# Essays in Market Microstructure

## Inter-Market Competition, Algorithmic Trading, and Call Auctions

PETER HOFFMANN

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DIRECTOR DE LA TESI

Prof. Francisco Peñaranda

DEPARTAMENT D'ECONOMIA I EMPRESA

UNIVERSITAT POMPEU FABRA

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*To my family*

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## Abstract

This thesis covers three topics in Market Microstructure. Chapter 1 demonstrates that market access frictions may play a significant role in the competition between trading platforms. Analyzing a recent dataset of the trading activity in French and German stocks, we provide evidence that the incumbent markets dominate because the sole market entrant exposes liquidity providers to an excessive adverse selection risk due to a lack of noise traders. Chapter 2 presents a theoretical model of price formation in a dynamic limit order market with slow human traders and fast algorithmic traders. We show that in most cases, algorithmic trading has a detrimental effect on human traders' welfare. Finally, Chapter 3 empirically analyzes the role of pre-trade transparency in call auctions. Comparing the trading mechanisms in place on the French and German stock exchanges, we find that transparency is associated with higher trading volume, greater liquidity, and better price discovery.

## Resumen

Esta tesis estudia tres temas diferentes de la microestructura de los mercados financieros. El capítulo 1 demuestra que fricciones en el acceso al mercado pueden desempeñar un papel significativo en la competencia entre plataformas de negociación de activos. El análisis de un conjunto de datos recientes de la actividad en acciones francesas y alemanas demuestra que los mercados primarios dominan debido a que el único mercado satélite expone los proveedores de liquidez a un riesgo excesivo de selección adversa, causado por una falta de *noise traders*. El capítulo 2 presenta un modelo teórico de formación de precios en un mercado dinámico con *limit order book* poblado por agentes humanos lentos y agentes algorítmicos rápidos. Se demuestra que, en la mayoría de los casos, la negociación algorítmica tiene un efecto negativo sobre el bienestar de agentes humanos. Por último, el capítulo 3 analiza empíricamente el papel de la transparencia pre-negociación en las subastas de apertura y de cierre. Comparando los mecanismos en las bolsas francesas y alemanas, encontramos que la transparencia está asociada con un volumen mayor, una liquidez mayor y un mejor *price discovery*.

# Preface

The topic unifying the three chapters of this thesis is market microstructure, a research field that can be loosely defined as the detailed study of the trading process in financial markets. When thinking of a stock exchange, most people picture a large hall populated by shouting men. While this could somehow be considered to mirror reality until at least the mid 1970's, the landscape has changed dramatically since then. Today, most trading floors are little more than a stage for TV journalists, as virtually all exchanges have become purely electronic marketplaces.

As the three chapters of this thesis show, the widespread use of modern communications technology has had a tremendous effect on the way financial assets are traded. Electronic trading platforms allow for the design of different market mechanisms that are tailored towards different clienteles and serve different purposes. Geographically distant markets are connected via high-speed computer networks, enabling competition between market centers that have transformed from mutual enterprises into profit-maximizing corporations. More recently, the technological revolution within the financial industry has accelerated dramatically, as more and more trading is conducted via sophisticated computer algorithms, rendering human intervention superfluous.

The first chapter is motivated by the recent establishment of the pan-European Markets in Financial Instruments Directive (MiFID) and examines the interplay adverse selection and transaction fees in the context of inter-market competition. Based on the seminal work by Glosten and Milgrom (1985), we develop a simple model of trading in a fragmented financial market. We show that if so called trade-throughs (transactions at prices inferior to the best available quote) are not prohibited, imperfections in traders' market access may lead to differences in the adverse selection risk faced by liquidity providers across trading venues. Consequently, the main (primary) market may dominate in equilibrium even if it charges higher fees than its competitors. The empirical analysis of a recent dataset of trading in French and German stocks suggests that trades on Chi-X, a recently launched low-cost trading platform, carry significantly more private information than those executed in the Primary Markets. Additionally, we present evidence that much of this difference appears to be driven by the inability of uninformed traders to choose the market they trade in. Consistent with our theory, we find a negative relationship between the competitiveness of Chi-X's quotes and this *excess* adverse selection risk faced by liquidity providers in the cross-section. In conclusion, our results suggest that trade-throughs may constitute a serious obstacle to inter-market competition.

Without doubt, algorithmic trading is the most intensively debated topic in current market microstructure research. Building on prior work by Foucault (1999), Chapter 2 contributes to the ongoing controversy by presenting a stylized model of price formation in a dynamic limit order market populated by slow human traders (HTs) and fast algorithmic traders (ATs). In particular, we assume that their increased speed allows ATs to avoid the winner's curse associated with limit orders when trading with HTs. We show how algorithmic trading may lead both to increases and decreases in trading volume (and thus aggregate welfare). An analysis on individual traders' welfare shows that the presence of ATs has a detrimental effect on HTs' welfare for most parameter constellations. Intuitively, the reduced risk incurred when placing limit orders (their *endogenous* outside option) induces ATs do demand better terms of trade for market orders. As a consequence, HTs may either increase the aggressiveness of their quotes or accept a decreased probability of execution, leading to lower profits from limit orders in both cases. In turn, this makes HTs willing to accept lower profits from market orders.

Finally, Chapter 3 (which is joint work with Jos van Bommel) studies call auctions, a widely used trading mechanism to determine the opening and closing prices on most stock exchanges across the globe. Using a matched sample approach, we compare the performance of the trading protocols in place on the French (Euronext) and German (Deutsche Börse) stock exchanges. While the former market discloses the limit order book to market participants during its auctions, the latter only provides indicative information about the clearing price and the associated trading volume. Using a matched sample approach over a sample period of two years, we find that the auctions in the transparent market exhibit a significantly higher trading volume, better liquidity (measured as the price impact of order flow) and improved price discovery. Overall, these findings are consistent with a) transparency enabling liquidity traders to signal their trading intentions and b) an informative limit order book.

# Table of Contents

# List of Figures

# List of Tables

**Page**

# 1. ADVERSE SELECTION, TRANSACTION FEES, AND MULTI-MARKET TRADING

## 1.1. Introduction

The introduction of the Markets in Financial Instruments Directive (MiFID) in late 2007 has spawned competition among stock exchanges across Europe. Under the new legislation, alternative trading platforms (so-called Multilateral Trading Facilities, henceforth MTFs) may directly compete with the national stock exchanges (Primary Markets) for customer order flow. Ultimately, MiFID aims at creating a level playing field that promotes competition between market centers and fosters innovation.

One issue that has received a great deal of attention in the context of inter-market competition is the design of best execution policies. Under MiFID, intermediaries such as banks and brokers bear the entire responsibility for obtaining “the best possible result” for their clients’ orders. Importantly, best execution is not only based on prices but rather permits the consideration of a wide array of additional execution characteristics such as liquidity, order size, and the likelihood of execution, among others (see e.g. Petrella (2009) and Gomber and Gsell (2006) for details). Consequently, MiFID does not formally enforce inter-market price priority, and orders are permitted to execute at a price that is inferior to the best *available* price across venues (“trade-throughs”). This differs considerably from the rules that are in place in the United States under Reg NMS, which mandates exchanges to re-route orders to other market centers if those are offering a better price (“trade-through rule”).

In this article, we argue that allowing for trade-throughs may benefit the Primary Markets and therefore limit inter-market competition. To this end, we study how adverse selection and transaction fees interact in a fragmented financial market where trade-throughs are not prohibited. Inspired by the current market setting in Europe, we develop an extension of the Glosten and Milgrom (1985) sequential trade model where liquidity providers post quotes in two separate trading platforms, the Primary Market and a low-cost MTF. A key ingredient in our model is the existence of market access frictions. Following Foucault and Menkveld (2008), we assume that the Primary Market is accessible by all agents in the economy, while trading on the MTF requires a so-called smart order routing system that is only available to a subset of the trader population. Due to the absence of a trade-through rule, this access friction gives rise to inter-market differences in the adverse selection risk faced by liquidity providers. If informed traders are more likely than uninformed traders to be "smart routers", situations can arise where the Primary Market offers better quotes frequently despite charging higher transaction fees.

The analysis of a recent sample of transactions and quote data for German and French stocks confirms the existence of imperfections in traders' routing abilities, as only about every second trade originates from agents with access to Chi-X, a recently launched MTF. Moreover, we find that trades executed on this new trading platform carry significantly more private information than their counterparts on the Primary Markets, while trade-throughs are particularly uninformative. This implies that liquidity providers on the MTF incur a higher adverse selection risk precisely because an important fraction of the uninformed order flow is held captive in the Primary Markets. Cross-sectional regressions provide empirical support for our theory, as we find that this excess adverse selection risk is negatively related to Chi-X's presence at the inside quote.

These results have important implications for the design of best executions policies. Allowing for trade-throughs benefits the Primary Markets because captive traders constitute a stable customer basis that is not subject to competition from other exchanges. Additionally, liquidity providers on alternative trading venues are exposed to a higher adverse selection risk because smart routers are more likely to be informed than the average trader. This excess risk frequently results in poor quotes and therefore diverts additional order flow from smart routers to the Primary Markets. Therefore, trade-throughs constitute an important obstacle for inter-market competition as the cheaper market (in terms of transaction fees) may end up with very little order flow, even from agents that have access to it. In this sense, our model supports the idea that the enforcement of inter-market price priority may foster competition between exchanges.

Our findings are in line with existing concerns about MiFID's best execution policy. In the absence of inter-market linkages, market fragmentation increases the costs of monitoring markets in real-time, as it requires intermediaries to adopt a smart order routing system. For smaller market participants, the substantial costs associated with such an infrastructure may well exceed the expected benefits. Consistent with this view, a recent article in the Financial Times[1] reports that much retail order flow is routinely routed towards the Primary Markets as small brokers shy away from investments in technology. Additionally, Ende and Lutat (2010) document a sizeable fraction of trade-throughs in European stocks, which confirms the existence of market access imperfections post-MiFID.

Moreover, recent anecdotal evidence appears consistent with our view that a higher adverse selection risk negatively affects the competitiveness of MTFs. On September 8th, 2008, the market opening on the London Stock Exchange was delayed until 4 pm in the afternoon due to a technical problem. While Chi-X and Turquoise were still available for trading in UK

[1] See Financial Times, "Private investors fail to see MiFID benefits", March 7th 2010.

stocks, the market activity ceased almost completely during the LSE's system outage. A similar event occurred on Euronext on April 20th, 2009, when trading commenced with a delay of one hour. Again, trading did not migrate to the MTFs. Presumably, excessive adverse selection risk lead to a market breakdown. Yet another outage on the LSE during the afternoon of November 9th, 2009 saw some trading migrate to alternative venues. The fact that the outage occurred late in the trading day may have helped the MTFs, as much of the day's price discovery had already occurred prior to the LSE's breakdown.

This chapter contributes to the existing literature on inter-market competition. While early theoretical papers (e.g. Pagano (1989) and Chowhdry and Nanda (1991)) argue that markets display a natural tendency to consolidate as a consequence of liquidity externalities, there is a large empirical literature that empirically documents the existence of fragmented financial markets (e.g. Bessembinder (2003), Boehmer and Boehmer (2003), Goldstein et al. (2008), Biais et al. (2010a)).

Most closely related to our work, Foucault and Menkveld (2008) develop and test a theory of competition between two markets in an environment that allows for trade-throughs. In their model, which abstracts from uncertainty about the asset's fundamental value, risk-neutral competitive agents trade off the expected revenue from liquidity provision against order submission fees. They find that the share of liquidity provided on the alternative trading platform (weakly) increases in the proportion of smart routers. While our work shares their assumption of heterogeneity in traders' routing abilities, we consider a model with a risky asset and asymmetric information. We therefore contribute to the literature by studying the interplay of adverse selection risk and transaction fees in the context of inter-market competition under the absence of price priority across trading venues.

Naturally, our work is also closely related to a number of papers that study differences in informed trading across markets. One strand of this literature analyzes the effects of “cream-skimming” and payment for order flow (e.g. Easley et al. (1996a), Bessembinder and Kaufman (1997), Battalio et al. (2002), Parlour and Rajan (2003)). In our context, the competitiveness of alternative trading platforms is hampered by the concentration of uninformed order flow on the Primary Markets due to trade-throughs generated by captive traders. This contrasts strongly with the standard paradigm within this literature, where uninformed order flow is directed *away* from the main market center due to so-called preferencing agreements[2]. Other papers (e.g. Grammig et al. (2001), Barclay et al. (2003), Goldstein et al. (2008)) document differences in informed trading between dealer markets and anonymous electronic trading systems. Generally, these studies find order flow in electronic markets to be more informative, presumably because informed traders value the higher speed of execution offered by these venues and try to prevent information leakage due to interacting with intermediaries such as market makers. In contrast, we show that differences in informed trading across exchanges may also arise through the absence of inter-market price priority paired with frictions in traders’ market access.

Finally, our model also accommodates the results of Hengelbrock and Theissen (2009), who study the market entry of the Turquoise MTF in late 2008 and find that the trading activity in larger and less volatile stocks tends to fragment more.

This chapter is organized as follows. Section 1.2 introduces our theoretical model, while Section 1.3 describes the institutional environment and

[2] Preferencing agreements usually establish a relation between a broker and a trading platform, where brokers receive a payment for directing the entire order flow to a particular venue. This practice was pioneered by Bernhard Madoff in the 1980’s.

presents the data. Section 1.4 introduces estimates for differences in informed trading between the Primary Markets and Chi-X, and Section 1.5 presents evidence on the model's empirical implication. Section 1.6 concludes, while proofs and tables are relegated to Appendix A.

## 1.2. The Model

There is a single risky asset with liquidation value $V \in \left\{ \overline{V}, \underline{V} \right\}$, where we set $\Pr(V = \overline{V}) \equiv d_0 = 1/2$ for simplicity. The asset can be traded on two separate trading platforms, which we denote by C(hi-X) and P(rimary Market). These markets are populated by $N^P \geq 2$ and $N^C \geq 2$ identical, risk neutral market makers, respectively, who post bid and ask quotes for a single unit of the risky asset. Market P charges a cost $c > 0$ per trade to market makers, while the cost charged by market C is normalized to zero. We assume that $c$ is very small in comparison to the asset's fundamental uncertainty, i.e. $c << (\overline{V} - \underline{V})/2$.

There is a continuum of traders, who arrive sequentially at time points $t = 1, \ldots, \mathrm{T}$. Unlike Dennert (1993), we assume that each trader may buy or sell at most one unit of the asset. Moreover, agents may only trade once, directly upon entering the market. A proportion $m$ of the trader population is perfectly informed about the liquidation value $V$, while the remaining traders are uninformed. Whereas all agents can trade in market P, trading in market C requires a smart order routing system that is not available to everyone in the economy. Denote the proportion of informed and uninformed traders with smart order routing technology by $q^I$ and $q^U$, respectively. We call those traders smart routers, while agents that can only trade in market P are named captive traders. Figure A.1 in Appendix A.2 graphically depicts the structure of the trader population for the case $q^I > q^U$.

The overall proportion of smart routers is given by $q = mq^I + (1-m)q^U$. Let $m^{SR}$ and $m^{CT}$ denote the proportion of informed traders among smart routers and captive traders, respectively, which are given by

$$m^{SR} \equiv \frac{q^I m}{q} \qquad m^{CT} \equiv \frac{(1-q^I)m}{1-q}$$

It is easy to see that $q^I > q^U$ ($q^I < q^U$) implies $m^{SR} > m > m^{CT}$ ($m^{SR} < m < m^{CT}$).

[Insert Figure A.1 about here]

Uninformed traders buy or sell with equal probability. Informed traders buy if $V = \overline{V}$ and the best ask at which they can trade is less or equal to $\overline{V}$ at the time of their arrival, and sell if $V = \underline{V}$ and the best available bid is higher or equal to $\underline{V}$. Otherwise, they do not trade. Traders always choose to trade in the market that offers the better price given their trading interest and market access.

One issue arises in situations where both venues display identical quotes, such that smart routers are indifferent between markets. Given that market makers are not required to place their quotes on a discrete grid (i.e. the tick size is zero), such ties may arise even if both trading platforms charge different fees for market orders, because ultimately all fees are borne by the market order traders (liquidity providers simply pass them on). Clearly, a positive tick size can break traders' indifference, as ties will only occur before transaction costs. Then, smart routers will rationally trade in the market that demands lower fees for market orders. As the introduction of a tick size comes at the expense of additional notation without providing further insights, we opt for a reduced-form approach and assume that smart routers always trade in market C in the case of an inter-market tie.

**Assumption (Tie-breaking rule):**

In the case of an inter-market tie, smart routers always trade in market C.

On the other hand, if several market makers post the same price in the same market, we assume that one of them is randomly selected (with equal probability) as a trading partner for the incoming trader. After each trading round *t*, market makers update their beliefs about the probability of the high outcome of the asset's liquidation value using Bayes' rule and revise their quotes accordingly. We assume that they observe each other's trades[3], which implies that they hold identical beliefs about the liquidation value at all times. Let $d_{t-1}$ denote this common belief prior to the arrival of the *t*-th trader.

For simplicity, we restrict our analysis to the bid side. Results for the ask side can be derived following exactly the same logic. Let $b_t^{i,P}$ and $b_t^{j,C}$ the quotes of market makers *i* and *j* in markets P and C, respectively, where $i = 1, \ldots, N^P$ and $j = 1, \ldots, N^C$. Moreover, define the best bid in market *k* as $b_t^k = \max\left\{b_t^{1,k}, ..., b_t^{N^k,k}\right\}$ for $k \in \{P, C\}$.

Given that captive traders can only trade in the Primary Market, the probability of a sell occurring in market P is always strictly positive. On the other hand, trade may only occur in market C if the bid quote at least matches the bid prevailing in market P. This leads us to the following definition of market co-existence.

**Definition (Market co-existence):**

Markets co-exist if and only if $b_t^C \geq b_t^P$.

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[3] This can be interpreted as markets being subject to post-trade transparency. See Madhavan (1995) for a model without post-trade transparency.

We are now ready to state our main result, which provides a condition for market co-existence to obtain in equilibrium. We restrict our attention to symmetric equilibria in pure strategies.

**Proposition 1:**

Let $N^P \to \infty$. Then, in equilibrium, markets co-exist if and only if

$$c \geq (\overline{V} - \underline{V})(m^{SR} - m)\Psi_B(d_{t-1}) \quad (1.1)$$

where

$$\Psi_B(d_{t-1}) = \frac{2d_{t-1}(1-d_{t-1})}{\left[1+m^{SR}(1-2d_{t-1})\right]\left[1+m(1-2d_{t-1})\right]}$$

**Proof:**

See Appendix A.1.

To understand the intuition behind this result, first consider the case where $q^I \leq q^U$. This implies that $m^{SR} \leq m$, i.e. the proportion of informed traders among smart routers is no greater than the proportion of informed traders in the overall trader population, such that market makers on platform C face a (weakly) lower adverse risk. Additionally, market C does not charge any transaction fees, so that the best bid on Chi-X is always strictly higher than the best bid in the Primary Market. In this case, condition (1.1) is necessarily satisfied as the right-hand side is always negative, such that markets co-exist.

Now consider the converse situation, where $q^I > q^U$, or equivalently $m^{SR} > m$. In this case, liquidity providers in market C face an excess adverse selection risk due to a higher proportion of informed traders among smart routers, which is captured by the right-hand side in (1.1). On the other hand, they do not incur the transaction fee $c$ that is payable for transactions in market P. Clearly, the best bid on Chi-X can only match or improve upon the Primary Market if the fee savings compensate for the excess adverse selection risk. The function $\Psi_B(d_{t-1})$ captures the behaviour of the adverse selection differential over time: As the order flow is informative about the asset's liquidation value, market makers' beliefs $d_{t-1}$ converge to either zero or one as the number of trading rounds becomes large, such that $\Psi_B(d_{t-1})$ approaches zero. As the adverse selection risk diminishes, differences in quotes across markets are entirely determined by the difference in transaction fees, and the market co-existence condition is necessarily satisfied.

Proposition 1 has an empirical implication for Chi-X's quote competitiveness in the cross-section. In order to see this, define $\Delta AS = \max_{d_{t-1}} \Delta AS(d_{t-1})$, where $\Delta AS(d_{t-1}) = (\overline{V} - \underline{V})(m^{SR} - m)\Psi_B(d_{t-1})$. Now consider two assets, $A$ and *B*, and suppose that $\Delta AS_A(d_{t-1}) > \Delta AS_B(d_{t-1})$ for all market maker beliefs $d_{t-1}$, i.e. the cross-market adverse selection differential (Chi-X minus Primary Market) is always strictly greater for asset A. If $c \geq \Delta AS_A$, the market co-existence condition (1.1) is satisfied for all possible beliefs and the best bid on Chi-X will match or improve upon the best bid in the Primary Market throughout the entire trading day for both assets. On the other hand, condition (1.1) is not always satisfied if $c < \Delta AS_A$. Moreover, as $\Delta AS_A(d_{t-1}) > \Delta AS_B(d_{t-1})$ for all $d_{t-1}$, there exist beliefs for which Chi-X matches or improves on the Primary Market for asset *B*, while the Primary Market displays a strictly better quote for asset $A$. The converse never holds. This leads us to the following empirical prediction.

**Corollary 1:**

In the cross-section, Chi-X's presence at the inside quote (weakly) decreases in the adverse selection risk differential (Chi-X minus Primary Market).

## 1.3. Institutional details and data

In the remainder of the chapter, we empirically analyze a sample of transaction data from Chi-X and two Primary Markets, Euronext (Paris) and Xetra (Frankfurt), in order to validate the empirical prediction of our model (Corollary 1). Before we turn to the description and a preliminary analysis of our dataset, we provide a brief overview of the institutional details that pertain to our sample period (May – April 2008).

### 1.3.A. Institutional details

Chi-X was launched on March 30th, 2007, when it started to offer trading in German and Dutch blue chips. Later the same year, trading was extended to the largest British (June 29th), French (September 28th), and Swiss (November 23rd) equities. Several other European markets were added subsequently, and starting in late 2008, the spectrum of available stocks was extended to mid-caps. As of November 2010, almost 1,400 stocks from 15 European countries could be traded on Chi-X. According to Fidessa[4], Chi-X's market share during the first six months of 2010 exceeded 20% for Belgian, Dutch, French, German, and British blue chips.

Like virtually all European stock markets, Chi-X is organized as a continuous, fully electronic limit order market (LOM). During trading hours, participants can continuously submit, revise and cancel limit and market orders. Non-executed limit orders are stored in the limit order book, and incoming market orders execute against those. Trading is fully anonymous, both pre- and post-trade.

[4] See http://fragmentation.fidessa.com.

Chi-X offers a very simple fee structure, which is asymmetric (a so-called make/take fee scheme): Passive executions (limit orders) receive a rebate of 0.2 bps, while aggressive executions (market orders) are charged 0.3 bps. Therefore, the platforms overall revenue per trade amounts to 0.1 bps. In the US market, these make/take fee schemes have proven key to success for the ECNs.

As opposed to Chi-X, the Primary Markets under consideration in this chapter (Euronext Paris and Deutsche Boerse's Xetra) do not distinguish between active and passive executions, i.e. their fee structures are symmetric. Euronext charges €1.20 plus 0.055 bps per executed order, which amounts to 0.455 bps for an average trade size of ~€30,000 (see Table A.2). Xetra charges 0.552 bps per trade (subject to a minimum charge of €0.69 with a cap at €20.70), which is somewhat more expensive for the average trade size but cheaper for smaller trades. Both exchanges offer different rebate schemes for particularly active members subject to minimum activity charges. Overall, the transaction fees in both Primary Markets are relatively similar and significantly exceed those on Chi-X, particularly for orders providing liquidity[5].

Besides charging considerably lower fees, Chi-X distinguishes itself from the Primary Markets in several other aspects. Most prominently, the MTF specifically targets high frequency traders via an ultra-low system latency, which according to the platform[6] is "up to ten times faster than the fastest European primary exchange". Moreover, Chi-X offers a wider range of admissible order types such as hidden and pegged orders. While the first

[5] Both Xetra and Euronext have designated market makers that are committed to maintain a minimum spread and a certain depth for individual stocks. Those market participants are usually exempt from transaction fees.

[6] "Chi-X celebrates its first anniversary", Chi-X press release, 07.04.2008

order type is completely invisible until executed[7], the latter type is a limit order where the limit price is "pegged" to a reference price, e.g. the best bid in the Primary Market, and is updated continuously. Finally, at the time of our sample (April-May 2008), Chi-X facilitated the undercutting of Primary Markets' quotes by offering a lower tick size for most securities. The entry of additional MTFs triggered a race for lower tick sizes in early 2009, which was ended with an agreement brokered by the Federation of European Securities Exchanges (FESE), after which the MTFs adopted the tick sizes used by the respective Primary Market.

While Chi-X only offers trading in a continuous LOM, both Xetra and Euronext additionally hold call auctions to set the opening and closing prices. Xetra also has an intraday call auction at 13:00 CET, which nevertheless generates only negligible trading volume except on days where derivative contracts expire (see Hoffman and van Bommel (2010)). Moreover, unlike Chi-X, the Primary Markets have a fixed set of rules that triggers an automatic call auction in times of extreme price movements (so-called volatility interruptions).

## 1.3.B. Data and preliminary analysis

Chi-X Ltd. generously provided us with a very detailed dataset for the months of April and May 2008, comprising a total of 43 trading days. The data contains information on the entire order traffic generated during this period, listing limit order additions, cancellations/modifications as well as trades separately. Timestamps are rounded to the nearest millisecond. From this data, we reconstruct the entire limit order book as well as the best bid and offer (BBO) at each point in time. Data for trades and quotes of French and German stocks on their respective Primary Markets (Euronext and

[7] Hidden orders usually must meet minimum size requirements under MiFID.

Xetra) during the corresponding time period was obtained from Reuters. Again, timestamps are rounded to the nearest millisecond. While the Chi-X data always contains a qualifier that tells us whether a market order was a buy or a sell, we sign trades on the Primary Markets using the Lee and Ready (1991) algorithm. As opposed to trades in a dealer market such as the NYSE, the risk of order misclassification is very small in a pure limit order book. Merging the BBO data from Chi-X and the Primary Markets, we obtain the European Best Bid and Offer (EBBO). We restrict our analysis to the continuous trading phase, which spans the time between 9:00 and 17:30 CET.

At the time of our sample, Chi-X was the only existing MTF and only offered trading in blue chips, such that our analysis is limited to the constituents of the CAC40 and DAX30 indices. We drop three French stocks (Arcelor, EADS and Dexia) from our sample as they are simultaneously traded on other European markets (Amsterdam, Frankfurt, and Brussels, respectively), such that our final sample comprises of 67 stocks.

[Insert Table A.1 and Table A.2 about here]

Table A.1 lists the stocks contained in our sample, while Table A.2 contains summary statistics on the trading activity during our sample period. Overall, trading on Chi-X accounts on average for 5.95% of total trading volume, or 12.31% in terms of trades. Consequently, the average trade size on the Primary Markets (€28,990) is more than twice as large as the average transaction value on Chi-X (€12,620). This is in line with Chi-X being largely dominated by algorithmic traders, who have been shown to employ much smaller trade sizes than human traders (see e.g. Hendershott and Riordan (2009)). Moreover, it is consistent with small orders being particularly cheap to execute on the MTF due to the Primary Markets' minimum fixed fees per order (see Section 1.3.A).

We also report the results for terciles based on stocks' average trading volume. Chi-X has a considerably larger market share (both in terms of trades and traded value) for the most active stocks, which is consistent with the evidence presented in Hengelbrock and Theissen (2009) for the Turquoise MTF.

Panel A of Table A.3 contains statistics about the quality of Chi-X's quotes. The MTF is frequently present at the EBBO (around 49% for either bid or ask), and often even improves on the Primary Markets' quotes (ca. 26% for bid or ask). Nevertheless, the frequency with which the MTF is simultaneously present at both sides of the inside quote (alone) is considerably lower with approximately 24% (7%), indicating that the activity on Chi-X is often restricted to one side of the market. While the Primary Markets are naturally present at the inside quote more often, they frequently face competition for at least one side of the market as they only spend roughly 26% of the time alone at the EBBO. Investigating the individual terciles, one can see that the MTF's quote competitiveness is somewhat higher for more active stocks, which is in line with the higher market shares in those stocks.

[Insert Table A.3 about here]

Panel B of Table A.3 reports the average available market depth for each trading venue conditional on being present at the inside quote. Overall, the available depth in the Primary Markets is roughly three times the depth on Chi-X, which may in part explain the observed cross-market differences with respect to the average trade size. Nevertheless, the MTF displays considerable depth for its quotes.

Based on its presence at the best quotes, Chi-X's market share (in terms of trades or trading volume) seems strikingly low. This is consistent with the market access friction in our model, which forces captive traders to trade in the Primary Market irrespectively of the quotes prevailing elsewhere. In order to quantify this friction, we follow Foucault and Menkveld (2008), who suggest estimating the proportion of smart routers ($q$) by the percentage of trades being executed on Chi-X conditional on the Primary Market offering a *strictly* worse quote. We additionally require that the depth on the MTF is sufficient to get the order filled entirely because it is natural to assume that traders take quantities into account when deciding on where to route their orders. Given that Chi-X offers a considerably lower market depth on average, some agents may avoid splitting up their orders and therefore prefer the Primary Market. This will particularly be the case if the marginally better price on the MTF is only available for a small fraction of the total order size.

The results in Table A.4 (first column) strongly confirm the importance of imperfect order routing. Conditional on Chi-X offering a better quote with sufficient depth, every second order is still executed in the Primary Market, such that the proportion of smart routers is roughly 50%. Interestingly, the routing friction varies little across the different activity terciles, which is in contrast to the results in Foucault and Menkveld (2008), who report a marked drop in smart order routing once moving beyond the most active stocks.

[Insert Table A.4 about here]

An additional item of great interest is the tie-breaking rule assumed in the theoretical model of Section 1.2. Given the data at hand, we can actually compute an estimate of the tie-breaking rule, again following Foucault and Menkveld (2008). In particular, the probability of a trade occurring on the

Primary Market conditional on equal quotes across venues and sufficient depth on Chi-X to fill the order completely is equal to

$$p = (1 - q) + qt$$

where $q$ is the proportion of smart routers and $t$ denotes the parameter of the tie-breaking rule. This equation simply states that all captive traders plus a fraction $t$ of the smart routers will trade on the Primary Market in the case of a tie, while the remaining agents trade on Chi-X. The last two columns of Table A.4 contain the estimates for the proportion of trades executing in the Primary Market under an inter-market tie ($p$) and the tie-breaking rule ($t$) for the entire sample as well as the individual terciles. We find that our assumption regarding the tie-breaking rule in Section 1.2 is clearly confirmed, as we cannot reject the null hypothesis that $t$ is equal to zero, indicating that smart routers always choose to trade on Chi-X if it at least matches the quotes in the Primary Market. Given that Chi-X charges lower fees for market orders (except for very large orders), this result is not very surprising.

## 1.4. Estimating differences in informed trading

The implications of our model from Section 1.2 regarding market co-existence crucially depend on whether or not informed traders are more likely than noise traders to have access to the alternative trading platform (i.e. Chi-X). It is important to notice that traders' routing abilities directly translate into the adverse selection risk faced by market makers and the price impact of trades. We can therefore filter out the relevant case for our setting by testing for differences in informed trading between Chi-X and the Primary Markets.

## 1.4.A. Effective spread decomposition

One of the most widely used measures for the assessment of trading costs is the percentage effective half-spread, which is defined as

$$ES_t = q_t \frac{p_t - m_t}{m_t} \tag{1.2}$$

where $p_t$ denotes the transaction price at time *t*, $m_t$ is the contemporaneously prevailing EBBO mid-quote, and $q_t$ is a trade direction indicator that takes the value of 1 for buys and -1 for sells. Compared to the quoted spread, this measure has the advantage that it measures trading costs only at the actual time of a trade, taking into account that liquidity demanders will attempt to time the market and trade when the bid-ask spread is relatively narrow.

