either as an infinite time horizon, or as a stochastic one. The idea for implementing cooperative solutions is based on the fact that now the players can “punish” each other for not cooperating, by using competitive strategies in the future. Since both players have an interest in future cooperation, because it gives them higher payoffs, then cooperation is sustainable as a long-term strategy, as long as players attach enough weight to the future.

We will illustrate this result by describing a pair of strategies which actually constitute a subgame-perfect Nash equilibrium, when future payoffs are not discounted at a very high rate. The simplest (among many other) strategies that lead to equilibria that support cooperation are the so-called *trigger strategies*, by which each player imposes on the other the harshest possible punishment when the latter has broken the cooperation: whenever a player detects a noncompliance with cooperation, then it reverts to the competitive strategy *for all future periods.* Let us show that if both players follow trigger strategies, then a subgame-perfect Nash equilibrium ensues (if the discount factor is large enough, i.e. the interest rate is low enough).

Let us assume that no deviation has taken place in the past. From one player’s viewpoint, the decision about whether to deviate in the current period implies a trade-off: there will be immediate gains in the current period (because deviating while the other player cooperates yields the highest possible gain), but the gains in all future periods will be those of competition, instead of the higher gains the player would get if both continued to cooperate. How future gains (land rents) are discounted –together with the number of cities in competition– is one of the key factors among the conditions determining the equilibrium solution and the optimal strategy to be followed by each city.

Given that cities will compete forever whenever a failure to cooperate is detected, the present value of total land rents –PVTR– can be calculated for the different scenarios that can occur. Namely, the PVTR has been calculated in the following three instances:

- When cities always cooperate
- When cities always compete
- When one city deviates in the first period while the other one cooperates, and both compete from that period on.

Let $PVTR_{coop}$ denote the present value of land rents when both cities cooperate; $PVTR_{comp}$, the present value of land rents when competition takes place; $PVTR_{dev}$, the present value of rents resulting from deviating in the first period and competing in the subsequent ones; and $PVTR_{coop'}$, the present value of rents when the city cooperates in the first period when the other one deviates, and both compete in the remaining periods. In figure 2.3 we represent a static game in which the choice of strategies by each player corresponds to the problem faced by a player in the dynamic game with infinitely many periods, when confronted with another player that is using a trigger strategy; thus, the payoffs in this static game represent the present value of the flow of land rents in the dynamic game, when a player is considering whether to deviate from cooperation, when its opponent is using a trigger strategy.

		City 2: Cooperate	City 2: Compete
City 1	Cooperate	$PVTR_{coop}, PVTR_{coop}$	$PVTR_{coop'}, PVTR_{dev}$
	Compete	$PVTR_{dev}, PVTR_{coop'}$	$PVTR_{comp}, PVTR_{comp}$

Figure 2.3. Static game corresponding to a dynamic game with population controls when cooperation is allowed.

The worst possible scenario for a city in terms of the aggregate total land rents occurs when it chooses the cooperative population control while the other city deviates in the first period, since $PVTR_{coop'} < PVTR_{comp}$.

Suppose first that city 1 chooses the population control according to its best response function, that is, it chooses the strategy "compete". In this instance, the best strategy for city 2 is to compete as well, since doing so yields higher land rents. Similarly, that applies symmetrically to city 1 when it is city 2 the one that competes. Then, the set if strategies (*compete, compete*) constitute a Nash equilibrium.

Suppose now that city 1 cooperates. City 2 can either cooperate as well, or deviate from the cooperative solution and compete in all the remaining periods. To determine which strategy is best, we must know which of the two leads to the highest payoff. Comparing $PVTR_{coop}$ against $PVTR_{dev}$, we find that the optimal solution will depend on the value of the discount factor. Cooperating is optimal whenever

$$PVTR_{coop} > PVTR_{dev},$$

and comparing the two values of present value of land rents:

