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Improving Urban Deliveries via Collaboration

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Abstract

Distribution of goods is essential for the economic development of cities but at the same time it entails several problems to the urban systems and different stakeholders. Carriers spend a significant portion of their cost in the last-mile distribution due to traffic congestion and lack of available loading/unloading facilities. In turn, citizens undergo environmental effects like pollution, noise or space competition.

Collaborative transportation is currently one of the major trends in transportation research due to its potential benefits with little need for big infrastructure or costly investments. This thesis deals with three different situations that appear repeatedly in the urban context, which can be improved by means of collaboration among private companies and/or public authorities.

The first part of the thesis studies a mildly disruptive collaboration approach, which is based on sharing loading/unloading urban facilities via an in-advance booking system, managed by local public authorities. In this context, the Parking Slot Assignment Problem is the mathematical problem that finds assignments of carriers to parking places that satisfy their time windows requests. We propose a feasibility model first, and then four other models with various objective functions that penalize the deviation from the requested time windows in different ways. We propose and compare two different formulations: one with time as a continuous variable and a second one with time discretization. Finally, we evaluate and compare the different proposals with extensive computational experiments in a set of test instances.

An intermediate level of collaboration among carriers is studied in the second part of this thesis. Urban areas have high customers density and usually there are shared customers (customers with demand from different carriers in the same time horizon). We propose an innovative problem: the Shared Customer Collaboration Vehicle Routing Problem, where several carriers are willing to collaborate transferring part of the demand of their shared customers, if the overall transportation cost is reduced. A vehicle-based and a load-based formulation are studied, and experimented over a specifically-generated instance set.

The highest level of collaboration in urban deliveries resorts to Urban Consolidation Centers, which are normally led by public authorities but need the collaboration of carriers for a successful implementation. Urban Consolidation Centers are urban terminals where the load from different carriers is consolidated and then a unique neutral carrier performs last-mile deliveries. In the third part of the thesis we propose continuous models that analyze the improvement in efficiency of urban distribution with the use of Urban Consolidation Centers under different assumptions. Continuous approximation models are known to produce robust solutions, which are useful to provide guidelines for general cases through sensitive analysis.

In the three parts of the thesis, innovative models and approaches are proposed and validated on experiments that use data from real scenarios.

Resum

La distribució urbana de mercaderies és una activitat essencial pel desenvolupament de les ciutats. Al mateix temps, però, comporta diversos problemes als nuclis urbans i als diferents actors involucrats. Els costos de la distribució urbana resulten una part molt significativa dels costos dels transportistes, especialment a causa de la congestió i la manca de zones de càrrega i descàrrega. Per altre banda, els ciutadans pateixen els efectes de la pol·lució, el soroll o la competició per l'espai públic.

El transport col·laboratiu és actualment una de les principals tendències de recerca en transport, doncs ofereix beneficis atractius amb poca inversió. Aquesta tesi tracta tres situacions que trobem repetidament en el context urbà, situacions on diverses formes de col·laboració poden representar una millora, i que consideren tant col·laboració entre empreses privades com la col·laboració conjunta d'empreses privades amb les administracions.

La primera part de la tesi estudia un nivell de col·laboració baix, basat en compartir les zones de càrrega i descàrrega gràcies a un sistema de reserves gestionat per l'administració. En aquest context, sorgeix el *Parking Slot Assignment Problem* (Problema d'assignació de places de parking), com el problema matemàtic que assigna transportistes a places de parking satisfent els seus requeriments a través de finestres temporals. En primer lloc proposem un model de factibilitat, i després proposem quatre models amb funcions objectius desiguals que penalitzen la desviació de les finestres temporals de formes diferents. Es proposen i comparen dues formulacions: una amb el temps com una variable contínua, i la segona amb discretització temporal. Finalment, s'avaluen i es comparen les diferents propostes a través d'uns extensos experiments computacionals en un conjunt de test basat en dades reals.

Un nivell intermedi de col·laboració entre transportistes s'analitza en la segona part d'aquesta tesi. Les àrees urbanes presenten una alta densitat de clients i és comú trobar clients compartits (és a dir, clients que reben mercaderies a través de diferents transportistes en el mateix interval temporal). Proposem un problema innovador: el *Shared Customer Collaboration Vehicle Routing Problem* (Problema de rutes de vehicles amb col·laboració de clients compartits), on diferents transportistes estan disposats a col·laborar transferint part de la demanda dels seus clients compartits, si el cost total de transport es redueix. S'estudien dues formulacions: una basada en els vehicles i una altra basada en la càrrega, i s'experimenta en un conjunt d'instàncies generades.

El màxim nivell de col·laboració en distribució urbana de mercaderies és l'ús de centres de consolidació urbana. Aquests centres estan normalment liderats per l'administració pública però necessiten l'activa col·laboració dels transportistes per aconseguir una implantació amb èxit. Els centres de consolidació urbana són terminals urbanes on es consolida la càrrega dels diferents transportistes i després, un únic transportista neutral realitza la distribució d'última milla. En aquesta tercera part de la tesi proposem models continus que analitzen la millora de l'eficiència en la distribució urbana a través de l'ús de centres de consolidació urbana amb diferents hipòtesis. Els models continus produeixen solucions robustes, que són útils per proporcionar guies en casos genèrics a través de l'anàlisi de sensibilitat.

En les tres parts de la tesi es proposen nous enfocs i models que es validen a través d'experiments utilitzant dades obtingudes d'escenaris reals.

Resumen

La distribución urbana de mercancías es una actividad esencial para el desarrollo de las ciudades, aunque al mismo tiempo conlleva diversos problemas en los núcleos urbanos y los distintos actores involucrados. Los costes de la distribución urbana resultan una parte muy significativa de los costes de los transportistas, especialmente a causa de la congestión y la falta de zonas de carga y descarga. Por otro lado, los ciudadanos sufren los efectos de la contaminación, el ruido y la competición por el espacio público.

El transporte colaborativo es actualmente una de las principales tendencias en la investigación en transporte, pues ofrece beneficios atractivos con poca inversión. Esta tesis trata tres situaciones que se reproducen repetidamente en el contexto urbano, donde distintas formas de colaboración (tanto entre compañías privadas como con administraciones) pueden representar una mejora.

La primera parte de la tesis estudia un nivel de colaboración bajo, basado en compartir las zonas de carga y descarga a través de un sistema de reservas gestionado por la administración. En este contexto surge el *Parking Slot Assignment Problem* (Problema de asignación de plazas de parking), como el problema matemático que asigna transportistas a plazas de parking satisfaciendo sus requerimientos a través de ventanas temporales. En primer lugar proponemos un modelo de factibilidad, y después cuatro modelos con funciones objetivo que penalizan la desviación de las ventanas temporales de formas distintas. Se proponen y comparan dos formulaciones: una con el tiempo como una variable continua, y la segunda con discretización temporal. Finalmente, se evalúa y compara las distintas propuestas a través de unos extensos experimentos computacionales en un conjunto de test basado en datos reales.

Un nivel intermedio de colaboración entre transportistas se analiza en la segunda parte de esta tesis. Las áreas urbanas presentan una alta densidad de clientes, y es común encontrar clientes compartidos (es decir, clientes que reciben mercancías a través de distintos transportistas en el mismo intervalo temporal). Proponemos un problema innovador: el *Shared Customer Collaboration Vehicle Routing Problem* (Problema de rutas de vehículos con colaboración de clientes compartidos), donde los distintos transportistas están dispuestos a colaborar transfiriendo parte de la demanda de sus clientes compartidos, si el coste total del transporte se reduce. Estudiamos dos formulaciones: una basada en los vehículos y otra basada en la carga, y se experimenta en un conjunto de instancias generadas.

El máximo nivel de colaboración en distribución urbana de mercancías es el uso de centros de consolidación urbana. Estos centros, normalmente liderados por la administración pública, necesitan la activa colaboración de los transportistas para conseguir una exitosa implantación. Se trata de terminales urbanas donde se consolida la carga de distintos transportistas y, después, un único transportista neutral realiza la distribución de última milla. En esta tercera parte de la tesis proponemos modelos continuos que analizan la mejora de la eficiencia en la distribución urbana a través del uso de centros de consolidación urbana con distintas hipótesis. Los modelos continuos producen soluciones robustas, que son útiles para proporcionar guías en casos genéricos a través del análisis de sensibilidad.

En las tres partes de la tesis se proponen nuevos enfoques y modelos que se validan con experimentos utilizando datos obtenidos en escenarios reales.

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1 Preface

Distribution of goods currently entails several problems to urban systems, including stakeholders and urban fabric. The urban fabric of most major cities is dense and complex. Furthermore, there is a high heterogeneity of zones: residential, commercial, industrial, recreational and parks, along with different requirements for the transportation of goods. Streets are unequal, unidirectional or bidirectional, giving different accessibility to the city, and with small space to unload/load goods. The most emblematic stakeholders are carriers, who spend most part of their time and a significant portion of their cost in the last-mile distribution. The main reasons are the increasing levels of traffic congestion, the lack of unloading/loading zones and other inefficiencies. In turn, citizens as passive stakeholders undergo environmental effects like pollution, noise or space competition. The other important stakeholders are customers, that demand good level of service: compliance of time windows, high frequencies and competitive prices. Although disturbances caused are discouraging, it is necessary to bear in mind that urban distribution is crucial to the city's economic development.

The above problems are challenging because the market is heterogeneous with multiple types of products, roles, and opposite objectives. Although they cause great inconveniences in most of the cities and several types of policies have been tested in different municipalities, there is still a lack of clear guidelines on the best policies for each context, problem or situation. City authorities are also becoming aware of the environmental impact of freight activities in their areas and try to push carriers to find new solutions that improve city logistics. Historical centers and highly dense commercial areas are specially sensitive and are trying to be protected from massive freight operations. Alternatives are found through innovative solutions. Thus, proposing, modeling and evaluating solutions involving public and private agents that produce attractive benefits for companies and for citizens is essential to overcome the mentioned difficulties.

Nowadays, *collaborative* transportation is one of the major trends in transportation research. Not only in freight transportation but also in passenger transportation, with multiple tools that allow passengers to contact other passengers or drivers to share the ride. In general, the collaborative economy is becoming more popular worldwide due to its potential benefits with little need for organization or facilities. In the case of freight transportation the increasing costs for carriers combined with a great market competition motivate transport companies to explore new and more efficient solutions. Some elements of urban systems where collaboration may lead to considerable improvements are highlighted below.

From a merely private business perspective, carriers could take advantage of some type of collaboration. Agreements between two or more carriers can be reached to transfer some specific customers. For instance, the location of a given customer might render the interchanged solution more profitable for both carriers, who then will have to agree on the economic value. Alternatively, when common customers exist, i.e. customers receiving goods from more than one carrier, transfering them among carriers can potentially save costs in the last-mile distribution.

Support from public authorities can further stimulate collaboration if public space is devoted to goods operation activities. Public space is scarce and needs to be properly assigned and shared. In the case of loading and unloading areas, public authorities offer private carriers public space where they can operate. The use of this space can be organized in a collaborative manner, where all private carriers express their needs in advance and a fair allocation is provided that guarantees space to all of them.

The above forms of collaboration can be enhanced if combined with the *consolidation* concept. The basic idea of consolidation in urban environments is to gather demand from multiple origins, which was planned to enter the city with different vehicles with low load factor, and to ship it together with a lower number of vehicles with higher load factor. Without consolidation, vehicles tend to run with low load factors due to small time windows, market split among carriers and backhauling. Consolidation allows to cut down the number of vehicles-kilometer to fulfill the freight demand of a particular urban parcel. Reducing the vehicles-kilometers is beneficial for carriers, who save cost and time, and for citizens who undergo less impacts. Due to the above issues, consolidation of demand is crucial to reduce the number of vehicles-kilometer.

This thesis is aimed at proposing, studying, validating, and analyzing consolidation and collaborative solutions to three particular problems that appear repeatedly at different levels in an urban context. The main three problems investigated are not directly related among them. However, the framework where they arise is similar and the purpose is the same: to propose solutions that enhance urban deliveries with different levels of collaboration. To that end, some strategies will be designed and compared to quantify the costs and the benefits, and the attributes of the problems and the solutions will be examined.

As already mentioned, one conflictive issue in urban distribution is the management of loading and unloading areas. Administrations must face the following related problems: How many areas to reserve and where they should be allocated; How to assure that carriers make a good use of the allocated space; and, How to avoid illegal parking, among others. At present, the evolution of technology and the decrease of the price of technological devices allows new possible solutions with collaboration among carriers, for the rational and fair use of the space. In this thesis we propose a prebooked system for loading/unloading areas for carriers. The system operates in the following way: carriers request some time period for the use of the loading/unloading areas, and the problem arises trying to comply with all the service requests for a limited number of parking slots. The objective is to efficiently assign requests to time slots in order to assure that the system can be implemented in reality. In Part I of this thesis, we propose different models and objective functions to compare and decide the most suitable one for the problem. Moreover, the essential problem that we address can be applied to other transportation problems of resource allocation with time windows. For instance, the administration of public rechargeable points for electrical vehicles or the use of docks in a freight terminal. Therefore, the study is not interesting only from its theoretic point of view but also for its potential applications.

Alternatives for collaboration from a private business perspective may range from punctual agreements between two companies to cover peaks of demand to the use of a unique common carrier in the last-mile distribution. At one intermediate level, we propose a strategy with high potential benefits, which consists of transferring shared customers among carriers. In urban environments, where density is high, we find several customers visited by more than one carrier, or reduced areas with several customers that receive goods from different carriers. The transfer in advance of this demand between carriers, can greatly reduce the last-mile delivery cost, since we allow customers to be visited only once per time horizon. Collaboration strategies for shared customers are proposed and analyzed in Part II of this thesis.

Consolidation in urban environments can be performed through Urban Consolidation Centers (UCCs). UCCs operate as transshipment and consolidation points and usually consist of a small urban terminal that shelters and facilitates load operations between carriers. Multiple carriers with demand destined to the urban area visit the consolidation center, and transfer their demand to a unique carrier that performs the last-mile deliveries with higher demand density. This operation avoids a great number of vehicles entering the urban area to deliver their loads. In [78] consolidation is seen as one of the major solutions for the city logistics problematics. In Part III, UCCs are analyzed from a strategic point of view for the improvement of urban deliveries.

In all three parts of the thesis, the current situation as well as alternative systems, potentially more efficient, are modeled in order to quantify the potential advantages of the proposed solutions. In the case of the UCC, the use of a continuous model provides a general insight of the performance in a general situation. These types of models allow to detect key variables, understand relationships among parameters and draw general conclusions about the situation.

Mixed integer mathematical programming models are used to analyze the other problems studied in this thesis: collaboration among carriers when it is possible to transfer among a reduced set of customers and the fair allocation of loading/unloading requests in a public space devoted to carriers operations inside the city. These models provide detailed insight for each specific situation. Nevertheless, depending on the properties of the problem and the specific formulation, the general solution methodology can become very complex or computationally unaffordable.

Since the problems addressed in the thesis can be frequently detected in most cities, all proposed models have been tested computationally on instances based on real life data. Data was collected, mainly from the city of Barcelona, to generate a set of benchmark instances that could reliably represent real situations. Indeed, the proposed models could be applied to other context or cities providing particular detailed solutions for other cases. Furthermore, additional experiments have been run with sets of randomly generated benchmark instances.

1.1 Contributions

The major contributions of this thesis are described below grouped together for each of the parts.

In Part I, we propose a new problem, the Parking Slot Assignment Problem, that arises in prebooking systems for loading/unloading facilities and we propose five different models which mainly differ in the objective function. We implement two different formulations, one that considers time as a continuous variables and one with time discretization. We prove with computational experiments that the earliness/tardiness criterion produces solutions which are also good for the other models. From a more theoretical point of view, we propose a condition to check unfeasibility, and we compare the domains of the different models.

In Part II, we propose a new problem for collaboration among carriers, the Shared Customer Collaboration Vehicle Routing Problem. For this problem, we propose two different formulations: a vehicle based and a load based formulation. We prove experimentally that the load based formulation outperforms the vehicle based formulation. The numerical results from the computational experiments allow us to evaluate the cost reduction that can be attained using this form of collaboration.

In Part III, we propose an analytic approach to evaluate the efficiency of the UCCs. We prove that the estimation of benefits assuming equal-market share among carriers is valid since the effects of different market shares among carriers is not significant. Also, we provide a tool to limit the participation of small carriers on UCCs by controlling the minimum savings contributions.

1.2 Publications and conferences

Some of the results of this thesis have been published in international journals or presented at conferences or workshops. The publications and conference participations is listed here:

Publications

- M. Roca-Riu, E. Fernández, and M. Estrada, *Parking slot assignment for urban distribution: models and formulations*, in OMEGA - The International Journal of Management Sciences 57, 2015.
- M. Roca-Riu, M. Estrada, and E. Fernández, *An Evaluation of Urban Consolidation Centers through continuous analysis with non-equal market share companies*, in Procedia - Social and Behavioral Sciences, 2016. Ninth International Conference on City Logistics, Tenerife, Spain.
- M. Roca-Riu and M. Estrada, *An evaluation of urban consolidation centers through logistics systems analysis in circumstances where companies have equal market shares*, in Procedia - Social and Behavioral Sciences, 2012. Seventh International Conference on City Logistics, Mallorca, Spain.
- M. Roca-Riu, E. Fernández, and M.G. Speranza, *The Shared Customer Collaboration Vehicle Routing Problem*, in preparation.

Conferences

- M. Roca-Riu, E. Fernández, and M.G. Speranza, *The Vehicle Routing Problem with Shared Customers*, in The fifth Workshop on Combinatorial Optimization, Routing and Location (CORAL), 2015. Salamanca, Spain.
- M. Roca-Riu and E. Fernández, *Parking Slot Assignment for urban distribution: models and formulations*, in The fourth meeting of the EURO Working Group on Vehicle Routing and Logistics Optimization (VEROLOG), 2015. Vienna, Austria.
- M. Roca-Riu, E. Fernández, and M. Estrada, *Parking slot assignment for urban distribution: models and formulations*, in IFORS-Conference of the International Federation of Operational Research Societies, Invited session on City Logistics, 2014. Barcelona, Spain.

Part I

The Parking Slot Assignment Problem

Introduction

Major cities face multiple problems due to delivery operations. Delivery operations are the final step in the chain of urban distribution. They normally occur on public space and are known to be a crucial part of urban deliveries [49]. They greatly impact cities, neighborhoods and sensible areas in terms of air quality, visibility, safety and occupation of space.

In many cases city councils regulate the conditions under which carriers may operate, and then each carrier acts as its own decision maker, scheduling its operations according to established rules and its own resources limitations. In contrast, city councils must act as common decision makers for all operations carried out with the use of public space, since a criterion on how to allocate public space among carriers is needed. The adequate management of public space is crucial for a successful urban distribution. The lack of parking facilities has been pointed out among the aspects with higher impact in urban delivery (see, for instance, Ma et al. [49]).

On the other hand, the advance of technological systems and the decrease of the price of technological equipment allow the implementation of new solutions, which at reasonable cost can control and organize loading and unloading areas. Recently, the city of Barcelona has changed the control system of the loading and unloading areas. From the year 2001, transport companies were using a timely cardboard placed in the vehicle to indicate the beginning of the operation activity, which had to be completed within 30 minutes. From March 2015, the control is done by a mobile app in combination with the geolocation of the phone. The new system allows remote control of the users, and the collection of more information for a better planning.

Apart from controlling the correct use of the offered resources, the ultimate goal of the city council is to regulate the use of public space in order to prevent carriers from illegal parking and to improve urban distribution. Currently in most of the cities, some public space, consisting of a set of parking places, is already allocated for loading and unloading operations at a given area during some hours each day. This part of the thesis deals with the problem of allocating public parking space in the streets during the loading and unloading hours for goods distribution from an operations research prespective. In particular, we propose a system in which the city council would ask carriers to express in advance their requests for a parking time interval and to inform about the duration of their operations. These durations will take into account not only loading and unloading activities but also movement times between the parking space and the operation site. Then, each carrier would be assigned a time interval, based on his preference. Since carriers will know in advance their assigned time interval, they will be able to re-optimize their routes beforehand so as to arrive on time to the assigned parking space. Thus, we assume that carriers will accept and respect the assigned intervals, even if they do not fit their requests.

In the proposed system carriers can only park at designated parking areas and *during previously assigned time periods*, which are organized in advance by the city council. Such a system would eliminate the very negative effects in traffic flow due to carriers double-lane parking and would also benefit carriers greatly, since available parking space would be guaranteed at designated time periods. The system is based on the collaboration between the regulating public entity, and the different carriers to make the best use of a public facility. The public entity offers space and the allocation of carriers to a spot, and carriers participate obeying assignments and respecting other carriers in their time slots. Some practical studies have proven the benefits of establishing a booking system for allocating in advance carriers to time slots [62, 54, 25]. To the best of our knowledge, however, the existing works have used priority lists to assign slots to requests. The problem described in this part of the thesis considers alternative criteria for fairly assigning requests to time slots.

We propose several alternatives to model the Parking Slot Assignment Problem (PAP) that we introduce. For all of them we discuss alternative mathematical programming formulations, study some of their properties and design a simple heuristic. Extensive computational experiments have been run to analyze and compare the performance of each of the proposed models and to evaluate their requirements in terms of computing times.

Chapter 2 revises the existing literature relevant to the problem. The PAP is formally introduced in Chapter 3, and several modeling alternatives for the PAP are also discussed. While the first model focuses on the feasibility of the problem, the remaining alternatives consider other optimization criteria. In Chapter 4 two different formulations are proposed and the simple heuristic is described. Section 4.1 presents a Mixed Integer Linear Programming (MILP) formulation for the feasibility model, based on the Vehicle Routing Problem (VRP), which can also be adapted to the alternative models, and some of their properties are studied. In Section 4.2 MILPs are proposed for the different models, based on the structure of the Assignment Problem (AP). In Section 4.3 the greedy heuristic is described. Chapter 5 describes the computational experiments we have run, and presents the obtained results together with an extensive analysis and comparison. The results from both formulations and the heuristic on the different proposed model are analyzed and compared. We close this part of the thesis in Chapter 6 with some comments and possible avenues for further research.

Part of this work has given rise to a journal article in OMEGA - The International Journal of Management Science, see [71].

2 Literature Review

Several works point out the advantages of allocating specific parking facilities for carriers in order to reduce the negative impact of distribution operations. As an example, the results of [62] illustrate the positive response of most drivers and truck operators to one such initiative in Kobe (Japan) in 2001. It is however clear that, even if a set of parking places is allocated, negative effects will prevail when carriers arrive to delivery areas but find no available parking space. The benefit of establishing some booking system for allocating in advance places has been assessed in several practical studies. The results of a pilot test carried out in 2005 in Dos Hermanas, Sevilla (Spain) [54] confirm the effectiveness of the internet booking system implemented for the assignment of parking space in load areas of the city center. An in-advance booking system was theoretically studied in [51] for the city of Winchester (UK) in the High-street area. Supported by an EU project [25], a recent eight months pilot test in Bilbao (Spain) successfully trialled a booking system for carriers in four zones of the city. Free areas could be assigned to users without pre-booking, and pre-booked carriers could be reassigned if they were out of schedule, as well.

In the above referenced works priority lists have been used for deciding the assignment of places to requests. However, alternative criteria or techniques can be applied for establishing the allocation of parking space to carriers. In this part of the thesis we propose the use of mathematical programming optimization models for solving the PAP. To the best of our knowledge this problem has not been addressed so far in the literature in the context of urban distribution. Nevertheless, we can find some similarities with other problems studied in the literature. For instance, the PAP can be seen as a particular case of a scheduling problems with time windows (see, for instance, [42]). Further, the concept of earliness/tardiness, as it has been used in scheduling problems with time windows [41, 86] or other contributions in flow shop scheduling problems based on the manufacturing industry [64, 85] can also be exploited in our case as we will see later on.

Apart from the practical focus of the studies mentioned before, the approach of our study is basically to solve a scheduling problem with time windows. A review of works addressing this problem or problems with similar features is presented below. The references related to problems involving scheduling with time windows can be divided according to three considerations: problems with features similar to those of the PAP, scheduling models where the basic objective is to process the work on-time, or scheduling problems where it is considered that not doing a task within a given time window incurs an earliness/tardiness penalty, which has to be minimized.

2.1 Problems with features similar to the PAP

Problems like the Berth Allocation Problem (BAP) [52], the Aircraft-Gate Allocation Problem (AGAP) [6], or the allocation of trains to platforms at rail stations (Train Platforming Problem [10]) also have some similarities with the PAP. In all these problems it is assumed that the arrival times of the vehicles as well as the durations of their operations are known in advance, and the vehicles have to be assigned to some facility for a given time. The BAP aims to optimally schedule and assign vessels to berthing areas along a quay. The most common objective in the BAP is to minimize total service time. This objective favors the assignment of higher priorities to vessels with smaller handling volumes than to vessels with larger handling volumes [66]. Because this type of solution may not satisfy the ocean carrier' preferences, another studied objective is the minimization of the deviation from the preferred berth [65]. Some other works consider, in addition, objective functions with penalties for unsatisfied time windows [32]. Similar characteristics are present in the AGAP [6], in which the gates where aircrafts will stop are planned taking into account different criteria: efficiency of flight schedules, passenger walking distance, or robust use of the gates in front of disruptions [56].

The distinctive feature of the PAP with respect to the above problems is that the carriers time windows are flexible, in the sense that the parking times assigned to the carriers by the city council may not coincide with the requested ones. Still we assume that carriers accept and respect the assigned intervals, provided these are known in advance. The reason for this assumption is that carriers can adapt their routes in advance so as to arrive on time to the assigned parking space. This assumption does not hold in the BAP, the AGAP or the allocation of trains to platforms, whose time windows are not flexible and thus must be respected when making the assignment. In the BAP, while advancing the arrival date to port is usually not feasible, postponing it typically implies very high costs. The same happens with the departure dates from the port due to contractual agreements between port operators and ocean carriers. Something similar happens with aircrafts, where indeed flight schedules are not planned according to the availability of gates at the airports.

2.2 On-time jobs problems

The work [26] focuses on on-time jobs while other works like [31] or [42] present models avoiding direct formulations. The work [9] simplifies the problem to the case with equal processing times, although in reality problems are usually more complex. In practice, scheduling problems are often tied to other problems, as, for instance, a subsequent delivery phase or the use of machines with family setup times. Several authors, see for instance [27, 73, 74], have studied scheduling problems combined with related problems.

The problem of scheduling non-preemptive jobs processed within time windows on identical parallel machines is presented in [26]. The paper shows first that the problem of determining if a particular set of jobs can be completed by the available machines is NP-complete. Then, a model is presented using graph theory and heuristics are used to solve two variations of the problem: Fixed job Scheduling Problem (FSP), where each job has to be completed at a fixed start and end times; and Variable job Scheduling Problem (VSP), where each job can be completed within a time window larger than its processing time. The solution of VSP is based on the FSP results for a restricted problem.

A simplification of the problem with equal processing times and time windows is faced in [9], where Brucker and Kravchenko showed that the problem was solvable in polynomial time. The algorithm presented is also useful for other problems: the minimization of the mean flow time or the minimization of the weighted sum of completion times but without deadline restriction.

Another approach [31] is to find the minimum number of resources needed for fixed and variable job schedules, i.e. the objective is to minimize the number of machines needed to carry out all the jobs. Solving this problem determines the feasibility of the problem with a fixed number of resources. The problem of minimizing the number of machines is introduced as a special case of Dilworth's problem [22], and an adaptation of an approximation solution method based on entropy principle of informational smoothing is presented. Then, the problem is formulated as a pure integer programming problem and an exact algorithm is given. The algorithm examines a sequence of feasibility capacitated transportation problems with job splitting elimination side constraints.

The work of Koulamas in [42] formulates the problem maximizing the weighted number of on-time jobs in single machine scheduling with time windows. The NP-completeness is proven, and two lemmas validate the proposed problem decomposition. Regarding the solution methodology, a heuristic is proposed, which proved to give solutions in a reasonable computational time where the average deviation from the upper bound proposed is about 10%.

Finally, we mention two references that combine scheduling with time windows with additional features. In [27] the main problem of delivery with time windows is merged with a production problem in a manufacturing plant. The overall problem is decomposed into two phases: a production phase and a delivery phase which must be coordinated. The extended problem is formulated as a MILP, and a tabu search heuristic is implemented and compared to an exact solution method derived from [3], which is applied only to small problems due to computation time requirements.

The works [73, 74], based on a real problem, consider that machines can perform different jobs, but a setup task must be carried before the first task of each job type. The concept of family setup times is introduced, where the idea is to assume that doing similar jobs together saves time for setting up the machine. The authors propose a branch-and-bound algorithm for solving both problems (single machine [74] and parallel machines [73]).

2.3 Earliness/Tardiness problems

The main characteristic of this family of problems is the idea of penalizing the deviation from the target time (or time window) for a job. In general, the penalty is proportional to the deviation.

A primary problem is considered in [28], which is to schedule tasks with a specified length and a preferred starting time with non-preemption in one-processor. Two different cost functions are examined: the sum of the absolute discrepancies (difference) from the preferred starting times and the maximum discrepancy of any task. The article also contains the NP proof of the total discrepancy problem, which is used to show the computational difficulty of more complex problems, like ones similar to the PAP. Finally, an efficient algorithm proposed for finding the minimum cost schedule in two cases: when all the tasks all have the same length or when the tasks are required to be executed in a given fixed sequence.

The work [28] is used in [41] to show the complexity of the single-machine earliness/tardiness penalties with arbitrary time windows problem. The problem is defined as follows: no costs are incurred if the job is completed within the time window, but earliness/tardiness penalties are incurred otherwise. Penalties are defined proportional to the distance between the assigned time and the preferred time window. Then, the problem is decomposed into two subproblems: first, finding a good job sequence, and second, optimally inserting idle time into a given sequence. Heuristics are proposed for the former, and an optimal algorithm adapted from [28] for the latter.

Later, in 2003 [76] presented a branch-and-bound algorithm, which solves efficiently the earliness/tardiness problem based on a Lower Bound (LB) formulation. The formulation of a LB for the problem is also valid for a generic formulation of cost penalties, and the authors provide a pseudo-polynomial time algorithm to compute the LB.

A more complex problem is proposed in [4]. It considers a set of jobs and each job consists of more than one task. Since jobs are chains of ordered operations to be processed in a set of machines, precedence constraints appear inside a job. The authors apply Lagrangean relaxations, in particular the relaxations of the precedence and the resource constraints, and evaluate their efficiency. The Lagrangean relaxation of the precedence constraints can be decomposed into independent single-machine scheduling subproblems with earliness/tardiness penalty costs. Although a single machine scheduling problem with earliness/tardiness penalty costs is NP-hard, it can be solved efficiently with an advanced branch-and-bound procedure designed in [76]. This allows to find efficiently the vector that maximizes the Lagrangian function for a given multipliers vector and, thus, to solve the Lagrangean dual with a subgradient algorithm. The relaxation of the resource constraints becomes a problem where single subproblems consist of scheduling operations of the same job. Then, each problem can be solved by dynamic programming in time linear on the number of jobs and the temporal horizon.

Table 2.1 summarizes the above references.

First Author	Year	Reference	Practical or Theoretical	Objective	Solution Methodology	Comment
Gertsbakh	1978	[31]	Theoretical	Minimal resources	Exact algorithm	
Garey	1988	[28]	Theoretical	Min absolute difference and min max difference	Algorithm for particular cases	Same length or executed in a fixed sequence
Gabrel	1995	[26]	Theoretical	On-time jobs	Heuristic	Graph theory formulation
Schutten	1996	[74]	Theoretical	Family setup single machine	Exact algorithm	
Schutten	1996	[73]	Theoretical	Family setup parallel machine	Branch-and-bound	
Koulamas	1996	[41]	Theoretical	Earliness/tardiness TW	Heuristic and exact algorithm	Decomposition
Koulamas	1997	[42]	Theoretical	Maximum weighted on-time jobs	Heuristic	Decomposition
Ma	2001	[49]	Practical	Capture data	-	Detect problems
Odani	2001	[62]	Practical	Experimental problem	-	
Bolat	2001	[6]	Theoretical	Aircraft-gate assignment	Genetic algorithms	
Sourd	2003	[76]	Theoretical	Earliness/tardiness	Lower Bound (LB)	Pseudo-polynomial algorithm to compute LB
Park	2003	[65]	Theoretical	Berth scheduling	Lagrangean relaxation and subgradient optimization	Two phase
Moccia	2004	[52]	Theoretical	Maritime container terminals	Several approaches	BAP
Garcia	2005	[27]	Theoretical	Delivery phase	Tabu search	
Pinedo	2005	[66]	Theoretical	Scheduling	Several approaches	Reference book
Golias	2007	[32]	Theoretical	Berth allocation	Branch-and-bound	Service deadlines
Brucker	2008	[9]	Theoretical	On-time jobs with equal-processing times	Exact algorithm	Solvable in polynomial time
Baptiste	2008	[4]	Theoretical	Job and task formulation	Lagrangean relaxation and problem decomposition	Branch-and-bound or dynamyic programming
Caprara	2011	[10]	Theoretical and practical	Train platforming	Branch-and-bound	Case study Italian infrastructure manager
Mercedes	2015	[56]	Theoretical	Gate allocation	Coloured Petri networks	Delay effects

Table 2.1: Summary of references relevant for the Parking Slot Assignment Problem

3 Problem definition and modelling alternatives

We assume that the city council decides to implement a system where a set of parking places will be assigned to different carriers at given time slots. Carriers would express in advance their requests for a parking time interval, and inform about the duration of their operations. Carriers can be flexible in their time windows, since they will know in advanced their assigned time interval, and they will be able to re-optimize their routes to take advantage of the reserved time slot. The duration of the operations will take into account all the activities that take place while carrier is parked at the loading/unloading area. This time will cover all the reserved time of parking place for the given carrier.

The main question that we address in this chapter is how to assign the list of carriers requests to the available places and time slots. Any solution satisfying all requests within their time windows would be optimal. From this point of view, one would think that the problem we face reduces to a feasibility problem. However, a given instance may not have an assignment satisfying all requests. What should the outcome be in this case? What can the decision maker do if there is no parking slot for everyone? Some fair criterion is needed in order to allocate carriers requests when their needs can not be satisfied.

As just mentioned, suitable and fair criteria are not easy to decide. There is a public resource that should be allocated fairly to several operators. If not every request can be satisfied, meaning that the problem is unfeasible, we will allow solutions with non-accomplished requests. Non-accomplished requests are requests not scheduled within their indicated time windows. This naturally leads to alternatives aiming to reduce the degree of non-accomplishment of requests in a fair way. The concept of fairness has been addressed in optimization, associated with various types of problems, particularly when resources have to be allocated [45]. In our case, we incorporate fairness by resorting to objective functions that penalize unfeasible solutions in alternative ways, by using different criteria to quantify their degree of non-accomplishment of requests. For instance, we can minimize the overall non-accomplishment by somehow weighting the earliness or tardiness of the requests assignments relative to their respective time windows, measured in time units. Alternatively, the objective may focus on the number of non-accomplished requests. When considering the following alternative objectives, we will extend the domain for feasible solutions allowing for the violation of time windows constraints, possibly, introducing additional particular constraints.

The objective functions we propose try to cover different aims when minimizing the overall non-accomplishment. If we consider relevant how far served requests are from the requested time windows, the earliness/tardiness penalty seems to be the more general and widely used criterion in the scheduling literature. Even in this case the weighted sum is the most efficient approach from a system level, even if this criterion might seem unfair from a more individual perspective, as, for instance, if a small group of requests is largely penalized to make it possible to satisfy the remaining requests. For that reason, the minimization of the maximum earliness/tardiness penalty is considered to represent a more fair criterion from an individual perspective. When trying to combine both previous objectives together, the maximum earliness/tardiness penalty can be limited to guarantee that there are not large penalties to individual requests, and maintain the weighted sum as the general criteria in the benefit of the system. On the other hand, if we only take into account whether or not the request is satisfied in the requested time windows, the most straightforward and general formulation is to minimize the number of unsatisfied requests. Finally, we propose a cost formulation. This formulation can be useful when the costs of non-accomplished requests could be faithfully approximated. In all the previous proposals weights in the considered criterion can be used to refine the different criteria and prioritize requests of different nature or from different companies. The detailed optimization criteria are described below. In the next chapter we formulate the different proposed models with particular formulations.

MOD 0. Feasibility PAP. Solution that fulfills all time windows imposed by requests.

MOD 1. Earliness/tardiness minimization. Minimize an objective in which time windows violations are penalized in proportion to the earliness or tardiness of the assigned time in relation to the requested time window. The criteria is to minimize the overall non-accomplishment.

MOD 2. Minimization of maximum earliness/tardiness. This model focuses again on a measure of the earliness/tardiness of solutions. Now, instead of considering the overall non-accomplishment, we focus on the maximum non-accomplishment, measured as the maximum earliness or tardiness from the requested time windows, among all requests. This objective can be useful if small deviations from the requested time windows are not considered important, and the relevant measure of the quality of a solution is the maximum earliness/tardiness among all requests.

MOD 3. Earliness/tardiness minimization subject to maximum displacement. This model tries to somehow address jointly the two main concerns of MOD1 and MOD2. MOD3 has the same objective function as in MOD1, the overall non-accomplishment, but limiting the earliness or tardiness in the assignment of any request from its requested time window to a maximum value fixed in advance, named maximum displacement.

MOD 4. Minimization of number of requests scheduled outside the time window. Models MOD1, MOD2, MOD3 quantify the time deviations of the solutions from the requests time windows, but ignore if this non-accomplished demand affects to a small or large number of requests. In MOD4 we focus on the number of requests scheduled outside the asked time window, rather than on the magnitude of the non-accomplishment.

MOD 5. Cost minimization. Except for MOD0, all previous models allow solutions where some request is not scheduled within its time window. An alternative to such relaxed models would be to impose time window constraints, and to outsource additional parking space at the time periods when the city council parking space is insufficient to satisfy the carriers demand. In a realistic scenario, this additional space could be obtained from a nearby public parking or from parking areas for non-commercial private use, next to the city council parking places. The outsourced additional parking places would be reserved for loading/unloading operations for only some hours of the total time horizon.

We restrict our study to the framework of one one single loading and unloading area with several parking places and we assume that data is deterministic. This is indeed a simplifying assumption as the nature of the problem involves some uncertainty, particularly with respect to vehicle arrival times. However, as we will see, the deterministic formulations proposed are already complex and difficult to solve.

The PAP is mathematically defined as follows. We consider one loading and unloading area with c common parking places that can be used by carriers for their loading/unloading operations. Let $[0, T]$ denote the time interval when loading/unloading operations must be scheduled. Let also Q, with $|Q| = q$, denote the index set of loading/unloading operations, each of them with a request for parking assignment within time period $[0, T]$. Associated with each request $i \in Q$, the parameters $[a_i, b_i]$ and s_i respectively denote the time window for the beginning of the operation i and its duration. Since b_i is the latest instant when the beginning of operation i can be scheduled, operation i can last until $b_i + s_i$.

