Universitat Autònoma de Barcelona

Modeling and Predictive Control of a Cash Concentration and Disbursements System

Carlos Antonio Herrera Cáceres

Universitat Autònoma de Barcelona

Modeling and Predictive Control of a Cash Concentration and Disbursements System

Carlos Antonio Herrera Cáceres

PhD Thesis

directed by

Dr. Asier Ibeas Hernández

Programa de doctorat en Telecomunicació i Enginyeria de Sistemes

Departament de Telecomunicació i Enginyeria de Sistemes

Escola d’Enginyeria, Universitat Autònoma de Barcelona

September 2016

Dr. Asier Ibeas, Associate Professor at the Department of Telecommunications and Systems Engineering, Universitat Autònoma de Barcelona

CERTIFIES

That the thesis entitled Modeling and Predictive Control of a Cash Concentration and Disbursements System by Carlos Antonio Herrera Cáceres, presented in partial fulfillment of the requirements for the degree of Doctor, in the doctoral program named *Telecomunicació i Enginyeria de Sistemes*, Escola d'Enginyeria, Universitat Autònoma de Barcelona, has been developed and written under my supervision.

Dr. Asier Ibeas Hernández

Bellaterra, September 2016.

For

Juan Carlos.	The first time I saw you, my life changed radically. The world was no longer for me. Since that moment, the world is all for you.
Margely.	You are the miracle of my life, who makes life outbreak. Your presence makes everything new and beautiful.
Francisco.	I asked God you were born and here you are with us. I see in your eyes that you want to explore and understand the world. More than anyone, you can do it.
Lucía.	Your voice is music to my ears. You are my treasure, your time will come soon.
Juan Pablo.	You are the tenderest of my life, our future. You will do great things.
Isabella.	You have sprouted into this world with the opportunity to carry out this project.

Thanks to

Asier Ibeas — You have been a stalwart partner in this project, your character is implicit in its content. Thanks for being there. Were it not for you, the project would not have materialized.

Universidad Autónoma de Barcelona.

Thank you for being my host in Spain and allowing the project to come true: Illustrious University.

Juan Carlos, Margely and Francisco.

Unconditional allies forever, thank you. This is also your work. You are part of the required effort for completion.

Emma — Your sacrifice is also implicit in the content of this work. Thanks for your understanding and patience. Thanks for the love you profess me.

Fidelina — You have given me the life and, thanks to that, all this exists.

I know God is with me, so I am sure He has helped me to achieve this goal.

Contents

Glossary

This is a summary of the notation, symbols, variables and acronyms used along the thesis.

Notation and symbols

$\tau (= 1)$	Basis time interval.
t	Variable indexing time (of the form $h\tau$, $h \in \mathbb{N}$).
z	Forward shift operator used in Z-transform.
z^{-1}	Backward shift operator used in Z-transform.
$L_{a,b}, L_m, L_p$	Time delay (of the form $h\tau$, $h \in \mathbb{N}$).
$x(t)$	Value of x in time domain (x at time t).
$X(z)$	Value of X in z domain, which is the transform of x in time domain.
$x(t+k\|t)$	Value of x as decision variable or control action, at stage k, with available information at time t.
$\hat{x}(t+k)$	Forecasted value of $x(t+k)$, at stage k.
$\hat{x}(t+k\|t)$	Predicted value of $x(t+k)$, at stage k, with available information at time t.
$\tilde{x}(t+k\|t)$	Expected value of $x(t+k)$, at stage k, without considering any control action, with available information at time t.
x_{min} or $\underline{x}$	Minimum value of x.
x_{max} or $\overline{x}$	Maximum value of x.
x^*	Optimal value of x.
(s, S)	Parameters of the cash transfers policy.
K	Linear feedback gain to stabilize the system.
X	Matrix or vector **X**.
$\mathbf{1}_N$	Column vector of dimension N with all entries equal to one.
$\mathbf{I}_{\mathrm{n}x\mathrm{m}}$	Identity matrix of size nxm.

$\mathbf{0}_{nxm}$	Matrix of size nxm with all entries equal to zero.
$[\cdot]^t$ or $\mathbf{X}^t$	Transposed matrix/vector or transposed of **X**.
$E(\cdot)$	Expectation operator.
$Var(\cdot)$	Variance operator.
$\lfloor x \rfloor$	The floor function.
$\lceil x \rceil$	The ceil function.
$frac(x)$	Fractional part of x ($x - \lfloor x \rfloor$).
$\underset{u \in \mathbb{U}}{min}\, J$ or $\underset{u \in \mathbb{U}}{min}\{\cdot\}$	Min function: Minimum value of J for all values of $u \in \mathbb{U}$.
$arg[\cdot]$	Function argument. It is used in combination with "Min function" in order to indicate the value of u for which J is minimum.

Model variables and parameters

$BASE$	Number of time intervals t per year used for calculate interest or fees for a single time interval t (Time of a year $= BASE.\tau$).
CCB	Daily change in the cash balance.
i_I	Annual passive interest rate.
i_O	Annual overdraft interest rate.
i_E	Annual active interest rate.
MT	Minimum size of a cash transfer.
Ʀ	Upper bound of the credit line.
$d_j^c(t)$	Disbursements of the j agency during discrete time interval t.
$d_j^a(t)$	Total cash required to cover the overdraft of the disbursements account of the j agency during interval t.
b_m^d	Portion of funds paid to suppliers according to the transit time L_m.
$r_j^c(t)$	Total revenue of the j agency during t (deposits in the revenue account).
$r_j^a(t)$	Money made available in the revenue account of the j agency at time t (based on deposits).
b_m^r	Portion of cash collection made according to the form of payment m.

L_m	Transit time (delay) of money paid to suppliers or collected from customers.
L_p	Time it takes (transit time or delay) for money to be available in the main account (p) of CCDS as a result of cash transfer order $u_{p,j}^{p}(t)$ effected from the agency's revenue account (or vice versa $u_{j,p}^{p}(t)$).
$L_{a,b}$	Time it takes to be available in the account a the money transferred from the account b.
$u_{a,b}^{p}(t)$	Order of available cash transfer made at the end of t from the b account to the a account of CCDS (amount of money that a requires b).
$u_{b,a}^{q}(t)$	Cash transferred from b account, which will be available in the a account at the end of discrete interval t (due to the cash transfer order $u_{a,b}^{p}(t)$ effected $L_{a,b}$ before).
$y_j^T(t)$	Actual balance of the j account at the end of time t.
$y_j^D(t)$	Deferred balance of the j account at the end of time t.
$y_j^L(t)$	Frozen balance of the j account at the end of time t.
$y_j^A(t)$	Available cash balance of the j account at the end of time t (after cash transfer).
$y_j^B(t)$	Available cash balance at end of t, recorded in the books of the company for j account (after cash transfer).
$y_j^C(t)$	Available cash balance of the j account at the end of time t, but before assessing any cash transfer policy.
$A(z^{-1}), B(z^{-1})$	Process polynomial matrices.
$C(z^{-1}), D(z^{-1})$	ARMA model polynomial matrices.
$e(z)$	White noise series with zero-mean and standard deviation one.
$n(z)$	Random value representing uncertainty, which affect the available cash level in the revenue account (equivalent to $r_j^a(t)$) or the overdraft level in the disbursement account (equivalent to $d_j^a(t)$).

$P(z)$	Rational part of the model, integrator ($P(z) = 1/(1 - z^{-1})$).
s	On the cash transfers policy, the amount of cash over which the agency should make the transfer.
S	On the cash transfers policy, the level at which returns the available cash after the transfer is performed.
M	Reference for cash balance in the main account
N	Prediction horizon.
$w(t + k)$	Reference signal that acts as a guide in the control process.
J	Objective function of the prediction process, which measures the total error in the prediction horizon between the predicted output and the predicted trajectory reference.
J_y	Part of the objective function of the prediction process: sum of squared deviations.
J_u	Part of the objective function of the prediction process: sum of the control effort.
J_k	Part of the objective function of the prediction process: sum until the stage k.
$\delta(k)$	On the objective function of DP model, it is the coefficient of the squared deviation for the control increment at stage k, taking into account the future behavior of the system.
$\lambda(k)$	On the objective function of DP model, it is the coefficient of the control effort for the control increment at stage k, taking into account the future behavior of the system.
$\hat{c}(t + k\|t)$	Minimum accumulated error (cost) between the predicted output and the predicted trajectory reference until the prediction period $t + k$.

Acronyms

ADP	Approximate Dynamic Programming.
ARMA	Autoregressive Moving Average Process.
BIBO	Bounded-Input, Bounded-Output.

CAPM	Capital Asset Pricing Model,
CARIMA	Controlled Auto-Regressive and Integrated Moving-Average.
CCDS	Cash Concentration and Disbursements System.
CFP	Corporate Financial Planning
CSGPC	Constrained Stable Generalized Predictive Control.
DCF	Discounted Cash Flow.
DFT	Discrete Fourier Transform.
DP	Dynamic Programming.
EPSAC	Extended Prediction Self-Adaptive Control.
ERP	Enterprise Resource Planning.
FFT	Fast Fourier Transform.
GPCL	Generalized Predictive Control with Learning.
ICB	Impulse Control Band.
MBPC	Model Based Predictive Control.
MILP	Mixed Integer Linear Programming.
MPC	Model Predictive Control.
MRP	Material Requirements Planning.
MWLS	Mixed-Weights Least Squares.
NPV	Net Present Value.
QPGPC	Quadratic Programming Generalized Predictive Control.
SME	Small and Medium Enterprise.
SOM	Self-Organizing Map.
SVR	Support Vector Regression.
ZBA	Zero Balance Accounts.

Chapter 1

Introduction

Corporate Financial Planning is a relatively complex subject that brings together diverse aspects generally treated as distinct issues, which have been fundamentally developed in the last sixty years. Indeed, whenever a researcher writes about any of these particular issues, explicitly or implicitly, he is doing it on corporate financial planning. That is why, in order to clarify its meaning, a definition is provided below[(1.1)]:

> *Corporate Financial Planning (CFP) is a systematic process guided by financial executives, which is designed and carried out with the purpose of establishing financial policies for the future and the goals to ensure the growth or survival of the organization. This process takes as inputs on the one hand, the current financial state of the organization and on the other, the possible scenarios. As a result, through an aggregation mechanism with the participation of different organizational divisions, a comprehensive corporate financial plan is established.*

The corporate financial plan is a manual containing, firstly, the financial objectives and policies, which go hand in hand with the general philosophical and strategic approach of the organization. Secondly, the manual incorporates a summary of the consolidated financial result of all investment projects and business units belonging to the investment portfolio. This includes the most outstanding features in terms of opportunities, risks assumed, interactions between corporate financing and investment decisions, and capacity to cope with expected and unexpected situations. Thirdly, the corporate financial plan contains the capital budgeting itself. That is, the investment plan including, in addition to a list of investments, an implementation schedule and the respective funding source in each case. Financing is subject to the options on the market, the cost of capital, capital structure, projected discounted cash flow (DCF) and organizational policies. Individually or in groups, investment projects are based on a budgetary analysis including, on the one hand, both the analysis of the ordinary

[(1.1)] Definition of the author.

activities using techniques for the revenue forecasting, and proper determination of the cost structure, by applying the relevant technical and administrative studies. On the other hand, details regarding financial incomes and expenses are expressed, as well as depreciation, amortization and debt service. As is known, this analysis makes a special examination of the tax aspect, depending on the country concerned. Finally, the corporate financial plan includes a consolidated cash flow projection (comprehensive and detailed), particularly noting the organization's ability to grow or survive.

Ideally, the corporate financial plan includes pro forma financial statements, consolidated and by business unit or project. This format is of great help for the completion of the financial analysis of short and long term. The corporate financial plan clearly differentiates between:

a) Objectives and policies for cash management and treasury.
b) Corporate short-term financial plans.
c) Objectives, policies and plans in the long term.

When the corporate financial plan is carried out, the organization adapts to its surroundings, adjusting to market fluctuations and economic trends in the short and long term. Depending on the size and characteristics of the organization, corporate financial planning uses sophisticated methods to choose from the best investment options under uncertainty and raise the needed funds, by deciding appropriately the origin of them and making an adequate assessment of the capital cost. In the CFP process, financial managers must establish policies on income distribution among dividends, retained earnings and taxes. Without neglecting their innate personal skills, they are compelled to deal with capital budgeting problems, taking into account interdependence and risk characteristics in each case. In this task, discounted cash flow (DCF) analysis has its recognized importance. Specifically, a good corporate financial plan should ensure balanced allocation of resources, determine the optimal financial structure and, ultimately, find a positive impact on firm value, based on the establishment of consistent financial policies. For this, CFP uses conceptual and technological tools based traditionally on empirical and theoretical models and currently in innovative intelligent models. In countries with highly developed capital markets, financial managers must understand and evaluate sufficiently its functioning and behavior. In this scenario, an explicit recognition of the interrelationships between assets and liabilities should be performed. In countries without capital markets or major flaws in them, financial manager's decisions on financing are subject to banking voracity and rigidity or flexibility of government political system.

In this regard, a model for CFP is a *logical-mathematical description about the financial dynamic of an organization, built to support the process of CFP and facilitate the assessment of its performance*. The objectives of a model for CFP are diverse. In general, any of the following or combination of them:

- ✓ To establish relationships between investment options and financing.
- ✓ To predict the future behavior based on projections of exogenous and endogenous financial variables.
- ✓ To support decisions on the best options.
- ✓ To prepare pro forma financial statements.

On the one hand, a model for CFP includes the corresponding sensitivity analysis, considering various scenarios and their financial implications. Furthermore, a model for CFP facilitates the analysis of the actual behavior of financial variables in comparison with established objectives and plans. Finally, a model for CFP should be robust. That is, a model that produce feasible results for a wide range of potential financial scenarios, subject to the particular context of decisions and working interactively with those directly involved in the effective construction of the corporate financial plan, assisted by one or more financial analysts. In the presence of uncertainty, the entire creation process should be robust and that condition must remain in time according to the planning horizon, including the method, the set of solutions and conclusions drawn.

According to the characteristics and objectives, there are several types of models for CFP. In this sense, it is worth highlighting research contributions that have direct theoretical-practical implications on CFP and the formulation of corporate financial models, which are also generally applied in financial economics. Besides, it is necessary to explore other issues directly related to modeling CFP or that help in its formulation. First, CFP is largely designed as a mirror of the accounting. This is because in an organization, the events measurable in monetary units are processed according to accounting standards. Thus, a basis is provided for comparing the real behavior with the goals and plans when they are executed. Consequently, research on models for business accounting is also noteworthy at this thesis. In addition, assuming that financial planning is an integral part of the general corporate planning, researches concerning financial planning combined to corporate strategic planning are taken into consideration, which is also justified since it is necessary to formalize different aspects or functions in order to successfully perform planning activities. Certainly, this work is framed within CFP models and their applications. Somehow, some of other issues closely linked to CFP are dealt with in so far as it necessary. Among these topics, it is worth mentioning: Capital budgeting, capital structure, financial structure, leverage and debt, cost of capital, cash management or working capital management, cash flow, prediction and forecasting, risk analysis, valuation, financial control and interactions between corporate financing and investment decisions.

The subject is as extensive as interesting. However, trying to delimit this research, it has been wanted to address a fundamental issue of CFP. That is, the study of cash management and the short-term financial planning through the movement of cash between bank accounts involved in the important financial decisions of a firm (item b on page 8). In this way, the work focuses on Cash Concentration and Disbursements Systems (CCDS), which are used by firms for the purpose of improving the planning and control of current assets and cash management. The aim of a CCDS is to concentrate available cash in a master bank account in order to make best use of money in large amounts to support investment and financing operations. Consequently, the object of study, the main motivation of this research, is to achieve an accurate representation of a CCDS, allowing its numerical simulation, analysis and evaluation, as well as the subsequent possibility of exploring new researches and the development of algorithms for the financial decision support, based on tools of control theory.

1.1 Problem statement and novel contributions of the thesis

Specifically, the thesis proposes three things. Firstly, designing a mathematical model of a CCDS to support, on the one hand, the financial planning of a company in the short term and, on the other, the operations of cash concentration and disbursements of the company in real time. This support is achieved by adding a set of automatic modules, based on the previously said mathematical model, as part of the information systems of the company. Using this model as a basis, at second instance, the thesis proposes formulating a model predictive control (MPC) to automate the process of cash transfers between elements of CCDS considering optimization criteria in managing the bank accounts of the company.

This perspective is novel in the sense that the automatic control has not been previously considered as part of a CCDS, which is reinforced, since the thesis considers the development of a versatile predictive controller under a decentralized approach that is applied primarily in bank accounts with greater movement. Is worth to say, bank accounts directly related to customers and suppliers. In this sense, the controller is developed to automate accounts receiving money from customers, after which the necessary adaptations are made, in order to use the controller to automate the accounts through which money is paid to suppliers. Thirdly, based on concepts regularly accepted in the corporate financial field, the thesis proposes using optimal control for bank accounts related to invest cash surplus or fund the cash deficits. This combination results in a comprehensive model involving the entire cash management of the company, giving rise to the support required by the financial management of the company to achieve its goals. In this regard, the complete functioning of the CCDS is demonstrated through a hypothetical case, designed to illustrate the potentiality and versatility of the model.

1.2 Structure of the thesis

From the above, the thesis has been structured as follows:

- ✓ ***Chapter 2*** is devoted to presenting a literature review (historical and current) about scientific and academic contributions on modeling corporate financial planning. This review focuses mainly on theoretical proposals. However, some relevant empirical researches, or directly linked to theoretical models presented are taken into account. Its analysis includes the approach adopted and the practical application, if any. At the end of the chapter, a literature review about CCDS as a specific application of models for CFP is presented.
- ✓ ***Chapter 3*** presents the mathematical modeling of the Cash Concentration and Disbursements System, which is made by using difference equations to analyze the behavior of CCDS in discrete-time and, then using the *Z*-transform to obtain a transformed model suitable to apply tools arising from the systems theory and control to make managerial decisions. The problem is seen as a particular problem of supply chain management, for which a decentralized approach is adopted. The control for each one of bank accounts is focused as an inventory problem.

- ✓ ***Chapter 4*** presents the control problem of the revenue bank account contained in the CCDS, which is addressed by proposing a Model Predictive Control (MPC). The solution for this problem of Model Predictive Control is made by constructing a backward Dynamic Programming (DP) model, which is formulated as a base for application in other bank accounts of the system, with the necessary adjustments.
- ✓ ***Chapter 5*** is engaged in the formulation of a MPC for a disbursements bank account, also contained in the CCDS, from the proposed model in Chapter 4. The proposal considers cash transfer time delay (transit time) from the viewpoint of the agency, which means that the control strategy is different in several aspects regarding to the revenue account's model.
- ✓ ***Chapter 6*** presents a case study whose purpose is to demonstrate the ability of the model and proposed controller to achieve the intended purposes. The approach applied for the control of the main account of the CCDS keeps the cash balance in a band system, subject to revenue and disbursements of the firm as well as the availability of cash invested and the credit line.
- ✓ ***Chapter 7*** contains the conclusions of the research and expectations of related future researches. Moreover, various appendixes are included with the purpose of improving the comprehension of the work content.
- ✓ ***Appendixes A to E*** complement the contents of the thesis.

The main result of this thesis is that it opens a very fruitful path for countless research by combining systems engineering, control theory and corporate financial planning.

Chapter 2

A survey on models for Corporate Financial Planning: State of the art

This chapter presents a literature review about scientific and academic contributions on modeling Corporate Financial Planning (CFP), mainly corresponding to theoretical proposals, including some relevant empirical researches, or directly linked to theoretical models presented. It has been wanted to give special credit to those who have proposed important theories and models in corporate finance. A literature review about Cash Concentration and Disbursements Systems (CCDS) is also performed as part of the corporate short-term financial planning, particularly in the context of cash management, as well as the treatment of cash balances using the concepts of inventory management. Hence, the study begins (Section 2.1) with those pioneer researchers whose theories have direct theoretical-practical implications on CFP and the formulation of corporate financial models, which are also generally applied in financial economics. With the purpose of exploring other issues directly related to modeling CFP or that help in its formulation, the rest of the chapter is structured as follows. Section 2.2 presents the contributions or researches corresponding to Models in Business Accounting. Section 2.3 is dedicated to publications concerning financial planning joined to corporate strategic planning. Because it directly addresses the objective of this chapter (CFP models), Section 2.4 is the bulk of the content. Section 2.5 presents publications devoted to the application of models for CFP. Section 2.6 makes a literature review about CCDS framed within the corporate short-term financial planning. Finally, Section 2.7 includes the concluding remarks of the chapter.

2.1 Financial theories

This section presents theories that have had direct theoretical and practical implications on the development of corporate financial models. In addition to the topics already mentioned, it discusses research that addresses issues such as: portfolio selection, conditions of perfect / imperfect capital markets, investment diversification, pricing of share, theory of capital optimal accumulation, market interdependencies, Tobin's q-theory, single-period or multi-period model, equilibrium model, uncertainty, critical rate of return and financially constrained firms, among others. In general, this section includes twenty-four research papers relevant for all corporate finance literature.

The starting point of literature review is fixed in Markowitz (1952, 1959), due to his important contributions in the portfolio selection process. He establishes the premise of risk interdependence among all investment projects in a firm, which proposes the use of the decision-making process of an investor's portfolio to address the problems of corporate capital budgeting. In the same decade, Lintner (1956) presents his research on corporate dividend policy. In turn, Modigliani and Miller (1958, 1961) lay the groundwork of the modern theory of capital structure and valuation theory based on the dividend policy, which stimulates the interest of financial economists on the subject. Their controversial theory, according to which the value of firm does not depend on the mode of financing under conditions of perfect capital markets, really has marked a milestone in modern finance theory. Again, Lintner (1962) defines necessary and sufficient conditions for the valuation of assets and the determination of the effects of leverage. Gordon (1962) develops a mathematical model that allows the company to determine the investment and financing policies that maximizes its value by pricing of its shares. Also, in the assumption that replacement investment is proportional to capital stock, Jorgenson (1963) presents a theory with the respective empirical tests of investment behavior based on the neoclassical theory of capital optimal accumulation. The decade of the 60's was very productive in terms of formulating theories. Based on a singular experiment, Brainard and Tobin (1968) present a reasoning about the need for considering the market interdependencies when formulating theoretical and empirical financial models. On the other hand, Tobin (1969) suggests that investment is a function of the ratio of the market value of new additional investment goods to their replacement cost (marginal q, Tobin´s q theory), that is, the firm's optimal capital accumulation problem with adjustment costs[2.1]. With the same significance level, Stiglitz (1969, 1974, 1988) argues for importance of financial policy to determine the firm's value in the framework of a general equilibrium model. The work of Stiglitz is based on a re-examination of the Modigliani and Miller (1958) model and its extension to a multi-period case.

[2.1] q represents the firm´s equilibrium (preferably $q \approx 1$). If $q > 1$, additional investment means that benefits are greater than the cost of the assets. If $q < 1$, it is better that the firm put up for sale its assets instead of using them.

Back in the early 70's, Schall (1971) examines the relevance of the investment diversification in the case of imperfect capital markets and shows that, under certain conditions, the investment diversification is also irrelevant with imperfect capital markets. A year later, from a general approach Leland (1972) establishes the necessary and sufficient conditions that determine the impact of uncertainty on corporate decision: principle of increasing uncertainty. Then, Korkie (1975) shows that principle of increasing uncertainty established by Leland does not hold for certain values of risk. Furthermore, Scott (1976) specifies parameters, which determine the firm´s optimal capital structure, particularly under conditions of imperfect information.

Taking up the Tobin´s approach, Hayashi (1982) presents a general model for the firm's maximization present value by integrating both theories of investment, Jorgenson´s and Tobin´s. Furthermore, he derives the optimal rate of investment as a function of q, and an exact relationship between marginal q and average q[(2.2)]. As for the assumption of Modigliani and Miller under conditions of perfect capital markets, Gordon (1989) suggests that the complexity of the reality of modern society cannot be interpreted in the light of that theory. But nevertheless, Detemple, Gottardi and Polemarchakis (1995) make their contribution to the debate arguing for the relevance and effectiveness of the financial policy relative to the value of the firm.

More recently, Hau (2004) discusses the conditions under which the principle of increasing uncertainty of Leland (1972) is violated. Dasgupta and Sengupta (2007) present a multi-period model of moral hazard by examining the relationship between the critical rate of return for the investment, the level of equity of financially constrained firms and interest rates. Lane (2009) shows a slight correction about the work of Modigliani and Miller under specific conditions of leverage. Lastly, Bolton, Chen and Wang (2011) propose a dynamic model of investment, financing and risk management, which emphasizes the central importance of endogenous marginal value of liquidity for corporate decisions with financial constraints.

In general, financial theories are applied in financial economics. However, when they are combined with accounting and budgeting schemes, they give rise to models for CFP. For this reason, the next section addresses the issue of business accounting models.

(2.2) According to Hayashi, marginal q is the ratio of the market value of an additional unit of capital to its replacement cost. Average q is the ratio of the market value of existing capital to its replacement cost.

2.2 Models in Business Accounting

In an organization, the events measurable in monetary units are processed according to accounting standards. This treatment involves recording, classifying and summarizing data. Hence, when performing a corporate financial plan, specifically, budgeting is built like a mirror of accounting. In this way, the real behavior can be compared to the objectives and plans when executed. Below are described research efforts to represent the accounting process by using mathematical models, as well as its relationship with the planning process.

The first contributions in this topic correspond to Mattessich (1957, 1958, 1964a), who in his time, directs attention to the growing use of formal mathematical methods in the field of accounting. He proposes a matrix formulation of an accounting system, the mathematical representation of a whole accounting cycle and the application of analytical methods in accounting. Also, Mattessich (1964b) publishes the details of a computer program that uses a model to simulate the budget process of a hypothetical industrial firm, which includes some adaptive control mechanisms. Demski (1967) represents the process of business planning through a linear programming model of accounting systems in the framework of a structured approach. For practical purposes and seeking new perspectives in the field of accounting and budgeting, Ijiri and Thompson (1970) apply mathematical control theory to analyze the accounting and budget problems. In order to study the behavioral aspects of the small business environment, Thornton-Trump and Fu (2000) use a continuous feedback financial model in the conceptual framework of accounting described in the Handbook of the Canadian Institute of Chartered Accountants.

Such works could be considered as the natural evolution of the technical-academic link between financial management based on historical data and financial management based on a vision of the future. That is, the use of prediction tools to support financial and strategic planning.

2.3 Financial planning within Strategic planning

Financial planning is an integral part of the overall planning of the firm, so it is necessary to formalize diverse aspects to successfully execute the task of planning. Some researchers have studied this relationship. For instance, Myers (1984) defends the integration of financial analysis and strategic planning, discusses the factors about the gap between it and the theory of finance. From Myers´s exposure until today, this integration is much more recognized in the technical and academic community. In this section a review is made on relevant publications and most recent both theoretical and empirical research of this issue. In addition, a classification of all inspected references on this topic is presented.

The relevant theoretical research about the role of the finance function within Strategic planning starts with Moag, Carleton and Lerner (1967). They suggest the notion of enterprise as a complex system and emphasize the need for applying technology with a view to create a set of integrated models as a single system. As part of its activities, the finance officer should take into consideration control between the different parts of the system, through the establishment and rationalization of performance standards and operational controls. Under this approach, financial control is justified because the basic function of finance is the system's growth. Hamilton and Moses (1974) describe a computer-based corporate planning system, which combines the optimization analytical power with simulation capabilities, pointing out the importance of making the maximum contribution to the corporate planning process. As explained before, Myers (1984) defends the integration of financial analysis and strategic planning and discusses three factors about the gap between it and the theory of finance: the language and culture, the misuse of the concept of DCF in strategic definitions, and the fact that the analysis of DCF may fail in certain circumstances. Barton and Gordon (1987) discuss the grounds on which should be unified the concepts of finance and strategy, describing the factors that help to understand the capital structure and have impact on the decisions about it. Gupta and Li (2003) develop a scheme that incorporates several aspects of behavioral preferences in the decision making process of saving and investing for long-term financial planning. These performance characteristics are incorporated in an optimization framework that enables the creation of robust systems for decision support. With the aim of achieving better integration between strategic management and financial theory, Vlachy (2009) proposes a model based on option theory to solve the problem of inconsistency between the carrying value of corporate assets and market value. In a general sense, Conneely (2011) examines the purpose and phases of financial management in relation to the strategic plan. Finally, based on the theory of corporate life cycle, Wang (2011) examines the financial characteristics of Small and Medium Enterprises (SME's) in different states and proposes to create a good business environment to undertake financial strategies, specify the behaviors and strengthen budgetary control.

Similarly, there exist significant empirical researches. Elliott (1972a) presents evidence on differences between the financial results of the firm when it is directed by a manager-owner, compared to when it is managed by a non-owner-manager. Walker and Petty (1978) present a study in which contrast different financial indicators and variables between small and large companies. Based on an extensive literature review, McInnes and Carleton (1982) analyze in detail the contributions to a normative theory useful for formulating corporate and financial models. Supported by a field study, they focus on the proposal providing a basis for comparing practice of financial modeling and identify differences between theory and practice. Robinson and Pearce (1983) examine the relationship between the formalization of procedures for planning and financial performance. Brooke and Duffy (1986) evaluate the effectiveness of information technology application to support the use of models for financial strategic planning into certain industrial and commercial organizations. Their results show that many problems are solved when this technology is based on direct participation in modeling activity of management responsible for the strategic management function. Rhyne ((1986) studies the relationship between financial performance and characteristics of corporate planning systems. Sandberg, Lewellen and Stanley (1987) present an operational framework that helps in the decision to embed decisions about the degree of financial leverage within corporate strategic plan. They show evidence to test the operational framework. In a meta-analytic study, Boyd (1991) discloses results that underestimate the relationship between strategic planning and financial performance. Capon, Farley and Hulbert (1994) conduct a critical review of meta-analytic results presented by Boyd (1991) concluding that there is a small positive relationship between strategic planning and financial performance. Kochhar and Hitt (1998) examine the relationship between strategic and financial management. Particularly, they develop and test the theory that links the diversification strategy of the firm with its capital structure. To conclude, Beech (2001) outlines the importance of considering market-based demand forecasting for develop future scenarios and evaluate each situation to determine its potential effect on certain financial and operational measures. This study uses a particular case of strategic planning applied to the provision of health services.

To complement, certain theoretical researches have been added which include some empirical evidence in order to prove their theories or hypotheses. In this regard, Schendel and Patton (1978) present a simultaneous equation model for corporate strategy to be used as a means to overcome the multiple objectives of firm performance. Through a case study, Kunsch, Chevalier and Brans (2001) validate the methodology of adaptive control seen as a tool for planning and online control of complex socioeconomic processes, combined with system dynamics and group multi-criteria decision aids. Then this scheme is compared with the recurrent planning and control of financial processes of the organization under study. Mercier (2002) uses the logic of real options theory to justify the integration between financial planning and strategic planning in oil and gas companies. More recently, based on a critical analysis of the difficulties and limitations when implementing financial subsystems of Enterprise Resource Planning (ERP), Zhu (2006) studies the case from a systemic viewpoint, with emphasis on system optimization and better integration with its surroundings.

Appendix A contains three tables (A.1, A.2 and A.3) showing complete results of the inspection to all publications classified as Strategic planning. These references are categorized according to the subject matter: financial strategy, planning and financial management, modeling for enterprise, Methodology / preferences and Systems. In accordance with the inspection, the type of research (theoretical or empirical) corresponding to each reference is indicated. Also, techniques, models and tools were specified, as well as any other observations, for example: empirical data analysis, testing, comparative study or literature review.

It can be concluded that the use of empirical techniques has been very important in these researches as well as simulation techniques using simultaneous equations, especially in the task of formulating firm models. On the other hand, the inclusion of critical analysis of Zhu (2006) on the implementation of ERP systems does not mean that the use of this technology is limited. On the contrary, there is enough experience in this field, but from academic viewpoint, the relationship between ERP systems and financial issue has been little studied.

2.4 Corporate financial planning models

This section is the center of the chapter. It presents a historical and current literature review around scientific and academic contributions to the formulation of models for CFP, mainly corresponding to theoretical propositions. The proposals have been diverse, guided by the efforts of several outstanding researchers. For this reason, trying to make a comprehensive analysis of the literature review, publications have been classified in four major categories according to techniques, tools, or type of model used in each case:

- ✓ Classical theoretical or analytical formulation,
- ✓ Mathematical programming,
- ✓ Simulation,
- ✓ New technologies.

It does not mean that those categories are pure in content, since in several cases they overlap between them. What is attempted here is to assign the publications within the category in which the author focused his study. Additionally, another classification (sub-categories) is used depending on the topic addressed in each publication: Capital budgeting models, cash flow patterns, cash management, earnings-per-share (EPS) growth models, equilibrium analysis, financial control, financial decisions, financial management, financial planning models, financial policy, financial strategy, financial structure, financing decisions, forecasting, investment planning, and simulation for financial planning.

Without making a judgment about their merits or weaknesses, a detailed description is made below of the most relevant researches in chronological order including the most current. A table listing all references inspected (see Appendix A) is also presented. This list indicates the topic (sub-category), the type of research (theoretical, empirical), techniques, models and tools used and any other important observations, for example, empirical data analysis, empirical evidence, comparative study and literature review.

2.4.1 Theoretical / Analytical formulations

This subsection includes relevant researches about formulating CFP models, which have based their discussion on classical mathematical tools, financial theories and firm´s theory, among others.

In the decade of the 60's, Teichroew, Robichek and Montalbano (1965) analyze the criteria and procedures of decision making under uncertainty. They use the concepts of Discounted Cash Flow (DCF) and internal rate of return, for the acceptance or rejection of investment and financing options available in companies. Moreover, together with a discussion about the entrepreneurial decision problems and the Gordon and Lintner models for valuing dividend capitalization, Lerner and Carleton (1966a) present an analytical solution to the price maximization problem for a company faced with restrictions both in product market and in financial market. The aim of the model is to maximize the share price of a company under uncertainty facing restrictions in product market, factor market and financial market. In the same year, Miller and Orr (1966) develop a simple analytical model for cash management, inventory management and other applications, incorporating characteristics of cash movements produced by commercial operations, together with the lumpy transfer cost feature of the Baumol model (Baumol, 1952). A decade later, Hite (1977) assesses the impact of leverage on the optimization of the company when the interest is tax deductible. For this purpose, he uses the classic Capital Asset Pricing Model (CAPM), which links the financial policy and the firm's actual decisions. Based on the optimal capital structure theory in a general context, Taggart (1977) presents a comprehensive model of corporate financing patterns, taking into account the restrictions of the balance sheet. Back in the 90's, Lewis (1990), discusses the impact of taxation on corporate financial policy by using a multi-period model and assuming that the only imperfection in the environment are taxes without considering other imperfections, such as costs of agency and bankruptcy.

Table A.4 (Appendix A) presents a complete list of the proposed CFP models based on classical theoretical or analytical formulations.

2.4.2 Mathematical programming

Researches on CFP models based on mathematical programming are the most numerous. The programming tools used are quite diverse, among them: linear, nonlinear, quadratic, integer linear and mixed integer linear; multiple-criteria decision making and its variants: goal and fractional programming; multiple-objective; linear and nonlinear stochastic programming; network and stochastic dynamic network programming; dynamic programming and global optimization. In some cases, mathematical programming has been combined with another type of tool or model such as simulation, mathematical control, stochastic control, multi-stage or heuristic models or by making empirical tests to verify results. Also in some instances, a single research includes two or more different mathematical programming models for comparative purposes. Below, are discussed in historical order the most relevant publications that have proposed CFP models based on mathematical programming.

In the late 50's, Charnes, Cooper and Miller (1959) apply theorems and tools of linear programming to support the decisions that have to do with:

- ✓ Allocating funds within a company.
- ✓ Determining the best operational program jointly with the financial planning.
- ✓ Evaluating projects based on their rates of return.
- ✓ Identifying the interactions between proposed investments.
- ✓ Determining opportunity costs, bearing in account the situations where there is capital rationing and liquidity constraints.

In the first half of the '60s, Ijiri, Levy and Lyon (1963) present an approach by combining linear programming and double-entry bookkeeping. Based on an initial balance sheet and relevant objectives, they explore planning options to lead the company to better balance sheet at the end of the period, taking into account other aspects of management policy and technological limitations. Thereby, useful information for management so as to support financial planning process is provided. It is also important the work of Weingartner (1963), who performs a revision to the methods proposed by Lorie-Savage on capital budgeting, and addresses the problem of selecting investment projects using a model based on integer linear programming. The aim of his model is to maximize the sum of the present values, subject to funding constraints because the firm does not have access to external sources of financing. Additionally, Weingartner (1966) conducts a review about the types of models needed to deal with situations of capital rationing. In the absence of a universally valid method, he suggests how to develop linear programming models useful as a guide for investment decisions. Although his models do not consider the uncertainty, Weingartner makes an explicit recognition of the need to take it into account. Likewise, Lintner (1964) proposes a stochastic dynamic programming model of business growth and equity values, to establish the optimal size of capital budgeting, dividends, retentions and the expected rates of growth over time. Baumol and Quandt (1965) use linear programming models basically like Weingartner, in finding a solution to the problems of selecting investments and discount rates under capital rationing. Meanwhile, Robichek, Teichroew and Jones (1965) present a linear programming approach in which raise the problem of short-term financing in companies under certainty determining optimal solutions for a number of cases, including an analysis based on marginal costs.

In the second half of the 60's, Chambers (1967) uses linear programming to provide a method for allocating funds among the projects of a company, preferably using internal sources and takes into consideration the way funding affects other financial statements besides cash flow. The proposal makes reasonable predictions about investment opportunities in a planning horizon of multiple periods and develops decision criteria for borrowing.

One of the main authors using mathematical programming models for CFP is Willard Carleton, who has proposed various models, e.g. for investment selection problem, which considers among other restrictions, borrowing and taxes using dynamic programming (Carleton, 1968). Also, (Carleton, 1969) presents a corporate financial model using linear programming based on the theory of capital budgeting, which considers the most important aspects of both proposals, Weingartner´s (1963, 1966) and Baumol´s *et al*. (1965). In his model the problem of selecting investment is only one part. That is, it includes a macro model to determine the capital cost, which is then used to determine the net present value of each individual investment project. This approach pays special attention to three aspects: (a) the investor's requirements, (b) the accounting principle considering the firm as an ongoing business, and (c) characteristics of each project evaluated as part of a whole. According to Carleton, the capital budgeting decisions must be consistent with the overall objective of corporate financial decisions. Carleton (1970) discusses about the interdependence among the investment and financing decisions as limiting the implementation of corporate financial models which creates a gap between theory and practice. Carleton points to the fact that to date the literature emphasizes mainly on the conceptual domain. Proceeding from this analysis, he proposes a deterministic linear programming model, whose application helps enterprises for long-term financial planning. The model maximizes the discounted flow of future dividends and the final price of the shares, taking into account the structure of the accounting and the financial constraints.

In the early 70's, Pogue and Bussard (1972a) suggest a reformulation of short-term financial planning model proposed by Robichek, Teichroew and Jones (1965), incorporating the concept of uncertainty associated with forecasting of cash requirements and including financing options, such as, commercial papers and multi-period investment options. Likewise, using mixed integer programming for investment selection and the financing scheduling, Hamilton and Moses (1973) describe the development and effective implementation of a multi-period optimization model for corporate strategic planning. This model considers the whole spectrum of financial decisions, including internal capital budgeting, acquisitions, disinvestments, creating and paying debt, issuance and repurchase of shares and dividend payments. Furthermore, Myers and Pogue (1974) also use mixed integer linear programming to develop a financial planning model, which sets as its goal the enterprise market value maximization, based on capital market theory and financial theory. As an interesting contribution at that moment, Crum, Klingman and Tavis (1979) develop a framework for conceptualizing and formulating financial planning models using a network-shaped mathematical structure, written starting from an initial formulation based on mixed integer linear programming (MILP). This approach includes facilities for interaction between financial executive and his model. It is also relevant the contribution of Ashton and Atkins (1979a, 1979b), who examine possible approaches for solving linear programming problems applied to the medium-term financial planning using multi-objective programming methods. The central theme of their research is the firm valuation problem, with independence from the planning horizon based on perfect information about future opportunities.

In the 80's, it is worth mentioning the work of Kallberg, White and Ziemba (1982), that uses stochastic linear programming to propose a model considering the fundamental factors of uncertainty on short-term CFP problem. This model incorporates issues such as forecasting cash requirements and costs of liquidation and termination, among others.

In the decade of 90s, Mulvey and Vladimirou (1992) propose a multi-scenario network dynamic model with stochastic parameters to represent the asset allocation problem in selecting investment portfolio, while showing evidence about the benefits of the networking stochastic approach. Five years later, Maranas, Androulakis, Floudas, Berger and Mulvey (1997) address the problem of financial assets allocation under uncertainty for different categories of investment in a long-term horizon. In this proposal a global optimization algorithm keeping constant the ratio of assets in the portfolio composition is used. The following year Güven and Kaynarca (1998) use mixed integer linear programming to introduce an integrated model for investment and financial planning, whose aim is to maximize the cash flows after taxes in environments where incentives are provided to the investment and tax exemptions.

Recently, Mulvey and Shetty (2004) describe a framework for modeling significant financial planning problems based on multi-stage optimization under uncertainty, including stochastic programming, nonlinear programming and dynamic stochastic control. Their work takes into account risk management and combines investment options with borrowing strategies. Xu and Birge (2006) propose an optimization model using stochastic integer programming with nonlinear constraints for integrated CFP, based on a valuation framework of the simultaneous decisions under uncertainty on production and finance. Also Badell, Fernandez, Guillen and Puigjaner (2007) raise the problem of integration between production scheduling and financial planning, for which, combine a deterministic cash flow management model with a formulation based on mixed integer linear programming (MILP). Finally, Uspuriene, Sakalauskas, Ginevicius, Rutkauskas, Pocs and Stankeviciene (2010) discuss the problems of financial management and decision-making for financial planning with the purpose of maximize *profit* in nonprofit organizations. To do so they present a two-stage stochastic linear programming model for short-term CFP.

Table A.5 (Appendix A) shows a complete list of researches concerning corporate financial planning models based on mathematical programming.

2.4.3 Simulation

Researches on CFP using simulation concept are also numerous. Frequently are raised from an experimental point of view or case studies. Also, simulation techniques are used for comparative studies and combined with other techniques such as: Empirical data analysis, empirical test, mathematical programming, simultaneous equations, recursive equations, interactive algorithms, least squares forecasting models, Monte-Carlo, system dynamics, filtering theory and intelligent models, among others. The most relevant publications that have proposed CFP models using the concept of simulation are commented below in historical order. Then, a detailed list of all publications inspected is shown.

Besides research presented previously in Section 2.2, Mattessich (1961) develops a model of simulation and experimentation based on simultaneous equations, combined with models of aggregation, for a Generalized Periodic Budgeting System. This approach is applicable to the firm as a whole, as well as a combination of models of suboptimization for individual departments.

In the 70's, Warren and Shelton (1971) propose a CFP model based on a system of simultaneous equations, which shows the overall operating performance of the company. The model does not provide optimal responses; instead, it corresponds to a simulation technique for the financial planning decisions and sensitivity analysis generating pro forma financial statements. Likewise, Elliott (1972b) depicts the main elements of the overall business financial performance through the development and evaluation of a model based on simultaneous equations. This work presents evidence on the potential usefulness of simultaneous equations models to forecast and analyze corporate financial performance. Moreover, Carleton, Dick and Downes (1973) make a contrast between financial modeling contemporary practice and contemporary finance theory with a view to determine their differences. Whereupon, the authors design a financial policy model with better features accepted for practice, to be used as a simulation model whose outputs are simultaneously determined using linear programming. In addition, they discuss about the gap between theory and practice and its implications for future academic activities in modeling financial policies. As an extension of the Warren *et al*. (1971) model, Francis and Rowell (1978) suggest a simultaneous equations model which aims to generate pro-forma financial statements to describe the future corporate financial condition based on the pattern of sales performance.

In the 80's, as well as Carleton *et al*. (1973), Crum, Klingman and Tavis (1983) suggest a scheme in which an optimization model is used in conjunction with a simulation model. Accordingly, both models are mutually reinforcing, which allows overcoming technical and conceptual difficulties when using these approaches separately. Their proposal seeks the integration of three functional areas of working capital management: marketing, production and finance.

Later, Xie and Xie (2007, 2008) propose planning models based on system dynamics with applications to financial forecasting and financial scenario planning. Their proposal uses causal loop diagram and stock and flow diagram to represent the following functional areas of the company by way of sub-systems: market and order, sales and accounts receivable, production and inventory, purchase and accounts payable, workforce, investment, financing, profit and cash flow. Furthermore, Cai, Chen and Hu (2009) produce a financial decisions support system from the description of a system dynamics model aiming to provide a method for simulate the business growth, controlling its speed, and assessing its impact on cash flow integrated with the factors that affect it. Finally, Gryglewicz (2011) presents a model of a company in the framework of the trade-off theory (Fischer, Heinkel and Zechner, 1989). His model allows choosing the optimal capital structure, cash holdings and dividends, under the impact of flow cash affected by long-term uncertainty and liquidity crisis in the short term.

Table A.6 (Appendix A) shows a complete list of researches concerning corporate financial planning models based on simulation concept.

2.4.4 New technology

For the purposes of this research, the concept of *new technology* refers to [(2.3)]:

> *Those techniques or models using artificial intelligence or the machine learning paradigm, which rely on high capacity of processing of current computers combined with theoretical, mathematical or conceptual developments (quantitative and qualitative) for purposes of diagnosis, prediction, identification or representation.*

It includes: Fuzzy logic, Fuzzy binary equations, Possibility distribution model, Fuzzy probability model, Neural networks, Back-propagation neural network, Support vector regression, Expert systems, Principle of scenario aggregation, Genetic programming, Agent-based simulation model, Grey relational analysis, Clustering technique, Logistic regression, Self-organizing map and Quantum-behaved Particle Swarm Optimization Algorithm, among other related techniques.

In this way, Tarrazo and Gutierrez (2000) present a methodology to develop practice-oriented CFP models, supported on a case study, by using fuzzy mathematical programming under uncertainty based on expectations. Malagoli, Magni and Mastroleo (2007) construct a formal model (fuzzy expert system), taking into account the experience of decision maker by combining logic and intuition to rank the firm under studio within a sector, in order to assess its capacity to create value and achieve its purpose. The fuzzy expert system addresses both qualitative and quantitative variables, integrates financial variables and strategic management. Kundisch and Dzoienziol (2008) present a model that takes into account the uncertainty and risk for the intelligent

[(2.3)] Definition of the author.

processing of financial problems and their solution, applicable beyond the domain of financial planning. In the formulation of the model, probability constraints are transformed into scenario-specific minimum payment constraints. Meanwhile, based on a cause-effect simple scheme, Terceño and Vigier (2011) develop a model for the firm´s financial economic diagnosis (ratios, bankruptcy prediction model) which, in turn, uses fuzzy binary equations resolution models for determining a financial economic knowledge matrix. In this way, the action of a financial analyst in his task of diagnosis is simulated. To simulate the causes and processes of financial difficulties at different stages of enterprise life cycle, Cao and Chen (2012) propose an agent-based model, concluding on the model validity by comparing simulation results with the actual situation of enterprise. Likewise, using data collected about six hundred enterprises in China, Pan (2012) conducts a complete study to deeply understand and determine the business operational performance and financial situation of enterprises, which applies Grey relational analysis to investigate the business operational performance in each case. Then, based on financial characteristic, he uses a clustering technique to divide enterprises into two groups. As a result, the author adopts three models (Genetic programming, Back-Propagation Neural Network and Logistic Regression) to construct an Operational Enterprise Performance model and an Enterprise Finance Characteristic model. Finally, Chen (2012) proposes a multi-phased and dynamic evaluation model of corporate financial structure by integrating both, self-organizing map (SOM) and support vector regression (SVR) techniques. This approach uses financial indicators extracted from financial statements, which determines the position of the company in the SOM. Several dynamic patterns of company behavior are displayed and recognized and the future trend of its financial structure is predicted.

Table A.7 (Appendix A) shows a complete list of research relating to CFP addressed from the perspective of new technologies. Additionally, Table A.8 (Appendix A) shows publications referring to financial planning combined with mathematical control theory, stochastic optimal control theory or stochastic optimal control problem (according to authors mention it) and other less popular techniques.

2.5 Financial Planning Applications

In addition to the models proposed by the authors mentioned in Section 2.4, many applications of models for CFP have been published. Without being exhaustive, and by means of particularly studied cases, the following is a list of references describing the experiences in the application of models for CFP categorized according to the nature of the organization concerned. The list does not include applications of financial planning models for banks.

- ✓ Health organizations: Coleman and Kaminsky (1977); Branson, Helmrath, Reck and Gerhardt (1981); Hopkins, Heath and Levin (1982); Cleverley (1987); Babock (1997).
- ✓ Public utilities or organizations: Gardiner and Ward (1974); Blanning and Crandall (1978); Rychel (1982); Linke and Whitford (1983); Nolan and Foran (1983); Moreau (1986); Miyajima and Nakai (1986).
- ✓ Processing enterprises: Tinsley and Hoglund (1973); Orthlieb (1979); Kirca and Koksalan (1996); Satir (2003); Collan (2004).
- ✓ Educational organizations: Sinuanystern (1984); Mavrotas, Caloghirou and Koune (2005).
- ✓ Insurers / Leasing company: Gentry (1972); Carino and Ziemba (1998); Folcut, Ciocirlan, Serban and Katalinc (2008).
- ✓ Other applications: Cook (1984); Aba-Bulgu, Islam, Helander, Xie, Jaio and Tan (2007); Hahn and Kuhn (2012).

Due to the development of modern financial systems, it is obvious that all operations and financial decisions of a company are reflected in their bank accounts as well as it is done in the accounting records. Then it can be assumed that "in bank accounts there is sufficient information" to establish mechanisms to support financial operations and financial planning of the firm (short and long term). Hence, by using the firm's bank accounts like information source, it becomes feasible apply the appropriate financial theories and develop models for corporate financial planning, including models based on mathematical programming, as well as simulation techniques, new technologies and modern control theories. Thereby, under the assumption that a Cash Concentration and Disbursements System (CCDS) includes all bank accounts of the company, this thesis takes this kind of system as subject of study developing a comprehensive model for its representation. Then, the system is simulated by adding predictive control techniques applied on bank accounts belonging to the CCDS. As previously stated, the CCDS can be used in different categories of organization for improving the planning and control of current assets and cash management. From there, a model for a CCDS can be considered a general representation for various categories of companies. Considering the importance of this approach, the next section addresses the issue of literature dedicated to the CCDS.

2.6 Cash Concentration and Disbursements Systems

Cash management is generally framed within the corporate short-term financial planning. Depending on the organizational structure, firms usually use a cash concentration and disbursements system (CCDS), which has been treated by different researchers. In this regard, Anvari and Mohan (1980) document an informatics system for decisions support, whose purpose is to introduce efficiency in the process of transferring the cash deposited in local banks toward the main accounts of the company, as well as issuing transfer orders based on a model of inventory decisions. For their part, Stone and Hill (1980, 1981) conduct a review about transfer techniques used to date (management about a target, anticipation, dual balances timing) and their shortcomings. They focus on the major decisions for structuring a cash concentration system. That is, the selection and assignment of both the bank of concentration and the transfer method. They propose a model using MILP in order to minimize the cost of the transfer program. To do this, they combine the most general versions of transfer techniques and properly conclude on the use of them to establish the transfer program. Anvari (1981) also examines a method of transferring revenues toward central accounts via depositary transfer checks for companies operating outlets. The procedure assumes the inventory policy (s, S) as the optimal transfer policy, where s is the re-order point and S is the re-order level (Beyer and Sethi, 1999). Similarly, under a deterministic approach, Anvari and Goyal (1985) describe a procedure to find an optimal relation between transfer and investment in a decentralized company. Finally, Anvari (1987) proposes a model whose objective is to determine an optimal procedure for issuing transfer orders based on inventory policy (s, S) applied to the cash concentration process.