$$\begin{aligned}
&\frac{t}{450k}[20\alpha^2(N^A)^2 + 90\alpha N^A N^B + 45(N^B)^2 + 25\alpha N^A] \\
&+ \frac{t}{r(1+r)450k}[20\alpha(N^A)^2 + 90\alpha N^A N^B + 45(N^B)^2 + 25\alpha N^A] > \\
&\frac{t}{450k}[23\alpha^2(N^A)^2 + 96\alpha N^A N^B + 48(N^B)^2 + 25\alpha(N^A)^2] \\
&+ \frac{t}{288r(1+r)k}[11\alpha^2(N^A)^2 + 54\alpha N^A N^B + 27(N^B)^2 + 16\alpha(N^A)^2],
\end{aligned} \tag{2.25}$$

where r here is the interest rate. The expression can be simplified to

$$\frac{(\alpha N^A + N^B)^2}{160r(1+r)} > \frac{(\alpha N^A + N^B)^2}{150}. \tag{2.26}$$

Concluding, the present value of land rents under cooperation exceeds that of deviation as long as

$$r < \frac{-2+\sqrt{19}}{4} \approx 0.59 \tag{2.27}$$

Equivalently, the discount factor $1/(1+r)$ should be larger than approximately 0.63.

Let us remark that the same argument shows that, if the discount factor exceeds the critical value, cooperation can be sustained as a *subgame perfect* equilibrium with trigger strategies. In order to show that a Nash equilibrium is subgame perfect, we must show that the choice of strategies following any possible past history constitutes a Nash equilibrium for the remainder of the game. Now, whenever failure to cooperate has occurred in the past, then the pair of strategies call for each player to choose competition forever, which is always a Nash equilibrium. On the other hand, if both players have always cooperated in the past, the situation is similar to the one we just analyzed.

We have illustrated here that trigger strategies can sustain cooperation as an equilibrium, provided the discount factor is large enough. However, in real life situations trigger strategies are far from reasonable. The motive is that they are not robust to small uncertainties that might result, for instance, in one player interpreting that the other has deviated when this has actually not happened. But in this case there are other far more reasonable strategies which can be applied and allow cooperation to be sustained as an equilibrium as well. For a nonformal discussion, see Dixit and Nalebuff (1991).

Estimations performed in different countries and for different time periods, always result in real interest rates very close to zero. Therefore, if the discount factors in our model are those that derive from real interest rates, we can conclude that cooperation is always sustainable as an equilibrium. On the other extreme, one might consider self-interested politicians who care nothing about what happens when they quit office, in which case the discount factor would be much smaller, most likely below the critical level we have found. Most practical situations one might consider would be between those two extreme cases.

If cooperation is sustainable as an equilibrium, this calls for cities to choose smaller sizes, that is, to set more stringent population controls. On the other hand, more competitive equilibria would result in larger city sizes, that is, less restrictive population controls.

4. The effects of a tax on housing consumption

Other possible instruments to constrict city size are taxes that modify housing bid-rents of households, and consequently distort landowners' decisions of converting land from rural to urban. In this section we will consider a tax on housing. The rationale for introducing such a tax is to levy revenues that the community might want to use to finance public goods or services. We find that using this price instrument instead of a quantity instrument (size controls) as before, has distributional consequences.

City i will introduce a tax h_i, $0 < h \leq 1$, per unit of housing consumption. The residents' budget constraints will change to:

$$Y^j = z^j + tr + s^j\, R_i^j(r) + s^j\, h_i, \tag{2.1}$$

or, expressed in terms of the housing bid-rents:

$$R_i^j(r) = \frac{Y^j - tr - z^j}{s^j} - h_i. \tag{2.2}$$

The land bid-rent functions become:

$$L_i^B(r) = k\, \left[Y^B - tr - z^B - h_i - P\right] \tag{2.3}$$

$$L_i^A(r) = k\left[\frac{Y^A - tr - z^A}{\alpha} - h_i - P\right]. \tag{2.3'}$$

4.1 Equilibrium values when one city uses taxes

The new land rent functions differ from the one in the market situation because of the new tax on housing, h_i. In equilibrium, it must be true that $L_i^B(r_i) = 0$, independently of whether or not the city uses a tax. Assume first that only 1 introduces the tax h_1.

