Discusión y Conclusiones.

En esta sección presentamos las contribuciones originales del trabajo realizado, y por otro lado, las conclusiones que se pueden derivar del mismo. Las contribuciones más relevantes del presente trabajo, las podremos clasificar en dos grandes grupos, de acuerdo a la naturaleza de las leyes de control desarrolladas, ya sean continuas o discretas.

modelo continuo:

- Usando el modelo promedio PWM para los convertidores DC-DC, se aportarón diversas estrategias y algoritmos de control para la estabilización del voltaje de salida hacia un valor constante pre-especificado. Estas se describen a continuación:

  1. Se ha innovado en el campo del modelo matemático de los convertidores al demostrar que los modelos promedio de estos sistemas responden a la formulación de Euler–Lagrange (E-L) clásica. El resultado final coincide totalmente con los trabajos publicados por Middlebrook y Cuk, y proporciona una visión que esta más ligada a las propiedades físicas del sistema. El área se puede extender hasta agregar nuevos modelos dinámicos de los convertidores DC–DC, tomando en cuenta características parásitas del circuito de cebado.
  2. Se aportó la estrategia que permite realizar la estabilización directa del voltaje de salida en convertidores de corriente contínua, incluso cuando esta salida es de fase no–mínima, por medio de régimenes "pseudo-deslizantes".

     Además se le agrega una nueva aportación al esquema, al ser posible asociar la estrategia del control estabilizante, con un esquema adaptativo, aunque este estuviera en base al modelo nominalmente linealizado del convertidor, y el cual necesariamente requiría un conocimiento perfecto de los parámetros del circuito.
  3. Se aportó una nueva visión en el análisis teórico y el diseño del control para convertidores DC–DC, que se desarrollo a partir de la Forma Canónica de Retroalimentación Pura y el método del "paso atrás, definidos por Kanellakopoulos [22, 23] y viceversa. Al proponer un esquema adaptable estabilizante dinámico, para los convertidores de corriente contínua, que permite el ajuste del voltaje de salida y además la sintonía de parámetros desconocidos del convertidor.

4. Apoyados en la naturaleza de Euler–Lagrange de los modelos promedio de los convertidores DC–DC, se procedió a proponer el análisis y diseño de controladores dinámicos basados en pasividad. Los controladores fueron sintetizados usando el método de la modificación de la energía y la inyección de amortiguamiento extensamente utilizada en sistemas mecánicos de fase mínima.
5. Los esquemas de control "pasivos" fueron extendidos al caso de resistencia de carga desconocida, para cada uno de los convertidores e inclusive para el caso de tres convertidores "Boost" en cascada, aportando el análisis y diseño del esquema adaptativo. El control retroalimentado de estos convertidores basados en la salida de fase no–mínima, resultan ser más interesantes que las aplicadas al control de sistemas robóticos.

modelo discreto:

- En el estudio de los modelos "derivados" de los convertidores DC-DC, se logró desarrollar una serie de estrategias de análisis y diseño de control, que permite:

  1. Aportar un modelo basado en discretización exacta de los convertidores *derivados*, sobre la base de un esquema de regulación por modulación de anchura de pulsos.
  2. Se aportó una nueva solución al problema de control no–lineal resultante que condujo a una nueva clase de controladores estáticos del tipo implícito, al mismo tiempo que se reveló la naturaleza no–kalmaniana de los modelos de tales convertidores. En este contexto se diseñaron novedosos esquemas con políticas de control que permiten la estabilización e inclusive el seguimiento de señales a la salida del convertidor.

     Es necesario resaltar que en el seguimento de señales fué posible obtener controladores en forma explícita, usando los modelos promedio discretos, con la heurísitica de estar en base nuevamente a la regulación de la corriente de entrada en el inductor.
  3. Se obtuvo el esquema que contempla la posibilidad de obtener controladores basados en la modulación de frecuencia de pulsos, obteniendosé igualmente un control implícito para llevar a cabo la estabilización del voltaje a la salida de los modelos derivados DC–DC, exactamente discretizados.

Como tópico de investigaciones futuras, pareciera interesante y es de particular interes práctico, el desarrollo de esquemas de controles adaptativos no–lineales, mediante discretización exacta, aplicados a lo modelos tradicionales de segundo orden, que como es de esperarse serán de mayor dificultad.

□ □ □

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Design of pulse width modulation controllers for stabilization and tracking in derived DC-to-DC power converters

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Feedback controllers, based on pulse-width-modulation (PWM), are derived for the average input current stabilization and tracking problems in derived DC-to-DC power supplies of the buck and boost types. The stabilization problems are solved on the basis of steady-state considerations about the current 'ripple' and exactly discretized nonlinear models describing the sampled PWM regulated input current trajectories. In the boost-derived converter, the stabilization problem leads to an implicit static nonlinear feedback controller, or duty ratio synthesizer, which requires online solutions of a transcendental equation at each sampling instant. The signal tracking problems are solved on the basis of discrete-time, non-kalmanian state representation models describing the average PWM regulated input current. These generalized state models naturally allow for explicit dynamical, rather than static, feedback regulators. Computer simulations, including unmodelled load variations and external stochastic perturbation inputs, are presented which test the robustness of the proposed PWM controller performances.

1. Introduction

Simplified versions of DC-to-DC power converters may be obtained by removing the storing capacitors on the output circuits. Corresponding to the buck, the boost, and the buck–boost converters (see Severns and Bloom 1983), the obtained converters are known, respectively, as 'choppers', step-up and step up-down converters. Generically, they may also be addressed as 'derived' converters (see Rashid 1993, for details).

The feedback regulation of DC-to-DC power supplies is customarily accomplished, through pulse-width-modulation (PWM) feedback strategies (see Kassakian *et al.* 1991, Rashid 1993). Typically, control objectives include input current stabilization, around a given constant value, or time-varying reference input signal tracking. In this context, infinite frequency average models (see Middlebrook and Cúk 1976) or equivalent control models are frequently invoked, at the controller design stage, in order to obtain a smooth feedback specification of the computed duty ratio

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function (see also Sira-Ramírez 1989, 1991, Sira-Ramírez and Lischinsky-Arenas 1991, and Sira-Ramírez *et al.* 1993). The performance features of the actual PWM controlled circuit responses, with respect to those predicted by the average PWM model, depend on the magnitude of the sampling frequency associated with the pulse width modulator. For low sampling frequencies, the closed-loop precision deteriorates allowing substantial errors in the stabilization and tracking tasks.

The use of average models, however, may not be entirely justified for derived DC-to-DC power supplies. First, the appealing simplicity of the dynamic models does not seem to require further simplification through a questionable smooth approximation and, secondly, the possibilities for exact discretization, which is certainly a much more involved process in the traditional version of the converters, make it reasonable to attempt a direct PWM controller design based on an exact discrete-time model of the derived converter (see Kassakian *et al.* 1991). The exact discretization circumvents all problems related to the approximation involved in the finite magnitude of the sampling frequency used for the pulse width modulator.

It is the purpose of this paper to explore, in detail, the feasibility of PWM stabilizing controller designs for stabilization and tracking in derived DC-to-DC power converters. The approach is based on exact discretization of the sampled input current. Discrete-time regulation policies based on approximate discretization and approximate linearization were explored by Ehsani *et al.* (1983). The outline of an approximate discretization approach for the stabilization of more complex DC-to-DC power supplies can also be found in Kassakian *et al.* (1991). Related developments, from a viewpoint different to that of feedback control, are found in Rashid (1993).

In this article we present the fundamentals of an exact discretization approach for the input current stabilization and tracking problems in the derived versions of the buck and the boost DC-to-DC power supplies. The results, however, can also be extended to include the buck–boost derived converter. The linearity in the input, associated with the traditional infinite frequency average models of the converters, is effectively destroyed by the exact discretization procedure. Nevertheless, the obtained models still remain linear in the state. The proposed approach offers no special difficulties for the stabilization problem in converters of the buck-derived type. For such a class of derived converters, it is also possible to obtain an explicit expression relating steady-state average input current values to steady-state sampled input current values. This key fact allows us to solve explicitly the average current stabilization problem in terms of an equivalent sampled current stabilization problem. However, in the stabilization problem for the boost-derived converter, the resulting nonlinear discrete-time duty ratio synthesizers (controllers) are of the implicit type, i.e. at each sampling instant, the feedback duty ratio function is given by the numerical solution of a transcendental equation. Similar transcendental equations allow for the offline computation of the desired steady-state average input current in terms of the steady-state sampled input current. The signal tracking problems are addressed by introducing discrete-time average models for the PWM regulated input current trajectories. The discrete-time models naturally result in generalized, i.e. non-kalmanian, state representations of the systems (see Fliess 1992). In both cases, the proposed average models naturally lead to nonlinear explicit dynamical feedback duty ratio synthesizers.

Section 2 presents an exact discretization approach for PWM feedback regulator designs solving stabilization problems defined on derived DC-to-DC power supplies of the buck and boost types. In this section we also present simulations of the proposed

feedback control schemes for the derived converters. Section 3 presents the nonlinear discrete-time average models, and the corresponding solutions to the signal tracking problems, for the two types of derived converters. The conclusions and suggestions for further research in this area are presented in the final section.

2. Feedback stabilization of derived DC-to-DC power converters via an exact discretization scheme

2.1. *The buck-derived converter*

Consider the buck-derived converter circuit shown in Fig. 1 (Rashid 1993). The switch regulated model describing the behaviour of the input current, denoted by x, is given by

$$\dot{x} = -\frac{R}{L}x + \frac{E}{L}u$$

$$y = Rx \tag{2.1}$$

where y is the output load voltage and the parameters R, L and E stand, respectively, by the load resistance, the inductance of the input circuit, and the constant input source voltage. The variable u denotes the switch position function taking values on the discrete set $\{0, 1\}$.

A regulation strategy, based on a PWM specification of the switch position function, may be, generally speaking, specified by (see Sira-Ramírez *et al.* 1993)

$$u(t) = \begin{cases} 1 & \text{for } t_k \leqslant t < t_k + \mu(t_k)T \\ 0 & \text{for } t_k + \mu(t_k)T < t < t_k + T \end{cases} \tag{2.2}$$

$$t_{k+1} = t_k + T; \quad k = 0, 1, 2, \ldots$$

where $\mu(\cdot)$ is known as the actual duty ratio function, taking values in the interval $[0, 1]$ of the real line. T is the sampling period and t_k is the sampling instant. A typical example of a PWM commanded switch position function trajectory is depicted in Fig. 2.

Since the duty ratio μ is specified online in a feedback manner, i.e. computed as a function explicitly depending on the sampled value of the input current $x(t_k)$ at each instant t_k, one may obtain values of μ which lie outside the closed interval $[0, 1]$. We must, therefore, make a distinction between the computed duty ratio function, denoted by $\mu_c(\cdot)$ and the actual duty ratio function, denoted by $\mu(\cdot)$. The relation between these variables is simply given by

$$\mu(t) = \begin{cases} 1 & \text{for } \mu_c(t) > 1 \\ \mu_c(t) & \text{for } 0 \leqslant \mu_c(t) \leqslant 1 \\ 0 & \text{for } \mu_c(t) < 0 \end{cases} \tag{2.3}$$

The actual duty ratio function is thus the forceful limitation of the computed duty ratio function to the closed interval $[0, 1]$.

The buck-derived converter owes its popular name 'chopper' to the fact that the input current is limited to taking values on the interval $[0, E/R]$, as can easily be verified from the circuit equations. The corresponding (positive) output load voltages delivered by the converter cannot, therefore, exceed the value E of the external source voltage.

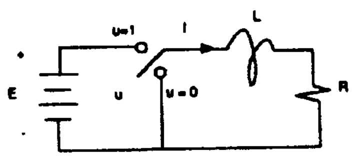

Figure 1. The buck-derived converter.

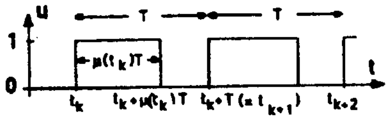

Figure 2. PWM commanded switch position function.

2.2. *An exact discretization of the PWM regulated buck-derived converter*

The linear nature of the two possible topologies of the converter circuit facilitate the derivation of an exact discrete-time model for the evolution of the sampled values of the input current in the buck-derived converter (2.1), when subject to a switching policy of the form (2.2). Indeed, given the value of x at time t_k, denoted by $x(t_k)$, the value of the input current at the end of the 'pulse', of width $\mu(t_k)T$, is obtained as

$$x(t_k+\mu(t_k)T) = \exp\{-\theta_1\mu(t_k)T\}x(t_k)+\frac{\theta_2}{\theta_1}[1-\exp\{-\theta_1\mu(t_k)T\}] \qquad (2.4)$$

where we have let the parameter θ_1 denote the quotient R/L and θ_2 denote E/L.

The sampled value of the input current at the end of the sampling interval is obtained, after some further computations, as

$$x(t_k+T) = \exp\{-\theta_1 T\}x(t_k)+\frac{\theta_2}{\theta_1}\exp\{-\theta_1 T\}[\exp\{\theta_1\mu(t_k)T\}-1] \qquad (2.5)$$

If we denote $\Psi_1 = e^{-\theta_1 T}$ and $\Psi_2 = \theta_2/\theta_1$, the discrete-time model for the evolution of the input current, depicted at the sampling instants, is given by the following model

$$x(t_{k+1}) = \Psi_1 x(t_k)+\Psi_1\Psi_2[\Psi_1^{-\mu(t_k)}-1] \qquad (2.6)$$

where the value of the duty ratio function at time t_k, $\mu(t_k)$, must now be effectively regarded as the 'control input' variable, to be specified at the beginning of each sampling period. The discrete-time model for the sampled input current is, therefore, nonlinear in the new control input, $\mu(t_k)$.

