# Strategic Pricing in Oligopoly Markets

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# Contents

*Pentru Elena, Anca şi Ion*

The closer you come to the end, the more there is to say.

The end is only imaginary, a destination you invent to keep yourself going,

but a point comes when you realize you will never get there.

You might have to stop, but this is only because you have run out of time.

You stop, but that does not mean you have come to the end.

(Paul Auster, In the Country of the Last Things, 1987, p.183)

# Acknowledgements

I owe a personal debt to my family and to many friends here in Barcelona, to those back home and to those scattered all over the world: Thanks for the great time we had together, for e-mails and phonecalls, for putting effort in seeing each other again as often as possible, for being so close, even if far away. Last, but not the least,

# Chapter 1

# Introduction

A representative firm in the economy faces competition, however it is not a price taker like a competitive firm. It acts in an imperfectly competitive environment and, though it is not a monopolist, it retains some degree of market power. Oligopoly market structure is characterized by the existence of relatively few competitors. Each firm knows the identity of its rivals and is aware that its choices affect their profits. Essentially, there is strategic interaction among firms. This interdependence and the lack of ability to make binding agreements make non-cooperative game theory the most appropriate tool for the analysis of oligopoly markets. Noncooperative oligopoly theory studies situations in which each producer maximizes his own profits taking as given the strategies of the rivals.

The present thesis analyses oligopolistic industries, focusing on situations where product market competition (in prices or quantities) is preceded by other strategic decisions, like product positioning through vertical integration, advertising or R&D investments. The analysis considers the number of firms in the market as exogenously given, though admittedly, is more realistic to believe that structure depends on the nature of competition.

The second chapter proposes a model of persuasive advertising motivated by the impressive amounts spent on this activity, and by its extensive use. Two major views

on advertising emerged in the economic literature. The first view states that advertising is persuasive, meant to change consumers' tastes, to induce subjective product differentiation and brand loyalty. It increases the brand or selective demand and softens price competition. According to the second view, the informative one, firms use advertising to inform consumers about existence, price or characteristics of their products, especially when product search is costly. Thus, advertising increases total industry demand, attracting more consumers to the market, and favors competition. This analysis embraces the persuasive view on advertising, focusing on homogenous product markets, and proposes a model of two dimensional competition in non-price persuasive advertising and prices. The model shows how advertising can be used to sustain high prices, and predicts an asymmetric advertising expenditure profile. It also gives a new interpretation of price dispersion in advertising intensive industries. Many consumer goods offer examples of nearly homogenous, heavily advertised products. In the US market for cola soft drinks the goods are close substitutes, and firms like Coca-Cola and Pepsico invest large amounts to differentiate their products.

The third chapter deals with product bundling, a widespread practice. Product tying or bundling describes a situation in which several products are sold together in the form of a package at a unique price. Firms bundle to increase efficiency, to guarantee the quality of complementary products, and also for strategic reasons. Typically a major concern is the use of bundling or tying - when one product is sold by a single firm - as an exclusionary strategy or as a barrier to entry. For example, tying allows a firm to commit to very aggressive competition and induce rivals to exit. Or, it is a tool to decrease the threat of entry in the monopolized market, by excluding rivals at the tied good level or increasing their costs. More recently, economic literature pointed out that bundling may be used to achieve a dominant market position and depress rival's profits, especially when the number of components is large. This is true even when firms face competition in all component markets (Nalebuff, 2000). The present analysis points out that bundling may be anticompetitive, but the conditions

under which it is depend on the market parameters and can be very restrictive. If incumbent firms bundle, the rivals have or do not not incentives to bundle depending on the price-elasticity of the demand. My results help to mark out the frontiers of anticompetitive bundling, and emphasize the importance of good knowledge of market conditions in antitrust decisions. Product tying was often present in the antitrust courts. Recent examples are EC v. Microsoft Corp. (2000), subject to severe remedies, and US v. Microsoft Corp. (1998) which nurtured a long-lasting controversy.

The last chapter is devoted to the analysis of innovation investments in a duopoly model. Research and development (R&D) occupies a central place in the analysis of a firm or of an industry. On top of that, the crucial role of technological progress as a determinant of growth, stresses its importance for the economy overall. Economic literature differentiated between product and process innovation. The former one refers to the creation of new goods or to quality improvement of existing products. Process innovation aims at increasing production efficiency and is typically modeled as an investment in cost reduction. The present thesis concentrates on process R&D, though part of its message is that many results go beyond this taxonomy. Another distinction originates from the nature of R&D competition. In tournament models, the time of innovation is uncertain and the first to succeed in the R&D wins the patent race. However, non-tournament models, where innovation is deterministic and not patent-oriented, seem to receive more empirical support. Chapter 4 compares the outcomes and dynamic efficiency of Bertrand and Cournot competition, in a non-tournament model with asymmetric innovation abilities. It presents a situation where both consumer surplus and profits can be larger under quantity competition.

More specifically, chapter 2 studies the strategic effect of persuasive advertising in homogenous product markets. It proposes a model in which an oligopoly first invests in advertising in order to induce brand loyalty within consumers who would otherwise purchase the cheapest alternative on the market, and then competes in prices

for the remaining brand indifferent consumers. I define the outcome of the two-stage game and show that equilibrium prices exhibit price dispersion being random draws from asymmetric distributions. The expected profits of the firms and the advertising spending profile are asymmetric. There is one firm choosing a lower advertising level, while the remaining firms choose same higher investment. For the equilibrium advertising expenditure, there are a family of pricing equilibria with at least two firms randomizing on prices. One of the limiting equilibria has all firms randomizing; the other one has only two firms randomizing, while the others choose monopoly pricing with probability one. The setting offers a way of modelling homogenous product markets where persuasive advertising creates subjective product differentiation and changes the nature of subsequent price competition. The kind of advertising investment asymmetry that the model proposes is more adequate for small oligopolies and corresponds to the empirical evidence that advertising intensive markets have a two tier-structure. Finally, the pricing stage of the model can be regarded as a variant of the Model of Sales by Varian (1980). His article analyses equilibrium pricing in a market with two types of buyers: informed (aware of all prices) and uninformed ones (who shop at random and are equally split among the firms). The two stage game offers a way of endogeneizing consumers heterogeneity and raises a robustness question to Varian's symmetric setting.

In chapter 3, I develop a model of imperfect competition where duopolists horizontally differentiated à la Hotelling compete in complementary product markets. The analysis focuses on the effects of bundling on price competition, and identifies the incentives to bundle for two sizes of the bundle and two types of demand function, an inelastic and an elastic one. Complementary component sellers first decide whether to bundle or not, and then compete in prices. With an inelastic demand, for bundles of two and three components, there are no incentives to bundle. Nalebuff (2000) showed that for systems of more than three components the best response to bundling is not to bundle. Therefore, bundling may be used to achieve a dom-

inant market position and reduce rivals' profits. I show that this is no longer true when an elastic demand is considered. The incentives to bundle are stronger (they already exist for bundles of three components) and, whenever they exist, the market outcome is symmetric bundling, the most competitive one. The welfare analysis shows that the incentives to bundle are socially excessive. However, bundle against bundle competition (the market outcome with an elastic demand) generates higher consumer surplus and lower profits than bundle against component competition (the market outcome with an inelastic demand). Finally, this research suggests that the anticompetitiveness of bundling is particularly sensitive to the price elasticity of the demand and, under certain conditions, it may foster competition.

In Chapter 4, a joint research with Uğur Akgün, we consider a differentiated duopoly market with substitute goods, where only one firm can reduce marginal cost of production before product market competition takes place. The model uses a linear demand, a decreasing returns to scale R&D technology and allows innovation outcome to spill to the rivals. We compare the equilibria of quantity and price competition in the second stage. We show that, with high substitutability and low innovation costs: a) R&D investment can be higher under Bertrand competition if spillovers are low, and b) output, consumer surplus and total welfare can be larger under Cournot if spillovers are high. A new result is that, with process innovation, both consumers and producers can be better off under quantity competition. We also conclude that our ranking of the innovation level is robust to the consideration of product instead of process R&D. More generally, all our comparison results can be encountered under both types of innovation. This follows from contrasting our dynamic efficiency ordering (with cost-reduction R&D) with the exiting literature on product innovation.

Each chapter is self-contained and suitable for independent reading.

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**Chapter 2**

# Advertising, Brand Loyalty and Pricing

## 2.1 Introduction

The interest in the economic analysis of advertising is continuously resuscitated by the amazing diversity of media[1] and by the large amounts invested in advertising. The US 100 largest advertisers spent between USD 0.312-3.6 billions on advertising in 2002.[2] The most advertised segments include many consumer goods: beer, cigarettes, cleaners, food products, personal care, and soft drinks. In many of these markets the goods are nearly homogenous, and the high amounts mentioned above seem to suggest that advertising, rather than increasing the demand, redistributes the buyers among sellers.

In the present paper, I study the strategic effect of persuasive advertising in homogenous product markets. For this purpose, I construct a model of two-dimensional competition in non-price advertising and prices. Firms first invest in advertising in order to induce brand loyalty to consumers who would otherwise purchase the cheapest

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[1] Magazines, newspapers, television, radio, internet or outdoor ads.
[2] Advertising Age, June 23, 2003, 100 Leading National Advertisers.

alternative on the market, and then compete in prices for the remaining brand indifferent consumers. At equilibrium, prices exhibit dispersion being random draws from asymmetric distributions. The variation in the price distributions is reflected by the expected profits and, in consequence, the advertising levels chosen by the firms are asymmetric. There is one firm choosing a lower advertising level, while the remaining firms choose the same higher advertising. For this profile of advertising expenditure, there are a family of pricing equilibria with at least two firms randomizing on prices. One limiting equilibrium has all firms randomizing; the other one has only two firms randomizing and the others choosing monopoly pricing with probability 1. As the number of rivals increases, more firms prefer to price less aggressively, counting on their loyal bases rather than undercutting in order to capture the indifferent market.

The setting proposes a way of modelling homogenous product markets where persuasive advertising creates subjective product differentiation and changes the nature of subsequent price competition. It also offers a new perspective on the coexistence of advertising and price dispersion. The market outcome turns out to be asymmetric, despite a priori symmetry of the firms.

The model predicts an asymmetric advertising expenditure profile, especially adequate for small oligopolies. The results relate to the carbonated cola drinks market in the US, where Coca Cola and Pepsico invest similar large amounts in advertising, while Cadbury-Schweppes spends less on advertising. Similarly, in the US sport drinks market, Pepsico highly advertises its product Gatorade, while Coca Cola promotes less its product Powerade.

The basic model in this paper draws from the persuasive view on advertising that goes back to Kaldor (1950).[3] More recently, Friedman (1983) and Schmalensee (1972,1976) dealt with oligopoly competition in models where advertising increases selective demand.[4] Schmalensee (1972) explores the role played by promotional com-

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[3]Robinson (1933), Braithwaite (1928), Galbraith (1958, 1967) and Packard (1957,1969) also contributed to this view.

[4]Von der Fehr and Stevik (1998) and Bloch and Manceau (1999) analyze the role of persuasive

petition in differentiated oligopoly markets where price changes are infrequent. This article departs from his work assuming that competition takes place in both advertising and prices. It turns out that advertising can significantly relax price competition.

Much empirical work explored whether homogenous goods advertising is informative and affects the primary (industry) demand, or is persuasive and affects only the selective (brand) demand. The results are often contradictory, and they seem to vary across industries. The persuasive view was supported by Baltagi and Levin (1986) in the US cigarette industry, and by Kelton and Kelton (1982) for US brewery industry. Using inter-industry data, they report a strong effect of advertising on selective demand.[5] More recent studies, using disaggregated data (at industry or brand level), show that advertising is meant to decrease consumers' price sensitivity. For instance, Krishnamurthi and Raj (1985) find that brand demand becomes more inelastic once advertising increases.[6]

Although taking a different view on advertising, my article shares a number of technical features with part of the informative advertising literature dealing with price dispersion phenomena. A seminal article by Stigler (1961) revealed the role of informative advertising in homogenous product markets, and related it to price dispersion.[7]

The pricing stage of the game can be viewed as a modified version of the model of sales by Varian (1980), in the sense that the total base of captured consumers is

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advertising in differentiated duopoly markets, and Tremblay and Polasky (2002) show how persuasive advertising may affect price competition in a duopoly with no real product differentiation.

[5]Lee and Tremblay (1992) find no evidence that advertising promotes beer consumption in the US. Nelson and Moran (1995), in an inter-industry study of alcoholic beverages, conclude that advertising serves to reallocate brand sales.

[6]Pedrick and Zufryden (1991) report a strong direct effect of advertising exposure over brand choice in the yogurt industry. However, some recent empirics point out towards the dominance of experience over advertising. See Bagwell (2003) for a review.

[7]Butters (1977), Grossman and Shapiro (1984), McAfee (1994), Robert and Stahl (1993), Roy (2000) construct price dispersion models where oligopolists use targeted advertising to offer information about their products to consumers who are completely uninformed or incur costly search to collect information.

shared asymmetrically instead of being evenly split among the firms. The two-stage game offers a way of endogeneizing consumers' heterogeneity: It turns out that the symmetric outcome does not obtain, raising a robustness question to Varian's (1980) symmetric model.

Baye, Kovenock and de Vrier (1992) present an alternative way of checking the robustness of Varian's symmetric setting. They construct a metagame where both firms and consumers are players, and asymmetric price distributions cannot form part of a subgame perfect equilibrium. Here, only firms are making decisions and the subgame perfect equilibria of the game are asymmetric.

Narasimhan (1988) derived the mixed pricing equilibrium in a price-promotions model where two brands act as monopolists on loyal consumer markets and compete in a common market of brand switchers. The pricing stage of my model extends his setting to oligopoly, when the switchers are extremely price sensitive.

Section 2.2 describes the model, while section 2.3 derives the equilibrium in the pricing stage and comments on the chosen strategies. Section 2.4 presents the equilibrium emerging in the advertising stage, and defines the outcome of the sequential game. In section 2.5, I propose an alternative interpretation of the model. In sections 2.6 and 2.7, I discuss the setting and present some concluding remarks. All the proofs missing from the text are relegated to an Appendix.

## 2.2 The Model

There are $n$ **firms** selling a homogenous product. All firms have the same constant marginal cost. In the first period, firms choose simultaneously and independently an advertising expenditure and, in the second period, they compete in prices. Let $\alpha_i$ be the advertising expenditure chosen by firm $i$.

This model deals with non-price advertising. Each firm promotes its product

to induce subjective differentiation and generate brand loyalty.[8] The fraction of consumers that are loyal to firm $i$ depends on the advertising expenditure profile. At the end of the first stage, the advertising choices become common knowledge.

In the second stage, firm $i$ chooses the set of prices that are assigned positive density in equilibrium and the corresponding density function. Let $F_i(p)$ be the cumulative distribution function of firm $i$'s offered prices. The price charged by a firm is a draw from its price distribution.

I assume that there is a continuum of **consumers**, with total measure 1, who desire to purchase one unit of the good whenever its price does not exceed a common reservation value $r$. After advertising takes place, part of the consumers remain indifferent (possibly because no advertising reached them or, alternatively, no advertising convinced them) and the remaining ones become loyal to one brand or another. The indifferent consumers view the alternatives on the market as perfect substitutes and all purchase from the lowest price firm. The loyal consumers are split amongst the advertised brands. The size of each group is determined by the total advertising investment on the market. The total number of loyal consumers, $U$, is assumed to be an increasing and concave function of the aggregate advertising expenditure of the firms, $U = U(\Sigma_i \alpha_i)$, with $\lim_{\Sigma_i \alpha_i \to \infty} U(\Sigma_i \alpha_i) = 1$.[9] Thus, with higher advertising more consumers join the captured (loyal) group. One may think that more advertising would be more convincing. The captured consumers are split amongst the firms according to a market sharing function depending on the advertising expenditure of the firms. Let it be $S(\alpha_i, \alpha_{-i}) = S_i \in [0, 1]$, satisfying the following properties:[10]

1. $\Sigma_{i=1,...n} S_i = 1$;

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[8] Advertising may carry some emotional content that potentially touches people and makes them develop loyalty feelings for the product. However, I assume that there is no real differentiation among the products. See Section 6 for a more detailed discussion on this view.

[9] The empirical studies often present evidence that advertising is subject to diminishing returns to scale. See the reviews by Scherer and Ross (1990) and Bagwell (2003).

[10] This function is used by Schmalensee (1976). An example of loyal market share function satisfying these conditions is $S(\alpha_i, \alpha_{-i}) = \frac{\alpha_i}{\Sigma_j \alpha_j}$.

2. $\dfrac{\partial S_i}{\partial \alpha_i} \geq 0$ with strict inequality if $\exists j \neq i$ s.t. $\alpha_j > 0$ or if $\alpha_j = 0$, for all $j$;

3. $\dfrac{\partial S_i}{\partial \alpha_j} \leq 0$ with strict inequality if $\alpha_i > 0$;

4. $S(\alpha_i, 0) = 1$ if $\alpha_i > 0$;

5. $S(0, \alpha_{-i}) = 0$ if $\exists j \neq i$ s.t. $\alpha_j > 0$ and $S(0,0) = 0$;

6. $S(\alpha_i, \alpha_{-i})$ is homogeneous of degree 0;

7. $S(\alpha_i, \alpha_{-i}) = S(\alpha_i, \widetilde{\alpha}_{-i})$ whenever $\widetilde{\alpha}_{-i}$ is obtained from $\alpha_{-i}$ by permutation.

The symmetry of the loyal market sharing function follows from the symmetry of the firms. Conditions 1, 4 and 5 require that all loyal consumers be split amongst the firms with positive advertising expenditure. Conditions 2 and 3 require that a higher own advertising increase the market share of a firm, whenever it is below 1, and a higher rival advertising decrease the share of a firm whenever it is above 0. Condition 6 requires that the share profile remain unchanged to multiplications of the advertising by the same factor.[11] Condition 7 states that the market share of a firm is determined by rivals' advertising levels and not by their identity.

Finally, the remaining buyers form the base of indifferent consumers, denominated by $I$. Hence, each firm faces an indifferent base $I = 1 - U(\Sigma_i \alpha_i)$ and a particular captured or locked-in base $U_i = S(\alpha_i, \alpha_{-i}) U(\Sigma_i \alpha_i)$. Firms cannot price discriminate between these two types of consumers.

The timing of the game can be justified by the fact that I deal with non-price advertising. Firms are building-up an image through advertising expenditure. A change in the brand advertising takes time, whereas firms can modify their prices almost continuously.

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[11]However, for instance, doubling the advertising expenditure leads to an increase in the *size* of the loyal market, although the sharing rule does not change. Hence, escaladating advertising would increase the captured base of each firm, all else equal.

The concept of equilibrium used in the model is subgame perfect Nash equilibrium. The two stage advertising-pricing game is solved backwards.

## 2.3 The Pricing Stage

In the first stage, firms simultaneously choose the advertising investment. In the second stage, knowing the whole profile of advertising expenditure, firms choose prices. In this section I solve the pricing game for an arbitrary weakly ordered profile of advertising spending. Let $U_i$ be the loyal base of consumers captured by firm $i$, and let $I = 1 - U = 1 - \Sigma_i U_i$ be the group of brand switchers who buy the lowest priced brand. Without loss of generality, assume $U_i \geq U_j$ whenever $i \leq j$ with $i, j \in N = \{1, 2, ...n\}$.

For any firm $i \in N$ only the prices in the interval $A_i = [c, r]$ are relevant, with $c$ being the common constant marginal cost of production. Pricing at $p_i < c$ firms would make negative profits and pricing at $p_i > r$ firms would make zero profits. The profit function of firm $i$, $i = 1, 2...n$ is given by:

$$\pi_i (p_i, p_{-i}) = \begin{cases} (p_i - c)(U_i + I) \text{ if } p_i \leq r \text{ and } p_i < p_j, \ \forall j \neq i \\ (p_i - c)(U_i + I\varphi) \text{ if } p_i \leq r \ , \ p_i = p_j, \ j \in M \subseteq N \\ \qquad \text{and } p_i < p_k, \forall k \in N \backslash M \text{ with } \varphi = \frac{1}{|M|} \\ (p_i - c)U_i \text{ if } p_i \leq r \text{ and } \exists j \neq i \text{ s.t. } p_j < p_i. \end{cases} \quad (2.1)$$

The firms choose the prices that maximize their payoffs taking as given the pricing strategies of the rivals.

**Proposition 1.** *The game* $(A_i, \pi_i; i \in N)$ *has no pure strategy Bertrand-Nash equilibrium.*

*Proof.* Assume $(p_i^*, p_{-i}^*)$ is a pure strategy Nash equilibrium profile. Then, by the definition of such equilibrium, $\nexists p_i$ such that $\pi_i (p_i, p_{-i}^*) > \pi_i (p_i^*, p_{-i}^*)$. Let $i$ be such that $p_i^* \leq p_k^*, \forall k \neq i$. Consider first the case where $p_i^* = p_j^* \neq c$, where $p_j^* =$

$\{\min p_k^* \mid k \in N \setminus \{i\}\}$. Then $\pi_i\left(p_i^*, p_{-i}^*\right) = \left(p_i^* - c\right)\left(U_i + I\varphi\right) < \pi_i\left(p_i^* - \varepsilon, p_{-i}^*\right) = \left(p_i^* - c - \varepsilon\right)\left(U_i + I\right)$. Hence, $\exists p_i = p_i^* - \varepsilon$, for $0 < \varepsilon < \frac{\left(p_i^* - c\right)\left(1 - \varphi\right)I}{U_i + I}$, such that $\pi_i\left(p_i, p_{-i}^*\right) > \pi_i\left(p_i^*, p_{-i}^*\right)$. This argument fails at $p_i^* = p_j^* = c$, but $\pi_i\left(c, p_{-i}^*\right) = 0 < \pi_i\left(r, p_{-i}^*\right)$. Consider, finally, the case $p_i^* < p_j^*$, where $p_j^* = \{\min p_k^* \mid k \in N \setminus \{i\}\}$. Then $\pi_i\left(p_i^*, p_{-i}^*\right) = \left(p_i^* - c\right)\left(U_i + I\right) < \pi_i\left(p_i^* + \varepsilon, p_{-i}^*\right) = \left(p_i^* - c + \varepsilon\right)\left(U_i + I\right)$ whenever $\varepsilon < p_j^* - p_i^*$. Then, $\exists p_i = p_i^* + \varepsilon$, for $0 < \varepsilon < p_j^* - p_i^*$, such that $\pi_i\left(p_i, p_{-i}^*\right) > \pi_i\left(p_i^*, p_{-i}^*\right)$. The cases presented above complete the proof of nonexistence of a pure strategy Bertrand-Nash equilibrium in the game $\left(A_i, \pi_i; i \in N\right)$. $\square$

Existence of a mixed strategy equilibrium can be proven by construction.[12] This approach gives also the functional forms of the equilibrium pricing strategies of the firms.