Then $r_2 = r_3 = r$, and since in equilibrium z^B is common to all cities, we have:

$$r = \frac{\alpha N^A + N^B}{2k} - \frac{r_1}{2}. \tag{2.4}$$

Since $L^B(r_1) = 0$, we can use 2.1 to express the size of 1 in terms of the tax h_1:

$$r_1 = \frac{\alpha N^A + N^B}{3k} - \frac{2h_1}{3t}. \tag{2.5}$$

There is a linear and negative relationship between the tax and the size of the city. Introducing a tax on housing consumption also reduces housing rents, and consequently land rents, and therefore city 1 will be smaller. Now we can find the expressions for z^B and z^A in terms of the housing tax h_1:

$$z^B = Y^B - P - \frac{t}{3k}[\alpha N^A + N^B] - \frac{h_1}{3}, \tag{2.6}$$

and

$$z^A = Y^A - \alpha P - \frac{\alpha t}{3k}[N^A + N^B] - \frac{\alpha h_1}{3}. \tag{2.7}$$

We have again that $\widehat{r_i} = \widehat{r} = \frac{\alpha N^A}{3k}$. Examining the effect of h_1 on z^A and z^B, we see that as the housing tax increases the consumption of z^A and z^B decreases, as does utility.

A population constraint and a tax on housing consumption that result in the same city size have the same negative effect on households, who in either situation reach identical utility levels and the same consumption of z. The result is natural, since both interventions affect households in the same way: the housing consumption tax acts as an additional expense, while the population control causes housing rents to increase. In both scenarios, the income that can be dedicated to non-land goods shrinks.

On the other hand, a tax on land instead of on housing would cause the city to become smaller as well, and residents to attain higher utility levels, but land owning would become less profitable in city 1.

Notice, however, that population controls and taxes have different distributional consequences. With population controls, landowners of developed land receive higher land rents, while residents experience a reduction in their utility levels. When the housing consumption tax is used, resident households experience a comparable decrease in z and the utility level, but all landowners in the city lose too. Instead, the local authority benefits from all aggregate housing consumption taxes.

What tax level would the local government choose if its objective were to maximize the sum of aggregate taxes (Rh_1) levied from residents in city 1? Since the tax affects both the city size and the number of households, the objective function of the local authority is

$$Rh_1 = h_1 k r_1 = h_1 k \left[\frac{\alpha N^A + N^B}{3k} - \frac{2h_1}{3t} \right]. \tag{2.8}$$

Maximizing the above expression with respect to h_1 yields the optimal value, h_1^*, which is

$$h_1^* = \frac{t}{4k} \left[\alpha N^A + N^B \right]. \tag{2.9}$$

This optimal tax corresponds to a city size

$$r_1^* = \frac{1}{6k} \left[\alpha N^A + N^B \right]. \tag{2.10}$$

After substituting 2.9 in the expressions of z^B and z^A in 2.6 and 2.7, we obtain:

$$z^B(h_1^*) = Y^B - P - \frac{5t}{12k} [\alpha N^A + N^B], \tag{2.11}$$

and

$$z^A(h_1^*) = Y^A - \alpha P - \frac{\alpha t}{12k} [5N^B - (4+\alpha)N^A]. \tag{2.12}$$

Both levels of (private goods) consumption are smaller compared to the ones achieved in the market situation. The effects on z would be identical if directly using a population control leading to the city size achieved when using h_1^*. Residents lose in a similar way both with taxes and population controls. On the contrary, landowners gain with the introduction of the population control, but are worse off with the tax on housing that ultimately diminishes land rents. Likewise, landowners of undeveloped land become worse with the tax. Local communities benefit, because they receive the tax revenues.