Besides its simplicity, this measure has the additional advantage that it can be decomposed into an adverse selection (price impact) component

$$AS_t = q_t \frac{m_{t+\Delta t} - m_t}{m_t} \tag{1.3}$$

and an order processing component, usually termed realized half-spread

$$RS_t = q_t \frac{p_t - m_{t+\Delta t}}{m_t} \tag{1.4}$$

where $m_{t+\Delta t}$ is the mid-quote $\Delta t$ minutes after the transaction, the time at which the market maker is assumed to cover her position. While these measures constitute extreme simplifications of reality (e.g. trades between *t* and $t + \Delta t$ are ignored), they have become a benchmark for assessing trading costs. Moreover, this spread decomposition also allows us to compare market maker revenues before transaction fees across markets through the realized spread.

For both markets, we calculate all three measures for each stock and trading day and then calculate averages across stock-days for the separate activity terciles. Given that even the least active stocks in our sample have a considerable trading volume, this procedure delivers relatively conservative standard errors. Moreover, trade-weighted statistics would bias the results in favour of Chi-X, as it has a larger market share in the most active stocks, which generally exhibit lower effective spreads. In all calculations, we exclude trades that occur when the market is locked or crossed, i.e. when the EBBO spread is non-positive (see Shkilko et al. (2008)). Nevertheless, including these observations does not alter the results qualitatively.

The results are listed in Table A.5. Overall, trading on Chi-X is not cheaper before fees: Across all stocks and days, the effective spread on Chi-X averages 2.67 bps, compared to 2.64 bps in the Primary Market (Panel A). The difference of 0.03 bps is very small in economic terms (roughly 1%) and statistically insignificant. Given that Chi-X charges lower fees for market orders (except for very large trade sizes, see Section 1.3.A), the difference in effective spreads can be expected to be slightly negative net of fees. Exact calculations are not possible because participants in the Primary Markets may be granted rebates depending on their trading activity. Overall, the results suggest that trading on Chi-X is at most marginally cheaper than on the Primary Markets net of fees.

[Insert Table A.5 about here]

Nevertheless, a look at the spread decomposition[8] (Panels B and C) reveals important differences between both markets. Chi-X displays a significantly larger adverse selection component (2.68 bps compared to 2.27 bps), but markedly lower realized spreads (-0.01 bps vs. 0.37 bps). Importantly, the differences are both statistically and economically very significant. Liquidity

[8] We set $\Delta t$ = 5 minutes. Setting the interval to 15 or 30 minutes delivers qualitatively similar results.

providers on Chi-X are exposed to a much greater adverse selection risk, while their gross revenues are essentially equal to zero. Nevertheless, as Chi-X grants a 0.2 bps rebate per executed limit orders, they still net a small profit after fees. Given that liquidity providers on the Primary Markets face a positive transaction fee[9], market makers' revenues appear to be very similar across markets. Additionally, the fact that revenues from liquidity provision are very close to zero indicates a very competitive market.

Overall, the effective spread decomposition clearly suggests that liquidity providers on Chi-X face a higher adverse selection risk. This result appears very robust as it holds across all activity terciles, and we observe a higher price impact on Chi-X for all but three stocks[10] (these negative differences are not statistically different from zero). Moreover, the realized spreads nicely illustrate that the liquidity rebate on the MTF helps market makers to sustain this excess risk.

In our model, cross-market differences in adverse selection risk arise because the proportion of informed traders differs between smart routers and captive traders. While a higher price impact for orders executed on Chi-X indicates that smart routers are more likely than captive traders to be informed ($q^I > q^U$ or equivalently $m^{SR} > m$), it also implies that we should observe a lower price impact for trade-throughs, as those stem exclusively from less informed captive traders ($m^{CT} < m$). In order to verify this, we separate the Primary Market trades into trade-throughs and non-trade-throughs and calculate the effective spread decomposition for both types of transactions.

[Insert Table A.6 about here]

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[9] Except for designated market makers.

[10] For the sake of parsimony, we do not report the results for individual stocks (available upon request).

The results in Table A.6 strongly confirm that trades violating inter-market price priority are more likely to stem from uninformed traders than transactions that occur while the Primary Market is at the inside quote. The estimated average price impact of a trade-through is 0.91 bps, which is less than half of the 2.39 bps price impact of non-trade-throughs. Naturally, trade-throughs display a significantly larger effective spread (4.53 bps), as they leave money on the table by trading through a better available quote. Paired with the lower price impact, this boosts the realized spread (3.62 bps), which is pocketed by the market maker. Overall, these results indicate that the observed excess adverse selection risk on Chi-X is driven by the absence of mainly uninformed captive traders.

## 1.4.B. Hasbrouck's structural VAR

In a seminal contribution, Hasbrouck (1991) suggests a structural VAR model to estimate the permanent price impact of a trade. Since then, this measure has emerged as one of the most frequently employed procedures in the empirical market microstructure literature. The basic idea behind Hasbrouck's model is that there exists a dynamic linear relationship between price (quote) changes and trades, where current trades have an impact on current *and* future price changes, while current price changes can only trigger future trades. In our context, the model can be written as

$$r_t = \sum_{i=1}^{K} a_i r_{t-i} + \sum_{i=0}^{K} b_i x_{t-i}^{P} + \sum_{i=0}^{K} c_i x_{t-i}^{C} + e_t^{r} \tag{1.5}$$

$$x_t^{P} = \sum_{i=1}^{K} d_i r_{t-i} + \sum_{i=1}^{K} e_i x_{t-i}^{P} + \sum_{i=1}^{K} f_i x_{t-i}^{C} + e_t^{P} \tag{1.6}$$

$$x_t^{C} = \sum_{i=1}^{K} g_i r_{t-i} + \sum_{i=1}^{K} h_i x_{t-i}^{P} + \sum_{i=1}^{K} i_i x_{t-i}^{C} + e_t^{C} \tag{1.7}$$

where $r_t$ denotes log changes in the EBBO mid-quote and the $x_t^k$, $k \in \{P, C\}$ are discrete variables that take the value of 1 for a buy, -1 for a sell, and 0 otherwise. As detailed by Hasbrouck (1991), the discrete nature of the $x_t^k$ does not constitute any obstacle for the structural VAR. We estimate the model in tick time, such that trades across markets are necessarily uncorrelated[11], and truncate the VAR after 10 lags[12]. At the beginning of each trading day, all lags are set to zero. To judge the long-term (permanent) price impact of a trade, the VAR is inverted to obtain the VMA representation

$$\begin{pmatrix} r_t \\ x_t^P \\ x_t^C \end{pmatrix} = \begin{pmatrix} A(L) & B(L) & C(L) \\ D(L) & E(L) & F(L) \\ G(L) & H(L) & I(L) \end{pmatrix} \begin{pmatrix} e_t^r \\ e_t^P \\ e_t^C \end{pmatrix} \tag{1.8}$$

where *A(L) – I(L)* are lag polynomials. This is the impulse response function, and the long-term price responses to an unexpected trade in either market are given by the coefficient sums $PI^P = \sum_{i=0}^{\infty} B_i$ and $PI^C = \sum_{i=0}^{\infty} C_i$ .

[Insert Table A.7 about here]

Table A.7 contains the permanent price impacts for both markets (impulse responses are truncated after 20 periods), where we again report stock-day averages for the entire sample and the activity terciles. The results are in line with those from the effective spread decomposition. On average, the permanent price impact of a trade on Chi-X amounts to 1.86 bps, compared to 1.61 bps for Primary Market trades. The difference is statistically significant at the 1% level. As in the previous section, we observe a higher

[11] Estimating the model with data aggregated to 5-second intervals delivers qualitatively similar results but requires placing upper and lower bounds on a venue's price impact as the order flows are no longer uncorrelated.
[12] The inclusion of additional lags does not alter our conclusions.

price impact on Chi-X for each activity tercile, which underlines the robustness of our findings. For individual stocks, we find a higher price impact for Primary market trades in only 6 cases, and none of these differences are statistically significant[13]. Overall, these results provide further evidence for liquidity providers facing a higher adverse selection risk on Chi-X.

In order to check whether the difference in adverse selection across markets is indeed due to captive traders being mainly uninformed, we modify the VAR from equations (1.5) – (1.7) and split the Primary Market order flow into trade-throughs and non-trade-throughs. This results in the following VAR system

$$r_t = \sum_{i=1}^{K} a_i r_{t-i} + \sum_{i=0}^{K} b_i x_{t-i}^{P,NTT} + \sum_{i=0}^{K} c_i x_{t-i}^{P,TT} + \sum_{i=0}^{K} d_i x_{t-i}^{C} + e_{r,t} \qquad (1.9)$$

$$x_t^{P,NTT} = \sum_{i=1}^{K} e_i r_{t-i} + \sum_{i=1}^{K} f_i x_{t-i}^{P,NTT} + \sum_{i=1}^{K} g_i x_{t-i}^{P,TT} + \sum_{i=1}^{K} h_i x_{t-i}^{C} + e_{NTT,t} \qquad (1.10)$$

$$x_t^{P,TT} = \sum_{i=1}^{K} i_i r_{t-i} + \sum_{i=1}^{K} j_i x_{t-i}^{P,NTT} + \sum_{i=1}^{K} k_i x_{t-i}^{P,TT} + \sum_{i=1}^{K} l_i x_{t-i}^{C} + e_{TT,t} \qquad (1.11)$$

$$x_t^{C} = \sum_{i=1}^{K} m_i r_{t-i} + \sum_{i=1}^{K} n_i x_{t-i}^{P,NTT} + \sum_{i=1}^{K} o_i x_{t-i}^{P,TT} + \sum_{i=1}^{K} p_i x_{t-i}^{C} + e_{C,t} \qquad (1.12)$$

where $x_{t-i}^{P,TT}$ and $x_{t-i}^{P,NTT}$ refer to Primary Market order flow due to trade-throughs and non-trade-throughs, respectively. Table A.8 reports the permanent price impacts obtained from the corresponding VMA representation. The results are qualitatively similar to those obtained from the effective spread decomposition. The permanent price impact of a trade-through is 0.44 bps, which is significantly lower than 1.72 bps impact of a non-trade-through, with a t-statistic of around 15. This supports the view that market makers on Chi-X face a higher adverse selection risk precisely

[13] The results for individual stocks, which we do not report for brevity, are available upon request.

because their quotes are not exposed to the relatively uninformed captive traders.

[Insert Table A.8 about here]

### 1.4.C. PIN

Given our theoretical framework, the PIN model by Easley et al. (1996b) is a natural choice for assessing differences in informed trading between Chi-X and the Primary Markets. Nevertheless, a number of recent papers (e.g. Duarte and Young (2009) and Aktas et al. (2007)) have cast doubt on the model's ability to capture the presence of informed traders. Moreover, it is well-known that the PIN model is subject to numerical problems, particularly for stocks with high trading activity (see e.g. Yan and Zhang (2010) and Easley et al. (2010)). In our case, these problems are additionally amplified by the relatively short sample (43 trading days) and the need to estimate additional parameters for a two-market PIN model as in Easley et al. (1996a) or Grammig et al. (2001). Consequently, we find that the numerical maximization of the likelihood function is not successful for most stocks in our sample. In Appendix A.3, we provide an alternative method for estimating differences in informed trading between two markets via the PIN model and present the associated results.

## 1.5. Differences in adverse selection and Chi-X's quote competitiveness

The results of the previous section suggest that trades on Chi-X carry more private information than those executing place in the Primary Markets. From the perspective of our theoretical model, this corresponds to the case where informed traders have a higher likelihood of being smart routers than captive traders (i.e. $q^I > q^U$ or equivalently $m^{SR} > m$ ). Recall from the discussion of

Proposition 1 in Section 1.2 that in this case, liquidity providers on Chi-X are only able to match the primary market's quotes if their cost advantage from transaction fees ($c$) exceeds the excess adverse selection risk they face.

In order to validate our model empirically, we adopt a cross-sectional perspective. According to Corollary 1, Chi-X's presence at the inside quote is expected to decrease (weakly) in the adverse selection risk differential. As the empirical evidence suggests that the adverse selection risk differential is positive for almost all stocks, we actually expect to observe a strictly negative relationship.

We begin by calculating, for each stock, the fraction of time during which Chi-X is present at the EBBO, taking the average of both sides of the market[14]. We then regress this measure of Chi-X's quote competitiveness on measures that capture the difference in adverse selection across trading venues and additional control variables, i.e. we estimate the cross-sectional regression

$$Chi\_at\_best_i = a_0 + a_1 1_{[Euronext]} + g(\Delta AS_i) + f'X_i + e_i \qquad (1.13)$$

where $1_{[Euronext]}$ is an indicator variable that takes the value of 1 if the stock is listed on Euronext and 0 otherwise, $\Delta AS_i$ denotes the excess adverse selection risk on Chi-X for stock *i*, and $X_i$ is a vector of control variables.

We employ three different variables in order to quantify the excess adverse selection risk on Chi-X. The first two are the stock-specific cross-market differences of the price impact measures from Sections 1.4.A and 1.4.B, denoted $\Delta AS_i^{SD}$ and $\Delta AS_i^{HB}$, respectively. For the third variable, we ignore any cross-sectional variation in the proportion of informed traders and

[14] Considering only one side of the market (either bid or ask) delivers qualitatively similar results.

simply proxy $(m^{SR} - m)(\overline{V} - \underline{V})$ by $s_i$, which denotes annualized return volatility based on closing prices for the calendar year prior to our sample period. Table A.9 contains the cross-sectional correlation matrix of our three explanatory variables. Unsurprisingly, we find a strong cross-sectional correlation of 0.63 between $\Delta AS_i^{SD}$ and $\Delta AS_i^{HB}$. More interestingly, both measures are highly correlated with stock price volatility (between 0.42 and 0.45), which indicates that all three variables are picking up similar effects.

[Insert Table A.9 about here]

We include a number of control variables that we expect to influence Chi-X's presence at the best quote.

While we have not incorporated the effect of tick sizes in our model for tractability reasons, it is known that a discrete pricing grid leads to rounding errors and therefore artificially inflates the bid-ask spread (see e.g. Harris (1994)). As a consequence, we expect Chi-X's quote competitiveness to increase in the tick size differential[15], which we define as the average[16] difference in tick sizes (Primary market minus Chi-X) for stock *i* scaled by the stock's average transaction price. We furthermore include the proportion of smart routers as a control variable in order to disentangle our story from that of Foucault and Menkveld (2008). Additionally, we also control for the log of trading volume and a stock's return synchronicity (based on the R-

[15] For some stocks, the difference in tick sizes is considerable. For example, during most of the sample, the tick size for Infineon is €0.01 on Xetra, compared to €0.001 on Chi-X. Given the stock's low price level (below €10), the bid-ask spread on the Primary Market is frequently equal to the tick size. Consequently, Chi-X is particularly attractive for trading in this stock as it allows the placement of orders within the primary quotes. A few days before the end of the sample period, Deutsche Börse reduced the tick size to €0.005.

[16] A total of 9 stocks experience a change in tick sizes during our sample period, all of them corresponding to a reduction in the Primary Market tick size. One stock (STMicroelectronics) experiences two changes.

Square of a market model regression[17]). While trading volume is simply a variable outside our model, the results in Hengelbrock and Theissen (2009) suggest that trading in more active stocks has a higher tendency to fragment. Return synchronicity may capture effects of algorithmic traders, which often engage in index arbitrage trades and have been shown to be particularly quick in reacting to "hard" information (Jovanovic and Menkveld (2010)).

[Insert Table A.10 about here]

The coefficient estimates are listed in Table A.10. As predicted by our model, the results indicate that an increase in the adverse selection risk differential is associated with Chi-X being less frequently at the inside quote. The coefficients on $\Delta AS_i^{SD}$, $\Delta AS_i^{HB}$ and $s_i$ are all negative and strongly significant (t-statistics ranging from 3.3 to 8.2). Importantly, the observed effects are also economically important. For example, a one standard deviation increase in $\Delta AS_i^{SD}$ (~0.37 bps) is associated with a decrease of around 4.35% in Chi-X's presence at the inside quote. The other variables have marginal effects of similar magnitude (7.07% and 3.95% for $\Delta AS_i^{HB}$ and $s_i$, respectively).

All control variables carry the expected sign. Particularly the difference in tick sizes across venues plays an important role for Chi-X's quote competitiveness. Increasing the tick size differential by one standard deviation (~2.2 bps) leads Chi-X's presence at the best quote to increase by 7-11%, depending on the specification. Different from Foucault and Menkveld (2008), we find that the proportion of smart routers is not significantly related to Chi-X's quote competitiveness. This is likely due to

[17] We use the transformation $SYNCH = \ln(R^2 / (1-R^2))$, see e.g. Teoh et al. (2009). Market model regressions are estimated using 1 year of daily data prior to our sample period.

the fact that exchanges do not charge any fees for order submission, which is an important feature of their model and data. For the other control variables, we find that both higher trading volume and higher return synchronicity are associated with Chi-X being at the inside quote more frequently. Finally, there is some weak evidence for Chi-X offering worse quotes in stocks listed on Euronext. This may be due to the staggered entry of the MTF across countries. While Chi-X entered the German market roughly one year before the start of our sample period, it did not offer trading in French stocks until half a year later.

While our theoretical model is strictly speaking about quotes, it has very similar implications regarding Chi-X's actual market share. Given a fixed number of trading rounds, a higher excess adverse selection risk leads to less trade on Chi-X after controlling for the proportion of smart routers. We therefore re-estimate equation (1.13), but replace Chi-X's presence at the best quote with the MTF's market share in terms of trades. The results (Table A.11) are very similar than the results for quotes. All variables capturing the adverse selection risk differential are negative and statistically significant. Again, the economic effects are substantial. For example, a one standard deviation increase in $\Delta AS_i^{SD}$ is associated with an increase of 1.10% in Chi-X's market share. Unsurprisingly, the explanatory effect of the proportion of smart routers is strongly significant. The coefficients on the remaining control variables are, by and large, similar to the results using Chi-X's presence at the inside quote.

[Insert Table A.11 about here]

Overall, the cross-sectional evidence suggests that Chi-X's competitiveness is significantly hampered by excess adverse selection risk. These findings strongly support our theoretical model.

## 1.6. Conclusion

Motivated by the current regulatory framework in Europe set forth under MiFID, we analyze how adverse selection risk and transaction fees interact in a fragmented financial market where trade-throughs are not prohibited. We argue that liquidity providers on alternative trading platforms will be subject to an increased adverse selection risk if informed traders are more likely to have access to this market via a smart order routing system. Consequently, the Primary Market will dominate (display better quotes) most of the trading day despite charging higher transaction fees. We formalize this argument with an extension of the Glosten and Milgrom (1985) sequential trade model.

The analysis of a recent sample of transactions and quote data for German and French stocks reveals that liquidity providers on Chi-X (a recently launched trading platform) face a significantly greater adverse selection risk. Moreover, trade-throughs that execute "by default" in the Primary Markets are particularly uninformed. In line with our theoretical model, we find a negative relationship between the excess adverse selection risk and Chi-X's presence at the inside quote. Moreover, our view is additionally supported by anecdotal evidence from Primary Market outages.

Our findings have some implications for the design of best execution policies. Allowing for trade-throughs favors the Primary Markets by ensuring that the least informative order flow does not reach the MTFs, thereby hampering liquidity provision on these platforms due to an increased adverse selection risk. Our findings suggest that protecting orders from trade-throughs in the spirit of RegNMS may foster competition between trading venues as it helps to level the playing field.

There are some interesting avenues for future research. In our theoretical analysis, we have taken exchanges' transaction fees and investors' routing technologies as given. This choice follows from noise traders' willingness to trade at any price and the assumption that agents do not have the chance to trade multiple times. Clearly, a more realistic model would aim to determine these variables endogenously.

# 2. ALGORITHMIC TRADING IN A DYNAMIC LIMIT ORDER MARKET

## 2.1. Introduction

In recent years, the number of financial market transactions taking place without any human intervention has increased dramatically. Recent estimates suggest that algorithmic trading now accounts for more than 70% of the volume traded in US equity markets[18]. Many of these computer-driven trading strategies, generally dubbed as high-frequency trading, rely on the execution of a vast number trades within a very short time, spurring an arms race within the financial industry for the most sophisticated technology in an effort to be faster than the competition.

The rise of algorithmic trading is being accompanied by a heated debate among financial economists, practitioners, and regulators about the pros and cons of the increasing computerization of the trading process. While proponents argue that technology increases market efficiency via improved liquidity[19], critiques claim that high-frequency traders (HFT) make profits at the expense of other market participants and have the potential to destabilize markets[20].

This chapter contributes to this debate by presenting a model of trading in a limit order market with two types of agents, fast algorithmic traders (ATs) and slow human traders (HTs). While limit orders potentially enable traders to lower their execution costs by earning instead of paying the bid-ask

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[18] See Financial Times, "High-frequency trading under scrutiny", July 28th, 2009.

[19] See e.g. Optiver, "High Frequency Trading", Position Paper, 2011, http://fragmentation.fidessa.com/wp-content/uploads/High-Frequency-Trading-Optiver-Position-Paper.pdf

[20] See SEC Chairman Mary Schapiro's speech in front of the Security Traders Association "Remarks Before the Security Traders Association", www.sec.gov/news/speech/2010/spch092210mls.htm.

spread, they may become stale upon the arrival of new information unless being updated or cancelled in a timely fashion (Copeland and Galai (1983)). Consequently, the use of sophisticated computer programs that enable the continuous monitoring and updating of limit orders may reduce this risk of being "picked off" substantially.

In order to model such usage of automated trading in a tractable way, we propose an extension of Foucault's (1999) dynamic limit order market. In the original model, traders face the mentioned risk of being picked off in the case of adverse price movements (also called the winner's curse), as they are not able to cancel or revise their limit orders. We classify these agents as HTs and introduce a new type of trader, ATs. While the use of cutting edge IT technology facilitates the monitoring of limit orders and allows for fast reactions to changing market conditions, it only constitutes an advantage over those market participants that do not have this technology. In other words, ATs exert a negative externality on each other. To capture this idea in the most parsimonious way, we assume that ATs may revise/cancel their limit orders after the arrival of new public information, but only in case the next arriving agent is a HT. We analyze the equilibrium in this dynamic limit order market and compare it to the baseline case without ATs studied by Foucault (1999). Our findings are as follows.

First, algorithmic trading may lead to both increases and decreases in trading volume, depending on the magnitude of the winner's curse problem. If the volatility of the fundamental asset value is high, HTs engage in order shading due to the risk of being picked off. ATs may avoid this order shading when trading with HTs due to their ability to revise limit prices after news arrivals, which leads to an increase in trading volume. On the other hand, HTs do not engage in any order shading for low levels of fundamental volatility in the absence of ATs. But given their comparative advantage at posting limit orders, ATs are not willing to accept the same quotes as HTs when using market orders. Thus, if the proportion of ATs is low, it can be optimal for HTs to post limit with a lower execution probability, leading to a decrease in trading volume.

Second, the introduction of algorithmic trading decreases HTs' expected profits when trading via limit orders. Given ATs' higher outside option (due to their ability to cancel/revise limit orders), HTs either need to post more aggressive quotes in order to make them attractive for ATs (otherwise it is optimal for them to submit a limit order), or accept a lower execution probability by posting limit orders only aimed at HTs. Although the second choice allows them to submit lower bids and higher asks than in the absence of ATs, the net effect on their expected profits from limit orders is always negative.

Third, the expected trading costs of HTs when using market orders may increase or decrease with the introduction of algorithmic trading. On the one hand, the quotes posted by ATs always reflect the latest fundamental information once they are encountered by HTs, rendering a relatively high trading cost for market orders. On the other hand, HTs may benefit from the increased aggressiveness of other HTs' limit orders, which are also directed at ATs. Nevertheless, this effect diminishes as the proportion of ATs increases and their comparative advantage shrinks, such that HTs' trading costs fall below the level that obtains in the absence of ATs.

We close our analysis by examining traders' welfare. We show that if algorithmic trading is widespread, HTs are strictly worse off than under no algorithmic trading. Additionally, even low levels of algorithmic trading have a detrimental effect on HTs' welfare in most cases, as the potentially lower costs for market orders do not fully compensate the decrease in expected gains from limit orders. Nevertheless, for some intermediate values of volatility, a low level of algorithmic trading can increase HTs' welfare, as it induces HTs to post limit orders with a higher execution probability and only marginally affects the profits for human limit order traders.

Our work is closely related to several recent studies concerning the impact of algorithmic trading on market performance and investor welfare. An excellent and comprehensive overview can be found in Biais and Woolley (2011). Jovanovic and Menkveld (2010) study the emergence of competitive middlemen that intermediate between early limit order traders and late market order traders. Similar to our findings, these middlemen may increase or reduce trade by either solving or creating an adverse selection problem. In Biais et al. (2010b), algorithmic trading helps to reap gains from trade by facilitating the search for trading opportunities, but at the same time increases adverse selection for slow traders. In equilibrium, investment in algorithmic trading is above its social optimum. Finally, Cartea and Penalva (2011) propose a model where their increased speed allows HFT to impose a haircut on liquidity traders, which increases trading volume and price volatility, but lowers the welfare of liquidity traders.

There are several empirical studies of algorithmic trading. Hendershott et al. (2011) use the introduction of NYSE's autoquote system as an exogenous instrument and show that algorithmic trading causally improves liquidity. Brogaard (2010) and Hendershott and Riordan (2009) study proprietary datasets that allow them to distinguish between HTs and ATs, and both report that ATs contribute substantially to price discovery and market liquidity. The former paper additionally confirms the finding by Chaboud et al. (2009), that trading strategies by ATs are more correlated than those of HTs. The results in Hendershott and Riordan (2009) additionally indicate that algorithmic traders are more informed than humans and manage to time their trades as to demand (supply) liquidity when it is cheap (expensive). Menkveld (2011) studies trading of a large HFT market maker in Dutch stocks and shows that this activity is highly profitable, particularly if positions are held for a very short time. Finally, Kirilenko et al. (2011) study the recent "flash crash" in U.S. equity markets and find that HFT may have exacerbated volatility during this brief liquidity crisis, although they are not to blame for the crash itself.

This chapter is organized as follows. Section 2.2 provides the setup of the model, while we solve for the equilibrium in Section 2.3. Section 2.4 analyzes this equilibrium, and Section 2.5 concludes. All proofs, tables and figures are relegated to Appendix B.

## 2.2. A dynamic limit order market with fast algorithmic traders and slow human traders

### 2.2.A. The Limit Order Market

We consider an infinite-horizon[21] version of Foucault's (1999) dynamic limit order market. There is a single risky asset whose fundamental value follows a random walk, i.e.

$$v_{t+1} = v_t + e_{t+1}$$

where the innovations can take values of $+s$ and $-s$ with equal probability and are independent over time. Trading takes place sequentially at time points $t = 1, 2, \ldots$ and the order size is fixed at one unit. In this model,

[21] This assumption is merely for convenience, as it simplifies the algebra and in particular the calculation of trader welfare in Section 2.4. Foucault (1999) derives a stationary equilibrium by assuming that the terminal date is stochastic, as the trading process stops after each period with constant probability $1 - r > 0$. While assuming an infinite horizon implies that the asset never pays off, it can be readily interpreted as the limiting case where $r \to 1$.

trading arises due to differences in private values. Specifically, we assume that at time $t'$, the reservation price of a trader arriving at time $t \le t'$ is given by

$$R_{t'} = v_{t'} + y_t$$

which is the sum of the asset's fundamental value and the time-invariant private valuation $y_t$. We assume that this private valuation can take two values, $y_h = +L$ and $y_1 = -L$, with equal probability, where $L > 0$. The *y*'s are independent and identically distributed across traders, and moreover independent from the asset value innovations. All traders are risk-neutral and maximize their expected utility. The utility obtained by an agent purchasing or selling the asset is given by

$$U(y_t) = (v_{t'} + y_t - P_{t'})q_{t'}$$

where $t' \ge t$ denotes the time of the transaction, $P_{t'}$ is the transaction price, and $q_{t'}$ is a trade direction indicator that takes the value of $+1$ for buy transactions and -1 for sell transactions. The utility of an agent that does not trade is normalized to zero.

Besides their private valuations, agents differ in their trading technology, denoted $q_t$. They can either be algorithmic traders (ATs,$q_t = 1$) or human traders (HTs, $q_t = 0$). Let $a \in [0,1]$ denote the probability that an agent is an AT. Again, we assume that *q* is identically distributed across traders and independent of *y* and *e*. We will comment on the differences between ATs and HTs below. We call $q_t$ a trader's type.

Trading is organized as a limit order market. Consider a buyer (i.e. an agent with private valuation $y_h$). Upon his arrival, he can either a) submit a market buy order or b) submit a buy limit order for one unit of the asset.[22] We assume that he decides to submit a limit order if he is indifferent between both choices. Similarly, sellers choose between market and limit sell orders. All limit orders are valid for one period, i.e. they expire unless being executed by the following agent.

In real life, agents face the risk of being picked off unless they are able to monitor their limit orders perfectly and revise them instantaneously after the arrival of new information (Foucault et al. (2003) and Liu (2009) study the cost of monitoring limit orders). Clearly, the use of sophisticated computer algorithms allows for a virtually continuous monitoring of outstanding limit orders and therefore reduces picking off risks. In order to model this difference between HTs and ATs in the most parsimonious way, we allow ATs to cancel/revise their limit orders after the arrival of new information (i.e. the realization of $e_{t+1}$), but before the arrival of the next agent provided he is a HT. If the next trader is an AT as well, the order cannot be cancelled/revised. HTs can never cancel their orders.

Let $s_t = (A_t^m, B_t^m)$ denote the best bid and ask quote in the market. If there is no bid (ask) quote posted, we write $B_t^m = -\infty$ ($A_t^m = \infty$). Upon entering the market, a trader learns his type $q_t$ and private valuation $y_t$, and observes the state of the limit order book $s_t$ as well as the current fundamental value of the asset $v_t$. Call $S_t = (s_t, v_t)$ the state of the market.

[22] Foucault (1999) assumes that traders always submit a buy *and* sell limit order, which is without loss of generality as limit prices can always be chosen such that limit orders have a zero execution probability. In fact, in equilibrium, the ask (bid) quotes of buyers (sellers) are never executed, such that we directly assume that buyers (sellers) only submit buy (sell) limit orders.