A mixed strategy for firm $i$ is defined by a function, $f_i : A_i \to [0, 1]$, which assigns a probability density $f_i(p) \geq 0$ to each pure strategy $p \in A_i$ so that $\int_{A_i} f_i(p)\, dp = 1$.

Let $\widehat{S}_i = [L_i, H_i]$ be the support of the equilibrium distribution of prices chosen by firm $i$. This means that $f_i(p) > 0$ for all $p \in \widehat{S}_i$. Denominate by $F_i(p)$ the cumulative distribution function related to $f_i(p)$.

At a price $p$, a firm sells to its loyal market and is the winner of the indifferent market provided that $p$ is the lowest price. Consequently, its expected demand is given by:

$$D_i(p) = \left(1 - \Sigma_i U_i\right)\Pi_{j \neq i}\left(1 - F_j(p)\right) + U_i.$$

**Lemma 1.** *Let* $L = L_k = \min_i \{L_i\}$. *Then* $\exists j \neq k$, *such that* $L_j = L_k = L$. *Moreover for each firm*

$$L_i - c \geq (r - c)\frac{U_i}{U_i + I}.$$

*Proof.* If $L_k = \min_i \{L_i\}$ and $\nexists j$, s.t. $L_j = L_k$, choosing a lower bound in the interval $(L_k, L')$, with $L' = \min_{i \neq k} \{L_i\}$, does not decrease firm $k$'s probability of

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[12]It is guaranteed as the pricing game satisfies the conditions found by Dasgupta and Maskin (1986, p. 14, Thm.5).

being the winner of the indifferent consumers, and strictly increases its expected profits. Then, $\exists\ j \neq k$, such that $L_j = L_k$. Suppose firm $i$ chooses a price $p$ such that $p - c < (r - c)\frac{U_i}{U_i + I}$. Then, the maximal profits of firm $i$ when pricing at $p$ are $(p - c)(I + U_i) < (r - c)U_i$. The RHS represents the certain profit of firm $i$ when it sells only to its loyal base at the monopoly price (its minmax value). Then, $L_i - c \geq (r - c)\frac{U_i}{U_i + I}$ for all $i$. $\qquad\square$

**Lemma 2.** $H_i = r,\ \forall\ i = 1, ..., n.$

*Proof.* At any price $p_i > r$, $\pi_i(p_i, p_{-i}) = 0$. This implies $H_i \leq r$. Let $H_m = \min_{j \in N}\{H_j\}$. Suppose by contradiction that $H_m < r$.

I show first that (a) $\nexists j \in N$ such that $f_j(p) > 0$ for $p \in (H_m, r)$. Suppose such $j$ existed. Notice that $(1 - F_m(p)) = 0$ for $p \in (H_m, r)$. Then, $\pi_j(p) = (p - c)U_j < (r - c)U_j$, and firm $j$ would deviate.

To complete the proof I show that (b) $A = \{i \mid i \in N \text{ and } H_i = H_m\} = \emptyset$.

That is, $H_m < r \Leftrightarrow A \neq \emptyset$. By (a), $H_j = r$ for $j \in N \setminus A$. (b1) Let $|A| = 1 \Leftrightarrow A = \{m\}$. Then, because $\Pi_{j \neq m}(1 - F_j(H_m)) = \Pi_{j \neq m}(1 - F_j(r - \varepsilon))$,

$(H_m - c)(U_m + I\Pi_{j \neq m}(1 - F_j(H_m))) <$

$(r - c - \varepsilon)(U_m + I\Pi_{j \neq m}(1 - F_j(r - \varepsilon)))$,

and firm $m$ would deviate. (b2) Let $|A| > 1$. If $\exists j \in A$ s.t. $pr(p_j = H_m) = 0$, then the previous argument applies to $i \in A$, $i \neq j$ and there are incentives to deviate. If $\forall j \in A$, $pr(p_j = H_m) \neq 0$, there is positive probability of a tie at $H_m$ and any firm $j$ has incentives to deviate to $H_m - \varepsilon$ for $\varepsilon > 0$ small. $\qquad\square$

**Lemma 3.** *There is at least one firm that does not have an atom at $r$.*

*Proof.* Assume to the contrary that all firms assign positive probability to pricing at $r$, equal to $\phi_i \in (0, 1]$. Then by pricing at $r$, firm $k$ makes profits of $(r - c)(U_k + I\phi)$, where $\phi = \frac{1}{n}\Pi_{i \neq k}\phi_i$. But, pricing at $r - \varepsilon$, firm $k$ makes profits of $(r - c - \varepsilon)(U_k + I) >$

$(r - c)(U_k + I\phi)$ for $0 < \varepsilon < \frac{I(1-\phi)}{I+U_k}(r - c)$. This proves that $\exists k \in N$, s.t. $F_k(r) = 1$. $\qquad\square$

**Lemma 4.** $F_i(p)$ *is continuous on* $[L_i, r)$.

*Proof.* A heuristic proof follows, and a formal one is presented in the Appendix. If firm $i$ has a jump at $p \in [L_i, r)$, then there is $\varepsilon > 0$ such that for $j \neq i$, $F_j(p) = F_j(p + \varepsilon)$. Otherwise, firm $j$ would increase its expected profit by choosing $p - \delta$, instead of $p_j \in (p, p + \varepsilon)$. This contradicts the optimality of $p$ given that firm $i$ would only increase its profits by moving the mass to $p + \varepsilon$. Notice that this argument fails at $p = r$ because profits at $r + \varepsilon$ are equal to zero. $\qquad\square$

**Remark 1.** *Lemmas 1-4 do not rule out the existence of degenerate distributions for some firms.*

I propose the following asymmetric pricing equilibrium, valid for any weakly ordered profile of loyal bases.

**Conjecture 1.** *For* $U_1 \geq ... \geq U_{n-1} \geq U_n$, *the supports are* $\widehat{S}_n = [L, r]$, $\widehat{S}_{n-1} = [L, r]$ *and* $\widehat{S}_k = \{r\}$ *for* $k = 1, 2...n - 2$. *In addition,* $F_k(p) = 0$ *for* $p < r$ *and* $F_k(p) = 1$ *for* $p \geq r$. *The cdf's* $F_n(p)$ *and* $F_{n-1}(p)$ *are continuous on* $[L_i, r)$ *and* $F_{n-1}(r) < 1$.

Note that by Lemma 3, $L_i - c \geq (r - c)\frac{U_i}{I+U_i}$ for all $i$. Also $L_k - c \geq (r - c)\frac{U_{n-1}}{I+U_{n-1}}$ for any $k \leq n - 2$. Then firm $n$ has no incentives to put positive densities to prices $p \in [(r - c)\frac{U_n}{I+U_n} + c, (r - c)\frac{U_{n-1}}{I+U_{n-1}} + c)$ because all these prices are strictly dominated by $p = (r - c)\frac{U_{n-1}}{I+U_{n-1}} + c$, which does not decrease the probability of winning the indifferent consumers $(I)$, but strictly increases firm's expected profits. Therefore, $L = (r - c)\frac{U_{n-1}}{I+U_{n-1}} + c$.

In equilibrium a firm should be indifferent among all the strategies (prices) that form the support of its distribution function. Firm $n$ should be indifferent between

any price $p \in [L, r)$ and pricing at the lower bound of its support, $L$. It follows that:

$$(p - c)\left[(1 - F_{n-1}(p))I + U_n\right] = (L - c)(I + U_n) \Rightarrow$$
$$F_{n-1}(p) = \frac{(p - L)(U_n + I)}{I(p - c)}.$$

Similarly, firm $n - 1$ should be indifferent between any price $p \in [L, r)$ and pricing at the upper bound of its support, $r$. This gives:

$$(p - c)\left[(1 - F_n(p))I + U_{n-1}\right] = (r - c)U_{n-1} \Rightarrow$$
$$F_n(p) = \frac{(p - c)(U_{n-1} + I) - (r - c)U_{n-1}}{I(p - c)}.$$

For the conjectured strategies to be an equilibrium: i) expected profit of any firm should be constant at all prices in the support of its distribution, ii) distribution functions should be well defined and, iii) no firm should have incentives to price outside its support.

i) *Constant profit conditions.*

The expected profits are:

$$\pi_k(r) = (r - c)U_k \qquad \forall k \in N \backslash \{n - 1, n\},$$
$$E\pi_{n-1}(p) = (r - c)U_{n-1} \qquad \forall p \in \widehat{S}_{n-1},$$
$$E\pi_n(p) = (L - c)(U_n + I) \qquad \forall p \in \widehat{S}_n.$$

Consider pricing in the interval $[L, r]$. Only firms $n$ and $n - 1$ choose these prices. Using the distribution functions it can be easily shown that this requirement is fulfilled.

ii) *Properties of the distribution functions.*

The distribution functions are increasing $\left(F'_{n-1}(p) > 0, \ F'_n(p) > 0\right)$ with $F_{n-1}(L) = F_n(L) = 0$. Firm $n - 1$ puts positive probability on pricing at $r$ equal to $\frac{U_{n-1} - U_n}{U_{n-1} + I}$, while the cdf of firm $n$ is continuous on $[L_{n-1}, r]$. For any firm $k = 1, 2 ... n - 2$, the degenerate distribution functions are well defined.

iii) *Deviation outside the support.*

For firms $n$ and $n-1$, deviating outside the support means pricing above $r$ or pricing below $L$. But, all such prices are strictly dominated by pricing in the interval $[L, r]$. Consider firm $k = 1, 2...n-2$. Deviating to price $p < r$, it makes profits:

$$(p-c)\left(U_k + I\left(1 - F_n\left(p\right)\right)\left(1 - F_{n-1}\left(p\right)\right)\right) =$$
$$(p-c)\,U_k + (r-p)\,U_k \frac{\left[(L-c)\,I - (p-L)\,U_n\right]U_{n-1}}{I\left(p-c\right)U_k}.$$

Its expected profit at $r$ is $(r-c)\,U_k = (r-p)\,U_k + (p-c)\,U_k$.

Let $g\left(p\right) = \frac{\left[(L-c)I-(p-L)U_n\right]U_{n-1}}{I(p-c)U_k}$ and notice that $g'\left(p\right) = -\frac{(L-c)U_{n-1}(I+U_n)}{p-c} < 0$ and $g\left(L\right) = \frac{U_{n-1}}{U_k} \leq 1$. It follows that deviation to prices in the interval $[L, r)$ is not profitable. Deviation to prices below $L$ is trivially unprofitable. Hence, no firm has incentives to price outside its support. This completes the proof of next result.

**Proposition 2.** *The following distribution functions represent a mixed strategy Nash equilibrium of the pricing subgame* $(A_i, \pi_i; i \in N)$.

$$F_n\left(p\right) = \begin{cases} 0 \; for \; p < L = (r-c)\,\dfrac{U_{n-1}}{I+U_{n-1}} + c \\[2mm] \dfrac{(I+U_{n-1})}{I} - \dfrac{(r-c)\,U_{n-1}}{I\left(p-c\right)} \; for \; L \leq p \leq r \\[2mm] 1 \; for \; p \geq r \end{cases} \quad,$$

$$F_{n-1}\left(p\right) = \begin{cases} 0 \; for \; p < L = (r-c)\,\dfrac{U_{n-1}}{I+U_{n-1}} + c \\[2mm] \dfrac{(I+U_n)}{I} - \dfrac{(r-c)\,U_{n-1}\left(I+U_n\right)}{I\left(p-c\right)\left(I+U_{n-1}\right)} \; for \; L \leq p \leq r \\[2mm] 1 \; for \; p \geq r \end{cases} \quad,$$

$$F_k\left(p\right) = \begin{cases} 0 \; for \; p < r \\ 1 \; for \; p \geq r \end{cases} \quad for \; \forall k = 1, 2...n-2.$$

The price of firm $k$ $(k = 1, 2...n-2)$ first order stochastically dominates the price of firm $n-1$, while the latter stochastically dominates the price of firm $n$.
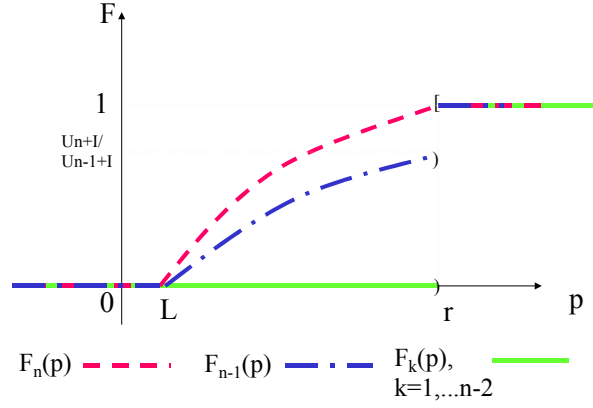
Figure 2.1: Price distributions in Proposition 2

When charging a price $p < r$, a firm $i$ with $U_i > U_j$, $i, j \in N$, looses $\Delta^- = (r - p) U_i > (r - p) U_j$, while facing the same potential gain as firm $j$ in the indifferent market $\Delta^+ = (p - c) I$. Thus, firm $i$ is less aggressive than firm $j$ because it tends to loose more. Given that with $n \geq 3$ the price rivalry augments, it turns out that the firms with higher loyal base $U_k \geq U_{n-1}$, $k = 1, ...n - 2$, choose to maintain monopoly pricing, $r$, with probability 1. The mass point at $r$ in the distribution of firm $n - 1$ increases in the difference between the two lowest loyal bases, $F_{n-1}(r) = 1 - \frac{U_{n-1} - U_n}{U_{n-1} + I}$.

The firms that choose degenerate distributions make deterministic profits $\pi_k(r) = (r - c) U_k$ for $k = 1, ...n - 2$. The expected profits of the remaining firms are $E\pi_{n-1}(p) = (r - c) U_{n-1}$ and $E\pi_n(p) = (L - c)(U_n + I) = (r - c) U_{n-1} \frac{U_n + I}{U_{n-1} + I}$.

Notice that $(r - c) U_{n-1} \geq E\pi_n(p) \geq (r - c) U_n$. Then, $E\pi_i(p) \geq E\pi_j(p)$ whenever $U_i \geq U_j$ (or $i \leq j$). Firm $n - 1$, despite of choosing a mixed pricing strategy, makes in equilibrium expected profit equal to the profit it would make on its loyal base if it chose monopoly pricing. By contrast, at equilibrium, firm $n$ has expected profit higher than its monopoly profit on its loyal base.

The equilibrium in Proposition 2 predicts price dispersion in relatively small mar-

kets. When the number of firms increases, so does the number of firms that permanently choose monopoly pricing, and the dispersion in prices tends to become insignificant. The higher the number of competitors the lower the chances of an individual firm to win the indifferent market. When the number of competitors is higher, more firms prefer to rely on their locked-in markets and act as monopolists rather than engage in aggressive pricing.

Narasimhan (1988) offers an explanation for price dispersion in competitive markets based on consumer loyalty. He restricts attention to a duopoly. The present paper offers an extension of his setting to oligopoly. With arbitrary weakly ordered profiles of loyal groups, only the two lowest firms engage in price promotions[13] and, for this reason, the potential of this model to explain market wide price promotions is limited when the number of firms increases.

The next Proposition presents a uniqueness result.

**Proposition 3.** *If firms employ convex supports, the equilibrium stated in Proposition 2 is the unique equilibrium that applies to* **any** *weakly ordered profile of loyal bases.*

However, when $n > 2$, for particular profiles of loyal groups, there are other pricing equilibria, as well. *In all equilibria the firms make the same expected payoffs.* This allows to solve the reduced form game in the first stage. I present another equilibrium for the profile $U_n < U_{n-1} = ... = U_1$, which turns out to be relevant for the present analysis.

**Proposition 4.** *If the loyal bases, $U_i$, $i = 1, 2...n$, satisfy $U_i = \overline{U}$ for $i = 1, 2...n-1$ and $U_n < \overline{U}$, there exists a pricing equilibrium with all firms randomizing over the*

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[13]Pricing below the monopoly level with positive probability is interpreted as a price promotion.

*same convex support. The firms choose prices according to the following distributions:*

$$
F_i\left(p\right) = \begin{cases} 0 \ for\ p < L = (r - c)\,\dfrac{\overline{U}}{I + \overline{U}} + c \\[2mm] 1 - \left[\dfrac{(L - c)\,(I + U_n)}{I\,(p - c)} - \dfrac{U_n}{I}\right]^{\frac{1}{n-1}} \ for\ L \le p < r \\[2mm] 1 \ for\ p \ge r \end{cases}
$$

$$
for\ i = 1, 2 ... n - 1,
$$

$$
F_n\left(p\right) = \begin{cases} 0 \ for\ p < L = (r - c)\,\dfrac{\overline{U}}{I + \overline{U}} + c \\[2mm] 1 - \left[\dfrac{(r - p)\,\overline{U}}{I\,(p - c)}\right]\left[\dfrac{L\,(I + U_n)}{I\,(p - c)} - \dfrac{U_n}{I}\right]^{\frac{2-n}{n-1}} \ for\ L \le p \le r \\[2mm] 1 \ for\ p \ge r. \end{cases} \quad .
$$

*Proof.* Directly follows from the constant profit condition. □

**Remark 2.** *For this specific profile of loyal bases, there are other equilibria in-between the ones in Propositions 2 and 4. They have a number $k < n - 2$ of firms choosing $r$ with probability 1 and the rest randomizing over the same convex support. Firms $n - 1$ and $n$ necessarily randomize.*

It can be seen from Proposition 4 and Figure 2.2 that price of firm $n$ is stochastically dominated by price of firm $i$ $(i = 1, ..., n-1)$. The same intuition as before holds, but given that $U_i = \overline{U}$ for $i = 1, 2 ... n - 1$, it is possible that all firms randomize at equilibrium.

The pricing subgame maybe understood as a variant of Varian's "Model of Sales", with asymmetric captured consumer bases. "A Model of Sales" is meant to describe markets which exhibit price dispersion, despite the existence of at least some rational consumers.[14] The model interprets sales as a way to discriminate between consumers who are assumed to come in two types, informed and uninformed. All consumers have

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[14]Varian is concerned with understanding "temporal price dispersion" rather than "spatial price dispersion". That is, intertemporal changes in the pricing of a given firm rather than cross-sectional price volatility.
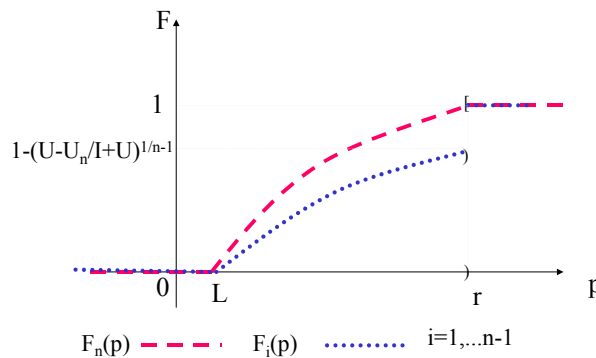
Figure 2.2: Price distributions in Proposition 4

a common reservation value and they purchase a unit of the good whenever the price does not exceed the valuation, the uninformed ones choose randomly a shop and the informed ones buy from the cheapest seller. The paper by Varian restricts attention to the symmetric equilibrium of the symmetric game (with uninformed consumers evenly split amongst the firms).[15] However, there exists a family of asymmetric equilibria of the symmetric game.[16]

The present setting offers a way of endogenizing the creation of locked-in consumers, and it raises a robustness question to Varian's symmetric setting because it turns out that at equilibrium the captured bases are asymmetric.

Extending Varian's model, Baye, Kovenock and de Vries (1992) construct a metagame in which consumers are also players. In the first stage, uninformed consumers and firms move simultaneously. Firms choose a price distribution and the uninformed consumers decide from which firms to purchase. In the second stage, the

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[15] If the optimal pricing distributions are continuous and the supports are convex, then the equilibrium is symmetric (Proposition 9 of Varian (1980), p.658). See the Appendix for a revision of this Proposition.

[16] A comprehensive analysis of all the asymmetric equilibria of the symmetric game is provided by Baye, Kovenock and deVries (1992).

informed consumers choose the seller they will buy from. Given that the asymmetric price distributions can be ranked by first-order stochastic dominance, they show that the unique subgame perfect equilibrium of the extensive game is the symmetric one. However, this follows from the equilibrium consistency requirement that a firm with higher expected price cannot have a larger uninformed consumer base.