4.2 Equilibrium values when two cities use taxes

As with the population growth control case, consider now that all cities in the system except for the passive city 3 impose taxes on housing consumption. Thus, 1 and 2 enact taxes h_1 and h_2. In order to maximize the tax revenue, city 1 must now consider the behaviour of all other *active* cities, and so must city 2. The expression for 2.8 in page 20 will determine the level of z^B in the system, common to the three cities. Applying the condition that $L^B(r_1) = L^B(r_2) = 0$, z^B can be expressed exclusively in terms of the taxes applied by cities 1 and 2. Thus

$$z^B(h_1, h_2) = Y^B - P - \frac{t}{3k}[\alpha N^A + N^B] - \frac{1}{3}[h_1 + h_2]. \tag{2.13}$$

And from the expressions in 2.8, we find that

$$z^A(h_1, h_2) = Y^A - \alpha P - \frac{\alpha t}{3k}[N^A + N^B] - \frac{\alpha}{3}[h_1 + h_2]. \tag{2.14}$$

The expression for r_1 is computed similarly:

$$r_1 = \frac{\alpha N^A + N^B}{3k} + \frac{1}{3t}[h_2 - 2h_1]. \tag{2.15}$$

The objective of city 1 will be again to maximize tax revenues, but now taking into account the decision adopted by city 2. Using the expression of r_1 in 2.15, we can write the maximization problem for 1:

$$\max_{h_1} \; h_1 k \left[\frac{\alpha N^A + N^B}{3k} + \frac{1}{3t}(h_2 - 2h_1) \right], \tag{2.16}$$

The solution is the best response function for city 1:

$$h_1(h_2) = \frac{t(\alpha N^A + N^B)}{4k} + \frac{h_2}{4}. \tag{2.17}$$

By symmetry, we can see that the best response function for city 2 is:

$$h_2(h_1) = \frac{t(\alpha N^A + N^B)}{4k} + \frac{h_1}{4}. \tag{2.18}$$

Notice that the sign of the partial derivatives of the reaction functions is positive. Contrary to what happened in the population control game, now the best response functions slope upward in the tax game. This means that taxes as growth control instruments act as *strategic complements* rather than substitutes.[3] When 2 chooses a relatively high tax, more households are diverted to the remaining cities in the system, including 1, since the decision is made considering that h_1 remains fixed, but not the population level in 1. By imposing a higher h_1 it is possible to increase the revenue levied from those diverted households.

The equilibrium is found by solving the system of equations formed by the two reaction functions. The equilibrium taxes are:

$$h_{comp} = h_1 = h_2 = \frac{t}{3k}[\alpha N^A + N^B]. \tag{2.19}$$

The tax revenue that corresponds to the equilibrium taxes is

$$Rh(h_{comp}) = \frac{2t}{27k}[\alpha N^A + N^B]^2 \tag{2.20}$$

for any of the cities enacting taxes. The resulting city sizes are

$$r(h_{comp}) = \frac{2}{9k}[\alpha N^A + N^B]. \tag{2.21}$$

This city size is larger than in the one controlling city case, and it is smaller than the size obtained in the population control game. However, comparisons must be carefully

[3] Like the strategies of Bertrand competitors. See Bulow et al. (1985).

made, since different objective functions have been used for the population and the tax control games. Alternatively, the total level of revenues should be compared.[4]

Other studies have shown that price instruments lead to higher equilibrium populations (Helsley and Strange, 1995). As for the levels of z^A and z^B which ultimately affect the levels of utility in the system, we find that

$$z^B(h_{comp}) = Y^B - P - \frac{5t}{9k}\,[\alpha N^A + N^B], \tag{2.22}$$

and

$$z^A(h_{comp}) = Y^A - \alpha P - \frac{\alpha t}{9k}\,[5N^B + (3+2\alpha)N^A]. \tag{2.23}$$

The consumption of non-land goods diminishes when introducing strategic interaction between cities, for both types of households, again compared to the outcome in which there is a single controlling city. As the number of cities using taxes increases, the negative effects on z are more important, because the number of households that are diverted from the controlling cities is larger. This causes housing and land rents in the system to increase.

4.3 A cooperative framework with taxes

As in the population control scenario, we consider the possibility of cooperation between jurisdictions in their tax decisions. Given the symmetry of the set up, the optimal collusive choice will result in equal tax rates in both cities. Therefore, we can simplify and express aggregate revenues from taxes for any of the active cities as:

$$Rh(h_{coop}) = h_{coop}k\,[\frac{\alpha N^A + N^B}{3k} - \frac{1}{3t}h_{coop}]. \tag{2.24}$$

[4]This comparison will be further explored in an upcoming section.

