# Chapter 2

# A Strategic Analysis of Two Different Types of Planning Restrictions

## 1. Introduction

The use of urban planning instruments is common in most modern societies. Among European countries, planning systems vary a great deal, but the presence of the public sector is habitual along the several stages of the planning process. In Spain there has been a long tradition of intervening the land market and the planning process, and instruments such as density levels and the delimitation of land suitable for development are jointly used. The role of land-use controls as a means to guide urban development and restrict urban growth has been long and widely analyzed in the literature. From the viewpoint of resident households, the economic justification for the introduction of growth controls lies mostly on the alleged relationship between urban size and the existence of external costs, related for instance to the appearance of congestion or to the loss of outer landscapes. In this sense, restricting the urban size may lead to increases in welfare. This is the approach followed by the so-called *amenity–creation models*, in which planning restrictions tend to improve urban amenities, which ultimately translates into increases in land rents [Brueckner (1990); Engle et al. (1992)].

However, in the theoretical urban economics literature it seems to predominate the idea that actual planning restrictions are welfare–worsening, even though they may correct externalities [Fischel (1990b); Anas et al. (1998)]. Even when they are ultimately successful in preserving the urban environment, they are supposed to achieve that end at too high a cost compared to alternative instruments such as taxes or impact fees, that truly distort residents' decisions [Brueckner (1997); Brueckner (2001)]. Recently, a new line of research has regarded urban planning decisions as the result of the strategic interaction among cities. This approach allows for the emergence of restricted city sizes even though urban growth does not involve external costs [Helsley and Strange (1995), Brueckner (1998)].

In this chapter we analyze the welfare effects of planning restrictions, under different scenarios. It uses the bid-rent framework to analyze two types of growth controls: population regulations and a tax on housing consumption. It is assumed that the utility function of residents is not affected by any urban characteristics such as density or city size. Thus, utility only depends upon the consumption of land and all other private non-land goods. The model consists of a closed system of three interdependent cities where utility is determined endogenously. There are two types of households depending on their income levels, and both types can freely and at zero cost migrate among cities. The effects when one or two of the cities impose some type of regulation are analyzed, and quantity and price instruments are considered, in the form of population controls and a tax on housing consumption. In particular, cities may impose *endogenous* land use regulations, in the sense that they maximize a given objective function for the local planner. Local communities maximize the fiscal revenue arising either from population controls or taxes.

Special attention is paid to the scenarios in which two of the cities may impose some form of controls, which will be strategically chosen. The equilibrium strategies will be obtained for the cases in which cities use population controls and taxes. Other authors have analyzed this case in a static setting; we extend here the analysis to a dynamic context and, additionally, allow for the possibility of cooperation between jurisdictions.

The chapter is organized as follows. Section 2 describes the main features of the model, and shows the equilibrium conditions in the case without planning restrictions. In section 3, the effects of endogenous population controls are analyzed, distinguishing between two scenarios: in the first case, a single city imposes the control; in the second case, cities take into account the decisions of rival communities and decide strategically. We characterize the equilibrium strategies both for a one-period and a multi-period scenario. Section 4 considers the effects of price controls in the form of taxes on housing. It covers again the case when decisions are taken separately or considering other cities' choices, in static and dynamic frameworks. Section 5 shows the differences in tax collection outcomes that result from using population controls or taxes. The final section summarizes the main results and conclusions of the analysis.

## 2. The basic model

The benchmark model here has 3 cities, denoted by subscript $i$, $i = 1, 2, 3$. Cities 1 and 2 may impose growth restrictions, while in all cases city 3 simply accommodates all coming residents. Cities are supposed to be linear and with a width of 1. All residents work at the Central Business District (CBD), located at an extreme of the city. Individuals in city $i$ must commute to the city centre at a cost $T_i(r)$, where $r$ is the distance from the residence to the CBD. Transportation costs are the same for all cities and increase linearly with distance so that, for all $i$, $T_i(r) = tr$. We will abstract from differences in housing size, and assume that all households with the same income level will end up by renting housing of the same size (which will be the same across cities), and respond to utility differentials by costlessly migrating from one city to another.

