \ . \tag{A.24}$$

Notice that only operators of tree-level type are shifted. Using this shift freedom, we could eliminate 7 out of the 10 operators $\{\mathcal{O}_H, \mathcal{O}_r, \mathcal{O}_6, \mathcal{O}_T, \mathcal{O}_B, \mathcal{O}_W, \mathcal{O}_{2W}, \mathcal{O}_{2B}, \mathcal{O}_{K4}, \mathcal{O}_{2G}\}$ by choosing appropriately the α_i's and leave only $\mathcal{O}_H$, $\mathcal{O}_T$ and $\mathcal{O}_6$. As we discussed in Sec. 6.1, however, it is convenient to keep the operators $\mathcal{O}_W$ and $\mathcal{O}_B$ in the basis, in which we could also keep $\mathcal{O}_{2W}$, $\mathcal{O}_{2B}$ and $\mathcal{O}_{2G}$. If we do not use 5 of these shifts to remove $\mathcal{O}_W$, $\mathcal{O}_B$, $\mathcal{O}_{2W}$, $\mathcal{O}_{2B}$ and $\mathcal{O}_{2G}$, they can be used later on to remove 5 other operators involving fermions. We will discuss such operators next.

---

[1] Shifts of order m^2/Λ^2 induced on the renormalizable dimension-4 SM operators play no role. There are also shifts in the coefficients of the operators made of fermions that we show below.

Besides the bosonic redundant operators discussed above, there are redundant operators that involve Higgs and fermion fields. For instance, we have the following first-class operators:

$$\widetilde{\mathcal{O}}_L^F = (i\bar{F}_L \overset{\leftrightarrow}{\slashed{D}} F_L)|H|^2 \, , \quad \widetilde{\mathcal{O}}_L^{(3)F} = (i\bar{F}_L\sigma^a \overset{\leftrightarrow}{\slashed{D}} F_L)(H^\dagger\sigma^a H) \, , \quad \widetilde{\mathcal{O}}_R^f = (i\bar{f}_R \overset{\leftrightarrow}{\slashed{D}} f_R)|H|^2 \, , \tag{A.25}$$

as well as the second-class operators

$$\mathcal{O}_{yH}^u = y_u D_\mu(\bar{Q}_L u_R)D^\mu\widetilde{H} \, , \qquad \mathcal{O}_{yR}^u = y_u D_\mu(\bar{Q}_L\widetilde{H})\gamma^\mu\gamma^\nu D_\nu u_R \, ,$$

$$\mathcal{O}_{yL}^u = y_u D_\mu\bar{Q}_L\gamma^\mu\gamma^\nu D_\nu(\widetilde{H}u_R) \, , \qquad \mathcal{O}_{yLR}^u = y_u(D_\mu\bar{Q}_L)\gamma^\mu\gamma^\nu(D_\nu u_R)\widetilde{H} \, , \tag{A.26}$$

(and similar operators for down-type quarks and leptons). In addition, there are (second-class) operators involving fermions and gauge bosons:

$$\mathcal{O}_{BL}^F = g'(\bar{F}_L\gamma^\mu F_L)\partial^\nu B_{\mu\nu} \, , \ \mathcal{O}_{BR}^f = g'(\bar{f}_R\gamma^\mu f_R)\partial^\nu B_{\mu\nu} \, , \ \mathcal{O}_{WL}^F = g(\bar{F}_L\sigma^a\gamma^\mu F_L)D^\nu W_{\mu\nu}^a \, ,$$

$$\mathcal{O}_{GL}^q = g_s(\bar{Q}_L T^A\gamma^\mu Q_L)D^\nu G_{\mu\nu}^A \, , \ \mathcal{O}_{GR}^f = g_s(\bar{f}_R T^A\gamma^\mu f_R)D^\nu G_{\mu\nu}^A \, . \tag{A.27}$$

To see that the operators (A.25)-(A.27) can indeed be removed from the Lagrangian, consider the following field redefinitions that involve fermions:

$$\begin{aligned}
Q_L &\rightarrow Q_L(1 + g_*^2\alpha_L J_H/\Lambda^2) + [g_*^2\alpha_L^{(3)} J_H^a\sigma^a Q_L + i\alpha_{yL}^u \slashed{D} J_{y_u\bar{Q}_L} + y_u\alpha_{yLR}^u\widetilde{H}J_{Du_R} \\
&\quad + i\alpha_{yL}^d \slashed{D} J_{y_d\bar{Q}_L} + y_d\alpha_{yLR}^d H J_{Dd_R}]/\Lambda^2 \, , \\
u_R &\rightarrow u_R(1 + g_*^2\alpha_R^u J_H/\Lambda^2) + i\alpha_{yR}^u \slashed{D} J_{y_u\bar{u}_R}/\Lambda^2 \, , \\
d_R &\rightarrow d_R(1 + g_*^2\alpha_R^d J_H/\Lambda^2) + i\alpha_{yR}^d \slashed{D} J_{y_d\bar{d}_R}/\Lambda^2 \, , \\
B_\mu &\rightarrow B_\mu + g'[\alpha_{F_L}^B J_{F\mu} + \alpha_{f_R}^B J_{ff\mu}]/\Lambda^2 \, , \\
W_\mu^a &\rightarrow W_\mu^a + g\alpha_{f_L}^W J_{L\mu}^a/\Lambda^2 \, , \\
G_\mu^A &\rightarrow G_\mu^A + g_s[\alpha_{F_L}^G J_{F\mu}^A + \alpha_{f_R}^G J_{f\mu}^A]/\Lambda^2 \, , \\
\widetilde{H} &\rightarrow \widetilde{H} + \alpha_{Ht}J_{y_tH}^\dagger/\Lambda^2 \, , \\
H &\rightarrow H + \alpha_{Hb}J_{y_bH}^\dagger/\Lambda^2 \, , \tag{A.28}
\end{aligned}$$

under which the Wilson coefficients shift as follows: For the Higgs-fermion operators of Eq. (7.4), plus the straightforward generalization to the down-type fermions, and the up-down mixed operator of Eq. (6.17), we get (for third-generation quarks):

$$\begin{aligned}
c_{y_t} &\rightarrow c_{y_t} - \alpha_1 - 2\frac{\lambda}{g_*^2}\alpha_{Ht} - \alpha_R^t - \alpha_L + \alpha_L^{(3)} \, , \\
c_{y_b} &\rightarrow c_{y_b} - \alpha_1 - 2\frac{\lambda}{g_*^2}\alpha_{Hb} - \alpha_R^b - \alpha_L - \alpha_L^{(3)} \, , \\
c_R &\rightarrow c_R + \frac{|y_t|^2}{g_*^2}\alpha_{yL}^t + \frac{g'^2}{2g_*^2}(2Y_R^t\alpha_B + \alpha_{t_R}^B) \, ,
\end{aligned}$$

$$c_L^{q3} \quad \to \quad c_L^{q3} - \frac{1}{2}\frac{|y_t|^2}{g_*^2}\alpha_{yR}^t + \frac{g'^2}{2g_*^2}(2Y_L^q\alpha_B + \alpha_{Q_L}^B) \,,$$

$$c_L^{(3)q3} \quad \to \quad c_L^{(3)q3} + \frac{1}{2}\frac{|y_t|^2}{g_*^2}\alpha_{yR}^t + \frac{g^2}{2g_*^2}(\alpha_W + \alpha_{Q_L}^W) \,,$$

$$c_R^{tb} \quad \to \quad c_R^{tb} - \frac{1}{2}(\alpha_{yL}^t + \alpha_{yL}^b) \,. \tag{A.29}$$

The Higgs-fermion redundant operators of Eq. (A.25) can be eliminated by the shifts:

$$\tilde{c}_R^t \quad \to \quad \tilde{c}_R^t + \alpha_R^t - \frac{|y_t|^2}{g_*^2}(\alpha_{yL}^t + \alpha_{yLR}^t) \,,$$

$$\tilde{c}_R^b \quad \to \quad \tilde{c}_R^b + \alpha_R^b \,,$$

$$\tilde{c}_L^{q3} \quad \to \quad \tilde{c}_L^{q3} + \alpha_L - \frac{1}{2}\frac{|y_t|^2}{g_*^2}\alpha_{yR}^t \,,$$

$$\tilde{c}_L^{(3)q3} \quad \to \quad \tilde{c}_L^{(3)q3} + \alpha_L^{(3)} + \frac{1}{2}\frac{|y_t|^2}{g_*^2}\alpha_{yR}^t \,, \tag{A.30}$$

(where, from here on, we neglect $|y_b|^2$ and $|y_\tau|^2$ contributions) while the redundant Higgs-fermion operators of Eq. (A.26) can be eliminated by the shifts:

$$c_{yH}^f \quad \to \quad c_{yH}^f + \alpha_{Hf} + \alpha_2 \,,$$

$$c_{yR}^f \quad \to \quad c_{yR}^f + \alpha_{yR}^f \,,$$

$$c_{yL}^f \quad \to \quad c_{yL}^f + \alpha_{yL}^f \,,$$

$$c_{yLR}^f \quad \to \quad c_{yLR}^f + \alpha_{yLR}^f \,. \tag{A.31}$$

All the redundant gauge-fermion operators of Eq. (A.27) can be removed by the shifts:

$$c_{BR}^f \quad \to \quad c_{BR}^f + Y_R^f\alpha_{2B} - \alpha_{f_R}^B \,,$$

$$c_{BL}^F \quad \to \quad c_{BL}^F + Y_L^F\alpha_{2B} - \alpha_{F_L}^B \,,$$

$$c_{WL}^F \quad \to \quad c_{WL}^F + \frac{1}{2}\alpha_{2W} - \alpha_{F_L}^W \,,$$

$$c_{GL,GR}^q \quad \to \quad c_{GL,GR}^q + \alpha_{2G} - \alpha_{q_{L,R}}^G \,. \tag{A.32}$$

Finally, the coefficients of four-fermion operators will also be shifted but we will not need such shifts and we do not list them.

Using all the shift freedom to remove these redundant operators we end up (say, for the third family) with the following Higgs-fermion Wilson coefficients: $y_f c_{y_f}, c_R^f, c_L^F, c_L^{(3)F}$ and c_R^{tb}, with $f = t, b, \tau$ and $F = q, l$, in agreement with the operators listed in Tab. 6.2.

B. Anomalous dimensions of $d = 6$ Wilson coefficients

In the analysis of Chapter 8 we were interested in potentially large radiative effects in the running of the $d = 6$ Wilson coefficients c_i from the scale Λ of new physics to the electroweak scale. To study such effects we computed the one-loop anomalous dimensions γ_{c_i} for the Wilson coefficients, which are functions of the coefficients themselves, that is:

$$\gamma_{c_i} = \frac{dc_i}{d\log\mu} = \gamma_{c_i}(c_j) \ , \tag{B.1}$$

where μ is the renormalization scale.

When redundant operators are removed from the Lagrangian some care has to be taken in computing anomalous dimensions of the operators left in the basis. The reason is that redundant operators can be generated through RG evolution by operator mixing with non-redundant operators. In other words, the γ_{c_i}'s of redundant operators are not zero in general.

