Essays on Growth through Creative Destruction

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Contents

Chapter 1

Introduction

The process of economic growth is much more complex than the simple accumulation of wealth or capital. Economies today have new products to satisfy new needs and new means of production that make previous technologies obsolete. As Romer (1990) emphasizes, technological change lies at the heart of economic growth and this implies that economies are always subject to change and innovation. However, technological progress is not only creative, it is also destructive since the design of new goods and production technologies will displace old ones. This process of "creative destruction" is set in motion by the private search for profits and represents one of the sources of economic growth:

> "...The fundamental impulse that sets and keeps the capitalist engine in motion comes from the new consumers' goods, the new methods of production or transportation, the new markets, the new forms of industrial organization that capitalist enterprise creates.... The opening up of new markets...and the organizational development from the craft shop and factory to such concerns as U.S. Steel illustrate the same process of industrial mutation...that incessantly revolutionizes the economic structure *from within*, incessantly destroying the old one, incessantly creating a new one. This process of Creative Destruction is the essential fact about capitalism..." [Joseph Schumpeter, Capitalism, Socialism and Democracy (1942).]

Schumpeter's view of the economic system was left aside for a long time until it was recovered by the new growth theorists in the last decade. The first attempts to construct a growth theory had already identified technological progress as the most important source of growth (Abramovitz 1956; Kendrick 1956; Solow 1957). However, it was modelled as something exogenous, as manna fallen from heaven. A key feature of technological change is that it arises from within the economic system, from the action of private agents in search of higher rents. As Schumpeter writes "...a theoretical construction which neglects...[the process of creative destruction]...neglects all that is most typically capitalist about it; even if correct in logic as well as in fact, it is like Hamlet without the Danish prince." The purpose of endogenous growth theory is thus, to integrate technological change into the process of economic growth, modelling it as a partially excludable, non rival good arising from "intentional actions taken by people who respond to market incentives." (Romer 1990).

Growth theory has experienced a long evolution since the work of Solow (1957), in which technological progress was the exogenous source of growth and the saving rate was constant. Models like the Ramsey-Cass-Koopmans one[1] endogenized the saving behavior but long run growth continued to be out of the picture due to the assumption of decreasing returns. Only in the last decades appeared models with long run endogenous growth. The first attempt was performed by the so-called Ak models which overcame the presence of decreasing returns to capital accumulation with the introduction of externalities arising from government spending (Barro 1990), learning by doing or the stock of knowledge (Romer 1986). These models were able to generate long run growth based on capital accumulation but they ignored the role of technological change induced by private innovation. Conversely,

[1] Ramsey (1928), Cass (1965) and Koopmans (1965).

the work by Romer (1990), Grossman and Helpman (1991) and Aghion and Howitt (1992) opened a new research line for the theory of economic growth. This time, the focus was on the modelling of endogenous technological change. These authors modelized the behavior of research firms that invest resources in order to get valuable innovations. These innovations have value because even though knowledge is non-rival it is partially excludable which, coupled with the introduction of some kind of imperfect competition, allowed researchers to obtain rents from innovation. Two major objections are normally raised against these models. First, that they ignore capital accumulation as a source of growth, and second, that they present scale effects (Jones 1995). Building on the ideas introduced by the previous three seminal papers, other models have introduced modifications that avoid the existence of scale effects and that restore capital accumulation in its role as an important source of growth. We are going to focus on this type of models and, in particular, on the framework presented in Howitt and Aghion (1998), Howitt (1999) and Aghion and Howitt (1998). The reason for this choice is threefold: First, the framework is sufficiently simple to obtain analytical results. Second, it modelizes the R&D and manufacturing sectors of the economy in a way that is complex enough to analyze the most important interactions among economic variables. And third, it is a flexible model that allows us to introduce the departures that we want to consider. The basic model is characterized by the presence of a continuum of intermediate goods used as inputs in the production of a consumption good. For each of these intermediate products there exists a research sector competing in a patent race to obtain a better production technology or a new version of the good that increases efficiency in production and that will permit the owner of the patent become the monopolist producer of the new version. These advances in productivity are the main source of economic growth. A very important feature of the model is the presence of intersectoral and intertemporal spillovers that affect the gains in productivity finally reached in any given sector. Moreover, the existence of a continuum of sectors eliminates uncertainty at the aggregate and, therefore, we can perform non-stochastic steady state analysis. Another characteristic of the model is that research activity requires capital apart from labor to produce innovations. The first models of endogenous technological change assumed that labor was the only input to research. Even though R&D could be seen as a labor (or human capital) intensive activity, assuming that no physical capital is used in research is a strong simplification. Aghion and Howitt (1998) argue that this assumption is the cause of an implication at odds with empirical evidence. Namely, that capital accumulation could not contribute to long run growth. In summary, this model provides a suitable framework for the study of many different policies that may affect the growth performance of the economy but taking into account the existence of side effects of specific policy measures on other sectors of the economy. In addition, the multisector approach makes this model a very appropriate framework to analyze the distributional effects of technological change.

This dissertation has three chapters exploring different aspects of the endogenous growth literature. Chapter 2 focuses on the influence of financial intermediation on the research process analyzing the impact of the need for external finance of the creative process. Chapter 3 considers the role of the public sector on research activity, introducing the various policy instruments that can be used by the government in order to influence the level of private and total research of an economy. Finally, chapter 4 explores how technological progress can generate inequality in the productive sector of an economy, and how changes in the determinants of the growth behavior affect the distribution of profits and productivities across sectors.

The idea that innovation is crucial for economic growth and development is not new. Indeed, we can trace it back to the work of Schumpeter at the beginning of the twentieth century. However, technological innovation is the result of research, an activity which is costly and has an uncertain outcome. Modelizing R&D under the assumption of perfect credit markets leaves aside a very important feature of this activity, namely, that it is plagued with problems of asymmetric information. It appears more interesting to modelize research activity in an environment in which those that carry out the research project are not the ones providing the funds for its development. The second chapter of this thesis proposes a model in which I explicitly modelize the relationship between the researcher and the provider of the funds, allowing for moral hazard on the part of the researcher. In addition, the financial intermediary may influence the behavior of the researcher through monitoring of her research activity. As a result, the level of funds devoted to monitoring by the financial sector will influence R&D productivity and thus, the growth rate of the economy. In terms of policy, we will be able to compare different instruments at the disposal of the public sector. The usual direct subsidies to research will now see their effects undercut by the existence of moral hazard and as a result, their influence on research performance may not be beneficial to the extent that under some conditions, they could be growth reducing. In addition, the introduction of a monitoring technology suggests that a policy that stimulates the investment of intermediaries in monitoring could have positive effects on growth. Intuitively, more intense monitoring will increase research productivity and this should boost the rate of innovation. However, increasing monitoring intensity may have a negative impact on the level of research intensity or on the incentives to accumulate capital, so that the final effect on economic growth could be negative. Comparative statics analysis shows that this will not be the case. Indeed, a policy that promotes the provision of financial services may be preferable to a direct subsidy to research in terms of the growth effects of both policies. Concerning welfare, the fact that a policy that promotes growth will generally reduce the level of consumption per efficiency unit makes the analysis extremely complex. In the case of financial services, the presence of various externalities affecting both the research sector and the process of capital accumulation implies a generally non-optimal level of financial intensity. Depending on the characteristics of the economy considered, this level may be too large or too small and, consequently, there may be a role for policies trying to bring financial intensity closer to its optimal level.

An important feature of the R&D sector is that a large share of total research is publicly financed. The public good nature of technological knowledge and the existence of productivity spillovers from research are normally used as the reason for public intervention in the R&D sector. However, there are several ways in which research policy can be performed and assessing the effects on the economy of these different instruments is an open field. The third chapter of this thesis addresses these issues by means of an endogenous growth model in which the public sector performs an active research policy. I consider several instruments, as research subsidies, publicly performed research and research performed at private firms financed by public funds which try to cover most of the actual policy parameters used by developed countries. We find different effects on growth and welfare and on the level of private research intensity induced by these policies. In particular, while direct research subsidies and publicly financed research have unambiguously positive effects on growth, research performed at public institutions may damage economic growth. This is due to the crowding out of private research caused by public research when it competes with private firms in the "patent race". Another important feature of research is introduced in this chapter. Namely, the difference between basic and applied research. In consonance with the literature on this topic, applied research is concerned with projects aimed at the obtention of an innovation with market applications and, thus, they will normally give rise to a patent. In contrast, basic research is devoted to projects whose outcomes do not initially have market applications though they add to the knowledge base and allow for the development of future projects with applied components. Both the public and the private sector will be allowed to perform both types of research. We will observe that the growth effects of public research will be very different depending on whether it is oriented to applied or basic fields. Namely, I find that while the effect on growth of public basic research is unambiguously positive, the influence of public applied research may be growth reducing due to the large negative impact of this type of intervention on private research. Finally, a welfare evaluation of all the policies considered suggests that welfare may be improved with research policy though an excessive or badly designed intervention may damage consumer welfare.

The process of technological change does not affect all sectors uniformly. Innovative sectors gain productivity and profits relative to the rest of the economy while non-innovative sectors see their technology become obsolete and their profits shrink. There exists thus a distribution of productivities and profits that may be affected by the determinants of economic growth. How changes in these determinants may influence this distribution is the object of the last chapter of the thesis. I find that when an economy is growing faster due to a larger productivity of research, or to a tax policy that stimulates capital accumulation, inequality will decrease. However, when faster growth is due to tax incentives to research in high technology sectors or to structural changes that allow a better absorption of spillovers, inequality among productive sectors will increase. Similarly, changes affecting the distribution of productivities may also be associated with specific changes in the growth behavior of the economy. If the scope of technological spillovers is sufficiently broad, a distribution with a larger mass of high-tech sectors will be associated with a higher growth rate. Nevertheless, a larger mass of research intensive sectors is not necessarily associated with faster growth when spillovers are technology specific or narrow in scope. In this case, the size of the leading group will not affect the growth rate because the increased probability of innovation due to the larger mass of high-tech products is completely offset by the reduction in the marginal impact of an individual innovation.

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Chapter 2

Financial Intermediation in a Model of Growth through Creative Destruction

2.1 Introduction

The renewed interest on growth and their determinants has pointed at the financial structure as one of the key factors in the development of nations. This paper introduces a financial sector in one of the more recent models of growth, the one first presented in Howitt and Aghion (1998). This framework allows us to explicitly model how the R&D activity is financed by means of contracts designed to reduce the incidence of researcher's moral hazard. As a consequence, the financial sector will have real effects on the economy.

Analyzing the interaction between financial and economic activity has been the aim of a rather prolific literature. The first remarkable reference is the work of Schumpeter at the beginning of the twentieth century. He suggested that financial institutions are important for economic activity because they evaluate and finance entrepreneurs in their research and development projects. Similarly, development economists like Gurley and Shaw (1955), Goldsmith (1969), and McKinnon (1973) defended the idea that financial development encourages growth because it increases the level of investment and improves its allocation. In addition, they argued that faster growing economies require higher amounts of financial services and that the richer the economy, the sooner it is able to pay for financial superstructures. Unfortunately, a lack of formal analysis is common to all these papers on development. This is probably because previous to the formulation of a rigorous framework on the relationship between finance and growth it was necessary to develop further the theory of economic growth.

Neoclassical exogenous growth theory did not offer the appropriate frame of reference because financial variables could only have level effects. The appearance of the first works on endogenous growth determined the starting point of the literature on growth and finance. Classic references of this first line of research are Greenwood and Jovanovic (1990), Bencivenga and Smith (1991, 1993), Levine (1991, 1992) and Saint Paul (1992). They used the basic Ak framework combined with credit market models of financial intermediation. In these papers, financial markets are considered as institutions intended to provide services of risk pooling and collection of information about borrowers. They also facilitate the flow of resources from savers to investors in the presence of market imperfections. Papers on this area introduce several devices to fight against adverse selection, moral hazard or liquidity shocks in order to make intermediaries arise endogenously. The role of intermediation is thus, to reduce the inefficiency caused by these imperfections. Consequently, financial institutions promote growth because their activity implies a more efficient allocation of resources. With respect to the backward link from growth to finance suggested by empirical evidence, they follow the basic argument of earlier work. Namely, that there exists a fixed component in the cost of financial services and that some limit of wealth must be trespassed before the establishment of a financial structure is affordable.

New developments in the theory of economic growth have led to another line of research. Grossman and Helpman (1991b) and Romer (1990) suggested that economic growth comes mainly from the invention and development of new products rather than from the accumulation of physical or human capital. Recovering the Schumpeterian view of the role of financial institutions in economic activity, some authors tried to explain how financing of innovation can affect the growth process. Good exponents of this literature are King and Levine (1993a), De la Fuente and Marín (1996) and Blackburn and Hung (1998). Using this new framework they introduce informational frictions in the credit market, providing a rationale for the appearance of intermediaries. King and Levine consider financial intermediaries that act as evaluators of prospective entrepreneurs and as providers of insurance for innovators. However they do not introduce incentive problems. This type of problems can arise because risk averse innovators will try to get full insurance. That is, they will try to get the same payment no matter whether they innovate or not. If this payment is positive, researchers do not have incentives to innovate, especially, if to innovate they must exert effort. The papers by De la Fuente and Marín, and Blackburn and Hung take this moral hazard problem into account though from different perspectives. The first pair of authors provides banks with an imperfect monitoring technology that reveals the innovator's level of effort with a certain probability, while Blackburn and Hung use the costly state verification paradigm, that is, that innovators have incentives to declare that they have not been successful so as to avoid payment. At some cost, investors can verify the result of the project. The common message of this group of papers is that financing of innovation is crucial for economic growth, and that the more efficient is the financial sector the faster the economy will grow. Concerning the feedback effects of growth on finance, these models provide a natural link without recurring to fixed costs assumptions. De la Fuente and Marín argue that growth causes changes in factor prices which increase the return to information gathering and hence favor financial intermediation activities.

The above growth models used by the latter line of research ignore capital accumulation as a source of growth. Aghion and Howitt (1998) argue that they ignore capital accumulation because it is assumed that labor is the only input into research and that labor is inelastically supplied. Therefore, a rise in capital intensity will have two opposite effects. On one hand, it will make payoffs to innovation greater but on the other hand, it will increase labor's productivity, making the input to research more expensive. These two effects cancel each other out so that capital accumulation leaves innovative activity unaffected and thus, it cannot influence long run growth.[1] However, it is arguable that the only source of growth is innovation and, accordingly, Aghion and Howitt propose another model of creative destruction with capital accumulation. They assume that research is produced out of labor and intermediate inputs. In their model, both R&D activities and capital

[1]For details see Aghion and Howitt (1998) pages 99-102.

accumulation determine growth and moreover, they are complementary. Growth cannot go on forever if there were no innovation because diminishing returns would reduce investment while without capital accumulation the rising cost of capital would choke off innovation.

This paper explicitly models the contractual relationship between the researcher and the provider of funds for the project in a model of endogenous technological change in the spirit of Howitt and Aghion (1998). Financial intermediaries are endowed with a monitoring technology that allows them to force researchers to exert a higher level of effort than the one they would choose in the absence of monitoring. Hence, research productivity is determined in the credit market and thus, may be affected by financial variables. In particular, the promotion of financial activities will enhance the economy's growth performance. That is, subsidies to financial intermediation will increase R&D productivity moving the economy to a faster growing balanced growth path. In addition, a subsidy to financial intermediation may be more effective than a direct subsidy to research. The latter policy induces a higher research intensity that rises the growth rate. However, the tax change reduces researchers' incentives to exert effort, which implies higher monitoring costs and a lower R&D productivity. This undercuts the positive growth effects of the research subsidy to the point that for a high enough subsidy rate, the growth effect can become negative.

It is also shown that there exists a negative relationship between the equilibrium level of financial services and capital accumulation. The intuition for this comes from the fact that a policy that promotes financial activity will increase research productivity and thus, reduce the incentives to accumulate capital due to the business stealing effect.

The effect of financial activity on research productivity causes two external effects of opposite sign. On one hand, its positive effect on the productivity of the research project will spillover to the other sectors of the economy and it will increase their productivity. On the other hand, the increase in R&D productivity will raise the arrival rate of innovations and consequently, the probability that an incumbent producer is replaced by the latest innovator. The higher probability of being replaced and thus, of losing the flow of profits, discourages capital accumulation. This is the so-called business stealing effect, or creative destruction process. The interaction of these externalities makes the no-tax equilibrium level of financial services inefficient. Consequently, there exists a role for policies aimed at bringing the provision of financial services closer to its efficient level.

The paper is divided in 6 sections. Section 2 presents the model, sections 3 and 4 study the steady state and the dynamics of the system respectively, section 5 performs the welfare analysis and section 6 concludes the paper.

2.2 The model

I consider a model of creative destruction with capital accumulation and technological spillovers.[2] In the basic model without intermediation, capital accumulation and investment in R&D are the key variables for long run growth. In the present model however, they are not the only ones. This is due to the fact that research productivity is no longer an exogenous parameter. It will be determined by the amount of resources devoted to the financial sector of the economy. The availability of financial services increases the success probability of projects and, hence, the productivity of research. Thus, financial activities

[2]The growth model is based on the work of Howitt and Aghion (1998).

will also be relevant for the determination of long run growth.

2.2.1 Consumers

There is a representative consumer who maximizes the present value of utility

$$V(C_t) = \int_0^\infty \ln(C_t) e^{-\rho t} dt. \tag{2.1}$$

I use the logarithmic functional form for simplicity. As usual C_t is consumption at date t and ρ is the rate of discount of consumption.

2.2.2 Final good sector

The consumption good is produced in a competitive market out of labor and intermediate goods. Labor is represented by a continuous mass of individuals L, and it is assumed to be inelastically supplied. Intermediate goods are produced by a continuum of sectors of mass 1, being m_{it} the supply of sector i at date t. The production function is a Cobb-Douglas with constant returns on intermediate goods and efficiency units of labor

$$Y_t = L^{1-\alpha} \int_0^1 A_{it} m_{it}^{\alpha} di,$$

where Y_t is final good production and A_{it} is the productivity coefficient of each sector. I assume equal factor intensities to simplify calculations.

2.2.3 Intermediate goods

The intermediate sector has a monopolistic structure. In order to become the monopolist producer of an intermediate good, the entrepreneur has to buy the patent of the latest version of the product. This patent gives him the right to produce the good until an innovation occurs and the monopolist is displaced by the owner of the new technology.

The only input in the production of intermediate goods is capital. In particular, it is assumed that A_{it} units of capital are needed to produce one unit of intermediate good i at date t. As we will see, this assumption is necessary in order to obtain stability. The evolution of each sector's productivity coefficient, A_{it} is determined in the research sector.

Capital is hired in a perfectly competitive market at the rental rate ζ_t. Hence, the cost of one unit of intermediate good is $A_{it}\zeta_t$. On the other hand, the equilibrium price of the intermediate good, $p(m_{it})$ will be its marginal product

$$p(m_{it}) = \alpha L^{1-\alpha} A_{it} m_{it}^{\alpha-1},$$

where m_{it} is production of intermediate good i at date t. Thus, the monopolist's profit maximization problem is the following:

$$\begin{aligned} \pi_{it} &= \max_{m_{it}} [p(m_{it})m_{it} - A_{it}\zeta_t m_{it}] \\ s.t. \quad p(m_{it}) &= \alpha L^{1-\alpha} A_{it} m_{it}^{\alpha-1}, \end{aligned}$$

from where we obtain the profit-maximizing supply and the flow of profits as

$$\begin{aligned} m_{it} &= L\left(\frac{\alpha^2}{\zeta_t}\right)^{\frac{1}{1-\alpha}} \\ \pi_{it} &= \alpha(1-\alpha)L^{1-\alpha} A_{it} m_{it}^{\alpha}. \end{aligned}$$

Thanks to the assumption of equal factor intensity, supply of intermediate goods is equal in all sectors, $m_{it} = m_t$. Thus, the aggregate demand of capital is equal to $\int_0^1 A_{it} m_t di$. Let $A_t = \int_0^1 A_{it} di$, be the aggregate productivity coefficient. Then, equilibrium in the capital market requires demand to equal supply

$$A_t m_t = K_t,$$

or equivalently, the flow of intermediate output must be equal to capital intensity k_t,

$$m_t = \frac{K_t}{A_t} \equiv k_t.$$

With this notation we can express the equilibrium rental rate in terms of capital intensity

$$\zeta_t = \alpha^2 L^{1-\alpha} k_t^{\alpha-1}. \tag{2.2}$$

2.2.4 Research sector

Innovations are produced using the same technology of the final good. Hence, it needs physical capital (embodied in the intermediate goods) apart from labor to be produced. Technology is assumed to be increasingly complex and hence further innovations will require higher investments. Accordingly, if N_t is the amount invested in research, the Poisson arrival rate of innovations will be $\lambda_t n_t$, where $n_t = \frac{N_t}{A_t^{\max}}$ is the productivity adjusted level of research and λ_t is research productivity. The total amount of investment in research is divided by $A_t^{\max}$ in order to take into account the effect of increasing technological complexity since $A_t^{\max}$ is the leading edge coefficient that represents the aggregate state of knowledge. We approximate aggregate technological development by the productivity coefficient of the most advanced technology in the economy. When an innovation occurs, the productivity coefficient of that sector jumps discontinuously to $A_t^{\max}$. The leading edge coefficient grows gradually, at a rate that depends on the aggregate flow of innovations. The flow of profits to a monopolist who started producing at t, $\alpha(1-\alpha) L^{1-\alpha} A_t^{\max} m_t^{\alpha}$, is the payoff to innovators if they succeed. Because this payment does not depend on the sector, the level of research will be the same across sectors and the aggregate flow of innovations is thus $\lambda_t n_t$. We will assume that $A_t^{\max}$ grows at a rate proportional to this aggregate flow of innovations

$$\frac{\dot{A}_t^{\max}}{A_t^{\max}} = \sigma\lambda_t n_t, \quad \sigma > 0.$$

It can be proved (see Appendix A) that the long-run cross-sectorial distribution of the relative productivity parameters, $a_{it} = \frac{A_{it}}{A_t^{\max}}$, is time invariant and equal to

$$H(a) = a^{\frac{1}{\sigma}}, \quad 0 \leq a \leq 1. \tag{2.3}$$

To simplify, it is assumed that the initial distribution of a is also $H(a)$.

Consider the arbitrage equation of the research sector. This equation establishes the equality between the expected value of an innovation and its cost at the margin. The value of an innovation at t, V_t, must be the present value of the future flow of profits to the incumbent producer until a new technology displaces the monopolist. This flow of profits is $(1-\alpha)\alpha A_t^{\max} L^{1-\alpha} k_t^{\alpha}$, so the present value is given by

$$V_t = \int_t^{\infty} e^{-\int_t^{\tau}[r_s+\lambda_s n_s]ds}(1-\alpha)\alpha A_t^{\max} L^{1-\alpha} k_\tau^{\alpha} d\tau.$$

The expected marginal revenue of the innovation must equal its marginal cost. The cost of one unit of research in terms of output is 1. Therefore, since $n_t = \frac{N_t}{A_t^{\max}}$, the cost of one unit of research intensity is $A_t^{\max}$. I assume that there is a proportional tax on innovation that increases its cost.[3] Thus, the marginal cost of increasing research intensity is $(1+\tau_n)A_t^{\max}$

[3]Perhaps, this is better understood if we consider a negative tax, i.e. a subsidy. The subsidy would reduce the cost of innovation.

units of output, where τ_n is the tax to innovative activity. Hence, the research arbitrage condition may be written as

$$1 + \tau_n = \lambda_t \frac{(1-\alpha)\alpha L^{1-\alpha} k_t^{\alpha}}{r_t + \lambda_t n_t}. \tag{2.4}$$

Equation (2.4) gives the research intensity as a function of capital intensity and the endogenously determined arrival rate of innovations, λ_t. Thus, the equilibrium level of research is a function of capital intensity and, indirectly, of financial intensity.[4]

2.2.5 Capital market

Capital is used as a factor of production in the intermediate goods sector. We have seen that equilibrium in the capital market requires the rental rate to satisfy equation (2.2). The owner of a unit of capital will obtain ζ_t for it. This amount must be enough to cover the cost of capital. This includes the rate of interest (r_t), the depreciation rate (δ), and the tax rate on capital accumulation (τ_k). Hence, the capital market arbitrage equation is

$$r_t + \delta + \tau_k = \alpha^2 L^{1-\alpha} k_t^{\alpha-1}, \tag{2.5}$$

which establishes a decreasing relationship between the interest rate and capital intensity.

2.2.6 Financing of research

Financial intermediaries channel savings both for its use as capital in production and to finance research projects. I assume that each intermediary has access to deposits at the market determined rate of interest. There is no risk of bankruptcy because they hold a perfectly diversified portfolio of production loans and research financing contracts.

[4] The arrival rate of innovations, or R&D productivity, is positively related to monitoring intensity.

No imperfection is introduced in the provision of production loans. However, I will consider some degree of informational asymmetry in the design of research financing contracts. In particular, I assume that researchers have no funds to invest in the project and, therefore, they have to look for external finance. The limited liability constraint implies that there will exist a potential problem of moral hazard on the part of the researcher. The funds needed for the project will be provided by intermediaries which are endowed with a monitoring technology that allows them to increase the effort of the researcher. Moreover, I assume that the intensity with which the intermediary monitors the researcher determines the additional effort that the former can force the latter to exert, as in Besanko and Kanatas (1993). It is assumed that there exists a one-to-one relationship between effort and probability of success. Therefore, the monitoring services of the financial intermediaries determine R&D productivity.

Consider a research project that requires an initial investment of one unit of output and that will yield a return v with probability λ. Given the research sector outlined in the previous section, the return per unit of output invested, v, must be equal to $\frac{V}{A^{\max}}$. The researcher obtains the funds from the intermediary and in exchange she will pay a fix amount p in case of success and nothing otherwise.[5]

The expected profits for the researcher are given by

$$\lambda(v-p) - D(\lambda),$$

where $D(\lambda)$ is the disutility caused by the effort necessary to obtain a probability of success equal to λ. We will assume that it takes the following form, which is borrowed from the

[5]This is a consequence of the limited liability constraint.

work of Besanko and Kanatas (1993):

$$D(\lambda) = \frac{\lambda^2}{2\beta}.$$

If the researcher received no monitoring at all, the level of effort he would exert would be $\lambda_0 = \beta(v - p)$. This *no-monitoring level of effort* is implementable at no cost for the intermediary. However, if the intermediary wishes to impose a higher level of effort, he will have to face a cost which I assume increasing and convex in the difference between the desired level of effort and λ_0.[6] In particular, I assume that in order to obtain a success probability of λ, the investment required is given by the following expression:

$$M(\lambda - \lambda_0) = \frac{(\lambda - \lambda_0)^2}{2s},$$

and therefore, the profits of the intermediary can be written as

$$\Pi_I = \lambda p - (1 + \tau_f)M(\lambda - \lambda_0) - 1,$$

where τ_f is a tax on the monitoring activities of intermediaries. Notice that imposing taxation on monitoring activities implies that we are assuming that the monitoring costs of the intermediary are observable. Thus we are considering moral hazard only on the part of the researcher. This different treatment can be justified by the nature of the *effort* that intermediaries and researchers do. The disutility caused to the researcher by this effort is non-pecuniary while the monitoring effort of banks can be measured in monetary units, a feature that makes it easier to observe, especially when we are talking about financial intermediaries, one of the most regulated sectors in developed economies.

[6]See Besanko and Kanatas (1993) for details.

There exists a large number of intermediaries that compete in the provision of financial services. A researcher will choose one of them on the basis of his supply of financial services since it will determine the probability of success of her project. However, once the researcher chooses an intermediary to finance her project, she will not be able to break this contract and ask another bank for finance. This assumption can be justified by the existence of switching costs or by the reluctancy of research firms to reveal information about their project. In addition, the fact that once the choice is made the researcher cannot turn to another intermediary implies that the bank is placed in a position of power in its relationship with the researcher. In particular, for a given λ, the intermediary will be able to impose the payment that maximizes his profits, i.e.

$$p(v,\lambda) = v - \frac{\lambda\left[\beta(1+\tau_f) - s\right]}{\beta^2(1+\tau_f)}. \tag{2.6}$$

The fact that the intermediary is able to impose the payment that maximizes his profits does not mean that the researcher is not going to gain with the contract. Indeed, the nature of the limited liability constraint implies that the researcher is always going to obtain a positive payment in expected terms.[7] Notice also that this payment scheme implies a negative relationship between p and λ. This is optimal for the intermediary because p is positively related to the monitoring cost of obtaining a given level of effort. Additionally, if the researcher is subject to an intensive control, she will have to pay less to the intermediary while there is a higher probability that the project succeeds. This may compensate the

[7] Recall that the payment is positive in case of success and zero in case of failure, which yields a positive payment in expected terms. In order to guarantee that the expected payment is positive we have to impose some restrictions on the parameters. In particular, we require

$$s < \frac{\beta(1+\tau_f)}{2}.$$

researcher for the intensive monitoring. In fact, if the relationship between p and λ is given by (2.6), the expected profits of the researcher become monotonically increasing in λ. Hence, this contract makes monitoring desirable for the researcher, since it will reduce the share of the intermediary in the project's return and increase the probability that the project succeeds. As a consequence, a researcher will choose the intermediary that offers the highest level of monitoring services. Therefore, no λ that implies a positive amount of profits will be an equilibrium since any intermediary can attract all the researchers by marginally increasing the degree of monitoring intensity and hence the probability of success. If the number of intermediaries is sufficiently large to impede agreements that limit competition, in equilibrium bank profits will be zero. Therefore, the equilibrium probability of success will be the highest value of λ that implies zero profits. That is, it is the positive root of

$$\lambda p(v, \lambda) - (1 + \tau_f)M(\lambda - \lambda_0(v, p(v, \lambda))) - 1 = 0$$

which yields a positive relationship between the productivity of research and the value of the project, as expressed by

$$\lambda = \tilde{\lambda}(v). \tag{2.7}$$

2.2.7 Equilibrium

Equations (2.4), (2.5) and (2.7) determine partial equilibrium in each market. These equations can be combined in order to obtain the following equilibrium conditions for each market:

(a) Research market equilibrium

$$1 + \tau_n = \lambda_t(v_t - p(v_t, \lambda_t)). \tag{2.8}$$

(b) Capital market equilibrium

$$r_t + \delta + \tau_k = \alpha^2 L^{1-\alpha} k_t^{\alpha-1}. \tag{2.9}$$

(c) Credit market equilibrium

$$\lambda_t = \tilde{\lambda}(v_t). \tag{2.10}$$

Notice that the research arbitrage condition has been modified to take into account the payment to the intermediary.

Equations (2.6) and (2.8) imply the following equilibrium expression for λ:

$$\lambda = \left[\frac{\beta^2(1+\tau_f)(1+\tau_n)}{\beta(1+\tau_f)-s}\right]^{\frac{1}{2}}. \tag{2.11}$$

Hence, research productivity is time invariant and depends only upon the research and credit markets' structural parameters.

Using (2.11), equation (2.10) may be written in the following form:

$$v_t = \frac{\lambda}{\Phi\left(\tau_f, \tau_n\right)},$$

where

$$\Phi\left(\tau_f, \tau_n\right) = \frac{2\beta^2(1+\tau_f)(1+\tau_n)}{(1+\tau_n)\left[2\beta(1+\tau_f)-s\right]+2\left[\beta(1+\tau_f)-s\right]}.$$

Thus, the system formed by equations (2.8), (2.9) and (2.10) can be reduced to the following system:[8]

$$\lambda = \left[\frac{\beta^2(1+\tau_f)(1+\tau_n)}{\beta(1+\tau_f)-s}\right]^{\frac{1}{2}}.$$

[8]Notice that in equation (2.12) we are just substituting v_t by its expression in equilibrium.

$$\frac{\lambda}{\Phi\left(\tau_f,\tau_n\right)}=\frac{\alpha(1-\alpha)L^{1-\alpha}k_t^{\alpha}}{r_t+\lambda n_t} \tag{2.12}$$

$$r_t+\delta+\tau_k=\alpha^2 L^{1-\alpha}k_t^{\alpha-1}, \tag{2.13}$$

which determines the equilibrium values of k_t and n_t. Notice also that from equations (2.12) and (2.13) one can obtain the equilibrium relationship between n_t and k_t as given by

$$n_t=n^d(k_t)=\frac{\Phi\left(\tau_f,\tau_n\right)}{\lambda^2}\frac{(1-\alpha)\alpha L^{1-\alpha}k_t^{\alpha}}{1+\tau_n}-\frac{\alpha^2 L^{1-\alpha}k_t^{\alpha-1}-\delta-\tau_k}{\lambda}. \tag{2.14}$$

With this equilibrium relationship the model can be reduced to a dynamic system of two differential equations in capital and consumption. The law of motion of capital is given by

$$\dot{K}_t=Y_t-C_t-N_t-E_t-\delta K_t,$$

where E_t is the total amount of resources invested in monitoring. If $M(\lambda-\lambda_0)$ is the monitoring cost per unit of output invested in research, then E_t must equal $M(\lambda-\lambda_0)N_t$. Notice that in equilibrium $M(\lambda-\lambda_0)$ is a constant. Thus, in order to simplify, let us denote it by $e=M(\lambda-\lambda_0)=\frac{s(1+\tau_n)}{2\left(1+\tau_f\right)\left[\beta\left(1+\tau_f\right)-s\right]}$ so that E_t will be equal to eN_t.

The law of motion for consumption comes from utility maximization

$$\dot{C}_t=(r_t-\rho)C_t.$$

In order to obtain a system with steady state, express all variables in terms of efficiency units [9]

$$\dot{k}_t = L^{1-\alpha}k_t^{\alpha} - c_t - (1+\sigma)(1+e)n_t - (\delta + g_t)k_t \tag{2.15}$$

$$\dot{c}_t = (r_t - \rho - g_t)c_t, \tag{2.16}$$

and substitute the equilibrium expressions for r_t, g_t and n_t in equations (2.15) and (2.16) to express the system in terms of capital intensity and consumption per efficiency unit

$$\begin{aligned} \dot{k}_t &= L^{1-\alpha}k_t^{\alpha} - c_t - (1+\sigma)(1+e)n^d(k_t) - (\delta + g^d(k_t))k_t \\ \dot{c}_t &= (\alpha^2 L^{1-\alpha}k_t^{\alpha-1} - \delta - \tau_k - \rho - g^d(k_t))c_t. \end{aligned}$$

where

$$g^d(k_t) = \sigma\lambda n^d(k_t).$$

Due to the non-linearity of the system it must be linearized around the steady state in order to analyze the local dynamics. Accordingly, we will study the system at the steady state in the next section.

2.3 Steady State Analysis

In a steady state all variables grow at a constant rate. If we substitute the equilibrium values $m_{it} = k_t = \frac{K_t}{A_t}$ in the aggregate production function, we obtain the usual Cobb-Douglas functional form at the aggregate level

$$Y_t = (A_t L)^{1-\alpha} K_t^{\alpha}.$$

[9] Note that

$$A_t = \int_0^1 A_{it} di = A_t^{\max} \int_0^1 \frac{A_{it}}{A_t^{\max}} di = A_t^{\max} \int_0^1 a h(a) da = A_t^{\max} E(a) = \frac{A_t^{\max}}{1+\sigma}.$$

Therefore, $\frac{N_t}{A_t} = \frac{(1+\sigma)N_t}{A_t^{\max}} = (1+\sigma)\, n_t$.

This expression implies that the rate of growth of output will be that of the aggregate productivity coefficient and, given that A_t is proportional to the leading edge coefficient, the growth rate of the economy will be

$$g = \sigma\lambda n,$$

where λ and n are constant and determined jointly with k through the equilibrium conditions of research, capital and credit markets.[10] These conditions, evaluated at the steady state, are the following:

$$\frac{\lambda}{\Phi} = \frac{\alpha(1-\alpha)L^{1-\alpha}k^{\alpha}}{\rho + (1+\sigma)\lambda n}$$

$$\rho + \sigma\lambda n + \delta + \tau_k = \alpha^2 L^{1-\alpha}k^{\alpha-1} \tag{2.17}$$

$$\lambda = \left[\frac{\beta^2(1+\tau_f)(1+\tau_n)}{\beta(1+\tau_f) - s}\right]^{\frac{1}{2}},$$

from where we obtain

$$n = \frac{\Phi(\tau_f, \tau_n)}{\lambda^2}\frac{\alpha(1-\alpha)L^{1-\alpha}k^{\alpha}}{(1+\sigma)} - \frac{\rho}{(1+\sigma)\lambda}, \tag{2.18}$$

and the equation that implicitly determines the steady state value of k, which is the result of plugging (2.18) into (2.17)

$$F(k) = \frac{\rho}{(1+\sigma)} + \frac{\Phi(\tau_f, \tau_n)}{\lambda}\frac{\sigma}{(1+\sigma)}\alpha(1-\alpha)L^{1-\alpha}k^{\alpha} + \delta + \tau_k - \alpha^2 L^{1-\alpha}k^{\alpha-1} = 0. \tag{2.19}$$

The steady state growth rate can be expressed in terms of capital intensity using equation (2.17) to obtain

$$g = \alpha^2 L^{1-\alpha}k^{\alpha-1} - \rho - \delta - \tau_k.$$

[10]Variables without time suscript denote steady state values.

