PhD THESIS

Medium-term power planning in electricity markets with renewable generation sources

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ABSTRACT

The problem addressed in this thesis is the medium-term generation planning over a yearly horizon of a generation company participating in a liberalized electricity market with pool auction of generation and consumption and with bilateral contracts between generation companies and distributions companies or big consumers. It is assumed that the generation portfolio of several generation companies includes a significant proportion of dispatchable renewables (hydro generation with storage reservoirs and pumping schemes) and non-dispatchable renewables as wind power and solar photovoltaic generation. It is also assumed than more than one generation company are able to influence market-price levels through their bidding in the auction so that the market could be oligopolistic. The results obtained are of interest to price-maker generation companies, but also to price-taker generators, and to the market operator to check whether the participants in the market behave as a cartel or seeking an equilibrium.

The stochasticity of parameters in the medium-term planning is modeled in two ways. Regarding consumers load and generation unit outages, through the use of the probabilistic method of load matching: by representing the load through predicted load-duration curves of each period into which the yearly horizon is subdivided, by considering the capacity and an outage probability of each generation unit and by using the existing convolution techniques and the linear-inequality load-matching constraints.

Regarding renewable energy sources, stochastic programming is used. Hydro-generation scenarios of inflows are developed for each period. As for non-dispatchable renewables (wind power and solar photo-voltaic generation), a novel model of representing them through two pseudo-units: one base unit with small outage probability and a crest unit with large outage probability is proposed, and scenarios are developed for the relevant parameters of the pseudo-units. The solar photo-voltaic generation model requires splitting each period into three subperiods with the dark hours, with the medium-light hours and with the bright hours.

Quasi-Monte Carlo techniques have been employed to create a large scenario fan later reduced to a scenario tree with a reduced number of scenarios.

Market prices are taken into account through an endogenous linear market-price function of load duration whose intercept depends on total hydro generation level and on wind power and solar photovoltaic level in each node of the scenario tree. With such market price function, the endogenous cartel solution and the equilibrium solutions to the medium-term planning can be obtained. To avoid having to consider the total exponential

number of load-matching constraints, a load matching heuristic has been employed where small batches of new load matching constraints are generated after successive optimizations considering only the generated load matching constraints. For equilibrium solutions, the Nikaido-Isoda relaxation algorithm of successive solutions is employed using the successive optimizations of the load-matching heuristic.

In mixed-market systems with auction and bilateral contracts, a time-share hypothesis is formulated and the profits function for generation companies with the generation left after honoring their bilateral contracts is formulated. The profit function obtained is non-convex, and a direct global optimization solver was tried, but proved not to be practical for the size of problem to be solved. A non-linear interior-point constrained optimization solver, also employed for problems in pure pool markets, was tried with several special techniques to circumvent the troubles caused by the non-convexity of the objective function and satisfactory results were obtained.

A novel model of multi-period medium-term pumping was presented and employed. Results for several realistic test cases having different generation settings have been presented and analyzed.

CONTENTS

GLOSSARY OF SYMBOLS AND ACRONYMS

This orientation table indicates whether a symbol is an acronym (ACR), or a dimension (DIM), or a function (FUN), or an index (IND), or a parameter (PAR), or a set (SET), or a variable to be optimized (VAR) or a variable not to be optimized (NOP).

0	IND
A	PAR
$\mathcal{A}(\lambda)$	SET
B	SET
b^i, b^ν	PAR
BC	ACR
BCDC	ACR
brSPV	IND
$\mathrm{brSPVbs}$	IND
$\mathrm{brSPVcr}$	IND
C, C^i, C_λ	PAR
c_j, c^i_j	PAR
CBC	ACR
Cmp	ACR
CVaR	ACR
D_λ	PAR
d	PAR
d_ϑ	PAR
DC	ACR
DcxQ	ACR
DistCo	ACR
DR	ACR
$\widehat{e}^i$	PAR
eDcxQ	ACR
ee	IND
eff	PAR
$F(z)$	FUN
f_j, $\widetilde{f}_j$	PAR
g	IND
GDC	ACR

GenCo	ACR
H	SET
h	IND
H_k	SET
i	IND
$i(\nu)$	IND
iid	ACR
j	IND
K	SET
k	IND
L	PAR
$L_{k,\lambda}$	FUN
L_t	SET
$\mathcal{L}$	SET
l^i, $l^{i(\nu)}$	PAR
LDC	ACR
llSPV	IND
LMC	ACR
LMH	ACR
LRCC	ACR
m	DIM
MC	ACR
MIBEL	ACR
mLMH	ACR
MO	ACR
MTGP	ACR
$\mathcal{N}$	SET
n	DIM
n_c	DIM
n_D	DIM

n_i	DIM
n_u	DIM
N-DR	ACR
NIRA	ACR
non-LMC	ACR
p	IND
$\widetilde{P}$	PAR
P_{sp90}	PAR
$\overline{p}^i$	PAR
$\underline{p}_i$	PAR
$p(z)$	FUN
$p^\nu(t, x_h)$	FUN
Phyd	ACR
q_j, q^i_j	PAR
QMC	ACR
r, r_λ	PAR
$r^\nu_j(x^\nu_j, \widetilde{x}^\nu_j)$	FUN
RCC	ACR
rhs	ACR
RoP	ACR
$S(z)$	FUN
$S_\emptyset(z)$	FUN
$S_\omega(z)$	FUN
$s(\omega)$	FUN
SGC	ACR
SO	ACR
SPV	ACR
T	DIM
t	NOP
t^i, T^i, $T^{i(\nu)}$	PAR

u_h^ν	NOP
v_h^0	PAR
v_h^i	PAR
$\bar{v}_P$	PAR
ν_{RC}	NOP
VaR	ACR
VAR	ACR
w	IND
w_h^i	PAR
$\overline{w}_h^i$	PAR
w_{bs}	IND
w_{cr}	IND
WP	ACR
$\overline{X}^i$	PAR
$\mathcal{X}$	SET
x_j^i	VAR
$\tilde{x}_j^i$	VAR
x_{gj}^ν	VAR
$\mathbf{x}_k$	VAR
$\bar{x}$	PAR
$\underline{x}_k$	PAR
$\overline{x}_k$	PAR
y_h^i	VAR
z	NOP

$^-$	IND
α_k	NOP
γ	PAR
δ_P	PAR
ζ	PAR
η	PAR
κ	PAR
λ	PAR
$^\nu$	IND
ν	PAR
ν^-	PAR
ξ	PAR
π_ν	PAR
ρ	PAR
Σ	PAR
$\tau_{k,\lambda}$	NOP
$\Upsilon(\mathring{u}, x)$	FUN
$\phi_k(\mathbf{x})$	FUN
$\phi_k(x_k \vert \widehat{\mathbf{x}})$	FUN
$\Psi(\widehat{\mathbf{x}}, \mathbf{x})$	FUN
Ω	SET
Ω_k	SET
$\widetilde{\Omega}$	SET
ω	SET

Acronyms (ACR)

BC	Bilateral Contract
BCDC	Bilateral Contract Duration Curve
CBC	Component-by-Component algorithm
Cmp	Compensating unit in model of pumping scheme
CVaR	Conditional Value at Risk
DC	Difference of two convex functions
DcxQ	Difference of convex quadratic function
DistCo	Distribution Company
DR	Dispatchable renewable (hydro generation)

eDcxQ	Equilibrium difference of convex quadratic
GDC	Generation Duration Curve
GenCo	Generation Company
iid	Independent and identically distributed
LDC	Load Duration Curve
LMC	Load-Matching Constraint
LMH	Load-Matching Heuristic
LRCC	Linearized Reverse Convex Constraint
MC	Monte Carlo
MIBEL	Portuguese-Spanish electricity market
mLMH	Modified Load-Matching Heuristic
MO	Market Operator
MTGP	Medium-term generation planning
N-DR	Non-dispatchable renewable (SPV and WP)
NIRA	Nikaido-Isoda relaxation algorithm
non-LMC	Non-Load-Matching Constraints
Phyd	Hydro generation unit from pumped water
QMC	Quasi-Monte Carlo
RCC	Reverse Convex Constraint
rhs	Right-hand side of a constraint or equality expression
RoP	Rest of Participants (other than the SGC)
SGC	Specific Generation Company participating in a power pool
SO	System Operator
SPV	Solar Photo Voltaic (energy or power)
VaR	Value at Risk
VAR	Vector Auto Regressive
WP	Wind Power

Dimensions (DIM)

m	Number of stochastic variables
n	Number of scenarios ($n = card(\mathcal{L})$)
n_c	Number of non-load matching constraints
n_D	Total dimension of the scenario tree
n_i	Number of periods

n_u	Number of generation units
T	Number of time periods

Functions (FUN)

$F(z)$	Distribution function
$L_{k,\lambda}$	Profit losses function of participant k over scenario λ
$p(z)$	Density function of a load
$p^{\nu}(t, x_h)$	Market-price function
$r_j^{\nu}(x_j^{\nu}, \widetilde{x}_j^{\nu})$	Generation profit of unit j in node ν (47)
$S(z)$	Load survival function (1)
$S_{\emptyset}(z)$	Load survival function when no unit has been loaded
$S_{\omega}(z)$	Load survival function of still-unsupplied load after having loaded units in ω
$s(\omega)$	Expected unsupplied energy after loading units in set ω
$\Upsilon(\mathring{u}, x)$	A decision-dependent integrand
$\phi_k(\mathbf{x})$	Utility function of player k
$\phi_k(x_k \vert \widehat{\mathbf{x}})$	Utility function of participant k generating x_k while RoP generate $\widehat{\mathbf{x}}$
$\Psi(\widehat{\mathbf{x}}, \mathbf{x})$	Nikaido-Isoda function

Indices (IND)

0	Subscript that is related to the external energy
$brSPV$	Subscript that is related to the single SPV unit in bright-hours
$brSPVbs$	Subscript that is related to the bright-hours SPV base unit
$brSPVcr$	Subscript that is related to the bright-hours SPV crest unit
ee	Subscript that is related to the energy equivalent unit (4.2.2)
g	Subscript that indicates the load type($g \in L_t$) (4.2.4)
h	Subscript that indicates a hydro unit
i	Superscript that indicates the period
$i(\nu)$	Superscript that indicates the period associated to a node ν
j	Subscript that indicates the generation unit
k	Subscript that indicates the generation company
$llSPV$	Subscript that is related to the low-light hours SPV unit
P	Subscript that represents the pumped-storage unit

w	Subscript that is related to the single WP unit equivalent to crest plus base WP units
w_{bs}	Subscript that is related to the WP base unit
w_{cr}	Subscript that is related to the WP crest unit
ν	Superscript that indicates the node of the scenario tree
$-$	Superscript that represents the predecessor of a node

Parameters (PAR)

A	Matrix of coefficients of the non-load-matching constraints
b^i, b^ν	Price-intercept of the market-price function in period i, node ν (€/MWh)
C, C^i, C_λ	Left-hand side of non-load-matching constraints
c_j, c_j^i	Capacity of unit j (MW)
D_λ	Left-hand side of non-load-matching constraints
d	Correlation of the market-price with the hydro generation per time unit
d_ϑ	Vector of 0 or 1 coefficients of the left-hand side of the LMC built with the set of units ϑ
$\widehat{e}^i$	Total energy in the LDC of period i
eff	Efficiency coefficient of pumping generation
f_j, $\widetilde{f}_j$	Linear Generation Cost of unit j (€/MWh)
L	Lower triangular matrix of Cholesky's decomposition
l^i, $l^{i(\nu)}$	Slope of the market-price function in period i
$\widetilde{P}$	Expected profit
$\overline{p}^i$	Peak Load Power in period i
$\underline{p}_i$	Base Load Power in period i
P_{sp90}	Profit spread
q_j, q_j^i	Outage probability of unit j
r, r_λ	Right-hand side (rhs) of non-load-matching constraints
t^i, T^i, $T^{i(\nu)}$	Period duration (h)
ν_h^0	Energy stored in the reservoir system h before the start of the first period
ν_h^i	Expected energy stored in the reservoir system h at the end of period i
w_h^i	Water inflows in hydro basin h over period i
$\overline{w}_h^i$	Expected water inflows in hydro basin h over period i
$\overline{\nu}_P$	Energy capacity of the pumping-station upper reservoir
$\overline{X}^i$	Historical mean of the stochastic process at period i

$\bar{x}$	Initial point or the solution to a former problem
$\underline{x}_k$	Lower bound of expected generation x_k
$\bar{x}_k$	Upper bound of expected generation x_k
γ	Predetermined fraction of the energy of the natural inflows over period $i(\nu)$
δ_P	Efficiency coefficient of the pumping generation
η	Vector of QMC points
κ	Diagonal matrix of mean reverting coefficients
λ	Node that represents the leaf of the scenario tree
ν	Node of a scenario tree
ν^-	Predecessor of node ν
ξ	Normal vector
π_ν	Probability of node ν
ρ	Parameter that changes the non-convexity of the objective function (see §6.4.1)
Σ	Variance matrix
ζ	Realization of an independent N(0,1) variable

Sets (SET)

$\mathcal{A}(\lambda)$	Path from the root to the leaf λ
B	Set of participants in the market ($B = \{SGC, RoP\}$)
H	Set of hydro units ($H \subset \Omega$)
H_k	Set of hydro generations associated to company k
K	Set of generation companies
$\mathcal{L}$	Set of leafs
L_t	Set of load type
$\mathcal{N}$	Set of nodes
$\mathcal{X}$	Set of decision vectors
Ω	Set of generation units
Ω_k	Set of generation units of GenCo k
ω	Subset of generation units ($\omega \subset \Omega$)
$\widetilde{\Omega}$	Set of generation units that match the bilateral LDC

Variables to be optimized (VAR)

x_j^i	Expected energy generated by unit j over the period i
$\tilde{x}_j^i$	Expected energy devoted to match BCs by unit j over the period i
x_{gj}^{ν}	Expected energy generated by unit j over the period corresponding to node ν of a stochastic scenario tree for matching load type g (MWh)
$\mathbf{x}_k$	Decision vector with the expected generated energies of units of GenCo k
y_h^i	Expected energy generated by regulation hydro system h over period i (MWh)

Variables not to be optimized (NOP)

t	Load duration of LDC (h)
u_h^{ν}	Representation of the generation of an equivalent regulation reservoir (34)
ν_{RC}	Extra variable used in RCC formulation (51)
z	LDC power (MW)
α_k	Value-at-Risk of market participant k
$\tau_{k,\lambda}$	Non-negative variables used in CVaR constraints (see (54))

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1 INTRODUCTION

1.1 DEFINITION AND MOTIVATION

Given a temporary horizon including or coinciding with a yearly period, the medium-term generation planning for a generation company (GenCo) is the study of how to organize the generation of each of the GenCos' units over the horizon so that the profits (revenue minus costs) are maximized. This planning, formerly applied to a company with generation and distribution to its own consumers, is addressed in this thesis to a GenCo participating in a liberalized electricity market, which is a problem more complicated than that applied to the case of a company with generation and its own consumers.

The problem addressed in this thesis is the medium-term generation planning of a GenCo participating in a market with pool auction of generation and consumption and with bilateral contracts between GenCos and Distributions Companies (DistCos) or big consumers. It is assumed that the generation portfolio of several GenCos includes a significant proportion of dispatchable renewables (hydro generation with storage reservoirs and pumping schemes) and non-dispatchable renewables as wind power and solar photovoltaic generation.

The research group GNOM [65], within which this thesis has been developed, has been working on the optimization of electricity generation planning since well before the change in Spain from a regulated electrical system to a liberalized electrical market, and has continued its research on this subject in the frame of liberalized markets. One of the main paths of research in this area has been medium-term planning [67, 81], that this thesis is pursuing with new challenges. One main challenge has been singling out the model of non-dispatchable energy sources —wind power (WP) and solar photovoltaic (SPV) power— in order to accurately appraising the effects on market price and on the use of other technologies due to the penetration of WP and SPV in the generation portfolios. Another challenge is the medium-term planning solution in mixed markets, where participation of GenCos in the market auction is combined with bilateral contracts (BCs) between GenCos and DistCos, which leads to a non-convex problem. Using Stochastic Programming [9] is necessary for taking into account the medium-term stochasticity of many parameters. There are many sub models and computational procedures to be employed in the computational implementation, which are described in this thesis, and, in what regards the size of problem solved and the number of scenarios considered, this thesis presents the computational requirements to solve in reasonable time the medium-term planning using the models and procedures proposed.

Price-makers solve the medium-term generation planning (MTGP) in order to obtain maximum profits. Moreover, they get to know the expected profits and its dispersion; they can also make plans on fuel procurements. Price-takers solve the MTGP to obtain a prediction of market prices and of profits. By solving the MTGP, the Market Operator (MO) can know whether participants in the market behave as a cartel or as equilibrium. In long-term, one can use the MTGP in order to determine which type of generation is needed to build (and its capacity) and which old unit has to be removed. It is important to have a reliable simulation of what would be the impact on current unit generation and which economic effects would be produced. In mixed electricity markets, GenCos have to make an important decision: which of its units will be devoted to honor the BC agreements, and for how long in each period will be generating for matching the BC load, leaving the rest of the time to participate in the market (understanding that while meeting the BCs, units cannot participate in the market). The MTGP, which can be solved as often as necessary, may have as its first period the next week. The medium-term results for this first period provide target values for the short-term total generations over the next week, since short-term planning should not disregard medium-term results.

Due to previous research that has been done in GNOM, there are some methodological principles that are clear and have been taken into account in the design of new methodologies employed in this thesis: the use of the *probabilistic matching* (considering the load duration curve (LDC) for the consumption and the capacity and the outage probability for the generation units), to consider the system load and the use of a linear market price function of the load duration instead of using market price forecasts, the use of the Bloom & Gallant formulation of the optimization of cost satisfying the load-matching and other constraints and the use of stochastic programming techniques to represent stochastic parameters other than load variability and unit outages.

Medium-Term Generation Planning is a very important issue to the operation of electricity generation companies. Because of its many uses (such as that of budgeting for and the planning of fuel acquisitions) several methodologies have been studied and modeled.

1.2 INITIAL OBJECTIVES

The objectives formulated at the beginning of the thesis were:

- Building a MTGP model for a pure pool market with a single-price market function, based on the fact that all participants are considered in the market, using the probabilistic load matching (use of the LDC and the capacity and outage probability of all units). The yearly horizon is subdivided into few periods of variable length (the further away the longer). Use of stochastic programming with scenario tree of renewable energy parameters is needed, and single-period and multi-period medium-term non-load-matching constraints have to be considered.

- Use of the Bloom & Gallant formulation plus the load-matching heuristic (LMH) [68] or a better alternative to account for load-matching constraints.
- To determine updated data of MIBEL.
- To determine the use of new forms of representing WP and SPV within the probabilistic matching.
- To investigate a new model of medium-term pumping and how to add it to the model used.
- Verify in the new model developed, the Nikaido-Isoda Relaxation Algorithm (NIRA) for equilibrium solutions using endogenous function with respect to hydro and successive optimizations within LMH (or a better alternative) while adding load-matching constraints (LMCs).
- Expand the current model in order to introduce the use of Bilateral Contracts (BC), which are considered through time-share hypothesis and matching three LDCs (system LDC, SGC BCDC and RoP BCDC) in each period with non-convex objective function.
- To use initially of Ipopt solver (IP with filter step-length determination) or alternative procedures and, if necessary, to develop an interior point method with optimal analytical determination of step size (because the objective function is quadratic, it will be non-convex if BCs are employed).
- Consideration of the risk of profit loss according to the profit distribution in scenarios of the scenario tree created using stochastic programming methods.

1.3 SUMMARY OF CONTRIBUTIONS

This thesis contributes to medium-term electricity generation planning models in several aspects. These are the following:

- A model for the representation of non-dispatchable renewables based on a two-unit system with different parameters of capacity and outage probability for WP and for SPV. A graphical justification of such model is presented.
- The development of scenario trees using QMC techniques for WP and SPV, and for the dispatchable hydro inflows taking into account the different parameters of units

in each scenario node, as justified in this thesis, and the influence on market-price level in each scenario node due to WP and SPV; the decomposition of the LDC of each period into three LDC for the dark, the low-light, and the bright hours.

- The extension of the medium-term single-period pumping model presented in [11] to the multi period case using extra variables and extra constraints.
- The stochastic medium-term mixed-market model with BCs, renewables and pumping and the characterization of its non-convexity. A procedure based on the use of a conventional nonlinear programming solver has been presented, where a solution strategy was employed to circumvent the likely trouble caused by non-convexity of the sequence of optimizations of the mLMH.
- The use of the mLMH in all the models described above to account for the matching the LDCs of all periods considered without having to use an exponential number of LMCs.
- The experience of the unsuccessful attempt to use a commercial global optimization software for solving the BC non-convex problem; the use of a conventional solver for this non-convex problem and the monitoring of the results of the successive optimizations modifying if necessary the convexity of the problem in order to obtain a solution with proper match of the LDCs.

1.4 THESIS CONTENTS

This thesis has seven chapters and is divided into two parts, the first part contains some conceptual backgrounds and the literature is reviewed, while the second one has the development of the main contributions of this thesis. The chapters have the following structure: the first chapter outlines the definition and the motivation behind this study, its objectives and contributions and a summary of the contents. Chapter 2 defines some basic concepts that are involved in medium-term generation planning: the load duration curve that models the demand, the generation duration curve, the linear minimum-cost function, the load-matching constraints and the market-price function. It also describes the different market types that exists, the Spanish electricity system and different types of agent and behaviors in the electricity markets. It is also defined the equilibrium behavior and the algorithm to obtain an equilibrium solution, the stochastic programming and the data that has been used. In Chapter 3 there is a review of the existing literature about medium-term generation planning. There is also the description of the heuristics LMH and mLMH, the representation of the hydro generation and the pumping model. In Chapter 4, there is the characterization of the renewable generation with stochastic formulations of hydro generation, WP generation and SPV generation. It is also described the scenario generation method and the scenario reduction procedure employed. Chapter 5 describes the medium-term planning model with dispatchable and non-dispatchable renewable sources for both behaviors, the endogenous and the equilibrium solution. In Chapter 6 is presented the medium-term model with auction and bilateral contracts. Finally, Chapter 7 contains the main results obtained, suggestions for further research and publications and presentations generated by this thesis. There are also two Appendices where there are the real data used in a test case and the results obtained.

2 CONCEPTUAL BACKGROUNDS

This chapter presents basic modeling concepts already developed and tested before the start of this thesis. This set of concepts includes the load duration curve (LDC), which models the medium-term demand, the loading order with which the expected generations can be calculated, the generation-duration curve that contains the expected generation that matches the LDC, the load-matching constraints (LMC) and the linear minimum-cost medium-term planning. It goes on with a description of the electricity markets and presents the medium-term market-price function from which the expected revenue is computed. It also presents the endogenous cartel model, the equilibrium behavior and an algorithm to obtain the equilibrium solution.

Furthermore, it is necessary to present some basic concepts of stochastic programming, which are used in order to create a scenario tree, the description of the stochastic parameters in medium-term planning and some tools that are useful to compute the profit distribution.

The last sections introduce the data and the solver that have been employed in this thesis.

2.1 THE LOAD-SURVIVAL FUNCTION AND THE LOAD DURATION CURVE

For a given time period, the *load-survival function* ($S(z)$) represents the probability of there being a load greater than z (see Figure 1, below). Given the density function $p(z)$ of a load (see Figure 1, above), the load-survival function is calculated as:

$$S(z) = 1 - F(z) = 1 - \int_0^z p(y)\, dy \,, \tag{1}$$

where $F(z)$ is the distribution function of $p(z)$ and it is defined for positive values ($z \geqslant 0$).

Some properties of this function are:

- $S(0) = 1$ and $S(\infty) = 0$
- $S(z)$ is a monotone decreasing function

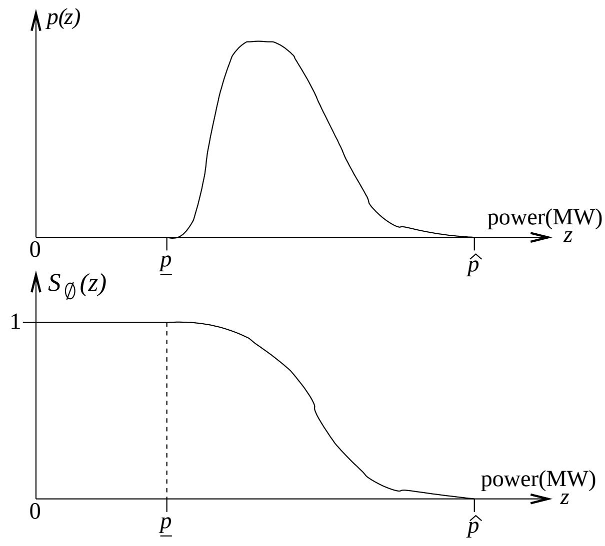


Figure 1: Probability density function of load $p(z)$ (above), and load survival function $S(z)$ (below)

For a given time period, the LDC is the curve that gives, in the abscissa axis, the duration of the load bigger or equal to a given power in the ordinate (see Figure 2, right). The LDC corresponds to the chronological load of the given period, a certain week in Figure 2 (left), ordered in descending load power in the LDC. For a future period, the LDC must be predicted. One of the most important constraints in a generation planning model is the matching of the LDC at each period. The LDC retains all the variability for each load and it is characterized by

- $\widehat{e}^i$: the total energy over period i
- t^i : the duration of period i
- $\underline{p}^i$: the base load
- $\overline{p}^i$: the peak load
- the shape

The load-survival function and the LDC are equivalent since by exchanging the LDC axis and scaling the total duration T^i to probability 1, the load-survival function results.

2.1.1 Loading order

When the generation units have a linear generation cost, and unit generations have no constraint to satisfy other than matching the LDC, there is no point to start loading a given unit until all units with a lower cost have been fully loaded. The loading order from less

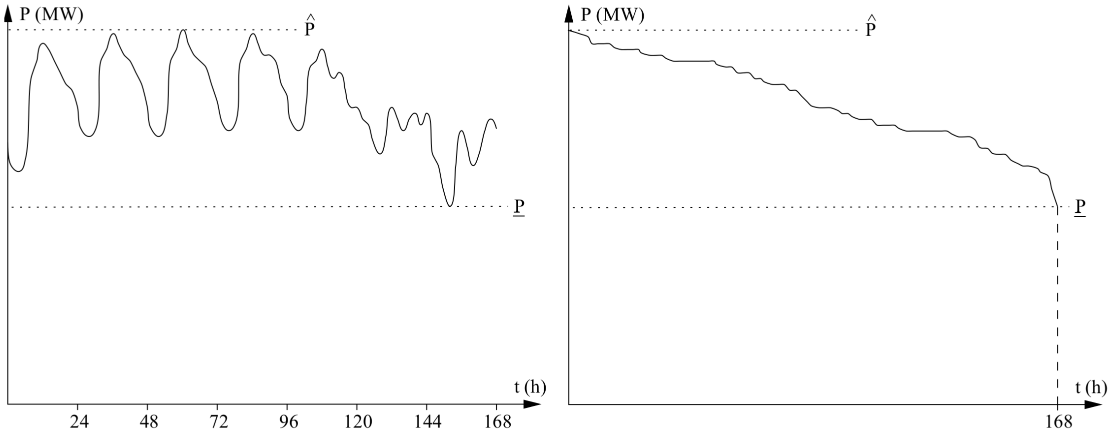


Figure 2: Load Duration Curve over a week

to more expensive units is called *merit order*. See Nabona and Díez [57] for a proof that the merit order is optimal under the conditions exposed.

Should unit generations have to satisfy constraints other than LDC matching, or should unit generations have to maximize some utility function other than a linear function (as the summation of linear costs), there may be an optimal loading pattern called *constrained loading order* in which there may be subsets of units loaded at the same time: where a unit is loaded up to less than its full capacity when one or several units start its or their loading. This lump of units interleaving their loading are said to *split* their loading.

2.1.2 The modification through convolution of the load survival function when loading generation units

The exact matching of the LDC by generation units having each an outage probability was proposed in 1967 by Bałériaux et al. [3].

Let

- c_j, the power capacity of unit j (in MW)
- q_j, outage probability, and $1 - q_j$ in service probability of loaded unit j
- f_j, linear cost of unit j (in €/MWh)
- Ω, set of all available units having $n_u = |\Omega|$ units
- ω, subset $\omega \in \Omega$ of generation units such that $j \notin \omega$
- $S_\omega(z)$, the load survival function of still-unsupplied load after having loaded units in ω.

- $S_\emptyset(z)$, load survival function when no unit has been loaded

The modification of the load survival function through convolution proposed by Balériaux et al. [3] expresses the change to the load-survival function caused by loading unit j and it is computed as:

$$S_{\omega \cup j}(z) = q_j S_\omega(z) + (1 - q_j) S_\omega(z + c_j)\,. \tag{2}$$

The unit energies obtained using the convolution procedures are true expected values.

Let ω be the subset of units loaded before unit j. The expected generation of unit j ordered after units in ω is

$$x_{j|\omega} = T(1 - q_j) \int_0^{c_j} S_\omega(z)\, dz\,, \tag{3}$$

where $S_\omega(z)$ is the load-survival function of still-unsupplied load after loading units in ω.

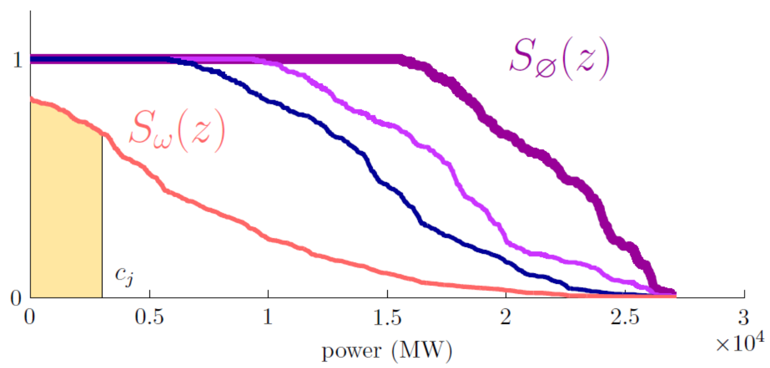


Figure 3: Real load-survival function after loading three units and contribution of the fourth unit

In Figure 3, there is an example of the modification through convolution of a load survival function after having loaded three units and the contribution of another unit. The yellow area in Figure 3 corresponds to the integral in (3), and must be multiplied by $T(1 - q_j)$ to produce x_j.

Let x_j be the expected energy generated by unit j over a period of length t. An alternative deduction of 3 is

$$x_j = t(1 - q_j) \int_0^{c_j} zp(z)dz = t(1 - q_j) \int_0^{c_j} (1 - F(z))dz = t(1 - q_j) \int_0^{c_j} S(z)dz\,, \tag{4}$$

where $p(z)$ is the density function of the demand (see Figure 1) and $F(z)$ is the distribution function of $p(z)$. Furthermore, the last equality holds because z is continuous and non-negative (for $z < 0$, $p(z) = 0$).

The load-survival function corresponding to the LDC (without having loaded any unit) is denoted by $S_{\emptyset}(z)$. If the convolution (2) is applied successively, the unsupplied load in terms of $S_{\emptyset}(z)$ is:

$$S_{\omega}(z) = S_{\emptyset}(z) \prod_{m \in \omega} q_m + \sum_{\phi \subseteq \omega} \Big(S_{\emptyset}(z + \sum_{j \in \phi} c_j) \prod_{j \in \phi} (1 - q_j) \prod_{j \in \omega \setminus \phi} q_j \Big) \,, \tag{5}$$

where ϕ represents any subset of ω. It is possible to say from (5) that the survival function $S_{\omega}(z)$ of the unsupplied load is the same regardless of the order in which the units in ω have been loaded. However, the expected generation of each unit depends on its position in the loading order.

It can be seen from (2) that $S_{\omega \cup j}(z) \leqslant S_{\omega}(z) \,, \forall z$. And then it is easy to verify from (3) that loading a unit j after loading unit k will yield a lower expected generation x_j than if unit j was loaded just before unit k.

The upper and the lower bound, $\overline{x}_k$ and $\underline{x}_k$, to the expected generation of unit k correspond to its loading in the first and in the last position of the loading order:

$$\overline{x}_k = (1 - q_k) t \int_0^{c_k} S_{\emptyset}(z) \, dz \,, \tag{6a}$$

$$\underline{x}_k = (1 - q_k) t \int_0^{c_k} S_{\Omega \setminus k}(z) \, dz \,. \tag{6b}$$

The expected unsupplied energy, $s(\omega)$, after loading units in ω can be computed as

$$s(\omega) = t \int_0^{\overline{p}} S_{\omega}(z) \, dz \,. \tag{7}$$

2.1.3 External energy

The probability that there may be time lapses where a number of units are unavailable due to random outages, such that the load in these time lapses cannot be met, is not null. In this case, external units connected to the load to be satisfied would supply the part of the load that is not satisfied. The *external energy* is represented by the index 0. The *load balance* equation relates the unit generations with the external x_0:

$$\sum_{j=0}^{n_u} x_j = \widehat{e} \,. \tag{8}$$

It is assumed that external energy is always available and that its costs are higher than the most expensive unit of the load to be satisfied. So using (8) the *external energy* can be calculated as

$$x_0 = \widehat{e} - \sum_{j=1}^{n_u} x_j \, . \tag{9}$$

2.2 THE GENERATION DURATION CURVE

The generation duration curve (GDC) is the ensemble of the expected generations of the units over the time period to which the LDC refers. The energy generated by each unit j is the slice of area under the generation-duration curve, which corresponds to the capacity of the thermal unit c_j.

Figure 4 shows an optimal loading order in a real test case solved in [58] (unit U13 is loaded first, unit F03 is the second, etc.) and it also shows an example of *splitting*: here unit D11 is split by units G11 and G02. Although the LDC and the GDC do not have the same shape due to the outage probabilities, the area under the LDC and the area under the GDC must coincide.
The GDC goes further up than the peak load $\overline{p}$, and what is beyond $\sum_{j\in\Omega} c_j$ corresponds to the external energy. The total vertical length of the GDC is $\overline{p} + \sum_{j\in\Omega} c_j$. The irregular shape of the right side of the GDC comes from the fact that the generation duration may not be uniform for all generated power.

2.2.1 The Probabilistic Matching

In Figure 5, there is an example of the *probabilistic matching*. This method uses the convolutions (2) successively, and it is a way to combine exactly the load survival function and the outage probability of the units. In this example, there is the load survival function $S_\emptyset(z)$ and there are three units with capacities C_1, C_2 and C_3 and with outage probabilities 10%, 20% and 5%, respectively. At first step, unit 1 is loaded and the convolution is made obtaining the load survival function after loading the first unit, $S_1(z)$. The same has been done for the other two units, thus obtaining the corresponding load survival functions, $S_{1,2}$ and $S_{1,2,3}$. At the bottom of the figure it can be seen the GDC and the load survival function $S_{1,2,3}$, which it corresponds to the external energy.

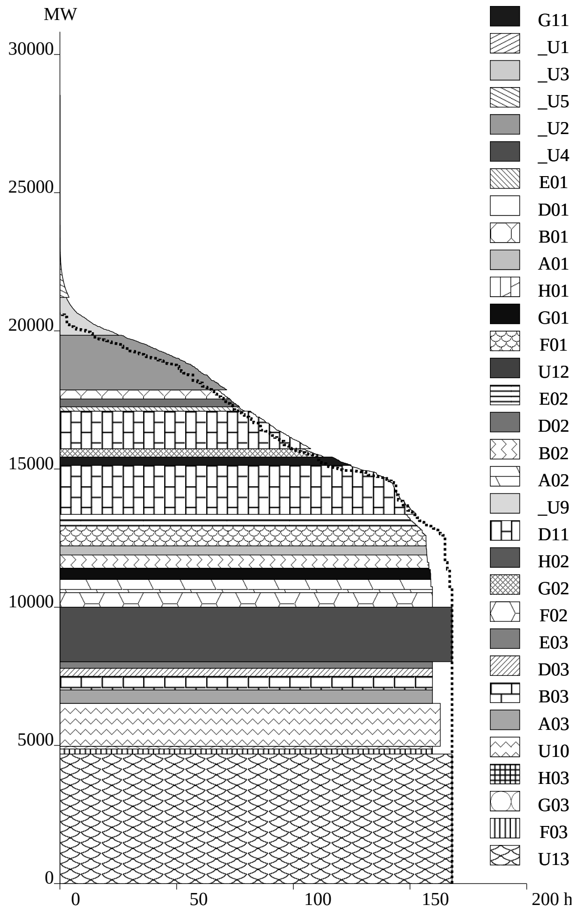


Figure 4: Load duration curve (dotted line) and generation duration curve for a week in a real case with 32 units

2.3 THE LINEAR MINIMUM-COST MEDIUM-TERM PLANNING

Bloom and Gallant [11] presented the matching of a given LDC for a single period using the expected energies x_j generated by the units and the expected unsupplied energy $s(\omega)$ defined in (7) while satisfying other constraints and minimizing a linear cost function. They established that the *load-matching constraints (LMCs)* given by

$$\sum_{j\in\omega} x_j \leqslant \widehat{e} - s(\omega) \qquad \forall\, \omega \subset \Omega\,, \tag{10}$$

must be satisfied. There are an exponential number of LMCs because there are $2^{n_u} - 1$ subsets ω of the set Ω of available units, with $n_u = |\Omega|$.

The overall Bloom and Gallant planning model that minimizes production cost is the linear optimization problem:

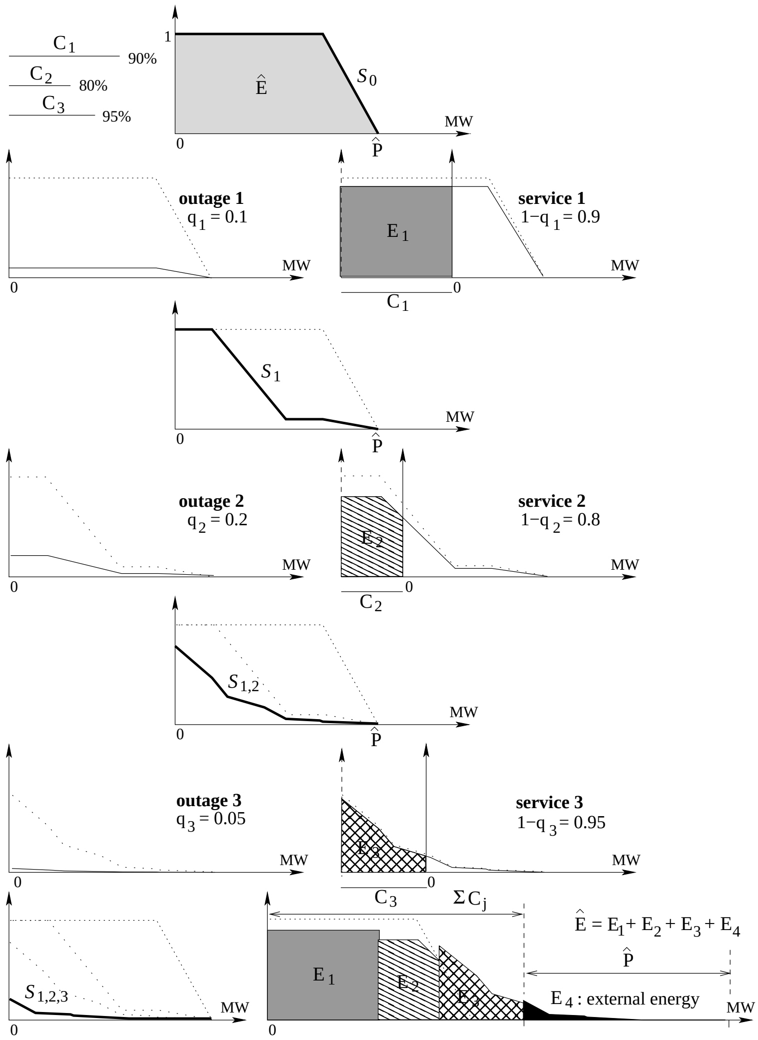


Figure 5: Example of *Probabilistic Matching* for a three unit case

$$\min_{x_j} \sum_{j=0}^{n_u} \widetilde{f}_j x_j \tag{11a}$$

$$\text{s.t.:} \sum_{j \in \omega} x_j \leqslant \widehat{e} - s(\omega) \qquad \forall\, \omega \subset \Omega \tag{11b}$$

$$Ax \geqslant r \tag{11c}$$

$$\sum_{j=0}^{n_u} x_j = \widehat{e} \tag{11d}$$

$$x_j \geqslant 0 \qquad j = 0, 1, ..., n_u \tag{11e}$$

where (11b) are the exponential number of LMCs, A and r are the coefficients matrix and the right-hand side (rhs) of the *non-load-matching constraints (non-LMCs)* (11c). Expression

(11d) is the load-balance equation from which external energy can be calculated.

The Bloom and Gallant model for cost minimization was used in Leppitsch and Hobbs [45] for a study on NO_x regulation effects, and was extended to multi-period problems in Pérez-Ruiz and Conejo [71] and Nabona et al. [58] using different techniques.