Feasible solutions to the PAP consist of assignments of requests to parking places within the time period $[0, T]$, that satisfy the time window for the beginning of each request and such that at each time slot no more than c parking places are occupied. Figure 3.1 presents a graphical representation of the problem over a morning time horizontal interval.

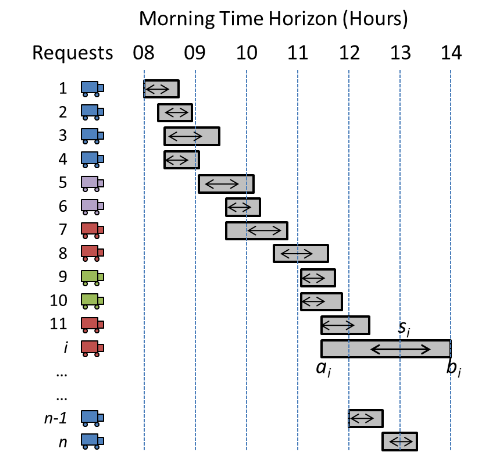

Figure 3.1: Requests of different companies along morning time horizon

3.1 An unfeasibility condition for MOD0

Below we present a sufficient unfeasibility condition for MOD0. As we have discussed, the problem can be modeled with the only aim to find a feasible solution, i.e, to find a solution that satisfies all the time windows. However, as we will see in the computational experiments section, detecting its unfeasibility is not always easy, even if large computing times are allowed. For this reason an effective condition can be very useful for detecting unfeasible instances.

The main idea is to analyze the requests that must be completed within different time intervals and to compare the time needed to satisfy these requests and the overall time offered in this interval, which depends on the available places. Indeed, if the total time needed for the requests is greater than the time offered, the instance is unfeasible. Before presenting the sufficient condition we introduce some additional notation. Let $Q^{\alpha,\beta} \subset Q$ denote the subset of requests whose time window is contained in the interval $[\alpha,\beta]$. That is, $Q^{\alpha,\beta} = \{i \in Q \mid [a_i, b_i] \subseteq [\alpha,\beta]\}$.

A Lower Bound (LB) on the time requested within a given interval $[\alpha,\beta]$ is the sum of the durations of the requests that must be completed inside the given time interval, i.e. $L^{\alpha,\beta} = \sum_{i\in Q^{\alpha,\beta}} s_i$. This bound may not be tight, since there may be additional requests for the time interval $[\alpha,\beta]$, when the time window of some request not in $Q^{\alpha,\beta}$ overlaps it, i.e. $[\alpha,\beta] \cap [a_i, b_i] \neq \emptyset$ for some $i \notin Q^{\alpha,\beta}$.

On the other hand, an upper bound of the time that is offered in the interval $[\alpha,\beta]$ is $U^{\alpha,\beta}$, computed as the sum of the duration of the time interval multiplied by the number of places offered, $c(\beta-\alpha)$, plus some extra time. For the extra time we take into account that even if some requests are assigned at the very end of the time interval (time instant β), the assignment will still be feasible since time windows limit the beginning of the service. Indeed, the operations corresponding to such assignments will totally take place after time interval $[\alpha,\beta]$. Thus, to compute the extra time we assume that exactly c requests are assigned just at the end of the interval (one at each place), and that these assignments correspond to the c largest durations of the requests indexed in set $Q^{\alpha,\beta}$. That is, the extra time coincides with the c-centrum [1, 29, 75, 77] of the durations of the requests indexed in $Q^{\alpha,\beta}$, that we denote by $c\text{-}s^{\alpha,\beta}$. Hence, if $s_{i_1} \geq s_{i_2} \geq \cdots \geq s_{i_{|Q^{\alpha,\beta}|}}$ are the sorted duration values of the requests indexed in $Q^{\alpha,\beta}$, then $c\text{-}s^{\alpha,\beta} = \sum_{r=1}^{c} s_{i_r}$. Therefore, our upper bound on the time that is offered in the interval $[\alpha,\beta]$ can be expressed as $U^{\alpha,\beta} = \quad c(\beta-\alpha) + c\text{-}s^{\alpha,\beta}$.
Our unfeasibility sufficient condition is then as follows:

Proposition 1. *Let $[\alpha,\beta] \subseteq [0,T]$. If $L^{\alpha,\beta} > U^{\alpha,\beta}$ then MOD0 is unfeasible.*

Example 1. *The following example illustrates the performance of Proposition 1.*

Consider a small instance with $c = 1$, $q = 10$, i.e. one loading/unloading area and ten requests, and the values of a_i, b_i, s_i shown in Table 3.1.

	1	2	3	4	5	6	7	8	9	10
a_i	480	500	520	520	520	520	640	640	720	720
b_i	500	520	560	560	560	560	680	700	740	740
s_i	20	21	25	18	20	22	15	16	18	22

Table 3.1: Data for an example instance for unfeasibility condition

Let us analyze the interval $[\alpha,\beta] = [520, 560]$. The subset of requests whose time window is contained in the interval $[\alpha,\beta]$, is $Q^{\alpha,\beta} = \{3,4,5,6\}$. The proposed lower bound on the time requested within the given interval is $L^{\alpha,\beta} = 25+18+20+22 = 85$, i.e., the sum of the durations of requests in $Q^{\alpha,\beta}$. Then, the instance can only be if feasible at least 85 units of time are offered to service the requests in the interval $[\alpha,\beta]$.

The proposed upper bound of the time that is offered in the interval is $U^{\alpha,\beta} = c(\beta-\alpha)+c\text{-}s^{\alpha,\beta}$. To compute $c\text{-}s^{\alpha,\beta}$, we order the durations of the requests indexed in $Q^{\alpha,\beta}$, $25 \geq 22 \geq 20 \geq 18$, and add the first c values. Since $c = 1$, in this case we only consider the largest one. Thus, $c\text{-}s^{\alpha,\beta} = \sum_{r=1}^{1} s_{i_r} = 25$, and the upper bound is $U^{\alpha,\beta} = 40 + 25 = 65$. example we see that $L^{\alpha,\beta} > U^{\alpha,\beta}$, so the proposed instance is unfeasible. As in the interval $[520, 560]$, the total time needed to serve the requests in this time period is greater than the total time offered.

4 Formulations for the Parking Slot Assignment Problem

The formulations for new combinatorial problems as the one proposed here are usually inspired by existing formulations of classical problems in the literature. The basic formulation depends on the properties and features of the problem, as well as on the objective function. Based on this, decision variables, constraints and objective functions are chosen for the basic structure. Then, if necessary, specific features are introduced by new variables or constraints. The formulation must reliably represent the underlying model. At the same time, good properties are desirable when testing these formulations under standard solving procedures.

In order to exploit the flexibility of time requests in the described problem, we first consider that requests can be assigned continuously in time. The different time durations of the loading/unloading operations, and the flexibility of the time windows suggest the suitability of using assignment times as a continuous variables. If requests are allowed to be assigned to a continuous point in time, formulations provide more accurate solutions to the problem. Thus, the PAP is first considered as a scheduling problem with time windows. I.e., what loading/unloading tasks to schedule at each parking place, and at what times. Following previous work on time constrained routing and scheduling [20], the PAP is formulated as a Vehicle Routing Problem with Time Windows (VRPTW), where time is considered a continuous variable. This formulation is detailed in Section 4.1.

Given the results of some tests with the VRPTW formulations, we consider a second formulation with time discretization. The coarse approximation of time discretization entails some loss of accuracy, and it increases the problem dimension notably. But with time discretization, PAP can be formulated as a variation of an Assignment Problem (AP), (See Section 4.2). As will be seen, the results obtained with these type of formulations are very good.

Finally, Section 4.3 presents a simple heuristic for the PAP. The aim is not to provide optimal results but to use the results of the heuristic to assess the results obtained with both formulations.

4.1 The Parking Slot Assignment Problem as a VRP

As mentioned earlier, the loading/unloading requests are considered as tasks that have to be fulfilled by one of the parking places available, and time is considered as a continuous variable. If possible, satisfying the time windows required. Then, the problem can be seen as a scheduling problem with time windows. Based on [20], scheduling problems with time windows can be formulated as VRPTWs [79]. Customers represent the carriers requests and each vehicle represents a parking place. Thus, each *route* is equivalent to a sequence of carriers requests which are served consecutively at the same parking space.

To formulate the problem we define an auxiliary complete directed network $N = (V, A)$, with set of vertices $V = Q \cup \{v_d\}$, where v_d plays the role of the depot for these fictitious routes, but has no real meaning. The arcs of A can be classified in the following types: (a) (v_d, i) with $i \in Q$; (b) (i, j) with $i, j \in Q$; and, (c) (i, v_d) with $i \in Q$. We define two sets of decision variables. Binary variables x_{ij}, for all $(i, j) \in A$ indicate whether or not arc (i, j) is used. The meaning of these variables is the following. When both $i, j \in Q$, $x_{ij} = 1$ if and only if the requests of carriers i and j are assigned consecutively to the same parking place. When $i = v_d$, $x_{v_d j} = 1$ indicates that the request of carrier $j \in Q$ is the first one in the sequence assigned to some parking space. Finally, when $j = v_d$, $x_{i v_d} = 1$ indicates that the request of carrier $i \in Q$ is the last one in the sequence assigned to some parking place. We also define a second set of continuous decision variables t_i, with $i \in Q$, to indicate the starting time for the parking request of carrier $i \in Q$ in the schedule. Then a formulation for the PAP is as follows:

$$\text{minimize} \quad z(x, t) \tag{4.1}$$

$$\text{subject to} \quad \sum_{j \in Q} x_{v_d j} \le c \tag{4.2}$$

$$\sum_{(i,j) \in A} x_{ij} - \sum_{(j,i) \in A} x_{ji} = 0 \qquad i \in Q \tag{4.3}$$

$$\sum_{(i,j) \in A} x_{ij} = 1 \qquad i \in Q \tag{4.4}$$

$$t_i + s_i - t_j \le (1 - x_{ij})M \qquad i, j \in Q,\ (i, j) \in A \tag{4.5}$$

$$a_i \le t_i \le b_i \qquad i \in Q \tag{4.6}$$

$$0 \le t_i \le T \qquad i \in Q \tag{4.7}$$

$$x_{ij} \in \{0, 1\} \qquad (i, j) \in A. \tag{4.8}$$

The objective function $z(x, t)$ will be discussed in the following section depending on the criteria used. In all cases, when the domain defined by Constraints (4.2)-(4.8) contains some feasible solution the optimal value will be zero. The meaning of the constraints is as follows. Constraints (4.2)–(4.4) define the *routes*. In particular, Constraint (4.2) limits the maximum number of parking places to c, by restricting the number of routes starting at the depot. Constraints (4.3) guarantee the flow balance along the hypothetical routes. Constraints (4.4) ensure that all requests are allocated to a parking place, by imposing that exactly one route serves each customer. Constraints (4.5) relate time variables t with flow variables x, to guarantee that time values respect the sequence of service. M is a big enough value that makes the constraint redundant when $x_{ij} = 0$. Note that Constraints (4.5) also prevent subtours. The time window constraint of each request is imposed in (4.6). Finally, (4.7) and (4.8) define the domain for the t and x variables. Note that if data parameters are integer values, and x are binary, then t are also integer without the need of explicitly imposing it. The reader is addressed to [79] for further details of this type of models. The above formulation has $(q+1)q$ binary variables and q continuous variables. The number of constraints is $4q + q(q-1) + 1$.

Because it is known that constraints with big M values produce weak linear programming (LP) relaxation bounds, it is convenient to find tight estimations of M. For instance, we can use $M = \max_i b_i + \max_i s_i - \min_j a_j$, (in Section 5.1 alternative values are proposed).

In formulation (4.2)–(4.8) the time when each of the requests is allocated to some parking place, t_i, is explicit, while the specific parking place to which it is allocated is not explicit. Note, however, that the set of requests allocated to each parking space can be easily identified by tracing the set of requests of each of the routes. This allocation could have been made explicit in the formulation by defining decision variables x^k_{ij} with $(i, j) \in A$, $k \in \{1, \ldots, c\}$ indicating whether or not request i and j are allocated to parking place k, and request j is scheduled immediately after request i, at the expenses of a considerably larger number of binary variables.

In principle, the LP relaxation of the formulation (4.2)–(4.8) could be reinforced with the classical subtour elimination constraints (SEC):

$$\sum_{\substack{(i,j)\in A\\ i,j\in W}} x_{ij} \le |W|-1 \qquad W \subset V \setminus \{v_d\}. \tag{4.9}$$

However, as shown below, for usual values of the data and the parameter M, the addition of the SECs (4.9) does not reinforce the LP relaxation of Constraints (4.2)-(4.8), since the resulting domain will always contain some feasible solution, even if the original domain with the integrality constraints is not feasible.

A feasible solution to the LP relaxation of the domain defined by Constraints (4.2)–(4.8) plus the SEC Constraints (4.9) can be obtained in the following way (see Figure 4.1):

- The flow through arcs connecting the depot with each other node (dashed arcs in Figure 4.1) is c/q in both directions, i.e. $x_{v_d j} = x_{i v_d} = c/q$, $\forall i,j \in Q$.
- The flow through any other arc connecting two nodes $i,j \in Q$ (solid lines in Figure 4.1) is $\frac{q-c}{q(q-1)}$ in both directions, i.e. $x_{ij} = x_{ji} = \frac{q-c}{q(q-1)}$, $\forall i,j \in Q$.
- All time variables are set to the upper end of their time window, i.e. $t_i = b_i$, $\forall i \in Q$.

By construction it is clear that flow constraints (4.3) are satisfied at the depot and at the rest of the nodes. To see that constraints (4.9) are always satisfied, let S be a subset of $Q \setminus \{v_d\}$ with $|S| = r$. Since each of the arcs with both endnodes in S has value $\frac{q-c}{q(q-1)}$, the left hand side of the associated constraint (4.9) is $r(r-1)\frac{q-c}{q(q-1)}$. Thus the constraint is satisfied if and only if $r(r-1)\frac{q-c}{q(q-1)} \le r-1$, which always holds since $r/q \le 1$ and $\frac{q-c}{q-1} \le 1$.

Finally, Constraints (4.5) are satisfied if:

$$b_j \ge b_i + s_i - M\left(1 - \frac{q-c}{q(q-1)}\right). \tag{4.10}$$

We have checked that with the data used in the computational experiments (see Chapter 5) $1-(q-c)/[q(q-1)] \ge 0.95$. Thus, if we use the tightest value for M, which is $M_{ij} = b_i + s_i - a_j$, the above condition is satisfied when $b_j \ge 0.95a_j + 0.05(b_i + s_i)$, which is intuitively true in most cases. In particular, this condition is always fulfilled by all the realistic data we have used in the computational section.

Therefore the LP value of the different models we will consider will always be zero, independently of whether or not Constraints (4.9) are used. For this reason in the following Constraints (4.9) are omitted.

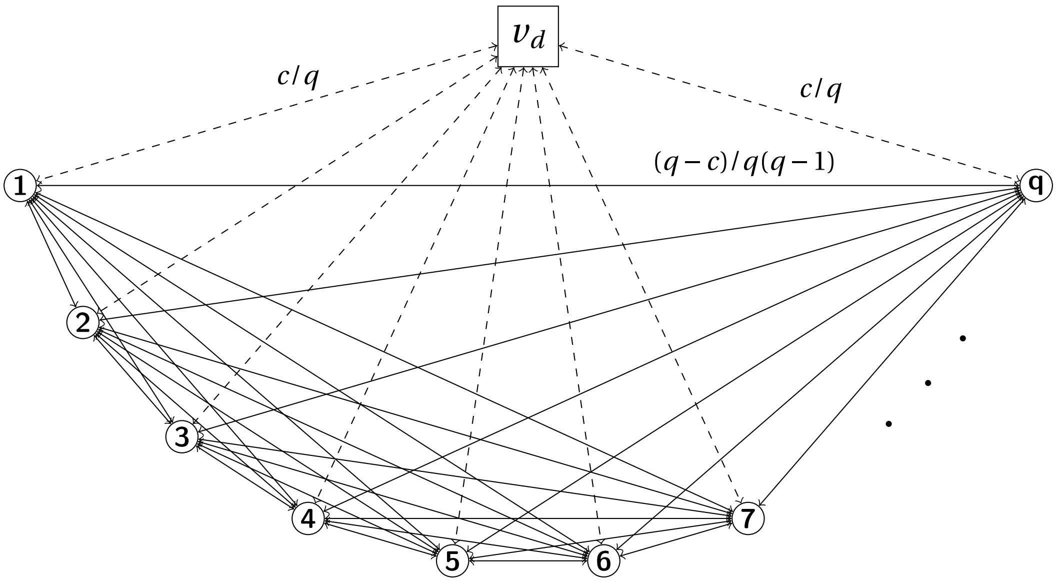

Figure 4.1: Feasible solution for LP relaxation of Constraints (4.2)–(4.8)

4.1.1 Alternative models for the PAP

Below we present formulations for the different criteria for the PAP proposed in Chapter 3. The different optimization criteria considered may require additional modeling changes. For instance, the value of the big constant M, which often has to be re-computed, since the proposed value above is no longer valid when Constraints (4.6) are relaxed. Instead, we can take $M = T + \max_i s_i$, as an upper bound on the maximum allowed time for loading/unloading operations.

The particular formulation of the objective function of each model is the following:

MOD0: Feasibility PAP Here we just look for a feasible solution to Formulation (4.2)–(4.8), ignoring any other aspect. Any constant objective function $z^0(x,t) = \kappa$ is appropriate, so throughout we use $\kappa = 0$.

In the following models, Constraints (4.6) are omitted whereas all other constraints, (4.2)–(4.5) and (4.7)–(4.8), are maintained.

MOD1: Earliness/tardiness minimization In this model we minimize an objective in which time windows violations are penalized in proportion to their earliness or tardiness. To this end we introduce one new set of decision variables e_i, $i \in Q$, that represent the earliness/tardiness in the assignment of each request. For $i \in Q$, e_i is defined as the time deviation $a_i - t_i$ between the lower limit a_i and the actual scheduling time of request $i \in Q$, if it is scheduled before a_i, or as the deviation $t_i - b_i$ between the actual scheduling time of request $i \in Q$ and its upper limit b_i if it is scheduled after b_i. That is, e_i can be determined as the maximum of the three values: 0, $a_i - t_i$ and $t_i - b_i$. This expression is not linear, although it can be easily linearized by inclusion of the additional sets of Constraints (4.11)–(4.13).

$$e_i \geq a_i - t_i \qquad i \in Q \tag{4.11}$$
$$e_i \geq t_i - b_i \qquad i \in Q \tag{4.12}$$
$$e_i \geq 0 \qquad i \in Q. \tag{4.13}$$

The objective in MOD1 is the minimization of the overall non-accomplishment, which is a weighted sum of the earliness/tardiness (4.14). That is:

$$z^1(x,t) = \sum_{i \in Q} w_i e_i = \sum_{i \in Q} w_i \max\{0, a_i - t_i, t_i - b_i\} \tag{4.14}$$

where w_i is the weight associated to request $i \in Q$.

MOD2: Minimization of maximum earliness/tardiness The measure of the earliness/ tardiness is considered by the maximum value. This model focuses on the maximum non-accomplishment, measured as the maximum earliness or tardiness from the requested time windows, among all requests. This objective can be expressed as:

$$z^2(x,t) = \max_{i \in Q} \max\{0, a_i - t_i, t_i - b_i\}. \tag{4.15}$$

MOD2 is a bottleneck min-max optimization problem. We minimize the objective function $z^2(x,t)$, defined as the maximum deviation from its requested time window among all carriers. In its turn, for each carrier $i \in Q$, the value of its deviation is $\max\{0, a_i - t_i, t_i - b_i\}$. This inner *max* only guarantees that the deviation from its preferred time window is computed correctly. Note that when a request is computed inside its requested time window, both $a_i - t_i < 0$ and $t_i - b_i < 0$. In this case, however, the correct value of the deviation is 0.

As before, this objective function is not linear, but can be easily linearized by extending the set of variables and constraints of MOD1 with one additional variable and one additional set of constraints. Let m denote the maximum non-accomplishment. Thus, the objective in MOD2 is the minimization of m. That is:

$$z^2(x,t) = m. \tag{4.16}$$

Variable m must be related to the remaining variables in the formulation. This can be done, for instance, by means of constraints:

$$m \geq e_i \qquad i \in Q. \tag{4.17}$$

It is possible to simplify the above formulation by removing all the e_i variables, and using directly m in Constraints (4.11)–(4.13). With this we reduce the q continuous variables e_i, and the q Constraints (4.17). We call MOD2b to the resulting model, where the specific set of constraints is:

$$m \geq a_i - t_i \qquad i \in Q \tag{4.18}$$
$$m \geq t_i - b_i \qquad i \in Q \tag{4.19}$$
$$m \geq 0 \qquad . \tag{4.20}$$

MOD3: Earliness/tardiness minimization subject to maximum displacement This model has the same objective function as MOD1, the overall non-accomplishment, but limiting the earliness or tardiness in the assignment of any request from its requested time window to a maximum value fixed in advance, d. That is, in MOD3 the objective is:

$$z^3(x,t) = \sum_{i\in Q} w_i e_i = \sum_{i\in Q} w_i max(0, a_i - t_i, t_i - b_i). \tag{4.21}$$

Now, in addition to Constraints (4.11)–(4.13), which establish the values of variables e_i, $i \in Q$, we include one new constraint for each request, limiting its maximum possible earliness or tardiness:

$$e_i \leq d \qquad i \in Q. \tag{4.22}$$

Note that in this formulation, the value of M can be set to $M = \max_i b_i + 2d + \max_i s_i - \min_j a_j$, because of the new constraint.

MOD4: Minimization of number of requests scheduled outside the time window The number of requests scheduled outside the requested time window is the objective in MOD4. The number of affected requests is important in this case, rather than the magnitude of the non-accomplishment. To compute this value, associated with each request $i \in Q$ we define a binary decision variable β_i indicating whether or not request i is scheduled outside its time window. Now the objective that we consider is:

$$z^4(x,t) = \sum_{i\in Q} \beta_i. \tag{4.23}$$

In order to activate the new indicator variables we include the set of constraints:

$$K\beta_i \geq e_i \qquad i \in Q \tag{4.24}$$

where K is a parameter that must be bigger than e_i for all $i \in Q$. For instance, we can set $K = \max_{i\in Q} \max\{a_i, T - b_i\}$.

Note that, similarly to MOD2 the e_i variables can be removed from Constraints (4.11)–(4.13) and use the β_i variables instead. By doing so q continuous variables e_i and the q Constraints (4.24) are eliminated. We will call the resulting model MOD4b. The constraints that activate the β_i variables are:

$$K\beta_i \geq a_i - t_i \qquad i \in Q \tag{4.25}$$
$$K\beta_i \geq t_i - b_i \qquad i \in Q. \tag{4.26}$$

An extension of MOD4 and MOD4b arises when each non-accomplished request is weighted by the duration of its associated operation. Then, the objective is:

$$z^{4'}(x,t) = \sum_{i\in Q} s_i \beta_i. \tag{4.27}$$

MOD5: Cost minimization In this model we impose again time window Constraints (4.6), and we consider costs to outsource additional parking space at the time periods when the city council parking space is insufficient to satisfy the carriers demand. The idea of MOD5 is to minimize outsourcing costs:

$$\sum_{t=0}^{T} \mu_t n_t. \tag{4.28}$$

In (4.28) n_t represents the excess (relative to c) in the number of requests that are scheduled at time slot t and μ_t is the unit cost for each outsourced parking place at time slot $t \in [0, T]$. Therefore, (4.28) represents the cost for making available additional parking places during some hours of the day so as to eliminate non-accomplishment. Below we present an extension of formulation (4.2)–(4.8) suitable for MOD5.

Consider a new set of decision variables y that will be used to represent the *routes* associated with outsourced requests. That is $y_{ij} = 1$ if request $j \in Q$ is scheduled immediately after request $i \in Q$ in some outsourced parking place. Then the extended formulation is:

$$\begin{aligned}
\text{minimize} \quad & z^5(x, y, t) && && (4.29)\\
\text{subject to} \quad & \sum_{j \in Q} x_{v_d j} \le c && && (4.2)\\
& \sum_{(i,j)\in A} x_{ij} - \sum_{(j,i)\in A} x_{ji} = 0 && i \in Q && (4.3)\\
& t_i + s_i - t_j \le (1 - x_{ij})M && (i,j) \in A \text{ with } i, j \in Q && (4.5)\\
& a_i \le t_i \le b_i && i \in Q && (4.6)\\
& 0 \le t_i \le T && i \in Q && (4.7)\\
& \sum_{(i,j)\in A} y_{ij} - \sum_{(j,i)\in A} y_{ji} = 0 && i \in Q && (4.30)\\
& \sum_{(i,j)\in A} (x_{ij} + y_{ij}) = 1 && i \in Q && (4.31)\\
& t_i + s_i - t_j \le (1 - y_{ij})M && (i,j) \in A \text{ with } i, j \in Q && (4.32)\\
& x_{ij}, y_{ij} \in \{0, 1\} && (i,j) \in A. && (4.33)
\end{aligned}$$

Constraints (4.2)–(4.7) have been explained before. The new set of flow constraints (4.30) which are associated with outsourced requests. Constraints (4.31) guarantee that all requests are scheduled, either to a reserved parking place or to an outsourced place. The assigned time slots of the outsourced requests are now regulated by means of the set of Constraints (4.32).

Note that with the new set of decision variables, for $i \in Q$ the sum $\sum_{(i,j)\in A} y_{ij}$, takes the value 0 or 1, and indicates whether or not the request of customer $i \in Q$ is outsourced. If we assume that $\mu_t = 1$ for all $t \in \{0, \ldots, T-1\}$, i.e. the unit cost for outsourced parking space is the same for all time periods, then

$$z^5(x, y, t) = \sum_{t=0}^{T} \mu_t n_t = \sum_{t=0}^{T} n_t = \sum_{i \in Q} \sum_{(i,j)\in A} y_{ij} \tag{4.34}$$

which coincides with the objective of MOD4. That is, objective (4.34) plus Constraints (4.2)–(4.3), (4.5)–(4.7), (4.30)–(4.33) is also an alternative formulation for MOD4.

The above formulation, however, as it stands, does not allow to compute the objective function value $z^5(x, y, t) = \sum_{t=0}^{T} \mu_t n_t$ for the general case when the outsourcing costs μ_t may vary among time periods. In this case we need to know the exact number of requests that are occupying an outsourced parking place at a given time period t. For this we can define an additional set of binary variables O_{it}, $i \in Q$, $t \in \{0, \ldots, T-1\}$ to identify the requests that are occupying an outsourced place at a given time period. These new variables are related to the original y variables by means of the constraints:

$$O_{jt'} \geq O_{it} - (1 - y_{ij}) \quad i, j \in Q, t, t' \in \{0, \ldots, T-1\} \text{ with } t' = t + s_i \tag{4.35}$$

$$O_{it} \leq O_{it'} \quad t \in \{0, \ldots, T-1\}, t+1 \leq t' \leq t + s_i. \tag{4.36}$$

While Constraints (4.35) activate the outsourcing indicator variables at the time period when a request is outsourced, Constraints (4.36) guarantee that the indicator variable of an outsourced request is activated during all the time interval in which the operation associated with this request takes place. Now we can express the objective function as:

$$z^{5'}(x, y, t) = \sum_{t=0}^{T} \mu_t \sum_{i \in Q} O_{it}. \tag{4.37}$$

This formulation has qT new binary variables, and $q(q-1)T + qT\sum_i s_i$ constraints, resulting in many more variables and constraints than the previous ones. Also, the objective function weights should be defined.

In the Appendix, several tables summarize the details of the presented models and objectives. Table A.1 presents the parameters and the sets used and Table A.2 summarizes the objective functions. Table A.3 and A.4, respectively, present all the variables and constraints, grouped by the formulation they belong to. Finally, Table A.5 counts the number of variables and constraints of each formulation.

4.1.2 Relationship among models

The models proposed in Section 4.1.1 focus on solving the same problem under different criteria. Therefore, they present some relationships that will be analyzed in this section through the comparison of the domains of their respective formulations. We use Ω^i to denote the feasible domain of MODi. In particular,

$\Omega^0 = \{(x, t)$ satisfying (4.2)–(4.8)$\}$

$\Omega^1 = \{(x, t, e)$ satisfying (4.2)–(4.5), (4.7)–(4.8), (4.11)–(4.13)$\}$

$\Omega^2 = \{(x, t, e, m)$ satisfying (4.2)–(4.5), (4.7)–(4.8), (4.11)–(4.13), (4.17)$\}$

$\Omega^3 = \{(x, t, e)$ satisfying (4.2)–(4.5), (4.7)–(4.8), (4.11)–(4.13), (4.22)$\}$

$\Omega^4 = \{(x, t, e, \beta)$ satisfying (4.2)–(4.5), (4.7)–(4.8), (4.11)–(4.13), (4.25)–(4.26)$\}$

$\Omega^5 = \{(x, t, y)$ satisfying (4.2)–(4.3), (4.5)–(4.7), (4.30)–(4.33)$\}$

Observe that, in general, the above domains are defined in different spaces so they cannot be compared. The exception being Ω^1 and Ω^3, for which we have $\Omega^3 \subset \Omega^1$. In order to compare the above domains we consider their respective projections onto the Ω^1 space being:

$$\Omega^1_{(x,t)} = \Omega^1 \bigcap \{e_i = 0 \quad i \in Q\}$$

$$\Omega^2_{(x,t)} = \Omega^2 \bigcap \{m = 0,\ e_i = 0 \quad i \in Q\}$$

$$\Omega^3_{(x,t)} = \Omega^3 \bigcap \{e_i = 0 \quad i \in Q\}$$

$$\Omega^4_{(x,t)} = \Omega^4 \bigcap \{e_i = 0,\ \beta_i = 0 \quad i \in Q\}$$

$$\Omega^5_{(x,t)} = \Omega^5 \bigcap \{y_{ij} = 0 \quad (i,j) \in A \quad i \in Q\}$$

Note that when $e_i = 0$ for $i \in Q$, Constraints (4.11)–(4.13) reduce to (4.6) and $\Omega^0 = \Omega^1_{(x,t)}$. With a similar reasoning we can relate Ω^0 to the other domains defined above.

Proposition 2. *For the restricted domains the following relationships hold:*

(1) $\Omega^0 = \Omega^1_{(x,t)}$

(2) $\Omega^0 = \Omega^2_{(x,t)}$

(3) $\Omega^0 = \Omega^3_{(x,t)}$

(4) $\Omega^0 = \Omega^4_{(x,t)}$

(5) $\Omega^0 = \Omega^5_{(x,t)}$.

Moreover, given a solution $(x,t) \in \Omega^0$, then the extended solution $(x,t,0) \in \Omega^1$, and similar extended solutions can be built for the rest of the domains. Thus, we have:

Corollary 1. *If* $\Omega^0 \neq \emptyset$ *then,* $\Omega^i \neq \emptyset$ *for* $i = 1,\ldots,5$.

As a consequence of the previous proposition, we can obtain further relations. For instance, suppose $\Omega^1 \neq \emptyset$ and its optimal value is 0. Let $(x,t,e) \in \Omega^1$ be an optimal solution. Then $e = 0$, and (x,t) is a solution in Ω^0. With a similar reasoning, we have:

Corollary 2. *If* $\Omega^i \neq \emptyset$ *for some* $i = 1,\ldots,5$ *and its optimum value is* 0*, then*

(1) $\Omega^0 \neq \emptyset$*, and*

(2) $\Omega^j \neq \emptyset$ *and its objective value is* 0 *for* $j = 1,\ldots,5 \quad j \neq i$.

4.2 The Parking Slot Assignment Problem as an AP

As already mentioned in the introduction, when we discretize time, the PAP can be considered as a modified Assignment Problem (AP). The AP is one of the fundamental combinatorial optimization problems [55]. The most general form is as follows. Suppose n tasks are to be carried out and each must be assigned to a single person. We have a staff of n people available and each person can be assigned only one task. For each staff member and task, there is a cost to match them. The problem consists in finding the matching between staff members and tasks that provides the minimum total cost. Assignment formulations present good properties when no further constraints are present. It is well-known that the coefficients matrix of the AP is Totally Unimodular (TU) and, thus, can be solved in polynomial time. The more intuitive formulation of the different PAP models as variations of APs presents some advantages:

we are able to reduce the symmetry and we can avoid the use of big M parameters, however, at the expenses of increasing notably the size of the formulations.

If we divide the whole time interval in a given number of time slots, then for each pair (parking place, time slot), we have a place and time slot (staff) that can be assigned to a given request (task). This allows to see the PAP as an AP in which the number of requests does not coincide with that of time slots, so some time slots will remain empty.

Let $t \in [0,1,\dots,T]$ be the discretized set of possible time moments where parking requests can be assigned to start. In order to formulate the PAP as a variation of an AP, for each $i \in N$, $t \in [0,\dots,T]$, we define a decision variable h_{it}, which is equal to 1 if request $i \in Q$ is assigned to start at time t or 0 otherwise. Given these elements, the following constraints are necessary:

$$\sum_{t\in[0,\dots,T]} h_{it} = 1 \qquad i \in Q \tag{4.38}$$

$$\sum_{i\in Q} \sum_{t'\in\{\hat{t}-(s_i-1),\hat{t}-(s_i-2),\dots,\hat{t}|t'\geq 0\}} h_{it'} \leq c \qquad \hat{t} \in [0,\dots,T] \tag{4.39}$$

(4.38) impose that for every request one assignment is active at a given time t from the whole time interval. (4.39) guarantee that at given time interval $\hat{t}$, at most c requests are being served simultaneously. Note that in the sum $\sum_{t'\in\{\hat{t}-(s_i-1),\hat{t}-(s_i-2),\dots,\hat{t}|t'\geq 0\}} h_{it'}$, for each time interval $\hat{t}$ and request i, we add up all the h_{it} with $t \in \{\hat{t}-(s_i-1), \hat{t}-(s_i-2),\dots,\hat{t}|t' \geq 0\}$, meaning that we consider if request i would still be being served at time interval t, if it has been assigned to start s_i units of time before t. We also need to limit the sum to values of t' greater than zero, in order to make the sum of (4.39) valid for any value of $\hat{t} \in [0,\dots,T]$. This would be the basic constraint structure that needs to be adapted to each optimization criteria.

4.2.1 Alternative models

Similar to the previous formulation, in this section we adapt the objective function of each model to the new AP formulation.

MOD0: Feasibility PAP Again, here we just look for a feasible solution to formulation (4.38)–(4.39). A constant objective function $z^0(h) = 0$ is appropriate.

For the rest of the models, we can define a parameter $\hat{e}_{it}$ which corresponds to the penalty value if request i is assigned to start at time t. Note that in this section $\hat{e}_{it}$ is a parameter, not a variable, so for any given i and t we can pre-compute the value of the parameter.

MOD1: Earliness/tardiness minimization The objective of this model considered the overall non-accomplishment, expressed as a weighted sum of earliness/tardiness penalty. In this case, the parameter $\hat{e}_{it}$ will take the value of the earliness/tardiness penalty of assigning request i at time t. Then, the objective function is as follows:

$$z^1(h) = \sum_{i\in Q} \sum_{t'\in[0,\dots,T]} \widehat{e}_{it} h_{it'} \tag{4.40}$$

MOD2: Minimization of maximum earliness/tardiness In this model the measure of the earliness/tardiness is given by the maximum value. Then, the parameter $\hat{e}_{it}$ will take the same value of the previous model, but the objective function will change slightly. Instead of minimizing the weighted sum of the earliness/tardiness penalty, the minimization will be over the maximum penalty.

$$z^2(h) = \max_{i \in Q} \max_{t' \in [0,\dots,T]} \widehat{e}_{it} h_{it'} \tag{4.41}$$

Also, we could use an extra variable m directly in the objective function representing the maximum earliness/tardiness value, which would be determined by the following equations.

$$\widehat{e}_{it} h_{it'} \leq m \qquad i \in Q,\ t' \in [0,\dots,T] \tag{4.42}$$

MOD3: Earliness/tardiness minimization subject to maximum displacement For MOD3, we can use the same value of parameters $\hat{e}_{it}$ as in MOD1, for the earliness/tardiness criterion. But we need to impose that if the parameter is greater than the given maximum displacement for some time moments ($\hat{e}_{it} \geq d$), then, the assignment can only be done at time points when the parameter is less than d. That is achieved adapting (4.38) for (4.43)

$$\sum_{t \in [0,\dots,T] | \widehat{e}_{it} \leq d} h_{it} = 1 \qquad i \in Q \tag{4.43}$$

MOD4: Minimization of number of requests scheduled outside the time window In case of MOD4 the value of the penalty parameter $\hat{e}_{it}$ will be computed differently. In this case, the parameter will be binary, 1 if the assignment is made outside the time window or 0 otherwise. But, the objective function will be the same as MOD1.

$$z^4(h) = \sum_{i \in Q} \sum_{t' \in [0,\dots,T]} \widehat{e}_{it} h_{it'} \tag{4.44}$$

4.2.2 The coefficient matrix of the assignment formulation

In order to check the properties of the proposed formulation, in this section we will show the coefficient matrix of the formulation. In the following matrix, we have the coefficients of the constraints. Each column is associated to a given variable h_{it}, each row is associated with a given constraint. The first block of rows correspond to Constraints (4.38), that guarantee that each request is assigned to start at a given time interval. The second block of rows correspond to Constraints (4.39), which limits the number of requests that can be using a parking spot at a given time interval to c. At each given time interval, we make sure that there are at most c requests being served.

$$
\begin{array}{cccccc|cccccc|cccccc|c|cccccc}
h_{10} & h_{11} & h_{12} & h_{13} & \cdots & h_{1T} & h_{20} & h_{21} & h_{22} & h_{23} & \cdots & h_{2T} & h_{10} & h_{31} & h_{32} & h_{33} & \cdots & h_{3T} & & h_{Q0} & h_{Q1} & h_{Q2} & h_{Q3} & \cdots & h_{QT} \\
1 & 1 & 1 & 1 & \cdots & 1 & & & & & & & & & & & & & & & & & & & \\
& & & & & & 1 & 1 & 1 & 1 & \cdots & 1 & & & & & & & & & & & & & \\
& & & & & & & & & & & & 1 & 1 & 1 & 1 & \cdots & 1 & & & & & & & \\
& & & & & & & & & & & & & & & & & & \cdots & & & & & & \\
& & & & & & & & & & & & & & & & & & & 1 & 1 & 1 & 1 & \cdots & 1 \\
\hline
1 & & & & & & 1 & & & & & & 1 & & & & & & & 1 & & & & & \\
1 & 1 & & & & & 1 & 1 & & & & & 1 & 1 & & & & & & 1 & 1 & & & & \\
1 & 1 & 1 & & & & 1 & 1 & 1 & & & & 1 & 1 & 1 & & & & & 1 & 1 & 1 & & & \\
1 & 1 & 1 & 1 & & & 1 & 1 & 1 & 1 & & & 1 & 1 & 1 & 1 & & & & 1 & 1 & 1 & 1 & & \\
& & & & & & & & & & & & & & & & & & \vdots & & & & & & \\
& 1 & 1 & \cdots s_1 \cdots & 1 & & & 1 & 1 & \cdots s_2 \cdots & 1 & & & 1 & 1 & \cdots s_3 \cdots & 1 & & & & 1 & 1 & \cdots s_Q \cdots & 1 & \\
& & & \ddots & \ddots & & & & & \ddots & \ddots & & & & & \ddots & \ddots & & & & & & \ddots & \ddots & \\
& & & 1 & \cdots s_1 \cdots & 1 & & & & 1 & \cdots s_2 \cdots & 1 & & & & 1 & \cdots s_3 \cdots & 1 & & & & & 1 & \cdots s_Q \cdots & 1
\end{array}
$$

Unfortunately, even all the coefficients have value 1, matrix is not TU. For instance, we select columns h_{10}, h_{1s_1+1} and h_{2s_1}, and to rows 1, s_1 s_1+1, where $s_1 < s_2$, and the associated sub matrix will be

$$\begin{array}{c} \\ 1 \\ s_1 \\ s_1+1 \end{array} \begin{array}{c} \begin{array}{ccc} h_{10} & h_{1s_1+1} & h_{2s_1} \end{array} \\ \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \end{bmatrix} \end{array} \tag{4.45}$$

which has determinant equal to -2. The presented formulation is compact and intuitive but we can not guarantee the integrality property when using it.