In Baumol (1952) and Tobin (1956) is found the pioneering argument that allows applying to cash balances an analogous treatment (theoretical and conceptual) like inventory levels of goods. Meanwhile, Miller and Orr (1966, 1968) incorporate the randomness to Baumol model. Also, Girgis (1968) and Eppen and Fama (1968, 1969), separately, use a mathematical programming approach to present forecasting cash models under uncertainty. In addition to maintenance costs and shortage costs in the account management, they suggest a linear relationship of the cost of cash transfer regarding the transfer size. Furthermore, Marquis and Witte (1989) examine the implications on the demand for money in order to reduce the average or the variability on transfer requirements, by means of a stochastic decisions model for the selection of the optimal cash management program. The Marquis and Witte model focuses on minimizing the total expected cost, which includes illiquidity costs, opportunity cost due to idle cash, and operating costs and management. More currently, Gupta and Dutta (2011) study the flow in a supply chain upstream in order to schedule payments, subject to the restrictions in receiving the money. This latest research does not refer exactly CCDS; however, it provides a viewpoint more current on the treatment of money flows within organizations by using a supply chain and inventory approach, which is of interest to the present thesis.

2.7 Concluding remarks of the chapter

The analysis of the surveyed researches in this chapter suggests a logical sequence showing that academic and scientific production occurs in three waves from the 60s to the 80s: creating theories, creating models and applying of models for different economic sectors. In any case, there is a significant drop of academic production after these waves in the 90's, which resurfaces again in the 2000s. As it can be observed, this occurs due to the emergence of new technologies. Moreover, there exists a possible link between the simulation's issue and the emergence of new technologies. These two approaches are complementary and usually employed together. Probably, this is the reason why resurges the use of models for CFP in the 2000s. The application of new technologies to formulate CFP models is a promising field to develop new research. Likewise, it calls attention the limited use of the concepts of control and other tools and techniques in this area, which also could open up possibilities for starting new research projects.

Result of this literature review helps to arouse interest in the emergence of some research lines, placing at center of them the formulation of models for CFP both short and long term. There have been some research on the subject of control, however, it is possible to expand perspectives on the matter by combining several techniques to propose models and tools for CFP viewed as a controlled system, excluding or including optimization criteria. The scope is higher due to the variety of methods and theories of control (stochastic optimal control theory, basic control or various forms of advanced control). In addition, there are research opportunities if new technologies are used with the purposes of diagnosis, prediction, identification or representation, by leveraging the optimization analytical power when joining the simulation capabilities.

In particular, the issue of CCDS models addressed in this thesis, is seen as a relatively unexplored field. That being the case, focusing the analysis of financial systems from the point of view of dynamic systems could open doors for research. Seen this way, control techniques can be applied following the approach of supply chains and inventory problems. Justly, the next chapter proposes a simulation model for a CCDS, whose formulation opens up prospects for the application of rigorous techniques of automatic control.

Chapter 3

A simulation model for a Cash Concentration and Disbursements System

Cash management generally refers to planning, use and control of financial resources of an organization. It includes all activities of financial management aimed at ensuring cash availability for ordinary operations, avoiding the risk of insolvency, which is framed within the corporate short-term financial planning, as introduced in previous chapters 1 and 2. Cash management usually includes functions such as collection, concentration, management and disbursement of cash. Because of its importance, researchers have contributed diverse theoretical and practical tools for improving the critical decisions that globally affect the results. Depending on the organizational structure, in particular, different firms use a cash concentration and disbursements system (CCDS) for the purpose of improving the planning and control of current assets and cash management, as was explained in Chapters 1 and 2. This scheme is commonly used in companies whose structure is scattered nationally in different regions, in the form of agencies or distributors. Through agencies or distributors, the cash collection is performed, as well as the payments of supplies or services required for operation are channeled. The aim of the system is to concentrate cash money in a master bank account, in order to have complete control of cash, and provide greater investment opportunities, when there surpluses large amounts of money. Namely, to maximize the availability of cash not invested in fixed assets or inventories, and thus, avoid the risk of insolvency and improve profitability. As the case may in each region, revenue bank accounts are designated, in which all cash coming from the collection is deposited. Similarly, disbursement bank accounts are allocated in order to cover paychecks issued to suppliers and other commitments. According to a pre-established policy, the cash in revenue accounts is transferred periodically to the main account, and from there, the necessary cash amounts are transferred to the disbursements accounts, in order to cover the demand for cash.

As part of the framework in which this thesis is inserted, the previous chapter discussed about the researches that originally have addressed the cash concentration and disbursements systems using management inventory concepts. In this thesis, an inventory approach is also adopted. However, this research presents a basic difference with respect to such publications, since the proposed model uses difference equations to represent the entire CCDS with discrete-time handling. Furthermore, the purpose is to establish the conditions for the application of modeling techniques, arising from the systems theory and control, through a model according to García et al. (2012, 2013).

Several reasons justify the use of difference equations in the financial field. First, because in many situations it is convenient to focus financial problems as dynamic systems with the discrete time independent variable. This, without denying that in other situations is appropriate using time as a continuous variable to represent systems by using differential equations. Second, financial events can be represented intuitively through recurrence relations. In this sense, the incognito in difference equations makes up a succession and consequently, several of the terms of the sequence appear in the same equation. Third, the nature of the financial problems, joined to the need to analyze different scenarios, obliges keep handy the general solution of a model based on difference equations (which represents the set of all particular solutions) for analysis by financial managers. Fourth, a model based on difference equations may include deterministic or random variables. Thus, several authors have used difference equations to treat various problems of economy and finance; see Cagan (1956), Khan (1975), Obstfeld and Rogoff (1995, 1996), Woodford (1998) and Christev (2005). However, the most widespread application is found in financial engineering. It is common to use models based on difference equations for the valuation of financial profits, bank deposits and amortization systems (Tenorio *et al*., 2013). Contrasting with those investigations, this proposal addresses the problem as a dynamic system in which several components operate by exchanging monetary flows. The purpose is to centralize the decisions about cash to make the best use if there is surplus and cover deficits effectively and efficiently. In other words, the focus is on control decisions seeking to transfer cash towards the elements according to the requirements, with relevant cost considerations. This approach also analyzes the decision variables related to the frequency of cash transfers and the amount of cash transferred. Concretely, this chapter proposes a model for a CCDS, which facilitate the establishment of cash transfer rules, the control criteria and a comprehensive cash management program. Based on García *et al*. (2012), the model assumes the existence of delays due to banking procedures. Moreover, regarding the elements of the system, a bank account plays the role of internal customer or supplier, but instead of a supply chain, bank accounts make up a network. Nonetheless, external customers of the company are at one end of the network and external suppliers are at the other end. This approach opens real prospects for research on the application of systems engineering techniques to problems in the field of finance, seen as dynamic systems whose elements exchange goods or cash. In this regard, the proposals can combine techniques and tools of control theory, applied to short and long term corporate financial planning.

This introduction has been focused on the general description of the research and the proposed solution. In order to present a model representing the CCDS, in what follows, the chapter has been structured into four sections. Before, it is good to clarify that the fundamental contents of this chapter have been published in Herrera and Ibeas (2015) and Herrera-Cáceres and Ibeas (2016a). Section 3.1 provides the assumptions and mathematical details of the proposed model using difference equations. Section 3.2 includes an equivalent version of the model represented by algebraic equations based on the application of *Z*-transform. Section 3.3 contains a basic simulation of the CCDS in different scenarios in order to show the performance and functionality of the proposed model. Finally, Section 3.4 provides the conclusions of the chapter.

3.1 Mathematical modeling of the CCDS using difference equations

In the financial area, banks offer to companies the service for cash concentration and disbursements. Usually, this service includes the operation of revenue accounts and expense accounts with zero balance (*Zero Balance Account* – ZBA). The proposed model can be applied to this particular case. However, it can be applied in general circumstances. For instance, when the bank's policies do not allow ZBA or due to that the company does not want to (or cannot) use the services of a single bank for implementing the CCDS.

To this end, this research presents the case of a generic company whose agencies are distributed geographically in different regions. On the one hand, the company carries out the cash collection through its agencies, and customers can use different forms of payment (cash, check, debit card, credit card and direct debit). On the other hand, for operation in each agency, payments to suppliers are channeled through checks for the concept of supplies or services required. The model assumes the existence of a centrally operated main account with policy of cash balance oscillating between pre-established bands. Nevertheless, this does not detract generality to the model since the policy of cash balance must be tailored according to the specific reality of each company. Likewise, the model includes an investment account to which the cash surplus of the main account is transferred, and a line of credit to cover deficits in that account. In addition, a revenue account and a disbursements account are assigned to each agency. Furthermore, the model takes into consideration the different financial charges for maintenance of cash, illiquidity and transfer of funds, as well as operating and management of the system. Also, interest earned on the investment account are credited. The model represents the flow of money between the identified elements of CCDS, and the flow of requirements or cash transfer orders. All subject to the rules of operation and cash transfer between accounts.

3.1.1 Problem formulation

Figure 3.1 shows how operates a CCDS. In it, the elements involved in the system are identified by j $(= 0,1, \dots ,13)$. External elements represented are: (a) $j = 0$, suppliers or payees of checks issued by firm for payment of supplies or services. The process is activated when checks are cashed or deposited (box office or clearinghouse); then, the bank responds immediately in order to meet this requirement; (b) $j = 12$, customers who pay goods or services to the firm, which for its part also activate the process; (c) $j = 13$, a bank (the banking) that provides financial services to the company.

The internal elements are the different bank accounts operated by the company. The main account is represented by $j = 5$, which operates centrally; receives cash transfers from the revenue accounts of each agency and, also from it, funds to cover overdrafts incurred in the expense accounts of agencies are transferred. For its part, $j = 6$ represents an investment account to which the cash surplus from the main account is transferred. Moreover, $j = 7$ identifies the movement of the loan, which is a line of credit to cover deficits in in the main account. As it can be seen, CCDS could be modeled considering a company generally composed of K agencies. However, the graph in Figure 3.1 assumes a firm with only $K = 4$ agencies, which is taken for the further development of the model in this chapter. Each agency is identified in two ways, according to the account referred, because it operates two accounts (revenues and disbursements). With $j = 1,2,3,4$ the disbursements account of agency j is identified, and $j = 8,9,10,11$ represents the respective revenue account. Moreover, the representation of the payment of financial charges on the main account and the disbursements accounts has also been considered in Figure 3.1. Also the investment account receives credits, due to interest earned. Thick lines of the graphic show the flow of money between the system elements, and clear lines correspond to flow of requirements or cash transfer orders.

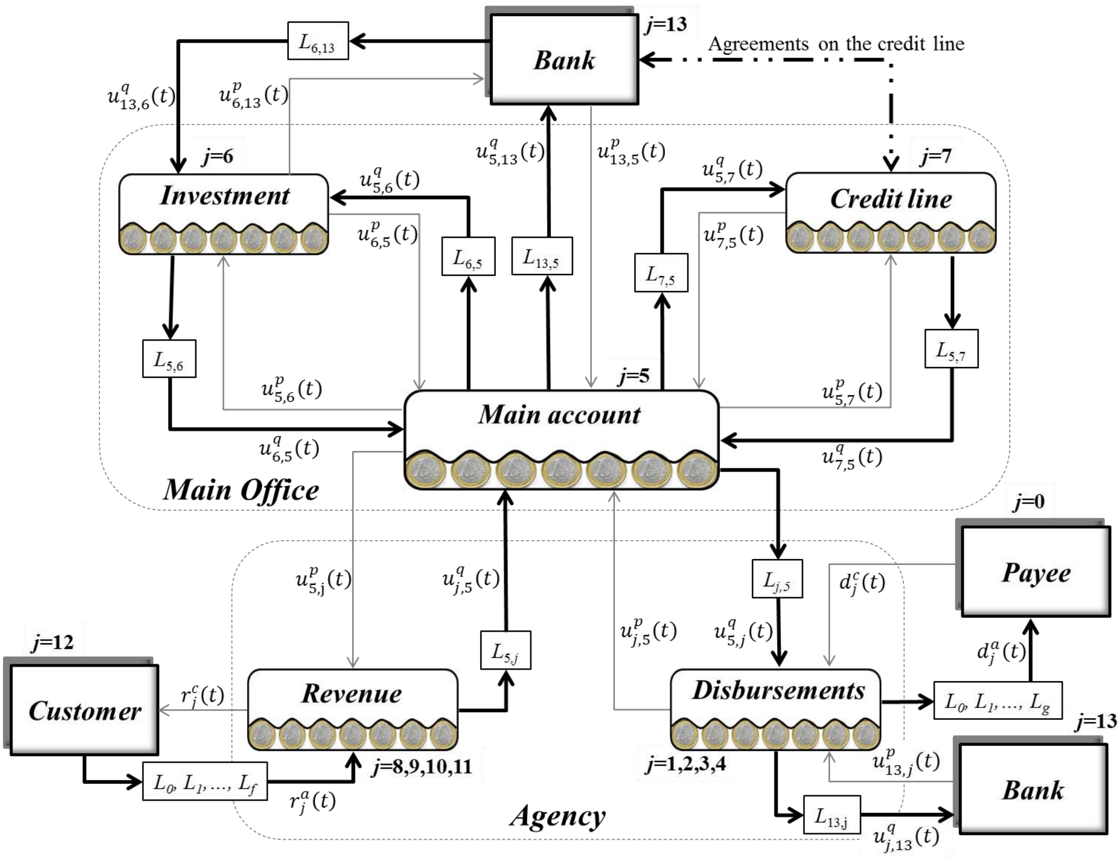


Figure 3.1. Graphical representation of a Cash Concentration and Disbursements System (CCDS)

For simplicity, the following considerations have been made prior to its explanation. First, a basis time interval $\tau = 1$ is assumed, usually a day, but it can also refer to a week or a month, or other time interval used in financial practice, according to the dynamics of the CCDS. Second, overall between two accounts a and b, $u^{p}_{a,b}(t)$ (clear line on the graph) represents the amount of money that a requires b at the end of any discrete time interval t $(= h\tau,$ with $h \in \mathbb{N})$, according to the rules of operation of the CCDS. That is, a transfer by the amount demanded from b to a, is required. Furthermore, $u^{q}_{b,a}(t)$ represents the amount of money effectively transferred from b to a at the end of discrete interval t. Also, $L_{a,b}$ (of the form $h\tau$ with $h \in \mathbb{N}$) represents the time it takes to be available in the account a the money transferred from the account b. Thus, assuming an equivalent nomenclature to that used by García *et al*. (2012, 2013), in general, it is fulfilled:

$$u_{b,a}^{q}(t) = u_{a,b}^{p}(t - L_{a,b}) \quad (3.1)$$

Exceptions to this representation are given in cases of cash collections from customers and payments to suppliers and other services, which are explained below. It is also common to establish criteria regarding the minimum size of cash transfers, depending on banking and enterprise policies, which in turn depends on the costs. Therefore, if MT is the minimum size of a transfer, hereinafter it is assumed that any order of transfer between two accounts a and b must meet $u_{a,b}^{p}(t) \geq MT > 0$. Otherwise $u_{a,b}^{p}(t) = 0$.

In general, the mathematical details relating to each of the elements or accounts of the CCDS correspond to movements and balances. The movements are debits and credits on a bank account and refer to the cash collection, deposits, payments, withdrawals, transfers, etc. Balances in bank accounts are usually: available cash balance, deferred balance, frozen balance and total balance. The latter, sometimes called current or actual balance of account. In this respect and for the purposes of this thesis, the following generic variables are defined:

- ✓ Available balance ($y_a^A(t)$) a is the amount of money in the bank account a at the end of a discrete time t, as a result of movements that occurred during that time interval, excluding the deferred and frozen balances.
- ✓ Frozen balance ($y_a^L(t)$) is the amount of money in the account a at the end of a discrete time t, which has been debited from this account in order to be transferred toward the account b, in accordance with the transfer order $u_{b,a}^{p}(t)$ (compensable / not completed). Once this transfer order is completed ($u_{a,b}^{q}(t + L_{b,a})$), the amount of money becomes available cash in the account b, and it is subtracted from the frozen balance in the account a.
- ✓ Deferred balance ($y_a^D(t)$) is the amount of money that reflects the bank account a at the end of a discrete time t, as a result of one or more deposits of money made in a manner other than cash, about which the banking procedures have not guaranteed the effective availability of money they represent. Once this guarantee is met, along

with the banking procedures, the deferred balance disappears, being added to the available cash balance.

- ✓ Total balance ($y_a^T(t)$) is the sum of the available, blocked and deferred balances ($y_a^T(t) = y_a^A(t) + y_a^D(t) + y_a^L(t)$).
- ✓ Book balance ($y_a^B(t)$) is the balance in the bank account a at the end of a discrete time t according to the accounting records of the company.

Differences normally found between book balances and balances in the bank have very diverse causes, but they are not the subject of this thesis. However, for formal reasons, later an allusion to the book balances of the disbursements accounts is done since they could be a controllable reference for the analysis of the proposed model, because the company issues paychecks to their suppliers. In what follows, the mathematical development of the elements of CCDS is presented. That is, the mathematical relationships describing the behavior of each of the accounts included in the model. Namely, revenue account, disbursement account, main account, investment account and credit line. Subsections 3.1.2 to 3.1.6 show the equations relating to the available balance ($y_a^A(t)$) on each account, which is of interest in this thesis. However, Herrera-Cáceres and Ibeas (2016a) develop the entire model including the equations for deferred balance ($y_a^D(t)$), frozen balance ($y_a^L(t)$) and current balance ($y_a^T(t)$) in each case.

3.1.2 Movements and balance on the revenue accounts

The revenues $r_j^c(t)$ of the agency j $(= 8,9,10,11)$ at time t (usually a day) correspond to payments made by its customers during t, which are deposited in a special revenues account j, at the end of that time interval. In principle, before the closing of the interval t, these available cash balances of revenue accounts are transferred to the main account. Commonly, clients make their payments in the following forms: cash, debit card, credit card, check, automatic debit authorization or bank transfers. However, for purposes of simplification and generalization, $f + 1$ forms of payment are considered, each one identified by $m = 0,1,2, \dots, f$, and L_m (of the form $h\tau$ with $h \in \mathbb{N}$) is the transit time or delay to make available the money deposited in the revenue account, according to the form of payment m. In this case, $m = 0$ corresponds to cash payments or whose availability is immediate in the account at time in which the deposit is made, that is, transit time $L_0 = 0$. Whilst, according to banking procedures, the availability of deposits related to other forms of payment is $L_m \geq 0$ time intervals after making the deposit or transaction. Also for simplicity, the model assumes that revenues by check and cash during the interval t are deposited at the end of that interval. Furthermore, according to the form of payment m, the model assumes that the following discrete deterministic scheme represents the collection's behavior:

Let b_m^r denote the portion of cash collection made in accordance with the form of payment m, for which is fulfilled: $0 \leq b_m^r \leq 1$ and $\sum_{m=0}^{f} b_m^r = 1$. Thus, total revenues in the agency j during t are given by:

$$r_j^c(t) = \sum_{m=0}^{f} I_{m,j}(t), j = 8,9,10,11 \qquad (3.2)$$

where $I_{m,j}(t) = b_m^r r_{j,12}^c(t)$ is the total money income in the revenue account j according to the form of payment m during the interval t. Hence, the cash collection will be available in the revenue account in accordance with the following expression:

$$r_j^a(t) = \sum_{m=0}^{f} \left[b_m^r r_j^c(t - L_m) \right], j = 8,9,10,11 \qquad (3.3)$$

Note that it has not been supposed anything regarding the stochastic or deterministic nature of $r_j^c(t)$, but during testing and application of the model, criteria in this regard may be taken.

Furthermore, subject to the rules of operation of CCDS and in accordance with established policies, in principle at end of time interval t, transfer cash available ($u_{5,j}^p(t)$) from the revenue account j to the main account is ordered. According to (3.1), this cash will be available in the main account $L_{5,j} \geq 0$ time intervals after the cash transfer order. So that, considering (3.3), in the revenue account j the new cash balance at end of interval t is:

$$y_j^A(t) = y_j^A(t-1) + \sum_{m=0}^{f} \left[b_m^r r_j^c(t - L_m) \right] - u_{5,j}^p(t), j = 8,9,10,11 \qquad (3.4)$$

Because of this, at the end of interval t the available cash balance on the main account is increased according to (3.1).

3.1.3 Movements and balance on the disbursements accounts

The disbursements in agencies during the interval t are represented by $d_j^c(t)$ ($j = 1,2,3,4$). These disbursements correspond to checks issued to payees for the time interval t against a special disbursements bank account of the agency. There exist an interesting variability in relation to the transit time of a check issued by the firm, which is not controlled process. Consequently, here it has been assumed that payees present the check deposit according to the following discrete deterministic scheme: (a) Number of time intervals in transit: L_m ($m = 0,1,2, \dots, g$, L_m of the form $h\tau$ with $h \in \mathbb{N}$); (b) Portion of funds in transit according to L_m: b_m^d, with $0 \leq b_m^d \leq 1$ and $\sum_{m=0}^{g} b_m^d = 1$. Because of this assumption, the cash required to cover checks, through either box office or clearinghouse, answers the following expression:

$$d_j^a(t) = \sum_{m=0}^{g}\left[b_m^d d_j^c(t - L_m)\right], j = 1,2,3,4 \quad (3.5)$$

This expression determines the overdraft in interval t due to checks presented by payees against the j agency disbursements account. In turn, at least in principle, at the end of t the overdraft is covered with a cash transfer order ($u_{j,5}^p(t)$) from the main account. According to (3.1), the amount transferred will be available in j disbursements account after $L_{j,5}$ time intervals. As a result, the available cash balance at end of t, recorded in the books of the company for j disbursements account, is given by:

$$y_j^B(t) = y_j^B(t-1) + u_{j,5}^p\left(t - L_{j,5}\right) - d_j^c(t), j = 1,2,3,4 \quad (3.6)$$

However, for purposes of bank reconciliation, the available cash balance registered in the bank for the j disbursements account is:

$$y_j^A(t) = y_j^A(t-1) + u_{5,j}^q(t) - d_j^a(t) - u_{13,j}^p(t), \; j = 1,2,3,4 \quad (3.7)$$

Being $u_{13,j}^p(t)$ the finance charge due to overdrawn in the disbursements account (see Appendix B). Note that for this concept $y_j^B(t)$ is only updated when the bank informs the finance charge to the firm through the respective debit note.

Substituting (3.5) and considering (3.1) to (3.7), it is obtained for $j = 1,2,3,4$:

$$y_j^A(t) = y_j^A(t-1) + u_{j,5}^p\left(t - L_{j,5}\right) - \sum_{m=0}^{g}\left[b_m^d d_j^c(t - L_m)\right] - u_{13,j}^p(t) \quad (3.8)$$

3.1.4 Movements and balance on the main account of CCDS

At the end of each time interval t, the main account balance increases due to cash transfers from the revenue accounts, and decreases because of cash transfers to cover the overdrafts of the disbursements accounts. Therefore, after considering (3.1), as a result of ordinary operations of the firm, the new balance on the main account has the expression:

$$D(t) = y_5^A(t-1) + \sum_{j=8}^{11}\left[u_{5,j}^p\left(t - L_{5,j}\right)\right] - \sum_{j=1}^{4} u_{j,5}^p(t) \quad (3.9)$$

where $y_5^A(t-1)$ is the available cash balance in the main account at the end of the previous interval time.

However, there exist other movements in this account. On the one hand, the interaction with the investment account and credit line. On the other hand, the company establishes the following policy regarding the cash balance on the main account: M for return level, M_{max} for higher limit and M_{min} for lower limit. Details about the analysis of the central financial charges involved are showed in Appendix B. The total of these charges during t is represented by $u_{13,5}^p(t)$; which when it is considered, expression (3.9) can be adjusted as follows:

$$D(t) = y_5^A(t-1) + \sum_{j=8}^{11}\left[u_{5,j}^p\left(t-L_{5,j}\right)\right] - \sum_{j=1}^{4} u_{j,5}^p(t) - u_{13,5}^p(t) \qquad (3.10)$$

In this way, when $D(t) > M_{max}$, it is said that there is a cash surplus due to ordinary operations of firm, with respect to minimum balance. Otherwise, $D(t) < M_{min}$ means that there is a deficit.

A surplus in the main account allows two movements: (a) A transfer to the investment account ($u_{6,5}^p(t)$) for a profit instead of holding idle cash. (b) A payment of the credit's line debt ($u_{7,5}^p(t)$), in order to reduce the financial cost. These decisions shall take effect in accordance with (3.1).

As can be seen, a surplus determines there is no movement of cash towards the main account from the investment account or from the credit line. That is, $u_{5,6}^p(t) = 0$ and $u_{5,7}^p(t) = 0$. Moreover, a deficit in the main account can be covered from the investment account ($u_{5,6}^p(t)$) or from the line of credit ($u_{5,7}^p(t)$). In this case, (3.1) also determines the availability of these transfers in the main account.

Also, a deficit determines there is no movement of cash from the main account to the investment account or to the credit line. That is, $u_{6,5}^p(t) = 0$ and $u_{7,5}^p(t) = 0$. In sum, the new available cash balance in the main account at the end of *t* is given by:

$$y_5^A(t) = D(t) + u_{6,5}^q(t) + u_{7,5}^q(t) - u_{6,5}^p(t) - u_{7,5}^p(t) \qquad (3.11)$$

Then, the following final expression for available cash balance in the main account is obtained by using (3.1) and replacing by (3.10):

$$y_5^A(t) = y_5^A(t-1) + \sum_{j=8}^{11}\left[u_{5,j}^p\left(t - L_{5,j}\right)\right] - \sum_{j=1}^{4} u_{j,5}^p(t) - u_{13,5}^p(t)$$
$$+u_{5,6}^p\left(t - L_{5,6}\right) + u_{5,7}^p\left(t - L_{5,7}\right) - u_{6,5}^p(t) - u_{7,5}^p(t) \quad (3.12)$$

3.1.5 Movements and balance on the investment account:

Financial income during the time interval t due to investment in the investment account are given by $u_{6,13}^p(t)$ (see Appendix B), whereby, at the end of t, the cash balance in the investment account is given by:

$$y_6^A(t) = y_6^A(t-1) + u_{6,5}^p\left(t - L_{6,5}\right) + u_{6,13}^p\left(t - L_{6,13}\right) - u_{5,6}^p(t) \quad (3.13)$$

3.1.6 Movements and balances on the credit line

The loan balance is: $y_7^A(t) = y_7^A(t-1) - u_{5,7}^a(t) + u_{5,7}^p(t)$. But, by applying (3.1) it becomes:

$$y_7^A(t) = y_7^A(t-1) - u_{7,5}^p\left(t - L_{7,5}\right) + u_{5,7}^p(t) \quad (3.14)$$

Finally, if Ɽ is the upper bound of the credit line, its availability is expressed by:

$$A_7(t) = \text{Ɽ} - y_7^A(t) \quad (3.15)$$

This model of CCDS, which is composed of several difference equations, has been solved in the time domain by the author. That is, functionality tests of the model have been made through the computer and the results have been satisfactory. See Section 3.3 where the model behavior is shown by means of several basic numerical simulations. However, the next section presents an equivalent version based on *Z*-transform, which is made in order to promote its use in combination with diverse control techniques. Moreover, this model is the framework within which are proposed the models presented in Chapters 4 and 5. Later, it is integrally applied by simulating a case study in Chapter 6.

3.2 Equivalent model using Z-transform

Hereinafter, the *Z*-transform (Ogata, 1996) is applied in order to obtain an equivalent model consisting of algebraic equations, which represents the available balances of the model proposed in the previous section. Whereupon, it is intended to open the possibility of applying rigorous and advanced techniques of Control Theory as well as make a contribution in the field of finance. Originally, modern control theory has been applied to the field of finance by Sethi y Thompson (1970). Also, Vasconcellos (1988), Premachandra (2004), Bar-Ilan *et al*. (2004), Baccarin (2009), Liang y Sun (2011), Cerqueti (2012) y Song *et al*. (2013) focus their proposals on cash management as an optimal control problem.

This research show the technical and conceptual feasibility of applying control theories and techniques to the financial field. Especially, greater perspectives are opened if considering the use of *Z*-transform together with the kind of model proposed here. The issue of transformation techniques has been discussed in previous works to developing financial models; mainly, the Fourier transform, widely used in models for valuing financial options. On one hand, see the work of Buser (1986), who introduces equivalent simplified models of the present value equation and shows how increases the representation and resolution capacity of such mathematical models based on the Laplace transform. Furthermore, by adopting the principle of Net Present Value (NPV), the Laplace transform has been used together with the input-output analysis for the theoretical representation and the treatment of problems of Material Requirements Planning (MRP) (Grubbström, 1998, 1999; Grubbström and Tang, 1999) and production-inventory systems (Grubbström and Huynh, 2006). Also, Naim *et al*. (2007) use Laplace transforms along with the NPV concept to select the parameters in the ordering policy of a production planning and control system. Moreover, Duffy (2006) widely discusses the Laplace and Fourier transformation techniques applied to the heat equation, given its popularity in financial engineering and its relationship with the equation of Black-Scholes, which has fundamental importance for the formulation of models for valuing financial options in real time. Similarly, based on its computational importance, Cerny (2006, 2009) explains the operation of the Discrete Fourier Transform (DFT) and its implementation algorithm (the Fast Fourier Transform - FFT), in the binomial option pricing model. Further, Cerny refers to the DFT as a special case of the *Z*-transform through some applications of the FFT. Among other works using the Fourier transform for the valuation of financial options, are included: Carr and Madan (1999), Lee (2004), Carr and Wu (2004) and Chourdakis (2005), as well as for the spread option valuation: Dempster and Hong (2002) and Hurd and Zhou (2010). Albanese *et al*. (2004) deal on the calculation of risk values and Cerny (2004) addresses the risk assessment. The *Z*-transform approach has also been tackled by Fusai *et al*. (2012) in solving equations on the financial options valuation, regarding which they consider different quadrature procedures allowing the solution of a linear system using iterative algorithms instead of recursive algorithms. In their book, Fusai and Roncoroni (2008) illustrate the use of the Laplace transform and *Z*-transform for the valuation of financial options and the FFT for procedures on pricing swaps. Additionally, Boissard (2012) applies the *Z*-transform properties in researching investment strategies. His work aims to determine trends in time series for forecasting variance through moving averages; then calculates the value at risk. Finally, Cui *et al*. (2013) developed a method based on the *Z*-transform to solve multidimensional "linearized" discrete-time systems, which can be used to analyze the effects of policies on the economy.

None of these works refers to the use of transformation techniques for formulating models applied to cash management and planning and control of current assets. For this reason, the use of systems engineering techniques in simulation models for cash concentration and disbursements systems (CCDS), in combination with the application of the *Z*-transform, has not been addressed previously in the literature. Consequently, this proposal is considered by the author as a novel contribution, which opens perspectives to study the application of control techniques in new researches.

Seen this literature review, the equivalent model after applying the properties of the *Z*-transform is presented below. Thus, the use of the time shifting property allows obtaining, in summary:

Available cash balance in the revenue account (when applying *Z*-transform in (3.4)):

$$Y_j^A(z) = \left[\sum_{m=0}^{f} (b_m^r z^{-L_m}) \,.\, R_j^c(z)\right] / (1 - z^{-1}) - U_{5,j}^p(z)/(1 - z^{-1}),\ j = 8,9,10,11 \qquad (3.16)$$

Available cash balance in the disbursements account (when applying *Z*-transform in (3.8)):

$$Y_j^A(z) = \left[z^{-L_{j,5}} . U_{j,5}^p(z)\right] / (1 - z^{-1}) - \left[\sum_{m=0}^{g} (b_m^d z^{-L_m}) \,.\, D_j^c(z)\right] / (1 - z^{-1}) - U_{13,j}^p(z)/(1 - z^{-1}),\ j = 1,2,3,4 \qquad (3.17)$$

Available cash balance in the main account (when applying *Z*-transform in (3.12)):

$$Y_5^A(z) = \sum_{j=8}^{11} \left[z^{-L_{5,j}} . U_{5,j}^p(z)/(1 - z^{-1})\right] - \sum_{j=1}^{4} \left[U_{j,5}^p(z)/(1 - z^{-1})\right] - U_{13,5}^p(z)/(1 - z^{-1})$$

$$+ \left[z^{-L_{5,6}}.\,U_{5,6}^{p}(z)\right]/(1-z^{-1}) + \left[z^{-L_{5,7}}.\,U_{5,7}^{p}(z)\right]/(1-z^{-1})$$
$$- U_{6,5}^{p}(z)/(1-z^{-1}) - U_{7,5}^{p}(z)/(1-z^{-1}) \qquad (3.18)$$

Balance in the investment account (when applying Z-transform in (3.13)):

$$Y_6^A(z) = \left[z^{-L_{6,5}}.\,U_{6,5}^{p}(z)\right]/(1-z^{-1})$$
$$+ \left[z^{-L_{6,13}}.\,U_{6,13}^{p}(z)\right]/(1-z^{-1}) - U_{5,6}^{p}(z)/(1-z^{-1}) \qquad (3.19)$$

Loan balance - credit line (when applying Z-transform in (3.14)):

$$Y_7^A(z) = -\left[z^{-L_{7,5}}.\,U_{7,5}^{p}(z)\right]/(1-z^{-1}) + U_{5,7}^{p}(z)/(1-z^{-1}) \qquad (3.20)$$

Clearly, different criteria could be used to establish the role played by each variable in this model. However, for the purposes of this research, the central objective of a CCDS is assumed as determinant, which is: *Once the money is collected, to concentrate the cash in a unique bank account, called main account, with the aim to have overall control of cash management, providing greater investment opportunities with large sums of available surplus cash. That is, maximizing the availability of cash not invested in fixed assets or inventory, and thus avoid the risk of insolvency and improve profitability.*

This objective clarifies several aspects of the model. First, the role of each variable is defined in accordance with the use of each account. Second, it allows determining the reference variables regarding the desired cash level on each account. Third, the objective suggests the model dynamics regarding control measures, and finally clarifies the nature of the model inputs such as disturbance variables that affect in different ways the levels of available cash in the accounts. With this approach, it is possible to establish a scheme for the classification of the involved variables, which is sufficiently exposed in Herrera-Cáceres and Ibeas (2016a). The available cash balance is the control variable in each account, the reference variable is the desired value for each account type: usually zero on revenue accounts and disbursements accounts, as well as on credit line; M in the main account, and a sufficiently large value in the investment account, depending on the expected profit of firm. For its part, the control actions correspond to the cash transfer orders seeking return each account balance to desired level. While the disturbances respond to dependent actions of external elements, in each case. In Lin *et al*. (2004) arguments that allow grouping are given, one hand, all the customers revenues during t as a single aggregate revenue, and on the other, as a single aggregate payment, all payments to payees during time interval t.

Note that equations (3.16) to (3.20) represent unstable relationships between variables, which is one of the problems in the financial management system under consideration. It also appears in other problems such as inventory management in supply chains (García *et al*., 2012). For simplicity, in the remainder of the chapter the following relationship will be used:

$$P(z) = 1/(1 - z^{-1}) \tag{3.21}$$

Also, due to the deterministic assumption of the transit times for both the collection as payments, for the moment, the expressions $b^d(z) = \sum_{m=0}^{g}(b_m^d z^{-L_m})$ and $b^r(z) = \sum_{m=0}^{f}(b_m^r z^{-L_m})$ do not significantly compromise model complexity. These expressions represent only a deterministic distribution of the input variable value in various time intervals in the future.

Considering the above, the following model in matrix format ((3.22) to (3.46)) is obtained to represent available cash balances in the CCDS, in which some of the matrices have been divided into blocks due to their size, being $\mathbf{I}_{4x4}$ an identity matrix of size $4x4$ and $\mathbf{0}_{nxm}$ a nxm matrix with all entries equal to zero:

$$\mathbf{Y}^A(z) = \mathbf{M}(z)\mathbf{U}(z) + \underbrace{\mathbf{B}(z)\mathbf{O}(z)}_{\text{Disturbances}} \tag{3.22}$$

wherein $\mathbf{Y}^A(z)$ is the observed output, $\mathbf{U}(z)$ is the set of control variables, $\mathbf{M}(z)$ is the system dynamics matrix representing the system behavior and $\mathbf{B}(z)\mathbf{O}(z)$ is the disturbances. $\mathbf{M}(z)$ is factorized using $P(z)$ and the Schur product (or component-wise product) (Dennis, 2009) $\mathbf{E} \circ \mathbf{L}(z)$.

$$\mathbf{Y}^A(z) = (Y_1^A(z) \quad Y_2^A(z) \quad \cdots \quad Y_{11}^A(z))^t \tag{3.23}$$

$$\mathbf{M}(z) = P(z)[\mathbf{E} \circ \mathbf{L}(z)] \tag{3.24}$$

$$\mathbf{U}(z) = \left(U_{1,5}^p(z) \quad \cdots \quad U_{4,5}^p(z) \quad U_{5,6}^p(z) \quad \ldots \quad U_{5,11}^p(z) \quad U_{6,5}^p(z) \quad U_{7,5}^p(z)\right)^t \tag{3.25}$$

The factors on (3.24) are:

$$\mathbf{E} = \begin{pmatrix} \mathbf{E_1}(z) & \mathbf{0}_{4x2} & \mathbf{0}_{4x4} & \mathbf{0}_{4x2} \\ \mathbf{0}_{3x4} & \mathbf{E_2}(z) & \mathbf{0}_{3x4} & \mathbf{E_4}(z) \\ \mathbf{0}_{4x4} & \mathbf{0}_{4x2} & \mathbf{E_3}(z) & \mathbf{0}_{4x2} \end{pmatrix} \tag{3.26}$$

$$\mathbf{E_1}(z) = \mathbf{I}_{4x4} \qquad \mathbf{E_2}(z) = \begin{pmatrix} 1 & 1 \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \tag{3.27}$$

$$\mathbf{E_3}(z) = \mathbf{I}_{4x4} \qquad \mathbf{E_4}(z) = \begin{pmatrix} 1 & 1 \\ 1 & 0 \\ 0 & 1 \end{pmatrix} \tag{3.28}$$

$$\mathbf{L}(z) = \begin{pmatrix} \mathbf{L_1}(z) & \mathbf{0}_{4x2} & \mathbf{0}_{4x4} & \mathbf{0}_{4x2} \\ \mathbf{0}_{3x4} & \mathbf{L_2}(z) & \mathbf{0}_{3x4} & \mathbf{L_4}(z) \\ \mathbf{0}_{4x4} & \mathbf{0}_{4x2} & \mathbf{L_3}(z) & \mathbf{0}_{4x2} \end{pmatrix} \tag{3.29}$$

$$\mathbf{L_1}(z) = \begin{pmatrix} z^{-L_{1,5}} & 0 & 0 & 0 \\ 0 & z^{-L_{2,5}} & 0 & 0 \\ 0 & 0 & z^{-L_{3,5}} & 0 \\ 0 & 0 & 0 & z^{-L_{4,5}} \end{pmatrix} \tag{3.30}$$

$$\mathbf{L_2}(z) = \begin{pmatrix} z^{-L_{5,6}} & z^{-L_{5,7}} \\ 0 & 0 \\ 0 & 0 \end{pmatrix} \tag{3.31}$$

$$\mathbf{L_3}(z) = -\mathbf{I}_{4x4} \tag{3.32}$$

$$\mathbf{L_4}(z) = \begin{pmatrix} -1 & -1 \\ z^{-L_{6,5}} & 0 \\ 0 & -z^{-L_{7,5}} \end{pmatrix} \tag{3.33}$$

$$\mathbf{B}(z) = P(z)\mathbf{D}(z) \tag{3.35}$$

$$\mathbf{O}(z) = (\boldsymbol{\theta_1}(z) \quad \boldsymbol{\theta_2}(z) \quad \boldsymbol{\theta_3}(z) \quad \boldsymbol{\theta_4}(z) \quad \boldsymbol{\theta_5}(z))^t \tag{3.36}$$

$$\boldsymbol{\theta_1}(z) = (D_1^c(z) \quad \cdots \quad D_4^c(z))^t \tag{3.37}$$

$$\boldsymbol{\theta_2}(z) = \left(U_{1,5}^p(z) \quad \cdots \quad U_{4,5}^p(z) \quad U_{5,6}^p(z) \quad \cdots \quad U_{5,11}^p(z) \quad U_{6,13}^p(z)\right)^t \tag{3.38}$$

$$\boldsymbol{\theta_3}(z) = (R_8^c(z) \quad \cdots \quad R_{11}^c(z))^t \tag{3.39}$$

$$\boldsymbol{\theta_4}(z) = \left(U_{13,1}^p(z) \quad \cdots \quad U_{13,4}^p(z)\right)^t \tag{3.40}$$

$$\boldsymbol{\theta_5}(z) = U_{13,5}^p(z) \tag{3.41}$$

$$\mathbf{D}(z) = \begin{pmatrix} \mathbf{D_1}(z) & \mathbf{0}_{4x11} & \mathbf{0}_{4x4} & \mathbf{D_4}(z) & \mathbf{0}_{4x1} \\ \mathbf{0}_{3x4} & \mathbf{D_2}(z) & \mathbf{0}_{3x4} & \mathbf{0}_{3x4} & \mathbf{D_5}(z) \\ \mathbf{0}_{4x4} & \mathbf{0}_{4x11} & \mathbf{D_3}(z) & \mathbf{0}_{4x4} & \mathbf{0}_{4x1} \end{pmatrix} \tag{3.42}$$

$$\mathbf{D_1}(z) = -b^d(z)\mathbf{I}_{4x4} \tag{3.43}$$

$$\mathbf{D_2}(z) =$$

$$\begin{pmatrix} -1 & -1 & -1 & -1 & 0 & 0 & z^{-L_{5,8}} & z^{-L_{5,9}} & z^{-L_{5,10}} & z^{-L_{5,11}} & 0 \\ 0 & 0 & 0 & 0 & -1 & 0 & 0 & 0 & 0 & 0 & z^{-L_{6,13}} \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} \tag{3.44}$$

$$\mathbf{D_3}(z) = b^r(z)\mathbf{I}_{4x4} \qquad \mathbf{D_4}(z) = -\mathbf{I}_{4x4} \tag{3.45}$$

$$\mathbf{D_5}(z) = (-1 \quad 0 \quad 0)^t \tag{3.46}$$

The model is completed by considering two conditions: (a) The minimum transfer size; that is, generally any cash transfer order between two accounts a and b must meet $0 < MT \leq U_{a,b}^p(z)$, otherwise $U_{a,b}^p(z) = 0$. (b) In order to maintain the available cash balance at desired level, any transfer from the main account toward the investment account or loan, or vice versa, is subject to the existence of surplus or deficit (on the main account).

This is a discrete, multivariate model consisting of multiple delays, whose construction seeks an equilibrium between being manageable and its proximity to reality. Therefore, the approach may be properly evaluated when control techniques be applied. For instance, regarding delays that occur due to banking procedures, there are diverse realities. For a CCDS fully managed by a single bank, it is possible to schedule transfers without delay in the money availability. In this case, the bank has control of everything concerning the transit of money between agencies, making it transparent for the firm; however, the costs of contracted banking services reflect this fact. In a CCDS in which local banks participate, delays in cash transfers between accounts could vary from one to three days, depending on the territory and environmental conditions, which determine their knowledge. Other hand, when it comes to firms represented in different countries enter in play issues of concern whose treatment requires an additional analysis (Anvari, 1986), which are beyond the scope of this thesis. Nevertheless, it is valid to evaluate the possibility of incorporating criteria for identifying delays when adopting any control approach.

As discussed at the beginning of the chapter, addressing the problem of cash management with the inventory approach has provided interesting proposals for financial evaluation, as well as for determining the frequency and size of cash transfers, and delays of transfer orders. All this provides ample criteria that, combined with the model proposed here, complement aspects for its development and implementation. In the next section, the results of several simulations are shown. For reasons of space, only the description of behavior of the system central accounts in different scenarios is included. The simulation shown in Chapter 6 presents enough details about the behavior of the entire model.

3.3 Simulation of the CCDS based on time domain

With the purpose of showing the functionality and behavior of the model, this section is intended for the presentation of simulations based on time domain. In this regard, in Figures 3.2 through 3.6 some graphics resulting from its application in different scenarios are presented. These graphics illustrate the behavior of system central accounts and show its functioning with respect to what is expected of results. In order to demonstrate the basic behavior of the system, all graphics (except Figure 3.6) refer to cases without delay by banking procedures.

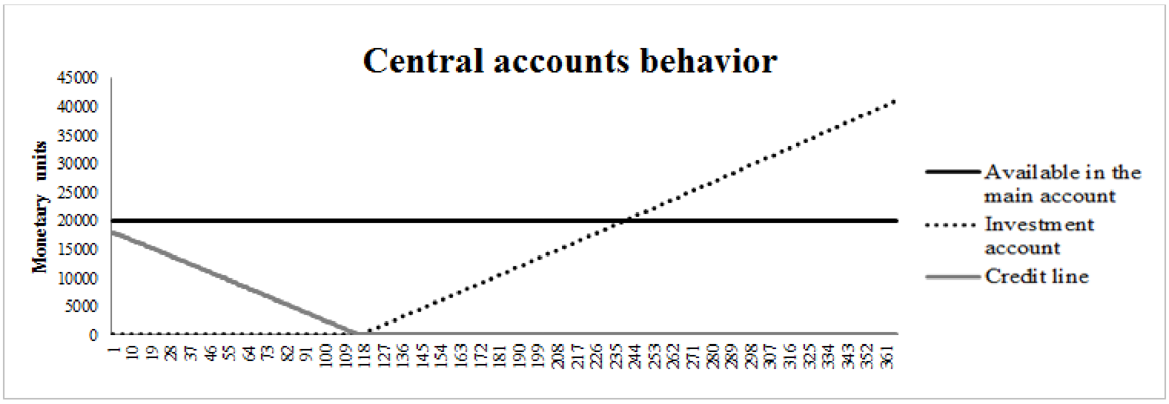


Figure 3.2: Behavior with constant positive net income.

The graphs show the relationship between the different variables. On the one hand, the main account maintains the condition of minimum balance and, on the other, the investment account and the credit line give support to it or room for maneuver. When the latter does not occur, because the credit line is maxed out and the investment account to zero, the system cannot maintain the minimum balance on the main account.

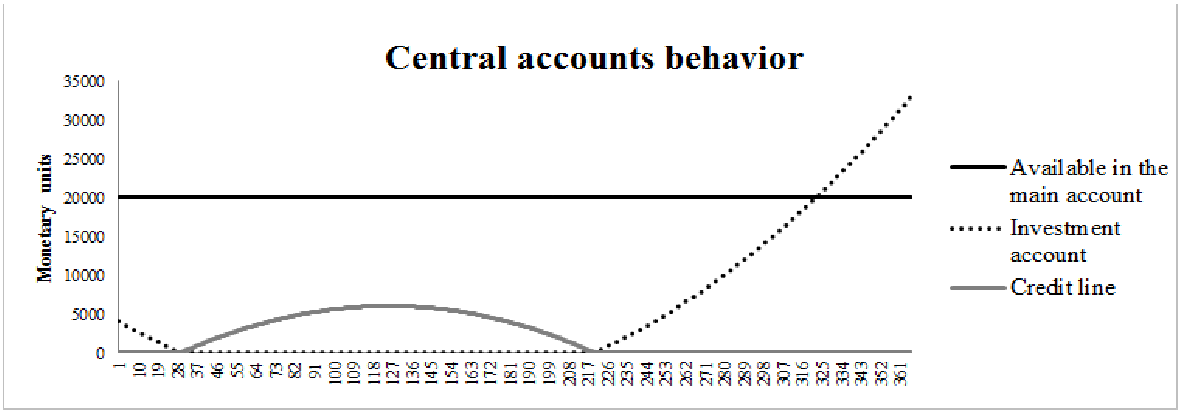


Figure 3.3: Behavior with increasing linear net income.

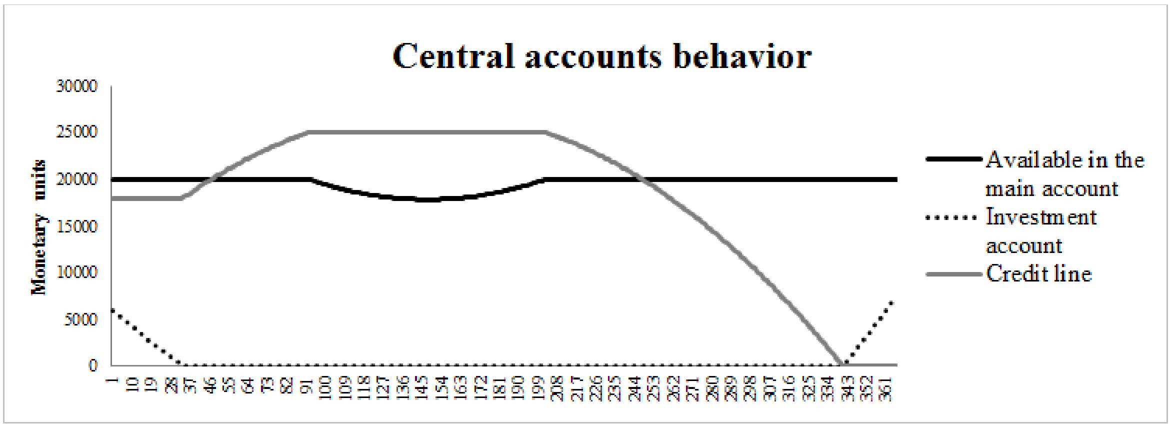


Figure 3.4. Behavior with increasing linear net income, initially negative and the credit line demanded.

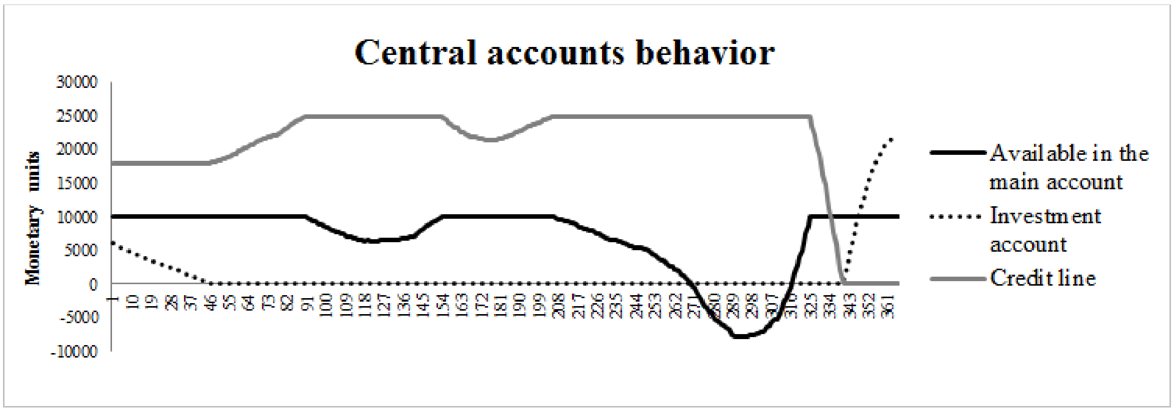


Figure 3.5. Behavior with variable net income. Eventually, the company reaches insolvency.

