Universitat Autònoma de Barcelona

# Essays on fiscal policy

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*A sa meva família, de tot cor.*

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All errors are my own.

# Abstract

This doctoral thesis develops three topics on fiscal policy in the field of macroeconomics. The first paper assesses the quantitative implications of optimal fiscal policy in a model with tax evasion. The Chamley-Judd result of a zero capital tax in the long run does not hold due to the presence of both labor income and consumption tax evasions, not even if we expand the model by introducing capital tax evasion in closed economy. In contrast to Coleman (2000), we find that subsidizing labor is not optimal. In the quantitative part, we show that the Ramsey planner cuts consumption, labor, and capital taxes; that the optimal tax rate on consumption is considerably lower than the optimal tax rate on labor income; and that lower taxes on labor and consumption reduce their respective levels of tax evasion. In addition, the welfare gains associated with this policy experiment are substantial.

The second paper extends previous macroeconomic frameworks on shadow economy with consumption tax evasion. The model presented here is the first one to jointly tackle both labor income and consumption tax evasions. By introducing a TFP shock that affects equally both the declared and undeclared sectors, we are able to produce a countercyclical evolution of tax evasion and improve the fit of the model to the data. We also explore the consequences for taxation of our framework; specifically, we show that it may be unfeasible to implement tax shifts implying significant reductions of income tax in favor of higher excises on consumption. Laffer curves are flatter than in the model without tax evasion. The limits imposed by tax evasion are stricter for consumption tax, for which the slippery slope of the Laffer curve starts roughly at a mere rate of 10%.

Finally, the last paper aims to study what would have been the overall and heterogeneous effects of implementing the so-called Eurobonds in the European sovereign debt crisis (2009-12). Specifically, we focus on the proposal by Delpla and von Weizsäcker (2010). The Euro area is divided in four groups according to their initial government debts and their respective bond yields during the sovereign debt crisis, and a small open economy model is

set up for each one without (baseline model) and with Eurobonds. In addition, we consider three scenarios for Eurobond yields: low, medium, and high. GIIPS (groups I and II) are able to reduce the crowding-out effect on productive investment, taxes, and debt as well as increase GDP and welfare in all scenarios. The rest of countries (groups III and IV) lose in terms of GDP, welfare, and debt in the medium and high yields. In the low, all groups are better off. Therefore, the key message is that Eurobonds could be a good policy to address times of soaring sovereign spreads, but their degree of success depends on the level of commitment.

# Contents

# Chapter 1

# Optimal fiscal policy under tax evasion

(joint with Adrián Pino Alcalde)

This paper assesses the quantitative implications of optimal fiscal policy in a model with tax evasion. The Chamley-Judd result of a zero capital tax in the long run does not hold due to the presence of both labor income and consumption tax evasions, not even if we expand the model by introducing capital tax evasion in closed economy. In contrast to Coleman (2000), we find that subsidizing labor is not optimal. In the quantitative part, we show that the Ramsey planner cuts consumption, labor, and capital taxes; that the optimal tax rate on consumption is considerably lower than the optimal tax rate on labor income; and that lower taxes on labor and consumption reduce their respective levels of tax evasion. In addition, the welfare gains associated with this policy experiment are substantial.

## 1.1 Introduction

The objective of this paper is to study the optimal fiscal policy implications of dealing with an economy suffering from tax evasion. The tax evasion phenomenon has been extensively documented for developed and emerging economies, e.g. the survey by Alm (2012) and several empirical works by Schneider et al. (2000, 2010, 2012), among others.

In this paper, tax evasion is modeled in such a way that mitigates tax distortions. Agents can evade taxes by reallocating consumption, labor, and capital from declared to undeclared activities, which are enforced only partially. Hence, the main consequence for the government is that tax evasion leads to higher elasticities of declared income and declared consumption with respect to tax rates due to this additional substitution effect.[1] On the other hand, our model also captures the view that tax evasion creates inefficiency because it is a mean for less productive sectors to cancel out the absolute advantage of more productive sectors, i.e. underground sectors unfairly compete, by evading taxes, with formal sectors. As a result, productive efficiency (e.g. output per unit of labor) is lower than if there were no evasion.[2]

Taking into account the size and these consequences of tax evasion on the economy, our work contributes to the literature that tries to ascertain what are the best combination of taxes between capital, labor, and consumption.

A very general point of this paper is that tax evasion goes against extreme and well-known results in the literature of optimal fiscal policy.

We show that the Chamley-Judd result does not hold in a model with both labor income and consumption tax evasions. Moreover, we demonstrate that the optimal tax rate on capital income is always positive when we introduce capital tax evasion as well as consumption and labor tax evasions in closed economy (see Subsection 1.5.2). However, optimal capital taxes depend on their level of tax enforcement in small open economy.

The model with only capital tax evasion yields the same optimal tax mix as in the standard model without tax evasion, i.e. taxing consumption at a constant rate, subsidizing labor at the rate imposed on consumption, and setting the capital tax to zero in all periods.[3]

In contrast to the result of Coleman (2000), subsidizing labor is not optimal. On the one hand, in our model with only labor income tax evasion, the optimal labor tax is zero for $t > 0$, whereas the consumption tax is positive. Therefore, there is no need of constraining labor taxes to be non-negative, as is done in Coleman (2000), Laczó and Rossi (2014), or Correia (2000); in order to get realistic tax rates. On the other hand, in our model with only consumption tax evasion, the optimal consumption tax is zero for $t > 0$, while the

[1] Laffer curves are flatter with tax evasion, pointing out the difficulty to raise government revenues. See Busato and Chiarini (2013), Orsi et al. (2014), and Forcades and Pino (2015).

[2] Several previous papers also point out these views of tax evasion and shadow economy. See, for example, Doligalski and Rojas (2015), La Porta and Shleifer (2014), Gillman and Kejak (2014), Orsi et al. (2014), Ihrig and Moe (2004), Busato et al. (2004, 2012, 2013), and Virmani (1989).

[3] See Coleman (2000).

labor tax is positive.

On this branch of the literature, there is a general consensus that taxation of consumption is preferred over capital and labor. The reasons are the following. First, the demand of capital is completely elastic in the long run, therefore the long-run tax rate on capital income should be the minimum possible.[4] Second, given that consumption and labor taxes affect the consumption-leisure margin, government should use the less distortive tax and set the other to zero.[5] One way to show that taxing consumption is more efficient than taxing labor is that consumption taxes create a lower tax wedge than the labor income taxes, for instance, assume that the policy $\tau_t^c = 0$ and $\tau_t^n = 0.4$ is replaced by $\tau_t^c = 0.4$ and $\tau_t^n = 0$, then $1 - (1 - 0.4) > 1 - \frac{1}{(1+0.4)}$, which imply a lower distortion of the consumption tax with respect to the labor income tax.[6] Another way to illustrate this is that the Laffer curve of labor income tax equals zero for $\tau_t^n = 1$, while that of consumption tax is positive for $\tau_t^c = 1$. Third, the feasibility of fiscal policy requires that the larger tax base (consumption) is taxed and the lower tax base (labor income) is not, the reason is that to finance with only one tax rate the same expenditure it is required a lower tax rate on consumption than on labor income because the former collects more than the latter, therefore this implies an additional motive to the one pointed out before that shows that consumption tax is more efficient.

In this line of reasoning, Coleman (2000) show that replacing income taxes with consumption taxes would lead to a large welfare gain in the United States. Correia (2010) show that this result holds even with heterogeneous agents. Laczó and Rossi (2014) extend the study of this tax mix to the case in which governments cannot credibly commit to fiscal policies, their result is that the welfare gains from taxing consumption are much larger under discretion than under commitment, and the result of taxing only consumption still hold in their framework.

Our paper demonstrates analytically that this preference for the consumption tax over the labor income tax is reversed when consumption tax evasion is introduced to the standard model without tax evasion. In addition, in our model with both labor income and

[4] For example, in the standard neoclassical model, the inverse of demand function in the long run reads $r^k = \frac{1}{(1-\tau^k)}\left(\frac{1}{\beta} - 1 + \delta\right)$, which implies a perfectly elastic demand of capital since the price does not depend on the quantity.

[5] In the standard neoclassical model, the consumption-leisure margin reads $\frac{-u_{n_t}}{u_{c_t}} = \frac{\left(1-\tau_t^n\right)}{\left(1+\tau_t^c\right)} w_t$.

[6] We define the tax wedge as the deviation from the Pareto optimum as a consequence of taxation, i.e. $1 - \frac{\left(1-\tau_t^n\right)}{\left(1+\tau_t^c\right)}$.

consumption tax evasions, one of the main quantitative findings is that, in fact, optimal consumption taxation is considerably lower than optimal labor income taxation. Hence, when we add labor income tax evasion as well as consumption tax evasion, the preference for consumption taxation is still reversed as the quantitative results show.

Furthermore, another interesting result is that the spike in the capital tax rate in period 1 is much lower than in Chari et al. (1994), which find an optimal tax rate on capital income around 1000% in period 1. In addition, we do not find a subsidy to labor income in period 0. Specifically, we find that the optimal capital tax in period 1 is around 197% and the optimal labor tax in period 0 equals 0.09%, for the benchmark calibration; and besides, these tax rates are around 66% and 1.27%, respectively, for the alternative utility function (with an extra cost of undeclared labor) presented in Subsection 1.5.1. Therefore, our welfare gains do not rely comparatively so much on a very large initial tax rate on capital.[7] Moreover, we show that the optimal tax rate on capital income is lower than 100% in our model with all three kinds of tax evasion for $t > 0$,[8] see Subsection 1.5.2.

This work also contributes to the strand of studies about macroeconomics and public policy that take into account tax evasion. Nicolini (1998) studies how increasing inflation can be used to indirectly tax underground sectors operating in cash that otherwise would be untaxed. We, like him, assume that there is a demand of evaded consumption goods. Caballé and Panadés (2004) endogeneize tax evasion, that is, study the consequences that inflation has on income tax evasion. In their scenario not paying is a risky activity because you may get inspected, depending on what you declared yesterday. This delay of inspection will increase tax evasion in economies with high inflation. In our paper, on the other hand, we do not use a state variable reflecting whether the agent has been inspected or not since we have a representative agent. Busato et al. (2012, 2013) study the quantitative effects of underground production and income tax evasion on Laffer curves and aggregate output in the long run. The main difference with respect to them is that we also contemplate the possibility of consumption tax evasion. Olovsson (2014) includes the home production of services, which cannot be taxed and can be a stylized way to model tax evasion in the consumption of services. The optimal policy amounts to taxing the capital used in the production of home services and lowering the tax rate on market services below the tax rate on goods.

[7] As it is common in the literature, our initial capital tax, $\tau_0^k$, is given. Even though $\tau_1^k$ is a distortionary tax, it is a way of taxing initial capital, $k_0$, in the end.

[8] Namely, consumption, labor and capital tax evasions.

In our quantitative model, we find optimal paths of taxes in a model in which agents are allowed to buy goods with and without consumption tax, and are able to choose whether to work paying labor income tax or not.[9] These two forms of tax evasion have significant implications on the constraints faced by government when designing fiscal policy, and the leeway it has to adjust taxes. In a previous paper, Forcades and Pino (2015), we propose a DGSE model trying to replicate some stylized facts of tax evasion; namely, the countercyclical evolution of shadow economy and the inverse relationship between tax evasion and tax rates. In addition, in that paper, we explore the implications that tax evasion have on aggregate variables, such as labor, investment, consumption, GDP, and government revenues; both in and out of steady state. Here, on the other hand, the objective is to assess what are the quantitative effects on the mix of taxes (consumption, labor, and capital taxes), on allocations, and on welfare gains of switching from the current tax system present in a model calibrated for Spain to the Ramsey optimal one. We calibrate our model using Spanish data because its estimates of tax evasion and underground economy are above the European average. Hence, the proportions of labor income and consumption tax evasions will be key targets for our calibration.

In the quantitative part, we show that the Ramsey planner should cut all tax rates in the long run. The optimal tax rate on capital is equal to 16.03 percent. Even though the optimal tax rate is 8.92 percentage points lower than the current tax rate, the results found by Chamley (1986) and Judd (1985) do not hold. On the other hand, the optimal consumption tax is set to 13.24 percent, while the optimal tax rate on labor income is equal to 36.44 percent. In addition, tax evasion falls with respect to the initial level; specifically, the proportions of evaded consumption and undeclared labor decrease by 16.44 and 2.90 percent due to lower taxes on consumption and labor, respectively.[10] The welfare gain from implementing these optimal taxes is 3.31 percent.

We define the level of tax enforcement as the probability of detection times the punishment, therefore what matters is the combination of both, not their particular values separately. Our sensitivity analysis highlights that probabilities and penalties are key variables in the design of any tax system. For instance, the optimal consumption and

[9] We abstract from capital tax evasion in the quantitative model since the qualitative results do not change, see Subsection 1.5.2.

[10] The optimal steady-state labor and consumption tax rates are 1.10 and 1.56 percentage points lower than the current rates, respectively.

labor taxes decrease by around 3-4 percentage points when their respective probabilities of detection fall from 10 percent to zero.

The outline of the paper is the following. Section 1.2 introduces the model used. In Section 1.3, we describe the optimal fiscal policy problem faced by the government, and besides, we establish some qualitative properties of optimal taxation. In Section 1.4, we show the quantitative results of implementing optimal tax rates for the benchmark calibration and a sensitivity analysis to various alternative parameter values is performed. Section 1.5 presents two alternative modeling assumptions, i.e. capital tax evasion and extra disutility of underground labor, so as to perform some robustness checks. Finally, Section 1.6 concludes.

## 1.2 The model

There is continuum of identical agents that live indefinitely, which chooses how much to consume, work, and invest. The difference with respect to standard models is that they are able to do these choices evading taxes.[11] That is, they can work in an underground sector in which they do not pay labor income tax, and consume a good that does not pay consumption tax. Obviously, there is a risk of incurring in these illegal activities. Following Allingham and Sandmo (1974), if the worker is caught not declaring labor income tax, she will have to pay a surcharge $s^n > 1$ over the tax rate, event that happens with probability $p^n$. Let us denote $w_t^i$ and $n_t^i$, $i \in \{M, U\}$, the wages and hours worked in the market ($i = M$) and underground ($i = U$) labor markets. Therefore their expected income from undeclared work is given by $p^n(1 - s^n \tau_t^n) w_t^U n_t^U + (1 - p^n) w_t^U n_t^U$. In order to simplify the formulation and do not have to carry a state variable reflecting whether the worker was caught or not, we assume that all agents pool their income together and this amounts to saying that the representative agent gets an average income weighted by the probabilities that labor income tax is inspected.

On the consumption side, firms sell goods with tax $c_t^V$ and without invoice/tax $c_t^{NV}$ (e.g. non-VAT), being the relative price of the latter $p_t^{NV}$. Government inspects the agents to see if they comply with their consumption tax.[12] In the case agents bought goods without

[11] We leave capital tax evasion for Subsection 1.5.2 since the qualitative results do not change.

[12] In reality government inspects firms rather than agents. However, we use this simplification assumption since agents are owners of the firms in the model.

consumption tax, they are inspected with probability $p^c$, and the tax agency charges them the unpaid consumption tax rate plus a surcharge $s^c > 1$. Thus, the representative agent's expected evaded consumption expenditure is given by $p^c(1 + s^c\tau_t^c)c_t^{NV} + (1 - p^c)c_t^{NV}$.

In addition, the expected tax payment when evading, $p^j s^j \tau_t^j$, should be less than the statutory one, $\tau_t^j$, otherwise there would not be incentives to evade taxes. Hence, tax enforcement is lower than one, $p^j s^j < 1$, $j \in \{n, c\}$.

Agents can save in capital $k_{t+1}$, and in government bonds $b_{t+1}$. Therefore, the problem that agents solve is the following:

$$\max_{\{c_t^V, c_t^{NV}, n_t^M, n_t^U, k_{t+1}, b_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(c_t, l_t),$$

subject to,

$$\begin{gathered} c_t = \left[(c_t^V)^\rho + \phi\,(c_t^{NV})^\rho\right]^{\frac{1}{\rho}}, \\ (1 + \tau_t^c)c_t^V + (1 + p^c s^c \tau_t^c)p_t^{NV} c_t^{NV} + i_t + b_{t+1} = (1 - \tau_t^n)w_t^M n_t^M \\ +(1 - p^n s^n \tau_t^n)w_t^U n_t^U + (1 - \tau_t^k)r_t k_t + (1 + r_t^b)b_t, \\ i_t = k_{t+1} - (1 - \delta)k_t, \\ l_t + n_t^M + n_t^U \leq 1, \\ c_t^V, c_t^{NV}, n_t^M, n_t^U, k_{t+1}, b_{t+1} \geq 0. \end{gathered} \tag{1.1}$$

The utility function is CRRA and separable between consumption and leisure. Consumption is an aggregate of goods with and without consumption tax in a CES fashion,[13] with $\rho \leq 1$ determining the elasticity of substitution between the two goods.[14] Parameter $\rho$ indicates how substitutable the evaded consumption is compared to the legal one. We use this parameter because when agents buy a good/service without invoice, even if the two consumptions are exactly the same, there might be some sort of substitutability between them due to the fact that, for example, they might not have the same warranty, they might not have the same post-sale services, etc. The parameter $\phi$ represents all factors that, as well as government policies ($s^c$, $p^c$, and $\tau^c$), affect the size of consumption tax evasion, for

[13] Olovsson (2004) use the same kind of aggregation between market services, which are taxed, and home-produced services, which of course are not taxed.

[14] With $\rho = 1$ goods would be perfect substitutes. We discard $\rho = 1$ because agents would consume only the cheapest good, and consumption tax evasion would be zero or 100% in that economy. We are interested to see intermediate consumption bundles that resemble real economies.

example, it may be interpreted as tax morality and reputation costs, see Gordon (1989), or as social norms, see Besley et al. (2015), among others. We will use it to match the size of consumption tax evasion.

Leisure is what is left in the agent's time after working in the market and underground economy. This utility function allows to study how a household allocates its labor supply between the regular and underground sectors.

Using the notation $U_{i_t}(\cdot)$ for the partial derivative of the utility with respect to variable $i$, the equilibrium conditions of the model are given by

$$\frac{U_{c_t^V}(\cdot)}{(1+\tau_t^c)} = \beta\left(1+(1-\tau_{t+1}^k)r_{t+1}-\delta\right)\frac{U_{c_{t+1}^V}(\cdot)}{\left(1+\tau_{t+1}^c\right)}, \tag{1.2}$$

$$r_t^b = (1-\tau_t^k)r_t - \delta, \tag{1.3}$$

$$\frac{U_{c_t^V}(\cdot)}{U_{c_t^{NV}}(\cdot)} = \frac{(1+\tau_t^c)}{(1+p^c s^c \tau_t^c)}, \tag{1.4}$$

$$\frac{-U_{n_t^M}(\cdot)}{U_{c_t^V}(\cdot)} = \frac{(1-\tau_t^n)w_t^M}{(1+\tau_t^c)}, \tag{1.5}$$

$$\frac{U_{n_t^M}(\cdot)}{U_{n_t^U}(\cdot)} = \frac{(1-\tau_t^n)w_t^M}{(1-p^n s^n \tau_t^n)w_t^U}, \tag{1.6}$$

where (1.2) is the Euler equation, (1.3) the non-arbitrage condition between capital and bonds, (1.4) the marginal rate of substitution (MRS) between evaded and non-evaded consumption, (1.5) the MRS between declared consumption and labor, and (1.6) the MRS between taxed and evaded work.

### 1.2.1 Firms

Firms maximize profits choosing how much capital $k_t$, declared labor $n_t^M$, and undeclared labor $n_t^U$ to hire:

$$\max_{\left\{k_t, n_t^M, n_t^U\right\}_{t=0}^{\infty}} \left\{Y_t - w_t^M n_t^M - w_t^U n_t^U - r_t k_t\right\}.$$

Following Busato and Chiarini (2004), total output is the sum of market and underground productions, $Y = F(k_t, n_t^M, n_t^U) = Y_t^M + Y_t^U$, where $Y_t^M$ is associated with the market sector, $Y_t^M = k_t^\alpha (n_t^M)^{1-\alpha}$, and $Y_t^U$ with the hidden sector, $Y_t^U = A^U k_t^\alpha (n_t^U)^{1-\alpha}$, respectively. As in Orsi et al. (2014), firms combine evaded hours of work, $n_t^U$, with capital to produce underground output. The first-order conditions of the firms are:

$$w_t^M = F_{n_t^M} = (1-\alpha) k_t^\alpha (n_t^M)^{-\alpha}, \tag{1.7}$$

$$w_t^U = F_{n_t^U} = A^U (1-\alpha) k_t^\alpha (n_t^U)^{-\alpha}, \tag{1.8}$$

$$r_t = F_{k_t} = \alpha k_t^{\alpha-1} \left( (n_t^M)^{1-\alpha} + A^U (n_t^U)^{1-\alpha} \right). \tag{1.9}$$

These are standard conditions but with the only particularity that capital enters in both sectors. The parameter $A^U$ is the idiosyncratic total factor productivity (TFP) of underground sector relative to the market sector,[15] it represents all factors that, as well as government policies ($s^n$, $p^n$, and $\tau^n$ ), affect the size of labor income tax evasion, e.g. technology, human capital, management practices, workplace organization, etc. This parameter captures the fact that less productive sectors tend to evade more. For instance, Marcelli et al. (1999) and Gallaway and Bernasek (2002) document a clear negative correlation between educational (or skill) level and participation in the irregular labor market, Ihrig and Moe (2004) show that there is a negative relationship between real GDP per worker and the size of shadow economy, and La Porta and Shleifer (2014) find that, in developing countries, unregistered firms are small and extremely unproductive, compared even to the small formal firms, and especially relative to the larger formal firms. In the end, this parameter will be useful to match a reasonable size of labor income tax evasion.[16]

The relative price of evaded consumption is equal to one, $p_t^{NV} = 1$, since both kind of consumptions are produced identically.

### 1.2.2 Government

Government raises revenues from given taxes $\tau_t^c, \tau_t^n$ and $\tau_t^k$, and issue new debt $d_{t+1}$ to repay outstanding debt and finance an exogenous sequence of useless government consumption $g_t$ (it is as if it was thrown to the sea). The expression of those revenues is

[15] It could also be written as a labor-augmenting productivity, i.e. $A^U = z_n^{1-\alpha}$ so that $Y^U = k_t^\alpha (z_n n_t^U)^{1-\alpha}$. It does not imply any difference.

[16] In Section 1.5, we match this size by using a alternative theory: agents suffer an extra disutility when working in the underground economy.

$$R_t = \underbrace{\tau_t^c\left(c_t^V + p^c s^c c_t^{NV}\right)}_{\text{Consumption Tax}} + \underbrace{\tau_t^n\left(w_t^M n_t^M \; + \; p^n s^n w_t^U n_t^U\right)}_{\text{Labor Income Tax}} + \underbrace{\tau_t^k r_t k_t}_{\text{Capital income tax}}, \tag{1.10}$$

where the first term on the RHS is the revenue coming from consumption tax, the second, labor income tax, and lastly, the revenues generated by the tax on capital. The government budget constraint is given by

$$g_t - b_{t+1} = R_t - (1 + r_t^b)b_t. \tag{1.11}$$

### 1.2.3 Competitive equilibrium

Given $k_0$, $b_0$, and $\{\tau_t^c, \tau_t^n, \tau_t^k, g_t\}_{t=0}^{\infty}$, a competitive equilibrium in this economy is a sequence of quantities, $\{c_t^V, c_t^{NV}, n_t^M, n_t^U, k_{t+1}, b_{t+1}\}_{t=0}^{\infty}$, and prices, $\{w_t^M, w_t^U, r_t, r_t^b\}_{t=0}^{\infty}$, such that,

- Given prices, optimality conditions of the agent (1.2) to (1.6) and the agent's budget constraint (1.1) are satisfied.
- Given prices, optimality conditions of the firm (1.7) to (1.9) hold.
- Aggregate resource constraint clears:

$$c_t^V + c_t^{NV} + g_t + k_{t+1} - (1 - \delta)k_t = Y_t \tag{1.12}$$

- Labor market clears

$$n_t^M + n_t^U + l_t = 1 \tag{1.13}$$

Any allocations and prices that satisfy the agent's budget constraint (1.1) and the feasibility constraint (1.12) must also satisfy the government budget constraint (1.11).

## 1.3 Ramsey equilibrium

In this section we lay out the Ramsey problem that government faces. The objective of the benevolent government is to maximize the lifetime utility of the agents, given an exogenous and deterministic level of government consumption $g_t$, subject to the feasibility constraint and the competitive equilibrium conditions. We assume that this benevolent planner has access to a commitment technology so that there is no possibility of deviating from policies set from $t = 0$ onwards. Therefore, we do not deal with the issue of time inconsistency.

We assume that at $t = 0$ our economy departs from the steady state of the competitive equilibrium or *status quo*, given the current tax rates. In addition, the level of government consumption is held fixed throughout the transition.

Hence, the Ramsey problem amounts to find the optimal taxes, $\{\tau_t^c, \tau_t^n\}_{t=0}^{\infty}$ and $\{\tau_t^k\}_{t=1}^{\infty}$, that maximize the welfare of the agent, subject to the private budget constraint (1.1), the optimality conditions (1.2)-(1.6), the feasibility constraint (1.12) and the labor market clearing condition (1.13). As usual, $\tau_0^k$ is given because it is a non-distortionary tax on given initial conditions.

We rewrite the problem in terms of quantities. The wage rates and the rental rate on capital are expressed in terms of quantities using the marginal product conditions (1.7)-(1.9). The tax rate on capital is expressed from the Euler equation (1.2). The tax rates on labor income and consumption can be obtained from equations (1.4)-(1.6). Therefore, we have two taxes that should satisfy three conditions. In contrast to the standard model without tax evasion, this reflects that, when consumption and labor taxes are chosen optimally, the government not only should take into account the static distortion of these taxes on the consumption-leisure margin, equation (1.5), it also ought to consider the substitution effect that these taxes generated between taxed and evaded consumption and labor, equations (1.4) and (1.6), respectively.

These expressions of prices and tax rates are substituted away into the agent's budget constraint (1.1), which is substituted forward in order to obtain the implementability constraint,

$$\sum_{t=0}^{\infty}\beta^t \left(c_t^V U_{c_t^V}(\cdot) + c_t^{NV} U_{c_t^{NV}}(\cdot) + n_t^M U_{n_t^M}(\cdot) + n_t^U U_{n_t^U}(\cdot)\right) = \frac{W_0 U_{c_0^V}}{(1+\tau_0^c)}, \tag{1.14}$$

where the term $W_0$ is the initial wealth, $\big(1 + (1-\tau_0^k)F_{k_0} - \delta\big)(b_0 + k_0)$, which depends on the initial conditions.[17] Let us use $\Lambda$ to represent the Lagrange multiplier of condition (1.14).

[17]Since $\tau_0^k$ is not decided optimally by the government, we assume that initial bond yields, $r_0^b$, adjust so that the non-arbitrage condition (1.3) is satisfied at $t = -1$. Otherwise, $W_0$ should read $\big(1 + (1-\tau_0^k)F_{k_0} - \delta\big)k_0 + (1 + r_0^b)b_0$.

As we have just seen, the presence of both tax evasions implies that (1.14) is not a sufficient condition to guarantee the existence of a competitive equilibrium. This means that if consumption and labor income taxes are expressed in terms of quantities by equations (1.4) and (1.6), respectively, condition (1.5) of the competitive equilibrium must be imposed explicitly in the Ramsey problem. Let $\mu_t$ denote the shadow price associated with this condition.

Therefore, in the presence of consumption and labor tax evasions, the Ramsey problem can be formulated as[18]

$$\begin{aligned}\max_{\{c_t^V, c_t^{NV}, n_t^M, n_t^U, k_{t+1}\}_{t=0}^{\infty}} \quad & \sum_{t=0}^{\infty} \beta^t \Bigg\{ U\left(c_t, 1 - n_t^M - n_t^U\right) \\ & + \Lambda \left(c_t^V U_{c_t^V}(\cdot) + c_t^{NV} U_{c_t^{NV}}(\cdot) + n_t^M U_{n_t^M}(\cdot) + n_t^U U_{n_t^U}(\cdot)\right) \\ & - \lambda_t \left(c_t^V + c_t^{NV} + g_t + k_{t+1} - (1-\delta)k_t - F(k_t, n_t^U, n_t^M)\right) \\ & + \mu_t \left(\frac{-U_{n_t^M}(\cdot)}{U_{c_t^V}(\cdot)} - \frac{(1-\tau_t^n)F_{n_t^M}}{(1+\tau_t^c)}\right)\Bigg\} \\ & - \Lambda \frac{W_0 U_{c_0^V}}{(1+\tau_0^c)},\end{aligned} \tag{1.15}$$

where $\Lambda > 0$ is the multiplier of the implementability constraint, and $\lambda_t, \mu_t > 0$; $\forall t$. As we mentioned before, the expression of the tax rate on labor income is obtained from the MRS between market and underground labor (1.6) and is given by

$$\tau_t^n = \frac{U_{n_t^U}(\cdot) F_{n_t^M} - U_{n_t^M}(\cdot) F_{n_t^U}}{U_{n_t^U}(\cdot) F_{n_t^M} - p^n s^n U_{n_t^M}(\cdot) F_{n_t^U}}, \tag{1.16}$$

which depends on the marginal utilities and productivities of market and underground labor, respectively, and the policy parameters, probabilities, $p^n$, and penalties, $s^n$.

In the same way, from the MRS between taxed and evaded consumption (1.4), it follows that the tax rate on consumption, $\tau_t^c$, is given by

$$\tau_t^c = \frac{U_{c_t^V}(\cdot) - U_{c_t^{NV}}(\cdot)}{U_{c_t^{NV}}(\cdot) - p^c s^c U_{c_t^V}(\cdot)}, \tag{1.17}$$

which depends on the marginal utilities of consumption (evaded and non-evaded) and the

[18] In fact, you can decide which one of conditions (1.4)-(1.6) is added as an extra constraint to the Ramsey problem provided that you express (in quantities) $\tau_t^c$ and $\tau_t^n$ accordingly with the other two constraints. For instance, if $\tau_t^n$ and $\tau_t^c$ are recovered (expressed) by (1.6) and (1.5), respectively, you should add equation (1.4) as additional constraint to the Ramsey problem.

policy parameters ($p^c s^c$).

The equations that characterize the Ramsey equilibrium are the feasibility constraint (1.12), the agent's MRS between declared consumption and labor (1.5), the implementability constraint (1.14), and the first-order conditions to (1.15).[19]

In Appendices B and C we present the Ramsey problem when there is only consumption tax evasion and only labor income tax evasion, respectively. These cases are simpler, but they imply some important qualitative results compared to the standard model without any kind of tax evasion.

### 1.3.1 Qualitative results

We start by showing what is the optimal tax mix in a model with only labor income tax evasion. Proposition 1 describes the properties of this optimal tax scheme.

**Proposition 1.** In the presence of labor income tax evasion, after a transitional period,[20] the optimal tax rate on labor income is set equal to zero and the optimal tax rate on consumption is positive and sizeable. On the other hand, the optimal tax rate on capital income converges to zero in the long run.

The proof of this proposition is taken from the setup developed in Appendix B. First, comparing the MRS between market and underground labors of the competitive equilibrium (1.6) and the same MRS of the Ramsey equilibrium (1.54), we can verify that $\tau_t^n = 0$, $t > 0$. Therefore, in contrast to Coleman (2000), it is not optimal to subsidize labor for $t > 0$, which implies that we do not need to constrain our Ramsey problem in order to avoid negative and unrealistic labor income tax rates.

Second, comparing the Euler equation of the competitive equilibrium (1.2) at the steady state,

$$1 = \beta(1 + F_k - \delta) - \beta\tau^k F_k, \tag{1.18}$$

[19] The first-order conditions of the Ramsey problem (1.15) are presented in the Appendix A.

[20] As in Chari et al. (1994), the transitional period is characterized by a high spike in the tax rate on capital in period 1 together with a fall of the tax rate on labor in period 0 with respect to the current tax rate. In addition, the tax rate on consumption in period 0 is larger than the current tax rate as in Coleman (2000).

and the Euler equation of the Ramsey equilibrium (1.55) at the steady state, $1 = \beta(1+F_k-\delta)$,[21] we can conclude that $\tau^k = 0$ at the steady state. Finally, by a process of elimination, some tax rate should be positive in order to satisfy the implementability constraint, in this case the consumption tax.

According to Proposition 1, it is optimal in the long run to tax only consumption. This result is the same as the one obtained in several models without tax evasion in which labor, capital, and consumption taxes are chosen optimally, e.g. Coleman (2000), Correia (2010), and Laczó and Rossi (2014).

Now, we analyze the optimal tax mix in a model in which there is only consumption tax evasion. Proposition 2 summarizes the properties of this optimal tax mix.

**Proposition 2.** In the presence of consumption tax evasion, after a transitional period,[22] the optimal tax rates on consumption and capital are set equal to zero and the optimal tax rate on labor income is positive and sizeable.

To prove Proposition 2 we follow the expressions obtained in Appendix C. First, using conditions (1.4) and (1.58), we can show that $\tau_t^c = 0$, $t > 0$. Second, the Euler equation of the competitive equilibrium (1.2) and and the Euler equation of the Ramsey equilibrium (1.59), since $\tau_t^c = 0$, $t > 0$, imply that $\tau_t^k = 0$, $t \geq 2$. Finally, by a process of elimination, some tax rate should be positive in order to satisfy the implementability constraint, in this case the labor tax.

These optimal taxation schemes imply the following proposition:

**Proposition 3.** The first-best allocation is not attainable when there is evasion at least on consumption tax or on labor income tax.

[21] As Straub and Werning (2014) show, we cannot assume that Lagrange multipliers have a finite steady state. Therefore, following Marcet et al. (2015), we have first checked by solving the model numerically that optimal consumption $c_t \to c$ as $t \to \infty$, where $c$ is positive and constant, so that, now, we do know that Lagrange multipliers have a finite steady state ($\lambda_t = \lambda_{t+1}$). Note that when solving the model numerically, we do not rely on any assumption.

[22] See footnote 16.

In the case where we consider only labor income tax evasion, the optimal policies $\tau_t^n = 0$ and $\tau_t^c > 0$, for $t > 0$, does not eliminate the distortion, $(1 - \tau_t^n) / (1 + \tau_t^c)$, that affects the MRS between consumption and leisure (1.51). Thus, Proposition 3 follows.

In the case where there is only consumption tax evasion, the tax wedge in equation (1.56) does not vanish given the optimal tax mix, since $\tau_t^c = 0$ and $\tau_t^n > 0$, for $t > 0$.

We use these simple cases to prove, without loss of generality, that of course, the first-best allocation is not attainable either in our model with both tax evasions. Later, in Proposition 4, we will show that, under the presence of both tax evasions, it is optimal in the steady state to tax income from capital, which implies *per se* that the Pareto Optimal allocation is not attainable. In addition, we also show, in the quantitative part of the paper, that optimal taxes on labor and consumption do not eliminate the distortions affecting conditions (1.4)-(1.6).

It has been pointed out by the literature that in this kind of fiscal policy problems, the optimal policy is to tax capital heavily in the first periods, and then decrease this tax rate to zero. It is worth making some comments about the optimal steady-state tax rate on capital in our model with both tax evasions, see Proposition 4.

**Proposition 4.** Under the presence of both labor income and consumption tax evasions, the optimal steady-state tax rate on capital income is positive.

