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***MODELING AND NUMERICAL STUDY OF
NONSMOOTH DYNAMICAL SYSTEMS.***

Applications to Mechanical and Power Electronics Systems.

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**MODELING AND NUMERICAL STUDY OF
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Iván Merillas Santos

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Resumen

Esta tesis trata sobre el modelado y el estudio numérico de sistemas dinámicos no suaves (SDNS). La primera parte de esta tesis consiste en el modelado de algunos convertidores de potencia dc-dc usando el formalismo complementario. Este marco teórico matemático permite asegurar existencia y unicidad de soluciones de una manera natural y sintética. Específicamente funciona muy bien para convertidores electrónicos de potencia porque incorporan modos generalizados de conducción discontinua, caracterizados por una reducción de la dimensión de la dinámica efectiva. Para sistemas con un sólo diodo, se han presentado condiciones analíticas para las variables de estado para la presencia de modos generalizados de conducción discontinua y resultados de simulaciones mostrando una variedad de comportamientos como persistencia o reentrada de modos generalizados de conducción discontinua. Además se ha modelado, analizado y simulado un convertidor de potencia en paralelo, el cual tiene cuatro diodos e ilustra la conveniencia del formalismo complementario para simular sistemas eléctricos con un gran número de diodos ideales. Para terminar esta primera parte se ha presentado la simulación de un convertidor boost usando un control modo deslizante a pesar de que no se ha desarrollado todavía una teoría general de control para sistemas complementarios.

La segunda parte de la tesis se centra en el análisis de bifurcaciones en SDNS, y concretamente en sistemas mecánicos con impactos o fricción. Es conocido que sistemas no suaves o discontinuos pueden exhibir las bifurcaciones exhibidas en sistemas suaves tales como bifurcaciones de doblamiento de periodo, silla-nodo, etc. Además de estas, hay también algunas nuevas transiciones llamadas bifurcaciones inducidas por discontinuidades que son únicas de estos sistemas. En el Capítulo 3 de esta tesis se ha estudiado el comportamiento complejo de un sistema “cam-follower”, el cual es una clase de sistema mecánico con impactos. Bajo variaciones de la velocidad rotacional del “cam” se han analizado diferentes bifurcaciones inducidas por discontinuidades, tales como bifurcaciones de impacto con esquinas y transiciones de “chattering” completo a incompleto. Además se han expuesto condiciones necesarias para órbitas periódicas con un sólo impacto y se ha continuado diferentes órbitas periódicas. Para concluir este capítulo se han analizado regiones de coexistencia de soluciones usando diagramas de dominio de atracción con un método de mapeado de celda a celda.

Otro tipo de bifurcaciones inducidas por discontinuidades, recientemente clasificadas, son las llamadas bifurcaciones de deslizamiento. Dichas bifurcaciones son un comportamiento característico de los llamados sistemas de Filippov. Se pueden identificar cuatro posibles casos: “crossing-sliding”, “grazing-sliding”, “switching-sliding” y “adding-sliding”. En el Capítulo 4 se ha presentado detallados ejemplos de todos los posibles escenarios en un oscilador con fricción seca usando una característica de fricción medida experimentalmente e introducida por Popp [124]. Además, se ha presentado una “switching-sliding” bifurcación degenerada de codimensión dos. En este caso dos curvas de bifurcación de deslizamiento de codimensión una, una “crossing-sliding” y una “adding-sliding”, nacen del punto de codimensión dos. Por otro lado se ha mostrado una bifurcación suave de codimensión dos llamada cúspide y se ha estudiado la coexistencia de órbitas periódicas en la región comprendida entre ambas bifurcaciones silla-nodo.

En el Capítulo 5 se ha investigado la dinámica del modelo Burridge-Knoppoff con dos bloques para la simulación de terremotos. Ciertos estudios numéricos previamente realizados por Nussbaum and Ruina [113] verificaron que, con una fuerza de fricción de tipo Coulomb (esto es que el coeficiente de fricción dinámica es constante), el sistema presenta sólo comportamiento periódico. Sin embargo, se ha mostrado que también pueden ser observadas regiones caóticas en una configuración simétrica, incluso si una fricción de Coulomb es utilizada, si uno de los supuestos usualmente utilizados en la literatura de sismología no es considerado. Por otro lado, se ha estudiado el comportamiento del sistema en una configuración asimétrica. Variando la asimetría del sistema se han observado diferentes soluciones periódicas y regiones de caos. Con respecto al punto de vista del análisis de bifurcaciones, se han analizado varias bifurcaciones suaves e inducidas por discontinuidades en este sistema.