Maximizing with respect to the tax level h_{coop}, we find the optimal tax under cooperation:

$$h_{coop} = \frac{t}{2k}\,[\alpha N^A + N^B], \tag{2.25}$$

The taxes are now larger than the Nash equilibrium tax level of the noncooperative scenario, $h_{coop} > h_{comp}$. Accordingly, the resulting city size for the controlling cities is

$$r(h_{coop}) = \frac{1}{6k}\,[\alpha N^A + N^B], \tag{2.26}$$

which is smaller than the size under competition.

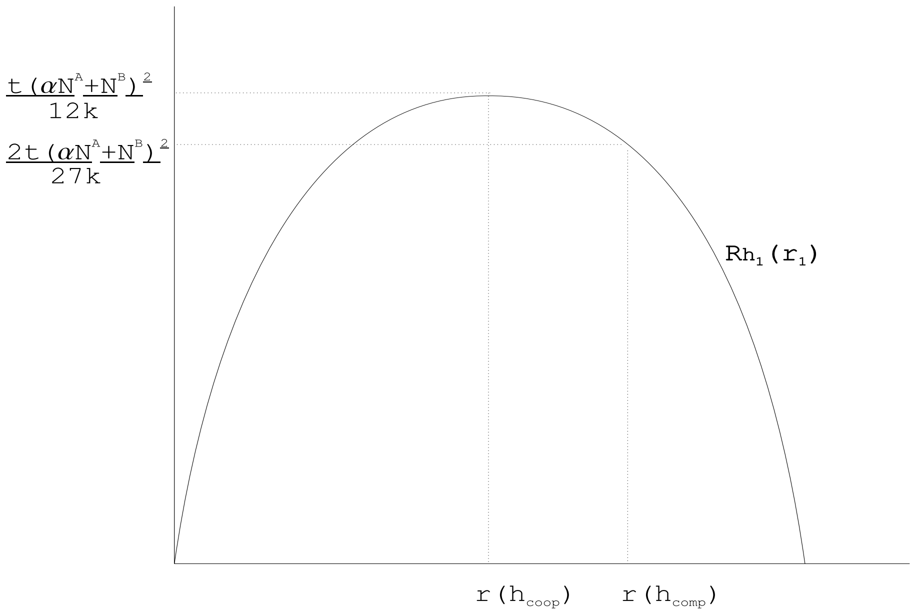

Figure 2.1. Tax revenues under symmetrical tax levels

Let $Rh(h_{coop})$ denote the tax revenue when both cities cooperate. Likewise, $Rh(h_{comp})$ in equation 2.20 denotes the revenue under competition. It can be shown that $Rh(h_{coop}) > Rh(h_{comp})$, that is, tax revenues are larger under the cooperative framework. Assuming that one city enacts the cooperative tax, the best tax level for the other city can be calculated from the city's best response function. Let h_{dev} denote the tax chosen when deviating. Then,

$$h_{dev} = \frac{3t}{8k}\,[\alpha N^A + N^B]. \tag{2.27}$$

That is, the city that deviates from the agreement gains from imposing a lower tax rate. The gains from the attraction of a larger number of residents offset the loss associated with the collection of a smaller per capita tax. The overall tax revenue for the city that breaks the cooperation agreement is

$$Rh(h_{dev}) = \frac{3t}{32k}\left[\alpha N^A + N^B\right]^2, \tag{2.28}$$

while the city that enacts h_{coop} obtains a diminished tax revenue of

$$Rh(h_{coop'}) = \frac{t}{16k}[\alpha N^A + N^B]^2, \tag{2.29}$$

where $Rh(h_{coop'})$ denotes overall tax revenue for the city that chooses h_{coop}. If only revenues from a single period are considered, then the highest revenues are obtained by the city that deviates from the cooperative agreement, while the other maintains the cooperative tax h_{coop}. Thus,

$$Rh(h_{dev}) > Rh(h_{coop}) > Rh(h_{comp}) > Rh(h_{coop'}) \tag{2.30}$$

Equilibrium in a static context As in the case of population controls, we will find that cooperation is not self-enforcing in a static one-period framework.