The use of implicit differentiation allows us to analyze the effect on k of parameter changes, and to obtain the following comparative statics results:

Proposition 1 *The steady state growth rate increases with subsidies to capital accumulation and to financial activity. The growth rate is decreasing (increasing) in τ_n when $\tau_n > -\frac{s}{2\beta(1+\tau_f)-s}$ $\left(\tau_n < -\frac{s}{2\beta(1+\tau_f)-s}\right)$.*

Proof. See Appendix. ■

Proposition 2 *The steady state growth rate is increasing in σ (the size of innovations), decreasing in ρ and δ and increasing in s (the scale parameter of the monitoring costs) and β (the scale parameter of the disutility of effort).*

Proof. See Appendix. ■

Proposition 1 establishes a marginal positive relation between financial activity and growth. This relation may be understood because a subsidy to financial activity (or equivalently a reduction in τ_f) implies a lower monitoring cost. Thus, monitoring intensity increases. Accordingly, the positive growth effect of this policy is due to the externality that financial activity causes on the accumulation of public knowledge. Promoting financial activity is equivalent to increase the productivity of R&D and thus, to make a better use of the resources allocated to research.

The result obtained for the growth effects of research subsidies reflects the moral hazard problem of R&D. The smaller cost of research represents an increase of the expected return for researchers that does not depend on the effort they exert. It can be shown that a lower τ_n reduces the no monitoring level of effort.[11] This implies a higher monitoring cost and, thus, λ falls. Therefore, even though we expect a positive effect on research intensity, the R&D productivity reduction may be enough to cause a negative effect on the growth rate.

Aghion and Howitt (1998) argue that capital accumulation and innovation are complementary factors for long run growth. To illustrate this assertion, they reduce the capital tax, a measure that directly affects the capital market, and study the reaction of the economy. The reduction of the cost of capital rises the equilibrium value of capital intensity making the flow of profits accruing to a successful innovator grow. Consequently, investment in the research sector will increase. Thus a policy that directly favors capital accumulation also incentives innovation and economic growth. The same argument can be applied in the present model. Therefore, innovation and capital accumulation continue being complementary factors for long run growth. Furthermore, this policy has no negative effects either on λ_0 or on λ. Thus, a subsidy to capital accumulation may be preferable in terms of growth to a direct subsidy to research.

We can perform the same experiment on financial activity. Thus, let us reduce the financial tax. The lower monitoring cost stimulates the production of financial services, inducing a rise in the arrival rate of innovations and, consequently, a larger rate of creative destruction. This discourages capital investment because the incumbent monopolist faces a larger probability of being replaced. Thus, the effect on capital accumulation is negative.

[11]The equilibrium expression for λ_0 is given by

$$\lambda_0 = \left[\frac{(1+\tau_n)\left[\beta\left(1+\tau_f\right)-s\right]}{(1+\tau_f)} \right]^{\frac{1}{2}}. \tag{2.20}$$

Thus, the result follows immediately.

That is, a policy that incentives financial activity will make the economy grow faster even though it will discourage capital investment. Therefore, capital and financial intensity should be considered substitutive factors for long run growth. Notice that this negative effect of research financing on capital accumulation undercuts the growth effects of intermediation promoting policies.

At the no-tax equilibrium a marginal reduction of any of the three taxes would increase the growth rate. In order to identify the most effective policy, the tax changes are made equivalent in terms of the amount of resources generated for the government budget. The budget constraint of the government is given by

$$\tau_n N_t + \tau_k K_t + \tau_f E_t = T,$$

where T is the lump-sum transference or tax used to balance the budget when we introduce a policy change. In order to make two policy changes equivalent, the change induced on T must be the same. Therefore, to compare the growth effects of τ_k, τ_f and τ_n, we must compare the following expressions:

$$\left.\frac{dg}{dT}\right|_{dT=K_t d\tau_k} = \frac{dg}{d\tau_k}\frac{d\tau_k}{dT} = \frac{dg}{d\tau_k}\frac{1}{K_t}$$

$$\left.\frac{dg}{dT}\right|_{dT=E_t d\tau_f} = \frac{dg}{d\tau_f}\frac{d\tau_f}{dT} = \frac{dg}{d\tau_f}\frac{1}{E_t}$$

$$\left.\frac{dg}{dT}\right|_{dT=N_t d\tau_n} = \frac{dg}{d\tau_n}\frac{d\tau_n}{dT} = \frac{dg}{d\tau_n}\frac{1}{N_t},$$

all evaluated at $\tau_f = \tau_k = \tau_n = 0$. This allows us to establish the following propositions:

Proposition 3 *At the no-tax equilibrium, the growth effect of τ_f is stronger than the growth effect of τ_n, i.e., $\frac{dg}{d\tau_f}\frac{1}{E_t} < \frac{dg}{d\tau_n}\frac{1}{N_t}$.*

Proof. See Appendix. ■

Proposition 4 *At the no-tax equilibrium, the growth effect of* τ_f *is stronger than the growth effect of* τ_k*, i.e.* $\frac{dg}{d\tau_f}\frac{1}{E_t} < \frac{dg}{d\tau_k}\frac{1}{K_t}$*, whenever*

$$\alpha(1-\alpha)L^{1-\alpha}k^{\alpha} < \frac{\lambda}{\Phi}\frac{2[\beta - s]}{s}\rho. \tag{2.21}$$

Proof. See Appendix. ■

Proposition 3 implies that, at the no-tax equilibrium, subsidizing the financial sector will be more growth promoting than directly subsidizing research. Similarly, Proposition 4 implies that the financial tax may have larger effects on growth than the capital tax. Therefore, there exist situations in which subsidizing financial activity is the most effective policy in order to improve the growth performance of the economy. Notice that in the case of Proposition 4, condition (2.21) is expressed in terms of k which is an endogenous variable. Consequently, it could happen that the condition is never satisfied. However, by means of calibration, it is relatively easy to find sets of parameters for which the condition is satisfied. Notice also that the effectiveness of the financial tax depends upon s, the scale parameter for monitoring costs. A small s means a large monitoring cost and a low monitoring intensity, e. Therefore, the lower the s, the smaller the relative amount of resources allocated to financial services in equilibrium and the stronger the marginal effect we can induce on monitoring intensity. To sum up, this result proposes the use of subsidies or tax cuts to financial activity as an alternative instrument to promote innovation without the moral hazard problems of direct research subsidies.

2.4 Dynamics

After analyzing the behavior of the economy at its long run equilibrium, the system can now be linearized so as to study the dynamics of the model around the steady state. Recall that the system is formed by the following equations:

$$\begin{aligned}
\dot{k}_t &= L^{1-\alpha}k_t^{\alpha} - c_t - (1+\sigma)(1+e)n^d(k_t) - (\delta + g^d(k_t))k_t \\
\dot{c}_t &= (\alpha^2 L^{1-\alpha}k_t^{\alpha-1} - \delta - \tau_k - \rho - g^d(k_t))c_t.
\end{aligned}$$

The linearized system is obtained computing the Jacobian of the system and evaluating it at the steady state. In order to simplify notation let us express the system as follows

$$\begin{aligned}
\dot{k}_t &= \varphi(k_t, c_t; \tau_k, \tau_f, \tau_n) \\
\dot{c}_t &= \phi(k_t, c_t; \tau_k, \tau_f, \tau_n).
\end{aligned}$$

Then the derivatives needed are the following:

$$\begin{aligned}
\varphi_k(k,c) &= \alpha L^{1-\alpha}k^{\alpha-1} - (1+\sigma)(1+e)n^{d\prime}(k_t) - (\delta+g) - k(g^{d\prime}(k)) \\
\varphi_c(k,c) &= -1 \\
\phi_k(k,c) &= c(-\alpha^2(1-\alpha)L^{1-\alpha}k^{\alpha-2} - g^{\prime d}(k)) \\
\phi_c(k,c) &= 0.
\end{aligned}$$

With this notation the linearized system will be

$$\begin{aligned}
\dot{k}_t &= \varphi_k(k,c)(k_t - k) - (c_t - c) \\
\dot{c}_t &= \phi_k(k,c)(k_t - k).
\end{aligned}$$

The determinant of the matrix of the system is equal to the function $\phi_k(k,c)$ evaluated at the steady state, which can be proved to be negative. Therefore the system presents local saddle path stability. For future reference, let λ_1 be the negative eigenvalue and λ_2 the positive one.

2.5 Welfare analysis

Now that we have characterized the dynamics of the system we can analyze the welfare implications of changes in tax parameters.

From equation (2.1) we can express utility at the steady state in terms of the stationary level of consumption and the long-run growth rate

$$V_s(c,g) = \int_0^{\infty} \ln(cA_t)e^{-\rho t}dt = \frac{\ln(cA_0)}{\rho} + \frac{g}{\rho^2}.$$

The change in steady state welfare is a combination of the change in steady state consumption and the change in steady state growth

$$\frac{\partial V_s(c,g)}{\partial \tau_i} = \frac{1}{\rho c}\frac{\partial c}{\partial \tau_i} + \frac{1}{\rho^2}\frac{\partial g}{\partial \tau_i} \qquad \text{for } i = k, f, n. \tag{2.22}$$

This measure of welfare is valid to compare two situations of long run equilibrium. However, it does not consider the periods of transition during which the economy moves from one equilibrium to another. In order to reflect the transition we must analyze the effect on lifetime utility. Rewrite equation (2.1) to obtain the following expression for lifetime utility as a function of the different tax rates (τ_i where $i = k, f, n.$):

$$V(\tau_i) = \frac{\ln(A_0)}{\rho} + \int_0^{\infty} \left[\int_0^t g_s(\tau_i)ds\right] e^{-\rho t}dt + \int_0^{\infty} \ln(c_t(\tau_i))e^{-\rho t}dt$$

where $g_t(\tau_i)$ and $c_t(\tau_i)$ are the time paths of the growth rate and the level of consumption per efficiency unit after a change in one of the tax parameters. The effect on utility will thus be given by the effects on the paths of growth and consumption. I will obtain first the effect on the paths of consumption and capital intensity and then use the latter to get the effect on the path of the growth rate.

Let $c = p(k, \tau_i)$ be the saddle path of the system which can be interpreted as the graph of a policy function relating consumption and capital. Then, we know that its slope p_k, is positive and equal to $\frac{\phi_k}{\lambda_1}$. Substituting the policy function into the law of motion of k, the equilibrium dynamics of the system can be characterized by a single differential equation which describes the evolution of the state variable along the stable manifold.

$$\dot{k} = \varphi(k, c) = \varphi(k, p(k, \tau_i)) = \Psi(k, \tau_i).$$

The solution to this equation, $k_t(\tau_i)$, gives the equilibrium value of k as a function of time and the tax parameter. Using $k_t(\tau_i)$ in the policy function we would obtain the time path of c

$$c_t(\tau_i) = p(k_t(\tau_i), \tau_i).$$

To calculate the change in welfare we need the derivative of the whole time path of c with respect to τ_i

$$\frac{dc_t(\tau_i)}{d\tau_i} = p_k \frac{dk_t(\tau_i)}{d\tau_i} + p_{\tau_i}, \tag{2.23}$$

where p_{τ_i} is the derivative of the policy function with respect to the tax or graphically, the shift in the saddle path caused by the policy change.

In order to compute $\frac{dk_t(\tau_i)}{d\tau_i}$, notice that $k_t(\tau_i) = k(t, \tau_i)$ must satisfy identically the original equation

$$\dot{k}(t, \tau_i) \equiv \varphi\left(p(k(t, \tau_i), \tau_i), k(t, \tau_i), \tau_i\right),$$

differentiate both sides with respect to τ_i

$$\dot{k}_{\tau_i} = \frac{dk_{\tau_i}}{dt} = \left[\varphi_c p_k + \varphi_k\right] k_{\tau_i} + \varphi_c p_{\tau_i} + \varphi_{\tau_i}.$$

Hence k_{τ_i} satisfies a linear differential equation. Moreover, when we start from a steady state, the coefficients of this equation are constant and we can write

$$\dot{k}_{\tau_i} = \lambda_1 k_{\tau_i} - p_{\tau_i} + \varphi_{\tau_i}.$$

The general solution is given by

$$k_{\tau_i}(t) = \exp\left(\lambda_1 t\right) k_{\tau_i}(0) + (1 - \exp\left(\lambda_1 t\right)) k_{\tau_i}(\infty).$$

Since k is a predetermined variable, the change at the date of the policy change $k_{\tau_i}(0)$ must be zero. The long run effect, $k_{\tau_i}(\infty) = \lim_{t\to\infty} k_{\tau_i}(t)$, is in fact the derivative of the steady state value of k with respect to the tax parameter, and can be expressed as

$$k_{\tau_i}(\infty) = \frac{p_{\tau_i} - \varphi_{\tau_i}}{\lambda_1}.$$

The equilibrium time path of the derivative of k with respect to τ_i is thus given by

$$k_{\tau_i}(t) = (1 - \exp(\lambda_1 t)) \left[\frac{p_{\tau_i} - \varphi_{\tau_i}}{\lambda_1}\right],$$

that is, k will gradually reach its new steady state value at a rate equal to the negative eigenvalue.

Substitute now in equation (2.23) to obtain the final expression for the derivative of the time path of consumption with respect to the tax parameter

$$\frac{dc_t(\tau_i)}{d\tau_i} = p_k (1 - \exp(\lambda_1 t)) \left[\frac{p_{\tau_i} - \varphi_{\tau_i}}{\lambda_1}\right] + p_{\tau_i}.$$

As before, we can identify the immediate change and the long run effect

$$\begin{aligned}\frac{dc_0(\tau_i)}{d\tau_i} &= p_{\tau_i}, \\ \frac{dc_\infty(\tau_i)}{d\tau_i} &= p_k\left[\frac{p_{\tau_i}-\varphi_{\tau_i}}{\lambda_1}\right]+p_{\tau_i},\end{aligned}$$

where the first represents the necessary jump of consumption to get on the new saddle path and the second is the effect on the steady state value of consumption. Thus, consumption will initially jump to the new saddle path and then it will approach its new steady state value at a rate equal to λ_1.

The derivative of the growth rate and consumption per efficiency unit at date t are given by

$$\frac{dg_t(\tau_i)}{d\tau_i} = \frac{dg^d(k)}{dk}\left(1-\exp(\lambda_1 t)\right)\frac{\partial k}{\partial \tau_i}+\frac{\partial g^d(k)}{\partial \tau_i} \tag{2.24}$$

$$\frac{dc_t(\tau_i)}{d\tau_i} = \frac{\partial c}{\partial \tau_i}-p_k\exp(\lambda_1 t)\frac{\partial k}{\partial \tau_i}. \tag{2.25}$$

Notice that the derivatives of g^d are evaluated at the steady state because we consider the stationary equilibrium as the situation before the tax change.

Expressions (2.24) and (2.25) allow us to write the change in welfare as follows:

$$\frac{\partial V(\tau_i)}{\partial \tau_i}=\frac{\partial V_s(\tau_i)}{\partial \tau_i}+\left[\frac{\frac{\rho-\lambda_1}{\rho}\frac{dg^d(k)}{dk}+\frac{(1-\alpha)\zeta}{k}}{\lambda_1\left(\rho-\lambda_1\right)}\right]\frac{\partial k}{\partial \tau_i}. \tag{2.26}$$

Equations (2.22) and (2.26) give the general expressions for the effect of the three taxes on the different measures of welfare. Let us see now the specific results for each policy.

2.5.1 Tax on capital

The effect on welfare of the capital tax is given by

$$\frac{\partial V_s(c,g)}{\partial \tau_k}=\frac{1}{\rho c}\frac{\partial c}{\partial \tau_k}+\frac{1}{\rho^2}\frac{\partial g}{\partial \tau_k}$$

$$\frac{\partial V(c,g)}{\partial \tau_k} = \frac{\partial V_s(c,g)}{\partial \tau_k} + \left[\frac{\frac{\rho - \lambda_1}{\rho} \frac{dg^d(k)}{dk} + \frac{(1-\alpha)\zeta}{k}}{\lambda_1 (\rho - \lambda_1)} \right] \frac{\partial k}{\partial \tau_k}, \tag{2.27}$$

where the first expression represents the effect on welfare if the transition is excluded. Both the expression in square brackets in equation (2.27) and $\frac{\partial k}{\partial \tau_k}$ are negative. Therefore, the effect on welfare using the second measure will always be larger than the effect if we use the first measure.

Proposition 1 shows that $\frac{\partial g}{\partial \tau_k}$ is negative. However, the effect on consumption is ambiguous. The derivative of consumption with respect to the capital tax is given by

$$\frac{\partial c}{\partial \tau_k} = \frac{k}{1 + \frac{\Phi}{\lambda} \frac{\sigma}{1+\sigma} k} \left(-\frac{1}{\alpha} + \frac{(1+e)\Phi}{\lambda^2} - \frac{\rho + \tau_k}{(1-\alpha)\zeta} + \frac{\Phi}{\lambda} \frac{\sigma}{1+\sigma} k \right).$$

The functional form of this derivative implies that for large enough values of steady state capital intensity, the derivative will be positive while it may be negative for smaller values of k. Since the relationship between k and the capital tax is negative, this suggests that for negative or small values of τ_k we might expect a positive effect on consumption while for large values of the tax, $\frac{\partial c}{\partial \tau_k}$ may become negative. Therefore, we may roughly represent the relationship between consumption and the capital tax as an inverted U-shaped curve whose maximum shifts right or left depending on the structural characteristics of the economy. In summary, there may exist a consumption maximizing value of τ_k but whether it is a subsidy or a tax depends upon the economy considered. These results can also be applied to the relationship between welfare and this tax. I have calibrated the model for a usually accepted set of parameters obtaining that in every case, the welfare maximizing rate of this policy instrument was a subsidy.[12] Consequently, in economies with a positive capital tax

[12] The set of parameters used includes $\rho = 0.02$, $\delta = 0.05$, $\sigma = \ln(1.1)$ and $L = 1$. The values of β and s were chosen so that the resulting steady state values of the growth rate and the probability of success lay in a reasonable interval. The computer program used for calibration is available upon request.

rate, a tax reduction will generally cause a welfare improvement.

2.5.2 Tax on financial services

The welfare derivatives for the financial tax are

$$\frac{\partial V_s(c,g)}{\partial \tau_f} = \frac{1}{\rho c}\frac{\partial c}{\partial \tau_f} + \frac{1}{\rho^2}\frac{\partial g}{\partial \tau_f}$$

$$\frac{\partial V(c,g)}{\partial \tau_f} = \frac{\partial V_s(c,g)}{\partial \tau_f} + \left[\frac{\frac{\rho-\lambda_1}{\rho}\frac{dg^d(k)}{dk} + \frac{(1-\alpha)\zeta}{k}}{\lambda_1\left(\rho-\lambda_1\right)}\right]\frac{\partial k}{\partial \tau_f},$$

and given that $\frac{\partial k}{\partial \tau_f}$ is positive, the effect on welfare of this tax will always be smaller if we consider the transition.

As before, we know that the derivative of the growth rate with respect to this tax is negative. The effect on consumption is given by

$$\begin{aligned} \frac{\partial c}{\partial \tau_f} &= (1-\alpha)\,\zeta\frac{\partial k}{\partial \tau_f}\left(\frac{1+\alpha}{\alpha} - \frac{(1+e)\Phi}{\lambda^2} + \frac{\rho+\tau_k}{(1-\alpha)\,\zeta}\right) + \\ &\quad + \left[-\frac{\partial}{\partial \tau_f}\frac{\Phi\,(1+e)}{\lambda^2}\right]\alpha\,(1-\alpha)\,L^{1-\alpha}k^{\alpha} + \rho\left[\frac{\partial}{\partial \tau_f}\frac{1+e}{\lambda}\right]. \end{aligned} \tag{2.28}$$

In order to simplify the analysis, the range of values of the tax parameters is restricted so that we can give an unambiguous sign to this derivative. To this end, we will not consider values of the capital tax rate below $-\rho$ nor subsidy rates to the research sector above $\frac{5}{7}$. Under these assumptions, we can establish the following proposition:

Proposition 5 *If $\tau_k > -\rho$ and $\tau_n > -\frac{5}{7}$, the derivative of steady state consumption per efficiency unit with respect to the financial tax is positive.*

Proof. See Appendix ■

Consequently, a marginal change in the financial tax will cause opposite effects on growth and consumption, depending the final change in welfare on which effect dominates. Obviously, the value of the discount rate is determinant for the sign of $\frac{\partial V_s(c,g)}{\partial \tau_f}$. This derivative will be positive whenever $\frac{\partial c}{\partial \tau_f} + \frac{c}{\rho}\frac{\partial g}{\partial \tau_f}$ is positive. A small ρ means that consumers weight more heavily the growth effect of the tax. Thus, if ρ is small enough, welfare will increase with reductions of the financial tax. Notice also that for a given discount rate, increases in τ_f make steady state consumption per efficiency unit grow. Therefore, we may expect positive effects on welfare for low values of the tax though they may disappear as the tax rate increases. Hence, we also find the inverted U-shaped curve representing the relationship between welfare and the financial tax.

A calibration of the model gives a rough idea of how can financial policies improve welfare. At the no tax equilibrium and for the same set of parameters used before, I obtain the following results:

Table 2.1
Welfare effect of τ_f

$\boldsymbol{\alpha}$	$\frac{\partial V_s}{\partial \tau_f}$	$\frac{\partial V}{\partial \tau_f}$
0.80	-0.014	-0.031
0.75	-0.010	-0.023
0.70	-0.005	-0.015
0.65	-0.002	-0.007
0.60	0.001	-0.002
0.55	0.004	0.002
0.50	0.005	0.004
0.45	0.005	0.005
0.40	0.005	0.005
0.35	0.003	0.003

A negative sign of the welfare derivative means that the optimal policy is to reduce the financial tax. Conversely, a positive entry implies that the optimal policy is a tax increase. This calibration suggests that financial services will be underprovided in a relatively capital intensive economy while in less capital intensive economies, a reduction of its provision could increase welfare. Recall that the financial sector has real effects on the economy only because it can modify the productivity of research. A high α means a relatively high equilibrium value of k which in turn implies a high research intensity. Therefore, a policy that favors monitoring and thus, increases the productivity of research, will have larger growth effects in an economy with a relatively higher research intensity. This larger growth effect will be able to compensate for the reduction in steady state consumption per efficiency unit. On the contrary, if α is small, so is equilibrium research intensity and thus, the higher productivity in this case will not be able to induce a large enough increase in the growth rate.

2.5.3 Tax on research activity

The welfare derivatives for the research tax are

$$\frac{\partial V_s(c,g)}{\partial \tau_n} = \frac{1}{\rho c}\frac{\partial c}{\partial \tau_n} + \frac{1}{\rho^2}\frac{\partial g}{\partial \tau_n}$$

$$\frac{\partial V(c,g)}{\partial \tau_n} = \frac{\partial V_s(c,g)}{\partial \tau_n} + \left[\frac{\frac{\rho-\lambda_1}{\rho}\frac{dg^d(k)}{dk} + \frac{(1-\alpha)\zeta}{k}}{\lambda_1\left(\rho-\lambda_1\right)}\right]\frac{\partial k}{\partial \tau_n},$$

and as with the financial tax, the fact that $\frac{\partial k}{\partial \tau_n}$ is positive makes the effect on welfare of this tax smaller if we consider the transition.

The derivative of steady state consumption per efficiency unit is given by the following expression:

$$\frac{\partial c}{\partial \tau_n} = (1-\alpha)\,\zeta \frac{\partial k}{\partial \tau_n}\left(\frac{1+\alpha}{\alpha} - \frac{(1+e)\Phi}{\lambda^2} + \frac{\rho+\tau_k}{(1-\alpha)\,\zeta}\right) + \\ + \left[-\frac{\partial}{\partial \tau_n}\frac{\Phi\,(1+e)}{\lambda^2}\right]\alpha\,(1-\alpha)\,L^{1-\alpha}k^{\alpha} + \rho\left[\frac{\partial}{\partial \tau_n}\frac{1+e}{\lambda}\right].$$

The effect of the research tax on consumption is established in the next proposition:

Proposition 6 *If $\tau_n > -\frac{s}{2\beta(1+\tau_f)-s}$ and $\tau_k > -\rho$, the derivative of steady state consumption per efficiency unit with respect to the research tax is positive.*

Proof. See Appendix. ■

Given that the effect on growth of this tax is negative, the final effect on welfare will depend upon the discount rate.[13] As with the financial tax, if ρ is small enough, welfare may increase with a reduction of research taxation. In general though, we expect the typical inverted-U relationship in the sense that increases of the research tax may initially improve welfare though further increases could finally harm it.

If the government were considering whether to subsidize the research or the financial sector, we know that the financial tax will have larger effects on growth and in this sense it would be preferable.[14] However, we must consider also the effect on consumption. We would like to have the result that the effect on consumption of the financial subsidy is smaller since consumption will be reduced. However, we find the opposite result. That is, a financial subsidy will cause a larger reduction in steady state consumption per efficiency unit than a research subsidy. Consequently, whether one policy is preferable to the other

[13] I will restrict the rest of the welfare analysis of this tax to $\tau_n > -\frac{s}{2\beta(1+\tau_f)-s}$, because the sign of the derivative of consumption for smaller values of τ_n is ambiguous.

[14] In what follows, I assume that the initial situation is the no-tax equilibrium. Therefore, the effect on growth of the two subsidies is positive being the financial tax more effective.

in terms of welfare will depend upon the discount rate of the economy. A calibration of the model for $\rho = 0.02$, yields the following results:

Table 2.2
Welfare effects of τ_f and τ_n

$\boldsymbol{\alpha}$	$\frac{\partial V_s}{\partial \tau_f}\frac{1}{e}$	$\frac{\partial V_s}{\partial \tau_n}$	$\frac{\partial V}{\partial \tau_f}\frac{1}{e}$	$\frac{\partial V}{\partial \tau_n}$
0.80	-14.0	5.7	-30.9	5.7
0.75	-10.0	5.4	-22.9	5.4
0.70	-5.0	5.1	-15.0	5.1
0.65	-2.0	4.8	-7.5	4.8
0.60	1.0	4.3	-1.6	4.3
0.55	4.0	3.8	2.4	3.8
0.50	5.0	3.1	4.6	3.1
0.45	5.0	2.4	5.2	2.4
0.40	5.0	1.6	4.7	1.6
0.35	3.0	0.8	3.3	0.8

Notice that the sign of the welfare derivative with respect to the research tax is positive in every case. This means that a subsidy (a marginal reduction of the tax) would reduce welfare. In other words, the positive growth effect is not enough to compensate for the negative effect on steady state consumption per efficiency unit. Therefore, if the government wishes to increase welfare, the appropriate policy is a research tax increase. With respect to the other policy instrument, the financial tax, the effect on welfare of the latter is larger when α is either very large or very small. Thus, if we consider $\alpha = 0.75$ as a proxy for the capital intensity of a developed economy, a policy that promotes the financing of research projects by intermediaries dominates a direct subsidy to research both in terms of growth and welfare.

2.6 Conclusions

Innovation is nowadays recognized as one of the most important factors of economic growth. However, the presence of informational asymmetries and the difficult appropriation of R&D's external effects cause inefficiencies that may reduce the private production of innovation. This paper analyses the consequences on economic growth of the activity of financial intermediaries that try to reduce the incidence of moral hazard on research. There exists moral hazard because in the absence of monitoring, researchers choose the amount of effort that maximizes their expected utility, a smaller level of effort than the one that would maximize the expected value of the project. The no-monitoring level of effort is smaller because the researcher receives only a part of the value of the innovation while the rest goes to the intermediary. However, the intermediary is provided with a monitoring technology that enables him to impose a higher effort. The monitoring intensity will determine the amount of effort affordable and the probability of success of the research project. This paper shows that a policy that incentives monitoring is able to improve the growth performance of the economy due to its positive effect on R&D productivity. Furthermore, it is shown that directly subsidizing research may reduce the growth rate of the economy. The negative effect on growth of a research subsidy may appear because it accentuates the incidence of moral hazard. As a consequence, this paper proposes subsidies to capital accumulation and to financial activity as alternative growth promoting policies. The advantage of these policies with respect to the research subsidy is that they do not see their effects undercut by a reduction of R&D productivity.

A subsidy to financial activity increases the growth rate of the economy. However, its effect on steady state consumption per efficiency unit is negative. Therefore, the actual value of the discount rate will determine the sign of the welfare effect in each case. Nevertheless, for a typical value of the discount rate, it is obtained that financial services will be underprovided in relatively capital intensive economies while they will be overprovided in less capital intensive economies. This may be due to the interaction of two externalities of opposite sign. On the one hand, the positive effect of financial activities on R&D productivity makes the whole economy more productive since the growth rate of aggregate productivity depends positively on the arrival rate of innovations. However, the magnitude of this positive effect depends upon the relative importance of the research sector which in turn is determined by capital intensity. Thus, the more capital intensive the economy, the greater this effect will be. On the other hand, a higher probability of success due to a more intense monitoring implies a higher probability of replacement for the incumbent producer. This discourages capital accumulation. Whether the reduction in the equilibrium level of capital causes a large or a small effect depends upon the initial situation of the economy. If capital intensity was relatively low, the initial equilibrium level of capital is relatively small and a further reduction will have large negative effects on the economy. On the contrary, if the economy was in an equilibrium with a large level of capital per efficiency unit, a reduction will not represent a big damage. Thus, the positive externality is stronger when capital intensity is high, while the negative externality has larger effects when the economy is less capital intensive. Therefore, policies aimed at balancing the effects of the two externalities will be welfare improving.

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2.7 Proofs of propositions

Proof that $H(a)$ is the limiting distribution of relative productivities.

(Adapted from Aghion and Howitt (1998))

Let $F(\cdot, t)$ denote the cumulative distribution of the absolute productivity parameters, A, across sectors at date t. Pick any $A > 0$ and let it be the leading edge coefficient at $t_0 \geq 0$. Define $\Phi(t) = F(A, t)$. Then

$$\begin{aligned} \Phi(t_0) &= 1 \\ \frac{d\Phi(t)}{dt} &= -\Phi(t)\lambda_t n_t \text{ for all } t \geq t_0. \end{aligned} \tag{2.29}$$

Equation (2.29) gives the rate at which the fraction of sectors with a productivity coefficient smaller than A falls. This rate is given by the flow of innovations occurred in the sectors behind A, i.e. $\Phi(t)\lambda_t n_t$. The solution to this differential equation is

$$\Phi(t) = e^{-\int_{t_0}^{t} \lambda_s n_s ds} \text{ for all } t \geq t_0.$$

Recall that

$$\frac{dA_t^{\max}}{dt} = \sigma A_t^{\max} \lambda_t n_t$$

and that $A = A_{t_0}^{\max}$, therefore

$$\frac{A}{A_t^{\max}} = e^{-\sigma \int_{t_0}^{t} \lambda_s n_s ds},$$

or equivalently

$$\Phi(t) = \left(\frac{A}{A_t^{\max}} \right)^{\frac{1}{\sigma}}.$$

Define a to be the relative productivity $\frac{A}{A_t^{\max}}$. By construction, $\Phi(t)$ is the fraction of sectors in which the productivity coefficient is less than A. Hence, the last equation establishes that this fraction is given by equation (2.3) at date t if a is the relative productivity at t of a sector that innovated on or after date t_0. If t is large enough, this will include almost all values of a between 0 and 1. ■

Proof of Proposition 1. The signs of the derivatives of the growth rate depend upon the signs of the derivatives of the steady state capital intensity. Consider equation (2.19) which defines the steady state value of k. Straightforward differentiation yields

$$\begin{aligned}
\frac{\partial F(k)}{\partial k} &= \alpha^2(1-\alpha)L^{1-\alpha}k^{\alpha-2}\left[1+\frac{\sigma}{(1+\sigma)}\frac{\Phi}{\lambda}k\right] \\
\frac{\partial F(k)}{\partial \tau_k} &= 1 \\
\frac{\partial F(k)}{\partial \tau_f} &= \frac{\partial}{\partial \tau_f}\left(\frac{\Phi(\tau_f,\tau_n)}{\lambda}\right)\frac{\sigma\alpha(1-\alpha)L^{1-\alpha}k^{\alpha}}{(1+\sigma)} \\
\frac{\partial F(k)}{\partial \tau_n} &= \frac{\partial}{\partial \tau_n}\left(\frac{\Phi(\tau_f,\tau_n)}{\lambda}\right)\frac{\sigma\alpha(1-\alpha)L^{1-\alpha}k^{\alpha}}{(1+\sigma)},
\end{aligned} \tag{2.30}$$

where

$$\frac{\partial}{\partial \tau_f}\left(\frac{\Phi(\tau_f,\tau_n)}{\lambda}\right)=\frac{\Phi e}{\lambda(1+\tau_n)}\frac{(1+\tau_n)\,s-2\left[\beta(1+\tau_f)-s\right]}{(1+\tau_n)\left[2\beta(1+\tau_f)-s\right]+2\left[\beta(1+\tau_f)-s\right]},$$

expression which is negative for the range of values assumed for the parameters. The sign of the derivative in (2.30) depends upon $\frac{\partial}{\partial \tau_n}\left(\frac{\Phi(\tau_f,\tau_n)}{\lambda}\right)$ given by

$$\frac{\partial}{\partial \tau_n}\left(\frac{\Phi(\tau_f,\tau_n)}{\lambda}\right)=\frac{\Phi}{2\lambda(1+\tau_n)}\frac{2\left[\beta(1+\tau_f)-s\right]-(1+\tau_n)\left[2\beta(1+\tau_f)-s\right]}{(1+\tau_n)\left[2\beta(1+\tau_f)-s\right]+2\left[\beta(1+\tau_f)-s\right]}.$$

This derivative is negative if and only if $\tau_n > -\frac{s}{2\beta(1+\tau_f)-s}$. Therefore,

$$\frac{\partial k}{\partial \tau_k} = -\frac{\frac{\partial F(k)}{\partial \tau_k}}{\frac{\partial F(k)}{\partial k}} < 0,$$

$$\frac{\partial k}{\partial \tau_f} = -\frac{\frac{\partial F(k)}{\partial \tau_f}}{\frac{\partial F(k)}{\partial k}} > 0,$$

$$\frac{\partial k}{\partial \tau_n} = -\frac{\frac{\partial F(k)}{\partial \tau_n}}{\frac{\partial F(k)}{\partial k}} \geq 0 \qquad \text{for } \tau_n \geq -\frac{s}{2\beta(1+\tau_f)-s}$$

and

$$\frac{\partial k}{\partial \tau_n} = -\frac{\frac{\partial F(k)}{\partial \tau_n}}{\frac{\partial F(k)}{\partial k}} < 0 \qquad \text{for } \tau_n < -\frac{s}{2\beta(1+\tau_f)-s}.$$

Given the signs of the derivatives of k with respect to the different taxes, the effects on growth can be obtained recalling that the following equation must hold in equilibrium:

$$g = \alpha^2 L^{1-\alpha} k^{\alpha-1} - \rho - \delta - \tau_k.$$

Consequently, the derivative of the growth rate with respect to the capital tax is given by

$$\frac{\partial g}{\partial \tau_k} = -\left(1-\alpha\right)\alpha^2 L^{1-\alpha} k^{\alpha-2} \frac{\partial k}{\partial \tau_k} - 1,$$

or equivalently

$$\frac{\partial g}{\partial \tau_k} = \frac{-\frac{\sigma}{(1+\sigma)}\frac{\Phi(\tau_f,\tau_n)}{\lambda}k}{\left[1+\frac{\sigma}{(1+\sigma)}\frac{\Phi(\tau_f,\tau_n)}{\lambda}k\right]},$$

which is unambiguously negative. Therefore, the growth rate depends negatively on the capital tax and thus, a subsidy increase or a reduction of the tax would enhance growth.