To prove Proposition 4 we combine the steady-state version of the Euler equation of the competitive equilibrium (1.18) together with the Euler equation of capital of the Ramsey equilibrium (1.45) in the steady state,[23,24]

$$1 = \beta\,(1 + F_k - \delta) - \beta \frac{\mu}{\lambda} \frac{1}{(1 + \tau^c)} \left((1 - \tau^n) F_{n^M k} - \tau_k^n F_{n^M}\right). \tag{1.19}$$

If $\mu$ were equal to zero in the steady state, then the usual Chamley-Judd result would hold, that is, the tax on capital income would equal zero in the long run. In our case $\mu > 0$ due

[23]This Euler equation of the Ramsey problem with respect to $k_{t+1}$ is provided in Appendix A. Let $F_{n^M k}$ and $\tau_k^n$ denote the derivatives of the expressions of the marginal productivity of market labor, (1.7), and of the labor tax, (1.16), with respect to capital, respectively. If $\alpha$ is the same in market and underground production then $\tau_k^n = 0$.

[24]As in Proposition 1, we have first checked by solving the model numerically that there exists an interior steady state for allocations and $\mu_t$ converges so that $\lambda_t = \lambda_{t+1}$. This is the right way to proceed, see, for example, Straub and Werning (2014) or Marcet et al. (2015).

to the presence of both labor income and consumption tax evasions. Therefore, conditions (1.18) and (1.19) imply that the optimal steady-state tax rate on capital income is given by

$$\begin{aligned} \tau^k &= \frac{1}{F_k}\frac{\mu}{\lambda}\frac{(1-\tau^n)F_{n^M k}}{(1+\tau^c)} \\ &= \frac{\mu}{\lambda}\frac{(1-\tau^n)}{(1+\tau^c)}\frac{(1-\alpha)}{[n^M + A^U(n^M)^\alpha(n^U)^{1-\alpha}]}, \end{aligned} \tag{1.20}$$

which is positive since $\lambda > 0$, $\alpha < 1$, $(1-\tau_t^n)\,/\,(1+\tau_t^c) > 0$, and the rest of variables are positive.[25] Hence, it is verified that the Chamley-Judd result does not hold in our framework.

Below, we will see that the optimal steady-state tax rate on capital income is sizeable for realistic calibrations of our model.

In Subsection 1.5.2, we study the optimal taxation of capital income when capital tax evasion is also allowed. In addition, we propose several ways to model this kind of evasion: closed and small open economy.

## 1.4 Quantitative analysis

In this section we study the quantitative implications of implementing the optimal tax mix implied by the Ramsey problem (1.15).

We explain the calibration of our model in the first subsection. The second subsection reports optimal paths of taxes, allocations, and welfare gains that are associated with transitioning to a Ramsey tax policy. And finally, in the last subsection, we consider alternative values for some important parameters in order to evaluate their quantitative effects on tax rates and welfare gains.

### 1.4.1 Calibration

For the quantitative part of the paper, the representative agent is assumed to have the following preferences:

[25] Condition (1.5) implies that $\frac{(1-\tau_t^n)}{(1+\tau_t^c)} > 0$.

$$U(c_t, 1 - n_t^M - n_t^U) = log(c_t) + \Phi_l \frac{(1 - n_t^M - n_t^U)^{1-\gamma}}{1-\gamma}, \tag{1.21}$$

where the aggregate consumption, $c_t$, is a basket of evaded and taxed consumption, as explained in Section 1.2.

We calibrate this economy to match Spanish data. We use Spain because all the cross-country statistics show that tax evasion is a very prevalent phenomenon in its economy in the last decades and nowadays.

Following Benhabib et al. (1991) and Ragan (2005), who consider a parameter of substitutability equal to 0.8 and 0.9, respectively, between home produced and market consumption, we set a $\rho = 0.95$, as we consider that our kind of consumptions are even more substitute, but a lower value is considered in the sensitivity analysis.

The weight on evaded consumption, $\phi$, is set to 0.83 in order to match a ratio of evaded consumption to total consumption equal to 23.25% in accordance to the average value of the period 2008-2011 estimated by the study of the Taxation and Customs Union Directorate for the EU-Comission (2013). Nam et al. (2001) also estimated it, finding a value of 22.6%, we use the former target because is newer.[26]

According to the Working Time Survey published by the INE (1996), the share of disposable time allocated to working in the market is 32.2%. Therefore, we choose a weight parameter on leisure, $\Phi_l$, equal to 0.13 in order to match this fact.

The parameter $\gamma$ is set to 4, which implies a Frisch elasticity of labor supply around 0.5 for market labor, which is a standard value in the microeconomic literature.[27]

The parameter $A^U$ is set to 0.48 to get a proportion of underground production to total GDP equal to 22.8%, which is the size of the shadow economy as % of GDP, average over 1999-2010, estimated by Schneider (2012).

We set $\beta$ to the standard value 0.96, which implies a steady-state interest rate around 4%, see, for example, Chari et al. (1994), Correia (2010), and Olovsson (2014). We use a standard value of $\delta$, specifically, the one calibrated by Díaz-Giménez and Díaz-Saavedra (2009), which equals 0.078. The capital and labor income shares are calculated in the

[26] These estimates use the National Accounts and the Input-Output Tables. They compare the actual and theoretical revenues in order to get an approximated measure of Value Added Tax (VAT) evasion.

[27] This value is in line with microeconomic estimates, which tend to be lower than unity. See, for example, Pencavel (1986) and Browning et al. (1999).

standard way with data of 2009-2012 from the National Accounts, particularly following Gollin (2002), thus we set $\alpha = 0.3734$.

The current tax rates are estimated following Mendoza et al. (1994), in the case of Spain the average effective tax rates (2009-2012) obtained are $\tau^n = 37.54\%$, $\tau^k = 24.95\%$, and $\tau^c = 14.80\%$.[28] Remember that $\tau_0^k$ in the policy transition is also set equal to 24.95%.

The estimated probability of being caught when undeclaring work, $p^N$ , is equal to 0.1, we have followed the same procedure as Busato and Chiarini (2004) using data of 2013. We set the same value for $p^c$due to there is not data available. According to the Spanish Law, the fiscal surcharges, $s^i$, are equal to 1.5.

The initial value of debt-to-GDP is 54%, it is taken from National Accounts in 2009. Hence, the government consumption, $g_t$, is residually chosen to satisfy the steady-state version of government budget constraint (1.11).

This will be the *status quo* of the optimal policy problem. From that point on, the government must spend the constant amount $g$ every period, and the initial value of capital and debt will be those of the steady-state competitive equilibrium, i.e. $k_0$ and $b_0$. A summary of our benchmark calibration of the parameters is presented in Table 1.1.

### 1.4.2 Results

After reviewing the qualitative properties of the Ramsey equilibrium, we show the quantitative effects on optimal tax rates, on allocations, and on welfare gains for the benchmark calibration.

One of the main conclusions we can extract from this quantitative part is that tax evasion goes against extreme optimal tax results which are well-known in the literature.

The optimal tax rate on labor income is positive and significant for $t > 0$, in contrast to Coleman (2000). Figure 1.2 graphs full transitions of optimal tax rates. Therefore, the standard result of subsidizing labor or setting zero tax rate on labor, when $\tau_t^n$ is restricted to be non-negative, in all the periods does not hold. The reason is that, setting a very low tax rate (or even a subsidy) on labor would mean that agents would choose to work less in the underground economy (not to evade labor taxes). However, to compensate for this low tax (or subsidy), a greater consumption tax would be required and, given that agents can also evade consumption taxes, this is not optimal.

[28] These estimates are similar to those found by Trabandt and Uhlig (2012) for 2009.

Another striking result is that it is preferable to tax labor income over consumption, even when we consider that their levels of enforcement are the same. In Table 1.2, we observe that the optimal steady-state tax rate on consumption, 13.24%, is quite lower than the optimal steady-state tax rate on labor, 36.44%. Table 1.2 also shows that the Ramsey planner reduces the steady-state labor tax rate by 1.10 percentage points with respect to the current rate but also cuts the steady-state consumption tax rate by 1.56 percentage points.

On the other hand, we find that optimal tax rates on labor and consumption undergo a significant period of transition and then are roughly constant, see Figure 1.2. Specifically, the labor tax rate in period 0 is almost zero (0.09%) and consumption tax rate in period 0 is equal to 19.39 percent. These taxes are like this in order to compensate the high capital tax rate in period 1 (197.05%) and guarantee enough production to satisfy the spike in consumption in period 0. The full transitions of allocations are shown in Figure 1.1.

As it was pointed out in the optimal capital tax formula (1.20), the Chamley-Judd result does not hold in our setup. Table 1.2 shows that the optimal steady-state tax rate on capital income is 16.03%. Therefore, even though capital tax rate is cut by 8.92 percentage points with respect to the current rate, it is far away from zero. Figure 1.2 illustrates that optimal tax rates on capital income, after the transitional period 1, are decreasing and converges to its positive steady-state value.

Since our focus is on the long-run properties of optimal taxation under tax evasion, we do not restrict optimal tax rates on capital to be up to 100% because a high initial capital tax rate in period 1 simply have the effect of speeding up the transitional dynamics of optimal tax rates. However, our effects of a very high capital tax rate in period 1 on welfare gains, i.e. the proportion of welfare gains due to the tax rate $\tau_1^k$, are much lower than those in the literature because we find a $\tau_1^k = 197.05\%$, and for instance, Chari et al. (1994) and Olovsson (2014) report optimal tax rates on capital in period 1 around 1000%. Therefore, our welfare gains do not rely comparatively so much on a very large initial tax rate on capital.[29]

To sum up the results on optimal taxation, the Ramsey planner should cut all three taxes below the current tax rates. The optimal capital tax experiences the largest reduction with respect to the current taxes; specifically, it is 8.92 percentage points lower. On the other hand, the optimal consumption tax is 23.20 percentage points lower than the optimal

[29] In fact, for the alternative utility function proposed in Subsection 1.5.1, we will find a welfare gain of 1.33 percent given a tax rate on capital in period 1 equal to 65.68 percent.

tax rate on labor.

Table 1.3 shows that, except for underground labor and evaded consumption, all variables are higher in the Ramsey equilibrium than in the *status quo*. The investment is stimulated by a lower tax rate on capital, thus the stock of capital is higher in the Ramsey allocation. A lower labor tax encourage the supply of labor, the declared hours are 1.63 percent higher. A lower consumption tax in combination with a higher income raises consumption, non-evaded consumption increases by 13.26 percent. Finally, the debt-to-GDP ratio falls to 7.87 percent.

In order to study the effect of optimal taxes on tax evasion, we compute the key ratios describing the level of tax evasion on consumption and labor income. Table 1.4 shows that tax evasion falls with the implementation of the optimal mix of taxes. Specifically, the ratio of evaded consumption to total consumption falls by 16.44 percent with respect to the initial as a consequence of a lower consumption tax, and, in the same direction, the ratio of undeclared hours of labor (undeclared output) decreases by 2.90 (2.05) percent as a consequence of a lower labor tax.

Now, we focus on the welfare gains of implementing the optimal fiscal policy. The welfare measure used is the constant percentage amount by which consumption must be increased in all periods in the status quo economy so as to yield the same utility as under the Ramsey equilibrium, taking into account the full transition. The welfare gains associated with the policy experiment are large and equal to 3.31 percent (see Table 1.2).

### 1.4.3 Sensitivity analysis

In this section, we check the sensitivity of the results to the potentially key parameters $A^U$, $\rho$, $\alpha$, and probabilities $p^c$ and $p^n$. We evaluate the quantitative effects on tax rates and welfare. Table 1.5 summarizes the results for the alternative parameters considered.[30]

The change of parameter values affect marginal productivities and utilities, which decrease or increase the substitution effect between declared and undeclared allocations due to changes in tax rates. In other words, $\partial c_t^{NV}/\partial\tau_t^c > 0$ and $\partial n_t^U/\partial\tau_t^n > 0$ become higher or lower. Hence, it is possible to use the optimal tax formulas in Section 1.3 to understand the effects of the parameters on the optimal tax rates.

Now, we consider the same TFP in the two productions, $A^U = 1$. This higher value increases the supply of labor in the shadow economy, $n_t^U$. Hence, the government reduces

[30]This table reports the optimal tax rates and welfare gains relative to the change of one parameter. This implies that the initial economy no longer exactly matches all calibration targets for the Spanish economy.

labor tax to encourage market work in order to compensate this higher labor tax evasion and consequently fulfill its budget constraint. From equation (1.16), we can see that government must cut labor tax when $A^U$ exogenously increases the marginal productivity of underground labor relative to in the benchmark calibration because the numerator falls more than the denominator in this equation. The welfare gain is 5.14%, which is higher than in the benchmark calibration, thanks to a huge reduction in labor tax to 24.53% and a decrease in consumption and capital tax rates to 13.81% and 20.55%, respectively. Hence, the overall cut in taxation is more notable in this case than in the benchmark calibration.

We also consider a lower elasticity of substitution between taxed consumption and evadable consumption, e.g. $\rho = 0.85$, which is the intermediate value of the two mentioned above in the calibration section. This lower elasticity implies a higher tax rate on consumption with respect to the current tax rate (14.80%) because evaded consumption becomes a less attractive alternative with respect to taxed consumption (relative to in the benchmark calibration).[31] In this case, the welfare gain is equal to 2.89%, which is large but slightly lower than that of the benchmark calibration due to the increase in consumption tax to 18.04% despite the small decrease in labor and capital tax rates to 35.14% and 14.82%, respectively.

The underground production may be associated with technologies that are more labor intensive, i.e. $Y^U = A^U k^{\alpha^U} (n^U)^{1-\alpha^U}$ where $\alpha^U < \alpha$, for example, Orsi et al. (2014) estimate an output elasticity of capital for the underground production lower than for the market production. In this section, we consider a capital income share, $\alpha^U$, equal to 0.2. The reduction of this parameter makes the marginal productivity of underground labor, $F_{n_t^U}$, lower with respect to the benchmark calibration and therefore reduces the supply of this kind of labor,[32] the contrary of what we show that happens when $A^U$ increases. Hence, from equation (1.16), we can see that optimal labor tax increases because the numerator falls less than the denominator in this equation as a consequence of the reduction of $F_{n_t^U}$. The welfare gains associated with this parameter change are still significant, 1.43%, but much lower than in the benchmark calibration, the reason is that although capital tax rate is cut to 11.43%, optimal tax rates on consumption and, especially, on labor are even raised above the current tax rates, being equal to 15.37% and 41.91%, respectively.

The result that the optimal tax rate on consumption is lower than the optimal tax rate

[31] In particular, the derivative $\frac{\partial c_t^{NV}}{\partial \tau_t^c} > 0$ is lower than with higher elasticity of substitution.

[32] The derivative $\frac{\partial n_t^U}{\partial \tau_t^n} > 0$ is lower than with higher capital intensity.

on labor income is robust to all changes in the parameters considered in this section.

We will use the probabilities of detection as a proxy of how "easy" is to evade, i.e. as the enforcement level of such a tax. It is interesting to contrast what is the importance of the perception of audit rates in the tax mix structure of an economy. Intuitively, lower probabilities make tax evasion a more attractive alternative. Hence, the expected result is that optimal tax rates have a positive relationship with probabilities, i.e. the government should set lower taxes when they are easier to evade. We confirm this relationship from (1.16) and (1.17), the optimal tax rate on labor income (consumption) is lower relative to in the benchmark economy when $p^n$ ($p^c$) decreases, and vice versa. Results for different probabilities are displayed in Tables 1.6 and 1.7.

When the probability of detection, $p^c$, is reduced to 0%, the proportion of consumption tax evasion in the status quo increases from 23.29 percent to 32.01 percent (compare Tables 1.4 and 1.7). As a result, the optimal tax rate on consumption (10.15%) decreases with respect to in the benchmark calibration, whereas labor income optimal tax rate slightly increases, being equal to 36.90%. On the other hand, if what decreases is $p^n$ (to 0%), the proportion of labor income tax evasion in the status quo increases from 30.39 percent to 33.77 percent. Thus, labor tax rate is cut to 32.46%, while consumption tax rate rises a bit (to 13.78%) relative to in the benchmark calibration.

The optimal tax rate on capital income increases in front of any reduction of probability, because the other two taxes become less enforceable comparatively. We can see it in Column 3 of Table 1.6.

Hence, these results suggest that probabilities of detection have an important influence over optimal tax rates. Therefore, a margin for future research would be the inquiry of which is the optimal level of enforcement, given that it is costly for the government to increase tax enforcement. For instance, it would allow us to study the trade-off between tax and enforcement rates. This extension would connect the literature of optimal fiscal policy with the literature of state (and fiscal) capacity, see Besley and Persson (2009).

However, it is interesting to notice that the result that the labor income tax is preferred to the consumption tax in terms of optimality is robust to changes in probabilities. In Table 1.6, we can observe that $\tau^c$ is always quite lower than $\tau^n$, even in the scenario where $p^n$ is equal to zero (being $p^c = 0.1$).

In the same table (Column 4), we can see that the lower are the probability pairs the higher are the welfare gains. We are not saying that low probabilities imply more efficiency, only that welfare gains of implementing optimal taxes are larger. The model is not designed for computing which are the optimal probabilities of detection.

For instance, when both $p^n$ and $p^c$ are equal to zero (Row 3), we find the highest welfare gain (5.68%). This result is expected since tax evasion *per se* increases the efficiency of the economy because it offers an opportunity to diminish the distortionary effect of taxes via reallocation of consumption and labor. Therefore, when probabilities are lower, agents have more room to evade and this forces the Ramsey planner to cut taxes more, which implies higher welfare gains. We can see this again in Row 3, consumption, labor, and capital taxes are reduced to 10.58%, 32.98%, and 18.73%, respectively, which represents the lowest overall level of taxation in the sensitivity analysis of probabilities.

Changes in penalties would have the same effects on optimal tax rates and welfare gains as changes in probabilities, for this reason we don't report the outcomes here.

## 1.5 Alternative modeling assumptions

In this section, we present two modifications of our main framework. First, we introduce an extra utility cost of underground labor instead of an idiosyncratic productivity for underground production. Second, we add capital tax evasion in order to study the qualitative properties of the optimal tax mix, especially to study the optimal long-run tax rate on capital income in order to see whether or not it is zero.

### 1.5.1 Extra disutility of underground labor

Instead of using an idiosyncratic productivity,[33] we could match the size of labor income tax evasion by incorporating an extra disutility of underground labor. This formulation is common in the literature of underground economy, see, for example, Busato et al. (2004, 2012, 2013) or Orsi et al. (2014). Specifically, this approach incorporates an asymmetry in the utility function between taxed and evaded labor, which we have considered of the form

$$U(c_t, n_t^M, n_t^U) = log(c_t) + \Phi_l \frac{(1-n_t^M-n_t^U)^{1-\gamma}}{1-\gamma} - \Phi_u n_t^U,$$

where the last term in the utility function $\Phi_u n_t^U$, which is also used by Busato and Chiarini (2012), reflects an idiosyncratic cost of working in the underground sector.

The extra cost in terms of utility may be associated with the lack of social and health insurance in the underground sector. On the other hand, $\Phi_u$, like $\phi$, might also be interpreted as individual and social motives.

[33]Therefore, in this section, $A^U = 1$.

The parameter $\Phi_u$ is set to 3.58 to match a proportion of underground output to total output equal to 22.8%, as before. Since we want our economy to resemble the previous one, the weight parameter on leisure should also be recalibrated to match its respective target. Therefore, in this case, $\Phi_l$ is equal to 0.27.

The current and optimal tax rates for this alternative utility function are displayed in Table 1.8. In addition, Table 1.10 reports the levels of tax evasion.

The main qualitative intuition behind these tax rates has already been provided in Section 1.4. Despite this, there are some important results to be highlighted. Intuitively, the fact that agents dislike more working in the underground sector makes this kind of labor a less attractive alternative, i.e agents work less in the underground economy. A way to see this, it is comparing the ratio of undeclared hours of labor in this case and in the benchmark calibration, the former is equal to 12.49 percent, whereas the latter is equal to 30.39 percent. In addition, we can verify that $U_{n_t^U}(\cdot)$ increases in equation (1.16), which implies a higher optimal labor tax. Specifically, the optimal tax rate on labor income in the long run rises to 47.28 percent, whereas optimal tax rates on consumption and capital income in the long run fall to 11.2 and 11.00 percent, respectively. Hence, the Ramsey planner cuts the optimal steady-state tax rate on consumption (capital income) by around 3.5 (14) percentage points with respect to the current tax rate, but raises the tax rate on labor income by around 10 percentage points above its current rate. Full transitions of allocations and optimal taxes are shown in Figures 1.3 and 1.4, respectively.

In addition, the capital tax rate in period 1 is equal to 65.68 percent, which represents a much lower tax rate than in Chari et. al. (1994), and the optimal labor tax in period 0 equals 1.27%. The welfare gains associated with this case are equal to 1.33 percent, which are very similar to the welfare gains found by Olovsson (2014) under the restriction that the tax rate on capital is not allowed to increase (1.63 percent).[34]

These findings emphasize that this additional utility cost of underground labor, which is not present in capital and consumption tax evasions, e.g. the fact that you do not contribute to your future pension or that you do not have insurance in front of accidents in the shadow economy, should be taken into account when designing the system of taxation because these factors reduce the substitution effect between declared and undeclared work due to changes in tax rates.[35]

[34] Recall that we do not restrict any tax rate in this paper.

[35] That is, the derivative $\frac{\partial n_t^U}{\partial \tau_t^n} > 0$ is lower than without the extra disutility of undeclared labor.

### 1.5.2 Adding capital tax evasion

As we said before, the main point of this part is to see whether or not the optimal steady-state tax rate on capital is equal to zero when we expand our model by introducing tax evasion on capital income.

In this part of the paper, in addition to labor income and consumption tax evasions, we allow agents to evade capital taxes by investing in another asset. We propose two alternative ways to model capital tax evasion. First, we assume that agents can evade capital taxation by investing in another productive capital that enters in the underground production. Second, we assume that agents can evade by hiding income through deposits of a tax haven, hence we consider small open economy in this last case.

**Optimal tax rates on capital in closed economy**

Here, we assume that capital gains from underground production are evaded. We define this capital as $k_t^U$ and its return as $r_t^U$, and besides, we assume that the probability of being caught is $p^k$ and, in such a case, the government will charge a fine $s^k$. Here, the tax enforcement $p^k s^k$ is also lower than 1.

Thus, the agent's budget constraint reads

$$(1+\tau_t^c)c_t^V + (1+p^c s^c \tau_t^c)c_t^{NV} + i_t^M + i_t^U + b_{t+1} = (1-\tau_t^n)w_t^M n_t^M + (1-\tau_t^k)r_t^M k_t^M$$
$$+(1-p^n s^n \tau_t^n)w_t^U n_t^U + (1-p^k s^k \tau_t^k)r_t^U k_t^U + (1+r_t^b)b_t,$$

and the laws of motion for capital are

$$i_t^M = k_{t+1}^M - (1-\delta)k_t^M,$$

$$i_t^U = k_{t+1}^U - (1-\delta)k_t^U.$$

From firm's problem we get the following prices of factors

$$w_t^M = F_{n_t^M} = (1-\alpha)(k_t^M)^\alpha (n_t^M)^{-\alpha},$$

$$w_t^U = F_{n_t^U} = A^U(1-\alpha)(k_t^U)^\alpha (n_t^U)^{-\alpha},$$

$$r_t^M = F_{k_t^M} = \alpha(k_t^M)^{\alpha-1}(n_t^M)^{1-\alpha},$$

$$r_t^U = F_{k_t^U} = A^U\alpha(k_t^U)^{\alpha-1}(n_t^U)^{1-\alpha}.$$

The first-order conditions of the competitive equilibrium with respect to $k_{t+1}^{M}$ and $k_{t+1}^{U}$ consist of:

$$\frac{U_{c_t^V}(\cdot)}{(1+\tau_t^c)} = \beta\left(1+(1-\tau_{t+1}^k)F_{k_{t+1}^M}-\delta\right)\frac{U_{c_{t+1}^V}(\cdot)}{(1+\tau_{t+1}^c)}, \tag{1.22}$$

$$\frac{U_{c_t^V}(\cdot)}{(1+\tau_t^c)} = \beta\left(1+(1-p^k s^k\tau_{t+1}^k)F_{k_{t+1}^U}-\delta\right)\frac{U_{c_{t+1}^V}(\cdot)}{(1+\tau_{t+1}^c)}. \tag{1.23}$$

Before going to the details of optimal taxation with capital tax evasion as an additional element, we point out that the first-best allocation is attainable when there is only capital income tax evasion. In this case, the optimal tax mix achieving the Pareto optimum is the same as in Coleman (2000). Proposition 5 summarizes it.

**Proposition 5.** In a model with only capital tax evasion, an optimal tax policy is to set a constant tax on consumption, a constant subsidy to labor income equal to the consumption tax, and a tax on capital income equal to zero in all periods.

The proof of Proposition 5 is the following. The competitive equilibrium of a model with capital tax evasion is characterised by three marginal conditions, the MRS between consumption and leisure

$$\frac{-U_{n_t}(\cdot)}{U_{c_t}(\cdot)} = \frac{(1-\tau_t^n)\,w_t}{(1+\tau_t^c)} \tag{1.24}$$

and the Euler equations

$$\frac{U_{c_t}(\cdot)}{(1+\tau_t^c)} = \beta\left(1+(1-\tau_{t+1}^k)F_{k_{t+1}^M}-\delta\right)\frac{U_{c_{t+1}}(\cdot)}{(1+\tau_{t+1}^c)} \tag{1.25}$$

$$\frac{U_{c_t}(\cdot)}{(1+\tau_t^c)} = \beta\left(1+(1-p^k s^k\tau_{t+1}^k)F_{k_{t+1}^U}-\delta\right)\frac{U_{c_{t+1}}(\cdot)}{(1+\tau_{t+1}^c)}. \tag{1.26}$$

If we set $\tau_t^c=\bar{\tau}$, $\tau_t^n=-\tau_t^c$, and $\tau_t^k=0$, $\forall t$, this tax mix eliminates the distortions affecting (1.24)-(1.26), which then become the marginal conditions that characterize the Pareto-optimal allocation for this economy. Hence, as formally proved in Coleman (2000), this tax policy is optimal since it satisfies the intertemporal government's budget constraint. Furthermore, he shows that $\bar{\tau}>0$ provided that the present value of consumption is larger than the present value of labor income. Note that in the data of most countries consumption exceeds wage income.

Going back to the model with consumption, labor, and capital tax evasions; we use the non-arbitrage condition of the competitive equilibrium to express the capital tax rate as

$$\tau_t^k = \frac{F_{k_t^M} - F_{k_t^U}}{F_{k_t^M} - p^k s^k F_{k_t^U}}, \; t > 0. \tag{1.27}$$

which depends on the marginal productivities of capital and the policy parameters. The labor and consumption taxes are expressed by (1.16) and (1.17), respectively.

In this case, the Ramsey problem can be formulated as[36]

$$\begin{aligned}
\max_{\{c_t^V, c_t^{NV}, n_t^M, n_t^U, k_{t+1}^M, k_{t+1}^U\}_{t=0}^\infty} \quad & \sum_{t=0}^{\infty} \beta^t \Bigg\{ U\big(c_t, 1 - n_t^M - n_t^U\big) \\
& + \Lambda \left( c_t^V U_{c_t^V}(\cdot) + c_t^{NV} U_{c_t^{NV}}(\cdot) + n_t^M U_{n_t^M}(\cdot) + n_t^U U_{n_t^U}(\cdot) \right) \\
& - \lambda_t \left( c_t^V + c_t^{NV} + g_t + i_t^M + i_t^U - F(k_t^U, k_t^M, n_t^M, n_t^U) \right) \\
& + \mu_t \left( \frac{-U_{n_t^M}(\cdot)}{U_{c_t^V}(\cdot)} - \frac{(1 - \tau_t^n) F_{n_t^M}}{(1 + \tau_t^c)} \right) \\
& + \theta_{t+1} \left( \frac{U_{c_t^V}(\cdot)}{(1 + \tau_t^c)} - \beta \left( 1 + (1 - \tau_{t+1}^k) F_{k_{t+1}^M} - \delta \right) \frac{U_{c_{t+1}^V}(\cdot)}{(1 + \tau_{t+1}^c)} \right) \Bigg\} \\
& - \Lambda \frac{W_0 U_{c_0^V}}{(1 + \tau_0^c)},
\end{aligned} \tag{1.28}$$

where $W_0 = \left[1 + (1 - \tau_0^k) F_{k_0^M} - \delta\right] k_0^M + \left[1 + F_{k_0^U} - \delta\right] k_0^U + (1 + r_0^b) b_0$ and $\tau_0^k$, $k_0^U$, $k_0^M$, $b_0$, and $r_0^b$ are given. Since the resource constraint, the MRS of substitution between consumption and leisure (1.24), and the Euler equation of non-evaded capital (1.25) bind in each period, the Lagrange multipliers $\lambda_t$, $\mu_t$, and $\theta_{t+1}$ are strictly positive, $\forall t$.

In the next proposition we show how tax rates on capital should look like if government chooses them optimally.

**Proposition 6.** In a closed economy with tax evasion on consumption, labor, and capital; the optimal tax rate on capital income is always positive.

[36]Instead, we could add the first-order condition of the competitive equilibrium with respect to $k_{t+1}^U$ as an extra constraint to the Ramsey problem.

To prove Proposition 6 we compute the first-order conditions of (1.28) with respect $k_{t+1}^M$ and $k_{t+1}^U$

$$\begin{aligned}\lambda_t = \;& \beta\lambda_{t+1}(1+F_{k_{t+1}^M}-\delta)-\beta\mu_{t+1}\frac{(1-\tau_{t+1}^n)F_{n_{t+1}^M k_{t+1}^M}-\tau_{k_{t+1}^M}^n F_{n_{t+1}^M}}{(1+\tau_{t+1}^c)} \\ & -\beta\theta_{t+1}\frac{U_{c_{t+1}^V}(\cdot)}{(1+\tau_{t+1}^c)}\left[(1-\tau_{t+1}^k)F_{k_{t+1}^M k_{t+1}^M}-\tau_{k_{t+1}^M}^k F_{k_{t+1}^M}\right]\end{aligned} \tag{1.29}$$

$$\lambda_t = \beta\lambda_{t+1}(1+F_{k_{t+1}^U}-\delta)+\beta\mu_{t+1}\frac{\tau_{k_{t+1}^U}^n F_{n_{t+1}^M}}{(1+\tau_{t+1}^c)}+\beta\theta_{t+1}\frac{U_{c_{t+1}^V}(\cdot)}{(1+\tau_{t+1}^c)}\tau_{k_{t+1}^U}^k F_{k_{t+1}^M}. \tag{1.30}$$

Combining (1.29) and (1.30), we get

$$\left(F_{k_t^M}-F_{k_t^U}\right) = \left.\begin{aligned}& \frac{\mu_t}{\lambda_t}\frac{1}{(1+\tau_t^c)}\left[\tau_{k_t^U}^n F_{n_t^M}+(1-\tau_t^n)F_{n_t^M k_t^M}-\tau_{k_t^M}^n F_{n_t^M}\right] \\ & +\frac{\theta_t}{\lambda_t}\frac{U_{c_t^V}(\cdot)}{(1+\tau_t^c)}\left[\tau_{k_t^U}^k F_{k_t^M}+(1-\tau_t^k)F_{k_t^M k_t^M}-\tau_{k_t^M}^k F_{k_t^M}\right]\end{aligned}\right., t>0.$$

Then, substituting $\tau_t^k$ for expression (1.27) and $\tau_{k_t^M}^k$, which is the derivative of (1.27) with respect to $k_t^M$, gives

$$\left(F_{k_t^M}-F_{k_t^U}\right) = \left.\begin{aligned}& \frac{\mu_t}{\lambda_t}\frac{1}{(1+\tau_t^c)}\left[\tau_{k_t^U}^n F_{n_t^M}+(1-\tau_t^n)F_{n_t^M k_t^M}-\tau_{k_t^M}^n F_{n_t^M}\right] \\ & +\frac{\theta_t}{\lambda_t}\frac{U_{c_t^V}(\cdot)}{(1+\tau_t^c)}\left[\tau_{k_t^U}^k F_{k_t^M}-\frac{p^k s^k F_{k_t^M k_t^M}(1-p^k s^k)\left(F_{k_t^U}\right)^2}{\left(F_{k_t^M}-p^k s^k F_{k_t^U}\right)^2}\right]\end{aligned}\right., t>0, \tag{1.31}$$

where $(1+\tau_t^c)>0$; $(1-\tau_t^n)/(1+\tau_t^c)>0$; $\tau_{k_t^U}^n, \tau_{k_t^U}^k, F_{n_t^M k_t^M}>0$; $\tau_{k_t^M}^n, F_{k_t^M k_t^M}<0$; and the rest of elements are positive so that $F_{k_t^M}-F_{k_t^U}>0$.[37] Hence, comparing (1.31) together with equation (1.27), we can conclude that $\tau_t^k>0$, $\forall t$, since $\tau_0^k$ is given or when chosen optimally is positive.[38] Therefore, the Chamley-Judd result does not hold either when we introduce capital tax evasion as well as consumption and labor tax evasions to the model. Furthermore, conditions (1.27) and (1.31) imply that $\tau_t^k<1$, $t>0$.

[37] As in Coleman (2000), $(1+\tau_t^c)>0$, otherwise the price of consumption is negative, which is impossible in equilibrium.

[38] Clearly, if the numerator of equation (1.27) is positive, the denominator too.

**Optimal tax rates on capital in small open economy (SOE)**

Let $k_t^{TH}$ denote the savings deposited in a tax haven (TH) with a exogenous return equal to $r_t^{TH}$. The government catch this evasion with probability $p^k$ and it will charge a fine $s^k$ in such a case. This framework is very similar to the setup described by Correia (1996), with the exception of the part regarding labor income and consumption tax evasions. In SOE models, we will show that the fact that government is able to caught with some probability the evasion of capital income is key to determine optimal capital taxes.

The agent's budget constraint is then

$$\begin{gathered}(1+\tau_t^c)c_t^V + (1+p^c s^c \tau_t^c)c_t^{NV} + i_t + k_{t+1}^{TH} + b_{t+1} = (1-\tau_t^n)w_t^M n_t^M + (1-\tau_t^k)r_t k_t \\ +(1-p^n s^n \tau_t^n)w_t^U n_t^U + \left(1+(1-p^k s^k \tau_t^k)r_t^{TH}\right)k_t^{TH} + (1+r_t^b)b_t,\end{gathered}$$

and the firm's problem is the same as in Subsection 1.2.1.

The first-order conditions of the competitive equilibrium with respect to $k_{t+1}$ and $k_{t+1}^{TH}$ consist of:

$$\frac{U_{c_t^V}(\cdot)}{(1+\tau_t^c)} = \beta\left(1+(1-\tau_{t+1}^k)r_{t+1}-\delta\right)\frac{U_{c_{t+1}^V}(\cdot)}{\left(1+\tau_{t+1}^c\right)}, \tag{1.32}$$

$$\frac{U_{c_t^V}(\cdot)}{(1+\tau_t^c)} = \beta\left(1+(1-p^k s^k \tau_{t+1}^k)r_{t+1}^{TH}\right)\frac{U_{c_{t+1}^V}(\cdot)}{\left(1+\tau_{t+1}^c\right)}. \tag{1.33}$$

Before going to the details of optimal taxation in the model with all three tax evasions, it is worth noting that the optimal tax mix implied by Proposition 5 also applies to small open economy with only capital tax evasion. The proof is the same as before.

Going back to the model with all three tax evasions, we use previous equations (1.32) and (1.33) to express the capital tax rate as

$$\tau_t^k = \frac{F_{k_t} - \delta - r_t^{TH}}{F_{k_t} - p^k s^k r_t^{TH}}, t > 0,$$

which depends on the marginal productivity of non-evaded capital, the interest rate of evaded capital, and the policy parameters. The labor and consumption taxes are expressed by (1.16) and (1.17), respectively.