## 2.4  Advertising Expenditure Choices

In this section, I derive the equilibrium of the reduced form game in the first stage where oligopolists simultaneously choose an advertising expenditure. Their payoffs are the profits emerging in the pricing stage minus the chosen advertising expenditure. The *gross* of advertising cost profits are:

$$E\pi_j(p) = (r - c) U_j, \qquad \forall p \in \widehat{S}_j \text{ and } \forall j \in N \setminus \{n\};$$

$$E\pi_n(p) = (L - c)(U_n + I) = (r - c) U_{n-1} \frac{(U_n + I)}{(U_{n-1} + I)}, \qquad \forall p \in \widehat{S}_n.$$

The loyal consumer group of firm $i$ $(U_i)$ is defined as a share $S(\alpha_i, \alpha_{-i})$ of the total number of loyal consumers given by $U(\Sigma_i \alpha_i)$. That is,

$$U_i(\alpha_i, \alpha_{-i}) = S(\alpha_i, \alpha_{-i}) U(\Sigma_j \alpha_j).$$

Each firm may invest in generating loyal consumers and the total number of brand loyals on the market is determined by the aggregate expenditure. The advertising technology is imperfect, so that there is always a fraction of the consumers who are not persuaded (or reached) by advertising. This fraction forms the brand indifferent group $(I)$ and buys the cheapest product:

$$I = 1 - \Sigma_i U_i(\alpha_i, \alpha_{-i}) = 1 - U(\Sigma_i \alpha_i).$$

Under the assumptions made so far, the loyal group of a firm is increasing in own advertising. The incremental consumers may proceed from the brand indifferent group $(U'(\Sigma_i \alpha_i) > 0)$ or from the rival loyal groups $\left( \frac{\partial S(\alpha_i, \alpha_{-i})}{\partial \alpha_i} \geq 0 \right)$. The last source

leaves open the possibility of reciprocal cancellation across brands.[17] An increase in rival advertising has two conflictive effects on the loyal base of a firm. There is a positive effect due to the increase in the total number of loyals ($U'\left(\Sigma_i \alpha_i\right) > 0$) and a negative one due to a decrease in the share of the firm $\left(\frac{\partial S(\alpha_i, \alpha_{-i})}{\partial \alpha_j} \leq 0\right)$. I assume that the overall effect of rival advertising is negative.[18]

I assume that:

$$\frac{\partial^2 U_i}{\partial \alpha_i \partial \alpha_j} < 0; \quad \frac{\partial^2}{\partial \alpha_n \partial \alpha_j} \left(\frac{U_{n-1}\left(U_n + I\right)}{\left(U_{n-1} + I\right)}\right) < 0 \text{ and } \frac{\partial}{\partial \alpha_i} U_i \left(0, \alpha_{-i}\right) > \frac{1}{r - c}.$$

The first and second conditions are sufficient for an equilibrium to exist. The last condition is necessary for a non-trivial result. In the remainder of this paper, the first subscript refers to the identity of the firm, and the second to the partial derivative. That is, $S_{i,j} = \frac{\partial S_i}{\partial \alpha_j} = \frac{\partial S(\alpha_i, \alpha_{-i})}{\partial \alpha_j}$ and $U_{j,j} = \frac{\partial U_j}{\partial \alpha_j} = \frac{\partial(S_j(\alpha_j, \alpha_{-j})U(\Sigma_i \alpha_i))}{\partial \alpha_j}$.

Firms $j = 1, 2...n-1$ maximize the expected profit net of advertising expenditure:

$$\pi_j^{net} = \left(r - c\right) S_j U \left(\Sigma_i \alpha_i\right) - \alpha_j.$$

Firm $n$ maximizes its expected profit net of advertising costs:

$$\pi_n^{net} = \left(r - c\right) S_{n-1} U \left(\Sigma_i \alpha_i\right) \frac{\left(S_n U \left(\Sigma_i \alpha_i\right) + I\right)}{\left(S_{n-1} U \left(\Sigma_i \alpha_i\right) + I\right)} - \alpha_n.$$

The FOC of the maximization problems above implicitly define $\alpha_n^* \left(\alpha_{-n}\right)$ and $\alpha_j^* \left(\alpha_{-j}\right)$ for $j = 1, 2...n - 1$. In addition, $\alpha_l^* = \alpha_j^* = \alpha^*, \forall j, l \in N \setminus \{n\}$.

A symmetric equilibrium $\left(\alpha_n^* = \alpha_j^* = \alpha^*\right)$ exists if, when choosing $\alpha^*$, firm $n$ does not have incentives to decrease, and the other firms do not have incentives to increase.

$$\frac{\partial}{\partial \alpha_n} \pi_n^{net} \left(\alpha^*\right) \geq 0 \text{ and } \frac{\partial}{\partial \alpha_j} \pi_j^{net} \left(\alpha^*\right) \leq 0 \text{ for all } j = 1, 2, ...n - 1.$$

In Appendix B it is shown that this requirement leads to a contradiction. Together with the optimization problem above, this proves the following result.

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[17] Metwally (1975,1976) and Lambin (1976) find empirical evidence in this sense.

[18] The market size is normalized to 1. It is possible that advertising raises all firms' market shares, but seems more reasonable to be so if it generates an expansion of market size.

Table 2.1:

| $n$ | $r - c$ | $\alpha_j,$ $j \neq n$ | $\alpha_n$ | $U_j,$ $j \neq n$ | $U_n$ | $I$ | $\pi_j^{net},$ $j \neq n$ | $\pi_n^{net}$ |
|---|---|---|---|---|---|---|---|---|
| 5 | 100 | 16.56 | 11.79 | .209 | .149 | .012 | 4.39 | 3.46 |
| 5 | 25 | 4.48 | .21 | .234 | .011 | .052 | 1.37 | 1.08 |
| 4 | 100 | 19.22 | 15.37 | .259 | .207 | .013 | 6.74 | 5.64 |
| 4 | 25 | 5.09 | 1.58 | .285 | .088 | .055 | 2.03 | 1.43 |
| 3 | 100 | 22.53 | 19.62 | .343 | .298 | .015 | 11.76 | 10.43 |
| 3 | 25 | 5.82 | 3.11 | .369 | .197 | .063 | 3.41 | 2.45 |

**Proposition 5.** *In any pure strategy Nash equilibrium of the reduced form advertising game $\alpha_n < \alpha_i = \alpha_j$ for $\forall i, j \in \{1, 2 ... n - 1\}$. The values of $\alpha_n$ and $\alpha_i$ ($\forall i \in \{1, 2 ... n - 1\}$) are implicitly defined by the FOCs.*

**Proposition 6.** *The advertising expenditure in Proposition 5, together with any of the pricing strategy profiles in Proposition 2, Proposition 4, or Remark 2 give the subgame perfect equilibria of the two stage game.*

To illustrate Proposition 5, let $S\left(\alpha_i, \alpha_{-i}\right) = \frac{\alpha_i}{\Sigma_j \alpha_j}$ for $\forall i \in N$, $U\left(\Sigma_i \alpha_i\right) = \frac{\Sigma_i \alpha_i}{1 + \Sigma_i \alpha_i}$, and $c = 0$. Table 2.1 presents the equilibrium advertising expenditure, the size of the indifferent group, and the net profits of the firms, for different values of of $n$ and $r$.[19]

The sharing rule in the example, $S\left(\alpha_i, \alpha_{-i}\right) = \frac{\alpha_i}{\Sigma_j \alpha_j}$ maybe interpreted as a probability that an arbitrary loyal consumer chooses firm $i$.[20]

Although they are identical, at equilibrium, firms choose asymmetric advertising expenditure. This asymmetry follows from the choice of mixed pricing strategies. There is one firm with a strictly lower advertising level (firm $n$) and with the lowest loyal group. All other firms choose the same higher level of advertising and have equal larger loyal consumer bases. It is difficult to rank the *net* profits of the firms; however, in the examples above, the low advertiser makes lower profits than its rivals.

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[19] For computational convenience, the chosen advertising technology is a very effective one, resulting in a large loyal market $(U)$.

[20] This probability is nonincreasing in $i$ and thus a firm with higher advertising is more likely to be chosen by a loyal consumer.

This need not be always the case: The levels of net profits depend on the number of firms, on the advertising technology, and on the monopoly margins.

In Table 2.1, the individual advertising spending ($\alpha_i$) is decreasing in the number of firms and increasing in the monopoly margin $r - c$.

The lowest loyal group firm prices more aggressively, has higher probability of being the winner of the indifferent market, and makes lower *gross* expected profits. All other firms price less aggressive and make equal gross of advertising expenditure profits equal to monopoly profit on their loyal market.

## 2.5   A Random Utility Interpretation

In this section I offer a possible microfoundation to the demand system considered in the case of a duopoly. In particular, I argue that a random utility model  can lead to the proportional market sharing function $S_i = \frac{\alpha_i}{\Sigma_j \alpha_j}$ used in the examples. With a duopoly there is a unique mixed pricing equilibrium where both firms randomize, and $E\left(p_i \mid p_i < r\right) = E\left(p_j \mid p_j < r\right).$ In the marketing literature price dispersion models are used to explain price promotions.  The expected price conditional on a brand being being priced lower than $r$ is thought of as the average discounted price, while $1 - F\left(r\right)$ measures the frequency of discounts. Considering that loyal consumers are myopic and care only about the average discount and advertising, the proportional market sharing function, $S_i = \frac{\alpha_i}{\Sigma_j \alpha_j}$, can be derived from a random utility model.[21]

## 2.6   Discussion

While there is no doubt that advertising plays an important informative role in the economy, not less numerous are the occasions in which it does not provide any relevant information on price or product characteristics. In many homogenous product mar-

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[21]Consumers have idiosyncratic preferences over the brands and each consumer chooses the one that has the greatest brand advertising-average discount differential (see Anderson, dePalma and Thisse (1992)).

kets, advertising only operates a redistribution of the consumers among the sellers. Many advertising campaigns and a great deal of the TV spot advertising have rather an emotional content and try to attract consumers associating the product with attitudes or feelings that have no relevant relation to the product or its consumption.[22] In the U.S. market for cola soft drinks, where there is little genuine differentiation between Coke and Pepsi, heavy advertising generates subjective differentiation. Coke relies on more traditional values in its "Coca Cola...Real" campaign, while Pepsi addresses to the "GeneratioNext".[23] Similarly, Perrier significantly strengthened its position in the French mineral water market with its advertising campaign directed to the young generation, under the slogan "Perrier c'est fou" ("Perrier is crazy") making the product be perceived as very fashioned.[24] These considerations support the persuasive view on advertising and offer a justification for the stylized model here.

The present setting suggests that high advertisers tend to have higher prices. Often, blind tests show that consumers perceive highly advertised brand names as different.[25] The model also predicts the existence of a group of heavy advertisers and of a low advertiser. This is compliant with the empirical evidence that markets

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[22]Commenting on the new trends in television advertising, James Twitchell Professor of advertising at the University of Florida noted that "Advertising is becoming art. You don't need it, but it's fun to look at" (see Herald Tribune, January 10, 2003 p.7). Explaining the content of Chevrolet TV ads for automobiles "Heartbeat of America", broadcasted in 1988, General Motors advertising executive Sean Fitzpatrick observed that they "may look disorganized, but every detail is cold-heartedly calculated. People see the scenes they want to identify with.[...] It's not verbal. It's not rational. It's emotional, just the way people buy cars." (see Scherer and Ross 1990 p.573, originally from "On the Road again, with a Passion", New York Times, October 10, 1988)

[23]In the early part of the 1997, Pepsi launched its GeneratioNext campaign: "GeneratioNext is about everything that is young and fresh, a celebration of the creative spirit. It is about the kind of attitude that challenges the norm with new ideas, at every step of the way" (see www.pepsi.com). See also Tremblay and Polasky (2002).

[24]See J. Sutton (1991), p.253. Even more relevant for the present discussion is the subsequent campaign of Perrier under the (only literary) meaningless slogan "Ferrier c'est pou", a partial toggle of its initial slogan.

[25]"Double-blind experiments have repeatedly demonstrated that consumers cannot consistently distinguish premium from popular-priced beer brands, but exhibit definite preferences for the premium brands when labels are affixed-correctly or not." (see Scherer and Ross, 1990 p.582)

with significant advertising have a two-tier structure. The endogenous asymmetric advertising profile is more suitable for small oligopolies. In the US sport drinks market, Coca Cola and Pepsico are the two major suppliers. In 2002, Pepsico invested 125 millions to promote its product Gatorade, while Coca Cola invested 11 millions to advertise its product Powerade.

Several extensions to this work are worth mentioning. A major limitation of the present model is the extreme post-advertising heterogeneity of consumers: Loyal consumers are extremely advertising responsive, while the indifferent ones continue to be extremely price sensitive. One may consider that the loyals are willing to pay a price-premium for their most preferred brand, so that they become *less* price sensitive. Also, here, indifferent consumers are aware about existence of all products. It is more realistic to assume that the indifferent consumers know only the prices of some sellers.

## 2.7    Conclusions

The present article proposes a way to model the effects of persuasive advertising on price competition in a homogenous product market. I solve a two stage game in which an oligopoly competes, first, in persuasive advertising and, then, in prices. Advertising results in the creation of a loyal group attached to the firm. In the pricing stage, firms compete for remaining brand indifferent consumers. The equilibrium outcome exhibits price dispersion, although it is possible to have up to $n-2$ firms choosing monopoly pricing with probability 1. The advertising choices of the firms reflect the asymmetry in the mixed pricing strategies, and, at equilibrium, there is one firm that chooses a lower level of advertising and the remaining ones choose the same higher advertising. The model predicts an asymmetric market outcome despite initial symmetry of firms, and suggests how persuasive advertising may be successfully used to relax price competition.

## 2.8 Appendix

### 2.8.1 Appendix A

*Proof of Lemma 4.* I prove that $F_i(p)$ is continuous on $[L_i, r)$ by contradiction. Assume there is a mass point in the equilibrium distribution of firm $i$ at price $p < r$ and let the mass be equal to $\xi$. For simplicity, let $c = 0$.

Case 1. Suppose $\exists\, j$ pricing in the interval $(p, p + \varepsilon]$. Then, the variation in the profit of firm $j$ when pricing at $p - \varepsilon$ instead of $p + \varepsilon$ is:

$$(p - \varepsilon)\left[U_j + I\Pi_{k \neq j}(1 - F_k(p - \varepsilon))\right] - (p + \varepsilon)\left[U_j + I\Pi_{k \neq j}(1 - F_k(p + \varepsilon))\right] \geq$$

$$-2\varepsilon U_j + \Pi_{k \neq i,j}(1 - F_k(p - \varepsilon))\begin{bmatrix} I(p - \varepsilon)(1 - F_i(p - \varepsilon)) \\ -I(p + \varepsilon)(1 - F_i(p + \varepsilon)) \end{bmatrix} =$$

$$-2\varepsilon U_j + \Pi_{k \neq i,j}(1 - F_k(p - \varepsilon))\begin{bmatrix} I(p + \varepsilon)(F_i(p + \varepsilon) - F_i(p - \varepsilon)) \\ -2\varepsilon I(1 - F_i(p - \varepsilon)) \end{bmatrix}.$$

Then, $E\pi_j(p - \varepsilon) - E\pi_j(p + \varepsilon) \to \Pi_{k \neq i,j}(1 - F_k(p))Ip\xi > 0$ when $\varepsilon \to 0$. This contradicts the optimality of the equilibrium strategy of firm $j$.

Case 2. Suppose $\nexists j$ pricing in the interval $(p, p + \varepsilon]$, then $E\pi_i(p + \varepsilon) = (p + \varepsilon)U_i + I(p + \varepsilon)\Pi_{k \neq i}(1 - F_k(p)) > pU_i + Ip\Pi_{k \neq i}(1 - F_k(p))$. This contradicts the optimality of the equilibrium strategy of firm $i$ and completes the proof. □

*Proof of Proposition 3.* Suppose there exists a Nash equilibrium in prices different than the one in Proposition 2. Let $\widehat{S}_i^*$ be the associated supports of the price distributions. For simplicity, let $c = 0$.

By Lemma 2, $\{r\} \subseteq \widehat{S}_i^*$ for all $i$. Let $K = \left\{ i \mid \{r\} \neq \widehat{S}_i^* \right\}$.

By Proposition 1 and Lemma 1, $|K| \geq 2$.

a) Assume, first, $|K| > 2$.

By Lemma 3, $\exists l \in N$, such that $f_l^*(r) = 0$. Notice that for all $i \in N \setminus K$, $f_i^*(r) = 1$. Hence, $l \in K$.

By Lemma 1, $\exists i, k \in N$, s.t. $L_i = \min \widehat{S}_i^* = L_k = \min \widehat{S}_k^*$

a1) Suppose that $l = i$. Let $k, h \in K$, $k \neq l \neq h$ and $L_l = L_k$.

1.1. Consider, first, the case $L_h > L_l = L_k$. Then,

$p\left(U_h + I\Pi_{j\neq h}\left(1 - F_j\left(p\right)\right)\right) = rU_h$ and $p\left(U_k + I\Pi_{j\neq k}\left(1 - F_j\left(p\right)\right)\right) = rU_k$.

It follows that $\frac{1-F_k(p)}{1-F_h(p)} = \frac{U_h}{U_k}$ or $F_k\left(p\right) = \frac{U_k-U_h}{U_k} + \frac{U_h}{U_k}F_h\left(p\right)$.

By Lemma 4, $F_h\left(L_h\right) = 0 \Rightarrow F_k\left(L_h\right) = \frac{U_k-U_h}{U_k}$. Then, because $L_h > L_k$ $F_k\left(L_h\right) > 0 \Leftrightarrow U_k > U_h$. If firm $h$ deviates to $L_k$, then for sure wins and makes profits of $L_k\left(U_h + I\right)$. But, $L_k \geq \frac{U_k r}{U_k+I} > \frac{U_h r}{U_h+I} \Rightarrow L_k\left(U_h + I\right) > \frac{U_h r}{U_h+I}\left(U_h + I\right) = U_h r$, where the RHS of the equality is the profit of firm $h$ when it prices on its support. This proves that this cannot be an equilibrium.

1.2. Consider that $L_h = L_l = L_k$. Following the equilibrium conditions above $F_h\left(L_h\right) = 0 \Rightarrow F_k\left(L_h\right) = \frac{U_k-U_h}{U_k} \Rightarrow U_k = U_h$. This proves that this cannot be an equilibrium for **any** profile of weakly ordered loyal bases.

a2) Suppose that $i \neq l \neq k$. Then, $L_l > L_i = L_k$. Then, using the above equilibrium condition, follows that $U_k = U_i$. So this cannot be an equilibrium for **any** profile of weakly ordered loyal bases.

b) Finally, by construction, the equilibrium presented in Proposition 2 is unique for $|K| = 2$. $\qquad\square$

**Proposition (Varian (1980), p.658):** If each store's optimal strategy involves zero probability of a tie, and $f\left(p\right) > 0$ for all $p^* \leq p < r$, then each store must choose the same strategy.

Notation: $f\left(p\right)$ represents the pricing density, $p^*$ represents the average cost associated with serving the whole informed market plus the proportional part of the uninformed one and $r$ is the common reservation value of the consumers.

### 2.8.2 Appendix B

*Proof of Proposition 5.* The advertising choices, in the reduced form game, follow from the FOC's of the maximization problems (see Proposition 5):

$\frac{\partial \pi_i^{net}}{\partial \alpha_i} = 0 \Leftrightarrow U_{i,i} = \frac{1}{(r-c)}$

$\frac{\partial \pi_n^{net}}{\partial \alpha_n} = 0 \Leftrightarrow$

$\frac{(U_n+I)(U_{n-1}+I)U_{n-1,n}+U_{n-1}\left[\left(U_{n,n}-U'\right)(U_{n-1}+I)-\left(U_{n-1,n}-U'\right)(U_n+I)\right]}{(U_{n-1}+I)^2} = \frac{1}{(r-c)}$

Suppose to the contrary that $\alpha_n^* = \alpha_j^* = \alpha^*$ for all $j \neq n$. Then, should hold that:

$\frac{\partial}{\partial \alpha_n} \pi_n^{net}(\alpha^*) \geq 0$ and $\frac{\partial}{\partial \alpha_j} \pi_j^{net}(\alpha^*) \leq 0$ for all $j = 1, 2, ... n-1$.

The above inequalities imply the following one:

$U_{i,i} \leq \frac{U_{n-1,n}(U_n+I)}{U_{n-1}+I} + \frac{U_{n-1}[(U_{n,n}-U')(U_{n-1}+I)-(U_{n-1,n}-U')(U_n+I)]}{[U_{n-1}+I]^2}$

Notice that $S_i(\alpha^*) = S_j(\alpha^*)$ and $S_{i,i}(\alpha^*) = S_{j,j}(\alpha^*)$ for $\forall i, j$. In effect, $U_i(\alpha^*) = U_j(\alpha^*)$ and $U_{i,i}(\alpha^*) = U_{j,j}(\alpha^*)$ for $\alpha_n^* = \alpha_j^* = \alpha^*$, given that $U_i = S_i U$ and $U_{i,i} = S_{i,i}U + S_i U'$. Then, the last inequality becomes: $U_{i,i} \leq U_{n-1,n} + \frac{U_{n-1}[U_{n,n}-U_{n-1,n}]}{(U_{n-1}+I)} \Leftrightarrow$

$S_{i,i}U + S_i U' \leq S_{n-1,n}U + S_{n-1}U' + S_{n-1}U\frac{S_{n,n}U+S_nU'-S_{n-1,n}U-S_{n-1}U'}{S_{n-1}U+1-U} \Leftrightarrow$

$S_{i,i}U \leq S_{n-1,n}U + S_{n-1}U\frac{S_{n,n}U-S_{n-1,n}U}{S_{n-1}U+1-U} \Leftrightarrow$

$(S_{i,i} - S_{n-1,n})U(1-U) \leq 0 \Leftrightarrow$

As $S_{i,i} > 0$ and $S_{i,j} < 0$ then the inequality holds only if $U = 0$ or $U = 1$. But, $\lim_{\Sigma_i \alpha_i \to \infty} U(\Sigma_i \alpha_i) = 1$ and $\frac{\partial U_i(0,\alpha_{-i})}{\partial \alpha_i} > 0$. Then, $\alpha_n < \alpha_j = \alpha, \forall j \neq n$.  □

*Proof of Proposition 6.* To make sure that the candidate maximum defines the first stage strategies in a subgame perfect equilibrium, firm $j = 1, ... n-1$ should not have incentives to leapfrog firm $n$, and firm $n$ should not have incentives to leapfrog its rivals.

