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The Maastricht Convergence Criteria and Monetary and Fiscal Policies for the EMU Accession Countries

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To my Father

Chapter 1

Introduction

My PhD dissertation concentrates on the theoretical analysis of the way monetary and fiscal policies should be conducted in the European Monetary Union (EMU) accession countries. Despite diverse macroeconomic experience and structural differences these economies share common characteristics: they are small open economies with rapid productivity growth, infrastructure improvements and are vulnerable to external disturbances. Importantly their fiscal and monetary policies are requires to stabilize their economies in accordance with the membership requirements of the EMU summarized in the Maastricht Treaty. In reality, we observe that the choice of monetary and fiscal policies and also the progress in economic stabilization differ substantially between the accession countries. That said, many of the EMU accession countries do not satisfy at the moment some of the Maastricht convergence criteria.

My interest lies in identifying the implications of different monetary and fiscal policies on the compliance with the Maastricht criteria. I investigate whether structural differences can affect monetary choices. I characterize the optimal monetary policy and also optimal interaction between monetary and fiscal policy in the EMU accession countries. Finally, I study how the Maastricht criteria affect the design of optimal policies and their ability to stabilize business cycle fluctuations.

In order to address all these issues I take advantage of the new open macroeconomics literature. In particular I perform the whole analysis in the framework of dynamic stochastic general equilibrium model of a small open economy incorporating frictions such as price stickiness and distortionary taxation, which provide a role for monetary and fiscal policy. As far as the structure of the economy is concerned I introduce two goods sectors: nontradable goods and tradable goods. I take into account recent empirical literature, both on OECD and EMU Accession countries, that highlights the role of sector specific shocks in explaining international business cycle fluctuations (see e.g. Canzoneri et al. (1999), Marimon and Zilibotti (1998) and

Mihaljek and Klau (2004)). The model can be seen as an extension of the one-sector small open economy model developed by De Paoli (2004). It is also similar in its structure to the two-country models of Altissimo et al (2004), Benigno and Thoenisen (2003) and Liu and Pappa (2005).

Finally, the model aims at incorporating important characteristics of the EMU accession countries: productivity growth, diverse degrees of exchange rate pass through and a volatile stochastic environment. The model is calibrated to match the moments of a chosen EMU accession country, i.e. the Czech Republic.

In Chapter 2 I study the ability of different monetary regimes to satisfy the Maastricht convergence criteria. I analyze regimes that reflect the policy choices observed in the EMU accession countries, i.e. a peg regime, a managed float and a flexible exchange rate regime with CPI inflation targeting. In particular, I study responses of the Maastricht variables (i.e. CPI inflation, nominal interest and nominal exchange rate) under different regimes to both domestic supply and demand shocks and also external shocks. I discuss the implications of openness, trade specialization pattern and the degree of exchange rate pass through on the choice of the monetary regime that would satisfy the Maastricht criteria. Importantly, I provide a quantitative framework to evaluate whether a given monetary regime can satisfy the Maastricht criteria.

I find that there exists a significant trade-off between compliance with the CPI inflation criterion and the nominal interest rate criterion. Under the benchmark parameterization none of the regimes satisfies all the criteria. The sensitivity analysis reveals that the probability that some of the regimes will satisfy all the criteria increases with openness of the economy and degree of substitution between home and foreign traded goods. However the ultimate choice of the regime which satisfies all the criteria depends on the degree of exchange rate-pass through. Low degree of pass through discriminates between regimes: when the economy gets more open, variances of the Maastricht variables under the peg and managed float regime diminish while the contrary is true for the CPI targeting regime. If degree of exchange rate pass through is high, then higher openness enables all the regimes to meet the Maastricht criteria.

Chapter 3 focuses on characterization of optimal monetary policy for EMU accession countries in the framework of the already developed model. I also investigate how the monetary Maastricht criteria affect the optimal monetary policy. First, I characterize the optimal monetary policy from a timeless perspective (Woodford (2003)). I derive the micro founded loss function using the second order approximation methodology developed by Rotemberg and Woodford (1997) and Benigno and Woodford (2005). The derived loss function can be seen as a generalization of the previous studies encompassing both the closed (Aoki (2001), Benigno (2004), Rotemberg and Woodford (1997)) and open economy frameworks (Gali and Monacelli (2005) and

De Paoli (2004)). I find that the optimal monetary policy in a two-sector small open economy should not only target inflation rates in the domestic sectors and aggregate output fluctuations, but also domestic and international terms of trade.

In order to characterize the optimal monetary policy constrained by the monetary Maastricht criteria I reformulate the criteria using the methodology developed by Rotemberg and Woodford (1997, 1999) for the analysis of the zero bound problem of the nominal interest rate. The optimal constrained policy differs in two aspects from the optimal unconstrained policy (stabilization and deterministic components): it restricts fluctuations of the Maastricht variables and also sets new deterministic targets for these variables that serve as additional buffers to comply with the criteria.

Under the chosen parameterization optimal monetary policy does not satisfy the CPI inflation and the nominal interest rate criteria. The optimal constrained policy induces smaller variability of the CPI inflation and of the nominal interest rate. At the same, it is also characterized by a deflationary bias which results in targeting CPI inflation rate and nominal interest rate that are 0.7% p.a. lower than their equivalents in the reference countries. As a result, such a policy induces additional welfare costs that amount to 30% of the initial deadweight loss of the optimal unconstrained policy.

While the first two chapters focus on monetary policy issues in the EMU accession countries, fiscal requirements set out in the Maastricht Treaty (as a part of the Stability and Growth Pact) indicate that fiscal policy in these countries could be also constrained.

That said, in Chapter 4 I incorporate fiscal policy by endogenizing tax and debt decisions and restricting taxes to only distortionary ones. Bearing in mind that monetary policy in the EMU accession countries is constrained by the Maastricht criteria, I investigate whether fiscal policy can serve as an additional stabilization tool and how its ability to stabilize business cycles changes when it is subject to the fiscal Maastricht criteria. In general, I analyze the properties of the optimal monetary and fiscal policy constrained by the Maastricht criteria and the Stability and Growth Pact (SGP) requirements. Additionally, I study the relative importance of monetary and fiscal criteria in shaping the stabilization pattern of the constrained policy.

I find that targets of the unconstrained optimal monetary and fiscal policy are similar to those of the optimal monetary policy alone. Similarly to the findings of Chapter 3, the constrained policy not only restricts fluctuations of the Maastricht variables but creates an additional buffer through new deterministic targets of the Maastricht variables. Under the chosen parameterization, the optimal monetary and fiscal policy violate three Maastricht criteria: on the CPI inflation rate, the nominal interest rate and deficit to GDP ratio. Since monetary criteria play a dominant role in affecting the stabilization process of the constrained policy, CPI inflation and the nominal interest rate are characterized by a smaller variability (than under the unconstrained policy) at the expense of a higher variability of deficit to GDP ratio. The constrained policy is characterized by a deflationary bias which results in targeting the CPI inflation rate and the nominal interest rate that are lower by 1.3% p.a. than their equivalents in the countries taken as a reference. The constrained policy is also characterized by targeting surplus to GDP ratio at around 3.7%. As a result the policy constrained by the Maastricht criteria induces additional welfare costs that amount to 60% of the initial deadweight loss associated with the optimal policy.

Summing up, Maastricht criteria have a significant effect on the way monetary and fiscal policies should be conducted in the EMU accession countries. They induce serious trade-offs for policymakers (as it is analyzed in Chapter 2). Based on the analysis undertaken in Chapters 3 and 4, I draw the following policy recommendation: in order to satisfy the Maastricht criteria policymakers should reconsider both the stabilization and deterministic component of the monetary and fiscal policy.

Acknowledgements

The idea for this thesis originates from the real life challenges that monetary and fiscal policy makers face in Poland - my home country - and also in other EMU accession countries. I address the main issue - compliance with the Maastricht criteria taking advantage of the recent developments in the new open macroeconomics. This would have not been possible if I did not meet several extraordinary people during the course of my thesis writing. My greatest gratitude goes towards my supervisor, Kosuke Aoki. I thank him not only for his excellent supervision and encouragement but also for sharing his economic knowledge with me. Special thanks also goes to Evi Pappa, Gianluca Benigno and Hugo Rodriguez for many useful discussions. I would like also to thank various Directors and Coordinators of my Ph.D. program IDEA (Enriqueta Aragonés, Carmen Beviá, Inés Macho-Stadler, Pau Olivella, David Pérez-Castrillo) for the economic support during my Ph.D. and giving me the freedom in pursuing my research interests. The grant provided by Ministerio de Educación y Ciencia is gratefully acknowledged.

Last but not least, this thesis would not be finished if I did not have constant support and belief in me from my family (my parents, Marta and Zygmunt, and my brother Maciej) and especially from Dario. Dziękuję Wam Kochani!

Chapter 2

Maastricht Criteria and Monetary Regimes

2.1 Introduction

A common objective and also an obligation of the new entrant countries to the European Union (EU) is the accession to the European Monetary Union (EMU).[1] These economies share common characteristics: they are small open economies with a rapid productivity growth, infrastructure improvements and are vulnerable to external disturbances At the same time their monetary policies are obliged to satisfy the Maastricht convergence criteria which stand for the prerequisites to enter the EMU. All the EMU accession countries should achieve a high and durable degree of price stability, which in quantitative terms is reflected in low inflation rates and low long-term interest rates. Additionally, nominal exchange rates of the EMU accession countries versus the euro should stay within the normal fluctuation margins provided for by the Exchange Rate Mechanism of the European Monetary System.

The choice of the monetary policy regime in these countries is crucial for their compliance with the Maastricht criteria. We observe heterogeneity in the choice of the regime among the EMU accession countries. Baltic countries (i.e. Estonia, Latvia and Lithuania) and Bulgaria chose to peg to the euro. The Czech Republic, Hungary and Slovakia decided for the managed floating regime, Poland and Romania went for the flexible regime with CPI strict targeting. Interestingly, many EMU accession

[1] On the 1st of May 2004 10 Central and Eastern European countries, i.e. Cyprus, the Czech Republic, Estonia, Hungary, Latvia, Lithuania, Malta, Poland, Slovakia and Slovenia, entered the European Union. Additionally, Bulgaria and Romania entered the EU on 1st of January 2007. Importantly all these countries are entitled to enter the EMU as it was stated in their accession agreement with the EU. Slovenia is the first country in this group that joined the European Monetary Union on January 1, 2007. Cyprus and Malta joined the EMU on January 1, 2008.

countries do not fulfill some of the Maastricht criteria. Bulgaria, Estonia, Hungary, Latvia, Lithuania, Romania and Slovakia fail to fulfill the CPI inflation criterion. Hungary and Romania violate the nominal interest rate criterion. Poland, Slovakia and Romania do not comply with the nominal exchange rate criterion.

The goal of this paper is to study the ability of different monetary regimes to comply with the Maastricht criteria. To provide a proper framework for the analysis, we build a small open economy model with nominal rigidities and two production sectors: a nontraded and traded good sector. In this way we want to take account of the empirical literature that emphasizes the role of sector productivity shocks in shaping inflation and real exchange rate dynamics in the EMU accession countries (see Mihaljek and Klau (2004)). We perform policy experiments by changing the monetary regimes and analyzing their implications on the compliance with the Maastricht criteria. We study monetary regimes that are currently adopted in the EMU accession countries, i.e. peg regime, managed float and flexible exchange rate regime with CPI inflation targeting.

The interaction between the Maastricht requirements and the monetary regimes has attracted the interest of academics. For example, Buiter and Grafe (2003) and Coricelli (2002) call for adopting the peg regime in these countries as it enhances the credibility of the monetary policies and also strengthens links with the EU and EMU. Similarly, Ravenna (2005) finds that the gain from a credible adoption of the fixed regime can outweigh the loss of monetary policy independence. At the same time, all these authors acknowledge that the peg regime can prevent from the compliance with the Maastricht CPI inflation criterion and suggest that this criterion should be revised. Devereux (2003) and Natalucci and Ravenna (2007) find that the monetary regime characterized by flexible inflation targeting with some weight on exchange rate stability should comply with the Maastricht criteria.

There are several caveats of the previous studies that this paper aims to eliminate. First of all, the studies concentrate mainly on the Balassa-Samuelson effect (Balassa (1964)) and therefore on the implications of only one type of shocks, i.e. productivity shocks in the home traded sector, on the choice of monetary regime. We analyze responses of the Maastricht variables (CPI inflation rate, nominal interest rate and nominal exchange rate) under different monetary regimes to both domestic supply and demand shocks and also external shocks. Policy recommendations could be sensitive to structural differences among the EMU accession countries. We discuss thoroughly implications of openness, trade specialization pattern and degree of exchange rate pass through on the choice of monetary regime that would satisfy the Maastricht criteria. Finally, most of the exisitng studies discuss the ability of different monetary regimes to satisfy the criteria in qualitative terms. We provide a quantitative framework that enables us to evaluate whether a given monetary regime can satisfy the Maastricht criteria.

Our results can be summarized as follows. There exist significant trade-offs between compliance with the CPI inflation criterion and the nominal interest rate criterion. Under the benchmark specification (which aims to reflect the Czech Republic economy) none of the regimes satisfies all the criteria. The sensitivity analysis reveals that the probability that some of the regimes will satisfy all the criteria increases with openness of the economy and the degree of substitution between traded goods. However the ultimate choice of the regime which satisfies all the criteria depends on the degree of exchange rate pass through. Low degree of pass through discriminates between regimes: when economy gets more open, variances of the Maastricht variables under the peg and managed float regime diminish while the contrary is true for the CPI targeting regime. If degree of exchange rate pass through is high, then higher openness enables all the regimes to meet the Maastricht criteria.

The paper is organized as follows. Section 2.2 reviews some stylized facts on the EMU accession countries based on the empirical literature. Section 2.3 describes the model and contrasts it with the existing theoretical literature. Section 2.4 and 2.5 focus on the determinants of the macroeconomic volatility in the long run and in the short run. Section 2.6 presents comparison of the monetary regimes under the benchmark parameterization. Section 2.7 reports the sensitivity analysis results on the structural parameters and their impact on the monetary regime performance. Section 2.8 concludes indicating further research directions.

2.2 Stylized facts on the EMU accession economies

Our aim is to detect important characteristics of the EMU accession countries which affect the choice of the monetary regime in these countries. Importantly we study the determinants of macroeconomic volatility in these countries. Moreover we have a close look at some structural parameters which can be indicative for the choice of the monetary regime. Finally we analyze briefly economic performance of the EMU accession economies on the basis of their monetary regime choice.

All the EMU accession countries can be treated as small open economies. Their real GDP do not exceed 1% of the nominal GDP of the euro area (except for Poland for which the ratio amounts to 3%). However structure of these economies varies as far as share of nontraded sector and degree of openness are concerned.[2] In particular the ratio of imports in their nominal GDP ranges from 37% (for Poland) up to 83% (for Estonia). Importantly the euro area countries are the biggest trading partner of these countries with the share on average of 50% in their total trade.

[2] Detailed data can be found in Appendix A.

As far as the stochastic environment of the EMU accession countries is concerned, Sueppel (2003) finds that these countries are characterized by higher growth and wider output fluctuations than the euro area and other EU countries.[3] Moreover he identifies that the degree of synchronization of their business cycles with the euro area is smaller and heterogenous than of the United Kingdom, Sweden and Denmark. This a consequence of the stabilization process taking place in these countries and reflected in the structural reforms, infrastructure improvements and a high productivity growth.

Having in mind the restrictions set on the monetary policy in the accession countries we find important to identify the main determinants of the real exchange rate dynamics which summarize the pressures on inflation, nominal interest rate and nominal exchange rate.

Since all the EMU accession countries are characterized by a high productivity growth (especially in the tradable sector) many researchers test the hypothesis of the Balassa - Samuelson effect for these countries. According to the Balassa -Samuelson effect (Balassa (1964)) a country which experiences a higher productivity growth in the traded sector will face higher consumer prices and subsequently real exchange rate appreciation. An existence of the strong Balassa - Samuelson effect could endanger the attempts of keeping low inflation differential between these countries and the euro area. We can list the following empirical studies analyzing the Balassa-Samuelson effect in the EMU accession countries: Cipriani (2001), de Broeck and Slok (2001), Egert et al. (2002), Fisher (2002), Halpern and Wyplosz (2001), Coricelli and Jazbec (2001), Arratibel et al. (2002) and Mihaljek and Klau (2004). The main findings of these papers are rather diverse. The estimates indicate that the Balassa - Samuelson effect can explain from 0 - 3.5% per annum of the existing difference between inflation rates in the transition countries and the euro area.[4]

The original formulation of the Balassa - Samuelson theory totally neglects the role of the demand side of an economy in affecting the real exchange rate dynamics.[5] Some authors such as de Gregorio et el. (1994), de Broeck and Slok (2001), Cova (2004) and Astrov (2005) and Dufrenot et al. (2003) point out that in reality also demand side shocks can lead to real exchange rate appreciation and inflationary pressures. According to de Broeck and Slok (2001) observed growth of incomes in the EMU accession countries can increase the demand for nontradable goods and subse-

[3] See Figure A.1 in Appendix A.

[4] These different results come from the varied methodologies used and also diverse treatment of the data: especially the share of nontradable goods in the economies and inclusion of the regulated prices in it. Moreover many studies neglected also a significant rise in productivity of nontradables and existence of the nontradable component in tradable goods.

[5] This is due to very restrictive assumptions such as the law of one price for tradables, perfect mobility of production factors and perfect competition.

quently their price. Additionally since government expenditures fall predominantly on the nontraded goods they lead to a rise of price of nontradables. Moreover de Gregorio and Wolf (1994), Cova (2004) and Astrov (2005)[6] argue that demand shocks in the accession countries can lead to terms of trade improvements and through the income effect to real exchange rate appreciation and inflation. Astrov (2005) finds that real exchange rate in the EMU accession countries is affected positively by terms of trade (depreciation effect) and negatively by the share of government expenditures (appreciation effect) in the gross domestic product.[7] Additionally Dufrenot et al. (2003)[8] report that public finances and current account influence the real exchange rate dynamics. Their substantial deterioration is reflected in the real exchange rate depreciation.

The described demand side and supply side shocks constitute qualitatively for the common factors shaping the macroeconomic volatility in the EMU accession countries. Still there exist initial conditions, i.e. inflationary environment and structural parameters such as degree of openness and degree of exchange rate pass-through which make the countries to choose different monetary regimes.

Interestingly as far as the initial conditions are concerned Klyuev (2001) in his model of exchange rate regime choice in the EMU accession countries[9] finds the nonlinear relationship between the rate of inflation and the degree of exchange rate flexibility. The panel study indicates that a rise in inflation from a low level suggests introduction of more flexible exchange rate regimes while an increase in already high inflation is a sign to implement a rather fixed regime. The fixed regime present in the environment of considerable rigidities in both labour and goods market may lead to a decrease in the competitiveness of a country. That is why several Central and Eastern European countries (i.e. the Czech Republic, Hungary and Poland) have decided recently to introduce more flexible exchange rate arrangements.

Moreover the traditional Optimum Currency Area theory indicates that countries that are more open and therefore more vulnerable to nominal exchange rate movements

[6] The authors argue that these demand shocks are reflected in an increased demand for the tradables due to quality improvements (consistent with a changing composition of the tradables in the EMU Accession countries). In that way the Balassa - Samuelson effect can be replicated as long as the productivity increase consists in a quality improvement and a rise in the price of tradables.

[7] It is a panel regression study. The countries included in the sample are: Bulgaria, Croatia, Czech Republic, Hungary, Poland, Slovakia, Slovenia and Romania. The sample period for the study is 1990-2001. In this study one can also find the summary of some of the previous results.

[8] The authors of this study use the structural VAR and Behavioral Equilibrium Exchange Rate methodology. The study is is developed for 5 countries: the Czech Republic, Hungary, Poland, Slovakia and Slovenia.

[9] His study includes 13 Central and Eastern European economies: Albania, Bulgaria, Croatia, the Czech Republic, Estonia, Hungary, Latvia, LIthuania, Macedonia, Poland, Romania, Slovakia and Slovenia.

should opt for the fixed regime. This can be somewhat explanatory for the case of Estonia which chose to peg and on the other pole for Poland which opted for the flexible regime.[10]

The degree of exchange rate pass through in an economy, i.e. the degree to which extent nominal exchange rate fluctuations feed into the domestic prices and affect the rate of inflation in the economy is especially crucial for small, open economies. According to Calvo and Reinhart (2002) and also empirical studies by Chaudry and Hakura (2002) and Devereux and Yetman (2003) exchange rate shocks in the emerging economies tend to feed into aggregate inflation at a much faster pace than in the industrialized economies. This fact influences the choice of monetary policy which should be used to adjust to external shocks. Moreover it raises the question of how important the exchange rate adjustment should be in the chosen monetary rule.

Importantly the large pass through together with observed rigidities in the labour and goods market endanger the effectiveness of monetary policy and suggest implementation of strict exchange rate targeting. Additionally Coricelli and Jazbec (2004) in their study on the four EMU accession countries find that managed float policies aimed at accommodating the adverse shocks on the real exchange rate can actually induce the strong exchange rate pass-through.[11] That is why Slovenia and Hungary (opting for more fixed regimes) are reported to experience perfect pass-through while in case of the Czech Republic and Poland (opting for more flexible regimes) this degree is much smaller.

Summing up the EMU accession economies experience common driving forces affecting their macroeconomic volatility. Still they differ in some structural parameters and ultimate choices of the monetary regimes. The natural question which arises now how the choice of the monetary regime can influence the macroeconomic volatility of a country and compliance with the Maastricht criteria. A quick look at the data presented in Appendix A indicates that countries following monetary regimes that entails some degree of the nominal exchange rate stabilization are characterized by strong productivity growth but at the same time experience higher inflation rates.

2.3 The Model

We build a small scale model of an accession economy with the aim to study how different monetary regimes perform in stabilizing the Maastricht variables, i.e. inflation, nominal interest rate and nominal exchange rate in the stochastic environment. We

[10] see Appendix B.

[11] The reaction function of such a policy responds to disequilibria in the real exchange rate rather than deviations from the inflation and/or nominal exchange rate target.

present an EMU accession economy as a small open economy interacting with the rest of the world economy - proxied as the euro area. We model a small open economy as the limiting case of a two country model where the size of one of the countries is set to zero. In each of the economies there are two good sectors: nontraded and traded goods. We consider highly integrated two economies where asset markets are complete. The structure of labour markets is such that labour is mobile between sectors in each country and immobile between the countries. We assume existence of home bias in consumption which is a function of the relative size of an economy and its degree of openness.[12]

Purchasing power parity (PPP) is violated due to three reasons: existence of the nontraded sector, home bias in consumption and also local currency pricing in the traded good sector which violates law of one price. Moreover, in order to study a role of the monetary policy, we introduce monopolistic competition and price rigidities with staggered Calvo contracts in all the good sectors. However we abstract from any monetary frictions by assuming cashless limiting economies. Importantly existence of market power in the traded good sector opens up role for terms of trade in transmission of the shocks. Additionally, local currency pricing in the traded good sector induces the imperfect exchange rate pass - through into domestic prices.[13] The stochastic environment of the small open economy is characterized by asymmetric productivity shocks originating in both domestic sectors, preference shocks and foreign consumption shocks.

The model can be seen as an extension of a one-sector small open economy model of De Paoli (2004). Moreover, it is also similar in its structure to two-country models of Benigno and Thoenisen (2003) and Altissimo et al (2004).[14] As far as the literature on monetary policy in the EMU accession economies is concerned our model is closely related to Devereux (2003), Natalucci and Ravenna (2003). Importantly, their specification of the traded good sector (i.e. domestic firms are price takers) implies exogeneity of terms of trade.

[12] This assumption enables us to consider a limit case of the zero size of the home economy and concentrate on the small open economy.

[13] In Section 2.7.2 we discuss an alternative pricing of firms, i.e. producer currency pricing.

[14] In both papers the assumption regarding a two-sector structure of an economy plays a crucial role. Benigno and Thoenisen (2003) examine the real exchange rate fluctuations between United Kingdom and the euro area and analyze whether Balassa-Samuelson effect could explain the real exchange rate appreciation of the British pound in the nineties. Altissimo et al. (2004) focus their analysis on the determinants of inflation differentials in a currency area.

2.3.1 Households

The world economy consists of a measure one of agents: $[0, n)$ belonging to the small country (home) and $[n, 1]$ belonging to the rest of the world - the euro area (foreign). There are two types of differentiated goods produced in each country: traded and nontraded goods. Home traded goods are indexed on the interval $[0, n)$ and foreign traded goods on the interval $[n, 1]$ respectively. The same applies to the nontraded goods. Since our focus is on the limiting case of a two-country model we show only equations of the home economy. Foreign variables are indexed with $*$.

Households are assumed to be infinitely lived and they behave according to the permanent income hypothesis. Moreover in each country they can choose between three types of goods: nontraded, domestic traded and foreign traded goods. C_t^i represents consumption at period t of a domestic consumer i and L_t^i stands for his labour supply. Each agent i maximizes the following utility function:[15]

$$\max E_{t_0} \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[U\left(C_t^i, B_t\right) - V\left(L_t^i\right) \right] \right\} \tag{2.1}$$

where E_t denotes the expectation conditional on the information set at date t, β is the intertemporal discount factor and $0 < \beta < 1$, $U(\cdot)$ stands for flows of utility from consumption and $V(\cdot)$ represents flows of disutility from supplying labour.[16] C is a composite consumption index. We define consumers' preferences over the composite consumption index C_t of tradable goods $(C_{T,t})$ (domestically - produced and foreign ones) and nontradable goods $(C_{N,t})$:

$$C_t \equiv \left[\mu^{\frac{1}{\phi}} C_{N,t}^{\frac{\phi-1}{\phi}} + (1-\mu)^{\frac{1}{\phi}} C_{T,t}^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}} \tag{2.2}$$

where $\phi > 0$ is the elasticity of substitution between traded and nontraded goods and $\mu \in [0, 1]$ is the share of the nontraded goods in the overall consumption. The traded good consumption is a composite of the domestically - produced traded goods (C_H) and foreign produced traded goods (C_F):

$$C_{T,t} \equiv \left[\nu^{\frac{1}{\theta}} C_{H,t}^{\frac{\theta-1}{\theta}} + (1-\nu)^{\frac{1}{\theta}} C_{F,t}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \tag{2.3}$$

where $\theta > 0$ is elasticity of substitution between home traded and foreign traded goods, ν - home bias being the function of the relative size of the small economy with

[15] In general we assume that U is twice differentiable, increasing and concave in C_t and V is twice differentiable, increasing and convex in L_t.

[16] We assume specific functional forms of the consumption utility $U\left(C_t^i\right)$, and disutility from labour $V\left(L_t^i\right)$: $U\left(C_t^i, B_t\right) \equiv \frac{B_t^{\rho}\left(C_t^i\right)^{1-\rho}}{1-\rho}$, $V\left(L_t^i\right) \equiv \varphi_l \frac{\left(L_t^i\right)^{1+\eta}}{1+\eta}$ with ρ $(\rho > 0)$ - the inverse of the intertemporal elasticity of substitution in consumption and η $(\eta \geq 0)$ - the inverse of the labour supply elasticity.

respect to the foreign one and its degree of openness λ such that $(1-\nu) = (1-n)\lambda$ and $\lambda \in [0,1]$.[17] Let us notice that degree of openness is related to degree of home bias, i.e. the higher degree of openness the smaller degree of home bias. Finally, C_j (where $j = H, N$) is a consumption sub-index of the continuum of differentiated goods:

$$C_{j,t} \equiv \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma}} \int_0^n c_t\,(j)^{\frac{\sigma^j - 1}{\sigma^j}}\, dj \right]^{\frac{\sigma}{\sigma-1}} \tag{2.4}$$

where $\sigma > 1$ represent elasticity of substitution between differentiated goods in each of the sectors. The consumption - based price indices expressed in the units of currency of the respective country are the following ones:

$$P_t \equiv \left[\mu P_{N,t}^{1-\phi} + (1-\mu) P_{T,t}^{1-\phi} \right]^{\frac{1}{1-\phi}} \tag{2.5}$$

$$P_{T,t} \equiv \left[\nu P_{H,t}^{1-\theta} + (1-\nu) P_{F,t}^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{2.6}$$

with

$$P_{j,t} \equiv \left[\left(\frac{1}{n}\right) \int_0^n p_t\,(j)^{1-\sigma}\, dj \right]^{\frac{1}{1-\sigma}}. \tag{2.7}$$

The existence of the nontraded goods, assumed home bias and also possibility of local currency pricing cause the deviations from purchasing power parity. So $P \neq SP^*$ (where S stands for the nominal exchange rate). The real exchange rate can be defined in the following manner: $RS = \frac{SP^*}{P}$. Moreover we define the terms of trade as $T = \frac{P_F}{P_H}$ and the domestic terms of trade as $T^d = \frac{P_N}{P_T}$ domestic terms of trade).

In order to represent the small open economy limiting case we use the definition of ν and set $n \to 0$. From consumers' preferences we can derive total demand of the generic goods - n (home nontraded goods) and h (home traded goods):

$$y^d(n) = \left[\frac{p(n)}{P_N}\right]^{-\sigma} \left[\frac{P_N}{P}\right]^{-\phi} \mu C \tag{2.8}$$

$$y^d(h) = \left[\frac{p(h)}{P_H}\right]^{-\sigma} \left[\frac{P_H}{P_T}\right]^{-\theta} (1-\lambda) C_T + \left[\frac{p^*(h)}{P_H^*}\right]^{-\sigma} \left(\frac{P_H^*}{P_T^*}\right)^{-\theta} \lambda C_T^* \tag{2.9}$$

Households get disutility from supplying labour to all the firms present in each country. Each individual supplies labour to both sectors, i.e. traded and nontraded one:

[17] This specification is based on De Paoli (2004).

$$L_t^i = L_t^{i,H} + L_t^{i,N}. \tag{2.10}$$

We assume that consumers have the access to a complete set of securities - contingent claims traded internationally. Each household faces the following budget constraint:

$$P_t C_t^i + E_t\{Q_{t,t+1} D_{t+1}\} \leq D_t + P_t TR_t^i + W_{H,t}^i L_{H,t}^i + W_{N,t}^i L_{N,t}^i + \frac{\int\limits_0^n \Pi_{N,t}^i di}{n} + \frac{\int\limits_0^n \Pi_{H,t}^i di}{n} \tag{2.11}$$

where at date t: D_{t+1} - nominal payoff of the portfolio held at the end of period (t), $Q_{t,t+1}$ - the stochastic discount factor for one-period ahead nominal payoffs relevant to the domestic household, $\Pi_{H,t}$ and $\Pi_{N,t}$ - nominal profits from the domestic firms and TR_t^i - nominal lump sum transfers from the domestic government to the household i. The similar budget constraint can be written for the foreign economy. Moreover in both countries consumers face no Ponzi game restriction. The short term interest rate (R_t) is defined as the price of the portfolio which delivers one unit of currency in each contingency that occurs next period:

$$R_t^{-1} = E_t\{Q_{t,t+1}\} \tag{2.12}$$

The maximization problem of any household consists in maximizing discounted stream of utility (2.1) subject to the budget constraint (2.11) in order to determine the optimal path of the consumption index, labour index and contingent claims at all times. The solution to the household decision problem gives a set of first order conditions.[18] Optimization of the portfolio holdings leads to the following Euler equations for the home and foreign economy:

$$U_C(C_t, B_t) = \beta E_t \left\{ U_C(C_{t+1}, B_{t+1}) Q_{t,t+1}^{-1} \frac{P_t}{P_{t+1}} \right\} \tag{2.13}$$

$$U_C(C_t^*) = \beta E_t \left\{ U_C(C_{t+1}^*) Q_{t,t+1}^{-1} \frac{S_t P_t^*}{S_{t+1} P_{t+1}^*} \right\} \tag{2.14}$$

There is a perfect sharing in this setting meaning that marginal rates of consumption in nominal terms are equalized between countries in all states and at all times:

$$\frac{U_C(C_{t+1}^*)}{U_C(C_t^*)} \frac{P_{t+1}}{P_t} = \frac{U_C(C_{t+1}, B_{t+1})}{U_C(C_t, B_t)} \frac{S_{t+1} P_{t+1}^*}{S_t P_t^*} \tag{2.15}$$

[18] We suppress here subscript i as we assume that in equilibrium all the agents are identical. Therefore we represent optimality conditions for a representative agent.

Moreover choosing appropriately the distribution of initial wealth we obtain that:

$$\frac{U_C(C_t, B_t)}{U_C(C_t^*)} = \upsilon \frac{P_t}{S_t P_t^*} \tag{2.16}$$

where $\upsilon > 0$ and depends on the initial wealth distribution. We have to point out here that although the assumption of complete markets conveniently simplifies the model it neglects a possibility of wealth effects as a result of the shocks.

The optimality condition for the labour supply is the following one:

$$\frac{W_t^k(i)}{P_t} = \frac{V_L(L_t^i)}{U_C(C_t^i, B_t)} \tag{2.17}$$

where $W^k(i)$ - nominal wage of the consumer i in the sector k ($k = H, N$). So the real wage is equal to the marginal rate of substitution between labour and consumption.[19]

2.3.2 Firms

All the firms are owned by consumers. Both traded and nontraded sectors are monopolistically competitive. Since firms use only labour as their output the production function for firm i in k ($k = H, N$) sector is the following one:

$$Y_{k,t}(i) = A_t^k L_t^k(i) \tag{2.18}$$

Subsequently the nominal marginal cost for the firm i in the k sector is:

$$MC_t^k(i) = \frac{W_t^k(i)}{A_t^k}. \tag{2.19}$$

Nontraded sector

Prices are set according to Calvo pricing scheme. Each period a fraction of firms $(1 - \alpha_N)$ decides their price maximizing the future expected profits.

The maximization problem of any firm in the nontraded sector at time t is given by:

$$\max_{P_{N,t_0}(i)} E_{t_0} \sum_{t=t_0}^{\infty} (\alpha_N)^t Q_{t_0,t} \left[(1 - \tau_N) P_{N,t_0}(i) - MC_t^N(i) \right] Y_{N,t_0:t}^d(i) \tag{2.20}$$

$$subject\ to\ Y_{N,t_0:t}^d(i) = \left(\frac{P_{N,t_0}(i)}{P_{N,t}} \right)^{-\sigma} Y_{N,t} \tag{2.21}$$

[19] Notice that wages are equalised between the sectors inside each of the economies due to perfect labour mobility and perfect competition in the labour market.

where $Y^d_{N,t_0:t}(i)$ - demand for the individual good produced by firm i at time t conditional on keeping the price $P_{N,t_0}(i)$ fixed at the level chosen at time t_0, MC_t^N - nominal marginal cost in the nontraded sector at time t, τ_N - revenue taxes in the nontraded sector.

Let us notice that in the flexible price equilibrium the optimal price in the nontraded sector is set at any time according to the following relation:

$$\frac{\widetilde{P}_{N,t}(i)}{P_t} = \mu_N MC_t^{N,r}\frac{P_t}{P_{N,t}} \tag{2.22}$$

where $\mu_N = \frac{\sigma}{(\sigma-1)(1-\tau_N)}$ and $MC_t^{N,r} = \frac{W_t^N(i)}{P_t A_t^N}$.

In the sticky price environment we obtain the following inflation equation:

$$\widehat{\pi}_{N,t} = k_N(-\widehat{A}_t^N + \widehat{\omega}_t - (1-b)\widehat{T}_t^d) + \beta E_t \widehat{\pi}_{N,t+1}, \tag{2.23}$$

where $k_N = \frac{(1-\alpha_N\beta)(1-\alpha_N)}{\alpha_N}$ and $b = \mu(\overline{T^d}\,\overline{p_T})^{1-\theta}$ - represents a share of nontraded goods in the consumption basket of the small open economy evaluated at the steady state.

According to equation (2.23) the sectorial inflation ($\widehat{\pi}_{N,t}$) depends on changes in the real marginal cost which and the relative prices. Real marginal cost decreases due to productivity increases ($\widehat{A}_t^N$) and raises in result of higher real wages ($\widehat{\omega}_t$). Additionally a rise in the relative price of nontraded goods generates a substitution effect away from this sector and leads to deflationary pressures. The magnitude of this effect depends inversely on the share of nontraded goods in the domestic consumption basket.

Traded sector

As far as the traded goods are concerned we assume a possibility of price discrimination between domestic market and a foreign one. We study two alternative pricing decisions: local currency pricing (LCP) and producer currency pricing (PCP). The first one implies delayed pass-through while the second one implies perfect exchange pass-through. As a benchmark scenario we choose LCP pricing. In Section 7 we discuss thoroughly implications of higher pass-through and PCP on performance of alternative monetary regimes.

Under LCP firms in the traded good sector decide their prices maximizing the expected profits subject to the demand schedule in a given market, i.e. domestic or foreign one:[20]

[20] We can separate pricing decisions depending on the market since our production function is linear.

- domestic market

$$\max_{P_{H,t_0}(i)} E_{t_0} \sum_{t=t_0}^{\infty} (\alpha_H)^s Q_{t_0,t} \left[(1-\tau_H) P_{H,t_0}(i) - MC_t^H(i) \right] Y_{H,t_0:t}^d(i) \quad (2.24)$$

$$subject\ to\ Y_{H,t_0:t}^d(i) = \left(\frac{P_{H,t_0}(i)}{P_{H,t}} \right)^{-\sigma} C_{H,t}; \quad (2.25)$$

- foreign market

$$\max_{P^*_{H,t_0}(i)} E_t \sum_{s=0}^{\infty} (\alpha_H)^s Q_{t_0,t} \left[(1-\tau_H) S_t P^*_{H,t_0}(i) - MC_t^H \right] Y_{H,t_0:t}^{*d}(i) \quad (2.26)$$

$$subject\ to\ Y_{H,t_0:t}^{*d}(i) = \left(\frac{P^*_{H,t_0}(i)}{P^*_{H,t}} \right)^{-\sigma} Y^*_{H,t}. \quad (2.27)$$

where $Y_{H,t_0:t}^d(i)$, $Y_{H,t_0:t}^{*d}(i)$ - demands for the individual good produced by firm i at time t in the domestic and export home traded sector conditional on keeping, respectively, the prices $P_{H,t_0}(i)$ and $P^*_{H,t_0}(i)$ fixed at the level chosen at time t_0, MC_t^H - nominal marginal cost in the home traded sector at time t, τ_H - revenue taxes in the home traded sector.

When prices are flexible the optimal prices in the home traded sector, i.e. the internal price $\widetilde{P}_{H,t}$ and export price $\widetilde{P}^*_{H,t}$ are set at any time according to the following relations:

$$\frac{\widetilde{P}_{H,t}(i)}{P_t} = \mu_H MC_t^{H,r} \frac{P_t}{P_{H,t}}, \quad (2.28)$$

$$\frac{\widetilde{P}^*_{H,t}(i)}{P^*_t} = \mu_H MC_t^{H,r} \frac{1}{RS_t} \frac{P^*_t}{P^*_{H,t}} \quad (2.29)$$

where $\mu_H = \frac{\sigma}{(\sigma-1)(1-\tau_H)}$ and $MC_t^{H,r} = \frac{W_t^H(i)}{P_t A_t^H}$.

In the sticky price environment we obtain two sector inflation equations for goods in the traded sector, i.e. home traded inflation $\widehat{\pi}_{H,t}$ and export traded inflation $\widehat{\pi}^*_{H,t}$:

$$\widehat{\pi}_{H,t} = k_H(-\widehat{A}_t^H + \widehat{\omega}_t + b\widehat{T}_t^d + a\widehat{T}_t) + \beta E_t \widehat{\pi}_{H,t+1} \quad (2.30)$$

$$\widehat{\pi}^*_{H,t} = k^*_H(-\widehat{A}_t^H + \widehat{\omega}_t - \widehat{RS}_t + \widehat{T}^*_t + b^*\widehat{T}_t^{*d}) + \beta E_t \widehat{\pi}^*_{H,t+1} \quad (2.31)$$

where $k_H = \frac{(1-\alpha_H\beta)(1-\alpha_H)}{\alpha_H}$, $k^*_H = \frac{(1-\alpha_H\beta)(1-\alpha_H)}{\alpha_H}$, and a, b, a^*, b^* are the steady state ratios.

As in the case of nontraded sector, sector inflation is driven by changes in the real marginal cost and relative prices. As far as the export sector is concerned, inflation dynamics are also affected by real exchange rate changes, e.g. real exchange rate depreciation leads to deflationary pressures in this sector.

Similarly, we can derive the optimal prices for the both markets of the foreign traded good sector. As a result, we obtain the following inflation equation for the import sector of our small open economy:

$$\widehat{\pi}_{F,t} = k_F(-\widehat{A}_t^{*F} + \widehat{\omega}_t^* + \widehat{RS}_t - (1-a)\widehat{T}_t + b\widehat{T}_t^d) + \beta E_t \widehat{\pi}_{F,t+1}. \tag{2.32}$$

Under PCP inflation in the import and export sector of the small open economy is driven entirely by domestic inflation of a given sector and nominal exchange rate movements, i.e. inflation in the export sector: $\widehat{\pi}_{H,t}^* = \widehat{\pi}_{H,t} - \Delta\widehat{S}_t$, inflation in the import sector: $\widehat{\pi}_{F,t} = \widehat{\pi}_{F,t}^* + \Delta\widehat{S}_t$.

2.3.3 Monetary and fiscal policies

The government in this small open economy occupies with collecting revenue taxes which are later redistributed to households in the form of lump sum transfers in such a way that each period there is a balanced budget:

$$\int_0^n \tau_t \left(P_{H,t}(i)Y_{H,t}(i) + P_{N,t}(i)Y_{N,t}(i)\right) di = \int_0^n TR_t^j dj \tag{2.33}$$

The existence of price stickiness and also other rigidities in the model such as deviations from PPP provide a role for the monetary policy. The distortion caused by monopolistic competition is offset by setting the appropriate output subsidies for each of the domestic sectors in the steady state so that output in the flexible price equilibrium is efficient.[21]

The monetary authority uses a short-term interest rate as its instrument. The general form of the interest rate feedback rule is the following one:

$$\widetilde{R}_t = \left(\frac{\pi_{t-1}}{\bar{\pi}}\right)^{\mu_\pi} \left(\frac{S_{t-1}}{\bar{S}}\right)^{\mu_S} \overline{R} \tag{2.34}$$

where μ_π, μ_S are the feedback coefficients to CPI inflation around a target rate $\bar{\pi}$ ($\bar{\pi}$ is the steady state value of CPI inflation), nominal exchange rate around a target level of $\bar{S}$ ($\bar{S}$ is the steady state value of the nominal exchange rate), $\overline{R}$ - the steady state value of the nominal interest rate. We also assume the interest rate smoothing:

[21] See Appendix B.1 for a derivation of the efficient steady state.
As in Rotemberg and Woodford (1998) we assume that the average level of output is optimal and independent of monetary policy.

$$R_t = \widetilde{R}_t^{1-\kappa} R_{t-1}^{\kappa} \varepsilon_t^{mp} \tag{2.35}$$

where κ - the rate of interest rate smoothing, ε_t^{mp} - the monetary policy shock (exogenous).

The loglinearized (around the steady state) version of equation (2.34) is the following:

$$\widehat{R}_t = \mu_\pi(1-\kappa)\widehat{\pi}_t + \mu_S(1-\kappa)\widehat{S}_t + \kappa\widehat{R}_{t-1} + \widehat{\varepsilon}_t^{mp} \tag{2.36}$$

where $\widehat{R}_t = \ln \frac{R_t}{R}$.

This form of the feedback rule allows us to study different regimes chosen by the EMU accession countries.[22] We follow here approach presented by Natalucci and Ravenna (2003). In particular the flexible exchange rate regime with the CPI targeting is characterized by a strong feedback coefficient to fluctuations in the aggregate inflation ($\mu_\pi \to \infty$). On the other side the fixed regime is characterized by a strong feedback coefficient to fluctuations in the nominal exchange rate ($\mu_S \to \infty$).[23] The managed float involves both nonzero feedback coefficients to fluctuations in the nominal exchange rate and inflation.

2.4 Macroeconomic volatility in the long run

This section analyzes the long run effects of the stochastic shocks in the presented small open economy environment. We solve the model by taking first order approximation around the steady state in the flexible price environment. Importantly the flexible price environment can be considered as the long run equilibrium towards which the sticky price equilibrium converges. Subsequently the solution of the model will provide us with the representation of the variables as functions of the stochastic shocks.

We focus on the real exchange rate dynamics as it can give us insight on the dynamics of the Maastricht variables in the sticky price environment.

From the supply relations in the flexible price environment ((2.22)) and the definition of the real exchange rate we can obtain the following relations between relative prices:

$$\widehat{T_t^{d,n}} = \widehat{A}_t^H - \widehat{A}_t^N - a\widehat{T_t^n}, \tag{2.37}$$

[22] The monetary rule used for parameterization has slightly more general form as it involves also response coefficient to aggregate output fluctuations (see Section 2.6.1).

[23] Notice that combining the Euler conditions for both economies and risk sharing condition we obtain an uncovered interest rate parity condition that directly links changes in the nominal exchange rate to nominal interest rates in both countries: $\Delta\widehat{S}_{t+1} = \widehat{R}_t - \widehat{R}_t^*$. Therefore in the case of the domestic shocks under the peg regime the nominal interest rate does not react.

$$\widehat{RS_t^n} = -b\widehat{T_t^{d,n}} + (1-a)\widehat{T_t^n}. \tag{2.38}$$

where $\widehat{T_t^{d,n}}$, $\widehat{T_t^n}$, $\widehat{RS_t^n}$ - fluctuations (around the steady) of domestic terms of trade, terms of trade and real exchange rate in the flexible price environment.

As a consequence the real exchange rate can be represented as a function of the productivity shocks and terms of trade:

$$\widehat{RS_t^n} = -b\widehat{A}_t^H + b\widehat{A}_t^N + (1-a(1-b))\widehat{T_t^n} \tag{2.39}$$

The above equation indicates that real exchange rate depends positively on the productivity shocks occurring in the domestic nontraded sector and negatively on the productivity shocks occurring in the domestic traded sector. However also terms of trade have to be taken into account when analyzing the overall effect of the stochastic shocks on the real exchange rate dynamics.

The assumption of imperfect substitution and the existence of market power in the domestic traded sector appears to be crucial when analyzing the validity of the Balassa Samuelson hypothesis and its inflationary impact on the EMU accession countries. In our framework when productivity shock in the domestic traded sector occurs we observe a rise in the ratio of domestic terms of trade through a unified labour market channel and increased real wages in the whole economy. Moreover the higher the share of nontraded goods the higher this appreciation effect on the real exchange rate. However since the home and foreign traded goods are imperfect substitutes we observe a lower price of the home traded goods in relation to the foreign ones. This worsens the terms of trade and has a depreciation effect on the real exchange rate.[24] This effect is stronger the smaller the degree of openness, the higher share of nontraded goods and a smaller degree of substitutability between home and foreign traded goods. So overall effect of the home traded productivity shocks on the real exchange rate is not certain.[25]

Importantly productivity shocks in the home nontraded sector lead to real exchange rate depreciation due to a decline in the domestic terms of trade accompanied by a rise in terms of trade. Moreover domestic demand shocks result in real exchange rate

[24] as in Benigno and Thoenisen (2003) and Altissimo et al (2004).

[25] A recent empirical literature sheds some light on this uncertain effect of home tradable productivity shocks. In particular Arratibel et al. (2002) report that inflation in the EMU accession countries is negatively affected by labour productivity increases in the manufacturing sector (in many empirical studies the sectorial productivity is proxied by labour productivity). This finding is based on the panel study on determinants of dual inflation (in tradable and nontradable goods) in the chosen EMU accession countries. The regression equation (with inflation as the dependent variable) is based on the hybrid new Phillips curve equation with some other explanatory variables such as: exchange rate regime, productivity growths, liberalisation index, oil prices, government deficit ratios, unemployment ratios, GDP, euro area GDP growth and terms of trade.

appreciation through its positive effect on the domestic terms of trade and negative effect on terms of trade.[26]

These conclusions are in contrast with Devereux (2003) and Natalucci and Ravenna (2003) who base their analyses on the assumption that home traded prices are fixed internationally. This supposition is based on the argument that EMU accession countries cannot affect their terms of trade. As a result terms of trade are treated exogenously and cannot act as transmitters and absorbers of shocks. That is why in their framework we observe a strong real exchange appreciation in presence of the home traded productivity shocks (see (2.39) when $\widehat{T_t^n}$ - exogenous). It is important to note that in such a framework there is no role for demand shocks as real exchange rate dynamics are determined entirely by productivity shocks in both domestic sectors.

Summing up the real exchange rate and therefore inflation movements can be a result of both demand and supply side shocks. In our analysis we identify a set of the crucial structural parameters which influence the way real exchange rate responds to the shocks. These are: share of nontraded goods in the aggregate consumption, degree of openness and degree of substitutability between home and foreign traded goods.

2.5 Macroeconomic volatility in the short run

In the short run when prices are sticky the real exchange rate adjustment to the new steady state depends on the chosen monetary rule, i.e. behavior of the nominal interest rate. Combining international risk sharing condition and Euler condition we obtain that the real exchange rate is a function of the current and future real interest rate differentials between the small domestic economy and the foreign one (see (2.13), (2.14), (2.16)):

$$\widehat{RS}_t = E_t \sum_{i=0}^{\infty} \left[\left(\widehat{R}^*_{t+i} - \widehat{\pi}^*_{t+i+1} \right) - \left(\widehat{R}_{t+i} - \widehat{\pi}_{t+i+1} \right) \right] \tag{2.40}$$

However on the contrary to the flexible price environment where the real interest rates are functions of the shocks the real interest rates are formed by the chosen monetary rule.

The current and future decisions on the real interest rates are reflected in the current consumption. In order to understand the effects of each of the monetary regimes on the stabilization of the domestic variables it is useful to introduce a new variable: the consumption gap defined as the difference between the current consumption in the sticky price environment and the consumption under the flexible price environment.

[26] Domestic demand shocks lead to a higher relative price of home goods which results in a decline of terms of trade.

We can write the log - linearized (around the efficient steady state) Euler condition in terms of consumption gaps:

$$\widehat{Cgap}_t = \widehat{Cgap}_{t+1} - \frac{1}{\rho}\left(\widehat{R}_t - \widehat{\pi}_{t+i+1} - \widehat{RR}^n_t\right) \tag{2.41}$$

where: $\widehat{Cgap}_t = \widehat{C}_t - \widehat{C}^n_t$, $\widehat{C}^n_t-$ natural rate of consumption, i.e. consumption in the flexible price equilibrium, $\widehat{RR}^n_t-$ the natural real interest rate, i.e. the real interest rate in the flexible price equilibrium. Performing infinite recursions on (2.41) we obtain that the current consumption gap differential is determined by current and future real interest rate gap differentials in the sticky and flexible price environment:

$$\widehat{Cgap}_t = -E_t \sum_{i=0}^{\infty} \frac{1}{\rho}\left[\left(\widehat{R}_{t+i} - \widehat{\pi}_{t+i+1}\right) - \widehat{RR}^n_{t+i}\right] \tag{2.42}$$

Additionally by combining equations (2.40) and (2.41) current real exchange rate can be represented as:

$$\widehat{RS}_t = E_t \sum_{i=0}^{\infty}\left[\widehat{RR}^{n\,*}_{t+i} - \widehat{RR}^n_{t+i}\right] + \rho\left(\widehat{Cgap}_t - \widehat{Cgap}^*_t\right) \tag{2.43}$$

The above relation gives us very useful insights concerning the nature of any monetary rule studied as compared to the flexible price equilibrium where the monetary rule cannot affect the economy.

If the real interest rates were above the natural ones in the domestic economy then this would have an additional appreciation effect on the real exchange rate, which is associated with deflation or/and nominal appreciation of the currency. On the other hand if the real interest rates were below the natural ones in the domestic economy this would lead to an additional depreciation effect on the current real exchange rate, which is associated with inflation or/and nominal depreciation of the currency.

2.6 Monetary regimes comparison

2.6.1 Parameterization

We follow the previous literature on the EMU accession economies (i.e. Laxton and Pesenti (2003), Natalucci and Ravenna (2003)) we calibrate the model to match moments of the variables for the Czech Republic economy.

The degree of openness of the small open economy , λ, is assumed to be 0.4 which implies that the imported consumption constitutes for around 40% of the tradable consumption. The share of nontradables in the aggregate consumption, μ, is assumed to be 0.42. These values are in accordance with the corresponding weights in CPI index for the Czech Republic over the period 2000-2005 (see Table A.1 in Appendix A). Moreover, the share of nontradable consumption in the foreign aggregate consumption (μ^*) is assumed to be 0.6, consistent with the value chosen by Benigno and Thoenisen (2003) for the euro area economy.

The discount factor, β, equals 0.99 implying the annual interest rate of around 4 percent. Following Stockman and Tesar (1995) we assume that inverse of intertemporal elasticity of substitution, ρ, is set to 2. As in Laxton and Pesenti (2003) we assume that inverse of labour supply elasticity, η, is equal to 4. The elasticity of substitution between tradable and nontradable consumption, ϕ, is set to 0.5 as in Stockman and Tesar (1995) and the elasticity of substitution between home and foreign tradables, θ, is assumed to be 1.5 following Backus et al (1995). The elasticity of substitution between differentiated goods, σ, is equal 10, which together with the revenue tax of 0.1[27] implies a markup of 1.23[28].

The degree of price rigidity in the nontraded sector, α_N, is chosen to be 0.85. The degree of price rigidity in the tradable sectors, α_H and α_F, are slightly smaller and equal 0.8. These values are a bit higher than the values reported in the micro and macro studies for the euro area countries.[29] At the same time, they are in accordance with Smets and Wouters (2003) who calibrate their model to the euro area data and Natalucci and Ravenna (2003)[30] who choose these values for the EMU accession countries.

The shock processes are assumed to follow autoregressive processes AR(1). The parameters of the shocks are chosen to match the historical moments of the variables. Following Natalucci and Ravenna (2003) and Laxton and Pesenti (2003), the productivity shocks in both domestic sectors are characterized by a strong persistence parameter equal to 0.85. Standard deviations of productivity shocks are set to 1.6% (nontraded sector) and 1.8% (traded sector).These values are consistent in magnitude with values chosen by Natalucci and Ravenna (2003), i.e. 1.8% (nontraded sector) and 2% (traded sector). Additionally, we assume that productivity shocks are strongly correlated, their correlation coefficient is set to 0.7.[31] Other shocks are independent of

[27] This value represents the average share of Taxes less Subsidies in the Gross Domestic Product at 1995 constant prices in the Czech Republic for the years 1995-2006 (source: Eurostat).

[28] Martins et al. (1996) estimate the average markup for manufacturing sectors at around 1.2 in most OECD countries over the period 1980-1992. Some studies (Morrison (1994), Domowitz et al (1988)) suggest that the plausible estimates range between 1.2 and 1.7.