4.3 A heuristic for the Parking Slot Assignment Problem

As we will see in the following sections, CPLEX can be quite time consuming with the VRP formulations. Thus, heuristic methods can be of interest, as potentially they could produce good solutions in smaller computing times. Next we present a simple heuristic to obtain feasible solutions to the PAP. The heuristic consists of a greedy constructive phase, followed by a simple local search.

For the constructive phase, first requests are ordered by non-decreasing values of their earliest start time, a_i. Ties are broken by non-decreasing values of their latest start time, b_i. If ties remain, they are broken by non-decreasing values of the requests durations s_i. Possible remaining ties are broken arbitrarily. Hence we assume that

$$\begin{cases} (i) & a_i \leq a_{i+1}, \quad i \in Q; \\ (ii) & b_i \leq b_{i+1} \quad \text{if } a_i = a_{i+1}; \\ (iii) & s_i \leq s_{i+1} \quad \text{if } a_i = a_{i+1} \text{ and } b_i = b_{i+1}. \end{cases} \tag{4.46}$$

In the constructive phase parking places are considered in turn. When a parking place is selected, unassigned requests are explored by increasing order of their indices, and assigned to the current place provided that they preserve the feasibility of the current assignment to the parking place. Requests with time window conflict with the current assignment remain unassigned and will be considered for assignment to subsequent parking places.

When all parking places have been considered the constructive phase enters a final step. Now unassigned requests are considered in turn and assigned to some parking place outside their preferred time windows. Each such request is scheduled either at the very begining or the very end of some parking place schedule, depending on the alternative which incurs the smallest earliness or tardiness. For a given request, the parking place is selected so as to minimize the resulting associated penalty. In this final step unassigned requests are considered by increasing values or their time duration, s_i. At the end of the algorithm all the requests are assigned to some parking place. However, the assignment need not be feasible, as some requests may be scheduled outside their time windows. A pseudocode of the algorithm is presented in Algorithm 1. $A(p)$ contains the indices of all the requests assigned to parking place p. $Init_p$ and End_p respectively denote the starting times of the first and last requests assigned to parking place p.

In the local search we try to interchange the schedule of two requests. Again the index order of the requests is the one indicated by (4.46). We consider two different pairs of requests $i, j \in Q$, $i \neq j$: (i) i and j are adjacent in Q, independently of the scheduled parking place p, (ii) i and

j are not adjacent but in Q there is at most one request between i and j, i.e., $|i-j| \leq 2$, again independent of the scheduled parking place. All possible interchanges are considered until no more improvements are possible.

```
Constructive Phase
Input: c, Q, a, b, s,
Initialize list UNASSIGNED with elements i ∈ Q in non-decreasing order of a_i
with ties broken as explained in (4.46);
Initialize list of requests assigned to each parking place: A(p) ← ∅,
p = 1...c
for (p ← 1...c) do
    i ← FIRST(UNASSIGNED);
    Init_p ← a_i
    t ← Init_p
    while (i ≠ nil) do
        A(p) ← A(p) ∪ {i}        (Assign request i to parking place p)
        t ← max(t + s_i, a_i)
        UNASSIGNED ← UNASSIGNED \ {i}
        i ← NEXT(UNASSIGNED)
        while (b_i < t and i ≠ nil) do
            i ← NEXT(UNASSIGNED)
        end
    end
    End_p ← t
end
Sort elements of UNASSIGNED by non-decreasing values of s_i
while (UNASSIGNED ≠ ∅) do
    i ← FIRST(UNASSIGNED);
    for (p ← 1...c) do
        δ_p ← Min{|a_i − Init_p|, |b_i − End_p|}
    end
    p̄ ← argmin_{p=1...c}{δ_p}
    A(p̄) ← A(p̄) ∪ {i}
    Update Init_p̄, or End_p̄ as appropriate
end
```

Algorithm 1: Constructive Phase.

5 Computational Experiments

In this chapter we present the results we have obtained in a series of computational experiments we have run to analyze and compare the different models and formulations proposed and studied in the previous chapters. First, data generation is described in Section 5.1 and then results are presented in the six subsequent sections. The results from each formulation are presented separately, in Section 5.2 for the VRP formulation and in Section 5.3 for the AP formulation. Since the primary objective is to solve the feasibility problem, in both sections we show the results obtained after one hour of execution of MOD0, plus the evaluation of the unfeasibility condition of Section 3.1. Furthermore, we analyze the effectiveness of the models MOD1-MOD4 within the same maximum computing time. In Section 5.4 the results of the heuristic are presented. In Section 5.5 we compare the results of both formulations and the heuristic. Then, in Section 5.6 we compare the solutions given by each of the models MOD1-MOD4 with the best formulation. To this end, we cross-evaluate the solutions. That is, for each instance and model, the best solutions obtained by the other models in the same instance are evaluated. Finally, in Section 5.7 a sensitivity analysis of the maximum displacement is carried out with models MOD1 and MOD3, which share the overall non-accomplishment criteria.

Models have been implemented in the Optimization Programming Language `OPL` [63] and solved with the commercial software `CPLEX 12.1` [13] . All experiments have been run on a PC limited to 1 thread running at 2.6GHz and 16GB of RAM. Computing time has been limited to one hour, so the aim is to compare both formulations under the same computational effort with `CPLEX`. We consider that the computing effort is moderate, and it would also be realistic in practice, when one solution should be obtained by the city council each working day.

5.1 Data generation

Since we are not aware of any benchmark instances that could be used in our experiments, we generated a set of 60 test instances which follow the patterns observed in an experimental study in the city of Barcelona (Spain) [67]. All instances have a similar structure with parameters randomly generated. Each instance represents an area during a whole day [0,T]. The number of parking places for each instance is uniformly drawn from [2,8], and we assume requests are only made within a subinterval $[\widehat{a},\widehat{b}] \subseteq [0,T]$ corresponding to morning hours from 8h to 14h or afternoon hours from 16h to 20h. We use minutes as time unit. This provides enough precision and it is operative in practice. Thus, $[0,T] = [0,1440]$ and $[\widehat{a},\widehat{b}] = [480,840]$ for the morning or $[\widehat{a},\widehat{b}] = [960,1200]$ if we also consider the afternoon period.

Then, the total number of requests of each instance is computed. Following the patterns observed in reality [67], requests are distributed according to a triangular pattern around a peak hour that is located either in the center or at the beginning of the morning or afternoon interval, depending on the goods type. At peak hour, demand is usually higher than the number of available places. Given these distributions and usage levels, a unique parameter defining the overall demand density determines the total number of requests. The overall demand density γ is the number of requests per hour-place, and for each instance it is uniformly drawn from [1.5, 3.5]. For each instance, given the interval $[\widehat{a}, \widehat{b}]$, γ, and the number of places c, the total number of requests is computed as $|Q| = \gamma(\widehat{b} - \widehat{a})c$.

Finally, the data for the requests of each instance are generated: duration of the operation, and requested time window for the beginning of the service. Following [67], the duration of an operation is drawn from a normal distribution N(18,5). We assume that the duration of the operation includes all the time needed for the operator to perform the delivery, including the time to cover the distance from the parking place to the customer, or the specific activities related to the shipments that might involve more or less time. The process for generating the time windows is more complex. As mentioned earlier, [67] shows that requests are distributed according to a triangular pattern around peak hours, where the peak hour is located either at the center or at the beginning of the morning or afternoon intervals. Thus, instances have been divided in three types, depending on the distribution of requests along time intervals: *(a) triangular centered*, *(b) triangular asymmetric*, and *(c) double peak*. In the first group, only the morning interval is considered, and the peak hour is located at the middle of the interval (11h). The second group also considers the morning interval but the peak hour is skewed earlier in the morning (9h30). Finally, double peak considers morning and afternoon subintervals and peak hours are centered in the middle of the respective intervals (11h and 18h). Once the type of demand has been set for an instance, for each of its requests we set the midpoint for its time window as well as its width. The midpoint is drawn according to the distribution of its type of demand. Finally, the width of the time windows is set to 20, 40, 60, 80 minutes, with probability 0.2, 0.2, 0.5, 0.1 respectively. Time windows are built symmetrically, centered at the middle of the interval, depending on the time window width. Table 5.1 summarizes the characteristics of the instances. In the Appendix D one instance of each type (triangular centered, triangular asymmetric and double peak) is graphically represented.

Note that the overall demand density γ and the duration of the required requests entail different levels of saturation around peak hour. Parameter γ is related to the unfeasibility of the instance, since the higher the density of demand, the more difficult it becomes to give service to all requests. In all the experiments the weights coefficients in the objective functions of MOD1 and MOD3 have been set at value one.

5.2 Numerical results with the VRP formulation

Before presenting the results of the VRP formulation in the different proposed models, we discuss the different values of M tested for each of the models. For MOD0, four different values of parameter M in Constraints (4.5) were preliminarily considered: *(a)* the unique value for each instance proposed in Section 4.1.1, $M = \max_i b_i + \max_i s_i - \min_j a_j$. Adapted values for each subset of constraints: *(b)* $M_i = b_i + s_i - \widehat{a}$ and *(c)* $M_{ij} = b_i + s_i - a_j$. A more general value for all instances based on generation instance parameters: *(d)* $\overline{M} = \widehat{b} + \max_i s_i - \widehat{a}$. No significant differences were observed neither in the results nor in the computational times, so the more general value $\overline{M}$ was used in all the experiments presented in this section.

Four different values of M were also tested for MOD3. A more general value for all instances: *(a)* $\overline{M} = \widehat{b} - \widehat{a} + 2d + \max_i s_i$; two constraint related values: *(b)* $M_i = b_i + 2d + s_i - \widehat{a}$ and *(c)*

General data				
#	Places (c)	Requests interval $[\hat{a}, \hat{b}]$	Demand density (γ)	Distribution Pattern
20	$U[2,8]$	$[480, 840]$	$U[1.5, 3.5]$	Centered (11h)
20		$[480, 840]$		Skewed (9h30)
20		$[480, 840] \bigcup [960, 1200]$		Centered (11h,18h)
Requests data				
	Duration of operation (s_i)		TW width	
	$N[18,5]$		20 min with probability 0.2 40 min with probability 0.2 60 min with probability 0.5 80 min with probability 0.1	

Table 5.1: Instances characteristics

$M_{ij} = b_i + 2d + s_i - a_j$; and the unique value proposed for each instance in Section 4.1.1: *(d)* $M = \max_i b_i + 2d + \max_i s_i - \min_j \underline{a_j}$. Given that only small differences were obtained with the different values, the general value $\overline{M}$ was chosen.

In the case of MOD1, MOD2 and MOD4 no tighter values for M are possible and the value used was $M = T + \max_i s_i$. As for the parameter K of MOD4, we experimented with value $K_i = \max(a_i, T - b_i)$ but not significant differences were observed and the general value ($K = \max_i \max\{a_i, T - b_i\}$) was used.

As we will see, the outcome of MOD0 is closely related to the outcome of MOD1-MOD4, so we start this section by analyzing the effect of some of the instances parameters in the results of MOD0, and the effectiveness of the unfeasibility check of Proposition 1. Figure 5.1 relates the status of MOD0 at termination to the value of the demand density parameter γ and the type of requests distribution (triangular centered, triangular asymmetric and double peak). For this, benchmark instances are partitioned in three sets: *(a) Feasible* (top); *(b) Time Limit* (middle), when time limit was reached without knowing whether or not the instance is feasible; and *(c) Unfeasible* (bottom), detected either by `CPLEX` or by the sufficient condition of Proposition 1. Several intervals $[\alpha, \beta]$ were used in the unfeasibility condition check. The center of all intervals coincides with the peak hour, whereas the interval widths range from a minimum of 40 minutes to a maximum of 180 minutes, with checks every 20 minutes. The computational burden of these tests is negligible as it never never exceeds 0.01 seconds. Slight differences can be observed in Figure 5.1 among the three types of benchmark instances (triangular centered, triangular asymmetric and double peak). In the first two groups, there are more instances solved to optimality than in the the double peak ones, where more instances reached the maximum time limit.

As it was expected, Figure 5.1 shows that instances become more difficult as demand density increases. At the extreme values of the parameter γ instances are either optimally solved when $\gamma \in [1.5, 1.75]$, or unfeasibility is proven when $\gamma \in [3.07, 3.5]$. Moreover, none of the instances with $\gamma \in [2.38, 3.07]$ was found to be feasible. Observe the effectiveness of the unfeasibility check, which was able to detect the unfeasibility of nine instances for which `CPLEX 12.1` terminated within the time limit without detecting neither feasibility or unfeasibility.

Next we analyze the numerical results obtained with `CPLEX` for models MOD0-MOD4 with the

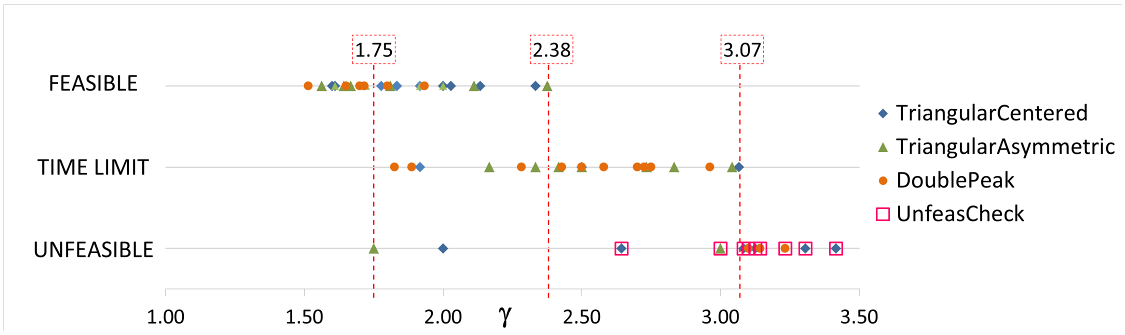

Figure 5.1: Results from MOD0 with the VRP formulation

VRP formulation. The results obtained for each model and benchmark instance are presented in Table 5.2. The meaning of the first five columns is the following. In the first column *(Id)*, each instance is identified with a numerical label. Column *Rq* gives the number of requests, *c* the number of parking places, *Hop* the length of the time horizon, i.e. the number of operating hours, and γ the density.

Column *Type* under MOD0, indicates the status of MOD0 at termination: *F* when a provable optimal solution was found, *U* when unfeasibility was proven by CPLEX, *UC* when unfeasibility was proven by the check of Proposition 1, and *TL* when the time limit was reached but the instance could not be classified in any of the former groups. For each model MODj, $j = 0, \ldots, 4$, column *SOL* gives the value of the optimal solution, when the value is bold, or the value of the best solution found when the time limit was reached, otherwise. If no solution was found within the allowed computing time the entry in column *SOL* is empty. Negative entries in column *SOL* of MOD0 indicate that the unfeasibility of the instance was proven. This entry is -1 when unfeasibility was detected by CPLEX within the CPU time limit (instances 15 and 30), or -2 when unfeasibility was detected with the test of Proposition 1. For each model, column *TIME* gives the CPU time to termination in seconds or 3,600 when time limit was reached.

Table 5.2 does not include information on LBs at termination because, for all models MOD0-MOD4, these bounds were always zero in all the cases that optimality could not be proven.

As can be seen, MOD0 found a provable optimal solution for 28 instances and proved that the instance was not feasible in two cases. The average CPU time for the instances that terminated with a certificate of optimality or unfeasibility is 148.1 seconds. However, for the remaining 30 instances it terminated without knowing whether or not the instance was feasible, even if the feasibility check indicates that 9 such instances are unfeasible. The average computing time over the complete set of benchmark instances rises up to 1,874 seconds. According to Corollary 1 when MOD0 is feasible, the optimal value of MOD1-MOD4 will be zero since there is an assignment of parking places that satisfies the time window requests of all the carriers. Conversely, if MOD0 is not feasible the optimal values to MOD1-MOD4 will be strictly positive. In this respect, the results in Table 5.2 confirm that, computationally, the outcome MOD0 gives valuable information not only with respect to MOD0 itself but also with respect to MOD1-MOD4. Indeed, all the 28 instances that are feasible for MOD0 were optimally solved by MOD1-MOD3, and 23 such instances were also optimally solved by MOD4. Furthermore, no model MODj, $j = 1, \ldots, 4$ was able to optimally solve any of the unfeasible instances within

					MOD0			MOD1		MOD2		MOD3		MOD4	
Id	Rq	c	Hop	γ	Type	SOL	TIME	SOL	TIME	SOL	TIME	SOL	TIME	SOL	TIME
1	111	7	6	2.64	UC	-2	3,600	1,352	3,600	48	3,600		3,600	30	3,600
2	75	4	6	3.13	UC	-2	3,600	1,391	3,600	54	3,600	1,359	3,600	24	3,600
3	60	4	6	2.50	TL		3,600	200	3,600	12	3,600	142	3,600	9	3,600
4	58	6	6	1.61	F	**0**	1	**0**	61	**0**	29	**0**	32	**0**	32
5	119	6	6	3.31	UC	-2	3,600	2,602	3,600	73	3,600		3,600	55	3,600
6	82	4	6	3.42	UC	-2	3,600	1,951	3,600	71	3,600		3,600	26	3,600
7	92	5	6	3.07	TL		3,600	629	3,600	28	3,600	675	3,600	16	3,600
8	42	3	6	2.33	F	**0**	466	**0**	3,248	**0**	1,682	**0**	2,017	2	3,600
9	48	5	6	1.60	F	**0**	5	**0**	510	**0**	49	**0**	46	**0**	122
10	73	6	6	2.03	F	**0**	9	**0**	933	**0**	187	**0**	555	**0**	420
11	66	6	6	1.83	F	**0**	1	**0**	97	**0**	146	**0**	392	**0**	154
12	74	4	6	3.08	UC	-2	3,600	1,169	3,600	44	3,600	1,111	3,600	19	3,600
13	64	5	6	2.13	F	**0**	45	**0**	3,573	**0**	489	**0**	3,373	2	3,600
14	36	3	6	2.00	F	**0**	2	**0**	41	**0**	9	**0**	8	**0**	18
15	24	2	6	2.00	U	-1	394	13	3,600	3	3,600	13	3,600	1	3,600
16	87	6	6	2.42	TL		3,600	428	3,600	19	3,600	409	3,600	13	3,600
17	104	8	6	2.17	TL		3,600	253	3,600	15	3,600	245	3,600	14	3,600
18	146	8	6	3.04	TL		3,600	1,885	3,600	76	3,600		3,600	49	3,600
19	76	7	6	1.81	F	**0**	156	**0**	3,261	**0**	720	**0**	422	**0**	1,703
20	30	3	6	1.67	F	**0**	0	**0**	6	**0**	9	**0**	2	**0**	3
21	69	7	6	1.64	F	**0**	7	**0**	65	**0**	124	**0**	94	**0**	62
22	76	6	6	2.11	F	**0**	26	**0**	3,304	**0**	543	**0**	3,258	**0**	1,437
23	136	8	6	2.83	TL		3,600	1,446	3,600	71	3,600		3,600	34	3,600
24	54	3	6	3.00	UC	-2	3,600	495	3,600	35	3,600	531	3,600	11	3,600
25	57	4	6	2.38	F	**0**	11	**0**	3,369	**0**	2,651	**0**	3,291	1	3,600
26	82	5	6	2.73	TL		3,600	995	3,600	44	3,600	885	3,600	19	3,600
27	75	8	6	1.56	F	**0**	2	**0**	367	**0**	73	**0**	222	**0**	482
28	45	3	6	2.50	TL		3,600	9	3,600	3	3,600	13	3,600	2	3,600
29	98	7	6	2.33	TL		3,600	12	3,600	14	3,600	90	3,600	7	3,600
30	21	2	6	1.75	U	-1	1,022	7	3,600	2	3,600	7	3,600	1	3,600
31	72	4	10	1.80	F	**0**	2,074	**0**	3,313	**0**	190	**0**	1,402	2	3,600
32	109	4	10	2.73	TL		3,600	1,166	3,600	44	3,600	829	3,600	36	3,600
33	50	2	10	2.50	TL		3,600	307	3,600	33	3,600	306	3,600	10	3,600
34	106	7	10	1.51	F	**0**	3	**0**	971	**0**	714	**0**	223	**0**	805
35	129	5	10	2.58	TL		3,600	1,431	3,600	221	3,600	1,145	3,600	40	3,600
36	62	2	10	3.10	UC	-2	3,600	730	3,600	42	3,600	652	3,600	18	3,600
37	51	3	10	1.70	F	**0**	2	**0**	71	**0**	22	**0**	7	**0**	132
38	137	6	10	2.28	TL		3,600	529	3,600	205	3,600	352	3,600	20	3,600
39	97	3	10	3.23	UC	-2	3,600	1,764	3,600	69	3,600		3,600	38	3,600
40	135	5	10	2.70	TL		3,600	1,989	3,600	121	3,600	1,531	3,600	60	3,600
41	55	2	10	2.75	TL		3,600	361	3,600	41	3,600	357	3,600	11	3,600
42	170	7	10	2.43	TL		3,600	1,573	3,600	317	3,600	1,217	3,600	65	3,600
43	100	4	10	2.50	TL		3,600	274	3,600	22	3,600	279	3,600	18	3,600
44	220	7	10	3.14	UC	-2	3,600	21,537	3,600	438	3,600		3,600	168	3,600
45	237	8	10	2.96	TL		3,600	27,701	3,600	419	3,600		3,600	186	3,600
46	77	7	6	1.83	F	**0**	23	**0**	333	**0**	258	**0**	323	**0**	618
47	46	4	6	1.92	TL		3,600	**0**	839	**0**	98	**0**	459	**0**	2,010
48	22	2	6	1.83	F	**0**	0	**0**	2	**0**	2	**0**	1	**0**	2
49	23	2	6	1.92	F	**0**	1	**0**	21	**0**	3	**0**	3	**0**	5
50	32	3	6	1.78	F	**0**	2	**0**	86	**0**	6	**0**	8	**0**	68
51	31	3	6	1.72	F	**0**	0	**0**	16	**0**	3	**0**	4	**0**	4
52	29	3	6	1.61	F	**0**	1	**0**	16	**0**	3	**0**	4	**0**	4
53	23	2	6	1.92	F	**0**	1	**0**	34	**0**	4	**0**	11	**0**	45
54	60	5	6	2.00	F	**0**	7	**0**	142	**0**	68	**0**	208	**0**	53
55	41	4	6	1.71	F	**0**	3	**0**	69	**0**	23	**0**	17	**0**	23
56	116	6	10	1.93	F	**0**	115	**0**	3,385	**0**	3,281	**0**	623	1	3,600
57	103	6	10	1.72	F	**0**	58	**0**	1,329	**0**	3,264	**0**	3,287	**0**	3,322
58	66	4	10	1.65	F	**0**	4	**0**	173	**0**	71	**0**	17	**0**	270
59	73	4	10	1.83	TL		3,600	2	3,600	2	3,600	3	3,600	1	3,600
60	151	8	10	1.89	TL		3,600	286	3,600	194	3,600	273	3,600	17	3,600

Table 5.2: Results of MOD0-MOD4 in the set of instances with the VRP formulation

the maximum CPU time: neither the ones detected by CPLEX nor the ones detected by the unfeasibility check. On the other hand, by solving MOD1-MOD4, the feasibility/unfeasibility with respect to MOD0 was disclosed for only one instance with status *TL* (instance 47). This instance was optimally solved by all MOD1-MOD4 and is feasible with respect to MOD0 as its optimal value with respect to MOD1-MOD4 is zero.

MOD1, MOD2 and MOD4 always produced some solution, even if its optimality was not proven. In contrast, for seven instances MOD3 consumed the allowed computing time without finding a feasible solution. Recall that instances can be unfeasible for MOD3 because of the maximum displacement constraint (4.22). Four of the instances without a solution for MOD3 (1, 5, 6, 44) are known to be unfeasible for MOD0, whereas for instances 18, 24 and 39 its condition with respect to MOD0 is unknown.

The average CPU times required by MOD1-MOD4 are considerably larger that those of MOD0, even if we restrict to the benchmark instances that terminate with a certificate of optimality. It seems, however, that the min-max objective of MOD2 is somehow less demanding than the sum-type objectives of MOD1, MOD3 and MOD4.

The effectiveness of MOD1-MOD4 is summarized in Table 5.3, where benchmark instances have been partitioned relative to their status with respect to MOD0 in *Feasible*, *Unfeasible*, detected either by CPLEX or by the sufficient condition of Proposition 1, and *Time Limit*, when the time limit reached without knowing whether or not the instance is feasible. For each tested model we further partition each of the above groups of instances according to the possible outcomes of the tested model: optimum found (*OPT*), time limit reached with a feasible solution (*TL/S*), and time limit reached without a feasible solution (*TL/NS*). The entries in the table give the number of instances in each class and the average CPU times over the set of instances in the class. No clear conclusion can be drawn from Table 5.3 about the effectiveness of the models in terms of their capability for finding good quality solutions. For instances in *Feasible*, MOD4 performs slightly worse than the other models, since in 5 instances non-optimal solutions were found. But as already mentioned, in terms of their capability of proving feasibility, the other models work exactly as MOD0. For *Unfeasible* and *Time Limit* instances, the performance of MOD1, MOD2 and MOD4 is the same, and a little better than that of MOD3.

MOD0			Status	MOD1		MOD2		MOD3		MOD4	
Status	No.	CPU		No.	CPU	No.	CPU	No.	CPU	No.	CPU
Feasible	28	108	*OPT*	28	1,028	28	522	28	709	23	425
			TL/S	-		-		-		5	3,600
			TL/NS	-		-		-		-	
Unfeasible	11	3,512	*OPT*	-		-		-		-	
			TL/S	11	3,600	11	3,600	6	3,600	11	3,600
			TL/NS	-		-		5	3,600	-	
Time Limit	21	3,600	*OPT*	1	839	1	98	1	459	1	2010
			TL/S	20	3,600	20	3,600	17	3,600	20	3,600
			TL/NS	-		-		3	3,600	-	

Table 5.3: Results of MOD1-MOD4 in the set of instances with the VRP formulation

For the reasons explained before and justified in Section 4.1, the experiments described above were run without taking into account the SECs (4.9). Still, we made some tests to confirm their potential usefulness empirically. First, we checked that, indeed, all 60 benchmark instances satisfied the condition that ensures that the LP relaxation of MOD0 will be feasible, even with the addition of the SECs. (For a detailed statement of the condition see expression

(4.10)). Note that, in turn, this means that the LP values of MOD1-MOD4 are all zero. Then, we implemented a callback with a separation procedure for the SECs and run a second set of experiments. The rationale for these additional tests was that the addition of the SECs could reinforce the LP relaxation of some nodes of the enumeration tree, even if they had no effect on the root node. In our second set of experiments, constraints (4.9) were separated at all the nodes of the enumeration tree with depth up to 15. This strategy was hopeless: while we appreciated no difference in the number of instances whose unfeasibility was proven, the number of instances that terminated with a certificate of optimality decreased notably for all five models MOD0-MOD4. Again, in all cases when optimality could not be proven the LB at termination was zero.

5.3 Numerical results with the AP formulation

As we will see, the AP formulation provides more optimal solutions under the same computing time than the VRP based formulations. In particular, within one hour of computing time all instances of MOD0 are solved to optimality. Then, we can analyze the feasibility of the instances (with the outcome of MOD0) related to some of their features. Figure 5.2 relates for the different instances the status of MOD0 at termination to the value of the demand density parameter γ and the type of requests distribution (triangular centered, triangular asymmetric and double peak). For this, benchmark instances are now partitioned in only two sets: *(a) Feasible* (top); and *(c) Unfeasible* (bottom), detected by CPLEX with formulation AP. We also maintain a special symbol in the instances where the sufficient condition of Proposition 1 could prove their unfeasibility.

Slight differences can be observed in Figure 5.2 among the three types of benchmark instances (triangular centered, triangular asymmetric and double peak). In the first two groups, there are more instances solved to optimality than in the the double peak ones, where more instances are unfeasible.

As it was expected, Figure 5.2 shows that instances tend to be unfeasible as demand density increases. For the extreme values of the parameter γ, instances are either optimally solved when $\gamma \in [1.5, 1.75]$, or unfeasibility is proven when $\gamma \in [2.38, 3.5]$.

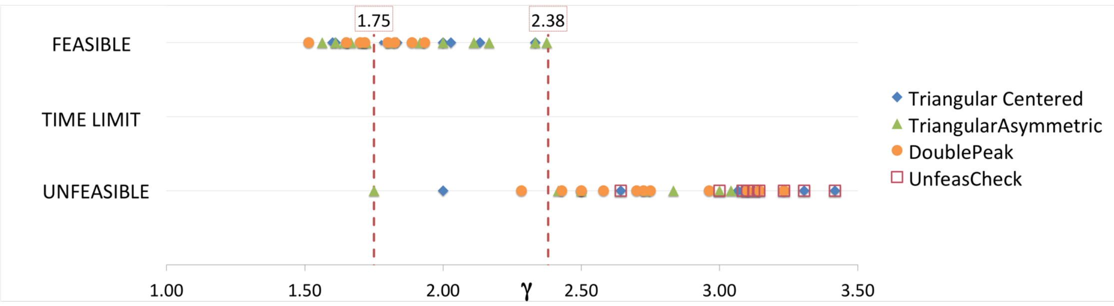

Figure 5.2: Results from MOD0 with the AP formulation

The numerical results obtained with CPLEX and the AP formulation are presented next. The results obtained for each model and benchmark instance are presented in Table 5.4. The meaning of the columns is as in Table 5.2.

					MOD0			MOD1		MOD2		MOD3		MOD4	
Id	Rq	c	Hop	γ	Type	SOL	TIME	SOL	TIME	SOL	TIME	SOL	TIME	SOL	TIME
1	111	7	6	2.64	UF	**-1**	5	**551**	44	52	3,600	**551**	48	**11**	69
2	75	4	6	3.13	UF	**-1**	3	950	3,600	48	3,600	**999**	80	**12**	40
3	60	4	6	2.50	UF	**-1**	2	**64**	15	**6**	867	**64**	12	**2**	31
4	58	6	6	1.61	F	**0**	3	**0**	6	**0**	5	**0**	8	**0**	12
5	119	6	6	3.31	UF	**-1**	5	**1,565**	52	568	3,600	**1,686**	110	**21**	82
6	82	4	6	3.42	UF	**-1**	4	**1,380**	92	523	3,600	**1,494**	716	**15**	49
7	92	5	6	3.07	UF	**-1**	6	**181**	43	27	3,600	**183**	33	**3**	49
8	42	3	6	2.33	F	**0**	4	**0**	5	**0**	15	**0**	8	**0**	13
9	48	5	6	1.60	F	**0**	3	**0**	5	**0**	4	**0**	7	**0**	12
10	73	6	6	2.03	F	**0**	5	**0**	8	**0**	6	**0**	11	**0**	15
11	66	6	6	1.83	F	**0**	4	**0**	6	**0**	5	**0**	10	**0**	17
12	74	4	6	3.08	UF	**-1**	5	**794**	50	**35**	1,888	**797**	21	**12**	65
13	64	5	6	2.13	F	**0**	5	**0**	10	**0**	23	**0**	11	**0**	15
14	36	3	6	2.00	F	**0**	2	**0**	5	**0**	3	**0**	5	**0**	7
15	24	2	6	2.00	UF	**-1**	2	**13**	5	**4**	101	**13**	5	**1**	8
16	87	6	6	2.42	UF	**-1**	5	**87**	43	**6**	1,858	**87**	63	**3**	54
17	104	8	6	2.17	F	**0**	7	**0**	15	**0**	43	**0**	19	**0**	32
18	146	8	6	3.04	UF	**-1**	9	**842**	61	828	3,600	**852**	46	**15**	114
19	76	7	6	1.81	F	**0**	4	**0**	9	**0**	9	**0**	12	**0**	15
20	30	3	6	1.67	F	**0**	2	**0**	3	**0**	3	**0**	5	**0**	6
21	69	7	6	1.64	F	**0**	5	**0**	6	**0**	8	**0**	10	**0**	9
22	76	6	6	2.11	F	**0**	6	**0**	8	**0**	8	**0**	12	**0**	11
23	136	8	6	2.83	UF	**-1**	7	**432**	46	139	3,600	**432**	49	**9**	49
24	54	3	6	3.00	UF	**-1**	4	**422**	36	**30**	1,273	**432**	120	**7**	10
25	57	4	6	2.38	F	**0**	4	**0**	8	**0**	23	**0**	9	**0**	7
26	82	5	6	2.73	UF	**-1**	6	**553**	51	**27**	2,716	**557**	39	11	62
27	75	8	6	1.56	F	**0**	6	**0**	8	**0**	8	**0**	10	**0**	9
28	45	3	6	2.50	UF	**-1**	3	**9**	9	**2**	342	**9**	6	**1**	13
29	98	7	6	2.33	F	**0**	7	**0**	13	**0**	39	**0**	9	**0**	17
30	21	2	6	1.75	UF	**-1**	2	**7**	4	**2**	55	**7**	3	**1**	3
31	72	4	10	1.80	F	**0**	5	**0**	10	**0**	29	**0**	7	**0**	11
32	109	4	10	2.73	UF	**-1**	6	**427**	24	**28**	1,757	**430**	41	**11**	3,600
33	50	2	10	2.50	UF	**-1**	3	**294**	10	**33**	332	**299**	8	**6**	10
34	106	7	10	1.51	F	**0**	6	**0**	11	**0**	10	**0**	9	**0**	12
35	129	5	10	2.58	UF	**-1**	8	**422**	36	897	3,600	**422**	18	**11**	124
36	62	2	10	3.10	UF	**-1**	4	**611**	32	**42**	946	**613**	11	**14**	56
37	51	3	10	1.70	F	**0**	4	**0**	6	**0**	18	**0**	5	**0**	7
38	137	6	10	2.28	UF	**-1**	8	**62**	24	1,038	3,600	**62**	20	**3**	27
39	97	3	10	3.23	UF	**-1**	6	**1,118**	53	64	3,600	**1,165**	41	**18**	26
40	135	5	10	2.70	UF	**-1**	7	**715**	51	41	3,600	**715**	42	**15**	35
41	55	2	10	2.75	UF	**-1**	4	**330**	17	**41**	505	**344**	17	**7**	13
42	170	7	10	2.43	UF	**-1**	8	**224**	54	1,047	3,600	**224**	39	**6**	35
43	100	4	10	2.50	UF	**-1**	5	**153**	28	**14**	998	**153**	16	**4**	21
44	220	7	10	3.14	UF	**-1**	10	**1,733**	185	1,034	3,600	**1,822**	113	**30**	150
45	237	8	10	2.96	UF	**-1**	10	**1,338**	1,157	1,011	3,600	**1,345**	183	**25**	196
46	77	7	6	1.83	F	**0**	4	**0**	9	**0**	6	**0**	7	**0**	9
47	46	4	6	1.92	F	**0**	3	**0**	8	**0**	15	**0**	5	**0**	6
48	22	2	6	1.83	F	**0**	2	**0**	4	**0**	3	**0**	3	**0**	4
49	23	2	6	1.92	F	**0**	2	**0**	4	**0**	8	**0**	3	**0**	4
50	32	3	6	1.78	F	**0**	3	**0**	6	**0**	4	**0**	4	**0**	5
51	31	3	6	1.72	F	**0**	3	**0**	6	**0**	4	**0**	3	**0**	4
52	29	3	6	1.61	F	**0**	3	**0**	6	**0**	11	**0**	4	**0**	5
53	23	2	6	1.92	F	**0**	2	**0**	4	**0**	8	**0**	3	**0**	4
54	60	5	6	2.00	F	**0**	4	**0**	10	**0**	6	**0**	6	**0**	9
55	41	4	6	1.71	F	**0**	3	**0**	6	**0**	12	**0**	4	**0**	5
56	116	6	10	1.93	F	**0**	6	**0**	16	**0**	9	**0**	11	**0**	14
57	103	6	10	1.72	F	**0**	6	**0**	15	**0**	35	**0**	10	**0**	15
58	66	4	10	1.65	F	**0**	4	**0**	10	**0**	6	**0**	6	**0**	8
59	73	4	10	1.83	F	**0**	5	**0**	13	**0**	29	**0**	7	**0**	12
60	151	8	10	1.89	F	**0**	8	**0**	22	**0**	52	**0**	14	**0**	18

Table 5.4: Results of MOD0-MOD4 in the set of instances with the AP formulation

As can be seen, MOD0 found the optimal solution in all the instances, 33 instances being feasible and 27 proved not feasible. The average CPU time is 4.81 seconds. According to Corollary 1 when MOD0 is feasible, the optimal value of MOD1-MOD4 will be zero since there is an assignment of parking places that satisfies the time window requests of all the carriers. Conversely, if MOD0 is not feasible the optimal values to MOD1-MOD4 will be strictly positive. In this respect, the results in Table 5.4 confirm that, computationally, the outcome MOD0 gives valuable information not only with respect to MOD0 itself but also with respect to MOD1-MOD4. Indeed, all the 33 instances that are feasible for MOD0 were optimally solved by MOD1-MOD4, but MOD0 was fastest in finding the optimal solution with an average of 4.34 seconds, followed by MOD3, MOD1, MOD4 and MOD2 with averages of: 7.84, 8.37, 10.53 and 14.14 seconds respectively. Regarding unfeasible instances, MOD1, MOD3 and MOD4 performed very well. MOD3 was able to provide the optimal solution for all of the instances; MOD1 and MOD4 all the optimal solutions except for 1 instance. MOD2, however, even if tested under different `CPLEX` parameters it is not able to provide the optimal solution in 14 out of the 27 unfeasible instances.

Even if MOD3 can be unfeasible because its maximum displacement constraint (4.22), this did not happen. The average CPU times required by MOD1-MOD4 are slightly larger that those of MOD0, even if we restrict to the benchmark instances that terminate with a certificate of optimality. Contrary to what happened with the VRP formulation, the AP formulation with the min-max objective of MOD2 is somehow more demanding than with the sum-type objectives of MOD1, MOD3 and MOD4.