Consider firm $n$. Its profits are:

$\pi_n(\alpha) = \begin{cases} (r-c)U_{n-1}(\alpha, \alpha_i^*)\frac{U_n(\alpha,\alpha_i^*)+I}{U_{n-1}(\alpha,\alpha_i^*)+I} - \alpha & \text{if } \alpha \leq \alpha_i^* \\ (r-c)U_n(\alpha, \alpha_i^*) - \alpha & \text{if } \alpha > \alpha_i^* \end{cases}$,

where $\alpha$ is the choice of firm $n$ and $\alpha_i^*$ is the equilibrium choices of firms $j = 1, ... n-1$. Consider the case $\alpha > \alpha_i^*$, then the following is true:

$(r-c)\frac{\partial U_n}{\partial \alpha_n}(\alpha, \alpha_i^*) - 1 \leq (r-c)\frac{\partial U_n}{\partial \alpha_n}(\alpha, \alpha_i^*, \alpha_n^*) - 1 = 0.$

The LHS is the first derivative of the profit with respect to own choice and the RHS equality is the FOC of firm $j$ in equilibrium. The inequality follows from strict concavity of $U_i$ $\left(\frac{\partial U_j}{\partial \alpha_j}(\alpha_i^*,.) > \frac{\partial U_j}{\partial \alpha_j}(\alpha,.)\right)$ and from strategic substitutability of ad-

vertising choices $\left(\frac{\partial U_i}{\partial \alpha_i}(.,\alpha_i^*) > \frac{\partial U_i}{\partial \alpha_i}(.,\alpha_n^*)\right)$. Then, firm $n$ has incentives to decrease. When $\alpha \leq \alpha_i^*$, the profit function is strictly concave and, hence, maximized at $\alpha = \alpha_n^*$. Consider firm $j = 1,...n-1$. Its profits are:

$$\pi_j(\alpha) = \begin{cases} (r-c)\,U_n(\alpha,\alpha_i^*) - \alpha & \text{if } \alpha \geq \alpha_n^* \\ (r-c)\,U_n(\alpha,\alpha_i^*,\alpha_n^*)\frac{U_j(\alpha,\alpha_i^*,\alpha_n^*)+I}{U_n(\alpha,\alpha_i^*,\alpha_n^*)+I} - \alpha & \text{if } \alpha < \alpha_n^* \end{cases}$$

where $\alpha$ is the choice of firm $n$, and $\alpha_i^*$ and $\alpha_n^*$ are the equilibrium choice of firms $j = 1,...n-1$ and, respectively, $n$.

Consider $\alpha < \alpha_n^*$, then the following is true:

$\frac{\partial \pi_j(\alpha)}{\partial \alpha_j}(\alpha,\alpha_i^*,\alpha_n^*) > \frac{\partial \pi_n(\alpha)}{\partial \alpha_n}(\alpha_n^*,\alpha_i^*,\alpha_i^*) = 0.$

The LHS is the first derivative of the profit with respect to own choice and the RHS equality is the FOC of firm $n$ in equilibrium. The inequality follows from strict concavity of $\pi_j$ $\left(\frac{\partial \pi_i}{\partial \alpha_i}(\alpha_n^*,.) < \frac{\partial \pi_i}{\partial \alpha_i}(\alpha,.)\right)$ and from the strategic substitutability of the advertising choices $\left(\frac{\partial \pi_j}{\partial \alpha_j}(.,\alpha_i^*) < \frac{\partial \pi_j}{\partial \alpha_j}(.,\alpha_n^*)\right)$. Then, firm $j$ has incentives to increase.

When $\alpha \geq \alpha_n^*$, the profit function is strictly concave and, hence, maximized at $\alpha = \alpha_j^*$. $\qquad\qquad\square$

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**Chapter 3**

# Is Bundling Anticompetitive?

## 3.1 Introduction

Bundling is a vertical practice and consists of selling together several products in the form of a package, regardless of whether consumers want to buy components from another seller. Examples of bundles are the desktop application packages Microsoft Office or Sun's Star Office.[1] Firms often tie to increase efficiency. However, there are also strategic reasons like market foreclosure or extension of monopoly power that raise concerns about the anticompetitive effects of the practice.[2] Recent economic theory also views bundling as a way of acquiring an advantageous market position when competing against uncoordinated component sellers, explaining in this manner

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[1] A quick glance at the IT industries reveals that bundling is a common practice. Apple's i-Mac is an all-in-one computer, display and operating system (Mac OS X), the later being itself a bundle of traditional OS functions and DVD player, Media Player (QuickTime) and MS Internet Explorer. Real Networks offers various bundles of media capture, creation, presentation and delivery. In fact, Real One Player is a bundle of audio and video player, jukebox and media browser, like its main competitor, Windows Media Player. The examples do not limit to IT industries. Bundling of cars with radios or air conditioners was central to antitrust cases like *Town Sound and Custom Tops, Inc. v. Chrysler Motors Corp. (1992)* or *Heatransfer Corp. v. Volkswagenwerk AG* (1977).

[2] More recently, tying was subject to harsh treatment in the antitrust courts, in cases like, *Eastman Kodak v. Image Technical Service Inc. (1992),* and *EC v. Microsoft Corp. (2000)* where bundling of Windows OS with Windows Media Player was central to the case. (The decision in this case was appealed in June 2004.) Also, bundling of Windows OS with Internet Explorer was part of the controversial case *US v. Microsoft Corp. (1998).*

the success of application bundles like MS Office.

The increasingly frequent use of bundling raises questions about its impact on competition and points out toward the necessity of economic analysis able to provide sound grounds for antitrust enforcement policies. In this paper I show that the potential of bundling to create a dominant market position crucially depends on the parameter conditions: A best response to rival bundling may be to bundle or not depending on the elasticity of demand. I focus on the implications of bundling on complementary goods markets. In a model of imperfect competition with spatial product differentiation, I consider product bundles formed of two and, respectively, three components. Each component is produced by differentiated firms that differ across markets. The firms choose whether or not to bundle. Using two types of demand function, an elastic and an inelastic one, I identify how price elasticity affects the incentives to bundle.

Bundling increases price competition by decreasing product differentiation on the market. However, it has also a positive effect due to the internalization of the positive externality that a price decrease of one component has on the demand for the complementary ones. I show that the positive effect of bundling increases with the size of the system and, in the three component systems case, with elastic demand, is strong enough to offset the negative effects of an increase in price competition.

My model relates to the work of Nalebuff (2000), who analyses the effects of bundling on price competition in an inelastic demand setting. Once there are four or more items, the bundle seller does better than when he sells components individually and, moreover, bundle against component competition is a stable market outcome. This suggests that bundling can be an effective anticompetitive tool giving a firm that sells a bundle of complementary products a substantial advantage over rivals who sell the component products separately.

The present analysis shows that the *elastic demand setting* no longer supports these results. Here, *whenever there are incentives to bundle, the market outcome is*

*bundle against bundle competition and bundling cannot be viewed as an anticompetitive tool.* With elastic demand, the changes in prices affect, not only the market sharing, but also the level of the demand at each location. In a zero elasticity setting, the demand for a product depends only on the price difference between the rival alternatives on the market. Then, the effects of a price decrease are partially offset by the response of the competitor. In a positive elasticity setting, the demand depends both on the price difference and the own price. Even though a price decrease generates a market sharing effect altered by the reaction of the rival, it has a direct positive market size effect unaltered by the competitors, because at each location more consumers buy.

I perform the related welfare analysis and show that the incentives to bundle are socially excessive. However, bundle against bundle competition (the market outcome with an elastic demand) generates higher consumer surplus than bundle against component competition (the market outcome with inelastic demand).

The present article closely relates to the work of Matutes and Regibeau (1992). They were the first to extend the monopoly bundling framework by considering a duopoly that produces in two complementary markets. They mainly focus on the compatibility decision and the incentives to bundle. While my model is sharing certain features with their article, some of the most important differences are the consideration of duopolies varying across markets (resulting in a price coordination problem) and of three component systems, besides from the two component ones (allowing to analyze how the number of tied items affects the incentives to bundle and the market outcome).

Section 3.2 presents the model and section 3.3 examines the market equilibria in all considered cases. In section 3.4, I perform the welfare analysis. Finally, Section 3.5 discusses the results and section 3.6 draws the final conclusions. Proofs are presented in the Appendix.

## 3.2 The Model

I consider bundles of two ($n = 2$) and, respectively, three ($n = 3$) components. In each component market, there are two rival brands, produced by firms $A_i$ and, respectively, $B_i$ ( $i = \{1, ..., n\}$ ).[3] The $A$ and $B$ components are imperfect substitutes, while $i$ and $j$ ($i \neq j$) are complementary components. Consumers get their valuation of the product only if all components are purchased. The product differentiation in each component market is modeled à la Hotelling. For each component I use a coordinate axis, where $A$ brand is located at 0 and the $B$ one is located at 1. The locations of all the systems form the vertices of a square (when $n = 2$) or cube (when $n = 3$) of volume 1. The transportation cost per unit of length is assumed to be 1. The production cost is assumed to be zero.[4]

Consumer locations are uniformly distributed on the square (cube). For a specific component, a consumer location belongs to the interval $[0, 1]$. The coordinate gives the linear transportation cost incurred by the consumers at this specific location. Let $x$ be the axis for the first component, $y$ for the second one and $z$ for the third component (this comes into play only for $n = 3$). Considering $n = 2$, a consumer located at $(x, y)$, incurs a transportation cost of $x$ if he buys the first component from the $A$ firm, or, respectively, $1 - x$ if he buys the first component from the $B$ firm, and a transportation cost of $y$ if he buys the second component from the $A$ firm or, respectively, $1 - y$ if he buys it from the $B$ firm. When $n = 3$ a consumer located at $(x, y, z)$ incurs, in addition, a cost of $z$, when buying the third component from $A$ or $1 - z$, when buying it from $B$.[5]

Each consumer chooses one system that minimizes his perceived price ($\theta$) equal to the price of the system plus the transportation cost. He buys only if this sum does not exceed his valuation of the product.

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[3] Duopolists serving different component markets are different firms.

[4] With positive constant marginal cost the results would still hold as mark-ups over the cost.

[5] For example, the transportation cost is $x + 1 - y$ for bundle $(A_1, B_2)$ and $x + 1 - y + z$ for bundle $(A_1, B_2, A_3)$.

At each consumer location, there is a linear demand $D(\theta) = b - a\theta$, depending on the perceived price at this location.[6] I parametrize the slope to analyze how incentives to bundle change with price elasticity.[7] To study the effects of bundling on price competition, I restrict attention to covered market equilibria where firms directly interact on the market. For this, at each location in the unit square (cube) there must be consumers who buy.[8] Consumers' valuation of the product (when $a = 0$) and the maximal valuation (when $a = 1$) should be high enough. When $a = 1$, not necessarily all consumers can buy, due to their heterogeneous valuations. For the simulation, the value $b = 10$ guarantees that the market is covered in equilibrium.[9]

For both inelastic and elastic demand: a) I identify the incentives to bundle and, b) I determine how the incentives to bundle change with the size of the bundle. For the first purpose (a), I compare the equilibrium outcomes of the three possible competition modes when pure bundling is available: component versus component competition (CvsC), bundle versus component competition (BvsC), and bundle versus bundle competition (BvsB). To see how the number of components affects the incentives to bundle (b), I compare the results for two- and three-component systems.

I assume that, whenever firms bundle, the tied system is incompatible with the competing components, and that pure bundling is the only available strategy: A bundler does not sell separate components. In section 3.5, I discuss the robustness of the results when these requirements are relaxed.

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[6] "The way to justify a downward sloping demand at a given location is to envision a large number of consumers with different tastes for the system at this location." (J. Tirole, 1988, footnote 64, p. 335.) When $a = 0$, all consumers at a given location have the same valuation of the product.

[7] When $a = 0$, the demand is inelastic and when $a = 1$, the demand is elastic.

[8] That is, at the most remote locations where a system is preferred (i.e., on the indifference line (plane)) there should be consumers who buy.

[9] In the inelastic case a change of $b$ only changes the size of the market. In the elastic demand case, the same qualitative results can be obtained for any value of $b$, whenever the market is still fully covered at equilibrium.

## 3.3   The incentives to bundle

In this section I analyze the incentives to bundle for two types of demand function and two sizes of the bundle. There are incentives to bundle if the profit of the bundler in BvsC is higher than the aggregate profits of the potential bundlers in CvsC. If, in addition, aggregate profits of the component selling competitors of the bundler are higher than their profits in BvsB, then BvsC is a stable market outcome.

In CvsC, each consumer buys $n$ complementary components, and he can mix and match to form his own system. Let $p_i$ and $q_i$ be the prices charged by the producers of component $i$, $A_i$ and, respectively, $B_i$.

In the presence of bundling, there are two possible cases. In BvsB, all $A$ and, respectively, $B$ firms coordinate their pricing decision and sell their components only as a bundle. In BvsC, only the $A$ firms sell a bundle, and $B$ firms continue to sell separate components. Consumers choose one of the two bundles (systems). Let $p$ be the price of the coordinated $A$ firm, $q$ be the price of the $B$ firms (in BvsB it is the price of the bundle, and in BvsC it is the sum of component prices, $\Sigma_i q_i$) and let $\Delta$ be the price difference between the two systems $q - p$.

In BvsC, the bundler internalizes the positive externality that one component seller has on the seller of a complementary good, while its competitors, selling separate components, neglect this effect. Then, $p < q = \Sigma q_i$ and $\Delta > 0$.[10]

At equilibrium should hold that $|q_i - p_i| \leq 1$ and $|\Delta| \leq n$, otherwise all consumers prefer to buy the lower price system.[11]

Each consumer buys the system with the smallest total cost, and the locations of the consumers that buy the same system form an adjacent polygon (polyhedron) within the unit square (cube). When $a = 0$, at each such location the demand is

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[10] This is the result of the Cournot's dual problem and it still holds in a differentiated duopoly setting (Singh & Vives, 1984.)

[11] There can be no equilibrium with only one active firm. The competitor would make zero profits, while undercutting the monopolist, can make positive profits.

equal to 10. When $a = 1$, the demand at each location depends on the perceived price.

Subsection 3.3.1. is devoted to the case of two-component systems and subsection 3.3.2. deals with three-component systems. In the Appendix, the results are grouped by competition mode.

### 3.3.1 Two-component system case

In the **absence of bundling** there are two $A$ firms $(A_1, A_2)$ and two $B$ firms $(B_1, B_2)$, each selling the corresponding individual component. There are four systems available on the market: $(A_1, A_2), (A_1, B_2), (B_1, A_2), (B_1, B_2)$. The perceived prices of a consumer located at $(x, y)$ are: $p_1 + p_2 + x + y$ for system $(A_1, A_2)$, $q_1 + q_2 + 1 - x + 1 - y$ for system $(B_1, B_2)$, $q_1 + p_2 + 1 - x + y$ for system $(B_1, A_2)$ and $p_1 + q_2 + x + 1 - y$ for system $(A_1, B_2)$.

The vertical line $x = x_0 \equiv \frac{1 + q_1 - p_1}{2}$ separates the locations of consumers who buy different first component, and the horizontal line $y = y_0 \equiv \frac{1 + q_2 - p_2}{2}$ separates the locations of consumers who buy different second component.

For example, (see Figure 3.1A) locations of the consumers who buy system $(A_1, A_2)$ lie inside square $MNOP$. When $a = 1$, demand at a location is highest at point $O$ (transportation cost is minimal), and lowest at point $M$ (transportation cost is maximal).[12] Demand for system $(A_1, A_2)$ is given by the volume depicted in Figure 3.1B.

From the computation of the demands and the profit maximization problem, follow the equilibria.

**Proposition 7.** *With two-component systems, in the absence of bundling, the equilibrium prices and the corresponding profits are:*

*a) when $a = 0$, $p_i = q_i = 1$ and $\pi_{Ai} = \pi_{Bi} = 5$.*

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[12]In a symmetric equilibrium, the demand at points $N$ and $P$, and along the line that connects them is equal.

A                                                    B
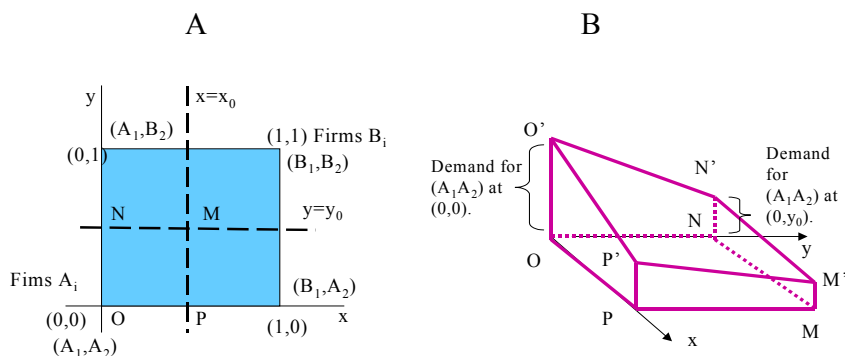


Figure 3.1: A. Determination of locations of consumers who prefer same system. B. Computation of the demand for system $(A_1, A_2)$ in the elastic demand case.

b) when $a = 1$, $p_i = q_i = 0.91101$ and $\pi_{Ai} = \pi_{Bi} = 3.4974$

In the **presence of bundling**, locations where consumers are indifferent between the two systems are given by the intersection of the unit square with line $x + y = \frac{\Delta+2}{2}$. This line divides the unit square in two areas each formed by consumers who prefer the same system (see Figure 3.2A). Locations in the unit square that lie below the indifference line are served by firm $A$ and the locations in the unit square that lie above are served by the $B$ firms.

To have a covered market, when $a = 1$, the zero demand line should lie further from the seller's location than the indifference line.[13]

When both $A$ and $B$ firms bundle, there is a symmetric equilibrium. The indifference line coincides with $x + y = 1$, as $\Delta = 0$. For instance, in Figure 3.2A, locations of consumers who buy bundle $A$ lie within triangle $ORQ$ and, at equilibrium, locations are split equally between firms. When $a = 1$, demand is highest at

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[13]Then at all locations of consumers who prefer a certain bundle, there are buyers who purchase. Zero demand line of a system is given by $x + y = 10 - p_s$, where $p_s$ is the system price.

Figure 3.2: A. Determination of locations of consumers who prefer same system in the presence of bundling. B. Demand for bundle A, with elastic demand, under symmetric bundling.

point $O$ (transportation cost is minimal) and lowest at points $R$ and $Q$ (and along the indifference line that connects them where transportation cost is constant and minimal). Figure 3.2B depicts the volume that gives total demand for bundle $A$ in this setting.

Using the volumes I compute the demands for the two bundles. From the corresponding first order conditions of the profit maximization problem I obtain the equilibrium prices.

**Proposition 8.** *With two-component bundles, when both A and B firms bundle, the equilibrium prices and the corresponding profits are:*
*a) when $a = 0$, $p = q = 1$ and $\pi_A = \pi_B = 5$.*
*b) when $a = 1$, $p = q = 0.9265$ and $\pi_A = \pi_B = 3.8946$.*

**Asymmetric bundling** is the last competition mode to consider. Figure 3.2A depicts the market sharing in this case, where the indifferent consumers locations lie on segment $ST$.

Using the market sharing between the two available systems, demands faced by firm $A$ and by the $B$ firms can be computed.[14] While $A$ firms coordinate their pricing decision, each $B$ firm chooses separately the price of the component it sells.[15] The profit maximization problem gives the equilibrium in this case.

**Proposition 9.** *With two-component bundles, under asymmetric bundling, the equilibrium prices and the corresponding profits are:*

*a) when $a = 0$, $p = 1.4533$, $q_i = 0.86332$ ($\Sigma q_i = 1.7266$) and $\pi_A = 9.117$, $\pi_{Bi} = 3.2173$ ($\Sigma_i \pi_{Bi} = 6.4346$).*

*b) when $a = 1$, $p = 1.3045$, $q_i = 0.7971$ ($\Sigma q_i = 1.5943$) and $\pi_A = 6.5736$, $\pi_{Bi} = 2.2849$ ($\Sigma_i \pi_{Bi} = 4.5697$).*

With two-component bundles, independent of the elasticity of demand, the highest profits are obtained when all firms sell separate components and consumers are allowed to mix and match. There are no incentives to bundle in this case. Moreover, assuming that $A$ firms bundled, the $B$ firms do not have incentives to bundle as their profits in BvsC are higher than their profits in BvsB. Symmetric bundling is the strongest competition attainable in this setting and makes the price of the bundle fall to about half of the price of the system in CvsC. It also makes the profits of the bundlers be almost half of their aggregate profits in the absence of bundling.

### 3.3.2   Three-component system case

In this part I consider bundles formed of three goods. **In the absence of bundling, there are three $A$ firms ($A_1, A_2, A_3$) and three $B$ firms ($B_1, B_2, B_3$), each selling one component. Thus, there are eight systems available on the market. The perceived**

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[14]When $a = 0$, the demand for the bundle is the volume of the triangular prism with base $SOT$ (see Fig. 3.2B) and height 10 minus the volumes of two triangular prisms with same height whose bases lie beyond the unit square. When $a = 1$, the geometric figure is similar to the one in Fig. 3.2B, requiring the above-mentioned adjustments.

[15]Each $B$ firm maximizes its profits only with respect to own price. The resulting price of the $B$ system is the sum of the individual prices of the two components.

price of a consumer located at $(x, y, z)$ is, for example, $p_1 + p_2 + q_3 + x + y + 1 - z$ for buying $(A_1, A_2, B_3)$.

The planes $x = x_0 \equiv \frac{1+q_1-p_1}{2}$, $y = y_0 \equiv \frac{1+q_2-p_2}{2}$, and $z = z_0 \equiv \frac{1+q_3-p_3}{2}$ separate the locations of consumers who prefer different first, second and, respectively, third component. The locations of consumers that buy the same system form an interior parallelepiped with three faces adjacent to the unit cube. Intuitively, the geometric representation is an extension of Figure 3.1 to three dimensions. When $a = 1$, total demand for a system is given by the volume of a four dimensional figure.[16] After deriving the functional forms of the demands, next result follows.