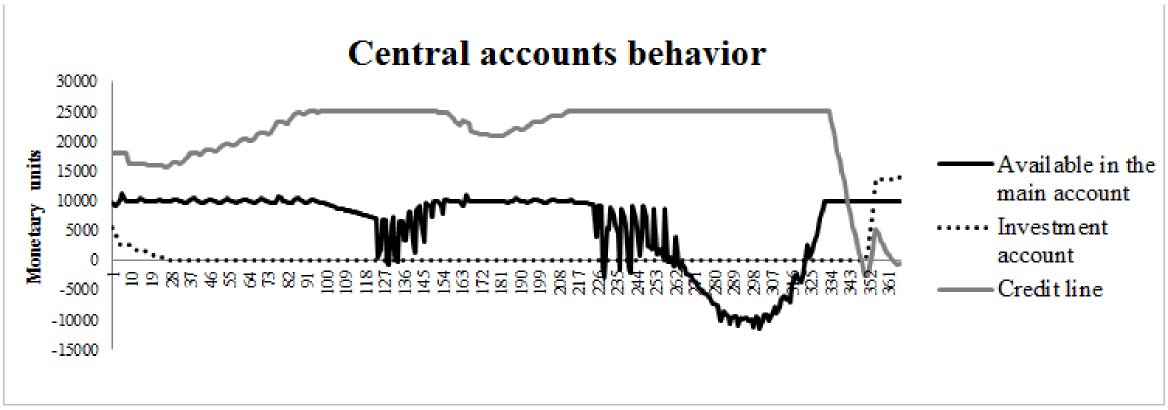


Figure 3.6. Behavior with variable net income, and overall delay of two time intervals.

The diversity of scenarios is virtually unlimited. However, in the examples, it has been tried to show graphically that the model generates the expected results, according to hypothetical entries. The possibility remains open to make adjustments in order to adapt the model to the structure of a particular company and, thus, apply real data for definitive tests of model.

3.4 Concluding remarks of the chapter

This chapter presented the mathematical model for a Cash Concentration and Disbursements System. The model was built from a detailed analysis of the monetary flows between the bank accounts implicated. The analysis resulted in a difference equations system, which represents quantitatively the system behavior. In some cases of interest, the numerical simulation carried out has shown coherent behavior with that expected, from the intuitive point of view. Finally, the system was depicted using the framework of the *Z*-transform, which is typical in analyzing and controlling discrete-time systems. The scheme based on difference equations included assessing the various balances related to each of the accounts (see Herrera-Cáceres and Ibeas, 2016a).

However, the outcome of the model based on the *Z*-transform is only about available cash balances. Furthermore, it is only matter of fitting the model to determine the deferred balances, frozen balances and current balances for each case by using the pertinent relations. Another important aspect refers the establishment of cash balance policy in the main account. Previously, it has been assumed as a range of oscillation, which it is according to the model of Miller and Orr.

In this chapter, it has not been applied decision criteria or optimization techniques. This is because initially the author has wanted to test the basic functionality of the model. Nonetheless, the window for using control-engineering tools is opened to address relevant research. Additionally, there is the expectation of testing the model with real data in a case study. Nevertheless, in Chapter 6, a case study (hypothetical but realistic) is examined showing the potential of the model. Its use is valued from the point of view of the support offered to decision makers. On the other hand, there exist several control techniques and theories over which would be interesting to explore its applicability to the proposal of the research. Particularly with the presentation of the model in terms of *Z*-transform, a range of possibilities is deployed in order to address advanced control techniques in combination with systems engineering techniques, by applying them on CFP, especially, to tackle the cash management problem. In this regard, the following are some useful modeling techniques to shed light on the matter: predictive control, adaptive control, inverse-model-based control, internal model control, direct control, indirect control, supervised control and multivariable control. Precisely, the next chapter of this thesis proposes a model predictive control under a decentralized approach in order to controlling cash balance in the revenue accounts for CCDS model presented in this chapter. This proposal or any other must produce feasible results for a wide range of probable financial scenarios, subject to the particular context of decisions. In the presence of uncertainty, the whole creation process must be robust; condition required throughout the planning horizon, including the method, the set of solutions and conclusions drawn.

Chapter 4

Model Predictive Control for a revenue account of a CCDS

The proposed model in Chapter 3 describes a Cash Concentration and Disbursements System (CCDS) using difference equations. An equivalent version of the model is included represented by algebraic equations based on the application of Z-transform. The aim of the CCDS is to concentrate cash money in a main bank account in order to make best use of money in large amounts to support investment and financing operations. The system also includes a revenue bank account and a disbursements bank account in each agency. According to a pre-established policy, the cash in revenue accounts of the agencies is transferred periodically to the main account, and from there, the necessary cash amounts are transferred to the disbursements accounts. A CCDS is liable to be approached from the point of view of control theory, by which automatic control techniques can be applied to control the balances in the accounts of the system. In other words, the model in Chapter 3 focuses on control decisions aimed at transfer money toward the system elements in accordance with the requirements. Such decisions take into account relevant costs, the proper frequency and the amount of cash transferred. This is why that the aim of the this research is to control the available cash balance in the accounts involved, for which systems theory can provide mechanisms to make decisions autonomously. As noted above, control actions are the frequency of transfers and the amounts transferred.

This chapter presents a model predictive control (MPC) for a revenue bank account contained in the CCDS model proposed in Chapter 3. In general, the problem is cast as a supply chain management particular problem, and in each bank account, control is focused as an inventory problem. Model predictive control or model-based predictive control (MBPC) is a control technique through which the control action in each sample time is determined by solving an optimization problem involving one or more cost functions on a moving prediction horizon, preferably finite. The current state of the plant is used as an initial state for the optimal control problem, then the first optimal control signal of the sequence calculated for the control horizon is applied to the plant as a control action. The proposal considers MPC as a viable strategy for decision-making in a CCDS because it allows taking into account the desired reference values for future that are the result of planning or scheduling according to the policies of the company and the stated objectives. Then, the controller determines the control actions to achieve these objectives based on the reference values, the system's past behavior and the established constraints. Although for most companies is not easy to invest in advanced control techniques, in general, some other advantages of the MPC can be pointed out. For instance, the fact that it offers the possibility of achieving optimum operating levels because inherently includes an optimization problem, as explained above. Also, because of its versatility, it is applicable to various types of processes: stable or unstable, multivariable, with delays and, particularly, slow processes such as the related to this work. Finally, MPC might react to in advance the changes caused by the disturbances.

On predictive control, Clarke *et al*. (1987a, 1987b), Tsang & Clarke (1988) and Clarke & Mohtadi (1989) presented the initial ideas. Robinson & Clarke (1991) also apply the robust stability concept to the generalized model of predictive control. Likewise, Bone (1995) develops an iterative learning control formulation termed generalized predictive control with learning (GPCL) based on a Controlled Auto-Regressive and Integrated Moving-Average (CARIMA) plant model. Moreover, De Keyser (2003) proposes the methodological ideas on EPSAC (Extended Prediction Self-Adaptive Control), as part of the MPC family, applied to linear and nonlinear systems. In the same way, in Mayne *et al*. (2005, 2006 and 2009) robust approaches on restricted linear systems are found. More related to this thesis, Dong, Zheng & Li (2011) applied MPC to the inventory management in the supply chain planning problem. Lastly, Fu *et al*. (2014) focus their work on establishing a general method to mitigate the bullwhip effect in supply chain management using MPC. Also, in Bemporad & Morari (1999) it is located an extensive literature review about this topic.

With regard to the MPC proposed here, it is supposed that the firm has established an adequate level of autonomy in the financial decisions of each agency. This assumption means that, besides a general cash transfers policy consistent with the applied model, it is assumed that there are no restrictions to decide independently the amounts transferred from the agency's revenue account towards the CCDS's main account, and vice versa. Because of this, a decentralized approach for the CCDS is adopted, which makes possible to propose: (a) a control model for agency's revenue account, (b) a control model for agency's disbursement account, and (c) a control model for central accounts of the CCDS, as a whole. In particular, the chapter is devoted to the formulation of a model predictive control for the revenue account of an agency. That is, a control model for a type of account, which later is applied to all agencies of the CCDS. Furthermore, a prediction model solved in the time domain through dynamic programming (DP) is proposed.

Dynamic programming deals with problems related to processes that can be represented in multiple stages. That is, problems in which a bounded or unbounded sequence of operations is required to be performed in order to achieve a desired result (Bellman, 1952). This approach allows transforming the representation of a problem from the space of decisions to the space of functions (Bellman, 1954a, 1954b). Early on, Bellman (1966) exposed the feasibility of using DP on specific control problems, and other authors have directly applied DP on inventory control problems and simulation (Bensoussan, 2011, and Cardoso *et al.*, 2013). However, in his work Bellman himself and other authors have noted the problem of dimensionality (Wijngaard, 1979), which is presented as one of its major drawbacks as well as the need for discretization of models (Bertsekas, 1975). Meadows (1997) establishes a new criterion for assessing the stability of nonlinear models predictive control based on monotonicity of its cost function. He decomposes a general cost equation formulated for MPC as a problem of N-stages into a series of sub-problems, allowing represent a MPC under the DP approach. Also, Diehl & Björnberg (2004) proposed the application of robust dynamic programming method to predictive control problems in discrete time. However, their proposal is not oriented to an optimization model with respect to the worst-case of the system behavior, which normally leads to the formulation of a min-max model. In contrast, they use a joint representation of the feasible set and cost-to-go, which guarantees robust stability for a closed-loop MPC. This condition is demonstrated through standard techniques of polyhedral computation. On the other hand, Bertsekas (2005) embeds rollout and MPC based on a concept of restricted structure policies, by starting with a suboptimal/heuristic policy, after which it is improved by using its cost-to-go as an approximation to the optimal cost-to-go. Belarbi & Megri (2007) introduce a stable predictive controller based on fuzzy model combined with fuzzy dynamic programming with branch and bound. The controller aim is to drive the system status to a terminal region, where a closed-loop linear local controller of stabilization is invoked solving a Lyapunov equation. In a series of papers (Lee & Wong, 2010, Wong & Lee, 2011 and Lee, 2011), an interesting discussion about limitations of model predictive control (MPC) is presented. In this regard, they analyze the possibility of reducing these limitations by combining MPC and Approximate Dynamic Programming (ADP) in which the robustness condition is achieved through the use of a penalty function. They suggest that applying ADP in control models can replace or complement MPC in order to reduce the online computational load, as well as with the uncertainty associated to the disturbance. Among other matters of interest, they also claim it is more convenient to use a post-decision state formulation, compared to a pre-decision state formulation. That is how, they propose the use of ADP based on a post-decision state formulation and argue that this approach allows the use of deterministic dynamic programming solvers, off-line and on-line. As a complement, their work proposes ways to integrate both methods to alleviate each other's shortcomings, after analyzing pros and cons of MPC and ADP. Nevertheless, they conclude that combining ADP and MPC has offered limited effectiveness. In any case, the author have not found references about research works addressing the cash management issue under the approach of MPC together with DP, therefore, the approach can be considered as a novel contribution, which opens perspectives on the application of these techniques to the field of finance.

The solution presented in thesis combines DP and MPC according to the approach of Camacho & Bordons (2007), for whom, robust control refers to design of a controller by preserving stability and performance despite the presence of modeling inaccuracies or uncertainties. That is, explicitly considering the discrepancies between the model and the real process. Compared with the proposals described above, particularly Lee & Wong (2010), this proposal does not consider any learning technique neither the approximate dynamic programming approach, see also Sui *et al*. (2010). In this regard, it helps the argument of Lee & Wong (2010), for whom the main concern should be the safety and economy of operation in progress and not in effective learning. Rather, this work focuses on the idea of closed-loop prediction, that is, it is based on the stabilizing regulator in cascade fashion to reduce uncertainty ranges using a linear feedback gain (Bemporad, 1998, Chisci *et al*., 2001). As will be seen later, a condition for process stability is determined, regardless of the forecast/control horizon. Most of the researchers emphasize the importance of choosing the prediction horizon in terms of the solution quality and of computational burden (Belarbi & Megri, 2007). On their part, Rossiter *et al*. (1995) refer to the treatment of stable MPC with constraints which are supposed to be feasible: quadratic programming generalized predictive control (QPGPC), mixed-weights least squares (MWLS), problem in constrained stable generalized predictive control (CSGPC).

In this respect, although the approach using a closed-loop linear feedback itself does not solve the DP dimensionality problem, here it is proved that under certain conditions the size of prediction horizon is a minor problem for process stability. Another feature of the model proposed here is the use of a smoothing standard model for forecasting the disturbance, which is modified to create a forecasted series covering the entire forecasting horizon. With this approach, it is possible to address the formulation of MPC in the context of deterministic DP problem, partially relieving the problem of dimensionality of DP. Resolved this, and after proposing an equivalent or extended model for the rest of the accounts of CCDS, it will be possible to configure a strategy that facilitates the establishment of the transfer automatic program between different accounts. In this way, the benefit-cost plan provided by the financial directors is achieved. The control criteria are defined, and the comprehensive cash management plan is supported.

The remainder of the chapter is divided into five sections. Section 4.1 makes the presentation of the problem. Afterwards, Section 4.2 shows the prediction model for control. Section 4.3 includes the matrix representation of dynamic programming model and its stability analysis, followed by Section 4.4, in which the simulation results are presented and, finally, Section 4.5 deals with the concluding remarks. Furthermore, two appendices provide additional details on several of the issues discussed in the chapter. Appendix C: a reviewing of the objective function behavior of the proposed MPC. Appendix D: a segment of the algorithm version applied for simulation. Also, it is good to clarify that the content of this chapter has been published in Herrera-Cáceres and Ibeas (2016b, 2016c).

4.1 Problem formulation

As mentioned earlier, a decentralized approach for the CCDS is adopted, whereby, a model predictive control (MPC) for agency's revenue account is proposed. Figure 4.1 shows the information and money flow related to the agency's revenue account, affected by the MPC controller. As in Chapter 3, a basis time interval $\tau = 1$ is assumed, usually a day, but it can also refer to a week or a month, or other time interval used in financial practice, according to the dynamics of the CCDS. In addition, in the remainder of this chapter, the agency (and its relation to other system elements) is not identified with a subscript in the nomenclature used to differentiate it from other agencies; instead, the same mathematical treatment to each one of the agencies composing the entire company is assumed.

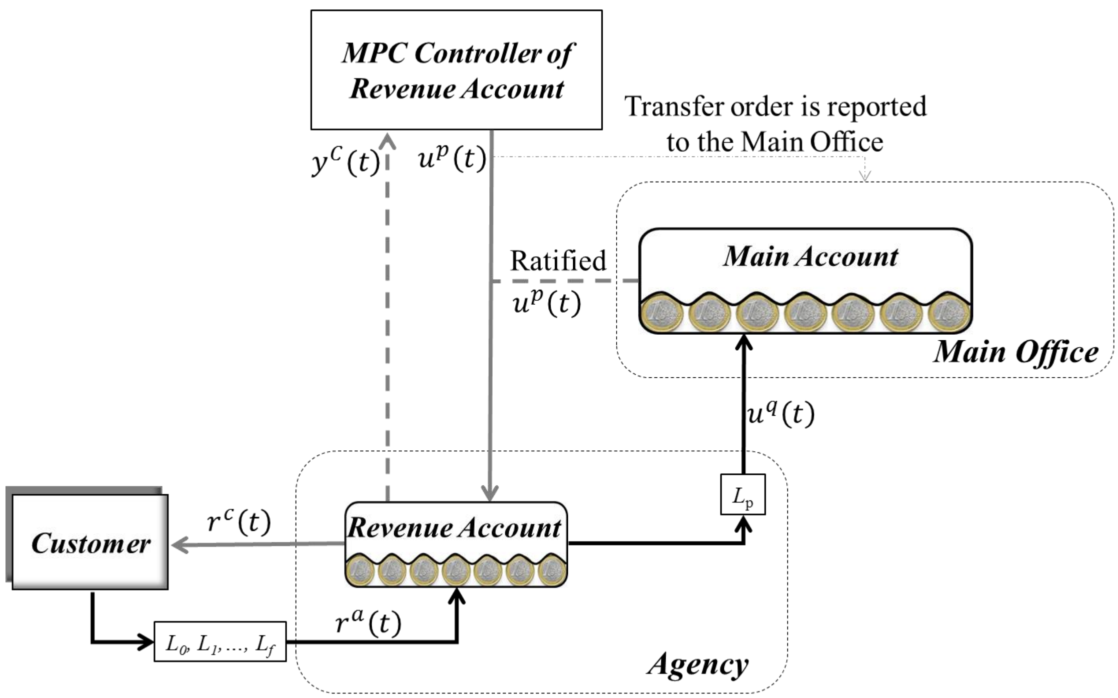


Figure 4.1: Decentralized MPC strategy for CCDS – Revenue account of Agency

4.1.1 Modeling the revenue account

The cash collection process has been described in Chapter 3, but below is repeated for convenience. The cash collection is the start of the process. The agency demands to its customers the payment for goods sold or services rendered. Thus, $r^c(t)$ refers to cash deposits in agency's revenue account at any discrete time interval $t = h\tau$ ($h \in \mathbb{N}$). Deposits are made latest at the end of t (for instance, at the end of the day), which correspond to the payments made by customers during that interval. Customers make their payments according to $f+1$ different forms of payment, identified by $m=0,1,2,\ldots,f$. Each form of payment is associated with a transit time or delay represented by L_m (of the form $h\tau$, $h\in\mathbb{N}$), after which money will be available in agency's revenue account. In this case, $m=0$ corresponds to cash payments, or other forms of payment involving immediate availability of money in the account, at time in which the deposit or transaction is made; that is, transit time $L_0=0$. Moreover, based on bank procedures, deposits relative to other forms of payment are available $L_m>0$ after making the deposit or transaction. Furthermore, for the purposes of this document, the behavior of the cash availability as a function of the payment method m is foreseen according to the following discrete deterministic scheme:

Let b_m^r be the portion of cash collection made according to the form of payment m, for which it is true: $0\le b_m^r\le 1$ and $\sum_{m=0}^{f} b_m^r=1$. Thus, total revenue in the agency during t is given by:

$$r^c(t)=\sum_{m=0}^{f} r^{cm}(t) \tag{4.1}$$

Being $r^{cm}(t)=b_m^r r^c(t)$ the total deposit in the revenue account of the agency in accordance with the form of payment m during the time interval t. Hence, the money will be available in the revenue account in accordance with the following expression:

$$r^a(t)=\sum_{m=0}^{f}[b_m^r r^c(t-L_m)] \tag{4.2}$$

Moreover, an available cash transfer ($u^p(t)$) from the agency's revenue account to the CCDS's main account is ordered. This order is made at the end of t, subject to the operating rules of the CCDS, in accordance with previously established policies. **The local MPC controller determines the transferred amount $\boldsymbol{u^p(t)}$**, following which it is reported to the main office. It will be available $L_p\ge 0$ after the transfer order in the main account. That is, $u^q(t)=u^p\left(t-L_p\right)$ and L_p is of the form $h\tau$, $h\in\mathbb{N}$.

The difference equations representing the movement and balance of the agency's revenue account provide the basis for formulating the model predictive control (MPC). In this regard, the following difference equations satisfy the conservation law of both money and information:

Available cash balance:

$$y^A(t) = y^A(t-1) + \sum_{m=0}^{f}[b_m^r r^c(t - L_m)] - u^p(t) \tag{4.3}$$

Actual balance:

$$y^T(t) = y^A(t) + y^D(t) + y^L(t) \tag{4.4}$$

Or, explicitly:

$$y^T(t) = y^T(t-1) + r^c(t) - u^p\big(t - L_p\big) \tag{4.5}$$

Since cash transfers between accounts can only be made based on available balances, in this model consisting of equations (4.1) to (4.3), what it is wanted is to control the $y^A(t)$ variable. Deferred balance, frozen balance and actual balance are mentioned here only for administrative reference and because of their importance in the bank conciliation.

The following sequence of events is defined:

a. At discrete time t, the agency's customers make payments, which gives rise to deposits amounting to $r^c(t)$ in the respective revenue account, later than at the end of t.

   Arguments that allow grouping all the customers' revenue during t as a single aggregate amount without affecting the dynamics of the system are given in Lin *et al*. (2004). This is complemented by the deterministic discrete scheme explained above about the cash availability, which depend on the form of payment used by customers.

b. The operative available balance $y^C(t)$ is observed, which is calculated by summing $r^a(t)$ and $y^A(t-1)$ (the available balance at the end of previous period, $t-1$). Hence:

$$y^C(t) = y^A(t-1) + r^a(t) \tag{4.6}$$

c. In accordance with established policy, the amount of cash transferred ($u^p(t)$) to the CCDS's main account is decided and ordered. Also, $u^p(t)$ is reported to the firm main office. From the agency's point of view, this is the control action based on the MPC controller decision. However, in the main account, this cash money will be available L_p after t.

d. Again, the available balance $y^A(t)$ is measured. The other balances in the revenue account (frozen, deferred and total) are also measured. This is because, for purposes of bank conciliation, the firm's management needs to know the entire account balances scheme.

There is an argument of physical and economic order that is given in this type of system to differentiate between operating cash balance ($y^C(t)$) and cash balance at the close of a discrete interval t ($y^A(t)$), with $y^A(t) = y^C(t) - u^p(t)$. A financial system operates differently than other systems on which control techniques are applied. In many systems, you can handle certain aspects such as deciding the control action the beginning or at the end of a discrete time interval. In a financial system, the usual thing is decide at the end of the time interval because a part of continuous time is not incorporated into a discrete model. During this "dead" time (nonworking hours for other systems), financial systems decide investments based on available cash determined during working hours. Also, financial systems are connected to other financial systems with different time zones. For this reason, decisions are made at closing. At the beginning of the next discrete time interval, everything looks the same but the money has moved behind racks. This is one of the reasons why the decision to transfer in our CCDS is made at the end instead of at the beginning of a discrete interval t.

In the events sequence described above, it is important to highlight that the transfer order ($u^p(t)$) is performed at the end of t, so that the money transferred will be available in the main account L_p after t (item c). Indeed, the company must establish the cash transfer policy represented in a reference signal $w(t)$**,** whose parameters explicitly include the criterion for deciding the amount to be transferred and the frequency. Such that, once the $y^C(t)$ value is determined, the control system calculates the necessary transfers ($u^p(t)$) at every time t in order to achieve the control objective. **Consequently, an MPC controller is proposed for autonomously determining the necessary transfers given the value of $\boldsymbol{y^C(t)}$, based on a reference signal $\boldsymbol{w(t)}$.** Details regarding the design of the controller will be discussed theoretically and through simulation in the rest of this chapter to show compliance with the objective of control. It should be clear that the aim is to control the available cash balance ($y^A(t)$). Other variables (balances) are mentioned only due to their relationship with it and because they are useful in the company administrative area, as mentioned above.

4.1.2 Transfer function

Modeling the revenue bank account is important to pin down the dynamic response of the system. For this reason, an equivalent model by using the *Z*-transform is presented considering the sharing of information and cash between the customer, the revenue account and the main account of CCDS. Namely, the transfer function, which is essential for the simulation and the basis for building the prediction model later in Section 4.2.

Particularly, applying the time shifting property of *Z*-transform on (4.3) and (4.6) (Ogata, 1996), they become:

$$Y^A(z) = \frac{1}{1-z^{-1}}\left(-U^p(z) + \sum_{m=0}^{f}(b_m^r z^{-L_m})\, R^c(z)\right) \tag{4.7}$$

$$Y^C(z) = z^{-1}Y^A(z) + R^a(z) \tag{4.8}$$

And, by considering (4.2), (4.7) can be written as:

$$Y^A(z) = \frac{1}{1-z^{-1}}\left(-U^p(z) + R^a(z)\right) \tag{4.9}$$

wherein:

$$R^a(z) = X(z).R^c(z) \text{ and } \quad X(z) = \sum_{m=0}^{f}(b_m^r z^{-L_m}) \tag{4.10}$$

Note that equations (4.7) or (4.9) represent an unstable relation between the variables (through the integrator $P(z) = 1/(1 - z^{-1})$), which is one of the problems in financial management control system under consideration, and which also appear in other problems such as supply chain inventory management (García *et al.*, 2012). In this case, the accumulator $P(z)$ is the plant (the system). That is, a SISO system with transfer function of degree 1 in both numerator and denominator, which allows to define the model concerned as biproper (relative degree 0) or, at most, proper (not strictly). Stated another way, the output ($y^A(t)$) of the system at t time interval is determined by the input ($u_p(t)$) at the same t time interval. Moreover, as shown in (4.9) and from the viewpoint of the agency, there is not dead time or delay (z^{-L} con $L = 0$). For the purpose of better understand the issue of biproper models, Teixeira et al. (2009) argue regarding the determination of the root-locus of a feedback system with biproper transfer function in open-loop from the root-locus of an equivalent system with strictly proper transfer function in open-loop. Furthermore, because of its nature, the system may be considered a slow system, when compared with the systems usually studied in the field of control. This feature and the adoption of a decentralized approach allowing simplify the problem in general, significantly reduces the drawbacks regularly found in studying biproper systems.

4.1.3 Cash transfer policy

A cash transfer policy establishes the amount of money transferred and the frequency of transfers. Its formulation provides the basis for the complete representation of the system dynamics. Due to the characteristics of the system, the available balance in the revenue account of the agency behaves according to an inventory policy or equivalent criterion. On the one hand, there are different inventory policies useful when establishing the cash transfer policy. By way of example and for comparative purposes, it has been assumed in this thesis a continuous inventory monitoring system of the type (s, S), which does not diminish generality to the model. Once assumed a criterion, it is used to design the reference signal for the prediction model. On the (s, S) policy, s is the amount of cash over which the agency should make the transfer based on the optimal (s, S)-policy (Beyer & Sethi, 1999), and S is the level at which returns the available cash after the transfer is performed. Which means that, theoretically, $u^p(t) = |s - S|$. On the other, the proposal in this chapter is inspired by the concept of Zero Balance Accounts (ZBAs) in each agency, as explained in Chapter 3. That is, the available cash in the agency's revenue account periodically is transferred to the main account, whereupon the available balance returns to zero. Which also does not diminish generality to the model. Thereby, if $S = 0$, in practice the cash transfer policy is defined as:

$$u^p(t) = \begin{cases} 0 & if\ y^C(t) < s \\ y^C(t) & if\ y^C(t) \geq s \geq MT \end{cases} \tag{4.11}$$

Being $MT(\geq 0)$ the minimum transfer size, established according to banking agreement or firm's policy. For the purpose of simplification, in the subsequent development of the chapter $MT = 0$ is assumed, which mean the minimum transfer policy not exist. As discussed below, this policy is used to justify the application of sawtooth model like reference signal in the proposed model. However, it can also be used in the application of a traditional inventory control model by directly applying the (s, S)-strategy with periodic review. Hence, the results in both cases are comparable. Nevertheless, the predictive control itself offers more versatility in defining policies and establishing strategies by adding some complexity elements. This can be done by modifying the parameters of the policy, changing the inventory policy or incorporating other aspects not considered so far, such as for example, delay time. In Chapter 5 the application of a generalized form of this policy is discussed. That is, if $|y^C(t)| \geq |s|$, the value of the transferred amount is $\hat{y}^C(t+k|t) - S$. In this case, the decision maker has the choice to set the level at which the available cash balance returns when the zero balance account concept is not applied.

4.1.4 Uncertainty process

The agency's revenue acts as disturbance because they introduce uncertainty affecting the available cash level in the revenue account. Following De Keyser & Ionescu (2003), if (4.9) is rewritten as follows:

$$A(z^{-1})Y^A(z) = B(z^{-1})U^p(z) + n(z) \tag{4.12}$$

where $A(z^{-1}) = 1 - z^{-1}$ and $B(z^{-1}) = -1$, the cash revenue model ($n(z) = R^a(z)$) is equivalent to an Autoregressive Moving Average Process (ARMA) of the form:

$$D(z^{-1})n(z) = C(z^{-1})e(z) \tag{4.13}$$

And, renaming $X(z)$:

$$C(z^{-1}) = X(z) = \sum_{m=0}^{f}(b_m^r z^{-L_m}) \tag{4.14}$$

Because this research is limited to stationary scenarios, the model does not incorporate any deviations in the process caused by other external perturbations. The $e(z)$ series is a white noise with zero-mean and standard deviation one. Also, to complete the agency's revenue model, $D(z^{-1}) = K_r,$ where K_r is a constant in the interval $(0, 1)$ inversely proportional to the average revenue level of the agency, and:

$$e(z)/D(z^{-1}) = R^c(z) \tag{4.15}$$

4.2 Prediction model for control

Literature presents various alternatives to address the control problem described by (4.8), (4.9) and (4.10). In this chapter, a model predictive control (MPC) according to Camacho & Bordons (2007) is assumed. With this approach, the sequence of operations described in Section 4.1 is analyzed, whereby the established policy focuses on deciding the amount of cash to be transferred ($u^p(t)$) to the main account of CCDS at the end of t. That is how, the system response fits to a reference signal ($w(t)$) representing the cash transfer policy. The cash transferred will be available L_p later in this account. In this way, based on past inputs and outputs and the future control signals, the prediction model is in charge of determining the sequence of actions that minimizes at least a cost function at each sampling instant. Then, using the receding horizon strategy, the first component of this sequence is applied, with which the value $u^p(t)$ is set, integrating thereby the prediction model and the control objective in a single process whose guide is the reference signal. To this end, a dynamic programming model resolved in the time domain is proposed. With this technique, the purpose is achieved by determining a functional equation whose solution is equivalent to solving the original problem. This equation is based on the recursion concept application and the fundamental approach of Bellman (1954b) optimality principle, according to which an optimal policy has the property that, regardless of the initial conditions, "*... the remaining policies constitute an optimal policy with respect to state resulting of the first decisions*". The concept can be applied in both directions, forward and backward (Verdú & Poor, 1984). Here, a dynamic programming model in backward recursion version is applied, which has several advantages, besides the problem of dimensionality noted earlier. On the one hand, DP allows dynamically analyzing a range of options in the state space (exploring exhaustively the state space). Furthermore, the Dynamic Programming backward version directly, properly and efficiently, determines the optimal value raised by the objective of MPC, based on the systems' recursion property by dividing a complex problem into sub-problems easy to solve. Also, it is possible to combine the principle of optimality with the recursion property, starting at the end of control horizon until the value of $u^p(t)$ is determined as an optimal solution at the beginning of control horizon. Similarly, with DP it is dynamically possible to handle the problem of control horizon and finally, after analyzing the problem, an appropriate strategy for the convergence and the stability of the process can be determined.

4.2.1 Prediction model

Regarding the features of the MPC applied to the cash balance control problem in the agency's revenue account of a CCDS, the following is proposed. The prediction horizon N $(t+1, t+2, \ldots, t+N)$ determines $N+1$ stages $(t+0, t+2, \ldots, t+N)$ for the dynamic programming model. This means, there are N forecasted instants for which the future outputs of the model are predicted. Then, the value of $u^p(t)$ at step 0 is calculated (call it $u^p(t+k|t)$, with $k=0$, or $u^p(t|t)$), which is sent to the process. Similarly, the predicted outputs $\hat{y}^A(t+k|t)$ (cash balance at the end of time $t+k$, with $k = 0{,}1 \ldots N$) depend on the known values (inputs and outputs) until time t and the set of future control signals $u^p(t+k|t)$. The prediction model based on dynamic programming considers $u^p(t+k|t)$ as the decision variable representing the alternatives at stage $t+k$. That is, the amount to be transferred to the CCDS's main account from the agency's revenue account at the end of $t+k$. The operating cash balance (just before transferring) is the state variable:

$$\hat{y}^C(t+k|t) = \hat{y}^A(t+k-1|t) + \hat{r}^a(t+k) \tag{4.16}$$

That is, a pre-decision state formulation, wherein $\hat{r}^a(t+k)$ is the forecasted amount of cash revenue that becomes available during $t+k$, based on the process represented by (4.2). Moreover, the predicted values of the output variable are established by:

$$\hat{y}^A(t+k|t) = \hat{y}^A(t+k-1|t) + \hat{r}^a(t+k) - u_p(t+k|t) \tag{4.17}$$

Particularly, $\hat{y}^A(t+N|t)$ is referred to the cash balance of the agency's revenue account at the end of the forecast period $t+N$, which prevents entering an infinite loop (Camacho & Bordons, 2007).

4.2.2 Revenue forecasting

For the revenue forecasting, the standard exponential smoothing method N-N (no trend, no seasonal) was considered (Gardner Jr., 2006), whose expression is of the form:

$$\hat{n}(t+1) = \alpha . n(t+1) + (1-\alpha)\hat{n}(t) \tag{4.18}$$

where $\alpha \in (0{,}1)$ is a smoothing constant or smoothing parameter for the level of the series, and $\hat{n}(t+1)$ is the expected value of data at the end of $t+1$, or the smoothed level of the series, calculated after the $n(t+1)$ value is observed. When developing the series, it can be seen that the $\hat{n}(t+k)$ forecast mainly depends on the observed values sum and, lesser extent, on the first forecast of the series:

$$\hat{n}(t+k) = \alpha \sum_{j=0}^{k-1}(1-\alpha)^j n(t+k-j) + (1-\alpha)^k \hat{n}(t), \text{ for } k = 1,2,\dots,N \quad (4.19)$$

Which, if it comes to a process with constrained uncertainties ($\underline{n} \leq n(t) \leq \overline{n}$), values are also bounded according to:

$$[1-(1-\alpha)^k]\underline{n} < \hat{n}(t+k) < [1-(1-\alpha)^k]\overline{n} \quad (4.20)$$

And, for sufficiently large values of k, forecasting boundaries correspond to the bounds of the observed values ($\underline{n} \leq \hat{n}(t+k) \leq \overline{n}$).

Now, what does *observed value* means in MPC? It is known that the prediction model is invoked at time t, the elapsed time of simulation. From t, the forecasting model for the disturbance is used, which regularly generates a single predicted value, at time $t+1$. The reviewed literature in this research does not explain how it is used locally regarding the observed values to generate the predicted values remaining in the MPC along the entire horizon $(1,2,\dots,N)$. See e.g.:

- ✓ Under the approach of ADP, with the aim of identify relevant regions of the state space in order to make random exploration or construct well-controlled trajectories (Lee & Lee, 2006; Wong & Lee, 2011).
- ✓ To establish a balance between exploration in the state space and robustness based on ADP (Lee & Wong, 2010).
- ✓ With the objective of optimizing the performance of a nominal model or solving a min-max problem to optimize the robust performance (Robinson & Clarke, 1991, Bemporad & Morari, 1999; Camacho & Bordons, 2007).
- ✓ Just as it has been done in the present work, by establishing the forecasting model of disturbance in supply chain management (Fu *et al*., 2014; Dong *et al*., 2011).

Typically, only it refers to a forecasting model being used for the entire forecasting horizon. Also, multivariate random walk procedures are used in not simulated processes to estimate the parameters by means maximum likelihood methods, or univariate random walk procedures which approximate to maximum likelihood methods (Gardner Jr., 2006). In this work, a pseudo-random generator with equivalent parameters to those of the simulation is used to generate locally values observed.

4.2.3 Reference signal

In accordance with the objective of a CCDS and within the framework of the decentralized approach used in the controller's proposal, it would be desirable to keep the revenue accounts at a predetermined level: zero, if a ZBA criterion is used, or other desired value, S if a policy (s, S) is assumed, as explained in Subsection 4.1.3. Whatever the case, due to the nature of the system, the available balance in the revenue account always behaves according to a triangular function, since cash balances are preferably transferred to the main account once assessed its use. Hence, a stationary scenario (Subsection 4.1.4) allows establishing a periodic reference trajectory, piecewise linear, commonly called sawtooth wave [(4.1)]. Then, following Trott (2004) and assuming the cash transfer policy expressed in (4.11), the reference signal can be generated as:

$$w(t+k) = A.frac\left(\frac{t+k}{T} + \phi\right) \qquad k = 0,1 \dots N \tag{4.21}$$

wherein:

- ✓ $frac(x) = x - \lfloor x \rfloor$ (fractional part of x),
- ✓ $A = T * E\big(d_a(t)\big)$ is the wave amplitude,
- ✓ $T = \big\lceil s/E\big(d_a(t)\big)\big\rceil$ is the wave period, and $\phi = 0$ is its phase,
- ✓ $E\big(d_a(t)\big)$ is the expected value of available revenues,
- ✓ s is the amount of cash over which the agency should make the transfer in accordance with the optimum transfer policy, and
- ✓ $\lfloor x \rfloor$, $\lceil x \rceil$ are the floor and ceil functions, respectively.

This wave can also be generated using Matlab® *sawtooth* and $pi(= \pi)$ functions with the following expression: $w(t) = A * (sawtooth(2 * pi * t/\mathrm{T}) + 1)/2$.

The periodic behavior of the reference signal is equivalent to the cash transfer policy described in Subsection 4.1.3. That is, the cash balance represented by the state variable $\hat{y}^C(t+k|t)$ in step $t+k$ is evaluated, after which a decision is made: either the cash

(4.1) The sawtooth signal is very well suited to the established policy. However, the predictive control method has the capability to adapt to reference signal variations, making it more versatile.

balance is transferred to the main account or not. Accordingly, the decision variable in the dynamic programming model takes the values $u^p(t+k|t) = 0$ or $u^p(t+k|t) = \hat{y}^C(t+k|t) \geq MT,\ k = 0,1,2,\ldots,N$. Because it is a Dynamic Programming backward strategy, the dynamic process begins at step $k = N$, ending at step $k = 0$, when the $u^p(t|t)$ value is determined, corresponding to the minimum value of the cost function of model. This is $u^p(t|t) = 0$ or $u^p(t|t) = \hat{y}^C(t|t)$ $(= y^C(t) \geq MT)$.

4.2.4 Bellman equation

Typically, the design of a control model includes an optimum criterion according to which the aim of keeping the process as closely to the reference signal as possible is fulfilled. This criterion is usually expressed in the form of a quadratic function of the error between the predicted output and the predicted reference trajectory. For this reason, it has been considered the following objective function for the prediction process:

$$J = \underbrace{\sum_{k=0}^{N} \delta(k)[\hat{y}^A(t+k|t) - w(t+k)]^2}_{J_y} + \underbrace{\sum_{k=1}^{N} \lambda(k)[\Delta u^p(t+k-1|t)]^2}_{J_u} \qquad (4.22)$$

where:

- ✓ $\hat{y}^A(t+k|t)$ is the predicted outputs at stage $t+k$.
- ✓ $w(t+k)$ is the reference signal at stage $t+k$.
- ✓ $\Delta u^p(t+k-1|t)$ is the variation of control signal at stage $t+k$.

The coefficients $\delta(k)$ and $\lambda(k)$ are sequences that take into account the system future behavior. Usually, they are considered constant values or exponential sequences, which is set according to each particular case. Lee & Lee (2006) incorporate a discount factor (as $\delta(k)$) that handles the tradeoff between the immediate and delayed costs. For this study, $\delta(k)$ and $\lambda(k)$ are of the form $\delta(k) = {\alpha_y}^{N-k}$ and $\lambda(k) = {\alpha_u}^{N-k}$. Values of α_y and α_u in the range (0,1) give rise to a smoothed control, which is not wanted in this work. The proposed control model requires faster control, which occurs if $\alpha_y, \alpha_u > 1$.

The formulation in (4.22) is the cost function of model, which measures the total error in the prediction horizon between the predicted output and the predicted reference trajectory. Because it is a pre-decision state formulation, the initial state of the dynamic process (step $k = 0$) is preset. However, the control action ($u^p(t|t)$) and the process output

($\hat{y}^A(t|t)$) at this step are values to be determined by the process. Hence, although the forecast horizon is from 1 to N, the objective function (4.22) includes the value of stage $k = 0$. See also the reference signal (4.21). Applying DP, the aim is to minimize the error expressed in (4.22). On one hand, its variability is a function of the decision variable ($u^p(t+k|t)$, future control signals) which, in turn, is subject to the cash transfer policy represented in the referential signal. Moreover, variability in (4.22) also depends on the predicted disturbance ($\hat{r}^a(t+k)$), whose stability, per se, determines the process stability.

Due as well to the nature of the problem, the foregoing leads to the model simplification in several respects (Bertsekas, 2005), aiming to reduce some of the disadvantages of application of DP. For purposes of generalization, the second term in (4.22), which considers the control effort, is optionally included as a measure of the increments of u^p (Δu^p), Camacho & Bordons (2007). Given the characteristics of u^p series, theoretically $\Delta u^p \in \{-u^p, 0, u^p\}$, or in accordance with Subsection 4.2.3, $\Delta u^p \in \{-y^C, 0, y^C\}$. Likewise, the series $\hat{y}^A(t+k|t) - w(t+k)$ is a sawtooth wave with period T. Then, if limits for r^a and y^C are assumed as well $0 \le \underline{r}^a \le r^a \le \overline{r}^a < +\infty$ and $0 \le \underline{y}^C \le y^C \le \overline{y}^C < +\infty$, respectively, both terms in (4.22) (called J_y and J_u), can be written as:

$$0 \le J_y \le A_y^2 \sum_{k=0}^{N} \left[{\alpha_y}^{N-k} frac^2\left(\frac{k}{T} + \phi\right)\right] \tag{4.23}$$

$$\left(\underline{y}^C\right)^2 \sum_{k=1}^{N} {\alpha_u}^{N-k} \le J_u \le \left(\overline{y}^C\right)^2 \sum_{k=1}^{N} {\alpha_u}^{N-k} \tag{4.24}$$

where A_y, T and ϕ are, respectively, the amplitude, the period and the phase of the sawtooth wave within the prediction process.

These conditions have been used to study the behavior of J_y and J_u as described in Appendix C and, in this manner, deciding on whether or not to include the control effort (J_u) in the objective function. The main conclusion of the study states that in both cases, smoothed control ($\alpha_y, \alpha_u < 1$) and faster control ($\alpha_y, \alpha_u > 1$), the relationship between the factors $[\hat{y}^A(t+k|t) - w(t+k)]^2$ and $[\Delta u^p(t+k-1|t)]^2$ has a leading role in the outcome. Nevertheless, an appropriate combination of α_y and α_u can be chosen in order to handle the result of the objective function J. The issue focuses on selecting a combination of weights such that, on the one hand, the control objective is met and, on the other hand, the model is simplified. These requirements are not met when α_y and α_u are relatively similar values or when α_u is greater than α_y. The most convenient condition, as concluded in Appendix C, occurs when α_y is sufficiently greater than α_u, and, hence, it is possible to dispense with the control effort term (J_u). If smooth control were the case, with a combination of values of α_y and α_u close to 1, J_u dominates the value of J and has a decisive role on the control action. On the other hand, with a combination of values for α_y and α_u close to zero, the interaction between J_y and J_u is higher. It is worth observing that the objective function behavior is determined by the periodic shape of the curve (with period T), when the size of the control horizon varies, which is most noticeable for values of α_y and α_u near zero. In contrast, if a prediction model with faster control is proposed ($\alpha_y, \alpha_u > 1$), it is obtained that the remarkable thing is the exponential growth of J_y and J_u when growing their respective weighting, N or both. However, in this case J_u grows faster than J_y, which agrees with the previously indicated importance about the factors $[\hat{y}^A(t+k|t) - w(t+k)]^2$ and $[\Delta u^p(t+k-1|t)]^2$ in (4.22). As already mentioned, selecting a suitable combination of α_u and α_y helps when setting the parameters of the objective function of the proposed model, including adding or not the control effort term. Based on this analysis, hereinafter, the choice of α_y values sufficiently greater than α_u is assumed, making negligible the control effort term (including $\alpha_u = 0$). However, henceforth the following expression of the objective function will be used, working best in the aim of simplifying the model predictive control and meeting the control objective:

$$J = \sum_{k=0}^{N} \delta(k)[\hat{y}^A(t+k|t) - w(t+k)]^2 \qquad (4.25)$$

Another simplification of this formulation is because the N values of $\hat{r}^a(t+k)$ are known once they are established using the forecasting model. This allows handling dynamic programming model as an equivalent deterministic discrete model (Bertsekas, 2005). Moreover, other authors justify the establishment of different conditions in order to reduce the dimensionality problem of DP, Wijngaard (1979).

For the purpose of establish the optimality, if $\mathbb{U}$ is the set of allowable sequences of control actions ($\mathbb{U}_p = \{0, \hat{y}^C(t+k|t)\}$, with $k = 0,1, \dots, N$), the minimum accumulated cost $\hat{c}(t+k|t)$ until the prediction period $t + k$ is defined, which is expressed in terms of $\hat{c}(t+k+1|t)$, using the following recursive function based on (4.25):

$$\hat{c}(t+k|t) = \min_{u^p \in \mathbb{U}} \{\hat{c}(t+k+1|t) + \delta(k)[\hat{y}^A(t+k|t) - w(t+k)]^2\}, k = 0,1,\dots,N \quad (4.26)$$

$$u^p(t+k|t) = arg[\hat{c}(t+k|t)], k = 0,1,\dots,N \quad (4.27)$$

whereupon:

$$\hat{c}(t|t) = \min_{u^p \in \mathbb{U}} J \quad \text{y} \quad u^p(t) = u^p(t|t) = arg[\hat{c}(t|t)] \quad (4.28)$$

And, to establish a limit to the prediction horizon, it is set $\hat{c}(t+N+1|t) = 0$. Which prevents going into an infinite loop (Camacho & Bordons, 2007).

In fact, (4.26) to (4.28) configure the control problem formulation that allows determining the future actions sequence, from which $u^p(t)$ is chosen.

4.3 Matrix representation of DP model

A matrix representation of DP model offers a broader view of the process dynamics and facilitates stability analysis. Based on (4.8) and (4.9), and their equivalents (4.16) and (4.17) on the DP model, the process is displayed graphically as shown in Figure 4.2.

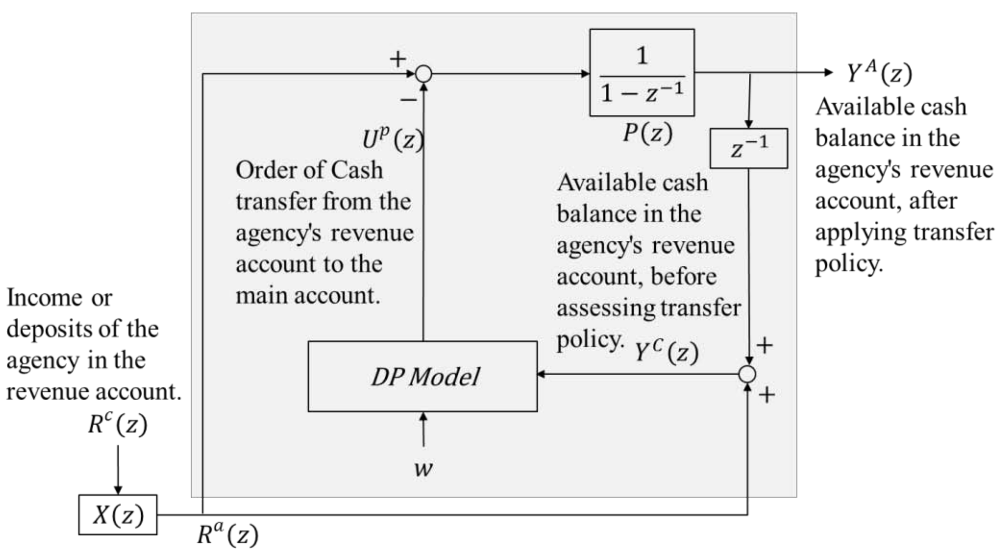


Figure 4.2: System structure including a DP model.