All households have identical tastes, but may differ in income level. There are two levels of income: $N^A$ individuals have an income $Y^A$, and $N^B$ individuals have $Y^B$, with $Y^A < Y^B$. Income levels are supposed to be exogenous and the role of firms in the city is not considered. Individuals spend their income between a composite good $z$, housing space $s$, and transportation. The utility of an individual with income $Y^j$ ($j \in \{A, B\}$) that resides in city $i$ ($i \in \{1,2,3\}$) is

$$u_i^j = u(s_i^j, z_i^j) \tag{2.1}$$

The composite good $z$ will be taken as the numéraire. The utility is strictly increasing and strictly concave in both arguments (the first partial derivatives are strictly positive and the hessian matrix is negative definite).

Housing consumption will be the same for individuals with the same income level, regardless of the city they reside in. The housing size of higher income individuals is normalized to $s_i^B = s^B = 1$; lower income individuals will have a housing size $s_i^A = s^A = \alpha$, with $0 < \alpha < 1$. Since housing space is determined exogenously, the only variables that affect the utility level achieved by households will be the consumption of all other private goods different from housing, $z_i^j$. Residents in each city pay housing rents to absentee landowners; the rental price of housing per period of time, which depends on the size and location of the residence, is denoted $R_i^j(r)$. The housing market is assumed to be competitive.

Given the fact that individuals have perfect mobility, in equilibrium all individuals with the same income level will end up by achieving the same utility level, regardless of the city they reside in; this in turn implies that their consumption of the composite commodity, $z_i^j$, will be the same. Our analysis will be greatly simplified if we introduce those equilibrium conditions from the start. That is, all individuals with income level $j$ will end up by consuming $z_i^j = z^j$, and achieving a utility level $u^j$. Taking this into account, the budget constraint of a household with income level $j$ residing in city $i$ at a distance $r$ from the CBD can be expressed as:

$$Y^j = z^B(s^j, u^j) + s^j\, R_i^j(r) + t\, r \tag{2.2}$$

That is,

$$Y^B = z^B(1, u^B) + R_i^B(r) + t\, r \tag{2.3}$$

$$Y^A = z^A(\alpha, u^A) + \alpha R_i^A(r) + t\, r, \tag{2.3$'$}$$

In terms of the housing bid-rents, the constraints can be expressed as:

$$R_i^B(r) = Y^B - tr - z^B \tag{2.4}$$

$$R_i^A(r) = \frac{Y^A - tr - z^A}{\alpha}. \tag{2.4$'$}$$

Since transportation costs increase proportionally with distance, the housing rent or housing *bid-rent* decreases linearly with distance to the CBD. For each income level, there exists a family of housing bid-rent functions that correspond to different utility levels. For individuals to be in equilibrium and indifferent among locations within the city, housing rents must vary as described by the housing-bid rent function above. Thus, at a larger distance from the CBD, higher transportation costs are compensated by a smaller housing rent, so that all individuals belonging to the same income group can attain identical utility levels independently of the particular location. The fact that $R_i^A$ is steeper than $R_i^B$ implies that the low income individuals locate closer to the CBD. The inner segment where the low income households locate has a radius of $\widehat{r_i}$. High income residents locate in the outer segment comprised between radius $\widehat{r_i}$ and $r_i$, where $r_i$ represents the edge of the city.

Housing is produced from land and capital, according to the production function $H(l, k) = lk$; given a fixed amount of land $l$, the (restricted) production function thus obtained displays constant returns to scale with respect to capital. Combining $k$ units of capital and $l$ units of land yields $lk$ units of housing [1]. The rental price of capital is

[1] Accordingly, the variable $k$ denotes density, since it refers to the number of housing units per unit of land.

denoted by $P$. It will be assumed that both types of housing require the same amount of capital investment. Let $L_i^j(r)$ represent the rental price of land, which will also vary with distance. If in one unit of land $k$ units of housing are built, this requires $k$ units of capital; hence, the total rental price of housing can be assigned to the land and capital factors:

$$k\,R_i^j(r) = L_i^j(r) + P\,k. \tag{2.5}$$

That is,

$$R_i^j(r) = \frac{L_i^j(r)}{k} + P, \tag{2.6}$$

or

$$L_i^j(r) = k\,\left[R_i^j(r) - P\right] = k\,\left[\frac{Y^j - tr - z^j}{s^j} - P\right]. \tag{2.7}$$

In all cities, at radius $\widehat{r}_i$ (the dividing point between low and high income housing) the land rents must coincide, that is

$$k\,\left[\frac{Y_i^A - t\widehat{r}_i - z^A}{\alpha} - P\right] = k[Y^B - t\widehat{r}_i - z^B - P]. \tag{2.8}$$

At all locations, land is allocated to that activity yielding the highest return.