**Proposition 10.** *With three-component systems, in the absence of bundling, the equilibrium prices and the corresponding profits are:*
*a) when $a = 0$, $p_i = q_i = 1$ and $\pi_{Ai} = \pi_{Bi} = 5$.*
*b) when $a = 1$, $p_i = q_i = 0.89735$ and $\pi_{Ai} = \pi_{Bi} = 2.9424$.*

In the **presence of bundling**, with an elastic demand at each location, the covered market condition requires the indifference plane to lie closer to the location of the system than the zero demand plane.[17] The set of locations where consumers are indifferent between the two bundles is given by the intersection of the unit cube with the plane $x + y + z = \frac{\Delta+3}{2}$. The locations in the unit cube that lie below the indifference plane are served by $A$ firm and the locations above are served by the $B$ firm(s).

**When both $A$ and $B$ firms bundle** there is a symmetric equilibrium. The equation of the indifference plane is $x + y + z = \frac{3}{2}$. When $a = 1$, the demand faced by any of the bundlers is given by the volume of a four-dimensional figure.[18]

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[16] Sectioning horizontally, these adjacent parallelepipeds, I obtain a rectangle. At each location in this rectangle there is an elastic demand. The demand corresponding to such section is given by the volume of the flat top pyramid similar to $MNOPM'N'P'O'$ in Figure 3.1B. Integrating over all sections, I compute the demands for the systems.

[17] The zero demand plane of a system is given by $x+y+z = 10 - p_s$, where $p_s$ is the price charged for the system.

[18] Using the sectioning method presented before and integrating over all sections, the demands for

**Proposition 11.** *With three-component bundles, when both A and B firms bundle, the equilibrium prices and the corresponding profits are:*
*a) when $a = 0$, $p = q = 1.333$ and $\pi_A = \pi_B = 6.6665$.*
*b) when $a = 1$, $p = q = 1.1903$ and $\pi_A = \pi_B = 4.5922$.*

Under **BvsC**, $\Delta = \Sigma_i q_i - p > 0$ and $a = 1$, the demand is given by the volume of a four dimensional figure to be computed using the steps described in BvsB case. The profit maximization problem gives the equilibrium in this case, where each independent seller $(B_i)$ maximizes with respect to its own price, without considering the impact of his pricing strategy on the other complementary component sellers, $B_j$ $(i \neq j)$.

**Proposition 12.** *With three-component bundles, under asymmetric bundling, the equilibrium prices and the corresponding profits are:*
*a) when $a = 0$, $p = 2.094$, $q_i = 0.88493$ $(\Sigma_i q_i = 2.6548)$ and $\pi_A = 14.72$, $\pi_{Bi} = 2.6287$ $(\Sigma_i \pi_{Bi} = 7.8861)$.*
*b) when $a = 1$, $p = 1.7671$, $q_i = 0.7898$ $(\Sigma_i q_i = 2.3694)$ and $\pi_A = 8.8302$, $\pi_{Bi} = 1.5066$ $(\Sigma_i \pi_{Bi} = 4.5198)$.*

With an *inelastic demand* and three-component bundles, still there are no incentives to bundle: While aggregate profits of potential bundlers, in CvsC are equal to 15, in BvsC, the bundler makes a profit of 14.72. Moreover, if $A$ firms bundle, the $B$ firms do not have incentives to bundle because their profits are higher in BvsC than in BvsB. Symmetric bundling is the strongest attainable competition mode, profits of the bundlers fall by more than half of their aggregate profits in the absence of bundling. Nalebuff (2000) shows that for bundles of more than 4 components there are incentives to bundle, and the component seller competitors of a bundler do not have incentives to bundle. This makes bundling be an efficient tool to depress rivals' profits, while increasing the profits of the bundler, when large bundles are involved.

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the two bundles can be computed. In this case, the demand corresponding to an arbitrary section is the volume of a flat top pyramid resembling $OQRO'R'Q'$ in Figure 3.2B.

The competitors, moreover, are worse off if they decide to bundle their components as well.

But, when *demand is elastic*, for bundles of three-components, the profits of a bundler competing against components exceed aggregate profits that complementary component sellers obtain in the absence of bundling. In effect, the incentives to bundle are stronger in this setting. In addition, when there are incentives to bundle, BvsC is not a stable outcome, because component sellers have incentives to bundle as well. The market outcome is BvsB, the most competitive one. Hence, once an elastic demand is considered, bundling cannot be viewed as an efficient anticompetitive tool. With elastic demand, for larger systems, there are incentives to bundle, and the market outcome is characterized by strong competition, unlike the inelastic demand case.

CvsC, where consumers are allowed to mix and match components, counts with highest product differentiation and, therefore, with weakest price competition. Bundling decreases product differentiation and leads to stronger price competition, reducing profits. But, it also helps to internalize the positive price externality that complementary goods sellers have on each other, augmenting profits. Incentives to bundle result from the trade off between these two opposed effects. The positive impact on profits gets stronger as bundle size increases because a decrease in the price of a component favors a larger number of complementary components. Similarly, symmetric bundling affects positively previously uncoordinated competitors of a bundler, making them internalize the positive price externality they have on each other. Also, with inelastic demand, under BvsB, the higher incentives to undercut prices[19] hurt both firms and make the competitors of a bundler worse off than under BvsC.

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[19]Starting from BvsC, BvsB generates a price decrease relatively to the price of the system previously sold by independent sellers. This decrease, further decreases $\Delta$, makes the initial bundler loose market share and determines him to undercut prices. In his turn the last bundler has incentives to undercut and so on.

With *inelastic demand*, whenever there are incentives to bundle, the market outcome of the bundling game (BvsC) favors the bundler and is detrimental to his competitors. In the presence of bundling, the demands depend only on the price difference $(\Delta)$ and not on the own price. A decrease in price affects demand only to the extend to which it acts upon the price difference $(\Delta)$. Then, under symmetric bundling, part of the internalized price externality is offset by the price cut of competitors.

With *elastic demand*, incentives to bundle are stronger and the best response to bundling is to bundle. An important difference is that aggregate demand depends on both the price difference and own price. As before, the decrease in price due to bundling affects the demand through the price difference $(\Delta)$, *but it also has a direct effect*. This makes the incentives to bundle be stronger with an elastic demand (they already exist for bundles of three components, unlike the inelastic setting). In addition, the incentives to undercut prices created by symmetric bundling, are attenuated by the increase in the demand at each location due to lower prices. This makes the competitors of a bundler better off when selling a bundle. As a result, the outcome of the bundling game resembles a Prisoners' Dilemma.

Although, for computational reasons, I performed the analysis only for bundles of two and three components, I believe that same result holds for arbitrarily large bundles.

## 3.4   Welfare analysis

To complete the analysis, I assess the impact of bundling on social welfare, computed as the sum of the consumer surplus and profits.

With *inelastic demand*, given that the prices are just a transfer from consumers to firms and that market is fully covered, total surplus depends on the total transportation cost incurred by the consumers. Social welfare is maximized when the total

transportation cost is minimized, case that turns out to be CvsC, independently of bundle size. Moreover, total surplus in CvsC is equal to the socially optimal level. BvsB results in the highest level of consumers surplus and, although, firms obtain their lowest profits, total welfare exceeds the one created by BvsC. With bundles of two and three components, there are no incentives to bundle and equilibrium welfare is socially optimal. Once larger bundles are considered, the market outcome is BvsC, and total surplus is lowest.

With *elastic demand,* the price directly affects the levels of social welfare and even surplus maximizing competition mode is below the welfare optimal levels. When demand is elastic, CvsC creates highest total surplus. BvsB, the market outcome when larger bundles are considered, results in a higher welfare than BvsC. BvsB depresses profits, but consumers' gain offsets producers losses. Hence, when there are incentives to bundle and demand at each location is elastic, the market outcome creates the highest welfare attainable in a bundling setting.[20]

Tables 3.1 and 3.2 present consumer surplus, profits and total welfare, for inelastic and, respectively, elastic demand, and Table 3.3 presents optimal welfare levels.[21] The computation can be found in the Appendix.

Table 3.1: Welfare levels with inelastic demand

| No. of components | $n = 2$ | | | $n = 3$ | | |
|---|---|---|---|---|---|---|
| Competition | CvC | BvB | BvC | CvC | BvB | BvC |
| Cons. Surplus | 75 | 83.33 | 77.61 | 62.5 | 75.72 | 66.10 |
| Total Profits | 20 | 10 | 15.52 | 30 | 13.33 | 22.60 |
| Total Surplus | 95 | 93.33 | 93.16 | 92.5 | 89.06 | 88.70 |

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[20]Still, this level of welfare is below the one created under CvsC, and below optimal level.

[21]I also present optimal welfare in the presence of bundling (in BvsB), though is always below the levels in CvsC (to which I refer in the text whenever the welfare optimal levels are mentioned).

Table 3.2: Welfare levels with elastic demand

| No. of components | $n = 2$ | | | $n = 3$ | | |
|---|---|---|---|---|---|---|
| Competition | CvC | BvB | BvC | CvC | BvB | BvC |
| Cons. Surplus | 29.49 | 35.36 | 31.26 | 21.53 | 29.8 | 23.89 |
| Total Profits | 13.99 | 7.78 | 11.14 | 17.65 | 9.18 | 13.35 |
| Total Surplus | 43.48 | 43.14 | 42.41 | 39.18 | 38.98 | 37.24 |

Table 3.3: Welfare optimal levels

| Demand | Inelastic | | | | Elastic | | | |
|---|---|---|---|---|---|---|---|---|
| No. of comp. | $n = 2$ | | $n = 3$ | | $n = 2$ | | $n = 3$ | |
| Competition | CvC | BvB | CvC | BvB | CvC | BvB | CvC | BvB |
| Welfare | 95 | 93.33 | 92.5 | 89.06 | 45.14 | 43.58 | 42.81 | 39.70 |

## 3.5   Results and discussion

Using the equilibrium profits corresponding to each of the cases studied in Sections 3.3 and 3.4, I can construct the normal form representation of the bundling game. This may be related to a one shot game or to a sequential move one. The players of the hypothetical game are the $A$ firms and the $B$ firms, they can choose between two possible actions *Bundle* (BU) and *Don't bundle* (DB) and the resulting payoffs are their profits. Table 3.4 and 3.5 present the payoff matrices.

With two and three component bundles, in the inelastic demand setting, the outcome of the game is CvsC.[22] With elastic demand and two component bundles, there are no incentives to bundle. But, with three component bundles, market outcome is BvsB. This last case resembles a Prisoner's Dilemma and shows that, when demand is elastic and there are incentives to bundle, the market outcome is the most competitive one. These results assess that once an elastic demand is considered, bundling

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[22]Nalebuff (2000) shows that for larger bundles the outcome is BvsC.

can no longer be regarded as an anticompetitive device used to create a favorable market position, as it leads to the strongest possible competition.

Table 3.4: Normal forms with inelastic demand

| | | $n = 2$ | | | | $n = 3$ | |
| --- | --- | --- | --- | --- | --- | --- | --- |
| | | $A$ | | | | $A$ | |
| | | BU | DB | | | BU | DB |
| $B$ | BU | $(5, 5)$ | $(9.11, 6.43)$ | $B$ | BU | $(6.66, 6.66)$ | $(14.72, 7.88)$ |
| | DB | $(6.43, 9.11)$ | $(10, 10)$ | | DB | $(7.88, 14.72)$ | $(15, 15)$ |

Table 3.5: Normal forms with elastic demand

| | | $n = 2$ | | | | $n = 3$ | |
| --- | --- | --- | --- | --- | --- | --- | --- |
| | | $A$ | | | | $A$ | |
| | | BU | DB | | | BU | DB |
| $B$ | BU | $(3.89, 3.89)$ | $(6.57, 4.56)$ | $B$ | BU | $(4.59, 4.59)$ | $(8.83, 4.51)$ |
| | DB | $(4.56, 6.57)$ | $(6.99, 6.99)$ | | DB | $(4.51, 8.83)$ | $(8.82, 8.82)$ |

This work focuses on extending the inelastic demand setting to an elastic one. However, for this purpose uses several other restrictions that maybe important for the results. Although a more flexible setting would make the analysis more general, the complexity of the underlying computation required some simplifications of the model. For tractability, I assume that the bundle is incompatible with rival components in BvsC, and that pure bundling is the only strategy available to a bundler. If these assumptions are relaxed, the profits of all firms are higher in BvsC. Then, incentives to bundle increase when competing against separate components, and decrease when competing against a bundle. Whether the results are robust to these extensions depends on the magnitude of the gain in profit and on how it changes with bundle size. The overall effect is rather ambiguous.[23]

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[23]See subsection 3.7.4 for some partial results. However, typical examples of incompatible bundles are physically integrated TV sets and DVD players, or monitors and computers. Pure bundling may be found in the car industry, due to the practice of adding new facilities to the basic product, or in IT industry where many applications are not available outside the bundle.

Though I deal with effects of bundling in a *post entry* set-up, present analysis can provide intuition on the potential of the practice as a barrier to entry. Monopolists selling complementary components do better coordinating their price decision and, under the threat of entry, competition with a bundle decreases entrant's profits. Then, bundling has an anticompetitive effect, making the incumbent look tougher and increasing the range of fixed costs where entry is deterred. With elastic demand and larger bundles, firms can credibly commit to bundling, in order to discourage entry. With inelastic demand, commitment to bundling is not credible when the incumbent is facing coordinated entry, but bundling offers an important first mover advantage in front of uncoordinated entry.

## 3.6 Conclusions

The present paper uses a model of imperfect competition with product differentiation in complementary goods markets in order to determine the effects of bundling on the nature of competition. It considers two types of demands and two bundle sizes. The incentives to bundle increase with the size of the system, and they are stronger when demand is elastic than when it is inelastic.

For three component bundles, with elastic demand, there are already incentives to bundle against rivals selling separate components, unlike the inelastic demand case where there are no such incentives, although this size of the bundle is close to their existence. Whenever there are incentives to bundle in the elastic demand case, the stable market outcome is bundle against bundle competition, leading to lowest prices. This is contrary to inelastic demand case where bundling can be used to depress rivals profits, and the stable market outcome is bundle versus components competition.

This paper suggest that, unless entry decisions are at stake, bundling cannot be considered a way to achieve an advantageous position, once elastic demands are allowed. Not only is potential anticompetitiveness of bundling particularly sensitive

to demand elasticity, but, under certain conditions, the practice may foster price competition. The results contribute to determine the boundaries of anticompetitive product bundling and may be useful for competition policy in information technology industries, or wherever there is evidence of this practice.

## 3.7 Appendix

### 3.7.1 Tables of results by competition mode

**Component versus component**

| Demand | Inelastic | | Elastic | |
|---|---|---|---|---|
| No. of comp. | $n = 2$ | $n = 3$ | $n = 2$ | $n = 3$ |
| $p_i = q_i =$ | 1 | 1 | 0.911 | 0.897 |
| $\Sigma_i p_i = \Sigma_i q_i =$ | 2 | 3 | 1.822 | 2.692 |
| $D_{Ai} = D_{Bi} =$ | 5 | 5 | 3.839 | 3.279 |
| $\pi_{Ai} = \pi_{Bi} =$ | 5 | 5 | 3.497 | 2.942 |
| $\Sigma_i \pi_{Ai} = \Sigma_i \pi_{Bi} =$ | 10 | 15 | 6.994 | 8.827 |

**Bundle versus component**

| Demand | Inelastic | | Elastic | |
|---|---|---|---|---|
| No. of comp. | $n = 2$ | $n = 3$ | $n = 2$ | $n = 3$ |
| $p =$ | 1.453 | 2.094 | 1.304 | 1.767 |
| $q_i =$ | 0.863 | 0.884 | 0.797 | 0.789 |
| $\Sigma_i q_i =$ | 1.726 | 2.654 | 1.594 | 2.369 |
| $D_A =$ | 6.273 | 7.029 | 5.039 | 4.997 |
| $D_{Bi} =$ | 3.726 | 2.970 | 2.864 | 1.907 |
| $\pi_A =$ | 9.117 | 14.72 | 6.573 | 8.830 |
| $\pi_{Bi} =$ | 3.217 | 2.628 | 2.283 | 1.506 |
| $\Sigma_i \pi_{Bi} =$ | 6.434 | 7.886 | 4.567 | 4.519 |

**Bundle versus bundle**

| Demand | Inelastic | | Elastic | |
|---|---|---|---|---|
| No. of comp. | $n = 2$ | $n = 3$ | $n = 2$ | $n = 3$ |
| $p = q =$ | 1 | 1.333 | 0.926 | 1.190 |
| $D_A = D_B =$ | 5 | 5 | 4.203 | 3.858 |
| $\pi_A = \pi_B =$ | 5 | 6.666 | 3.894 | 4.592 |

### 3.7.2 Proofs of Propositions

*Proof of Proposition 7.* The demand for $A_1$ will be given by:

$$\int_0^{x_0} \left[ \begin{array}{l} \int_0^{y_0} 10 - a\left(p_1 + p_2 + x + y\right) dy + \\ \int_{y_0}^1 10 - a\left(p_1 + q_2 + x + 1 - y\right) dy \end{array} \right] dx =$$

$$-\frac{-1 - q_1 + p_1}{8} \left( -2a - aq_1 - 3ap_1 - 2ap_2 - 2ap_2q_2 + ap_2^2 - 2aq_2 + aq_2^2 + 40 \right).$$

The demand for $B_1$ will be given by:

$$\int_{x_0}^1 \left[ \begin{array}{l} \int_0^{y_0} 10 - a\left(p_2 + q_1 + 1 - x + y\right) dy + \\ \int_{y_0}^1 10 - a\left(q_1 + q_2 + 1 - x + 1 - y\right) dy \end{array} \right] dx =$$

$$\frac{1 - q_1 + p_1}{8} \left( -2a - 3aq_1 - ap_1 - 2aq_2 - 2ap_2 + aq_2^2 - 2ap_2q_2 + 40 + ap_2^2 \right).$$

The demand for $A_2$ will be given by:

$$\int_0^{y_0} \left[ \begin{array}{l} \int_0^{x_0} 10 - a\left(p_1 + p_2 + x + y\right) dx + \\ \int_{x_0}^1 10 - a\left(p_2 + q_1 + 1 - x + y\right) dx \end{array} \right] dy =$$

$$-\frac{1 + q_2 - p_2}{8} \left( 2a + aq_2 + 3ap_2 + 2ap_1 + 2ap_1q_1 - ap_1^2 + 2aq_1 - aq_1^2 - 40 \right).$$

The demand for $B_2$ will be given by:

$$\int_{y_0}^1 \left[ \begin{array}{l} \int_{x_0}^1 10 - a\left(q_1 + q_2 + 1 - x + 1 - y\right) dx + \\ \int_0^{x_0} 10 - a\left(p_1 + q_2 + x + 1 - y\right) dx \end{array} \right] dy =$$

$$\frac{-1 + q_2 - p_2}{8} \left( 2a + 3aq_2 + ap_2 + 2aq_1 + 2ap_1 - 40 - aq_1^2 + 2ap_1q_1 - ap_1^2 \right).$$

The system of equations formed by the FOCs and the symmetric equilibrium conditions lead to the candidate equilibrium

prices.  □

*Proof of Proposition 8.* I compute the demand functions for more general values of $\Delta$. They are also used to determine the equilibrium in BvsC. For their derivation I use Figure 3.2.

$D_A = -\frac{1}{24}\left(\Delta+2\right)^2\left(a\Delta-30+3ap+2a\right)-$
$2\max\left\{0,-\frac{1}{24}\Delta^2\left(a\Delta+3a-30+3ap\right)\right\}=$
$$= \begin{cases} -\frac{1}{24}\left(\Delta+2\right)^2\left(a\Delta-30+3ap+2a\right) & \text{if} \quad \Delta<0 \\ \left[\begin{array}{l} -\frac{1}{24}\left(\Delta+2\right)^2\left(a\Delta-30+3ap+2a\right) \\ -2\left(-\frac{1}{24}\Delta^2\left(a\Delta+3a-30+3ap\right)\right) \end{array}\right] & \text{if} \quad \Delta\geq 0 \end{cases}$$
$D_B = \frac{1}{24}\left(\Delta-2\right)^2\left(a\Delta+30-3aq-2a\right)-$
$2\max\left\{0,\frac{1}{24}\Delta^2\left(a\Delta-3a+30-3aq\right)\right\}=$
$$= \begin{cases} \frac{1}{24}\left(\Delta-2\right)^2\left(a\Delta+30-3aq-2a\right) & \text{if} \quad \Delta\geq 0 \\ \left[\begin{array}{l} \frac{1}{24}\left(\Delta-2\right)^2\left(a\Delta+30-3aq-2a\right) \\ -2\left(\frac{1}{24}\Delta^2\left(a\Delta-3a+30-3aq\right)\right) \end{array}\right] & \text{if} \quad \Delta<0 \end{cases}$$

At $p=q: D_A = -\frac{4}{24}\left(-30+3ap+2a\right)$ and $D_B = -\frac{4}{24}\left(-30+3aq+2a\right)$.

The side derivatives of $D_A$ and $D_B$ are equal at $\Delta=0$. $\left(D'_A=D'_B=\frac{1}{2}ap-5.\right)$ The functions are continuous and differentiable.

At a symmetric equilibrium, by the profit maximization, the FOC becomes:

$p\left(\frac{1}{2}ap-5\right)-\frac{4}{24}\left(-30+3ap+2a\right)=0 \Leftrightarrow \frac{1}{2}ap^2-5p+5-\frac{1}{2}ap-\frac{1}{3}a=0.$

Substituting for $a$ I obtain the candidate equilibria.  □

*Proof of Proposition 9.* From the Cournot's dual problem it follows that $\Delta>0$.