The only eigenvalue associated with the linear sampled state dynamics, given by Ψ_1, is evidently positive and strictly smaller than unity. The steady-state value of the sampled input current, denoted by $\bar{x}_\infty$, corresponding to constant duty ratio function of value μ_∞, is then readily obtained from (2.6) as

$$\bar{x}_\infty = \frac{\Psi_1\Psi_2}{1-\Psi_1}(\Psi_1^{-\mu_\infty}-1) \qquad (2.7)$$

The overbar in (2.7) refers to the 'lower' portion of the actual zigzagged trajectory

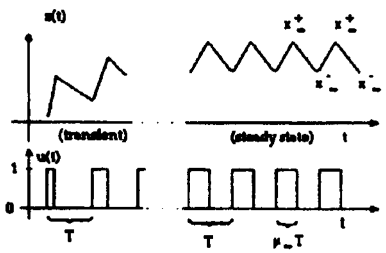

Figure 3. Transient and steady-state PWM controlled state trajectory.

partially described by (2.6) (see Fig. 3). Evidently, a feedback regulation policy, which specifies the duty ratio function $\mu(t_k)$ solely on the basis of the sampled state $x(t_k)$, is by no means satisfactory. The reason for such a statement stems from the fact that the 'ripple', unavoidably associated with the switch-regulated evolution of $x(t_k)$, is not taken into account by the model (2.6) alone. One must also take into account the values of $x(t)$ at the end of each width-modulated control input pulse occurring within the sampling period of length T. In other words, one must take into account the values of $x(t)$ at the instants $t = t_k+\mu(t_k)T$; $k = 0, 1, 2, \ldots$ (see Fig. 3).

We now relate the values of x at times $t_{k+1}+\mu(t_{k+1})T$ and $t_k+\mu(t_k)T$ so as to obtain the values of the 'upper' corners of the zigzagged input current trajectory. One obtains, after some algebraic manipulations

$$x(t_{k+1}+\mu(t_{k+1})T) = \Psi_1^{\mu(t_{k+1})}\Psi_1^{1-\mu(t_k)}x(t_k+\mu(t_k)T)+\Psi_2(1-\Psi_1^{\mu(t_{k+1})}) \qquad (2.8)$$

The eigenvalue associated with the above linear state dynamics is clearly given by the product $\Psi_1^{\mu(t_{k+1})}\Psi_1^{1-\mu(t_k)}$. This quantity is strictly positive and smaller than unity for values of μ bounded by the unit interval [0, 1]. The steady-state value of the 'upper' corners of the state trajectory, described by (2.8), corresponding to a constant value μ_∞ of the duty ratio function, is given by (see Fig. 3)

$$x^+_\infty = \frac{\Psi_2}{1-\Psi_1}(1-\Psi_1^{\mu_\infty}) \qquad (2.9)$$

The relation between the steady-state values x^+_∞ and x_∞ can be obtained from (2.7) and (2.9) as

$$\bar{x}_\infty = x^+_\infty\Psi_1^{1-\mu_\infty} \qquad (2.10)$$

Since Ψ_1 is a positive number, which is strictly less than 1, one can conclude that $\bar{x}_\infty < x^+_\infty$ for $\mu_\infty \in [0, 1]$.

The steady state 'ripple', denoted by r_∞, may then be described as the following difference

$$r_\infty = x^+_\infty - \bar{x}_\infty = \frac{\Psi_2}{1-\Psi_1}(1-\Psi_1^{\mu_\infty})(1-\Psi_1^{1-\mu_\infty}) \qquad (2.11)$$

We define a steady-state average value for the input current trajectory as

$$x_{av}(\infty) = \bar{x}_\infty+\tfrac{1}{2}r_\infty = \tfrac{1}{2}(\bar{x}_\infty+x^+_\infty) \qquad (2.12)$$

Using the expressions (2.7) and (2.9) in (2.12) one obtains

$$x_{ss}(\infty) = \frac{1}{2}\left(\frac{\Psi_2}{1-\Psi_1}\right)(1-\Psi_1^{\mu_\infty})(1+\Psi_1^{1-\mu_\infty}) \qquad (2.13)$$

We proceed to express the steady-state value of the sampled input current trajectory x_∞^-, in terms of the average steady-state input current $x_{ss}(\infty)$. This relation allows us to define a suitable stabilizing feedback duty ratio (control) policy on the basis of the sampled states of the discrete-time model (2.6). The feedback policy properly takes into account the ripple associated with the controller trajectory, and asymptotically achieves a pre-specified desired steady-state value for the average input current. To achieve this goal one simply eliminates the steady-state value of the duty ratio, μ_∞ from the expressions (2.7) and (2.13). One then obtains,

$$x_\infty^- = -\Psi_2\left[\left(\frac{1}{2}\left(1-\frac{2x_{ss}(\infty)}{\Psi_2}\right)+\frac{\Psi_1}{1-\Psi_1}\right)-\left(\frac{1}{4}\left(1-\frac{2x_{ss}(\infty)}{\Psi_2}\right)^2+\frac{\Psi_1}{(1-\Psi_1)^2}\right)^{1/2}\right] \qquad (2.14)$$

2.3. *A stabilizing PWM control policy for the buck-derived converter*

The stabilization problem for the buck-derived converter consists of specifying a PWM feedback regulation policy of the form (2.2) such that the steady-state average value of the controlled input current trajectory $x(t)$ reaches a desired constant value $x_{ss}(\infty) = X$.

A stabilizing feedback regulation policy $\mu(t_k)$ can then be explicitly obtained on the basis of the sampled states of the discrete-time model (2.6) by forcing $x(t_k)$ to stabilize asymptotically around the value x_∞, corresponding to X, which we denote by $x_\infty^-(X)$ and rewrite as

$$x_\infty^-(X) = -\Psi_2\left[\left(\frac{1}{2}\left(1-\frac{2X}{\Psi_2}\right)+\frac{\Psi_1}{1-\Psi_1}\right)-\left(\frac{1}{4}\left(1-\frac{2X}{\Psi_2}\right)^2+\frac{\Psi_1}{(1-\Psi_1)^2}\right)^{1/2}\right] \qquad (2.15)$$

We impose on the sampled controlled system the following linear asymptotically stable closed-loop behaviour

$$x(t_{k+1}) = \alpha(x(t_k) - x_\infty^-(X)) + x_\infty^-(X); \quad |\alpha| < 1 \qquad (2.16)$$

Substituting the right-hand side of expression (2.6) on (2.16) and solving for the duty ratio function $\mu(t_k)$ one obtains the following nonlinear computed duty ratio feedback control policy

$$\mu_c(t_k) = -\frac{1}{\ln \Psi_1}\ln\left[1+\frac{(\alpha-\Psi_1)x(t_k)+(1-\alpha)x_\infty^-(X)}{\Psi_1\Psi_2}\right]; \quad k = 0,1,2,\ldots \qquad (2.17)$$

The actual duty ratio function $\mu(t_k)$ may be readily obtained from expression (2.3). Figure 4 depicts the PWM feedback regulation scheme based on the exact discrete-time dynamics model of the sampled input current.

Expression (2.17) allows for the determination of the region of non-saturation of the actual duty ratio function. Indeed, the double inequality: $0 < \mu_c < 1$, yields the following corresponding region for the sampled state

$$0 < (\alpha-\Psi_1)x(t_k)+(1-\alpha)x_\infty(X) < \Psi_2(1-\Psi_1) \qquad (2.18)$$

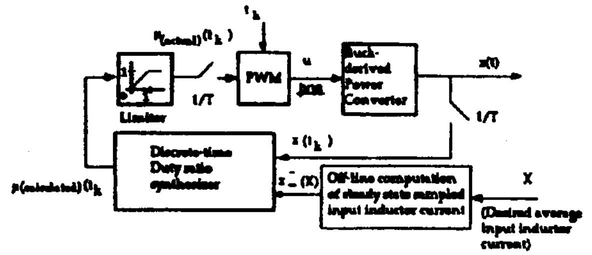

Figure 4. A PWM feedback regulation scheme for the buck-derived converter based on exact discretization.

2.4. *Simulation results*

In order to test the robustness of the previously proposed PWM feedback regulation policy we carried out simulations on the following noise perturbed model of the buck-derived converter

$$\dot{x} = -\frac{R}{L}x + \left(\frac{E+v(t)}{L}\right)u \qquad (2.19)$$

where $v(t)$ is a (computer generated) stochastic perturbation signal representing an unmodelled additive noisy voltage source affecting the behaviour of the circuit. The values for the parameters defining the converter were taken to be

$$R = 2{\cdot}8\times10^{-2}\,\Omega; \quad L = 1{\cdot}0\times10^{-2}\,\text{mH}; \quad E = 126\ \text{V}$$

The sampling period was chosen to be $T = 0{\cdot}125$ ms ($1/T = 8$ kHz) and the desired steady-state value of the average dynamics was set to be $X = 1237$ A. The eigenvalue for the closed loop linear dynamics, α, was set to be 0·3. The corresponding value of the steady-state input current was found to be $x_\infty^-(1237) = 1080{\cdot}7$ A. The required steady-state average value of the input current as well as the steady-state values x_∞^+ and x_∞^- are well within the allowable range which guarantees non-saturation of the actual duty ratio function.

Figure 5 depicts a typical simulated PWM feedback controlled trajectory for the input current arising from the perturbed model (2.19). This figure also shows the actual duty ratio function $\mu(t)$ and the corresponding switch position function $u(t)$. At the end of the figure we show the perturbation signal $v(t)$. As shown, in spite of the influence of the unmodelled perturbation signal, $v(t)$ the derived nonlinear discrete-time duty ratio controller performs remarkably well.

The robustness of the proposed feedback control scheme was also tested with respect to a class of modelling errors, represented by significant, but temporary, circuit parameter variations. We performed several simulations, which included a sudden, unmodelled, load resistance variation. Figure 6 depicts the responses of the closed-loop regulated inductor current to variations of 0%, 20%, 40% and 80%, above the nominal load resistance value R, while the plant was still being affected by the external stochastic perturbation noise $v(t)$. Such variations were left to occur in the time

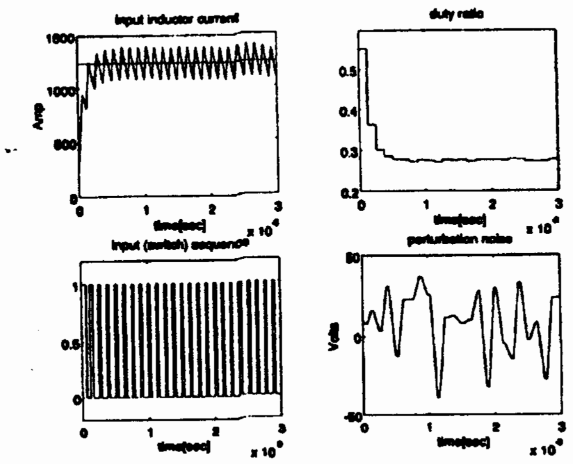

Figure 5. Simulation results of PWM regulation of perturbed buck-derived converter.

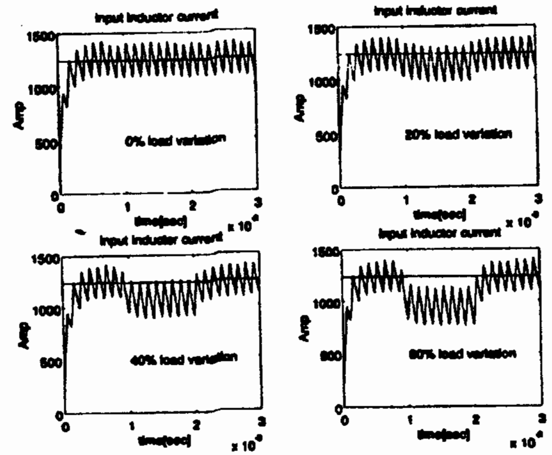

Figure 6. Simulation results of PWM regulation of perturbed buck-derived converter subject to several load variations.

interval between 0·001 s and 0·002 s. It can be seen that, up to a 20% load variation, the performance of the controller is quite robust. The controller is seen to drive the input current response to its original preassigned value, right after the load perturbation is over.

2.5. *The boost-derived converter*

In this section we briefly summarize the derivation of an implicit nonlinear feedback regulator for the indirect output voltage stabilization of the boost derived converter, shown in Fig. 7 (see Rashid 1993). The feedback loop synthesizing the required duty ratio function is based on a desired steady-state average input current value X.

The boost-derived switch regulated model

$$\dot{x} = -\frac{R}{L}x(1-u)+\frac{E}{L}$$
$$= -\theta_1(1-u)x+\theta_2 \quad (2.20)$$

PWM feedback regulation strategy for the switched position

$$u(t) = \begin{cases} 1 & \text{for } t_k \leqslant t < t_k+\mu(t_k)T \\ 0 & \text{for } t_k+\mu(t_k)T \leqslant t < t_k+T \end{cases} \quad (2.21)$$

$$t_{k+1} = t_k + T; \quad k = 0, 1, 2, \ldots$$

An exact discretization of the PWM regulated boost-derived dynamics

$$x(t_k+\mu(t_k)T) = \theta_2\mu(t_k)T + x(t_k) \quad (2.22)$$

$$x(t_k+T) = \exp\{-\theta_1 T(1-\mu(t_k))\}[\theta_2\mu(t_k)T + x(t_k)] + \frac{\theta_2}{\theta_1}[1-\exp\{\theta_1 T(1-\mu(t_k))\}]$$
$$= \Psi_1^{(1-\mu(t_k))}x(t_k) + \Psi_1^{(1-\mu(t_k))}[\mu(t_k)\Psi_3 - \Psi_2] + \Psi_2 \quad (2.23)$$

with $\Psi_1 = \exp\{-\theta_1 T\}$, $\Psi_2 = \theta_2/\theta_1$ and $\Psi_3 = \theta_2 T$.

Steady-state value of the sampled input current

$$x_\infty^- = \frac{\Psi_1^{(1-\mu_\infty)}[\mu_\infty\Psi_3 - \Psi_2] + \Psi_2}{1-\Psi_1^{(1-\mu_\infty)}} \quad (2.24)$$

Discrete-time dynamics of the 'upper corners' of the PWM regulated input current trajectory

$$X(t_{k+1}+\mu(t_{k+1})T) = \Psi_1^{(1-\mu(t_k))}x(t_k+\mu(t_k)T) + \Psi_3\mu(t_{k+1}) + \Psi_2(1-\Psi_1^{(1-\mu(t_k))}) \quad (2.25)$$

Steady-state value of the 'intersampling' peaks of the input current

$$x_\infty^+ = \frac{\mu_\infty\Psi_3 + \Psi_2(1-\Psi_1^{(1-\mu_\infty)})}{1-\Psi_1^{(1-\mu_\infty)}} \quad (2.26)$$

Steady-state 'ripple'

$$r_\infty = x_\infty^+ - x_\infty^- = \Psi_3\mu_\infty \quad (2.27)$$

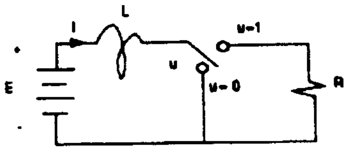

Figure 7. The boost-derived converter.