## 2.2.B. Payoffs

Consider a buyer that arrives at time $t$ when the state of the market is $S_t$. If he chooses to submit a buy market order, his payoff is equal to

$$U_{t,k}^{B,MO}(A_t^m) = v_t + L - A_t^m \quad k \in \{HT, AT\}$$

Instead, he can choose to submit a buy limit order. The expected payoff of a HT buyer submitting a buy limit order with bid price $B_{t,HT}$ is given by

$$E(U_{t,HT}^{B,LO}(B_{t,HT})) = h_t^B(B_{t,HT}) E_{Ex}(v_{t+1} + L - B_{t,HT}) \tag{2.1}$$

where $h_t^B(B_{t,HT})$ denotes the execution probability of a buy limit order with bid price $B_{t,HT}$ and $E_{Ex}(\cdot)$ is an expectation conditional on the execution of the respective limit order. Given that an AT may revise his limit order in case the next arriving trader is a HT, an AT buyer chooses three different bid prices, $(B_{t,AT}, B_{t,AT}^{-s}, B_{t,AT}^{+s})$, and his payoff is given by

$$\begin{aligned} E(U_{t,AT}^{B,LO}(B_{t,AT}, B_{t,AT}^{-s}, B_{t,AT}^{+s})) &= a\, h_t^B(B_{t,AT} \mid q_{t+1} = 1) E_{Ex}(v_{t+1} + L - B_{t,AT}) \\ &+ (1-a)[\frac{1}{2} h_t^B(B_{t,AT}^{+s} \mid q_{t+1} = 0, e_{t+1} = +s)(v_t + s + L - B_{t,AT}^{+s}) \\ &+ \frac{1}{2} h_t^B(B_{t,AT}^{-s} \mid q_{t+1} = 0, e_{t+1} = -s)(v_t - s + L - B_{t,AT}^{-s})] \end{aligned} \tag{2.2}$$

where the $h_t^B(\cdot \mid q_{t+1}, e_{t+1})$ denote execution probabilities conditional on the realization of next period's trader type and asset value innovation. Similarly, a seller submitting a sell market order obtains

$$U_{t,k}^{S,MO}(B_t^m) = B_t^m - (v_t - L) \quad k \in \{HT, AT\}$$

while the payoffs for HTs and ATs from posting sell limit orders with ask prices equal to $A_{t,HT}$ and $(A_{t,AT}, A_{t,AT}^{-s}, A_{t,AT}^{+s})$, respectively, are given by

$$E(U_{t,HT}^{S,LO}(A_{t,HT})) = h_t^S(A_{t,HT}) E_{Ex}(A_{t,HT} - (v_{t+1} - L))$$

$$E(U^{S,LO}_{t,AT}(A_{t,AT}, A^{-s}_{t,AT}, A^{+s}_{t,AT})) = a\, h^S_t(A_{t,AT} \mid q_{t+1}=1) E_{Ex}(A_{t,HT} - (v_{t+1} - L))$$
$$+ (1-a)[\frac{1}{2} h^S_t(A^{+s}_{t,AT} \mid q_{t+1}=1, e_{t+1}=+s)(A^{+s}_{t,AT} - (v_t + s - L))$$
$$+ \frac{1}{2} h^S_t(A^{-s}_{t,AT} \mid q_{t+1}=1, e_{t+1}=-s)(A^{-s}_{t,AT} - (v_t - s - L))]$$

## 2.3. Equilibrium

Let $B^*_{t,HT}$ denote the optimal bid price chosen by a HT buyer that decides to place at limit order at time *t*. Thus, upon arrival, a HT buyer chooses between a) a buy market order at ask price $A^m_t$ and b) a buy limit order with bid price $B^*_{t,HT}$. We call his choice the HT buyer's order placement strategy $O^B_{HT}(S_t) \in \left\{ b^m_t, B^*_{t,HT} \right\}$, where $b^m_t$ denotes a market buy order at time *t*. Similarly, let $(B^*_{t,AT}, B^{-s\,*}_{t,AT}, B^{+s\,*}_{t,AT})$ be the optimal bid prices for an AT buyer that opts for limit orders when arriving at time *t*. He then chooses between a) a buy market order at ask price $A^m_t$ and b) a buy limit order with bid price $B^*_{t,AT}$, which, unless the next trader is an AT, is cancelled after the arrival of positive (negative) fundamental information and followed by the submission of a new buy limit order with bid price $B^{+s\,*}_{t,AT}$ ($B^{-s\,*}_{t,AT}$). Hence, $O^B_{AT}(S_t) \in \left\{ b^m_t, (B^*_{t,AT}, B^{-s\,*}_{t,AT}, B^{+s\,*}_{t,AT}) \right\}$. The choices of HT and AT sellers are completely symmetric, i.e. they choose between a) a market sell at $B^m_t$ and b) limit sell orders with ask prices equal to $A^*_{t,HT}$ and $(A^*_{t,AT}, A^{-s\,*}_{t,AT}, A^{+s\,*}_{t,AT})$, respectively, such that their order placement strategies are $O^S_{HT}(S_t) \in \left\{ s^m_t, A^*_{t,HT} \right\}$ and $O^S_{AT}(S_t) \in \left\{ s^m_t, (A^*_{t,AT}, A^{-s\,*}_{t,AT}, A^{+s\,*}_{t,AT}) \right\}$, where $s^m_t$ denotes a market sell. As in Foucault (1999) and Colliard and Foucault (2011), we focus on stationary Markov-perfect equilibria, which is natural because trader's profits do not depend on the history of the game but only on the state of the market upon their arrival.

**Definition**: A Markov-perfect equilibrium of the limit order market consists of order placement strategies $O^{B*}_{HT}(\cdot)$, $O^{S*}_{HT}(\cdot)$, $O^{B*}_{AT}(\cdot)$ and $O^{S*}_{AT}(\cdot)$ such that, for each possible state of the market $S_t$, i) $O^{B*}_{HT}(S_t)$ $(O^{B*}_{AT}(S_t))$ maximizes the expected utility of a HT (AT) buyer arriving in state $S_t$ if all other traders follow the strategies $O^{B*}_{HT}(\cdot)$, $O^{S*}_{HT}(\cdot)$, $O^{B*}_{AT}(\cdot)$ and $O^{S*}_{AT}(\cdot)$ and ii) $O^{S*}_{HT}(S_t)$ $(O^{S*}_{AT}(S_t))$ maximizes the expected utility of a HT (AT) seller arriving in state $S_t$ if all other traders follow the strategies $O^{B*}_{HT}(\cdot)$, $O^{S*}_{HT}(\cdot)$, $O^{B*}_{AT}(\cdot)$ and $O^{S*}_{AT}(\cdot)$.

Foucault (1999) elegantly shows that it is possible to characterize traders' optimal decisions by means of cutoff prices that depend on a trader's private valuation and the current fundamental value of the asset. The buy (sell) cutoff price is the highest (lowest) ask (bid) price at which an arriving buyer (seller) submits a market buy (sell) order instead of a buy (sell) limit order. Let $V^{LO*}_{HT}(y_t)$ and $V^{LO*}_{AT}(y_t)$ denote equilibrium expected profits from posting limit orders for HTs and ATs, respectively, that is

$$V^{LO*}_{HT}(y_t) = \begin{cases} E(U^{B,LO}_{t,HT}(B^*_{t,HT})) & if \;\; y_t = y_h \\ E(U^{S,LO}_{t,HT}(A^*_{t,HT})) & if \;\; y_t = y_l \end{cases}$$

$$V^{LO*}_{AT}(y_t) = \begin{cases} E(U^{B,LO}_{t,AT}(B^*_{t,AT}, B^{-s\,*}_{t,AT}, B^{+s\,*}_{t,AT})) & if \;\; y_t = y_h \\ E(U^{S,LO}_{t,AT}(A^*_{t,AT}, A^{-s\,*}_{t,AT}, A^{+s\,*}_{t,AT})) & if \;\; y_t = y_l \end{cases}$$

Then the buy and sell cutoff prices are given by

$$C^{S*}_{HT}(v_t, y_t) - (v_t + y_t) = V^{LO*}_{HT}(y_t)$$
$$C^{S*}_{AT}(v_t, y_t) - (v_t + y_t) = V^{LO*}_{AT}(y_t)$$
$$(v_t + y_t) - C^{B*}_{HT}(v_t, y_t) = V^{LO*}_{HT}(y_t)$$
$$(v_t + y_t) - C^{B*}_{AT}(v_t, y_t) = V^{LO*}_{AT}(y_t)$$

Intuitively, the expected profits from submitting limit orders constitute an endogenous outside option for the arriving trader. Therefore, he will only submit a market buy (sell) order when the best available ask (bid) price is below (above) his cutoff buy (sell) price. The above system of equations can be solved for the equilibrium cutoff prices, which in turn give rise to traders' equilibrium quotation strategy.

The following proposition provides a closed-form characterization of the equilibrium quotation strategies.

**Proposition 1 (Equilibrium Quotes):**

For fixed parameters $(a, s, L)$, there exists a unique Markov-perfect equilibrium. The type of equilibrium is as follows.

Type 1: If $a \le a_1^*$ and $s \ge s_1^*$, equilibrium quotes are

$$B_{t,HT}^* = v_t - s - L + (2L)\frac{1-a}{5-a} \qquad A_{t,HT}^* = v_t + s + L - (2L)\frac{1-a}{5-a}$$

$$B_{t,AT}^{+s\,*} = v_t + s - L + (2L)\frac{1-a}{5-a} \qquad A_{t,AT}^{+s\,*} = v_t + s + L - (2L)\frac{1-a}{5-a}$$

$$B_{t,AT}^{-s\,*} = v_t - s - L + (2L)\frac{1-a}{5-a} \qquad A_{t,AT}^{-s\,*} = v_t - s + L - (2L)\frac{1-a}{5-a}$$

$$B_{t,AT}^* = v_t - s - L + (2L)\frac{8-a(3+a)}{(5-a)(4+a)} \qquad A_{t,AT}^* = v_t + s + L - (2L)\frac{8-a(3+a)}{(5-a)(4+a)}$$

The execution probabilities of buy limit orders are given by

$$h_t^B(B_{HT}^*) = (1-a)/4,\ h_t^B(B_{t,AT}^* \mid q_{t+1} = 1) = 1/4,\ h_t^B(B_{t,AT}^{+s\,*} \mid q_{t+1} = 0, e_{t+1} = +s) = 1/2$$

Type 2: If $a_1^* < a \le a_2^*$ and $s \ge s_4^*$ or $a_2^* < a$ and $s \ge s_5^*$, equilibrium quotes are

$$B_{t,HT}^* = v_t - s - L + (2L)\frac{3-a}{7+3a} \qquad A_{t,HT}^* = v_t + s + L - (2L)\frac{3-a}{7+3a}$$

$$B_{t,AT}^{+s\,*} = v_t + s - L + (2L)\frac{1+a}{7+3a} \qquad A_{t,AT}^{+s\,*} = v_t + s + L - (2L)\frac{1+a}{7+3a}$$

$$B_{t,AT}^{-s\,*} = v_t - s - L + (2L)\frac{1+a}{7+3a} \qquad A_{t,AT}^{-s\,*} = v_t - s + L - (2L)\frac{1+a}{7+3a}$$

$$B_{t,AT}^{*} = v_t - s - L + (2L)\frac{3-a}{7+3a} \qquad A_{t,AT}^{*} = v_t + s + L - (2L)\frac{3-a}{7+3a}$$

The execution probabilities of buy limit orders are given by

$h_t^B(B_{t,HT}^*) = 1/4$, $h_t^B(B_{t,AT}^* \mid q_{t+1} = 1) = 1/4$, $h_t^B(B_{t,AT}^{+s\,*} \mid q_{t+1} = 0, e_{t+1} = +s) = 1/2$ and $h_t^B(B_{t,AT}^{-s\,*} \mid q_{t+1} = 0, e_{t+1} = -s) = 1/2$.

Type 3: If $a \le a_1^*$ and $s_3^* \le s < s_1^*$ or $a_1^* < a \le a_2^*$ and $s_3^* \le s < s_4^*$, equilibrium quotes are

$$B_{t,HT}^{*} = v_t - L + (2L)\frac{2-a}{6-a} + s\,\frac{4-a}{6-a} \qquad A_{t,HT}^{*} = v_t + L - (2L)\frac{2-a}{6-a} - s\,\frac{4-a}{6-a}$$

$$B_{t,AT}^{+s\,*} = v_t - L + (2L)\frac{2-a}{6-a} + s\,\frac{4-a}{6-a} \qquad A_{t,AT}^{+s\,*} = v_t + L - (2L)\frac{2-a}{6-a} + s\,\frac{8-a}{6-a}$$

$$B_{t,AT}^{-s\,*} = v_t - L + (2L)\frac{2-a}{6-a} - s\,\frac{8-a}{6-a} \qquad A_{t,AT}^{-s\,*} = v_t + L - (2L)\frac{2-a}{6-a} - s\,\frac{4-a}{6-a}$$

$$B_{t,AT}^{*} = v_t - L + (2L)\frac{8-a(2+a)}{(6-a)(4+a)} - s\,\frac{20+a(6-a)}{(6-a)(4+a)}$$

$$A_{t,AT}^{*} = v_t + L - (2L)\frac{8-a(2+a)}{(6-a)(4+a)} + s\,\frac{20+a(6-a)}{(6-a)(4+a)}$$

The execution probabilities of buy limit orders are given by $h_t^B(B_{t,HT}^*) = (2-a)/4$, $h_t^B(B_{t,AT}^* \mid q_{t+1} = 1) = 1/4$, $h_t^B(B_{t,AT}^{+s\,*} \mid q_{t+1} = 0, e_{t+1} = +s) = 1/2$ and $h_t^B(B_{t,AT}^{-s\,*} \mid q_{t+1} = 0, e_{t+1} = -s) = 1/2$.

Type 4: If $a \le a_2^*$ and $s_2^* \le s < s_3^*$, equilibrium quotes are

$$B_{t,HT}^{*} = v_t - L + (2L)\frac{2-a}{6-a} + s\,\frac{4-a}{6-a} \qquad A_{t,HT}^{*} = v_t + L - (2L)\frac{2-a}{6-a} - s\,\frac{4-a}{6-a}$$

$$B_{t,AT}^{+s\,*} = v_t - L + (2L)\frac{2-a}{6-a} + s\,\frac{4-a}{6-a} \qquad A_{t,AT}^{+s\,*} = v_t + L - (2L)\frac{2-a}{6-a} + s\,\frac{8-a}{6-a}$$

$$B_{t,AT}^{-s\,*} = v_t - L + (2L)\frac{2-a}{6-a} - s\,\frac{8-a}{6-a} \qquad A_{t,AT}^{-s\,*} = v_t + L - (2L)\frac{2-a}{6-a} - s\,\frac{4-a}{6-a}$$

$$B_{t,AT}^{*} = v_t - L + (2L)\frac{4+a(2-a)}{(6-a)(2+a)} + s\,\frac{14-4a}{(6-a)(2+a)}$$

$$A_{t,AT}^{*} = v_t + L - (2L)\frac{4+a(2-a)}{(6-a)(2+a)} - s\,\frac{14-4a}{(6-a)(2+a)}$$

The execution probabilities of buy limit orders are given by $h_t^B(B^*_{t,HT}) = (2-a)/4$, $h_t^B(B^*_{t,AT} \mid q_{t+1}=1) = 1/2$, $h_t^B(B^{+s*}_{t,AT} \mid q_{t+1}=0, e_{t+1}=+s) = 1/2$ and $h_t^B(B^{-s*}_{t,AT} \mid q_{t+1}=0, e_{t+1}=-s) = 1/2$.

Type 5: If $a \le a_2^*$ and $s < s_2^*$ or $a_2^* < a$ and $s < s_5^*$, equilibrium quotes are

$$B^*_{t,HT} = v_t - L + (2L)\frac{1}{3} + s\,\frac{4}{3(1+a)} \qquad A^*_{t,HT} = v_t + L - (2L)\frac{1}{3} - s\,\frac{4}{3(1+a)}$$

$$B^{+s*}_{t,AT} = v_t - L + (2L)\frac{1}{3} + s\,\frac{1+3a}{3(1+a)} \qquad A^{+s*}_{t,AT} = v_t + L - (2L)\frac{1}{3} + s\,\frac{5+3a}{3(1+a)}$$

$$B^{-s*}_{t,AT} = v_t - L + (2L)\frac{1}{3} - s\,\frac{5+3a}{3(1+a)} \qquad A^{-s*}_{t,AT} = v_t + L - (2L)\frac{1}{3} - s\,\frac{1+3a}{3(1+a)}$$

$$B^*_{t,AT} = v_t - L + (2L)\frac{1}{3} + s\,\frac{4}{3(1+a)} \qquad A^*_{t,AT} = v_t + L - (2L)\frac{1}{3} - s\,\frac{4}{3(1+a)}$$

The execution probabilities of buy limit orders are given by $h_t^B(B^*_{t,HT}) = 1/2$, $h_t^B(B^*_{t,AT} \mid q_{t+1}=1) = 1/2$, $h_t^B(B^{+s*}_{t,AT} \mid q_{t+1}=0, e_{t+1}=+s) = 1/2$ and $h_t^B(B^{-s*}_{t,AT} \mid q_{t+1}=0, e_{t+1}=-s) = 1/2$.

For each type of equilibrium, the execution probabilities of sell limit orders are given by $h_t^S(A^*_{t,HT}) = h_t^B(B^*_{t,HT})$, $h_t^S(A^*_{t,AT} \mid q_{t+1}=1) = h_t^B(B^*_{t,AT} \mid q_{t+1}=1)$, $h_t^S(A^{+s*}_{t,AT} \mid q_{t+1}=0, e_{t+1}=+s) = h_t^B(B^{-s*}_{t,AT} \mid q_{t+1}=0, e_{t+1}=-s)$ and $h_t^S(A^{-s*}_{t,AT} \mid q_{t+1}=0, e_{t+1}=-s) = h_t^B(B^{+s*}_{t,AT} \mid q_{t+1}=0, e_{t+1}=+s)$. The variables $a_1^*, a_2^*, s_1^*, s_2^*, s_3^*, s_4^*$ and $s_5^*$ are defined in Appendix B.1.A.

**Proof :**

See Appendix B.1.A.

In the following, we briefly describe the different equilibria, focusing on the type of limit orders chosen by HTs and ATs. For brevity, we just focus on the behavior of buyers (sellers behave symmetrically).

Type 1:

In the type-1 equilibrium, a HT buyer that chooses to submit a limit buy order sets the bid price such that the order is only executed if the agent arriving subsequently is a HT seller and the fundamental asset value has decreased. An AT buyer posting a buy limit order initially sets a bid price slightly above the sell cutoff price of an AT seller for the case of a price decrease, $B^{*}_{t,AT} = C^{S*}_{AT}(v_t - s, -L)$. If the next agent turns out to be a HT, the AT will be fast enough to cancel his initial limit order after observing the innovation in the asset value. The bid price of the new buy limit order is slightly above the sell cutoff price of a HT seller and therefore depends on the realization of the observed innovation, i.e. $B^{+s*}_{t,AT} = C^{S*}_{HT}(v_t + s, -L)$ and $B^{-s*}_{t,AT} = C^{S*}_{HT}(v_t - s, -L)$.

Type 2:

In this equilibrium, HT buyers that post limit orders choose the bid price such that the order is executed if the next agent is either a HT or an AT seller and the asset value has decreased. AT buyers post the same limit orders as in a type-1 equilibrium.

Type 3:

HT buyers submit buy limit orders whose bid price is slightly above the sell cutoff price of a HT seller after a price increase, that is $B^{*}_{t,HT} = C^{S*}_{HT}(v_t + s, -L)$. Therefore, such orders are executed if a) the next arriving agent is a HT seller independently of the asset value innovation, or b) the next trader is an AT seller and the asset value has decreased. AT buyers post the same limit orders as in a type-1 equilibrium.

Type 4:

HT buyers post the same limit orders as in a type-3 equilibrium. An AT buyer posting a buy limit order initially sets a bid price slightly above the sell cutoff price of an AT seller for the case of a price increase, $B^{*}_{t,AT} = C^{S*}_{AT}(v_t + s, -L)$. Therefore, this order is executed if the next arriving agent is an AT seller, independently of the evolution of the fundamental asset value. If the next agent turns out to be a HT, the AT buyer behaves as in a type-1 equilibrium.

Type 5:

HT buyers that opt for a limit order choose a bid price such that the order is executed if the next agent is a seller. AT buyers posting limit orders behave as in a type-4 equilibrium.

Figure B.1 in Appendix B.2 graphically depicts the regions in the $(a, s)$-space that correspond to the different equilibria, where we have set $L = 1$.

## 2.4. Order flow composition, trading costs, and welfare

### 2.4.A. Order flow composition

At each point in time, the arriving agent can be 1) a HT submitting a limit order, 2) a HT submitting a market order, 3) an AT submitting a limit order, or 4) an AT submitting a market order. Let $f^i = (f^i_1, f^i_2, f^i_3, f^i_4)$ be the corresponding stationary probability distribution in a type-*i* equilibrium, where $i \in \{1, 2, 3, 4, 5\}$. Given this distribution, one can easily deduce the equilibrium order flow composition. Call the probability of a randomly arriving investor submitting a market (limit) order the trading (make) rate, which are given by

$$TR^i = f_2^i + f_4^i$$
$$MR^i = f_1^i + f_3^i = 1 - TR^i$$

Similarly, we can calculate the trading and make rates for HTs and ATs separately, which are given by

$$TR_{HT}^i = f_2^i / (f_1^i + f_2^i)$$
$$MR_{HT}^i = f_1^i / (f_1^i + f_2^i) = 1 - TR_{HT}^i$$
$$TR_{AT}^i = f_4^i / (f_3^i + f_4^i)$$
$$MR_{AT}^i = f_3^i / (f_3^i + f_4^i) = 1 - TR_{AT}^i$$

Let $TR^*$ denote the equilibrium trading rate (e.g. $TR^* = TR^1$ if $a \le a_1^*$ and $s \ge s_1^*$), and let $TR^*\big|_{a=0}$ be the equilibrium trading rate without ATs (i.e. the trading rate in the Foucault (1999) model). Then, we obtain the following results.

**Proposition 2 (Order flow composition):**

Let $a \in (0,1)$. Then, $TR_{HT}^* \ge TR_{AT}^*$. Moreover, if $s \ge s_1^*(0)$ $(s < s_1^*(0))$, then $TR^* > TR^*\big|_{a=0}$ $(TR^* \le TR^*\big|_{a=0})$.

**Proof:** See Appendix B.1.B.

First, in equilibrium, HTs use market orders more frequently than ATs. This result is hardly surprising given ATs' comparative advantage at posting limit orders. The intuition for the second result is as follows. Consider the Foucault (1999) model, i.e. suppose there are no ATs. If volatility (and therefore the picking off risk) is high $(s \ge s_1^*(0))$, buyers (sellers) submit limit orders that only execute in case the asset value decreases (increases). Now, introducing ATs reduces such order shading, because ATs do not risk being picked off when trading with humans. Consequently they can condition their quotes on the most recent innovation in the asset value, which enables some trade after asset value increases (decreases) and thus leads to a higher trading volume.

Now suppose that volatility is low ($s < s_1^*(0)$). In the absence of ATs, it is not optimal for limit order traders to engage in order shading, as the risk of being picked off is very small. Then, introducing ATs may actually reduce trade. The reason behind this is that, for a low value of $a$, the outside option of placing a limit order is very high for ATs (they only face the risk of being picked off when trading with other ATs), and therefore HTs find it optimal to engage in a low level of order shading (they place buy (sell) limit orders that AT sellers (buyers) only find worth executing in the case of a price decrease (increase)). Once $a$ rises above a certain threshold, it becomes optimal for HT buyers (sellers) to submit limit orders that are executed by AT sellers (buyers) irrespectively of the asset value innovation, such that the trading rate is back to its level without ATs.

For illustration, Figure B.2 in Appendix B.2 depicts $TR_{HT}^*$, $TR_{AT}^*$ and $TR^*$ as a function of $a$ for different values of $s$. Notice that different values of $a$ may give rise to different equilibria for a fixed level of $s$ (see Figure B.1). Moreover, the exact values of $a$ for which we move from one equilibrium to another depend on $s$ itself in most cases. Therefore, for each possible equilibrium combination that can arise as $a$ moves from zero to one, we choose the mean level of $s$ that gives rise to the respective combination of equilibria. For example, for $0 \leq s < s_3^*(0)$, we may end up in a type-4 or a type-5 equilibrium, depending on $a$. Therefore, we plot the graph for $s = s_3^*(0)/2$. Table B.4 lists the employed values for $s$ together with the associated values of $a$ (we call them "switching points") where we move from one equilibrium to another[23].

Our results are somehow similar to those derived in Jovanovic and Menkveld (2010), where ATs intermediating between human traders can either increase trade by solving an adverse selection problem or decrease trade by creating an adverse selection problem. Empirically, they find that the entry of Chi-X, a trading platform particularly catering to the algorithmic trading community was associated with a reduction in trading volume for Dutch stocks of about 13%.

## 2.4.B. Gains from posting limit orders

Given the equilibrium quotes and execution probabilities, it is straightforward to compute the expected profits from posting limit orders. For simplicity, we here focus on the expected profits of buyers (in equilibrium, they are equal to the expected profits of sellers due to symmetry). Let $p_{j,k}^{B,+s}$ ($p_{j,k}^{B,-s}$) denote the equilibrium probability that the asset value increases (decreases) and subsequently a buy limit order posted by a type-$k$ trader is executed by a sell market order from a type-$j$ trader, where $j,k \in \{HT, AT\}$. Then, using equations (2.1) and (2.2), the equilibrium expected profits from posting limit orders can be written as

$$E(U_{t,HT}^{B,LO}(B_{t,HT}^{*})) = (p_{HT,HT}^{B,-s} + p_{AT,HT}^{B,-s})(v_t - s + L - B_{t,HT}^{*}) + (p_{HT,HT}^{B,+s} + p_{AT,HT}^{B,+s})(v_t + s + L - B_{t,HT}^{*}) \tag{2.3}$$

$$\begin{aligned} E(U_{t,AT}^{B,LO}(B_{t,AT}^{*}, B_{t,AT}^{-s\,*}, B_{t,AT}^{+s\,*})) = {} & p_{AT,AT}^{B,-s}(v_t - s + L - B_{t,AT}^{*}) + p_{AT,AT}^{B,+s}(v_t + s + L - B_{t,AT}^{*}) \\ & + p_{HT,AT}^{B,-s}(v_t - s + L - B_{t,AT}^{-s\,*}) + p_{HT,AT}^{B,+s}(v_t + s + L - B_{t,AT}^{+s\,*}) \end{aligned} \tag{2.4}$$

23 We use $s = 1$ for the case where $s \geq s_1^{*}(a_1^{*})$, as there exists no upper bound.

Let $E(U_{HT}^{LO*}) = E(U_{t,HT}^{B,LO}(B_{t,HT}^{*}))$ and $E(U_{AT}^{LO*}) = E(U_{t,AT}^{B,LO}(B_{t,AT}^{*}, B_{t,AT}^{-s*}, B_{t,AT}^{+s*}))$ be the equilibrium expected utilities obtained by HTs and ATs, respectively, when posting limit orders[24]. Moreover, let $E(U^{LO*})\big|_{a=0}$ denote the expected utility obtained from limit orders in the absence of ATs.

**Proposition 3 (Expected profits from limit orders):**

For any $a \in (0,1)$, we have $E(U_{AT}^{LO*}) > E(U^{LO*})\big|_{a=0} > E(U_{HT}^{LO*})$.

**Proof:** See Appendix B.1.C.

This result follows directly from ATs' ability to revise limit orders when trading with HTs, which reduces their risk of being picked off in the case of adverse price movements. There are two possible ways HTs may react to the improved outside option of ATs (which of the two is optimal depends on the level of $a$). They may i) post more aggressive limit orders than in the absence of ATs or ii) accept a reduced probability of execution for their limit orders (i.e. they decide to post quotes that are not (always) executed by ATs). It is immediate that the first reaction always harms HTs' expected profits from limit orders, as they simply offer better quotes, but the execution probability of their orders is as in the case where $a = 0$. On the other hand, choosing ii) allows HTs to post less aggressive quotes than in the absence of ATs, as the outside option of other HTs' has suffered. Nevertheless, the effect of a reduced execution probability dominates, such that their expected profits are also lower in this case.

## 2.4.C. Costs of submitting market orders

Following Foucault (1999), we define the trading cost as the signed difference between the transaction price and the fundamental asset value. In

[24] Notice that we may drop the time subscripts due to stationarity.

equilibrium, the cost incurred by a seller submitting a sell market order at time *t* is given by the fundamental value of the asset minus the best available bid price.

$$TC_t^S = v_t - B_t^m$$

It is important to note that the best available bid crucially depends on the type of both the market order trader and the limit order trader whose quote is executed. This is due to the assumption that ATs may revise their quotes, but only in the case the agent arriving after him is a HT. Additionally, the trading cost may also depend on the most recent realization of the fundamental asset value due to the picking off risk faced by the limit order trader.

Let $TC_{t,j,k}^S$ denote the trading cost for a type-*j* seller that arrives at time *t* and submits a sell market order that executes against a buy limit order posted by a type-*k* buyer at time $t-1$, where $j,k \in \{HT, AT\}$. In particular, consider a HT seller who arrives at time *t* and submits a sell market order that executes against the best available bid. If the bid stems from a HT, the trading cost is given by

$$TC_{t,HT,HT}^{S,+s} = (v_{t-1} + s) - B_{t-1,HT}^{*} \quad (2.5)$$

for $e_t = +s$ and

$$TC_{t,HT,HT}^{S,-s} = (v_{t-1} - s) - B_{t-1,HT}^{*} \quad (2.6)$$

for $e_t = -s$ . In case the best available bid was posted by an AT buyer (and therefore was revised before the arrival of the market order trader), the trading cost is given by

$$TC_{t,HT,AT}^{S,+s} = (v_{t-1} + s) - B_{t-1,AT}^{+s\,*} \quad (2.7)$$

for $e_t = +s$ and

$$TC_{t,HT,AT}^{S,-s} = (v_{t-1} - s) - B_{t-1,AT}^{-s\,*} \quad (2.8)$$

for $e_t = -s$ . In order to calculate the expected trading costs, we simply have to weight the trading costs for each possible event by its stationary probability. Then, submitting time subscripts, we have

$$E(TC^S_{HT}) = \frac{f_1(p^{B,+s}_{HT,HT} TC^{S,+s}_{HT,HT} + p^{B,-s}_{HT,HT} TC^{S,-s}_{HT,HT}) + f_3(p^{B,+s}_{HT,AT} TC^{S,+s}_{HT,AT} + p^{B,-s}_{HT,AT} TC^{S,-s}_{HT,AT})}{f_1(p^{B,+s}_{HT,HT} + p^{B,-s}_{HT,HT}) + f_3(p^{B,+s}_{HT,AT} + p^{B,-s}_{HT,AT})} \tag{2.9}$$

Following exactly the same logic, the expressions for the trading costs of AT sellers are given by

$$TC^{S,+s}_{t,AT,HT} = (v_{t-1} + s) - B^*_{t-1,HT} \tag{2.10}$$

$$TC^{S,-s}_{t,AT,HT} = (v_{t-1} - s) - B^*_{t-1,HT} \tag{2.11}$$

$$TC^{S,+s}_{t,AT,AT} = (v_{t-1} + s) - B^*_{t-1,AT} \tag{2.12}$$

$$TC^{S,-s}_{t,AT,AT} = (v_{t-1} - s) - B^*_{t-1,AT} \tag{2.13}$$

and the expected trading cost for an AT seller is consequently given by

$$E(TC^S_{AT}) = \frac{f_1(p^{B,+s}_{AT,HT} TC^{S,+s}_{AT,HT} + p^{B,-s}_{AT,HT} TC^{S,-s}_{AT,HT}) + f_3(p^{B,+s}_{AT,AT} TC^{S,+s}_{AT,AT} + p^{B,-s}_{AT,AT} TC^{S,-s}_{AT,AT})}{f_1(p^{B,+s}_{AT,HT} + p^{B,-s}_{AT,HT}) + f_3(p^{B,+s}_{AT,AT} + p^{B,-s}_{AT,AT})} \tag{2.14}$$

Finally, the average expected trading cost for sellers is given by

$$E(TC^S) = \frac{f_2}{f_2 + f_4} E(TC^S_{HT}) + \frac{f_4}{f_2 + f_4} E(TC^S_{AT}) \tag{2.15}$$

By symmetry, we have that $E(TC^B_{HT}) = E(TC^S_{HT}) \equiv E(TC_{HT})$, $E(TC^B_{AT}) = E(TC^S_{AT}) \equiv E(TC_{AT})$ and $E(TC^B) = E(TC^S) \equiv E(TC)$.

Similar to the previous section, let $E(TC^*)$ denote the equilibrium average expected trading cost, and let $E(TC^*)\big|_{a=0}$ be the equilibrium average expected trading cost without ATs. One can show the following.

**Proposition 4 (Expected trading costs):**

$E(TC^*_{HT}) > E(TC^*) > E(TC^*_{AT})$ for all $a \in (0,1)$. Define $\bar{a} = (7-\sqrt{33})/2$ and let $\bar{\bar{a}}$ be such that $s_4^*(\bar{\bar{a}}) = s_1^*(0)$. Then, the following holds.

**i)** Let $s \geq s_1^*(a_1^*)$. Then
- a) If $a \leq a_1^*$, then $E(TC^*_{HT}) > E(TC^*) > E(TC^*)\big|_{a=0}$.
- b) If $\bar{a} \geq a > a_1^*$, then $E(TC^*) < E(TC^*_{HT}) \leq E(TC^*)\big|_{a=0}$.
- c) If $a > \bar{a}$, then $E(TC^*_{HT}) > E(TC^*)\big|_{a=0} > E(TC^*)$.

**ii)** Let $s_1^*(a_1^*) > s \geq s_1^*(0)$. Then
- a) If $a \leq \bar{\bar{a}}$, then $E(TC^*)$ and $E(TC^*_{HT})$ may be higher or lower than $E(TC^*)\big|_{a=0}$.
- b) If $\bar{a} \geq a > \bar{\bar{a}}$, then $E(TC^*) < E(TC^*_{HT}) \leq E(TC^*)\big|_{a=0}$.
- c) If $a > \bar{a}$, then $E(TC^*_{HT}) > E(TC^*)\big|_{a=0} > E(TC^*)$.