The demand for bundle A is given by:

$D_A = -\frac{1}{24}\left(q_1+q_2-p+2\right)^2\left(a\left(q_1+q_2-p\right)-30+3ap+2a\right)$
$+\frac{2}{24}\left(q_1+q_2-p\right)^2\left(a\left(q_1+q_2-p\right)+3a-30+3ap\right).$

The demand for the second firms' system is given by:

$D_B = \frac{1}{24}\left(q_1+q_2-p-2\right)^2\left(a\left(q_1+q_2-p\right)+30-3a\left(q_1+q_2\right)-2a\right).$

The two $B$ firms are maximizing separately, with respect to their own price.

The system of FOC of the profit maximization problem gives the candidates for equilibrium prices. $\qquad\square$

*Proof of Proposition 10.* The total demand of firm $A_1$ is:

$$D_{A1} = -\frac{-1 - q_1 + p_1}{8}\begin{pmatrix} -3ap_1 - 2aq_2 + 40 - 2ap_3q_3 - 2ap_2q_2 + aq_2^2 + \\ ap_2^2 - 2aq_3 + aq_3^2 - aq_1 + ap_3^2 - 2ap_2 - 2ap_3 - 3a \end{pmatrix}.$$

The total demand of firm $A_2$ will be:

$$D_{A2} = \frac{-q_2 + p_2 - 1}{8}\begin{pmatrix} 3ap_2 + 2ap_1q_1 + 2aq_3 - aq_3^2 + 2ap_3 - ap_1^2 + 2ap_1 \\ +aq_2 + 2ap_3q_3 + 2aq_1 - ap_3^2 - 40 - aq_1^2 + 3a \end{pmatrix}.$$

The total demand of firm $A_3$ will be:

$$D_{A3} = -\frac{-q_3 + p_3 - 1}{8}\begin{pmatrix} -3ap_3 + ap_1^2 - 2ap_2q_2 + 40 - 2aq_2 - 2ap_2 - 3a - \\ 2ap_1q_1 + aq_2^2 + aq_1^2 - 2aq_1 - 2ap_1 - aq_3 + ap_2^2 \end{pmatrix}.$$

The demand for firm $B_1$ will be:

$$D_{B1} = \frac{1 - q_1 + p_1}{8}\begin{pmatrix} -ap_1 - 2aq_2 + 40 - 2ap_3q_3 - 2ap_2q_2 - 3a - 3aq_1 \\ +ap_2^2 - 2aq_3 + aq_2^2 - 2ap_3 + ap_3^2 - 2ap_2 + aq_3^2 \end{pmatrix}.$$

The demand for firm $B_2$ will be given by:

$$D_{B2} = -\frac{-q_2 + p_2 + 1}{8}\begin{pmatrix} ap_2 + 3a - ap_3^2 - 40 - aq_1^2 + 2ap_3 - ap_1^2 + 3aq_2 + \\ 2aq_3 - aq_3^2 + 2aq_1 + 2ap_1 + 2p_3aq_3 + 2p_1aq_1 \end{pmatrix}.$$

The demand for firm $B_3$ will be given by:

$$D_{B3} = \frac{1 - q_3 + p_3}{8}\begin{pmatrix} -ap_3 + ap_1^2 + 40 - 3a - 3aq_3 + aq_2^2 + ap_2^2 - 2aq_2 \\ -2aq_1 - 2q_1ap_1 - 2p_2aq_2 + aq_1^2 - 2ap_2 - 2ap_1 \end{pmatrix}.$$

The solution to the system of FOC of the profit maximization problem gives the candidates to equilibrium prices. $\qquad\square$

*Proof of Proposition 11.* I present the demands for more general values of $\Delta$, so they serve also to determine the equilibrium in BvsC.

$$D_A\left(p,q\right) = \begin{cases} -\frac{1}{384}\left(\Delta+3\right)^3\left(3a\Delta+8ap+9a-80\right) & \text{if} \quad \Delta \leq -1 \\[2ex] \left[ \begin{array}{l} -\frac{1}{384}\left(\Delta+3\right)^3\left(3a\Delta+8ap+9a-80\right) \\[1ex] +\frac{3}{384}\left(\Delta+1\right)^3\left(3a\Delta+11a+8ap-80\right) \end{array} \right] & \text{if} \quad -1 \leq \Delta \leq 1 \\[3ex] \left[ \begin{array}{l} -\frac{1}{384}\left(\Delta+3\right)^3\left(3a\Delta+8ap+9a-80\right) \\[1ex] +\frac{3}{384}\left(\Delta+1\right)^3\left(3a\Delta+11a+8ap-80\right) \\[1ex] -\frac{3}{384}\left(\Delta-1\right)^3\left(3a\Delta-80+13a+8ap\right) \end{array} \right] & \text{if} \quad \Delta \geq 1 \end{cases}$$

$$D_B\left(p,q\right) = \begin{cases} -\frac{1}{384}\left(\Delta-3\right)^3\left(3a\Delta+80-9a-8aq\right) & \text{if} \quad -\Delta \leq -1 \\[2ex] \left[ \begin{array}{l} -\frac{1}{384}\left(\Delta-3\right)^3\left(3a\Delta+80-9a-8aq\right) \\[1ex] +\frac{3}{384}\left(\Delta-1\right)^3\left(3a\Delta+80-11a-8aq\right) \end{array} \right] & \text{if} \quad -1 \leq -\Delta \leq 1 \\[3ex] \left[ \begin{array}{l} -\frac{1}{384}\left(\Delta-3\right)^3\left(3a\Delta+80-9a-8aq\right) \\[1ex] +\frac{3}{384}\left(\Delta-1\right)^3\left(3a\Delta+80-11a-8aq\right) \\[1ex] -\frac{3}{384}\left(\Delta+1\right)^3\left(3a\Delta-13a-8aq+80\right) \end{array} \right] & \text{if} \quad -\Delta \geq 1 \end{cases}$$

I look for a symmetric equilibrium. At $\Delta = 0$ the demand is continuous and differentiable:

$D_A\left(\Delta=0\right) = -\frac{1}{2}ap - \frac{35}{64}a + 5$

$D_A'\left(\Delta=0\right) = D_B'\left(\Delta=0\right) = \frac{1}{16}a - \frac{15}{4} + \frac{3}{8}ap$

FOC becomes: $\frac{\partial\pi_A}{\partial p} = 5 - \frac{35}{64}a - \frac{7}{16}ap - \frac{15}{4}p + \frac{3}{8}p^2a = 0$

Substituting for $a$, gives the equilibrium price candidates.      $\square$

*Proof of Proposition 12.* I use throughout the maximization problem the fact that $1 \geq \Delta > 0$. I show that there is no other equilibrium in the following two lemmas.

When $0 < \Delta < 1$ (that is, $-1 < \Delta < 1$), the demands of the two firms become:

$D_A = -\frac{1}{384}\left(q_1+q_2+q_3-p+3\right)^3\left(3a\left(q_1+q_2+q_3-p\right)+8ap+9a-80\right)$

$+\frac{3}{384}\left(q_1+q_2+q_3-p+1\right)^3\left(3a\left(q_1+q_2+q_3-p\right)+11a+8ap-80\right),$

$D_B = -\frac{1}{384}\left(q_1+q_2+q_3-p-3\right)^3\left( \begin{array}{c} 3a\left(q_1+q_2+q_3-p\right) \\ +80-9a-8a\left(q_1+q_2+q_3\right) \end{array} \right)$

$$+\tfrac{3}{384}\left(q_1+q_2+q_3-p-1\right)^3\begin{pmatrix}3a\left(q_1+q_2+q_3-p\right)+80\\-11a-8a\left(q_1+q_2+q_3\right)\end{pmatrix}.$$

The system of FOC of the profit maximization problem gives the candidates for equilibrium prices. $\qquad\square$

**Lemma 5.** *For $n=3$, in BvsC, there is no equilibrium when $\Delta=1$.*

*Proof.* I prove for the case of $a=1$. The proof for $a=0$, follows the same steps.

$\Delta=q-p=q_1+q_2+q_3-p=1\Rightarrow q=p+1$

$\pi_A=p\left(-\tfrac{1}{384}\left(4\right)^3\left(3-71+8p\right)+\tfrac{3}{384}\left(2\right)^3\left(3+8p-69\right)\right)=\tfrac{173}{24}p-\tfrac{5}{6}p^2$

Consider a small deviation of firm $A$ to a higher price, $p'=p+\varepsilon$, for $\varepsilon>0$, very small. Then, $-1\leq\Delta<1$, and demand faced by firm $A$ is:

$D_A=-\tfrac{1}{384}\left(q_1+q_2+q_3-p+3\right)^3\left(3\left(q_1+q_2+q_3-p\right)-71+8p\right)$

$+\tfrac{3}{384}\left(q_1+q_2+q_3-p+1\right)^3\left(3\left(q_1+q_2+q_3-p\right)+8p-69\right).$

Notice that $\Delta'=q-p'=1-\varepsilon$. Then,

$\pi'_A=(p+\varepsilon)\begin{pmatrix}-\tfrac{1}{384}\left(1-\varepsilon+3\right)^3\left(3\left(1-\varepsilon\right)-71+8\left(p+\varepsilon\right)\right)\\+\tfrac{3}{384}\left(1-\varepsilon+1\right)^3\left(3\left(1-\varepsilon\right)+8\left(p+\varepsilon\right)-69\right)\end{pmatrix}=$

$\tfrac{173}{24}p-\tfrac{5}{6}p^2+\varepsilon\left(\tfrac{173}{24}-\tfrac{11}{3}p+\tfrac{1}{4}p^2\right)+\varepsilon^2\left(-\tfrac{17}{6}-\tfrac{9}{16}p+\tfrac{1}{8}p^2\right)+$

$\varepsilon^3\left(-\tfrac{13}{16}+\tfrac{13}{24}p-\tfrac{1}{24}p^2\right)+\varepsilon^4\left(\tfrac{5}{12}-\tfrac{13}{192}p\right)-\tfrac{5}{192}\varepsilon^5.$

In order for such deviation to be profitable should hold that $\pi'_A-\pi_A>0\Leftrightarrow$

$\left(\tfrac{173}{24}-\tfrac{11}{3}p+\tfrac{1}{4}p^2\right)+\varepsilon\left(-\tfrac{17}{6}-\tfrac{9}{16}p+\tfrac{1}{8}p^2\right)+$

$\varepsilon^2\left(-\tfrac{13}{16}+\tfrac{13}{24}p-\tfrac{1}{24}p^2\right)+\varepsilon^3\left(\tfrac{5}{12}-\tfrac{13}{192}p\right)-\tfrac{5}{192}\varepsilon^4>0.$

Or, when $\varepsilon\to 0$, in the limit the above expression becomes:

$\tfrac{173}{24}-\tfrac{11}{3}p+\tfrac{1}{4}p^2=\tfrac{1}{4}\left(p-\tfrac{22}{3}+\tfrac{\sqrt{898}}{6}\right)\left(p-\tfrac{22}{3}-\tfrac{\sqrt{898}}{6}\right)>0.$

Then, there are incentives to deviate whenever $p<\tfrac{22}{3}-\tfrac{\sqrt{898}}{6}=2.338\,9.$ (1)

Consider a small deviation of firm $A$ to a smaller price, $p''=p-\varepsilon$, for $\varepsilon>0$, very small. Then, $\Delta>1$, and demand faced by firm $A$ is:

$D_A=-\tfrac{1}{384}\left(\Delta+3\right)^3\left(3\Delta-71+8p\right)+\tfrac{3}{384}\left(\Delta+1\right)^3\left(3\Delta+8p-69\right)$

$-\tfrac{3}{384}\left(\Delta-1\right)^3\left(3\Delta-67+8p\right).$

Notice that $\Delta'' = 1 + \varepsilon$. Then,

$$\pi_A'' = (p - \varepsilon) \begin{pmatrix} -\frac{1}{384}(1 + \varepsilon + 3)^3 (3(1 + \varepsilon) - 71 + 8(p - \varepsilon)) \\ +\frac{3}{384}(1 + \varepsilon + 1)^3 (3(1 + \varepsilon) + 8(p - \varepsilon) - 69) \\ -\frac{3}{384}(1 + \varepsilon - 1)^3 (3(1 + \varepsilon) - 67 + 8(p - \varepsilon)) \end{pmatrix} =$$

$\frac{173}{24}p - \frac{5}{6}p^2 + \varepsilon\left(-\frac{173}{24} + \frac{11}{3}p - \frac{1}{4}p^2\right) + \varepsilon^2\left(-\frac{17}{6} - \frac{9}{16}p + \frac{1}{8}p^2\right) +$
$\varepsilon^3\left(\frac{13}{16} - \frac{1}{24}p - \frac{1}{48}p^2\right) + \varepsilon^4\left(-\frac{1}{12} + \frac{13}{384}p\right) - \frac{5}{384}\varepsilon^5.$

In order for the deviation to be profitable, should be that $\pi_A'' - \pi_A > 0$

$\pi_A'' - \pi_A = \left(-\frac{173}{24} + \frac{11}{3}p - \frac{1}{4}p^2\right) + \varepsilon\left(-\frac{17}{6} - \frac{9}{16}p + \frac{1}{8}p^2\right) +$
$\varepsilon^2\left(\frac{13}{16} - \frac{1}{24}p - \frac{1}{48}p^2\right) + \varepsilon^3\left(-\frac{1}{12} + \frac{13}{384}p\right) - \frac{5}{384}\varepsilon^5 > 0.$

When $\varepsilon \to 0$, in the limit, the above expression becomes:

$-\frac{173}{24} + \frac{11}{3}p - \frac{1}{4}p^2 > 0 \Leftrightarrow \frac{1}{4}\left(\frac{22}{3} - \frac{\sqrt{898}}{6} - p\right)\left(p - \frac{22}{3} - \frac{\sqrt{898}}{6}\right) > 0.$

So, there are incentives to deviate whenever $p \in (2.3389, 10]$. (2)

By (1) and (2), I am left with checking the incentives to deviate at $p = 2.3389$, where

$q = 3.3389$ and $q_i = 1.113$, $i = 1, 2, 3$. Then, at $\Delta = 1$:

$\pi_{B1} = \left[-\frac{1}{384}(-2)^3[3 + 71 - 8(q_1 + q_2 + q_3)]\right]q_1 =$
$\frac{37}{24}q_1 - \frac{1}{6}q_1^2 - \frac{1}{6}q_1q_2 - \frac{1}{6}q_1q_3.$

Consider a deviation of firm $B_1$ from $q_1$ to $q_1' = 1.113 - \varepsilon$. As $p = 2.3389$ and

$\Delta = 1 - \varepsilon$:

$$\pi_{B1}' = (q_1 - \varepsilon)\left[\begin{array}{l} -\frac{1}{384}(1 - \varepsilon - 3)^3 (3(1 - \varepsilon) + 71 - 8(q_1 + q_2 + q_3 - \varepsilon)) \\ +\frac{3}{384}(1 - \varepsilon - 1)^3 (3(1 - \varepsilon) + 69 - 8(q_1 + q_2 + q_3 - \varepsilon)) \end{array}\right] =$$

$\left(\frac{37}{24}q_1 - \frac{1}{6}q_1^2 - \frac{1}{6}q_1q_2 - \frac{1}{6}q_1q_3\right) +$
$\varepsilon\left(-\frac{37}{24} + \frac{31}{12}q_1 + \frac{1}{6}q_2 + \frac{1}{6}q_3 - \frac{1}{4}q_1^2 - \frac{1}{4}q_1q_3 - \frac{1}{4}q_1q_2\right) +$
$\varepsilon^2\left(-\frac{29}{12} + \frac{25}{16}q_1 + \frac{1}{4}q_2 + \frac{1}{4}q_3 - \frac{1}{8}q_1^2 - \frac{1}{8}q_1q_3 - \frac{1}{8}q_1q_2\right) +$
$\varepsilon^3\left(-\frac{21}{16} - \frac{1}{6}q_1 + \frac{1}{8}q_2 + \frac{1}{8}q_3 + \frac{1}{24}q_1^2 + \frac{1}{24}q_1q_3 + \frac{1}{24}q_1q_2\right) +$
$\varepsilon^4\left(+\frac{7}{24} - \frac{13}{192}q_1 - \frac{1}{24}q_2 - \frac{1}{24}q_3\right) + \frac{5}{192}\varepsilon^5.$

For firm $B_1$ to deviate, should hold that:

$\pi_{B1}' - \pi_{B1} > 0 \Leftrightarrow$
$\left(-\frac{37}{24} + \frac{31}{12}q_1 + \frac{1}{6}q_2 + \frac{1}{6}q_3 - \frac{1}{4}q_1^2 - \frac{1}{4}q_1q_3 - \frac{1}{4}q_1q_2\right) +$
$\varepsilon\left(-\frac{29}{12} + \frac{25}{16}q_1 + \frac{1}{4}q_2 + \frac{1}{4}q_3 - \frac{1}{8}q_1^2 - \frac{1}{8}q_1q_3 - \frac{1}{8}q_1q_2\right) +$

$\varepsilon^2 \left( -\frac{21}{16} - \frac{1}{6}q_1 + \frac{1}{8}q_2 + \frac{1}{8}q_3 + \frac{1}{24}q_1^2 + \frac{1}{24}q_1q_3 + \frac{1}{24}q_1q_2 \right) +$

$\varepsilon^3 \left( +\frac{7}{24} - \frac{13}{192}q_1 - \frac{1}{24}q_2 - \frac{1}{24}q_3 \right) + \frac{5}{192}\varepsilon^4 > 0.$

Or, when $\varepsilon \to 0$, in the limit:

$-\frac{37}{24} + \frac{31}{12}q_1 + \frac{1}{6}q_2 + \frac{1}{6}q_3 - \frac{1}{4}q_1^2 - \frac{1}{4}q_1q_3 - \frac{1}{4}q_1q_2 > 0.$

Thus, $p = 2.3389$ and $q_1 = q_2 = q_3 = 1.113$ cannot be an equilibrium, because any of the $B_i$ firms has incentives to deviate to a price higher than 1.113. (3)

When $q_2 = q_3 = 1.113$

$-\frac{37}{24} + \frac{31}{12}q_1 + \frac{1}{6}q_2 + \frac{1}{6}q_3 - \frac{1}{4}q_1^2 - \frac{1}{4}q_1q_3 - \frac{1}{4}q_1q_2 > 0$

$\Leftrightarrow 2.026\,8q_1 - 0.25q_1^2 - 1.170\,7 > 0 \Leftrightarrow 7.481\,3 > q_1 > 0.625\,94.$

So, there are incentives to deviate whenever $q_i \in (0.625\,94, 7.481\,3]$. (3)

$(1), (2)$ and $(3)$ complete the proof that there can be no equilibrium when $\Delta = 1$. $\square$

**Lemma 6.** *For $n = 3$, in BvsC, there is no equilibrium when $\Delta > 1$.*

*Proof.* The demand functions are:

$D_A = -\frac{1}{384}\left( q_1 + q_2 + q_3 - p + 3 \right)^3 \left( 3a\left( q_1 + q_2 + q_3 - p \right) + 8ap + 9a - 80 \right)$

$\quad +\frac{3}{384}\left( q_1 + q_2 + q_3 - p + 1 \right)^3 \left( 3a\left( q_1 + q_2 + q_3 - p \right) + 11a + 8ap - 80 \right)$

$\quad -\frac{1}{384}\left( q_1 + q_2 + q_3 - p - 1 \right)^3 \left( 3a\left( q_1 + q_2 + q_3 - p \right) - 80 + 13a + 8ap \right),$

$D_B = -\frac{1}{384}\left( q_1 + q_2 + q_3 - p - 3 \right)^3 \left( \begin{array}{c} 3a\left( q_1 + q_2 + q_3 - p \right) + 80 - 9a \\ -8a\left( q_1 + q_2 + q_3 \right) \end{array} \right).$

Using the demand and deriving the candidates for equilibrium prices from the system of FOC of the profit maximization problem, can be shown that there is no equilibrium consistent with the covered market conditions and with $\Delta > 1$. $\square$

**Remark 3.** *Using the same steps as the ones in Lemma 5, it can be shown that there is no equilibrium with $\Delta = -1$.*

### 3.7.3 Welfare analysis: Consumer surplus

I present the derivation of the consumer surplus results for an elastic demand. Similarly, results in Table 3.1 can be derived.

*Two components and elastic demand.* Table 3.2 for $n = 2$.

*1. CvsC:* Consumer surplus corresponding to one system at a symmetric equilibrium is:

$CS(p) = \frac{1}{2} \int_0^{\frac{1}{2}} \int_0^{\frac{1}{2}} (10 - p_i - x - y)^2 \, dxdy = \frac{1}{2} \left( \frac{2167}{96} - \frac{19}{4} p_i + \frac{1}{4} p_i^2 \right).$

At equilibrium, total consumer surplus is $29.497$.

*2. BvsB:* Consumer surplus corresponding to one firm is:

$CS = \frac{1}{2} \int_0^1 \int_0^{1-x} (10 - p - x - y)^2 \, dydx = \frac{523}{24} - \frac{14}{3} p + \frac{1}{4} p^2.$

At equilibrium, total consumer surplus is $35.364$.

*3. BvsC:* Firm A bundles. At equilibrium, the indifference line is $x + y = \frac{q_1 + q_2 - p + 2}{2} = 1.1449$.

The consumer surplus generated by the bundler is:

$CS = \frac{1}{2} \int_0^1 \int_0^{1.1449 - y} (10 - p - x - y)^2 \, dxdy -$

$\frac{1}{2} \int_1^{1.1449} \int_0^{1.1449 - x} (10 - p - x - y)^2 \, dydx =$

$27.149 - 5.8665p + .3172p^2.$

B firms sell separate components. The consumer surplus generated by the component sellers is:

$CS = \frac{1}{2} \int_0^{0.8551} \int_0^{0.8551 - y} (10 - p - x - y)^2 \, dxdy =$

$32.525 - 6.8953p + .3656p^2.$

At equilibrium, total consumer surplus is $31.267$. $\qquad\qquad \square$

*Three components and elastic demand.* Table 3.2 for $n = 3$.