Steady-state average value of the input current trajectory

$$x_{av}(\infty) = x_\infty^- + \tfrac{1}{2} r_\infty \qquad (2.28)$$

$$x_{av}(\infty) = \frac{[\Psi_2 \mu_\infty (1+\Psi_1^{(1-\mu_\infty)}) + 2\Psi_3(1-\Psi_1^{(1-\mu_\infty)})]}{2(1-\Psi_1^{(1-\mu_\infty)})} \qquad (2.29)$$

Existence of steady-state duty ratio function for the desired value of steady-state average input current

$$\Psi_1^{1-\mu_\infty} = \frac{2X - \Psi_2\mu_\infty - 2\Psi_3}{2X+\Psi_2\mu_\infty - 2\Psi_3} \qquad (2.30)$$

The existence of a unique solution of (2.30), for μ_∞, follows from the fact that, as functions of μ_∞, the graph of the function on the left-hand side of (2.30) continuously increases in [0, 1], while the graph of the function on the right-hand side continuously decreases on such an interval. The graphs can be shown always to intersect each other, at most once, within the interval [0, 1].

Desired linear asymptotically stable closed loop dynamics

$$x(t_{k+1}) = \alpha(x(t_k) - x_\infty^-(X)) + x_\infty^-(X); \quad |\alpha| < 1 \qquad (2.31)$$

Implicit nonlinear feedback duty ratio synthesizer

$$\Psi_1^{(1-\mu_c(t_k))} = \frac{\alpha x(t_k) - \Psi_3 + (1-\alpha)x_\infty^-(X)}{x(t_k) - \Psi_3 + \mu_c(t_k)\Psi_2} \qquad (2.32)$$

A necessary and sufficient condition for the existence of a unique solution for the duty ratio

$$x_\infty^-(X) < x(t_k) + \frac{\Psi_3}{1-\alpha} \qquad (2.33)$$

Remark: Implicit feedback controllers of the form (2.32) demand an online numerical solution of the corresponding transcendental equation for the computed duty ratio function $\mu_c(t_k)$, once the sampled state, $x(t_k)$, is available. The calculation time required at each sampling instant may be quite significant, thus introducing an important limitation in the implementation of the feedback control law. In the simulations carried out below a provision for such a computational time may be easily incorporated by further restricting the duty ratio function to be higher than a certain fixed positive lower bound, say of value μ_L. In other words, instead of the hard limiting condition $\mu \in [0, 1]$ on the computed duty ratio function μ, one enforces the limitation, $\mu \in [\mu_L, 1]$, with $\mu_L > 0$. At the beginning of the sampling period, and during the fraction of the sampling interval, given by $\mu_L T$, control calculations must be performed. □

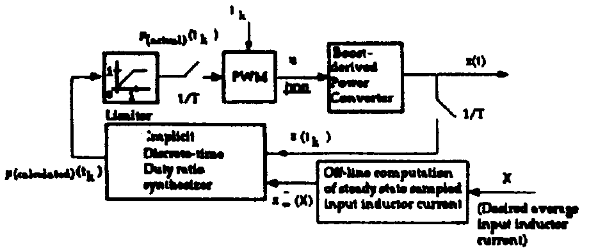

Figure 8. A PWM feedback regulation scheme for the boost-derived converter based on exact discretization.

2.6. *Simulation results*

In order to test the robustness of the previous derived PWM feedback regulation policy, based on exact discretization, we used the following noise perturbed model of the boost-derived converter

$$\dot{x} = -\frac{R}{L}(1-u)x + \left(\frac{E+v(t)}{L}\right) \qquad (2.34)$$

where $v(t)$ represented an unmodelled computer generated stochastic perturbation signal representing a noisy voltage source affecting the behaviour of the circuit. The values for the parameters defining the converter were taken to be the same as in the buck-derived case

$$R = 2{\cdot}8 \times 10^{-3}\,\Omega; \quad L = 1{\cdot}0 \times 10^{-3}\,\text{mH}; \quad E = 126\,v$$

The sampling period was chosen to be $T = 0{\cdot}125$ ms ($1/T = 8$ kHz) and the desired steady-state value of the average dynamics was set to be $X = 6000$ A. The eigenvalue for the closed loop linear dynamics, α, was set to be 0·3. The corresponding value of the steady-state input current was found to be $x_\infty^-(6000) = 5804$ A. The required steady-state average value of the input current as well as the steady-state values x_∞^+ and x_∞^- are well within the allowable range, which guarantees non-saturation of the actual duty ratio function.

Figure 9 depicts a typical simulated PWM feedback controlled trajectory for the input current arising from the perturbed model (2.34). This figure also shows the actual duty ratio function $\mu(t)$ and the corresponding switch position function $u(t)$. As can be seen from the duty ratio trajectory, in this case, the control calculation time could have been accommodated within the time interval $\mu_L T = 0{\cdot}0250$ ms, with $\mu_L = 0{\cdot}2$. At the end of the figure we show the perturbation signal $v(t)$. As shown, in spite of the unmodelled perturbation signal the derived nonlinear discrete-time duty ratio controller performs quite satisfactorily. Load variations similar to those carried out in the previous example were also performed, with similar results. The simulations are not shown in the interest of brevity.

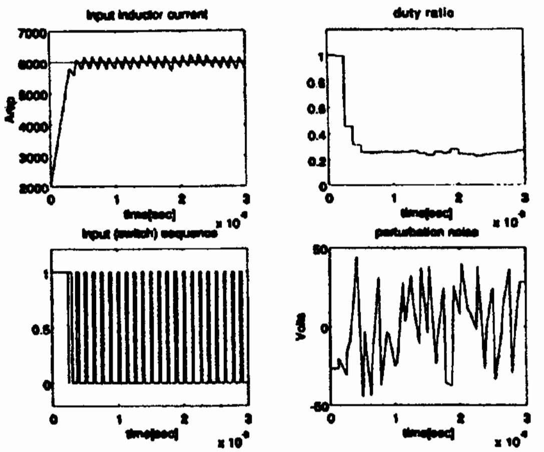

Figure 9. Simulation results of PWM stabilization of perturbed boost-derived converter.

3. Feedback signal tracking for derived DC-to-DC power converters via a discrete-time average model

In this section we present a discrete-time model for the average input currents of the derived DC-to-DC power converters presented in the previous section. The average models are obtained by a state elimination procedure on the expressions describing the mean value of the input current at the sampling times and at the end of the pulse associated with the switch position function. The state elimination leads to a non-kalmanian representation of the average input current dynamics. The models allow for a rather direct specification of the duty ratio synthesizers as dynamical feedback regulators.

We define the average value of the input current trajectory, at time t_k, as

$$z(t_k) = x(t_k) + \tfrac{1}{2}[x(t_k+\mu(t_k)T) - x(t_k)] = \tfrac{1}{2}[x(t_k) + x(t_k+\mu(t_k)T)] \tag{3.1}$$

Remark: Note that the association of the average value $\frac{1}{2}[x(t_k)+x(t_k+\mu(t_k)T)]$ with the time instant t_k, as $z(t_k)$, is clearly quite arbitrary. In fact one could choose an intermediate instant in the interval, $[t_k, t_k+\mu(t_k)T]$, to represent the corresponding value of time. However, this convention only complicates the presentation and does not substantially differ from the one we have chosen. □

Similarly, the average of value of the input current at time t_k+T is given by

$$z(t_{k+1}) = \tfrac{1}{2}[x(t_{k+1}) + x(t_{k+1}+\mu(t_{k+1})T)] \tag{3.2}$$

Average PWM dynamics can be obtained from (3.1) and (3.2) by means of a state elimination procedure. Indeed, note that for each one of the treated converters, the expression (3.1) for $z(t_k)$ can be rewritten in terms of the sampled state $x(t_k)$ and the duty ratio $\mu(t_k)$ at time t_k. In a similar fashion, expression (3.2) for $z(t_{k+1})$ may also be,

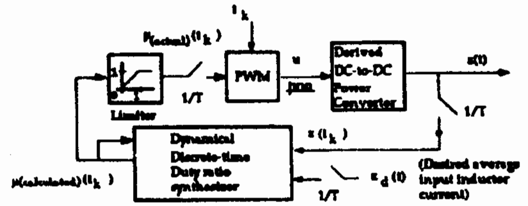

Figure 10. A discrete-time based PWM feedback regulation scheme for the signal tracking problem in derived DC-to-DC power converters.

ultimately, written in terms of $x(t_k)$ and the duty ratio function μ at times t_k and t_{k+1}, i.e. $\mu(t_k)$ and $\mu(t_{k+1})$. From the resulting equations for $z(t_k)$ and $z(t_{k+1})$ one may proceed to eliminate the state $x(t_k)$ thus obtaining $z(t_{k+1})$ as a function of $z(t_k)$ and the duty ratio function $\mu(t_k)$, $\mu(t_{k+1})$. Because of the linearity in the state associated with all of the invoked expressions, the resulting average PWM dynamics are also linear in the state $z(t_k)$. One obtains dynamics of the form

$$z(t_{k+1}) = \Phi_1(\mu(t_k), \mu(t_{k+1}))\, z(t_k) + \Phi_2(\mu(t_k), \mu(t_{k+1})) \tag{3.3}$$

which, evidently constitutes a non-kalmanian state representation, or more properly, a generalized state-space representation for the average input current dynamics (Fliess 1992).

By imposing desired linear discrete-time tracking error dynamics on the proposed average input current dynamics (3.3) a dynamical duty ratio synthesizer is readily obtained.

The dynamical feedback regulation scheme to be used for the PWM solution of the tracking problems, associated with the derived DC-to-DC power converters, as shown in Fig. 10.

In the following sections we present the average discrete-time input current model along with the duty ratio synthesizer for the solution of the corresponding tracking problem for the two derived converters studied in this article.

3.1. *The buck-derived converter*

The dynamics of the average value of the input current $z(t_k)$ is obtained by following the state elimination procedure outlined at the beginning of this section. The average model results in

$$z(t_{k+1}) = \Psi_1\left(\frac{1+\Psi_1^{\mu(t_{k+1})}}{1+\Psi_1^{\mu(t_k)}}\right) z(t_k) + \tfrac{1}{2}\Psi_2\left\{\frac{\Psi_1^{1-\mu(t_k)}(1-\Psi_1^{\mu(t_k)})(1+\Psi_1^{\mu(t_{k+1})}) + (1-\Psi_1^{\mu(t_{k+1})})(1+\Psi_1^{\mu(t_k)})}{1+\Psi_1^{\mu(t_k)}}\right\} \tag{3.4}$$

An interesting fact, which makes the average model (3.4) non-traditional, is that the resulting recursion formula, obtained for the evolution of the average current values, requires the values of the duty ratio function, i.e. of the control input variable, at two consecutive sampling instants.

Suppose $\lim_{k\to\infty}\mu(t_k)$ exists, and assume it is given by the constant value μ_∞, then, evidently, $\lim_{k\to\infty}\mu(t_k) = \lim_{k\to\infty}\mu(t_{k+1}) = \mu_\infty$ and the corresponding steady-state value, z_∞, of the average input current, as computed from (3.4), is given by

$$z_\infty = \frac{1}{2}\left(\frac{\Psi_3}{1-\Psi_1}\right)(1-\Psi_1^{\mu_\infty})(1+\Psi_1^{1-\mu_\infty}) \qquad (3.5)$$

which exactly corresponds to the steady-state value $x_{av}(\infty)$ computed in (2.13) and found from slightly different considerations.

3.2. *An average input current tracking problem for the buck-derived converter*

Let $z_d(t)$ represent a desired time-varying reference input signal. It is desired to have the average input current $z(t_k)$ asymptotically track the sampled values, $z_d(t_k)$ of the reference input signal.

Let $e(t_k)$ denote the average tracking error at time t_k, given by $e(t_k) = z(t_k) - z_d(t_k)$. The following tracking error dynamics may then be imposed on the closed loop system

$$e(t_{k+1}) = \alpha e(t_k); \quad |\alpha| < 1 \qquad (3.6)$$

In terms of the average input current, such dynamics result in the following expression

$$z(t_{k+1}) = \alpha(z(t_k) - z_d(t_k)) + z_d(t_{k+1}) \qquad (3.7)$$

Since $z_d(t_{k+1})$ is assumed to be known beforehand, the preceding equation does not have the connotation of an acausal system.

Substituting the right-hand side of expression (3.4) into (3.7), and solving for $\mu(t_{k+1})$ one obtains the following time-varying dynamical nonlinear feedback controller, or duty ratio synthesizer, for the average input current tracking problem

$$\mu(t_{k+1}) = \frac{1}{\ln\Phi_1}\ln\left\{\frac{2(1+\Psi_1^{\mu(t_k)})}{\Psi_3[\Psi_1^{1-\mu(t_k)}(1-\Psi_1^{\mu(t_k)})-(1+\Psi_1^{\mu(t_k)})]+2\Psi_1 z(t_k)}\right.$$
$$\times\left[1-\frac{1}{2}\Psi_3\left(\frac{\Psi_1^{1-\mu(t_k)}(1-\Psi_1^{\mu(t_k)})+(1+\Psi_1^{\mu(t_k)})}{1+\Psi_1^{\mu(t_k)}}\right)\right.$$
$$\left.\left.-\left(\frac{\Psi_1}{1+\Psi_1^{\mu(t_k)}}-\alpha\right)z(t_k)-\alpha z_d(t_k)+z_d(t_{k+1})\right]\right\} \qquad (3.8)$$

Remark: The initialization of the above controller requires the specification of $z(t_0)$ which, in fact, involves knowledge of both $x(t_0)$ and $\mu(t_0)$. The initial input current $x(t_0)$ may be measured, or simply set by previously discharging the energy initially stored in the inductor. The initial duty ratio, $\mu(t_0)$, must then be arbitrarily chosen. This implies that the quantities $x(t_0)$ and $x(t_0+\mu_0 T)$ are assumed to be initially known. Note also that a PWM control policy based on the average model (3.3) necessarily requires online measurements of the average input current, i.e. values of $x(t)$ have to be measured both at the end of each pulse, within the sampling interval, and at the beginning of each sampling period. □

The computed duty ratio function must never exceed the natural limiting values of the actual duty ratio function, represented by the closed interval [0, 1]. Possibilities for saturation of the actual duty ratio function depend on the time derivative of the desired input reference signal, on the imposed closed-loop eigenvalue and on the

circuit parameters themselves. By more conventional averaging techniques (see Sira-Ramirez *et al.* 1993) one can obtain an estimate of the tracking limitations of the circuit.