### 2.1 Equilibrium without planning restrictions

Equilibrium in the land market involves several conditions. Firstly, total population $N^A$ and $N^B$ must be accommodated within the boundaries of the cities.

In city $i$, $\widehat{r}_i$ units of land are allocated to low income households. If each unit of land has $k$ units of housing, $k/\alpha$ households can be accomodated in it, since each household occupies $1/\alpha$ units of housing. Therefore, the total number of low income households that will reside in $i$ will be

$$\frac{k\,\widehat{r}_i}{\alpha}$$

Since the housing space occupied by each income category is the same across cities, in equilibrium it must be the case that all $N^A$ low income households get accomodation:

$$\widehat{r_1} + \widehat{r_2} + \widehat{r_3} = \frac{\alpha N^A}{k} \tag{2.9}$$

The equilibrium condition for the high income households will be:

$$N^B = k\ [(r_1 - \widehat{r_1}) + (r_2 - \widehat{r_3}) + (r_3 - \widehat{r_3})]$$

Using the equilibrium condition for the low income households, we can rewrite the last equality as

$$r_1 + r_2 + r_3 = \frac{\alpha N^A + N^B}{k}. \tag{2.10}$$

Secondly, if residents are perfectly mobile, the utility level achieved by each type of household will be the same in all cities. Since housing consumption is fixed and identical for individuals in the same income range, for them to be indifferent between cities their consumption of non-housing goods must also be the same, that is $z_i^j = z^j$.

Finally, in a context without planning restrictions it is required that in all cities the urban land rent equals the value of the best alternative use at the city limit, usually considered to be agriculture. For simplicity, the value of land in agricultural use will be normalized to zero. Then $L^B(r_i) = 0$, or

$$Y^B - tr_i - z^B - P = 0. \tag{2.11}$$

Equation 2.11 implies that $r_1 = r_2 = r_3 = r$, and from 2.10, it results:

$$r = \frac{\alpha N^A + N^B}{3k}. \tag{2.12}$$

Thus, in the non-restricted equilibrium, population is equally distributed across cities, and the low income households occupy an identical inner radius of

$$\hat{r} = \frac{\alpha N^A}{3k}. \tag{2.13}$$

Combining 2.11 and 2.12, we find the amount of $z$ consumed by individuals with income $Y^B$, which is:

$$z^B = Y^B - P - \frac{t}{3k}[\alpha N^A + N^B]. \tag{2.14}$$

To obtain $z^A$, we use conditions 2.8, 2.13, and 2.14:

$$z^A = Y^A - \alpha P - \frac{\alpha t}{3k}[N^A + N^B]. \tag{2.15}$$

Notice that, in the absence of negative externalities caused by crowding, the higher the density level the better households are, since higher density allows savings in transportation costs and does not provoke any external costs. As we mentioned before, in our model density is measured by the variable $k$.

The equilibrium utility levels are:

$$u_m^A = u\left(\alpha, Y^A - \alpha P - \frac{\alpha t}{3k}[N^A + N^B]\right) \tag{2.16}$$

and

$$u_m^B = u\left(1, Y^B - P - \frac{t}{3k}\left[\alpha N^A + N^B\right]\right). \tag{2.17}$$

In the following sections we will ignore all externalities, and therefore the consumption $z$ of the composite good will allow us to measure how planning controls affect utility levels.

## 3. The effects of population controls

In this section the planning instruments are population controls that restrict the city size. The choice of the appropriate city size is endogenous in the sense that it maximizes aggregate urban land rents, an objective function commonly considered in the urban literature. Two scenarios are considered. Firstly, the case where only one city in the system restricts its size; secondly, two of the cities impose population controls and they decide strategically.

