Contractual Agreements and Endogenous Partnerships in Moral Hazard Economies

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To my family

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Chapter 1

Introduction

The main goal of this thesis is to analyse contractual agreements and partnership formation in two-sided markets that are characterised by moral hazard problems.

Throughout this thesis we will analyse markets where two different types of individuals are engaged in trading relationships. A two-sided market may consists of firms and investors, depository institutes and depositors, landowners and tenants, etc. Individuals in each side of the market assume distinct tasks while engaging in trade with those of the other side. Usually individuals tend to form partnerships by writing binding contracts, which are enforceable by law.

When decision making authority is delegated from one individual to another, contractual arrangements determine the allocation of surplus produced in a partnership. *Principal-agent* relationships are often employed to analyse such allocation problem.

In a principal-agent relationship, there are two types of individuals: the *principal*, usually assumed to design a contract, and the *agent*, who is supposed to take an action in order to produce an output. An action might be effort choice, investment decision, etc. The contract specifies the sharing of output between the principal and the agent. Often, the action of the agent is not verifiable by the principal, and hence a contract which is contingent on these actions cannot be enforced. In this case, we say that the relationship is subject to *moral hazard* problem. Such trading relationship often assumes that this action is costly for the agent. Therefore, a contract must provide proper incentives for the agent such that he chooses the action desired by the principal.

Often *limited liability* is seen as an origin of moral hazard problem. Limited liability implies that some legal precept prohibits a payment (to the agent) less than some minimum level. Therefore, the principal cannot offer something less than this minimum even if she would like to do so in order to achieve a desired outcome.

A two-sided market can be analysed by using *matching games.* A matching assigns an individual of one side to another individual of the other side. When two individuals are matched, they write binding contracts. The equilibrium of this game can be analysed by using *stability* as a solution concept. Stability in a two-sided market implies that there is no individual or no pair (matched or unmatched) who can do (strictly) better by changing the current configuration. In the first part of this thesis (Chapters 2 and 3), we analyse principal-agent relationships as a matching game. Also, a matching game allows us to deal with endogenous partnerships which in turn implies endogenous sharing of surplus.

In Chapter 2, we focus on the principal agent problem. Economic theory often concentrates on partnership between one principal and one agent, disregarding the presence of other principals and agents which might influence the optimal contractual arrangements in a *given* relation. In this chapter, we focus on an economy where there are more than one principal and more than one agent. We call this a *principal-agent market.* We model the market as a two-sided matching game between principals and agents. The main objective is to determine matching between principals and agent, and the set of optimal contracts for the principal-agent pairs. we consider an economy with several identical principals and several agents differentiated only with respect to their initial wealth. A pair of individuals, one principal and one agent, can enter into a relationship by signing a contract. This contract specifies the contingent payments that are to be made by the agent. Also it sets the level of investment, which together with a non-verifiable effort made by the agent, determines the probability of having a high return from the project the agent operates on. The initial wealth of the agent may not cover the amount to be invested and hence, the wealth differences imply differences in liability.

In chapter 3, we analyse an incentive model of a financial economy consisting of investors and firms. The chapter is analysed with a similar tool we develop in Chapter 2. In this financial economy, the firms need external sources to fund the projects they own. The investors are the potential source of funds to the firms. We analyse the equilibrium of the financial market using *stability* as the solution concept. A financial contract specifies that the intermediary finances the project and receives state-contingent claims on the project return. Each firm operates on his project after he obtains external finance and chooses a non-contractible effort level. Choice of effort influences the probability of having a high return from the project. Firm's liability is limited to his current income.

In chapter 4, we study the risk taking behaviour of banks when they can invest in a prudent asset or in a gambling asset. The two-sided market consists of banks and depositors. Banks mobilise deposit by offering deposit rates. We analyse the deposit market using a model based on spatial competition, and characterise the equilibrium. The banking sector described here consists of a finite number of banks. Banks are identical with respect to their equity capital. They choose between a *prudent asset* and a *gambling asset* to invest their total fund (equity plus deposit). The gambling asset on average yields lower return than the prudent asset, but if the gambling is successful it gives higher return. There is a continuum of depositors, each having one unit of monetary fund apiece, which they may place in a bank to earn deposit rate offered. The depositors are not insured in case the gamble fails.

Each chapter is self-contained, and thus can be read independently.

Chapter 2

The Principal-Agent Matching Market

(This chapter is jointly written with David Pérez-Castrillo)

2.1 Introduction

A large set of literature contributing to the theory of incentives analyses optimal contracts in principal-agent relationships when there exist asymmetries of information. When this asymmetry concerns an action, or a decision to be made by an agent, a *moral hazard* problem emerges. Several works analyse optimal contracts when only one principal and one agent interact, including the seminal works by Pauly [22], Mirrlees [20], and Harris and Raviv [15]. The *principal-agent contracts* involve the provision of incentives and typically lead to inefficiency due to the informational asymmetry.

The main goal of this chapter is to propose a useful framework to analyse the relationship between each principal-agent pair not as an isolated entity but as a part of an entire market where several principals and agents interact. In this framework, the utilities obtained by each principal and each agent are determined endogenously in the market. This allows us to improve over the previous approach where the agents' utilities are exogenously given and the principals assume all the bargaining power. We consider the simultaneous determination of the identity of the parties who meet (i.e., which agent is contracted by which principal) and the contracts they sign in an environment where each relationship is subject to moral hazard.[1]

We model the principal-agent economy as a *two-sided matching game*. An outcome of this economy is an endogenous matching and a set of contracts, one for each principal-agent pair under the matching. Roughly speaking, an outcome is said to be stable if there is no individual or no relevant pair objecting the existing outcome. The chapter studies the set stable outcomes of this principal-agent matching market.

In particular, we consider an economy with several identical principals and several agents differentiated only with respect to their initial wealth. A pair of individuals, one principal and one agent, can enter into a relationship by signing a contract. This contract specifies the contingent payments that are to be made by the agent. Also it sets the level of investment, which together with a non-verifiable effort made by the agent, determines the probability of having a high return from the project the agent operates on. The initial wealth of the agent may not cover the amount to be invested and hence, the wealth differences imply differences in liability.