Let us explain how this effect can be taken care of in a simple way. Consider a basis formed by a set of coefficients $\{c_i\}$, after removing a set of redundant coefficients $\{c_i^r\}$. The procedure to remove the c_i^r is straightforward and has been illustrated in the previous Appendix. One starts from the shifts induced by field-redefinitions with arbitrary parameters α_k, which have the form

$$c_i \to c_i'(\alpha_j) = c_i + \sum_k s_{ik}\alpha_k \ , \quad c_i^r \to c'^r_i(\alpha_j) = c_i^r + \sum_k s_{ik}^r \alpha_k \ . \tag{B.2}$$

Then the α_k's are chosen so as to remove the redundant operators,

$$c'^r_i(\alpha_j^*) = 0 \quad \Rightarrow \quad \alpha_j^* = -\sum_i (s^r)_{ji}^{-1} c_i^r \ . \tag{B.3}$$

It is then convenient to define the following combinations

$$C_i \equiv c_i'(\alpha_j^*) = c_i - \sum_{kl} s_{ik}(s^r)_{kl}^{-1} c_l^r \ , \tag{B.4}$$

which are invariant under the arbitrary shifts of Eq. (B.2) and correspond to a more physical definition of the Wilson coefficients.

The anomalous dimensions of these shift-invariant C_i's are simply

$$\gamma_{C_i} = \gamma_{c_i} - \sum_{kl} s_{ik}(s^r)_{kl}^{-1} \gamma_{c_l^r} = \gamma_i(c_k; c_k^r) \ , \tag{B.5}$$

where the last expression just indicates some function of the Wilson coefficients and we distinguish in its argument between coefficients in the basis and coefficients of redundant operators. The key property of this function is that it must depend on the Wilson coefficients only through the shift-invariant combinations. That is, it satisfies

$$\gamma_i(c_k; c_k^r) = \gamma_i(C_k; 0) \ . \tag{B.6}$$

This implies that setting $c_k^r = 0$ in these $\gamma_i(c_k; c_k^r)$ functions is now a consistent procedure to obtain the anomalous dimensions after removing redundant operators. An explicit example of this is given at the end of this Appendix. We applied this procedure to calculate the anomalous dimensions used in Sec. 8.1. In the next Subsections we will list the required shift-invariant C_i combinations and present the γ_{c_i}'s necessary to complete the calculation.

B.1   Shift-invariant combinations of Wilson coefficients

In order to simplify the expressions for the shift-invariant combinations C_i of Wilson coefficients, we present them first in a basis that treats as redundant the operators $\mathcal{O}_B$, $\mathcal{O}_W$, $\mathcal{O}_{2B}$, $\mathcal{O}_{2W}$ and $\mathcal{O}_{2G}$. We explain afterwards how to express these combinations in other bases, as those that keep these operators. As we are not interested in calculating

the anomalous dimensions of 4-fermion operators, below we restrict our C_i's to non-4-fermion current-current operators. We find:

$$
\begin{aligned}
C_H &\equiv c_H - c_r - \frac{3g^2}{4g_*^2}(2c_W - c_{2W}) \,, \\[6pt]
C_T &\equiv c_T - \frac{g'^2}{4g_*^2}(2c_B - c_{2B}) \,, \\[6pt]
C_6 &\equiv c_6 + 2c_r + \frac{g^2}{g_*^2}(2c_W - c_{2W}) + 4\frac{\lambda}{g_*^2}c_{K4} \,, \\[6pt]
C_{y_t} &= c_{y_t} + \frac{1}{2}c_r + 2\frac{\lambda}{g_*^2}\left(c_{yH}^t + c_{K4}\right) + \tilde{c}_R^t + \tilde{c}_L^{q3} - \tilde{c}_L^{(3)q3} + \frac{g^2}{4g_*^2}(2c_W - c_{2W}) \\[6pt]
&\quad + \frac{|y_t|^2}{g_*^2}(c_{yR}^t + c_{yL}^t + c_{yLR}^t) \,, \\[6pt]
C_{y_b} &= c_{y_b} + \frac{1}{2}c_r + 2\frac{\lambda}{g_*^2}\left(c_{yH}^b + c_{K4}\right) + \tilde{c}_R^b + \tilde{c}_L^{q3} + \tilde{c}_L^{(3)q3} + \frac{g^2}{4g_*^2}(2c_W - c_{2W}) \,, \\[6pt]
C_{y_\tau} &= c_{y_\tau} + \frac{1}{2}c_r + 2\frac{\lambda}{g_*^2}\left(c_{yH}^\tau + c_{K4}\right) + \tilde{c}_R^\tau + \tilde{c}_L^{l3} + \tilde{c}_L^{(3)l3} + \frac{g^2}{4g_*^2}(2c_W - c_{2W}) \,, \\[6pt]
C_R^t &= c_R^t - \frac{|y_t|^2}{g_*^2}c_{yL}^t + \frac{g'^2}{2g_*^2}[Y_R^t(c_B - c_{2B}) + c_{BR}^t] \,, \\[6pt]
C_R^b &= c_R^b + \frac{g'^2}{2g_*^2}[Y_R^b(c_B - c_{2B}) + c_{BR}^b] \,, \\[6pt]
C_R^\tau &= c_R^\tau + \frac{g'^2}{2g_*^2}[Y_R^\tau(c_B - c_{2B}) + c_{BR}^\tau] \,, \\[6pt]
C_L^{q3} &= c_L^{q3} + \frac{1}{2}\frac{|y_t|^2}{g_*^2}c_{yR}^t + \frac{g'^2}{2g_*^2}[Y_L^q(c_B - c_{2B}) + c_{BL}^{q3}] \,, \\[6pt]
C_L^{l3} &= c_L^{l3} + \frac{g'^2}{2g_*^2}[Y_L^l(c_B - c_{2B}) + c_{BL}^{l3}] \,, \\[6pt]
C_L^{(3)\,q3} &= c_L^{(3)q3} - \frac{1}{2}\frac{|y_t|^2}{g_*^2}c_{yR}^t + \frac{g^2}{4g_*^2}(c_W - c_{2W} + 2c_{WL}^{q3}) \,, \\[6pt]
C_L^{(3)\,l3} &= c_L^{(3)l3} + \frac{g^2}{4g_*^2}(c_W - c_{2W} + 2c_{WL}^{l3}) \,, \\[6pt]
C_R^{tb} &= c_R^{tb} + \frac{1}{2}(c_{yL}^t + c_{yL}^b) \,.
\end{aligned}
\tag{B.7}
$$

Out of the 59 independent operators for a single family, 20 are of one-loop type and 25 are 4-fermion tree-level operators. The remaining 14 are tree-level operators whose number corresponds to the 14 physical C_i's in Eq. (D.22).

Let us now discuss how these C_i's would be modified in other bases. For example, if we keep $\mathcal{O}_B$ and $\mathcal{O}_W$ in the basis instead of the leptonic operators $\mathcal{O}_L^l$ and $\mathcal{O}_L^{(3)\,l}$, then one should remove C_L^l and $C_L^{(3)l}$ from the list of C_i's. This is accomplished by making

the replacements

$$c_B \quad \to \quad c_{2B} - \frac{1}{Y_L^l} c_{BL}^l - \frac{2g_*^2}{Y_L^l g'^2} c_L^l \;,$$

$$c_W \quad \to \quad c_{2W} - 2c_{WL}^l - 4\frac{g_*^2}{g^2} c_L^{(3)\,l} \;, \tag{B.8}$$

in all the C_i's (obtaining in particular $C_L^l = C_L^{(3)\,l} = 0$) and then add to the list the following two new C_i's:

$$C_B \quad = \quad c_B - c_{2B} + \frac{1}{Y_L^l} c_{BL}^l + \frac{2g_*^2}{Y_L^l g'^2} c_L^l \;,$$

$$C_W \quad = \quad c_W - c_{2W} + 2c_{WL}^l + 4\frac{g_*^2}{g^2} c_L^{(3)\,l} \;. \tag{B.9}$$

The replacement in Eq. (B.8) introduces a dependence on $\gamma_{c_{BL}^l}$, $\gamma_{c_{c_L}^l}$, $\gamma_{c_{WL}^l}$ and $\gamma_{c_L^{(3)l}}$ in the calculation of the anomalous dimensions of the C_i's, but the only non-redundant coefficients that appear in those anomalous dimensions depend on leptonic Yukawa couplings that we neglect.

In a similar way, $\mathcal{O}_{2B}$, $\mathcal{O}_{2W}$ and $\mathcal{O}_{2G}$ can be kept in the basis instead of three 4-fermion operators of the first family, e.g. $\mathcal{O}_{RR}^e$, $\mathcal{O}_{LL}^l$ and $\mathcal{O}_{RR}^{(8)d}$. In this basis c_{2B}, c_{2W} and c_{2G} have to be replaced by linear combinations of c_{RR}^e, c_{LL}^l and $c_{RR}^{(8)d}$ in the C_i's above. However, this replacement has no impact on the anomalous dimensions of the C_i's if we only keep the coefficients of Eq. (8.3) and neglect small Yukawas. Indeed, it is simple to realize that Eq. (8.3) can only renormalize $c_{2B,2W,2G}$ or c_{RR}^e, c_{LL}^l and $c_{RR}^{(8)d}$ through lepton or down Yukawa, which are terms we neglect in our RGEs. Therefore, whether we keep $\mathcal{O}_{2B,2W,2G}$ or 4-fermion operators, the RGEs given in Chapter 8 are unaffected.

B.2 Anomalous dimensions before removing redundant operators

To calculate the anomalous dimensions γ_{C_i}'s, following Eq. (B.5), we need to calculate the anomalous dimensions of the Wilson coefficients entering in the C_i's, including those that are redundant.

We have calculated these anomalous dimensions to linear order in the c_j's of Eq. (8.3), the only exception being c_r, which we keep for illustrative purposes here. Parametrically one has $\gamma_{c_i} \sim g_j^2 c_j / 16\pi^2$ and we only keep $g_j^2 = \{y_t^2, g_s^2, g^2, g'^2, \lambda\}$, dropping $g_j^2 =$

$\{y_b^2, y_\tau^2, ...\}$. The anomalous dimensions, calculated in Landau gauge, are:

$$\gamma_{c_H} = \frac{1}{4\pi^2}\left\{N_c y_t^2[c_{y_t} + c_L^{(3)}] + \lambda(7c_H - c_r) + \frac{3}{8}\left[g^2(c_H + 2c_r) + g'^2 c_r\right]\right\}$$
$$-4\gamma_h c_H, \tag{B.10}$$

$$\gamma_{c_T} = \frac{1}{16\pi^2}\left[4N_c y_t^2(c_R - c_L) + \frac{3}{2}g'^2(c_H - c_r)\right] - 4\gamma_h c_T, \tag{B.11}$$

$$\gamma_{\lambda c_6} = \frac{1}{8\pi^2}\left\{54\lambda^2 c_6 - 4N_c y_t^4 c_{y_t} + 12\lambda^2(3c_H + 2c_r) - \frac{3}{8}\left[2g^4 + (g^2 + g'^2)^2\right]c_r\right\}$$
$$-6\gamma_h \lambda c_6, \tag{B.12}$$

$$\frac{\gamma_{y_t c_{y_t}}}{y_t} = \frac{1}{16\pi^2}\left\{4\lambda\left[c_R - c_L + 3c_L^{(3)} + 6c_{y_t}\right] - g'^2\left[c_R + 4c_L - 4c_L^{(3)} + \frac{2}{3}c_{y_t}\right]\right.$$
$$\left. -3g^2 c_R - 8g_s^2 c_{y_t} + 2y_t^2\left[4c_{LR} + 4C_F c_{LR}^{(8)} + c_R - c_L + c_L^{(3)} + 2c_{y_t} + c_H\right]\right\}$$
$$-(3\gamma_h + \gamma_{Q_L} + \gamma_{t_R})c_{y_t}, \tag{B.13}$$

