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Influence of reverse logistics on optimal manufacturing, remanufacturing, and storage capacities

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Dissertation

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by

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Abstract

The purpose of this thesis is to study the influence of reverse logistics in optimal manufacturing, remanufacturing and storage capacities of an industrial system.

The interest in reverse logistics has grown in recent years in parallel with the increasing concern about environmental issues in the industrialized world. In chapter 2, we provide an introduction to reverse logistics: explaining the definition of reverse logistics and the reasons for which has been created as a differentiated area of management of traditional logistics, describing the types of products involved in reverse logistics and the different processes to recover its value, and examining the behavioral characteristics of a reverse logistics systems, compared with the traditional logistics system.

In chapter 3 we review the literature both in the field of capacity management in traditional systems and in the field of reverse logistics.

To meet the objective of the thesis, we study three models of a system in which the recovered product is indistinguishable from the new product. The process followed for the study was the same in each of the models presented: firstly, we determine the optimal production policy for every value of capacities; the second step is determining the optimal value of the capacities when optimal policies are applied and third we study the dependency of optimal capacities on some parameters related with reverse logistics.

In chapter 4 we study a system with uniform demand and random returns to show the influence of the randomness of returns in the optimal capacities. The cost function to optimize is the expected value of cost in a period.

In chapter 5 we study a model in which demand and returns are known functions, continuous and periodic. The cost function to optimize is the cost incurred in the period and the problem of determining the optimal production policy is an optimal control problem. Using this model, we analyze the dependence of optimal capacity on the time between sales and product returns.

In chapter 6 we present a stochastic model where demand and returns are sequences of random variables. The cost function to optimize is the expected value of cost in a period. To perform the calculation of optimal policies we assume that returns are stochastically independent of demand (this assumption is commonly used in the literature). The hypothesis is validated by simulation after optimal capacity is calculated. The model is used to study the dependence of the optimal capacities on the probability that the product is returned and also on the variable costs of remanufacturing.

Finally in chapter 7 we discuss the conclusions and future research topics.

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Contents

Notation and Acronyms

Capacities:

P	Manufacturing capacity (units of output)
R	Remanufacturing capacity (units of output)
S	Storage capacity (units of product)

Variable costs:

c_p	Variable manufacturing cost (per unit of output)
c_r	Variable remanufacturing cost (per unit of output)
c_s	Variable cost of alternative supplier (per unit)
c_{ec}	Variable cost of external channel (per unit)
c_{rc}	Cost of collecting end-of-life product (per unit)
h	Holding cost of a product (per unit and period)
f	Manufacturing order cost (per manufacturing order)

Fixed costs:

$C_p(P)$	Manufacturing cost depending on manufacturing capacity (per period)
$C_r(R)$	Remanufacturing cost depending on remanufacturing capacity (per period)
$H(S)$	Storage cost depending on storage capacity (per period)

Demand and returns in stochastic models with discrete time:

D	Demand (units per period).
T_1, T_2	End-of-life of a product occurs between T_1 and T_2 after it is sold
p_i	Probability that the end-of-life of a product occurs *i* periods after it is sold (i = $T_1,...,T_2$)
ρ	Probability of and end-of-life product being returned

Demand and returns in deterministic model with continuous time:

t	Time
$d(t)$	Demand in time *t* (units per time unit).
τ	Return lag period (i.e. time between the moment at which the product sold and the moment at which is returned
ρ	Return rate (i.e. units returned/units sold)

Chapter 1

Justification and aim of thesis

Interest in reverse logistics has increased in recent years with the growing concern for the environment in the industrialized world. Companies have recognized that their customers are increasingly seeking products and services that are environmentally sound. The management of products that have completed their useful life is now a key factor in business decision-making processes and the use of reverse logistics can provide companies the tools they need to act efficiently.

Much of the research in the field of reverse logistics has focused on tactical and operational rather than strategic aspects, with the bulk of studies examining production planning and inventory management (Rubio *et al.*, 2008). Few studies have analyzed aspects related to capacity planning in systems with remanufacturing capabilities (Georgiadis *et al.*, 2006).

Decisions regarding manufacturing capacity are generally taken in the context of strategic planning, whereas production and inventory management decisions are considered to be of a more tactical nature, meaning that they might be less than optimal if not integrated into the decision-making process as a whole (Hax and Candea, 1984). The problem of jointly managing capacities and inventory levels has been dealt with by numerous studies (Van Mieghem, 2003). This type of management approach consists of optimizing a function that contemplates manufacturing capacity acquisition and maintenance costs and production and inventory management costs. A key factor when addressing this problem is whether demand is stochastic or deterministic. Deterministic demand is not very realistic but may be of use for drawing conclusions regarding the behaviour of systems, simply because it is easier to analyze. Models that analyze joint capacity and inventory management can be designed alongside models of systems without reverse logistics, and adapted accordingly.

The aim of this thesis is to study the influence of reverse logistics on optimal manufacturing, remanufacturing, and storage capacities.

In order to study the influence of reverse logistics on optimal manufacturing, remanufacturing, and storage capacities, we studied three models of a system in which new and recovered products are indistinguishable from each other. The first of these is a model with uniform demand and random returns, the second is a model with known demand and known returns, and the third is a model with random demand and random returns. In each of the cases, we studied the impact of different reverse logistics factors on optimal capacities. The first model shows how the random nature of returns influences optimal capacities, the second model shows how optimal capacities vary with variations in the time between when a product is sold and returned, and the third model shows how capacities are dependent on the probability of return.

Chapter 2

Introduction

According to de Britto and Dekker (2004), the European Working Group on Reverse Logistics (REVLOG) defined reverse logistics as:

> "The process of planning, implementing and controlling backward flows of raw materials, in-process inventory, packaging and finished goods, from a manufacturing, distribution or use point, to a point of recovery or point of proper disposal"

According to this definition, reverse logistics consists of three distinct phases: planning, implementation, and the control of material flows. Thus, using the process of reverse logistics requires the taking of strategic decisions (to resolve planning aspects) and operational decisions (to resolve issues related to implementation and the control of material flows).

Material flows in reverse logistics are flows that take place in production and/or distribution processes. They essentially involve raw material, in-process products, packaging, and finished products. The source of flow can be any point at which these materials are located and destinations include points at which they can be recovered and disposed of adequately. Product recovery refers to the tasks required to ensure that a product or its components can be reused.

Closed-loop supply chains (CLSCs) are closely related to reverse logistics. CLSCs are supply chains with a reverse logistic process with a backward flow of material towards the manufacturer. We will study the concept of CLSCs in more detail later in this paper but the following questions immediately arise. Under what circumstances does it make sense to plan, implement, and control return flows? What materials should be considered? Why would a company be interested in recovering or disposing of certain materials or products? In brief, why should a company implement a reverse logistics system?

Fernández (2004) compiled a long list of reasons for implementing reverse logistics systems from the literature (Thierry *et al*. 1995; Guide *et al*. 2000; Tan and Kumar 2003, Tan *et al*. 2003, de Brito and Dekker 2004), of which the following can be highlighted:

- *Legal requirements*. In recent decades, there has been a proliferation of legal measures aimed at protecting the environment from the potentially harmful effects of products that have completed their useful life. In many cases, manufacturers and distributors can now be held accountable for harm caused by waste generated by their products. In the European Union, for example, companies are responsible for recovering or correctly disposing of any waste generated by products they produce or distribute.

- *Growing concern for the environment by both consumers and companies*. The increased social awareness of the need to protect the environment has led to increasing demands for environmentally responsible behaviour by companies, particularly in terms of carbon emissions and waste generation. Companies, for their part, wish to reinforce their image of environmentally responsible enterprises.

- *Profitability*. Product recovery can generate both direct benefits (reduction in use of raw material and waste disposal costs and recovery of value of end-of-life products) and indirect benefits (demonstration of environmentally responsible behaviour and improved customer relations).

- *New direct distribution channels*. Several links in the supply chain have been eliminated in certain sectors thanks to the use of electronic mail. Tasks related with product devolutions, that were traditionally distributed among various operators are now performed by new direct distribution channels.

The next question is what type of materials should be recovered and why. As stated by de Brito and Dekker (2004), products are returned or disposed of either because they do not work properly or because they are no longer of use. We can differentiate between the following types of returns:

- *Production returns*. Products that are recovered in the production phase, e.g., surplus raw material, in-process or finished products that do not meet quality standards, damaged products.

- *Distribution returns*. Finished products returned during the distribution phase. There are several reasons why a product is returned to a manufacturer:

    - Product recalls, due to defects that could affect safety or interfere with correct usage. Such products are normally returned within a recall campaign launched by the manufacturer and/or distributor.

    - Commercial returns, i.e. products returned by retailers to the supplier. These returns can include defective products, products damaged prior to delivery, short-life products, and unsold products.

    - Stock adjustments.

    - Products used throughout the supply chain (e.g. pallets).

- *Customer returns* i.e. finished products returned by the customer/end user. Examples are commercial returns, products under guarantee, products in need of repair, products that have reached the end of their period of use (e.g., leased products), and end-of-life products.

As we can see, the types of products returned, and the reasons for these returns, are very varied. The flows they generate, however, have certain characteristics that distinguish them from traditional material flows. Fleischmann *et al*. (1997) and Tibben-Lembke and Rogers (2002) described these differences:

- *Uncertainty surrounding the quality and quantity of products returned*. The main source of uncertainty in traditional supply channels is related to demand variations. Supply, in contrast, is considered to be controlled and reliable. Traditional logistics systems contain numerous management tools to offset the effects of uncertainty regarding demand levels. Examples are management of existing stock and demand forecast tools. Reverse logistics systems, in contrast, are affected by uncertainties regarding the quality and quantity of returned products.

- *Several points of origin but a single destination point.* Traditional material flows move from a single point (point of manufacture) to many destinations (points of consumption or use). In reverse logistics systems, however, the flows move from these points towards a single remanufacturing or disposal point.

- *Product and packaging quality*. The quality of returned products may differ from that of a new product. For example, if a product has completed its useful life or is returned because it is defective, it will be of a lower quality than a new product. Quality also varies from one returned product to the next, and this influences the processing required and the associated costs.

- *Unclear destination and/or path*. When a product is returned, it is not immediately known if it is going to be processed or disposed of. It must therefore be stored until it has been inspected and a decision taken. In traditional logistics, products have clear destinations and their movement depend on demand-related factors.

- *Production control and inventory management*. Production control and inventory management in traditional logistics systems assumes that suppliers deal with orders in a predictable manner. There is a certain control over the behaviour of the supply chain. In the case of product recovery, however, the behaviour of a return channel is very difficult to control.

All of these differences mean that traditional logistics solutions are not directly applicable to reverse logistics systems.

In a reverse logistics system, returned products are inspected to decide if they should be directly reused/resold, disposed of, or recovered. The processes required to recover a product depend on the complexity of the tasks to be performed, the extent of product transformation required, and the amount of value added during the transformation. Thierry *et al*. (1995) mentioned the following product recovery processes:

- *Repair*. Processes required to make a defective product work properly. The tasks involved are product disassembly, repair of damaged parts, and reassembly.

- *Refurbishment*. Processes applicable to used products that still work but have lost performance quality. The tasks involved are inspection, disassembly, repair/replacement of necessary parts, and reassembly. The resulting product does not have the same quality as a new product.

- *Remanufacturing*. Processes involving disassembly, classification, refurbishment, and reassembly to create an as-new product.

- *Cannibalization*. Recovery of a small part of a returned product to be used in the repair, refurbishment, or remanufacture of other products.

- *Recycling*. Recovery of material from returned products to be transformed into raw material for new processes.

The final destination of a recovered product depends on its condition after completion of the recovery tasks listed above. A recovered product may be distinguishable from a “new” one. Recovered products, however, may also be indistinguishable from new products. In such cases, they can be returned to the market using the same channels as those used for new products. Production systems, are therefore, affected by reverse logistics, and the aim of this thesis, as mentioned at the beginning, is to analyze this influence.

Before analyzing the aspects of CLSC management in more detail, let us look at some basic characteristics of a standard supply chain. A supply chain is characterized by flows of material and information moving in opposite directions, passing successively through the different participants in the chain. Material flowing from a supplier to a customer moves forward (downstream) whereas information on orders between a customer and a supplier moves backwards (upstream). From a very general perspective, supply chain processes can be divided into two subprocesses.

- *Production planning and inventory management.* This consists of the design and management of the production process (planning of needs, acquisition of raw material and components, design and planning of the production process and design and control of material management system) and the management of inventories (design and inventory policies for raw materials, components, in-process material, and finished products).

- *Distribution.* This consists of all the steps relating to the transport and delivery of material from the wholesaler to the retailer. Several options exist. Material can be transported, for example, either directly to the retailer or stored in warehouses from where it is then shipped.

Supply chain management can be divided into 11 different areas:

1. *Location*. Decisions regarding the location of the supply chain take into account quantitative and qualitative aspects such as basic supply and transport infrastructures, local legislation, government incentives, and taxes.

2. *Transport and logistics*. This includes all aspects related to the flow of materials within the supply chain, including the transport, storage, and handling of materials.

3. *Inventory management and demand forecast*. Inventory-related costs tend to be easy to identify and reduce by analyzing the problems affecting a supply chain. Simple stochastic inventory models can be used to generate potential savings related to the sharing of information among the participants in the chain.

4. *Marketing and channel structures*. This includes management aspects related to the structure of the supply chain and supplier-customer relationships. While inventory management focuses on the quantitative aspects of this relationship, marketing and channel structure management deals with customer relations, negotiations, legal issues, and the influence of the management of channels and supply chain structure on the bullwhip effect.

5. *Management of suppliers and supply sources*. This involves the management of supplier relations and decisions regarding their location.

6. *Information systems*. This category involves decisions regarding information technology systems designed to optimize inventory levels.

7. *Product design and launching of new products*. This deals with aspects that should be taken into account to facilitate the creation of new products and shorten time to market.

8. *Aftersales services and support.* Management of repair services for sold products and replacement part supplies.

9. *Outsourcing*. This examines the impact of outsourcing logistics services on the supply chain. In many cases, it is necessary to establish strategic alliances when outsourcing key services such as the use of external logistics suppliers.

10. *Metrics and incentives*. This involves the design and use of metrics to analyze supply chain management aspects.

11. *Global aspects.* This examines the impact of a company's international operations on the above categories.

CLSC is a relatively new concept within the area of supply chain management. Guide and Van Wassenhove (2006) defined CLSC management as “the design, control, and operation of a system to maximize value creation over the entire life cycle of a product with the dynamic recovery of value from different types and volumes of return over the time”. Souza (2008) provided a list of strategic, tactical, and operational aspects related to CLSC management that can be used to provide solutions to the above-mentioned problems, linked to the behaviour of reverse logistics material flows.

- *Strategic aspects*. Location and capacity of returned product facilities, recycling facilities, and remanufacturing facilities, for example. Product recovery strategy, prices of recovered products.

- *Tactical aspects*. Quality and quantity of returned products that are going to be processed for recovery. Planning of remanufacturing programme, taking into account the uncertainties surrounding return quantities and quality. Return inventory management.

- *Operational aspects*. Operational decisions regarding tasks aimed at recovering the value of returned products.

CLSCs can also be classified into different groups (see, for example, Flapper *et al*. 2005). We have decided to classify them according to whether or not the recovered product is distinguishable from the new product.

- *Distinguishable*. In this case, the design of the CLSC should be as shown in Figure 1. There is no interaction between the direct supply chain and the flow of materials in the reverse logistics system. The CLSC loop is closed through the customers.

- *Indistinguishable*. In this case, the production system is fed by the reverse logistics system. Reverse logistics has a considerable effect on production system dynamics as returns form a new supply channel and generate a series of factors that complicate inventory management such as uncertainty and lack of control over the channel, existence of multiple sources of supply, supply capacity constraints, and a lack of monotonicity in return inventory levels. This model is shown in Figure 2.

As can be seen, the behaviour of the reverse logistics system influences that of the direct supply chain. The management of the traditional production and storage system should therefore take into account flows of material from the reverse logistics system.

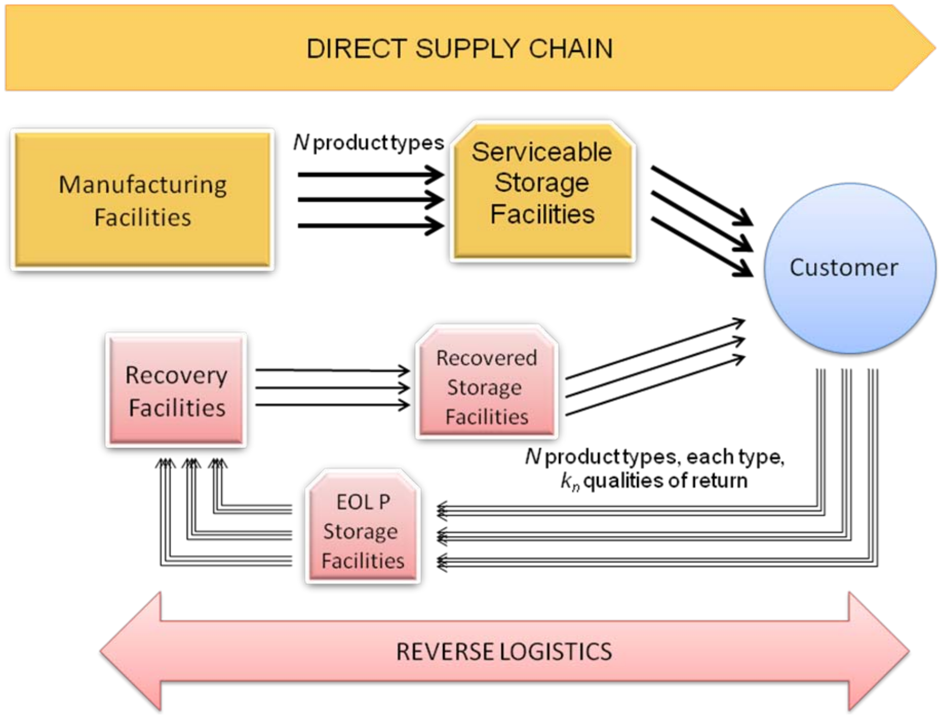


Figure 1. Closed-loop supply chain model in which recovered products are distinguishable from new products. Source: authors.

The growing interest in reverse logistics and CLSCs in the business community (Díaz *et al*. 2004) is evidenced by the increase in the level of related activities in leading sectors such as the transport sector, the consumer electronics sector, the textile sector, and the press and media, to name but a few (Verstrepen *et al.*, 2007). Interest is also increasing within academic spheres, with the publication of many articles analyzing how reverse logistics systems work in companies such as Canon, Philip Morris, Esteé Lauder, Kodak, and Nortel Networks (de Brito *et al*. 2004). The following table 1, taken from Fernández (2004), shows some of the studies conducted in this area.

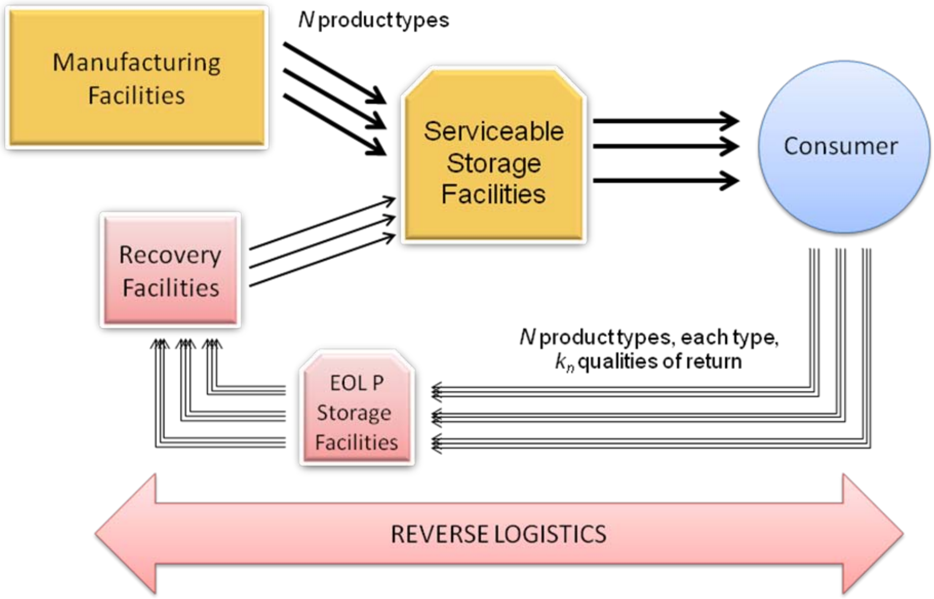


Figure 2. Closed-loop supply chain model in which recovered products are indistinguishable from new products. Source: authors.