We begin by providing a complete characterisation of the set of stable outcomes of the principal-agent economy. The first simple property we prove is that all the principals earn the same profit in a stable outcome. In particular, if the principals constitute the long side of the market, their profits are zero. The second feature is that the contracts offered in a stable outcome are optimal, i.e., it is not possible to increase the utility level of the principal without making the agent strictly worse-off. More interestingly, in a stable outcome, the matching itself is efficient, in the sense that it is the one that maximises productive efficiency. For example, if the agents are in the long side of the market, only the wealthier ones, i.e., the more attractive ones are matched. Third, the productive efficiency of a contract signed in a stable outcome increases with the wealth of a matched agent. That is, the richer the agent, closer his contract to the first-best. The additional surplus generated due to this increase in efficiency accrues to the agent. Finally, the contracts signed in a stable outcome of this economy are more efficient than principal-agent contracts, i.e., the contracts signed when the principals assume all the bargaining power.

The previous characteristics of the set of stable outcomes have very relevant policy implications when applied to particular environments. For example, consider an economy where landowners (principals) contract with tenants (agents) who are subject to limited liability. Suppose that the government would like to improve the situations of the tenants by endowing the agents with some additional money. Our analysis suggests that the government will be interested in creating wealth asymmetries among tenants since otherwise, the landowners would appropriate all the incremental surplus intended to the tenants.

From the point of view of matching theory, one can see our model as a generalization of the *assignment game* with several buyers and sellers described by Shapley and Shubik [30].[3] In the current model, a relationship is established through a contingent contract, rather than a price. The first distinguishing feature is that the surplus of each principal-agent pair, in our model, is determined endogenously. Next, the utility cannot be transferred between a principal and an agent on a one-to-one basis. In other words, unlike the assignment game, our model is a non-transferable utility game. We consolidate stability as a reasonable solution concept for this principal-agent matching market by proposing a simple mechanism in which each of the agents proposes a contract and each principal chooses an agent. We show that the equilibrium outcomes of this mechanism coincide with the set of stable outcomes of the matching market.

Serfes [29] analyses an economy where the agents have different attitudes towards risk and the principals own assets which are subject to different exogenous variability. He also models the economy as a two-sided matching game and characterises the stable outcome where the principals have all the bargaining power. In his model, a principal-agent pair cannot block an outcome with any contract, rather it is the principal who proposes a contract once a blocking pair is formed. The predictions of the model by Serfes [29] are different from those of the standard risk model where an isolated principal-agent pair is studied. In particular, there can be a positive, negative, or non-monotonic relationship between risk and incentives.

[3] [illegible]

A few other papers study agency problems with several principals and agents. In a tenancy relation Shetty [31], and Ray and Singh [23] propose a model where a set of principals compete for a continuum of agents in the presence of limited liability. Restricting themselves to linear contracts, they show that if the tenant s (agent) crop-share is unconstrained, wealthier tenants receive fixed-rent contracts, while poorer tenants receive sharecropping contracts.[2] Also in an economy with a continuum of (heterogeneous) participants in both sides, Legros and Newman[19] present sufficient conditions for matchings to be monotone when utility between partners is not fully transferable. In contrast with the above two papers, our framework can accommodate the analysis of economies with a few participants as well as those with a large number of participants. Mookherjee and Ray [21] analyse the optimal short term contracts in an infinitely repeated interaction among principals and agents who are randomly matched at each period. Finally, the work of Barros and Macho-Stadler [4] looks into a situation where several principals compete for an agent. They also find that the competition among the principals make the incentive contracts more efficient.

2.2 The Model

2.2.1 Principals and Agents

We consider an economy with a (finite) set of risk neutral *principals*, $\mathcal{P} = \{P, P_2, \ldots, P_n\}$ and a (finite) set of risk neutral *agents*, $\mathcal{A} = \{A, A^2, \ldots, A^m\}$. A principal might be a landowner, a lender or an employer. An agent is a tenant, a borrower or a worker. Principals are of identical characteristics. Agents differ with respect to their initial wealth. An agent A^i has an initial wealth w^i, which is known to the principals. Without any loss of generality, we order the wealth level as $w \geq w^2 \geq \ldots \geq w^m \geq 0$. The principals and the agents are matched in pairs and a contract is signed by each pair. We allow for the possibility that a principal or an agent can seek for an alternative partner and can sign a different contract. Hence, the matching is endogenous rather than being exogenous.

[2] T x 5
S 28 S 9, G P 4
s

2.2.2 Projects

When a principal-agent pair is formed,[4] the agent operates on a project, chooses effort level e from the set $\{0, 1\}$, and investment K is made, which is financed entirely by the principal. An agent incurs a disutility of e when he chooses the effort level e. The effort exerted is not contractible but the level of investment is.[5] Effort and investment influence the return of each project which is uncertain. Given an effort level e and investment K, let $\pi_e(K)$ be the probability of the event of success (denoted by S) and $1 - \pi_e(K)$, the probability of failure (denoted by F). Each project generates a return $y > 0$ in case of success. In case of failure, the return is 0. We assume (a) $\pi(K) > \pi_0(K)$, for all $K > 0$, (b) $0 \leq \pi_e(K) \leq 1$, for all $K > 0$ and $\pi_0(0) = 0$ and (c) $\pi_e^{I}(K) > 0 > \pi_e^{II}(K)$ for all $K > 0$ and $\lim_{K \to \infty} \pi_e^{I}(K) = 0$. Part (c) guarantees that the solution in K is interior. We denote by $\mathcal{M} = \{\mathcal{P}, \mathcal{A}, w, \pi\}$ the market, where $w = (w, \ldots, w^{m})$ denotes the vector of initial wealth of the agents in $\mathcal{A}$ and π represents the technology.