**iii)** Let $s_1^*(0) > s \geq s_3^*(0)$. Then $E(TC^*_{HT}) > E(TC^*) > E(TC^*)\big|_{a=0}$.

**iv)** Let $s_3^*(0) > s$.
- a) If $a \leq 1/4$, then $E(TC^*)$ and $E(TC^*_{HT})$ may be higher or lower than $E(TC^*)\big|_{a=0}$.
- b) If $1/4 < a \leq 1/3$, then $E(TC^*_{HT}) > E(TC^*)\big|_{a=0}$. $E(TC^*)$ may be higher or lower than $E(TC^*)\big|_{a=0}$.
- c) If $1/3 < a$, then $E(TC^*_{HT}) > E(TC^*) > E(TC^*)\big|_{a=0}$.

**v)** If $s_1^*(0) > s \geq s_3^*(a_2^*)$ and $a > a_1^*$, then $E(TC^*_{AT})$ may be higher or lower than $E(TC^*)\big|_{a=0}$. Otherwise, we have $E(TC^*)\big|_{a=0} > E(TC^*_{AT})$.

**Proof:** See Appendix B.1.D.

ATs enjoy lower trading costs than HTs because the latter may only pick off stale quotes when trading with other HTs. Moreover, the outside option of posting limit orders is more profitable to ATs, such that they are not willing to incur the same trading costs as HTs. This result is very much in line with the findings reported by Hendershott and Riordan (2009), who report that ATs "consume liquidity when it is cheap", i.e. they incur lower trading costs than HTs.

We find that both the expected trading costs for HTs and average expected trading costs in the presence of algorithmic traders may be higher or lower than in an economy with only HTs. This is also illustrated in Figure B.3, where we plot equilibrium expected trading costs as a function of $a$ for different values of $s$.

To see how the presence of ATs can increase expected trading costs for HTs, consider for example the case where $s \geq s_1^*(a_1^*)$. Without ATs ($a = 0$), the equilibrium bid (ask) prices are set such that a buy (sell) limit order is only executed in case the next agent is a seller (buyer) and the asset value has decreased (increased). In other words, the picking off risk is sufficiently high to induce order shading by limit order traders. Not consider what happens if we introduce a small proportion of ATs ($a \leq a_1^*$). Given that $a$ is small, it is optimal for HTs to submit buy (sell) limit orders that are only executed by other HTs, but not by ATs. As described in the previous section, this allows HTs to reduce the aggressiveness of their quotes, because their limit orders only target other HTs, whose outside option has become less valuable due to a decreased execution probability. As a consequence, the expected trading costs for HTs increase below their level in the absence of ATs. Given that there are relatively few ATs, the increase in the expected trading cost for HTs also leads to higher average expected trading costs.

To see how algorithmic trading can lead to a decrease in HTs' expected trading costs, consider what happens in the above scenario if we increase $a$. Once we have $a > a_1^*$, it becomes optimal for HTs to post buy (sell) limit orders that can also be executed by AT sellers (buyers) in the case of a price decrease (increase). This happens for two reasons. First, given that $a$ is now relatively large, the new strategy leads to a considerably higher execution probability of HTs' limit orders. Second, ATs exert a negative externality on each other by increasing the picking off risk (note that ATs may not cancel their limit orders if the next trader is also an AT), resulting in a lower outside option. The increased aggressiveness of HTs' limit orders benefits other HTs and decreases their expected trading costs below the level obtained when $a = 0$. Nevertheless, this effect diminishes as $a$ increases further. First, HTs that arrive to the market are less and less likely to find a quote submitted by another HT (note that ATs can discriminate between HTs and ATs, offering worse quotes to the former), and second, the aggressiveness of the HTs' quotes diminishes as ATs face an increasingly higher picking off risk and therefore are willing to accept worse quotes. Given that $a$ is relatively high, average expected trading costs are nevertheless lower than in the absence of ATs.

It is worth mentioning that HTs' expected trading costs may also decrease for very low values of $a$ under some circumstances. Consider the economy without ATs and suppose that $s$ is such that it is optimal for traders to engage in order shading, but they are almost indifferent to not doing so (i.e. $s_1^*(a_1^*) > s \geq s_1^*(0)$). Thus in equilibrium, HT buyers (sellers) submit buy (sell) limit orders that are only executed by HTs in the case of a price decrease (increase). Now, introduce a small proportion of ATs, which leads to a decrease (increase) in HTs' sell (buy) cutoff prices. Suddenly, it becomes optimal for HTs to submit limit orders that may also be executed if the asset value increases (decreases), albeit only by HTs. As these limit orders now face the risk of being picked off (which is profitable for market order traders), HTs' expected trading costs experience a drop. Once $a$ increases, this effect disappears as the execution probability of HTs' limit orders decreases and they are better off not incurring any picking off risk.

Several empirical studies (e.g. Hendershott et al. (2011), Jovanovic and Menkveld (2010)) report that algorithmic trading improves liquidity as it leads to lower trading costs (as measured e.g. by the effective spread), supporting the claim of HFT advocates. While we indeed find that the presence of ATs can lead to decreases in average expected trading costs, this may well come along with an increase in HTs' expected trading costs, in particular for high levels of algorithmic trading.

### 2.4.D. Welfare

Consider a HT. In equilibrium, his order type choice depends on the state of the limit order book upon his arrival. For example, a HT seller will opt for a market sell order if the best available bid price is above his sell cutoff price and otherwise post a sell limit order. Thus his ex-ante (before arriving to the market, but after learning his type) expected utility can be written as

$$EU_{HT} = TR_{HT} E(U_{HT}^{MO}) + MR_{HT} E(U_{HT}^{LO})$$

where we have omitted the buyer/seller superscripts as both have the same expected utilities. Similarly, we can write the ex-ante expected utility of an AT as

$$EU_{AT} = TR_{AT} E(U_{AT}^{MO}) + MR_{AT} E(U_{AT}^{LO})$$

It is easy to see that $E(U_{HT}^{MO}) = L - E(TC_{HT})$ and $E(U_{AT}^{MO}) = L - E(TC_{AT})$, i.e. the expected utility from submitting a market order is equal to the private gains from trade $|y_t| = L$ minus the expected trading cost. Given the results from the previous two sections, the calculation of the ex-ante expected utilities is straightforward. Finally, the ex-ante expected utility of the average trader is given by

$$\overline{EU} = (1-a)EU_{HT} + a\,EU_{AT} = TR \times (2L)$$

which is simply the trading rate multiplied by the overall gains from trade.

Similar to the previous sections, let $EU^*_{HT}$ and $EU^*_{AT}$ be the equilibrium expected utilities of HTs and ATs, respectively. Moreover, let $\overline{EU}^*$ denote the equilibrium expected utility of the average trader and write $\overline{EU}^*\big|_{a=0}$ for the expected utility in the absence of ATs.

**Proposition 5 (Welfare):**

$EU^*_{AT} > EU^*_{HT}$ for all $a \in (0,1)$. Moreover

**i)** If $s \geq s^*_1(0)$ ($s < s^*_1(0)$), then $\overline{EU}^* > \overline{EU}^*\big|_{a=0}$ ($\overline{EU}^* \leq \overline{EU}^*\big|_{a=0}$).

**ii)** If $s < s^*_1(0)$, then $EU^*_{HT} < \overline{EU}^*\big|_{a=0}$.

**iii)** If $s \geq s^*_1(a^*_1)$, then $EU^*_{HT} < \overline{EU}^*\big|_{a=0}$.

**iv)** Let $s^*_1(0) \leq s < s^*_1(a^*_1)$.

a) If $a > \overline{\overline{a}}$, then $EU^*_{HT} < \overline{EU}^*\big|_{a=0}$.

b) If $a \leq \overline{\overline{a}}$, then $EU^*_{HT}$ may be higher or lower than $\overline{EU}^*\big|_{a=0}$.

**v)** If $s^*_1(0) > s \geq s^*_3(a^*_2)$ and $a > a^*_1$, then $EU^*_{AT}$ may be higher or lower than $\overline{EU}^*\big|_{a=0}$. Otherwise, we have $EU^*_{AT} > \overline{EU}^*\big|_{a=0}$.

**Proof:** See Appendix B.1.E.

The fact that ATs obtain a strictly higher expected utility than HTs directly follows from Propositions 3 and 4, as they reap greater expected profits from limit orders and incur lower expected trading costs for market orders. Similarly, the fact that aggregate welfare (i.e. the ex-ante expected utility of the average trader) increases in the presence of ATs for high asset value volatility, but may decrease otherwise directly follows from Proposition 2 and the fact that aggregate welfare is equal to the overall gains from trade multiplied by the trading rate.

Moreover, combining these results, it is immediate that algorithmic trading decreases HTs' welfare in the case where $s < s_1^*(0)$. As HTs obtain a strictly lower expected utility than ATs, and the average trader is worse off as under no algorithmic trading, HTs must incur some loss in welfare.

For $s \geq s_1^*(0)$, the overall effect of algorithmic trading on HTs' welfare is more ambiguous, as aggregate welfare increases. While ATs reap the entire welfare gain in most cases, a low level of algorithmic trading may actually lead to an increase in HTs' welfare (see Panel 2 in Figure B.4 in Appendix B.2). This is because the presence of ATs may reduce order shading by HTs (as discussed in Section 2.4.C). Nevertheless, this scenario unfolds only if volatility is slightly above the level where HTs are almost indifferent between submitting limit orders with high or low execution probabilities for $a = 0$. In any case, HTs' welfare under a high level of algorithmic trading is strictly below its level reached with only human market participants.

Finally, part v) of the proposition shows that for some parameter constellations, the welfare loss from algorithmic trading may be so severe that even the equilibrium expected utility of ATs falls below the welfare obtained by traders in an economy without any algorithmic trading. This is illustrated in Panel 3 of Figure B.4.

## 2.5. Conclusion

Extending the model of Foucault (1999), we have studied a dynamic limit order market with slow human traders and fast algorithmic traders. The use of modern IT-infrastructure allows market participants to manage limit orders more efficiently and therefore reduces their risk of being picked off.

Although we assume (for tractability) that the presence of ATs does not increase HTs' risk of being picked off, the latter are still affected by the introduction of algorithmic trading. This occurs because agents' choice between limit and market orders is endogenous. While the sequential structure of the model endows limit order traders with some level of market power, their advantage is limited by the ability of the next trader to post a limit order himself. Thus, intuitively, their increased speed endows ATs with an improved outside option compared to HTs, which is detrimental to the welfare of HTs in most cases, although aggregate welfare (which is proportional to trading volume in our setting) may actually increase.

There are several interesting avenues for further research. For example, many electronic trading platforms have established asymmetric fee structures in recent years, now charging different costs to limit and market orders (so-called maker/taker schemes). Given that ATs have a comparative advantage at using limit orders, it may be very interesting to study the interplay of algorithmic trading and the employed fee structure. Nevertheless, recent work by Colliard and Foucault (2011) suggests that the fee structure may only matter in the case of a discrete pricing grid.

# 3. PRE-TRADE TRANSPARENCY IN CALL AUCTIONS

## 3.1. Introduction

Today, trading in most of the world's equity markets takes place via an electronic limit orders book. Nevertheless, the daily opening and closing prices are usually determined via a uniform price call auction. Whereas the merits of call auctions have been widely documented, their design has received relatively little attention.

In this chapter, we empirically investigate the call auctions of two of the world's leading electronic stock exchanges, Euronext (Paris Bourse) and Xetra (Frankfurt). Although the auctions in both markets share many important features such as the price setting mechanism, they crucially differ in the degree of pre-trade transparency, i.e. the information disclosed to market participants during the auction. While Euronext continues to display the limit order book as during the continuous trading phase, Xetra only discloses an indicative clearing price and the associated trading volume. Additionally, Euronext auctions last for a fixed time period, whereas Xetra randomizes the crossing time within a given corridor. Given that both exchanges have been employing these different mechanisms for more than a decade without making major changes, we believe that a comparison of both systems is overdue.

There is a large literature on the role of call auctions on stock exchanges. Garbade and Silber (1979) point to an auction's enhanced price efficiency and reduced price impact, which is traded off against the cost of delay and loss of order flow information. Madhavan (1992) shows how call auctions can jump-start continuous trade, which may otherwise stagnate due to adverse selection.

Most of the work related to the opening auction has focused on the role of price discovery after the arrival of new information in the non-trading period between days. In a seminal paper, Biais et al. (1999) study the opening auction on the Paris Bourse and find evidence for price discovery during the call phase, despite the fact that orders are non-binding. Cao et al. (2000) report similar evidence for the pre-opening period on NASDAQ.

Price discovery generally plays a less prominent role during the closing auction, as it is not preceded by an extended non-trading period with the arrival of new information. Instead it allows market participants to execute large orders at the closing price with a relatively low price impact[25]. This is particularly interesting for large institutional investors, as the closing price is of special importance to them, serving for the calculation of net asset values (NAVs), portfolio returns and subsequently performance evaluation. Moreover, changes in index weights and constituents become usually effective at the market close, inducing a particular need by passive investors to trade at these prices. Cushing and Madhavan (2000) provide anecdotal evidence on the inclusion of shares of "Safeway" into the S&P 500 index, which was accompanied by a substantial inflation of the closing price due to large demand by passive investors replicating the index. Overall, the related literature has largely endorsed the use of closing auctions. Pagano and Schwartz (2003) find that the introduction of closing call auctions on the Paris Bourse lowered execution costs, increased liquidity, and sharpened price discovery. Comerton-Forde et al. (2007) find similar findings for the Singapore Stock Exchange.

An additional motive for the use of call auctions to set closing prices is the prevention of price manipulation. Empirical regularities such as the day-end

[25] As opposed to trading in a continuous market, a call auction allows for the simultaneous execution of a large number of orders, allowing traders to increase liquidity by pooling as in Admati and Pfleiderer (1988).

effect, first documented by Harris (1989), and the volatility spike during the last minutes of trading reported by Chan et al. (1995) and others have cast doubt on whether closing prices represent equilibrium valuations. Comerton-Forde and Putniņš (2011) provide a plethora of anecdotal evidence by studying a sample of 160 court cases dealing with price manipulation. Overall, most cases relate to trading of illiquid stocks at the market close. Motives for manipulating the closing price (also known as *marking the close*) can arise from principal-agent relations such as broker intermediation (Hillion and Suominen (2004)), delegated investment management (e.g. Sias and Starks (1997)), or the gaming of derivatives cash settlements (Kumar and Seppi (1992), Pirrong (2001) and Xiaoyan Ni et al. (2005)). Comerton-Forde et al. (2007) show that the introduction of a closing auction on the Singapore Stock Exchange has reduced the skewness and kurtosis of closing returns, suggesting less price manipulation.

While there seems to be a widespread consensus among both academics and practitioners in favour of the use of call auctions, their actual design has not been studied intensively. Although the issue of pre-trade transparency has generally experienced a great deal of attention within the market microstructure literature, most research has focused on limit-order and dealer markets (see Madhavan (2000) and Biais et al. (2005) for an excellent overview), as call auctions are a more recent phenomenon. One notable exception is the paper by Baruch (2005), who models the NYSE opening procedure and shows that an open limit-order book fosters competition among informed liquidity providers (which act as limit-order traders) and thus improves liquidity. Chakraborty et al. (2010) develop a model of call auctions where order shading due to uncertainty about potential counterparties may result in inefficiencies. They show that disclosure of the order book may restore efficiency as it allows for signalling of trading intentions in the absence of private information. This closely relates to the concept of “sunshine trading” (Admati and Pfleiderer (1991)). Dia and Pouget (2004) study the transparent but illiquid call auctions of the West African stock exchange and find that a transparent system is critical for liquidity and price discovery. Similarly, Gerace et al. (2009) find evidence of increased investor participation during the pre-opening session on the Shanghai Stock Exchange after an increase in market transparency.

On the other hand, experimental studies largely argue against transparency in call auction settings. Friedman (1993) finds that order book disclosure results in lower price discovery, while his results on trading volume are mixed. Similarly, Arifovic and Ledyard (2007) compare repeated open book and closed book auctions, and find that the latter yield higher allocative efficiency, both with artificial agents and in laboratory experiments. Transparency may also encourage bluffing and therefore reduce gains from trade, as found by Biais et al. (2009).

Given the mixed evidence, the co-existence of different transparency regimes in real markets is not very surprising and likely to be a manifestation of the ongoing controversy regarding the optimal degree of transparency. We contribute to this debate by analyzing a matched sample of German and French stocks traded on Xetra and Euronext, respectively. Both markets have pioneered the use of call auctions in modern equity markets, but operate under very different transparency regimes.

Overall, our findings suggest that pre-trade transparency in the form of order book disclosure is beneficial. Compared to Xetra, call auctions on Euronext display significantly higher trading volume both at the open and at the close. While we observe this pattern across the board, the observed difference is most pronounced for trading in blue chips at the market close (10.34% vs. 7.87% of total trading volume). Overall, we interpret these findings as being consistent with an open order book facilitating the location of trading partners by signalling the willingness to trade, particularly in a setting where trades are not motivated by private information but rather due to liquidity trading (e.g. by passive investors). The analysis of cross-sectional correlations in trading volume (Sias and Starks (1997)) provides additional support for the concentrated presence of institutional investors in Euronext closing auctions for the most heavily traded stocks. Moreover, Euronext's call auctions are more liquid (as measured by the inverse of the Amihud ratio), i.e. liquidity seekers enjoy a lower price impact on average.

Turning to price discovery, our findings indicate that an open order book also fosters price discovery, particularly at the open. Except for the most active stocks, the opening auction on Euronext contributes significantly more to price discovery (between 24% and 38%) than on Xetra (18% and 28%). This evidence is consistent with an informative limit order book (see e.g. Cao et al. (2009) and Hendershott and Jones (2005)). As expected, the closing auctions contribute very little to price discovery (3-4%) due to the absence of news arrivals, and we find no significant differences across markets. Euronext's dominance in terms of price discovery at the opening is furthermore confirmed by the analysis of quoted spread within the first 60 minutes of the continuous trading period: Stocks on Xetra start the day with wider quoted spread (as compared to the average daily spread), but converge to the level of the spreads found on Euronext within the first 15 minutes.

Additionally, we examine the efficiency of opening and closing prices. Overall, closing prices appear to contain large transitory components of up to 50%, with no significant differences across markets. We show that these transitory effects are purely idiosyncratic and most likely to represent an increased inventory risk faced by liquidity providers in the face of the illiquid overnight period. Opening prices display relatively small transitory components, with Euronext displaying smaller inefficiencies consistent with improved price discovery. Finally, we briefly examine the possibility of sniping (the practice of entering orders at the very last moment so as to prevent any reaction from other market participants) during Euronext's closing auction, given that its ending time is deterministic as opposed to Xetra. Overall, sniping seems to be a relatively minor issue.

The remainder of this chapter is organized as follows: Section 3.2 gives an overview of the auction mechanisms in place at both exchanges, while Section 3.3 describes the data and sample construction. Section 3.4 investigates trading volume and liquidity and Section 3.5 examines price discovery and efficiency. Section 3.6 concludes, and all tables and figures are relegated to the Appendix C.

## 3.2. Description of the Auction Mechanisms

This section provides a brief description of the auction mechanisms in place on Xetra and Euronext, respectively. For the sake of brevity, we only touch the aspects that are relevant to our analysis. For a more comprehensive overview of the detailed rules and regulations governing each market, we refer the reader to the respective trading manuals that are available on the exchanges' websites[26]. A much more detailed account can also be found in Kasch-Haroutounian and Theissen (2009).

### 3.2.A. Euronext

Euronext starts displaying its electronic limit order book every morning at 07:15. During the *call phase*, the system continuously displays the indicative clearing price and five levels of unfilled buy and sell orders. At any point in

[26] www.euonext.com and www.deutscheboerse.com, respectively.

time, market participants may submit, cancel and modify orders at their convenience until exactly 09:00, when the book is briefly frozen and the clearing price is calculated as to maximize trading volume and minimize the resulting surplus (in the case of multiple prices leading to the same trading volume). After orders are matched and removed from the limit order book, continuous trade starts. At 17:30[27] continuous trading is halted and the closing call phase commences. The disclosure and order submission rules are identical to those for the opening auction. The call phase terminates at exactly 17:35, when orders are assigned and filled. Subsequently, the resulting surplus is offered during a “trading at last session”, which lasts for ten minutes. During this period, where all trades are executed at the closing price, traders can also submit new buy and sell orders. In all auctions, the call phase may be extended at the discretion of the exchange, and there are static and dynamic price ranges whose violation can trigger time extensions.

### 3.2.B. Xetra

Every morning at 07:30, the Xetra system starts collecting limit buy and sell orders, with cancellations and modifications possible at any point in time. From 08:50 onwards it starts displaying the indicative clearing price and the associated trading volume, while the limit order book remains undisclosed. At 09:00[28], a thirty second random ending period starts, during which the call phase is terminated at random (the call phase is ended for each stock individually). Upon the ending of the call phase, the clearing price is set using the same algorithm as on Euronext. Subsequently, orders are crossed and removed from the limit order book, which is now being displayed, and continuous trading commences. The closing auction (starting at 17:30 and

[27] Previously to February 19th 2007, the closing auction was held from 17:25 to 17:30. The timing was changed in order to align the auction with other European equity markets.

[28] 9:00 am for members of DAX and TecDAX, 9:02 am otherwise.

lasting until 17:35 plus a maximum of 30 seconds) follows exactly the same rules as the opening auction, except that it starts directly in the call phase. As on Euronext, call phases may be extended due to high volatility and other reasons at the discretion of the exchange. Xetra also offers a noon auction, which usually lasts 2 minutes plus a maximum of 30 seconds and is held launched at 13:00. In our analysis, we discard the noon auction, as it attracts very little trading volume and there is no direct counterpart on Euronext[29].

## 3.3. Data description and sample selection

Our sample spans the period 2006-2007. We obtain intraday prices and quotes for French and German stocks from SIRCA. The files contain all executed trades together with the traded volume and execution time (up to 1/1000 of a second) as well as the best bid and ask quotes with their respective depth at any point in time. Additional data on stock characteristics, dividends, capital structure changes, stock splits, etc. are obtained from Datastream.

Following the literature on cross-exchange comparisons (e.g. Huang and Stoll (1996), Venkataraman (2001), and Kasch-Haroutounian and Theissen (2009)), we control for differences in stock characteristics by constructing a matched sample.

We start by defining an initial pool of securities for each of the two exchanges. For Euronext we choose the members of the SBF 250, which is composed of the 250 largest and most heavily traded stocks that are listed on the Paris Bourse. The stocks in this index are subdivided into the CAC40

[29] The main objective of the noon auction is the setting of the settlement price for index futures and options on expiry dates, when the auction lasts 5 minutes. On these days, which we discard from our sample (see Section 3.3), trading volume in the noon auction is considerable (up to 30% of the daily total).

(large caps), the NEXT20 (next 20 largest stocks after CAC40), the MID100 (mid-caps) and the SMALL90 (small-caps). For Xetra, we select the stocks comprising the following main indices: Dax30 (large caps), MDax (mid-caps), SDax (small-caps) and TecDax (technology stocks, mid- or small cap). As we suspect that index membership might be an important determinant of call auction trading (in particular due to the rising importance of passive investment strategies), we restrict our sample to those stocks that have been member of any of the above indices during the whole sample period[30]. This leaves us with a sample of 213 French stocks and 129 German stocks. We furthermore exclude all stocks that are traded on less than 495 days[31] of the sample in order to ensure sufficiently regular trading activity. Finally we exclude shares of EADS, as the company is listed on both exchanges but Euronext is clearly the main market (with a market share of roughly 90%), as well as shares of BAINS MER MONACO, whose continuous trading period is shorter than the regular trading hours. This reduces our sample to 199 French and 125 German stocks.

We consider three characteristics in order to create a matched sample: trading volume ($x_1$), free float capitalization ($x_2$), and volatility as measured by the standard deviation of daily returns ($x_3$). We then compute the following distance measures between German stocks *i* and French stocks *j*:

$$DIST_{i,j} = \sum_{k=1}^{3} \left( \frac{x_k^{\,i} - x_k^{\,j}}{\left(x_k^{\,i} + x_k^{\,j}\right)/2} \right)^2 \tag{3.1}$$

For each German stock we select the French stock which generates the lowest $DIST_{i,j}$. If a French stock is matched to more than one German stock,

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[30] This means that we allow a stock to change the index (e.g. moving from MDax to SDax).

[31] There are a total of 507 trading days for Xetra and 510 trading days for Euronext. So we effectively exclude stocks that trade at least on roughly 97.5% of all trading days. While this cut-off is somewhat arbitrary, our results are not sensitive to small changes in the cutoff.

we keep the match with the lowest distance and drop the other ones. Finally, we require that the differences in the individual characteristics do not exceed a threshold by imposing

$$\left| \frac{x_k^{\,i} - x_k^{\,j}}{\left(x_k^{\,i} + x_k^{\,j}\right)/2} \right| \leq 0.75 \quad \forall k \tag{3.2}$$

This procedure leaves us with a final sample of 63 pairs of stocks, which we further divide in three size groups according to trading volume (in Euros) - large caps, mid caps, and small caps. For robustness, we additionally conduct our analysis on samples obtained with several variations of the matching procedure. None of the presented results are qualitatively different under alternative matching procedures.[32]

[Insert Table C.1 and Table C.2 about here]

Table C.1 lists the stock pairs resulting from our matching procedure, while the associated descriptive statistics regarding the variables employed are tabulated in Table C.2. The quality of the matching is evident from the small and non-systematic differences in matching characteristics. The only notable exception is that German large cap stocks tend to have higher trading volumes. This finding is in line with Kasch-Haroutounian and Theissen (2009), who find a higher turnover for Xetra stocks.

## 3.4. Trading volume and liquidity

### 3.4.A. Trading Volume

We start our analysis by looking at the trading volumes executed during the call auctions. For each day $t$ and stock $i$ in the sample, we compute the

[32] Among others, we changed the order of selection (starting with French instead of German stocks), and used different cutoff values (of data availability and distance). We also matched on trading volume and volatility only.

relative trading volume as the proportion of total daily trading volume, $Vol_{i,t}^{d}$, that is executed during an auction

$$RV_{i,t}^{x} = \frac{Vol_{i,t}^{x}}{Vol_{i,t}^{d}} \tag{3.3}$$

where $x \in \{o(pen), c(lose)\}$. We asses the difference across exchanges by calculating, for each pair $(i, j)$ and trading day *t* the difference in $RV$ as

$$\Delta RV_{i,j,t}^{x} = RV_{i,t}^{x} - RV_{j,t}^{x} \tag{3.4}$$

where stock *i* is traded on Euronext and stock *j* is the matched stock traded on Xetra.

One important difference between Xetra and Euronext lies in the procedures used for setting the settlement prices for options and futures: On Euronext, the settlement prices for all derivative contracts are determined as the prices of the closing call auction, while on Xetra the index options and futures are settled with the prices determined in the noon auction (the individual stock options are also settled with the closing prices). As expiry effects may potentially lead to extremely high trading volume, we exclude the third Friday of each calendar month[33] when calculating $RV$ and $\Delta RV$ in order not to pick up expiry effects.

[Insert Table C.3 about here]

The results can be found in Table C.3. On average, the Euronext (Xetra) opening auction accounts for 2.6% (1.4%) of daily trading volume, while the closing auction represents roughly 6.8% (5.5%) of the transacted value. The differences of 1.2% and 1.3% are statistically significant at any conventional

[33] The third Friday of the month is the expiry date for exchange-traded stock and index options. Every third month, this day is also the expiry date for Index Futures. Those dates are common for both exchanges.

level using a simple t-test on the paired differences $\Delta RV$. The fact that we observe a higher $RV$ for Euronext across both auctions and all individual volume terciles underlines the robustness of these results. Overall, the closing auction attracts considerably more trading activity than the opening auction, except for the least active stocks, which is in contrast to the results obtained earlier studies. Kehr et al. (2001) study a sample of 15 large German stocks and find that approximately 10% of the day's trading volume takes place during the *opening* auction. Amihud and Mendelson (1987) report that on the NYSE 5.6% of the daily volume is negotiated during the opening, 8.4 times as much as at the close. Lehmann and Modest (1994) find that on the Tokyo stock exchange, approximately 27% of trade is executed during the opening and only 3% during the closing auctions. Given the enormous growth of passive investment strategies, this migration of transactions volume is not very surprising. After all, index trackers face the constant need to adjust their portfolios towards the current index weights. Given that these weights are usually based on closing prices, the closing auction is the optimal point in time to rebalance their portfolios (see e.g. Cushing and Madhavan (2000)). In line with this argument, we find that the closing auction appears to be particularly important for stocks with high trading volume, accounting for 10.30% (7.87%) of the total trading activity on Euronext (Xetra). Given that we observe the largest absolute difference $\Delta RV$ (roughly 2.5%) for these stocks, this suggests that the transparent Euronext system may facilitate institutional liquidity traders' search for liquidity by enabling them to signal their demands, as proposed in the theoretical model of Chakraborty et al. (2010). As most passive strategies focus mainly on blue chips (this is almost by definition because most benchmarks are constructed using market capitalization as portfolio weightings), this would additionally explain why we observe a smaller $\Delta RV$ for mid and small caps.

## 3.4.B. Liquidity / Price Impact

Given that we document a higher trading volume for Euronext's call auctions, a natural question that arises in our context is whether this translates into higher liquidity (a lower price impact of trading). While many widely used liquidity measures either rely on the bid-ask spread or strings of transaction prices (for a recent overview of liquidity measures, see Goyenko et al. (2009)) and are therefore not applicable in our setting, we can resort to Amihud's illiquidity measure (Amihud (2002)), which is based on Kyle's concept of liquidity (Kyle (1985)), the response of prices to order flow. The inverse of this measure can then be taken as a measure of liquidity and is calculated as

$$AMH_i^{x-1} = \left( \frac{1}{T_i} \sum_{t=1}^{T} \frac{\left| r_{i,t}^x \right|}{V_{i,t}^x} \right)^{-1} \tag{3.5}$$

where $T_i$ denotes the number of trading days with non-zero trading volume for stock *i* and $r_{i,t}^x$ is the return associated with the trading volume $V_{i,t}^x$. The return of the opening (closing) auction is defined as the return from the previous day's close (same day's midquote right prior the start of the closing auction) to the opening (closing) auction's clearing price[34]. Similarly, the return of the continuous trading session is the return from the opening price to the last midquote before the closing auction.

We calculate this measure for the opening and closing auctions as well as for the continuous trading phase. Table C.4 reports the results. We assess the statistical significance of cross-market differences using a paired t-test and (given the small sample size) a non-parametric Wilcoxon signed-rank test. First, a look at the results for the continuous trading phase (Panel C)

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[34] In the calculation of the overnight return (close-to-open), we adjust the opening price for dividends, stock splits, and rights issues.

confirms that our matching algorithm has eliminated any important cross-market differences in liquidity. The exception is the large cap tercile, for which the Xetra stocks are slightly more liquid.

[Insert Table C.4 about here]

The results for the opening and closing auctions (Panels A and B) indicate that, by and large, Euronext is the more liquid market, as we find a higher inverse Amihud ratio on Euronext across the board. Overall, these results confirm the picture that has emerged from our analysis of trading volumes, suggesting that the elevated auction trading volume on Euronext does not come at the cost of higher price volatility and can therefore be interpreted as being the more efficient mechanism.

## 3.4.C. Cross-sectional volume correlations

Sun (2008) argues that similarities in institutional portfolios lead to positively correlated trading volume. Pollet and Wilson (2008) show that institutional investors scale their portfolio up or down as a reaction to in/outflows from their principals. Finally, rebalancing by index funds automatically creates correlated trading volume as it involves trading simultaneously in several stocks. Thus, correlated trading volume across assets can be interpreted as a sign of institutional trading. Sias and Starks (1997) have looked at correlations in trading volume to gauge institutional presence.

Call auctions provide relatively easy access to a high volume, competitively determined price. We suspect that institutional investors such as index funds, mutual funds, and pension funds, are particularly interested in trading in the closing auction, as its clearing price is a popular benchmark for portfolio valuations.[35]

To gauge the concentrated presence of institutional (liquidity) traders, we therefore study the correlation of trading volume across stocks during the auctions and the continuous trading session. Since we have total of 21 stocks in each size group, we obtain ½·21·20 = 210 pairwise correlations coefficients for each market and trading activity tercile. In order to examine the presence of institutions in the different auctions, we calculate the ratio of the average cross-sectional volume correlation in the respective auction to the average cross-sectional volume correlation during the continuous trading phase. Standard errors for this ratio are easily obtained using the delta method. Finally, we assess differences in the concentration of institutional traders by comparing the ratios across markets, where we base inference on a simple t-test assuming independence across samples.