*1. CvsC:* Consumer surplus corresponding to an arbitrary system is:

$CS(p) = \frac{1}{2} \int_0^{\frac{1}{2}} \int_0^{\frac{1}{2}} \int_0^{\frac{1}{2}} (10 - p - x - y - z)^2 \, dxdydz = \frac{685}{64} - \frac{37}{16} p + \frac{1}{8} p^2$

Total consumer surplus is $21.534$.

*2. BvsB:* At equilibrium, $x + y + z = 1.5$.

The consumer surplus generated by one firm is:

$CS\left(p\right) = \frac{1}{2} \int_0^1 \int_0^{1.5-z} \int_0^{1.5-z-x} \left(10 - p - x - y - z\right)^2 dy dx dz -$
$\int_1^{1.5} \int_0^{1.5-z} \int_0^{1.5-z-x} \left(10 - p - x - y - z\right)^2 dy dx dz.$

Total consumer surplus is $29.8$.

*3. BvsC:* Firm A bundles. The indifference line is given by $x + y + z = 1.8012$.

The consumer surplus related to the bundle is:

$CS_A = \frac{1}{2} \int_0^1 \int_0^{1.8012-z} \int_0^{1.8012-z-x} \left(10 - p - x - y - z\right)^2 dy dx dz -$
$\int_1^{1.8012} \int_0^{1.8012-z} \int_0^{1.8012-z-x} \left(10 - p - x - y - z\right)^2 dy dx dz.$

B firms sell separate components. The consumer surplus corresponding to B system is:

$CS_B = \frac{1}{2} \int_0^1 \int_0^{1.1988-z} \int_0^{1.1988-z-x} \left(10 - p - x - y - z\right)^2 dy dx dz -$
$\int_1^{1.1988} \int_0^{1.1988-z} \int_0^{1.1988-z-x} \left(10 - p - x - y - z\right)^2 dy dx dz.$

At equilibrium, total consumer surplus is $23.893$. □

## 3.7.4 Compatibility of the components

I present in this subsection the equilibrium that would emerge in bundle versus component competition, were the products compatible. Some of the consumers buy both the bundle and a component and, free disposing a component of the bundle, create a new system closer to their ideal one.

**Proposition 13.** *When products are compatible:*
*a) With inelastic demand and $n = 2$, , at equilibrium $p = 1.3895$ and $q_1 = q_2 = 0.80805$. Corresponding profits are $\pi_A = 8.5606$ and $\pi_{Bi} = 3.2644$.*
*b) With elastic demand and $n = 2$, at equilibrium, $p = 1.2648$ and $q_1 = q_2 = 0.75287$. Corresponding profits are $\pi_A = 6.3512$ and $\pi_{Bi} = 2.3913$.*
*c) With inelastic demand and $n = 3$, at equilibrium, $p = 1.8512$ and $q_1 = q_2 = q_3 = .75824$. Corresponding profits are $\pi_{Bi} = 2.7964$ and $\pi_A = 12.459$.*

## 3.8 References

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**Chapter 4**

# Innovation in an Asymmetric Setting: Comparing Cournot and Bertrand Equilibria

## 4.1  Introduction

The present note compares the outcomes and the dynamic efficiency of Cournot and Bertrand equilibria in a differentiated duopoly where only one firm can invest in cost reduction. *We show that output and consumer surplus can be larger under quantity competition, and that Bertrand firms may invest more in R&D than Cournot ones. These results differ from the existing ones in the process innovation literature.*

Singh and Vives (1984) show that when a duopoly interacts only in the product market Bertrand equilibrium results in larger output, consumer surplus and welfare, and lower prices than Cournot equilibrium. Vives (1985) shows that in a differentiated products oligopoly prices are lower under Bertrand competition. These results support the view that price competition is more efficient than quantity competition when a static market is considered. However, in a dynamic setting, where firms make

some strategic choices before market competition, the situation might be different.

A number of more recent contributions compare Bertrand and Cournot competition modes in differentiated duopolies, when strategic investments in research and development (for process or product innovation) precede the market game. Qiu (1997) considers a symmetric duopoly and allows R&D outcomes to spill over. He shows that Cournot firms invest more in innovation than Bertrand firms. He also demonstrates that while quantity and consumer surplus are still larger under price competition, total welfare may be larger under quantity competition if the spillovers are large and the substitutability is high. *In this paper, we find well defined examples, in the symmetric setting, outside Qiu's parameter restrictions where Cournot quantities and consumer surplus are larger than Bertrand ones.*

Bester and Petrakis (1993) use an asymmetric setting where only one firm can pay a fixed amount to achieve a discrete cost reduction. They show that the incentives to invest in process innovation can be larger under price competition if the goods are close substitutes. Their analysis does not allow for spillovers and, as they work with global methods, does not provide market outcome or efficiency comparisons.

Symeonidis (2003) complements Qiu's analysis working in a symmetric setting with quality improvement instead of cost reduction investment. In his setting, R&D outcome directly enters consumer's utility unlike cost reduction that has only an indirect effect through the quantities. Qiu's results on innovation levels and total welfare are still valid with this new type of R&D. Symeonidis shows that it is possible to have larger quantities and consumer surplus under Cournot competition if spillovers are high and goods are close substitutes. *As he points out, product R&D boosts demand and helps this result. However, the result can be obtained under process R&D, as well.*

In a tournament model, Delbono and Denicolo (1990) consider a homogenous good oligopoly where firms first engage in an R&D race for a cost-reducing patent and then compete in the market in prices or in quantities. They find that, with linear

demand, the R&D investment is larger when the market competition takes place in prices. However, as they show, this investment may be too excessive compared to the socially optimal level leading welfare to be lower than the quantity competition case.

In a setting similar to that of Qiu we allow only one firm to invest in cost reduction and show that:

a. Innovation maybe larger in Bertrand than in Cournot competition if goods are close substitutes, spillovers are low and efficiency of cost reduction is high;

b. Quantities of both firms are larger in Cournot than in Bertrand competition if goods are close substitutes, and spillovers and efficiency of cost reduction are high;

c. Consumer surplus and total welfare might be higher under quantity competition than under price competition if goods are close substitutes, and spillovers and efficiency of cost reduction are high.

Our first result (a), confirms the findings of Bester and Petrakis when innovation is chosen optimally, and extends them for low, but positive spillovers. In Bertrand competition, only the level of output has a positive effect on innovation. Spillovers, strategic complementarity of the prices, and the cost of R&D negatively affect innovation. Asymmetry in the R&D abilities and low differentiation favor the output effect, while low spillovers and R&D cost decrease the negative effects, so that under these conditions Bertrand firms may innovate more than Cournot ones.

A new result is the ranking of quantities (b). Unlike the previous papers dealing with process R&D, we report that the Cournot quantities may exceed the Bertrand ones. This happens in a region where Cournot firms innovate more than Bertrand ones and where the spillovers are high. Interestingly, this result does not depend on the asymmetry of the model or on the nature of innovation (process vs. product). In the asymmetric setting, in addition, it is possible to have the quantity of the non-innovator larger under quantity competition in cases where the output of the innovator is larger under price competition.

The dynamic efficiency comparison (c) shows that, consumer surplus is higher in Cournot than in Bertrand. This result is driven by the quantity ranking. Total welfare can be higher under quantity competition, like in the symmetric case. In fact, we point out that both consumers and producers can be better off under quantity competition.

Comparison of Bertrand and Cournot equilibria can be interpreted as an analysis of the effects of increased competition on innovation and dynamic efficiency. Our analysis reveals that with a high level of product substitutability and efficient R&D technology, both the innovation level and dynamic efficiency can be ranked in any order across different types of competition, depending on the level of spillovers.

Section 4.2 introduces our linear-quadratic model with asymmetric process innovation; Section 4.3 presents the market outcomes and the efficiency measures under Cournot and Bertrand competition. The comparisons between different competition modes follow in Section 4.4. Some final conclusions are contained in Section 4.5, and all proofs missing from the text are relegated to an appendix.

## 4.2   The Model

Consider a differentiated duopoly producing substitute goods. In the first stage, one of the firms can invest in marginal cost reduction. The outcome of the innovation is deterministic and it may spillover to the rival. In the second stage, firms compete in the product market. We consider two alternative competition modes, allowing firms to choose quantities and, respectively, prices. The timing of the game can be justified by the fact that R&D investment is a long term decision related to the production technology, while firms can change their output level or prices faster.

Following Singh and Vives (1984) and Qiu (1997) we work in a partial equilibrium setting and assume that the utility function of the representative consumer is given

by:

$$U(q_1, q_2) = \alpha(q_1 + q_2) - \frac{1}{2}\left(q_1^2 + 2\gamma q_1 q_2 + q_2^2\right).$$

Then, $q_i$ is the quantity of product $i$, and $\gamma \in (0,1)$ is a measure of product substitutability: Product differentiation decreases with $\gamma$.

The inverse demand function is given by:

$$p_i = \alpha - q_i - \gamma q_j \qquad i, j = 1, 2,$$

and the direct demand is given by:

$$q_i = \frac{1}{1-\gamma^2}\left[\alpha(1-\gamma) - p_i + \gamma p_j\right] \qquad i, j = 1, 2.$$

Prior the R&D investment in the first stage, the duopolists share the same production technology, having equal constant marginal cost, $c < \alpha$. The innovation capabilities are asymmetric: Only one firm can invest an amount $V(x) = \frac{vx^2}{2}$ to achieve a cost reduction of $x$. The parameter $v$ is inversely related to the efficiency of the R&D activity. Notice that the innovation technology exhibits decreasing returns to scale. This is necessary for concavity of the first stage profits. The innovation outcome spills over to the rival at a rate $\rho \in [0,1]$. Thus, at the end of the first stage the innovator has a marginal cost $c - x$, and the rival has a cost $c - \rho x$.

We solve by backward induction for the Subgame Perfect Equilibrium of the two stage game. First, we consider quantity competition in the second stage and, then, price competition. We compare the market outcomes (innovation, quantities, prices) and the dynamic efficiency (consumer surplus, profits, total welfare) of the two competition modes.

## 4.3   Cournot and Bertrand equilibria

Consider, first, **quantity competition in the second stage**. Firms choose an output level to maximize their profits.

$$\pi_i = q_i \left( p_i - c_i \right) = q_i \left( \alpha - q_i - \gamma q_j - c_i \right) \qquad i,j = 1,2, \text{ and } i \neq j.$$

The Cournot-Nash equilibrium, and the corresponding profits and prices are given by:

$$q_i^C = \frac{(2 - \gamma)\, \alpha + \gamma c_j - 2c_i}{4 - \gamma^2}, \ \pi_i^C = \left[ \frac{(2 - \gamma)\, \alpha + \gamma c_j - 2c_i}{4 - \gamma^2} \right]^2$$

$$\text{and } p_i^C = \frac{(2 - \gamma)\, \alpha + \gamma c_j - \left( \gamma^2 - 2 \right) c_i}{4 - \gamma^2} \qquad i,j = 1,2, \text{ and } i \neq j.$$

Let firm 1 be the innovator. In the first stage firm 1 chooses a cost reduction level, $x$, to maximize its overall profit, $\Pi_1^C = \left[ \frac{(2-\gamma)(\alpha-c)+(2-\gamma\rho)x}{4-\gamma^2} \right]^2 - \frac{vx^2}{2}$. The equilibrium R&D level is:

$$x^C = \frac{2 \left( \alpha - c \right) \left( 2 - \gamma \right) \left( 2 - \gamma\rho \right)}{v \left( 4 - \gamma^2 \right)^2 - 2 \left( 2 - \gamma\rho \right)^2}. \tag{1}$$

The second order condition for an interior maximum requires:

$$v \left( 4 - \gamma^2 \right)^2 - 2 \left( 2 - \gamma\rho \right)^2 > 0.$$

The equilibrium quantities and prices are given by:

$$q_1^C = \frac{v \left( \alpha - c \right) \left( 4 - \gamma^2 \right) \left( 2 - \gamma \right)}{v \left( 4 - \gamma^2 \right)^2 - 2 \left( 2 - \gamma\rho \right)^2} > 0 \text{ and} \tag{2}$$

$$q_2^C = \frac{v \left( 4 - \gamma^2 \right) \left( 2 - \gamma \right) \left( \alpha - c \right) - 2 \left( 2 - \gamma\rho \right) \left( \alpha - c \right) \left( 1 - \rho \right)}{v \left( 4 - \gamma^2 \right)^2 - 2 \left( 2 - \gamma\rho \right)^2}.$$

**Lemma 7.** *In the reduced form game,*

$$v > \frac{\alpha}{c} \frac{2 \left( 2 - \gamma\rho \right)}{\left( 2 + \gamma \right) \left( 4 - \gamma^2 \right)} + \frac{2 \left( 2 - \gamma\rho \right) \gamma \left( 1 - \rho \right)}{\left( 4 - \gamma^2 \right)^2}$$

*is necessary and sufficient for positive post-innovation costs, and is sufficient for the second order condition of the maximization problem, while*

$$v > \frac{2 \left( 2 - \gamma\rho \right) \left( 1 - \rho \right)}{\left( 4 - \gamma^2 \right) \left( 2 - \gamma \right)}$$

*is necessary and sufficient for $q_2^C > 0$.*

Equilibrium consumer surplus and total welfare are given by:

$$CS^C = \frac{\left(q_1^C\right)^2 + 2\gamma q_1^C q_2^C + \left(q_2^C\right)^2}{2},$$

$$W^C = \frac{3\left(q_1^C\right)^2 + 2\gamma q_1^C q_2^C + 3\left(q_2^C\right)^2}{2} - \frac{v\left(x^C\right)^2}{2}.$$

Finally, consider **price competition in the second stage**. Firms choose a price to maximize their profits.

$$\pi_i = q_i\left(p_i - c_i\right) = \frac{\left(p_i - c_i\right)}{1 - \gamma^2}\left[\alpha\left(1 - \gamma\right) - p_i + \gamma p_j\right] \qquad i, j = 1, 2 \text{ and } i \neq j.$$

The Bertrand-Nash equilibrium, and the related equilibrium profits and quantities are:

$$p_i^B = \frac{\left(1 - \gamma\right)\left(2 + \gamma\right)\alpha + \gamma c_j + 2c_i}{4 - \gamma^2},$$

$$\pi_i^B = \frac{1}{1 - \gamma^2}\left[\frac{\left(1 - \gamma\right)\left(2 + \gamma\right)\alpha + \gamma c_j - \left(2 - \gamma^2\right)c_i}{4 - \gamma^2}\right]^2,$$

$$q_i^B = \frac{\left(1 - \gamma\right)\left(2 + \gamma\right)\alpha + \gamma c_j - \left(2 - \gamma^2\right)c_i}{\left(1 - \gamma^2\right)\left(4 - \gamma^2\right)} \qquad i, j = 1, 2 \text{ and } i \neq j.$$

In the first stage, firm 1, the innovator, chooses an R&D level, $x$, to maximize its overall profit, $\Pi_1^B = \frac{1}{1-\gamma^2}\left[\frac{(1-\gamma)(2+\gamma)\alpha+\gamma c_j-(2-\gamma^2)c_i}{4-\gamma^2}\right]^2 - \frac{vx^2}{2}$. The equilibrium innovation is:

$$x^B = \frac{2\left(\alpha - c\right)\left(\gamma + 2\right)\left(1 - \gamma\right)\left(2 - \gamma^2 - \gamma\rho\right)}{v\left(1 - \gamma^2\right)\left(4 - \gamma^2\right)^2 - 2\left(2 - \gamma^2 - \gamma\rho\right)^2}. \tag{3}$$

The second order condition of the maximization problem requires:

$$v\left(1 - \gamma^2\right)\left(4 - \gamma^2\right)^2 - 2\left(2 - \gamma^2 - \gamma\rho\right)^2 > 0.$$

The equilibrium quantities and prices are:

$$q_1^B = \frac{v\left(\alpha - c\right)\left(4 - \gamma^2\right)\left(2 + \gamma\right)\left(1 - \gamma\right)}{v\left(1 - \gamma^2\right)\left(4 - \gamma^2\right)^2 - 2\left(2 - \gamma\rho - \gamma^2\right)^2} > 0 \text{ and} \tag{4}$$

$$q_2^B = \frac{v\left(\alpha - c\right)\left(4 - \gamma^2\right)\left(2 + \gamma\right)\left(1 - \gamma\right) - 2\left(\alpha - c\right)\left(2 - \gamma\rho - \gamma^2\right)\left(1 - \rho\right)}{v\left(1 - \gamma^2\right)\left(4 - \gamma^2\right)^2 - 2\left(2 - \gamma\rho - \gamma^2\right)^2}.$$

**Lemma 8.** *In the reduced form game,*

$$v > \frac{\alpha}{c} \frac{2\,(\gamma + 2)\,(1 - \gamma)\,\left(2 - \gamma^2 - \gamma\rho\right)}{(1 - \gamma^2)\,(4 - \gamma^2)^2} + \frac{2\gamma\,(1 - \rho)\,\left(2 - \gamma^2 - \gamma\rho\right)}{(1 - \gamma^2)\,(4 - \gamma^2)^2}$$

*is necessary and sufficient for positive post-innovation costs, and is sufficient for the second order condition of the maximization problem, while*

$$v > \frac{2\,\left(2 - \gamma\rho - \gamma^2\right)(1 - \rho)}{(4 - \gamma^2)\,(2 + \gamma)\,(1 - \gamma)}$$

*is necessary and sufficient for $q_2^B > 0$.*

Notice that these conditions require the efficiency of R&D to be quite low when the goods are very close substitutes and spillovers are not very strong: If the costs of innovation are not high enough, firm 2 would be pushed out of the market.

In equilibrium, consumer surplus, profits and welfare are:

$$CS^B = \frac{\left(q_1^B\right)^2 + 2\gamma q_1^B q_2^B + \left(q_2^B\right)^2}{2},$$

$$W^B = \frac{\left(3 - 2\gamma^2\right)\left(q_1^B\right)^2 + 2\gamma q_1^B q_2^B + \left(3 - 2\gamma^2\right)\left(q_2^B\right)^2}{2} - \frac{v\left(x^B\right)^2}{2}.$$

Finally, using Lemma 7 and 8 we can write the necessary condition for the equilibrium innovation to be well defined under both types of product market competition.

**Assumption 1:**

$$v > 2\max\left[A, B, C\right] \tag{A1}$$

where

$$A = \frac{\alpha}{c} \frac{(2 - \gamma\rho)}{(2 + \gamma)\,(4 - \gamma^2)} + \frac{(2 - \gamma\rho)\,\gamma\,(1 - \rho)}{(4 - \gamma^2)^2},$$

$$B = \frac{\left(2 - \gamma\rho - \gamma^2\right)(1 - \rho)}{(4 - \gamma^2)\,(2 + \gamma)\,(1 - \gamma)},$$

$$C = \frac{\alpha}{c} \frac{(2 + \gamma)\,(1 - \gamma)\,\left(2 - \gamma^2 - \gamma\rho\right)}{(1 - \gamma^2)\,(4 - \gamma^2)^2} + \frac{\gamma\,(1 - \rho)\,\left(2 - \gamma^2 - \gamma\rho\right)}{(1 - \gamma^2)\,(4 - \gamma^2)^2}.$$

## 4.4 Comparisons

### 4.4.1 Innovation comparison

We start the comparison of the outcomes under quantity and price competition with the R&D levels given by (1) and (3).

**Proposition 14.** *Suppose A1 holds, then given $\gamma$:*

$$a)\ x^C\ >\ x^B\ \ if\ \gamma < 1 - c/\alpha,$$

$$b)\ If\ \gamma\ >\ 1 - c/\alpha\ then\ there\ exists\ v^*\left(\gamma\right)\ such\ that:$$

*i) $x^C > x^B$ for all $v > v^*\left(\gamma\right)$, and*
*ii) for any $v < v^*\left(\gamma\right)$ there exists $\rho^*\left(\gamma\right) \in [0,1]$ with $x^C < x^B$ for $\rho < \rho^*$, and $x^C > x^B$ for $\rho > \rho^*$.*
*Furthermore, $v^*\left(\gamma\right)$ and $\rho^*\left(\gamma\right)$ increase with $\gamma$.*

This result is consistent with the findings of Bester and Petrakis (1993). In their linear-quadratic model, one of the duopolists in a differentiated market can buy a fixed level cost-reduction, and the rival firm does not benefit from any spillovers. They show that the incentives to innovate -the gain in profit due to the decrease in cost- might be larger in Bertrand than in Cournot if differentiation is low.[1] Then, it is intuitive that when the innovating firm has the option to choose cost-reduction optimally, it might invest more in R&D under Bertrand competition. Our findings confirm this intuition and extend the result to the case of low but strictly positive spillovers. When the products are close substitutes, and the efficiency of R&D activity is high enough, a threshold spillover, $\rho^*$, can be defined such that for levels below this, Bertrand firms innovate more. This threshold value increases with the substitutability.

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[1] This happens in a region where a social planner does not produce both varieties. They also show that the incentives to innovate are socially excessive under market competition if the products are sufficiently close-substitutes and the fixed cost-reduction is low. On the contrary, market competitors underinvest in process R&D if the efficiency gain is large or the products are sufficiently differentiated. In the absence of spillovers, these results should also hold in our setting.