3.3. *Simulation results*

The previously derived PWM feedback tracking policy, based on exact discretization and the introduced average model was used on the same noise perturbed model (2.19) of §2.4.

It is desired to track a 'trapezoidal' reference input signal, as shown in Fig. 11, with $z_{max} = 1237$ A. This reference signal is expressed as

$$z_d(t) = \begin{cases} 1237t & \text{for } 0 < t \leq 1 \text{ ms} \\ 1237 & \text{for } 1 < t \leq 2 \text{ ms} \\ 1237-1237(t-2) & \text{for } 2 < t \leq 3 \text{ ms} \end{cases}$$

Figure 12 depicts the simulated PWM feedback controlled trajectory for the input current arising from the controlled perturbed model (2.19). This figure also shows the actual duty ratio function $\mu(t)$ and the corresponding switch position function $u(t)$. In this figure we also show the perturbation signal $\iota(t)$. As shown, in spite of the unmodelled perturbation signal, the derived nonlinear discrete-time duty ratio controller tracks remarkably well the desired reference signal, as long as the computed duty ratio function does not take values outside the interval [0, 1]. Note that close to the end of the tracking horizon the actual duty ratio function μ is seen to 'saturate' to the value of zero. The problem is circumvented by suitably lowering the maximum value of the desired reference signal $z_d(t)$ as well as its slope on the 'descending' portion of the trapezoid.

3.4. *The boost-derived converter*

Generalized state representation of average input current dynamics

$$z(t_{k+1}) = \Psi_1^{1-\mu(t_k)}z(t_k) - \Psi_1^{1-\mu(t_k)}\Psi_2 + \Psi_2 + \tfrac{1}{2}\Psi_3(\Psi_1^{1-\mu(t_k)}\mu(t_k) + \mu(t_{k+1}) \qquad (3.9)$$

Steady-state value of average input current

$$z_\infty = \frac{[\Psi_3\mu_\infty(1+\Psi_1^{(1-\mu_\infty)})+2\Psi_2(1-\Psi_1^{(1-\mu_\infty)})]}{2(1-\Psi_1^{(1-\mu_\infty)})} \qquad (3.10)$$

which coincides with (2.29).

Desired closed loop linear dynamics

$$z(t_{k+1}) = \alpha(z(t_k) - z_d(t_k)) + z_d(t_{k+1}) \qquad (3.11)$$

Dynamical duty ratio synthesizer

$$\mu(t_{k+1}) = \frac{2}{\Psi_3}\left[(\alpha-\Psi_1^{1-\mu(t_k)})z(t_k) - \Psi_1^{1-\mu(t_k)}\left(\frac{1}{2}\Psi_3\mu(t_k)-\Psi_2\right)-\Psi_2-\alpha z_d(t_k)+z_d(t_{k+1})\right] \qquad (3.12)$$

Remark: The initialization of the above controller is carried out in a manner similar to that corresponding to the buck-derived converter (see the Remark of §3.2).

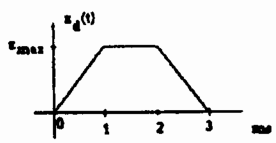

Figure 11. Desired trapezoidal input current signal.

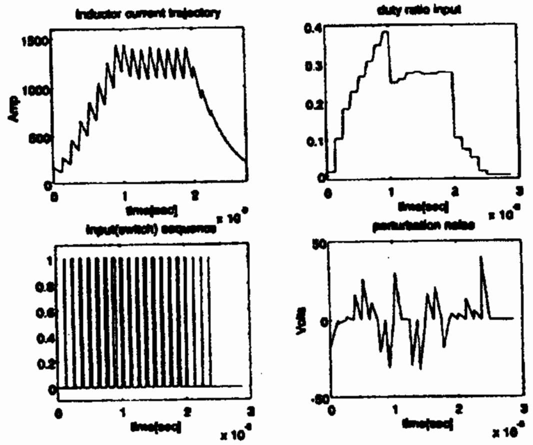

Figure 12. Simulation results for trapezoidal signal tracking problem for the perturbed buck-derived converter.

3.5. *Simulation results*

The previously derived PWM feedback tracking policy, based on exact discretization and the introduced average model, was used on the noise perturbed model (2.34) of the boost-derived converter presented in §2.8.

It was required to track a 'trapezoidal' reference input signal, similar to that shown in Fig. 11, with $z_{max} = 6000$ A. In accordance with the 'step-up' character of the derived-boost converter, the reference signal $z_d(t)$ to be tracked was specified by the following expression

$$z_d(t) = \begin{cases} 4500 + 1500t & \text{for } 0 < t \leqslant 1 \text{ ms} \\ 6000 & \text{for } 1 < t \leqslant 2 \text{ ms} \\ 6000 - 1500(t-2) & \text{for } 2 < t \leqslant 3 \text{ ms} \end{cases}$$

Figure 13 depicts the simulated PWM feedback controlled trajectory for the input current arising from the controlled perturbed model (2.34) of the boost-derived converter. This figure also shows the actual duty ratio function $\mu(t)$ and the corresponding switch position function $u(t)$. In this figure we also show the

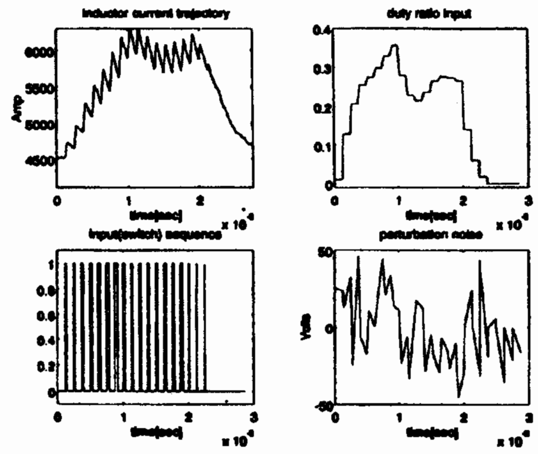

Figure 13. Simulation results for trapezoidal signal tracking problem for the perturbed boost-derived converter.

perturbation signal $v(t)$. As shown, in spite of the unmodelled perturbation signal, the derived nonlinear discrete-time duty ratio controller tracks remarkably well the desired reference signal, as long as the computed duty ratio function does not take values outside the interval [0, 1]. Note that close to the end of the tracking horizon the actual duty ratio function μ is seen to 'saturate' to the value of zero. The problem may be circumvented by suitably lowering the maximum value of the desired reference signal $z_d(t)$ as well as the slope on the 'descending' portion of the trapezoid.

4. Conclusions

In this article an exact discretization scheme has been proposed for the input current stabilization and tracking tasks in perfectly known derived DC-to-DC power supplies of the buck and boost types. The complexities arising in the stabilization problem associated with such devices are related, fundamentally, to the highly nonlinear form of the derived duty ratio compensators. For the boost converter, the controller cannot be found explicitly and a transcendental equation must be solved online, at each sampling instant, on the basis of the (sampled) state of the converter circuit. The signal tracking problem is solved by means of a non-kalmanian state representation of the average input current. Explicit dynamical controllers could then be found from exact discrete-time linearization schemes imposed on the average controlled models. The results are appropriate in high power systems where the sampling frequency may be limited.

Some of the difficulties encountered in the one-dimensional converter cases treated here become, not surprisingly, much harder when dealing with the two-dimensional models of traditional DC-to-DC power converters. In such cases, the symbolic manipulation tasks associated with the solution of the stabilization problems become particularly intricate, even with the help of very efficient computer packages such as Maple, or Mathematica.

Simulation studies have revealed a certain degree of sensitivity of the proposed exact control schemes to sudden, unmodelled, load parameter variations. As a topic for further research, the case of stabilization and tracking problems for derived converters with uncertain parameters is, therefore, of particular practical interest and one for which efficient nonlinear discrete-time adaptive and robust control techniques must be developed.

Acknowledgments

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Dynamical adaptive pulse-width-modulation control of DC-to-DC power converters: a backstepping approach

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Adaptive regulation of pulse-width-modulation (PWM) controlled DC-to-DC power supplies is proposed using a suitable combination of dynamical input-output linearization and the 'backstepping' controller design method. A nominal parameter, input-dependent, state coordinate transformation of the average PWM converter models leads to a type of pure parameter feedback canonical form associated with the Fliess generalized observability canonical form of such average models. A backstepping design procedure can then be immediately devised which leads to a dynamical adaptive regulation scheme for the generation of the stabilizing duty ratio function. The validity of the proposed approach, regarding control objectives and robustness with respect to unmodelled, yet unmatched, and bounded stochastic perturbation inputs, is tested through digital computer simulations.

1. Introduction

Feedback regulation of switchmode DC-to-DC power converters is usually accomplished by means of pulse-width-modulation (PWM) feedback strategies. For the fundamental background of this important subject the reader is referred to conference proceedings (such as the yearly Power Specialist Conference Records, the multi-volume series edited by Middlebrook and Cúk (1981), or the remarkable collection of articles recently edited by Bose (1992). Also, useful material may be found in specialized books such as Kassakian *et al.* (1991), Severns and Bloom (1985) and Csaki *et al.* (1983).

PWM feedback regulation strategies for DC-to-DC power converters are usually based on perfect knowledge of the converter parameters (see, among many other authors, the articles by Sira-Ramírez and co-workers (1989, 1991, 1992). This fundamental assumption is sometimes invalid due to imprecise knowledge of the values of the converter circuit components as well as of the external voltage source. The situation is often due to either measurement errors, or unavoidable ageing effects on the circuit components. Automatic control problems which efficiently handle uncertainty in the system parameter values usually require adaptive solutions employing different forms of the so-called 'uncertainty equivalence principle' (Sastry and Bodson 1989). In other words, the controller is designed as if the system parameters were perfectly known, and then the values of the parameters appearing in the controller expression are regarded as tunable, in an online fashion. Parameter

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tuning is accomplished by the specification of an updating, or parameter adaptation, law designed to simultaneously guarantee the demands of the regulation objectives and the stability of the adaptation process.

Adaptive feedback control techniques for PWM controlled DC-to-DC power supplies have been explored by Sira-Ramirez *et al.* (1993 a, b). The approach in these contributions incorporated an extension of the results found in Sastry and Isidori (1989), for the adaptive stabilization of partially linearizable, minimum-phase, discontinuously controlled nonlinear systems.

In this paper a rather different adaptive feedback strategy is adopted by resorting to an approach inspired by the recently introduced adaptive backstepping controller design methodology. Backstepping adaptation was developed for the regulation of a large class of state linearizable nonlinear systems exhibiting constant, but otherwise unknown, parameter values. The basic ideas and rather useful variations, of the backstepping adaptive design procedure can be found in the excellent research articles of Kanellakopoulos *et al.* (1991 a, b, c) and by Krstić *et al.* (1992), to which the reader is referred for enlightening details.

We specifically assume that the circuit converter components are only nominally known and that their constant discrepancies from the given nominal values are totally unknown. The use of an input-dependent, nominal parameter based, input–output linearizing state coordinate diffeomorphism for the unknown system produces an imperfectly transformed system in generalized phase variables (Fliess 1989). The structure of the resulting systems, in generalized phase variables, strongly resembles the pure parameter feedback canonical form, presented by Kanellakopoulos (1991 a), except for the presence of the control inputs in some of the 'regressor vectors', as well as the control input time derivative in the transformed system equations.

Computation of the feedback controllers, and of the associated incremental parameter update laws, is then carried out by resorting to a backstepping calculation procedure applied to the obtained generalized pure incremental parameter feedback canonical form. The net result is that one yields adaptive dynamical duty ratio synthetizers, rather than traditional static feedback compensators. The advantage of dynamically generated duty ratio control signals lies in the enhanced smoothed character of this important feedback regulation signal during the actual (i.e. discontinuous) operation of the converters. Smoothing of the duty ratio function increases the precision, and qualitative performance features, of the closed-loop behaviour of the DC-to-DC power converter circuits.

Section 2 is devoted to revisiting, via a 'boost' converter example, the fundamentals of the input–output linearization of PWM controlled DC-to-DC power converters by means of a dynamical feedback duty ratio synthesizer. We assume that all parameters in the system are perfectly known. The input–output linearization scheme achieves indirect regulation of the average output capacitor voltage by means of average input inductor current regulation. This strategy, which essentially involves a 'change of output' effectively avoids the non-minimum phase problem in the direct regulation of the output capacitor voltage variable. This method has already been used by Sira-Ramirez *et al.* (1991, 1993) and it was later justified, from a general viewpoint, by Benvenuti *et al.* (1992) for nonlinear systems and by Fliess and Sira-Ramirez (1993) for linear systems.

Section 3 presents the developments leading to a dynamical adaptive PWM control strategy for DC-to-DC power supplies of the 'boost' and 'buck-boost' types with unknown incremental parameter values. Computer simulations are presented which

clearly indicate the effectiveness, and robustness, of the proposed adaptive feedback regulation scheme with respect to unmodelled, and unmatched, external stochastic perturbation inputs of bounded nature. Section 4 contains the conclusions and suggestions for further work in this area.

2. A nominal input–output linearization strategy for DC-to-DC power converters

This section contains the developments leading to dynamical feedback duty ratio synthesizers for the PWM stabilization of nominal average models of DC-to-DC power converters. The scheme, already exploited by Sira-Ramirez (1991), is presented here only for the purpose of making the article self-contained. The fundamental idea is to obtain a generalized nonlinear phase variable representation of the input–output behaviour of the average circuit for which the control synthesis problem is straightforward. Due to non-minimum phase problems associated with the output capacitor voltage variable, the regulated output is chosen as the input inductor current. Thus, indirect output voltage regulation is achieved.