An algebraic handling of (4.16) and (4.17) allows obtaining:

$$\hat{y}^C(t+k|t) = \hat{y}^C(t+k-1|t) - u^p(t+k-1|t) + \hat{r}^a(t+k) \quad (4.29)$$

$$\hat{y}^A(t+k|t) = \hat{y}^C(t+k|t) - u^p(t+k|t) \tag{4.30}$$

And, the prediction equations of the entire process can be expressed in a compact form as:

$$\mathbf{Y}^C = \mathbf{1}_N \hat{y}^C(t|t) - \mathbf{M}_u[\mathbf{U}^p - \mathbf{R}^a] \tag{4.31}$$

where $\mathbf{1}_N$ is a column vector of dimension N with all entries equal to one, and:

$$\mathbf{Y}^C = [\hat{y}^C(t+1|t) \quad \hat{y}^C(t+2|t) \quad \cdots \quad \hat{y}^C(t+k|t) \quad \cdots \quad \hat{y}^C(t+N-1|t) \quad \hat{y}^C(t+N|t)]^t \tag{4.32}$$

$$\mathbf{M}_u = \begin{bmatrix} 1 & 0 & \cdots & 0 & \cdots & 0 & 0 \\ 1 & 1 & \cdots & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ 1 & 1 & \cdots & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ 1 & 1 & \cdots & 1 & \cdots & 1 & 0 \\ 1 & 1 & \cdots & 1 & \cdots & 1 & 1 \end{bmatrix} \tag{4.33}$$

$$\mathbf{U}^p = [u^p(t|t) \quad u^p(t+1|t) \quad \cdots \quad u^p(t+k-1|t) \quad \cdots \quad u^p(t+N-2|t) \quad u^p(t+N-1|t)]^t \tag{4.34}$$

$$\mathbf{R}^a = [\hat{r}^a(t+1) \quad \hat{r}^a(t+2) \quad \cdots \quad \hat{r}^a(t+k) \quad \cdots \quad \hat{r}^a(t+N-1) \quad \hat{r}^a(t+N)]^t \tag{4.35}$$

That is:

$$\begin{bmatrix} \hat{y}^C(t+1|t) \\ \hat{y}^C(t+2|t) \\ \vdots \\ \hat{y}^C(t+k|t) \\ \vdots \\ \hat{y}^C(t+N-1|t) \\ \hat{y}^C(t+N|t) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ \vdots \\ 1 \\ 1 \end{bmatrix} \hat{y}^C(t|t) - \begin{bmatrix} 1 & 0 & \cdots & 0 & \cdots & 0 & 0 \\ 1 & 1 & \cdots & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ 1 & 1 & \cdots & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ 1 & 1 & \cdots & 1 & \cdots & 1 & 0 \\ 1 & 1 & \cdots & 1 & \cdots & 1 & 1 \end{bmatrix} \left\{ \begin{bmatrix} u^p(t|t) \\ u^p(t+1|t) \\ \vdots \\ u^p(t+k-1|t) \\ \vdots \\ u^p(t+N-2|t) \\ u^p(t+N-1|t) \end{bmatrix} - \begin{bmatrix} \hat{r}^a(t+1) \\ \hat{r}^a(t+2) \\ \vdots \\ \hat{r}^a(t+k) \\ \vdots \\ \hat{r}^a(t+N-1) \\ \hat{r}^a(t+N) \end{bmatrix} \right\} \tag{4.36}$$

Also, from (4.30) it is obtained:

$$\mathbf{Y}^A = \mathbf{Y}^C - \mathbf{U}^{p1} \tag{4.37}$$

wherein:

$$\mathbf{Y}^A = [\hat{y}^A(t+1|t) \quad \hat{y}^A(t+2|t) \quad \cdots \quad \hat{y}^A(t+k|t) \quad \cdots \quad \hat{y}^A(t+N-1|t) \quad \hat{y}^A(t+N|t)]^t \tag{4.38}$$

$$\mathbf{U}^{p1} = [u^p(t+1|t) \quad u^p(t+2|t) \quad \cdots \quad u^p(t+k|t) \quad \cdots \quad u^p(t+N-1|t) \quad u^p(t+N|t)]^t \tag{4.39}$$

And, by substituting (4.31) into (4.37) it is obtained:

$$\mathbf{Y}^A = \mathbf{1}_N[\hat{y}^C(t|t) - u^p(t|t)] - \mathbf{M}_u[\mathbf{U}^{p1} - \mathbf{R}^a] \tag{4.40}$$

Or, equivalently:

$$\begin{bmatrix} \hat{y}^A(t+1|t) \\ \hat{y}^A(t+2|t) \\ \vdots \\ \hat{y}^A(t+k|t) \\ \vdots \\ \hat{y}^A(t+N-1|t) \\ \hat{y}^A(t+N|t) \end{bmatrix} = \begin{bmatrix} 1 \\ 1 \\ \vdots \\ 1 \\ \vdots \\ 1 \\ 1 \end{bmatrix} [\hat{y}^C(t|t) - u_p(t|t)] - \begin{bmatrix} 1 & 0 & \cdots & 0 & \cdots & 0 & 0 \\ 1 & 1 & \cdots & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ 1 & 1 & \cdots & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ 1 & 1 & \cdots & 1 & \cdots & 1 & 0 \\ 1 & 1 & \cdots & 1 & \cdots & 1 & 1 \end{bmatrix} \left\{ \begin{bmatrix} u^p(t+1|t) \\ u^p(t+2|t) \\ \vdots \\ u^p(t+k|t) \\ \vdots \\ u^p(t+N-1|t) \\ u^p(t+N|t) \end{bmatrix} - \begin{bmatrix} \hat{r}^a(t+1) \\ \hat{r}^a(t+2) \\ \vdots \\ \hat{r}^a(t+k) \\ \vdots \\ \hat{r}^a(t+N-1) \\ \hat{r}^a(t+N) \end{bmatrix} \right\} \quad (4.41)$$

4.3.1 Stability of DP model

For the purposes of this research, it is assumed that a system is stable if the limits of its response are a finite function of the limits of the system input (BIBO stability, Swamy *et al*., 1985). Based on it, in the sequel the stability analysis of the system proposed above is carried out. Previously, it should be clarified that the constraints on (4.26) and (4.40) are established in accordance with the limits for $\hat{r}^a(t+k)$. Under this premise, it is expected to maintain the prediction process constrained by exploiting the convexity of the objective function. See Bemporad *et al*. (2003) and Camacho & Bordons (2007) in whose works objective functions with equivalent characteristics are analyzed. So, to analyze the system stability, from (4.31) a general expression for the predicted operating cash balance at the kth stage is obtained:

$$\hat{y}^C(t+k|t) = \hat{y}^C(t|t) + \sum_{j=1}^{k}[\hat{r}^a(t+j) - u^p(t+j-1|t)] \quad (4.42)$$

From where the uncertainty in $t+k$ is given by

$$\tilde{y}^C(t+k|t) = \hat{y}^C(t|t) + \sum_{j=1}^{k}\hat{r}^a(t+j) \quad (4.43)$$

If a band for the uncertainty is established in $t+k$ according to:

$$0 \le r_{min} \le \hat{r}^a(t+k) \le r_{max} < +\infty \quad \text{and}$$
$$0 \le y_{min} \le \hat{y}^A(t+k|t) \le y_{max} < +\infty \quad \text{with } k = 1,2,\dots,N \quad (4.44)$$

Whereby also $y_{min} + r_{min} \le \hat{y}^C(t+k|t) \le y_{max} + r_{max}$, and if the extremes of these feasibility intervals for $\hat{y}^C(t+k|t)$ and $\hat{r}^a(t+k)$ are considered, it is obtained:

$$y_{min} + (N+1)r_{min} \le \tilde{y}^C(t+N|t) \le y_{max} + (N+1)r_{max} \quad (4.45)$$

Because the extremes of interval in (4.45) are increasing functions of N, there is no guarantee of stability or convergence for the process. Moreover:

$$\lim_{N\to+\infty}[y_{max} + (N+1)r_{max}] = +\infty \quad \text{and} \quad \lim_{N\to+\infty}[y_{min} + (N+1).\, r_{min}] = +\infty \tag{4.46}$$

However, Camacho & Bordons (2007) suggest the use of a stabilization controller in a cascade fashion (Linear feedback) to reduce the uncertainty prediction bands. In this case, the control actions are equivalent to increases in the input provided by the stabilizing regulator, providing a closed-loop reaction to the uncertainties in the prediction horizon. In this way, the following change of variable is established:

$$u^p(t+k|t) = \mathrm{K}\hat{y}^C(t+k|t) + v^p(t+k|t) \tag{4.47}$$

where $\mathrm{K} \in (0,1)$ (preferably, but not necessary, close to 1) is a linear feedback gain that stabilizes the system. The auxiliary variable $v^p(t+k|t)$ is the reference signal for the inner loop controller (see Figure 4.3).

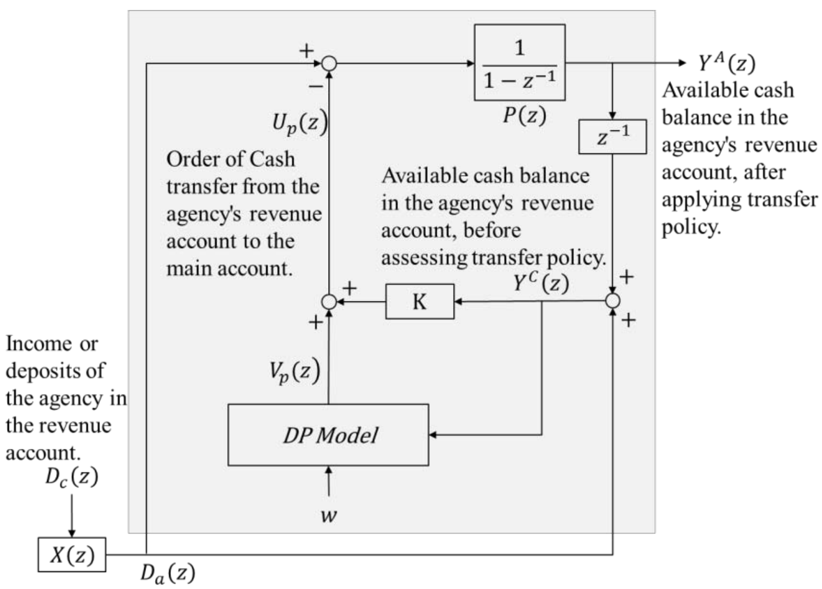


Figure 4.3: System structure including the DP model with a linear feedback gain.

With this variable change, (4.29) and (4.30) becomes, respectively:

$$\hat{y}^C(t+k|t) = (1-\mathrm{K})\hat{y}^C(t+k-1|t) - v^p(t+k-1|t) + \hat{r}^a(t+k) \tag{4.48}$$

$$\hat{y}^A(t+k|t) = (1-\mathrm{K})\hat{y}^C(t+k|t) - v^p(t+k|t) \tag{4.49}$$

Then, the matrix representation is:

$$\mathbf{Y}^C = \mathbf{M}_y\hat{y}^C(t|t) - \mathbf{M}_v(\mathbf{V}^{\boldsymbol{p}} - \mathbf{R}^{\boldsymbol{a}}) \tag{4.50}$$

wherein:

$$\mathbf{M}_y = [(1-\mathrm{K}) \quad (1-\mathrm{K})^2 \quad \cdots \quad (1-\mathrm{K})^k \quad \cdots \quad (1-\mathrm{K})^{N-1} \quad (1-\mathrm{K})^N]^t \tag{4.51}$$

$$\mathbf{M}_v = \begin{bmatrix} 1 & 0 & \cdots & 0 & \cdots & 0 & 0 \\ (1-\mathrm{K}) & 1 & \cdots & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ (1-\mathrm{K})^{k-1} & (1-\mathrm{K})^{k-2} & \cdots & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ (1-\mathrm{K})^{N-2} & (1-\mathrm{K})^{N-3} & \cdots & (1-\mathrm{K})^{N-k-1} & \cdots & 1 & 0 \\ (1-\mathrm{K})^{N-1} & (1-\mathrm{K})^{N-2} & \cdots & (1-\mathrm{K})^{N-k} & \cdots & (1-\mathrm{K}) & 1 \end{bmatrix} \tag{4.52}$$

$$\mathbf{V}^p = [v^p(t|t) \quad v^p(t+1|t) \quad \cdots \quad v^p(t+k-1|t) \quad \cdots \quad v^p(t+N-2|t) \quad v^p(t+N-1|t)]^t \tag{4.53}$$

That is:

$$\begin{bmatrix} \hat{y}^C(t+1|t) \\ \hat{y}^C(t+2|t) \\ \vdots \\ \hat{y}^C(t+k|t) \\ \vdots \\ \hat{y}^C(t+N-1|t) \\ \hat{y}^C(t+N|t) \end{bmatrix} = \begin{bmatrix} (1-\mathrm{K}) \\ (1-\mathrm{K})^2 \\ \vdots \\ (1-\mathrm{K})^k \\ \vdots \\ (1-\mathrm{K})^{N-1} \\ (1-\mathrm{K})^N \end{bmatrix} \hat{y}^C(t|t) - \begin{bmatrix} 1 & 0 & \cdots & 0 & \cdots & 0 & 0 \\ (1-\mathrm{K}) & 1 & \cdots & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ (1-\mathrm{K})^{k-1} & (1-\mathrm{K})^{k-2} & \cdots & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ (1-\mathrm{K})^{N-2} & (1-\mathrm{K})^{N-3} & \cdots & (1-\mathrm{K})^{N-k-1} & \cdots & 1 & 0 \\ (1-\mathrm{K})^{N-1} & (1-\mathrm{K})^{N-2} & \cdots & (1-\mathrm{K})^{N-k} & \cdots & (1-\mathrm{K}) & 1 \end{bmatrix} \left\{ \begin{bmatrix} v^p(t|t) \\ v^p(t+1|t) \\ \vdots \\ v^p(t+k-1|t) \\ \vdots \\ v^p(t+N-2|t) \\ v^p(t+N-1|t) \end{bmatrix} - \begin{bmatrix} \hat{r}^a(t+1) \\ \hat{r}^a(t+2) \\ \vdots \\ \hat{r}^a(t+k) \\ \vdots \\ \hat{r}^a(t+N-1) \\ \hat{r}^a(t+N) \end{bmatrix} \right\} \tag{4.54}$$

Also, by (4.49) it is obtained:

$$\mathbf{Y}^A = (1-\mathrm{K})\mathbf{Y}^C - \mathbf{V}^{\boldsymbol{p1}} \tag{4.55}$$

being:

$$\mathbf{V}^{\boldsymbol{p1}} = [v^p(t+1|t) \quad v^p(t+2|t) \quad \cdots \quad v^p(t+k|t) \quad \cdots \quad v^p(t+N-1|t) \quad v^p(t+N|t)]^t \tag{4.56}$$

Which after reordering, becomes:

$$\mathbf{Y}^A = \mathbf{M}_y[(1-\mathrm{K})\hat{y}^C(t|t) - v^p(t|t)] - \mathbf{M}_v(\mathbf{V}^{\boldsymbol{p1}} - (1-\mathrm{K})\mathbf{R}^a) \tag{4.57}$$

Or, also:

$$\begin{bmatrix} \hat{y}^A(t+1|t) \\ \hat{y}^A(t+2|t) \\ \vdots \\ \hat{y}^A(t+k|t) \\ \vdots \\ \hat{y}^A(t+N-1|t) \\ \hat{y}^A(t+N|t) \end{bmatrix} = \begin{bmatrix} (1-\mathrm{K}) \\ (1-\mathrm{K})^2 \\ \vdots \\ (1-\mathrm{K})^k \\ \vdots \\ (1-\mathrm{K})^{N-1} \\ (1-\mathrm{K})^N \end{bmatrix} [(1-\mathrm{K})\hat{y}^C(t|t) - v^p(t|t)] - \begin{bmatrix} 1 & 0 & \cdots & 0 & \cdots & 0 & 0 \\ (1-\mathrm{K}) & 1 & \cdots & 0 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ (1-\mathrm{K})^{k-1} & (1-\mathrm{K})^{k-2} & \cdots & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \ddots & \vdots & \vdots \\ (1-\mathrm{K})^{N-2} & (1-\mathrm{K})^{N-3} & \cdots & (1-\mathrm{K})^{N-k-1} & \cdots & 1 & 0 \\ (1-\mathrm{K})^{N-1} & (1-\mathrm{K})^{N-2} & \cdots & (1-\mathrm{K})^{N-k} & \cdots & (1-\mathrm{K}) & 1 \end{bmatrix} \left\{ \begin{bmatrix} v^p(t+1|t) \\ v^p(t+2|t) \\ \vdots \\ v^p(t+k|t) \\ \vdots \\ v^p(t+N-1|t) \\ v^p(t+N|t) \end{bmatrix} - (1-\mathrm{K}) \begin{bmatrix} \hat{r}^a(t+1) \\ \hat{r}^a(t+2) \\ \vdots \\ \hat{r}^a(t+k) \\ \vdots \\ \hat{r}^a(t+N-1) \\ \hat{r}^a(t+N) \end{bmatrix} \right\} \tag{4.58}$$

Owing to the linear feedback, can be seen in (4.50) or (4.54) that the uncertainty in $t+k$ is given by:

$$\tilde{y}^C(t+k|t) = (1-\mathrm{K})^k \hat{y}^C(t|t) + \sum_{j=1}^{k} \left[(1-\mathrm{K})^{k-j} \hat{r}^a(t+j)\right], \text{ with } k = 1,2,\dots,N \tag{4.59}$$

Considering the extremes of the feasible intervals of $\hat{y}^C(t+k|t)$ and $\hat{r}^a(t+k|t)$ in (4.44), it is obtained:

$$(1-\mathrm{K})^k(y_{min}+r_{min})+\frac{1-(1-\mathrm{K})^k}{\mathrm{K}}r_{min} \le \tilde{y}^C(t+k|t) \le$$

$$(1-\mathrm{K})^k(y_{max}+r_{max})+\frac{1-(1-\mathrm{K})^k}{\mathrm{K}}r_{max} \quad (4.60)$$

If in addition to this result, an upper and a lower bound for the uncertainty, consistent with the feasible intervals of $\hat{y}^A(t+k|t)$ and $\hat{r}^a(t+k)$ in (4.44), a valid interval for process stability is determined regardless of the value of N:

$$\begin{aligned} &0 < \max\left\{\frac{r_{min}}{y_{min}+r_{min}}, \frac{r_{max}}{y_{max}+r_{max}}\right\} \le \mathrm{K} < 1 \quad &\text{if } y_{min}+r_{min} > 0 \\ &0 < \frac{r_{max}}{y_{max}+r_{max}} \le \mathrm{K} < 1 &\text{if } y_{min}+r_{min} = 0 \end{aligned} \quad (4.61)$$

Also, for sufficiently large values of N:

$$0 \le \frac{1}{\mathrm{K}}r_{min} \le \tilde{y}^C(t+N|t) \le \frac{1}{\mathrm{K}}r_{max} \quad (4.62)$$

It can be seen that the process stability is ensured by incorporating the concept of linear feedback (closed-loop stabilization) suggesting that the prediction horizon is a minor problem by setting a range for the linear feedback gain, which provides new options to reduce the DP dimensionality problem. This result is condensed in the following theorem:

Theorem 1: Let the prediction process described by (4.29) and (4.30), which is modified by including the linear feedback gain according to (4.47). If bounds for disturbance and the predicted output variable are set according to (4.44), then (4.61) is a valid interval for process stability regardless of the prediction horizon size.

Corollary 2: In order to stabilize the prediction process, (4.62) is a bounded interval for the uncertainty in the closed-loop for a sufficiently large size of prediction horizon.

This result can be interpreted as the possibility of reducing the closed-loop system reaction to the uncertainties by introducing a stabilizing regulator in cascade fashion, which significantly improves the control system stability. The stability analysis of prediction process is complemented with several graphs presented in Figure 4.4, which show different behaviors of the uncertainty prediction bands. For $k = 0$, each lower bound curve on the graphics starts in $y_{min} + r_{min}$, then asymptotically converges to r_{min}/K for $k = 1,2,\dots,N$. Similarly, the upper bound starts in $y_{max} + r_{max}$ and asymptotically converges to r_{max}/K. Uncertainty prediction bands are shown in graphic (a) for a K value less than the interval

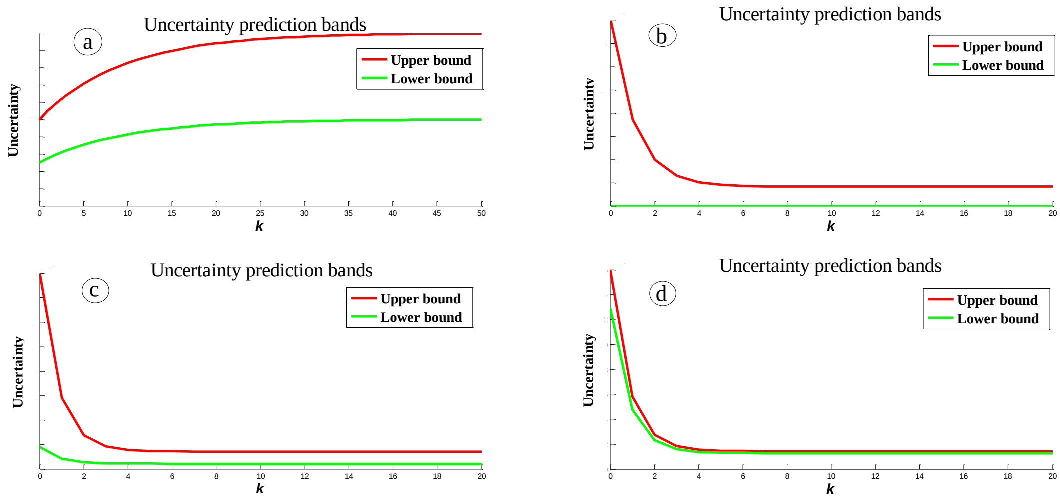


Figure 4.4: Behaviors of the uncertainty prediction bands

(4.61), which determines a range, which is widened when k increases. By contrast, in the rest of the graphic the assigned K value is inside the interval (4.61), which determines a range that is narrowed when k increases. Graphic (b) shows the behavior of the uncertainty prediction bands when the lower bound of (4.61) is zero ($y_{min} + r_{min} = 0$). Likewise, graphics (c) and (d) show reasonable conditions for process stability with K in the range (4.61) for two different scenarios expressed by (4.44). These results complement the conclusion of Theorem 1. The uncertainty prediction bands asymptotically converge

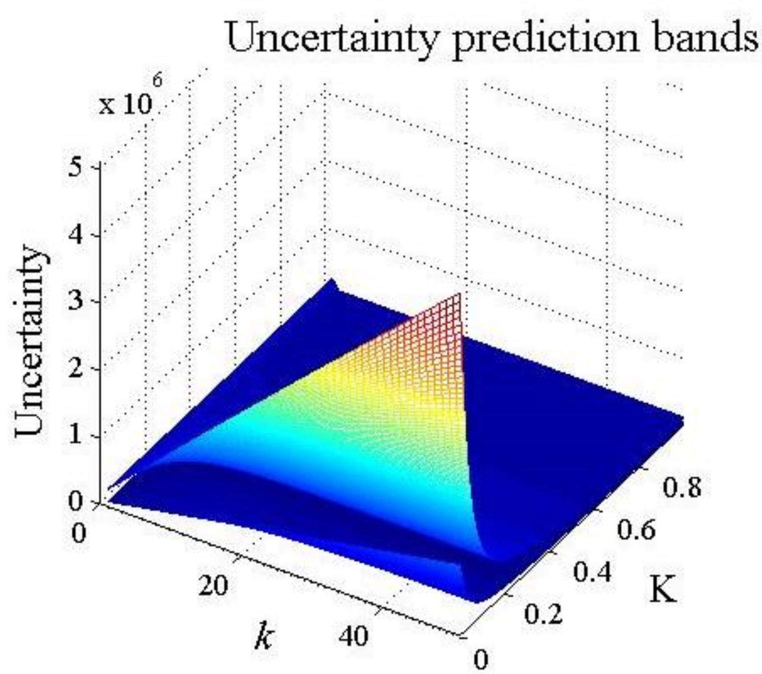


Figure 4.5: How the uncertainty prediction bands evolve for K values in (0,1)

towards $[r_{min}/\mathrm{K}, r_{max}/\mathrm{K}]$. Finally, Figure 4.5 illustrates how the uncertainty prediction bands evolve for K values in $(0, 1)$. The graph shows that such bands are narrower when K is close to 1, even for small values of k. In practice, this means that from a finite value of N, and subsequent, equivalent results can be achieved for the prediction process.

4.3.2 Set of information for the prediction process

The complete set of information for the prediction process in the revenue account of the agency is presented here, which is in the records of balance represented in the different variables. In this regard, the feasible values of the state variable $\hat{y}^C(t+k|t)$, at each stage k are established. Consequently, the set of values for the decision variable $(u^p(t+k|t))$ and the output variable $(\hat{y}^A(t+k|t))$ are also established. Furthermore, the solution algorithm for the dynamic program is outlined.

Let $j(\leq k)$ denotes the next stage after the last cash transfer carried from revenue bank account to the main bank account. Then, the feasible values of $\hat{y}^C(t+k|t)$ (total cash available at the stage $k(=0,1,\dots,N)$ to be transferred to the main account) are given by:

$$\hat{y}^C(t+k|t) = \begin{cases} \sum_{i=j}^{k} \hat{r}^a(t+i) & j = 1,\dots,k \\ y^C(t|t) + \sum_{i=1}^{k} \hat{r}^a(t+i) & j = 0 \text{ (there have been no transfers)} \end{cases} \tag{4.63}$$

Accordingly, (4.63) represents the set of dynamic program for $\hat{y}^C(t+k|t)$ at the stage k. That is, $k+1$ feasible values, each one identified by j $(=0,1,\dots,k)$. Knowing this, $k+2$ feasible values for $u^p(t+k|t)$ are set. This is, $u^p(t+k|t) = 0$ if cash transfer is not made, else $u^p(t+k|t) = \hat{y}^C(t+k|t)$. In the first case, by (4.47) and (4.49) it is obtained, respectively:

$$v^p(t+k|t) = -\mathrm{K}\hat{y}^C(t+k|t) \tag{4.64}$$

$$\hat{y}^A(t+k|t) = (1-\mathrm{K})\hat{y}^C(t+k|t) - \left(-\mathrm{K}\hat{y}^C(t+k|t)\right) = \hat{y}^C(t+k|t) \tag{4.65}$$

Also, the kth term of the cost function is:

$$J_k = \delta(k)[\hat{y}^A(t+k|t) - w(t+k)]^2 = \delta(k)[\hat{y}^C(t+k|t) - w(t+k)]^2 \tag{4.66}$$

And the next or new value of the state variable (stage $k+1$) is:

$$\hat{y}^C(t+k+1|t) = \hat{y}^A(t+k|t) + \hat{r}^a(t+k+1) = \hat{y}^C(t+k|t) + \hat{r}^a(t+k+1) \tag{4.67}$$

Otherwise, when the cash transfer is carried out, it is obtained, respectively:

$$v^p(t+k|t) = (1-\mathrm{K})\hat{y}^C(t+k|t) \tag{4.68}$$

$$\hat{y}^A(t+k|t) = (1-\mathrm{K})\hat{y}^C(t+k|t) - (1-\mathrm{K})\hat{y}^C(t+k|t) = 0 \tag{4.69}$$

$$J_k = \delta(k)[-w(t+k)]^2 \tag{4.70}$$

$$\hat{y}^C(t+k+1|t) = \hat{r}^a(t+k+1) \tag{4.71}$$

The above provides the basis for building the algorithm corresponding to the prediction model. Appendix D shows a segment of the algorithm version based on Matlab®, in which limits are set only for disturbance. According to the corollary 2, in this case the stability criterion is valid for sufficiently large values of N. To better understanding of the algorithm, a description of each variable is included, together with its corresponding equivalence in mathematical notation.

4.4 Simulation results

This simulation was performed to show the results in a single agency revenue bank account of a hypothetical CCDS. The money managed in the simulation refers to a generic currency unit called MU. The overall impact on the system main account is obtained by the arithmetic sum of all agencies of the firm. Chapter 6 includes the complete simulation for all agencies in a case study. Figure 4.3 shows in summary how the simulation results are determined. The value of the state variable (y^C) is set by solving (4.8), which is the DP model input to calculate the value of the auxiliary variable (v^p). Then, through (4.47) the value of u^p is obtained, whereby the output value (y^A) is found through (4.9). The graph in Figure 4.6 shows that the cash level faithfully follows the sawtooth signal despite the input variations for a time 20τ. Different functional tests were conducted varying the cash transfer policy parameters, for different prediction horizons. In each of them, results were obtained consistent with the expected outcomes (sawtooth behavior). Particularly, the result of Figure 4.6 (like Figures 4.7 and 4.8) was obtained using $\alpha_y = 10$ and $N = 5$. Nevertheless, equally satisfactory results are achieved when using any $\alpha_y > 1$ and $N \geq 5$, taking care that this value covers at least a period T ($N \geq T$). Figure 4.7 shows the behavior of accumulated and punctual cost due to managing (operation and maintenance) of the revenue account, comparing it with the expected cost (sawtooth behavior). These costs are calculated in accordance with Appendix B. Figure 4.8 compares the amounts of cash transferred from the revenue account to the main account with respect to the equivalent reference calculated from the sawtooth signal. Figures 4.9 and 4.10 respectively show the

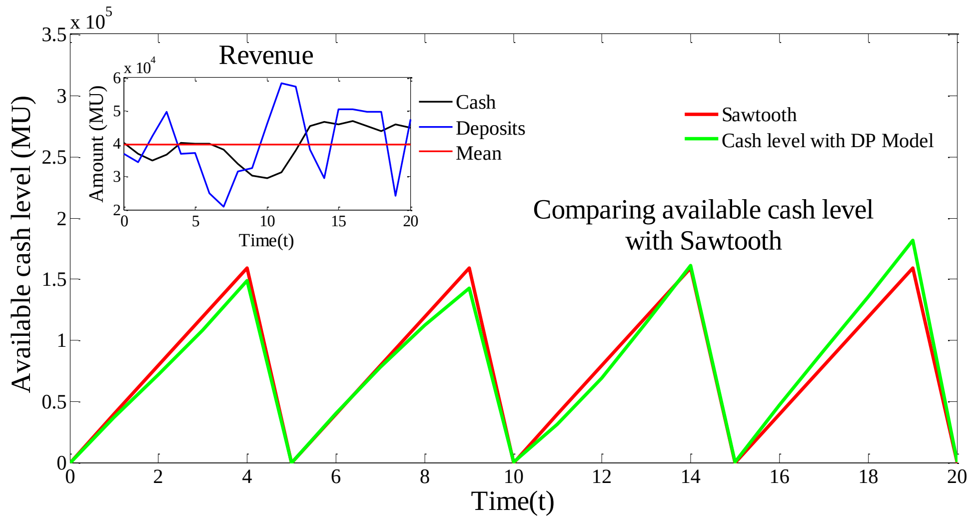


Figure 4.6: Comparing available cash level using the DP model with regard to the sawtooth model.

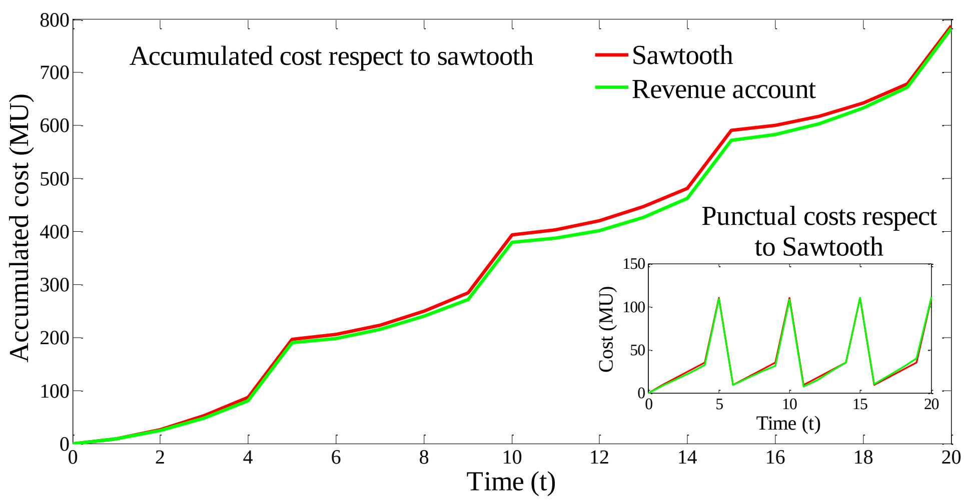


Figure 4.7: Comparing costs using the DP model with regard to the sawtooth model.

level of cash available in the revenue account and the amounts transferred when the wave period is one. Which means that, each sampling time the cash available is transferred to the main account. Figure 4.11 presents simulation results with different conditions for uncertainty and forecasting horizon keeping the same cash transfer policy parameters. The aim of this graph is to show that, even though the uncertainty dispersion is reflected in the variability of the result, the control model takes care that the output retakes each time the sawtooth shape of the reference signal, even for different sizes of the control horizon.

Figure 4.12 shows the simulation result when a smooth control criterion is used ($0 < \alpha_y < 1$). It is noted that the control is not guaranteed, because not always the reference signal is followed.

Moreover, Figure 4.13 shows the simulation result when considering the control effort with a relatively high value of α_u. Here again, it can be seen that the control is not guaranteed. In tests with higher values of α_u, control is lost since the beginning of simulation. Also, for a sufficiently large value of α_y and a value of α_u close to one (but greater than), or if $0 \leq \alpha_u < 1$, the control is achieved. However, in this case the effect of J_u in the objective function is negligible.

Finally, Figure 4.14 shows three simulations using the same random series with the aim of comparing different strategies. Identical results were observed, which have been presented in 3D for better visualization. In the graph, the simulation with two different prediction horizons ($N = 5$ and $N = 20$) are compared. The difference is focused on runtime of the model: between 0.03 and 0.08 seconds for $N = 5$, (1,260 executions of the recursive function (4.26)), and 1,324.2 seconds for $N = 20$ (approximately $4.2x10^8$

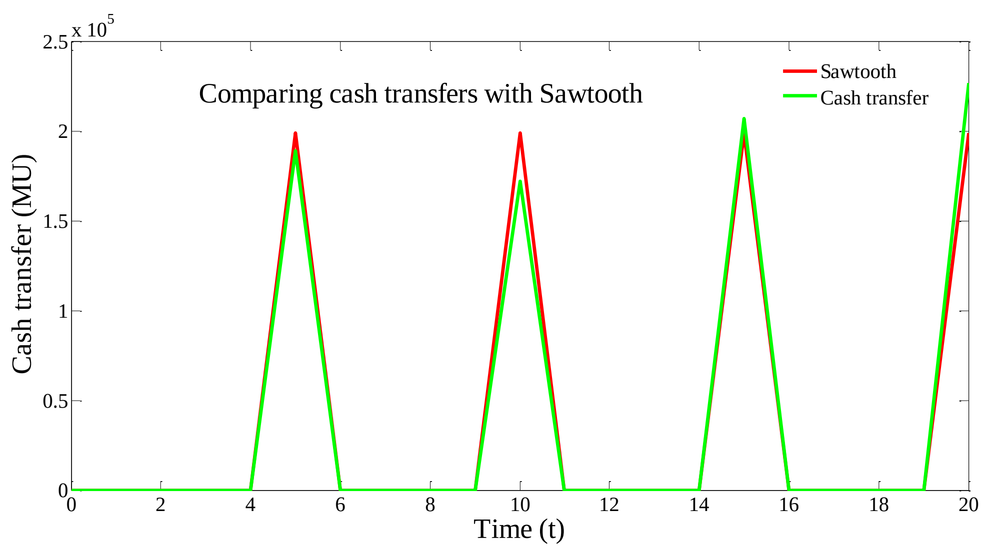


Figure 4.8: Comparing the amounts of cash transferred with respect to the sawtooth model.

executions of the recursive function (4.26)). These results, together with what is shown in Figure 4.11, are in favor of Theorem 1. In each case, the greater or lesser variability of the output depends on the greater or lesser dispersion of uncertainty. In no case, greater variability indicates a worse outcome. Also, in the graph (Figure 4.14) the execution of the DP Model is compared with respect to the application of a model using the traditional strategy (s, S) with periodic review, which invites further study of the proposed model predictive control.

The different simulations were made using different statistical parameters, by which some results may present greater or lesser variability. This is important because it shows that the model works in each case. Justly, the emphasis of this chapter is to show that the model is functional and useful for the scenarios that serve as input.

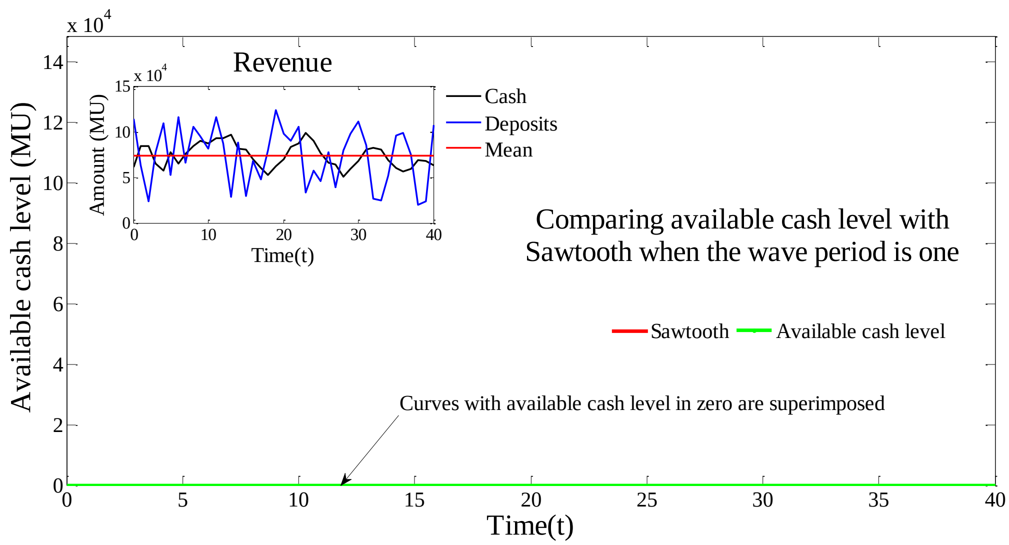


Figure 4.9: Comparing available cash level using the DP model with regard to the sawtooth model when the wave period is one.

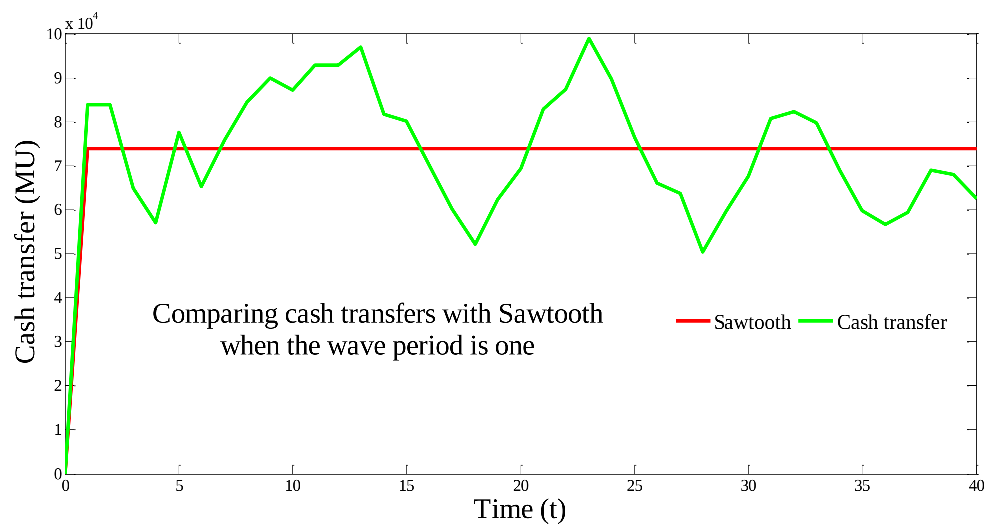


Figure 4.10: Comparing cash transfers using the DP model with regard to the sawtooth model when the wave period is one.

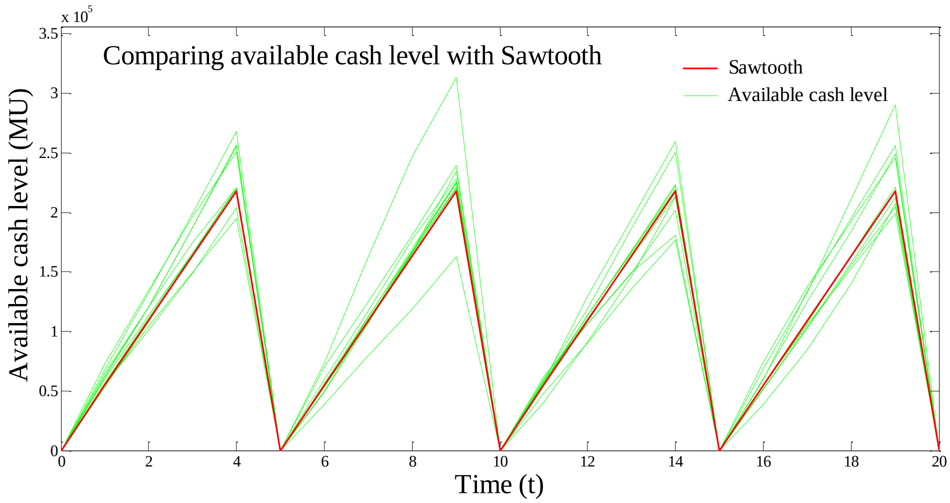


Figure 4.11: Results using different inputs with similar expected revenue

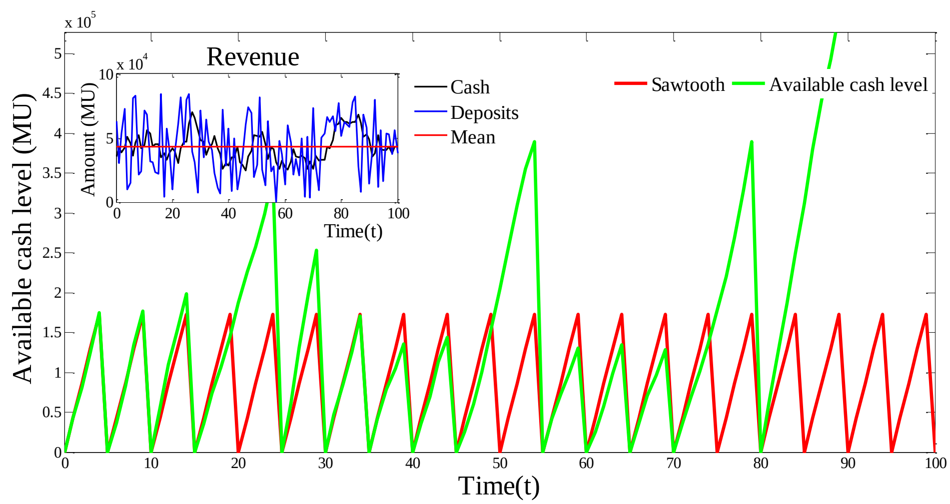


Figure 4.12: Comparing available cash level using the DP model with regard to the sawtooth model when smoothed control is used

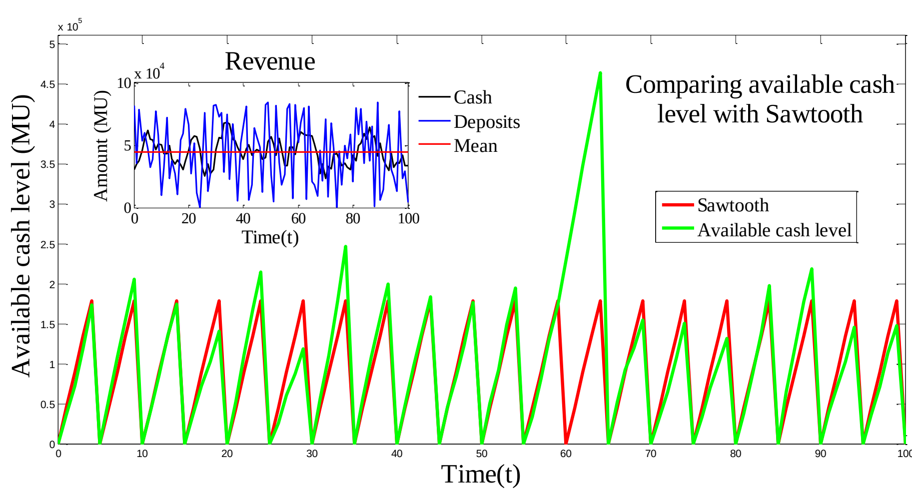


Figure 4.13: Comparing available cash level using the DP model with regard to the sawtooth model when faster control is used including a very large value of α_u.

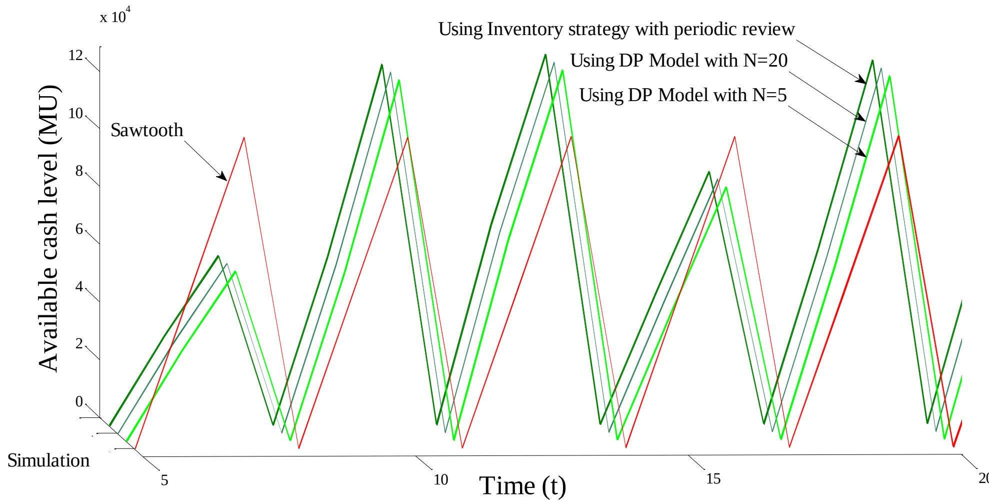


Figure 4.14: Comparing available cash level using the DP model (for $N = 5$ and $N = 20$) and sawtooth model with regard to traditional inventory control model applying the (s,S) strategy with periodic review.

4.5 Concluding remarks of the chapter

This chapter has presented a model predictive control for a revenue account belonging to a cash concentration and disbursements system, based on the application of inventory policies to the cash balance. The model is applied to all revenue accounts of the CCDS as can be seen in Chapter 6. Dynamic programming was used for the prediction model, for which several simplifications have been proposed seeking alleviate some of the known problems when dynamic programming is applied under uncertainty. For this purpose, a standard forecasting model for uncertainty was included, which means that, for this particular study, options that use smart scan on the state space or an optimization model min-max based on the worst-case were not considered. However, a band for the uncertainty is established to narrow the input of the DP model, together with a stabilizing regulator in cascade fashion using a linear feedback gain (closed-loop). This combination allows determining a range for the system stability regardless of size of the prediction horizon. Also, an objective function based on a finite horizon quadratic criterion is used, which excludes the control effort. But, as a generalization, a discount dynamic parameter in the respective cost function is included. In the carried tests, equivalent results are achieved for various behaviors of that discount parameter, which is determined off-line. The reference signal used is a sawtooth function, which conveniently adapts to the applied inventory policy. However, the predictive control method has the capability to adapt to reference signal variations, making it more versatile.

The results achieved are equivalent to those obtained by using a traditional inventory control model by directly applying the (s, S) strategy with periodic review. With this, it is possible to support the conclusions in Lee & Wong (2010), Wong & Lee (2011) and Lee (2011) regarding the limited effectiveness when combining ADP and MPC. Nonetheless, the proposal is much more versatile, since it responds automatically to different scenarios (inputs and reference signals) that the user wishes to incorporate. Additionally, the method leaves open the possibility of obtaining promising results if some complexity elements are added to model, among which are mentioned in particular the use of a seasonal or non-stationary scheme for uncertainty (not based on white noise) as was raised in this chapter. In addition, there is the possibility of including based models smart scan the state-space, also, models that comprehensively involve the complete set of state space, e.g., an optimization model min-max, which takes into account the worst case. Another feature of the model used here is that the transfer function does not include delay or dead time, which would add another element of complexity. However, Chapter 5 incorporates this feature on a model for a disbursements account of the CCDS.

Chapter 5

Model Predictive Control of the overdraft coverage problem

In Chapter 4, a model predictive control (MPC) based on dynamic programming (DP) was presented, under a decentralized approach. The aim of this model is to control available cash balances on revenue accounts belonging to a CCDS. The shape of the model reference signal is inspired in the periodic behavior of an inventory policy that, by way of example, it was assumed of the type (s, S), with a continuous inventory monitoring system and transfers to the main account.

In this chapter, a model predictive control (MPC) is proposed for a disbursement bank account also contained in the CCDS, based on the model for this system formulated in Chapter 3. In general, the problem looks like a supply chain management special case and, in particular, the control of each bank account is focused as an inventory problem where cash is the good supplied, with periodic transfers from the main account covering its overdraft. To this end, the model assumes that the company issues payments to suppliers, causing an overdraft of the disbursement account. Periodically, the overdraft is covered by transferring cash from the main account. It is also supposed that the firm has established an adequate level of autonomy in the financial decisions of each agency. Therefore, besides a general cash transfers policy consistent with the applied model, it is assumed that there are no restrictions to decide independently the level of overdrawn, as well as the amounts transferred from the main account of CCDS toward the disbursements account of the agency. Furthermore, the proposed prediction model considers cash transfer time delay (transit time) from the viewpoint of the agency. This feature means that the control strategy is different in several aspects with regard to the model proposed in Chapter 4.

First, it is known that the information about the overdraft in the disbursement account originates in the agency. From this information, it is possible to decide on the cash transfer in order to cover the overdraft. Justly, the aim of the model proposed is to support this decision using the MPC controller. However, the decision (when and how much) is taken at the firm's main office. Of course, considering the anticipation needed to cover disbursements during the delay time and the balance of overdraft itself. This requires to introduce into the model the concept called (for the purposes of this chapter) overdraft coverage, which includes both the overdraft balance and the money in transit to cover it (cash transfers pending for availability on the expense account). Because this, the MPC controller must consider the time lag between the decision taken and its real effect on the controlled account at the agency. In this sense, the question about the variable to be controlled is raised, which in this case may be the overdraft balance or, alternatively, the overdraft coverage, as well as the relationship between them in function of the control action. This analysis may also refer to the state variable. In trying to answer this question, it has been posed the possibility of making two proposals for model predictive control, as well as the corresponding analysis to determine the relationship between them. Whereupon, the objective in this chapter is embodied and developed below. Starting of a pre-decision-state formulation, briefly it is as follows.

The first approach considers overdraft coverage as state variable just before deciding a cash transfer. With this state value, the system orders a cash transfer to keep the overdraft coverage within the desired reference parameters. To achieve the control objective, this action "*indirectly*" controls the overdraft balance after delay time due to money in transit.

The second approach considers the overdraft balance like state variable also just before deciding a cash transfer. The system orders a cash transfer using this state value, with which allows "*directly*" reach the control objective after delay time, maintaining the overdraft balance within the desired reference parameters. It is novel present both approaches and perform their theoretical and experimental comparison, since usually other authors (Fu *et al*., 2014) try this kind of models using the second approach, because the first one excludes the delay time of the model. It has been wanted to compare both proposals finding conclusions of interest for further research. The controller described in this chapter, along with the controller presented in Chapter 4, completes the proposal of automatic control of CCDS, which then is used in the simulation of a case study in Chapter 6.

The chapter is divided as follows. Section 5.1 makes the problem formulation. Section 5.2 shows two prediction models for control. Section 5.3 includes a comparison between the two proposals with the respective analysis of the results. Finally, Section 5.4 deals with concluding remarks. Is worth clarifying that the content of this chapter is being addressed in Herrera-Cáceres and Ibeas (2016d).

5.1 Problem formulation

Figure 5.1 shows the information flow and money flow related to the disbursements account of agency, affected by the MPC controller. For simplicity, the following considerations are made prior to its explanation. First, as in Chapters 3 and 4, a basis time interval ($\tau = 1$) is assumed, usually a day, but it can also refer to a week or a month, or other time interval used in financial practice, according to the dynamics of the CCDS. Second, in the remainder of the chapter, the agency is not identified with a subscript in the nomenclature used to differentiate it from other agencies; instead, the same mathematical treatment to each one of the agencies composing the entire company is assumed.

5.1.1 Modeling the disbursements account

The process of payment to suppliers has been described in Chapter 3, but below is repeated for convenience. The disbursements in agencies during discrete time interval $t(= h\tau$ con $h \in \mathbb{N})$ are represented by $d^c(t) \leq 0$. They correspond to payments made to payees (checks or direct debit) for the interval t against a special disbursements bank account of the agency. Variability related to the transit time of a payment made by the firm is a non-controlled process. Consequently, the following discrete deterministic scheme is assumed for it: (a) Number of periods in transit: L_m (of the form $h\tau, h \in \mathbb{N}$), with $m = 0,1,2, \dots, g$; (b) Portion of funds in transit according to L_m: b_m^d, with $0 \leq b_m^d \leq 1$ and $\sum_{m=0}^{g} b_m^d = 1$. Because of this assumption, the cash required to cover the overdraft of the agency's disbursements account during interval t answers the following expression:

$$d^a(t) = \sum_{m=0}^{g}[b_m^d d^c(t - L_m)] \quad (5.1)$$

In turn, at the end of t, a cash transfer order to cover this overdraft is requested to the firm's main office. Thereupon, subject to previously established policies, a transfer order ($u^p(t)$) from the CCDS's main account to the agency's disbursements account is issued. The ordered amount $u^p(t)$ is determined by the MPC controller and it will be available in the agency's disbursements account L_p after the transfer order ($L_p \geq 0$, also of the form $h\tau, h \in \mathbb{N}$). That is $u^q(t) = u^p\left(t - L_p\right)$. As a result, considering the delay of transfer order, the available balance registered in the bank for the disbursement account of an agency at end of t, is given by:

$$y^A(t) = y^A(t-1) - u^p\left(t - L_p\right) + d^a(t) \tag{5.2}$$

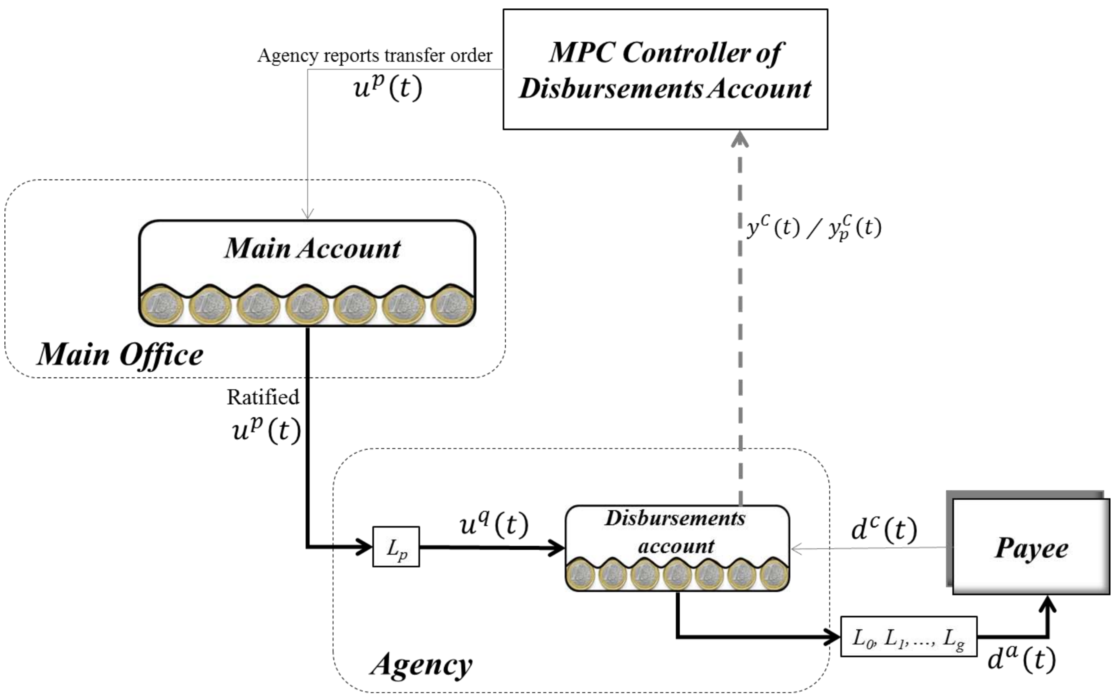


Figure 5.1: Decentralized MPC strategy for CCDS – Disbursements account of Agency.