2.2.3 Contracts and Payoffs

A principal-agent pair (P_i, A^j) signs a contract, c, which is a three dimensional vector (θ_S, θ_F, K). We take the convention that the agent keeps the output. Then the first component of the contract, θ_S is the transfer to the principal in the event of *success* and the second component, the transfer in case of *failure*. The third component of c is the level of investment. Given a contract $c = (\theta_S, \theta_F, K)$ signed by a pair (P_i, A^j), let e_c be defined as the effort that maximises the agent's utility:[6]

$$e_c = \underset{e}{argmax} \{\pi_e(K)(y - \theta_S) - (1 - \pi_e(K))\theta_F - e\}. \tag{IC}$$

For a contract c, the effort chosen by the agent will be e_c given that the effort is not contractible. This is the *incentive compatibility* constraint. Moreover, we normalise the per unit opportunity cost of financing a project to 1. Then the expected utilities

[4] T

[5] G w w G

[6] C $e_c =$ 0 x w x

of the principal P_i and the agent A^j when they sign the contract c will be:

$$\begin{aligned} u_{P_i}(A^j, c) &= \pi_{e_h}(K)\theta_S + (1 - \pi_{e_h}(K))\theta_F - K \\ u_{A^j}(P_i, c) &= \pi_{e_h}(K)(y - \theta_S) - (1 - \pi_{e_h}(K))\theta_F - e_h. \end{aligned}$$

Notice that we have defined the expected utility of A^j *net* of the wealth w^j. The gross expected utility of A^j would be $u_{A^j}(P_i, c) + w^j$. For future notational convenience, we denote by $c^{null} = (0, 0, 0)$, the *null contract*. Under c^{null}, $u_{P_i}(A^j, c^{null}) = u_{A^j}(P_i, c^{null}) = 0$. We assume that for an agent, signing a contract c^{null} is equivalent to the situation where he is not contracted by any principal, i.e., his *reservation utility* equals 0. Agent s liability is limited to his current wealth. This imposes restrictions on the set of contracts. *Limited liability* implies

$$\theta_S \le y + w^j \,, \tag{LS}$$

$$\theta_F \le w^j \,. \tag{LF}$$

The assumption of risk neutrality together with limited liability makes the incentive compatibility constraint costly and hence, it *gives rise* to *moral hazard* in agent s effort choice. A sensible contract for a principal-agent pair must satisfy the incentive compatibility and limited liability constraints. Furthermore, neither an agent nor a principal would accept a contract with negative expected utility. That is, a contract for a pair (P_i, A^j) has to be acceptable to each member of the pair. We say that a contract c is *acceptable* for (P_i, A^j) if $u_{P_i}(A^j, c) \ge 0$ and $u_{A^j}(P_i, c) \ge 0$. We club all these natural restrictions into the following definition.[7]

Definition 1. *A contract is* **feasible for an agent** A^j *if it satisfies the restrictions of limited liability and acceptability.*

Denote by $\mathcal{X}^j$ the set of contracts feasible for agent A^j. From now on we will concentrate only on feasible contracts.

The incentive compatibility constraint implies that the agent may choose any of the two effort levels (high or low). In order to deal with interesting situations, we will assume, *from now on*, that the output y in case of success is high enough so that it is always optimal first, to establish a relationship and second, to set a contract

[7] N

that induces the agent to exert high effort. Hence, one can substitute the incentive compatibility constraint (IC) by the following:

$$(\pi(K) - \pi_0(K))(y - \theta_S + \theta_F) \geq 1. \tag{IC'}$$

We will denote by $\mathcal{Z}^j \subset \mathcal{X}^j$ the set of feasible contracts that also satisfy the incentive compatibility constraint (4.19). One particular class of contracts are the *principal-agent* contracts, where the principal assumes all the bargaining power. The principal-agent contract for the pair (P_i, A^j), denoted c^{ij}, solves the following programme:

$$\max_{c \in \mathcal{Z}^j} u_{P_i}(A^j, c). \tag{P1}$$

Given the limited liability constraints, the moral hazard problem is typically costly for the principal, i.e., she earns lower profits compared to the *first best* situation, where she does not face any moral hazard problem. This happens if agent's wealth is below the level which makes the limited liability constraints no longer binding. Denote by w^B this threshold level of initial wealth. Next, we show that if the principal has all the bargaining power, she strictly prefers hiring an agent with higher wealth if the *first best* has not already been reached.

Proposition 1. *If $w^j > w^k$ and $w^j < w^B$, then $u_{P_i}(A^j, c^{ij}) > u_{P_i}(A^k, c^{ik})$.*

Proof. See Appendix A. □

2.2.4 Matching

Principals and agents are matched in pairs and when a pair is formed, a contract is signed. The following three definitions describe a matching and a relevant outcome of this principal-agent economy.

Definition 2. *A (one-to-one)* **matching** *for $\mathcal{M}$ is a mapping $\mu : \mathcal{P} \cup \mathcal{A} \to \mathcal{P} \cup \mathcal{A}$ such that (i) $\mu(P_i) \in \mathcal{A} \cup \{P_i\}$ for all $P_i \in \mathcal{P}$, (ii) $\mu(A^j) \in \mathcal{P} \cup \{A^j\}$ for all $A^j \in \mathcal{A}$ and (iii) $\mu(A^j) = P_i$ if and only if $\mu(P_i) = A^j$ for all $(P_i, A^j) \in \mathcal{P} \times \mathcal{A}$.*

The definition implies that a matching for a market $\mathcal{M}$ is a mapping which specifies that either each individual of one side of the market is assigned to another individual of the other side or, the individual remains alone. We say that the pair (P_i, A^j) is *matched* under μ if $\mu(P_i) = A^j$ (or, equivalently, $\mu(A^j) = P_i$).