Examples of non-LMCs are:

- Maximum hydro generation: water availability is limited and the use of stored water is sometimes restricted by demands for irrigation.
- Bonus-scheme coal: some generation units that burn national coal benefit up to a certain limit, of a reduction on the coal cost according to a government act.
- Minimum generation time: generation units in the Spanish system are paid for their availability.
- Specific Generation Company (SGC) market-share: in medium-term planning it is reasonable to consider the SGC market-share.
- Take-or-pay constraints: the model should include constraints for any agreement settled by the company such as fuel supply contracts.
- Other constraints, such as CO_2 emission limits.

2.3.1 Nested LMCs

When no non-LMC of $Ax \geqslant r$ is active at the optimizer of the linear problem (11), the solution to the problem corresponds to loading units in the order of ascending costs, which is called the *merit order*. Then the active LMCs are the following triangular system, where there are as many active LMCs as units there are:

$$\begin{aligned} x_1 = \; & \widehat{e} - s(1) \\ x_1 + x_2 = \; & \widehat{e} - s(1,2) \\ x_1 + x_2 + x_3 = \; & \widehat{e} - s(1,2,3) \\ \dots \; & \\ x_1 + x_2 + x_3 + \ldots + x_{n_u} = \; & \widehat{e} - s(1,2,\ldots,n_u)\,, \end{aligned} \tag{12}$$

where it is assumed that the unit indices j and k and the unit costs f_j and f_k are such that, for $j < k$ we have $f_j \leqslant f_k$.

It is to be noted that the energy x_1 of the first unit loaded appears in all active LMCs (12); the energy of the 2nd unit to be loaded x_2 appears in the active LMCs from the second

to the last; and the energy of the last unit to be loaded x_{n_u} only appears in the last active LMC of (12).
The coefficients in the left-hand side of the constraints is a vector formed by ones and zeros, depending on which units there are in the subset that the LMC refers to. The coefficient will be 1 if the unit is in the constraint and it will be 0 otherwise. Regarding these zeros and ones of the active constraints, it is possible to see that the ones must be nested at the solution point. The merit order loading corresponds to the pure staircase shape of their one coefficients.

For non-linear objective functions or whenever there are active non-LMCs, the active LMCs may not correspond to the merit order and may be less than n_u in number. Nevertheless, the active LMCs at the optimizer will always correspond to a loading order and the active LMCs will be nested.

Let d_ϑ represent the row vector of coefficients of the left-hand side of the LMC built with the set of units ϑ. Then d_ϑ is *nested* into d_ζ (being ϑ and ζ any subset of units) if $\vartheta \subset \zeta$. For example, if $\vartheta = \{2, 4\}$ and $\zeta = \{1, 2, 4, 5, 6\}$, then d_ϑ is nested into d_ζ because $\vartheta \subset \zeta$.

In general, a set of constraints is nested if it is possible to order the constraints in the set in such a way that every constraint is nested in the next. For example,

	u_1	u_2	u_3	u_4	u_5	u_6	
d_η	.	.	.	1	.	.	$\eta = 4$
d_ϑ	.	1	.	1	.	.	$\vartheta = 2, 4$
d_ζ	1	1	.	1	1	1	$\zeta = 1, 2, 4, 5, 6$
d_Ω	1	1	1	1	1	1	$\Omega = 1, 2, 3, 4, 5, 6$

where d_η is nested into d_ϑ, d_ζ and d_Ω; d_ϑ is nested into d_ζ and d_Ω and, finally, d_ζ is only nested into d_Ω.

If units are reordered in loading order, one can observe

	u_4	u_2	u_1	u_5	u_6	u_3
d_η	1	.	.	.	.	.
d_ϑ	1	1	.	.	.	.
d_ζ	1	1	1	1	1	.
d_Ω	1	1	1	1	1	1

It can be seen that this example has a *flight-of-stairs* structure with a landing and that the load order of these units must be 4, 2, (1,5,6) and 3.

(1,5,6) is a group of units that are loaded at the same time. This means that the loading of one of these units is initiated before the loading of any of these units is finished. These three units form a landing in the flight-of-stairs structure of the ones in the active LMCs, which is called a *splitting* in loading order (see §2.1.1) and the relative order in which they have to start loading depends on the values of x_1, x_5 and x_6 at the solution.

2.4 TIME HIERARCHY IN MEDIUM-TERM PLANNING

For planning and optimization purposes in power systems there are several hierarchic time scales, each having a different function. In long-term power planning the horizon is between 2 and 10 years. The purpose is to decide which capacity will have the new generation units and when to start operation in order to match future loads. In the short-term power planning the scope is from 1 day to 1 week. The aim of this planning is decide when to start-up and shutdown generation units and the amount of power to generate at each hour by each unit while meeting several constraints.

2.4.1 Horizon, periods and subperiods employed in the test cases solved in this thesis

The horizon in medium-term power planning is normally one year, but in some cases, it could be 2 years. Usually the horizon that is chosen is annual because within a year there is a weather cycle, there is also a demand cycle and a water inflows cycle. Normally the medium-term horizon is split into several shorter periods with different length, which increases as the time moves away from the initial period (which usually corresponds to the current week). This division is made because some constraints may apply to periods shorter than the whole horizon and because some of the parameters change over time.

The uneven six periods employed in this thesis span a full year, and they are shorter close to the start and three-month long the last three. The load data employed in the test cases of this thesis correspond to the consumptions in the MIBEL during a year from October 2010 to October 2011. The lengths of the successive periods are of 7, 24, 60, 92, 92 and 90 days; the initial date being October 4.

When solving weekly the yearly medium-term planning from next week, the first period provides target values for the generations that can be used for coordinating the medium and the weekly short-term planning. The first plus the second, which span one month, and the third: two-month long, will provide the expected generations for the first and for the first three months (and their corresponding expected fuel consumptions, profits, etc.). This is also obtained for the six-, nine-, and twelve-month horizons.

In §4.2.3 there is in detail the consideration of solar photovoltaic generation and its characterization. As it will be explained, when this generation is introduced in the optimization problem, periods (and also the data) should be split into three subperiods: dark subperiod, low-light subperiod and bright subperiod.

In this thesis superscript ^{i} will denote the period and n_i will indicate the number of periods that the problem considers.

2.4.2 Bloom & Gallant formulation extended to n_i periods

The Bloom & Gallant formulation was extended into a multi-period version in [71] and in [58] where

$$\min_{x_j} \sum_{i=1}^{n_i} \sum_{j=0}^{n_u} \widetilde{f}_j x_j^i \tag{13a}$$

$$\text{s.t.:} \sum_{j \in \omega} x_j^i \leqslant \widehat{e}^i - s^i(\omega) \qquad \forall\, \omega \subset \Omega^i, \qquad i = 1, \ldots, n_i \tag{13b}$$

$$A^i x^i \geqslant r^i \qquad i = 1, \ldots, n_i \tag{13c}$$

$$\sum_{i=1}^{n_i} A^{0i} x^i \geqslant r^0 \tag{13d}$$

$$\sum_{j=0}^{n_u} x_j^i = \widehat{e}^i \qquad i = 1, \ldots, n_i \tag{13e}$$

$$x_j^i \geqslant 0 \qquad j = 0, 1, \ldots, n_u \qquad i = 1, \ldots, n_i\,. \tag{13f}$$

In problems with separable (by periods) objective function, as (13) is, only constraints (13d) link all period subproblems together.

2.5 LIBERALIZED ELECTRICITY MARKETS OF DIFFERENT TYPES

In *regulated* electricity systems, generation and distribution are integrated in a single supplier company, consumers have no choice of the supplier so the integrated electricity company has a load of its own to satisfy. The electricity price is fixed by a regulator (Government or Energy Institution).

In *liberalized* electricity markets, GenCos and DistCos are separate and independent companies (with no collusion among them). The electricity is traded between GenCos and

DistCos either through an auction or directly between one GenCo and one DistCo through a Bilateral Contract (BC), or in a mixed type market (with auction and BCs). Aside of the BCs (if there are any) GenCos have no load of their own. The auction in a market is controlled by a Market Operator (MO) who selects the lowest price among units of bidding GenCos to match the highest priced consumption bids of DistCos until the bid price of GenCos' unmatched units and the bid price of DistCos unmatched loads meet.

The transmission network in a market is owned by a third party that, through a System Operator (SO), must control that the transmission network is able to withstand the transactions agreed in the auction or through BCs. There could also be a third party transmission network in a regulated market.

2.5.1 Electricity market types

There are three main types of liberalized electricity markets:

- *Pure pool markets*, where all electricity is exchanged through a MO. GenCos bid their generation, for each day and for each hour, to the pool. DistCos bid their demand. The MO clears the market. The price of the last accepted bid for each hour is the market price for this hour.
- *Pure bilateral markets*, where all electricity is traded directly between a given GenCo and a given DistCo or consumer for each day and hour for a price agreed among them. Each consumer is free to choose its distributor and there is no auction.
- *Mixed system of pool and bilateral contracts*, where part of the load is traded as a bilateral contract between generation companies and distributors or big consumers, and the rest of the load is bid to the pool where the market operator clears the market each hour.

In all three markets, there is the system operator (which is also responsible for the transportation network) that checks the feasibility and the security of the proposed exchanges and introduces modifications to the schedule when it is necessary.

In a bilateral system, producer and consumer trade a specific amount of power for a specific period (from weeks to a year) and hourly zone (base, or peak) at a fixed price. Bilateral agreements can be settled in organized futures markets, through brokers, or in capacity rental auctions. The Nordpool in Scandinavia and the Iberian Electricity Market (MIBEL) are examples of mixed trading system.

There are many papers in the literature that deal with several electricity market types in different countries; a sample of these follows. [76] proposes a model for the mixed market in the Nordpool optimizing water allocation resources and the forward contracts, and in [69] a procedure for determining, also in the Nordpool, short-term and medium-term hourly spot prices based solely on records of past spot prices and on future forward prices is presented. For the MIBEL, a medium-term energy procurement model for a large consumer with access to several types of BCs with known tariffs and bounds is addressed in [13], and [15] refers to a mixed market with pool auction and bilateral contracts. In [16] a prediction of scenarios of future pool prices is developed from past pool prices and from the prices of the futures-market products at the European Energy Exchange in Germany. [20] presents the oligopolistic model in an American market, where a different market price is determined for each node of a transportation network.

In medium-term operation, all accepted bids in a time period (a week, or a month) must match the LDC of this period for the whole market. There is no specific load to be matched by a SGC. The only known loads are the predicted LDC's for the whole market. As all generation companies pursue their maximum profit, it is natural to attempt to maximize the profit of all generation companies combined, which is the problem that will be first described. It is called the *generators' surplus* maximization and means a degree of *collusion* among producers.

A SGC solving the generators' surplus problem may introduce its own operational constraints (fuel and emission limits, contracts, etc.) and may also introduce a *market-share constraint* for its units in one or several periods. The medium-term results will indicate how the SGC should program its units so that its profit is maximized while meeting all constraints.

2.5.2 The MIBEL market

Before 1997, in Spain there was a regulated electricity system where each company had its own generation and its own customers. The generation company had to satisfy the consumption of each of its clients (both individual customers and businesses). The liberalization of the electricity market was in January 1998. However, additional regulations were introduced in 2006 and it was when the MIBEL started up. In this market almost all the energy is auctioned. With this liberalization, companies that held a monopoly in its area had to split into three separate businesses: generation, distribution, and commercialization.

The MIBEL offers two possibilities for trading the electricity: an organized power-exchange market that is also called *pool* and through a Bilateral Contract (BC) which is an agreement between one supplier and one consumer for a fixed quantity at a fixed negotiated price for a given time period. Before 1998, BCs already existed but were hardly used by companies because they had to pay a penalization in order to use them. In July 2006, there was a market regulation that expanded the concept of BCs providing new possibilities to exchange energy inside and outside of the market.

To ensure the proper functioning of the electricity market, the market operator, the system operator and the regulator were created with specific functions:

- Market Operator (MO): The institution OMEL (Compañía Operadora del Mercado Español de la Electricidad, [89]) was set up. It manages daily and future markets and it makes the matching of selling and buying offers.
- System Operator (SO): Red Eléctrica (REE, [88]) guarantees that the electricity is supplied with safety, quality, and reliability. REE is the owner of the high-voltage transmission network.
- Regulator: Comisión Nacional de Energía (CNE, [87]) guarantees the functioning of the market based on the free competition, it prevents abuses of dominant positions and it determines the minimum requirements of quality and safety.

The agents that can participate in the MIBEL or market participants are companies authorized to participate in the electricity market as buyers or sellers.

2.5.3 Types of agent and of behavior in liberalized electricity markets

Regarding the ability of a GenCo of provoking an increase of the market price by increasing the price-bids of its generation, it is possible to distinguish two types of company:

- A *price-taker* is a company that is unable to produce a change in market price. Normally there are small companies.
- A *price-maker* is a company that is able to alter market prices. The increase of market price then comes as a consequence of bidding generation at prices higher than marginal production prices while having a non-negligible share of the generation capacity of the market (above 3%).

A market where there are several price-maker GenCos is called an *oligopolistic market*. GenCos are no longer interested in generating at the lowest cost but in obtaining the maximum profit (which is the revenue from the market for all accepted bids minus generation costs). There are many models for maximizing the profits of a GenCo in an electricity market. They vary depending on market type and behavior, type of company and the type of risk considered.

This thesis is devoted to pure pool and mixed market type with a single market price.

2.6 MARKET-PRICE FUNCTION OF LOAD DURATION AND THE CARTEL MEDIUM-TERM PLANNING

From the records of past market prices and load series (see Figure 6) it is possible to calculate a market-price function with respect to load duration for each time period. This function has to be used with expected generations that match the LDC of each time period.

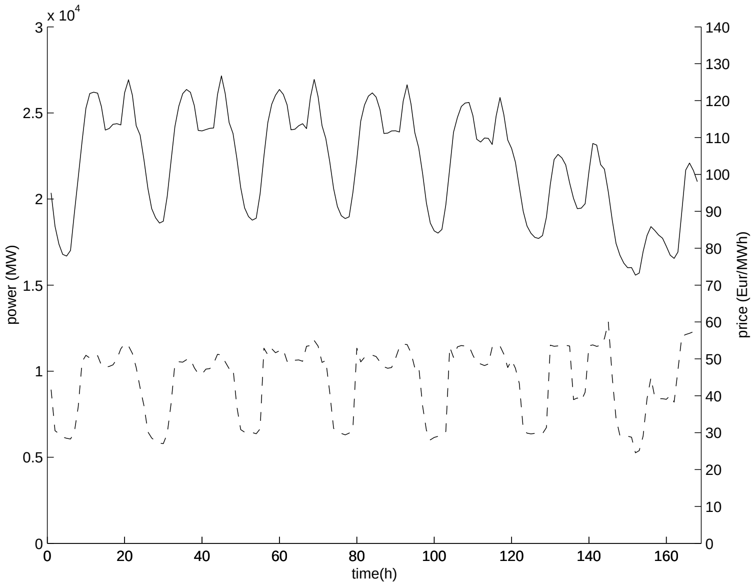


Figure 6: Hourly loads (continuous curve) and market prices (dashed) in a weekly period

If we sort the market prices and loads in a decreasing loading order, we obtain a price-duration curve that may be non-smooth and non-decreasing. However, it shows a decreasing trend (see Figure 7) that can be adjusted by a linear function (as proposed in [59]): $b^i + l^i t$, being t the load duration over period i, b is a constant and l is the (negative) slope. The oscillations of market price can be smoothed considering the linear regression employed.

In order to determine the maximum profit of unit j we can use the simplifying assumption regarding the shape of unit contributions in the GDC (see Figure 8). The contribution of all units will be assumed to have a rectangular shape with height c_j (the capacity of unit j) and base length $\frac{x_j^i}{c_j}$ (the expected generation duration), instead of an irregular right side [59].

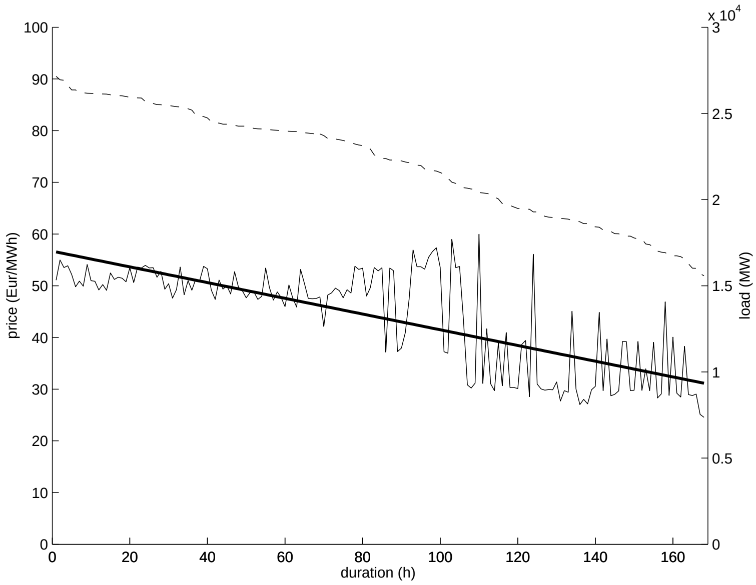


Figure 7: Market prices ordered by decreasing load power (thin continuous curve) in weekly period, market price linear function with respect to the load duration (thick line) and LDC (dashed)

The profit (revenue at market prices minus costs) of unit j over period i will be

$$\int_0^{\frac{x_j^i}{c_j}} c_j\{b^i + l^i t - f_j\}\,dt = (b^i - f_j)x_j^i + \frac{l^i}{2c_j}x_j^{i\,2}, \tag{14}$$

and adding for all periods and units, and taking into account the external energy, we get the profit function to be maximized:

$$\sum_i^{n_i}\left[\sum_j^{n_u}\left\{\left(b^i - \widetilde{f}_j\right)x_j^i + \frac{l^i}{2c_j}x_j^{i\,2}\right\} - \widetilde{f}_0 x_0^i\right]$$

which is quadratic in the generated energies. Using the load balance equation (9) we are led to the equivalent expression:

$$\sum_i^{n_i}\left[\sum_j^{n_u}\left\{\left(b^i - f_j\right)x_j^i + \frac{l^i}{2c_j}x_j^{i\,2}\right\} - \widetilde{f}_0 \widehat{e}^i\right]$$

with $f_j = \widetilde{f}_j - \widetilde{f}_0$.

In liberalized electricity markets, GenCos offer their generation in the auction of the market operator. The market price is determined in the clearing process, which is based on the construction of an offer curve and a demand curve for each hour and the price is fixed by the intersection of these two curves [17]. In medium-term operations, all accepted bids in

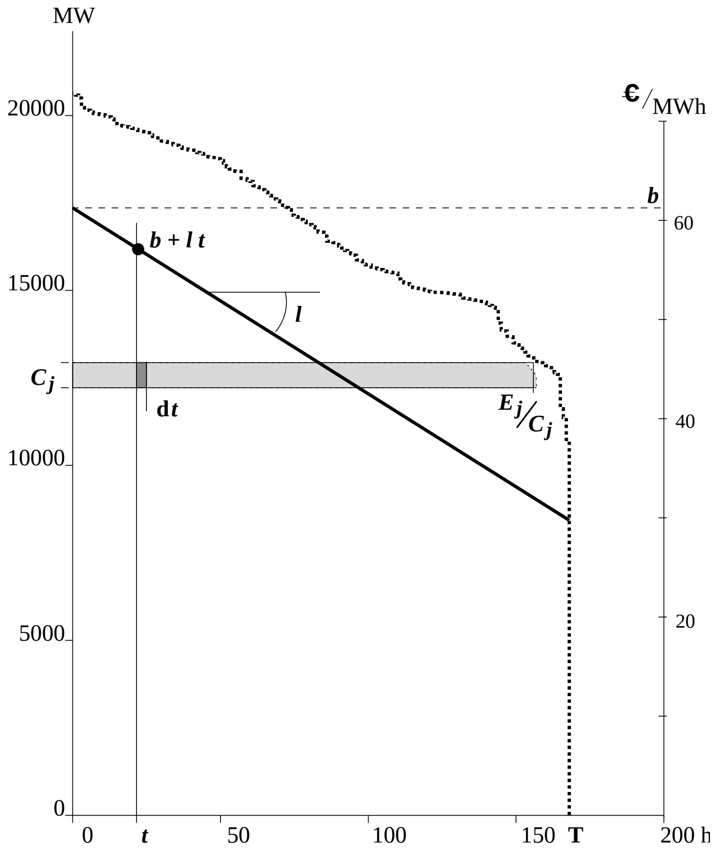


Figure 8: Medium-term price function for a time period and contribution of unit j

a certain period of time (normally a week or a month) must match the LDC of that period for the whole market. As all GenCos pursue their maximum profit, it is natural to attempt to maximize the profit of all GenCos combined, which could be called the *generators' surplus (or cartel) maximization*. A SGC solving this problem may introduce its own operational constraints, such as fuel and emission limits or contracts, and may introduce a *market-share* constraint for its units in one or several periods.

The *cartel medium-term generation planning model* can be formulated as follows

$$\max_{x_j^i} \sum_{i=1}^{n_i} \left[\sum_{j \in \Omega} \left\{ \left(b^i - \tilde{f}_j\right) x_j^i + \frac{l^i}{2c_j} \left(x_j^i\right)^2 \right\} - \tilde{f}_0 x_0^i \right] \tag{15a}$$

$$\text{s.t.:} \sum_{j \in \omega} x_j^i \leqslant e^i - s^i(\omega) \quad \forall\, \omega \subset \Omega \quad \forall\, i \tag{15b}$$

$$\sum_{j \in \Omega} x_j^i + x_0^i = e^i \quad \forall\, i \tag{15c}$$

$$\sum_{i=1}^{n_i} A^i x^i \geqslant r \tag{15d}$$

$$x_j^i \geqslant 0 \quad j \in \Omega \quad \forall\, i \tag{15e}$$

where constraints (15d) makes (15) non-separable by periods.

2.7 THE ENDOGENOUS MARKET-PRICE FUNCTION AND THE ENDOGENOUS CARTEL MODEL

The most obvious endogenous modification of the market-price function is that due to hydro generation. It can be clearly observed from historical records as that in Figure 9 that when the hydro generation level increases, market prices tend to decrease.

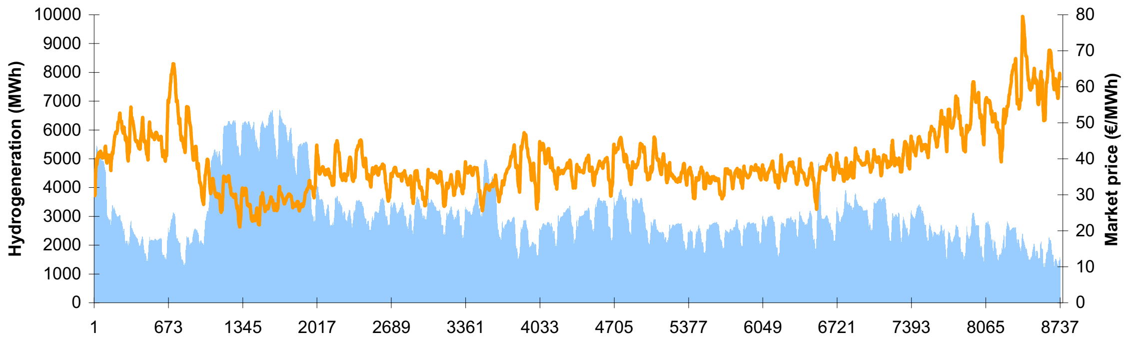


Figure 9: Weekly moving average of the market price (orange) and of hydro generation (blue area) during 2007 in the Spanish power pool

Given that both the peak-power and the base-power demand prices appear to be equally affected by the hydro generation level, a linear change in the basic coefficient b^i was introduced in [67]

$$b^i = b_0^i + \frac{d}{T^i} \sum_{k \in H} x_k^i, \tag{16}$$

where H is the set of all hydro units in the pool and d the (negative) correlation coefficient of the market price with the hydro generation per time unit.

Substituting b^i of (16) in (14), integrating and simplifying, the profit maximization function obtained is

$$\sum_i^{n_i}\Big[\sum_j^{n_u}\{(b_0^i - \tilde{f}_j)x_j^i + \frac{d}{T^i}\sum_{k\in H} x_k^i x_j^i + \frac{l^i}{2c_j}{x_j^i}^2\} - \tilde{f}_0 x_0^i\Big]\,, \tag{17}$$

which is still quadratic, but its matrix is no longer diagonal and it may be indefinite for values l^i and d found in practice.

Taking $\widetilde{f}_j - \widetilde{f}_0$ as f_j and removing the constants terms from the objective function the *generators' surplus problem with endogenous influence of hydro* is presented as follows

$$\min_{x_j^i} \sum_i^{n_i}\sum_j^{n_u}\Big\{\big(f_j - b^i\big)x_j^i - \frac{d}{T^i}\sum_{k\in H} x_k^i x_j^i - \frac{l^i}{2c_j}{x_j^i}^2\Big\} \tag{18a}$$

$$\text{subject to: } \sum_{j\in\omega} x_j^i \leqslant \widehat{e}^i - s^i(\omega) \quad \forall\omega \subset \Omega^i, \quad i = 1,\dots,n_i \tag{18b}$$

$$A_{\geqslant}^i x^i \geqslant r_{\geqslant}^i \qquad i = 1,\dots,n_i \tag{18c}$$

$$\sum_i A_{\geqslant}^{0i} x^i \geqslant r_{\geqslant}^0 \tag{18d}$$

$$A_{=}^i x^i = r_{=}^i \qquad i = 1,\dots,n_i \tag{18e}$$

$$\sum_i A_{=}^{0i} x^i = r_{=}^0 \tag{18f}$$

$$x_j^i \geqslant \underline{0} \qquad j = 1,\dots,n_u, \quad i = 1,\dots,n_i \tag{18g}$$

2.8 THE EQUILIBRIUM BEHAVIOR AND THE NIRA ALGORITHM TO OBTAIN IT

A behavioral principle different from the generators' surplus maximization, which is monopolistic on the part of the generation companies, is the oligopolistic Nash equilibrium in a game with Cournot competition type, which means a higher degree of competition than the generators' surplus maximization. In a Nash-Cournot equilibrium it is possible to assume either two (the SGC and the RoP), or more players (K generation companies, whose units are $\Omega_k \,|\, \Omega := \{\Omega_1, \Omega_2, \dots, \Omega_K\}$). Furthermore, in the Cournot model of competition it is assumed that the decision (generation) of one player is conditioned by the decisions (generations) of the rest of the players and that the market price is a function of the overall decisions (total expected generation). In a Nash equilibrium, no player can increase its revenue by unilaterally changing its decision (generation). It is not sure that a given pool behaves more like a Nash-Cournot equilibrium than like a monopolistic generators' surplus maximization.

For the medium-term planning to have *Cournot competition* (and an equilibrium solution) it is necessary to consider that the players mutually condition each other generations. This is so in case we consider the *endogenous model* explained, where the hydro generation of each player influences the market price.

Let us assume a game with K players. Let $\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_K) \in \mathcal{X}$ be a decision vector of the set $\mathcal{X}$ of decision vectors. The utility surplus function of player k is denoted by $\phi_k(\mathbf{x})$ and can be defined as

$$\phi_k(\mathbf{x}) = \sum_i \sum_{j \in \Omega_k^i} \left\{ (f_j - b^i)x_j^i - \frac{l^i}{2c_j}(x_j^i)^2 - \frac{d}{T^i}\left[\sum_{l \in H_k} x_l^i x_j^i + \sum_{l \in H_{m|m \neq k}} x_l^i x_j^i\right]\right\}, \tag{19}$$

where H_k is the set of hydro generators associated to player k and $H_{m|m \neq k}$ is the set of hydro generators associated to players m other than player k.

Given the joint decision vectors $\mathbf{x} = (\mathbf{x}_1, \mathbf{x}_2, \ldots, \mathbf{x}_K) \in \mathcal{X}$ and $\widehat{\mathbf{x}} = (\widehat{\mathbf{x}}_1, \widehat{\mathbf{x}}_2, \ldots, \widehat{\mathbf{x}}_K) \in \mathcal{X}$, a new decision vector where all the agents $s \neq k$ play $\widehat{\mathbf{x}}$, while the agent k plays $\mathbf{x}$ is defined as follows:

$$(\mathbf{x}_k | \widehat{\mathbf{x}}) = (\widehat{\mathbf{x}}_1, \ldots, \mathbf{x}_k, \ldots, \widehat{\mathbf{x}}_K) \in \mathcal{X}$$

Using the definition (19), the utility function of player k, which is playing $\mathbf{x}$, while other players play $\widehat{\mathbf{x}}$ is determined by

$$\phi_k(x_k | \widehat{\mathbf{x}}) = \sum_i \sum_{j \in \Omega_k^i} \left\{ (f_j - b^i)x_j^i - \frac{l^i}{2c_j}(x_j^i)^2 - \frac{d}{T^i}\left[\sum_{l \in H_k} x_l^i x_j^i + \sum_{l \in H_{m|m \neq k}} \widehat{x}_l^i x_j^i\right]\right\}. \tag{20}$$

A point $\mathbf{x}^*$ is a Nash equilibrium point if the following holds:

$$\phi_k(\mathbf{x}^*) = \max_{(\mathbf{x}_k | \mathbf{x}^*) \in \mathcal{X}} \phi_k(\mathbf{x}_k | \mathbf{x}^*) \quad \forall k \in \{1, \ldots, K\}. \tag{21}$$

The Nikaido-Isoda relaxation algorithm (NIRA) [63, 90] is a successive-optimization-based procedure to obtain a Nash-Cournot equilibrium point in a constrained game. In [18], there is an application of the NIRA algorithm to a short-term market equilibrium problem. GenCos in oligopolistic markets should behave as players in equilibrium, each being unable to unilaterally increase its profits. The endogenous cartel model is also the basic kernel for the market equilibrium model when the equilibrium is obtained through the NIRA procedure and the utility function is the profit calculated with an endogenous price function that changes with the amount of generation of different technologies, as done in [84, 56] for a medium-term pure-pool generation planning problem.

The Nikaido-Isoda function is:

$$\Psi(\widehat{\mathbf{x}}, \mathbf{x}) := \sum_{k=1}^{K} \big(\phi_k(x_k|\widehat{\mathbf{x}}) - \phi_k(\widehat{\mathbf{x}})\big)\,. \tag{22}$$

An equivalent formulation of a Nash equilibrium is that $\mathbf{x}^*$ is an equilibrium point if:

$$\max_{\mathbf{x}\in\mathcal{X}} \Psi(\mathbf{x}^*, \mathbf{x}) = 0\,.$$

The *optimal response* function Z is defined as:

$$Z(\widehat{\mathbf{x}}) := \arg\max_{\mathbf{x}\in\mathcal{X}} \Psi(\widehat{\mathbf{x}}, \mathbf{x})\,, \tag{23}$$

and the NIRA algorithm-updating rule is:

$$\mathbf{x}^{\text{new}} \leftarrow (1-u)\widehat{\mathbf{x}} + uZ(\widehat{\mathbf{x}}) \quad u \in \mathbb{R},\ 0 < u \leqslant 1$$

In order for the NIRA procedure to converge to an equilibrium point, $\Psi(\widehat{\mathbf{x}}, \mathbf{x})$ should be *weakly convex* w.r.t. $\widehat{\mathbf{x}}$ and *weakly concave* w.r.t. $\mathbf{x}$, and function $Z(\cdot)$ should be single valued.

The structure of the NIRA procedure is:

- $\widehat{\mathbf{x}} \leftarrow \mathbf{x}_0\,, \quad 0 < u \leqslant 1\,, \quad (\text{e.g.}\, u \leftarrow 0.4)$
- **repeat until** $\Psi^* \leqslant \epsilon$

  obtain $Z(\widehat{\mathbf{x}}) = \mathbf{x}^*$ by solving $\max_{\mathbf{x}\in\mathcal{X}} \Psi(\widehat{\mathbf{x}}, \mathbf{x})$

  compute $\Psi^* = \Psi(\widehat{\mathbf{x}}, Z(\widehat{\mathbf{x}})) = \sum_{k=1}^{K} \big(\phi_k(x_k^*|\widehat{\mathbf{x}}) - \phi_k(\widehat{\mathbf{x}})\big)$

  $\widehat{\mathbf{x}} \leftarrow uZ(\widehat{\mathbf{x}}) + (1-u)\widehat{\mathbf{x}}$

Note that, given $\widehat{\mathbf{x}}$, $\phi_k(\widehat{\mathbf{x}})$ is a constant that need not be optimized, the maximization of $\Psi(\widehat{\mathbf{x}}, \mathbf{x})$ is equivalent to solving

$$\begin{aligned} \max_{x_j, y} \quad & \sum_{k=1}^{K} \phi_k(\mathbf{x}_k|\widehat{\mathbf{x}}) \qquad \phi_k(\mathbf{x}_k|\widehat{\mathbf{x}}) \text{ as in } (20) \\ \text{s.t.:} \quad & \text{LMCs } (15b),\ \text{nonLMCs } (15d) \\ & \text{balance } (15c) \text{ and bounds } (15e) \end{aligned} \tag{24}$$

and using (20) the problem could be recast as

$$
\begin{aligned}
\max_{x_j^i} \quad & \sum_{i=1}^{n_i} \sum_{k=1}^{K} \sum_{j \in \Omega_k^i} \left\{ (b^i - f_j)x_j^i + \frac{l^i}{2c_j}(x_j^i)^2 + \frac{d}{T^i}[\sum_{l \in H_k} x_l^i x_j^i - \sum_{l \in H_{m|m \neq k}} \widehat{x}_l^i x_j^i] \right\} \\
\text{s.t.:} \quad & \text{LMCs (15b), nonLMCs (15d)} \\
& \text{balance (15c) and bounds (15e)}
\end{aligned} \tag{25}
$$

It should be noted that the NIRA objective $\phi_k(\mathbf{x}_k|\widehat{\mathbf{x}})$ is better conditioned than the endogenous cartel one because many of the cross products $x_j^i x_l^i$ (j being a unit of a certain GenCo and l being a hydro unit of a different GenCo), which may bring about non convexity, now become linear terms $x_j^i \widehat{x}_l^i$ where $\widehat{x}_l^i$ takes a constant value.

The medium-term planning is a *constrained game* because there are constraints that link the generation of several players. Many of the LMCs and the load balance equation, which includes the external energy, link the generations of different players, e.g., the load balance equations (15c), which could be recast as:

$$
\sum_{k=1}^{K} \sum_{j \in \Omega_k^i} x_k^i + x_0^i = \widehat{e}^i
$$

As the mLMH (see §3.2.2 and §3.6) is a successive optimization procedure adding more LMCs, one may as well change its objective function as required by the NIRA procedure in order to find the equilibrium solution. This can be done by introducing some changes in the mLMH procedure as described in [56].

2.9 STOCHASTIC PARAMETERS IN MEDIUM-TERM PLANNING AND STOCHASTIC PROGRAMMING

Some of the parameters used in the medium-term model are stochastic. As an example, we have the water inflows w_h^i in hydro basin h, for which as a first approximation one can use its expected value. However, the stochastic programming [9] recommends the use of a *scenario tree* where every uncertainty at each period is represented and has an associated probability. The optimization over the scenario tree provides a solution for every scenario taking into account the non-anticipativity principle (one variable in a node cannot use information of nodes that correspond to future periods).

Regarding the uncertainty of the load and the stochasticity of the unit outages, there is no need to create scenarios for the unit failures and for the load variation, whenever the probabilistic matching (see §2.2.1) is used.

2.9.1 Scenario Tree: characteristics and notation

A scenario tree is a discrete representation of the possible states that our random parameters could take at each period i over the time horizon. Therefore, a tree is a subset of nodes $\mathcal{N}$ that are linked together in a hierarchical way.

The main characteristics of a node ν are

- Random parameters take predetermined values at each node, which results in specific values of the variables x_j^ν that correspond to that node.
- It belongs to a certain time period, $i(\nu)$.
- It has a specific probability, π_ν.
- It has a single predecessor (or father) node, ν^-.

Summarizing, one scenario tree consists of a set of nodes $\mathcal{N}$ connected to each other. Each node $\nu \in \mathcal{N}$ has an associated time period $i(\nu)$, one predecessor node ν^- and represents the probability of realization of that period. Each node also has a probability π_ν and it holds that:

$$\sum_{\forall \nu | i(\nu) = \tilde{i}} \pi_\nu = 1 \quad \tilde{i} = 1, 2, ..., n_i$$

We define $\mathcal{L} := \{\nu \in \mathcal{N} | i(\nu) = n_i\}$ as the set of *leaves* (nodes at final period) and $\mathcal{A}(\geqslant) := \{1, ..., \lambda^-, \lambda\}$ as the *path* from the root node to the node λ which is the scenario λ with the same probability as the node λ, π_λ. For more information, see [36].

In our study, the root node represents the present (normally it will be of a length of one week). At each period, we will have optimistic realizations, normal realizations and pessimistic ones, thus all possibilities will be taken into account. The scenario tree will be created using a mixture of a multidimensional vector auto regressive model and Quasi-Monte Carlo methods.

2.9.2 The Vector Auto-Regressive model

Vector Auto-Regressive (VAR) models are usually used for multivariate time series. Its essence is a system of equations, with as many equations as series to analyze or predict. But in which there is no distinction between endogenous and exogenous variables. Thus, each variable is explained by past lags of itself (like an Auto-Regressive model) and past lags of other variables.

We assume that renewable generations follow a n_D dimensional modified VAR model of type:

$$X_{i+1} = \kappa_{i+1}(\overline{X}_{i+1} - X_i) + X_i + \xi\,, \tag{26}$$

where X_{i+1} is the stochastic variable vector at period $i+1$, κ_{i+1} is the diagonal matrix of the mean reverting coefficients, $\overline{X}_{i+1}$ is the vector of the historical mean series and ξ is a normal vector with zero mean and covariance matrix Σ, i.e., $\xi \sim N(\underline{0}, \Sigma)$.

2.9.3 The Quasi-Monte Carlo technique for creating a scenario tree

There are many options that could be used for the scenario generation such as Monte Carlo (MC) sampling, optimal quantization of probability distributions, quadrature rules based on sparse grids or Quasi-Monte Carlo (QMC) methods. *MC* methods are normally used for generating uniformly distributed random variables and for transforming these variables to other distributions.

A *pseudorandom* number generator produces a finite sequence of numbers $u_1, u_2, ..., u_s$ in the $[0, 1]$ interval. A *linear congruential generator* is a recurrence of the following form:

$$g_{s+1} = a g_s \bmod q \tag{27a}$$
$$u_{s+1} = g_{s+1}/q\,. \tag{27b}$$

Here, the multiplier a and the modulus q are integer constants that determine the values generated, given an initial value g_0. This initial value, called *seed*, normally is defined by the user and must be an integer between 1 and $m-1$. Due to its simplicity, this type of generator is the most used in practice.

Quasi-Monte Carlo or *low-discrepancy* methods are alternatives to MC methods (see [61, 78]). They seek to increase accuracy specifically by generating points that are too evenly distributed to be random. These methods have the potential to accelerate convergence from the $O(1/\sqrt{n})$ rate associated with MC to nearly $O(1/n)$ convergence. Niederreiter [62] provides a thorough treatment of the theory.

The problem now is to approximate the integral of a function f as the average of the function evaluated at a set $t_1, ..., t_N$ of points.

$$\int_{[0,1]^{n_D}} f(u)du \simeq \frac{1}{n_D} \sum_{i=1}^{n_D} f(t_i)\,. \tag{28}$$

In a MC method, the set $t_1, ..., t_{n_D}$ is a subsequence of pseudorandom numbers obtained as in (27). In a QMC method, the set is a subsequence of a low-discrepancy sequence (or quasi-random sequences). These sequences are used to generate representative samples from the probability distributions simulated in these problems.

A *low-discrepancy sequence* is one with the property that for all values of n_D, its subsequence $t_1, ..., t_{n_D}$ has a low discrepancy. The discrepancy will be low if the proportion of points in the subsequence falling into an arbitrary set is close to proportional to the measure (intuitively interpreted as its size) of that set.

An example of a low-discrepancy sequence is the following one:

$$Q_{n,n_D}(f) = \frac{1}{n}\sum_{k=0}^{n-1} f\big(\{\frac{k\tau}{n} + \Delta\}\big)\,, \tag{29}$$

where $\tau \in \mathbb{Z}^{n_D}$, $\{\tau\}$ means component-wise the fractional part of y, and the 'shift' Δ is a uniformly distributed random variable in $[0,1)^{n_D}$. This sequence is a quadrature rule invented for the integration of periodic functions over the n_D-dimensional cube. A quadrature rule is an approximation of the definite integral of a function, usually stated as a weighted sum of function values at specified points within the domain. (see [44, 77])

Let's now consider a stochastic program of the following form

$$\min\{\int_{\Xi} \Upsilon(\xi, x)P(d\xi) : x \in X\}, \tag{30}$$

where $X \subseteq \mathbb{R}^m$ is a constraint set, P a probability distribution on $\Xi \subseteq \mathbb{R}^{n_D}$, and $f = \Upsilon(\mathring{u}, x)$ is a decision-dependent integrand.

The idea of QMC methods is to replace those random samples in MC methods by deterministic points that are uniformly distributed in $[0,1]^{n_D}$. Having n_i time periods and m stochastic variables, which are the natural water inflows, and WP and SPV generations, the total dimension will be $n_D = m \times n_i$.