[Insert Table C.5 about here]

Table C.5 details the results. First, we notice that average volume correlations are higher for Xetra stocks across the board, which may be due to higher correlations in fundamentals or other exogenous reasons. Overall, we find that average volume correlations for the opening auction are significantly lower than during the continuous trading phase, which is likely to be due to cross-sectional heterogeneity in news arrivals. While the same pattern obtains for the closing auction in mid and small caps, we find an increase in the average cross-sectional volume correlation for large caps. This is consistent with the concentrated presence of institutional traders that trade simultaneously in several blue chips in order to rebalance their portfolios.

[35] Net Asset Values are computed at closing prices and changes in index weights become effective at the close. These are strong incentives for risk averse fund managers to trade at the close.

While we see a marginal increase on Xetra (7.4% with statistical significance at the 10% level), we find a strong increase of 26.8% on Euronext with a p-value of less than 0.01. Assuming independent distributions, we find that this difference of 19.4% is statistically significant at the 1% level, which is in line with the increased trading volume for the most active Euronext stocks during the closing auction. Overall, our results suggest that much of the trading at the close is driven by institutional investors and there is some evidence that the transparent Euronext system may help institutions to execute their trades more efficiently.

## 3.5. Price Discovery and efficiency

In this section, we turn to the analysis of price discovery and efficiency. We begin by computing several measures of price discovery for both markets and then investigate whether auction prices are subject to transitory effects.

### 3.5.A. Price discovery

We begin by examining the contribution of call auctions to price discovery. To this end, we compute three different measures of price discovery, the variance ratio ($VR$), the weighted price contribution ($WPC$) and the $R^2$ of unbiasedness regressions. van Bommel (2009) shows that all three measures are asymptotically equivalent if prices follow a driftless martingale. The $VR$ is defined as the ratio of the variance of auction returns to the variance of daily returns, i.e.

$$VR_i = \frac{\hat{s}_x^2}{\hat{s}_d^2} \tag{3.6}$$

where $x \in \{o(pen), c(lose)\}$ and $d$ denotes daily close-to-close returns.

The second measure of price discovery is the weighted price contribution (*WPC*), introduced by Barclay and Warner (1993), which is defined as

$$WPC_{i,x} = \sum_{t=1}^{T_i} \frac{\left|r_{i,t}^d\right|}{\sum_{t=1..T}\left|r_{i,t}^d\right|} \frac{r_{i,t}^x}{r_{i,t}^d} \tag{3.7}$$

where $r_{i,t}^d$ denotes the close to close return on day *t*, and $r_{i,t}^x$ denotes the return during auction *x* on day *t*. Intuitively, the *WPC* measures the proportion of a period return that is due to a sub-period, weighted by the absolute period return.

Finally, we consider the $R^2$ of the regression of the close-to-close returns on the auction returns. Biais et al. (1999) first used such *unbiasedness regressions* to assess price discovery during the pre-opening auction of the Paris Bourse.

[Insert Table C.6 about here]

Unsurprisingly, the results (Table C.6) show that the opening auctions play a vital part in the process of price discovery, as they contribute between 18% and 38% to total price discovery, depending on the measure and market considered. Given their trading volume, closing auctions hardly contribute anything to price discovery, averaging roughly 3% to 4%. Together with the findings from the previous section, these results highlight the different focus of the opening and closing auctions.

Taking a glance at the differences across markets, we find that opening auctions on Euronext constitute a significantly higher contribution to overall price discovery than the auctions conducted on Xetra. For example, we find an average $R^2$ of 27.0% for French stocks compared to 17.9% for German equities. The difference is statistically significant at the 1% level. The results for the other two measures are qualitatively very similar. In contrast, the overall picture suggests that there are no significant differences across markets regarding price discovery at the close. In summary, these results suggest that transparency appears not only to foster trading volume by facilitating the location of suitable counterparties, but additionally may improve price discovery. This finding is consistent with an informative limit order book, as e.g. in Cao et al. (2009) or Hendershott and Jones (2005).

## 3.5.B. Post-open quoted spreads

Among others, McInish and Wood (1992) report a pattern of declining quoted spreads throughout the first hours of continuous trading, which is consistent with a gradual reduction in uncertainty about the asset's true fundamental value as equilibrium prices are discovered. This suggests that we can judge the extent of price discovery occurring in the opening auction by looking at the quoted spread relative to its average value throughout the trading day, as a low (high) degree of price discovery should lead to wider (tighter) spreads right after the open. In order to investigate this issue, we divide the first hour of continuous trading into twelve 5-minute intervals. Then, for each interval $t = 1,...,12$, we compute the relative quoted spread, which is simply the time-weighted bid-ask spread for stock *i* on day *t* during interval $\tau$ normalized by the average bid-ask spread throughout the entire trading day

$$RQS_{t,t}^{i} = \frac{QS_{t,t}^{i}}{QS_{t}^{i}} \tag{3.8}$$

We assess the difference across exchanges for a particular auction by computing the differences for each trading day *t* and stock pair $(i, j)$ as

$$\Delta RQS_{t,t}^{i,j} = RQS_{t,t}^{i} - RQS_{t,t}^{j} \tag{3.9}$$

and base inference on a simple paired t-test.

[Insert Table C.7 and Figure C.1 around here]

The results can be found in Table C.7 and are also depicted in Figure C.1 for illustration. Overall, we find that stocks traded on Euronext display considerably lower relative quoted spreads at the start of the continuous trading session than the matched German stocks. For example, French large caps start the day with a quoted spread that is roughly 2.5 times the intraday average, while German large exhibit a bid-ask spread that is three times as high as its average value throughout the day. The results for mid and small caps are qualitatively very similar. Across the board, the average difference $\Delta RQS$ is highest for the first 5-minute interval, and then gradually converges towards zero. For the most and least active stocks, full convergence is reached within the first 15 minutes, while the convergence for mid caps is somewhat slower, taking roughly 30 minutes. Overall, this strongly confirms the picture that has emerged from the previous section.

## 3.5.C. Price efficiency

In market microstructure research, prices are called efficient if they follow a random walk, i.e. returns do not display any autocorrelation (see e.g. Hasbrouck (2007)). In order to examine the issue of price efficiency in our context, we run panel regressions of post-auction returns on the returns realized in the call auctions.

To assess the efficiency of the opening auction we regress the returns during the first fifteen minutes of continuous trading on the returns established at the open. In order to estimate differences in price efficiency across markets, we include a dummy variable that takes the value of 1 for stock listed on Xetra and zero otherwise, such that the linear model is given by

$$r_{i,t}^{po} = a + b r_{i,t}^{o} + d D^{X} + g D^{X} r_{i,t}^{o} + e_{i,t} \qquad (3.10)$$

where $r^{o}_{i,t}$ denotes the return during the opening auction (measured versus yesterday's close) for stock *i* on day *t*, $r^{po}_{i,t}$ denotes the corresponding return during the first fifteen minutes of the continuous trading phase (measured versus the clearing price of the opening auction), and $D^X$ is the aforementioned dummy variable. Similarly, we regress the return overnight return $r^{o}_{i,t}$ on the return from the previous day's closing auction $r^{c}_{i,t-1}$ (measured versus the midquote prior to the start of the closing auction's call phase)

$$r^{o}_{i,t} = a + b r^{c}_{i,t-1} + d D^X + g D^X r^{c}_{i,t-1} + e_{i,t} \qquad (3.11)$$

The estimated slope coefficients $b$ and $g$ are tabulated in Panel A of Table C.8. In order to account for cross-sectional and serial correlation, we employ standard errors clustered by stocks and trading days (Rogers (1993) and Petersen (2009)). The inclusion of time and stock-specific fixed effects does not yield qualitatively different estimates, so we simply report the results for the pooled OLS estimates.

[Insert Table C.8 about here]

For both the opening and the closing auctions, we find $b$ to be negative and statistically significant. While opening prices are relatively close to following a random walk ($b = -0.090$), the returns from the closing auctions experience a sharp overnight reversal ($b = -0.446$). This suggests that returns at the close contain a large transitory component, which is most likely due to an increased inventory risk for liquidity providers in the face of the illiquid overnight period (see e.g. Stoll (1978)). These results are in line with those from other studies such as Michayluk and Sanger (2006) and Kandel et al. (Forthcoming). Regarding differences in price efficiency across markets, we find that opening prices on Euronext are slightly more efficient than those set on Xetra ($g = -0.034$), while there is no significant difference at the close. These results are in line with our findings regarding price discovery (Sections 3.5.A and 3.5.B). In order to distinguish whether the observed return reversals are of rather idiosyncratic or systematic nature, we re-estimate the above regressions with returns for equal-weighted portfolios and abnormal returns, where the latter are calculated as the raw returns for stock *i* minus the equal weighted portfolio from the corresponding trading activity tercile. The estimates (Panels B and C) reveal that the return reversals constitute purely idiosyncratic effects. We find that the reversal coefficients for the equal weighted portfolios are either slightly positive (opening auction) or statistically insignificant (closing auction), while the coefficients for the idiosyncratic reversal regressions are very similar to the result obtained when using raw returns. Nevertheless, the difference across markets for the opening auction has become more pronounced ($g = -0.068$ and significant at the 1% level), suggesting that opening prices set on Euronext are significantly more efficient than those resulting from Xetra's opening auction.

One potential concern regarding the overnight reversal regression is the fact that both opening *and* closing prices are set via call auction. Thus, the observed difference across markets with respect to price discovery may affect our results for the closing auction's price efficiency. In order to circumvent this issue, we alternatively calculate the overnight return in equation (3.11) as the return from the previous day's close until the midquote prevailing at 10:00 am. The results, which we do not report for brevity, are qualitatively very similar to those obtained using the opening prices.

### 3.5.D. Sniping

As described in Section 3.2, the ending time of the call auctions on Xetra is randomized within a 30-second corridor. According to the exchange, the main motive for this randomization is the avoidance of potential price manipulation[36]. In contrast, the ending time on Euronext is deterministic. Roth and Ockenfels (2002) study online auctions for items with private and common values and find overwhelming evidence for sniping in the presence of a deterministic deadline. In our context, the deterministic ending on Euronext may induce a potential manipulator to submit a price-changing order in the very last second of the call auction, leaving no chance for other participants to react to his action. This kind of behaviour may particularly obtain for the closing auction, where there is a considerable incentive to manipulate prices.

[Insert Table C.9 about here]

In order to investigate such potential price manipulations, we calculate the proportion of closing auctions on Euronext where the indicative price has changed at least once in the last $s$ seconds, $s \in \{1,3,5\}$. The results are in Panel A of Table C.9. As can be seen, last-second changes in the indicative price are quite common: The indicative price is altered in about 42% of all auctions within the last 5 seconds, and around 19% of all times in the very last second. Moreover, last-second price changes are by far more common for more active stocks, where the indicative price changes in the last second in almost 65% of all closing auctions, compared to only 10.5% for low-volume stocks. This may simply be due to a much more competitive behaviour of traders for high-volume stocks. In order to investigate whether

[36] Xetra Market Model, Release 7.1

these price changes are the result of considerable price manipulation via sniping, we estimate

$$r_{i,t}^{o} = a + b r_{i,t-1}^{c} + d D^{s} + g D^{s} r_{i,t-1}^{c} + e_{i,t} \qquad (3.12)$$

where $D^{s}$ is a dummy variable taking the value 1 if the indicative price has changed within the last $s$ seconds. If manipulation takes place, the observed overnight reversal should be stronger, i.e. we expect $\gamma$ to be negative. Panel B of Table C.9 contains the results, where we only report the coefficient $\gamma$ to conserve space. We find that $\gamma$ is negative and statistically significant at the 5% level for the least active stocks with $s = 5$ and $s = 3$, while it is not significantly different from zero for $s = 1$. While it is intuitive that manipulation in less active stocks should be more feasible, the fact that the coefficient turns insignificant for the last second is counterintuitive. All other coefficients are not significant. Thus, there is at most weak evidence for sniping on Euronext in less active stocks, such that we conclude that sniping seems to be a relatively minor issue.

## 3.6. Conclusion

Motivated by the ongoing controversy about the optimal degree of pre-trade transparency, we examine the call auctions on two major European equity markets, Euronext (Paris) and Xetra (Frankfurt). While the former trading system displays the limit order book during its auctions, the latter only provides indicative information about the clearing price and the associated trading volume. In our analyses, we focus on two aspects, namely liquidity and price discovery.

Our findings suggest that transparency in the form of order book disclosure enhances liquidity, as it fosters trading volume without leading to higher absolute price changes. The analysis of trading volume correlations additionally points at the concentrated presence of institutional investors in the closing auction, particularly on Euronext. Hence, our results are consistent with transparency enabling agents to signal their willingness to trade as in Chakraborty et al. (2010). Additionally, an open order book also appears to improve price discovery at the market opening, which is consistent with an informative limit order book as in Cao et al. (2009) and Hendershott and Jones (2005). Our findings regarding price discovery are furthermore supported by higher post-open bid-ask spreads on Xetra, which gradually converge to the levels found on Euronext within the first 15 minutes of continuous trading. Additionally, we provide evidence that price changes occurring during the closing auctions carry significant transitory components that are largely idiosyncratic and are most likely to constitute a premium for providing liquidity in the face of overnight inventory risk. Finally, we provide some evidence that the deterministic ending time of Euronext's auction does not give rise to systematic price manipulation via sniping.

Additionally, our findings confirm that the opening and the closing auction have very different objectives. While the opening auction mainly serves as a tâtonnement process that facilitates the formation of equilibrium prices after the arrival of overnight information, the closing auction is rather concerned minimizing trading costs for institutional (liquidity) traders. It is very interesting that both markets under consideration (as well as many other exchanges) are using the same mechanisms to set the opening and closing prices. In fact, either market additionally uses the same mechanism as so-called "circuit-breaker" in the case of abnormally high volatility. While exchanges in practice will be very reluctant to make even minor changes to an established mechanism, it is an interesting question for further research to investigate whether this "one size fits all" paradigm is the optimal solution.

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# A. APPENDICES TO CHAPTER 1

## A.1. Proof of Proposition 1

Before we state the proof of Proposition 1, it is useful to introduce some additional notation.

As trader populations differ across markets, so does the function that maps market makers (henceforth MMs) prior into their posterior beliefs. Let $d_t\left(sell_t, d_{t-1}\right)$ denote the posterior belief about the probability of a high realization of the liquidation value after a sell by the *t*-th trader, given the prior $d_{t-1}$. Then, using Bayes' rule,

$$d_t(sell_t, d_{t-1}) = \frac{(1-m)d_{t-1}}{1+m(1-2d_{t-1})} \qquad \text{if sell in P and } b_t^P > b_t^C \qquad \text{(A.1)}$$

$$d_t(sell_t, d_{t-1}) = \frac{(1-m^{CT})d_{t-1}}{1+m^{CT}(1-2d_{t-1})} \qquad \text{if sell in P and } b_t^P \le b_t^C \qquad \text{(A.2)}$$

$$d_t(sell_t, d_{t-1}) = \frac{(1-m^{SR})d_{t-1}}{1+m^{SR}(1-2d_{t-1})} \qquad \text{if sell in C} \qquad \text{(A.3)}$$

From this follows that the expected liquidation value of the asset conditional on the arrival of a sell order is given by

$$B(m, d_{t-1}) \equiv E(V \mid \text{sell}, d_{t-1}) = \frac{(1-m)d_{t-1}\overline{V} + (1+m)(1-d_{t-1})\underline{V}}{(1+m(1-2d_{t-1}))} \quad \text{if sell in P and } b_t^P > b_t^C \qquad \text{(A.4)}$$

$$B(m^{CT}, d_{t-1}) \equiv E(V \mid \text{sell}, d_{t-1}) = \frac{(1-m^{CT})d_{t-1}\overline{V} + (1+m^{CT})(1-d_{t-1})\underline{V}}{(1+m^{CT}(1-2d_{t-1}))} \quad \text{if sell in P and } b_t^P \le b_t^C \qquad \text{(A.5)}$$

$$B(m^{SR},d_{t-1}) \equiv E(V \mid sell, d_{t-1})$$
$$= \frac{(1-m^{SR})d_{t-1}\overline{V} + (1+m^{SR})(1-d_{t-1})\underline{V}}{(1+m^{SR}(1-2d_{t-1}))} \quad \text{if sell in C}$$

(A.6)

Clearly, $m^{SR} > m > m^{CT}$ $(m^{SR} < m < m^{CT})$ implies $B(m^{CT},d_{t-1}) \geq B(m,d_{t-1}) \geq B(m^{SR},d_{t-1})$ $(B(m^{CT},d_{t-1}) \leq B(m,d_{t-1}) \leq B(m^{SR},d_{t-1}))$, where equality applies to the limiting cases of $d_{t-1} = 0$ and $d_{t-1} = 1$.

We are now ready to state the proof of Proposition 1. For notational simplicity, we omit the time subscripts on MMs beliefs.

**Proof of Proposition 1:**

**Part A:** $q^I > q^U$

**Case A1:** $B(m,d) - c > B(m^{SR},d)$

In this case, $b_t^{i,P}* = B(m,d) - c$ for $i = 1,\ldots,N^P$ and $b_t^{j,C}* < b_t^P*$ for $j = 1,\ldots,N^C$ constitutes a Nash equilibrium. All trade occurs in market P, and MMs obtain zero expected profits in both markets. Unilaterally decreasing the bid in market P does not improve on zero expected profits, as such a quote does not attract any market order, given the other MMs quote. On the other hand, $B(m,d) - c$ is the maximum bid that does not lead to expected losses, thereby ruling out any unilateral increases in the quote. Concerning market C, any bid $b_t^{j,C} \geq B(m,d) - c$ will yield an expected loss for market maker j as the expected liquidation value conditional on the sell of a smart router is $B(m^{SR},d) < B(m,d) - c$. Lowering the bid unilaterally does not improve on zero expected profits.

To see that we must have $b_t^P* > b_t^C*$ in equilibrium, first consider the case where $b_t^P* < b_t^C*$. Clearly, equilibrium requires $b_t^C* \leq B(m^{CT}, d)$. Given $b_t^P* < b_t^C*$, market makers in market P make expected profits. Thus for every $b_t^P* < b_t^C*$, there exists some $b_t^{i,P} = b_t^P* + e < b_t^C*$ that allows market maker *i* to overbid her rival in the same market and thereby increase her profits. Hence, this cannot be an equilibrium. Now consider the situation where $b_t^P* = b_t^C*$. Given that MMs in market P (C) face proportions $m^{CT}$ ($m^{SR}$) of informed traders, we must have $b_t^P* \leq B(m^{CT}, d) - c$ and $b_t^C* \leq B(m^{SR}, d)$ and therefore $b_t^P* \leq B(m^{SR}, d)$. By symmetry, we have $b_t^{i,P}* = b_t^P*$ and $b_t^{j,C}* = b_t^C*$ for $i = 1, \ldots, N^P$ and $j = 1, \ldots, N^C$. If $b_t^P* = b_t^C* < B(m^{SR}, d)$, then market maker *j* in market C can capture the market by posting $b_t^{j,C} = b_t^C* + e$, such that this is not an equilibrium. On the other hand, if $b_t^P* = b_t^C* = B(m^{SR}, d)$, market maker *i* in market P makes an expected profit equal to $\Pi = (1-q)(B(m^{CT}, d) - B(m^{SR}, d) - c) / N^P > 0$. Increasing her bid marginally to $b_t^{i,P} = B(m^{SR}, d) + e < B(m, d) - c$, her profit is equal to $B(m, d) - B(m^{SR}, d) - c - e$, which is always greater than $\Pi$ for $N^P \to \infty$.

**Case A2:** $B(m^{CT}, d) - c \geq B(m^{SR}, d) \geq B(m, d) - c$

For this case, $b_t^{j,C}* = b_t^{i,P}* = B(m^{SR}, d)$ for $i = 1, \ldots, N^P$ and $j = 1, \ldots, N^C$ is a Nash equilibrium. Given our assumption regarding inter-market ties, smart routers trade in market C and captive traders in market P. MMs in market C earn zero expected profits and MMs in market P make expected profits as $B(m^{CT}, d) - c - B(m^{SR}, d) \geq 0$. Unilaterally decreasing any bid leads to zero profits, as it attracts no market order, given the other quotes. On the other hand, a unilateral increase in the bid quote in market P (C) generates expected losses as such a quote then faces a proportion $m$ ($m^{SR}$) of informed traders.

To see that any equilibrium must satisfy $b_t^C* = b_t^P*$, first assume that $b_t^C* > b_t^P*$. Clearly, we must have that $b_t^C* = B(m^{SR}, d)$. Given the quotes in market C, market makers in market P face a proportion $m^{CT}$ of informed traders. As long as $b_t^C* > b_t^P*$, there always exists some $b_t^{i,P} = b_t^P* + e < b_t^C*$ that allows market maker *i* to overbid her rivals in the same market and thereby increase her profits. Hence, this cannot be an equilibrium. Now consider the converse situation, i.e. $b_t^P* > b_t^C*$. Given that market P displays the best quotes across markets, the adverse selection risk is defined by a proportion $m$ of informed traders, such that we must have $b_t^P* = B(m, d) - c$. But then, market maker *j* in market C faces a proportion $m^{SR}$ of informed traders, such that she can obtain an expected profit by increasing her bid to $b_t^{j,C} = B(m, d) - c$. Hence, this cannot an equilibrium either.

**Case A3:** $B(m^{SR}, d) > B(m^{CT}, d) - c$

Under this constellation, $b_t^{i,P}* = B(m^{CT}, d) - c$ and $b_t^{j,C}* = B(m^{SR}, d)$ for $i = 1, \ldots, N^P$ and $j = 1, \ldots, N^C$ constitutes a Nash equilibrium. Smart routers trade in market C, captive traders in market P, and all MMs obtain zero expected profits. Unilaterally lowering a bid quote in either does not attract any market orders, given the other quotes. Increasing the quote in market P leads to expected losses because such a quote faces proportions $m^{CT}$ (if $b_t^{i,P} \in (B(m^{CT}, d) - c, B(m^{SR}, d)]$) or $m$ (if $b_t^{i,P} > B(m^{SR}, d)$) of informed traders. Similarly, higher bid quotes in market C lead to expected losses as the expected liquidation value conditional on a sell order in market C is equal to $B(m^{SR},d)$.

To see that we must have $b_t^C* > b_t^P*$ in equilibrium, first consider the case where $b_t^C* < b_t^P*$. Under this constellation, markets makers in market P face a proportion $m$ of informed traders and equilibrium requires that $b_t^P* = B(m,d) - c$. As in the previous case, market maker *j* in market C can make an expected profit by posting $b_t^{j,C}* = B(m,d) - c$, such that this is not an equilibrium. Now suppose that $b_t^C* = b_t^P*$. Clearly, we must have that $b_t^P* \leq B(m^{CT},d) - c$, because any higher bid in market P will lead to losses given a proportion $m^{CT}$ of informed traders. But then, any market maker *j* in market C has an incentive to increase her bid marginally to some $b_t^{j,C} = b_t^C* + e$, thereby capturing the market and increasing her profits.

Combining cases A1-A3, we find that $b_t^C* \geq b_t^E*$ if and only if $B(m^{SR},d) \geq B(m,d) - c$. Using equations (A.4) and (A.6), the market co-existence condition (1.1) follows.

**Part B:** $q^U \geq q^I$

In this case, the inequality $B(m^{SR},d) > B(m^{CT},d) - c$ is always satisfied because $m^{SR} < m^{CT}$. It follows from case A3 that we must have $b_t^{j,C}* = B(m^{SR},d)$ for $j = 1,\ldots,N^C$ and $b_t^{i,P}* = B(m^{CT},d) - c$ for $i = 1,\ldots,N^P$ in equilibrium, such that markets co-exist. Condition (1.1) is satisfied because $m^{SR} - m < 0$.

Q.E.D.

## A.2. Tables and Figures

**Table A.1 Sample Stocks**

This table contains a list of the 67 French and German stocks contained in our sample, separated into terciles based on their average trading volume (in €).

| High Volume Stocks (N=22) | Medium Volume Stocks (N=23) | Low Volume Stocks (N=22) |
|---|---|---|
| Total | Carrefour | Postbank |
| Deutsche Bank | BMW | Linde |
| Allianz | Deutsche Post | Michelin |
| Siemens | Vivendi | Bouygues |
| Daimler | ThyssenKrupp | Pernod Ricard |
| E.ON | Credit Agricole | Alcatel Lucent |
| Societe Generale | EDF | PPR |
| BNP Paribas | Lafarge | Accor |
| Deutsche Telekom | Renault | Adidas |
| France Telecom | Schneider | Cap Gemini |
| RWE | Vallourec | Gaz de France |
| Volkswagen | L'Oreal | STMicroelectronics |
| AXA | Veolia | Merck |
| SAP | Lufthansa | Metro |
| Bayer | Danone | Unibail Rodamco |
| BASF | LVMH | Hypo Real Estate |
| Deutsche Börse | MAN | Air France - KLM |
| Suez | Alstom | Henkel |
| Munich Re | Saint Gobain | TUI |
| Sanofi Synthelabo | Vinci | Fresenius Medical Care |
| Continental | Peugeot | Essilor |
| Commerzbank | Air Liquide | Lagardere |
| | Infineon | |

**Table A.2 Sample Statistics**

This table contains summary statistics of the trading activity on Chi-X and the Primary Markets for our sample of 67 French and German stocks, aggregated into terciles based on trading activity. MS Chi-X denotes the market share of Chi-X for trades and trading volume as a percentage of the consolidated market (Chi-X plus primary market). Ratio C/P denotes the average trade size on Chi-X as percentage of the average trade size in the Primary Market.

| Panel A: Avg. daily # of trades (1,000 trades) | | | |
|---|---|---|---|
| | Chi-X | Primary | MS Chi-X (%) |
| High Volume | 1.06 | 5.85 | 14.98 |
| Medium Volume | 0.52 | 3.99 | 11.47 |
| Low Volume | 0.30 | 2.59 | 10.52 |
| All | 0.63 | 4.14 | 12.31 |
| **Panel B: Avg. daily trading volume (Mio. €)** | | | |
| | Chi-X | Primary | MS Chi-X (%) |
| High Volume | 18.33 | 244.31 | 6.99 |
| Medium Volume | 5.83 | 96.85 | 5.53 |
| Low Volume | 2.96 | 51.54 | 5.36 |
| All | 8.99 | 130.39 | 5.95 |
| **Panel C: Average trade size (€1,000)** | | | |
| | Chi-X | Primary | Ratio C/P (%) |
| High Volume | 17.38 | 42.26 | 41.82 |
| Medium Volume | 10.99 | 24.86 | 43.63 |
| Low Volume | 9.55 | 20.04 | 47.94 |
| All | 12.62 | 28.99 | 44.45 |

**Table A.3 Quote competitiveness and market depth**

This table contains statistics on the quote competitiveness and the available market depth for our sample of 67 French and German stocks, aggregated into terciles based on trading activity and reported separately for Chi-X and the Primary Markets. Panel A reports the average frequency with which a given market is present (alone) at the inside quote, while Panel B reports the average market depth (in €10,000) for each market conditional on being present (alone) at the inside quote.

| | At best bid | At best ask | At both | At best bid alone | At best ask alone | At both alone |
|---|---|---|---|---|---|---|
| Panel A: Presence (%) at the inside quote | | | | | | |
| Chi-X | | | | | | |
| High Volume | 52.65 | 53.56 | 27.45 | 27.75 | 28.18 | 7.11 |
| Medium Volume | 49.77 | 48.59 | 23.84 | 27.28 | 25.66 | 6.54 |
| Low Volume | 45.13 | 45.63 | 20.98 | 24.99 | 23.00 | 6.21 |
| All | 49.30 | 49.25 | 24.08 | 26.68 | 25.62 | 6.62 |
| Primary market | | | | | | |
| High Volume | 72.25 | 71.82 | 51.18 | 47.35 | 46.44 | 21.24 |
| Medium Volume | 72.72 | 74.34 | 53.59 | 50.23 | 51.41 | 25.48 |
| Low Volume | 75.01 | 77.00 | 58.22 | 54.87 | 54.37 | 30.22 |
| All | 73.32 | 74.38 | 54.32 | 50.81 | 50.75 | 25.64 |
| Panel B: Depth (in €10,000) conditional on presence at the inside quote | | | | | | |
| Chi-X | | | | | | |
| High Volume | 31.02 | 31.02 | 30.64 | 27.08 | 27.27 | 26.94 |
| Medium Volume | 17.69 | 17.53 | 17.45 | 16.43 | 15.51 | 15.94 |
| Low Volume | 14.49 | 15.29 | 14.77 | 14.06 | 13.29 | 13.86 |
| All | 21.02 | 21.22 | 20.90 | 19.15 | 18.64 | 18.88 |
| Primary market | | | | | | |
| High Volume | 91.74 | 102.41 | 103.04 | 74.62 | 87.42 | 95.70 |
| Medium Volume | 50.42 | 53.35 | 55.78 | 43.20 | 46.94 | 50.78 |
| Low Volume | 39.25 | 42.46 | 42.91 | 35.16 | 38.28 | 39.20 |
| All | 60.32 | 65.88 | 67.07 | 50.88 | 57.39 | 61.73 |

**Table A.4 The proportion of smart routers and the tie-breaking rule**

This table contains estimates for the proportion of smart routers ($\theta$), the proportion of executions in the Primary Market under inter-market ties ($\pi$), as well as for the tie-breaking rule parameter ($\tau$) for our sample of 67 French and German stocks, aggregated into terciles based on trading activity. All variables are defined in Section 1.3. Standard errors are robust to serial and cross-sectional correlation. The standard errors for the tie-breaking rule parameter are based on the delta method.

| | Proportion of smart routers | Proportion of Primary Market Trades under inter-market ties | Tie-breaking rule parameter |
|---|---|---|---|
| High Volume | 0.514 | 0.510 | 0.046 |
| | (0.018) | (0.022) | (0.035) |
| Medium Volume | 0.495 | 0.513 | 0.015 |
| | (0.018) | (0.018) | (0.033) |
| Low Volume | 0.494 | 0.525 | 0.039 |
| | (0.020) | (0.027) | (0.041) |
| All | 0.501 | 0.516 | 0.033 |
| | (0.013) | (0.015) | (0.025) |

**Table A.5 Effective spread decomposition (Chi-X vs. Primary Market Trades)**

This table contains the average effective spreads as well as the decomposition into the adverse selection (price impact) and realized spread components for trades on Chi-X and the Primary Markets, respectively, following equations (1.2) to (1.4) in Section 1.4.A. Averages are based on stock-days and aggregated into terciles based on trading activity. For differences, statistical significance at the 10%, 5%, and 1% level is denoted by *, **, and ***, respectively. Standard errors are robust to serial and cross-sectional correlation.

| | | | |
|---|---|---|---|
| | Panel A: Effective Spread | | |
| | Chi-X | Primary | Difference |
| High Volume | 1.999 | 1.996 | 0.003 |
| | (0.126) | (0.138) | (0.020) |
| Medium Volume | 2.672 | 2.705 | -0.033 |
| | (0.151) | (0.245) | (0.035) |
| Low Volume | 3.341 | 3.217 | 0.124** |
| | (0.217) | (0.219) | (0.052) |
| All | 2.671 | 2.640 | 0.031 |
| | (0.128) | (0.140) | (0.074) |
| | Panel B: Price Impact | | |
| | Chi-X | Primary | Difference |
| High Volume | 2.088 | 1.628 | 0.460*** |
| | (0.149) | (0.119) | (0.073) |
| Medium Volume | 2.693 | 2.306 | 0.387*** |
| | (0.190) | (0.161) | (0.091) |
| Low Volume | 3.270 | 2.877 | 0.393*** |
| | (0.210) | (0.163) | (0.122) |
| All | 2.684 | 2.271 | 0.413*** |
| | (0.139) | (0.122) | (0.076) |
| | Panel C: Realized Spread | | |
| | Chi-X | Primary | Difference |
| High Volume | -0.090 | 0.367 | - 0.457*** |
| | (0.102) | (0.098) | (0.074) |
| Medium Volume | -0.021 | 0.399 | - 0.419** |
| | (0.104) | (0.152) | (0.191) |
| Low Volume | 0.071 | 0.340 | -0.269 |
| | (0.114) | (0.158) | (0.168) |
| All | -0.013 | 0.369 | - 0.382*** |
| | (0.078) | (0.094) | (0.104) |