[29] Stahl (2004) estimates that the average duration between price adjustment in the manufacturing sector is 9 months (i.e. degree of price rigidity is 0.67). On the other hand, Gali et al (2001) and Benigno and Lopez-Salido (2003) estimate the aggregate supply relations for the European countries and find that overall degree of price rigidity for these countries to be 0.78.

[30] They argue that the existence of a high share of regulated prices in the EMU accession countries justifies such a high value of price stickiness.

[31] Empirical evidence suggests that sector productvity shocks are strongly correlated (see e.g. Backus

each other. Parameters defining the preference shock are, 0.72% (standard deviation) and 0.95 (persistence parameter).[32] Parameters of the foreign consumption shock are estimated using quarterly data on aggregate consumption in the euro area over the period 1990-2005 (source: Eurostat). The standard deviation of the foreign consumption shock is equal to 0.23% and its persistence parameter is 0.85.

In order to match the historical moments of the Czech Republic economy, we parameterize the monetary policy rule, i.e. the nominal interest rate follows the rule described by: $\widehat{R}_t = 0.9\widehat{R}_{t-1} + 0.1(\widehat{\pi}_t + 0.2\widehat{Y}_t + 0.3\widehat{S}_t) + \widehat{\varepsilon}_{R,t}$, where $\widehat{\varepsilon}_{R,t}$ is the monetary policy innovation with a standard deviation) 0.4%. In Table B.3 in Appendix B.3 we present comparison of the model moments with the historical moments.

Finally, for the purpose of our analysis regarding performance of the monetary regimes, we specify each of the regimes by assigning specific values of the feedback coefficients in the monetary rule (see (2.36)). In particular:

- a fixed exchange rate regime (a strict peg to the currency of the foreign economy) is described as the monetary rule with $\mu_\pi = 0$, $\mu_S \to \infty$,
- a flexible exchange rate regime in which the monetary rule stabilizes CPI inflation is described as the monetary rule with $\mu_\pi \to \infty$, $\mu_S = 0$,
- a managed float exchange rate regime in which the monetary rule stabilizes CPI inflation and nominal exchange rate is described as the monetary rule with $\mu_\pi = 2$, $\mu_S = 0.025$ and the smoothing parameter $\kappa = 0.9$.[33]

Based on the theoretical discussion in the previous sections we analyze performance of the monetary regimes in response to domestic and foreign shocks. We focus on the Maastricht variables, i.e. nominal interest, aggregate inflation and nominal exchange rate. At the same time, we also compare the overall stabilization pattern of each of the regimes by observing evolution of consumption gap.

2.6.2 Impulse responses to the domestic and foreign shocks

We study how the small domestic economy responds to the domestic and foreign shocks. First we identify the common patterns of responses of the key domestic variables that are present in the flexible price environment and under all the considered regimes. Next we identify the sources of differences in the response of each of the

et al (1992)).

[32] These values are similar to the values chosen by Laxton and Pesenti (2003), 0.4% (standard deviation) and 0.7 (persistence parameter).

[33] The specific values of the feedback coefficients are taken from Natalucci and Ravenna (2003) and represent estimates of Taylor rules for the OECD countries.

regimes by analyzing behavior of the consumption gap (see (2.43)). Finally we evaluate the monetary regimes taking as a point of reference their ability to comply with the Maastricht criteria.

Domestic supply shocks

We examine the effects of domestic productivity shocks in both sectors (see Figure (2.1) and (2.2) in Appendix 2.A). Both productivity shocks result in the real exchange rate depreciation in the flexible price environment and also under all the regimes. An imperfect substitution between all types of goods leads to a decline in domestic prices and the real exchange depreciation. Moreover we observe a decline in the natural real interest rate which is associated with the increase in the domestic aggregate consumption. Subsequently the expenditure switching effect leads to an increase in the domestic aggregate output.

Importantly the magnitude of the real exchange rate depreciation differs for the two shocks analyzed. This can be understood by observing the changes in relative prices (see (2.38)). Productivity shocks in the nontraded sector lead to a decline in the domestic terms of trade and a rise in international terms of trade. Both changes have a depreciation effect on the real exchange rate. On the other hand productivity shocks in the traded productivity sector result in a rise of both types of relative prices with the opposing effects on the real exchange rate.

The differences in response of the economy under the alternative regimes are summarized by the consumption gap (see equations (2.41), (2.43)). Since the productivity shocks entail deflationary pressures the magnitude of a change in the nominal interest will depend on the importance which is attached to inflation changes in each of the alternative monetary rules and also to the fluctuations in the nominal exchange rate. Not surprisingly CPI targeting results in the strongest decline of the nominal interest rate and a positive consumption gap. On the other hand the peg regime, not able to use the nominal interest rate to stabilize the economy, is characterized by the strongest deflation followed later by inflation and a negative consumption gap.

The stabilization under CPI targeting regime involves a high response of the nominal interest rate and a nonstationary depreciation of the nominal exchange rate.[34] On the other hand peg regime guarantees stabilization of the nominal exchange rate but at the expense of deflation and a fall in real wage. Importantly complete stabilization of the nominal exchange rate guarantees the stationarity of aggregate price level which is reflected in the pattern of aggregate inflation: first it declines and then it rises after

[34] This finding is consistent with the study of Benigno and Benigno (2004), i.e. nonstationary behaviour of the nominal exchange rate can be generated by the real shocks drawn from the stationary distribution in the flexible exchange rate regimes.

several quarters. The managed float is characterized by the intermediate responses: the smoothed character of the Taylor rule and moderate response coefficients towards inflation and nominal exchange rate result in the muted hump - shaped response of the nominal interest rate. Consistent with the findings of Benigno&Benigno (2004) we observe depreciation followed by appreciation under this regime. Similarly we also report deflation (of the magnitude similar to the peg regime) followed by small inflation. The magnitudes of these short run effects depend on respectively response coefficient towards inflation and response coefficient towards nominal exchange rate. Finally persistence of deflation under this regime depends on the smoothing parameter.

Notice that these results are on the contrary to the findings of Devereux (2003) and Natalucci and Ravenna (2003) who report that CPI inflation targeting leads to excessive recession when responding to domestic supply shocks in the tradable sector. The main difference in results originates from the assumption on the endogeneity of terms of trade.

Domestic demand shocks

Now we analyze the response of the domestic economy to the government expenditure shocks in the nontraded sector (see Figure (2.3) in Appendix 2.A). The domestic preference shock leads to a direct increase in domestic consumption. Natural rate of interest rate increases resulting in the real exchange rate appreciation. An additional domestic demand boosts production in both domestic sectors and subsequently leads to a rise in real wages and higher real marginal cost. Domestic goods become relatively more expensive which is reflected in improved terms of trade and also a rise in domestic terms of trade.

We identify the differences between the alternative regimes by examining the behavior of the consumption gap. Note that domestic demand shocks lead to inflationary pressures and the real exchange rate appreciation. The CPI targeting is characterized by the highest increase in the nominal interest as this regime aims at stabilizing inflation. This response results in a negative consumption gap and a higher real exchange rate appreciation leading to a smaller expansion in the economy. On the other hand the peg regime allowing for inflation (which is later balanced by a small deflation so that aggregate price level is stationary) and also the highest rise in real wage reports a positive consumption gap resulting in a smaller real exchange rate appreciation and a boom in the economy. The managed float regime is characterized by moderate change in the nominal interest rate which stabilizes partially nominal exchange rate (depreciation followed by appreciation) and inflation (followed by deflation). However the change in inflation under this regime is of the same magnitude as under the peg regime.

It is worth pointing out that since in our setting the domestic demand shocks lead to the real exchange rate appreciation and inflation we face the same evaluation of the regimes as in Devereux (2003) and Natalucci and Ravenna (2003) for the domestic traded productivity shocks.

Foreign shocks

The general pattern of response of the domestic economy to the foreign shocks depends on the way foreign aggregate consumption and also foreign real interest rate are affected. In particular foreign supply shocks lead to an increase in the foreign consumption and decline in the foreign real interest rate. Foreign demand shocks result in a decrease in the foreign consumption and an increase in the foreign real interest rate.[35] A change in the foreign consumption leads to a change of the same sign in the domestic aggregate consumption. At the same time we also observe a change in the real exchange rate (induced by a change in the foreign real interest rate) which affects adversely aggregate output through the expenditure switching effect. [36] As a result the domestic natural rate of interest changes to a lesser extent than the foreign one.

Importantly the peg regime totally accommodates all the foreign shocks by setting the same nominal interest as the foreign one which leads to a high volatility in the domestic variables (see Figure (2.4) in Appendix 2.A). As a result of the foreign aggregate consumption increase we observe a significant inflation and a negative consumption gap.

The remaining regimes allowing for some degree of the nominal exchange rate flexibility choose a different response in the domestic nominal interest as both of them, to a different extent, are concerned with the inflationary pressures which arise through the changes in inflation of the import sector and real exchange rate movements. That is why their responses are muted in comparison to the flexible price economy and lead to a negative consumption gap in result of the foreign supply shocks and a positive consumption gap in the case of the foreign demand shocks.

An overall evaluation of the monetary regimes performance

Till now, we have analyzed how monetary regimes respond to domestic and foreign shocks. But how these different responses affect ability of monetary regimes to comply with the Maastricht criteria? In order to answer this question we reformulate the

[35] The mechanisms of the effects of the foreign shocks on the foreign variables are similar to the ones explained in the previous subsections.

[36] The strength of the expenditure switching effect depends on the structural parameters, i.e. elasticity of demand between home and foreign tradables and also the domestic monetary policy.

Maastricht criteria in two important dimensions.[37] First, we state the Maastricht criteria in quarterly terms. Second, we reformulate the upper bounds on levels into the upper bounds on variances of the Maastricht variables. The upper bounds on variances are calculated in such a way that compliance with the reformulated criterion gives 95% probability that the original criterion on levels is satisfied.[38] Subsequently, a criterion will be satisfied (violated) when the variance of the respective Maastricht variable is lower (higher) than the upper bound.

In Table 2.1 we present the variances of the Maastricht variables under alternative monetary regimes. We find that none of the regimes satisfies all the Maastricht criteria. While the nominal exchange rate criterion is satisfied by all the regimes there exists a trade-off between compliance with nominal interest rate criterion and CPI inflation criterion. Not surprisingly, CPI targeting regime fails to satisfy the nominal interest rate criterion. On the other hand, peg regime fails to satisfy the CPI inflation criterion. The above trade-off is well reflected in variances induced by the managed float regime. Under this regime, variance of the nominal interest rate almost hits the upper bound of the criterion. But still it is not enough to guarantee stabilization of the CPI inflation rate in accordance with the Maastricht criterion.

Which of the regimes performs the best with respect to Maastricht criteria? Overall, managed float guarantees moderate variances of all Maastricht variables. Interestingly, this regime also induces the smallest variance of the consumption gap (as shown in Table 2.1). This indicates that both from the points of view of compliance with the Maastricht criteria and at the same time efficiency monetary regime in the EMU Accession countries should allow for some flexibility in stabilization of CPI inflation and the nominal exchange rate.[39]

	CPI inflation	nominal interest rate	nominal exchange rate	consumption gap
CPI targeting	0	0.60	21.03	0.35
managed float	0.15	0.06	6.10	0.20
peg regime	0.27	0.02	0	0.63
bound	0.04	0.06	58.57	-
note: Variances and bounds are multiplied by 100^2 (in $(\%)^2$)				

Table 2.1: Variances of the Maastricht variables and consumption gap under alternative regimes (LCP)

Next we study whether our findings can be subject to the chosen set of the struc-

[37] This reformulation methodology of the Maastricht criteria is explained in Appendix B.4. Chapter 3 provides a thorough discussion regarding this reformulation.

[38] A similar appraoch of reformulating the criteria was undertaken by Natalucci and Ravenna (2007).

[39] This result is similar to Devereux (2003) and Natalucci and Ravenna (2003).

tural parameters describing the small domestic economy.

2.7 Sensitivity analysis

The theoretical analysis of the real exchange rate determination in the long and short run enabled us to identify the structural parameters that can affect the responses of the small domestic economy to different shocks. In the long run perspective we discussed that a share of nontraded goods, a degree of openness and also a degree of substitution between home and foreign goods affect the magnitude of a change in the real exchange rate. Additionally in the short run a degree of exchange rate pass through in the domestic economy can alter the performance of the small domestic economy.

2.7.1 The long run analysis - openness of economy

Share of nontradables and degree of openness (defined as the share of imports in the tradable consumption) give us the insight on how open the economy is: a high share of nontradables together with small degree of openness indicate a relatively closed economy and a small share of nontradables together with a high degree of openness describe a more open economy. Changes in the degree of openness, share of nontradables and also degree of substitution between home and foreign goods affect the magnitude of the movements in the flexible price equilibrium real exchange rate (see equation (2.38)). Importantly the more open economy is the stronger interdependence between nominal exchange rate movements and the inflationary pressures. The higher the degree of substitutability between home and foreign goods the smaller movements in the terms of trade and traded inflation. In Figures (2.5), (2.6), (2.7) we present variances of the Maastricht variables and also consumption gap as functions of the share of nontraded consumption, degree of openness and degree of substitution between home and foreign goods.

We find that the ability of the monetary regimes to comply with the Maastricht criteria depends in a substantial way on openness of domestic economy and the degree of substitutability of traded goods. Importantly, managed float regime and peg regime can satisfy the CPI inflation criterion provided that share of nontradables is small and/or degree of openness is high and/or home and foreign goods and good substitutes. On the other hand, CPI targeting regime does not satisfy nominal interest rate criterion no matter how open the economy is.[40] Finally, nominal exchange rate criterion is always satisfied by all the regimes. Not surprisingly, variance of the nominal exchange

[40] Variance of the nominal interest rate under this regime remains above the upper bound for all the parameter configurations (μ, λ).

rate decreases for the CPI targeting and managed float regime with the more open economy and the higher degree of substitutability of traded goods.

Additionally, we control on how close different regimes are with respect to the flexible price equilibrium by studying variance of consumption gap. It appears that the more open economy is managed float and peg regime not only are characterized by an increased ability to satisfy the Maastricht criteria but also they are closer to the efficient flexible price equilibrium.

2.7.2 The short run analysis- exchange rate pass through

Our benchmark model assumes that there is a delayed pass through reflected in the local currency pricing (LCP). Importantly, the delayed pass-through diminishes the expenditure switching role of the nominal exchange rate. That is why, the managed exchange rate regimes outperform[41] the flexible exchange rate regimes in such an environment (Devereux and Engel (2003)). On the other hand, when exchange rate pass-through is high then nominal exchange rate movements enable necessary relative price adjustments in the environment where prices are sticky and the country faces real country-specific shocks (Friedman (1953)). Having these results in mind, we study how the assumption of instead high-pass through affects the relative performance of monetary regimes, i.e. the ability of alternative monetary regimes to comply with the Maastricht criteria. We compare local currency pricing environment with producer currency pricing (PCP).

In Table 2.2 we present variances of the Maastricht variables under alternative regimes. First of all, none of the regimes satisfies all the criteria. Interestingly, variances of the Maastricht variables under the CPI targeting and the managed regime are smaller than under LCP. The high pass-through of the nominal exchange rate under PCP enables fast relative price adjustment under these regimes. Thanks to this, both the nominal exchange rate and nominal interest rate are characterized by a smaller variance than under LCP.[42] Note that, in accordance with the discussion above, CPI targeting regime is characterized by the smallest variance of the consumption gap.

Finally, we control whether these results are dependent on how open the domestic economy is. In Figures (2.9) and (2.10) we present variances of the Maastricht variables and consumption gap as a function of the share of nontraded consumption and degree of openness. Interestingly, all the regimes can satisfy the Maastricht criteria provided that the degree of openness of the economy is high. Variances of all the Maastricht

[41] are characterised by higher welfare.

[42] Compare Figures (2.1) and (2.8) that represent impulse responses to the domestic productivity shock in the nontraded sector under LCP and PCP respectively.

	CPI inflation	nominal interest rate	nominal exchange rate	consumption gap
CPI targeting	0	0.15	8.97	0.27
managed float	0.09	0.05	5.51	0.35
peg regime	0.27	0.02	0	0.63
bound	0.04	0.06	58.57	-
note: Variances and bounds are multiplied by 100^2 (in $(\%)^2$)				

Table 2.2: Variances of the Maastricht variables and consumption gap under alternative regimes (PCP)

variables diminish as the economy is more open. Importantly, CPI targeting regime is the closest to the efficient flexible price equilibrium as it implies the smallest variance of the consumption gap. This result is robust to all the parameter specification of the share of nontraded consumption and degree of openness.

2.8 Conclusions

This paper studies the ability of different monetary regimes adopted by the EMU Accession countries (i.e. peg regime, managed float and flexible exchange rate regime with CPI inflation targeting) to satisfy the monetary Maastricht criteria. We identify some common characteristics of these countries regarding both the structure of the economy and its stochastic environment which can influence the choice of the monetary regime. Then we build a two-country dynamic stochastic general equilibrium model representing a small open economy - one of the EMU accession countries, and a big country - the euro area. This framework enables us to conduct policy experiments consisting in analyzing the effects of different monetary regimes on the way a small open economy responds to the set of domestic and foreign shocks.

The analysis suggests that the ability of regimes to satisfy the Maastricht criteria depends on the openness of an economy and the substitutability of home and foreign goods. At the same time, the degree of exchange rate pass through plays an important role as it affects to a great extent variances of the Maastricht variables. There exists a trade-off between satisfying the nominal interest rate and inflation criterion. We find that for many parameter specifications there is no regime which complies with all the Maastricht criteria. Higher degree of openness and strong substitutability of traded goods enables some of the regimes to comply with the criteria. However the ultimate choice of the regime which satisfies all the criteria depends on the exchange rate-pass through. Moreover, we obtain for some parameterizations that regimes that satisfy all the criteria are also characterized by small consumption gap. That suggests that in this situation there is no trade-off between fulfillment of the Maastricht criteria and desirability of the efficient outcome. However in order to address this issue properly, we need to perform the welfare analysis together with the derivation of the optimal policy constrained by the Maastricht criteria. This is the topic of Chapter 3.

2.A Figures

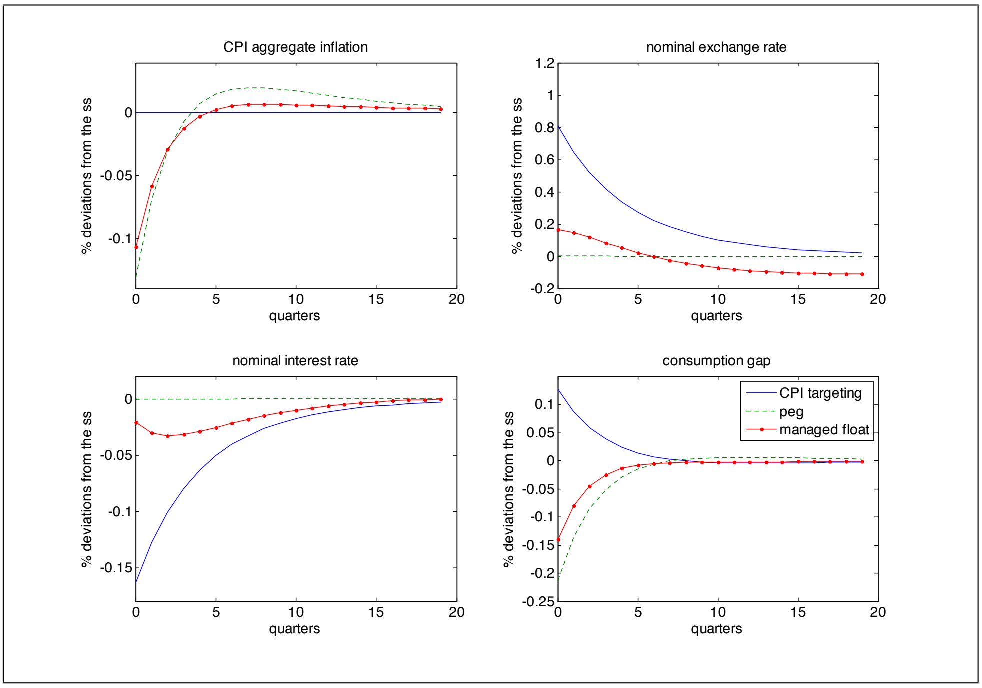

Figure 2.1: Impulse responses of the Maastricht variables and the consumption gap to the domestic nontradable productivity shock (LCP)

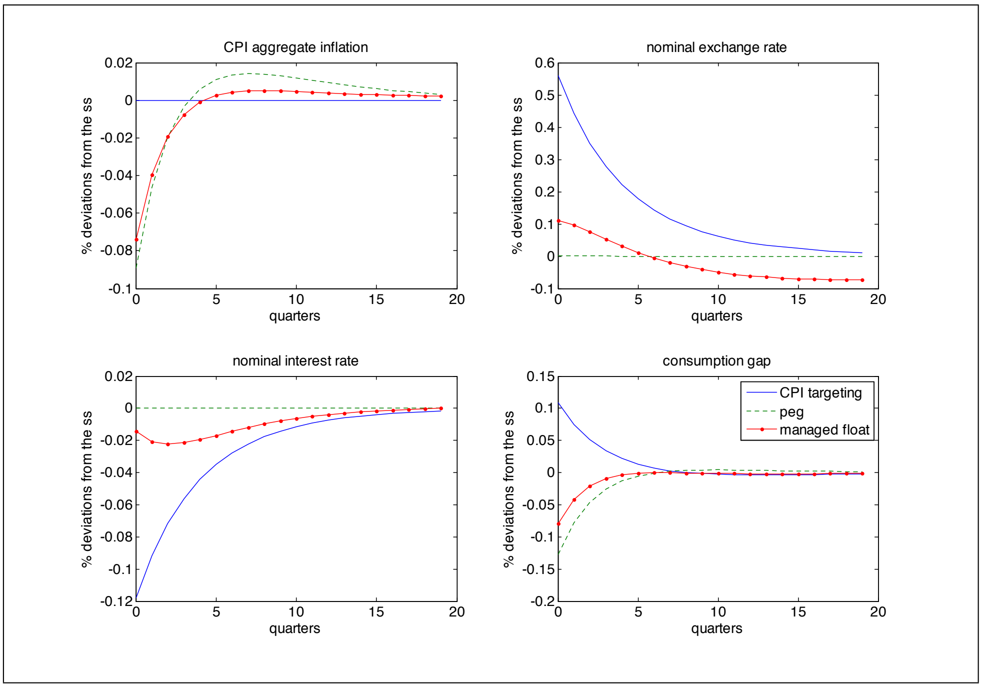

Figure 2.2: Impulse responses of the Maastricht variables and the consumption gap to the domestic tradable productivity shock

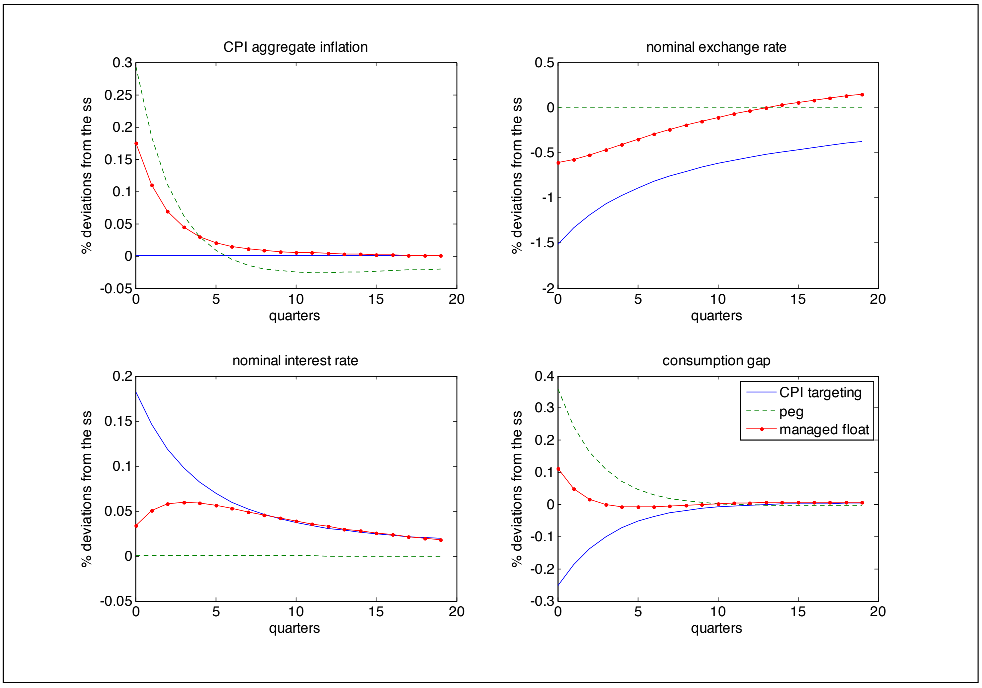

Figure 2.3: Impulse responses of the Maastricht variables and the consumption gap to the domestic preference shock (LCP)

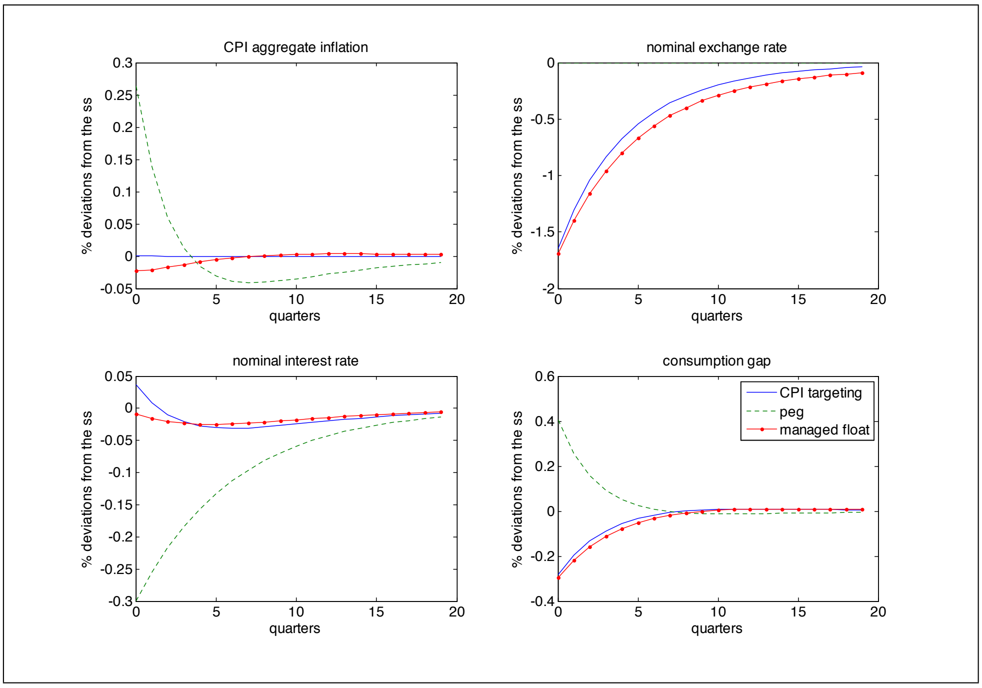

Figure 2.4: Impulse responses of the Maastricht variables and the consumption gap to the foreign consumption shock

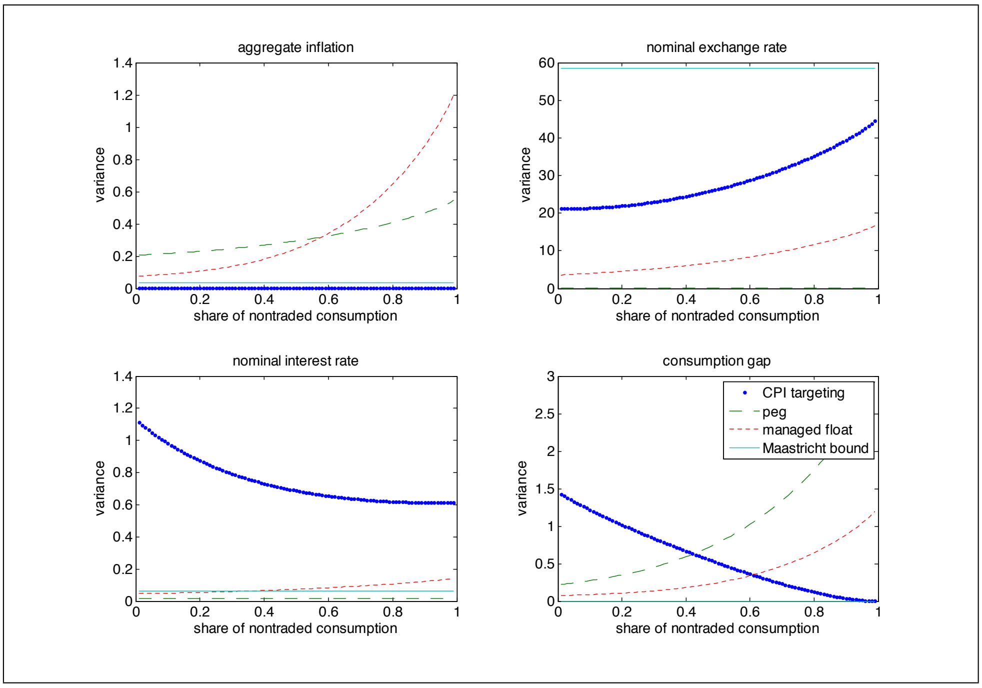

Figure 2.5: Variances of the Maastricht variables and the consumption gap as a function of the share of nontraded consumption (LCP)

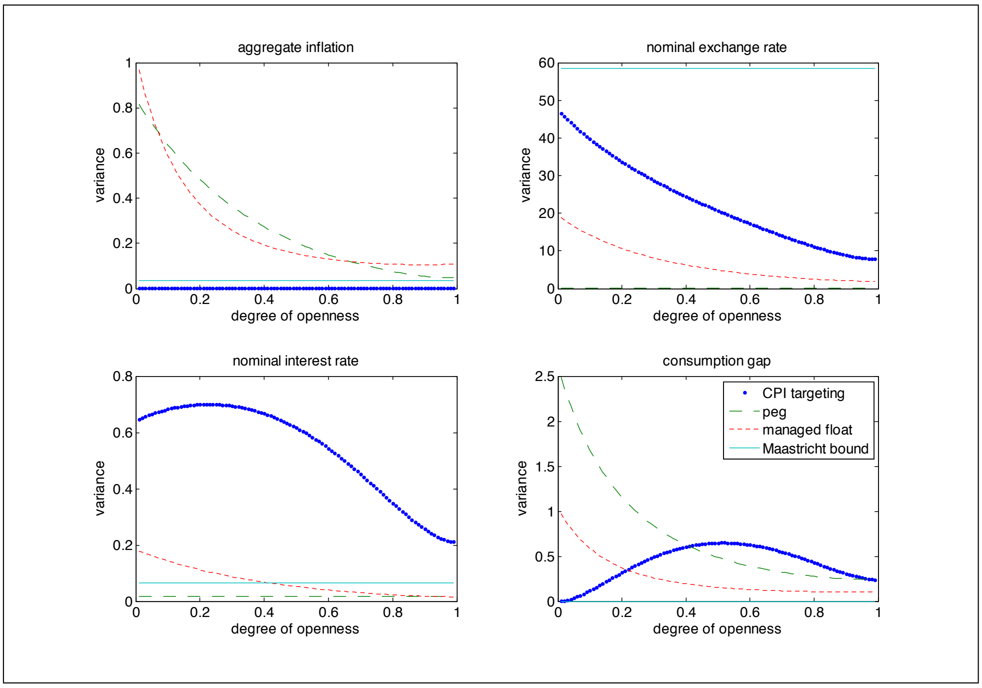

Figure 2.6: Variances of the Maastricht variables and the consumption gap as a function of the degree openness (LCP)

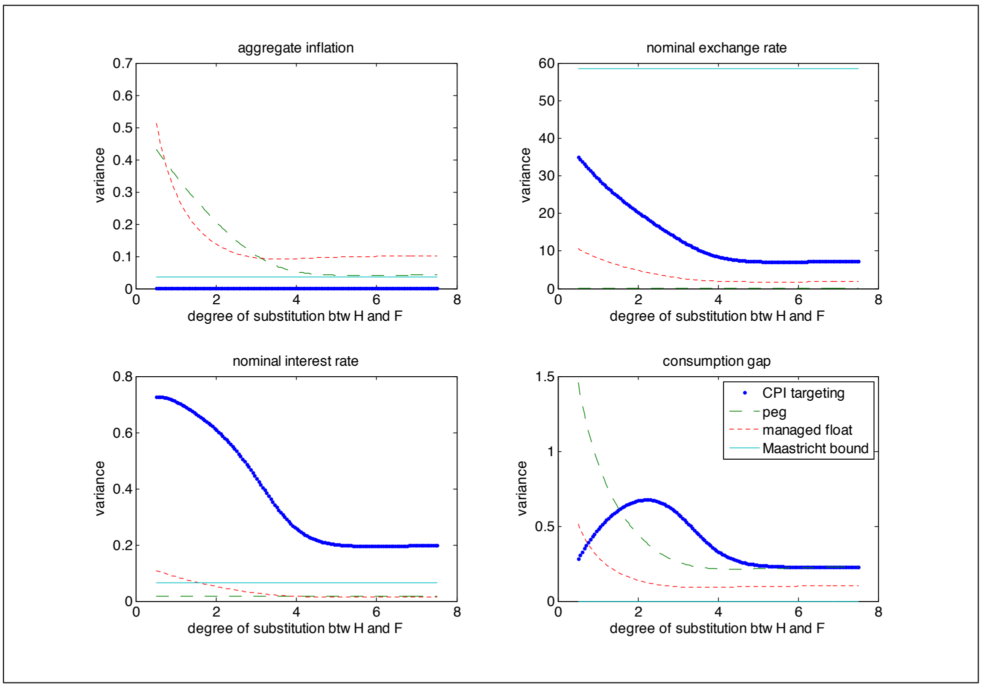

Figure 2.7: Variances of the Maastricht variables and the consumption gap as a function of the degree of substitutability of home and foreign goods (LCP)

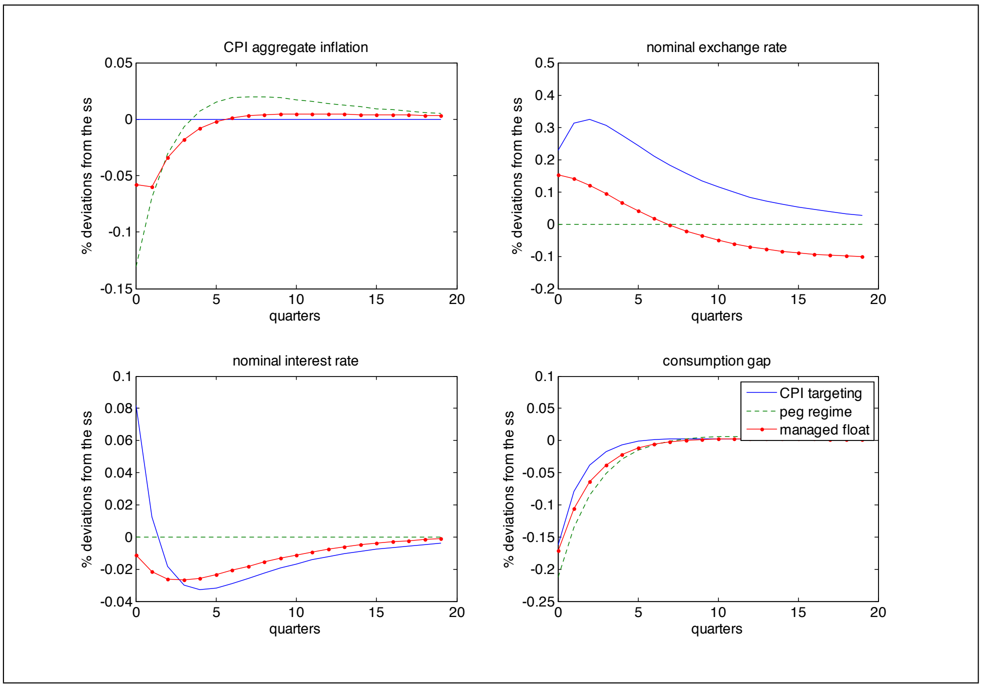

Figure 2.8: Impulse responses of the Maastricht variables and the consumption gap to the domestic nontradable productivity shock (PCP)

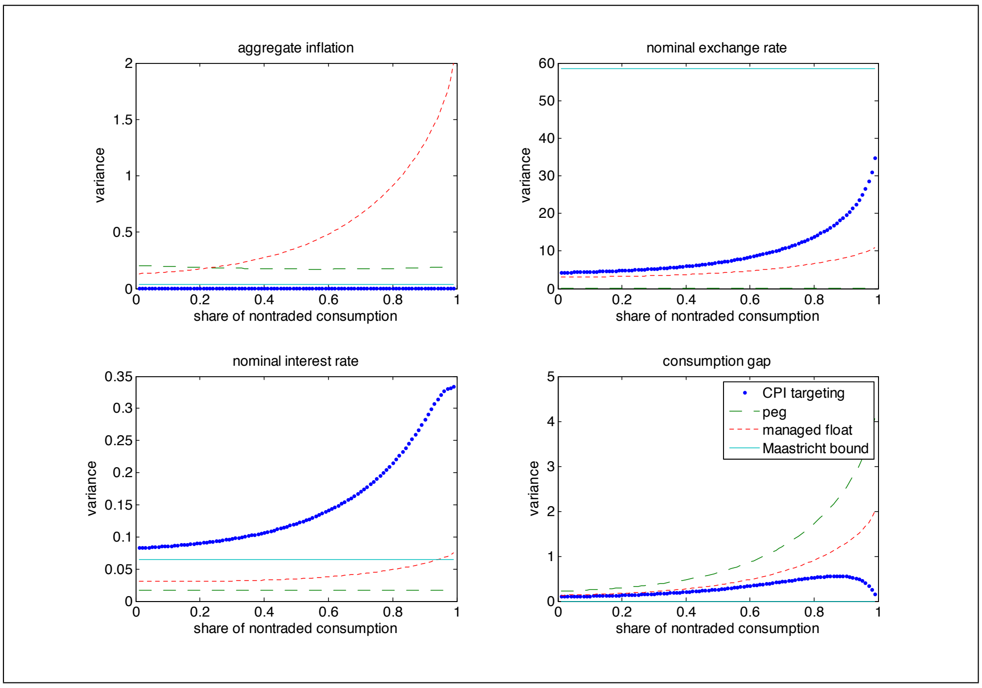

Figure 2.9: Variances of the Maastricht variables and the consumption gap as a function of the share of nontraded consumption (PCP)

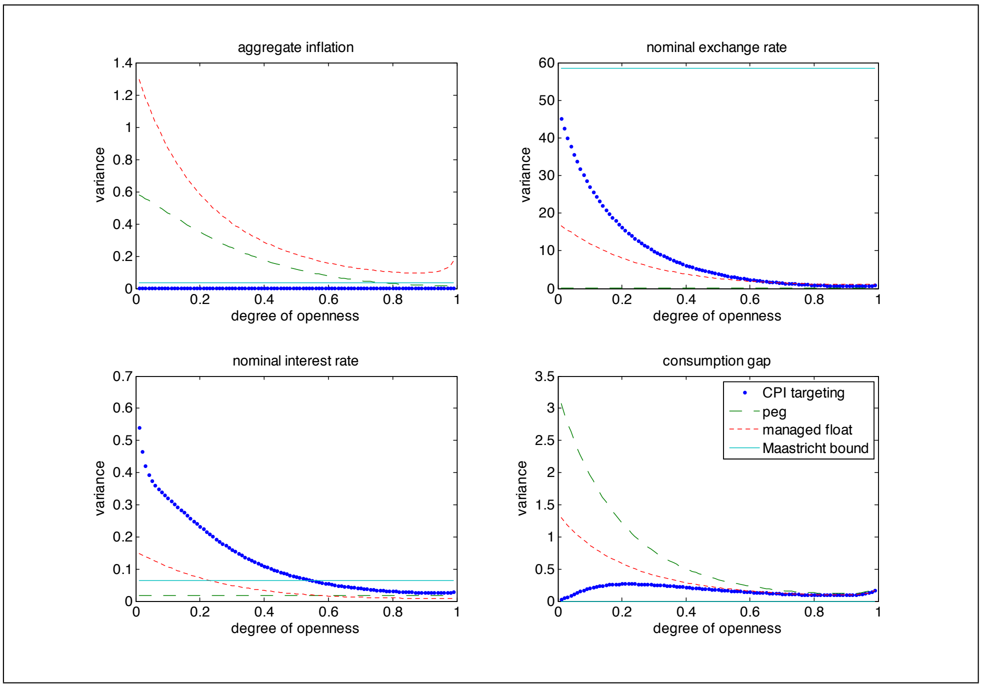

Figure 2.10: Variances of the Maastricht variables and the consumption gap as a function of the degree of openness (PCP)

Chapter 3

Maastricht Criteria and Optimal Monetary Policy

3.1 Introduction

The European Union Accession Treaty signed by the Central and Eastern European countries[1] includes the obligation to participate in the third stage of the economic and monetary union, i.e. the obligation to enter the European Monetary Union (EMU) in the near future.[2] In order to enter the EMU these countries are required to satisfy the Maastricht convergence criteria (for details see Appendix A). The criteria are designed to guarantee that prior to joining the European Monetary Union, countries attain a high degree of economic convergence not only in real but also in nominal terms. The countries should achieve a high and durable degree of price stability, which is reflected in low inflation rates and low long-term interest rates. Quantitatively, over a period of one year before the examination the average rate of CPI inflation should not exceed that of the three best performing Member States by more than 1.5% points, while the average nominal long-term interest rate should not exceed by more than 2% that of the best performing Member States in terms of price stability. Additionally, nominal exchange rates of the EMU accession countries versus the euro should stay within normal fluctuation margins (i.e. 15% bound around the central parity) provided for by the Exchange Rate Mechanism of the European Monetary System for at least two years.[3]

[1] Cyprus, Czech Republic, Estonia, Hungary, Latvia, Lithuania, Malta, Poland, Slovakia and Slovenia signed the EU Accession Treaty on May 1, 2004. Bulgaria and Romania joined this group on January 1, 2007.

[2] Slovenia is the first country in this group that joined the European Monetary Union on January 1, 2007. Cyprus and Malta joined the EMU on January 1, 2008.

[3] The Maastricht Treaty also imposes the fiscal criterion, i.e. the sustainability of the government financial position which refers to a government budgetary position without an excessive deficit (Article

By setting constraints on the monetary variables, these criteria affect the way monetary policy should be conducted in the EMU accession countries. Monetary policy plays a crucial role in the stabilization process of an economy exposed to shocks. The obligation to fulfill the Maastricht convergence criteria by the EMU accession countries can restrict the stabilization role of the monetary policy. At the moment, many EMU accession countries do not satisfy some of the Maastricht convergence criteria. Bulgaria, Estonia, Hungary, Latvia, Lithuania, Romania and Slovakia fail to fulfill the CPI inflation rate criterion (see Figure A.3 in Appendix A). Hungary and Romania also violate the nominal interest rate criterion (see Figure A.5 in Appendix A). Finally the nominal exchange rate fluctuations of Polish Zloty, Slovakian Koruna and Romanian Lei versus the euro exceed the band set by the nominal exchange rate criterion (see Figure A.6 in Appendix A).[4]

Keeping this in mind, a natural question arises. How do the Maastricht convergence criteria modify the optimal monetary policy in an economy that faces domestic and external shocks?

To answer this question, we develop a DSGE model of a small open economy with nominal rigidities exposed to both domestic and external shocks. Our model assumes fully credible macroeconomic policies. We take as a point of reference the small open economy model developed by De Paoli (2004). The production structure of the economy is composed of two sectors: a nontraded good sector and a traded good sector. The same production structure is present, among others, in the two country model of Liu and Pappa (2005) and the currency area model of Altissimo et al (2005). There are several reasons to impose such a structure in our model. According to the literature, the existence of the nontraded sector helps us to explain international business cycle fluctuations and especially real exchange rate movements (e.g. Benigno and Thoenisen (2003), Corsetti et al. (2003), Stockman and Tesar (1994)). Moreover, the empirical studies regarding the OECD countries find that a major part of the aggregate fluctuations rather have their source in sector-specific than country-wide shocks (e.g. Canzoneri et al. (1999), Marimon and Zilibotti (1998)). Finally we want to match our model with the empirical literature on the EMU accession countries that emphasizes the role of sector productivity shocks in shaping inflation and real exchange rate patterns in these countries (e.g. Mihaljek and Klau (2004)).

In this framework we characterize the optimal monetary policy from a timeless perspective (Woodford (2003)). We derive the micro founded loss function using the second-order approximation methodology developed by Rotemberg and Wood-

104c(6) of the Maastricht Treaty). However, in this paper, we focus on the monetary aspects of the Maastricht convergence criteria.

[4]For the purpose of this exercise we take the bilateral exchange rate from the day of the EU accession to be the central parity.

ford (1997) and Benigno and Woodford (2005). We find that the optimal monetary policy (unconstrained policy) should not only target inflation rates in the domestic sectors and aggregate output fluctuations, but also domestic and international terms of trade. Since the Maastricht convergence criteria are not easily implementable in our model, we reformulate them using the methodology developed by Rotemberg and Woodford (1997, 1999) for the analysis of the zero bound problem of the nominal interest rate. This method enables us to verify whether a given criterion is satisfied by only computing first and second moments of a variable for which the criterion is set. We focus on the criteria imposed on the CPI inflation rate, the nominal interest rate and the nominal exchange rate as we do not explicitly model the fiscal policy. We present how the loss function changes when the monetary policy is constrained by the Maastricht convergence criteria. Finally, we derive the optimal monetary policy that satisfies all Maastricht convergence criteria (constrained policy).

Under the chosen parameterization (which aims at reflecting the economy of the Czech Republic), the unconstrained optimal monetary policy violates the CPI inflation rate and the nominal interest rate criteria. The optimal policy which satisfies these two criteria also guarantees the satisfaction of the nominal exchange criterion. Both the stabilization component and the deterministic component of the constrained policy are different from the unconstrained optimal policy. The constrained policy leads to a smaller variability of the CPI inflation, the nominal interest rate and the nominal exchange rate than under optimal monetary policy. Moreover, this policy is characterized by a deflationary bias which results in targeting a CPI inflation rate and a nominal interest rate that are 0.7% lower (in annual terms) than the CPI inflation rate and the nominal interest rate in the reference countries. As a result, the constrained policy induces additional welfare costs which amount to 30% of the initial deadweight loss associated with the optimal monetary policy. These losses need to be offset against the potential benefits from complying with the criteria which are not analyzed in this paper.

The Maastricht convergence criteria can serve as a commitment technology that improves the credibility of macroeconomic policies in the accession countries (more on this in Ravenna (2005)). Intuitively, the CPI inflation criterion serves to prevent inadequate policies that could lead to higher production costs thus lowering growth. The nominal interest rate restricts the long run inflation expectations to guarantee sustainability of inflation convergence. Finally, the exchange rate criterion aims at preventing possible tensions in the foreign exchange rate market and precludes excessive devaluation before adoption of the euro (more on this in the studies of the National Bank of Hungary (2002), the National Bank of Poland (2004) and the ECB Convergence Report (2006)).

The literature has so far concentrated on two aspects of monetary policy in the EMU accession countries: the appropriate monetary regime in the light of the future accession to the EMU and the ability of alternative monetary regimes to comply with the Maastricht convergence criteria. The first stream of literature represented by, among others, Buiter and Grafe (2003), Coricelli (2002), calls for adopting the peg regime to the euro in these countries, as it enhances credibility of the monetary policy and strengthens the links with the EU and the EMU. Using a DSGE model with nominal rigidities and imperfect credibility, Ravenna (2005) finds that the gain from a credible adoption of the fixed regime towards the euro can outweigh the loss of resignation from the independent monetary policy. Nevertheless, Buiter and Grafe (2003) also claim that an adoption of the fixed regime can seriously endanger the fulfillment of the CPI inflation criterion and therefore call for a change in this criterion. Their reasoning is based on the empirical studies regarding sources of the CPI inflation and real exchange rate developments in the EMU accession countries. A majority of the studies[5] concentrate on the Balassa–Samuelson effect (Balassa (1964)), which predicts that countries experiencing a higher productivity growth in the traded sector are also characterized by a higher CPI inflation rate and real exchange rate appreciation. Others (e.g. Mihaljek and Klau (2004)) also highlight the role of productivity shocks in the nontraded sector in affecting the CPI inflation rate and the real exchange rate appreciation in the EMU accession countries.[6]

The second stream of the literature builds an analysis in the framework of open economy DSGE models. Devereux (2003) and Natalucci and Ravenna (2003) find that the monetary regime characterized by flexible inflation targeting with some weight on exchange rate stability succeeds in fulfilling the Maastricht criteria. In Chapter 2 of this thesis we show that probability of a given monetary regime to comply with the Maastricht criteria increases with the degree of openness of an economy and the substitutability of home and foreign goods. The degree of exchange rate pass through determines which of the regimes can comply with the criteria. Low degree of pass through discriminates between regimes: when economy gets more open, variances of the Maastricht variables under the peg and managed float regime diminish while the contrary is true for the CPI targeting regime. If degree of exchange rate pass through is high, then higher openness enables all the regimes to meet the Maastricht criteria.

[5] We can list the following empirical studies that analyze CPI inflation and real exchange rate developments in the EMU accession countries: Cipriani (2001), de Broeck and Slok (2001), Egert et al. (2002), Fisher (2002), Halpern and Wyplosz (2001), Coricelli and Jazbec (2001), Arratibel et al. (2002), Mihaljek and Klau (2004).

[6] This study goes in line with a recent paper by Altissimo et al (2004) on the sources of inflation differentials in the euro area. The authors find that the nontraded sector (proxied as the service sector) contributes the most to price dispersion among member countries.

Two other studies are also worth noting: Laxton and Pesenti (2003) and Ferreira (2006). The authors of the first paper study how different interest rate rules perform in stabilizing variability of inflation and output in a small open economy. The second paper focuses on the calculation of the welfare loss that the EMU accession countries will face when they join the EMU. However, contrary to our study, it does not provide the micro founded welfare criterion.

In contrast to previous studies, our analysis is characterized by the normative approach. We construct the optimal monetary policy for a small open economy and contrast it with the optimal policy that is restricted to satisfy the Maastricht convergence criteria. Therefore, our framework enables us to set guidelines on the way in which monetary policy should be conducted in the EMU accession countries.

The rest of the paper is organized as follows. The next section introduces the model and derives the small open economy dynamics. Section 3.3 describes derivation of the optimal monetary policy. Section 3.4 presents the way we reformulate the Maastricht convergence criteria in order to implement them in our framework. Section 3.5 is dedicated to the derivation of the optimal policy constrained by the Maastricht convergence criteria. Section 3.6 compares the optimal monetary policy with the optimal monetary policy constrained by the Maastricht convergence criteria under the chosen parameterization of the model. Section 3.7 concludes.

3.2 The model

Our modelling framework is based on a one-sector small open economy model of De Paoli (2004) where all goods, i.e. home and foreign ones, are tradable. We extend this model by incorporating two domestic sectors, i.e. a nontraded and a traded sector. Our model is also closely related to the studies of Devereux (2003) and Natalucci and Ravenna (2003). However, we relax an assumption present in their studies regarding perfect competition and homogeneity of goods in the traded sector, which enables us to discuss a role of terms of trade in the stabilization process of a small open economy. In that way our modelling framework is similar to a two-country model with two production sectors of Liu and Pappa (2005).

Following De Paoli (2004), we model a small open economy as the limiting case of a two-country problem, i.e. where the size of the small open economy is set to zero. In the general framework, the model represents two economies of unequal size: a small open home economy and a foreign large economy (which is proxied as the euro area). We consider two highly integrated economies where asset markets are complete. In each of the economies, there are two goods sectors: nontraded goods and traded goods. Moreover, we assume that labour is mobile between sectors in each country and immobile between countries. We assume the existence of home bias in consumption which, in turn, depends on the relative size of the economy and its degree of openness. This assumption enables us to consider a limiting case of the zero size of the home economy and concentrate on the small open economy.

Purchasing power parity (PPP) is violated for two reasons: existence of the non-traded sector and home bias in consumption. Furthermore, in order to study the role of monetary policy in this framework, we introduce monopolistic competition and price rigidities with staggered Calvo contracts in all goods sectors. However, we abstract from any monetary frictions by assuming cashless limiting economies.[7] The stochastic environment of the small open economy is characterized by asymmetric productivity shocks originating in both domestic sectors, preference shocks and foreign consumption shocks.

3.2.1 Households

The world economy consists of a continuum of agents of unit mass: $[0, n)$ belonging to a small country (home) and $[n, 1]$ belonging to the rest of the world, i.e. the euro area (foreign). There are two types of differentiated goods produced in each country: traded and nontraded goods. Home traded goods are indexed on the interval $[0, n)$ and foreign traded goods on the interval $[n, 1]$, respectively. The same applies to nontraded goods. In order to simplify the exposition of the model, we explain in detail only the structure and dynamics of the domestic economy. Thus, from now on, we assume the size of the domestic economy to be zero, i.e. $n \to 0$.

Households are assumed to live infinitely and behave according to the permanent income hypothesis. They can choose between three types of goods: nontraded, domestic traded and foreign traded goods. C_t^i represents consumption at period t of a consumer i and L_t^i constitutes his labour supply. Each agent i maximizes the following utility function:[8]

$$\max E_{t_0} \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[U\left(C_t^i, B_t\right) - V\left(L_t^i\right) \right] \right\}, \tag{3.1}$$

where E_{t_0} denotes the expectation conditional on the information set at date t_0, β is the intertemporal discount factor and $0 < \beta < 1$, $U(\cdot)$ stands for flows of utility from consumption and $V(\cdot)$ represents flows of disutility from supplying labour.[9] C is a

[7] See Woodford (2003).

[8] In general, we assume U to be twice differentiable, increasing and concave in C_t and V to be twice differentiable, increasing and convex in L_t.