The derivatives of the growth rate with respect to the financial tax and to the innovation tax are

$$\frac{\partial g}{\partial \tau_f} = -(1-\alpha)\alpha^2 L^{1-\alpha} k^{\alpha-2} \frac{\partial k}{\partial \tau_f},$$

and

$$\frac{\partial g}{\partial \tau_n} = -(1-\alpha)\alpha^2 L^{1-\alpha} k^{\alpha-2} \frac{\partial k}{\partial \tau_n}.$$

Given the signs of the derivatives of k we have previously obtained, the corresponding results of Proposition 1 follow. ■

Proof of Proposition 2. The derivative of k with respect to σ is given by the following expression:

$$\frac{\partial k}{\partial \sigma} = \frac{-\lambda n}{(1+\sigma)\,\alpha^2(1-\alpha)L^{1-\alpha}k^{\alpha-2}\left[1+\frac{\sigma}{(1+\sigma)}\frac{\Phi(\tau_f,\tau_n)}{\lambda}k\right]},$$

which is negative. Thus, capital intensity at the steady state is negatively related to σ. In consequence, the derivative of g with respect to σ is positive.

The other two results are immediate since the derivative of g with respect to δ is equal to the derivative with respect to τ_k and the derivative of k with respect to ρ satisfies

$$\frac{\partial k}{\partial \rho} = \left[\frac{1}{1+\sigma}\right]\frac{\partial k}{\partial \tau_k}.$$

Therefore, if the derivative of g with respect to τ_k is negative, so is the derivative of g with respect to ρ.

Regarding the effect on the growth rate of changes in s and β, notice that

$$\frac{\partial F(k)}{\partial s}=\frac{\sigma\alpha(1-\alpha)L^{1-\alpha}k^{\alpha}}{(1+\sigma)}\frac{\partial\left(\frac{\Phi}{\lambda}\right)}{\partial s},$$

and

$$\frac{\partial F(k)}{\partial \beta}=\frac{\sigma\alpha(1-\alpha)L^{1-\alpha}k^{\alpha}}{(1+\sigma)}\frac{\partial\left(\frac{\Phi}{\lambda}\right)}{\partial \beta},$$

where

$$\frac{\partial}{\partial s}\left(\frac{\Phi}{\lambda}\right)=\frac{\Phi}{\lambda}\frac{[2\beta(1+\tau_f)-(3+\tau_n)s]}{2\left[\beta(1+\tau_f)-s\right]\left[(1+\tau_n)\left[2\beta(1+\tau_f)-s\right]+2\left[\beta(1+\tau_f)-s\right]\right]}$$

$$\frac{\partial}{\partial \beta}\left(\frac{\Phi}{\lambda}\right)=\frac{\Phi}{\lambda\beta}\frac{[\beta(1+\tau_f)-2s]+(1+\tau_n)\frac{\left[\beta(1+\tau_f)\left[2\beta(1+\tau_f)-3s\right]+2s^2\right]}{2\left[\beta(1+\tau_f)-s\right]}}{\left[(1+\tau_n)\left[2\beta(1+\tau_f)-s\right]+2\left[\beta(1+\tau_f)-s\right]\right]},$$

are both positive. Therefore, $\frac{\partial F(k)}{\partial s}$ and $\frac{\partial F(k)}{\partial \beta}$ are also positive, which implies that $\frac{\partial k}{\partial s}$ and $\frac{\partial k}{\partial \beta}$ are negative. Therefore, the derivatives of the growth rate with respect to these parameters are both positive. ■

Proof of Proposition 3. $\frac{dg}{d\tau_f}\frac{1}{E_t}<\frac{dg}{d\tau_n}\frac{1}{N_t}$ holds if and only if $\frac{dg}{d\tau_f}\frac{1}{e}<\frac{dg}{d\tau_n}$. At the no tax equilibrium this inequality is given by the following expression:

$$-\frac{(1-\alpha)\alpha^2L^{1-\alpha}k^{\alpha-2}}{e}\frac{\partial k}{\partial\tau_f}<-(1-\alpha)\alpha^2L^{1-\alpha}k^{\alpha-2}\frac{\partial k}{\partial\tau_n},$$

or equivalently

$$\frac{1}{e}\frac{\partial k}{\partial\tau_f}>\frac{\partial k}{\partial\tau_n}.$$

This inequality holds whenever

$$\frac{1}{e}\frac{\partial}{\partial \tau_f}\left(\frac{\Phi(\tau_f,\tau_n)}{\lambda}\right) < \frac{\partial}{\partial \tau_n}\left(\frac{\Phi(\tau_f,\tau_n)}{\lambda}\right).$$

Evaluating both derivatives at the no-tax equilibrium and simplifying we obtain that the condition for the inequality to hold is

$$s < \frac{4}{7}\beta.$$

The parameters involved in the last expression (s and β) must be positive and satisfy the following condition:

$$s < \frac{\beta}{2}(1+\tau_f),$$

which is necessary to guarantee a positive expected value of the project for the researcher. Therefore, at the no-tax equilibrium, the growth effect of τ_f is larger than the growth effect of τ_n. ■

Proof of Proposition 4. The growth effect of τ_f is larger in absolute value than the growth effect of τ_k when $\frac{dg}{d\tau_f}\frac{1}{E_t} < \frac{dg}{d\tau_k}\frac{1}{K_t}$ which at the steady state is equivalent to require that $\frac{dg}{d\tau_f}\frac{1}{(1+\sigma)en} < \frac{dg}{d\tau_k}\frac{1}{k}$. Evaluating both derivatives at the no-tax equilibrium and simplifying yields the desired expression, i.e., $\alpha\left(1-\alpha\right)L^{1-\alpha}k^{\alpha} < \frac{\lambda}{\Phi}\frac{2[\beta-s]}{s}\rho$. ■

Proof of Proposition 5. The derivative of c with respect to τ_f is given by equation (2.28). In order to obtain positive values of steady state consumption, we assume that the parameters are such that $\frac{1+\alpha}{\alpha} - \frac{(1+e)\Phi}{\lambda^2} > 0$. Under this assumption, the first term of this expression is positive and so is the second. However, the last term may be positive or negative depending on the actual values of τ_f and τ_n. Nevertheless, from equation (2.14) we can express this derivative as follows:

$$\frac{\partial c}{\partial \tau_f} = (1-\alpha)\,\zeta \frac{\partial k}{\partial \tau_f}\left(\frac{1+\alpha}{\alpha} - \frac{(1+e)\Phi}{\lambda^2} + \frac{\rho+\tau_k}{(1-\alpha)\,\zeta}\right) + \tag{2.31}$$
$$+\left[-\frac{\partial}{\partial \tau_f}\left(\frac{\Phi\,(1+e)}{\lambda^2}\right)\right]\frac{\lambda^2}{\Phi}(1+\sigma)\,n + \rho\left[\frac{\partial}{\partial \tau_f}\left(\frac{1+e}{\lambda}\right) - \frac{\partial}{\partial \tau_f}\left(\frac{\Phi\,(1+e)}{\lambda^2}\right)\frac{\lambda^2}{\Phi}\right],$$

where the first term is positive because $\frac{\partial k}{\partial \tau_f}$ is positive, $\rho + \tau_k$ is positive under the assumptions of the proposition and we had previously assumed that the parameters must be such that $\frac{1+\alpha}{\alpha} > \frac{(1+e)\Phi}{\lambda^2}$ in order to guarantee a positive level of consumption in equilibrium.

The second term of (2.31) will be positive whenever $\frac{\partial}{\partial \tau_f}\left(\frac{\Phi(1+e)}{\lambda^2}\right)$ is negative. This derivative is given by the following expression, which is negative when $\tau_n > -\frac{5}{7}$:

$$\frac{\partial}{\partial \tau_f}\left(\frac{\Phi\,(1+e)}{\lambda^2}\right) = \frac{\Phi e}{\lambda^2}\frac{2\beta(1+\tau_f)^2 - (1+\tau_n)\,[4\beta(1+\tau_f)-s] - 2\,[2\beta(1+\tau_f)-s]}{(1+\tau_f)\,[(1+\tau_n)\,[2\beta(1+\tau_f)-s] + 2\,[\beta(1+\tau_f)-s]]}.$$

The third term of (2.31) may be expressed as follows:

$$\frac{\rho e}{(1+\tau_f)}\left[\frac{2\,(\tau_f-\tau_n)}{(1+\tau_n)} + \frac{(1+\tau_n)\,[4\beta(1+\tau_f)-s] + 2\,[2\beta(1+\tau_f)-s] - 2\beta(1+\tau_f)^2}{(1+\tau_n)\,[2\beta(1+\tau_f)-s] + 2\,[\beta(1+\tau_f)-s]}\right]. \tag{2.32}$$

For $\tau_n > -\frac{5}{7}$ and $\tau_f \geq \tau_n$, this expression is positive. However, if $\tau_f < \tau_n$ the sign of the whole expression is not so obvious. When $\tau_f < \tau_n$, the second term of expression (2.32) is increasing in s. Therefore, it will approach its minimum value when s goes to zero. This implies that

$$\frac{(1+\tau_n)\left[4\beta(1+\tau_f)-s\right]+2\left[2\beta(1+\tau_f)-s\right]-2\beta(1+\tau_f)^2}{(1+\tau_n)\left[2\beta(1+\tau_f)-s\right]+2\left[\beta(1+\tau_f)-s\right]}>\frac{2\left(1+\tau_n\right)+2-(1+\tau_f)}{(2+\tau_n)},$$

or equivalently that the term in brackets of equation (2.32) is larger than $\frac{(1+\tau_f)(3+\tau_n)}{(1+\tau_n)(2+\tau_n)}$ which is positive for all values of τ_f and τ_n between -1 and 1.

In summary, it has been shown that the three terms are positive for the range of values of τ_n and τ_k considered. Therefore, the derivative in (2.31) is positive. ■

Proof of Proposition 6. The derivative of steady state consumption per efficiency unit with respect to the research tax is given by the following expression:

$$\begin{aligned}\frac{\partial c}{\partial \tau_n} &= (1-\alpha)\,\zeta\frac{\partial k}{\partial \tau_n}\left(\frac{1+\alpha}{\alpha}-\frac{(1+e)\Phi}{\lambda^2}+\frac{\rho+\tau_k}{(1-\alpha)\,\zeta}\right)+\\ &\quad+\left[-\frac{\partial}{\partial \tau_n}\left(\frac{\Phi\,(1+e)}{\lambda^2}\right)\right]\alpha\,(1-\alpha)\,L^{1-\alpha}k^{\alpha}+\rho\left[\frac{\partial}{\partial \tau_n}\left(\frac{1+e}{\lambda}\right)\right],\end{aligned} \tag{2.33}$$

where the first term is positive since we have imposed $\frac{1+\alpha}{\alpha}>\frac{(1+e)\Phi}{\lambda^2}$. The second term is also positive since

$$\frac{\partial}{\partial \tau_n}\left(\frac{\Phi\,(1+e)}{\lambda^2}\right)=\frac{\Phi}{\lambda^2}\frac{s-(1+\tau_f)\left[2\beta\,(1+\tau_f)-s\right]}{(1+\tau_f)\left[(1+\tau_n)\left[2\beta\,(1+\tau_f)-s\right]+2\left[\beta\,(1+\tau_f)-s\right]\right]},$$

is negative. However, the last term has an ambiguous sign. The derivative in brackets may be expressed as

$$\frac{\partial}{\partial \tau_n}\left(\frac{1+e}{\lambda}\right)=\frac{e-1}{2\lambda\,(1+\tau_n)}.$$

Thus, the sum of the second and third term of (2.33) yields

$$\frac{-\frac{\Phi}{\lambda^2}\alpha\,(1-\alpha)\,L^{1-\alpha}k^{\alpha}\left[s-(1+\tau_f)\left[2\beta\,(1+\tau_f)-s\right]\right]}{(1+\tau_f)\left[(1+\tau_n)\left[2\beta\,(1+\tau_f)-s\right]+2\left[\beta\,(1+\tau_f)-s\right]\right]}+\rho\frac{e-1}{2\lambda\,(1+\tau_n)}. \tag{2.34}$$

Next, use (2.18) in order to write expression (2.34) as follows:

$$\frac{\Phi n(1+\sigma)\left[\frac{2\beta\left(1+\tau_f\right)-s}{\left(1+\tau_f\right)}-\frac{s}{\left(1+\tau_f\right)^2}\right]}{2\beta^2\left(1+\tau_f\right)\left(1+\tau_n\right)}+\frac{\rho\frac{\Phi}{\lambda}\left[\frac{(1+\tau_n)\left[2\beta\left(1+\tau_f\right)-s\right]}{\beta\left(1+\tau_f\right)-s}-2\right]\left[\frac{2\left[\beta\left(1+\tau_f\right)-s\right]}{(1+\tau_n)}+\frac{s}{\left(1+\tau_f\right)}\right]}{8\beta^2\left(1+\tau_f\right)\left(1+\tau_n\right)}$$

The first term is positive while the sign of the second term is determined by

$$\frac{(1+\tau_n)\left[2\beta\left(1+\tau_f\right)-s\right]}{\beta\left(1+\tau_f\right)-s}-2,$$

expression that happens to be positive for $\tau_n>-\frac{s}{2\beta\left(1+\tau_f\right)-s}$. ■

Chapter 3

Research Policy and Endogenous Growth

3.1 Introduction

The objective of this paper is to study the effect of public research policy on both the productivity of private R&D and the growth performance of the economy. In order to do so, we will consider different research policies in the context of an endogenous growth model, where we make explicit the difference between basic and applied research.

Previous literature on public intervention in the research sector is mainly undertaken from the industrial organization perspective. Papers on this area are generally concerned with the microeconomic effects of research subsidies and patent policy. Some attention has been paid, however, to public research. The papers by Mamuneas and Nadiri (1996), Ham and Mowery (1998) and Mamuneas (1999) provide microeconomic foundations for the hypothesis that public R&D causes positive external effects on private productivity. In addition, Mamuneas and Nadiri (1996) find econometric evidence that publicly financed R&D induces cost savings but crowds out privately financed R&D investment.

There are few papers that consider public research investment from a macroeconomic perspective. Glomm and Ravikumar (1994) present a model in which the economy grows thanks to public research. However, this paper is focussed on distributional problems and, therefore, the presence of public research in this model is just a simplifying assumption in order to obtain endogenous technological innovation without the difficulties that would imply the introduction of a private R&D sector. Pelloni (1997) allows the government to invest in public research so as to improve the growth performance of the economy but does not allow for private research. On the contrary, Park (1998) considers both public and private research. This author introduces public research in the model of expanding variety of products first presented in Romer (1990). He assumes that public research indirectly contributes to economic growth because it causes a positive external effect on the knowledge accumulation of the private sector. However, the paper is mainly concerned with open economies issues and international spillovers rather than with public research policy. This last paper does not distinguish between basic and applied research. Indeed, the difference between basic and applied research is absent from all the papers previously mentioned. Very few authors have tried to address the issue of basic versus applied research, especially in a macroeconomic context. The paper by David (2000) reviews the literature and establishes the main debates on the issue of public science, focusing on the differences between basic and applied research and the need for public provision of basic knowledge. Similarly, the work of David and Hall (2000) analyzes the effects of the various public research policies on private R&D expenditures, though the analysis is performed by means of a simple, partial equilibrium, static model. Regarding empirical studies on the influence of R&D expenditures on productivity growth, Griliches (1986) finds evidence of the positive effects of both publicly financed R&D and basic research while Mansfield (1995) analyses the interaction between academic research and industrial innovation. The most recent econometric work on the relationship between public and private research is surveyed in David, Hall and Toole (2000). However, there still exists a need for a theoretical model able to modelize the effects of research policy on economic growth.

In order to bring the analysis closer to reality, we have considered the main policy responses that actual governments use to prevent private underprovision of research. These policies are usually classified in two groups. The first one concerns the direct procurement of research in public facilities, while the second includes policies consisting on giving incentives for a greater amount of private investment. These incentives can take the form of tax reductions intended to reduce the cost of R&D but they can also involve direct funding of specific R&D programs. We will modelize these types of policies in the framework of an endogenous growth model.

There exists a growing debate concerning whether public research should take the form of basic or applied research and whether it should be performed at public institutions or in close coordination with private firms. We will make explicit the difference between basic and applied research and explore the different effects that the various policies available could have on the R&D sector and the economy as a whole.

The basic model we propose as framework for this analysis is the one first presented in Aghion and Howitt (1998). In this model, the economy grows thanks to both capital accumulation and technological change. Therefore, this model overcomes one of the main objections traditionally raised against technological change models, namely, that capital accumulation was ignored as a source of growth. Furthermore, the presence of a continuum of research sectors eliminates uncertainty at the aggregate, allowing for the use of non-stochastic steady state analysis at the macroeconomic level. In Aghion and Howitt model, firms invest in research projects that yield a new product or a new production technology with a certain probability. In the present model, we want to introduce a distinction between basic and applied research. Intuitively, applied research is aimed at obtaining innovations able to improve a particular production technology or that can give raise to a new product or variety. On the contrary, basic research is usually concerned with projects whose outcomes do not normally have a direct market application, though they add to the knowledge base. This does not necessarily mean that private firms will not perform basic research, since we consider that even though basic research alone would not be able to produce a marketable innovation, it is able to increase the productivity of applied research. This is due to the fact that basic science allows researchers in applied fields to understand previous knowledge or to adapt innovations from other fields to their own sector.

We find that subsidies to private research increase R&D investment, both in applied and basic fields, and that this policy is beneficial for long term growth. However, the effect on steady state consumption is generally negative and, therefore, the final effect on welfare results from the trade off between consumption and growth. Due to this fact, the sign of the effect on welfare is ambiguous. For a empirically acceptable set of parameter values, a marginal subsidy to research would have positive effects though excessively high values of the subsidy could harm welfare rather than improve it.

Concerning the other available policy instruments, we will differentiate public production of research from direct funding of R&D projects in the following manner: when research is performed at public institutions, any innovation with a market application that arises from public research will compete with private research in the concession of patents. On the other hand, direct funding of research consists of public aids to private projects which, if successful, will keep the patent in the private sector. Consider thus first the case in which the public sector performs exclusively basic research at public institutions. This type of research increases aggregate knowledge and will affect private firms only through the spillovers created by the faster growing base of knowledge. In other words, the faster accumulation of non-rival and non-excludable knowledge will induce a more important technology improvement when an innovation occurs in the private sector. The growth effect of a higher public budget for research is unambiguously positive while the welfare effect, calibrated for empirically acceptable values of the parameters, seems to be also positive for small values of public research.

When the public sector is allowed to perform both applied and basic research, it may happen that a public project gains the patent in a given sector. In this sense, the public sector behaves as a direct competitor of private research firms and therefore, the public investment in research should be taken into account when computing the sector's rate of replacement. This rate is given by the probability that an innovation occurs in a given sector, which in turn is determined by the amount of research invested in that sector. A higher rate of replacement implies a lower value of the innovation because it reduces the expected life of the patent. Therefore, even though public research will add to the accumulation of knowledge, it also causes this "business-stealing" effect that crowds out private research. Consequently, the net result on the growth rate will depend on which effect dominates and on the actual values of private and public research. We find that in order to have a positive effect on growth of either type of public R&D, the amounts of public applied and basic research must keep certain proportions. In particular, we find that increasing public applied research from zero, the crowding out of private research is so large that the effect on growth will initially be negative for any given value of public basic research. However, if we keep increasing public applied research, the effect on growth will become positive. This is due to the fact that public research is actually substituting private R&D as the source of innovative activity. Indeed, it is relatively easy to crowd out completely private research when the public sector performs both applied and basic research. With regard to the welfare change induced by this policy, again the opposite behavior of consumption and growth forces a calibration in order to obtain a sign. The calibration suggests that welfare may be improved with both types of public research though the introduction of applied public research will initially reduce welfare. Similarly, excessive amounts of public research will determine low levels of steady state consumption per efficiency unit which eventually, will impede further welfare improvements.

An alternative policy instrument is direct funding of specific research projects. In order to simplify and to differentiate it clearly from the previous policy, we assume that the government provides a given amount of output to be used in a specific project, either applied or basic, but that in case of success, the patent remains with the research firm. The implications of this type of financing differ from the previous policy in the sense that the amount of research financed by the public sector increases the productivity of private research. This is so because in order to obtain a given probability of success, the private investment required is smaller the larger the amount financed by the government. We still find the "business stealing" effect of the previous policy but it is now softened by the increase of research productivity. Consequently, the effects on growth of both applied and basic research are unambiguously positive. Remarkably, we find that in equilibrium, the effects on growth and research intensity of public basic and applied research are identical, which suggests that if the research policy is developed through direct funding of private projects, the relevant amount is the total research investment and not whether it has been devoted to applied or basic projects. Another relevant difference with respect to public production of research is the impact on the amount of private investment in research. While research performed at public institutions causes a clear crowding out of private R&D investment, the effect of direct funding is ambiguous. Depending on the actual values of the parameters, we can even find that public and private research behave as complements at the steady state. The econometric evidence is not clear at this point. While some studies identify public and private research as substitutes, other works find that an increase in public research may cause a parallel increase in private R&D investment.[1] The fact that data on R&D expenditures usually include together both public research and public funding of private projects may be one of the causes of the present difficulties to settle the question.

[1]See David *et al* (2000).

With respect to the welfare effects of this policy, again the trade off between consumption and growth determines a positive impact on welfare for small values of public research investment that may become negative for higher values as consumption per efficiency unit diminishes.

In summary, the results suggest that while tax incentives to private research always have a positive growth effect, public research may not be the appropriate policy in some circumstances. We find that research performed at public institutions is always beneficial if it is only concerned with basic research. However, if public institutions do investigate also in applied fields, the impact on long run growth may be negative when some conditions are met. The condition for a positive growth effect requires that the relative amounts of applied and basic research lie between some limits, and that one of them is not excessive with respect to the other. For instance, if the amounts of basic and applied research are chosen so as to maximize the probability of success for a given amount of total investment, the effect on growth is always positive, though the crowding out of private research is so important that it would be relatively easy to crowd it out completely. We also find that direct funding of research has unambiguously positive growth effects.

The rest of the paper is divided into the following sections: section 2 presents the model, sections 3 and 4 present the steady state and welfare analysis and section 5 concludes the paper.

3.2 The model

We consider a growth model with endogenous technological change in which research may be performed by both the private and the public sector. Long run economic growth comes from both technological innovation and capital accumulation. There exist two types of research projects depending on whether they are concerned with basic or applied issues. Successful applied projects produce a new technology that will generate monopoly rents for the owner of the patent. Research projects focused exclusively on basic fields are not able to generate a new product or variety though they contribute to the accumulation of general knowledge. In combination with applied research, basic research is able to increase R&D productivity because it facilitates the absorption of intersectoral and intertemporal spillovers. As a consequence, private firms will only engage in projects with an applied component though they may find it optimal to devote some additional resources to basic research in order to increase the productivity of their own research.

3.2.1 Consumers

There exists an infinitely lived representative consumer whose utility function is assumed to be logarithmic for the sake of simplicity. Consequently, the lifetime utility of the consumer will be given by the following expression:

$$V(C_t) = \int_0^\infty \ln(C_t) e^{-\rho t} dt, \tag{3.1}$$

where C_t is consumption at time t and ρ is the rate of discount.

3.2.2 Final good sector

The consumption good is produced in a competitive market out of labor and intermediate goods. Labor is represented by a continuous mass of individuals L, and it is assumed to be inelastically supplied. Intermediate goods are produced by a continuum of sectors of mass 1, being m_{it} the supply of sector i at date t. The production function is a Cobb-Douglas with constant returns on intermediate goods and efficiency units of labor

$$Y_t = L^{1-\alpha} \int_0^1 A_{it} m_{it}^{\alpha} di, \tag{3.2}$$

where Y_t is final good production and A_{it} is the productivity coefficient of each sector. The evolution of each sector's productivity coefficient A_{it} is determined in the research sector. I assume equal factor intensity to simplify calculations.

3.2.3 Intermediate goods

Intermediate goods are used as factors of production in the final good sector. Each sector has a monopolistic structure. In order to become the monopolist producer of an intermediate good, the entrepreneur has to buy the patent of the latest version of the product. This patent gives him the right to produce the good until an innovation occurs and the monopolist is displaced by the owner of the new technology.

The only input in the production of intermediate goods is capital. In particular, it is assumed that A_{it} units of capital are needed to produce one unit of intermediate good i at date t. This implies that more productive intermediate inputs are more capital intensive, an assumption that simplifies the analysis and has no important implications under the Cobb-Douglas conditions.

Capital is rented in a perfectly competitive market at rate ζ_t. Hence, the cost of one unit of intermediate good is $A_{it}\zeta_t$. On the other hand, the equilibrium price of the intermediate good, $p(m_{it})$ will be its marginal product

$$p(m_{it}) = \alpha L^{1-\alpha} A_{it} m_{it}^{\alpha-1},$$

where m_{it} is production of intermediate good i at date t. Thus, the monopolist's profit maximization problem is the following:

$$\begin{aligned} \pi_{it} &= \max_{m_{it}} [p(m_{it})m_{it} - A_{it}\zeta_t m_{it}] \\ \textit{subject to} \quad p(m_{it}) &= \alpha L^{1-\alpha} A_{it} m_{it}^{\alpha-1}, \end{aligned}$$

from where we obtain the profit-maximizing supply and the flow of profits as

$$\begin{aligned} m_{it} &= L\left(\frac{\alpha^2}{\zeta_t}\right)^{\frac{1}{1-\alpha}} \\ \pi_{it} &= \alpha(1-\alpha)L^{1-\alpha} A_{it} m_{it}^{\alpha}. \end{aligned}$$

Due to the assumption of equal factor intensity, supply of intermediate goods is equal in all sectors, $m_{it} = m_t$. Thus, the aggregate demand of capital is equal to $\int_0^1 A_{it} m_t di$. Let $A_t = \int_0^1 A_{it} di$, be the aggregate productivity coefficient. Then, equilibrium in the capital market requires demand to equal supply

$$A_t m_t = K_t,$$

or equivalently, the flow of intermediate output must be equal to capital intensity, k_t

$$m_t = k_t = \frac{K_t}{A_t}.$$

With this notation we can express the equilibrium rental rate in terms of capital intensity

$$\zeta_t = \alpha^2 L^{1-\alpha} {k_t}^{\alpha-1}. \tag{3.3}$$

3.2.4 Research sector

For each of the above intermediate sectors, there is a number of research firms competing in a patent race to get the next innovation. Innovations are produced using the same technology of the final good. Hence, it needs physical capital (embodied in the intermediate goods) apart from labor to be produced. Technology is assumed to be increasingly complex and hence further innovations will require higher investments. Accordingly, the amount invested in research in each sector N_{it} will be adjusted by a coefficient representing the aggregate state of knowledge. This coefficient will be given by $A_t^{\max}$, the productivity parameter of the leading edge technology. Hence, we may define $n_{it} = \frac{N_{it}}{A_t^{\max}}$ as the productivity adjusted level of research.

The arrival rate of innovations at each sector is given by the following expression:

$$\lambda \left[n_a \left(1 + b n_b\right) \right]^{\frac{1}{2}}, \tag{3.4}$$

where λ is a positive parameter representing the productivity of research, n_a and n_b are the levels of research intensity devoted to applied and basic issues respectively, and b is a positive parameter that measures the influence of basic research on the total private research productivity.[2] This functional form tries to capture the idea that basic research is not essential in order to obtain an innovation with market applications as opposed to applied research, which is assumed to be essential. Given the total amount n_{it} of research, the firm will choose n_a and n_b in order to maximize the probability of obtaining an innovation. The

[2]For the functional form of the contributions of basic and applied research to the probability of success we follow Cassiman, Pérez-Castrillo and Veugelers (2001).

optimal shares are thus,

$$n_a = \begin{cases} \frac{n_{it}}{2} + \frac{1}{2b} & \text{if } n_{it} > \frac{1}{b} \\ n_{it} & \text{otherwise.} \end{cases} \tag{3.5}$$

$$n_b = \begin{cases} \frac{n_{it}}{2} - \frac{1}{2b} & \text{if } n_{it} > \frac{1}{b} \\ 0 & \text{otherwise.} \end{cases} \tag{3.6}$$

In order to simplify the analysis, we will consider only those situations in which private firms invest in basic research, that is, $n_{it} > \frac{1}{b}$.[3] The results when this assumption is not satisfied are presented in Appendix 3.9.

In equilibrium, the arrival rate of innovations in sector i will be given by the following expression

$$\lambda p\left(n_{it}\right) = \lambda\left(\frac{1 + bn_{it}}{2\sqrt{b}}\right),$$

which may be obtained substituting (3.5) and (3.6) in (3.4).

The payoff to innovators if they succeed is the flow of profits obtained from the monopolistic exploitation of the new technology. The value of this payoff is identical for any researcher innovating at t and therefore, research intensity will be the same across sectors. Consequently, we drop the i subindex from research intensity.

When an innovation occurs in a given sector, the productivity parameter of that sector jumps discontinuously to $A_t^{\max}$, the leading edge productivity coefficient. Thus, advances in other sectors spillover to the rest of the economy making the technology improvement induced by the next innovation more important. The evolution of $A_t^{\max}$ is determined by the evolution of the aggregate state of knowledge. While for a particular firm we assumed

[3] See Appendix 3.9 for the parameter restrictions necessary to guarantee this condition.

that basic research was not essential in order to obtain an innovation, in the case of the aggregate state of knowledge, we are going to assume that both basic and applied research are essential factors. This assumption reflects the extended belief that in the long run, the knowledge base cannot go on growing if basic knowledge is not further developed.[4] Consequently, we assume that the rate of growth of $A_t^{\max}$ is given by the following expression:

$$\frac{\dot{A}_t^{\max}}{A_t^{\max}} = \sigma\lambda \left(n_A\right)^{\beta} \left(n_B\right)^{1-\beta},$$

where n_A and n_B are total applied and basic research intensity, that is, including both public and private research. Under these assumptions, the distribution of productivity parameters across sectors will change as $A_t^{\max}$ grows. However, if we define the relative productivity parameter of a sector as $a_{it} = \frac{A_{it}}{A_t^{\max}}$, one can prove that the distribution of a_{it} converges to a stationary distribution. In addition, the stationarity of the distribution of a implies that the aggregate and the leading edge productivities are proportional.[5]

In order to determine private research intensity, consider the value of obtaining an innovation at time t. When the innovation occurs, a new technology with a productivity parameter $A_t^{\max}$ is available for the owner of the patent. The new producer will force the previous incumbent out of the market and will start producing as a monopolist. Therefore, the flow of profits will be given by the following expression:

$$\alpha \left(1-\alpha\right) L^{1-\alpha} A_t^{\max} k_t^{\alpha}.$$

The new producer will be able to keep its monopolistic position until a new innovation

[4] See David (2000).

[5] For the distribution of relative productivities across sectors see Appendix 3.6.

Let $h\left(a\right)$ be the density function of a. Then, by definition, $A_t = \int_0^1 A_{it} di = A_t^{\max} \int_0^1 a_{it} di = A_t^{\max} \int_0^1 a h\left(a\right) da = A_t^{\max} E\left(a\right).$

occurs in that sector. Therefore, the present value of the innovation at time t is given by

$$V_t = \int_t^{\infty} e^{-\int_t^{\tau}[r_s+\lambda p(n_{it})]ds}(1-\alpha)\alpha A_t^{\max} L^{1-\alpha} k_\tau^\alpha d\tau,$$

where $\lambda p\left(n_{it}\right)$ is the flow probability that an innovation occurs in that sector.

The cost of one unit of research in terms of output is 1. Therefore, since $n_t = \frac{N_t}{A_t^{\max}}$, the cost of one unit of research intensity is $A_t^{\max}$. We assume that there exists a proportional subsidy to innovation that reduces its cost. Thus, the marginal cost of increasing research intensity is $(1-s_n)A_t^{\max}$ units of output, where s_n is the subsidy to innovative activity. Hence, the research arbitrage condition is

$$1 - s_n = \left(\frac{\lambda p\left(n_t\right)}{n_t}\right)\left(\frac{(1-\alpha)\alpha L^{1-\alpha} k_t^\alpha}{r_t + \lambda p\left(n_t\right)}\right). \tag{3.7}$$

Notice that this arbitrage condition establishes a relationship between the equilibrium values of capital and research intensity.

3.2.5 Capital market

Capital is used as a factor of production in the intermediate goods sector. We have seen that equilibrium in the capital market requires the rental rate to satisfy equation (3.3). The owner of a unit of capital will obtain ζ_t for it. This amount must be enough to cover the cost of capital. This includes the rate of interest (r_t), the depreciation rate (δ), and the tax rate on capital accumulation (τ_k). Hence, the capital market arbitrage equation is

$$r_t + \delta + \tau_k = \alpha^2 L^{1-\alpha} {k_t}^{\alpha-1}, \tag{3.8}$$

which establishes a decreasing relationship between the interest rate and capital intensity.

3.2.6 Research policy

There exist three major types of public intervention in the research sector. The first one, already introduced in the model, consists of tax incentives to reduce the private cost of research production. In addition, the government may directly modify the total amount of output invested in research. We will assume that it can do so in two different ways. It can produce research at public institutions without any kind of collaboration with private firms and in direct competition with them. This policy is dubbed *public provision* of research. On the other hand, the government may fund private projects, acting in close collaboration with private firms. To simplify, the government is assumed to act altruistically in this case, which implies that the patent remains with the private firm. To differentiate it from the previous policy, we will refer to this one as *public funding.* Let us now analyze the implications for the basic model of these two types of policies.

Public provision of research.

Assume the government can perform research in the same conditions as private firms and define $\tilde{\Gamma}_a$ and $\tilde{\Gamma}_b$ as the amounts of output invested in applied and basic research by public institutions.[6] Thus, public applied and basic research intensity will be given by $\Gamma_a = \frac{\tilde{\Gamma}_a}{A_t^{\max}}$ and $\Gamma_b = \frac{\tilde{\Gamma}_b}{A_t^{\max}}$. Therefore, the probability that the public sector gets an innovation will be given by $\lambda \left[\Gamma_a \left(1 + b\Gamma_b\right)\right]^{\frac{1}{2}}$. The additional research implies that the total probability of an innovation occurring in a given sector will now be $\lambda \left[n_a \left(1 + bn_b\right)\right]^{\frac{1}{2}} + \lambda \left[\Gamma_a \left(1 + b\Gamma_b\right)\right]^{\frac{1}{2}}$.

[6] We assume that the amount invested in each sector is the same so that aggregate and sectoral amounts coincide.

Consequently, the research arbitrage equation will be given by

$$1 - s_n = \left(\frac{\lambda p\,(n)}{n}\right)\left(\frac{(1-\alpha)\alpha L^{1-\alpha}k_t^{\alpha}}{r_t + \lambda p\,(n) + \lambda\left[\Gamma_a\,(1+b\Gamma_b)\right]^{\frac{1}{2}}}\right). \tag{3.9}$$

Notice that public research in this case induces a higher rate of creative destruction, i.e. a higher probability that the owner of the patent is replaced. Therefore, the research activity of the public sector reduces the present value of an innovation for a private researcher. In the case that the public sector gets the patent, it will be sold to an intermediate good producer and the value of the patent will be transferred to consumers in the form of a lump sum transfer.

Public funding of research.

This type of research policy directly affects the microeconomic decision of the research firm about the amounts to be invested in basic and applied research. Consequently, we must rewrite the problem of the firm as follows:

$$\max_{n_a, n_b} \lambda\left[(n_a + \Gamma_a)\,(1 + b\,(n_b + \Gamma_b))\right]^{\frac{1}{2}}, \tag{3.10}$$

subject to the following constraints:

$$\begin{aligned} n_{it} &= n_a + n_b \\ n_a &\geq 0 \\ n_b &\geq 0. \end{aligned}$$

The optimal choices for n_a and n_b are

$$n_a = \begin{cases} \frac{n_{it}+\Gamma_b-\Gamma_a}{2} + \frac{1}{2b} & \text{if } n_{it}+\Gamma_a-\Gamma_b \geq \frac{1}{b} \\ n_{it} & \text{otherwise,} \end{cases}$$

$$n_b = \begin{cases} \frac{n_{it}+\Gamma_a-\Gamma_b}{2} - \frac{1}{2b} & \text{if } n_{it}+\Gamma_a-\Gamma_b \geq \frac{1}{b} \\ 0 & \text{otherwise.} \end{cases}$$

We will consider only situations with n_b positive in the main text. The results when private firms do not perform basic research may be found in Appendix 3.9. For n_a and n_b positive, the probability of obtaining an innovation given n_{it} is

$$\lambda p\left(n_{it}, \Gamma\right) = \frac{\lambda\left[1 + b\left(n_{it} + \Gamma\right)\right]}{2\sqrt{b}}, \tag{3.11}$$

where $\Gamma = \Gamma_a + \Gamma_b$. Again, the symmetric behavior of the sectors in equilibrium allows us to drop the i subindex of n_{it} in (3.11). The probability of the project being successful per unit of research intensity is thus

$$\frac{\lambda p\left(n_t, \Gamma\right)}{n_t}.$$

Therefore, the research arbitrage equation is given by

$$1 - s_n = \left(\frac{\lambda p\left(n_t, \Gamma\right)}{n_t}\right)\left(\frac{(1-\alpha)\alpha L^{1-\alpha} k_t^{\alpha}}{r_t + \lambda p\left(n_t, \Gamma\right)}\right).$$

The main difference with the previous policy in terms of the implications for the research arbitrage equation, is that even though the effect on the rate of creative destruction still remains, there is an additional effect on the productivity of research. This effect is represented by the fact that in the presence of public funding, the probability of obtaining an innovation per unit of private research intensity is now given by $\frac{\lambda(1+b(n+\Gamma))}{2\sqrt{b}n}$ rather than by $\frac{\lambda(1+bn)}{2\sqrt{b}n}$.