$$\frac{\gamma_{y_b c_{y_b}}}{y_b} = \frac{1}{8\pi^2}\left\{2\lambda[c_L + 3c_L^{(3)} + 6c_{y_b}] + y_t^2[2c_L^{(3)} - c_{y_t}] + \left(\frac{1}{6}g'^2 - 4g_s^2\right)c_{y_b}\right.$$
$$+ \frac{y_t^2}{g_*^2}[3g^2 - 2y_t^2 - 4\lambda]c_R^{tb} + g'^2[c_L + c_L^{(3)}]$$
$$\left. - \frac{y_t^4}{g_*^2}\left[(2N_c + 1)c_{y_t y_b} + C_F c_{y_t y_b}^{(8)}\right]\right\} - (\gamma_{Q_L} + \gamma_{b_R} + 3\gamma_h)c_{y_b}, \tag{B.14}$$

$$\frac{\gamma_{y_\tau c_{y_\tau}}}{y_\tau} = \frac{1}{16\pi^2}\left[3\left(8\lambda - g'^2\right)c_{y_\tau} + 2N_c \frac{y_t^2}{g_*^2}(\lambda - y_t^2)(2c_{y_t y_\tau} + c'_{y_t y_\tau})\right]$$
$$-(\gamma_{L_L} + \gamma_{\tau_R} + 3\gamma_h)c_{y_\tau}, \tag{B.15}$$

$$\gamma_{c_R} = \frac{1}{8\pi^2}\left\{y_t^2\left[N_c c_{LR} - 2(N_c + 1)c_{RR} + 2c_R - c_L + \frac{1}{4}(c_H - c_r)\right]\right.$$
$$\left. - \frac{3}{4}(3g^2 + g'^2)c_R\right\} - 2(\gamma_h + \gamma_{t_R})c_R, \tag{B.16}$$

$$\gamma_{c_L} = \frac{1}{8\pi^2}\left\{y_t^2\left[-N_c c_{LR} - \frac{1}{2}c_R + c_L - 3c_L^{(3)} - \frac{1}{8}(c_H - c_r) + (2N_c + 1)c_{LL}\right.\right.$$
$$\left.\left. -C_F c_{LL}^{(8)}\right] - \frac{3}{4}(3g^2 + g'^2)c_L\right\} - 2(\gamma_h + \gamma_{Q_L})c_L, \tag{B.17}$$

$$\gamma_{c_L^{(3)}} = \frac{-1}{8\pi^2}\left\{y_t^2\left[c_L + c_L^{(3)} - \frac{1}{8}(c_H - c_r) + c_{LL} + C_F c_{LL}^{(8)}\right] + \frac{3}{4}(g^2 + g'^2)c_L^{(3)}\right\}$$
$$-2(\gamma_h + \gamma_{Q_L})c_L^{(3)} \tag{B.18}$$

$$\gamma_{c_r} = \frac{1}{4\pi^2}\left\{N_c y_t^2\left[c_{y_t} - 2c_L^{(3)}\right] + \lambda(c_H + 5c_r) + \frac{3}{8}\left[(5g^2 + g'^2)c_r - 2g^2 c_H\right]\right\}$$
$$-4\gamma_h c_r,$$

$$\frac{1}{y_t}\gamma_{y_t c_{yH}} = -\frac{g_*^2}{4\pi^2}\left[c_{LR} + C_F\, c_{LR}^{(8)}\right] - (\gamma_h + \gamma_{Q_L} + \gamma_{t_R})c_{yH}\,, \tag{B.19}$$

$$\frac{1}{y_t}\gamma_{y_t c_{yR}} = \frac{g_*^2}{8\pi^2}\left[c_L - 3c_L^{(3)}\right] - (\gamma_h + \gamma_{Q_L} + \gamma_{t_R})c_{yR}\,, \tag{B.20}$$

$$\frac{1}{y_t}\gamma_{y_t c_{yL}} = -\frac{g_*^2}{8\pi^2}c_R - (\gamma_h + \gamma_{Q_L} + \gamma_{t_R})c_{yL}\,, \tag{B.21}$$

$$\frac{1}{y_t}\gamma_{y_t c_{yLR}} = \frac{g_*^2}{16\pi^2}\left[c_R - c_L + 3c_L^{(3)}\right] - (\gamma_h + \gamma_{Q_L} + \gamma_{t_R})c_{yLR}\,, \tag{B.22}$$

$$\frac{1}{y_b}\gamma_{y_b c_{yH}^b} = \frac{1}{16\pi^2}y_t^2\left[(2N_c + 1)\,c_{y_t y_b} + C_F\, c_{y_t y_b}^{(8)}\right] - (\gamma_h + \gamma_{Q_L} + \gamma_{b_R})c_{yH}^b\,, \tag{B.23}$$

$$\gamma_{c_{K4}} = \frac{y_t^2}{4\pi^2}N_c c_{yH} - 2\gamma_h c_{K4}\,, \tag{B.24}$$

$$\gamma_{\tilde{c}_R} = \frac{y_t^2}{16\pi^2}\left[3c_{y_t} + c_R + \frac{1}{2}(c_H + 2c_r)\right] - 2(\gamma_h + \gamma_{t_R})\tilde{c}_R\,, \tag{B.25}$$

$$\gamma_{\tilde{c}_L} = \frac{y_t^2}{32\pi^2}\left[3c_{y_t} - c_L + 3c_L^{(3)} + \frac{1}{2}(c_H + 2c_r)\right] - 2(\gamma_h + \gamma_{Q_L})\tilde{c}_L\,, \tag{B.26}$$

$$\gamma_{\tilde{c}_L^{(3)}} = \frac{y_t^2}{32\pi^2}\left[-c_{y_t} + c_L + c_L^{(3)} - \frac{1}{2}c_H\right] - 2(\gamma_h + \gamma_{Q_L})\tilde{c}_L^{(3)}\,, \tag{B.27}$$

$$\gamma_{c_W} = \frac{g_*^2}{48\pi^2}\left[16N_c c_L^{(3)} - (c_H + c_T)\right] - \left(2\gamma_h + \gamma_W + \frac{1}{g}\beta_g\right)c_W\,, \tag{B.28}$$

$$\gamma_{c_B} = \frac{g_*^2}{48\pi^2}\left[\frac{8N_c}{3}(2c_R + c_L) - (c_H + 5c_T)\right] - 2\gamma_h c_B\,, \tag{B.29}$$

$$\gamma_{c_{BR}} = \frac{g_*^2}{12\pi^2}\left\{\frac{1}{3}\left[4(N_c + 1)c_{RR} + N_c c_{LR}\right] + c_R\right\} - 2\gamma_{t_R}c_{BR}\,, \tag{B.30}$$

$$\gamma_{c_{BL}} = \frac{g_*^2}{12\pi^2}\left\{\frac{1}{3}\left[(2N_c + 1)c_{LL} + C_F c_{LL}^{(8)} + N_c c_{LR}\right] + 2c_L\right\} - 2\gamma_{Q_L}c_{BL}\,, \tag{B.31}$$

$$\gamma_{c_{WL}} = \frac{g_*^2}{12\pi^2}\left[c_{LL} + C_F c_{LL}^{(8)} + c_L^{(3)}\right] - \left(2\gamma_{Q_L} + \gamma_W + \frac{1}{g}\beta_g\right)c_{WL}\,, \tag{B.32}$$

where $C_F = (N_c^2 - 1)/(2N_c)$, $\beta_g = \frac{dg}{d\log\mu}$ and

$$\gamma_h = \frac{1}{16\pi^2}\left[-N_c y_t^2 + \frac{3}{4}(3g^2 + g'^2)\right]\,, \quad \gamma_{Q_L} = \frac{1}{16\pi^2}\left[-\frac{1}{2}y_t^2\right]\,, \quad \gamma_{t_R} = -\frac{y_t^2}{16\pi^2}\,,$$

$$\gamma_W = -\frac{1}{g}\beta_g - \frac{3}{16\pi^2}g^2 = \frac{1}{16\pi^2}\frac{g^2}{6}\,, \tag{B.33}$$

are the wave-function renormalization terms. The corresponding wave-function terms for leptons and b_R ($\gamma_{L_L}, \gamma_{\tau_R}$ and γ_{b_R}) are proportional to small Yukawa couplings squared that we are neglecting. Notice that in the above results we have included some dependence on Wilson coefficients beyond those of Eq. (8.3) and c_r. In particular, we have
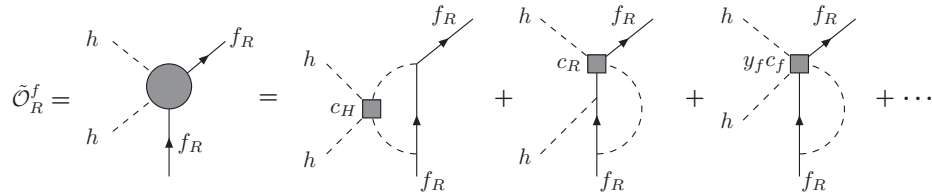
FIGURE B.1: *Diagrams that generate at one-loop the redundant operator $\tilde{\mathcal{O}}_R^f$.*
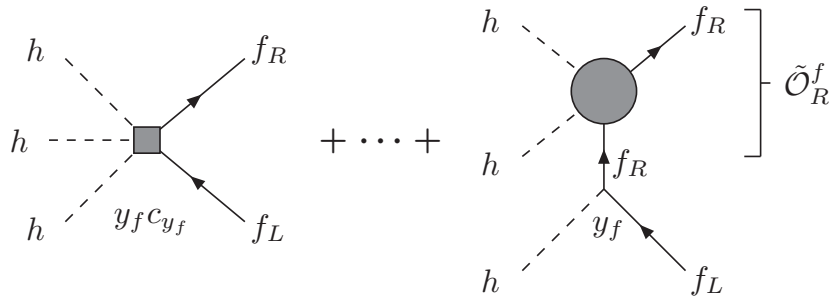
FIGURE B.2: *Contributions to the process $hhh \to \bar{f}_L f_R$ at order $1/\Lambda^2$, including the one-loop corrections shown in Fig. B.1.*

kept the contributions from wave function renormalization (which are trivial to take into account) in all cases, and we also kept the contributions from c_T in γ_{c_W} and γ_{c_B} that were already calculated in Ref. [5]. These anomalous dimensions have been calculated through the (divergent pieces of the) one-loop effective action.

Using Eqs. (B.10)-(B.32) we can calculate the anomalous dimensions γ_{C_i}'s for the shift-invariant Wilson coefficients. These are given in Sec. 8.1. We have cross-checked those RGEs by calculating them in an alternative way. We have looked at the one-loop radiative corrections to some particular physical processes and required the corresponding amplitudes to be independent of the renormalization scale. In order to find agreement between both methods, it is crucial to include in the amplitude for the physical process non-1PI contributions. In the effective action approach, such diagrams are in one-to-one correspondence with the redundant operators being eliminated.

As an illustrative example of the previous point, consider the contribution of the redundant operator $\tilde{\mathcal{O}}_R^f$ to the renormalization of $\mathcal{O}_{y_f}$. One-loop radiative corrections do generate $\tilde{\mathcal{O}}_R^f$ in the one-loop effective action, as shown in Fig. B.1, even if we remove $\tilde{\mathcal{O}}_R^f$ from the (tree-level) Lagrangian. The physical combination C_{y_f} [see Eq. (D.22)] depends on $\tilde{c}_R^f$ and, therefore, the anomalous dimension $\gamma_{C_{y_f}}$ also depends on $\gamma_{\tilde{c}_R^f}$. The same result for $\gamma_{C_{y_f}}$ can be obtained by looking at the physical process $hhh \to \bar{f}_L f_R$. The $1/\Lambda^2$ diagrammatic contributions to this process are shown in Fig. B.2. Besides

the tree-level contribution through c_{y_f} shown on the left, there are one-loop corrections, among which we just show the ones related to the redundant operator $\tilde{\mathcal{O}}_R^f$. Having removed the redundant $\tilde{\mathcal{O}}_R^f$ from the basis, there is no tree-level contribution from $\tilde{c}_R^f$ to $hhh \to \bar{f}_L f_R$ and the divergences from the one-loop blob shown in Fig. B.2 have to be absorbed by $c_{y_f}(\mu)$ to obtain an amplitude that is independent of the renormalization scale μ.