Personal computers	Ashayeri, J., Heuts, R., Jansen, A. (1999) Knemeyer, A., Ponzurick, T., Logar, C. (2002) Krikke, H. Harten, A., Schuur, P. (1999) Fleischmann, M., van Nunen, J., Gräve, B. (2002) Tan, A., Yu, W., Kumar, A. (2003) White, Ch., Masanet, E., Rosen, Ch., Beckman, S. (2003)
Vehicles	Bellmann, K. and Kahre, A. (2000) Purohit, D. (1992)
Packaging and containers	Bloemhof-R. J., van Nunen, J, Vroom, J, van der Linden, A. (2001) Del Castillo, E. and Cochran, J. (1996) Duhaime, R., Riopel, D., Langevin, A. (2000) Giuntini, R. and Andel, T. (1994) Kroon, L. and Vrijens, G. (1995)
Carpets	Ammons, J., Realff, M., Newton, D. (1997) Louwers, D., Kip, B., Peters, E., Souren, F., Flapper, S. (1999)
Power tools	Klausner, M. and Hendrickson, C. (2000)
Electronic equipment	de Ron, Ad. and Penev, K. (1995) Fleischmann, M., Beullens, P., Bloemhof-R., J., Wassenhove, L. (2001) Maslennikova, I. and Foley, D. (2000)
Domestic appliances	Krikke, H., Bloemhof-R., J., Wassenhove, L. (2003)
Paper	Madu, Ch., Kuei, Ch., Madu, I. (2002) Fleischmann, M., Beullens, P., Bloemhof-R., J., Wassenhove, L. (2001)
Plastic	Pohlen, T. and Farris, M. (1992)
Medical equipment	Ritchie, L., Burnes, B., Whitlle, P., Hey, R. (2000) Rudi, N., Pycke, D., Sporsheim, P. (2000)
Batteries	Stavros, E., Costas, P., Theodore, G. (2003)

Table 1. Key publications on case studies of reverse logistics and closed-loop supply chains.

Chapter 4

System with deterministic uniform demand

In this chapter we study a production system with constant demand and stochastic returns for a single product and analyze the effects of stochastic remanufacturing factors on system performance.

In Section 4.1 we describe the system and outline the conditions of the parameters involved, considering two scenarios. In the first one the company meets all demand and in the second scenario not all demand is necessarily met. In section 4.2 we describe the manufacturing and remanufacturing policy for the first scenario, provide an approximation of the probability distribution used to determine the amount and rate of returned products, present an algorithm for calculating the optimal manufacturing and remanufacturing capacities, and calculate optimal values for a specific case study. In section 4.3 we give an iterative process to determine the manufacturing and remanufacturing capacities for the second scenario. In section 4.4 we describe how to determine optimal manufacturing and remanufacturing capacities when there are *n* different quality types of returned products. We present several examples to illustrate how are calculated the manufacturing and remanufacturing policies and the manufaturing and remanufacturing capacities. The examples shown are solved using MATLAB. Finally in section 4.5 we present the main conclusions of the chapter.

4.1 Description of the system

We consider a system that produces and sells a single product. The product is returned to the company once it has completed its useful life. The returned product can then be remanufactured and resold as new or disposed of. The system has the following features:

- The time horizon of the system is discrete with periods of equal length.
- The company makes the decisions at the end of each period.
- The demand *D* (units/period) is known and is the same in each period.
- It is a just-in-time production system, so there should be no inventories.
- The system has maximum manufacturing and remanufacturing capacities of *P* and *R* units per period respectively. It is assumed that there is sufficient manufacturing capacity to supply the demand, i.e. $P + R \geq D$. It is also assumed that $P \leq D$ and $R \leq D$ because capacities greater than *D* will never be used in order to meet demand.

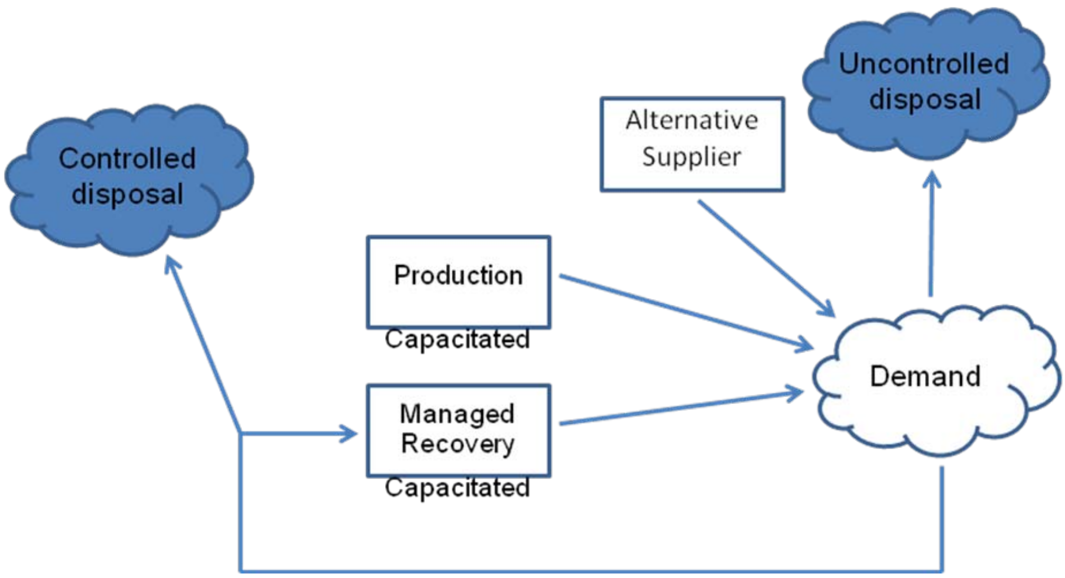


Figure 1. Schematic representation of the system

The manufacturing costs of the original production system and the remanufacturing system are composed of fixed costs C_p and C_r (which depend on the installed capacity and, therefore, do not vary provided that the manufacturing and remanufacturing capacities remains constant) and variable costs (per unit of output) c_p and c_r. It is assumed that C_p is an increasing function of P and C_r is an increasing function of R.

The returns have the following characteristics:

- The end-of-life of the product occurs between periods T_1 and T_2 after it is sold. p_i is the probability that the end-of-life of the product occurs i periods after it is sold ($i = T_1,\ldots,T_2$).

- ρ is the probability of an end-of-life product being returned. Therefore, $\rho \cdot p_i$ is the probability that a unit sold in period t will be returned in period $t+i$.

- There is only one quality type for returned products. Therefore, each unit of returned product undergoes the same remanufacturing process.

- A remanufactured product has the same life expectancy and return quality as a manufactured product.

- Each returned unit has a cost of c_{rc}.

- The cost of disposing a returned product is zero.

If we assume that there is no product returns, the optimal inventory policy is such that the inventory costs are zero. Therefore, the costs for each period would be $C_p(D) + c_p \cdot D$. When products are returned and recovered, the company can sell units from either the original production system or the remanufacturing system.

Since there is an inherent degree of uncertainty in the availability of returns, we analyze two different scenarios. In the first one there is a supplier with sufficient capacity that enables the company meet all demand with a cost per unit of c_s; it is assumed that c_s is greater than c_p and c_r. In the second scenario the company's inventory policy can sometimes cause supply interruptions; in this case, the unmet demand is lost at a shortage cost per unit, b; it is assumed that b is greater than c_p and c_r.

4.2 Determining optimal manufacturing and remanufacturing capacities in a system with an alternative supplier

The costs incurred by the company during each period depend on the quantity of goods manufactued, recovered, and remanufactured by the company and on the goods purchased from the supplier. These amounts will be limited by the installed manufacturing and remanufacturing capacities and by the quantity of returned products, which is a random value.

The optimal manufacturing and remanufacturing capacities are calculated by minimizing the expected value of the cost incurred in each period according to the following process: first, we determine the optimal manufacturing and remanufacturing policy for a period and the associated cost for a given capacity and a given return; next, we calculate the expected value of the associated cost and determine the capacities that produce the lowest value.

4.2.1 Optimal manufacturing and remanufacturing policy

The manufacturing and remanufacturing policy is obtained by optimizing the equation shown below, given the manufacturing and remanufacturing capacities, P and R, and the units of returned product available during each period, r:

$$[\text{MIN}]\ c = C_p(P) + C_r(R) + c_p \cdot x + c_r \cdot y + c_s \cdot (D - x - y) + c_{rc} \cdot r$$

s.t.:

$$\begin{aligned} & x + y \leq D \\ & x \leq P \\ & y \leq \min\{R, r\} \\ & x, y \geq 0 \end{aligned}$$

Where x and y are the quantities of product to manufacture and remanufacture respectively. The optimal solution depends on the values of r, P, R and D, and also on the relation between c_r and c_p.

When $c_r < c_p$, we prefer remanufacturing to manufacturing. The optimal y is the highest value allowed by constraints (i.e. $y = \min\{R, r\}$) and the optimal x is $\min\{P, D - y\}$. Three cases can be distinguished:

1. $r < D - P$: The company meets total demand using the alternative supplier. The optimal values and costs incurred are:

   $x = P$, $y = r$

   $c = C_p(P) + C_r(R) + (c_p - c_s) \cdot P + c_s \cdot D + (c_r - c_s + c_{rc}) \cdot r$

2. $D - P \leq r < R$: The optimal values and costs incurred are:

   $x = D - r$, $y = r$

   $c = C_p(P) + C_r(R) + c_p \cdot D + (c_r - c_{p+} c_{rc}) \cdot r$

3. $r \geq R$: The returns are higher than R. The optimal values are:

   $x = D - R$, $y = R$

   $c = C_p(P) + C_r(R) + c_p \cdot D + (c_r - c_p) \cdot R + c_{rc} \cdot r$

When $c_r \geq c_p$ we prefer manufacturing to remanufacturing. The optimal x is the highest value allowed by constraints (i.e. $x = P$) and the optimal y is $\min\{r, D - x\}$. We have two cases:

1. $r < D - P$. The optimal values and costs incurred are:

   $x = P$, $y = r$

   $c = C_p(P) + C_r(R) + (c_p - c_s) \cdot P + c_s \cdot D + (c_r - c_s + c_{rc}) \cdot r$

2. $D - P \leq r$. In this case, the optimal values and costs incurred are:

   $x = P$, $y = D - P$

   $c = C_p(P) + C_r(R) + (c_p - c_r) \cdot P + c_r \cdot D + c_{rc} \cdot r$

4.2.2 Probability distribution of returned product quantity

The quantity of product returned during a given period from the quantity of product sold in the *i*-th previous period follows a binomial distribution B(D, $\rho \cdot p_i$), where p_i is the probability that the product will come to the end of its useful life during the *i*-th period after its sale; ρ is the probability that the product will be returned once it has completed its useful life and D is the quantity of product sold during the *i*-th previous period.

The quantity of product returned during a given period is equal to the sum of the returned products from each of the previous periods. The probability that this value will be r is denoted by $p(r)$.

The expected value of combined manufactured and remanufactured products from the company is called PM and is calculated using the following expression:

$$PM = D - \sum_{r=0}^{D-P} \left(D - P - r \right) p(r) \qquad (4.1)$$

When $\rho \cdot p_i$ is sufficiently small, we can approximate the probability distribution of returns from a given period to a Poisson distribution with parameter $D \cdot \rho \cdot p_i$. Therefore, the total quantity of product returned during a given period follows a Poisson distribution with parameter $\rho \cdot D$ (since the sum of p_i is 1). In this case we obtain:

$$p(r) = \frac{e^{-\rho D} (\rho D)^r}{r!} \qquad (4.2)$$

4.2.3 Calculating optimal manufacturing and remanufacturing capacities

If we assume the manufacturing and remanufacturing policy established in section 4.2.1 and the probability distribution of product returns defined in section 4.2.2, we can determine the expected value of the cost function by using the following expression:

$$E(c(r)) = \sum_{r=0}^{\infty} c(r) \cdot p(r) \tag{4.3}$$

Case $c_r < c_p$:

$$E(c(r)) = C_p(P) + C_r(R) + \sum_{r=0}^{D-P} \left[(c_p - c_s) \cdot P + c_s \cdot D + (c_r - c_s + c_{rc}) \cdot r \right] \cdot p(r) +$$

$$\sum_{r=D-P}^{R} \left[c_p \cdot D + (c_r - c_p + c_{rc}) \cdot r \right] \cdot p(r) + \sum_{r=R}^{\infty} \left[c_p \cdot D + (c_r - c_p) \cdot R + c_{rc} \cdot r \right] \cdot p(r)$$

By reordering the terms we obtain:

$$E(c(r)) = c_p \cdot D + c_{rc} \cdot E(r) + C_p(P) + \left(c_s - c_p\right) \sum_{r=0}^{D-P} \left(D - P - r\right) \cdot p(r) +$$

$$C_r(R) - \left(c_p - c_r\right) \cdot \left[R - \sum_{r=0}^{R} \left(R - r\right) \cdot p(r) \right] \tag{4.4}$$

Where $E(r)$, the expected value of r is equal to $\rho \cdot D$ because the entire used product is recovered. We can then define the following functions for determining the optimal solution:

$$g_1(P) = C_p(P) + \left(c_s - c_p\right) \sum_{r=0}^{D-P} \left(D - P - r\right) p(r) \tag{4.5}$$

$$g_2(R) = C_r(R) - \left(c_p - c_r\right) \cdot \left[R - \sum_{r=0}^{R} \left(R - r\right) p(r) \right] \tag{4.6}$$

$$g(P, R) = c_p \cdot D + c_{rc} \cdot \rho \cdot D + g_1(P) + g_2(R) \tag{4.7}$$

Therefore, the desired values of P and R are the solution of the following problem PROBL:

[MIN] $g(P, R)$

s.t.:

$$
\begin{aligned}
&P \le D \\
&R \le D \\
&P + R \ge D \\
&P, R \ge 0
\end{aligned}
$$

Case $c_r \ge c_p$:

The desired values of the capacities P and R are the solution of the problem PROBL but now with the following expressions for g_1, g_2 and g :

$$g_1(P) = C_p(P) + (c_p - c_r) \cdot P + (c_s - c_r) \cdot \sum_{r=0}^{D-P} \left(D - P - r\right) p(r) \tag{4.8}$$

$$g_2(R) = C_r(R) \tag{4.9}$$

$$g(P, R) = c_r \cdot D + c_{rc} \cdot \rho \cdot D + g_1(P) + g_2(R) \tag{4.10}$$

Both cases are non-linear programming problems.

4.2.4 Numerical example

We analyze a company that produces and sells a product with the following features:

- Demand D = 100 u/period.
- Variable cost of manufacturing c_p = €10/u.
- Variable cost of remanufacturing c_r = €5/u.
- Variable recovery cost c_{rc} = €1/u.
- Fixed manufacturing costs according to the capacity P: $C_p(P) = 15 \cdot P - 0{,}05 \cdot P^2$.
- Fixed remanufacturing costs according to the capacity R: $C_r(R) = 3 \cdot R - 0{,}01 \cdot R^2$.

- Unitary cost of supply c_s = €30/u.

- Probability of product returns $\rho = 0.3$.

- Probability distribution of product returns: the company configuration meets the conditions for using a Poisson distribution with parameter $\rho \cdot D$.

The system without returns will have a manufacturing capacity of 100 units with a cost of €2000 per period. When the remanufacturing system is included, the minimum of *g* is reached at (*P*, *R*) = (72, 30) and its value is *g (P*, *R)* = €1818.70. This gives a *PM* of 98.70.

Figure 2 shows the graph of the function *g(P, R)*.

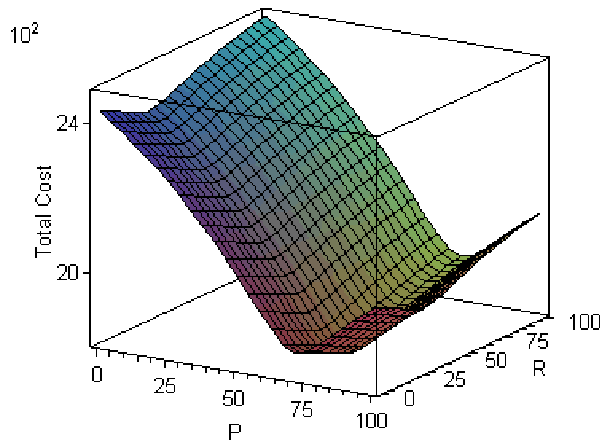


Figure 2. Total cost depending on the manufacturing and remanufacturing capacities.

4.3 Determining optimal manufacturing and remanufacturing capacities in a system without an alternative supplier

The optimal manufacturing and remanufacturing policy is calculated as in section 4.2.1, using the same expressions and changing the unit cost of supply c_s for the unit cost of shortage *b*.

The quantity of product returned during a given period from the quantity of product sold in the *i*-th previous period follows a binomial distribution $B(v_i, \rho \cdot p_i)$, where v_i is the quantity sold during the *i*-th previous period and ρ and p_i are defined as in section 4.2.2.

Since in this case there is a possibility of inventory shortage, the value of v_i behaves randomly and is less than *D*. We suppose that the system is in a stationary state and therefore the probability distributions of sales are the same in each period. The probability distribution of returned product quantity, $p(r)$, depends on $q(v)$, the probability distribution of the quantity sold in any period, which, in turn, depends on $p(r)$. In order to solve this cyclic dependency we use the following iterative process (IP1) to compute $p(r)$:

Step 0: Start the process with

$$q_0(v) = \begin{cases} 1 & v = D \\ 0 & v \neq D \end{cases}$$

Where $q(v)$ is the probability that the sales in a period will be *v*.

Step 1: Compute $\upsilon_n(r_i)$, approximation, in the n-th iteration, of the probability of the number of returned units corresponding to the sales of the *i*-th preceding period is equal to r_i as follows, for $i = T_1,\dots,T_2$ and $r_i = 0,\dots,D$:

$$\upsilon_n(r_i) = \sum_{v=r_i}^{D} \upsilon_n(r_i \mid v) \cdot q_{n-1}(v) = \sum_{v=r_i}^{D} \binom{v}{r_i} (\rho \cdot p_i)^{r_i} \cdot (1 - \rho \cdot p_i)^{v-r_i} \cdot q_{n-1}(v) \quad (4.11)$$

Compute $p_n(r)$, approximation, in the n-th iteration, of the probability of the total number of returned units is equal to *r,* for $r = 0,\dots,(T_2 - T_1 + 1) \cdot D$:

$$p_n(r) = \sum_{\sum_{i=T_1}^{T_2} r_i = r} \prod_{i=T_1}^{T_2} \upsilon_n(r_i) \tag{4.12}$$

Step 2: Calculate the PD of product sold each period $q_n(v)$ using $p_n(r)$ calculated in step 1:

$$q_n(v) = \begin{cases} 0 & v < P \\ p_n(v-P) & P \le v < D \\ 1 - \sum_{r=0}^{D-P} p_n(r) & v = D \end{cases} \tag{4.13}$$

Step 3: Calculate the difference between $q_{n-1}(v)$ and $q_n(v)$ where difference means some measure of how far one distribution is from the other (for example the quantity $\left|E\left(q_n(v)\right) - E\left(q_{n-1}(v)\right)\right|$ can be used as a measure of the difference). If the difference is greater than a tolerance, add 1 to n and go to step 1; otherwise take $p(r) = p_n(r)$.