En el Capítulo 6 se presenta la plataforma de “software” SICONOS dedicada a la simulación de SDNS. Primeramente se ha dado una visión general de este “software” y se ha explicado la manera en la que SDNS son modelados y simulados dentro de esta plataforma. Además se ha explicado en detalle las rutinas para análisis (estabilidad, bifurcaciones, variedades invariantes,...) de SDNS implementadas en la plataforma. Para concluir esta parte, varios ejemplos representativos han sido mostrados para ilustrar las posibilidades de la plataforma SICONOS.

Finalmente, en el último capítulo se presentan las conclusiones de esta tesis y algunos problemas aún abiertos para futuras líneas de investigación.

Summary

This thesis is concerned with the modeling and numerical study of non-smooth dynamical systems (NSDS). The first part of the thesis deals with the modeling of some DC-DC power converters using the complementarity formalism. This mathematical theoretical framework allows us to ensure existence and uniqueness of solutions in a “natural” and synthetic way. Specifically, it works pretty well in power electronic converters because it incorporates generalized discontinuous conduction modes (GDCM), characterized by a reduction of the dimension of the effective dynamics. For systems with a single diode, analytical state-space conditions for the presence of a GDCM are stated and simulation results, showing a variety of behaviours, such as persistent or re-entering GDCM, are also presented. Furthermore, the modeling, analysis and simulation of a parallel resonant converter (PRC) which has four diodes illustrate the convenience of the complementarity formalism to simulate electrical systems with a large number of ideal diodes. We also present the simulation of a boost converter with a sliding mode control, even though a general control theory for complementarity systems is not still developed.

In the second part of the thesis we focus on the study of changes of structural stability under parameter variations (bifurcation analysis) in NSDS. We have studied different mechanical systems which involve impacts and dry-friction. It is known that nonsmooth or discontinuous dynamical systems can exhibit most of the bifurcations also exhibited by smooth systems such as period-doublings, saddle-nodes, etc. In addition to these, there are also some novel transitions so-called discontinuity-induced bifurcations (DIBs) which are unique to these systems. We have investigated the complex behaviour occurring in a cam-follower system, which is a class of impacting mechanical system. DIBs such as corner impact bifurcations and transitions from complete to uncomplete chattering motions have been analysed under variations of the rotational speed of the cam. Furthermore, necessary conditions for single impact periodic orbits are stated and continuations of different periodic orbits are also presented. Regions of coexisting solutions have been also analysed by mean of domain of attraction diagrams using a cell-to-cell mapping method.

Another type of DIBs recently classified are the so-called sliding bifurcations. Such bifurcations are a characteristic feature of so-called Filippov systems. Basically, four distinct cases can be identified: crossing-

sliding, grazing-sliding, switching-sliding and adding-sliding. We present detailed examples of all these different bifurcation scenarios in a dry friction oscillator using a measured friction characteristic firstly introduced by Popp [124]. Furthermore, a codimension-two degenerate switching-sliding bifurcation is displayed. In this case of degenerate switching-sliding bifurcation two curves of codimension-one sliding bifurcations, crossing-sliding and adding-sliding, branch out from the codimension-two point. Also, a cusp smooth codimension-two bifurcation is shown and coexistence of periodic orbits in the region between both fold codimension-one curves is studied.

We have also investigated the dynamic behaviour of the two-block Burridge model for earthquake simulations. Previous numerical studies investigated by Ruina [113] verified that, with a friction force of Coulomb type (that is with a constant dynamic friction coefficient), the system presents only periodic behaviour. We show that chaotic regions can be observed in a symmetric configuration even if a Coulomb friction is considered with the relaxation of the assumption that the driving block does not move during the slipping events. Furthermore, we have studied the behaviour of the system with asymmetric configuration. Different periodic solutions and regions of chaos can be observed varying the asymmetry of the system. With respect to the bifurcation point of view, we have analysed several smooth and discontinuity-induced bifurcations observed in this system.

The next to last chapter of this thesis presents the SICONOS software platform dedicated to simulation of NSDS. We give an overview of the SICONOS software and the way NSDS are modeled and simulated within the platform. Routines for analysis (stability, bifurcations, invariant manifolds, ...) of NSDS implemented in the platform are explained in detail. To conclude this part, several representative samples are shown in order to illustrate the SICONOS platform abilities.

Conclusion and some open problems are presented in the last chapter.