Because the disbursement account is usually overdrawn, being periodically covered with cash transfers from the main account, from now on, $y^A(t)$ will be named as: overdraft balance. In addition to this, the record into the account books of the company must be considered, which is useful for bank reconciliation (see Chapter 3). Furthermore, it has been envisaged that the finance charge due to the overdraft in the disbursements account of agency is included in one of the terms in (5.1). Likewise, due to how this account operates, theoretically it has no deferred balances or frozen balances. For this reason, the total balance is equal to the available balance. However, if the company`s main office reports the agency about the transfer ordered to cover the overdraft in the disbursements account, it is possible to establish an expression for the cash position (or overdraft coverage) in the account:

$$y_p^A(t) = y_p^A(t-1) - u^p(t) + d^a(t) \tag{5.3}$$

Overdraft coverage is the sum of the overdraft balance and the money in transit to cover it. In addition, because cash transfers between accounts can only be made to cover overdraft, in this model consisting of equations (5.1) to (5.3), the aim is to control the overdraft balance ($y^A(t)$).

Why control the overdraft balance? After an analysis regarding the financial costs (interest rates, spreads between interest rates, bank fees, taxes due to financial transactions), the company establishes a cash transfer policy (see Chapter 3). This cash transfer policy takes into consideration a fixed cost per transfer, a variable cost depending on the transfer size, and costs due to negative balances or overdrafts. Typically, banks offer a particular program about overdraft facilities, which must be evaluated by the company in accordance with its negotiating capacity, since it is a basic aim to minimize the total financial cost expressed in the cash transfer policy.

The following sequence of events is defined:

a. At the discrete time t, debits are made in the agency's disbursements account. Then, the required cash is depicted like a single aggregate amount $d^a(t)$. This does not affect the dynamics of the system. Lin *et al*. (2004) offer arguments that allow grouping all of disbursements during t as a single aggregate amount.

b. The overdraft balance $y^C(t)$ of the agency's disbursements account is observed before transfer order. $y^C(t)$ is calculated by adding $d^a(t)$ to overdraft balance $y^A(t-1)$ at the end of $t-1$. Similarly, the overdraft coverage $y_p^C(t)$ is observed (before transfer order), which is also calculated by adding $d^a(t)$ to the overdraft coverage $y_p^A(t-1)$ at the end of $t-1$. This is:

$$y^C(t) = y^A(t-1) + d^a(t) \tag{5.4}$$

$$y_p^C(t) = y_p^A(t-1) + d^a(t) \tag{5.5}$$

c. Both $y^C(t)$ and $y_p^C(t)$ are reported to the firm's main office. Whereupon, in accordance with the established policy, the amount of money transferred ($u^p(t)$) from the main account of CCDS toward the agency's disbursements account is ordered. Also, $u^p(t)$ is reported to the agency management. From the main office point of view, this is the control action based on the MPC controller decision. However, $u^p(t)$ will be available after L_p time intervals in the disbursements account of the agency.

d. Again, the overdraft coverage $y_p^A(t)$ is measured. The overdraft balance $y^A(t)$ is also adjusted depending on the amount transferred L_p ago.

$$y_p^A(t) = y_p^C(t) - u^p(t) \tag{5.6}$$

$$y^A(t) = y^C(t) - u^p(t - L_p) \tag{5.7}$$

Again in this sequence, it is important to highlight that the cash transfer order ($u^p(t)$) is performed at the end of t, so that the money transfered will be available in the disbursements account L_p after t (item c, above). Indeed, the company must establish the cash transfer policy represented in a reference signal**,** whose parameters explicitly include the criterion for deciding the amount to be transferred and the frequency. So that, once $y^C(t)$ and $y_p^C(t)$ values are determined, the control system calculates the necessary transfers ($u^p(t)$) at every time t in order to achieve the control objective. It may be noted that both $y^C(t)$ and $y_p^C(t)$ can be used as state variables, which results in two possible ways to propose a control model. **Consequently, in this chapter two MPC controllers are proposed, which autonomously determine the necessary transfers. The first, given the value of $y^C(t)$ based on a reference signal $w(t)$ representing the cash transfer policy. The second, given the value of $y_p^C(t)$ based on a reference signal $w_p(t)$ representing the cash transfer policy**. All of which will be discussed theoretically and through simulation in the rest of this chapter to show compliance with the control objective. It should be clear that the aim is to control the overdraft balance ($y^A(t)$). Let it remember that, in principle, the cash transfer policy is a true reflection of the logical objective according to which the cost of management the account is minimal, as it was clarified previously.

5.1.2 Transfer function

Modeling the disbursement bank account is important to pin down the dynamic response of the system. For this reason, an equivalent model by using the Z-transform is presented considering the sharing of information and cash between the customer, the disbursement account and the main account of CCDS. Namely, the transfer function, which is essential for the simulation and the basis for building the prediction model later. Consequently, applying the time shifting property of Z-transform on (5.1) to (5.5) (Ogata, 1996), they become:

$$Y_p^C(z) = z^{-1}Y_p^A(z) + D^a(z) \tag{5.8}$$

$$Y_p^A(z) = \frac{1}{1-z^{-1}}\left(-U^p(z) + D^a(z)\right) \tag{5.9}$$

$$Y^C(z) = z^{-1}Y^A(z) + D^a(z) \tag{5.10}$$

$$Y^A(z) = \frac{1}{1-z^{-1}}\left(-z^{-L_p}U^p(z) + D^a(z)\right) \tag{5.11}$$

wherein:

$$D^a(z) = X'(z)D^c(z) \text{ and } \quad X'(z) = \sum_{m=0}^{g}[b_m^d z^{-L_m}] \tag{5.12}$$

Note that equations (5.9) and (5.11) represent unstable relations between the variables (through the integrator $P(z) = 1/(1-z^{-1})$), which is one of the problems in financial management control system under consideration, and which also appear in other problems such as supply chain inventory management (García *et al*., 2012). Moreover, as shown in (5.9) and (5.11) from the viewpoint of the agency, the dead time or delay of control action only applies for $Y^A(z)$ (z^{-L_p} with $L_p \geq 0$).

5.1.3 Cash transfer policy and uncertainty process

Just like in Chapter 4, the cash transfer policy is determined by the operation criterion of the account. That is, the overdraft balance in the disbursements account of the agency behaves according to an inventory policy or equivalent criterion. The criterion established is used to design the reference signal for the prediction model. Accordingly, a cash transfer from the main account periodically should cover the overdraft balance seeking that the balance returns to desired level. As in Chapter 4, it has been assumed a continuous inventory monitoring system of the type (s, S), without intending to diminish the generality of the model. Herein, s (< 0) is the overdraft level under which the agency should make the transfer, and S is the level at which returns the balance account after the overdraft is covered by cash transfer. Making a generalization of (4.11), the cash transfer policy is as follows:

$$u^p(t) = \begin{cases} 0 & \text{if } |y^C(t)| < |s| \\ \tilde{u}^p(t) & \text{if } |y^C(t)| \geq |s| \end{cases} \quad (5.13)$$

As previously was discussed, this policy is used to justify the application of sawtooth model like reference signal in the proposed model. However, the predictive control itself offers more versatility in defining policies and establishing strategies by modifying of some parameters or by adding some complexity elements. In any case, the decision maker has the choice to set the return level and the cash transfer policy, on which the controller has the capability to adapt.

On the other hand, the disbursements in agencies act as disturbance because they introduce uncertainty affecting the balance in the disbursements account. Following De Keyser & Ionescu (2003), (5.9) and (5.11) can be rewritten respectively as follows:

$$A(z^{-1})Y_p^A(z) = B_p(z^{-1})U^p(z) + n(z) \quad (5.14)$$

$$A(z^{-1})y^A(z) = B(z^{-1})U^p(z) + n(z) \quad (5.15)$$

where $A(z^{-1}) = 1 - z^{-1}$, $B_p(z^{-1}) = -1$ and $B(z^{-1}) = -z^{-L_p}$, the disbursements model ($n(z) = D^a(z)$) is equivalent to an Autoregressive Moving Average Process (ARMA) of the form:

$$D(z^{-1})n(z) = C(z^{-1})e(z) \quad (5.16)$$

And, renaming $X'(z)$:

$$C(z^{-1}) = X'(z) = \sum_{m=0}^{g}[b_m^d z^{-L_m}] \quad (5.17)$$

Because this research is limited to stationary scenarios, the model does not incorporate any deviations in the process caused by other external perturbations. The $e(z)$ series is a white noise with zero-mean and standard deviation one. In addition, to complete the agency's disbursements model, $D(z^{-1}) = K_d$, where K_d is constant in the interval $(-1, 0)$ inversely proportional to the average disbursements level of the agency, and:

$$e(z)/D(z^{-1}) = D^c(z) \quad (5.18)$$

5.2 Two prediction models for control

As stated previously, the nature of the problem expressed in the transfer function offers the opportunity to develop two alternative ways for the MPC. The basic difference between the two proposals lies in choosing the state variable and the output variable; however, they use common elements, such as the cash transfer policy and the uncertainty process. Equally, as discussed below, the reference signal shape as well as the forecasting model for the required cash to cover the overdraft, are used in both proposals. They are based on the model predictive control (MPC) formulated in Chapter 4, but analyzing the sequence of operations described in Section 5.1, and applying an equivalent backward dynamic programming model. That is how, the system response fits to a reference signal representing the cash transfer policy. This policy can be represented either by a reference signal for overdraft coverage ($w_p(t)$) or by a reference signal for overdraft balance ($w(t)$) on the disbursement account of agency, depending on the model selected. In any case, the transferred cash will be available L_p later in this account. Likewise, the standard exponential smoothing method N-N (no trend, no seasonal) expressed in (4.18) is used in order to forecast the required cash to cover the overdraft at time $t + 1$.

The following common features are established regarding both MPC proposals based on DP, applied to the control problem of overdraft coverage in a disbursements account:

- ✓ A prediction horizon N and control horizon N_u, which may coincide depending on considerations about delay in the control action (in any case $N_u \leq N$).
- ✓ $N + 1$ stages are established for the dynamic programming model ($t + 0, t + 2, \dots, t + N$).
- ✓ N forecast instants for which the future outputs of the model are predicted.
- ✓ u^p is the decision variable representing the alternatives at each stage: the amount to be transferred from the CCDS's main account toward the agency's disbursements account at the end of each stage.
- ✓ N_u values of u^p are determined, of which the first ($u^p(t)$, at stage 0) is sent to the plant.
- ✓ The N predicted outputs depend on the known values (inputs and outputs) until time t and the set of future control signals u^p.

5.2.1 Control model for overdraft coverage

The first proposal is as shown in Figure 5.2, which contains a diagram based on the control model of Chapter 4, including a stabilization controller in a cascade. The model uses the information about overdraft coverage in the disbursements account, represented by (5.8) and (5.9), as well as the amount of cash required to cover payments presented by payees at time t (5.12). Then, the MPC is formulated without using delay time. In this way the model formulation is identical to the proposal made in Chapter 4, but with the difference that the reference signal used represents the cash transfer policy applied to the overdraft coverage in the account, instead of the available cash balance. With this result, (5.11) is used to determine the overdraft balance including the delay time of the transfer ordered L_p ago.

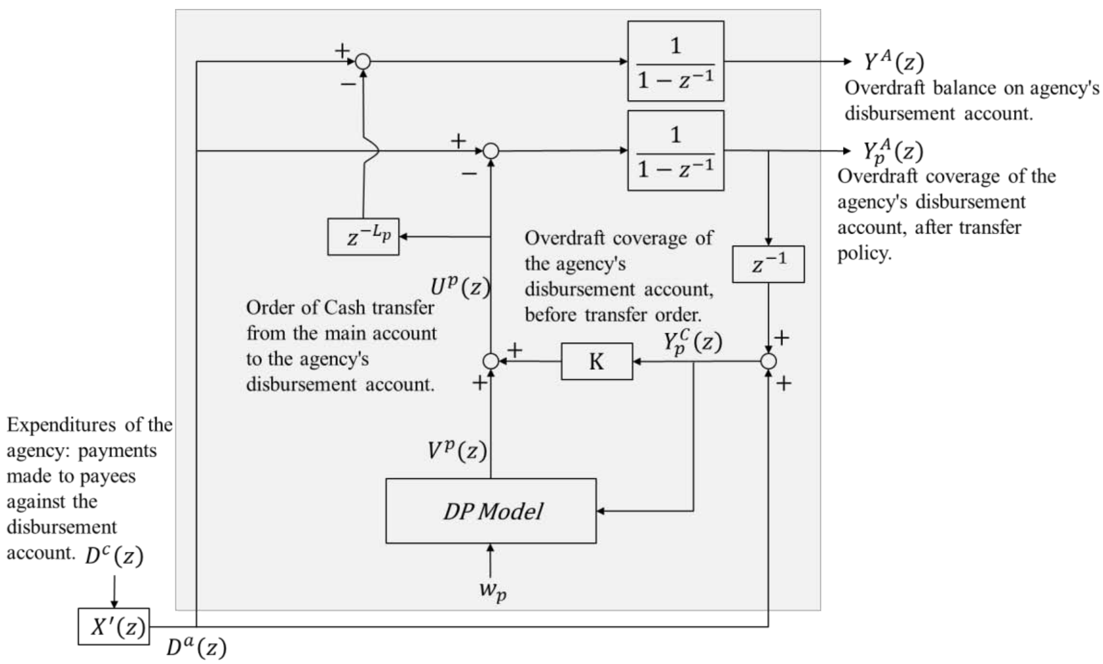


Figure 5.2: System structure including the DP model control for overdraft coverage with linear feedback gain.

The features of this proposal are:

- ✓ The prediction horizon is N $(t+1, t+2, \dots, t+N)$, which coincides here with the control horizon N_u $(N = N_u)$.
- ✓ $N_u + 1$ stages $(t+0, t+2, \dots, t+N_u)$ for the dynamic programming model.
- ✓ N_u forecast instants for which the future outputs $(\hat{d}^a(t+k))$ of the model are predicted.

- ✓ The decision variable is u^p, then $N_u + 1$ values of it are determined ($u^p(t+k|t), k = 0, \dots N_u$).
- ✓ The first value of u^p ($u^p(t|t) = u^p(t)$, at stage 0) is sent to the process.
- ✓ The $N_u + 1$ predicted outputs $\hat{y}_p^A(t+k|t)$ (overdraft coverage at the end of time $t+k$, with $k = 0{,}1 \dots N_u$) depend on the known values (inputs and outputs) until time t and the set of future control signals $u^p(t+k|t)$
- ✓ The overdraft coverage (just before transferring) is the state variable:

$$\hat{y}_p^C(t+k|t) = \hat{y}_p^A(t+k-1|t) + \hat{d}^a(t+k) \tag{5.19}$$

That is, a pre-decision-state formulation, wherein $\hat{d}^a(t+k)$ is the forecasted amount of cash required to cover demanded payments during $t+k$, based on the process represented by (5.1). Moreover, the predicted values of the output variable are established by:

$$\hat{y}_p^A(t+k|t) = \hat{y}_p^A(t+k-1|t) + \hat{d}^a(t+k) - u^p(t+k|t) \tag{5.20}$$

Particularly, $\hat{y}_p^A(t+N_u|t)$ is referred to the overdraft coverage of the agency's disbursements account at the end of the forecast period $t+N_u$, which prevents entering an infinite loop (Camacho & Bordons, 2007).

In accordance with an equivalent reasoning to the revenue account in Subsection 4.2.3, the balance in the disbursement account of the agency behaves following a triangular function, since the overdraft balance must be covered periodically from the main account. Thus, the assumption of a stationary scenario (Subsection 5.1.3) allows establishing a sawtooth wave reference trajectory. Then, following Trott (2004) and under assuming the cash transfer policy expressed in (5.13), the reference signal can be generated as:

$$w_p(t+k) = A.frac\left(\frac{t+k}{T} + \phi\right) - S \qquad k = 0{,}1 \dots N_u \tag{5.21}$$

That is, a generalized formulation of the triangular function (sawtooth wave) with respect to (4.21), wherein:

- ✓ $frac(x) = x - \lfloor x \rfloor$ (fractional part of x),
- ✓ $A = T.E\left(d^a(t)\right)$ is the wave amplitude,
- ✓ $T = \left\lceil s/E\left(d^a(t)\right)\right\rceil$ is the wave period

- ✓ $\phi = L_p/T$ is the wave phase, meaning the delay of the cash availability in the disbursement account of the agency, with respect to the moment when the transfer is ordered at the main office.
- ✓ In this case, $S = L_p . E\left(d^a(t)\right)$ is the expected overdraft during delay. However, in order to handle any level desired for return, S can be set as an external decision.
- ✓ $E\left(d^a(t)\right)$ is the expected value of overdraft,
- ✓ s is the amount of overdraft over which the agency should require the transfer in accordance with the optimum transfer policy, and
- ✓ $\lfloor x \rfloor$, $\lceil x \rceil$ are the floor and ceil functions, respectively.

The periodic behavior of the reference signal is equivalent to the cash transfer policy described in Subsection 5.1.3. That is, the overdraft coverage represented by the state variable $\hat{y}_p^C(t+k|t)$ in step $t+k$ is evaluated, after which a decision is made: either $\hat{y}_p^C(t+k|t) - S$ is transferred from the main account or not. Accordingly, the decision variable in the dynamic programming model takes the values $u^p(t+k|t) = 0$ or $u^p(t+k|t) = \tilde{u}^p(t+k|t) = \hat{y}_p^C(t+k|t) - S,\ k = 0,1,2,\ldots,N_u$. Because it is a backward Dynamic Programming strategy, the dynamic process begins at step $k = N_u$, ending at stage $k = 0$, when the $u^p(t|t)$ value is determined, corresponding to the minimum value of the cost function of model. This is $u^p(t|t) = 0$ or $u^p(t|t) = \tilde{u}^p(t|t) = \hat{y}_p^C(t|t) - S$ $(= \hat{y}_p^C(t) - S)$.

The criterion for fulfill the aim of keeping the control process close to the reference signal is equivalent to (4.22) and (4.25), with considerations, expressed in Appendix C, regarding whether or not to include the control effort in the objective function. Then, the objective function used hereinafter for this model is:

$$J_p = \sum_{k=0}^{N_u} \delta(k)\left[\hat{y}_p^A(t+k|t) - w_p(t+k)\right]^2 \quad (5.22)$$

In this case, it has also wanted to handle the dynamic programming model as an equivalent deterministic discrete model, so it assumes that the N_u values of $\hat{d}^a(t+k)$ are given by known once they are established using the forecasting model. This allows simplify the DP model formulation, so it is handled as an equivalent deterministic discrete model (Bertsekas, 2005).

The optimality criterion is subject to the set of allowable sequences of control actions ($\mathbb{U}_p = \{0, \hat{y}_p^C(t+k|t) - S\}$, with $k = 0,1,\ldots,N_u$). In such a case, the minimum accumulated cost $\hat{c}_p(t+k|t)$ until the prediction period $t+k$ is defined, which is expressed in terms of $\hat{c}_p(t+k+1|t)$, using the following recursive function:

$$\hat{c}_p(t+k|t) = \min_{u^p \in \mathbb{U}_p} \left\{\hat{c}_p(t+k+1|t) + \delta(k)\left[\hat{y}_p^A(t+k|t) - w_p(t+k)\right]^2\right\}, \quad (5.23)$$

$$u_p(t+k|t) = arg\left[\hat{c}_p(t+k|t)\right] \quad (5.24)$$

whereupon $k = 0,1,\ldots,N_u$, and:

$$\hat{c}_p(t|t) = \min_{u^p \in \mathbb{U}_p} J_p \quad \text{y} \quad u^p(t) = u^p(t|t) = arg\left[\hat{c}_p(t|t)\right] \quad (5.25)$$

Furthermore, $\hat{c}_p(t+N_u+1|t) = 0$ allows establishing a limit to the prediction horizon, which prevents going into an infinite loop (Camacho & Bordons, 2007).

In fact, (5.23) to (5.25) configure the control problem formulation that allows determining the future actions sequence, from which $u^p(t)$ is chosen. As it can be seen, this model is very similar to the model in Chapter 4, with the difference that the values represented by the state variable ($\hat{y}_p^C(t+k|t)$) and the output variable ($\hat{y}_p^A(t+k|t)$) herein refer to the overdraft coverage, but not to balance available in the account. That is, $\hat{y}_p^A(t+k|t)$ measures the overdraft coverage, do not overdraft balance. The account's aim is to cover demanded payments, whereupon for a time t, the account is overdrawn until a cash transfer from the main account (ordered L_p ago) covers the overdraft. More clearly, on the one hand, revenue bank accounts usually keep positive balances. The money periodically is transferred to the main account in order to take the revenue account back to desired level (normally zero). Thereby, the costs involved managing these accounts are mainly: opportunity cost due to money tied, and transfer costs. Theoretically, revenue accounts never have negative balances, therefore do not incur cost overdraft. On the other hand, disbursement accounts normally keep negative balances. Theoretically, by periodic transfers from the main account taking them to zero (or other desired level). However, occasionally in them appear positive balances when the overdraft is covered in excess. In this case, the costs involved are usually: overdraft costs, transfer costs and opportunity costs when balances are positive. Further, transferred money belatedly covers the overdraft (L_p), as stated before.

Like in the case of the revenue bank account, a stabilization controller in a cascade fashion (Linear feedback) is added. In this way, the following change of variable is established:

$$u^p(t+k|t) = \mathrm{K}\hat{y}_p^C(t+k|t) + v^p(t+k|t) \tag{5.26}$$

With this, an algebraic handling of (5.19) and (5.20) allows obtaining:

$$\hat{y}_p^C(t+k|t) = (1-\mathrm{K})\hat{y}_p^C(t+k-1|t) - v^p(t+k-1|t) + \hat{d}^a(t+k) \tag{5.27}$$

$$\hat{y}_p^A(t+k|t) = (1-\mathrm{K})\hat{y}_p^C(t+k|t) - v^p(t+k|t) \tag{5.28}$$

And, the prediction equations of the entire process can be expressed in a compact form as:

$$\mathbf{Y}_p^C = \mathbf{M}_y\hat{y}_p^C(t|t) - \mathbf{M}_v(\mathbf{V}^p - \mathbf{D}^a) \tag{5.29}$$

$$\mathbf{Y}_p^A = \mathbf{M}_y\left[(1-\mathrm{K})\hat{y}_p^C(t|t) - v^p(t|t)\right] - \mathbf{M}_v(\mathbf{V}^{p1} - (1-\mathrm{K})\mathbf{D}^a) \tag{5.30}$$

Under the assumption that $N = N_u$, this is an equivalent model to (4.50) y (4.57), wherein:

$$\mathbf{Y}_p^C = \left[\hat{y}_p^C(t+1|t) \quad \hat{y}_p^C(t+2|t) \quad \cdots \quad \hat{y}_p^C(t+k|t) \quad \cdots \quad \hat{y}_p^C(t+N_u-1|t) \quad \hat{y}_p^C(t+N_u|t)\right]^t \tag{5.31}$$

$$\mathbf{V}^p = [v^p(t|t) \quad v^p(t+1|t) \quad \cdots \quad v^p(t+k-1|t) \quad \cdots \quad v^p(t+N_u-2|t) \quad v^p(t+N_u-1|t)]^t \tag{5.34}$$

$$\mathbf{D}^a = [\hat{d}^a(t+1|t) \quad \hat{d}^a(t+2|t) \quad \cdots \quad \hat{d}^a(t+k|t) \quad \cdots \quad \hat{d}^a(t+N_u-1|t) \quad \hat{d}^a(t+N_u|t)]^t \tag{5.35}$$

$$\mathbf{Y}_p^A = \left[\hat{y}_p^A(t+1|t) \quad \hat{y}_p^A(t+2|t) \quad \cdots \quad \hat{y}_p^A(t+k|t) \quad \cdots \quad \hat{y}_p^A(t+N_u-1|t) \quad \hat{y}_p^A(t+N_u|t)\right]^t \tag{5.36}$$

$$\mathbf{V}^{p1} = [v^p(t+1|t) \quad v^p(t+2|t) \quad \cdots \quad v^p(t+k|t) \quad \cdots \quad v^p(t+N_u-1|t) \quad v^p(t+N_u|t)]^t \tag{5.37}$$

So the following is a general expression for the predicted operating cash balance in the kth stage:

$$\hat{y}_p^C(t+k|t) = (1-\mathrm{K})^k\hat{y}_p^C(t|t) - \sum_{j=1}^{k}(1-K)^{k-j}v^p(t+j-1|t) + \sum_{j=1}^{k}(1-K)^{k-j}\hat{d}^a(t+j|t) \tag{5.38}$$

And, this is a general expression for the overdraft coverage (output variable) in the kth stage:

$$\hat{y}_p^A(t+k|t) = (1-\mathrm{K})^{k+1}\hat{y}_p^C(t|t) - (1-K)^k v^p(t|t)$$

$$-\sum_{j=1}^{k}(1-K)^{k-j}v^{p}(t+j|t)+\sum_{j=1}^{k}(1-K)^{k-j+1}\hat{d}^{a}(t+j|t) \qquad (5.39)$$

In regards this model, remember that in Chapter 4 it was determined: (a) a valid interval for process stability regardless of the prediction horizon size, (b) a bounded interval for the uncertainty in the closed-loop for a sufficiently large size of prediction horizon, and (c) solution algorithm.

In summary, the DP model determines the $v^p(t)$ value. Then, $u^p(t)$ is obtained by using (5.26) and applied to the plant, whose output is $y_p^A(t)$. Finally, $y^A(t)$ is determined through (5.11), which is the final controlled variable including the time delay (L_p).

5.2.2 Control model for overdraft balance

The second proposal of a model predictive control combined with dynamic programming is as shown in Figure 5.3. The graph also shows a diagram based on the proposal in Chapter 4 including a stabilization controller in a cascade. However, this time the model uses the information about the overdraft balance in the disbursements account, represented by (5.10) and (5.11), as well as the amount of cash required to cover demanded payments at time t (5.12). Then, the MPC is formulated using delay time. In this way, the model formulation considers the reference signal representing the transfer policy applied to the overdraft balance in the account. The direct result is the overdraft balance including the delay time of the transfer ordered L_p ago.

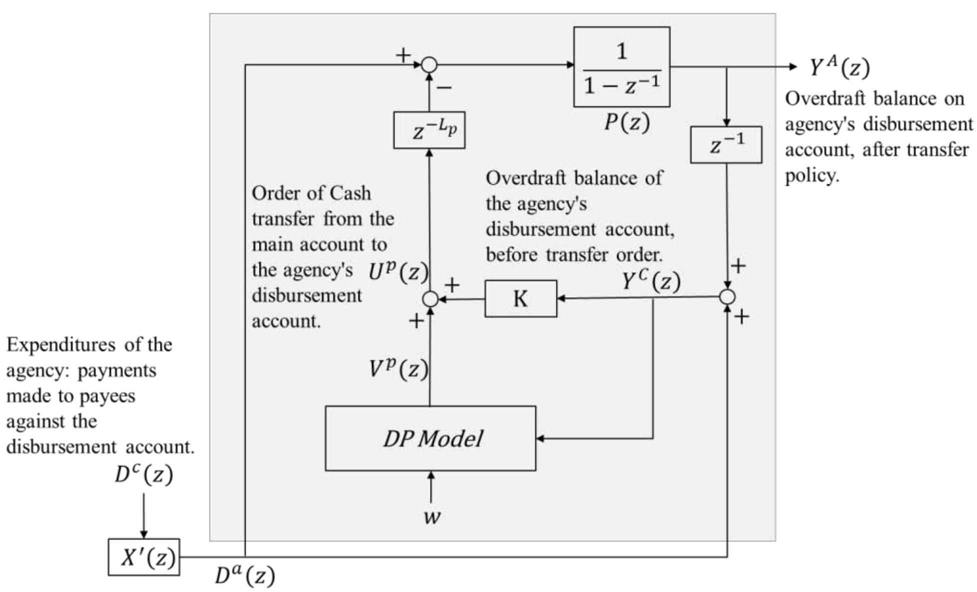


Figure 5.3: System structure including the DP model control for overdraft balance with linear feedback gain.

The features of this proposal are:

- ✓ The prediction horizon is N $(t+1, t+2, \ldots, t+N)$ and the control horizon is N_u, with $N_u = N - L_p$.
- ✓ $N_u + 1$ stages $(t+L_p+0, t+L_p+2, \ldots, t+N)$ for the dynamic programming model.
- ✓ N forecast instants for which the future outputs $(\hat{d}^a(t+k))$ of the model are predicted.
- ✓ The decision variable is u^p, about which $N_u + 1$ values are determined $(u^p(t+k|t), k = 0, \ldots N_u)$.
- ✓ The first value of u^p $(u^p(t|t) = u^p(t)$, at stage 0) is sent to the process.
- ✓ The $N+1$ predicted outputs $\hat{y}^A(t+k|t)$ (overdraft balance at the end of time $t+k$, with $k = 0,1 \ldots N$) depend on the known values (inputs and outputs) until time t and the set of future control signals $u^p(t+k|t)$.
- ✓ The overdraft balance (just before transferring) is the state variable:

$$\hat{y}^C(t+k|t) = \hat{y}^A(t+k-1|t) + \hat{d}^a(t+k) \tag{5.40}$$

That is also a pre-decision-state formulation with $\hat{d}^a(t+k)$ as the forecasted amount of cash required to cover demanded payments during $t+k$, based on the process represented by (5.1). Moreover, the predicted values of the output variable are established by:

$$\hat{y}^A(t+k|t) = \hat{y}^A(t+k-1|t) + \hat{d}^a(t+k) - u^p\left(t-L_p+k\middle|t\right) \tag{5.41}$$

Similarly, $\hat{y}^A(t+N|t)$ is referred to the overdraft balance of the agency's disbursements account at the end of the forecast period $t+N$, which prevents entering an infinite loop (Camacho & Bordons, 2007).

For practical purposes (5.40) and (5.41) can be rewritten as follows, because the control over the output variable, in effect, is applied from L_p after t, and the control action to be determined by the MPC is $u^p(t+k|t)$, with $k = 0$. That is $u^p(t|t) = u_p(t)$:

$$\hat{y}^C\left(t+L_p+k\middle|t\right) = \hat{y}^A\left(t+L_p+k-1\middle|t\right) + \hat{d}^a\left(t+L_p+k\right) \tag{5.42}$$

$$\hat{y}^A\left(t+L_p+k\middle|t\right) = \hat{y}^A\left(t+L_p+k-1\middle|t\right) + \hat{d}^a\left(t+L_p+k\right) - u^p(t+k|t) \tag{5.43}$$

with: $k = 0,1 \ldots N_u$

The state variable's value (before the control action) is:

$$\hat{y}^C(t+L_p|t) = \hat{y}^C(t|t) - \sum_{j=1}^{L_p} u^p(t+j-1-L_p|t) + \sum_{j=1}^{L_p} \hat{d}^a(t+j|t) \tag{5.44}$$

And the output variable's value (after the control action) is:

$$\hat{y}^A(t+L_p|t) = \hat{y}^C(t+L_p|t) - u^p(t|t) \tag{5.45}$$

In consequence, the periodic reference trajectory used is generated by:

$$w(t+k) = A.frac\left(\frac{t+k}{T}+\phi\right)+S, \qquad k = N_1, \dots, N \tag{5.46}$$

with $N_1 = L_p$ and $\phi = 0$, because the output signal ($\hat{y}^A$) at stage $t+k$ must be compared with respect to the reference signal calculated for the same stage $t+k$. Further, $S = 0$, if it assumes that the level at which returns the overdraft is zero after the transfer order.

Therefore, a decision is made after the overdraft balance represented by the state variable $\hat{y}^C(t+L_p+k|t)$ at stage $t+L_p+k$ is evaluated: either $\hat{y}^C(t+L_p+k|t) - S$ is transferred from the main account or not. In this way, the decision variable in the dynamic programming model takes the values $u^p(t+k|t) = 0$ or $u^p(t+k|t) = \tilde{u}^p(t+k|t) = \hat{y}^C(t+L_p+k|t) - S,\ k = 0,1,2,\dots,N_u$. Because it is a backward Dynamic Programming strategy, the dynamic process begins at step $k = N_u$, ending at step $k = 0$, when the $u^p(t|t)$ value is determined, corresponding to the minimum value of the cost function of model. This is $u^p(t|t) = 0$ or $u^p(t|t) = \tilde{u}^p(t|t) = \hat{y}^C(t+L_p|t) - S$ as it (5.13).

The objective function for the prediction process, expressed here in the form of a quadratic function is as follow:

$$J = \sum_{k=0}^{N_u} \delta(k)\left[\hat{y}^A(t+L_p+k|t) - w(t+L_p+k)\right]^2 \tag{5.47}$$

Or its equivalent: $J = \sum_{k=N_1}^{N} \delta(j)[\hat{y}^A(t+k|t) - w(t+k)]^2$, with $N_1 = L_p$.

This is, the total error in the prediction horizon between the predicted output and the predicted reference trajectory. In this case, three criteria used in the previous proposal have been considered in the same manner: the $\delta(k)$ series, the control effort of the objective function, and handling the formulation as a deterministic discrete model, once the $\hat{d}^a(t+k)$ series is established. Then, the minimum accumulated cost $\hat{c}(t+k|t)$ until the prediction period $t+k$ is defined as follow:

$$\hat{c}(t+k|t) = \min_{u^p \in \mathbb{U}} \left\{ \hat{c}(t+k+1|t) + \delta(k)\left[\hat{y}^A\left(t+L_p+k\middle|t\right) - w\left(t+L_p+k\right)\right]^2 \right\} \tag{5.48}$$

$$u^p(t+k|t) = arg[\hat{c}(t+k|t)],\ k = 0,1,\dots,N_u \tag{5.49}$$

whereupon, $k = 0,1,\dots,N_u$, and:

$$\hat{c}(t|t) = \min_{u^p \in \mathbb{U}} J \quad \text{y} \quad u^p(t) = u^p(t|t) = arg[\hat{c}(t|t)] \tag{5.50}$$

Also, $\mathbb{U} = \left\{0, \hat{y}^C\left(t+L_p+k\middle|t\right) - S\right\}$ is the set of allowable sequences of control actions. $\hat{c}_p(t+k|t)$ is the minimum accumulated cost until the control period $t+k$, expressed in terms of $\hat{c}_p(t+k+1|t)$, and $\hat{c}_p(t+N_u+1|t) = 0$.

The same procedure is followed to add a stabilization controller in a cascade fashion (Linear feedback) to this model. The following change of variable is made:

$$u^p(t+k|t) = \mathrm{K}\hat{y}^C\left(t+L_p+k\middle|t\right) + v^p(t+k|t) \tag{5.51}$$

With this variable change and an algebraic handling, (5.42) and (5.43) becomes, respectively:

$$\hat{y}^C\left(t+L_p+k\middle|t\right) = (1-\mathrm{K})\hat{y}^C\left(t+L_p+k-1\middle|t\right) - v^p(t-1+k|t) + \widehat{d}^a\left(t+L_p+k\right) \tag{5.52}$$

$$\hat{y}^A\left(t+L_p+k\middle|t\right) = (1-\mathrm{K})\hat{y}^C\left(t+L_p+k\middle|t\right) - v^p(t+k|t) \tag{5.53}$$

So the prediction equations of the entire process can be expressed in a compact form as:

$$\mathbf{Y}^C = \mathbf{M}_y\hat{y}^C\left(t+L_p\middle|t\right) - \mathbf{M}_v(\mathbf{V}^p - \mathbf{D}^a) \tag{5.54}$$

$$\mathbf{Y}^A = \mathbf{M}_y\left[(1-\mathrm{K})\hat{y}^C\left(t+L_p\middle|t\right) - v^p(t|t)\right] - \mathbf{M}_v(\mathbf{V}^{p1} - (1-\mathrm{K})\mathbf{D}^{a1}) \tag{5.55}$$

wherein:

$$\hat{y}^C\left(t+L_p\middle|t\right) = (1-\mathrm{K})^{L_p}\hat{y}^C(t|t) \\ -\sum_{j=1}^{L_p}(1-\mathrm{K})^{L_p-j}v^p\left(t+j-1-L_p\middle|t\right) + \sum_{j=1}^{L_p}(1-\mathrm{K})^{L_p-j}\hat{d}^a(t+j|t) \tag{5.56}$$

$$\mathbf{Y}^C = \left[\hat{y}^C\left(t+L_p+1\middle|t\right) \quad \hat{y}^C\left(t+L_p+2\middle|t\right) \quad \cdots \quad \hat{y}^C\left(t+L_p+k\middle|t\right) \quad \cdots \quad \hat{y}^C\left(t+L_p+N_u-1\middle|t\right) \quad \hat{y}^C\left(t+L_p+N_u\middle|t\right)\right]^t \tag{5.57}$$

$$\mathbf{Y}^A = \left[\hat{y}^A\left(t+L_p+1\middle|t\right) \quad \hat{y}^A\left(t+L_p+2\middle|t\right) \quad \cdots \quad \hat{y}^A\left(t+L_p+k\middle|t\right) \quad \cdots \quad \hat{y}^A\left(t+L_p+N_u-1\middle|t\right) \quad \hat{y}^A\left(t+L_p+N_u\middle|t\right)\right]^t \tag{5.58}$$

$$\mathbf{D}^{a1} = \left[\hat{d}^a\left(t+L_p+1\middle|t\right) \quad \hat{d}^a\left(t+L_p+2\middle|t\right) \quad \cdots \quad \hat{d}^a\left(t+L_p+k\middle|t\right) \quad \cdots \quad \hat{d}^a\left(t+L_p+N_u-1\middle|t\right) \quad \hat{d}^a\left(t+L_p+N_u\middle|t\right)\right]^t \tag{5.59}$$

It can be seen that, with the change of variable (5.51), (5.44) is equivalent to (5.56).

From the above, the following is a general expression for the overdraft balance (state variable) in the kth stage:

$$\hat{y}^C\left(t+L_p+k\middle|t\right)=(1-\mathrm{K})^k\hat{y}^C\left(t+L_p\middle|t\right) \\ -\sum_{j=1}^{k}(1-\mathrm{K})^{k-j}v^p(t+j-1|t)+\sum_{j=1}^{k}(1-\mathrm{K})^{k-j}\hat{d}^a\left(t+L_p+j\middle|t\right) \quad (5.60)$$

And, the general expression for the overdraft (output variable) in the kth stage:

$$\hat{y}^A\left(t+L_p+k\middle|t\right)=(1-\mathrm{K})^{k+1}\hat{y}^C\left(t+L_p\middle|t\right)-(1-K)^k v^p(t|t) \\ -\sum_{j=1}^{k}(1-\mathrm{K})^{k-j}v^p(t+j|t)+\sum_{j=1}^{k}(1-\mathrm{K})^{k-j+1}\hat{d}^a\left(t+L_p+j\middle|t\right) \quad (5.61)$$

The Theorem 1 and Corollary 2 of Chapter 4 are also valid for this proposal. That is, a valid interval for process stability can be established regardless of the prediction horizon size, as well as a bounded interval for the uncertainty in the closed-loop for a sufficiently large size of prediction horizon.

In summary, the DP model determines the value of $v^p(t)$. Then, using (5.51) $u^p(t)$ is obtained and applied to the plant, then the value of $y^A(t)$ is got, which is the final desired controlled variable.

5.3 Results analysis

The idea of presenting two approaches to the overdraft control problem has its origin in the fact that the problem can be stated in two different ways, depending on the state and output variable chosen. Moreover, because it is interesting to answer the question of whether the controller described in Chapter 4 can be used in any of these proposals or both. In this regard, a mathematical and experimental comparison becomes necessary between the two proposals.

5.3.1 Comparing both models, overdraft balance and overdraft coverage

First, the comparison begins at the base. This is, from (5.20) and (5.43) it can be seen that:

$$\hat{y}^A\left(t+L_p+k\middle|t\right)-\hat{y}_p^A(t+k|t) \\ =\hat{y}^A\left(t+L_p+k-1\middle|t\right)-\hat{y}_p^A(t+k-1|t)+\hat{d}^a\left(t+L_p+k\middle|t\right)-\hat{d}^a(t+k|t) \quad (5.62)$$

This is a basic expression for $\hat{y}^A\left(t+L_p+k\middle|t\right)-\hat{y}_p^A(t+k|t)$ with which, developing the successive differences, after an algebraic handling, it is obtained:

$$\hat{y}^A\left(t+L_p+k\middle|t\right)-\left[\hat{y}^C\left(t+L_p\middle|t\right)+\sum_{j=1}^{k}\hat{d}^a\left(t+L_p+j\middle|t\right)\right]=\hat{y}_p^A(t+k|t)-\left[\hat{y}_p^C(t|t)+\sum_{j=1}^{k}\hat{d}^a(t+j|t)\right] \quad (5.63)$$

Or:

$$\hat{y}^A\left(t+L_p+k\middle|t\right)-\tilde{y}^C\left(t+L_p+k\right)=\hat{y}_p^A(t+k|t)-\tilde{y}_p^C(t+k) \quad (5.64)$$

where $\tilde{y}^C\left(t+L_p+k\right)$ is the accumulated uncertainty at the kth stage for process on overdraft balance and $\tilde{y}_p^C(t+k)$ is the accumulated uncertainty at the kth stage for process on overdraft coverage.

If the accumulated control action at the kth stage is:

$$u_{accu}(k)=u^p(t|t)+\sum_{j=1}^{k}u^p(t+j|t) \quad (5.65)$$

Adding and subtracting $u_{accu}(k)$ into (5.63) and applying (5.26) allows obtaining (5.39), the output variable $\hat{y}_p^A(t+k|t)$. Moreover, if (5.28) is applied to this result the state variable $\hat{y}_p^C(t+k|t)$ in (5.38) is obtained and therefore, the corresponding matrix representation (5.29) and (5.30) of the MPC for the overdraft coverage.

Equally, by adding and subtracting $u_{accu}(k)$ into (5.63) and applying (5.51) allows obtaining (5.61), the output variable $\hat{y}^A\left(t+L_p+k\middle|t\right)$. Also, if (5.53) is applied to this result the state variable $\hat{y}^C\left(t+L_p+k\middle|t\right)$ in (5.60) is obtained and therefore, the corresponding matrix representation (5.54) and (5.55) of the MPC for the overdraft balance.

A similar reasoning can be used for (5.22) and (5.47) in order to ensure that both objective functions are equivalent, if:

$$\sum_{j=1}^{L_p}\hat{d}^a(t+j|t)=L_p.E\left(d^a(t)\right) \quad (5.66)$$

In this case, must be done the respective substitution in (5.44) according to this condition.

From the above, it is shown that both proposals “(5.22), (5.29), (5.30), (5.39), (5.40)” and “(5.47), (5.54), (5.55), (5.60), (5.61)” are theoretically equivalent, since the difference between their outputs (y^A) is null if (5.66) is met, because it does not depend on the control action. That is, the cash transfers delay is out of the control process, and the output variability respect to controlled process only depends on the variability of uncertainty during delay time, after control action. These results are condensed in the next theorem:

Theorem 3: Let the control model described by (5.8) to (5.12), on which a MPC based on DP is formulated. If (5.66) is met, the system output (y^A) is the same regardless of whether the MPC is formulated using y_p^A as partial output variable without considering the delay time in the model or, alternatively, the MPC is formulated directly using y^A as output variable considering the delay time as part of the model.

5.3.2 Simulation results

Several simulations were performed in order to test both proposals. As expected, identical results are obtained if (5.66) is met. Simulation shows results in a single agency disbursement bank account of a hypothetical CCDS. The overall impact on the system main account is obtained by the arithmetic sum of all agencies of the firm. Figure 5.4 allows see how the simulation results are determined. The value of the state variables (y^C and y_p^C) is set by solving (5.4) and (5.5), respectively, which are inputs of the DP models to calculate the value of the auxiliary variable (v_p), depending on which one is used: controlling overdraft coverage or controlling overdraft balance. If the overdraft coverage controller is used, the value of u^p is obtained through (5.26), while if the overdraft balance controller is used, u^p is obtained through (5.51). In both cases, the graphs obtained are identical; due to this, only one of them is included in Figure 5.4. The output values (y^A and y_p^A) are found through (5.6) and (5.7). Note that delay of the cash transfer only affects the overdraft balance (y^A).

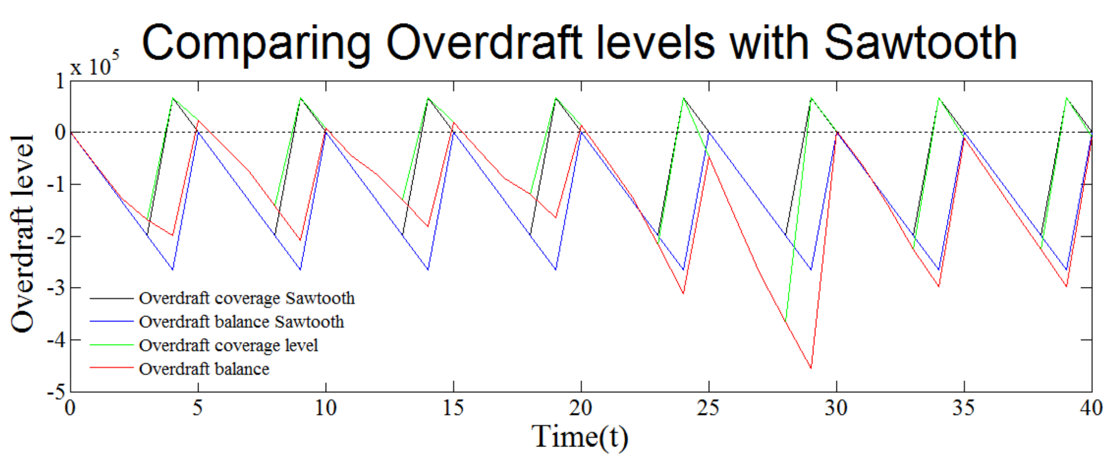


Figure 5.4: Comparing overdraft coverage level and overdraft balance using the DP model with regard to sawtooth models.

It may be noted that by applying the control directly on the overdraft coverage, an indirect control over the overdraft balance is achieved. Justly, the differences obtained respect reference signal are due to the disturbance variations during delay time. For cases in which the delay is small, control is more satisfying. Otherwise, when the delay is greater, differences enlarge. Still pending is combine the control model with selecting a better forecasting method for disturbance.

5.4 Concluding remarks of the chapter

This chapter has presented two MPC proposals for a disbursement account belonging to a CCDS, based on the application of inventory policies to the overdraft balance and the overdraft coverage. Dynamic programming was used in the prediction model, for which several features and simplifications of the proposed model in the previous chapter were considered, as well as a stabilizing regulator in cascade fashion using a linear feedback gain (closed-loop) is included. Also, an objective function based on a quadratic criterion of finite horizon is used with which the cost is optimized, with a dynamic parameter of discount but without the control effort term. However, delay was added to the problem with regard to the model of Chapter 4. Finally, a sawtooth shaped function like reference signal is used, which is generalized with respect to the model in Chapter 4.

Because the MPC controller proposed in this chapter has generalized some characteristics with respect to the formulation of Chapter 4, the versatility in this case is greater in the sense that it adapts to a wide range of scenarios by varying its different parameters.

The main conclusion refers to the fact of proposing two MPC models that fulfill the same aim, which are theoretically equivalent. That is, one can be derived from the other if the expected uncertainty during the delay time is assumed. In this case, the system output (y^A) is the same regardless of whether the MPC is formulated using y_p^A as partial output variable (output variable of DP model) or, alternatively, the MPC is formulated directly using y^A as output variable (both, DP model and entire system). With this, it is also concluded that the time delay is out of the control process, and the output variability respect to controlled process only depends on the variability of uncertainty during delay time, after control action. The foregoing suggests that research should focus on combining this proposal with better methods of prediction uncertainty or its recognition during the delay time.

The next chapter uses the results provided by the model predictive control proposed in Chapters 4 and 5. This is made in the context of a case study, highlighting the fact that, on the one hand, the predictive control fulfills its purpose and, on the other, its operation is transparent to the decision maker.

Chapter 6

Simulating a CCDS: a case study

Chapter 3 presents the simulation model of the CCDS, based on difference equations and an equivalent model after applying *Z*-transform. Specifically, Section 3.4 shows a basic simulation of the system in order to test the functionality of the model, which shows the expected results. Moreover, Chapters 4 and 5 propose a model predictive control based on dynamic programming, in order to control the available cash of the revenue accounts and the overdraft in disbursements accounts. This chapter presents a case study using hypothetical data in order to perform a comprehensive analysis of results and the potentialities of the model.

For this purpose, Section 6.1 presents the case study. Next, Section 6.2 shows the results obtained when running the model in various scenarios. Then, Section 6.3 discusses the results taken together and provides recommendations relevant to the case study. Finally, Section 6.4 presents the conclusions and observations.

6.1 Case study

The case study refers to DISPRONLI, Ltd., which is a fictitious company that distributes mass consumer products through seven agencies located in different cities to the east of Spain. However, the company plans to expand its operations to other regions. In order to organize its finances, DISPRONLI has designated two bank accounts to each agency as shown in Table 6.1. In short, the company has seven revenue accounts, seven disbursements accounts, a main account, an investment account and a credit line. These last three are managed in the main office of the company. Table 6.1 also shows interest rates as well as the costs and delays associated with bank transfers. As can be seen, the company has seventeen bank accounts in four different banks.

Bank	Office / Agency	Accounts	Interest rate (Short term)			Bank transfer order			
						Between accounts of the same bank		Toward accounts of other banks	
			Passive	Active	Overdraft	Fixed cost (euros)	Delay (days)	Fixed cost (euros)	Delay (days)
A	Main office	Main, Investment, Credit line	2,50%	8,20%	9,20%	125,00	0	250,00	1
	A1, A2, A3	Revenues, Disbursements							
B	B1, B2	Revenues, Disbursements	2,30%	8,10%	9,30%	100,00	0	240,00	1
C	C1	Revenues, Disbursements	2,10%	8,20%	9,30%	120,00	0	230,00	1
D	D1	Revenues, Disbursements	2,00%	9,00%	9,50%	120,00	0	240,00	1

Table 6.1. Basic data of the concentration system DISPRONLI.