The main steps of the QMC procedure in order to generate n scenarios are:

- Determine QMC points by a component-by-component algorithm (see Nuyens and Cools [64] and Kuo [43]): $\eta^j \in [0,1]^{n_D}$, $j = 1, \ldots, n$ with probability $\frac{1}{n}$, where $\{\eta^j\}_{j=1}^n$ approximates the uniform distribution. In order to determine the QMC points, an algorithm is used that in each step minimizes the worst-case error for increasing dimensions while keeping all previous z_j fixed.

  In order to obtain these QMC points in this thesis, it has been used the implementation of the CBC algorithm, CBC_{poly}, made by Neder [60].

Algorithm 1 Component-by-component (CBC)

for $s = 1 \to s_{max}$ **do**
  for all $z_s \in \mathcal{Z}_n$ **do**
    $e_s^2(z_s) = -1 + \frac{1}{n} \sum_{k=0}^{n-1} \prod_{j=1}^{s} \left(1 + \gamma_j \omega \left(\left\{\frac{kz_j}{n}\right\}\right)\right)$
  end for
  $z_s = \arg\min_{z \in \mathcal{Z}_n} e_s^2(z)$
end for

- Most simulation entails sampling random variables from distributions other than the uniform. A typical simulation uses methods for transforming samples from the uniform distribution to samples from other distributions. The method employed in this thesis is the *Inverse Transform Method* (see Marsaglia [52]).
  Generate n realizations ζ_i^j, $i = 1,..,n$, $j = 1,..,n_D$ of n_D independent N(0,1) random variables using the inverse transform method with the QMC points η^j obtained before. This transform method computes the random number ζ^j such that $F(\zeta^j) = \eta^j$.

- Compute the Cholesky decomposition of the covariance matrix $\Sigma = LL^T$ (L^T being the transposed L matrix and L a lower triangular matrix of dimension $n_D \times n_D$

- Compute the n realizations $\xi^j = L\zeta^j + r$, $j = 1,..,n$ of the original $N(r, \Sigma)$ random vector. Since in this model the value of the mean is $r = 0$, the formula to use is $\xi^j = L\zeta^j$, $j = 1,..,n$.

Finally, the stages to generate a scenario tree are:

1. Generate n scenarios ξ^j, $j = 1, \ldots, n$ by quasi-Monte Carlo techniques with probability $\frac{1}{n}$, without tree structure, where n is taken to be a prime number that will represent the number of scenarios prior to reduction.

2. Generation of a scenario tree out of the n scenarios: a fan of data scenarios is created using the n realizations obtained with quasi-Monte Carlo procedures and using the stochastic model described before in §2.9.2

$$X_{i+1}(\nu) = \kappa_{i+1}(\overline{X}_{i+1} - X_i) + X_i + \xi(\nu), \forall \nu \in \mathcal{N}$$

2.9.4 The reduction to a scenario tree of a given size

The generated scenario tree must be reduced because it is a dense tree and may be too big to be used in stochastic programming formulations. The software *Scenred* (Heitsch and W. Römisch [32]), which uses an algorithm that deletes some of the nodes and redistributes their probability among the remaining ones using the Fortet-Mourier probability metric (see Dupačová et al. [22] and Heitsch and Römisch [31]) has been employed in this work.

2.9.5 Non-anticipativity constraints

When considering Stochastic Programming, it is necessary to introduce the concept of *non-anticipativity*. This implies that in, for example, a multi-period financial investment problem, no decision can be made at period t using any information that is revealed after period t. So mathematical models should include constraints that enforce that fact.

However, in this thesis the formulation of the stochastic problems is made with variables that are associated to nodes, and not to scenario paths, i.e., the variable associated to a specific node is common to all scenario paths that include the specific node. This avoids having to introduce *non-anticipativity constraints* in the formulation, because the decision variable in period i is the same no matter which decision follows in successive periods in different scenario paths.

2.10 PROFITS' DISTRIBUTION, PROFIT SPREAD, PROFIT LOSSES AND VALUE AT RISK

In order to assess the effect of N-DR generation on the overall GenCos' profit it is necessary to consider the expected medium-term profit and also a measure of the associated profit dispersion for both the case with N-DRs, and for the case without it, but having instead a conventional generation unit providing the same expected energy as that obtained from the N-DRs. The conventional equivalent unit will be a dispatchable one with an efficient technology, e.g.: combined cycle, and with a generation capacity such that it produces the same expected generation as the N-DRs over the medium-term horizon.

The expected profit objective function maximized is the weighted sum of the profit over each scenario path using the probability of each path, which is that of the corresponding leaf node. This expected profit value is equivalent to that calculated with the weighted sum of the profit in each node using the probability of each node. The distribution of profit is that of the profits in each scenario path with the corresponding probability of each path.

There are widely accepted rules of graphical representation of discrete distributions (as a bar chart) from a reduced number of observations. These rules refer to the number of bars of equal width with which to cover the range of values spanned by the available observations:

- For up to 50 observations: divide the observation value range into 7 adjacent intervals.
- For 51 to 100 observations: divide the observation value range into 10 adjacent intervals.

- For 101 to 200 observations: divide the observation value range into 12 adjacent intervals.

The number of observations in case of representing the medium-term profits obtained through a Stochastic Programming solution correspond to the number of scenarios considered (each with its scenario probability), and the numbers of scenarios in the test cases employed are 21, 59 and 75.

The discrete distribution of the profit value of each scenario with its own probability may be multimodal and discontinuous as it can be seen in Figure 10 for case 18r21 (with 21 scenarios) of endogenous cartel in a pure pool market (see §5.1 and §5.5.1). Fitting a parametric statistical distribution to distribution to the discrete distribution of Figure 10 (such as a Weibull or a Normal distribution) may entail an important mismatch.

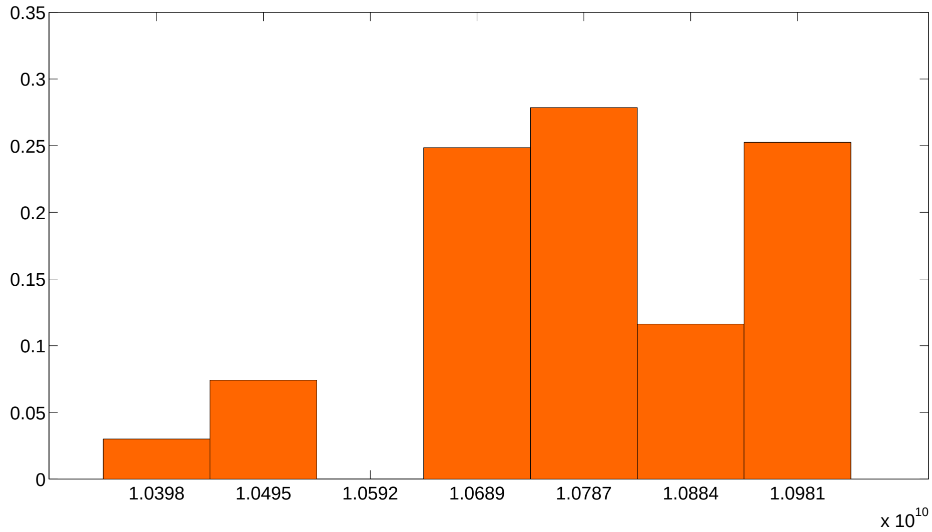


Figure 10: Profit distribution of case 18r21 (see §5.5.1)

Therefore, it is better to employ a nonparametric measure of profit dispersion, as the 5% to 95% profit range, which contains the central 90% of *profit spread*, denoted by P_{sp90}. This measure is created by sorting in an ascending way the profits of each scenario. Then the profit that leaves behind a 5% of the total area of the histogram and the profit that leaves behind a 95% of the total area are chosen to calculate the profit spread. It is to be noted that, when considering as losses the profits below the expected profit $\widetilde{P}$, the 5% profit value corresponds to the value at risk (VaR) leaving at its right an area of the profit distribution of $1 - \beta$, with β being a 95% confidence level.

2.11 DIFFERENCE OF CONVEX OBJECTIVE FUNCTION AND THE REVERSE-CONVEX CONSTRAINT

One of the standard techniques in the minimization of an indefinite function is to decompose it as the difference of two convex functions and the use of an equivalent formulation that in practice proves to be more effective (see [34, 25]). For this decomposition, it will be used that given two variables v and w their product $v \times w$ is

$$v \times w = \frac{(u+w)^2 - (u-w)^2}{4}, \tag{31}$$

which is a difference of convex quadratic functions.

2.11.1 Equivalent formulation

Optimization problems with a non-convex objective function that can be decomposed as a difference of two convex (DC) functions, have an equivalent formulation using an extra variable v_{RC} as a minimization of a convex objective function subject to a reverse-convex constraint (RCC) [34]. It is possible to show this fact by way of an example. The following DC problem is considered:

$$\min_{x} (x+1)^2 - x(x-4) \tag{32a}$$

$$\text{s.t.: } \frac{5}{6} \leqslant x \leqslant \frac{7}{2} \tag{32b}$$

The objective function (32a) is equivalent to a linear function $(x+1)^2 - x(x-4) = 1 + 6x$. Thus, the problem results in the minimization of a linear function subject to the variable bounds. Therefore, the solution obtained is $x^* = 5/6$ (because the equivalent linear function is increasing in x). Following [34], it is possible to solve the equivalent problem having the additional variable v_{RC} and a reverse-convex constraint:

$$\min_{x, v_{RC}} (x+1)^2 - v_{RC} \tag{33a}$$

$$\text{s.t.: } v_{RC} \leqslant x(x-4) \tag{33b}$$

$$\frac{5}{6} \leqslant x \leqslant \frac{7}{2} \tag{33c}$$

In Figure 11, it can be seen in black the decomposed DC function to be optimized (33a) and in bold black the RCC (33b). It can be appreciated that the solution obtained using this method is the same as obtained before, $x^* = 5/6$.

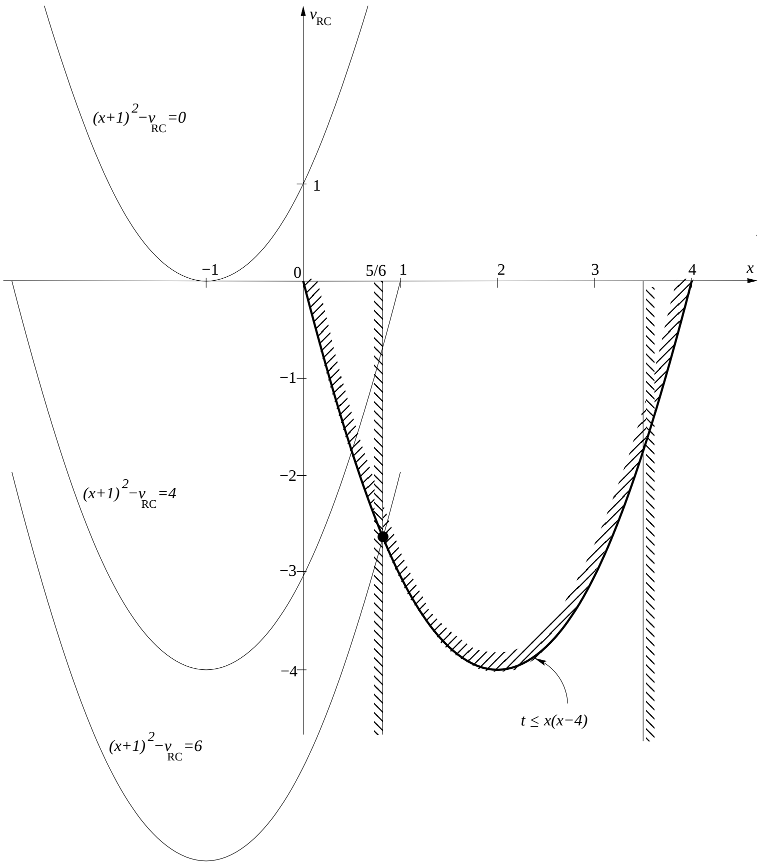


Figure 11: One-dimensional example of RCC formulation

2.11.2 Solver for a DC problem

In order to solve DC problems, direct methods of global optimization [34, 25] are the most appropriate ones. However, these methods are not very efficient for problems with many variables.

2.12 MIBEL DATA SOURCES EMPLOYED

There are different types of generation systems, the ones that are employed in MIBEL are hydro systems (with or without pumping scheme), thermal systems (such as coal, nuclear, fuel and gas units) and non-dispatchable renewable sources like solar photovoltaic generation and wind power.

The data employed in this thesis have been downloaded, basically, from the website of the system operator [88] and from the market operator [89]. The MO supplied the demand, market prices and wind power bid data for the dates chosen. The SO also tracks the WP and the SPV generation in Spain and has a depository where historic data can be obtained. However, as WP and SPV are relatively new sources there is not as much available data as there is for other generation units, such as hydro. For example, while for the hydraulic data one can obtain 20 years of information, for renewable sources there is only 5 years of information.

2.12.1 Load data in periods and subperiods

Load data have been downloaded from the website of the system operator [88]. There it is possible to obtain hourly data from the past. Since the scope of this thesis is for one year, the chosen data is from October 2010 until October 2011. The data matrix has 365 rows corresponding to the days of the year and 24 columns corresponding to the hours of each day. However, the load data matrix has to be split into periods and, in the case of using SPV, into subperiods.

2.12.2 Generation data for different settings and for renewable sources

Some of the available generation units will belong to the Specific Generation Company (SGC) with detailed information and the other ones will belong to the Rest of Participants (RoP), for these ones the units of similar characteristics are usually merged into a single unit in order to reduce the number of variables and thus the problem size.

Let us denote by M the set of units merged into a given pseudo-unit, and let ι be the index of the composing units. The parameters of the pseudo-unit can be calculated as

- maximum power capacity $c_M = \sum\limits_{\iota \in M} c_\iota$
- linear generation cost $f_M = \frac{\sum\limits_{\iota \in M} c_\iota f_\iota}{\sum\limits_{\iota \in M} c_\iota}$
- outage probability $q_M = \frac{\sum\limits_{\iota \in M} c_\iota q_\iota}{\sum\limits_{\iota \in M} c_\iota}$

Hydro generation reservoirs, simplified reservoir systems and run-of-the-river schemes behave as a pseudo-unit subject to additional constraints. Thermal unit generation may also be subject to constraints such as fuel availability and emission limits. Past data series for natural water inflows in reservoirs and the hourly wind-power and solar photovoltaic generation can be obtained from System Operator's web site [88]. As these generations in medium-term planning are considered stochastic, time series will be used as a priori information for scenario generation. In this thesis data from 2009 until 2012 will be employed as historical data in order to create the scenario tree.

2.13 SOLVER EMPLOYED

The solver used is the publicly available Interior Point code *Ipopt* version 3.10.3 (Wächter and Biegler [94]). This code implements a filter method to determine step sizes and allows nonlinear objective functions and nonlinear constraints.

Ipopt finds a direction to modify the primal and the dual problem variables by applying Newton's method to the nonlinear Karush-Kuhn-Tucker conditions (see Chapter 3 of Bertsekas [8]) of the barrier function and constraints. A specific feature of *Ipopt* is that it uses *Inertia Corrections*, if necessary, in the Newton's system matrix to ensure descent of the direction.

A maximum step size along the direction obtained is found that keeps the primal and the dual variables strictly within bounds. *Ipopt* then determines by backtracking exploring a decreasing sequence of trial steps using acceptance criteria based on a Flechter-Leyffer filter [26] and on additional requirements. The filter is based on improving either, or both, of a barrier-objective function value and of an infeasibility norm. *Ipopt* applies a second order correction to the direction calculation in case the filter is not passed. In case the filter is passed, *Ipopt* puts some additional requirements whenever the infeasibility norm is below a predetermined minimum value and the directional derivative of the barrier objective function is negative enough: the step size is accepted if the new point satisfies an Armijo 1 condition (see Chapter 1 of Bertsekas [8]) for the descent of the barrier objective function.

This is of relevance for the difference of convex quadratic objective function of the medium-term generation planning problem described in Chapter 6 of this thesis, and for the equivalent problem of minimizing the positive definite quadratic part of the problem subject to the constraints and also subject to a reverse convex constraint made with the concave quadratic part of the original objective function (see §2.11.1). Note that the non-convexity is in the barrier objective function, while the constraints are linear and the infeasibility function is a positive definite square norm, in case of not using the equivalent formulation. In case of using the reverse convex constraint, the barrier objective function would be well behaved and the feasibility norm would depend on the successive linearizations of the reverse convex constraint. It may be convenient to switch problem formulations from non-convex objective to convex objective plus reverse convex constraint depending on results obtained in successive optimizations within the process of solving the medium-term planning, as explained in §6.4 and §6.6.

Possible outcomes from the optimization of an indefinite problem are:

- *Optimal solution* (the normal solution within tolerances)
- *Acceptable solution* (a solution within slightly relaxed tolerances)
- *Restoration phase failure* (a solution with acceptable primal and dual feasibilities that does not satisfy optimality conditions due to near singularity of the objective function Hessian)
- *Iteration limit exceeded* (no convergence within iteration limit; default limit 3000 iterations)
- *Infeasibility* or other anomalous solution (no feasible point reached).

3 THE STATE OF THE ART IN MEDIUM-TERM GENERATION PLANNING

In this chapter the most relevant treatments that have been proposed for solving medium-term generation planning problems are discussed.

3.1 FEATURES OF THE MEDIUM-TERM GENERATION PLANNING OF A GENCO IN A LIBERALIZED MARKET

The main features of the medium-term generation planning of a GenCo in a liberalized electricity market are:

- The stochasticity in the load, in the outages of generation units of different types, in the availability of the dispatchable and of the non-dispatchable energy sources, such as hydro generation, WP and SPV, and in the market price.
- The matching of the system load by all participants in the market.
- The satisfaction of other medium-term constraints such as emission caps over time, fuel availabilities.
- The consideration of pumping schemes.
- The influence on market price of the generation by different GenCos and the consideration of different market behaviors.
- The profit distribution and the measures of the risk of profit loss.

The available papers on medium-term generation planning will be appraised regarding the features mentioned.

3.1.1 Consideration of Load Matching

Taking into account the matching of the load, Balériaux et al. [3] proposed a method for computing the expected generation of each unit given a loading order and taking into account the capacity and the outage probability of the units for matching an LDC using convolutions. Note that in [3] the uncertainty in the load is considered by using the LDC to represent it, and the stochastic unit outages are considered through the outage probability of each unit; and the exact probabilistic match of the expected unit generation and the expected load is obtained.

Bloom and Gallant [11] proposed a linear programming model of the probabilistic LDC matching of [3], satisfying other linear operational constraints, while a linear cost function was optimized. The variables employed were the expected unit energies x_j, $j = 1, \dots, n_u$ generated, where n_u is the number of units. The LDC matching is expressed through linear inequality constraints (11b), their number being the exponential $2^{n_u} - 1$. Using the Bloom and Gallant formulation there are a number of procedures to consider, for example, column generation methods and the active set method. Examples of Column Generation Methods are the Ford-Fulkerson [27] and Dazing-Wolfe methods [19].

In Pérez-Ruiz and Conejo [71] it can be found an example of using a column-generation method in medium-term planning with the Bloom and Gallant formulation [11]. These methods work by splitting the problem into two problems: the master problem and the subproblem. The subproblem is solved to obtain a new variable (a vertex of the polyhedron of feasible points) and then it is necessary to solve the master problem to obtain the best convex combination of the generated vertices.

Algorithms that implement the active set method consider a subset of the set of all constraints. As the optimization proceeds, these methods remove constraints from this set and add new ones. The method also requires an initial feasible point satisfying all constraints, a procedure to find this initial feasible point is given in Nabona et al. [58]. The Bloom and Gallant formulation has been extended to multi-period planning problems in liberalized electricity markets by [59] using either the Dantzig-Wolfe [19, 71], the Ford-Fulkerson [27] column generation method or the active set method (see Nabona et al. [58]). They use an exogenous price function of load duration.

In [67, 68, 81, 84, 51, 50], the LDC is exactly matched through the Bloom and Gallant's formulation and a heuristic is employed to avoid having to generate an exponential number of LMCs.

There is an alternative way to represent the LDC that will be called here the multi-block model of load matching. It consists in approximating the LDC with a number of rectangular blocks (normally two to five blocks) of constant power and a given duration as it can be shown in Reneses et al. [72], in which case the number of variables is increased but the numbers of constraints that is needed to represent the matching of the LMC is considerably reduced. In the literature it can be found under different names, see Cabero et al. [12] and Escudero et al. [24]. Generation satisfies load merely by using a load balance equation with slack energy that is penalized in the objective function, and no LDC-matching constraints are taken into account. Uncertainties in load and unit outages are simulated through the use of a number of scenarios. In [7, 21] load is also considered through blocks.

In [56] a six-block representation with a decreasing height (as is the load in the LDC) is employed. A new variable is needed in order to match the block loads of a period LDC. It is the power produced by each unit for matching each specific block in each period. In addition, a set of constraints to prevent the expected unit energies from exceeding its maximum value is needed. In that study there was a comparison between using the multi-block model, the load-matching heuristic (see §3.2) and the modified load-matching heuristic (§3.2.2). Computational cases showed that although using multi-block the problem was much easier to implement, the method lost accuracy in the results.

In Park et al. [70] annual loads are known, the authors use peak load and base load and a simple energy balance constraint for successive years. In [53] there is an hourly load matching and in [75] the load is forecasted and load scenarios are developed. In [4, 5] the load is matched for periods and subperiods. In [91] the load-survival function after loading units in merit order is computed using the approximation of cumulants method.

3.1.2 Consideration of a market-price function, of price elasticity, of a conjectural supply function and of market behavior

Regarding on how to model the electricity market-price in liberalized electricity markets, it is possible to consider, on the one hand, that market-prices can be predicted for every single period, in which case the optimization of the profits of a GenCo can be made without taking into account the rest of GenCos (this is the case of price-takers) and on the other hand, market-prices cannot be predetermined and all GenCos must be taken into account.

Eichhorn et al. [23] is an example of the first case, where authors formulate a stochastic programming model that takes into account the fact that the uncertainty factors (modeled as multivariate ARMA) and the decision variables have different time scales. In Liu and Wu [47], the authors address the analysis of the market by formulating it as an optimal control problem, which simulates the operation of the market. In there, electricity demand, which has some price elasticity, can be modeled by some convex function of electricity price. The authors propose a procedure for finding short-term bidding strategies that are based on an analysis of the market in the long-term. Its main result is a prediction of the market-price.

In Centeno et al. [14] a stochastic approach when modeling a medium-term planning model is presented with inelastic demand and system's marginal clearing price for each period and branch is obtained from the Lagrange multiplier of the constraint. Here, both the electricity market and a number of important decisions that companies must make (such as hydro resources allocation, fuel purchases, emissions allowance, and bilateral contracts) are included. In [4] marginal price is directly obtained from the results of the optimization problem. Both [75, 76] use a prediction of market prices at each considered period.

Authors in [95] obtain an estimate of the hourly market-clearing price by cutting the total strategic supply-bid function with the power of the corresponding block. [91] presents a procedure for determining the price-duration curve for a sample day in a future period in an oligopolistic energy market under the Bertrand model of perfect competition and under a supply function equilibrium model where the competing firms are identical and the load has zero elasticity. They determine the electricity-price duration curve for a future period in the same way as is determined the LDC.

In [37, 15] the hourly market price of the entire medium-term horizon are previously determined by a forecast. While in [12, 10] the authors consider the price as an uncertain parameter and it is taken into account via a scenario tree. In [55] Markov chains are used considering the set of scenarios modeled as a Markov model. In this case historical data is available and some statistical techniques are used in order to model, estimate or forecast market prices.

There are procedures based on the simulation of a market-clearing algorithm, but most of them are for describing systems in the short-term. As an example, there is Villar and Rudnick [93] and Otero-Novas et al. [66] where these methodologies are applied to many successive hourly bids with the consideration of generation and demand uncertainty. The demand model covers a day, divided into chronological hourly blocks. It is an inelastic demand (it does not respond to price). In [7] a supply-bid function is employed for an hourly interval for an SGC in an oligopolistic market. A total strategic supply-bid function is obtained by aggregating each supply-bid function of each GenCo participating in the market. An estimate of the hourly market-clearing price is obtained and the estimates of the market-prices of the successive blocks of the LDC produce a market-price duration curve for each period.

In the case of sufficient information on the GenCos forming the pool and on the market auction (as there is in the MIBEL [89]), it is possible to set up price models using estimated market price functions that express the change in price level with the generation duration, as in the abscissas of the LDC of each period, see [59, 68]. In [68] the authors present a heuristic which defines a small subset of the LMCs likely to contain all the active LMCs at the optimizer. In addition, in [51, 56, 67, 50] is presented a pure pool model where the price function employed is linear and endogenous, as it depends on the hydro generation level. In [84] two market-price functions are used: the first is endogenous with respect to the hydro generation and with respect to other generation technologies; the second market-price function is endogenous with respect to the generation level of different GenCos, as it also is in [14].

If we take into account the oligopoly-based models, where market price is an endogenous variable, so many equilibrium models have been proposed for modeling medium-term planning. The most popular equilibrium model is the Nash-Cournot equilibrium. The Cournot oligopoly model assumes that the participants compete in quantities. Another model is the Bertrand one. It is a pure competition because this model assumes that the participants compete in price.

In [92] the Cournot model for predicting price duration functions is presented. They have developed analytical models and procedures based on generation system and load data for probabilistically representing two different market competition models, one that is based on perfect competition and another one is based on asymmetric oligopoly. In [39] the market price is defined through an inverse demand function.

In Day et al. [20] an extensive survey on possible participant strategies and the resulting equilibrium of the models are proposed. As pure Cournot equilibrium models tend to provide higher prices than those observed in reality, the authors introduced the concept of conjectured variations, which aims to represent the fact that rivals do react to high prices by producing more. In [4] and in [72] the market equilibrium is presented by means of a conjectural variation that relies on the modeling of the companies' strategic variables adjustment. They first formulate the equations of an equilibrium point and then use a minimization problem as an equivalent formulation. The market equilibrium is solved with stochastic programming techniques. In [4] under some assumptions, the solution of the market equilibrium conditions is equivalent to the solution of a minimization problem with a structure that strongly resembles classical optimization of hydro-thermal coordination.

In [5, 14] there is the extension of these models taking into account the uncertainties about the inflows and the demand. In [5] the market equilibrium is represented by means of a conjectural variation, where each GenCo bids are a linear function of the price and it is solved with stochastic programming techniques. The uncertainties considered are hydro inflows, fuel prices, system demand, generating units' failures and competence behavior. Pumped-hydro units are included in the model. In [14] the results are analyzed based on a conjectured-price-response market equilibrium representation in the Spanish electricity market. A pumped-hydro unit is considered.

In [29, 41] there are supply function equilibrium methods that have been applied in short-term. Authors in [73] presented a decomposition approach where a dual stochastic program mixed with a Cournot model is proposed for medium-term hydro generation policies. [73] is one of the first works on modeling hydro operation in deregulated markets using Dynamic Programming.

The equilibria in power planning can be calculated with so many algorithms: the explicit use of complementary conditions, some simulation methods and, as in the Nikaido-Isoda Relaxation Algorithm (NIRA, [63]), the numerical derivation of the agents' response function. As an example of the first algorithm, there is the direct exploitation of complementary conditions presented in Hobbs [33] which leads to mixed complementary problems. In these models most functions are linear or quadratic and constraints are linear. They simulate bilateral markets. This method depends on the existence of a solution to a system of equations and inequalities, which result from mixed complementarity conditions.

Contreras et al. [18] were the first to apply the NIRA algorithm into the short-term electricity market modeling. In [56] the NIRA algorithm is used in order to find the equilibrium solution. In Tesser et al. [84] a heuristic to avoid having to use an exponential number of LMCs is used and the consideration of exogenous stochastic factors is included. An endogenous price function of load duration is used as input of an oligopoly problem solved using NIRA.

Finally, in [40, 24] there is no market, the authors only consider the minimum cost problem.

3.1.3 The consideration of non-LMCs

Regarding the existence of non-LMCs in the optimization problem, [4, 5] include constraints related to hydro generation, where the final level for each hydro unit is taken into account and there is an energy balance constraint for each period considered and for each hydro unit, they also include the performance of pumping for hydro unit. In [10] there are single-unit constraints for thermal plants and balance equations for reservoirs.

In [70] is presented a pumped storage hydro plant and a reservoir capacity of pumped-storage generators constraint is introduced. They also consider emission limit constraints for CO_2, SO_x and NO_x. The authors in [12] use take-or-pay contracts constraints for each gas unit and for each time period and, as they employ hydro generation, there is a constraint regarding the water reservoir management that limits the amount of water that is stored in the reservoir. Hydro systems constraints over different time periods and environmental constraints intra-periods are considered in [71]. Although in [24] there is no market consideration, authors have fuel procurement, fuel consumption and fuel stock restrictions, and some constraints related to water flow balancing and hydro generation. In [76] it is also considered the water balance reservoir.

In [84, 81] the authors use other constraints such as those that include the minimum annual generation time for capacity payments in the MIBEL, time-dependent upper bounds on the generation of "special regime" pseudo-units (e.g. renewable sources) based on historical records, and the possibility of considering take-or-pay gas contracts for fuel-gas units. In [59, 56, 67, 68, 51, 50] the expected hydro generation in several basins is included, a limit on the availability of some fuel types is considered, and there is a constraint on the minimum generation time over a period by certain units and special-regime minimum generation limits. Furthermore, in [51, 50] pumping balance constraints are also considered.

3.1.4 Consideration of stochasticity, of risk, of Bilateral Contracts and of Renewable energies

There are different methodologies related to the scenario tree generation for multi-period processes. As an example, there is the bootstrapping where resampling and statistical estimation are used to predict the behavior of systems or processes and, thus, to represent scenarios. In the literature, there are several examples using this methodology, however, most of them are dedicated to economics, for example see [1, 54]. In addition, in [79] the authors propose a model to generate natural inflow energy scenarios in long-term operation planning using Periodic Autoregressive Models estimated via Bootstrap.

Another technique is the use of some sophisticated statistical models plus the use of discretization methods. Given the statistical model of the uncertainties, a scenario tree can be generated by different approaches: random sampling, adjusted random sampling, moment matching, etc. Høyland and Wallace [35] is an example of deterministic moment matching. They solve a non-linear optimization model where the decision variables are the returns and the probabilities of the event tree while the constraints and the objective function enforce the desired statistical properties: the distance between the moments of the stochastic process to be approximated and the moments of the approximating event tree are minimized, and the scenario tree is directly generated with the desired number of nodes. As an example of random sampling we have Kouwenberg [42] where a method is suggested to simplify the tree fitting problem by applying it recursively at each stage. The author also compares moment matching, random sampling and adjusted sampling. Li-Yong et al. [46] present a survey of each method.

Heitsch and Römisch [30] and Dupačová et al. [22] proposed other methodologies based on a scenario reduction heuristic that generate decision-stable event trees. They start from a dense event tree and they present a backward and a forward algorithm that produces a reduced event tree characterized by a probability space close to the original one (under a given probability metric). In Tesser [81] it is developed a technique to generate event trees: starting from a VAR model, a dense event tree is generated by a mixture of random sampling and tree fitting. Then this tree is reduced to the desired size by using the scenario reduction proposed in [30]. Monte Carlo discretization (see Glasserman [28]) is used in order to generate the realizations of the random variables.

Regarding the risk of profit losses, many studies have been published. In [12] a three-step procedure is presented for solving a formulation of the risk-management problem. Some scenarios are defined for fuel prices, water inflows and demand. The risk is measured with the CVaR. The basic formulation through the CVaR of limiting the risk of profit losses of the market participants can be found in [83], where an application to a realistic MTGP equilibrium problem is also presented. Further developments to contemplate multi-period risk measures (in order to adopt decisions in each period as new information arrives) is described and demonstrated in [81].

Forward contracts and Bilateral Contracts become a tool to hedge against price volatility in the medium term, due to uncertain hydro inflows and market prices. A medium-term energy procurement model for a consumer with self-production and access to BCs can be found in [15] where the authors propose a mixed-integer linear programming model, and for a large consumer with access to several types of BCs with known tariffs and bounds in [13]. The authors minimize the expected value of the procurement cost while limiting its risk by including risk aversion through the CVaR methodology. In [39] it is assumed that a Nash-Cournot equilibrium with an inelastic demand is reached in hydrothermal systems by the players. Bilateral Contracts are used for reducing market power.

In [38, 95] risk-benefit functions are defined and an iterative negotiation is proposed before the agreement on a BC is sealed. They propose a practical process in which the bargainers take both benefits and risks into account. The authors claim that their process will lead to agreement on a mutually beneficial and risk-tolerable forward bilateral contract. In [49] a multi-agent negotiation scheme is proposed to analyze the behavior of a bilateral market. In [76] a mixed model is proposed that optimizes both the water allocation resources and the forward contracts for a small producer that owns only hydro generation. A risk-averse decision maker would be willing to accept a solution with a lower expected revenue. Here the probability of different scenarios is considered and a scenario tree is constructed. In this formulation, favorable decisions are rewarded and unfavorable ones are punished. Uncertainties are considered, such as water inflows or market price, which is exogenous to the company's generation. They are represented through scenarios. Bjørkvoll et al. [10] present an approach for generation planning and risk management as two-phase decisions. The SGC is considered a price-taker company. The model is decomposed by units, without any constraint that relates their generation, so the model can be optimized independently for each unit. In it, the risk is considered by introducing the bilateral contracts markets and the future ones.

Attention has also been given to pumping together with BCs, as both divert generation resources from satisfying the auctioned load. There are works on the short-term operation of pumped storage units in electricity markets selling its generation and buying energy for pumping based on a previously determined market-price hourly forecast, for a week in [48], and for a day and establishing a BC price in [37].

Renewable sources is an active area of research due to the fact that there are many published studies that are concerned with the impacts of N-DRs generation in electricity planning, for example [86, 74, 6, 53]. However, most of them deal with short-term applications. In [6] integration of WP has impacts on the resulting prices at the electricity market. During hours with high wind power, electricity prices decrease. When WP can cover all the demand, prices are zero. In [74] a new methodology to analyze the operation planning with an emphasis on keeping reserves is presented.

Some examples of medium or long-term analysis are [70] where WP and solar cells are considered as renewable sources. Another example is [21] where the LDC of a future year is modified while wind is used. It combines analysis of generation economics, load and wind-generation characteristics.

In [40] is described the process to calculate the benefit of the integration of WP plants in a power system: the effect of WP is deducted from the LDC and then the remaining LDC is matched with the other generation units. In [85] convolution techniques are employed to assess the future performance of a grid-linked hybrid system of WP and SPV. Meibom et al. [53] describe a method to study the operational impacts of WP penetrations in Ireland's power system. They solve a stochastic mixed integer linear optimization scheduling model with a succession of several short-term problems. This approach does not consider any non-LMC constraints and uses a 3-day rolling-horizon over a year employing short-term procedures. Forced outages of power plants are considered through time series of mean time to failure and mean time to repair.

3.2 THE LOAD-MATCHING HEURISTIC

Efficient procedures to account for the LMCs using a direct solution method (as a quadratic programming solver) in the solution of the medium-term planning (18) entails having to generate and to handle all the LMCs (18b), which is not efficient given the very large number of these linear inequality constraints. However, through the unit loading order principles (see §2.3.1) it is known that only a reduced subset of, at most n_u LMCs, will be active at the optimizer for each LDC to be matched.

The load-matching heuristic (LMH) presented in [68] is a means of avoiding having to create and employ an exponential number of (linear inequality) LMCs in medium-term planning optimizations. Its objective is to determine a subset of LMCs that contain all LMCs active (satisfied as equalities) at the optimizer. The heuristic is a succession of optimizations employing a very reduced subset of LMCs, which is enlarged after each optimization with some more LMCs selected taking into account the last optimal results. The optimizations referred to before is the medium-term generation planning model (15) except in the LMCs (15b). It is based on the fact that each solution corresponds to a loading order, and the loading order can be univocally represented by a subset of *nested* LMCs.

3.2.1 The first load-matching heuristic

Computational test cases in Pagès and Nabona [68] show that solutions from the heuristic were generated much faster than when using some column generation method and are good enough as they reaches a value close enough to the actual value. Thus, the LMH can be considered a worthwhile alternative.
The outline of the heuristic is as follows:

1. **Initialization part**: Initialize the list of LMCs with the all-one constraint $L = \{1, 2, ..., n_u\} = \{\Omega\}$ plus the upper-bounds (6a) that are in the subset of LMCs (11b) with $|\omega| = 1$, and solve a recast version of problem (11) with just the all-one LMC (i.e. the energy balance constraint where all units have a coefficient 1).

2. **Self-ordering**: where the subset ϑ of units j whose generation in the former solution is at (or close to) its upper bound, $x_j \approx \overline{x}_j$, is formed. The LMCs made with any subset of units in ϑ is added to the list L and, again, the problem is solved. For notation purpose, let $|\vartheta_0|$ indicate the cardinality of ϑ as it is now.

3. **Step-by-step order**: while set ϑ has less elements than Ω, find the unit k that is not yet in ϑ and that is nearest to its upper bound. The set ϑ and the list L are updated and the problem is resolved.

The experience with the application of the heuristic shows that it is highly efficient, allowing the solution of problems with big n_u value in short execution time, and it is reliable as its solution corresponds to that found using the full Bloom and Gallant formulation.
The total number of LMCs generated using the LMH is $2^{|\vartheta_0|} + n_u - |\vartheta_0|$ while using the complete Bloom and Gallant formulation there are $2^{n_u} - 1$ of LMCs.

3.2.2 The modified load-matching heuristic

Sometimes the heuristic LMH fails to generate some of the LMCs that are active at the solution. In Nabona [56] the modified load-matching heuristic (mLMH) is presented as an elaborated heuristic to improve the LMH.

The mLMH for dealing efficiently with the LMCs (11b) is a finite multistage procedure to generate a reduced subset of LMCs that will most likely contain the *active* LMCs (satisfied as equalities) at the optimizer. The strategy of this heuristic is to use the same number of sets as LMH uses but with the addition of other sets.

The *all unit but one* and the *all unit but two* sets will be added at first stage of the heuristic (the initialization part) because it will restrict the number of units that are or close to its maximum value in the cartel solution. The *all unit but three* set of the reduced set of the nine less loaded unit (UN9) plus all units in the complementary set of all units not in the set of nine less loaded units (UX9). And all combinations of the subset of the seven less loaded units (UN7) plus all units of its complementary set (UX7).

The outline of the mLMH heuristic is:

1. **Initialization part**: Initialize the list of LMCs with the all-one constraint, all-but-one unit and all-but-two units, $L = \{\Omega\} \cup \binom{\Omega}{n_u-1} \cup \binom{\Omega}{n_u-2}$ plus the upper bounds and solve the problem.

2. **Self-ordering**: where the subset ϑ of units j whose generation in the former solution is at (or close to) its upper bound, $x_j \approx \overline{x}_j$, is formed. The LMCs made with any subset of units in ϑ is added to the list L and, again, the problem is solved. For notation purpose, let $|\vartheta_0|$ indicate the cardinality of ϑ as it is now.

3. **Low-load unit ordering using sets UN7 and UN9**: only units j with ratio $\rho_j < \frac{1}{2}$ are included in UN7 and UN9. This produces that these two sets may contain less than 7 and less than 9 elements respectively. Update list L and solve the problem.

4. **Step-by-step order**: while set ϑ has less elements than Ω, find the unit k that is not yet in ϑ and that is nearest to its upper bound. The set ϑ and the list L are updated and the problem is resolved.

All stages in the mLMH are associated to LMC generation followed by an optimization solution producing fresh unit generation values.

In comparison with the LMH, the mLMH performs better because more LMCs that are active at the optimizer are generated by the modified heuristic in all stages.

3.3 THE REPRESENTATION OF HYDRO GENERATION

In the medium-term is common to represent an entire basin with several hydraulic reservoirs as a single equivalent reservoir with constant head. This reservoir has a generation that is proportional to water discharge in order to avoid using too many variables. In the resulting replicated network, natural hydro inflows are transformed in electricity that goes through a network constituted by a single node in each subperiod as it is shown in Figure 12.