Definition 3. *A menu of contracts C* **compatible with a matching** *μ for $\mathcal{M}$ is a vector of contracts, $C = (c_1, ..., c_n, c^1, ..., c^m)$ such that (a) $c_i = c^j$ if $\mu(P_i) = A^j$ and c^j is feasible for (P_i, A^j), (b) $c_i = c^{null}$ if $\mu(P_i) = P_i$ and (c) $c^j = c^{null}$ if $\mu(A^j) = A^j$.*

Definition 4. *An* **outcome** *(μ, C) for the market $\mathcal{M}$ is a matching μ and a menu of contracts C compatible with μ.*

The outcomes of the market we describe here are endogenous. This endogeneity has two aspects. First, the contracts signed by the principals and the agents are endogenous. In the principal-agent theory, considerable attention has been paid in order to analyse the contracts that prevail in a given (isolated) principal-agent relationship. The second aspect is that the matching itself should be endogenous. We will approach this perspective in the same vein as the matching theory. We require that a *reasonable* outcome should be immune to the possibility of being *blocked* by any principal-agent pair (as well as by any single individual). Consider an outcome (μ, C). If there is a principal-agent pair which can sign a feasible contract such that both the principal and the agent are strictly better-off under the new arrangement compared to their situation in the outcome (μ, C), then such an outcome is not reasonable. This idea corresponds the notion of stability.

Definition 5. *An outcome (μ, C) for the market $\mathcal{M}$ is* **stable** *if there does not exist any pair (P_i, A^j) and any contract $c^{ij} \in X^{ij}$ such that $u_{P_i}(A^j, c^{ij}) > u_{P_i}(\mu(P_i), c_i)$ and $u_{A^j}(P_i, c^{ij}) > u_{A^j}(\mu(A^j), c^j)$.*

The above definition makes sure that there does not exist any principal-agent pair that can *block* the current outcome, signing a feasible contract c^{ij} between them. Moreover, since all the contracts in a stable outcome are feasible, this implies that a stable outcome is also *individually rational*.

2.3 The Set of Stable Outcomes

In this section we characterise the set of stable outcomes of the market $\mathcal{M}$. We start by stating two important properties of a stable outcome.

First, all the contracts in a stable outcome are optimal. By optimality we mean that there is no possibility of improving the utility of one individual in a principal-agent pair without making the other individual worse-off. The following lemma states the optimality property.

Lemma 1. *All the contracts in a stable outcome are optimal.*

Proof. Suppose (μ, C) is stable, but the contract $c \in C$ signed by P_i and A^j, where $\mu(A^j) = P_i$, is not optimal. Then there exists a contract c^d, feasible for (P_i, A^j) such that (i) $u_{P_i}(A^j, c^d) > u_{P_i}(A^j, c)$ and (ii) $u_{A^j}(P_i, c^d) > u_{A^j}(P_i, c)$. In that case (P_i, A^j) will block (μ, C) with c^d. This contradicts the fact that (μ, C) is initially stable. □

It is interesting to notice that the optimality of a contract between a principal and an agent in any stable outcome is guaranteed by the possibility that the same pair can block the initial outcome with a different contract. Another property of stable outcomes is that no principal can gain more than any of her counterpart does. The profits of all the principals are equal. Lemma 2 proves this assertion.

Lemma 2. *In any stable outcome* (μ, C), $u_{P_1}(\mu(P_1), c_1) = u_{P_2}(\mu(P_2), c_2)$ *for any* $P_1, P_2 \in \mathcal{P}$.

Proof. Suppose (μ, C) is a stable outcome and $u_{P_1}(\mu(P_1), c_1) > u_{P_2}(\mu(P_2), c_2)$. We show that there exists a contract $c^d \in C$ such that $(P_2, \mu(P_1))$ blocks the outcome with c^d. First, note that $\mu(P_1) \in \mathcal{A}$, otherwise $u_{P_1}(\mu(P_1), c_1) = 0$. Suppose $c_1 = (\theta_S, \theta_F, K)$ and consider $c^d = (\theta_S - \varepsilon, \theta_F - \varepsilon, K)$ with $\varepsilon > 0$.[8] It is easy to check that $c_{e_1} = c_{e^d}$. Hence, for ε small enough, $u_{P_2}(\mu(P_1), c^d) = u_{P_1}(\mu(P_1), c_1) - \varepsilon > u_{P_2}(\mu(P_2), c_2)$ and $u_{\mu(P_1)}(P_2, c^d) = u_{\mu(P_1)}(P_1, c_1) + \varepsilon > u_{\mu(P_1)}(P_1, c_1)$. Therefore, $(P_2, \mu(P_1))$ blocks (μ, C) with c^d and hence the lemma. □

The above lemma states the intuitive property that, when the principals are identical, they must obtain the same profits in a stable outcome. This property is no longer valid if we consider some heterogeneity among the principals.

Lemma 1 implies that the contracts in a stable outcome must be optimal. Hence, a contract signed by a matched pair (P_i, A^j) must maximize the expected utility of one party taking into account that the other gets at least a certain utility level. One

[8] In w w $c - \varepsilon$ $[\theta_S - \varepsilon, \theta_F - \varepsilon, K]$, w $c = [\theta_S, \theta_F, K]$

particular class of optimal contracts are the *principal-agent* contracts, which have been discussed in Section 2.2.3.

The utility possibility frontier for any principal-agent pair is the set of utilities generated by the contracts that solve a programme similar to (P) where the reservation utility of the agent can take value not only equal to zero as in (P), but any number. The same set of optimal contracts results in if one maximises agent's utility subject to a participation constraint of the principal (PCP). We will denote by $c^j(\bar{u})$ the optimal contract that solves the following programme (as before we take agent's utility net of his wealth w^j):

$$\begin{cases} \max_{c \in Z^j} u_{A^j}(P_i, c) \\ s.t. \quad u_{P_i}(A^j, c) \geq \bar{u} \ (PCP). \end{cases} \tag{P2}$$

Notice that the contract that solves (P2) is acceptable for A^j only if $\bar{u}$ is not too high. More precisely, $u_{A^j}(P_i, c^j(\bar{u})) \geq 0$ if and only if $\bar{u} \leq u_{P_i}(A^j, c^{j*})$. In the following theorems we characterise completely the set of stable outcomes. The properties that the contracts in a stable outcome are optimal and that all principals earn equal profits provide a partial characterisation. These help us complete the description of the set of stable outcomes. We distinguish among different cases. In Theorem 1, we consider the situation where there are more agents than principals ($m > n$) in the economy. In Theorem 2, we analyse the situations where there are same number of principals and agents and there are more principals than agents. Notice that the two lemmas stated above hold irrespective of the cardinalities of the set of principals and the set of agents.