Table 6.2 shows the estimated revenue for 2017. This is uniformly distributed within the ranges shown in the table. Revenues are higher between the months of April to September, as shown in the table. Moreover, due to the kind of product offered by the company, the payment methods used by customers are cash, debit cards and credit cards. Therefore, the money from sales is available in the revenue accounts as shown in Table 6.3.

	Estimated revenues (euros / day)					
	From April to September			From January to March and October to December		
Agency	Maximum	Minimum	Mean	Maximum	Minimum	Mean
A1	150.685,00	94.520,00	122.602,50	90.411,00	56.712,00	73.561,50
A2	89.041,00	53.425,00	71.233,00	53.424,60	32.055,00	42.739,80
A3	109.589,00	61.644,00	85.616,50	65.753,40	36.986,40	51.369,90
B1	68.493,00	45.205,00	56.849,00	41.095,80	27.123,00	34.109,40
B2	75.342,00	49.315,00	62.328,50	45.205,20	29.589,00	37.397,10
C1	130.137,00	78.082,00	104.109,50	78.082,20	46.849,20	62.465,70
D1	61.644,00	28.767,00	45.205,50	36.986,40	17.260,20	27.123,30
DISPRONLI	684.931,00	410.958,00	547.944,50	410.958,60	246.574,80	328.766,70

Table 6.2. Estimated revenues of DISPRONLI per agency and time of the year.

In all agencies, disbursements are estimated at 90% compared to revenue between the months of April to September. The rest of the year, disbursements are approximately 98% with respect to revenues. Disbursements are also uniformly distributed within the ranges indicated. Payment policy of DISPRONLI is to pay in two basic forms to suppliers: checks and direct transfer. Furthermore, according to statistical, the debit in the respective disbursements accounts is performed according to the behavior shown in Table 6.4.

Form of payment	Cash	Debit card			Credit card	
	0	1	2	3	4	5
Delay (days)	0	0	1	2	4	5
Behavior (%)	26%	19%	29%	6%	8%	12%

Table 6.3 Transit time of the money in the cash collection process.

In the last report submitted to DISPRONLI's Executive Board, the Director of Finance have planned to start in 2017 using the 80% of the capacity of the credit line, which is capped at twelve million euros. However, it estimates that the investment account will be in the order of two million euros at that time. Meanwhile, the cash balance on the main account of the company will be 1.5 million euros at the beginning of 2017.

Form of payment	Check					Direct transfer	
	0	1	2	3	4	5	6
Transit time (days)	0	1	2	3	4	0	1
Behavior (%)	11%	29%	18%	9%	3%	12%	18%

Table 6.4 Form in which the money paid to payees is debited to the disbursements account.

With this information, the Director of Finance has been asked to prepare a financial plan for 2017 evaluating some options that the Executive Director has under negotiation. On the one hand, there is the possibility of replacing the investment account by an investment fund in which net financial income (risk free) are six points higher than what is got in the investment account of Bank A. Nonetheless, with this option, at the time of to need cash to cover any shortfall in the main account, its availability will be delayed one day. Also, due to financial procedures, availability of any new deposit made in the investment fund, will have one day effect after the deposit is made. The fixed cost for each transfer order from the Investment Fund toward the main account is 180 euros. According to the Executive Director, any decision taken on the investment account will take effect from the second half of 2017.

Another aspect that the Director of Finance should evaluate is regarding the transfer costs. This is because the Executive Director is negotiating the possibility of reducing these costs up to ninety percent, in all banks with which the company relates. Finally, it is known that the Director of Finance will use an intuitive criterion regarding the cash balance on the main account: two hundred thousand euros for return level, three hundred thousand euros for higher limit and ten thousand euros for lower limit. The latter is the minimum balance set by the Executive Director. However, it has been requested the possibility of using a technical approach in this respect. Thus, it will be evaluated the possibility of fixing bands for the cash balance in main account according to Miller & Orr (1966). For accounts of revenues and disbursements of the agencies, the company adopts a policy of type (s, S), where s is the optimum level to decide a cash transfer and $S = 0$.

6.2 Results of executing the model

The Director of Finance has organized data in order to enter them into the model through a special interface. The first thing observed when the model is run is the behavior of sales revenue and disbursements of all agencies. Figure 6.1 shows an example of the behavior of deposits due to sales of one of the agencies, and the availability of cash in the respective revenue account. This result is obtained by applying the ARMA model on the revenue uncertainty process represented by (4.12) to (4.15). In this case, for $f = 5$: $b_0^r = 0.45$, $b_1^r = 0.29$, $b_2^r = 0.06$, $b_3^r = 0.00$, $b_4^r = 0.08$, $b_5^r = 0.12$.

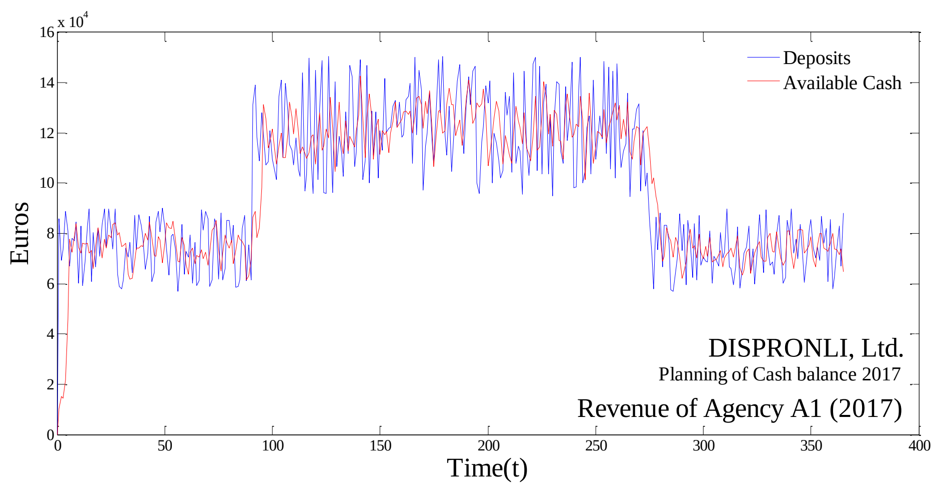


Figure 6.1. An example of revenues of agencies: Agency A1.

Figure 6.2 is also an example of the behavior of disbursements of one of the agencies. The graph shows payments made, which are then debited to the disbursements account. This result is obtained by applying the ARMA model on the disbursements uncertainty process represented by (5.14) to (5.18). In this case, for $g = 4$: $b_0^d = 0.23$, $b_1^d = 0.47$, $b_2^d = 0.18$, $b_3^d = 0.09$, $b_4^d = 0.03$. For its part, Figure 6.3 shows the coverage overdraft and the overdraft balance because of disbursements and the controller action to cover the overdraft through cash transfers from the main account. In this regard, it is clear that the MPC applied in each of the accounts of both revenue and disbursement, in a comprehensive manner, meets the objective of control. In the case of revenue accounts, the result is obtained by applying the controller represented by (4.25) to (4.28), (4.48) and (4.49), while in the case of disbursements accounts, the result is obtained by applying the controller represented by (5.22) to (5.25), (5.27), (5.28) and then (5.11).

This case study refers to an example in which there are two different seasons in the year for input to the model, which was designed only for stationary disturbance behavior. What has been done is to run the stationary model several times, depending on the behavior of the disturbance throughout the year (revenue and disbursements of each agency). The final conditions of a run are used as initial conditions for the next run. To do this it has been added a module allowing to the model adapt to changes in the surrounding conditions. This approach can be used, for example, if one takes a case in which each month of the year behaves differently. That is, the model runs twelve consecutive times. Thus, the user can establish how the simulation is performed to adapt the model to its needs. Another way to see this is that, although the model was designed for stationary conditions, the stability and robustness feature of the MPC could facilitate its use in non-stationary conditions.

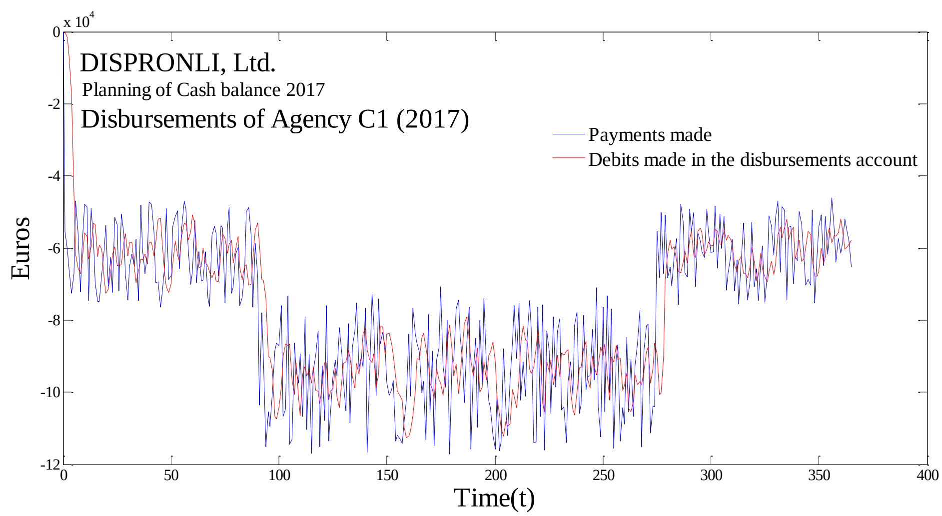


Figure 6.2. An example of disbursements in agencies: Agency C1.

With the purpose of making valid comparisons, each time the model is run, it is done with the same random number seed, which means that the same input data are used for all the results obtained. First, the model is executed under the financial conditions initially

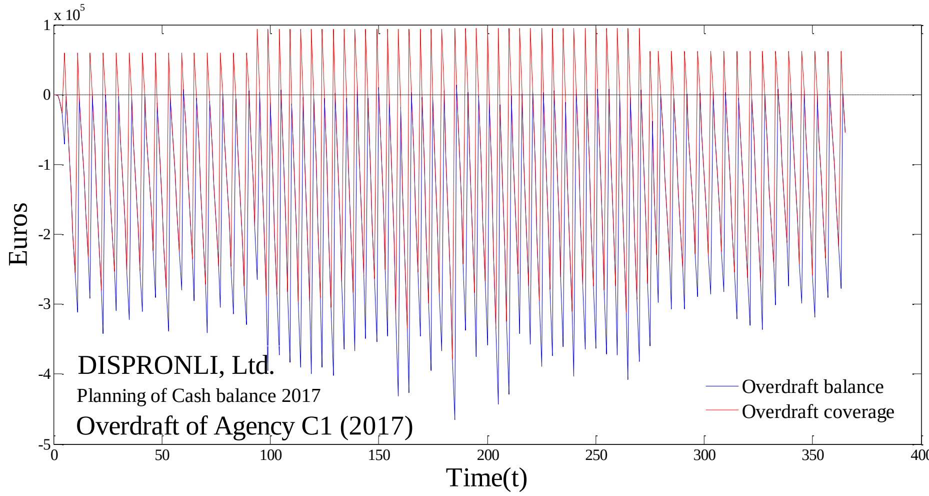


Figure 6.3. Coverage and balance of overdraft in agencies as a result of disbursements and transfers from the main office: Agency C1.

foreseen by the Director of Finance. Figure 6.4 shows the behavior of managed accounts at the main office of DISPRONLI. It is noted that, given the initial conditions (availability of the credit line, the balance in the investment account and balance in the main account), the values are corrected from the beginning of the year according to the policy of minimum balance and the relationship between interest rates. That is, the cash money flows through the main account from the investment account toward the credit line. It may also be noted the stability of the cash balance on the main account. Moreover, DISPRONLI begins again save money in the investment account from the second half of the year, when the loan is paid in full. Figure 6.5 shows the dynamics of transfers between accounts of the main office, from the point of view of the main account. The results in Figures 6.4 and 6.5 have been achieved by applying the model of the CCDS represented by (3.16) to (3.20) or (3.22). Figure 6.6 presents the behavior of transfer costs and financing (interest payments), as well as the interest income (interest earned). Transfer costs vary within the expected range. The financial cost decreases according to how the balance of the credit line decreases. Financial income only occurs when the balance in the investment account is positive. Figure 6.7 shows the evolution of the accumulated costs as well as interest income. Finally, Figure 6.8 compares the CCDS total management costs (559,628.28 euros/year) regarding financial income (17,924.67 euros/year). A net result of -541,703.61 euros/year. It is worth to clarify also that the results of Figures 6.6 to 6.8 have been obtained by applying the expressions on financial incomes and charges of a CCDS (Appendix B).

The Director of Finance makes a second run of the model to observe its behavior in the same conditions as the previous run, this time applying the criterion of Miller and Orr for determining the limits of available cash balance on the main account. Before that, he performs the following calculations based on the fact that Miller and Orr set their criterion according to the daily change in the cash balance. So there are different calculations according to revenue levels.

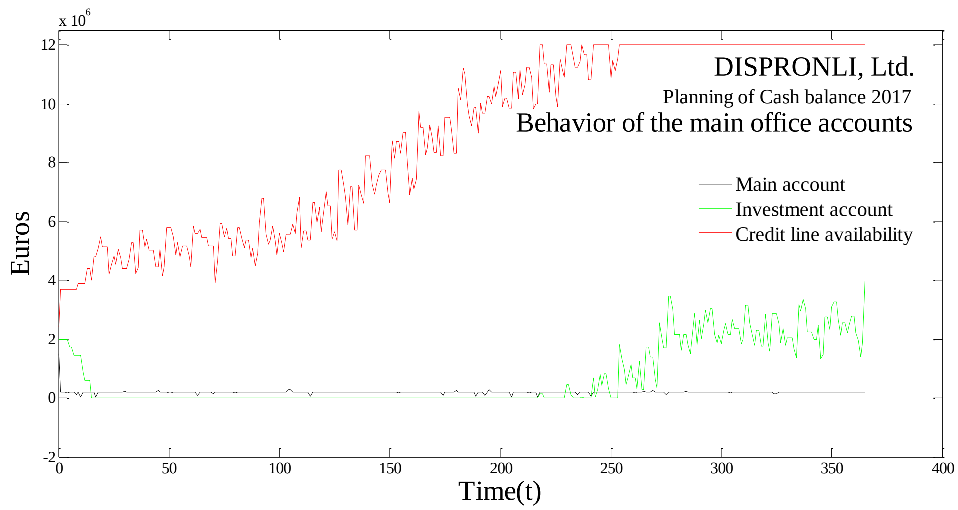


Figure 6.4. Behavior of the main office accounts with intuitive limits for the main account.

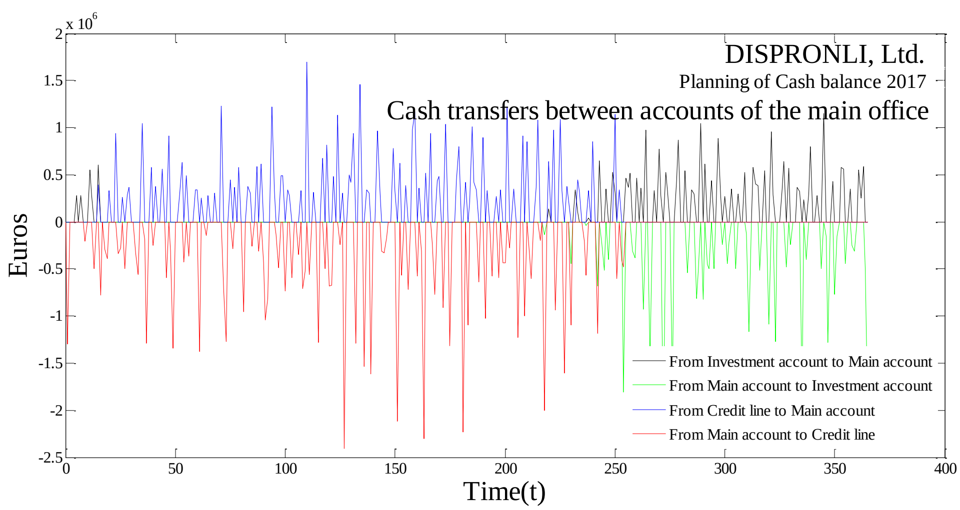


Figure 6.5. Cash transfers between accounts of the main office.

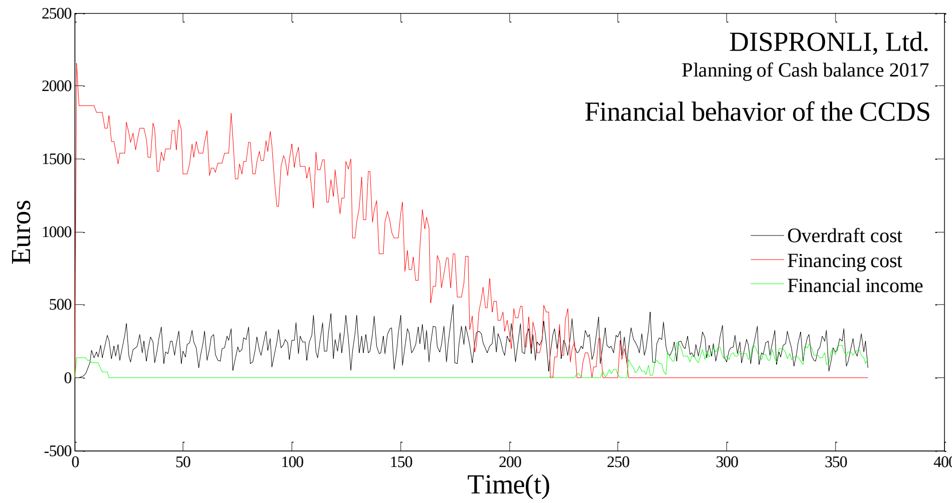


Figure 6.6. Financial behavior of the CCDS. It only includes the overdraft and financing costs, and financial income.

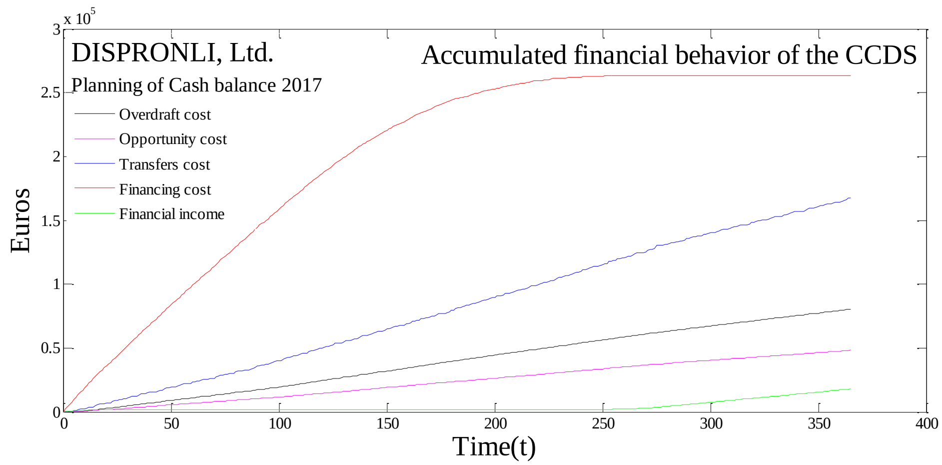


Figure 6.7. Accumulated financial behavior of the CCDS.

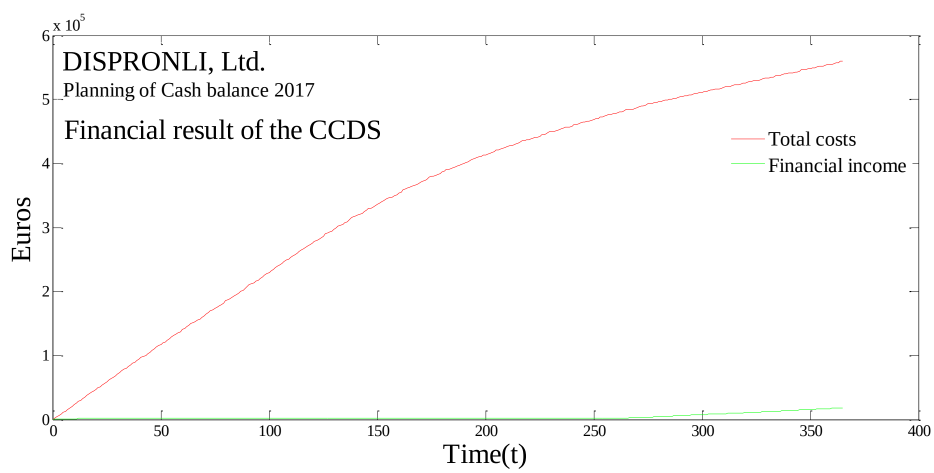


Figure 6.8. Comparing the total costs of CCDS with regard to financial income.

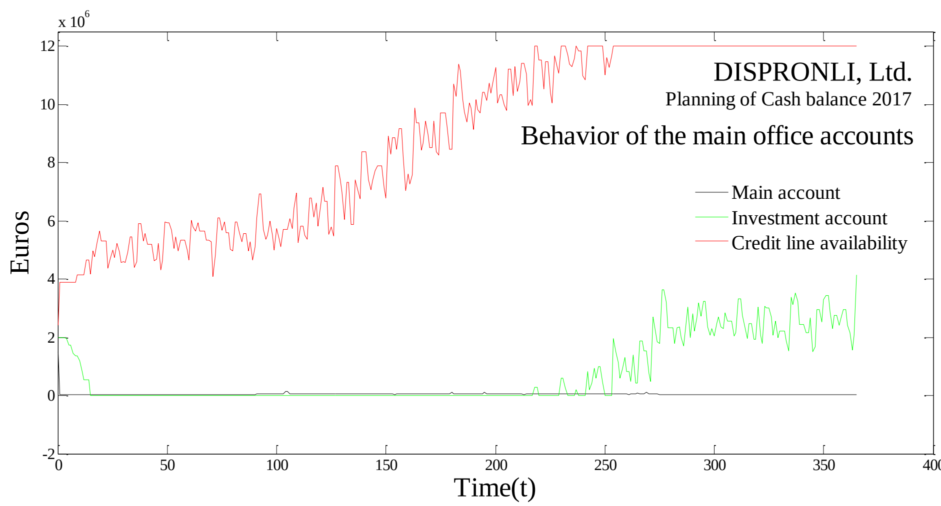
Miller and Orr calculate the cash upper limit and the return level (optimal cash balance) by using an expression equivalent to this:

$$M^* = \sqrt[3]{\frac{3.FCT.Var(CCB)}{4.i_I/BASE}} + M_{min} \tag{6.1}$$

$$M^*_{max} = 3M^* - 2M_{min} \tag{6.2}$$

where (see Appendix B):

- ✓ M^* is the optimal cash balance.
- ✓ M_{min} is the cash lower limit (given by the decision maker).
- ✓ M^*_{max} is the optimal cash upper limit.
- ✓ FCT is the fixed cost per transfer. Because transfers are made to and from the investment account to vary the cash balance on the main account, the fixed transfer cost is averaged if they are different.
- ✓ $Var(CCB)$ is the variance of the daily change in the cash balance. The variance of a uniform distribution between x_1 and x_2 is: $(x_2 - x_1)^2/12$.
- ✓ i_I is the type of passive annual interest due to investment (free of risk).
- ✓ $BASE$ is the number of days per year ($BASE = 365$).



✓

✓ Figure 6.9. Behavior of the main office accounts with optimal limits for the main account.

After these calculations, the results shown in Table 6.5 were obtained. With this, Figure 6.9 shows the output of simulation using the criterion of Miller and Orr to set the limits of the cash balance on the main account. This graphic is similar to Figure 6.4, with the difference of the limits for cash balance. This time it has been obtained: total management costs of 550,153.22 euros/year, financial income of 19,369.68 euros/year. Resulting net due to the CCDS management: -530,783.53 euros/year.

In both cases, DISPRONLI has a negative net result. Nevertheless, noting that using the Miller and Orr criterion slightly improves the financial result, the Director of Finance considers this option to analyze a scenario with the investment fund that currently the Executive Director has under negotiation. Table 6.6 shows the limits for the cash balance of the main account taken into account the investment fund. For its part, Figures 6.10 and 6.11 show the behavior of the central accounts in both cases, the intuitive criterion of the Director of Finance and the Miller and Orr criterion. In these graphs, it can notice the consequence of having a higher deposit rate to loan rate. In this scenario from the second semester 2017, the money flows from the credit line toward the investment account through the main account. It may also be noted that the delay for the availability of cash in the main account implies significant instability in its cash balance, which in turn causes an increase of costs due to frequent overdraft. The matter now is whether the new interest rate on the

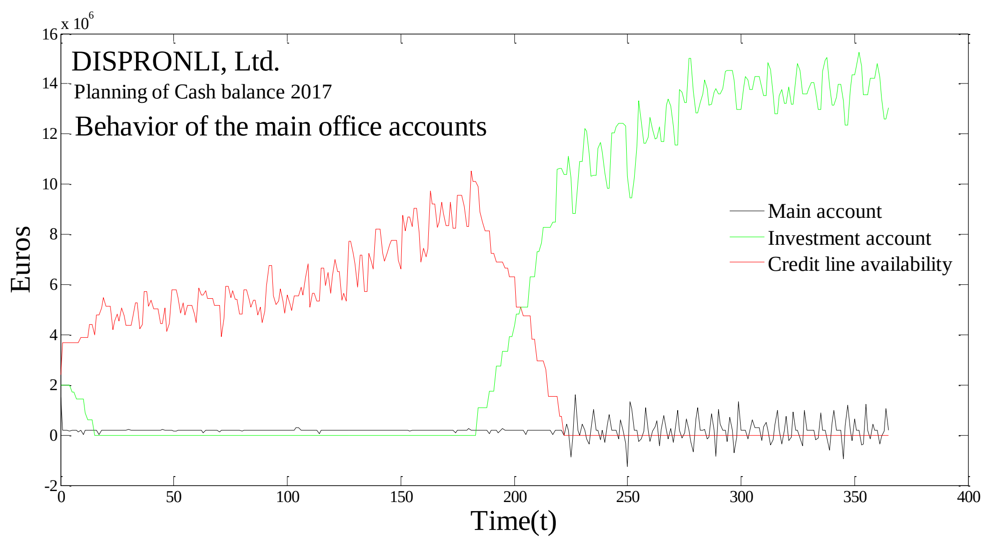


Figure 6.10. Behavior of the main office accounts with intuitive limits for the main account and the new investment fund.

investment fund really offset the rising of overdraft cost. In this regard, Table 6.7 presents a summary of the financial results with the different options. The conclusion is that less financial expenditure is obtained considering the new investment fund combined with the criterion of Miller and Orr bands.

Figures 6.12 to 6.14 refer to the case combining the criterion of Miller and Orr and the investment fund under negotiation, which starts operating in the second half of 2017. These graphics show, respectively: (a) the behavior of transfer costs and financing (interest payments), as well as the interest income (interest earned), (b) the evolution of the accumulated costs as well as interest income, (c) the comparison of the total management costs of CCDS, financial income and the net result.

	From April to September	From January to March and October to December
Daily change of cash balance		
From:	41.095,80	4.931,50
To:	68.493,10	8.219,17
Mean:	54.794,45	6.575,33
Variance:	62.551.003,94	900.734,46
Fixed cost per transfer.	125,00	125,00
Type of liable annual interest	0,025	0,025
Cash balance		
Lower limit:	10.000,00	10.000,00
Optimal:	54.074,37	20.722,77
Upper limit:	142.223,12	42.168,31

Table 6.5. Balances optimal reference in the main account under the original conditions assumed for simulation.

	From July to September	From October to December
Daily change of cash balance (mean)		
From:	41.095,80	4.931,50
To:	68.493,10	8.219,17
Mean:	54.794,45	6.575,33
Variance:	62.551.003,94	900.734,46
Fixed cost per transfer.	215,00	215,00
Type of liable annual interest	0,085	0,085
Cash balance		
Lower limit:	10.000,00	10.000,00
Optimal:	45.118,50	18.543,91
Upper limit:	115.355,51	35.631,74

Table 6.6. Balances optimal reference in the main account by assuming the new investment fund.

	Criterion:	Intuitive	Miller and Orr
Financial conditions initially foreseen	CCDS total management costs	559.628,28	550.153,22
	Financial income	17.924,67	19.369,68
	Net result	**-541.703,61**	**-530.783,54**
Considering the new fund of investment	CCDS total management costs	1.012.557,12	1.005.482,87
	Financial income	477.264,21	484.685,75
	Net result	**-535.292,91**	**-520.797,12**

Table 6.7. Summary of the financial results of simulation using the original parameters.

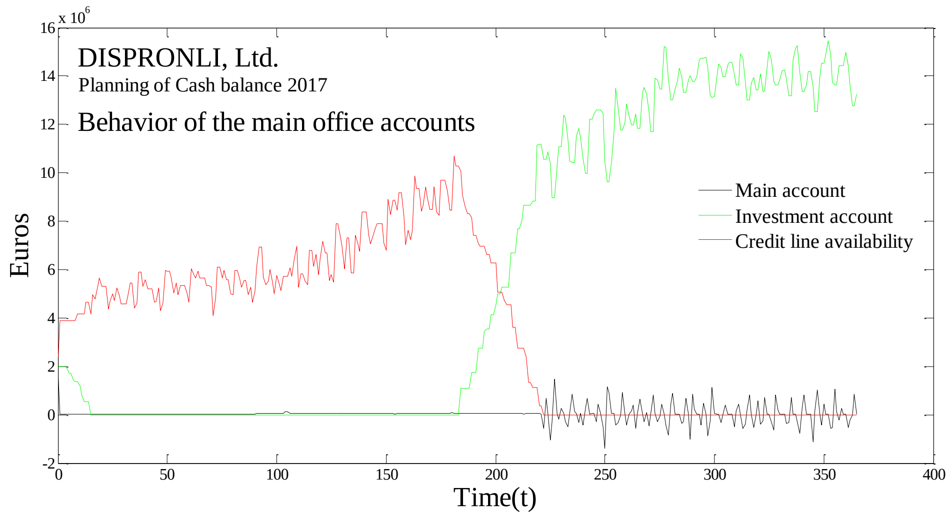


Figure 6.11. Behavior of the main office accounts with optimal limits for the main account and the new investment fund.

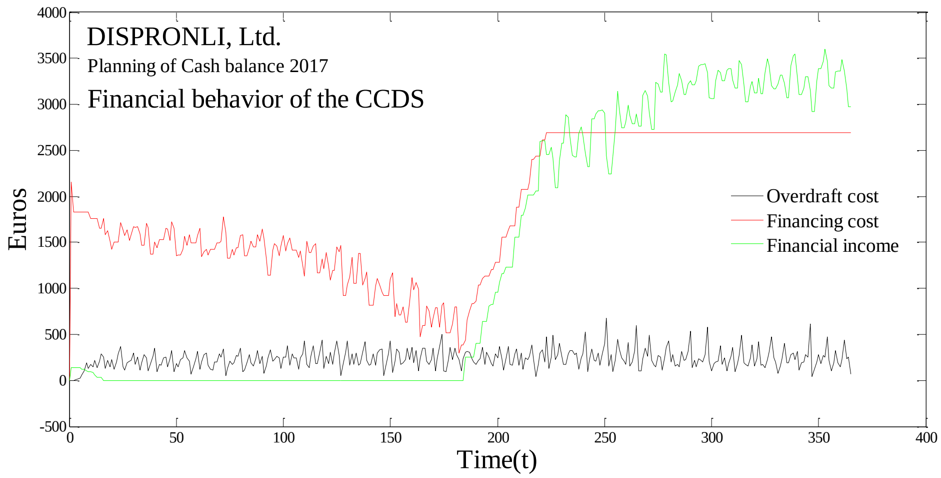


Figure 6.12. Financial behavior of the CCDS with the investment fund and optimal limits in the main account.

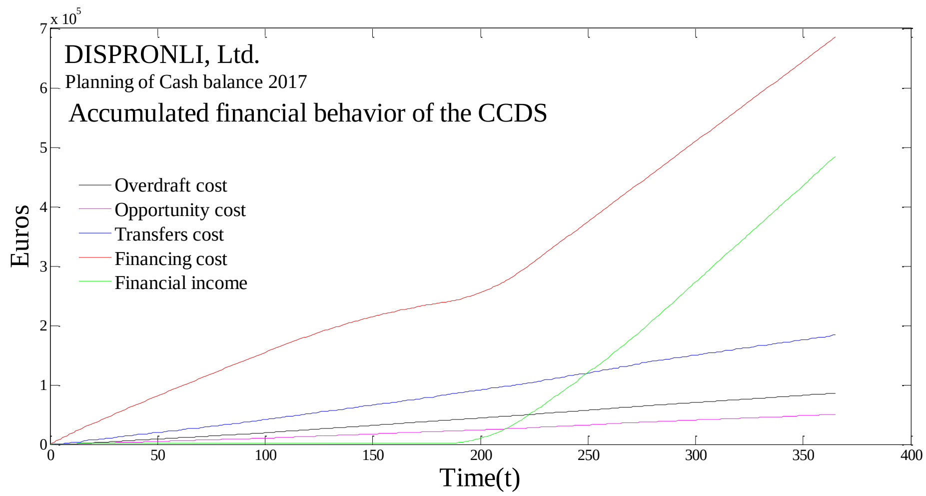


Figure 6.13. Accumulated financial behavior of the CCDS with the investment fund and optimal limits in the main account.

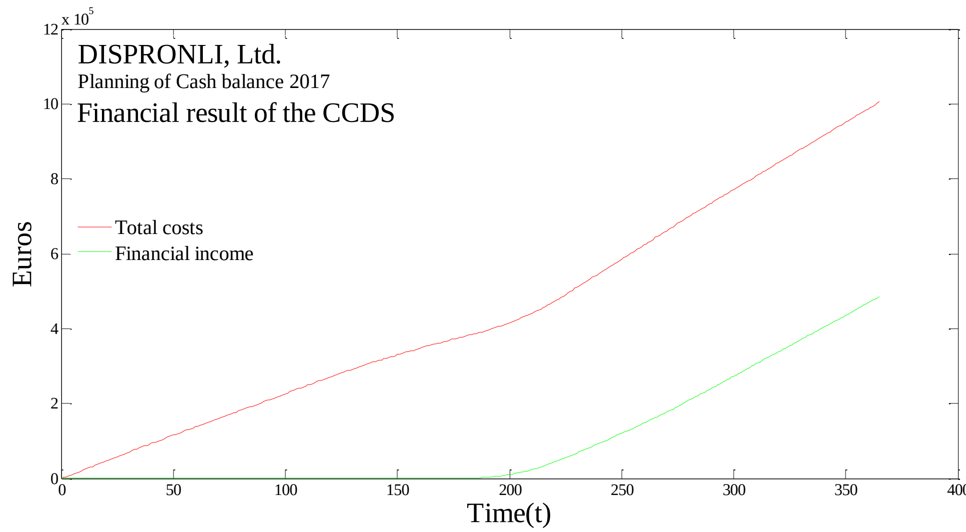


Figure 6.14. Comparing the total costs of CCDS with regard to financial income, considering the investment fund and optimal limits for the main account.

The Director of Finance makes a last run considering the investment fund combined with the optimal cash balance and optimal cash upper limit of Miller and Orr, this time taken into account a decrease of 90% in the fixed cost per transfer (see Figure 6.15). Table 6.8 shows a summary of the results obtained for the different options in this scenario. If the Executive Director is successful in negotiations on fixed costs per transfer, it can achieve an overall savings of up to 46%, which is important in this type of financial systems.

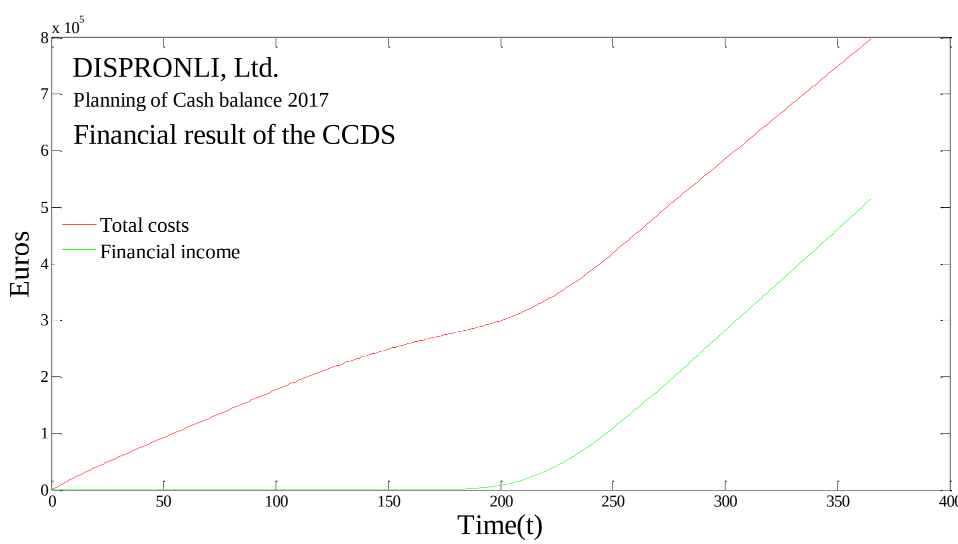


Figure 6.15. Comparing the total costs of CCDS with regard to financial income considering 90% off in cost per transfer.

Criterion:		Intuitive	Miller and Orr
Financial conditions initially foreseen	CCDS total management costs	393.412,35	382.219,83
	Financial income	31.644,77	33.255,94
	Net result	**-361.767,58**	**-348.963,89**
Considering the new fund of investment	CCDS total management costs	807.653,12	797.895,70
	Financial income	506.666,02	515.575,19
	Net result	**-300.987,10**	**-282.320,51**

Table 6.8. Summary of the financial results of simulation with 90% off in cost per transfer.

6.3 Discussion of results and recommendations

The graphs show that the determinants on the result are the financing costs, the costs per transfers and the costs of overdraft. This means that the firm's management should give priority to its analysis, in that order. Terms of negotiating must be found through which these costs are substantially reduced, combining this with increased financial income. In particular, the cost per transfer is determined by the company structure, forcing it to hold accounts in different banks. The option to unify the entire system in a single bank must also be evaluated with priority. Moreover, management of the firm should decide between getting higher gain on investments or greater stability in the cash balance. An alternative is to find an option in which both aspects are improved simultaneously. This means getting a better passive interest rate coupled with the least possible delay for transfers between the investment account and the main account of CCDS. In this regard, it can explore the possibility of advancing the deal with the investment fund starting to operate from the beginning of the year. Another aspect to evaluate is to set a technical financial criterion on the capital structure of the company. This means, to consider other financial factors such as long-term debt and its relation to total assets and equity of the company, deciding the best combination. The CCDS is part of an analysis of this type, including the size of the credit line, interest rates, cost structure of the company and, in general, from the outside of the company, the structure of financial markets and the banking system. The types of risk and tax rates must be assessed as an integral part of the financial plan. The criterion of Miller and Orr has been criticized by not considering some characteristics of financial reality of the firm. The Director of Finance should be documented to establish a modern approach in this respect.

In particular, the author points out that the criterion of Miller and Orr (and others mentioned in Chapter 2) does not take into account two important elements. First, the cost per transfer from the main account toward the investment account is not necessarily the same as in reverse. Hence, a criterion for determining the correct parameter should be set. The case study of DISPRONLI has used the average; however, this criterion must be validated. Secondly, the company will eventually be obliged to finance a cash deficit not from the investment account, but from the credit line. Or vice versa, if there is surplus, pay off debt rather than investing money. Hence, in this case, it must also set a criterion for determining the applicable interest rate in a decision involving: stop paying interest versus earning interest and pay interest versus stop earning interest.

6.4 Concluding remarks of the chapter

In literature, there are many authors who excel by propose models for the management of cash balance. It is not the purpose of this case study conduct a comprehensive analysis on the subject, about which Chapter 2 extends widely. Criterion of Miller and Orr has been used as one of the most frequently mentioned. However, the author believes it is enough with this example to test the operation of the CCDS model. Any other approach would mean only a minor adaptation of the system.

The importance of this chapter is that it shows very clearly the versatility of the simulation model of a CCDS to suit different realistic scenarios. Some parameters that originally are assumed constant, the dynamic of test allows handling them as semi variables or piecewise constant. It also opens perspectives if new complexity features are added without losing effectiveness. For instance, it is possible to adapt the model considering the change of some of parameters constant turning them into variables. Of course, this changes the nature of the system and, consequently, a linear model becomes a nonlinear model. The case study on the hypothetical DISPRONLI firm allows focusing the usefulness and potential of the model by using stationary variations within financial planning horizon. That is, the planning horizon is divided into smaller segments within which the model runs under the assumptions originally envisaged. Really, several consecutive simulations were performed. Then, the procedure unifies all model execution as a single simulation. Another aspect that highlights of this case study refers to the existence of a predictive local controller, which operates in a transparent manner with respect to the viewpoint of the decision maker. Moreover, the results are analyzed with the confidence that this controller fully complies its purpose. It is also good to note that the case study focuses on the planning function (future), demonstrating the versatility of the model. However, it can insert the controller and all other devices of the model in a real time system for direct support of company operations and financial decisions day to day. In other words, the model has been used in this case study for analysis, however, in itself can be implemented as an automatic tool for decision making. Therefore, it is evident that the purpose of CCDS model formulated in Chapter 3, which includes the MPC developed in chapters 4 and 5, achieves the purpose of automation. Even more, it faithfully fulfills the general objective of representation of the reality studied.

Chapter 7

General conclusions and future work

In this thesis, the issue of cash management is addressed, framed on the subject of models for Corporate Financial Planning. The study is made focusing on the movement of bank accounts involved in the important financial decisions of a firm. For this purpose, the research has raised as its greatest motivation to achieve an accurate representation of a Cash Concentration and Disbursements System (CCDS) allowing its numerical simulation as well as its analysis and evaluation. The work provides the basis for the subsequent possibility of exploring new research and the development of algorithms for the financial decision support, based on tools of control theory.

Concretely, the thesis has accomplished three things:

- ✓ A novel formulation of a mathematical model for a CCDS to support the financial planning of a company in the short term, which can also be used to support the operations of cash concentration and disbursements of the company in real time. The use of difference equations facilitates the mathematical representation of the CCDS intuitively, which is complemented with its representation through the Z-transform. In this way, the model has been particularly linked to the automatic control subject.
- ✓ A novel formulation of a model predictive control (MPC) to automate the cash transfer decisions by using optimization criteria in managing the revenue and disbursements bank accounts of the agencies. As it has been showed, the MPC is adaptable and versatile, which has been applied on bank accounts with greater movement, that is, accounts related to customers and suppliers.
- ✓ The formulation of a model for the optimal control of bank accounts related to invest cash surplus or fund the cash deficits, keeping the cash balance of the main bank account of the system within a bands scheme previously established.

This combination is structured in a comprehensive model encompassing the entire cash management of the company. Thereby, managers have a tool that provides the needed support to achieve their goals. The complete functioning of the CCDS is demonstrated through a case study (hypothetical but realistic), designed to illustrate about the potentiality and versatility of the model.

With regard to the MPC, a backward version of Dynamic Programming is used for the prediction model under a decentralized approach. The process model is based on a pre-decision state formulation, in which several simplifications have been proposed seeking alleviate some of the known problems when dynamic programming is applied under uncertainty. The approach of Supply Chain Management combined with inventory concepts has served as basic to this formulation. Nevertheless, the MPC offers greater adaptability and versatility when establishing different policies and strategies, by varying the parameters or by adding some complexity elements.

Because the MPC has been developed to automate the cash transfer decisions in the revenue accounts and the disbursements accounts of a CCDS, it is necessary to emphasize the differences between both uses. On the one hand, cash transfer orders in the revenue accounts originate from the agencies, which determines that, from their point of view, the MPC does not consider delay. Moreover, the balances of these accounts are usually positive. On the other hand, cash transfer orders in the disbursements accounts originate from the main office of the company, which determines that, from the point of view of the agency, the MPC should consider the delay. In addition, the balances of these accounts are generally negative, expressing the overdraft in the respective account. Aside from these differences, there are common elements that should be mentioned:

- ✓ In both cases, it has been possible to represent the uncertainty process through an autoregressive moving average model (ARMA) which interprets, on the one hand, the cash collection process and, on the other, the payment process to suppliers of the company.
- ✓ For the revenue forecasting and the disbursements forecasting, the standard exponential smoothing method N-N (no trend, no seasonal) was considered.

- ✓ The reference signal used in both proposals is a sawtooth function, which conveniently adapts to the inventory policy (s, S) that has been used for comparative purposes. However, the predictive control method has the capability to adapt to reference signal variations, which supports what has been said before about its versatility, responding automatically to different scenarios (inputs and reference signals) that the user wishes to incorporate.

The proposal made for disbursement accounts has offered the opportunity to provide two versions of the MPC, which have resulted to be theoretically equivalent, as has been shown in Chapter 5 (Theorem 3).

A special consideration deserves having found an interval for process stability regardless of the prediction's size horizon as has been shown in Chapter 4 (Theorem 1, Corollary 2). Which is accomplished by establishing a band for the uncertainty to narrow the input of the DP model, together with a stabilizing regulator in cascade fashion using a linear feedback gain (closed-loop). Another aspect highlighted is the fact that, in using an objective function based on a finite horizon quadratic criterion, the thesis provides formal arguments to exclude or include the control effort. For this purpose, a detailed study (analytical and graphic) has been made about the behavior of the objective function under different combinations of the discount dynamic parameter in the cost function, as well as the control horizon.

The case study of Chapter 6 shows very clearly the versatility of the simulation model of a CCDS to suit a wide range of realistic financial scenarios. As already mentioned, the model has been made under the assumption of stationary behavior of the uncertainty process. Nevertheless, the case study shows that it is possible focusing the usefulness and potential of the model by using stationary variations within financial planning horizon. That is, the planning horizon is divided into smaller segments within which the model runs under the assumptions originally envisaged. Then, the procedure unifies all model execution as a single simulation. Another aspect that highlights is the transparent manner with which the MPC operates with respect to the viewpoint of the decision maker. Whereby, simulation results are analyzed with the confidence that this controller fully complies its purpose. It is also evident, the usefulness of the model for the short-term financial planning, as well as for the operational role of the CCDS in real time.

From the point of view of the financial manager, the way in which the CCDS simulation model has been formulated offers the opportunity to evaluate the process from various angles. Firstly, analyzing the various cost elements involved in the process, as well as financial income. Secondly addressing the issue of bargaining power of the company, linked to its importance in the financial market. Thirdly, the evaluation of the firm's structure, combined with its strategic capacity within the financial market in which it operates, offers the opportunity not only to assess the financial costs but also the delays that significantly decide the overall financial outcome of the CCDS. Fourthly, whether the model is applied in real time or in financial planning process, it facilitates deciding the appropriate time to make a specific decision. Finally, it would be a topic of interest to assess how the financial risk factor and the tax rates are influencing the system as an integral part of the financial plan.

The case study has provided the possibility of including the analysis on the source of funding for cash deficits or the destination of the surplus funds when determining the optimal balance and the maximum cash balance. This, considering that conventional models evaluate only the need to exchange cash between the main account and investment portfolio.

Despite the objectives achieved in this thesis, the research leaves open a number of issues of interest for future work. Below, a list of topics that could complement the work done is provided.

- ✓ There exist several control techniques and theories with which would be interesting to explore its applicability to the proposal of the thesis. Particularly, due to the model has been expressed in terms of Z-transform.
- ✓ With respect to the formulated MPC-DP, options that use smart scan on the state space or an optimization model min-max based on the worst-case can be considered. It remains the expectation on how add new complexity features for the model without losing effectiveness. A new research should focus on combining the proposal of this

thesis with better methods of prediction uncertainty or its recognition during the delay time.

- ✓ According to literature review, combining simulation methods and new technologies makes the subject of corporate financial planning arouses interest in researchers.
- ✓ There is also the expectation of testing the model in a case study with real data.

Appendix A

Tables of state of the art

This appendix contains several tables completing the result of the bibliographical inspection on modeling corporate financial planning. The research was made with the purpose of to know scientific and academic contributions about this subject during the last sixty years. The information is complemented in Section A.2 with survey papers and books or textbooks, with a view to extend the range of the bibliographical review offered. Finally, in Section A.3 a comprehensive discussion about the state of the art is made.

A.1 Tables completing the literature review

This section includes tables A.1 to A.9 mentioned in Chapter 2.

Researches about financial strategy

Reference	Type of research - Technique / Model / Tool
Welter (1973)	Theoretical: Reasoning; Organizational logic approach / methodological framework.
Duhaime and Thomas (1983)	Empirical: Reasoning; Literature or application review.
Robinson and Pearce (1983)	Empirical: Reasoning; Data analysis (1); Comparative study.
Myers (1984)	Theoretical: Reasoning.
Brooke and Duffy (1986)	Empirical: Data analysis (1).
Rhyne (1986)	Empirical: Data analysis (1).
Sandberg, Lewellen and Stanley (1987)	Empirical: Data analysis (1).
Whipple (1988)	Theoretical: Reasoning.
Pivnicny (1989)	Theoretical: Reasoning; Organizational logic approach / methodological framework.
Boyd (1991)	Empirical: Data analysis (1).
Capon, Farley and Hulbert (1994)	Empirical: Data analysis (1).
Kleinmuntz, Kleinmuntz, Stephen and Nordlund (1999)	Theoretical: Reasoning; Organizational logic approach / methodological framework.
Beech (2001)	Empirical: Data analysis (1).
Mercier (2002)	Theoretical; Empirical: Reasoning; Data analysis (1).
Vlachy (2009)	Theoretical: Reasoning.
Conneely (2011)	Theoretical: Reasoning.

(1) Data analysis involves one of the following: Statistical analysis, Field research, Case study, Database or Collected data

Table A.1. Complete results of the inspection to all publications categorized as financial strategy: Research in which mainly discusses the relationship between corporate strategy and financial planning.

Researches about planning and financial management, modeling for enterprise, systems

Reference	Type of research - Technique / Model / Tool
Research on various aspects of financial management as a framework for determining financial policies and decisions	
Dohrn and Salkin (1969)	Theoretical: Reasoning; Organizational logic approach / methodological framework.
Singhvi (1972)	Theoretical: Reasoning.
Walker and Petty (1978)	Empirical: Data analysis (1); Comparative study.
McInnes and Carleton (1982)	Empirical: Reasoning; Data analysis (1); Comparative study; Literature or application review.
Morgan and Martin (1986)	Theoretical: Reasoning.
Barton and Gordon (1987)	Theoretical: Reasoning; Literature or application review.
Kochhar and Hitt (1998)	Empirical: Data analysis (1).
Zaharia (2009)	Theoretical: Reasoning.
Research using different techniques or tools for attempt to model an enterprise; however, the result highlights the importance of financial planning	
Hamilton and Moses (1974)	Theoretical: Reasoning; Simulation; Organizational logic approach / methodological framework.
Zettergren (1975)	Theoretical: Simulation; Worksheet.
Schendel and Patton (1978)	Theoretical; Empirical: Data analysis (1); Simultaneous equations; Empirical test.
de Kluyver and McNally (1982)	Theoretical; Applicative: Reasoning; Simulation; Organizational logic approach / methodological framework.
Laínez Gadea and Bellostas Pérez-Grueso (1991)	Theoretical: Reasoning; Organizational logic approach / methodological framework.
Patz (1993)	Theoretical: Linear programming.
Five papers showing details of the implementation and using information technology combined with financial planning	
Schmaltz (1980)	Theoretical; Empirical: Reasoning; Data analysis (1).
Finlay (1985)	Theoretical; Empirical: Reasoning; Data analysis (1); Organizational logic approach / methodological framework.
Cowen and Middaugh (1988)	Theoretical; Empirical: Reasoning; Data analysis (1); Organizational logic approach / methodological framework.
Zhu (2006)	Theoretical; Empirical: Reasoning; Organizational logic approach / methodological framework.
Li, Jiang, Chen, Jin and Lin (2012)	Empirical: Reasoning; Data analysis (1).