The effectiveness of MOD1-MOD4 is summarized in Table 5.5, where benchmark instances have been partitioned relative to their status with respect to MOD0 in *Feasible* or *Unfeasible*. For each tested model we further partition each of the above groups of instances according to the possible outcomes of the tested model: optimum found (*OPT*) or time limit reached with a feasible solution (*TL/S*). The entries in the table give the number of instances in each class and the average CPU times over the set of instances in the class. It is clear that MOD1, MOD3 and MOD4 provide optimal solutions in nearly all of the instances, so the AP formulation is effective in finding optimal solutions for these models. However, MOD2 has some problems for finding optimal solutions when the instance is not feasible.

MOD0			Status	MOD1		MOD2		MOD3		MOD4	
Status	No.	CPU		No.	CPU	No.	CPU	No.	CPU	No.	CPU
Feasible	33	4.34	*OPT*	33	8.37	33	14.14	33	7.84	33	10.53
			TL/S	-		-		-		-	
Unfeasible	27	5.39	*OPT*	26	85.42	13	1,048	27	70.35	26	53.59
			TL/S	1	3,600	14	3,600	-		1	3,600

Table 5.5: Results of MOD1-MOD4 in the set of instances with the AP formulation

5.4 Numerical results with the heuristic

As already mentioned, the results of the heuristic are not aimed at providing optimal solutions. The aim is to assess the results of the previous formulations, especially for the VRP formulation, where `CPLEX` is quite time-consuming for some of the models. Detailed results are given in Table 5.6, which contains several columns. The first five columns summarize the characteristics of the instances. Columns labeled *C-HEUR* give the objective values obtained with the constructive phase of the heuristic. Empty entries in column *C-HEUR* of MOD0 or

MOD3 indicate that the heuristic was not able to find a feasible solution for the corresponding instance. Columns labeled *I-HEUR* give the values obtained with the local search, when it improved the solution of the constructive phase. Empty entries corresponding to instances when the constructive heuristic found a feasible solution, indicate that the local search was not able to improve the solution of the heuristic phase. The constructive heuristic is able to provide 8 feasible solutions for MOD0 and 25 solutions for MOD3, for the rest of the models it provides a feasible solution for all the instances. The improvement heuristic is able to provide better solutions in some of the instances for MOD1-MOD3. In paricular, some of the non-feasible solutions for MOD3 build for the constructive heuristics are made feasible with the improvement. The compared quality of the solutions is analyzed in the following section. The computing times required by the heuristic are, in general, small. This is not surprising giving its simplicity. They are on average 0.003 seconds for the first phase and 0.03 seconds for the improvement phase.

					MOD0	MOD1		MOD2		MOD3		MOD4	
Id	Rq	c	Hop	γ	C_HEUR	C_HEUR	I_HEUR	C_HEUR	I_HEUR	C_HEUR	I_HEUR	C_HEUR	I_HEUR
1	111	7	6	2.64		1,360		80				24	
2	75	4	6	3.13		1,610		109				21	
3	60	4	6	2.50		652		97				9	
4	58	6	6	1.61	**0**	**0**		**0**		**0**		**0**	
5	119	6	6	3.31		2,577		135				36	
6	82	4	6	3.42		2,402		132				26	
7	92	5	6	3.07		1,435		115				17	
8	42	3	6	2.33		361		95				5	
9	48	5	6	1.60		87		28		87		5	
10	73	6	6	2.03		97		53	45	97		2	
11	66	6	6	1.83	**0**	**0**		**0**		**0**		**0**	
12	74	4	6	3.08		1,476		125				18	
13	64	5	6	2.13		366		57		366		9	
14	36	3	6	2.00		84		52		84		2	
15	24	2	6	2.00		182		73	55			3	
16	87	6	6	2.42		830		92				14	
17	104	8	6	2.17		309		69				11	
18	146	8	6	3.04		2,048		98				33	
19	76	7	6	1.81		52	44	29	28	52	44	2	
20	30	3	6	1.67	**0**	**0**		**0**		**0**		**0**	
21	69	7	6	1.64	**0**	**0**		**0**		**0**		**0**	
22	76	6	6	2.11		136	81	75	39		162	2	
23	136	8	6	2.83		1,435		100				27	
24	54	3	6	3.00		997		115				12	
25	57	4	6	2.38		314		82				6	
26	82	5	6	2.73		1,218		104				19	
27	75	8	6	1.56	**0**	**0**		**0**		**0**		**0**	
28	45	3	6	2.50		238		75	53		392	5	
29	98	7	6	2.33		358		68				8	
30	21	2	6	1.75		99		51	49	99		2	
31	72	4	10	1.80		336		84	81			6	
32	109	4	10	2.73		2,436		149				27	
33	50	2	10	2.50		843		147				9	
34	106	7	10	1.51	**0**	**0**		**0**		**0**		**0**	
35	129	5	10	2.58		2,493		125				30	
36	62	2	10	3.10		1,683		164				18	
37	51	3	10	1.70		32		32	18	32		1	
38	137	6	10	2.28		1,110		107				17	
39	97	3	10	3.23		3,294		182				28	
40	135	5	10	2.70		2,787		147				34	
41	55	2	10	2.75		933		156				10	
42	170	7	10	2.43		2,333		110				32	
43	100	4	10	2.50		1,098		137				11	
44	220	7	10	3.14		5,313		164				56	
45	237	8	10	2.96		5,171		154				59	
46	77	7	6	1.83		21	1	21	1	21	1	1	
47	46	4	6	1.92		75		36	32	75		3	
48	22	2	6	1.83	**0**	**0**		**0**		**0**		**0**	
49	23	2	6	1.92		97	48	54	18	97	48	2	
50	32	3	6	1.78		57	35	31	23	57	35	2	
51	31	3	6	1.72		65	51	34	24	65	51	2	
52	29	3	6	1.61		102		57	45	102		2	
53	23	2	6	1.92		90		34		90		3	
54	60	5	6	2.00		45		45	25	45		1	
55	41	4	6	1.71		7		7		7		1	
56	116	6	10	1.93		217		62	42		245	5	
57	103	6	10	1.72		120		45		120		4	
58	66	4	10	1.65	**0**	**0**		**0**		**0**		**0**	
59	73	4	10	1.83		422		100	70			8	
60	151	8	10	1.89		496		69				10	

Table 5.6: Solution obtained with proposed heuristic

5.5 Comparison of the VRP and AP formulations, and the heuristic

In this section the results of both formulations, and the results provided by the heuristic will be presented together and compared. The comparison will be done in terms of number of instances solved to optimality, quality of the solution when optimality has not been proved and computing time. Tables 5.7 and 5.8 combine together the results presented in the previous sections. The first five columns correspond to information related to each instance and described in the previous chapters. Then, for each model we have five columns, two for the VRP formulation: the value of the obtained solution and the computing time; two more for the AP formulation: again, the value of the obtained solution and the computing time; and one last column with the value of the solution obtained with the heuristic. Entries in bold correspond to values of solutions proven optimal. Shaded entries indicate values whose optimality was not proven with the corresponding formulation/method, but which correspond either to best-known values or to values proven optimal by another formulation/ method.

The AP formulation clearly outperforms the VRP formulation. For MOD0, MOD1, MOD3 and MOD4, the AP formulation is nearly always able to provide an optimal solution, and in most of the cases this solution is obtained in less than 200 seconds. MOD2 has a different performance: formulation AP gives 46 optimal solutions for MOD2, while the VRP formulation only obtains 29. The VRP formulation is able to provide, however, 10 best-known solutions, the heuristic 5, and the AP formulation only 3. Note that with this model, the AP formulation finds an optimal solution for instances 15 and 33, even if VRP formulation provides a solution with lower value of the objective function without proving its optimality. This is possible due to time discretization. The solutions obtained with the VRP formulations have non-integer values for time variables, which are unfeasible for the AP formulation. This model presents an extra difficulty for solving it. Due to the symmetry of the solutions, there are multiple solutions with the same value of the objective function, which makes the solution process more complex. Solutions obtained with the heuristic are nearly always of poor quality, which was expected since it is a very simple heuristic with short computing time. The heuristic only performs best in some of the instances for MOD2. It also outperforms the VRP formulation in MOD4.

					MOD0							MOD1					MOD2					MOD3					MOD4				
					VRP			AP			HEUR	VRP		AP		HEUR	VRP		AP		HEUR	VRP		AP		HEUR	VRP		AP		HEUR
Id	Rq	c	Hop	γ	Type	SOL	TIME	Type	SOL	TIME	SOL	SOL	TIME	SOL	TIME	SOL	SOL	TIME	SOL	TIME	SOL	SOL	TIME	SOL	TIME	SOL	SOL	TIME	SOL	TIME	SOL
1	111	7	6	2.64	UC	-2	3,600	U	**-1**	5		1,352	3,600	**551**	44	1,360	48	3,600	52	3,600	80		3,600	**551**	48		30	3,600	**11**	69	24
2	75	4	6	3.13	UC	-2	3,600	U	**-1**	3		1,391	3,600	950	3,600	1,610	54.5	3,600	48	3,600	109	1,359	3,600	**999**	80		24	3,600	**12**	40	21
3	60	4	6	2.50	TL		3,600	U	**-1**	2		200	3,600	**64**	15	652	12	3,600	**6**	867	97	142	3,600	**64**	12		9	3,600	**2**	31	9
4	58	6	6	1.61	F	**0**	1	F	**0**	3	**0**	**0**	61	**0**	6	**0**	**0**	29	**0**	5	**0**	**0**	32	**0**	8	**0**	**0**	32	**0**	12	**0**
5	119	6	6	3.31	UC	-2	3,600	U	**-1**	5		2,602	3,600	**1,565**	52	2,577	73.5	3,600	568	3,600	135		3,600	**1,686**	110		55	3,600	**21**	82	36
6	82	4	6	3.42	UC	-2	3,600	U	**-1**	4		1,951	3,600	**1,380**	92	2,402	71	3,600	523	3,600	132		3,600	**1,494**	716		26	3,600	**15**	49	26
7	92	5	6	3.07	TL		3,600	U	**-1**	6		629	3,600	**181**	43	1,435	28.5	3,600	27	3,600	115	675	3,600	**183**	33		16	3,600	**3**	49	17
8	42	3	6	2.33	F	**0**	466	F	**0**	4		**0**	3,248	**0**	5	361	**0**	1,682	**0**	15	95	**0**	2,017	**0**	8		2	3,600	**0**	13	5
9	48	5	6	1.60	F	**0**	5	F	**0**	3		**0**	510	**0**	5	87	**0**	49	**0**	4	28	**0**	46	**0**	7	87	**0**	122	**0**	12	5
10	73	6	6	2.03	F	**0**	9	F	**0**	5		**0**	933	**0**	8	97	**0**	187	**0**	6	45	**0**	555	**0**	11	97	**0**	420	**0**	15	2
11	66	6	6	1.83	F	**0**	1	F	**0**	4	**0**	**0**	97	**0**	6	**0**	**0**	146	**0**	5	**0**	**0**	392	**0**	10	**0**	**0**	154	**0**	17	**0**
12	74	4	6	3.08	UC	-2	3,600	U	**-1**	5		1,169	3,600	**794**	50	1,476	44	3,600	**35**	1,888	125	1,111	3,600	**797**	21		19	3,600	**12**	65	18
13	64	5	6	2.13	F	**0**	45	F	**0**	5		**0**	3,573	**0**	10	366	**0**	489	**0**	23	57	**0**	3,373	**0**	11	366	2	3,600	**0**	15	9
14	36	3	6	2.00	F	**0**	2	F	**0**	2		**0**	41	**0**	5	84	**0**	9	**0**	3	52	**0**	8	**0**	5	84	**0**	18	**0**	7	2
15	24	2	6	2.00	U	-1	394	U	**-1**	2		13	3,600	**13**	5	182	3.5	3,600	**4**	101	55	13	3,600	**13**	5		1	3,600	**1**	8	3
16	87	6	6	2.42	TL		3,600	U	**-1**	5		428	3,600	**87**	43	830	19.5	3,600	**6**	1,858	92	409	3,600	**87**	63		13	3,600	**3**	54	14
17	104	8	6	2.17	TL		3,600	F	**0**	7		253	3,600	**0**	15	309	15.5	3,600	**0**	43	69	245	3,600	**0**	19		14	3,600	**0**	32	11
18	146	8	6	3.04	TL		3,600	U	**-1**	9		1,885	3,600	**842**	61	2,048	75.5	3,600	828	3,600	98		3,600	**852**	46		49	3,600	**15**	114	33
19	76	7	6	1.81	F	**0**	156	F	**0**	4		**0**	3,261	**0**	9	44	**0**	720	**0**	9	28	**0**	422	**0**	12	44	**0**	1,703	**0**	15	2
20	30	3	6	1.67	F	**0**	0	F	**0**	2	**0**	**0**	6	**0**	3	**0**	**0**	9	**0**	3	**0**	**0**	2	**0**	5	**0**	**0**	3	**0**	6	**0**
21	69	7	6	1.64	F	**0**	7	F	**0**	5	**0**	**0**	65	**0**	6	**0**	**0**	124	**0**	8	**0**	**0**	94	**0**	10	**0**	**0**	62	**0**	9	**0**
22	76	6	6	2.11	F	**0**	26	F	**0**	6		**0**	3,304	**0**	8	81	**0**	543	**0**	8	39	**0**	3,258	**0**	12	162	**0**	1,437	**0**	11	2
23	136	8	6	2.83	TL		3,600	U	**-1**	7		1,446	3,600	**432**	46	1,435	70.5	3,600	139	3,600	100		3,600	**432**	49		34	3,600	**9**	49	27
24	54	3	6	3.00	UC	-2	3,600	U	**-1**	4		495	3,600	**422**	36	997	35	3,600	**30**	1,273	115	531	3,600	**432**	120		11	3,600	**7**	10	12
25	57	4	6	2.38	F	**0**	11	F	**0**	4		**0**	3,369	**0**	8	314	**0**	2,651	**0**	23	82	**0**	3,291	**0**	9		1	3,600	**0**	7	6
26	82	5	6	2.73	TL		3,600	U	**-1**	6		995	3,600	**553**	51	1,218	44	3,600	**27**	2,716	104	885	3,600	**557**	39		19	3,600	11	62	19
27	75	8	6	1.56	F	**0**	2	F	**0**	6	**0**	**0**	367	**0**	8	**0**	**0**	73	**0**	8	**0**	**0**	222	**0**	10	**0**	**0**	482	**0**	9	**0**
28	45	3	6	2.50	TL		3,600	U	**-1**	3		9	3,600	**9**	9	238	3	3,600	**2**	342	53	13	3,600	**9**	6	392	2	3,600	**1**	13	5
29	98	7	6	2.33	TL		3,600	F	**0**	7		12	3,600	**0**	13	358	14.5	3,600	**0**	39	68	90	3,600	**0**	9		7	3,600	**0**	17	8
30	21	2	6	1.75	U	-1	1,022	U	**-1**	2		7	3,600	**7**	4	99	2	3,600	**2**	55	49	7	3,600	**7**	3	99	1	3,600	**1**	3	2

Table 5.7: Solutions obtained with the VRP, the AP formulations and the heuristic

					MOD0							MOD1					MOD2					MOD3					MOD4				
					VRP			AP			HEUR	VRP		AP		HEUR	VRP		AP		HEUR	VRP		AP		HEUR	VRP		AP		HEUR
Id	Rq	c	Hop	γ	Type	SOL	TIME	Type	SOL	TIME	SOL	SOL	TIME	SOL	TIME	SOL	SOL	TIME	SOL	TIME	SOL	SOL	TIME	SOL	TIME	SOL	SOL	TIME	SOL	TIME	SOL
31	72	4	10	1.80	F	**0**	2,074	F	**0**	5		**0**	3,313	**0**	10	336	**0**	190	**0**	29	81	**0**	1,402	**0**	7		2	3,600	**0**	11	6
32	109	4	10	2.73	TL		3,600	U	**-1**	6		1,166	3,600	**427**	24	2,436	44.5	3,600	**28**	1,757	149	829	3,600	**430**	41		36	3,600	**11**	3,600	27
33	50	2	10	2.50	TL		3,600	U	**-1**	3		307	3,600	**294**	10	843	32.5	3,600	**33**	332	147	306	3,600	**299**	8		10	3,600	**6**	10	9
34	106	7	10	1.51	F	**0**	3	F	**0**	6	**0**	**0**	971	**0**	11	**0**	**0**	714	**0**	10	**0**	**0**	223	**0**	9	**0**	**0**	805	**0**	12	**0**
35	129	5	10	2.58	TL		3,600	U	**-1**	8		1,431	3,600	**422**	36	2,493	221	3,600	897	3,600	125	1,145	3,600	**422**	18		40	3,600	**11**	124	30
36	62	2	10	3.10	UC	-2	3,600	U	**-1**	4		730	3,600	**611**	32	1,683	41.5	3,600	**42**	946	164	652	3,600	**613**	11		18	3,600	**14**	56	18
37	51	3	10	1.70	F	**0**	2	F	**0**	4		**0**	71	**0**	6	32	**0**	22	**0**	18	18	**0**	7	**0**	5	32	**0**	132	**0**	7	1
38	137	6	10	2.28	TL		3,600	U	**-1**	8		529	3,600	**62**	24	1,110	205	3,600	1,038	3,600	107	352	3,600	**62**	20		20	3,600	**3**	27	17
39	97	3	10	3.23	UC	-2	3,600	U	**-1**	6		1,764	3,600	**1,118**	53	3,294	69.5	3,600	64	3,600	182		3,600	**1,165**	41		38	3,600	**18**	26	28
40	135	5	10	2.70	TL		3,600	U	**-1**	7		1,989	3,600	**715**	51	2,787	121	3,600	41	3,600	147	1,531	3,600	**715**	42		60	3,600	**15**	35	34
41	55	2	10	2.75	TL		3,600	U	**-1**	4		361	3,600	**330**	17	933	41	3,600	**41**	505	156	357	3,600	**344**	17		11	3,600	**7**	13	10
42	170	7	10	2.43	TL		3,600	U	**-1**	8		1,573	3,600	**224**	54	2,333	317	3,600	1,047	3,600	110	1,217	3,600	**224**	39		65	3,600	**6**	35	32
43	100	4	10	2.50	TL		3,600	U	**-1**	5		274	3,600	**153**	28	1,098	22.5	3,600	**14**	998	137	279	3,600	**153**	16		18	3,600	**4**	21	11
44	220	7	10	3.14	UC	-2	3,600	U	**-1**	10		21,537	3,600	**1,733**	185	5,313	438	3,600	1,034	3,600	164		3,600	**1,822**	113		168	3,600	**30**	150	56
45	237	8	10	2.96	TL		3,600	U	**-1**	10		27,701	3,600	**1,338**	1,157	5,171	419	3,600	1,011	3,600	154		3,600	**1,345**	183		186	3,600	**25**	196	59
46	77	7	6	1.83	F	**0**	23	F	**0**	4		**0**	333	**0**	9	1	**0**	258	**0**	6	1	**0**	323	**0**	7	1	**0**	618	**0**	9	1
47	46	4	6	1.92	TL		3,600	F	**0**	3		**0**	839	**0**	8	75	**0**	98	**0**	15	32	**0**	459	**0**	5	75	**0**	2,010	**0**	6	3
48	22	2	6	1.83	F	**0**	0	F	**0**	2	**0**	**0**	2	**0**	4	**0**	**0**	2	**0**	3	**0**	**0**	1	**0**	3	**0**	**0**	2	**0**	4	**0**
49	23	2	6	1.92	F	**0**	1	F	**0**	2		**0**	21	**0**	4	48	**0**	3	**0**	8	18	**0**	3	**0**	3	48	**0**	5	**0**	4	2
50	32	3	6	1.78	F	**0**	2	F	**0**	3		**0**	86	**0**	6	35	**0**	6	**0**	4	23	**0**	8	**0**	4	35	**0**	68	**0**	5	2
51	31	3	6	1.72	F	**0**	0	F	**0**	3		**0**	16	**0**	6	51	**0**	3	**0**	4	24	**0**	4	**0**	3	51	**0**	4	**0**	4	2
52	29	3	6	1.61	F	**0**	1	F	**0**	3		**0**	16	**0**	6	102	**0**	3	**0**	11	45	**0**	4	**0**	4	102	**0**	4	**0**	5	2
53	23	2	6	1.92	F	**0**	1	F	**0**	2		**0**	34	**0**	4	90	**0**	4	**0**	8	34	**0**	11	**0**	3	90	**0**	45	**0**	4	3
54	60	5	6	2.00	F	**0**	7	F	**0**	4		**0**	142	**0**	10	45	**0**	68	**0**	6	25	**0**	208	**0**	6	45	**0**	53	**0**	9	1
55	41	4	6	1.71	F	**0**	3	F	**0**	3		**0**	69	**0**	6	7	**0**	23	**0**	12	7	**0**	17	**0**	4	7	**0**	23	**0**	5	1
56	116	6	10	1.93	F	**0**	115	F	**0**	6		**0**	3,385	**0**	16	217	**0**	3,281	**0**	9	42	**0**	623	**0**	11	245	1	3,600	**0**	14	5
57	103	6	10	1.72	F	**0**	58	F	**0**	6		**0**	1,329	**0**	15	120	**0**	3,264	**0**	35	45	**0**	3,287	**0**	10	120	**0**	3,322	**0**	15	4
58	66	4	10	1.65	F	**0**	4	F	**0**	4	**0**	**0**	173	**0**	10	**0**	**0**	71	**0**	6	**0**	**0**	17	**0**	6	**0**	**0**	270	**0**	8	**0**
59	73	4	10	1.83	TL		3,600	F	**0**	5		2	3,600	**0**	13	422	2	3,600	**0**	29	70	3	3,600	**0**	7		1	3,600	**0**	12	8
60	151	8	10	1.89	TL		3,600	F	**0**	8		286	3,600	**0**	22	496	194	3,600	**0**	52	69	273	3,600	**0**	14		17	3,600	**0**	18	10

Table 5.8: Solutions obtained with the VRP, the AP formulations and the heuristic

	MOD0	MOD1		MOD2		MOD3		MOD4	
	Opt	Opt	Best	Opt	Best	Opt	Best	Opt	Best
VRP	28	29	32	29	40	29	31	24	26
AP	33	59	60	46	49	60	60	59	60
Heur	8	8	8	8	13	8	8	8	8

Table 5.9: Summary of methods results

A summary of the results is given in Table 5.9. Entries in rows *VRP*, *AP* and *Heur* of Table 5.9 give, for each model, the number of instances when each formulation produce an optimal solution (Opt) and the number of instances where the solution is the best-known (Best). Formulation AP always produces the best results, with optimal solutions in nearly all of the instances of all models. Formulation VRP gives not so good results with 24-29 optimal solutions, and an extra number of best-known solutions. In some cases the VRP formulation is able to find the optimal solution but not to prove its optimality. For example, in instance 15 the VRP formulation obtained a solution of value 15 which was not proven optimal. Nonetheless, we now know that this solution is optimal given the solution of the AP formulation. Also, in MOD2, the VRP formulation is able to give some best-known solutions.

The results of *Heur* are very modest, with 8 optimal solution, and some best-known solutions for MOD2. Nevertheless, for some instances the heuristic solution was better than that produced by the VRP formulation for some model. This happens particularly with MOD4, where the heuristic outperformed VRP formulation for 19 instances. We attribute this poor performance to the symmetry of its objective function. Note that in MOD4 the objective just counts the number of requests scheduled outside their preferred time window. This means that many alternative optimal solutions may exist and thus, pruning nodes in the enumeration tree may become an extremely arduos task.

5.6 Cross-evaluation of solutions

In the remainder of this section we further analyze models MOD1-MOD4. In particular, we evaluate the goodness of the solutions produced by each of the models relative to the other models. For this comparison we use the AP formulation, which generally outperforms the VRP formulation as we have seen in the previous section. We restrict our analysis to the 27 unfeasible instances, because the feasible instances are solved by all models and their optimal values are zero. First we introduce some additional notation that we will use to present our results. For $j = 1,\dots 4$, let z^j denote the objective function of model MODj, so for any feasible solution δ, $z^j(\delta)$ is the objective function value of solution δ for MODj. Let also δ_{jk} be the best solution obtained with MODj for instance k, when it found some optimal or feasible solution, with value $z^j(\delta_{jk})$. As we will see, for a given instance k, sometimes a solution δ_{jk} produced by MODj is good for another model MODi, with $i \neq j$. Hence, we will also use the notation v_{jk} to denote the best-known value for MODj and instance k. That is:

$$v_{jk} = \min_{i=1,\dots,4} \{z^j(\delta_{ik})\}.$$

Furthermore for each model MODj and instance k we compute the percentage deviations of the objective values of the best solutions produced by all four models models, $z^j(\delta_{ik})$, $i = 1,\dots 4$, with respect to the best-known values v_{jk}, which are denoted by Δ^j_{ik}. These deviations are computed as

$$\Delta_{ik}^{j} = 100\frac{z^{j}(\delta_{ik}) - v_{jk}}{(1+v_{jk})},$$

where a “1” has been added in the denominator to prevent dividing by zero.

Table 5.10 gives for each model MODj and instance k, its best-known value, v_{jk}, in bold if this value is known to be the optimal solution. The table also contains Δ_{ik}^{j}, $i = 1,\ldots,4$. Solutions are always feasible in MOD1, MOD2, and MOD4, but sometimes the solutions produced by MOD1, MOD2, MOD4, are not be feasible for MOD3. When the solution produced by MODi for instance k was not feasible for MOD3, the entry Δ_{ik}^{3} is -1. The last two rows of Table 5.10 give average percentage gaps Δ_{ik}^{j}, over the set of all the instances k for which MODi obtained a feasible solution for MODj, and the total number of best-known solutions for MODj produced by MODi.

The information of Table 5.10 is summarized in Table 5.11. The entry in row MODi and column MODj gives the number of tested instances for which the solution produced MODi gives the best-known value for MODj, i.e. $v_{jk} = z^{j}(\delta_{ik})$. Row labeled *Total Best* gives the sum of the above rows. For each of the models, this is the overall number of best-known solutions obtained for this model, with any of the models MODi, $i = 1,\ldots,4$. Indeed this value can be bigger than the number of instances, when more than one model produced a best-known/optimal solution for some instance k. In the case of MOD2 this may also happen because some other model produced a better solution than the one produced by MOD2 itself. This happens with several instances where the best solution found by MOD2 is much worse than the optimal solution of MOD3. MOD3 with the maximum displacement constraint fixed at 60 minutes, guarantees a solution for MOD2 with objective value at most 60. Column *ALL* gives the total number of best-known solutions produced by each of the models.

From the results presented in Tables 5.10 and 5.11, it is clear that each model usually produces the highest number of best-known/optimal solutions for it. Some models give best-known solutions for some of the other models as well. MOD1 and MOD3 are closely related, although some solutions produced by MOD1 are not feasible for MOD3 since they not fulfill the maximum displacement constraint. Optimal solutions from MOD3 are sometimes not optimal for MOD1, but usually very close to the optimal ones, as illustrated by the fact that the average percent deviation is 1.62. The behavior of MOD4 is independent of that of the other models. In fact the solutions produced by the other models are never optimal, or even close, for MOD4. This could be expected as its objective function is very different from that of the other models. MOD2 presents some difficulties, since for 14 instances we did not obtain an optimal solution. Then, sometimes MOD1 or MOD3 produce better solutions than the one originally obtained by MOD2 itself. As already said, the symmetry of this model makes it more complex to solve.

MOD3 performs in general better than any other model, giving optimal or near-optimal solutions for MOD1, and some competitive solutions for MOD2, as indicated by the percent deviation gaps. This reinforces the interest of MOD3, which, on the other hand, can be seen as a good compromise from the modeling point of view, as it guarantees a maximum deviation from the preferred time window for all requests. Since the application framework that motivates the study of the PAP aims at producing “fair” solutions, the good performance of MOD3 with respect to the other models increases its relevance.

Id	Type	MOD1 best	Δ^1_{1k}	Δ^1_{2k}	Δ^1_{3k}	Δ^1_{4k}	MOD2 best	Δ^2_{1k}	Δ^2_{2k}	Δ^2_{3k}	Δ^2_{4k}
1	UF	**551**	0	373	0	895	52	26	0	13	1,409
2	UF	950	0	81	5	303	48	114	0	24	841
3	UF	**64**	0	132	0	1,477	**6**	600	0	614	10,900
5	UF	**1,565**	0	640	8	474	60	113	833	0	1,144
6	UF	**1,380**	0	478	8	431	60	151	759	0	1,197
7	UF	**181**	0	623	1	803	27	161	0	118	2,204
12	UF	**794**	0	83	0	595	**35**	122	0	69	2,011
15	UF	**13**	0	93	0	1,650	**4**	60	0	120	4,800
16	UF	**87**	0	136	0	2,401	**6**	171	0	271	11,900
18	UF	**842**	0	6,232	1	725	59	53	1,282	0	1,207
23	UF	**432**	0	1,546	0	861	48	0	186	6	1,396
24	UF	**422**	0	100	2	486	**30**	219	0	94	1,855
26	UF	**553**	0	95	1	772	**27**	207	0	118	2,864
28	UF	**9**	0	180	0	2,500	**2**	200	0	33	8,567
30	UF	**7**	0	75	0	8,588	**2**	167	0	100	23,067
32	UF	**427**	0	250	1	1,402	**28**	124	0	110	2,838
33	UF	**294**	0	126	2	690	**33**	94	0	79	1,976
35	UF	**422**	0	8,749	0	1,212	60	0	1,372	0	1,449
36	UF	**611**	0	105	0	973	**42**	84	0	37	1,740
38	UF	**62**	0	81,759	0	1,765	17	6	5,672	0	3,061
39	UF	**1,118**	0	257	4	711	60	70	7	0	1,270
40	UF	**715**	0	196	0	737	41	79	0	45	2,126
41	UF	**330**	0	182	4	873	**41**	107	0	45	2,081
42	UF	**224**	0	25,976	0	1,323	41	0	2,395	7	2,062
43	UF	**153**	0	125	0	1,127	**14**	40	0	73	5,980
44	UF	**1,733**	0	4,469	5	737	60	64	1,597	0	1,577
45	UF	**1,338**	0	6,629	1	777	60	44	1,559	0	1,449
Avrg			0	5,173.73	1.62	1,306.91		113.98	580.04	73.31	3,813.74
No. Best			27	0	12	0		3	17	8	0

Id	Type	MOD3 best	Δ^3_{1k}	Δ^3_{2k}	Δ^3_{3k}	Δ^3_{4k}	MOD4 best	Δ^4_{1k}	Δ^4_{2k}	Δ^4_{3k}	Δ^4_{4k}
1	UF	**551**	-1	373	0	-1	**11**	217	600	225	0
2	UF	**999**	-1	73	0	-1	**12**	192	308	231	0
3	UF	**64**	0	132	0	-1	**2**	233	1,133	200	0
5	UF	**1,686**	-1	-1	0	-1	**21**	145	427	159	0
6	UF	**1,494**	-1	-1	0	-1	**15**	144	413	188	0
7	UF	**183**	-1	615	0	-1	**3**	475	2,050	600	0
12	UF	**797**	-1	82	0	-1	**12**	169	362	185	0
15	UF	**13**	0	93	0	-1	**1**	150	400	100	0
16	UF	**87**	0	136	0	-1	**3**	325	1,075	275	0
18	UF	**852**	-1	-1	0	-1	**15**	244	788	225	0
23	UF	**432**	0	-1	0	-1	**9**	310	1,030	310	0
24	UF	**432**	-1	95	0	-1	**7**	188	463	188	0
26	UF	**557**	-1	94	0	-1	11	125	408	167	0
28	UF	**9**	0	180	0	-1	**1**	50	700	200	0
30	UF	**7**	0	75	0	-1	**1**	0	350	50	0
32	UF	**430**	-1	247	0	-1	**11**	75	592	125	0
33	UF	**299**	-1	122	0	-1	**6**	100	400	71	0
35	UF	**422**	0	-1	0	-1	**11**	217	925	233	0
36	UF	**613**	-1	104	0	-1	**14**	87	253	80	0
38	UF	**62**	0	-1	0	-1	**3**	150	3,150	175	0
39	UF	**1,165**	-1	-1	0	-1	**18**	89	353	111	0
40	UF	**715**	-1	196	0	-1	**15**	169	450	181	0
41	UF	**344**	-1	171	0	-1	**7**	75	425	100	0
42	UF	**224**	0	-1	0	-1	**6**	271	2,243	300	0
43	UF	**153**	0	125	0	-1	**4**	220	1,100	220	0
44	UF	**1,822**	-1	-1	0	-1	**30**	139	597	155	0
45	UF	**1,345**	-1	-1	0	-1	**25**	181	781	192	0
Avrg			0	171.40	0			175.54	806.43	194.25	0
No. Best			10	0	27	0		1	0	0	27

Table 5.10: Percent deviations from best-known/optimal solutions

	MOD1	MOD2	MOD3	MOD4	ALL
MOD1	27	3	10	1	41
MOD2	0	17	0	0	17
MOD3	12	8	27	0	47
MOD4	0	0	0	27	27
Total Best	39	28	37	28	

Table 5.11: Cross evaluation of solution

5.7 Sensitivity analysis of maximum displacement

MOD1 and MOD3 only differ in Constraints (4.22), which limit the maximum non-accomplishment per request in MOD3. In all the experiments described above the value of this parameter was set to $d = 60$ minutes. In this section, we present a sensitivity analysis on the value of the maximum displacement per request. To see the effect of Constraints (4.22), MOD3 which nearly always provides the best solution, has also been solved with the AP formulation for varying values of the maximum displacement parameter $d \in \{40, 50, 70, 80, \infty\}$ We use MOD3(d) to refer to MOD3 with a parameter value d. Note that MOD3(∞)=MOD1. A summary of the obtained results is presented in Table 5.12. Rows correspond to different values of d.

The first two columns respectively indicate the number of instances for which the optimality of the obtained solution was proven (*#OPT*) and was not proven (*#TL/S*) within the maximum time limit. In all but one instance MOD3(∞) was solved to optimality. The next column indicates the number of feasible instances (*Feas*). Note that there are six unfeasible instances for MOD3(40), which have no feasible solution where all requests are assigned less than 40 minutes outside their requested time windows. The last column under *quality index (qi)* has been computed as follows: For each instance $k \in Feas$ and parameter value d, we denote by v_k^d its optimal objective function value for MOD3(d). Then, for each value d, we compute the quality index as the following average of the optimal values:

$$qi(d) = \frac{\sum_{k \in Feas} v_k^d}{|Feas|} \tag{5.1}$$

which is an average measure of the quality of the solutions of model MOD3(d) when parameter d is used. These values are given in the last column of Table 5.12.

	d	*#OPT*	*# TL/S*	*Feas*	*quality index (qi(d))*
MOD3	40	60	0	54	169.09
	50	60	0	60	162.91
	60	60	0	60	159.87
	70	60	0	60	158.63
	80	60	0	60	158.41
	∞	59	1	60	158.15

Table 5.12: Comparison of MOD1 and MOD3

The average results presented in Table 5.12 show that, in general, models perform as expected. When the maximum deviation parameter decreases MOD3 becomes unfeasible for some instances. However, the quality of the solutions obtained when the parameter is bigger increases, as we allow more flexibility in the solutions. For instance, MOD3(40) has an average optimal objective function of 162.91, that decreases to 158.41 in MOD3(80).

6 Conclusions

In this part of the thesis we have addressed the Parking Slot Assignment Problem (PAP), a novel problem motivated by the need of providing parking space to carriers for their loading/unloading operations. The PAP is to find assignments of carriers to parking places that satisfy their time window requests. We have studied different modeling alternatives for the PAP, including a feasibility model, which looks for an assignment satisfying all the carriers time windows requests, and several other models, which allow deviations from the requested time windows, that evaluate the degree of non-accomplishment with different criteria. An unfeasibility sufficient condition for the feasibility model has been given, which has proven to be effective in practice.

Two different formulations have been proposed and implemented for each model. The first formulation assumes time as a continuous variable, and requests can be assigned to start at any point in time. Then, the problem is considered as a scheduling problem with time windows that can be formulated as a VRPTW. The domains of the proposed formulations have been compared. The second formulation discretizes time, which entails some loss of accuracy and also makes the problem grow in terms of the number of variables. In that case, the problem can be considered as a variation of an AP, which is known to have good theoretical properties. Unfortunately, in our case, the integrality property is lost, due to the additional constraints characteristic of our specific models.

We have evaluated and compared experimentally the proposed models and formulations by solving a set of test instances using `CPLEX`. In our experiments the computing time has been limited to one hour, to avoid very long runs, but also to adapt to the nature of the PAP, which demands a solution daily. The obtained results have been presented together with a detailed analysis and comparison of various indicators.

The results of the two formulations, with continuous and discrete time, have been analyzed separately and then compared between them and with the results of a simple heuristic. Empirical results have showed that the discrete time formulation outperforms the continuous time formulation in the tested instances. The discrete time formulation has been able to provide optimal results in most of the instances, and generally better quality solutions than the continuous formulation. As could be expected, in general, the quality of the solutions produced by `CPLEX` with both formulations is considerably better than that of the heuristic solutions. Nevertheless, in a few instances of MOD2 and MOD4 the heuristic solution outperformed `CPLEX` with the continuous time formulation. We attribute this to the objective functions of these two models, which may produce very many alternative optima.

The feasibility model and the other models, which evaluate differently the degree of non-accomplishment, have been compared among them. The solutions produced by each model have been evaluated for the other models. Furthermore, for MOD3 a sensitivity analysis has been performed on the maximum allowed displacement from the requested time window. Broadly speaking the obtained results indicate that the earliness/tardiness criterion tends to produce solutions which are also good for the other models.

A promising avenue for research is to further explore some of the more general models that we have also proposed. This includes considering weights in the terms of the objective function and testing the cost minimization formulation. On the other hand, incorporating uncertainty to our data will certainly lead to more realistic models. Moreover, other loading/unloading planning decisions can be investigated (sizing or location) linked to the optimal results of the PAP. Finally, the proposed formulations can be exploited for other similar transportation problems of resource allocation with time windows, like the administration of public rechargeable points for electrical vehicles or the use of docks in a freight terminal.

Part II

The Shared Customer Collaboration Vehicle Routing Problem

Introduction

Most major cities present a dense and complex urban fabric, which difficults considerably last-mile deliveries. Different transport carriers offer transport services located all over the city, which, in the end, implies multiple number of trips from the different companies to the same areas to perform their deliveries. From a system point of view, this operation is likely to be non-efficient. Performing several trips in the same area means more vehicle kilometers. In turn, this generates negative effects to all stakeholders involved in last-mile deliveries. It is more costly for the whole set of transport companies, and, at the same time, it generates more pollution, traffic and space occupancy in the city, which is a nuisance for citizens.