When in a power pool GenCos have hydro generation resources, its use over the periods is also optimized. There are several ways of representing hydro generation in medium-term planning. A simple model is to consider the generation x_h^v of one or of several hydro basins

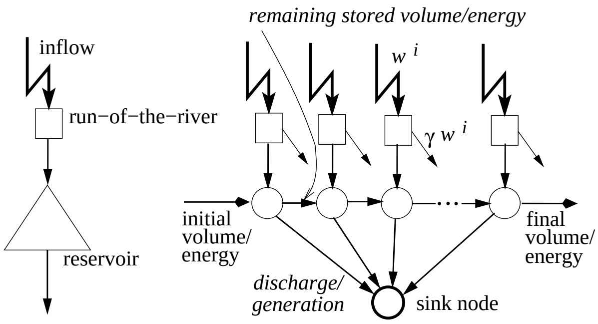


Figure 12: Simplification of reservoir systems

over period $i(\nu)$ as if it were from a pseudo-unit with zero cost, zero outage probability and estimated capacity c_h, whose generation is subject to:

$$\left.\begin{array}{l}\sum\limits_{\nu\in\mathcal{A}_\lambda}\left(u_h^\nu - w_h^\nu\right) = v_h^0 - v_h^\lambda \quad \forall\lambda\in\mathcal{L}\\ \sum\limits_{\substack{\nu\in\mathcal{A}_\lambda|\\ i(\nu)\leqslant k}}\left(u_h^\nu - w_h^\nu\right) \leqslant v_h^0 \; \forall k\in 1,\dots,n_i-1,\ \forall\lambda\\ u_h^\nu \geqslant 0 \quad \forall\nu\in\mathcal{N}\\ x_h^\nu = u_h^\nu + \gamma w_h^\nu \quad \forall\nu\in\mathcal{N}\end{array}\right\} \forall h\in H \tag{34}$$

where it is used the auxiliary variables u_h^ν to represent the generation of an equivalent regulation reservoir, and the hydro generation x_h^ν corresponds to the addition of u_h^ν and the upstream run-of-the-river generation γw_h^ν, which is a predetermined fraction γ of the energy of the natural inflows w_h^ν over period $i(\nu)$. The parameter v_h^0 is the energy stored in the reservoir system before the start of the first period and v_h^λ that stored at the end of the last period in path λ. $H\subset\Omega$ is the set of hydro pseudo-units considered.

Concerning how to take into account the hydro generation in medium-term problems, in Tesser et al. [84] are presented the energy/hydro balance constraints. These constraints ensure that, for each scenario, the initial volume at the reservoir must be equal to the final volume. However, in this thesis, as it is presented in (39), the final stored energy (i.e. the final volume at the reservoir for each scenario) may be different depending on whether hydro inflows have been higher or lower than the expected inflow.

3.4 THE MULTI-PERIOD MEDIUM-TERM PUMPING MODEL

The aim of hydroelectric plants is to convert the potential energy of stored water into electricity. They can be reservoir, run-of-the-river or pumped-storage hydroelectric plants. A pumping plant usually has two separate reservoirs located at two different levels, an upper and a lower one and its main characteristic is that its machinery can be used either for pumping water from the lower to the higher reservoir consuming electricity, or for releasing water from the upper reservoir through the turbine generating electricity. Reversible turbine units allow the pumping and the generation using the same turbine and the same pipe joining the higher and the lower reservoir. Generation companies use this type of electric energy storage for load balancing. During periods of low demand (and low market price), for example at night, they pump the water to the upper reservoir by using electricity (at a low cost). During periods of high electricity demand (and high market price), power is generated by releasing the stored water through turbines in the same way as conventional hydropower station.

In Bloom and Gallant [11] is described and demonstrated the procedure for modeling pumped-storage systems and conventional units in a single medium-term period of generation planning with pumped storage and conventional units while matching the LDC through the Bloom and Gallant formulation.

This procedure was extended in Marí and Nabona [51] to a multiperiod stochastic problem with N-DRs, which involves matching three different LDCs in each period (see §5.3), and it will be formulated here for problems (with no SPV generation) where a single LDC is to be matched in each period. It requires the increase of the LDC load of each period by the capacity c_P of the pumped-storage unit, the use of two extra units of capacity c_P, zero failure probability and zero cost: a pseudo-unit, not receiving revenue from the market, for compensating the extra load, with expected generation x^{ν}_{Cmp} and the other with expected generation x^{ν}_{Phyd} for hydro generation from pumped water, and the consideration of a variable v^{ν}_P with the pumped energy stored in the upper reservoir with $0 \leqslant v^{\nu}_P \leqslant \bar{v}_P \, \forall \nu$ with $\bar{v}_P$ being the energy capacity of the pumping-station upper reservoir. The energy spent in pumping is transformed into stored energy through an efficiency coefficient $\mathrm{eff}_P < 1$ ($\mathrm{eff}_P = 0.75$ in our data).

The total extra energy $c_P T^{i(\nu)}$ in the LDC of node ν will be split in three parts, each of which produced or consumed in non-overlapping hours within the period length $T^{i(\nu)}$: the energy of the compensating unit x^{ν}_{Cmp}, the hydro energy generated in the pumping station x_{Phyd}, and the pumping by the other units. This pumped energy is

$$c_P T^{i(\nu)} - x^{\nu}_{Cmp} - x^{\nu}_{Phyd} \geqslant 0 \quad \forall \, \nu \in \mathcal{N} \, . \tag{35}$$

In Figure 13, the left picture represents the LDC for a month, on the right picture it can be seen the same LDC but with the capacity c_P increased. The green slice on the left picture represents the total extra energy added to the LDC and it will be split as it is shown in the right picture. The order in which these generations are situated within the extra load is relevant. GenCos will pump water to the upper reservoir in the base-load power hours because they have a low market price, while in the peak load or peak market price hours, they will release hydro from pumped water. The compensating unit will be in the middle of these two other generations in order to satisfy the extra load in hours with neither pumping nor generation from pumped hydro.

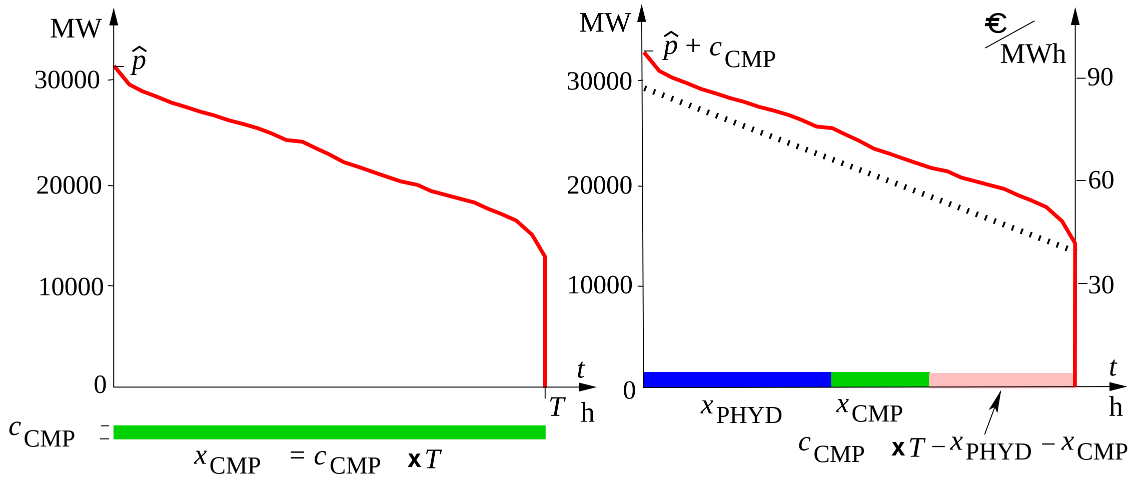


Figure 13: Representation of the LDC without pumping storage for one period (left), the LDC extended with the extra pumping load (right) and the linear market-price (dotted line)

The stored energy in the upper reservoir could only be a fraction of the pumped energy, $\mathrm{eff_P}(c_P T^{i(\nu)} - x^{\nu}_{Cmp} - x^{\nu}_{Phyd})$. Thus, assuming that the initial, and final, energy stored in the upper reservoir is ν^0_P, the energy stored in the pumping-station upper reservoir must satisfy respectively for the root node, numbered one, for nodes ν not being the root nor the leaf nodes, and for the leaf nodes $\lambda \in \mathcal{L}$:

$$
\begin{aligned}
&\nu^1_P = \mathrm{eff_P}\left(c_P T^1 - x^1_{Cmp} - x^1_{Phyd}\right) - x^1_{Phyd} + \nu^0_P\,,\\
&\nu^{\nu}_P = \mathrm{eff_P}\left(c_P T^{i(\nu)} - x^{\nu}_{Cmp} - x^{\nu}_{Phyd}\right) - x^{\nu}_{Phyd} + \nu^{\mathrm{pred}(\nu)}_P \quad \forall \nu > 1 \text{ and } \nu \notin \mathcal{L}\,,\\
&\mathrm{eff_P}\left(c_P T^{i(\lambda)} - x^{\lambda}_{Cmp} - x^{\lambda}_{Phyd}\right) - x^{\lambda}_{Phyd} + \nu^{\mathrm{pred}(\lambda)}_P = \nu^0_P \quad \forall \lambda \in \mathcal{L}\,,
\end{aligned}
\tag{36}
$$

where $\mathrm{pred}(\nu)$ is the predecessor node of node ν in the scenario tree, which is in the former period of that of ν.

Both (35) and (36) are included in the model as non-LMCs.

3.4.1 The pumping load fee

Even though the pumped-storage scheme means a net energy consumption overall, the pumped-storage operation should produce a profit increase provided that both the satisfaction by the conventional units of the extra load (35) that the pumping means for the conventional units and the hydro generation that is produced from it are remunerated at market price. It is assumed that the extra load (35) is also paid at market price, thus requiring that this fee be deducted from the profit function to maximize.

The pumping extra load fee, to be deducted from the profit objective function of each node ν in problems considering medium-term pumping, can be calculated as:

$$c_P \int_{\frac{x^{\nu}_{Cmp}+x^{\nu}_{Phyd}}{c_P}}^{T^{i(\nu)}} \left(b^{\nu} + l^{i(\nu)} t\right) dt = \begin{aligned}[t] & b^{\nu}\left(c_P T^{i(\nu)} - x^{\nu}_{Cmp} - x^{\nu}_{Phyd}\right) \\ & + \frac{l^{i(\nu)}}{2c_P}\left\{\left(c_P T^{i(\nu)}\right)^2 - \left(x^{\nu}_{Cmp} + x^{\nu}_{Phyd}\right)^2\right\}, \end{aligned} \tag{37}$$

where b^{ν} is as in (40). The integration limits in (37) have been taken so that the pumping load-duration segment corresponds to the lowest market prices.

The model description above is for a single pumping-scheme in the system. It is easy to extend the model to several different pumping-schemes in the system, each having a different set of generation units devoted to pumping for each different scheme.

3.5 THE PROFIT FUNCTION

In Tesser et al. [84], the dependence of the market-price function on endogenous factors was studied. There the market price intercept is linearly dependent on hydro generation and other technologies, such as nuclear, coal and special regime. However in order to compare it with the profit function presented in §5.1 in Chapter 5 it has been simplified here (38) taking into account only the hydro generation (for simplification purposes notation from [84] has been changed to fit the notation in this thesis).

$$b^i = b^i_0 - d^i_0 \sum_{h \in H} x^i_h . \tag{38}$$

In this case, the market-price function b^i only depends linearly on the hydro generation while in (40) the term b^{ν}_0 has the influence of the SPV and the WP generation. These small differences between these two models cause remarkable changes in the results obtained.

3.6 THE NIRA ALGORITHM WITHIN THE MODIFIED LOAD-MATCHING HEURISTIC

As explained in §2.8 the NIRA procedure is a successive maximization of $\max_{x_j,y} \sum_{k=1}^{K} \phi_k(x_k|\widehat{\mathbf{x}})$ subject to LMCs, non-LMCs and bounds, and an update of the fixed values $\widehat{\mathbf{x}}$. As the LMH and the mLMH are a successive optimization procedure adding more LMCs, in order to find the equilibrium solution it is necessary to change the objective function. In Nabona [56] there is described a way to introduce some changes in the mLMH procedure in order to take into account the modifications required by the NIRA procedure.

The mLMH must have the changes listed below:

- At the end of first step (the initialization part), it is necessary to solve the endogenous cartel instead of solving the cartel one.
- At the end of the self-ordering part, again it is necessary to solve the endogenous cartel instead of solving the cartel problem.
  Additionally it is necessary to store in an auxiliary variable the value of the hydro generation of the SGC $\widehat{x}_{H_{SGC}} = \sum_{l \in H_{SGC}} x_l$ and the value of the hydro generation corresponding to the RoP $\widehat{x}_{H_{RoP}} = \sum_{l \in H_{RoP}} x_l$.
- At the end of the low-load unit ordering using sets UN7 and UN9, set the NIRA coefficient $u : 0 < u \leqslant 1$ (e.g. $u = 0.4$). Resolve the equilibrium problem and update the former hydro solution $\widehat{x}_{H_{SGC}} := u \times x_{H_{SGC}} + (1-u) \times \widehat{x}_{H_{SGC}}$, $\widehat{x}_{H_{RoP}} := u \times x_{H_{RoP}} + (1-u) \times \widehat{x}_{H_{RoP}}$ and set the number of NIRA iterations at 1 $ite_{NIRA} := 1$.
- In each loop of the step-by-step order, it is necessary to solve the equilibrium problem instead of solving the cartel problem and update $\widehat{x}_{H_{SGC}}$, $\widehat{x}_{H_{RoP}}$ as in the previous item in this list and $ite_{NIRA} := ite_{NIRA} + 1$.
- Finally, it is necessary an extra loop for completion of NIRA iterations. If the number of NIRA iterations is less than 15 then the equilibrium problem has to be solved and $\widehat{x}_{H_{SGC}}$, $\widehat{x}_{H_{RoP}}$ and ite_{NIRA} must be updated.

3.7 CONVERGENCE STRATEGIES OF THE NIRA ALGORITHM

Due to the fact that the objective function is indefinite, it is possible that NIRA algorithm does not converge to an equilibrium point. In order to accelerate the convergence in the algorithm some procedures have been proposed such as a dynamic adaptation of step length u. This adaptation should be included in the algorithm.

The structure of the NIRA procedure with dynamic adaptation of step length u is:

- $\widehat{\mathbf{x}} \leftarrow \mathbf{x}_0\,, \quad \Psi^{old} \leftarrow +\infty\,, \quad u \leftarrow 0.7\,, \quad \widehat{u} \leftarrow 0.01$
- **repeat until** $\Psi^* \leqslant \epsilon$

    obtain $Z(\widehat{\mathbf{x}}) = \mathbf{x}^*$ by solving $\max\limits_{\mathbf{x} \in \mathcal{X}} \Psi(\widehat{\mathbf{x}}, \mathbf{x})$

    compute $\Psi^* = \Psi(\widehat{\mathbf{x}}, Z(\widehat{\mathbf{x}})) = \sum\limits_{k=1}^{K} \big(\phi_k(x_k^* | \widehat{\mathbf{x}}) - \phi_k(\widehat{\mathbf{x}})\big)$

    $\Delta\Psi \leftarrow \Psi^* - \Psi^{old}\,, \quad \Psi^{old} \leftarrow \Psi^*$

    if $\Delta\Psi \leqslant 0$

    then (we are approximating the solution)

        $\widehat{\mathbf{x}} \leftarrow uZ(\widehat{\mathbf{x}}) + (1-u)\widehat{\mathbf{x}}$

        if $u \leqslant 0.5$ **then** $u \leftarrow 1.01u$

    else (we are moving away from the solution)

        $\widehat{\mathbf{x}} \leftarrow uZ(\widehat{\mathbf{x}}) + (1-u)\widehat{\mathbf{x}}$

        $u \leftarrow 0.9u$

4 THE GENERATION OF AN EVENT TREE FOR THE RENEWABLE ENERGY SOURCES

4.1 INTRODUCTION

Renewable sources are an important aspect of sustainability because the energy that they produce is naturally refilled or regenerated. The most frequently used sources are water-courses, wind and solar power, and biomass. These types of generations are also called *green energy* as they are non-polluting, i.e. the emissions from the electricity generation, if any, do not affect the environment. Their importance is currently growing due to their null greenhouse effect, which mean no global warming, and due to the greater international awareness and concern about the problem.

The dispatchable generation refers to electricity sources that can be turned on or off at the request of the power system operator. We can establish a classification within this type of generation depending on whether these energies are dispatchable or not. As an example of dispatchable renewable sources (DR), we have the hydro generation. The natural water inflows are stored in a reservoir and the electricity comes from the potential energy of stored water driving a water turbine and generator. Non-dispatchable renewable resources (N-DR) are, for example, wind and solar photovoltaic generation. While we can regulate hydroelectric generation by discharging more or less, or no water, wind and solar photovoltaic generation cannot be regulated. If there is sunlight, the solar cells will generate electricity without the possibility to turn them off (the same happens in the wind farms).

In recent years, non-dispatchable renewable resources have had an increasing penetration in the MIBEL (in 2013, N-DR resources covered the 26% of the electricity demand in Spain) so other technologies have had to reduce its generation share. Specific models to represent the N-DR sources along with the classical dispatchable sources must be developed and tested. The specific models proposed in this thesis should:

- use Stochastic Programming (see §2.9), [51], and
- employ the probabilistic method of load matching for generating units described in §2.1.2 [3, 11, 51].

4.2 CHARACTERIZATION OF THE RENEWABLE GENERATION

4.2.1 Stochastic formulation of Hydro Generation

Hydro generation is one of the most favored means of electricity production, because water is free. However, water inflows are uncertain and the use of stored water can be restricted. Besides being uncertain, water inflows (in Spain) show a similar pattern through the year as it can be seen in Figure 14. Scenarios will be developed for water inflows w_h^ν (see (34) in §3.3) and then will be introduced into the stochastic model as it can be seen in §2.9.3.

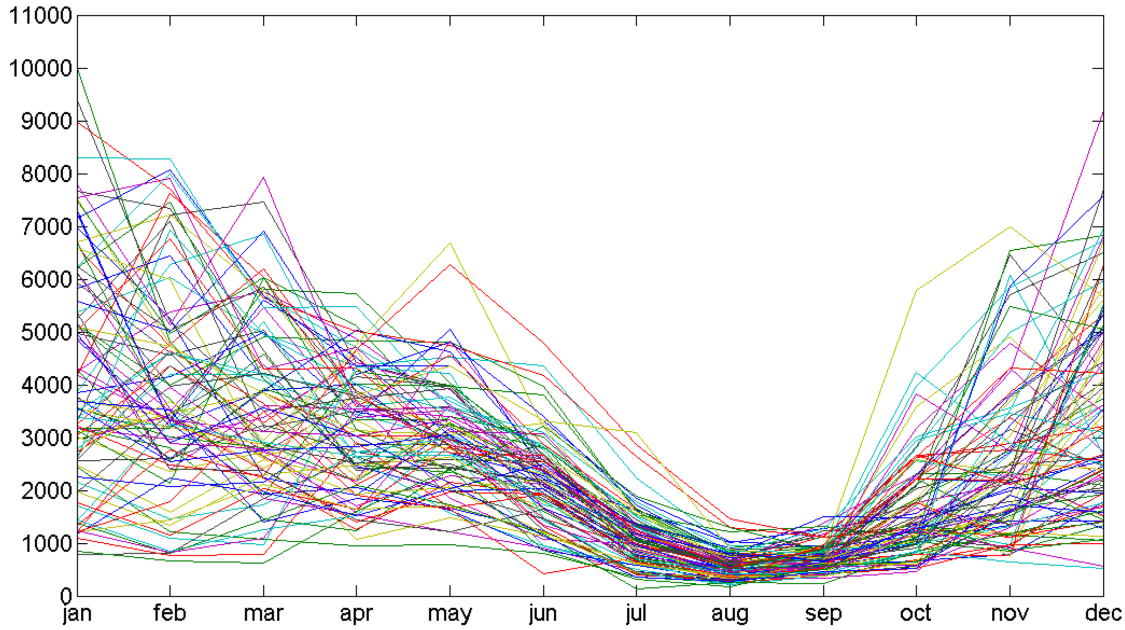


Figure 14: Available Monthly Hydro Generation (GWh) in Spain between 1920 and 2012

4.2.2 Stochastic formulation of Wind Power Generation

The Wind Power (WP) generation is obtained by the effect of air currents. It is an abundant, renewable and clean resource and that contributes in the reduction of harmful emissions, and does not produce any type of pollution. However, its biggest drawback is its intermittency, which makes it difficult to predict its generation.

In previous studies [67, 68, 84], WP is included inside a special regime unit when formulating the model. The generation within this special regime unit has a specific treatment in the Spanish power pool as all its power production is accepted in each hourly auction.

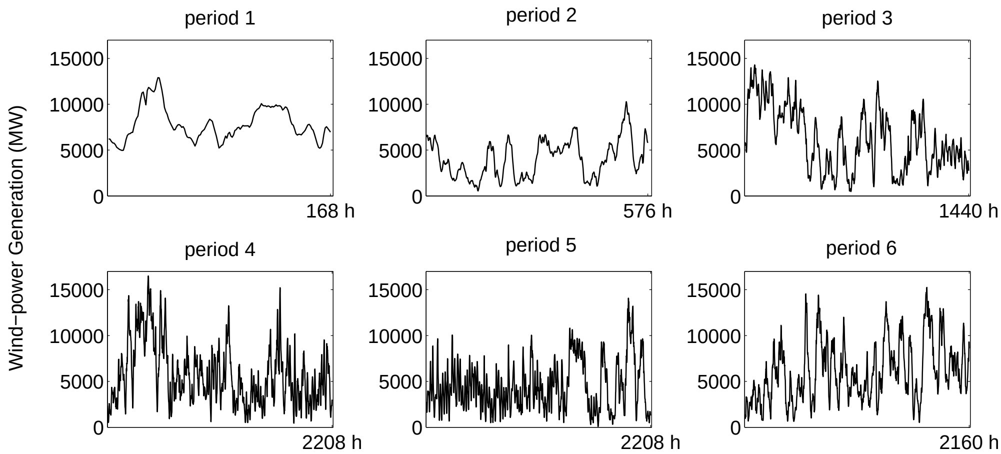


Figure 15: Hourly WP generation in Spain split into 6 successive periods of different length from October 2012 to October 2013

In Figure 15, there is the WP generation in Spain from October 2012 to October 2013. In each period, it can be seen that there is some variability and it is difficult to see any pattern. Due to the variability of WP, we propose a more realistic approach when incorporating WP: WP generation is considered as a zero-cost pseudo-unit whose capacity varies at each period; furthermore, the failure probability associated with the unit is modeled using concepts unique to WP generation.

The analysis of WP data can be shown in Figure 16. The left graphic depicts the evolution of WP generation over March 2012 in Spain. In order to adapt it to the medium-term planning procedures, we have split it into three segments as in the figure. The lower segment corresponds to a generation capacity that leaves out a 15% of the energy, which would be equivalent to a failure probability of 15%; we will call it the *base* WP unit. The upper part contains WP spikes that only amount to 1.5% of the total WP energy, and this generation will be ignored. The middle segment has the capacity from the top of the lower segment to where the third segment starts; its corresponding failure probability is high (66% for this month). This middle segment will be referred to as the *crest* WP unit.

The use of the base and the crest unit described above is a compromise between model accuracy and computational efficiency. In this regard, using a single unit would be inaccurate on two counts: because a single failure probability of about 40% would not adequately represent the 65% failure rate of the crest section nor the 15% base section, and because the profit calculation of WP through the approximation as a rectangle of width x_W/c_W (14), with $c_W = c_{Wbs} + c_{Wcr}$, where subindex $_W$ refers to the single WP unit, of the WP energy area in the generation duration curve is less accurate than the approximation as a rectangle of width x_{Wbs}/c_{Wbs} plus another one of width x_{Wcr}/c_{Wcr}, especially if the rectangles correspond to units with wide different failure probabilities. Discarding the 1.5% energy spikes is a loss of accuracy compensated by a gain of accuracy due to a much lower c_{Wcr},

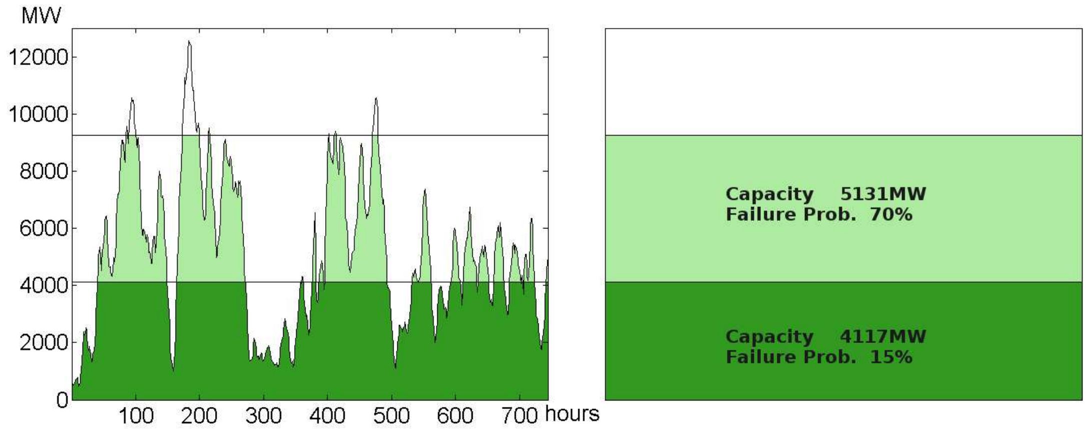


Figure 16: Hourly WP generation during March 2012 in Spain (left) and capacity and failure probability of equivalent units (right)

and a lower q_{Wcr}.

Using two (or more) units for representing WP incurs in the error of not considering that the 15% unavailability hours of the base unit are always a part of the 65% unavailability of the crest unit (see Figure 16, left), while the assumption of the probabilistic technique is that the failures of units are independent events. This error is less important than the failure probability mismatch associated to modeling WP with a single unit. Figure 17 illustrates how using the base and the crest WP units fits much better the LDC that results from deducting the WP generation than when using a single WP unit with equivalent parameters $c_W = c_{Wbs} + c_{Wcr}$ and

$$q_W = 1 - \frac{c_{Wbs}(1 - q_{Wbs}) + c_{Wcr}(1 - q_{Wcr})}{c_W}.$$

The convolutions (2), scaled with the duration $T = 744$ h (of March), have been used with March 2012 MIBEL LDC and WP base and crest capacities 4117 MW and 5131 MW, and failure rates 15% and 70% respectively of Figure 16, to produce the resulting modified LDCs when having loaded the base and crest units or the single equivalent unit shown in Figure 17 (right). Even though the case with the single equivalent WP unit is error free from the viewpoint of the independence of the unit failures, the fit of the base plus crest units is much closer to the real LDC minus reordered WP (approximated by a seventh degree polynomial in Figure 17) than the one with the single WP unit.

On the other hand, the larger the number of units, the larger the problem is and the longer it takes to solve (see §5.5). Using two units instead of a single one to represent WP means a larger problem. The use of three units to represent WP would still be more accurate but the marginal increase in accuracy may not compensate the increase in the use of computational resources for scenario generation and problem solving.

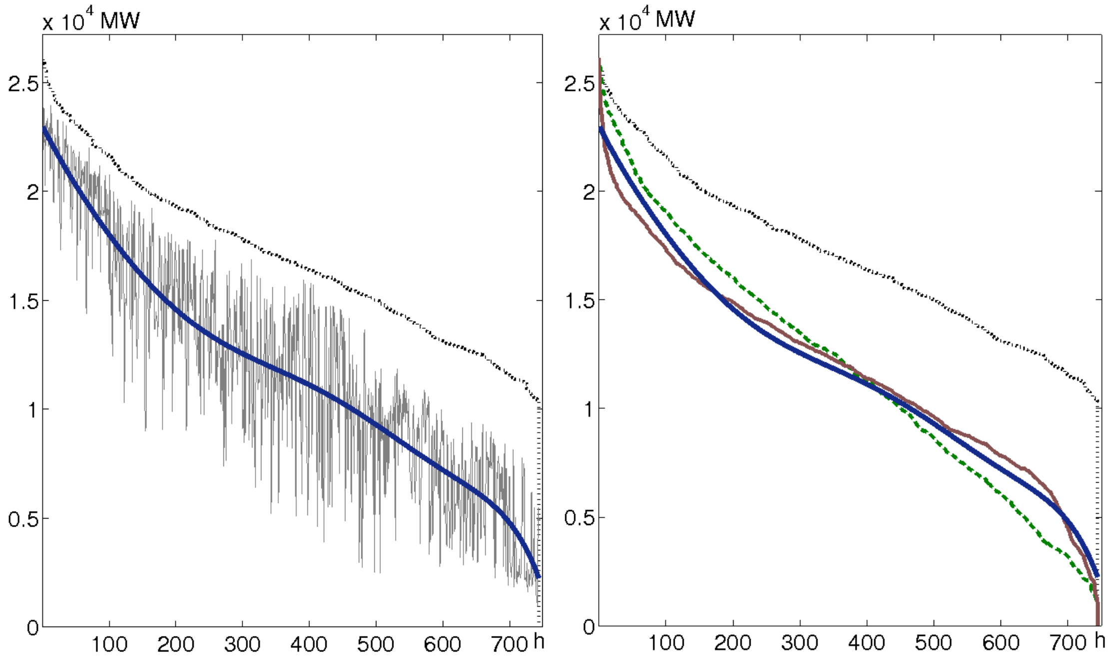


Figure 17: MIBEL's March 2012 LDC –dotted– and thick blue polynomially approximated LDC minus WP (left), and LDCs after having loaded base and crest WP units –continuous–, or the single equivalent WP unit –dashed–, together with LDC minus WP polynomial (right)

It is important to remark the effect that a unit such as the crest WP has on the generation of the rest, given the high failure probability of the crest WP unit (its high unavailability). To appreciate it we consider that the probabilistic matching of an LDC is independent of the order in which the generation units are loaded, but the expected generation of each unit does depend on the order of loading. Cost-free (N-DR) WP units will be loaded first and the rest of units would have to match the remaining load still to be matched. We could thus compare the load-survival function after loading (using (2)) the crest WP (with high q_{Wcr}) and compare it with the loading of an energy-equivalent unit (with lower capacity c_{ee} and lower failure probability q_{ee}, such that $c_{Wcr}(1 - q_{Wcr}) = c_{ee}(1 - q_{ee})$. Figure 18 shows the difference on the load-survival function of a simple polygonal LDC.

The expected generations of the rest of units for matching either the left load-survival function of the bottom of Figure 18 or the right one are going to have a significantly lower generation duration than those in the bottom-left figure for most of the units.

4.2.3 Stochastic formulation of Solar Photovoltaic Generation

The Solar Photovoltaic (SPV) generation is created by the conversion of sunlight into electricity using solar panels. As WP generation, SPV is a renewable and clean resource but it

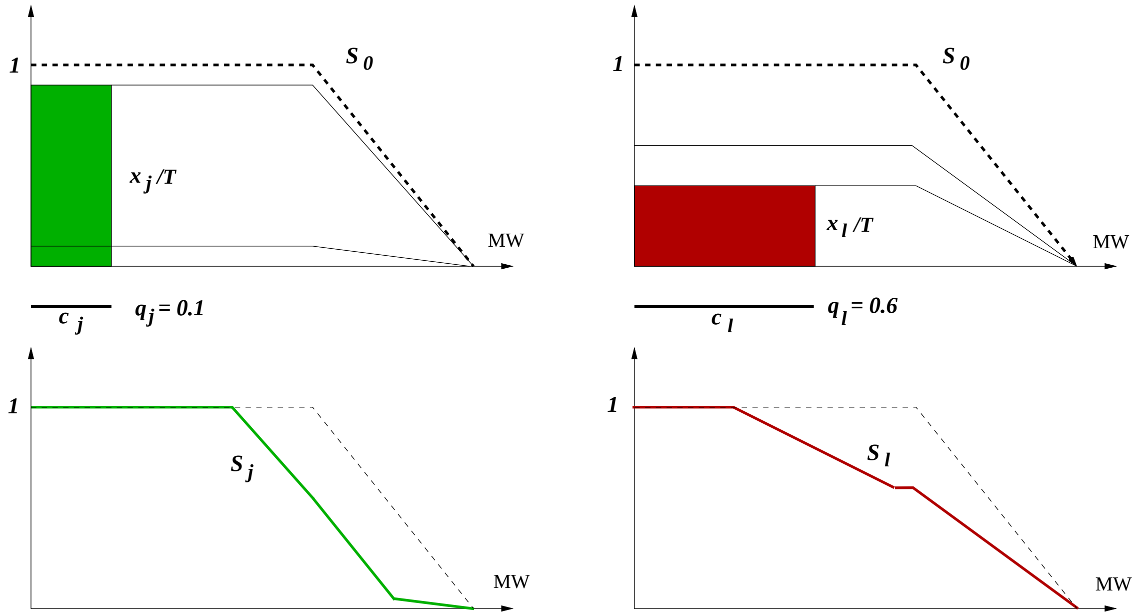


Figure 18: Simple load-survival function after loading of normal energy-equivalent unit (left) and after loading of crest WP unit (right)

is not as abundant as WP in the MIBEL due to government regulations that have led to a much lower installed capacity of SPV. In Figure 19 is represented the overlaid daily SPV generation for each month over 2009 and 2010.

For each period the distribution of the SPV hourly generation has a bell-shape, where in the early daily hours and in the late hours there is no generation (or a small one), in the midday is when the maximum generation is possible and the rest of the time there is like a medium sun light.

Considering these characteristics entails having to break up each time period into three subperiods: the *dark* subperiod, where there cannot be any SPV production, the *low-light* subperiod where there are the hours with only some SPV generation, and the *bright* subperiod with the hours of higher SPV generation. These subperiods are not chronologically consecutive, but will be treated as such with no important loss of precision in the probabilistic matching of the load.

Figure 20 shows the daily SPV generation in March 2012 in Spain. The separation into three subperiods of a specific period is done with the following criteria: the daily hours with not enough light to generate go to the dark subperiod (the white area in Figure 20); with the rest of the daily hours we order them from highest to lowest average SPV generation, and we pick up the highest average generation daily hours that amount to 60% of the total SPV generation of the period to be the bright subperiod (the yellow area), and the remaining hours will be the low-light subperiod (the blue area). The bright-subperiod hourly limits

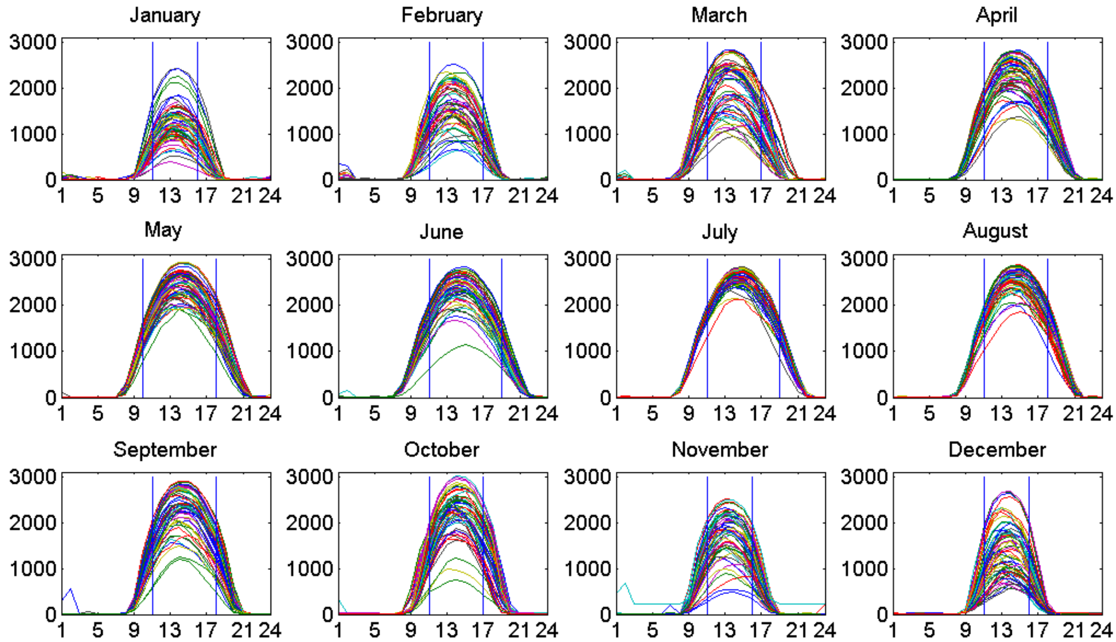


Figure 19: Overlaid daily SPV generations in Spain for 2009 and 2010

are also marked in the monthly periods of Figure 19.

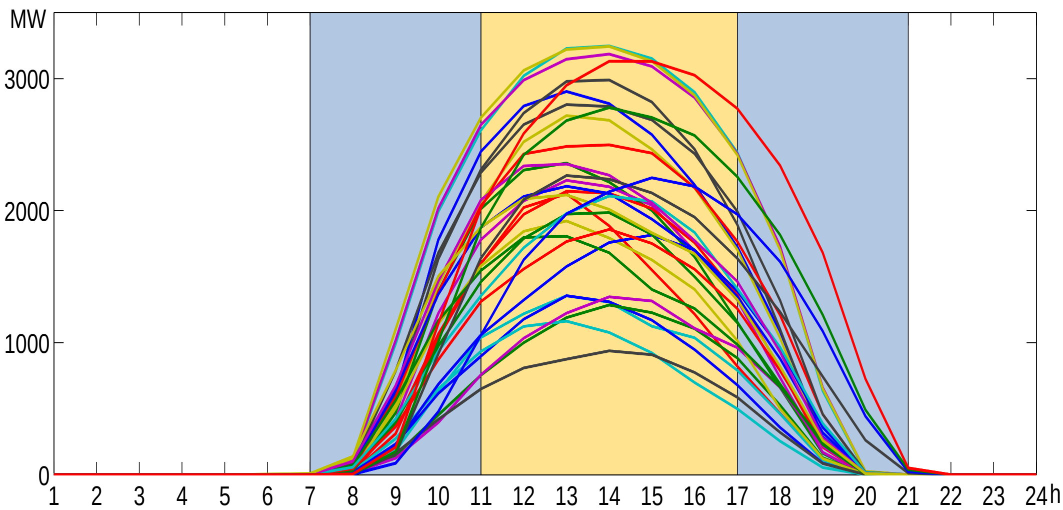


Figure 20: Overlaid daily SPV generations in Spain during March 2012, showing the subdivision of daily hours into dark, low-light, and bright subperiods

Considering the plot of SPV generation during the bright subperiods only, results in a randomness similar to that of WP and the same subdivision as in WP into a base-SPV unit and a crest-SPV unit will be considered. As with WP, only scenarios with respect to the capacity of the base-SPV units will be taken into account. The crest-SPV units will have capacity and failure parameters specific and constant for each bright subperiod, as with WP.

During the low-light subperiod hours, we will only consider a crest-SPV unit also with parameters specific and constant for each subperiod. There is no SPV unit in the dark subperiods.

4.2.4 The three LDCs in each node when considering SPV Generation

When addressing SPV power, each node of the scenario tree will require matching three different LDCs corresponding to the dark, the medium light, and the bright hours (whose durations change over a yearly horizon in non-tropical areas). The subset of units that represent the SPV power within the set of units that match each of the three LDCs is different. This entails having to employ three different sets of LMCs, each with its specific variables, in each node.

Each load type is denoted by $g \in L_t$, where L_t is the set of load types (of dark, low-light, and bright hours), and each variable and parameter will have an extra subscript g to consider what load type corresponds to. Some of the parameters that characterize units that represent SPV or WP power are different in different periods or nodes, which further complicate the coding of the medium-term planning with non-dispatchable renewables.

4.3 STOCHASTIC PARAMETERS IN EACH SCENARIO NODE

In order to include the dispatchable and non-dispatchable renewable (DR and N-DR) sources in our optimization problem, scenarios will be developed as follows.

4.3.1 Stochastic hydro generation parameters in the nodes

For hydro generation, giving values to the inflows w_h^ν for all nodes, and defining a final stored energy v_h^λ in line with the expected inflow $\widetilde{w}_h^i$ in each period of the scenario path:

$$v_h^\lambda = v_h^0 + \delta \sum_{\nu \in \mathcal{A}_\lambda} (w_h^{i(\nu)} - \widetilde{w}_h^{i(\nu)}) \quad \frac{1}{5} < \delta < 1 \quad \forall \lambda \in \mathcal{L}\,, \tag{39}$$

where $\delta = 0.4$ in the data employed, and $\widetilde{w}_h^{i(\nu)}$ is the expected inflow value of period $i(\nu)$.

As the difference of stored energy $v_h^\lambda - v_h^0$ is predetermined for each scenario path, its value is a constant and needs not be optimized. There is plenty of hydro generation inflow data in the MIBEL (almost a 100-year historic record).

4.3.2 Stochastic WP parameters in each scenario node

The issue to solve is to choose a number of stochastic parameters to represent WP given the reduced number of years with coherent WP data in the MIBEL (2009-2012). Although there has been WP generation in the MIBEL for many years, only for the few recent years the WP generation portfolio has had little changes from one year to the next. Extracting three parameters c_{Wbs}, c_{Wcr} and q_{Wcr} from the available data could be risky and it has been decided to deduct a single stochastic parameter and give to the rest fixed values for each period i (see §4.2.2).