### *3.1 Equilibrium with one controlling city*

Assume now that city 1 imposes an urban population control that restricts city size, and that all excluded households can be accommodated in cities 2 and 3. There, the condition that urban land rent equals zero at the city limit continues to be valid. In equilibrium, since low income households get accomodation closer to the CBD, for all three cities land rents must still be equal at $\widehat{r_i}$, so 2.8 in page 14 still applies in the restricted case. Now, using 2.10 it results

$$z^B = Y^B - P - \frac{t}{2}\left[\frac{\alpha N^A + N^B}{k} - r_1\right], \tag{2.1}$$

where $r_1$ is now a choice variable for the local government in city 1.

Since the introduction of the population control does not alter the size of the inner segment where the low income households live, we have that, for all cities

$$\widehat{r_i} = \frac{\alpha N^A}{3k}. \tag{2.2}$$

In our model, when city 1 imposes an urban population control, the low income residents continue to be equally split between the controlled and uncontrolled cities, and the size of the inner segments does not vary. Although it has been assumed that both types of households are mobile, the previous result implies that population controls may affect unequally different types of households. In our case, population controls affect only high income households, while low income households are not affected at all in their actions, though, as we show next, their equilibrium utility will change. Since the relative steepness of land-rents functions does not change in the regulated situation, then $\widehat{r_i}$ does not vary with the introduction of population controls.

Using 2.9, we obtain

$$z^A = Y^A - \alpha P - \frac{\alpha t}{2k}\left[\frac{(\alpha+2)}{3}N^A + N^B - k\,r_1\right]. \tag{2.3}$$

With a population growth control in city 1, both $z^A$ and $z^B$ are negatively affected. The positive signs of the partial derivatives of $z^j$ with respect to $r_1$ show that the consumption of goods other than housing increases with $r_1$, that is, the less restrictive the control is. Since housing consumption is exogenously determined, the population control makes residents worse off in this simple context without environmental externalities.

The above results apply whatever the values of $r_1$. Consider now the particular case when the population control introduced is endogenous, in the sense that it maximizes a particular objective function chosen by the local government. The objective function will be the sum of all land rents in city 1, $TR_1$. Remember that the land rents benefit the absentee landowners. The rationale for this objective function can be that higher land rents imply higher collection from property taxes, so the cities have more resources available. The local planner chooses the value of $r_1$ that maximizes the objective function $TR_1$, that is

$$\max_{r_1} \int_0^{\widehat{r_1}} k\left[\frac{Y^A - tr - z^A}{\alpha} - P\right]dr + \int_{\widehat{r_1}}^{r_1} k[Y^B - tr - z^B - P]dr. \tag{2.4}$$

The first order conditions of this maximization problem (obtained applying Leibniz's rule to differentiate the integral) result in a city limit $r_1$ smaller than the market equilibrium city size,

$$r_1* = \frac{1}{4k}[\alpha N^A + N^B]. \tag{2.5}$$

Finally, $z^B$ and $z^A$ can be expressed in terms of the parameters:

$$z^B = Y^B - P - \frac{3t}{8k}[\alpha N^A + N^B], \tag{2.6}$$

and

$$z^A = Y^A - \alpha P - \frac{\alpha t}{8k}\left[\frac{(\alpha+8)}{3}N^A + 3N^B\right]. \tag{2.7}$$

It can be shown that, as should be expected, introducing the endogenous population control makes both types of residents consume smaller amounts of $z^A$ and $z^B$, and as a result they attain smaller utility levels.

### *3.2 Equilibrium with two controlling cities*

Assume now that cities 1 and 2 impose population controls so as to maximize total land rents, and that each is aware of the other city's policy and objective. That is, there will be strategic interaction between both cities, which try to maximize land rents (a proxy for property tax collection). In equilibrium, all households will end up by being accommodated. This is possible thanks to the passive role played by city 3, which just accomodates all households that go there.