The Ramsey problem in this case would be[39]

$$
\begin{aligned}
\max_{\{c_t^V, c_t^{NV}, n_t^M, n_t^U, k_{t+1}, k_{t+1}^{TH}\}_{t=0}^{\infty}} \quad & \sum_{t=0}^{\infty} \beta^t \Bigg\{ U\big(c_t, 1 - n_t^M - n_t^U\big) \\
& + \Lambda \left( c_t^V U_{c_t^V}(\cdot) + c_t^{NV} U_{c_t^{NV}}(\cdot) + n_t^M U_{n_t^M}(\cdot) + n_t^U U_{n_t^U}(\cdot) \right) \\
& - \lambda_t \left( c_t^V + c_t^{NV} + g_t + k_{t+1} - (1-\delta) k_t + k_{t+1}^{TH} \right. \\
& \left. - F(k_t, n_t^M, n_t^U) - \left(1 + r_t^{TH}\right) k_t^{TH} \right) \\
& + \mu_t \left( \frac{-U_{n_t^M}(\cdot)}{U_{c_t^V}(\cdot)} - \frac{(1-\tau_t^n) F_{n_t^M}}{(1+\tau_t^c)} \right) \\
& + \varphi_{t+1} \left( \frac{U_{c_t^V}(\cdot)}{(1+\tau_t^c)} - \beta \left( 1 + (1 - p^k s^k \tau_{t+1}^k) r_{t+1}^{TH} \right) \frac{U_{c_{t+1}^V}(\cdot)}{\left(1+\tau_{t+1}^c\right)} \right) \Bigg\} \\
& - \Lambda \frac{W_0 U_{c_0^V}}{(1+\tau_0^c)},
\end{aligned}
\tag{1.34}
$$

where $W_0 = \left[1 + (1-\tau_0^k) F_{k_0} - \delta\right] k_0 + (1 + r_0^{TH}) k_0^{TH} + (1 + r_0^b) b_0$ and $\tau_0^k$, $k_0$, $k_0^{TH}$, $b_0$, $r_0^b$, $r_0^{TH}$ are given.

In the presence of international capital tax evasion, the optimal tax rates on capital income are described by the following proposition.

**Proposition 7.** In a small open economy and in the presence of consumption, labor, and capital tax evasions; the optimal steady-state tax rate on capital income is zero whenever government has the chance to enforce, at least partially, the capital income evaded abroad through tax inspection. However, whenever the level of enforcement is ineffective (probability of detection or punishment equal to zero), the optimal tax rate on capital income is always positive.

The first-order conditions of (1.34) with respect to $k_{t+1}$ and $k_{t+1}^{TH}$ are

[39] Instead, we could add the first-order condition of the competitive equilibrium with respect to $k_{t+1}$ as an extra constraint to the Ramsey problem.

$$\lambda_t = \beta\lambda_{t+1}(1 + F_{k_{t+1}} - \delta) - \beta\mu_{t+1}\frac{(1-\tau^n_{t+1})F_{n^M_{t+1}k_{t+1}} - \tau^n_{k_{t+1}}F_{n^M_{t+1}}}{(1+\tau^c_{t+1})} + \beta\varphi_{t+1}p^k s^k \frac{U_{c^V_{t+1}}(\cdot)}{\left(1+\tau^c_{t+1}\right)} r^{TH}_{t+1}\tau^k_{k_{t+1}}, \tag{1.35}$$

$$\lambda_t = \beta(1 + r^{TH}_{t+1})\lambda_{t+1}. \tag{1.36}$$

Therefore, if we compare conditions (1.33) and (1.36) at steady state,[40] it is easy to verify that $\tau^k$ is equal to zero. Therefore, the optimal tax rate on capital income is zero in the steady state, independently of the existence of the other tax evasions: labor and consumption. The reason for this result was pointed out by Correia (1996) in her "worldwide tax system", the optimal steady-state tax rate on capital is zero since influences the steady-state growth rate of the economy, i.e. it introduces dynamic distortions (affects the intertemporal marginal rate of substitution).[41] That is, when government is able to tax both capital gains, optimal capital taxation is the same as in the standard closed economy, thus the Chamley-Judd result holds.

On the other hand, if we assume that $p^k$ or $s^k$ is equal to zero the resulting problem is very similar to what Correia (1996) calls the "territorial tax system", with the exception of the part regarding labor income and consumption tax evasions. In this case, the first-order condition of competitive equilibrium with respect to $k_{t+1}$ remains unchanged, (1.32), and that with respect to $k^{TH}_{t+1}$ is

$$\frac{U_{c^V_t}(\cdot)}{(1 + \tau^c_t)} = \beta\left(1 + r^{TH}_{t+1}\right)\frac{U_{c^V_{t+1}}(\cdot)}{\left(1 + \tau^c_{t+1}\right)}. \tag{1.37}$$

This condition, together with (1.32), implies that the tax rate on capital income can be expressed as

$$\tau^k_t = \frac{F_{k_t} - \delta - r^{TH}_t}{F_{k_t}},\ t > 0. \tag{1.38}$$

In this case, the Ramsey problem would be the same as before, (1.34), but with $p^k s^k = 0$. Combining (1.35), without the term associated with $p^k s^k$, and (1.36) gives

[40] The latter reads $1 = \beta(1 + r^{TH})$ provided that allocations converge to an interior steady state and Lagrange multipliers $\mu_t$ and $\varphi_t$ converge. As shown by Straub and Werning (2014), these assumptions can be verified to hold in a representative-agent framework with intertemporally-separable utility.

[41] In other words, it affects the Euler equation of capital income evaded abroad, and therefore the interest rate of the economy.

$$F_{k_t} - \delta - r_t^{TH} = \frac{\mu_t}{\lambda_t} \frac{(1-\tau_t^n) F_{n_t^M k_t} - \tau_{k_t}^n F_{n_t^M}}{(1+\tau_t^c)}, \; t > 0. \tag{1.39}$$

Equations (1.38) and (1.39) taken together imply that

$$\begin{aligned} \tau_t^k &= \frac{1}{F_{k_t}} \frac{\mu_t}{\lambda_t} \frac{(1-\tau_t^n) F_{n_t^M k_t} - \tau_{k_t}^n F_{n_t^M}}{(1+\tau_t^c)} \\ &= \frac{\mu_t}{\lambda_t} \frac{(1-\tau_t^n)}{(1+\tau_t^c)} \frac{(1-\alpha)}{\left[ n_t^M + A^U (n_t^M)^\alpha (n_t^U)^{1-\alpha} \right]} \end{aligned}, \; t > 0, \tag{1.40}$$

which is positive given that $\mu_t, \lambda_t, > 0$; $\alpha < 1$; $(1-\tau_t^n)/(1+\tau_t^c) > 0$; and the rest of variables are positive. This result is based on the same reason explained by Correia (1996), the optimum is to minimize the static distortion since the tax rate does not affect the intertemporal marginal rate of substitution. Hence, the existence of consumption and labor tax evasions play a role. Furthermore, conditions (1.38) and (1.39) imply that $\tau_t^k < 1$, $t > 0$.

These results show that the level of enforcement on capital income evaded abroad is crucial to determine what is the optimal tax rate on capital income. We have seen that when government can affect the return of income evaded abroad, the long-run capital tax is optimally set equal to zero. On the other hand, when government cannot affect this return, the optimal capital tax rate is always positive. Hence, this issue is closely related to the current effort of policymakers so as to sign information exchange treaties with tax havens, e.g. G20 countries during the financial crisis. Johannesen and Zucman (2014) attempt to address the question of whether or not treaties significantly raise the probability of detecting tax evasion and greatly improve tax collection. They document, using a panel data for the period 2003-2011, that tax evaders shifted deposits to havens not covered by a treaty with their home country rather than repatriating funds.

Their empirical evidence might be associated with the case in which $p^k$ or $s^k$ is equal to zero. In other words, agents tend to minimize enforcement by reallocating income to less transparent countries. Remember that, in this case, optimal tax rates on capital income are determined by the presence of static distortions; namely, the presence of both consumption and labor tax evasions; and therefore the optimal capital tax rate is always positive.

## 1.6 Conclusions

In this paper, we have studied the design of the optimal tax mix in the presence tax evasion. We have shown that tax evasion goes against extreme and well-known results in the literature of optimal fiscal policy.

The optimal tax on labor is positive and significant in contrast to Coleman (2000), Correia (2010), or Laczó and Rossi (2014). Related to this point, we find that labor income tax is preferred to consumption tax in optimal terms, given that $\tau_t^c < \tau_t^n$, $t > 0$. Specifically, optimal tax rates on consumption and labor income are equal to 13.24% and 36.44%, respectively. In addition, this result is robust to large changes in parameters and to the alternative treatment of underground labor in the utility function.

The Chamley-Judd result does not hold due to the presence of labor income and consumption tax evasions; namely, it is optimal to decrease taxation on capital but it is not zero in the long run. Specifically, the optimal steady-state tax rate on capital income is 16.03%. In Subsection 1.5.2, we show that the Chamley-Judd result does not hold either when our framework also incorporates capital tax evasion in closed economy. On the other hand, in small open economy, the optimal long-run tax rate on capital is positive only when the government cannot affect the capital income evaded abroad, i.e. when probability of detection or penalty is equal to zero.

The results also show that tax evasion falls with respect to the initial level; that is, the proportions of evaded consumption and underground labor decrease by 16.44 and 2.90 percent due to lower taxes on consumption and labor, respectively.

Our sensitivity analysis points out that the enforcement level is a key variable in the design of an optimal tax system. The optimal tax rates on consumption and labor reduce by around 3-4 percentage points relative to in the benchmark calibration when their respective probabilities fall.

To sum up, the Ramsey planner must cut all taxes below the current tax rates and the welfare gains from switching to optimal taxes are large. Specifically, the welfare gain is 3.31 percent.

These findings show that it is very important that government takes tax evasion into account when designing the optimal tax system. The take-home message is that government should reduce the tax burden so as to improve welfare and discourage tax evasion.

Finally, our paper computes the optimal tax rates given a level of tax enforcement. Hence, an interesting point for future research would be the study of which is the optimal level of enforcement when government has to spend resources in order to increase tax enforcement.[42] Specifically, we could analyze the trade-off between tax and enforcement rates, i.e. if government needed to raise revenues, would it be optimal to increase taxes or to increase enforcement? What is the optimal combination of tax and enforcement rates in order to finance a certain level of public expenditure? This extension would link the literature of optimal fiscal policy with the literature of state (or fiscal) capacity, see, for example, Besley and Persson (2009).

[42] If enforcement were costless, the optimal choice of tax and enforcement rates would be trivial. That is, government should set $p^i s^i = 1$ and apply Coleman's result as long as the first-best is attainable.

# Appendices

## A First-order conditions of the Ramsey problem

We assume that the utility function is separable in consumption and leisure, hence the second cross-derivatives between these two arguments are zero. On the other hand, the second cross-derivatives between consumptions and between labors are not zero. We will use the notation $U_{i_t j_t}(\cdot)$ for the second derivatives of the utility with respect to variable $j$, the notation $\tau^i_{j_t}$ for the first derivative of tax $i$ with respect to variable $j$, and the notation $F_{i_t j_t}$ for the second derivative of the production function with respect to variable $j$.

The first-order conditions of (1.15) with respect to $c_t^V$, $c_t^{NV}$, $n_t^M$, $n_t^U$, and $k_{t+1}$, for $t > 0$, are:

$$\begin{aligned} &U_{c_t^V}(\cdot) + \Lambda\left(U_{c_t^V}(\cdot) + c_t^V U_{c_t^V c_t^V}(\cdot) + c_t^{NV} U_{c_t^{NV} c_t^V}(\cdot)\right) = \lambda_t \\ &-\mu_t\left(\frac{U_{n_t^M}(\cdot)U_{c_t^V c_t^V}(\cdot)}{U_{c_t^V}(\cdot)^2} + \frac{(1-\tau_t^n)F_{n_t^M}\tau^c_{c_t^V}}{(1+\tau_t^c)^2}\right), \end{aligned} \tag{1.41}$$

$$\begin{aligned} &U_{c_t^{NV}}(\cdot) + \Lambda\left(c_t^V U_{c_t^V c_t^{NV}}(\cdot) + U_{c_t^{NV}}(\cdot) + c_t^{NV} U_{c_t^{NV} c_t^{NV}}(\cdot)\right) = \lambda_t \\ &-\mu_t\left(\frac{U_{n_t^M}(\cdot)U_{c_t^V c_t^{NV}}(\cdot)}{U_{c_t^V}(\cdot)^2} + \frac{(1-\tau_t^n)F_{n_t^M}\tau^c_{c_t^{NV}}}{(1+\tau_t^c)^2}\right), \end{aligned} \tag{1.42}$$

$$\begin{aligned} &U_{n_t^M}(\cdot) + \Lambda\left(U_{n_t^M}(\cdot) + n_t^M U_{n_t^M n_t^M}(\cdot) + n_t^U U_{n_t^U n_t^M}(\cdot)\right) + \lambda_t F_{n_t^M} \\ &-\mu_t\left(\frac{U_{n_t^M n_t^M}(\cdot)}{U_{c_t^V}(\cdot)} + \frac{(1-\tau_t^n)F_{n_t^M n_t^M} - F_{n_t^M}\tau^n_{n_t^M}}{(1+\tau_t^c)}\right) = 0, \end{aligned} \tag{1.43}$$

$$\begin{aligned} &U_{n_t^U}(\cdot) + \Lambda\left(n_t^M U_{n_t^M n_t^U}(\cdot) + U_{n_t^U}(\cdot) + n_t^U U_{n_t^U n_t^U}(\cdot)\right) + \lambda_t F_{n_t^U} \\ &-\mu_t\left(\frac{U_{n_t^M n_t^U}(\cdot)}{U_{c_t^V}(\cdot)} - \frac{F_{n_t^M}\tau^n_{n_t^U}}{(1+\tau_t^c)}\right) = 0, \end{aligned} \tag{1.44}$$

$$\lambda_t = \beta\lambda_{t+1}(1 + F_{k_{t+1}} - \delta) - \beta\mu_{t+1}\frac{(1-\tau^n_{t+1})F_{n^M_{t+1}k_{t+1}} - \tau^n_{k_{t+1}}F_{n^M_{t+1}}}{(1+\tau^c_{t+1})}. \tag{1.45}$$

For $t = 0$ the first-order conditions with respect to $c_0^V, c_0^{NV}, n_0^M, n_0^U$, and $k_1$ are:

$$U_{c_0^V}(\cdot) + \Lambda\left(U_{c_0^V}(\cdot) + c_0^V U_{c_0^V c_0^V}(\cdot) + c_0^{NV} U_{c_0^{NV} c_0^V}(\cdot)\right) = \lambda_0$$
$$-\mu_0\left(\frac{U_{n_0^M}(\cdot)U_{c_0^V c_0^V}(\cdot)}{U_{c_0^V}(\cdot)^2} + \frac{(1-\tau_0^n)F_{n_0^M}\tau^c_{c_0^V}}{(1+\tau_0^c)^2}\right) + \Lambda W_0 \frac{U_{c_0^V c_0^V}(1+\tau_0^c) - U_{c_0^V}\tau^c_{c_0^V}}{(1+\tau_0^c)^2}, \tag{1.46}$$

$$U_{c_0^{NV}}(\cdot) + \Lambda\left(c_0^V U_{c_0^V c_0^{NV}}(\cdot) + U_{c_0^{NV}}(\cdot) + c_0^{NV} U_{c_0^{NV} c_0^{NV}}(\cdot)\right) = \lambda_0$$
$$-\mu_0\left(\frac{U_{n_0^M}(\cdot)U_{c_0^V c_0^{NV}}(\cdot)}{U_{c_0^V}(\cdot)^2} + \frac{(1-\tau_0^n)F_{n_0^M}\tau^c_{c_0^{NV}}}{(1+\tau_0^c)^2}\right) + \Lambda W_0 \frac{U_{c_0^V c_0^{NV}}(1+\tau_0^c) - U_{c_0^V}\tau^c_{c_0^{NV}}}{(1+\tau_0^c)^2}, \tag{1.47}$$

$$U_{n_0^M}(\cdot) + \Lambda\left(U_{n_0^M}(\cdot) + n_0^M U_{n_0^M n_0^M}(\cdot) + n_0^U U_{n_0^U n_0^M}(\cdot)\right) + \lambda_0 F_{n_0^M}$$
$$-\mu_0\left(\frac{U_{n_0^M n_0^M}(\cdot)}{U_{c_0^V}(\cdot)} + \frac{(1-\tau_0^n)F_{n_0^M n_0^M} - F_{n_0^M}\tau^n_{n_0^M}}{(1+\tau_0^c)}\right) - \Lambda \frac{U_{c_0^V}(k_0+b_0)}{(1+\tau_0^c)}(1-\tau_0^k)F_{k_0 n_0^M} = 0, \tag{1.48}$$

$$U_{n_0^U}(\cdot) + \Lambda\left(n_0^M U_{n_0^M n_0^U}(\cdot) + U_{n_0^U}(\cdot) + n_0^U U_{n_0^U n_0^U}(\cdot)\right) + \lambda_0 F_{n_0^U}$$
$$-\mu_0\left(\frac{U_{n_0^M n_t^U}(\cdot)}{U_{c_0^V}(\cdot)} - \frac{F_{n_0^M}\tau^n_{n_0^M}}{(1+\tau_0^c)}\right) - \Lambda \frac{U_{c_0^V}(k_0+b_0)}{(1+\tau_0^c)}(1-\tau_0^k)F_{k_0 n_0^U} = 0, \tag{1.49}$$

$$\lambda_0 = \beta\lambda_1(1 + F_{k_1} - \delta) - \beta\mu_1 \frac{(1-\tau_1^n)F_{n_1^M k_1} - \tau^n_{k_1}F_{n_1^M}}{(1+\tau_1^c)}. \tag{1.50}$$

# B The model with labor income tax evasion

Now, we have two static optimal conditions and a dynamic optimal condition, which is equal to (1.2). The MRS between the two labors is the same as in our main framework, but now the MRS between consumption and leisure is equal to

$$\frac{-U_{n_t^M}(\cdot)}{U_{c_t}(\cdot)} = \frac{(1-\tau_t^n)w_t^M}{(1+\tau_t^c)} \tag{1.51}$$

Therefore, this implies that, if we combine this new condition with the expression of labor income tax (1.16), the tax rate on consumption reads

$$\tau_t^c = \frac{U_{c_t}(\cdot)F_{n_t^U}\left(p^n s^n - 1\right)F_{n_t^M}}{U_{n_t^U}(\cdot)F_{n_t^M} - p^n s^n U_{n_t^M}(\cdot)F_{n_t^U}} - 1. \tag{1.52}$$

The Ramsey problem in this case would be

$$\begin{aligned}\max_{\{c_t, n_t^M, n_t^U, k_{t+1}\}_{t=0}^{\infty}} \quad & \sum_{t=0}^{\infty}\beta^t \Bigg\{ U\left(c_t, 1 - n_t^M - n_t^U\right) \\ & + \Lambda\left(c_t U_{c_t}(\cdot) + n_t^M U_{n_t^M}(\cdot) + n_t^U U_{n_t^U}(\cdot)\right) \\ & - \lambda_t\left(c_t + g_t + k_{t+1} - (1-\delta)k_t - F(k_t, n_t^U, n_t^M)\right)\Bigg\} \\ & - \Lambda\,\frac{W_0 U_{c_0}}{(1+\tau_0^c)},\end{aligned} \tag{1.53}$$

and besides, for utility functions of the form (1.21), the combination of the first-order conditions of the Ramsey problem (1.53) with respect to $n_t^M$ and $n_t^U$ would be

$$\frac{U_{n_t^M}(\cdot)}{U_{n_t^U}(\cdot)} = \frac{w_t^M}{w_t^U}, t > 0. \tag{1.54}$$

In addition, the first-order condition with respect to $k_{t+1}$ reads

$$\lambda_t = \beta\lambda_{t+1}(1 + F_{k_{t+1}} - \delta), \forall t,$$

or

$$U_{c_t}(\cdot) = \beta U_{c_{t+1}}(\cdot)(1 + F_{k_{t+1}} - \delta), t > 0. \tag{1.55}$$

# C The model with consumption tax evasion

Now, we have two static optimal conditions and a dynamic optimal condition, which is the equal to (1.2). The MRS between the two consumptions is the same as in our main framework, but now the MRS between consumption and leisure is equal to

$$\frac{-U_{n_t}(\cdot)}{U_{c_t^V}(\cdot)} = \frac{(1-\tau_t^n)w_t}{(1+\tau_t^c)} \tag{1.56}$$

Therefore, this implies that, if we combine this new condition with the expression of consumption tax (1.17), the tax rate on labor income reads

$$\tau_t^n = 1 + \frac{U_{n_t}(\cdot)(1-p^c s^c)}{F_{n_t}\left(U_{c_t^{NV}}(\cdot) - p^c s^c U_{c_t^V}(\cdot)\right)}.$$

The Ramsey problem in this case consists of

$$\begin{aligned}\max_{\{c_t^V, c_t^{NV}, n_t, k_{t+1}\}_{t=0}^{\infty}} \quad & \sum_{t=0}^{\infty} \beta^t \Bigg\{ U(c_t, 1-n_t) \\ & + \Lambda \left( c_t^V U_{c_t^V}(\cdot) + c_t^{NV} U_{c_t^{NV}}(\cdot) + n_t U_{n_t}(\cdot) \right) \\ & - \lambda_t \left( c_t^V + c_t^{NV} + g_t + k_{t+1} - (1-\delta)k_t - F(k_t, n_t) \right) \Bigg\} \\ & - \Lambda \frac{W_0 U_{c_0^V}}{(1+\tau_0^c)},\end{aligned} \tag{1.57}$$

and besides, for utility functions of the form (1.21), the first-order conditions of the Ramsey problem (1.57) with respect to $c_t^V$ and $c_t^{NV}$ imply that

$$U_{c_t^V}(\cdot) = U_{c_t^{NV}}(\cdot), t > 0, \tag{1.58}$$

and the first-order conditions with respect to $c_t^V$ and $k_{t+1}$ imply that

$$U_{c_t^V}(\cdot) = \beta(1 + F_{k_{t+1}} - \delta) U_{c_{t+1}^V}(\cdot), t > 0. \tag{1.59}$$

## Tables and figures

Table 1.1: Calibration parameters

| Parameter | Description | Value |
|---|---|---|
| $\delta$ | Depreciation | 0.078 |
| $\beta$ | Discount factor | 0.96 |
| $\rho$ | Affects the elasticity of subs. $c_t^V$ and $c_t^{NV}$ | 0.95 |
| $\phi$ | Weight on evaded consumption | 0.8388 |
| $\Phi_l$ | Weight parameter on leisure | 0.1366 |
| $\gamma$ | Affects the elasticity of labor supply | 4 |
| $\alpha$ | Share of capital | 0.3734 |
| $A^U$ | TFP of underground production | 0.4857 |
| $p^c$ | Prob. of detection evading $\tau_t^c$ | 0.1 |
| $s^c$ | Surcharge when caught evading $\tau_t^c$ | 1.5 |
| $p^n$ | Prob. of detection evading $\tau_t^n$ | 0.1 |
| $s^n$ | Surcharge when caught evading $\tau_t^n$ | 1.5 |

Table 1.2: Optimal steady-state tax rates and welfare gains

| | $\tau^c$ | $\tau^n$ | $\tau^k$ | Welfare gain |
|---|---|---|---|---|
| Current tax rates | 14.80 | 37.54 | 24.95 | - |
| Optimal steady-state tax rates | 13.24 | 36.44 | 16.03 | 3.31 |

Note: all numbers are in percent.

Table 1.3: Steady-state allocations

| | $c^V$ | $c^{NV}$ | $n^M$ | $n^U$ | $k$ | $d/Y$ | $Y^M$ | $Y^U$ | $Y$ |
|---|---|---|---|---|---|---|---|---|---|
| Status quo allocation | 30.78 | 9.34 | 32.45 | 14.17 | 188.69 | 54% | 62.61 | 18.09 | 80.71 |
| Steady-state Ramsey allocation | 34.86 | 8.42 | 32.98 | 13.80 | 227.27 | 7.87% | 67.80 | 19.08 | 86.88 |

Table 1.4: Key ratios of tax evasion

| | $c^{NV}/c$ | $n^U/n$ | $Y^U/Y$ |
|---|---|---|---|
| Status quo allocation | 23.29 | 30.39 | 22.42 |
| Steady-state Ramsey allocation | 19.46 | 29.51 | 21.96 |

Note: all numbers are in percent.

Table 1.5: Optimal steady-state tax rates and welfare gains for alternative parameters

| | $\tau^c$ | $\tau^n$ | $\tau^k$ | Welfare gain |
|---|---|---|---|---|
| $A^U = 1$ | 13.81 | 24.53 | 20.55 | 5.14 |
| $\rho = 0.85$ | 18.03 | 35.14 | 14.82 | 2.89 |
| $\alpha^U = 0.2$ | 15.37 | 41.91 | 11.43 | 1.43 |

Note: all numbers are in percent.

Table 1.6: Optimal steady-state tax rates and welfare gains for alternative probabilities

| | $\tau^c$ | $\tau^n$ | $\tau^k$ | Welfare gain |
|---|---|---|---|---|
| $p^c = 0$ | 10.15 | 36.90 | 16.48 | 4.35 |
| $p^n = 0$ | 13.78 | 32.46 | 18.13 | 4.72 |
| $p^c$, $p^n = 0$ | 10.58 | 32.98 | 18.73 | 5.68 |

Note: all numbers are in percent.

Table 1.7: Key ratios of tax evasion for alternative probabilities (percent)

| | | $c^{NV}/c$ | $n^U/n$ | $Y^U/Y$ |
|---|---|---|---|---|
| $p^c = 0$ | Status quo allocation | 32.01 | 30.39 | 22.42 |
| | Steady-state Ramsey allocation | 17.10 | 29.88 | 22.15 |
| $p^n = 0$ | Status quo allocation | 23.29 | 33.77 | 24.16 |
| | Steady-state Ramsey allocation | 20.76 | 29.26 | 21.84 |
| $p^c$, $p^n = 0$ | Status quo allocation | 32.01 | 33.77 | 24.17 |
| | Steady-state Ramsey allocation | 18.23 | 29.69 | 22.06 |

Table 1.8: Optimal steady-state tax rates and welfare gains for the alternative utility function

| | $\tau^c$ | $\tau^n$ | $\tau^k$ | Welfare gain |
|---|---|---|---|---|
| Current tax rates | 14.80 | 37.54 | 24.95 | - |
| Optimal steady-state tax rates | 11.25 | 47.28 | 11.00 | 1.33 |

Note: all numbers are in percent.

Table 1.9: Steady-state allocations for the alternative utility function

| | $c^V$ | $c^{NV}$ | $n^M$ | $n^U$ | $k$ | $d/Y$ | $Y^M$ | $Y^U$ | $Y$ |
|---|---|---|---|---|---|---|---|---|---|
| Status quo allocation | 30.78 | 9.34 | 32.15 | 4.59 | 188.48 | 54% | 62.23 | 18.38 | 80.62 |
| Steady-state Ramsey allocation | 36.36 | 6.54 | 29.83 | 5.39 | 243.01 | 65.60% | 65.28 | 22.36 | 87.65 |

Table 1.10: Key ratios of tax evasion for the alternative utility function

| | $c^{NV}/c$ | $n^U/n$ | $Y^U/Y$ |
|---|---|---|---|
| Status quo allocation | 23.29 | 12.49 | 22.8 |
| Steady-state Ramsey allocation | 15.24 | 15.32 | 25.51 |

Note: all numbers are in percent.

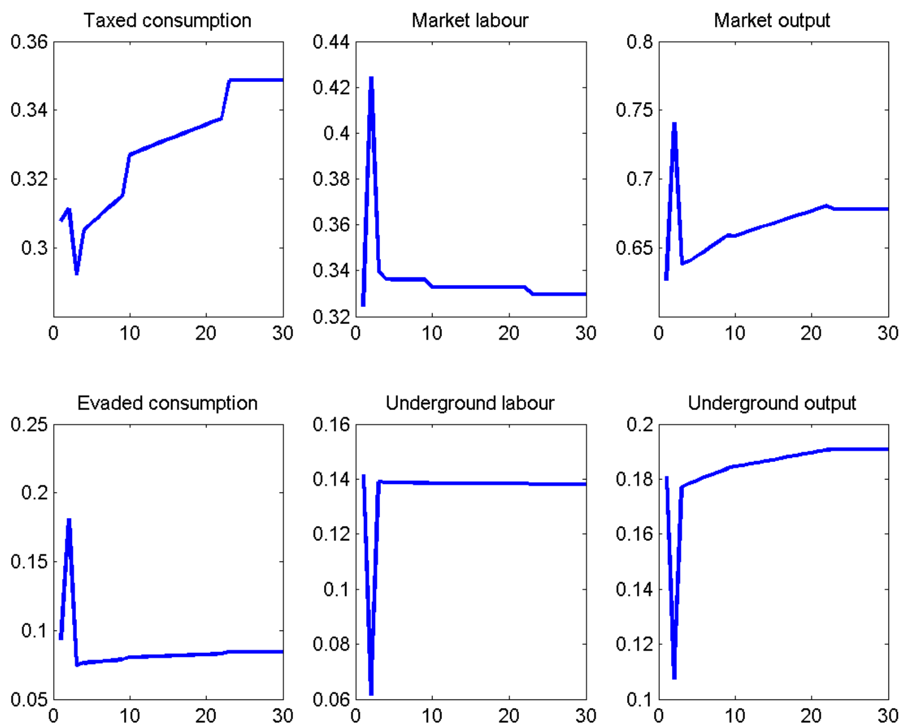



Figure 1.1: Full transition of Ramsey-equilibrium allocations
Note: the initial observation corresponds to the status quo allocation.

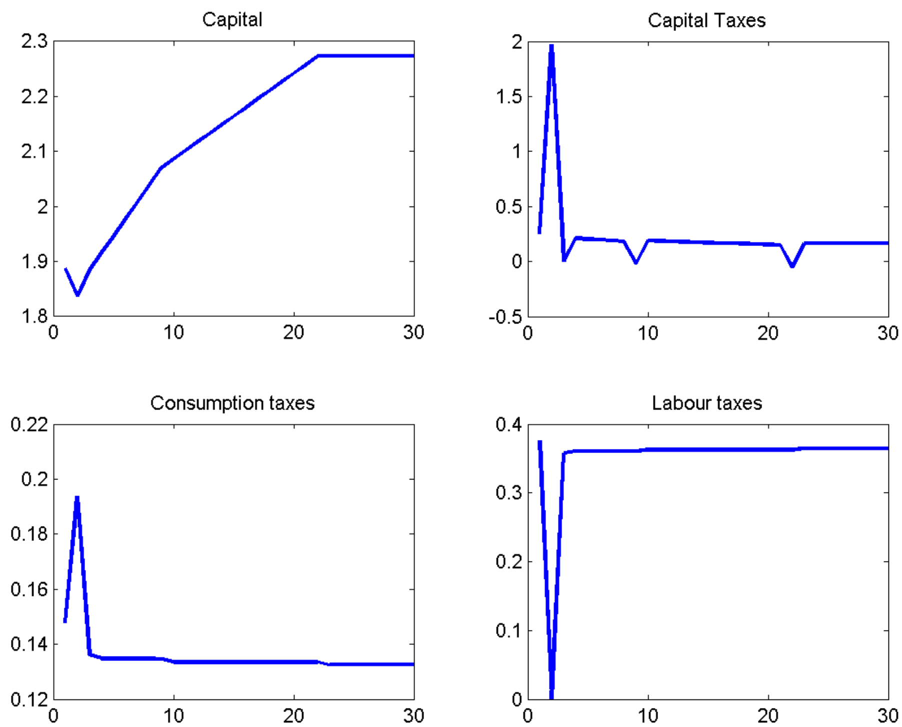



Figure 1.2: Full transition of optimal tax rates and capital
Note: the initial observation corresponds to the status quo allocation. All tax rates are in percent.

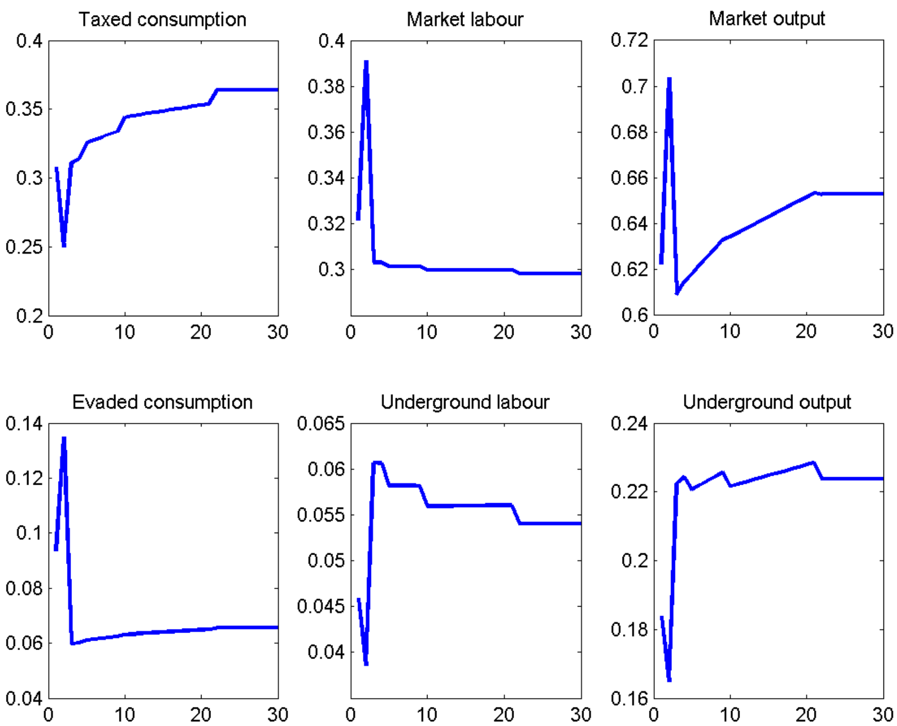



Figure 1.3: Full transition of Ramsey-equilibrium allocations for the alternative utility function

Note: the initial observation corresponds to the status quo allocation.

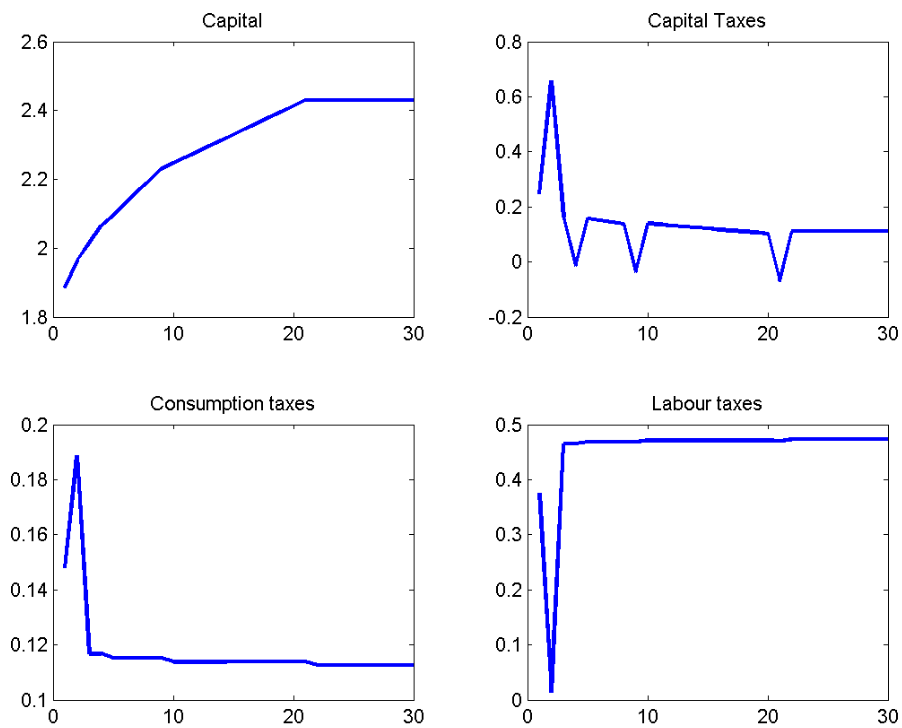



Figure 1.4: Full transition of optimal tax rates and capital for the alternative utility function

Note: the initial observation corresponds to the status quo allocation. All tax rates are in percent.