3.2.7 Equilibrium

General equilibrium is defined by the two equations determining equilibrium in the capital and research sectors. These equations are

$$r_t + \delta + \tau_k = \alpha^2 L^{1-\alpha} {k_t}^{\alpha - 1}, \tag{3.12}$$

for the capital market and

$$1 - s_n = \left(\frac{\lambda p\,(n_t)}{n_t} \right) \left(\frac{(1-\alpha)\alpha L^{1-\alpha} k_t^{\alpha}}{r_t + \lambda p\,(n_t) + \lambda \left[\Gamma_a \left(1 + b\Gamma_b \right) \right]^{\frac{1}{2}}} \right), \tag{3.13}$$

for the research market in the case of public provision or,

$$1 - s_n = \left(\frac{\lambda p\,(n_t, \Gamma)}{n_t} \right) \left(\frac{(1-\alpha)\alpha L^{1-\alpha} k_t^{\alpha}}{r_t + \lambda p\,(n_t, \Gamma)} \right), \tag{3.14}$$

for the case of public funding.

The systems formed by equations (3.12) and (3.13), and (3.12) and (3.14) define the equilibrium values for k_t and n_t in each case. These systems implicitly determine a relationship between capital and research intensity that allows us to analyze the dynamics of the model in terms of capital and consumption. The laws of motion for capital and consumption are given by

$$\dot{K}_t = Y_t - C_t - N_t - \delta K_t,$$

and

$$\dot{C}_t = (r_t - \rho) C_t, \tag{3.15}$$

where (3.15) is derived from the consumer's optimization problem. These expressions can be written in efficiency units as follows:

$$\dot{k}_t = L^{1-\alpha}k_t^{\alpha} - c_t - \frac{1}{E(a)}n_t - (\delta+\gamma_t)k_t \quad (3.16)$$

$$\dot{c}_t = (r_t - \rho - \gamma_t)c_t, \quad (3.17)$$

where γ_t is the growth rate of $A_t^{\max}$ and therefore is given by $\sigma\lambda\,(n_A)^{\beta}\,(n_B)^{1-\beta}$ which, ultimately is a continuous function of n_t. Let $n^d(k_t)$ be the dynamic relationship between capital and research intensity defined by equations (3.12) and (3.13) or (3.12) and (3.14).[7] Then, we can express equations (3.16) and (3.17) in terms of k_t and c_t exclusively

$$\dot{k}_t = L^{1-\alpha}k_t^{\alpha} - c_t - \frac{1}{E(a)}n^d(k_t) - (\delta+\gamma^d(k_t))k_t \quad (3.18)$$

$$\dot{c}_t = (\alpha^2 L^{1-\alpha}k_t^{\alpha-1} - \delta - \tau_k - \rho - \gamma^d(k_t))c_t. \quad (3.19)$$

Due to the non-linearity of the system we proceed with its linearization around the steady state in order to analyze the local dynamics of the model. It can be proved that the system exhibits local saddle path stability around the steady state. Therefore, we can perform comparative statics analysis at the long run equilibrium.

3.3 Steady state

In equilibrium the production function is simplified due to the fact that the equilibrium value of intermediate input is the same for every sector. Consequently, we may

[7] Specifically, $n^d(k_t)$ is obtained as follows: equation (3.12) defines the interest rate as a function of k_t. Therefore, we can substitute in either (3.13) or (3.14) so as to obtain n_t as an implicit function of capital intensity. Depending on whether we are considering public provision or direct funding, $n^d(k_t)$ is defined by equation (3.13) or (3.14). Consequently, we should use a different notation for each function. However, for the sake of simplicity and because the implications for the dynamics of the model are equivalent, we denote the two functions by $n^d(k_t)$.

write equation (3.2) as

$$Y_t = A_t L^{1-\alpha} k_t^{\alpha},$$

which implies that in a steady state, the rate of growth of output will be the rate of growth of aggregate productivity. That is

$$\gamma = \sigma\lambda \left(n_A\right)^{\beta} \left(n_B\right)^{1-\beta}. \tag{3.20}$$

Using this result, and the fact that in a steady state k and n are constant we may write equations (3.12), (3.13) and (3.14) as follows:

$$\gamma + \rho + \delta + \tau_k = \alpha^2 L^{1-\alpha} k^{\alpha-1}, \tag{3.21}$$

$$1 - s_n = \left(\frac{\lambda p\left(n\right)}{n}\right)\left(\frac{(1-\alpha)\alpha L^{1-\alpha}k^{\alpha}}{\gamma + \rho + \lambda p\left(n\right) + \lambda\left[\Gamma_a\left(1 + b\Gamma_b\right)\right]^{\frac{1}{2}}}\right), \tag{3.22}$$

$$1 - s_n = \left(\frac{\lambda p\left(n, \Gamma\right)}{n}\right)\left(\frac{(1-\alpha)\alpha L^{1-\alpha}k^{\alpha}}{\gamma + \rho + \lambda p\left(n, \Gamma\right)}\right). \tag{3.23}$$

Equations (3.21) and (3.22) on one hand and (3.21) and (3.23) on the other determine the steady state values of k and n for the two alternative assumptions. Let us consider the two research policies separately.

3.3.1 Public provision

If research is performed at public institutions, in direct competition with private firms, the equations determining k and n are (3.21) and (3.22). The growth effect of giving tax incentives to private research firms is established in the following proposition:

Proposition 7 *The long run growth rate increases when the subsidy rate to private research is raised.*

Proof. See Appendix 3.7.1. ■

The cost reduction induced by the subsidy increases the optimal choice of private research intensity. The higher investment in research implies a larger productivity growth and hence, the economy will grow faster.

Concerning the effect of public provision of research, notice that if no public applied research is performed, the amount of basic research produced at public institutions does not affect the rate of creative destruction. However, this research adds to the stock of knowledge and will make private research more productive via spillovers, both of the intertemporal and intersectoral varieties. Therefore, the effect on growth of increasing public investment in basic research should be positive. However, a higher value of Γ_b will reduce private research intensity. This crowding out of private research is due to the increase in factor prices induced by the higher public investment. Nevertheless, the reduction in private research is not large enough to compensate for the positive effect of the public investment and the final net result on the growth rate is positive. On the contrary, if public applied research is positive then basic research has an additional effect. Namely, that it will increase the probability that the public sector gains a patent. This will induce a larger crowding out of private research and reduce the expected life of any future innovation, because the rate of replacement will be higher. Whether the final impact on growth will be positive or negative depends upon the levels of public basic and applied research and on the parameter values. Due to the ambiguity in the sign of the growth effect we proceed to define parameter subspaces for which the growth derivative shows the desired sign. Let us define the following vectors of parameters: $\theta \equiv (\alpha, \delta, \rho, \lambda, s_n, \tau_k, \sigma, L) \in \Theta$ where $\Theta \equiv [0,1]^6 \times (0,\infty)^2$, $\psi \equiv (\beta, b, \Gamma_a, \Gamma_b) \in \Psi$ where $\Psi \equiv [0,1] \times [0,\infty)^3$ and $\omega \equiv \theta \times \psi \in \Theta \times \Psi$. Denote the parameter space by $\Omega \equiv \Theta \times \Psi$ and define the following subspaces of Ψ and Ω :

$$\Psi_1 = \left\{ \psi \in \Psi \,\middle|\, \text{either } \frac{1}{2(1-\beta)} \left(\frac{b\Gamma_a}{1+b\Gamma_b}\right)^{\frac{1}{2}} < 1 \text{ and } 1 + b\left(\Gamma_a - \Gamma_b\right) > 0 \right.$$
$$\left. \text{or } \frac{1}{2}\left(\frac{b\Gamma_a}{1+b\Gamma_b}\right)^{\frac{1}{2}} \left(1 + \left(\frac{\beta}{1-\beta}\right)\frac{b\Gamma_b}{1+b\Gamma_a}\right) < 1 \text{ and } 1 + b\left(\Gamma_a - \Gamma_b\right) < 0 \right\},$$

$$\Psi_2 = \left\{ \psi \in \Psi \,\middle|\, \text{either } \frac{1}{2\beta} \left(\frac{1+b\Gamma_b}{b\Gamma_a}\right)^{\frac{1}{2}} < 1,\ \Gamma_a > 0 \text{ and } 1 + b\left(\Gamma_a - \Gamma_b\right) < 0 \right.$$
$$\left. \text{or } \frac{1}{2}\left(\frac{1+b\Gamma_b}{b\Gamma_a}\right)^{\frac{1}{2}} \left(1 + \left(\frac{1-\beta}{\beta}\right)\frac{1+b\Gamma_a}{b\Gamma_b}\right) < 1,\ \Gamma_a > 0 \text{ and } 1 + b\left(\Gamma_a - \Gamma_b\right) > 0 \right\},$$

$$\Omega_1 = \left\{ \omega \in \Omega \,\middle|\, \frac{1}{2}\left(\frac{b\Gamma_a}{1+b\Gamma_b}\right)^{\frac{1}{2}} > 1 + \epsilon,\ \chi_1 > \chi_2 \text{ and } \chi_3 < \chi_4 \right\},$$

and

$$\Omega_2 = \left\{ \omega \in \Omega \,\middle|\, \frac{1}{2}\left(\frac{1+b\Gamma_b}{b\Gamma_a}\right)^{\frac{1}{2}} > 1 + \epsilon,\ \chi_1 > \chi_2 \text{ and } \chi_3 < \chi_4 \right\},$$

where ϵ, χ_1, χ_2, χ_3 and χ_4 are defined in Appendix 3.7.1. The following propositions establish the effect of public basic and applied research on growth:

Proposition 8 *If either $\Gamma_a = 0$ or $\psi \in \Psi_1$ then, the effect on growth of public basic research is positive. Conversely, if $\omega \in \Omega_1$, the growth effect of Γ_b is negative.*

Proof. See Appendix 3.7.1. ■

Proposition 8 implies that the effect on growth of public basic research is ambiguous when there exists a positive level of public applied research. Intuitively, a larger public investment in basic research will make the economy grow faster when the existing level of public applied research is not too large and Γ_b keeps in a certain range relative to Γ_a. On the contrary, in order to find a negative effect on growth, the amount of public applied research must be very large relative to the amount of public basic research. In any case, when both Γ_a and Γ_b are very large, the effect on growth will generally be positive, due to the fact that for high levels of public research intensity, the level of private research will be so low that the relevant variables for the growth rate of the economy will be the amounts of public investment.

Proposition 9 *If $\psi \in \Psi_2$ then the effect on growth of public applied research is positive. On the contrary, if $\omega \in \Omega_2$ then $\frac{d\gamma}{d\Gamma_a} < 0$.*

Proof. See Appendix 3.7.1. ■

If the public sector decides to increase public applied research from zero, the most relevant effect will be a large crowding out of private research. As a consequence, the rate of growth of the economy will generally fall when the levels of public applied research are close to zero. However, if public investment in applied research keeps growing the effect on growth may be inverted. This is so because the crowding out of private research is smaller as Γ_a grows. The conditions in Proposition 9 require large values of both basic and applied public research in order to have a positive effect on growth of Γ_a and small values of public applied research or large differences between basic and applied investments in order to have a negative effect on growth.

In summary, what the previous propositions require is that the amounts invested in applied and basic research keep certain proportions. If the investment in one of the two types of research is too large or too small relative to the other then the effect on growth will be negative. Accordingly, it appears interesting to analyze the implications of public R&D when it is divided into basic and applied research following a certain rule. Given that this type of public intervention depicts the public sector behaving as a private research firm, we want to consider also the effect of public research if the amounts of public basic and applied research are chosen so as to get the maximum probability of obtaining an innovation for a given amount of public investment in research. In other words, let $\Gamma = \Gamma_a + \Gamma_b$, $\Gamma_a = \frac{1+b\Gamma}{2b}$ and $\Gamma_b = \frac{b\Gamma-1}{2b}$ for $\Gamma \geq \frac{1}{b}$. Then, $(\Gamma_a(1+b\Gamma_b))^{\frac{1}{2}} = p(\Gamma)$ and the comparative statics results of marginal changes in Γ are as follows:

Proposition 10 *The effect on the steady state growth rate of a marginal increase in* Γ *is positive.*

Proof. See Appendix 3.7.1. ■

The result established in the previous proposition implies that the public sector can actually substitute the private research sector and, since we have assumed the same productivity for the private and the public sector, this would be beneficial for the growth performance of the economy. However, this result is due to the assumption that the amount of research invested by the public sector is not limited by profitability conditions, since it may be financed by lump sum taxes. If we assumed instead that the public sector must look for finance in the credit market, then it would be constrained by the same research arbitrage equation as private firms, and there would exist a maximum level of research at which its marginal cost equals the marginal benefit.

3.3.2 Public funding

If research policy consists on the provision of funds for private firms' research projects, the relevant equations in order to determine the steady state values of n and k are (3.21) and (3.23). In this case, the following propositions apply:

Proposition 11 *A higher subsidy rate to private research increases the steady state growth rate of the economy.*

Proof. See Appendix 3.7.2. ■

Proposition 11 shows that the effect of a subsidy to private research is not affected by the assumption on whether public research is performed at public institutions or in coordination with private firms. Thus, the concession of tax incentives to private research continues having a positive effect on long run growth, since it increases the amount of private research intensity. Notice also that an increase in n reduces the ratio of total applied research to total basic research. This may suggest that the privately chosen amounts of applied and basic research are biased towards applied research, while the economy could benefit from a reduction of this ratio. With respect to the effects of the amounts devoted to public research, we find that they are quite different to public provision, as the next proposition establishes:

Proposition 12 *A higher research intensity in either applied or basic fields implies a larger rate of growth in the long run. In equilibrium, the effects of marginal changes of applied and basic research on the growth rate are identical.*

Proof. See Appendix 3.7.2. ■

The positive effect on private research productivity of this type of research policy outweighs the negative effect of the higher probability of replacement induced by public research, which makes the crowding out of private R&D smaller or even, in some cases cause the opposite effect. That is, we can find situations in which an increase of public research implies a higher amount of private R&D investment. The effect on private research is thus ambiguous, as opposed to the previous case, in which private R&D always decreases after an increase in public research. Concerning the result that the effects of public applied and basic research are identical in terms of growth, it is due to the fact that private firms internalize the funds provided by the public sector in such a way that if for instance, the amount of public basic research is increased, the firm will reduce its own investment in basic research and devote more resources to applied research. The same applies for public applied research. Therefore, the behavior of the firm neutralizes the possibility of having different effects on growth of these two types of public R&D. In addition, we find that an increase in either applied or basic public research is going to reduce the ratio of total applied research to total basic research and from Proposition 12 we know that this is going to have a positive effect on growth. Thus, we find again, as in the case of research subsidies, that a reduction of $\frac{n_A}{n_B}$, with n_A and n_B increasing, is beneficial for the rate of growth of the economy.

One of the main differences between public funding of research and direct R&D subsidies is that with public funding the government may choose the amounts devoted to basic and applied fields. The result established in Proposition 12 indicates that this difference will not be relevant for the growth performance of the economy. However, this does not imply that both policies are equivalent. If we want to compare the growth effects of research subsidies and public research, we can take as reference the no intervention equilibrium and compare the growth and private research derivatives with respect to the policy instruments. The following propositions compare the effects of the introduction of these policies:

Proposition 13 *If $s_n = 0$ and $\Gamma = 0$, then the growth effects of equivalent changes in public funding of research and the research subsidy are equal.*

Corollary 14 *If $s_n = 0$ and $\Gamma = 0$, then*

$$\frac{dn}{d\Gamma} = \frac{dn}{ds_n}\left(\frac{1}{n}\right) - 1.$$

Proofs. See Appendix 3.7.2. ■

The previous results compare the effects of the two policies at $s_n = 0$ and $\Gamma = 0$ because at this point they can be made equivalent in terms of the public budget. Two policies are equivalent in terms of the public budget if they imply the same fiscal effort.[8] Thus, if the government's budget is given by

$$T_t = s_n N_t + \Gamma A_t^{\max} - \tau_k K_t,$$

any two policies that we wish to compare must imply the same marginal change in the lump sum tax T_t used to balance the budget. We find that the growth effects are identical, however the effects on private research differ, since the subsidy will always induce a larger increase in this variable. Intuitively, the research subsidy provokes a larger investment from the private sector, while the increase in public funding provides an extra investment that

[8] See the proof of Proposition 13 for the adjustment necessary to make the changes in the instruments equivalent in terms of the public budget.

allows the private sector to reduce their investment effort. Therefore, even though their effect on growth is equivalent, they have different effects on research intensity and probably on consumption and welfare. The choice of policy will thus depend on how the authorities want to influence private research investment.

In summary, we find that both tax incentives and public funding of private research have unambiguously positive effects on long run growth and therefore, are research policies that can be undertaken without fear of damaging the growth performance of the economy. However, public provision of research is a more dangerous tool, since under some conditions, public research can be harmful for the private R&D sector and the economy as a whole. Nevertheless, if public provision of research were exclusively confined to basic fields, or if basic and applied research are kept in the right proportions, the negative effects of this type of policy would be avoided.

Concerning the debate on whether public research should be more market oriented or be devoted only to the accumulation of basic knowledge, the model predicts different results depending on which specific policy is carried over. If we are considering public funding of private research and we take funds from basic research to use them in applied fields, the effect on long run growth will be null due to the accommodating behavior of private research firms. However, in the case of public provision, a redirection of funds from basic to applied fields will have positive or negative effects depending on the initial situation of the economy.

3.4 Welfare analysis

From equation (3.1) we can express utility at the steady state in terms of the stationary level of consumption and the long-run growth rate

$$V_s(c,\gamma) = \int_0^\infty \ln(cA_t)e^{-\rho t}dt = \frac{\ln(cA_0)}{\rho} + \frac{\gamma}{\rho^2}.$$

The change in steady state welfare is a combination of the change in steady state consumption and the change in steady state growth

$$\frac{\partial V_s(c,\gamma)}{\partial x} = \frac{1}{\rho c}\frac{\partial c}{\partial x} + \frac{1}{\rho^2}\frac{\partial \gamma}{\partial x}, \tag{3.24}$$

where x represents any of the three policy instruments, s_n, Γ_a and Γ_b.

This measure of welfare is valid to compare two situations of long run equilibrium. However, it does not consider the periods of transition during which the economy moves from one equilibrium to another. In order to reflect the transition we must analyze the effect on lifetime utility. Rewrite equation (3.1) to obtain the following expression for lifetime utility:

$$V(x) = \frac{\ln(A_0)}{\rho} + \int_0^\infty \left[\int_0^t \gamma_s(x)ds\right] e^{-\rho t}dt + \int_0^\infty \ln(c_t(x))e^{-\rho t}dt$$

where $\gamma_t(x)$ and $c_t(x)$ are the time paths of the growth rate and the level of consumption per efficiency unit after a change in one of the policy parameters. The effect on utility will thus be given by the effects on the paths of growth and consumption. I will obtain first the effect on the paths of consumption and capital intensity and then use the latter to get the effect on the path of the growth rate.

Let $c = p(k,x)$ be the saddle path of the system which can be interpreted as the graph of a policy function relating consumption and capital. Then we know that its slope, p_k its positive and equal to $\frac{\phi_k}{\lambda_1}$.[9] Substituting the policy function into the law of motion of capital, which we denote by $\varphi(k,c)$, the equilibrium dynamics of the system can be characterized by a single differential equation which describes the evolution of the state variable along the stable manifold.

$$\dot{k} = \varphi(k,c) = \varphi(k, p(k,x)) = \Psi(k,x).$$

The solution to this equation, $k_t(x)$, gives the equilibrium value of k as a function of time and the policy parameter. Using $k_t(x)$ in the policy function we would obtain the time path of c

$$c_t(x) = p(k_t(x), x).$$

To calculate the change in welfare we need the derivative of the whole time path of c with respect to x

$$\frac{dc_t(x)}{dx} = p_k \frac{dk_t(x)}{dx} + p_x, \tag{3.25}$$

where p_x is the derivative of the policy function with respect to the policy instrument or graphically, the shift in the saddle path caused by the policy change.

In order to compute $\frac{dk_t(x)}{dx}$, notice that $k_t(x) = k(t,x)$ must satisfy identically the original equation

$$\dot{k}(t,x) \equiv \varphi\left(p(k(t,x),x), k(t,x), x\right),$$

where $\varphi\left(c,k;x\right)$ is the law of motion of capital given by equation (3.18). Differentiate both

[9]We denote by ϕ_k the derivative with respect to capital of the law of motion for consumption evaluated at the steady state, and λ_1 is the negative eigenvalue of the system formed by (3.18) and (3.19) also evaluated at the steady state.

sides with respect to x

$$\dot{k}_x = \frac{dk_x}{dt} = [\varphi_c p_k + \varphi_k] \, k_x + \varphi_c p_x + \varphi_x.$$

Hence k_x satisfies a linear differential equation. Moreover, when we start from a steady state, the coefficients of this equation are constant and we can write

$$\dot{k}_x = \lambda_1 k_x - p_x + \varphi_x.$$

The general solution is given by

$$k_x(t) = \exp(\lambda_1 t) \, k_x(0) + (1 - \exp(\lambda_1 t)) k_x(\infty).$$

Since k is a predetermined variable, the change at the date of the policy change $k_x(0)$ must be zero. The long run effect, $k_x(\infty) = \lim_{t\to\infty} k_x(t)$, is in fact the derivative of the steady state value of k with respect to the policy parameter, and can be expressed as

$$k_x(\infty) = \frac{p_x - \varphi_x}{\lambda_1}.$$

The equilibrium time path of the derivative of k with respect to x is thus given by

$$k_x(t) = (1 - \exp(\lambda_1 t)) \left[\frac{p_x - \varphi_x}{\lambda_1} \right],$$

that is, k will gradually reach its new steady state value at a rate equal to the negative eigenvalue.

Substitute now in equation (3.25) to obtain the final expression for the derivative of the time path of consumption with respect to the policy parameter

$$\frac{dc_t(x)}{dx} = p_k (1 - \exp(\lambda_1 t)) \left[\frac{p_x - \varphi_x}{\lambda_1} \right] + p_x.$$

As before, we can identify the immediate change and the long run effect

$$\begin{aligned}\frac{dc_0(x)}{dx} &= p_x, \\ \frac{dc_\infty(x)}{dx} &= p_k\left[\frac{p_x-\varphi_x}{\lambda_1}\right]+p_x,\end{aligned}$$

where the first represents the necessary jump of consumption to get on the new saddle path and the second is the effect on the steady state value of consumption. Thus, consumption will initially jump to the new saddle path and then it will approach its new steady state value at a rate equal to λ_1.

The derivative of the growth rate and consumption per efficiency unit at date t are given by[10]

$$\begin{aligned}\frac{d\gamma_t(x)}{dx} &= \frac{d\gamma^d(k)}{dk}\left(1-\exp(\lambda_1 t)\right)\frac{dk}{dx}+\frac{d\gamma^d(k)}{dx} \\ \frac{dc_t(x)}{dx} &= \frac{dc}{dx}-p_k\exp(\lambda_1 t)\frac{dk}{dx}.\end{aligned}$$

Hence, the change in welfare will be given by the following expression:

$$\frac{dV(x)}{dx}=\frac{dV_s(x)}{dx}+\left[\frac{\left(\frac{\rho-\lambda_1}{\rho}\right)\frac{d\gamma^d(k)}{dk}+\frac{(1-\alpha)\zeta}{k}}{\lambda_1\left(\rho-\lambda_1\right)}\right]\frac{dk}{dx}. \tag{3.26}$$

Equations (3.24) and (3.26) give the general expressions for the effect of the three policies on the different measures of welfare. Given that the expression in square brackets is negative, the relationship between the two measures of welfare will be determined by the sign of $\frac{\partial k}{\partial x}$ in each case.

Consider the change in welfare excluding the periods of transition, that is, equation (3.24). If steady state consumption and growth evolve in opposite directions, the actual

[10]The derivatives of γ^d are evaluated at the steady state because we consider the stationary equilibrium as the situation before the tax change.

value of the discount rate ρ will be determinant for the sign of the welfare change and we will not be able to give an unambiguous sign to the change in welfare without assuming a specific value for the discount rate. Unfortunately, this will normally be the case. Just for illustrative purposes, a calibration was made for empirically acceptable values of the parameters.[11] Table 1 suggests that the research subsidy may have positive effects on welfare though only for low values of the policy instrument. However, if we introduce a slight change in the parameters, the range of values for which an increase of s_n is welfare improving is substantially enlarged. In summary thus, the calibration is suggesting that the research subsidy may increase welfare, though for high values of s_n the change in welfare becomes negative. Figure 3.1 illustrates the effect on the two measures of welfare taking as reference the level of welfare attained at $s_n = 0$.

Regarding the effects on consumption of public research, we cannot give an unambiguous sign to the relevant derivatives, which implies that very little can be said about the effect on welfare of policies affecting the level of public research. Nevertheless, the simulation results presented in tables 3.1 to 3.7 suggest that the effect on welfare of marginally increasing public R&D may be positive. In particular, Tables 3.2 and 3.3 present the effect on consumption and welfare of marginal changes in public basic research. If public applied research is zero, a marginal increase in basic R&D is negative for steady state consumption. However, the effect on the measure of welfare that includes the periods of transition is initially positive. In addition, we found that this result is quite robust to changes in parameters other than the discount rate. Figure 3.2 shows the relationship between welfare and public basic research. Observe that as the amount of basic research increases its effect on

[11] Refer to Appendix 3.8 for the calibration results.

the growth rate diminishes while consumption per efficiency unit decreases due to the taxes necessary to finance this policy. As a consequence, the effect on welfare becomes negative for high values of public R&D investment. Table 3.3 shows the results when public applied research is positive. Notice that the introduction of basic research reduces the growth rate, implying an initial reduction of the measure of welfare that includes the periods of transition. Further increases in Γ_b will make the economy grow faster as we move into the set of policy parameter combinations that increases γ. Nevertheless, the two measures of welfare fall for these higher values of public basic research as shown in Figure 3.3. Similarly, Table 3.4 shows the results obtained for changes in public applied research. They show that there exists a range of values of Γ_a for which the effect on welfare may be positive. The results obtained for the case in which the public sector chooses the amounts of basic and applied research as if it were a private firm (table 3.5) are very similar and show a remarkable crowding out of private research.

The results under the public funding assumption, presented in tables 3.6 and 3.7, and Figures 3.6 and 3.7 indicate that the effect on welfare of research subsidies and public R&D may be positive but only for low values of the policy parameters.

3.5 Conclusions

This paper has addressed the issue of the need for an active research policy and has discussed the implications of the different alternatives that actual governments have at their disposal. The analysis has been performed in the context of an endogenous growth model with technological change in which private firms may invest both in applied and basic research. The difference between these two types of research is relevant due to the existing debate on whether public research should limit itself to basic knowledge or, on the contrary, should be directed to projects with market applications. In addition, it has been found that subsidies to private research will make the economy grow faster and may increase consumer welfare since this policy increases private research investment.

Public research performed at public institutions has different implications depending on whether this research is directed towards basic or applied fields. When public research is exclusively concerned with basic knowledge, the effect on growth and welfare of this type of public investment is positive. This is due to the fact that innovations arising from public basic research will add to the stock of knowledge and spillover to the rest of the economy. These spillovers translate into more important technological improvements when private R&D is successful, which determines a higher growth rate of aggregate technology and hence, of the economy. However, if public institutions do research also in applied fields, any innovation arising from this type of research will be patentable. This implies that public institutions will compete with private firms in the patent race and thus, public research will have to be included when considering the rate of replacement of a sector. This rate is given by the probability that an innovation occurs in a given sector and determines the expected life of an existing patent. The effect of public applied research on the rate of replacement represents a negative externality for private research firms, since the expected value of an innovation falls when the rate of replacement increases. However, public applied research also adds to the stock of knowledge and in consequence, causes a positive external effect. The interaction of these opposing forces determines an ambiguous effect of public applied research on growth.

On the other hand, we have found that public funding of private projects has an unambiguously positive growth effect. This is mainly due to the higher productivity of private research induced by this policy and to the fact that it does not necessarily crowd out private research. Indeed, whether public funding of research projects induces more private investment or not depends upon the initial situation of the economy and on the actual values of the parameters though, for a set of plausible parameter values, it is easy to obtain the result that private and public research behave as complements rather than substitutes. We observe that in equilibrium, the funding of either applied or basic projects have identical effects on private research and growth. Therefore, if research policy is instrumented through the funding of private projects, it is irrelevant whether the funds are used for basic or applied projects. This is turn implies that moving funds from basic projects to more applied fields, as proposed by the "new economic instrumentalism",[12] would have a null effect on long run growth. Additionally, we have compared the effects on growth of subsidizing research as opposed to public funding of projects. We have found that the effect on growth of both policies is equal when evaluated at the no intervention equilibrium.

[12] See David (2000).

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3.6 Distribution of relative productivities across sectors

Let $F(\cdot, t)$ be the cumulative distribution function of absolute productivity coefficients A_{it} at any given date t. Define $\Phi(t) = F\left(A_{t_0}^{\max}, t\right)$. Then, $\Phi(t)$ gives us the mass of sectors with a productivity coefficient below $A_{t_0}^{\max}$ at date t. Therefore,

$$\begin{aligned} \Phi(t_0) &= 1 && (3.27) \\ \frac{d\Phi(t)}{dt} &= -\Phi(t)\, d(t), && (3.28) \end{aligned}$$

where $d(t)$ is the probability that a sector innovates. Thus, depending on the assumption that we are considering, it will be given by $\lambda p(n_t) + \lambda \Gamma_a (1 + b\Gamma_b)$ in the case of public provision or $\lambda p(n_t, \Gamma)$ in the case of public funding. Equation (3.27) holds because at t_0 no sector can have a productivity parameter above $A_{t_0}^{\max}$ and equation (3.28) gives us the rate at which the sectors behind $A_{t_0}^{\max}$ innovate and get a productivity parameter larger than $A_{t_0}^{\max}$. These two equations define a differential equation whose solution is given by

$$\Phi(t) = \exp\left(-\int_{t_0}^{t} d(s)\, ds\right). \qquad (3.29)$$

We also know that $\frac{\dot{A}_t^{\max}}{A_t^{\max}} = \gamma(t)$, therefore

$$A_t^{\max} = A_{t_0}^{\max} \exp\left(\int_{t_0}^{t} \gamma(s)\, ds\right).$$

Define $a_0 = \frac{A_{t_0}^{\max}}{A_t^{\max}}$, then

$$a_0 = \exp\left(-\int_{t_0}^{t} \gamma(s)\, ds\right). \qquad (3.30)$$

Equation (3.30) defines an implicit function relating t with a_0, the relative productivity parameter of a sector that innovated on date t_0. Let $t = \tilde{t}(a_0)$ be this function, and use it to perform a change of variable in (3.29) so that we will now have

$$\Phi\left(\tilde{t}\left(a_0\right)\right) = \exp\left(-\int_{\tilde{t}^{-1}(t_0)}^{\tilde{t}^{-1}(t)} d\left(\tilde{t}\left(a_0\right)\right)\tilde{t}'\left(a_0\right)da_0\right).$$

Notice that this function is giving us the mass of sectors with a productivity parameter smaller or equal than $A_{t_0}^{\max}$ and that this is equivalent to the mass of sectors with a relative productivity parameter a_{it} below a_0. Therefore, we can redefine $\Phi\left(\tilde{t}\left(a_0\right)\right) = H\left(a_0\right)$ as the value of the distribution function for a sector that innovated on date t_0. After a long enough period of time, all sectors will have innovated at least once and therefore, $H\left(a\right)$ will be the distribution function of any sector with $a \in (0,1)$. Therefore, the long run distribution of relative productivity parameters across sectors will be given by

$$H\left(a\right) = \exp\left(\int_a^1 d\left(\tilde{t}\left(u\right)\right)\tilde{t}'\left(u\right)du\right),$$

where we are using $\tilde{t}^{-1}\left(t\right) = a$ and $\tilde{t}^{-1}\left(t_0\right) = 1$. Notice that this distribution is time invariant.