[9] We assume specific functional forms of consumption utility $U\left(C_t^i\right)$, and disutility from labour $V\left(L_t^i\right)$: $U\left(C_t^i\right) \equiv \frac{\left(C_t^i\right)^{1-\rho} B_t^{\rho}}{1-\rho}$, $V\left(L_t^i\right) \equiv \varphi_l \frac{\left(L_t^i\right)^{1+\eta}}{1+\eta}$ with ρ $(\rho > 0)$, the inverse of the intertemporal

composite consumption index. We define consumers' preferences over the composite consumption index C_t of traded goods ($C_{T,t}$) (domestically produced and foreign ones) and nontraded goods ($C_{N,t}$):

$$C_t \equiv \left[\mu^{\frac{1}{\phi}} C_{N,t}^{\frac{\phi-1}{\phi}} + (1-\mu)^{\frac{1}{\phi}} C_{T,t}^{\frac{\phi-1}{\phi}}\right]^{\frac{\phi}{\phi-1}}, \tag{3.2}$$

where $\phi > 0$ is the elasticity of substitution between traded and nontraded goods and $\mu \in [0,1]$ is the share of the nontraded goods in overall consumption. Traded good consumption is a composite of the domestically produced traded goods (C_H) and foreign produced traded goods (C_F):

$$C_{T,t} \equiv \left[(1-\lambda)^{\frac{1}{\theta}} C_{H,t}^{\frac{\theta-1}{\theta}} + \lambda^{\frac{1}{\theta}} C_{F,t}^{\frac{\theta-1}{\theta}}\right]^{\frac{\theta}{\theta-1}}, \tag{3.3}$$

where $\theta > 0$ is the elasticity of substitution between home traded and foreign traded goods, and λ is the degree of openness of the small open economy ($\lambda \in [0,1]$).[10] Finally, C_j (where $j = N,\ H,\ F$) are consumption sub-indices of the continuum of differentiated goods:

$$C_{j,t} \equiv \left[\left(\frac{1}{n}\right)^{\frac{1}{\sigma}} \int_0^n c_t\,(j)^{\frac{\sigma-1}{\sigma}}\, dj\right]^{\frac{\sigma}{\sigma-1}}, \tag{3.4}$$

where $\sigma > 1$ represents elasticity of substitution between differentiated goods in each of the sectors. Based on the above presented preferences, we derive consumption-based price indices expressed in the units of currency of the domestic country:

$$P_t \equiv \left[\mu P_{N,t}^{1-\phi} + (1-\mu) P_{T,t}^{1-\phi}\right]^{\frac{1}{1-\phi}}, \tag{3.5}$$

$$P_{T,t} \equiv \left[\nu P_{H,t}^{1-\theta} + (1-\nu) P_{F,t}^{1-\theta}\right]^{\frac{1}{1-\theta}} \tag{3.6}$$

with

$$P_{j,t} \equiv \left[\left(\frac{1}{n}\right) \int_0^n p_t\,(j)^{1-\sigma}\, dj\right]^{\frac{1}{1-\sigma}}. \tag{3.7}$$

elasticity of substitution in consumption and η ($\eta \geq 0$), the inverse of labour supply elasticity and B_t , preference shock.

[10] Following de Paoli (2004) and Sutherland (2002), we assume home bias (ν) of the domestic households to be a function of the relative size of the home economy with respect to the foreign one (n) and its degree of openness (λ) such that $(1-\nu) = (1-n)\lambda$ where $\lambda \in [0,1]$. Importantly, the higher is the degree of openness, the smaller is the degree of home bias. Since $n \to 0$, we obtain that $\nu = 1 - \lambda$.

Although we assume the law of one price in the traded sector (i.e. $p(h) = Sp^*(h)$ and $p(f) = Sp^*(f)$ where S is the nominal exchange rate), both the existence of the nontraded goods and the assumed home bias cause deviations from purchasing power parity, i.e. $P \neq SP^*$. The real exchange rate can be defined in the following manner: $RS \equiv \frac{SP^*}{P}$. Moreover, we define the international terms of trade as $T \equiv \frac{P_F}{P_H}$ and the ratio of nontraded to traded goods' prices (domestic terms of trade) as $T^d \equiv \frac{P_N}{P_T}$.

From consumer preferences, we can derive total demand for the generic goods – n (home nontraded ones), h (home traded ones), f (foreign traded ones):

$$y^d(n) = \left[\frac{p(n)}{P_N}\right]^{-\sigma} \left[\frac{P_N}{P}\right]^{-\phi} \mu C, \tag{3.8}$$

$$y^d(h) = \left[\frac{p(h)}{P_H}\right]^{-\sigma} \left[\frac{P_H}{P_T}\right]^{-\theta} (1-\lambda)C_T + \left[\frac{p^*(h)}{P_H^*}\right]^{-\sigma} \left(\frac{P_H^*}{P_T^*}\right)^{-\theta^*} \lambda C_T^*, \tag{3.9}$$

$$y^d(f) = \left[\frac{p^*(f)}{P_F^*}\right]^{-\sigma} \left[\frac{P_F^*}{P_T^*}\right]^{-\theta} C_T^* \tag{3.10}$$

where variables with an asterisk represent the foreign equivalents of the domestic variables. Importantly, since the domestic economy is a small open economy, demand for foreign traded goods does not depend on domestic demand. However, at the same time, demand for domestic traded goods depends on foreign demand.

Households get disutility from supplying labour to all firms present in each country. Each individual supplies labour to both sectors, i.e. the traded and the nontraded sector:

$$L_t^i = L_t^{i,H} + L_t^{i,N}. \tag{3.11}$$

We assume that consumers have access to a complete set of securities-contingent claims traded internationally. Each household faces the following budget constraint:

$$P_t C_t^i + E_t\{Q_{t,t+1}D_{t+1}\} \leq D_t + TR_t^i + W_{H,t}^i L_{H,t}^i + W_{N,t}^i L_{N,t}^i + \frac{\int_0^n \Pi_{N,t}^i di}{n} + \frac{\int_0^n \Pi_{H,t}^i di}{n}, \tag{3.12}$$

where at date t, D_{t+1} is nominal payoff of the portfolio held at the end of period (t), $Q_{t,t+1}$ is the stochastic discount factor for one-period ahead nominal payoffs relevant to the domestic household, $\Pi_{H,t}$ and $\Pi_{N,t}$ are nominal profits from the domestic firms and TR_t^i are nominal lump-sum transfers from the domestic government to household i. Moreover, consumers face no Ponzi game restriction.

The short-term interest rate (R_t) is defined as the price of the portfolio which delivers one unit of currency in each contingency that occurs in the next period:[11]

$$R_t^{-1} = E_t\{Q_{t,t+1}\}. \tag{3.13}$$

The maximization problem of any household consists of maximizing the discounted stream of utility (3.1) subject to the budget constraint (3.12) in order to determine the optimal path of the consumption index, the labour index and contingent claims at all times. The solution to the household decision problem gives a set of first-order conditions.[12] Optimization of the portfolio holdings leads to the following Euler equations for the domestic economy:

$$U_C(C_t, B_t) = \beta E_t \left\{ U_C(C_{t+1}, B_{t+1}) Q_{t,t+1}^{-1} \frac{P_t}{P_{t+1}} \right\}. \tag{3.14}$$

There is a perfect sharing in this setting, meaning that marginal rates of consumption in nominal terms are equalized between countries in all states and at all times.[13] Subsequently, appropriately choosing the distribution of initial wealth, we obtain the risk sharing condition:

$$\frac{U_C(C_t, B_t)}{U_C(C_t^*, B_t^*)} = \upsilon \frac{P_t}{S_t P_t^*} = \upsilon RS_t^{-1}, \tag{3.15}$$

where $\upsilon > 0$ and depends on the initial wealth distribution. The risk sharing condition implies that the real exchange rate is equal to the marginal rate of substitution between domestic and foreign consumption.

The optimality condition for labour supply in the domestic economy is the following:

$$\frac{W_t^k}{P_t} = \frac{V_L(L_t)}{U_C(C_t, B_t)}, \tag{3.16}$$

where W^k is the nominal wage of the representative consumer in sector k ($k = H, N$).[14] So the real wage is equal to the marginal rate of substitution between labour and consumption.

[11] Following the literature, we assume one period to be one quarter.

[12] We here suppress subscript i as we assume that in equilibrium, all agents are identical. Therefore, we represent optimality conditions for a representative agent.

[13] We have to point out here that although the assumption of complete markets conveniently simplifies the model, it neglects a possibility of wealth effects in response to different shocks (Benigno (2001)).

[14] Notice that wages are equalized between sectors inside each of the economies, due to perfect labour mobility and perfect competition in the labour market.

3.2.2 Firms

All firms are owned by consumers. Both traded and nontraded sectors are monopolistically competitive. The production function is linear in labour which is the only input. Consequently, its functional form for firm i in sector k ($k = N,\ H$) is the following:

$$Y_{k,t}(i) = A_t^k L_t^k(i). \tag{3.17}$$

Price is set according to the Calvo (1983) pricing scheme. In each period, a fraction of firms $(1-\alpha_k)$ decides its price, thus maximizing the future expected profits. The maximization problem of any firm in sector k at time t_0 is given by:

$$\max_{P_{k,t_0}(i)} E_{t_0} \sum_{t=t_o}^{\infty} (\alpha_k)^s Q_{t_0,t} \left[(1-\tau_k)P_{k,t_0}(i) - MC_t^k(i)\right] Y_{k,t_0:t}^d(i)$$

$$\textit{subject to } Y_{k,t_0:t}^d(i) = \left(\frac{P_{k,t_0}(i)}{P_{k,t}}\right)^{-\sigma} Y_{k,t}, \tag{3.18}$$

where $Y_{k,t_0:t}^d(i)$ is demand for the individual good in sector k produced by producer i at time t conditional on keeping the price $P_{k,t_0}(i)$ fixed at the level chosen at time t_0, $MC_t^k = \frac{W_t^k(i)}{A_t^k}$ is the nominal marginal cost in sector k at time t, and τ_k are revenue taxes in sector k.

Given this setup, the price index in sector k evolves according to the following law of motion:

$$(P_{k,t})^{1-\sigma} = \alpha_k (P_{k,t-1})^{1-\sigma} + (1-\alpha_k)(P_{k,t_0}(i))^{1-\sigma}. \tag{3.19}$$

3.2.3 Fiscal and monetary policies

The government in the domestic economy is occupied with collecting revenue taxes from firms that are later redistributed to households in the form of lump-sum transfers in such a way that each period, there is a balanced budget:

$$\int_0^n \left(\tau_N P_{N,t}(i) Y_{N,t}(i) + \tau_H P_{H,t}(i) Y_{H,t}(i)\right) di = \int_0^n TR_t^j dj. \tag{3.20}$$

A role for the monetary policy arises due to existing nominal and real rigidities in the economy: price stickiness (together with monopolistic competition), home bias and the nontraded good sector, which lead to deviations from PPP. The system is therefore closed by defining appropriate monetary rule for the domestic economy.

3.2.4 A loglinearized version of the model

This section presents a system of the equilibrium conditions for the small open economy in the loglinear form, which is derived through the first-order approximation around the deterministic steady state with zero inflation defined in Appendix C.1. Here, we characterize the dynamic features of this model where the variables with a hat stand for the log deviations from the steady state. Additionally, the variables with an asterisk represent the foreign equivalents of the domestic variables.

The supply-side of the economy is given by two Phillips curves, one for the nontraded and one for the domestic traded sector, respectively, which are derived from (3.18):

$$\widehat{\pi}_{N,t} = k_N(\rho\widehat{C}_t + \eta\widehat{L}_t - \widehat{A}_{N,t} - \rho\widehat{B}_t - \widehat{p}_{N,t}) + \beta\widehat{\pi}_{N,t+1}, \tag{3.21}$$

$$\widehat{\pi}_{H,t} = k_H(\rho\widehat{C}_t + \eta\widehat{L}_t - \widehat{A}_{H,t} - \rho\widehat{B}_t - \widehat{p}_{H,t}) + \beta\widehat{\pi}_{H,t+1} \tag{3.22}$$

where $\widehat{p}_{N,t} \equiv \ln(\frac{P_{N,t}}{P_t})$, $\widehat{p}_{H,t} \equiv \ln(\frac{P_{H,t}}{P_t})$, $\widehat{\pi}_{N,t} \equiv \ln(\frac{P_{N,t}}{P_{N,t-1}})$, $\widehat{\pi}_{H,t} \equiv \ln(\frac{P_{H,t}}{P_{H,t-1}})$, $k_N \equiv \frac{(1-\alpha_N)(1-\alpha_N\beta)}{\alpha_N}$, $k_H \equiv \frac{(1-\alpha_H)(1-\alpha_H\beta)}{\alpha_H}$ and aggregate labour supply ($\widehat{L}_t$) is defined through the labour market clearing condition ((3.11), (3.17)):

$$\widehat{L}_t = \widetilde{d}_{Y_N}(\widehat{Y}_{N,t} - \widehat{A}_{N,t}) + \widetilde{d}_{Y_H}(\widehat{Y}_{H,t} - \widehat{A}_{H,t}), \tag{3.23}$$

where $\widetilde{d}_{Y_N} \equiv \frac{\overline{Y}_N}{\overline{Y}_N+\overline{Y}_H}$, $\widetilde{d}_{Y_H} \equiv \frac{\overline{Y}_H}{\overline{Y}_N+\overline{Y}_H}$ are ratios evaluated in the steady state.

It is worth underlining that inflation dynamics in both domestic sectors do not only depend on the real marginal costs in a given sector, but also on the relative prices of goods. In particular, a higher relative price of goods in one sector in relation to other goods induces a substitution away effect and leads to deflationary pressures in this sector.

The demand side of the small open economy is represented by the market clearing conditions in both nontraded and domestic traded sectors ((3.8), (3.9)):

$$\widehat{Y}_{N,t} = \widehat{C}_t - \phi\widehat{p}_{N,t}, \tag{3.24}$$

$$\begin{aligned}\widehat{Y}_{H,t} = d_{CH}\widehat{C}_t - \theta\widehat{p}_{H,t} + b(\phi-\theta)d_{CH}\widehat{T}_t^d + (1-d_{CH})\theta\widehat{RS}_t +& \\ + (1-d_{CH})\widehat{C}_t^* + b^*(\phi-\theta)(1-d_{CH})\widehat{T}_t^{d*}&\end{aligned} \tag{3.25}$$

where $d_{CH} \equiv (1-\lambda)(1-\mu)\frac{\overline{C}}{\overline{Y_H}}\overline{p}_H^{-\theta}\overline{p}_T^{\theta-\phi}$, $b \equiv \mu(\overline{p_N})^{1-\phi}$, $b^* \equiv \mu^*(\overline{p_N^*})^{1-\phi}$ are ratios evaluated in the steady state. Additionally, we define aggregate output as the sum of sector outputs:

$$\widehat{Y}_t = d_{YN}(\widehat{p}_{N,t} + \widehat{Y}_{N,t}) + d_{YH}(\widehat{p}_{H,t} + \widehat{Y}_{H,t}), \tag{3.26}$$

where $d_{YN} \equiv \frac{\overline{p_N}\,\overline{Y_N}}{\overline{Y}}$ and $d_{YH} \equiv \frac{\overline{p_H}\,\overline{Y_H}}{\overline{Y}}$ are ratios evaluated in the steady state.

The complete asset market assumption (3.15) gives us the following risk sharing condition:

$$\widehat{C}_t = \widehat{B}_t + \frac{1}{\rho}\widehat{RS}_t + \widehat{C}^*_t - \widehat{B}^*_t. \tag{3.27}$$

From the definition of price indices ((3.5), (3.6)), we obtain the following relations between relative prices, terms of trade, domestic terms of trade and real exchange rate:

$$(a-1)\widehat{p}_{H,t} = b\widehat{T^d_t} + a\widehat{RS_t} - b^* a\widehat{T^{d*}_t}, \tag{3.28}$$

$$\widehat{p}_{N,t} = (1-b)\widehat{T^d_t}, \tag{3.29}$$

$$\widehat{p}_{H,t} = -b\widehat{T^d_t} - a\widehat{T}_t, \tag{3.30}$$

where $a \equiv \lambda\left(\frac{\overline{RSp^*_F}}{\overline{p_T}}\right)^{1-\theta}$ is the ratio evaluated in the steady state. We also derive the laws of motion for the international terms of trade and the domestic terms of trade from their definitions:

$$\widehat{T}_t = \widehat{\pi}_{F,t} - \widehat{\pi}_{H,t} + \widehat{T}_{t-1}, \tag{3.31}$$

$$\widehat{T}^d_t = \widehat{\pi}_{N,t} - \widehat{\pi}_{T,t} + \widehat{T}^d_{t-1}, \tag{3.32}$$

where $\widehat{\pi}_{T,t} = (1-a)\widehat{\pi}_{H,t} + a\widehat{\pi}_{F,t}$ and $\widehat{\pi}_{F,t} = \widehat{\pi}^*_{F,t} + (\widehat{S}_t - \widehat{S}_{t-1})$ with $\widehat{\pi}_{T,t} \equiv \ln(\frac{P_{T,t}}{P_{T,t-1}})$, $\widehat{\pi}_{F,t} \equiv \ln(\frac{P_{F,t}}{P_{F,t-1}})$, $\widehat{\pi}^*_{F,t} \equiv \ln(\frac{P^*_{F,t}}{P^*_{F,t-1}})$.

Finally, we present equations defining the Maastricht variables: the CPI inflation rate ($\widehat{\pi}_t$), the nominal interest rate ($\widehat{R}_t$) and the nominal exchange rate ($\widehat{S}_t$). First, the nominal interest rate can be derived from the loglinearized version of the Euler condition (3.14):

$$\widehat{R}_t = \rho(\widehat{C}_{t+1} - \widehat{B}_{t+1}) - \rho(\widehat{C}_t - \widehat{B}_t) + \widehat{\pi}_{t+1}, \tag{3.33}$$

where $\widehat{\pi}_t \equiv \ln(\frac{P_t}{P_{t-1}})$. CPI aggregate inflation is a weighted sum of the sector inflation rates:

$$\widehat{\pi}_t = b\widehat{\pi}_{N,t} + (1-a)(1-b)\widehat{\pi}_{H,t} + a(1-b)\widehat{\pi}^*_{F,t} + a(1-b)(\widehat{S}_t - \widehat{S}_{t-1}). \tag{3.34}$$

Notice that CPI aggregate inflation does not only depend on the domestic sector inflation rates, but also on the foreign traded inflation rate and changes in the nominal exchange rate. For example, a nominal exchange rate depreciation puts an upward pressure on the CPI inflation rate.

The nominal exchange rate can be derived from the definition of the real exchange rate:

$$\widehat{S}_t = \widehat{S}_{t-1} + \widehat{\pi}_t - \widehat{\pi}_t^* + \widehat{RS}_t - \widehat{RS}_{t-1}. \tag{3.35}$$

The law of motion of the nominal exchange rate depends on the real exchange rate fluctuations and differences in the aggregate inflation rates between the home and the foreign economy. Additionally, by combining the international risk sharing condition (3.27) and Euler conditions for the domestic and foreign economy (3.33), we obtain a relation between the nominal interest rate and the nominal exchange rate:

$$\widehat{S}_t = \widehat{R}_t^* - \widehat{R}_t + \widehat{S}_{t+1}. \tag{3.36}$$

This equation represents a version of the uncovered interest rate parity, which implies that changes in the nominal exchange rate result from differences between the domestic and foreign monetary policy. Let us point out that although very intuitive, this equation does not constitute an independent equilibrium condition.

The system is closed by specifying a monetary rule. In this paper, we derive the optimal monetary policy rule which maximizes welfare of the society subject to the structural equations of the economy. The optimal rule is specified as a rule where the monetary authority stabilizes the target variables in order to minimize the welfare loss of society and provide the most efficient allocation.[15] Apart from the optimal monetary derivation in this framework, we also consider the optimal monetary policy which is additionally constrained by the Maastricht convergence criteria.

Summing up, the dynamics of the small open economy are summarized by the following variables, $\widehat{\pi}_{N,t}$, $\widehat{\pi}_{H,t}$, $\widehat{C}_t$, $\widehat{L}_t$, $\widehat{Y}_{H,t}$, $\widehat{Y}_{N,t}$, $\widehat{p}_{N,t}$, $\widehat{p}_{H,t}$, $\widehat{Y}_t$, $\widehat{RS}_t$, $\widehat{T}_t^d$, $\widehat{T}_t$, $\widehat{S}_t$, $\widehat{\pi}_t$, $\widehat{R}_t$ which are determined by equations (3.21)–(3.35), given the evolution of the stochastic shocks $\widehat{A}_{N,t}$, $\widehat{A}_{H,t}$, $\widehat{B}_t$ and the foreign variables $\widehat{C}_t^*$, $\widehat{T}_t^{d^*}$, $\widehat{\pi}_t^*$, $\widehat{\pi}_{F,t}^*$.[16]

[15] Giannoni and Woodford (2003) call these type of rules flexible inflation targeting rules.

[16] For simplicity, we choose to consider only one type of external shocks, foreign consumption shocks ($\widehat{C}_t^*$). As a result, $\widehat{T}_t^{d^*}$, $\widehat{\pi}_t^*$, $\widehat{\pi}_{F,t}^*$ are assumed to be zero. Moreover, all shocks follow an AR(1) process with normally distributed innovations.

3.3 The optimal monetary policy

This section characterizes the optimal monetary policy, i.e. the policy maximizing welfare of society subject to the structural equations of an economy. The micro foundations of our model give us a natural welfare measure, i.e. a discounted sum of expected utilities for the agents in the economy (see equation (3.1)).

We use a linear quadratic approach (Rotemberg and Woodford (1997, 1999)) and define the optimal monetary policy problem as a minimization problem of the quadratic loss function subject to the loglinearized structural equations (presented in the previous section). First, we present the welfare measure derived through a second-order Taylor approximation of equation (3.1):

$$W_{t_0} = U_C \overline{C} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [z_v' \widehat{v}_t - \frac{1}{2} \widehat{v}_t' Z_v \widehat{v}_t - \widehat{v}_t' Z_\xi \widehat{\xi}_t] + tip + O(3), \tag{3.37}$$

where $\widehat{v}_t' = \begin{bmatrix} \widehat{C}_t & \widehat{Y}_{N,t} & \widehat{Y}_{H,t} & \widehat{\pi}_{N,t} & \widehat{\pi}_{H,t} \end{bmatrix}$; $\widehat{\xi}_t' = \begin{bmatrix} \widehat{A}_{N,t} & \widehat{A}_{H,t} & \widehat{B}_t & \widehat{C}_t^* \end{bmatrix}$; $z_v' = \begin{bmatrix} 1 & -s_{CY_N} & -s_{CY_H} & 0 & 0 \end{bmatrix}$ with $s_{CY_N} \equiv \frac{\overline{\omega Y_N}}{\overline{C}}$ - steady state share of nontraded labour income in domestic consumption, $s_{CY_H} \equiv \frac{\overline{\omega Y_H}}{\overline{C}}$ - steady state share of home traded labour income in domestic consumption and matrices Z_v, Z_ξ are defined in Appendix C.4; *tip* stands for *terms independent of policy* and $O(3)$ includes terms that are of a higher order than the second in the deviations of variables from their steady state values.

Notice that the welfare measure (3.37) contains the linear terms in aggregate consumption and sector outputs. These linear terms result from the distortions in the economy. First, monopolistic competition in both domestic sectors leads to inefficient levels of sector outputs and subsequently, an inefficient level of aggregate output. Second, since the domestic economy is open, domestic consumption and aggregate output are not equalized. Importantly, their composition depends on the domestic and international terms of trade. Third, there exists a(n) (international) terms of trade externality (see Corsetti and Pesenti (2001)) according to which monetary policy has an incentive to generate a welfare improving real exchange rate appreciation which leads to a lower disutility from labour without a corresponding decline in the utility of consumption.

The presence of linear terms in the welfare measure (3.37) means that we cannot determine the optimal monetary policy, even up to first order, using the welfare measure subject to the structural equations (3.21)–(3.35) that are only accurate to first order. Following the method proposed by Benigno and Woodford (2005) and Benigno and Benigno (2005), we substitute the linear terms in the approximated welfare function (3.37) by second moments of aggregate output, domestic and international terms of trade using a second-order approximation to the structural equations of the economy.[17] As a result, we obtain the fully quadratic loss function which can be represented as a function of aggregate output ($\widehat{Y}_t$), domestic and international terms of trade ($\widehat{T}_t^d$, $\widehat{T}_t$) and domestic sector inflation rates ($\widehat{\pi}_{H,t}$, $\widehat{\pi}_{N,t}$). Its general expression is given below:

$$L_{t_0} = U_C \overline{C} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [\frac{1}{2}\Phi_Y (\widehat{Y}_t - \widehat{Y}_t^T)^2 + \frac{1}{2}\Phi_{T^d}(\widehat{T}_t^d - \widehat{T}_t^{d^T})^2 +$$
$$\frac{1}{2}\Phi_T(\widehat{T}_t - \widehat{T}_t^T)^2 + \Phi_{TT^d}\widehat{T}_t^d\widehat{T}_t + \Phi_{YT^d}\widehat{Y}_t\widehat{T}_t^d + \Phi_{YT}\widehat{Y}_t\widehat{T}_t +$$
$$+\frac{1}{2}\Phi_{\pi_H}\widehat{\pi}_{H,t}^2 + \frac{1}{2}\Phi_{\pi_N}\widehat{\pi}_{N,t}^2] + tip + O(3), \quad (3.38)$$

where $\widehat{Y}_t^T$, $\widehat{T}_t^{d^T}$, $\widehat{T}_t^T$ are target variables which are functions of the stochastic shocks and, in general, are different from the flexible price equilibrium processes of aggregate output, domestic terms of trade and international terms of trade.[18] The coefficients Φ_Y, Φ_{T^d}, Φ_T, Φ_{TT^d}, Φ_{YT^d}, Φ_{YT}, Φ_{π_H}, Φ_{π_N} are functions of the structural parameters of the model. The term *tip* stands for *terms independent of policy.*

Our loss function can be seen as a generalization of the previous studies encompassing both the closed (Aoki (2001), Benigno (2004), Rotemberg and Woodford (1997)) and open economy frameworks (Gali and Monacelli (2005), De Paoli (2004)).[19] Notice that if we set the size of the nontraded sector to zero and therefore obtain a one-sector small open economy, the loss function becomes identical to the loss function derived by De Paoli (2004).[20] In this case, the loss function is a function of the variances of aggregate output, terms of trade[21] and home traded inflation. On the other hand, if we set the degree of openness to zero, we obtain the case of a two-sector closed economy which was studied by Aoki (2001) and Benigno (2004). Here, the loss function is a

[17]Details of the derivation can be found in Appendix C.4.

[18]As previously shown in papers by Gali and Monacelli (2005) and De Paoli (2004), in the small open economy framework the target variables will be identical to the flexible price allocations only in some special cases, i.e. an efficient steady state, no markup shocks, no expenditure switching effect (i.e. $\rho\theta = 1$) and no trade imbalances.

[19]Rotemberg and Woodford (1998) present a one-sector closed economy model. Aoki (2001) presents a two-sector closed economy model with sticky prices only in one of the sectors. Benigno (2004) analyses the case of a monetary union comprised of two countries, which can be interpreted as a two-sector closed economy with totally segmented labour markets (interpretation as in Woodford (2003)). Gali and Monacelli (2005) and De Paoli (2004) study the case of a one-sector small open economy.

[20]In our representation, there is a covariance term between terms of trade and aggregate output which can be represented as the weighted sum of the variances of aggregate output and terms of trade.

[21]In the analysis of De Paoli (2004), it is actually the variance of the real exchange rate. However, it must be kept in mind that in a one-sector small open economy model, terms of trade and real exchange rate are proportional.

function of the variances of aggregate output, domestic terms of trade, the covariance term between the two and variances of the sector inflation rates. Additionally, our loss function is closely related to the loss function derived for a national policymaker in a two-country model with two sectors of Liu and Pappa (2005). Interestingly, since the monopolistic competition distortion, trade imbalances and also expenditure switching effect are not present in their model, the loss function of a national policymaker depends only on the sector inflation rates and the sector output fluctuations around the flexible price targets.

We characterize the optimal plan under commitment where the policy maker chooses the set of variables $\{\widehat{\pi}_{N,t}, \widehat{\pi}_{H,t}, \widehat{C}_t, \widehat{L}_t, \widehat{Y}_{H,t}, \widehat{Y}_{N,t}, \widehat{p}_{N,t}, \widehat{p}_{H,t}, \widehat{Y}_t, \widehat{RS}_t, \widehat{T}^d_t, \widehat{T}_t, \widehat{S}_t, \widehat{\pi}_t, \widehat{R}_t\}_{t=t_0}^{\infty}$ in order to minimize the loss function (3.38) subject to constraints (3.21)–(3.35), given the initial conditions on this set of variables. The initial conditions (that refer only to period t_0) guarantee the timeless perspective of the problem and make the first-order conditions of the problem time invariant (see Woodford (2003)).

To simplify the exposition of the optimal plan, we reduce the number of variables to the set of five domestic variables which determine the loss function (3.38), i.e. $\widehat{Y}_t$, $\widehat{T}^d_t$, $\widehat{T}_t$, $\widehat{\pi}_{N,t}$, $\widehat{\pi}_{H,t}$. Therefore, we represent the structural equations of the two-sector small open economy (3.21)–(3.35) in terms of these variables. The coefficients are defined in Appendix C.4.2.

The supply side of the economy is represented by two Phillips curves which are derived from equations (3.21) and (3.22) through a substitution of aggregate consumption, aggregate labour and relative prices:

$$\begin{aligned}\widehat{\pi}_{N,t} = k_N(m_{N,Y}\widehat{Y}_t + m_{N,T^d}\widehat{T}^d_t + m_{N,T}\widehat{T}_t + m_{N,A_N}\widehat{A}_{N,t} + \\ + m_{N,A_H}\widehat{A}_{H,t} + m_{N,B}\widehat{B}_t) + \beta\widehat{\pi}_{N,t+1}, \quad (3.39)\end{aligned}$$

$$\begin{aligned}\widehat{\pi}_{H,t} = k_H(m_{H,Y}\widehat{Y}_t + m_{H,T^d}\widehat{T}^d_t + m_{H,T}\widehat{T}_t + m_{H,A_N}\widehat{A}_{N,t} + \\ + m_{H,A_H}\widehat{A}_{H,t} + m_{H,B}\widehat{B}_t) + \beta\widehat{\pi}_{H,t+1}. \quad (3.40)\end{aligned}$$

The equation describing the demand side of the economy is derived from the market clearing conditions ((3.24), (3.25)) and the risk sharing condition (3.27):

$$\widehat{C}^*_t = \widehat{Y}_t + n_{T^d}\widehat{T}^d_t + n_T\widehat{T}_t + n_B\widehat{B}_t, \quad (3.41)$$

where aggregate consumption, relative prices and real exchange rate were substituted out.

The last structural equation represents the law of motion of the domestic and international terms of trade:

$$\widehat{T}_t^d - \widehat{T}_{t-1}^d = \widehat{\pi}_{N,t} - \widehat{\pi}_{H,t} - a(\widehat{T}_t - \widehat{T}_{t-1}). \tag{3.42}$$

Finally, the policy maker following the optimal plan under commitment chooses $\{\widehat{Y}_t, \widehat{T}_t^d, \widehat{T}_t, \widehat{\pi}_{H,t}, \widehat{\pi}_{N,t}\}_{t=t_0}^{\infty}$ in order to minimize the loss function (3.38) subject to the constraints (3.39)–(3.42), given the initial conditions on nonpredetermined variables: $\widehat{Y}_{t_0}, \widehat{T}_{t_0}^d, \widehat{T}_{t_0}, \widehat{\pi}_{H,t_0}, \widehat{\pi}_{N,t_0}$. In accordance with the definition of the optimal plan from a timeless perspective (see Woodford (2003), p.538) the first-order conditions of the problem for all $t \geq t_0$ are the following (where $\gamma_{i,t}$ with $i = 1, 2, 3, 4$ are accordingly the Lagrange multipliers with respect to (3.39)– (3.42)):

- with respect to $\widehat{\pi}_{N,t}$:

$$\Phi_{\pi_N}\widehat{\pi}_{N,t} + \gamma_{1,t} - \gamma_{1,t-1} - \gamma_{4,t} = 0, \tag{3.43}$$

- with respect to $\widehat{\pi}_{H,t}$:

$$\Phi_{\pi_H}\widehat{\pi}_{H,t} + \gamma_{2,t} - \gamma_{2,t-1} + \gamma_{4,t} = 0, \tag{3.44}$$

- with respect to $\widehat{Y}_t$:

$$\Phi_Y(\widehat{Y}_t - \widehat{Y}_t^T) + \Phi_{YT^d}\widehat{T}_t^d + \Phi_{YT}\widehat{T}_t - k_N m_{N,Y}\gamma_{1,t} - k_H m_{H,Y}\gamma_{2,t} - \gamma_{3,t} = 0, \tag{3.45}$$

- with respect to $\widehat{T}_t^d$:

$$\begin{aligned}\Phi_{T^d}(\widehat{T}_t^d - \widehat{T}_t^{d^T}) + \Phi_{TT^d}\widehat{T}_t + \Phi_{YT^d}\widehat{Y}_t - k_N m_{N,T^d}\gamma_{1,t} - k_H m_{H,T^d}\gamma_{2,t} - \\ - n_{T^d}\gamma_{3,t} + \gamma_{4,t} - \beta\gamma_{4,t+1} = 0,\end{aligned} \tag{3.46}$$

- with respect to $\widehat{T}_t$:

$$\begin{aligned}\Phi_T(\widehat{T}_t - \widehat{T}_t^T) + \Phi_{TT^d}\widehat{T}_t^d + \Phi_{YT}\widehat{Y}_t - k_N m_{N,T}\gamma_{1,t} - k_H m_{H,T}\gamma_{2,t} - \\ - n_T\gamma_{3,t} + a\gamma_{4,t} - \beta a\gamma_{4,t+1} = 0\end{aligned} \tag{3.47}$$

Equations (3.43)–(3.47) and constraints (3.39)–(3.42) fully characterize the behavior of the economy under the optimal monetary policy.

3.4 The Maastricht convergence criteria – a reinterpretation

Including the Maastricht criteria in their original form as additional constraints of the optimal monetary policy requires computationally demanding techniques. In particular, it results in solving the minimization problem of the loss function (3.38) subject to additional nonlinear constraints. On the other hand, the linear quadratic approach has two important advantages that make us decide to reformulate the criteria. First, it provides us with the analytical and intuitive expression for the loss function which can also serve as a welfare measure to rank alternative suboptimal policies. Second, the linear quadratic approach makes it easy to check second-order conditions (which would otherwise be quite difficult) for local optimality of the derived policy.

Therefore, the purpose of this section is to describe the way in which we reformulate the Maastricht criteria in order to implement them as additional constraints faced by the monetary policy in our linear quadratic framework.

First, we summarize the Maastricht criteria (described in Appendix A) by the following inequalities:

- CPI aggregate inflation criterion

$$\pi_t^A - \pi_t^{A,*} \leq B_\pi, \tag{3.48}$$

  where $B_\pi = 1.5\%$, π_t^A is annual CPI aggregate inflation in the domestic economy, $\pi_t^{A,*}$ is the average of the annual CPI aggregate inflations in the three lowest inflation countries of the European Union.

- nominal interest rate criterion

$$R_t^L - R_t^{L,A*} \leq C_R \tag{3.49}$$

  where $C_R = 2\%$, R_t^L is the annul interest rate for ten-year government bond in the domestic economy, $R_t^{L,A*}$ is the average of the annual interest rates for ten-year government bonds in the three countries of the European Union with the lowest inflation rates.

- nominal exchange rate criterion

$$(1 - D_S)S \leq S_t \leq (1 + D_S)S, \tag{3.50}$$

  where $D_S = 15\%$ and S is the central parity between euro and the domestic currency and S_t is the nominal exchange rate.

In order to adjust the criteria to the structure of the model, we assume that the variables $\pi_t^{A,*}$ and $R_t^{L,A*}$, respectively, represent foreign aggregate inflation and the foreign nominal interest rate, i.e. $\widehat{\pi}_t^*$, $\widehat{R}_t^*$ (which are proxied to be the euro area variables). Here, we implicitly assume that the aggregate inflation rate and the nominal interest rate of the euro area do not differ to any great extent from the average of the three lowest inflation countries of the European Union.[22]

Second, we impose some simplifying assumptions regarding the criteria to adjust them to the quarterly nature of the model. The CPI inflation rate criterion is stated in annual terms. We decide to reformulate this criterion into the criterion on the quarterly CPI inflation rate with an appropriately changed upper bound, i.e. $B_\pi \equiv ((1.015)^{0.25} - 1)$. Notice that the criterion on the quarterly CPI inflation rate is stricter than the criterion set on the annual CPI inflation rate.[23] As far as the nominal interest rate criterion is concerned, we also decide to reformulate it into the criterion on the quarterly nominal interest rate. So, our reformulated criterion with the adjusted upper bound, i.e. $C_R \equiv ((1.02)^{0.25} - 1)$, is stricter than the original criterion.[24] Still, to keep the exposition of both criteria simple, we decide to use the reformulated criteria.

Moreover, the nominal exchange rate criterion is stated in terms of the quarterly nominal exchange rate movements. Additionally, we define the central parity of the nominal exchange rate as the steady state value of the nominal exchange rate ($S = \overline{S}$).

In order to implement the already adjusted criteria into the linear quadratic framework, we take advantage of the method proposed by Rotemberg and Woodford (1997, 1999) and Woodford (2003) which is applied to the zero bound constraint for the nominal interest rate. The authors propose to approximate the zero bound constraint for the nominal interest rate by restricting the mean of the nominal interest rate to be at least k standard deviations higher than the theoretical lower bound, where k is a sufficiently large number to prevent frequent violation of the original constraint. The main advantage of this alternative constraint over the original one is that it is much less computationally demanding and it only requires computation of the first and second moments of the nominal interest rate, while the original one would require checking whether the nominal interest rate is negative in any state which, in turn, depends on the distribution of the underlying shocks.

Importantly, to further simplify the exposition of the criteria, we assume that the

[22] We are aware of the CPI inflation rate dispersion among the EMU member countries. Still the framework of the model does not allow us to consider the criteria strictly in their original form.

[23] This means that it is possible that the original criterion can be still satisfied, even though the quarterly CPI inflation rate violates the reformulated criterion. On the other hand, if the quarterly CPI inflation satisfies the criterion, the original criterion is also satisfied.

[24] If we assume that the expectations hypothesis holds, an upper bound restriction on the quarterly nominal interest rate implies an upper bound criterion on the ten-year government bond yield. However, the reverse is not true.

foreign economy is in the steady state, so that foreign CPI inflation and the nominal interest rate ($\widehat{\pi}_t^*$, $\widehat{R}_t^*$) are zero. We find that this case adequately represents the policy problem in the EMU accession countries as the majority of the shocks in these countries has a domestic nature (see Fidrmuc and Kirhonen (2003)).[25]

Similarly to Woodford (2003), we redefine the criteria using discounted averages in order to conform with the welfare measure used in our framework. Let us remark that the average value of any variable (x_t) is defined as the discounted sum of the conditional expectations, i.e.:

$$\widehat{m}(x_t) \equiv E_{t_0} \sum_{t=t_0}^{\infty} \beta^t x_t. \tag{3.51}$$

Accordingly, its variance is defined by:

$$\widehat{var}(x_t) \equiv E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (x_t - \widehat{m}(x_t))^2. \tag{3.52}$$

Below, we show the reformulated Maastricht convergence criteria.[26] Each criterion is presented as a set of two inequalities:

- CPI aggregate inflation criterion:

$$(1-\beta) E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (B_\pi - \widehat{\pi}_t) \geq 0, \tag{3.53}$$

$$(1-\beta) E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (B_\pi - \widehat{\pi}_t)^2 \leq K \left((1-\beta) E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (B_\pi - \widehat{\pi}_t) \right)^2; \tag{3.54}$$

- nominal interest rate criterion:

$$(1-\beta) E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (C_R - \widehat{R}_t) \geq 0 \tag{3.55}$$

$$(1-\beta) E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (C_R - \widehat{R}_t)^2 \leq K \left((1-\beta) E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (C_R - \widehat{R}_t) \right)^2 \tag{3.56}$$

- nominal exchange rate criterion must be decomposed into two systems of the inequalities, i.e. the upper bound and the lower bound:

[25] In section 3.6 we discuss the consequences of relaxing this assumption (e.g. a departure from the steady state of the foreign economy or a suboptimal foreign monetary policy) for the nature of optimal policy constrained by the Maastricht criteria and the associated welfare loss.

[26] The detailed derivation of the Maastricht convergence criteria can be found in Appendix C.5.

– upper bound

$$(1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t(D_S-\widehat{S}_t)\geq 0 \tag{3.57}$$

$$(1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t(D_S-\widehat{S}_t)^2\leq K\left((1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t(D_S-\widehat{S}_t)\right)^2 \tag{3.58}$$

– lower bound

$$(1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t(D_S+\widehat{S}_t)\geq 0 \tag{3.59}$$

$$(1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t(D_S+\widehat{S}_t)^2\leq K\left((1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t(D_S+\widehat{S}_t)\right)^2 \tag{3.60}$$

where $K = 1 + k^{-2}$ and $D_S = 15\%$, $B_\pi = (1.015)^{0.25} - 1$, $C_R = (1.02)^{0.25} - 1$, $k = 1.96$.

The first inequality means that the average values of the CPI inflation rate, the nominal interest rate and the nominal exchange rate, respectively, should not exceed the bounds, B_π, C_R and D_S. The second inequality further restrains fluctuations in the Maastricht variables by setting an upper bound on their variances. This upper bound depends on the average values of the Maastricht variables and the bounds, B_π, C_R and D_S. Importantly, it also depends on parameter K which guarantees that the original constraints on the Maastricht variables ((3.48)–(3.50)) are satisfied with a high probability. Under a normality assumption, by setting $K = 1 + 1.96^{-2}$, we obtain that fulfillment of inequalities (3.53)–(3.60) guarantees that each of the original constraints should be met with a probability of 95%.

Summing up, the set of inequalities (3.53)–(3.60) represent the Maastricht convergence criteria in our model.

3.5 Optimal monetary policy constrained by the Maastricht criteria

This section presents how to construct the loss function of the optimal monetary policy constrained by the Maastricht convergence criteria summarized by inequalities (3.53)–(3.60) (constrained optimal policy). In this respect, we follow Woodford (2003). Specifically, the loss function of the constrained optimal monetary policy is augmented by the new elements which describe fluctuations in CPI aggregate inflation, the nominal interest rate and the nominal exchange rate.

We state the following proposition, which is based on Proposition 6.9 (p. 428) in Woodford (2003):

Proposition 1 *Consider the problem of minimizing an expected discounted sum of quadratic losses:*

$$E_{t_0}\left\{(1-\beta)\sum_{t=t_0}^{\infty}\beta^t L_t\right\}, \tag{3.61}$$

subject to (3.53) - (3.60). Let $m_{1,\pi}$, $m_{1,R}$, $m_{1,S}^U$, $m_{1,S}^L$ *be the discounted average values of* $(B_\pi - \widehat{\pi}_t)$, $(C_R - \widehat{R}_t)$, $(D_S - \widehat{S}_t)$, $(D_S + \widehat{S}_t)$ *and* $m_{2,S}^U$, $m_{2,S}^L$, $m_{2,\pi}$, $m_{2,R}$ *be the discounted means of* $(B_\pi - \widehat{\pi}_t)^2$, $(C_R - \widehat{R}_t)^2$, $(D_S - \widehat{S}_t)^2$, $(D_S + \widehat{S}_t)^2$ *associated with the optimal policy. Then, the optimal policy also minimizes a modified discounted loss criterion of the form (3.61) with* L_t *replaced by:*

$$\widetilde{L}_t \equiv L_t + \Phi_\pi(\pi^T - \widehat{\pi}_t)^2 + \Phi_R(R^T - \widehat{R}_t)^2 + \Phi_{S,U}(S^{T,U} - \widehat{S}_t)^2 + \Phi_{S,L}(S^{T,L} - \widehat{S}_t)^2, \tag{3.62}$$

under constraints represented by the structural equations of an economy. Importantly, $\Phi_\pi \geq 0$, $\Phi_R \geq 0, \Phi_{S,U} \geq 0$, $\Phi_{S,L} \geq 0$ *and take strictly positive values if and only if the respective constraints (3.54), (3.56), (3.58), (3.60) are binding. Moreover, if the constraints are binding, the corresponding target values* π^T, R^T, $S^{T,U}$, $S^{T,L}$ *satisfy the following relations:*

$$\pi^T = B_\pi - Km_{1,\pi} < 0 \tag{3.63}$$

$$R^T = C_R - Km_{1,R} < 0 \tag{3.64}$$

$$S^{T,U} = D_S - Km_{1,S} < 0 \tag{3.65}$$

$$S^{T,L} = -D_S + Km_{1,S} > 0. \tag{3.66}$$

Proof can be found in Appendix C.6.

In the presence of binding constraints, the optimal monetary policy constrained by the Maastricht convergence criteria do not only lead to smaller variances of the Maastricht variables, it also assigns target values for these variables that are different from the steady state of the optimal monetary policy.

In particular, if the constraints on the nominal interest rate or CPI inflation are binding, the target values for these variables are negative. This means that the constrained optimal monetary policy should target the CPI inflation rate or the nominal interest rate that is actually lower than the foreign CPI inflation or the foreign nominal interest rate, respectively. Therefore, this policy results in a deflationary bias. Finally, the deflationary bias together with a decrease in the nominal interest rate lead to a nominal exchange rate appreciation. Notice that if the upper bound criterion on the nominal exchange rate is binding, the constrained optimal policy is also characterized by a nominal exchange rate appreciation and negative averages of the nominal interest rate and the CPI inflation rate.

3.6 Numerical exercise

The purpose of this section is twofold, to characterize the optimal monetary policy for the EMU accession countries, given their obligation to satisfy the Maastricht convergence criteria and analyze whether and how it differs from the optimal monetary policy not constrained by the criteria (the unconstrained optimal monetary policy). To this end, in the first step, we present the optimal monetary policy and identify whether such a policy violates any of the Maastricht convergence criteria. Second, based on the results, we construct the optimal policy that satisfies all the criteria (the constrained optimal monetary policy). Third, we compare both policies by studying their welfare costs and analyzing their response pattern to the shocks.

3.6.1 Parameterization

Following the previous literature on the EMU accession countries (i.e. Laxton and Pesenti (2003), Natalucci and Ravenna (2003)) and also Chapter 2 we decide to calibrate the model to match the moments of the variables for the Czech Republic economy.

The discount factor, β, equals 0.99, which implies an annual interest rate of around four percent. The coefficient of risk aversion in consumer preferences is set to 2 as in Stockman and Tesar (1995) to get an intertemporal elasticity of substitution equal to 0.5. As far as labour supply elasticity ($\frac{1}{\eta}$) is concerned, the micro data estimates of η consider $[3, 20]$ as a reasonable range. We decide to set η to 4. The elasticity of substitution between nontradable and tradable consumption, ϕ, is set to 0.5 as in Stockman and Tesar (1994) and the elasticity of substitution between home and foreign tradable consumption, θ, is set to 1.5 (as in Chari et al. (2002) and Smets and Wouters (2004)). The elasticity of substitution between differentiated goods, σ, is equal to 10, which together with the revenue tax of 0.1[27] implies a markup of 1.23.[28]

The share of nontradable consumption in the aggregate consumption basket, μ, is assumed to be 0.42, while the share of foreign tradable consumption in the tradable consumption basket, λ, is assumed to be 0.4. These values correspond to the weights in CPI reported for the Czech Republic over the period 2000–2005.[29] As far as the foreign economy is concerned, we set the share of nontradable consumption in the foreign aggregate consumption basket, μ^*, to be 0.6, which is consistent with the value chosen by Benigno and Thoenisen (2003) regarding the structure of euro area

[27] This value represents the average share of Taxes less Subsidies in the Gross Domestic Product at 1995 constant prices in the Czech Republic for the years 1995-2006 (source: Eurostat).

[28] Martins et al. (1996) estimate the average markup for manufacturing sectors at around 1.2 in most OECD countries over the period 1980-1992. Some studies (Morrison (1994), Domowitz et al (1988)) suggest that the plausible estimates range between 1.2 and 1.7.

[29] Source: Eurostat.

consumption.

Following Natalucci and Ravenna (2003), we set the degree of price rigidity in the nontraded sector, α_N, to 0.85. The degree of price rigidity in the traded sector, α_H, is slightly smaller and equals 0.8. These values are somewhat higher than the values reported in the micro and macro studies for the euro area countries.[30] Still, Natalucci and Ravenna (2003) justify them by a high share of the government regulated prices in the EMU accession countries.

All shocks that constitute the stochastic environment of the small open economy follow the AR(1) process. The parameters of the shocks are chosen to match the historical moments of the variables. Similarly to Natalucci and Ravenna (2003) and Laxton and Pesenti (2003), the productivity shocks in both domestic sectors are characterized by a strong persistence parameter equal to 0.85. Standard deviations of the productivity shocks are set to 1.6% (nontraded sector) and 1.8% (traded sector). These values roughly reflect the values chosen by Natalucci and Ravenna (2003), 1.8% (nontraded sector) and 2% (traded sector). Moreover, the productivity shocks are strongly correlated, their correlation coefficient is set to 0.7.[31] All other shocks are independent of each other. Parameters defining the preference shock are, 0.72% (standard deviation) and 0.95 (persistence parameter). These values are similar to the values chosen by Laxton and Pesenti (2003), 0.4% (standard deviation) and 0.7 (persistence parameter). Parameters of the foreign consumption shock are estimated using quarterly data on aggregate consumption in the euro area over the period 1990-2005 (source: Eurostat). The standard deviation of the foreign consumption shock is equal to 0.23% and its persistence parameter is 0.85.

Following Natalucci and Ravenna (2003), we parametrize the monetary policy rule, i.e. the nominal interest rate follows the rule described by: $\widehat{R}_t = 0.9\widehat{R}_{t-1} + 0.1(\widehat{\pi}_t + 0.2\widehat{Y}_t + 0.3\widehat{S}_t) + \widehat{\varepsilon}_{R,t}$, where $\widehat{\varepsilon}_{R,t}$ is the monetary policy innovation with a standard deviation equal to 0.45%. Such a parametrization of the monetary policy rule enables us to closely match the historical moments of the Czech economy.

We summarize all parameters described above in Table C.1 (Structural parameters) and Table C.2 (Stochastic environment) in Appendix C.3. Moreover Table C.3 (Matching the moments) in Appendix C.3 compares the model moments with the historical moments for the Czech Republic economy.

[30] Stahl (2004) estimates that the average duration between price adjustment in the manufacturing sector is nine months (which corresponds to the degree of price rigidity: 0.67). On the other hand, Gali et al (2001) and Benigno and Lopez-Salido (2003) estimate the aggregate supply relations for the European countries and find the overall degree of price rigidity for these countries to be 0.78.

[31] Empirical evidence shows that productivity shocks are highly persistent and positively correlated (see Backus et al (1992)).

3.6.2 Unconstrained optimal monetary policy

Now, we characterize the optimal monetary policy under the chosen parameterization. First, we analyze what the main concern of the optimal monetary policy is by studying the coefficients of the loss function given by (3.38).[32] In Table 3.1, we present these coefficients.

Φ_{π_N}	Φ_{π_H}	Φ_Y	Φ_{T^d}	Φ_T	Φ_{TT^d}	Φ_{YT^d}	Φ_{YT}
117.81	28.62	3.51	0.11	0.11	-0.05	-0.31	-0.28

Table 3.1: Loss function coefficients under the optimal monetary policy

The highest penalty coefficient is assigned to fluctuations in nontradable sector inflation and home tradable inflation. Therefore, the optimal monetary policy mainly stabilizes domestic inflation. This finding is in line with the literature on core inflation targeting (Aoki (2001)). Apart from that, the optimal monetary policy faces a trade off between stabilizing the output gap and the sector inflations which is reflected in the positive values of the penalty coefficients assigned to fluctuations in domestic and international terms of trade.

Next, we check whether the optimal monetary policy satisfies the Maastricht convergence criteria. Since the means of all variables under the optimal monetary policy are zero, we can reduce constraints (3.53)–(3.60) to the following set of inequalities:

$$\widehat{var}(\widehat{\pi}_t) \leq (K-1)B_\pi^2 \tag{3.67}$$

$$\widehat{var}(\widehat{R}_t) \leq (K-1)C_R^2 \tag{3.68}$$

$$\widehat{var}(\widehat{S}_t) \leq (K-1)D_S^2, \tag{3.69}$$

where $\widehat{var}(x_t)$ with $x_t = \widehat{\pi}_t,\ \widehat{R}_t,\ \widehat{S}_t$ is defined by (3.52). Notice that these constraints set the upper bounds on the variances of the Maastricht variables. In Table 3.2, we present variances of these variables under optimal monetary policy and the respective upper bounds that represent the right-hand side of equations (3.67)–(3.69). We write that a criterion is violated (satisfied) when the variance of the respective Maastricht variable is higher (smaller) than the upper bound.

[32] Following Benigno and Woodford (2005), we check whether the second-order conditions of the policy problem are satisfied in order to guarantee that there is no alternative random policy that could improve the welfare of society. This consists in checking whether all eigenvalues of the matrix representing the loss function (3.38) are nonnegative (see Proposition 3, p.23 in Benigno and Woodford (2004)).

	CPI inflation	nominal interest rate	nominal exchange rate
mean (in %)	0	0	0
variance	0.2638	0.3525	16.6195
bound	0.0356	0.0651	58.57
criterion	*violated*	*violated*	*satisfied*
note: Variances and bounds are multiplied by 100^2 (in $(\%)^2$)			

Table 3.2: Moments of the Maastricht variables under the optimal monetary policy

The optimal monetary policy violates two of the Maastricht convergence criteria, the CPI inflation criterion and the nominal interest rate criterion. The nominal exchange rate criterion is satisfied.[33] Therefore, the loss function of optimal monetary policy for the EMU accession countries must be augmented by additional terms.

3.6.3 Constrained optimal policy

Now, we construct the optimal monetary policy that satisfies all Maastricht criteria. First, we augment the loss function of the optimal monetary policy with additional terms reflecting fluctuations of CPI inflation and the nominal interest rate and solve the new policy problem.[34] Second, we check whether such a policy also satisfies the nominal exchange rate criterion.

The loss function of the optimal policy that satisfies two additional constraints on CPI inflation and the nominal interest rate is given below:

$$\widetilde{L}_t = L_t + \frac{1}{2}\Phi_\pi(\pi^T - \widehat{\pi}_t)^2 + \frac{1}{2}\Phi_R(R^T - \widehat{R}_t)^2, \tag{3.70}$$

where $\Phi_\pi > 0$, $\Phi_R > 0$ and $\pi^T < 0$, $R^T < 0$. Values of the penalty coefficients (Φ_π, Φ_R) and targets (π^T, R^T) can be obtained from the solution to the minimization problem of the original loss function constrained by structural equations (3.39)–(3.42) and also the additional constraints on the CPI inflation rate (3.53)–(3.54) and the nominal interest rate (3.55)–(3.56).[35] These values are presented in Table 3.3:

Notice that values of the penalty coefficients on the CPI inflation rate and nominal interest rate fluctuations are of the same magnitude as the penalty coefficients on the domestic inflation rates. The negative target value for the CPI inflation rate means

[33] Note that currently, the Czech Republic economy satisfies the Maastricht criteria regarding CPI inflation, the nominal interest rate and the nominal exchange rate. See Figures A.3 (the CPI inflation criterion), A.5 (the nominal interest rate criterion) and A.6 (the nominal exchange criterion) in Appendix A.

[34] First-order conditions of this policy are presented in Appendix C.7.

[35] Special thanks to Michael Woodford for explaining the algorithm to find the parameters of the constrained policy problem (see p. 427–435 in Woodford (2003)).