Theorem 1. *If $m > n$, then an outcome (μ, C) is stable for the market $\mathcal{M}$ if and only if the following three conditions hold:*
(a) $\mu(P_i) \in \mathcal{A}$ *for all* $P_i \in \mathcal{P}$, $\mu(A^j) \in \mathcal{P}$ *if* $w^j > w^{n+1}$ *and* $\mu(A^j) = A^j$ *if* $w^j < w^n$,
(b) $u_{P_i}(\mu(P_i), c_i) = \bar{u} \in [u_{P_i}(A^{n+1}, c^{(n+1)*}), u_{P_i}(A^n, c^{n*})]$ *for all* $P_i \in \mathcal{P}$, *and*
(c) $c^j = c^j(\bar{u})$ *if* $\mu(A^j) \in \mathcal{P}$ *and* $c^j = c^{null}$ *if* $\mu(A^j) = A^j$.

Proof. We first prove that **(a)**-**(c)** are necessary conditions for any stable outcome.

(a) Suppose first, that in a stable outcome (μ, C) any principal P_i is not matched. Then $u_{P_i}(\mu(P_i), c_i) = 0$. Now consider an agent A^j who is initially unmatched under μ. Then the contract $c^{j*} - \varepsilon \in Z^j$ yields strictly higher payoffs to both P_i and A^j.

Hence, (P_h, A^j) with $c^{jn} - \varepsilon$ blocks (μ, C). Second we show that A^j is matched if $w^j > w^{n+1}$. Suppose, on the contrary, that A^j is unmatched under μ and hence, $u_{A^j}(A^j, c^j) = 0$. Because of the previous proof, under μ there are n agents matched. Suppose, A^k is a matched agent such that $w^k \leq w^{n+1}$. Following Proposition 1, $u_{\mu(A^k)}(A^j, c^{jn}) > u_{\mu(A^k)}(A^k, c^{kn})$. Given that $u_{A^k}(\mu(A^k), c^k) \geq 0$ (since, the contract is feasible), $u_{\mu(A^k)}(A^k, c^k) \leq u_{\mu(A^k)}(A^k, c^{kn}) < u_{\mu(A^k)}(A^j, c^{jn})$. Take $c^s = c^{jn} - \varepsilon$, with ε small enough. It is easy to see that $(\mu(A^k), A^j)$ with the contract c^s will block the outcome which is a contradiction. For the last part of (a), suppose on the contrary that A^j is matched under μ and $w^j < w^n$. Since n agents are matched, take A^k such that this agent is not matched in a stable outcome and $w^k > w^n$. Applying the same argument as before, it is easy to show that $(\mu(A^j), A^k)$ with the contract $c^{kn} - \varepsilon$ will block the current outcome.

(b) We know that in all the stable outcomes the profits of the principals must be equal. Denote by Π the common profit of the principals. First we will show that in a stable outcome (μ, C), $\Pi \geq u_{P_h}(A^{n+1}, c^{(n+1)n})$. Suppose on the contrary, $\Pi < u_{P_h}(A^{n+1}, c^{(n+1)n})$. From part (a) of the theorem we know that any agent with less wealth than w^n cannot be matched in a stable outcome. Suppose this is A^{n+1} and consider any principal P_h. Then there is a contract $c^s = c^{(n+1)n} - \varepsilon$, with ε small enough, such that (1) $u_{P_h}(A^{n+1}, c^s) = u_{P_h}(A^{n+1}, c^{(n+1)n}) - \varepsilon > \Pi$ and (2) $u_{A^{n+1}}(P_h, c^s) \geq \varepsilon > 0 = u_{A^{n+1}}(\mu(A^{n+1}), c^{n+1})$. Hence, (P_h, A^{n+1}) blocks the outcome. Second, from Proposition 1 we know that $u_{P_h}(A^j, c^{jn}) > u_{P_h}(A^k, c^{kn})$ if and only if $w^j > w^k$. In a stable outcome (μ, C), an agent with wealth greater than w^{n+1}, say A^n is matched with some principal, say P_h. Then $u_{P_h}(A^n, c_n) = \Pi > u_{P_h}(A^n, c^{nn})$ implies that $u_{A^n}(P_h, c_n) < u_{A^n}(P_h, c^{nn})$. This is not possible in a stable outcome.

(c) Let (μ, C) be a stable outcome. By Lemma 1, any contract $c \in C$ is optimal and c^j is such a contract. So, given the stability of (μ, C), $c^j = c^j(\Pi)$ if $\mu(A^j) \in \mathcal{P}$.

We now prove that any outcome (μ, C) satisfying (a)-(c) is indeed stable. Suppose $\mu(A^j) \in \mathcal{P}$ and consider any principal P_h who, because of part (a), is matched. Clearly, (P_h, A^j) cannot block the outcome with any contract. Indeed, there does not exist a contract such that P_h gets more than Π and A^j gets more than $u_{A^j}(\mu(A^j), c^j)$ since $c^j(\Pi)$ is optimal by (c). Now suppose $\mu(A^j) = A^j$ and choose any arbitrary P_h (we can do so, since all principals have the same profit). By (a), we know that $w^j \leq w^{n+1}$. Then the maximum utility P_h can get by contracting A^j such that $u_{A^j}(\_) \geq 0$ is $u_{P_h}(A^j, c^{jn}) \leq u_{P_h}(A^{n+1}, c^{(n+1)n})$. Given that $\Pi \geq u_{P_h}(A^{n+1}, c^{(n+1)n})$ (because of (b)), there is no room for the pair (P_i, A^j) to block (μ, C). ∎

We have already established that in a stable outcome all the principals get the same utility. When there are too many agents, this uniform utility cannot be less than the surplus that can be created by the richest unmatched agent and it cannot be more than the surplus that can be created by the poorest matched agent. In the following theorem, we restate Theorem 1 in cases where there are same number of principals and agents and where there are more principals than agents.