Dedication

*A mis padres,
por su amor, cariño e infinito apoyo.
Gracias por enseñarme a tomar siempre mis propias decisiones.*

*A mi hermano,
por su desparpajo, vitalidad y alegría.
Siempre asomas una sonrisa en mi boca. No cambies nunca tato.*

*A mis abuelos,
por su cariño, dulzura y sencillez.
Los veranos de mi infancia a su lado me traen recuerdos inolvidables.*

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por sus consejos, amor y comprensión.
Gracias por regalarme maravillosos momentos a tu lado.*



*One day Alice came to a fork in the road and
saw a Cheshire cat in a tree.
“Would you tell me, please, which way I ought to go from here?”,
she asked.
“That depends a good deal on where you want to get to”,
replied the Cat.
“I don’t much care where”, said Alice.
“Then”, said the Cat, “it doesn’t matter which way you go.”
“As long as I get somewhere”, Alice added as an explanation.
“Oh, you’re sure to do that”, said the Cat, “if you only walk long enough”.*

Alice’s Adventures in Wonderland.

Lewis Carroll (1832-1898)

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Table of Contents

Resumen	i
Summary	iii
Acknowledgements	iii
Agradecimientos	v
Contents	x
List of Figures	xiv
1 Introduction	1
1.1 Motivation	1
1.2 Nonsmooth dynamical systems	3
1.2.1 Introduction	3
1.2.2 Classification of nonsmooth dynamical systems	4
1.2.3 Literature Survey of discontinuity-induced bifurcations	5
1.3 Objective and Scope of the Thesis	7
1.3.1 General Objective	7
1.3.2 Specific Objectives	8
1.3.3 Scope	8
1.4 Outline of the Thesis and Contributions	9
2 Modeling switched power converters using the complementarity formalism	13
2.1 Introduction	14

TABLE OF CONTENTS

2.2	Modeling using complementarity formalism	17
2.2.1	Complementarity systems	17
2.2.2	Passivity in Linear Systems	20
2.3	Power converters as complementarity systems	21
2.3.1	Boost converter	22
2.3.2	Buck converter	23
2.3.3	Buck-boost converter	24
2.3.4	Čuk converter	24
2.4	Generalized discontinuous conduction modes for systems with a single diode	25
2.5	Application to the power converters with single diode.	28
2.5.1	Boost converter	29
2.5.2	Buck converter	30
2.5.3	Buck-boost converter	30
2.5.4	Čuk converter	31
2.5.5	Simulations	33
2.6	Parallel Resonant Converter as LCS	36
2.6.1	Initial and Local Well-Posedness	37
2.7	Simulation of a boost converter as LCS with SMC	41
2.7.1	Dynamical equation	41
2.7.2	Simulation	44
2.8	Conclusions	44
3	Cam-follower system	49
3.1	Introduction	49
3.2	Cam Follower Systems: Mechanical modeling.	52
3.3	Notation for periodic orbits	56
3.4	Numerical methods.	57
3.5	Complex dynamics on cam follower systems	58
3.5.1	Observed dynamics	59
3.5.2	Coexistence of Attractors	60
3.6	Mathematical Analysis	62
3.6.1	First detachment	63
3.6.2	Chattering	64
3.6.3	Periodic Orbits	67
3.7	Conclusions	75
4	Sliding Bifurcations in a dry friction oscillator	77
4.1	Introduction	78
4.2	Description of the model	80
4.3	Filippov systems	82
4.4	Sliding Bifurcations: an overview	84
4.4.1	Crossing-sliding	84
4.4.2	Grazing-sliding	85

TABLE OF CONTENTS

4.4.3	Switching-sliding	85
4.4.4	Adding-sliding	86
4.5	Numerical simulations methods	86
4.6	Numerical analysis of the system	89
4.7	Two-parameter Nonsmooth Bifurcation	96
4.8	Cusp bifurcation and coexistence of attractors	97
4.9	Conclusions	99
5	Bifurcations in a two-block stick-slip system	103
5.1	Introduction	103
5.2	The Mechanical Model	107
5.3	Dynamics of the model	109
5.4	One-dimensional event map	112
5.5	Chaos in a symmetric configuration	114
5.6	Observed behaviours in an asymmetric model	114
5.7	Description of some bifurcations	117
5.7.1	Flip bifurcation	118
5.7.2	Fold bifurcation	118
5.7.3	Crossing-sliding bifurcation	121
5.7.4	Grazing-sliding bifurcation	121
5.8	Conclusions	123
6	SICONOS Platform	125
6.1	Introduction and motivation	125
6.2	NSDS in SICONOS platform	127
6.2.1	Overview of SICONOS platform	127
6.2.2	NSDS modeling in SICONOS platform	128
6.2.3	Simulation techniques in SICONOS platform	131
6.3	Routines for analysis in SICONOS	132
6.3.1	Domain of attraction routines	132
6.3.2	Bifurcation diagram routines	137
6.4	Benchmarks	142
6.4.1	Buck converter	142
6.4.2	Forced Harmonic Oscillator	147
6.4.3	Parallel Resonant Converter	149
7	Conclusions and future research	155
7.1	Overview and Summary of Contributions.	155
7.2	Publications	160
7.3	Future research	161
	Appendices	163
	Appendix A: Moreau's Time-stepping	165