Finally, the reader can check, using the previous anomalous dimensions which include the dependence on the redundant coefficient c_r, that the anomalous dimensions of the shift invariant combinations $c_H - c_r$, $c_6 + 2c_r$, $c_{y_t} + c_r/2$, $c_{y_b} + c_r/2$, plus all the other Wilson coefficients, are functions of these same combinations, so that one can take $c_r = 0$ in a consistent way.

C. Custodial symmetry of $\mathcal{L}_{d=6}$

The $d = 6$ operators of the basis of Sec. 6.1 can be made invariant under the custodial $SU(2)_L \otimes SU(2)_R$ by promoting their coefficients to (non-propagating) spurion fields transforming under this symmetry. In this Appendix we present these transformation rules.

The bosonic sector of the SM Lagrangian can be made custodial invariant by promoting the gauge coupling g' to transform as a triplet under $SU(2)_R$:

$$g'_a \sigma_a \rightarrow g'_a\, R\sigma_a R^\dagger \,, \tag{C.1}$$

whose nonzero VEVs, given by $\langle g'_a \rangle = g'\delta_{a,3}$, define how the custodial symmetry is explicitly broken by this coupling. For the Higgs field, that transforms as a $(\mathbf{2}_L, \mathbf{2}_R)$, it is convenient to use the matrix field

$$\Sigma = \frac{1}{\sqrt{2}} \left(\tilde{H}\,,\, H \right) \,, \tag{C.2}$$

that transforms under the custodial group as $\Sigma \rightarrow L\Sigma R^\dagger$, and therefore its covariant derivative is given by $D_\mu\Sigma = \partial_\mu\Sigma - igW^a_{L\,\mu}\sigma^a\Sigma/2 + ig'^a B_\mu\Sigma\sigma^a/2$.

To make the Yukawa sector of the SM invariant under the custodial symmetry, we can promote the Yukawa couplings to transform as a doublet under $SU(2)_R$:

$$Y_u \rightarrow RY_u \,, \tag{C.3}$$

where $\langle Y_u \rangle = (y_u, 0)^T$, and similarly for the other Yukawas. The Yukawa term is then written as $\sqrt{2}\bar{Q}_L\Sigma Y_u u_R$, where the SM fermions transform as singlets under $SU(2)_R$. To define the proper hypercharge assignment for the SM fermions, we have to enlarge the global group to $SU(2)_L \otimes SU(2)_R \otimes U(1)_X$ and define the hypercharge as $Y = T^3_R + X$. This means that the $U(1)_Y$ is not only contained in $SU(2)_R$ but also in $U(1)_X$ and therefore g' also has a singlet component under the custodial group.

Using the above definitions we can write the Lagrangian $\mathcal{L}_6$ as an invariant under $SU(2)_L \otimes SU(2)_R$. This requires to promote a few of the coefficients to spurion fields transforming non-trivially. The result is shown in Tab. C.1. Only c_T and $c_{L,R}$ transform non-trivially, being then, together with g' and the Yukawa couplings, the only sources of custodial breaking. This information is useful to deduce what combinations of coefficients and couplings can contribute at the one-loop level to a given anomalous dimension. For example, contributions to γ_{c_T} can only come from terms that transform as $\mathbf{5}_R$: $\langle g'_a g'_b c_H \rangle = g'^2 c_H \delta_{a,3} \delta_{b,3}$ and $\langle Y_u^\dagger \sigma^a Y_u c_{L,R}^b \rangle = -y_u^2 c_{L,R} \delta^{a,3} \delta^{b,3}$, as the explicit calculation shows. In the same way it can be understood why γ_{c_H} depends on $y_t^2 c_L^{(3)}$ but not on $y_t^2 c_L$, being c_H a singlet under the custodial symmetry.

Useful information can also be derived from the transformations under the parity P_{LR} that interchanges $L \leftrightarrow R$. In the bosonic sector, we have

$$
\begin{aligned}
\Sigma &\leftrightarrow \Sigma^\dagger \\
\frac{g'^a}{g'} B_\mu &\leftrightarrow W_{L\,\mu}^a \\
g' &\leftrightarrow g \\
c_H &\leftrightarrow c_H \\
c_W &\leftrightarrow c_B \\
\kappa_{HW} &\leftrightarrow \kappa_{HB} \,,
\end{aligned}
\tag{C.4}
$$

while c_T and κ_{BB} do not have a well-defined transformation property inside the operator basis. For this reason it could be convenient to work with the operator $\mathcal{O}_{WB}$ instead of $\mathcal{O}_{BB}$ [both related by Eq. (7.7)] that is even under P_{LR}, and therefore $\kappa_{WB} \leftrightarrow \kappa_{WB}$.

For operators involving SM fermions, we have several possibilities for the transformation properties under P_{LR}, see Ref. [157]. The two simplest ones are to consider (for the up-type quark)

$$
\text{I)} \ Q_R \equiv \frac{1}{y_u} Y_u u_R \quad \text{and} \quad Q_L
\tag{C.5}
$$

that transform respectively as $(\mathbf{1}, \mathbf{2}_R)_{1/6}$ and $(\mathbf{2}_L, \mathbf{1})_{1/6}$ under $SU(2)_L \otimes SU(2)_R \otimes U(1)_X$, or, alternatively,

$$
\text{II)} \ \mathcal{Q}_L \equiv \frac{1}{y_u} Q_L \otimes Y_u^\dagger \quad \text{and} \quad u_R
\tag{C.6}
$$

transforming as $(\mathbf{2}_L, \mathbf{2}_R)_{2/3}$ and $(\mathbf{1}, \mathbf{1})_{2/3}$ respectively. For the first case, Eq. (C.5), we can write the operators $\mathcal{O}_R$ and $\mathcal{O}_L^{(3)}$ in the following way:

$$
- i c_R \text{tr}[\sigma_a \Sigma^\dagger \overset{\leftrightarrow}{D}_\mu \Sigma] \bar{Q}_R \sigma_a \gamma^\mu Q_R \ \text{and} \ i c_L^{(3)} \text{tr}[\Sigma^\dagger \sigma_a \overset{\leftrightarrow}{D}_\mu \Sigma] \bar{Q}_L \sigma_a \gamma^\mu Q_L \,,
\tag{C.7}
$$

such that under P_{LR} we can define $Q_L \leftrightarrow Q_R$ and

$$\text{I) } c_R \leftrightarrow c_L^{(3)} . \tag{C.8}$$

For the second case, Eq. (C.6), we can write the operators $\mathcal{O}_L$ and $\mathcal{O}_L^{(3)}$ as

$$ic_L \text{tr}[\sigma_a \Sigma^\dagger \overset{\leftrightarrow}{D}_\mu \Sigma] \text{tr}[\bar{\mathcal{Q}}_L^T \epsilon^T \sigma_a \gamma^\mu \epsilon \mathcal{Q}_L^T] \text{ and } ic_L^{(3)} \text{tr}[\Sigma^\dagger \sigma_a \overset{\leftrightarrow}{D}_\mu \Sigma] \text{tr}[\bar{\mathcal{Q}}_L \sigma_a \gamma^\mu \mathcal{Q}_L] , \tag{C.9}$$

and define $\mathcal{Q}_L \leftrightarrow \epsilon \mathcal{Q}_L^T \epsilon^T$ under P_{LR} that gives the transformation rule

$$\text{II) } c_L \leftrightarrow -c_L^{(3)} . \tag{C.10}$$

In this latter case, invariance under P_{LR} implies $c_L + c_L^{(3)} = 0$, and therefore no corrections to the $Zb_L\bar{b}_L$ coupling.

Operator	Spurion	$SU(2)_L \otimes SU(2)_R$	vev
$\frac{1}{2}c_T^{a,b}\text{tr}[\sigma^a \Sigma^\dagger \overset{\leftrightarrow}{D}_\mu \Sigma]\text{tr}[\sigma^b \Sigma^\dagger \overset{\leftrightarrow}{D^\mu} \Sigma]$	$c_T^{a,b}$	$(\mathbf{3}_R \otimes \mathbf{3}_R)_s = \mathbf{5}_R + \mathbf{1}$	$c_T \delta^{a,3}\delta^{b,3}$
$\frac{1}{2}c_H \left(\partial_\mu \text{tr}[\Sigma^\dagger \Sigma]\right)^2$	c_H	$\mathbf{1}$	c_H
$c_6 \left(\text{tr}[\Sigma^\dagger \Sigma]\right)^3$	c_6	$\mathbf{1}$	c_6
$-\frac{i}{2}c_B g'^a \text{tr}[\sigma^a \Sigma^\dagger \overset{\leftrightarrow}{D}_\mu \Sigma]\partial_\nu B^{\mu\nu}$	c_B	$\mathbf{1}$	c_B
$\frac{i}{2}c_W g\text{tr}[\Sigma^\dagger \sigma_a \overset{\leftrightarrow}{D}_\mu \Sigma]D_\nu W_a^{\mu\nu}$	c_W	$\mathbf{1}$	c_W
$c_y \text{tr}[\Sigma^\dagger \Sigma]\sqrt{2}\bar{Q}_L \Sigma Y_u u_R$	c_y	$\mathbf{1}$	c_y
$-ic_R^a \text{tr}[\sigma^a \Sigma^\dagger \overset{\leftrightarrow}{D}_\mu \Sigma]\bar{f}_R \gamma^\mu f_R$	c_R^a	$\mathbf{3}_R$	$c_R \delta^{a,3}$
$-ic_L^a \text{tr}[\sigma^a \Sigma^\dagger \overset{\leftrightarrow}{D}_\mu \Sigma]\bar{f}_L \gamma^\mu f_L$	c_L^a	$\mathbf{3}_R$	$c_L \delta^{a,3}$
$ic_L^{(3)}\text{tr}[\Sigma^\dagger \sigma_a \overset{\leftrightarrow}{D}_\mu \Sigma]\bar{f}_L \sigma_a \gamma^\mu f_L$	$c_L^{(3)}$	$\mathbf{1}$	$c_L^{(3)}$
$-4ic_R^{ud}\text{tr}[\Sigma Y_d Y_u^\dagger D_\mu \Sigma^\dagger]\bar{u}_R \gamma^\mu d_R$	c_R^{ud}	$\mathbf{1}$	c_R^{ud}
$\kappa_{BB} g'^a g'^a \text{tr}[\Sigma^\dagger \Sigma]B_{\mu\nu}B^{\mu\nu}$	κ_{BB}	$\mathbf{1}$	κ_{BB}
$-i\kappa_{HB} g'^a \text{tr}[\sigma^a D_\mu \Sigma^\dagger D_\nu \Sigma]B^{\mu\nu}$	κ_{HB}	$\mathbf{1}$	κ_{HB}
$i\kappa_{HW} g\text{tr}[D_\mu \Sigma^\dagger \sigma_a D_\nu \Sigma]W_a^{\mu\nu}$	κ_{HW}	$\mathbf{1}$	κ_{HW}
$\kappa_{DB}\sqrt{2}\bar{Q}_L \Sigma Y_u \sigma^{\mu\nu}u_R B_{\mu\nu}$	κ_{DB}	$\mathbf{1}$	κ_{DB}
$\kappa_{DW}\sqrt{2}\bar{Q}_L \sigma^a \Sigma Y_u \sigma^{\mu\nu}u_R W_{\mu\nu}^a$	κ_{DW}	$\mathbf{1}$	κ_{DW}
$\kappa_{DG}\sqrt{2}\bar{Q}_L \Sigma Y_u T^A \sigma^{\mu\nu}u_R G_{\mu\nu}^A$	κ_{DG}	$\mathbf{1}$	κ_{DG}