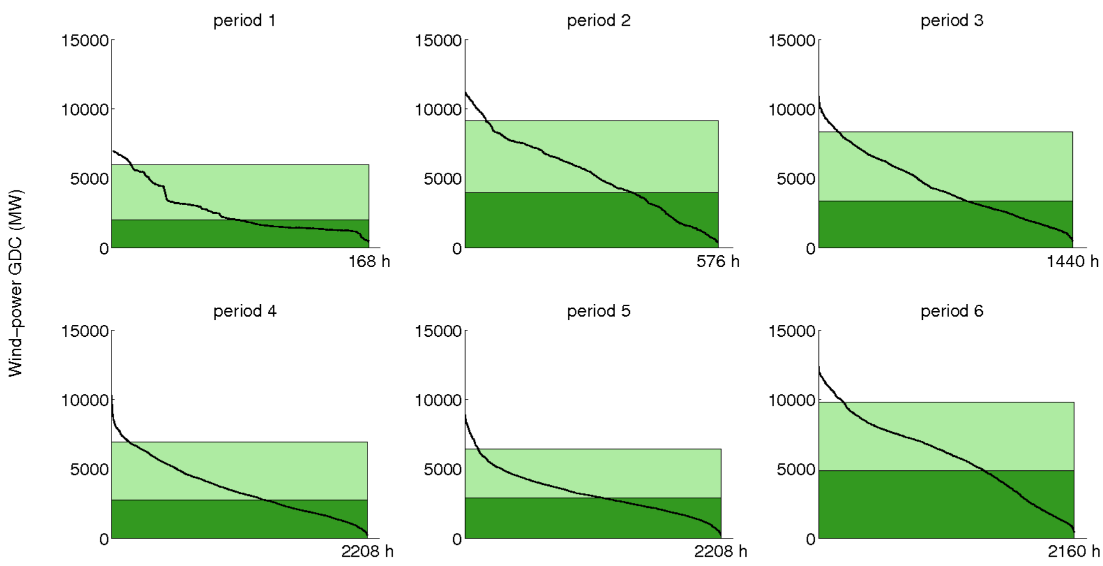
Figure 21 shows the WP generation for two years split into 6 periods of different length. For a period, hourly generation is sorted decreasingly (black line), thereby obtaining the WP GDC. The darker area represents the capacity of the base unit, while the lighter one is the capacity of the crest unit.

Standard deviations of base capacities (with 15% failure probability) and of crest units (with variable failure probabilities in the 55-85% range) over the 4 years (2009-2012) for each of the 6 periods are similar (see Figure 21 for 2009 and 2010). Although results reveal that base capacity has less deviation and it would be preferable to create scenarios for the crest unit, it was decided to generate a scenario tree for the base capacity, because the WP base unit has a much stronger influence than the crest WP unit given its much lower failure probability (15%). Therefore, it is reasonable to use c^{ν}_{WPbs} as a single node stochastic parameter for each node ν and take the c^{i}_{WPcr} and q^{i}_{WPcr} as period expected values calculated from the current WP records. (This is to be reviewed in the near future as more WP data from the MIBEL become available, using a new scenario formation contemplating more N-DRs parameters per node).

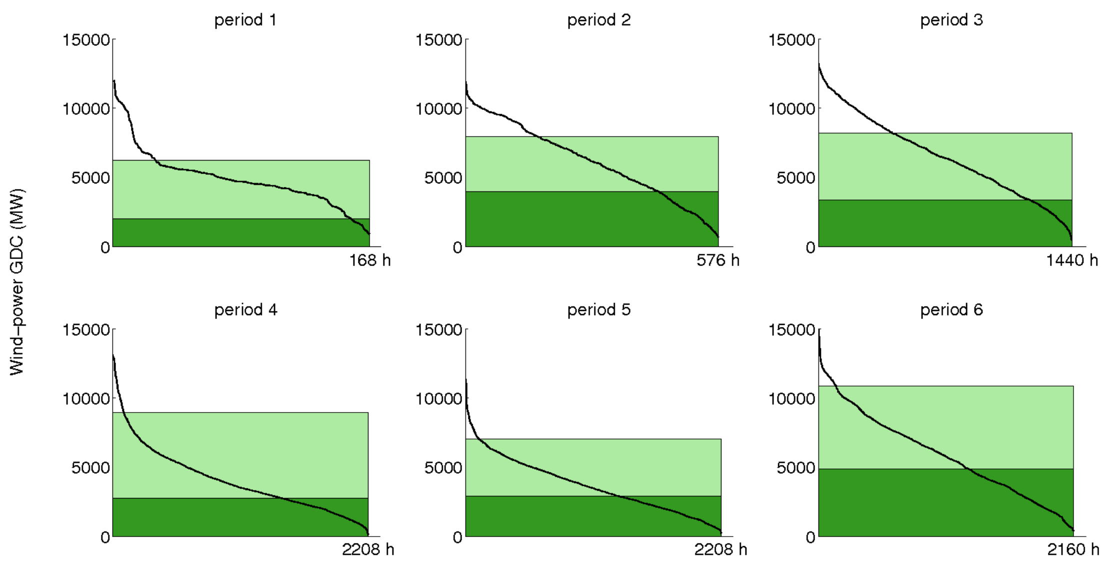
Scenarios will be first generated for the total WP generation x^{ν}_{W} at each node ν. Since WP characterization is made by using two pseudo-units, it is possible to state that the total WP generation in a node is the generation of the base-WP unit plus the generation of the crest-WP unit, $x^{\nu}_{W} = x^{\nu}_{Wbs} + x^{\nu}_{Wcr}$.

Due to the fact that the crest-WP unit has been taken to depend on the period, it is easy to see that the generation of base-WP unit is $x^{\nu}_{Wbs} = x^{\nu}_{W} - x^{i(\nu)}_{Wcr}$. Furthermore, the generation of a unit is computed by the formula $x^{\nu}_{Wbs} = c^{\nu}_{Wbs}(1 - q_{Wbs})T^{i(\nu)}$. So the capacity of base-WP unit is directly obtained as

$$c^{\nu}_{Wbs} = \frac{x^{\nu}_{W} - x^{i(\nu)}_{Wcr}}{(1 - q_{Wbs})T^{i(\nu)}} \, .$$



(a) 2009



(b) 2010

Figure 21: Representation of the WP GDC (black line) for 2 years (2009 and 2010) split into 6 periods of different length, the capacity of crest unit (light area) and the capacity of base unit (dark area)

4.3.3 Stochastic SPV parameters in each scenario node

For SPV, something similar has been done. SPV generation has been split into 3 parts, no sun, low-light and bright subperiods. As the bright hours represent the 80% of the total SPV generation, scenarios will be focused on them. The scarcity of SPV data in the MIBEL is similar, if no worse, than that of WP. Therefore, a single SPV parameter will be taken for each node.

As in WP, first of all, scenarios are developed for the total SPV generation in bright subperiods. This generation is also divided in two pseudo-units, $x^{\nu}_{\mathrm{brSPV}} = x^{\nu}_{\mathrm{brSPVbs}} + x^{\nu}_{\mathrm{brSPVcr}}$. The generation of the crest unit depends also on the period, so the capacity of the base unit is computed

$$c^{\nu}_{\mathrm{brSPVbs}} = \frac{x^{\nu}_{\mathrm{brSPV}} - x^{i(\nu)}_{\mathrm{brSPVcr}}}{(1 - q_{\mathrm{brSPVbs}})T^{i(\nu)}_{\mathrm{brSPV}}} .$$

However, in low-light hours there is also SPV generation. From historical records, it has been seen, as expected, that there is a correlation between SPV generation in bright hours and the one in low-light hours. In this case, a linear function is used to relate SPV generation in these two subperiods:

$$x^{\nu}_{\mathrm{llSPV}} = 559.19 + 0.253x^{\nu}_{\mathrm{brSPV}} .$$

As seen in §4.2.3, SPV generation in low-light subperiod is characterized with a single crest unit. The historical data show that the outage probability of this pseudo-unit does not vary much within a subperiod of several years, so it was decided that this probability will only depend on the period, $q^{i(\nu)}_{\mathrm{llSPV}}$. Thus, the capacity of this pseudo-unit will depend on the node ν and is calculated as

$$c^{\nu}_{\mathrm{llSPV}} = \frac{x^{\nu}_{\mathrm{llSPV}}}{(1 - q^{i(\nu)}_{\mathrm{llSPV}})T^{i(\nu)}_{\mathrm{llSPV}}} .$$

Regarding WP and SPV, values b^{ν}_{0} are given to the market-price level at peak load in each node ν, given the expected value of WP and SPV associated to that node.

4.4 SCENARIO GENERATION USING A QUASI-MONTE CARLO PROCEDURE

In §2.9.3 it was explained how to use QMC methods to generate a scenario tree. Here, the application to the DR and N-DRs will be described. Having $n_i = 6$ time periods and $m = 3$

stochastic variables, which are the natural water inflows, and WP and SPV generations, the total dimension will be $n_D = m \times n_i = 18$.

Since the only stochastic parameter in our VAR model (26) is χ, n scenarios $\chi^j,\ J = 1, \ldots, n$ with probability $\frac{1}{n}$ will be generated by QMC techniques, without tree structure, where $n = 499$ is taken to be a prime number that will represent the number of scenarios prior to reduction. There will be the generation of a scenario tree out of the $n = 499$ scenarios $\xi^j \in \mathbb{R}^{n_D}$, $\xi^j = \begin{pmatrix} \xi_1^j \\ \vdots \\ \xi_{n_i}^j \end{pmatrix}$, $\xi_k^j \in \mathbb{R}^m,\ k = 1, \ldots, n_i$

The steps in the QMC procedure are:

- Determine QMC points by a component-by-component algorithm [64]: $\eta^j \in [0,1]^{n_D}$, $j = 1, \ldots, n$ with probability $\frac{1}{n}$, where $\{\eta^j\}_{j=1}^n$ approximates the uniform distribution.
- Generate n realizations ζ_i^j, $i = 1, .., n$, $j = 1, .., n_D$ of n_D independent N(0,1) random variables using an inverse method [52] with the QMC points η^j obtained before.
- Compute the Cholesky decomposition of the covariance matrix $\Sigma = LL^T$
- Compute the n realizations $\xi^j = L\zeta^j + r$, $j = 1, .., n$ of the original $N(r, \Sigma)$ random vector. Since in this model the value of the mean is $r = 0$, the formula to use is $\xi^j = L\zeta^j$, $j = 1, .., n$.

In the second stage, a fan of data scenarios is created using the $n = 499$ realizations obtained with QMC procedures and using the stochastic model described before:

$$X_{i+1}(\nu) = \kappa_{i+1}(\overline{X}_{i+1} - X_i) + X_i + \xi(\nu), \forall \nu \in \mathcal{N}$$

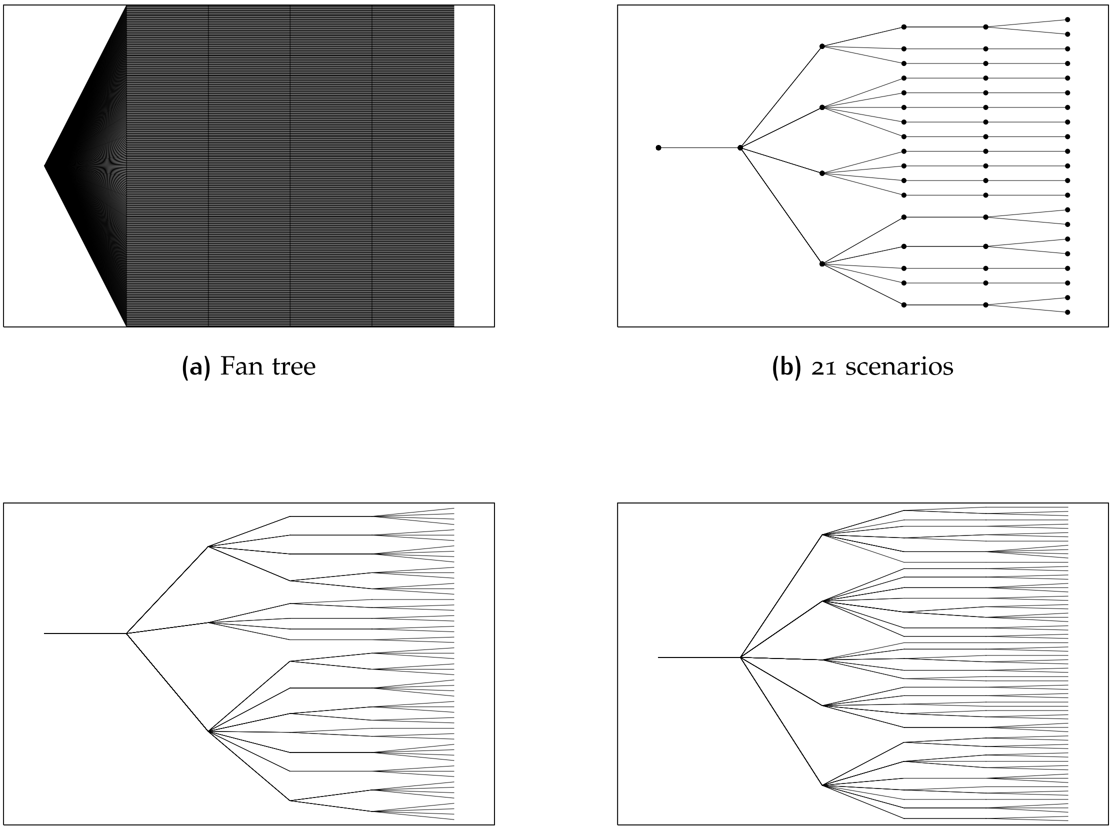
The fan scenario tree created has 499 scenarios and 2496 nodes; see (a) in Figure 22

4.5 SCENARIO REDUCTION

In order to have a scenario tree that is both reliable and solvable with a reasonable CPU time in medium-term optimization, the original scenario fan created has been reduced into 21, 59 and 75 scenarios. The procedure for scenario reduction is that described in §2.9.4. Figure 22 shows the reduction of the scenario fan (a) into (b) 21, (c) 59 and (d) 75 scenarios.

As it can be seen, the shape of each reduced scenario tree is different so it is not possible to obtain a 21-scenario tree through one of 59 or 75 scenarios. Each tree has to be always obtained by reducing the original FAN scenario tree into the desired size.

The different scenario reductions for the three stochastic variables considered (hydro, WP and SPV) are presented together in Figure 23.



(a) Fan tree

(b) 21 scenarios

(c) 59 scenarios

(d) 75 scenarios

Figure 22: (a) FAN scenario tree with 499 scenarios for 3 stochastic variables. (b-d) Reduction of the scenario tree into 21, 59 and 75 scenarios

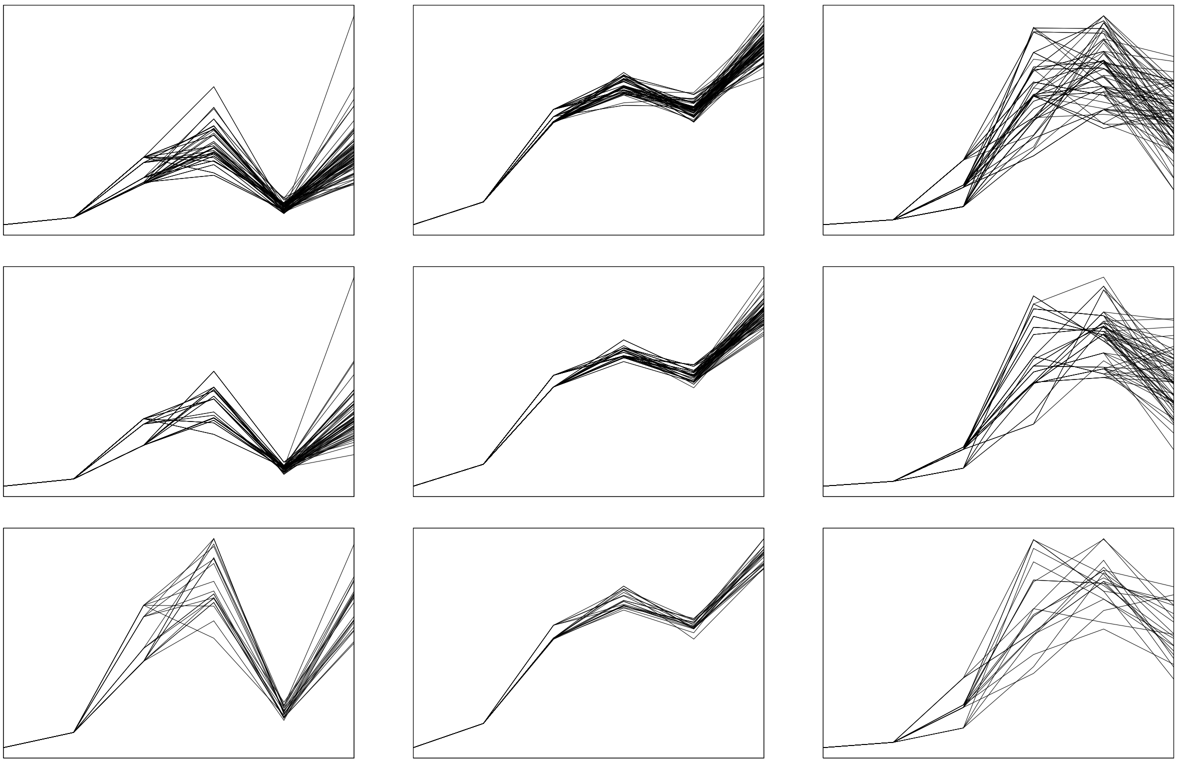


Figure 23: Scenario Reduction: in the first column the hydro inflows are presented, in the second the WP generation and in the third the bright subperiod SPV generation. In the rows are represented different reductions: from top to bottom 75, 59 and 21 scenarios

5 MEDIUM-TERM PLANNING WITH DISPATCHABLE AND NON-DISPATCHABLE RENEWABLES

Nowadays, the use of renewable sources has been increased significantly in order to generate electricity. Therefore, it is necessary to update previous models (see §3.1) introducing this kind of generation into them. The medium-term model proposed here uses probabilistic matching of the pool LDCs using the capacities and the failure rates of units of all GenCos participating in the market.

In this chapter, it can be seen that this model employs an endogenous market-price function influenced by hydro and N-DRs, and that it represents each N-DR generation with two units: one having a normal failure rate and another with a large one. It also takes into account medium-term constraints, which relate unit generations over several periods over the year, such as those for hydro generation with regulation reservoirs, take-or-pay contracts for fuels, pumping with medium-term storage capacity, or emission caps over several months.

5.1 THE ENDOGENOUS CARTEL SOLUTION

The analysis of the historical records of generations and market prices in the MIBEL [88, 89] shows a clear negative correlation between hydro generation and market price. In light of the effect that hydro generation has on the average market price, a correction in the market-price intercept b^i in (14) was proposed [67, 84, 56]. When considering N-DRs (WP and SPV) we have an influence of WP and SPV on the basic market price of the same type. Given that each node ν of the scenario tree will have a given level of WP and SPV (see §4.3), we can specify the influence of hydro generation, which is dispatchable, on the market price as:

$$b^{\nu} = b_0^{\nu} + \frac{d}{T^{i(\nu)}} \sum_{h \in H} x_h^{\nu}, \tag{40}$$

where b_0^ν is established in terms of the levels of WP and SPV in node ν and a decrease factor for price level due to WP and SPV, and $d < 0$ is to be estimated from historical data. The new cartel profit function to be maximized,

$$\sum_{\nu \in \mathcal{N}} \pi^\nu \left[\sum_{j \in \Omega} \left\{ \left(b_0^\nu - f_j + \frac{d}{T^{i(\nu)}} \sum_{h \in H} x_h^\nu \right) x_j^\nu + \frac{l^{i(\nu)}}{2c_j} \left(x_j^\nu \right)^2 \right\} - f_0 x_0^\nu \right], \tag{41}$$

where f_j is the generation cost of the j^{th} unit, x_0^ν is the external energy and f_o its price, is also quadratic, and is endogenous as price depends on the level of hydro generation of all GenCos, and b_0^ν includes the effect of the fixed levels of WP and SPV in the node on the basic market price.

In the case of the MIBEL, with the value of the parameters estimated, the quadratic part of (41) is indefinite in some of the nodes. Therefore, the usual procedures that maximize concave problems may find only a local maximizer, though the computational results obtained do not point in this direction.

WP and SPV generations have similar effect to that of hydro generation on market-price levels. Contrary to hydro generation from regulation reservoirs, WP and SPV levels cannot be controlled. Their effect on the LDC and on price levels must be taken into account but it cannot be optimized. The value of the market price for peak load b_0^ν in each node ν of a scenario tree of WP and SPV parameters, will take into account the expected WP and SPV associated to each node, which are known, using a formula as that in (40), using an appropriate value of d for WP and SPV, which appear to be independent events in the MIBEL.

The presence or absence of a nuclear unit has an influence on market price levels. It can be easily taken into account for given maintenance schedules. Random outages of nuclear units could be taken into account within stochastic programming scenario trees, but it has not been contemplated in this thesis. As to other thermal generation technologies (coal, combined cycle, etc.), their influence on market-price level in the MIBEL is of lesser importance and they have not been taken into account in this thesis. These influences were described and used in [84].

Medium-term endogenous cartel model with N-DRs

$$\max_{x^{\nu}_{gj}, y^{\nu}} \sum_{\nu \in \mathcal{N}} \pi^{\nu} \left\{ \sum_{g \in Lt} \sum_{j \in \Omega_g} \left[\left(b^{\nu}_0 + \frac{d}{T^{i(\nu)}_g} \sum_{h \in H} x^{\nu}_{gh} - f_j \right) x^{\nu}_{gj} + \frac{l^{i(\nu)}}{2c_j} \left(x^{\nu}_{gj} \right)^2 \right] - f_0 x^{\nu}_{g0} \right\} \tag{42a}$$

$$\text{s.t.:} \sum_{j \in \omega} x^{\nu}_{gj} \leqslant e^{\nu}_g - s^{\nu}_g(\omega) \quad \forall g \in Lt,\ \forall \omega \subseteq \Omega_g,\ \forall \nu \in \mathcal{N} \tag{42b}$$

$$\sum_{j \in \Omega_g} x^{\nu}_{gj} + x^{\nu}_{g0} = e^{\nu}_g \quad \forall g \in Lt,\ \forall \nu \in \mathcal{N} \tag{42c}$$

$$\sum_{\nu \in \mathcal{A}_\lambda} \Big(\sum_{g \in Lt} C_{g\lambda} x^{\nu}_g \Big) + \sum_{\nu \in \mathcal{A}_\lambda} D_\lambda y^{\nu} \geqslant r_\lambda \quad \forall \lambda \in \mathcal{L} \tag{42d}$$

$$\sum_{\nu \in \mathcal{A}_\lambda} \left(u^{\nu}_h - w^{\nu}_h \right) = v^0_h - v^{\lambda}_h \quad \forall \lambda \in \mathcal{L},\ \forall h \in H \tag{42e}$$

$$\sum_{\substack{\nu \in \mathcal{A}_\lambda | \\ i(\nu) \leqslant k}} \left(u^{\nu}_h - w^{\nu}_h \right) \leqslant v^0_h \ \forall k \in 1, \dots, n_i - 1,\ \forall \lambda,\ \forall h \in H \tag{42f}$$

$$u^{\nu}_h \geqslant 0 \quad \forall \nu \in \mathcal{N},\ \forall h \in H \tag{42g}$$

$$x^{\nu}_h = u^{\nu}_h + \gamma w^{\nu}_h \quad \forall \nu \in \mathcal{N},\ \forall h \in H \tag{42h}$$

$$x^{\nu}_{gj} \geqslant 0 \quad j \in \Omega_g \quad y^{\nu} \geqslant 0, \quad \forall g \in Lt, \quad \forall \nu \in \mathcal{N}. \tag{42i}$$

The difference between this model and the model (18) presented in §2.7 is, basically, that this one takes into account the characterization of the renewable generation and this fact forces the use of three LDCs to match (that of dark, of low light, and of bright hours) within stochastic programming. The problem to be solved now is more complex and needs more effort to be solved.

5.2 THE EQUILIBRIUM MODEL WITH DISPATCHABLE AND NON-DISPATCHABLE RENEWABLES

The stochastic formulation of the equilibrium problem with renewable and N-DRs is similar to that employed in [84], but with endogenous terms referred only to hydro generation, and using b^{ν}_0 that takes into account the influence on prices of WP and SPV:

$$
\begin{aligned}
\max_{x^{\nu}_{gj}, y^{\nu}} \sum_{\nu \in \mathcal{N}} \pi^{\nu} \Bigg\{ & \sum_{g \in Lt} \sum_{j \in \Omega_g} \Big[\big(b^{\nu}_0 - f_j\big) x^{\nu}_{gj} + \frac{l^{i(\nu)}}{2c_j} \big(x^{\nu}_{gj}\big)^2 \\
& + \frac{d}{T^{i(\nu)}_g} \sum_{k=1}^{K} \big(\sum_{l \in H_k} x^{\nu}_{gl} + \sum_{\substack{l \in H_m | \\ m \neq k}} \tilde{x}^{\nu}_{gl} \big) x^{\nu}_{gj} \Big] - f_0 x^{\nu}_{g0} \Bigg\}
\end{aligned} \tag{43a}
$$

$$
\text{s.t.: LMCs } (42b\text{–}42c) \text{ and non} - \text{LMCs } (42d - 42i)
$$

In the model presented the generation x^{ν}_{Phyd} of pumped hydro should there be a pumping storage scheme is not considered as part of the hydro generation that alters the basic market price b^{ν} (40).

5.3 CONSIDERATION OF A PUMPING STORAGE SCHEME IN MEDIUM-TERM PLANNING WITH N-DRS

When considering the pumping storage scheme with N-DRs (see §3.4) it is necessary to match three different LDCs in each period. So in this case the expected generation of the pumping storage in some node ν will be $x^{\nu}_{g\,Cmp}$, for the LDC of each load type $g \in Lt$: for dark, medium-light and bright hours.

Considering this fact, it is also necessary to update and extend the pumping load fee described in §3.4.1 to take into account these three different LDCs. Thus,

$$
\begin{aligned}
c_P \sum_{g \in Lt} \int_{\frac{x^{\nu}_{g\,Cmp} + x^{\nu}_{g\,Phyd}}{c_P}}^{T^{i(\nu)}_g} \big(b^{\nu} + l^{i(\nu)} t\big) dt = \; & b^{\nu} \sum_{g \in Lt} \big(c_P T^{i(\nu)}_g - x^{\nu}_{g\,Cmp} - x^{\nu}_{g\,Phyd}\big) + \\
& \frac{l^{i(\nu)}}{2c_P} \sum_{g \in Lt} \Big\{ \big(c_P T^{i(\nu)}_g\big)^2 - \big(x^{\nu}_{g\,Cmp} + x^{\nu}_{g\,Phyd}\big)^2 \Big\},
\end{aligned} \tag{44}
$$

where b^{ν} is as in (40). The integration limits in (44) have been taken so that the pumping load-duration segment corresponds to the lowest market prices. Note that the generation of the compensating unit is not to be considered in profit calculation, and that using the compensating unit for generating the entire extra load added to the LDCs would lead to the same result as with no pumped storage.

Finally, the *equilibrium medium-term generation planning model with pumping generation* is presented:

$$\max_{x^{\nu}_{g\,j}, y^{\nu}} \sum_{\nu\in\mathcal{N}} \pi^{\nu} \left\{ \begin{array}{l} \sum\limits_{g\in Lt}\sum\limits_{j\in\Omega_g} \Big[\big(b^{\nu}_0 - f_j\big)x^{\nu}_{g\,j} + \frac{l^{i(\nu)}}{2c_j}\big(x^{\nu}_{g\,j}\big)^2 + \frac{d}{T^{i(\nu)}_g} \sum\limits_{k=1}^{K} \big(\sum\limits_{l\in H_k} x^{\nu}_{g\,l} + \sum\limits_{\substack{l\in H_m| \\ m\neq k}} \tilde{x}^{\nu}_{g\,l} \big) x^{\nu}_{g\,j} \Big] \\ -f_0 x^{\nu}_{g\,0} - \sum\limits_{g\in Lt} \big[(b^{\nu}_0 + \frac{d}{T^{i(\nu)}_g} \sum\limits_{h\in H} x^{\nu}_{g\,h})(c_P T^{i(\nu)}_g - x^{\nu}_{g\,Cmp} - x^{\nu}_{g\,Phyd}) \big] \\ -\frac{l^{i(\nu)}}{2c_P} \sum\limits_{g\in Lt} \Big\{ \big(c_P T^{i(\nu)}_g\big)^2 - \big(x^{\nu}_{g\,Cmp} + x^{\nu}_{g\,Phyd}\big)^2 \Big\} \end{array} \right\} \tag{45a}$$

s.t.: LMCs (42b–42c) and non-LMCs (42d–42i)

$$c_P T^{i(\nu)}_g - x^{\nu}_{g\,Cmp} - x^{\nu}_{g\,Phyd} \geqslant 0 \quad \forall\, \nu \in \mathcal{N}, \quad \forall\, g \in Lt \tag{45b}$$

$$\nu^1_P = \sum_{g\in Lt} \Big\{ \delta_P \big(c_P T^1_g - x^1_{g\,Cmp} - x^1_{g\,Phyd}\big) - x^1_{g\,Phyd} \Big\} + \nu^0_P \tag{45c}$$

$$\nu^{\nu}_P = \sum_{g\in Lt} \Big\{ \delta_P \big(c_P T^{i(\nu)}_g - x^{\nu}_{g\,Cmp} - x^{\nu}_{g\,Phyd}\big) - x^{\nu}_{g\,Phyd} \Big\} + \nu^{\nu^-}_P \quad \forall \nu > 1 \text{ and } \nu \notin \mathcal{L} \tag{45d}$$

$$\sum_{g\in Lt} \Big\{ \delta_P \big(c_P T^{i(\lambda)}_g - x^{\lambda}_{g\,Cmp} - x^{\lambda}_{g\,Phyd}\big) - x^{\lambda}_{g\,Phyd} \Big\} + \nu^{\lambda^-}_P = \nu^0_P \quad \forall \lambda \in \mathcal{L} \tag{45e}$$

where the second and the third line of the objective function (45a) is the pumping extra load fee. The model has the same LMCs and non-LMCs as the medium-term endogenous cartel model with N-DRs (42), and the constraints that are referred to pumping scheme (see (35) and (36) in §3.4) here extended in (45b) to (45e) to the loads in the dark, the medium-light and the bright hours of each period.

5.4 TEST CASES

As described in §2.12.1 and in §2.12.2, the LDCs of periods and subperiods, and the generation units that match these loads define the test cases solved. In the test cases where there are DR (hydro) and N-DRs (WP and SPV), randomness is represented through stochastic scenario trees (see §4.4).

The numbers of conventional generation units of cases are 18, and 24, where units of the same technology are from less to more disaggregated, and a few more units, N-DR and/or related to pumping are also considered in different generation settings. The unit numbers will identify the test case. Cases with 18 units consider that there are three participants in the market, the SGC and two other GenCos with hydro generation, and cases with 24 units consider that there are five participants in the market, the SGC and 4 GenCos with hydro generation. In §2.4.1 is defined the six uneven periods in to which the yearly horizon considered is subdivided.

Table 2 has the fixed dimensions of each case with the generation setting identified by one or two letters: rR for the presence of N-DRs (WP and SPV) using a single small r when solving the endogenous cartel model and the capital R for the equilibrium one, RP for N-DRs plus pumping, and with letter C to refer to having an extra conventional unit instead of N-DRs. The tree size is indicated with the number of scenarios after the generation setting letters in the case identifier. The generation setting is specified in columns "Rnw" (N-DRs), "Pmp" (pumped storage) and "Cnr" (conventional unit replacing N-DRs). Problem size is specified through the numbers n_u of units, n_v of tree nodes, and n_{var} of variables. The number of equality and inequality non-LMC is also indicated.

The number of generated LMCs during the solution using the mLMH is given in the tables with the results.

Table 2: Fixed Dimensions of Test Cases

	generation setting			problem size			non-LMCs	
case	Rnw	Pmp	Cnr	n_u	n_v	n_{var}	$=$	$\geqslant$
18rR21	√			18	61	3471	654	315
18rR59	√			18	100	5694	1195	885
18rR75	√			18	153	8715	1752	1125
18RP21	√	√		20	61	3877	715	498
18RP59	√	√		20	100	6335	1295	1185
18RP75	√	√		20	153	9711	1905	1584
18C21			√	14	61	1035	288	315
18C59			√	14	100	1698	595	885
18C75			√	14	153	2599	834	1125
24rR21	√			24	61	5118	1266	630
24rR59	√			24	100	8394	2272	1770
24rR75	√			24	153	12846	3354	2250
24RP21	√	√		26	61	5524	1327	813
24RP59	√	√		26	100	9035	2372	2070
24RP75	√	√		26	153	13842	3507	2709
24C21			√	20	61	1584	534	630
24C59			√	20	100	2598	1072	1770
24C75			√	20	153	3976	1518	2250

In Appendix A, there are the details of the data for the equilibrium case with pumping generation, 24 units and 21 scenarios.

5.5 COMPUTATIONAL RESULTS

Two types of utility function are considered: the endogenous cartel behavior (maximization of generators surplus taking into account the effect of hydro generation on market prices), and the equilibrium behavior considering two or more players.

5.5.1 Results with the endogenous cartel model

Table 3 contains the endogenous cartel results with N-DRs. The first column $\widetilde{P}$ shows the expected profit value of the solution, and P_{sp90} its 5 to 95% profit spread (see §2.10). Column LMCs indicates the number of LMCs generated by the mLMH, the *Ipopt* exe column indicates the number of different executions of the solver (in the application of the mLMH), and the ite column the number of iterations. The CPU sec. column contains the AMPL plus solver execution times.

Table 3: Endogenous Cartel Solutions for different Generation Settings

	optimal profit (10^6) €			*Ipopt*		CPU
case	$\widetilde{P}$	P_{sp90}	LMCs	exe	ite	(sec)
18r21	10836.106	537.946	53699	17	2294	281
18r59	10807.713	540.613	88328	17	2843	604
18r75	10804.554	609.673	135096	17	3299	1089
24r21	10857.759	538.221	77857	21	4497	1324
24r59	10834.745	539.481	127448	22	5558	2856
24r75	10831.064	609.043	195169	22	6112	5062

As it can be seen in Table 3 for the same number of units the profit spread, P_{sp90}, is increasing as the number of scenarios increase. It is a reasonable result because if the number of scenarios is increasing then the variability increases too.

Another logical result is that as the number of scenarios increases, the value of the objective function is smaller (although it is quite similar for all of the cases) and both the number of constraints and the CPU time increases.

In Appendix B there are, in detail, the results obtained for the equilibrium case using pumping units and with 21 scenarios and 24 units. In it, there is the evolution of the objective function along the successive optimizations of the problem and it is presented the optimal values for the expected generation by each unit j over each node ν. Finally, it is included Figure 36 where it can be appreciated the evolution of the objective function until it leads to stable solutions.

5.5.2 Results with the equilibrium model

Table 4 contains the results of the equilibrium planning solutions obtained with the NIRA algorithm for the test cases using different generation settings including the presence or not of a pumped storage scheme. It also contains the results using a conventional unit instead of the N-DR generation. This table has the same structure as Table 3.

Table 4: Equilibrium Solutions for different Generation Settings

	optimal profit (10^6) €			*Ipopt*		CPU
case	$\widetilde{P}$	P_{sp90}	LMCs	exe	ite	(sec)
18R21	10817.996	543.420	53699	19	10932	2478
18R59	10789.223	554.017	88328	18	13229	5513
18R75	10784.936	618.641	135096	19	18118	12702
18RP21	11297.488	563.945	61286	24	9221	2263
18RP59	11267.881	556.280	100317	24	11728	5423
18RP75	11262.825	637.761	153679	24	15977	13034
18C21	9336.594	117.264	15471	17	1336	51
18C59	9320.608	207.450	25352	17	1342	88
18C75	9322.401	216.256	38687	17	1547	148
24R21	10830.967	543.164	77857	22	17231	9412
24R59	10806.115	543.022	127448	22	21193	20925
24R75	10801.476	613.566	195169	24	32212	53761
24RP21	11463.659	565.906	88086	28	13172	7505
24RP59	11436.825	535.899	143858	28	16133	17113
24RP75	11433.911	627.287	221017	28	22507	41335
24C21	9333.432	117.920	22511	17	1628	155
24C59	9322.668	207.888	36460	17	1559	261
24C75	9323.883	216.929	56088	17	1822	460

The differences between the endogenous cases and the corresponding equilibrium ones illustrate the important increase in CPU time of the equilibrium solution with respect to the endogenous cartel solution owing to the changing objective function (20) in the equilibrium model, which requires a much higher number of iterations, while the number of LMCs generated by the mLMH remains the same.

Equilibrium profits are systematically lower than the endogenous cartel ones. It has been shown that the distribution of hydro generation along the successive periods in equilibrium solutions is different from that in cartel solutions, with that of the equilibrium model being closer to the minimum cost solution (where there is more hydro generation during higher demand periods) than that of the endogenous cartel [84].

Table 5, with the peak load $\bar{p}^i$ of each period in the 4th column, shows this for the endogenous cartel case 24r21 and the equilibrium case 24R21. In [84], it was also described how other thermal generation technologies influenced market-price level in the MIBEL some years ago. This influence appears to be less important now with a higher share of N-DRs, and it has not been taken into account in this thesis.

Table 5: Comparison of Hydro Generation in Cases 24r21 and 24R21

	load			24r21	24R21
i	T^i h	$\widehat{e}^i$ MWh	$\bar{p}^i$ MW	x_h MWh	x_h MWh
1	168	3190062	27857	159674	250544
2	576	10238656	27034	1339422	1045161
3	1440	26727324	30983	2284373	2589853
4	2208	43464615	32432	2767199	4281843
5	2208	35495820	26417	5545843	3582622
6	2160	37838776	26711	2884320	3230809

5.5.3 Comparison of different types of generation setting

The difference between having or not N-DRs as part of the generation portfolio can be appreciated in Table 4 and in Figure 24. In that figure the darker (orange) distribution of profits of case 24R59, with renewables, and that lighter (pink) of case 24C59, with an extra conventional unit able to provide the same energy as the non-dispatchable renewables, are plotted on the same graphic.

Note the difference in expected profits and the larger profit spread P_{sp90} between cases 24R59 and 24C59, where the null operating cost of the renewables becomes important, though subject to a higher risk of profit loss. Handling the renewable model, having to use three different LDCs at each tree node requires much more CPU time. The difference in expected profits comes from the costs of the extra conventional unit. The different shape of the distributions and the different length of the profit spread P_{sp90} indicate that.

The difference of the profit spread indicates that using renewable sources produces more variation into the problem. This result is as it was expected due to the fact that when N-DRs are used, it is necessary to create a scenario tree for 3 variables, whereas when they are not used this tree is created only for a single variable.

In Figure 24 the (black) solid vertical line on the profit distribution indicates the expected profit $\widetilde{P}$, while the (red) dashed line indicates the 5% VaR profit limit and the (green) dotted line the 95% upper profit limit of the P_{sp90} profit spread. Note also that the histogram plotted shows the irregular shape of the profit distributions obtained. Larger number of scenarios would produce smoother profit histograms but would increase the execution time so the problem would become intractable.

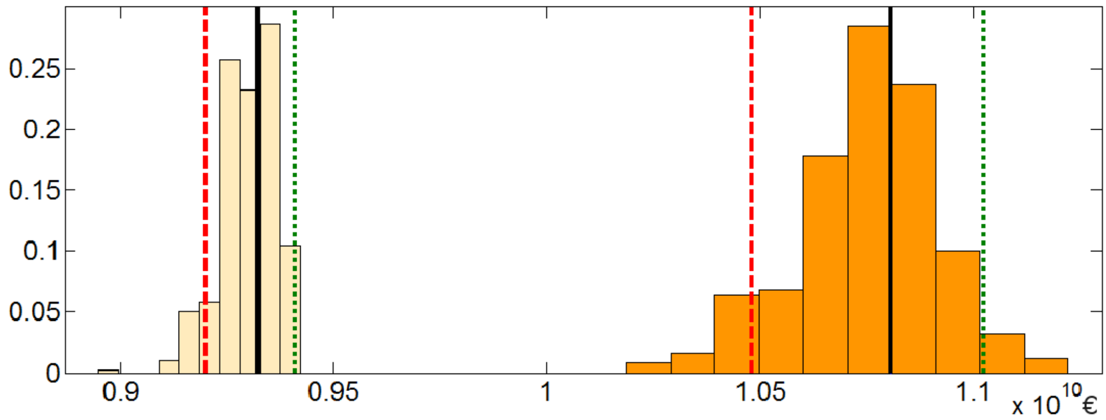


Figure 24: Profit distribution of cases 24C59 (left) with an equivalent dispatchable unit and 24R59 (right) with N-DRs.

The availability or not of pumped storage units means a difference in medium term profits as explained at sections §3.4 and 5.3. Part of the difference in profits between the cases RP and R in Table 4 is due to the extra generation by conventional units that the pumping load means, which is partly offset by the pumping load fee and by the extra hydro generation (with no cost) made possible by the pumping.

In the equilibrium solution of case 18RP21 the expected generation dedicated to pumping is 6742.3 GWh and the expected extra hydro obtained is 5056.7 GWh. The other part comes from the different shape of the equivalent LDC. Figure 25 (top) shows the generations by technology (thermal, hydro, hydro from pumping, WP and SPV), and the pumping extra load (in negative) in each subperiod of dark, low-light and bright hours over the successive periods.