TABLE OF CONTENTS

Appendix B: Sliding Mode Control	169
---	------------

List of Figures

1.1	Sudden transition from a regular periodic solution to chaos.	3
1.2	Classification of non-smooth dynamical systems.	5
2.1	The boost converter	21
2.2	The buck converter	21
2.3	The buck-boost converter	22
2.4	The Čuk converter	22
2.5	GDCM for the boost converter with switch open. Upper: x_1 on the horizontal axis and Γ on the vertical one. Middle: u as a function of time. Lower: y as a function of time. The GDCM has a finite duration and it is not re-entrant for the parameters and initial conditions chosen.	34
2.6	GDCM for the Cuk converter with switch closed. Upper: x_3 on the horizontal axis and Γ on the vertical one. Middle: u as a function of time. Lower: y as a function of time. The GDCM has a finite duration, but it is re-entrant for the parameters and initial conditions chosen.	35
2.7	GDCM for the Cuk converter with switch open. Upper: $x_1 - x_2$ on the horizontal axis and Γ on the vertical one. Middle: u as a function of time. Lower: y as a function of time. For the parameters and initial conditions used, the GDCM lasts indefinitely.	35
2.8	A Power Resonant Convert diagram	36
2.9	Voltage on the capacitor	45
2.10	Current in the inductance	46
2.11	Sliding surface	46

LIST OF FIGURES

2.12	Zoom of the voltage on the capacitor	47
2.13	Zoom of the current in the inductance	47
2.14	Zoom of the sliding surface	48
3.1	Cam-Follower schematics. (a) $t=0$ (b) $t=\tau$	51
3.2	Three different cam profiles. From left to right: Cam profiles, Constraint positions, Velocities and Accelerations.	54
3.3	(a) Cam profile definition. (b) Constraint position $c(t)$, velocity $\dot{c}(t)$ and acceleration $\ddot{c}(t)$	55
3.4	Cam profile definition (a) $\theta = 0$ (b) $\theta = \pi$	56
3.5	Time response for 190 <i>rpm</i> . (a) Follower position vs. Cam position. (b) Relative position. (c) Phase space, relative position vs. relative velocity.	58
3.6	Observed dynamics. (a) Impact Bifurcation diagram for cam velocity $rpm \in [114, 200]$. (b) Impact Bifurcation diagram for cam velocity $rpm \in [190, 300]$. (c) Impact Bifurcation diagram for cam velocity $rpm \in [357.5, 361.5]$. (d) Stroboscopic bifurcation diagram for cam velocity $rpm \in [357.5, 361.5]$ sampling the states at $\Pi_s = \pi$. (e) Impact Bifurcation diagram for cam velocity $rpm \in [660, 760]$. (f) Stroboscopic bifurcation diagram for cam velocity $rpm \in [670, 750]$ sampling the states at $\Pi_s = b$	61
3.7	Domain of attraction $rpm=358.5$	62
3.8	Zoom of the first chattering part. (a) Numerical simulation. (b) Analytical calculations	66
3.9	Transition from complete to uncomplete chattering. (a) Time evolution for $rpm = 198.4$, (b) Time evolution for $rpm = 198.56$, (c) Stroboscopic bifurcation diagram in range $rpm [198, 200]$ (d) Zoom of the stroboscopic bifurcation diagram.	67
3.10	Stable P(1,1) bifurcation diagram using a continuation method.	70
3.11	Eigenvalues of the stable P(1;1).	71
3.12	Unstable P(1;1) bifurcation diagram using a continuation method.	72
3.13	Eigenvalues of the unstable P(1,1).	73
3.14	Corner impact of the bifurcation of a P(2;1:1) orbit.	74
4.1	Block subject to a spring force, a friction force with a moving belt and an harmonic force of angular frequency ω	80
4.2	Stable sliding mode	83
4.3	Four possible scenarios of sliding bifurcations.	85
4.4	Influence of c_1 for the friction approximation given by (4.30).	88
4.5	Bifurcation diagram: (a) Event-driven method (b) Smoothing method	90