Now 1 and 2 want to maximize their respective aggregate land rents, $TR_1$ and $TR_2$, but taking into account the rival's choice of city size. This situation gives rise to a game between the two cities, in which a Nash equilibrium will imply the choice of strategies by the two cities which are best replies to each other. The decision variables for both cities are their population controls, that is, the maximum size the city will be allowed to have.

In the case in which only one city imposes population controls, we saw that the size of the population of low income households was not affected by that policy. In this case the same thing will happen, and exactly for the same reasons. That is, the size of the low income household segment will be, for all cities, $\widehat{r} = \frac{\alpha N^A}{3k}$. Besides, in city 3 equation 2.11 in page 15 holds, and hence the equilibrium levels of $z^B$ and $z^A$ will be given by:

$$z^B = Y^B - P - t\left[\frac{\alpha N^A + N^B}{k} - (r_1 + r_2)\right] \tag{2.8}$$

$$z^A = Y^A - \alpha P - \frac{\alpha t}{k}\left[\frac{(1+2\alpha)}{k}N^A + N^B - k(r_1 + r_2)\right] \tag{2.9}$$

The expressions imply that, as in the case in which only one city imposes controls, less stringent population controls lead to higher values of $z^B$ and $z^A$, that is, to higher utilities.

The objective of city 1 is to maximize aggregate land rents $TR_1$. However, 1 has to consider 2's choice of $r_2$. This is achieved by combining $z^B$ and $z^A$ in 2.8 and 2.9 together with the expression of $TR_1$ in 2.4. From the maximization of that expression, we find the best reply function for city 1:

$$r_1(r_2) = \frac{\alpha N^A + N^B}{3k} - \frac{r_2}{3}. \tag{2.10}$$

By symmetry, the expression of the best reply function of city 2 is:

$$r_2(r_1) = \frac{\alpha N^A + N^B}{3k} - \frac{r_1}{3}. \tag{2.11}$$

Notice that the best reply $r_1$ for city 1 is decreasing in $r_2$, that is, the strategies of both players are *strategic substitutes.*[2] Thus, if city 2 fixes a not too stringent (i.e. large) $r_2$, then city 1 benefits from choosing a smaller $r_1$. The land rent sacrificed by excluding a household is smaller the larger is the size chosen by the rival. A more stringent control increases the opportunity cost of losing population, and as a result cities choose larger sizes.

Solving the system with the best response functions for cities 1 and 2 yields smaller sizes for cities 1 and 2, but a larger $r_3$. The expressions for the Nash equilibrium $r_1$ and

[2] Like the strategies of Cournot competitors. See Bulow, Geanakoplos and Klemperer (1985).

$r_2$ coincide with the optimal choice of one city if it alone were imposing the control:

$$r_{comp} = r_1 = r_2 = \frac{1}{4k}\left[\alpha N^A + N^B\right]. \tag{2.12}$$

However, the equilibrium utilities achieved are smaller. Since more cities in the system impose population controls, this leads to a larger number of residents diverted to city 3, and consequently to higher land rents. The equilibrium values of $z^B$ and $z^A$ in this simultaneous population control game are:

$$z^B = Y^B - P - \frac{t}{2k}[\alpha N^A + N^B], \tag{2.13}$$

and

$$z^A = Y^A - \alpha P - \frac{\alpha t}{2k}\left[N^B + \frac{(2+\alpha)}{3}N^A\right]. \tag{2.14}$$

Finally, we find the equilibrium land rents. Substituting 2.12 back into the expression for land rent in city 1 in 2.4, and considering that both 1 and 2 use optimal growth controls, we obtain the expression for total land rents, denoted by $TR_{comp}$ (where *comp* stands for *competitive*):

$$TR_{comp} = \frac{t}{288k}[11\alpha(N^A)^2 + 54\alpha N^A N^B + 27(N^B)^2 + 16\alpha N^A]. \tag{2.15}$$

### *3.3 A cooperative framework*

This subsection introduces the possibility of cooperation among jurisdictions. The literature based on the use of game theory as a means to explain urban growth controls has so far used a noncooperative approach. Nevertheless, given the mutual advantages of achieving cooperation, it is interesting to explore under what circumstances is cooperation sustainable. In this subsection these issues are considered, both in a static and in a dynamic context.