# Chapter 2

# The macroeconomics of consumption and labor tax evasions

(joint with Adrián Pino Alcalde)

This paper extends previous macroeconomic frameworks on shadow economy with consumption tax evasion. The model presented here is the first one to jointly tackle both labor income and consumption tax evasions. By introducing a TFP shock that affects equally both the declared and undeclared sectors, we are able to produce a countercyclical evolution of tax evasion and improve the fit of the model to the data. We also explore the consequences for taxation of our framework; specifically, we show that it may be unfeasible to implement tax shifts implying significant reductions of income tax in favor of higher excises on consumption. Laffer curves are flatter than in the model without tax evasion. The limits imposed by tax evasion are stricter for consumption tax, for which the slippery slope of the Laffer curve starts roughly at a mere rate of 10%.

## 2.1 Introduction

In recent times, international observers such as the European Union have been recommending southern European economies with fiscal straits to undertake tax reforms consisting in increasing the fiscal pressure on indirect taxation together with a reduction in income taxes,[1] policy commonly referred as tax shift.[2] Apart from being a requirement for achieving a harmonization in fiscal policies and the eventual creation of a fiscal union in the Eurozone, the economic rationale for such reforms are the claimed improvements in efficiency and growth, thereby bringing government budgets back to balance. The details are the following. First, the demand of capital is completely elastic in the long run, therefore the long-run tax rate on capital income should be the minimum possible.[3] Second, given that consumption and labor taxes affect the consumption-leisure margin, government should use the less distortive tax and set the other to zero.[4] One way to show that taxing consumption is more efficient than taxing labor is that consumption taxes create a lower tax wedge than labor income taxes, for instance, assume that the policy $\tau_t^c = 0$ and $\tau_t^n = 0.4$ is replaced by $\tau_t^c = 0.4$ and $\tau_t^n = 0$, then $1 - (1 - 0.4) > 1 - \frac{1}{(1+0.4)}$, which imply a lower distortion of the consumption tax with respect to the labor income tax.[5] Another way to illustrate this is that the Laffer curve of labor income tax equals to zero for $\tau_t^n = 1$, while that of consumption tax is positive for $\tau_t^c = 1$. Third, the feasibility of fiscal policy requires that the larger tax base (consumption) is taxed and the lower tax base (labor income) is not, the reason is that to finance with only one tax rate the same expenditure it is required a lower tax rate on consumption than on labor income because the former collects more than the latter, therefore this implies an additional motive to the one pointed out before that shows that consumption tax is more efficient.

For the case of Spain, for example, quantitative evaluations from both European institutions and from experts of the Spanish government praise the benefits that a tax shift would have. From the former, Rossini et al. (2014) study the macroeconomic implications of a revenue-neutral tax cut on labor income of the low skilled workers of 1% GDP, compensated with a raise in VAT. They argue that such a measure would reduce unemployment by 1 percentage point (pp). In march 2014, a commission of experts from the

[1] By indirect taxation we mean consumption taxes. Throughout the article we refer as Value Added Tax (VAT), sales tax and consumption tax indistinctively.

[2] This is mentioned in the 2012 Annual Growth Survey. Full text `http://ec.europa.eu/europe2020/pdf/annualgrowthsurveyen.pdf`

[3] For example, in the standard neoclassical model, the inverse of demand function in the long run reads $r^k = \frac{1}{(1-\tau^k)}\left(\frac{1}{\beta} - 1 + \delta\right)$, which implies a perfectly elastic demand of capital since the price does not depend on the quantity.

[4] In the standard neoclassical model, the consumption-leisure margin reads $\frac{-u_{n_t}}{u_{c_t}} = \frac{\left(1-\tau_t^n\right)}{\left(1+\tau_t^c\right)} w_t$.

[5] We define the tax wedge as the deviation from the Pareto optimum as a consequence of taxation, i.e. $1 - \frac{\left(1-\tau_t^n\right)}{\left(1+\tau_t^c\right)}$.

Spanish Treasury issued a report looking into this issue.[6] They argued that an increase in consumption tax of 1 pp, coupled with a reduction of both 0.7 pp in labor tax and 0.3 pp in capital tax, will produce a grow of GDP of 0.3 pp, and in employment of 0.2 pp in a three-year period.

This conclusion is also frequent in the literature of optimal fiscal policy. Coleman (2000) show that replacing income taxes with consumption taxes would lead to a large welfare gain in the United States. Correia (2010) finds for the US that eliminating the current progressive income tax, and establishing a flat tax on consumption would reduce inequality and improve the welfare of the currently poor. Finally, Laczó and Rossi (2014) extend the study of the optimal tax mix to the case in which governments cannot credibly commit to fiscal policies, their result is that the welfare gains from taxing consumption are much larger under discretion than under commitment, and the result of taxing only consumption still hold in their framework.

All these analyses rest on the assumption that agents will pay these tax increases religiously, which is very unrealistic. Therefore, the contribution of this paper is twofold. First, it lays out a theoretical model where both direct and indirect tax evasions coexist as a result of optimal decision of agents. Up to our knowledge, there is no study that combines both. Such a model is able to shed light on the steady state effects that changes in taxation will have on the amount evaded by the agents, both on consumption and income, allowing us to see how governments are limited when setting fiscal policies in the presence of tax evasion.

We will show that tax evasion matters a lot for Laffer curves. As we shall see in Section 2.6, tax evasion leads to flatter Laffer curves of consumption and labor taxes than in the model without evasion, i.e. to higher elasticities of declared income and declared consumption with respect to tax rates. Thus, pointing out an extra difficulty to raise government revenues due to tax evasion. In fact, under tax evasion, the Laffer curve of consumption tax peaks at a rate between 0 and 1 in contrast to the standard result.

These new Laffer curves should lead to better policy design for countries in which tax evasion is widespread. This is very important since current analyses of fiscal policy in DGSE models ignore tax evasion, see, for example, Trabandt and Uhlig (2011, 2012). Therefore, extreme and well-known tax results in the literature of optimal fiscal policy will

[6] *Comisión de Expertos para la Reforma del Sistema Tributario Español.* Full text: `http://www.fettaf.com/system/application/documentos/documentosvarios/InformeExpertosReformaSistemaTributarioEspanol.pdf`

disappear; namely, Coleman's result of taxing only consumption, and not taxing income (from capital and labor) at all, will not be feasible.

Moreover, this paper is used as a foundation when designing an optimal fiscal system in the presence of direct and indirect tax evasions. In Forcades and Pino (2015), we study what would be the optimal tax mix chosen by the benevolent Ramsey planner in a model with evasion of consumption, labor, and capital taxes.

The second contribution is that, when we extend the model to the business cycle, we are able to replicate the cyclical properties of tax evasion with very little structure. Our model produces a countercyclical evolution of tax evasion, an empirical regularity that has been pointed out by the literature. In addition, adding labor income and consumption tax evasions improves the empirical performance of the model along the business cycle.

Now, we explain the main empirical evidence and how our model performs. Tax evasion is a widespread phenomenon in all the developed world.[7] Estimates from Schneider et al. (2000, 2005, 2010, 2012) show that the extent of the shadow economy in Europe range from 10 to almost 30% of the GDP. This shadow economy mainly implies two kinds of tax evasion.[8] First, we have undeclared work, which are all the labor activities that do not pay labor income tax, and second, there is underreporting, which is the practice normally done by small business by which they only declare a part of the sales, thus reducing their profit figures. In the first case, we are talking about income tax evasion. The second case involves profit and consumption tax evasions.

There are plenty of empirical studies looking at the issue of tax evasion, we report here some of this evidence. An interesting macroeconomic fact has been frequently found is that, all else equal, the higher the tax rates, the more sizeable the tax evasion becomes. Table 2.1 shows regression results of a couple of studies, the first column taking VAT gap as dependent variable,[9] and the second column using the whole shadow economy (in percentage of GDP), taken from Schneider (2012). We see in both cases that higher tax rates reduce compliance. Our model produces the inverse relationship between tax evasion

[7] Our focus is restricted to developed economies. Developing countries face the problem of lacking a well functioning system to screen and collect money from tax payers. For such issues of *state capacity*, see Besley and Persson (2009).

[8] See Schneider (2013).

[9] The *VAT gap* is a measure of VAT non-compliance obtained from national accounts. It measures the difference between the theoretical revenues the government expects (from the tax base) and the actual revenue raised, and the fraction of disparity between the two is called VAT gap. Hence, the VAT gap is expressed as a percentage of theoretical liability (potential VAT collections). See the report for the Taxation and Customs Union Directorate, European Commission (2013).

and tax rates observed in the data.

| | Dependent variables | |
|---|---|---|
| | VAT non-compliance | Shadow economy |
| Unemployment rate | 0.90*** (0.30) | 0.63*** (5.96) |
| Indirect tax rate | 0.74* (0.44) | 0.31*** (3.85) |
| Income tax | - | 0.37*** (4.30) |
| Sample: | EU-27 (2000-2011) | 39 OECD (1998-2010) |

Table 2.1: OLS regressions of consumption tax evasion and shadow economy

Another issue is the evolution of the shadow economy along the business cycle. The empirical literature has pointed out that it is countercyclical, i.e. in bad economic times there is more tax evasion. See, for example, Tanzi (1983), Mare (1996), Bovi (1999), and Giles (1999), among others. In Table 2.1, we report positive and highly significant coefficients from regressing the VAT gap and the size of shadow economy against unemployment rate.[10] In addition, Figure 2.1 shows the Hodrick-Prescott-filtered cyclical components of GDP (solid line) and shadow economy (dashed) for some European countries.[11] It is visually clear that shadow economy grows during the troughs of the business cycle, and decreases in its peaks. One possible cause is that agents, in an attempt to smooth consumption, and given their lower income during crises, have to incur in some illegal activities to achieve it. This can be done by buying some goods without consumption tax, or working in an underground sector, because during crises legal firms reduce their activity. Our main novelty comes in the evasion of consumption tax. We will explain the mechanisms in detail in Section 2.3, but to give a tease of how the model works, we model this good without consumption tax in a way that it becomes an inferior good, i.e. ceteris paribus, the lower the income, the higher the consumption. Up to our knowledge, we have not seen inferior goods on a dynamic macroeconomic model. The way we achieved it was to assume that, in order to acquire the good without taxes, agents will have to spend time shopping for them. This is not an unearthly assumption, in reality not all the firms are willing to get

[10] The VAT gap report (2013) and Schneider (2012) argue that unemployment rate is a better proxy for the business cycle than output gap or GDP growth rate in the analysis of tax evasion or shadow economy.

[11] The data on shadow economy is also taken from Schneider (2012). The size of shadow economy is expressed as a percentage of GDP.

into trouble for selling goods under the counter, and agents seeking these illegal substitutes will be likely to spend time shopping them around.

Nevertheless, we also contribute to the other part of shadow economy, i.e. labor income tax evasion, because, in constrast to Busato and Chiarini (2004) and Orsi et al. (2014), we are able to replicate the countercyclicality of undeclared labor without imposing an idiosyncratic productivity shock for the underground production different from the one experienced by official economy. We will show, in Section 2.5, that it is enough to have an underground sector with a capital intensity lower than that in the official economy. Related to this, it is a stylized fact that labor intensive sectors (such as construction, agriculture, or household services) have the highest share of undeclared labor.[12]

The layout of the paper is the following. The next section offers a short review of related literature. In Section 2.3, we lay out the foundation for the model of tax evasion. We explain the calibration strategy followed in Section 2.4. Specifically, we calibrate our model using Spanish data because its estimates of tax evasion and underground economy are above the European average. We take our model to the business cycle in Section 2.5. In Section 2.6, we present some comparative static results showing how tax evasion limits the ability of the governments to raise taxes, problem specially acute for the case of consumption tax. Finally, Section 2.7 concludes.

## 2.2 Literature review

There is a vast literature talking about tax evasion, approaching the problem from a variety of angles. Earlier studies tackled the issue from a microeconomic perspective, dealing with models in which agents/firms optimally choose whether to evade/not to reveal their types in order to pay less to the government. The literature started with the seminal paper of Alligham and Sandmo (1972), that provided a simple yet appealing explanation on tax evasion borrowing from models of economics of crime and optimal portfolio theory. There have been multiple extensions to their framework,[13] but grosso modo, tax evasion is modelled as a risky asset, and at the end risk aversion is what drives tax evasion, a rather limited result to explain the wide differences in tax evasion across countries. Cremer and Pestieu (2001) study the optimal tax mix between commodity and income taxation in the presence of moral hazard. When individual characteristics such as income or labor

[12]See, for example, the estimates in the report "The Shadow Economy in Europe, 2013" by Schneider, F., and A.T. Kearney.

[13]For a historical review of the literature, see Alm (2012).

endowment are unobservable to the government, and setting different taxes on different commodities plays a role in encouraging agents to truthfully reveal their type.

Looking more specifically into the issue of consumption tax evasion, there is not that much done. McLaren (1998), in the spirit of "black markets" where the goods are smuggled into a country, uses a model of sales tax evasion where government choose the effort to curtail these activities. He shows that assuming a feasible plan of taxes that induce black markets, there exists always another plan that does not induce black markets and yields as much utility as the black market equilibrium, in the spirit that lowering taxes on tobacco can do away with cigarette smuggling.

There are also papers studying the profit tax evasion behavior on firms. Bayer and Cowell (2009) look at the strategic interdependence of firms in a same industry when reporting profits. For instance, if one firm declares lower profits than their competitors in the industry, it can expect to be audited with a higher likelihood. These interdependencies shape how a government should design optimal audit rules. There are even experimental approaches to the tax mix problem under the problem of tax evasion, which are part of a new economic discipline known as behavioral public finance. Watrin and Ullmann (2008) show that even when tax rates and enforcement efforts are the same in both income and sales tax, there is lower compliance when the tax payment decision is framed as a consumption tax, rather than an income tax.

There is a growing literature dealing with tax evasion from a macroeconomic perspective. Busato and Chiarini (2004) and Busato et al. (2012, 2013) study the general equilibrium implications of income tax evasion. They show that tax evasion changes the Laffer curve of labor income tax and is welfare improving for agents so long as it helps them to smooth the effect of distortionary fiscal policies. Also, tax evasion in their article helps to improve the fit of the RBC model to macroeconomic variables such as the hours worked. Our paper uses the same approach but we also contemplate the possibility of consumption tax evasion. In a model in which evaded consumption goods are bought with cash, Nicolini (1998) studies what would be the optimal monetary policy in this framework. He claims that even though there are theoretical grounds to set a positive inflation rate to "tax" the informal sector, the welfare gain of it would be very small. We, like him, assume that there is a demand of evaded consumption goods. Caballé and Panadés (2004) look into the opposite causality, that is, analyze the impact of inflation on tax evasion. Their novelty is the dynamic setting, in which that tax inspections occur depending on the previous decisions to evade. This produce a non-degenerate distribution of assets even though agents have identical income. The delay of inspection will increase tax evasion in economies with high inflation. Here, on the other hand, we do not use a state variable reflecting whether the agent has been inspected or not since we have a representative agent. Orsi et al. (2014) use Busato and Chiarini (2004) model and take it to a more quantitative level, adding a rich set of shocks in order to structurally estimate the Italian economy and be able to find the size of the underground economy. Their conclusion is that, from mid eighties to 2000s tax evasion has not stop increasing, coinciding with a period of continuous increases in income tax in Italy. They do not account for consumption tax evasion either.

## 2.3 The model

Here, we develop a macroeconomic model with both labor income and consumption tax evasions. Our work is close to Ihrig and Moe (2004) and Busato and Chiarini (2004) in terms of the modelling of labor income tax evasion. From the consumption tax evasion, our model is close to the above mentioned work of Nicolini (1998) on the side of the demand for the evaded goods. On the supply side, the approach to model the incentives of the firms is novel.

Let us look at the incentives to undertake these two kinds of illegal activities. On the income side, the reasons to evade are evident. Not paying the corresponding labor income tax could be understood as a lottery for the agent. If she is not inspected by the government, the agent is rewarded with her untaxed income, and in the case of not being that lucky, the agent will have to pay the amount due plus an administrative sanction. Hence agents evade income until the marginal benefit of evading equals the marginal cost, which is the tax plus penalty.

For consumption tax evasion, we will opt for a simple representation in which firms are selling a market good for which the agents will have to pay taxes, and another undeclared good without them.[14] In this scenario, the evasion of consumption taxes arises from a double incentive, both for consumers and firms. From the consumer perspective, if the good that pays taxes and the good that does not are reasonably good substitutes, the

[14] In this paper we do not want to explain *why* does tax evasion exist, but rather *how* does it change. Hence, we assume that agents derive utility from consuming evaded goods. This will create a demand for undeclared goods, which in turn will encourage firms to supply these goods. A study attempting to explain why evasion exist would have to take into account factors such as the driving factors of *tax morale* of the individuals, and the *capacity* that the state has to indeed enforce the established taxes. Despite being an extremely interesting topic, these issues are out of the scope of this study.

agent will have an incentive to spend at least part of their their income on these cheaper evaded goods.[15] Agents have nothing to lose in this business, because governments normally inspect firms (and not consumers) to check that they are collecting the consumption tax from agents, and if they do not, they are prosecuted and fined.[16]

On the firms side, incentives are more complex. Business have to comply with a profit income tax. One way to avoid this disbursement to the state is simply hiding part of their sales, which will purportedly reduce their revenues and profits, thus diminishing the tax base. Agents are aware that they are buying these goods under the counter, so they will not pay the consumption tax that firms are complied to collect, which makes a double fraud, both in profit and consumption taxes. The existence of a demand for these undeclared goods justify its supply. So, if firms do not participate in selling these, it means losing revenues for them.

### 2.3.1 Agents

There is continuum of agents that live indefinitely, which chooses how much to consume, work, and invest. The difference with respect to standard models is that they are able to do these choices evading taxes. On their labor income side, they can work in an underground sector in which they do not pay labor income tax, and consume a good that does not pay consumption tax. Obviously, there is a risk of incurring in these illegal activities. If the worker is caught not declaring its income, she will have to pay a the corresponding tax $\tau^n$ on these activities, plus a administrative surcharge $s^n$ over the tax rate. For example, if income tax is 10%, and $s^n$ is 1.5, the agent will have to pay an income tax of 15% over her undeclared income. Agents are inspected with probability $p^n$. Let us denote $w^i$ and $n^i$, $i \in \{M, U\}$, the wages and hours worked in the market ($i = M$) and underground ($i = U$) labor market. Therefore their expected income from undeclared work is given by $p^n(1 - s^n\tau^n)w^n n^U + (1 - p^n)w^U n^U$. Moreover, as we will see explicitly in Section 2.4, there is also an extra disutility of working in the underground sector, apart from the leisure that the agent is giving up. The economic reason is that this underground labor market does not provide the same protections than its market counterpart, like not having unemployment

[15] Obviously, if the goods are perfect substitutes, agents will spend all of their income in the untaxed goods.

[16] It is important here to bear in mind how consumption tax operates in real world. Abstracting from intermediate production, consumption tax is a tax that is collected by the firms and transferred to the government. That is why in fiscal policy it is said that, for example, VAT is *neutral* to the firm in terms of costs. However, if we contemplate the possibility of evasion, it will not be, as we will see.

benefits, health insurance, or a publicly provided pension. On the other hand, this extra cost in terms of utility might also be interpreted as individual and social motives.[17] In the end, this extra disutility of undeclared labor is useful to match reasonable levels of the underground labor market.

On the consumption tax side, agents can buy goods with tax, $c^V$, and without tax, $c^{NV}$ (e.g. non-VAT), being $p^{NV}$ the relative price of the undeclared good with respect to its market counterpart. Unlike the labor tax evasion, the government does not inspect whether the agents buy or not the good with tax.[18] Instead, inspection will occur on the firm side, which makes sense since firms are the "collectors" of consumption tax. The difference between $c^V$ and $c^{NV}$ is that agents have to spend time "shopping" for the latter. The rate at which they transform time into $c^{NV}$ is given by

$$c^{NV} = f(n^S) = A^S n^S,$$

where $n^S$ is the amount of time spent shopping, and $A^S$ is the linear technology or rate of transformation between time and goods. As we have said in the introduction, it is not an extremely unrealistic assumption. In real life one can expect that not all firms will be willing to get into trouble selling without consumption tax, either because their moral standards, or just the deterrence that the penalties produce. We intend to capture the time tax offenders would spend going around stalls, trying to find a crooked seller willing to offer $c^{NV}$. As we will see, this shopping time makes the good without consumption tax an inferior good, and thus its consumption will be countercyclical. This parameter will be used to match the observed size of consumption tax evasion.

What agents really care about in their utility is an aggregate consumption, which is like a basket of the two goods. To combine them we use a CES aggregator in the following way

$$c_t = \left[(c_t^V)^\rho + \phi(c_t^{NV})^\rho\right]^{\frac{1}{\rho}},$$

where $\phi$ determines the weight of the consumption without tax in the aggregator. Its value will be below one, since we assume that agents like $c^{NV}$ less than $c^V$. Parameter $\rho$

[17] For example, as tax morality and reputation costs, see Gordon (1989), or as social norms, see Besley et al. (2015), among others.

[18] This is what happens in real life. The government has no means to know whether a good is sold with or without VAT, and if it had, it would be very complicated.

determines the degree of substitutability between the two types of consumptions. When $\rho = 1$ we are in the case of perfect substitutes, and when it approaches negative infinity the goods became perfect complements (Leontief utility). It is worth doing some remarks about this. There is no economic reason to assume that there should be some degree of complementarity between the goods, people do not usually like to consume goods that do not pay taxes, they do it because they have to. We do it just as a mathematical simplification that allow us to have interior solutions for the consumption of both goods. Moreover, as we will see in the calibration section, this complementarity is very small.

We assume that all agents pool their income together, so we do no not have to keep a state variable reflecting whether the agent has been inspected in the current period. This is equivalent to saying that there exists a representative agent that gets an average income weighted by the probabilities that labor income tax is inspected. This agent maximize their discounted lifetime utility, that depends on consumption and leisure, choosing demand for evaded and non-evaded consumption, and the supply of hours in the market and underground sector, that is

$$\max_{\{c_t^V, c_t^{NV}, n_t^M, n_t^U, k_{t+1}\}_{t=0}^{\infty}} \sum_{t=0}^{\infty} \beta^t U(c_t, l_t)$$

subject to the budget constraint

$$\begin{aligned}(1+\tau_t^c)c_t^V + p_t^{NV} c_t^{NV} + i_t = & \; (1-\tau_t^n)\, w_t^M n_t^M + (1-\tau_t^k) r_t k_t + \\ & (1-p^n)\, w_t^U n_t^U + p^n (1 - s^n \tau_t^n)\, w_t^U n_t^U + \pi_t \end{aligned}, \tag{2.1}$$

the law of motion for capital,

$$i_t = k_{t+1} - (1-\delta)k_t, \tag{2.2}$$

the time constraint

$$l_t + n_t^M + n_t^U + n_t^S \leq 1, \tag{2.3}$$

the appropriate non-negativity constraints, $c_t, k_{t+1} \geq 0$, initial condition for capital $k_0$ and a shopping time condition for goods purchased without consumption tax

$$c_t^{NV} = A^S n_t^S. \tag{2.4}$$

As we can see from (2.1), the investment good is expressed in the price of the good with consumption tax. The households' first-order conditions can be summarized in the following system of equations, given by the the marginal rate of substitution (MRS) between between legal and illegal consumption

$$\frac{U_{c_t^V}(\cdot)}{U_{C_t^{NV}}(\cdot)} = \frac{(1+\tau_t^c)}{p_t^{NV}}, \tag{2.5}$$

the MRS between legal and illegal work

$$\frac{U_{n_t^M}(\cdot)}{U_{n_t^U}(\cdot)} = \frac{(1-\tau_t^n)w_t^M}{(1-p^n s^n \tau_t^n)w_t^U}, \tag{2.6}$$

the MRS between consumption and leisure

$$\frac{-U_{n_t^M}(\cdot)}{U_{c_t^V}(\cdot)} = \frac{(1-\tau_t^n)w_t^M}{(1+\tau_t^c)}, \tag{2.7}$$

and the Euler equation

$$U_{c_t^V}(\cdot) = \beta\big(1+(1-\tau_{t+1}^k)r_{t+1}-\delta\big)U_{c_{t+1}^V}(\cdot), \tag{2.8}$$

where $U_i(\cdot)$ is the marginal utility with respect to variable $i$, $\frac{\partial U(\cdot)}{\partial i}$.

### 2.3.2 Firms

Firms are competitive, risk neutral, and maximize expected profits. We assume that firms operate two production functions, one that use declared work (for which the agents will have to pay income tax), and another production function that uses undeclared work. The novel part is that, from this total production, they decide what to sell with consumption tax and what without it. Let us explain in detail the rationale behind this.

Suppose that the firms have the legal activities in the front door, and a sweatshop running in the back door, so to speak. Whatever that comes out of these two production functions is treated as total production, and it is not distinguishable one from another. Once the production is done, in the counter of the shop, the sales assistant surreptitiously asks the client whether she wants to buy the good without tax and save some money. If the agent say yes, then the sales assistant wraps the good in a paper that says $c^{NV}$, and the production becomes the good without consumption tax. The agent pays $p^{NV}$ for it. If the agent's morale standards are higher, the assistant says sorry and wrap the good in a $c^V$ paper, and charges $(1+\tau^c)$ for the good. Even though both goods are produced exactly in same way, they are not the same good. The intuition behind is that if the consumer has some problems with the good and want to ask for a refund, the shop is not going to offer it if the good was $c^{NV}$, because it is as if this transaction never occurred. Similarly for customer service, post-sale service, etc. This leads us to model $c^V$ and $c^{NV}$ as different goods. The proportion of sales that the firm chooses to offer without tax is denoted with $\theta_t \in [0, 1]$. All the sales done without consumption tax will not enter into the accounting of the firm, and will not be taken into account when the firm has to pay the profit tax. So consumption tax evasion for the firm plays two roles. First, it allows to satisfy the demand agents have for $c^{NV}$ and second, saves them some money in the profit tax, obviously under case that they have not been caught.

The government is obviously aware that some firms evade taxes, and inspects them with probability $p^c$. Recall that firms are responsible for collecting taxes, and if they operate according to the law, this tax will be neutral for them. However, if they happen to sell $c^{NV}$, they will be in trouble when caught by the government, that will demand all the corresponding consumption tax of $c^{NV}$, which the firm indeed does not have it because it was not collected from the consumers, and will have to disburse it from its own profits. Similarly to the case of the agents, the government will also demand an administrative sanction $s^c$ on top of the standard tax rate. In terms of the model, this amount is $(1 - s^c\tau_t^c)\theta_t p_t^{NV}(Y_t^M + Y_t^U)$. We will consider that in case of inspection of the firm, the government does not care that they are operating with illegal work $n^u$, since this activity is already prosecuted on the agent's side.[19] On top of this, if the firm is inspected, all the revenues generated by sales without consumption tax will be taken into account when paying the profit tax. This effect is reflected in $\pi^i$ and $\pi^{ni}$, which we will see below in equations (2.9) and (2.10), denoting profits when inspected and not inspected, respectively. Being precise, the problem that firms solve is given by

$$\max_{\theta_t, n_t^M, n_t^U, k_t} \left\{ p^c \, \pi_t^i + (1 - p^c) \, \pi_t^{ni} \right\},$$

subject to,

$$\begin{aligned} Y_t^M &= f(k_t, n_t^M, z_t) = e^{z_t} \, (k_t)^\alpha (n_t^M)^{1-\alpha} \\ Y_t^U &= f(k_t, n_t^U, z_t) = e^{z_t} \, (n_t^U)^\sigma \\ Y_t &= Y_t^M + Y_t^U \\ \theta_t &\in [0, 1], \end{aligned}$$

[19]This is obviously a simplification.

where

$$\pi^i = (1-\tau^\pi)\Big[(1-\theta_t)(Y_t^M + Y_t^U) + (1 - s^c\tau_t^c)\theta_t p_t^{NV}(Y_t^M + Y_t^U) - w_t^M n_t^M - w_t^U n^U - r_t k_t\Big] \tag{2.9}$$

$$\pi^{ni} = (1-\tau^\pi)\Big[(1-\theta_t)(Y_t^M + Y_t^U) - w_t^M n_t^M - w_t^U n_t^U - r_t k_t\Big] + p_t^{NV}\theta_t(Y_t^M + Y_t^U), \tag{2.10}$$

and the productivity shock evolves according to the $AR(1)$ process

$$z_t = \rho_\varepsilon z_{t-1} + \varepsilon_t$$

where $\rho_\varepsilon \in (0,1)$ and $\varepsilon_t$ is independent and normally distributed with mean zero and standard deviation $\sigma_\varepsilon$.

We model the production using undeclared work as labor intensive. In addition, capital used in the underground sector is mainly fixed, such as the building of the firm and so on. This simplification has been used in the literature of informal/underground economy, for example, see Ihrig and Moe (2004) and Busato and Chiarini (2012). Orsi et al. (2014) structurally estimate the share of capital in the underground, and find it to be smaller than its official counterpart. It is intuitive that installing or changing the productive capacity for the illegal sector may attract the attention of the state, leading to the firm eventually being caught.

Government collects revenues from four taxes: consumption taxes $\tau^c$, labor income taxes $\tau^n$, capital taxes $\tau^k$, and the profit tax for the firms $\tau^\pi$. Moreover, it also raises revenues from the penalties that it imposes on tax evaders, both agents and firms. This revenue is used to finance a useless expenditure $g_t$:

$$\begin{aligned} g_t = & \underbrace{\tau_t^c\big(c_t^V + p_t^{NV} p^c s^c \theta_t Y_t\big)}_{\text{Consumption tax}} + \underbrace{\tau_t^n\big(w_t^M n_t^M \; + \; p^n s^n w_t^U n_t^U\big)}_{\text{Labor tax}} + \underbrace{\tau_t^k r_t k_t}_{\text{Capital tax}} \\ & \underbrace{+\tau^\pi\Big[\,\big((1-\theta_t) + p^c(1 - s^c\tau_t^c)\theta_t p_t^{NV}\big)\, Y_t - r_t k_t - w_t^M n_t^M - w_t^U n_t^U\Big]}_{\text{Profit tax}}, \end{aligned} \tag{2.11}$$

where in each tax, we have first, the legal revenues collected from the tax, and after the part caught in inspections. As we said before, when the firm is caught evading, it is also forced to take into account the illegal revenues made from selling without consumption tax at the calculation of their payment of profit tax. This is given by $p^c(1 - s^c\tau_t^c)\theta_t p_t^{NV} Y_t$, as it can be seen in equation (2.11).

The first-order conditions of optimality for firm gives us the following equations for the price of the good without tax

$$p_t^{NV} = \frac{(1-\tau_t^\pi)}{p^c(1-\tau^\pi)(1-s^c\tau_t^c)+(1-p^c)} \tag{2.12}$$

and price of the productive factors

$$r_t = \alpha e^{z_t} k_t^{\alpha-1} (n_t^M)^{1-\alpha} \tag{2.13}$$

$$w_t^M = (1-\alpha) e^{z_t} k_t^{\alpha} (n_t^M)^{-\alpha} \tag{2.14}$$

$$w_t^U = \sigma e^{z_t} (n_t^U)^{\sigma-1} \tag{2.15}$$

Let us make a quick comment about the equilibrium price $p^{NV}$. As it is seen from (2.12), its value is determined by parameters of the economy such as tax rates and the strictness of inspections, meaning that the price does not depend on quantities. This is the result of having a linear choice in the proportion of evaded goods. The firm is faced by an arbitrage condition, and sets a price that just compensate it for the risk taken in the activity. The inverse demand of the firm $p^{NV}(C^{NV})$ is flat, and all adjustments to achieve the equilibrium will occur in the demand side. We thought of different structures for the choice of tax evasion in the firm side that would not give rise to this phenomenon, but all of them implied more complicated modelizations, in which the extra complications implied outweighed the minor problem of having a fixed price.

### 2.3.3 Equilibrium

A Competitive Equilibrium in this economy are sequences of quantities $\{c_t^V, c_t^{NV}, n_t^M, n_t^U, n_t^S, k_{t+1}, \theta_t\}_{t=0}^{\infty}$ and prices $\{r_t, w_t^M, w_t^U, p_t^{NV}\}_{t=0}^{\infty}$ such that, given prices, agents decisions satisfy first order conditions (2.5) to (2.8), for firms, equations (2.12) to (2.15) hold, and markets clear, i.e.
market for the good with taxes

$$c_t^V + g_t + i_t = (1-\theta_t)\, Y_t, \tag{2.16}$$

market for the good without consumption tax

$$c_t^{NV} = \theta_t\, Y_t, \tag{2.17}$$

and labor market

$$l_t + n_t^M + n_t^U + n_t^S = 1. \tag{2.18}$$

## 2.4 Calibration

In this section, we describe the functional forms and parameters assigned to the model. As we said in the introduction, our objective is to make it resemble the Spanish economy. Ours is not a quantitative exercise, meaning that we do not want to precisely match targets of the economy, but rather qualitatively explain the evolution of variables with respect to the cycle. Our vector of parameters is $\left(\phi, \rho, A^S, \beta, \gamma, \alpha, \tau^c, \tau^n, \tau^k, p^c, p^n, s^c, s^n, \Phi_l, \Phi_u, \rho_\varepsilon, \sigma_\varepsilon\right)$. The first two of them are free parameters. Values for $\beta$, $\gamma$, $\alpha$, $s^c$ and $s^n$ will be taken from the literature. Parameters $\tau^c$, $\tau^n$, $\tau^k$, $p^c$, $p^n$, $A^S$, $\Phi_l$ and $\Phi_u$ will be calibrated to match the specific requirements of our model. We will start from the agent side. The representative consumer is assumed to have the following preferences:

$$U(c_t, l_t) = log(c_t) + \Phi_l \frac{(1 - n_t^M - n_t^U - n_t^S)^{1-\gamma}}{1-\gamma} - \Phi_u n_t^U,$$

where the aggregate consumption, $c_t$, is a basket that aggregates both taxed and untaxed goods, with an aggregator as explained in Subsection 2.3.1.

Benhabib et al. (1991) and Ragan (2005) have a similar aggregator to combine home and market produced goods. Their estimates for the substitutability parameters range from 0.8 and 0.9. Provided the explanation we have given above about the nature of how substitutable are the two goods between each other (they are nearly the same good), we set a slightly higher value, $\rho = 0.95$. We have experimented with different values of the parameter and this choice is without loss of generality. The weight on evadable consumption $\phi$ is taken from the model without shopping time and is set to 0.83.[20]

The parameter of the shopping time $A^S$, is set to 2.8182 in order to match a ratio of evadable consumption to total consumption equal to 23.25%, the average value of the period 2008-2011 estimated by the study of the Taxation and Customs Union Directorate for the EU-Comission (2013). It estimates the share of evaded consumption or VAT gap from National Accounts and Input-Output Tables.

On the labor side, the parameter $\gamma$ is set to 0.5, which implies a Frisch elasticity of labor

[20]See Forcades and Pino (2015).

supply around 3.5 for labor, which is a standard value in the macroeconomic literature; specifically, is the average value between King and Rebelo (1999) and Trabandt and Uhlig (2011).[21] We set the discount factor $\beta$ to the standard value 0.96, implying a long-run interest rate around 4%. We use a discount factor $\delta = 0.07$, taken from Díaz-Giménez and Díaz-Saavedra (2009). Capital share is set to $\alpha = 0.3734$, calculated from the Spanish National Accounts in 2009-2012, see Gollin (2002). The underground output elasticity of labor, $\sigma$, is set to to the same value as in the market production function, i.e. $\sigma = 1 - \alpha$. We have no available data in order to estimate such parameter, so we prefer to set it to a standard value. This parameter affects the profits that the firm earns from illegal activities. For this value of $\sigma$, the ratio of profits to GDP is a 4%, which is not extremely unreasonable. Effective tax rates are estimated following Mendoza et al. (1994),[22] except profit tax that is set to 0.25, which is the rate that most firms pay in Spain.