In general, we will not be able to obtain the functional form of $H\left(a\right)$ for any economic equilibrium. Nevertheless, in order to study the dynamics of the economy it is enough to know that the distribution is time invariant. However, we can get the expression of $H\left(a\right)$ when the economy is in a steady state, since in that case both the growth rate of the economy γ, and the probability of innovation d, are constant and thus (3.30) becomes

$$a = \exp\left(-\gamma\left(t - t_0\right)\right),$$

from where we can obtain the expression for $\tilde{t}\left(a\right)$ as given by

$$t = -\frac{\ln a}{\gamma} + t_0,$$

which allows us to obtain the distribution function as

$$H(a) = a^{\frac{d}{\gamma}}.$$

3.7 Proofs of propositions

3.7.1 Propositions under the public provision assumption

Proof of Proposition 7. The effect on growth of s_n is given by the following expression:

$$\frac{d\gamma}{ds_n} = \frac{1}{2}\left(\frac{\beta\gamma}{n_A}\right)\left(1+\left(\frac{1-\beta}{\beta}\right)\frac{n_A}{n_B}\right)\frac{dn}{ds_n}.$$

Therefore, in order to find the sign of $\frac{d\gamma}{ds_n}$ we need first the sign of $\frac{dn}{ds_n}$. This derivative can be obtained from the system determining steady state equilibrium using implicit differentiation techniques. Consider the case of public provision of research. The relevant system of equations is the one formed by (3.21) and (3.22). Rewrite these equations in the following form:

$$f_1(k,n) = (1-s_n)\left[\gamma+\rho+\lambda p(n)+\lambda\left[\Gamma_a\left(1+b\Gamma_b\right)\right]^{\frac{1}{2}}\right] - \frac{\lambda p(n)(1-\alpha)\alpha L^{1-\alpha}k^{\alpha}}{n} = 0 \tag{3.31}$$

$$f_2(k,n) = \gamma+\rho+\delta+\tau_k-\alpha^2 L^{1-\alpha}k^{\alpha-1} = 0, \tag{3.32}$$

so that we may define the function $F:(0,\infty)\times(0,\infty)\to R^2$ whose components are $f_1(\cdot,\cdot)$ and $f_2(\cdot,\cdot)$ and use the implicit function theorem to find the derivatives needed. The Jacobian of F will be given by

$$J_F(k,n) = \begin{pmatrix} -\frac{\lambda p(n)(1-\alpha)\zeta}{n} & (1-s_n)\left(\lambda p'(n) + \frac{d\gamma}{dn}\right) - \lambda \frac{d}{dn}\left(\frac{p(n)}{n}\right)(1-\alpha)\alpha L^{1-\alpha}k^{\alpha} \\ \frac{(1-\alpha)\zeta}{k} & \frac{d\gamma}{dn} \end{pmatrix},$$

where $\frac{d\gamma}{dn} = \frac{1}{2}\left(\frac{\beta\gamma}{n_A}\right)\left(1 + \left(\frac{1-\beta}{\beta}\right)\frac{n_A}{n_B}\right)$. The Jacobian may be inverted to obtain

$$[J_F]^{-1} = \frac{1}{\det(J_F)} \begin{pmatrix} \frac{d\gamma}{dn} & -(1-s_n)\left(\lambda p'(n) + \frac{d\gamma}{dn}\right) + \lambda \frac{d}{dn}\left(\frac{p(n)}{n}\right)(1-\alpha)\alpha L^{1-\alpha}k^{\alpha} \\ -\frac{(1-\alpha)\zeta}{k} & -\frac{\lambda p(n)(1-\alpha)\zeta}{n} \end{pmatrix},$$

where

$$\begin{aligned} \det(J_F) = & -(1-\alpha)\zeta\frac{d\gamma}{dn}\left(\frac{\lambda p(n)}{n} + \frac{(1-s_n)}{k}\right) \\ & -(1-\alpha)\zeta\left(\frac{(1-s_n)\lambda p'(n)}{k} - \lambda\frac{d}{dn}\left(\frac{p(n)}{n}\right)\alpha(1-\alpha)L^{1-\alpha}k^{\alpha-1}\right), \end{aligned}$$

is negative. The derivatives of F with respect to s_n are given by

$$\begin{aligned} \frac{df_1}{ds_n} &= -\left[\gamma + \rho + \lambda p(n) + \lambda\left[\Gamma_a(1+b\Gamma_b)\right]^{\frac{1}{2}}\right] \\ \frac{df_2}{ds_n} &= 0. \end{aligned}$$

Therefore, $\frac{dn}{ds_n}$ will be given by

$$\frac{dn}{ds_n} = \frac{-1}{\det(J_F)}\left(\frac{(1-\alpha)\zeta}{k}\right)\left[\gamma + \rho + \lambda p(n) + \lambda\left[\Gamma_a(1+b\Gamma_b)\right]^{\frac{1}{2}}\right],$$

which is positive. Therefore, the derivative of the steady state rate of growth with respect to s_n is also positive. ■

Proof of Proposition 8. The effect on growth of public basic research is given by

$$\frac{d\gamma}{d\Gamma_b} = \left(\frac{(1-\beta)\,\gamma}{n_B}\right)\left(1+\frac{1}{2}\left(1+\left(\frac{\beta}{1-\beta}\right)\frac{n_B}{n_A}\right)\frac{dn}{d\Gamma_b}\right).$$

Accordingly, let us find $\frac{dn}{d\Gamma_b}$. The Jacobian of F is not modified but we have to compute the derivatives of F with respect to Γ_b. They are given by the following expressions:

$$\begin{aligned}
\frac{df_1}{d\Gamma_b} &= (1-s_n)\left[\frac{(1-\beta)\,\gamma}{n_B}+\frac{\lambda b^{\frac{1}{2}}}{2}\left(\frac{b\Gamma_a}{1+b\Gamma_b}\right)^{\frac{1}{2}}\right]\\
\frac{df_2}{d\Gamma_b} &= \frac{(1-\beta)\,\gamma}{n_B},
\end{aligned}$$

which implies that the derivative of private research with respect to public basic research, as expressed by

$$\frac{dn}{d\Gamma_b} = \frac{(1-\alpha)\zeta\left[\left(\frac{1-s_n}{k}\right)\left(\frac{\lambda b^{\frac{1}{2}}}{2}\left(\frac{b\Gamma_a}{1+b\Gamma_b}\right)^{\frac{1}{2}}\right)+\left(\frac{\lambda p(n)}{n}+\frac{1-s_n}{k}\right)\left(\frac{(1-\beta)\gamma}{n_B}\right)\right]}{\det\left(J_F\right)},$$

is negative. The derivative of γ with respect to Γ_b is therefore,

$$\frac{d\gamma}{d\Gamma_b} = \frac{\left(\frac{(1-\beta)\gamma}{n_B}\right)\left(\left(\frac{1-s_n}{k}\right)\left(\frac{\lambda b^{\frac{1}{2}}}{2}\right)\left(1-\frac{1}{2}\left(1+\left(\frac{\beta}{1-\beta}\right)\frac{n_B}{n_A}\right)\left(\frac{b\Gamma_a}{1+b\Gamma_b}\right)^{\frac{1}{2}}\right)+\frac{\lambda\alpha(1-\alpha)L^{1-\alpha}k^{\alpha-1}}{2\sqrt{b}n^2}\right)}{\frac{d\gamma}{dn}\left(\frac{\lambda p(n)}{n}+\frac{(1-s_n)}{k}\right)+\frac{(1-s_n)}{k}\left(\frac{\lambda b^{\frac{1}{2}}}{2}\right)+\frac{\lambda\alpha(1-\alpha)L^{1-\alpha}k^{\alpha-1}}{2\sqrt{b}n^2}}.$$

If public applied research is zero, $\frac{d\gamma}{d\Gamma_b}$ is positive. However, if Γ_a is positive, the effect on growth of public basic research will be positive whenever

$$\left(\frac{1-s_n}{k}\right)\left(\frac{\lambda b^{\frac{1}{2}}}{2}\right)\left(1-\frac{1}{2}\left(1+\left(\frac{\beta}{1-\beta}\right)\frac{n_B}{n_A}\right)\left(\frac{b\Gamma_a}{1+b\Gamma_b}\right)^{\frac{1}{2}}\right)+\frac{\lambda\alpha\,(1-\alpha)\,L^{1-\alpha}k^{\alpha-1}}{2\sqrt{b}n^2}>0. \tag{3.33}$$

A sufficient condition for $\frac{d\gamma}{d\Gamma_b} > 0$ would be

$$\frac{1}{2}\left(1+\left(\frac{\beta}{1-\beta}\right)\frac{n_B}{n_A}\right)\left(\frac{b\Gamma_a}{1+b\Gamma_b}\right)^{\frac{1}{2}} \leq 1, \tag{3.34}$$

but this expression still depends upon n. Recall that $\frac{n_B}{n_A} = \frac{bn-1+2b\Gamma_b}{1+bn+2b\Gamma_a}$. For given values of public basic and applied research, $\frac{n_B}{n_A}$ is a function of n whose derivative is given by $\frac{d}{dn}\left(\frac{n_B}{n_A}\right) = \frac{2b(1+b(\Gamma_a-\Gamma_b))}{(1+bn+2b\Gamma_a)^2}$. Therefore, $\frac{n_B}{n_A}$ is an increasing function of n when $1+b\left(\Gamma_a-\Gamma_b\right) > 0$ and a decreasing function when $1+b\left(\Gamma_a-\Gamma_b\right) < 0$. Consider $\frac{n_B}{n_A}$ increasing. Then it will take its maximum value when n goes to infinity. Since $\lim_{n\to\infty}\frac{n_B}{n_A} = 1$, a sufficient condition for (3.34) to be satisfied is

$$\frac{1}{2\left(1-\beta\right)}\left(\frac{b\Gamma_a}{1+b\Gamma_b}\right)^{\frac{1}{2}} \leq 1.$$

Consider now the case when $\frac{n_B}{n_A}$ is decreasing in n, that is when $1+b\left(\Gamma_a-\Gamma_b\right) < 0$. In this case, $\frac{n_B}{n_A}$ will take its maximum value at $n = \frac{1}{b}$ (for the range of values that we are considering) and condition (3.34) will be satisfied if

$$\frac{1}{2}\left(1+\left(\frac{\beta}{1-\beta}\right)\frac{b\Gamma_b}{1+b\Gamma_a}\right)\left(\frac{b\Gamma_a}{1+b\Gamma_b}\right)^{\frac{1}{2}} \leq 1.$$

It follows that if $\psi \in \Psi_1$, condition (3.33) is satisfied and $\frac{d\gamma}{d\Gamma_b} > 0$.

In order to prove the third part of the proposition, we have to find a sufficient condition for $\frac{d\gamma}{d\Gamma_b} < 0$. Notice that in equilibrium $f_1(k,n) = 0$ and therefore

$$\frac{\lambda\alpha\left(1-\alpha\right)L^{1-\alpha}k^{\alpha-1}}{2\sqrt{b}n^2} = \left(\frac{1-s_n}{k}\right)\left(\frac{\rho+\gamma+\lambda p\left(n\right)+\lambda\left(\Gamma_a\left(1+b\Gamma_b\right)\right)^{\frac{1}{2}}}{n\left(1+bn\right)}\right),$$

which recalling (3.33) allows us to state that $\frac{d\gamma}{d\Gamma_b}$ will be negative whenever

$$\left[\frac{1}{2}\left(\frac{b\Gamma_a}{1+b\Gamma_b}\right)^{\frac{1}{2}}\right] > 1+\frac{\gamma+\rho+\lambda p\left(n\right)+\lambda\left[\Gamma_a\left(1+b\Gamma_b\right)\right]^{\frac{1}{2}}}{\lambda p\left(n\right)bn}. \tag{3.35}$$

Notice that the numerator of the last expression of (3.35) is the discount rate of the flow of profits, which for reasonable values of the parameters should be smaller than 1. In order to impose this condition, define n^1 as the level of research intensity implying a discount rate of 1. Then, a sufficient condition for the discount rate to be smaller than 1 is $n < n^1$ or, equivalently,[13]

$$\chi_1 > \chi_2, \tag{3.36}$$

where

$$\chi_1 = \left(\frac{(1-s_n)\left(\gamma\left(n^1\right) + \rho + \lambda p\left(n^1\right) + \lambda\left(\Gamma_a\left(1+b\Gamma_b\right)\right)^{\frac{1}{2}}\right)}{\lambda b p\left(n^1\right)(1-\alpha)\,\alpha L^{1-\alpha}} \right)^{\frac{1}{\alpha}},$$

and

$$\chi_2 = \left(\frac{\alpha^2 L^{1-\alpha}}{\gamma\left(n^1\right) + \rho + \delta + \tau_k} \right)^{\frac{1}{1-\alpha}}.$$

After having imposed this upper bound for n, a sufficient condition for (3.35) to hold is

$$\left[\frac{1}{2} \left(\frac{b\Gamma_a}{1+b\Gamma_b} \right)^{\frac{1}{2}} \right] > 1 + \frac{1}{\lambda p\left(n\right) bn}.$$

This expression implies that if Γ_a is large relative to Γ_b, the effect on growth of public basic research will be negative as long as the level of private research intensity is not so small that $\frac{1}{\lambda p(n) bn}$ becomes excessively large. Therefore, what we are requiring is that Γ_a is large relative to Γ_b but also that they both are not too large. If we want to find a sufficient condition that depends only on the values of the parameters we have to impose a lower bound for n. Let n^0 be the level of research intensity that satisfies $\frac{1}{\lambda p(n) bn} = \epsilon$, where ϵ is a

[13]This condition is obtained from the equations that determine the equilibrium value of n, that is from (3.31) and (3.32).

real number.[14] Then, if $n > n^0$, condition (3.35) will be satisfied when

$$\left[\frac{1}{2}\left(\frac{b\Gamma_a}{1+b\Gamma_b}\right)^{\frac{1}{2}}\right] > 1+\epsilon. \tag{3.37}$$

In addition, we have to impose the following restriction on the parameters in order to guarantee $n > n^0$:

$$\chi_3 < \chi_4, \tag{3.38}$$

where

$$\chi_3 = \left(\frac{(1-s_n)\left(\gamma\left(n^0\right)+\rho+\lambda p\left(n^0\right)+\lambda\left(\Gamma_a\left(1+b\Gamma_b\right)\right)^{\frac{1}{2}}\right)}{\lambda bp\left(n^0\right)(1-\alpha)\,\alpha L^{1-\alpha}}\right)^{\frac{1}{\alpha}},$$

and

$$\chi_4 = \left(\frac{\alpha^2 L^{1-\alpha}}{\gamma\left(n^0\right)+\rho+\delta+\tau_k}\right)^{\frac{1}{1-\alpha}}.$$

Thus, if $\omega \in \Omega_1$ then $\frac{d\gamma}{d\Gamma_b} < 0$. ■

Proof of Proposition 9. The derivative of the growth rate with respect to public applied research is given by

$$\frac{d\gamma}{d\Gamma_a} = \left(\frac{\beta\gamma}{n_A}\right)\left(1+\frac{1}{2}\left(1+\left(\frac{1-\beta}{\beta}\right)\frac{n_A}{n_B}\right)\frac{dn}{d\Gamma_a}\right).$$

As in the previous propositions we compute first the derivative of private research intensity with respect to public applied research. In order to do so we need the derivatives of the component functions of F, i.e.

$$\frac{df_1}{d\Gamma_a} = (1-s_n)\left[\frac{\beta\gamma}{n_A}+\left(\frac{\lambda b^{\frac{1}{2}}}{2}\right)\left(\frac{1+b\Gamma_b}{b\Gamma_a}\right)^{\frac{1}{2}}\right]$$

$$\frac{df_2}{d\Gamma_a} = \frac{\beta\gamma}{n_A}.$$

[14] The choice of ϵ must take into account that if it is either too large or too small the set of parameter values satisfying the condition may be empty. For a standard set of parameter values $\epsilon = 1$, for instance, yields a non-empty set.

Next, pre-multiply $\begin{pmatrix} \frac{df_1}{d\Gamma_a} \\ \\ \frac{df_2}{d\Gamma_a} \end{pmatrix}$ by the second row of $-\left[J_F\right]^{-1}$ to obtain

$$\frac{dn}{d\Gamma_a} = \left(\frac{(1-\alpha)\zeta}{\det\left(J_F\right)}\right)\left[\left(\frac{(1-s_n)}{k}\right)\left(\frac{\lambda b^{\frac{1}{2}}}{2}\right)\left(\frac{1+b\Gamma_b}{b\Gamma_a}\right)^{\frac{1}{2}} + \left(\frac{\lambda p\left(n\right)}{n} + \frac{(1-s_n)}{k}\right)\left(\frac{\beta\gamma}{n_A}\right)\right].$$

Notice that this expression is also negative. Now we can write $\frac{d\gamma}{d\Gamma_a}$ as follows:

$$\frac{d\gamma}{d\Gamma_a} = \frac{\left(\frac{\beta\gamma}{n_A}\right)\left(\left(\frac{1-s_n}{k}\right)\left(\frac{\lambda b^{\frac{1}{2}}}{2}\right)\left(1-\frac{1}{2}\left(1+\left(\frac{1-\beta}{\beta}\right)\frac{n_A}{n_B}\right)\left(\frac{1+b\Gamma_b}{b\Gamma_a}\right)^{\frac{1}{2}}\right) + \frac{\lambda\alpha(1-\alpha)L^{1-\alpha}k^{\alpha-1}}{2\sqrt{b}n^2}\right)}{\frac{d\gamma}{dn}\left(\frac{\lambda p(n)}{n} + \frac{(1-s_n)}{k}\right) + \frac{(1-s_n)}{k}\left(\frac{\lambda b^{\frac{1}{2}}}{2}\right) + \frac{\lambda\alpha(1-\alpha)L^{1-\alpha}k^{\alpha-1}}{2n^2\sqrt{b}}}. \tag{3.39}$$

Therefore, public applied research will have a positive effect on growth only if

$$\left(\frac{1-s_n}{k}\right)\left(\frac{\lambda b^{\frac{1}{2}}}{2}\right)\left(1-\frac{1}{2}\left(1+\left(\frac{1-\beta}{\beta}\right)\frac{n_A}{n_B}\right)\left(\frac{1+b\Gamma_b}{b\Gamma_a}\right)^{\frac{1}{2}}\right) + \frac{\lambda\alpha\left(1-\alpha\right)L^{1-\alpha}k^{\alpha-1}}{2n^2\sqrt{b}} \tag{3.40}$$

is positive.

A sufficient condition for the expression in (3.40) to be positive is

$$1-\frac{1}{2}\left(1+\left(\frac{1-\beta}{\beta}\right)\frac{n_A}{n_B}\right)\left(\frac{1+b\Gamma_b}{b\Gamma_a}\right)^{\frac{1}{2}} > 0. \tag{3.41}$$

Given that $\frac{n_A}{n_B}$ as a function of n is increasing when $1+b\left(\Gamma_a-\Gamma_b\right) < 0$ and decreasing when $1+b\left(\Gamma_a-\Gamma_b\right) > 0$, it will take its maximum value when n goes to infinity in the first case and when $n=\frac{1}{b}$ in the second case. Therefore, sufficient conditions for (3.41) to be satisfied are

$$\begin{array}{rl} \frac{1}{2\beta}\left(\frac{1+b\Gamma_b}{b\Gamma_a}\right)^{\frac{1}{2}} < 1 & for \quad 1+b\left(\Gamma_a-\Gamma_b\right) < 0 \\ \frac{1}{2}\left(\frac{1+b\Gamma_b}{b\Gamma_a}\right)^{\frac{1}{2}}\left(1+\left(\frac{1-\beta}{\beta}\right)\frac{1+b\Gamma_a}{b\Gamma_b}\right) < 1 & for \quad 1+b\left(\Gamma_a-\Gamma_b\right) > 0 \end{array}.$$

Under these conditions, $\frac{d\gamma}{d\Gamma_a} > 0$. Thus, if $\psi \in \Psi_2$, the growth effect of public applied research will be positive.

The expression of $\frac{d\gamma}{d\Gamma_a}$ in equation (3.39) implies that this derivative will be negative whenever

$$\left(\frac{1-s_n}{k}\right)\left(\frac{\lambda b^{\frac{1}{2}}}{2}\right)\left(1-\frac{1}{2}\left(1+\left(\frac{1-\beta}{\beta}\right)\frac{n_A}{n_B}\right)\left(\frac{1+b\Gamma_b}{b\Gamma_a}\right)^{\frac{1}{2}}\right)+\frac{\lambda\alpha\left(1-\alpha\right)L^{1-\alpha}k^{\alpha-1}}{2\sqrt{b}n^2}<0. \tag{3.42}$$

Therefore, following the same reasoning as in the previous proofs, the effect on growth of public applied research will be negative when $\omega \in \Omega_2$. ■

Proof of Proposition 10. The derivatives of $f_1(k,n)$ and $f_2(k,n)$ with respect to Γ are given by the following expressions:

$$\frac{df_1}{d\Gamma} = (1-s_n)\left(\frac{d\gamma}{dn}+\lambda p'(\Gamma)\right) \tag{3.43}$$

$$\frac{df_2}{d\Gamma} = \frac{d\gamma}{dn}, \tag{3.44}$$

where we are using the fact that under these assumptions, the derivative of the growth rate with respect to Γ keeping n constant is equal to the derivative of the growth rate with respect to n. Given (3.43) and (3.44), the derivative of private research with respect to public research is

$$\frac{dn}{d\Gamma}=\frac{(1-\alpha)\zeta}{\det(J_F)}\left(\frac{d\gamma}{dn}\left(\frac{1-s_n}{k}+\frac{\lambda p(n)}{n}\right)+\frac{1-s_n}{k}\lambda p'(\Gamma)\right),$$

which is negative. Given $\frac{dn}{d\Gamma}$, we can express the growth derivative as

$$\frac{d\gamma}{d\Gamma}=\frac{d\gamma}{dn}\left(1+\frac{dn}{d\Gamma}\right)=\frac{d\gamma}{dn}\left(-\frac{(1-\alpha)\zeta}{\det(J_F)}\right)\left(\frac{-\lambda\tilde{\pi}}{k}\frac{d}{dn}\left(\frac{p(n)}{n}\right)\right),$$

where $\tilde{\pi}=\frac{\pi}{A_t^{\max}}$. Since $\frac{d}{dn}\left(\frac{p(n)}{n}\right)$ and $\det(J_F)$ are negative, then $\frac{d\gamma}{d\Gamma}>0$. ■

3.7.2 Propositions under the public funding assumption

Proof of Proposition 11. The relevant equations under this assumption are (3.21) and (3.23) so that the component functions of F are now

$$f_1(k,n) = (1-s_n)\left[\gamma+\rho+\lambda p\left(n,\Gamma\right)\right] - \frac{\lambda p\left(n,\Gamma\right)}{n}(1-\alpha)\alpha L^{1-\alpha}k^{\alpha} = 0$$
$$f_2\left(k,n\right) = \gamma+\rho+\delta+\tau_k-\alpha^2L^{1-\alpha}k^{\alpha-1} = 0.$$

Hence, the Jacobian and its inverse are given by the following matrices:

$$J_F\left(k,n\right) = \begin{pmatrix} -\frac{\lambda p(n,\Gamma)(1-\alpha)\zeta}{n} & (1-s_n)\left(\lambda p'\left(n,\Gamma\right)+\frac{d\gamma}{dn}\right)-\lambda\frac{d}{dn}\left(\frac{p(n,\Gamma)}{n}\right)(1-\alpha)\alpha L^{1-\alpha}k^{\alpha} \\ \frac{(1-\alpha)\zeta}{k} & \frac{d\gamma}{dn} \end{pmatrix}$$

and

$$[J_F]^{-1} = \frac{1}{\det\left(J_F\right)}\begin{pmatrix} \frac{d\gamma}{dn} & -(1-s_n)\left(\lambda p'\left(n,\Gamma\right)+\frac{d\gamma}{dn}\right)+\lambda\frac{d}{dn}\left(\frac{p(n,\Gamma)}{n}\right)(1-\alpha)\alpha L^{1-\alpha}k^{\alpha} \\ -\frac{(1-\alpha)\zeta}{k} & -\frac{\lambda p(n,\Gamma)(1-\alpha)\zeta}{n} \end{pmatrix},$$

where

$$\det\left(J_F\right) = -(1-\alpha)\zeta\frac{d\gamma}{dn}\left[\frac{(1-s_n)}{k}+\frac{\lambda p\left(n,\Gamma\right)}{n}\right] - $$
$$-(1-\alpha)\zeta\left(\frac{(1-s_n)\,\lambda p'\left(n,\Gamma\right)}{k}-\lambda\frac{d}{dn}\left(\frac{p\left(n,\Gamma\right)}{n}\right)(1-\alpha)\alpha L^{1-\alpha}k^{\alpha-1}\right)$$

is also negative.

The derivatives of the component functions of F with respect to s_n are

$$\frac{df_1}{ds_n} = -\left[\gamma+\rho+\lambda p\left(n,\Gamma\right)\right]$$
$$\frac{df_2}{ds_n} = 0.$$

Thus, $\frac{dn}{ds_n}$ is given by

$$\frac{dn}{ds_n} = -\left(\frac{(1-\alpha)\,\zeta}{\det(J_F)}\right)\left(\frac{\gamma+\rho+\lambda p\,(n,\Gamma)}{k}\right).$$

Therefore, private research intensity increases with subsidies to research and so does the growth rate of the economy. ■

Proof of Proposition 12. Recall that $\Gamma = \Gamma_a + \Gamma_b$. Given that both applied and basic public research enter the component functions in equivalent positions, the derivatives of private research intensity with respect to Γ_a and Γ_b will be identical. In addition, the equilibrium expression for the growth rate under this assumption is given by

$$\gamma = \sigma\lambda\left(\frac{1+b\,(n+\Gamma_a+\Gamma_b)}{2b}\right)^{\beta}\left(\frac{b\,(n+\Gamma_a+\Gamma_b)-1}{2b}\right)^{1-\beta}.$$

Consequently,

$$\frac{d\gamma}{d\Gamma_a} = \frac{d\gamma}{d\Gamma_b} = \frac{d\gamma}{d\Gamma} = \frac{d\gamma}{dn}\left(1+\frac{dn}{d\Gamma}\right). \tag{3.45}$$

Therefore, we can talk about Γ exclusively.

In order to obtain $\frac{dn}{d\Gamma}$ we compute the derivatives of the component functions of F as follows:

$$\begin{aligned}
\frac{df_1}{d\Gamma} &= (1-s_n)\left[\frac{d\gamma}{dn}+\lambda p'\,(n,\Gamma)\right]-\frac{\lambda p'\,(n,\Gamma)\,\tilde{\pi}}{n} \\
\frac{df_2}{d\Gamma} &= \frac{d\gamma}{dn},
\end{aligned}$$

where $\tilde{\pi} = \frac{\pi}{A_t^{\max}}$. Therefore, the derivative of private research with respect to public research is given by

$$\frac{dn}{d\Gamma} = \frac{-\frac{d\gamma}{dn}\left(\frac{(1-s_n)}{k}+\frac{\lambda p(n,\Gamma)}{n}\right)+\frac{\lambda p'(n,\Gamma)\tilde{\pi}}{nk}-\frac{(1-s_n)\lambda p'(n,\Gamma)}{k}}{\frac{d\gamma}{dn}\left[\frac{(1-s_n)}{k}+\frac{\lambda p(n,\Gamma)}{n}\right]+\frac{(1-s_n)\lambda p'(n,\Gamma)}{k}-\lambda\frac{d}{dn}\left(\frac{p(n,\Gamma)}{n}\right)\frac{\tilde{\pi}}{k}}.$$

Notice that the sign of this derivative is ambiguous, which implies that whether public research crowds out private research or not, depends on the values of the parameters and the initial situation of the economy.

Given $\frac{dn}{d\Gamma}$, it is immediate from (3.45) that the derivative of the growth rate with respect to public funding of research is positive and equal to

$$\frac{d\gamma}{d\Gamma} = \frac{\left(\frac{\lambda\tilde{\pi}}{k}\right)\frac{d\gamma}{dn}\left(\frac{p'(n,\Gamma)}{n} - \frac{d}{dn}\left(\frac{p(n,\Gamma)}{n}\right)\right)}{\frac{d\gamma}{dn}\left[\frac{(1-s_n)}{k} + \frac{\lambda p(n,\Gamma)}{n}\right] + \frac{(1-s_n)\lambda p'(n,\Gamma)}{k} - \lambda\frac{d}{dn}\left(\frac{p(n,\Gamma)}{n}\right)\frac{\tilde{\pi}}{k}}.$$

■

Proof of Proposition 13. In order to compare the growth effects of s_n and Γ, they must have an equivalent impact on the public budget. So let us consider as the initial situation the equilibrium corresponding to $s_n = 0$ and $\Gamma = 0$. In this situation, we must compare $\frac{d\gamma}{d\Gamma}$ and $\frac{d\gamma}{ds_n}\left(\frac{1}{n}\right)$. Recall that

$$\begin{aligned}
\frac{d\gamma}{ds_n} &= -\frac{d\gamma}{dn}\left(\frac{(1-\alpha)\zeta}{\det(J_F)}\right)\left(\frac{\gamma+\rho+\lambda p(n,\Gamma)}{k}\right) \\
\frac{d\gamma}{d\Gamma} &= \frac{d\gamma}{dn}\left(\frac{(1-\alpha)\zeta}{\det(J_F)}\right)\left(\lambda\frac{d}{dn}\left(\frac{p(n,\Gamma)}{n}\right)\frac{\tilde{\pi}}{k} - \frac{\lambda p'(n,\Gamma)\tilde{\pi}}{nk}\right).
\end{aligned}$$

Notice also that from $f_1(k,n) = 0$, $\lambda\frac{\tilde{\pi}}{k}\left(\frac{p'(n,\Gamma)}{n} - \frac{d}{dn}\left(\frac{p(n,\Gamma)}{n}\right)\right) = (1-s_n)\left(\frac{\gamma+\rho+\lambda p(n,\Gamma)}{nk}\right)$, therefore

$$\frac{d\gamma}{d\Gamma} = -\frac{d\gamma}{dn}\left(\frac{(1-\alpha)\zeta}{\det(J_F)}\right)(1-s_n)\left(\frac{\gamma+\rho+\lambda p(n,\Gamma)}{nk}\right).$$

Since we are considering $s_n = 0$, $\frac{d\gamma}{ds_n}\left(\frac{1}{n}\right) = \frac{d\gamma}{d\Gamma}$. ■

Proof of Corollary 14. Recall that

$$\begin{aligned}
\frac{d\gamma}{ds_n} &= \frac{d\gamma}{dn}\left(\frac{dn}{ds_n}\right) \\
\frac{d\gamma}{d\Gamma} &= \frac{d\gamma}{dn}\left(1+\frac{dn}{d\Gamma}\right).
\end{aligned}$$

Therefore, if $\frac{d\gamma}{ds_n}\left(\frac{1}{n}\right) = \frac{d\gamma}{d\Gamma}$, then $\frac{dn}{ds_n}\left(\frac{1}{n}\right) = 1 + \frac{dn}{d\Gamma}$ and $\frac{dn}{d\Gamma} = \frac{dn}{ds_n}\left(\frac{1}{n}\right) - 1$. ■

3.8 Calibration

Tables 3.1 to 3.7 show the results of the calibration of the model for the following set of parameters:

- Capital intensity, $\alpha = 0.7$. Therefore, we are considering a broad concept of capital that could include human capital.
- Contribution of applied research to technological change, $\beta = 0.55$.
- The discount rate and the depreciation rate are the standard values of $\rho = 0.02$ and $\delta = 0.05$.
- Other parameter values are: $\lambda = 0.05$, $b = 5$, $\sigma = \ln(1.2)$, $\tau_k = 0$, $L = 1$. They were chosen so that the rate of growth of the economy, the interest rate and the level of consumption were positive and in a reasonable range.
- Regarding the choice of the range of values for public research, we took as reference the value of private research intensity when $s_n = \Gamma_a = \Gamma_b = 0$. At this equilibrium, $n = 7.143$.
- The following tables and figures present the welfare effects of the different policy instruments. By default, the policy instruments that are not being analyzed are set to zero, except when indicated.

3.8.1 Public provision of research

Table 3.1
Welfare effect of the research subsidy

1a) $\lambda = 0.05$

s_n	$\frac{dc}{ds_n}$	$\frac{dV_s}{ds_n}$	$\frac{dV}{ds_n}$
0.0	-15.35	-1.53	13.77
0.1	-16.93	-5.34	12.17
0.2	-18.90	-12.5	7.88
0.3	-21.34	-26.8	-2.76
0.4	-24.59	-59.8	-30.59
0.5	-29.10	-156.9	-120.2
0.6	-35.70	-748.2	-699.8

1b) $\lambda = 0.1$

s_n	$\frac{dc}{ds_n}$	$\frac{dV_s}{ds_n}$	$\frac{dV}{ds_n}$
0.0	-11.55	8.81	27.08
0.1	-12.58	6.34	27.11
0.2	-13.83	0.77	24.73
0.3	-15.41	-12.00	16.14
0.4	-17.50	-44.30	-10.4
0.5	-20.30	-148.6	-106.4
0.6	-24.50	-920.0	-864.7

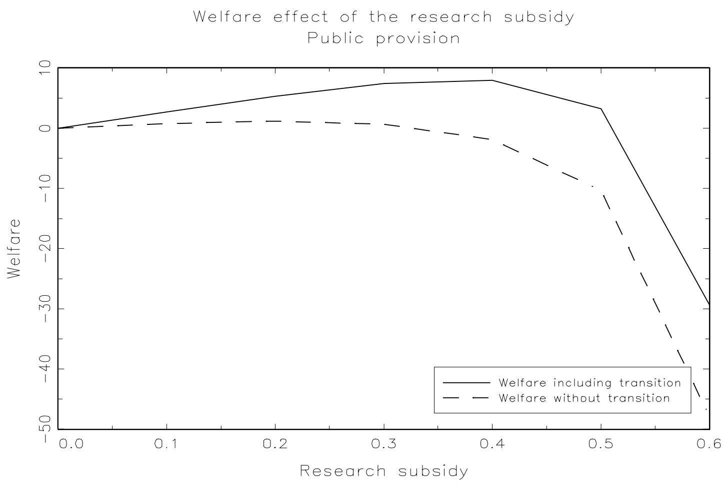

Figure 3.1: Welfare effect of the research subsidy

Table 3.2 Welfare effect of Γ_b when $\Gamma_a = 0$				
Γ_b	$\frac{dc}{d\Gamma_b}$	$\frac{dV_s}{d\Gamma_b}$	$\frac{dV}{d\Gamma_b}$	n
0	-1.92	-0.65	1.10	7.14
0.5	-1.85	-1.15	0.51	6.93
1	-1.79	-1.65	-0.08	6.74
1.5	-1.75	-2.18	-0.68	6.56
2	-1.70	-2.74	-1.29	6.40
2.5	-1.67	-3.34	-1.94	6.25
3	-1.64	-3.99	-2.63	6.11
4	-1.59	-5.54	-4.23	5.85
6	-1.52	-10.66	-9.29	5.43

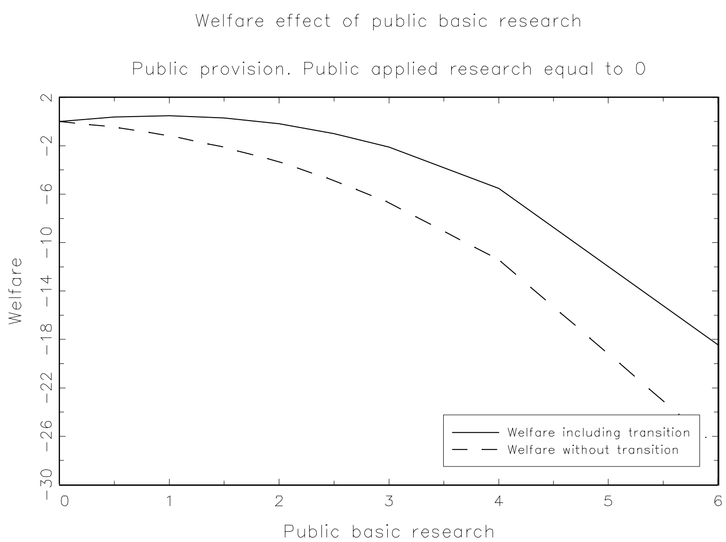

Figure 3.2: Welfare effect of public basic research ($\Gamma_a = 0$).

Table 3.3
Welfare effect of Γ_b when $\Gamma_a = 0.5$

Γ_b	$\frac{dc}{d\Gamma_b}$	$\frac{dV_s}{d\Gamma_b}$	$\frac{dV}{d\Gamma_b}$	γ	n
0	1.05	0.20	-0.78	0.03241	6.57
0.1	0.50	0.05	-0.44	0.03232	6.46
0.2	0.20	-0.05	-0.25	0.03228	6.35
0.5	-0.31	-0.28	-0.05	0.03230	6.07
1	-0.64	-0.59	-0.12	0.03253	5.67
1.5	-0.78	-0.88	-0.34	0.03285	5.33
2	-0.86	-1.16	-0.60	0.03320	5.02
2.5	-0.90	-1.44	-0.89	0.03354	4.75
3	-0.92	-1.72	-1.19	0.03387	4.49

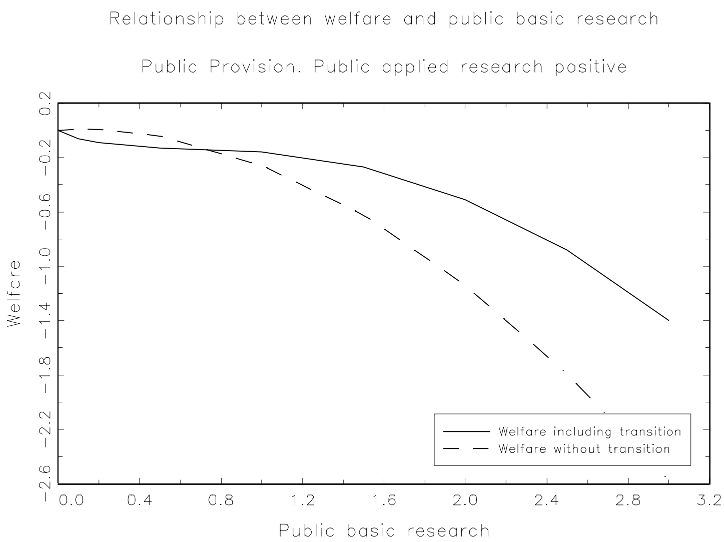

Figure 3.3: Welfare effect of public basic research ($\Gamma_a = 0.5$).