In particular, we can identify different situations where the same customer has to be visited by more than one carrier in the given time horizon. Such customers will be referred to as *shared customers*. In these cases, the expected potential saving due to collaboration among these carriers is likely to be attractive since carriers should not modify their own routes to accommodate the service of demand corresponding to other carriers. Furthermore, we believe that carriers could be willing to collaborate in the presence of shared customers, since the customers they would not visit are already customers of some competing carrier. In this case, carrier collaboration considers the opportunity to transfer the demand that one carrier has to serve to a given customer, to a carrier that is already visiting the same customer. For instance, two carriers can agree to collaborate in the following way: instead of visiting a shared customer twice in the same time horizon, it could be visited by only one carrier who would serve the load from both carriers. Hence, the corresponding load should be transferred between the depots of the carriers before the beginning of the tours, and carriers must agree on an economic value.

Urban areas usually have high customer density. There are many customers located in a reduced area, specially in commercial neighborhoods. Thus, even in situations where there are not strictly shared costumers from different carriers, we can use an aggregation of different customers. We can build a macro-customer, which is an aggregation of some customers located very close among them, in the same street section or in the same block. In terms of routing visiting this macro-customer can easily give service to all the implied customers in one single stop. We can use the term *nearby customer*, defined as a set of customers with a given maximum distance between any pair of them. Furthermore, in regular but non-urgent deliveries, the time horizon can be considered to be more than one day, or even a week. Then, it's even more common to have shared customers (or nearby customers) receiving goods from different carriers, which could be sent in the same route by a unique carrier.

Collaboration among carriers can be a controversial topic since it considers collaboration among stakeholders that inherently compete. The research performed in this part of the thesis aims to prove and quantify the benefits of this type of collaboration. This way, carriers (or administrations) have accurate approximations of the benefits in order to decide to take part in (or incentive) this type of collaboration. Agreements among companies on how to compensate for transferred costumers is a very interesting topic, but it is out of the scope of this thesis. Nonetheless, it is important to have accurate approximations of the costs and savings for each company, in order to have a fair allocation of the costs.

Potential applications to real situations can be regular deliveries to bars and restaurants with non-perishable goods (drinks, dry food), daily delivery to offices with parcel deliveries, which receive several documents, deliveries of textile, shoe or gift shops. As an example, [15] describes the real case of three Dutch companies of distribution of frozen products. The companies had a considerable amount of overlap between customers, on average 68 %.

From a more general perspective, different levels of collaborative strategies can be considered. The following framework offers alternatives that can benefit all the stakeholders. The list is ordered by increasing level of collaboration. In all cases it is crucial that companies share information.

- One company decides to outsource some customers to another carrier to save some costs. This problem has been widely studied in the literature, see for instance [21, 7, 30], and the term *Common Carrier* has been coined.
- Customers that have to be served by several companies consolidate their demand in only one carrier. The demand should be transferred between their depots. The transfer is performed in order to reduce the number of stops, and potentially reduces the total travelled distance. This is the type of intermediate collaboration that we study in this part of the thesis. To the best of our knowledge it has not yet been studied in the literature.
- An Urban Consolidation Center (UCC) for several companies is created. Such a center would be near the core of the area where demand is concentrated. Every company brings the goods to the center and common vehicles do the transport jointly between all companies in the urban environment.

UCCs are studied in the third part of the thesis. As mentioned, in this part the Collaboration among carriers where there are shared customers is proposed as an intermediate level of collaboration. We consider that in a given area there are some *shared customers* or *nearby customers*, who have demand for several carriers. We consider that these customers can be transferred before the routing, and the costs of all carriers can be reduced.

This part of the thesis is organized as follows, Chapter 7 revises the existing literature relevant to the problem. Chapter 8 describes the problem and two different formulations are proposed. The numerical results of computational experiments with both formulations are analyzed on a set of test instances, based on Cordeau instances [12] in Chapter 9. Some preliminary conclusions are presented in Chapter 10 .

7 Literature Review

There is a wide and varied literature related to collaboration between carriers in urban distribution problems. Collaboration among companies at same level of the supply chain is referred to as *horizontal cooperation*. As opposed to *vertical collaboration*, when collaboration happens among actors from different levels of the supply chain: shippers, carriers and/or customers. There are few works focusing only on the operational problem of collaborating carriers, and different approaches devoted to the cost allocation problem; i.e., how to split among participants the benefits of the collaboration. Even tough the cost allocation problem is not addressed in this part of the thesis, the cost allocation approach is often related to the solution of the fundamental operational problem, so we review it here.

The idea of networks of collaborating carriers is used in practice in the European less-than-truckload market. Six out of the top ten carrier organizations are actually networks of small- and medium- sized collaborating companies [39]. However, most of the reviewed works are oriented to collaboration among carriers in line-haul environments. We are not aware of works focused on intermediate collaboration on last-mile deliveries.

Recently, in [16] an extensive survey focused on the potential benefits and impediments for horizontal cooperation in the Flandes region of Belgium. Most of Logistic Service Providers believe in the potentials of horizontal collaboration, in particular the improvement of profitability and/or the improvement on the level of service. At the same time, the impediments that they agree on the most are finding a reliable party to lead the cooperation and the construction of a fair allocation mechanism for benefits. It is therefore reasonable that most of the work is devoted to the cost allocation problem.

To the best of our knowledge, from the operational point of view, very few works are available. The work [83] integrates transshipment into conventional pickup and delivery problem with collaboration. A MIP model is formulated for a problem that allows exchanging requests between carriers and the possibility of demand transshipment at pre-specified transshipment points. One of the more complex features of the problems is the synchronization of vehicles at transshipment points.

As mentioned, an usual approach to collaboration strategies combines the solution of the operational model with the cost allocation problem, i.e. how the cost differential of the collaborative process is distributed amongst the participants. An extensive review of operational planing of horizontal cooperation among road transportation carriers, from the carriers perspective can be found in [82]. Not only theoretical works are discussed, but it also covers an overview of several implementations. Works are divided in order sharing or capacity sharing. In order sharing, most of the works focus on joint route planning or auction based mechanisms, but also bilateral lane changes, load swapping and dispatching policies are discused. In vehicle sharing, most approaches make use of mathematical programming or negotiating protocols.

In the following paragraphs we revise some of the most relevant works. Works of [15, 23, 53] are oriented from a joint route planning perspective. Instead[43, 44, 81] use the settling of game theory to model the behavior of collaborating partners, and the solution to the cost allocation problem. The last group [5, 18, 24, 46] uses different auctions systems to model the collaboration among transport companies.

7.1 Joint route planning

The work [15] estimates the synergy that can be provided by the combination of outsourcing and horizontal cooperation. Based on a joint route planning assumption, two situations are compared: original situation where all entities perform their orders individually or the use of a unique depot. The problem is modeled as a Vehicle Routing Problem with Time Windows (VRPTW). Both situations are solved based on heuristics of random insertion criteria considering time windows tightness, demand, distance and criticality of a given customer. Then two local search operators (ICROSS and IOPT) are applied. The article also describes the case of three Dutch companies of distribution of frozen products as a case study. The companies had a considerable amount of overlap between customers, on average 68 %. In the given case, they were able to provide a reduction of 30.8% of travelled distance and 50% reduction of fleet. However, in other initiatives mentioned the benefits range from 15% to 30%.

The work of [23] focuses on the identification of repeatable, dedicated truckload continuous move tours for companies that regularly send truckload shipments. The fundamental problem is based on the Lane Covering Problem (LCP). Since time considerations are critical to the practical viability of the tours, an extension of LCP is solved by specifically developed heuristics. The methodology is applied to a set of randomly generated Euclidean instances, and also a real-world case study is evaluated.

Also from joint route planning point of view, [53] compares collaborative versus non-collaborative scenarios from the last-mile perspective. Problems are modeled as Capacitated Vehicle Routing Problems (CVRP). The objective is to minimize the travel distance, but other metrics are also evaluated. The solution is based on two hierarchical heuristic approach. Benefits of the collaboration are clear and are transferred to all indicators evaluated. The methodology is applied with real data from the city of Bogotá, Colombia.

7.2 Game theory

In [43] the authors proposed a collaboration process between freight forwarding entities in three phases: preprocessing, profit optimization and profit sharing. In the preprocessing, each company evaluates its own requests and other collaborating companies requests. In the profit optimization phase the best joint solution is obtained. Finally, in the profit sharing phase the benefits of the coalition are distributed among companies based on the collaboration-advantage-index and the individual residual profit. The game theory formulation guarantees that collaboration will not imply losses to participants and, usually, benefits should be expected.

Later, in 2008, [44] proposed another solution for collaborating freight carriers, solely combining features of routing and scheduling problems with cooperative game theory. First, collaborative and non-collaborative scenarios are solved based on a Pickup and Delivery Problem with Time Windows (PDPTW). In the non-collaborative scenario, for each carrier a single depot PDPTW is solved by means of a heuristic. For the collaborative scenario, the multi-depot PDPTWs where all requests from all companies are merged, is solved in the same way. The solutions are compared to illustrate the potential of collaboration. Secondly, based on game theory, the use of the Shapely value is justified to propose an structure of profit sharing amongst collaborating carriers. Its good properties are the fairness, the uniqueness, the ease of implementation, and the stability of the solution. Finally, the computational results are applied to three artificial instances and one instance based on real data from a German freight forwarder with several autonomous profit centers which operate independently of each other.

Recently, [38] has proposed a three step approach to model collaborative logistics systems. First, a conceptual agent based model is described, second, a game theory method is applied for supplier and carrier collaboration independently. The correctness of the system is verified by formulating the problem mathematically.

Finally, [81] extends the study of horizontal logistics alliance when partners adopt a flexible attitude (allow changes to the terms of their deliveries). Apart from the cost decrease of the collaboration, the total cost can be further decreased if partners adopt a flexible attitude. The theoretical proposal is evaluated with a case study in Belgium, where cost gain is allocated with the Shapely value. Individual gains range from 20% to 37%. The literature review indicates that mistrust about the fairness of the chosen allocation rule has already caused many alliances to fail. Transparency and simplicity of uncomplicated rules of thumb have proven to be more likely to be adopted. In the long run, however, participants become frustrated and leave the collaboration. For that reason, the authors use game theory to provide decision support. Concepts like Nucleolus [72] or Shapley value [44] are used. The design of the decision support system, with the use of the Shapely value is proved to give incentives to partners to behave in a manner that is beneficial for the alliance.

7.3 Auction systems

An auction is a protocol that facilitates agents to indicate their interests in one or more resources. Based on agent indications the protocol determines both an allocation of resources and an allocation of payments among the agents. Different types of auctions have been proposed for horizontal collaboration in transportation.

The first use of a combined-value auction for transportation services, back in 1993, is reviewed in [47]. Combined-value auctions allow participants to make a bid of a single amount for a collection of items. The work combines the LCP with experimental economics. Sears Logistics Services[1] was the first company that sought to consolidate its use of trucking services and reduce costs. The company was using bilateral negotiations, which were time consuming and expensive. Instead, their experiment proved that combined-value auctions worked and it opened a door to subsequent applications.

In the case of [5], they propose a framework to maximize the overall profit of the network with few information and without decreasing the profit of the carriers. The paper concentrates on a model for pickup and delivery services simplified without considering capacity, which reduces to a Traveling Salesmen Problem (TSP) with precedence constraints, even though more

[1] http://www.slslogistics.com/

general models could be integrated in the framework. The collaborative framework is based on a decentralized control with auction based exchange mechanism. It consists of a mechanism to exchange requests between carriers and a corresponding cash flow model for the possible network transactions. It includes a scheme for calculating revenue, cost and profit, but a simple uniform sharing of the collaboration gains is applied. The framework is based on the reassignment of requests, based on the following 5-step procedure: form a request candidate set; compose bundles of candidate set; determine of marginal profits; assignment of bundles to carriers; and, profit sharing. These steps are repeated until no further improvement is possible. Steps 1 and 3 are individually autonomously performed by the individual carriers, and then the rest are centrally performed. Two algorithms are derived from the procedure. The collaborative situation is compared to the situation without collaboration and with a centralized control with full information. Results are evaluated through two performance indicators: the collaboration gain and the decentralization cost. A set of 90 instances are built and evaluated under the three scenarios. The work concludes that decentralized planning clearly diminishes the drawback of individual planning, although the cost of decentralization remains considerable compared to central planning.

Later, [84] presents a similar work where collaborating partners have limited capacities in their fleets. The route-based exchange mechanism is based on three steps: preprocessing, route generation process, and final winner determination. They assume, however, that all carriers offer all their requests for exchange, which is unrealistic in practice.

Based on [33], two main drivers for carrier collaboration are cost reduction and privacy preservation. In their article a double auction mechanism is proposed to enable consolidation through order sharing, based on single-round sealed-bid double action. It is assumed that the bidding is thruthful and no collusion is allowed. The auction starts and the carriers: i) post information about shearable orders (volume, destination,...); ii) inform the auctionneer their offer prices and the spare capacity in their vehicles; and, iii) search and bid for posted orders specifying a price. Then the winner determination is performed via a maximization of the actual cost saving attained by all participating carriers. Sensible information is kept only for auctioneers but orders are public for all other carriers.

A multi-agent and auction based framework approach is proposed in [18]. The authors state that the two main issues for carrier collaboration are the optimal reassignment of transportation requests among carriers and the fair allocation of the profit among the carriers. The problem each carrier faces is broken down in two problems: the selection of outsourcing requests (based on minimum profit margin) and the bidding of the outsourcing requests. Then, multiple auction processes of multiple outsourcing requests can happen at the same time, which are solved as an outsourcing requests selection problem. The performance of the framework is evaluated and compared through simulation on 20 random generated instances. Again, three scenarios are considered: no collaboration, decentralized collaboration and central collaboration. Results show that the proposed approach can increase the profit of each carrier and the total profit of all carriers in most instances through reducing transportation costs.

The work [24] focuses on making the collaborative framework dynamic and incentive compatible. The underlying problem is based on the pickup and delivery problem. It is assumed that shipments, reservation values, and reservation costs are submitted and processed by the collaborative mechanism in real time. The proposed mechanism is called second-price based dynamic collaborative, inspired by the one-item second-price auction. This type of auction is incentive compatible and achieves perfect efficiency. The results are simulated in a hypothetical square geographic region with four carriers. Three different performance measures are used to evaluate the performance of the carriers: average empty distance, number of shipments served and sum of individual carriers profit. The collaborative system clearly generates more profit than the system with independent carriers. Some assumptions were made, however, that can be difficult to match in practice: all vehicles and loads are compatible, collaborative mechanism implementation and operation costs are not considered; implementation and communication is faultless; and, in real-time, service areas of carriers overlap completely. In any case, it is clear that there could be substantial benefits from dynamic collaboration.

Stochastic bid price optimization for a carriers is proposed in [46]. They approach the problem with a simultaneous, independent, single-round, sealed-bid, first-price auctions of lanes aimed at maximizing the carrier's expected profit. Each carriers bidding strategy is supposed to take into account competitors bid pricing based on past bid statistics. The lowest competitor bid on each lane is modeled as a random variable with a particular probability distribution. Carriers cost are determined by the Length Constraint Lane Covering Problem (LCLCP). The results of the extension simulation study are promising, both in scientifically but also in a practical environment.

8 Problem definition and formulations

8.1 Problem definition

Collaborative economy is currently one of the major trends in innovation and development, and collaborative transportation is one of the proposals to attain cost reductions in the competitive market of urban transportation. From the private business perspective, collaborative agreements among transport operators can reduce the overall system cost, which somehow should be transferred to collaborating transport operators. In particular, with the existence of shared or nearby customers, the potential savings are promising.

We assume that a set of transport companies that operate in the same area are willing to collaborate to save distribution costs of the overall system. We also consider that the set of customers is placed in a urban environment with a high customer density. Thus, there will be a group of shared customers or nearby customers. I.e., there will be either a group of customers that receive goods from more than one transport company in the same time horizon, or groups of customers from different transport companies located close enough so one transport company can serve each group with one stop in the delivery route. In the described scenario, we consider that each company is willing to transfer a part of its loads to other companies, specifically, the part of its loads corresponding to shared or nearby customers. Customers will only be transferred when the transfer decreases the overall distribution costs. For the overall distribution costs we take into account two different costs: distribution costs of all carriers performing their routes and the routing costs due to the transportation of the loads of the transferred customers between the depots of the implied carriers.

We propose formulations to represent the described problem and branch-and-cut algorithms for solving them. Applying the previous methodology to a set of test instances, reductions can be estimated to quantify the potentialities of the solutions to the new models in different scenarios.

In the remaining of this section we give a formal definition of the problem and describe in detail the necessary assumptions. Let C denote a set of carriers operating in the same area and willing to transfer shared or nearby customers. Let also N denote the total group of costumers in the area. We assume that for each customer $u \in N$ the subset $C_u \subseteq C$ is the set of carriers that serve this customer. If $|C_u| = 1$ for $u \in N$ customer u is only served by carrier $i = C_u$. On the contrary, if $|C_u| > 1$ there are several carriers that serve customer u. In that case, we assume that the load for customer u can be served by any carrier $j \in C_u$. We call d_{ui} the demand of costumer u with respect to carrier i ($i \in C_u$). We consider that the demand d_{ui} for one customer u, from one carrier i has to be entirely served by one single carrier $j \in C_u$. This means that demand splitting is not allowed. I.e., if the demand for one customer from one carrier is transferred to another carrier, its total demand is served by only the new carrier. We also define the set of customers of carrier $j \in C$, $N_j = \{u \in N, j \in C_u\}$.

As an illustrative example, Figure 8.1 presents a simple case with only two companies (A, B). In this case we have three types of customers: customers with demand only from company A, customers with demand only from company B, and customers with demand from both.

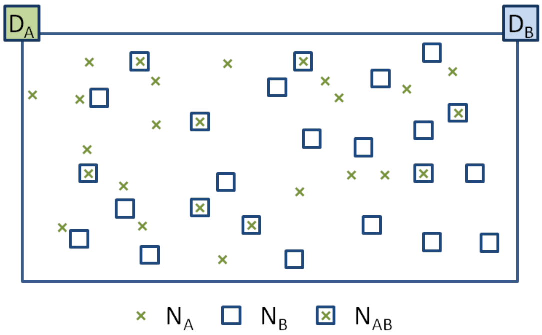

Figure 8.1: Market distribution with common costumers

We consider that each carrier has two types of costumers: a group of exclusive customers and a group of shared customers. Exclusive customers are customers that only have demand from one carrier and therefore they are not considered for collaboration, i.e, only their own carrier can serve them. Shared customers are costumers whose demand can be transferred and served by only one of the companies for which the customer has demand. The problem is an extension of the classical VRP, in which exclusive costumers can only be served by one single company, from its depot and with its vehicles, and a group of shared customers can be served by a subset of companies, from their depots and vehicles. In case the demand of a customer is transferred between two carriers, then there is the need to use some vehicle to transfer between companies depots the demand before the daily routing. The objective of the problem is to minimize the total distribution costs of all the companies participating in the collaboration. This includes: routing costs for each of the carriers and transfer costs among carriers depots to previously place the transferred demand in the depot of the company that will finally serve the load to the customer. We call this problem, the Shared Customer Collaboration Vehicle Routing Problem (SCC-VRP).

We propose two different formulations for the SCC-VRP. The first one is the most intuitive and it is based on the use of variables that describe explicitly the arcs traversed by the vehicles routes to serve the demand. This formulation needs to include Subtour Elimination Constraints (SEC). Due to the exponential number of this family of constraints, in the proposed branch-and-cut algorithm they are included in the formulation as lazy constraints. The first formulation is also strengthened with the reinforced capacity cuts. The second formulation avoids the use of specific variables for each vehicle and is based on the carriers load at each visited vertex. This avoids the need of SECs. For the sake of clarity, we first present a load based model for the single company Capacitated Vehicle Routing Problem (CVRP), and then the load based formulation is extended for the SCC-VRP.

8.2 Vehicles based formulation for the SCC-VRP

As already mentioned, the problem is a modification of a VRP. We adapt the formulation for the VRPTW proposed in [20] to the SCC-VRP. The model is based on variables defining the arcs traversed by each of the vehicles.

We consider an unlimited fleet of homogeneous vehicles K with capacity Q. Specifically, we have a set of vehicles K_i for each carrier i, and $K = \bigcup_{i \in C} K_i$. This is indeed a simplifying assumption as carriers only have a limited number of vehicles. Nonetheless, in this first approach to the problem we make this assumption to disregard this constraint. To formulate the problem we define an auxiliary complete directed network $G = (V, A)$. V is the set of customers and the depots, and A is the set of arcs connecting each pair of customers and each customer with the depots. The vehicles of each company $(i \in C)$ have its origin and destination in depot $d(i)$ so $V = N \cup_{i \in C} \{d(i)\}$ and A is the set $V \times V$. Given $u,\ v \in N$, c_{uv} are the travel costs between customers u and v. Finally, we denote by c the cost per in advance transfer between any pair of depots. The main variables are routing variables x^k_{uv}, $(u, v) \in A$, $k \in K$, which take the value 1 if arc (u, v) is used by vehicle k and 0 otherwise. Note that this variable is indirectly related with a carrier, since each route $k \in K_j$ for a given carrier. Note also that this variable can not take value 1 when customers do not have at least one common carrier, i.e., $x^k_{uv} = 0$ if $C_u \cap C_v = \emptyset$.

To adapt the formulation to our problem we need to define some additional variables: z^k_{uij} with value 1 if demand of costumer $u \in N$ from carrier $i \in C_u$ is served by company $j \in C_u$ in route $k \in K_j$ and 0 otherwise. We consider that there are some vehicles that carry out round trips between pairs of companies depots with transferred demand. We use the following variables to check whether or not a trip takes place between an specific pair of depots. Let y_{ij} with $i, j \in C : j > i$ be a binary variable that is activated if there is some demand of carrier i served by carrier j or vice versa . We call v_c the overall number of in advance transfers between pairs of carriers depots. Then, the formulation based on vehicles variables for the CSS-VRP:

$$\text{minimize} \quad \sum_{k\in K}\sum_{(u,v)\in A} c_{uv}x^k_{uv} + c v_c \tag{8.1}$$

$$\text{subject to} \quad \sum_{i\in C_u}\sum_{k\in K_i}\sum_{v\in N\cup\{d(i)\}} x^k_{uv} \geq 1 \qquad u\in N \tag{8.2}$$

$$\sum_{v\in N\cup\{d(i)\}} x^k_{d(i)v} \leq 1 \qquad i\in C,\ k\in K \tag{8.3}$$

$$\sum_{u\in N\cup\{d(i)\}} x^k_{uv} - \sum_{u\in N\cup\{d(k)\}} x^k_{vu} = 0 \qquad i\in C, k\in K,\ v\in N \tag{8.4}$$

$$\sum_{j\in C_u}\sum_{k\in K_j} z^k_{uij} = 1 \qquad u\in N,\ i\in C_u \tag{8.5}$$

$$\sum_{v\in N\cup\{d(j)\}} x^k_{uv} \geq z^k_{uij} \qquad u\in N,\ i,j\in C,\ k\in K_j \tag{8.6}$$

$$\sum_{u\in N}\sum_{i\in C_u} z^{k-1}_{uij} \geq \sum_{u\in N}\sum_{i\in C_u} z^k_{uij} \qquad j\in C,\ k\in K_j\setminus\{\min_k K_j\} \tag{8.7}$$

$$\sum_{u\in N}\sum_{i\in C_u} d_{ui}z^k_{uij} \leq Q \qquad j\in C,\ k\in C_j \tag{8.8}$$

$$z^k_{uij} + z^k_{uji} \leq y_{ij} \qquad u\in N,\ i,j\in C, j>i,\ k\in K_j \tag{8.9}$$

$$v_C = \sum_{i\in C}\sum_{j\in C, j>i} y_{ij} \tag{8.10}$$

$$x^k(\delta(W)) \geq x^k(\delta^-(u)) \qquad W\subset V\setminus\{v_d\},\ k\in K,\ u\in W \tag{8.11}$$

$$x^k_{uv} \text{ binary}, \qquad j\in C,\ u,v\in N_j,\ k\in K_j \tag{8.12}$$

$$z^k_{uij} \text{ binary}, \qquad u\in N, i,j\in C_u, k\in K_j \tag{8.13}$$

$$y_{ij} \text{ binary}, \qquad i,\ j\in C\ ,\ j>i \tag{8.14}$$

$$v_C \text{ integer} \tag{8.15}$$

This is a MILP formulation where the objective function (8.1) represents the total cost. Total cost is the sum of the distribution costs of all companies, plus a fixed unit cost (c) per transfer between each pair of carriers depots. Constraints (8.2) impose that each costumer is visited at least once by the a vehicle of some company serving the customer. Constraints (8.3)–(8.4) describe the flow on the path used by vehicle k. Constraints (8.5)–(8.6) relate variables x and z. They guarantee that the demand for customer u from company i is assigned to one company j and vehicle k. If demand for customer u from company i is served by its own company, then z^k_{uii} is equal to one, and there is no transfer. Furthermore, we guarantee that if the assignment is made to a given company and route (z^k_{uij}), some corresponding routing variable (x^k_{uv}) is activated. Constraints (8.7) avoid symmetry in the solution. We have an ordered set of vehicles per company, and we do not allow to use a new one if the previous one is not being used. Moreover, we order the routes by the number of customers visited. We force that if a vehicle is used, the next vehicle of the company has fewer customers. Constraints (8.8) ensure that the capacity of the vehicles is not exceeded. Constraints (8.9)–(8.10) check the need of number of in advance transfer between pairs of carriers depots between depots in order to compute this transfer cost in the objective function. Constraints (8.11) eliminate subtours. The domains of the variables, and binary/integer conditions are given in (8.12)–(8.15). In the Appendix B, several tables summarize the details of the presented formulation. Table B.1 presents the parameters and the sets used and Table B.2 summarizes the variables. Finally, Table B.3 counts the number of variables and constraints of the formulation.

8.3 The single company VRP

The solution for a single company VRP is the classical CVRP with only one depot. In the following formulation, we avoid the use of index for vehicles (k). Instead, we use variables that control the load of the routes at the different arcs. This was inspired by the work of [48, 2], with different formulations for the CVRP and the Multi-period Vehicle Routing Problem with Due-dates.

Again, we define an auxiliary complete directed network $G = (V, A)$, where V is the set of customers plus the depot (D), in this case a unique depot for the single company, and A is the set of arcs connecting each pair of customers and each customer with the depot. The main design variables are x_{uv}, $(u, v) \in A$, which takes the value one if some route uses arc (u, v). Then, l_{uv} is the load of the vehicle through arc (u, v). We use load variables to keep track of the load on the routes. The CVRP can be formulated as follows:

$$\text{minimize} \quad \sum_{(u,v)\in A} c_{uv} x_{uv} \tag{8.16}$$

$$\text{subject to} \quad \sum_{v\in N\cup D} x_{uv} = 1 \qquad u \in N \tag{8.17}$$

$$\sum_{u\in N\cup D} x_{uv} - \sum_{u\in N\cup D} x_{vu} = 0 \qquad v \in N \tag{8.18}$$

$$l_{uv} \leq Q x_{uv} \qquad (u, v) \in A \tag{8.19}$$

$$\sum_{v\in N\cup D} l_{vu} - \sum_{v\in N\cup D} l_{uv} = d_u \qquad u \in N \tag{8.20}$$

$$\sum_{v\in N} l_{Dv} - \sum_{v\in N} l_{vD} = \sum_{u\in N} d_u \tag{8.21}$$

$$x_{uv} \text{ binary,} \qquad (u, v) \in A \tag{8.22}$$

$$l_{uv} \geq 0 \qquad u, v \in N \tag{8.23}$$

This is a MILP where the objective function (8.16) represents the distribution cost. Constraints (8.17) impose that each costumer has to be visited exactly once. Constraints (8.18) describe the flows of the routes for the single company. Constraints (8.19) make sure via load variables that the capacity of the vehicles is not exceeded. Constraints (8.20)–(8.21) guarantee the fulfillment of the demand for each customer, and relate load variables arriving and leaving each node with the demand at the given node. Definition of the variables, and binary conditions and bounds are given in (8.22)–(8.23). The summary of the parameters and sets of this formulation can be found in the Appendix B in Table B.6.

8.4 Load based formulation for the SCC-VRP

The load formulation of the previous section is extended below to the SCC-VRP. We avoid the use of an specific index for vehicles (k), and use instead load variables to control the load of the routes. In addition, we use assignment variables that keep track of the company that carries out the service to a given customer demand.

Again, we define an auxiliary complete directed network $G = (V, A)$, where V is the set of customers plus depots, and A is the set of arcs connecting each pair of customers and each customer with the depot. Each company has its own depot, and its vehicles have their origin and destination at this given depot ($d(i)$), $i \in C$. Then, $N = V \bigcup \{\cup_{i\in C}\{d(i)\}\}$ and A is $V \times V$. Again, c_{uv} denotes the cost of traveling between customers $u, v \in N$.

The main design variables are x^i_{uv}, $(u, v) \in A$, $i \in C$, which take the value 1 if arc (u, v) is used by a route of company i and 0 otherwise. Note again, that if customer u and customer v do not have common companies serving them ($C_u \cap C_v = \emptyset$), this variable can not take value 1. We use load variables to keep track of the load on the arcs. Let l^i_{uv} be the load through arc $(u, v) \in A$ of a vehicle from company $i \in C_u \cap C_v$. We also use variables that assign the demand of customer u from company i, to the company that finally serves that demand j. Let these variables be: z_{uij}, where $u \in N$ and $i, j \in C_u$.

Finally, similarly to the vehicle based formulation, we consider that there are some vehicles that perform a round trip between the depots of pairs of companies that need to transport in advance the transferred demand. We use such variables to check if the trip between depots is performed or not. Let y_{ij} with $i, j \in C_i \cap C_j \neq \emptyset : j > i$ be a binary variable that is activated if part of the demand of carrier i will be served by carrier j or vice versa. As before, we denote by v_C the number of transfers needed between depots to transport goods in advance. The load based formulation is the following:

$$\text{minimize} \quad \sum_{(u,v)\in A} c_{uv} \sum_{i\in C} x^i_{uv} + c v_C \tag{8.24}$$

$$\text{subject to} \quad \sum_{i\in C_u} \sum_{v\in N\cup\{d(i)\}} x^i_{uv} \geq 1 \qquad u \in N \tag{8.25}$$

$$\sum_{v\in N\cup\{d(i)\}} x^i_{uv} \leq 1 \qquad u \in N,\ i \in C_u \tag{8.26}$$

$$\sum_{u\in N\cup\{d(i)\}} x^i_{uv} - \sum_{u\in N\cup\{d(i)\}} x^i_{vu} = 0 \qquad v \in N,\ i \in C_v \tag{8.27}$$

$$\sum_{j\in C_u} z_{uij} = 1 \qquad u \in N,\ i \in C_u \tag{8.28}$$

$$\sum_{v\in N\cup\{d(j)\}} x^j_{uv} \geq z_{uij} \qquad u \in N,\ i, j \in C, \tag{8.29}$$

$$l^i_{uv} \leq Q x^i_{uv} \qquad (u, v) \in A,\ i \in C \tag{8.30}$$

$$\sum_{v\in N\cup\{d(i)\}} l^i_{vu} - \sum_{v\in N\cup\{d(i)\}} l^i_{uv} = \sum_{j\in C_u} d_{uj} z_{uji} \qquad u \in N,\ i \in C \tag{8.31}$$

$$\sum_{v\in N} l^i_{d(i)v} - \sum_{v\in N} l^i_{vd(i)} = \sum_{u\in N} \sum_{j\in C_u} d_{uj} z_{uji} \qquad i \in C \tag{8.32}$$

$$z_{uij} + z_{uji} \leq y_{ij} \qquad u \in N,\ i \in C,\ j \in C | j > i \tag{8.33}$$

$$v_C = \sum_{i\in C} \sum_{j\in C|j>i} y_{ij} \tag{8.34}$$

$$x^i_{uv} \text{ binary,} \qquad i \in C,\ u, v \in N_i, \tag{8.35}$$

$$z_{uij} \text{ binary,} \qquad u \in N,\ i, j \in C_u \tag{8.36}$$

$$l^i_{uv} \geq 0 \qquad i \in C, u, v \in N_i, \tag{8.37}$$

$$y_{ij} \text{ binary,} \qquad i, j \in C,\ j > i \tag{8.38}$$

$$v_C \text{ integer} \tag{8.39}$$

This is a MILP where the objective function (8.24) represents the total distribution cost. Constraints (8.25)–(8.26) impose that each costumer is visited at least once by some company, and that each company can visit each customer at most one. Constraints (8.27) describe the flows on the routes for each company. Constraints (8.28)–(8.29) relate variables x and z. Furthermore, we guarantee that if the assignment is made to a given company (z_{uij}), some of the corresponding flow variables (x^i_{uv}) is activated. Constraints (8.30–8.32) make sure that the capacity of the vehicles is not exceeded, using load variables. Constraints (8.33)–(8.34) check the need of vehicles connecting depots, in order to compute this transfer cost in the objective function. The domains of the variables is given in (8.35)–(8.39). In the Appendix B, several tables summarize the details of the presented formulation. Table B.1, similarly to the previous formulation presents the parameters and the sets used, except sets K and KT that are not used here. Table B.4 summarizes the variables and Table B.5 counts the number of variables and constraints of the formulation.

9 Computational Experiments

In this chapter we describe the computational experiments we have run to analyze and compare the two formulations proposed in the previous chapter. First, data generation is described in Section 9.1. Then, the branch-and-cut for the vehicle based formulation is explained in Section 9.2, where we explain the separation procedure of capacity constraints. The results are analyzed in Section 9.3. In Section 9.4 the numerical results obtained with the vehicle based formulation and the load based formulation are presented and analyzed. Finally, the optimal results with collaboration are compared to the solutions without collaboration in Section 9.5.

Models have been implemented in the Optimization Programming Language `OPL` and solved with the commercial software `CPLEX` 12.1. All experiments have been run on a PC limited to 1 thread running at 2.6GHz and 60GB of RAM. In all the experiments the computing time has been limited to one or two hours. We consider that the computing effort is low, given our problem is a variation of the well-known complex VRP. In this case our priority is not to solve instances to optimality but to provide a context for comparison with no collaboration policy.

9.1 Data generation

Since we are not aware of any benchmark instances that could be used in our experiments, we generated a set of 12 test instances based on the instances proposed by Cordeau [12] for the Multiple Depot Vehicle Routing Problem (MDVRP). For the experiments, we used two carriers, thus, we selected the instances with two depots, to assign one depot to each carrier. To obtain instances of sizes that we could afford, we limited the number of customers to values between 18 and 30, and used the corresponding data from the original instances. Then, we set 25% of customers as shared. Their demand is split between the two carriers. The first half of the remaining customers is assigned to the first carrier, and the second half to the second carrier. The characteristics of the instances are summarized in Table 9.1, and Figures E.1 and E.2 in the Appendix E give the plots of the 12 instances.

The above set of 12 instances has been used to test the different formulations and the efficiency of the procedures applied to separate the large families of constraints. To further discuss the potential savings under different situations a larger set of instances was randomly generated with different characteristics, as described below.

We consider two types of instances: a set of instances where customers are randomly located in a square of 100 units of side, and a set of instances with clustered customers, where each instance has between 3 and 5 clusters in a square of 100 units and customers are located around one of the clusters. The total number of customers is either 10, 15, 20, 25 or 30, and each customer has a probability of 0.25 of being a shared customer. Each customer has an integer

Instance	Q	$\sum_{i\in N} d_{iA}$	$\sum_{i\in N} d_{iB}$	$\sum_{i\in N} d_{iA}+d_{iB}$	$\|N\|(\|A\|,\|B\|)$	Shared
1	100	152 (42)	176 (39)	328	23 (12, 17)	6
2	100	201 (55)	196 (53)	397	29 (16, 19)	6
3	100	163 (15)	110 (13)	273	20 (14, 9)	3
4	100	176 (37)	136 (34)	312	23 (16, 14)	7
5	100	179 (53)	173 (52)	352	24 (16, 16)	8
6	200	61 (25)	213 (22)	274	21 (8, 17)	4
7	200	93 (50)	180 (48)	273	20 (10, 16)	6
8	200	189 (57)	210 (54)	399	30 (17, 19)	6
9	500	585 (113)	325 (111)	910	20 (14, 9)	3
10	500	524 (56)	386 (54)	910	20 (14, 10)	4
11	60	56 (20)	112 (20)	168	18 (8, 14)	4
12	60	40 (20)	136 (20)	176	20 (8, 17)	5

Table 9.1: Data summary of the instances

uniform demand between 5 and 50, with groups of instances with lower demand per customer and groups of instances with higher demand per customer. In particular, the demand of customers has been generated randomly from a uniform distribution. Three different types of instances have been generated, uniform in [5, 20], [10, 35] and [25, 50]. Accordingly, capacity is: 100, 200, 300, 400 or 500. Depots are located at two different extremes of the square, i.e. one is located at position (0,0) and the other one at (100,100). The transport costs between two given customers is the Euclidean distance and the transfer costs between the depots is fixed at 50. The specific details of each of the instances can be found in Tables E.1 and E.2 of the Appendix E.

9.2 Branch-and-cut for the vehicle based formulation

The vehicle based formulation for the SCC-VRP will be solved with a branch-and-cut algorithm where the exponential family of inequalities will be sequentially separated when they are identified. Also capacity constraints will be separated as we next explain.

9.2.1 Separation of Subtour Elimination Constraints

In the vehicle based formulation, we need to impose constraints (8.11) to avoid the creation of cycles in the solution. There is, however, an exponential number of such constraints, so they are not included in the initial formulation, but they are separated as lazy constraints whenever an integer solution is found. Then, in order to know whether a subtour exists, it is enough to check the connected components of each route. If some connected component does not contain the corresponding depot, there is a violated SEC. An algorithm based on Recursive Deep First Search is used for computing the connected components. Being S a connected component of route k that does not contain the depot, the associated lazy constraint is:

$$\sum_{u,v\in S} x^k_{uv} =< |S|-1 \tag{9.1}$$

9.2.2 Separation of capacity inequalities

The vehicle formulation contains capacity constraints (8.8) which can be seen as knapsack constraints. The usual procedure to separate these constraints has been implemented. We define the knapsack problem and some necessary concepts. Then, the procedure to separate cover inequalities based on knapsack constraints is presented. And finally the application of the procedure in the constraints of the formulation is discussed.

The knapsack problem is a classical problem in combinatorial optimization. Given a set of items, each with a mass and a value, determine the items to include in a collection so that the total weight is less than or equal to a given limit and the total value is as large as possible.