Theorem 2. *(i) If $m = n$, then an outcome (μ, C) is stable for the market $\mathcal{M}$ if and only if the following three conditions hold:*
(a) *All principals and agents are matched,*
(b) $u_{P_i}(\mu(P_i), c_i) = \bar{u} \in [0, u_{P_i}(A^n, c^{n*})]$ *for all $P_i \in \mathcal{P}$, and*
(c) $c^j = c^j(\bar{u})$ *for any A^j.*
(ii) If $m < n$, then an outcome (μ, C) is stable for the market $\mathcal{M}$ if and only if the following three conditions hold:
(a) *Only m principals and all the agents are matched,*
(b) $u_{P_i}(\mu(P_i), c_i) = 0$ *for all $P_i \in \mathcal{A}$, and*
(c) $c^j = c^j(0)$ *for any A^j.*

Proof. Similar to the proof of Theorem 1. ∎

Part (i) describes the situation where there are as many principals as agents. Since all principals consume the same utility, they can obtain as low as zero utility but no more than the maximal utility that can be consumed by the principal matched with the poorest agent. Part (ii) concerns the situation where there is an abundance of principals. Since each principal gets the same utility level and since the unmatched ones necessarily obtain zero utility, each principal shall consume a zero utility too.

The above theorems characterise the stable outcomes for this principal-agent economy. First important thing to note is the optimality property of the contracts in the stable outcome. Optimality in this market has in fact two aspects. The contracts signed are optimal for the parties involved. This was a property already established in Lemma 1. On the other hand, part (a) in both theorems makes sure that the matching itself is optimal too. This is the case because, in a stable outcome, all the individuals in the short side of the market are matched and, when there are more agents than principals, only the *best* (wealthier) agents are the ones who get contracted.

The second important property is that the profits of the principals are equal. In a stable outcome there emerges competition among the principals for the wealthier agents. In particular, when there are more principals than agents (Theorem 2(ii)), the profit of each principal is driven down to zero.

Third, in a stable outcome, all the agents whose wealth level is above the wealth of the poorest agent contracted obtain a strictly higher utility than that under a principal-agent contract. In fact, there are stable outcomes where the same is true even for the poorest agent contracted. To understand the reason for this property, notice that had the agents been symmetric, i.e., if they had equal initial wealth, and they were large in number, the principals would assume all the bargaining power. In this case, the stable outcome would involve a principal-agent contract for each agent hired. The asymmetry among the agents does not let the principals appropriate all the incremental surplus generated in a principal-agent relationship, even when there are more agents than principals. Rather, the competition among principals makes the incremental surplus accrue to the agents. This competition is even more acute when there is an abundance of principals. In this case, it follows from Theorem 2 that the entire surplus generated in a relation accrues to the agent.

Finally, as is usual in the classical matching models, the set of stable outcomes in our economy has a nice structure. First, if (μ, C) is a stable outcome and μ' is an efficient matching, then (μ', C) is also stable. That is, the set of stable outcome is the Cartesian product of the set of efficient matchings and a set of menus of optimal contracts. Second, if one stable outcome (μ, C) is better for an agent than another stable outcome (μ', C'), then (μ, C) is better than (μ', C') for all the agents hired and worse for all the principals matched. In particular, out of all the stable outcomes there exists a stable outcome which is the best from the principals' point of view and similarly for the agents. In this economy, these two extreme points in the set of stable outcomes correspond to the outcomes in which the utilities of the principals are $\bar{u} = u_{P_i}(A^m, c^{mm})$ and $\underline{u} = u_{P_i}(A^{m+1}, c^{(m+1)m})$.[8] The first point is the principals' optimal stable outcome (we refer to this as *P-optimum*), while the second is the agents' optimal stable outcome (call this *A-optimum*).

[8] If $a^m = a^{m+1}$, , $\bar{u}$, q $u_{P_i}(A^m, c^{mm}) =$ $u_{P_i}(A^{m+1}, c^{(m+1)m})$ M .

In our framework, transactions occur via contracts. The major difference between this economy and a market where transactions go through prices (as in the assignment game analysed by Shapley and Shubik [30]) is that the total surplus produced in a particular relation does depend on the way in which the surplus is shared between the principal and the agent and on the design of the contract. The size of the surplus that accrues to the agent influences the extent to which the limited liability constraints are binding and hence the total surplus.

2.4 Characteristics of the Contracts in a Stable Outcome

In this section, we provide the characteristics of the contracts signed in a stable outcome. We have already shown that any such contract solves the maximisation programme (P2). Now we turn on to analyse the characteristics of the solution to this programme. We will develop the analysis under the following assumption. In the Appendix A we comment on the qualitative changes if the opposite assumption holds.[10]

Assumption 1. $\pi_1(K)\pi_0'(K) - \pi_1'(K)\pi_0(K) > 0$ *for all* $K > 0$.

Assumption 1 implies that the derivative of $\frac{\pi_0}{\pi_1}$ with respect to K is positive. That is, the higher the level of investment, the lower is the difference between π_0 and π_1, and hence, the influence of making a high effort.