Table 3.4 Welfare effect of applied research					
Γ_a	$\frac{dc}{d\Gamma_a}$	$\frac{dV_s}{d\Gamma_a}$	$\frac{dV}{d\Gamma_a}$	γ	n
0	–	–	–	0.03264	7.14
10^{-8}	8628	1940	-6314	0.03264	7.14
0.01	6.58	1.58	-4.63	0.03246	7.09
0.1	0.68	0.31	-0.27	0.03223	6.95
0.2	-0.10	0.13	0.27	0.03221	6.84
0.5	-0.78	-0.14	0.61	0.03241	6.58
1	-1.75	-0.48	0.49	0.03297	6.21
2	-1.20	-1.17	-0.15	0.03424	5.61
4	-1.23	-2.66	-1.76	0.03658	4.69
10	-1.14	-11.50	-10.85	0.04105	3.03

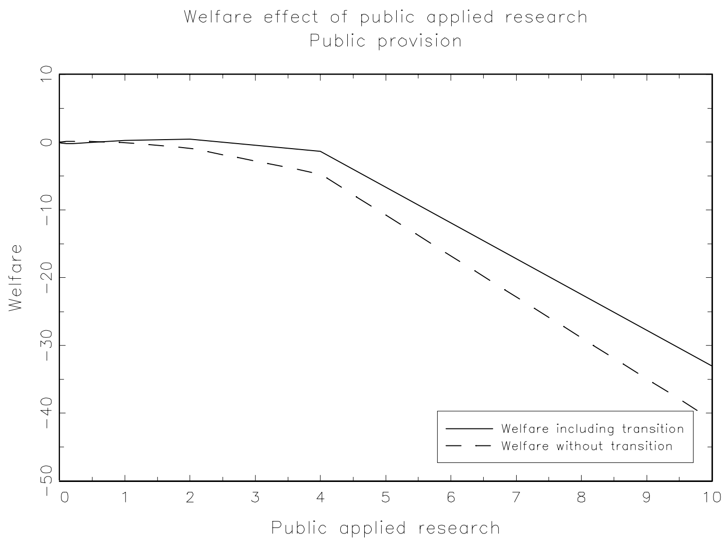

Figure 3.4: Welfare effect of public applied research

Table 3.5 Welfare effect of Γ when $\Gamma_b = \frac{b\Gamma-1}{2b}$ and $\Gamma_a = \frac{b\Gamma+1}{2b}$				
Γ	$\frac{dc}{d\Gamma}$	$\frac{dV_s}{d\Gamma}$	$\frac{dV}{d\Gamma}$	n
0.2	-0.06	-0.00	0.06	6.84
0.5	-0.07	-0.00	0.06	6.55
1	-0.08	-0.00	0.07	6.05
1.5	-0.09	0.00	0.09	5.57
2	-0.11	0.01	0.12	5.08
2.5	-0.13	0.01	0.15	4.60
3	-0.16	0.01	0.18	4.12
4	-0.27	0.02	0.30	3.18
5	-0.47	0.02	0.50	2.28
6	-0.91	-0.03	0.92	1.47
7	-1.67	-0.35	1.44	0.85
8	-2.30	-1.42	1.29	0.48
9	-2.49	-3.62	-0.07	0.30
10	-2.45	-7.68	-3.00	0.20

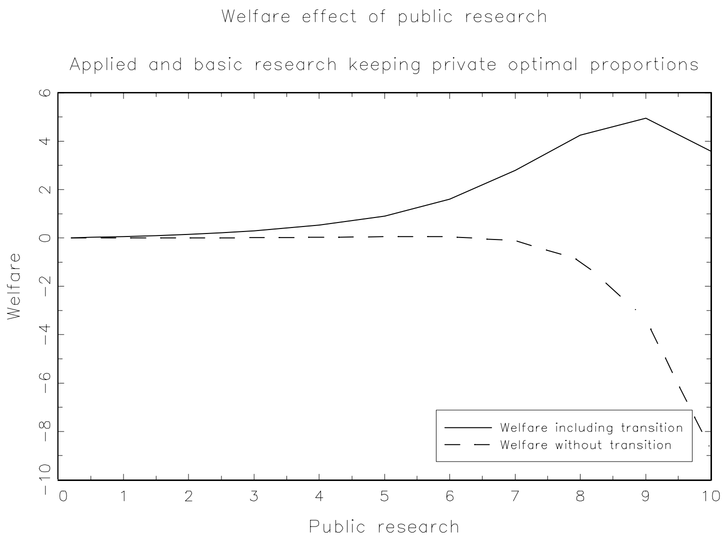

Figure 3.5: Welfare effect of public research when $\Gamma_a = \frac{1+b\Gamma}{2b}$ and $\Gamma_b = \frac{b\Gamma-1}{2b}$.

3.8.2 Public funding of research

The calibration is made with the same set of parameters as before but using the corresponding equations for this assumption.

Table 3.6 Welfare effect of s_n			
s_n	$\frac{dc}{ds_n}$	$\frac{dV_s}{ds_n}$	$\frac{dV}{ds_n}$
0.0	-15.35	-1.53	13.77
0.1	-16.93	-5.35	12.17
0.2	-18.88	-12.74	7.88
0.3	-21.34	-26.85	-2.76
0.4	-24.59	-59.80	-30.60
0.5	-29.06	-156.9	-120.2
0.6	-35.72	-748.2	-699.8

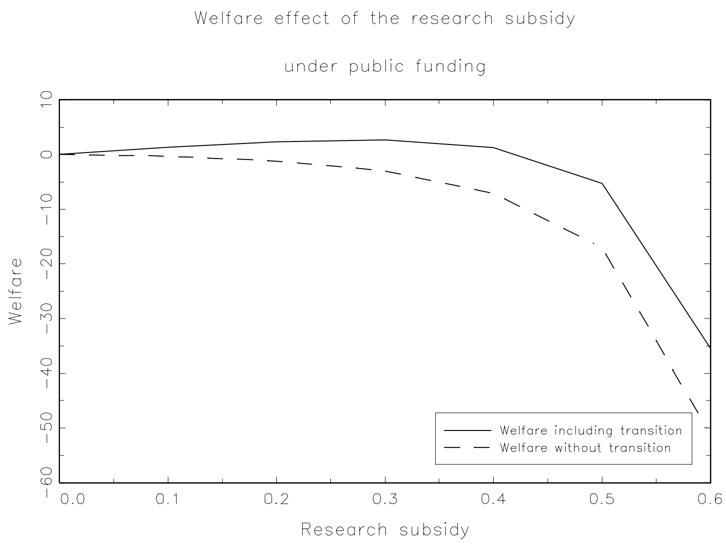

Figure 3.6: Welfare effect of the research subsidy under public funding.

Table 3.7 Welfare effect of Γ			
Γ	$\frac{dc}{d\Gamma}$	$\frac{dV_s}{d\Gamma}$	$\frac{dV}{d\Gamma}$
0	-2.15	-0.21	1.93
0.5	-2.11	-0.49	1.66
1	-2.08	-0.83	1.35
1.5	-2.04	-1.23	0.99
2	-2.01	-1.72	0.54
2.5	-1.98	-2.31	0.00
3	-1.95	-3.04	-0.65
4	-1.89	-5.11	-2.51
6	-1.78	-15.52	-11.9

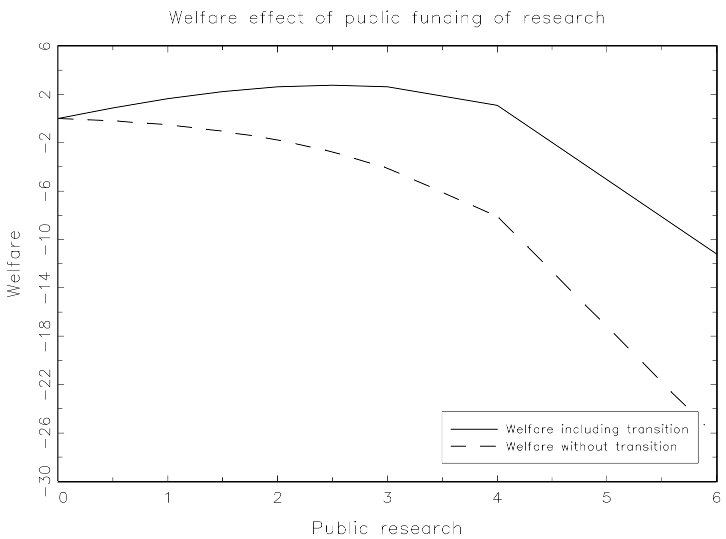

Figure 3.7: Welfare effect of public R&D funding.

3.9 Results when private firms do not invest in basic research

In order to guarantee that the equilibrium value of basic research intensity n_b is positive we have to impose some restrictions on the parameters. For the general case without any type of public research, the equations determining n and k in a steady state would be (3.7) and (3.8) but substituting the interest rate by its steady state expression

$$r = \rho + \gamma.$$

Solve for k in (3.7) and (3.8) and note that the first one is increasing in n while the second is decreasing. Therefore, the following condition implies $n > \frac{1}{b}$:

$$\chi_5 < \chi_6, \tag{3.46}$$

where

$$\begin{aligned} \chi_5 &= \left(\frac{(1 - s_n) \left(\gamma \left(\frac{1}{b} \right) + \rho + \lambda p \left(\frac{1}{b} \right) \right)}{\lambda b p \left(\frac{1}{b} \right) (1 - \alpha) \alpha L^{1-\alpha}} \right)^{\frac{1}{\alpha}} \\ \chi_6 &= \left(\frac{\alpha^2 L^{1-\alpha}}{\gamma \left(\frac{1}{b} \right) + \rho + \delta + \tau_k} \right)^{\frac{1}{1-\alpha}}, \end{aligned}$$

and where $\gamma\left(\frac{1}{b}\right)$ is the growth rate (see equation (3.20) for its functional form) associated to $n = \frac{1}{b}$ and $p(n) = \frac{1+bn}{2\sqrt{b}}$.

Notice that the introduction of the public research will modify the condition for positive private basic research given in equation (3.46). If we consider public provision of research, the condition is

$$\chi_7 < \chi_8, \tag{3.47}$$

where

$$\chi_7 = \left(\frac{(1-s_n)\left(\gamma\left(\frac{1}{b}\right)+\rho+\lambda p\left(\frac{1}{b}\right)+\lambda\left(\Gamma_a\left(1+b\Gamma_b\right)\right)^{\frac{1}{2}}\right)}{\lambda bp\left(\frac{1}{b}\right)(1-\alpha)\,\alpha L^{1-\alpha}}\right)^{\frac{1}{\alpha}}$$

$$\chi_8 = \left(\frac{\alpha^2 L^{1-\alpha}}{\gamma\left(\frac{1}{b}\right)+\rho+\delta+\tau_k}\right)^{\frac{1}{1-\alpha}}.$$

Similarly, if we consider public funding of research projects, the condition for n_b positive is obtained from equations (3.21) and (3.23) obtained in section 3.3. Following the same reasoning used to derive condition (3.46) a sufficient condition for n_b to be positive is

$$\chi_9 < \chi_{10},$$

where

$$\chi_9 = \left(\frac{(1-s_n)\left(\gamma\left(\frac{1}{b}+\Gamma_b-\Gamma_a\right)+\rho+\lambda p\left(\frac{1}{b},\Gamma\right)\right)}{\lambda bp\left(\frac{1}{b},\Gamma\right)(1-\alpha)\,\alpha L^{1-\alpha}}\right)^{\frac{1}{\alpha}}$$

$$\chi_{10} = \left(\frac{\alpha^2 L^{1-\alpha}}{\gamma\left(\frac{1}{b}+\Gamma_b-\Gamma_a\right)+\rho+\delta+\tau_k}\right)^{\frac{1}{1-\alpha}},$$

and where $\gamma\left(\frac{1}{b}+\Gamma_b-\Gamma_a\right)$ is the rate of growth associated to a level of research intensity $n=\frac{1}{b}+\Gamma_b-\Gamma_a$. The following subsections show the results when these conditions are not satisfied.

3.9.1 Public provision of research

If the equilibrium level of private research is not large enough (i.e. $n \leq \frac{1}{b}$), firms do not invest in basic research and therefore, $n_a = n$ and $n_b = 0$. Consequently, the arrival rate of innovations in the private sector will be given by $\lambda p(n)$ where $p(n) = n^{\frac{1}{2}}$ and the research arbitrage equation will now be

$$1-s_n = \left(\frac{\lambda}{n_t^{\frac{1}{2}}}\right)\left(\frac{(1-\alpha)\alpha L^{1-\alpha}k_t^{\alpha}}{r_t+\lambda n_t^{\frac{1}{2}}+\lambda\left(\Gamma_a(1+b\Gamma_b)\right)^{\frac{1}{2}}}\right).$$

The marginal benefit of one unit of research intensity is the product of the value of the innovation V_t and the private marginal effect of research on that sector's rate of innovation $\lambda\frac{p(n_t)}{n_t}$. Thus, the research firm sees itself as facing a constant arrival rate $\lambda\frac{p(n_t)}{n_t}n_{jt}$ per unit of research expenditure where n_t is the sector's R&D expenditure and n_{jt} is the firm's research intensity.[15]

The equations determining the steady state value of n and k are given by

$$\begin{aligned}
\gamma+\rho+\delta+\tau_k &= \alpha^2 L^{1-\alpha}k^{\alpha-1} \\
1-s_n &= \left(\frac{\lambda}{n_t^{\frac{1}{2}}}\right)\left(\frac{\lambda(1-\alpha)\alpha L^{1-\alpha}k_t^{\alpha}}{r_t+\lambda n_t^{\frac{1}{2}}+\lambda\left(\Gamma_a(1+b\Gamma_b)\right)^{\frac{1}{2}}}\right).
\end{aligned}$$

Following the same steps as in the proofs of propositions 7, 8 and 9 we can compute the derivatives of private research and growth with respect to the policy variables. We find that these derivatives are given by

$$\begin{aligned}
\frac{dn}{ds_n} &= -\frac{\left(\frac{(1-\alpha)\zeta}{k}\right)\left(\gamma+\rho+\lambda n^{\frac{1}{2}}+\lambda\Gamma_a(1+b\Gamma_b)\right)}{\det(J_F)} \\
\frac{dn}{d\Gamma_a} &= \frac{(1-\alpha)\zeta\left(\frac{\beta\gamma}{n_A}\left(\frac{\lambda}{n^{\frac{1}{2}}}+\frac{(1-s_n)}{k}\right)+\left(\frac{(1-s_n)\lambda}{k}\right)\left(\frac{1+b\Gamma_b}{\Gamma_a}\right)^{\frac{1}{2}}\right)}{\det(J_F)} \\
\frac{dn}{d\Gamma_b} &= \frac{(1-\alpha)\zeta\left(\frac{(1-\beta)\gamma}{n_B}\left(\frac{\lambda}{n^{\frac{1}{2}}}+\frac{(1-s_n)}{k}\right)+\left(\frac{(1-s_n)\lambda b}{2k}\right)\left(\frac{\Gamma_a}{1+b\Gamma_b}\right)^{\frac{1}{2}}\right)}{\det(J_F)},
\end{aligned}$$

[15] When research firms do not perform basic research, the function $p(n)$ shows decreasing returns. Following Aghion and Howitt (1998) we assume that it results from research congestion within a product which implies that the research firm sees itself as facing a constant arrival rate per unit of R&D expenditure.

and

$$\begin{aligned}
\frac{d\gamma}{ds} &= \left(\frac{\beta\gamma}{n_A}\right)\frac{dn}{ds} \\
\frac{d\gamma}{d\Gamma_a} &= \left(\frac{\beta\gamma}{n_A}\right)\left(\frac{(1-\alpha)\zeta}{-\det(J_F)}\right)\left(\frac{\lambda(1-s_n)}{2kn^{\frac{1}{2}}}+\frac{\lambda\pi}{2kn^{\frac{3}{2}}}-\left(\frac{\lambda(1-s_n)}{2k}\right)\left(\frac{1+b\Gamma_b}{\Gamma_a}\right)^{\frac{1}{2}}\right) \\
\frac{d\gamma}{d\Gamma_b} &= \left(\frac{(1-\beta)\gamma}{n_B}\right)\left(\frac{(1-\alpha)\zeta}{-\det(J_F)}\right) \\
&\quad \left(\frac{\lambda(1-s_n)}{2kn^{\frac{1}{2}}}+\frac{\lambda\pi}{2kn^{\frac{3}{2}}}-\left(\frac{\beta n_B}{(1-\beta)n_A}\right)\left(\frac{\lambda b(1-s_n)}{2k}\right)\left(\frac{\Gamma_a}{1+b\Gamma_b}\right)^{\frac{1}{2}}\right).
\end{aligned}$$

Since $\det(J_F)$ is negative, the derivatives of private research with respect to the subsidy rate and basic and applied public research are respectively positive, negative and negative. This implies that the derivative of the growth rate with respect to the research subsidy is positive. The relationship between steady state growth and public research is ambiguous. We find that $\frac{d\gamma}{d\Gamma_a}$ will be positive whenever

$$\Gamma_a > \frac{1+b\Gamma_b}{b}, \tag{3.48}$$

since under this condition $\frac{\lambda(1-s_n)}{2kn^{\frac{1}{2}}}-\left(\frac{\lambda(1-s_n)}{2k}\right)\left(\frac{1+b\Gamma_b}{\Gamma_a}\right)^{\frac{1}{2}}$ is positive. A sufficient condition for negativity would be obtained forcing

$$\frac{\lambda(1-s_n)}{2kn^{\frac{1}{2}}}+\frac{\lambda\pi}{2kn^{\frac{3}{2}}}-\left(\frac{\lambda(1-s_n)}{2k}\right)\left(\frac{1+b\Gamma_b}{\Gamma_a}\right)^{\frac{1}{2}}<0. \tag{3.49}$$

Of course, a necessary condition is

$$\Gamma_a \leq \frac{1+b\Gamma_b}{b}.$$

Requiring (3.49) is equivalent to require

$$\left(\frac{\Gamma_a}{1+b\Gamma_b}\right)^{\frac{1}{2}} < \frac{\lambda n}{2\lambda n^{\frac{1}{2}}+\rho+\gamma+\lambda\left(\Gamma_a\left(1+b\Gamma_b\right)\right)^{\frac{1}{2}}}. \tag{3.50}$$

Let $\bar{n}$ be the equilibrium value of research intensity when $\Gamma_a = 0$. Then if the right hand side of (3.50) is positive when evaluated at 0, there will exist a range of values of Γ_a for which the condition is satisfied. However, the left hand side of (3.50) grows with Γ_a and the right hand side decreases (because n is negatively related to Γ_a and the function is increasing in n). This implies that we will reach a value of Γ_a smaller than $\frac{1+b\Gamma_b}{b}$ for which the condition is no longer satisfied.

With respect to the growth derivative of public basic research, notice that if $\Gamma_a = 0$, it will be positive for any positive value of public basic research. If Γ_a is positive, then a sufficient condition for $\frac{d\gamma}{d\Gamma_b} > 0$ is the following:

$$\Gamma_b \leq \frac{1 + \left(1 + \left(\frac{\beta}{1-\beta}\right)^2\right)^{\frac{1}{2}}}{\frac{b}{2}\left(\frac{\beta}{1-\beta}\right)^2}. \tag{3.51}$$

In order to obtain $\frac{d\gamma}{d\Gamma_b} < 0$ the next inequality must be satisfied:

$$\frac{\lambda\left(1-s_n\right)}{2kn^{\frac{1}{2}}} + \frac{\lambda\pi}{2kn^{\frac{3}{2}}} - \left(\frac{\beta}{1-\beta}\right)\frac{n_B}{n_A}\left(\frac{\lambda b\left(1-s_n\right)}{2k}\right)\left(\frac{\Gamma_a}{1+b\Gamma_b}\right)^{\frac{1}{2}} < 0,$$

which is equivalent to require

$$b\Gamma_b\left(\frac{\beta}{1-\beta}\right)\left(\frac{\Gamma_a}{1+b\Gamma_b}\right)^{\frac{1}{2}} > \frac{\left(2\lambda n^{\frac{1}{2}} + \rho + \gamma + \lambda\left(\Gamma_a\left(1+b\Gamma_b\right)\right)^{\frac{1}{2}}\right)\left(n+\Gamma_a\right)}{n}. \tag{3.52}$$

Of course, for (3.52) to be satisfied it is necessary that (3.51) is not. The right hand side of (3.52) is decreasing in n if $\Gamma_a > n$. Therefore, if we impose $\Gamma_a > \frac{1}{b}$ and recall that we are just considering equilibria with $n < \frac{1}{b}$, we can consider this expression decreasing in n. Therefore, a sufficient condition for $\frac{d\gamma}{d\Gamma_b} < 0$ would be

$$b\Gamma_b\left(\frac{\beta}{1-\beta}\right)\left(\frac{\Gamma_a}{1+b\Gamma_b}\right)^{\frac{1}{2}} > \frac{\left(2\lambda\left(\hat{n}\right)^{\frac{1}{2}} + \rho + \gamma\left(\hat{n}\right) + \lambda\left(\Gamma_a\right)^{\frac{1}{2}}\right)\left(\hat{n}+\Gamma_a\right)}{\hat{n}},$$

where $\hat{n}$ is the equilibrium value of research intensity when $\Gamma_b = 0$.

Therefore, if initially private firms do not perform basic research, the appropriate policy to induce them to do so consists of research subsidies that will increase the level of private research since the effect of public R&D, though positive on growth under some conditions, reduces the total amount of private research and thus, will not induce a positive level of private basic research.

3.9.2 Public funding of research

When private research does not reach a high enough level, private firms may not devote resources to basic research. In the case of the *public funding* assumption this decision depends also on the level of public research. If $n < \frac{1}{b}+\Gamma_b-\Gamma_a$, then all the research resources of private firms are devoted to applied projects of research. In this case, the arrival rate of innovations will be

$$\lambda\left((n+\Gamma_a)(1+b\Gamma_b)\right)^{\frac{1}{2}},$$

while the research arbitrage equation will be given by

$$1-s_n = \left(\frac{\lambda\left((n_t+\Gamma_a)(1+b\Gamma_b)\right)^{\frac{1}{2}}}{n_t}\right)\left(\frac{(1-\alpha)\alpha L^{1-\alpha}k_t^{\alpha}}{r_t+\lambda\left((n_t+\Gamma_a)(1+b\Gamma_b)\right)^{\frac{1}{2}}}\right).$$

Therefore, the derivatives of private research and the rate of growth may be computed to obtain

$$\begin{aligned}
\frac{dn}{ds} &= -\frac{\left(\frac{(1-\alpha)\zeta}{k}\right)\left(\gamma+\rho+\lambda\left((\Gamma_a+n)(1+b\Gamma_b)\right)^{\frac{1}{2}}\right)}{\det(J_F)} \\
\frac{dn}{d\Gamma_a} &= \frac{(1-\alpha)\zeta\left(\left(\frac{\beta\gamma}{n_A}\right)\left(\frac{\lambda((n+\Gamma_a)(1+b\Gamma_b))^{\frac{1}{2}}}{n}+\frac{(1-s_n)}{k}\right)-\frac{(1-s_n)(\rho+\gamma)}{2k(n+\Gamma_a)}\right)}{\det(J_F)} \\
\frac{dn}{d\Gamma_b} &= \frac{(1-\alpha)\zeta\left(\frac{(1-\beta)\gamma}{n_B}\right)\left(\frac{\lambda((n+\Gamma_a)(1+b\Gamma_b))^{\frac{1}{2}}}{n}+\frac{(1-s_n)}{k}\right)}{\det(J_F)}+ \\
&\quad+\frac{(1-\alpha)\zeta\left(\left(\frac{(1-s_n)\lambda b}{2k}\right)\left(\frac{\Gamma_a+n}{1+b\Gamma_b}\right)^{\frac{1}{2}}-\left(\frac{b\lambda\pi}{2nk}\right)\left(\frac{\Gamma_a+n}{1+b\Gamma_b}\right)^{\frac{1}{2}}\right)}{\det(J_F)},
\end{aligned}$$

and

$$\begin{aligned}
\frac{d\gamma}{ds_n} &= \left(\frac{\beta\gamma}{n_A}\right)\frac{dn}{ds} \\
\frac{d\gamma}{d\Gamma_a} &= \left(\frac{\beta\gamma}{n_A}\right)\left(\frac{(1-\alpha)\,\zeta}{-\det(J_F)}\right)\left(\frac{\lambda\,(1-s_n)}{k}\left(\frac{dp\,(n)}{dn}\right)-\lambda\frac{d}{dn}\left(\frac{p\,(n)}{n}\right)\frac{\tilde{\pi}}{k}+\frac{(1-s_n)\,(\rho+\gamma)}{kn_A}\right) \\
\frac{d\gamma}{d\Gamma_b} &= \left(\frac{(1-\beta)\,\gamma}{n_B}\right)\left(\frac{(1-\alpha)\,\zeta}{-\det(J_F)}\right) \\
&\quad\left(\left(\frac{\lambda\,(1-s_n)}{k}\right)\left(\frac{dp\,(n)}{dn}\right)-\lambda\frac{d}{dn}\left(\frac{p\,(n)}{n}\right)\frac{\tilde{\pi}}{k}+\left(\frac{\beta}{1-\beta}\right)\left(\frac{(1-s_n)\,b\Gamma_b\,(\rho+\gamma)}{2kn_A\,(1+b\Gamma_b)}\right)\right),
\end{aligned}$$

where

$$\begin{aligned}
p\,(n) &= ((n+\Gamma_a)(1+b\Gamma_b))^{\frac{1}{2}}, \\
\frac{dp\,(n)}{dn} &= \frac{1}{2}\left(\frac{1+b\Gamma_b}{n+\Gamma_a}\right)^{\frac{1}{2}}, \\
\frac{d}{dn}\left(\frac{p\,(n)}{n}\right) &= -\frac{1}{2}\left(\frac{(n+2\Gamma_a)\,(1+b\Gamma_b)^{\frac{1}{2}}}{n^2\,(n+\Gamma_a)^{\frac{1}{2}}}\right).
\end{aligned}$$

Therefore, the three derivatives are positive as we obtained for the case in which private firms performed basic research.

Chapter 4

Technological Progress and the Distribution of Productivities across Sectors

4.1 Introduction

Does technological progress increase or reduce inequality in the profitability of productive activities? How is the distribution of productivities related to the growth process? Do growth promoting policies induce different degrees of inequality among productivities? In this paper we try to provide answers to these questions by means of an endogenous growth model in which the distribution of productivities across sectors affects and is affected by the characteristics of the process of technological change.

We take as reference the Aghion and Howitt (1998) model of endogenous technological change in which the distribution of relative productivities is time invariant and is not affected by changes in most of the parameters except for the size of innovations. By means of the introduction of a punishment to obsolescence, we develop a model in which both technological parameters and policy instruments will be able to modify the distribution of productivities. We will find that in some cases, faster growth can induce more inequality, introducing a wider gap between the technological leaders of the economy and the less innovative sectors.

R&D based models of growth were initially divided into horizontal models of product development (as in Romer 1990) and models of growth through creative destruction (Aghion and Howitt 1992). The introduction of the schumpeterian concept of creative destruction allows for the existence of obsolescence of old intermediate products but technological improvements in other sectors can also cause relative obsolescence. However, Aghion and Howitt (1992) considered only one intermediate sector producing improved varieties of the same good as technology evolved. When a multisector approach is taken, as in Caballero and Jaffé (1993) and Howitt and Aghion (1998), a wide variety of new considerations appear. In this new framework, growth promoting policies will make aggregate productivity grow faster but different policies may have distinct effects on the distribution of productivities across the economy. Empirical studies detect relevant changes in the distribution of productivities in the last decades. Cameron *et al.* (1997) find that the distribution of productivity levels across UK manufacturing sectors exhibits an increase in dispersion and becomes increasingly skewed during the period 1973-1989. They find evidence of convergence of a number of industries just below the mean while productivity levels in a few sectors persistently remain above and rise away from mean values. This divergence in productivity levels between high-tech industries and traditional sectors and the formation of technological clusters has been observed in most developed countries.[1] In addition, there exists a wide array of policies that try to affect the productive performance of different sectors. Research subsidies are predominantly devoted to high-tech sectors while most countries develop programs to support the competitiveness of traditional sectors or to increase research productivity.[2] The model we propose allows to analyze the distributional implications of these different policies in a theoretical framework.

The distribution of productivities considered in this model differs from the one used in leap-frogging neo-Schumpeterian literature in the following aspect: In the standard model, the occurrence of a sole innovation would take the productivity of the sector to the leading edge, no matter how long ago occurred the last innovation or how obsolete was the previous technology. In our model, the introduction of a *punishment* to obsolescence creates two classes of sectors. If the relative productivity of a sector falls below a given threshold, it will not be able to reach the technological frontier with just one innovation. Instead, the productivity increase will only be a fraction of the gap existing between the previous productivity and the most advanced technology of the economy. We will refer to these sectors as the *lagging group.* Conversely, the *leading group* will be formed by those sectors with a relative productivity parameter above that threshold. These sectors are able to reach the leading edge if they innovate, but if they do not, their relative productivity will fall and will enter into the lagging class. The resulting distribution will be affected

[1] See Bergeron *et al.* (1998) or Boschma (1999).

[2] See Ford, R. and W. Suyker (1990) and Eaton *et al.* (1998).

by policy variables and technological parameters. We find that a larger productivity of research or an increase in the incentives to accumulate capital will make the economy grow faster and reduce the mass of technological laggards, improving thus the distribution of productivities and profits across sectors. Conversely, a larger size of innovations or a higher influence of individual innovations on the aggregate state of knowledge will increase the size of the lagging group, and therefore, there will exist a larger mass of firms earning relatively low profits with respect to the technological leaders. Whether this increase in the size of innovations is growth enhancing or not will depend on the assumption we make about what determines the growth rate of productivity. Similarly, a research subsidy to high-tech sectors will also reduce the mass of the leading group since it will induce a higher research intensity and thus a faster rate of decay of non-innovating sectors. Again, the effect on growth of this subsidy depends on how the size of the leading group affects the evolution of aggregate productivity. We have also found that a subsidy to less research intensive sectors will reduce the size of the lagging group and may increase the rate of growth of the economy.

In summary, this model establishes a set of links between the process of technological progress and the distribution of productivities and profits across economic sectors. We find that if technological progress affects high-tech and traditional sectors differently, the impact of changes in the determinants of economic growth may be very different depending on which is the source of faster growth.

The rest of the paper is organized as follows: Section 4.2 presents the model, sections 4.3 and 4.4 perform the equilibrium and steady state analysis and section 4.5 concludes the paper.

4.2 The model

This paper presents a model in which the nature of the process of innovation will affect the distribution of productivities across sectors. The paper is based on the work of Aghion and Howitt (1998) but their model is modified in such a way that changes in the technological parameters will influence the distribution of profits across economic sectors.

4.2.1 Consumers

There exists a representative consumer who gets utility from the consumption of a final good. He therefore, will maximize the present value of utility

$$V(C(t)) = \int_0^\infty \ln(C(t))e^{-\rho t}dt, \tag{4.1}$$

where $C(t)$ is consumption at time t and ρ is the rate of discount.

4.2.2 Final good sector

The consumption good is produced in a competitive sector out of labor L, that is assumed to be exogenously given, and a continuum of mass one of intermediate goods. Let $m_i(t)$ be the supply of sector i at date t. The production function is a Cobb-Douglas with constant returns on intermediate goods and efficiency units of labor as given by

$$Y(t) = L^{1-\alpha} \int_0^1 A_i(t) [m_i(t)]^\alpha di, \tag{4.2}$$

where $Y(t)$ is final good production and $A_i(t)$ is the productivity coefficient of each sector. The evolution of each sector's productivity coefficient $A_i(t)$ is determined in the research sector. I assume equal factor intensities to simplify calculations.

4.2.3 Intermediate goods

Intermediate goods are produced in a sector formed by a continuum of monopolies each producing one good. They are monopolies because their production technology is protected by a patent. The only input in the production of intermediate goods is capital. In particular, it is assumed that $A_i(t)$ units of capital are needed to produce one unit of intermediate good i at date t. As we will see, this assumption is necessary to obtain stability. Capital is hired in a perfectly competitive market at rate $\zeta(t)$. Therefore, the cost of one unit of intermediate good i is $A_i(t)\zeta(t)$. Because the final good sector is assumed to be competitive, the equilibrium price $p(m_i(t))$ of intermediate good i will be its marginal product

$$p(m_i(t)) = \alpha L^{1-\alpha} A_i(t)\left[m_i(t)\right]^{\alpha-1}.$$

Consequently, the monopolist's profit maximization problem will be

$$\begin{aligned} \pi_i(t) &= \max_{m_i(t)} [p(m_i(t))m_i(t) - A_i(t)\zeta(t)m_i(t)] \\ \textit{subject to} \quad p(m_i(t)) &= \alpha L^{1-\alpha} A_i(t)\left[m_i(t)\right]^{\alpha-1}, \end{aligned}$$

from where we obtain the profit-maximizing supply and the flow of profits as

$$\begin{aligned} m_i(t) &= L\left(\frac{\alpha^2}{\zeta(t)}\right)^{\frac{1}{1-\alpha}} \\ \pi_i(t) &= \alpha(1-\alpha)L^{1-\alpha}A_i(t)\left[m_i(t)\right]^{\alpha}. \end{aligned}$$

Due to the assumption of equal factor intensities, supply of intermediate goods is equal in all sectors, $m_i(t) = m(t)$. Thus, the aggregate demand of capital is equal to $\int_0^1 A_i(t)\, m(t)\, di$. Let $A(t) = \int_0^1 A_i(t)\, di$, be the aggregate productivity coefficient. Then, equilibrium in the capital market requires demand to equal supply

$$A(t)\, m(t) = K(t),$$

or equivalently, the flow of intermediate output must be equal to $\frac{K(t)}{A(t)}$ which we will call capital intensity and denote by $k(t)$. That is,

$$m(t) = \frac{K(t)}{A(t)} \equiv k(t).$$

With this notation we can express the equilibrium rental rate in terms of capital intensity

$$\zeta(t) = \alpha^2 L^{1-\alpha} [k(t)]^{\alpha-1}. \tag{4.3}$$

4.2.4 Research sector

Innovations are produced using the same technology of the final good sector. Hence, they need capital apart from labor to be produced. Let $n_i(t) \equiv \frac{N_i(t)}{A^{\max}(t)}$ be the productivity adjusted level of research or research intensity of sector i at date t. It is defined as the total amount of output invested in research by that sector $N_i(t)$, divided by $A^{\max}(t)$, the productivity coefficient of the most advanced technology in the economy. Investment in research is adjusted by $A^{\max}(t)$ in order to take into account the effect of increasing technological complexity. Thus, as technology evolves and becomes more complex, an ever increasing amount of research will be necessary in order to obtain further technological improvements. The Poisson arrival rate of innovations in each sector is assumed to be $\lambda n_i(t)$, where λ is a positive parameter representing the productivity of research.

Let us define $a_i(t)$ as the relative productivity parameter of sector i at date t. This relative productivity is given by the productivity coefficient $A_i(t)$ of that sector, divided by the productivity coefficient $A^{\max}(t)$ of the leading edge technology, and this ratio measures the technological level of the sector with respect to the most advanced technology of the economy. We will assume that $A^{\max}(t)$ will grow due to the flow of innovations in the economy. Therefore, if $A_i(t)$ does not change, the relative productivity parameter will gradually fall as the sector's technology becomes obsolete. This process of obsolescence can be avoided if an innovation occurs in the sector since then, its productivity coefficient will increase. In order to take into account the effect of intertemporal and intersectoral spillovers, we assume that $A_i(t)$ will jump to $A^{\max}(t)$. That is, the final increase in productivity depends upon the evolution of innovations in the rest of the economy and the technological gain will arise from the adoption of new technologies created in other sectors and the absorption of spillovers. However, consider a sector with a very low relative productivity parameter. A low value of $a_i(t)$ implies that the sector's technology has fallen far behind the leading edge and that no recent innovations have taken place. Let us call this type of sectors *lagging sectors.* In Aghion and Howitt's model, a sole innovation would take the productivity coefficient of this sector to the leading edge. In the present model, we will introduce a punishment for having lagged behind, in the sense that if the relative productivity parameter has fallen below a given threshold, innovating once will not allow the sector to reach the top of the distribution. We will thus assume that if an innovation occurs in a lagging sector, the productivity coefficient attained will only be a fraction of $A^{\max}(t)$. Specifically, we assume that if $a_i(t)$ falls below β, the relative productivity parameter attainable by an innovation will be γ instead of 1, where $0 \leq \beta < \gamma < 1$. In order to analyze the implications of this assumption, we will consider first the determination of the equilibrium level of research investment.

There exists a number of research firms in each sector competing in a patent race to get the next innovation for a specific production technology. The first innovating firm gains the patent and it either becomes the monopolist producer of the new variety or sells the patent to an established firm. In any case, the reward to the innovation will be the present value of the flow of profits arising from the monopolistic exploitation of the patent. Let us denote the value of the innovation by $V(t)$. On the other hand, the cost of one unit of research is one unit of output. If a firm invests one unit of research it will have a probability of obtaining the innovation equal to $\frac{\lambda}{A^{\max}(t)}$. The research arbitrage equation establishes that the cost of one unit of research must be equal to the expected revenue from this research. Therefore,

$$1 - s_i = \frac{\lambda V(t)}{A^{\max}(t)}, \tag{4.4}$$

where s_i is the subsidy rate to research in sector i. Consider now the determination of the value of the innovation $V(t)$. The flow of profits will depend on whether the innovating sector was a leading or a lagging sector. If the innovation has occurred in a leading sector, then the productivity coefficient achieved is $A^{\max}(t)$ and the flow of profits will be given by $\alpha(1-\alpha) L^{1-\alpha} A^{\max}(t) [k(t)]^{\alpha}$ and equation (4.4) may be written as

$$1 - s_i = \frac{\lambda\alpha(1-\alpha) L^{1-\alpha} [k(t)]^{\alpha}}{r(t) + \lambda n_i(t)},$$

where $r(t)$ is the interest rate. Notice that in order to compute the present value of the flow of profits, the rate of discount includes $\lambda n_i(t)$ in addition to the rate of interest. The term $\lambda n_i(t)$ represents the probability that the incumbent monopolist is replaced by the owner of a new patent and it is also known as the rate of creative destruction.