We choose $\Phi_l$ equal to 1.0091 to reach a value of $n^M$ in the model that matches the share of time allocated to work in the market in Spain, which is a 32.2%, following the Working Time Survey, published by the INE in (1996).[23] The extra disutility of the underground labor, $\Phi_u$, is set to 1.6727 to match a share of underground labor over the total labor equal to 21.9%, which is the average value, over 1997-1998, of this ratio for Spain, target that we obtain from Schneider (2000).

Finally, let us have a word about the parameters determining the penalties of tax evasion. Governments are very reluctant to disclose information about tax inspections and procedures, since such information may encourage tax offenders to outsmart the system. To set the probability of being caught evading income tax, we use data on the number of inspected tax filings. We set $p^N$ to 0.1, which is simply a proportion of inspected tax filings with respect to the number of registered firms.[24] This is the same procedure that Busato and Chiarini (2004) use. We set the same value for $p^c$, since we have been unable to find any data on VAT inspections. For the surcharges in tax evasions, a good representation of

[21]Therefore, we are using a value in line with macroeconomic estimates. However, we also present our results for a Frisch elasticity equal to 0.5, which is in line with microeconomic estimates (tend to be lower than unity). See, for example, Pencavel (1986) and Browning et al. (1999).

[22]These estimates are similar to those found by Trabandt and Uhlig (2012) for Spain in 2009.

[23]Instituto Nacional de Estadística.

[24]Source: *Ministerio de Trabajo. Dirección General de la Inspección de Trabajo y Seguridad Social*. The number of firms is taken from the INE. We use data on labor inspections on firms, even though we have maintained throughout the whole article that income tax evasion is done on the agent side. This is still true, but we have not find any data on the number of inspections on the agent side. Moreover, the data we use is about labor inspections, so it has to be the same as if we have data on the inspections on Income Tax filings.

the Spanish tax law is to set $s^i$, equal to 1.5.

The parameters for the $AR(1)$ process followed by productivity shock are set as in Prescott (1986). Thus, the persistence parameter, $\rho_\varepsilon$, is equal to 0.95 and the standard deviation of the innovation, $\sigma_\varepsilon$, is equal to 0.0072.

| Parameter | Description | Value |
|---|---|---|
| $\delta$ | Depreciation | 0.0782 |
| $\beta$ | Discount factor | 0.96 |
| $\rho$ | Affects the elasticity of subs. $c_t^V$ and $c_t^{NV}$ | 0.95 |
| $\phi$ | Weight on evaded consumption | 0.83 |
| $\Phi_l$ | Weight parameter on leisure | 1.0091 |
| $\Phi_u$ | Extra disutility of undeclared work | 1.6727 |
| $\gamma$ | Affects the elasticity of labor supply | 0.5 |
| $\alpha$ | Share of capital in $Y^M$ | 0.3734 |
| $\sigma$ | Share of labor in $Y^U$ | 0.6266 |
| $A^S$ | Shopping time | 2.8182 |
| $\rho_\varepsilon$ | Autocorrelation of shock | 0.95 |
| $\sigma_\varepsilon$ | Standard deviation of shock | 0.0072 |
| $p^c$ | Prob. of detection evading $\tau_t^c$ | 0.1 |
| $s^c$ | Surcharge when caught evading $\tau_t^c$ | 1.5 |
| $p^n$ | Prob. of detection evading $\tau_t^n$ | 0.1 |
| $s^n$ | Surcharge when caught evading $\tau_t^n$ | 1.5 |
| $\tau^c$ | Consumption tax | 14.80% |
| $\tau^n$ | Labor income tax | 37.54% |
| $\tau^k$ | Capital income tax | 24.95% |
| $\tau^\pi$ | Profit tax | 0.25% |

Table 2.2: Calibration parameters

## 2.5 Business Cycle

In this section, we study how our model performs in the business cycle. Table 2.3 reports second moments from the model. We HP-filter the log-series produced by the model, to make our results comparable with the literature. We take simulations that are $t = 1000$ periods long, and calculate standard deviations and correlations for each simulation. We do 1000 simulations, and report the average across simulations. Column 1 shows standard deviations (percent) of the variables. Column 2 reports the relative deviations with respect to total output. Column 3 shows the correlations of the variable in question with respect to total output. Finally, Column 4 displays the correlations with respect to productivity.[25] $n$ is the total hours of work (market and underground) and $c$ is aggregate consumption (evaded and non-evaded).

| Variable ($j$) | $\sigma_j$ (%) | $\sigma_j/\sigma_Y$ | $corr(j,Y)$ | $\sigma_j/\sigma_{Y/n}$ | $corr(j,\, Y/n)$ |
|---|---|---|---|---|---|
| $Y$ | 1.5747 | 1 | 1 | 1.8542 | 0.9238 |
| $n$ | 0.8540 | 0.5423 | 0.9249 | 1.0056 | 0.7089 |
| $i$ | 6.4865 | 4.1191 | 0.8659 | 7.6378 | 0.6159 |
| $c$ | 0.7747 | 0.4920 | 0.7596 | 0.9122 | 0.9474 |
| $Y/n$ | 0.8493 | 0.5393 | 0.9238 | 1 | 1 |

Table 2.3: Business cycle properties

Given the parameters of the shock process, which are the values used by Prescott (1986), King and Rebelo (1999), and Hansen and Wright (1992); our model produces a very similar volatility of output. In these papers, standard deviation of output ranges between 1.3-1.79, whereas in our model is equal to 1.5747.[26]

Our model generates higher volatility of consumption than standard models.[27] For instance, they find that consumption is 0.25-0.44 percent as variable as output, respectively, whereas our model produces a volatility of consumption 0.49 times output, respectively. The higher volatility of aggregate consumption is explained by the substitution effect between evaded and non-evaded consumption. Even though evaded consumption is countercyclical, both consumptions are so volatile, since they are highly substitutable, that aggregate consumption is more volatile than in the standard model. Specifically, the relative standard deviations $\sigma_j/\sigma_Y$ of evaded and non-evaded consumption are equal to 1.5984 and 1.0552, respectively. Therefore, our value is closer to actual data.

However, we find a correlation of aggregate consumption with respect to output equal to 0.76,[28] which is lower than those in the literature (around 0.95). This coefficient is lower than in the standard model because evaded consumption is countercyclical (see Table 2.4). Hence, our correlation of consumption with output is more accurated when compared with data.

We find a standard deviation of labor equal to 0.85. This value is larger than in King

[25] Productivity is computed dividing total output by total hours of labor.
[26] The standard deviation of official output$\sigma_{YM}$ is equal to 1.7425.
[27] Using the same intertemporal elasticity of substitution (log-utility function).
[28] The correlation of non-evaded consumption and output is equal to 0.8490.

and Rebelo (1999), where they find a value being equal to 0.67. In addition, they use a Frisch elasticity of labor supply equal to 4.

In line with $\sigma_n$, Hansen and Wright (1992) and King and Rebelo (1999) find volatilities relative to output around 0.49 and 0.48, respectively, while we find this volatility equal to 0.54.[29] Moreover, our relative standard deviation with respect to productivity is equal to 1.0056, whereas Hansen and Wright (1992) find a value equal to 0.94. Therefore, these two ratios are higher to previous papers and closer to data.

The higher volatilities of labor are explained by the substitution effect between declared and undeclared sectors, despite the fact that undeclared labor is countercyclical, which indeed would reduce this volatility. For example, the standard deviations of declared and undeclared labor are equal to 0.8924 and 1.4281, respectively, which are higher than the standard deviation of total labor. Hence, the presence of underground sector implies an additional reallocation effect to the intratemporal equilibrium condition between leisure and labor. We explain this mechanism below when we focus on the variables related to tax evasion.

As with labor and consumption, our model produces higher volatility of investment with respect to output; specifically, we find a relative standard deviation around 4.12, whereas this relative standard deviation in the literature is around 2.95-3.15,[30] which fits better to the data. The additional reallocation of resources between both consumptions and labors affects the intertemporal condition of equilibrium, and therefore makes investment more volatile. For instance, Busato and Chiarini (2004) also find a higher volatility of investment with respect to the standard model, specifically, it is equal to 6.64 in their case.

Correlations of labor hours with productivity and output are equal to 0.71 and 0.92,[31] respectively, which are lower to those found in the literature. For instance, Hansen and Wright (1992) report a value of 0.95 for the correlation between productivity and hours. In addition, King and Rebelo (1999) find that the correlation between hours of labor and output is equal to 0.97. Thus, our values are closer to actual data, where the first is around zero or even negative and the second is around 0.8.[32] Our ratios are lower than in the standard model because undeclared labor is countercyclical (see Table 2.4).

We find a correlation of investment with respect to output equal to 0.87, which is lower

[29]The relative standard deviations of declared and undeclared labor are equal to 0.5667 and 0.9069, respectively.

[30]We use the same intertemporal elasticity of consumption (log-utility) as in the papers mentioned.

[31]These correlations for declared labor are 0.8332 and 0.9543, respectively.

[32]See Hansen and Wright (1992) and King and Rebelo (1999).

than those in the literature (around 0.99), but more similar to data. For example, Hansen and Wright (1992) and King and Rebelo (1999) report values from data equal to 0.73 and 0.80, respectively.

The correlation of productivity and output is equal around 0.92 and it is similar to King and Rebelo (1999), who find a value of 0.98. All these values are quite higher than data (around 0.5 or below). In addition, the relative standard deviation of productivity with respect to output is around 0.54, which coincides with Hansen and Wright (1992) and King and Rebelo (1999). In this case, ours/their model matches data on this ratio.

To sum up this part of real business cycle statistics with respect to standard models, we find that consumption, investment, and labor are more volatile and their correlations with output are lower. In addition, the correlation of hours with productivity is also lower.

The variables related with undeclared activities are countercyclical. There are two reasons for this. First, on the consumption tax evasion, shopping for the undeclared good requires time, and when the economy is in the middle of an expansion, labor productivity, and hence wages are higher. This makes the opportunity cost of shopping for the illegal good (which is not being working in the market) very expensive, and instead of spending time looking for $c^{NV}$, agents work more and start buying the market $c^{V}$ good. This is a demand effect. On the supply side, firms see that the business of $c^{NV}$ is no longer profitable, so $\theta$ falls. Second, on the income tax evasion, a good productivity shock increases the demand of both labors but, since market sector uses capital more intensively, the demand of capital also increases. This extra demand of capital raise even more the demand of market labor (since these inputs are complements). Hence, market labor demand rises more than underground demand, thus pushing up market wages above underground wages. Finally, market labor supply increases due to this larger wages, while underground labor supply falls. To sum up, in front of an unexpected increase in $z_t$, the increase in wages, *ceteris paribus* labor supply $n_t^i$, is higher for market labor compared to underground labor due to capital intensity, which encourages agents to reallocate labor from underground to official sector. As a consequence, real interest rate increases more due to both large demand and supply of market labor relative to the case without underground sector, and therefore supply and demand of investment. And, the contrary when productivity falls unexpectedly. Therefore, this explains the higher volatilities of investment and labor pointed out before, as well as in part of consumption.[33]

[33]Remember that the volatility of consumption is also affected by the high volatilities of evaded and non-evaded consumption, as we explained before.

We show all these effects in Figure 2.2, where we plot scatterplots of the productivity shock $z_t$ vs the variable of interest across a point in time for all simulations. The solid black lines are linear regressions. In addition, Table 2.4 displays the correlations of the variable in question with respect to total output.[34]

| Variable ($j$) | $corr(j, Y)$ |
|---|---|
| $Y^U$ | 0.4372 |
| $Y^U/Y$ | -0.8802 |
| $n^U$ | -0.6378 |
| $n^U/n$ | -0.8655 |
| $c^{NV}$ | -0.9877 |
| $c^{NV}/c$ | -0.9813 |
| $\theta$ | -0.9916 |

Table 2.4: Correlations of tax evasion variables

Despite a rise in productivity shock increases both outputs, market output increases more than underground output because the demand and supply of capital and market labor are higher than the demand of underground labor and of course supply of this labor since, in fact, decreases. Specifically, $corr\left(Y^M, z_t\right) = 0.9582$ and $corr\left(Y^U, z_t\right) = 0.5539$. Therefore, as we can see in Table 2.4, the size of underground production $Y^U/Y$ is reduced over the cycle. In particular, its coefficient of correlation with respect to total output is equal to -0.88.

Underground labor and evaded consumption are negatively correlated with total output, both in levels and as proportions of total labor and consumption, respectively. Specifically, the size of undeclared work presents a negative coefficient of correlation with output, which is equal to -0.86. And, the size of evaded consumption is negatively correlated with total output, specifically, its coefficient of correlation is equal to -0.98.

In line with $c^{NV}$, the proportion of undeclared sales $\theta$ is also countercyclical. We find that its coefficient of correlation with total output is equal to -0.99.

We report in Table 2.5 the business cycle statistics for the lower Frisch elasticity (0.5). The main difference with respect to the case of a large elasticity of labor supply is related to the volatility of hours of work. Now, the standard deviation of hours is equal to 0.3387 and the relative standard deviation with respect to output is around 0.27.

[34]These variables are not transformed neither in logs nor detrended since we cannot compare these outcomes with actual time-series.

| Variable ($j$) | $\sigma_j$ (%) | $\sigma_j/\sigma_Y$ | $corr(j,Y)$ | $\sigma_j/\sigma_{Y/n}$ | $corr(j, Y/n)$ |
|---|---|---|---|---|---|
| $Y$ | 1.2665 | 1 | 1 | 1.2980 | 0.9877 |
| $n$ | 0.3387 | 0.2675 | 0.8933 | 0.3472 | 0.8122 |
| $i$ | 5.5754 | 4.4022 | 0.7896 | 5.7141 | 0.6876 |
| $c$ | 0.7936 | 0.6266 | 0.6945 | 0.8133 | 0.7968 |
| $Y/n$ | 0.9757 | 0.7704 | 0.9877 | 1 | 1 |

Table 2.5: Business cycle properties for the alternative Frisch elasticity (0.5)

Moreover, Table 2.6 displays correlations of tax evasion and shadow economy with respect to output for the lower Frisch elasticity of labor supply. We observe that these values are very similar to those in Table 2.4.

| Variable ($j$) | $corr(j,Y)$ |
|---|---|
| $Y^U$ | 0.5358 |
| $Y^U/Y$ | -0.8372 |
| $n^U$ | -0.6470 |
| $n^U/n$ | -0.7815 |
| $c^{NV}$ | -0.9991 |
| $c^{NV}/c$ | -0.9952 |
| $\theta$ | -0.9977 |

Table 2.6: Correlations of tax evasion variables for the alternative Frisch elasticity (0.5)

## 2.6 Laffer Curves

In this section, we move on to see some comparative statics in terms of taxes, and analyze what happens with revenues in both consumption and labor taxation. We compare these results with the model without evasion, in which we take a standard model with one consumption good that always pay taxes, and only one production function that employs only legal work, whose returns pay diligently its income tax.[35] All results reported in this section are taken from the steady state of the model. In Figures 2.3a and 2.3b we plot total revenues when changing labor income and consumption taxes, respectively. Both graphs compare the case with and without evasion. The remaining taxes that are not varying are set to its estimated values. Tax rates are indicated in the x-axis, and revenues in the y-axis. Blue plots for the case of tax evasion, and green when there is not.

[35] In this case, since profits are zero, $\tau^\pi$ becomes irrelevant.

As we can see, for the case of no tax evasion, there is no slippery slope in the Laffer curve of consumption tax, i.e. the government can increase taxation to whatever level it wants, and revenues will continue increasing. On the other hand, Laffer curve of labor income tax equals to zero for $\tau_t^n = 1$. These two forms of the Laffer curve are very standard in the literature.[36]

However, we show that under tax evasion both Laffer curves peak at a rate between 0 and 1. Therefore, evasion limits tax policies the government can set in order to raise revenues. For income tax, the peak of the Laffer curve occurs before, specifically, roughly above a tax rate equal to 30% revenues fall. What is more surprising is the case of consumption tax, where taxes above 10% decrease revenues because agents give up $c^V$ for its illegal counterpart $c^{NV}$. These two graphs are our word of warning for policy reforms calling for a total substitution of income taxation in favor of indirect taxation.

Now, we focus on the labor income tax revenues and consumption tax (and profit tax) revenues. We observe that labor income tax revenues start falling around a 40% tax rate when tax evasion is considered, see Figure 2.3c. In the case of consumption tax, Figure 2.3d displays the consumption tax revenues, the red line is the addition of both consumption and profit tax revenues. We can see that profit tax losses are an important element when we analyze the effect of consumption tax evasion on government revenues. Specifically, consumption tax revenues decrease beyond a tax rate of 22%, but it does around 12% when profit tax revenues are taken into account.

We present in Figure 2.4 Laffer curves for a alternative Frisch elasticity equal to 0.5.[37] In this case, Laffer curves of labor income tax shift out their peaks around 5-10 percentage points of tax rates. For example, in Figures 2.3a and 2.4a, we observe that, in the case of non-evasion, Laffer curve peak at around 0.75 and, for lower elasticity of labor supply, it does around 0.85. Hence, as expected, government is able to set higher taxes in order to raise revenues due to the lower elasticity of labor supply.

## 2.7 Conclusion

In this article, we extend previous macroeconomic frameworks on shadow economy with consumption tax evasion. The model presented here is the first one to jointly tackle both labor income and consumption tax evasions.

[36] See, for instance, Trabandt and Uhlig (2011).

[37] We need to adjust some parameters so as to work with an economy that resembles the previous one. Therefore, we set $\Phi_l$ equal to 0.1390, $\phi$ to 0.78, $\Phi_u$ to 1.7091, and $A^S$ to 3.8636.

Two main results arise when we explore the cyclical properties of our framework by introducing an aggregate productivity shock that affects equally both the declared and undeclared sectors. First, we find that consumption, investment, and labor are more volatile and their correlations with output are lower compared to the standard model. Therefore, including labor income and consumption tax evasions improves the ability of the standard RBC models to match key statistics of data. Second, we are able to produce a countercyclical evolution of the shadow economy, an empirical regularity that has been pointed out by the literature.

Given the business cycle properties of our model, we push this study further to explore what are the fiscal policy implications of considering consumption tax evasion as well as labor income tax evasion. We show that the consequences for taxation are significant. In particular, we produce Laffer curves in which revenues fall quicker than in the standard model without tax evasion when tax rates are pushed beyond reasonable levels, specially for the case of consumption tax in which the slippery slope of the Laffer curve starts roughly at a mere 10%, rate way below the 20% that most advanced economies have.

These new Laffer curves should lead to better policy design for countries in which tax evasion is widespread. This is very important since current analyses of fiscal policy in DGSE models ignore tax evasion, see, for example, Trabandt and Uhlig (2011, 2012).

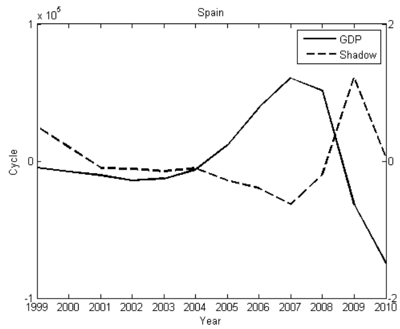
Consequently, our paper casts doubt on the possibility of implementing tax reforms suggested by international institutions and scholars, implying significant reductions of income tax, or its elimination altogether, in favor of higher rates over consumption. Therefore, extreme and well-known tax results in the literature of optimal fiscal policy will disappear; namely, Coleman's result of taxing only consumption, and not taxing income (from capital and labor) at all, will not be feasible.[38]

According to the results, we argue that tax evasion should be taken into account in the analysis of fiscal policy. Moreover, not only income tax evasion should be added to the standard model, as in Busato and Chiarini (2013), but also consumption tax evasion.

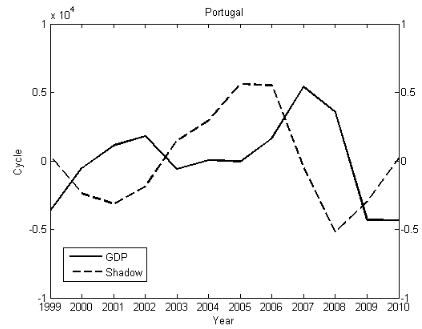
Finally, the structure developed in this paper is the building block for Forcades and Pino (2015), where we take this framework and attempt the normative question of studying what would be the optimal tax mix (between consumption, labor, and capital) given the fiscal limitations that tax evasion imposes.

[38] See Coleman (2000).

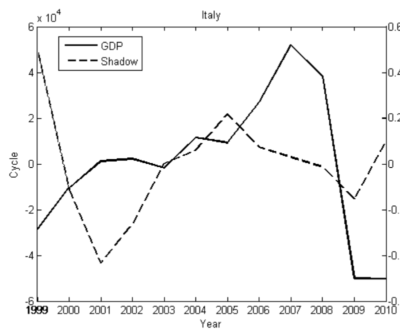
# Figures



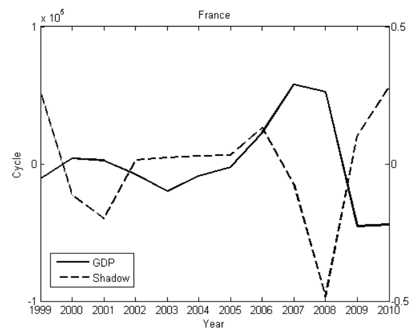

(a) Spain



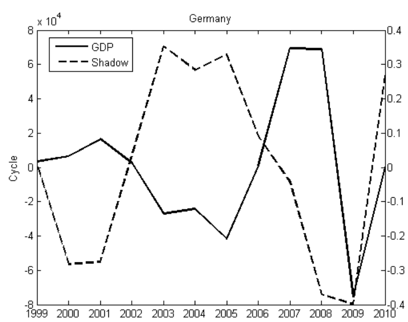

(b) Portugal



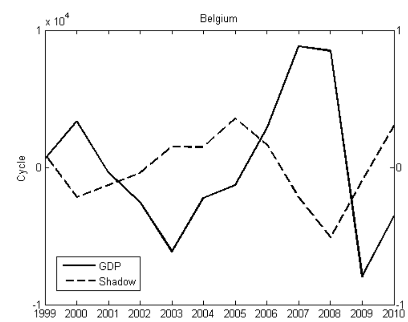

(c) Italy




(d) France

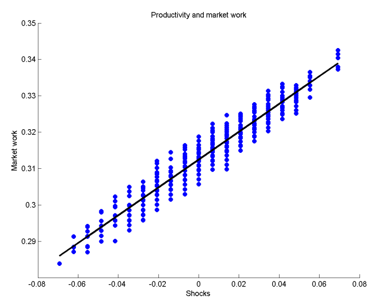



(e) Germany

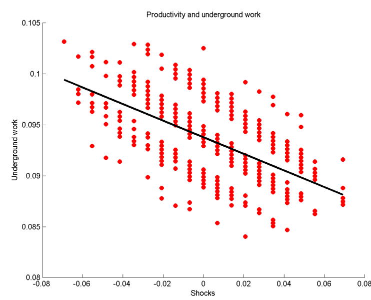



(f) Belgium

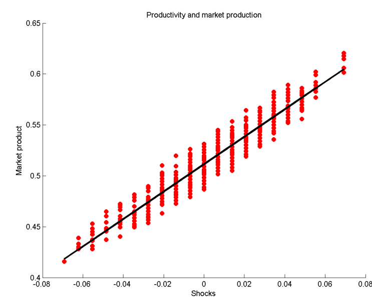
Figure 2.1: Cyclical components: GDP and the shadow economy. Source: own calculations using data from Schneider (2012).



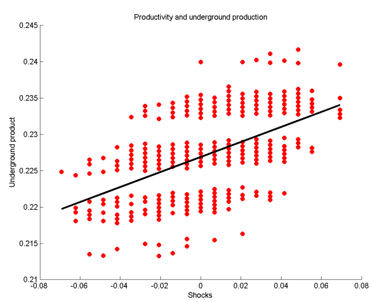

(a) Shock and $n^M$



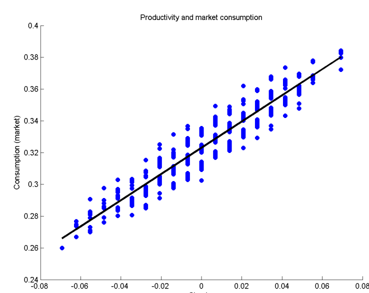

(b) Shock and $n^U$



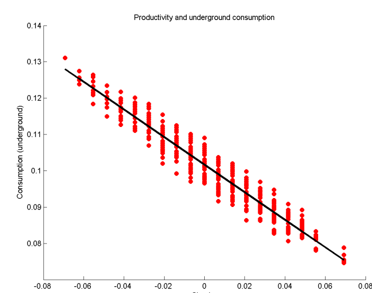

(c) Shock and $Y^M$



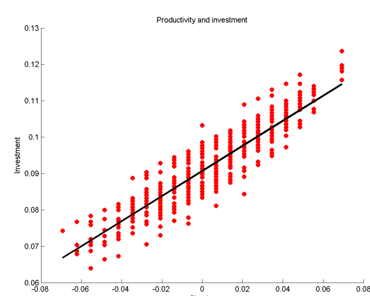

(d) Shock and $Y^U$




(e) Shock and $c^V$

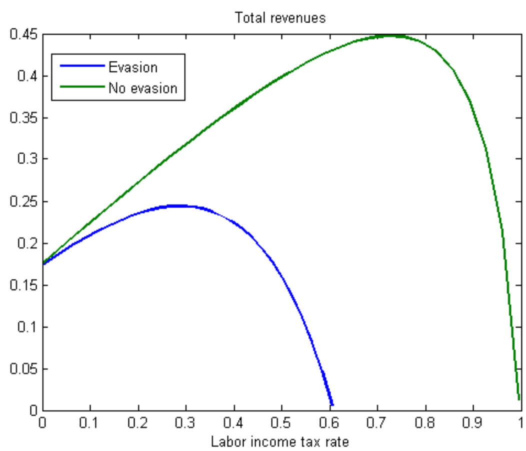



(f) Shock and $c^{NV}$

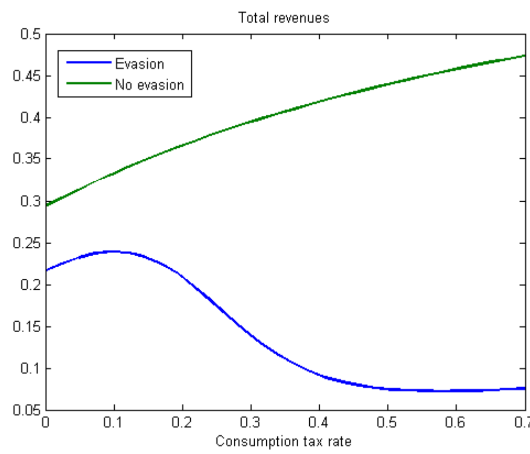



(g) Shock and $i$

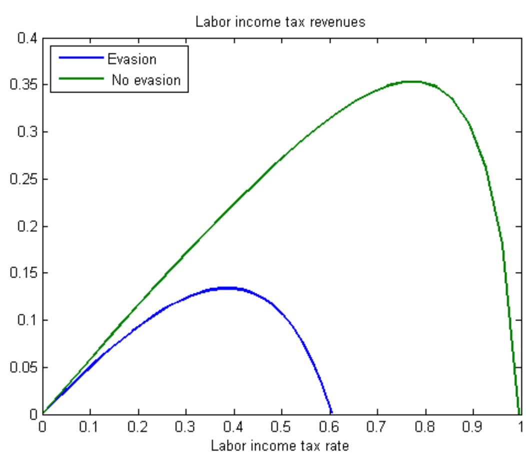
Figure 2.2: Outcomes vs TFP shock



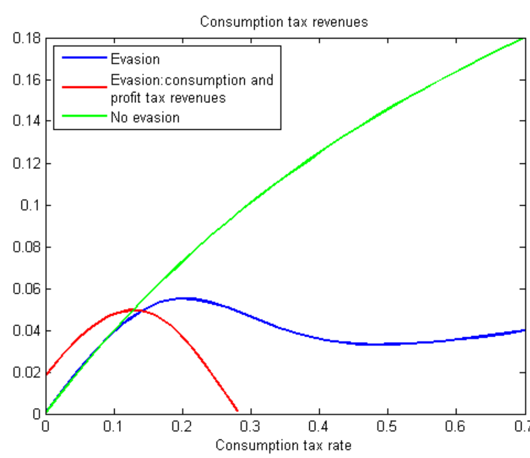

(a) Laffer curve of labor income tax




(b) Laffer curve of consumption tax

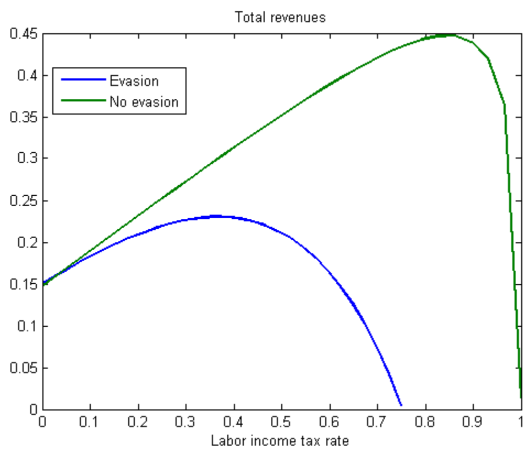



(c) Laffer curve for income tax revenues

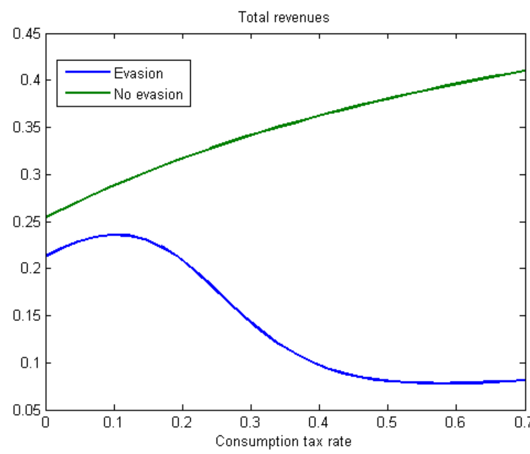



(d) Laffer curve for consumption tax revenues

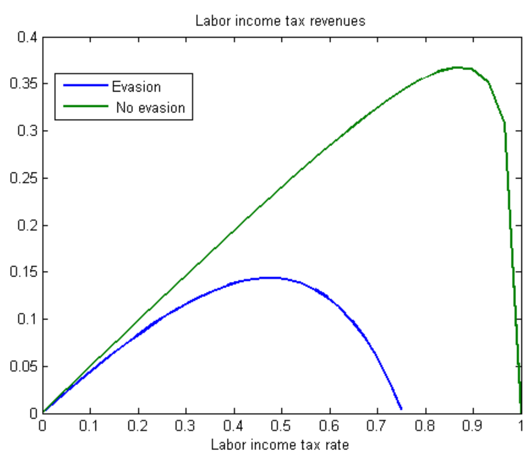
Figure 2.3: Laffer Curves



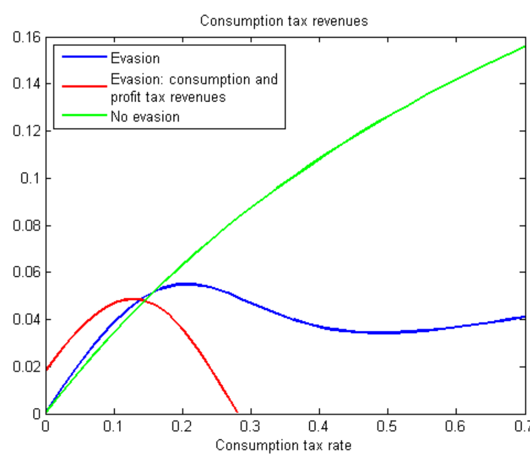

(a) Laffer curve of labor income tax




(b) Laffer curve of consumption tax




(c) Laffer curve for income tax revenues




(d) Laffer curve for consumption tax revenues

Figure 2.4: Laffer Curves for alternative Frisch elasticity (0.5)

# Chapter 3

# A small open economy analysis of Eurobonds

This paper aims to study what would have been the overall and heterogeneous effects of implementing the so-called Eurobonds in the European sovereign debt crisis (2009-12). Specifically, we focus on the proposal by Delpla and von Weizsäcker (2010). The Euro area is divided in four groups according to their initial government debts and their respective bond yields during the sovereign debt crisis, and a small open economy model is set up for each one without (baseline model) and with Eurobonds. In addition, we consider three scenarios for Eurobond yields: low, medium, and high. GIIPS (groups I and II) are able to reduce the crowding-out effect on productive investment, taxes, and debt as well as increase GDP and welfare in all scenarios. The rest of countries (groups III and IV) lose in terms of GDP, welfare, and debt in the medium and high yields. In the low, all groups are better off. Therefore, the key message is that Eurobonds could be a good policy to address times of soaring sovereign spreads, but their degree of success depends on the level of commitment.

## 3.1 Introduction

The European sovereign debt crisis (2009-12) is a period characterized by two features: a continuation of the recession started in 2007 with the international financial crisis and a soar in sovereign bond spreads in GIIPS (Greece, Ireland, Italy, Portugal, and Spain) but not in the rest of the Euro area, which is called the fiscal fragmentation. The first feature has been somehow general in all countries of the Monetary Union, the second has highlighted specific economic consequences for GIIPS that have not taken place in the rest of Euro-area members. Figure 3.1 shows how GIIPS experienced a high increase on their government borrowing rates compared to the rest of countries.

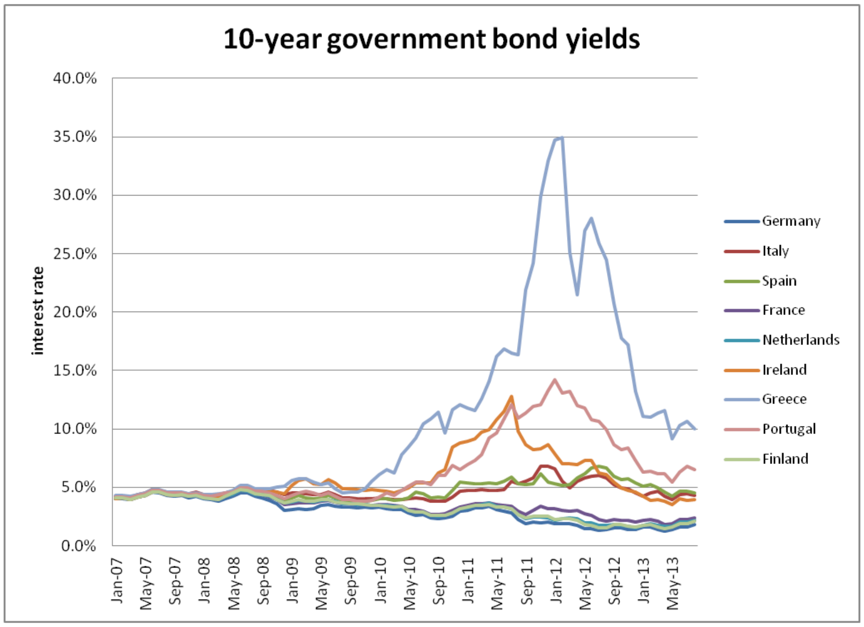



Figure 3.1: 10-year government bond yields. Source: Eurostat.

In this context, several unified fiscal policies have been proposed to address the fiscal fragmentation, most of them around the idea of creating Eurobonds. The idea is that countries pool their debts as Eurobonds in order to avoid self-fulfilling debt crisis in a context of over-indebtedness but, at the same time, they have incentives to pursue fiscal discipline, i.e. a policy that also faces the potential moral hazard problem. The first Eurobonds proposal that takes into account both sides of the coin was written by Jacques Delpla and Jakob von Weizsäcker (2010).