2.1. *Boost converter*

Consider the boost converter circuit shown in Fig. 1. This circuit is described by the state equation model

$$\left.\begin{aligned} \dot{I}(t) &= -\frac{1}{L}(1-u)\,V(t)+\frac{E}{L} \\ \dot{V}(t) &= \frac{1}{C}(1-u)\,I(t)-\frac{1}{RC}V(t) \\ y(t) &= I(t) \end{aligned}\right\} \tag{1}$$

where I and V represent, respectively, the input inductor current and the output capacitor voltage variables. The positive quantity E is the external input voltage. The variable u denotes the switch position function, acting as a control input which takes values in the discrete set $\{0, 1\}$. The output y of the system is represented by the input inductor current I.

We define

$$\theta_1 = \frac{1}{L}, \quad \theta_2 = \frac{1}{C}, \quad \theta_3 = \frac{1}{RC}, \quad \theta_4 = \frac{E}{L} \tag{2}$$

as the system parameters assumed to be nominally known.

A PWM feedback control strategy for the regulation of the boost converter circuit is typically given by the following prescription of the switch position function (Sira-Ramírez 1989 a, Sira-Ramírez *et al.* 1991):

$$\left.\begin{aligned} u &= \begin{cases} 1, & \text{for } t_k \leqslant t < t_k+\mu(t_k)T \\ 0, & \text{for } t_k+\mu(t_k)T \leqslant t < t_k+T \end{cases} \\ t_{k+1} &= t_k+T, \quad k=0,1,\ldots \end{aligned}\right\} \tag{3}$$

where t_k represents a sampling instant; the parameter T is the fixed sampling period, also called the duty cycle; and the sampled values of the state vector $x(t)$ of the

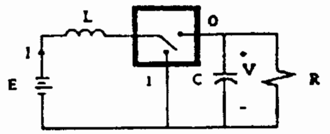

Figure 1. Boost converter circuit.

converter are denoted by $x(t_k)$. The function $\mu(\cdot)$ is the duty ratio function acting as a truly feedback policy. The value of the duty ratio function $\mu[t_k]$ determines at every sampling instant t_k the width of the upcoming 'pulse' (switch at the position $u = 1$) as $\mu[t_k]T$. The actual duty ratio function $\mu(\cdot)$ is evidently a function limited to the closed interval [0, 1] on the real line.

The control problem associated with the stabilization of the discontinuously controlled system (1) and (3) towards some (feasible) prespecified constant desired equilibrium point, consists of specifying the duty ratio function μ as a static, or dynamical, feedback control policy, i.e. as a function of the state vector x, or as the solution of a time-varying differential equation based on the measured values of the state x. As formulated, the problem of synthesizing a suitable duty ratio function μ is quite involved, owing to the difficulty in performing an exact discretization of the PWM system model (1) and (3). A conceptually useful, and practical, alternative consists of resorting to the infinite frequency average PWM model, also known as the state space average model of the PWM controlled converter (1) and (2) (Kassakian *et al.* 1991, Middlebrook and Ćuk 1981). The assumption of an infinite sampling frequency results in a smooth linear average system model of (1) in which the duty ratio function μ is readily interpreted as a control input to the average system in formal replacement of the switch position function u. In fact, the duty ratio function becomes the equivalent control input in the corresponding sliding mode (Utkin 1978) interpretation of the obtained idealization (Sira-Ramírez 1989b).

The above idealization has the fundamental advantage of reducing the duty ratio synthesis problem to a standard nonlinear feedback control design problem in which the duty ratio function acts as the required feedback control input. Any of the well known static, or dynamical, feedback controller design procedures established in the recent literature (Isidori 1989, Rugh 1986, Fliess 1989) can be applied to the nonlinear average model of the circuit to obtain the required duty ratio function as a nonlinear feedback control law.

Consider, then, the following nominal average PWM model of the boost converter circuit:

$$\left.\begin{aligned} \dot{\zeta}_1 &= -\Theta_1(1-\mu)\zeta_2+\Theta_4 \\ \dot{\zeta}_2 &= \Theta_2(1-\mu)\zeta_1-\Theta_3\zeta_2 \\ \eta &= \zeta_1 \end{aligned}\right\} \quad (4)$$

where ζ_1 and ζ_2 represent the averaged values of the original state variables I and V. The average output variable ζ_1 is here denoted by η.

For a given constant value $\mu = U$ of the duty ratio function, the correponding equilibrium values of the average state variables of the circuit are obtained as

$$\zeta_1(U) = \frac{\Theta_3\Theta_4}{\Theta_1\Theta_2(1-U)^2}, \quad \zeta_2(U) = \frac{\Theta_4}{\Theta_1(1-U)} \quad (5)$$

Note that by straightforward elimination of the constant parameter U, in the set of equations (5), the equilibrium values for ζ_1 and ζ_2 are related by

$$\zeta_1(U) = \frac{\theta_3\theta_1}{\theta_2\theta_4}\zeta_2^2(U)$$

Hence, the prescription of a desired steady-state value for the average output capacitor voltage $\zeta_2(U)$ uniquely determines both the required constant value of the duty ratio function U and the corresponding average value for the input inductor current $\zeta_1(U)$. This simple fact allows the indirect regulation of the average output capacitor voltage of the converter through regulation of the average input inductor current. For this reason our control problem will be formulated in terms of achieving a desired steady-state equilibrium value, denoted by Y, for the average input inductor current ζ_1.

Consider, then, the following, locally invertible, nominal input-dependent state coordinate transformation of the nonlinear average model (4):

$$x_1 = \zeta_1, \quad x_2 = -\Theta_1(1-\mu)\zeta_2+\Theta_4 \quad (6)$$

and its associated inverse transformation is

$$\zeta_1 = x_1, \quad \zeta_2 = \frac{\Theta_4 - x_2}{\Theta_1(1-\mu)} \quad (7)$$

Using the above state coordinate transformation (6) and (7), on the average circuit equations (4), one obtains the following Fliess generalized observability canonical form of the average boost converter model:

$$\left.\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -\theta_1\theta_2(1-\mu)^2x_1-\theta_3(x_2-\theta_4)-\frac{\dot{\mu}}{1-\mu}(x_2-\theta_4) \\ \eta &= x_1 \end{aligned}\right\} \quad (8)$$

The zero dynamics associated with the equilibrium point $x_1 = \zeta_1(U)$ and $x_2 = 0$ are given by the first-order dynamics,

$$\dot{\mu} = -\frac{\theta_3}{(1-U)^2}(1-\mu)(\mu-U)(2-\mu-U) \quad (9)$$

The zero dynamics exhibits two unstable equilibrium points, $\mu = U-2 < 0$ and $\mu = 1$. The only stable equilibrium point $\mu = U$ makes the average system minimum phase in the region of interest.

Let the desired behaviour of the transformed system (8) be prescribed by the following asymptotically stable second-order linear time-invariant dynamics:

$$\left.\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= -2\xi\omega_n x_2-\omega_n^2(x_1-Y) \\ \eta &= x_1 \end{aligned}\right\} \quad (10)$$

where ξ and ω_n are design parameters that reflect our need for particular transient features of the average regulated output $\eta = x_1$. The constant Y represents a desired steady-state closed-loop equilibrium value of the average output variable η. In our case of perfectly known parameter values, the desired output is obtained as $Y = \zeta_1(U) = \Theta_3\Theta_4/\Theta_1\Theta_2(1-U)^2$.

From (8) and (10) one readily obtains an expression for the dynamical feedback controller yielding the required linearizing and stabilizing duty ratio function. In terms of the transformed variables x_1 and x_2 the duty ratio function μ is obtained as the solution of the time-varying differential equation

$$\dot{\mu} = \frac{1-\mu}{x_2-\theta_4}[2\xi\omega_n x_2 + \omega_n^2(x_1 - Y) - \theta_1\theta_2(1-\mu)^2 x_1 - \theta_3(x_2-\theta_4)] \quad (11)$$

In terms of the average state variables ζ_1 and ζ_2, the dynamical duty ratio synthesizer is equivalently obtained as

$$\dot{\mu} = \frac{1}{\theta_1\zeta_2}[(\theta_1\theta_2(1-\mu)^2 - \omega_n^2)\zeta_1 + (2\xi\omega_n\theta_1 - \theta_1\theta_3)(1-\mu)\zeta_2 - 2\xi\omega_n\theta_4 + \omega_n^2 Y] \quad (12)$$

The values of μ, obtained from the on-line solution of equation (11), actually represent the computed duty ratio function, which we still denote by μ. However, in order to implement this dynamical feedback control strategy on the actual (i.e. discontinuously regulated) converter system the values of μ must be necessarily limited to the closed interval [0, 1]. We then define the actual duty ratio function denoted by μ_a as

$$\mu_a(t) = \begin{cases} 1, & \text{for } \mu(t) \geq 1 \\ \mu(t), & \text{for } 0 < \mu(t) < 1 \\ 0, & \text{for } \mu(t) \leq 0 \end{cases} \quad (13)$$

Finally, it should be noted that when the sampling period T is sufficiently small, the actual values of the state variables V and I, rather than their average values, ζ_1 and ζ_2, may be used for the on-line solution of the computed duty ratio funtion μ. This procedure is precisely at the heart of the state average method for PWM designs. A theoretical justification of this procedure has been given by Sira-Ramírez *et al.* (1993).

Summarizing, a dynamical PWM controller achieving the asymptotic stabilization of the average input inductor current of the boost converter circuit (1) to a desired constant equilibrium value $y = Y$, is given by

$$\left.\begin{aligned} u &= \begin{cases} 1, & \text{for } t_k \leq t < t_k + \mu_a(t_k)T \\ 0, & \text{for } t_k + \mu_a(t_k)T \leq t < t_k + T \end{cases} \\ t_{k+1} &= t_k + T, \quad k = 0, 1, \ldots \end{aligned}\right\} \quad (14)$$

where $\mu_a(t_k)$ represents the sampled values of the actual duty ratio function at time t_k, given by (13). The computed duty ratio function $\mu(t)$ is obtained from the on-line solution of

$$\dot{\mu} = \frac{1}{\theta_1 V(t)}[(\theta_1\theta_2(1-\mu)^2 - \omega_n^2)I(t) + (2\xi\omega_n\theta_1 - \theta_1\theta_3)(1-\mu)V(t) - 2\xi\omega_n\theta_4 + \omega_n^2 Y] \quad (15)$$

3. An adaptive feedback control strategy for indirect output voltage regulation in DC-to-DC power converters

3.1. *Boost converter*

With reference to the boost converter circuit, consider the following version of the average boost converter model:

$$\left.\begin{aligned} \dot{\zeta}_1 &= -\theta_1(1-\mu)\zeta_2 + \theta_4 \\ \dot{\zeta}_2 &= \theta_2(1-\mu)\zeta_1 - \theta_3\zeta_2 \\ y &= \zeta_1 \end{aligned}\right\} \quad (16)$$

where the θ_i, $i = 1, 2, 3, 4$, represent the actual parameter values, modelled by

$$\theta_i = \Theta_i + \Delta\theta_i, \quad i = 1, 2, 3, 4 \quad (17)$$

with Θ_i, $i = 1, 2, 3, 4$, being the nominal parameters, assumed to be perfectly known. The quantities $\Delta\theta_i$, $i = 1, 2, 3, 4$, denote the corresponding constant, but unknown, incremental variations of the parameters from their nominal values.

Consider, then, the nominal input-dependent state coordinate transformation, used for exact input output linearization of the average boost converter model in the preceding section:

$$\left.\begin{aligned} x_1 &= \zeta_1 \\ x_2 &= -\Theta_1(1-\mu)\zeta_2 + \Theta_4 \end{aligned}\right\} \quad (18)$$

Clearly this control-parametrized transformation is invertible everywhere, except when the duty ratio function μ is identically equal to one. The associated inverse transformation is readily found to be

$$\left.\begin{aligned} \zeta_1 &= x_1 \\ \zeta_2 &= \frac{\Theta_4 - x_2}{\Theta_1(1-\mu)} \end{aligned}\right\} \quad (19)$$

When the state coordinate transformation (18) and (19) is applied to the actual boost converter model (16) and (17), the transformed system is not quite in the Fliess generalized observability canonical form (8), but rather in what we call the generalized pure incremental parameter feedback canonical form. The transformed system is easily shown to be given by

$$\begin{aligned} \dot{x}_1 &= x_2 + \Delta\theta^T\gamma_1(x_1, x_2) \\ \dot{x}_2 &= \dot{\mu}\left(\frac{-x_2+\Theta_4}{(1-\mu)}\right) - \Theta_1\Theta_2(1-\mu)^2 x_1 - \Theta_3(x_2 - \Theta_4) + \Delta\theta^T\gamma_2(x_1, x_2, \mu) \\ y &= x_1 \end{aligned} \quad (20)$$

where

$$\begin{aligned} \gamma_1^T(x_1, x_2) &= \left[\frac{x_2 - \Theta_4}{\Theta_1} \quad 0 \quad 0 \quad 1\right] \\ \gamma_2^T(x_1, x_2, \mu) &= [0 \quad -\Theta_1(1-\mu)^2 x_1 \quad -x_2 + \Theta_4 \quad 0] \\ \Delta\theta^T &= [\Delta\theta_1 \quad \Delta\theta_2 \quad \Delta\theta_3 \quad \Delta\theta_4] \end{aligned} \quad (21)$$

For ease of reference we let

$$f(x,\mu,\dot{\mu},\Theta) = \dot{\mu}\left(\frac{-x_2+\Theta_4}{1-\mu}\right) - \Theta_1\Theta_2(1-\mu)^2 x_1 - \Theta_3(x_2-\Theta_4) \tag{22}$$

The transformed system (20) strongly resembles the more traditional pure parameter feedback canonical form developed by Kanellakopoulos *et al.* (1991), except for the presence of the control input (duty ratio) μ in the regressor vector γ_2 and the presence of the first-order time derivative $\dot{\mu}$ of the control input in the second differential equation. This control input derivative will, in fact, act as the actual control input, whereas the control input μ may be regarded as playing the role of an additional state variable.

We now proceed to apply the adaptive backstepping algorithm, as developed by Kanellakopoulos *et al.* (1991), to the transformed model (20) and (21).