Φ_π	Φ_R	π^T (in %)	R^T (in %)
42.65	23.87	-0.1779	-0.1877

Table 3.3: The augmented loss function coefficients under the constrained monetary policy

that now, monetary policy targets the CPI inflation rate and the nominal interest rate that in annual terms are 0.7% smaller than their foreign counterparts.

Finally, we check whether this policy also satisfies the nominal exchange rate criterion. In Table 3.4, we present the first and second discounted moments of all Maastricht variables and evaluate whether each of the criteria is satisfied. A criterion is satisfied when the respective set of inequalities that describes this criterion holds. In particular, the CPI inflation criterion is described by the set of inequalities (3.53)–(3.54), the nominal interest criterion is explained by (3.55)–(3.56) and the nominal exchange rate criterion by (3.57)–(3.60).

	CPI inflation	nominal interest rate	nominal exchange rate
mean (in %)	-0.0572	-0.0576	-5.7226
variance	0.0475	0.0809	14.6207
criterion	$satisfied$	$satisfied$	$satisfied$
note: Variances are multiplied by 100^2 (in $(\%)^2$)			

Table 3.4: Moments of the Maastricht variables under the constrained optimal policy

Importantly, the nominal exchange rate criterion is satisfied. Not surprisingly, variances of the CPI inflation rate and the nominal interest rate are smaller than under the optimal policy. Notice that the variance of the nominal exchange rate is smaller than the one under the optimal monetary policy. This is due to the fact that the nominal exchange rate changes are, apart from the domestic sector inflation rates, one of the components of the CPI inflation rate (see (3.34)). So the policy that targets domestic inflation rates and the CPI aggregate inflation rate at the same time also decreases the nominal exchange rate variability. Let us remark that the negative targets for the nominal interest rate and the CPI aggregate inflation lead to negative means of all Maastricht variables. Therefore, a central bank choosing such a policy commits itself to a policy resulting in the average CPI inflation rate and the nominal interest rate being 0.2% smaller in annual terms than their foreign counterparts. Additionally, this policy is characterized by an average nominal exchange rate appreciation of nearly 6%.

Summing up, the optimal monetary policy constrained by additional criteria on the CPI inflation and the nominal interest rate is the policy satisfying all Maastricht convergence criteria.

3.6.4 Comparison of the constrained and unconstrained optimal policy

Now, we focus on the comparison of the optimal monetary policy and the optimal policy constrained by the convergence criteria. First, we calculate the welfare losses associated with each policy and second, we analyze differences between the policies in their stabilization pattern when responding to the shocks.

In Table 3.5, we present the expected discounted welfare losses for both policies:

	UOP	COP
loss	7.1533	9.2956
note: Losses are multiplied by 100^2 (in $(\%)^2$)		

Table 3.5: Welfare losses for the unconstrained and constrained optimal policy

where UOP is the unconstrained optimal policy and COP is the constrained optimal monetary policy.

The obligation to comply with the Maastricht convergence criteria induces additional welfare costs equal to 30% of the optimal monetary policy loss. These welfare costs are mainly explained by the deterministic component of the constrained policy. Although the constrained optimal policy reduces variances of the Maastricht variables, it must also induce negative targets for the CPI inflation rate and the nominal interest rate to satisfy the criteria. These negative targets result in the negative means of all variables.

The welfare loss associated with the constrained optimal policy crucially depends on the foreign economy and the way its monetary policy is conducted. In our benchmark case, we assume the foreign economy to be in the steady state. This helps us simplify the exposition of the constrained optimal monetary policy problem. However, by allowing the foreign economy to be hit by stochastic shocks and, moreover, its monetary policy to be suboptimal, we obtain different targets and also penalty coefficients for domestic CPI inflation and the nominal interest rate. It can be shown that in such a situation, the targets on the CPI inflation rate and the nominal interest rate will not only depend on the average values of their foreign counterparts, but also on their fluctuations. However, a deflationary bias feature of the constrained policy is preserved.[36] These different values of targets and penalty coefficients will alter the welfare loss associated with the constrained optimal monetary policy. Importantly, the

[36] See Proposition 4 in Appendix C.8.

more volatile is the foreign economy (due to suboptimal policy or a volatile stochastic environment of the foreign economy) the smaller is the welfare loss associated with the constrained optimal policy.

Now, we investigate how the two policies, constrained optimal monetary policy and unconstrained optimal monetary policy, differ when responding to the shocks. First, we analyze which shocks are most important in creating fluctuations of the Maastricht variables. In the table below, we present variance decomposition results for CPI aggregate inflation, the nominal interest rate and the nominal exchange rate.[37] Since the variance decomposition structure does not change to any considerable extent with the chosen policy, we report results for the constrained policy.

	shocks:			
variables:	A_N	A_H	B	C^*
CPI inflation	80%	2%	11%	7%
nominal interest rate	86%	7%	4%	3%
nominal exchange rate	75%	3%	20%	2%

Table 3.6: Variance decomposition of the Maastricht variables under the constrained monetary policy

Around 80% of the total variability of CPI aggregate inflation, the nominal interest rate and the nominal exchange rate are explained by domestic nontradable productivity shocks. This result is consistent with the literature on the sources of inflation differentials in the euro area (Altissimo et al (2005), Duarte and Wolman (2003), Canzoneri et al. (2005)). We have to stress that the nontraded productivity shocks play such an important role due to a small number of shocks that explain the stochastic environment. Subsequently a portion of variance of the Maastricht variables explained by the nontraded productivity shocks can implicitly reflect other sources of shocks (compare variance decomposition for the euro area economy in the model of Smets and Wouters (2004)).

Notice that although parameters describing productivity shocks are similar in our setup, each of the productivity shocks has a different impact on the real exchange rate and therefore, on the Maastricht variables. This can easily be understood by analyzing the following equation, which relates the real exchange rate to domestic and international terms of trade (see (3.28), (3.30)):

$$\widehat{RS_t} = b^* \widehat{T_t^{d*}} - b\widehat{T_t^d} + (1-a)\widehat{T}_t. \tag{3.71}$$

[37] We calculate variance decomposition using discounted variances of the variables.

Both domestic productivity shocks result in real exchange rate depreciation. However, the magnitude of the real exchange rate depreciation differs between the two shocks. Nontradable productivity shocks lead to a decline in the domestic terms of trade and a rise in the international terms of trade. Both changes have a depreciation effect on the real exchange rate. On the other hand, domestic tradable productivity shocks result in a rise of both types of terms of trade. From equation (3.71) we see that increases in both types of terms of trade cancel out and lead to a small change in the real exchange rate. As a result, domestic nontradable productivity shocks lead to a stronger real exchange rate depreciation and therefore, larger changes in the nominal interest rate and the CPI inflation rate.

Having all this in mind, we decide to study the stabilization pattern of both policies in response to domestic nontradable productivity shocks (see Figure 3.1).

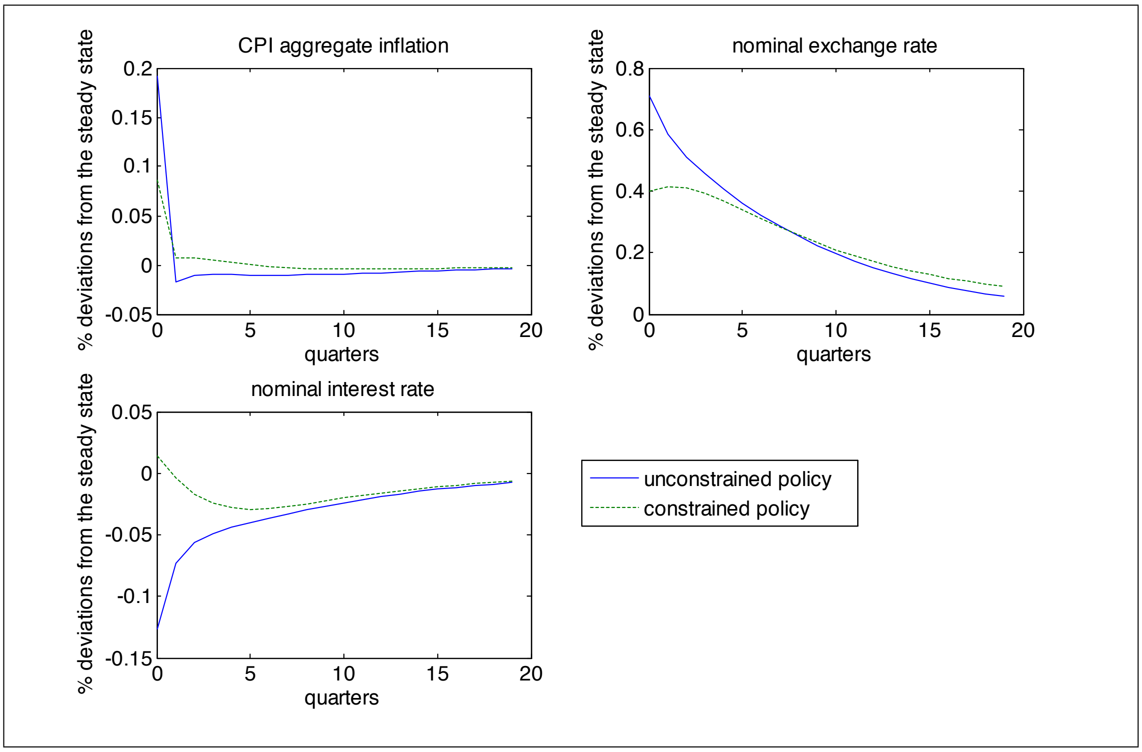

Figure 3.1: Impulse responses of the Maastricht variables to a positive domestic nontradable productivity shock

Under the unconstrained optimal policy, a positive domestic nontradable productivity shock leads to a fall in the nominal interest rate. This decrease of the nominal interest partially stabilizes deflationary pressures in the domestic nontraded sector and supports an increase in domestic aggregate output and consumption (not shown here). Since the foreign nominal interest rate remains constant, the uncovered interest rate parity induces a nominal exchange rate depreciation followed by an expected appreciation. The initial nominal exchange rate depreciation results in a strong initial increase of CPI inflation, which declines in subsequent periods, reverting to its mean.

The constrained policy is characterized by both CPI targeting and nominal interest rate targeting. To reduce the initial CPI increase (observed under the unconstrained policy), such a policy induces a more muted response of the real exchange rate and a stronger fall in domestic nontraded prices. These two effects are achieved through a more contractionary policy, i.e. a higher nominal interest rate as compared with the unconstrained optimal policy. Such behavior of the nominal interest rate is in line with the nominal interest rate targeting feature of the constrained optimal policy. As a result, an initial increase of the CPI inflation is smaller. Moreover, a higher domestic nominal interest rate leads to a smaller depreciation of the nominal exchange rate through the uncovered interest rate parity.

Summing up, in response to domestic nontradable productivity shocks, the constrained optimal policy leads to smaller fluctuations in all three Maastricht variables than unconstrained optimal monetary policy. However, it must be kept in mind that the constrained optimal policy commits to the inflation rate and the nominal interest rate that are lower than their foreign counterparts which results in substantial welfare costs.

3.6.5 Robustness analysis

The characteristics of the unconstrained and constrained optimal policy critically depend on the structural parameters of an economy and also the volatility of the stochastic environment. The purpose of this section is to investigate how changes in values of the parameters describing the structure and the stochastic environment of the small open economy affect our main findings.

As far as the structure of the small open economy is concerned, we identify two crucial parameters: share of nontradables (μ) and degree of openness (λ). We derive the unconstrained and constrained optimal policy for different values of these parameters. Our findings can be summarized as follows:

- for all possible combinations of (μ, λ),[38] the nominal exchange rate criterion is satisfied under the unconstrained optimal policy,

[38] All combinations of (μ, λ) for which the second-order conditions of the unconstrained policy problem are satisfied.

- for all possible combinations of (μ, λ), the nominal interest rate criterion is not satisfied under the unconstrained optimal policy,
- the CPI inflation rate criterion is satisfied under the unconstrained optimal policy for small values of λ and/or high values of μ, i.e. for economies that are relatively closed and have a high share of nontradables (see Table 3.7 in Appendix 3.A),
- for small values of λ and high values of μ, the constrained policy that satisfies the CPI inflation rate criterion and the nominal interest rate criterion fails to satisfy the nominal exchange rate criterion (the lower bound constraint is not satisfied, i.e. the nominal exchange rate appreciates too much, see Table 3.8 in Appendix 3.A).

Under our chosen parameterization of the stochastic environment, the productivity shocks are characterized by the highest standard deviation. Not surprisingly, elimination of the preference and foreign consumption shocks does not alter our results, i.e. the unconstrained optimal policy fails to satisfy the CPI inflation rate criterion and the nominal interest rate criterion and the optimal policy constrained by these two criteria also satisfies the nominal exchange rate criterion. The results do not change, even if we eliminate one of the productivity shocks, i.e. in the traded or nontraded sector. Finally, the unconstrained optimal policy satisfies all Maastricht convergence criteria provided that the standard deviations of the productivity shocks in both sectors are reduced by at least 80% of the original values (see Table 3.8 in Appendix 3.A). We have to stress here that it is the magnitude of the shocks and not its source that determines whether the Maastricht criteria are satisfied under the optimal policy. Our parameterization of the shocks aims to match the historical moments of the variables in the Czech Republic economy. Thus introduction of more shocks would not alter the results. It would rather change the importance of a given type of shock in creating fluctuations of the Maastricht variables.

Summing up, both the structure and the stochastic environment of the small open economy affect the characteristics of the unconstrained and constrained optimal policy. In relatively closed economies and/or with a high share of nontradables, there is a trade off between complying with the CPI inflation rate and the nominal interest rate criteria and the nominal exchange rate criterion. Moreover, volatility of productivity shocks plays a crucial role in determining whether the unconstrained optimal monetary policy is compatible with the Maastricht convergence criteria.

3.7 Conclusions

This paper characterizes the optimal monetary policy for the EMU accession countries, taking into account their obligation to meet the Maastricht convergence criteria. We perform our analysis in the framework of a two-sector small open economy DSGE model.

First, we derive the micro founded loss function which represents the policy objective function of the optimal monetary policy using the second-order approximation method. We find that the optimal monetary policy should not only target inflation rates in the domestic sectors and aggregate output fluctuations, but also domestic and international terms of trade. This results originates from the distortions present in the economy: monopolistic competition that implies inefficient sector outputs, price stickiness in both sectors that leads to an inefficient path of the domestic terms of trade and the international terms of trade externality that can affect the wedge between marginal disutility from labour and utility of consumption. All these distortions lead to the introduction of new elements in the loss function: domestic and international terms of trade.

Second, we reformulate the Maastricht convergence criteria taking advantage of the method developed by Rotemberg and Woodford (1997, 1999) to address the zero bound nominal interest rate problem. We show how the loss function changes when the monetary policy is subject to the Maastricht convergence criteria: the CPI inflation rate criterion, the nominal interest rate criterion and the nominal exchange rate criterion. The loss function of such a constrained policy is characterized by additional elements that penalize fluctuations of the CPI inflation rate, the nominal interest rate and the nominal exchange rate around the new targets different from the steady state of the unconstrained optimal monetary policy.

Under the chosen parameterization (which roughly represents the Czech Republic), optimal monetary policy violates the CPI inflation and the nominal interest rate criteria. The optimal policy that instead satisfies these two criteria also satisfies the nominal exchange rate criterion. Both the deterministic component and the stabilization component of the constrained policy are different from the unconstrained optimal policy. The constrained policy leads to a lower variability of the CPI inflation, the nominal interest rate and the nominal exchange rate. At the same time, this policy targets the CPI inflation rate and the nominal interest rate that are 0.7% lower (in annual terms) than their counterparts in the reference countries. This produces additional welfare costs that amount to 30% of the optimal monetary policy loss. As discussed earlier, these costs do not take into account the credibility gains related to compliance with the Maastricht criteria.

The tools developed in this paper can be used to describe the optimal policy which faces additional constraints that are exogenously decided and do not form part of the structural constraints of an economy. Importantly, the Maastricht Treaty also sets restrictions on the debt and deficit policy of the EMU accession countries. Therefore a natural extension of the analysis involves the introduction of fiscal policy by endogenizing tax and debt decisions. Including all the restrictions faced by the fiscal and monetary policies in the EMU accession countries would enable us to investigate the effects of these restrictions on the interaction between the two policies. These are the topics which we cover in Chapter 4.

3.A Robustness analysis results

We provide the results of our robustness analysis.

	The unconstrained optimal policy and the Maastricht criteria									
$\mu\backslash\lambda$	0.01	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	CPI	nCPI	nCPI	nCPI	nCPI	nCPI	nCPI	nCPI	nCPI	nSOC
0.1	CPI	nCPI	nCPI	nCPI	nCPI	nCPI	nCPI	nSOC	nSOC	nSOC
0.2	CPI	nCPI	nCPI	nCPI	nCPI	nCPI	nSOC	nSOC	nSOC	nSOC
0.3	CPI	nCPI	nCPI	nCPI	nCPI	nSOC	nSOC	nSOC	nSOC	nSOC
0.4	CPI	CPI	nCPI	nCPI	nCPI	nSOC	nSOC	nSOC	nSOC	nSOC
0.5	CPI	CPI	nCPI	nCPI	nSOC	nSOC	nSOC	nSOC	nSOC	nSOC
0.6	CPI	CPI	nCPI	nCPI	nSOC	nSOC	nSOC	nSOC	nSOC	nSOC
0.7	CPI	CPI	CPI	nCPI	nSOC	nSOC	nSOC	nSOC	nSOC	nSOC
0.8	CPI	CPI	CPI	nSOC	nSOC	nSOC	nSOC	nSOC	nSOC	nSOC
0.9	CPI	CPI	CPI	nSOC	nSOC	nSOC	nSOC	nSOC	nSOC	nSOC
note: CPI - CPI criterion satisfied; nCPI - CPI criterion not satisfied;										
nSOC - second order conditions not satisfied										

Table 3.7: Robustness analysis - structural parameters under the optimal monetary policy

	The constrained optimal policy and the Maastricht criteria									
$\mu \backslash \lambda$	0.01	0.1	0.2	0.3	0.4	0.5	0.6	0.7	0.8	0.9
0.01	nNER	nNER	nNER	nNER	nNER	NER	NER	NER	NER	nSOC
0.1	nNER	nNER	nNER	nNER	NER	NER	NER	nSOC	nSOC	nSOC
0.2	nNER	nNER	nNER	nNER	NER	NER	nSOC	nSOC	nSOC	nSOC
0.3	nNER	nNER	nNER	nNER	NER	nSOC	nSOC	nSOC	nSOC	nSOC
0.4	nNER	nNER	nNER	nNER	NER	nSOC	nSOC	nSOC	nSOC	nSOC
0.5	nNER	nNER	nNER	nNER	nSOC	nSOC	nSOC	nSOC	nSOC	nSOC
0.6	nNER	nNER	nNER	nNER	nSOC	nSOC	nSOC	nSOC	nSOC	nSOC
0.7	nNER	nNER	nNER	nNER	nSOC	nSOC	nSOC	nSOC	nSOC	nSOC
0.8	nNER	nNER	nNER	nSOC	nSOC	nSOC	nSOC	nSOC	nSOC	nSOC
0.9	nNER	nNER	nNER	nSOC	nSOC	nSOC	nSOC	nSOC	nSOC	nSOC
note: NER - nominal exchange rate (NER) criterion satisfied;										
nNER - NER criterion not satisfied; nSOC - second order conditions not satisfied										

Table 3.8: Robustness analysis - structural parameters under the constrained monetary policy

	variances of the Maastricht variables under the unconstrained policy (in $(\%)^2$)		
shocks	CPI inflation	nominal interest rate	nominal exchange rate
A_N, A_H, B, C^*	0.2638	0.3525	16.6195
A_N, B, C^*	0.1252	0.1028	8.5792
A_H, B, C^*	0.0649	0.1063	5.7338
A_N, A_H	0.2372	0.3424	13.2265
A_N^r, A_H^r, B, C^*	0.0356	0.0228	3.9607
bound	0.0356	0.0651	58.57
note: $SD(A_N^r) = 0.2SD(A_N)$; $SD(A_H^r) = 0.2SD(A_H)$			
where SD - standard deviation			

Table 3.9: Robustness analysis - stochastic environment

Chapter 4

Maastricht Criteria and Optimal Monetary and Fiscal Policy

4.1 Introduction

Monetary policy has been used as the main stabilization tool in many countries. EMU accession countries, on their way to EMU, face the Maastricht criteria that put serious constraints on their monetary policies (as it was analyzed in Chapter 2 and 3). These countries should achieve a high and durable degree of price stability which is, in quantitative terms, reflected in low inflation rates and low long term interest rates. Additionally, nominal exchange rates of the EMU accession countries versus the euro should stay within normal fluctuation margins. At the same time fiscal policy that could be seen as an additional stabilization tool is bound by the restrictions imposed in the Stability and Growth Pact (SGP). Accordingly these countries should be characterized by a sustainable government financial position which is defined in terms of upper limits on deficit to GDP ratio and debt to GDP ratio. At the moment many EMU accession countries do not satisfy some of the criteria.[1]

In this context a number of questions arise: Can fiscal policy serve as an additional stabilization tool in the presence of the restrictions set on monetary policy? How does this ability of fiscal policy to mitigate business cycles change when faced with fiscal constraints as well? What should be the optimal monetary and fiscal policy that

[1] Bulgaria, Estonia, Hungary, Latvia, Lithuania, Romania and Slovakia fail to fulfill the CPI inflation rate criterion (see Figure A.3 in Appendix A). Moreover, Hungary and Romania also violate the nominal interest rate criterion (see Figure A.5 in Appendix A). Moreover, the nominal exchange rate fluctuations of Polish Zloty, Slovakian Koruna and Romanian Lei versus the euro exceed the band set by the nominal exchange rate criterion (see Figure A.6 in Appendix A). Additionally, deficit to GDP criterion is not satisfied by the Czech Republic, Hungary, Poland and Slovakia (see Figure A.7 in Appendix A). Finally, only Hungary is characterised by an excessive debt to GDP ratio according to the limits set by the Maastricht criteria.

satisfies all the Maastricht criteria? And finally, which criteria: fiscal or monetary ones put stronger constraints on stabilization role of macroeconomic policies?

To this purpose, we develop a DSGE model of a small open economy with nominal rigidities, distortionary taxation and government debt exposed to both domestic and external shocks. The model is an extension of the framework developed in Chapter 3 where fiscal policy does not issue any debt and taxes are assumed to be lump sum. As in Chapter 3 the production structure is composed of two sectors: nontraded and home traded sector. In that way, we want to take into account recent empirical literature both on OECD and EMU Accession countries that highlights the role of sector specific shocks in explaining international business cycle fluctuations (see e.g. Canzoneri et al. (1999), Marimon and Zilibotti (1998), Mihaljek and Klau (2004)). Finally, following Benigno and Woodford (2003) and Benigno and De Paoli (2006) monetary and fiscal policy is conducted in a fully coordinated way by a single policy maker.

In this framework we characterize the optimal monetary and fiscal policy from a timeless perspective (Woodford (2003)). As in Chapter 3, we derive the micro founded loss function using the second order approximation methodology developed by Rotemberg and Woodford (1997) and Benigno and Woodford (2005). We find that the optimal monetary and fiscal policy (unconstrained policy) should not only target inflation rates in the domestic sectors and aggregate output fluctuations but also domestic and international terms of trade. Subsequently, we present how the loss function changes when the monetary and fiscal policy is constrained by the Maastricht convergence criteria. We derive the optimal monetary and fiscal policy that satisfies all the Maastricht convergence criteria (constrained policy). The Maastricht convergence criteria are not easily implementable in our model. Here we take advantage of the methodology developed by Rotemberg and Woodford (1997, 1999) for the analysis of the zero bound problem and adapted in Chapter 2 and 3 for the analysis of the monetary criteria. This method enables us to verify whether a given criterion is satisfied by only computing first and second moments of a variable for which the criterion is set.

Under the chosen parameterization (which aims to reflect the Czech Republic economy) the optimal monetary and fiscal policy violates the CPI inflation rate, the nominal interest rate and deficit to GDP ratio criteria. Both the stabilization component and deterministic component of the constrained policy are different from the unconstrained optimal policy. The constrained policy leads to a smaller variability of the CPI inflation and of the nominal interest rate. At the same time it causes higher variability of deficit to GDP ratio. This reflects an active stabilization role of fiscal policy in the presence of direct constraints on monetary instrument. As in Chapter 3 the constrained policy is characterized by a deflationary bias which results in targeting the CPI inflation rate and the nominal interest rate that are in annual terms lower by 1.3% than the CPI inflation rate and the nominal interest rate in the countries taken as a reference. The constrained policy is also characterized by targeting surplus to GDP ratio at around 3.7%. This result is determined by a relative dominance of the monetary criteria over the fiscal ones in affecting the stabilization process. The constrained policymaker uses actively fiscal instruments and in order to comply with all the criteria has to assign a relatively high surplus to GDP ratio target. As a result, the policy constrained by the Maastricht convergence criteria induces additional welfare costs that amount to 60% of the initial deadweight loss associated with the optimal unconstrained policy. These welfare costs have their origin in conflicting interest of monetary and fiscal criteria and also relatively poor performance of fiscal policy as an additional stabilization tool.

The literature on the macroeconomic policies in the EMU accession countries concentrated so far on the analysis of the monetary criteria and their impact on the appropriate choice of the monetary regime (as it was done in Chapter 2). In particular, Devereux and Lane (2003), Ferreira (2006), Laxton and Pesenti (2003) and Natalucci and Ravenna (2007) study this issue in a framework of open economy DSGE models.[2]

The issue of a proper design of fiscal policy in the EMU Accession countries has not been studied up till now. However, theoretical literature addressed the problem of a joint optimal monetary and fiscal policy both in the closed, open economy, and monetary union environment. Schmitt-Grohe and Uribe (2004) and Benigno and Woodford (2003) study the design of optimal monetary and fiscal policy in the closed economy environment, while Benigno and De Paoli (2006) derive the optimal monetary and fiscal policy for a small open economy. These papers find that variations in fiscal instruments should serve the same objectives as those in the optimal monetary policy design, i.e. stabilization of inflation and the output gap that measures the total distortion of the level of economic activity. Ferrero (2005), Gali and Monacelli (2008) and Pappa and Vassilatos (2005) are examples of papers that study optimal monetary and fiscal policy in a monetary union. In particular, Ferrero (2005) shows that regional fiscal policies that respond to a measure of real activity perform better in terms of welfare than balanced budget rules. Gali and Monacelli (2005) find that the lack of regional monetary instrument generates a stabilization role for regional fiscal policies. Pappa and Vassilatos (2006) examine how general fiscal rules that are designed to satisfy fiscal criteria affect macroeconomic stability and welfare in a two-region monetary union. They find that some flexibility in compliance with fiscal criteria can be welfare

[2] Chapter 2 and 3 provide a detailed discussion of both empirical and theoretical papers on the monetary policy in the EMU accession countries.

improving.

We take advantage of these theoretical studies and characterize an optimal monetary and fiscal policy mix in a model which tries to reflect some of the characteristics of the EMU Accession countries. Then we analyze the effects of the Maastricht criteria on the optimal policies. In that way we can set guidelines on the way monetary and fiscal policy should be conducted in the EMU Accession countries.

The rest of the paper is organized as follows: the next section describes the model. Section 4.3 explains the derivation of the optimal monetary and fiscal policy. Section 4.4 presents the way we reformulate the Maastricht convergence criteria in order to implement them into our framework. Section 4.5 is dedicated to the derivation of the optimal policy constrained by the Maastricht convergence criteria. Section 4.6 compares the unconstrained optimal monetary and fiscal policy with the optimal monetary and fiscal policy constrained by the Maastricht convergence criteria under the chosen parameterization of the model. Section 4.7 concludes.

4.2 The model

Our modelling framework is based on a two-sector SOE model of Chapter 3 and one-sector SOE models of De Paoli (2004) and Benigno and De Paoli (2006). Following De Paoli (2004), we model a small open economy as the limiting case of a two-country problem, i.e. where the size of the small open economy is set to zero. We consider two highly integrated economies where asset markets are complete. In each of the economies, there are two goods sectors: nontraded goods and traded goods. Each of the sectors (domestic and foreign) features heterogeneity of goods and monopolistic competition. Labour is the only factor of production and is mobile between sectors in each country and immobile between countries. We assume the existence of home bias in consumption which, in turn, depends on the relative size of the economy and its degree of openness. Although the law of one price holds, existence of home bias leads to deviations from the purchasing power parity.

As far as the monetary and fiscal policy is concerned we follow Benigno and Woodford (2003) and Benigno and De Paoli (2006) and assume that the policies are conducted in a fully coordinated way by a single policymaker. The role for monetary policy arises through the introduction of monopolistic competition and price rigidities with staggered Calvo contracts in all goods sectors. The model features complete pass-through as prices are set in the producer's currency. We abstract from any monetary frictions by assuming cashless limiting economies.[3] The fiscal policy issues a

[3] See Woodford (2003).

one period nominal non-state contingent debt which is financed only through the distortionary revenue taxes collected in both domestic sectors. On the contrary to the model in Chapter 3 the lump sum taxes are not available.

Finally, the stochastic environment of the small open economy is characterized by asymmetric productivity shocks originating in both domestic sectors, preference shocks, foreign consumption shocks and government expenditure shocks.

4.2.1 Households

The world economy consists of a continuum of agents of unit mass: $[0, n)$ belonging to a small country (home) and $[n, 1]$ belonging to the rest of the world, i.e. the euro area (foreign). There are two types of differentiated goods produced in each country: traded and nontraded goods. Home traded goods are indexed on the interval $[0, n)$ and foreign traded goods on the interval $[n, 1]$, respectively. The same applies to nontraded goods. In order to simplify the exposition of the model, we explain in detail only the structure and dynamics of the domestic economy. Thus, from now on, we assume the size of the domestic economy to be zero, i.e. $n \to 0$.

Households are assumed to live infinitely and behave according to the permanent income hypothesis. They can choose between three types of goods: nontraded, domestic traded and foreign traded goods. C_t^i represents consumption at period t of a consumer i and L_t^i constitutes his labour supply. Each agent i maximizes the following utility function:[4]

$$\max E_{t_0} \left\{ \sum_{t=t_0}^{\infty} \beta^{t-t_0} \left[U\left(C_t^i, B_t\right) - V\left(L_t^i\right) \right] \right\}, \tag{4.1}$$

where E_{t_0} denotes the expectation conditional on the information set at date t_0, β is the intertemporal discount factor and $0 < \beta < 1$, $U(\cdot)$ stands for flows of utility from consumption and $V(\cdot)$ represents flows of disutility from supplying labour.[5] C is a composite consumption index. We define consumers' preferences over the composite consumption index C_t of traded goods $(C_{T,t})$ (domestically produced and foreign ones) and nontraded goods $(C_{N,t})$:

$$C_t \equiv \left[\mu^{\frac{1}{\phi}} C_{N,t}^{\frac{\phi-1}{\phi}} + (1-\mu)^{\frac{1}{\phi}} C_{T,t}^{\frac{\phi-1}{\phi}} \right]^{\frac{\phi}{\phi-1}}, \tag{4.2}$$

[4] In general, we assume U to be twice differentiable, increasing and concave in C_t and V to be twice differentiable, increasing and convex in L_t.

[5] We assume specific functional forms of consumption utility $U\left(C_t^i\right)$, and disutility from labour $V\left(L_t^i\right)$: $U\left(C_t^i\right) \equiv \frac{\left(C_t^i\right)^{1-\rho} B_t^{\rho}}{1-\rho}$, $V\left(L_t^i\right) \equiv \varphi_l \frac{\left(L_t^i\right)^{1+\eta}}{1+\eta}$ with ρ $(\rho > 0)$, the inverse of the intertemporal elasticity of substitution in consumption and η $(\eta \geq 0)$, the inverse of labour supply elasticity and B_t , preference shock.

where $\phi > 0$ is the elasticity of substitution between traded and nontraded goods and $\mu \in [0,1]$ is the share of the nontraded goods in overall consumption. Traded good consumption is a composite of the domestically produced traded goods (C_H) and foreign produced traded goods (C_F):

$$C_{T,t} \equiv \left[(1-\lambda)^{\frac{1}{\theta}} C_{H,t}^{\frac{\theta-1}{\theta}} + \lambda^{\frac{1}{\theta}} C_{F,t}^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}}, \tag{4.3}$$

where $\theta > 0$ is the elasticity of substitution between home traded and foreign traded goods, and λ is the degree of openness of the small open economy ($\lambda \in [0,1]$).[6] Finally, C_j (where $j = N$, H, F) are consumption sub-indices of the continuum of differentiated goods:

$$C_{j,t} \equiv \left[\left(\frac{1}{n} \right)^{\frac{1}{\sigma}} \int_0^n c_t(j)^{\frac{\sigma-1}{\sigma}} \, dj \right]^{\frac{\sigma}{\sigma-1}}, \tag{4.4}$$

where $\sigma > 1$ represents elasticity of substitution between differentiated goods in each of the sectors. Based on the above presented preferences, we derive consumption-based price indices expressed in the units of currency of the domestic country:

$$P_t \equiv \left[\mu P_{N,t}^{1-\phi} + (1-\mu) P_{T,t}^{1-\phi} \right]^{\frac{1}{1-\phi}}, \tag{4.5}$$

$$P_{T,t} \equiv \left[\nu P_{H,t}^{1-\theta} + (1-\nu) P_{F,t}^{1-\theta} \right]^{\frac{1}{1-\theta}} \tag{4.6}$$

with

$$P_{j,t} \equiv \left[\left(\frac{1}{n} \right) \int_0^n p_t(j)^{1-\sigma} \, dj \right]^{\frac{1}{1-\sigma}}. \tag{4.7}$$

Although we assume the law of one price in the traded sector (i.e. $p(h) = Sp^*(h)$ and $p(f) = Sp^*(f)$ where S is the nominal exchange rate), both the existence of the nontraded goods and the assumed home bias cause deviations from purchasing power parity, i.e. $P \neq SP^*$. The real exchange rate can be defined in the following manner: $RS \equiv \frac{SP^*}{P}$. Moreover, we define the international terms of trade as $T \equiv \frac{P_F}{P_H}$ and the ratio of nontraded to traded goods' prices (domestic terms of trade) as $T^d \equiv \frac{P_N}{P_T}$.

From consumer preferences, we can derive total demand for the generic goods – n (home nontraded ones), h (home traded ones), f (foreign traded ones):

[6] Following de Paoli (2004) and Sutherland (2002), we assume home bias (ν) of the domestic households to be a function of the relative size of the home economy with respect to the foreign one (n) and its degree of openness (λ) such that $(1-\nu) = (1-n)\lambda$ where $\lambda \in [0,1]$. Importantly, the higher is the degree of openness, the smaller is the degree of home bias. Since $n \to 0$, we obtain that $\nu = 1 - \lambda$.

$$y^d(n) = \left[\frac{p(n)}{P_N}\right]^{-\sigma}\left[\frac{P_N}{P}\right]^{-\phi}\mu(C+G), \tag{4.8}$$

$$y^d(h) = \left[\frac{p(h)}{P_H}\right]^{-\sigma}\left[\frac{P_H}{P_T}\right]^{-\theta}(1-\lambda)(C_T+G_T) + \left[\frac{p^*(h)}{P_H^*}\right]^{-\sigma}\left(\frac{P_H^*}{P_T^*}\right)^{-\theta^*}\lambda(C_T^*+G_T^*), \tag{4.9}$$

$$y^d(f) = \left[\frac{p^*(f)}{P_F^*}\right]^{-\sigma}\left[\frac{P_F^*}{P_T^*}\right]^{-\theta}(C_T^*+G_T^*) \tag{4.10}$$

where variables with an asterisk represent the foreign equivalents of the domestic variables. Moreover, G and G^* denote exogenous aggregate government expenditures which have the same composition as the private consumption. Accordingly, G_T and G_T^* denote government expenditure in the tradable sector. Importantly, since the domestic economy is a small open economy, demand for foreign traded goods does not depend on domestic demand. However, at the same time, demand for domestic traded goods depends on foreign demand.

Households get disutility from supplying labour to all firms present in each country. Each individual supplies labour to both sectors, i.e. the traded and the nontraded sector:

$$L_t^i = L_t^{i,H} + L_t^{i,N}. \tag{4.11}$$

We assume that consumers have access to a complete set of securities-contingent claims traded internationally. Each household faces the following budget constraint:

$$P_tC_t^i + E_t\{Q_{t,t+1}D_{t+1}\} \leq D_t + W_{H,t}^iL_{H,t}^i + W_{N,t}^iL_{N,t}^i + \frac{\int_0^n \Pi_{N,t}^i di}{n} + \frac{\int_0^n \Pi_{H,t}^i di}{n}, \tag{4.12}$$

where at date t, D_{t+1} is nominal payoff of the portfolio held at the end of period (t), $Q_{t,t+1}$ is the stochastic discount factor for one-period ahead nominal payoffs relevant to the domestic household, $\Pi_{H,t}$ and $\Pi_{N,t}$ are nominal profits from the domestic firms. Moreover, consumers face no Ponzi game restriction.

The short-term interest rate (R_t) is defined as the price of the portfolio which delivers one unit of currency in each contingency that occurs in the next period:[7]

$$R_t^{-1} = E_t\{Q_{t,t+1}\}. \tag{4.13}$$

[7]Following the literature, we assume one period to be one quarter.

The maximization problem of any household consists of maximizing the discounted stream of utility (4.1) subject to the budget constraint (4.12) in order to determine the optimal path of the consumption index, the labour index and contingent claims at all times. The solution to the household decision problem gives a set of first-order conditions.[8] Optimization of the portfolio holdings leads to the following Euler equations for the domestic economy:

$$U_C(C_t, B_t) = \beta E_t \left\{ U_C(C_{t+1}, B_{t+1}) Q_{t,t+1}^{-1} \frac{P_t}{P_{t+1}} \right\}. \tag{4.14}$$

There is a perfect sharing in this setting, meaning that marginal rates of consumption in nominal terms are equalized between countries in all states and at all times.[9] Subsequently, appropriately choosing the distribution of initial wealth, we obtain the risk sharing condition:

$$\frac{U_C(C_t, B_t)}{U_C(C_t^*, B_t^*)} = \upsilon \frac{P_t}{S_t P_t^*} = \upsilon RS_t^{-1}, \tag{4.15}$$

where $\upsilon > 0$ and depends on the initial wealth distribution. The risk sharing condition implies that the real exchange rate is equal to the marginal rate of substitution between domestic and foreign consumption.

The optimality condition for labour supply in the domestic economy is the following:

$$\frac{W_t^k}{P_t} = \frac{V_L(L_t)}{U_C(C_t, B_t)}, \tag{4.16}$$

where W^k is the nominal wage of the representative consumer in sector k $(k = H, N)$.[10] So the real wage is equal to the marginal rate of substitution between labour and consumption.

4.2.2 Firms

All firms are owned by consumers. Both traded and nontraded sectors are monopolistically competitive. The production function is linear in labour which is the only input. Consequently, its functional form for firm i in sector k $(k = N,\ H)$ is the following:

[8] We here suppress subscript i as we assume that in equilibrium, all agents are identical. Therefore, we represent optimality conditions for a representative agent.

[9] We have to point out here that although the assumption of complete markets conveniently simplifies the model, it neglects a possibility of wealth effects in response to different shocks (Benigno (2001)).

[10] Notice that wages are equalized between sectors inside each of the economies, due to perfect labour mobility and perfect competition in the labour market.

$$Y_{k,t}(i) = A_t^k L_t^k(i). \tag{4.17}$$

Price is set according to the Calvo (1983) pricing scheme. In each period, a fraction of firms $(1-\alpha_k)$ decides its price, thus maximizing the future expected profits. The maximization problem of any firm in sector k at time t_0 is given by:

$$\max_{P_{k,t_0}(i)} E_{t_0} \sum_{t=t_o}^{\infty} (\alpha_k)^s Q_{t_0,t} \left[(1-\tau_{R,t}^k) P_{k,t_0}(i) - MC_t^k(i)\right] Y_{k,t_0:t}^d(i)$$

$$\textit{subject to } Y_{k,t_0:t}^d(i) = \left(\frac{P_{k,t_0}(i)}{P_{k,t}}\right)^{-\sigma} Y_{k,t}, \tag{4.18}$$

where $Y_{k,t_0:t}^d(i)$ is demand for the individual good in sector k produced by producer i at time t conditional on keeping the price $P_{k,t_0}(i)$ fixed at the level chosen at time t_0, $MC_t^k = \frac{W_t^k(i)}{A_t^k}$ is the nominal marginal cost in sector k at time t, and $\tau_{k,t}$ are revenue taxes in sector k.

Given this setup, the price index in sector k evolves according to the following law of motion:

$$(P_{k,t})^{1-\sigma} = \alpha_k (P_{k,t-1})^{1-\sigma} + (1-\alpha_k)(P_{k,t_0}(i))^{1-\sigma}. \tag{4.19}$$

4.2.3 Government budget constraint

The government issues a nominal, non-state contingent debt denominated in domestic currency and taxes the revenue income of firms in the nontraded sector at rate $\tau_{N,t}$ and also in the home traded sector at rate $\tau_{H,t}$. The revenues are spent on government expenditures (G_t) and interest payments on outstanding nominal debt. We assume that there are no seigniorage revenues.

Government debt, D_t, expressed in nominal terms, follows the law of motion:

$$D_t = R_{t-1} D_{t-1} - P_t sr_t \tag{4.20}$$

where sr_t is the real primary budget surplus:

$$sr_t = \tau_{N,t} p_{N,t} Y_{N,t} + \tau_{H,t} p_{H,t} Y_{H,t} - G_t \tag{4.21}$$

and $p_{N,t} \equiv \frac{P_{N,t}}{P_t}$ and $p_{H,t} \equiv \frac{P_{H,t}}{P_t}$ denote relative prices. We define:

$$d_t \equiv \frac{D_t R_t}{P_t} \tag{4.22}$$

in order to rewrite the government budget constraint as:

$$d_t = d_{t-1}\frac{R_t}{\Pi_t} - R_t sr_t. \tag{4.23}$$

The rational-expectations equilibrium requires that the expected path of government surpluses must satisfy an intertemporal solvency condition :

$$\frac{d_{t_0-1}}{\Pi_{t_0}}U_C(C_{t_0}, B_{t_0}) = E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}U_C(C_t, B_t)sr_t. \tag{4.24}$$

in each state of the world that may be realized at date t_0. This condition restricts the possible paths that may be chosen for the tax rates $\{\tau_{N,t}, \tau_{H,t}\}$. Moreover, monetary policy can affect this condition by influencing inflation in period t_0 and also affecting the discount factors in subsequent periods. This condition is derived from the household optimization condition (4.14) and law of motion of debt (4.23). As discussed in Woodford (2001) this condition serves as one of the constraints in choosing an optimal plan among possible rational-expectations equilibria.

4.2.4 Monetary and fiscal policy

A role for the macroeconomic policy arises due to existing nominal and real rigidities in the economy: price stickiness (together with monopolistic competition), home bias and the nontraded good sector, which lead to deviations from PPP. The policy maker has three instruments: two fiscal ones - revenue tax rates in both domestic sectors and a monetary one - nominal interest rate. The system is therefore closed by defining appropriate monetary and fiscal policy rules for the domestic economy.

We approximate the model around a steady state in which exogenous shocks take constant values. Moreover steady state inflation is zero and tax rates are chosen in order to maximize welfare of the agents. The loglinearized version of the model is available in the Appendix D.2.

4.3 The optimal fiscal and monetary policy

Since our model is microfounded the optimal policy is defined as the policy that maximizes welfare of society subject to the structural equations of an economy.

We use a linear quadratic approach (Rotemberg and Woodford (1997, 1999)) and define the optimal monetary policy problem as a minimization problem of the quadratic loss function subject to the loglinearized structural equations. First, we present the welfare measure derived through a second-order Taylor approximation of equation (4.1):

$$W_{t_0} = U_C \overline{C} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [z_v' \widehat{v}_t - \frac{1}{2} \widehat{v}_t' Z_v \widehat{v}_t - \widehat{v}_t' Z_\xi \widehat{\xi}_t] + tip + O(3), \tag{4.25}$$

where $\widehat{v}_t' = \begin{bmatrix} \widehat{C}_t & \widehat{Y}_{N,t} & \widehat{Y}_{H,t} & \widehat{\pi}_{N,t} & \widehat{\pi}_{H,t} \end{bmatrix}$; $\widehat{\xi}_t' = \begin{bmatrix} \widehat{A}_{N,t} & \widehat{A}_{H,t} & \widehat{B}_t & \widehat{C}_t^* & \widehat{G}_t \end{bmatrix}$; $z_v' = \begin{bmatrix} 1 & -s_{CY_N} & -s_{CY_H} & 0 & 0 \end{bmatrix}$ with $s_{CY_N} \equiv \frac{\overline{w} \overline{Y_N}}{\overline{C}}$ - steady state share of nontraded labour income in domestic consumption, $s_{CY_H} \equiv \frac{\overline{w} \overline{Y_H}}{\overline{C}}$ - steady state share of home traded labour income in domestic consumption, and matrices Z_v, Z_ξ are defined in Appendix D.4; *tip* stands for *terms independent of policy* and $O(3)$ includes terms that are of a higher order than the second in the deviations of variables from their steady state values.

Notice that the welfare measure (4.25) contains the linear terms in aggregate consumption and sector outputs. These linear terms originate from different distortions in the economy. First, monopolistic competition together with distortionary revenue taxes in both domestic sectors lead to inefficient levels of sector outputs and also an inefficient level of aggregate output. Second, since government spends its revenues on government expenditures domestic consumption and aggregate output are not equalized. Third, openness of the economy can also result in trade imbalances, i.e. domestic consumption can be different from the aggregate output. Importantly, their composition depends on the domestic and international terms of trade. Finally, similarly to De Paoli (2007) and Chapter 3 there exists an international terms of trade externality that creates an incentive for policy to generate a welfare improving real exchange rate appreciation, i.e. disutility from labour decreases without an accompanying decrease in the utility of consumption.

The presence of linear terms in the welfare measure (4.25) means that we cannot determine the optimal policy, up to first order, using the welfare measure subject to the structural equations that are only accurate to first order. Following the method proposed by Benigno and Woodford (2005) and Benigno and Benigno (2005), we substitute the linear terms in the approximated welfare function (4.25) by using a second order approximation to some of the structural conditions.[11] As a result, we obtain the fully quadratic loss function which can be represented as a function of aggregate output ($\widehat{Y}_t$), domestic and international terms of trade ($\widehat{T}_t^d$, $\widehat{T}_t$), domestic sector inflation rates ($\widehat{\pi}_{H,t}$, $\widehat{\pi}_{N,t}$) and revenue tax rates in the nontraded and home traded sector ($\widehat{\tau}_{N,t}$, $\widehat{\tau}_{H,t}$). Its general expression is given below:

[11] Details of the derivation can be found in Appendix D.4.2.

$$\min L_{t_0} = U_C\overline{C}\sum_{t=t_0}^{\infty}\beta^{t-t_0}[\frac{1}{2}\Phi_y(\widehat{Y}_t-\widehat{Y}_t^T)^2+\frac{1}{2}\Phi_{T^d}(\widehat{T}_t^d-\widehat{T}_t^{dT})^2+ \tag{4.26}$$

$$\frac{1}{2}\Phi_T(\widehat{T}_t-\widehat{T}_t^T)^2+\frac{1}{2}\Phi_{\tau_N}(\widehat{\tau}_{N,t}-\widehat{\tau}_{N,t}^T)^2+\frac{1}{2}\Phi_{\tau_H}(\widehat{\tau}_{H,t}-\widehat{\tau}_{H,t}^T)^2 \tag{4.27}$$

$$+\Phi_{YT^d}\widehat{Y}_t\widehat{T}_t^d+\Phi_{YT}\widehat{Y}_t\widehat{T}_t+\Phi_{T^dT}\widehat{T}_t^d\widehat{T}_t+ \tag{4.28}$$

$$+\widehat{\tau}_{N,t}(\Phi_{Y\tau_N}\widehat{Y}_t+\Phi_{T\tau_N}\widehat{T}_t+\Phi_{T^d\tau_N}\widehat{T}_t^d)+ \tag{4.29}$$

$$+\widehat{\tau}_{H,t}(\Phi_{Y\tau_H}\widehat{Y}_t+\Phi_{T\tau_H}\widehat{T}_t+\Phi_{T^d\tau_H}\widehat{T}_t^d) \tag{4.30}$$

$$+\frac{1}{2}\Phi_{\pi_N}\widehat{\pi}_{N,t}^2+\frac{1}{2}\Phi_{\pi_H}\widehat{\pi}_{H,t}^2]+tip+O(3) \tag{4.31}$$

where $\widehat{Y}_t^T$, $\widehat{T}_t^{d^T}$, $\widehat{T}_t^T$, $\widehat{\tau}_{N,t}^T$, $\widehat{\tau}_{H,t}^T$ are target variables which are functions of the stochastic shocks and, in general, are different from the flexible price equilibrium processes of aggregate output, domestic terms of trade and international terms of trade.[12] The term *tip* stands for *terms independent of policy.* The coefficients Φ_Y, Φ_{T^d}, Φ_T, Φ_{τ_N}, Φ_{τ_H}, Φ_{TT^d}, Φ_{YT^d}, Φ_{YT}, Φ_{π_H}, Φ_{π_N}, $\Phi_{Y\tau_N}$, $\Phi_{T\tau_N}$, $\Phi_{T^d\tau_N}$, $\Phi_{Y\tau_H}$, $\Phi_{T\tau_H}$, $\Phi_{T^d\tau_H}$, are functions of the structural parameters of the model.

Similarly to Chapter 3 our loss function generalizes previous studies regarding optimal monetary policy characterization in both closed economy environments (Aoki (2001), Benigno (2004), Rotemberg and Woodford(1997)) and open economy environments (Gali and Monacelli (2005), De Paoli (2007)). Moreover this loss function is also related to the literature on optimal monetary and fiscal policy in the sticky price environment (Benigno and Woodford (2003), Benigno and De Paoli (2007), Schmitt - Grohe and Uribe (2003)). As in Benigno and Woodford (2003) and Benigno and De Paoli (2007) we obtain that variations in distortionary taxation should be chosen to serve the same objectives as those emphasized in the literature on monetary stabilization policy. Interestingly, our loss function also involves some stabilization of taxes.[13]

To simplify the exposition of the optimal plan we reduce number of variables to a set of eight domestic variables which determine the loss function (4.26), i.e. $\widehat{Y}_t$, $\widehat{T}_t^d$, $\widehat{T}_t$, $\widehat{\pi}_{H,t}$, $\widehat{\pi}_{N,t}$, $\widehat{\tau}_{N,t}$, $\widehat{\tau}_{H,t}$, $\widehat{d}_t$. In Appendix we present the structural equations of the two-sector small open closed economy in terms of these variables (see D.4.3). Finally, the policy maker following the optimal plan under commitment

[12] As previously shown in papers by Gali and Monacelli (2005) and De Paoli (2007), in the small open economy framework the target variables will be identical to the flexible price allocations only in some special cases, i.e. an efficient steady state, no markup shocks, no expenditure switching effect (i.e. $\rho\theta = 1$) and no trade imbalances. Moreover, as shown by Benigno and Woodford (2003) a non-zero steady state share of government expenditures in output affects the target variables.

[13] As it is going to be seen in our numerical analysis, this stabilisation feature is insignificant in quantitative terms.

chooses $\{\widehat{Y}_t, \widehat{T}_t^d, \widehat{T}_t, \widehat{\pi}_{H,t}, \widehat{\pi}_{N,t}, \widehat{\tau}_{N,t}, \widehat{\tau}_{H,t}, \widehat{d}_t\}_{t=t_0}^{\infty}$ in order to minimize the loss function (4.26) subject to the constraints (D.183)–(D.189), given the initial conditions on non-predetermined variables: $\widehat{Y}_{t_0}, \widehat{T}_{t_0}^d, \widehat{T}_{t_0}, \widehat{\pi}_{H,t_0}, \widehat{\pi}_{N,t_0}, \widehat{\tau}_{N,t_0}, \widehat{\tau}_{H,t_0}, \widehat{d}_{t_0}$. In accordance with the definition of the optimal plan from a timeless perspective (see Woodford (2003), p.538) we derive the first-order conditions of the problem for all $t \geq t_0$ (we present them in Appendix in order not to overload the main text, see equations (D.193)–(D.207)). Equations that represent first order conditions (D.193)–(D.207) and constraints (D.183)–(D.189) fully characterize behavior of the economy under the optimal policy.

4.4 The Maastricht convergence criteria – a reinterpretation

The Maastricht convergence criteria have a nonlinear nature as they set specific bounds on both monetary and fiscal variables. Subsequently, derivation of the optimal policy constrained by the Maastricht criteria would involve solving a nonlinear optimization problem that requires computationally demanding techniques. On the other hand, as already emphasized in Chapter 3 the linear quadratic approach has two important advantages: an analytical and intuitive expression for the loss function and also easy to check second-order conditions for local optimality of the derived policy. As a result, following Chapter 3 we reformulate the Maastricht criteria in order to introduce them as additional constraints faced by the optimal policy in the linear quadratic approach.

We consider three monetary criteria regarding CPI inflation rate, nominal interest rate and nominal exchange rate and also one fiscal criterion that sets an upper bound on deficit to GDP. We neglect the debt to GDP criterion as almost all the EMU accession countries are characterized by moderate debt to GDP ratios (see Figure (A.8)) that are smaller than the upper limit of 60% quoted in the Stability and Growth Pact. Moreover, in the past failure in compliance with debt to GDP criterion (on the contrary to the deficit to GDP criterion) was not treated as an obstacle to enter to the EMU (e.g. case of Belgium or Italy). This is in accordance with the stipulates of the Maastricht Treaty Article (see Appendix A).

Subsequently, we summarize the Maastricht criteria (described in Appendix A) by the following inequalities:

- CPI aggregate inflation criterion

$$\pi_t^A - \pi_t^{A,*} \leq B_\pi, \tag{4.32}$$

where $B_\pi = 1.5\%$, π_t^A is annual CPI aggregate inflation in the domestic economy,

$\pi_t^{A,*}$ is the average of the annual CPI aggregate inflations in the three lowest inflation countries of the European Union.

- nominal interest rate criterion

$$R_t^L - R_t^{L,A*} \leq C_R \tag{4.33}$$

where $C_R = 2\%$, R_t^L is the annul interest rate for ten-year government bond in the domestic economy, $R_t^{L,A*}$ is the average of the annual interest rates for ten-year government bonds in the three countries of the European Union with the lowest inflation rates.

- nominal exchange rate criterion

$$(1 - D_S)S \leq S_t \leq (1 + D_S)S, \tag{4.34}$$

where $D_S = 15\%$ and S is the central parity between euro and the domestic currency and S_t is the nominal exchange rate.

- deficit to GDP criterion

$$df_t \leq F_{df} \tag{4.35}$$

where $F_{df} = 3\%$ and df_t - annual deficit to GDP. In our framework deficit is defined as a sum of interest payments on outstanding debt minus the primary surplus that consists of tax revenues and government expenditures.