Before going to the details of the policy and the analysis, we summarize the main economic consequences of this fragmented increase in sovereign risks:

1. Weak fiscal performance. In some countries, especially in GIIPS, the inertia of past government expenditure was not compensated by tax collection since the interna-

tional financial crisis in 2007. Despite a countercyclical fiscal policy was necessary in order to raise revenues, the fiscal effort of taxpayers of these countries has been much stronger because of the soaring sovereign spreads. Before the debt crisis, tax collection financed the government expenditure, nowadays, it must also repay the higher interest rates. This is very problematic when debt is very high, as it is now the case in the Euro area, because although a country reached sustainable primary deficits, the debt service itself would not allow reductions of debt. Figure 3.2 evidences that GIIPS have not been able to reduce the growth rate of debt in spite of the austerity measures since 2010; in fact, growth rates are even increasing since 2009.

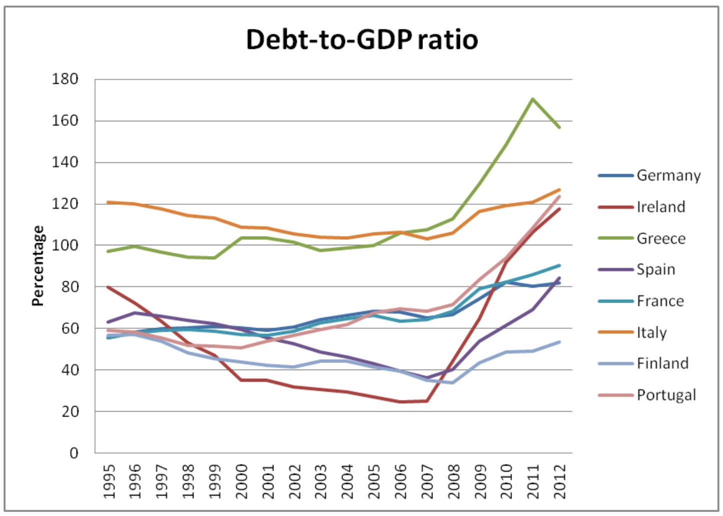



Figure 3.2: Debt-to-GDP ratio. Source: Eurostat.

2. Sovereign debt has crowded out or displaced productive investment. Broner et al. (2014) pointed out that as sovereign spreads rose, domestic residents, above all banks, increased significantly their exposure to their own sovereign debt in GIIPS and therefore domestic credit shifted from the private to the public sector in GIIPS. They collected information on the lending by domestic banks, through loans and bonds. Figure 3.3 graphs the ratio of public credit (government of the country) over private credit (non-financial corporations and households), it gives us evidence of the credit reallocation from the private to the public sector that took place in GIIPS coinciding with the increase in sovereign spreads. The figure shows that banks significantly reduced the loans to the private productive sector, non-financial corporations and

households, and raised holdings of government bonds since 2009.

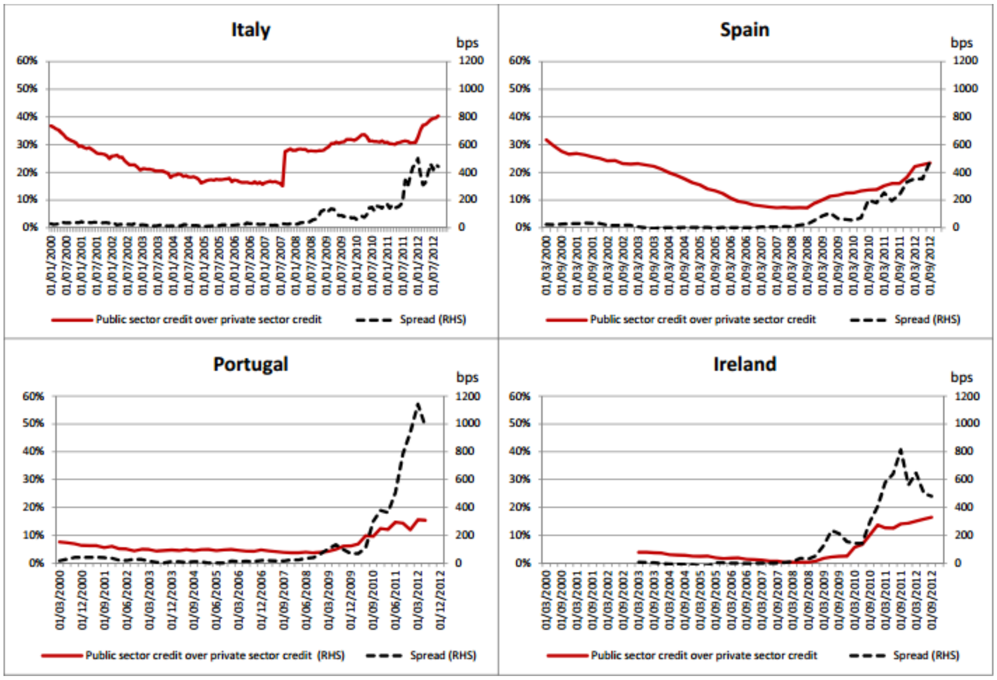



Figure 3.3: Credit allocation in GIIPS. Source: Broner et al. (2014).

On the other hand, sovereign debt spreads of the rest of Euro area were low (Figure 3.1). Figure 3.4 takes France and Germany as control group to illustrate that, comparatively, no credit reallocation took place in the rest of countries, we can see that the ratio was much more constant during the sovereign debt crisis (2009-2012).

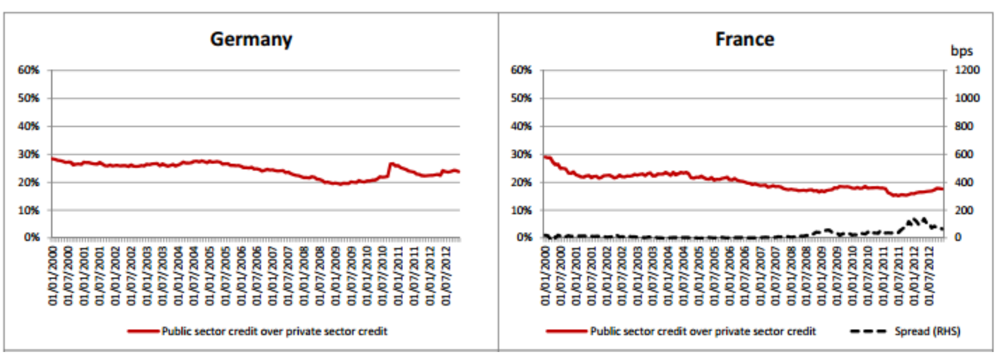



Figure 3.4: Credit allocation in the rest of countries. Source: Broner et al. (2014).

3. From a fragmented national debt market (fiscal fragmentation) to a financial fragmentation of the Euro-area credit markets. Figure 3.5 evidences that the transmission of ECB's policy to borrowing in the real economy was broken in GIIPS during the sovereign debt crisis, it shows that borrowing rate of non-financial corporations (NFC) are in line with sovereign bond yields, not with the ECB base rate,[1] which we can see that in previous periods it was the case. On the other hand, borrowing rates evolve according to the reference interest rate, before and after the debt crisis, in the rest of countries (Finland, France, Germany, and Netherlands). In the words of Broner et al. (2014), the fact that private spreads increased as well in GIIPS *"suggests that the credit reallocation from the private to the public sector pointed out above led to crowding out and more difficult access to credit for domestic firms and consumers."*

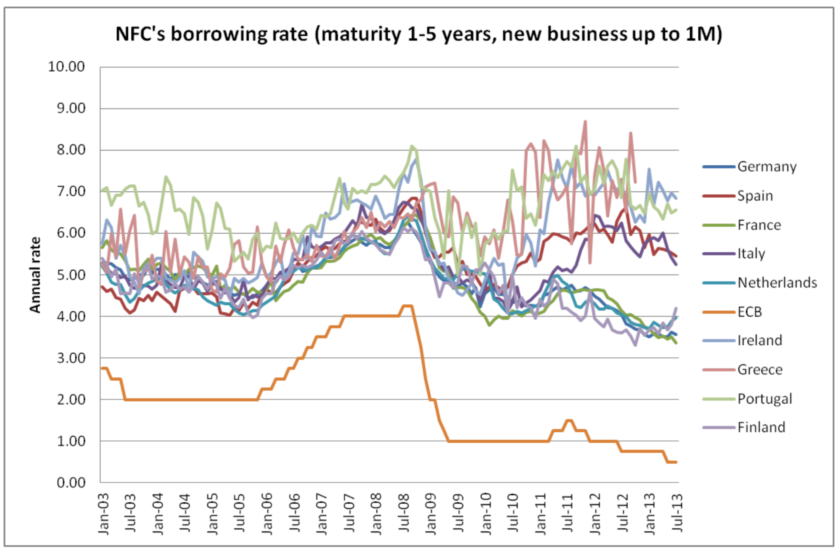



Figure 3.5: NFC's borrowing rate and ECB base rate (monthly average). Source: ECB database.

4. Political and institutional crisis, at European and country level. The monetary union is not accompanied by a political union, the European sovereign debt crisis has pointed out that the status quo was not able to handle this kind of crisis. For

[1]NFC's borrowing rates refer to all new interest rates on loans to NFC of up to 1 million euros and with a maturity of 1-5 years.

this motive, the European Financial Stabilisation Mechanism (EFSM) and European Financial Stability Facility (EFSF) were created, which together with the IMF provided rescue packages to some governments: Greece (2010 and 2012), Portugal (2011), and Ireland (2010). Later, they were integrated in the European Stability Mechanism (ESM), which became the permanent crisis resolution mechanism for the Euro area and gave financial aid to banking systems of Spain and Cyprus (2012).

Another source of discussion has been the disagreement about the role of ECB, for example, Germany defended that its only purpose was to control inflation and a lot of countries wanted the ECB to promote growth and employment as the Federal Reserve.

This political scenario, lately reactive (only corrective) measures and disagreement in very important points of the Euro area, feed back sovereign risks, it added fear of systemic consequences on top of the current increasing uncertainty of unsustainable fiscal paths in some countries. Even, doubts around the sustainability of the euro and the functioning of the Euro area as a system have been largely extended.

Therefore, Euro-area institutions lack of proactive (automatic) mechanisms and more integration that ensures that if any country struggles the system will have a proper plan to address this event. We are referring to the necessity of implementing Eurobonds and creating a banking union for the unique money in order to avoid another financial fragmentation in the future.

These consequences have represented a second hit to GIIPS after the international financial crisis of 2007, thus deepening the recession even more.

This paper develops a theory (baseline model) in order to capture the first two consequences: weak fiscal performance and crowding-out effect of public debt on productive investment. Linked to the latter, we want to see whether the lending behavior of banks in different countries can be explained by differences in government bond yields. To account for these facts, we set up a small open economy in which government debt, whose rate of return is exogenously determined abroad, can be bought by resident and foreign investors.[2] Therefore, domestic agents have a trade-off between purchasing government

[2] A small open economy model in which government debt can be bought only by domestic agents but these, at the same time, can borrow from abroad is equivalent in terms of results. For instance, see Correia (1996) and Broner et al. (2014).

bonds and investing in firms.

The baseline model is used to assess the implications of bond yields on the feasibility of fiscal policy (taxes) and on the stock of debt. In addition, we show that our baseline model is enough to generate a credit reallocation from the private to the public sector when bond yields increase.

On top of that, I evaluate the implementation of Eurobonds in the baseline model. Particularly, taking as given the proposal by Delpla and von Weizsäcker (2010), from now on "the Eurobonds proposal" or "Eurobonds". In their own words, sovereign debt of Euro-area countries should be split as follows,

> *"Blue Bonds: EU countries should pool up to 60 percent of GDP of their national debt under joint and several liability as senior sovereign debt, thereby reducing the borrowing cost for that part of the debt. Red debt: any national debt beyond a country's Blue Bond allocation should be issued as national and junior debt with sound procedures for an orderly default, thus increasing the marginal cost of public borrowing and helping to enhance fiscal discipline."*

Therefore, the Eurobonds proposal defines a policy for weak and strong fiscal countries because the fact of pooling debt will reduce the possibility of self-fulfilling debt crisis, the senior pooled debt in Eurobonds will be a very liquid and safe asset, and an orderly default on the junior national debt will impose fiscal discipline, avoiding the moral hazard problem. For these reasons, when Eurobonds are introduced in the baseline model, I abstract from modeling moral hazard. Specifically, default risk is treated as an exogenous interest rate for Eurobonds and, for national bonds (debt beyond the 60% of GDP), the extra default risk is treated as an endogenous interest-rate premium. In addition, Eurobond yields, for which we consider three scenarios, represent the quality of Euro-area institutions.

Delpla and von Weizsäcker (2010) analysis is qualitative, focuses on defining the instrument, on developing the institutional set-up in order to guarantee a suitable implementation, and on guessing the possible general fiscal policy implications and the heterogeneity of fiscal benefits and costs on different countries. On the other hand, my paper develops a theory so as to verify their hypotheses, given the assumptions of the proposal and later debates; and besides testing the idea, it also studies the effects of the policy on fiscal performance and on productive credit or investment. Related to the latter, although Delpla and von Weizsäcker (2011) propose that national bonds should be kept out of the banking system, the investors are not restricted to buy them in our model.

Specifically, our model is used (i) to see what would be the fiscal implications of this coordinated policy in order to check the Eurobonds proposal, (ii) to see what would be the fiscal performance with and without the policy, (iii) to analyze whether Eurobonds would be a good solution to the crowding-out effect on GIIPS, (iv) to evaluate the output and welfare gains implied by Eurobonds, and (v) to evaluate the overall outcomes of the policy. All of this considering that there is another group of countries of the monetary union that may have costs; that is, taking into account the heterogeneous effects of the policy.

In Section 3.2, the baseline model, which is based on small open economy, is presented. Then, the Eurobonds scheme is theoretically introduced in the baseline model. Section 3.3 describes the calibration, the estimation, and the data used in the quantitative part. In Section 3.4, I firstly evaluate the replication capability of the baseline model with respect to government debt, bond holdings of agents, and productive investment for the period of interest (2009-2012). Secondly, as I said above, the model is used to study the effects of implementing Eurobonds for the same period, simulations are performed, and the results are discussed. Finally, Section 3.5 highlights the main conclusions, it discusses the validity of results and also evaluates the policy from a general perspective considering the overall results.

Now, a brief literature review follows.

### 3.1.1 Literature review

As it is explained in the introduction, I analyze the Eurobonds proposal by Delpla and von Weizsäcker (2010) along the paper. Their proposal suggests that sovereign debt in Euro-area countries should be split into two parts. The first part, the senior "Blue" tranche of up to 60 percent of GDP, would be pooled among participating countries and jointly and severally guaranteed. The second part, the junior "Red" tranche, would keep debt in excess of 60 percent of GDP under a purely national responsibility, leading to a differentiation in interest rates. The disciplining effect of the higher marginal cost of borrowing is the most important distinction between *The Blue Bond proposal* and the first generation of proposals to pool the debt of EU countries in a Eurobond, Bonnevay (2010), Leterme (2010), or the concerns voiced by Issing (2009).

Broner et al. (2014) proposed a small open economy version of the Diamond model with credit frictions. Their paper develops a theory in order to explain, among other things, why domestic residents buy their own sovereign debt in turbulent times and why foreigners sell their holdings of bonds. They argue that the cause is credit discrimination: the expected return of sovereign bonds is higher for residents than for foreigners because the probability of repayment (default) is higher (lower) for the former. In addition, they show that these purchases of government bonds displace productive investment due to financial frictions. On the other hand, our model assumes that residents and foreigners have the same return because the purpose of this paper is completely different. Particularly, we want to see whether the lending behavior of banks in different countries can be explained by differences in government bond yields, not by credit discrimination.

I introduce country-specific spreads in a very similar way as Schmitt-Grohé and Uribe (2003). They proposed a small open economy model with a debt-elastic interest-rate premium, based on the empirical evidence of Akitoby and Stratmann (2008), with a technical purpose: to induce stationarity of the equilibrium dynamics. Under their formulation the deterministic steady state of the small open economy model is independent of the initial net foreign asset position of the economy, specifically external debt and capital, which allows to apply approximation techniques to get a solution of the model. I use a variation of this kind of spread in order to represent the Eurobonds policy, a spread that is zero if government debt of the country does not go beyond the 60% of GDP but generates positive and increasing spreads when government debt is larger than this threshold. I introduce it because reflects the paradigm that Eurobonds want to impose, shifting from a situation where spread drivers are uncertain to more objective causes based on perceived default risk, where debt level would play a center role. That is, I argue that this formulation serves as a good theoretical representation of an increasing borrowing rate at the margin.

## 3.2 The model

We set up four small open economies (SOE) according to the following classification based on the debt-to-GDP ratio at the beginning of the sovereign debt crisis and on the two patterns of bond yields described in the introduction (see Figure 3.1):

| $Bond\ yields_{(2009-2012)} \setminus \frac{DEBT_{2009}}{GDP_{2009}}$ | Debt-to-GDP<60% (low) | Debt-to GDP>60% (high) |
|---|---|---|
| Sovereign crisis, GIIPS (high) | I :Spain, Ireland | II:Italy, Portugal, Greece |
| No sovereign crisis (low) | III:Finland, Luxembourg | IV:Germany, France, Austria, and Netherlands |

These two dimensions, initial government debt level and actual bond yields, have been chosen because are crucial in the posterior policy evaluation.

This division is done because these four groups of countries are a good simplification of the whole Euro area, in the rest of the paper letter "$j$" will refer to each I, II, III, and IV groups or economies. Hereafter, "GIIPS" will refer to groups I and II together, and "the rest of countries" to groups III and IV.

In every Small Open Economy $j$ there are the following agents: representative agent (resident/domestic investors or households), risk-neutral international investors, firms, and government. Since households are the last owner of banks in the real world, when a household invests in a firm or buys debt in the model, it is equivalent to a bank giving credit to a productive firm or buying government bonds.

I assume that in each period the government have the ability to borrow in an internationally traded bond which can be bought by domestic and foreign investors. The equilibrium conditions of the international financial market, or rest of the world, determine a law of motion for the interest rate (bond yields), which is exogenous to every economy $j$.

In every SOE, I consider two different cases: baseline model and Eurobonds scheme. Therefore, these are the descriptions of both cases for all economies:

### 3.2.1 Baseline model

The behaviors of the agents in the baseline model are the following:

Households choose consumption, $c_t^j$, labor, $l_t^j$, investment, $i_t^j$, and bond holdings, $b_t^j$, in order to maximize the discounted utility, $\sum_{t=0}^{\infty} \beta_j^t U(c_t^j)$, where $U(c_t^j) = log(c_t^j)$, subject to the budget constraint,

$$(1+\tau_t^{cj})c_t^j + i_t^j + b_{t+1}^j = (1-\tau_t^{kj})r_t^{kj}k_t^j + (1-\tau_t^{wj})w_t^j l_t^j + b_t^j(1+r_t^j), \qquad (3.1)$$

the law of motion for capital,

$$i_t^j = k_{t+1}^j - (1-\delta^j)k_t^j,$$

the appropriate non-negativity constraints, $c_t^j, k_{t+1}^j \geq 0$, initial conditions for capital and bond holdings, $k_0^j$ and $b_0^j$, and the market clearing condition of labor,

$$l_t^j \in [0,1]. \tag{3.2}$$

$b_{t+1}^j$ are the government bonds bought at the end of period $t$ yielding a interest rate, $r_{t+1}^j$, in period $t+1$. $i_t^j$ denotes investment in new units of capital ($k_t^j$) in period $t$, $\tau_t^{cj}$ the consumption tax rate, $\tau_t^{kj}$the capital tax rate, and $\tau_t^{wj}$ the labor tax rate.

The agent is also subject to a No-Ponzi-game condition,

$$\lim_{T\to\infty} \frac{b_{T+1}^j}{\prod_{s=0}^T (1+r_s^j)} \geq 0. \tag{3.3}$$

The government balances its budget by selling bonds,

$$g_t^j - d_{t+1}^j = \tau_t^{kj} r_t^{kj} k_t^j + \tau_t^{cj} c_t^j + \tau_t^{wj} w_t^j l_t^j - d_t^j (1+r_t^j), \tag{3.4}$$

where $d_t^j$ and $d_{t+1}^j$ denotes the debt position assumed at the beginning and end of period $t$, respectively. Government spending $g_t^j$, which is useless, and taxes$\left\{\tau_t^{kj}, \tau_t^{wj}, \tau_t^{cj}\right\}_{t=0}^{\infty}$ are exogenously given.[3]

The government also satisfies a No-Ponzi-game condition,

$$\lim_{T\to\infty} \frac{d_{T+1}^j}{\prod_{s=0}^T (1+r_s^j)} \leq 0. \tag{3.5}$$

The government debt is bought by residents and foreigners,

$$d_t^j = b_t^j + b_t^{f,j}, \tag{3.6}$$

where $b_t^{f,j}$ is the quantity of government bonds bought by international investors.

[3] One of these government instruments is chosen to satisfy the government's intertemporal budget constraint (3.19). I specify which one in Section 3.3.

Firms take the prices as given, so we have perfect competition. The firms choose capital, $k_t^j$, and labor, $l_t^j$, to maximize profits,

$$\Pi_t^j(k_t^j, l_t^j) = y_t^j - r_t^{kj} k_t^j - w_t^j l_t^j, \tag{3.7}$$

subject to a Cobb-Douglas technology,

$$y_t^j = \left(k_t^j\right)^{\alpha^j} (l_t^j)^{1-\alpha^j},$$

where $y_t^j$ is the output produced by the factors: $k_t^j$, initial physical capital in period $t$, and $l_t^j$, quantity of labor at $t$. In the side of costs, $r_t^{kj}$ is the return of physical capital and $w_t^j$ is the real wage.

The feasibility constraint is given by

$$c_t^j + g_t^j + k_{t+1}^j - (1-\delta^j)k_t^j - b_{t+1}^{f,j} = y_t^j - b_t^{f,j}(1+r_t^j). \tag{3.8}$$

**Computation of equilibrium**

**Definition.** Given $k_0^j$, $d_0^j$, $b_0^j$, $b_0^{f,j}$, and $\left\{g_t^j, \tau_t^{kj}, \tau_t^{cj}, \tau_t^{wj}, r_t^j\right\}_{t=0}^{\infty}$, the competitive equilibrium is a sequence of quantities, $\left\{c_t^j, k_{t+1}^j, l_t^j, d_{t+1}^j, b_{t+1}^j, b_{t+1}^{f,j}\right\}_{t=0}^{\infty}$, and prices, $\left\{r_t^{kj}, w_t^j\right\}_{t=0}^{\infty}$, such that

- Given $\left\{r_t^{kj}, w_t^j\right\}_{t=0}^{\infty}$, households, choosing $\left\{c_t^j, k_{t+1}^j, b_{t+1}^j\right\}_{t=0}^{\infty}$, maximize their utility subject to the budget constraint (3.1), the non-negativity constraints, and the No-Ponzi-game condition (3.3).
- Given $\left\{r_t^{kj}, w_t^j\right\}_{t=0}^{\infty}$, firms, choosing $\left\{k_t^j, l_t^j\right\}_{t=0}^{\infty}$, maximize profits (3.7).
- Public sovereign debt, $\left\{d_{t+1}^j\right\}_{t=0}^{\infty}$, satisfies the government budget constraint (3.4) and the No-Ponzi-game condition (3.5).
- $\left\{c_t^j, k_{t+1}^j, l_t^j, b_{t+1}^{f,j}\right\}_{t=0}^{\infty}$ satisfy the market clearing conditions of goods (3.8), $\left\{l_t^j\right\}_{t=0}^{\infty}$ of labor (3.2) and $\left\{d_{t+1}^j, b_{t+1}^j, b_{t+1}^{f,j}\right\}_{t=0}^{\infty}$ of assets (3.6) .

### Equilibrium conditions

The first-order conditions (FOC) of households' maximization problem are the Euler equation of bonds,

$$\frac{u_c(c_t^j)}{(1+\tau_t^c)} = \beta_j \left(1 + r_{t+1}^j\right) \frac{u_c(c_{t+1}^j)}{(1+\tau_{t+1}^c)}, \tag{3.9}$$

the Euler equation of capital,

$$\frac{u_c(c_t^j)}{(1+\tau_t^c)} = \beta_j \left(1 + (1-\tau_t^{kj}) r_{t+1}^k - \delta^j\right) \frac{u_c(c_{t+1}^j)}{(1+\tau_{t+1}^c)}, \tag{3.10}$$

and the Transversality conditions,

$$\lim_{T\to\infty} \frac{b_{T+1}^j}{\prod_{s=1}^T (1+r_s^j)} = 0 \tag{3.11}$$

$$\lim_{T\to\infty} \lambda_T^j k_{T+1}^j = 0. \tag{3.12}$$

Since resident investors have no desutility from labor, the optimal labor choice is $l_t^j = 1$.

From the firms' maximization problem we find that the competitive prices of factors are equal to the marginal productivity,

$$r_t^{kj} = \alpha^j \frac{y_t^j}{k_t^j} \tag{3.13}$$

$$w_t^j = (1-\alpha^j) \left(k_t^j\right)^{\alpha^j}. \tag{3.14}$$

Combining (3.9) and (3.10) we get the no-arbitrage condition,

$$(1 + r_{t+1}^j) = \left(1 + (1-\tau_t^{kj}) r_{t+1}^k - \delta^j\right). \tag{3.15}$$

Using the perfect-competition price of capital (3.13), the capital next period is:

$$k^j{}_{t+1} = \left(\frac{\alpha^j (1-\tau_{t+1}^{kj})}{r_{t+1}^j + \delta^j}\right)^{\frac{1}{1-\alpha^j}}. \tag{3.16}$$

Using the household budget constraint (3.1) and prices of productive factors in equilibrium (3.13)-(3.14), I solve for $c_t^j$, $r_t^{kj}$ and $w_t^j$, respectively, in the Euler equation of bonds (3.9), which can be rewritten as

$$\frac{1}{(1-\tau_t^{kj})\alpha^j k_t^{j\alpha^j}+(1-\tau_t^{wj})(1-\alpha^j)k_t^{j\alpha^j}+b_t^j(1+r_t^j)+(1-\delta^j)k_t^j-k_{t+1}^j-b_{t+1}^j}=\beta_j\left(1+r_{t+1}^j\right)$$

$$\frac{1}{(1-\tau_{t+1}^{kj}))\alpha^j k_{t+1}^{j\alpha^j}+(1-\tau_{t+1}^{wj})(1-\alpha^j)k_{t+1}^{j\alpha^j}+b_{t+1}^j(1+r_{t+1}^j)+(1-\delta^j)k_{t+1}^j-k_{t+2}^j-b_{t+2}^j}. \tag{3.17}$$

In addition, from the government budget constraint (3.4) substituting forward, and using the transversality condition,

$$\lim_{T\to\infty}\frac{d_{T+1}^j}{\prod_{s=0}^{T}(1+r_s^j)}=0, \tag{3.18}$$

the government's intertemporal budget constraint is

$$d_0^j=\sum_{t=0}^{\infty}\frac{1}{\prod_{n=0}^{t}(1+r_n^j)}\left[\tau_t^{cj}c_t^j+\tau_t^{kj}r_t^{kj}k_t^j+\tau_t^{wj}w_t^j-g_t^j\right]. \tag{3.19}$$

Therefore, the equations that characterize the competitive equilibrium are (3.4), (3.6), (3.8), (3.11)-(3.12), (3.16)-(3.17), and (3.19). I solve the model following a FOC approach and numerical methods. The system (3.4), (3.6), (3.8), (3.16)-(3.17), and (3.19) involves an infinite number of equations and unknowns. To make the computation of the equilibrium feasible, I assume that the economy converges to a steady state at some date $T$, where $T$ is 100 periods ahead, which besides ensures that the Transversality conditions, (3.11)-(3.12), are satisfied. From the Euler equations of bonds and capital at the steady state,

$$\frac{1}{\beta^j}=1+r_{ss}^j \tag{3.20}$$

$$\frac{1}{\beta^j}=1+(1-\tau_{ss}^{kj})\alpha^j k_{ss}^{\alpha^j-1}-\delta^j, \tag{3.21}$$

we see that steady state level of capital is a function of the parameters of the model, hence it is unique. On the other hand, as shown by Schmitt-Grohe and Uribe (2003), the steady state levels of bond holdings and consumption depend on initial conditions, such as the initial level of bond holdings itself.

Hence, given the initial conditions, $d_t^j$, $b_0^j$, $b_0^{f,j}$, and $k_0^j$, where $0 = T_0 = 2009$, and the terminal condition at $T$, I use the Newton-Raphson algorithm to solve the square system of equations (3.4), (3.6), (3.8), and (3.16)-(3.17) for $t = 2009, 2010, ..., T$, where $\left\{c_t^j, k_{t+1}^j, d_{t+1}^j, b_{t+1}^j, b_{t+1}^{f,j}\right\}_{2009}^{T}$ are the unknowns. In addition, the government's intertemporal budget constraint (3.19) must be satisfied, thus one of the government instruments is freely chosen to fulfill this condition. I specify which instrument when explaining the calibration and estimation.

### 3.2.2 Eurobonds scheme

Now, the policy is implemented in the baseline model at the initial period ($T_0$). Each of our economies starts with the same initial conditions as in the baseline case, $k_{T_0}^j$, $d_{T_0}^j$, $b_{T_0}^j$, $b_{T_0}^{f,j}$, and $r_{T_0}^j$ but now the government, in order to balance budget, is able to issue bonds ($d_{t+1}^j \, for \, t = T_0, ..., T.$) of two types in the following periods: Eurobonds and national bonds. We assume that all groups participate in the policy, thus the government of each group issue Eurobonds up to 60% of its GDP and the debt in excess of this threshold, if necessary, is issued as national bonds. Eurobond yields are represented by $r_{s,t}^e$ and the yields of national bonds by $r_{s,t}^{n,j}$.

Thus, the government budget constraint is given by

$$g_t^j - d_{t+1}^j = \tau_t^{kj} r_t^{kj} k_t^j + \tau_t^{cj} c_t^j + \tau_t^{wj} w_t^j l_t^j - d_t^j (1 + r_t^j), t = T_0, \tag{3.22}$$

$$g_t^j - d_{t+1}^j = \tau_t^{kj} r_t^{kj} k_t^j + \tau_t^{cj} c_t^j + \tau_t^{wj} w_t^j l_t^j - min\left\{d_t^j, \phi y_t^j\right\}(1 + r_{s,t}^e) - max\left\{0, d_t^j - \phi y_t^j\right\}(1 + r_{s,t}^{n,j}), t > T_0, \tag{3.23}$$

where $\phi = 60\%$, and $s = \{l, m, h\}$ are the several scenarios considered for Eurobond yields: low, medium, and high. The debt position in Eurobonds of government is $min\left\{d_t^j, \phi y_t^j\right\}$ and $max\left\{0, d_t^j - \phi y_t^j\right\}$ is the quantity of national bonds, which corresponds to the debt issued beyond the 60% of GDP. Therefore, the market clearing conditions of bonds are given by

$$min\{d_t^j, \phi y_t^j\} = b_t^{e,j} + b_t^{f,e,j}, t > T_0, \tag{3.24}$$

$$max\{0, d_t^j - \phi y_t^j\} = b_t^{n,j} + b_t^{f,n,j}, t > T_0, \tag{3.25}$$

where $b_t^{f,e,j}$ and $b_t^{f,n,j}$ are the quantity of Eurobonds and national bonds bought by foreign investors, respectively. And, $b_t^{e,j}$ and $b_t^{n,j}$ are the quantity of Eurobonds and national bonds bought by domestic investors, respectively.

With respect to Eurobond yields, a scenario analysis is performed because there has been a large debate on this part of the policy, thus three scenarios are taken as given for $r_{s,t}^e$. I consider the low and high yields that have been mentioned more, and also the average of these two extremes. This rate should be understood as an indicator of the quality of the Euro-area institutions, as it is mentioned in the introduction. I define the quality of institutions as the addition of two elements: degree of integration and degree of automaticity of fiscal mechanisms. In this context of public debt analysis, the degree of integration refers to pool or not debts under a joint and several guarantee. The degree of automaticity depends on having or not automatic mechanisms, for this concrete policy we refer to have or not an orderly planned mechanism of default (Eurobonds as senior debt and national bonds as junior). Particularly, $r_{l,t}^e$ represents the optimal quality of institutions, $r_{m,t}^e$ and $r_{h,t}^e$ the intermediate and the worst quality, respectively.

Low yields, $r_{l,t}^e$, consider that Eurobonds is an extremely safe asset equivalent to the US treasury bonds or benchmark German bonds due to two motives: liquidity and safety. The first because pooling debt under joint and several guarantee would mean that Eurobonds would be a very high liquid asset for world investors, and the second, because the 60% of GDP as senior debt is an easily sustainable level that would make that the default risk was very low. On the contrary, high yields ($r_{h,t}^e$) consist of a weighted average of bond yields of all the Euro-area members.[4] In this case, the policy is implemented without the optimal institutional reforms that would force a qualitative change from the uncertainty of the status quo. And, $r_{m,t}^e$ is the medium yield of $r_{l,t}^e$ and $r_{h,t}^e$.

The interest rate of national debt is

$$r_{s,t}^{n,j} = r_{s,t}^e + \psi_s^j(e^{\frac{d_t^j}{y_t^j} - \phi} - 1),$$

where $\psi_s^j \geq 0$, otherwise we would be making senseless assumptions, for example, like assuming a negative relationship between overindebtness and spreads, i.e. that the first lowers the second. The interest rate $r_{s,t}^{n,j}$ is simply the interest rate of Eurobonds in case plus

[4]Weighting by debt in Eurobonds, i.e. debt below or equal to 60% of GDP.

an interest-rate premium. The second term of the sum reflects that beyond the 60% of GDP, the country issue national debt, what the authors called “red tranche”. This interest-rate premium is taken from Schmitt-Grohé and Uribe (2003) although with public debt and for other purpose, I argue that having an interest-rate premium that is an increasing function with respect to the debt-to-GDP ratio is a good theoretical representation of having an increasing borrowing cost at the margin, which is a key element in the definition of the Eurobonds proposal.[5] In the words of the authors, *“In case of a partial default, the red tranche will be hit first and the blue tranche will only be affected by that part of the default (if any) that is not absorbed by the junior tranche. In other words, any government funds used to service and repay government debt will always first be used to satisfy the claims of the blue bond holders...From an investor's perspective, the prospect of a less disruptive default on the junior tranche increases the risk of default, thereby calling for an additional risk premium”.* As I said in the introduction, the extra default risk of national bonds is treated as an endogenous interest-rate premium, which depends on the debt-to-GDP ratio. This formulation implies that when a country follows an unsustainable fiscal path, with a debt moving away from the Eurobonds threshold, default risk increases and so the borrowing rate at the margin.

Hence, the agent's budget constraint is

$$(1+\tau_t^{cj})c_t^j + i_t^j + b_{t+1}^{e,j} + b_{t+1}^{n,j} = (1-\tau_t^{kj})r_t^{kj}k_t^j + (1-\tau_t^{wj})w_t^j l_t^j + (1+r_t^j)b_t^j, t = T_0, \quad (3.26)$$

$$(1+\tau_t^{cj})c_t^j + i_t^j + b_{t+1}^{e,j} + b_{t+1}^{n,j} = (1-\tau_t^{kj})r_t^{kj}k_t^j + (1-\tau_t^{wj})w_t^j l_t^j + b_t^{e,j}(1+r_{s,t}^e) + b_t^{n,j}(1+r_{s,t}^{n,j}), t > T_0. \quad (3.27)$$

Finally, the feasibility constraint reads

$$c_t^j + g_t^j + i_t^j - (b_{t+1}^{f,e,j} + b_{t+1}^{f,n,j}) = y_t^j - b_t^{f,j}(1+r_t^j), t = T_0, \quad (3.28)$$

$$c_t^j + g_t^j + i_t^j - (b_{t+1}^{f,e,j} + b_{t+1}^{f,n,j}) = y_t^j - b_t^{f,e,j}(1+r_{s,t}^e) - b_t^{f,n,j}(1+r_{s,t}^{n,j}), t > T_0. \quad (3.29)$$

[5]See the definition of the policy in the Introduction.