Consider the case in which the effects of the introduction of taxes are going to be realized for a single period only. Then, as it happened in the population control case, it is a dominant strategy for both cities to always choose to compete, and therefore the equilibrium results in tax levels h_{comp} for both cities. The unique equilibrium is Pareto dominated by the cooperative agreement, that is, although communities would be able to attain higher revenues if they maintained the cooperative tax level, cooperating is not an equilibrium.

The game is depicted in table 2.2. The fact that competition constitutes a dominant strategy is a direct consequence of the inequalities among revenues shown in equation 2.30. In equilibrium, both cities achieve an identical tax revenue of $R(h_{comp})$.

		City 2: Cooperate	City 2: Compete
City 1	Cooperate	$Rh(h_{coop}), Rh(h_{coop})$	$Rh(h_{coop'}), Rh(h_{dev})$
	Compete	$Rh(h_{dev}), Rh(h_{coop'})$	$Rh(h_{comp}), Rh(h_{comp})$

Figure 2.2. Static game with taxes when allowing for cooperation.

Equilibrium in a dynamic context We refer the reader to the discussion we made in section 3.3, about the analysis of the possibilities for cooperation in the population controls in a dynamic setting, because most of it applies here as well.

Again, a well-defined end period will cause competition to be the only (subgame-perfect) equilibrium. But an indefinite time horizon allows sophisticated punishment strategies in the case of deviations from a cooperative agreement, and therefore cooperation may be sustained as an equilibrium, provided both cities attach enough value to future revenues.

As in the population controls case, we will show how trigger strategies allow cooperation to be sustained as an equilibrium, provided the discount factor is large enough. In order to do that, we compute the present values of tax revenues in the three following cases:

- Both cities cooperate for ever: $PVR_h(h_{coop})$.
- Both cities compete for ever: $PVR_h(h_{comp})$
- One city cooperates and the other deviates in the present period, and then both compete for ever after: $PVR_h(h_{coop'})$ and $PVR_h(h_{dev})$, respectively.

For each case, the expressions for aggregate tax collections are shown below.

$$Rh(h_{coop}) = \frac{t}{12k}[\alpha N^A + N^B]^2 + \frac{t}{12kr(1+r)}[\alpha N^A + N^B]^2 \tag{2.31}$$

$$Rh(h_{comp}) = \frac{2t}{27k}[\alpha N^A + N^B]^2 + \frac{2t}{27kr(1+r)}[\alpha N^A + N^B]^2 \tag{2.32}$$

$$Rh(h_{dev}) = \frac{3t}{32k}[\alpha N^A + N^B]^2 + \frac{2t}{27kr(1+r)}[\alpha N^A + N^B]^2 \tag{2.33}$$

$$Rh(h_{coop'}) = \frac{t}{16k}[\alpha N^A + N^B]^2 + \frac{2t}{27k}[\alpha N^A + N^B]^2 \tag{2.34}$$

To find out whether cooperation can be sustained as an equilibrium, we must compare aggregate tax revenues if cooperation prevails with those obtained by one player when deviating. Deviation results in a larger revenue in the current period, but leads to the smaller competition revenue levels from then on.

Once the cooperation agreement has been abandoned by one of the cities, competing becomes the Nash equilibrium strategy in the subgame that results. The cooperative solution is a Nash equilibrium as well if the present value of revenues under cooperation exceeds the present value of revenues when deviating, that is

$$Rh(h_{coop}) > Rh(h_{dev}), \tag{2.35}$$

or

$$\begin{aligned} &\frac{t}{12k}[\alpha N^A + N^B]^2 + \frac{t}{12kr(1+r)}[\alpha N^A + N^B]^2 > \\ &\frac{3t}{32k}[\alpha N^A + N^B]^2 + \frac{2t}{27kr(1+r)}[\alpha N^A + N^B]^2 \end{aligned} \tag{2.36}$$

Operating the expression above and solving for r it results that cooperation is the equilibrium solution as long as the interest rate satisfies

$$r < \frac{-3+\sqrt{41}}{6} \approx 0.567. \tag{2.37}$$

Equivalently, the discount factor should be larger than, approximately, 0.638.