The optimal manufacturing and remanufacturing capacities are calculated by solving problem PROBL from section 4.2.3 but replacing the unit cost of supply c_s with the unit cost of shortage b in the expression of $g_1(P)$ and replacing $c_{rc}\cdot\rho\cdot D$ with $c_{rc}\cdot\rho\cdot V$ in the expression of $g(P,R)$, where V is the expected value of the product sold:

$$V = D - \sum_{r=0}^{D-P} (D - P - r)\, p(r) \tag{4.14}$$

In the case $c_r < c_p$, using the expression of V, we have:

$$g_1(P) = C_p(P) + (b - c_p)\cdot(D - V) \tag{4.15}$$

$$g(P, R) = C_p(P) + b\cdot D + (c_p + c_{rc}\cdot\rho - b)\cdot V + g_2(R) \tag{4.16}$$

Analogously, in the case $c_r \ge c_p$ we have:

$$g(P, R) = C_p(P) + (c_p - c_r)\cdot P + b\cdot D + (c_r + c_{rc}\cdot\rho - b)\cdot V + g_2(R) \tag{4.17}$$

When solving the problems it is important to take into account that the PD of product returns $p(r)$ depends on P. Therefore, we define an iterative process (IP2) to find the optimal values:

Step 0: Start the process with $P_0 = (1 - \rho)\cdot D$.

Step 1: Compute the PD of returned products using the iterative process IP1 described above.

Step 2: Determine (P_n, R_n) by solving problem PROBL, which optimizes the value of the expected cost $g_n(P_n,R_n)$.

Step 3: If the desired accuracy in $g_n(P_n, R_n)$ is not achieved, then go to step 1; otherwise finish the process.

We recalculate the numerical example of section 4.2.4 but replacing the unit cost of supply c_s with the unit cost of shortage b = €30/u and with product end-of-life occurring between periods 1 and 6 with probabilities $p_1 = 0.1$, $p_2 = 0.2$, $p_3 = 0.2$, $p_4 = 0.25$, $p_5 = 0.15$, $p_6 = 0.1$.

The minimum of g is reached at (P, R) = (72, 30) and its value is $g(P, R)$ = €1820.90. This gives a value of V = 98.61.

We have used the following tolerances in step 3 of each iterative process:

For IP1: $$\frac{\left|E\left(q_n(v)\right)-E\left(q_{n-1}(v)\right)\right|}{E\left(q_{n-1}(v)\right)}<0.001$$

For IP2: $$\frac{\left|g_n\left(P_n,R_n\right)-g_{n-1}\left(P_{n-1},R_{n-1}\right)\right|}{g_{n-1}\left(P_{n-1},R_{n-1}\right)}<0.001$$

The main process (IP2) converges in 3 iterations and for each iteration IP1 converges in 3 iterations.

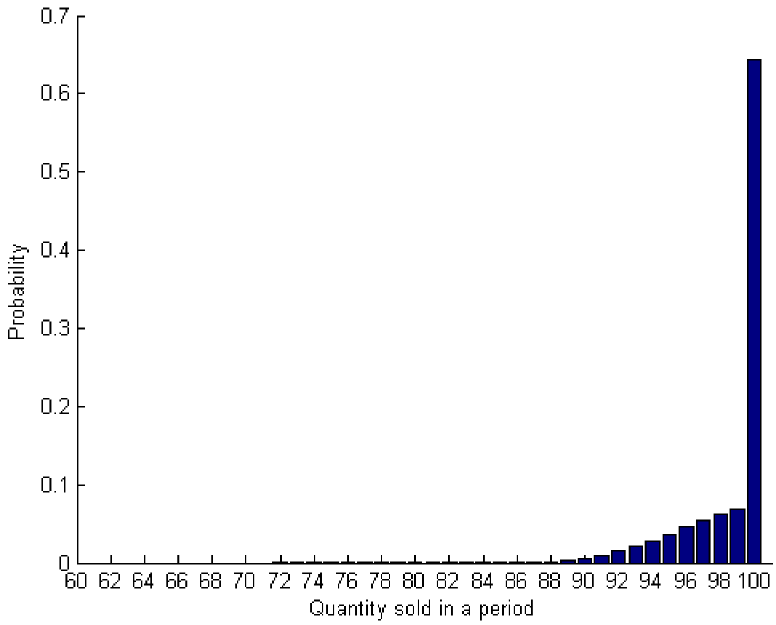


Figure 3. Probability distribution of quantity sold in a period.
Mean value = 98.61, standard deviation = 2.44.

4.4 System with n quality types of returned products

In this section we consider a specific case in which the returned product is defined according to a series of quality types and we calculate the optimal manufacturing and remanufacturing policy. The procedure outlined in this section can be considered a generalization of the one described in the previous section, 4.2.

The configuration is similar to that of a system in which all returned products are of the same quality. The variable remanufacturing costs are c_j, $j=1,\ldots,n$. and the returns have the following characteristics:

- p_i and ρ are defined in the same way as for a single quality.
- There are n different quality types for returned products.
    - π_j, $j=1,\ldots,n$ is the probability that a returned product is of quality type *j*.

- o a_j units of remanufacturing resources are required to remanufacture one unit of returned product of quality type *j (j=1,…,n).*

It is assumed that c_s is greater than c_p and c_j (*j=1,…,n*).

The manufacturing and remanufacturing policy is obtained by optimizing the linear equation shown below, given the manufacturing and remanufacturing capacities, *P* and *R*, and the units of returned product of quality type *j* (*j=1,…,n*) available in each period, r_j:

$$[\text{MIN}]\ c = C_p(P) + C_r(R) + c_p \cdot x + \sum_{j=1}^{n} c_j \cdot y_j + c_s \cdot \left(D - x - \sum_{j=1}^{n} y_j \right)$$

s.t.:

$$x + \sum_{j=1}^{n} y_j \le D$$
$$x \le P$$
$$\sum_{j=1}^{n} a_j \cdot y_j \le R$$
$$y_j \le r_j \quad j = 1,...,n$$
$$x, y_1,..., y_n \ge 0$$

Where *x* is the quantity of product to manufacture and y_j are the quantities of returned product of quality *j* (*j=1,…,n*) to remanufacture. By modifying the notation slightly, we obtain the following formula:

$$[\text{MAX}]\ \sum_{j=1}^{n+1} S_j \cdot y_j$$

s.t.:

$$\sum_{j=1}^{n+1} y_j \le D$$
$$\sum_{j=1}^{n} a_j \cdot y_j \le R$$
$$y_j \le r_j \quad j = 1,...,n+1$$
$$y_1,..., y_{n+1} \ge 0$$

Where the objective function has been reversed and the notation has been changed as follows:

- The variable x is redefined as $y_{n+1} = x$
- The objective function parameters are compacted:
  - $S_j = c_s - c_j$ for $j=1,\ldots,n$
  - $S_{n+1} = c_s - c_p$
  - $r_{n+1} = P$
  - $a_{n+1} = 0$

By using the constraints of the problem, the dual problem and the complementary slackness theorem we obtain the following expressions:

$$\sum_{j=1}^{n+1} y_j \leq D$$

$$\sum_{j=1}^{n} a_j \cdot y_j \leq R$$

$$y_j \leq r_j \quad j = 1,\ldots,n+1$$

$$\mu_D + a_j \cdot \mu_Y + \mu_j \geq S_j \quad j = 1,\ldots,n+1$$

$$\left(y_j - r_j\right)\mu_j = 0 \quad j = 1,\ldots,n+1$$

$$\left(\sum_{j=1}^{n+1} y_j - D\right)\mu_D = 0$$

$$\left(\sum_{j=1}^{n} a_j \cdot y_j - R\right)\mu_Y = 0$$

$$\left(\mu_D + a_j \cdot \mu_Y + \mu_j - S_j\right)y_j = 0 \quad j = 1,\ldots,n+1$$

$$y_j, \mu_j, \mu_D, \mu_Y \geq 0 \quad j = 1,\ldots,n+1$$

Where $\mu_j, \mu_D, \mu_Y \geq 0 \quad j = 1,...,n+1$ are the dual variables. Four different cases can be distinguished depending on the values of r_j ($j=1,\ldots,n+1$), R and D:

1. The company is unable to cover all demand and all returned products can be remanufactured. Then,

   $$\sum_{j=1}^{n+1} r_j < D \text{ and } \sum_{j=1}^{n} a_j \cdot r_j < R$$

   And the optimal values are:

   - $\mu_D = \mu_Y = 0$
   - $y_j = r_j \quad \mu_j = S_j \quad j = 1,...,n+1$

2. The company is unable to cover all demand and not all returned products can be remanufactured. Then ,

   $$\sum_{j=1}^{n+1} r_j < D \text{ and } \sum_{j=1}^{n} a_j \cdot r_j \geq R$$

   The optimal values are:

   - $y_{n+1} = r_{n+1}, \; \mu_{n+1} = S_{n+1}$

   Defining:

   - $\alpha_j = S_j / a_j$

   There is a subscript k such that the optimal solution is:

   - $\mu_D = 0$
   - $y_j = r_j, \; \mu_j = a_j\left(\alpha_j - \alpha_k\right)$ if $\alpha_j > \alpha_k$

- $y_k = \frac{1}{a_k}\left(Y - \sum_{j=1}^{k-1} a_j \cdot r_j\right) \le r_k$, $\mu_k = 0$

- $y_j = 0$, $\mu_j = 0$ if $\alpha_j \le \alpha_k$

3. The company can cover all demand and all returned products can be remanufactured. Then

$$\sum_{j=1}^{n+1} r_j \ge D \text{ and } \sum_{j=1}^{n} a_j \cdot r_j < R$$

Optimal values: there is a subscript *k* such that the optimal solution is:

- $\mu_D = S_k$, $\mu_Y = 0$

- $y_j = r_j$, $\mu_j = S_j - S_k$ if $S_j > S_k$

- $y_k = D - \sum_{j=1}^{k-1} r_j \le S_k$, $\mu_k = 0$

- $y_j = 0$, $\mu_j = 0$ if $S_j \le S_k$

4. The company can cover all demand but not all returned products can be remanufactured. Then

$$\sum_{j=1}^{n+1} r_j \ge D \text{ and } \sum_{j=1}^{n} a_j \cdot r_j \ge R$$

Optimal values: no analytical expression can be found for the optimal solution and must be calculated case by case.

4.5 Conclusions

In this chapter we studied the behavior of a system with reverse logistics for manufacturing and remanufacturing a product under steady demand. The optimal manufacturing policy is constant when there is no reverse logistics, the company satisfies all the demand and no inventories are required.

We can draw several conclusions about the effects of uncertainty on the amount and rate of returns in the system and use them to compare it with an equivalent system without reverse logistics. First of all we saw that the optimal manufacturing policy becomes more complex when the system has to take into account product returns. Also, using the method that has been described for calculating the optimal manufacturing and remanufacturing capacities, we found that the manufacturing capacity can be set at a lower value than the demand and so the demand could not be totally met unless we use an alternative supplier. Finally, if the company could operate with inventories, the optimal capacities could change, so the uncertainty on returns also influences the inventory system.

In the last section, we described a system with n different return qualities and determined the optimal policy for a given period. We saw that the complexity increases and that could be optimal to remanufacture although the cost of remanufacture were higher than the original manufacturing costs.

Chapter 3

Literature review

In this chapter, we will review key articles that have been published in the field of interest and that aid us in meeting the aim of this thesis. We have divided these articles into two types: those that deal with manufacturing and storage capacity management in production systems and those that use mathematical models to study reverse logistics systems.

The aim of this chapter is to shed light on aspects that should be taken into account when managing production system capacities and to describe show mathematical models that have been used to study systems with reverse logistics.

3.1 Manufacturing and storage capacity management

Several reviews have summarized studies dealing with capacity management (Luss, 1982; Van Mieghem, 2003; Wu *et al*. 2005). Van Mieghem (2003), for example, described the different types of problems related to capacities—

increases/decreases, choice of technology, acquisition, and location—and discussed how these problems were addressed in the literature.

We are going to focus on optimal management strategies based on capacity acquisition and increases/decreases. Rajagopalan and Swaminathan (2001) explored the interaction between production planning and capacity acquisition decisions in an environment with deterministic demand growth. Atamtürg and Hochbaum (2001) studied optimal solutions in an environment with non-stationary deterministic demand and production needs that could be covered through the acquisition of new capacities, subcontracting, and the use of existing inventories. Bradley and Arntzen (1999) used an approach aimed at maximizing return on assets in an aggregate planning model and concluded that a production strategy based on minimizing unit cost and maximizing equipment use can generate less-than-optimal financial results. Queuing theory and newsvendor network models have been used to study systems with stochastic demand (Van Mieghem, 2003). Bradley and Glynn (2002) used a queuing-like model to demonstrate that the impact of capacity decisions on optimal inventory policies should be taken into account when taking such decisions. Newsvendor network models are used when the function to optimize and the corresponding constraints are linear (Van Mieghem and Rudi, 2002; Angelus and Porteus, 2002), allowing manufacturing capacity and inventory policy to be optimized simultaneously. Alp and Tan (2008) presented a dynamic programming model to resolve the problem of determining permanent manufacturing capacity levels and optimal adjustments with contingency resources (e.g. via the use of overtime) to meet demand. Their study can be classified as tactical or operational as they proposed resolving production-capacity problems with the temporary hiring of workers or the use of overtime, contrasting with strategic-type studies whose purpose is to optimize the acquisition of permanent resources.

3.2 Reverse logistics and CLSCs

Rubio *et al.* (2008) analyzed the main characteristics of articles in the area of reverse logistics. Based on the methodology used, they reported that 30% of the studies were case reports, literature reviews, or surveys, 65% were studies of mathematical models, and the remaining 5% were theoretical studies on the management of CLSCs. Of the studies that used mathematical models, 7.5% dealt with problems related to the recovery and distribution of end-of-life products, 80% dealt with problems related to production planning and inventory management, and 12.5% dealt with problems related to the supply chain.

The aim of this thesis, which is to study the influence of reverse logistics on production system capacities, falls within the area of CLSC management. It is, however, also related to inventory management because we consider that such systems should operate optimally.

3.2.1 Mathematical CLSC models

Jarayaman *et al*. (1999) presented a mixed integer programming model that resolves the problem of designing a CLSC by simultaneously taking into account the location of remanufacturing/distribution facilities, transport, and the optimal production and storage of remanufactured products.

Majumder and Groenevelt (2001) presented a system in which the remanufactured returned product was indistinguishable from the new product and a model in which a manufacturer and a remanufacturer were competing to sell new and remanufactured products. Using the model, they drew conclusions about incentives that existed in the system to increase the quantity of products to be remanufactured.

A model presented by Linton *et al.* (2002) that took into account the stochastic behaviour of useful life and the probability of return to estimate cathode ray tube televisions returns showed the importance of estimating returns when designing a CLSC.

Bufardi *et al*. (2004) proposed a multicriteria decision aid (MCDA) approach for deciding how to deal with end-of-life products. They analyzed key factors that should be taken into account including the formulation of a set of alternatives, the selection of criteria to evaluate these alternatives, and the choice of an appropriate MCDA method.

Fandel and Stammen (2004) designed a mixed-integer linear programming model that analyzed the business process during the entire life cycle of a product, including recycling. The main contribution of this study is that the model can be used as a strategic decision-making tool when designing a CLSC.

Georgiadis and Vlachos (2004) used a system dynamics approach to estimate stock and return flows in a reverse logistics supply chain in which variations in remanufacturing capacity were allowed. They considered that demand depended on the green image factor, which, in turn, depended on model variables related to the recovery of products.

Hesse *et al.* (2005) proposed a model for the hospital bed market in the United States based on the game theory. The market was dominated by two companies that sold new products (primary market) and could repurchase used products to resell in the secondhand market (secondary market). The model provided the quantity of products that should be recovered and the price at which they should be resold.

Horvath *et al.* (2005) studied the influence of reverse logistics on the financial management of a retail chain. The random nature of the quality and quantity of product returns from customers affects retailer cash flow management. The article presented a model to calculate the expected retailer holding time (time from when the returned product is received to the time it can be resold). Using this model, they drew several conclusions on cash flow management strategies in retail chains.

Nagurney and Toyasaki (2005) proposed a model for the integrated management of the CLSC that can be used to analyze and calculate material flows and prices in the electronic product recycling sector.

Finally, Corbacioglu and van der Laan (2007) showed that holding costs for remanufactured and manufactured products cannot be calculated in the same way and that the method used to calculate the former is not trivial.

3.2.2 Mathematical inventory management models in reverse logistics systems

Inventory management in a reverse logistics system differs from that in a traditional logistics system when the recovery system interacts with the existing manufacturing system, i.e. in cases where the recovered and the new product are identical. In practically all the articles that present mathematical inventory management models for reverse logistics systems, it is assumed that new and recovered products are indistinguishable from each other. It is also assumed, in practically all the models, that the system has unlimited resource capacities (production, recovery, and storage). The main differences between the models can be seen in Tables 1 and 2.

Following the system used by Fleischmann and Minner (2004), we have classified these models into deterministic and stochastic models.

Deterministic models:

Constant demand models. Richter and Dobos (2004) performed a comparative study of the most important reverse logistic models with inventory management based on the economic order quantity. The differences they observed were due to the fact that the models analyzed different systems with different optimization criteria, both in terms of the function to optimize and the set of manufacturing and remanufacturing policies permitted. These policies depended on the design of the model. For models with production setup costs, for example, manufacturing and remanufacturing were performed in batches and separately (i.e. products were either manufactured or remanufactured at a given moment). Such policies are typical in systems that share manufacturing and remanufacturing resources.

The following table 1 shows the key articles that have analyzed deterministic models.

	Schrady (1967)	Mabini *et al.* (1992)	Richter (1996)	Teunter (2001)	Teunter and van der Laan (2002)	Dobos and Richter (2004)	Choi *et al.* (2007)	Rubio and Corominas (2008)	Kiesmüller *et al.* (2000)	Minner and Kleber (2001)	Dobos (2003)	Richter and Weber (2001)
Time	Continuous											Discrete
Demand	Constant								Variable			
Returns	Constant								Variable			
Disposal?	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes	Yes
Cost of disposal?	Yes	No	Yes	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes	Yes
Returned product stock?	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes
Serviceable stock?	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes
Identical unit storage costs for new and remanufactured products?	Yes	Yes	Yes	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Identical recovered and new products?	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Manufacturing and remanufacturing setup costs?	Yes	Yes	Yes	Yes	Yes	Yes	Yes	No	No	No	No	Yes
Supply interruptions admitted?	No	Yes	No	No	No	No	No	No	Yes	No	No	No
Manufacturing lead time?	No	Yes	No	No	No	No	No	No	No	No	No	No
Remanufacturing lead time?	No	Yes	No	No	No	No	No	No	No	No	No	No

Table 1. Key articles describing deterministic models, with characteristics of systems analyzed.

In the model presented by Teunter (2001), “the class of policies Π considered are those with fixed batch sizes Q_m for manufacturing and Q_r for recovery, where M manufacturing batches and R recovery batches succeed each other”. Within class Π, only policies in which $M = 1$ *or* $R = 1$ are considered. In the study by Richter (1996), in order to meet demand, products in the recoverable product warehouse are recovered until the maximum batch size is met and then new products are manufactured. Minner (2001) showed that manufacturing and remanufacturing in batches of the same size is not necessarily an optimal policy. The model presented by Teunter and van der Laan (2002) showed that optimal order quantity calculation based on the optimization of average costs are different from those based on net present value costs. Choi *et al.* (2007) extended Richter's model to contemplate a wider set of manufacturing/remanufacturing policies. Acceptable policies were those that alternated manufacturing and remanufacturing batches in order to meet demand. The authors came to the conclusion that optimal policies were not necessarily those considered by Teunter (2001) or Richter (1996). Nonetheless, they performed a numerical study of 8,100,000 cases in which only 0.2% of cases had an optimal solution outside the set of Teunter (2001).