The *first-best* level of investment, K^{fb} is given by the following equation:

$$\pi_1'(K^{fb})y = 1. \tag{2.1}$$

In the first-best contract, K^{fb} is the level of investment that would be chosen if there was no moral hazard problem, or equivalently, if the limited liability would not have any bite. In order to analyse the programme (P2), one can identify two disjoint ranges of values of w^A where the optimal solutions are different. First, for a very high level of agent's wealth both the incentive compatibility constraint and limited

[10]Appendix A provide [P2]

liability constraint (in the event of failure) are not binding.[20] This is equivalent to saying that there is no moral hazard problem. The threshold level of initial wealth, $w(\bar{u})$, beyond which the optimal investment reaches its first–best level K^{*} depends on the utility of the principal, $\bar{u}$, and is:

$$w(\bar{u}) = -\pi_1(K^{*})y + K^{*} + \bar{u} + \frac{\pi_1(K^{*})}{\pi_1(K^{*}) - \pi_0(K^{*})}.$$

For low levels of initial wealth, $w^j \leq w(\bar{u})$, both the incentive constraint and the limited liability constraint bind. In this region the moral hazard problem becomes important and hence, the optimal investment is lower than its first-best level. The optimal investment $\tilde{K}(w^j;\bar{u})$ is implicitly defined by the following equation:

$$-\pi_1(K)y + K + \bar{u} + \frac{\pi_1(K)}{\pi_1(K) - \pi_0(K)} = w^j.$$

Given Assumption 1, the optimal investment increases with agents' wealth. The optimal investment is summarised in the following equation:

$$K = \begin{cases} \tilde{K}(w^j;\bar{u}) & \text{if } w^j < w(\bar{u}) \\ K^{*} & \text{if } w^j \geq w(\bar{u}). \end{cases}$$

We also describe in brief the characteristics of the state contingent transfers. Notice that, for $w^j \geq w(\bar{u})$, any combination of (θ_S, θ_F) that satisfies the constraints can be candidate for the optimum. One possible optimum corresponds to $\theta_F = w^j$. In case where the constraints (IC) and (LF) are binding (for $w^j \leq w(\bar{u})$), $\theta_F = w^j$ is also an optimum. Using the participation constraint of the principal, one can then easily calculate the optimal transfer in case of success which is given by the following.

$$\theta_S = \begin{cases} \frac{\bar{u}+\tilde{K}(w^j;\bar{u})-\bar{u}-\pi_0(\tilde{K}(w^j;\bar{u}))w^j}{\pi_1(\tilde{K}(w^j;\bar{u}))} & \text{if } w^j < w(\bar{u}) \\ \frac{\bar{u}+K^{*}-\bar{u}-\pi_0(K^{*})w^j}{\pi_1(K^{*})} & \text{if } w^j \geq w(\bar{u}) \end{cases}$$

Once we know the characteristics of the solutions to program (P2), we use theorems 1 and 2 to provide a description of the contracts in a stable outcome. Consider first

[20] O

$\bar{u}$ θ_S (PCP)

a situation with many agents where the wealth of most of them is zero, i.e., $m > n$ and $w^n = w^{n+1} = 0$. In this economy, the contracts signed in all the stable outcomes are the same. The contract signed by the hired agents with zero wealth will be the corresponding principal-agent contract, while the contract signed by the richer agents will correspond to the solution of program (P2), for $\overline{u} = u_{P_1}(A^n, e^{n})$. Figure 1 depicts the level of investments in the stable outcome. [12] For comparison, the diagram also includes the level of investments $K(w^i)$ that would be made if all the agents would sign a principal-agent contract. In this figure, $\overline{K}$ is the minimum level that would be invested by the agents with very low level of wealth (say, less than $\overline{w}$). The investment level is closer to the first-best level K^{fb} as the wealth of an agent is higher. That is, the productive efficiency of the relationship increases with the agent's wealth. The investment level coincides with the first-best level if the agent, say agent A_i, is rich enough, i.e., $w^i \geq w(u_{P_1}(A^n, e^{nn}))$. It is worth noting also that these investments are always higher than those under principal-agent contracts, unless the agent's wealth is very large, $w \geq w^{fb}$.

[Insert Figure 1 about here]

For the same economy, Figure 2 depicts agents' net and gross utility levels (the common principals' utility is $u_{P_1}(A^n, e^{nn})$). Agents' net utility increases with the wealth level (unless the level of wealth is already above $w(u_{P_1}(A^n, e^{nn}))$). The utility of wealthier agents is not only higher because of the initial wealth levels. They also profit from the increase in the surplus due to the more efficient (i.e., closer to the first-best) contracts.

[Insert Figure 2 about here]

For completeness, Figure 3 depicts the set of investment levels in the stable outcomes when $m > n$, $w^n > w^{n+1}$, and w^{n+1} is large. The line corresponding to the level of investments in a particular stable outcome, say $K^\mu(w^i)$ is quite similar to that in Figure 1 (although it starts from a level higher than $\overline{K}$). This line will be placed at a higher (or a lower) position depending if we are in a stable outcome closer to (or further from) the A-optimum. In particular, the lowest line (that starts from $K(w^{n})$) corresponds to the investment levels in the P-optimum.

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[12]For ... , ... w ... $\pi[K] = \frac{K}{4K}$ ... $\pi_0[K] = \frac{K}{2+K}$. O ...

[Insert Figure 3 about here]

The graphical representation of an economy with more principals than agents is very similar to that in figures 1 and 2. The levels of investment and of net and gross utilities are as in figures 1 and 2, with the only difference that they all start at a higher level than $\overline{K}$ and $\overline{w}$.

2.5 Implementing the Set of Stable Outcomes

In this section we further argue about stability as a reasonable solution concept for the market we analyse. We show that the set of stable outcomes that we have characterised in theorems 1 and 2 are also the equilibrium outcomes of a very simple and natural non-cooperative interaction between the principals and the agents. The simple mechanism that we propose, called Γ^A, is a two-stage game where in the first stage each agent proposes a contract. In the second stage of the game, each principal contracts an agent.[13] Formally, at the first stage of the mechanism, agents send their messages simultaneously. The message of each agent is an element of the set of feasible contracts. A message $x^j \in X^j$ of agent A^j should be understood as the contract he demands. At the second stage, knowing the messages of the agents, each principal P_i sends a message $s_i \in \mathcal{A} \cup \{P_i\}$. A message of a principal should be understood as the agent she wants to hire or she wants to stay unmatched. The outcome function $g(.)$ associates to each vector of messages, $s = (s_1, ..., s_n, x^1, ..., x^m)$ a matching, μ^s, and a menu of contracts, $C(s)$, such that $\mu^s(A^j)$ is the smallest indexed principal of the set $\mathcal{P}^j = \{P_i \in \mathcal{P} \mid s_i = A^j\}$ if $\mathcal{P}^j \neq \emptyset$ and $\mu^s(A^j) = A^j$, otherwise. Moreover,

$$c^j(s) = \begin{cases} x^j & \text{if } \mu^s(A^j) \in \mathcal{P} \\ c^{null} & \text{,otherwise.} \end{cases}$$

The natural solution concept used here is *Subgame Perfect Equilibrium*. We will analyse the Subgame Perfect Equilibria in pure strategies (SPE).