(1) Data analysis involves one of the following: Statistical analysis, Field research, Case study, Database or Collected data

Table A.2. Complete results of the inspection to all publications categorized as planning and financial management, modeling for enterprise, systems.

Researches about Methodology and preferences

Reference	Type of research - Technique / Model / Tool
Earley and Carleton (1962)	Empirical: Data analysis (1); Comparative study.
Moag, Carleton and Lerner (1967)	Theoretical: Reasoning; Organizational logic approach / methodological framework.
Elliott (1972a)	Empirical: Data analysis (1); Comparative study.
García Bermejo (1984)	Theoretical: Reasoning; Organizational logic approach / methodological framework.
Zuckerman (2000)	Theoretical; Empirical: Reasoning; Data analysis (1).
Kunsch, Chevalier and Brans (2001)	Theoretical; Empirical: Data analysis (1); Comparative study; Organizational logic approach / methodological framework.
Gupta and Li (2003)	Theoretical: Reasoning; Organizational logic approach / methodological framework.
Wang (2011)	Theoretical: Reasoning.

(1) Data analysis involves one of the following: Statistical analysis, Field research, Case study, Database or Collected data

Table A.3. Complete results of the inspection to all publications categorized as methodology and preferences: Researches that deal with financial planning from the point of methodological perspective, organizational and preferences or managerial styles

Theoretical and analytical formulations

Reference	Sub-category	Type of research - Technique / Model / Tool
Teichroew, Robichek and Montalbano (1965)	Cash flow pattern	Theoretical: Analytical formulation.
Lerner and Carleton (1966a)	Financing decisions	Theoretical: Analytical formulation.
Miller and Orr (1966)	Cash management	Theoretical: Analytical formulation.
Carleton and Lerner (1967)	Investment planning	Empirical: Data analysis (1); Theory of the firm.
Inselbag (1973)	Financing decisions	Theoretical: Dynamic theory of the firm.
Hite (1977)	Financial policy / firm's real decisions	Theoretical: Analytical formulation; Theory of the firm.
Taggart (1977)	Financing decisions	Theoretical; Empirical: Data analysis (1); Analytical formulation; Empirical test.
Tapiero and Zuckerman (1980)	Cash management	Theoretical: Analytical formulation.
Barnea, Talmor and Haugen (1987)	Equilibrium analysis	Theoretical: Analytical formulation.
Lewis (1990)	Financial policy	Theoretical: Analytical formulation.

(1) Data analysis involves one of the following: Statistical analysis, Field research, Case study, Database or Collected data

Table A.4. Complete results of the inspection to all researches categorized as theoretical and analytical formulations for corporate financial planning models.

Mathematical programming formulations

Reference	Sub-category	Type of research - Technique / Model / Tool
Charnes, Cooper and Miller (1959)	Capital budgeting models	Theoretical: Linear programming.
Weingartner (1966)	Investment planning	Theoretical: Linear programming; Integer linear programmin.
Ijiri, Levy and Lyon (1963)	Financial planning model	Theoretical; Experimental: Linear programming.
Lintner (1964)	Financial decisions	Theoretical: Dynamic programming.
Baumol and Quandt (1965)	Investment planning	Theoretical: Linear programming.
Robichek, Teichroew and Jones (1965)	Cash management	Theoretical: Linear programming.
Weingartner (1966)	Investment planning	Theoretical: Linear programming.
Chambers (1967)	Financial decisions	Theoretical: Linear programming.
Carleton (1968)	Financial planning model	Theoretical: Dynamic programming.
Verge (1969)	Financing decisions	Theoretical: Dynamic programming; Empirical test.
Carleton (1969)	Capital budgeting models	Theoretical: Linear programming.
Carleton (1970)	Financial planning model	Theoretical: Linear programming.
Krouse (1972)	Financial planning model	Theoretical: Multiple-criteria decision making: Multiple-objective; Multi-stage model; Optimal systems control.
Pogue and Bussard (1972)	Financial planning model	Theoretical: Linear programming.
Osteryoung (1972)	Financial planning model	Theoretical: Reasoning; Linear programming.
Hamilton and Moses (1973)	Financial planning model	Theoretical; Applicative: Data analysis (1); Mixed integer linear problem (MILP); Simulation.
Myers and Pogue (1974)	Financial planning model	Theoretical: Mixed integer linear problem (MILP).
Merville and Tavis (1974)	Financial planning model	Theoretical: Multiple-criteria decision making: Goal programming.

(1) Data analysis involves one of the following: Statistical analysis, Field research, Case study, Database or Collected data

Table A.5. Complete results of the inspection to all researches categorized as mathematical programming formulations for corporate financial planning models.

Mathematical programming formulations

Reference	Sub-category	Type of research - Technique / Model / Tool
Machado and Carleton (1978)	Financial planning model	Theoretical; Experimental: Data analysis (1); Linear programming.
Sealey (1978)	Financial planning model	Theoretical: Linear programming; Multiple-criteria decision making: Goal programming, Multiple-objective; Comparative study.
Buhler and Gehring (1978)	Financial planning model	Theoretical: Linear programming.
Crum, Klingman and Tavis (1979)	Financial planning model	Theoretical: Network programming; Interactive algorithm; Mixed integer linear problem (MILP).
Ashton and Atkins (1979a)	Valuation	Theoretical; Empirical: Data analysis (1); Linear programming; Multiple-criteria decision making: Multiple-objective.
Ashton and Atkins (1979b)	Valuation	Theoretical: Linear programming.
Kvanli (1980)	Financial planning model	Theoretical: Multiple-criteria decision making: Goal programming.
Kallberg, White and Ziemba (1982)	Financial planning model	Theoretical: Data analysis (1); Stochastic linear programming.
Brick, Mellon, Surkis and Mohl (1983)	Financial structure	Theoretical; Experimental: Data analysis (1); Linear programming.
Cook (1984)	Financial planning model	Theoretical; Applicative: Multiple-criteria decision making: Goal programming.
Ashton (1985)	Simulation for financial planning	Theoretical: Multiple-criteria decision making: Goal programming.
Mulvey and Vladimirou (1992)	Financial planning model	Theoretical; Empirical: Data analysis (1); Stochastic dynamic network programming; Multi-stage model; Empirical test.
Goedhart and Spronk (1995a)	Financial planning model	Theoretical: Multiple-criteria decision making: Goal programming; Multiple-criteria decision making: Fractional programming.
Goedhart and Spronk (1995b)	Financial management	Theoretical: Multiple-criteria decision making: Goal programming; Heuristic modeling.
Maranas, Androulakis, Floudas, Berger and Mulvey (1997)	Financial planning model	Theoretical; Empirical: Data analysis (1); Stochastic nonlinear programming; Global optimization; Multi-stage model, Empirical test.
Carino, Myers and Ziemba (1998)	Financial planning model	Theoretical; Experimental; Applicative: Linear programming; Multiple-criteria decision making: Multiple-objective; Simulation.

(1) Data analysis involves one of the following: Statistical analysis, Field research, Case study, Database or Collected data

Table A.5. (Cont.)

Mathematical programming formulations

Reference	Sub-category	Type of research - Technique / Model / Tool
Güven and Kaynarca (1998)	Financial planning model	Theoretical; Experimental: Data analysis (1); Mixed integer linear problem (MILP).
Mulvey and Shetty (2004)	Financial planning model	Theoretical: Nonlinear programming; Stochastic nonlinear programming; Dynamic stochastic control; Multi-stage model.
Xu and Birge (2006)	Valuation	Theoretical: Stochastic nonlinear programming.
Badell, Fernandez, Guillen and Puigjaner (2007)	Valuation	Theoretical; Empirical: Data analysis (1); Mixed integer linear problem (MILP).
Uspuriene, Sakalauskas, Ginevicius, Rutkauskas, Pocs and Stankeviciene (2010)	Financial planning model	Theoretical; Experimental: Stochastic linear programming; Simulation; Monte-Carlo.

(1) Data analysis involves one of the following: Statistical analysis, Field research, Case study, Database or Collected data

Table A.5. (Cont.)

Simulation formulations

Reference	Sub-category	Type of research - Technique / Model / Tool
Mattessich (1961)	Capital budgeting models	Theoretical; Experimental: Simulation; Simultaneous equations.
Kingshott (1968)	Simulation for financial planning	Theoretical: Simulation.
Warren and Shelton (1971)	Simulation for financial planning	Theoretical; Applicative: Simulation; Simultaneous equations.
Elliott (1972b)	Forecasting	Theoretical; Experimental; Empirical: Data analysis (1); Simulation; Simultaneous equations; Comparative study; Empirical test.
Carleton, Dick and Downes (1973)	Financial planning model	Theoretical; Experimental: Analytical formulation; Simulation; Linear programming.
Stone (1973)	Simulation for financial planning	Theoretical: Nonlinear programming; Interactive algorithm; Simulation.
Pappas and Huber (1973)	Cash flow pattern	Theoretical: Simulation; Least squares forecasting model.
Gentry and Pyhrr (1973)	EPS growth model	Theoretical: Analytical formulation; Simulation; Monte-Carlo.
Francis and Rowell (1978)	Financial planning model	Theoretical; Empirical: Data analysis (1); Simultaneous equations; Empirical test.
Gentry (1979)	Financial planning model	Theoretical: Simulation.
Hayen (1983)	Financial planning model	Theoretical: Simulation; Simultaneous equations.
Crum, Klingman and Tavis (1983)	Cash management	Theoretical: Linear programming; Network programming; Simulation.
Moroto Acín and Mascareñas (1986)	Simulation for financial planning	Theoretical: Simulation; Simultaneous equations.
Stahl, Evans, Mollaghasemi, Russell and Biles (1993)	Simulation for financial planning	Theoretical: Simulation.
Martínez Lobato and Ferrando Bolado (1997)	Simulation for financial planning	Theoretical: Simulation; Recursive equations.
Bach, Verbraeck and Hlupic (2003)	Simulation for financial planning	Theoretical: System Dynamics; Simulation.
Xie and Xie (2007)	Forecasting	Theoretical; Applicative: Data analysis (1); System Dynamics; Simulation; Empirical test.

(1) Data analysis involves one of the following: Statistical analysis, Field research, Case study, Database or Collected data

Table A.6. Complete results of the inspection to all researches categorized as Simulation formulations for corporate financial planning models.

Simulation formulations

Reference	Sub-category	Type of research - Technique / Model / Tool
Xie and Xie (2008)	Financial planning model	Theoretical; Applicative: Data analysis (1); System Dynamics; Simulation.
Cai, Chen and Hu (2009)	Cash management	Theoretical; Experimental: System Dynamics; Simulation.
Lin, Cai and Zhou (2011)	Forecasting	Theoretical; Empirical: Data analysis (1); System Dynamics; Simulation.
Gryglewicz (2011)	Financial decisions	Theoretical: Analytical formulation; Simulation; filtering theory.

(1) Data analysis involves one of the following: Statistical analysis, Field research, Case study, Database or Collected data

Table A.6. (Cont.)

New technology formulations

Reference	Sub-category	Type of research - Technique / Model / Tool
Back and Back (1995)	Financial planning model	Theoretical; Experimental: Optimization by trial and error; Fuzzy logic / expert systems; Empirical test.
Martínez Lobato (1999)	Simulation for financial planning	Theoretical: Fuzzy logic / expert systems; Simulation; Recursive equations.
Tarrazo and Gutierrez (2000)	Financial planning model	Theoretical: Linear programming; Fuzzy logic / expert systems; Possibility distribution model.
Zhang, Torra, Narukawa, Valls and Domingo-Ferrer (2006)	Investment planning	Theoretical: Quadratic programming; Principle of scenario aggregation; Fuzzy logic / expert systems; Fuzzy probability model; Possibility distribution model; Genetic programming; Neural networks.
Sun, Xu and Fang (2006)	Simulation for financial planning	Theoretical; Experimental: Quantum-behaved Particle Swarm Optimization (QPSO) Algorithm; Simulation.
Malagoli, Magni and Mastroleo (2007)	Valuation	Theoretical: Fuzzy logic / expert systems.
Kundisch and Dzoienziol (2008)	Financial decisions	Theoretical; Experimental: Principle of scenario aggregation; Simulation.
Terceno and Vigier (2011)	Forecasting	Theoretical: Fuzzy binary equations; Simulation.
Cao and Chen (2012)	Simulation for financial planning	Theoretical; Empirical: Data analysis (1); Simulation; Agent-based simulation model; Empirical test.
Pan (2012)	Financial management	Theoretical; Empirical: Data analysis (1); Grey relational analysis; Clustering technique ; Genetic programming; Back-propagation neural network ; Logistic regression .
Chen (2012)	Financial structure	Theoretical; Empirical: Data analysis (1); Multi-phased and dynamic evaluation model; Self-organizing map; Support vector regression.

(1) Data analysis involves one of the following: Statistical analysis, Field research, Case study, Database or Collected data

Table A.7. Complete results of the inspection to all researches categorized as New technology formulations for corporate financial planning models.

Mathematical control theory / Stochastic control and Other techniques or tools formulations

Reference	Sub-category	Type of research - Technique / Model / Tool
		Mathematical control theory / Stochastic control formulations
Sethi and Thompson (1970)	Cash management	Theoretical: Continuous and discrete maximum principle, Hamiltonian function ; Stochastic optimal control theory
Brennan and Schwartz (1984)	Valuation	Theoretical: Stochastic optimal control theory
Vasconcellos (1988)	Financial control	Theoretical: Stochastic optimal control theory
Liang and Sun (2011)	Financial control	Theoretical: Stochastic optimal control theory.
Cerqueti (2012)	Financial decisions	Theoretical: Dynamic programming; Stochastic optimal control theory.
		Other techniques or tools formulations
Powell and Vergin (1975)	Financing decisions	Theoretical; Experimental: Heuristic modelin.
Reid (2003)	Valuation	Theoretical; Empirical: Data analysis (2); Agency theory; Empirical test.
Siegmann and Lucas (2005)	Financial decisions	Theoretical: Analytical formulation; Multi-stage model.
Schwienbacher (2007)	Financial strategy	Theoretical: Analytical formulation; Agency theory.

(1) Control involves one of the following: Mathematical control theory or Stochastic control

(2) Data analysis involves one of the following: Statistical analysis, Field research, Case study, Database or Collected data

Table A.8. Complete results of the inspection to all researches categorized as Mathematical control theory / Stochastic control and Other techniques or tools formulations for corporate financial planning models.

Financial Planning Applications

Category	Reference
Health organizations	Coleman and Kaminsky (1977); Branson, Helmrath, Reck and Gerhardt (1981); Hopkins, Heath and Levin (1982); Cleverley (1987); Babock (1997).
Public utilities or organizations	Gardiner and Ward (1974); Blanning and Crandall (1978); Rychel (1982); Linke and Whitford (1983); Nolan and Foran (1983); Moreau (1986); Miyajima and Nakai (1986).
Processing enterprises	Tinsley and Hoglund (1973); Orthlieb (1979); Kirca and Koksalan (1996); Satir (2003); Collan (2004).
Educational organizations	Sinuanystern (1984); Mavrotas, Caloghirou and Koune (2005).
Insurers / Leasing company	Gentry (1972); Carino and Ziemba (1998); Folcut, Ciocirlan, Serban and Katalinc (2008).
Other applications	Cook (1984); Aba-Bulgu, Islam, Helander, Xie, Jaio and Tan (2007); Hahn and Kuhn (2012).

Table A.9. Complete results of the inspection to all publications classified within Financial Planning Applications subject.

A.2 Books, textbooks and surveys

This section complements the provided information with survey papers and books or textbooks addressing the corporate financial planning issue, directly or indirectly. On the one side, this revision extends the scope of the literature review here offered. On the other, it provides an opportunity to compare other research whose purpose might coincide with this literature review.

It has been wanted to present a categorized list of published books and textbooks that address the issue of the formulation of models for corporate financial planning, explicitly or implicitly. Table A.10 shows five categories used in which such books and textbooks are classified. Special reference is made to productive work that Richard A. Brealey and Stewart C. Myers have done, along with other authors. They have published several books about corporate finance linked to financial planning, which have been useful for academics and practitioners for many years. See e.g. Brealey and Myers (1988), Brealey, Myers, and Marcus (2007), Brealey, Myers and Allen (2006).

Books and textbooks related to corporate financial planning

Category	Reference
Models in business accounting	Mattessich (1964a); Mattessich (1964b).
Financial management	Dauten (1956); Kent (1960); Brandt (1965); Robichek and Myers (1965); Cooke and Bomeli (1967); Engler (1975), Brealey and Myers (1988), Brealey, Myers and Allen (2006), Brealey, Myers, and Marcus (2007).
Theory of finance	Markowitz (1959); Gordon (1962); Weingartner (1963); Vickers (1968); Haley and Schall (1973); Crum and Derkinderen (1981).
Financial analysis, planning and Control	Tse (1960); Walker and Baughn (1961); Lerner and Carleton (1966b); Wilson (1974); Chambers and Lacey (1994).
Model Formulation	Hallsten (1966); Bhaskar (1978); Grinyer and Wooller (1978); Bryant (1982); van der Hoek and Elliott (2006).

Table A.10. Complete results of the inspection to books and textbooks that, explicitly or implicitly, address the issue of corporate financial planning.

With respect to the survey papers, it has been wanted to include a rich set of references whose purpose is equivalent to this bibliographical research, which is explore the uses, tools, techniques, approaches and facets associated to corporate financial planning, in different kinds of organizations or researches. However, this work differs enough of these survey papers. Firstly, it provides an historical perspective on the development of models for corporate financial planning, which is circumscribed within the last sixty years, from its origins to the present. In this context, following a uniform criterion, a large number of publications have been selected and categorized. This is, the selection does not focus on the experience about creating or using models for corporate financial planning in a specific economic sector, nor does about a particular technique, tool, region, country or epoch. The study tries to equitably encompass all the research that exists on corporate financial planning in the last sixty years. Secondly, the work discriminates according to the approach used in each research (theoretical or empirical) and makes a distinction between theorizing, modeling and application of models. It also appreciates the experimental and demonstrative sense of the proposals considered. In this research, it has been recognized the breadth of the topic of corporate financial planning and its close relationship with other subjects.

Surveys or review related to corporate financial planning

Category	Reference
General classification of corporate financial planning and linked topics	Clowes and Marshall (1972); [Anonymous] (1982); Shim and McGlade (1984); Brealey and Edwards (1991); Clarke and Tobias (1995); Domenech, Blandon and Sales (2009); Sytnyk (2012).
Functional or aplicative classification	Grinyer and Wooller (1975); Naylor and Gattis (1976); Naylor and Schauland (1976a) ; Coleman, Kaminsky and McGee (1978); Coleman, Kaminsky and McGee (1980); Kerr (1981); Grinyer (1983); Lee and Junkus (1983); Kumar and Vrat (1989); Harris and Raviv (1991); Bellovary, Giacomino and Akers (2007); Denis (2011).
Operational research	Gershefski (1970); Ashford, Berry and Dyson (1988); Batson (1989); Lin and O'leary (1993); Zopounidis (1999); Mulvey (2001); Steuer and Na (2003).
Simulation	Grinyer (1973); Grinyer and Batt (1974); Naylor and Schauland (1976b).
Forecasting	Francis (1983); Ramnath, Rock and Shane (2008).
Continuous-time methods	Sundaresan (2000).

Table A.11. Complete results of the inspection to references whose purpose is explore the uses, tools, techniques, approaches and facets of corporate financial planning, in different kinds of organizations or researches.

Table A.11 presents six categories in which these surveys are classified, as a complement to the original objective. It can be seen in the table that some of the authors devote their research to a specific topic, e.g. Mulvey (2001) and Steuer and Na (2003) on operations research, Grinyer (1973) about simulation, Ramnath, Rock and Shane (2008) about prognosis. In other cases, the authors have wanted to make a general review, however, present their work from a particular approach, e.g. Clowes and Marshall (1972) from a viewpoint of control engineering and systems. Brealey and Edwards (1991) provide a historical review of the topic of finances in general since the eighteenth century, but their study requires an update, mainly due to the theoretical and technological advances of today. The approach in this thesis attempts to be comprehensive, covering all contributions on the subject of corporate financial planning, from the conceptual and technological perspective.

A.3 Comprehensive discussion about the state of the art

By studying the number of publications in each decade, according to the different topics covered, allows observing that theorizing has a peak in the 60s, as well as models for business accounting. While modeling for corporate financial planning has the peak in the 70s and researches about strategies and application of models for corporate financial planning have the peak in the 80s. Production of survey papers peaks in the 80s, which could have an explanation equivalent to the case of application of models: surveys and applications occur after the models have been formulated. All the above suggests a logical sequence, which shows that academic and scientific production occurs in three waves: creating theories, creating models, applying models for different economic sectors. In any case, there is a significant drop of academic production after these waves, which resurfaces again in the 2000s. As it can be observed, this occurs due to the emergence of new technologies.

On the other hand, the number of theoretical models for corporate financial planning based on mathematical programming and simulation determine the global behavior, each with its peak in the 70s. Moreover, there exists a possible link between the emergence of new technologies and simulation. These two approaches are complementary and usually employed together. Probably, this is the reason why resurges the use of models for corporate financial planning in the 2000s. The application of new technologies to formulate corporate financial planning models is a promising field to develop new research. Likewise, it calls attention the limited use of the concepts of control and other tools and techniques in this area, which also could open up possibilities for starting new research projects.

There are three determining factors when designing a model for corporate financial planning, which are uncertainty, the time variable and the planning horizon. Firstly, most of the inspected corporate financial planning models incorporate at least one exogenous variable under uncertainty. A little less, corresponds to models under certainty and some proposals raise a discussion of how their model can be adjusted when including, or when excluding uncertainty conditions. The time variable is largely treated as a discrete variable, regardless of whether other variables are considered continuous in the model. The relationship is nearly nine to one, the number of models that use discrete time variable with respect to those using continuous time variable. The planning horizon may be short, medium or long, short term generally corresponds to financial planning models for cash management. Table A.12 shows a summary about the incidence in percentages of models that use uncertainty concept, time variable and planning horizon.

Uncertainty / Certainty - Time – Term

Percentage of research with models under Uncertainty / Certainty:				
Uncertainty	Certainty	Certainty / Uncertainty		
70,0%	26,0%	4,0%		
Percentage of research by use of the time variable:				
Discrete	Continuous			
89,0%	11,0%			
Percentage of research by the planning horizon:				
Short	Short / Mid	Short / Mid / Long	Mid / Long	Long
28,0%	1,0%	14,0%	4,0%	53,0%

Table A.12. Percentage of models that use uncertainty condition, time variable and planning horizon.

The literature review helps arousing interest in the emergence of some research lines, placing at center of them the formulation of models for corporate financial planning both short and long term. It has been seen that there is some research on the control subject. However, it is possible to widen by combining several techniques to propose models and tools for corporate financial planning viewed as a controlled system, excluding or including optimization criteria. The scope is higher due to the variety of methods and theories of control (stochastic optimal control theory, basic control or various forms of advanced control). In addition, there are research opportunities if new technologies are used with the purposes of diagnosis, prediction, identification or representation, by leveraging the optimization analytical power when joining the simulation capabilities. This does not exclude the possibility of modeling using the concept and tools of system dynamics. The outlook could be broadened by applying these schemes on firms of different size under diverse financial scenarios. Moreover, it suggest the creation of capabilities that enable financial executives dynamically generate their own models of corporate financial planning. Finally, it is attractive the idea to develop options for translation or conversion of control schemes used in simulation models for the purpose of apply them into administrative proceedings, so that the firm may make adjustments in decision-making in real time.

Appendix B

Financial incomes and charges of a CCDS

This appendix presents several expressions for calculating earned interests and financial charges of a CCDS, under the following assumptions: (a) the bank makes debits or credits in the account on a t'ly basis, calculating interest or fees based on the balance at the end of the previous time interval ($t - 1$). (b) In each case, the respective annual interest rate is split according to the number of time intervals t per year ($BASE$), which is used to calculate interest or fees for a single time interval.

Financial income during the time interval t due to investment in the investment account are given by:

$$I_F(t) = y_6^A(t-1)(i_I/BASE) \tag{A.1}$$

where $y_6^A(t-1)$ is the available balance in the investment account at the end of the previous time interval ($t-1$), and i_I is the type of passive annual interest paid by the bank due to investment.

The finance charge due to overdrawn in the disbursements account ($u_{13,j}^p(t)$) has the following expression:

$$u_{13,j}^p(t) = Max\{-y_j^A(t-1)(i_O/BASE); 0\},\ j = 1,2,3,4 \tag{A.2}$$

where $y_j^A(t-1)$ is the available balance in the disbursements account (really overdraft) at the end of previous time interval, and i_O is the annual interest rate charged by the bank due to overdraft.

The central financial charges of the CCDS include: (a) charge due to overdrawn in the main account, (b) active interest due to loan balance (credit line), and (c) charge by cash transfers. The central financial charges of the CCDS are given by:

$$O_F(t) = Max\{-y_5^A(t-1)(i_O/BASE); 0\}$$

$$+Max\{y_7^A(t-1)(i_E/BASE);0\}$$
$$+\sum_{\forall \theta_{a,b}(t)>0} CT(u_{a,b}^p(t)) \quad (A.3)$$

where: $y_5^A(t-1)$ is the available balance in the main account at the end of the previous time interval. $y_7^A(t-1)$ is the total loan balance at the end of the previous time interval based on the agreements on the credit line. i_O is the annual interest rate charged by the bank due to overdraft. i_E is the annual interest rate charged by the bank due to the balance on the credit line. $CT(u_{a,b}^p(t))$ is a linear function of the type:

$$CT\left(u_{a,b}^p(t)\right) = FCT + VCT.u_{a,b}^p(t) \quad (A.4)$$

being FCT a fixed cost per cash transfer, and VCT a variable cost per monetary unit transferred.

If FCT is interpreted as a minimum transfer cost, (A.4) can be rewritten as follows:

$$CT\left(u_{a,b}^p(t)\right) = Max\left(FCT,\ VCT.u_{a,b}^p(t)\right) \quad (A.5)$$

The total cost of CCDS ($CTS(t)$) is completed by adding an overall cost of operation and management of the system, plus the opportunity cost of positive balances in the accounts, which in principle are given only the revenue accounts and the main account:

$$CTS(t) = O_F(t) + \left\{GC + i_I \sum_{\forall j\neq 6,7} Max\left[y_j^A(t-1);0\right]\right\}/BASE \quad (A.6)$$

where: GC is the annual fixed cost of operation and management of the system (in current monetary units), and $y_j^A(t-1)$ is the available balance in the j account at the end of the previous time interval.

Appendix C

Reviewing the behavior of the objective function

This appendix presents a reviewing of the objective function behavior for the proposed model predictive control, by studying the interaction between the squared deviations and the control effort.

Regarding the control effort, the characteristics of the u^p series have been considered, with which $\Delta u^p \in \{-y^C, 0, y^C\}$ and limits for y^C $(\in [\underline{y}^C, \overline{y}^C])$, as it was proposed in Subsection 4.2.4. If the cyclic behavior (with period T) of u^p series is also considered, and t $(= 0,1,\dots,T-1)$ is assumed as the time instant at which the first cash transfer is performed, (4.24) can be rewritten as follows, after removing zeros in the Δu^p series:

$$\left(\underline{y}^C\right)^2 S_u \leq J_u \leq \left(\overline{y}^C\right)^2 S_u, \tag{C.1}$$

where $\left(\underline{y}^C\right)^2$ and $\left(\overline{y}^C\right)^2$ may be considered as the minimum amplitude and maximum amplitude of the curve, respectively, and:

$$S_u = \sum_{j=0}^{N_1} {\alpha_u}^{N-N_{i1}-jT} + \sum_{j=0}^{N_2} {\alpha_u}^{N-1-t-jT}, \tag{C.2}$$

$$N_1 = (N_{f1} - N_{i1})/T, \qquad N_{i1} = \lfloor (T-t)/T \rfloor T + t, \tag{C.3}$$

$$N_{f1} = \lfloor (N-t)/T \rfloor T + t \qquad \text{and} \qquad N_2 = \lfloor (N-t-1)/T \rfloor. \tag{C.4}$$

Then, it is obtained that:

$$S_u = \frac{{\alpha_u}^{N-t}}{1-{\alpha_u}^{-T}} \left({\alpha_u}^{t-N_{i1}} - {\alpha_u}^{t-N_{f1}-T} + {\alpha_u}^{-1} - {\alpha_u}^{-(N_2+1)T-1}\right), \tag{C.5}$$

which helps to understand the J_u behavior, whether for values of α_u less than one as for values of it greater than one.

In regards to the squared deviations, (4.23) can be particularized by making the amplitude $A_y = T.\Delta\overline{y}^A$ and $\phi = (T - t)/T$, being:

$$\Delta\overline{y}^A = \max\left[\left|\underline{r}^a - E\left(r^a(t)\right)\right|, \left|\overline{r}^a - E\left(r^a(t)\right)\right|\right] \text{ (The greatest variation)} \quad \text{(C.6)}$$

This is:

$$0 \leq J_y \leq \left(T.\Delta\overline{y}^A\right)^2 S_y \quad \text{(C.7)}$$

wherein:

$$S_y = \sum_{k=0}^{N} \left[\alpha_y{}^{N-k} frac^2\left(\frac{k}{T} + \phi\right)\right] \quad \text{(C.8)}$$

Thereby, it is possible to make a comparative graphical analysis about the behavior of both terms in (4.22), the squared deviations and the control effort, by observing the interaction between them to achieve the overall result of the objective function. Figures C.1 to C.6 show the behavior in each case. They are surface plots of S_y and S_u or J_y and J_u, by varying with respect to size of the prediction horizon (N) and the weighting values (α_y and α_u). Figures C.1 and C.2 are included to show the shape of the curves. They correspond to weighting values less than one. Particularly, for α_y and α_u values close to zero, observe the periodic behavior (with period T). Figure C.3 compares the behavior of S_y and S_u for $\alpha_y, \alpha_u < 1$, as if $T.\Delta y^A = 1$ and $\overline{y}^C = 1$. With this graph, it can be seen a potential interaction between the squared deviations and the control effort. In this case, that gives rise to smoothed control, the interaction between the squared deviations and the control effort is determined by the relationship between their respective factors $[\hat{y}^A(t+j|t) - w(t+j)]^2$ and $[\Delta u^p(t+j-1|t)]^2$. Indeed, Figure C.4 shows that by applying a realistic situation ($T.\Delta\overline{y}^A > 1$ and $\overline{y}^C > 1$, satisfying (16) and (17)), the J_u exponential growth is higher than J_y when their respective weightings or N, or both, grow. This is reasonable, as long as it complies $\left(\overline{y}^C\right)^2 > \left(T.\Delta\overline{y}^A\right)^2$ and $0 \leq \left(\underline{y}^C\right)^2$, which is necessarily true.

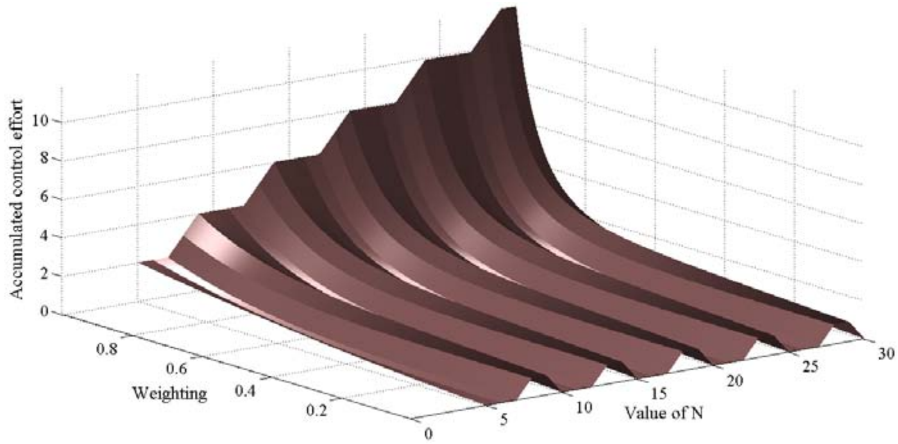


Figure C.1: Behavior of S_u for α_u values less than one.

Figures C.5 and C.6 show the behavior of both terms (squared deviations and control effort) when $\alpha_y, \alpha_u > 1$, which is of interest in this thesis, due to the proposed MPC requires faster control. Figure C.5 shows the difference of S_y and S_u (as if $T.\Delta\overline{y}^A = 1$ and $\overline{y}^C = 1$), where is clearly seen that, S_y is greater than S_u when their respective weightings or N, or both, grow. Nonetheless, the difference is reversed if $T.\Delta\overline{y}^A > 1$ and $\overline{y}^C > 1$. That is, J_u growth is greater than J_y under the same conditions (Figure C.6). Which means that the relationship between the factors $[\hat{y}^A(t+j|t) - w(t+j)]^2$ and $[\Delta u^p(t+j-1|t)]^2$ is decisive in the outcome. However, a suitable combination of α_y and α_u helps when setting the parameters of the objective function of the proposed model. In effect, the analysis presented is an important tool for that purpose, including add or not the control effort term.

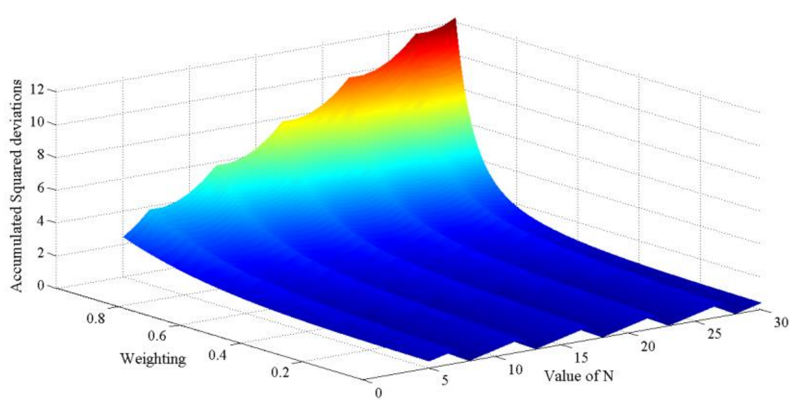


Figure C.2: Behavior of S_y for α_y values less than one.

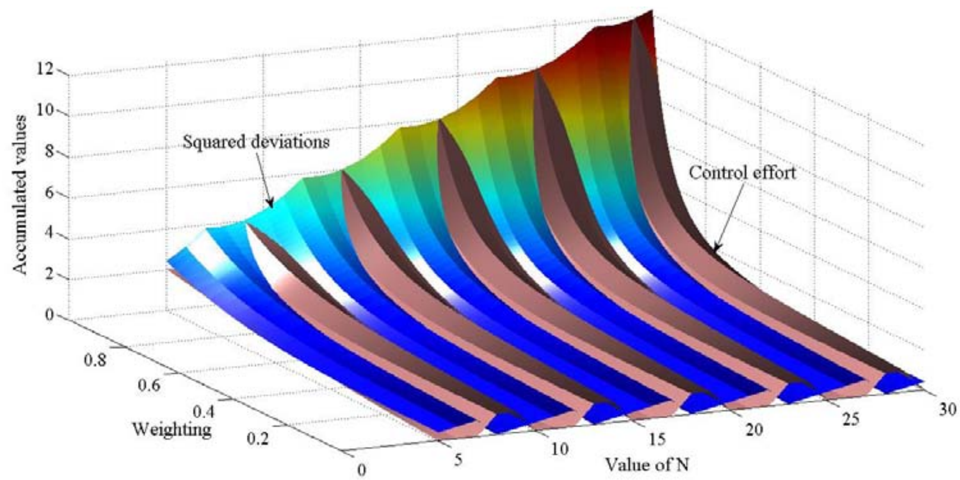


Figure C.3: Comparing squared deviations and control effort (values α_y and α_u less than one). Here, the behavior of S_y and S_u.

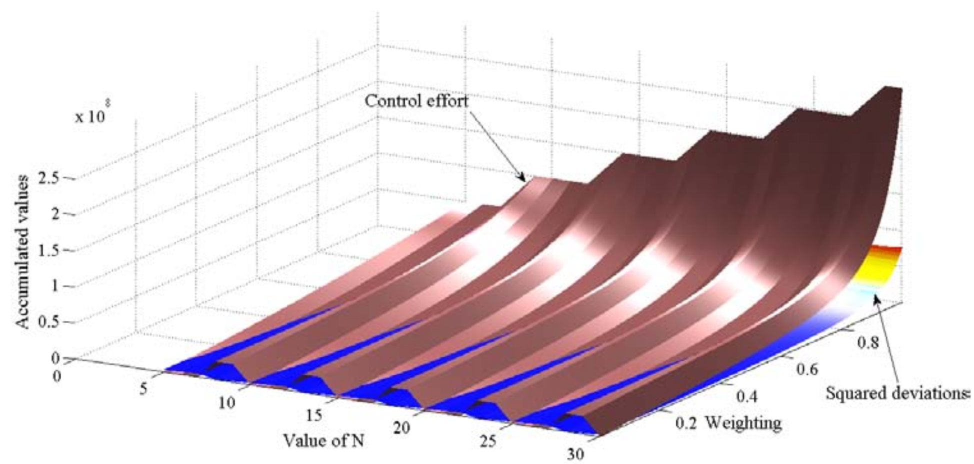


Figure C.4: Comparing squared deviations and control effort (values α_y and α_u less than one). Here, the behavior of J_y and J_u.

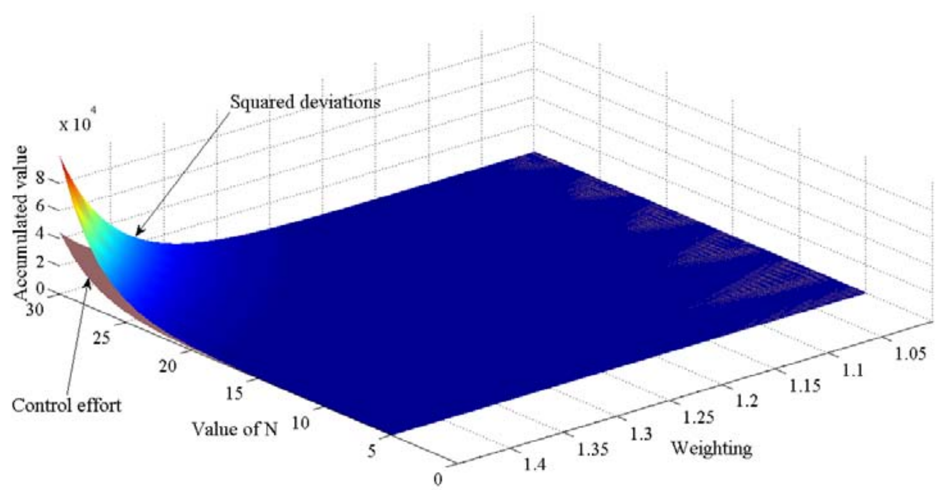


Figure C.5: Comparing squared deviations and control effort (values α_y and α_u greater than one). Here, the behavior of S_y and S_u.

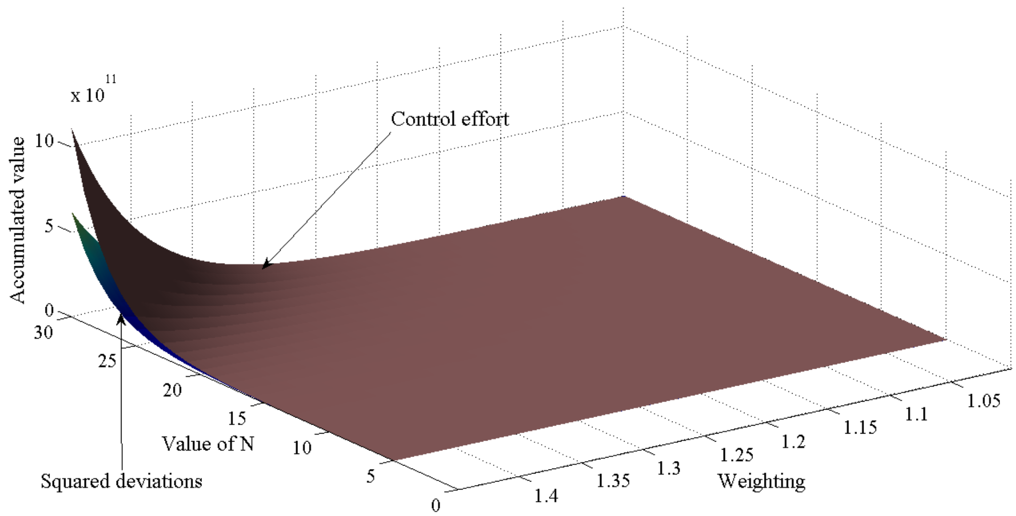


Figure C.6: Comparing squared deviations and control effort (values α_y and α_u greater than one). Here, the behavior of J_y and J_u.

Appendix D

Algorithm of the prediction model

This appendix presents a segment of the algorithm version based on Matlab®, in which limits are set only for disturbance.

Segment of the simulation main algorithm:

Description of variables and their corresponding equivalence in mathematical notation:

- ✓ $t \equiv t$: In the difference equations model, time sequence
- ✓ $ra(t) \equiv r^a(t), R^a(z)$: Available money in the revenue account at time t
- ✓ $up(t) \equiv u^p(t), U^p(z)$: Control action at time t
- ✓ $ya(t) \equiv y^A(t), Y^A(z)$: Available cash balance at the end of t after applying the cash transfer policy
- ✓ $yc(t) \equiv y^C(t), Y^C(z)$: Available cash balance at the end of t before assessing any cash transfer policy
- ✓ $Alpha \equiv \alpha$: In the revenue forecasting, smoothing parameter for the level of the series
- ✓ $rmax \equiv r_{max}$: Upper limit for the uncertainty
- ✓ $rmin \equiv r_{min}$: Lower limit for the uncertainty
- ✓ $N \equiv N$: Prediction horizon
- ✓ $n \equiv$ n: In the revenue forecasting, sequence of observed values
- ✓ $nh \equiv \hat{n}$: In the revenue forecasting, sequence of expected values of data
- ✓ $Enh \equiv E\left(r^a(t)\right)$: Expected value of available revenues
- ✓ $Phi \equiv \phi$: Phase of wave period
- ✓ $s \equiv s$: Amount of cash over which the agency should make the transfer based on the optimal (s, S)-policy
- ✓ $T \equiv T$: Wave period

- ✓ $cht \equiv \hat{c}(t+k|t)$: Minimum accumulated cost (in this case with $k = 0$, $\hat{c}(t|t)$)
- ✓ $vpt \equiv v^p(t+k|t)$: Reference signal (auxiliary variable) for the inner loop controller
- ✓ $Delta \equiv \delta$: Sequence of discount factors handling the tradeoff between the immediate and delayed costs
- ✓ K ≡ K: Linear feedback gain stabilizing the system

```
%
yc(t) = ya(t − 1) + ra(t);
[nh] = Rev_Forecast (N, ra(t), Alpha, rmin, rmax);
Enh = mean(ra);
T = ceil(s/Enh);
[vpt, cht] = DP_Model (yc(t), nh, Enh, T, t, 0, N, Delta, K, Phi);
up(t) = K ∗ yc(t) + vpt;
ya(t) = yc(t) − up(t);
%
```

Rev_Forecast function:

```
%
function [nh] = Rev_Forecast (N, rat, Alpha, dmin, dmax);
        n = rmin + (rmax − rmin) ∗ rand(1, N);
        nh_aux(1) = rat;
        for j = 1: N
                nh(j) = Alpha ∗ n(j) + (1 − Alpha) ∗ nh_aux(j);
                nh_aux(j + 1) = nh(j);
        end;
end
%
```

DP_Model function:

Description of new or local variables and their corresponding equivalence in mathematical notation:

- ✓ $yct, Yyct, Nyct \equiv \hat{y}^C(t+k|t)$: State variable value at kth stage. Notation includes the cases when the cash transfer at stage k is made or not made
- ✓ $k \equiv k$: Subscript pointing kth stage
- ✓ $w \equiv w(t), w(t+k)$: Value of reference signal
- ✓ $Delta(k+1) \equiv \delta(k)$: Discount factors at kth stage handling the tradeoff between the immediate and delayed costs. The subscript position is shifted due to its use in Matlab®
- ✓ $YCTran, NCTran \equiv J_k$: kth term of the cost function. Notation includes the cases when the cash transfer at stage k is made or not made

- ✓ $NewYyct, NewNyct \equiv \hat{y}^C(t+k+1|t)$: State variable value for the next stage ($k+1$). Notation includes the cases when the cash transfer at stage k is made or not made
- ✓ $nh(k+1) \equiv \hat{n}(k)$: Forecasted value of revenue at stage kth. The subscript position is shifted due to its use in Matlab®
- ✓ $vpt, Yvpt, Nvpt \equiv v^p(t+k|t)$: Reference signal value for the inner loop controller. Notation includes the resulting cases at stage $k+1$ when the cash transfer at stage k is made or not made
- ✓ $cht, Ycht, Ncht \equiv \hat{c}(t+k|t)$: Minimum accumulated cost. Notation includes the resulting cases at stage $k+1$ when the cash transfer at stage k is made or not made

```
%
function [vpt, cht] = DP_Model (yct, nh, Enh, T, t, k, N, Delta, K, Phi)
        [w] = SawtoothWave (T * Enh, t + k, T, Phi);
        YCTran = Delta(k + 1) * (− w)^2;
        NCTran = Delta(k + 1) * (yct − w)^2;
        if k == N
                [vpt, cht] = Decision (YCTran, NCTran, (1 − K) * yct, −K * yct);
        else
                NewYyct = nh(k + 1);
                [Yvpt, Ycht] = DP_Model (NewYyct, nh, Enh, T, t, k + 1, N, Delta, K, Phi);
                NewNyct = yct + nh(k + 1);
                [Nvpt, Ncht] = DP_Model (NewNyct, nh, Enh, T, t, k + 1, N, Delta, K, Phi);
                [vpt, cht] = Decision (YCTran + Ycht, NCTran + Ncht, (1 − K) * yct, −K * yct);
        end;
end
%
```

SawtoothWave function:

```
%
function [w] = SawtoothWave (A, t, T, Phi);
        x = Phi + t/T;
        w = A * (x − floor(x));
end
%
```

Decision function:

```
%
function [vpt, cht] = Decision (Ycht, Ncht, Yyct, Nyct)
        if Ycht <= Ncht
                vpt = Yyct;
                cht = Ycht;
        else
                vpt = Nyct;
                cht = Ncht;
        end;
end
```

Appendix E

Publications

This appendix provides a list of publications generated by the thesis.

Conferences

- ✓ Herrera, C. A. and Ibeas, A., 2015. A simulation model for a Cash Concentration and Disbursements System. 23rd. Mediterranean Conference on Control and Automation (MED), Torremolinos, Spain, 935-942.
- ✓ Herrera-Cáceres, C. A. and Ibeas, A., 2016b. Model predictive control for a revenue account of a Cash Concentration and Disbursements System. 24th. Mediterranean Conference on Control and Automation (MED), Athens, Greece, 118-124.

Papers:

Published:

- ✓ Herrera-Cáceres, C. A. and Ibeas, A., 2016a. Mathematical Modeling of a Cash Concentration and Disbursements System. RIAI - Revista Iberoamericana de Automática e Informática Industrial, ISSN: 1697-7912, 13(3):339-349, doi: 10.1016/j.riai.2015.07.008.
- ✓ Herrera-Cáceres, C. A. and Ibeas, A., 2016c. Model Predictive Control of cash balance in a Cash Concentration and Disbursements System. Journal of the Franklin Institute. In press.

In progress:

- ✓ Herrera-Cáceres, C. A. and Ibeas, A., 2016d. Model Predictive Control of the overdraft coverage problem.

References

[Anonymous], 1982, Bibliography on corporate financial models 1964-1980, IEEE Transactions on Power Apparatus and Systems 101(9), 3084-3090.

Aba-Bulgu, M.; Islam, S. M. N.; Helander, M.; Xie, M.; Jaio, M. and Tan, K. C., 2007, Modelling corporate financial crisis management: Optimal cashflow management in SMES, 2007 IEEE International Conference On Industrial Engineering And Engineering Management, Vols 1-4 , 922-925.

Albanese, C., Jackson, K. and Wiberg, P., 2004. A new Fourier transform algorithm for value-at-risk. Quantitative Finance 4(3), 328-338.

Anvari, M. and Goyal, S. K., 1985. Optimization of the decentralized cash management problem. European Journal of Operational Research 20(2), 198-205.

Anvari, M. and Mohan, N., 1980. A computerized cash concentration system. Omega-International Journal of Management Science 8(4), 459-464.

Anvari, M., 1981. An application of inventory theoretical models to cash collection. Decision Sciences 12, 126–135, DOI: 10.1111/j.1540-5915.1981.tb00067.x.

Anvari, M., 1986. Efficient scheduling of cross-border cash transfers, Financial Management 15(2), 40-49.

Anvari, M., 1987. Cash transfer scheduling for concentrating noncentral receipts. Management Science 33(1), 25-38.

Ashford, R. W.; Berry, R. H. and Dyson, R. G., 1988, Operational-research and financial management, European Journal Of Operational Research 36(2), 143-152.

Ashton, D. J. and Atkins, D. R., 1979a, Multicriteria programming for financial-planning, Journal of The Operational Research Society 30(3), 259-270.

Ashton, D. J. and Atkins, D. R., 1979b, Horizon valuations for mathematical-programming in financial-planning, Journal Of The Operational Research Society 30(7), 625-631.

Ashton, D. J., 1985, Goal Programming and Intelligent Financial Simulation Models Part I—Some Problems in Goal Programming, Accounting and Business Research 16(61), 3 - 10.

Babock, C., 1997, Building an effective financial model for strategic planning., Cost & Quality Quarterly Journal : CQ 3(4), 43-5.

Baccarin, S., 2009. Optimal impulse control for a multidimensional cash management system with generalized cost functions. European Journal of Operational Research 196(1), 198-206.

Bach, M. P.; Verbraeck, A. and Hlupic, V., 2003, The small business management flight simulator in an environment of financial indiscipline, Simulation In Industry, 237-246.

Back, B. and Back, R. J. R., 1995, Financial statement planning in the presence of tax constraints, European Journal of Operational Research 85(1), 66-81.

Badell, M.; Fernandez, E.; Guillen, G. and Puigjaner, L., 2007, Empowering financial tradeoff with joint financial and supply chain planning models, Mathematical And Computer Modelling 46(1- 2), 12-23.

Bar-Ilan, A., Perry, D. and Stadje, W., 2004. A generalized impulse control model of cash management. Journal of Economic Dynamics & Control 28(6), 1013-1033.

Barnea, A.; Talmor, E. and Haugen, R. A., 1987, Debt and taxes - a multiperiod investigation, Journal of Banking & Finance 11(1), 79-97.

Barton, S. L. and Gordon, P. J., 1987, Corporate-strategy - useful perspective for the study of capital structure, Academy of Management Review 12(1), 67-75.

Batson, R. G., 1989, Financial-planning using goal programming, Long Range Planning 22(5), 112-120.

Baumol, W. J. and Quandt, R. E., 1965, Investment and discount rates under capital rationing - a programming approach, Economic Journal 75(298), 317-329.

Baumol, W. J., 1952. The transactions demand for cash: an inventory theoretic approach. Journal of Finance, LXVI.

Beech, A. J., 2001, Market-based demand forecasting promotes informed strategic financial planning, Healthcare Financial Management: Journal of The Healthcare Financial Management Association 55(11), 46-54, 56.