Rubio and Corominas (2008) studied optimal policies in a lean production environment and concluded that an optimal production strategy combined manufacturing, remanufacturing, and disposal. The model is extended to analyze a system with limited manufacturing and remanufacturing capacities.

Variable demand/continuous time models. Minner and Kleber (2001) presented a linear cost model, formulated an optimal control problem, and resolved it using Pontryagin's maximum principle. Their model was extended by Kiesmüller *et al*. (2000), who introduced the possibility of stock shortage, backlogging unmet demand. Kleber *et al*. (2002) also extended the model of Minner and Kleber (2001) by taking into account multiple product return options. Dobos (2003) also presented a similar model to that proposed by Minner and Kleber (2001) but the function to be optimized was quadratic rather than linear.

Variable demand/discrete time models. Richter and Weber (2001) extended the classical Wagner-Within model by including the possibility of returned products. They first presented a model for remanufactured products from the moment they are returned to the manufacturer to the moment they are returned to the market. They then modelled a system with both manufacturing and remanufacturing facilities, and finally introduced the option of disposing of recovered products. With constant costs over time and zero setup costs, the model proposed by Richter and Weber (2001) was equivalent to the discrete version of the model used by Minner and Kleber (2001).

Stochastic models:

Van der Laan and Salomon (1997) proposed a production planning and inventory control model in a system with both remanufacturing and disposal. The aim was to create a system that was both stable and robust. They proposed two types of inventory policies: a push-disposal policy and a pull-disposal policy, which, while not necessarily optimal, reduced variations in inventory levels. They showed that the expected cost of the system with the option of product disposal was lower than that of the system without this option. To perform the calculations, they used a definition of inventory position that did not include either the returned product or the product to be returned.

Kiesmüller and van der Laan (2001) showed that assuming that demand and returns are independent can lead to the use of less-than-optimal inventory policies. The main characteristics of the model used are shown in Table 2. To perform the calculations, they defined an inventory position that took into account the product yet to be returned and order-up-to inventory policies (although they acknowledged that these may not be optimal). Although they admitted the possibility of stock shortage, they computed the probability distribution of returns under the assumption that demand is fully met (arguing that the service level would be high). The model did not admit product disposal policies.

Fleischmann *et al*. (2002) present a continuous-time model in which both demand and returns were independent Poisson processes. They calculated the optimal production policy by considering net demand (demand less returns) and extending the results of Federguren and Zheng (1992).

	van der Laan and Salomon (1997)	Inderfurth (1997)	Buchanan and Abad (1998)	Kiesmüller and van der Laan (2001)	Fleischmann *et al.* (2002)	Fleischmann and Kuik (2003)	Inderfurth (2004)
Time	Continuous	Discrete	Discrete	Discrete	Continuous	Discrete	Discrete
Demand	Coaxian-2	General	General	Poisson	Poisson process	General	General
Returns	Coaxian-2	General	General	Stochastic	Poisson process	General	General
Recovery costs?	Yes	Yes	No	No	No	No	Yes
Disposal?	Yes	Yes	No	No	No	No	Yes
Cost of disposal?	Yes	Yes	No	No	No	No	Yes
Returned product stock?	Yes	Yes	No	No	No	No	Yes
Serviceable stock?	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Identical unit storage costs for new and remanufactured products?	Yes	Yes	Yes	Yes	Yes	Yes	No
Identical recovered and new products?	Yes	Yes	Yes	Yes	Yes	Yes	No
Manufacturing and remanufacturing setup costs?	Yes	No	No	No	Prod.	Prod.	No
Supply interruptions admitted?	Yes	Yes	Yes	Yes	Yes	Yes	No
Manufacturing lead time?	Yes	Yes	No	Yes	No	No	Yes
Remanufacturing lead time?	Yes	Yes	No	Yes	No	No	Yes

Table 2. Key articles describing stochastic models, with characteristics of systems analyzed.

Fleischmann and Kuik (2003) studied a discrete-time system with random, independent demands and returns. The cost structure was formed by fixed order costs and convex stock shortage and holding costs. They showed that an inventory control policy of the type (s,S) is optimal when the optimization criterion is the minimization of the expected cost value.

The system proposed by Inderfurth (2004) was different to those described above as he assumed that the remanufactured product and the new product were different and therefore sold in different markets. When there were no remanufactured products, the company offered new products. The model was used to calculate the company's manufacturing, remanufacturing, and disposal policies based on optimizing the expected cost value per period.

On reviewing the literature, we can conclude that few studies have analyzed the problem of jointly determining capacity and inventory. As Vlachos *et* al. (2007, p. 368), stated, "Capacity planning is an extremely complex issue, since each time a company considers expanding productive capacity, it must consider a myriad of possibilities". Examples include the duration and type of product life cycle and the uncertainty that surrounds the return process in terms of how many products will be returned, in what condition, and when and where (Georgiadis *et al.*, 2006). Using the system dynamics approach, Vlachos *et al*. (2007) modelled the long-term behaviour of a CLSC in a remanufacturing environment with efficient capacity expansion policies for product recovery and remanufacturing. They included in their analysis certain environmental and legislative factors that influence profitability calculations. Although they also used the system dynamics approach, Georgiadis *et al.* (2006) embarked on a more ambitious analysis in terms of both objectives and structure, allowing for wider applicability of results. Specifically, the authors investigated the most suitable capacity planning policies for products with different life cycles and return flow characteristics. Rubio and Corominas (2008) studied a system with deterministic demand and adjustable manufacturing and remanufacturing capacities.

Other studies performed in the area of reverse logistics have analyzed systems with capacity constraints. Kiesmüller *et al*. (2004) and Kleber (2006) proposed a deterministic continuous-time, finite horizon model for inventory management. Using the theory of optimal control, this model determined the structure of the optimal policy for a system without constraints. The model was extended to include systems with manufacturing and remanufacturing capacity constraints and to include an update factor to calculate the total cost for a long-term horizon.

Chapter 5

System with periodic demand

In this chapter we describe a method for calculating manufacturing and storage capacity in a reverse logistics system in which demand is deterministic and product returns depend on demand. Using the method described, we will show how the returns function influences both manufacturing and storage capacities.

Section 5.1 describes the system we are going to study. Section 5.2 describes a method for calculating an optimal manufacturing policy in an environment with fixed manufacturing and storage capacities. In section 5.3, we apply this method to calculate optimal manufacturing and storage capacities, and in section 5.4, we apply it to study the impact of the product return lag period on optimal manufacturing and storage capacities. We present several examples to illustrate how are calculated the manufacturing policies and the manufaturing and storage capacities. The examples shown are solved using MAPLE. Finally in section 5.5 we present the main conclusions of the chapter.

5.1 Description of the system

We consider a company that in addition to manufacturing and selling a particular product, can also recover it when it has reached the end of its life and sell it again as new by remanufacturing or reusing it. We have defined the following time-dependent variables:

$d(t)$: product demand (product units per time unit). Demand is periodic with fundamental period T.

$u_r(t)$: returned products to be remanufactured (product units per time unit). Returns depend on demand, with $u_r(t) = \rho \cdot d(t-\tau)$, where ρ is the return rate and τ is the return lag period, i.e. the time between the moment at which the product is sold and the moment at which it is returned.

$p(t)$: production of new products (product units per time unit).

$r(t)$: remanufacture of returned products (product units per time unit).

$I(t)$: finished product inventory (product units).

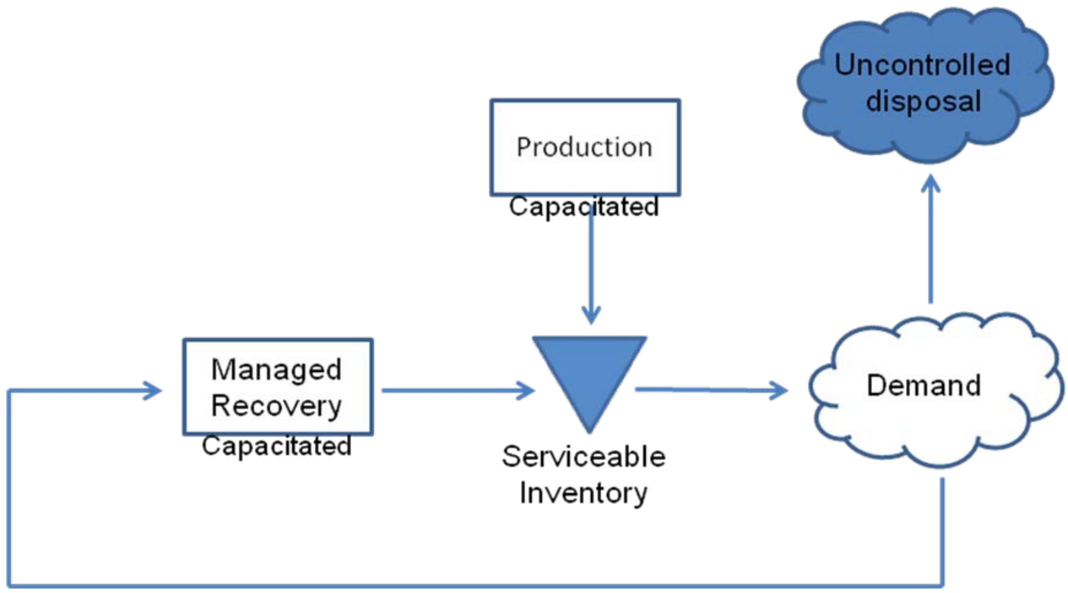


Figure 1. Schematic representation of the system

Product demand is a non-negative deterministic function with a continuous first derivative and is greater than returns $d(t) \geq \rho \cdot d(t-\tau)$. The system is in a stationary mode, i.e., all the time-dependent functions have the same value in t

and in $t+T$. To satisfy demand, the company has an inventory of finished products $I(t)$ which is fed by the manufacturing system at a rate of $p(t)$ and by the returned product remanufacturing system at a rate of $r(t)$.

The company remanufactures all returned products as soon as they are received, i.e. $r(t) = u_r(t) = \rho \cdot d(t-\tau)$ and does not accept product supply interruptions.

Cost function

The cost function variables are defined as follows:

c_p: variable manufacturing cost per one new product unit

c_r: variable remanufacturing cost per unit

h: final inventory cost (cost per time unit of having one finished product unit in stock).

$C_p(P)$: fixed manufacturing cost per one new product; this depends on manufacturing capacity P (maximum attainable manufacturing rate)

$C_r(R)$: fixed remanufacturing cost per returned product; this depends on remanufacturing capacity $R = \max\{r(t) \mid 0 \le t \le T\}$

$H(S)$: fixed storage cost per finished product; this depends on the finished product storage capacity S

It is assumed that c_p, c_r and h are constant and that $C_p(P)$, $C_r(R)$ and $H(S)$ are continuous functions with a continuous, non-negative first derivative.

The costs incurred in period T are:

$$c_T = C_p(P) + C_r(R) + H(S) + \int_0^T \left(c_p p(t) + c_r r(t) + h \cdot I(t) \right) dt$$

The following constraints apply:

$$\frac{dI}{dt} = p(t) + r(t) - d(t)$$

$$0 \le p(t) \le P$$

$$0 \le r(t) \le R$$

$$0 \le I(t) \le S$$

To determine optimal manufacturing, remanufacturing, and storage capacities, we will solve the cost function minimization c_T subject to the previous constraints.

Net demand is defined as follows:

$$\hat{d}(t) = d(t) - \rho \cdot d(t - \tau)$$

$\hat{d}(t)$ is a non-negative deterministic function that is periodic with fundamental period T and has a continuous first derivative. Integrating the first constraint between t = 0 and t = T gives

$$\int_0^T p(t)\,dt = \int_0^T \hat{d}(t)\,dt \qquad (5.1)$$

Thus, the variable manufacturing cost in period T depends only on net demand, which means that it will not influence c_T minimization. In view of the above conditions, optimal manufacturing and storage conditions can be determined by solving the following mathematical program:

$$[MIN]\, c_T = C_p(P) + H(S) + \int_0^T hI(t)dt$$

s. t.:

$$\frac{dI}{dt} = p(t) - \hat{d}(t) \qquad (5.2)$$

$$0 \le p(t) \le P$$

$$0 \le I(t) \le S$$

The problem is solved in two stages: first, we calculate the optimal manufacturing policy for given capacities *P*, *S*, and then we calculate optimal manufacturing and storage capacities for the finished product.

5.2 Optimal manufacturing policy

In this section, we will determine the optimal manufacturing policy for a company with fixed manufacturing capacity *P* and fixed storage capacity *S*. We will first describe a simplified case and then extend our findings to a production system with periodic demand.

5.2.1 Simplified case

Let us consider the case of a company with variable demand over 52 weeks and an average demand of 100 units per week (Figure 2). To satisfy demand at all times, the company must product at least 100 units per week. It will therefore need to always produce at maximum capacity if it is to have sufficient stocks to cover demand between t = 26 and t = 52, which is when demand exceeds manufacturing capacity. To calculate storage capacity, it suffices to observe that maximum inventory levels will be reached in week t = 26 (from this moment on manufacturing will not cover demand) and minimum levels in week t = 52. The difference between stock levels between these two times is the area bounded by the demand curve and the manufacturing capacity curve between t = 26 and t = 52, and its value is 827.6. Storage capacity, therefore, must be at least S = 827.6 units.

Let us now assume that manufacturing capacity is 120 units per week. As can be seen in Figure 3, maximum inventory levels are reached in t_2 = 29.4 and minimum levels in t_3 = 48.6. Similar calculations to above show that the minimum storage capacity must be S = 374.7 units. At a point in time before t_2, the company will start to produce at maximum levels in order to accumulate stock to cover demand after t_2. Let this point in time be t_1, which is calculated by matching S to the manufacturing surplus between t_1 and t_2. As can be seen in Figure 2, t_1 = 17.9. Following an optimal manufacturing plan, the company should produce as much as possible between t_1 and t_3 and match manufacturing to demand at all other times.

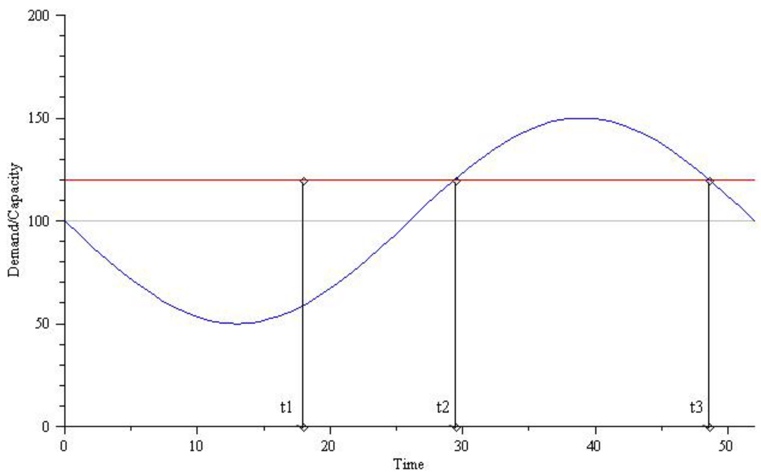


Figure 2. Demand over 52 weeks. The points shown correspond to t_1 = 17.9, t_2 = 29.4, and t_3 = 48.6. The red horizontal line represents manufacturing capacity and the grey horizontal line, average demand.

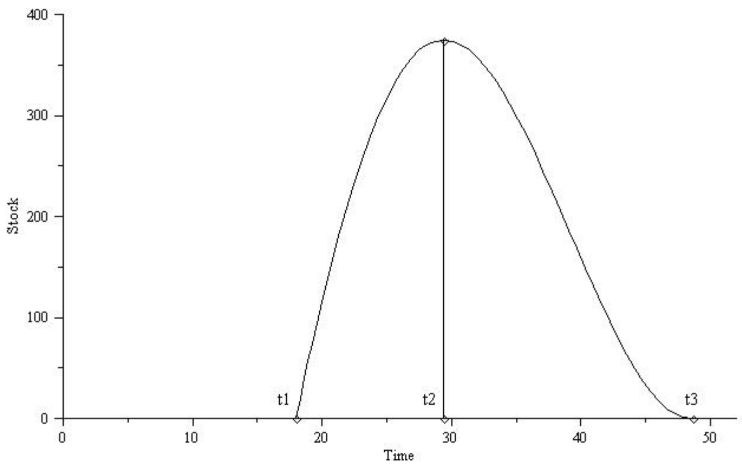


Figure 3. Inventory levels with an optimal manufacturing policy and a manufacturing capacity of 120 units per week. The points shown correspond to t_1 = 17.9, t_2 = 29.4, and t_3 = 48.6

Let us now assume that a returned product is remanufactured and sold as new and that τ is the number of weeks from the moment at which the product is sold to the moment at which it is ready to be resold. Let us also assume that 20% of all products sold are returned (ρ = 0.2) and that we want to calculate the necessary storage capacity. To do this, we will analyze the net demand curve (demands – returns), just as we did in the above scenarios, in which there were no returns. As can be seen in Figures 4(a), 4(b), and 4(c), for each τ value, there is a corresponding storage capacity requirement and a period of time between t_1 and t_3 in which manufacturing is at its maximum capacity. In the three cases studied, we established a manufacturing capacity of 96 units (20% higher than the average net demand).

Table 1 shows a summary of the results for different return lag periods.

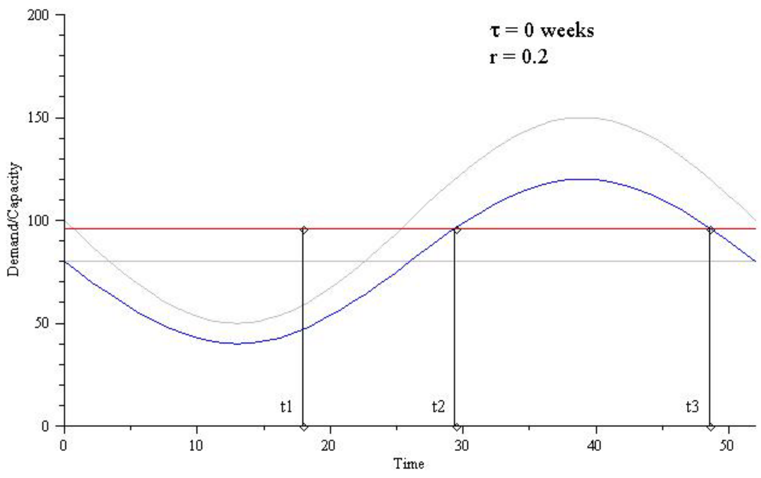
The following conclusions can be made for a periodic demand function with a peak and a trough:

- Minimum storage requirements are determined by manufacturing capacity.
- In a system with product returns, storage requirements depend on manufacturing capacity and on the length of time between when the product was sold and when it was returned. Storage capacity is twice as high for a return lag period of 26 weeks as for a period of 0 weeks.

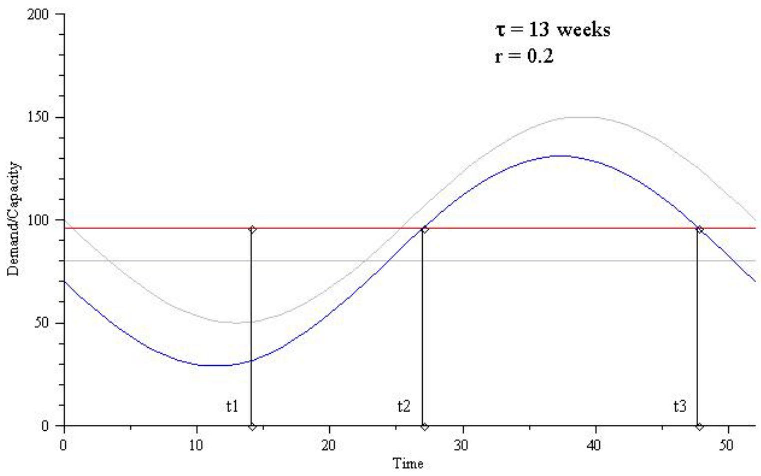
Return lag period (τ)	t_1	t_2	t_3	S
0	17.9	29.4	48.6	299.8
13	14.1	27.0	47.7	470.0
26	14.4	28.2	49.8	612.6

Table 1. Times and storage capacities required for a system with variable demand, a return rate of 20%, and varying return lag periods (τ).