Step 0: Let Y be the desired steady-state equilibrium value of the output variable x_1 and define the stabilization error z_1 as

$$z_1 = x_1 - Y \tag{23}$$

Step 1: Consider the stabilization error equation

$$\dot{z}_1 = x_2 + \Delta\theta^T \gamma_1(x_1, x_2) \tag{24}$$

Suppose that the transformed variable x_2 can be used as a 'pseudo-control' in (24) and proceed to compute the required value of x_2 which stabilizes the error variable z_1 to zero. Computation of x_2 requires the unknown vector $\Delta\theta$. Using the 'certainty equivalence principle' (Kanellakopoulos *et al.* 1991) we replace the vector $\Delta\theta$ by an estimate in the fictitious stabilizing 'control law'. We proceed to devise also a parameter update law for the hypothesized estimate of the incremental parameter vector $\Delta\theta$, denoted here by $\widehat{\Delta\theta}^1$. This specification must result in simultaneous stable adaptation and convergence to zero of the error variable z_1. The superscript 1 will denote a first estimate of $\Delta\theta$. Let c_1 be a strictly positive design parameter. We then have as a plausible 'pseudo-control' action the following expression for x_2:

$$x_2 = -c_1 z_1 - \frac{\widehat{\Delta\theta}_1^1}{\Theta_1}(x_2 - \Theta_4) - \widehat{\Delta\theta}_4^1, \quad c_1 > 0 \tag{25}$$

where $\widehat{\Delta\theta}_i^1, i = 1, 4$, denotes a first estimate of $\Delta\theta_i, i = 1, 4$. A simple Lyapunov stability argument shows that the pseudo controller (25) and the update law

$$\dot{\widehat{\Delta\theta}}{}^1_\theta = {}_1\gamma_1(x_1, x_2) \tag{26}$$

yield a closed-loop stable system for which z_1 is guaranteed to converge to zero.

Since x_2 is not really a control input, one defines the pseudo-control error variable z_2 as the difference between x_2 and its required value, computed in (25). Let

$$z_2 = x_2 - \left[-c_1 z_1 - \frac{\widehat{\Delta\theta}_1^1}{\Theta_1}(x_2 - \Theta_4) - \widehat{\Delta\theta}_4^1\right] \tag{27}$$

By solving for x_2 from (27) and using (23), one obtains a new state coordinate

transformation defining, respectively, the stabilization error and the pseudo-control error variables, z_1 and z_2

$$z_1 = x_1 - Y$$

$$z_2 = c_1 z_1 + x_2\left(1 + \frac{\widehat{\Delta\theta}_1^1}{\Theta_1}\right) - \frac{\widehat{\Delta\theta}_1^1}{\Theta_1}\Theta_4 + \widehat{\Delta\theta}_4^1 \tag{28}$$

The corresponding inverse transformation is simply obtained as

$$x_1 = z_1 + Y$$

$$x_2 = \frac{\Theta_1}{\Theta_1 + \widehat{\Delta\theta}_1^1}\left[z_2 - c_1 z_1 + \frac{\widehat{\Delta\theta}_1^1}{\Theta_1}\Theta_4 - \widehat{\Delta\theta}_4^1\right] \tag{29}$$

The first equation of the transformed system may then be written as

$$\dot{z}_1 = z_2 - c_1 z_1 + (\Delta\theta_1 - \widehat{\Delta\theta}_1^1)\left[\frac{1}{\Theta_1 + \widehat{\Delta\theta}_1^1}\left[z_2 - c_1 z_1 + \frac{\widehat{\Delta\theta}_1^1}{\Theta_1}\Theta_4 - \widehat{\Delta\theta}_4^1\right] - \frac{\Theta_4}{\Theta_1}\right] + (\Delta\theta_4 - \widehat{\Delta\theta}_4^1) \tag{30}$$

which can be briefly expressed as

$$\dot{z}_1 = z_2 - c_1 z_1 + (\Delta\theta - \widehat{\Delta\theta}^1)^T w_1(z_1, z_2, \widehat{\Delta\theta}^1) \tag{31}$$

where

$$w_1^T(z_1, z_2, \widehat{\Delta\theta}^1) = \left[\left(\frac{1}{\Theta_1 + \widehat{\Delta\theta}_1^1}\left[z_2 - c_1 z_1 + \frac{\widehat{\Delta\theta}_1^1}{\Theta_1}\Theta_4 - \widehat{\Delta\theta}_4^1\right] - \frac{\Theta_4}{\Theta_1}\right) \quad 0 \quad 0 \quad 1\right] \tag{32}$$

Note that the update laws corresponding to $\widehat{\Delta\theta}_2^1$ and $\widehat{\Delta\theta}_3^1$ will yield constant values for such estimates. Note, moreover, that these two estimated parameters are not needed in this first step of the backstepping calculation.

We let W_1^1 denote the first component of the regressor vector $w_1(z_1, z_2, \widehat{\Delta\theta}_1^1)$.

The first adaptation law (26) may then be rewritten, in terms of the new error variables z_1 and z_2, as

$$\dot{\widehat{\Delta\theta}}{}^1_\theta = z_1 w_1(z_1, z_2, \widehat{\Delta\theta}^1) \tag{33}$$

Step 2: We proceed to complete the state coordinate transformation (28) and (29) of the original phase variables by considering now the differential equation for the pseudo controller error z_2. Using the definition of z_2 and z_1 and the first incremental parameter update laws for the involved components of the vector $\Delta\theta$, one obtains, after long but straightforward manipulations, the following expression:

$$\begin{aligned}\dot{z}_2 = {} & \left(1 + \frac{\widehat{\Delta\theta}_1^1}{\Theta_1}\right)\Bigg[-\frac{\dot{\mu}}{-\mu}(x_2(z_1, z_2, \widehat{\Delta\theta}^1) - \Theta_4) - \Theta_1\Theta_2(1-\mu)^2(z_1 + Y) \\ & - \Theta_3(x_2(z_1, z_2, \widehat{\Delta\theta}^1) - \Theta_4) - \Delta\theta_2\Theta_1(1-\mu)^2(z_1 + Y) - \Delta\theta_3(x_2(z_1, z_2, \widehat{\Delta\theta}^1) - \Theta_4)\Bigg] \\ & + c_1[z_2 - c_1 z_1 - \widehat{\Delta\theta}_1^1 W_1^1 - \widehat{\Delta\theta}_4^1] + z_1(1 + (W_1^1)^2) + c_1(\Delta\theta_1 W_1^1 + \Delta\theta_4)\end{aligned} \tag{34}$$

where $x_2(z_1, z_2, \widehat{\Delta\theta}^1)$ is given by the second equation of (29), which we do not substitute just to avoid lengthy intermediate equations. In the rest of this section x_2 stands for $x_2(z_1, z_2, \widehat{\Delta\theta}^1)$.

If we now equate the dynamics obtained in (34) for z_2 to the dynamics of an asymptotically stable behaviour for z_2, given by

$$\dot{z}_2 = -c_2 z_2, \quad c_2 > 0 \tag{35}$$

one can immediately solve for the required control input derivative $\dot{\mu}$ upon invoking, once more, the certainty equivalence principle. In this instance the unknown value of the vector $\Delta\theta$ will be replaced by a new vector of parameter estimates, denoted by $\widehat{\Delta\theta}^2$. One then obtains

$$\dot{\mu} = \frac{(1-\mu)\Theta_1}{(\Theta_1 + \widehat{\Delta\theta}_1^1)(x_2 - \Theta_4)} \Big\{ c_2 z_2 + c_1[z_2 - c_1 z_1 - (\widehat{\Delta\theta}^1)^T w_1(z_1, z_2, \widehat{\Delta\theta}^1)]$$
$$+ z_1[1 + (W_1^1)^2] + c_1(\widehat{\Delta\theta}^2)^T w_1(z_1, z_2, \widehat{\Delta\theta}^1) - \left(1 + \frac{\widehat{\Delta\theta}_1^1}{\Theta_1}\right)[\Theta_1 \Theta_2 (1-\mu)^2 (z_1 + Y)$$
$$+ \Theta_3(x_2 - \Theta_4) + \widehat{\Delta\theta}_2^2 \Theta_1 (1-\mu)^2 (z_1 + Y) + \widehat{\Delta\theta}_3^2 (x_2 - \Theta_4)] \Big\} \tag{36}$$

where $\widehat{\Delta\theta}_j^2$, $j = 2, 3$, represent the new estimates of the incremental parameter vector components $\Delta\theta_j$, $j = 2, 3$, and x_2 is given by (29). The expression for the dynamically controlled error variable z_2 (i.e. the closed-loop behaviour of z_2) is found to be

$$\dot{z}_2 = -c_2 z_2 + \left(1 + \frac{\widehat{\Delta\theta}_1^1}{\Theta_1}\right)[-(\Delta\theta_2 - \widehat{\Delta\theta}_2^2)\Theta_1 (1-\mu)^2 (z_1 + Y)$$
$$-(\Delta\theta_3 - \widehat{\Delta\theta}_3^2)(x_2 - \Theta_4)] + c_1[(\Delta\theta_1 - \widehat{\Delta\theta}_1^2) W_1^1 + (\Delta\theta_4 - \widehat{\Delta\theta}_4^2)] \tag{37}$$

which can also be briefly expressed as

$$\dot{z}_2 = -c_2 z_2 + (\Delta\theta - \widehat{\Delta\theta}^2)^T w_2(z_1, z_2, \mu, \widehat{\Delta\theta}^1, \widehat{\Delta\theta}^2) \tag{38}$$

The regressor vector for the new estimation process is thus given by

$$w_2(z_1, z_2, \mu, \widehat{\Delta\theta}^1, \widehat{\Delta\theta}^2) = \left(1 + \frac{\widehat{\Delta\theta}_1^1}{\Theta_1}\right) \gamma_2(z_1, z_2, \mu, \widehat{\Delta\theta}^1, \widehat{\Delta\theta}^2) + c_1 w_1(z_1, z_2, \widehat{\Delta\theta}^1) \tag{39}$$

Note that the dependence of w_2 on $\widehat{\Delta\theta}^2$ is implicit through its dependence on μ, as given by the solution of (36).

As in the previous step, an incremental parameter adaptation law for the vector of new estimates $\widehat{\Delta\theta}^2$ can be devised to achieve simultaneously a stable adaptation process and an asymptotic convergence to zero of the pseudo-control error variable z_2. Such a new incremental parameter update law is given by

$$\dot{\widehat{\Delta\theta}}{}^2 = z_2 w_2(z_1, z_2, \mu, \widehat{\Delta\theta}^1, \widehat{\Delta\theta}^2) \tag{40}$$

3.1.1. *Summary of adaptive controller expressions for the boost converter.* The adaptive PWM controller is next summarized in terms of the original state variables of the system. The constant Y stands for the desired value of the input inductor current $I(t)$. The constants c_1 and c_2 are positive design constants, satisfying $c_1, c_2 > 2$.

The adaptive feedback regulated switch position function is synthesized as

$$u = \begin{cases} 1, & \text{for } t_k \leq t < t_k + \mu_a(t_k)T \\ 0, & \text{for } t_k + \mu_a(t_k)T \leq t < t_k + T \end{cases} \qquad t_k + T = t_{k+1}, \quad k = 0, 1, 2, \ldots \tag{41}$$

where $\mu_a(t)$ is obtained from a bounding operation carried out on the computed duty ratio μ in the following manner:

$$\mu_a(t) = \begin{cases} 1, & \text{if } \mu(t) \leq 1 \\ \mu(t), & \text{if } 0 < \mu(t) < 1 \\ 0, & \text{if } \mu(t) \leq 0 \end{cases} \tag{42}$$

The duty ratio function μ is obtained as the solution of the following time-varying differential equation from an initial condition which does not cause permanent saturation of the actual duty ratio $\mu_a(t)$.

$$\dot{\mu} = \frac{1}{V(t)(\Theta_1 + \widehat{\Delta\theta}_1^1)} \Big\{ -(I(t) - Y)(1-\mu)^2 V^2(t) + (\Theta_1 + \widehat{\Delta\theta}_1^1)(1-\mu)$$
$$\times [(\Theta_2 + \widehat{\Delta\theta}_2^2)(1-\mu) I(t) - (\Theta_3 + \widehat{\Delta\theta}_3^2) V(t)] - (I(t) - Y)$$
$$- c_1[-(\Theta_1 + \widehat{\Delta\theta}_1^2)(1-\mu) V(t) + (\Theta_4 + \widehat{\Delta\theta}_4^2)]$$
$$+ c_2[(\Theta_1 + \widehat{\Delta\theta}_1^1)(1-\mu) V(t) - (\Theta_4 + \widehat{\Delta\theta}_4^1) - c_1(I(t) - Y)] \Big\} \tag{43}$$

The estimated values of the controller parameters are obtained as the online solution of the following system of differential equations:

$$\begin{aligned}
\dot{\widehat{\Delta\theta}}{}_1^1 &= -(I(t) - Y)(1-\mu) V(t) \\
\dot{\widehat{\Delta\theta}}{}_2^1 &= 0 \\
\dot{\widehat{\Delta\theta}}{}_3^1 &= 0 \\
\dot{\widehat{\Delta\theta}}{}_4^1 &= -(I(t) - Y) \\
\dot{\widehat{\Delta\theta}}{}_1^2 &= [-(Y_1 + \widehat{\Delta\theta}_1^1)(1-\mu) V(t) + (\Theta_4 + \dot{\widehat{\Delta\theta}}{}_4^1) + c_1(I(t) - Y)][-c_1(1-\mu) V(t)] \\
\dot{\widehat{\Delta\theta}}{}_2^2 &= [-(Y_1 + \widehat{\Delta\theta}_1^1)(1-\mu) V(t) + (\Theta_4 + \widehat{\Delta\theta}_4^1) + c_1(I(t) - Y)][-(\Theta_1 + \widehat{\Delta\theta}_1^1)(1-\mu)^2 I(t)] \\
\dot{\widehat{\Delta\theta}}{}_3^2 &= [-(Y_1 + \widehat{\Delta\theta}_1^1)(1-\mu) V(t) + (\Theta_4 + \widehat{\Delta\theta}_4^1) + c_1(I(t) - Y)][(\Theta_1 + \widehat{\Delta\theta}_1^1)(1-\mu) V(t)] \\
\dot{\widehat{\Delta\theta}}{}_4^2 &= [-(\Theta_1 + \widehat{\Delta\theta}_1^1)(1-\mu) V(t) + (\Theta_4 + \widehat{\Delta\theta}_4^1) + c_1(I(t) - Y)] c_1
\end{aligned} \tag{44}$$

3.1.2. *Simulation results.* Simulations were carried out for a perturbed version of the boost converter model in conjunction with the adaptive controller described by (41)–(44). An unmodelled stochastic but bounded, yet unmatched, uncertain signal (denoted by $\eta(t)$) was hypothesized to be acting on the circuit through the external source voltage E. The designed dynamical adaptive PWM controller (41)–(44) was then directly used for the regulation of the input inductor current variable $I(t)$ of the converter using the actual discontinuously regulated state variables $I(t)$ and $V(t)$, rather than the averaged values ζ_1 and ζ_2.