TABLE C.1: *Transformation of the spurion Wilson coefficients of the $d = 6$ operators under the custodial symmetry and their corresponding VEV. We are dropping fermion indices in the coefficients.*

D. Dealing with redundant operators

In this appendix we explain in detail the anomalous dimension matrix presented in the main body of the paper, Tabs. 9.2-9.3 and 9.7. As remarked in Sec. 9, a common effect encountered in the computation of the scaling of the dim-6 operators is the appearance of counter-terms that correspond to operators not included in our basis, i.e. operators that are redundant for the description of physical processes. In particular, the set of 13 operators we were interested in Chapter 9,

$$\{\mathcal{O}_H, \mathcal{O}_T, \mathcal{O}_B, \mathcal{O}_W, \mathcal{O}_{2B}, \mathcal{O}_{2W}, \mathcal{O}_{BB}, \mathcal{O}_{WW}, \mathcal{O}_{WB}, \mathcal{O}_{3W}, \mathcal{O}_{2G}, \mathcal{O}_{GG}, \mathcal{O}_{3G}\} \ , \qquad \text{(D.1)}$$

not only mix among themselves under the RG flow but also generate redundant operators that are not included in our basis (defined in Sec. 9). In this appendix we first give a pedagogic example of radiatively generated redundant operators, Sec. ??. Then, we present the set of redundant operators generated by those in Eq. (D.1), together with their anomalous dimensions, Sec. D.1. In Sec. D.2 we explain how the redundant operators are redefined back into our basis and what is their effect on the anomalous dimensions of the operator set in Eq. (D.1) [6].

D.1   Anomalous dimension matrix

The relevant redundant operators that are radiatively generated by those in Eq. (D.1) are:

$$\begin{aligned}
&\mathcal{O}_r = |D_\mu H|^2 |H|^2 \ , &&\mathcal{O}_{K4} = \left|D^2 H\right|^2 \ , \\
&\mathcal{O}_{LL}^{(3)L_1} = (\bar{L}_L \sigma^a \gamma^\mu L_L)(\bar{L}_L \sigma^a \gamma^\mu L_L) \ , &&\mathcal{O}_L^{(3)L_1} = i(H^\dagger \sigma^a \overset{\leftrightarrow}{D}_\mu H)\bar{L}_L^1 \sigma^a \gamma^\mu L_L^1 \ , \\
&\mathcal{O}_{LL}^{L_1} = (\bar{L}_L \gamma^\mu L_L)(\bar{L}_L \gamma^\mu L_L) \ , &&\mathcal{O}_R^{e_1} = i(H^\dagger \overset{\leftrightarrow}{D}_\mu H)(\bar{e}_R^1 \gamma^\mu e_R^1) \ , \\
&\mathcal{O}_{RR}^{(8)u_1 d_1} = (\bar{u}_R \gamma^\mu T^A u_R)(\bar{d}_R \gamma^\mu T^A d_R) \ , &&\mathcal{O}_{RR}^{e_1} = (\bar{e}_R \gamma^\mu e_R)(\bar{e}_R \gamma^\mu e_R) \ ,
\end{aligned} \qquad \text{(D.2)}$$

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$$\mathcal{O}_{K3L}^{F_i} = \tfrac{1}{2}\bar{F}_L^i \left(\slashed{D}D^2 + D^2\slashed{D}\right) F_L^i\,, \quad \mathcal{O}_{K3R}^{f_i} = \tfrac{1}{2}\bar{f}_R^i \left(\slashed{D}D^2 + D^2\slashed{D}\right) f_R^i\,,$$

$$\mathcal{O}_{WL}^{F_i} = gD^\nu W_{\mu\nu}^a (\bar{F}_L^i \sigma^a \gamma^\mu F_L^i)\,, \quad \mathcal{O}_{WL}'^{F_i} = g\widetilde{W}_{\mu\nu}^a i\bar{F}_L^i \sigma^a \gamma^\mu D^\nu F_L^i\,,$$

$$\mathcal{O}_{BL}^{F_i} = g'D^\nu B_{\mu\nu}(\bar{F}_L^i \gamma^\mu F_L^i)\,, \quad \mathcal{O}_{BL}'^{F_i} = g'\widetilde{B}_{\mu\nu} i\bar{F}_L^i \gamma^\mu D^\nu F_L^i\,,$$

$$\mathcal{O}_{BR}^{f_i} = g'D^\nu B_{\mu\nu}(\bar{f}_R^i \gamma^\mu f_R^i)\,, \quad \mathcal{O}_{BR}'^{f_i} = g'\widetilde{B}_{\mu\nu} i\bar{f}_R^i \gamma^\mu D^\nu f_R^i\,,$$ (D.3)

$$\mathcal{O}_{GL}^{Q_i} = g_s D^\nu G_{\mu\nu}^A (\bar{Q}_L^i T^A \gamma^\mu Q_L^i)\,, \quad \mathcal{O}_{GL}'^{Q_i} = g_s \widetilde{G}_{\mu\nu}^A i(\bar{Q}_L^i T^A \gamma^\mu D^\nu Q_L^i)\,,$$

$$\mathcal{O}_{GR}^{q_i} = g_s D^\nu G_{\mu\nu}^A (\bar{q}_R^i T^A \gamma^\mu q_R^i)\,, \quad \mathcal{O}_{GR}'^{q_i} = g_s \widetilde{G}_{\mu\nu}^A i(\bar{q}_R^i T^A \gamma^\mu D^\nu q_R^i)\,,$$

By *relevant* we mean those radiatively generated redundant operators that modify the Wilson coefficients of the operators in Eq. (D.1) when the former operators are redefined into operators in our basis, defined in Sec. 9.

Below we present in three different tables the anomalous dimension matrix of the operators in Eq. (D.1) as well as the relevant redundant operators generated by them, Eq. (D.3), at the order stated in Eq. (9.6). We work with arbitrary ξ in the background field gauge (see Eq. (9.7)) and use dimensional regularization. All the contributions given in Tabs. D.1, D.2 and D.3 below arise from one-particle-irreducible Feynman diagrams, i.e. it is the one-loop renormalization of the Effective Action.

In Tab. D.1 we display the contributions of $\mathcal{O}_H$, $\mathcal{O}_r$ and $\mathcal{O}_T$ to the running of the Wilson coefficients of the operators in Eq. (D.1). We have defined

$$\gamma_{c_i} = 16\pi^2 \frac{dc_i}{d\log\mu}\,, \quad \beta_g = \frac{dg}{d\log\mu}$$ (D.4)

	c_H	c_r	c_T
γ_c	$28\lambda + 12y_t^2 - 3\left(\tfrac{5}{2}g^2 + g'^2\right)$	$\tfrac{3}{2}\left(2g^2 + g'^2\right) - 4\lambda$	$8\lambda - 6g^2 - \tfrac{3}{2}g'^2$
γ_c	$\tfrac{3}{2}g'^2$	$-\tfrac{3}{2}g'^2$	$12\lambda + 12y_t^2 + \tfrac{9}{2}g^2$
γ_c	$-\tfrac{1}{3}$	$\tfrac{1}{3}$	$-\tfrac{5}{3}$
γ_c	$-\tfrac{1}{3}$	$\tfrac{1}{3}$	$-\tfrac{1}{3}$
γ_c	$4\lambda - 3g^2$	$20\lambda + 12y_t^2 - \tfrac{3}{2}\left(g^2 + g'^2\right)$	$-4\lambda + 3g^2 - 6g'^2$

TABLE D.1: *Anomalous dimension matrix. Further contributions of $\mathcal{O}_H$, $\mathcal{O}_r$ and $\mathcal{O}_T$ to other operators in Eq. (D.1) and Eq. (D.3) are either zero or proportional to the Yukawa coupling of any fermion lighter than the top. The dashed line separates the anomalous dimension of the operators in our basis from that of the redundant operators.*

and

$$\gamma_H = -N_c y_t^2 + \frac{1}{4}\left(3[3-\xi_W]g^2 + [3-\xi_B]g'^2\right) \ , \quad \gamma_W = -\frac{1}{g}\beta_g = \left(\frac{43}{6} - \frac{4}{3}N_G\right)g^2 \ ,$$

$$\gamma_G = -\frac{1}{g_s}\beta_{g_s} = \left(11 - \frac{4}{3}N_G\right)g_s^2 \ , \quad \gamma_B = -\frac{1}{g'}\beta_{g'} = \left(-\frac{1}{6} - \frac{20}{9}N_G\right)g'^2 \ ,$$

$$\text{(D.5)}$$

in the background field gauge. $N_G = 3$ is the number of generations. The contributions not shown are either zero or proportional to the Yukawa coupling y_l of any fermion lighter than the top. Notice that in Tab. D.1 we have gone beyond the strictly necessary computations to obtain the anomalous dimension matrix and also included the contributions of the operator $\mathcal{O}_r$, that is redundant with respect to our basis; their contributions are used for a crosscheck in Sec. D.3.

In Tab. D.2 we show the contributions of $\mathcal{O}_{BB}, \mathcal{O}_{WW}, \mathcal{O}_{WB}$ and $\mathcal{O}_{3W}$ to the running of the operators in Eq. (D.1). The c_{3W} self-renormalization has been extracted from the result of Ref. [132]. Their contribution to the running of the redundant operators in Eq. (D.3) that we have not written are either zero or proportional to y_l.

	c_{BB}	c_{WW}	c_{WB}	c_{3W}
γ_c	$6g'^4$	$18g^4$	$6g'^2 g^2$	0
γ_c	0	0	0	0
γ_c	0	0	0	0
γ_c	0	0	0	$2g^2$
γ_c	0	0	0	0
γ_c	0	0	0	$4g^2$
γ_c	$\frac{g}{2} - \frac{9g}{2} + 6y_t^2 + 12\lambda$	0	$3g^2$	0
γ_c	0	$-\frac{3g}{2} - \frac{5g}{2} + 6y_t^2 + 12\lambda$	g'^2	$\frac{5}{2}g^2$
γ_c	$2g'^2$	$2g^2$	$-\frac{g}{2} + \frac{9g}{2} + 6y_t^2 + 4\lambda$	$-\frac{g}{2}$
γ_c	0	0	0	$24g^2 - 2\gamma_W$
γ_c	$6g'^4$	$18g^4$	$6g'^2 g^2$	0
γ_c	0	0	0	g^2

TABLE D.2: *Anomalous dimension matrix. Further contributions of $\mathcal{O}_{BB}, \mathcal{O}_{WW}, \mathcal{O}_{WB}$ and $\mathcal{O}_{3W}$ to other operators in Eq. (D.1) and Eq. (D.3) are either zero or proportional to the Yukawa coupling of fermions lighter than the top. The dashed line separates the anomalous dimension of the operators in our basis from that of the redundant operators.*