If the innovating sector was a lagging sector, then the flow of profits arising from the innovation will be $\alpha(1-\alpha)L^{1-\alpha}\gamma A^{\max}(t)[k(t)]^{\alpha}$. Consequently, equation (4.4) will now be given by

$$1-s_i=\gamma\left(\frac{\lambda\alpha(1-\alpha)L^{1-\alpha}[k(t)]^{\alpha}}{r(t)+\lambda n_i(t)}\right).$$

It is thus obvious that research intensity in lagging and leading sectors will be generally different. In particular, we can establish that the relationship between research intensities will be

$$\lambda n_l(t)=\gamma\tau\lambda n_h(t)-(1-\gamma\tau)r(t),$$

where $n_l(t)$ and $n_h(t)$ are research intensity in lagging and leading sectors, respectively, and $\tau=\frac{1-s_h}{1-s_l}$, where s_l and s_h are the corresponding subsidies to lagging and leading sectors. Notice that in equilibrium, the research intensity performed in all the sectors belonging to the same group will be equal given that they will obtain the same reward. Notice also that if $\tau\leq\frac{1}{\gamma}$, research intensity in the lagging group will not be larger than research intensity in the leading group. In what follows we will restrict the analysis to subsidy values satisfying this condition, namely, that the subsidy to lagged sectors may increase research intensity up to but not above the level of leading sectors. Thus we will not consider subsidies that would make lagged sectors more research intensive than the technological leaders.

For the sake of simplicity, we will assume that aggregate knowledge and, hence, $A^{\max}(t)$ will only grow thanks to innovations in the leading group. Intuitively, this implies that lagged sectors only adapt technological improvements from other sectors, but do not add to the growth of the technological frontier. Indeed, data on the contribution of traditional sectors to knowledge creation suggest that this assumption is not too far from reality.[3] We will consider two alternative assumptions for the growth behavior of $A^{\max}(t)$. The first assumption simply states that the rate of growth of the knowledge frontier is proportional to the aggregate probability of innovation in leading sectors, that is

$$\frac{\dot{A}^{\max}(t)}{A^{\max}(t)} = \sigma(1-\phi)\lambda n_h(t), \tag{4.5}$$

where $\sigma > 0$ is a parameter that measures the effect of individual innovations on the leading edge productivity coefficient. This parameter is traditionally interpreted as measuring the size of innovations, but it can also represent the degree of interrelation between sectors or the capacity to absorb spillovers from other sectors. The parameter ϕ measures the size of the lagging group. We will refer to this assumption as the *aggregate assumption.*

In models where technological progress is due to both vertical and horizontal innovations, it is generally assumed that an increase in the mass of available technologies reduces the effect of an innovation on the aggregate economy. In particular, it is assumed that the increasing probability of innovation due to the larger mass of products is completely offset by the reduction in the marginal impact of an individual innovation.[4] In this case the rate of growth of aggregate knowledge would be proportional to the average probability of innovation in leading sectors. Consequently, we will refer to this assumption as the *average assumption* and the rate of growth of $A^{\max}(t)$ would be given by

$$\frac{\dot{A}^{\max}(t)}{A^{\max}(t)} = \theta\lambda n_h(t), \tag{4.6}$$

[3]Cameron *et al.* (1997) report that only seven industries out of nineteen accounted for 95% of TFP growth in the UK economy in the last decades. Among these industries, Computing, Pharmaceuticals and Aerospace, the highest productivity attainers, accounted for a 42% of the total growth in productivity.

[4]See Aghion and Howitt (1998), chapter 12 or Howitt (1999).

where $\theta > 0$ is a parameter measuring the effect on the rate of growth of aggregate knowledge of a change in the average probability of innovation in leading sectors.

The key difference between these two assumptions lies on whether we consider that the technological frontier is formed by all the production technologies in the economy or only by those sectors innovatively active enough to reach the frontier with just one innovation. In the first case, an increase in the mass of the leading group should make the economy grow faster because the sectors in this group are more research intensive. In the second case, even though there will be more research, there will also exist more technologies to improve and thus, research efforts will have to be distributed among more different fields.

The lagging group is formed by sectors with obsolete technologies in which no innovation has occurred for a considerably long period of time. Productivity increases in these sectors are generally due to the adoption of technologies from other sectors. Therefore, ignoring them as part of the technological frontier should not represent a problem, at least when there exists a large distance between traditional and high-tech sectors. In very developed economies we may expect a wide gap between the leading-edge production technologies and the most traditional sectors of the economy. In these cases, spillovers from the high-tech sectors will probably be technology specific and narrower in scope.[5] This picture of the technological system is better fit by the average assumption. On the other hand, consider an economy in the early phases of development or with a nearly non-existing high-tech sector. Then, the difference between the leading-edge and the more obsolete sectors will not be so large and technological improvements in the leading group will not be so

[5]Indeed, Cameron et al. (1997) find informal evidence suggesting that for at least a small subsector of industries, the development of technology is quite specific to the individual sector and does not spill over rapidly into many other manufacturing sectors.

specific that the whole mass of technologies cannot benefit from it. In this case, the most appropriate assumption would be the aggregate assumption.

Trying to connect these theoretical discussion with empirical findings, let us mention the paper by Caballero and Jaffé (1993) in which the authors observe a decline over the twentieth century in a parameter representing the "potency of spillovers emanating from each cohort of ideas or the intensity of use of old ideas by new ideas". This decline could be interpreted, in the authors' words, as a process by which "research is steadily becoming narrower and hence generates fewer spillovers because each new idea is relevant to a smaller and smaller set of technological concerns". The authors estimate that the average idea at the beginning of the century generated about 5 times the level of spillovers as the average recent idea. This narrower scope for spillovers could be supporting the average assumption, by which the relevant set of technologies that conform the technological frontier is the leading group and an increase in the size of this group would induce a smaller effect of innovations on the enlarged set of technologies.

We will develop the model first under the average assumption because this assumption allows us to identify the effect of growth determinants on the distribution of relative productivities. In fact, under the average assumption we could abstract from the complications arising from the interaction between the productivities distribution and the growth rate. When considering the aggregate assumption, we will have to take into account the relationship between changes in the mass of the lagging group and changes in the growth rate.

In addition to the effect on the research investment of firms, the introduction of the assumption that lagged sectors will not be able to reach the leading edge with a sole innovation has another important implication. Without this assumption, the long run distribution of relative productivity parameters is time invariant and does not depend on the growth behavior of the economy. Specifically, the long run distribution of relative productivities is described by the following distribution function[6]

$$H(a) = a^{\frac{1}{\sigma}}.$$

In the present model however, the distribution of relative productivities will depend upon the growth rate of the economy and will be affected by changes in the determinants of equilibrium.

4.2.5 Capital market

Capital is used as a factor of production in the intermediate goods sector. We have seen that equilibrium in the capital market requires the rental rate to satisfy equation (4.3). The owner of a unit of capital will obtain $\zeta(t)$ for it. This amount must be enough to cover the cost of capital. This includes the interest rate $r(t)$, the depreciation rate δ, and the tax rate on capital accumulation τ_k which is introduced in order to parametrize the incentives to accumulate capital. Hence, the capital market arbitrage equation is

$$r(t) + \delta + \tau_k = \alpha^2 L^{1-\alpha} [k(t)]^{\alpha-1},$$

which establishes a decreasing relationship between the interest rate and capital intensity.

[6] See Aghion and Howitt (1998).

4.2.6 Public sector

The role of the government in this model will be confined to the concession of subsidies to leading and lagging sectors s_h and s_l, respectively and the imposition of the tax on capital accumulation τ_k. The public budget will be balanced through a lump-sum tax or transfer T which will help us to isolate the effects of the different policy instruments. Therefore, the government budget is given by the following equation:

$$T(t) = s_h N_h(t) + s_l N_l(t) - \tau_k K(t).$$

4.2.7 Distribution of relative productivity coefficients

The existence of a lagging group that behaves differently after an innovation determines a distribution of relative productivities that will be affected by changes in the technological and policy parameters. The next proposition provides the distribution function of a under the average assumption:

Proposition 15 *The long run distribution of relative productivity coefficients under the average assumption is time invariant and is described by the following cumulative distribution function:*

$$H(a) = \begin{cases} \phi + (1-\phi) a^{\frac{1}{\theta}} & if \quad \gamma \leq a \leq 1 \\ \phi + \left(\frac{a}{\gamma}\right)^{\frac{1}{\theta}} \left((1-\phi)\gamma^{\frac{1}{\theta}} + \phi \int_a^{\gamma} \lambda n_l \left(\tilde{t}(a)\right) \left(\frac{a}{\gamma}\right)^{-\frac{1}{\theta}} \tilde{t}'(a)\, da \right) & if \quad \beta \leq a \leq \gamma \\ \phi \exp\left(\int_a^{\beta} \lambda n_l \left(\tilde{t}(a)\right) \tilde{t}'(a)\, da \right) & if \quad a \leq \beta \end{cases}, \tag{4.7}$$

where $\tilde{t}(a)$ is a differentiable and decreasing function relating date t and the relative productivity a of a given sector which is implicitly defined by equation (4.23) in Appendix 4.6.

Proof. See Appendix 4.6. ■

Similarly, Proposition 16 gives the distribution function of a under the aggregate assumption.

Proposition 16 *The distribution of relative productivity coefficients under the aggregate assumption is time invariant and may be characterized by the following distribution function:*

$$H(a) = \begin{cases} \phi + (1-\phi)\, a^{\frac{1}{\sigma(1-\phi)}} & if \quad \gamma \leq a \leq 1 \\ \begin{array}{l} \phi + (1-\phi)\, a^{\frac{1}{\sigma(1-\phi)}} + \\ + \left(\frac{a}{\gamma}\right)^{\frac{1}{\sigma(1-\phi)}} \phi \int_a^\gamma \lambda n_l\left(\tilde{t}(a)\right) \left(\frac{a}{\gamma}\right)^{-\frac{1}{\sigma(1-\phi)}} \tilde{t}'(a)\, da \end{array} & if \quad \beta \leq a \leq \gamma \\ \phi \exp\left(\int_a^\beta \lambda n_l\left(\tilde{t}(a)\right) \tilde{t}'(a)\, da\right) & if \quad a \leq \beta \end{cases}. \tag{4.8}$$

Proof. See Appendix 4.6. ■

The distribution of relative productivity coefficients will thus be affected by policy changes and the characteristics of the process of technological change. In order to analyze the implications of changes in these parameters we solve the model in the following section.

4.3 Equilibrium

4.3.1 Equilibrium under the average assumption.

General equilibrium is defined by the following equations:

$$1 - s_h = \frac{\lambda\alpha(1-\alpha) L^{1-\alpha} [k(t)]^\alpha}{r(t) + \lambda n_h(t)}, \tag{4.9}$$

$$\lambda n_l(t) = \gamma\tau\lambda n_h(t) - (1-\gamma\tau)\, r(t), \tag{4.10}$$

$$r(t) + \delta + \tau_k = \alpha^2 L^{1-\alpha} [k(t)]^{\alpha-1}, \tag{4.11}$$

where (4.9) is the arbitrage equation for research in a leading sector, (4.10) gives the relationship between lagged and leading sectors research intensity and (4.11) is the capital market arbitrage equation. The last expression implies that the interest rate is a function of the equilibrium value of capital intensity. Thus, from (4.9) we can view $n_h(t)$ as a function of $k(t)$, while (4.10) gives us the research intensity $n_l(t)$ in lagged sectors as a function of capital intensity $k(t)$. Consequently, using these equations we may denote $n_h(t) = n_h(k(t))$, $n_l(t) = n_l(k(t))$ and $r(t) = r(k(t))$. Therefore, we can express the dynamics of the model in terms of capital and consumption. The laws of motion for these two variables are

$$\dot{K}(t) = Y(t) - C(t) - (N_h(t) + N_l(t)) - \delta K(t),$$

and

$$\dot{C}(t) = (r(t) - \rho)C(t), \tag{4.12}$$

where (4.12) is derived from the consumer's optimization problem. These expressions can be written in efficiency units as follows:

$$\begin{aligned} \dot{k}(t) &= L^{1-\alpha} k(t)^{\alpha} - c(t) - \frac{1}{E(a)} (n_h(k(t)) + n_l(k(t))) - (\delta + g(t))k(t) && (4.13) \\ \dot{c}(t) &= (r(t) - \rho - g(t))c(t), && (4.14) \end{aligned}$$

where $g(t)$ is the rate of growth of aggregate knowledge and $E(a)$ is the mean of the distribution of relative productivity parameters.[7] The rate of growth of aggregate knowledge

[7] We are using the relationship between aggregate and leading edge productivity since $A_t = \int_0^1 A_{it} di = A_t^{\max} \int_0^1 \frac{A_{it}}{A_t^{\max}} di = A_t^{\max} \int_0^1 a h(a)\, da = A_t^{\max} E(a)$, where $h(a)$ is the density function of a.

and $E(a)$ can also be viewed as functions of $k(t)$ since $E(a)$ will depend upon $n_h(t)$ and $r(t)$, while $g(t)$ is given by the following expression:

$$g(t) = \frac{\dot{A}(t)}{A(t)} = \frac{\dot{A}^{\max}(t)}{A^{\max}(t)} + \frac{\dot{E}(a)}{E(a)} = \theta\lambda n_h(t) + \frac{\dot{E}(a)}{E(a)}.$$

Since the distribution of a is time invariant in the long run, so is $E(a)$. Therefore, $g(t) = \theta\lambda n_h(t)$.

Due to the non-linearity of the system, we linearize it around the steady state in order to analyze the local dynamics of the model. We find local saddle path stability around the steady state.[8] Therefore, we can perform comparative statics analysis at the long run equilibrium.

4.3.2 Equilibrium under the aggregate assumption.

The equations determining equilibrium under this assumption are the same as for the average assumption except that the rate of growth of aggregate technology is now given by

$$g(t) = \sigma(1-\phi)\lambda n_h(t), \tag{4.15}$$

where ϕ is implicitly defined as a function of k by equation (4.29) in Appendix 4.6. Therefore, the dynamic system defined by equations (4.13) and (4.14) with $g(t)$ defined by (4.15) presents also local saddle path stability. See Appendix 4.7 for a proof.

[8] See Appendix 4.7 for a proof.

4.4 Steady state analysis

4.4.1 Steady state analysis under the average assumption

In equilibrium, the production function is simplified due to the fact that the equilibrium value of intermediate input is the same for every sector. Consequently, we may write equation (4.2) as

$$Y(t) = A(t) L^{1-\alpha} [k(t)]^{\alpha},$$

which implies that in a steady state, the rate of growth of output will be the rate of growth of aggregate productivity. That is

$$g = \theta \lambda n_h.$$

Using this result, and the fact that in a steady state k and n_h are constant we may write equations (4.9), (4.10) and (4.11) as follows:

$$1 - s_h = \frac{\lambda \alpha (1-\alpha) L^{1-\alpha} k^{\alpha}}{\rho + (1+\theta) \lambda n_h}, \tag{4.16}$$

$$\lambda n_l = \gamma \tau \lambda n_h - (1 - \gamma \tau)(\rho + \theta \lambda n_h), \tag{4.17}$$

$$\rho + \theta \lambda n_h + \delta + \tau_k = \alpha^2 L^{1-\alpha} k^{\alpha - 1}, \tag{4.18}$$

where we are using the steady state relationship between the interest rate and the growth rate, i.e. $r = \rho + \theta \lambda n_h$. Equations (4.16) and (4.18) determine the steady state values for k and n and allow us to perform comparative statics on the different parameters of the model. The following proposition establishes the steady state relationships between some of the parameters and the growth rate:

Proposition 17 *The steady state growth rate is increasing in θ, λ and s_h and decreasing in τ_k.*

Proof. See Appendix 4.6 ■

These results were already obtained in the standard model. They are relevant however, because we want to look at the relationship between growth and the distribution of profits across sectors. The next lemma establishes the relationship between the mass of the lagging group and the previous parameters:

Lemma 18 *The mass of the lagging group ϕ is increasing in θ, τ_k and s_h and decreasing in λ.*

Proof. See Appendix 4.6 ■

The result established in Lemma 18 allows us to rank distribution functions. A change in these parameters will have the following effects on the distribution of relative productivities:

Proposition 19 *a) Let $\theta_1 < \theta_2$ and let $H_{\theta_i}(a)$ be the distribution function of relative productivities associated to θ_i for $i = 1, 2$. Then, $H_{\theta_1}(a) < H_{\theta_2}(a)$ for $a \in (0, 1)$.*

b) Let $\lambda_1 < \lambda_2$ and let $H_{\lambda_i}(a)$ be the distribution function of relative productivities associated to λ_i for $i = 1, 2$. Then, $H_{\lambda_1}(a) > H_{\lambda_2}(a)$ for $a \in (0, 1)$.

c) Let $s_{h1} < s_{h2}$ *and let* $H_{s_{hi}}(a)$ *be the distribution function of relative productivities associated to* s_{hi} *for* $i = 1, 2$*. Then,* $H_{s_{h1}}(a) < H_{s_{h2}}(a)$ *for* $a \in (0, 1)$*.*

d) Let $\tau_{k1} < \tau_{k2}$ *and let* $H_{\tau_{ki}}(a)$ *be the distribution function of relative productivities associated to* τ_{ki} *for* $i = 1, 2$*. Then,* $H_{\tau_{k1}}(a) < H_{\tau_{k2}}(a)$ *for* $a \in (0, 1)$*.*

Proof. See Appendix 4.6 ■

Proposition 19 implies first degree stochastic dominance of $H_{\theta_1}(a)$ over $H_{\theta_2}(a)$, of $H_{\lambda_2}(a)$ over $H_{\lambda_1}(a)$, of $H_{s_{h1}}(a)$ over $H_{s_{h2}}(a)$ and of $H_{\tau_{k1}}(a)$ over $H_{\tau_{k2}}(a)$. Consequently, the Generalized Lorenz curves for the distribution of relative productivities associated to $\theta_{1,}$ s_{h1}, τ_{k1} and λ_2 dominate the Generalized Lorenz curves associated to θ_2, s_{h2}, τ_{k2} and λ_1 respectively.[9] Accordingly, an increase in λ or a reduction in θ, s_h or τ_k reduces the inequality induced by the distribution of relative productivities across sectors. In other words, an increase in the growth rate due to a larger value of θ or s_h will shift $H(a)$ upwards and therefore, make the generalized Lorenz curve shift downwards. Figure 4.1 illustrates the effect of an increase in any of these two parameters. Observe that the shift in the distribution function implies that after the change, there exists a larger mass of sectors with smaller relative productivity coefficients and that the mass of the leading group[10] is reduced. Conversely, a higher growth rate due to a larger value of λ or to a reduction in τ_k will shift $H(a)$ downwards and make the generalized Lorenz curve shift upwards. This implies that the relationship between growth and the distribution of productivities can be positive or negative depending on the cause of faster growth. The effect of an increase in

[9]See Shorrocks (1983) for a proof of these results and a definition of the Generalized Lorenz Curve.

[10]In the figure, the leading group is formed by those sectors with $a > \gamma$, where γ is set to 0.8 just for illustrative purposes.

θ due for instance to a higher ability of firms to absorb externalities is a larger growth rate. However, it will also induce an increase in the mass of firms that lag behind and that consequently, have smaller relative profits while the leading group, the one with higher relative profits, is reduced. Similarly, a higher subsidy to research in leading sectors, will make the economy grow faster due to the higher research intensity of these sectors, but the gap between the leading and the lagging group will be wider. However, when faster growth is due to a larger productivity of research or to a tax reduction that stimulates capital accumulation, the result is the opposite. That is, the mass of lagging sectors is reduced while the number of sectors in the high-technology group increases, which reduces the inequality among relative productivities. Consequently, faster growth due to an increase in θ or s_h will induce a more unequal distribution of productivities and profits. On the other hand, if the cause of faster growth is an improvement in the productivity of research that affects all sectors or a policy change that stimulates capital accumulation, productive inequality will decrease. Observe that we are considering a set of parameters that includes proper policy instruments like subsidies to R&D and taxes on capital accumulation on one hand and exogenous technological parameters like the scope of spillovers θ and research productivity λ on the other. Strictly speaking, θ and λ are not policy instruments that can be changed at the discretion of the public sector. However, one can think of policies oriented at influencing their values. Empirical studies have found evidence that investment in infrastructure and education or the performance of public research can improve private research productivity and the absorptive capacity of private firms (see Eaton *et al.* 1998).

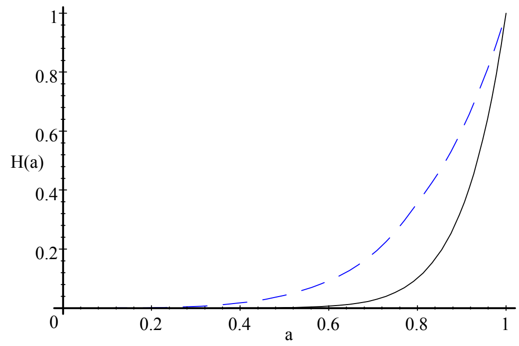

Figure 4.1: Shift in $H(a)$ caused by an increase in either θ or s_h.

4.4.2 Steady state analysis under the aggregate assumption

Under the assumption that the rate of growth of the leading edge technology is determined by the aggregate probability of innovation in the leading group, the rate of growth of the economy will be given by

$$g = \sigma\lambda(1-\phi)\, n_h.$$

Therefore, the equations determining the steady state values of k, n_h, n_l and ϕ are

$$1 - s_h = \frac{\lambda\alpha(1-\alpha)\, L^{1-\alpha}k^{\alpha}}{\rho + (1+\sigma(1-\phi))\,\lambda n_h}, \tag{4.19}$$

$$\lambda n_l = \gamma\tau\lambda n_h - (1-\gamma\tau)\,(\rho + \sigma(1-\phi)\,\lambda n_h)\,, \tag{4.20}$$

$$\rho + \sigma(1-\phi)\,\lambda n_h + \delta + \tau_k = \alpha^2 L^{1-\alpha}k^{\alpha-1}, \tag{4.21}$$

$$(1-\phi)\,\beta^{\frac{1}{\sigma(1-\phi)}} - \phi\frac{n_l}{n_h}\left(1 - \left(\frac{\beta}{\gamma}\right)^{\frac{1}{\sigma(1-\phi)}}\right) = 0, \tag{4.22}$$

where (4.19), (4.20), and (4.21) are the research arbitrage equation in leading sectors, the relationship between lagged and leading sectors research intensity and the capital market equilibrium condition, respectively, all expressed for the steady state. Equation (4.22) is derived from the steady state distribution function of a. It establishes that ϕ must be such that the distribution function is continuous at $a = \beta$. The following proposition establishes the steady state relationships between some of the parameters and the growth rate:

Proposition 20 *The steady state growth rate is increasing in both λ and s_l, decreasing in σ and τ_k and the effect of s_h on growth is ambiguous.*

Proof. See Appendix 4.6 ■

Observe that the effects on growth of research productivity and the tax on capital accumulation are not altered by the assumption that the growth rate depends upon the size of the leading group. However, this is not the case for the other three parameters. The next lemma presents the effect of these parameters on the size of the lagging group, which will help us understand the cause of the new results:

Lemma 21 *The mass of the lagging group ϕ is increasing in σ, τ_k and s_h and decreasing in both λ and s_l.*

Proof. See Appendix 4.6 ■

Lemma 21 implies that a larger λ will increase the productivity of research on one hand, and on the other, it will reduce the mass of lagging sectors. Therefore, a larger productivity of research is both growth enhancing and promotes less inequality among productivities across sectors. A similar effect is induced by a reduction of τ_k, that is, by an increase in the incentives to accumulate capital. With respect to s_l, notice that under the average assumption it had no effect on the rate of growth. However, under the aggregate assumption we observe that it reduces the mass of lagging sectors. This is a positive effect on growth that is able to compensate the reduction induced on the research intensity of leading sectors. The cases of the other two parameters are more complex to understand. Consider the effect of having a larger σ. Recall that this parameter measures the size of innovations or the influence of individual innovations on the leading edge productivity. When σ increases, research intensity falls due to the rise in the interest rate that makes the inputs to research more expensive. However, σ has a positive direct effect on the growth rate, which made the total growth effect positive under the average assumption. Under the aggregate assumption, we observe that the size of innovation has an additional effect on ϕ which will make the final impact on growth negative. The larger size of innovation makes the relative productivity parameter of the non-innovating sectors fall faster and therefore, there will exist a larger probability of entering the lagging group. Something similar happens when we increase the subsidy to research in high-tech sectors. The subsidy provides incentives to perform a higher research intensity in the leading sectors which will induce large productivity increases for innovators. However, those sectors that were not successful, will lag behind more rapidly and enlarge the lagging group. Consequently, the net effect on growth is ambiguous. Thus, under the aggregate assumption the influence of policy parameters on the mass of the lagging group affects the growth rate finally achieved and introduces important changes in the effectiveness of intendedly growth promoting policies. Only those policies that influence positively both R&D investments and the mass of the leading group will unambiguously promote growth. On the contrary, those policies that induce a larger lagging group will see their growth effectiveness undercut due to their distributional effects.

The complexity of the system under the aggregate assumption prevents us from establishing a ranking of distribution functions similar to the one presented in Proposition 19. Nevertheless, the results for the value of ϕ provide a partial characterization of the effects on the distribution function.

4.5 Conclusions

This paper has analyzed the effects of technological progress on the distribution of relative productivities across sectors. In particular, we have observed how changes in the characteristics of the process of technological change induce modifications on the distribution of productivities and profits across economic activities and how they may influence the growth performance of the economy. We have found that increases in research productivity, in the incentives to accumulate capital and larger subsidies to technological laggards will increase the mass of research intensive sectors and improve the growth rate of the economy. However, higher subsidies to technological leaders and a larger size of innovations or a higher degree of spillovers will increase the mass of the lagging class, which may in some cases reduce the growth rate of the economy.

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4.6 Proofs

Proof of Proposition 15. In order to derive the distribution of relative productivities define $A^{\max}(t_0)$ to be the absolute productivity coefficient of a sector that innovated on date t_0 and achieved the leading edge productivity. Then, from equation (4.6) we may write

$$\frac{A^{\max}(t_0)}{A^{\max}(t)} = \exp\left(-\int_{t_0}^{t} \theta\lambda n_h(s)\, ds\right), \tag{4.23}$$

which establishes that as $A^{\max}(t)$ grows, the relative productivity parameter of this sector will fall at a rate $\theta\lambda n_h(t)$. Define $F(\cdot, t)$ as the cumulative distribution of the absolute productivity coefficients A across sectors at any arbitrarily given date t. Define $\Phi(t) = F(A^{\max}(t_0), t)$. Then,

$$\Phi(t_0) = 1$$

and

$$\frac{d\Phi(t)}{dt} = \begin{cases} -(\Phi(t) - \Phi(t_2))\lambda n_h(t) & if \quad t_0 \le t < t_1 \\ -(\Phi(t) - \Phi(t_2))\lambda n_h(t) - \Phi(t_2)\lambda n_l(t) & if \quad t_1 \le t < t_2 \\ -\Phi(t)\lambda n_l(t) & if \quad t_2 \le t \end{cases}, \tag{4.24}$$

where t_1 and t_2 are, respectively, the dates at which $a_0 = \frac{A^{\max}(t_0)}{A^{\max}(t)}$ equals γ and β. Thus t_1 and t_2 are implicitly defined by the following equations which in turn, are derived from

(4.23):

$$\exp\left(-\theta\int_{t_0}^{t_1}\lambda n_h(s)\,ds\right) = \gamma \tag{4.25}$$

$$\exp\left(-\theta\int_{t_0}^{t_2}\lambda n_h(s)\,ds\right) = \beta. \tag{4.26}$$

The time derivative of $\Phi(t)$ gives us the rate at which the sector that innovated at date t_0 is left behind by other innovating sectors. Notice that while $t > t_1$, $a_0 > \gamma$ and the sector will only be overtaken by those sectors belonging to the leading group and having an absolute productivity parameter below $A^{\max}(t_0)$. Those sectors have a flow probability of innovation $\lambda n_h(t)$ and a mass of $\Phi(t) - \Phi(t_2)$. However, when $t_1 \leq t < t_2$, the relative productivity coefficient a_0 has fallen below γ and consequently, it may be overtaken by all innovating sectors having an absolute productivity coefficient below $A^{\max}(t_0)$. Therefore, we have a number of sectors which belong to the leading group, $\Phi(t) - \Phi(t_2)$ with a flow probability of innovation equal to $\lambda n_h(t)$ and all the sectors in the lagging group $\Phi(t_2)$, with a flow probability of $\lambda n_l(t)$. When $t \geq t_2$, all the sectors with an absolute productivity coefficient below $A^{\max}(t_0)$, that is $\Phi(t)$, belong to the lagging group and therefore, have a flow probability of innovation of $\lambda n_l(t)$. Equation (4.24) defines a differential equation whose solution is given by the following expression:

$$\Phi(t) = \begin{cases} \Phi(t_2) + (1-\Phi(t_2))\exp\left(-\int_{t_0}^{t}\lambda n_h(s)\,ds\right) & if \quad t_0 \leq t < t_1 \\ \Phi(t_2) + (1-\Phi(t_2))\exp\left(-\int_{t_1}^{t}\lambda n_h(s)\,ds\right)\gamma^{\frac{1}{\theta}} - \\ -\exp\left(-\int_{t_1}^{t}\lambda n_h(s)\,ds\right)\Phi(t_2)\int_{t_1}^{t}\lambda n_l(v)\exp\left(\int_{t_1}^{v}\lambda n_h(s)\,ds\right)dv & if \quad t_1 \leq t < t_2 \\ \Phi(t_2)\exp\left(-\int_{t_2}^{t}\lambda n_l(s)\,ds\right) & if \quad t_2 \leq t \end{cases}, \tag{4.27}$$

where

$$\Phi\left(t_2\right) = \frac{\beta^{\frac{1}{\theta}}}{\beta^{\frac{1}{\theta}} + \left(\frac{\beta}{\gamma}\right)^{\frac{1}{\theta}} \int_{t_1}^{t_2} \lambda n_l\left(t\right) \exp\left(\int_{t_1}^{t} \lambda n_h\left(s\right) ds\right) dt}.$$

Equation (4.23) implicitly defines t as a function of a_0. Let $\tilde{t}\left(a_0\right)$ be this function and use it to perform a change of variable in (4.27). The function that we obtain is

$$\Phi\left(\tilde{t}\left(a_0\right)\right) = \begin{cases} \Phi\left(t_2\right) + \left(1 - \Phi\left(t_2\right)\right)\left(a_0\right)^{\frac{1}{\theta}} & if \quad \gamma \leq a_0 \leq 1 \\ \begin{array}{l} \Phi\left(t_2\right) + \left(1 - \Phi\left(t_2\right)\right)\left(a_0\right)^{\frac{1}{\theta}} + \\ + \left(\frac{a_0}{\gamma}\right)^{\frac{1}{\theta}} \Phi\left(t_2\right) \int_{a_0}^{\gamma} \lambda n_l\left(\tilde{t}\left(a_0\right)\right) \left(\frac{a_0}{\gamma}\right)^{-\frac{1}{\theta}} \tilde{t}'\left(a_0\right) da_0 \end{array} & if \quad \beta \leq a_0 \leq \gamma \\ \Phi\left(t_2\right) \exp\left(\int_{a_0}^{\beta} \lambda n_l\left(\tilde{t}\left(a_0\right)\right) \tilde{t}'\left(a_0\right) da_0\right) & if \quad a_0 \leq \beta \end{cases}.$$

From the definition of $\Phi\left(t\right)$ we know that this function gives the mass of sectors with an absolute productivity parameter below $A^{\max}\left(t_0\right)$ at date t. In terms of relative productivity coefficients, $\Phi\left(\tilde{t}\left(a_0\right)\right)$ gives us the mass of sectors with a relative productivity coefficient below a_0 and therefore, it is giving us the value of the distribution function of relative productivity parameters for a sector that innovated on date t_0. In the long run, almost all sectors will have innovated at least once and therefore $\Phi\left(\tilde{t}\left(a_0\right)\right)$, which can now be renamed $H\left(a_0\right)$, represents the cumulative distribution function of any sector with a relative productivity parameter between 0 and 1. The expression for $H\left(a\right)$ in (4.7) can be obtained replacing the size of the lagging group $\Phi\left(t_2\right)$ by a parameter ϕ, whose definition in terms of a is given by

$$\phi = \frac{\beta^{\frac{1}{\theta}}}{\beta^{\frac{1}{\theta}} - \left(\frac{\beta}{\gamma}\right)^{\frac{1}{\theta}} \int_{\beta}^{\gamma} \lambda n_l(\tilde{t}\left(a\right)) \exp\left(-\int_{a}^{\gamma} \lambda n_h\left(\tilde{t}\left(a\right)\right) \tilde{t}'\left(a\right) da\right) \tilde{t}'\left(a\right) da}.$$

Observe that $H\left(a\right)$ does not depend on t and therefore it is time invariant. ■

Proof of Proposition 16. The distribution function in (4.8) is obtained following the same steps as in the previous proof except that in this case, the relationship between a_0 and t is given by

$$a_0 = \exp\left(-\int_{t_0}^{t} \sigma(1-\phi)\lambda n_h(s)\,ds\right), \tag{4.28}$$

and ϕ is implicitly defined by the following equation:

$$(1-\phi)\beta^{\frac{1}{\sigma(1-\phi)}} + \phi\left(\frac{\beta}{\gamma}\right)^{\frac{1}{\sigma(1-\phi)}} \int_{\beta}^{\gamma} \lambda n_l\left(\tilde{t}(a)\right)\exp\left(-\int_{a}^{\gamma} \lambda n_h\left(\tilde{t}(a)\right)\tilde{t}'(a)\,da\right)\tilde{t}'(a)\,da = 0. \tag{4.29}$$

■

Proof of Proposition 17. In order to find the derivatives of the growth rate with respect to θ, λ and s_h let us express equations (4.16) and (4.18) as follows:

$$(1-s_h)(\rho + (1+\theta)\lambda n_h) - \lambda\alpha(1-\alpha)L^{1-\alpha}k^{\alpha} = 0$$

$$\rho + \theta\lambda n_h + \delta + \tau_k - \alpha^2 L^{1-\alpha}k^{\alpha-1} = 0,$$

and denote then by $f_1(k, n_h; \theta, \lambda, s_h)$ and $f_2(k, n_h; \theta, \lambda, s_h)$ respectively. These functions may be considered as the components of a function $F : (0,\infty)\times(0,\infty) \to R^2$ and use the implicit function theorem to find the derivatives needed. The Jacobian of F with respect to k and n_h will be given by

$$J_F(k, n_h) = \begin{pmatrix} -\lambda\alpha^2(1-\alpha)L^{1-\alpha}k^{\alpha-1} & (1-s_h)(1+\theta)\lambda \\ \alpha^2(1-\alpha)L^{1-\alpha}k^{\alpha-2} & \theta\lambda \end{pmatrix},$$

and its inverse is equal to the following expression:

$$[J_F(k,n_h)]^{-1} = \frac{1}{\det[J_F(k,n_h)]}\begin{pmatrix} \theta\lambda & -(1-s_h)(1+\theta)\lambda \\ -\alpha^2(1-\alpha)L^{1-\alpha}k^{\alpha-2} & -\lambda\alpha^2(1-\alpha)L^{1-\alpha}k^{\alpha-1} \end{pmatrix},$$

where $\det[J_F(k,n_h)] = -\lambda(1-\alpha)\zeta\left(\theta\lambda + \frac{(1-s_h)(1+\theta)}{k}\right)$. The Jacobian of F with respect to the parameters is given by

$$\begin{aligned} J_F(\theta,\lambda) &= \begin{pmatrix} (1-s_h)\lambda n_h & (1-s_h)(1+\theta)n_h - \alpha(1-\alpha)L^{1-\alpha}k^{\alpha} \\ \lambda n_h & \theta n_h \end{pmatrix}, \\ J_F(s_h,\tau_k) &= \begin{pmatrix} -\rho-(1+\theta)\lambda n_h & 0 \\ 0 & 1 \end{pmatrix}. \end{aligned}$$

Implicit differentiation implies the following expressions for the derivatives of n_h and k with respect to the parameters:

$$\frac{dn_h}{d\theta} = -\frac{\left(\lambda + \frac{(1-s_h)}{k}\right)n_h}{\theta\lambda + \frac{(1-s_h)(1+\theta)}{k}} \tag{4.30}$$

$$\frac{dn_h}{d\lambda} = \frac{\left(\frac{1-s_h}{k}\left(\frac{\rho}{\lambda}\right) - \theta\lambda n_h\right)}{\lambda\left(\theta\lambda + \frac{(1-s_h)(1+\theta)}{k}\right)} \tag{4.31}$$

$$\frac{dn_h}{ds_h} = \frac{\rho + (1+\theta)\lambda n_h}{\lambda k\left(\theta\lambda + \frac{(1-s_h)(1+\theta)}{k}\right)} \tag{4.32}$$

$$\frac{dn_h}{d\tau_k} = \frac{-1}{\theta\lambda + \frac{(1-s_h)(1+\theta)}{k}}$$

$$\begin{aligned}
\frac{dk}{d\theta} &= \frac{(1-s_h)\,\lambda^2 n_h}{\det(J_F)} \\
\frac{dk}{d\lambda} &= \frac{\theta\lambda\pi}{\det(J_F)} \\
\frac{dk}{ds_h} &= \frac{\theta\lambda\left(\rho+(1+\theta)\,\lambda n_h\right)}{\det(J_F)} \\
\frac{dk}{d\tau_k} &= \frac{-(1-s_h)(1+\theta)}{(1-\alpha)\,\zeta\left(\theta\lambda+\frac{(1-s_h)(1+\theta)}{k}\right)}.
\end{aligned}$$

Recall that the rate of growth is given by $g = \theta\lambda n_h$, but also, from $f_2(k, n_h) = 0$, we know that

$$g = \alpha^2 L^{1-\alpha} k^{\alpha-1} - \delta - \rho - \tau_k.$$

Therefore

$$\begin{aligned}
\frac{dg}{d\theta} &= -(1-\alpha)\,\alpha^2 L^{1-\alpha} k^{\alpha-2} \frac{dk}{d\theta} = \frac{\lambda n_h \frac{(1-s_h)}{\theta k}}{\lambda+\frac{(1-s_h)(1+\theta)}{\theta k}} \\
\frac{dg}{d\lambda} &= -(1-\alpha)\,\alpha^2 L^{1-\alpha} k^{\alpha-2} \frac{dk}{d\lambda} = \frac{\theta\tilde{\pi}}{k\left(\lambda+\frac{(1-s_h)(1+\theta)}{\theta k}\right)} \\
\frac{dg}{ds_h} &= \theta\lambda\frac{dn_h}{ds_h} = \frac{\rho+(1+\theta)\,\lambda n_h}{k\left(\lambda+\frac{(1-s_h)(1+\theta)}{\theta k}\right)} \\
\frac{dg}{d\tau_k} &= \theta\lambda\frac{dn_h}{d\tau_k} = \frac{-\theta\lambda}{\theta\lambda+\frac{(1-s_h)(1+\theta)}{k}},
\end{aligned}$$

where $\tilde{\pi} = \frac{\pi}{A^{\max}(t)}$. The first three derivatives are positive and the last one is negative. Thus, steady state growth is increasing in θ, λ and s_h and decreasing in τ_k. ■

Proof of Lemma 18. In a steady state, the distribution of relative productivity coefficients will be given by

$$H(a) = \begin{cases} \phi + (1-\phi)\, a^{\frac{1}{\theta}} & for \quad \gamma \leq a \leq 1 \\ \phi + (1-\phi)\, a^{\frac{1}{\theta}} - \phi \frac{n_l}{n_h}\left(1 - \left(\frac{a}{\gamma}\right)^{\frac{1}{\theta}}\right) & for \quad \beta \leq a \leq \gamma \\ \phi \left(\frac{a}{\beta}\right)^{\frac{n_l}{\theta n_h}} & for \quad 0 \leq a \leq \beta \end{cases}. \tag{4.33}$$

where ϕ in a steady state is given by

$$\phi = \frac{\beta^{\frac{1}{\theta}}}{\beta^{\frac{1}{\theta}} + \left(1 - \left(\frac{\beta}{\gamma}\right)^{\frac{1}{\theta}}\right)\frac{n_l}{n_h}}. \tag{4.34}$$

The derivative of ϕ with respect to λ will be determined by $\frac{d}{d\lambda}\left(\frac{n_l}{n_h}\right)$. Accordingly, let us perform this derivative first. From equation (4.17) $\frac{n_l}{n_h} = \gamma\tau - (1-\gamma\tau)\left(\frac{\rho}{\lambda n_h} + \theta\right)$. Therefore, $\frac{d}{d\lambda}\left(\frac{n_l}{n_h}\right) = \frac{(1-\gamma\tau)\rho}{(\lambda n_h)^2}\left(\lambda \frac{dn_h}{d\lambda} + n_h\right)$. Equation (4.31) allows us to write $\lambda \frac{dn_h}{d\lambda} + n_h = \frac{\frac{(1-s_h)}{k}(\rho + \lambda(1+\theta)n_h)}{\lambda\left(\theta\lambda + \frac{(1-s_h)(1+\theta)}{k}\right)}$ which is positive. If $\frac{n_l}{n_h}$ increases with λ, then ϕ necessarily decreases.