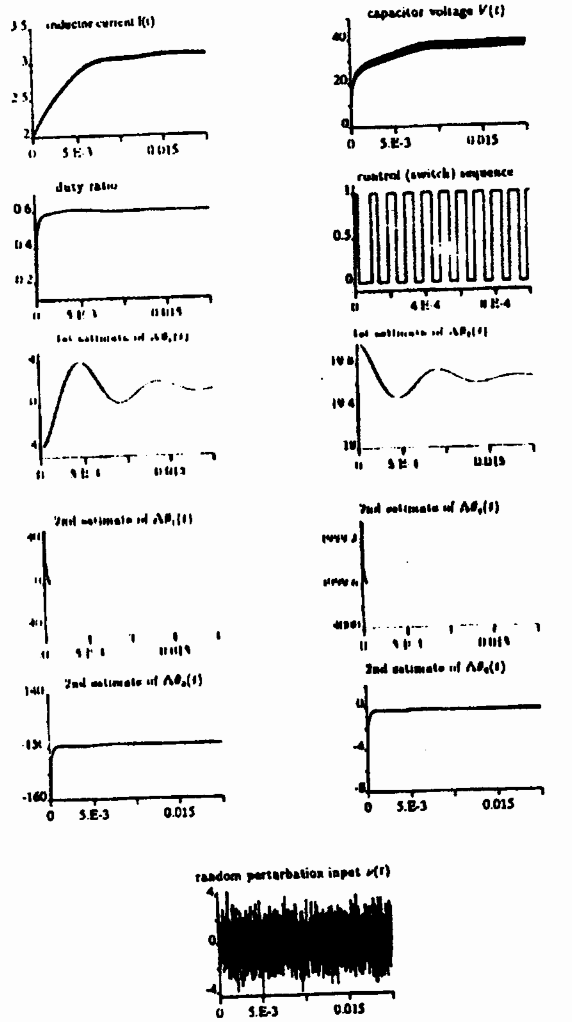

Figure 2. Adaptively controlled state trajectories of perturbed boost converter, evolution of controller incremental parameter estimates and perturbation noise signal.

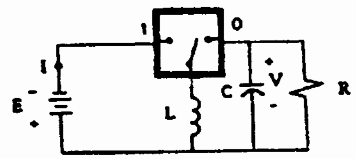

Figure 3. Buck–boost converter circuit.

The perturbed circuit model, used in the computer simulations, were taken to be

$$\left.\begin{aligned} \dot{I}(t) &= -\frac{1}{L}(1-u)V(t)+\left(\frac{E+v(t)}{L}\right) \\ \dot{V}(t) &= \frac{1}{C}(1-u)I(t)-\frac{1}{RC}V(t) \\ y &= I(t) \end{aligned}\right\} \tag{45}$$

The simulation results, depicting the behaviour of the controlled converter, are shown in Fig. 2. The nominal values of the converter parameters were chosen as $L = 20$ mH, $C = 20$ μF, $R = 30\,\Omega$ and $E = 15$ V. These values rendered: $\theta_1 = 50$; $\theta_2 = 5.0 \times 10^4$; $\theta_4 = 1.667 \times 10^4$ and $\theta_5 = 750$. The actual parameter values used in the simulations were set to $\theta_1 = 55$, $\theta_2 = 4.5 \times 10^4$, $\theta_4 = 1.5 \times 10^4$ and $\theta_5 = 825$. These values were, however, assumed to be completely unknown in the controller implementation. In Fig. 2 the response of the input inductor current $I(t)$ evolves towards the preassigned (nominal) equilibrium value $I(t) = Y = 3.125$ A, which corresponds to a nominal output capacitor voltage $V(t) = 37.5$ V and a duty ratio $U = 0.6$ V. The PWM sampling frequency was set to 10 kHz. The asymptotically stable evolution of the duty ratio function $\mu(t)$ towards its equilibrium value, $\mu = U = 0.6$, along with a small portion (0.1 ms) of the PWM switching actions is also presented in this figure. The trajectories of the estimated incremental parameter values, $\Delta\hat{\theta}_1(t)$, $\Delta\hat{\theta}_2(t)$, $\Delta\hat{\theta}_4$, $\Delta\hat{\theta}_5$ $\Delta\hat{\theta}_1$ and $\Delta\hat{\theta}_4$, are also depicted in this figure. Finally, a sample of the computer generated stochastic perturbation input $v(t)$ is shown at the end of Fig. 2.

3.2. *The buck–boost converter*

In this section we briefly summarize the controller expressions obtained from the backstepping calculation procedure applied to a nominally transformed parameter uncertain average PWM buck–boost converter model.

Consider the buck–boost converter circuit shown in Fig. 3.

State-space model of the buck–boost converter

$$\left.\begin{aligned} \dot{I}(t) &= \frac{1}{L}(1-u)V(t)+\frac{E}{L}u \\ \dot{V}(t) &= -\frac{1}{C}(1-u)I(t)-\frac{1}{RC}V(t) \\ y(t) &= I(t) \end{aligned}\right\} \tag{46}$$

where $I(t)$ and $V(t)$ represent, respectively, the input inductor current and the output capacitor voltage variables. The positive quantity E is the constant external input voltage. The variable u denotes the switch position function taking values in the discrete set $\{0, \}$. The output $y(t)$ of the system is represented by the input inductor current $I(t)$.

Nominal parameters definitions

$$\Theta_1 = \frac{1}{L}, \quad \Theta_2 = \frac{1}{C}, \quad \Theta_3 = \frac{1}{RC}, \quad \Theta_4 = \frac{E}{L} \tag{47}$$

Uncertainty model for the parameters

$$\theta_i = \Theta_i + \Delta\theta_i, \quad i = 1,2,3,4 \tag{48}$$

Average PWM model of the buck–boost converter

$$\left.\begin{aligned} \dot{\zeta}_1 &= \theta_1(1-\mu)\zeta_2 + \theta_4\mu \\ \dot{\zeta}_2 &= -\theta_2(1-\mu)\zeta_1 - \theta_3\zeta_2 \\ \eta &= \zeta_1 \end{aligned}\right\} \tag{49}$$

Nominal transformation of average PWM buck–boost converter model to Fliess' generalized observability canonical form

$$\left.\begin{aligned} x_1 &= \zeta_1 \quad \zeta_2 = \Theta_1(1-\mu)\zeta_2 + \Theta_4\mu, \\ \zeta_1 &= x_1, \quad z_2 = \frac{x_2 - \Theta_4\mu}{\Theta_1(1-\mu)} \end{aligned}\right\} \tag{50}$$

Uncertain buck–boost converter model transformed to generalized phase variables

$$\left.\begin{aligned} \dot{x}_1 &= x_2 + \Delta\theta^T\gamma_1(x_1, x_2) \\ \dot{x}_2 &= f(x_1, x_2, \mu, \dot{\mu}) + \Delta\theta^T\gamma_2(x_1, x_2, \mu) \end{aligned}\right\} \tag{51}$$

where

$$\Delta\theta^T = [\Delta\theta_1 \quad \Delta\theta_2 \quad \Delta\theta_3 \quad \Delta\theta_4]$$

$$\gamma_1^T(x_1, x_2) = \left[\frac{x_2 - \Theta_4\mu}{\Theta_1} \quad 0 \quad 0 \quad \mu\right]$$

$$\gamma_2^T(x_1, x_2, \mu) = [0 \quad -\Theta_1(1-\mu)^2 x_1 \quad -x_2 + \Theta_4\mu \quad 0]$$

$$f(x_1, x_2, \mu, \dot{\mu}) = \dot{\mu}\left(\Theta_4 - \frac{x_2 - \Theta_4\mu}{1-\mu}\right) - \Theta_1\Theta_2(1-\mu)^2 x_1 - \Theta_3(x_2 - \Theta_4\mu) \tag{52}$$

3.2.1. *Summary of adaptive controller expressions for the buck–boost converter*

$$\left.\begin{aligned} u &= \begin{cases} 1, & \text{for } t_k \leqslant t < t_k + \mu_a(t_k)T \\ 0, & \text{for } t_k + \mu_a(t_k)T \leqslant t < t_k + T \end{cases} \\ t_k + T &= t_{k+1}, \quad k = 0,1,2,\ldots \end{aligned}\right\} \tag{53}$$

where $\mu_a(t)$ is obtained from the following bounding operation:

$$\mu_a(t) = \begin{cases} 1, & \text{if } \mu(t) \leqslant 1 \\ \mu(t), & \text{if } 0 < \mu(t) < 1 \\ 0, & \text{if } \mu(t) \leqslant 0 \end{cases} \tag{54}$$

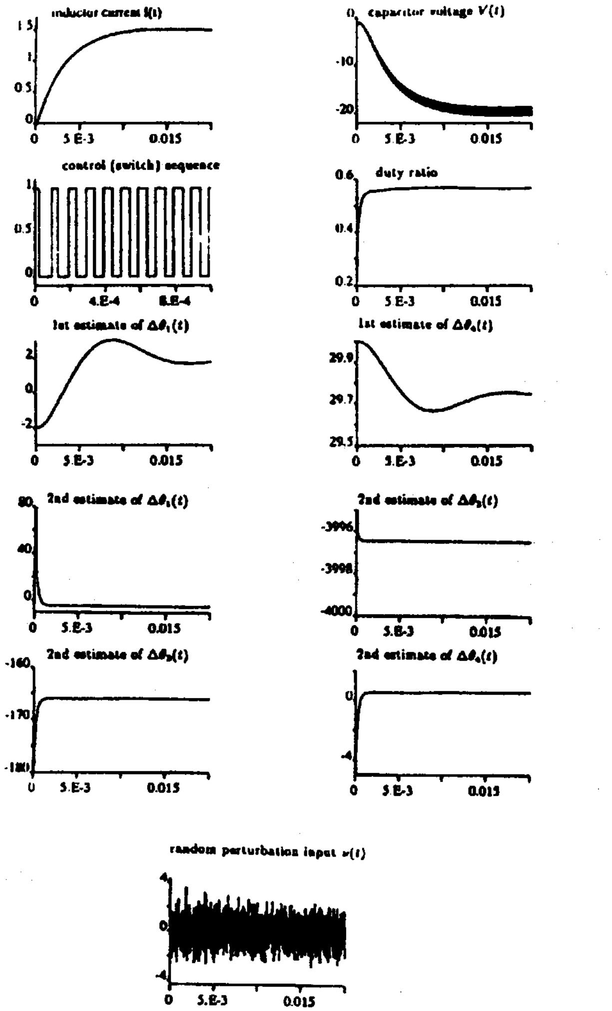

Figure 4. Adaptively controlled state trajectories of perturbed buck–boost converter, evolution of controller incremental parameter estimates and perturbation noise signal.

We let $\hat{\theta}_i^j$, $i = 1, \ldots, 4$, $j = 1, 2$, stand for $\Theta_i + \Delta\hat{\theta}_i^j$ in the following expressions.

$$\dot{\mu} = \frac{1}{(\hat{\theta}_4^1 - \hat{\theta}_1^1 V(t))}\{-(I(t)-Y)(1-\mu)^2 V^2(t) + \hat{\theta}_1^1 \hat{\theta}_3^2 (1-\mu)^2 I(t) + \hat{\theta}_1^1 \hat{\theta}_2^2 (1-\mu) V(t) - (I(t)-Y)\mu - c_1(\hat{\theta}_1^2(1-\mu)V(t) + \hat{\theta}_4^2\mu) + c_2[\hat{\theta}_1^1(1-\mu)V(t) + \hat{\theta}_4^1 + c_1(I(t)-Y)]\} \quad (55)$$

$$\left.\begin{aligned}
\dot{\hat{\theta}}_1^1 &= (I(t)-Y)(1-\mu)V(t)\\
\dot{\hat{\theta}}_2^1 &= 0\\
\dot{\hat{\theta}}_3^1 &= 0\\
\dot{\hat{\theta}}_4^1 &= (I(t)-Y)\mu\\
\dot{\hat{\theta}}_1^2 &= [\hat{\theta}_1^1(1-\mu)V(t) + \hat{\theta}_4^1\mu + c_1(I(t)-Y)][c_1(1-\mu)V(t)]\\
\dot{\hat{\theta}}_2^2 &= [\hat{\theta}_1^1(1-\mu)V(t) + \hat{\theta}_4^1\mu + c_1(I(t)-Y)][-\hat{\theta}_1^1(1-\mu)^2 I(t)]\\
\dot{\hat{\theta}}_3^2 &= [\hat{\theta}_1^1(1-\mu)V(t) + \hat{\theta}_4^1\mu + c_1(I(t)-Y)][-\hat{\theta}_1^1(1-\mu)V(t)]\\
\dot{\hat{\theta}}_4^2 &= [\hat{\theta}_1^1(1-\mu)V(t) + \hat{\theta}_4^1\mu + c_1(I(t)-Y)]c_1\mu
\end{aligned}\right\} \quad (56)$$

3.2.2. *Simulation results*. Simulations were carried out for the following perturbed version of the buck–boost converter model:

$$\left.\begin{aligned}
\dot{I}(t) &= \frac{1}{L}(1-u)V(t) + \left(\frac{E+v(t)}{L}\right)u\\
\dot{V}(t) &= -\frac{1}{C}(1-u)I(t) - \frac{1}{RC}V(t)\\
y &= I(t)
\end{aligned}\right\} \quad (57)$$

in conjunction with the adaptive controller described by (53)–(56).