The same reasoning we made in the case of population controls applies as well here. To the extent that the legislatures are not myopic and look further enough into the future, cooperation can be sustained as a (subgame-perfect) equilibrium, when the time horizon is indefinite.

5. Comparison of results with population controls and taxes

We have so far assumed that the housing tax and the population controls were endogenous, in the sense that they maximize the respective objective functions set by the local communities. In this section, aggregate taxes levied are compared depending on the instrument used. Because households' utility levels do not depend upon any local public good or urban amenity, the expenditure side of the tax collection is being ignored. It can be likewise argued that the positive effect on households' utility of this expenditure would be the same for a constant level of tax revenues, and then only the negative effects associated with the particular source of the fiscal revenue matter.

To compare the different effects of using population controls or taxes for collection purposes, we make the following assumption. Taxes on housing directly yield a certain amount of tax revenue. As for the population control effect on fiscal revenues, two extreme scenarios can be considered. Firstly, it could be the case that only increased land rents were taxed, for instance if the whole increase in land rents was captured by the local government. The second possibility consists of assuming that the local community imposes a certain tax p on total land rents, not only on value increases. (For instance, with $p = 1$ the local community would appropriate all land rents. Obviously, such an extreme tax would cause landowners to lose all incentives to efficiently allocate each plot of land to the highest bidder.)

The comparison of tax revenues under each instrument will be simpler if we represent each revenue level against the associated city size r_1, as in figure 2.1. The graph plots the revenue size associated with each city size, under three different scenarios: housing tax competition, population controls such that the tax revenue equals the increased land rents, and population controls such that the tax revenue equals *all* land rents. In all cases only symmetric solutions are considered.

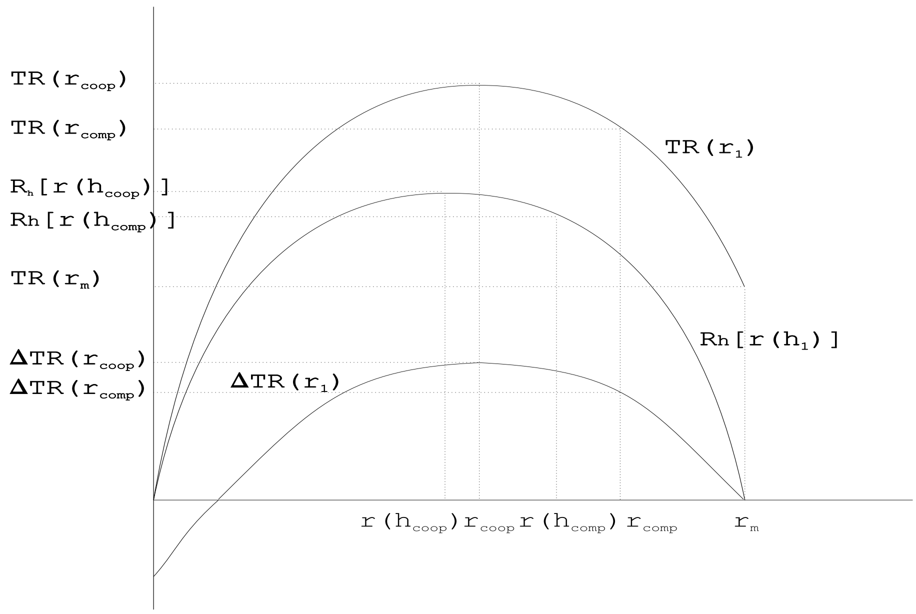

Figure 2.1. Comparison of tax revenues with population controls and housing taxes