In order to look for the derivative of ϕ with respect to θ, let us write ϕ as follows:

$$\phi = \frac{1}{1 + \left(\beta^{-\frac{1}{\theta}} - \gamma^{-\frac{1}{\theta}}\right)\frac{n_l}{n_h}}.$$

Then,

$$\frac{d\phi}{d\theta} = -\phi^2 \left[\frac{d}{d\theta}\left(\beta^{-\frac{1}{\theta}} - \gamma^{-\frac{1}{\theta}}\right)\frac{n_l}{n_h} + \left(\beta^{-\frac{1}{\theta}} - \gamma^{-\frac{1}{\theta}}\right)\frac{d}{d\theta}\left(\frac{n_l}{n_h}\right)\right],$$

where

$$\frac{d}{d\theta}\left(\beta^{-\frac{1}{\theta}} - \gamma^{-\frac{1}{\theta}}\right) = \frac{1}{\theta^2}\beta^{-\frac{1}{\theta}}\ln(\beta) - \frac{1}{\theta^2}\gamma^{-\frac{1}{\theta}}\ln(\gamma) \tag{4.35}$$

$$\frac{d}{d\theta}\left(\frac{n_l}{n_h}\right) = -(1-\gamma\tau)\left(1 - \frac{\rho\frac{dn_h}{d\theta}}{\lambda (n_h)^2}\right). \tag{4.36}$$

Since $\beta < \gamma$ and $\frac{dn_h}{d\theta} < 0$, both (4.35) and (4.36) are negative, which implies that $\frac{d\phi}{d\theta}$ is positive.

The sign of $\frac{d\phi}{ds_h}$ will be determined by the sign of $\frac{d}{ds_h}\left(\frac{n_l}{n_h}\right)$. Hence, if the level of research intensity in lagged sectors relative to research intensity in leading sectors falls, then $\frac{d\phi}{ds_h}$ will be positive. From (4.32) n_h increases with s_h. Therefore, in order to prove that $\frac{d}{ds_h}\left(\frac{n_l}{n_h}\right)$ is negative, it is enough to show that $\frac{dn_l}{ds_h}$ is negative. Consider thus this derivative

$$\frac{dn_l}{ds_h}=\frac{-\left(\rho+(1+\theta)\,\lambda n_h\right)\left((1-s_h)+\gamma\tau\theta\lambda k\right)}{\lambda\,(1-s_h)\left(\theta\lambda k+(1-s_h)\,(1+\theta)\right)},$$

which is negative. Hence, since $\frac{dn_l}{ds_h}$ and $\frac{d}{ds_h}\left(\frac{n_l}{n_h}\right)$ are negative, $\frac{d\phi}{ds_h}$ is positive.

Similarly, the sign of $\frac{d\phi}{d\tau_k}$ will the determined by $\frac{d\left(\frac{n_l}{n_h}\right)}{d\tau_k}$ which is given by

$$\frac{d\left(\frac{n_l}{n_h}\right)}{d\tau_k}=(1-\gamma\tau)\left(\frac{\rho}{\lambda n_h^2}\frac{dn_h}{d\tau_k}\right),$$

a negative expression. Therefore, $\frac{d\phi}{d\tau_k}$ is positive. ■

Proof of Proposition 19. Consider the steady state distribution of relative productivities given by equation (4.33). The effect of θ on $H(a)$ may be computed as

$$\frac{dH(a)}{d\theta}=\begin{cases}\frac{d\phi}{d\theta}\left(1-a^{\frac{1}{\theta}}\right)+(1-\phi)\left(\frac{-\ln a}{\theta^2}\right)a^{\frac{1}{\theta}} & if \quad \gamma<a\leq 1\\ \frac{d\phi}{d\theta}\left(1-a^{\frac{1}{\theta}}-\left(1-\left(\frac{a}{\gamma}\right)^{\frac{1}{\theta}}\right)\frac{n_l}{n_h}\right)+(1-\phi)\,a^{\frac{1}{\theta}}\left(\frac{-\ln a}{\theta^2}\right)+ & \\ +\phi\frac{n_l}{n_h}\left(\frac{a}{\gamma}\right)^{\frac{1}{\theta}}\left(\frac{-\ln\left(\frac{a}{\gamma}\right)}{\theta^2}\right)-\left(1-\left(\frac{a}{\gamma}\right)^{\frac{1}{\theta}}\right)\phi\frac{d}{d\theta}\left(\frac{n_l}{n_h}\right) & if \quad \beta<a\leq\gamma\\ \phi\left(\frac{a}{\beta}\right)^{\frac{n_l}{\theta n_h}}\left[\frac{\frac{d\phi}{d\theta}}{\phi}+\ln\left(\frac{a}{\beta}\right)\frac{d}{d\theta}\left(\frac{n_l}{\theta n_h}\right)\right] & if \quad 0\leq a\leq\beta\end{cases}.$$

The three pieces of this function are positive since $\left(1-a^{\frac{1}{\theta}}-\left(1-\left(\frac{a}{\gamma}\right)^{\frac{1}{\theta}}\right)\frac{n_l}{n_h}\right)$ is positive and both $\frac{d}{d\theta}\left(\frac{n_l}{\theta n_h}\right)$ and $\frac{d}{d\theta}\left(\frac{n_l}{n_h}\right)$ are negative.[11] This implies that if we increase σ, the

[11]The expression $\left(1-a^{\frac{1}{\theta}}-\left(1-\left(\frac{a}{\gamma}\right)^{\frac{1}{\theta}}\right)\frac{n_l}{n_h}\right)$ is positive if $\frac{n_l}{n_h}\leq 1$. A sufficient condition for $\frac{n_l}{n_h}\leq 1$ is $\gamma\tau\leq 1$, which is an assumption we have already made.

resulting distribution will attach a higher value to any $a \in (0,1)$. Therefore, if $\theta_1 < \theta_2$ then, $H_{\theta_1}(a) < H_{\theta_2}(a)$ for $a \in (0,1)$.

Similarly, the effect of λ on $H(a)$ will be given by

$$\frac{dH(a)}{d\lambda} = \begin{cases} \frac{d\phi}{d\lambda}\left(1 - a^{\frac{1}{\theta}}\right) & if \quad \gamma < a \leq 1 \\ \frac{d\phi}{d\lambda}\left(1 - a^{\frac{1}{\theta}} - \left(1 - \left(\frac{a}{\gamma}\right)^{\frac{1}{\theta}}\right)\frac{n_l}{n_h}\right) - \phi\left(1 - \left(\frac{a}{\gamma}\right)^{\frac{1}{\theta}}\right)\frac{d}{d\lambda}\left(\frac{n_l}{n_h}\right) & if \quad \beta < a \leq \gamma \\ \phi\left(\frac{a}{\beta}\right)^{\frac{n_l}{\theta n_h}}\left[\frac{\frac{d\phi}{d\lambda}}{\phi} + \frac{\ln\left(\frac{a}{\beta}\right)}{\theta}\frac{d}{d\lambda}\left(\frac{n_l}{n_h}\right)\right] & if \quad 0 \leq a \leq \beta \end{cases}.$$

The three pieces are negative since $\frac{d\phi}{d\lambda}$ is negative, $\frac{d}{d\lambda}\left(\frac{n_l}{n_h}\right)$ is positive and $\ln\left(\frac{a}{\beta}\right)$ for $a < \beta$ is negative. Consequently, $\frac{dH(a)}{d\lambda}$ is negative for all values of a between 0 and 1. Therefore, if $\lambda_1 < \lambda_2$, the distribution function associated to λ_2 will give smaller values to any $a \in (0,1)$ than the distribution function associated to λ_1. Therefore, $H_{\lambda_1}(a) > H_{\lambda_2}(a)$ for $a \in (0,1)$.

The proof for s_h is similar. Consider the derivative of $H(a)$ with respect to s_h

$$\frac{dH(a)}{ds_h} = \begin{cases} \frac{d\phi}{ds_h}\left(1 - a^{\frac{1}{\theta}}\right) & if \quad \gamma < a \leq 1 \\ \frac{d\phi}{ds_h}\left(1 - a^{\frac{1}{\theta}} - \left(1 - \left(\frac{a}{\gamma}\right)^{\frac{1}{\theta}}\right)\frac{n_l}{n_h}\right) - \phi\left(1 - \left(\frac{a}{\gamma}\right)^{\frac{1}{\theta}}\right)\frac{d}{ds_h}\left(\frac{n_l}{n_h}\right) & if \quad \beta < a \leq \gamma \\ \phi\left(\frac{a}{\beta}\right)^{\frac{n_l}{\theta n_h}}\left[\frac{\frac{d\phi}{ds_h}}{\phi} + \frac{\ln\left(\frac{a}{\beta}\right)}{\theta}\frac{d}{ds_h}\left(\frac{n_l}{n_h}\right)\right] & if \quad 0 \leq a \leq \beta \end{cases}.$$

Again, the three pieces are positive, since $\frac{d\phi}{ds_h}$ is positive, $\frac{d}{ds_h}\left(\frac{n_l}{n_h}\right)$ is negative and $\ln\left(\frac{a}{\beta}\right)$ is negative for $a < \beta$. Consequently, if $s_{h1} < s_{h2}$ then, $H_{s_{h1}}(a) < H_{s_{h2}}(a)$ for $a \in (0,1)$.

Similarly,

$$\frac{dH(a)}{d\tau_k}=\begin{cases}\frac{d\phi}{d\tau_k}\left(1-a^{\frac{1}{\theta}}\right) & for \quad \gamma\leq a\leq 1\\ \frac{d\phi}{d\tau_k}\left(1-a^{\frac{1}{\theta}}\right)-\left(1-\left(\frac{a}{\gamma}\right)^{\frac{1}{\theta}}\right)\left(\frac{d\phi}{d\tau_k}\frac{n_l}{n_h}+\phi\frac{d\left(\frac{n_l}{n_h}\right)}{d\tau_k}\right) & for \quad \beta\leq a\leq\gamma\\ \frac{d\phi}{d\tau_k}\left(\frac{a}{\beta}\right)^{\frac{n_l}{\theta n_h}}+\phi\left(\frac{a}{\beta}\right)^{\frac{n_l}{\theta n_h}}\frac{1}{\theta}\frac{d\left(\frac{n_l}{n_h}\right)}{d\tau_k}\ln\left(\frac{a}{\beta}\right) & for \quad 0\leq a\leq\beta\end{cases}$$

is also positive because $\frac{d\phi}{d\tau_k}\frac{n_l}{n_h}+\phi\frac{d\left(\frac{n_l}{n_h}\right)}{d\tau_k}=\phi^2\frac{d\left(\frac{n_l}{n_h}\right)}{d\tau_k}$ which is negative. ■

Proof of Proposition 20 and Lemma 21. The distribution function of relative productivity parameters in a steady state under the aggregate assumption is given by

$$H(a)=\begin{cases}\phi+(1-\phi)\,a^{\frac{1}{\sigma(1-\phi)}} & for \quad \gamma\leq a\leq 1\\ \phi+(1-\phi)\,a^{\frac{1}{\sigma(1-\Phi)}}-\phi\frac{n_l}{n_h}\left(1-\left(\frac{a}{\gamma}\right)^{\frac{1}{\sigma(1-\phi)}}\right) & for \quad \beta\leq a\leq\gamma\\ \phi\left(\frac{a}{\beta}\right)^{\frac{n_l}{\sigma(1-\phi)n_h}} & for \quad 0\leq a\leq\beta\end{cases}.$$

where ϕ in this case is implicitly defined by the following expression:

$$(1-\phi)\,\beta^{\frac{1}{\sigma(1-\phi)}}-\phi\frac{n_l}{n_h}\left(1-\left(\frac{\beta}{\gamma}\right)^{\frac{1}{\sigma(1-\phi)}}\right)=0.$$

Under the aggregate assumption, the system determining the steady state values of k, n_h and ϕ may be expressed as follows:

$$F(k,n_h,\phi)=\begin{pmatrix}f_1(k,n_h,\phi)\\ f_2(k,n_h,\phi)\\ f_3(k,n_h,\phi)\end{pmatrix}=\begin{pmatrix}0\\0\\0\end{pmatrix},$$

where

$$f_1(k,n_h,\phi)=(1-s_h)(\rho+(1+\sigma(1-\phi))\lambda n_h)-\lambda\alpha(1-\alpha)L^{1-\alpha}k^{\alpha}$$

$$f_2(k,n_h,\phi)=\rho+\sigma(1-\phi)\lambda n_h+\delta+\tau_k-\alpha^2L^{1-\alpha}k^{\alpha-1}$$

$$f_3(k,n_h,\phi)=(1-\phi)\,\beta^{\frac{1}{\sigma(1-\phi)}}-\phi\frac{n_l}{n_h}\left(1-\left(\frac{\beta}{\gamma}\right)^{\frac{1}{\sigma(1-\phi)}}\right).$$

The Jacobian of this function is given by

$$J_F(k, n_h, \phi) = \begin{pmatrix} -\lambda(1-\alpha)\zeta & (1-s_h)(1+\sigma(1-\phi))\lambda & -(1-s_h)\sigma\lambda n_h \\ \frac{(1-\alpha)\zeta}{k} & \sigma(1-\phi)\lambda & -\sigma\lambda n_h \\ 0 & -\frac{\phi(1-\gamma\tau)\rho\omega}{\lambda n_h^2} & \Psi \end{pmatrix},$$

where $\omega = \left(1 - \left(\frac{\beta}{\gamma}\right)^{\frac{1}{\sigma(1-\phi)}}\right)$ and

$$\Psi = -\beta^{\frac{1}{\sigma(1-\phi)}}\left(1 - \frac{\ln\beta}{\sigma(1-\phi)}\right) - \frac{n_l}{n_h}\omega + \phi\frac{n_l}{n_h}\left(\frac{\beta}{\gamma}\right)^{\frac{1}{\sigma(1-\phi)}}\frac{\ln\left(\frac{\beta}{\gamma}\right)}{\sigma(1-\phi)^2} - \phi\omega(1-\gamma\tau)\sigma$$

Notice that Ψ is negative. Consequently, the determinant of the Jacobian, given by

$$\det(J_F) = -(1-\alpha)\zeta\left(\lambda\Psi\left(\sigma(1-\phi)\left(\frac{1-s_h}{k}+\lambda\right)+\frac{1-s_h}{k}\right) - \left(\frac{1-s_h}{k}+\lambda\right)\frac{\sigma\phi(1-\gamma\tau)\rho\omega}{n_h}\right),$$

is positive. In order to compute the derivatives for comparative statics we need the inverse of the Jacobian, that is

$$[J_F]^{-1} = \frac{-1}{\det(J_F)}\begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix},$$

where

$$
\begin{aligned}
a_{11} &= -\sigma\left(1-\phi\right)\lambda\Psi + \frac{\sigma\phi\left(1-\gamma\tau\right)\rho\omega}{n_h} \\
a_{12} &= \left(1-s_h\right)\left[\left(1+\sigma\left(1-\phi\right)\right)\lambda\Psi + \frac{\sigma\phi\left(1-\gamma\tau\right)\rho\omega}{n_h}\right] \\
a_{13} &= \left(1-s_h\right)\lambda^2\sigma n_h \\
a_{21} &= \frac{\left(1-\alpha\right)\zeta\Psi}{k} \\
a_{22} &= \lambda\left(1-\alpha\right)\zeta\Psi \\
a_{23} &= \lambda\sigma n_h\left(1-\alpha\right)\zeta\left(\lambda + \frac{1-s_h}{k}\right) \\
a_{31} &= \left(\frac{\left(1-\alpha\right)\zeta}{k}\right)\left(\frac{\phi\left(1-\gamma\tau\right)\rho\omega}{\lambda n_h^2}\right) \\
a_{32} &= \frac{\left(1-\alpha\right)\zeta\phi\left(1-\gamma\tau\right)\rho\omega}{n_h^2} \\
a_{33} &= \left(1-\alpha\right)\zeta\lambda\left(\sigma\left(1-\phi\right)\left(\lambda + \frac{\left(1-s_h\right)}{k}\right) + \frac{\left(1-s_h\right)}{k}\right).
\end{aligned}
$$

Consider now the derivatives of the component functions with respect to the relevant parameters.

$$
J_F\left(\lambda,\sigma,\tau_k\right) = \begin{pmatrix} \frac{-\left(1-s_h\right)\rho}{\lambda} & \left(1-s_h\right)\left(1-\phi\right)\lambda n_h & 0 \\ \sigma\left(1-\phi\right)n_h & \left(1-\phi\right)\lambda n_h & 1 \\ -\frac{\phi\left(1-\gamma\tau\right)\rho\omega}{\lambda^2 n_h} & X & 0 \end{pmatrix},
$$

where

$$
X = \beta^{\frac{1}{\sigma(1-\phi)}}\left(\frac{-\ln\beta}{\sigma^2}\right) + \phi\left(1-\phi\right)\left(1-\gamma\tau\right)\omega + \phi\frac{n_l}{n_h}\left(\frac{\beta}{\gamma}\right)^{\frac{1}{\sigma(1-\phi)}}\left(\frac{-\ln\left(\frac{\beta}{\gamma}\right)}{\sigma^2\left(1-\phi\right)}\right),
$$

is positive. Applying the rules of implicit differentiation, we obtain the derivatives needed to establish the results of the proposition. With respect to the productivity of research the relevant derivatives are

$$\begin{aligned}
\frac{dk}{d\lambda} &= \frac{(1-s_h)\,\sigma\phi\,(1-\gamma\tau)\,\rho\omega\left(\sigma\,(1-\phi)-\frac{\rho}{\lambda n_h}-1\right)+\tilde{\pi}\sigma\,(1-\phi)\,\lambda\Psi}{\det\left(J_F\right)} \\
\frac{d\phi}{d\lambda} &= -\frac{(1-\alpha)\,\zeta}{\det\left(J_F\right)}\left(\frac{(1-s)\,\phi\,(1-\gamma\tau)\,\rho\omega\,(\rho+\lambda n_h\,(1+\sigma\,(1-\phi)))}{\lambda^2 n_h^2 k}\right),
\end{aligned}$$

where $\tilde{\pi}=\frac{\pi(t)}{A^{\max}(t)}$. The derivative of capital intensity with respect to λ gives us the effect on growth because from $f_2\left(k,n_h,\phi\right)=0$, we know that $g=\alpha^2L^{1-\alpha}k^{\alpha-1}-\delta-\tau_k-\rho$ and therefore,

$$\frac{dg}{d\lambda}=-\left(1-\alpha\right)\alpha^2L^{1-\alpha}k^{\alpha-2}\frac{dk}{d\lambda}.$$

The sign of $\frac{d\phi}{d\lambda}$ is immediate. With respect to the sign of $\frac{dk}{d\lambda}$, it will be negative if $\sigma\left(1-\phi\right)-\frac{\rho}{\lambda n_h}-1\leq 0$. Recall that we are assuming that the subsidy structure must be such that the research intensity of lagging sectors will never be larger than the research intensity of high-tech sectors. This implied an upper bound for $\gamma\tau$ of 1. Thus, if $\gamma\tau\leq 1$ then $\frac{n_l}{n_h}\leq 1$ which implies $\sigma\left(1-\phi\right)\leq 1$ and consequently $\frac{dk}{d\lambda}$ is negative and $\frac{dg}{d\lambda}$ is positive.

The derivatives with respect to σ are as follows:

$$\frac{dn_h}{d\sigma}=\frac{n_h\,(1-\phi)\left(\frac{1-s_h}{k}+\lambda\right)\left(\beta^{\frac{1}{\sigma(1-\phi)}}+\frac{n_l}{n_h}\omega\right)}{\Psi\left(\lambda\sigma\,(1-\phi)+\frac{(1-s_h)}{k}\,(1+\sigma\,(1-\phi))\right)-\left(\frac{(1-s_h)}{k}+\lambda\right)\frac{\sigma\phi(1-\gamma\tau)\rho\omega}{\lambda n_h}}$$

$$\frac{d\phi}{d\sigma}=\frac{(1-\alpha)\,\zeta\left(\frac{(1-\phi)\phi(1-\gamma\tau)\rho\omega}{n_h}\left(\frac{(1-s_h)}{k}+\lambda\right)+X\lambda\left(\sigma\,(1-\phi)\left(\lambda+\frac{(1-s_h)}{k}\right)+\frac{(1-s_h)}{k}\right)\right)}{\det\left(J_F\right)}$$

$$\frac{dg}{d\sigma}=\lambda n_h\left(1-\phi\right)+\sigma\lambda\left(1-\phi\right)\frac{dn_h}{d\sigma}-\sigma\lambda n_h\frac{d\phi}{d\sigma}.$$

The derivative of research intensity with respect to σ is negative and $\frac{d\phi}{d\sigma}$ is positive. Thus, the sign of $\frac{dg}{d\sigma}$ is not immediate. Nevertheless, notice that

$$n_h + \sigma \frac{dn_h}{d\sigma} = \frac{\sigma\left(1-\phi\right)\left(\lambda + \frac{1-s_h}{k}\right)\left(\Psi + \beta^{\frac{1}{\sigma(1-\phi)}} + \frac{n_l}{n_h}\omega\right) + \frac{\Psi(1-s_h)}{k} - \left(\frac{1-s_h}{k} + \lambda\right)\frac{\sigma\phi(1-\gamma\tau)\rho\omega}{\lambda n_h}}{\Psi\left(\lambda\sigma\left(1-\phi\right) + \frac{(1-s_h)}{k}\left(1+\sigma\left(1-\phi\right)\right)\right) - \left(\frac{(1-s_h)}{k} + \lambda\right)\frac{\sigma\phi(1-\gamma\tau)\rho\omega}{\lambda n_h}},$$

is negative because

$$\Psi + \beta^{\frac{1}{\sigma(1-\phi)}} + \frac{n_l}{n_h}\omega = \beta^{\frac{1}{\sigma(1-\phi)}}\frac{\ln\beta}{\sigma\left(1-\phi\right)} + \phi\frac{n_l}{n_h}\left(\frac{\beta}{\gamma}\right)^{\frac{1}{\sigma(1-\phi)}}\frac{\ln\left(\frac{\beta}{\gamma}\right)}{\sigma\left(1-\phi\right)^2} - \phi\omega\left(1-\gamma\tau\right)\sigma$$

is negative. Hence, $\frac{dg}{d\sigma}$ is also negative.

The derivative of n_h with respect to τ_k is negative while $\frac{d\phi}{d\tau_k}$ is positive. Therefore, $\frac{dg}{d\tau_k}$ is negative.

Let us consider now the effect of the two subsidies. The derivatives of the component functions with respect to s_h and s_l are given by

$$J_F\left(s_h, s_l\right) = \begin{pmatrix} -\left(\rho + \left(1+\sigma\left(1-\phi\right)\right)\lambda n_h\right) & 0 \\ 0 & 0 \\ -\frac{(\rho+(1+\sigma(1-\phi))\lambda n_h)\phi\omega\gamma\frac{\partial\tau}{\partial s_h}}{\lambda n_h} & -\frac{(\rho+(1+\sigma(1-\phi))\lambda n_h)\phi\omega\gamma\frac{\partial\tau}{\partial s_l}}{\lambda n_h} \end{pmatrix}.$$

Notice that $\frac{\partial\tau}{\partial s_h} = -\frac{1}{1-s_l}$, therefore, $\frac{dn_h}{ds_h} > 0$. The derivative of ϕ with respect to s_h is not so immediate but it can be shown that it is equal to

$$\frac{d\phi}{ds_h} = \left(\frac{\left(\rho + \left(1+\sigma\left(1-\phi\right)\right)\lambda n_h\right)}{\det\left(J_F\right)}\right)\left(\frac{\left(1-\alpha\right)\zeta\phi\omega\left[\frac{1}{k}\left(\frac{n_l}{n_h} + \sigma\left(1-\phi\right)\right) + \frac{\sigma(1-\phi)\lambda\gamma\tau}{1-s_h}\right]}{n_h}\right).$$

The derivative of the growth rate with respect to this subsidy is given by

$$\frac{dg}{ds_h} = -\left(\frac{\sigma\lambda\left(\rho + \left(1+\sigma\left(1-\phi\right)\right)\lambda n_h\right)\left(1-\alpha\right)\zeta}{\det\left(J_F\right)}\right)\chi,$$

where

$$\chi = \frac{(1-\phi)\,\beta^{\frac{1}{\sigma(1-\phi)}}\left(\ln\beta^{\frac{1}{\sigma(1-\phi)}} - \frac{1}{\phi}\right)}{k} - \frac{\phi\,(1-\phi)\,(1-\sigma)\,\gamma\tau\omega\left(\lambda + \frac{1-s_h}{k}\right)}{1-s_h} + \\ + \left(\frac{\phi}{k}\right)\frac{n_l}{n_h}\left(\left(\frac{\beta}{\gamma}\right)^{\frac{1}{\sigma(1-\phi)}}\left(\ln\left(\left(\frac{\beta}{\gamma}\right)^{\frac{1}{\sigma(1-\phi)}}\right) - 1\right) + 1\right).$$

The first two terms are negative but the last term is positive, which implies that the sign of this derivative will generally be ambiguous. However, the last term goes to zero as β approaches γ while the first term is increasing (in absolute value) in β. Therefore, if β is "sufficiently" close to γ, the whole derivative will be negative.

Regarding the steady state effects of an increase in s_l, we observe that

$$\frac{dn_h}{ds_l} < 0, \frac{d\phi}{ds_l} < 0 \; and \; \frac{dg}{ds_l} > 0.$$

The sign of the first two derivatives is immediate and the sign of the derivative of the growth rate with respect to this subsidy is obtained from

$$\frac{dg}{ds_l} = \sigma\lambda\left((1-\phi)\frac{dn_h}{ds_l} - n_h\frac{d\phi}{ds_l}\right) = \frac{\sigma\lambda\phi\omega\gamma\tau^2\left(\rho + (1+\sigma(1-\phi))\,\lambda n_h\right)(1-\alpha)\,\zeta}{k\det\left(J_F\right)},$$

therefore, a subsidy to lagged sectors will make the economy grow faster and reduce the mass of the lagging group. ■

4.7 Dynamics

Proposition 22 *The dynamic system under the average assumption, defined by equations (4.13) and (4.14), presents local saddle path stability.*

Proof. In order to analyze the dynamics of the system let us express equations (4.13) and (4.14) as follows:

$$\begin{aligned}\dot{k}(t) &= \varphi(k(t), c(t)) \\ \dot{c}(t) &= \psi(k(t), c(t)).\end{aligned}$$

With this notation, we can compute the Jacobian of the system and evaluate it at the steady state. The derivatives needed are the following:

$$\begin{aligned}&\varphi_k(k,c) = \alpha L^{1-\alpha}k^{\alpha-1} - \frac{1}{E(a)}\left(\frac{dn_h(k)}{dk} + \frac{dn_l(k)}{dk}\right) + \frac{[n_h(k)+n_l(k)]\frac{dE(a)}{dk}}{[E(a)]^2} - \\ &\qquad - (\delta+g) - k(\frac{dg(k)}{dk}) \\ &\varphi_c(k,c) = -1 \\ &\psi_k(k,c) = c(-\alpha^2(1-\alpha)L^{1-\alpha}k^{\alpha-2} - \frac{dg(k)}{dk}) \\ &\psi_c(k,c) = 0.\end{aligned}$$

The determinant of the Jacobian is equal to $\psi_k(k,c)$ which is negative since $\frac{dg(k)}{dk} = \theta\lambda\frac{dn_h(k)}{dk}$ and $\frac{dn_h(k)}{dk}$ is positive. Recall that $n_h(k(t))$ was defined by equations (4.9) and (4.11) as

$$n_h(k(t)) = \frac{(1-\alpha)\,\alpha L^{1-\alpha}[k(t)]^{\alpha}}{1-s_h} - \frac{\alpha^2 L^{1-\alpha}[k(t)]^{\alpha-1}}{\lambda}.$$

Therefore,

$$\frac{dn_h(k(t))}{dk(t)} = \frac{(1-\alpha)\,\alpha^2 L^{1-\alpha}[k(t)]^{\alpha-1}}{1-s_h} + \frac{\alpha^2(1-\alpha)\,L^{1-\alpha}[k(t)]^{\alpha-2}}{\lambda},$$

is positive for every positive value of k.

Given that the determinant of the Jacobian is negative, the system presents local saddle path stability. ■

Proposition 23 *The dynamic system formed by equations (4.13) and (4.14) under the aggregate assumption presents local saddle path stability.*

Proof. Since the equations of the system are the same as in Proposition 22, we know that the system will be local saddle path stable if the determinant of the Jacobian is negative. The determinant is given by

$$\psi_k(k,c) = c(-\alpha^2(1-\alpha)L^{1-\alpha}k^{\alpha-2} - \frac{dg(k)}{dk}),$$

where

$$\frac{dg(k(t))}{dk(t)} = \sigma\lambda\left(1-\phi(k)\right)\frac{dn_h(k(t))}{dk(t)} - \sigma\lambda n_h(k)\frac{d\phi(k(t))}{dk(t)}.$$

Thus, if $\frac{d\phi(k(t))}{dk(t)}$ is negative, $\frac{dg(k(t))}{dk(t)}$ will be positive, and $\psi_k(k,c)$ will be negative as we want to prove. The implicit function that defines ϕ as a function of k is given by (4.29), so let

$$F(k,\phi) = (1-\phi)\beta^{\frac{1}{\sigma(1-\phi)}} - \phi\left(\frac{\beta}{\gamma}\right)^{\frac{1}{\sigma(1-\phi)}}\int_{t_1}^{t_2}\lambda n_l(t)\exp\left(\int_{t_1}^{t}\lambda n_h(s)\,ds\right)dt = 0.$$

Then

$$\frac{dF}{d\phi} = \left(\frac{\beta}{\gamma}\right)^{\frac{1}{\sigma(1-\phi)}}\left(-\gamma^{\frac{1}{\sigma(1-\phi)}} + \gamma^{\frac{1}{\sigma(1-\phi)}}\frac{\ln\gamma}{\sigma(1-\phi)} - \int_{t_1}^{t_2}\lambda n_l(t)\exp\left(\int_{t_1}^{t}\lambda n_h(s)\,ds\right)dt\right)$$

and

$$\frac{dF}{dk} = -\phi\lambda\left(\frac{\beta}{\gamma}\right)^{\frac{1}{\sigma(1-\phi)}}$$
$$\int_{t_1}^{t_2}\left(\frac{dn_l(t)}{dk}\exp\left(\int_{t_1}^{t}\lambda n_h(s)\,ds\right) + n_l(t)\exp\left(\int_{t_1}^{t}\lambda n_h(s)\,ds\right)\int_{t_1}^{t}\lambda\frac{dn_h(s)}{dk}ds\right)dt.$$

Since $\frac{dn_l(k(t))}{dk(t)}$ and $\frac{dn_h(k(t))}{dk(t)}$ are both positive, $\frac{dF}{dk}$ is negative, and so is $\frac{dF}{d\phi}$ which implies that $\frac{d\phi(k(t))}{dk(t)}$ as given by

$$\frac{d\phi(k(t))}{dk(t)} = -\frac{\frac{dF}{dk}}{\frac{dF}{d\phi}},$$

is negative. Consequently, $\psi_k(k, c)$ is negative. ■