We decide to impose a number of adjustments on the original form of the Maastricht criteria. First, these adjustments originate from the structure of the model which assumes that there are two countries in the world and dynamics are explained in quarters.[14] As a result, we assume that the variables $\pi_t^{A,*}$ and $R_t^{L,A*}$, respectively, represent foreign aggregate inflation and the foreign nominal interest rate, i.e. $\widehat{\pi}_t^*$, $\widehat{R}_t^*$ (which are proxied to be the euro area variables). Subsequently, all the four criteria are reformulated in quarterly terms. We change appropriately upper bounds regarding the CPI inflation rate and the nominal interest rate, i.e. $B_\pi \equiv ((1.015)^{0.25} - 1)$, $C_R \equiv ((1.02)^{0.25} - 1)$. Additionally, we define the central parity of the nominal exchange rate as the steady state value of the nominal exchange rate ($S = \overline{S}$). Moreover, taking into account the evidence on the predominance of domestic shocks in the EMU accession countries (see Fidrmuc and Kirhonen (2003)) we assume that foreign economy is in

[14] A detailed explanation regarding the reformulation of monetary criteria can be found in Chapter 3.

the steady state (i.e. foreign inflation and foreign nominal interest rate $(\widehat{\pi}_t^*, \widehat{R}_t^*)$ are zero).[15]

Second, using the method proposed by Rotemberg and Woodford (1997, 1999) and Woodford (2003) we approximate the Maastricht criteria in order to implement them into the linear quadratic framework. The authors propose to approximate the zero bound constraint for the nominal interest rate by restricting the mean of the nominal interest rate to be at least k standard deviations higher than the theoretical lower bound, where k is a sufficiently large number to prevent frequent violation of the original constraint. The main advantage of this alternative constraint over the original one is that it is much less computationally demanding and it only requires computation of the first and second moments of the nominal interest rate.

Similarly to Woodford (2003), we redefine the criteria using discounted averages in order to conform with the welfare measure used in our framework. Let us remark that the average value of any variable (x_t) is defined as the discounted sum of the conditional expectations, i.e.:

$$\widehat{m}(x_t) \equiv E_{t_0} \sum_{t=t_0}^{\infty} \beta^t x_t. \tag{4.36}$$

Accordingly, its variance is defined by:

$$\widehat{var}(x_t) \equiv E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (x_t - \widehat{m}(x_t))^2. \tag{4.37}$$

Below, we show the reformulated Maastricht convergence criteria.[16] Each criterion is presented as a set of two inequalities:

- CPI aggregate inflation criterion:

$$(1-\beta) E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (B_\pi - \widehat{\pi}_t) \geq 0, \tag{4.38}$$

$$(1-\beta) E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (B_\pi - \widehat{\pi}_t)^2 \leq K \left((1-\beta) E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (B_\pi - \widehat{\pi}_t) \right)^2; \tag{4.39}$$

- nominal interest rate criterion:

$$(1-\beta) E_{t_0} \sum_{t=t_0}^{\infty} \beta^t (C_R - \widehat{R}_t) \geq 0 \tag{4.40}$$

[15] In Chapter 3 we discuss the consequences of relaxing this assumption (e.g. a departure from the steady state of the foreign economy or a suboptimal foreign monetary policy) for the nature of optimal policy constrained by the Maastricht criteria and the associated welfare loss.

[16] The detailed derivation of the Maastricht convergence criteria can be found in Appendix D.5.

$$(1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t(C_R-\widehat{R}_t)^2 \leq K\left((1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t(C_R-\widehat{R}_t)\right)^2 \tag{4.41}$$

- nominal exchange rate criterion must be decomposed into two systems of the inequalities, i.e. the upper bound and the lower bound:
  - upper bound

$$(1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t(D_S-\widehat{S}_t) \geq 0 \tag{4.42}$$

$$(1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t(D_S-\widehat{S}_t)^2 \leq K\left((1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t(D_S-\widehat{S}_t)\right)^2 \tag{4.43}$$

  - lower bound

$$(1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t(D_S+\widehat{S}_t) \geq 0 \tag{4.44}$$

$$(1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t(D_S+\widehat{S}_t)^2 \leq K\left((1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t(D_S+\widehat{S}_t)\right)^2 \tag{4.45}$$

- deficit to GDP criterion:

$$(1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t(F_{df}-\widehat{df}_t) \geq 0, \tag{4.46}$$

$$(1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t(F_{df}-\widehat{df}_t)^2 \leq K\left((1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t(F_{df}-\widehat{df}_t)\right)^2; \tag{4.47}$$

where $K = 1 + k^{-2}$ and $D_S = 15\%$, $B_\pi = (1.015)^{0.25} - 1$, $C_R = (1.02)^{0.25} - 1$, $F_{df} = 3\%$ and $k = 1.96$.

The first inequality means that the average values of the CPI inflation rate, the nominal interest rate, the nominal exchange rate and deficit to GDP, respectively, should not exceed the bounds, B_π, C_R and D_S and F_{df}. The second inequality further restrains fluctuations in the Maastricht variables by setting an upper bound on their variances. This upper bound depends on the average values of the Maastricht variables and the bounds, B_π, C_R, D_S and F_{df}. Importantly, it also depends on parameter K which guarantees that the original constraints on the Maastricht variables ((4.32)–(4.35)) are satisfied with a high probability. Under a normality assumption, by setting $K = 1+1.96^{-2}$, we obtain that fulfillment of inequalities (4.38)–(4.47) guarantees that each of the original constraints should be met with a probability of 95%.

Summing up, the set of inequalities (4.38)–(4.47) represent the Maastricht convergence criteria in our model.

4.5 Optimal policy constrained by the Maastricht criteria

Following Woodford (2003) and Chapter 3 we present the loss function of the optimal policy constrained by the Maastricht convergence criteria summarized by inequalities (4.38)–(4.45) (constrained optimal policy). The loss function of the constrained optimal policy is augmented by the new elements which describe fluctuations in monetary and fiscal variables, i.e.: CPI aggregate inflation, the nominal interest rate, the nominal exchange rate and deficit to GDP ratio.

We state a proposition which can be seen as an extension of the Proposition 1 in Chapter 3:[17]

Proposition 2 *Consider the problem of minimizing an expected discounted sum of quadratic losses:*

$$E_{t_0}\left\{(1-\beta)\sum_{t=t_0}^{\infty}\beta^t L_t\right\}, \tag{4.48}$$

subject to (4.38) - (4.47). Let $m_{1,\pi}$, $m_{1,R}$, $m^U_{1,S}$, $m^L_{1,S}$, $m_{1,df}$ *be the discounted average values of* $(B_\pi-\widehat{\pi}_t)$, $(C_R-\widehat{R}_t)$, $(D_S-\widehat{S}_t)$, $(D_S+\widehat{S}_t)$, $(F_{df}-\widehat{df}_t)$ *and* $m_{2,\pi}$, $m_{2,R}$, $m^U_{2,S}$, $m^L_{2,S}$, $m_{2,df}$ *be the discounted means of* $(B_\pi-\widehat{\pi}_t)^2$, $(C_R-\widehat{R}_t)^2$, $(D_S-\widehat{S}_t)^2$, $(D_S+\widehat{S}_t)^2$, $(F_{df}+\widehat{df}_t)^2$ *associated with the optimal policy. Then, the optimal policy also minimizes a modified discounted loss criterion of the form (4.48) with* L_t *replaced by:*

$$\widetilde{L}_t \equiv L_t+\Phi_\pi(\pi^T-\widehat{\pi}_t)^2+\Phi_R(R^T-\widehat{R}_t)^2+\Phi_{S,U}(S^{T,U}-\widehat{S}_t)^2 \tag{4.49}$$

$$+\Phi_{S,L}(S^{T,L}-\widehat{S}_t)^2+\frac{1}{2}\Phi_{df}(df^T-df_t)^2, \tag{4.50}$$

under constraints represented by the structural equations of an economy. Importantly, $\Phi_\pi \geq 0$, $\Phi_R \geq 0, \Phi_{S,U} \geq 0$, $\Phi_{S,L} \geq 0$, $\Phi_{df} \geq 0$ *and take strictly positive values if and only if the respective constraints (4.39), (4.41), (4.43), (4.45), (4.47) are binding. Moreover, if the constraints are binding, the corresponding target values* π^T, R^T, $S^{T,U}$, $S^{T,L}$, df^T *satisfy the following relations:*

$$\pi^T = B_\pi - Km_{1,\pi} < 0 \tag{4.51}$$

$$R^T = C_R - Km_{1,R} < 0 \tag{4.52}$$

$$S^{T,U} = D_S - Km_{1,S} < 0 \tag{4.53}$$

$$S^{T,L} = -D_S + Km_{1,S} > 0 \tag{4.54}$$

$$df^T = F_{df} - Km_{1,df} < 0. \tag{4.55}$$

[17] Both propositions are based on Proposition 6.9 (p.428) in Woodford (2003).

Proof can be found in Appendix D.6.

In the presence of binding constraints, the optimal policy constrained by the Maastricht criteria do not only lead to smaller variances of the Maastricht variables, it also assigns target values for these variables that are different from the deterministic steady state of the optimal policy. These targets reflect precautionary motive of the constrained policy.[18] In other words, the policy maker needs a buffer when it faces inequality constraints.

As in Chapter 3, if the monetary constraints on the CPI inflation or the nominal interest rate are binding, the constrained policy maker sets targets on these variables that are lower than their foreign equivalents. As far as the nominal exchange rate is concerned, depending on which: appreciation or depreciation constraint is binding, constrained policy maker will target, respectively, a more depreciated or more appreciated nominal exchange rate. Finally, when deficit to GDP criterion is binding, the constrained policy maker will target surplus to GDP.

4.6 Numerical exercise

This section characterizes the optimal monetary and fiscal policy for the economy bound to satisfy the Maastricht convergence criteria. First, we characterize the unconstrained optimal monetary and fiscal policy and control whether such a policy violates any of the Maastricht convergence criteria. Second, we characterize the optimal policy which is only constrained by monetary criteria or fiscal criterion. We analyze how the loss functions are augmented and also the stabilization pattern of the constrained policies. Finally, we describe the optimal policy constrained by all the criteria. We identify which criteria are in quantitative terms important in shaping the constrained policy and also compare the welfare losses among the constrained and unconstrained policy.

4.6.1 Parameterization

Our calibration follows to a great extent the previous analysis in Chapter 3 and also literature on the EMU accession countries (i.e. Laxton and Pesenti (2003) and Natalucci and Ravenna (2007)). We calibrate the model to match the moments of the variables for the Czech Republic economy.

The discount factor, β, equals 0.99, which implies an annual interest rate of around four percent. The coefficient of risk aversion in consumer preferences is set to 2 as in

[18]Similarly, Woodford (2003) shows that a policy maker constrained by the zero bound on the nominal interest rate targets a positive rate of the nominal interest rate.

Stockman and Tesar (1995) to get an intertemporal elasticity of substitution equal to 0.5. Inverse of the labour supply elasticity (η) is chosen to be 4 following the micro data evidence and also a small open economy model of Gali and Monacelli (2003). The elasticity of substitution between nontradable and tradable consumption, ϕ, is set to 0.5 as in Stockman and Tesar (1994) and the elasticity of substitution between home and foreign tradable consumption, θ, is set to 1.5 (as in Chari et al. (2002) and Smets and Wouters (2004)). The elasticity of substitution between differentiated goods, σ, is equal to 10, which together with the revenue tax of 0.19[19] implies a markup of 1.37.[20]

The share of nontradable consumption in the aggregate consumption basket, μ, is assumed to be 0.42, while the share of foreign tradable consumption in the tradable consumption basket, λ, is assumed to be 0.4. These values correspond to the weights in CPI reported for the Czech Republic over the period 2000–2005.[21] The steady state shares of the government expenditure to GDP (d_G) and also debt to GDP (b_D) correspond to the average values for the Czech republic economy over the period 1995-2006 and are set to 0.2 and 1.6 respectively. As far as the foreign economy is concerned, we set the share of nontradable consumption in the foreign aggregate consumption basket, μ^*, to be 0.6, which is consistent with the value chosen by Benigno and Thoenisen (2003) regarding the structure of euro area consumption. Finally, the steady state share of debt to GDP in the foreign economy is assumed to be 2.4 which reflects an average debt to GDP ratio in the euro area for the period 1995-2005. Taking government expenditure to GDP and debt to GDP shares as given we obtain that the steady state revenue tax rate should be 19,3%.[22]

Following Natalucci and Ravenna (2007), we set the degree of price rigidity in the nontraded sector, α_N, to 0.85. The degree of price rigidity in the traded sector, α_H, is slightly smaller and equals 0.8. These values are somewhat higher than the values reported in the micro and macro studies for the euro area countries.[23] Still, Natalucci and Ravenna (2007) justify them by a high share of the government regulated prices in the EMU accession countries.

[19] This value is calculated from the steady state budget constraint. Debt to GDP and steady state share of government expenditures to GDP are taken from the data on the Czech Republic economy.

[20] Martins et al. (1996) estimate the average markup for manufacturing sectors at around 1.2 in most OECD countries over the period 1980-1992. Some studies (Morrison (1994), Domowitz et al (1988)) suggest that the plausible estimates range between 1.2 and 1.7.

[21] Source: Eurostat.

[22] Note that the steady state which we present in the numerical exercise and in Appendix D.1 differs from the steady state used for calibration in two aspects: revenue tax rates can differ between the domestic sectors and are chosen in order to maximize welfare of the domestic consumers.

[23] Stahl (2004) estimates that the average duration between price adjustment in the manufacturing sector is nine months (which corresponds to the degree of price rigidity: 0.67). On the other hand, Gali et al (2001) and Benigno and Lopez-Salido (2003) estimate the aggregate supply relations for the European countries and find the overall degree of price rigidity for these countries to be 0.78.

All shocks that constitute the stochastic environment of the small open economy follow the AR(1) process. The parameters of the shocks are chosen to match the historical moments of the variables. Similarly to Natalucci and Ravenna (2007) and Laxton and Pesenti (2003), the productivity shocks in both domestic sectors are characterized by a strong persistence parameter equal to 0.85. Standard deviations of the productivity shocks are set to 1.6% (nontraded sector) and 1.8% (traded sector). These values roughly reflect the values chosen by Natalucci and Ravenna (2007), 1.8% (nontraded sector) and 2% (traded sector). Moreover, the productivity shocks are strongly correlated, their correlation coefficient is set to 0.7.[24] All other shocks are independent of each other. Parameters defining the preference shock are, 0.72% (standard deviation) and 0.95 (persistence parameter). These values are similar to the values chosen by Laxton and Pesenti (2003), 0.4% (standard deviation) and 0.7 (persistence parameter). Parameters of the foreign consumption shock are estimated using quarterly data on aggregate consumption in the euro area over the period 1990-2005 (source: Eurostat). The standard deviation of the foreign consumption shock is equal to 0.23% and its persistence parameter is 0.85. Similarly, parameters of the domestic government expenditure shock are estimated based on quarterly data on the final consumption of general government in the Czech Republic over the period 1995-2006. The standard deviation of the domestic government expenditure shock is equal 2% and its persistence parameter is 0.5.

Following Natalucci and Ravenna (2007), we parametrize the monetary policy rule, i.e. the nominal interest rate follows the rule described by: $\widehat{R}_t = 0.9\widehat{R}_{t-1} + 0.1(\widehat{\pi}_t + 0.2\widehat{Y}_t + 0.3\widehat{S}_t) + \widehat{\varepsilon}_{R,t}$, where $\widehat{\varepsilon}_{R,t}$ is the monetary policy innovation with a standard deviation equal to 0.44%. Such a parametrization of the monetary policy rule enables us to closely match the historical moments of the Czech economy. As far as the fiscal policy is concerned, we choose a fiscal rule described in Duarte and Wolman (2003). The rule takes a form of tax rate adjustment to debt to GDP dynamics: $\tau_t = \tau_{t-1} + \alpha_{b,\tau}(b_t - \overline{b}) + \alpha_{\Delta b,\tau}(b_t - b_{t-1})$. Parameters of the rule are taken from Mitchell et al (2002) and are set to $\alpha_{b,\tau} = 0.04/16$, $\alpha_{\Delta b,\tau} = 0.3/4$.

We summarize all parameters described above in Table D.1 (Structural parameters) and Table D.2 (Stochastic environment) in Appendix D.3. Moreover Table D.3 (Matching the moments) in Appendix D.3 compares the model moments with the historical moments for the Czech Republic economy.

[24] Empirical evidence shows that productivity shocks are highly persistent and positively correlated (see Backus et al (1992)).

4.6.2 The unconstrained optimal policy

The optimal monetary and fiscal policy is characterized around the optimal steady state. The optimal steady state is defined as a steady state in which revenue taxes in all the sectors are chosen to maximize welfare of the economies given exogenous share of government expenditure to GDP and debt to GDP. The optimal tax rates in the domestic sectors are equal respectively, in the nontraded sector: $\overline{\tau_N} = -0.38$ and in the home traded sector: $\overline{\tau_H} = 0.57$. The implied tax base (total tax revenue to GDP ratio) is equal to 21%.

As in Chapter 3 we obtain that the highest penalty coefficient is assigned to fluctuations in the nontradable sector inflation and home tradable inflation (see Table 4.1). The optimal monetary and fiscal policy is mainly concerned with stabilization of the domestic inflation, which is in line with core inflation targeting argument (Aoki (2001)) and also literature on the optimal monetary and fiscal policy (e.g. Benigno and Woodford (2003), Benigno and De Paoli (2006)). Moreover, the policy faces also trade-offs between stabilizing the output gap and sector inflation which is reflected in the positive values of the penalty coefficients associated with fluctuations of domestic and international terms of trade.

Similarly to the previous literature on optimal taxation (among others Barro (1979), Aiyagari et al (2002), Schmitt-Grohe and Uribe (2004), Benigno and Woodford (2003)) we find that the optimal policy does not stabilize taxes and debt which implies their nonstationary behaviour. We decide to induce stationarity into the model in order to be able to characterize properly the constrained policy. As already analyzed by Woodford (2003) and in Chapter 3, the constrained policy is characterized by two components: stabilization one (a coefficient that penalizes fluctuations of a variable of interest) and deterministic one (a target value for a variable of interest). While the stabilization component affects the way policy responds to the shocks, the deterministic component affects the steady state of the optimal policy and therefore also discounted means of the variables. Importantly, due to the nonstationarity of debt and taxes the steady state of the policy would not exist. In order to induce stationarity we add a new element to the original loss function which penalizes debt fluctuations.[25] Value of the coefficient (ϕ_d) is chosen to be quantitatively small in order not to affect dynamics of the model, i.e. $\phi_d = 10^{-4}$.[26]

[25] This additional element is a bit ad-hoc although it is motivated by an idea of model stationarization by Schmitt-Grohe and Uribe (2003). Alternatively, if one assumes that government debt is denominated in foreign currencies introduction of portfolio adjustment costs (presented by Schmitt-Grohe and Uribe (2003)) would also stationarize the model.

[26] For the purposes of sensitivity analysis we also present the results for $\phi_d = 10^{-5}$ and also for the unconstrained policy $\phi_d = 0$.

Φ_{π_N}	Φ_{π_H}	Φ_Y	Φ_{T^d}	Φ_T	Φ_{TT^d}	Φ_{YT^d}	Φ_{YT}
123.14	35.29	4.26	0.2	0.17	0.02	-0.58	-0.32

Table 4.1: Loss function coefficients under the optimal monetary and fiscal policy

In order to understand nature of the optimal policy we investigate how the optimal policy responds to the shocks. Based on variance decomposition of the Maastricht variables (presented in Table 4.2) we choose to analyze the impulse responses to a nontraded productivity shock.

The Maastricht variables:	A_N	A_H	B	C^*	G
CPI inflation	86%	4%	5%	2%	3%
nominal interest rate	87%	4%	1%	1%	7%
nominal exchange rate	79%	3%	15%	2%	1%
deficit to GDP	70%	1%	20%	1%	10%
debt to GDP	74%	1%	21%	1%	3%

Table 4.2: Variance decomposition of the Maastricht variables under the optimal monetary and fiscal policy

Similarly to Chapter 3, monetary instrument of the optimal policy - the nominal interest rate decreases in response to a positive nontraded productivity shock. This stabilizes the deflationary pressures in the domestic nontraded sector and at the same time supports increase in aggregate output. As a result, the nominal exchange rate depreciates (in accordance with the uncovered interest rate parity condition). Interestingly, fiscal component of the policy is characterized by a countercyclical behavior. Such a behavior of taxes has its origin in a specific structure of the economy, i.e. openness and two domestic sectors. First of all, as already studied by Benigno and De Paoli (2006) an open economy nature of the economy gives the optimal policy maker an incentive to use taxes in the countercyclical way thanks to existence of terms of trade externality.[27] By setting higher taxes in the sector where the shock occurred the optimal policy maker can engineer a welfare-improving real exchange rate appreciation. Secondly, two sector structure creates important trade-offs for the optimal policy maker. This trade-off was already studied by Gali and Monacelli (2005) in a

[27] This incentive is present under an assumption of the substitutability between home and foreign goods.

model of monetary union, where monetary and fiscal policy are set optimally under full coordination. In their model, each country's fiscal authority faces a trade-off between stabilization of domestic inflation as opposed to output and fiscal gap. Since the cost of inflation is higher than of the changes in distortionary taxation the optimal policy maker allows for fluctuations in the fiscal instruments.[28] As a result, in our model revenue taxes in the nontraded sector rise in order to stabilize the nontraded output and deflation in the nontraded sector. At the same time, revenue taxes in the home traded sector decrease to stabilize the home traded output and inflationary pressures in this sector.

Consequently, as in Gali and Monacelli (2005) domestic inflation stabilizes. Moreover, output increases in both domestic sectors. Finally, since the overall tax revenues rise deficit to GDP and debt to GDP decrease.

[*Figure* (4.1) *about here*]

Let us investigate now which Maastricht criteria are not satisfied by the optimal policy. Under the optimal policy means of all the variables are zero so the reinterpreted Maastricht criteria can be reduced to the constraints that set upper bounds on the variances of the Maastricht variables, i.e.:

$$\widehat{var}(\widehat{\pi}_t) \leq (K-1)B_{\pi}^2 \tag{4.56}$$

$$\widehat{var}(\widehat{R}_t) \leq (K-1)C_R^2 \tag{4.57}$$

$$\widehat{var}(\widehat{S}_t) \leq (K-1)D_S^2 \tag{4.58}$$

$$\widehat{var}(\widehat{df}_t) \leq (K-1)F_{df}^2 \tag{4.59}$$

where $\widehat{var}(x_t)$ with $x_t = \widehat{\pi}_t$, $\widehat{R}_t$, $\widehat{S}_t$, $\widehat{df}_t$ is defined by (4.37).

In the table below (Table 4.3) we present the variances of the Maastricht variables together with the upper bounds implied by the Maastricht criteria. We also show variance of debt to GDP and a respective bound for this variables in accordance with the limit set out in the Maastricht Treaty. Let us note that although variances of debt and deficit to GDP ratio do depend on the chosen value of the coefficient ϕ_d variances of other Maastricht variables do not.

[28] Notice however that on the contrary to our model Gali and Monacelli (2005) study a demand side fiscal instrument, i.e. government expenditures.

ϕ_d	$\widehat{\pi}_t$	$\widehat{R}_t$	$\widehat{S}_t$	$\widehat{df}_t$	$\widehat{b}_t$
0	0.5808	0.4173	22.3785	4.1889	2392.8873
ϕ_d	$\widehat{\pi}_t$	$\widehat{R}_t$	$\widehat{S}_t$	$\widehat{df}_t$	$\widehat{b}_t$
10^{-4}	0.5808	0.4088	22.8337	3.3054	702.2339
ϕ_d	$\widehat{\pi}_t$	$\widehat{R}_t$	$\widehat{S}_t$	$\widehat{df}_t$	$\widehat{b}_t$
10^{-5}	0.5808	0.4156	22.5169	4.0013	1758.4676
bound	0.0356	0.0651	58.5693	2.3428	650.7705
probability	0.69	0.78	1	0.92	0.99
criterion	*violated*	*violated*	*satisfied*	*violated*	*satisfied*
note: Variances and bounds are multiplied by 100^2 (in $(\%)^2$)					

Table 4.3: Variances of the Maastricht variables under optimal monetary and fiscal policy

The optimal unconstrained policy does not satisfy three of the Maastricht criteria: the CPI inflation criterion, the nominal interest rate criterion and the deficit to GDP criterion.[29] As a result, the loss function of optimal policy that satisfies the Maastricht criteria has to have some additional elements.

4.6.3 The constrained optimal policy

The optimal policy constrained by monetary criteria

Now we analyze the policy constrained only by monetary criteria: CPI inflation rate and the nominal interest rate. In particular, we examine whether in the presence of the monetary criteria fiscal policy can act as an additional stabilization tool.

First, we present parameters of the loss function associated with this constrained policy. The loss function takes the following form:

$$\widetilde{L}_t^m = L_t^s + \frac{1}{2}\phi_\pi(\pi^T - \widehat{\pi}_t)^2 + \frac{1}{2}\phi_R(R^T - \widehat{R}_t)^2 \tag{4.60}$$

where $\phi_\pi > 0$, $\phi_R > 0$ and $\pi^T < 0$, $R^T < 0$. Similarly to the policy constrained only by the fiscal criterion, values of parameters $(\phi_\pi, \phi_R, \pi^T, R^T)$ can be obtained from the solution to the minimization problem of the loss function L_t^s constrained by structural equations and also the monetary constraints. Table 4.4 provides the specific values for all the parameters for two different values of the penalty coefficient on debt fluctuations. It appears that values of the parameters of the constrained policy by

[29] Since debt follows a near nonstationary and also very persistent process we perform a Monte Carlo simulation exercise in which we simulate our model for $T = 50$ periods and repeat this simulation $J = 1000$ times. Based on this, we can calculate the average probabilities, for each of the Maastricht variables, of compliance with the criteria. We assume that a given criterion is not satisfied if the probability for a given variable is lower than 95% (which is in accordance with the parameter k). According to this methodology debt to GDP criterion is satisfied.

monetary criteria do not depend to a great extent on the degree of debt stabilization (ϕ_d). Importantly, values of the penalty coefficients on the nominal interest rate and CPI inflation rate are of the same magnitude as the penalty coefficients of the domestic inflation rates in the original loss function. Deterministic component of the constrained policy tells us that the policy maker constrained by monetary criteria should target CPI inflation rate and the nominal interest rate that are 0.8% *p.a.* lower than in the countries of reference.

ϕ_d	Φ_π	Φ_R	π^T (in %)	R^T (in %)
10^{-4}	36	31.1	-0.2082	-0.2331
10^{-5}	35.62	31.12	-0.2005	-0.2250

Table 4.4: The augmented loss function coefficients under the policy constrained by monetary criteria

Second, we show moments of the Maastricht variables under the policy constrained by monetary criteria (see Table 4.5). As far as discounted means are concerned, negative targets of the CPI inflation rate and the nominal interest rate lead to negative means in all the Maastricht variables, except for the mean of debt to GDP. A higher mean of debt to GDP results from a higher mean of surplus to GDP and higher means of revenue taxes (to be seen later in the analysis of the impulse responses). Variances of the nominal variables: CPI inflation rate, the nominal interest rate and nominal exchange rate are lower than under the optimal unconstrained policy. However, this smaller variability of nominal variables is accompanied by much higher variability of the fiscal variables: deficit to GDP and debt to GDP. Compliance with monetary criteria restricts usage of the nominal interest rate as a stabilization tool and requires stronger movements in taxes. These fiscal instruments have a direct impact on domestic inflation rates and also dampen changes in the aggregate output when responding to shocks. Subsequently, surplus to GDP is characterized by much higher variance and so does deficit to GDP and debt to GDP.

Third, we analyze how the policy constrained by monetary criteria differs from the optimal unconstrained policy in the stabilization process of an economy hit by a shock. We choose the shock that explains the most of variability of the Maastricht variables (see Table 4.2 on variance decomposition).

ϕ_d		$\widehat{\pi}_t$	$\widehat{R}_t$	$\widehat{S}_t$	$\widehat{df}_t$	$\widehat{b}_t$
10^{-4}	mean (in %)	-0.0667	-0.0688	-6.5996	-0.0969	1.5020
	variance	0.0836	0.0501	15.7296	15.9482	1419.7149
ϕ_d		$\widehat{\pi}_t$	$\widehat{R}_t$	$\widehat{S}_t$	$\widehat{df}_t$	$\widehat{b}_t$
10^{-5}	mean (in %)	-0.0641	-0.0662	-6.3404	-0.0760	2.5511
	variance	0.0828	0.0495	15.2364	17.1314	3137.7899
criterion		$satisfied$	$satisfied$	$satisfied$	$violated$	$violated$
note: Variances are multiplied by 100^2 (in $(\%)^2$)						

Table 4.5: Moments of the Maastricht variables under the policy constrained by monetary criteria

[$Figure$ (4.2) $about$ $here$]

The policy constrained by monetary criteria aims at stabilizing CPI inflation and restricts the nominal interest rate movements. Accordingly, the monetary policy increases nominal interest rate on impact. Thanks to this, nominal exchange rate depreciates by less dampening the inflationary impact of the import sector on the aggregate CPI. However such a contractionary behavior of the monetary policy leads to stronger deflationary pressures in the domestic sector. The domestic deflation is partly stabilized by the fiscal component of the constrained policy which is more countercyclical than the unconstrained policy, i.e. revenue taxes rise in both domestic sectors. This leads to a much stronger decrease in deficit to GDP, debt to GDP and also dampened increase in domestic aggregate output in comparison with the unconstrained policy.

The optimal policy constrained by fiscal criterion

Let us now present the constrained policy by the fiscal criterion: deficit to GDP criterion. We concentrate on how the fiscal criteria affect the ability of fiscal policy to stabilize business cycle fluctuations. The loss function of the policy constrained by the deficit to GDP criterion can be represented in the following way:

$$\widetilde{L}_t^f = L_t^s + \frac{1}{2}\phi_{df}(df^T - df_t)^2 \tag{4.61}$$

where $L_t^s = L_t + \phi_d \widehat{d}_t$ and $\phi_{df} > 0$ and $df^T < 0$. The solution to the minimization problem of the loss function L_t^s constrained by structural equations and also the constraint on deficit to GDP gives us values for the parameters ϕ_{df} and df^T. We present these values in Table 4.6 for two different values of the coefficient on debt stabilization.

ϕ_d	ϕ_{df}	df^T (%)
10^{-4}	0.0304	-1.58
10^{-5}	0.0219	-6

Table 4.6: The augmented loss function coefficients under the policy constrained by fiscal criterion

Values of the penalty coefficient on the deficit to GDP fluctuations are small in comparison with penalty coefficients associated with the variables present in the loss function. At the same time, deterministic component of the constrained policy involves targeting surplus to GDP equal to 1.6%. The sensitivity analysis reveals that parameters of the augmented loss function do depend on the chosen value of ϕ_d. However the general pattern of the constrained policy is the same. The goal of complying with the deficit to GDP criterion is achieved rather through deterministic component than the stabilization one.

Let us now check how the optimal policy constrained by deficit to GDP criterion affects compliance of the monetary criteria. In Table 4.7 we present the means and variances of all the Maastricht variables and also report whether each of the criteria is satisfied (based on the inequalities (4.32)–(4.35)).

ϕ_d		$\widehat{\pi}_t$	$\widehat{R}_t$	$\widehat{S}_t$	$\widehat{df}_t$	$\widehat{b}_t$
10^{-4}	mean (in %)	$-4 * 10^{-4}$	$2 * 10^{-6}$	-0.0447	-0.0586	-3.6984
	variance	0.5802	0.3951	23.7452	2.4352	582.6315
ϕ_d		$\widehat{\pi}_t$	$\widehat{R}_t$	$\widehat{S}_t$	$\widehat{df}_t$	$\widehat{b}_t$
10^{-5}	mean (in %)	-0.0001	0.0044	-0.0301	-0.4855	-30.6188
	variance	0.5801	0.4030	23.0651	3.1624	1473.1135
	criterion	*violated*	*violated*	*satisfied*	*satisfied*	*satisfied*
	note: Variances are multiplied by 100^2 (in $(\%)^2$)					

Table 4.7: Moments of the Maastricht variables under the policy constrained by fiscal criterion

Although the effects of a nonzero target for deficit to GDP are quantitatively small we can see that a negative target for deficit to GDP results in smaller discounted means of inflation, the nominal exchange rate and (by definition) also of debt to GDP. On the other hand, mean of the nominal interest rate (and also of aggregate output) increases as a result of the smaller means of revenue taxes (to be seen later when analyzing the impulse responses). Moreover, a smaller variance in the deficit to GDP triggers smaller variances of the CPI inflation and the nominal interest rate. At the same time, variance of the nominal exchange rate increases (this is in line with a higher variance of aggregate output - to be later seen in the analysis of impulse responses).

In order to understand how the nature of the policy constrained by deficit to GDP criterion differs from the optimal unconstrained policy, we analyze how both policies respond to the shocks. As previously, we concentrate on impulse responses to a positive nontraded productivity shock.

[*Figure* (4.3) *about here*]

The policy constrained by fiscal criterion restricts fluctuations of deficit to GDP. Accordingly, the fiscal component of the constrained policy has a more procyclical nature than the unconstrained policy, i.e. nontraded taxes increase by less and at the same time home traded taxes decrease by more. Interestingly, the monetary policy component of the constrained policy is more contractionary than under the unconstrained policy. Nominal interest rate decreases by less on impact than under the unconstrained policy leading to a smaller decline in debt interest payments. As a result, deficit to GDP decreases by less (surplus to GDP increases by less) and so does the debt to GDP. Moreover, the constrained policy is characterized by higher on impact nominal exchange rate depreciation than under the unconstrained policy. This is consistent with a slightly higher aggregate output (due to lower taxes). Finally, a higher nominal exchange rate depreciation leads to a higher on impact CPI inflation under the constrained policy.

The optimal policy constrained by all the Maastricht criteria

Having analyzed the impact of monetary and fiscal criteria separately on the optimal policy, we turn to the characterization of the optimal policy that complies at the same time with the monetary and fiscal criteria. In particular, we analyze which criteria: put more constraints on the optimal policy.

Similarly to previous sections, we present the parameters of such a policy, its moments and also response of the constrained policy to a positive nontraded productivity shock. Apart from that, we analyze welfare losses associated with the constrained policy and compare them with the loss of the optimal unconstrained policy. We also analyze which criteria: monetary or fiscal contribute the most to the generated loss under the constrained policy.

The loss function of the policy constrained by fiscal criterion: deficit to GDP and the monetary criteria: CPI inflation and the nominal interest rate can be represented in the following form:

$$\widetilde{L}_t = L_t^s + \frac{1}{2}\phi_\pi(\pi^T - \widehat{\pi}_t)^2 + \frac{1}{2}\phi_R(R^T - \widehat{R}_t)^2 + \frac{1}{2}\phi_{df}(df^T - df_t)^2 \quad (4.62)$$

where $\phi_\pi > 0$, $\phi_R > 0$, $\phi_{df} > 0$ and $\pi^T < 0$, $R^T < 0$, $df^T < 0$. Values of the parameters of such a constrained policy are obtained from the solution to the minimization problem of the loss function (L_t^s) constrained by the structural equations and fiscal and monetary criteria. As can be seen in Table 4.8, penalty coefficients of all the variables of interest are higher than under the policies that are only constrained by fiscal or monetary criteria. This feature reflects conflicting targets of each of the constrained stabilization policies. As far as targets are concerned we detect significant differences for deficit to GDP. As previously, we observe that although targets of deficit to GDP and debt do depend on the chosen value of the coefficient ϕ_d values of the targets of the monetary variables are not so much sensitive.

ϕ_d	Φ_π	Φ_R	Φ_{df}	π^T (in %)	R^T (in %)	df^T (in %)
10^{-4}	36	39	0.26	-0.3601	-0.3684	-3.7143
10^{-5}	38	39	0.06	-0.2045	-0.1884	-20.5072

Table 4.8: The augmented loss function coefficients under the policy constrained by all the criteria

In Table 4.9 we show moments of the Maastricht variables under the optimal policy constrained by monetary and fiscal criteria. As in the case of the policy constrained by monetary criteria, nominal variables have negative means. Moreover, a negative target of deficit to GDP results in negative means of deficit to GDP and also debt to GDP (the negative effect of deficit to GDP on the mean of debt to GDP is stronger than the positive effect of CPI inflation and the nominal interest rate). Variances of the CPI inflation and the nominal interest rate are not significantly higher than under the policy constrained only by monetary criteria. On the other hand, variance of deficit to GDP is much higher than under the policy constrained only by the fiscal criterion. Variance of the nominal exchange rate is also a bit higher than under the policy constrained by monetary criteria.

Importantly, under chosen reinterpretation of the Maastricht criteria, the nominal exchange rate does not satisfy the lower (appreciation) bound of the Maastricht criteria. As we know, nominal exchange rate movements depend on the nominal interest rate behavior through the uncovered interest rate parity. But since the nominal exchange rate has a nonstationary character a smaller variance of the nominal interest rate actually increases persistence of the nominal exchange rate movements (in the extreme situation when nominal interest rate does not change the nominal exchange rate jumps to the new level on impact and does not change for subsequent periods). As a result, variance of the nominal exchange rate can overvalue variability of the nominal exchange rate. That is why, since the variance of the nominal interest rate is 6 times smaller under the constrained optimal policy than under the unconstrained optimal policy, we assume that the nominal exchange rate criterion is satisfied under the optimal policy constrained by all the criteria.

ϕ_d		$\widehat{\pi}_t$	$\widehat{R}_t$	$\widehat{S}_t$	$\widehat{df}_t$	$\widehat{b}_t$
10^{-4}	mean (in %)	-0.1235	-0.1228	-12.2950	-1.0508	-53.6591
	variance	0.1012	0.0629	20.7093	4.2714	520.4854
ϕ_d		$\widehat{\pi}_t$	$\widehat{R}_t$	$\widehat{S}_t$	$\widehat{df}_t$	$\widehat{b}_t$
10^{-5}	mean (in %)	-0.0720	-0.0621	-7.3238	-3.7156	-227.5543
	variance	0.0852	0.0485	17.5175	11.7396	2178.8886
criterion		*satisfied*	*satisfied*	*satisfied*[30]	*satisfied*	*satisfied*
note: Variances are multiplied by 100^2 (in $(\%)^2$)						

Table 4.9: Moments of the Maastricht variables under the policy constrained by all the criteria

Now we compare the impulse responses of the Maastricht variables to a positive nontraded productivity shock under the policy constrained by all criteria with the unconstrained optimal policy. The fiscal component of the constrained policy has a more countercyclical nature than the unconstrained policy in the first quarters, i.e. taxes in the nontraded sector increase by more than under the unconstrained policy, while taxes in the home traded sector decrease by less than under the unconstrained policy. This reflects a higher importance of the monetary criteria over the fiscal criterion. Still, changes in taxes are not as pronounced as under the policy constrained only by monetary criteria. That is why aggregate output increases by more and deficit to GDP decreases by less than under the policy constrained only by monetary criteria. Finally, the monetary component of the constrained policy features a contractionary behavior as the policy constrained only by monetary criteria, i.e. nominal interest rate increases on impact to prevent an increase in CPI inflation.

[*Figure* (4.4) *about here*]

Now, we analyze the welfare losses associated with each policy. In Table 4.10 we report the expected discounted welfare losses for the policies constrained by fiscal criterion alone, by monetary criteria alone and by all the criteria. Importantly, values of the losses are not very much sensitive to the chosen value of debt stabilization coefficient.

Compliance with the fiscal criterion does not induce substantial welfare losses. The welfare cost associated with such a policy is equal to 0.15% of the optimal policy loss. On the other hand, compliance with the monetary criteria generates additional welfare loss that amounts to 43% of the optimal policy loss. This result reflects the fact fiscal policy performs relatively poorly as an additional stabilization tool. The welfare losses come mainly from a higher variability of the domestic inflation rates. Obligation to satisfy both monetary and fiscal criteria involves more welfare costs. An active use of revenue taxes is limited to meet the bound on deficit to GDP variability. As a result, the policy constrained by monetary and fiscal criteria produces an additional welfare cost equal to 60% of the optimal policy loss.

ϕ^d	UOP	COP-deficit to GDP	COP-monetary criteria	COP-all
0	6.6683		-	
10^{-4}	6.7242	6.7310	9.5872	10.6250
10^{-5}	6.6786	6.6877	9.4945	9.9510
note: Losses are multiplied by 100^2 (in $(\%)^2$)				

Table 4.10: Welfare losses for the unconstrained and constrained optimal monetary and fiscal policy

4.7 Conclusions

This paper studies the optimal monetary and fiscal policy constrained by the Maastricht convergence criteria in a small open economy exposed to domestic and external shocks. We develop a DSGE model of a small open economy with nominal rigidities and distortionary taxation.

First, we characterize the optimal monetary and fiscal policy from a timeless perspective using the linear quadratic approach. We find that the optimal monetary and fiscal policy (unconstrained policy) should not only target inflation rates in the domestic sectors and aggregate output fluctuations but also domestic and international terms of trade. Second, we analyze how the monetary and fiscal criteria affect the general properties of the optimal policy. We show that the policy constrained by the Maastricht criteria differs from the unconstrained policy along two dimensions: the stochastic and deterministic one. As expected the constrained policy restricts fluctuations of the Maastricht variables. Moreover, such a policy, using a precautionary motive, also changes deterministic targets of the Maastricht variables in order to create an additional buffer.

We also perform a numerical exercise in which we parameterize our model to match the variability of the Czech Republic economy. We find that the optimal monetary and fiscal policy violates three Maastricht convergence criteria: on the CPI inflation rate, the nominal interest rate and deficit to GDP ratio. Similarly to Chapter 3, the constrained policy leads to a smaller variability of the CPI inflation and the nominal interest rate. The policy is characterized by a deflationary bias which results in targeting the CPI inflation rate and the nominal interest rate that are lower in annual terms by 1.3% than the CPI inflation rate and the nominal interest rate in the countries taken as a reference.

The constrained policy induces a higher variability of deficit to GDP ratio than under the unconstrained policy. This reflects the fact that monetary criteria play a dominant role in affecting the stabilization process of the constrained policy. Fiscal policy actively uses its instruments to stabilize the economy in the presence of direct constraints on monetary instruments. At the same time it has to assign a relatively high surplus to GDP ratio target in order to comply with all the criteria. The welfare costs of the constrained policy are quite substantial and amount to 60% of the initial deadweight loss associated with the optimal unconstrained policy.

4.A Figures - Comparison of different policies

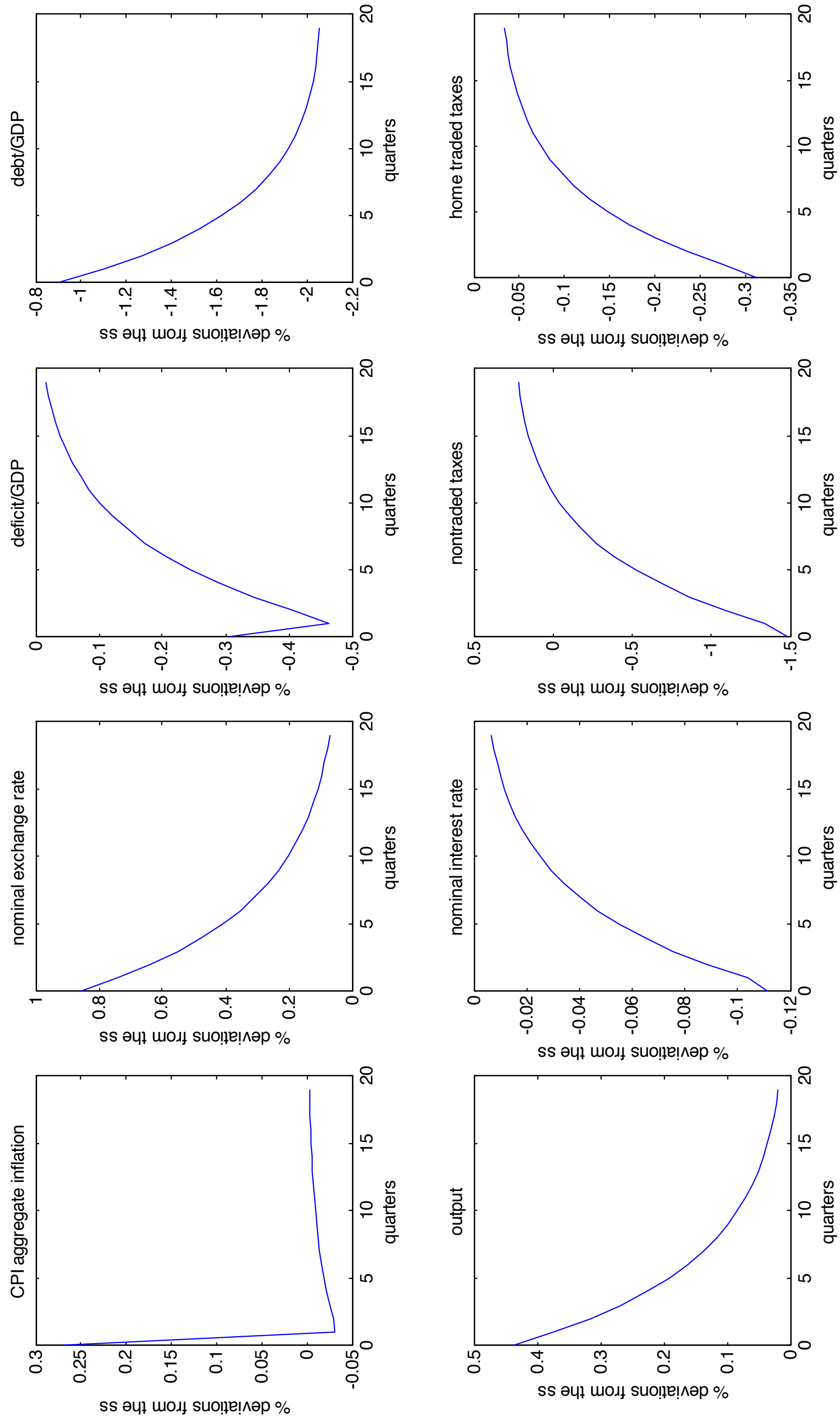

Figure 4.1: Impulse responses to a positive domestic nontraded productivity shock - unconstrained optimal policy

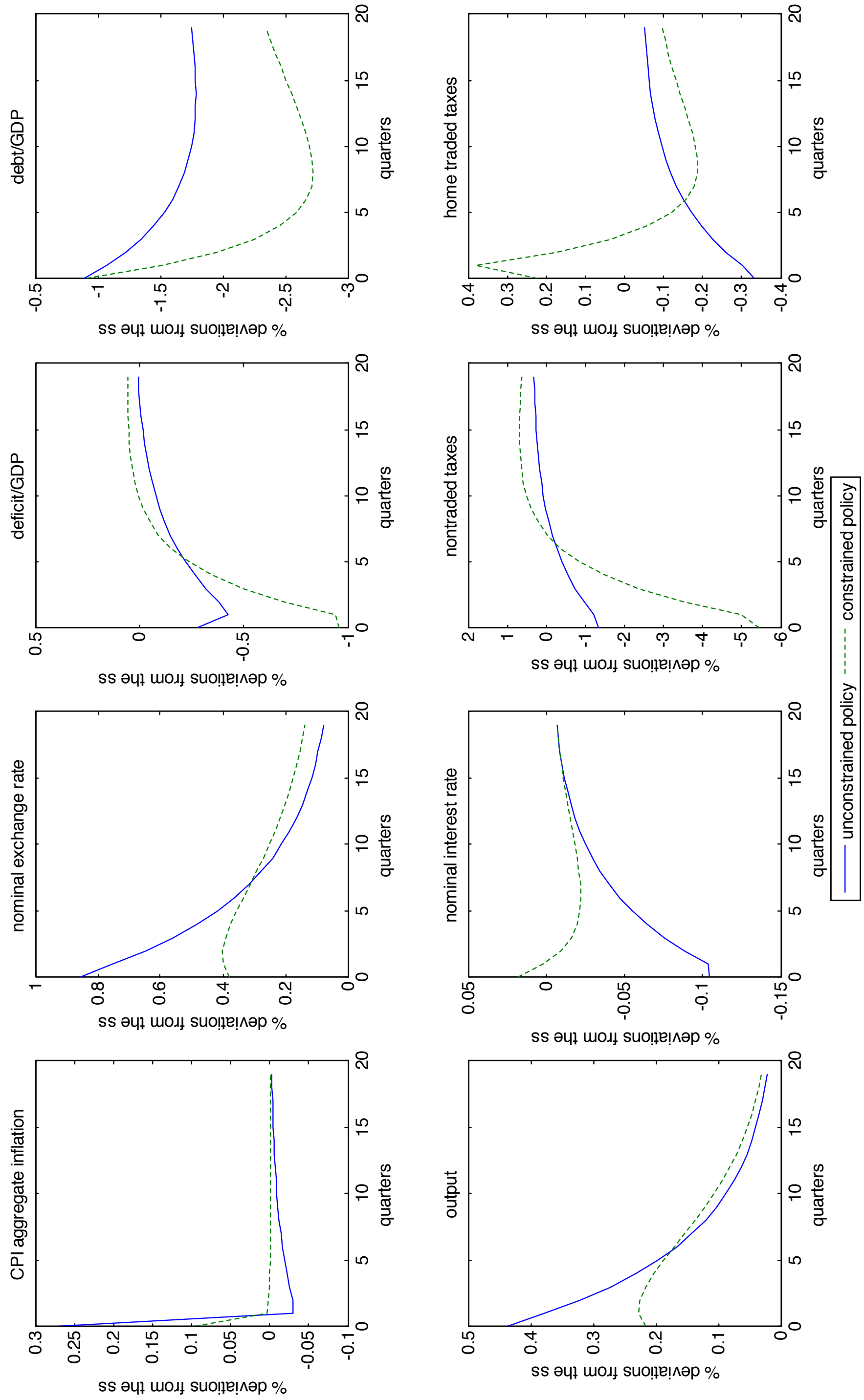

Figure 4.2: Impulse responses to a positive domestic nontraded productivity shock - monetary criteria

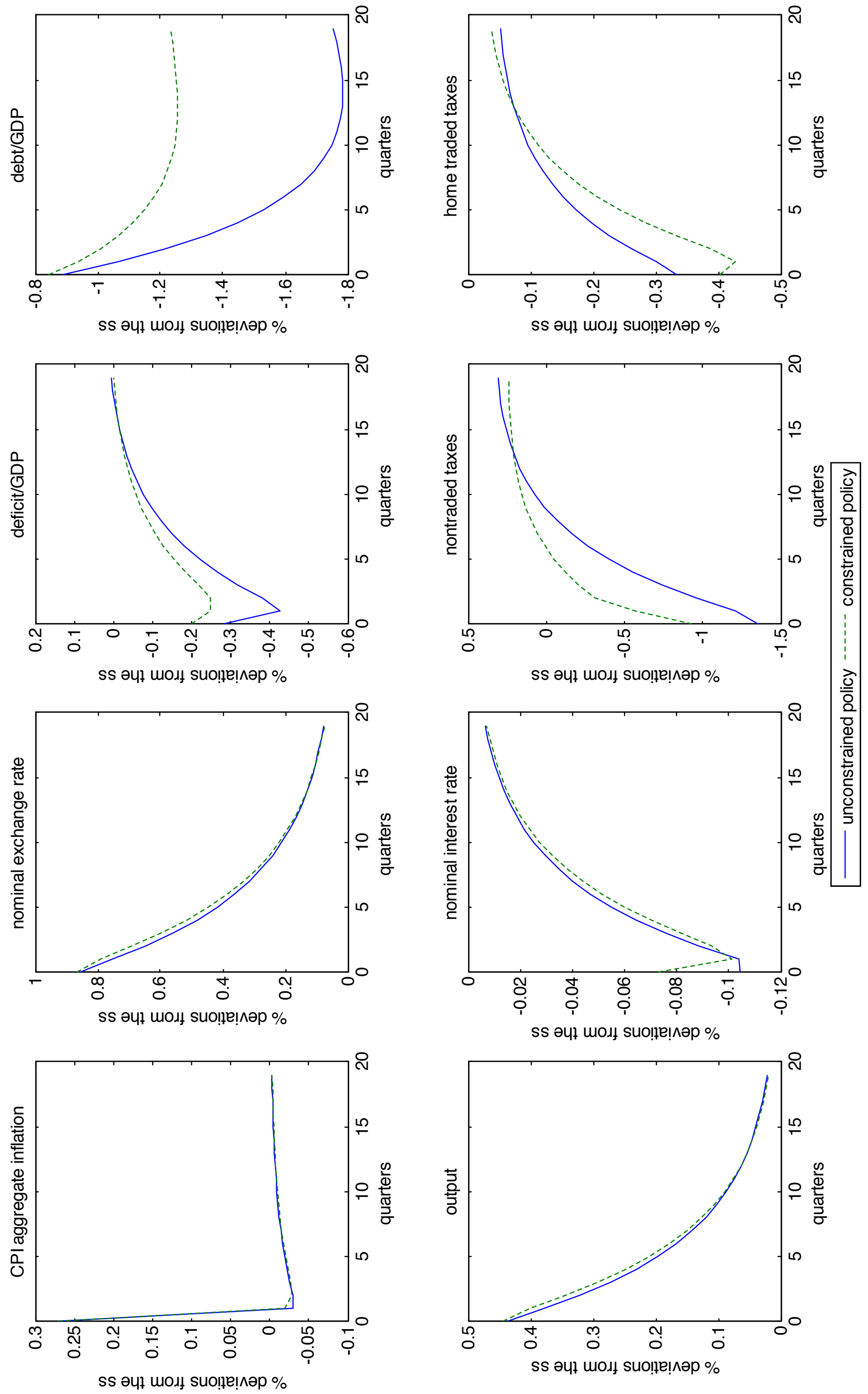

Figure 4.3: Impulse responses to a positive domestic nontraded productivity shock - fiscal criterion

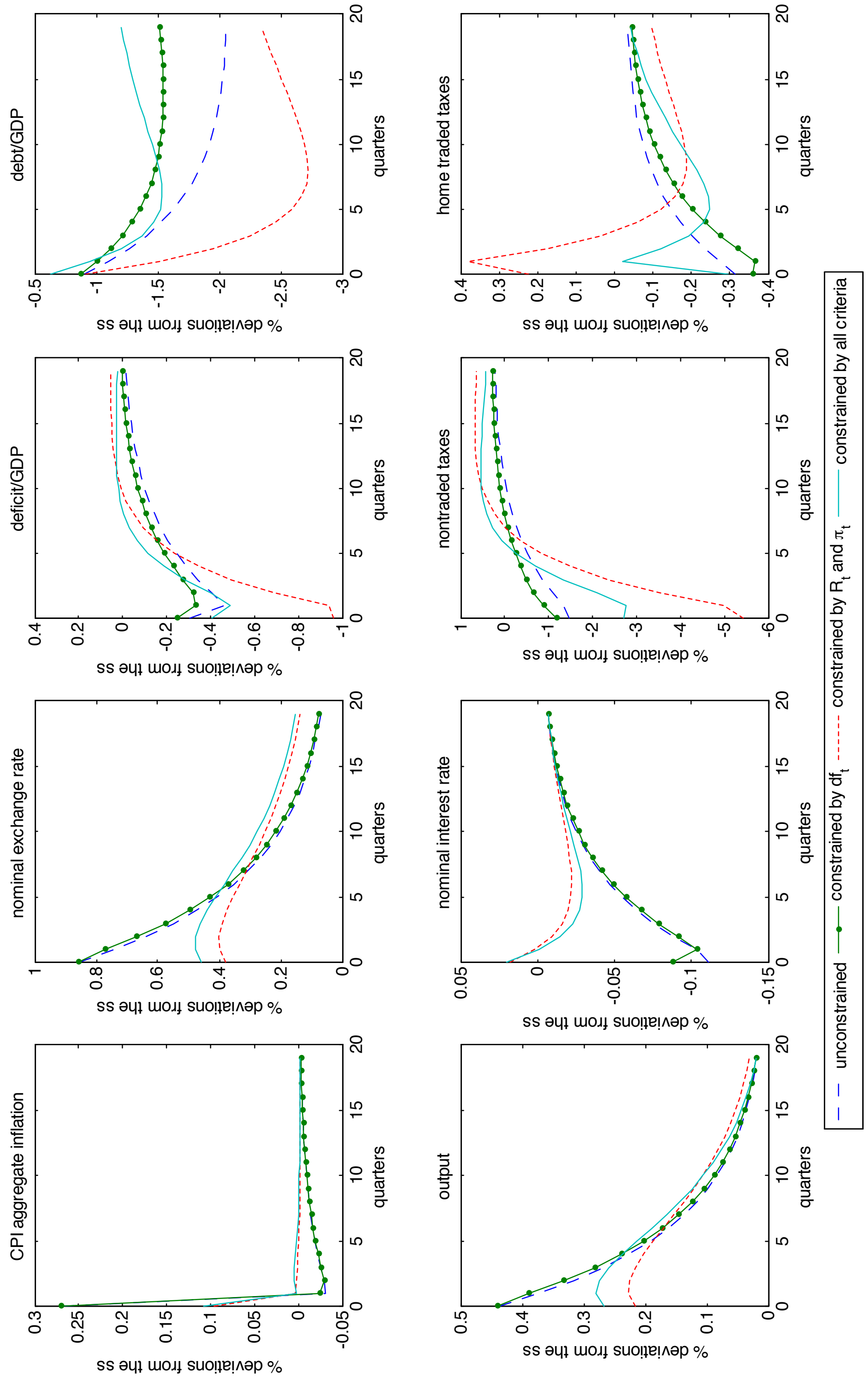

Figure 4.4: Impulse responses to a positive domestic nontraded productivity shock - comparison of different policies

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Appendix A

Maastricht Criteria and Data on EMU Accession Countries

A.1 The Convergence Criteria in the Maastricht Treaty

The Article 109j(1) of the Maastricht Treaty lays down the following monetary criteria as a prerequisite for entering the EMU:

- *the achievement of a high degree of price stability* which means that a Member State (of the EU) has a sustainable price performance and an average rate of inflation (the Consumer Price Index (CPI) inflation), observed over a period of one year before the examination, which does not exceed that of the three best performing Member States in terms of price stability by more than 1.5% points (the CPI inflation rate criterion);
- *the durability of the convergence ... reflected in the long term interest rate levels* which means that, over a period of one year before the examination, a Member State has an average nominal long-term interest rate that does not exceed that of the three best performing Member States in terms of price stability by more than 2% points (the nominal interest rate criterion);
- *the observance of the normal fluctuation margins provided for by the Exchange Rate Mechanism of the European Monetary System* (±15% bound around the central parity), for at least two years, without devaluing against the currency of any other Member State (the nominal exchange rate criterion).