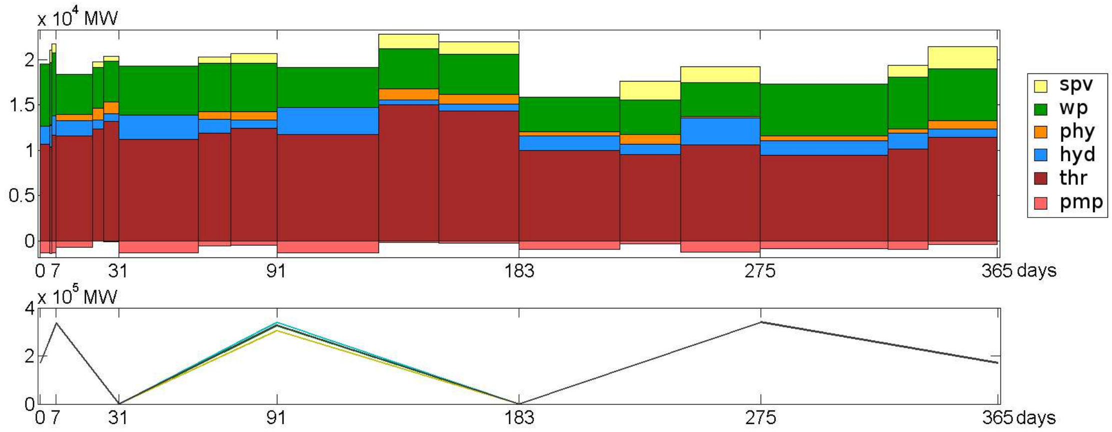


Figure 25: Generation by technologies: thermal, hydro, pumped hydro, WP and SPV, and negative pumping load in dark, low-light and bright hours of periods (above), and energy stored v_P^ν in pumping-scheme upper reservoir at end of each period in each scenario (below), of case 18RP21.

Pumping load along the different scenarios is fairly similar though not equal, but pumping hydro in a given period may be quite different in different scenarios. Figure 25 (bottom) shows the cyclic evolution of stored energy v_P^ν in the upper reservoir of the pumping scheme over the successive periods in each scenario path. (Note that the hours in the subperiods within a period are not chronologically successive.) For a given period, the positive or negative balance of eff_P times the pumping and the discharge yield the increase or decrease of v_P^ν in the period.

Regardless of the increasing or decreasing trend from one period to the next shown by v_P^ν (see Figure 25, bottom) there generally is positive pumping $c_P T_g^{i(\nu)} - x_{gCmp}^\nu - x_{Phyd}^\nu$ (in low price –or load– hours) and discharge x_{Phyd}^ν (in high price –or load– hours) within each period as long as it is feasible and profitable.

It should be noted that given unlimited availability of spare capacity by GenCos units and given the pumping capacity in the test cases $c_P = 1350$ MW, and the upper reservoir capacity of $\overline{v}_P = 340200$ MWh, it could be filled up from emptiness in 252 h, which means that there could be several cycles of pumping-generation within a long enough period, regardless of the increasing or decreasing trend of the variable v_P^ν, whose graphical representation in Figures 26 and 27 shows only the values at the end of each period.

Regarding the profit spread in equilibrium cases with and without pumped storage, it is systematically slightly higher in the solutions with pumped storage (see RP cases in Table 4) than in the corresponding ones without it (R cases). This can be seen in Figure 28 and this is so because in the objective function pumping and its generation contribute

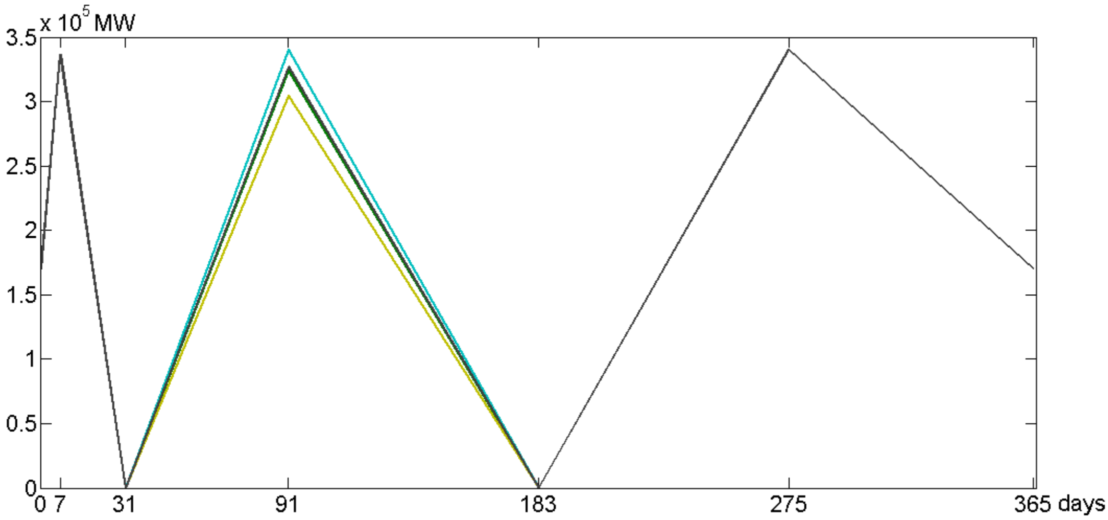


Figure 26: Energy stored v_P^y in pumping-scheme upper reservoir at end of each period in each scenario, of case 18RP21

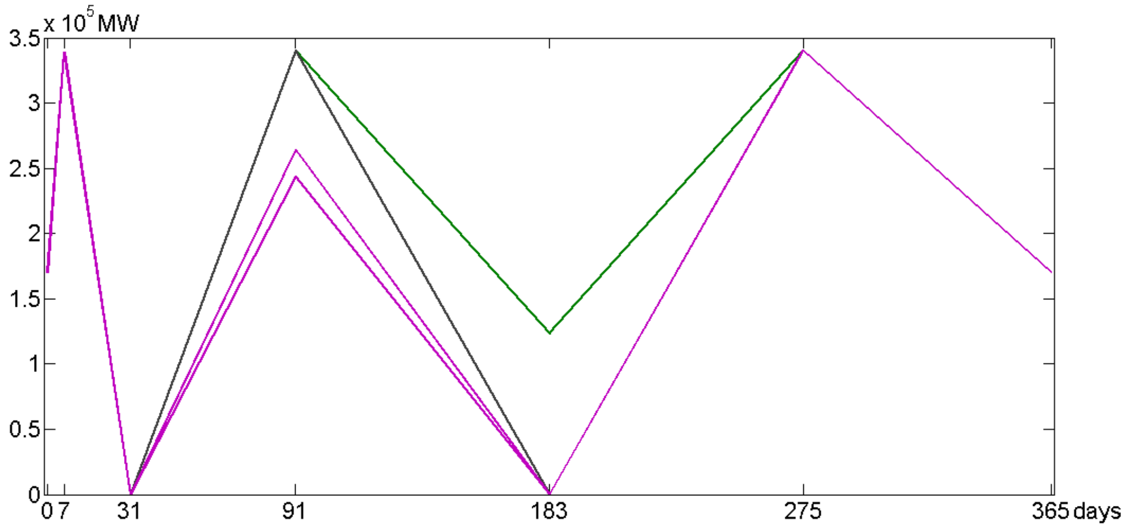


Figure 27: Energy stored v_P^y in pumping-scheme upper reservoir at end of each period in each scenario, of case 24RP75

to increase the expected profits but not to reduce profit spread. In this case, the distance between the two distributions is not as high as in the case shown in Figure 24 and this is because the effect produced in the objective function by N-DRs is much higher than the effect that pumping storage can produce.

In Figure 29 there are the profit distributions of the cases with 21, 59, 75 scenarios and 18 units with pumping scheme and renewable sources in the equilibrium behavior. As it can be seen these bar charts do not have the same number of bars and it is because the number of bars depend on the number of values that we have, as explained before in §2.10.

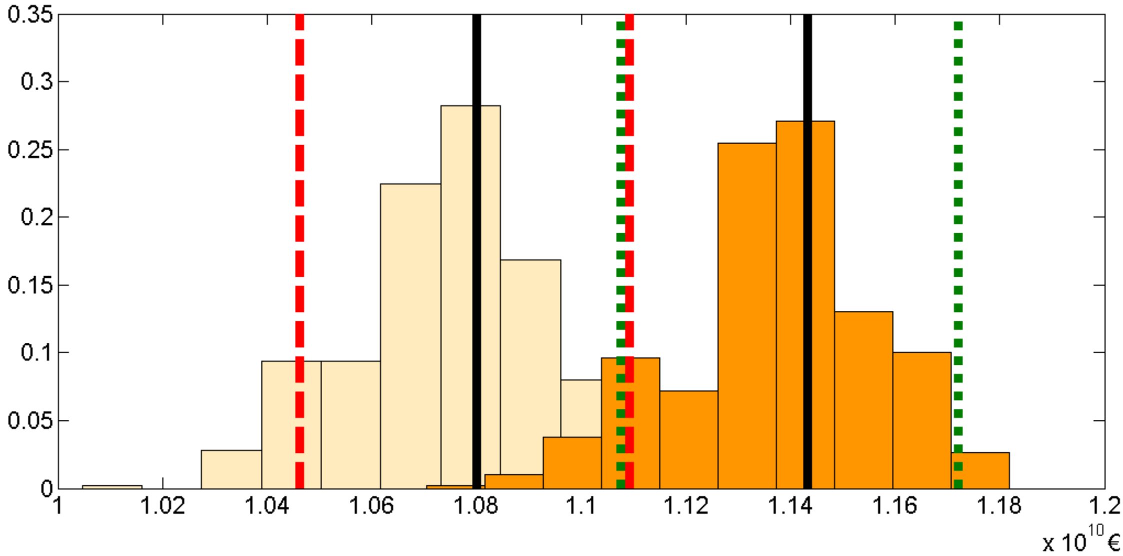


Figure 28: Profit distribution of cases 24R75 (left) with N-DRs and 24RP75 (right) with N-DRs and pumped storage.

In this example, the first graph has 7 bars; the second one, has 10 and the third has 12. As said before, having more scenarios produces smoother distributions but not enough to fit a statistical distribution. Therefore, it is not possible to calculate any classical procedure to measure profit losses or the value at risk (see §2.10).

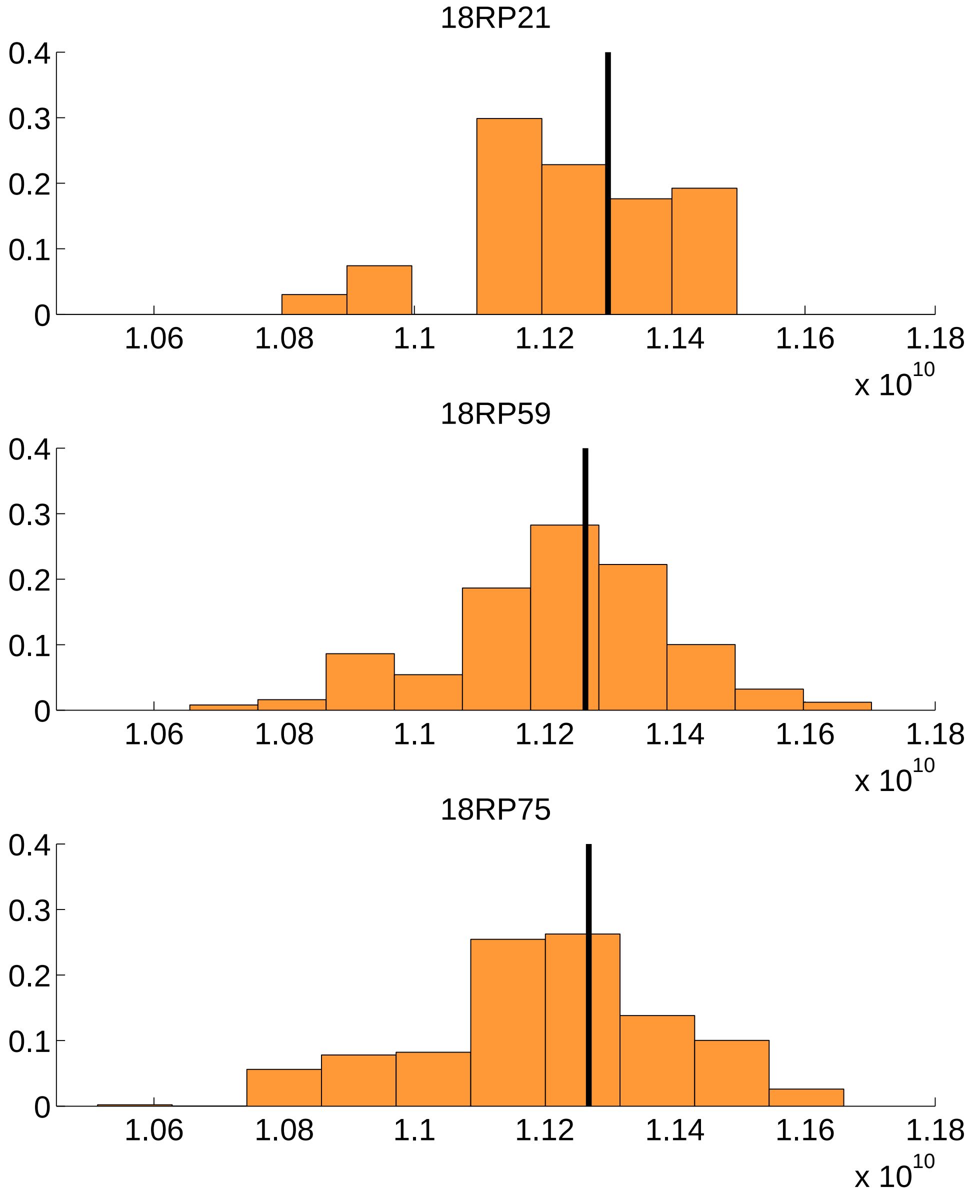


Figure 29: Profit distribution of cases 18RP21 (top), 18RP59 (middle) and 18RP75 (bottom)

6 MEDIUM-TERM PLANNING IN MIXED MARKETS WITH AUCTION AND BILATERAL CONTRACTS

As mentioned in §2.5 a bilateral contract (BC) is an agreement between two parties, normally a SGC and a big consumer or distributor, for supplying or exchange electric power under a set of specified conditions such as power (MW), energy amount (MWh), time of delivery, duration and price. BCs can take the form of futures and forward contracts.

One of the most extended electricity market types is the mixed market with pool auction and BCs. In it, GenCos may have BCs to supply energy in given amounts and instants, and they bid the remaining available generation capacity to the pool MO to get extra benefits.

BCs are risk-hedging instruments in markets to lock in future sale prices. However, it has been found that, without risk considerations, the problem of profit maximization in a mixed market is numerically much harder to solve than the pure-pool one, due to non-convexity in the objective function. It will be later shown that the envisaged inclusion of CVaR constraints with extra variables [83] would bring about further numerical trouble due to the indefiniteness of the profit-loss function of each market participant. Moreover, as shown in §5.5.3 the irregularity and discontinuity in the profits distribution could complicate the convergence. Therefore, the optimization of the BC size to check the risk has been excluded from consideration in this thesis and it is suggested as a matter for further research.

In this thesis the procedure for reaching BC agreements (auctions, capacity rentals,...) has not been taken into account. The problem considered is that of honoring the prospective BC agreements reached, while participating in the market auction with the remaining generation capacity to maximize the profits. The revenue from the BCs is known beforehand and cannot be optimized. The problem left is similar in formulation to that of medium-term generation planning in a pure pool market described in Chapter 5, but it is very different in solution difficulty as the mixed market objective is much more non convex than that of the pure pool, it has more variables and extra constraints, which leads to having to employ quite different procedures to solve it.

As the BCs considered in medium-term planning are a forecast (for the SGC, for the RoP and for the system), no risk consideration can be made because the BC forecasts are a fixed parameter. What is optimized is the contribution of each unit to honoring the BCs, while maximizing the profits from participating in the market (including the participation in pumping).

6.1 BILATERAL CONTRACT LOAD

The responsibility for matching the bilateral contract load and for satisfying other medium-term constraints, as the management of hydro resources or the emission caps, is on the GenCos. In mixed systems, the GenCos also decide which units will be devoted to supplying their bilateral contract load, and during which hours. For every hour, the System Operator (SO) in a mixed system market collects the information from the pool auction and from the bilateral exchanges, determines the system load, and checks that the transmission network can safely convey the generations and consumptions agreed, introducing corrective changes if necessary.

The analysis of the amount of energy traded through BCs for several months compared to the total energy produced in the MIBEL shows that the energy traded through BCs can be modeled as an LDC (see Figure 30). For future periods, BC duration curves (BCDC) could be forecasted in the same way as the system LDCs are estimated.

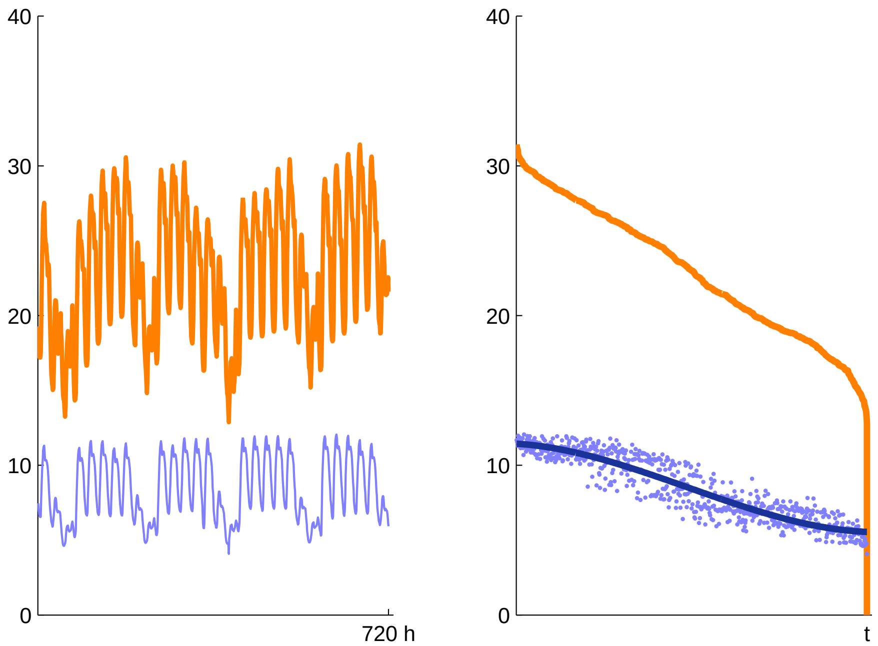


Figure 30: Series of the system demand and power traded through BCs (thin line) during June 2007 in GW (left); LDC, bilateral data ordered by decreasing load and non-increasing fitted polynomial (right).

Two types of BCs are contemplated by the regulations of the MIBEL: base contracts, which span the full length of the period, and peak contracts for several lengths of time. Piling up the peak contracts within a given period, and ordering the hours by decreasing system load, a generally decreasing BCDC will be obtained, as it is shown in Figure 30.

The information on the past bilateral hourly load is available to the participants in the market (see [88]), and from it, system BCDCs, as that in Figure 30 right, for future periods can be deducted. Moreover, a SGC will know from its own records, which are its forecasted BCDCs, and, by subtracting them from the forecasted BCDC of the system, they will have the BCDCs of the RoP. We have thus two types of BCDCs: that of the SGC and that of the RoP, each of which having its own generation units to satisfy its specific bilateral load. With the available data in the MIBEL, no other participant can be observed; therefore, the medium-term planning must be limited to considering a SGC and the RoP.

6.2 TIME-SHARE HYPOTHESIS

A time-share hypothesis is made to address the problem of a certain unit having the possibility of matching two different LDCs over a given period. This unit may devote all or a part of its generation to honor BCs as long as there is BC load still to be matched. The remaining available generation of that unit may participate in the market and will be rewarded using the market price function. For notation purposes, in this problem formulation it will be used the two-element set $B := \{\text{SGC, RoP}\}$, and the tilde $\tilde{}$ will be employed on parameter, variable and set symbols that refer to BCs. The set $\widetilde{\Omega}_\beta \subseteq \Omega$, $\forall \beta \in B$ will contain the units that may match the BCDC of either the SGC, or RoP.

It is supposed that the shaded part of Figure 31 (left) corresponds to BCs. Now it is assumed that the solution is given in Figure 31 (right), where each shaded slice represents the expected generation of a unit in $\widetilde{\Omega}$ (assuming null outage probability). For example, unit $\widetilde{u}_1$ devotes all of its generation to satisfy the BC agreements. But instead units $\widetilde{u}_2$ and $\widetilde{u}_3$ generate part of the time to satisfy the BCs and part of the time to the market. It is easy to see that the union of the darker areas conforms the BCDC. As in the model of the pure pool trading system (42), in order to compute the expected profit, two assumptions are taken: (i) a unit generates at its maximum capacity and (ii) the shape of the contribution is approximated to a rectangle.

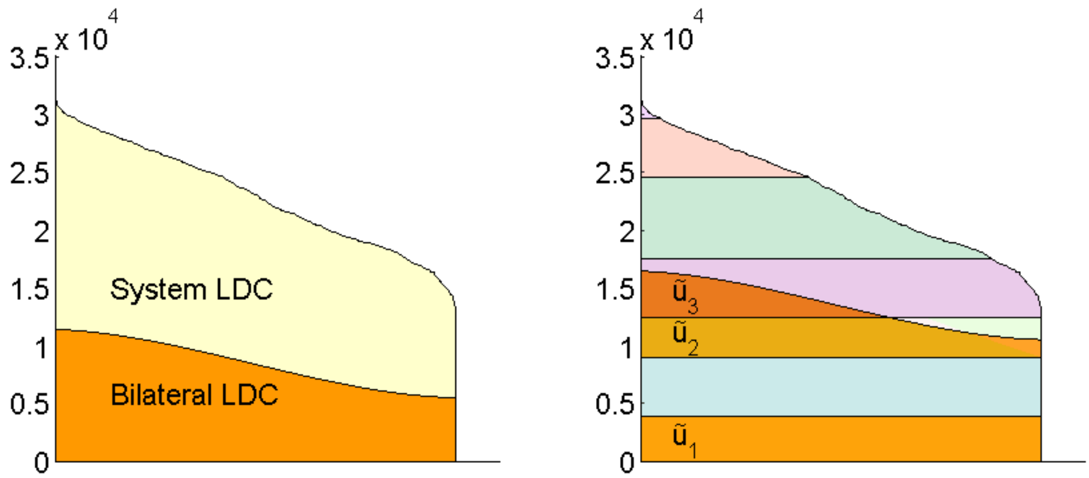


Figure 31: LDC of the system and part corresponding to the bilateral contracts LDC (shaded part, left); optimal load matching with production for bilateral contracts (right).

6.3 MAXIMIZATION OF STOCHASTIC MARKET PROFITS WITH ENDOGENOUS PRICE FUNCTION AND BILATERAL CONTRACTS

In order to calculate the profits in the mixed system, the market price function $p^\nu(t, x_h)$ is needed, with hydro and WP influence for each node ν of the scenario tree, whose period is denoted by $i(\nu)$:

$$p^\nu(t, x_h^\nu) = b_0^\nu + l^{i(\nu)}t + \frac{d}{T^{i(\nu)}} \sum_{h \in H} x_h^\nu, \tag{46}$$

where b_0^ν is the node WP-influenced market price at peak period load, decreased by the endogenous hydro generation term $\frac{d}{T^{i(\nu)}} \sum_{h \in H} x_h^\nu$ with d being an estimated negative factor, $h \in H$ are the hydro pseudo-units that represent hydro basins of set H, and $l^{i(\nu)}$ is the estimated negative slope of the market price with respect to the LDC load duration t.

From the total expected generation x_j^ν of unit j in node ν, the part $\widetilde{x}_j^\nu$ will be rewarded according to the agreed BCs and the rest $x_j^\nu - \widetilde{x}_j^\nu$ will be paid at market price. The market revenue is calculated multiplying the capacity of the unit by the integration of the price function during the unit expected generation time, which now starts at $\widetilde{x}_j^\nu/c_j$ and ends at the total expected time x_j^ν/c_j, as units matching BC peak load participate in the market in hours that come after the duration $\widetilde{x}_j^\nu/c_j$ of matched BC load. Subtracting the generation cost from the revenue, the generation unit profit $r_j^\nu(x_j^\nu, \widetilde{x}_j^\nu)$ will be obtained (where the revenue from the BC, which is fixed, is not included).

$$
\begin{aligned}
r_j^\nu(x_j^\nu, \widetilde{x}_j^\nu) &= c_j \int_{\frac{\widetilde{x}_j^\nu}{c_j}}^{\frac{x_j^\nu}{c_j}} p^\nu(t, x_h^\nu)\, dt - f_j x_j^\nu = \\
&= b_0^\nu(x_j^\nu - \widetilde{x}_j^\nu) + \frac{d}{T^{i(\nu)}}(x_j^\nu - \widetilde{x}_j^\nu) \sum_{h\in H} x_h^\nu + \frac{l^{i(\nu)}}{2c_j}\Big((x_j^\nu)^2 - (\widetilde{x}_j^\nu)^2\Big) - f_j x_j^\nu \,.
\end{aligned}
\tag{47}
$$

This function is indefinite, which means that when optimizing it, there may be more than one local optimum.

It is possible to extend the model (15) with the influence of the hydro generation on the price function, with stochasticity and with BCs.

$$\max_{x_j^\nu, \widetilde{x}_j^\nu} \sum_{\nu\in\mathcal{N}} \pi^\nu \Big\{ \sum_{\beta\in B} \sum_{j\in\widetilde{\Omega}_\beta} r_j^\nu(x_j^\nu, \widetilde{x}_j^\nu) - f_0 x_0^\nu \Big\} \tag{48a}$$

$$\text{s.t.: } \widetilde{x}_j^\nu \leqslant x_j^\nu \qquad \forall \beta \in B \quad \forall j \in \widetilde{\Omega}_\beta \quad \forall \nu \in \mathcal{N} \tag{48b}$$

$$\sum_{j\in\widetilde{\omega}_\beta} \widetilde{x}_j^\nu \leqslant \widetilde{e}_\beta^\nu - s^\nu(\widetilde{\omega}_\beta) \quad \forall \widetilde{\omega}_\beta \subset \widetilde{\Omega}_\beta \; \forall \beta \in B \quad \forall \nu \in \mathcal{N} \tag{48c}$$

$$\sum_{j\in\omega} x_j^\nu \leqslant e^\nu - s^\nu(\omega) \qquad \forall\, \omega \subseteq \Omega \quad \forall\, \nu \in \mathcal{N} \tag{48d}$$

$$\sum_{j\in\widetilde{\Omega}_\beta} \widetilde{x}_j^\nu = \widetilde{e}_\beta^\nu - s^\nu(\widetilde{\Omega}_\beta) \quad \forall \beta \in B \quad \forall \nu \in \mathcal{N} \tag{48e}$$

$$\sum_{j\in\Omega} x_j^\nu + x_0^\nu = e^\nu \qquad \forall\, \nu \in \mathcal{N} \tag{48f}$$

$$\sum_{\nu\in\mathcal{H}_\lambda} C_\lambda x^\nu + \sum_{\nu\in\mathcal{H}_\lambda} D_\lambda y^\nu \geqslant r_\lambda \quad \forall \lambda \in \mathcal{L} \tag{48g}$$

$$\widetilde{x}_j^\nu \geqslant 0 \qquad \forall \beta \in B \quad \forall j \in \widetilde{\Omega}_\beta \quad \forall \nu \in \mathcal{N} \tag{48h}$$

$$x_j^\nu \geqslant 0 \qquad \forall\, j \in \Omega \quad \forall\, \nu \in \mathcal{N} \tag{48i}$$

Because of $r_j^\nu(x_j^\nu, \widetilde{x}_j^\nu)$, defined in (47), the objective function is quadratic and indefinite. Constraints (48b), specific to the mixed-market problem, couple the total generation with the generation devoted to honor bilateral contracts. Constraints (48c) and (48d) are the LMCs for each LDC (BCDCs and system LDC). Constraints (48g) represent the single and multi-period non-LMCs, which are usually conditions over the total generation of some subsets of units. Constraints (48f) are the balance constraints between the LDC, the total energy e and the GDC taking into account the external energy. And, finally, constraints

(48h) and (48i) are the bounds of the variables.

The modeling of the bilateral contracts brings about two novelties: a unit can match two different load-duration curves (because of the time-share hypothesis), and the rewards that a unit will receive for its generation depend on the amount of energy produced for bilateral contracts.

6.4 THE DIFFERENCE OF CONVEX OBJECTIVE FUNCTION

Function (47) can be decomposed in the same way as (31) in §2.11 and therefore $r_j^\nu(x_j^\nu, \widetilde{x}_j^\nu)$ can be rewritten as:

$$r_j^\nu(x_j^\nu, \widetilde{x}_j^\nu) = b_0^\nu(x_j^\nu - \widetilde{x}_j^\nu) - f_j x_j^\nu \tag{49a}$$

$$- \left\{ \frac{d}{4T^{i(\nu)}} \Big(\sum_{h \in H} x_h^\nu - (x_j^\nu - \widetilde{x}_j^\nu) \Big)^2 + \frac{l^{i(\nu)}}{2c_j} \widetilde{x}_j^{\nu\,2} \right\} \tag{49b}$$

$$+ \left\{ \frac{d}{4T^{i(\nu)}} \Big(\sum_{h \in H} x_h^\nu + (x_j^\nu - \widetilde{x}_j^\nu) \Big)^2 + \frac{l^{i(\nu)}}{2c_j} x_j^{\nu\,2} \right\}, \tag{49c}$$

where the expression (49a) is linear, and given that both d and l^i are negative, (49c) is convex and (49b) is concave.

Taking into account §2.11.1, problem (48) can be recast as a minimization problem and its objective function (48a) could be rewritten as a difference of convex quadratic (DcxQ) function:

$$\min_{x_j^\nu, \widetilde{x}_j^\nu} \sum_{\nu \in \mathcal{N}} \pi^\nu \left\{ \sum_{\beta \in B} \sum_{j \in \widetilde{\Omega}_\beta} \left[f_j x_j^\nu - b_0^\nu (x_j^\nu - \widetilde{x}_j^\nu) - \frac{d}{4T^{i(\nu)}} \Big(\sum_{h \in H} x_h^\nu + (x_j^\nu - \widetilde{x}_j^\nu) \Big)^2 - \frac{l^{i(\nu)}}{2c_j} x_j^{\nu\,2} \right] + f_0 x_0^\nu \right\} \tag{50a}$$

$$+ \sum_{\nu \in \mathcal{N}} \pi^\nu \left\{ \sum_{\beta \in B} \sum_{j \in \widetilde{\Omega}_\beta} \left[\frac{d}{4T^{i(\nu)}} \Big(\sum_{h \in H} x_h^\nu - (x_j^\nu - \widetilde{x}_j^\nu) \Big)^2 + \frac{l^{i(\nu)}}{2c_j} \widetilde{x}_j^{\nu\,2} \right] \right\} \tag{50b}$$

s.t.: LMCs (48c, 48d, 48e, 48f)
and other constraints (48b, 48g, 48h, 48i)

where (50a) is a positive definite quadratic function and (50b) is a concave quadratic form.

Further to the contents of §2.11.2, it is possible to consider an alternative formulation of the former problem that uses an extra variable ν_{RC}:

$$
\begin{aligned}
&\min_{x_j^\nu, \widetilde{x}_j^\nu} (50a) \ -\nu_{RC} && (51a)\\
&\text{s.t.: } \nu_{RC} \leqslant \sum_{\nu\in\mathcal{N}} \pi^\nu \left\{ \sum_{\beta\in B} \sum_{j\in\widetilde{\Omega}_\beta} \left[\frac{-d}{4T^{i(\nu)}} \Big(\sum_{h\in H} x_h^\nu - (x_j^\nu - \widetilde{x}_j^\nu) \Big)^2 - \frac{l^i}{2c_j} \widetilde{x}_j^{\nu\,2} \right] \right\} && (51b)\\
&\qquad \text{LMCs } (48c, 48d, 48e, 48f)\\
&\qquad \text{and other constraints } (48b, 48g, 48h, 48i)
\end{aligned}
$$

where (51b) is a reverse convex constraint (RCC) because it makes the convex domain that it contains infeasible.

Both the DcxQ problem (50) and the RCC problem (51) are non-convex and hard to solve, especially from initial points far away from the solution.

The solution to the problem can be simplified by successively optimizing it with a linearized RCC (LRCC) about a former solution point obtained. Let $\bar{x}$ be an initial point, or the solution to a former problem with a LRCC, the problem to be solved would be linearly constrained with a positive definite objective function, having thus a unique solution:

$$
\begin{aligned}
&\min_{x_j^\nu, \bar{x}_j^\nu} (50a) \ -\nu_{RC} && (52a)\\
&\text{s.t.: LRCC} && (52b)\\
&\qquad \text{LMCs } (48c, 48d, 48e, 48f)\\
&\qquad \text{and other constraints } (48b, 48g, 48h, 48i)
\end{aligned}
$$

Moreover, the mLMH heuristic (see §3.2) should be employed to avoid having to generate an exponential number of LMCs, thus the linearization of the RCC and the application of the mLMH would be done at the same time.

Finally, it is viable to take into consideration an easier problem:

$$
\begin{aligned}
&\min_{x_j^\nu, \widetilde{x}_j^\nu} (50a) && (53a)\\
&\text{s.t.: LMCs } (48c, 48d, 48e, 48f)\\
&\qquad \text{and other constraints } (48b, 48g, 48h, 48i)
\end{aligned}
$$

where the objective function (53a) is the positive definite quadratic part (50a) of the objective function of the DcxQ problem (50). Problem (53) is convex, easy to solve and has a unique solution.

6.4.1 Changing the non-convexity of the objective function

Due to the non-convexity of the objective functions, the maximization of the profits for these problems may causes infeasibilities or other anomalous solutions. In order to correct this and make the progress of the results converge to the optimizer it is possible to do some changes into the objective function.

The degree of non-convexity of the objective functions can be easily changed by substituting the 4 in the denominator of the fraction $d/(4T^{i(\nu)})$ of (50b) and of (51b) by a parameter ρ and giving to ρ values different from 4: the lower the value, the less non-convex will be the objective functions. Starting with $\rho = 2$ in the early stages of the mLMH and changing it gradually to 4 in the later stages of it, the non-convexity of the problem is reduced during the first stages of the mLMH and recovers its proper degree towards the final stages.

Let us denote with mnm the number of iterations that have to be done in the step-by-step order phase of the mLMH. As said before, the change on the non-convexity is taken into account with the ρ parameter and it will be increased linearly at each iteration k from 2 to 4:

$$\rho = \frac{2mnm - 4}{mnm - 1} + \frac{2k}{mnm - 1}$$

6.4.2 Indefiniteness of the CVaR constraints

The inclusion of risk-aversion terms in the objective function in order to minimize the VaR of the market participants is implemented in practice [83] by considering new variables and by adding CVaR constraints such as:

$$\begin{aligned} &\tau_{k,\lambda} \geqslant L_{k,\lambda} - \alpha_k \quad \forall k \in B, \forall \lambda \in \mathcal{L} \\ &\tau_{k,\lambda} \geqslant 0 \quad \forall k \in B, \forall \lambda \in \mathcal{L} \end{aligned} \tag{54}$$

where α_k is the VaR (to be obtained) of each market participant $B = \{SGC, RoP\}$, λ is a scenario path, and $\tau_{k,\lambda}$ are new non-negative variables. $L_{k,\lambda}$ are the profit losses of each participant k over the realizations of scenario λ.

Given that the losses are minus the profits and the profits are here an indefinite function, the profit losses functions are also indefinite. The inclusion of the indefinite constraints (54) could further complicate the solution process. This inclusion has not been undertaken and is considered as an area for further research.

6.5 EQUILIBRIUM SOLUTION OF THE MEDIUM-TERM BILATERAL PLANNING

An extension to the equilibrium behavior similar to the one with renewable sources (see §5.2) has been made. The NIRA procedure (see §2.8) within the application of the mLMH (see §3.2.2) is initialized during its stage 3) and followed during the iterations of stage 4). The objective function of the NIRA optimization (24), based on the NIRA function (20) will be extended to the stochastic formulation taking into account the two observable participants in the market: the SGC and the RoP.

Using the constant terms

$$\widehat{x}_h^\nu, \ \forall h \in H | h \notin \Omega_\beta \, , \forall \beta \in B \, , \tag{55}$$

obtained as part of the results of a former solution, the equilibrium objective function expressed as a DcxQ function (eDcxQ) and the equilibrium problem are:

$$\begin{aligned}
\min_{x_j^\nu, \widetilde{x}_j^\nu} \sum_{\nu \in \mathcal{N}} \pi^\nu \{ f_0 x_0^\nu + \sum_{\beta \in B} \sum_{j \in \widetilde{\Omega}_\beta} \Big[f_j x_j^\nu - b_0^\nu (x_j^\nu - \widetilde{x}_j^\nu) - \frac{d}{T^{i(\nu)}} \Big(\sum_{h \in H | h \notin \widetilde{\Omega}_\beta} \widehat{x}_h^\nu \Big) (x_j^\nu - \widetilde{x}_j^\nu) \\
- \frac{d}{4T^{i(\nu)}} \Big(x_j^\nu - \widetilde{x}_j^\nu + \sum_{h \in H | h \in \widetilde{\Omega}_\beta} x_h^\nu \Big)^2 - \frac{l^{i(\nu)}}{2c_j} x_j^{\nu\,2} \Big] \} \qquad (56a) \\
+ \sum_{\nu \in \mathcal{N}} \pi^\nu \left\{ \sum_{\beta \in B} \sum_{j \in \widetilde{\Omega}_\beta} \left[\frac{d}{4T^{i(\nu)}} \Big(- x_j^\nu + \widetilde{x}_j^\nu \sum_{h \in H | h \in \widetilde{\Omega}_\beta} x_h^\nu \Big)^2 + \frac{l^{i(\nu)}}{2c_j} \widetilde{x}_j^{\nu\,2} \right] \right\} \qquad (56b) \\
\text{s.t.: LMCs } (48c, 48d, 48e, 48f) \\
\text{and other constraints } (48b, 48g, 48h, 48i)
\end{aligned}$$

Comparing the DcxQ problem (50) and the eDcxQ problem (56) it is possible to see that the later has more linear terms in the quadratic function and less quadratic terms both in the quadratic function and in the quadratic form, therefore the equilibrium problem is less non convex and should be easier to solve than the endogenous problem.

An alternative formulation to problem (56) using a RCC and an extra variable ν_{RC} would be:

$$\min_{x_j^\nu, \widetilde{x}_j^\nu} (56a) \; - \nu_{RC} \tag{57a}$$

$$\text{s.t.: } \nu_{RC} \leqslant \sum_{\nu \in \mathcal{N}} \pi^\nu \left\{ \sum_{\beta \in B} \sum_{j \in \widetilde{\Omega}_\beta} \left[\frac{-d}{4T^{i(\nu)}} \Big(-x_j^\nu + \widetilde{x}_j^\nu \sum_{h \in H | h \in \widetilde{\Omega}_\beta} x_h^\nu \Big)^2 - \frac{l^{i(\nu)}}{2c_j} \widetilde{x}_j^{\nu\,2} \right] \right\} \tag{57b}$$

LMCs (48c, 48d, 48e, 48f)

and other constraints (48b, 48g, 48h, 48i)

6.6 SOLUTION PROCEDURES

Direct methods of global optimization (see [34, 25]) are the most appropriate for solving an indefinite problem such as (48). However, the current implementations of these techniques, such as the available branch-and-reduce based code described in [80] have not provided an acceptable solution in reasonable time for the smallest test cases used in this work. Optimization solvers non specialized for global optimization may have difficulties with problem (48), but there are ways, as those presented in the following sections, that take advantage of the successive optimizations in the mLMH, leading to satisfactory results. These procedures are based on using different, though equivalent, formulations of (48) and on gradually changing the non-convexity of (48a) from less non-convex to its full non-convexity over the successive optimizations in the mLMH, where batches of LMCs are added.

Many tests have been done with the software BARON [80] through the modeling language GAMS (General Algebraic Modeling System, [2]). In each of them, one can observe that it takes hours to solve a reduced problem while other solvers, like *Ipopt*, solve them in minutes. Since no solution was obtained in a reasonable CPU time, the use of GAMS/BARON was rejected.

As stated before (see §2.13) the solver used is *Ipopt*. When trying to optimize a non-convex constrained problem, a non-global optimization algorithm, as *Ipopt* is, may determine that the problem is infeasible or unlimited, depending on the initial point, even if a finite solution exists.

The successive solutions in the application of the mLMH (see §3.2.2) use as initial point the solution of the former problem (plus default values for new variables associated to LMCs added). Default initial values are used in the first solution. To avoid a chain of infeasible solutions it is essential to avoid having infeasible or iteration exceeded solutions at the end of the first stage of the mLMH: *All unit, all-but-one unit and all-but-two unit LMCs*, or at the end of the second one: *LMCs with all combinations of subset of most loaded units*. To obtain that the positive definite problem (53) will be minimized in both stages, for the endogenous and for the equilibrium bilateral cases. Their solutions are always *Optimal*.

6.6.1 Solution strategies: definition and purpose

A solution strategy means a combination of successive problem modes, as described few sections before (see §6.4) along with changes in the non-convexity parameter ρ described before in §6.4.1.