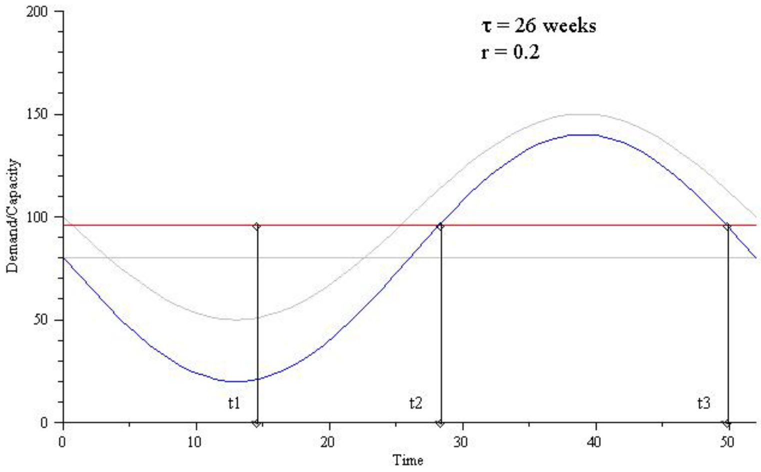
In the next section, we will extend our findings to a general case involving periodic demand.



(a) t_1 = 17.9, t_2 = 29.4, and t_3 = 48.6. S = 299.8



(b) t_1 = 14.1, t_2 = 27.0, and t_3 = 47.7. S = 470.0



(c) t_1 = 14.4, t_2 = 28.2, and t_3 = 49.8. S = 612.6

Figure 4: Net demand (demands – returns) for different return lag periods. The red horizontal line represents manufacturing capacity and the grey horizontal line, average net demand. The grey curve represents demand without returns.

5.2.2 General case

Let I_{max} and I_{min} be the maximum and minimum inventory levels, respectively.

$$I_{max} = \max_{t \in [0,T]} I(t)$$

$$I_{min} = \min_{t \in [0,T]} I(t)$$

For the problem to be solved, I_{min} must equal to 0 as if it was greater, there would be a solution with $\bar{I}(t) = I(t) - I_{min}$ for a lower cost.

Let $t_2 \in [0,T]$ be a point in time at which inventory levels are at their maximum ($I(t_2) = I_{max}$). If $I_{max} = 0$, the optimal policy would be $p(t) = \hat{d}(t)\ \forall t$, but this is only possible if $P \geq \hat{d}(t)\ \forall t$.

If $I_{max} \neq 0$, let t_1 and t_3, respectively, be the nearest points in time before and after t_2 when inventory levels are at their minimum ($I(t_1) = I(t_3) = I_{min}$). It is known that t_1, t_2, and t_3 exist because $I(t)$ is continuous and periodic.

5.2.2.1 Optimal policy in period $[t_1, t_3]$

The optimal manufacturing policy for the period $[t_1, t_3]$ is $p(t) = P$. To demonstrate this, we will show that no other policies in this period are optimal.

Suppose that we have a policy such that $p(t) < P$ in a given period $(t_4, t_5) \subset [t_1, t_3]$. Let $I(t)$ be the inventory function when this policy is applied; consider a point in time $\bar{t}_1$ such that

$$t_1 < \bar{t}_1 < t_4$$

$$I(\bar{t}_1) \le I(t) \quad t \in [\bar{t}_1, t_5]$$

$$I(\bar{t}_1) \le \int_{t_4}^{t_5} (P - p(t))dt$$

It is known that $\bar{t}_1$ exists because $I(t)$ is a continuous, strictly positive function in (t_1, t_3). Consider the point in time $\bar{t}_4$ such that

$$t_4 < \bar{t}_4 < t_5$$

$$I(\bar{t}_1) = \int_{\bar{t}_4}^{t_5} (P - p(t))dt$$

Then, the next policy

$$\bar{p}(t) = \begin{cases} \hat{d}(t) & \text{if } t \in [t_1, \bar{t}_1] \\ p(t) & \text{if } t \in (\bar{t}_1, \bar{t}_4) \\ P & \text{if } t \in [\bar{t}_4, t_5] \\ p(t) & \text{if } t \notin (t_1, t_5) \end{cases}$$

and the corresponding inventory function $\bar{I}(t)$

$$\bar{I}(t) = \begin{cases} 0 & \text{if } t \in [t_1, \bar{t}_1] \\ I(t) - I(\bar{t}_1) & \text{if } t \in (\bar{t}_1, \bar{t}_4) \\ I(t) - I(\bar{t}_1) + \int_{\bar{t}_4}^{t} (P - p(t)) & \text{if } t \in [\bar{t}_4, t_5] \\ I(t) & \text{if } t \notin (t_1, t_5) \end{cases}$$

satisfy the constraints of the problem. The cost of policy $\bar{p}(t)$ is lower than that of policy $p(t)$ since $\bar{I}(t) < I(t)$ in the period (t_1, t_5).

Thus, the optimal policy in period $[t_1, t_3]$ is $p(t) = P$, $I(t_1) = I(t_3) = 0$, and

$$I_{\max} = \int_{t_1}^{t_2} (P - \hat{d}(t))dt = \int_{t_3}^{t_2} (P - \hat{d}(t))dt \tag{5.3}$$

therefore

$$\int_{t_1}^{t_3}(P-\hat{d}(t))dt = 0 \qquad (5.4)$$

5.2.2.2 Calculating $I_{\max}$, t_1, t_2 and t_3

Let us assume that $p^*(P,t)$ is the optimal manufacturing policy when manufacturing capacity is P. Accordingly, $p(t) = p^*(P,t)$ is the solution to the mathematical program (5.2). By integrating the first constraint, we obtain:

$$I(t_b) - I(t_a) = \int_{t_a}^{t_b}(p^*(P,t)-\hat{d}(t))dt \quad \forall t_a, t_b \qquad (5.5)$$

For the second constraint, $p^*(P,t) \le P \quad \forall t$, which gives

$$I(t_a) - I(t_b) \ge \int_{t_a}^{t_b}(\hat{d}(t)-P)dt \quad \forall t_a \le t_b$$

and if we insert maximum inventory levels on each side of the inequality, we obtain

$$I_{\max} \ge \max_{t_a \le t_b}\{I(t_a)-I(t_b)\} \ge \max_{t_a \le t_b}\{\int_{t_a}^{t_b}(\hat{d}(t)-P)dt\} \qquad (5.6)$$

Given that $p^*(P,t)$ is the optimal policy, in section 5.2.2.1, we saw the existence of t_2 and t_3 satisfying (5.3); therefore

$$\max_{t_a \le t_b}\{\int_{t_a}^{t_b}(\hat{d}(t)-P)\,dt\} \ge \int_{t_2}^{t_3}(\hat{d}(t)-P)\,dt = I_{\max} \qquad (5.7)$$

By joining inequalities (5.6) and (5.7)

$$I_{\max} = \max_{t_a \le t_b}\int_{t_a}^{t_b}(\hat{d}(t)-P)\,dt \qquad (5.8)$$

Thus, $I_{\max}$ is obtained by solving the following non-linear program:

$$[MAX]\ f(x,y) = \int_{x}^{y}(\hat{d}(t)-P)\,dt \qquad (5.9)$$

s. t.:

$$x - y \leq 0 \quad (5.10)$$

Let us assume that $f(\bar{x},\bar{y})$ is the optimal value of the objective function (5.9), such that

$$I_{\max} = f(\bar{x},\bar{y})$$

To find point $(\bar{x},\bar{y})$ one can distinguish between 2 cases:

1. If $P - \hat{d}(t) \geq 0 \;\; \forall t \in [0,T]$, then $f(\bar{x},\bar{y}) = 0$ and $(\bar{x},\bar{y})$ can be any point on the boundary of the region defined by (5.10), i.e. $\bar{x} = \bar{y}$. Thus,

$$I_{\max} = 0$$

2. Otherwise, $(\bar{x},\bar{y})$ will be located inside the region defined by (5.10), i.e. $\bar{x} < \bar{y}$. Given that $(\bar{x},\bar{y})$ must be a local optimal point, it satisfies the following conditions:

$\nabla f(\bar{x},\bar{y}) = 0$. After algebra, we obtain the following conditions:

$$\hat{d}(\bar{x}) = P \qquad \hat{d}(\bar{y}) = P \quad (5.11)$$

The Hessian matrix of *f* at point $(\bar{x},\bar{y})$ is negative semi-definite

$$H(x,y) = \begin{pmatrix} \dfrac{\partial^2 f(x,y)}{\partial x^2} & \dfrac{\partial^2 f(x,y)}{\partial x \partial y} \\ \dfrac{\partial^2 f(x,y)}{\partial x \partial y} & \dfrac{\partial^2 f(x,y)}{\partial y^2} \end{pmatrix} = \begin{pmatrix} -\hat{d}'(x) & 0 \\ 0 & \hat{d}'(y) \end{pmatrix}$$

For this matrix to be negative semidefinite, the following must hold true:

$$\hat{d}'(\bar{x}) \geq 0 \qquad \hat{d}'(\bar{y}) \leq 0 \quad (5.12)$$

Thus, $(\bar{x},\bar{y})$ is a point that satisfies (5.11) and (5.12), and in addition, maximizes *f*.

To calculate the optimal value of f, it is not necessary to find all the points that satisfy (5.11) and (5.12); rather, one can restrict the search to a region of points (x,y) such that $x \in [0,T]$, $y \in [x, x+T]$, because f has the following properties:

(a) $f(x,y) = f(x+T, y+T)\ \forall x$ as $\hat{d}(t)$ is periodic of period T

(b) $f(x, y+y') = f(x,y) + f(y, y+y')\ \forall x,y,y'$

(c) $f(x, x+T) \leq 0\ \forall x$ because if this were not the case, using the previous properties, one would obtain

$$\int_0^T (\hat{d}(t) - P)\,dt = \int_x^{x+T} (\hat{d}(t) - P)\,dt > 0$$

in which case no manufacturing policies would satisfy (5.1).

When $x < y$, if $f(x+T,y+T)$ is the optimal value, then so is $f(x,y)$ (property a)

From the second property

$$f(x, y+T) = f(x,y) + f(y, y+T)$$

And from the third property, $f(x, y+T) \leq f(x,y)$. Therefore, when $x < y$, if $f(x,y+T)$ is optimal, then so is $f(x,y)$.

The following method can therefore be used to calculate $I_{\max}$**,** t_1, t_2 and t_3;

(a) Determine the sets U and V defined as:

$$U = \{x \in [0,T] \mid \hat{d}(x) = P,\ \hat{d}'(x) \geq 0\}$$

$$V = \{y \in [0,2T] \mid \hat{d}(y) = P,\ \hat{d}'(y) \leq 0\}$$

(b) Calculate $I_{\max}$ using the following expression:

$$I_{\max} = \max\{\int_x^y (\hat{d}(t) - P)\,dt \mid x \in U, y \in V, x < y < x+T\} \quad (5.13)$$

(c) t_2 and t_3, respectively, are the elements of U and V that satisfy (5.13)

(d) The point in time t_1 is the closest point to t_2 that satisfies both (5.4) and $t_1 \leq t_2$

5.2.2.3 Optimal policy in period [t_3,T+t_1]

Because the system is periodic, if the optimal policy for the period [t_1,T+t_1] is known, it is a simple matter to calculate it for [0,T]. The advantage of using the period [t_1,T+t_1] is that it only remains to calculate the optimal policy for the subperiod [t_3,T+t_1] that satisfies $I(t_3) = I(T+t_1) = 0$.

Let $t_3' \in [t_3, T+t_1]$ be the closest point in time to $T+t_1$ such that $\hat{d}(t_3) = P$, i.e.

$$t_3' = \max\{t \in V \cap [t_3, T+t_1]\}$$

The optimal manufacturing policy for the period $[t_3', T+t_1]$ is $p(t) = \hat{d}(t)$. To demonstrate this, it suffices to observe that $I(t_3') = 0$, because in this case using this policy would give $I(t) = 0$ for the period in question. Let us assume that we are using an optimal policy $p(t)$ and that $I(t_3') > 0$. Let $t_3'' \leq t_3'$ be the closest point in time to t_3' such that $I(t_3'') = 0$; if we integrate the first constraint of the mathematical programme (5.2) we have:

$$I(t_3') = -\int_{t_3''}^{t_3'} (\hat{d}(t) - p(t))dt$$

$$I(t_3') = \int_{t_3'}^{T+t_1} (\hat{d}(t) - p(t))dt$$

Because $I(t_3') > 0$, there exists a period $[t_4, t_5] \subset [t_3'', t_3']$ in which $p(t) > \hat{d}(t)$ and an interval $[t_4', t_5'] \subset [t_3', T+t_1]$ in which $\hat{d}(t) > p(t)$. The following policy is defined:

$$\overline{p}(t) = \begin{cases} p(t) & if \;\; t \in [t_3, t_4) \\ p(t) - \varepsilon_1 & if \;\; t \in [t_4, t_5] \\ p(t) & if \;\; t \in (t_5, t_4') \\ p(t) + \varepsilon_2 & if \;\; t \in [t_4', t_5'] \\ p(t) & if \;\; t \in (t_5', T+t_1] \end{cases}$$

Where ε_1 and ε_2 are chosen such that $\overline{p}(t)$ satisfies the constraints of the programme (5.2) and $(t_5 - t_4)\varepsilon_1 = (t'_5 - t'_4)\varepsilon_2$. The resulting inventory function $\bar{I}(t)$ is

$$\bar{I}(t) = \begin{cases} I(t) & \text{if } t \in [t_3, t_4) \\ I(t) - (t - t_4)\varepsilon_1 & \text{if } t \in [t_4, t_5) \\ I(t) - (t - t_4)\varepsilon_1 & \text{if } t \in (t_5, t'_4) \\ I(t) - (t_5 - t_4)\varepsilon_1 + (t - t'_4)\varepsilon_2 & \text{if } t \in [t'_4, t'_5] \\ I(t) & \text{if } t \in (t'_5, T + t_1] \end{cases}$$

The cost of policy $\overline{p}(t)$ is lower than that of policy $p(t)$, demonstrating that policy $p(t)$ cannot be optimal, which contradicts the initial hypothesis. Thus, $I(t'_3) = 0$ and the optimal policy in $[t'_3, T + t_1]$ is $p(t) = \hat{d}(t)$.

Next we calculate the optimal policy for the period $[t_3, t'_3]$. Let $t'_1 \in [t_3, t'_3]$ be the closest point in time to t'_3 such that

$$\int_{t'_1}^{t'_3} (P - \hat{d}(t))dt = 0 \qquad (5.14)$$

Then the optimal policy in $[t'_1, t'_3]$ is $p(t) = P$ because we can use the same argument as in section 5.2.2.1.

The optimal policy for the period $[t_3, t'_1]$ is calculated recursively using the reasoning presented in this section but replacing $T + t_1$ with t'_1.

5.3 Optimal manufacturing and storage capacities

5.3.1 Storage capacity S

Given a manufacturing capacity P, the optimal storage capacity S is the same as $I_{\max}$ (if it were higher, there would be surplus capacity and if it were lower, there would be supply interruptions).

Note that I_{max}, S, t_2, and t_3 depend on P as they all depend on the value of the elements of U and V and these depend on P.

5.3.2 Example

Consider this case with the following demand:

$$d(t) = 100\left[1 - 0.3\sin\left(\frac{\pi}{26}(t+4)\right) + 0.25\sin\left(\frac{\pi}{13}(t-13.3)\right)\right]$$

We want to determine storage capacity S with a return rate of $\rho = 0.2$, a return lag period of $\tau = 26$ weeks, and a manufacturing capacity of $P = 96$ units per week. See the example in Figure 5.

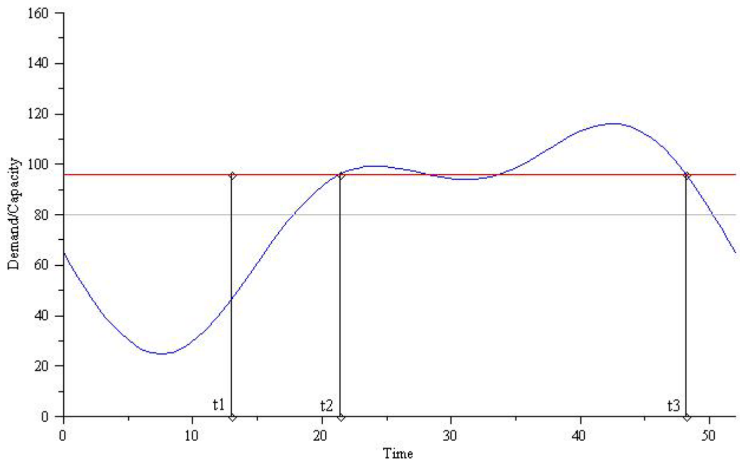


Figure 5: Net demand (demand – returns) for the example with a return lag period of 26 weeks ($\tau = 26$). The points shown correspond to $t_1 = 13.0$, $t_2 = 21.4$, and $t_3 = 48.2$. $P = 96$.

Then:

$U = \{21,4 \quad 33,7\}$ $V = \{28,0 \quad 48,2 \quad 80,0 \quad 100,19\}$

$S = 180.4$ units

$t_1 = 13.0$ $t_2 = 21.4$ $t_3 = 48.2$

5.3.3 Calculating optimal manufacturing capacity *P*

P_{min} and P_{max} are defined:

$$P_{min} = \frac{1}{T}\int_0^T \hat{d}(t)dt$$

$$P_{max} = \max_{t\in[0,T]} \hat{d}(t)$$

There are no manufacturing policies that satisfy (5.1) for *P* values of less than P_{min}, which means that the mathematical programme does not have a solution.

For $P \geq P_{max}$, $I_{max} = 0$. Because $C_p(P)$ is an increasing function, the minimum costs for $P \geq P_{max}$ are achieved when $P = P_{max}$.

Optimal manufacturing capacity, therefore, is found in the interval [P_{min}, P_{max}]. Given a value of $P \in [P_{min}, P_{max}]$, the cost c_T incurred in period [0,*T*] is calculated by following the steps below:

1. Calculate the optimal manufacturing policy $p^*(P,t)$ following the procedure described in section 5.2
2. Calculate *S* as described in section 5.3.1
3. Calculate $I(t)$ using (5.5)

$$I(t) = \int_{t_1}^{t} (p^*(P,t') - \hat{d}(t'))\, dt' \qquad (5.15)$$

4. Calculate the cost incurred in the period $[0,T]$

$$c_T = C_p(P) + H(S(P)) + h\int_0^T I(t)\, dt \qquad (5.16)$$

The expression $S(P)$ represents the dependent relationship between *S* and *P* described in section 5.3.1, and $p^*(P,t)$ is the optimal policy when manufacturing capacity is *P* described in section 5.2.

Optimal capacity is calculated numerically using the steps described above exploring $P \in [P_{\min}, P_{\max}]$ values.

5.3.4 *Example*

Let us consider another case with the same demand, the same return rate (ρ = 0.2), and the same return lag period (τ = 26 weeks) as above but with the following cost functions:

$$C_p(P) = 250(P - 80) + 14000$$

$$H(S) = 7\,S$$

Using the procedure described in section 5.2.2, we calculated the optimal policy for 25 P values in the interval $[P_{\min}, P_{\max}]$, where $P_{\min} = 80$ and $P_{\max} = 116.06$. Figure 6 shows the values for t_1, t_2 and t_3. Note that the dependency between t_2 and manufacturing capacity P is not continuous; in general, dependency between t_1, t_2 and t_3 in P is not continuous.