This model deals with constant marginal cost reduction, therefore a firm with a larger output has more incentives to innovate. This market size effect is positive regardless of the competition mode. The incentives to innovate are supported by the strategic effects in Cournot competition as the quantities are strategic substitutes. After a cost-reduction the innovator expands his output and this makes the rival contract his output increasing innovator's profits. Under Bertrand competition, where the prices are strategic complements, the strategic effect on innovation incentives is negative. A cost-reduction makes the innovator lower his price, inducing a price cut by the rival which decreases innovator's profits. The difference in the strategic effect is strong enough, so that symmetric firms invest more in R&D in Cournot competition. In our asymmetric setting, when spillovers are low, the cost reduction favors the innovator, whose market share is larger than in the symmetric setting. This makes the positive market size effect be stronger when only one firm innovates, and explains why Bertrand firms may invest more than Cournot ones.[2] However, this result depends on low differentiation, low spillovers and efficiency of the R&D activity. Both the spillovers and the costs of innovation have a negative effect on the levels of R&D, so they counteract the positive market size effect. If spillovers are high the market share advantage of the innovator gets smaller, and the results are similar to the symmetric setting. When the goods are close substitutes firms compete more and output is higher. Then, the market size effect helps Bertrand firms innovate more than the Cournot ones when spillovers are low. This still holds in the limit when the goods become perfect substitutes. An instance with perfect substitutes where Bertrand firms invest more in R&D is the tournament model considered by Delbono and Denicolo (1990).[3]

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[2]Cournot R&D investment is larger in the asymmetric setting compared to the symmetric one if $\gamma - 2\rho > 0$ and Bertrand R&D investment is larger in the asymmetric setting if $\gamma - 2\rho + \gamma^2\rho > 0$. Also $\gamma - 2\rho > 0$ is necessary for $x^B > x^C$ in the asymmetric game.

[3]Notice that when firms engage in an R&D race for a cost-reducing patent the resulting marginal costs are asymmetric and the winner can make profits in the market game even if the products are homogenous.

### 4.4.2 Quantity comparisons

Consider, first, the quantities of the innovator.

**Proposition 15.** *Suppose A1 holds, then given $\gamma$:*

$$a)\ q_1^B > q_1^C\ \text{if}\ \frac{\alpha}{c} > 3\ \text{or}\ \gamma < \gamma^* = \frac{2\left(\frac{\alpha}{c} - 1\right)}{\frac{\alpha}{c} + 1},$$

$$b)\ \text{If}\ \frac{\alpha}{c} > 3\ \text{and}\ \gamma > \gamma^*\ \text{then there exists}\ v^{**}\left(\gamma\right)\ \text{such that:}$$

*i) $q_1^B > q_1^C$ for all $v > v^{**}\left(\gamma\right)$, and*
*ii) for any $v < v^{**}\left(\gamma\right)$ there exists $\rho^{**}\left(\gamma\right) \in [0, 1]$ with $q_1^B > q_1^C$ for $\rho < \rho^{**}$, and $q_1^B < q_1^C$ for $\rho > \rho^{**}$.*

Several conditions are necessary for the innovator to produce more under Cournot competition:

- Marginal cost before innovation, $c$, has to be high enough relative to total market demand;

- Product differentiation should be sufficiently low;

- R&D costs should not be too high;

- Spillovers have to be strong.

In the absence of innovation Cournot firms produce less than Bertrand firms. In order for the dynamic effects to overturn this ordering, the marginal cost reduction under quantity competition should be sufficiently high compared to the reduction under price competition. Hence, initial marginal cost has to be large enough for Cournot firms to achieve a significant cost advantage over Bertrand ones. Similarly, for the innovation under Cournot to be significant, the R&D technology should be efficient, and the products should be close substitutes. Low differentiation leads to stronger competition, and makes cost reductions more valuable. Unlike the former

determinants, the spillovers have a negative effect on innovation. However, this negative effect is more detrimental in the case of price competition. For instance, in the extreme case of almost homogenous products and perfect spillovers, cost reduction is worthless for the innovator in the Bertrand market while it is still valuable in the Cournot one.

Next, we examine the quantities of firm 2, prices and consumer surplus when firm 1 produces more under Cournot competition.

**Proposition 16.** *If $q_1^C > q_1^B$ then $q_2^C > q_2^B$, and, consequently, $p_1^C < p_1^B$, $p_2^C < p_2^B$ and $CS^C > CS^B$.*

In a static model, for any given cost difference between duopolists, the quantity difference between low cost firm and high cost one is lower in Cournot competition. This is due to the low cost firm's less competitive behavior in quantity competition. In both types of competition, this quantity difference increases in cost difference at a decreasing rate with the rate being slower in Cournot. The innovator produces more in quantity competition when innovation is significantly larger for Cournot firms. However, the cost advantage gained is less significant since strong spillovers are necessary for this case. It turns out that at equilibrium innovation levels quantity difference is lower in quantity competition when $q_1^C > q_1^B$, and it follows that firm 2 is producing more as well. In fact, it is possible for this firm to be producing more in quantity competition even when the innovator produces more in price competition. For example, when $\alpha = 7$, $c = 3$, $\gamma = 0.9$, $\rho = 0.95$, $v = 0.63$ we have that $q_1^C = 2.338$, $q_1^B = 2.346$ together with $q_2^C = 2.213$, $q_1^B = 2.211$ at equilibrium.

When both quantities are larger under Cournot competition, it follows that prices are lower and consumer surplus is higher than in Bertrand competition. If only firm 2 produces more under quantity competition, consumer surplus ordering depends on the amplitude of $q_1^B - q_1^C$ relative to the amplitude of $q_2^B - q_2^C$. In the previous numeric example, consumer surplus is larger under price competition, $CS^B = 9.8666$ > $CS^C = 9.8202$, and both prices are higher under Cournot competition, $p_1^B = 2.6636$

$< p_1^C = 2.6745$ and $p_2^B = 2.6771 < p_2^C = 2.6866$. However, considering $v = 0.625$ and the other parameters same as before, we obtain $q_1^C = 2.3468$, $q_1^B = 2.3504$ and $q_2^C = 2.2243$, $q_1^B = 2.2143$ at equilibrium. Consumer surplus is larger under quantity competition, $CS^B = 9.8978 < CS^C = 9.9255$, and both prices are higher under Bertrand competition, $p_1^B = 2.6567 > p_1^C = 2.6513$ and $p_2^B = 2.6703 > p_2^C = 2.6636$.

The fact that both quantities can be larger under Cournot competition is not driven by the asymmetry of the model. In a symmetric setting, Qiu (1997) reports that Bertrand quantities are always larger than Cournot ones whenever a necessary condition for the social planner's problem to have an interior solution holds.[4] Nevertheless, there are parameter ranges where both Cournot and Bertrand equilibria are well defined, and symmetric output is larger under quantity competition than under price competition.[5] In the symmetric setting, the conditions for well defined equilibria are more restrictive.[6] Symmetric post-innovation marginal costs approach zero faster because spillovers flow in both directions. With asymmetric R&D abilities, it is possible to have positive post innovation costs for more efficient innovation technology. Then, the resulting higher innovation levels allow the asymmetric Cournot quantities to be larger.[7]

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[4]Under his condition optimal post-innovation costs are positive for *any $\gamma$ and $\rho$*. This condition is sufficient, but not necessary for the market equilibria to be well defined.

[5]For instance, with $\alpha = 7$, $c = 3$, $\gamma = 0.95$, $v = 1.25$ and $\rho = 0.99$, the symmetric quantity is larger in Cournot, $q^C = 2.1495 > q^B = 2.1208$. In fact, for these parameters, in the asymmetric game, quantities are larger in Bertrand, $q_1^C = 1.6682 < q_1^B = 2.0396$ and $q_2^C = 1.6595 < q_2^B = 2.0284$.

[6]The positive post innovation costs constraint does not allow for relatively more efficient R&D technology. There is a range of low values for $v$, where the asymmetric equilibria are well defined, but the symmetric ones are not.

[7]This happens when spillovers and substitutability are high, so that Cournot firms innovate significantly more than Bertand ones. Notice that in the example in footnote 5, despite high spillovers and substitutability, asymmetric quantities are larger in Bertrand. This is due to the relatively high R&D cost that is needed for interior symmetric equilibria.

### 4.4.3    Welfare and profit comparisons

Singh and Vives (1984) shows that Cournot duopolists make larger profits than Bertrand ones when they have the same profile of (possibly asymmetric) marginal costs. This means that, given a level of innovation, $x$, the profits of firm 1 are higher under quantity competition, $\Pi_1^C(x) > \Pi_1^B(x)$. In our model, firm 1 optimally chooses an innovation level, so that $\Pi_1^C(x^C) > \Pi_1^C(x^B)$ with $x^C$ and $x^B$ being the equilibrium R&D levels in Cournot and, respectively, in Bertrand. Then, it follows that $\Pi_1^C(x^C) > \Pi_1^C(x^B) > \Pi_1^B(x^B)$, the equilibrium profits of the innovator are larger under Cournot competition than under Bertrand competition. The rival does not choose an optimal R&D level, it only benefits costlessly from spillovers whenever $\rho > 0$. The ranking of his profits depends on $sign\left(q_2^C - \sqrt{1-\gamma^2}q_2^B\right)$. From Proposition 3 it follows that whenever $q_1^C > q_1^B$ at equilibrium, firm 2's profits are also larger under quantity competition than under price competition, $\Pi_2^C > \Pi_2^B$. These results together lead to $W^C = CS^C + \Sigma_i\Pi_i^C > W^B = CS^B + \Sigma_i\Pi_i^B$ and we have the following proposition.

**Proposition 17.** *Suppose A1 holds, then at equilibrium:*

$$i)\ \Pi_1^C\ >\ \Pi_1^B,$$
$$ii)\ if\ q_1^C\ >\ q_1^B\ then\ \Pi_2^C\ >\ \Pi_2^B\ and\ W^C > W^B.$$

Total welfare can be higher under Cournot even when both quantities are larger under price competition. This is due to the fact that, when innovation is higher in quantity competition, the benefits of larger cost-reduction may compensate the negative effect that lower output has on consumer surplus. This was observed by Qiu in his symmetric set-up. He showed that, in a dynamic model, quantity competition can produce more welfare even when quantities are larger in price competition. For our set-up, Proposition 17 already reports the possibility of Cournot competition being dynamically more efficient than the Bertrand one. However, the cases covered by this proposition do not conclude all situations where this occurs. For example,

when $\alpha = 7$, $c = 3$, $\gamma = 0.9$, $v = 0.65$, $\rho = 0.95$ we have $q_1^C = 2.2852 < q_1^B = 2.33$ and $q_2^C = 2.1705 < q_2^B = 2.2002$ , but $W^C = 17.294 > W^B = 11.516$.

## 4.5    Extensions and conclusions

R&D activity can focus on cost-reduction or, alternatively, on quality improvement. In Appendix B, we show that our ranking of the R&D levels extends to the case of product innovation. We consider a duopoly facing a linear quality-augmented demand following Symeonidis (2003). In the first stage, only one firm can buy a fixed quality increase, and in the second stage, competition takes place in quantities or prices. We identify a parameter equivalence that proves that all the results of Bester and Petrakis generalize to the case of product R&D. As our model suggests, these results should continue to hold when the firm can optimally choose a product R&D level. That is, *in an asymmetric model of product innovation, it is possible to have larger R&D levels under Bertrand competition than under Cournot if products are not too differentiated.* This contrasts with the results of Symeonidis who shows that, in a symmetric model, innovation is always larger under quantity competition.

In a model where only one of the duopolists engage in cost reducing R&D we have shown that under price competition the innovating firm can be reducing costs *more or less* than under quantity competition depending on the level of product differentiation, the rate of spillovers and the R&D efficiency. Furthermore, we show that the duopoly can produce more of both products under Cournot competition leading to a higher surplus both for consumers and producers. Thus, a priori, both the ordering of innovation and the market quantities between the two competition modes are ambiguous, and the previously mentioned parameters play a crucial role in their determination.

## 4.6   Appendix A

*Proof of Proposition 14.*

$$sign\left[\frac{x^C - x^B}{2(\alpha - c)}\right] =$$

$$sign\left[v\left(4 - \gamma^2\right)^2(1 - \gamma)(1 + \rho) - 2(1 - \rho)\left(2 - \gamma^2 - \gamma\rho\right)(2 - \gamma\rho)\right]$$

The condition for $x^B > x^C$ is

$$v < \frac{2(1 - \rho)\left(2 - \gamma^2 - \gamma\rho\right)(2 - \gamma\rho)}{(4 - \gamma^2)^2(1 - \gamma)(1 + \rho)} \equiv D.$$

For this condition to hold under $A1$ we need the following signs to be positive:

$$sign\left(D - 2A\right) = sign\left(D - 2C\right) = sign\left[\frac{(1 - \rho)(1 - \gamma\rho)}{(1 - \gamma)(1 + \rho)} - \frac{\alpha}{c}\right],$$

$$sign\left(D - 2B\right) = sign\left(\gamma - 2\rho\right).$$

First notice that if $\gamma < 1 - c/\alpha$ then the sign of $D - 2A$ is negative for any $\rho \in [0, 1]$ and, consequently $x^C > x^B$ whenever $A1$ holds. Second, when $\gamma > 1 - c/\alpha$ noticing that $D$ is decreasing in $\rho$ and letting $\rho = 0$ gives $v^*(\gamma) = \frac{4(2 - \gamma^2)}{(4 - \gamma^2)^2(1 - \gamma)}$ with $x^C > x^B$ for any $v > v^*(\gamma)$. For any $v < v^*(\gamma)$ define $\rho^* = \min[\gamma/2, y, z]$ where $y$ solves $v = \frac{2(1 - y)\left(2 - \gamma^2 - \gamma y\right)(2 - \gamma y)}{(4 - \gamma^2)^2(1 - \gamma)(1 + y)}$ (there exists such $y \in [0, 1]$ since $y = 0$ leads to $v < v^*(\gamma)$ and $y = 1$ leads to $v > 0$ and $v$ is continuous) and

$$z = \frac{1}{2\gamma}\left[\left(1 + \gamma + \frac{\alpha}{c}(1 - \gamma)\right) - \sqrt{\left(1 + \gamma + \frac{\alpha}{c}(1 - \gamma)\right)^2 - 4\gamma\left(1 - \frac{\alpha}{c}(1 - \gamma)\right)}\right].$$

For any $\rho > \rho^*$ either $D - 2B$ or $D - 2C$ is negative or $v > D$, thus $x^C > x^B$ whenever $A1$ holds. If $A1$ holds and $\rho < \rho^*$ then we have $x^B > x^C$.   □

*Proof of Proposition 15.*

$$sign\left(q_1^B - q_1^C\right) =$$

$$sign\left[v\left(4 - \gamma^2\right)^2(1 - \gamma) + 2(2 - \gamma)\gamma^2 + 2(2 - \gamma\rho)(2\gamma - 2 - \gamma\rho)\right]$$

The condition for $q_1^C > q_1^B$ is

$$v < \frac{2\left[(2 - \gamma\rho)(2 + \gamma\rho - 2\gamma) - (2 - \gamma)\gamma^2\right]}{(4 - \gamma^2)^2(1 - \gamma)} \equiv E$$

For this condition to be holding under $A1$ we need the following signs to be positive:

$$sign(E - 2A) = sign\left[2 - 2\gamma + \rho\gamma - \gamma^2 + \rho\gamma^2 - \rho^2\gamma^2 - \frac{\alpha}{c}(2 - \gamma\rho)(1 - \gamma)\right],$$

$$sign(E - 2B) = sign\left[4\rho - 2\gamma - \rho\gamma^2 - 2\rho^2\gamma + \rho\gamma^3\right],$$

$$sign(E - 2C) =$$
$$sign\left[2 - 2\gamma + \rho\gamma - 2\gamma^2 + \gamma^3 + \rho\gamma^2 - \rho^2\gamma^2 - \frac{\alpha}{c}(1 - \gamma)\left(2 - \gamma^2 - \gamma\rho\right)\right].$$

We have that if $sign(E - 2A)$ is positive $sign(E - 2C)$ is positive as well.

First, notice that if $\frac{\alpha}{c} > 3$ then $sign(E - 2A)$ is negative for any $\rho \in [0, 1]$, and for any $\gamma \in (0, 1)$ leading to $q_1^B > q_1^C$ under $A1$. Second if $\gamma < \frac{2\left(\frac{\alpha}{c} - 1\right)}{\frac{\alpha}{c} + 1} \equiv \gamma^*$ then $sign(E - 2A)$ is negative for any $\rho \in [0, 1]$, consequently $q_1^B > q_1^C$ under $A1$. For part b), noticing that $E$ increases in $\rho$ and letting $\rho = 1$ gives $v^{**}(\gamma) = \frac{2}{(4 - \gamma^2)}$ with $q_1^B > q_1^C$ for any $v > v^{**}(\gamma)$. For any $v < v^{**}(\gamma)$ define $\rho^{**}(\gamma) = \max[f, g, h]$ where

$$f \text{ solves } 2 - 2\gamma + f\gamma - \gamma^2 + f\gamma^2 - f^2\gamma^2 - \frac{\alpha}{c}(2 - \gamma f)(1 - \gamma) = 0,$$

$$g \text{ solves } 4g - 2\gamma - g\gamma^2 - 2g^2\gamma + g\gamma^3 = 0,$$

$$h \text{ solves } v\left(4 - \gamma^2\right)^2(1 - \gamma) - 2\left((2 - \gamma h)(2 + \gamma h - 2\gamma) - (2 - \gamma)\gamma^2\right) = 0.$$

For each equation LHS has different signs when $0$ and $1$ are substituted for the corresponding variable thus $f, g, h \in [0, 1]$.

For any $\rho < \rho^{**}$ either $E - 2A$ or $D - 2B$ is negative or $v > E$, and $q_1^B > q_1^C$ whenever $A1$ holds. If $A1$ holds and $\rho > \rho^*$ then we have $q_1^C > q_1^B$. $\qquad\square$

*Proof of Proposition 16.*

$$sign\left(q_2^B - q_2^C\right) = sign(F + G) \text{ where}$$

$F = v\gamma^2 \left(4 - \gamma^2\right) \left[v \left(4 - \gamma^2\right)^2 (1 - \gamma) + 2 \left(2 - \gamma\right) \gamma^2 + 2 \left(2 - \gamma\rho\right) \left(2\gamma - 2 - \gamma\rho\right)\right]$ and
$signF = sign \left(q_1^B - q_1^C\right),$
$G = -2v \left(4 - \gamma^2\right)^2 (1 - \rho) \gamma^2 (1 - \rho\gamma) + 4\gamma^2 (1 - \rho) \left(2 - \gamma\rho\right) \left(2 - \gamma^2 - \gamma\rho\right)$
Assume $q_1^C > q_1^B$ then

$$v \left(4 - \gamma^2\right)^2 (1 - \gamma) + 2 \left(2 - \gamma\right) \gamma^2 + 2 \left(2 - \gamma\rho\right) \left(2\gamma - 2 - \gamma\rho\right) < 0. \tag{5}$$

We claim that $G$ is strictly negative. Assume to the contrary that $G$ is non-negative. Then

$$v \left(4 - \gamma^2\right)^2 (1 - \rho\gamma) - 2 \left(2 - \gamma\rho\right) \left(2 - \gamma^2 - \gamma\rho\right) \le 0. \tag{6}$$

Summing up inequalities (5) and (6) gives a contradiction:

$$v \left(4 - \gamma^2\right)^2 (2 - \gamma - \rho\gamma) + 2 \left(2 - \gamma\right) \gamma^2 < 0.$$

Thus $G$ is strictly negative whenever $F$ is strictly negative. We conclude that $q_2^C > q_2^B$ if $q_1^C > q_1^B$. It is straightforward to check that if $q_2^C > q_2^B$ and $q_1^C > q_1^B$ then $p_2^C < p_2^B$, $p_1^C < p_1^B$ and $CS^C > CS^B$. $\qquad\square$

## 4.7   Appendix B

Consider the linear-quadratic model of Bester and Petrakis (1993). In the first stage, firm 1 can buy a cost reduction of $\Delta$ by paying a fixed amount. In the second stage, firms compete in prices or quantities. Using the profits in the reduced form game, we can compute the innovation incentives of the firms.

$$I_C = \pi_1^C \left(c_1 - \Delta, c_2\right) - \pi_1^C \left(c_1, c_2\right) =$$
$$\frac{4\beta^2\Delta\left[(2\beta - \gamma)\alpha - 2\beta c_1 + \gamma c_2 + \beta\Delta\right]}{\left(4\beta^2 - \gamma^2\right)^2},$$
$$I_B = \pi_1^B \left(c_1 - \Delta, c_2\right) - \pi_1^B \left(c_1, c_2\right) =$$
$$\frac{\left(2\beta^2 - \gamma^2\right)\beta\Delta\left[2\alpha\left(\beta - \gamma\right)\left(2\beta + \gamma\right) - \left(2c_1 - \Delta\right)\left(2\beta^2 - \gamma^2\right) + 2\beta\gamma c_2\right]}{\left(\beta^2 - \gamma^2\right)\left(4\beta^2 - \gamma^2\right)^2}.$$

Bester and Petrakis show that for high values of the substitutability parameter $(\gamma)$, it is possible to have $I^B > I^C$.

Consider now the quality augmented linear-quadratic model of Symeonidis (2003) with zero marginal cost, and a similar game. Firm 1 can buy a quality increase of $\Delta$ by paying a fixed amount. In the second stage, firms compete in prices or quantities. Using first stage profits, we can compute the innovation incentives of the firms.

$$I_C^* = \pi_1^C \left(u_1 + \Delta, u_2\right) - \pi_1^C \left(u_1, u_2\right) = \frac{16\Delta\left(4u_1 + 2\Delta - \sigma u_2\right)}{\left(16 - \sigma^2\right)^2},$$
$$I_B^* = \pi_1^B \left(u_1 + \Delta, u_2\right) - \pi_1^B \left(u_1, u_2\right) = \frac{2\left(8 - \sigma^2\right)\Delta\left[\left(8 - \sigma^2\right)\left(2u_1 + \Delta\right) - 4\sigma u_2\right]}{\left(4 - \sigma^2\right)\left(16 - \sigma^2\right)^2}.$$

Letting $c_i = 1 - u_i$, it can be shown that $I_C = I_C^*$ and $I_B = I_B^*$, for $\alpha = 1, \beta = 2, \gamma = \sigma$. Therefore, when substitutability is high, the incentive to invest in product quality may be higher under price competition. In fact, all results of Bester and Petrakis will continue to hold under product innovation, including comparisons with social incentives.

## 4.8 References

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