### Equilibrium conditions

Now, we have some different equilibrium conditions with respect to the baseline model. The rest of equilibrium conditions are equivalent to the baseline model and are also satisfied: optimal labor choice, equilibrium prices of capital (3.13) and labor (3.14), Euler equation of capital (3.10), and Transversality conditions (3.11)-(3.12) and (3.18).

From the point of view of an investor, national bonds dominate Eurobonds because they offer a higher return without uncertainty. Therefore, domestic investors always prefer to buy national bonds than Eurobonds, but they can only do it when government has the necessity to run up debt beyond a debt-to-GDP ratio higher than $\phi$. Since the domestic agents buy bonds at the highest possible interest rate, the Euler equation of bonds takes the following form

$$\frac{u_c(c_t^j)}{(1+\tau_t^c)} = \beta^j \left(1 + r_{s,t+1}^j\right) \frac{u_c(c_{t+1}^j)}{(1+\tau_{t+1}^c)}, \tag{3.30}$$

where $r_{s,t+1}^j$ is the interest rate at the margin of government borrowing,

$$r_{s,t+1}^j = r_{s,t+1}^e + \max\left\{0, \psi_s^j (e^{\frac{d_{t+1}^j}{y_{t+1}^j} - \phi} - 1)\right\}. \tag{3.31}$$

The interest rate at the margin is equal to Eurobond yields ($r_{s,t}^e$) when $d_t^j/y_t^j \leq \phi$ and, it is equal to the interest rate of national bonds ($r_{s,t}^{n,j}$) when government has the necessity to issue national debt ($d_t^j/y_t^j > \phi$). The function *max* represents this fact.

I define $b_{t+1}^j$ as the quantity of bonds bought by the residents, which is equal to

$$b_{t+1}^j = \begin{cases} b_{t+1}^{e,j} & if \quad \frac{d_{t+1}^j}{y_{t+1}^j} \leq \phi \\ b_{t+1}^{n,j} & if \quad \frac{d_{t+1}^j}{y_{t+1}^j} > \phi \end{cases}. \tag{3.32}$$

It reflects that residents buy Eurobonds or national bonds depending on the marginal interest rate. Therefore, when national bonds are an option, the debt in Eurobonds is entirely bought by the rest of the world (foreign investors).

Again, from the Euler equation of capital (3.10) together with the Euler of bonds (3.30), solving for $r_{s,t+1}^j$ by using (3.31), we can write the no-arbitrage condition as

$$1 + r^e_{s,t+1} + \max\left\{0, \psi^j_s(e^{\frac{d^j_{t+1}}{y^j_{t+1}} - \phi} - 1)\right\} = \left(1 + (1 - \tau^{kj}_t) r^k_{t+1} - \delta^j\right). \tag{3.33}$$

This equation shows that if the government has the necessity to issue debt beyond the 60% of GDP, the domestic investor will buy national bonds instead of Eurobonds, therefore the return of investment must be such that this agent is indifferent between investing and purchasing national bonds. On the other hand, when government debt is below the 60% of GPD, the return of investment is such that this agent is indifferent between investing and purchasing Eurobonds.

Solving for $r^{kj}_{t+1}$ in the no-arbitrage condition (3.33), by using (3.13), we get that capital next period is

$$k^j_{t+1} = \left( \frac{\alpha^j (1 - \tau^{kj}_{t+1})}{r^e_{s,t+1} + max\left\{0, \psi^j_s(e^{\frac{d^j_{t+1}}{y^j_{t+1}} - \phi} - 1)\right\} + \delta^j} \right)^{\frac{1}{1-\alpha^j}}. \tag{3.34}$$

Using the household budget constraints (3.26)-(3.27), the distinction between the kind of bonds purchased by domestic investors (3.32), and the prices of productive factors (3.13)-(3.14); I solve for $c^j_t$, $r^j_t$, and $w^j_t$, respectively, in the Euler equation of bonds (3.30), which can be rewritten, for $t = T_0$, as

$$\frac{1}{(1-\tau^{kj}_t)\alpha^j k^{j\alpha^j}_t + (1-\tau^{wj}_t)(1-\alpha^j) k^{j\alpha^j}_t + b^j_t(1+r^j_t) + (1-\delta^j)k^j_t - k^j_{t+1} - b^j_{t+1}} = \beta^j \left(1 + r^j_{s,t+1}\right)$$

$$\frac{1}{(1-\tau^{kj}_{t+1}))\alpha^j k^{j\alpha^j}_{t+1} + (1-\tau^{wj}_{t+1})(1-\alpha^j) k^{j\alpha^j}_{t+1} + b^j_{t+1}(1+r_{s,t+1}) + (1-\delta^j)k^j_{t+1} - k^j_{t+2} - b^j_{t+2}}, \tag{3.35}$$

and, for $t > T_0$, as

$$\frac{1}{(1-\tau^{kj}_t)\alpha^j k^{j\alpha^j}_t + (1-\tau^{wj}_t)(1-\alpha^j) k^{j\alpha^j}_t + b^j_t(1+r^j_{s,t}) + (1-\delta^j)k^j_t - k^j_{t+1} - b^j_{t+1}} = \beta^j \left(1 + r^j_{s,t+1}\right)$$

$$\frac{1}{(1-\tau^{kj}_{t+1}))\alpha^j k^{j\alpha^j}_{t+1} + (1-\tau^{wj}_{t+1})(1-\alpha^j) k^{j\alpha^j}_{t+1} + b^j_{t+1}(1+r^j_{s,t+1}) + (1-\delta^j)k^j_{t+1} - k^j_{t+2} - b^j_{t+2}}. \tag{3.36}$$

As in the baseline model, the government's intertemporal budget constraint is

$$d_0^j = \frac{1}{(1+r_0^j)}\left[\tau_0^{cj}c_0^j + \tau_0^{kj}r_0^{kj}k_0^j + \tau_0^{wj}w_0^j - g_0^j\right] +$$

$$\sum_{t=1}^{\infty}\frac{1}{\prod_{n=1}^{t}(1+r_{s,t}^j)}\left[\tau_t^{cj}c_t^j + \tau_t^{kj}r_t^{kj}k_t^j + \tau_t^{wj}w_t^j - g_t^j + \phi y_t^j max\left\{0, \psi_s^j(e^{\frac{d_t^j}{y_t^j}-\phi} - 1)\right\}\right]. \tag{3.37}$$

This equation is computed in the same way as in the baseline model, substituting forward the government budget constraint (3.22)-(3.23) and satisfying the transversality condition (3.18).

Thus, the system of equations that characterize the equilibrium, in this case, is defined by (3.11)-(3.12), (3.22)-(3.25), (3.28)-(3.29), and (3.34)-(3.37).

Given the same initial conditions and the steady-state condition at $T$, the equations (3.22)-(3.25), (3.28)-(3.29), and (3.34)-(3.36) for $t = 2009, 2010, ..., T$, where $\left\{c_t^j, k_{t+1}^j, d_{t+1}^j, b_{t+1}^j, b_{t+1}^{f,e,j}, b_{t+1}^{f,n,j}\right\}_{2009}^{T}$ are the unknowns, is a square system solved with the same method used to get the baseline solution. The steady state in this case has the same properties as in the baseline model.

Again, one of the government instruments is freely chosen to fulfill the condition (3.37), I specify which instrument in the next section.

## 3.3 Calibration and estimation

In this section, I explain how all the parameter values are computed, what are the values of data for the exogenous processes, and how effective tax rates are estimated. I use data of Spain, Italy, Finland, and France in order to calibrate and estimate parameters, effective tax rates, and exogenous processes in the small open economies I, II, III, and IV, respectively. All the data used is from Eurostat for the period 2009-2012: government bond yields, National Accounts (NA), and government finance statistics. The NA and government finance statistics are expressed in constant 2005 prices.

#### Effective tax rates and exogenous processes

I followed Mendoza, Razin, and Tesar (1994) in order to obtain estimates of the sequences of effective average tax rates $\left\{\tau_t^{kj}, \tau_t^{cj}, \tau_t^{wj}\right\}_{2009}^{2012}$. Mendoza (1994) use the government revenue statistics and NA from OECD (SNA93), here equivalent data from Eurostat (ESA95) is used. Figure 3.6 shows the effective tax rates and primary government expenditure of all groups.

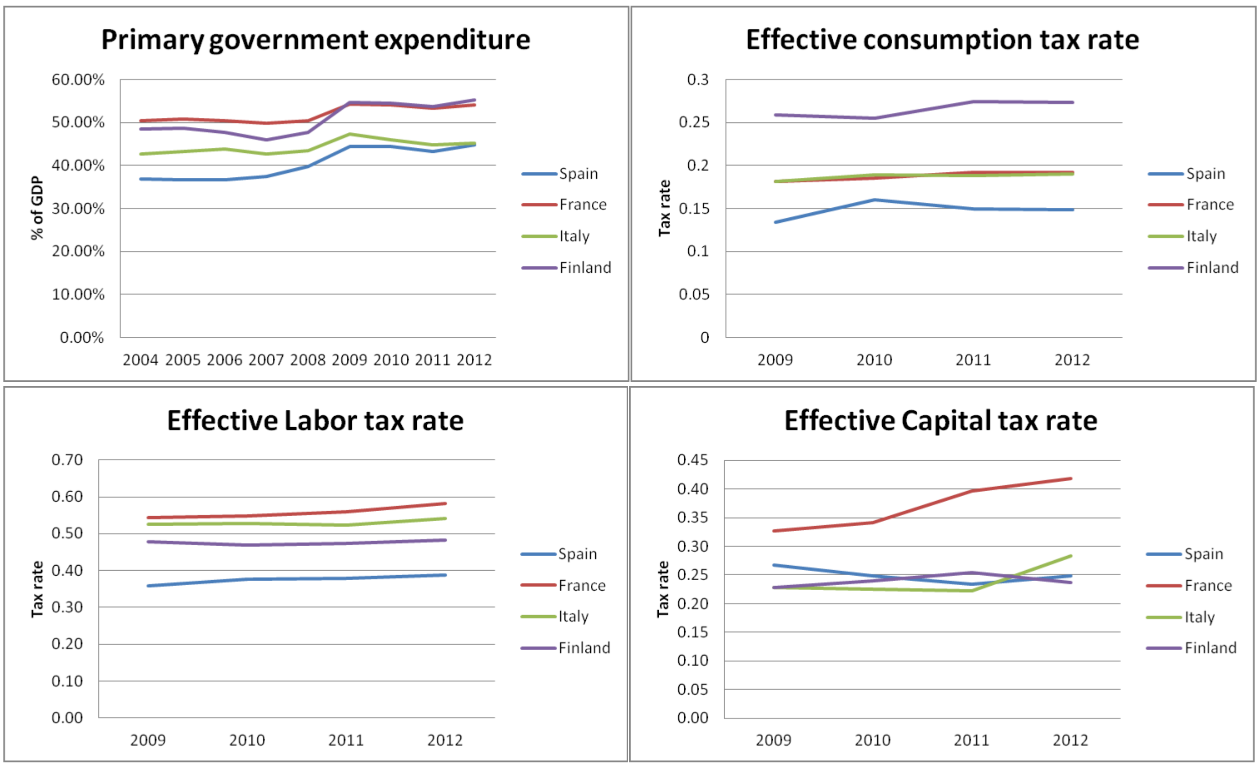



Figure 3.6: Primary government expenditure and effective tax rates

The exogenous government bond yields $\left\{r_t^j\right\}_{2009}^{2012}$ in the baseline model are taken from data (10-year government bond yields). The low, medium, and high scenarios for Eurobond yields, $\{r_{s,t}^e\}_{2010}^{2012}$, are: $r_{l,t}^e$ is the benchmark German bond, $r_{h,t}^e$ is the weighted average (weighted by debt below or equal to 60% of GDP) of all Euro-area countries, and $r_{m,t}^j$ is the medium interest rate between $r_{l,t}^e$ and $r_{h,t}^e$. Figure 3.7 graphs, for our period of interest, the interest rates in the baseline model and in the three different scenarios of Eurobonds.

After our period of interest, government instruments $\left\{g_t^j, \tau_t^{kj}, \tau_t^{wj}\right\}_{t=2013}^T$ are constant in their values of 2012. As mentioned above, a government instrument must be chosen to satisfy the government's intertemporal budget constraints (3.19) and (3.37), in this case $\left\{\tau_t^{cj}\right\}_{2013}^T$. Exogenous interest rates, $\left\{r_t^j, r_{s,t}^e\right\}_{t=2013}^T$, slowly return to $r_{2009}^j$ in 75 periods using a step function, and then remain constant in this value.

**Parameter values**

The capital and labor income shares are calculated in the standard way with data of 2009 from the NA, particularly following Gollin (2002). The depreciation rate and initial capital-output ratio are calibrated so that the equilibrium of the model in 2009 replicates the data in the NA. Following Kehoe and Ruhl (2009), multiplying the no-arbitrage condition (3.15) in 2009 by $k_{2009}^j$ in both sides,

$$k_{2009}^j r_{2009}^j = (1 - \tau_{2009}^{kj}) r_{2009}^{kj} k_{2009}^j - \delta k_{2009}^j,$$

and using the equilibrium price of capital (3.13), where in 2009 is $\alpha^j y_{2009}^j = r_{2009}^{kj} k_{2009}^j$, we obtain the following initial capital and depreciation rate:

$$k_0^j = k_{2009}^j = \frac{(1 - \tau_{2009}^{kj}) \alpha^j GDP_{2009}^j - CFC_{2009}^j}{r_{2009}^j}$$

$$\delta^j = \frac{CFC_{2009}^j}{k_{2009}^j},$$

where $GDP_{2009}^j$ is the gross domestic product and $CFC_{2009}^j$ is the consumption of fixed capital in NA.

The discount rate is computed from Euler equation of bonds (3.20) at steady state $\beta^j = 1/(1 + r_{ss}^j)$.

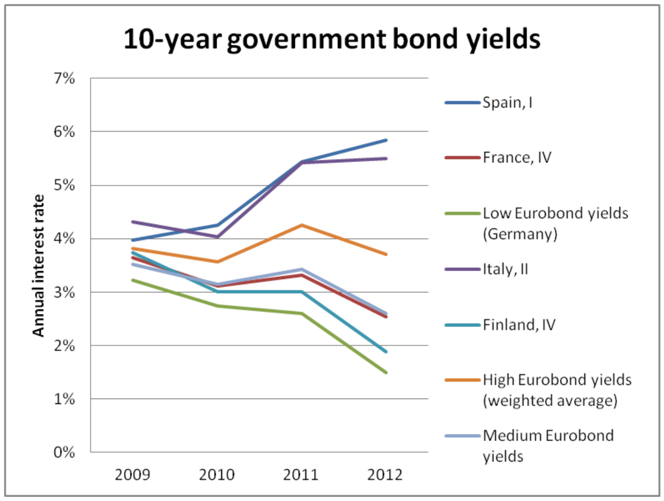



Figure 3.7: Interest rates. Source: Eurostat and own calculations.

The adjustment parameter of the debt-elastic interest-rate premium, $\psi_s^j$, is estimated by OLS, for each economy $j$, using real data on debt levels and spreads with respect to Eurobond yields in case $(r_{s,t}^e)$. A simple linear regression model is computed

$$(r_t^j - r_{s,t}^e) = \psi_s^j X_t^j, \qquad X_t^j = \max\left\{0, (e^{\frac{d_t^j}{y_t^j} - \phi} - 1)\right\}.$$

We use the Newey-West Standard Errors to study the significance of the coefficient because there is a problem of autocorrelation. We take the largest possible sample since the introduction of the Euro, monthly values from 2002 to 2012 (132 observations). Table 3.1 summarizes all these estimates. It is worth noting that there is probably another issue relative to the omission of relevant variables such as primary deficit or the level of confidence of investors, among others. This would make that our estimates present an upward biased, this estimation would assign more spread variability to the debt level than an unbiased estimator but for our purpose it is not a problem from a point of view of prudence. The estimates $\hat{\psi}_h^{IV}$ and $\hat{\psi}_m^{IV}$ are negative thus I set $\psi_h^{IV}, \psi_m^{IV} = 0$ since $\psi_s^j \geq 0$ in the model, which implies that no country when issue national debt could get a lower borrowing rate than Eurobonds, we are conservative again here. For group III, we can not estimate these parameters because the variable $X_t^{III}$ is always zero in the sample. Therefore, we assume that the parameters $\psi_s^{III}$, where $s = \{l, m, h\}$, are equal to those of the group with the most similar country-risk, namely group IV . The estimates of group I are not reliable because $X_t^I$ is only positive 34 out of 132 observations; specifically, this covariate generates extremly high estimates of $\psi_s^I$, see Table 3.1. Therefore, we set $\psi_s^I$ 5 times larger than $\psi_s^{II}$ in order to work with a conservative upper bound.[6]

The initial proportion of sovereign debt holdings between residents and non-residents has been taken from the "Bruegel Dataset of Sovereign debt holdings" by Merler (2014). There is a summary of all parameters and initial values in Table 3.2.

## 3.4 Simulations and results

The quantitative analysis consists of studying the competitive equilibrium outcomes without and with Eurobonds during the sovereign debt crisis (2009-2012). But, before implementing Eurobonds, I briefly study the baseline model.

The results plotted in all figures are the baseline model (red line) and the three scenarios of Eurobond yields: low (blue dashed line), medium (blue pointed line), and high (blue line). In addition, data appears represented by a black line in some figures.

### 3.4.1 Baseline model

First of all, I evaluate the replication capability of the baseline model. Specifically, I look for a theory that is able to do a reasonable replication of the government debt and of the lending behavior of resident investors between productive investment and bond holdings during the sovereign debt crisis (2009-2012).

Figure 3.8 presents the debt-to-GDP paths of the data and baseline model, as we can see in this figure the baseline debts are also increasing as in the data. In our model, the debt path of group I is very close to the data. For group II, the model generates higher debt-to-GPD ratios in 2011 and 2012 than in the data. In groups III and IV, the debt-to-GPD ratios of the model are lower with respect to the data in the last period.

As it is explained in the introduction, there is evidence (consequences 2 and 3) that public debt was crowding out productive investment in GIIPS during the European sovereign debt crisis. Therefore, the ratio of bond holdings over capital $(b_t^j/k_t^j)$ is computed in order to see what is the evolution of the credit allocation between the government and the

[6] Schmitt-Grohé and Uribe (2003), for example, use a calibration of this parameter equal to 0.000742.

private productive sector during this period. Accordingly, we collect data from domestic banks in order to get our proxy, computing the ratio of sovereign debt holdings over private productive credit.[7] Figure 3.9 plots the evolution of these ratios. First, we notice that the volatility is much higher in the model than in the data. Furthermore, we observe, in the data, that this ratio increases a lot in GIIPS during the sovereign crisis compared to previous years; specifically, in group I and II, this ratio increases by 10.5 and 8.56 % points during the period 2009-2012, respectively. In fact, there is a clear change in tendency. The results of the model show that agents reallocate from productive investment to sovereign bonds their part of income devoted to savings when interest rate increases. Therefore, these purchases crowd out or displace productive investment, and thus capital of the economy falls. In the model, we observe that the bond holdings-capital ratio increases by 47.5 and 57.6 % points during the same period for groups I and II, respectively.

On the other hand, the same empirical evidence (Figure 3.4) shows that the credit reallocation did not take place, or much less, in the rest of countries. Figure 3.9 shows that domestic banks of group III increased the ratio of public over private credit from 2.3% in 2009 to 4.7% in 2012. But, this ratio falls from 20.5% in 2009 to 13.5% in 2012 in group IV, following the tendency of previous years. In the model, group III experiences a reduction to the minimum of this ratio (0 %) in an opposite direction to the data; and for group IV, we can see that the difference between the data and model is much more noticeable, in the end it falls by 1.67% points during the sovereign crisis (2009-2012). Therefore, we can see that, for the rest of countries, the data and results are not so clearly marked in the same direction compared to GIIPS.

Relatedly, Figure 3.10 presents the proportion of public debt bought by residents ($b_t^j/d_t^j$), values of the data and baseline model.[8] The model presents much more volatility of this measure compared to the data. This figure shows how domestic investors significantly increased their bond holdings of public debt of the country in GIIPS during the period 2009-2012 as the interest rate were increasing, both in the data and model. Specifically, in the data, group I goes from 54.8% of debt in domestic hands in 2009 to 66.9% in 2012,

[7]The data on government debt holdings and private productive credit of domestic banks is taken from the ECB statistics (MFI balance sheets), and are outstanding amounts (stocks). Private productive credit refers to loans to NFC and households, excluding credit for consumption and lending for house purchase.

[8] Government debt holdings of residents are taken from the "Bruegel Dataset of Sovereign debt holdings" by Merler (2014).

and group II from 49.8% to 60.3%, respectively. In the model, they reach much higher proportions; for example, residents of group I and II buy all public debt in 2011 and 2012 (100%), increasing their holdings by 45.2 and 50.2 percentage points during the sovereign crisis, respectively.

On the contrary, as we can see in Figure 3.10, the proportion of public debt in domestic hands in the rest of countries does not experience a change in tendency as it does in GIIPS, in fact, it continues falling or stabilizing in the period 2009-2012. In the data, this proportion falls by 2.93 % points for group III and increases by 5 % points for group IV during this period. The changes of these proportions are -12.00 and -7.96 % points in groups III and IV, respectively, during the same period in the model.

According to the results, despite the high volatility of debt holdings with respect to bond yields, we can conclude, as in the data, that the crowding-out effect is much weaker in the rest of countries compared to GIIPS.

Hence, I consider that is a good model to do policy evaluation because it generally captures the fiscal performance, summarized by debt-to-GDP ratio, and the lending behavior of domestic residents in the several groups.

### 3.4.2 Model with Eurobonds

Henceforth, all the analysis is based on using the artificial economies to do policy evaluation for all the groups of countries. All the outcomes are presented for all scenarios, $s = \{l, m, h\}$, of Eurobond yields.

The first result is about the fiscal policy implications of Eurobonds. I test the key hypothesis of the Eurobonds proposal, the authors propose new bonds that mean a saving to taxpayers, but imply an incentive-driven policy at the same time. They argue that Eurobonds accomplish these two objectives thank to a resulting lower average borrowing costs, which would afford lower taxes, and an improved fiscal discipline due to increasing borrowing rates at the margin, respectively. Figure 3.11 illustrates the total annual interest rate payments. We can see that GIIPS win from the European solidarity because their total borrowing costs are reduced in all scenarios of Eurobond yields.

But, at the same time, the debt-elastic interest-rate premium guarantees an increasing borrowing cost at the margin. Figure 3.12 graphs marginal costs of public borrowing in all economies and scenarios. Group II for $r^e_{h,t}$ in 2010, for example, has a slightly higher borrowing rate at the margin than in the baseline model.[9] Specifically, even though the Eurobond yield $r^e_{h,2000}$ is equal to 3.57%, the marginal cost of borrowing $r^{II}_{h,2000}$ ( 4.22%) is larger than the actual bond yield $r^{II}_{2010}$ (4.04%) due to the endogenous interest-rate premium of the policy $\psi^{II}_h(e^{d^{II}_{2010}/y^{II}_{2010}-\phi}-1)$, which is equal to 0.65%. In addition, the interest-rate premiums in 2011 and 2012 are equal to 0.88% and 1.04%, respectively, thus increasing with unsustainable debt paths (see Figure 3.13). Therefore, countries with large debt-to-GDP ratios have the strongest incentive to undertake fiscal adjustments.

Group I is able to reduce the interest payments (see Figure 3.11) in all cases, and the interest rate at the margin is also below the baseline model is spite of the fact that the stock of debt surpasses the 60% of GDP; specifically, the debt-to-GDP ratio is equal to 67.68 and 71.88 percent in 2011 and 2012, respectively, when Eurobond yields are high (see Figure 3.13).

On the other hand, groups III and IV share risk from GIIPS if Eurobond yields are not so low, Figure 3.11 and 3.12 show that if Eurobond yields are high ($r^e_{h,t}$) and medium ($r^e_{m,t}$) both interest payments and borrowing rates at the margin are higher than actual rates. If Eurobond yields are low, both interest payments and marginal costs of borrowing for group III are lower than in the baseline model. In group IV, interest payments are lower than in the baseline model for $r^e_{l,t}$, but the interest rate at the margin is larger the actual rate in 2010 due to the interest-rate premium of national bonds, which is equal to 0.54%.

Thus, the direction of the results on these countries could reverse depending on the valuation of the debt in Eurobonds.

In order to close this point on fiscal implications we should study how Eurobonds affect the implementation of fiscal policy. That is, to corroborate if, apart of being an incentive-driven policy, Eurobonds will reduce the fiscal pressure. Table 3.3 delivers the resulting consumption tax rates, $\left\{\tau^{cj}_t\right\}^T_{2013}$, that fulfill the government's intertemporal budget constraints, (3.19) and (3.37), respectively. We can see that Eurobonds might ease the feasibility of fiscal policy because these tax rates are directly proportional to Eurobond yields. That is, taxes can be reduced when Eurobond yields fall, even being negative (subsidy) in some scenarios and groups. Actually, consumption taxes are below the baseline case in all groups when Eurobond yields are low. On the other hand, the rest of countries need higher taxes than the baseline ones for $r^e_{m,t}$ and $r^e_{h,t}$. Therefore, we see

[9] We can observe the same and for the same year in group II for $r^e_{m,t}$ and in group IV for $r^e_{l,t}$ .

that the policy means a saving to taxpayers of GIIPS in all scenarios $s$.

The second main result is about fiscal performance. The debt-to-GDP ratios of all the groups are plotted in Figure 3.13, the policy allows GIIPS to reach lower debt-to-GDP ratios with respect to the baseline model in all scenarios. The cause is a reduction of the debt burden together with a boost in economic activity, tax collection increases due to growing tax bases of consumption and income (Figures 3.15 and 3.16 display GDP and consumption, respectively). Table 3.4 summarizes the percentage point difference in the final debt-to-GDP for each scenario with respect to the baseline model. Group I gets a minimum (maximum) reduction of -14.96 (-42.61) percentage points. Group II gets a minimum (maximum) reduction of -10.64 (-59.04) percentage points.

On the contrary, groups III and IV end up with a debt-to-GDP ratio that is 24.19 and 15.43 (9.47 and 1.43) % points, respectively, higher than in the baseline model when Eurobond yields are high (low). However, for the low Eurobond yields, they reduce the final stock of debt by -6.29 and -7.09 % points , respectively.

The third main result is that the implementation of the policy would help to solve the crowding-out effect on productive investment in GIIPS. Figure 3.14 shows the effect of the policy on the credit allocation through the bond holdings-capital ratio, $b_t^j/k_t^j$. We observe that public over private credit under Eurobonds is much lower than in the baseline case for GIIPS. This ratio is always decreasing in GIIPS for low Eurobond yields, $r_{l,t}^e$, i.e. no crowding-out effect would take place. In group I, the government debt only displaces investment in 2011 for the high yields, but this group ends up with a bonds-capital ratio much lower than in the baseline model. In group II, the sovereing bonds crowd out investment in 2011 and 2012 when Eurobond yields are $r_{h,t}^e$. However, this group ends up with a bonds-capital ratio in 2012 much lower than in the baseline model when the medium and high scenarios are the case. In additon, we observe that this ratio for $r_{m,t}^e$ and $r_{h,t}^e$ in 2010 is larger with respect to the baseline ratio due to a higher marginal interest rate in this period compared to the actual rate, see Figure 3.12.

In the rest of countries, when Eurobond yields are high, the bonds-capital ratio is higher than in the baseline model. On the contrary, this ratio is always lower or equal when Eurobond yields are low. For $r_{m,t}^j$, it decreases from 1.6 percent to zero in group III and, in group IV, it is larger than in the baseline model.

Consequently, the output is in accordance to the evolution of productive investment. Figure 3.15 presents the GDP for all groups. It points out that group I reaches a higher GDP in all periods with Eurobonds. For group II, as in the credit allocation, GDP in 2010 is lower than in the baseline model for $r^{j}_{m,t}$ and $r^{j}_{h,t}$. Table 3.5 displays the percentage difference in output for each scenario with respect to the baseline model. Group I experiences a minimum (maximum) increase of 5.67% (18.85%) in the GDP, average over 2009-2012, and Group II, of 2.03% (16.40%).

However, group III reach a higher GDP in all the periods when $r^{e}_{l,t}$ occurs, and group IV only in 2012. Following Table 3.5, we can see that the average GDP grows by 2.74% and 1.00% above the baseline in group III and IV, respectively. In the medium (high) yields, the average GDP of group III experiences a reduction of 3.22% (8.22%) with respect to the baseline, and that of group IV falls by 0.5% (4.78%). Therefore, we observe that in the rest of countries the policy is beneficial in terms of output depending on Eurobond yields, $r^{e}_{s,t}$.

The last result is about welfare. The welfare measure used is the constant percentage amount by which consumption must be increased in all periods of 2009-2012 in the baseline economy so as to yield the same utility as under Eurobonds for the same years. The evolution of consumption in our period of interest is graphed in Figure 3.16. Table 3.6 summarizes the welfare gains in the three scenarios of Eurobonds with respect to the baseline model. We can see that GIIPS are better off in all scenarios of Eurobond yields considered in this paper. In the low scenario, there are large welfare gains for all groups. On the other hand, the welfare gains in groups III and IV are -12.38% (-26.70%) and -1.55% (-16.84%) when Eurobond yields are $r^{e}_{m,t}$ ($r^{e}_{h,t}$), respectively.

## 3.5 Conclusions

The results are robust because the analysis is very prudent when computing the return of Eurobond holdings and the debt-elastic interest-rate premium, in the sense that a large range of possible yields are considered for Eurobonds and the parameter of the national-debt interest-rate premium is computed in a conservative way, it is estimated, with an upward bias, considering the actual spreads with respect to Eurobond yields, and besides, we assume that no country can issue national debt at a borrowing rate below Eurobonds.[10] The first may not be the case because spreads could even be reduced below the actual with the new institutional set-up. The latter makes a lot of sense if Eurobonds are low

[10] Remember that our formulation requires to restrict the parameter $\psi^{j}_{s}$ to be non-negative, see Subsection 3.2.2.

but, if Eurobond yields are medium or high it would have been possible to consider that, for example, group IV could even have issued debt below these interest rates when issuing national debt because it has actual bond yields lower than medium and high Eurobond yields all the years of the sovereign debt crisis despite its debt-to-GDP is higher than 60% of GDP. Remember that this group represents countries like Germany, Netherlands, Austria, and France where bond yields were lower than $r^e_{m,t}$ and $r^e_{h,t}$ during the sovereign debt crisis. Hence, setting $\psi^{IV}_m, \psi^{IV}_h = 0$ when their estimates are negative may overstate the negative results for group IV under these scenarios. The same applies to group III.

In the same direction, it is worth noting that the study underestimates additional benefits from the fact that if Eurobonds had been implemented before, the disciplinary benefits would have affected sooner, which would have made that the initial debt levels could have been much lower at the beginning of the sovereign debt crisis. This is very important because as we have observed, the initial conditions matter a lot for the transitional implications of the policy.

The aim of the paper was to investigate the implications of Eurobonds taking into account that the effects would have been heterogeneous for the members of the Euro area depending on two dimensions: initial debt and actual bond yields. The four-group analysis has done this job in the previous section.

Our policy of interest is a coordinated fiscal policy; therefore, we must look at the general results to evaluate it. To do so, the results has also been computed for all the groups as a whole. I study the overall results on fiscal policy, output, and welfare, respectively.

The first overall result on fiscal policy is about the implications of Eurobonds on the feasibility of this kind of policies. Table 3.3, in the last row, reports the average consumption tax rate that guarantee the implementability of all the government's intertemporal budget constraints. We can see that the average tax rates in low and medium Eurobond yields are lower than in the baseline model. Specifically, tax rates can be cut from 39.58 percent to 20.96 and 33.70 percent, respectively. On the other hand, taxes should be raised to 48.43 percent for high Eurobond yields.

Secondly, we summarize the overall fiscal performance using the total final stock of debt-to-GDP ratio. This debt is lower than in the baseline when Eurobond yields are low and medium. We can see in the last row of Table 3.4 that total final debt-to-GPD is -28.67 and -11.45 percentage points below the baseline in the low and medium scenarios, respectively. This is a very appealing result because the policy is not only able to reduce debt in GIIPS, it also gets an overall reduction of debt. On the other hand, the final overall debt is 4.99 % points above the baseline when Eurobonds are high.

The total value added (GDP or output) is also another interesting measure for our analysis because it maximizes the possibilities of consumption, investment, and public expenditure at every period. The differences in total GDP appear in the last row of Table 3.5, we can observe that total GDP grows on average (2009-2012) significantly above the baseline when Eurobond yields are low and medium, in fact, it grows by 10.01% and 3.78% more, respectively. Under high yields, however, the overall output is on average -1.40% lower than in the baseline model.

The overall welfare remarkably rises with respect to the baseline model in the low and medium scenarios of Eurobond yields. Table 3.6 shows, in the last row, that total welfare gains for low and medium Eurobond yields are 42.05% and 14.89%, respectively. On the other hand, the overall welfare loss is equal to -5.52% for high Eurobond yields.

Up to this point, after having analyzed all the results, the most important conclusion we can get is that the Euro area has the opportunity to implement a policy that could be very profitable for all the groups of countries in order to address times of soaring sovereign spreads. Strictly speaking, following the analysis of total output and welfare, we observe that, if Eurobond yields are low, all the groups have a significant higher growth and welfare. Groups I, II, III, and IV have an output in the period 2009-2012 that is on average 18.85%, 16.40%, 2.74%, and 1.00% larger than in the baseline model (Table 3.5), respectively, and the welfare gains in the period 2009-2012 are 93.74%, 64.81%, 7.84%, and 18.54% (Table 3.6), respectively. In addition, the final debt-to-GDP ratio (2012) is -42.61, -59.04, -6.29, and -7.09 % points (Table 3.4) lower than in the baseline model in groups I, II, III, and IV, respectively. As it is assumed in the theory, this opportunity implies the optimal Euro-area institutions because this scenario ($r^e_{l,t}$) is only possible if all the members, without exception, commit to a more integration, pooling debt under joint and several liability as if they were a unique country, and to a more automatic Euro-area fiscal mechanisms, i.e. having an orderly planned mechanism of default (Eurobonds as senior debt and national bonds as junior). On the other hand, if the institutional reforms to a more integration and automatic mechanisms are not the case, Eurobond yields would be medium and high, which are not as good as the previous one or even worse than actual yields as the results point out. Particularly, we have seen that if this qualitative change in the Monetary Union institutions is not materialized, implying $r^e_{m,t}$ or $r^e_{h,t}$, the rest of countries lose in terms of GDP, welfare, and debt level.

In order to conclude, the degree of success of this unified policy strongly relies on the commitment of all groups to undertake the optimal institutional reforms, which are a suitable level of integration and a proper mechanisms, that make Eurobonds as close as possible to the benchmark German bonds or the US treasury bonds.

### 3.5.1 Extensions

This paper instead of solving an optimal government problem or an optimal fiscal policy problem (Ramsey equilibrium) is a competitive equilibrium analysis because its purpose was to analyze the overall and heterogeneous effects of Eurobonds on the several countries of the Euro area. Hence, it might have potential extensions in future essays. A first extension would be to study Eurobond proposals in the context of the literature on default and self-fulfilling debt crises, which is conformed by Calvo (1988), Cole and Kehoe (2000), Arellano (2008), Mendoza and Yue (2012), Corsetti and Dedola (2012), Aguiar et al. (2013), and Conesa and Kehoe (2013), among others. A second margin of research would link with the literature of optimal fiscal policy in small open economies in order to see what would be the role of Eurobonds, the main contributions to this literature are Correia (1996) and Razin and Sadka (1991a), among others. And finally, another extension could model the moral hazard, in a principal-agent problem when a country (agent) belongs to a monetary/fiscal union (principal), in order to study the implications of incentives when there are policies that affect all the members of the union, like Eurobonds.