For each P value, we calculated the optimal storage capacity using the method described in 5.3.1. Figure 7 shows the dependency between optimal storage capacity and manufacturing capacity, i.e. $S(P)$.

For each P value, we calculated the total cost incurred in period T using (5.16). Figure 8 shows the dependency between cost and manufacturing capacity when optimal manufacturing and storage policies are used.

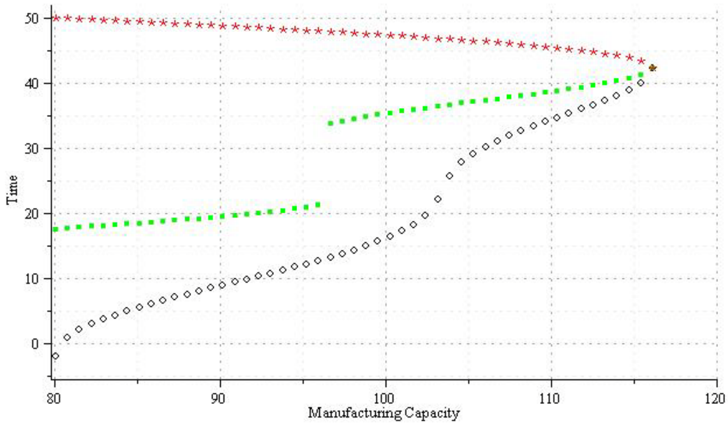


Figure 6: Dependency between manufacturing capacity and t_1 (black circles), t_2 (solid green circles), and t_3 (red asterisks).

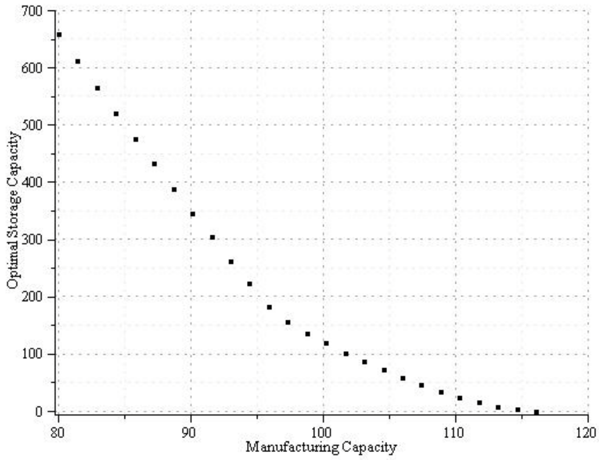


Figure 7. Optimal storage capacity with respect to manufacturing capacity.

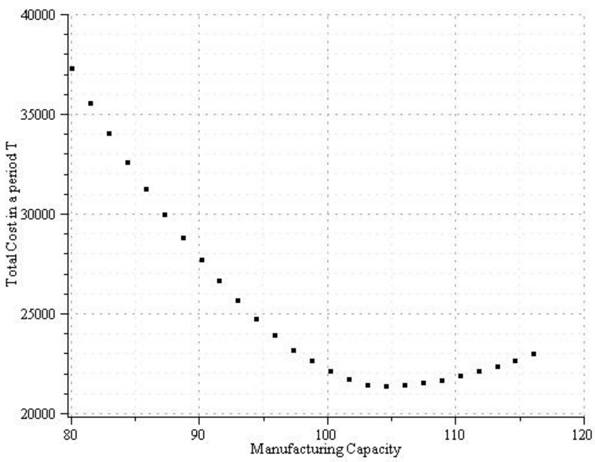


Figure 8. Total cost in a given period T with respect to manufacturing capacity using the optimal policy.

Finally, we numerically calculated the optimal manufacturing capacity (minimizing the total cost in period T) as P^* = 104.42. Using this capacity, we obtained the following values:

t_1 = 27.79 t_2 = 37.05 t_3 = 46.84

$S(P^*)$ = 73.29 $C_p(P^*)$ = 20105 $H(S(P^*))$ = 513 $c_T(P^*)$ = 21394

5.4 Dependency between optimal capacities and return period lag

In section 5.2.1, for a system with a fixed manufacturing capacity, we obtained different storage capacities for different return lag periods, demonstrating that storage capacity is dependent on this period.

This section presents the results of a study of the dependent relationship between optimal manufacturing and storage capacities and return lag periods in a system such as that described in section 5.3.4

We first calculated optimal manufacturing and storage capacities for a system without returns and obtained the following values:

P^* = 119.20 t_1 = 20.31 t_2 = 37.79 t_3 = 48.35

$S(P^*)$ = 118.61 $C_p(P^*)$ = 23800 $H(S(P^*))$ = 830 $c_T(P^*)$ = 25951

We then calculated optimal capacities and associated costs for a system with a return lag period that varied from 0 to 52 weeks. The results are shown in Figures 9, 10, and 11.

As can be seen in Figure 9, there are considerable differences (as high as 100%) between minimum and maximum levels. It can also be seen that optimal storage capacity may be higher in a system with returns than in one without.

Manufacturing capacity variations were minimum (close to 10%), as can be seen in Figure 10.

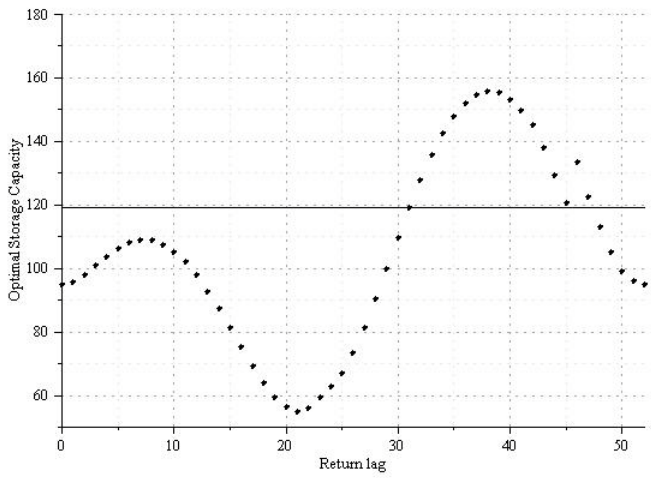


Figure 9. Dependency between optimal storage capacity *S* and return lag period for the scenario described in section 5.3.4. The points correspond to optimal storage capacities for the system with returns, and the continuous line to the optimal storage capacity for the system without returns.

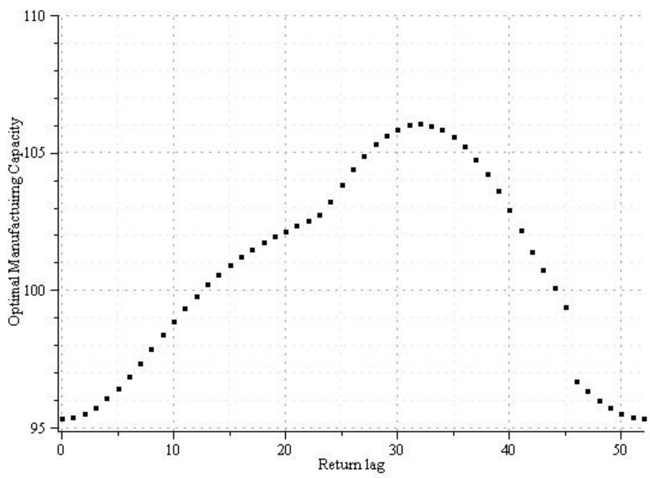


Figure 10. Dependency between optimal manufacturing capacity *P* and return lag period τ for the scenario described in section 5.3.4.

Finally, as can be seen in Figure 11, optimal costs varied considerably with lag period variations. It is worth noting that costs might ultimately be higher for the system with returns than that without (25951 in our case) once remanufacturing costs had been added.

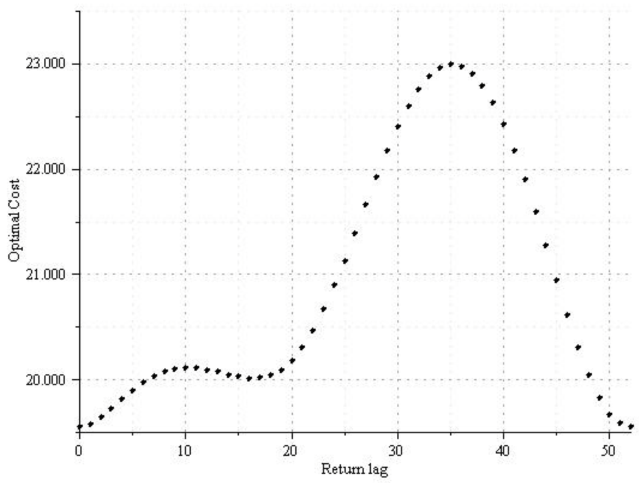


Figure 11. Dependency between optimal cost and return lag period τ for the scenario described in section 5.3.4.

5.5 Conclusions

We have presented a method for calculating optimal production and storage capacities in a reverse logistics system with periodic demand in which all returned products are manufactured.

Key aspects of method:

- By using fixed production and storage capacities, the method can be used to calculate optimal production policies.
- It can also be used to calculate optimal production and storage capacities.
- It is easy to apply and, as was seen in section 5.4, can be used to study the relationship between optimal capacities and product return lags.

Other important conclusions are those mentioned in section 5; the most relevant of these is that returns have a very strong impact on storage needs and overall profitability.

The method could also be used to study:

- The feasibility of implementing a reverse logistics system

- The advisability of investing in policies designed to modify product return lag periods
- The influence of product return rates ρ on production and storage capacities

Chapter 6

System with stochastic demand

In this chapter, we study a system with stochastic demand and returns in order to calculate optimal manufacturing and storage capacities. The model presented can be used to study the behaviour of optimal manufacturing and storage capacities when there are variations in manufacturing costs and return probability.

In section 6.1 we describe the system we are going to study and in section 6.2 we describe the method used to calculate the optimal manufacturing and remanufacturing policy under the assumption that manufacturing and storage capacities are known. We also explain how to calculate optimal capacities. In section 6.3 we present three numerical examples that have been solved using MATLAB and CPLEX: one shows how to calculate the optimal policy and the other two analyze how capacities change with variations in return probability and remanufacturing costs. Finally in section 6.4 we present the main conclusions of the chapter.

6.1 Description of the system

The system consists of a company that produces, sells, and recovers a product for which it has manufacturing, remanufacturing, and finished product storage systems. The remanufacturing system has sufficient capacity to remanufacture all the products returned.

Assumptions of model

- Time is discrete and the time horizon is infinite.
- Demand is random with a known probability distribution that is independent of the period; values are integers, with a maximum value of D.
- The remanufactured product is indistinguishable from the newly manufactured product.
- The useful life of the product ends between periods T_1 and T_2 after the product has been sold; it is a random variable and the probability distribution is independent of the sales period. π_τ is the probability that the useful life of a product has a duration of τ periods ($\tau = T_1,...T_2$).
- ρ is the probability of an end-of-life product being returned. Therefore, $\rho \cdot \pi_\tau$ is the probability that a unit sold in period t will be returned in period $t+\tau$.
- Demand that cannot be satisfied with manufactured or remanufactured products is met through an external supply channel with capacity $(T_2 - T_1 + 1)\cdot D$.
- Products that are manufactured and remanufactured in a given period are available for sale in the same period.

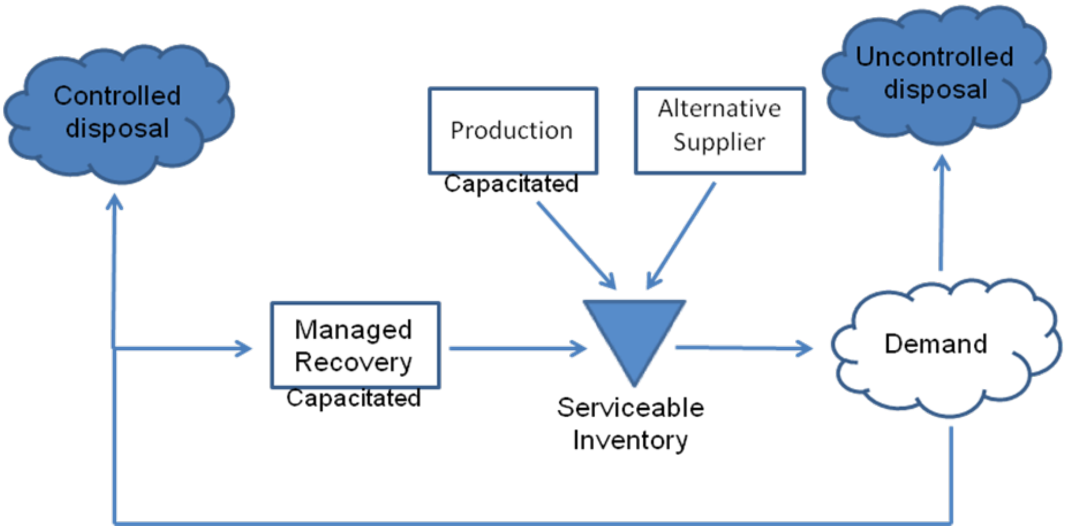


Figure 1. Schematic representation of system.

The costs for the company are as follows:

- The manufacturing system has a cost per period $C_p(P)$ (dependent on manufacturing capacity P) and a cost c_p per unit produced.
- The storage system has a cost per period $C_s(S)$, which is dependent on storage capacity S.
- e: unit cost of disposing of a returned product
- f: manufacturing order cost
- c_r: remanufacturing unit cost
- h: holding cost
- c_{ec}: external channel unit cost

It is assumed that functions $C_p(P)$ and $C_s(S)$ are continuous increasing functions.

The following variables are defined:

s_t: stock available at the end of period t

u_t: units manufactured in period t

v_t: units remanufactured in period t

d_t: product demand in period t; this is a random integer variable with $p_d = p(d_t = d)$, $(d = 0,...,D)$

r_t: units returned in period t; this is a random integer variable with $q_r = p(r_t = r)$, $(r = 0,\ldots,(T_2\text{-}T_1+1)\cdot D$.

The chronological order of events in period t is as follows:

1. Stock levels available at the end of the preceding period (s_{t-1}) are analyzed.

2. A decision is taken on how many products to manufacture (u_t), between 0 and $\min(P, S\text{-}s_{t-1})$.

3. Demand is satisfied with existing stock, newly manufactured products, and external channel supplies.

4. Returned products are remanufactured in this period as follows. If there are sufficient returns, these products are remanufactured until the warehouse is full and all other returns are disposed of. Otherwise, all returned products are remanufactured.

The quantity of products purchased from the external channel is $\max(0, d_t - s_{t-1} - u_t)$

The quantity of products to remanufacture is $v_t = \min(S - s'_t, r_t)$ where $s'_t = \max(0, s_{t-1} + u_t - d_t)$ is the stock level after demand has been met.

The stock at the end of the period will be $s_t = s'_t + v_t$. Therefore, s_t is a random variable that depends on previous stock levels s_{t-1}, random variables d_t and r_t, and the decision u_t. Note that the s_t variables have values of between 0 and S.

The cost incurred in period t is:

$$c_t = C_p(P) + C_s(S) + c_p \cdot u_t + c_r \cdot v_t + e \cdot \max(0, r_t - v_t) + h \cdot s_t +$$

$$+ c_{ec} \cdot \max(0, d_t - s_{t-1} - u_t) + f \cdot \max(0, \min(1, u_t)) \qquad (6.1)$$

Therefore, c_t is a random variable that depends on random variables s_{t-1}, s_t, d_t and r_t, on decision u_t, and on remanufacturing capacity P and storage capacity S.

6.2 Calculating optimal manufacturing and storage capacities

We want to calculate manufacturing capacities P and storage capacities S that minimize the expected cost in a period:

$$\min_{P,S} \min_{u_{P,S}} \left\{ E(c_t) \middle| u_{P,S}(s_{t-1}) \leq P \right\} \qquad (6.2)$$

The problem is resolved by calculating the P and S values that minimize the expected cost $E(c_t)$ when the optimal policy $u_{P,S}$ is used. To calculate the optimal manufacturing policy for fixed P and S values, the following problem must be resolved:

$$\min_{u_{P,S}} \left\{ E(c_t) \middle| u_{P,S}(s_{t-1}) \leq P \right\} \qquad (6.3)$$

Calculating the expected cost value will be more or less complicated depending on the behaviour of returns. If returns form a succession of independent random variables that are also independent of demand, the problem becomes considerably simpler. It is not surprising thus that the assumption that returns are independent of demand is common in studies designed to calculate optimal inventory policies in systems with reverse logistics and stochastic returns (e.g. Fleischmann *et al.*, 2002; van der Laan, 2003; and Fleischmann and Kuick, 2003). Fleischmann *et al*. (2002) argued the following: “Our assumption of independence of demand and returns is motivated by the fact that items are difficult to monitor once issued to the market. While at a first glance it may seem more appealing to model returns as a function of previous demand, estimating this correlation often appears to be difficult in practice. In many applications the issuing date of a given returned item is not known. Hence, the sojourn time in the market can only estimated on aggregated basis. Moreover, both the sojourn time and its variance may be rather large compared to demand inter-occurrence epochs, reducing observable correlation further. Hence, rejecting the assumption of independence of demand and returns appears to be hard in many cases". In this study, we therefore decided to resolve the problem of determining optimal capacities by first assuming that returns form a succession of independent random variables with a known probability distribution and then calculating the probability distribution of returns according to the useful life of the products and the probability of return to test the influence of the succession independence assumption (r_t) on the result.

Let us assume that returns (r_t) form a succession of random independent variables with probability distribution $q_r = p(r_t = r)$, $r = 0,\dots,(T_2 - T_1 + 1)\cdot D$. By fixing P and S, we can see that the problem of calculating the optimal policy is a Markov decision problem with an infinite horizon and remuneration, no actualization, and an optimization criterion consisting of minimizing the expected remuneration value.

The state in period t is determined by s_{t-1}, the state space is {0,1,…,S}, the actions in each period are defined by the manufacturing quantity u_t, the set of actions is {0,1,…,min(P,S)}, and the remuneration is related to the cost incurred in a given period, and is equal to $-(c_t - C_p(P) - C_s(S))$. The negative sign converts the cost function into a remuneration function; we subtract capacity costs from the cost per period to obtain a simpler expression of the remuneration function.