In this scenario without *competition*, the first information needed is the size that cities would choose to maximize aggregate land rents. Since the a priori parameters are symmetric, the maximizing choice of city sizes will also be symmetric, so we may simplify by assuming that $r_1 = r_2$ under the cooperation agreement. Let $r_{coop}$ denote each individual city size with cooperation. The expressions for $z^A$ and $z^B$ will now be

$$z^A = Y^A - \alpha P - \frac{\alpha t}{k}\left[\frac{(1+2\alpha)}{k}N^A + N^B - 2\,k\,r_{coop}\right] \tag{2.16}$$

and

$$z^B = Y^B - P - t\left[\frac{\alpha N^A + N^B}{k} - 2\,r_{coop}\right]. \tag{2.17}$$

To find the optimal cooperative city size it will be enough to maximize the total land rents in one of the cities, given the symmetry assumptions. Thus, we solve the problem

$$\max_{r_{coop}} \int_0^{\widehat{r}_{coop}} k\left(\frac{Y^A - tr - z^A}{\alpha} - P\right) dr + \int_{\widehat{r}_{coop}}^{r_{coop}} k\left(Y^B - tr - z^B - P\right) dr. \tag{2.18}$$

The first order conditions result in

$$r_{coop} = \frac{1}{5k}[\alpha N^A + N^B]. \tag{2.19}$$

The maximum value of the total land rents is obtained by substituting the maximizing value of $r_{coop}$ we just found:

$$TR_{coop} = \frac{t}{450k}[20\alpha^2(N^A)^2 + 90\,\alpha\,N^A N^B + 45(N^B)^2 + 25\alpha(N^A)^2]. \tag{2.20}$$

Although cooperation leads to the highest land rent attainable, it may well happen that the equilibrium solution is such that cities have incentives to compete rather than cooperate. Results vary when considering static or dynamic frameworks.

*Cooperation in a static context* For every city, the available strategies in the static case are “cooperating” or “competing”. The cooperative strategy implies the choice of $r_{coop}$

described in equation 2.19. The competitive strategy consists of choosing according to the best reply functions of the noncooperative framework, which can be found in equation 2.10.

Figure 2.1 shows the relationship between aggregate land rents and city size under the assumption that the population controls chosen by cities 1 and 2 are identical. Since we know that both the noncooperative equilibria and the collusive solution are symmetric, they are included in the graphic.

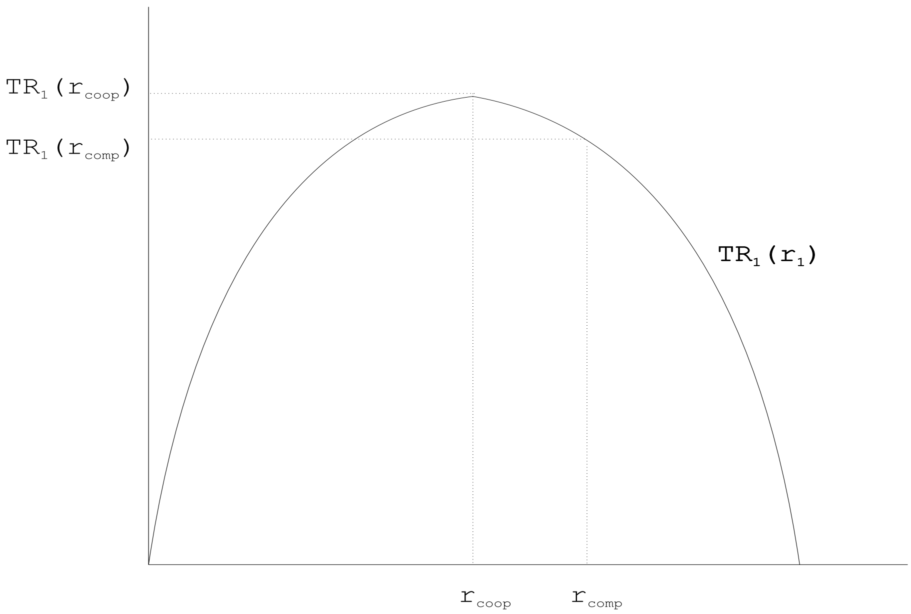


**Figure 2.1**. Total land rents with symmetric city sizes.

In order to formulate the "cooperation or competition" alternative as a game, we must consider what are the players' payoffs whenever one of them decides to compete and the other to cooperate. If we view cooperation as the "focal point" players would like to achieve, whenever one player chooses cooperation but the other is choosing its competitive strategy, we can say that the second player is "deviating" from cooperation.