The general formulation is as follows:

$$\underset{x}{\text{maximize}} \sum_{j \in N} c_j x_j \tag{9.2}$$

subject to:

$$\sum_{j \in N} a_j x_j \leq a_0 \tag{9.3}$$

$$x_j \in \{0,1\}, \; j \in N \tag{9.4}$$

where $N = \{1,2,3,\ldots,n\}$; $a = (a_1, a_2, \ldots, a_n)$, a_0. And, we assume, without loss of generality that:

$$a_j \leq a_0, \; j \in N \tag{9.5}$$

$$c_j \geq 0, \; a_j \geq 0, \; \forall j \in N \tag{9.6}$$

$$a_1 \geq a_2 \geq \ldots \geq a_n \tag{9.7}$$

A *cover* for N is a subset $S \subseteq N$ that fulfills the following condition $\sum_{j \in S} a_j > a_0$. The cover subset, generates a cover constraint. For each subset S, we have the following constraint: $\sum_{j \in S} x_j \leq |S| - 1$. Minimal covers are covers that if we delete any element of the set, they are no longer a cover. I.e., if S is a cover, we say it is minimal $\iff$ $S \setminus \{k\}$ is not a cover $\forall k \in S$. It is proven that inequalities associated to minimal covers dominate other covers. Given a minimal cover S, we can build an extension of this set, S, called $E(S)$. The set is enlarged by the elements where its associated weight is greater than the weight of all elements in the set. $E(S) = S \bigcup \{j \in N \setminus S : a_j \geq a_k \; \forall k \in S\}$. With this extension, the canonic inequalities can be builded: $\sum_{j \in E(S)} x_j \leq |S| - 1$.

In order to separate inequality covers from a given solution that has to fulfill a knapsack constraint, we have the following procedure based on [60]:

Given $\hat{x}$ a fractional solution, there is a subset $S \subseteq N$ that $\sum_{j \in S} \hat{x}_j \leq |S| - 1 \iff$

$$v = \underset{x}{\text{minimize}} \sum_{j \in N} (1 - \hat{x}_j) s_j < 1 \tag{9.8}$$

subject to:

$$\sum_{j \in N} a_j s_j \geq a_0 + 1 \tag{9.9}$$

$$s_j \in \{0,1\}, \; j \in N \tag{9.10}$$

So, if $s*$ is the optimal solution of the previous problem. When $v < 1$, any minimal cover $S \subseteq \{j \in N : s_j* = 1\}$ defines a violated cover inequality $\sum_{j \in S} x_j \leq |S| - 1$. From cover inequalities, we can further obtain facets. Given $S \subseteq N$ a minimal cover. Then, $\sum_{j \in S} x_j \leq |S| - 1$ is facet of $\text{conv}(P) \bigcap \{x \in \{0,1\}^n : x_j = 0, j \in N \setminus S\}$. We can find the facet of $\text{conv}(P)$, lifting the constraint, which needs the solving the following sequence of problems:

Let $S \subseteq N$ be minimal cover, and $N \setminus S = \{j_1, j_2, \ldots, j_s\}$ $(s = |N \setminus S|)$ arbitrarily ordered set. If we consider, the following set of problems:

$$z_{j_i} = \max_x \sum_{j \in S} x_j + \sum_{j=j_1}^{j=j_i-1} \beta_j x_j \tag{9.11}$$

subject to:

$$\sum_{j \in S} a_j x_j + \sum_{j=j_1}^{j=j_i-1} a_j x_j \leq a_0 - a_j \tag{9.12}$$

$$s_j \in \{0,1\}, \; j \in S \cup \{j_1, j_2, \ldots, j_{i-1}\} \tag{9.13}$$

The solutions determine the subsequence values $\beta_j = |S| - 1 - z_j$. Then, the following constraint is a facet of $\text{conv}(P)$.

$$\sum_{j \in S} x_j + \sum_{j=j_1}^{\mathbf{j=j_s}} \beta_j x_j \leq |S| - 1 \tag{9.14}$$

The vehicle formulation (8.1)–(8.15) contains capacity constraints (8.8), which can be seen as knapsack constraints. In order to separate these constraints in our problem, we use the previous procedure. Given a solution of the LP relaxation, CPLEX enters a user cut callback where we solve the associated problem to find a violated cover constraint. To solve the associated problem, we use a heuristic proposed by [14] where items are inserted in S, in non-decreasing order of $(1 - x_j)/a_j$ until a cover is obtained. The heuristic is proved to be $O(n\log(n))$. Then, the cover is checked for the minimal property, and if not, it is reduced. Once we have the minimal cover constraint, we try to lift it and obtain a facet of $\text{conv}(P)$ solving the sequence of small problems with CPLEX solver in the callback. Then, we add the facet and let CPLEX continue with the solving procedure.

9.3 Experiments for the vehicle based formulation

In this section we evaluate the effect of separation of knapsack constraints of the vehicle based model to decide how often the separation procedure should be applied. CPLEX tries to apply different types of cuts whenever it believes it might be useful. For that reason, we first check the effect of CPLEX cuts in the vehicle based model. In Table 9.2, the vehicle formulation with SECs implemented as lazy constraints is tested with different cut parameter values for CPLEX. The results where CPLEX applies cuts freely, are compared to the results where CPLEX does not use cover cuts, and the performance of CPLEX without the application of any type of cuts. Under the column OPT R, the status at the end of the optimization is given. In this case TL means that all instances reach the time limit before proving optimality of the current best solution. Under the column Obj, the value of the best feasible solution found is presented. As can be seen, better feasible solutions are obtained when generating no cuts at all. Therefore, for the following experiments CPLEX cuts are deactivated.

	Cuts applied freely		Cover cuts forbidden		All cuts forbidden	
Inst	Opt R	Obj	Opt R	Obj	Opt R	Obj
1	TL	650.82	TL	421.37	TL	416.97
2	TL	NoSolEx	TL	858.48	TL	578.22
3	TL	279.45	TL	NoSolEx	TL	NoSolEx
4	TL	824.36	TL	478.46	TL	466.65
5	TL	580.44	TL	NoSolEx	TL	430.87
6	TL	280.19	TL	289.83	TL	297.47
7	TL	174.75	TL	269.1	TL	162.22
8	TL	582.55	TL	870.64	TL	424.22
9	TL	837.01	TL	644.74	TL	716.98
10	TL	1040.29	TL	670.01	TL	727.81
11	TL	524.87	TL	522.93	TL	507.10
12	TL	839.02	TL	957.08	TL	820.92

Table 9.2: Results from vehicle based formulation with different cut parameter for `CPLEX`

Then, the vehicle based formulation is tested on the 12 instances with different strategies for separating the knapsack type constraints. Results are summarized in Table 9.3, where columns have the following information. Now the entries of Column Opt R are either TL, when the time limit was reached without proving optimality of the best solution found, or K, which means that the process was killed, in most of the cases due to lack of available memory. The second column (Obj) shows the value of the best solution found when the processed finished, or "Memo" when the process was killed due to lack of available memory. Column T(s) shows the computing time in seconds. The last column, UC, is the number of user cuts applied with `CPLEX` before the end of the process (including lazy constraints). Note that to perform this test, all `CPLEX` cuts have been deactivated. Results show that the best strategy is to only separate the knapsack type constraints at the root node.

We also compared value of the linear relaxation at the root node when knapsack constraints were separated and when they were not. We could even observe that for some instances the value of the linear relaxation did not change if knapsack constraints were separated. We can conclude that the best performance of vehicle based formulation is achieved when CPLEX cuts are deactivated and when we separate knapsack type constraints only at the root node.

	Every Node				Every 100 nodes				Every 500 nodes				Only root node			
Inst	Opt R	Obj	T (s)	UC	Opt R	Obj	T (s)	UC	Opt R	Obj	T (s)	UC	Opt R	Obj	T (s)	UC
1	K		71		K	Memo	1,016	9,420	TL	397.46	3,600	15,086	TL	368.15	3,600	13,862
2	K	Memo	106	278	K		998		K		2,640		TL	555.56	3,600	15,992
3	K	Memo	198	2,726	K	Memo	760		K	328.9	3,600	19,209	TL	345.29	3,600	20,274
4	K	Memo	123	700	TL	695.26	3,600	19,279	TL	610.55	3,600	22,552	TL	552.38	3,600	20,197
5	K	Memo	146	966	K		396		TL	Memo	2,113	10,219	TL	425.65	3,600	10,676
6	TL	334.34	3,600	9,901	TL	330.74	3,600	11,548	TL	330.74	3,600	12,209	TL	326.16	3,600	12,202
7	TL	162.40	3,600	5,806	TL	162.22	3,600	6,018	TL	162.22	3,600	6,155	TL	162.22	3,600	6,166
8	K		3,600		K		1,585		K	Memo	2,659	10,724	TL	424.22	3,600	11,239
9	K	Memo	289	2,907	K		1,265		K	732.87	3,600		TL	716.98	3,600	11,422
10	K	Memo	485	2,264	TL	854.13	3,600	9,969	TL	598.24	3,600	13,081	TL	727.81	3,600	11,351
11	K		1,534		TL	509.32	3,600	1,941	TL	509.06	3,600	2,066	TL	507.10	3,600	1,819
12	TL	801.68	3,600	8,817	TL	797.59	3,600	10,873	TL	820.92	3,600	10,937	TL	820.92	3,600	11,017

Table 9.3: Results from vehicle based formulation with different frequencies of knapsack separation constraints

9.4 Vehicle based formulation vs load based formulation

The results of the vehicle based formulation are compared to results of the load based formulation in Table 9.4. New columns r_A, r_B indicate the number of vehicles needed by each carrier in the optimal/best-known solution. The results of load based formulation clearly outperform the vehicle based formulation. The load based formulation is able to provide 10 optimal solutions, and better feasible solutions for the two instances where it reaches the time limit. The number of variables in the load based formulation is lower since we eliminate the (k) index, thus we decrease by k the number of variables. Furthermore, the load variables avoid the need of subtour elimination constraints. As we have seen, the number of subtour elimination constraints is exponential. As a consequence, if the formulation can avoid them but still produce optimal solutions, it is preferable. For the instances of the problem proposed, the load based formulation can with fewer variables produce optimal solutions.

	Vehicle based form					Load based form				
	Opt R	Obj	r_A	r_B	T(s)	Opt R	Obj	r_A	r_B	T (s)
1	TL	368.15	2	2	3,600	Opt	293.77	2	2	175.93
2	TL	555.56	2	2	3,600	TL	341.32	2	2	3,600
3	TL	345.29	2	2	3,600	Opt	253.02	2	1	195.91
4	TL	552.38	2	2	3,600	Opt	342.3	2	2	14.31
5	TL	425.65	2	2	3,600	Opt	340.28	2	2	843.39
6	TL	326.16	1	2	3,600	Opt	250.08		2	35.1
7	TL	162.22	1	1	3,600	Opt	170.5	1	1	4.92
8	TL	424.22	2	2	3,600	Opt	257.83	1	1	16.13
9	TL	716.98	2	2	3,600	Opt	412.06	1	1	4.97
10	TL	727.81	2	1	3,600	Opt	475.71	1	1	16.09
11	TL	507.10	1	2	3,600	Opt	506.9	1	2	4.41
12	TL	820.92	1	2	3,600	TL	769.79	1	2	3,600

Table 9.4: Solutions for vehicle based and load based formulation

9.5 Collaboration vs non-collaboration policies

Below we compare the results of the 12 instances solved by the load based formulation to the solutions of the same instances when no collaboration exists, so each company serves all its customers independently from the other company. The formulation proposed in Section 8.3 is used to solve the same instances with two individual carriers. Table 9.5 presents the solutions for the 12 instances with the load based formulation for the CSS-VRP (with an extended time limit of 6 hours for instances 2 and 12). The results of the load based formulation for the independent carrier are also presented in Table 9.5, solved independently for each company A and B. Cost savings range from a maximum of 19.1 % until a minimum of 0, when no collaboration happens even it is possible. The average cost reduction is 7.5%. In terms of number of vehicles, the collaborative solution allows to reduce the fleet size in several instances, never increasing it.

Inst	Non-collaboration (A,B)					Collaboration					
	Opt R	Obj	r_A	r_B	time (s)	Opt R	Obj	r_A	r_B	time (s)	- %
1	Opt, Opt	171.38+168.02 = 339.4	2	2	1.88+2.51	Opt	293.77	2	2	204.92	15.5
2	Opt, Opt	173.37+179.8 = 353.17	3	2	1.31+3.61	TL	337.42	2	2	21,600	4.7
3	Opt, Opt	158.56+113.45 = 272.01	2	2	25.74+0.18	Opt	253.02	2	1	218.7	7.5
4	Opt, Opt	194.97+173.14 = 368.11	2	2	1.56+0.72	Opt	342.3	2	2	16.9	7.5
5	Opt, Opt	171.94+191.66 = 363.6	2	2	0.26+7.32	Opt	340.28	2	2	59.42	6.9
6	Opt, Opt	107.48+145.87 = 253.35	1	2	0.03+0.10	Opt	250.08		2	21.23	1.3
7	Opt, Opt	65.8+104.7 = 170.5	1	1	0.04+0.42	Opt	170.5	1	1	6.52	0
8	Opt, Opt	118.65+161.41 = 280.06	1	2	0.12+0.12	Opt	257.83	1	1	26.74	8.6
9	Opt, Opt	296.37+194.45 = 490.82	2	1	0.18+0.04	Opt	412.06	1	1	2.64	19.1
10	Opt, Opt	293.45+230.38 = 523.83	2	1	0.14+0.16	Opt	475.71	1	1	12.31	10.1
11	Opt, Opt	203.49+329.21 = 532.7	1	2	0.02+0.75	Opt	506.9	1	2	7.73	5.1
12	Opt, Opt	243.27+556.01 = 799.28	1	3	0.05+3.46	Opt	769.8	1	2	6,117	3.8

Table 9.5: Solutions for collaboration and non-collaboration policies

The extended set of test instances has been solved under the same assumptions. Unfortunately in these instances, the formulation obtains less optimal solutions, especially when the size of the problem grows. Table 9.6 shows the aggregate results with the collaboration load based formulation. Each row corresponds to 20 instances generated with the number of customers given under the first column (N), 10 random and 10 clustered. Under the column Opt R there is the number of optimal obtained solutions with the formulation. Under Avrg. Obj, there is the average value of the objective function (either the optimal value or the best-known solution at termination time). The last column of each block contains the average time in seconds devoted to each instances.

	Random			Clustered		
N	Opt R	Avrg. Obj	T(s)	Opt R	Avrg. Obj	T (s)
10	10	586	15	8	442	1,438
15	10	666	59	8	470	4,400
20	4	796	4,696	0	539	7,200
25	2	931	6,100	0	700	7,200
30	0	1051	7,200	0	816	7,200

Table 9.6: Summary of solutions for the extended set

Random instances of 10 and 15 customers are always solved to optimality, but in the clustered set not all of them can be solved. Instances with a higher number of customers can not be always optimally solved. Only 4 and 2 random instances with respectively 20 and 25 customers have been solved to optimality. These results indicate that additional research should be dedicated to improve the proposed load formulation so as to make it possible to solve instances of bigger sizes.

Still, the obtained results allow us to compare the potential savings that could be obtained via collaboration. For this, the solutions obtained previously with the collaboration model are compared with the solutions of the individual companies. In the cases where no provable optimal solution is found, the value used for the comparison is the value of the best-known solution found within the time limit. The instances are nearly always optimally solved. The model without collaboration could optimally solve 194 of the 200 benchmark instances considered. Table 9.7 summarizes the percentage savings obtained in each group of instances. Note that this is a lower bound on the potential savings, since some collaborative solutions are not proven to be optimal. The number of optimal solutions obtained in each set of instances is again under column Opt R. Column Opt Indi gives the number of instances solved to optimality. For each collaborative instance, we need to solve two "individual" instances, one for each company. Column -% gives the average cost savings in the set of instances.

	Random			Clustered		
N	Opt R	Opt Indi	-%	Opt R	Opt Indi	-%
10	10	20	2.2	8	20	1.4
15	10	20	6.0	8	20	2.7
20	4	20	9.2	0	20	7.6
25	2	20	7.8	0	19	5.8
30	0	17	9.2	0	18	8.0

Table 9.7: Solutions for collaboration and non-collaboration policies in the extended set

The results of the smaller instances show small savings (1.2-6%). The number of customers is reduced and sometimes does not compensate to have transfers between depots. Potential savings increase, however, with the number of customers. Recall, in addition, that these are lower bounds on the potential savings since some solutions for the collaborative case were not been proven optimal.

10 Conclusions

In this part of the thesis we have presented the Shared Customer Collaboration Vehicle Routing Problem (SCC-VRP), an innovative problem in the context of collaboration in urban delivery. The SCC-VRP is an extension of the VRP with multiple carriers and depots (one associated with each carrier) and where a subset of customers receive demand from more than one carrier. The subset of customers receiving service from more than one carrier are called shared customers, and they can be transferred among carriers to save overall distribution costs. The objective is to minimize overall distribution costs taking into account that transferring customers among carriers will require additional transfers of goods between the depots of the involved carriers.

Two different formulations have been proposed for the SCC-VRP. The first one is based on the vehicle description of carriers routes. The formulation contains SECs which are implemented as lazy constraints. To improve the performance of the formulation a separation procedure is implemented for the capacity constraints based on the knapsack problem. The second formulation is based on a flow-type expression of the load on the routes of the carriers.

A small set of test instances has been generated, based on classical multi-depot vehicle routing instances to evaluate and compare the proposed formulations using branch-and-cut algorithms with `CPLEX`. The computation time has been limited to one hour to allow easy comparisons. The load based formulation clearly outperforms the results of the vehicle based formulation. Finally, the results of the SCC-VRP with the load formulation on the test instances have been compared with the solutions obtained when each carrier solves its CVRP independently. Preliminary results indicate that cost reductions can reach up to 19.1 %.

An extended set of larger instances randomly generated has been also tested with the load based formulation. Unfortunately, when instances are bigger the load formulation is also not able to produce optimal solutions. Therefore, we believe that the future work should focus on the improvement of such formulation, in order to be able to solve some medium-sized instances.

In any case, the obtained results allow us to estimate the potential savings that could be obtained when carriers collaboration is applied. When comparing best-known solutions to the model with collaboration to optimal solutions the with model without collaboration, we observe percentage savings of up to 18%. On average, the observed savings are around 9%. Note that these are only lower bounds on the potential savings, since some of the solutions for the collaboration model were not proven optimal.

Part III

Collaboration via Urban Consolidation Centers

Introduction

Urban Consolidation Centers (UCCs) are logistics facilities located closely to the geographic area that they serve, from which deliveries are carried out by a neutral carrier. UCCs combine consolidation of goods and collaboration between freight carriers through the use of a public freight terminal. The key element of UCCs is to avoid the need for all vehicles to deliver part loads into urban centers. Long haul transportation vehicles of different companies visit the UCC to unload their goods. Long haul operators can use bigger vehicles and have less time restrictions to perform their deliveries. Loads are then sorted and consolidated to other vehicles adapted for urban distribution. Finally, a neutral freight carrier does the local delivery in the area with higher costumer density.

UCCs are aimed to reduce, through collaboration, the negative effects of urban distribution. It is crucial, however, to obtain cost-efficiency in order to implement them in reality. Individual transportation companies are able to reduce operational costs when they deliver the goods to UCCs. These costs should at least compensate the costs of the neutral carrier in the last-mile distribution. In the following chapters a system without consolidation center is compared in terms of operational costs with a system where a consolidation center is implemented. Logistic System Analysis [17] is used to define an accurate model for estimating costs and benefits. Continuous approximation models with robust solutions provide tendencies in the sensitive analysis and give us more insights about the solutions in order to gain knowledge for general cases. To the best of our knowledge, this is a novel approach to the problem.

In our approach UCCs are analyzed as a strategic solution from an aggregated point of view. We first propose a model, which assumes that carriers have homogeneous market shares. We then consider the general case with heterogeneous (non-equal) market shares. Both formulations are presented in Chapter 12, with a tool that evaluates the trade-off between savings in the system and a minimum market share per company in a consolidation center. In Chapter 13, general results, and sensitivity analysis are discussed, and the case study of L'Hospitalet de Llobregat (Metropolitan Area of Barcelona) is analyzed. Results show that market share distribution does not affect cost savings significantly. Results from the case study show a 12-14% of operational cost savings in a general (non-homogeneous) case. From the case study we can also conclude that the commitment of the 40% of demand allows covering the costs of terminal implantation.

This work has given rise to presentations in the City Logistic Conference in 2011 and 2015. The full conference papers are publicly available in [69, 70].

11 Literature Review

Urban Consolidation Centers (UCCs) have been widely studied from both theoretical and practical points of view. We will review some of the relevant practical implementations and case studies as well as more theoretical or modeling works.

According to [8] a UCCs is *"a logistics facility that is situated in relatively close proximity to the geographic area that it serves, be a city center, an entire town or a specific site (e.g. shopping center), from which consolidated deliveries are carried out within that area. A range of other value-added logistics and retail services can also be provided at the UCCs."*

An extensive literature review of UCCs initiatives, an analysis of specific examples of different UCCs types, discussions related to the concept with a sample of interested supply chain parties, and a preliminary evaluation of different types of consolidation centers can be found in [8]. More recently [80] reproduced a similar work with particular focus on the UK Retail Sector.

11.1 Practical implementations

Works like [34] and [40] report practical experiences in Japan and Germany, respectively. As described in [34] a Multi-Carrier Joint Delivery Service (MCJDS) was started in central Fukuoka (Japan) in 1977, as an agreement-based activity among trucking companies. It was promoted by the Local Office of the Japanese Ministry of Transport in order to rationalize unnecessary movements of small trucks in each company and to reduce the traffic and environmental impacts in the city. This system was enlarged in 1987, and in 1994 it was restructured by a fully private company for providing a joint delivery/pick-up service. It now covers a pick-up/delivery area of 70 ha having around 5,600 potential customers. 36 trucking companies are committed to the MCJDS for the delivery and pick-up of their parcels in the area at a particular fare payment and consolidate around 100,000 parcels monthly. Ten large carriers take approximately 90% of the market. MCJDSs have been introduced in several other cities in Japan after the Fukuoka system. However, the transport share of the Fukuoka MCJDS is no more than one third of the parcels of the total distributed goods and it is still facing financial difficulties because of the insufficient number of parcels commissioned by trucking companies. Nevertheless, it seems beneficial from a traffic and an environmental viewpoint.

To analyze the possible reasons of the system inconveniences, [35] presents an economic model of the behaviour of the different transportation roles in the presence of a voluntary MCJDS, like the one in Fukukoa (Japan). Shippers, carriers and the MCJDS act in function of their own interests. Shippers are assumed to stochastically choose the carriers according to the utility function of the different carriers, which depends on: the level of fare, the frequency of service, and the sales potential of the carrier. Carriers try to maximize their expected profits subject to the number and size of trucks, and the drivers' shifts, offering the best possible frequency in order to maximize their number of customers. The MCJDS plays a role similar to carriers but with less motivation to maintain high frequency since their revenues are not directly related to carriers. The application of this model with some experimental data does not coincide with reality. Therefore, the authors assume that the motivation of carriers to commission parcels is social reputation, and include a negative logarithmic component that depends on the commissioned parcels in the objective function. Once the model has been adjusted to the real situation in Japan some conclusions are drawn. Even if MCJDSs are contributing to the improvement of traffic and environmental conditions, there is a lack of parcels commissioned to make it financially feasible. The commission fare should be adjusted accurately to attract more carriers and to minimize the financial loss. Alternatively, some public subsidies can be considered to extend the sweet zone [1]. Another issue that should be considered is the carriers sales potential: using drivers to also act as commercial agents is an impediment to MCJDSs, since this activity is blocked by the commission. The authors suggest that the use of the ITS can leave this motivation out and improve the performance of MCJDSs.

In [40] some German projects related to cooperation in distribution are described and some data is presented. Different systems were implemented: city terminal and neutral freight carrier in Freiburg, split of delivery area in Munich, and central neutral cooperation agency in Kassel amongst others. After describing the experiments, some economical conditions to achieve a positive result are described. The authors suggest that the local incentives for carriers are still too low to make cooperation viable.

In 2014 the city of Barcelona carried out a pilot test in the Ciutat Vella district. The last-mile deliveries of parcels and small shipments were performed by a new system that combined the use of electric tricycles and a transshipment terminal [59] . One of the main achievements of the pilot test was that different transport companies collaborated using a last-mile neutral company, as the close collaboration between competing carriers is still not common. Even tough the potential benefits have been assessed by different theoretical models. The key factors for the success of the pilot test are: the features of the area where it was implemented (dense commercial activity, access control, and historical quartier), the active involvement of all affected stakeholders, and the use of a neutral carrier for last-mile deliver, which did not enter in competition with the other carriers. The economic equilibrium was still not easy to reach, however.

11.2 Theoretical modeling

From a more theoretical viewpoint the work of Kawamura [37] analyzes UCCs from a prespective similar to the one proposed in this thesis. The paper presents an evaluation of Delivery Consolidation in U.S. Urban Areas with logistics cost analysis. The aim of the paper is to establish whether the consolidation through cooperation and coordination among business is able to overcome the current system based on peddling. The current system is characterized by retailers? distribution centers located far from stores, together with a distribution scheme with several "peddle-runs", covering different urban areas. The strategy may be effective from the companies viewpoint, but puts a large number of trucks in the urban areas creating social and environmental problems. The evaluated new system is based on the work of Köhler [40], and the strategy with urban consolidation consists of building a consolidation center in the city and using a neutral freight carrier in the urban distribution. Using continuous ap-

[1] Sweet zone: Ranges of values of the commission fare that can justify the social and financial viability.

Parameter	Ranges of Values (US)	Ranges of Values (Eur)
Population density	1500 to 20000 pax/mi^2	576 to 7692 pax/km^2
Demand[1]	200 to 800	200 to 800
Service Area	50 mi^2 to 500 mi^2	130 km^2 to 1300 km^2
Average distance to depot[2]	370 mi	595 km
Trucks (GVW)	Big 78000 lb[3]	35 Tn
	Medium 35000 lb	16 Tn
	Small 16500 lb	7.5 Tn
Average Speeds	Expressways 44 mph	70.8 km/h
	Arterial 17 mph	27.35 km/h
Value of Time	28.1 €$/h$	22.4 €$/h$

[1] In cartoons per store per week [2] Round trip [3] 1 lb = 0.45359237 kg

Table 11.1: Kawamura data

proximations from Logistics Cost Analysis, both strategies are analyzed. The input data was based on Europe and US cities. The most relevant data is presented in Table 11.1. The methodology to build the Logistics Cost Function is based on finding the optimal number of dispatches, and the optimal distance traveled, that minimize holding costs plus, in the case of consolidation, terminal costs. The results of the study show that the one-to-many-delivery-without-transshipment system using 78000lb Gross Vehicle Weight (GVW) trucks has the least expensive logistics costs, regardless of population density and service area. At higher densities, consolidation reduces the cost improvement relative to the peddle-run with small trucks. The cost advantage of consolidation, which mainly comes from economies of scale, is sufficient to overcome the additional cost associated with the terminal and transshipment. The authors suggest that only a combination of the following factors: severe congestion, narrow streets, large number of firms sharing the city terminal, high retail rent costs, and high demand rate, could make consolidation attractive to businesses.

Even if the work of Kawamura is similar to the approach of this part of the thesis with similar cost formulation, the ranges of the parameters may vary substantially to represent dense urban european areas. The main difference is that we assume that companies could use larger vehicles to bring the goods to the consolidation center and do it during the night, which are two essential elements for reducing costs.

Later in 2012, [68] presented a more general model considering the European context in terms of customer density, facility location or vehicle type. However, modelling hypotheses differences remain. In particular, it is assumed that all transport companies serve all customers, which seems unrealistic from a practical point of view. In contrast, our approach encompasses any market distribution among companies.

12 Analytical Formulation

The purpose of this section is to propose a compact model to quantify the effects of the implementation of a Urban Consolidation Center (UCC) to regulate urban distribution in an area of a city and to study under which circumstances it is globally beneficial for all participants. To do so, we propose a methodology which determines accurate approximations of all the costs involved.

The main idea behind our approach is that during certain time periods (preferably at night), carriers from all companies will bring the goods to the center with the possibility of using larger vehicles. During the day, with higher costumer density due to the consolidation of demand from several carriers, local deliveries will be performed more efficiently by a neutral freight carrier.

12.1 Assumptions and problem description

To easily describe the service area, we assume that several parameters of the zone are homogeneous, such as zone dimension, demand density, truck capacities, distance to closest depot, unit costs, number of carriers, and location of the center, among others. We further assume that the center will not be a warehouse, so goods will be received and shipped every day. We finally assume that carriers are willing to collaborate and use an UCC to perform last-mile deliveries.

The key definition variables are the dimensions of the vehicle zone delivery partition, which is assumed to be rectangular, vehicle load and the number of the trips per vehicle in the time horizon. The resulting cost decision function, traveled distance and vehicles-hour, can be simply formulated from the above variables and the parameters. Then, the model will be able to predict costs related to time and distance and any other interesting system metric.

We use the methodology of Logistic System Analysis [17] to define an accurate model for predicting costs and benefits of UCCs. Continuous approximation models produce robust solutions, particularly useful when dealing with strategic problems. This approach and the use of sensitive analysis allows us to study general tendencies and give more insight about the solutions.

In the following section, a basic model is formulated to find the optimal strategy that one company alone uses to serve from a depot, its clients spread over a delimited area. Then, the model is used as a basis to formulate and compare two alternative scenarios, (A) the independent transport companies performing last-mile delivery without UCC, and (B) a last-mile delivery system with collaboration among companies and consolidation through a UCC. In Section 12.2, we compare both situations where we further assume that companies have the same market share in the region, i.e. the number of customers per company is the same. Then, we relax the assumption in Section 12.3, and study the effect of non-homogeneous market-share. In Section 12.3.2, we model the trade-off between the participation of small carriers and its contribution to the savings of the UCC. The model can be used to determine a threshold on the minimum demand of a company to produce minimum desired savings in the operation costs of the consolidation center system. The general results, sensitivity analysis of the model and a case study are presented in Chapter 13

12.1.1 General Model Description

Let us assume that one company has to serve N customers located in a rectangular zone of area A from a depot located at distance ρ from the center of the service area, with vehicles of capacity C. Let δ be the customers density. The company designs the tours with the objective of minimizing a weighted sum of distance costs (minimum tours) and temporal costs (minimum time consumption). As typical routing strategies do, first the region is partitioned in groups of approximately S points each. Note that S will be limited by time and capacity constraints. Then vehicle tours within the time horizon are designed. Vehicles travel from the depot to some point in its zone, serve the clients and return to the depot. We will call *line-haul* distance to the distance from the depot to the nearest point in its zone plus the distance from the last visiting point to the depot. And *local distance* is the distance covered during the delivery of the items. Let m be the average number of vehicles trips. Given the density of streets in urban areas we use the square grid metric to determine distances in the service area. Thus, S points should be located in a connected area. We assume that zones are rectangles with sides $2w$ and P, which form a partition of the whole delivery region. We next compute the total distance traveled to give service to a costumer by computing two different components: local and line-haul. The local traveled distance is independent of the point where the route starts or ends the delivery. [61] proposed a simple (non-optimal) strategy for visiting the points in each zone and showed that if we use nearly rectangular partitions of the region, the partitions should be elongated towards the depot. If vehicles carry a full load, the number of points in the rectangle should be C, which expressed in terms of density should be equivalent to $2wP\delta$. However, we might be interested in reducing the average number of points to S. So we will denote the number of expected customers inside a rectangle as S and use the equality $2wP\delta = S$ to eliminate one variable. To estimate the local length of a tour as a function of w and S, we extend Daganzo's proposal [17] and divide the rectangle into two bands each of width w. Then, each route visits points in non-decreasing coordinate x along the length of the rectangle on the way out and decreasing x on the way back (See Figure 12.1). If points are randomly distributed in space, one can evaluate the expected total distance. We divide the distance into the traverse and the longitudinal, since we use square grid metric to approximate the distance. The average traverse travel distance per point is simply the average distance between two random points on an interval of width w, that is $w/3$. The total longitudinal travel in the rectangle is $2P$ or $2P/S$ per point and using the equality $2wP\delta = S$, we obtain $1/\delta w$. The average line-haul distance for a vehicle will be $2\rho - P$ or $((2\rho - P))/S$ per point. We subtract P to the distance of the depot due to the relative position of the service areas with the depot and, again replacing P using the equality, we obtain $((2\rho - S/2\delta w))/S$. Finally, adding the line-haul and local distances we obtain an approximation of the total distance traveled per point:

$$d = \frac{w}{3} + \frac{1}{\delta w} + \frac{2\rho - \frac{S}{2\delta w}}{S}.$$

Apart from costs derived from the covering of distance, costs also come from the time spent during the delivery. Using v_A, v_B and τ as urban speed inside the service area, interurban

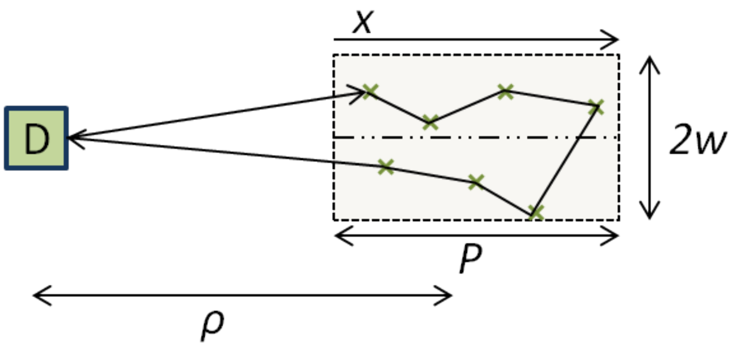

Figure 12.1: Local Strategy to cover all points

speed from the depot to the service area, and time lost per customer's unloading/loading of a vehicle respectively, the total time per vehicle trip is:

$$\frac{2\rho - P}{v_B} + \frac{2P}{v_A} + \frac{wS}{3v_A} + \tau S.$$

Then, the time devoted to each customer can be obtained from the above equation, dividing by the customers served by one vehicle (S).

$$\frac{2\rho - P}{v_B S} - \frac{1}{2w\delta v_B} + \frac{1}{w\delta v_A} + \frac{w}{3v_A} + \tau.$$

The objective function (12.1) is obtained as a weighted sum of distance costs and time related costs, with unit cost parameters c_d and c_t, . The resulting model is, thus:

$$\underset{w,S,m}{\text{minimize}} \quad N\left[\left(c_d + \frac{c_t}{v_B}\right)\frac{2\rho}{S} + \left(\frac{c_d}{2} + \frac{c_t}{v_A} - \frac{c_t}{2v_B}\right)\frac{1}{w\delta} + \left(c_d + \frac{c_t}{v_A}\right)\frac{w}{3} + c_t\tau\right] \tag{12.1}$$

$$S \le C \tag{12.2}$$

$$\frac{2\rho}{v_B} - \frac{S}{2w\delta v_B} + \frac{S}{w\delta v_A} + \frac{wS}{3v_A} + \tau S \le \frac{Y}{m} \tag{12.3}$$

$$w, S, m \ge 0 \qquad \text{continuous} \tag{12.4}$$

Constraints (12.2)–(12.3) model the limitations on the capacity of the vehicles, C, or on the time horizon, Y. We can solve the problem (12.1)–(12.4) analytically due to the nature of its variables. As m can become as small as needed, constraint (12.3) can be omitted making m equal to $Y/(2\rho/v_B - S/2w\delta v_B + S/w\delta v_A + wS/3v_A + \tau S)$ which is, obviously, positive. Then, in order to minimize the objective function, S should be as big as possible, which makes S equal to C, and from minimum conditions we obtain $w^* = \sqrt{(3(c_d/2 + c_t/v_A - c_t/2v_B)/\delta(c_d + c_t/v_A))}$. Let us call $\lambda = ((c_d/2 + c_t/v_A - c_t/2v_B))/((c_d + c_t/v_A))$, and so $w^* = \sqrt{3\lambda/\delta}$. The most important metrics are summarized in Table 12.1.

The costs of the system are proportional to N, the total number of customer of the system. The square root of customer density is dividing the terms in the total cost expression regarding local distribution, whereas capacity is dividing the terms regarding line-haul distribution. Finally, there is a constant time cost of stopping.

Local distance	$D_L = N\frac{\lambda+3}{3(\delta\lambda)^{1/2}}$
Line-haul distance	$D_{LH} = N\left[\frac{2\rho}{C} - \frac{(3\lambda)^{1/2}}{2\delta^{3/2}}\right]$
Time	$T = N\left[\frac{2\rho}{v_B C} + \frac{1}{(3\lambda\delta)^{1/2}}\left(\frac{1}{v_A} - \frac{1}{2v_B}\right) + \left(\frac{\lambda}{3\delta}\right)^{1/2}\frac{1}{v_A} + \tau\right]$
Cost	$c_d\left(D_L + D_{LH}\right) + c_t T$

Table 12.1: Summary of the most important metrics of the general model

12.2 UCCs with equal market share companies

In this section two models are proposed for each type of delivery strategy. The models approximate the costs of both situations: (A) the total costs of the system where each company performs their last-mile delivery independently, and, (B) the costs of a system where the companies collaborate and consolidate demand through a UCC. We assume that each company has the same market share in the delivery area. In the next section, this assumption is relaxed.

We assume that $M \geq 1$ equal-market share companies give service to N costumers, with the same number of customers each $\widehat{N} = N/M$. Therefore, $\widehat{\delta} = \delta/M$, which is smaller than δ. In the collaborating strategy, we consider that companies can use bigger trucks with capacity $B = k_C C$ ($k_C \geq 1$), and that the UCC is located at distance φ from the center of the service region ($\varphi = k_\rho \rho$ with $k_\rho \leq 1$). Note that k_C represents the enlargement of vehicle capacity in the line-haul distribution. We will call *capacity enlargement* to this parameter. Similarly, k_ρ represents the reduction of the distance from the closest depot to the final destinations. We call *depot distance reduction* to this parameter.

The two strategies will be compared in terms of costs, distance, and time consumption, assuming that all the companies try to minimize their costs. As mentioned before, in the system without a UCC it is assumed that each company carries the distribution to its customers independently with its own fleet. The total costs are the sum of the costs of each individual company. On the contrary, in the UCC system it is assumed that the costs are split in two delivery phases: the costs that each company undergoes to bring the goods to the consolidation center with its own fleet and the costs of the neutral freight carrier for last-mile distribution. It is further assumed that the distribution center is not a warehouse and that goods are received and shipped on the same day. Thus, no holding costs are incurred. Table 12.2 summarizes the results for strategy A, when each transport operator acts independently from the others, whereas Table 12.3 shows the results for strategy B, when operators act in collaboration through the use of a UCC.