To define the Markov decision problem, we need to determine $p_{ij}(u)$, the probability of transition between states i and j when decision u is taken. In other words $p_{ij}(u) = p(s_t = j \mid s_{t-1} = i, u_t = u)$ with $0 \le u \le \min(P, S - i)$. In the previous section, we saw that the variable state s_t was dependent on s_{t-1} and the random variables d_t and r_t. This dependence can be expressed as:

$$s_t = \max(0, s_{t-1} + u_t - d_t) + \min(S - \max(0, s_{t-1} + u_t - d_t), r_t) \qquad (6.4)$$

Therefore, the probability of transition between states is expressed by:

$$p_{ij}(u) = \sum_{(d,r)\in\Omega_{i+u,j}} p(d_t = d)p(r_t = r) \qquad (6.5)$$

Where the domains $\Omega_{i+u,j}$ contain the values (d,r) such that starting from state i and taking decision u, we progress to state j. In other words, if we make $k = i+u$, we define the domains as follows:

$$\Omega_{k,j} = \{(d,r)\in[0,D]\times[0,R] \mid j = \max(0,k-d) + \min(S-\max(0,k-d),r)\}$$

For $0 \le k \le P+S$ and $0 \le j \le S$. To calculate the domains $\Omega_{i+u,j}$, we distinguish between 3 cases:

Case 1: $j < S$ and $j \le i+u$

$$\Omega_{i+u,j} = \{(d,j) \mid i+u \le d \le D\} \cup \{(r-j+i+u,r) \mid 0 \le r \le \min(j-1, D+j-(i+u))\}$$

Case 2: $j < S$ and $j > i+u$

$$\Omega_{i+u,j} = \{(d,j) \mid i+u \le d \le D\} \cup \{(r-j+i+u,r) \mid j-i-u \le r \le \min(j-1, D+j-(i+u))\}$$

Case 3: $j = S$

$$\Omega_{i+u,j} = \{(d,r) \mid i+u \le d \le D, S \le r \le M\} \cup \{(d,r) \mid 0 \le d \le \min(D, i+u-1), S+d-i-u \le r \le M\}$$

Where $M = (T_2-T_1+1)\cdot D$. Hence

$$p_{ij}(u) = \begin{cases} \sum_{d=i+u}^{D} p(d_t = d)\cdot p(r_t = j) + \sum_{r=0}^{\min(j-1, D+j-(i+u))} p(d_t = i+u-j+r)\cdot p(r_t = r) & j < S \quad j \le i+u \le S \\ \sum_{d=i+u}^{D} p(d_t = d)\cdot p(r_t = j) + \sum_{r=j-(i+u)}^{\min(j-1, D+j-(i+u))} p(d_t = i+u-j+r)\cdot p(r_t = r) & j < S \quad j > i+u \\ \sum_{d=i+u}^{D}\sum_{r=S}^{M} p(d_t = d)\cdot p(r_t = r) + \sum_{k=S-(i+u)}^{\min(S-1, D+S-(i+u))} p(d_t = i+u-S+k)\cdot p(r_t \ge k) & j = S \quad i+u \le S \end{cases}$$

In other words,

$$p_{ij}(u)=\begin{cases}\sum_{d=i+u}^{D} p_d q_j + \sum_{r=0}^{\min(j-1,D+j-(i+u))} p_{i+u-j+r}\cdot q_r & j<S \quad j\le i+u\le S\\ \sum_{d=i+u}^{D} p_d q_j + \sum_{r=j-(i+u)}^{\min(j-1,D+j-(i+u))} p_{i+u-j+r}\cdot q_r & j<S \quad j>i+u\\ \sum_{d=i+u}^{D} p_d \sum_{r=S}^{M} q_r + \sum_{k=S-(i+u)}^{\min(S-1,D+S-(i+u))} p_{i+u-S+k}\cdot \sum_{r=k}^{M} q_r & j=S \quad i+u\le S\end{cases}$$

Note that $p_{ij}(u)$ is equal to $p\left((d,r)\in\Omega_{i+u,j}\right)$, the probability that $(d,r)\in\Omega_{i+u,j}$.

6.2.1 Calculating the optimal manufacturing policy

State transition costs will be the expected value of the costs of each of the possible paths towards the transitions.

Given manufacturing capacities P and storage capacities S, we want to calculate $c_{ij}(u)$: the expected cost of the transition from state i to j when decision u is taken, i.e. $c_{ij}(u) = E(c \mid i,j,u)$ where $c = c_t - C_p(P) - C_s(S)$. Defining

$$c(i,j,u,d,r) = c_p\cdot u + c_r\cdot\min(S - \max(0, i+u-d), r) + e\cdot\max(0, r - v_t) + h\cdot j + c_{ec}\cdot\max(0, d-i-u) + f\cdot\max(0, \min(1, u)) \qquad (6.6)$$

$$c_{ij}(u)=\sum_{(d,r)\in\Omega_{i+u,j}} c(i,j,u,d,r)p\left(d,r \mid (d,r)\in\Omega_{i+u,j}\right)=\sum_{(d,r)\in\Omega_{i+u,j}} c(i,j,u,d,r)\frac{p(d_t=d)p(r_t=r)}{p\left((d,r)\in\Omega_{i+u,j}\right)}$$

Hence,

$$c_{ij}(u)\cdot p_{ij}(u)=\sum_{(d,r)\in\Omega_{i+u,j}} c(i,j,u,d,r)p(d_t=d)p(r_t=r) \qquad (6.7)$$

Let us distinguish between 3 cases:

Case 1a: $j < S$ and $j \le i+u$:

$$c_{ij}(u)\cdot p_{ij}(u)=\sum_{r=0}^{\min(j-1,D+j-(i+u))}\left[c_p\cdot u + c_r\cdot r + h\cdot j + f\cdot\max\left(0,\min(1,u)\right)\right]\cdot p_{i+u-j+r}\cdot q_r +$$
$$+\sum_{d=i+u}^{D}\left[c_p\cdot u + c_r\cdot j + h\cdot j + c_{ec}\cdot(d-i-u) + f\cdot\max\left(0,\min(1,u)\right)\right]\cdot p_d\cdot q_j$$

Case 1b: $j < S$ and $j > i + u$:

$$c_{ij}(u) \cdot p_{ij}(u) = \sum_{r=j-(i+u)}^{\min(j-1, D+j-(i+u))} \left[c_p \cdot u + c_r \cdot r + h \cdot j + f \cdot \max\left(0, \min(1, u)\right) \right] \cdot p_{i+u-j+r} \cdot q_r +$$

$$+ \sum_{d=i+u}^{D} \left[c_p \cdot u + c_r \cdot j + h \cdot j + c_{ec} \cdot (d - i - u) + f \cdot \max\left(0, \min(1, u)\right) \right] \cdot p_d \cdot q_j$$

Case 2: $j = S$

$$c_{ij}(u) \cdot p_{ij}(u) = \sum_{k=S-(i+u)}^{\min(S-1, D+S-(i+u))} \sum_{r=k}^{M} \left[c_p \cdot u + c_r \cdot (k) + e \cdot (r - k) + h \cdot S + f \cdot \max\left(0, \min(1, u)\right) \right] \cdot p_{i+u-S+k} \cdot q_r$$

$$+ \sum_{d=i+u}^{D} \sum_{r=S}^{M} \left[c_p \cdot u + c_r \cdot S + e \cdot (r - S) + h \cdot S + c_{ec} \cdot (d - i - u) + f \cdot \max\left(0, \min(1, u)\right) \right] \cdot p_d \cdot q_r$$

For each manufacturing capacity P and storage capacity S, the optimal policy is calculated by resolving the following linear programme (Puterman, 1994, p. 391 and subsequent pages):

$$[\text{MIN}] \sum_{i=0}^{S} \sum_{u=0}^{P_i} c_i(u) \cdot y_{i,u}$$

s.t.:

$$\sum_{u=0}^{P_i} y_{i,u} - \sum_{j=0}^{S} \sum_{u=0}^{P_j} p_{ji}(u) \cdot y_{j,u} = 0 \qquad i = 0, \ldots, S$$

$$\sum_{i=0}^{S} \sum_{u=0}^{P_i} y_{i,u} = 1$$

$$y_{i,u} \geq 0 \qquad i = 0, \ldots, S, \; u = 0, \ldots, P_i$$

where $P_i = \min(P, S-i)$, $y_{i,u}$ are the variables, and $c_i(u)$ is:

$$c_i(u) = \sum_{j=0}^{S} p_{ij}(u) \cdot c_{ij}(u)$$

If $y^*_{i,u}$ is a basic optimal solution for the previous linear program, the optimal policy in state i will be to produce u if $y^*_{i,u} > 0$ and

$$\sum_{i=0}^{S}\sum_{u=0}^{P_i} c_i(u)\cdot y^*_{i,u}$$

is the expected cost of applying the above optimal policy. Therefore, the expected cost incurred in a period when the optimal policy is applied is

$$C_O(P,S) = C_p(P) + C_s(S) + \sum_{i=0}^{S}\sum_{u=0}^{U_i} c_i(u)\cdot y^*_{i,u} \tag{6.8}$$

6.2.2 Calculating optimal capacities

In the previous section, we described how to calculate the optimal policy and obtain the expected cost when this policy is applied with fixed manufacturing and storage capacities P and S. We also defined the function $C_O(P,S)$ which at each (P,S) point takes the expected cost value on applying the optimal policy when manufacturing capacity is P and storage capacity is S. The optimal capacities in this case will be those that minimize the function $C_O(P,S)$.

Given that $u_t \leq S$ and that $C_p(P)$ is an increasing function, the optimal value is achieved for a value of $P \leq S$. Note that S^*, the optimal storage capacity, is limited. From (6.1), we know that

$$C_s(S) \leq C_o(P,S) \quad \forall P,S$$

In particular, for optimal manufacturing and storage capacities (P^* and S^*, respectively),

$$C_s(S^*) \leq C_o(P^*,S^*)$$

We calculated $C_O(P_0,S_0)$ for some (P_0,S_0) and calculated S_{MAX} such that $C_s(S_{MAX}) = C_O(P_0,S_0)$. S_{MAX} exists as $C_s(S)$ will reach the value of $C_O(P_0,S_0)$. If not, $C_s(S)$ would be a limited function but storage capacity costs cannot be limited if capacity is increased indefinitely.

This gives

$$C_s(S^*) \leq C_o(P^*,S^*) \leq C_o(P_0,S_0) = C_s(S_{MAX})$$

And therefore $S^* \leq S_{MAX}$ as $C_s(S)$ is an increasing function.

6.2.3 Probability distribution of returns

In this section, we are going to calculate the probability distribution of returns based on the probability distribution of the useful life of the product and the probability of return. To do this, we defined the random variables $Z_{t,\tau}$: units returned in period t sold in period t-τ (τ = T_1,...T_2) and defined the following probability distributions related to these random variables:

- Distribution of probability of $Z_{t,\tau}$: $\eta_{\tau k}$ = p($Z_{t,\tau}$= k) k = 0,…,D.
- Distributions of probability of $Z_{t,\tau}$ conditioned by d_t = i (i = 0,...,D): given i we define $\upsilon_{\tau ik}$ = p($Z_{t,\tau}$= k | d_t = i) (τ = T_1,...T_2, k = 0,…,D).

We first calculated $\upsilon_{\tau ik}$, the conditioned probability distributions. We know that a product's useful life has a random duration of between T_1 and T_2 and once this has come to an end, the product has a probability ρ of being returned. Therefore, if the sales in a period are i, the probability distribution of returns they generate is:

$$\upsilon_{\tau i k} = p(Z_{t,\tau} = k \mid d_t = i) = \begin{cases} \binom{i}{k}(\rho\cdot\pi_\tau)^k \left(1\text{-}\ \rho\cdot\pi_\tau\right)^{i\text{-}k} & k \leq i \\ 0 & k > i \end{cases} \quad (6.9)$$

For i = 0,...,D and τ = T_1,...T_2. We have the values:

$$\eta_{\tau k} = p(Z_{t,\tau} = k) = \sum_{i=k}^{D} p_i \cdot \upsilon_{\tau i k} \quad \text{for } k = 0,\dots,D \text{ y } \tau = T_1,\dots T_2. \quad (6.10)$$

We are now able to calculate the probability distribution of returns as $r_t = \sum_{\tau=T_1}^{T_2} Z_{t,\tau}$, where the distribution is obtained from the probabilities of $Z_{t,\tau}$, as $p\left(r_t = r\right) = p\left(Z_{t,T_1} + \cdots + Z_{t,T_2} = r\right)$. Hence,

$$q_r = p(r_t = r) = \sum_{\sum_{\tau=1}^{T_2-T_1+1} k_\tau = r} \left(\prod_{\tau=1}^{T_2-T_1+1} \eta_{T_1+\tau-1,k_\tau} \right) \text{ for } r = 0,\dots,(T_2-T_1+1)\cdot D \qquad (6.11)$$

This expression can be calculated through the convolution of the Z_τ probability distributions, with the following recurrence relationship:

$$f(r,T) = \sum_{k=\max(0,r-(T-1)\cdot D)}^{\min(r,D)} \eta_{T_1+T-1,k} \cdot f(r-k,T-1) \text{ for } T>1 \text{ and } r = 0,\dots,T\cdot D:$$

This allows us to calculate $q_r = f(r, T_2\text{-}T_1+1)$ from $f(k,1) = \eta_{T_1,k}$ $k = 0,\dots,D$.

The random variables r_t (t = 1,2,…) form a succession of random variables that are dependent on demand and on each other, considerably complicating the calculation of the probability of transitions between states. We will see in the numerical example 2 how the assumption of return independence affects the solution obtained.

6.3 Numerical examples

6.3.1 Example 1

We wish to determine the manufacturing and storage capacities for a system with the following parameters and values:

D = 5; p_0=0.1, p_1=0.15, p_2=0.25, p_3=0.25, p_4=0.15, p_5=0.1

ρ = 0.3; T_1 = 1, T_2 = 3; π_1 = 0.25, π_2 = 0.50, π_3 = 0.25

c_p = 10, e = 1, f = 0.5, c_r = 5, h = 1, c_{ec} = 30, $C_p(P) = 10\cdot\sqrt{P}$, $C_s(S) = 2\cdot\sqrt{S}$

As explained in the preceding sections, the optimal policy is calculated for each manufacturing and storage capacity value (P and S, respectively).

Figure 2 shows the probability distribution of returns within a given period. Table 1 shows the optimal policies and associated cost for each manufacturing capacity value for S = 6. Note that when the system is in state $i = 6$, manufacturing is no longer taking place as the maximum quantity that can be produced in this case, P_6, is 0.

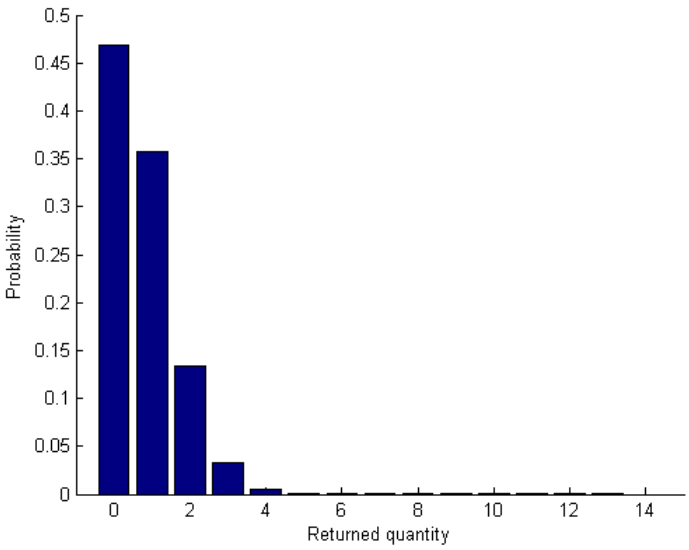


Figure 2. Probability distribution of returned products in a given period using the demand distribution and $\rho = 0.3$.

Manufacturing capacity	State							Costs
	0	1	2	3	4	5	6	
0	0	0	0	0	0	0	0	62.092
1	1	1	1	1	1	1	0	53.910
2	2	2	2	2	2	1	0	47.124
3	3	3	3	2	1	0	0	47.847
4	4	4	3	2	1	0	0	50.122
5	5	4	3	2	1	0	0	52.440
6	5	4	3	2	1	0	0	54.574

Table 1. Optimal policies for different manufacturing capacities with a storage capacity of S = 6.

The minimum cost is 47.124 and is achieved with a manufacturing capacity of 2.

Manufacturing costs are obtained by calculating the optimal policies for different *S* and *P* values. Table 2 shows the costs for different manufacturing and storage capacities.

Manufacturing capacity	Storage capacity: 0	1	2	3	4	5	6	7	8
0	75.750	65.275	61.766	61.137	61.320	61.693	62.092	62.476	62.839
1		67.673	58.689	55.384	54.227	53.889	53.910	54.091	54.345
2			57.378	50.875	48.272	47.252	47.124	47.311	47.574
3				51.629	48.425	47.716	47.847	48.149	48.491
4					50.590	49.999	50.122	50.418	50.758
5						52.323	52.440	52.733	53.072
6							54.574	54.867	55.206
7								56.830	57.169
8									58.996

Table 2. Production costs for different *S* and *P* values.

Note that the minimum cost is obtained when $P = 2$ and $S = 6$.

6.3.2 Example 2

In the following example, we will study optimal storage and manufacturing capacities and optimal cost when there are variations in the probability of returns ρ.

The following parameters are used:

$D = 10$; $p = (0, 0, 0, 0, 0, 0.1, 0.15, 0.25, 0.25, 0.15, 0.1)$

$T_1 = 1$, $T_2 = 3$; $\pi = (0.25, 0.50, 0.25)$

$c_p = 10$, $e = 1$, $f = 40$, $c_r = 5$, $h = 1$, $c_{ec} = 30$

$C_p(P) = 10 \cdot \sqrt{P}$, $C_s(S) = 2 \cdot \sqrt{S}$

In Figure 3, we show the probability distribution for demand.

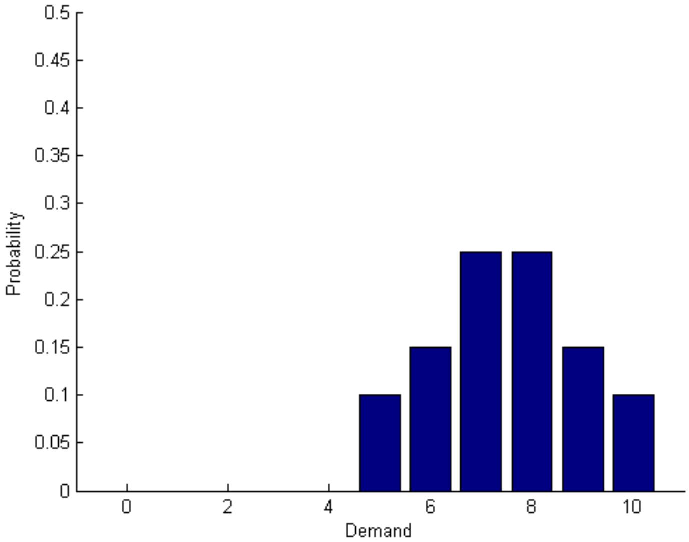
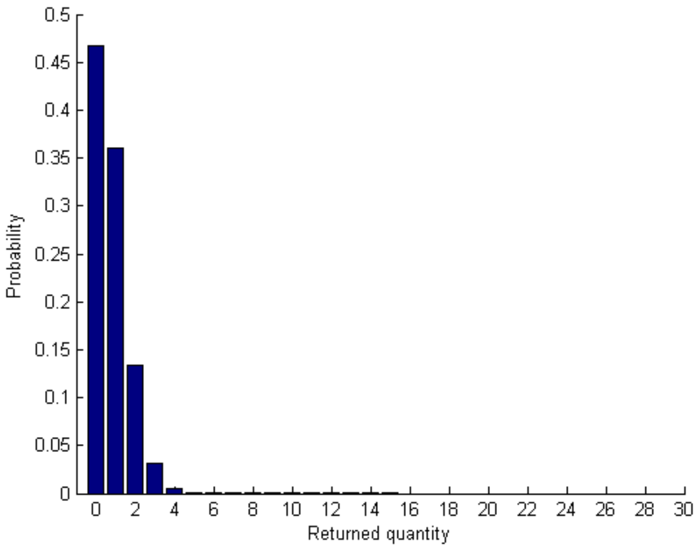
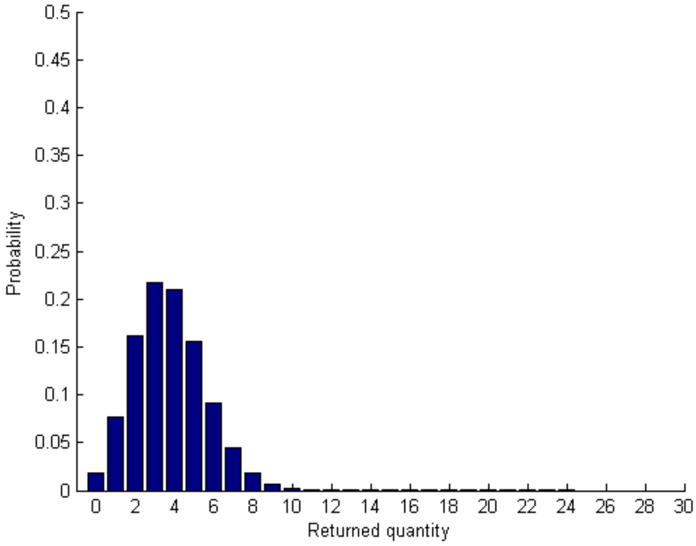


Figure 3. Probability distribution for demand from numerical example 2.