Belarbi, K. and Megri, F., 2007. A stable model-based fuzzy predictive control based on fuzzy dynamic programming. IEEE Transactions on Fuzzy Systems, Volume 15, Issue 4, 746-754.

Bellman, R., 1952. On The Theory of Dynamic Programming. Proceedings of the National Academy of Sciences of the United States of America, Volume 38, Issue 8, 716-719.

Bellman, R., 1954a. The Theory of Dynamic Programming. Bulletin of the American Mathematical Society, Volume 60, Issue 6, 503-515.

Bellman, R., 1954b. Some Problems in the Theory of Dynamic Programming. Econometrica, Volume 22, Issue 1, 37-48.

Bellman, R., 1966. Dynamic Programming. Science, Volume 153, Issue 3731, 34-&.

Bellovary, J.; Giacomino, D. and Akers, M., 2007, A Review of Bankruptcy Prediction Studies: 1930 to Present, Journal Of Financial Education 33, 1 - 42.

Bemporad, A. and Morari, M., 1999. Robust Model Predictive Control: A survey. Robustness in Identification and Control, Book Series Title: Lecture Notes in Control and Information Sciences, Volume 245, 207-226.

Bemporad, A., 1998. Reducing conservativeness in predictive control of constrained systems with disturbances. Proceedings of the 37th IEEE Conference on Decision and Control, volumes 1-4, 1384-1389.

Bemporad, A., Borrelli, F. and Morari., M., 2003 Min-max control of constrained uncertain discrete-time linear systems. IEEE Transactions on Automatic Control, Volume 48, Issue 9, 1600-1606.

Bensoussan, A., 2011. Dynamic Programming and Inventory Control: Volume 3 Studies in Probability, Optimization and Statistics. IOS press.

Bertsekas, D. P., 1975. Convergence of Discretization Procedures in Dynamic-Programming. IEEE Transactions on Automatic Control, Volume AC20, Issue 3, 415-419.

Bertsekas, D. P., 2005. Dynamic programming and suboptimal control: A survey from ADP to MPC. European Journal of Control, Joint IEEE Conference on Decision Control/European Control Conference, Volume 11, Issue 4-5, 310-334.

Beyer, D. and Sethi, S. P., 1999. The Classical Average-Cost Inventory Models of Iglehart and Veinott-Wagner Revisited. Journal of Optimization Theory and Applications 101(3), 523-555.

Bhaskar, K., 1978, Building financial models-a simulation approach, Associated Business Programmes, London , 345.

Blanning, R. W. and Crandall, R. H., 1978, Budget planning and heuristic models, Urban Systems 3(2-3), 101-116.

Boissard, J., 2012. Applications and uses of digital filters in finance. Master’s thesis, Swiss Federal Institute of Technology (ETHZ).

Bolton, P.; Chen, H. and Wang, N., 2011, A Unified Theory of Tobin's q, Corporate Investment, Financing, and Risk Management, Journal Of Finance 66(5), 1545-1578.

Bone, G. M., 1995. A Novel Iterative Learning Control Formulation of Generalized Predictive Control. Automatica, Volume 31, Issue 10, 1483-1487.

Boyd, B. K., 1991, Strategic-planning and financial performance - a meta-analytic review, Journal Of Management Studies 28(4), 353-374.

Brainard, W. C. and Tobin, J., 1968, Pitfalls in financial model building, American Economic Review 58(2), 99-122.

Brandt, L. K., 1965, Business finance-a management approach, Englewood Cliffs, N.J.: Prentice-Hall, Inc., vi, 633.

Branson, M. H.; Helmrath, T. A.; Reck, R. and Gerhardt, D., 1981, A financial planning model for ambulatory care settings, The Journal of Ambulatory Care Management 4(1), 15-28.

Brealey, R. A. and Edwards, H., 1991, A Bibliography of Finance, Cambridge, Ma: The MIT Press, xli + 822.

Brealey, R. A. and Myers, S. C., 1988, Principios de finanzas corporativas, 2da. Edición., Mcgraw-Hill Interamericana De España, S.A. , xxvi + 949.

Brealey, R. A.; Myers, S. C. and Allen, F., 2006, Principios de Finanzas Corporativas, 8va edición., Madrid: Mcgraw-Hill , 1138.

Brealey, R. A.; Myers, S. C. and Marcus, A. J., 2007, Fundamentos de Finanzas Corporativas, 5ta edición., Mcgraw-Hill / Interamericana De España, S.A. , xxviii + 764.

Brennan, M. J. and Schwartz, E. S., 1984, Optimal financial policy and firm valuation, Journal of Finance 39(3), 593-607.

Brick, I. E.; Mellon, W. G.; Surkis, J. and Mohl, M., 1983, Optimal capital structure - a multi-period programming-model for use in financial-planning, Journal of Banking & Finance 7(1), 45-67.

Brooke, G. M. and Duffy, N. M., 1986, The use of financial modeling in strategic-planning, Information & Management 11(1), 13-24.

Bryant, J. W., 1982, Financial Modelling in Corporate Management, Wiley, Chichester , 470.

Buhler, W. and Gehring, H., 1978, Short-term financial-planning with uncertain receipts and disbursements, European Journal of Operational Research 2(5), 313-326.

Buser, S. A., 1986. Laplace Transforms as Present Value Rules: A Note. The Journal of Finance 41(1), 243-247.

Cagan, P., 1956. The monetary dynamics of hyperinflation. In M. Friedman (Ed.), Studies in the Quantity Theory of Money. Chicago: University of Chicago Press.

Cai, Y. S.; Chen, J. X. and Hu, Z. B., 2009, Controlling the business growth speed from financial angle of view: using cash flow System dynamics model, First International Workshop On Database Technology And Applications, Proceedings, 610-613.

Camacho, E. F. and Bordons, C., 2007. Model predictive control - 2nd ed. Advanced Textbooks in Control and Signal Processing, Springer-Verlag London Limited, xxii + 405.

Cao, Y. and Chen, X. H., 2012, An agent-based simulation model of enterprises financial distress for the enterprise of different life cycle stage, Simulation Modelling Practice And Theory 20(1), 70-88.

Capon, N.; Farley, J. U. and Hulbert, J. M., 1994, Strategic-planning and financial performance - more evidence, Journal of Management Studies 31(1), 105-110.

Cardoso, R. T. N., Takahashi, R. H. C. and Cruz, F. R. B., 2013. Pareto Optimal Solutions for Stochastic Dynamic Programming Problems via Monte Carlo Simulation. Journal of Applied Mathematics, Article number 801734.

Carino, D. R. and Ziemba, W. T., 1998, Formulation of the Russell-Yasuda Kasai financial planning model, Operations Research 46(4), 433-449.

Carino, D. R.; Myers, D. H. and Ziemba, W. T., 1988, Concepts, technical issues, and uses of the Russell-Yasuda Kasai financial planning model, Operations Research 46(4), 450-462.

Carleton, W. T. and Lerner, E. M., 1967, Measuring corporate profit opportunities, Journal Of Financial And Quantitative Analysis 2(3), 225-240.

Carleton, W. T., 1968, A Dynamic Programming Model for Corporate Financial Planning, Paper Given At TIMS/ORSA Joint Meeting, San Francisco.

Carleton, W. T., 1969, Linear programming and capital budgeting models - new interpretation, Journal of Finance 24(5), 825-833.

Carleton, W. T., 1970, Analytical model for long-range financial planning, Journal of Finance 25(2), 291-327.

Carleton, W. T.; Dick, C. L. and Downes, D. H., 1973, Financial policy models - theory and practice, Journal of Financial and Quantitative Analysis 8(5), 691-709.

Carr, P. and Madan, D., 1999. Option valuation using the fast Fourier transform. The Journal of Computational Finance 2(4), 61–73.

Carr, P. and Wu, L. R., 2004. Time-changed Levy processes and option pricing. Journal of Financial Economics 71(1), 113-141.

Cerny, A., 2004. The risk of optimal, continuously rebalanced hedging strategies and its efficient evaluation via Fourier transform. London: Tanaka Business School, Series: Tanaka Business School discussion papers, pp. 1-41, ISSN 1744-6783, http://ssrn.com/abstract=559417.

Cerny, A., 2006. Introduction to Fast Fourier Transform in Finance. Cass Business School Research Paper, pp. 1-29.

Cerny, A., 2009. Mathematical Techniques in Finance: Tools for Incomplete Markets (Second Edition). Princeton University Press, Princeton and Oxford, p.p. xxii + 390.

Cerqueti, R., 2012. Financing policies via stochastic control: a dynamic programming approach. Journal Of Global Optimization 53(3), 539-561.

Chambers, D. R. and Lacey, N. J., 1994, Modern Corporate Finance: Theory and Practice, New York: Harpercollins College Publishers , xxxi + 640.

Chambers, D., 1967, Programming allocation of funds subject to restrictions on reported results, Operational Research Quarterly 18(4), 407-&.

Charnes, A.; Cooper, W. W. and Miller, M. H., 1959, Application of linear-programming to financial budgeting and the costing of funds, Journal of Business 32(1), 20-46.

Chen, M. Y., 2012, Visualization and dynamic evaluation model of corporate financial structure with self-organizing map and support vector regression, Applied Soft Computing 12(8), 2274-2288.

Chisci, L., Rossiter, J. A. and Zappa, G., 2001. Systems with persistent disturbances: predictive control with restricted constraints. Automatica, Volume 37, Issue 7, 1019-1028.

Chourdakis, K., 2005. Option pricing using the fractional FFT. The Journal of Computational Finance 8(2), 1-18.

Christev, A., 2005. The hyperinflation model of money demand: Some new empirical evidence from the 1990s. Centre for Economic Reform and Transformation, pp. 1-34.

Clarke, D. W. and Mohtadi, C., 1989. Properties of Generalized Predictive Control. Journal of International Federation of Automatic Control, Volume 25 Issue 6, Pages 859-875.

Clarke, D. W., Mohtadi, C. and Tuffs, P. S., 1987a. Generalized predictive control—Part I. The basic algorithm. Journal of International Federation of Automatic Control, Volume 23 Issue 2, Pages 137-148.

Clarke, D. W., Mohtadi, C. and Tuffs, P. S., 1987b. Generalized predictive control—Part II. The basic algorithm. Journal of International Federation of Automatic Control, Volume 23 Issue 2, Pages 149-160.

Clarke, S. and Tobias, A. M., 1995, Complexity in corporate modeling - a review, Business History 37(1), 17-44.

Cleverley, W. O., 1987, Strategic financial-planning - a balance-sheet perspective, Hospital & Health Services Administration 32(1), 1 - 20.

Clowes, G. A. and Marshall, S. A., 1972, Corporate financial models - survey, Proceedings Of The Institution Of Electrical Engineers-London 119(3), 369-&.

Coleman, J. R. and Kaminsky, F. C., 1977, A financial planning model for evaluating the economic viability of health maintenance organizations, Inquiry: A Journal Of Medical Care Organization, Provision And Financing 14(2), 176-88.

Coleman, J. R.; Kaminsky, F. C. and McGee, F., 1978, A survey of financial planning models for health care organizations, Journal of Medical Systems 2(1), 59-69.

Coleman, J. R.; Kaminsky, F. C. and McGee, F., 1980, The state of the art of financial modeling, Hospital & Health Services Administration 25(2), 31-55.

Collan, M., 2004, Fuzzy real investment valuation modelo for very large industrial real investment, Proceedings World Automation Congress, 2004, June July 2004, Vol.17, Pp.379-384.

Conneely, J. F., 2011, Strategic Planning and Financial Management, New Directions for Student Services, 2010, (132), P.51-61.

Cook, W. D., 1984, Goal programming and financial-planning models for highway rehabilitation, Journal Of The Operational Research Society 35(3), 217-223.

Cooke, G. W. and Bomeli, E. C., 1967, Business financial management, Boston: Houghton Mifflin Company , xii + 465.

Cowen, S. S. and Middaugh, J. K., 1988, Designing an effective financial-planning and control-system, Long Range Planning 21(6), 83-92.

Crum, R. L. and Derkinderen, F. G. J., 1981, Capital Budgeting Under Contitions of Uncertainty, Martinus Nijhoff Publishing, Boston , 238.

Crum, R. L.; Klingman, D. D. and Tavis, L. A., 1979, Implementation of large-scale financial-planning models - solution efficient transformations, Journal Of Financial And Quantitative Analysis 14(1), 137-152.

Crum, R. L.; Klingman, D. D. and Tavis, L. A., 1983, An operational approach to integrated working capital planning, Journal Of Economics And Business 35(3-4), 343-378.

Cui, X. Y., Gong, L. T., Zhao, X. J. and Zou, H. F., 2013. The Z-transform method for multidimensional dynamic economic systems. Applied Economics Letters 20(11), 1081-1088.

Dasgupta, S. and Sengupta, K., 2007, Corporate liquidity, investment and financial constraints: Implications from a multi-period model, Journal Of Financial Intermediation 16(2), 151-174.

Dauten, C. A., 1956, Business Finance: The Fundamentals of Financial Management - 2d ed., Englewood Cliffs, N.J.: Prentice-Hall , viii+530.

De Keyser, R. and Ionescu, C., 2003. The Disturbance Model in Model Based Predictive Control, IEEE Conference on Control Applications (CCA'2003), Istanbul, Turkey, CD paper CF-001472.

De Keyser, R., 2003. A 'Gent'le Approach to Predictive Control, "UNESCO Encyclopaedia of Life Support Systems (EoLSS)". Eolss Publishers Co Ltd, Oxford.

de Kluyver, C. A. and McNally, G. M. , 1982, Developing a corporate planning model for a small company, Long Range Planning, 1982, Vol.15(1), Pp.97-106.

Dempster, M. A. H. and Hong, S. S. G., 2002. Spread option valuation and the fast Fourier transform. Mathematical Finance - Bachelier Congress 2000, Springer Finance, pp. 203-220.

Demski, J. S., 1967, Accounting system structured on a linear programming model, Accounting Review 42(4), 701-712.

Denis, D. J., 2011, Financial flexibility and corporate liquidity, Journal of Corporate Finance 17(3), 667-674.

Dennis, S. B., 2009. Matrix mathematics: Theory, facts and formulas (Second Edition). Princeton University Press, Princeton, NJ, p.p. xlii+1139.

Detemple, J.; Gottardi, P. and Polemarchakis, H. M., 1995, The relevance of financial policy, European Economic Review 39(6), 1133-1154.

Diehl, M. and Bjornberg, J., 2004. Robust dynamic programming for min-max model predictive control of constrained uncertain systems. IEEE Transactions on Automatic Control, Volume 49, Issue 12, 2253-2257.

Dohrn, P. J. and Salkin, G. R., 1969, Use of financial models in long range planning, Long Range Planning 2(2), 27-31.

Domenech, M.C.I.; Blandon, J. G. and Sales, R. U., 2009, Analysis of scientific production in the Economics of Finance: 1995-2006, Revista Española De Documentación Científica 32(2), 83-104.

Dong, H., Zheng, H. and Li, Y. P., 2011. Model Predictive Control for inventory Management in Supply Chain Planning. International Conference on Advanced in Control Engineering and Information Science, CEIS 2011, Book Series Title: Procedia Engineering, Volume 15.

Duffy, D. J., 2006. Finite difference methods in financial engineering: A Partial differential equation approach. John Wiley & Sons, Ltd, 442 pages, ISBN: 978-0-470-85882-0.

Duhaime, I. M. and Thomas, H., 1983, Financial analysis and strategic management, Journal of Economics and Business 35(3-4), 413-440.
Earley, J. S. and Carleton, W. T., 1962, Budgeting and the theory of the firm - new findings, Journal of Industrial Economics 10(3), 165-173.
Elliott, J. W., 1972a, Control, size, growth and financial performance in firm, Journal of Financial and Quantitative Analysis 7(1), 1309-1320.
Elliott, J. W., 1972b, Forecasting and analysis of corporate financial performance with an econometric model of firm, Journal of Financial and Quantitative Analysis 7(2), 1499-1526.
Engler, G. N., 1975, Business financial management, Dallas, Texas: Business Publications, Inc. , xii+655.
Eppen, G. D. and Fama, E. F., 1968. Solutions for cash-balance and simple dynamic-portfolio problems. Journal of Business 41(1), 94-112.
Eppen, G. D. and Fama, E. F., 1969. Cash balance and simple dynamic portfolio problems with proportional costs. International Economic Review 10(2), 119-133.
Finlay, P. N., 1985, Classes of financial-planning package - a management perspective, Long Range Planning 18(6), 94-98.
Fischer, E., Heinkel, R., Zechner, J., 1989, Dynamic capital structure choice: theory and tests. Journal of Finance 44, 19–40.
Folcut, O.; Ciocirlan, D.; Serban, R. M. and Katalinc, B., 2008, Simulation model for the financing strategies of a leasing company, Annals Of DAAAM For 2008 & Proceedings Of The 19th International DAAAM Symposium , 509-510.
Francis, J. C. and Rowell, D. R., 1978, Simultaneous equation model of firm for financial analysis and planning, Financial Management 7(1), 29-44.
Francis, J. C., 1983, Financial-planning and forecasting models - an overview, Journal of Economics and Business 35(3-4), 285-300.
Fu, D. F., Ionescu, C. M., Aghezzaf, E. H. and De Keyser, R., 2014. Decentralized and centralized model predictive control to reduce the bullwhip effect in supply chain management. Computers & Industrial Engineering, Volume 73, 21-31.
Fusai, G. and Roncoroni, A., 2008. Implementing models in quantitative finance: Methods and cases. Springer, Series: Springer Finance, p.p. xxiii + 607.
Fusai, G., Marazzina, D., Marena, M. and Ng, M., 2012. Z-transform and preconditioning techniques for option pricing. Quantitative Finance 12(9), 1381-1394.
García Bermejo, M., 1984, Modelos de simulacion financiera. Una herramienta para la gestión, Revista Española De Financiación Y Contabilidad XIII(43), 101-116.
García Salcedo, C. A., Ibeas Hernández, A., Vilanova, R. and Herrera Cuartas, J., 2013. Inventory control of supply chains: Mitigating the bullwhip effect by centralized and decentralized Internal Model Control approaches. European Journal of Operational Research 224(2), 261-272.
García, C. A., Ibeas, A., Herrera, J. and Vilanova, R., 2012. Inventory control for the supply chain: An adaptive control approach based on the identification of the lead-time. Omega-International Journal of Management Science 40(3), 314-327.
Gardiner, C. and Ward, P.W., 1974, Long-range financial resource planning model for a local-authority, Operational Research Quarterly 25(1), 55-64.
Gardner Jr, E. S., 2006. Exponential smoothing: The state of the art - Part II. International Journal of Forecasting, Volume 22, Issue 4, 637-666.
Gentry, J. A. and Pyhrr, S. A., 1973, Simulating an EPS growth model, Financial Management 2(2), 68-75.
Gentry, J. A., 1972, Simulation of financial planning process of p-l insurers, Journal of Risk and Insurance 39(3), 383-396.
Gentry, J. A., 1979, Simulating the strategic financial-planning process, European Journal Of Operational Research 3(6), 441-449.

Gershefski, G. W., 1970, Corporate models - state of art, Management Science Series B-Application 16(6), B303-B312.

Girgis, N. M., 1968. Optimal cash balance levels. Management Science 15(3), 130-140.

Goedhart, M. H. and Spronk, J., 1995a, Financial-planning with fractional goals, European Journal Of Operational Research 82(1), 111-124.

Goedhart, M. H. and Spronk, J., 1995b, An interactive heuristic for financial-planning in decentralized organizations, European Journal Of Operational Research 86(1), 162-175.

Gordon, M. J., 1962, The Investment, Financing and Valuation of the Corporation, Homewood, Illinois: Richard D. Irwin, xiv, 256.

Gordon, M. J., 1989, Corporate-finance under the mm theorems, Financial Management 18(2), 19-28.

Grinyer, P. H, 1973, Corporate financial simulation models for top management, Omega, 1973, Vol. 1(4), Pp.465-482.

Grinyer, P. H. and Batt, C. D., 1974, Some tentative findings on corporate financial simulation-models, Operational Research Quarterly 25(1), 149-167.

Grinyer, P. H. and Wooller, J., 1975, Computer models for corporate planning, Long Range Planning 8(1), 14-25.

Grinyer, P. H. and Wooller, J., 1978, Corporate Models Today-A New Tool for Financial Management - 2nd edition, Institute of Chartered Accountants in England and Wales, London, 316.

Grinyer, P. H., 1983, Financial modeling for planning in the UK, Long Range Planning 16(5), 58-72.

Grubbstrom, R. W. and Huynh, T. T. T., 2006. Multi-level, multi-stage capacity-constrained production-inventory systems in discrete time with non-zero lead times using MRP theory. International Journal of Production Economics 101(1), 53-62.

Grubbstrom, R. W. and Tang, O., 1999. Further developments on safety stocks in an MRP system applying Laplace transforms and input-output analysis. International Journal of Production Economics 60(1), 381-387.

Grubbstrom, R. W., 1998. A net present value approach to safety stocks in planned production. International Journal of Production Economics 56(7), 213-229.

Grubbstrom, R. W., 1999. A net present value approach to safety stocks in a multi-level MRP system. International Journal of Production Economics 59(1-3), 361-375.

Gryglewicz, S., 2011, A theory of corporate financial decisions with liquidity and solvency concerns, Journal Of Financial Economics 99(2), 365-384.

Gupta, A.; Li, L. P.; Chen, S. H.; Cheng, H. D.; Chiu, D. K. Y.; Das, S.; Duro, R.; Kerre, E. E.; Leong, H. V.; Li, Q.; Lu, M.; Romay, M. G.; Ventura, D. and Wu, J., 2003, Behavioral modeling in optimal investment-consumption decisions for long-term financial planning, Proceedings Of The 7th Joint Conference On Information Sciences , 1017-1022.

Gupta, S. and Dutta, K., 2011. Modeling of financial supply chain. European Journal of Operational Research 211(1), 47-56.

Güven, S. and Kaynarca, A., 1998, An integrated investment and financial planning model, International Transactions In Operational Research, 1998, Vol. 5(2), Pp.123-136.

Hahn, G. J. and Kuhn, H., 2012, Simultaneous investment, operations, and financial planning in supply chains: A value-based optimization approach, International Journal Of Production Economics 140(2), 559-569.

Haley, C. W. and Schall, L. D., 1973, The Theory of Financial Decisions, New York: Mcgraw-Hill Book Company , xviii + 383.

Hallsten, B., 1966, Investment and Financing Decisions on Goal Formulation and Model Building, Economic Research Institute, Stockholm School Of Economics , 137 Sw. Kr.36..

Hamilton, W. F. and Moses, M. A., 1973, Optimization model for corporate financial planning, Operations Research 21(3), 677-692.

Hamilton, W. F. and Moses, M. A., 1974, Computer-based corporate planning system, Management Science Series B-Application 21(2), 148-159.

Harris, M. and Raviv, A., 1991, The theory of capital structure, Journal Of Finance 46(1), 297-355.

Hau, A ., 2004, Theory of the firm facing uncertain demand revisited, Economic Theory 24(2), 457-464.

Hayashi, F., 1982, Tobin marginal q and average q - a neoclassical interpretation, Econometrica 50(1), 213-224.

Hayen, R. L., 1983, How to design a financial-planning model, Long Range Planning 16(5), 111-122.

Herrera, C. A. and Ibeas, A., 2015. A simulation model for a Cash Concentration and Disbursements System. 23rd. Mediterranean Conference on Control and Automation (MED), Torremolinos, Spain, 935-942.

Herrera-Cáceres, C. A. and Ibeas, A., 2016a. Mathematical Modeling of a Cash Concentration and Disbursements System. RIAI - Revista Iberoamericana de Automática e Informática Industrial, ISSN: 1697-7912, 13(3):339-349, doi: 10.1016/j.riai.2015.07.008.

Herrera-Cáceres, C. A. and Ibeas, A., 2016b. Model predictive control for a revenue account of a Cash Concentration and Disbursements System. 24th. Mediterranean Conference on Control and Automation (MED), Athens, Greece, 118-124.

Herrera-Cáceres, C. A. and Ibeas, A., 2016c. Model Predictive Control of cash balance in a Cash Concentration and Disbursements System. Journal of the Franklin Institute. In press.

Hite, G. L., 1977, Leverage, output effects, and m-m theorems, Journal of Financial Economics 4(2), 177-202.

Hopkins, D. S. P.; Heath, D. and Levin, P. J., 1982, A financial-planning model for estimating hospital debt capacity, Public Health Reports 97(4), 363-372.

Hurd, T. R. and Zhou, Z. W., 2010. A Fourier transform method for spread option pricing. SIAM Journal on Financial Mathematics 1(1), 142-157.

Ijiri, Y. and Thompson, G. L., 1970, Applications of mathematical control theory to accounting and budgeting (continuous wheat trading model), Accounting Review 45(2), 246-258.

Ijiri, Y.; Levy, F. K. and Lyon, R. C., 1963, A linear-programming model for budgeting and financial-planning, Journal Of Accounting Research 1(2), 198-212.

Inselbag, I., 1973, Financing decisions and theory of firm, Journal Of Financial And Quantitative Analysis 8(5), 763-776.

Jorgenson, D. W., 1963, Capital theory and investment behavior, American Economic Review 53(2), 247-259.

Kallberg, J. G.; White, R. W. and Ziemba, W. T., 1982, Short-term financial-planning under uncertainty, Management Science 28(6), 670-682.

Kent, R. P., 1960, Corporate financial management, Homewood, Illinois: Richard D. Irwin, xv, 851.

Kerr, H. S., 1981, A bibliography for financial management, Financial Management 10(2), 123-134.

Khan, M., 1975. The monetary dynamics of hyperinflation: A note. Journal of Monetary Economics 1(3), 355-362.

Kingshott, A. L., 1968, Financial forecasting for corporate planning, Long Range Planning 1(2), 28-36.

Kirca, O. and Koksalan, M. M., 1996, An integrated production and financial planning model and an application, Iie Transactions 28(8), IIE TRANSACTIONS.

Kleinmuntz, D. N.; Kleinmuntz, C. E.; Stephen, R. G. and Nordlund, D. S., 1999, Measuring and managing risk improves strategic financial planning., Healthcare Financial Management: Journal of The Healthcare Financial Management Association 53(6), 50-8.

Kochhar, R. and Hitt, M. A., 1998, Linking corporate strategy to capital structure: Diversification strategy, type and source of financing, Strategic Management Journal 19(6), 601-610.

Korkie, R. M., 1975, Theory of the Firm Facing Uncertain Demand: Comment, The American Economic Review, 1975, Vol. 65(1), Pp.245-247.

Krouse, C. G., 1972, Model for aggregate financial planning, Management Science Series B-Application 18(10), B555-B566.

Kumar, R. and Vrat, P., 1989, Using computer-models in corporate-planning, Long Range Planning 22(2), 114-120.

Kundisch, D. and Dzoienziol, J., 2008, Decision Support for Financial Planning, Journal of Decision Systems 17(2), 175-209.

Kunsch, P.; Chevalier, A. and Brans, J. P., 2001, Comparing the adaptive control methodology (ACM) to the financial planning practice of a large international group, European Journal Of Operational Research 132(3), 479-489.

Kvanli, A.H., 1980, Financial-Planning Using Goal Programming, Omega-International Journal of Management Science 8(2), 207-218.

Laínez Gadea, J. A. and Bellostas Pérez-Grueso, A. J., 1991, La planificación y gestión estratégica en grupos multinacionales: Los modelos de simulación, Revista Española De Financiación Y Contabilidad, 1991, Issue 67, Pp.501-534.

Lane, D., 2009, The cost of capital, corporation finance and the theory of investment: a refinement, Applied Economics Letters 16(10), 1017-1019.

Lee, C. F. and Junkus, J. C., 1983, Financial analysis and planning - an overview, Journal of Economics and Business 35(3-4), 259-283.

Lee, J. H. and Lee, J.M., 2006. Approximate dynamic programming based approach to process control and scheduling. 7th. International Conference on Chemical Process Control (CPC 7), Computers & Chemical Engineering, Volume 30, Issue 10-12, 1603-1618.

Lee, J. H. and Wong, W., 2010. Approximate dynamic programming approach for process control. IFAC Symposium on Advanced Control of Chemical Processes (ADCHEM 2009), Journal of Process Control, Volume 20, Issue 9, 1038-1048.

Lee, J. H., 2011. Model Predictive Control and Dynamic Programming. 2011 11th International Conference on Control, Automation and Systems (ICCAS), Pages 1807-1809.

Lee, R. W., 2004. Option pricing by transform methods: extensions, unification and error control. The Journal of Computational Finance 7(3), 51-86.

Leland, H. E., 1972, Theory of the Firm Facing Uncertain Demand, The American Economic Review, 1972, Vol. 62(3), Pp.278-291.

Lerner, E. M. and Carleton, W. T., 1966a, Financing decisions of firm, Journal of Finance 21(2), 202-214.

Lerner, E. M. and Carleton, W. T., 1966b, A Theory of Financial Analysis, New York: Harcourt, Brace & World , xii, 281.

Lewis, C. M., 1990, A multiperiod theory of corporate financial policy under taxation, Journal Of Financial And Quantitative Analysis 25(1), 25-43.

Li, J.; Jiang, X. Y.; Chen, J. C.; Jin, D. and Lin, S., 2012, Considerations on Strengthening Enterprise Financial Control, Advances In Electronic Commerce, Web Application and Communication, Vol 1 148, 67-71.

Liang, Z. X. and Sun, B., 2011, Optimal control of a big financial company with debt liability under bankrupt probability constraints, Frontiers of Mathematics in China 6(6), 1095-1130.

Liang, Z. X. and Sun, B., 2011. Optimal control of a big financial company with debt liability under bankrupt probability constraints. Frontiers of Mathematics in China 6(6), 1095-1130.

Lin, C.; Cai, X. S. and Zhou, M., 2011, On Enterprise Financial Prediction Model Using System Dynamics, 2011 International Conference On Future Computers In Education (ICFCE 2011), Vol II , 53-56.

Lin, P. H., Wong, D. S. H., Jang, S. S., Shieh, S. S. and Chu, J. Z., 2004. Controller design and reduction of bullwhip for a model supply chain system using Z-transform analysis. Journal of Process Control 14(5), 487-499.

Lin, T. W. and O'Leary, D. E., 1993, Goal programming applications in financial management, Advances In Mathematical Programming And Financial Planning, Jai Press Inc 3, 211-229.

Linke, C. M. and Whitford, D. T., 1983, A multiobjective financial-planning model for electric utility rate regulation, Journal of Economics and Business 35(3-4), 313-330.

Lintner, J., 1956, Distribution of incomes of corporations among dividends, retained earnings, and taxes, American Economic Review 46(2), 97-118.

Lintner, J., 1962, Dividends, earnings, leverage, stock-prices and the supply of capital to corporations, Review Of Economics And Statistics 44(3), 243-269.

Lintner, J., 1964, Optimal dividends and corporate-growth under uncertainty, Quarterly Journal of Economics 78(1), 49-95.

Machado, E. L. and Carleton, W. T., 1978, Financial-planning in a regulated environment, Journal Of Financial And Quantitative Analysis 13(4), 759-777.

Malagoli, S.; Magni, C. A. and Mastroleo, G., 2007, The use of fuzzy logic and experto systems for rating and pricing firms; A new perspective on valuation, Managerial Finance, 2007, Vol.33 (11), P.836-852.

Maranas, C. D.; Androulakis, I. P.; Floudas, C. A.; Berger, A. J. and Mulvey, J. M., 1997, Solving long-term financial planning problems via global optimization, Journal Of Economic Dynamics & Control 21(8-9), 1405-1425.

Markowitz, H. M., 1952, Portfolio selection, The Journal of Finance, 1952, Vol. 7(1), Pp.77-91.

Markowitz, H. M., 1959, Portfolio selection: efficient diversification of investments, John Wiley & Sons, Inc., New York; Chapman & Hall, Limited, London , 344.

Marquis, M. H. and Witte, W. E., 1989. Cash management and the demand for money by firms. Journal of Macroeconomics 11(3), 333-350.

Martínez Lobato, F. and Ferrando Bolado, M., 1997, Un modelo de simulación borroso de planificación financiera, Revista Española De Financiación Y Contabilidad XXVI (93), 1091-1123.

Martínez Lobato, F., 1999, Un modelo de simulación para la planificación financiera, Revista Española De Financiación Y Contabilidad XXVIII (99), 443-495.

Mattessich, R., 1957, Towards a General and Axiomatic Foundation of Accountancy, Accounting Research, London 8(4), 328-355.

Mattessich, R., 1958, Mathematical-models in business accounting, Accounting Review 33(3), 472-481.

Mattessich, R., 1961, Budgeting models and system simulation, Accounting Review 36(3), 384-397.

Mattessich, R., 1964a, Accounting and Analytical Methods, Homewood, Illinois, Richard D. Irwin, Inc. , 552.

Mattessich, R., 1964b, Simulation of the firm through a budget computer program, Homewood, Illinois, Richard D. Irwin. Inc. .

Mavrotas, G.; Caloghirou, Y. and Koune, J., 2005, A model on cash flow forecasting and early warning for multi-project programmes: application to the Operational Programme for the Information Society in Greece, International Journal Of Project Management, 2005, Vol.23(2), Pp.121-133.

Mayne, D. Q., Rakovic, S. V., Findeisen, R. and Allgower, F., 2006. Robust output feedback model predictive control of constrained linear systems. Automatica, Volume 42, Issue 7, 1217-1222.

Mayne, D. Q., Rakovic, S. V., Findeisen, R. and Allgower, F., 2009. Robust output feedback model predictive control of constrained linear systems: Time varying case. Automatica, 45th IEEE Conference on Decision and Control, Volume 45, Issue 9, 2082-2087.

Mayne, D. Q., Seron, M. M. and Rakovic, S. V., 2005. Robust model predictive control of constrained linear systems with bounded disturbances. Automatica, Volume 41, Issue 2, 219-224.

McInnes, J. M. and Carleton, W. J., 1982, Theory, models and implementation in financial management, Management Science 28(9), 957-978.

Meadows, E. S., 1997. Dynamic programming and model predictive control. Proceedings of the 1997 American Control Conference, VOLS 1-6, 1635-1639.

Mercier, D., 2002, Integration of financial and strategic planning using a real options framework bolsters capital spending decisions, Oil & Gas Journal 100(12), 77-81.

Merville, L. J. and Tavis, L. A., 1974, Long-range financial planning, Financial Management 3(2), 56-63.

Miller, M. H. and Modigliani, F., 1961, Dividend policy, growth, and the valuation of shares, Journal of Business 34(4), 411-433.

Miller, M. H. and Orr, D., 1966. A Model of the demand for money by firms. The Quarterly Journal of Economics (Published by: Oxford University Press) 80(3), 413-435.

Miller, M. H. and Orr, D., 1968. Demand for money by firms - extensions of analytic results. Journal of Finance 23(5), 735-759.

Miyajima, M. and Nakai, M., 1986, The municipal financial-planning model - a simultaneous regression equations and goal programming approach, European Journal Of Operational Research 27(2), 158-167.

Moag, J. S.; Carleton, W. T. and Lerner, E. M., 1967, Defining finance function - model-systems approach, Journal of Finance 22(4), 543-555.

Modigliani, F. and Miller, M. H., 1958, The cost of capital, corporation finance and the theory of investment, American Economic Review 48(3), 261-297.

Moreau, D. H., 1986, Financial-planning model and its application, Journal of Water Resources Planning and Management-ASCE 112(4), 439-452.

Morgan, G. E. and Martin, J. D., 1986, A rationale for planning - implications from the economic-theory of the firm, Managerial and Decision Economics 7(4), 219-228.

Moroto Acín, J. A. and Mascareñas, J. M., 1986, PLAFIN-II: un modelo de simulacion para la planificacion financiera empresarial, Revista Espanola De Financiación Y Contabilidad XVI (49), 223-255.

Mulvey, J. M. and Shetty, B., 2004, Financial planning via multi-stage stochastic optimization, Computers & Operations Research 31(1), 1 - 20.

Mulvey, J. M. and Vladimirou, H., 1992, Stochastic network programming for financial-planning problems, Management Science 38(11), 1642-1664.

Mulvey, J. M., 2001, Introduction to financial optimization: Mathematical programming special issue, Mathematical Programming 89(2), 205-216.

Myers, S. C. and Pogue, G. A., 1974, Programming approach to corporate financial management, Journal of Finance 29(2), 579-599.

Myers, S. C., 1984, Finance theory and financial strategy, Interfaces 14(1), 126-137.

Naim, M. M., Wikner, J. and Grubbstrom, R. W., 2007. A net present value assessment of make-to-order and make-to-stock manufacturing systems. Omega-International Journal of Management Science 35(5), 524-532.

Naylor, T. H. and Gattis, D. R., 1976, Corporate-planning models, California Management Review 18(4), 69-78.

Naylor, T. H. and Schauland, H., 1976a, Survey of users of corporate-planning models, Management Science 22(9), 927-937.

Naylor, T. H. and Schauland, H., 1976b, Experience with corporate simulation-models - survey, Long Range Planning 9(2), 94-100.

Nolan, R. B. and Foran, R. E., 1983, Strategic financial-planning, Journal American Water Works Association 75(9), 436-440.

Obstfeld, M. and Rogoff, K., 1995. Exchange-rate dynamics Redux. Journal of Political Economy 103(3), 624-660.

Obstfeld, M. and Rogoff, K., 1996. Foundations of international macroeconomics. The MIT Press, p.p. xxiii + 804.

Ogata, K., 1996. Discrete time control systems. Prentice-Hall International.

Orthlieb, R. H., 1979, Corporate financial model - aid to better management, Journal of Canadian Petroleum Technology 18(1), 14-14.

Osteryoung, J. S., 1972, Integrated intertemporal linear programming model under risk for short-term financial planning, Journal Of Finance 27(5), 1176-1176.

Pan, W. T., 2012, The use of genetic programming for the construction of a financial management model in an enterprise, Applied Intelligence 36(2), 271-279.

Pappas, J. L. and Huber, G. P., 1973, Probabilistic short-term financial planning, Financial Management 2(3), 36-44.

Patz, A. L., 1993, A linear programming approach to open system total enterprise simulations, Developments In Business Simulation & Experiential Exercises 20, 86-89.

Pivnicny, V. C., 1989, Integrating financial and strategic planning, Health Care Strategic Management 7(9), 14 - 6.

Pogue, G. A. and Bussard, R. N., 1972, Linear programming model for short term financial planning under uncertainty, Sloan Management Review 13(3), 69-98.

Powell, J. R. P. and Vergin, R. C., 1975, Heuristic model for planning corporate financing, Financial Management 4(2), 13-20.

Premachandra, I. M., 2004. A diffusion approximation model for managing cash in firms: An alternative approach to the Miller-Orr model. European Journal of Operational Research 157(1), 218-226.

Ramnath, S.; Rock, S. and Shane, P., 2008, The financial analyst forecasting literature: A taxonomy with suggestions for further research, International Journal Of Forecasting 24(1), 34-75.

Reid, G. C., 2003, Trajectories of small business financial structure, Small Business Economics 20(4), 273-285.

Rhyne, L. C., 1986, The relationship of strategic-planning to financial performance, Strategic Management Journal 7(5), 423-436.

Robichek, A. A. and Myers, S. C., 1965, Optimal financing decisions, Englewood Cliffs, New Jersey: Prentice Hall, Inc., x + 166.

Robichek, A. A.; Teichroew, D. and Jones, J. M., 1965, Optimal short-term financing decision, Management Science 12(1), 1 - 36.

Robinson, R. B. and Pearce, J. A., 1983, The impact of formalized strategic-planning on financial performance in small organizations, Strategic Management Journal 4(3), 197-207.

Robinson, T. and Clarke, D., 1991. Robustness effects of a prefilter in receding-horizon predictive control. IEE PROCEEDINGS-D, Vol. 138, No. 1, 2–8.

Rossiter, J. A., Kouvaritakis, B. and Gossner, J. R., 1995. Feasibility and Stability Results for Constrained Stable Generalized Predictive Control. Automatica, Volume 31, Issue 6, 863-877.

Rychel, D. F., 1982, A case-history of financial and operational modeling in corporate-planning, Computers & Industrial Engineering 6(2), 125-129.

Sandberg, C. M.; Lewellen, W. G. and Stanley, K. L., 1987, Financial strategy - planning and managing the corporate leverage position, Strategic Management Journal 8(1), 15-24.

Satir, B., 2003, A general production and financial planning model for integrated poultry organizations, The Graduate School Of Natural And Applied Sciences Of The Middle East Technical University, 96.

Schall, L. D., 1971, Firm financial structure and investment, Journal Of Financial And Quantitative Analysis 6(3), 925-942.

Schendel, D. and Patton, G. R., 1978, Simultaneous equation model of corporate strategy, Management Science 24(15), 1611-1621.

Schmaltz, J. H., 1980, A management approach to a strategic financial-planning system, Sloan Management Review 21(2), 3 - 13.

Schwienbacher, A., 2007, A theoretical analysis of optimal financing strategies for different types of capital-constrained entrepreneurs, Journal of Business Venturing 22(6), 753-781.

Scott, J. H., 1976, A Theory of Optimal Capital Structure, The Bell Journal Of Economics, 1976, Vol. 7(1), Pp.33-54.

Sealey, C. W., 1978, Financial-planning with multiple objectives, Financial Management 7(4), 17-23.

Sethi, S. P. and Thompson, G. L., 1970. Applications of mathematical control theory to finance - modeling simple dynamic cash balance problems. Journal of Financial and Quantitative Analysis 5(4-5), 381-394.

Shim, J. K. and McGlade, R., 1984, The use of corporate-planning models - past, present and future, Journal Of The Operational Research Society 35(10), 885-893.

Siegmann, A. and Lucas, A., 2005, Discrete-time financial planning models under loss-averse preferences, Operations Research 53(3), 403-414.

Singhvi, S. S., 1972, Financial planning in a divisionalized firm, Long Range Planning, 1972, Vol. 5(4), Pp.15-20.

Sinuanystern, Z., 1984, A financial-planning model for a multi-campus college, Socio-Economic Planning Sciences 18(2), 135-142.

Song, N., Ching, W. K., Su, T. K. and Yiu, C. K. F., 2013. On optimal cash management under a Stochastic Volatility Model. East Asian Journal on Applied Mathematics 3(2), 81-92.

Stahl, I.; Evans, G. W.; Mollaghasemi, M.; Russell, E. C. and Biles, W. E., 1993, Discrete-event simulation for corporate financial-planning, 1993 Winter Simulation Conference Proceedings , 1264-1269.

Steuer, R.E. and Na, P., 2003, Multiple criteria decision making combined with finance: A categorized bibliographic study, European Journal Of Operational Research 150(3), 496-515.

Stiglitz, J. E., 1969, Re-examination of Modigliani-Miller Theorem, American Economic Review 59(5), 784-793.

Stiglitz, J. E., 1974, On the Irrelevance of Corporate Financial Policy, The American Economic Review, 1974, Vol. 64(6), Pp.851-866.

Stiglitz, J. E., 1988, Why financial structure matters, Journal Of Economic Perspectives 2(4), 121-126.

Stone, B. K. and Hill, N. C., 1980. Cash transfer scheduling for efficient cash concentration. Financial Management 9(3), 35-43.

Stone, B. K. and Hill, N. C., 1981. The design of a cash concentration system. Journal of Financial and Quantitative Analysis 16(3), 301-322.

Stone, B. K., 1973, Cash planning and credit-line determination with a financial statement simulator - case report on short-term financial planning, Journal of Financial and Quantitative Analysis 8(5), 711-729.

Sui, Z., Gosavi, A. and Lin, L., 2010. A Reinforcement Learning Approach for Inventory Replenishment in Vendor-Managed Inventory Systems with Consignment Inventory. EMJ-Engineering Management Journal, Volume 22, Issue 4, Special Issue SI, 44-53.

Sun, J.; Xu, W. B. and Fang, W., 2006, Solving multi-period financial planning problem via quantum-behaved particle swarm algorithm, Computational Intelligence, Pt 2, Proceedings 4114, 1158-1169.

Sundaresan, S. M., 2000, Continuous-time methods in finance: A review and an assessment, Journal of Finance 55(4), 1569-1622.

Swamy, M. N. S., Roytman, L. M. and Plotkin, E. I., 1985. On stability properties of three- and higher dimensional shift invariant digital filters. IEEE Trans. Circuirs Sysf., vol. CAS-32, pp. 888-892

Sytnyk, G. V., 2012, Genesis and contemporary paradigm of financial planning, Actual Problems of Economics (133), 33-48.

Taggart, R. A., 1977, Model of corporate financing decisions, Journal Of Finance 32(5), 1467-1484.

Tapiero, C. S. and Zuckerman, D., 1980, A note on the optimal-control of a cash balance problem, Journal Of Banking & Finance 4(4), 345-352.

Tarrazo, M. and Gutierrez, L., 2000, Economic expectations, fuzzy sets and financial planning, European Journal of Operational Research 126(1), 89-105.

Teichroew, D.; Robichek, A. A. and Montalbano, M., 1965, An analysis of criteria for investment and financing decisions under certainty, Management Science 12(3), 151-179.

Teixeira, M. C. M., Assuncao, E., Cardim, R., Silva, N. A. P. and Machado, E. R. M. D., 2009. On Complementary Root Locus of Biproper Transfer Functions. Mathematical Problems In Engineering. New York: Hindawi Publishing Corporation, p. 14. http://hdl.handle.net/11449/10522.

Tenorio, V. A. F., Martín C. A. M., Paralera, M. C. and Contreras R. I., 2013. Ecuaciones diferenciales y en diferencias aplicadas a los conceptos económicos y financieros. Revista de Métodos Cuantitativos para la Economía y la Empresa (16), 165-199. ISSN: 1886-516X. D.L: SE-2927-06. URL: http://www.upo.es/RevMetCuant/art.php?id=83.

Terceno, A. and Vigier, H., 2011, Economic-financial forecasting model of businesses using fuzzy relations, Economic Computation and Economic Cybernetics Studies and Research 45(1), 215-232.

Thornton-Trump, A. B. and Fu, W., 2000, Small business modeling within the financial accounting conceptual framework, Data Mining Ii 2, 35-43.

Tinsley, W. A. and Hoglund, C. R., 1973, Financial planning for dairy-herd expansions, Journal Of Dairy Science 56(6), 830-834.

Tobin, J., 1956. The interest-elasticity of transactions demand for cash. Review of Economics and Statistics 38(3), 241-247.

Tobin, J., 1969, A general equilibrium approach to monetary theory, Journal of Money, Credit And Banking, 1969, Vol. 1(1), Pp.15-29.

Trott, M., 2004. The Mathematica GuideBook for Programming. New York: Springer-Verlag. http://www.mathematicaguidebooks.org/.

Tsang, T. T. C. and Clarke, D.W., 1988. Generalized predictive control with input constraints. IEE Proc., Pt.D, Vol.135, No.6, pp.451–460.

Tse, J. Y. D., 1960, Profit Planning through Volume-Cost Analysis, New York: Macmillan Co. , x+240.

Uspuriene, A.; Sakalauskas, L.; Ginevicius, R.; Rutkauskas, A. V.; Pocs, R. and Stankeviciene, J., 2010, Two-stage continuous stochastic programming model for financial planning, 6th International Scientific Conference Business And Management 2010, Vols I And Ii , 263-269.

van der Hoek, J. and Elliott, R. J., 2006, Binomial Models in Finance, New York: Springer , xiii + 303.

Vasconcellos, G. M., 1988. On the application of optimal-control theory to financial-planning and forecasting. Journal Of Economics and Business 40(4), 309-318.

Verdú, S. and Poor, H. V., 1984. Backward, forward and backward-forward dynamic programming models under commutativity conditions. Proceedings of 23rd IEEE Conference on Decision Control, 1081-1086.

Verge, J. M., 1969, Sequential model of long-term financing in firm, Revue Economique 20(2), 301-336.

Vickers, D., 1968, The Theory of the Firm: Production, Capital, and Finance, New York, St. Louis, San Francisco, Toronto, London, Sydney: Mcgraw-Hill Book Company , xiv + 252.

Vlachy, J., 2009, Corporate strategy and financial theory, Politicka Ekonomie 57(2), 147-162.

Walker, E. W. and Baughn W. H., 1961, Financial Planning and Policy, New York: Harper & Brothers , xiv+508.

Walker, E. W. and Petty, J. W., 1978, Financial differences between large and small firms, Financial Management 7(4), 61-68.

Wang, Z. L., 2011, The financial strategy research for SME based on the theory Of corporate life cycle, 2011 International Conference On Business Management And Electronic Information, May 2011, Vol.3, Pp.626-628.

Warren, J. M. and Shelton, J. P., 1971, A Simultaneous Equation Approach to Financial Planning, The Journal Of Finance, 1971, Vol. 26(5), Pp.1123-1142.

Weingartner, H. M., 1963, Mathematical Programming and the Analysis of Capital Budgeting Problems, Englewood Cliffs, New Jersey: Prentice-Hall, Inc. , xiv + 200.

Weingartner, H. M., 1966, Criteria for programming investment project selection, Journal Of Industrial Economics 15(1), 65-76.

Welter, P., 1973, Financial aspects of company planning, Long Range Planning 6(1), 36-&.

Whipple, W., 1988, How finance can contribute to strategic-planning, Journal of Accountancy 165(1), 116-&.

Wijngaard, J., 1979. Decomposition for Dynamic-Programming in Production and Inventory Control. Engineering and Process Economics, Volume 4, Issue 2-3, 385-388.

Wilson, R. M. S., 1974, Financial control: a systems approach, Mcgraw Hill, London , 380.

Wong, W. C. and Lee, J. H., 2011. Postdecision-State-Based Approximate Dynamic Programming for Robust Predictive Control of Constrained Stochastic Processes. Industrial & Engineering Chemistry Research, Volume 50, Issue 3, 1389-1399.

Woodford, M., 1998. Control of the public debt: A requirement for price stability? Debt burden and its consequences for monetary policy. IEA Conference (118), 117-154.

Xie, L. H. and Xie, Y. L., 2007, An enterprise financial forecasting model based on system dynamics, Proceedings Of The 2007 Conference On Systems Science, Management Science And System Dynamics: Sustainable Development And Complex Systems, Vols 1-10, 2401-2408.

Xie, Y. L. and Xie, L. H., 2008, Application of System Dynamics to Enterprise Financial Scenario Planning, 2008 4th International Conference On Wireless Communications, Networking And Mobile Computing, Vols 1-31 , 10070-10074.

Xu, X. D. and Birge, J. R., 2006, Equity valuation, production, and financial planning: A stochastic programming approach, Naval Research Logistics 53(7), 641-655.

Zaharia, G. M., 2009, The budget - instrument of financial planning and control in firms, Metalurgia International 15(10), 182-184.

Zettergren, L., 1975, Financial issues in strategic planning, Long Range Planning 8(3), 23-32.

Zhang, Z. L.; Torra, V.; Narukawa, Y.; Valls, A. and Domingo-Ferrer, J., 2006, Decision aggregation in an agent-based financial investment planning system, Modeling Decisions For Artificial Intelligence 3885, 179-190.

Zhu, Z. Q., 2006, A systems approach to developing and implementing an ERP financial subsystem, Systems Research And Behavioral Science 23(2), 237-250.

Zopounidis, C., 1999, Multicriteria decision aid in financial management, European Journal Of Operational Research 119(2), 404-415.

Zuckerman, A., 2000, Leveraging strategic planning for improved financial performance, Healthcare Financial Management: Journal Of The Healthcare Financial Management Association 54(12), 54-7.