Local distance	$D_{LA} = M\widehat{N}\frac{\lambda+3}{3(\widehat{\delta}\lambda)^{1/2}} = N\frac{\lambda+3}{3(\frac{\delta}{M}\lambda)^{1/2}} = M^{1/2}N\frac{\lambda+3}{3(\delta\lambda)^{1/2}}$
Line-haul distance	$D_{LHA} = M\widehat{N}\left[\frac{2\rho}{C} - \frac{(3\lambda)^{1/2}}{2\widehat{\delta}^{3/2}}\right] = N\frac{2\rho}{C} - M^{3/2}N\frac{(3\lambda)^{1/2}}{2\delta^{3/2}}$
Time	$T_A = M\widehat{N}\left[\frac{2\rho}{v_B C} + \frac{1}{(3\lambda\widehat{\delta})^{1/2}}\left(\frac{1}{v_A} - \frac{1}{2v_B}\right) + \left(\frac{\lambda}{3\widehat{\delta}}\right)^{1/2}\frac{1}{v_A} + \tau\right] =$ $N\left[\frac{2\rho}{v_B C} + M^{1/2}\left(\frac{1}{(3\lambda\delta)^{1/2}}\left(\frac{1}{v_A} - \frac{1}{2v_B}\right) + \left(\frac{\lambda}{3\delta}\right)^{1/2}\frac{1}{v_A}\right) + \tau\right]$
Cost	$c_d(D_{LA} + D_{LHA}) + c_t T_A$

Table 12.2: Summary of for the model without consolidation (strategy A)

The critical parameters to decide if the consolidation is benefitting in terms of distribution

Local distance	$D_{LB} = N\frac{\lambda+3}{3(\delta\lambda)^{1/2}}$
Line-haul distance	$D_{LHB} = M\widehat{N}\frac{2\rho}{B} + N\left[\frac{2\varphi}{C} - \frac{(3\lambda)^{1/2}}{2\delta^{3/2}}\right] =$ $= N\frac{2\rho}{k_C C} + N\frac{2k_\rho \rho}{C} - N\frac{(3\lambda)^{1/2}}{2\delta^{3/2}} =$ $= N\frac{2\rho}{C}\left(k_\rho + \frac{1}{k_C}\right) - N\frac{(3\lambda)^{1/2}}{2\delta^{3/2}}$
Time	$T_B = N\left[\frac{2\rho}{v_B B} + \frac{2\varphi}{v_B C} + \frac{1}{(3\lambda\delta)^{1/2}}\left(\frac{1}{v_A} - \frac{1}{2v_B}\right) + \left(\frac{\lambda}{3\delta}\right)^{1/2}\frac{1}{v_A} + \tau\right] =$ $= N\left[\frac{2\rho}{v_B C}\left(k_\rho + \frac{1}{k_C}\right) + \frac{1}{(3\lambda\delta)^{1/2}}\left(\frac{1}{v_A} - \frac{1}{2v_B}\right) + \left(\frac{\lambda}{3\delta}\right)^{1/2}\frac{1}{v_A} + \tau\right]$
Cost	$c_d(D_{LB} + D_{LHB}) + c_t T_B$

Table 12.3: Summary of metrics for the model with consolidation (strategy B)

costs are: the number of companies collaborating, the capacity enlargement and the depot distance. It is clear that the total local distance is reduced by the proportion of the square root of the companies participating. The line-haul distance has two main components. The first one has been computed for different values of k_C and k_ρ. If $\left(k_\rho + 1/k_C\right) = 1$ the first line-haul distance component is kept. If the value is less than one, more savings can be obtained proportionally to this value. The second component is negligible as it is one order of magnitude less than the first one. Distribution time has a more complex formulation than distance; in fact, time is a reformulation of distance with speed parameters. In any case, most of the distance is traveled with interurban speed, so time is also reduced in the model for consolidation.

12.3 UCCs with non-equal market share companies

Similarly to the previous section, we assume there is a delivery zone of area A, where M companies operate and a total of N customers are uniformly distributed within the area A. We now assume that each company gives service to N_i customers, $i = 1,\ldots,M$, not necessarily equal, with $N = \sum_{i=1,..,M} N_i$. The customer density of each company is denoted by $\delta_i = N_i/A$, $i = 1,\ldots,M$. Note that δ_i is smaller than the overall demand density $\delta = N/A$. We use the same parameters as in the previous section. The distance from the depot to the geographical center of the service area is denoted by ρ. We further assume a fixed capacity C for the vehicles that perform urban distribution. In the case where no UCC exists, those vehicles traveling from the depot to the service area also have capacity C. Instead, we assume that companies use (bigger) trucks with capacity $B = k_C C$, ($k_C \geq 1$) when they use the consolidation center. The parameter k_C represents the relative increment of vehicle capacity in line-haul distribution and it is called *capacity enlargement*. We further assume that the UCC is located at distance φ from the geographic center of the service region with $\varphi = k_\rho \rho$, $k_\rho \leq 1$, i.e. the UCC is closer than the company's depot to the geographic center of the service region. The parameter k_ρ is called *depot distance reduction*. It relates two distances: the distance ρ from the geographical center of the service area to the depot of the carrier and the distance φ from the geographical center of the service area to the UCC.

Next, we present with the same methodology as in the previous section a continuous model for different market share distributions, which allows again the comparison of two alternative urban delivery strategies, when carriers have non-equal market-shares: A) a system without the UCC and B) a system with the UCC. Both strategies are compared in terms of costs, distance, and time consumption, under the assumption that all the companies are trying to minimize their costs. In strategy A the total costs are the sum of the costs of each individual company. On the contrary, in strategy B it is assumed that the costs are split in two delivery phases: the costs that each company undergoes to bring the goods to the consolidation center with its own fleet and the costs of the neutral freight carrier for last-mile distribution. Again, no holding costs are incurred since the distribution center is not a warehouse. Table 12.4 summarizes the results for strategy A, when each transport operator acts independently from the others, whereas Table 12.5 shows the results for strategy B, when operators act in collaboration through the use of a UCC. Note that we use D_{LHA}, D_{LA} and T_A, and D_{LHB}, D_{LB} and T_B to refer to the metrics of Strategy A and B respectively, but in this case with non-equal market share company.

Local distance	$D_{LA} = \frac{\lambda+3}{3\lambda^{1/2}} \sum_{i=1}^{M} \frac{N_i}{\delta_i^{1/2}}$
Line-haul distance	$D_{LHA} = N\frac{2\rho}{C} - \frac{(3\lambda)^{1/2}}{2} \sum_{i=1}^{M} \frac{N_i}{\delta_i^{3/2}}$
Time	$T_A = N\left[\frac{2\rho}{v_B C} + \tau\right] + \sum_{i=1}^{M} \frac{N_i}{\delta_i^{1/2}} \left(\frac{1}{(3\lambda)^{1/2}}\left(\frac{1}{v_A} - \frac{1}{2v_B}\right) + \left(\frac{\lambda}{3\delta}\right)^{1/2} \frac{1}{v_A}\right)$

Table 12.4: Summary of for the model without consolidation (strategy A)

Local distance	$D_{LB} = \frac{\lambda+3}{3\lambda^{1/2}} \frac{N}{\delta^{1/2}}$
Line-haul distance	$D_{LHB} = N\frac{2\rho}{C}\left(k_\rho + \frac{1}{k_C}\right) - \frac{(3\lambda)^{1/2}}{2} \frac{N}{\delta^{3/2}}$
Time	$T_B = N\left[\frac{2\rho}{v_B C}\left(k_\rho + \frac{1}{k_C}\right) + \tau\right] + \frac{N}{\delta^{1/2}} \left[\frac{1}{(3\lambda)^{1/2}}\left(\frac{1}{v_A} - \frac{1}{2v_B}\right) + \left(\frac{\lambda}{3}\right)^{1/2} \frac{1}{v_A}\right]$

Table 12.5: Summary of metrics for the model with consolidation (strategy B)

When analyzing the potential benefit of a consolidation center under the non-equal market share assumption, the new critical aspects in terms of distribution costs are the number of collaborating companies and the market share distribution, i.e. which portion of the customers has each of the carriers.

12.3.1 Analysis for several market share distributions

Some data of transport companies registered in Barcelona (Spain) was analyzed to check the heterogeneity of real markets [57]. Transport companies were classified by sales revenue and the number of employees, which are two variables closely related to market share. Figure 12.2 depicts the number of companies in each class . We can observe that the more significant group was the one of small companies, i.e., the group with smallest values both in sales and employees.

The difficulty to obtain reliable data of market share in a particular area lead us to analyze different possible distributions. In the following section, we formulate some possible market distributions based on some analytically simple functions. Since we cannot guarantee that in reality the market adjusts to some known functions, in the subsequent section an extended set of distributions is proposed, which covers any possible distribution of customers among carriers.

12.3.1.1 Non-equal market share structures.

We analyze four families of market share distributions based on the most simple functions. We assume that the total demand N is distributed among M companies. For each class, we

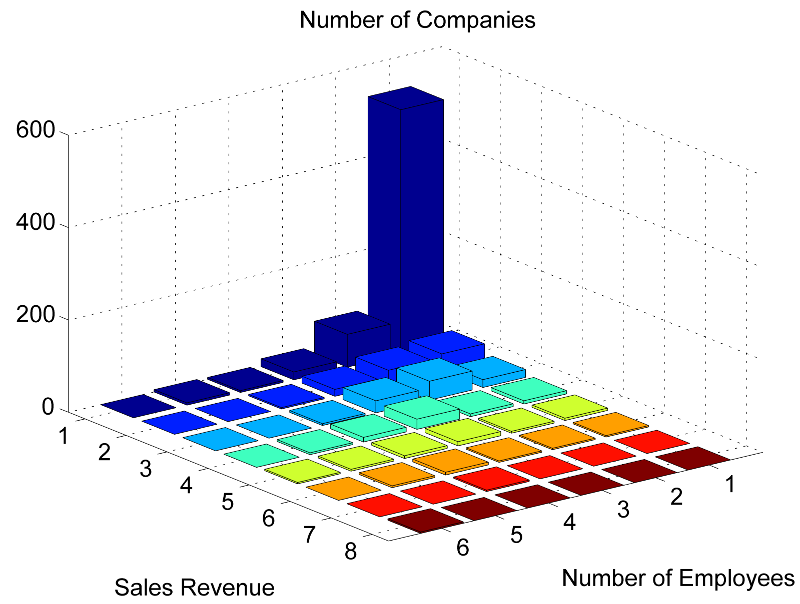

Figure 12.2: Number of transport companies in Barcelona (Spain) classified by sales revenue and number of employees

Sales revenue (in millions €)	**Number of employees**
1-until 0.3 €	1-from 1 to 5
2-from 0.3 to 0.6 €	2-from 6 to 10
3-from 0.6 to 1.5 €	3-from 11 to 25
4-from 1.5 to 3 €	4-from 26 to 50
5-from 3 to 6 €	5-from 51 to 100
6-from 6 to 15 €	6-from 101 to 250
7-from 15 to 30 €	
8-from 30 to 60 €	

Table 12.6: Legend of Figure 12.2

choose one particular representative by further setting an additional condition..

1. Grouped. There are two types of companies, one with many customers and the other with far fewer. Equations (12.5) describe this distribution, for a given set of parameters (a, b) . In our case, we assume that big companies have twice as many customers as smaller companies ($2a = b$).

$$\begin{cases} N_i = a = 2N/3M, & i = 1,..,M/2 \\ N_i = b = 4N/3M, & i = M/2+1,..,M \end{cases} \tag{12.5}$$

2. Lineal. Each company has a different number of customers and the distribution among them follows a lineal increase. (12.6) describes the distribution for a set of parameters (a, b). In our case, we assume that any other company smaller than the smallest of this

distribution, would have zero demand (i.e., imposing $a = 0$).

$$N_i = a + bi = (2N/M(M+1))\, i, i = 1,.., M \tag{12.6}$$

3. Exponential. Each company has a different number of customers and the distribution among them follows an exponential increase. The number of customers per company is more diverse than in the lineal case, and there can be bigger and smaller companies than in that case. (12.7) represents the distribution, and we set $b = 3/M$ to bend the distribution and distinguish it from the lineal one. Then $a = (N/M) / \sum_{i=1}^{M} \exp(3i/M)$.

$$N_i = a\exp(bi) = \frac{(N/M)}{\sum_{i=1}^{M} \exp(3i/M)} \exp(3i/M), i = 1,.., M \tag{12.7}$$

4. Uniform. All companies have the same number of customers. This is the equal market share assumption developed in the previous section that can be treated as a particular case. It is easy to check that the formulae presented in the previously mentioned work, coincide with the ones presented in (12.8) under uniform demand.

$$N_i = a = N/M, i = 1,.., M \tag{12.8}$$

In Figure 12.3 we plot the distribution of the customers demand for the proposed functions for N=400 customers and M=50 companies. Each value on the x axis represents one company, and each value on the y axis, the corresponding number of customers.

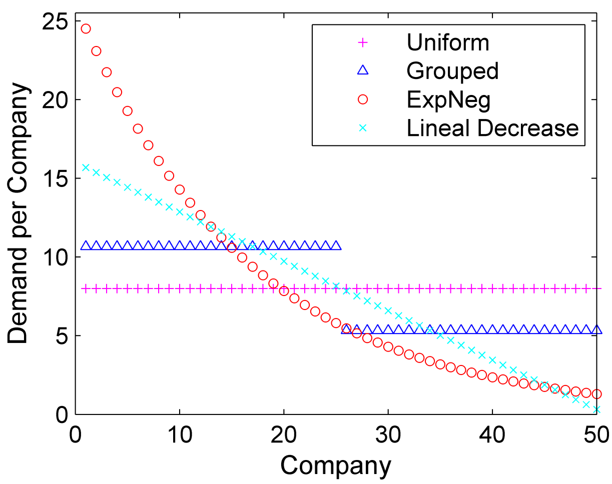

Figure 12.3: Distribution of customers among companies for functions proposed in 12.3.1.1

12.3.1.2 Extended set of potential market distributions

In a general case, the structure of the market is unknown. On the other hand, the structures presented previously can seem rather limited. Hence, we propose an extended set of distributions, which can represent any possible distribution of customers among carriers. For this we enumerate all the possibilities for distributing N customers among M companies. Using combinatorics, we can think of this as an assignment of an integer number in $\{1,\ldots,N\}$ to M carriers with some restrictions, as not all combinations represent meaningful distributions of customers. In particular, we should take into account the following:

- Exact coverage of the total demand. That is, each customer is assigned to one company.
- No combination of customers is repeated. The order in which the number of customers is assigned to a company is not important since the resulting distribution is the same.
- Assignment of at least one customer per company. Each company should have at least one customer, otherwise the assignment would not have M companies.

To obtain all possible distributions of customers among carriers, we have designed an iterative algorithm that enumerates and generates all of them. Each possible distribution is uniquely represented by a code defined by M integer numbers of customers sorted increasingly. For instance, if in one distribution the companies have 5,7,3,1, and 6 customers each, the unique code is: {1,3,5,6,7}. Thus, companies are implicitly ordered by increasing number of customers, so that the "first" and the "last" companies have the smallest and the largest number of customers, respectively.

The algorithm defines H_i, as the set of all possible combinations of numbers of customers among the first i companies, when N customers are distributed among M companies. H_1 is easy to generate. Its possible values range in $\{1,\ldots,\lfloor\frac{N}{M}\rfloor\}$. If we assign more than $\lfloor N/M\rfloor$ customers to the first company, we have to assign at least the same value to all the companies to keep the increasing order of the code. That would represent exceeding the total demand. $\lfloor x\rfloor$ denotes the maximum integer number no greater than x. Then, for $i = 1,\ldots M-2$, H_i is recursively built from H_{i-1}. Since we keep the non decreasing order in the values of the assigned customers, feasible assignments must leave enough unassigned demand for the $M-i$ remaining companies. Thus, the maximum demand we can assign to company i cannot exceed the demand not yet assigned among companies $1,\ldots,i-1$ divided by the number of remaining companies. Finally, H_M is obtained by completing each distribution in H_{M-1} with the remaining uncovered demand. Now the maximum possible value for the customers assigned to company i is which must be divided by the number of the remaining companies. See Algorithm 2 for details.

As an example of the results of Algorithm 2, in Figure 12.4 we show the 34 possible distributions of a total of 16 customers among 4 carriers (C1, C2, C3 and C4).

12.3.2 Trade-off between minimum carrier dimension and savings

Due to a highly competitive market, a system with a UCC could be objected by medium-size and large carriers. The reason for which large carriers would like to limit the participation of very small carriers is that small carriers do not significantly increase the number of customers, but are greatly benefitted from the consolidation center. We present a new tool to determine a threshold on the minimum demand of a company to join the UCC.

Let N_T, δ_T denote the total number of customers and demand density of the collaborating companies, respectively; and N_S, δ_S, the corresponding values for the individual company that wants to join the collaborative system. Using the same methodology of the previous section we can derive the formulae for this case. We will compute the costs of the two situations: C) the current situation where the individual company distributes independently, (see Table

```
Combination Generation
```
$H_1 \leftarrow \{\{1\},\{2\},\{3\},\ldots\{\lfloor N/M \rfloor\}\}$

for *($i \leftarrow 1 \ldots M-2$)* **do**

  for *($\bar{h} \in H_i$)* **do**

    Generate new possible distributions of H_{i+1} based on $\bar{h}$ (i.e. with the first i elements equal to $\bar{h}$).

    If $\bar{h} = \{h_1, h_2, h_3, \ldots, h_i\}$ with $h_1 \le h_2 \le h_3 \le \ldots \le h_i$ define all the possible r distributions of H_{i+1}.

    $\bar{h_1} = \{h_1, h_2, h_3, \ldots, h_i, h_i\}$

    $\bar{h_2} = \{h_1, h_2, h_3, \ldots, h_i, h_i + 1\}$

    $\bar{h_3} = \{h_1, h_2, h_3, \ldots, h_i, h_i + 2\}$

    $\ldots,$

    $\bar{h_r} = \left\{h_1, h_2, h_3, \ldots, h_i, \left\lfloor \frac{N-(\sum_{j=1\ldots i} h_j)}{M-i} \right\rfloor \right\}$

  end

end

for *($\bar{h} \in H_{M-1}$)* **do**

  $\bar{h} = \{h_1, h_2, h_3, \ldots, h_{M-1}\}$ generate the final distribution

  $\hat{h} = \left\{h_1, h_2, h_3, \ldots, h_{M-1}, N - \left(\sum_{j=1\ldots M-1} h_j\right)\right\}$

end

Algorithm 2: Combination Generation

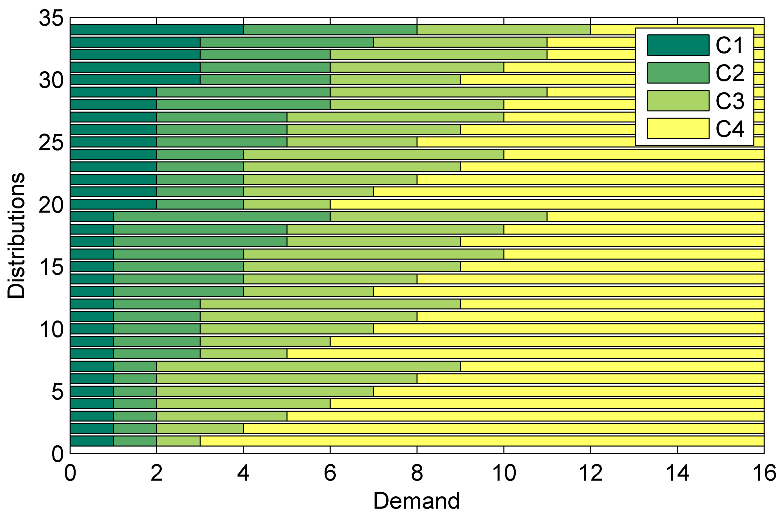

Figure 12.4: Potential distributions of customers among companies (16 customers, 4 carriers)

12.7), and D) the situation where the individual company joins the collaborative system, (see Table 12.8). Thus, we can establish a trade-off between the dimension of the individual carrier, and the savings of the hypothetic collaboration system. Note that we use subscript C to refer to the metrics of state C, and subscript D, respectively, for state D. Note that the formulae presented in the above tables allow us to decide the threshold for the minimum demand of a company to join the consolidation center, depending on its corresponding savings.

Local distance	$D_{LC} = \left[N_T \frac{\lambda+3}{3(\lambda\delta_T)^{1/2}}\right] + \left[N_S \frac{\lambda+3}{3(\lambda\delta_S)^{1/2}}\right]$
Line-haul distance	$D_{LHC} = \left[N_T \frac{2\rho}{C}\left(k_\rho + \frac{1}{k_C}\right) - \frac{(3\lambda)^{1/2}}{2}\frac{N}{\delta_T^{3/2}}\right] + \left[N_S \frac{2\rho}{C} - \frac{(3\lambda)^{1/2}}{2}\frac{N}{\delta_S^{3/2}}\right]$
Time	$T_C = \left[N_T \left(\frac{2\rho}{v_B C}\left(k_\rho + \frac{1}{k_C}\right) + \frac{1}{(3\lambda\delta_T)^{1/2}}\left(\frac{1}{v_A} - \frac{1}{2v_B}\right) + \left(\frac{\lambda}{3\delta_T}\right)^{1/2}\frac{1}{v_A} + \tau\right)\right] + \left[N_S \left(\frac{2\rho}{v_B C} + \left(\frac{1}{(3\lambda\delta_S)^{1/2}}\left(\frac{1}{v_A} - \frac{1}{2v_B}\right) + \left(\frac{\lambda}{3\delta_S}\right)^{1/2}\frac{1}{v_A}\right) + \tau\right)\right]$

Table 12.7: Summary of the metrics for strategy C

Local distance	$D_{LD} = (N_T + N_S)\frac{\lambda+3}{3(\lambda(\delta_T+\delta_S))^{1/2}}$
Line-haul distance	$D_{LHD} = (N_T + N_S)\frac{2\rho}{C}\left(k_\rho + \frac{1}{k_C}\right) - \frac{(3\lambda)^{1/2}}{2}\frac{N}{(\delta_T+\delta_S)^{3/2}}$
Time	$T_D = (N_T + N_S)\left[\frac{2\rho}{v_B C}\left(k_\rho + \frac{1}{k_C}\right) + \tau\right] + \frac{(N_T+N_S)}{(\delta_T+\delta_S)^{1/2}}\left[\frac{1}{(3\lambda)^{1/2}}\left(\frac{1}{v_A} - \frac{1}{2v_B}\right) + \left(\frac{\lambda}{3}\right)^{1/2}\frac{1}{v_A}\right]$

Table 12.8: Summary of the metrics for strategy D

13 General results, Sensitivity Analysis and Case Study

In this section we will present several of the previous models for various sets of parameter values based on European cities (see Table 13.1). The parameters that refer to the area (A) and distance to the nearest depot (ρ) have been evaluated within an interval of possible alternative situations. Speeds are approximate realistic values for urban trips based on [58] database. Capacity of vehicles and time lost per stop are taken from [36, 67]. Unit cost parameters are taken from [19, 11]. The study of the parameters k_C and k_ρ is detailed in the sensitivity analysis Section 13.2. These are average values and are realistic as design parameters for the existing UCC.

Parameters	Values	Units	Parameters	Values	Units
A	[1,20]	[km^2]	C	[5,15]	[stops]
δ	[2,100]	[stores/km^2]	c_d	0.3	[€/km]
ρ	[5,50]	[km]	c_t	26.36	[€/h]
v_A	25	[km/h]	k_C	1.4	[-]
v_B	50	[km/h]	k_ρ	0.1	[-]
τ	0.3	[h]	M	10	[-]

Table 13.1: Summary of the most important parameters regarding European cities

13.1 Non-equal market assumption

In this section we will analyze the effect of the non-equal market assumption of the previous model. First, we present the results of the trade-off between minimum carrier dimension and savings of the Section 12.3.2. The reason is that we will further use these results in the application of the extended set of potential market structures of Section 12.3.1. Then, we will analyze the results for several market share distributions. In Section 13.2 a sensitivity analysis of the parameters will be presented. Finally, general results for a given case study are discussed in Section 13.3

13.1.1 Trade-off between minimum carrier dimension and savings

This analysis will be done through the analysis of a parameter that relates N_T to N_S (number of customers of the consolidation center and number of customers of the carrier that wants to join the UCC, respectively). We use the expression $N_S = k_N N_T$ with $k_N \in [\varepsilon, 0.5]$, where $\varepsilon > 0$, i.e. the new carrier has a number of customers that is a portion of the customers that receive goods from the UCC. The model in Section 12.3.2 provides a useful tool that companies can use to limit the participation of very small carriers. For a new carrier to be admitted to join the system, the consolidation center may require that its incorporation will imply a minimum percentage reduction on the unit cost per customer. In Figure 13.1 (a), we present the unit cost of distribution per customer as a function of k_N for: i) the independent carrier, ii) the current UCC system, iii) the case where the independent carrier joins the UCC. In Figure 13.1 (b), we present the savings between the current costs of the UCC and the UCC plus the new carrier. We compare the unit cost per customer in both scenarios using the difference of these values expressed in percentage.

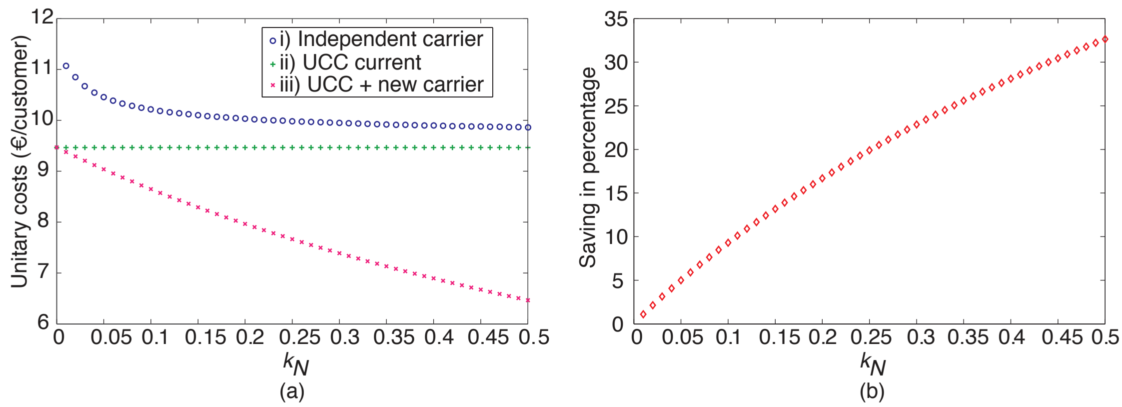

Figure 13.1: Cost and savings for several market share structures, as a function of k_N. (a) Total cost for each market structure and (b) Savings in percentage depending on the market structure and the number of companies.

In Figure 13.1 (a), we can observe the decrease in unitary costs as the number of customers of the independent carrier increases (k_N). Something similar happens as the new carrier increases the overall UCC demand, by incorporating its customers. Costs of the current UCC system remain fixed as they are not affected by the demand of the independent carrier. Figure 13.1 (b) shows the increase in savings of unitary costs as the demand that the new independent carrier brings to the system increases. Potential savings range from 5% if the independent carrier has 5% of the customers of the consolidation center up to a 32.5% if the independent carrier brings 50% of the demand of the UCC.

13.1.2 Results for several market share distributions

We now analyze how the number of companies affects the costs of the four market share distributions based on simple functions proposed in the previous section. In Figure 13.2 (a), we present the total costs for each of the considered market share structures, as well as the total distribution costs costs when the companies operate within the UCC. We can see that total costs are similar in the four market share distributions and that UCC costs are slightly lower. As could be expected, the difference in cost increases as more companies are considered. In Figure 13.2 (b), we present the percentage savings of UCC costs with respect to the costs of each market share structure. We observe that the uniform distribution is the one that provides more savings; other distributions, however, present the same range of savings, with a maximum difference that is smaller than 0.5%.

From the above figures, it is evident that the more companies collaborate, the more demand is consolidated, and thus, there are more savings. We can conclude that different market structures do not affect the operational savings of an UCC significantly. However, to extensively check this conclusion, we will present the results with the extended set of potential market distributions proposed in the previous section.

The size of the extended set of potential market distributions for N customers among M companies grows quickly with N. Thus, it is not possible to analyze the complete set, even when N and M take small values. In order to reduce the number of distributions, we have limited the participation of companies with a small number of costumers, as they would not be accepted by medium-large carriers. Using the analysis of the results of the previous section, we can determine the minimum demand per company to obtain a given percentage of savings. We set the threshold to a 5% of cost reduction, which limits the minimum demand per company to 5% of the total demand of the service area.

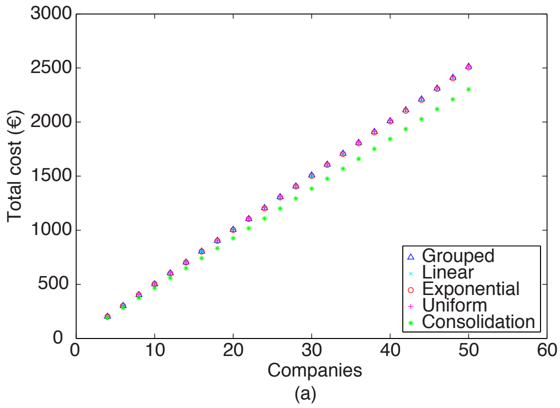

Figure 13.2: Costs and savings for several market share structures. (a) Total cost for each market structure and (b) Savings in percentage depending on the market structure and the number of companies.

We generate all market distributions with Algorithm 1 for a total of 80 customers distributed among 10 companies. The minimum demand per company is 4 customers, resulting in a total of 16,928 possible market distributions. For each distribution we compute the savings between the current situation with no collaboration and a total collaborative situation.

In Figure 13.3, we present a histogram of savings for the extended set of potential market distributions. The savings range between 6.3% and 7%. The differences in savings in the extended set of potential market distributions are not significant: the mean is 6.79 and the variance is 0.0067. Therefore, we can conclude that market structure does not affect savings in UCC strategy.

13.2 Sensitivity analysis

This section aims to provide insight on the effect of some parameters of the system, and their effect in the costs. The crucial parameters to assure savings when the consolidation centers used are k_C and k_ρ. Context parameters like A or δ are also considered, since they are set at a planning level and they have a great influence on the behavior of the resulting system. On the contrary, parameters like C, ρ, c_d, c_t, v_A, v_B, and τ are less dependent on planning decisions and tend to be linked to the characteristics of each commodity, city, or region.

Since we have seen that market share distribution does not significantly affect savings, in this section we will work with the equal market share distribution from Section 12.2

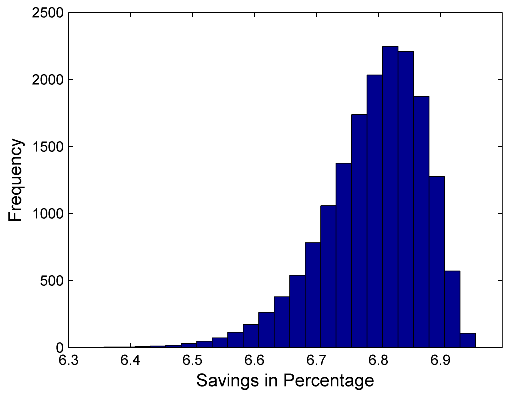

Figure 13.3: Savings expressed in percentage of the extended set of potential market distributions for 80 customers distributed among 10 companies.

Parameters will be analyzed one at a time, meaning that all other parameters will be fixed. In Figure 13.4, the effect of the number of companies is related to the percentage of local distance savings. The square root relationship can be clearly observed. The percentage savings are important for the first committing companies, but then the increments become rather small.

The predominant term of the total cost is line-haul cost, which can be significantly reduced by an appropriate combination of k_ρ and k_C. In Figure 13.5, line-haul distance savings in percentage are related to (k_ρ, k_C). For instance, a 0.2 reduction in the distance of the depot to the UCC and a capacity enlargement of factor 2 yields a line-length saving of approximately 20%. The difference between percentage savings and percentage losses can be clearly observed in the intersection of the surface with the plane $z = 0$. This is exactly the hyperbolic curve determined by $\left(k_\rho + 1/k_C\right) = 1$. Moreover, for smaller values of k_ρ and larger values of k_C, more savings can be obtained.

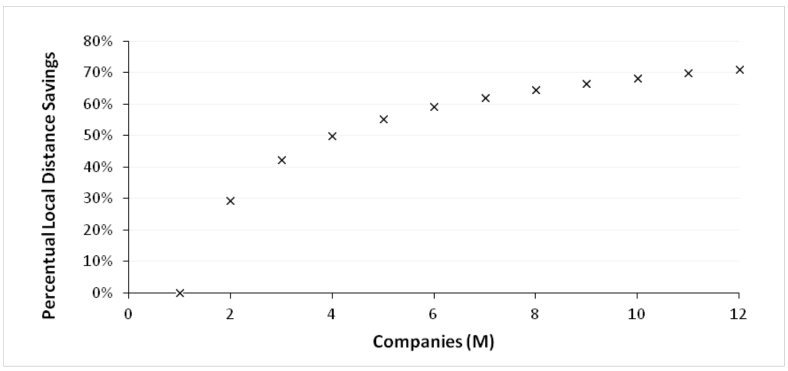

Figure 13.4: Local distance savings in percentage depending on the number of companies

As an example, in Figure 13.6 we present the unit cost for each of the two strategies with $k_\rho = 0.1$, $k_C = 1.4$. Note that in this case, such cost can be quite large since the data used is aggregated. The costs of the current system appear above and the values for the system with the consolidation center are below. We can observe savings around 12-14%. In general, operational cost savings in these ranges can be guaranteed.

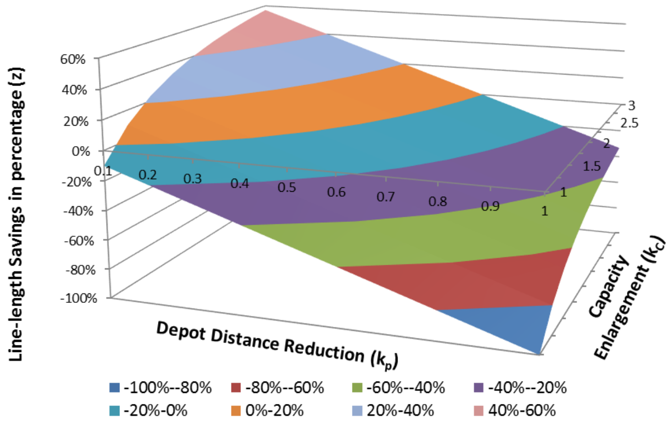

Figure 13.5: Line-haul distance savings in percentage depending on the value of the parameters k_ρ, k_C (Depot distance reduction, capacity enlargement)

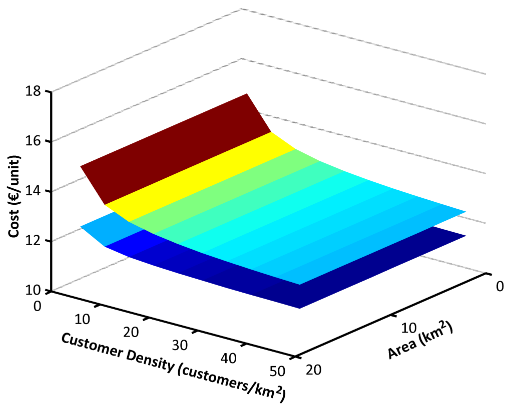

Figure 13.6: Unit cost of distribution depending on the area size and customer density

13.3 Case study

The city council of l'Hospitalet de Llobregat (Spain) expressed its interest in testing the potential improvements in a pilot test: the reduction in operation costs, the number of transportation vehicles in the area, congestion, pollution, noise, etc. thus the proposed methodology has been applied to define an optimal system and its benefits.

The main commercial area of L'Hospitalet de Llobregat, which is the most congested one and has narrow streets can be roughly delimited by a rectangular zone of approximately 0.64km^2. We consider that the city center is ρ=10 km away from the closest depot. Since the UCC can be built on the perimeter on the area, φ=0.08. Regarding trucks, we assume that the average capacity is 3 Tn of GVW for urban delivery (Madrid Municipality, 2004). With the UCC we assume that the capacity will be 5.4 Tn on average. The defining parameters of the zone are summarized in Table 13.2.

Different types of establishments exist, which can be classified in: personal consumption (A), hospitality and catering (B), leisure (C), construction or home materials (D), collective establishments (E), food stores (F), and others (G). We assume that each transport carrier only

Parameters	**Values**	**Units**	**Parameters**	**Values**	**Units**
A	0.64	[km^2]	c_d	0.3	[€/km]
ρ	10	[km]	c_t	26.36	[€/h]
v_A	25	[km/h]	k_C	1.8	[-]
v_B	50	[km/h]	k_ρ	0.08	[-]
Y	12	[h]			

Table 13.2: Summary of the most important metrics of the model with consolidation for the city of L'Hospitalet de Llobregat

Code	**Stores**	**Shipments**	**Companies**	M	$\widehat{N}$	N	δ	**Stops** (C)
A	80	2.5	52	11	4	43	67	7
B	126	2.7	51	11	7	74	115	7
C	128	2.5	14	3	23	69	107	10
D	133	5.3	28	6	25	151	236	15
E	136	1.9	30	6	9	52	81	4
F	154	2.2	53	11	6	71	111	15
G	252	1.9	30	6	16	97	151	4

Table 13.3: Basic features of the stores in L'Hospitalet de Llobregat

serves one of these types of establishments, so the savings analysis will be done separately. The city council of L'Hospitalet de Llobregat provided us with the number of stores of each type (Table 13.3). Table 13.3 has been completed using the information in the surveys of [50, 67] . For each type of establishment we have collected the following data: the average shipments received, the number of transport companies in the area, the number of companies participating in the UCC (M), the demand per company ($\widehat{N}$), the total demand in the area, the store density, and the stops per trip. (Note that stops per trip are equivalent to capacity).

The results are presented in Figure 13.7. As can be seen, depending on the percentage of companies participating in the UCC percentage savings between 10% and 12% can be achieved with respect to the current situation. We compare the total savings with an estimated cost for the UCC. The cost estimation should account for the infrastructural cost (rent, improvements, and maintenance), terminal personnel cost, technical machines, and information technologies. The results obtained for this case study indicate that with the participation of transport carriers that serve 40% of the demand, operational cost savings can be around 5% including the funding of the operation costs in the UCC.

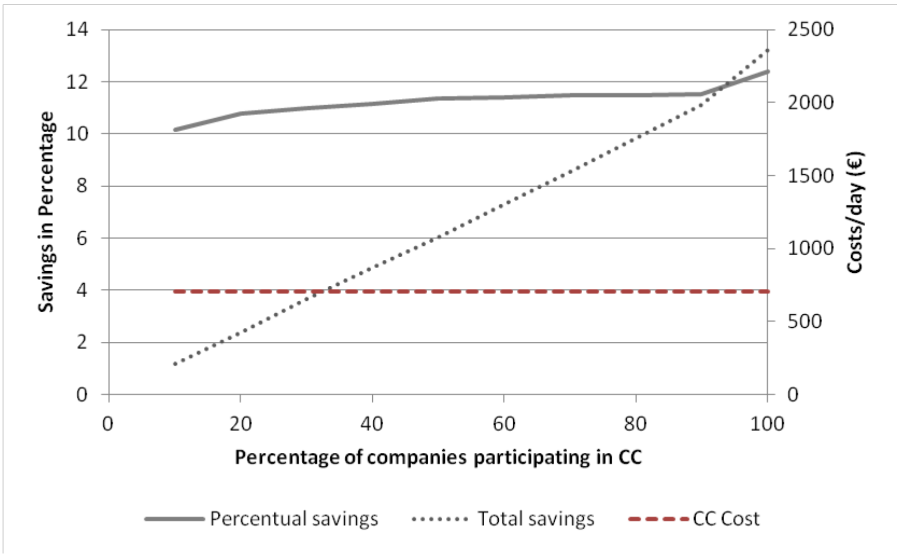

Figure 13.7: Results for L'Hospitalet de Llobregat

14 Conclusions

In this last part of the thesis we have proposed a continuous model that analyzes the improvement in the efficiency of urban distribution that can be achieved with the use of consolidation centers. The proposed methodology can be easily applied to more than one service area to obtain accurate approximations of the savings. Moreover, the sensitive analysis gives a better understanding of the tendencies with the evolution of the parameters, specifically with the key metrics involved: cost, distance and time.