**Table A.6 Effective spread decomposition (Trade-Throughs vs. Non-Trade-Throughs)**

This table contains the average effective spreads as well as the decomposition into the adverse selection (price impact) and realized spread components for both trade-throughs and non-trade-throughs on the Primary Markets, following equations (1.2) to (1.4) in Section 1.4.A. Averages are based on stock-days and aggregated into terciles based on trading activity. For differences, statistical significance at the 10%, 5%, and 1% level is denoted by *, **, and ***, respectively. Standard errors are robust to serial and cross-sectional correlation.

| Panel A: Effective Spread | | | |
|---|---|---|---|
| | Non-trade-throughs | Trade-throughs | Difference |
| High Volume | 1.883 | 3.173 | - 1.290*** |
| | (0.135) | (0.196) | (0.140) |
| Medium Volume | 2.527 | 4.659 | - 2.132*** |
| | (0.191) | (0.256) | (0.152) |
| Low Volume | 3.052 | 5.762 | - 2.710*** |
| | (0.196) | (0.436) | (0.294) |
| All | 2.488 | 4.533 | - 2.046*** |
| | (0.122) | (0.232) | (0.145) |

| Panel B: Price Impact | | | |
|---|---|---|---|
| | Non-trade-throughs | Trade-throughs | Difference |
| High Volume | 1.713 | 0.804 | 0.909*** |
| | (0.120) | (0.154) | (0.116) |
| Medium Volume | 2.453 | 0.690 | 1.763*** |
| | (0.200) | (0.247) | (0.171) |
| Low Volume | 2.995 | 1.253 | 1.742*** |
| | (0.170) | (0.201) | (0.200) |
| All | 2.388 | 0.912 | 1.476*** |
| | (0.130) | (0.151) | (0.124) |

| Panel C: Realized Spread | | | |
|---|---|---|---|
| | Non-trade-throughs | Trade-throughs | Difference |
| High Volume | 0.170 | 2.369 | - 2.199*** |
| | (0.085) | (0.248) | (0.242) |
| Medium Volume | 0.074 | 3.969 | - 3.896*** |
| | (0.079) | (0.286) | (0.286) |
| Low Volume | 0.057 | 4.509 | - 4.452*** |
| | (0.115) | (0.484) | (0.465) |
| All | 0.100 | 3.621 | - 3.521*** |
| | (0.068) | (0.262) | (0.247) |

**Table A.7 Permanent Price Impacts (Chi-X vs. Primary Market Trades)**

This table contains the average permanent price impact measures obtained from the VAR model in Section 1.4.B, equations (1.5) – (1.7). Averages are based on stock-days and aggregated into terciles based on trading activity. For differences, statistical significance at the 10%, 5%, and 1% level is denoted by *, **, and ***, respectively. Standard errors are robust to serial and cross-sectional correlation.

| | Chi-X | Primary | Difference |
|---|---|---|---|
| High Volume | 1.345 | 1.128 | 0.218*** |
| | (0.078) | (0.068) | (0.039) |
| Medium Volume | 1.883 | 1.610 | 0.273*** |
| | (0.094) | (0.071) | (0.056) |
| Low Volume | 2.337 | 2.106 | 0.232*** |
| | (0.138) | (0.110) | (0.081) |
| All | 1.856 | 1.614 | 0.241*** |
| | (0.087) | (0.078) | (0.043) |

**Table A.8 Permanent Price Impacts (Trade-Throughs vs. Non-Trade-Throughs)**

This table contains the average permanent price impact measures obtained from the VAR model in Section 1.4.B, equations (1.9) – (1.12). Averages are based on stock-days and aggregated into terciles based on trading activity. For differences, statistical significance at the 10%, 5%, and 1% level is denoted by *, **, and ***, respectively. Standard errors are robust to serial and cross-sectional correlation.

| | Non-trade-throughs | Trade-throughs | Difference |
|---|---|---|---|
| High Volume | 1.208 | 0.274 | 0.934*** |
| | (0.071) | (0.064) | (0.071) |
| Medium Volume | 1.745 | 0.326 | 1.419*** |
| | (0.113) | (0.091) | (0.114) |
| Low Volume | 2.196 | 0.725 | 1.471*** |
| | (0.109) | (0.129) | (0.109) |
| All | 1.717 | 0.440 | 1.277*** |
| | (0.083) | (0.087) | (0.085) |

**Table A.9 Correlation matrix of explanatory variables**

This table contains the correlation matrix for the explanatory variables capturing the excess adverse selection risk on Chi-X. All variables are described in Section 1.5. Statistical significance at the 10%, 5%, and 1% level is denoted by *, **, and ***, respectively.

| | ΔAS(SD) | ΔAS(HB) | σ |
|---|---|---|---|
| ΔAS(SD) | 1.000 | 0.635*** | 0.424*** |
| ΔAS(HB) | | 1.000 | 0.453*** |
| σ | | | 1.000 |

**Table A.10 Cross-sectional regressions**

This table contains estimates for the linear cross-sectional regression following equation (1.13). All variables are described in Section 1.5. Robust standard errors are given in parentheses. Statistical significance at the 10%, 5%, and 1% level is denoted by *, **, and ***, respectively.

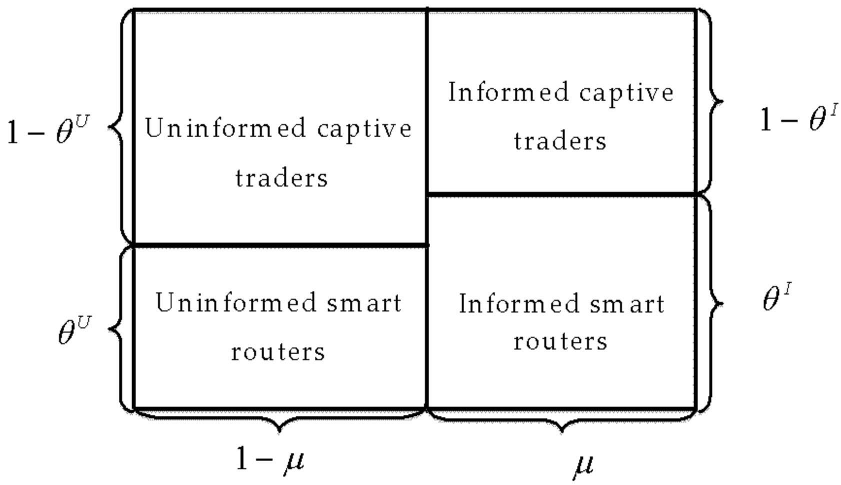
| Dependent Variable: Avg. Presence of Chi-X at the best quote | | | |
|---|---|---|---|
| | (1) | (2) | (3) |
| ΔAS(SD) | - 11.748***<br>(2.692) | | |
| ΔAS(HB) | | - 29.460***<br>(3.606) | |
| σ | | | - 0.564***<br>(0.169) |
| % Smart Routers | 0.092<br>(0.175) | -0.095<br>(0.152) | 0.040<br>(0.185) |
| ln(Volume) | 7.176***<br>(1.304) | 8.253***<br>(1.162) | 5.267***<br>(1.339) |
| Synch | 3.406**<br>(1.409) | 2.288**<br>(1.007) | 4.847***<br>(1.616) |
| Δtick (bps) | 4.044***<br>(0.790) | 4.705***<br>(0.460) | 3.449***<br>(0.750) |
| Euronext dummy | -2.280<br>(1.915) | 2.863<br>(1.726) | -2.624<br>(1.967) |
| Constant | - 86.932***<br>(27.592) | - 99.333***<br>(20.508) | -34.504<br>(30.037) |
| N | 67 | 67 | 67 |
| Adj. $R^2$ | 0.606 | 0.733 | 0.580 |

**Table A.11 Cross-sectional regressions**

This table contains estimates for the linear cross-sectional regression following equation (1.13), where we replace the independent variable by Chi-X's market share in terms of trades. All variables are described in Section 1.5. Robust standard errors are given in parentheses. Statistical significance at the 10%, 5%, and 1% level is denoted by *, **, and ***, respectively.

| Dependent Variable: Avg. Market Share Chi-X (# of Trades) | | | |
|---|---|---|---|
| | (1) | (2) | (4) |
| ΔAS(SD) | - 2.989***<br>(1.091) | | |
| ΔAS(HB) | | - 6.997***<br>(1.862) | |
| σ | | | - 0.137**<br>(0.059) |
| % Smart Routers | 0.318***<br>(0.059) | 0.273***<br>(0.056) | 0.305***<br>(0.061) |
| ln(Volume) | 2.491***<br>(0.419) | 2.735***<br>(0.403) | 2.020***<br>(0.419) |
| Synch | 1.390**<br>(0.549) | 1.106**<br>(0.500) | 1.726***<br>(0.575) |
| Δtick (bps) | 1.094***<br>(0.271) | 1.235***<br>(0.217) | 0.939***<br>(0.244) |
| Euronext dummy | - 3.472***<br>(0.748) | - 2.232***<br>(0.827) | - 3.542***<br>(0.757) |
| Constant | - 47.826***<br>(8.668) | - 50.598***<br>(7.702) | - 34.990***<br>(9.344) |
| N | 67 | 67 | 67 |
| Adj. $R^2$ | 0.588 | 0.643 | 0.572 |

**Figure A.1 Graphical representation of the trader population for** $q^I > q^U$



## A.3. A two-market PIN model using regression

As maximum likelihood estimation of the PIN model has become increasingly difficult, Easley et al. (2010) point out that that PIN can be approximated by

$$PIN^{Approx} = \frac{E\left(|I|\right)}{E\left(B+S\right)} \quad \text{(A.7)}$$

where $B$ and $S$ denote the number of buys and sells, and $I = B - S$ is the order imbalance. The intuition behind this approximation is straightforward. Total order flow is the sum of informed and uninformed trades, while the order imbalance is entirely attributable to informed trades. As a result, PIN can be approximated by the average imbalance divided by the average number of trades.

In our case, unfortunately, it is not sufficient to simply calculate this approximation for each market in isolation, as this would not impose equal event probabilities across markets. A two-market PIN model implies that the order imbalance in each market has the same sign (in expected terms), and the relative order imbalance is higher in the market with a larger proportion of informed traders. Let $I_t^k$ denote the order imbalances in market $k \in \{C, P\}$ on day $t$, such that the imbalance in the consolidated market is given by $I_t^T = I_t^C + I_t^P$. Moreover define venue $k$'s market share as $S_t^k = (B_t^k + S_t^k)/(B_t^T + S_t^T)$, where the superscript $T$ refers to the consolidated (total) market. Then, the product $I_t^T S_t^k$ is the expected order imbalance in market $k$ in the case where the probability of informed trading in this market is equal to the probability of informed trading in the consolidated market.

Then, we can test for differences in informed trading across venues via the simple linear regression

$$I_t^k = l^k (I_t^T S_t^k) + e_t \qquad \text{(A.8)}$$

If $l^k > 1 (l^k < 1)$, the probability of informed trading in market *k* is higher (lower) than in the consolidated market, as observed imbalances are of greater (lower) magnitude than expected from its actual market share.

Table A.12 details the estimation results for both Chi-X (Panel A) and the Primary Markets (Panel B). The first two columns report the mean and median coefficients from stock-specific OLS regressions for the entire sample and the different activity terciles, while the latter two columns contain the p-values from t-tests and Wilcoxon rank sum tests. Overall, the evidence indicates excess informed trading on Chi-X and a shortfall of informed traders on the Primary Markets. Except for the tercile with the most active stocks, the mean (median) regression coefficient for the MTF exceeds one, while the one for the Primary Markets is less than one.

We then use the product $(\hat{l}_i^C - 1)s_i$ as another measure of $\Delta AS_i$ in regression (1.13), where $\hat{l}_i^C$ is the regression coefficient from equation (A.8) using the Chi-X imbalance as independent variable, and $s_i$ denotes stock *i*'s annualized return volatility. The results can be found in Table A.13, where we also show the coefficients for the alternative specification using Chi-X's market share in terms of trades. The coefficients are negative and statistically significant, which is in line with the results obtained using the other measures of the adverse selection risk differential. Nevertheless, the effect's magnitude in terms of economic significance is considerably smaller than for the other variables employed in Section 1.5. For example, a one standard deviation increase in $(\hat{l}_i - 1)s_i$ (~15.44) is associated with a decrease of 1.96% in Chi-X's presence at the best quote, which is less than half of the effect for $\Delta AS_i^{SD}$.

**Table A.12 Imbalance Regressions**

This table contains summary statistics on the order imbalance regression in equation (A.8) for estimating differences in the probability of informed trading between Chi-X, the Primary Markets, and the consolidated market. Results are based on our sample of 67 French and German stocks, aggregated into terciles based on trading activity. The first two columns present the mean and median regression coefficients minus one, while the last two columns contain p-values from t-tests and non-parametric Wilcoxon signed-rank tests.

| Panel A: Imbalance Regressions for Chi-X | | | | |
|---|---|---|---|---|
| | λ-1 | | p-value (Ho: λ=1) | |
| | Mean | Median | T-test | Wilcoxon |
| High Volume | 0.095 | -0.003 | 0.419 | 0.518 |
| Medium Volume | 0.322 | 0.301 | <0.001 | <0.001 |
| Low Volume | 0.275 | 0.279 | 0.034 | 0.012 |
| All | 0.232 | 0.219 | <0.001 | <0.001 |
| **Panel B: Imbalance Regressions for Primary Markets** | | | | |
| | λ-1 | | p-value (Ho: λ=1) | |
| | Mean | Median | T-test | Wilcoxon |
| High Volume | 0.001 | 0.003 | 0.952 | 0.863 |
| Medium Volume | -0.034 | -0.032 | 0.002 | 0.002 |
| Low Volume | -0.031 | -0.030 | 0.026 | 0.018 |
| All | -0.021 | -0.027 | 0.012 | 0.008 |

**Table A.13 Cross-sectional regressions**

This table contains estimates for the linear cross-sectional regression following equation (1.13), where column headings denote the dependent variable. All variables are described in Section 1.5. Robust standard errors are given in parentheses. Statistical significance at the 10%, 5%, and 1% level is denoted by *, **, and ***, respectively.

| | % Chi-X at inside quote | Chi-X Market Share |
|---|---|---|
| (λ-1)σ | - 0.127* | - 0.061** |
| | (0.072) | (0.029) |
| % Smart Routers | 0.095 | 0.330*** |
| | (0.196) | (0.055) |
| ln(Volume) | 5.780*** | 1.971*** |
| | (1.804) | (0.588) |
| Synch | 2.009 | 0.965* |
| | (1.569) | (0.530) |
| Δtick (bps) | 3.066*** | 0.838*** |
| | (0.706) | (0.210) |
| Euronext dummy | -1.984 | - 3.589*** |
| | (2.538) | (0.833) |
| Constant | - 64.334* | - 39.253*** |
| | (37.762) | (12.077) |
| N | 67 | 67 |
| Adj. $R^2$ | 0.505 | 0.575 |

# B. APPENDICES TO CHAPTER 2

## B.1. Proofs

### B.1.A. Proof of Proposition 1

For each type of equilibrium, the proof proceeds in three steps:

***Step 1:*** Conjecture an ordering of cutoff prices.

***Step 2:*** Conjecture equilibrium strategies and solve for the equilibrium cutoff prices.

***Step 3:*** Verify that a) the assumed strategies are best replies (i.e. deviations are not profitable) and b) the cutoff prices satisfy the assumed ordering.

Define the following four orderings of sell cutoff prices.

Ordering 1:

$$C_H^{S*}(v_t - s, -L) \le C_A^{S*}(v_t - s, -L) \le C_H^{S*}(v_t - s, +L) \le C_A^{S*}(v_t - s, +L)$$
$$\le C_H^{S*}(v_t + s, -L) \le C_A^{S*}(v_t + s, -L) \le C_H^{S*}(v_t + s, +L) \le C_A^{S*}(v_t + s, +L)$$

Ordering 2:

$$C_H^{S*}(v_t - s, -L) \le C_A^{S*}(v_t - s, -L) \le C_H^{S*}(v_t - s, +L) \le C_H^{S*}(v_t + s, -L)$$
$$\le C_A^{S*}(v_t - s, +L) \le C_A^{S*}(v_t + s, -L) \le C_H^{S*}(v_t + s, +L) \le C_A^{S*}(v_t + s, +L)$$

Ordering 3:

$$C_H^{S*}(v_t - s, -L) \le C_A^{S*}(v_t - s, -L) \le C_H^{S*}(v_t + s, -L) \le C_H^{S*}(v_t - s, +L)$$
$$\le C_A^{S*}(v_t + s, -L) \le C_A^{S*}(v_t - s, +L) \le C_H^{S*}(v_t + s, +L) \le C_A^{S*}(v_t + s, +L)$$

Ordering 4:

$$C_H^{S*}(v_t - s, -L) \le C_A^{S*}(v_t - s, -L) \le C_H^{S*}(v_t + s, -L) \le C_A^{S*}(v_t + s, -L)$$
$$\le C_H^{S*}(v_t - s, +L) \le C_A^{S*}(v_t - s, +L) \le C_H^{S*}(v_t + s, +L) \le C_A^{S*}(v_t + s, +L)$$

For each ordering of sell cutoff prices, there is a corresponding ordering (due to symmetry) of buy cutoff prices. For example, for Ordering 1, we have

$$C_H^{B*}(v_t + s, +L) \ge C_A^{B*}(v_t + s, +L) \ge C_H^{B*}(v_t + s, -L) \ge C_A^{B*}(v_t + s, -L)$$
$$\ge C_H^{B*}(v_t - s, +L) \ge C_A^{B*}(v_t - s, +L) \ge C_H^{B*}(v_t - s, -L) \ge C_A^{B*}(v_t - s, -L)$$

**Table B.1 Cutoff price orderings and associated execution probabilities**

The following four tables contain the conditional and unconditional execution probabilities of buy limit orders according to the position of the limit price relative to the cutoff prices, separately for each employed ordering.

**Panel 1: Ordering 1**

| Bid Price | Execution Probability | Execution Probability conditional on $e_{t+1} = -s$ | Execution Probability conditional on $e_{t+1} = +s$ |
|---|---|---|---|
| $< C^{S*}_{HT}(v_t - s, -L)$ | 0 | 0 | 0 |
| $\in (C^{S*}_{HT}(v_t - s, -L), C^{S*}_{AT}(v_t - s, -L)]$ | $(1-a)/4$ | $(1-a)/2$ | 0 |
| $\in (C^{S*}_{AT}(v_t - s, -L), C^{S*}_{HT}(v_t - s, +L)]$ | ¼ | $1/2$ | 0 |
| $\in (C^{S*}_{HT}(v_t - s, +L), C^{S*}_{AT}(v_t - s, +L)]$ | $(2-a)/4$ | $(2-a)/2$ | 0 |
| $\in (C^{S*}_{AT}(v_t - s, +L), C^{S*}_{HT}(v_t + s, -L)]$ | ½ | 1 | 0 |
| $\in (C^{S*}_{HT}(v_t + s, -L), C^{S*}_{AT}(v_t + s, -L)]$ | $(3-a)/4$ | 1 | $(1-a)/2$ |
| $\in (C^{S*}_{AT}(v_t + s, -L), C^{S*}_{HT}(v_t + s, +L)]$ | ¾ | 1 | $1/2$ |
| $\in (C^{S*}_{HT}(v_t + s, +L), C^{S*}_{AR}(v_t + s, +L)]$ | $(4-a)/4$ | 1 | $(2-a)/2$ |
| $> C^{S*}_{AT}(v_t + s, +L)$ | 1 | 1 | 1 |

**Panel 2: Ordering 2**

| Bid Price | Execution Probability | Execution Probability conditional on $e_{t+1} = -s$ | Execution Probability conditional on $e_{t+1} = +s$ |
|---|---|---|---|
| $< C^{S*}_{HT}(v_t - s, -L)$ | 0 | 0 | 0 |
| $\in (C^{S*}_{HT}(v_t - s, -L), C^{S*}_{AT}(v_t - s, -L)]$ | $(1-a)/4$ | $(1-a)/2$ | 0 |
| $\in (C^{S*}_{AT}(v_t - s, -L), C^{S*}_{HT}(v_t - s, +L)]$ | $1/4$ | $1/2$ | 0 |
| $\in (C^{S*}_{HT}(v_t - s, +L), C^{S*}_{HT}(v_t + s, -L)]$ | $(2-a)/4$ | $(2-a)/2$ | 0 |
| $\in (C^{S*}_{HT}(v_t + s, -L), C^{S*}_{AT}(v_t - s, +L)]$ | $(3-2a)/4$ | $(2-a)/2$ | $(1-a)/2$ |
| $\in (C^{S*}_{AT}(v_t - s, +L), C^{S*}_{AT}(v_t + s, -L)]$ | $(3-a)/4$ | 1 | $(1-a)/2$ |
| $\in (C^{S*}_{AT}(v_t + s, -L), C^{S*}_{HT}(v_t + s, +L)]$ | $3/4$ | 1 | $1/2$ |
| $\in (C^{S*}_{HT}(v_t + s, +L), C^{S*}_{AT}(v_t + s, +L)]$ | $(4-a)/4$ | 1 | $(2-a)/2$ |
| $> C^{S*}_{AT}(v_t + s, +L)$ | 1 | 1 | 1 |

**Panel 3: Ordering 3**

| Bid Price | Execution Probability | Execution Probability conditional on $e_{t+1} = -s$ | Execution Probability conditional on $e_{t+1} = +s$ |
|---|---|---|---|
| $< C_{HT}^{S*}(v_t - s, -L)$ | 0 | 0 | 0 |
| $\in (C_{HT}^{S*}(v_t - s, -L), C_{AT}^{S*}(v_t - s, -L)]$ | $(1-a)/4$ | $(1-a)/2$ | 0 |
| $\in (C_{AT}^{S*}(v_t - s, -L), C_{HT}^{S*}(v_t + s, -L)]$ | $1/4$ | $1/2$ | 0 |
| $\in (C_{HT}^{S*}(v_t + s, -L), C_{HT}^{S*}(v_t - s, +L)]$ | $(2-a)/4$ | $1/2$ | $(1-a)/2$ |
| $\in (C_{HT}^{S*}(v_t - s, +L), C_{AT}^{S*}(v_t + s, -L)]$ | $(3-2a)/4$ | $(2-a)/2$ | $(1-a)/2$ |
| $\in (C_{AT}^{S*}(v_t + s, -L), C_{AT}^{S*}(v_t - s, +L)]$ | $(3-a)/4$ | $(2-a)/2$ | $1/2$ |
| $\in (C_{AT}^{S*}(v_t - s, +L), C_{HT}^{S*}(v_t + s, +L)]$ | $3/4$ | 1 | $1/2$ |
| $\in (C_{HT}^{S*}(v_t + s, +L), C_{AT}^{S*}(v_t + s, +L)]$ | $(4-a)/4$ | 1 | $(2-a)/2$ |
| $> C_{AT}^{S*}(v_t + s, +L)$ | 1 | 1 | 1 |

**Panel 4: Ordering 4**

| Bid Price | Execution Probability | Execution Probability conditional on $e_{t+1} = -s$ | Execution Probability conditional on $e_{t+1} = +s$ |
|---|---|---|---|
| $< C_{HT}^{S*}(v_t - s, -L)$ | 0 | 0 | 0 |
| $\in (C_{HT}^{S*}(v_t - s, -L), C_{AT}^{S*}(v_t - s, -L)]$ | $(1-a)/4$ | $(1-a)/2$ | 0 |
| $\in (C_{AT}^{S*}(v_t - s, -L), C_{HT}^{S*}(v_t + s, -L)]$ | $1/4$ | $1/2$ | 0 |
| $\in (C_{HT}^{S*}(v_t + s, -L), C_{AT}^{S*}(v_t + s, -L)]$ | $(2-a)/4$ | $1/2$ | $(1-a)/2$ |
| $\in (C_{AT}^{S*}(v_t + s, -L), C_{HT}^{S*}(v_t - s, +L)]$ | $1/2$ | $1/2$ | $1/2$ |
| $\in (C_{HT}^{S*}(v_t - s, +L), C_{AT}^{S*}(v_t - s, +L)]$ | $(3-a)/2$ | $(2-a)/2$ | $1/2$ |
| $\in (C_{AT}^{S*}(v_t - s, +L), C_{HT}^{S*}(v_t + s, +L)]$ | $3/4$ | 1 | $1/2$ |
| $\in (C_{HT}^{S*}(v_t + s, +L), C_{AT}^{S*}(v_t + s, +L)]$ | $(4-a)/4$ | 1 | $(2-a)/2$ |
| $> C_{AT}^{S*}(v_t + s, +L)$ | 1 | 1 | 1 |

**Type 1 equilibrium:**

Let $s_1^* = 4L/(5-a)$ and $a_1^* = \sqrt{5} - 2$.

**CASE A:**

***Step 1:*** Assume Ordering 1.

***Step 2:*** Conjecture the following equilibrium strategies: HT buyers submit a buy limit order with a bid price slightly above $C_{HT}^{S*}(v_t - s, -L)$, which has a probability of execution of $(1-a)/4$ (see Table B.1, Panel 1). AT buyers submit a buy limit order with a bid price slightly above $C_{AT}^{S*}(v_t - s, -L)$. If the next trader is not an AT, they cancel this order after observing the innovation in the fundamental value and set a new bid price slightly above $C_{HT}^{S*}(v_t - s, -L)$ $(C_{HT}^{S*}(v_t + s, -L))$ if $e_{t+1} = -s$ $(e_{t+1} = +s)$. The probability of execution for this strategy is $(1-a)/4 + (1-a)/4 + a/4 = (2-a)/4$ (see Table B.1, Panel 1). Moreover, conjecture the analogous strategies for HT and AT sellers, e.g. a HT seller submits a sell limit order with ask price slightly below $C_{HT}^{B*}(v_t + s, +L)$ with probability of execution equal to $(1-a)/4$. Thus, cutoff prices have to satisfy the following system of equations.

$$v_t + L - C_{HT}^{B*}(v_t, +L) = \frac{1-a}{4}(v_t - s + L - C_{HT}^{S*}(v_t - s, -L))$$

$$C_{HT}^{S*}(v_t, -L) - (v_t - L) = \frac{1-a}{4}(C_{HT}^{B*}(v_t + s, +L) - (v_t + s - L))$$

$$\begin{aligned} v_t + L - C_{AT}^{B*}(v_t, +L) = {} & \frac{1-a}{4}(v_t - s + L - C_{HT}^{S*}(v_t - s, -L)) \\ & + \frac{1-a}{4}(v_t + s + L - C_{HT}^{S*}(v_t + s, -L)) \\ & + \frac{a}{4}(v_t - s + L - C_{AT}^{S*}(v_t - s, -L)) \end{aligned}$$

$$\begin{aligned} C_{AT}^{S*}(v_t, -L) - (v_t - L) = {} & \frac{1-a}{4}(C_{HT}^{B*}(v_t + s, +L) - (v_t + s - L)) \\ & + \frac{1-a}{4}(C_{HT}^{B*}(v_t - s, +L) - (v_t - s - L)) \\ & + \frac{a}{4}(C_{AT}^{B*}(v_t + s, +L) - (v_t + s - L)) \end{aligned}$$

Tedious, but straightforward algebra yields the following cutoff prices:

$$C_{HT}^{S*}(v_t, -L) = v_t - L + (2L)\frac{1-a}{5-a}$$

$$C^{S*}_{AT}(v_t,-L)=v_t-L+(2L)\frac{8-a(3+a)}{(5-a)(4+a)}$$

$$C^{B*}_{HT}(v_t,+L)=v_t+L-(2L)\frac{1-a}{5-a}$$

$$C^{B*}_{AT}(v_t,+L)=v_t+L-(2L)\frac{8-a(3+a)}{(5-a)(4+a)}$$

***Step 3:*** Due to symmetry, it suffices to analyze the strategies of buyers. First, notice that the optimal bid price must be chosen such that it is slightly higher than the lower bound of any of the proposed intervals, because a higher bid can be decreased without reducing the execution probability. Moreover, it is easy to see that Ordering 1 implies that it is not optimal to post a bid price $B$ that lies in the interval$\in(C^{S*}_{HT}(v_t-s,+L),C^{S*}_{HT}(v_t+s,-L)]$. Such a limit order is only executed in the case of a price decrease, and therefore $B>C^{S*}_{HT}(v_t-s,+L)\geq v_t-s+L$. But then, the execution of this limit order cannot be profitable, because the bid price is above the reservation price of the trader posting the limit order. Moreover, it is clear that a bid price $B>C^{S*}_{HT}(v_t+s,+L)\geq v+s+L$ cannot be optimal, because it is higher the maximum valuation of any trader[37]. Thus, the proposed strategy for HT buyers is a best reply if and only if [38]:

$$\frac{1-a}{4}(v_t-s+L-C^{S*}_{HT}(v_t-s,-L))\geq\frac{1}{4}(v_t-s+L-C^{S*}_{AT}(v_t-s,-L))$$

$$\frac{1-a}{4}(v_t-s+L-C^{S*}_{HT}(v_t-s,-L))\geq\frac{1}{2}(v_t-s+L-C^{S*}_{HT}(v_t+s,-L))+\frac{1-a}{4}(v_t+s+L-C^{S*}_{HT}(v_t+s,-L))$$

$$\frac{1-a}{4}(v_t-s+L-C^{S*}_{HT}(v_t-s,-L))\geq\frac{1}{2}(v_t-s+L-C^{S*}_{AT}(v_t+s,-L))+\frac{1}{4}(v_t+s+L-C^{S*}_{AT}(v_t+s,-L))$$

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[37] Using the same kind of reasoning, we can reduce the possible equilibrium bid prices for other orderings of cutoff prices as well.

[38] In the following, we abuse notation by proceeding as if the bid prices were equal to the cutoff prices, as they can be chosen arbitrary close.

Similarly, AT buyers have no incentives to deviate if and only if:

$$\frac{a}{4}(v_t - s + L - C_{AT}^{S*}(v_t - s, -L)) \geq \frac{a}{2}(v_t - s + L - C_{AT}^{S*}(v_t + s, -L)) + \frac{a}{4}(v_t + s + L - C_{AT}^{S*}(v_t + s, -L))$$

$$\frac{a}{4}(v_t - s + L - C_{AT}^{S*}(v_t - s, -L)) \geq 0$$

Brute-force algebra reveals that these inequalities and the assumed ordering of cutoff prices are satisfied if and only $a \leq a_1^*$ and $s \geq \frac{24 + a(1-a)}{(5-a)(4+a)} L$ .

**CASE B:**

***Step 1:*** Assume Ordering 2.

***Step 2:*** Conjecture the same equilibrium strategies as in Case A, which implies identical cutoff prices.

***Step 3:*** The proposed strategies are best replies (for buyers) if and only if

$$\frac{1-a}{4}(v_t - s + L - C_{HT}^{S*}(v_t - s, -L)) \geq \frac{1}{4}(v_t - s + L - C_{AT}^{S*}(v_t - s, -L))$$

$$\frac{1-a}{4}(v_t - s + L - C_{HT}^{S*}(v_t - s, -L)) \geq \frac{2-a}{4}(v_t - s + L - C_{HT}^{S*}(v_t + s, -L)) + \frac{1-a}{4}(v_t + s + L - C_{HT}^{S*}(v_t + s, -L))$$

$$\frac{1-a}{4}(v_t - s + L - C_{HT}^{S*}(v_t - s, -L)) \geq \frac{1}{2}(v_t - s + L - C_{AT}^{S*}(v_t + s, -L)) + \frac{1}{4}(v_t + s + L - C_{AT}^{S*}(v_t + s, -L))$$

$$\frac{a}{4}(v_t - s + L - C_{AT}^{S*}(v_t - s, -L)) \geq \frac{a}{2}(v_t - s + L - C_{AT}^{S*}(v_t + s, -L)) + \frac{a}{4}(v_t + s + L - C_{AT}^{S*}(v_t + s, -L))$$

$$\frac{a}{4}(v_t - s + L - C_{AT}^{S*}(v_t - s, -L)) \geq 0$$

These inequalities together with the conjectured ordering of cutoff prices are satisfied if and only if $a \leq a_1^*$ and $\frac{24 + a(1-a)}{(5-a)(4+a)} L > s \geq L$ .