# Tables and figures

Table 3.1: Parameter estimations of the debt-elastic interest-rate premium

| Coefficient\Group $j$ | I:Spain | II:Italy | III:Finland | IV: France |
|---|---|---|---|---|
| $\hat{\psi}_l^j$ | 0.2757*** (0.0301184) | 0.0172*** (0.0036686) | - | 0.0235*** (0.0030874) |
| $\hat{\psi}_m^j$ | 0.2033*** (0.017941) | 0.0124*** (0.0026163) | - | $-0.0016$ (0.0013141) |
| $\hat{\psi}_h^j$ | 0.1310*** (0.0075826) | 0.0076*** (0.0015803) | - | $-0.0267$*** (0.0031947) |
| Note: Newey-West Standard Errors are reported in parentheses. * , **, *** indicate significance at the 10%, 5%, and 1% level, respectively. | | | | |

Table 3.2: Parameters and initial values

| Group\Parameter | $\alpha^j$ | $\delta^j$ | $k_o^j/y_o^j$ | $\beta^j$ | $\psi_l^j$ | $\psi_m^j$ | $\psi_u^j$ | $d_o^j/y_o^j$ | $b_o^j/d_o^j$ |
|---|---|---|---|---|---|---|---|---|---|
| I:Spain | 0.3578 | 0.0584 | 2.6704 | 0.9617 | 0.0860 | 0.0620 | 0.0380 | 54% | 54.8% |
| II:Italy | 0.4342 | 0.0446 | 3.8223 | 0.9587 | 0.0172 | 0.0124 | 0.0076 | 116.4% | 49.8% |
| III:Finland | 0.3726 | 0.0536 | 3.1609 | 0.9640 | 0.0235 | 0 | 0 | 43.5% | 11.9% |
| IV:France | 0.3381 | 0.0576 | 2.4205 | 0.9648 | 0.0235 | 0 | 0 | 79.2% | 40% |

Table 3.3: Consumption tax rate (percent)

| Group | Consumption Tax (constant $\tau_t^{cj}$, $t = 2013, ..., T$) | | | |
|---|---|---|---|---|
| | $r_{l,t}^e$ | $r_{m,t}^e$ | $r_{h,t}^e$ | Baseline |
| I | 15.08 | 20.72 | 27.93 | 37.27 |
| II | -12.48 | 1.09 | 17.00 | 23.74 |
| III | 77.29 | 99.56 | 123.89 | 84.81 |
| IV | 3.93 | 13.44 | 24.90 | 12.51 |
| Average | 20.96 | 33.70 | 48.43 | 39.58 |

Table 3.4: Final debt-to-GDP ratio (2012) and differences

| Group | Final Debt-to-GDP (%) | | | | Difference (% points) | | |
|---|---|---|---|---|---|---|---|
| | $r^e_{l,t}$ | $r^e_{m,t}$ | $r^e_{h,t}$ | Baseline | $r^e_{l,t}$ | $r^e_{m,t}$ | $r^e_{h,t}$ |
| I | 44.22 | 56.74 | 71.87 | 86.83 | -42.61 | -30.10 | -14.96 |
| II | 97.85 | 126.15 | 146.24 | 156.88 | -59.04 | -30.74 | -10.64 |
| III | 44.98 | 60.75 | 75.46 | 51.27 | -6.29 | 9.47 | 24.19 |
| IV | 77.47 | 86.00 | 100.00 | 84.57 | -7.09 | 1.43 | 15.43 |
| Total | 68.99 | 86.22 | 102.66 | 97.67 | -28.67 | -11.45 | 4.99 |

Table 3.5: Percentage change in GDP

| Production | 2010 | | | 2011 | | | 2012 | | |
|---|---|---|---|---|---|---|---|---|---|
| Group\Δ(%) | $r^e_{l,2010}$ | $r^e_{m,2010}$ | $r^e_{h,2010}$ | $r^e_{l,2011}$ | $r^e_{m,2011}$ | $r^e_{h,2011}$ | $r^e_{l,2012}$ | $r^e_{m,2012}$ | $r^e_{h,2012}$ |
| I | 9.44 | 6.60 | 3.97 | 17.48 | 11.46 | 4.50 | 29.63 | 19.62 | 8.53 |
| II | 0.02 | -1.16 | -1.60 | 14.42 | 6.49 | 2.09 | 34.77 | 15.43 | 5.60 |
| III | 1.95 | -1.02 | -3.76 | 2.94 | -2.98 | -8.08 | 3.35 | -5.66 | -12.82 |
| IV | -0.05 | -0.22 | -2.51 | 0.09 | -0.69 | -4.99 | 3.86 | -0.59 | -6.83 |
| Total | 2.28 | 0.68 | -1.15 | 9.25 | 3.65 | -1.45 | 18.48 | 7.00 | -1.60 |

| Production | Average (2009-2012) | | |
|---|---|---|---|
| Group\Δ(%) | $r^e_{l,t}$ | $r^e_{m,t}$ | $r^e_{h,t}$ |
| I | 18.85 | 12.56 | 5.67 |
| II | 16.40 | 6.92 | 2.03 |
| III | 2.74 | -3.22 | -8.22 |
| IV | 1.00 | -0.50 | -4.78 |
| Total | 10.01 | 3.78 | -1.40 |

Table 3.6: Welfare gains for the period of interest (2009-2012)

| Group | Welfare gains (percentage) | | |
|---|---|---|---|
| | $r^e_{l,t}$ | $r^e_{m,t}$ | $r^e_{h,t}$ |
| I | 93.74 | 57.00 | 24.54 |
| II | 64.81 | 28.94 | 5.16 |
| III | 7.84 | -12.38 | -26.70 |
| IV | 18.54 | -1.55 | -16.84 |
| Total | 42.05 | 14.89 | -5.52 |

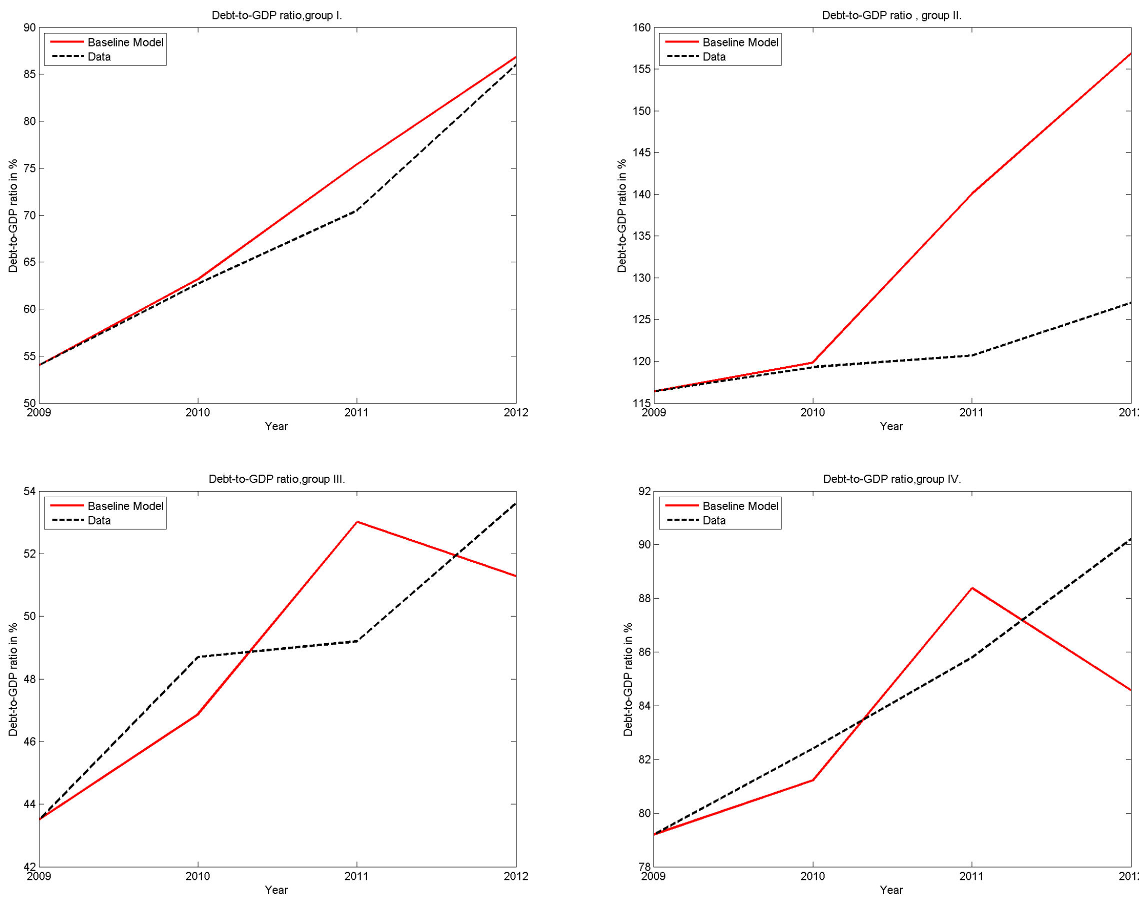



Figure 3.8: Debt-to-GDP ratio

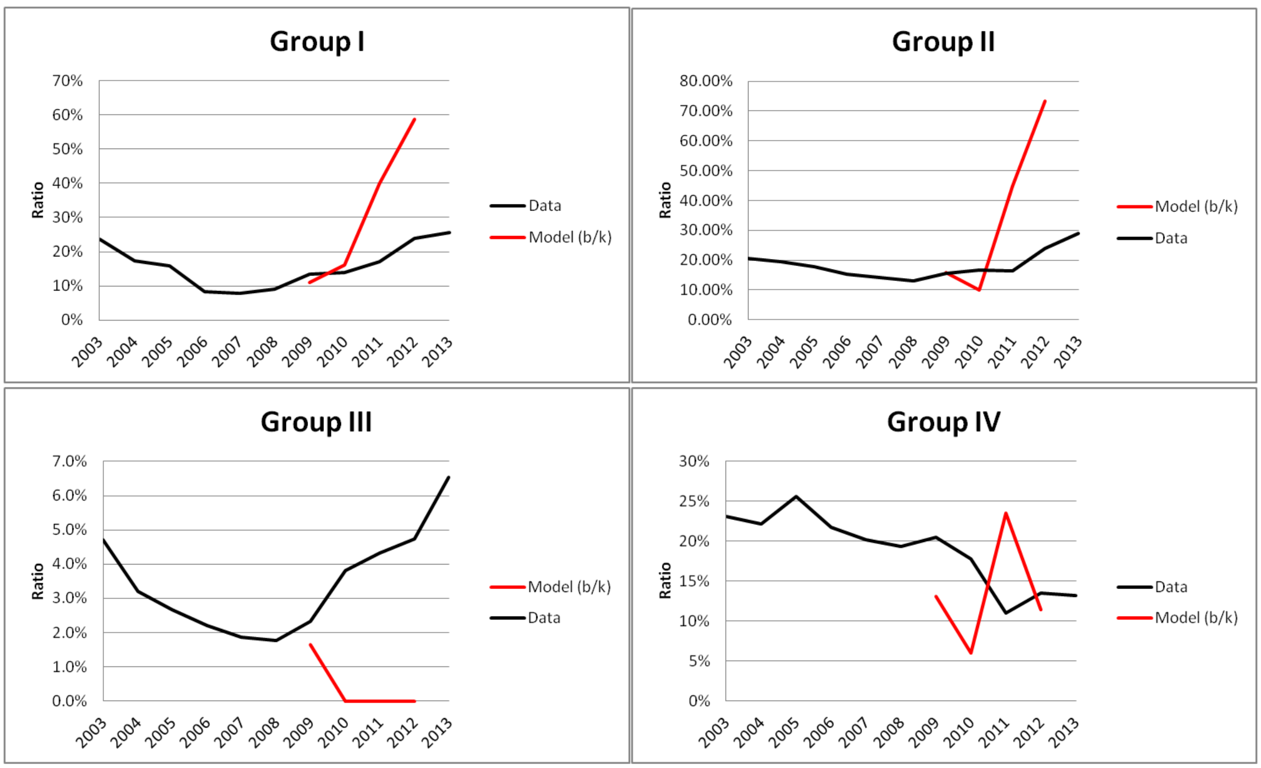



Figure 3.9: Sovereign debt holdings vs private productive credit

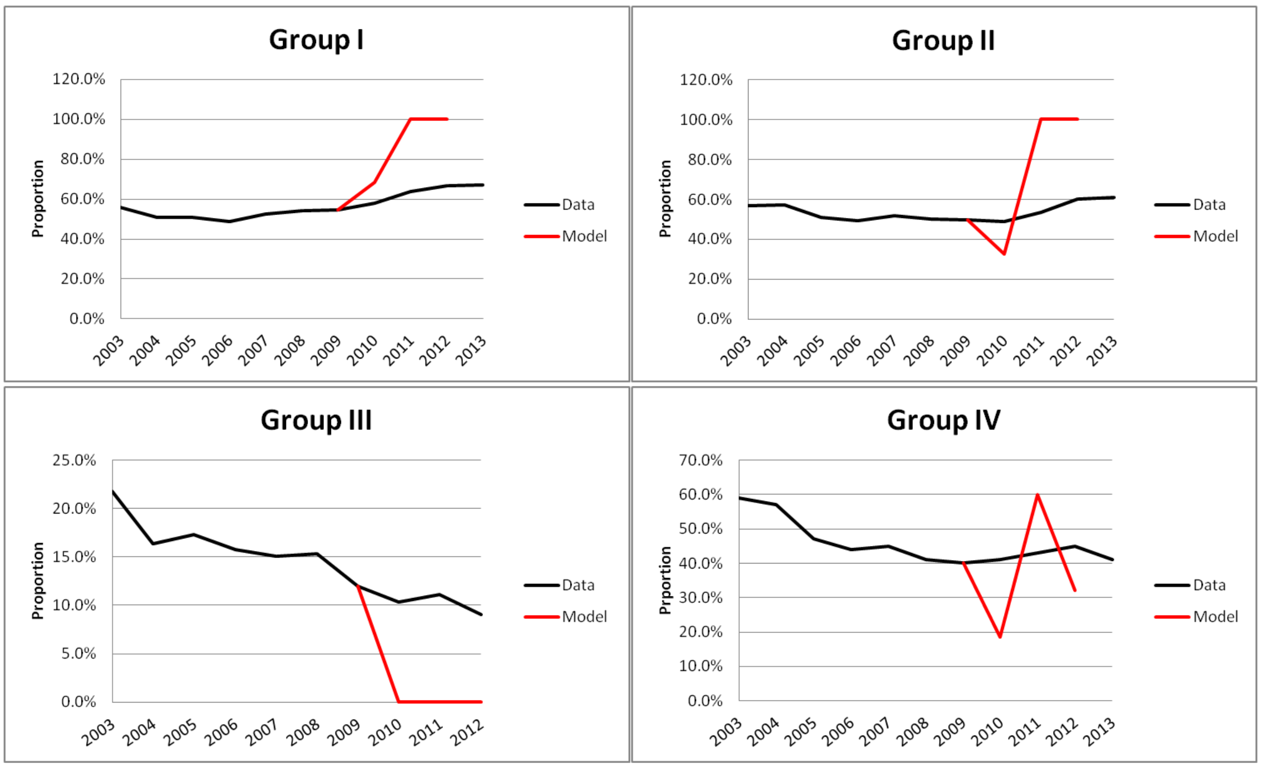



Figure 3.10: Resident and foreign holdings of government debt

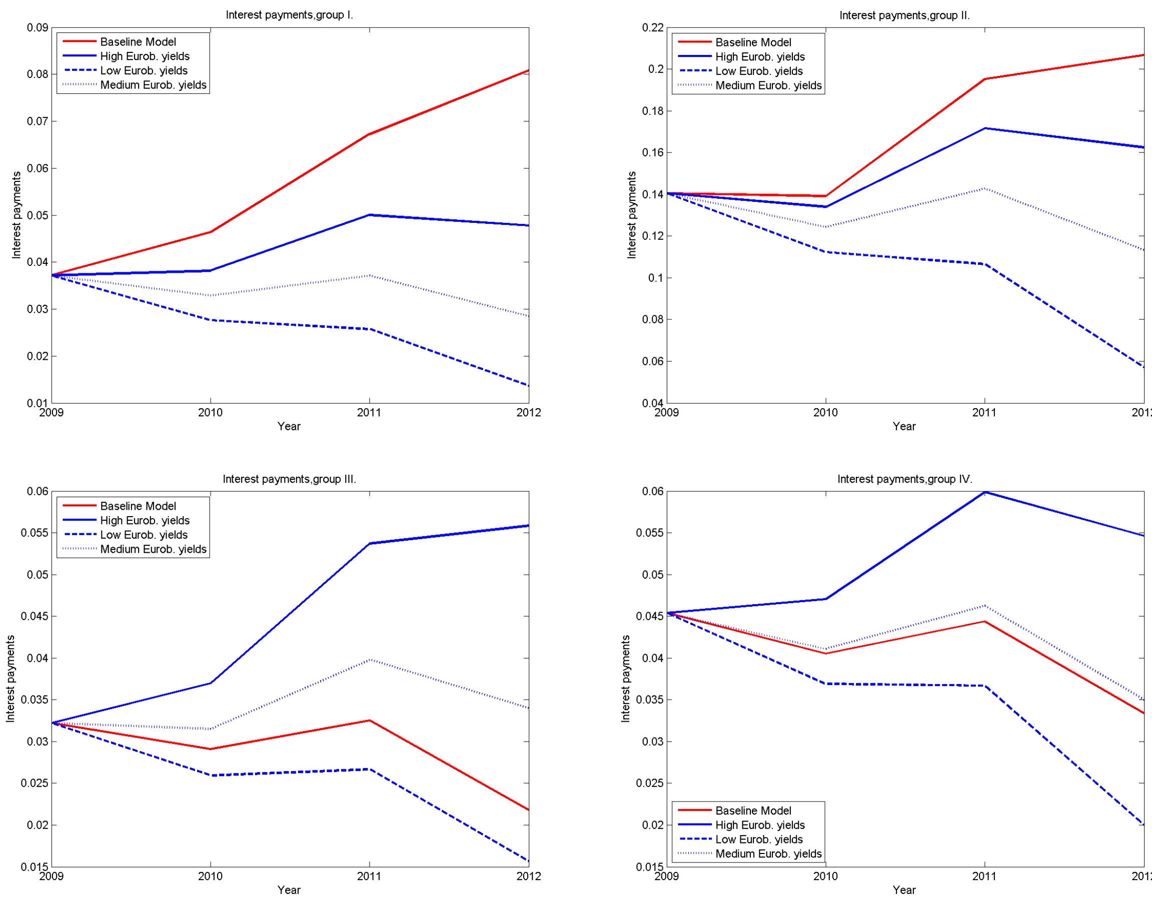



Figure 3.11: Comparison: interest rate payments

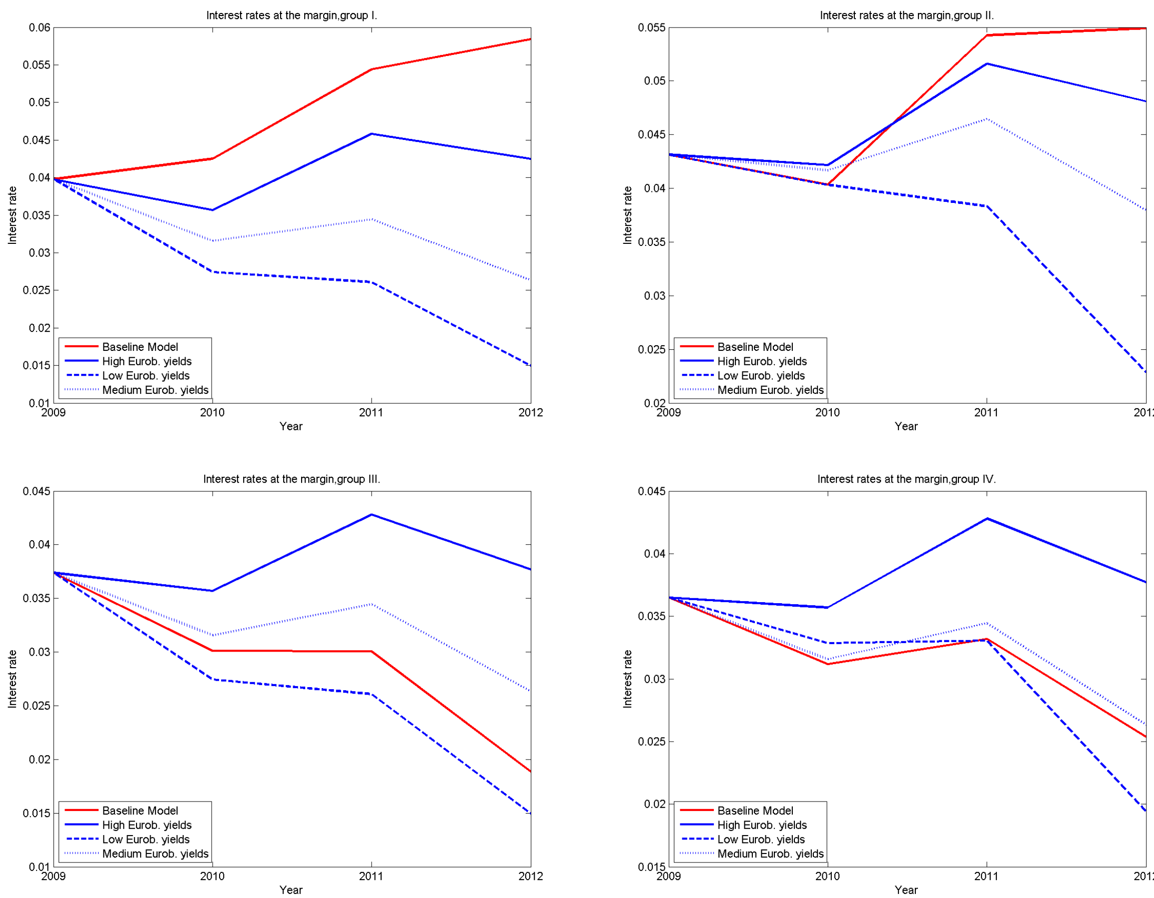



Figure 3.12: Comparison: interest rate at the margin

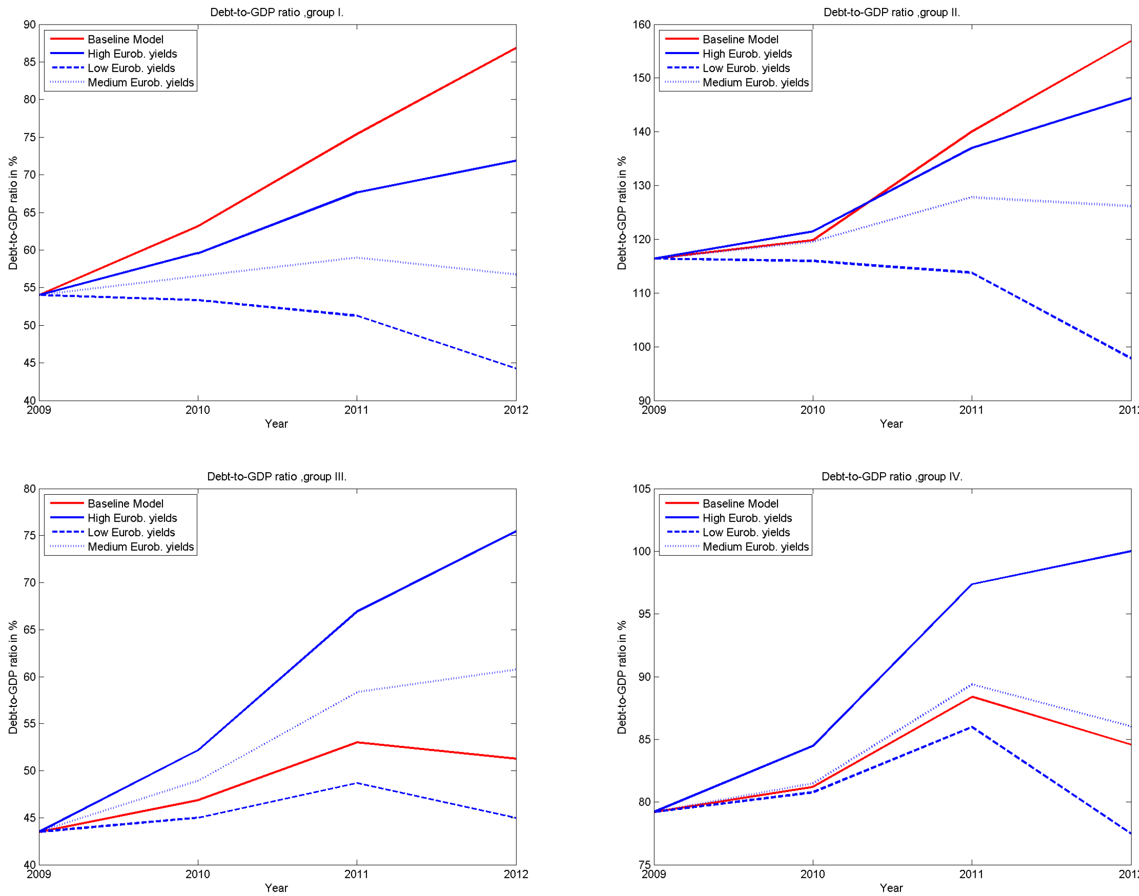



Figure 3.13: Comparison: debt-to-GDP ratio

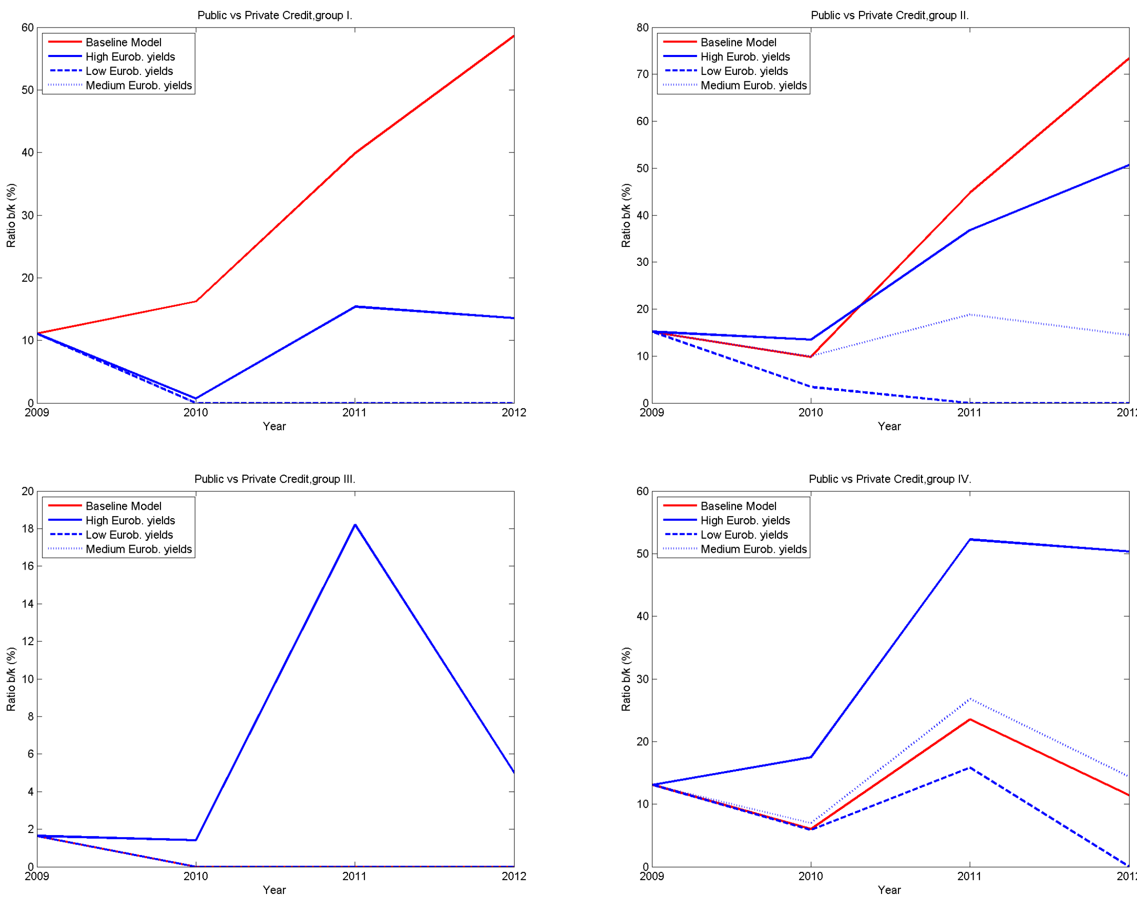



Figure 3.14: Comparison: bond holdings-capital ratio

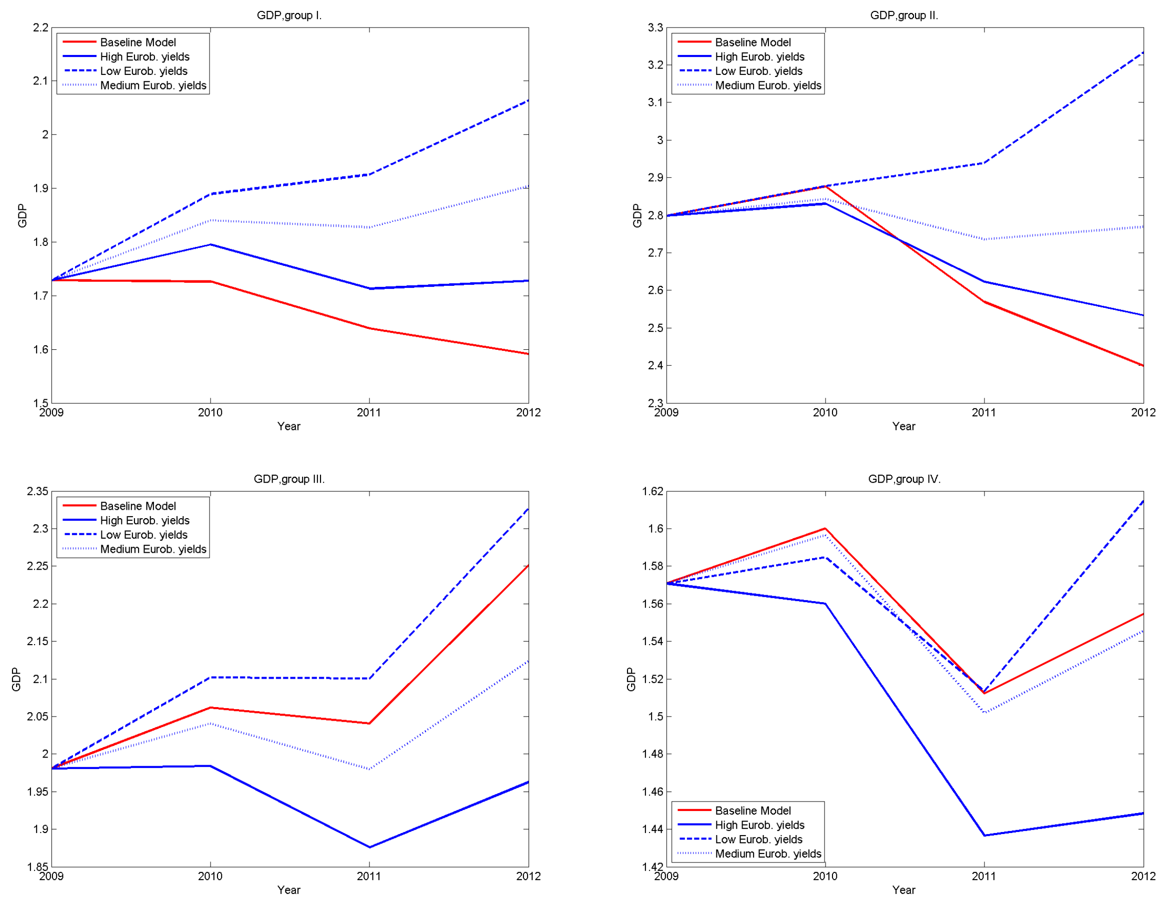



Figure 3.15: Comparison: GDP

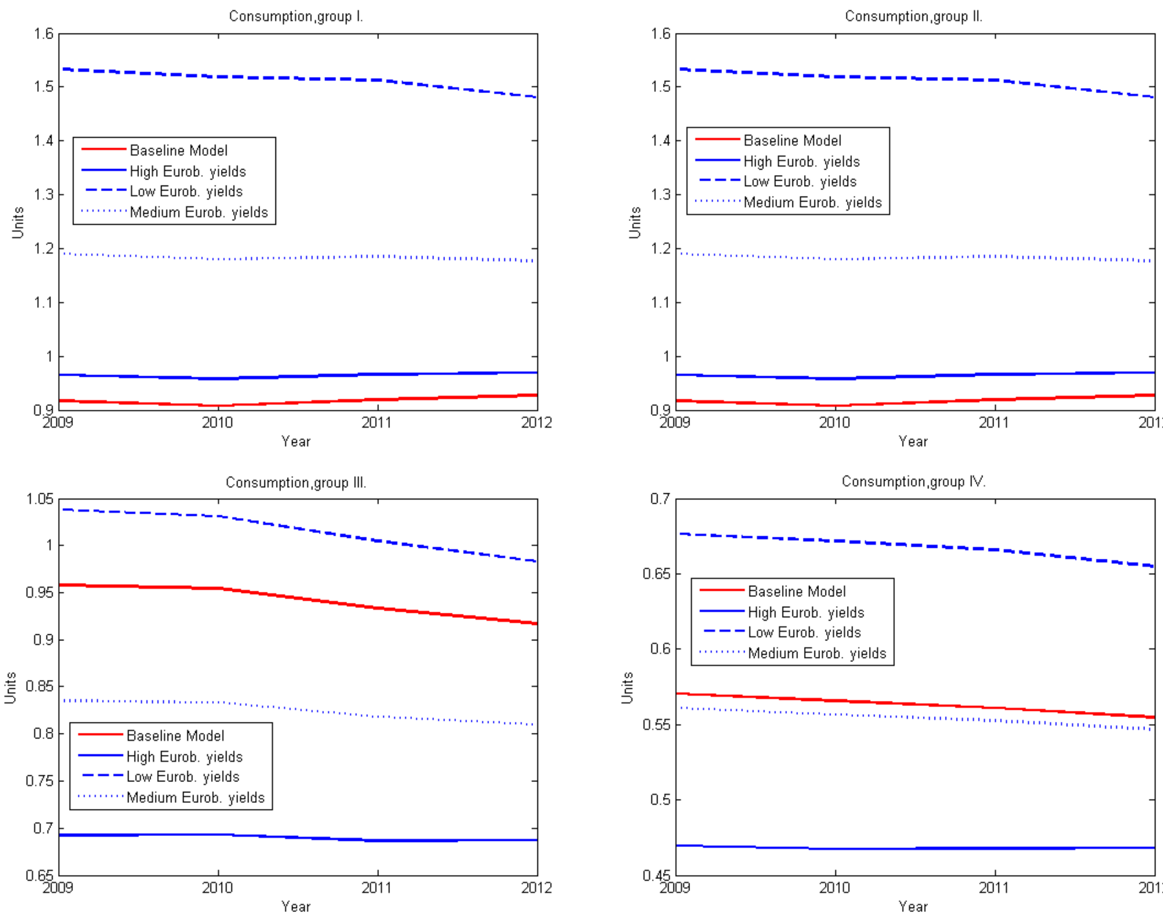



Figure 3.16: Comparison: consumption

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