Importantly, the Maastricht Treaty also imposes the criterion on the fiscal policy, i.e. the sustainability of the government financial position which refers to a government budgetary position without an excessive deficit (Article 104c(6) of the Maastricht

Treaty). The treaty stipulates: *The sustainability of the government financial position will be apparent from having achieved a government budgetary position without a deficit that is excessive.* In practice, the European Commission sets out two criteria:

- the annual government deficit: the ratio of the annual government deficit to gross domestic product must not exceed 3% at the end of the proceeding financial year. If this is not the case, the ratio must have declined substantially and continuously and reached a level close to 3% (interpretation in trend terms according to Article 104(2)) or alternatively, must remain close to 3% while representing only an exceptional and temporary excess

- government debt: the ratio of gross government debt to GDP must not exceed 60% at the end of the preceding financial year. If this is not the case, the ratio must have sufficiently diminished and must be approaching the reference value at a satisfactory pace (interpretation in trend terms according to Article 104(2)).

A.2 Data on EMU Accession Countries

We present figures and data regarding the EMU accession countries. All the data were collected from the Eurostat database and the European Commission webpage.

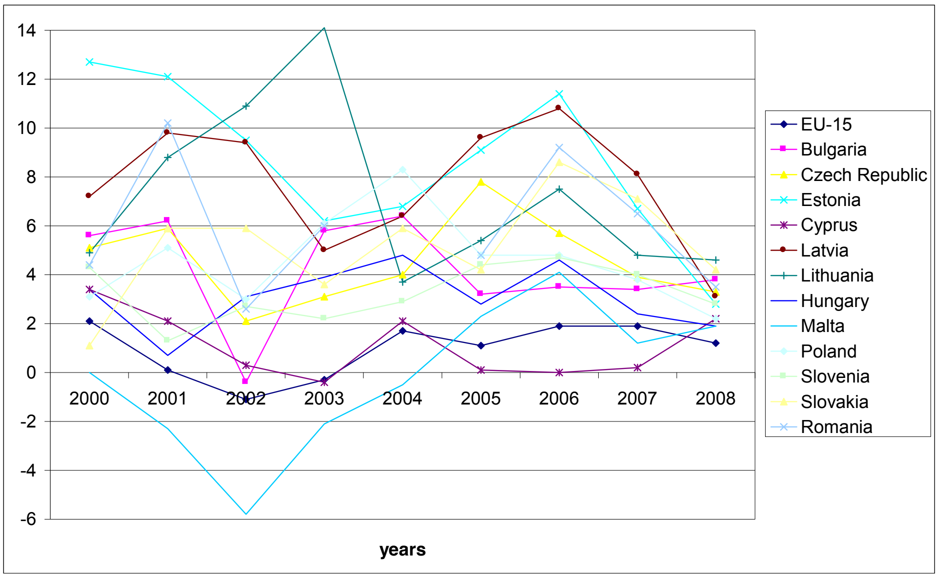

Figure A.1: Total annual labour productivity growth in the EMU accession countries and the EU 15 (annual rates in %) for the period 2000 - 2008. Values for 2007 and 2008 are forecasts.

countries	monetary regime	share of nontradables in consumption*	share of imports in GDP (at current places)#
Czech Republic	managed float	42%	68%
Estonia	peg	39%	86%
Hungary	managed float	44%	71%
Latvia	peg	37%	55%
Lithuania	peg	33%	58%
Poland	CPI targeting	37%	35%
Slovakia	managed float	41%	78%
average in the EU - 15		51%	63%
note: *– average 2000 - 2005; #– average 2000 - 2007; ˆ – average 2000 - 2004			

Table A.1: Structural characteristics of the EMU accession countries

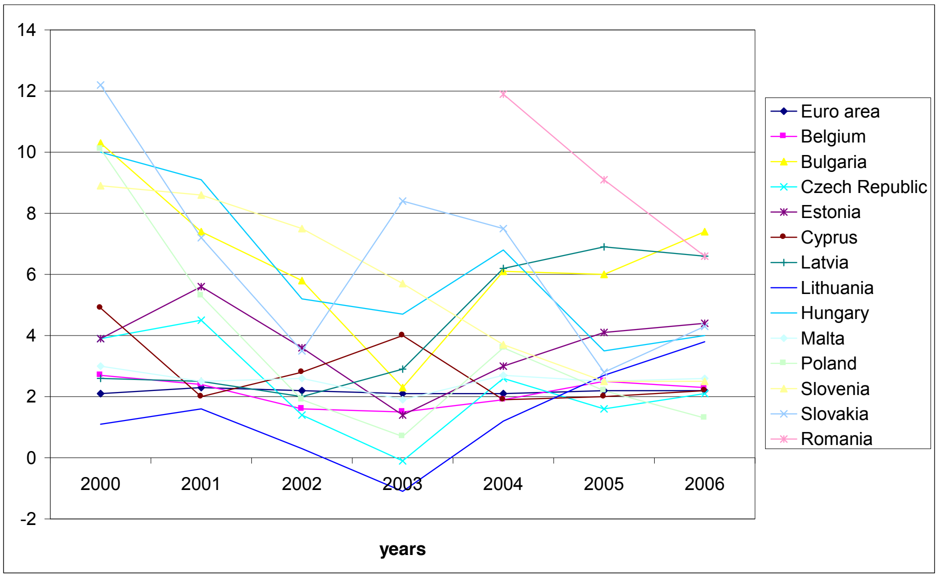

Figure A.2: CPI inflation in the EMU accession countries and the EU - 15 in 2000 - 2006 (annual % rates). For the purpose of clarity, CPI inflation rates in Romania are reported only for 2004 - 2006.

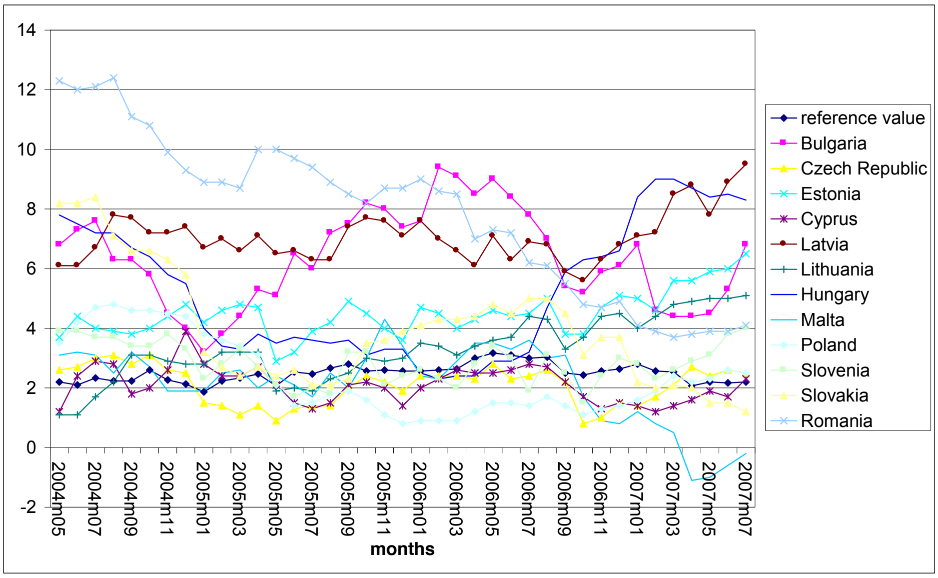

Figure A.3: CPI inflation rates in the EMU accession countries since their accession to the EU (annual rates in %)

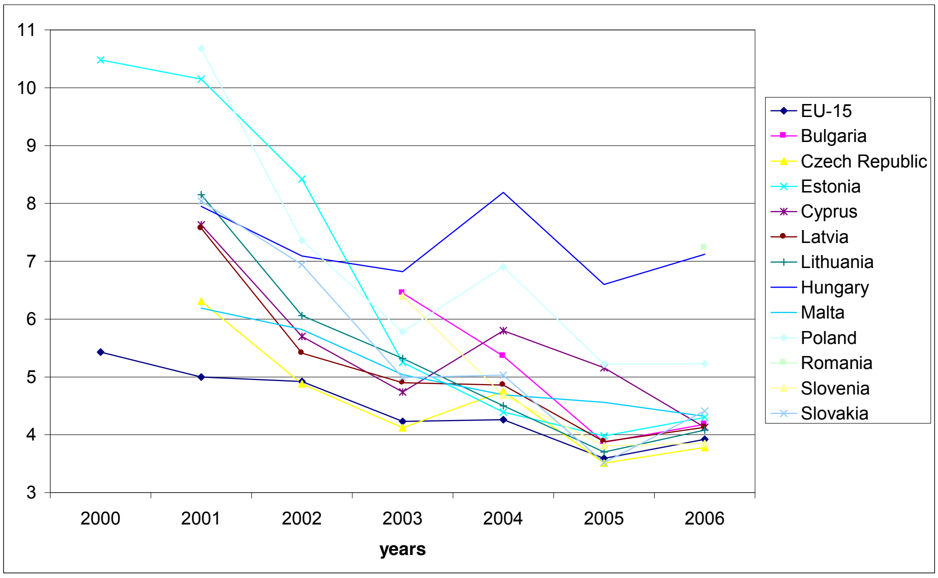

Figure A.4: EMU convergence criterion bond yields for the EMU accession countries and the euro area in 2001 - 2006 (annual % rates)

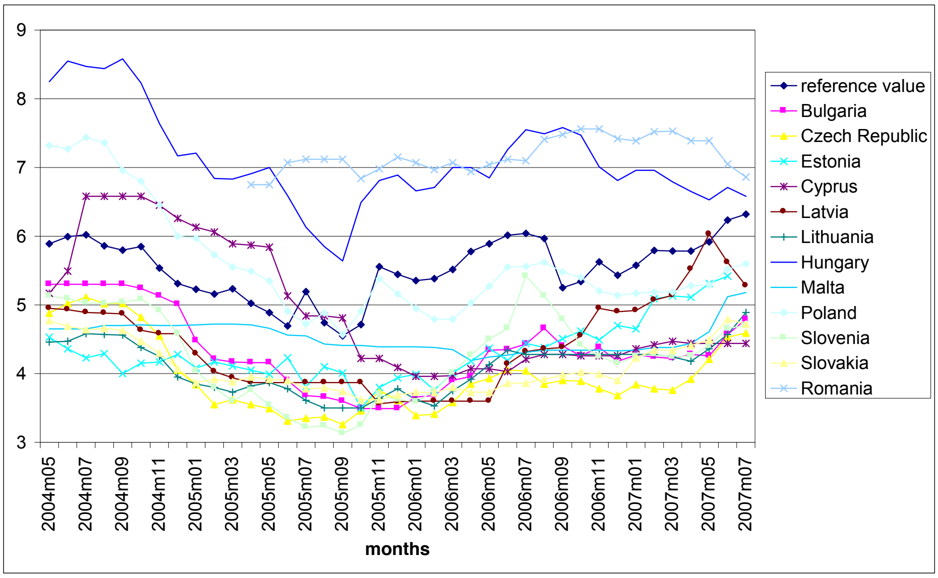

Figure A.5: EMU convergence criterion bond yields for the EMU accession countries since their accession to the EU (annual rates in %)

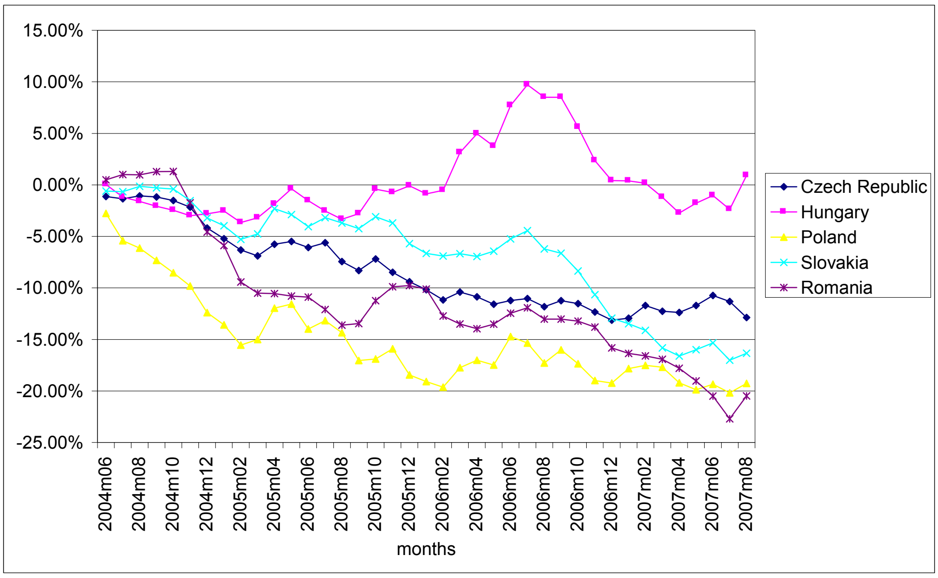

Figure A.6: Nominal exchange rate fluctuations vs. euro of the EMU accession countries since the accession to the EU (average monthly changes since the EU accession date)

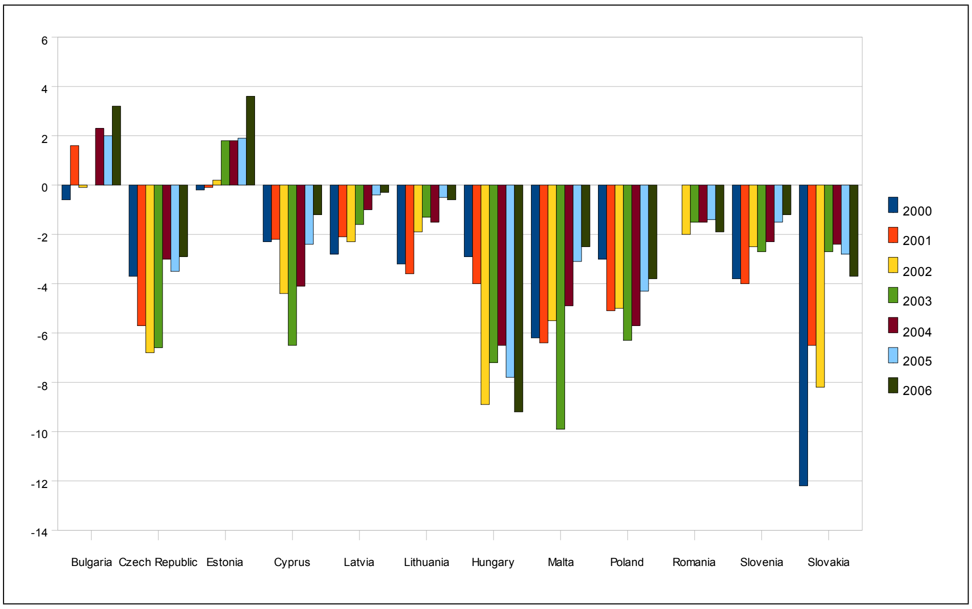

Figure A.7: Deficit to GDP ratio in the EMU Accession Countries in 2000 - 2006 (annual % rates)

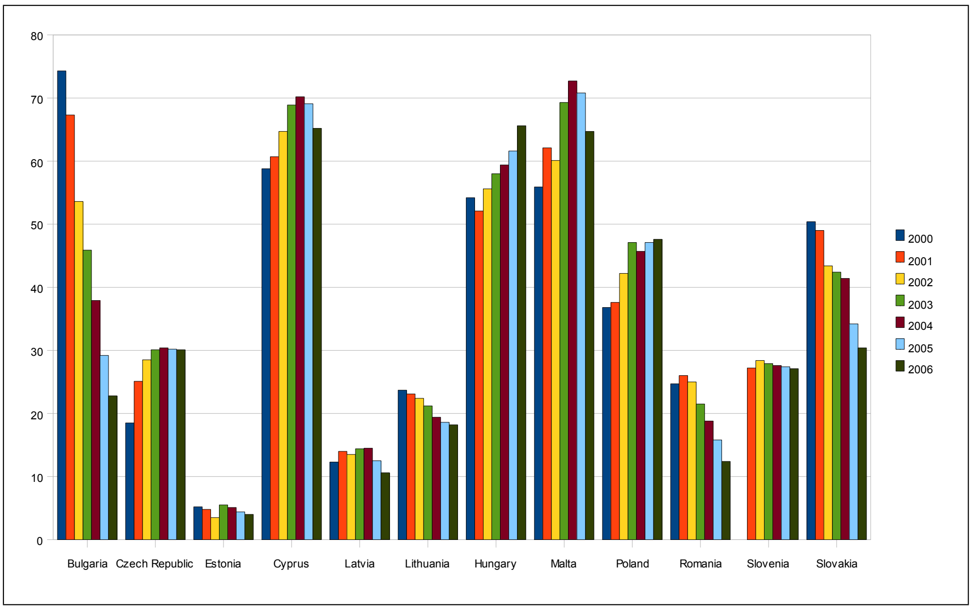

Figure A.8: Debt to GDP ratio in the EMU Accession Countries in 2000 - 2006 (annual % rates)

Appendix B

Model characteristics - chapter 2

B.1 Efficient steady state characterization

We define a deterministic steady state with zero inflation rate. There are no productivity shocks ($A^H = A^N = 1$). Other domestic shock: preference shock is assumed to take constant values ($B = \overline{B}$). In order to eliminate rigidities present as a result of the monopolistic competition in both domestic sectors we impose optimal subsidies (τ_N, τ_H) which values constitute for the solution of the social planner.

The social planner chooses subsidies (τ_N, τ_H) in order to maximize welfare of the domestic consumers subject to the aggregate constraints of the economy:

$$
\begin{aligned}
& \max(U(\overline{C}, \overline{B}) - V(\overline{L})) && \text{(B.1)}\\
& s.t. && \text{(B.2)}\\
\overline{C} &= \overline{RS}^{\frac{1}{\rho}} \overline{C}^{*} && \text{(B.3)}\\
1 &= \mu \overline{p}_N^{1-\phi} + (1-\mu)\overline{p}_T^{1-\phi} && \text{(B.4)}\\
\overline{p}_T^{1-\theta} &= (1-\lambda)\overline{p}_H^{1-\theta} + \lambda \overline{p}_F^{*1-\theta} \overline{RS}^{1-\theta} && \text{(B.5)}\\
\overline{Y_N} &= \mu \overline{p}_N^{-\phi} \overline{C} && \text{(B.6)}\\
\overline{Y_H} &= (1-\mu)(1-\lambda)\overline{p}_H^{-\theta} \overline{p}_T^{\theta-\phi} \overline{C} + \lambda(1-\mu)\overline{p}_H^{-\theta} \overline{RS}^{\theta} \overline{p}_F^{*\theta-\phi} \overline{C}^{*} && \text{(B.7)}
\end{aligned}
$$

Foreign variables: $\overline{p}_F^*$ and $\overline{C}^*$ are derived from the similar social planner's problem for the rest of the world assuming that such an economy is a closed one.

B.2 A log-linearized version of the model

We approximate the model around the above defined steady state. We present the loglinearized equations for the flexible price equilibrium and the sticky price equilibrium of the small open economy. Our set of shocks is composed of the domestic supply shocks: $A_{N,t}$, $A_{H,t}$, domestic demand shock: B_t and foreign shocks: C_t^*. We assume that $\widehat{\pi}_t^*$, $\widehat{\pi}_{F,t}^*$ and also $\widehat{T}_t^{d,*}$are zero (i.e. the rest of the world economy follows price stability policy).

B.2.1 The flexible price equilibrium

Supply

Nontraded sector:

$$(1-b)\widehat{T}_t^d = -\widehat{A}_t^N + \widehat{\omega}_t \tag{B.8}$$

where $b = \mu(\overline{T^d}\,\overline{p_T})^{1-\theta}$.

Domestic traded goods:

- internal consumption:

$$-b\widehat{T}_t^d - a\widehat{T}_t = -\widehat{A}_t^H + \widehat{\omega}_t \tag{B.9}$$

where $a = \lambda(\frac{\overline{RS}\,\overline{p}_F^*}{\overline{p_T}})^{1-\theta}$.

Domestic labour supply:

$$-\rho\widehat{C_t} + \widehat{\omega}_t - \eta(\frac{\overline{Y}_N}{\overline{Y}}\widehat{Y}_{N,t} + \frac{\overline{Y}_H}{\overline{Y}}\widehat{Y}_{H,t} - \frac{\overline{Y_H}}{\overline{Y}}\widehat{A}_t^H - \frac{\overline{Y_N}}{\overline{Y}}\widehat{A}_t^N) = 0 \tag{B.10}$$

Demand

Nontraded consumption:

$$\widehat{Y}_{N,t} = \widehat{C_t} - \phi(1-b)\widehat{T}_t^d \tag{B.11}$$

Domestic traded consumption:

$$\widehat{Y}_{H,t} = \theta d_{CT} a\widehat{T}_t + d_{CT}(\widehat{C}_t + \phi b\widehat{T}_t^d) + \theta(1-d_{CT})\widehat{T}_t + (1-d_{CT})\widehat{C}_t^* \tag{B.12}$$

where $d_{CT} = \frac{\overline{p}_H^{-\theta}\overline{p}_T^{\theta-\phi}(1-\lambda)(1-\mu)\overline{C}}{\overline{Y}_H}$.

Aggregate output definition:

$$\widehat{Y}_t = dyn(\widehat{Y}_{N,t} + (1-b)\widehat{T}_t^d) + dyh(\widehat{Y}_{H,t} - b\widehat{T}_t^d - a\widehat{T}_t) \tag{B.13}$$

where $dyn = \frac{\overline{p_N}\,\overline{Y_N}}{\overline{Y}}$; $dyh = \frac{\overline{p_H}\,\overline{Y_H}}{\overline{Y}}$.

Risk sharing:

$$\widehat{C}_t^* = \widehat{C}_t - \frac{1}{\rho}\widehat{RS}_t - \widehat{B}_t \tag{B.14}$$

Euler condition:

$$\widehat{RR}_t = \rho(\widehat{C}_{t+1} - \widehat{C}_t) - \rho(\widehat{B}_{t+1} - \widehat{B}_t) \tag{B.15}$$

where $\widehat{RR}_t = \widehat{R}_t - \widehat{\pi}_{t+1}$.

Definition of the real exchange rate:

$$\widehat{RS}_t = -b\widehat{T}_t^d + (1-a)\widehat{T}_t \tag{B.16}$$

B.2.2 The sticky price equilibrium

Supply

Nontraded sector:

$$\widehat{\pi}_{N,t} = k_N(-\widehat{A}_t^N + \widehat{\omega}_t - (1-b)\widehat{T}_t^d) + \beta E_t \widehat{\pi}_{N,t+1} \tag{B.17}$$

Domestic traded goods:

- internal consumption:

$$\widehat{\pi}_{H,t} = k_H(-\widehat{A}_t^H + \widehat{\omega}_t + b\widehat{T}_t^d + a\widehat{T}_t) + \beta E_t \widehat{\pi}_{H,t+1} \tag{B.18}$$

- export consumption:

$$\widehat{\pi}_{H,t}^* = k_H^*(-\widehat{A}_t^H + \widehat{\omega}_t - \widehat{RS}_t + \widehat{T}_t^*) + \beta E_t \widehat{\pi}_{H,t+1}^* \tag{B.19}$$

Foreign traded goods:

$$\widehat{\pi}_{F,t} = k_F(-\widehat{A}_t^{*F} + \widehat{\omega}_t^* + \widehat{RS}_t - (1-a)\widehat{T}_t + b\widehat{T}_t^d) + \beta E_t \widehat{\pi}_{F,t+1} \tag{B.20}$$

Labour supply:

$$-\rho\widehat{C}_t + \widehat{\omega}_t - \eta(\frac{\overline{Y}_N}{\overline{Y}}\widehat{Y}_{N,t} + \frac{\overline{Y}_H}{\overline{Y}}\widehat{Y}_{H,t} - \frac{\overline{Y_H}}{\overline{Y}}\widehat{A}_t^H - \frac{\overline{Y_N}}{\overline{Y}}\widehat{A}_t^N) = 0 \tag{B.21}$$

Demand

Nontraded consumption:

$$\widehat{Y}_{N,t} = \widehat{C_t} - \phi(1-b)\widehat{T}_t^d \tag{B.22}$$

Traded consumption:

$$\widehat{Y}_{H,t} = \theta d_{CT} a\widehat{T}_t + d_{CT}(\widehat{C}_t + \phi b\widehat{T}_t^d) + \theta(1-d_{CT})\widehat{T}_t^* + (1-d_{CT})\widehat{C}_t^* \tag{B.23}$$

where $d_{CT} = \frac{\overline{p}_H^{-\theta}\overline{p}_T^{\theta-\phi}(1-\lambda)(1-\mu)\overline{C}}{\overline{Y}_H}$.
Resource constraint:

$$\widehat{Y}_t = dyn(\widehat{Y}_{N,t} + (1-b)\widehat{T}_t^d) + dyh(\widehat{Y}_{H,t} - b\widehat{T}_t^d - a\widehat{T}_t) \tag{B.24}$$

where $dyn = \frac{\overline{p_N}\overline{Y_N}}{\overline{Y}}$; $dyh = \frac{\overline{p_H}\overline{Y_H}}{\overline{Y}}$.
Risk sharing:

$$\widehat{C}_t^* = \widehat{C}_t - \frac{1}{\rho}\widehat{RS}_t - \widehat{B}_t \tag{B.25}$$

Euler condition:

$$\rho E_t(\widehat{C}_{t+1} - \widehat{B}_{t+1}) = \rho(\widehat{C}_t - \widehat{B}_t) + \widehat{R}_t - E_t\widehat{\pi}_{t+1} \tag{B.26}$$

Monetary rule:

$$\widehat{R}_t = \mu_\pi(1-\kappa)\widehat{\pi}_t + \mu_S(1-\kappa)\widehat{S}_t + \kappa\widehat{R}_{t-1} + \widehat{\varepsilon}_t^{mp} \tag{B.27}$$

Prices

$$\widehat{\pi}_t = \widehat{\pi}_{H,t} + b(\widehat{T}_t^d - \widehat{T}_{t-1}^d) + a(\widehat{T}_t - \widehat{T}_{t-1}) \tag{B.28}$$

$$\widehat{T}_t^d - \widehat{T}_{t-1}^d = -\widehat{\pi}_{T,t} + \widehat{\pi}_{N,t} \tag{B.29}$$

$$\widehat{T}_t - \widehat{T}_{t-1} = \widehat{\pi}_{F,t} - \widehat{\pi}_{H,t} \tag{B.30}$$

$$\widehat{T}_t^* - \widehat{T}_{t-1}^* = \widehat{\pi}_{F,t}^* - \widehat{\pi}_{H,t}^* \tag{B.31}$$

$$\Delta\widehat{RS}_t = \Delta\widehat{S}_t + (\widehat{\pi}_t^* - \widehat{\pi}_t) \tag{B.32}$$

$$\Delta\widehat{S}_t = \widehat{S}_t - \widehat{S}_{t-1} \tag{B.33}$$

$$\Delta\widehat{RS}_t = \widehat{RS}_t - \widehat{RS}_{t-1} \tag{B.34}$$

B.3 Parameterization

We present values of the structural parameters and also values of the stochastic parameters chosen in the numerical exercise.

The parameter definition	value of the parameter	
inverse of the intertemporal elasticity of substitution	ρ	2
inverse of the labour supply elasticity	η	4
discount factor	β	0.99
intratemporal elasticity between variety of the goods	σ	10
elasticity of substitution between home and foreign tradables	θ	1.5
elasticity of substitution between tradables and nontradables	ϕ	0.5
share of nontradables	μ	0.42
degree of openness	λ	0.4
price rigidity in the nontradable sector	α_N	0.85
price rigidity in the tradable sectors	$\alpha_H, \alpha_H^*, \alpha_F$	0.8
share of nontradables in the foreign economy	μ^*	0.6

Table B.1: Structural parameters - model in Chapter 2

shocks	autoregressive parameter	standard deviation (in %)
nontradable productivity (A_N)	0.85	1.6
tradable productivity (A_H)	0.85	1.8
preference (B)	0.95	0.72
foreign consumption (C^*)	0.85	0.23
$corr(\widehat{A}_{N,t}, \widehat{A}_{H,t}) = 0.7$ where $corr$ - correlation coefficient		

Table B.2: Stochastic environment - model in Chapter 2

Note: The policy rule is calibrated following Natalucci and Ravenna (2003): $\widehat{R}_t = 0.9\widehat{R}_{t-1} + 0.1(\widehat{\pi}_t + 0.2\widehat{Y}_t + 0.3\widehat{S}_t) + \widehat{\varepsilon}_{R,t}$, where $SD(\widehat{\varepsilon}_{R,t}) = 0.45$.

Note: For comparison purposes the table shows also the results of the paper by Natalucci and Ravenna (2003). The model moments are theoretical.

Statistics	My framework		Natalucci and Ravenna (2003)	
Standard deviation in %	Model	Historical	Model	Historical
Output:	**1.74**	1.68	1.53	1.74
nontraded sector	**1.76**	1.56	2.72	1.55
traded sector	**3.68**	4.32	2.87	2.25
Consumption	**1.79**	1.93	2.28	2.29
Nominal interest rate	**0.52**	0.52	0.48	0.47
Nominal exchange rate	**2.60**	2.59	2.79	3.04
Real exchange rate	**3.19**	3.62	2.48	2.75
CPI inflation rate:	**0.56**	0.91	1.1	1.08
nontraded sector	**0.61**	0.97	0.79	2.61
traded sector	**0.94**	0.74	2.4	0.99

Table B.3: Matching the moments - model in Chapter 2

As far as the historical statistics are concerned our data sample for the Czech Republic is 1995:1 - 2006:2 (Natalucci and Ravenna (2003) database is 1994:1 - 2003:1). CPI inflation rate in the traded and nontraded sector data sample is 2000:1 - 2006:2. All series are logged (except for interest and inflation rates) and Hodrick - Prescott filtered. Rates of change are quarterly.

All data were collected from the Eurostat webpage (the data in Natalucci and Ravenna (2003) were collected from the OECD publication Statistical Compendium (2003) and the Czech Republic National Accounts (July 2003)). Data are seasonally adjusted where appropriate. We present the detailed data series. Output: Gross value added (GVA) at 1995 constant prices in national currency. Traded output is an aggregate of sectoral GVA for: Agriculture; Hunting; Forestry and Fishing; Total industry (excluding construction). Nontraded output is an aggregate of sectoral GVA for: Wholesale and retail trade, repair of motor vehicles, motorcycles and personal household goods; Hotels and restaurants; Transport, storage and communication; Financial intermediation, real estate, renting and business activities. Consumption: Final consumption expenditure of households at 1995 constant prices in national currency. Nominal interest rate: three months T - bill interest rate. Nominal exchange rate: Bilateral Koruny/euro exchange rate (quarterly average). Real exchange rate: CPI based real effective exchange rate (6 trading partners, quarterly average). CPI inflation rate: Harmonized Index of Consumer Prices (HICP). CPI inflation rate in the nontraded sector: HICP - Services. CPI inflation in the traded sector: HICP - Goods.

	shocks:			
variables:	A_N	A_H	B	C^*
CPI inflation	70%	3%	26%	0.5%
nominal interest rate	58%	3%	24%	15%
nominal exchange rate	32%	1.5%	27%	40%

Table B.4: Variance decomposition of the Maastricht variables under the benchmark rule - model in Chapter 2

B.4 Reinterpretation of the Maastricht convergence criteria

First, we summarize the Maastricht criteria by the following inequalities:

- CPI aggregate inflation criterion

$$\pi_t^A - \pi_t^{A,*} \leq B_\pi, \tag{B.35}$$

where $B_\pi = 1.5\%$, π_t^A is annual CPI aggregate inflation in the domestic economy, $\pi_t^{A,*}$ is the average of the annual CPI aggregate inflations in the three lowest inflation countries of the European Union.

- nominal interest rate criterion

$$R_t^L - R_t^{L,A*} \leq C_R \tag{B.36}$$

where $C_R = 2\%$, R_t^L is the annul interest rate for ten-year government bond in the domestic economy, $R_t^{L,A*}$ is the average of the annual interest rates for ten-year government bonds in the three countries of the European Union with the lowest inflation rates.

- nominal exchange rate criterion

$$(1 - D_S)S \leq S_t \leq (1 + D_S)S, \tag{B.37}$$

As explained in the main text, we restate the criteria in the quarterly terms. That means, that the bounds B_π and C_R have to be adjusted, i.e. $B_\pi = \sqrt[4]{1,015} - 1$ and $C_R = \sqrt[4]{1,02} - 1$. Assuming that shocks are normally distributed we can reformulate the original Maastricht criteria into the criteria which set upper bounds on variances of the Maastricht variables:

$$var(\widehat{\pi}_t) \leq (1 + k^{-2})B_\pi^2, \tag{B.38}$$

$$var(\widehat{R}_t) \leq (1 + k^{-2})C_R^2, \tag{B.39}$$

$$var(\widehat{S}_t) \leq (1 + k^{-2})D_S^2. \tag{B.40}$$

where $\widehat{var}$ $-$ variance defined by (3.52). Parameter $k = 1.96$ guarantees that compliance with the reformulated criteria gives 95% of probability that the original criteria are satisfied. These conditions are equivalent to the set of conditions in Chapter 3, i.e. (3.67) - (3.69).

Appendix C

Model characteristics - chapter 3

C.1 Steady state characterization

We define a deterministic steady state with zero inflation rate. We present a small open economy as the limiting case of a two country model, i.e. $n = 0$ and $\nu = 1 - \lambda$. All variables in the steady state are denoted with a bar. All the shocks take the constant values, in particular: $\overline{A_N} = \overline{A_H} = 1$, $\overline{B} = 1$.[1]

Moreover discount factors are:

$$Q_{t_0,t} = Q^*_{t_0,t} = \beta^{t-t_0} \tag{C.1}$$

First order conditions of the domestic firms are the following:

$$\overline{p}_N = \frac{\sigma_N}{(\sigma_N - 1)(1 - \overline{\tau_N})} \frac{\overline{W}^N}{\overline{P}}, \tag{C.2}$$

$$\overline{p}_H = \frac{\sigma_H}{(\sigma_H - 1)(1 - \overline{\tau_H})} \frac{\overline{W}^H}{\overline{P}} \tag{C.3}$$

where $\overline{p}_N = \frac{\overline{P_N}}{\overline{P}}$, $\overline{p}_H = \frac{\overline{P_H}}{\overline{P}}$. We define markups in the domestic nontraded and home traded sector: $\overline{\mu_N} = \frac{\sigma_N}{(\sigma_N - 1)(1-\overline{\tau_N})}$, $\overline{\mu_H} = \frac{\sigma_H}{(\sigma_H - 1)(1-\overline{\tau_H})}$.

Labour supply optimality conditions are presented below:

$$\overline{C}^{-\rho} \frac{\overline{W}^N}{\overline{P}} = \left(\overline{L}\right)^{\eta}, \tag{C.4}$$

$$\overline{C}^{-\rho} \frac{\overline{W}^H}{\overline{P}} = \left(\overline{L}\right)^{\eta}. \tag{C.5}$$

These two conditions imply that real wages are equalized in the domestic sectors:

[1] Foreign consumption is derived from the steady state relations of the foreign economy.

$$\frac{\overline{W}^N}{\overline{P}} = \frac{\overline{W}^H}{\overline{P}} = \overline{\omega}. \tag{C.6}$$

Moreover substituting first order conditions of the firms ((C.2), (C.3)) into the labour supply optimality conditions ((C.4), (C.5)) we obtain the following relation:[2]

$$\overline{p}_N \overline{\mu_N}^{-1} = \overline{p}_H \overline{\mu_H}^{-1}. \tag{C.7}$$

From the market clearing condition in the domestic labour market we know that:

$$\overline{L} = \overline{L_N} + \overline{L_H}. \tag{C.8}$$

Moreover from the production function we get that:

$$\overline{L_N} = \overline{Y_N}, \tag{C.9}$$

$$\overline{L_H} = \overline{Y_H}. \tag{C.10}$$

Demands for domestic traded and nontraded goods are presented below:

$$\overline{Y}_N = \overline{C}_N = \overline{p}_N^{-\phi} \mu \overline{C}, \tag{C.11}$$

$$\overline{Y}_H = \overline{C}_H + \overline{C}_H^* = \overline{p}_H^{-\theta} \overline{p}_T^{\theta-\phi} (1-\lambda)(1-\mu)\overline{C} + \overline{p}_H^{*-\theta} \overline{p}_F^{*\theta-\phi} \lambda (1-\mu^*)\overline{C}^* \tag{C.12}$$

where $\overline{p}_H^* = \frac{\overline{P_H^*}}{\overline{P^*}}$, $\overline{p}_F^* = \frac{\overline{P_F^*}}{\overline{P^*}}$.

We define aggregate output in the following way:

$$\overline{Y} = \overline{p}_N \overline{Y}_N + \overline{p}_H \overline{Y}_H. \tag{C.13}$$

Notice that since the law of one price holds for the traded goods: $\overline{p}_H^* = \overline{p}_H \overline{RS}^{-1}$.

We define the following steady state ratios for the home economy:

$$d_{CH} = (1-\lambda)(1-\mu)\frac{\overline{C}}{\overline{Y}_H} \overline{p}_H^{-\theta} \overline{p}_T^{\theta-\phi}, \tag{C.14}$$

$$s_C = \overline{\omega}\frac{\overline{L}}{\overline{\overline{C}}} = \frac{\overline{p}_N}{\overline{\mu}_N} \frac{\overline{Y_N} + \overline{Y_H}}{\overline{C}}, \tag{C.15}$$

$$d_{Y_N} = \frac{\overline{p}_N \overline{Y_N}}{\overline{Y}}, \tag{C.16}$$

[2]Notice that if markups in the nontraded and home traded sector are equal, i.e. $\overline{\mu_N} = \overline{\mu_H}$ then also the domestic relative prices of nontraded and home traded goods are equal, i.e. $\overline{p}_N = \overline{p}_H$.

$$d_{Y_H} = \frac{\overline{p}_H \overline{Y_H}}{\overline{Y}}, \tag{C.17}$$

$$\widetilde{d}_{Y_N} = \frac{\overline{Y_N}}{\overline{Y_N} + \overline{Y_H}}, \tag{C.18}$$

$$\widetilde{d}_{Y_H} = \frac{\overline{Y_H}}{\overline{Y_N} + \overline{Y_H}}. \tag{C.19}$$

From the complete asset market assumption and the assumption that the initial wealth distribution is such that $\upsilon = RS_0 \left(\frac{C_0}{C_0^*}\right)^{\rho} = 1$ we obtain:

$$\overline{C} = \overline{RS}^{\frac{1}{\rho}} \overline{C}^*. \tag{C.20}$$

Lastly from the definition of price indices and the assumption that the law of one price holds in the traded sector we get the following relations between relative prices:

$$\overline{p_T}^{1-\theta} \overline{p_H}^{\theta-1} = (1-\lambda) + \lambda (\overline{RS}\,\overline{p_H}^{-1} \overline{p_F^*})^{1-\theta}, \tag{C.21}$$

$$1 = \mu \overline{p_N}^{1-\phi} + (1-\mu) \overline{p_T}^{1-\phi}. \tag{C.22}$$

The set of the following conditions solves for the steady state values of domestic variables: $\overline{p}_N$, $\overline{p}_H$, $\overline{p}_T$, $\overline{C}$, $\overline{RS}$ ($\overline{C}^*$ and $\overline{p}_F^*$ are treated as exogenous and are obtained from a similar set of the conditions for the foreign economy):[3]

$$\overline{C} = \overline{RS}^{\frac{1}{\rho}} \overline{C}^*, \tag{C.26}$$

$$\overline{p}_N \overline{\mu_N}^{-1} = \overline{p}_H \overline{\mu_H}^{-1}, \tag{C.27}$$

$$\overline{C}^{-\rho} \overline{p}_H \overline{\mu_H}^{-1} = \left(\overline{p}_N^{-\phi} \mu \overline{C} + \overline{p}_H^{-\theta} \overline{p}_T^{\theta-\phi} (1-\lambda)(1-\mu) \overline{C} + \overline{p}_H^{-\theta} \overline{RS}^{\theta} \overline{p}_F^{*\theta-\phi} \lambda (1-\mu^*) \overline{C}^* \right)^{\eta}, \tag{C.28}$$

[3]The set of optimality conditions for the foreign economy which determines the steady state values of $\overline{C}^*$, $\overline{p}_F^*$, $\overline{p}_N^*$ is the following:

$$\overline{C}^{*-\rho-\eta} \overline{p}_N^* \overline{\mu}_N^{*-1} = \left(\overline{p}_N^{*-\phi} \mu^* + \overline{p}_F^{*-\phi} (1-\mu^*) \right)^{\eta} \tag{C.23}$$

$$\mu^* \overline{p}_N^{*1-\phi} + (1-\mu^*) \overline{p}_F^{*1-\phi} = 1 \tag{C.24}$$

$$\overline{p}_N^* \overline{\mu}_N^{*-1} = \overline{p}_F^* \overline{\mu}_F^{*-1} \tag{C.25}$$

$$\overline{p_T}^{1-\theta} = (1-\lambda)\overline{p_H}^{1-\theta} + \lambda(\overline{RSp_F^*})^{1-\theta}, \tag{C.29}$$

$$1 = \mu\overline{p_N}^{1-\phi} + (1-\mu)\overline{p_T}^{1-\phi}. \tag{C.30}$$

C.2 A loglinearized version of the model

We loglinearize the model around the above presented steady state. We present the structural equations that describe dynamics of the domestic economy. All the variables with hat represent the log deviations from the steady state. The system is closed by specifying the monetary rules for each of the economies.

$$\widehat{\pi}_{N,t} = k_N(\rho\widehat{C}_t + \eta\widehat{L}_t - \widehat{A}_{N,t} - \rho\widehat{B}_t - \widehat{p}_{N,t}) + \beta\widehat{\pi}_{N,t+1}, \tag{C.31}$$

$$\widehat{\pi}_{H,t} = k_H(\rho\widehat{C}_t + \eta\widehat{L}_t - \widehat{A}_{H,t} - \rho\widehat{B}_t - \widehat{p}_{H,t}) + \beta\widehat{\pi}_{H,t+1}, \tag{C.32}$$

$$\widehat{L}_t = \widetilde{d}_{Y_N}(\widehat{Y}_{N,t} - \widehat{A}_{N,t}) + \widetilde{d}_{Y_H}(\widehat{Y}_{H,t} - \widehat{A}_{H,t}), \tag{C.33}$$

$$\widehat{Y}_{N,t} = \widehat{C}_t - \phi\widehat{p}_{N,t}, \tag{C.34}$$

$$\begin{aligned}\widehat{Y}_{H,t} = d_{CH}\widehat{C}_t - \theta\widehat{p}_{H,t} + b(\phi-\theta)d_{CH}\widehat{T}_t^d + (1-d_{CH})\theta\widehat{RS}_t + \\ + (1-d_{CH})\widehat{C}_t^* + b^*(\phi-\theta)(1-d_{CH})\widehat{T}_t^{d*},\end{aligned} \tag{C.35}$$

$$\widehat{Y}_t = d_{YN}(\widehat{p}_{N,t} + \widehat{Y}_{N,t}) + d_{YH}(\widehat{p}_{H,t} + \widehat{Y}_{H,t}) \tag{C.36}$$

$$\widehat{C}_t = \widehat{B}_t + \frac{1}{\rho}\widehat{RS}_t + \widehat{C}_t^* - \widehat{B}_t^*, \tag{C.37}$$

$$(a-1)\widehat{p}_{H,t} = b\widehat{T_t^d} + a\widehat{RS_t} - b^*a\widehat{T_t^{d^*}}, \tag{C.38}$$

$$\widehat{p}_{N,t} = (1-b)\widehat{T_t^d}, \tag{C.39}$$

$$\widehat{p}_{H,t} = -b\widehat{T_t^d} - a\widehat{T}_t, \tag{C.40}$$

$$\widehat{T}_t = \widehat{\Delta S}_t - \widehat{\pi}_{H,t} + \widehat{T}_{t-1}, \tag{C.41}$$

$$\widehat{T}^d_t = \widehat{\pi}_{N,t} - \widehat{\pi}_{T,t} + \widehat{T}^d_{t-1}, \tag{C.42}$$

$$\widehat{\pi}_{T,t} = \widehat{\pi}_{H,t} + a(\widehat{T}_t - \widehat{T}_{t-1}). \tag{C.43}$$

The Maastricht variables can be derived from the following equations:

$$\widehat{S}_t = \widehat{\Delta S}_t - \widehat{S}_{t-1}, \tag{C.44}$$

$$\widehat{S}_t = \widehat{S}_{t-1} + \widehat{\pi}_t - \widehat{\pi}^*_t + \widehat{RS}_t - \widehat{RS}_{t-1}, \tag{C.45}$$

$$\widehat{R}_t = \rho(\widehat{C}_{t+1} - \widehat{B}_{t+1}) - \rho(\widehat{C}_t - \widehat{B}_t) + \widehat{\pi}_{t+1}, \tag{C.46}$$

$$\widehat{\pi}_t = b\widehat{\pi}_{N,t} + (1-a)(1-b)\widehat{\pi}_{H,t} + a(1-b)(\widehat{S}_t - \widehat{S}_{t-1}). \tag{C.47}$$

C.3 Parameterization

We present values of the structural parameters and also values of the stochastic parameters chosen in the numerical exercise.

The parameter definition	value of the parameter	
inverse of the intertemporal elasticity of substitution	ρ	2
inverse of the labour supply elasticity	η	4
discount factor	β	0.99
intratemporal elasticity between variety of the goods	σ	10
elasticity of substitution between home and foreign tradables	θ	1.5
elasticity of substitution between tradables and nontradables	ϕ	0.5
share of nontradables	μ	0.42
degree of openness	λ	0.4
price rigidity in the nontradable sector	α_N	0.85
price rigidity in the home tradable sector	α_H	0.8
steady state share of taxes in the nontradable sector	τ_N	0.1
steady state share of taxes in the tradable sector	τ_H	0.1
share of nontradables in the foreign economy	μ^*	0.6

Table C.1: Structural parameters - model in Chapter 3

shocks	autoregressive parameter	standard deviation (in %)
nontradable productivity (A_N)	0.85	1.6
tradable productivity (A_H)	0.85	1.8
preference (B)	0.95	0.72
foreign consumption (C^*)	0.85	0.23
$corr(\widehat{A}_{N,t}, \widehat{A}_{H,t}) = 0.7$ where $corr$ - correlation coefficient		

Table C.2: Stochastic environment - model in Chapter 3

Note: The policy rule is calibrated following Natalucci and Ravenna (2003): $\widehat{R}_t = 0.9\widehat{R}_{t-1} + 0.1(\widehat{\pi}_t + 0.2\widehat{Y}_t + 0.3\widehat{S}_t) + \widehat{\varepsilon}_{R,t}$, where $SD(\widehat{\varepsilon}_{R,t}) = 0.45$.

Statistics	My framework		Natalucci and Ravenna (2003)	
Standard deviation in %	Model	Historical	Model	Historical
Output:	**1.87**	1.68	1.53	1.74
nontraded sector	**1.95**	1.56	2.72	1.55
traded sector	**3.23**	4.32	2.87	2.25
Consumption	**1.94**	1.93	2.28	2.29
Nominal interest rate	**0.52**	0.52	0.48	0.47
Nominal exchange rate	**2.84**	2.59	2.79	3.04
Real exchange rate	**2.35**	3.62	2.48	2.75
CPI inflation rate:	**0.82**	0.91	1.1	1.08
nontraded sector	**0.59**	0.97	0.79	2.61
traded sector	**0.94**	0.74	2.4	0.99

Table C.3: Matching the moments - model in Chapter 3

Note: For comparison purposes the table shows also the results of the paper by Natalucci and Ravenna (2003). The model moments are theoretical.

As far as the historical statistics are concerned our data sample for the Czech Republic is 1995:1 - 2006:2 (Natalucci and Ravenna (2003) database is 1994:1 - 2003:1). CPI inflation rate in the traded and nontraded sector data sample is 2000:1 - 2006:2. All series are logged (except for interest and inflation rates) and Hodrick - Prescott filtered. Rates of change are quarterly.

All data were collected from the Eurostat webpage (the data in Natalucci and Ravenna (2003) were collected from the OECD publication Statistical Compendium (2003) and the Czech Republic National Accounts (July 2003)). Data are seasonally adjusted where appropriate. We present the detailed data series. Output: Gross value added (GVA) at 1995 constant prices in national currency. Traded output is an aggregate of sectoral GVA for: Agriculture; Hunting; Forestry and Fishing; Total industry (excluding construction). Nontraded output is an aggregate of sectoral

GVA for: Wholesale and retail trade, repair of motor vehicles, motorcycles and personal household goods; Hotels and restaurants; Transport, storage and communication; Financial intermediation, real estate, renting and business activities. Consumption: Final consumption expenditure of households at 1995 constant prices in national currency. Nominal interest rate: three months T - bill interest rate. Nominal exchange rate: Bilateral Koruny/euro exchange rate (quarterly average). Real exchange rate: CPI based real effective exchange rate (6 trading partners, quarterly average). CPI inflation rate: Harmonized Index of Consumer Prices (HICP). CPI inflation rate in the nontraded sector: HICP - Services. CPI inflation in the traded sector: HICP - Goods.

C.4 Quadratic representation of the optimal loss function

C.4.1 The second order approximation of the welfare function

We present a second order approximation to the welfare function (3.1):

$$W_{t_0} = U_C \overline{C} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [z_v' \widehat{v}_t - \frac{1}{2} \widehat{v}_t' Z_v \widehat{v}_t - \widehat{v}_t' Z_\xi \widehat{\xi}_t] + tip + O(3) \tag{C.48}$$

where $\widehat{v}_t' = \begin{bmatrix} \widehat{C}_t & \widehat{Y}_{N,t} & \widehat{Y}_{H,t} & \widehat{\pi}_{N,t} & \widehat{\pi}_{H,t} \end{bmatrix}$; $\widehat{\xi}_t' = \begin{bmatrix} \widehat{A}_{N,t} & \widehat{A}_{H,t} & \widehat{B}_t & \widehat{C}_t^* \end{bmatrix}$; *tip* stands for *terms independent of policy* and $O(3)$ includes terms that are of order higher than the second in the deviations of variables from their steady state values. The matrices z_v, Z_v, Z_ξ are defined below:

$$z_v' = \begin{bmatrix} 1 & -s_C \widetilde{d}_{Y_N} & -s_C \widetilde{d}_{Y_H} & 0 & 0 \end{bmatrix}, \tag{C.49}$$

$$Z_v = \begin{bmatrix} \rho - 1 & 0 & 0 & 0 & 0 \\ 0 & s_C \widetilde{d}_{Y_N}(1+\eta \widetilde{d}_{Y_N}) & \eta s_C \widetilde{d}_{Y_N} \widetilde{d}_{Y_H} \widehat{Y} & 0 & 0 \\ 0 & \eta s_C \widetilde{d}_{Y_N} \widetilde{d}_{Y_H} \widehat{Y} & s_C \widetilde{d}_{Y_H}(1+\eta \widetilde{d}_{Y_H}) & 0 & 0 \\ 0 & 0 & 0 & s_C \widetilde{d}_{Y_N} \frac{\sigma}{k_N} & 0 \\ 0 & 0 & 0 & 0 & s_C \widetilde{d}_{Y_H} \frac{\sigma}{k_H} \end{bmatrix}, \tag{C.50}$$

$$Z_\xi = \begin{bmatrix} 0 & 0 & -\rho & 0 \\ -s_C \widetilde{d}_{Y_N}(1+\eta \widetilde{d}_{Y_N}) & -\eta s_C \widetilde{d}_{Y_H} \widetilde{d}_{Y_N} & 0 & 0 \\ -\eta s_C \widetilde{d}_{Y_H} \widetilde{d}_{Y_N} & -s_C \widetilde{d}_{Y_H}(1+\eta \widetilde{d}_{Y_H}) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}. \tag{C.51}$$

C.4.2 Elimination of the linear terms

This section describes in detail how we eliminate the linear terms in the second order approximation to the welfare function in order to obtain a quadratic loss function. Moreover we reduce the number of structural variables that represent the policy problem by appropriate substitutions.