**CASE C:**

***Step 1:*** Assume Ordering 3.

***Step 2:*** Conjecture the same equilibrium strategies as in Case A, which implies identical cutoff prices.

***Step 3:*** The proposed strategies are best replies (for buyers) if and only if

$$\frac{1-a}{4}(v_t - s + L - C^{S*}_{HT}(v_t - s, -L)) \geq \frac{1}{4}(v_t - s + L - C^{S*}_{AT}(v_t - s, -L))$$

$$\frac{1-a}{4}(v_t - s + L - C^{S*}_{HT}(v_t - s, -L)) \geq \frac{1}{4}(v_t - s + L - C^{S*}_{HT}(v_t + s, -L)) + \frac{1-a}{4}(v_t + s + L - C^{S*}_{HT}(v_t + s, -L))$$

$$\frac{1-a}{4}(v_t - s + L - C^{S*}_{HT}(v_t - s, -L)) \geq \frac{2-a}{4}(v_t - s + L - C^{S*}_{AT}(v_t + s, -L)) + \frac{1}{4}(v_t + s + L - C^{S*}_{AT}(v_t + s, -L))$$

$$\frac{a}{4}(v_t - s + L - C^{S*}_{AT}(v_t - s, -L)) \geq \frac{a}{2}(v_t + L - C^{S*}_{AT}(v_t + s, -L))$$

$$\frac{a}{4}(v_t - s + L - C^{S*}_{AT}(v_t - s, -L)) \geq 0$$

These inequalities together with the conjectured ordering of cutoff prices are satisfied if and only if $a \leq a_1^*$ and $L > s \geq s_1^*$.

Combining Cases A, B and C, we conclude that the following quotation strategy and the associated order choice strategy constitute an equilibrium if and only if $a \leq a_1^*$ and $s \geq s_1^*$.

$$B^*_{t,HT}(v_t, +L) = C^{S*}_{HT}(v_t - s, -L) \qquad B^{-s*}_{t,AT}(v_t, +L) = C^{S*}_{HT}(v_t - s, -L)$$

$$B^{+s*}_{t,AT}(v_t, +L) = C^{S*}_{HT}(v_t + s, -L) \qquad B^*_{t,AT}(v_t, +L) = C^{S*}_{AT}(v_t - s, -L)$$

$$A^*_{t,HT}(v_t, -L) = C^{B*}_{HT}(v_t + s, -L) \qquad A^{-s*}_{t,AT}(v_t, -L) = C^{B*}_{HT}(v_t - s, -L)$$

$$A^{+s*}_{t,AT}(v_t, -L) = C^{B*}_{HT}(v_t + s, -L) \qquad A^*_{t,AT}(v_t, -L) = C^{B*}_{AT}(v_t + s, -L)$$

These quotes can be written as in Proposition 1 after substituting the cutoff prices obtained in Step 2 of Case 1. The execution probabilities follow directly from Table B.1, where the execution probabilities conditional on the next trader's type can be obtained easily by setting $a=0$ ($a=1$) for $q_{t+1}=0$ ($q_{t+1}=1$).

The proof for the remaining types of equilibria follows exactly the same logic. In order to conserve space, we will simply indicate the orderings that give rise to these equilibria and provide the equilibrium bid quotes in terms of the equilibrium sell cutoff prices (the ask quotes follow by symmetry). Define the following variables.

$$a_2^* = \frac{\sqrt{33}-5}{2} \qquad s_2^* = L\frac{2a(1+a)}{3-4a} \qquad s_3^* = L\frac{4(4+a)}{26-a^2}$$

$$s_4^* = L\frac{2(1-a)(4+a)}{7+3a} \qquad s_5^* = L\frac{4(1+a)}{7+3a}$$

**Type 2 equilibrium:**

This equilibrium arises under Orderings 1 – 4. Equilibrium bid quotes satisfy

$$B_{t,HT}^{*}(v_t,+L) = C_{AT}^{S*}(v_t - s, -L) \qquad B_{t,AT}^{-s\,*}(v_t,+L) = C_{HT}^{S*}(v_t - s, -L)$$

$$B_{t,AT}^{+s\,*}(v_t,+L) = C_{HT}^{S*}(v_t + s, -L) \qquad B_{t,AT}^{*}(v_t,+L) = C_{AT}^{S*}(v_t - s, -L)$$

**Type 3 equilibrium:**

This equilibrium arises under Orderings 3 and 4. Equilibrium bid quotes satisfy

$$B_{t,HT}^{*}(v_t,+L) = C_{HT}^{S*}(v_t + s, -L) \qquad B_{t,AT}^{-s\,*}(v_t,+L) = C_{HT}^{S*}(v_t - s, -L)$$

$$B_{t,AT}^{+s\,*}(v_t,+L) = C_{HT}^{S*}(v_t + s, -L) \qquad B_{t,AT}^{*}(v_t,+L) = C_{AT}^{S*}(v_t - s, -L)$$

**Type 4 equilibrium:**

This equilibrium arises only under Ordering 4. Equilibrium bid quotes satisfy

$$B_{t,HT}^{*}(v_t,+L) = C_{HT}^{S*}(v_t + s, -L) \qquad B_{t,AT}^{-s\,*}(v_t,+L) = C_{HT}^{S*}(v_t - s, -L)$$

$$B_{t,AT}^{+s\,*}(v_t,+L) = C_{HT}^{S*}(v_t + s, -L) \qquad B_{t,AT}^{*}(v_t,+L) = C_{AT}^{S*}(v_t + s, -L)$$

**Type 5 equilibrium:**

This equilibrium arises only under Ordering 4. Equilibrium bid quotes satisfy

$$B_{t,HT}^{-s*}(v_t,+L)=C_{AT}^{S*}(v_t+s,-L) \qquad B_{t,AT}^{-s*}(v_t,+L)=C_{HT}^{S*}(v_t-s,-L)$$
$$B_{t,AT}^{+s*}(v_t,+L)=C_{HT}^{S*}(v_t+s,-L) \qquad B_{t,AT}^{*}(v_t,+L)=C_{AT}^{S*}(v_t+s,-L)$$

Finally, it is possible to show that there exist no other equilibria than the ones just obtained, which yields uniqueness. The involved calculations are very long and tedious, such that they are omitted for brevity.

Q.E.D.

## B.1.B. Proof of Proposition 2

For each type of equilibrium, the transitions from one state to another follow a Markov chain with transition matrix $P_i$, $i \in \{1,2,3,4,5\}$. Using Table B.1 together with the equilibrium quotes in terms of cutoff prices (see the proof of Proposition 1), it is straightforward to obtain

$$P_1=\begin{bmatrix} 3(1-a)/4 & (1-a)/4 & a & 0 \\ 1-a & 0 & a & 0 \\ (1-a)/2 & (1-a)/2 & 3a/4 & a/4 \\ 1-a & 0 & a & 0 \end{bmatrix} \qquad P_2=\begin{bmatrix} 3(1-a)/4 & (1-a)/4 & 3a/4 & a/4 \\ 1-a & 0 & a & 0 \\ (1-a)/2 & (1-a)/2 & 3a/4 & a/4 \\ 1-a & 0 & a & 0 \end{bmatrix}$$

$$P_3=\begin{bmatrix} (1-a)/2 & (1-a)/2 & 3a/4 & a/4 \\ 1-a & 0 & a & 0 \\ (1-a)/2 & (1-a)/2 & 3a/4 & a/4 \\ 1-a & 0 & a & 0 \end{bmatrix} \qquad P_4=\begin{bmatrix} (1-a)/2 & (1-a)/2 & 3a/4 & a/4 \\ 1-a & 0 & a & 0 \\ (1-a)/2 & (1-a)/2 & a/2 & a/2 \\ 1-a & 0 & a & 0 \end{bmatrix}$$

$$P_5=\begin{bmatrix} (1-a)/2 & (1-a)/2 & a/2 & a/2 \\ 1-a & 0 & a & 0 \\ (1-a)/2 & (1-a)/2 & a/2 & a/2 \\ 1-a & 0 & a & 0 \end{bmatrix}$$

Given these transition matrices, the stationary probability distribution $f^i=(f_1^i,f_2^i,f_3^i,f_4^i)$ is given by the left eigenvector associated with the unit eigenvalue. Straightforward calculations reveal

$$f^1=\left(\frac{4(1-a)(4-a)}{(4+a)(5-a)},\frac{(1-a)(4+5a-a^2)}{(4+a)(5-a)},\frac{4a}{4+a},\frac{a^2}{4+a}\right)$$
$$f^2=\left(\frac{4(1-a)(4-a)}{20-a+a^2},\frac{(1-a)(4+3a+a^2)}{20-a+a^2},\frac{16a}{20-a+a^2},\frac{a(4-a+a^2)}{20-a+a^2}\right)$$
$$f^3=\left(\frac{(1-a)(4-a)}{6-a},\frac{2(1-a)}{6-a},\frac{a(5-a)}{6-a},\frac{a}{6-a}\right)$$
$$f^4=\left(\frac{8(1-a)}{12+a-a^2},\frac{(1-a)(4+a-a^2)}{12+a-a^2},\frac{2a(5-a)}{12+a-a^2},\frac{a(2+3a-a^2)}{12+a-a^2}\right)$$
$$f^5=\left(\frac{2(1-a)}{3},\frac{1-a}{3},\frac{2a}{3},\frac{a}{3}\right)$$

Using the definition of the trading rate, we obtain

$$TR^1=\frac{4+a(1-a)}{(4+a)(5-a)}\qquad TR^1_{HT}=\frac{4+a(5-a)}{(4+a)(5-a)}\qquad TR^1_{AT}=\frac{a}{4+a}$$
$$TR^2=\frac{4+3a(1-a)}{20-a(1-a)}\qquad TR^2_{HT}=\frac{4+a(3+a)}{20-a(1-a)}\qquad TR^2_{AT}=\frac{4-a(1-a)}{20-a(1-a)}$$
$$TR^3=\frac{2-a}{6-a}\qquad TR^3_{HT}=\frac{2}{6-a}\qquad TR^3_{AT}=\frac{1}{6-a}$$
$$TR^4=\frac{4-a(1-a)}{12+a(1-a)}\qquad TR^4_{HT}=\frac{4+a(1-a)}{12+a(1-a)}\qquad TR^4_{AT}=\frac{2+a(3-a)}{12+a(1-a)}$$
$$TR^5=\frac{1}{3}\qquad TR^5_{HT}=\frac{1}{3}\qquad TR^5_{AT}=\frac{1}{3}$$

It is immediate that we have $TR^i_{HT}\geq TR^i_{AT}$ for all *i*, such that $TR^*_{HT}\geq TR^*_{AT}$ follows. Foucault (1999) shows that $TR^*\big|_{a=0}=1/5$ for $s\geq s^*_1(0)$ and $TR^*\big|_{a=0}=1/3$ otherwise. If $s\geq s^*_1(0)$, a type-1, type-2 or type-3 equilibrium may arise. It is straightforward to verify that $TR^i>1/5$ for $i=1,2,3$. Similarly, if $s<s^*_1(0)$, a type-2, type-3, type-4 and type-5 equilibrium may arise. It is easy to check that in this case, we have $TR^i\leq 1/3$ for $i=2,3,4,5$ as required.

Q.E.D.

## B.1.C. Proof of Proposition 3

The event probabilities $p_{j,k}^{B,+s}$ and $p_{j,k}^{B,-s}$ for each type of equilibrium can be easily obtained using the execution probabilities in Table B.1 together with the equilibrium quotes in terms of cutoff prices (see the proof of Proposition 1).

Consider the type-1 equilibrium and recall that the asset value increases or decreases with probability $1/2$. First, we have $B_{HT}^{*} = C_{HT}^{S*}(v_t - s, -L)$, which is never executed by an AT and only executed by a HT if he is a seller and the asset value decreases, such that $p_{HT,HT}^{B,+s} = 0$, $p_{HT,HT}^{B,-s} = (1-a)/4$, $p_{AT,HT}^{B,+s} = 0$ and $p_{AT,HT}^{B,-s} = 0$. Similarly, we have $B_{AT}^{+s\,*} = C_{HT}^{S*}(v_t + s, -L)$ and $B_{AT}^{-s\,*} = C_{HT}^{S*}(v_t - s, -L)$, such that $p_{HT,AT}^{B,+s} = (1-a)/4$ and $p_{HT,AT}^{B,-s} = (1-a)/4$. Finally, $B_{AT}^{*} = C_{AT}^{S*}(v_t - s, -L)$ implies $p_{AT,AT}^{B,+s} = 0$ and $p_{AT,AT}^{B,-s} = a/4$. The probabilities for all other types of equilibria are obtained in exactly the same fashion. We collect them in Table B.2 below.

**Table B.2 Event Probabilities**

This table contains the event probabilities $p_{j,k}^{B,+s}$ and $p_{j,k}^{B,-s}$ for each type of equilibrium.

| Probability | Type 1 Equilibrium | Type 2 Equilibrium | Type 3 Equilibrium | Type 4 Equilibrium | Type 5 Equilibrium |
|---|---|---|---|---|---|
| $p_{HT,HT}^{B,+s}$ | 0 | 0 | $(1-a)/4$ | $(1-a)/4$ | $(1-a)/4$ |
| $p_{HT,HT}^{B,-s}$ | $(1-a)/4$ | $(1-a)/4$ | $(1-a)/4$ | $(1-a)/4$ | $(1-a)/4$ |
| $p_{AT,HT}^{B,+s}$ | 0 | 0 | 0 | 0 | $a/4$ |
| $p_{AT,HT}^{B,-s}$ | 0 | $a/4$ | $a/4$ | $a/4$ | $a/4$ |
| $p_{HT,AT}^{B,+s}$ | $(1-a)/4$ | $(1-a)/4$ | $(1-a)/4$ | $(1-a)/4$ | $(1-a)/4$ |
| $p_{HT,AT}^{B,-s}$ | $(1-a)/4$ | $(1-a)/4$ | $(1-a)/4$ | $(1-a)/4$ | $(1-a)/4$ |
| $p_{AT,AT}^{B,+s}$ | 0 | 0 | 0 | $a/4$ | $a/4$ |
| $p_{AT,AT}^{B,-s}$ | $a/4$ | $a/4$ | $a/4$ | $a/4$ | $a/4$ |

Now the equilibrium expected profits from posting limit orders can be obtained directly by substituting the above probabilities and the equilibrium quotes (Proposition 1) into equations (2.3) and (2.4). Let $E(U_{HT}^{LO,i})$ and $E(U_{AT}^{LO,i})$ denote the expected utility obtained by HTs and ATs, respectively, when posting limit orders in a type-i equilibrium. We find

$$E(U_{HT}^{LO,1}) = (2L)\frac{1-a}{5-a} \qquad E(U_{HT}^{LO,2}) = (2L)\frac{3-a}{7+3a}$$

$$E(U_{AT}^{LO,1}) = (2L)\frac{8-a(3+a)}{(5-a)(4+a)} \qquad E(U_{AT}^{LO,2}) = (2L)\frac{1+a}{7+3a}$$

$$E(U_{HT}^{LO,3}) = (2L)\frac{2-a}{6-a} - s\,\frac{2}{6-a} \qquad E(U_{AT}^{LO,3}) = (2L)\frac{8-a(2+a)}{(6-a)(4+a)} + s\,\frac{4(1-a)}{(6-a)(4+a)}$$

$$E(U_{HT}^{LO,4}) = (2L)\frac{2-a}{6-a} - s\,\frac{2}{6-a} \qquad E(U_{AT}^{LO,4}) = (2L)\frac{4+a(2+a)}{(6-a)(2+a)} + s\,\frac{2-a(8-a)}{(6-a)(2+a)}$$

$$E(U_{HT}^{LO,5}) = (2L)\frac{1}{3} - s\,\frac{2}{3(1+a)} \qquad E(U_{AT}^{LO,5}) = (2L)\frac{1}{3} + s\,\frac{1-3a}{3(1+a)}$$

It is easy to see that $E(U^{LO})\big|_{a=0} = (2L)/5$ for $s \geq s_1^*(0)$ and $E(U^{LO})\big|_{a=0} = (2L-s)/3$ otherwise. If $s \geq s_1^*(0)$, a type-1, type-2 or type-3 equilibrium may arise. It is straightforward to verify that in this case, $E(U_{AT}^{LO,i}) > (2L)/5 > E(U_{HT}^{LO,i})$ for $i = 1,2,3$. Similarly, if $s < s_1^*(0)$, a type-2, type-3, type-4 and type-5 equilibrium may arise. It is easy to check that in this case, we have $E(U_{AT}^{LO,i}) > (2L-s)/3 > E(U_{HT}^{LO,i})$ for $i = 2,3,4,5$ as required.

Q.E.D.

## B.1.D. Proof of Proposition 4

Using the equilibrium quotes from Proposition 1, we can calculate the trading costs for each combination of limit order trader and market order trader via equations (2.5) – (2.8) and (2.10) – (2.13). We collect them in Table B.3 below.

**Table B.3 Trading costs**

This table contains, for each type of equilibrium the trading costs for every possible combination of limit order trader and market order trader type.

| | **EQUILIBRIUM TYPES** | | | | |
|---|---|---|---|---|---|
| **Trading Cost** | **Type 1** | **Type 2** | **Type 3** | **Type 4** | **Type 5** |
| $TC^{S,+s}_{t,HT,HT}$ | - | - | $L\frac{2+a}{6-a}+s\frac{2}{6-a}$ | $L\frac{2+a}{6-a}+s\frac{2}{6-a}$ | $L\frac{1}{3}-s\frac{1-3a}{3(1+a)}$ |
| $TC^{S,-s}_{t,HT,HT}$ | $L\frac{3+a}{5-a}$ | $L\frac{1+5a}{7+3a}$ | $L\frac{2+a}{6-a}-s\frac{10-2a}{6-a}$ | $L\frac{2+a}{6-a}-s\frac{10-2a}{6-a}$ | $L\frac{1}{3}-s\frac{7+3a}{3(1+a)}$ |
| $TC^{S,+s}_{t,AT,HT}$ | - | - | - | - | $L\frac{1}{3}-s\frac{1-3a}{3(1+a)}$ |
| $TC^{S,-s}_{t,AT,HT}$ | - | $L\frac{1+5a}{7+3a}$ | $L\frac{2+a}{6-a}-s\frac{10-2a}{6-a}$ | $L\frac{2+a}{6-a}-s\frac{10-2a}{6-a}$ | $L\frac{1}{3}-s\frac{7+3a}{3(1+a)}$ |
| $TC^{S,+s}_{t,HT,AT}$ | $L\frac{3+a}{5-a}$ | $L\frac{5+a}{7+3a}$ | $L\frac{2+a}{6-a}+s\frac{2}{6-a}$ | $L\frac{2+a}{6-a}+s\frac{2}{6-a}$ | $L\frac{1}{3}+s\frac{2}{3(1+a)}$ |
| $TC^{S,-s}_{t,HT,AT}$ | $L\frac{3+a}{5-a}$ | $L\frac{5+a}{7+3a}$ | $L\frac{2+a}{6-a}+s\frac{2}{6-a}$ | $L\frac{2+a}{6-a}+s\frac{2}{6-a}$ | $L\frac{1}{3}+s\frac{2}{3(1+a)}$ |
| $TC^{S,+s}_{t,AT,AT}$ | - | - | - | $L\frac{4+a^2}{(6-a)(2+a)}-s\frac{2-a(8-a)}{(6-a)(2+a)}$ | $L\frac{1}{3}-s\frac{1-3a}{3(1+a)}$ |
| $TC^{S,-s}_{t,AT,AT}$ | $L\frac{4+a(7+a)}{(5-a)(4+a)}$ | $L\frac{1+5a}{7+3a}$ | $L\frac{8+a(6+a)}{(6-a)(4+a)}-s\frac{4(1+a)}{(6-a)(4+a)}$ | $L\frac{4+a^2}{(6-a)(2+a)}-s\frac{26-a^2}{(6-a)(2+a)}$ | $L\frac{1}{3}-s\frac{7+3a}{3(1+a)}$ |

Then, the expected trading costs for HTs and ATs, respectively, are obtained by straightforward substitution into equations (2.9) and (2.14). We obtain

$$E(TC_{HT}^{1}) = \frac{3+a}{5-a}L$$

$$E(TC_{AT}^{1}) = \frac{4+a(7+a)}{(5-a)(4+a)}L$$

$$E(TC_{HT}^{2}) = \frac{(1-a)(4-a)(1+5a)+8a(5+a)}{(7+3a)((4-a)(1-a)+8a)}L$$

$$E(TC_{AT}^{2}) = \frac{1+5a}{7+3a}L$$

$$E(TC_{HT}^{3}) = \frac{2+a}{6-a}L + \frac{2a(5-a)-(1-a)(4-a)^2}{4(6-a)}s$$

$$E(TC_{AT}^{3}) = \frac{(1-a)(4-a)(4+a)(2+a)+a(5-a)(8+a(6+a))}{(4+a)(6-a)}L$$
$$+\frac{2(5-a)((4+a)(4-a)+2a)}{(4+a)(6-a)}s$$

$$E(TC_{HT}^{4}) = \frac{2+a}{6-a}L + \frac{2a(5-a)-(1-a)(4-a)^2}{4(6-a)}s$$

$$E(TC_{AT}^{4}) = \frac{8(1-a)(2+a)^2+4a(5-a)(4+a^2)}{(2+a)(6-a)(8+4a(3-a))}L$$
$$+\frac{2(5-a)(8(1-a)(2+a)+a(28-8a)}{(2+a)(6-a)(8+4a(3-a))}$$

$$E(TC_{HT}^{5}) = \frac{1}{3}L - \frac{4-6a}{3(1+a)}s$$

$$E(TC_{AT}^{5}) = \frac{1}{3}L - \frac{4}{3(1+a)}s$$

Average expected trading costs for each type of equilibrium are then computed using equation (2.15).

It is easy to see that expected overall trading costs in the absence of ATs are given by $E(TC^*)\big|_{a=0} = \frac{3}{5}L$ for $s \geq s_1^*(0)$ and $E(TC^*)\big|_{a=0} = \frac{1}{3}L - \frac{2}{3}s$ otherwise.

**i)** Let $s \geq s_1^*(a_1^*)$, which implies that $E(TC^*)\big|_{a=0} = \frac{3}{5}L$ .

a) If $a \leq a_1^*$, we are always in a type-1 equilibrium such that $E(TC_{HT}^*) = E(TC_{HT}^1)$ and $E(TC^*) = E(TC^1)$. It is easy to check that $E(TC_{HT}^1) > E(TC^1) > \frac{3}{5}L$ .

b) If $\bar{a} \geq a > a_1^*$, we are always in a type-2 equilibrium and hence $E(TC_{HT}^*) = E(TC_{HT}^2)$ and $E(TC^*) = E(TC^2)$. Some algebra reveals that $E(TC^2) < E(TC_{HT}^2) \leq \frac{3}{5}L$ for the assumed range of $a$.

c) If $a > \bar{a}$ , we are always in a type-2 equilibrium and hence $E(TC_{HT}^*) = E(TC_{HT}^2)$ and $E(TC^*) = E(TC^2)$. It is straightforward to check that $E(TC^2) < \frac{3}{5}L < E(TC_{HT}^2)$ in this case.

**ii)** Let $s_1^*(a_1^*) > s \geq s_1^*(0)$, which implies that $E(TC^*)\big|_{a=0} = \frac{3}{5}L$ .

a) If $a \leq \bar{\bar{a}}$, we are either in a type-1, a type-3 or a type-2 equilibrium. It can be shown that, under the assumed range for the parameters, we have $E(TC_{HT}^1) > E(TC^1) > \frac{3}{5}L > E(TC_{HT}^i) > E(TC^i)$ for $i = 2,3$, which establishes the desired result.

b) If $\bar{a} \geq a > \bar{\bar{a}}$ , we are always is a type-2 equilibrium, such that the desired result follows from i) b).

c) If $a > \bar{a}$ , we are always is a type-2 equilibrium, such that the desired result follows from i) c).

**iii)** Let $s_1^*(0) > s \geq s_3^*(0)$, which implies $E(TC^*)\big|_{a=0} = \frac{1}{3}L - \frac{2}{3}s$ .

First, suppose that $s_1^*(0) > s \geq s_3^*(a_2^*)$. Depending on α, a type-1, type-2 or type-3 equilibrium may arise. Tedious algebra reveals that $E(TC_{HT}^i) > E(TC^i) > \frac{1}{3}L - \frac{2}{3}s$ for $i = 1,2,3$ in the assumed parameter range.

Now suppose that $s_3^*(a_2^*) > s \geq s_3^*(0)$, such that a type-3, type-4 or type-5 equilibrium may arise. One can show that $E(TC_{HT}^i) > E(TC^i) > \frac{1}{3}L - \frac{2}{3}s$ for $i = 3,4,5$ in the assumed parameter range.

**iv)** Let $s_3^*(0) > s$, which implies $E(TC^*)\big|_{a=0} = \frac{1}{3}L - \frac{2}{3}s$.

a) If $a \leq 1/4$, we are either in a type-4 or a type-5 equilibrium. One can show that under the assumed parameter configuration, $E(TC_{HT}^4) > \frac{1}{3}L - \frac{2}{3}s \geq E(TC_{HT}^5)$ and $E(TC^4) > \frac{1}{3}L - \frac{2}{3}s > E(TC^5)$, which leads to the desired result.

b) If $1/4 < a \leq 1/3$, we are either in a type-4 or a type-5 equilibrium. One can show that under the assumed parameter configuration, $E(TC_{HT}^4) > E(TC_{HT}^5) > \frac{1}{3}L - \frac{2}{3}s$ and $E(TC^4) > \frac{1}{3}L - \frac{2}{3}s \geq E(TC^5)$, which leads to the desired result.

c) Let $a > 1/3$.

(1) If $a_2^* \geq a > 1/3$, we are either in a type-4 or a type-5 equilibrium. One can show that $E(TC_{HT}^4) > E(TC_{HT}^5) > \frac{1}{3}L - \frac{2}{3}s$ and $E(TC^4) > E(TC^5) > \frac{1}{3}L - \frac{2}{3}s$ in this case, which yields the desired result.

(2) If $a > a_2^*$, we are in a type-5 equilibrium. Some quick algebra yields

$$E(TC_{HT}^5) > E(TC^5) > \frac{1}{3}L - \frac{2}{3}s \ .$$

**v)** If $s_1^*(0) > s \geq s_3^*(a_2^*)$ and $a > a_1^*$, we are either in a type-2 or a type-5 equilibrium. It is straightforward to show that in this case, $E(TC_{AT}^2) > \frac{1}{3}L - \frac{2}{3}s > E(TC_{AT}^5)$. On the other hand, one can show that for any other parameter configuration, $E(TC^*)\big|_{a=0} > E(TC_{AT}^*)$.

Q.E.D.

## B.1.E. Proof of Proposition 5

$EU_{AT}^* > \overline{EU}^* > EU_{HT}^*$ is a direct consequence of Propositions 3 and 4. Moreover,

**i)** This follows directly from Proposition 2 because $\overline{EU}^* = TR^* \times (2L)$.

**ii)** From Proposition 2, we have that $TR^* \leq TR^*\big|_{a=0}$ for $s < s_1^*(0)$. Combined with $EU_{AT}^* > \overline{EU}^* > EU_{HT}^*$, this implies $EU_{HT}^* < \overline{EU}^*\Big|_{a=0}$.

**iii)** For $s \geq s_1^*(a_1^*)$, $\overline{EU}^*\Big|_{a=0} = (2/5)L$. For this level of σ, a type-1 or type-2 equilibrium may arise, i.e. we have $EU_{HT}^* = EU_{HT}^1$ for $a \leq a_1^*$ and $EU_{HT}^* = EU_{HT}^2$ otherwise. Using the expressions for expected gains from limit orders and expected trading costs derived in the proofs of Propositions 3 and 4 together with the stationary distribution derived in the proof of Proposition 2, straightforward calculations yield the desired result.

**iv)** For $s_1^*(0) \leq s < s_1^*(a_1^*)$, $\overline{EU}^*\Big|_{a=0} = (2/5)L$.

a) Given that $a_1^* < \overline{\overline{a}}$, the result follows directly from iii).

b) For $a \leq \overline{\overline{a}}$, we can be in a type-1, type-2 or type-3 equilibrium. Straightforward, but tedious algebra reveals that for the assumed range of α, we have $EU_{HT}^3 > (2/5)L$, but $(2/5)L > EU_{HT}^i$ for i=1,2, which establishes the result.

**v)** For $s_1^*(0) > s \geq s_3^*(a_2^*)$ and $a > a_1^*$, one can verify that $EU_{AT}^*$ may be higher or lower than $\left.\overline{EU}^*\right|_{a=0}$. Combining Propositions 3 and 4, it is immediate that $EU_{AT}^* > \left.\overline{EU}^*\right|_{a=0}$ otherwise, because we have $\left.E(TC^*)\right|_{a=0} > E(TC_{AT}^*)$ and $E(U_{AT}^{LO*}) > \left.E(U^{LO*})\right|_{a=0}$.

Q.E.D.

## B.2. Tables and Figures

**Table B.4 Equilibrium combinations**

This table contains the equilibrium combinations for fixed levels of $s$ (details are described at the end of Section 2.4.A). We have set $L = 1$. The switching point levels of $a$ are denoted by $\tilde{a}_1$ and $\tilde{a}_2$.

| Equilibrium Combination | $s$ range | $s$ | $\tilde{a}_1$ | $\tilde{a}_2$ |
|---|---|---|---|---|
| 1: EQ 1, EQ 2 | $s \geq s_1^*(a_1^*)$ | 1 | 0.2361 | - |
| 2: EQ 1, EQ 3, EQ 2 | $s_1^*(a_1^*) > s \geq s_1^*(0)$ | 0.8198 | 0.1220 | 0.2521 |
| 3: EQ 3, EQ 2, EQ 5 | $s_1^*(0) > s \geq s_3^*(a_2^*)$ | 0.7381 | 0.3198 | 0.6544 |
| 4: EQ 3, EQ 4, EQ 5 | $s_3^*(a_2^*) > s \geq s_3^*(0)$ | 0.6458 | 0.1930 | 0.3647 |
| 5: EQ 4, EQ 5 | $s_3^*(0) > s \geq 0$ | 0.3077 | 0.2477 | - |

**Figure B.1 Equilibrium Map**

This graph depicts the different regions in the $(a, s)$-space that give rise to the respective equilibria. We have set $L = 1$.