Lastly, in Tab. D.3 we show the contributions of $\mathcal{O}_B, \mathcal{O}_W, \mathcal{O}_{2B}$ and $\mathcal{O}_{2W}$ to the running of any of the operators in Eq. (D.1) and Eq. (D.3). We have indicated by $\mathcal{O}\left(y_l\right)$ those contributions that *at most* are expected to be proportional to the Yukawa coupling of a fermion lighter than the top. As can be noted from Tab. D.3, the contribution of $\mathcal{O}_{2W}$ to the running of $\mathcal{O}_H$, $\mathcal{O}_r$, $\mathcal{O}_W$, $\mathcal{O}_{2W}$, $O_L^{(3)F_i}$, $\mathcal{O}_{WL}^{F_i}$ and $\mathcal{O}_{LL}^{(3)F_i}$ is ξ-dependent. This should not come as a surprise, even if we work in the background field gauge, where the counter-terms are gauge invariant. The reason is that at this point of the computation we still have redundant operators generated by the RG flow. By definition, in an over-complete basis that contains redundant operators only certain combinations of the Wilson coefficients enter in the physical observables. Hence, it is only after these physical combinations of the Wilson coefficients are taken, that the computation

	c	c	c	c
γ	$g\,(g + 4g)$	$g\,(3g + 4g) - 6\lambda g$	$-g\,(g + 4g)$	$-g\,(g\,(3 + 2\xi) + 4g) + 3\lambda g$
γ	$-g\,g - 6\lambda g$	$-g\,g$	$g\,g + 3\lambda g$	$g\,g$
γ	$\cdot + 6y$	$\cdot$	$-\cdot$	$-\cdot$
γ	$\cdot$	$g + 6y$	$-\cdot$	$-g\left(\cdot + 3\xi\right)$
γ	$-g$	0	-2γ	0
γ	0	$-g$	0	$g\left(\cdot - 3\xi\right) - 2\gamma$
γ	0	0	0	0
γ	0	0	0	0
γ	0	0	0	0
γ	0	0	0	0
γ	$g\,(2g - g) + 6\lambda g$	$g\,(6g - g) + 30\lambda g$	$g\,(g - 2g) - 3\lambda g$	$g\,(g - 2g\,(3 - \xi)) - 15\lambda g$
γ	$-g$	$-3g$	$\cdot$	g
γ	0	g	0	$g\,\xi$
γ	0	0	0	0
γ	0	0	0	0
γ	0	0	$-Y\,g$	$-g$
γ	0	0	$-Y\,g$	0
γ	$\mathcal{O}\left(y\right)$	$\mathcal{O}\left(y\right)$	$-Y\,g$	$-g - \xi\,g$
γ	$\mathcal{O}\left(y\right)$	$\mathcal{O}\left(y\right)$	$-Y\,g$	$-Y\,g$
γ	$\mathcal{O}\left(y\right)$	$\mathcal{O}\left(y\right)$	$-Y\,g$	0
γ	$\mathcal{O}\left(y\right)$	$\mathcal{O}\left(y\right)$	$-Y\,g$	$-g$
γ	$\mathcal{O}\left(y\right)$	$\mathcal{O}\left(y\right)$	$-Y\,g$	$-Y g$
γ	$\mathcal{O}\left(y\right)$	$\mathcal{O}\left(y\right)$	$-Y\,g$	0
γ	0	0	$-g\,(g\,Y)$	$g\,(g\,(1 + \xi) - 4(g\,Y))$
γ	0	0	$-6(g\,Y)$	$-g$
γ	0	0	$-6(g\,Y)$	0

TABLE D.3: *Contributions of the operators $\mathcal{O}_B, \mathcal{O}_W, \mathcal{O}_{2B}$ and $\mathcal{O}_{2W}$ to the anomalous dimension matrix of the operators in Eq. (D.1) and Eq. (D.3). By y_i we denote the Yukawa coupling of any fermion. The dashed line separates the anomalous dimension of the operators in our basis from that of the redundant operators.*

is guaranteed to be and should be gauge invariant. For instance, in Sec. D.2 we show that upon redefining the redundant operators in terms of operators in our basis the ξ dependence of the anomalous dimension vanishes. This subtlety is well known and, for instance, it also appears in the context of Non-Relativistic QCD, where the running of the Wilson coefficients is gauge independent only when the redundancy of different operators is taken into account [158]. This has also been recently stressed again in Ref. [135].

Tab. D.4 reports the contributions of $\mathcal{O}_{2G}, \mathcal{O}_{GG}, \mathcal{O}_{3G}, \mathcal{O}_{2B}$ and $\mathcal{O}_{2W}$ to the anomalous dimension of the (redundant) operators in Eq. (D.1) and Eq. (D.3), as needed to derive the anomalous dimension matrix of the dim-6 bosonic operators with gluons of our basis (see Tab. 9.7).

	c_{2G}	c_{GG}	c_{3G}	c_{2B}	c_{2W}
γ_c	$\frac{1}{2}g_s^2(59 - 9\xi_G) - 2\gamma_G$	0	$6g_s^2$	0	0
γ_c	0	$-\frac{3}{2}g'^2 - \frac{9}{2}g^2 + 12\lambda + 6y_t^2$	0	0	0
γ_c	0	0	$36g_s^2 - 2\gamma_G$	0	0
γ_c	$-12g_s^2(g'^2 Y_u Y_d)$	0	0	$-12(g'^2 Y_u Y_d)^2$	0
γ_c	$\frac{1}{2}g_s^4(9\xi_G - 1)$	0	0	$-12g_s^2(g'^2 Y_u Y_d)$	0
γ_c	$-\frac{4}{3}g_s^2$	0	0	Tab. D.3	Tab. D.3
γ_c	$-\frac{4}{3}g_s^2$	0	0	Tab. D.3	0
γ_c	$-g_s^2(\frac{9}{2}\xi_G + \frac{37}{9})$	0	$3g_s^2$	$-\frac{5}{6}(g'Y_{u,d})^2$	0
γ_c	$-g_s^2(\frac{9}{2}\xi_G + \frac{37}{9})$	0	$3g_s^2$	$-\frac{5}{6}(g'Y_Q)^2$	$-\frac{5}{8}g^2$
γ_c	$-\frac{5}{9}g_s^2$	0	0	Tab. D.3	Tab. D.3
γ_c	$-\frac{10}{9}g_s^2 Y_Q$	0	0	Tab. D.3	Tab. D.3
γ_c	$-\frac{10}{9}g_s^2 Y_{u,d}$	0	0	Tab. D.3	0
γ_c	$-\frac{4}{3}g_s^2$	0	0	$-(g'Y_{u,d})^2$	0
γ_c	$-\frac{4}{3}g_s^2$	0	0	$-(g'Y_Q)^2$	$-\frac{3}{4}g^2$
γ_c	$-\frac{2}{3}g_s^2$	0	0	Tab. D.3	Tab. D.3
γ_c	$-\frac{4}{3}g_s^2 Y_Q$	0	0	Tab. D.3	Tab. D.3
γ_c	$-\frac{4}{3}g_s^2 Y_{u,d}$	0	0	Tab. D.3	0

TABLE D.4: *Contributions of the operators $\mathcal{O}_{2G}, \mathcal{O}_{GG}, \mathcal{O}_{3G}, \mathcal{O}_{2B}$ and $\mathcal{O}_{2W}$ to the anomalous dimension of the operators in Eq. (D.1) and Eq. (D.3). The dashed line separates the anomalous dimension of the operators in our basis from that of the redundant operators.*

D.2   Removal of the radiatively-generated redundant operators

We now turn in to discuss how to deal with each operator in Eq. (D.3) and their effect on the operators of Eq. (D.1).

The easiest way to deal with the redundant operator $\mathcal{O}'^{f_i}_{BR} = g'\widetilde{B}_{\mu\nu}i\bar{f}^i_R\gamma^\mu D^\nu f^i_R$ [101] is by means of the identity[1]

$$\gamma^\mu\gamma^\nu\gamma^\rho = g^{\mu\nu}\gamma^\rho + g^{\nu\rho}\gamma^\mu - g^{\mu\rho}\gamma^\nu + i\epsilon^{\mu\nu\rho\sigma}\gamma_\sigma\gamma^5 \, ; \tag{D.6}$$

one finds

$$
\begin{aligned}
g'\widetilde{B}_{\mu\nu}\bar{f}_R\gamma^\mu iD^\nu f_R &= \frac{g'}{4}\bar{f}_R i\left(\gamma_\mu\gamma_\nu \overrightarrow{\slashed{D}} + \overleftarrow{\slashed{D}}\,\gamma_\mu\gamma_\nu\right) f_R g'\widetilde{B}^{\mu\nu} \\
&+ ig'\bar{f}_R\gamma_\rho\gamma_\mu\gamma_\nu f_R D^\rho \widetilde{B}^{\mu\nu} \, .
\end{aligned}
\tag{D.7}
$$

Then, using the fermion's EoM

$$
\begin{aligned}
\frac{g'}{4}\bar{f}_R i\left(\gamma_\mu\gamma_\nu \overrightarrow{\slashed{D}} + \overleftarrow{\slashed{D}}\,\gamma_\mu\gamma_\nu\right) f_R g'\widetilde{B}^{\mu\nu} &= \frac{1}{4}g'y_f i\bar{F}_L\sigma_{\mu\nu}f_R H g'\widetilde{B}^{\mu\nu} + \text{h.c.} \tag{D.8} \\
&= \frac{1}{4}g'y_f\bar{F}_L\sigma_{\mu\nu}f_R H g'B^{\mu\nu} + \text{h.c.} \equiv \frac{1}{4}\mathcal{O}^f_{DB} \, ,
\end{aligned}
$$

which is a *dipole* operator, where $\sigma^{\mu\nu} \equiv \frac{i}{2}[\gamma^\mu, \gamma^\nu]$; using again Eq. (D.6) in the second term of the right hand side of Eq. (D.7)

$$ig'\bar{f}_R\gamma_\rho\gamma_\mu\gamma_\nu f_R D^\rho \widetilde{B}^{\mu\nu} = 2g'\bar{f}_R\gamma_\sigma f_R D_\rho B^{\sigma\rho} = 2\mathcal{O}^f_{BR} \, . \tag{D.9}$$

Therefore, Eqs. (D.7)-(D.9) and analogous manipulations, are equivalent to the following shifts ($c_i \to c_i + \delta c_i$) in the following Wilson coefficients:

$$\delta c^F_{WL} = 2c'^F_{WL}, \quad \delta c^F_{BL} = 2c'^F_{BL}, \quad \delta c^f_{BR} = 2c'^f_{BR}, \quad \delta c^Q_{GL} = 2c'^Q_{GL}, \quad \delta c^q_{GR} = 2c'^q_{GR} \, . \tag{D.10}$$

The Wilson coefficient of the dipole operators are also shifted, see Eq. (D.8), however, we can not conclude that the dipoles are renormalized by the set of bosonic operators we considered because we did not compute direct contributions (those coming from one-particle-irreducible diagrams).

---

[1]We use the conventions of *Peskin & Schroeder* textbook [159].