The optimal monetary policy solves the welfare maximization problem with the constraints given by the structural equations of the economy (their loglinearized versions are (C.31) - (C.43)). The matrix representation of the second order approximation to the welfare function is the following:

$$W = U_C \overline{C} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [z_x' \widehat{x}_t - \frac{1}{2} \widehat{x}_t' Z_x \widehat{x}_t - \widehat{x}_t' Z_\xi \widehat{\xi}_t] + tip + O(3). \tag{C.52}$$

Similarly we present a second order approximation to all the structural equations in the matrix form:

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [\begin{bmatrix} A_1 \widehat{x}_t \\ A_2 \widehat{x}_t \\ \ldots \\ A_{13} \widehat{x}_t \end{bmatrix} + \frac{1}{2} \begin{bmatrix} \widehat{x}_t{}' B_1 \widehat{x}_t \\ \widehat{x}_t{}' B_2 \widehat{x}_t \\ \ldots \\ \widehat{x}_t{}' B_{13} \widehat{x}_t \end{bmatrix} + \begin{bmatrix} \widehat{x}_t{}' C_1 \widehat{\xi}_t \\ \widehat{x}_t{}' C_2 \widehat{\xi}_t \\ \ldots \\ \widehat{x}_t{}' C_{13} \widehat{\xi}_t \end{bmatrix}] + tip + O(3) = 0 \tag{C.53}$$

with

$$\widehat{x}_t' = \begin{bmatrix} \widehat{Y}_t & \widehat{L}_t & \widehat{C}_t & \widehat{Y}_{N,t} & \widehat{Y}_{H,t} & \widehat{p}_{N,t} & \widehat{p}_{H,t} & \widehat{T_t^d} & \widehat{T}_t & \widehat{RS_t} & \widehat{\Delta S_t} & \widehat{\pi_{H,t}} & \widehat{\pi_{N,t}} & \widehat{\pi_{T,t}} \end{bmatrix}, \tag{C.54}$$

$$\widehat{\xi}_t' \equiv \begin{bmatrix} \widehat{A}_{N,t} & \widehat{A}_{H,t} & \widehat{B}_t & \widehat{C}_t^* \end{bmatrix}$$

where *tip* means terms independent of policy.

Following the methodology of Benigno and Woodford (2005) in order to eliminate the linear terms in the welfare function we solve the system of linear equations:

$$\zeta A = z_x^{'} \tag{C.55}$$

where $A_{(13\times14)} = \begin{bmatrix} A_1 \\ A_2 \\ \ldots \\ A_{13} \end{bmatrix}$;

$\zeta_{(1\times13)} = \begin{bmatrix} \zeta_1 & \zeta_2 & \zeta_3 & \zeta_4 & \zeta_5 & \zeta_6 & \zeta_7 & \zeta_8 & \zeta_9 & \zeta_{10} & \zeta_{11} & \zeta_{12} & \zeta_{13} \end{bmatrix}$ and $z_{x(14\times1)}$.

As a result we obtain the loss function:

$$L_{t_0} = U_C \overline{C} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [\frac{1}{2} \widehat{x}_t' L_x \widehat{x}_t + \widehat{x}_t' L_\xi \widehat{\xi}_t] + tip + O(3) \tag{C.56}$$

where

$$L_x = Z_x + \zeta_1 B_1 + \zeta_2 B_2 + \zeta_4 B_4 + \zeta_6 B_6 + \zeta_9 B_9 + \zeta_{10} B_{10} + \zeta_{11} B_{11} \tag{C.57}$$

$$+\zeta_{12} B_{12} + \zeta_{13} B_{13}, \tag{C.58}$$

$$L_\xi = Z_\xi + \zeta_1 C_1 + \zeta_2 C_2 + \zeta_4 C_4 + \zeta_6 C_6 + \zeta_{13} C_{13}. \tag{C.59}$$

C.4.3 Substitution of the variables

We want to represent the loss function (**??**) and also the whole model just in terms of the following variables:

$$\widehat{y}_t' = \left[\begin{array}{ccccccc} \widehat{Y}_t & \widehat{T}_t^d & \widehat{T}_t & \Delta \widehat{S}_t & \widehat{\pi}_{H,t} & \widehat{\pi}_{N,t} & \widehat{\pi}_{T,t} \end{array} \right]. \tag{C.60}$$

In order to do this we define matrices $N_{x(14 \times 7)}$ and $N_{\xi(14 \times 4)}$ that map all the variables in the vector $\widehat{y}_t'$ in the following way:

$$\widehat{x}_t = N_x \widehat{y}_t' + N_\xi' \widehat{\xi}_t \tag{C.61}$$

where:

$$N_x = \left[\begin{array}{ccccccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 1 & ltd & lt & 0 & 0 & 0 & 0 \\ 1 & ctd & ct & 0 & 0 & 0 & 0 \\ 1 & yntd & ynt & 0 & 0 & 0 & 0 \\ 1 & yhtd & yht & 0 & 0 & 0 & 0 \\ 0 & pntd & 0 & 0 & 0 & 0 & 0 \\ 0 & phtd & pht & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & rstd & rst & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{array} \right], \tag{C.62}$$

$$N_x = \begin{bmatrix} 0 & 0 & 0 & 0 \\ lan & lah & lb & 0 \\ 0 & 0 & cb & 0 \\ 0 & 0 & ynb & 0 \\ 0 & 0 & yhb & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \tag{C.63}$$

with parameters defined below:

$$
\begin{aligned}
pntd &= 1-b && \text{(C.64)}\\
phtd &= -b && \text{(C.65)}\\
pht &= -a && \text{(C.66)}\\
rstd &= -b && \text{(C.67)}\\
rst &= 1-a && \text{(C.68)}\\
cb &= (1-d_{CH})d_{YH} && \text{(C.69)}\\
ct &= ad_{YH}(1-\theta)+d_{YH}(1-d_{CH})(\frac{1}{\rho}-\theta)(1-a) && \text{(C.70)}\\
ctd &= d_{YN}(1-b)(\phi-1)+bd_{YH}(1-\theta)+b(\theta-\phi)d_{CH}d_{YH}+ && \text{(C.71)}\\
&\quad +(1-d_{CH})(\theta-\frac{1}{\rho})d_{YH}b && \text{(C.72)}\\
yntd &= ctd-\phi*(1-b) && \text{(C.73)}\\
ynb &= cb && \text{(C.74)}\\
ynt &= ct && \text{(C.75)}\\
yhtd &= d_{YN}b\theta-d_{YN}d_{CH}b(\theta-\phi)-(1-d_{CH})(\theta-\frac{1}{\rho})bd_{YN}+ && \text{(C.76)}\\
&\quad -d_{YN}+b+d_{YN}\phi(1-b) && \text{(C.77)}\\
yht &= d_{YN}\theta a-d_{YN}(1-d_{CH})(\theta-\frac{1}{\rho})(a-1)+d_{YH}a && \text{(C.78)}\\
yhb &= -dyn(1-d_{CH}) && \text{(C.79)}\\
ltd &= \widetilde{d}_{YN}*yntd+\widetilde{d}_{YH}*yhtd && \text{(C.80)}\\
lt &= \widetilde{d}_{YN}*ynt+\widetilde{d}_{YH}*yht && \text{(C.81)}\\
lan &= -\widetilde{d}_{YN} && \text{(C.82)}\\
lab &= -\widetilde{d}_{YH} && \text{(C.83)}\\
lb &= \widetilde{d}_{YN}*ynb+\widetilde{d}_{YH}*yhb && \text{(C.84)}
\end{aligned}
$$

The loss function can be expressed now as:

$$L_{t_0} = U_C\overline{C}E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}[\frac{1}{2}\widehat{y}_t'L_y\widehat{y}_t+\widehat{y}_t'L_{\xi,y}\widehat{\xi}_t]+tip+O(3) \tag{C.85}$$

where:

$$
\begin{aligned}
L_y &= N_x^{'}L_xN_x, && \text{(C.86)}\\
L_{\xi,y} &= N_x^{'}L_xN_\xi+N_x^{'}L_\xi. && \text{(C.87)}
\end{aligned}
$$

Since variables $[\Delta S_t, \pi_{T,t}]$ do not appear in the original welfare objective function and in the second order terms of the structural equations we can further reduce the set of the variables which appear in the loss function to:

$$\widehat{y}_t' = \left[\begin{array}{ccccc} \widehat{Y}_t & \widehat{T}_t^d & \widehat{T}_t & \widehat{\pi}_{H,t} & \widehat{\pi}_{N,t} \end{array} \right] \tag{C.88}$$

The final set of the structural equations which represent the constraints of the maximization problem is:

$$\widehat{\pi}_{N,t} = k_N \widehat{mc}_t^{N,r} + \beta \widehat{\pi}_{N,t+1}, \tag{C.89}$$

$$\widehat{\pi}_{H,t} = k_H \widehat{mc}_t^{H,r} + \beta \widehat{\pi}_{H,t+1}, \tag{C.90}$$

$$\widehat{C}_t^* = \widehat{Y}_t + (ctd - \frac{1}{\rho} rstd)\widehat{T}_t^d + (ct - \frac{1}{\rho} rst)\widehat{T}_t + (cb-1)\widehat{B}_t, \tag{C.91}$$

$$\widehat{T}_t^d - \widehat{T}_{t-1}^d = \widehat{\pi}_{N,t} - \widehat{\pi}_{H,t} - a(\widehat{T}_t - \widehat{T}_{t-1}) \tag{C.92}$$

where:

$$\widehat{mc}_t^{N,r} = (\rho+\eta)\widehat{Y}_t + (\rho * ctd + \eta * ltd - pntd)\widehat{T}_t^d + \tag{C.93}$$

$$+(\rho * ct + \eta * lt)\widehat{T}_t - (1 + \eta * \widetilde{d}_{YN})\widehat{A}_{N,t} + \tag{C.94}$$

$$-\eta \widetilde{d}_{YH} \widehat{A}_{H,t} + (\rho * (cb-1) + \eta * lb)\widehat{B}_t \tag{C.95}$$

$$\widehat{mc}_t^{H,r} = (\rho+\eta)\widehat{Y}_t + (\rho * ctd + \eta * ltd - phtd)\widehat{T}_t^d + \tag{C.96}$$

$$+(\rho * ct + \eta * lt - pht)\widehat{T}_t - \eta \widetilde{d}_{YN} \widehat{A}_{N,t} + \tag{C.97}$$

$$-(1 + \eta * \widetilde{d}_{YH})\widehat{A}_{H,t} + (\rho * (cb-1) + \eta * lb)\widehat{B}_t \tag{C.98}$$

with:

$$m_{N,Y} = \rho + \eta \tag{C.99}$$

$$m_{N,T^d} = \rho * ctd + \eta * ltd - pntd \tag{C.100}$$

$$m_{N,T} = \rho * ct + \eta * lt \tag{C.101}$$

$$m_{N,A_N} = -(1 + \eta * \widetilde{d}_{YN}) \tag{C.102}$$

$$m_{N,A_H} = -\eta \widetilde{d}_{YH} \tag{C.103}$$

$$m_{N,B} = \rho * (cb-1) + \eta * lb \tag{C.104}$$

$$\begin{aligned}
m_{H,Y} &= \rho + \eta && \text{(C.105)}\\
m_{H,T^d} &= \rho * ctd + \eta * ltd - phtd && \text{(C.106)}\\
m_{H,T} &= \rho * ct + \eta * lt - pht && \text{(C.107)}\\
m_{H,A_N} &= -\eta \widetilde{d}_{YN} && \text{(C.108)}\\
m_{H,A_H} &= -(1 + \eta * \widetilde{d}_{YH}) && \text{(C.109)}\\
m_{H,B} &= \rho * (cb - 1) + \eta * lb && \text{(C.110)}
\end{aligned}$$

$$\begin{aligned}
n_{T^d} &= ctd - \frac{1}{\rho} rstd && \text{(C.111)}\\
n_T &= ct - \frac{1}{\rho} rst && \text{(C.112)}\\
n_B &= cb - 1 && \text{(C.113)}
\end{aligned}$$

Structural equations defining the Maastricht variables:

$$\widehat{R}_t = \widehat{\pi}_{t+1} - \rho(1-cb)(\widehat{B}_{t+1} - \widehat{B}_t) + \rho(\widehat{Y}_{t+1} - \widehat{Y}_t) + \rho * ctd(\widehat{T}^d_{t+1} - \widehat{T}^d_t) + \rho * ct(\widehat{T}_{t+1} - \widehat{T}_t), \tag{C.114}$$

$$\widehat{\pi}_t = \widehat{\pi}_{H,t} + b(\widehat{T}^d_t - \widehat{T}^d_{t-1}) + a(\widehat{T}_t - \widehat{T}_{t-1}), \tag{C.115}$$

$$\widehat{S}_t = \widehat{S}_{t-1} + \widehat{\pi}_t + rstd(\widehat{T}^d_t - \widehat{T}^d_{t-1}) + rst(\widehat{T}_t - \widehat{T}_{t-1}). \tag{C.116}$$

C.5 Reinterpretation of the Maastricht convergence criteria

We show how to reinterpret each of the Maastricht criteria in order to be able to use the method of Rotemberg and Woodford (1997, 1999).

C.5.1 Exchange rate criterion

We reinterpret the criterion on the nominal exchange rate (3.50) into two inequalities given below:[4]

$$E\left(\widehat{S}_t\right) - k * SD(\widehat{S}_t) \geq -15\%, \tag{C.117}$$

[4] E stands for the expectation operator and SD stands for the standard deviation operator.

$$E\left(\widehat{S}_t\right) + k * SD(\widehat{S}_t) \leq 15\%. \tag{C.118}$$

where k is large enough to prevent from violating the criterion (3.50) and SD refers to the standard deviation statistic.

These two inequalities can be represented as the following two sets of inequalities (to conform with the welfare measure we use discounted statistics):

$$\begin{cases} (1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t\left(\widehat{S}_t - (-15\%)\right) \geq 0 \\ (1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t\left(\widehat{S}_t - (-15\%)\right)^2 \leq K\left((1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t\left(\widehat{S}_t - (-15\%)\right)\right)^2, \end{cases} \tag{C.119}$$

$$\begin{cases} (1-\beta)E_{t_0}\sum_{t=0}^{\infty}\beta^t\left(15\% - \widehat{S}_t\right) \leq 0 \\ (1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t\left(15\% - \widehat{S}_t\right)^2 \leq K\left((1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t\left(15\% - \widehat{S}_t\right)\right)^2, \end{cases} \tag{C.120}$$

where $K = 1 + k^{-2}$.

C.5.2 Inflation criterion

We redefine the condition (3.48). We assume that the average inflation in the domestic economy should be at least k standard deviations smaller than the average inflation in the foreign economy plus a margin summarized by B_π (where $B_\pi = \sqrt[4]{1,015} - 1$):

$$E(\widehat{\pi}_t) \leq E(\widehat{\pi}^*_t) + B_\pi - kSD(\widehat{\pi}_t) \tag{C.121}$$

where $\widehat{\pi}_t$, $\widehat{\pi}^*_t$ are treated as deviations from the zero inflation steady state in the domestic economy and the foreign one accordingly (i.e. $\overline{\pi} = \overline{\pi}^* = 0$) and k large enough to prevent from violating criterion (3.48). We assume that the foreign economy is in the steady state so $\widehat{\pi}^*_t = 0\ \forall t$. As a result our restriction (C.121) becomes:

$$E(\widehat{\pi}_t) \leq B_\pi - kSD(\widehat{\pi}_t). \tag{C.122}$$

Since B_π is a constant we can use the following property of the variance: $Var(\widehat{\pi}_t) = Var(B_\pi - \widehat{\pi}_t)$. Our restriction becomes:

$$kSD(B_\pi - \widehat{\pi}_t) \leq E(B_\pi - \widehat{\pi}_t). \tag{C.123}$$

This restriction can be represented as a set of two restrictions:

$$(1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t\left(B_\pi-\pi_t\right)\geq 0, \tag{C.124}$$

$$(1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t\left(B_\pi-\widehat{\pi}_t\right)^2\leq K\left((1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t\left(B_\pi-\widehat{\pi}_t\right)\right)^2. \tag{C.125}$$

C.5.3 Nominal interest rate criterion

Similarly to the criterion on the CPI aggregate inflation we interpret the inequality (3.49):

$$E(\widehat{R}_t)\leq E(\widehat{R}_t^*)+C_R-kSD(\widehat{R}_t) \tag{C.126}$$

where k is large enough to prevent from frequent violating the criterion (3.49) and $C_R=\sqrt[4]{1,02}-1$.

As in the case of the foreign inflation we assume that $\widehat{R}_t^*=0\ \forall t$. So the restriction (C.126) becomes:

$$kSD(C_R-\widehat{R}_t)\leq E(C_R-\widehat{R}_t). \tag{C.127}$$

This inequality can be represented as a set of two inequalities:

$$(1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t\left(C_R-\widehat{R}_t\right)\geq 0, \tag{C.128}$$

$$(1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t\left(C_R-\widehat{R}_t\right)^2\leq K\left((1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t\left(C_R-\widehat{R}_t\right)\right)^2. \tag{C.129}$$

C.6 The constrained loss function

We provide the proof of the Proposition 1 stated in the main text. Since all the sets of the constraints have a similar structure the proof concerns the optimal monetary policy with only one constraint on the CPI inflation rate. The proof is based on the proof of Proposition 6.9 in Woodford (2003).

Proposition 3 *Consider the problem of minimizing an expected discounted sum of quadratic losses:*

$$E_{t_0}\left\{(1-\beta)\sum_{t=t_0}^{\infty}\beta^t L_t\right\} \tag{C.130}$$

subject to (3.53) - (3.54). Let $m_{1,\pi}$, $m_{2,\pi}$ *be the discounted average values of* $(B_\pi - \widehat{\pi}_t)$ *and* $(B_\pi - \widehat{\pi}_t)^2$ *associated with the optimal policy. Then the optimal policy also minimizes a modified discounted loss criterion of the form (C.130) with* L_t *replaced by:*

$$\widetilde{L}_t \equiv L_t + \Phi_\pi(\pi^T - \widehat{\pi}_t)^2 \tag{C.131}$$

under constraints represented by the structural equations. Importantly $\Phi_\pi \geq 0$ *and takes strictly positive value if and only if the constraint (3.54) binds. Moreover if the constraint (3.54) binds the corresponding target value* π^T *is negative and given by the following relation:*

$$\pi^T = B_\pi - K m_{1,\pi} < 0. \tag{C.132}$$

Proof. Let $m_{1,\pi}$ and $m_{2,\pi}$ be the discounted average values of $(B_\pi - \pi_t)$ and $(B_\pi - \pi_t)^2$ associated with the policy that solves the constrained optimization problem stated in the corollary. Let $m^*_{1,\pi}$ and $m^*_{2,\pi}$ be the values of these moments for the policy that minimizes (C.130) without additional constraints. Notice that since $m^*_{1,\pi} = B_\pi$ the constraint (3.53) does not bind.[5] We identify the deterministic component of policy, i.e. $m_{1,\pi}$ and also the stabilization component of policy which is: $m_{2,\pi} - (m_{1,\pi})^2$. Moreover we also conclude that $m_{1,\pi} \geq m^*_{1,\pi}$ since there is no advantage from choosing $m_{1,\pi}$ such that: $m_{1,\pi} < m^*_{1,\pi}$ - both constraints set only the lower bound on the value of $m_{1,\pi}$ for any value of the stabilization component of policy. If one chooses $m_{1,\pi}$ such that: $m_{1,\pi} > m^*_{1,\pi}$ then one can relax the constraint (3.54). So $m_{1,\pi} \geq m^*_{1,\pi}$. Based on the above discussion we formulate two alternative constraints to the constraints (3.53, 3.54):

$$(1-\beta)E_0 \sum_{t=0}^{\infty} \beta^t \left(B_\pi - \widehat{\pi}_t\right) \geq m_{1,\pi}, \tag{C.133}$$

$$(1-\beta)E_0 \sum_{t=0}^{\infty} \beta^t \left(B_\pi - \widehat{\pi}_t\right)^2 \leq m_{2,\pi}. \tag{C.134}$$

Observe that any policy that satisfies the above constraints satisfies also the weaker constraints: (3.53, 3.54). Now we take advantage of the Kuhn – Tucker theorem: the policy that minimizes (C.130) subject to (C.133, C.134) also minimizes the following loss criterion:

$$E_0 \left\{ (1-\beta) \sum_{t=0}^{\infty} \beta^t L_t \right\} - \mu_{1,\pi} E_0 \left\{ (1-\beta) \sum_{t=0}^{\infty} \beta^t \left(B_\pi - \widehat{\pi}_t\right) \right\} + \mu_{2,\pi} E_0 \left\{ (1-\beta) \sum_{t=0}^{\infty} \beta^t \left(B_\pi - \widehat{\pi}_t\right)^2 \right\} \tag{C.135}$$

[5] Means of all the variables under the unconstrained optimal policy are zero.

where $\mu_{1,\pi}$ and $\mu_{2,\pi}$ are the Lagrange multipliers which are nonnegative. If (3.54) binds then we obtain the following relation between the multipliers:

$$\mu_{1,\pi} = 2Km_{1,\pi}\mu_{2,\pi} \tag{C.136}$$

since $m_{2,\pi} = Km_{1,\pi}^2$.

Rearranging the terms in (C.135) we can define the new loss function as:

$$\widetilde{L}_t \equiv L_t + \mu_{2,\pi}\left((B_\pi - \widehat{\pi}_t) - \frac{\mu_{1,\pi}}{2\mu_{2,\pi}}\right)^2 \tag{C.137}$$

where the final term appears only when $\mu_{2,\pi} > 0$. Therefore $\Phi_\pi = \mu_{2,\pi} \geq 0$ and takes a strictly positive value only if (3.54) binds. Moreover for $\Phi_\pi > 0$ we have that:

$$\pi^T = B_\pi - \frac{\mu_{1,\pi}}{2\mu_{2.\pi}} = B_\pi - Km_{1,\pi}. \tag{C.138}$$

Notice that the target value for the CPI inflation is negative (since $K > 1$ and $m_{1,\pi} \geq B_\pi$):

$$\pi^T = B_\pi - Km_{1,\pi} < 0. \tag{C.139}$$

■

C.7 The constrained optimal monetary policy

We derive the first order conditions for the optimal monetary policy that satisfy the additional criteria on the nominal interest and the CPI aggregate inflation.

$$\begin{aligned}
\min L_{t_0} = U_C\overline{C}E_{t_0}\sum_{t=t_0}^{\infty}\beta^{t-t_0}[&\frac{1}{2}\Phi_Y(\widehat{Y}_t - \widehat{Y}_t^T)^2 + \frac{1}{2}\Phi_{T^d}(\widehat{T}_t^d - \widehat{T}_t^{d^T})^2 + \\
&\frac{1}{2}\Phi_T(\widehat{T}_t - \widehat{T}_t^T)^2 + \Phi_{TT^d}\widehat{T}_t^d\widehat{T}_t + \Phi_{YT^d}\widehat{Y}_t\widehat{T}_t^d + \Phi_{YT}\widehat{Y}_t\widehat{T}_t + \\
&\frac{1}{2}\Phi_{\pi_H}\widehat{\pi}_{H,t}^2 + \frac{1}{2}\Phi_{\pi_N}\widehat{\pi}_{N,t}^2 + \frac{1}{2}\Phi_R(\widehat{R}_t - R^T)^2 \\
&+ \frac{1}{2}\Phi_\pi(\widehat{\pi}_t - \pi^T)^2] + tip + O(3)
\end{aligned} \tag{C.140}$$

subject to :

$$\begin{aligned}
\widehat{\pi}_{N,t} = k_N(&m_{N,Y}\widehat{Y}_t + m_{N,T^d}\widehat{T}_t^d + m_{N,T}\widehat{T}_t + m_{N,A_N}\widehat{A}_{N,t} + m_{N,A_H}\widehat{A}_{H,t} \\
&+ m_{N,B}\widehat{B}_t) + \beta\widehat{\pi}_{N,t+1},
\end{aligned} \tag{C.141}$$

$$\begin{aligned}\widehat{\pi}_{H,t} = k_H(m_{H,Y}\widehat{Y}_t + m_{H,T^d}\widehat{T}^d_t + m_{H,T}\widehat{T}_t + m_{H,A_N}\widehat{A}_{N,t} + m_{H,A_H}\widehat{A}_{H,t} \\ + m_{H,B}\widehat{B}_t) + \beta\widehat{\pi}_{H,t+1},\end{aligned} \tag{C.142}$$

$$\widehat{C}^*_t = \widehat{Y}_t + n_{T^d}\widehat{T}^d_t + n_T\widehat{T}_t + n_B\widehat{B}_t, \tag{C.143}$$

$$\widehat{T}^d_t - \widehat{T}^d_{t-1} = \widehat{\pi}_{N,t} - \widehat{\pi}_{H,t} - a(\widehat{T}_t - \widehat{T}_{t-1}), \tag{C.144}$$

$$\begin{aligned}\widehat{R}_t = b\widehat{\pi}_{N,t+1} + (1-b)\widehat{\pi}_{H,t+1} - \rho(1-cb)(\widehat{B}_{t+1} - \widehat{B}_t) \\ + \rho(\widehat{Y}_{t+1} - \widehat{Y}_t) + \rho * ctd(\widehat{T}^d_{t+1} - \widehat{T}^d_t) + \\ (\rho * ct + a(1-b))(\widehat{T}_{t+1} - \widehat{T}_t),\end{aligned} \tag{C.145}$$

$$\widehat{\pi}_t = b\widehat{\pi}_{N,t} + (1-b)\widehat{\pi}_{H,t} + a(1-b)(\widehat{T}_t - \widehat{T}_{t-1}). \tag{C.146}$$

First order conditions of the minimization problem:

- wrt $\widehat{\pi}_{N,t}$:

$$0 = \Phi_{\pi_N}\widehat{\pi}_{N,t} + \gamma_{1,t} - \gamma_{1,t-1} - \gamma_{4,t} - b\beta^{-1}\gamma_{5,t-1} - b\gamma_{6,t}, \tag{C.147}$$

- wrt $\widehat{\pi}_{H,t}$:

$$0 = \Phi_{\pi_H}\widehat{\pi}_{H,t} + \gamma_{2,t} - \gamma_{2,t-1} + \gamma_{4,t} - (1-b)\beta^{-1}\gamma_{5,t-1} - (1-b)\gamma_{6,t}, \tag{C.148}$$

- wrt $\widehat{Y}_t$:

$$\begin{aligned}0 = \Phi_Y(\widehat{Y}_t - \widehat{Y}^T_t) + \Phi_{YT^d}\widehat{T}^d_t + \Phi_{YT}\widehat{T}_t - k_N m_{N,Y}\gamma_{1,t} \\ - k_H m_{H,Y}\gamma_{2,t} - \gamma_{3,t} + \rho\gamma_{5,t} - \rho\beta^{-1}\gamma_{5,t-1},\end{aligned} \tag{C.149}$$

- wrt $\widehat{T}^d_t$:

$$\begin{aligned}0 = \Phi_{T^d}(\widehat{T}^d_t - \widehat{T}^{d^T}_t) + \Phi_{TT^d}\widehat{T}_t + \Phi_{YT^d}\widehat{Y}_t - k_N m_{N,T^d}\gamma_{1,t} \\ - k_H m_{H,T^d}\gamma_{2,t} - n_{T^d}\gamma_{3,t} + \gamma_{4,t} \\ - \beta\gamma_{4,t+1} + \rho ctd\gamma_{5,t} - \rho ctd\beta^{-1}\gamma_{5,t-1},\end{aligned} \tag{C.150}$$

- wrt $\widehat{T}_t$:

$$0 = \Phi_T(\widehat{T}_t - \widehat{T}_t^T) + \Phi_{TT^d}\widehat{T}_t^d + \Phi_{YT}\widehat{Y}_t - k_N m_{N,T}\gamma_{1,t} - k_H m_{H,T}\gamma_{2,t} - n_T\gamma_{3,t} + a\gamma_{4,t} - \beta a\gamma_{4,t+1} + (\rho ct + a(1-b))\gamma_{5,t} - (\rho ct + a(1-b))\beta^{-1}\gamma_{5,t-1} - a(1-b)\gamma_{6,t} + a(1-b)\beta\gamma_{6,t+1}, \quad \text{(C.151)}$$

- wrt $\widehat{R}_t$:

$$0 = \Phi_R(\widehat{R}_t - R^T) + \gamma_{5,t}, \quad \text{(C.152)}$$

- wrt $\widehat{\pi}_t$:

$$0 = \Phi_\pi(\widehat{\pi}_t - \pi^T) + \gamma_{6,t}. \quad \text{(C.153)}$$

C.8 The constrained loss function - general case

We provide the proposition[6] that summarizes the discussion in Section 6 regarding the foreign economy. Since sets of the constraints for the CPI inflation rate and the nominal interest rate have the same structure the proposition concerns the optimal monetary policy with only one constraint on the CPI inflation rate.

Proposition 4 *Consider the problem of minimizing an expected discounted sum of quadratic losses:*

$$E_{t_0}\left\{(1-\beta)\sum_{t=t_0}^{\infty}\beta^t L_t\right\} \quad \text{(C.154)}$$

subject to

$$(1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t(B_\pi - \widehat{\pi}_t + \widehat{\pi}_t^*) \geq 0, \quad \text{(C.155)}$$

$$(1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t(B_\pi - \widehat{\pi}_t + \widehat{\pi}_t^*)^2 \leq K\left((1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t(B_\pi - \widehat{\pi}_t + \widehat{\pi}_t^*)\right)^2. \quad \text{(C.156)}$$

Let $n_{1,\pi}$, $n_{2,\pi}$ be the discounted average values of $(B_\pi - \widehat{\pi}_t + \widehat{\pi}_t^*)$ and $(B_\pi - \widehat{\pi}_t + \widehat{\pi}_t^*)^2$ associated with the optimal policy. Moreover assume that the average of the foreign CPI inflation rate (m_{1,π^*}) satisfies the following inequality: $m_{1,\pi^*} \geq -B_\pi$. Then the optimal policy also minimizes a modified discounted loss criterion of the form (C.154) with L_t replaced by:

$$\widetilde{L}_t \equiv L_t + \Phi_\pi(\pi_t^T - \widehat{\pi}_t)^2 \quad \text{(C.157)}$$

[6] We do not present the proof for this proposition since it resembles to a great extent the proof of Proposition 3.

under constraints represented by the structural equations. Importantly $\Phi_\pi \geq 0$ *and takes strictly positive value if and only if the constraint (C.156) binds. Moreover if the constraint (C.156) binds the corresponding target value* π_t^T *is given by the following relation:*

$$\pi_t^T = B_\pi + \widehat{\pi}_t^* - Kn_{1,\pi} < \widehat{\pi}_t^*. \tag{C.158}$$

Appendix D

Model characteristics - chapter 4

D.1 Steady state characterization

We define a deterministic steady state with zero inflation rate. We present a small open economy as the limiting case of a two country model, i.e. $n = 0$ and $\nu = 1-\lambda$. All variables in the steady state are denoted with a bar. All the shocks take the constant values, in particular: $\overline{A_N} = \overline{A_H} = 1$, $\overline{B} = 1$.[1] The discount factors are:

$$Q_{t_0,t} = Q^*_{t_0,t} = \beta^{t-t_0} \tag{D.1}$$

We assume that the the levels of debt to GDP ratio in both domestic and foreign economy (d_G, d^*_G) are exogenously given. We characterize the steady state with optimal tax rates, i.e. tax rates that maximize welfare given an exogenous level of debt to GDP ratio. Foreign variables are taken as given.

The optimal tax rates in our small open economy can be derived from the following constrained optimization problem:

$$\max_{\overline{C},\overline{Y_N},\overline{Y_H},\overline{p_N},\overline{p_H},\overline{p_T},\overline{RS},\overline{\tau_N},\overline{\tau_H}} (U(\overline{C}) - V(\overline{Y_N} + \overline{Y_H})) \tag{D.2}$$

subject to:

- goods' market clearing conditions:

$$\overline{Y_N} = \overline{p_N}^{-\phi}\mu(\overline{C} + \overline{G}), \tag{D.3}$$

$$\overline{Y_H} = \overline{p_H}^{-\theta}\overline{p_T}^{\theta-\phi}(1-\lambda)(1-\mu)(\overline{C}+\overline{G})+\lambda(1-\mu^*)\overline{p_H}^{-\theta}\overline{RS}^{\theta}\overline{p_F^*}^{\theta-\phi}(\overline{C^*}+\overline{G^*}), \tag{D.4}$$

[1] Foreign consumption is derived from the steady state relations of the foreign economy.

- relative prices:

$$1 = \mu \overline{p_N}^{1-\phi} + (1-\mu)\overline{p_T}^{1-\phi}, \tag{D.5}$$

$$1 = (1-\lambda)\overline{p_H}^{1-\theta} + \lambda \overline{p_F^*}^{1-\theta}\overline{RS}^{1-\theta}, \tag{D.6}$$

- risk sharing condition:

$$\overline{C}^{-\rho} = \overline{RS}^{-1}\overline{C^*}^{-\rho}, \tag{D.7}$$

- labour market clearing:

$$\overline{p_N} = \frac{\sigma}{\sigma-1}\frac{1}{1-\overline{\tau_N}}(\overline{Y_N}+\overline{Y_H})^{\eta}\overline{C}^{\rho}, \tag{D.8}$$

$$\overline{p_H} = \frac{\sigma}{\sigma-1}\frac{1}{1-\overline{\tau_H}}(\overline{Y_N}+\overline{Y_H})^{\eta}\overline{C}^{\rho}, \tag{D.9}$$

where domestic real wage is equal to the marginal rate of substitution between labour and consumption, i.e. $\overline{\omega} \equiv (\overline{Y_N}+\overline{Y_H})^{\eta}\overline{C}^{\rho}$.

One can define the markups in both sectors: $\overline{\mu_N} \equiv \frac{\sigma}{\sigma-1}\frac{1}{1-\overline{\tau_N}}$; $\overline{\mu_H} \equiv \frac{\sigma}{\sigma-1}\frac{1}{1-\overline{\tau_H}}$.

- government budget constraint:

$$(\beta^{-1}-1)b_D(\overline{p_N}\overline{Y_N}+\overline{p_H}\overline{Y_H}) = \overline{\tau_N p_N}\overline{Y_N} + \overline{\tau_H p_H}\overline{Y_H} - d_G\overline{Y}. \tag{D.10}$$

Similarly, we present the constrained maximization problem that solves for the foreign variables:

$$\max_{\overline{C},\overline{Y_N^*},\overline{Y_F^*},\overline{\tau_N^*},\overline{\tau_H^*},\overline{p_N^*},\overline{p_F^*}} U(\overline{C^*}) - V(\overline{L^*}) \tag{D.11}$$

subject to:

$$\overline{Y_N^*} = \overline{p_N^*}^{-\phi}\mu^*(\overline{C^*}+\overline{G^*}), \tag{D.12}$$

$$\overline{Y_F^*} = (1-\mu^*)\overline{p_F^*}^{-\phi}(\overline{C^*}+\overline{G^*}), \tag{D.13}$$

$$1 = \mu^*\overline{p_N^*}^{1-\phi} + (1-\mu^*)\overline{p_F^*}^{1-\phi}, \tag{D.14}$$

$$\overline{p_N^*} = \frac{\sigma}{\sigma-1}\frac{1}{1-\overline{\tau_N^*}}(\overline{Y_N^*}+\overline{Y_F^*})^{\eta}\overline{C^*}^{\rho}, \tag{D.15}$$

$$\overline{p_F^*} = \frac{\sigma}{\sigma-1}\frac{1}{1-\overline{\tau_F^*}}(\overline{Y_N^*}+\overline{Y_F^*})^{\eta}\overline{C^*}^{\rho}, \tag{D.16}$$

$$(\beta^{-1}-1)b_D^*(\overline{p_N^*Y_N^*}+\overline{p_F^*Y_F^*}) = \overline{\tau_N^* p_N^* Y_N^*} + \overline{\tau_F^* p_F^* Y_F^*} - d_G^*\overline{C^*}. \tag{D.17}$$

Note that for calibration purposes we derive a steady state in which tax rates are equal in both sectors. The steady state values of domestic variables $(\overline{C},\overline{Y_N},\overline{Y_H},\overline{p_N},\overline{p_H},\overline{p_T},\overline{RS},\overline{\tau})$ are solution to the following system of equations:

$$
\begin{cases}
\overline{Y_N} = \overline{p_N}^{-\phi}\mu(\overline{C}+\overline{G}) \\
\overline{Y_H} = \overline{p_H}^{-\theta}\overline{p_T}^{\theta-\phi}(1-\lambda)(1-\mu)(\overline{C}+\overline{G}) + \lambda(1-\mu^*)\overline{p_H}^{-\theta}\overline{RS}^{\theta}\overline{p_F^*}^{\theta-\phi}(\overline{C^*}+\overline{G^*}) \\
1 = \mu\overline{p_N}^{1-\phi} + (1-\mu)\overline{p_T}^{1-\phi} \\
1 = (1-\lambda)\overline{p_H}^{1-\theta} + \lambda\overline{p_F^*}^{1-\theta}\overline{RS}^{1-\theta} \\
\overline{C}^{-\rho} = \overline{RS}^{-1}\overline{C^*}^{-\rho} \\
\overline{p_N} = \frac{\sigma}{\sigma-1}\frac{1}{1-\overline{\tau}}(\overline{Y_N}+\overline{Y_H})^{\eta}\overline{C}^{\rho} \\
\overline{p_H} = \frac{\sigma}{\sigma-1}\frac{1}{1-\overline{\tau}}(\overline{Y_N}+\overline{Y_H})^{\eta}\overline{C}^{\rho} \\
(\beta^{-1}-1)b_D(\overline{p_N}\overline{Y_N}+\overline{p_H}\overline{Y_H}) = \overline{\tau_N}\overline{p_N}\overline{Y_N} + \overline{\tau_H}\overline{p_H}\overline{Y_H} - d_G\overline{Y}.
\end{cases}
\tag{D.18}
$$

D.2 A loglinearized version of the model

We present a system of the equilibrium conditions for the small open economy in the loglinear form, which is derived through the first-order approximation around the deterministic steady state with zero inflation described above. Here, we characterize the dynamic features of this model where the variables with a hat stand for the log deviations from the steady state. Variables with an asterisk represent the foreign equivalents of the domestic variables.

The supply-side of the economy is given by two Phillips curves, one for the non-traded and one for the domestic traded sector, respectively, which are derived from (4.18):

$$\widehat{\pi}_{N,t} = k_N(\rho\widehat{C}_t + \eta\widehat{L}_t - \widehat{A}_{N,t} - \rho\widehat{B}_t - \widehat{p}_{N,t} + \omega_N\widehat{\tau}_{N,t}) + \beta\widehat{\pi}_{N,t+1}, \tag{D.19}$$

$$\widehat{\pi}_{H,t} = k_H(\rho\widehat{C}_t + \eta\widehat{L}_t - \widehat{A}_{H,t} - \rho\widehat{B}_t - \widehat{p}_{H,t} + \omega_H\widehat{\tau}_{H,t}) + \beta\widehat{\pi}_{H,t+1} \tag{D.20}$$

where $\widehat{p}_{N,t} \equiv \ln(\frac{P_{N,t}}{P_t})$, $\widehat{p}_{H,t} \equiv \ln(\frac{P_{H,t}}{P_t})$, $\widehat{\pi}_{N,t} \equiv \ln(\frac{P_{N,t}}{P_{N,t-1}})$, $\widehat{\pi}_{H,t} \equiv \ln(\frac{P_{H,t}}{P_{H,t-1}})$, $k_N \equiv \frac{(1-\alpha_N)(1-\alpha_N\beta)}{\alpha_N}$, $k_H \equiv \frac{(1-\alpha_H)(1-\alpha_H\beta)}{\alpha_H}$, $\omega_N \equiv \frac{\overline{\tau_N}}{1-\overline{\tau_N}}$, $\omega_H \equiv \frac{\overline{\tau_H}}{1-\overline{\tau_H}}$ and aggregate labour supply ($\widehat{L}_t$) is defined through the labour market clearing condition ((4.11), (4.17)):

$$\widehat{L}_t = \widetilde{d}_{Y_N}(\widehat{Y}_{N,t} - \widehat{A}_{N,t}) + \widetilde{d}_{Y_H}(\widehat{Y}_{H,t} - \widehat{A}_{H,t}), \tag{D.21}$$

where $\widetilde{d}_{Y_N} \equiv \frac{\overline{Y}_N}{\overline{Y}_N+\overline{Y}_H}$, $\widetilde{d}_{Y_H} \equiv \frac{\overline{Y}_H}{\overline{Y}_N+\overline{Y}_H}$ are steady state ratios.

It is worth underlining that inflation dynamics in both domestic sectors do not only depend on the real marginal costs in a given sector, but also on the relative prices of goods. In particular, a higher relative price of goods in one sector in relation to other goods induces a substitution away effect and leads to deflationary pressures in this sector.

The demand side of the small open economy is represented by the market clearing conditions in both nontraded and domestic traded sectors ((4.8), (4.9)):

$$\widehat{Y}_{N,t} = d_{CN}\widehat{C}_t - \phi\widehat{p}_{N,t} + d_{GN}\widehat{G}_t, \tag{D.22}$$

$$\widehat{Y}_{H,t} = d_{CH}\widehat{C}_t - \theta\widehat{p}_{H,t} + b(\phi-\theta)d_{CH}\widehat{T}^d_t + (1-d_H)\theta\widehat{RS}_t + d_{C^*H}\widehat{C}^*_t + b^*(\phi-\theta)d_{C^*H}\widehat{T}^{d*}_t + d_{GH}\widehat{G}_t \tag{D.23}$$

where $d_{CN} \equiv \overline{p_N}^{-\phi}\mu\frac{\overline{C}}{\overline{Y}_N}$, $d_{GN} \equiv \overline{p_N}^{-\phi}\mu\frac{\overline{G}}{\overline{Y}_N}$, $d_{CH} \equiv (1-\lambda)(1-\mu)\frac{\overline{C}}{\overline{Y_H}}\overline{p}_H^{-\theta}\overline{p}_T^{\theta-\phi}$, $d_{C^*H} \equiv \lambda(1-\mu^*)\frac{\overline{C}^*}{\overline{Y_H}}\overline{p_H}^{-\theta}\overline{RS}^{\theta}\overline{p^*_F}^{\theta-\phi}$, $d_{GH} \equiv (1-\lambda)(1-\mu)\frac{\overline{G}}{\overline{Y_H}}\overline{p}_H^{-\theta}\overline{p}_T^{\theta-\phi}$, $b \equiv \mu(\overline{p_N})^{1-\phi}$, $b^* \equiv \mu^*(\overline{p^*_N})^{1-\phi}$ are steady state ratios. Additionally, we define aggregate output as the sum of sector outputs:

$$\widehat{Y}_t = d_{YN}(\widehat{p}_{N,t} + \widehat{Y}_{N,t}) + d_{YH}(\widehat{p}_{H,t} + \widehat{Y}_{H,t}), \tag{D.24}$$

where $d_{YN} \equiv \frac{\overline{p_N}\overline{Y_N}}{\overline{Y}}$ and $d_{YH} \equiv \frac{\overline{p_H}\overline{Y_H}}{\overline{Y}}$ are steady state ratios.

The complete asset market assumption (4.15) gives us the following risk sharing condition:

$$\widehat{C}_t = \widehat{B}_t + \frac{1}{\rho}\widehat{RS}_t + \widehat{C}^*_t - \widehat{B}^*_t. \tag{D.25}$$

From the definition of price indices ((4.5), (4.6)), we obtain the following relations between relative prices, terms of trade, domestic terms of trade and real exchange rate:

$$(a-1)\widehat{p}_{H,t} = b\widehat{T^d_t} + a\widehat{RS_t} - b^* a\widehat{T^{d*}_t}, \tag{D.26}$$

$$\widehat{p}_{N,t} = (1-b)\widehat{T^d_t}, \tag{D.27}$$

$$\widehat{p}_{H,t} = -b\widehat{T^d_t} - a\widehat{T}_t, \tag{D.28}$$

where $a \equiv \lambda\left(\frac{\overline{RSp^*_F}}{\overline{p_T}}\right)^{1-\theta}$ is the steady state ratio. We also derive the laws of motion for the international terms of trade and the domestic terms of trade from their definitions:

$$\widehat{T}_t = (\widehat{\pi}^*_{F,t} + \Delta\widehat{S}_t) - \widehat{\pi}_{H,t} + \widehat{T}_{t-1}, \tag{D.29}$$

$$\widehat{T}^d_t = \widehat{\pi}_{N,t} - \widehat{\pi}_{T,t} + \widehat{T}^d_{t-1}, \tag{D.30}$$

with $\widehat{\pi}_{T,t} \equiv \ln(\frac{P_{T,t}}{P_{T,t-1}})$, $\widehat{\pi}^*_{F,t} \equiv \ln(\frac{P^*_{F,t}}{P^*_{F,t-1}})$ and $\Delta\widehat{S}_t = \widehat{S}_t - \widehat{S}_{t-1}$. Tradable inflation ($\widehat{\pi}_{T,t}$) can be represented as:

$$\widehat{\pi}_{T,t} = \widehat{\pi}_{H,t} + a(\widehat{T}_t - \widehat{T}_{t-1}). \tag{D.31}$$

Dynamics of the government debt can be derived through the loglinearization of equation (4.23):

$$\widehat{d}_t = \frac{1}{\beta}(\widehat{d}_{t-1} + \widehat{R}_t - \widehat{\pi}_t) - d_{sr}(\widehat{R}_t + \widehat{sr}_t)$$

where $d_{sr} \equiv \frac{\overline{R}\overline{sr}}{\overline{d}}$ and real primary surplus ($\widehat{sr}_t$) evolves according to the loglinearized version of equation (4.21):

$$\widehat{sr}_t = s_{\tau_N}(\widehat{\tau}_{N,t} + \widehat{p}_{N,t} + \widehat{Y}_{N,t}) + s_{\tau_H}(\widehat{\tau}_{H,t} + \widehat{p}_{H,t} + \widehat{Y}_{H,t}) - s_G\widehat{G}_t$$

where $s_{\tau_N} \equiv \frac{\overline{\tau}_N\overline{p}_N\overline{Y}_N}{\overline{sr}}$; $s_{\tau_H} \equiv \frac{\overline{\tau}_H\overline{p}_H\overline{Y}_H}{\overline{sr}}$ and $s_G \equiv \frac{\overline{G}}{\overline{sr}}$.

Subsequently, an intertemporal government solvency condition has a following form:

$$\widehat{d}_{t-1} - \widehat{\pi}_t - \rho\widehat{C}_t + \rho\widehat{B}_t = (1-\beta)(-\rho\widehat{C}_t + \rho\widehat{B}_t + \widehat{sr}_t) + \beta E_t(\widehat{d}_t - \widehat{\pi}_{t+1} - \rho\widehat{C}_{t+1} + \rho\widehat{B}_{t+1}) \tag{D.32}$$

Additionally, we present equations defining monetary and fiscal variables that are constrained by the Maastricht criteria: the CPI inflation rate ($\widehat{\pi}_t$), the nominal interest rate ($\widehat{R}_t$) the nominal exchange rate ($\widehat{S}_t$), deficit to GDP ratio (df_t) and debt to GDP ratio (br_t). First, the nominal interest rate can be derived from the loglinearized version of the Euler condition (4.14):

$$\widehat{R}_t = \rho(\widehat{C}_{t+1} - \widehat{B}_{t+1}) - \rho(\widehat{C}_t - \widehat{B}_t) + \widehat{\pi}_{t+1}, \tag{D.33}$$

where $\widehat{\pi}_t \equiv \ln(\frac{P_t}{P_{t-1}})$. CPI aggregate inflation is a weighted sum of the sector inflation rates:

$$\widehat{\pi}_t = b\widehat{\pi}_{N,t} + (1-a)(1-b)\widehat{\pi}_{H,t} + a(1-b)\widehat{\pi}^*_{F,t} + a(1-b)(\widehat{S}_t - \widehat{S}_{t-1}). \tag{D.34}$$

Notice that CPI aggregate inflation does not only depend on the domestic sector inflation rates, but also on the foreign traded inflation rate and changes in the nominal exchange rate. For example, a nominal exchange rate depreciation puts an upward pressure on the CPI inflation rate.

The nominal exchange rate can be derived from the definition of the real exchange rate:

$$\widehat{S}_t = \widehat{S}_{t-1} + \widehat{\pi}_t - \widehat{\pi}_t^* + \widehat{RS}_t - \widehat{RS}_{t-1}. \tag{D.35}$$

The law of motion of the nominal exchange rate depends on the real exchange rate fluctuations and differences in the aggregate inflation rates between the home and the foreign economy. Additionally, by combining the international risk sharing condition (D.25) and Euler conditions for the domestic and foreign economy (D.33), we obtain a relation between the nominal interest rate and the nominal exchange rate:

$$\widehat{S}_t = \widehat{R}_t^* - \widehat{R}_t + \widehat{S}_{t+1}.$$

This equation represents a version of the uncovered interest rate parity, which implies that changes in the nominal exchange rate result from differences between the domestic and foreign monetary policy. Let us point out that although very intuitive, this equation does not constitute an independent equilibrium condition.

Deficit to GDP ratio depends on primary surplus and interest rate payments on debt:

$$df_t = \frac{\frac{d_{t-1}}{R_{t-1}}\pi_t(R_{t-1} - 1) - sr_t}{Y_t}.$$

From definition steady state ratio of deficit to GDP ratio is zero. Therefore the loglinearized version of the above equation is:

$$\widetilde{df}_t = \frac{\overline{d}}{\overline{Y}}(1-\beta)\widehat{d}_{t-1} + \frac{\overline{d}}{\overline{Y}}(1-\beta)\widehat{\pi}_t + \beta\frac{\overline{d}}{\overline{Y}}\widehat{R}_{t-1} - \frac{\overline{sr}}{\overline{Y}}\widehat{sr}_t, \tag{D.36}$$

where $\widetilde{df}_t = df_t$.

Finally, debt to GDP evolves according to the following equation:

$$\widehat{br}_t = \widehat{d}_t - \widehat{Y}_t - \widehat{R}_t. \tag{D.37}$$

The system is closed by specifying a monetary and fiscal rule. In this paper, we derive the optimal monetary and fiscal policy rule which maximizes welfare of the society subject to the structural equations of the economy. The optimal rule is specified as a rule where the monetary and fiscal authority stabilizes the target variables in order to minimize the welfare loss of society and provide the most efficient allocation.[2]

Summing up, the dynamics of the small open economy are summarized by the following variables, $\widehat{\pi}_{N,t}$, $\widehat{\pi}_{H,t}$, $\widehat{C}_t$, $\widehat{L}_t$, $\widehat{Y}_{H,t}$, $\widehat{Y}_{N,t}$, $\widehat{p}_{N,t}$, $\widehat{p}_{H,t}$, $\widehat{Y}_t$, $\widehat{RS}_t$, $\widehat{T}_t^d$, $\widehat{T}_t$, $\widehat{S}_t$, $\widehat{\pi}_t$, $\widehat{R}_t$, $\widehat{\tau}_{N,t}$, $\widehat{\tau}_{H,t}$, $\widehat{d}_t$, $\widehat{df}_t$, $\widehat{br}_t$ which are determined by equations (D.19)–(D.37), given the

[2] Giannoni and Woodford (2003) call these type of rules flexible inflation targeting rules.

evolution of the stochastic shocks $\widehat{A}_{N,t}$, $\widehat{A}_{H,t}$, $\widehat{B}_t$, $\widehat{G}_t$ and the foreign variables $\widehat{C}^*_t$, $\widehat{T}^{d^*}_t$, $\widehat{\pi}^*_t$, $\widehat{\pi}^*_{F,t}$.[3]

D.3 Parameterization

We present values of the structural parameters and also values of the stochastic parameters chosen in the numerical exercise.

The parameter definition	value of the parameter	
inverse of the intertemporal elasticity of substitution	ρ	2
inverse of the labour supply elasticity	η	4
discount factor	β	0.99
intratemporal elasticity between variety of the goods	σ	10
elasticity of substitution between home and foreign tradables	θ	1.5
elasticity of substitution between tradables and nontradables	ϕ	0.5
share of nontradables	μ	0.42
degree of openness	λ	0.4
price rigidity in the nontradable sector	α_N	0.85
price rigidity in the home tradable sector	α_H	0.8
steady state share of taxes in the nontradable sector	τ_N	0.19
steady state share of taxes in the tradable sector	τ_H	0.19
steady state share of government expenditure in GDP	d_G	0.2
steady state debt to GDP ratio	b_D	1.6
Foreign economy:		
steady state share of government expenditure in GDP	d^*_G	0.2
share of nontradables	μ^*	0.6
steady state debt to GDP ratio	b^*_D	2.4

Table D.1: Structural parameters - model in Chapter 4

shocks	autoregressive parameter	standard deviation (in %)
nontradable productivity (A_N)	0.85	1.6
tradable productivity (A_H)	0.85	1.8
preference (B)	0.95	0.72
foreign consumption (C^*)	0.85	0.23
government expenditure (G)	0.8	2
$corr(\widehat{A}_{N,t}, \widehat{A}_{H,t}) = 0.7$ where $corr$ - correlation coefficient		

Table D.2: Stochastic environment - model in Chapter 4

[3] For simplicity, we choose to consider only one type of external shocks, foreign consumption shocks ($\widehat{C}^*_t$). As a result, $\widehat{T}^{d^*}_t$, $\widehat{\pi}^*_t$, $\widehat{\pi}^*_{F,t}$ are assumed to be zero. Moreover, all shocks follow an AR(1) process with normally distributed innovations.

Note: The policy rule is calibrated following Natalucci and Ravenna (2007): $\widehat{R}_t = 0.9\widehat{R}_{t-1} + 0.1(\widehat{\pi}_t + 0.2\widehat{Y}_t + 0.3\widehat{S}_t) + \widehat{\varepsilon}_{R,t}$, where $SD(\widehat{\varepsilon}_{R,t}) = 0.44$.