The purpose of using a solution strategy is to avoid, as much as possible, obtaining abnormal solutions in the successive optimizations in the application of the mLMH. Abnormal solutions entail a risk of failing to generate LMCs that should be active in the optimizer in the complete problem. (Generating LMCs not active at the solution of the complete problem is not harmful though it means useless extra CPU time)

Many procedures have been tried. Most of them consisted on some variations in the problem mode depending on what kind of solution has been obtained (infeasibility, iteration limit, acceptable,...), for example, changing from the equilibrium problem eDcxQ (56) to problem eRCC (57) or from the endogenous one DcxQ (50) to the problem with the positive quadratic objective function (53). The following sections in this chapter contain the presentation of some strategies and their results.

6.7 TEST CASES

As described in §2.12.1 and in §2.12.2, the LDCs of periods and subperiods, and the generation units that match these loads define the test cases solved. Realistic data from the MIBEL have been employed. The planning horizon, as explained in §2.4.1, is one year decomposed into 6 periods of durations in days: 7, 23, 61, 92, 92 and 90, the starting date being October 4. Hydro generation is represented by a simplified model of several basins and has 13,153 GWh of expected inflows. A 1350 MW pumped storage scheme, whose upper reservoir can store up to 340200 MWh, and it is assumed to be half full both at the beginning and at the end of the yearly planning horizon is considered in several test cases. Loads (of 2010) to be satisfied amount to 157425 GWh, and generation units are considered in two degrees of disaggregation for different generation technologies. As for BCs, the units of the SGC considered have about 47% of system capacity.

Test cases are almost the same as in §5.4. The difference between them is that BC cases do not have an SPV unit. This is because with BCs there are already three LDCs and if the SPV unit is taken into account then it is necessary to introduce more LDCs and it would increase the difficulty of the problem. Instead of using so many LDCs, an approximation of SPV generation has been used. It has been calculated the average SPV generation along the scenarios and a pseudo-SPV unit with fixed capacity and fixed probability failure has been introduced.

The numbers of generation units of cases are 18, and 24. Results are given for three scenario-tree sizes with 21, 59 and 75 scenarios. Twenty-four possible problems (endogenous or equilibrium, with or without pumping, for 18 or for 24 units, using 21, 59, or 75 scenarios) have been successfully solved. The unit and the scenario numbers will identify the test case. Table 6 has the fixed dimensions of each case without or with a pumping scheme indicated by a letter "n" for absence, or "P" for presence of pumping, between the number of generation units and that of scenarios. Pumping schemes are included in the model, as it has been proposed in §3.4, adding the pumping load fee (37) into the objective function and constraints (35) and (36) included as non-LMCs. Problem size is specified with numbers n_u of units, n_v of scenario tree nodes, and n_{var} of variables. The number of equality and inequality non-LMC is also indicated. The number of generated LMCs during the solution using the mLMH is given in the tables with the results.

Table 6: Fixed Dimensions of Test Cases

		problem size			non-LMCs	
case	pumping	n_u	n_v	n_{var}	$=$	$\geqslant$
18n21		16	61	2131	410	1291
18n59		16	100	3496	795	2485
18n75		16	153	5351	1140	3573
18P21	√	18	61	2354	471	1535
18P59	√	18	100	3837	895	2885
18P75	√	18	153	5888	1293	4185
24n21		22	61	3046	656	1972
24n59		22	100	4996	1272	3970
24n75		22	153	7646	1824	5616
24P21	√	24	61	3269	717	2216
24P59	√	24	100	5337	1372	4370
24P75	√	24	153	8183	1977	6228

6.8 FIRST STRATEGY

The outline of the strategy applied in each stage (see §3.2.2) of the mLMH in the endogenous problem is:

1) Let $\rho := 2$, substitute the 4 in (50a) by ρ, and solve (53).

2) Solve (53).

3) Solve (51).

4) Set initial problem mode to (51).

   In each loop of addition of LMCs change gradually ρ from 2 to 4, and solve current problem mode;

   if in mode (50) the solution outcome is *Iteration limit exceeded* or *Infeasibility*, change mode to (51);

   if in mode (51) the solution outcome is *Optimal* or *Acceptable*, change mode to (50).

In the equilibrium problem, due to the different objective function, stages 3) and 4) are of the same type as in the endogenous problem but different.

3) Solve (51) and compute terms (55).

   To start NIRA, solve (57) and update terms (55).

4) Set initial problem mode to (57).

   In each loop of addition of LMCs, change gradually ρ from 2 to 4, solve current problem mode and update terms (55);

   if in mode (56) the solution outcome is *Iteration limit exceeded* or *Infeasibility*, change mode to (57);

   if in mode (57) the solution outcome is *Optimal* or *Acceptable*, change mode to (56).

To ensure that stages 1) and 2) will terminate with feasible points satisfying the basic LMCs incorporated, the convex problem (53) has been used in the first two stages of the application of the mLMH for endogenous and for the equilibrium mixed-market problem.

It has been seen that the solution after having added a large batch of LMCs, as in stage 3) of the mLMH, is numerically more stable for modes with RCC (51,57) than with a DcxQ (50,56), this being the reason for employing the RCC modes in stage 3) and to start stage 4) and for maintaining a low value for parameter ρ. At each loop of stage 4), only a small batch of LMCs is added after each solution. Increasing gradually ρ to 4 produces a restoration in the non-convexity. Employing this and the DcxQ modes, the *Restoration phase failure* outcome only occurs very occasionally and no corrective action appears to have any effect on the subsequent solutions.

The solution outcomes have been checked after the optimizations in the application of the mLMH to the mixed model 48 and it has been observed that the last four outcome types may occur. It must be stressed that it is important that the last optimization in the mLMH (with all LMC batches incorporated) finishes as Optimal, but having had previous non-standard solutions could entail a risk of having wrongly chosen the ensuing LMC batch, which could lead to solutions not matching properly the predicted LDCs and BCDCs. In that regard it has been observed that, with some of the test cases employed, the sequence of plain optimizations in the mLMH led to a series of infeasible outcomes terminating in a spurious solution. That is why the strategies described below are aimed at avoiding the infeasible or anomalous solutions, and at reducing as much as possible the exceeded iterations and the restoration failures.

6.8.1 Computational results with the First Strategy

Due to the fact that two types of utility function (the endogenous cartel behavior and the equilibrium behavior) are considered, in this section there will be presented results for both of them. In addition, it will be presented results for a case solved with an alternative procedure where it can be seen why this procedure has been rejected.

As explained before, two types of utility function are considered: the endogenous cartel behavior (48a or 50a+50b), and the equilibrium behavior (56a+56b) considering the two observable players (the SGC and the RoP). The effect of WP on market-price level is considered in the scenario-tree parameters b_0^{γ} [51].

$\widetilde{P}$ in Table 7 indicates the expected profit value of the solution, where the fixed revenue from the BCs is not included, and column LMCs the number of LMCs generated by the mLMH. The *Ipopt* exe column contains the number of different executions of the solver (in the application of the mLMH), columns "ac", "ei" and "rf" stand for the number of executions that terminate abnormally as an *acceptable*, *excess iteration*, or *restoration failure* solution, and the "ite" column indicates the number of iterations. The CPU sec. column contains the AMPL plus solver execution times.

The results for each generation setting and number of units are always given for the three-scenario tree sizes employed. The expected profit should be almost the same for the same case with different reduced tree sizes. Endogenous solutions will be later compared with the equilibrium results.

Table 8 contains the results of the equilibrium planning solutions obtained with the NIRA algorithm for the test cases using different generation settings including the presence or not of a pumped storage scheme.

The differences between the endogenous cases 24r21, 24r59 and 24r75 and the corresponding equilibrium ones 24R21, 24R59 and 24R75 illustrate the important increase in CPU time of the equilibrium solution with respect to the endogenous cartel solution owing to a

Table 7: Endogenous Cartel Solutions using First Strategy

	opt. profit				*Ipopt*			CPU
case	$\widetilde{P}$ 10^6 €	LMCs	exe	ac	ei	rf	ite	(sec)
18n21	4345.363	29849	14				10460	1069
18n59	4334.355	49175	14		1		13576	2386
18n75	4334.144	74831	16	1	1		17244	5066
18P21	4667.010	35272	18				6558	771
18P59	4654.505	57738	16				8793	1870
18P75	4655.574	88310	17	1			11585	4184
24n21	4478.882	50719	20	1			11913	2986
24n59	4473.304	83082	20				15210	6845
24n75	4460.351	127110	23	1			21004	15762
24P21	4800.822	54793	22				8706	2366
24P59	4794.489	89903	23	1			11798	5913
24P75	4784.174	137434	23				15062	12761

Table 8: Equilibrium Solutions using First Strategy

	opt. profit				*Ipopt*			CPU
case	$\widetilde{P}$ 10^6 €	LMCs	exe	ac	ei	rf	ite	(sec)
18n21	4262.275	29850	13			1	6642	602
18n59	4252.089	49172	14		1		11923	1937
18n75	4249.024	74829	14		1		10422	2715
18P21	4615.575	35308	18	1			6129	623
18P59	4599.129	57779	18	3			8239	1529
18P75	4601.778	78379	18				9388	3021
24n21	4406.553	50718	20				10925	2573
24n59	4400.160	83082	20				10863	4445
24n75	4387.324	127113	21				15214	10621
24P21	4755.211	54791	23				9777	2630
24P59	4738.464	89890	23	1		1	12565	6038
24P75	4731.780	137407	23	2			13365	11265

more complex objective function in the equilibrium model, which requires a much higher number of iterations, while the number of LMCs generated by the mLMH remains the same. Equilibrium profits are lower than the endogenous cartel ones, though not much as only three competing GenCos are considered in the cases with 24 units. The difference is not too significant as the hydro inflows are to be equally spent for generation over the time horizon in both models. It has been shown that the distribution of hydro generation along the successive periods is not the same [84], with that of the equilibrium model being closer to the minimum cost solution (where there is more hydro generation during higher demand periods) than that of the endogenous cartel.

The abnormal solutions that occur during the application of the proposed solution strategy appear not to trouble significantly the final results. The differences in expected profit for each case with different tree size present little variation, which can be caused by a non-absolutely perfect tree reduction. Having obtained widely different expected profits would have meant convergence to different local minimizers. However, this not being so, is no guarantee that better local optimizers may not exist.

6.9 SECOND STRATEGY

The Second Strategy corresponds to the same First Strategy but, instead of having a gradual change of non-convexity, having ρ fixed at 4 or, what it is the same, with no change in the convexity of the objective function.

6.9.1 Computational results with the Second Strategy

In Tables 9 and 10 there are the results obtained employing the alternative strategy of not altering the non-convexity of the objective functions. Tables in this section contain more or less the same results as Tables in section 6.8.1. The difference is that these new tables contain an extra column, noted by “inf", that gives information about the number of executions that terminate abnormally as *Infeasibility*. The expected profit value of the solutions $\widetilde{P}$, where the fixed revenue from the BCs is not included, is very similar in both solution strategies. But regarding the abnormal solutions obtained, it is easy to see that with this second strategy there are more solutions that end with an infeasible solution or with an excessive number of iterations. As said before, these solutions may include some LMCs that we have to avoid because they should not be active at the solution of the original problem. For example, in this second strategy 484 optimizations have been done, of which 16 acceptable, 11 exceeded iterations and 2 infeasible where using the first one, 451 optimizations have been done, of which 12 acceptable, 4 exceeded iterations and 2 restoration failure.

Table 9: Endogenous Solutions using Second Strategy

	opt. profit			*Ipopt*			CPU
case	$\widetilde{P}$ 10^6 €	exe	ac	ei	inf	ite	(sec)
18n21	4345.587	16				10703	1110
18n59	4336.520	16		1		13888	2423
18n75	4332.804	17	2	1		18626	5524
18P21	4663.949	18				8778	1046
18P59	4650.197	18				11340	2426
18P75	4648.146	20		1		12830	5337
24n21	4460.910	22				15164	3792
24n59	4448.311	22	1	1		19273	8988
24n75	4450.893	22	1	1	1	23199	17918
24P21	4790.566	22	1			12828	3688
24P59	4780.361	23	1	1		14767	7797
24P75	4775.638	24	1	1		19069	16857

Table 10: Equilibrium Solutions using Second Strategy

	opt. profit			*Ipopt*			CPU
case	$\widetilde{P}$ 10^6 €	exe	ac	ei	inf	ite	(sec)
18n21	4268.240	15				9771	945
18n59	4256.377	17		1		10311	1661
18n75	4252.940	16		1		11860	3214
18P21	4605.927	18				7612	978
18P59	4593.721	18	2			10441	2487
18P75	4596.079	19	2			10800	4511
24n21	4392.601	24				13108	3070
24n59	4380.010	22	1	1		14603	6145
24n75	4381.443	22	1	1		17485	12625
24P21	4741.286	24	1		1	11318	3960
24P59	4733.180	24				15903	8613
24P75	4732.864	25	2			16224	13554

6.10 THIRD STRATEGY

The Third Strategy presented differs from the first one in terms of changing some problems to solve. The outline of the strategy applied in each stage of the mLMH in the endogenous problem is:

1) Let $\rho := 2$, substitute the 4 in (50a) by ρ, and solve (53).

2) Solve (53). If the solution outcome is *Optimal* or *Acceptable*, change mode to (50). If the solution is *Iteration limit exceeded* or *Infeasibility*, change mode to (51).

3) If in mode (50) the solution outcome is *Acceptable* or *Iteration limit exceeded*, change mode to (51). If the solution is *Infeasibility*, change mode to (52);

   if in mode (51) the solution outcome is *Optimal*, change mode to (50). Else, if the solution outcome is *Iteration limit exceed* or *Infeasibility*, then change mode to (52).

4) In each loop of addition of LMCs change gradually ρ from 2 to 4, and solve current problem mode;

   if in mode (50) or (51) the solution outcome is *Infeasibility*, change mode to (52), in other case, change mode to (50);

   if in mode (52) the solution outcome is *Optimal* or *Acceptable*, change mode to (50). Else, if the solution outcome is *Iteration limit exceeded*, change mode to (51).

6.10.1 Computational results with the Third Strategy

Tables 11 and 12 have the results for both the equilibrium and the endogenous behaviors while modifying the parameter ρ between 2 and 4 and using the third strategy.

Although the expected profit $\widetilde{P}$ is similar between the first strategy and this one, it can be easily seen that this strategy is worse than the first one because the number of anomalous solutions is significantly higher than with the chosen strategy. Besides this, there is the existence of multiples infeasibilities. Due to these two facts, it is possible that the solution obtained is not the true optimal because abnormal solutions may have made the optimizer reach other solutions that are away from the optimal point. Comparing this strategy with the second, one can see that the third one is also worse than the second as for 456 optimizations, there are 8 acceptable, 14 exceeded iterations and 6 infeasible while in the second there are 16 acceptable, 11 exceeded iterations and 2 infeasible.

6.11 FOURTH STRATEGY

As done with the second strategy, this fourth one corresponds to the same as third strategy but, instead of having a gradually change for non-convexity, fixing rho at 4.

Table 11: Endogenous Solutions using Third Strategy

case	opt. profit $\widetilde{P}$ 10^6 €	*Ipopt* exe	ac	ei	inf	ite	CPU (sec)
18n21	4345.920	14		1		10704	1078
18n59	4338.360	15		1		13992	2460
18n75	4332.014	16		2	2	19448	5879
18P21	4667.322	16				7858	598
18P59	4654.042	16	1			7735	1586
18P75	4655.845	16	2			8596	3138
24n21	4464.484	21		1	1	15694	4065
24n59	4469.569	19	1	2	2	17849	8280
24n75	4450.431	22	3	1	1	24442	18720
24P21	4795.044	22				8286	2226
24P59	4781.426	23		1		11258	5612
24P75	4779.215	23		1		13800	11907

Table 12: Equilibrium Solutions using Third Strategy

case	opt. profit $\widetilde{P}$ 10^6 €	*Ipopt* exe	ac	ei	inf	ite	CPU (sec)
18n21	4258.640	15				4621	378
18n59	4250.181	15				7601	1139
18n75	4246.993	15				5938	1403
18P21	4616.145	18				6037	613
18P59	4599.786	18	1			9141	1745
18P75	4602.274	18				8949	2746
24n21	4394.785	23				10394	2383
24n59	4390.751	21		1		12076	5013
24n75	4374.533	21		1		14685	10405
24P21	4745.177	23				10361	2734
24P59	4734.109	23		1		14099	6841
24P75	4732.522	23		1		15850	12775

6.11.1 Computational results with the Fourth Strategy

Tables 13 and 14 have the results for the same behaviors but, instead of modifying ρ, there have been solved without parameter 4 (or fixing it to $\rho = 4$). Here, there were 480 optimizations, of which 18 acceptable, 41 exceeded iterations and 14 infeasible. Comparing with the other strategies similar expected profits were obtained but solutions that are more irregular were found. Consequently, this strategy was also rejected.

Table 13: Endogenous Solutions using Fourth Strategy

case	opt. profit $\widetilde{P}$ 10^6 €	*Ipopt* exe	ac	ei	inf	ite	CPU (sec)
18n21	4346.453	17				10425	1165
18n59	4337.560	16		1	1	13220	2677
18n75	4334.207	17	1		1	16374	5844
18P21	4664.728	18				8474	1115
18P59	4649.873	18	2		2	11045	2617
18P75	4648.655	18	2	1		13467	5484
24n21	4461.490	22	1	1	1	16206	4828
24n59	4448.874	22	1	1	1	17813	9833
24n75	4445.156	23	2	1	3	20215	16835
24P21	4791.166	22	2			12680	3995
24P59	4780.444	23	1	1	2	13926	9083
24P75	4774.412	24		1		17969	17551

Table 14: Equilibrium Solutions using Fourth Strategy

case	opt. profit $\widetilde{P}$ 10^6 €	*Ipopt* exe	ac	ei	inf	ite	CPU (sec)
18n21	4262.302	14				5740	589
18n59	4253.562	15		1		7483	1400
18n75	4248.075	15		1		7800	2633
18P21	4625.335	19	1	13	1	43562	6934
18P59	4594.970	18	3			9162	1721
18P75	4596.155	20	1	14	2	47654	22616
24n21	4391.292	23		1		12131	3476
24n59	4377.628	23		1		15487	8422
24n75	4373.529	21		1		15308	13558
24P21	4743.007	24				11581	3430
24P59	4732.926	24		1		14051	7898
24P75	4726.945	24	1	1		15961	13549

6.12 SUMMARY OF RESULTS WITH THE DIFFERENT STRATEGIES

In order to choose one strategy it is important to count the number of abnormal solutions and compare them. Strategy 1 has 12 acceptable solutions, 4 solutions with an excess of iterations and 2 restoration phase. Strategy 2 has 16 acceptable solutions, 11 with an excess of iterations and 2 infeasible solutions. Strategy 3 has 8 acceptable solutions, 14 solutions with an excess of iterations and 6 infeasible solutions. Finally, strategy 4 has 18 acceptable solutions, 41 solutions with an excess of iterations and 14 infeasible solutions. Many different strategies have been tried in order to find the best strategy but they have failed to improve the results presented in this thesis, so they are not included here.

First strategy is the best procedure because, while all strategies have similar results in the objective functions, this is the one with a lower number of abnormal solutions which means that is the one that minimizes the risk of not having generated LMCs that should be active in the optimal solution. The fact that, for a specific test case with different number of scenarios (21, 59 and 75), the results obtained are similar, it is a necessary, but not sufficient condition to think that the solution reached is near to the global optimizer. If the results had been very different among them, possibly, locally optimizers have been found.

In the light of the results, it seems reasonable and worthy to make a change in the non-convexity of the objective function. With it, some restoration phase solutions are obtained, but they are not as harmful as an infeasible solution due to they do not get away from the optimizer.

6.13 FURTHER ANALYSIS OF RESULTS OF THE FIRST STRATEGY

Figure 32 shows three aspects of the solution to equilibrium case 24P21: the evolution of the energy kept in the upper reservoir of the pumped storage scheme along the nodes of each scenario at the top, the expected power productions of different technologies in the middle, and the evolution of the expected market price at the bottom. Although the pumping capacity would allow filling up the upper reservoir in about 200 hours, there is not unlimited power available for pumping as units must satisfy system load and BCs resulting in a yearly mean spare power of 771 MW, which is fully used in all scenarios, given that the initial and final stored volumes are the same in all scenarios and pumping (and generation from pumping) is paid at market price.

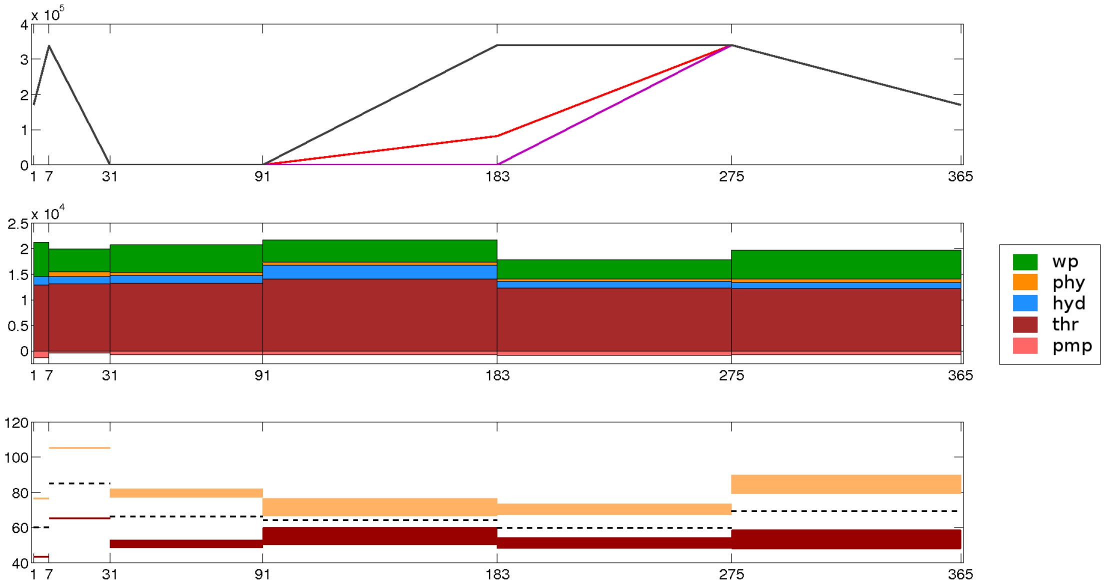


Figure 32: Solution to test case 24P21 showing the evolution over the periods of the energy stored in the pumped storage reservoir for the different scenarios (top), evolution of expected power productions of different technologies: "thr" thermal, "hyd" hydro, "phy" generation from pumped hydro, "wp" WP, and "pmp" pumping, in negative (middle), and evolution of market prices (bottom). Abscissae: time in days.

As for the expected power productions, the WP generations of each period are totally determined by the WP scenarios generated [51]. The rest of generations are the result of the optimization. It can be appreciated that the generation from pumped water corresponds to the 75% of the pumped energy by the rest of units, given the 75% efficiency considered of the pumping system. Given that the length of the periods is their duration in days, the areas of the rectangles in each period correspond to the energies generated by each technology. These results could be individualized for each generation unit in the data set used.

For the market prices, in orange there is the range of possible values of market prices at peak load over time periods, where in red is represented the range of values of market prices at base load. Finally, in black there is the expected market price. The market-price evolution obtained is influenced by all stochastic parameters and by all optimal generation policies obtained and by all constraints considered whether LMCs or non-LMCs.

Table 15 has the range of values (maximum and minimum) and the expected value ($\bar{x}_{CMP}$) of the generation of the compensatory unit (x_{CMP}) in the nodes of the scenario tree of each period for test case 24P21. The number of minutes, within parenthesis, that this pseudo-unit is working in each period is also included. As it can be seen, these values are not large but they are not null. This means that there are some minutes where there is neither generation from pumped hydro (because the upper reservoir is empty) nor generation capacity for pumping (because there is not unused generation capacity), and the pumping scheme

Table 15: Range and expected values of x_{CMP} in periods for equilibrium case 24P21

x_{CMP} in MWh (minutes)	1	2	3	4	5	6
max	82 (4)	418 (19)	1370 (61)	5017 (223)	243 (11)	402 (17)
$\bar{x}_{CMP}$	82 (4)	418 (19)	1316 (58)	4635 (206)	212 (9)	374 (16)
min	82 (4)	418 (19)	1181 (52)	4119 (183)	197 (8)	345 (15)

is idle.

Figure 33 shows the mean yearly energies of load, pumping, and of all technologies of the generation mix for the market and for the BCs in each scenario of the solution to equilibrium case 24P21. One important feature is that WP energy is split in two parts between market and BCs, the same as with thermal generation. Technologies with lower capacities, as hydro and pumped hydro generation, go almost entirely to either market or to BCs in the solution obtained.

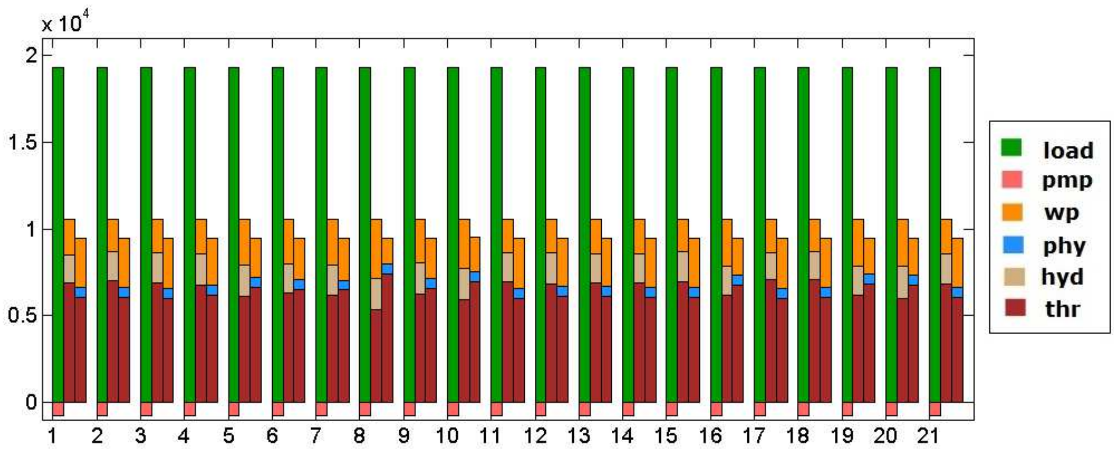


Figure 33: Yearly expected power of pumping "pmp", in negative, for each scenario in solution to equilibrium case 24P21, with system load (1st column), and generation mix by technologies "thr" thermal, "hyd" hydro, "phy" pumped hydro generation and WP, in market (2nd column) and in BCs (3rd column).

Regarding the difference between the endogenous and the equilibrium solution, in Figures 34 and 35 there are the evolutions of the energy kept in the upper reservoir of the pumped storage scheme. It can be seen that although there are different scenarios, endogenous solutions follow more or less the same pattern when using the pumping. In the equilibrium behavior, this fact is not as pronounced because there is more variation between the different scenarios.

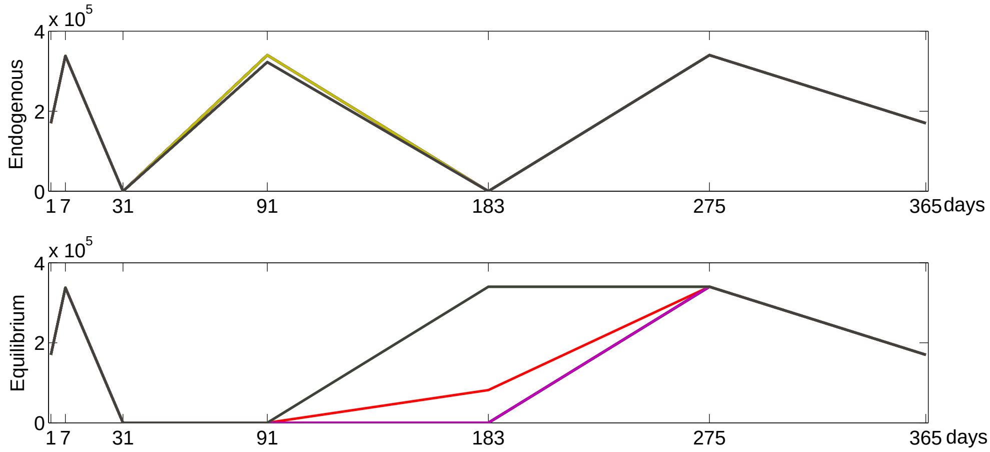


Figure 34: Endogenous solution (above) and equilibrium solution (below) to test case 24P21 showing the evolution over the periods of the energy stored in the pumped storage reservoir for the different scenarios.

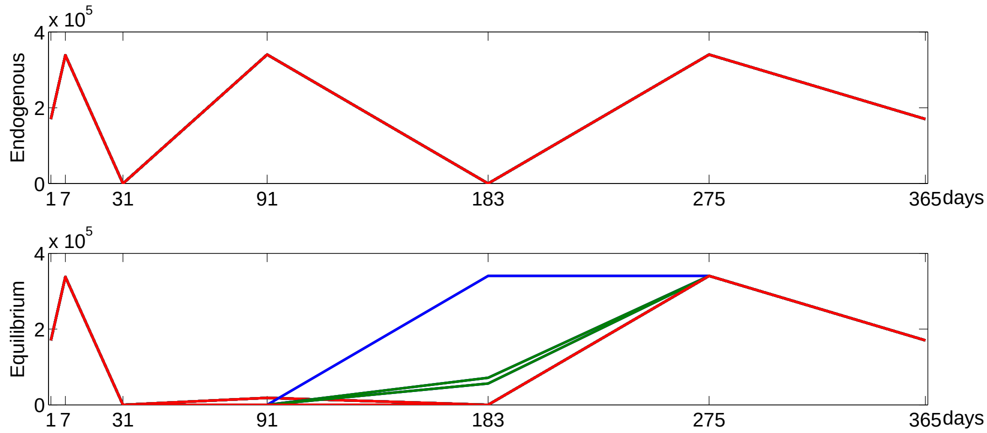


Figure 35: Endogenous solution (above) and equilibrium solution (below) to test case 24P59 showing the evolution over the periods of the energy stored in the pumped storage reservoir for the different scenarios.

7 CONCLUSIONS AND FURTHER RESEARCH

7.1 CONTRIBUTIONS AND IMPLEMENTATIONS

The list below summarizes the main contributions of this thesis:

- A model for the representation of non-dispatchable renewables based on a two-unit system with different parameters of capacity and outage probability for WP and for SPV. The graphical justification of such model through the similarity of the effect of subtracting the real WP generation from the LDC, and of the loading of the two units that represent WP has been presented.
- The development of scenario trees using QMC techniques for WP and SPV, and for the dispatchable hydro inflows taking into account the different parameters of units in each scenario node, as justified in this thesis, and the influence on market-price level in each scenario node due to WP and SPV; the decomposition of the LDC of each period into three LDCs for the dark, the medium light, and the bright hours. This entails having to employ three different sets of LMCs, each with its specific variables, in each node.
- The extension of the medium-term single-period pumping model presented in [11] to the multi period case using extra variables and extra constraints, and establishing the cost of the pumping energy at market price.
- The stochastic medium-term mixed-market model with BCs, renewables and pumping and the characterization of its non-convexity. A procedure based on the use of a conventional nonlinear programming solver has been presented, where a solution strategy was employed to circumvent the likely trouble caused by non-convexity of the sequence of optimizations of the mLMH, and the gradual variation of the non-convexity.
- The use of the mLMH in all the models described above to account for the matching the LDCs of all periods considered without having to use an exponential number of LMCs.
- The experience of the unsuccessful attempt to use a commercial global optimization software for solving the BC non-convex problem; the use of a conventional solver

for this non-convex problem and the monitoring of the results of the successive optimizations modifying if necessary the convexity of the problem in order to obtain a solution with proper match of the LDCs.

- The models presented in this thesis are useful for both price-makers, price-takers and for the market operator. For the price-maker, to obtain maximum profits and to have a generation scheduling of each generation unit. Moreover, they get to know the expected profits and its dispersion, so they can also make plans on fuel procurement. For the price-taker, to obtain a prediction of market-prices and of profits. Finally, the MO could obtain the endogenous cartel and the equilibrium solutions and appraise which behavior are the market participants closer to.

The following are the main implementations in this thesis:

- The coding in *AMPL* of all the models presented.
- The development of a test-case set with different dimensions and representing stochasticity with different numbers of scenarios so as to allow comparisons among different problem solutions and to establish trends in the increase of CPU time required for different problem sizes. The computational results for all test cases. The number of scenarios presented corresponds to problems that can be solved in reasonable CPU time on a 2010 server.
- The coding in *MATLAB* of the scenario tree generator using QMC techniques. It has been used the C++ algorithm $\mathrm{CBC}_{\mathrm{poly}}$ [60] in order to obtain the QMC points.
- Every code of each model uses several functions from the dynamic link library *RightHandSideCalculatorV3.dll* [82] in order to calculate the rhs of the LMCs. There are also 2 subroutines employed many times in each code. The first subroutine has 53 lines of code and the second one has 39 lines.
- Lines of code of some models: the case with the endogenous objective function and with renewable sources has 426 lines; the case with the equilibrium behavior and with pumping scheme using renewable sources has 593 lines; the case with the equilibrium behavior in a mixed market has 673 lines of code.

As for the initial objectives in the thesis project of §1.2, all objectives but the two last ones have been fulfilled. The objective of developing a specific interior point code taking into account that the objective function is the difference of two positive definite quadratic functions, with (positive or negative) step size in a given direction that can be readily calculated, was abandoned. This was because using the solver Ipopt and a convexification heuristic yielded satisfactory results. Regarding the last objective of consideration of the risk of profit loss, it was abandoned due to the difficulty of including non-convex inequality constraints as explained in §6.4.2, and is considered as an area for further research.

7.2 CONCLUSIONS

In the following list there are the principal conclusions of this thesis

- The probabilistic matching is an adequate procedure for medium-term power planning as it represents the exact probabilistic matching of load and avoids having to develop scenarios for load uncertainty and random failures of generation units, and it is able to adequately represent all renewable energy sources.
- A stochastic model has been presented for the medium-term planning of GenCos participating in a pure pool market where N-DRs have a significant share under the behavioral principles of endogenous cartel and equilibrium. Pumped storage has been also considered through a new formulation for multi-period market problems and multi-LDC matching. The results obtained with the two-unit model for the renewable energies (WP and SPV), including market shares by units and technologies, are reliable given the representation of the load, of the GenCos, of the market price, and of the stochasticity in the process.
- The probabilistic model of LDC matching based on considering all generation units represented by its capacity and forced outage probability has been employed and the mLMH has been used. This makes unnecessary to develop scenarios to account for load and units' forced-outage uncertainties. A novel way of representing N-DRs with two units: a base unit with a normal failure rate and a crest unit with a large one has been presented and justified.
- The scenario tree that represents the model stochasticity of hydro inflows, WP and SPV power was obtained using a quasi-Monte Carlo procedure. Considering SPV entails having to match three LDCs at each scenario tree node. The procedure presented has been applied to the solution of a number of test cases with realistic data from the MIBEL using scenario trees of different sizes, which provide valuable information on the computational requirements for solving these medium-term generation planning problems. The results obtained show that the solution procedure is reliable and coherent even though the objective functions for the endogenous cartel and for the equilibrium problems are non-convex. The convergence to a local optimizer by the interior-point solver (Ipopt) employed in each solution within the iterative procedure used, shows that these local optimizers exist. The CPU times required for large cases are long but affordable for medium-term planning.
- The stochastic medium-term planning described can be also employed as a reliable way to measure the effect of a given penetration of N-DRs on the market share by other generation technologies.
- A stochastic model has been presented for the medium-term planning of GenCos participating in a mixed pool and BC market including pumped storage under the

behavioral principles of endogenous cartel and equilibrium. The mLMH was employed for LDC matching and the NIRA for obtaining an equilibrium solution.

- The objective function of the medium-term mixed-market problem is non-convex, but its solution using a current implementation of global-optimization direct methods proved not to be practical. A procedure based on the use of a conventional nonlinear programming solver has been presented, where a solution strategy was employed to circumvent the likely trouble caused by non-convexity in the sequence of optimizations of the mLMH. Computational results show that the results are satisfactory. The results also provide the split between market and BCs in expected generation and duration of each unit.

- The models presented are a useful tool for medium-term generation planning. Expected profits and brackets of their dispersion for all participating GenCos are obtained from the results, together with the optimal hydro and pumping use over the horizon considered. From the optimal expected energies for each unit in each period the medium-term fuel procurement decisions can be readily calculated.

- BCs considered in short-term planning for risk curtailment, are not considered in this thesis because in medium-term they are a forecast, for the SGC and for the system (hence also for the RoP). This entails that no risk consideration can be made as the BC forecasts are fixed parameters. What is optimized is the contribution of each unit to honoring the BCs, while maximizing the profits from participating in the market (including the participation in pumping).

- The modeling of the bilateral contracts brings about two novelties: a unit can match two different load-duration curves (because of the time-share hypothesis), and the rewards that a unit will receive for its generation depend on the amount of energy produced for bilateral contracts.

7.3 AREAS FOR FURTHER RESEARCH

Possible directions for further research are:

1. The comparison of the NIRA procedure using the hydro endogenous function for finding the equilibrium solution with the alternative techniques based on the use of market-price elasticity or employing alternative optimality equilibrium constraints.

2. The development of formulations of the CVaR constraints (54) to circumvent its indefiniteness in order to constrain this risk in the medium-term solutions.

3. The use of direct global-optimization software to solve the non-convex medium-term BC problem, when this software becomes available and proves to be efficient enough.

4. The extension of the medium-term stochasticity by including the fuel price impact on generation costs and on market-price levels in every node of the scenario tree employed.
5. The development of specialized interior point codes for medium-term planning problems with DcxQ objective functions.
6. The extension of the MTGP model to include several different pumping-schemes in the system and subsets of generation units devoted to pumping for each different scheme.
7. The extension of the MTGP model to market types other than that of MIBEL.
8. The investigation of the effect of generation curtailments on non-dispatchable renewables.

7.4 PUBLICATIONS AND PRESENTATIONS RELATED TO THIS THESIS

- L. Marí and N. Nabona: *Endogenous model for medium-term electricity generation planning in liberalized mixed markets*, presented at 24th European Conference on Operational Research (Lisbon, July 2010)
- L. Marí and N. Nabona: *Medium-Term Generation Planning in Liberalized Mixed Electricity Markets*, presented at International Conference on Operations Research (Munich, September 2010)
- L. Marí and N. Nabona: *Medium-Term Generation Planning Optimization in Liberalized Electricity Markets*, presented at Research Seminar (Humboldt-Universität zu Berlin, June 2011)
- L. Marí and N. Nabona: *Renewable Energies in Medium-Term Power Planning*, [51] (DOI:10.1109/TPWRS.2014.2328033)
- L. Marí and N. Nabona: *Modeling renewable generation sources for medium-term electricity generation planning*, presented at 20th Conference of the International Federation of Operational Research Societies (Barcelona, July 2014)
- L. Marí, N. Nabona and A. Pagès: *Medium-Term Power Planning in Electricity Markets with Pool and Bilateral Contracts*, submitted in July 2015 to European Journal of Operational Research.