The relationship between land rents –or increased land rents– and city size is straightforward, by rearranging the general expression of land rents in equation 2.4. As for the relationship between housing tax revenues and city size, it can be easily obtained by combining the expression of housing tax revenues in equation 2.8 with the expression in equation 2.5 in page 31, that relates the housing tax level h_1 with the city size r_1. Each housing tax level is uniquely associated to a certain city size. As can be observed, the three revenue curves have a Laffer type shape. Thus, a small city size can represent either the utilization of a too stringent population control or the result of a relatively high tax on housing consumption. A smaller city is associated then either to a large increase in land rents or to to a higher housing tax. Both facts provoke a greater per capita revenue, but a reduction in the base of the revenue due to the fact that less residents remain in the city in equilibrium. Several city sizes have been highlighted: r_m, which represents the city size corresponding to the market situation; r_{comp}, the equilibrium city size when competing with population controls; $r(h_{comp})$, the city size obtained when competing with housing taxes; r_{coop}, the city size when cities set cooperatively their population controls; and $r(h_{coop})$, the resulting city size when cities cooperate to fix their housing taxes.

A city size of r_m corresponds to a situation where there is no population control or a tax $h_1 = 0$, and as a result $Rh = 0$ and $\Delta TR_1 = 0$. Total land rents equal the market value, that is TR_m.

When the increases in land rent values due to the introduction of population controls are fully taxed, housing taxes are always superior to population controls, because a fixed revenue level can be achieved at a smaller cost in terms of the decrease in residents' utility. The optimal city size when maximizing increased total land rents ΔTR_1 is $r_1 = (1/5k)[\alpha N^A + N^B]$ —the city size that maximizes aggregate land rents TR_1. The revenues arising from the implementation of a tax on housing leading to the same city size are greater. Alternatively, the revenue obtained with this city size, $\Delta TR_1 = \frac{t}{144k}[\alpha N^A + N^B]^2$, could be attained with a housing tax level leading to a greater city size –and as a result, with a smaller loss in residents' utility. For values of the city size greater than the optimal level r_1, the diverting of population caused by a direct population control provokes that total revenues begin to decline, up to the city size $r_1 = \frac{1}{6k}(\alpha N^A + N^B)$, which implies again that $\Delta TR_1 = 0$.

Secondly, consider a tax that levies total land rents, and not only land rent rises. Under this scenario, the comparison between housing taxes and population favors the population control instrument, since total land rents are always superior to taxes in terms of total revenue for identical city sizes. Under this total confiscation of land rents, the problem is that landowners have no incentive to efficiently allocate their land.

There exists an intermediate tax rate p that could be applied on total land rents, that would lead to identical outcomes in terms of tax revenues. Analytically, this p tax rate can be expressed in terms of the parameters, but its (complicated) expression adds no further intuition.

6. Conclusions

A recent trend in the urban economics literature has centered on the analysis of urban regulations as the result of strategic interaction among local jurisdictions. We develop in this chapter a framework to consider this problem. We use the framework to analyze the influence of price and quantity instruments as strategic variables to manage urban growth. We show that population controls are strategic substitutes, while taxes are strategic complements. This distinction lies at the base of the differential impact of those instruments.

We limit our analysis to two (active) cities, but it could easily be extended to n cities. In this case, it can be shown that the equilibrium utilities of the residents are smaller the larger is the number of cities using controls.

Measured in terms of residents' utility, competing with population controls is desirable because it leads to relatively greater city sizes and, consequently, to smaller negative impacts on utilities. If measured in terms of total revenues, population controls are superior to housing taxes only when all land rents are confiscated, but inferior when only increased land rents constitute the tax revenue.

An interesting contribution in this chapter is the consideration of the possibility of *cooperation among jurisdictions.* We show that, even though cities can gain by acting cooperatively, they may not be able to enforce cooperation if the city authorities do not care enough about the future, for instance if their planning horizon ends with their mandate. On the contrary, whenever city authorities are long-sighted enough, cooperation is self-enforcing.

Our framework opens up the possibility of future work to explore very interesting issues, like the influence of density, or the differential effect of policies on the distribution of income. We think that additional attention and a more careful analysis should be devoted to the distributional consequences of planning instruments. Another interesting possibility is to consider the influence of externalities. The presence of externalities is one of the reasons usually used to provide an economic justification of urban land controls. Regarding this aspect, the model is flexible enough so as to easily incorporate environmental externalities affecting households' welfare, for instance in the form of density levels.