	Statistics	
Standard deviation in %	Model	Historical
Output:	**1.79**	1.68
nontraded sector	**1.77**	1.56
traded sector	**3.25**	4.32
Consumption	**2.08**	1.93
Nominal interest rate	**0.51**	0.52
Nominal exchange rate	**2.83**	2.59
Real exchange rate	**2.36**	3.62
CPI inflation rate:	**0.81**	0.91
nontraded sector	**0.58**	0.97
traded sector	**0.92**	0.74

Table D.3: Matching the moments - model in Chapter 4

Note: The model moments are theoretical. As far as the historical statistics are concerned our data sample for the Czech Republic is 1995:1 - 2006:2. CPI inflation rate in the traded and nontraded sector data sample is 2000:1 - 2006:2. All series are logged (except for interest and inflation rates) and Hodrick - Prescott filtered. Rates of change are quarterly. All data were collected from the Eurostat webpage. Data are seasonally adjusted where appropriate. We present the detailed data series. Output: Gross value added (GVA) at 1995 constant prices in national currency. Traded output is an aggregate of sectoral GVA for: Agriculture; Hunting; Forestry and Fishing; Total industry (excluding construction). Nontraded output is an aggregate of sectoral GVA for: Wholesale and retail trade, repair of motor vehicles, motorcycles and personal household goods; Hotels and restaurants; Transport, storage and communication; Financial intermediation, real estate, renting and business activities. Consumption: Final consumption expenditure of households at 1995 constant prices in national currency. Nominal interest rate: three months T - bill interest rate. Nominal exchange rate: Bilateral Koruny/euro exchange rate (quarterly average). Real exchange rate: CPI based real effective exchange rate (6 trading partners, quarterly average). CPI inflation rate: Harmonized Index of Consumer Prices (HICP). CPI inflation rate in the nontraded sector: HICP - Services. CPI inflation in the traded sector: HICP - Goods.

D.4 Quadratic representation of the optimal loss function

D.4.1 The second order approximation of the welfare function

We present a second order approximation to the welfare function (4.1):

$$W_{t_0} = U_C \overline{C} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [z_v' \widehat{v}_t - \frac{1}{2} \widehat{v}_t' Z_v \widehat{v}_t - \widehat{v}_t' Z_\xi \widehat{\xi}_t] + tip + O(3) \tag{D.38}$$

where $\widehat{v}_t' = \left[\begin{array}{ccccc} \widehat{C}_t & \widehat{Y}_{N,t} & \widehat{Y}_{H,t} & \widehat{\pi}_{N,t} & \widehat{\pi}_{H,t} \end{array} \right]$; $\widehat{\xi}_t' = \left[\begin{array}{ccccc} \widehat{A}_{N,t} & \widehat{A}_{H,t} & \widehat{B}_t & \widehat{C}_t^* & \widehat{G}_t \end{array} \right]$; *tip* stands for *terms independent of policy* and $O(3)$ includes terms that are of order higher than the second in the deviations of variables from their steady state values. The matrices z_v, Z_v, Z_ξ are defined below:

$$z_v' = \left[\begin{array}{ccccc} 1 & -s_{CY_N} & -s_{CY_H} & 0 & 0 \end{array} \right], \tag{D.39}$$

$$Z_v = \left[\begin{array}{lllll} \rho - 1 & 0 & 0 & 0 & 0 \\ 0 & s_{CY_N}(1+\eta \widetilde{d}_{Y_N}) & \eta s_{CY_N} \widetilde{d}_{Y_H} & 0 & 0 \\ 0 & \eta s_{CY_N} \widetilde{d}_{Y_H} & s_{CY_H}(1+\eta \widetilde{d}_{Y_H}) & 0 & 0 \\ 0 & 0 & 0 & s_{CY_N} \frac{\sigma}{k_N} & 0 \\ 0 & 0 & 0 & 0 & s_{CY_H} \frac{\sigma}{k_H} \end{array} \right], \tag{D.40}$$

$$Z_\xi = \left[\begin{array}{llll} 0 & 0 & -\rho & 0 \\ -s_{CY_N}(1+\eta \widetilde{d}_{Y_N}) & -\eta s_{CY_N} \widetilde{d}_{Y_H} & 0 & 0 \\ -\eta s_{CY_N} \widetilde{d}_{Y_H} & -s_{CY_H}(1+\eta \widetilde{d}_{Y_H}) & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]. \tag{D.41}$$

where $s_{CY_N} = \frac{\varpi \overline{Y_N}}{\overline{C}}$; $s_{CY_H} = \frac{\varpi \overline{Y_H}}{\overline{C}}$.

D.4.2 Elimination of the linear terms

This section describes in detail how we eliminate the linear terms in the second order approximation to the welfare function in order to obtain a quadratic loss function. Moreover we reduce the number of structural variables that represent the policy problem by appropriate substitutions.

The optimal monetary and fiscal policy solves the welfare maximization problem with the constraints given by the structural equations of the economy (their loglinearized versions are (D.19) - (D.32)). The matrix representation of the second order approximation to the welfare function is the following:

$$W = U_C \overline{C} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [z_x' \widehat{x}_t - \frac{1}{2} \widehat{x}_t' Z_x \widehat{x}_t - \widehat{x}_t' Z_\xi \widehat{\xi}_t] + tip + O(3). \tag{D.42}$$

Similarly we present a second order approximation to all the structural equations in the matrix form:

$$E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [\begin{bmatrix} A_1 \widehat{x}_t \\ A_2 \widehat{x}_t \\ ... \\ A_{14} \widehat{x}_t \end{bmatrix} + \frac{1}{2} \begin{bmatrix} \widehat{x}_t{}' B_1 \widehat{x}_t \\ \widehat{x}_t{}' B_2 \widehat{x}_t \\ ... \\ \widehat{x}_t{}' B_{14} \widehat{x}_t \end{bmatrix} + \begin{bmatrix} \widehat{x}_t{}' C_1 \widehat{\xi}_t \\ \widehat{x}_t{}' C_2 \widehat{\xi}_t \\ ... \\ \widehat{x}_t{}' C_{14} \widehat{\xi}_t \end{bmatrix}] + tip + O(3) = 0 \tag{D.43}$$

with

$$\widehat{x}_t' = \begin{bmatrix} \widehat{Y}_t & \widehat{L}_t & \widehat{C}_t & \widehat{Y}_{N,t} & \widehat{Y}_{H,t} & \widehat{p}_{N,t} & \widehat{p}_{H,t} & \widehat{T}_t^d & \widehat{T}_t & \widehat{RS}_t & \widehat{\Delta S}_t & \widehat{\pi}_{H,t} & \widehat{\pi}_{N,t} & \widehat{\pi}_{T,t} & \widehat{\tau}_{N,t} & \widehat{\tau}_{H,t} \end{bmatrix}, \tag{D.44}$$

$$\widehat{\xi}_t' \equiv \begin{bmatrix} \widehat{A}_{N,t} & \widehat{A}_{H,t} & \widehat{B}_t & \widehat{C}_t^* & \widehat{G}_t \end{bmatrix}$$

where *tip* means terms independent of policy.

Following the methodology of Benigno and Woodford (2005) in order to eliminate the linear terms in the welfare function we solve the system of linear equations:

$$\zeta A = z_x^{'} \tag{D.45}$$

where $A_{(14\times16)} = \begin{bmatrix} A_1 \\ A_2 \\ ... \\ A_{14} \end{bmatrix}$;

$\zeta_{(1\times14)} = \begin{bmatrix} \zeta_1 & \zeta_2 & \zeta_3 & \zeta_4 & \zeta_5 & \zeta_6 & \zeta_7 & \zeta_8 & \zeta_9 & \zeta_{10} & \zeta_{11} & \zeta_{12} & \zeta_{13} & \zeta_{14} \end{bmatrix}$ and $z_{x(16\times1)}$.

As a result we obtain the loss function:

$$L_{t_0} = U_C \overline{C} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [\frac{1}{2} \widehat{x}_t' L_x \widehat{x}_t + \widehat{x}_t' L_\xi \widehat{\xi}_t] + tip + O(3) \tag{D.46}$$

where

$$L_x = Z_x + \zeta_1 B_1 + \zeta_2 B_2 + \zeta_4 B_4 + \zeta_6 B_6 + \zeta_9 B_9 + \zeta_{10} B_{10} + \zeta_{11} B_{11} + \zeta_{12} B_{12} + \tag{D.47}$$

$$\zeta_{13} B_{13} + \zeta_{14} B_{14}, \tag{D.48}$$

$$L_\xi = Z_\xi + \zeta_1 C_1 + \zeta_2 C_2 + \zeta_4 C_4 + \zeta_6 C_6 + \zeta_{13} C_{13} + \zeta_{14} C_{14}. \tag{D.49}$$

D.4.3 Substitution of the variables

We want to represent the loss function (D.46) and also the whole model just in terms of the following variables:

$$\widehat{y}_t' = \begin{bmatrix} \widehat{Y}_t & \widehat{T}_t^d & \widehat{T}_t & \Delta\widehat{S}_t & \widehat{\pi}_{H,t} & \widehat{\pi}_{N,t} & \widehat{\pi}_{T,t} & \widehat{\tau}_{N,t} & \widehat{\tau}_{H,t} \end{bmatrix}. \tag{D.50}$$

In order to do this we define matrices $N_{x(16\times9)}$ and $N_{\xi(14\times6)}$ that map all the variables in the vector y_t' in the following way:

$$\widehat{x}_t = N_x\widehat{y}_t' + N_\xi'\widehat{\xi}_t \tag{D.51}$$

where:

$$N_x = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ l_Y & ltd & lt & 0 & 0 & 0 & 0 & 0 & 0 \\ c_Y & ctd & ct & 0 & 0 & 0 & 0 & 0 & 0 \\ yny & yntd & ynt & 0 & 0 & 0 & 0 & 0 & 0 \\ yhy & yhtd & yht & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & pntd & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & phtd & pht & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & rstd & rst & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 \end{bmatrix}, \tag{D.52}$$

$$N_x = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 \\ lan & lah & l_B & 0 & l_G \\ 0 & 0 & c_B & 0 & c_G \\ 0 & 0 & ynb & 0 & yng \\ 0 & 0 & yhb & 0 & yhg \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \tag{D.53}$$

with parameters defined below:

$$pntd = 1 - b \tag{D.54}$$
$$phtd = -b \tag{D.55}$$
$$pht = -a \tag{D.56}$$
$$rstd = -b \tag{D.57}$$
$$rst = 1 - a \tag{D.58}$$
$$yhc = d_{CH} + d_{C^*H} \tag{D.59}$$
$$c_Y = \frac{1}{d_{YN}d_{CN} + d_{YH}yhc} \tag{D.60}$$
$$c_B = c_Y d_{C^*H} d_{YH} \tag{D.61}$$
$$c_G = -c_Y(d_{YN}d_{GN} + d_{YH}d_{GH}) \tag{D.62}$$
$$ct = c_Y(ad_{YH}(1-\theta) - d_{YH}(1-d_H)\theta(1-a) + \frac{1}{\rho}d_{YH}d_{C^*H}(1-a)) \tag{D.63}$$
$$ctd = c_Y(d_{YN}(1-b)(\phi-1) + bd_{YH}(1-\theta) + b(\theta-\phi)d_H d_{YH} + \tag{D.64}$$
$$+(1-d_H)\theta d_{YH}b - d_{C^*H}\frac{1}{\rho}d_{YH}b) \tag{D.65}$$
$$yny = c_Y d_{CN} \tag{D.66}$$
$$yntd = d_{CN}ctd - \phi * (1-b) \tag{D.67}$$
$$ynb = d_{CN}cb \tag{D.68}$$
$$yng = d_{GN} + d_{CN}cg \tag{D.69}$$
$$ynt = d_{CN}ct \tag{D.70}$$
$$yhy = \frac{1 - d_{YN}yny}{d_{YH}} \tag{D.71}$$
$$yhtd = -\frac{d_{YN}yntd - (d_{YN}(1-b) - bd_{YH})}{d_{YH}} \tag{D.72}$$
$$yht = -\frac{d_{YN}}{d_{YH}}ynt + a \tag{D.73}$$
$$yhb = -\frac{d_{YN}}{d_{YH}}ynb \tag{D.74}$$
$$yhg = -\frac{d_{YN}}{d_{YH}}yng \tag{D.75}$$
$$ltd = \widetilde{d}_{YN} * yntd + \widetilde{d}_{YH} * yhtd \tag{D.76}$$
$$lt = \widetilde{d}_{YN} * ynt + \widetilde{d}_{YH} * yht \tag{D.77}$$
$$lan = -\widetilde{d}_{YN} \tag{D.78}$$
$$lab = -\widetilde{d}_{YH} \tag{D.79}$$
$$l_B = \widetilde{d}_{YN} * ynb + \widetilde{d}_{YH} * yhb \tag{D.80}$$
$$l_G = \widetilde{d}_{YN} * yng + \widetilde{d}_{YH} * yhg \tag{D.81}$$
$$l_Y = \widetilde{d}_{YN} * yny + \widetilde{d}_{YH} * yhy \tag{D.82}$$

The loss function can be expressed now as:

$$L_{t_0} = U_C \overline{C} E_{t_0} \sum_{t=t_0}^{\infty} \beta^{t-t_0} [\frac{1}{2} \widehat{y}_t' L_y \widehat{y}_t + \widehat{y}_t' L_{\xi,y} \widehat{\xi}_t] + tip + O(3) \tag{D.83}$$

where:

$$L_y = N_x' L_x N_x, \tag{D.84}$$
$$L_{\xi,y} = N_x' L_x N_\xi + N_x' L_\xi. \tag{D.85}$$

Since variables $[\Delta S_t, \pi_{T,t}]$ do not appear in the original welfare objective function and in the second order terms of the structural equations we can further reduce the set of the variables which appear in the loss function to:

$$\widehat{y}_t' = \left[\begin{array}{ccccccc} \widehat{Y}_t & \widehat{T}_t^d & \widehat{T}_t & \widehat{\pi}_{H,t} & \widehat{\pi}_{N,t} & \widehat{\tau}_{N,t} & \widehat{\tau}_{H,t} \end{array} \right]. \tag{D.86}$$

The final set of the structural equations which represent the constraints of the maximization problem is:

$$\widehat{\pi}_{N,t} = k_N \widehat{mc}_t^{N,r} + \beta \widehat{\pi}_{N,t+1}, \tag{D.87}$$
$$\widehat{\pi}_{H,t} = k_H \widehat{mc}_t^{H,r} + \beta \widehat{\pi}_{H,t+1}, \tag{D.88}$$

$$0 = n_Y \widehat{Y}_t + n_{T^d} \widehat{T}_t^d + n_T \widehat{T}_t + n_B \widehat{B}_t + n_G \widehat{G}_t - \widehat{C}_t^*, \tag{D.89}$$

$$\widehat{T}_t^d - \widehat{T}_{t-1}^d = \widehat{\pi}_{N,t} - \widehat{\pi}_{H,t} - a(\widehat{T}_t - \widehat{T}_{t-1}) \tag{D.90}$$

$$\begin{aligned} \widehat{d}_{t-1} = \; & f_{\tau_N} \widehat{\tau}_{N,t} + f_{\tau_H} \widehat{\tau}_{H,t} + f_Y \widehat{Y}_t + f_{T^d} \widehat{T}_t^d + f_T \widehat{T}_t + f_{T^d(+1)} \widehat{T}_{t+1}^d + f_{T(+1)} \widehat{T}_{t+1} && \text{(D.91)} \\ & + f_{T(-1)} \widehat{T}_{t-1} + f_{\pi_N} \widehat{\pi}_{N,t} + f_{\pi_H} \widehat{\pi}_{H,t} + f_{\pi_{N(+1)}} \widehat{\pi}_{N,t+1} + f_{\pi_{H(+1)}} \widehat{\pi}_{H,t+1} && \text{(D.92)} \\ & + \beta \widehat{d}_t + f_G \widehat{G}_t + f_B \widehat{B}_t + f_{C^*} \widehat{C}_t^* - f_{C^*(+1)} \widehat{C}_{t+1}^* && \text{(D.93)} \end{aligned}$$

where:

$$\begin{aligned} \widehat{mc}_t^{N,r} = \; & m_{N,Y} \widehat{Y}_t + m_{N,T^d} \widehat{T}_t^d + m_{N,T} \widehat{T}_t + m_{N,\tau_N} \widehat{\tau}_{N,t} + && \text{(D.94)} \\ & + m_{N,A_N} \widehat{A}_{N,t} + m_{N,A_H} \widehat{A}_{H,t} + m_{N,B} \widehat{B}_t + m_{N,G} \widehat{G}_t && \text{(D.95)} \\ \widehat{mc}_t^{H,r} = \; & m_{H,Y} \widehat{Y}_t + m_{H,T^d} \widehat{T}_t^d + m_{H,T} \widehat{T}_t + m_{H,\tau_H} \widehat{\tau}_{H,t} + && \text{(D.96)} \\ & + m_{H,A_N} \widehat{A}_{N,t} + m_{H,A_H} \widehat{A}_{H,t} + m_{H,B} \widehat{B}_t + m_{H,G} \widehat{G}_t && \text{(D.97)} \end{aligned}$$

with:

$$m_{N,Y} = c_Y\rho + l_Y\eta \tag{D.98}$$
$$m_{N,T^d} = \rho * ctd + \eta * ltd - pntd \tag{D.99}$$
$$m_{N,T} = \rho * ct + \eta * lt \tag{D.100}$$
$$m_{N,\tau_N} = \omega_N \tag{D.101}$$
$$m_{N,A_N} = -(1 + \eta * \widetilde{d}_{YN}) \tag{D.102}$$
$$m_{N,A_H} = -\eta\widetilde{d}_{YH} \tag{D.103}$$
$$m_{N,B} = \rho * (c_B - 1) + \eta * lb \tag{D.104}$$
$$m_{N,G} = \rho c_G + \eta l_G \tag{D.105}$$

$$m_{H,Y} = c_Y\rho + l_Y\eta \tag{D.106}$$
$$m_{H,T^d} = \rho * ctd + \eta * ltd - phtd \tag{D.107}$$
$$m_{H,T} = \rho * ct + \eta * lt - pht \tag{D.108}$$
$$m_{H,\tau_H} = \omega_H \tag{D.109}$$
$$m_{H,A_N} = -\eta\widetilde{d}_{YN} \tag{D.110}$$
$$m_{H,A_H} = -(1 + \eta * \widetilde{d}_{YH}) \tag{D.111}$$
$$m_{H,B} = \rho * (c_B - 1) + \eta * l_B \tag{D.112}$$
$$m_{H,G} = \rho c_G + \eta l_G \tag{D.113}$$

$$n_Y = c_Y \tag{D.114}$$
$$n_{T^d} = ctd - \frac{1}{\rho} rstd \tag{D.115}$$
$$n_T = ct - \frac{1}{\rho} rst \tag{D.116}$$
$$n_B = cb - 1 \tag{D.117}$$
$$n_G = cg \tag{D.118}$$

$$\begin{aligned}
f_{\tau_N} &= (1-\beta)s_{\tau_N} && \text{(D.119)}\\
f_{\tau_H} &= (1-\beta)s_{\tau_H} && \text{(D.120)}\\
f_Y &= (1-\beta)(s_{\tau_N}yny + s_{\tau_H}yhy) && \text{(D.121)}\\
f_{T^d} &= (1-\beta)(s_{\tau_N}(yny+pntd) + s_{\tau_H}(yhy+phtd)) + \beta rstd && \text{(D.122)}\\
f_{T^d(+1)} &= \beta rstd && \text{(D.123)}\\
f_T &= (1-\beta)(s_{\tau_N}yny + s_{\tau_H}(yhy+pht) + a(1-b)(1+\beta)) + \beta rst && \text{(D.124)}\\
f_{T(+1)} &= \beta(a(1-b)+rst) && \text{(D.125)}\\
f_{T(-1)} &= -a(1-b) && \text{(D.126)}\\
f_{\pi_N} &= b && \text{(D.127)}\\
f_{\pi_H} &= (1-b) && \text{(D.128)}\\
f_{\pi_{N(+1)}} &= \beta b && \text{(D.129)}\\
f_{\pi_{H(+1)}} &= \beta(1-b) && \text{(D.130)}\\
f_G &= (1-\beta)(s_{\tau_N}yng + s_{\tau_H}yhg) && \text{(D.131)}\\
f_B &= (1-\beta)(s_{\tau_N}ynb + s_{\tau_H}yhb) && \text{(D.132)}\\
f_{C^*} &= \rho\beta && \text{(D.133)}\\
f_{C^*(+1)} &= -\rho\beta && \text{(D.134)}
\end{aligned}$$

Structural equations defining the Maastricht variables:

$$\begin{aligned}
\widehat{R}_t &= b\widehat{\pi}_{N,t+1} + (1-b)\widehat{\pi}_{H,t+1} - \rho(1-cb)(\widehat{B}_{t+1} - \widehat{B}_t) + \rho(\widehat{Y}_{t+1} - \widehat{Y}_t) + && \text{(D.135)}\\
&+\rho ctd(\widehat{T}^d_{t+1} - \widehat{T}^d_t) + (\rho ct + a(1-b))(\widehat{T}_{t+1} - \widehat{T}_t), && \text{(D.136)}
\end{aligned}$$

$$\widehat{\pi}_t = b\widehat{\pi}_{N,t} + (1-b)\widehat{\pi}_{H,t} + a(1-b)(\widehat{T}_t - \widehat{T}_{t-1}), \tag{D.137}$$

$$\widehat{S}_t = \widehat{S}_{t-1} + \widehat{\pi}_t + rstd(\widehat{T}^d_t - \widehat{T}^d_{t-1}) + rst(\widehat{T}_t - \widehat{T}_{t-1}), \tag{D.138}$$

$$\begin{aligned}
\widetilde{df}_t &= d_d\widehat{d}_{t-1} + d_\pi(b\widehat{\pi}_{N,t} + (1-b)\widehat{\pi}_{H,t} + a(1-b)(\widehat{T}_t - \widehat{T}_{t-1})) && \text{(D.139)}\\
&+d_R\widehat{R}_{t-1} + d_{sr}(sr_{\tau_N}\widehat{\tau}_{N,t} + sr_{\tau_H}\widehat{\tau}_{H,t} + sr_{T^d}\widehat{T}^d_t && \text{(D.140)}\\
&+sr_T\widehat{T}_t + sr_Y\widehat{Y}_t + sr_B\widehat{B}_t + sr_G\widehat{G}_t), && \text{(D.141)}
\end{aligned}$$

$$\widehat{br}_t = \widehat{d}_t - \widehat{Y}_t - \widehat{R}_t, \tag{D.142}$$

where the parameters are defined below:

$$
\begin{aligned}
d_d &= \frac{\overline{d}}{\overline{Y}}(1-\beta) \\
d_\pi &= \frac{\overline{d}}{\overline{Y}}(1-\beta) && \text{(D.143)} \\
d_R &= \beta\frac{\overline{d}}{\overline{Y}} && \text{(D.144)} \\
d_{sr} &= -\frac{\overline{sr}}{\overline{Y}} && \text{(D.145)} \\
sr_{T^d} &= s_{\tau_N}(pntd + yntd) + s_{\tau_H}(phtd + yhtd) && \text{(D.146)} \\
sr_T &= s_{\tau_N} ynt + s_{\tau_H}(pht + yht) && \text{(D.147)} \\
sr_Y &= s_{\tau_N} yny + s_{\tau_H} yhy && \text{(D.148)} \\
sr_B &= s_{\tau_N} ynb + s_{\tau_H} yhb && \text{(D.149)} \\
sr_G &= s_{\tau_N} yng + s_{\tau_H} yhg - s_G && \text{(D.150)}
\end{aligned}
$$

D.5 Reinterpretation of the Maastricht convergence criteria

We show how to reinterpret each of the Maastricht criteria in order to be able to use the method of Rotemberg and Woodford (1997, 1999).

D.5.1 Exchange rate criterion

We reinterpret the criterion on the nominal exchange rate (4.34) into two inequalities given below:[4]

$$E\left(\widehat{S}_t\right) - k * SD(\widehat{S}_t) \geq -15\%, \tag{D.151}$$

$$E\left(\widehat{S}_t\right) + k * SD(\widehat{S}_t) \leq 15\%. \tag{D.152}$$

where k is large enough to prevent from violating the criterion (4.34) and SD refers to the standard deviation statistic.

These two inequalities can be represented as the following two sets of inequalities (to conform with the welfare measure we use discounted statistics):

[4] E stands for the expectation operator and SD stands for the standard deviation operator.

$$\begin{cases} (1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t\left(\widehat{S}_t-(-15\%)\right)\geq 0 \\ (1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t\left(\widehat{S}_t-(-15\%)\right)^2\leq K\left((1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t\left(\widehat{S}_t-(-15\%)\right)\right)^2, \end{cases} \tag{D.153}$$

$$\begin{cases} (1-\beta)E_{t_0}\sum_{t=0}^{\infty}\beta^t\left(15\%-\widehat{S}_t\right)\leq 0 \\ (1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t\left(15\%-\widehat{S}_t\right)^2\leq K\left((1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t\left(15\%-\widehat{S}_t\right)\right)^2, \end{cases} \tag{D.154}$$

where $K = 1 + k^{-2}$.

D.5.2 Inflation criterion

We redefine the condition (4.32). We assume that the average inflation in the domestic economy should be at least k standard deviations smaller than the average inflation in the foreign economy plus a margin summarized by B_π (where $B_\pi = \sqrt[4]{1,015} - 1$):

$$E(\widehat{\pi}_t) \leq E(\widehat{\pi}_t^*) + B_\pi - kSD(\widehat{\pi}_t) \tag{D.155}$$

where $\widehat{\pi}_t$, $\widehat{\pi}_t^*$ are treated as deviations from the zero inflation steady state in the domestic economy and the foreign one accordingly (i.e. $\overline{\pi} = \overline{\pi}^* = 0$) and k large enough to prevent from violating criterion (4.32). We assume that the foreign economy is in the steady state so $\widehat{\pi}_t^* = 0\ \forall t$. As a result our restriction (D.155) becomes:

$$E(\widehat{\pi}_t) \leq B_\pi - kSD(\widehat{\pi}_t). \tag{D.156}$$

Since B_π is a constant we can use the following property of the variance: $Var(\widehat{\pi}_t) = Var(B_\pi - \widehat{\pi}_t)$. Our restriction becomes:

$$kSD(B_\pi - \widehat{\pi}_t) \leq E(B_\pi - \widehat{\pi}_t). \tag{D.157}$$

This restriction can be represented as a set of two restrictions:

$$(1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t\left(B_\pi - \pi_t\right) \geq 0, \tag{D.158}$$

$$(1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t\left(B_\pi - \widehat{\pi}_t\right)^2 \leq K\left((1-\beta)E_{t_0}\sum_{t=t_0}^{\infty}\beta^t\left(B_\pi - \widehat{\pi}_t\right)\right)^2. \tag{D.159}$$

D.5.3 Nominal interest rate criterion

Similarly to the criterion on the CPI aggregate inflation we interpret the inequality (4.33):

$$E(\widehat{R}_t) \leq E(\widehat{R}^*_t) + C_R - kSD(\widehat{R}_t) \tag{D.160}$$

where k is large enough to prevent from frequent violating the criterion (4.33) and $C_R = \sqrt[4]{1,02} - 1$.

As in the case of the foreign inflation we assume that $\widehat{R}^*_t = 0\ \forall t$. So the restriction (D.160) becomes:

$$kSD(C_R - \widehat{R}_t) \leq E(C_R - \widehat{R}_t). \tag{D.161}$$

This inequality can be represented as a set of two inequalities:

$$(1-\beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t \left(C_R - \widehat{R}_t\right) \geq 0, \tag{D.162}$$

$$(1-\beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t \left(C_R - \widehat{R}_t\right)^2 \leq K \left((1-\beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t \left(C_R - \widehat{R}_t\right)\right)^2. \tag{D.163}$$

D.5.4 Deficit to GDP criterion

Finally we interpret the inequality (4.35) that summarizes deficit to GDP criterion:

$$E(\widehat{df}_t) \leq F_{df} - kSD(\widehat{df}_t) \tag{D.164}$$

where k is large enough to prevent from frequent violating the criterion (4.35) and $F_{df} = 3\%$.

Subsequently, this inequality can be represented as a set of two inequalities:

$$(1-\beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t \left(F_{df} - \widehat{df}_t\right) \geq 0, \tag{D.165}$$

$$(1-\beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t \left(F_{df} - \widehat{df}_t\right)^2 \leq K \left((1-\beta)E_{t_0} \sum_{t=t_0}^{\infty} \beta^t \left(F_{df} - \widehat{df}_t\right)\right)^2. \tag{D.166}$$

D.6 The constrained loss function

We provide the proof of the Proposition 2 stated in the main text. Since all the sets of the constraints have a similar structure the proof concerns the optimal monetary policy with only one constraint on the CPI inflation rate. The proof is based on the proof of Proposition 6.9 in Woodford (2003).

Proposition 5 *Consider the problem of minimizing an expected discounted sum of quadratic losses:*

$$E_{t_0}\left\{(1-\beta)\sum_{t=t_0}^{\infty}\beta^t L_t\right\} \tag{D.167}$$

subject to (4.38) - (4.39). Let $m_{1,\pi}$, $m_{2,\pi}$ be the discounted average values of $(B_\pi - \widehat{\pi}_t)$ and $(B_\pi - \widehat{\pi}_t)^2$ associated with the optimal policy. Then the optimal policy also minimizes a modified discounted loss criterion of the form (D.167) with L_t replaced by:

$$\widetilde{L}_t \equiv L_t + \Phi_\pi(\pi^T - \widehat{\pi}_t)^2 \tag{D.168}$$

under constraints represented by the structural equations. Importantly $\Phi_\pi \geq 0$ and takes strictly positive value if and only if the constraint (4.39) binds. Moreover if the constraint (4.39) binds the corresponding target value π^T is negative and given by the following relation:

$$\pi^T = B_\pi - Km_{1,\pi} < 0. \tag{D.169}$$

Proof. Let $m_{1,\pi}$ and $m_{2,\pi}$ be the discounted average values of $(B_\pi - \pi_t)$ and $(B_\pi - \pi_t)^2$ associated with the policy that solves the constrained optimization problem stated in the corollary. Let $m^*_{1,\pi}$ and $m^*_{2,\pi}$ be the values of these moments for the policy that minimizes (D.167) without additional constraints. Notice that since $m^*_{1,\pi} = B_\pi$ the constraint (4.38) does not bind.[5] We identify the deterministic component of policy, i.e. $m_{1,\pi}$ and also the stabilization component of policy which is: $m_{2,\pi} - (m_{1,\pi})^2$. Moreover we also conclude that $m_{1,\pi} \geq m^*_{1,\pi}$ since there is no advantage from choosing $m_{1,\pi}$ such that: $m_{1,\pi} < m^*_{1,\pi}$ - both constraints set only the lower bound on the value of $m_{1,\pi}$ for any value of the stabilization component of policy. If one chooses $m_{1,\pi}$ such that: $m_{1,\pi} > m^*_{1,\pi}$ then one can relax the constraint (4.39). So $m_{1,\pi} \geq m^*_{1,\pi}$. Based on the above discussion we formulate two alternative constraints to the constraints (4.38, 4.39):

$$(1-\beta)E_0\sum_{t=0}^{\infty}\beta^t\left(B_\pi - \widehat{\pi}_t\right) \geq m_{1,\pi}, \tag{D.170}$$

[5] Means of all the variables under the unconstrained optimal policy are zero.

$$(1-\beta)E_0\sum_{t=0}^{\infty}\beta^t\left(B_\pi-\widehat{\pi}_t\right)^2\leq m_{2,\pi}. \tag{D.171}$$

Observe that any policy that satisfies the above constraints satisfies also the weaker constraints: (4.38, 4.39). Now we take advantage of the Kuhn – Tucker theorem: the policy that minimizes (D.167) subject to (D.170, D.171) also minimizes the following loss criterion:

$$E_0\left\{(1-\beta)\sum_{t=0}^{\infty}\beta^tL_t\right\}-\mu_{1,\pi}E_0\left\{(1-\beta)\sum_{t=0}^{\infty}\beta^t\left(B_\pi-\widehat{\pi}_t\right)\right\}+$$
$$\mu_{2,\pi}E_0\left\{(1-\beta)\sum_{t=0}^{\infty}\beta^t\left(B_\pi-\widehat{\pi}_t\right)^2\right\} \tag{D.172}$$

where $\mu_{1,\pi}$ and $\mu_{2,\pi}$ are the Lagrange multipliers which are nonnegative. If (4.39) binds then we obtain the following relation between the multipliers:

$$\mu_{1,\pi}=2Km_{1,\pi}\mu_{2,\pi} \tag{D.173}$$

since $m_{2,\pi}=Km_{1,\pi}^2$.

Rearranging the terms in (D.172) we can define the new loss function as:

$$\widetilde{L}_t\equiv L_t+\mu_{2,\pi}\left(\left(B_\pi-\widehat{\pi}_t\right)-\frac{\mu_{1,\pi}}{2\mu_{2,\pi}}\right)^2 \tag{D.174}$$

where the final term appears only when $\mu_{2,\pi}>0$. Therefore $\Phi_\pi=\mu_{2,\pi}\geq 0$ and takes a strictly positive value only if (4.39) binds. Moreover for $\Phi_\pi>0$ we have that:

$$\pi^T=B_\pi-\frac{\mu_{1,\pi}}{2\mu_{2.\pi}}=B_\pi-Km_{1,\pi}. \tag{D.175}$$

Notice that the target value for the CPI inflation is negative (since $K>1$ and $m_{1,\pi}\geq B_\pi$):

$$\pi^T=B_\pi-Km_{1,\pi}<0. \tag{D.176}$$

■

D.7 Unconstrained optimal monetary and fiscal policy

We derive the first order conditions for the unconstrained optimal monetary and fiscal policy.

$$\min L_{t_0} = U_C\overline{C}\sum_{t=t_0}^{\infty}\beta^{t-t_0}[\frac{1}{2}\Phi_y(\widehat{Y}_t-\widehat{Y}_t^T)^2+\frac{1}{2}\Phi_{T^d}(\widehat{T}_t^d-\widehat{T}_t^{dT})^2 \quad \text{(D.177)}$$

$$+\frac{1}{2}\Phi_T(\widehat{T}_t-\widehat{T}_t^T)^2+\frac{1}{2}\Phi_{\tau_N}(\widehat{\tau}_{N,t}-\widehat{\tau}_{N,t}^T)^2+ \quad \text{(D.178)}$$

$$\frac{1}{2}\Phi_{\tau_H}(\widehat{\tau}_{H,t}-\widehat{\tau}_{H,t}^T)^2+\Phi_{YT^d}\widehat{Y}_t\widehat{T}_t^d+\Phi_{YT}\widehat{Y}_t\widehat{T}_t+ \quad \text{(D.179)}$$

$$+\Phi_{T^dT}\widehat{T}_t^d\widehat{T}_t+\widehat{\tau}_{N,t}(\Phi_{Y\tau_N}\widehat{Y}_t+\Phi_{T\tau_N}\widehat{T}_t+\Phi_{T^d\tau_N}\widehat{T}_t^d) \quad \text{(D.180)}$$

$$+\widehat{\tau}_{H,t}(\Phi_{Y\tau_H}\widehat{Y}_t+\Phi_{T\tau_H}\widehat{T}_t+\Phi_{T^d\tau_H}\widehat{T}_t^d)+ \quad \text{(D.181)}$$

$$+\frac{1}{2}\Phi_{\pi_N}\widehat{\pi}_{N,t}^2+\frac{1}{2}\Phi_{\pi_H}\widehat{\pi}_{H,t}^2]+tip+O(3) \quad \text{(D.182)}$$

subject to :

$$\widehat{\pi}_{N,t} = k_N(m_{N,Y}\widehat{Y}_t+m_{N,T^d}\widehat{T}_t^d+m_{N,T}\widehat{T}_t+m_{N,\tau_N}\widehat{\tau}_{N,t}+m_{N,A_N}\widehat{A}_{N,t}+ \quad \text{(D.183)}$$

$$+m_{N,A_H}\widehat{A}_{H,t}+m_{N,B}\widehat{B}_t+m_{N,G}\widehat{G}_t)+\beta\widehat{\pi}_{N,t+1}, \quad \text{(D.184)}$$

$$\widehat{\pi}_{H,t} = k_H(m_{H,Y}\widehat{Y}_t+m_{H,T^d}\widehat{T}_t^d+m_{H,T}\widehat{T}_t+m_{H,\tau_H}\widehat{\tau}_{H,t}+m_{H,A_N}\widehat{A}_{N,t}+ \quad \text{(D.185)}$$

$$+m_{H,A_H}\widehat{A}_{H,t}+m_{H,B}\widehat{B}_t+m_{H,G}\widehat{G}_t)+\beta\widehat{\pi}_{H,t+1}, \quad \text{(D.186)}$$

$$\widehat{C}_t^*=n_Y\widehat{Y}_t+n_{T^d}\widehat{T}_t^d+n_T\widehat{T}_t+n_B\widehat{B}_t+n_B\widehat{B}_t, \quad \text{(D.187)}$$

$$\widehat{T}_t^d-\widehat{T}_{t-1}^d=\widehat{\pi}_{N,t}-\widehat{\pi}_{H,t}-a(\widehat{T}_t-\widehat{T}_{t-1}), \quad \text{(D.188)}$$

$$\widehat{d}_{t-1} = f_{\tau_N}\widehat{\tau}_{N,t}+f_{\tau_H}\widehat{\tau}_{H,t}+f_Y\widehat{Y}_t+f_{T^d}\widehat{T}_t^d+f_T\widehat{T}_t+f_{T^d(+1)}\widehat{T}_{t+1}^d \quad \text{(D.189)}$$

$$+f_{T(+1)}\widehat{T}_{t+1}+f_{T(-1)}\widehat{T}_{t-1}+f_{\pi_N}\widehat{\pi}_{N,t} \quad \text{(D.190)}$$

$$+f_{\pi_H}\widehat{\pi}_{H,t}+f_{\pi_{N(+1)}}\widehat{\pi}_{N,t+1}+f_{\pi_{H(+1)}}\widehat{\pi}_{H,t+1}+\beta\widehat{d}_t+ \quad \text{(D.191)}$$

$$+f_G\widehat{G}_t+f_B\widehat{B}_t+f_{C^*}\widehat{C}_t^*-f_{C^*(+1)}\widehat{C}_{t+1}^* \quad \text{(D.192)}$$

First order conditions of the minimization problem:

- wrt $\widehat{\pi}_{N,t}$:

$$0=\Phi_{\pi_N}\widehat{\pi}_{N,t}+\gamma_{1,t}-\gamma_{1,t-1}-\gamma_{4,t}-f_{\pi_N}\gamma_{5,t}-\beta^{-1}f_{\pi_{N(+1)}}\gamma_{5,t-1}, \quad \text{(D.193)}$$

- wrt $\widehat{\pi}_{H,t}$:

$$0 = \Phi_{\pi_H}\widehat{\pi}_{H,t} + \gamma_{2,t} - \gamma_{2,t-1} + \gamma_{4,t} - f_{\pi_H}\gamma_{5,t} - \beta^{-1} f_{\pi_{H(+1)}}\gamma_{5,t-1}, \quad \text{(D.194)}$$

- wrt $\widehat{Y}_t$:

$$\begin{aligned} 0 &= \Phi_Y(\widehat{Y}_t - \widehat{Y}_t^T) + \Phi_{YT^d}\widehat{T}_t^d + \Phi_{YT}\widehat{T}_t - k_N m_{N,Y}\gamma_{1,t} && \text{(D.195)} \\ & -k_H m_{H,Y}\gamma_{2,t} - n_Y\gamma_{3,t} - f_Y\gamma_{5,t}, && \text{(D.196)} \end{aligned}$$

- wrt $\widehat{T}_t^d$:

$$\begin{aligned} 0 &= \Phi_{T^d}(\widehat{T}_t^d - \widehat{T}_t^{d^T}) + \Phi_{TT^d}\widehat{T}_t + \Phi_{YT^d}\widehat{Y}_t - k_N m_{N,T^d}\gamma_{1,t} && \text{(D.197)} \\ & -k_H m_{H,T^d}\gamma_{2,t} - n_{T^d}\gamma_{3,t} + \gamma_{4,t} - \beta\gamma_{4,t+1} && \text{(D.198)} \\ & -f_{T^d}\gamma_{5,t} - \beta^{-1} f_{T^d(+1)}\gamma_{5,t-1}, && \text{(D.199)} \end{aligned}$$

- wrt $\widehat{T}_t$:

$$\begin{aligned} 0 &= \Phi_T(\widehat{T}_t - \widehat{T}_t^T) + \Phi_{TT^d}\widehat{T}_t^d + \Phi_{YT}\widehat{Y}_t - k_N m_{N,T}\gamma_{1,t} && \text{(D.200)} \\ & -k_H m_{H,T}\gamma_{2,t} + && \text{(D.201)} \\ & -n_T\gamma_{3,t} + a\gamma_{4,t} - \beta a\gamma_{4,t+1} - f_T\gamma_{5,t} && \text{(D.202)} \\ & -\beta^{-1} f_{T^d(+1)}\gamma_{5,t-1} - \beta f_{T(-1)}\gamma_{5,t+1}, && \text{(D.203)} \end{aligned}$$

- wrt $\widehat{d}_t$:

$$0 = -\beta\gamma_{5,t} + \beta\gamma_{5,t+1}, \quad \text{(D.204)}$$

- wrt $\widehat{\tau}_{N,t}$:

$$\begin{aligned} 0 &= \Phi_{\tau_N}(\widehat{\tau}_{N,t} - \widehat{\tau}_{N,t}^T) + \Phi_{Y\tau_N}\widehat{Y}_t + \Phi_{T\tau_N}\widehat{T}_t + \Phi_{T^d\tau_N}\widehat{T}_t^d + && \text{(D.205)} \\ & -k_N m_{N,\tau_N}\gamma_{1,t} - f_{\tau_N}\gamma_{5,t}, && \text{(D.206)} \end{aligned}$$

- wrt $\widehat{\tau}_{H,t}$:

$$\begin{aligned} 0 &= \Phi_{\tau_H}(\widehat{\tau}_{H,t} - \widehat{\tau}_{H,t}^T) + \Phi_{Y\tau_H}\widehat{Y}_t + \Phi_{T\tau_H}\widehat{T}_t + \Phi_{T^d\tau_H}\widehat{T}_t^d + && \text{(D.207)} \\ & -k_H m_{H,\tau_H}\gamma_{2,t} - f_{\tau_H}\gamma_{5,t}. && \text{(D.208)} \end{aligned}$$

D.8 Constrained optimal monetary and fiscal policy

We derive the first order conditions for the optimal policy that satisfies the additional criteria on the nominal interest, the CPI aggregate inflation and deficit to GDP ratio.

$$
\begin{aligned}
\min L^c_{t_0} &= U_C\overline{C}\sum_{t=t_0}^{\infty}\beta^{t-t_0}[\frac{1}{2}\Phi_y(\widehat{Y}_t-\widehat{Y}^T_t)^2+\frac{1}{2}\Phi_{T^d}(\widehat{T}^d_t-\widehat{T}^{dT}_t)^2 && \text{(D.209)}\\
&+\frac{1}{2}\Phi_T(\widehat{T}_t-\widehat{T}^T_t)^2+\frac{1}{2}\Phi_{\tau_N}(\widehat{\tau}_{N,t}-\widehat{\tau}^T_{N,t})^2+ && \text{(D.210)}\\
&+\frac{1}{2}\Phi_{\tau_H}(\widehat{\tau}_{H,t}-\widehat{\tau}^T_{H,t})^2+\Phi_{YT^d}\widehat{Y}_t\widehat{T}^d_t+\Phi_{YT}\widehat{Y}_t\widehat{T}_t+ && \text{(D.211)}\\
&+\Phi_{T^dT}\widehat{T}^d_t\widehat{T}_t+\widehat{\tau}_{N,t}(\Phi_{Y\tau_N}\widehat{Y}_t+\Phi_{T\tau_N}\widehat{T}_t+\Phi_{T^d\tau_N}\widehat{T}^d_t)+ && \text{(D.212)}\\
&+\widehat{\tau}_{H,t}(\Phi_{Y\tau_H}\widehat{Y}_t+\Phi_{T\tau_H}\widehat{T}_t+\Phi_{T^d\tau_H}\widehat{T}^d_t)+ && \text{(D.213)}\\
&+\frac{1}{2}\Phi_{\pi_N}\widehat{\pi}^2_{N,t}+\frac{1}{2}\Phi_{\pi_H}\widehat{\pi}^2_{H,t}+\frac{1}{2}\phi_\pi(\pi^T-\widehat{\pi}_t)^2+ && \text{(D.214)}\\
&+\frac{1}{2}\phi_R(R^T-\widehat{R}_t)^2+\frac{1}{2}\phi_{df}(df^T-df_t)^2]+tip+O(3) && \text{(D.215)}
\end{aligned}
$$

subject to :

$$
\begin{aligned}
\widehat{\pi}_{N,t} &= k_N(m_{N,Y}\widehat{Y}_t+m_{N,T^d}\widehat{T}^d_t+m_{N,T}\widehat{T}_t+m_{N,\tau_N}\widehat{\tau}_{N,t}+m_{N,A_N}\widehat{A}_{N,t}+ && \text{(D.216)}\\
&+m_{N,A_H}\widehat{A}_{H,t}+m_{N,B}\widehat{B}_t+m_{N,G}\widehat{G}_t)+\beta\widehat{\pi}_{N,t+1}, && \text{(D.217)}
\end{aligned}
$$

$$
\begin{aligned}
\widehat{\pi}_{H,t} &= k_H(m_{H,Y}\widehat{Y}_t+m_{H,T^d}\widehat{T}^d_t+m_{H,T}\widehat{T}_t+m_{H,\tau_H}\widehat{\tau}_{H,t}+m_{H,A_N}\widehat{A}_{N,t}+ && \text{(D.218)}\\
&+m_{H,A_H}\widehat{A}_{H,t}+m_{H,B}\widehat{B}_t+m_{H,G}\widehat{G}_t)+\beta\widehat{\pi}_{H,t+1}, && \text{(D.219)}
\end{aligned}
$$

$$\widehat{C}^*_t=n_Y\widehat{Y}_t+n_{T^d}\widehat{T}^d_t+n_T\widehat{T}_t+n_B\widehat{B}_t+n_B\widehat{B}_t, \tag{D.220}$$

$$\widehat{T}^d_t-\widehat{T}^d_{t-1}=\widehat{\pi}_{N,t}-\widehat{\pi}_{H,t}-a(\widehat{T}_t-\widehat{T}_{t-1}), \tag{D.221}$$

$$
\begin{aligned}
\widehat{d}_{t-1} &= f_{\tau_N}\widehat{\tau}_{N,t}+f_{\tau_H}\widehat{\tau}_{H,t}+f_Y\widehat{Y}_t+f_{T^d}\widehat{T}^d_t+f_T\widehat{T}_t+f_{T^d(+1)}\widehat{T}^d_{t+1} && \text{(D.222)}\\
&+f_{T(+1)}\widehat{T}_{t+1}+f_{T(-1)}\widehat{T}_{t-1}+f_{\pi_N}\widehat{\pi}_{N,t}+f_{\pi_H}\widehat{\pi}_{H,t} && \text{(D.223)}\\
&+f_{\pi_{N(+1)}}\widehat{\pi}_{N,t+1}+f_{\pi_{H(+1)}}\widehat{\pi}_{H,t+1}+ && \text{(D.224)}\\
&+\beta\widehat{d}_t+f_G\widehat{G}_t+f_B\widehat{B}_t+f_{C^*}\widehat{C}^*_t-f_{C^*(+1)}\widehat{C}^*_{t+1} && \text{(D.225)}
\end{aligned}
$$

$$\widehat{\pi}_t = b\widehat{\pi}_{N,t} + (1-b)\widehat{\pi}_{H,t} + a(1-b)(\widehat{T}_t - \widehat{T}_{t-1}), \tag{D.226}$$

$$\widehat{R}_t = b\widehat{\pi}_{N,t+1} + (1-b)\widehat{\pi}_{H,t+1} - \rho(1-cb)(\widehat{B}_{t+1} - \widehat{B}_t) + \rho(\widehat{Y}_{t+1} - \widehat{Y}_t) \tag{D.227}$$
$$+\rho ctd(\widehat{T}^d_{t+1} - \widehat{T}^d_t) + (\rho ct + a(1-b))(\widehat{T}_{t+1} - \widehat{T}_t), \tag{D.228}$$

$$\widetilde{df}_t = d_d\widehat{d}_{t-1} + d_\pi(b\widehat{\pi}_{N,t} + (1-b)\widehat{\pi}_{H,t} + a(1-b)(\widehat{T}_t - \widehat{T}_{t-1})) \tag{D.229}$$
$$+d_R\widehat{R}_{t-1} + \tag{D.230}$$
$$+d_{sr}(sr_{\tau_N}\widehat{\tau}_{N,t} + sr_{\tau_H}\widehat{\tau}_{H,t} + sr_{T^d}\widehat{T}^d_t + \tag{D.231}$$
$$+sr_T\widehat{T}_t + sr_Y\widehat{Y}_t + sr_B\widehat{B}_t + sr_G\widehat{G}_t), \tag{D.232}$$

First order conditions of the minimization problem:

- wrt $\widehat{\pi}_{N,t}$:

$$0 = \Phi_{\pi_N}\widehat{\pi}_{N,t} + \gamma_{1,t} - \gamma_{1,t-1} - \gamma_{4,t} - f_{\pi_N}\gamma_{5,t} - \beta^{-1}f_{\pi_{N(+1)}}\gamma_{5,t-1} \tag{D.233}$$
$$-b\gamma_{6,t} - \beta^{-1}b\gamma_{7,t} - bd_\pi\gamma_{8,t}, \tag{D.234}$$

- wrt $\widehat{\pi}_{H,t}$:

$$0 = \Phi_{\pi_H}\widehat{\pi}_{H,t} + \gamma_{2,t} - \gamma_{2,t-1} + \gamma_{4,t} - f_{\pi_H}\gamma_{5,t} - \beta^{-1}f_{\pi_{H(+1)}}\gamma_{5,t-1} \tag{D.235}$$
$$-(1-b)\gamma_{6,t} - \beta^{-1}(1-b)\gamma_{7,t} - (1-b)d_\pi\gamma_{8,t}, \tag{D.236}$$

- wrt $\widehat{Y}_t$:

$$0 = \Phi_Y(\widehat{Y}_t - \widehat{Y}^T_t) + \Phi_{YT^d}\widehat{T}^d_t + \Phi_{YT}\widehat{T}_t - k_N m_{N,Y}\gamma_{1,t} \tag{D.237}$$
$$-k_H m_{H,Y}\gamma_{2,t} - n_Y\gamma_{3,t} - f_Y\gamma_{5,t} + \tag{D.238}$$
$$+\rho\gamma_{7,t} - \rho\beta^{-1}\gamma_{7,t-1} - d_{sr}sr_Y\gamma_{8,t}, \tag{D.239}$$

- wrt $\widehat{T}^d_t$:

$$0 = \Phi_{T^d}(\widehat{T}^d_t - \widehat{T}^{d^T}_t) + \Phi_{TT^d}\widehat{T}_t + \Phi_{YT^d}\widehat{Y}_t - k_N m_{N,T^d}\gamma_{1,t} \tag{D.240}$$
$$-k_H m_{H,T^d}\gamma_{2,t} - n_{T^d}\gamma_{3,t} + \gamma_{4,t} - \beta\gamma_{4,t+1} \tag{D.241}$$
$$-f_{T^d}\gamma_{5,t} - \beta^{-1}f_{T^d(+1)}\gamma_{5,t-1} \tag{D.242}$$
$$+\rho ctd\gamma_{7,t} - \beta^{-1}\rho ctd\gamma_{7,t-1} - d_{sr}sr_{T^d}\gamma_{8,t}, \tag{D.243}$$

- wrt $\widehat{T}_t$:

$$
\begin{aligned}
0 \;=\; & \Phi_T(\widehat{T}_t - \widehat{T}_t^T) + \Phi_{TT^d}\widehat{T}_t^d + \Phi_{YT}\widehat{Y}_t - k_N m_{N,T}\gamma_{1,t} && \text{(D.244)}\\
& -k_H m_{H,T}\gamma_{2,t} + && \text{(D.245)}\\
& -n_T\gamma_{3,t} + a\gamma_{4,t} - \beta a\gamma_{4,t+1} - f_T\gamma_{5,t} - \beta^{-1} f_{T^d(+1)}\gamma_{5,t-1} && \text{(D.246)}\\
& -\beta f_{T(-1)}\gamma_{5,t+1} - a(1-b)\gamma_{6,t} + a(1-b)\beta\gamma_{6,t+1} + && \text{(D.247)}\\
& (\rho ct + a(1-b))\gamma_{7,t} - (\rho ct + a(1-b))\beta^{-1}\gamma_{7,t-1}, && \text{(D.248)}
\end{aligned}
$$

- wrt $\widehat{d}_t$:

$$0 = -\beta\gamma_{5,t} + \beta\gamma_{5,t+1} - \beta d_d\gamma_{8,t+1}, \tag{D.249}$$

- wrt $\widehat{\tau}_{N,t}$:

$$
\begin{aligned}
0 \;=\; & \Phi_{\tau_N}(\widehat{\tau}_{N,t} - \widehat{\tau}_{N,t}^T) + \Phi_{Y\tau_N}\widehat{Y}_t + \Phi_{T\tau_N}\widehat{T}_t + \Phi_{T^d\tau_N}\widehat{T}_t^d + && \text{(D.250)}\\
& -k_N m_{N,\tau_N}\gamma_{1,t} - f_{\tau_N}\gamma_{5,t} - d_{sr} sr_{\tau_N}\gamma_{8,t} && \text{(D.251)}
\end{aligned}
$$

- wrt $\widehat{\tau}_{H,t}$

$$
\begin{aligned}
0 \;=\; & \Phi_{\tau_H}(\widehat{\tau}_{H,t} - \widehat{\tau}_{H,t}^T) + \Phi_{Y\tau_H}\widehat{Y}_t + \Phi_{T\tau_H}\widehat{T}_t + \Phi_{T^d\tau_H}\widehat{T}_t^d + && \text{(D.252)}\\
& -k_H m_{H,\tau_H}\gamma_{2,t} - f_{\tau_H}\gamma_{5,t} - d_{sr} sr_{\tau_H}\gamma_{8,t} && \text{(D.253)}
\end{aligned}
$$

- wrt $\widehat{R}_t$:

$$0 = \Phi_R(\widehat{R}_t - R^T) + \gamma_{5,t} - d_R\beta\gamma_{8,t+1}, \tag{D.254}$$

- wrt $\widehat{\pi}_t$:

$$0 = \Phi_\pi(\widehat{\pi}_t - \pi^T) + \gamma_{6,t}, \tag{D.255}$$

- wrt $\widetilde{df}_t$:

$$0 = \Phi_{df}(\widetilde{df}_t - df^T) + \gamma_{8,t}. \tag{D.256}$$