A | DATA OF CASE 24P21

```
# data for stochastic medium-term endogenous planning
# with RE and pumping using the mLM heuristic. 21 scenarios. 24 units

# number of time periods
param nINT :=  6 ;

# last day number of each period // time period starts on 4/10/2010
param calendari :=
 1         7
 2        31
 3        91
 4       183
 5       275
 6       365
;

# slope of line that fits market-price function
param pen :=
 1   -0.1992
 2   -0.0697
 3   -0.0201
 4   -0.0075
 5   -0.0087
 6   -0.0145
;

# independent term of line that fits price-duration curve
param   b :=
  1      84.30855574             32      74.65439571
  2     112.50557085             33      76.15753331
  3      89.59929500             34      76.60063715
  4      84.85544840             35      78.13094791
  5      89.19888855             36      75.91129660
  6      89.23682103             37      73.30200237
  7      85.67420225             38      80.24843807
  8      81.29777165             39      74.53479449
```

```
  9      87.90666300            40      76.82914442
 10      87.72062237            41      85.42104309
 11      83.61308648            42      94.02872920
 12      81.56743336            43      90.85907262
 13      86.67000401            44      90.45606961
 14      83.81239504            45      91.62787296
 15      87.22878225            46      93.56135036
 16      82.63047218            47      87.92364691
 17      86.95835008            48      85.37569800
 18      88.60819897            49      95.61507409
 19      87.12198160            50      89.72751348
 20      85.34136613            51      89.92540508
 21      81.85706465            52      94.27893225
 22      87.28267880            53      87.82354185
 23      80.50241881            54      89.04118621
 24      76.34244872            55      95.36793047
 25      75.57047538            56      90.34348932
 26      76.82202938            57      93.83733868
 27      74.50609068            58      95.29257308
 28      73.90378793            59      88.92793395
 29      73.76318023            60      89.88527320
 30      75.10839459            61      92.93668625
 31      76.63554339
;

# endogenous coefficient of price level change with hydro generation
param d :=  -0.0048 ;

# number of nodes
param nNUS := 61 ;

# predecessor node (prd) and probability (prb) of each node
param:   prd     prb :=
  1        0      1                        32       15      0.050100200
  2        1      1                        33       16      0.052104208
  3        2      0.2004008                34       17      0.042084168
  4        2      0.234468936              35       18      0.042084168
  5        2      0.186372744              36       19      0.096192384
  6        2      0.378757512              37       20      0.096192384
  7        3      0.082164328              38       21      0.046092184
  8        3      0.044088176              39       22      0.092184368
  9        3      0.074148296              40       23      0.048096192
 10        4      0.040080160              41       24      0.032064128
```

```
 11        4      0.050100200             42       24      0.050100200
 12        4      0.030060120             43       25      0.044088176
 13        4      0.056112224             44       26      0.074148296
 14        4      0.058116232             45       27      0.040080160
 15        5      0.050100200             46       28      0.050100200
 16        5      0.052104208             47       29      0.030060120
 17        5      0.042084168             48       30      0.056112224
 18        5      0.042084168             49       31      0.058116232
 19        6      0.096192384             50       32      0.050100200
 20        6      0.096192384             51       33      0.052104208
 21        6      0.046092184             52       34      0.042084168
 22        6      0.092184368             53       35      0.042084168
 23        6      0.048096192             54       36      0.042084168
 24        7      0.082164328             55       36      0.054108216
 25        8      0.044088176             56       37      0.036072144
 26        9      0.074148296             57       37      0.060120240
 27       10      0.040080160             58       38      0.046092184
 28       11      0.050100200             59       39      0.092184368
 29       12      0.030060120             60       40      0.018036072
 30       13      0.056112224             61       40      0.030060120
 31       14      0.058116232
;

# generation units
set  UNI :=
  HYDR1  HYDR2  HYDR3  HYDR4  HYDR5  HYDR6  COAL1  COAL2  COAL3  COAL4
  FUEL1  FUEL2  FUEL3  GAS1   GAS2   GAS3   NUC1   NUC2   NUC3
  WNDPC  WNDPB  PVslC  PVslB  PVmsC  COMP   TURB;

# generation units with variable capacity and failure per period
set UNvIcq :=  WNDPC  PVslC ;

# generation units with variable capacity and failure per node
set UNvNcq :=  PVmsC ;

# generation units with variable capacity and fixed failure per node
set UNvNc :=  WNDPB  PVslB ;

# hydro units
set HdUNI :=  HYDR1  HYDR2  HYDR3 HYDR4  HYDR5  HYDR6 ;

# generation companies participating in the market
set GenCos :=  SGCUNI  RoPUNI1  RoPUNI2  RoPUNI3  RoPUNI4 ;
```

```
# units of SGCUNI
set UNIGC[SGCUNI] :=
  HYDR1  HYDR2  COAL1  FUEL1
  PVslC  PVslB  PVmsC  COMP   TURB;

# units of RoPUNI1
set UNIGC[RoPUNI1] :=
  HYDR3  COAL2  FUEL2  GAS2;

# units of RoPUNI2
set UNIGC[RoPUNI2] :=
  HYDR4  COAL3  GAS3   NUC1;

# units of RoPUNI3
set UNIGC[RoPUNI3] :=
  HYDR5  COAL4  NUC2   WNDPC  WNDPB;

# units of RoPUNI4
set UNIGC[RoPUNI4] :=
  HYDR6  GAS1   FUEL3  NUC3;

# initial volume/energy in hydro basin
param:   v0  :=
  HYDR1  1333691
  HYDR2   952636
  HYDR3  1905273
  HYDR4  2095800
  HYDR5  2000536
  HYDR6  2191064
;

# expected inflows
param expwhi:
          HYDR1      HYDR2      HYDR3      HYDR4      HYDR5      HYDR6 :=
    1     15578      11127      22254      24480      23367      25592
    2     65680      46914      93828     103211      98519     107902
    3    380641     271886     543773     598150     570962     625339
    4    570554     407539     815077     896585     855831     937339
    5    133256      95183     190366     209402     199884     218921
    6    477956     341397     682794     751074     716934     785213
;
```

```
# natural inflow in hydro basin over each node
param whi:
          HYDR1      HYDR2      HYDR3      HYDR4      HYDR5      HYDR6 :=
    1     15578      11127      22254      24480      23367      25592
    2     65680      46914      93828     103211      98519     107902
    3    343941     245672     491344     540479     515912     565046
    4    486388     347420     694840     764324     729582     799066
    5    448425     320303     640606     704667     672637     736697
    6    301243     215173     430346     473381     451864     494898
    7    523703     374074     748147     822962     785555     860369
    8    508464     363188     726376     799014     762695     835333
    9    688821     492015     984030    1082433    1033232    1131635
   10    639315     456653     913306    1004637     958972    1050302
   11    377331     269522     539044     592948     565996     619900
   12    483423     345302     690604     759664     725134     794194
   13    680790     486279     972557    1069813    1021185    1118441
   14    565537     403955     807909     888700     848305     929096
   15    624482     446058     892116     981328     936722    1025934
   16    510787     364848     729696     802665     766181     839150
   17    706790     504850    1009699    1110669    1060184    1161154
   18    533429     381020     762041     838245     800143     876347
   19    706715     504796    1009593    1110552    1060073    1161032
   20    495453     353895     707789     778568     743179     813958
   21    434882     310630     621260     683386     652323     714449
   22    511811     365579     731158     804274     767716     840832
   23    643415     459582     919164    1011080     965122    1057038
   24    116933      83523     167046     183751     175399     192103
   25    146682     104773     209546     230501     220024     240978
   26    155963     111402     222804     245084     233944     256224
   27    135366      96690     193380     212718     203049     222387
   28    123286      88061     176122     193734     184928     202540
   29    136692      97637     195274     214802     205038     224565
   30    147318     105227     210454     231499     220976     242022
   31    139396      99569     199137     219051     209094     229008
   32    132603      94716     189432     208375     198904     217847
   33    112338      80241     160483     176531     168507     184555
   34    128780      91985     183971     202368     193169     211566
   35    115476      82483     164966     181462     173214     189711
   36    126602      90430     180859     198945     189902     207988
   37    114742      81959     163917     180309     172113     188505
   38    105905      75646     151292     166421     158857     173986
```

```
  39     165842     118459     236917     260609     248763     272455
  40     153386     109561     219123     241035     230079     251991
  41     435456     311040     622079     684287     653183     715391
  42     515368     368120     736239     809863     773051     846675
  43     565997     404284     808567     889424     848996     929852
  44     511457     365326     730653     803718     767186     840251
  45     449578     321127     642254     706479     674366     738592
  46     523428     373877     747754     822529     785141     859917
  47     438466     313190     626380     689018     657699     720337
  48     404519     288942     577884     635672     606778     664566
  49     532648     380463     760926     837018     798972     875065
  50     526662     376187     752374     827612     789993     865230
  51     429293     306638     613276     674603     643940     705267
  52     414364     295974     591948     651143     621545     680740
  53     360742     257673     515346     566880     541113     592648
  54     401595     286853     573706     631077     602392     659762
  55     438472     313194     626388     689027     657707     720346
  56     571316     408083     816166     897782     856974     938591
  57     366289     261635     523269     575596     549433     601760
  58     520675     371910     743821     818203     781012     855394
  59     495717     354084     708167     778984     743576     814392
  60     688312     491651     983302    1081632    1032467    1130797
  61     581479     415342     830684     913753     872219     955287
;

# proportion of run-of-the-river generation from upstream natural inflows
param gamma := 0.16 ;

# type of LDC
set MCr := dark  low-light  bright ;

# hours of each subperiod
param mhp:  dark low-light  bright :=
 1          98     28        42
 2         336     96       144
 3         720    300       420
 4         920    552       736
 5         920    552       736
 6        1170    360       630
;

# units that match dark
set UNY[dark] :=
```

```
  HYDR1  HYDR2  HYDR3  HYDR4  HYDR5  HYDR6  COAL1  COAL2  COAL3  COAL4
  FUEL1  FUEL2  FUEL3  GAS1   GAS2   GAS3   NUC1   NUC2   NUC3
  WNDPC  WNDPB  COMP   TURB ;



# units in UNcv and in dark
set UNYcv[dark] :=
  WNDPC  WNDPB  ;

# units that match low-light
set UNY[low-light] :=
  HYDR1  HYDR2  HYDR3  HYDR4  HYDR5  HYDR6  COAL1  COAL2  COAL3  COAL4
  FUEL1  FUEL2  FUEL3  GAS1   GAS2   GAS3   NUC1   NUC2   NUC3
  WNDPC  WNDPB  COMP   TURB   PVmsC ;

# units in UNcv and in low-light
set UNYcv[low-light] :=
  WNDPC  WNDPB  PVmsC ;

# units that match bright
set UNY[bright] :=
  HYDR1  HYDR2  HYDR3  HYDR4  HYDR5  HYDR6  COAL1  COAL2  COAL3  COAL4
  FUEL1  FUEL2  FUEL3  GAS1   GAS2   GAS3   NUC1   NUC2   NUC3
  WNDPC  WNDPB  COMP   TURB   PVslC  PVslB ;

# units in UNcv and in bright
set UNYcv[bright] :=
  WNDPC  WNDPB  PVslC  PVslB ;

# maximum capacity, linear generation cost
# and % probability of failure of generation units
param:       mxc      lgc        q   :=
 HYDR1       700        0        0
 HYDR2       500        0        0
 HYDR3      1000        0        0
 HYDR4      1100        0        0
 HYDR5      1050        0        0
 HYDR6      1150        0        0
 COAL1      6000       28       20
 COAL2      3900       28       20
 COAL3      3200       28       19
 COAL4      5000       28       19
```

```
 FUEL1      1200       48       15
 FUEL2      1500       48       15
 FUEL3      1000       48       16
 GAS1        750       50        2
 GAS2        600       50        2
 GAS3        600       50        3
 NUC1       4000        5        5
 NUC2       2550        5        5
 NUC3       3500        5        5
 WNDPC         0        0        0
 WNDPB         0        0       15
 PVmsC         0        0        0
 PVslC         0        0        0
 PVslB         0        0       15
 COMP       1350        0        0
 TURB       1350        0        0
 ;

# max volume of the reservoir
param vmax := 340200 ;

# initial volume of the reservoir
param vini := 170100 ;

# efficiency parameter of pumping
param effp := 0.75 ;

# maximum capacity of units that change with periods
param mxcvI:    WNDPC    PVslC :=
        1        4099    510
        2        4851    585
        3        5551   1164
        4        5733   1080
        5        4460   1284
        6        6216   1490
;

# failure probability of units that change with periods
param qcvI:     WNDPC    PVslC :=
        1          75       64
        2          67       61
        3          68       56
        4          76       56
```

```
        5          74       54
        6          69       60
;

# maximum capacity of units that change with nodes
param mxcvN:  WNDPB      PVmsC      PVslB :=
   1          6917       1368        941
   2          3388        567        373
   3          4058       1284       1297
   4          4730        658        667
   5          4114        122        128
   6          4109        600        609
   7          3575       1316        861
   8          3980       2541       1660
   9          3368       1627       1064
  10          3386       1287        842
  11          3766       1983       1296
  12          3955       1019        667
  13          3483       2249       1469
  14          3747       1550       1013
  15          3431       2427       1586
  16          3857       1624       1062
  17          3456       1264        827
  18          3304        857        562
  19          3441       1528        999
  20          3606       2007       1312
  21          3928        769        504
  22          3426       1280        838
  23          4053       2544       1662
  24          3012       2107       1377
  25          3083       2203       1440
  26          2968       1562       1021
  27          3182       1872       1223
  28          3237       2665       1741
  29          3250       1475        965
  30          3126       1874       1225
  31          2985       2154       1408
  32          3168       2051       1340
  33          3029       2675       1747
  34          2988       2294       1499
  35          2846       2221       1451
  36          3052       2255       1474
  37          3293       2080       1360
```

```
  38           2651       1721       1125
  39           3179       2388       1560
  40           2967       1976       1292
  41           5045       1717       2820
  42           4231       1011       1663
  43           4531        951       1565
  44           4569       1270       2087
  45           4458       1206       1982
  46           4275       1720       2825
  47           4809        804       1323
  48           5049       1089       1790
  49           4081       1590       2612
  50           4638       1236       2032
  51           4619       1511       2483
  52           4208       1280       2103
  53           4818       1459       2397
  54           4703       2007       3295
  55           4105        761       1254
  56           4580       1784       2929
  57           4249       1097       1804
  58           4112       1883       3091
  59           4714        894       1472
  60           4623        576        950
  61           4334       1788       2936
;

# failure probability of units that change with nodes
param qcvN:  PVmsC :=
      1        67              32        56
      2        66              33        56
      3        70              34        56
      4        70              35        56
      5        70              36        56
      6        70              37        56
      7        56              38        56
      8        56              39        56
      9        56              40        56
     10        56              41        77
     11        56              42        77
     12        56              43        77
     13        56              44        77
     14        56              45        77
     15        56              46        77
```

```
      16        56               47        77
      17        56               48        77
      18        56               49        77
      19        56               50        77
      20        56               51        77
      21        56               52        77
      22        56               53        77
      23        56               54        77
      24        56               55        77
      25        56               56        77
      26        56               57        77
      27        56               58        77
      28        56               59        77
      29        56               60        77
      30        56               61        77
      31        56
;

# maintenance of thermal units
set icm :=
('dark', GAS1, 2)
('low-light', GAS1, 2)
('bright', GAS1, 2)
('dark', GAS2, 2)
('low-light', GAS2, 2)
('bright', GAS2, 2)
;

# price of external energy
param pex :=   94.2 ;

# number of >= (inequality) constraints
param nGEcons :=   0 ;

# terms and coefficients of >= inequality constraints
param: GEterms: coefGE := ;

# rhs of >= inequality constraints
param rhsGE := ;

# Number of equality constraints
param nEQcons :=  2 ;
```

```
# terms and coefficients of equality constraints
param: EQterms: coefEQ :=
     1 GAS2    1       1.0
     1 GAS2    2       1.0
     1 GAS2    3       1.0
     1 GAS2    4       1.0
     1 GAS2    5       1.0
     1 GAS2    6       1.0
     2 GAS3    1       1.0
     2 GAS3    2       1.0
     2 GAS3    3       1.0
     2 GAS3    4       1.0
     2 GAS3    5       1.0
     2 GAS3    6       1.0
;

# rhs of equality constraints
 param rhsEQ :=
1       3000000
2       3000000
;

# total energy in LDC of each type and period
param ein:     dark   low-light      bright :=
  1         1915648      589400      911814
  2         6189435     1894784     2932037
  3        13886243     6089926     8695155
  4        17627593    12615807    16202015
  5        14567301     9744803    14164516
  6        20268107     6978312    13508357
;

# peak load of each subperiod
param pmx:    dark    low-light      bright :=
  1          29207        28026       29158
  2          28384        26518       27140
  3          32333        31534       30533
  4          33782        33599       31433
  5          24842        25178       27767
  6          26883        26240       28061
;

# base load of each subperiod
```

```
param pmn:    dark    low-light      bright :=
  1          12216        13741       14831
  2          10450        10894       11121
  3          11389        11426       11750
  4          11286        11248       12059
  5           7999         8246        8530
  6          11169        11158       11493
;

# hourly loads
param crg :=
[1, dark, *, *]: 1 2 3 4 5 6 7 8 9 10 11 12 13 14 :=
1   14357   13211   12329   12216   12478   12824   14889
    18456   22322   23710   26175   25524   23727   20868
2   20214   19557   18217   17357   17364   18132   19994
    22993   23564   24395   26068   25801   23569   21502
3   21706   20430   19578   18695   18868   19741   21822
    24836   27484   27794   29207   27943   25384   23494
                              (...)
[6, bright, *, *]: 1 2 3 4 5 6 7 :=
276   25673   26167   27033   26659   24966   23629   22733
                              (...)
363   15894   15831   15907   16224   15484   15636   15363
364   14028   14143   14476   14777   14688   14169   14089
365   25040   23925   24218   24101   23908   23809   23693
;
```

B RESULTS FROM CASE 24P21

```
**** Cartel sol. with the all one and the all but one and but two constraints
     rel.cartel profit nd: -27170350780.65
     abs.cartel profit nd= 11271826010.46

**** Cartel sol. adding LMCs of combinations of units at/near max capac
     rel.cartel profit_nd: -27130683034.91
     abs.cartel profit_nd= 11232157992.14

     rel.equilibr. profit: -27588427089.40

**** Recalculation of prp and sets UNU(9) and UN7
     Start of NIRA iterations.

              abs.equilb profit  1: 11548188446.16

**** Iterative sol. adding an LMC for each unit not at its max.

              abs.equilb profit  2: 11523755546.20
              abs.equilb profit  3: 11504219865.38
              abs.equilb profit  4: 11497513265.03
              abs.equilb profit  5: 11493028242.30
              abs.equilb profit  6: 11488019900.14
              abs.equilb profit  7: 11484554283.54
              abs.equilb profit  8: 11482017757.73
              abs.equilb profit  9: 11479974864.18
              abs.equilb profit 10: 11477977665.57
              abs.equilb profit 11: 11476096931.73
              abs.equilb profit 12: 11474468387.97
              abs.equilb profit 13: 11472180108.89
              abs.equilb profit 14: 11470192369.72
              abs.equilb profit 15: 11468391256.40
              abs.equilb profit 16: 11467189238.31
              abs.equilb profit 17: 11466372794.43
              abs.equilb profit 18: 11465719382.93
              abs.equilb profit 19: 11465127193.20
              abs.equilb profit 20: 11464631582.53
```

```
            abs.equilb profit 21: 11464204124.41

 prf.dec: hNoGClow<--hNoGC*cc= 0.008   abs.equilb profit 22: 11463796902.69
 prf.dec: hNoGClow<--hNoGC*cc= 0.003   abs.equilb profit 23: 11463717762.72
 prf.dec: hNoGClow<--hNoGC*cc= 0.001   abs.equilb profit 24: 11463678365.07
 prf.dec: hNoGClow<--hNoGC*cc= 0.000   abs.equilb profit 25: 11463663975.29
 prf.dec: hNoGClow<--hNoGC*cc= 0.000   abs.equilb profit 26: 11463659182.90

CPU secs:  7504.7

**** Expected generation for each unit at each scenario, period and subperiod
**** D:dark, L: low-light, B: bright

scenario 1       GWh          1        2        3        4        5        6
  D    HYDR1:  1481.879     39171    68651   293370   461466   306641   312579
  D    HYDR2:  1058.928     27958    48976   208053   330545   219828   223568
  D    HYDR3:  1484.797     36478   103126   297277   509291   269866   268759
  D    HYDR4:  1559.871     42450    91196   336737   413163   322817   353508
  D    HYDR5:  1653.657      2181    96373   227383   620297   601958   105464
  D    HYDR6:  1562.790     43755   107365   376695   502214   219765   312995
  D    COAL1:  1541.633     35327   269156   441210   466585   208303   121052
  D    COAL2:  1114.351     26744   176270   300668   462492    73234    74943
  D    COAL3:   857.700     22425   147587   257031   315506    43817    71335
  D    COAL4:   886.422     37523   224610   361708   185734      216    76631
  D    FUEL1:     8.766        86      417     2040     4811      314     1098
  D    FUEL2:    21.493       249      899     3908    16101      118      218
  D    FUEL3:     7.085       117      966     4073     1647      152      129
  D     GAS1:     3.824        43        0     1060     2566       38      116
  D     GAS2:  1191.688     39033        0   306581    64463   383342   398269
  D     GAS3:  1499.419     37873   123895   292872   418902   266144   359735
  D     NUC1: 14702.456    336905  1204095  2533848  3496000  3282571  3849037
  D     NUC2:  9268.095    216044   764553  1594978  2228700  2027822  2435998
  D     NUC3: 12890.107    294095  1052160  2210957  3059000  2917235  3356660
  D    WNDPC:  6504.476    100426   537879  1278950  1265846  1066832  2254543
  D    WNDPB: 14195.581    576186   967613  2483496  2795650  2355384  5017252
  D     COMP:     1.722        48      320     1239        0        0      115
  D     TURB:   952.393       491   203066    70870     3097      849   674020
  L    HYDR1:   226.453      9826     1751   129202    20948    26896    37830
  L    HYDR2:   161.330      6997     1251    90790    14963    20010    27319
  L    HYDR3:   596.941     22619    11970   196465    29926   203794   132168
  L    HYDR4:   698.342     18309     2752   261414    32918   287130    95818
  L    HYDR5:   740.713       623    50540   209380    31422   353824    94923
  L    HYDR6:   783.387     15978    30048   211471   167480   170434   187976
```

L	COAL1:	1070.825	517	128639	8940	932506	60	164
L	COAL2:	875.116	56	93763	133915	606129	41130	122
L	COAL3:	692.680	7215	93647	911	497337	623	92947
L	COAL4:	854.572	15206	541	1395	777089	60179	163
L	FUEL1:	6.250	23	118	415	5472	30	193
L	FUEL2:	8.752	70	442	1203	6840	74	123
L	FUEL3:	3.191	28	94	517	2404	103	45
L	GAS1:	1.815	11	0	206	1560	12	26
L	GAS2:	855.495	9	0	131087	289629	223090	211680
L	GAS3:	775.227	9	40405	124216	257518	176818	176261
L	NUC1:	6755.753	106400	354074	1010736	2078344	1838199	1368000
L	NUC2:	4114.955	67830	183777	626411	1324944	1249724	662269
L	NUC3:	5751.843	86008	286739	997500	1726090	1742069	913438
L	WNDPC:	2808.581	28693	153680	532896	759508	640099	693706
L	WNDPB:	6110.266	164625	276461	1034790	1677390	1413230	1543770
L	PVmsC:	2985.552	38304	54432	385200	726432	1163064	618120
L	COMP:	1.929	0	87	0	1807	10	25
L	TURB:	1030.757	25	129513	447	645362	134195	121216
B	HYDR1:	90.115	16949	2627	16051	27931	6236	20321
B	HYDR2:	64.347	12085	1877	11465	19951	4455	14515
B	HYDR3:	487.469	26033	24774	103671	39901	149031	144058
B	HYDR4:	567.917	26045	14493	87680	43891	239735	156073
B	HYDR5:	303.300	935	12680	24076	41896	193231	30482
B	HYDR6:	608.412	22510	41385	66464	171886	88842	217326
B	COAL1:	2321.720	13474	200466	359582	1072972	393982	281243
B	COAL2:	1163.617	13527	123206	192379	697432	526	136547
B	COAL3:	1093.321	12109	100657	183043	580192	457	216864
B	COAL4:	2635.435	25833	164118	299652	924018	987444	234369
B	FUEL1:	6.339	111	628	996	3872	154	578
B	FUEL2:	6.480	103	294	470	4839	323	450
B	FUEL3:	3.868	334	331	531	1581	992	98
B	GAS1:	1.412	56	0	172	985	110	88
B	GAS2:	952.817	7413	0	132	416169	310530	218573
B	GAS3:	725.354	1893	63714	130	165762	290419	203436
B	NUC1:	9549.568	159600	500516	1557038	2796800	2356225	2179389
B	NUC2:	6325.880	101745	322568	999571	1782960	1782960	1336076
B	NUC3:	8520.061	139650	439459	1352016	2447200	2376765	1764971
B	WNDPC:	4099.741	43040	230520	746054	1012677	853466	1213985
B	WNDPB:	8932.759	246937	414691	1448706	2236520	1884307	2701598
B	PVslC:	1415.610	7711	32854	215107	349747	434711	375480
B	PVslB:	3452.481	33594	45655	463029	538642	861451	1510110
B	COMP:	1.623	0	171	183	1077	105	87
B	TURB:	3082.875	64	194229	566817	822191	947992	551582

```
**** values = [profits(scenario) probability(scenario)]

values = [
  11372502291.4    0.0320641280
  11660171297.8    0.0501002000
  11337957405.4    0.0440881760
  11572725304.5    0.0741482960
  11383994581.1    0.0400801600
  11372265917.9    0.0501002000
  11006458458.9    0.0300601200
  11139954685.9    0.0561122240
  11467093283.4    0.0581162320
  11417618137.4    0.0501002000
  11365708908.3    0.0521042080
  11635568439.2    0.0420841680
  11597530767.9    0.0420841680
  11507377871.1    0.0420841680
  11705861128.6    0.0541082160
  11403528134.9    0.0360721440
  11593950564.5    0.0601202400
  11714534038.9    0.0460921840
  11459423851.5    0.0921843680
  11202503910.8    0.0180360720
  11379897814.4    0.0300601200
];

**** Energy stored in upper reservoir (vb), generation of compensating unit (C),
**** hydro generation from pumped water (T), for each scenario (1-21),
**** for each period (1-6) and subperiod (D:dark, L:low-light, B:bright)

                1          2          3          4          5          6
 1  vb:   339148.9        0.0   340200.0        0.0   340200.0   170100.0
  D   C       47.5      320.2     1239.1        0.0        0.0      115.3
      T      491.3   203066.3    70869.5     3097.3      849.2   674019.5
  L   C        0.0       86.8        0.0     1807.0       10.4       24.5
      T       24.5   129513.2      446.7   645361.5   134195.3   121215.8
  B   C        0.0      170.6      182.6     1077.2      105.4       87.0
      T       64.4   194229.4   566817.4   822190.9   947991.5   551581.7
 2  vb:   339148.9        0.0   340200.0        0.0   340200.0   170100.0
  D   C       47.5      320.2     1239.1        0.0        0.0      130.8
      T      491.3   203066.3    70869.5     3097.3      849.2    89578.5
  L   C        0.0       86.8        0.0     1807.0       10.4       43.5
```

	T	24.5	129513.2	446.7	645361.5	134195.3	406848.3
B	C	0.0	170.6	182.6	1077.2	105.4	152.7
	T	64.4	194229.4	566817.4	822190.9	947991.5	850347.3
3	vb:	339148.9	0.0	340200.0	0.0	340200.0	170100.0
D	C	47.5	320.2	1239.1	0.0	0.0	124.0
	T	491.3	203066.3	70869.5	2805.2	830.1	171194.1
L	C	0.0	86.8	0.0	903.6	9.4	43.3
	T	24.5	129513.2	446.7	699579.6	745190.6	325235.0
B	C	0.0	170.6	182.6	706.7	100.1	151.4
	T	64.4	194229.4	566817.4	768810.8	337018.0	850348.6
4	vb:	339148.9	0.0	340200.0	0.0	340200.0	170100.0
D	C	47.5	320.2	1239.1	0.0	0.0	123.2
	T	491.3	203066.3	70869.5	3270.2	49079.7	255577.3
L	C	0.0	86.8	0.0	1651.4	0.0	34.9
	T	24.5	129513.2	446.7	644778.5	40478.5	314970.7
B	C	0.0	170.6	182.6	1050.2	127.0	121.3
	T	64.4	194229.4	566817.4	822679.2	993473.0	776246.5
5	vb:	339148.9	0.0	340200.0	0.0	340200.0	170100.0
D	C	47.5	320.2	0.0	0.0	0.0	125.5
	T	491.3	203066.3	8640.3	3254.5	804.8	228105.9
L	C	0.0	86.8	267.0	1928.8	12.0	37.0
	T	24.5	129513.2	251274.3	628497.1	737246.3	284897.7
B	C	0.0	170.6	209.7	1145.1	107.6	128.7
	T	64.4	194229.4	378623.9	838816.8	344983.4	833785.9
6	vb:	339148.9	0.0	340200.0	0.0	340200.0	170100.0
D	C	47.5	320.2	0.0	0.0	33.3	129.7
	T	491.3	203066.3	8640.3	2954.4	52006.5	372694.2
L	C	0.0	86.8	267.0	1252.3	6.1	26.9
	T	24.5	129513.2	251274.3	595341.1	642713.6	123708.5
B	C	0.0	170.6	209.7	858.1	83.5	97.1
	T	64.4	194229.4	378623.9	872685.8	388312.9	850402.9
7	vb:	339148.9	0.0	340200.0	0.0	340200.0	170100.0
D	C	47.5	320.2	0.0	0.0	33.1	118.9
	T	491.3	203066.3	8640.3	2821.9	140683.6	209738.1
L	C	0.0	86.8	267.0	1880.0	16.2	45.9
	T	24.5	129513.2	251274.3	647198.5	100167.4	287639.3
B	C	0.0	170.6	209.7	1057.6	119.6	161.7
	T	64.4	194229.4	378623.9	820606.3	842162.3	849396.9
8	vb:	339148.9	0.0	340200.0	0.0	340200.0	170100.0
D	C	47.5	320.2	0.0	0.0	0.0	115.3
	T	491.3	203066.3	8640.3	3172.1	818.9	489110.4
L	C	0.0	86.8	267.0	1186.7	12.2	37.2
	T	24.5	129513.2	251274.3	595074.2	725843.4	120780.9

B	C	0.0	170.6	209.7	864.2	109.4	128.4
	T	64.4	194229.4	378623.9	872760.5	356371.3	736902.6
9	vb:	339148.9	0.0	340200.0	0.0	340200.0	170100.0
D	C	47.5	320.2	0.0	0.0	0.0	135.0
	T	491.3	203066.3	8640.3	2968.0	856.7	178217.0
L	C	0.0	86.8	267.0	1546.5	10.1	30.3
	T	24.5	129513.2	251274.3	610229.1	745189.9	318188.4
B	C	0.0	170.6	209.7	966.3	104.8	108.4
	T	64.4	194229.4	378623.9	857611.7	336989.8	850391.6
10	vb:	339148.9	0.0	340200.0	0.0	340200.0	170100.0
D	C	47.5	320.2	0.0	0.0	33.9	121.9
	T	491.3	203066.3	9844.4	3215.6	337634.0	364905.8
L	C	0.0	86.8	402.7	1103.5	10.4	35.4
	T	24.5	129513.2	404597.3	590748.1	745189.6	202697.1
B	C	0.0	170.6	300.2	837.9	0.0	122.7
	T	64.4	194229.4	223999.9	877090.0	243.1	779191.3
11	vb:	339148.9	0.0	340200.0	0.0	340200.0	170100.0
D	C	47.5	320.2	0.0	0.0	35.3	122.3
	T	491.3	203066.3	9844.4	2889.5	22966.7	377885.8
L	C	0.0	86.8	402.7	1451.9	6.4	29.5
	T	24.5	129513.2	404597.3	623061.1	687566.0	118522.6
B	C	0.0	170.6	300.2	920.7	88.7	103.6
	T	64.4	194229.4	223999.9	844918.3	372497.2	850396.4
12	vb:	339148.9	0.0	340200.0	0.0	340200.0	170100.0
D	C	47.5	320.2	0.0	0.0	0.0	131.4
	T	491.3	203066.3	9844.4	3194.5	855.9	207108.3
L	C	0.0	86.8	402.7	1912.5	9.0	36.6
	T	24.5	129513.2	404597.3	627232.8	678815.5	289307.2
B	C	0.0	170.6	300.2	1129.3	100.4	128.1
	T	64.4	194229.4	223999.9	840154.8	403367.4	850371.9
13	vb:	339148.9	0.0	340200.0	0.0	340200.0	170100.0
D	C	47.5	320.2	0.0	0.0	0.0	118.8
	T	491.3	203066.3	9844.4	3326.9	898.5	293649.8
L	C	0.0	86.8	402.7	2405.4	10.0	29.9
	T	24.5	129513.2	404597.3	643067.6	134935.9	302702.0
B	C	0.0	170.6	300.2	1319.3	107.6	104.0
	T	64.4	194229.4	223999.9	823894.9	947201.0	750454.2
14	vb:	339148.9	0.0	264093.9	0.0	340200.0	170100.0
D	C	47.5	320.2	0.0	0.0	0.0	120.7
	T	491.3	203066.3	2219.5	3207.1	838.4	303994.8
L	C	0.0	86.8	316.5	1696.1	9.1	20.8
	T	24.5	129513.2	281075.0	743503.9	341716.1	244006.3
B	C	0.0	170.6	242.6	1056.2	99.5	78.0

	T	64.4	194229.4	398698.0	680505.9	740484.6	798819.1
15	vb:	339148.9	0.0	264093.9	0.0	340200.0	170100.0
D	C	47.5	320.2	0.0	0.0	0.0	134.3
	T	491.3	203066.3	2219.5	3207.1	838.4	45077.4
L	C	0.0	86.8	316.5	1696.1	9.1	52.4
	T	24.5	129513.2	281075.0	743503.9	341716.1	451364.0
B	C	0.0	170.6	242.6	1056.2	99.5	187.3
	T	64.4	194229.4	398698.0	680505.9	740484.6	850312.7
16	vb:	339148.9	0.0	264093.9	0.0	340200.0	170100.0
D	C	47.5	320.2	0.0	0.0	0.0	123.0
	T	491.3	203066.3	2219.5	3073.2	778.9	565181.7
L	C	0.0	86.8	316.5	1290.2	9.8	24.6
	T	24.5	129513.2	281075.0	621949.7	320087.5	273440.3
B	C	0.0	170.6	242.6	889.3	97.6	89.1
	T	64.4	194229.4	398698.0	802439.5	762173.2	508190.8
17	vb:	339148.9	0.0	264093.9	0.0	340200.0	170100.0
D	C	47.5	320.2	0.0	0.0	0.0	130.3
	T	491.3	203066.3	2219.5	3073.2	778.9	42077.5
L	C	0.0	86.8	316.5	1290.2	9.8	41.0
	T	24.5	129513.2	281075.0	621949.7	320087.5	454345.3
B	C	0.0	170.6	242.6	889.3	97.6	143.5
	T	64.4	194229.4	398698.0	802439.5	762173.2	850356.6
18	vb:	339148.9	0.0	264093.9	0.0	340200.0	170100.0
D	C	47.5	320.2	0.0	0.0	0.0	134.1
	T	491.3	203066.3	2219.5	2840.3	963.7	10423.3
L	C	0.0	86.8	316.5	2123.8	16.0	24.6
	T	24.5	129513.2	281075.0	629264.1	88592.0	485975.4
B	C	0.0	170.6	242.6	1140.2	134.5	91.7
	T	64.4	194229.4	398698.0	794893.2	993465.5	850408.3
19	vb:	339148.9	0.0	264093.9	0.0	340200.0	170100.0
D	C	47.5	320.2	0.0	0.0	0.0	120.5
	T	491.3	203066.3	2219.5	3219.9	805.5	174444.4
L	C	0.0	86.8	316.5	1913.9	7.8	43.9
	T	24.5	129513.2	281075.0	743286.1	745192.2	321987.0
B	C	0.0	170.6	242.6	1133.8	92.0	153.4
	T	64.4	194229.4	398698.0	680584.4	337045.2	850346.6
20	vb:	339148.9	0.0	264093.9	0.0	340200.0	170100.0
D	C	47.5	320.2	0.0	0.0	0.0	122.2
	T	491.3	203066.3	2219.5	2757.5	315897.4	649997.1
L	C	0.0	86.8	316.5	886.0	11.7	0.0
	T	24.5	129513.2	281075.0	744314.0	745188.3	126.5
B	C	0.0	170.6	242.6	694.2	0.0	196.9
	T	64.4	194229.4	398698.0	680647.8	21995.0	696653.9

```
21  vb:   339148.9        0.0   264093.9        0.0   340200.0   170100.0
  D   C       47.5      320.2        0.0        0.0        0.0      128.3
      T      491.3   203066.3     2219.5     2757.5   315897.4   190551.5
  L   C        0.0       86.8      316.5      886.0       11.7       25.4
      T       24.5   129513.2   281075.0   744314.0   745188.3   305849.8
  B   C        0.0      170.6      242.6      694.2        0.0       92.5
      T       64.4   194229.4   398698.0   680647.8    21995.0   850407.5

**** Total generation for each scenario (1-21) and for each type of generator
**** thm: thermal units, hyd: hydro units, pmp: pumped energy
**** phy: hydro generation from pumped water, wp: wind power generation
**** spv: solar photovoltaic generation

            1          2          3          4          5          6          7
thm   99065382  100956310   97931186   99515845   98919467   97570127   99205024
hyd   14130647   14607836   14996814   15803541   15866028   14670462   14876696
pmp    6754701    6754644    6755380    6754769    6755093    6755664    6755123
phy    5066026    5065983    5066535    5066076    5066320    5066748    5066342
wp    42651405   41156901   42601057   41306392   42360541   42840961   44200499
spv    7853643    6979909    8174290    7075317    6554267    8620745    5418688

            8          9         10         11         12         13         14
thm   96928538   98226379   98206415   98915251  100316299  101420623   98859684
hyd   15915999   15945583   15994558   14613201   15792645   14357795   14824057
pmp    6755683    6755427    6755661    6755379    6754986    6754616    6755277
phy    5066762    5066570    5066746    5066534    5066239    5065962    5066458
wp    43522566   41976165   41995218   42498976   40914834   41483015   41909497
spv    7334962    7553426    7505349    7673856    6676009    6437113    8107325


           15         16         17         18         19         20         21
thm  101278728   98627435  101309121  102214144  100483459   97588683   97257595
hyd   15044266   14505182   13280875   13788314   14456569   16318115   15680173
pmp    6755189    6755596    6755551    6754943    6755057    6755946    6755987
phy    5066392    5066697    5066663    5066207    5066293    5066959    5066990
wp    40811569   42445650   41837934   40985826   42139895   42751691   42221087
spv    6565810    8123355    7273597    6711701    6620029    7044156    8543979

**** Total generation for each period (1-6), for each superiod and
**** for each type of generator (thm: thermal, hyd: hydro, pmp: pumped energy,
**** phy: hydro generation from pumped water, wp: wind power generation and
**** spv: solar photovoltaic generation only in low-light and bright subperiods)
```

dark	1	2	3	4	5	6
thm	1046463	3964607	8175896	10725254	9142071	11113428
hyd	191993	515688	1811977	2812796	1913156	2167441
pmp	131761	250213	952848	1238915	1197632	1323647
phy	491	203066	18904	3085	44362	255727
wp	676612	1505492	3877084	4082955	3467658	6731300

low-light	1	2	3	4	5	6
thm	283380	1182238	3544168	8163068	5322722	3608738
hyd	74353	98313	490383	433879	699773	542749
pmp	37775	0	163872	88426	269828	198077
phy	25	129513	240871	655194	475363	287889
wp	193318	430140	1615452	2449773	2080595	2071169
spv	38304	54432	198476	910769	1166335	467711

bright	1	2	3	4	5	6
thm	475848	1915956	5111267	10557508	8042138	7049525
hyd	104557	97835	472119	555617	1486186	512018
pmp	56636	0	171643	196575	430192	47198
phy	64	194229	395124	796032	563314	803176
wp	289976	645211	2261632	3266364	2774127	3624546
spv	41305	78509	454593	1024656	1298582	1518885

Finally, in Figure 36 there is the evolution of the objective function over each iteration. First to solutions correspond to cartel ones, then the objective function increase its value and it corresponds to the equilibrium solution. As it can be seen, the objective function converges to an equilibrium point with 28 iterations.

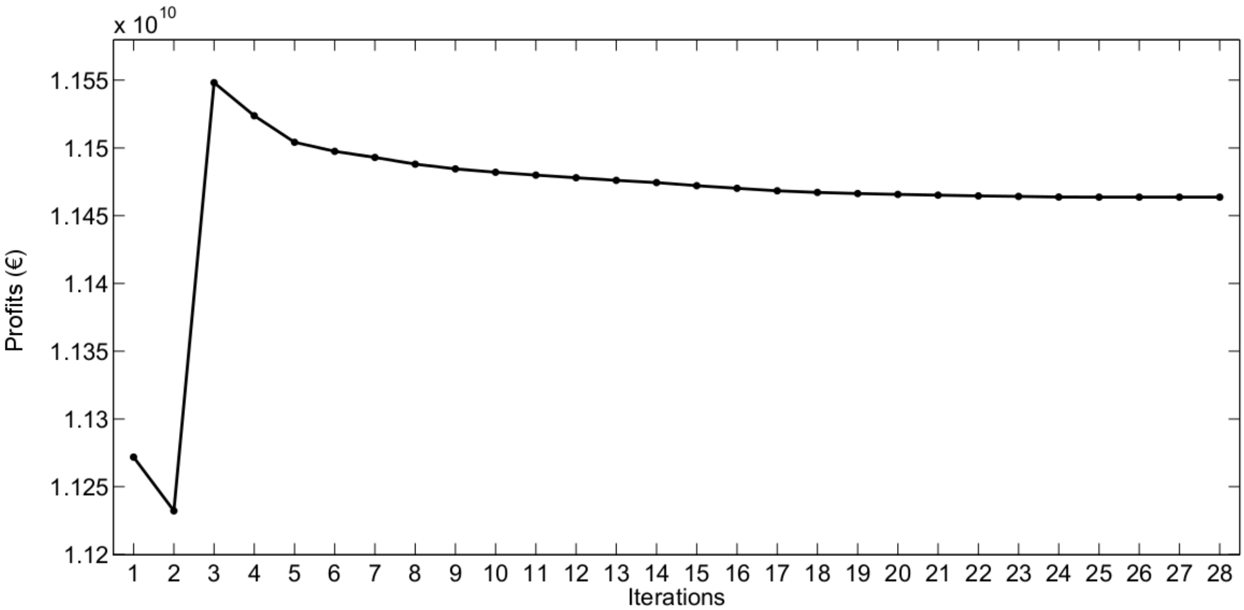


Figure 36: Profit evolution of case 24P21 with N-DRs and pumping

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