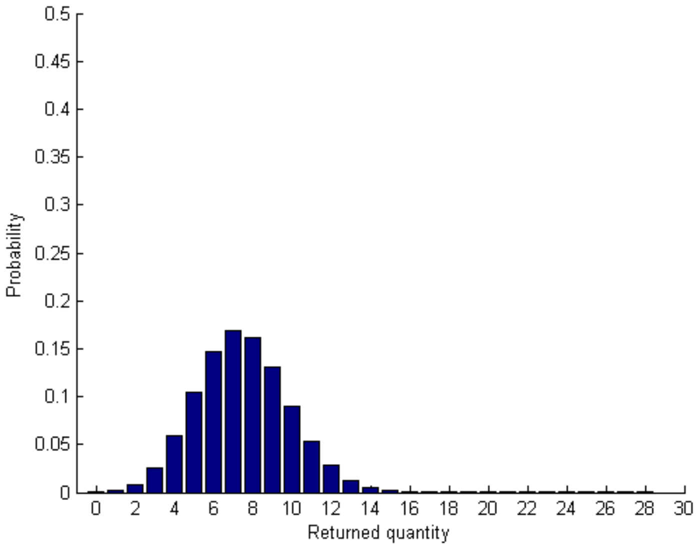
We took 10 return probability ρ values: 0, 0.1, 0.2, …, 1. Let us assume that returns form a succession of independent random variables whose probability distribution is calculated following the steps described in the preceding section. Figure 4 shows some of the probability distributions for returns, namely those corresponding to the return probabilities 0.1, 0.5, and 1.



(a) ρ: 0.1



(b) ρ: 0.5



(c) ρ: 1.0

Figure 4. Probability distribution of returns for different ρ values.

For each ρ value, we calculated the optimal manufacturing and storage capacities following the process explained in section 6.2. Table 3 and Figures 5 and 6 show the corresponding results.

ρ	S^*	P^*	Cost
0.0	18	12	149.99
0.1	17	11	144.44
0.2	17	10	138.66
0.3	17	10	132.67
0.4	16	9	126.38
0.5	15	8	119.82
0.6	14	7	112.97
0.7	14	6	105.77
0.8	13	0	89.94
0.9	16	0	75.09
1.0	16	0	65.29

Table 3. Optimal manufacturing capacity (P^*), storage capacity (S^*), and costs ($C_O(P^*,S^*)$) for different return probabilities ρ.

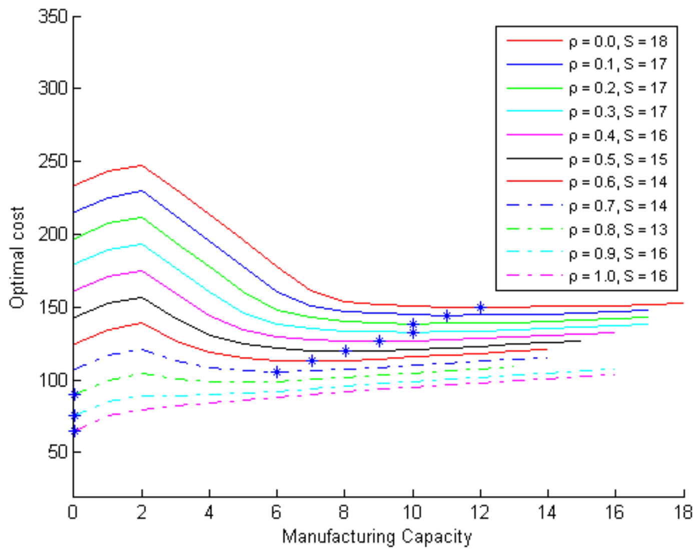


Figure 5. Expected cost versus manufacturing capacity with optimal policy for different ρ and S values. For each ρ value, the S value used is the optimal value. The asterisks show optimal cost.

<u>Observations</u>

In all cases, the optimal storage capacity is greater than demand.

For ρ values of close to 1, the optimal manufacturing capacity is 0. In other words, demand is met through returned products and external channel supplies.

For ρ values of close to 0, the optimal manufacturing capacity is greater than maximum demand. This means that in certain periods, it is financially worthwhile producing more products than there is demand for and storing the surplus. This is because order costs (f = 40) and external channel costs (c_{ec} = 30) are relatively high compared to manufacturing costs (c_p = 10) and storage costs (h = 5).

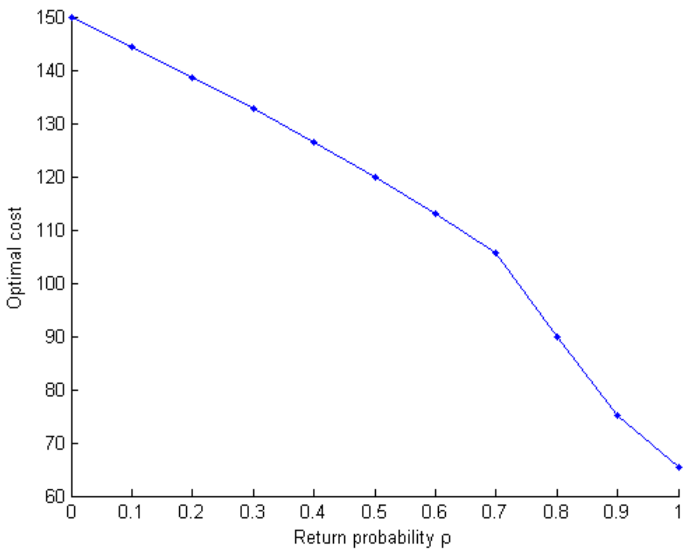


Figure 6. Expected cost value versus return probability ρ, using the optimal policy and optimal manufacturing and storage capacities.

The optimal cost is obtained for ρ = 1, S = 16, and P = 0. The reason for the extreme behaviour in this example is that return-related costs (e and c_r) are much lower than manufacturing-related costs (c_p, c_{ec}, and f).

To complete example 2, we will check how the return independence assumption affects the results obtained. To do this, we will simulate the system using optimal P and S values and the optimal policies obtained with the return independence assumption and then compare the results to those in Table 4.

For ρ values of between 0.1 and 1, we simulated 100 expected cost values. Each value was obtained by simulating the functioning of the system for 11,000 periods and calculating the average cost for periods 1001 to 11,000. In all of the cases, T_1 = 1. Table 4 shows a comparison of the data from the simulation and those from Table 3. For each ρ, the table shows the expected cost based on the return independence assumption (from Table 3), the average simulated cost (based on 100 samples), and the percentage difference between both amounts.

ρ	Expected cost value (calculated)	Mean cost value (simulated)	Difference (%)
0.1	144.4	144.5	-0.03%
0.2	138.7	138.7	0.00%
0.3	132.7	132.7	0.01%
0.4	126.4	126.4	0.01%
0.5	119.8	119.8	0.00%
0.6	113.0	112.9	0.03%
0.7	105.8	105.7	0.10%
0.8	89.9	89.1	0.90%
0.9	75.1	72.8	3.10%
1,0	65.3	59.4	9.06%

Table 4. Comparison of results from Table 3 and simulation results. The second column shows the data from Table 3, the third column shows the results of the simulation, and the fourth column shows the differences between both amounts expressed as a percentage.

We also simulated the expected cost for T_1 = 5 and 18, with ρ = 1. The difference between the cost obtained by simulation and that using the method explained in section 6.2. is 3.57% and 2.42%, respectively. It can therefore be seen that the longer the useful life of the product, the lower the effect of the return independence assumption.

6.3.3 Example 3

In this example, we study optimal cost for a system in which there are variations in remanufacturing cost c_r and return probability ρ.

The following parameters are used:

D = 10; p = (0, 0, 0, 0, 0.1, 0.15, 0.25, 0.25, 0.15, 0.1)

$T_1 = 1$, $T_2 = 3$; π = (0.25, 0.50, 0.25)

c_p = 10, e = 10, f = 40 , h = 5, c_{ec} = 30

$C_p(P) = \sqrt{P}$, $C_s(S) = 2 \cdot \sqrt{S}$

Table 5 and Figure 7 show the results obtained for c_r values of between 5 and 10. Note that for c_r values close to c_p (c_r = 9, 10), optimal costs are obtained when there are no returns; for values lower than 8, they are obtained when ρ = 1.

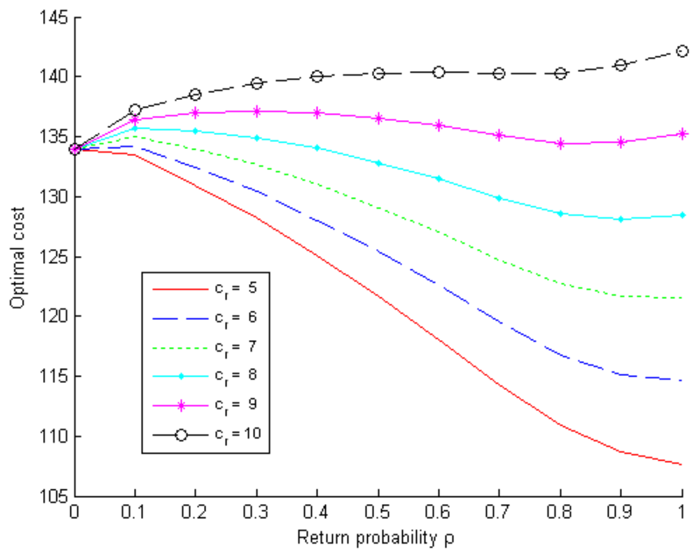


Figure 7. Expected cost value versus return probability ρ, using the optimal policy and optimal manufacturing and storage capacities for c_r values.

ρ	$c_r = 5$			$c_r = 6$			$c_r = 7$			$c_r = 8$			$c_r = 9$			$c_r = 10$		
	S^*	P^*	Cost	S^*	P^*	Cost	S^*	P^*	Cost	S^*	P^*	Cost	S^*	P^*	Cost	S^*	P^*	Cost
0.0	9	9	134.0	9	9	134.0	9	9	134.0	9	9	134.0	9	9	134.0	9	9	134.0
0.1	15	14	133.5	15	14	134.2	15	14	135.0	15	14	135.7	15	14	136.5	15	14	137.2
0.2	14	13	131.0	14	13	132.5	14	13	134.0	14	13	135.5	14	13	137.0	14	13	138.5
0.3	13	11	128.2	13	11	130.4	13	11	132.7	13	11	134.9	13	11	137.2	13	11	139.4
0.4	12	9	125.0	12	9	128.0	12	9	131.0	12	9	134.0	12	9	137.0	12	9	140.0
0.5	12	9	121.6	12	9	125.4	12	9	129.1	12	9	132.8	12	9	136.5	12	9	140.3
0.6	12	8	118.1	12	8	122.6	11	8	127.0	11	8	131.5	11	8	135.9	11	8	140.3
0.7	12	7	114.3	12	7	119.6	11	7	124.7	11	7	129.9	11	7	135.1	11	7	140.3
0.8	12	6	110.9	12	6	116.8	12	6	122.7	11	6	128.6	11	6	134.4	11	6	140.2
0.9	11	5	108.7	11	5	115.2	11	5	121.6	11	5	128.1	11	5	134.5	10	5	140.9
1.0	11	0	107.7	11	0	114.6	10	0	121.6	10	0	128.4	10	0	135.2	10	0	142.1

Table 5. Optimal manufacturing capacity (P^*), storage capacity (S^*), and costs ($C_O(P^*,S^*)$) for different remanufacturing costs *g* and return probabilities ρ.

6.4 Conclusions

In this chapter we have developed a model with reverse logistics, stochastic demand and returns, and limited manufacturing and storage capacities. Using a linear program we have calculated the optimal manufacturing policy when capacities are fixed and we have described the way to obtain the optimal capacities.

We have studied certain factors that are directly related to reverse logistics and affect the calculation of capacities:

1. Return independence assumption: in section 6.3, through a simulation study used to analyze the influence of return independence on optimal costs, we concluded that this influence, in most cases is not significant. Indeed, the calculations are simplified considerably. We also saw that the costs were very similar regardless of whether the return independence assumption was applied or not. Differences were only noticeable when the return probability was close to 1. Finally, we saw that the influence of the return independence assumption decreased with a longer return lag period.

2. Product return probability: we saw that manufacturing and storage capacities in a system with stochastic demand are strongly dependent on return probability and that optimal manufacturing capacities can vary greatly depending on the return probability values studied.

Chapter 7

Conclusions and future research topics

7.1 Conclusions

In our review of the literature, we saw numerous studies that have analyzed reverse logistics systems and others that have addressed the problem of determining optimal manufacturing and storage capacities in traditional systems. Only three of the studies analyzed (Kiesmüller *et al*. 2004, Vlachos *et al*. 2007, and Rubio and Corominas 2008), however, have taken into account the problem of determining optimal capacities in systems with reverse logistics. The difficulty of analyzing such systems probably explains why most studies have opted for simpler models. The aim of this thesis was to study optimal capacities in a reverse logistics environment to obtain results with real-life applications.

We studied three models, each from a category of systems dealt with in traditional inventory management studies: *1)* a system with uniform deterministic demand, *2)* a system with cyclically variable deterministic demand, and *3)* a system with stochastic demand. We then analyzed scenarios in which returns were known and unknown (stochastic) to assess how reverse logistics can influence optimal capacities in different demand and return scenarios.

In the introduction, we saw that return flows are different to flows in traditional logistics systems in several respects, of which the following influence to the models analyzed:

- Uncertainty surrounding the quality and quantity of products returned
- Varying quality from one returned product to the next
- Production control and inventory management

The random nature of returns is one reason why production control and inventory management are more complicated in reverse logistics systems, but it is not the only reason. Management strategies also become more complicated when there is interaction between reverse and traditional logistics systems, as is the case, for example, when new products and recovered products are indistinguishable from each other.

Thanks to the models presented, we were able to study how these characteristics influence optimal capacities. Specifically, we studied the influence of inventory management in chapter 5, the influence of inventory management and the uncertainty surrounding return quantities in chapter 6, and the influence of the varying quality from returned products, the uncertainty surrounding return quantities and the inventory management in chapter 4.

We used the same method to study the influence of reverse logistics on optimal capacities in each of the models studied. First of all, we calculated the optimal policy using a given cost function and assuming that capacities were fixed; second, we calculated the optimal capacity values that optimized cost; and third, we studied how optimal quantities varied with variations in reverse logistics parameters.

Only a few of the publications that have studied the behaviour of reverse logistics systems have calculated optimal policies when analyzing the influence of certain factors (such as the influence of production delays on cost). Instead, they tend to restrict the analysis to a set of a fixed policies. In our case, however, we had to calculate optimal policies as we needed these results to resolve the optimization problem in the second phase of our analysis. Had we restricted our search to a set of policies, the optimal costs calculated for some of the capacities might have been significantly different from the absolute optimal cost.

In the above chapters, we have studied certain factors that are directly related to reverse logistics and affect the calculation of capacities:

3. Product return probability

   In a system with constant demand and stochastic returns, we saw that optimal manufacturing capacity was lower than demand. Although the sum of manufacturing and remanufacturing capacities is higher than demand, the random behaviour of returns makes it impossible to always meet demand. We contemplated the possibility of using an alternative supply channel and saw that:

   - The new channel was used when returns were insufficient to cover demand even though supply costs were very high compared to manufacturing costs.

   - The quantity of products supplied by the new channel was small compared to total demand. The system was therefore capable of meeting practically all the demand.

   The system would behave similarly if an alternative supplier was not used. In other words, manufacturing and remanufacturing system would not be capable of covering all the demand but the level of service would be very high.

   We saw that a deterministic system becomes stochastic once returns are introduced, meaning that the management of such a system must be adapted accordingly.

   We saw in chapter 6 that manufacturing and storage capacities in a system with stochastic demand are strongly dependent on return probability and that optimal manufacturing capacities can vary greatly depending on the return probability values studied.

4. Return lag period

   On studying the system with deterministic demand and returns, we saw that optimal manufacturing capacity depends on the length of the return lag period. We also saw that storage capacity depends exclusively on manufacturing capacity, meaning that optimal storage capacity is also affected by the return lag period.

5. Return independence hypothesis

   In chapter 6, through a simulation study used to analyze the influence of return independence on optimal costs, we concluded that this influence is not significant. Indeed, the calculations are simplified considerably. We also saw that the costs were very similar regardless of whether the return independence assumption was applied or not. Differences were only noticeable when the return probability was close to 1. Finally, we saw that the influence of the return independence assumption decreased with a longer return lag period.

6. Remanufacturing costs

   The influence of remanufacturing costs on manufacturing capacities was analyzed in chapter 6. We saw that in a scenario with low remanufacturing costs compared to manufacturing costs, the optimal cost was obtained when the probability of return was 100%. When remanufacturing costs were high, the cost decreased with a decrease in return probability.

The following points should be taken into account when attempting to optimize a system to which a reverse logistics setup is added:

- Manufacturing and storage capacities should be adapted. We saw that optimal manufacturing and storage capacities are strongly dependent on several factors related with product returns. This means that when reverse logistic is implemented, the optimal capacities may be different of those capacities corresponding to the system without reverse logistics.

- Manufacturing policies and inventory management should be modified (note that these changes will affect the raw material purchase policy).

- Remanufacturing capacities should be implemented. When optimal remanufacturing capacity is not zero, the company must decide where remanufacturing system have to be installed and how reverse logistics is implemented (collection of end-of-life product and transportation to remanufacturing facilities or proper disposing).

- The relationship with suppliers should be modified as supply quantities and rhythms will change due to modifications on manufacturing policies and capacities.

7.2 Future research topics

The findings of this study could be used as a starting point for future studies. Such studies could:

1. Analyze the influence of other factors related to reverse logistics on capacities. Examples are the effect of manufacturing and remanufacturing lead times, remanufacturing and storage costs, and disposal costs on capacities. Design and run a computational experiment to validate the performance and the sensitivity of each model.

2. Analyze the behaviour of systems with unstable/nonseasonal demand and returns (average values, not constant over time).

3. Convert the models described into decision-aid tools, for both strategic decisions (determination of capacities) and operational decisions (determination of optimal inventory policies). Such studies could evaluate different alternatives and scenarios. Using the models presented in this study, further studies should analyze elements to be incorporated into these

models and in the cost function to be optimized. The inclusion of certain elements (e.g. the possibility of backorders, and manufacturing and remanufacturing lead times) would make these models more applicable to real-life situations.

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Appendix. Numerical results of examples

The compact disc attached to this thesis contains several files corresponding to the examples described in chapters 4 and 6. The files contained in CD are the following:

- Example_4.2.4.xls: optimal cost calculated for P = 0,…,100 ; R = 0,…, 100 in example shown in Figure 2 (section 4.2.4.).

- Example_6.3.2.xls: optimal cost for ρ = 0,…,1 P = 0,…, 20; S = 0,…, 20 in example shown in section 6.3.2.

- Example_6.3.2_SIM.xls: expected cost simulated in section 6.3.2.

- Example_6.3.3.c_r_5.txt: optimal cost calculated in example of section 6.3.3. for c_r = 5 and ρ = 0,…,1 P = 0,…, 15; S = 0,…, 15

- Example_6.3.3.c_r_6.txt: optimal cost calculated in example of section 6.3.3. for c_r = 6 and ρ = 0,…,1 P = 0,…, 15; S = 0,…, 15

- Example_6.3.3.c_r_7.txt: optimal cost calculated in example of section 6.3.3. for c_r = 7 and ρ = 0,…,1 P = 0,…, 15; S = 0,…, 15

- Example_6.3.3.c_r_8.txt: optimal cost calculated in example of section 6.3.3. for c_r = 8 and ρ = 0,…,1 P = 0,…, 15; S = 0,…, 15

- Example_6.3.3.c_r_9.txt: optimal cost calculated in example of section 6.3.3. for c_r = 9 and ρ = 0,…,1 P = 0,…, 15; S = 0,…, 15

- Example_6.3.3.c_r_10.txt: optimal cost calculated in example of section 6.3.3. for c_r = 10 and ρ = 0,…,1 P = 0,…, 15; S = 0,…, 15