4.6	(a) Map of the phase angle ϕ at the stick-slip transition ($\omega = 2.94$). (b) Second iterate of the map at the stick-slip transition ($\omega = 2.9$).	91
4.7	(a) Map of the phase angle ϕ at the stick-slip transition ($\omega = 2.94$). (b) Second iterate of the map at the stick-slip transition ($\omega = 2.9$).	91
4.8	(a) Second iterate of the map at the stick-slip transition ($\omega = 2.895$) (b) Second iterate of the map at the stick-slip transition ($\omega = 2.893$).	93
4.9	(a) Periodic orbit ($\omega = 1.6$) (b) Time evolution ($\omega = 1.6$) (c) Periodic orbit ($\omega = 1.72$) (d) Time evolution $\omega = 1.72$).	94
4.10	(a) Periodic orbit ($\omega = 2.5$) (b) Periodic orbit ($\omega = 2.8$).	95
4.11	Two parameter bifurcation diagram around the codimension-two points.	96
4.12	Phase space corresponding to: (a) Point 1, (b) Point 2, (c) Point 3, (d) Point 4, (e) Point 5.	98
4.13	Two-parameter bifurcation diagram showing the cusp bifurcation.	99
4.14	Different scenarios of the map P_{trans}^2 : (a) Region 1 (b) Cusp point (c) Curve Γ_{f1} (d) Curve Γ_{f2}	99
4.15	Domain of attraction using a cell-to-cell mapping method for $(A, \omega) = (3.6, 1.067)$ (a) Basins of attraction (b) Transient time.	100
4.16	Coexistence of periodic solutions for $(A, \omega) = (3.6, 1.067)$ with initial conditions (a) $(x_{10}, x_{20}) = (-0.685, 3.01)$ (b) $(x_{10}, x_{20}) = (-0.295, 0.99)$ (c) $(x_{10}, x_{20}) = (-0.445, 1.27)$	100
5.1	San Andreas fault	104
5.2	Illustration of the two blocks model with a constant velocity driver.	106
5.3	The open convex region of the plane limited by the blue lines is the global stick phase locus of the system.	109
5.4	Differences between considering or preventing assumption 1 ($\alpha = 3, \beta = 1, \phi = 1.25$ and $V_{dr} = 0.025$): (a) Periodic orbit considering assumption 1; (b) Period orbit being prevented assumption 1.	112
5.5	Illustration of the one-dimensional event map.	113
5.6	Bifurcation diagram	115
5.7	Two-dimensional bifurcation diagrams.	116
5.8	Bifurcation diagram	117
5.9	Flip bifurcation: (a) Second iterated map for $\alpha = 3.05$, (b) Fourth iterated map for $\alpha = 3.05$, (c) Second iterated map for $\alpha = 3.15$ and (d) Fourth iterated map for $\alpha = 3.15$	119

LIST OF FIGURES

5.10	Fold bifurcation: (a) Sixth iterated map for $\alpha = 3.19$ and (b) Sixth iterated map for $\alpha = 3.23$	120
5.11	Grazing-sliding bifurcation: (a) Fourth iterated map for $\alpha = 3.85$, and (b) Fourth iterated map for $\alpha = 3.86$	120
5.12	Bifurcation diagram	121
5.13	Crossing-sliding bifurcation: Difference between the relative velocities of both masses (a) $\alpha = 5.58$, and (b) $\alpha = 5.62$. . .	122
5.14	Grazing-sliding bifurcation: Relative velocities of mass 1 for $\alpha = 5.69$	122
6.1	Sketch of a cell mapping method.	133
6.2	The main organization of Cell Mapping method.	135
6.3	The main organization of Cell Mapping input data.	136
6.4	Examples of the graphical output.	137
6.5	Explanation of the bifurcation diagram program.	139
6.6	Bifurcation diagrams: Difference between to take a fixed initial condition (left picture) or a variable initial condition (right picture).	140
6.7	The main organization of Bifurcation input data.	141
6.8	Example of the graphical output.	142
6.9	Buck converter diagram	143
6.10	Time simulations of a Buck Converter in SICONOS Platform. (a) $V_{in} = 15V$. (b) $V_{in} = 25V$	145
6.11	Bifurcation simulations (a) Using SICONOS Platform. (b) Using an own MatLab code.	145
6.12	Domains of attraction simulations (a) Using SICONOS Platform. (b) Using an own MatLab code.	146
6.13	Bifurcation simulations (a) Using SICONOS Platform. (b) Using an own MatLab code.	148
6.14	Domains of attraction simulations (a) Using SICONOS Platform. (b) Using an own MatLab code.	149
6.15	A Parallel Resonant Convert diagram	150
6.16	Simulations in SICONOS Platform.	152
6.17	Simulations in SICONOS Platform 2.	153