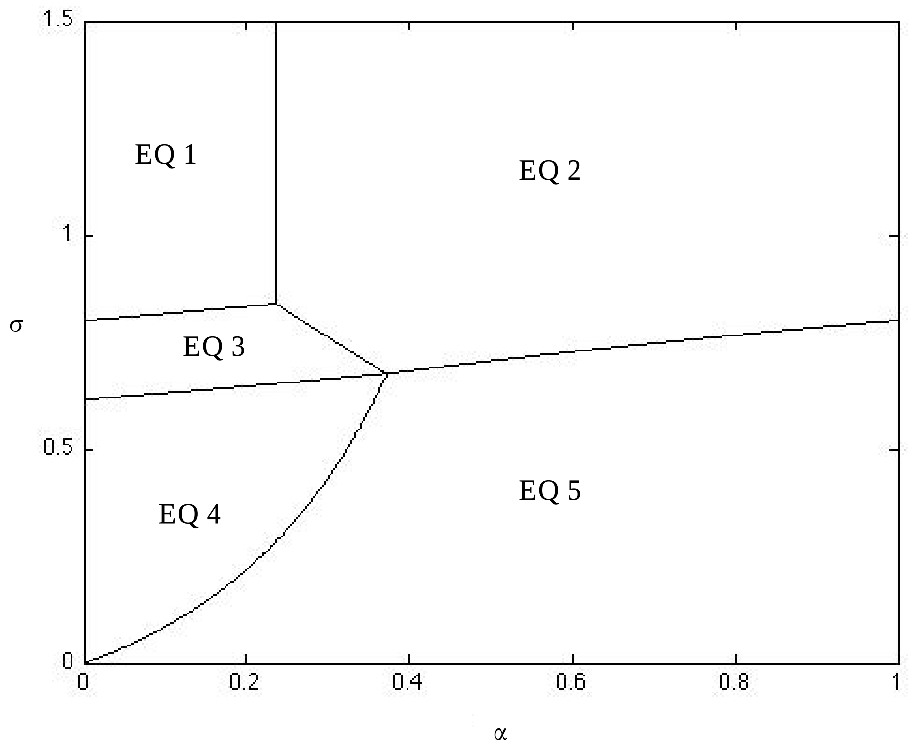
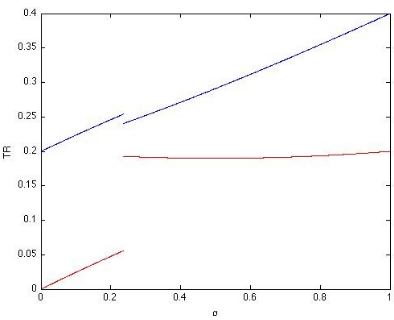
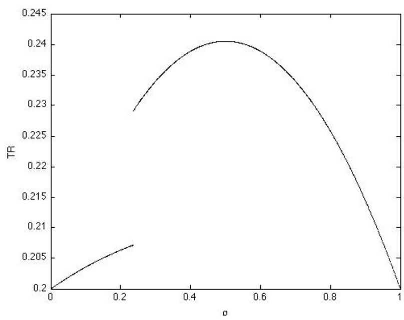


**Figure B.2 Equilibrium Trading Rate**

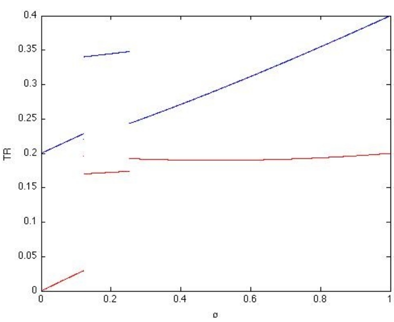
The figures in the left column (a) depict the equilibrium trading rates for HTs (blue) and ATs (red) for different levels of $s$ (descending from top to bottom, see Table B.4 as a function of $a$. The figures in the right column (b) show the equilibrium aggregate trading rate. Notice that the equilibrium trading rate in the absence of ATs is equal to the equilibrium trading rate for HTs in the case where $a = 0$.



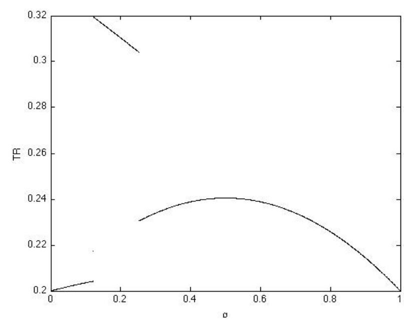

1a)



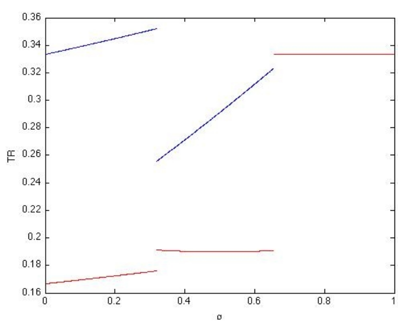

1b)



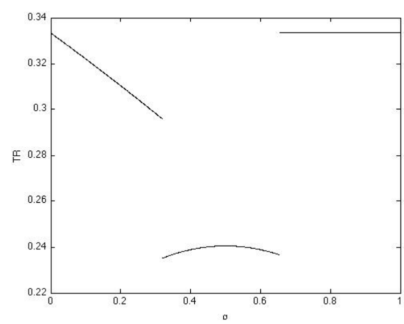

2a)



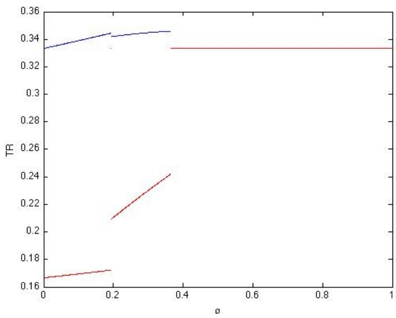

2b)



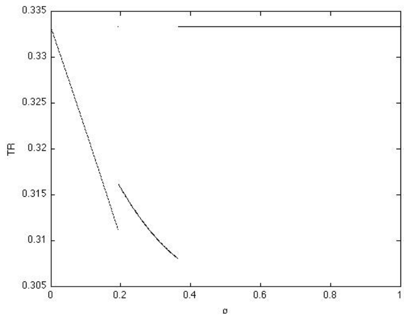

3a)



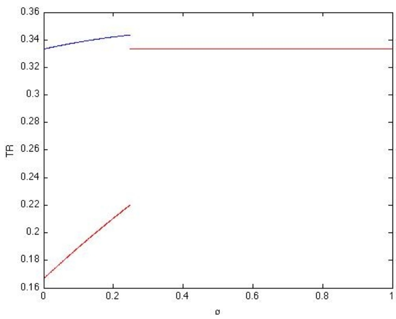

3b)


TR
0.36
0.34
0.32
0.3
0.28
0.26
0.24
0.22
0.2
0.18
0.16
0
0.2
0.4
0.6
0.8
1

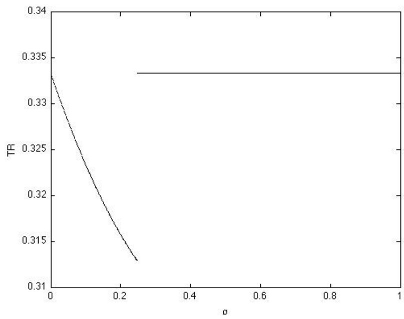
4a)


TR
0.335
0.33
0.325
0.32
0.315
0.31
0.305
0
0.2
0.4
0.6
0.8
1

4b)


TR
0.36
0.34
0.32
0.3
0.28
0.26
0.24
0.22
0.2
0.18
0.16
0
0.2
0.4
0.6
0.8
1

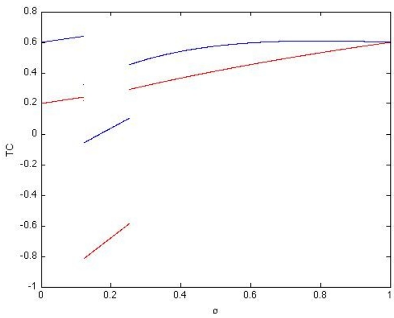
5a)


TR
0.34
0.335
0.33
0.325
0.32
0.315
0.31
0
0.2
0.4
0.6
0.8
1

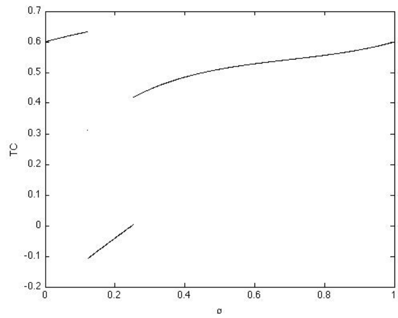
5b)

**Figure B.3 Equilibrium Trading Costs**

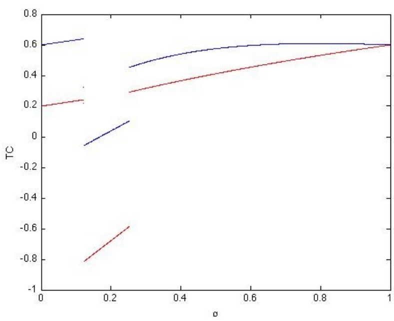
The figures in the left column (a) depict the equilibrium expected trading costs for HTs (blue) and ATs (red) for different levels of $s$ (descending from top to bottom, see Table B.4 as a function of $a$. The figures in the right column (b) show the equilibrium average expected trading cost. Notice that the equilibrium expected trading cost in the absence of ATs is equal to the equilibrium expected trading cost for HTs in the case where $a = 0$.



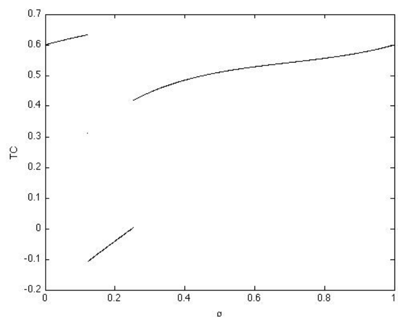

1a)



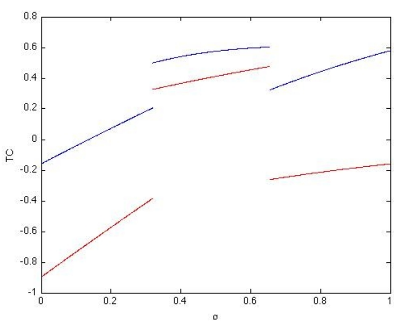

1b)



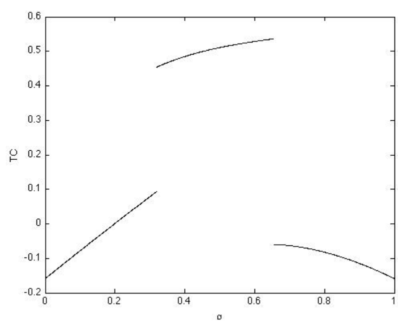

2a)



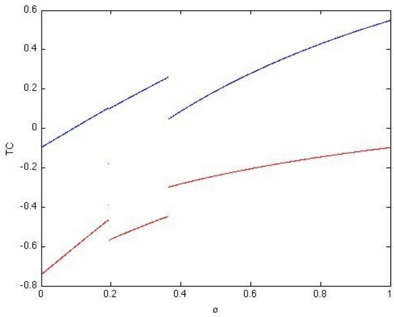

2b)



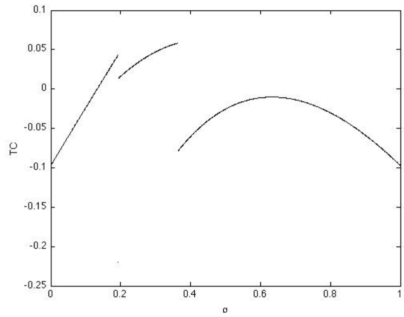

3a)



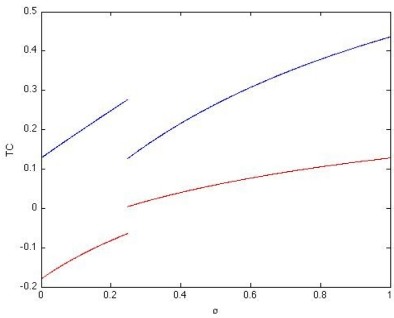

3b)



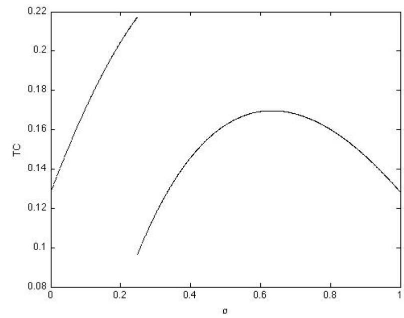

4a)




4b)



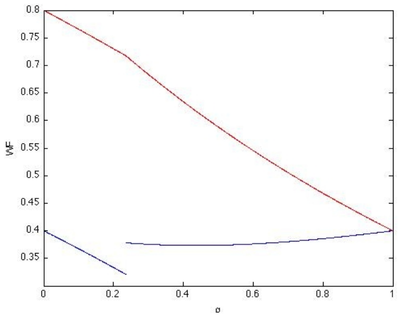

5a)



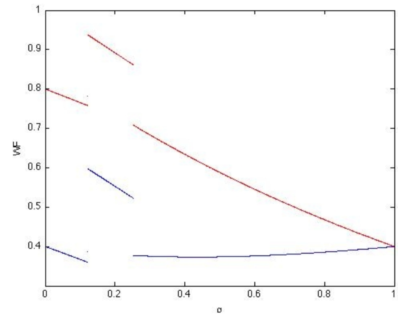

5b)

**Figure B.4 Welfare**

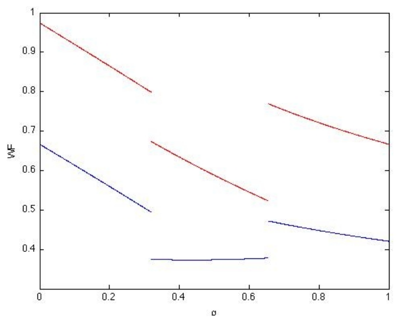
These graphs depict the equilibrium expected utilities for HTs (blue) and ATs (red) for different levels of $s$ (descending from top left to bottom right, see Table B.4) as a function of $a$. Notice that the equilibrium expected utility in the absence of ATs is equal to the equilibrium expected utility for HTs in the case where $a = 0$.



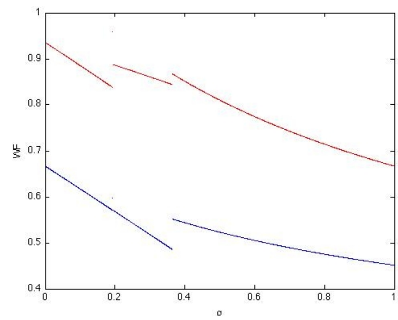
1)



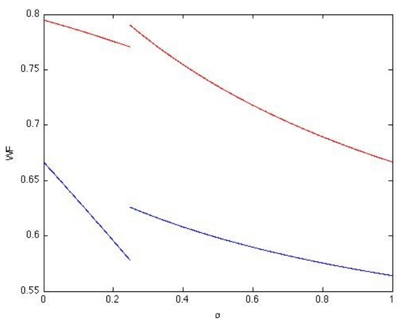
2)



3)



4)



5)

# C. APPENDIX TO CHAPTER 3

## C.1. Tables and Figures

**Table C.1 Matched Sample Stocks**

This table lists the stocks contained in the matched sample obtained by applying the matching algorithm described in Section 3.3. We further divide the sample into three terciles based on trading activity (Euro trading volume), denoted L (large caps), M (mid caps), and S (small caps).

| High turnover stocks (Large Caps, L) | | Medium turnover stocks (Mid Caps, M) | |
|---|---|---|---|
| Xetra | Euronext | Xetra | Euronext |
| ALTANA | CGG VERITAS | AMB GENERALI | BIC |
| BEIERSDORF | SODEXHO ALLIANCE | ARQUES INDUSTRIES | BULL REGPT |
| COMMERZBANK | RENAULT | BILFINGER BERGER | NEXANS |
| DAIMLER | SOCIETE GENERALE | CELESIO | CNP ASSURANCES |
| DEUTSCHE BOERSE | LAFARGE | DEUTSCHE EUROSHOP | CLARINS |
| DEUTSCHE POST AG | CREDIT AGRICOLE | DEUTZ | HAULOTTE GROUP |
| E.ON | BNP PARIBAS | DOUGLAS HOLDING | HAVAS |
| FMC VZ | EIFFAGE | HANNOVER RUECK | DASSAULT SYSTEMES |
| HENKEL VZ | GAZ DE FRANCE | HEIDELBERGCEMENT | EULER HERMES |
| HOCHTIEF | ATOS ORIGIN | HUGO BOSS VZ | TELEPERFORMANCE |
| HYPO REAL ESTATE | CAP GEMINI | IVG | PAGESJAUNES |
| INFINEON | PEUGEOT | KRONES | GROUPE STERIA |
| LANXESS | SAFRAN | MLP | UBISOFT ENTERTAIN |
| LUFTHANSA | STMICROELECTRONICS | MPC | ALTEN |
| MAN | ALSTOM | PREMIERE | SOITEC |
| MERCK | AIR FRANCE -KLM | RHOEN KLINIKUM | CIMENTS FRANCAIS |
| MUENCHENER RUECK | CARREFOUR | SOFTWARE | NEXITY |
| PUMA | TF1 | SUEDZUCKER | MAUREL ET PROM |
| RWE AG | FRANCE TELECOM | TELE ATLAS | FONC DES REGIONS |
| SALZGITTER | TECHNIP | VIVACON | NICOX |
| STADA | VALEO | WINCOR NIXDORF | BOURBON |

| Low turnover stocks (Small Caps, S) | |
|---|---|
| Xetra | Euronext |
| ADVA | DERICHEBOURG |
| BB BIOTECH | BIOMERIEUX |
| COMDIRECT BANK | IMS INTL METAL SCE |
| CTS EVENTIM | STALLERGENES |
| CURANUM | WAVECOM |
| DEUTSCHE BETEILIG | ASSYSTEM |
| DRAEGERWERK VZ | RODRIGUEZ GROUP |
| DYCKERHOFF VZ | DELACHAUX |
| ELEXIS | DEVOTEAM |
| ELRINGKLINGER | MANITOU BF |
| FIELMANN | SECHILIENNE SIDEC |
| FUCHS PETROLUB | IPSOS |
| GERRY WEBER | BOURSORAMA |
| GRAMMER | LISI |
| HCI CAPITAL | FAURECIA |
| JUNGHEINRICH VZ | EUROFINS SCIENT |
| KOENIG + BAUER | PLASTIC OMNIUM |
| PFEIFFER VACUUM | SECHE ENVIRONNEM |
| ROFIN SINAR | VILMORIN & CIE |
| SIXT | GFI INFORMATIQUE |
| TAKKT | SWORD GROUP |

**Table C.2 Matched Sample Descriptive Statistics**

This table reports summary statistics on the variables used for the construction of the matched sample, as detailed in Section 3.3.

| | | Trading Volume (daily in Mio. €) | | Volatility (%) | | Free Float Capitalization (Mio. €) | |
|---|---|---|---|---|---|---|---|
| Tercile | | Euronext | Xetra | Euronext | Xetra | Euronext | Xetra |
| L (N=21) | Mean | 105.1 | 121.8 | 1.78 | 1.75 | 14,835 | 14,155 |
| | Median | 71.1 | 62.8 | 1.65 | 1.79 | 7,089 | 6,216 |
| | StDev | 101.7 | 129.9 | 0.36 | 0.38 | 17,726 | 18,064 |
| M (N=21) | Mean | 8.4 | 8.5 | 2.12 | 2.05 | 1,321 | 1,262 |
| | Median | 6.7 | 6.0 | 1.95 | 2.08 | 1,218 | 1,048 |
| | StDev | 5.8 | 5.6 | 0.69 | 0.67 | 808 | 757 |
| S (N=21) | Mean | 1.2 | 1.2 | 1.96 | 2.09 | 354 | 387 |
| | Median | 1.2 | 1.2 | 1.90 | 2.07 | 275 | 275 |
| | StDev | 0.6 | 0.6 | 0.42 | 0.47 | 190 | 275 |
| All (N=63) | Mean | 38.2 | 43.8 | 1.95 | 1.96 | 5,504 | 5,268 |
| | Median | 6.7 | 6.0 | 1.90 | 1.98 | 1,218 | 1,178 |
| | StDev | 75.0 | 92.5 | 0.52 | 0.54 | 12,082 | 12,072 |

**Table C.3 Relative call auction trading volumes**

For each trading day and stock, we compute the trading volume during the opening and closing auction as a proportion of the trading volume during the day as defined in equation (3.3) of Section 3.4.A. The first two columns report the averages for each exchange and trading activity tercile. Inference regarding differences across markets is based either on a paired t-test (t) or a non-parametric Wilcoxon signed rank test (np). ***, **, and * denote statistical significance at the 1%, 5%, and 10% level, respectively. All numbers are given in percentage values.

| Panel A: Opening Auction | | | | |
|---|---|---|---|---|
| | Euronext | Xetra | *t* | *np* |
| L | 1.17 | 0.83 | *** | *** |
| M | 2.12 | 1.04 | *** | *** |
| S | 4.40 | 2.39 | *** | *** |
| All | 2.56 | 1.42 | *** | *** |
| **Panel B: Closing Auction** | | | | |
| | Euronext | Xetra | *t* | *np* |
| L | 10.30 | 7.87 | *** | *** |
| M | 5.93 | 5.27 | *** | *** |
| S | 4.06 | 3.39 | *** | *** |
| All | 6.76 | 5.51 | *** | *** |

**Table C.4 Call auction liquidity (Amihud Ratio)**

This table reports the average inverse Amihud Ratio as defined in equation (3.5) of Section 3.4.B. for the opening and closing auctions as well as the continuous trading phase separately for each exchange. Inference regarding differences across markets is based either on a paired t-test (t) or a non-parametric Wilcoxon signed rank test (np). ***, **, and * denote statistical significance at the 1%, 5%, and 10% level, respectively. For improved visibility, we adjust the average measure with a different scale for each trading activity tercile, which is given in the last column as Euro trading volume per basis point price change.

| Panel A: Opening Auction | | | | | |
|---|---|---|---|---|---|
| | Euronext | Xetra | *t* | *np* | € / BP |
| L | 14.6 | 14.1 | - | * | 1,000 |
| M | 10.7 | 2.5 | *** | *** | 100 |
| S | 12.2 | 7.2 | ** | ** | 10 |

| Panel B: Closing Auction | | | | | |
|---|---|---|---|---|---|
| | Euronext | Xetra | *t* | *np* | € / BP |
| L | 6.35 | 5.70 | ** | ** | 100,00 |
| M | 1.69 | 1.25 | - | ** | 10,000 |
| S | 0.19 | 0.12 | - | ** | 1,000 |

| Panel C: Continuous Trading | | | | | |
|---|---|---|---|---|---|
| | Euronext | Xetra | *t* | *np* | € / BP |
| L | 8.14 | 9.63 | * | - | 100,00 |
| M | 4.68 | 5.05 | - | - | 10,000 |
| S | 4.75 | 5.24 | - | - | 1,000 |

**Table C.5 Volume Cross Correlations**

This table reports the average cross-sectional correlation coefficient of Euro trading volume for each trading activity tercile on each exchange. We report the averages for the opening (Panel A) and closing (Panel B) auction as well as for the continuous trading phase (reported in both panels). Ratio A/C denotes the average correlation coefficient for the respective auction divided by the corresponding average for continuous trading, where standard errors are calculated using the delta method. The last column denotes the difference in the Ratio A/C across exchanges, where standard errors have been obtained under the assumption of independence. ***, **, and * denote statistical significance at the 1%, 5%, and 10% level, respectively.

| Panel A: Opening Auction | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Euronext | | | Xetra | | | Difference |
| | Cont. Trading | Auction | Ratio A/C | Cont. Trading | Auction | Ratio A/C | Ratio A/C |
| L | 0.179 | 0.067 | 0.375*** | 0.269 | 0.125 | 0.463*** | -0.088*** |
| | (0.009) | (0.006) | (0.028) | (0.009) | (0.007) | (0.019) | (0.034) |
| M | 0.128 | 0.079 | 0.615*** | 0.236 | 0.058 | 0.247*** | 0.369*** |
| | (0.009) | (0.007) | (0.043) | (0.012) | (0.005) | (0.019) | (0.047) |
| S | 0.088 | 0.081 | 0.921 | 0.143 | 0.098 | 0.682*** | 0.239*** |
| | (0.006) | (0.005) | (0.070) | (0.007) | (0.006) | (0.042) | (0.082) |

| Panel B: Closing Auction | | | | | | | |
|---|---|---|---|---|---|---|---|
| | Euronext | | | Xetra | | | Difference |
| | Cont. Trading | Auction | Ratio A/C | Cont. Trading | Auction | Ratio A/C | Ratio A/C |
| L | 0.179 | 0.227 | 1.268*** | 0.269 | 0.289 | 1.074* | 0.194*** |
| | (0.009) | (0.009) | (0.063) | (0.009) | (0.011) | (0.039) | (0.074) |
| M | 0.128 | 0.100 | 0.780*** | 0.236 | 0.221 | 0.936 | -0.156*** |
| | (0.009) | (0.007) | (0.039) | (0.012) | (0.012) | (0.050) | (0.063) |
| S | 0.088 | 0.075 | 0.849*** | 0.143 | 0.080 | 0.558*** | 0.291*** |
| | (0.006) | (0.005) | (0.069) | (0.007) | (0.005) | (0.037) | (0.078) |

**Table C.6 Price discovery**

This table reports the average values of the three price discovery measures presented in Section 3.5.A, separately for each exchange and trading activity tercile. Inference regarding differences across markets is based either on a paired t-test (t) or a non-parametric Wilcoxon signed rank test (np). ***, **, and * denote statistical significance at the 1%, 5%, and 10% level, respectively.

| Panel A: Opening Auction | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Variance Ratio | | | | unbiasedness $R^2$ | | | | *WPC* | | | |
| | Euronext | Xetra | *t* | *np* | Euronext | Xetra | *t* | *np* | Euronext | Xetra | *t* | *np* |
| L | 0.335 | 0.275 | - | * | 0.262 | 0.250 | - | - | 0.249 | 0.221 | ** | *** |
| M | 0.451 | 0.306 | ** | * | 0.329 | 0.165 | *** | *** | 0.244 | 0.202 | *** | *** |
| S | 0.353 | 0.247 | *** | *** | 0.218 | 0.122 | *** | ** | 0.233 | 0.174 | *** | *** |
| All | 0.380 | 0.276 | *** | *** | 0.270 | 0.179 | *** | *** | 0.242 | 0.200 | *** | *** |

| Panel B: Closing Auction | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Variance Ratio | | | | unbiasedness $R^2$ | | | | *WPC* | | | |
| | Euronext | Xetra | *t* | *np* | Euronext | Xetra | *t* | *np* | Euronext | Xetra | *t* | *np* |
| L | 0.015 | 0.015 | - | - | 0.026 | 0.015 | - | - | 0.013 | 0.011 | - | - |
| M | 0.029 | 0.019 | ** | * | 0.040 | 0.024 | * | ** | 0.036 | 0.026 | - | ** |
| S | 0.062 | 0.062 | - | - | 0.062 | 0.069 | - | - | 0.076 | 0.065 | - | * |
| All | 0.035 | 0.032 | - | - | 0.043 | 0.036 | - | - | 0.041 | 0.034 | - | - |

**Table C.7 Bid-ask spreads during the first hour of continuous trading**

This table reports the relative quoted spread (see equation (3.8) in Section 3.5.B for each exchange and trading activity tercile during the first hour of continuous trading, which we divide into 12 5-minute intervals. Inference regarding differences across markets is based either on a paired t-test (t). ***, **, and * denote statistical significance at the 1%, 5%, and 10% level, respectively.

| 5 minute interval | Large Caps | | | Mid Caps | | | Small Caps | | |
|---|---|---|---|---|---|---|---|---|---|
| | Euronext | Xetra | t | Euronext | Xetra | t | Euronext | Xetra | t |
| 1 | 2.500 | 2.994 | *** | 2.230 | 3.157 | *** | 1.621 | 1.887 | *** |
| 2 | 2.117 | 2.155 | ** | 2.162 | 2.529 | *** | 1.675 | 1.753 | *** |
| 3 | 1.788 | 1.819 | ** | 1.931 | 2.118 | *** | 1.609 | 1.631 | |
| 4 | 1.627 | 1.619 | | 1.741 | 1.822 | *** | 1.524 | 1.525 | |
| 5 | 1.520 | 1.498 | ** | 1.605 | 1.659 | *** | 1.453 | 1.445 | |
| 6 | 1.463 | 1.432 | *** | 1.533 | 1.554 | ** | 1.396 | 1.384 | |
| 7 | 1.394 | 1.371 | ** | 1.470 | 1.470 | | 1.358 | 1.339 | * |
| 8 | 1.322 | 1.333 | | 1.381 | 1.397 | | 1.312 | 1.288 | * |
| 9 | 1.262 | 1.254 | | 1.306 | 1.297 | | 1.258 | 1.235 | * |
| 10 | 1.204 | 1.211 | | 1.232 | 1.246 | | 1.210 | 1.205 | |
| 11 | 1.187 | 1.197 | | 1.205 | 1.206 | | 1.187 | 1.180 | |
| 12 | 1.175 | 1.160 | ** | 1.178 | 1.163 | * | 1.171 | 1.153 | |

**Table C.8 Reversal regressions**

This table contains the estimated regression coefficients from the models defined in equations (3.10) and (3.11) in Section 3.5.C, presented separately for each activity tercile and the entire sample. Standard errors clustered by stock and trading day are given in parentheses. ***, **, and * denote statistical significance at the 1%, 5%, and 10% level, respectively.

Panel A: Raw returns

| | Opening Auction | | | Closing Auction | |
|---|---|---|---|---|---|
| | β | γ | | β | γ |
| L | -0.082 *** | 0.017 | L | -0.231 *** | -0.085 |
| | *(0.021)* | *(0.026)* | | *(0.068)* | *(0.125)* |
| M | -0.069 *** | -0.079 *** | M | -0.482 *** | 0.167 ** |
| | *(0.017)* | *(0.030)* | | *(0.084)* | *(0.083)* |
| S | -0.141 *** | -0.010 | S | -0.477 *** | 0.023 |
| | *(0.021)* | *(0.028)* | | *(0.033)* | *(0.043)* |
| All | -0.090 *** | -0.034 * | All | -0.446 *** | 0.047 |
| | *(0.013)* | *(0.020)* | | *(0.033)* | *(0.040)* |

Panel B: Portfolio Returns

| | Opening Auction | | | Closing Auction | |
|---|---|---|---|---|---|
| | β | γ | | β | γ |
| L | 0.050 *** | -0.039 | L | 0.118 | -0.664 |
| | *(0.022)* | *(0.022)* | | *(0.373)* | *(0.524)* |
| M | 0.119 *** | -0.057 * | M | 0.398 | -0.309 |
| | *(0.030)* | *(0.032)* | | *(0.363)* | *(0.469)* |
| S | 0.010 | 0.116 | S | -0.013 | -0.107 |
| | *(0.020)* | *(0.071)* | | *(0.010)* | *(0.273)* |
| All | 0.068 *** | -0.008 | All | 0.158 | -0.218 |
| | *(0.011)* | *(0.015)* | | *(0.176)* | *(0.229)* |

Panel C: Abnormal returns

| | Opening Auction | | | Closing Auction | |
|---|---|---|---|---|---|
| | β | γ | | β | γ |
| L | -0.139 *** | 0.031 | L | -0.281 *** | -0.002 |
| | *(0.038)* | *(0.054)* | | *(0.058)* | *(0.131)* |
| M | -0.084 *** | -0.135 *** | M | -0.551 *** | 0.200 *** |
| | *(0.022)* | *(0.039)* | | *(0.076)* | *(0.084)* |
| S | -0.170 *** | -0.068 ** | S | -0.491 *** | 0.018 |
| | *(0.026)* | *(0.034)* | | *(0.033)* | *(0.044)* |
| All | -0.127 *** | -0.068 *** | All | -0.482 *** | 0.066 |
| | *(0.018)* | *(0.029)* | | *(0.031)* | *(0.043)* |

**Table C.9 Sniping**

Panel A of this table contains the proportion of closing auctions on Euronext that experience a change in the indicative clearing price in the last 5, 3, or 1 seconds, reported separately for the different terciles and the entire sample. Panel B reports the estimated regression coefficient γ of equation (3.12) in Section 3.5.D. Standard errors clustered by stock and trading day are given in parentheses. ***, **, and * denote statistical significance at the 1%, 5%, and 10% level, respectively.

| Panel A: Late Price Changes | | | |
|---|---|---|---|
| | 5 sec | 3 sec | 1 sec |
| L | 64.6% | 54.8% | 28.6% |
| M | 47.1% | 42.1% | 21.4% |
| S | 10.5% | 8.5% | 5.7% |
| ALL | 41.8% | 36.1% | 19.0% |

| Panel B: Regression coefficients | | | |
|---|---|---|---|
| | 5 sec | 3 sec | 1 sec |
| L | -0.05 | -0.08 | 0.03 |
| | *(-0.41)* | *(-0.77)* | *(0.29)* |
| M | 0.02 | 0.01 | -0.11 |
| | *(0.25)* | *(0.11)* | *(-0.94)* |
| S | -0.22** | -0.24** | -0.14 |
| | *(-2.07)* | *(-2.18)* | *(-0.91)* |
| ALL | 0.04 | 0.02 | 0.01 |
| | *(0.78)* | *(0.43)* | *(0.11)* |

**Figure C.1 Bid-ask spreads during the first hour of continuous trading**

This figure graphs the relative quoted spread (defined in equation (3.8) in Section 3.5.B for each exchange and trading activity tercile during the first hour of continuous trading, which we divide into 12 5-minute intervals. The exact figures are contained in Table C.7.