Then, for the operator $\mathcal{O}_{K3R}^{f_i}$, consider the field redefinition $\delta f_i = -\frac{c_{K3R}^{f_i}}{2\Lambda^2}D^2 f_i$, that removes $\mathcal{O}_{K3R}^{f_i}$ from the Lagrangian while generates the operator

$$-\frac{c_{K3R}^{f_i}y_{f_i}}{2\Lambda^2}D_\mu\bar{F}_{iL}D^\mu\left(f_{iR}H\right) + \text{h.c.} = -\frac{c_{K3R}^{f_i}y_{f_i}}{2\Lambda^2}[D_\mu\bar{F}_{iL}\gamma^\mu\gamma^\nu D^\nu\left(f_{iR}H\right)$$
$$-\frac{1}{2}\bar{F}_{iL}X_{\mu\nu}\sigma^{\mu\nu}f_{iR}H + \text{h.c.}],$$
(D.11)

where $X_{\mu\nu} = g'Y_{F_i}B_{\mu\nu} + gW_{\mu\nu}^a\tau^a + g_sG_{\mu\nu}^AT^A$, being τ^a and T^A the $SU(2)_L$ and $SU(3)_c$ generators in the fundamental representation, respectively. Then, by inserting the fermion's EoM in the first operator in the right hand side of Eq. (D.11) one gets operators of the type $\mathcal{L}_{\text{Yuk}}|H|^2$ and the operator $y_{f_i}\mathcal{O}_R^{f_i} \equiv y_{f_i}i(H^\dagger\overleftrightarrow{D}_\mu H)\bar{f}_R^i\gamma^\mu f_R^i$; we do not care about the latter (proportional to y_{f_i}) since our basis choice of Sec. 9 was to remove the operator $\mathcal{O}_R^{f_i}$ corresponding to a light fermion. Performing an analogous analysis for $\mathcal{O}_{K3L}^{F_i}$ we reach the same conclusion: neither of the two operator's scaling affects the anomalous dimension of the set of bosonic operators in Eq. (D.1). As in the case of $\mathcal{O}'_{WL,BL,BR}$, the same comment applies here: even-though the Wilson coefficient of the dipoles is shifted by the above manipulations, we do not conclude that they are renormalized by the bosonic operators.

Now, the remaining operators are redefined into our basis by performing field redefinitions. Consider the 37 independent field redefinitions

$$\Lambda^2\delta G_\mu^A = \alpha_{2G}(D^\nu G_{\mu\nu}^A) + g_S\sum_i\alpha_{QG}^i\bar{Q}_L^iT^A\gamma_\mu Q_L^i + g_S\sum_{i,q}\alpha_{qG}^i\bar{q}_R^iT^A\gamma_\mu q_R^i,,$$

$$\Lambda^2\delta W_\mu^a = ig\alpha_W(H^\dagger\sigma^a\overleftrightarrow{D}^\mu H) + \alpha_{2W}(D^\nu W_{\mu\nu}^a) + g\sum_{i,F}\alpha_{FW}^i\bar{F}_L^i\sigma^a\gamma_\mu F_L^i,$$

$$\Lambda^2\delta B_\mu = ig'\alpha_B(H^\dagger\overleftrightarrow{D}^\mu H) + \alpha_{2B}(\partial^\nu B_{\mu\nu}) + g'\sum_{i,F}Y_F\alpha_{FB}^i\bar{F}_L^i\gamma_\mu F_L^i + g'\sum_{i,f}Y_f\alpha_{fB}^i\bar{f}_R^i\gamma_\mu f_R^i,$$

$$\Lambda^2\delta H = \alpha_1 H|H|^2 + \alpha_2\left((D^2H) - y_e^{ij}\bar{e}_R^iL_L^j - y_d^{ij}\bar{d}_R^iQ_L^j - y_u^{ij}i\sigma^2(\bar{u}_R^iQ_L^j)^*\right),$$
(D.12)

with $F = \{L,Q\}$, $f = \{e,d,u\}$, $q = \{d,u\}$ and $i = 1,2,3$. These generate the following shifts for the Wilson coefficients of the dimension 6 operators:

$$\begin{aligned}
c_H &\to\quad c_H + 2(\alpha_1 + 2\lambda\alpha_2) - \alpha_W g^2 \qquad & c_r &\to\quad c_r + 2(\alpha_1 + 2\lambda\alpha_2) + \alpha_W g^2 \\
c_T &\to\quad c_T - \alpha_B g'^2 & c_{K4} &\to\quad c_{K4} - 2\alpha_2 \\
c_B &\to\quad c_B + \alpha_{2B} - 2\alpha_B & c_{WL}^{F_i} &\to\quad c_{WL}^{F_i} + \tfrac{1}{2}\alpha_{2W} - \alpha_{FW}^i \\
c_W &\to\quad c_W + \alpha_{2W} - 2\alpha_W & c_{BL}^{F_i} &\to\quad c_{BL}^{F_i} + Y_F(\alpha_{2B} - \alpha_{FB}^i) \\
c_{2B} &\to\quad c_{2B} + 2\alpha_{2B} & c_{BR}^{f_i} &\to\quad c_{BR}^{f_i} + Y_f(\alpha_{2B} - \alpha_{fB}^i)
\end{aligned}$$

$$c_{2W} \rightarrow \quad c_{2W} + 2\alpha_{2W} \qquad\qquad c_{LL}^{(3)F_i} \rightarrow \quad c_{LL}^{(3)F_i} + \tfrac{g^2}{2}\alpha_{FW}^i$$

$$c_{2G} \rightarrow \quad c_{2G} + 2\alpha_{2G} \qquad\qquad c_{LL}^{F_i} \rightarrow \quad c_{LL}^{F_i} + (Y_F g')^2 \alpha_{FB}^i$$

$$c_6 \rightarrow \quad c_6 - 4\alpha_1 \qquad\qquad c_{RR}^{f_i} \rightarrow \quad c_{RR}^{f_i} + (Y_f g')^2 \alpha_{fB}^i$$

$$c_{y_f}^i \rightarrow \quad c_{y_f}^i - \alpha_1 + 2\lambda\alpha_2 \qquad\qquad c_{LR}^{F_i f_j} \rightarrow \quad c_{LR}^{F_i f_j} + (Y_F Y_f g'^2)(\alpha_{fB}^i + \alpha_{FB}^i)$$

$$c_{y_f y_f}^{ij} \rightarrow \quad c_{y_f y_f}^{ij} + 2\alpha_2 \qquad\qquad c_L^{(3)F_i} \rightarrow \quad c_L^{(3)F_i} + \tfrac{g^2}{2}(\alpha_W + \alpha_{FW}^i)$$

$$c_{RR}^{u^i d^j} \rightarrow \quad c_{RR}^{u^i d^j} + g'^2 Y_u Y_d (\alpha_{uB}^i + \alpha_{dB}^j) \qquad\qquad c_L^{F_i} \rightarrow \quad c_L^{F_i} + Y_F g'^2 (\alpha_B + \tfrac{1}{2}\alpha_{FB}^i)$$

$$c_R^{f_i} \rightarrow \quad c_R^{f_i} + Y_f g'^2 (\alpha_B + \tfrac{1}{2}\alpha_{fB}^i)$$

$$c_{GL,R}^{q_i} \rightarrow \quad c_{GL,R}^{q_i} + \alpha_{2G} - \alpha_{qG}^i \text{ for } q = Q, u, d$$

$$c_{RR}^{(8)u^i d^j} \rightarrow \quad c_{RR}^{(8)u^i d^j} + g_s^2 (\alpha_{uG}^i + \alpha_{dG}^j). \tag{D.13}$$

Notice that using Fierz identities we can always trade the operator $\mathcal{O}_{LL}^{F_i}$ for $\mathcal{O}_{LL}^{(3)F_i}$: $\mathcal{O}_{LL}^{F_i} = \mathcal{O}_{LL}^{(3)F_i}$. This means that the shift in $c_{LL}^{F_i}$ can be recast as a shift in $c_{LL}^{(3)F_i}$, which becomes:

$$c_{LL}^{(3)F_i} \rightarrow c_{LL}^{(3)F_i} + \frac{g^2}{2}\alpha_{FW}^i + \left(c_{LL}^{F_i} + (Y_F g')^2 \alpha_{FB}^i \right). \tag{D.14}$$

We use the freedom given by the field redefinitions to set to zero the following 37 coefficients: $c_r, c_{K4}, c_{LL}^{(3)L_1}, c_{RR}^{e_1}, c_L^{(3)L_1}, c_R^{e_1}, c_{WL}^{F_i}, c_{BL}^{F_i}, c_{BR}^{f_i}, c_{GL}^{Q_i}, c_{GR}^{u_i}, c_{GR}^{d_i}, c_{RR}^{(8)u^1 d^1}$. This fixes all the shift parameters α_i and gives shift invariant combinations, under Eq. (D.13), of the Wilson coefficients of the operators in our basis:

$$c_H \rightarrow c_H - c_r + 6(c_L^{(3)L_1} - \tilde{c}_{LL}^{(3)L_1}) ,$$

$$c_T \rightarrow c_T + \frac{1}{Y_e}(c_R^{e_1} - \frac{1}{2Y_e}c_{RR}^{e_1}) ,$$

$$c_W \rightarrow c_W - 2c_{WL}^{L_1} - 4c_{WL}'^{L_1} + \frac{4}{g^2}(c_L^{(3)L_1} - 2\tilde{c}_{LL}^{(3)L_1}) ,$$

$$c_B \rightarrow c_B - \frac{1}{Y_e}c_{BR}^{e_1} - \frac{2}{Y_e}c_{BR}'^{e_1} + \frac{2}{Y_e g'^2}(c_R^{e_1} - \frac{1}{Y_e}c_{RR}^{e_1}) , \tag{D.15}$$

$$c_{2W} \rightarrow c_{2W} - 4c_{WL}^{L_1} - 8c_{WL}'^{L_1} - \frac{8}{g^2}\tilde{c}_{LL}^{(3)L_1} ,$$

$$c_{2B} \rightarrow c_{2B} - \frac{2}{Y_e}c_{BR}^{e_1} - \frac{4}{Y_e}c_{BR}'^{e_1} - \frac{2}{Y_e^2 g'^2}c_{RR}^{e_1} ,$$

$$c_6 \rightarrow c_6 + 2c_r + 4\lambda c_{K4} - 8(c_L^{(3)L_1} - \tilde{c}_{LL}^{(3)L_1}) ,$$

$$c_{2G} \rightarrow c_{2G} - c_{GR}^{d_1} - 2c_{GR}'^{d_1} - c_{GR}^{u_1} - 2c_{GR}'^{u_1} - \frac{1}{g_s^2}c_{RR}^{(8)u^1 d^1} ,$$

where

$$\tilde{c}_{LL}^{(3)L_1} = c_{LL}^{(3)L_1} + c_{LL}^{F_1} + g'^2 Y_L \left(c_{BL}^{L_1} + 2c_{BL}'^{L_1} - \frac{Y_L}{Y_e}(c_{BR}^{e_1} + 2c_{BR}'^{e_1} + \frac{1}{g'^2 Y_e}c_{RR}^{e_1}) \right) \tag{D.16}$$

and Eq. (D.10) has already been taken into account. This completes the removal of the operators in Eq. (D.3) in terms of the bosonic operators.