It can be shown that, as should be expected, $TR_{coop}$ is larger than $TR_{comp}$. Now, though $TR_{coop} > TR_{comp}$, cities have an incentive to deviate from the cooperative agreement, since $r_{coop}$ is not their best city size choice when the other one cooperates.

When 1 deviates while 2 cooperates, then 1's best choice is given by

$$r_1 = r_{dev} = \frac{4}{15k}[\alpha N^A + N^B], \tag{2.21}$$

and

$$r_2 = r_{coop} = \frac{1}{5k}[\alpha N^A + N^B]. \tag{2.22}$$

Likewise, if the corresponding values of aggregate land rents are denoted by $TR_{dev}$ and $TR_{coop'}$, then:

$$TR_{dev} = \frac{t}{450k}[23\alpha^2(N^A)^2 + 96\alpha N^A N^B + 48(N^B)^2 + 25\alpha(N^A)^2], \tag{2.23}$$

and

$$TR_{coop'} = \frac{t}{450k}[14\alpha(N^A)^2 + 78\alpha N^A N^B + 39(N^B)^2 + 25\alpha(N^A)^2]. \tag{2.24}$$

Logically, land rents are larger for the city that deviates from the cooperative agreement, and they are also larger with respect to the cooperative solution. Thus,

$$TR_{dev} > TR_{coop} > TR_{comp} > TR_{coop'}.$$

Figure 2.2 depicts the normal form for the static game just described.

| | | City 2: Cooperate | City 2: Compete |
|---|---|---|---|
| City 1 | Cooperate | $TR_{coop}, TR_{coop}$ | $TR_{coop'}, TR_{dev}$ |
| | Compete | $TR_{dev}, TR_{coop'}$ | $TR_{comp}, TR_{comp}$ |

**Figure 2.2**. Static game with population controls with cooperation.

In a static context, there exists a single Nash equilibrium in pure strategies in which both cities end up competing and do not cooperate. Actually, for each city if is better to choose its competitive strategy irrespective of what the other city chooses, that is, competition is a dominant strategy for each city. Notice that the equilibrium payoffs of the players are Pareto dominated by another (non-equilibrium) outcome:

the cooperative one. In this prisoners' dilemma kind of game, a static framework does not allow any possibilities for cooperation. That is, in a static context cooperation is not a self-enforcing strategy, given the incentives the players have to deviate, and the absence of punishments for breaking the cooperative agreements. However, in a dynamic context punishments appear endogenously, given that, at any moment in time, players have an interest in future cooperation, which raises their expected payoffs in the future. We consider this possibility next.

*Cooperation in a dynamic context* Whenever the choices of the cities take place in time, many new possibilities arise. The present choice of a city affects not only its current payoffs, but also future payoffs, depending on how the other city is going to react. This opens up possibilities for cooperation.

It is important to notice, nevertheless, the importance of having an *indefinite time horizon*. If there is a very definite time horizon (that is, an end period for the possible cooperation), which is commonly known to both parties, then the only reasonable (technically, subgame-perfect) equilibrium is the repetition of the competitive strategies in each period. This can be seen by using backward induction: in the last period, the players have no future opportunities for cooperation, and therefore it is optimal for them to compete; but then, in the previous period, knowing that what they do is not going to influence the actions in the last period, it is again optimal for the players to compete; the same procedure is repeated until we get a complete unravelling of the time sequence. In the case of population controls, this commonly known ending of the time horizon could be the end of the legislature, in case the current city government does not expect its mandate to be renewed.

On the other hand, if there is not such a well-definite ending to the possibilities of cooperation, then the backward induction argument cannot be made, and actually there are ample opportunities for cooperation. Formally, one can model this indefinite ending