We have proposed a tool to determine the minimum number of customers from all the companies participating in the consolidation center needed to reach target percentage savings. This can be very useful for planning purposes, when defining the companies that can participate in the consolidation center.

We have assumed that companies with different market shares within the service area act collaboratively through a UCC. However, the different market assumption does not substantially impact savings in percentage of the resulting center, when compared to a uniform market share distribution situation. Thus, we can conclude that the distribution of customers among the companies does not significantly affect the savings of the consolidation center.

The results obtained with data, based on densely populated urban areas such as European cities, show that benefits can reach up to a 12-14% in cost savings. The results are clearly differentiated from those in[37, 68], due to the differences in the modeling hypothesis. Moreover, a case study for L'Hospitalet de Llobregat has been presented, where we show with more detailed data that such savings can be reached. We have also seen that, in the collaborative strategy, the commitment of approximately 40% of the transport companies is needed to cover the UCC costs..

Future research on this topic could focus on by adding constraints to make the model even more realistic. Such constraints could include the time windows imposed by customers or the possibility of temporary storage in the consolidation center. Another possible avenue of research would study additional aspects of the system. For instance, the approximation of the costs incurred in UCCs and its financial possibilities. Or from a more general perspective, the distribution of the new costs and the benefits of a UCC system among all stakeholders involved.

15 General Conclusions

Transport collaboration in urban deliveries is the transversal topic. We have studied different forms collaboration, combining multiple perspectives and tackling different problems. The common aim was to study three particular problems, arising within different decision making frameworks, and provide models to quantify the potential benefits of the collaboration scenarios. We believe we have proved the potentialities of these types of collaboration. Ideally our analysis could encourage stakeholders (private companies, as well as public authorities) to establish or stimulate collaboration. From a methodological perspective, we have shown that the suitable formulation of the problem plays a key role on the problem approach.

Indeed, the research performed in this thesis is somehow limited. Unfortunately, private companies are extremely reluctant to share data. Nonetheless, the use of aggregate data to generate different instances gives a comprehensive test in the thesis. Furthermore, the different parts of the thesis are definitely connected but do not build on each other. Thus, the broader view of the thesis, with three different parts, somehow limits the depth in some sections.

We believe that the contributions of this thesis can be applied to other fields of research. The modeling of PAP provides a comprehensive approach to fair solutions in the presence of time windows. The different proposals and the obtained solutions can give some insight on how to address other problems with time windows when instances are not necessarily feasible. The use of time-discrete approach has proven to be valid for the PAP, which indicates an alternative for other similar problems when a time-continuous approach presents limitations. In turn, the modeling of the SCC-VRP with its different variants can inspire other related work. Finally, the continuous approach has proven to be valid in another field, which can contribute to make more visible the use of this type of models in different fields.

This thesis opens multiple opportunities for future work. The problems of the first two parts of the thesis, the PAP and the SCC-VRP are both new in the literature. Research can be continued in both topics from different perspectives: the proposal of other models that differently quantify the benefits of the collaboration, the assumption of other conditions that modify the problem, the design of exact-procedures or other algorithms to provide optimal solutions for the instances proposed or the study of the model properties, to mention just a few.

In the case of the PAP, the assignment problem is only a part of the whole urban planning challenges related to loading/unloading activities. The related problems (sizing, location, ...) can now be studied with the assumption that a fair allocation model exists to assign requests to parking places. In particular, the sizing of the loading/unloading areas can be revised since a more adjusted solution could be provided thanks to the in advanced assignment. In the case of the SCC-VRP, the problem can be extended under various assumptions. The conditions of the collaboration can be modified: possibility to transfer customers with low demand or inconvenient locations, possibility to transfer customers, etc. On the other hand, the way the benefits are split among the participants can be determinant for convincing stakeholders.

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A Parking Slot Assignment Problem. Summary Tables

A.1 The PAP as a VRP

Parameters and sets:	Description
$[0, T]$	Time interval for the assignment of parking slots
c	Number of parking places for vehicles to load/unload goods
Q	Index set of requests
q	Number of requests ($q = \|Q\|$)
a_i	Earliest starting time for request $i \in Q$
b_i	Latest starting time for request $i \in Q$
s_i	Duration of operation $i \in Q$
v_d	Fictitious depot
V	Set of vertices for the fictitious routes: $V = Q \cup \{v_d\}$
A	Set of arcs for the fictitious routes
d	Maximum allowed displacement from requested time window in MOD4
M	Big-M used in Constraints (4.5) relating variables
K	Big-M used in Constraints (4.25)–(4.26) of MOD3
w_i	Weight of request $i \in Q$ in the objective function of MOD1 and MOD3
μ_t	Unit cost for outsourced parking place at time slot $t \in \{0, ..., T-1\}$
κ	Constant value of objective function in MOD0

Table A.1: Parameters and sets of the models for PAP as a VRP

	Model	Objective function	
0	Feasibility	$z^0(x,t) = \kappa$	
1	Min ET	$z^1(x,t) = \sum_{i\in Q} w_i e_i$	(4.14)
2	Min-Max ET (a&b)	$z^2(x,t) = m$	(4.16)
3	Min ET st Max	$z^3(x,t) = \sum_{i\in Q} w_i e_i$	(4.21)
4	Min Num (a&b)	$z^4(x,t) = \sum_{i\in Q} \beta_i$	(4.23)
4'	Min Num weight(a&b)	$z^{4'}(x,t) = \sum_{i\in Q} s_i \beta_i$	(4.27)
5	Cost function	$z^5(x,y,t) = \sum_{t=0}^{T} n_t$	(4.34)
5'	Cost function time dependent	$z^{5'}(x,y,t) = \sum_{t=0}^{T} w_t \sum_{i\in Q} O_{it}$	(4.37)

Table A.2: Objective functions for PAP as a VRP

MOD	Common variables	Description
All	x_{ij}	If request j is performed exactly after request i (binary)
All	t_i	The time when request i begins being served (continuous)
MOD	**Specific variables**	**Description**
1/2/3/4	e_i	Earliness/tardiness associated with request i (continuous)
2/2b	m	Maximum earliness/tardiness (continuous)
4/4b	β_i	Indicator for request i if performed on time (binary)
5/5b	n_t	Number of outsourced requests scheduled at time slot t (integer)
5/5b	y_{ij}	If request j is performed exactly after request i in outsourced parking space (binary)
5b	O_{it}	Indicator for request i occupying an outsourced space at time slot t (binary)

Table A.3: Decision variables for the PAP as a VRP

MOD	Common Constraints			
All	Parking places	(4.2)	$\sum_{j\in Q} x_{v_d j} \le c$	
All	Connectivity	(4.3)	$\sum_{(i,j)\in A} x_{ij} - \sum_{(j,i)\in A} x_{ji} = 0$	$i \in Q$
All	Request completion	(4.4)	$\sum_{(i,j)\in A} x_{ij} = 1$	$i \in Q$
All	Time	(4.5)	$t_i + s_i - t_j \le (1 - x_{ij})M$	$i,j \in Q,\ (i,j) \in A$
All	Domain t	(4.7)	$0 \le t_i \le T$	$i \in Q$
All	Domain x	(4.8)	$x_{ij} \in \{0,1\}$	$(i,j) \in A$
None	Subtour elimination	(4.9)	$\sum_{\substack{(i,j)\in A \mid \\ i,j\in W}} x_{ij} \le \lvert W\rvert - 1$	$W \subset V$
MOD	**Particular Constraints**			
0 Feasibility	Time window	(4.6)	$a_i \le t_i \le b_i$	$i \in Q$
1 Min ET	ET	(4.11)	$e_i \ge a_i - t_i$	$i \in Q$
	"	(4.12)	$e_i \ge t_i - b_i$	$i \in Q$
	"	(4.13)	$e_i \ge 0$	$i \in Q$
2 Min-Max ET (a)	ET +			
	MET	(4.17)	$m \ge e_i$	$i \in Q$
2 Min-Max ET (b)	MaxET	(4.18)	$m \ge a_i - t_i$	$i \in Q$
	"	(4.19)	$m \ge t_i - b_i$	$i \in Q$
	"	(4.20)	$m \ge 0$	
3 Min ET st Max	ET +			
	Maximum displacement	(4.22)	$e_i \le d$	$i \in Q$
4 Min Num (a)	ET +			
	Num	(4.24)	$\beta_i K \ge e_i$	$i \in Q$
4 Min Num (b)	NumET	(4.25)	$\beta_i K \ge a_i - t_i$	$i \in Q$
	"	(4.26)	$\beta_i K \ge t_i - b_i$	$i \in Q$
5 Cost (a)	Connectivity y	(4.30)	$\sum_{(i,j)\in A} y_{ij} - \sum_{(j,i)\in A} y_{ji} = 0$	$i \in Q$
	Request assignment y	(4.31)	$\sum_{(i,j)\in A} (x_{ij} + y_{ij}) = 1$	
	Time y	(4.32)	$t_i + s_i - t_j \le (1 - y_{ij})M$	$i,j \in Q,\ (i,j) \in A$
	Domain y	(4.33)	$y_{ij} \in \{0,1\}$	$(i,j) \in A$
5 Cost (b)	Cost (a) +			
	Outsourcing	(4.35)	$O_{jt'} \ge O_{it} - (1 - y_{ij})$	$i,j \in Q, t, t' \in [0,T]$ with $t' = t + s_i$
	Outsourcing (b)	(4.36)	$O_{it} \le O_{it'}$	$t \in [0,T], t+1 \le t' \le t + s_i$

Table A.4: Constraints of the models for the PAP as a VRP

	MOD0	MOD1	MOD2		MOD3	MOD4		MOD5		
	Feasibility	Min ET	Min-Max ET		Min ET st Max	Min Num		Outsource		
			a	b		a	b	a	b	
Variables	q^2+2q	q^2+3q	q^2+3q+1	q^2+2q+1	q^2+3q	q^2+4q	q^2+3q	q^2+3q	$q^2+3q+qT$	
Common Constraints										Num Const
Parking places (4.2)	X	X	X	X	X	X	X	X	X	1
Connectivity (4.3)	X	X	X	X	X	X	X	X	X	q
Request completion (4.4)	X	X	X	X	X	X	X			q
Time (4.5)	X	X	X	X	X	X	X	X	X	$q(q-1)$
Domain t (4.7)	X	X	X	X	X	X	X	X	X	$[2q]$
Domain x (4.8)	X	X	X	X	X	X	X	X	X	$[2q(q-1)]$
Particular Constraints										
Time window (4.6)	X							X	X	$2q$
ET (4.11–4.13)		X	X		X	X				$2q$
MET (4.17)			X							q
MaxET (4.18–4.19)				X						$2q$
Maximum displacement (4.22)					X					q
Num (4.24)						X				q
NumET (4.25–4.26)							X			$2q$
Connectivity y (4.30)								X	X	q
Request y (4.31)								X	X	q
Time y (4.32)								X	X	q
Domain y (4.33)								X	X	$[2q]$
Outsource (4.35)									X	qT
Outsourceb (4.36)									X	$qT\sum_i s_i$
Constraints	q^2+3q+1	q^2+4q+1	q^2+5q+1	q^2+4q+1	q^2+5q+1	q^2+5q+1	q^2+3q+1	$2q^2+3q+1$	$q^2(2+T)+1+q(3-T+T\sum_i s_i)$	

Table A.5: Variable and constraint count for the PAP as a VRP

A.2 The PAP as an AP

Parameters and sets:	**Description**
$[0,1,\ldots,T]$	Discretized set of possible time for the assignment of parking slots
c	Number of parking places for vehicles to load/unload goods
Q	Index set of requests
s_i	Duration of operation $i \in Q$
d	Maximum allowed displacement from requested time window in MOD4
$\hat{e}_{it}$	Penalty value if request $i \in Q$ is assigned to start at time $t \in [0,1,\ldots,T]$

Table A.6: Parameters and sets of the models for PAP as an AP

MOD	**Common variables**	**Description**
All	h_{it}	If request i is performed at time interval t (binary)

Table A.7: Decision variable for the PAP as an AP

	Model	**Objective function**	
0	Feasibility	$z^0(h) = 0$	
1	Min ET	$z^1(h) = \sum_{i\in Q}\sum_{t'\in[0,\ldots,T]} \widehat{e}_{it} h_{it'}$	(4.40)
2	Min-Max ET	$z^2(h) = \max_{i\in Q}\max_{t'\in[0,\ldots,T]} \widehat{e}_{it} h_{it'}$	(4.41)
3	Min ET st Max	$z^3(h) = \sum_{i\in Q}\sum_{t'\in[0,\ldots,T]} \widehat{e}_{it} h_{it'}$	
4	Min Num	$z^4(h) = \sum_{i\in Q}\sum_{t'\in[0,\ldots,T]} \widehat{e}_{it} h_{it'}$	(4.44)

Table A.8: Objective functions for PAP as an AP

B Shared Customer Collaboration VRP. Formulations. Summary Tables

Parameters and sets:	**Description**
C	Set of carriers collaborating
N	Set of customers
C_u	Set of carriers serving customer $u \in N$
K_i	Set of unlimited vehicles for carrier $i \in C$
$K = \bigcup_{i \in C} K_i$	Total set of vehicles for all carriers
d_{ui}	Demand for customer $u \in N$ from company $i \in C_u$
$G = (V, A)$	Graph between customers
$d(k)$	Depot of route $k \in K$, same for all routes of same company
$V = N \cup_{k \in K} \{d(k)\}$	Set of nodes
$A \subseteq V \times V$	Set of arcs
c_{uv}	Cost of arc $(u, v) \in A$
Q	Capacity of vehicles $k \in K$
c	Fixed charge for one transfer between depots

Table B.1: Parameters and sets of the models

Variables:	**Description**
x^k_{uv}	equals 1 if vehicle $k \in K$ uses arc $(u, v) \in$A
z^k_{uij}	equals 1 if demand of customer $u \in N$ from carrier i is served by a vehicle of company j, in route $k \in K_j$. Note that z^k_{uii} equals 1 if demand from customer $u \in N$ from carrier i is served by the same company, not transferred.
y_{ij}	Binary variable that checks if there is the need to transfer demand between carriers i and j. $i \in C$ and $j \in C \| j > i$
v_C	the number of transfers needed between depots to transport goods

Table B.2: Variables of the vehicle based formulation

Variable:	**Count**
x^k_{uv}	$(n(n-1) + 2nc)k$
z^k_{uij}	$nc^2 k$
y_{ij}	$c + (c-1) + (c-2) + ... + 1$
v_C	1

Table B.3: Variable count of the vehicle based formulation

Variables:	**Description**
x^i_{uv}	equals 1 if a vehicle from company $i \in C$ uses arc $(u, v) \in$A.
z_{uij}	equals 1 if demand of customer $u \in N$ from carrier i is served by a vehicle of company j. Note that z_{uii} equals 1 if demand from customer $u \in N$ from carrier i is served by the same company, not transferred.
l^i_{uv}	equals the load of a vehicle from company $i \in C$ upon the arc $(u, v) \in$A
y_{ij}	Binary variable that checks if there is the need to transfer demand between carriers i and j. $i \in C$ and $j \in C \| j > i$
v_C	the number of transfers needed between depots to transport goods

Table B.4: Variables of the load based formulation

Variable:	**Count**
x^i_{uv}	(n(n-1)+2nc)c
z_{uij}	nc^2
l^i_{uv}	$(n(n-1)+2nc)c$
y_{ij}	$c+(c-1)+(c-2)+...+1$
v_C	1

Table B.5: Variable count of the load based formulation

Parameters and sets:	**Description**
N	Set of customers
C_u	Set of carriers serving customer $u \in N$
d_u	Demand for customer $u \in N$ (no need to specify carrier, only one)
$G = (V, A)$	Graph between customers
D	Depot of the company
$V = N \cup D$	Set of nodes
$A \subseteq V \times V$	Set of arcs
c_{uv}	Cost of arc $(u, v) \in A$
Q	Capacity of vehicles

Table B.6: Simplified parameters and sets for the single company VRP

C Abbreviations

Abbreviations

AGAP Aircraft-Gate Allocation Problem

AP Assignment Problem

BAP Berth Allocation Problem

CVRP Capacitated Vehicle Routing Problem

FSP Fixed job Scheduling Problem

GVW Gross Vehicle Weight

LB Lower Bound

LCP Lane Covering Problem

LCLCP Length Constraint Lane Covering Problem

LP Linear Programming

MCJDS Multi-carrier Joint Delivery Service

MDVRP Multiple Depot Vehicle Routing Problem

MILP Mixed Integer Linear Programming

NP Nondeterministic Polynomial time

PAP Parking Slot Assignment Problem

PDPTW Pickup and Delivery Problem with Time Windows

SEC Subtour Elimination Constraints

TSP Traveling Salesmen Problem

TU Totally Unimodular

TL Time Limit

TW Time Window

UCC Urban Consolidation Centers

VRP Vehicle Routing Problem

VRPSC Vehicle Routing Problem with Shared Customers

VRPTW Vehicle Routing Problem with Time Windows

VSP Variable job Scheduling Problem

D PAP. Test Instances

PAP. Test instances

In this Appendix we represent graphically three instances for the PAP, one of each type. We present instance 210 (triangular centered), instance 217 (triangular asymmetric) and instance 237 (double peak). Each request is represented in a row of the vertical axis with a blue rectangle and a red line. The rectangle shows the time window requested for starting the service. As described in Section 5.1 the durations are of 20, 40, 60 or 80 minutes. The red line is placed in the beginning of the interval and the length corresponds to the duration of the request. Note that since the time window corresponds to the request for starting the service, the duration can be longer than the time window.

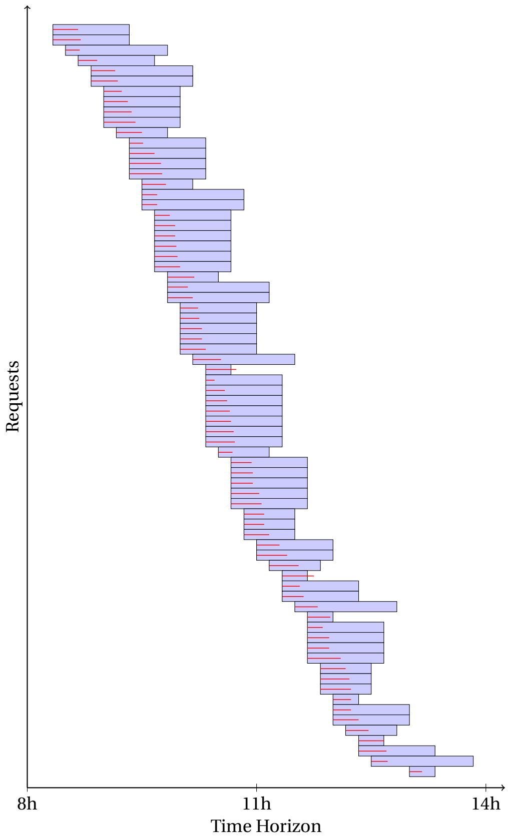

Figure D.1: Instance 210

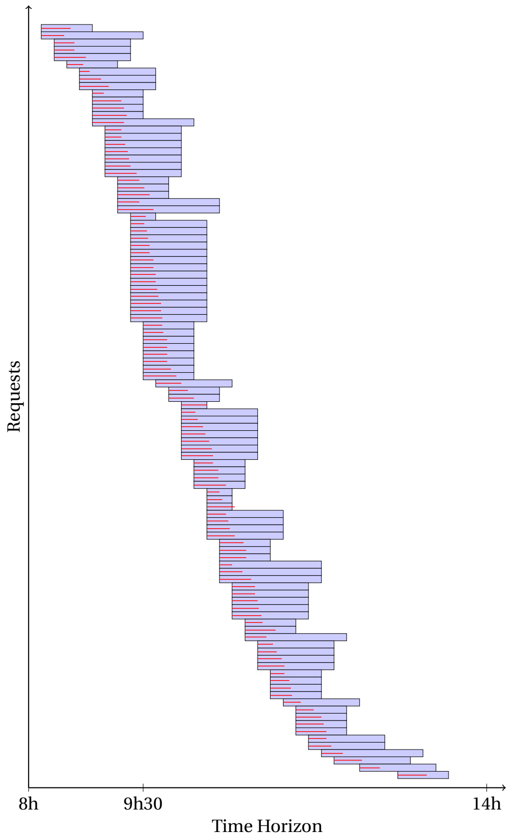

Figure D.2: Instance 217

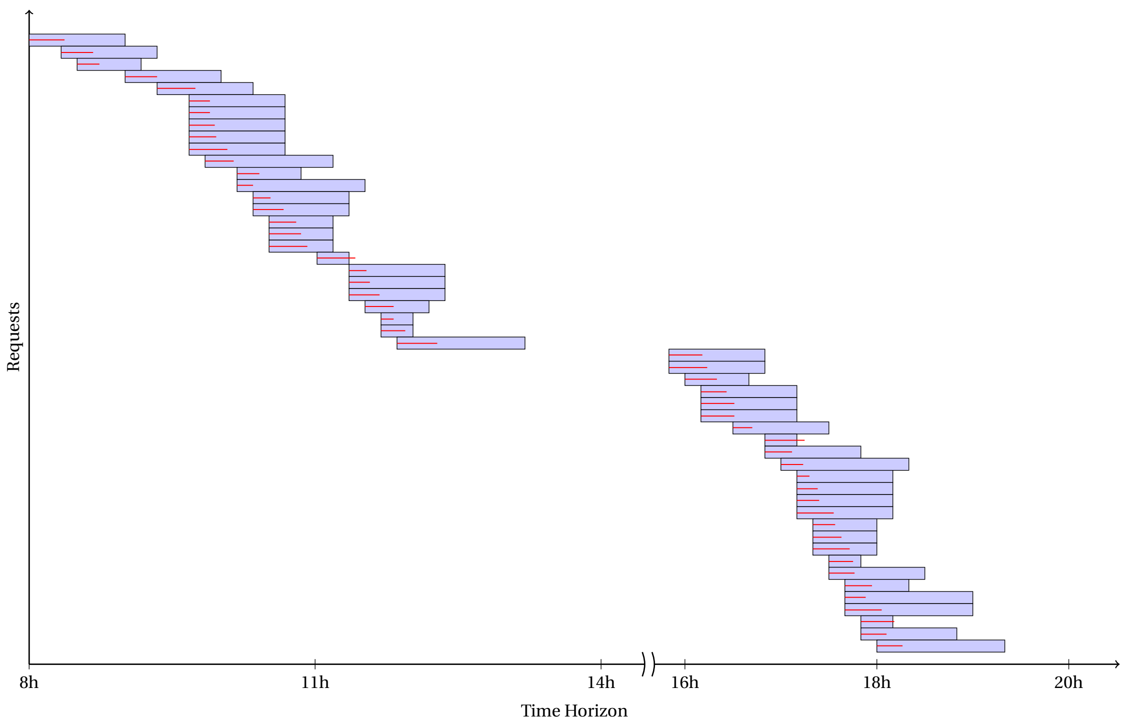

Figure D.3: Instance 237

E SCC-VRP. Test Instances

SCC-VRP. Test instances

In this Appendix additional information is given about the instances for the SCC-VRP used in the experiments of Section 9

Instance	R/C	Q	$\sum_{i\in N} d_{iA}$	$\sum_{i\in N} d_{iB}$	$\sum_{i\in N} d_{iA} + d_{iB}$	$\|N\|(\|A\|,\|B\|)$	Shared
1001	R	100	126(97)	250(111)	376	10 (5, 9)	4
1002	R	200	67(29)	164(38)	231	10 (4, 8)	2
1003	R	300	198(98)	271(104)	469	10 (6, 7)	3
1004	R	400	233(110)	202(95)	435	10 (7, 6)	3
1005	R	500	309(92)	267(91)	576	10 (6, 6)	2
1006	R	100	239(84)	116(87)	355	15 (13, 6)	4
1007	R	200	200(105)	166(40)	366	15 (9, 10)	4
1008	R	300	317(132)	347(108)	664	15 (8, 10)	3
1009	R	400	314(67)	309(77)	623	15 (9, 8)	2
1010	R	500	248(55)	499(48)	747	15 (5, 11)	1
1011	R	100	221(73)	249(77)	470	20 (12, 12)	4
1012	R	200	281(116)	239(88)	520	20 (13, 13)	6
1013	R	300	413(213)	505(219)	918	20 (12, 14)	6
1014	R	400	411(242)	594(268)	1005	20 (11, 16)	7
1015	R	500	681(357)	634(355)	1315	20 (15, 13)	8
1016	R	100	336(131)	328(128)	664	25 (16, 15)	6
1017	R	200	366(46)	258(79)	624	25 (16, 12)	3
1018	R	300	540(150)	536(159)	1076	25 (14, 15)	4
1019	R	400	545(169)	637(159)	1182	25 (13, 16)	4
1020	R	500	865(69)	529(28)	1394	25 (16, 10)	1
1021	R	100	447(195)	277(152)	724	30 (24, 15)	9
1022	R	200	460(212)	423(276)	883	30 (22, 19)	11
1023	R	300	816(143)	486(151)	1302	30 (21, 13)	4
1024	R	400	793(373)	776(344)	1569	30 (20, 19)	9
1025	R	500	1063(442)	863(425)	1926	30 (19, 19)	8
1026	C	100	126(97)	250(111)	376	10 (5, 9)	4
1027	C	200	67(29)	164(38)	231	10 (4, 8)	2
1028	C	300	198(98)	271(104)	469	10 (6, 7)	3
1029	C	400	233(110)	202(95)	435	10 (7, 6)	3
1030	C	500	309(92)	267(91)	576	10 (6, 6)	2
1031	C	100	239(84)	116(87)	355	15 (13, 6)	4
1032	C	200	200(105)	166(40)	366	15 (9, 10)	4
1033	C	300	317(132)	347(108)	664	15 (8, 10)	3
1034	C	400	314(67)	309(77)	623	15 (9, 8)	2
1035	C	500	248(55)	499(48)	747	15 (5, 11)	1
1036	C	100	221(73)	249(77)	470	20 (12, 12)	4
1037	C	200	281(116)	239(88)	520	20 (13, 13)	6
1038	C	300	413(213)	505(219)	918	20 (12, 14)	6
1039	C	400	411(242)	594(268)	1005	20 (11, 16)	7
1040	C	500	681(357)	634(355)	1315	20 (15, 13)	8
1041	C	100	336(131)	328(128)	664	25 (16, 15)	6
1042	C	200	366(46)	258(79)	624	25 (16, 12)	3
1043	C	300	540(150)	536(159)	1076	25 (14, 15)	4
1044	C	400	545(169)	637(159)	1182	25 (13, 16)	4
1045	C	500	865(69)	529(28)	1394	25 (16, 10)	1
1046	C	100	447(195)	277(152)	724	30 (24, 15)	9
1047	C	200	460(212)	423(276)	883	30 (22, 19)	11
1048	C	300	816(143)	486(151)	1302	30 (21, 13)	4
1049	C	400	793(373)	776(344)	1569	30 (20, 19)	9
1050	C	500	1063(442)	863(425)	1926	30 (19, 19)	8

Table E.1: Data summary of the extended set of instances 1001-1050

Instance	R/C	Q	$\sum_{i\in N} d_{iA}$	$\sum_{i\in N} d_{iB}$	$\sum_{i\in N} d_{iA}+d_{iB}$	$\|N\|(\|A\|,\|B\|)$	Shared
1051	R	100	90(52)	76(31)	166	10 (7, 7)	4
1052	R	200	80(17)	43(11)	123	10 (6, 5)	1
1053	R	300	114(26)	74(30)	188	10 (7, 5)	2
1054	R	400	76(41)	137(38)	213	10 (4, 8)	2
1055	R	500	151(100)	284(107)	435	10 (5, 8)	3
1056	R	100	118(38)	81(23)	199	15 (10, 8)	3
1057	R	200	133(56)	104(59)	237	15 (11, 8)	4
1058	R	300	186(64)	162(68)	348	15 (10, 8)	3
1059	R	400	241(88)	138(97)	379	15 (13, 7)	5
1060	R	500	227(125)	401(151)	628	15 (7, 12)	4
1061	R	100	127(55)	192(92)	319	20 (12, 14)	6
1062	R	200	136(62)	167(53)	303	20 (11, 14)	5
1063	R	300	242(82)	176(86)	418	20 (14, 11)	5
1064	R	400	190(82)	245(75)	435	20 (11, 14)	5
1065	R	500	373(87)	411(109)	784	20 (12, 11)	3
1066	R	100	228(61)	166(90)	394	25 (18, 13)	6
1067	R	200	172(15)	171(35)	343	25 (14, 13)	2
1068	R	300	345(192)	251(193)	596	25 (21, 15)	11
1069	R	400	291(130)	246(112)	537	25 (16, 16)	7
1070	R	500	547(189)	446(138)	993	25 (15, 15)	5
1071	R	100	323(144)	210(128)	533	30 (24, 16)	10
1072	R	200	282(119)	198(102)	480	30 (22, 17)	9
1073	R	300	470(177)	243(143)	713	30 (24, 15)	9
1074	R	400	286(135)	375(133)	661	30 (16, 22)	8
1075	R	500	518(151)	643(153)	1161	30 (16, 19)	5
1076	C	100	87(34)	56(37)	143	10 (8, 5)	3
1077	C	200	21(0)	91(0)	112	10 (2, 8)	0
1078	C	300	51(18)	129(11)	180	10 (3, 8)	1
1079	C	400	99(46)	115(27)	214	10 (5, 7)	2
1080	C	500	150(24)	174(31)	324	10 (6, 5)	1
1081	C	100	74(9)	97(8)	171	15 (7, 9)	1
1082	C	200	125(30)	102(21)	227	15 (9, 8)	2
1083	C	300	166(36)	140(30)	306	15 (9, 8)	2
1084	C	400	162(83)	211(71)	373	15 (8, 11)	4
1085	C	500	300(181)	337(147)	637	15 (9, 11)	5
1086	C	100	153(84)	151(89)	304	20 (14, 13)	7
1087	C	200	204(91)	119(86)	323	20 (16, 11)	7
1088	C	300	200(78)	233(95)	433	20 (13, 12)	5
1089	C	400	180(80)	230(66)	410	20 (10, 14)	4
1090	C	500	366(47)	375(69)	741	20 (11, 11)	2
1091	C	100	193(44)	154(37)	347	25 (14, 14)	3
1092	C	200	202(57)	212(44)	414	25 (14, 15)	4
1093	C	300	324(71)	185(57)	509	25 (18, 11)	4
1094	C	400	373(126)	167(119)	540	25 (22, 10)	7
1095	C	500	416(197)	597(230)	1013	25 (14, 18)	7
1096	C	100	210(61)	236(64)	446	30 (17, 19)	6
1097	C	200	223(86)	244(95)	467	30 (19, 19)	8
1098	C	300	254(96)	372(73)	626	30 (15, 21)	6
1099	C	400	390(139)	328(122)	718	30 (19, 18)	7
1100	C	500	543(144)	618(155)	1161	30 (17, 18)	5

Table E.2: Data summary of the extended set of instances 1051-1100

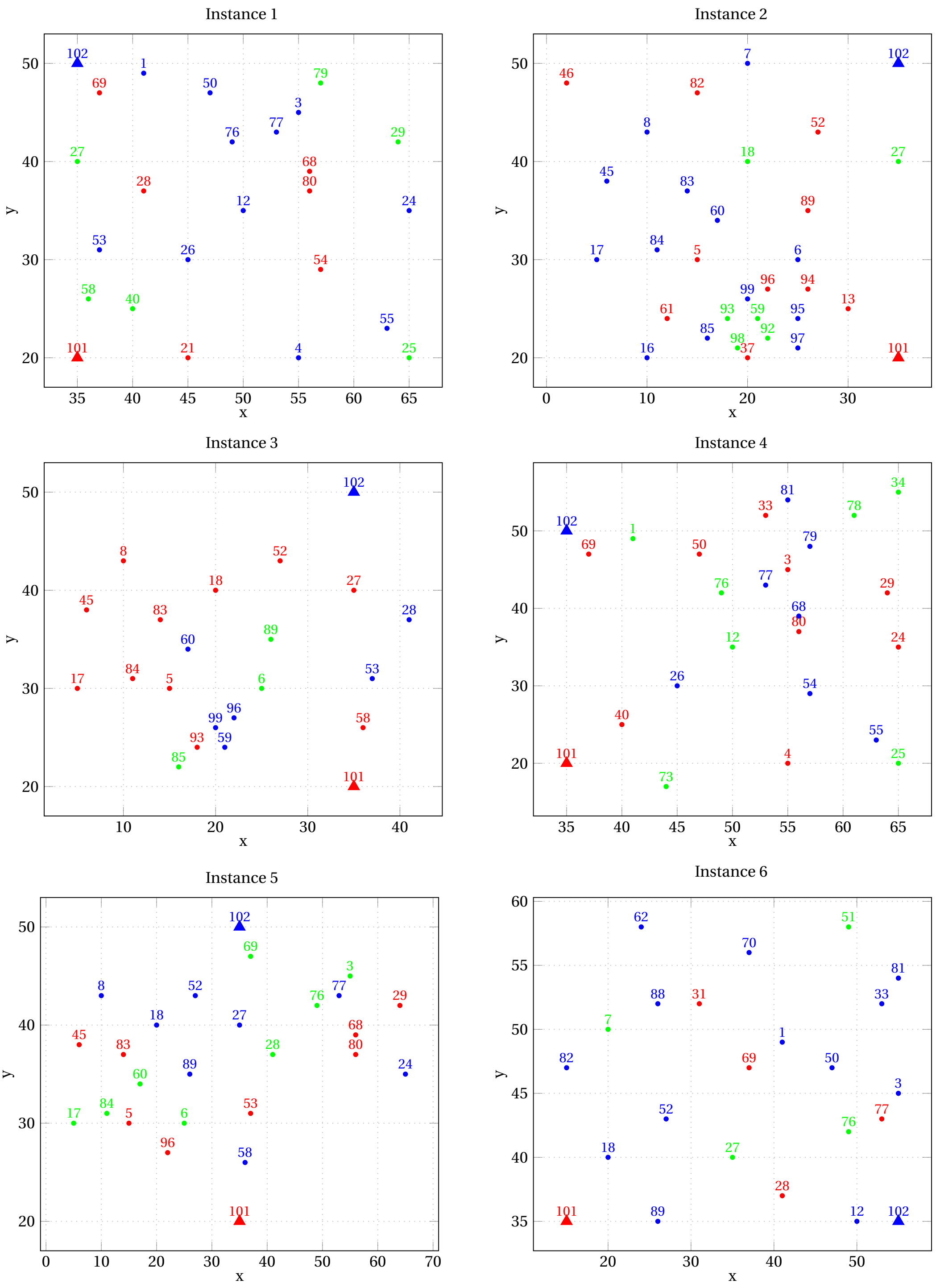

Figure E.1: SCC-VRP test instances 1-6 with 18-30 customer obtained from MDVRP [12]

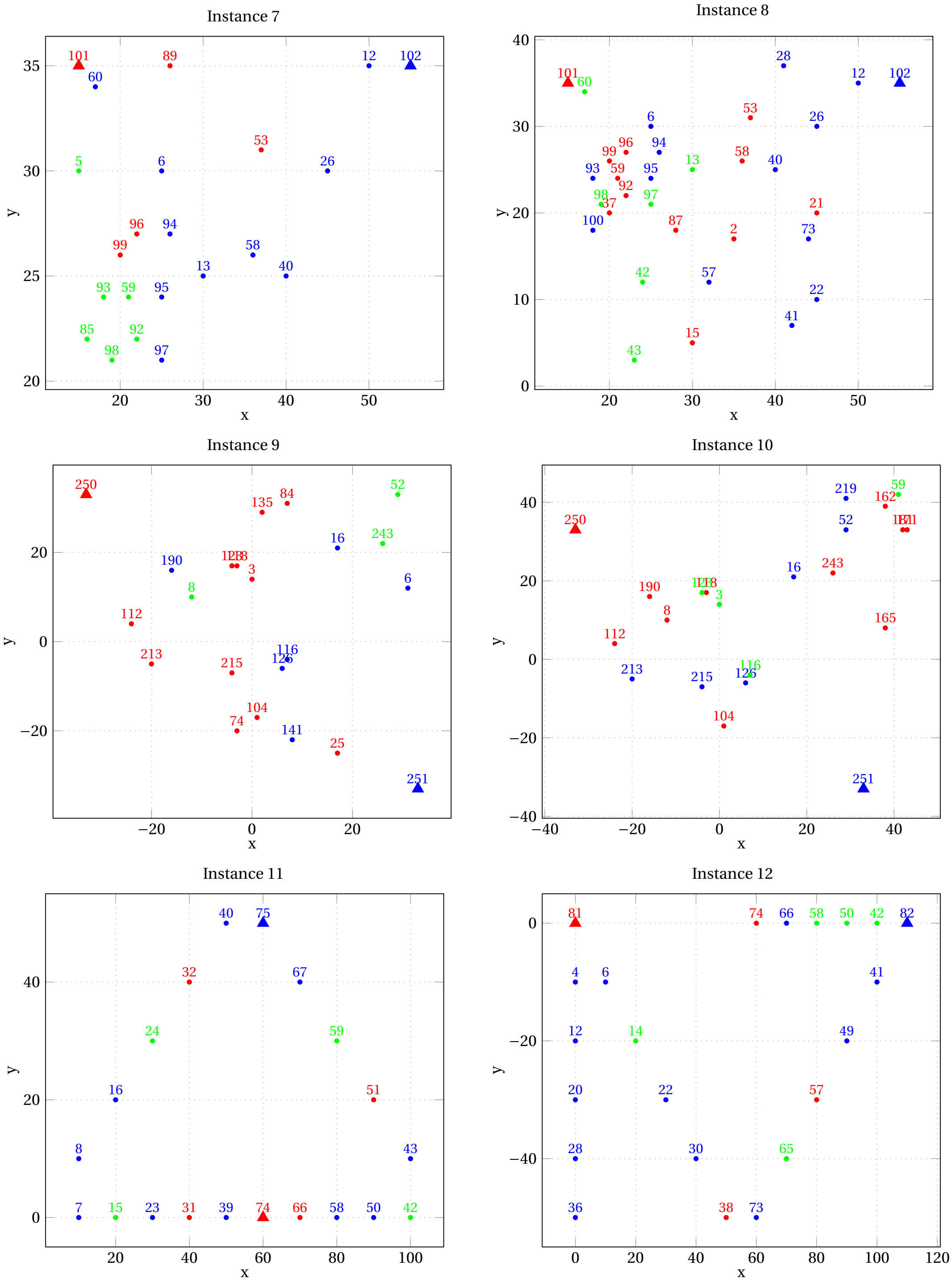

Figure E.2: SCC-VRP test instances 7-12 with 18-30 customer obtained from MDVRP [12]