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[13]T w , P' s-
C R. -M 2 T w if w
, x ,
w w

Theorem 3. **(i)** *When $n \neq m$, the set of SPE outcomes of the mechanism Γ^A coincides with the set of stable outcomes for the market $\mathcal{M}$.*
(ii) *When $n = m$, the set of SPE outcomes of the mechanism Γ^A coincides with the set of agents' optimal stable outcomes for the market $\mathcal{M}$*

Proof. See Appendix A. ∎

From the point of view of implementation, the above theorem shows that one can propose a very simple mechanism which makes it possible to implement the set of stable outcomes (or the agents' optimal stable outcomes) of this principal-agent economy.

2.6 An Application: A Landowner-Tenant Economy

In a seminal work, Shetty [31] shows that wealth differences among tenants play a key role in determining the credit contracts when there exists a possibility of default on the rental commitments. Difference in initial wealth implies difference in liability of the tenants. Hence, in the case where there is significant moral hazard problem due to limited liability, wealthier tenants are always preferred for a better contractual structure, since possibility of default is less with wealthier tenants. Our results can be used to analyse similar situations when a set of landowners interacts with a set of tenants through tenancy relations. One feature is to note that the kind of contracts we use can often be observed in the less developed economies. It is very common that the same person acts as landowner-cum-moneylender in the villages by leasing land and lending money to the same person (here, the tenant). The contracts described for the market $\mathcal{M}$ also capture these components. The state contingent transfers, (θ_S, θ_F) are the payments made to the landowners and K is the amount borrowed from the landowners that is invested eventually in land. In this economy, the tenants cannot seek loans from the formal credit sector due to lack of sufficient collateral, while the landowners can. Consequently, the landowners become the only sources of credit for the hapless borrowers.

With these interpretations, our results imply:

(i) In a stable outcome, all the contracts signed among landowners and tenants are optimal and all the landowners and only the wealthier tenants are matched. All the landowners earn the same profit and the contracts maximize the expected utility of the tenants for the common profit level of the landowners. Wealthier is the tenant, the more efficient the contract he signs (closer to first-best). The above findings also conform to the findings of Shetty [31], and Ray and Singh [28].

(ii) The investments made in a stable outcome are, in fact, closer to first-best than those that would be implemented if the tenants would sign principal-agent contracts. As landowners compete for the wealthier tenants, they are compelled to offer these tenants *better* contracts in order to attract them. Since the tenants obtain higher utility, the limited liability constraint is less stringent and hence the investment level approaches the first best. This phenomenon is described in figures 1 and 2. This comparison is relevant because the principal-agent contracts are the contracts that would have been offered, for example, if the landowners would collude.

The property highlighted in (ii) has important implications with respect to distributive (in)equality and efficiency. It suggests that for a very low level of aggregate wealth, more is the inequality in the distribution of tenant s wealth, higher is the total investment and more efficient the relationships. Indeed, as the wealth level of the poorest agent hired decreases, the market power of the other agents increases. Consequently, these other agents take more profit from a relationship and the contract terms are more efficient (i.e., the investment level is closer to the first-best.).

From a normative point of view, the analysis suggests that if the public authority has some money to distribute which could serve as collateral in tenancy relations, it may need to induce inequality among the tenants in order to increase both the efficiency of the contracts and the utility of (some of) the tenants. Suppose all the tenants have no initial wealth. If the public authority distributes to every tenant a small amount (less than $\overline{w}$ in Figure 1), then in the stable outcome, all the tenants will sign the principal-agent contracts investing a level $\overline{K}$ which is the same they would do with zero wealth. Hence, the efficiency of the relationship will remain the same as that prior to the distribution. Moreover, the gross utility of all the tenants hired will be the same as before. That is, the landowners will appropriate the additional amount distributed, which was intended to improve the welfare of the tenants. On the other hand, if the public authority distributes the money among a few tenants (a number smaller than the number of landowners), then the contracts signed by these tenants will be more efficient than before, and their gross utility will increase by more than the additional money they receive. Hence, targeting a small group rather than all the tenants improves the welfare of this group and overall efficiency.

2.7 Concluding Remarks

In this chapter we model a principal-agent economy as a two-sided matching market and characterise completely the set of stable outcomes of this economy. As we have mentioned earlier, our model can be seen as a generalisation of the assignment game described by Shapley and Shubik [30]. Our findings can easily be applied to various examples of principal-agent economies. We have already mentioned two of them in the previous section. The main task of this chapter lies in suggesting a general (competitive) equilibrium model of a principal-agent economy. Using the restriction of limited liability should be taken as a very simple way to tackle incentive problems. This work also consolidates stability as a reasonable solution concept. In this regard, we show that our results are not only the outcome of a cooperative game, but can be reached through very simple non-cooperative interactions between the principals and the agents.

This chapter leaves several avenues open to further research. First, we have assumed that the principals are identical. Although some of the conclusions of our analyses can immediately be extended to apply to economies with heterogenous principals, the characteristics of the contracts signed in the stable outcomes can be quite different from those identified in the current work. On the one hand, the results that the contracts signed in a stable outcome are optimal and the matching itself is efficient (in the sense that it maximises the total surplus) hold also in a framework with heterogenous principals. On the other, there is no unique way to model the differences among the principals and the contracts will be different depending on the type of heterogeneity one would like to introduce. Second, ours is a one-to-one matching model. If we consider the situation where several independent agents are matched with each principal, then the conclusions will remain unchanged. But these will be different in a more interesting situation where the action of an agent is dependent on that of others. This kind bears similarity with the agency problem in a multi-agent situation. A natural way to analyse this would be to make use of a many-to-one matching model.