As we have just shown, upon eliminating the redundant operators the Wilson coefficients of the operators of Eq. (D.1) are shifted in such a way that the anomalous dimensions are redefined as

$$
\begin{aligned}
\gamma_{c_H} &\rightarrow \gamma_{c_H} - \gamma_{c_r} + 6(\gamma_{c_L^{(3)L_1}} - \tilde{\gamma}_{c_{LL}^{(3)L_1}}) \,, \\
\gamma_{c_T} &\rightarrow \gamma_{c_T} + \frac{1}{Y_e}(\gamma_{c_R^{e_1}} - \frac{1}{2Y_e}\gamma_{c_{RR}^{e_1}}) \,, \\
\gamma_{c_W} &\rightarrow \gamma_{c_W} - 2\gamma_{c_{WL}^{L_1}} - 4\gamma_{c_{WL}^{\prime L_1}} + \frac{4}{g^2}(\gamma_{c_L^{(3)L_1}} - 2\tilde{\gamma}_{c_{LL}^{(3)L_1}}) \,, \\
\gamma_{c_B} &\rightarrow \gamma_{c_B} - \frac{1}{Y_e}\gamma_{c_{BR}^{e_1}} - \frac{2}{Y_e}\gamma_{c_{BR}^{\prime e_1}} + \frac{2}{Y_e g^{\prime 2}}(\gamma_R^{e_1} - \frac{1}{Y_e}\gamma_{c_{RR}^{e_1}}) \,, \\
\gamma_{c_{2W}} &\rightarrow \gamma_{c_{2W}} - 4\gamma_{c_{WL}^{L_1}} - 8\gamma_{c_{WL}^{\prime L_1}} - \frac{8}{g^2}\tilde{\gamma}_{c_{LL}^{(3)L_1}} \,, \\
\gamma_{c_{2B}} &\rightarrow \gamma_{c_{2B}} - \frac{2}{Y_e}\gamma_{c_{BR}^{e_1}} - \frac{4}{Y_e}\gamma_{c_{BR}^{\prime e_1}} - \frac{2}{Y_e^2 g^{\prime 2}}\gamma_{c_{RR}^{e_1}} \,, \\
\gamma_{c_6} &\rightarrow \gamma_{c_6} + 2\gamma_{c_r} + 4\lambda\gamma_{c_{K4}} - 8(\gamma_{c_L^{(3)L_1}} - \tilde{\gamma}_{c_{LL}^{(3)L_1}}) \,, \\
\gamma_{c_{2G}} &\rightarrow \gamma_{c_{2G}} - \gamma_{c_{GR}^{d_1}} - \gamma_{c_{GR}^{\prime d_1}} - \gamma_{c_{GR}^{u_1}} - \gamma_{c_{GR}^{\prime u_1}} - \frac{1}{g_s^2}\gamma_{c_{RR}^{(8)u^1 d^1}} \,, \quad\quad \text{(D.17)}
\end{aligned}
$$

where

$$
\tilde{\gamma}_{c_{LL}^{(3)L_1}} = \gamma_{c_{LL}^{(3)L_1}} + \gamma_{c_{LL}^{F_1}} + g^{\prime 2}Y_L\left(\gamma_{c_{BL}^{L_1}} + 2\gamma_{c_{BL}^{\prime L_1}} - \frac{Y_L}{Y_e}(\gamma_{c_B R^{e_1}} + 2\gamma_{c_{BR}^{\prime e_1}} + \frac{1}{g^{\prime 2}Y_e}\gamma_{c_{RR}^{e_1}})\right) .
$$
(D.18)

The anomalous dimensions of the remaining bosonic operators, that are not of the form *(SM current)×(SM current)*, are not redefined. In this way we can go to our original basis taking into account that some operators are generated radiatively even if we set their Wilson coefficient to zero at the matching scale. In the main body of the paper, Tabs.9.2-9.3 and 9.7, we gave the physical anomalous dimensions obtained using the right hand side of Eq. (D.17). As announced in Sec. D.1, the ξ dependence cancels out in the physical combinations of γ_{c_i}'s, which can be easily checked using Eq. (D.17).

D.3   Field Reparametrization-Invariance Crosscheck

There is a useful consistency check that can be done to the results presented in Tabs. D.1 and D.3. Consider the set of 9 operators

$$
\mathcal{B} = \{\mathcal{O}_{K4}, \mathcal{O}_6, \mathcal{O}_H, \mathcal{O}_r, \mathcal{O}_T, \mathcal{O}_B, \mathcal{O}_W, \mathcal{O}_{2B}, \mathcal{O}_{2W}\} \,. \quad\quad \text{(D.19)}
$$

By means of field redefinitions, these operators are related among themselves and to other operators that contain fermions, see Eq. (D.12). Therefore, in a hypothetical theory with no fermions, but otherwise equivalent to the SM, the operator set of Eq. (D.19) would be over-complete, i.e. there would be operators which could be removed using field redefinitions. Let us take this scenario as a working assumption for the rest of this Appendix. More concretely, consider the subset of field redefinitions of Eq. (D.12), parametrized by

$$\{\alpha_1, \alpha_2, \alpha_B, \alpha_{2B}, \alpha_W, \alpha_{2W}\} \tag{D.20}$$

and the shifts they produce on the operators of Eq. (D.19) given in Eq. (D.13). Using this shift freedom we can choose to remove all the operators in $\mathcal{B}$ except $\mathcal{O}_6$, $\mathcal{O}_H$ and $\mathcal{O}_T$. However, notice that the over-completeness[2] of $\mathcal{B}$ can be exploited to our advantage; physical observables are independent of the coordinates choice as long as such a choice is compatible with the assumed symmetries. Hence, physical observables can not depend on the arbitrary parameters α_i of Eq. (D.20) that we used to parametrize the field redefinitions. The following combinations of Wilson coefficients are invariant under such shifts:

$$\begin{aligned}
C_H &\equiv c_H - c_r - \frac{3}{4}g^2(2c_W - c_{2W}) \,, \\
C_T &\equiv c_T - \frac{1}{4}g'^2(2c_B - c_{2B}) \,, \\
C_6 &\equiv c_6 + 2c_r + g^2(2c_W - c_{2W}) + 4\lambda c_{K4} \,.
\end{aligned} \tag{D.21}$$

Physical observables depend on shift invariant combinations of couplings, which we denote by a capital C_i. Also, a key property is that the anomalous dimension of a shift invariant combination of couplings is a function of shift invariant combinations of couplings only [5]

$$\gamma_{C_i} = f(C_j) \,. \tag{D.22}$$

This is precisely the cross-check that can be done to the results computed in Tabs. D.1 and D.3. And indeed it is easy to check that:

$$\begin{aligned}
\gamma_{C_H} &= \left(24\lambda - 4g^2 - 3g'^2\right)C_H + \frac{1}{2}\left(24\lambda + 9g'^2 - 17g^2\right)C_T, \\
\gamma_{C_T} &= \frac{1}{6}\left(72\lambda + 5g'^2 + 27g^2\right)C_T + \frac{5}{3}g'^2 C_H \,,
\end{aligned} \tag{D.23}$$

as it should, given the fact that $\mathcal{O}_6$ does not renormalize $\mathcal{O}_r$, $\mathcal{O}_H$, $\mathcal{O}_T$. As it is clear from the discussion above, to compute Eq. (D.23) one has to insert the Higgs and gauge bosons anomalous dimensions and the gauge beta functions without the contributions

---

[2]Again, we stress that the set of operators in Eq. (D.19) is over-complete only in the absence of the SM fermions.

of the fermions:

$$\gamma_H^{nf} = \gamma_H|_{y_f=0} \ , \quad \gamma_W^{nf} = -\frac{1}{g}\beta_g^{nf} = \frac{43}{6}g^2 \ , \quad \gamma_B^{nf} = -\frac{1}{g'}\beta_{g'}^{nf} = -\frac{g'^2}{6} \ , \qquad \text{(D.24)}$$

in the background field gauge and the superscript nf stands for *no fermions*, to distinguish them from their SM counterparts. Notice also that in Eq. (D.23) the ξ dependence exactly cancels, as it should, rendering the result independent of the gauge fixing term of Eq. (9.7).

E. Components of supersymmetric operators

In this Appendix we show the expansion in component fields of some of the super-operators discussed in Part III. We work in the Wess-Zumino gauge. In particular, for the $U(1)$ case, we show the supersymmetry-preserving super-operator

$$\int d^4\theta \left(\Phi^\dagger e^{V_\Phi}\Phi\right)\left(Q^\dagger e^{V_Q}Q\right) = -|\tilde{q}|^2|D_\mu\phi|^2 - |\phi|^2|D_\mu\tilde{q}|^2 - \frac{1}{2}\partial_\mu|\tilde{q}|^2\partial^\mu|\phi|^2$$

$$+\frac{i}{2}|\phi|^2(q^\dagger\bar{\sigma}^\mu\overleftrightarrow{D}_\mu q) + \frac{1}{2}\left[(\psi^\dagger\bar{\sigma}^\mu q)(i\tilde{q}^*\overleftrightarrow{D}_\mu\phi) + h.c.\right] + \frac{1}{2}\left[\phi\tilde{q}^*(i\psi^\dagger\bar{\sigma}^\mu\overleftrightarrow{D}_\mu q) + h.c.\right]$$

$$-\frac{1}{2}\left(i\phi^*\overleftrightarrow{D}_\mu\phi - \psi^\dagger\bar{\sigma}^\mu\psi\right)\left(i\tilde{q}^*\overleftrightarrow{D}_\mu\tilde{q} - q^\dagger\bar{\sigma}^\mu q\right) + \frac{i}{2}|\tilde{q}|^2(\psi^\dagger\bar{\sigma}^\mu\overleftrightarrow{D}_\mu\psi)$$

$$-\left[(\psi^\dagger q^\dagger)\phi F_q + (\psi^\dagger q^\dagger)\tilde{q}F_\phi - \phi F_\phi^*\tilde{q}^* F_q + h.c.\right] + |\phi|^2|F_q|^2 + |\tilde{q}|^2|F_\phi|^2$$

$$-\sqrt{2}g(Q_\phi + Q_q)\left[|\phi|^2\tilde{q}\lambda^\dagger q^\dagger - |\tilde{q}|^2\phi\lambda^\dagger\psi^\dagger + h.c.\right] + g(Q_\phi + Q_q)|\phi|^2|\tilde{q}|^2 D, \quad \text{(E.1)}$$

where boundary terms have been dropped out after integration by parts rearrangements. The fields are embedded in the super-multiplets as $\Phi \sim \{\phi, \psi, F_\phi\}$, $Q \sim \{\tilde{q}, q, F_q\}$ and $V \sim \{\lambda, A_\mu, D\}$. The D and $F_{q,\phi}$ auxiliary fields are irrelevant in the discussion of the renormalization of loop-operators by JJ-operators because they are necessarily involved in vertices with too many scalar and/or fermion fields.

The loop-super-operators for the $U(1)$ case are given by

$$\int d^4\theta\, \eta^\dagger(\Phi^\dagger e^{V_\Phi}\Phi)\mathcal{W}^\alpha\mathcal{W}_\alpha = -\frac{1}{2}\mathcal{O}_{FF_+} + |\phi^2|\left(D^2 + 2i\lambda\sigma^\mu\partial_\mu\lambda^\dagger\right)$$

$$-\frac{1}{\sqrt{2}}\phi^*\lambda\sigma^{\mu\nu}\psi F_{\mu\nu} - \sqrt{2}\phi^*\psi\lambda D + \lambda\lambda\phi^* F_\phi, \quad \text{(E.2)}$$

$$\int d^4\theta \, \eta^\dagger \Phi(Q\overset{\leftrightarrow}{\mathcal{D}}_\alpha U)\mathcal{W}^\alpha \;=\; -\mathcal{O}_D + \Big\{ -\sqrt{2}i\phi\tilde{q}(u\sigma^\mu\partial_\mu\lambda^\dagger) + 2\tilde{u}\phi F_q D + 2\sqrt{2}\phi F_u\lambda q$$

$$+\sqrt{2}\tilde{u}F_\phi\lambda q + \sqrt{2}F_u\tilde{q}\psi\lambda + D\tilde{q}\psi u$$

$$-\frac{1}{2}\tilde{q}\psi\sigma_{\mu\nu}uF^{\mu\nu} + \sqrt{2}(\psi q)(\lambda u) - (u \leftrightarrow q)\Big\}. \qquad (E.3)$$

For the non-Abelian case, there is also the loop-super-operator

$$\int d^4\theta \, \eta^\dagger \text{tr}[\mathcal{D}^\beta\mathcal{W}^\alpha\mathcal{W}_\beta\mathcal{W}_\alpha] = \frac{1}{4}\mathcal{O}_{3F+} + i \, \text{tr}\left[\frac{1}{2}F_{\mu\nu}\lambda\sigma^{\mu\nu}(\sigma^\gamma\partial_\gamma\lambda^\dagger) + \lambda\sigma^\mu\partial_\mu\lambda^\dagger D\right]. \quad (E.4)$$

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