The simulation results, depicting the behaviour of the controller converter, are shown in Fig. 4. The nominal values of the converter parameters were chosen to be the same as for the boost converter case: $L = 20$ mH, $C = 20$ mF, $R = 30\ \Omega$, and $E = 15$ V. The actual parameter values used in the simulations were the same as before. The response of the input inductor current $I(t)$ is seen to evolve towards the preassigned (nominal) equilibrium value $I(t) = Y = 1{\cdot}5$ A, which corresponds to a nominal output capacitor voltage, $V(t) = -21{\cdot}38$ V. The PWM sampling frequency was also set at 10 kHz. The asymptotically stable evolution of the duty ratio function $\mu(t)$ towards its equilibrium value, $\mu = U = 0{\cdot}55$, along with a small portion of the switching action is also presented in this figure. The trajectories of the estimated incremental parameter values, $\Delta\hat{\theta}_1^1(t)$, $\Delta\hat{\theta}_4^1(t)$, $\Delta\hat{\theta}_1^2$, $\Delta\hat{\theta}_2^2$, $\Delta\hat{\theta}_3^2$ and $\Delta\hat{\theta}_4^2$, are also depicted in this figure. Finally, a sample of the computer-generated stochastic perturbation input $v(t)$ is shown at the bottom of Fig. 4.

4. Conclusions

An adaptive feedback control approach has been proposed which is based on nominal dynamical input–output linearization of the average model of PWM regulated DC-to-DC power converters and the backstepping algorithm. The approach achieves indirect average output capacitor voltage regulation by considering the input inductor current as the regulated output. This procedure sidesteps the non-minimum

phase problems associated with direct output capacitor voltage regulation. The simulated behaviour of the closed-loop system exhibits remarkable robustness with respect to unmatched and unmodelled external perturbation signals of bounded and stochastic nature.

Over-parametrization is implicit in the backstepping procedure when applied to systems in pure parameter feedback canonical form (Kanellakopoulos *et al.* 1991). This feature substantially contributes to increase the complexity of the controller. An alternative approach to the one presented here is constituted by the possibility of avoiding the over-parametrization associated with the incremental parameter update estimation process. The fundamental developments regarding this technique may be found in Krstić *et al.* (1992), and an alternative approach to that of this paper has been presented by Sira-Ramírez *et al.* (1995b).

Simulations show that the scheme is quite robust with respect to unmodelled stochastic, but bounded, external perturbation inputs of the unmatched type. This type of robust behaviour is inherited from: the underlying input–output viewpoint present in the generalized observability canonical form, used for the derivation of the dynamical feedback controller; and the robustness features traditionally associated with discontinuous feedback control policies of the pulse-width-modulation type.

A topic for further study is the direct output capacitor voltage regulation problem, which exhibits a non-minimum phase property, and hence an input–output linearization approach fails. In a recent work Sira-Ramírez *et al.* (1995a) proposed the possibility of handling the non-minimum phase case by means of a piecewise unstable dynamical compensator in which a controller output 'resetting' strategy is enforced. The adaptive version of this resetting controller did not use the backstepping algorithm.

Another very interesting development has been given by Karsenti and Lamnabhi-Lagarrigue (1995) in which the backstepping method is generalized to include sliding mode control strategies in systems with nonlinear parameter dependencies. Application of this latter technique to DC-to-DC power converters represents a welcome contribution, since a more realistic class of (nonlinear) incremental circuit parameter variations may be efficiently handled with such a method.

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Passivity-Based Controllers for the Stabilization of DC-to-DC Power Converters*

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Dynamical feedback controllers providing the synthesis of the duty ratio function in pulse-width-modulation (PWM) feedback controlled DC-to-DC power converters are derived using passivity-based considerations. The approach is shown to be naturally suited for the average-based regulation of several power supplies due to the Lagrangian nature of their average PWM models.

Key Words—Power supplies; passivity-based compensation; pulse-width modulation; Euler–Lagrange systems.

Abstract—Passivity-based feedback controllers are derived for the indirect stabilization of the average output voltage in pulse-width-modulation (PWM) controlled DC-to-DC power converters of the 'boost', 'buck–boost', and 'buck' types. The controller design is carried out on the basis of well-known average PWM models of such circuits. The average models are first shown to be Euler–Lagrange systems corresponding to a suitable set of average Euler–Lagrange parameters. The proposed regulators are based on an 'energy shaping plus damping injection' scheme, achievable through nonlinear dynamical feedback. The performance of the proposed passivity-based duty ratio synthesis policies is tested for the 'boost' converter case. The regulating feedback law, derived on the basis of a 'boost' model composed of ideal switches and ideal circuit components, is assessed, via computer simulations, on a realistic stochastically perturbed switched converter model, including parasitic resistances and parasitic voltage sources. © 1997 Elsevier Science Ltd.

1. INTRODUCTION

The feedback regulation of DC-to-DC power supplies is, broadly speaking, accomplished through either pulse-width-modulation (PWM) feedback strategies, or by inducing appropriate stabilizing sliding regimes. PWM control of these devices is treated in several books, among them those by Severns and Bloom (1982), Kassakian *et al.* (1991), and Rashid (1992). The topic has also been extensively treated by, among many other authors, Sira-Ramirez and Lischinsky-Arenas (1991). Sliding mode control of switched power supplies was first treated by Venkataramanan *et al.* (1985), and more recently by Sabanovic *et al.* (1993) in the context of motion control systems.

Both feedback controller design approaches entirely overlook the energy-related physical properties of either the original converter circuit or of its closed-loop structure. The controller design philosophy primarily insists on a mathematically motivated average closed-loop linearization, which is geared to solve the stabilization, or tracking, task.

In the context of PWM feedback policies, *state average models* of DC-to-DC power converters have been developed by Middlebrook and Ćuk (1976) from an approximate discretization viewpoint. An interesting refinement and generalization of this class of average PWM models has been proposed by Krein *et al.* (1990) using the analytical theory of averaging of differential equations. The same average models, known as *infinite switching frequency average models*, have been derived by Sira-Ramirez (1989), using as a justification Filippov's (geometric) averaging viewpoint (see Filippov, 1988).

All the average PWM models so far developed have been justified from a purely mathematical viewpoint, without due regard to their possible physical significance. It was one of our objectives in this study to demonstrate that average PWM models of power converters can be derived from the energy properties of the switched electrical circuit. The main advantage of underscoring the often overlooked physical properties of DC-to-DC power converters is that, in this way, we can advantageously exploit these properties at the feedback controller design stage. In particular, we would like to explore the relevance and implications of a 'passivity-based' approach in the feedback duty ratio synthesis problem (for background on the passivity-based methodology for controller design see Takegaki and Arimoto (1981), and for the subsequent developments see Ortega and Spong (1989), Berghuis and Nijmeijer (1993) and Brogliato *et al.* (1995).

A passivity-based controller design technique would be directly, and most naturally, applicable to the average-based PWM regulation of DC-to-DC power converters, provided one can demonstrate that such idealized, mathematically motivated, models actually correspond to systems derivable from classical Euler–Lagrange (EL) dynamics considerations.

In this paper, an EL dynamics modeling approach, which establishes the relevant physical characteristics of well-known average models of DC-to-DC power converters, is first presented. In particular, we prove that the traditional average PWM models of switched converters of the 'boost', 'buck–boost', and 'buck' types are indeed EL systems. The approach consists in establishing a suitable set of average EL *parameters* modulated by the duty ratio function. This average set of parameters is derived on the basis of, both a 'consistency' and an 'intermediacy' requirement with respect to the EL parameters of the two intervening electrical circuit topologies. Interestingly, the derived average PWM models entirely coincide with the well-known *state average models* of DC-to-DC power converters, introduced by Middlebrook and Ćuk (1976), and they also coincide with the *infinite switching frequency* models, derived in Sira-Ramirez (1989) and unified by Amran *et al.* (1991).

Due to the nonminimum phase nature of the average output voltage variable, a direct application of the passivity-based design method, aimed primarily at output-voltage regulation, leads to an unstable dynamical feedback controller. This is due to an underlying partial inversion of the average system model, carried out at the controller design stage. For this reason, an *indirect* approach, consisting of output-voltage regulation through inductor current stabilization is undertaken. Indirect controller design for nonminimum phase systems has been justified, for nonlinear systems, in the work of Benvenuti *et al.* (1992) and, in the context of DC-to-DC power converters, in the work of Sira-Ramirez and Lischinsky-Arenas (1991). The indirect control technique also naturally arises, from module-theoretic results, in sliding mode control of linear multivariable nonminimum phase systems, as inferred from the work of Fliess and Sira-Ramirez (1993).

The performance of the derived, indirect, dynamical state feedback controllers was successfully tested, via computer simulations, for the 'boost' converter example. The model used for the switched boost converter included an unmodeled stochastic perturbation input, directly affecting the external voltage source, as well as unmodelled parasitic resistances attaches to each one of the circuit elements. The model for the switching arrangement, usually consisting of a transistor and a diode, was taken, as proposed by Czarkowsi and Kazimierczuk (1993), to be an ideal switch, combined with lumped forward (i.e. ON) resistances and a parasitic voltage source, associated with the conducting state of the diode.

This paper is organized as follows. Section 2 presents an EL dynamics-based derivation of the average PWM models of the 'boost', 'buck–boost', and 'buck' converters. An ideal equivalent circuit realization is also provided for the three kinds of converter. Section 3 develops the passivity-based feedback controllers and demonstrates, for the 'boost' and 'buck–boost' converter cases, the nonminimum phase character of direct output voltage regulation options. The 'buck' converter case does not exhibit a nonminimum phase character and, therefore, direct and indirect output voltage regulation schemes are seen to be equally feasible. The simulation results are presented in Section 4. Section 5 contains the conclusions, and suggestions for further research in this area.

2. AVERAGE MODELS OF DC-to-DC POWER CONVERTERS AS EL SYSTEMS

The results of this section, regarding the EL nature of average PWM DC-to-DC power converters, extend the work found in Sira-Ramirez and Delgado de Nieto (1995), where only the 'boost' converter case is treated.

2.1. *Generalities about EL electric circuits*

The EL dynamics of an electric circuit, containing no magnetic couplings between its different branches, is classically characterized by the following set of nonlinear differential equations (see Meisel 1966):

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}}{\partial \dot{q}}\right) - \frac{\partial \mathcal{L}}{\partial q} = -\frac{\partial \mathcal{D}}{\partial \dot{q}} + \mathcal{F}_q, \qquad (1)$$

where $\dot{q}$ is the vector of flowing currents and q represents their time integrals, or *electric*

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charges. The vector of electric charges constitutes the *generalized coordinates* describing the circuit. This vector is assumed to have n components, represented by $q_1, \ldots, q_n$. The scalar function $\mathcal{L}$ is the *Lagrangian* of the system, defined as the difference between the *magnetic co-energy* of the circuit, denoted by $\mathcal{T}(\dot q, q)$, and the *electric field energy* of the circuit, denoted by $\mathcal{V}(q)$, i.e.

$$\mathcal{L}(\dot q, q) = \mathcal{T}(\dot q, q) - \mathcal{V}(q). \quad (2)$$

The function $\mathcal{D}(\dot q)$ is the *Rayleigh dissipation cofunction* of the system. The vector $\mathcal{F}_q = (\mathcal{F}_{q_1}, \ldots, \mathcal{F}_{q_n})$ represents the ordered components of the set of *generalized forcing functions*, or *voltage sources*, associated with the generalized coordinates.

EL circuits are thus generally represented by the set of equations

$$\frac{d}{dt}\left(\frac{\partial \mathcal{T}}{\partial \dot q}\right) - \frac{\partial \mathcal{T}}{\partial q} + \frac{\partial \mathcal{V}}{\partial q} = -\frac{\partial \mathcal{D}}{\partial \dot q} + \mathcal{F}_q \quad (3)$$

Following Ortega *et al.* (1995), we refer to the set of functions $(\mathcal{T}, \mathcal{V}, \mathcal{D}, \mathcal{F})$ as the *EL parameters* of the circuit, and simply express a circuit Σ by means of the ordered quadruple:

$$\Sigma = (\mathcal{T}, \mathcal{V}, \mathcal{D}, \mathcal{F}). \quad (4)$$

2.2. *The 'boost' converter*

2.2.1. *The switch-regulated model for the 'boost' converter.* Consider the switch-regulated 'boost' converter circuit in Fig. 1. The differential equations describing the circuit are

$$\dot x_1 = -(1-u)\frac{1}{L}x_2 + \frac{E}{L},$$
$$\dot x_2 = (1-u)\frac{1}{C}x_1 - \frac{1}{RC}x_2, \quad (5)$$

where x_1 and x_2 represent the input inductor current and the output capacitor voltage variables, respectively. The positive quantity E represents the constant-voltage value of the external voltage source. The variable u denotes the switch position function, acting as a control input. Such a control input takes place in the discrete set $\{0, 1\}$.

A PWM policy regulating the switch position function u may be specified as follows:

$$u(t) = \begin{cases} 1, & \text{for } t_k \le t < t_k + \mu(t_k)T \\ 0, & \text{for } t_k + \mu(t_k)T \le t < t_k + T \end{cases}$$
$$t_{k+1} = t_k + T;\ k = 0, 1, \ldots, \quad (6)$$

where t_k represents a sampling instant; the parameter T is the fixed sampling period, also called the *duty cycle*; the sampled values of the state vector $x(t)$ of the converter are denoted by $x(t_k)$. The function $\mu(\cdot)$ is the *duty ratio function*, truly acting as an external control input to the average PWM model of the converter (see Sira-Ramírez, 1989). The value of the duty ratio function $\mu(t_k)$ determines, at every sampling instant t_k, the width of the upcoming ON pulse as $\mu(t_k)T$ (during this period the switch is fixed at the position represented by $u = 1$). The actual duty ratio function $\mu(\cdot)$ is evidently a function limited to take values on the closed interval $[0, 1]$ of the real line.

2.2.2. *A Lagrangian formulation of the average PWM model.* We consider separately the Lagrangian dynamics formulation of the two circuits associated with each of the two possible positions of the regulating switch. Of course, the aim of carrying out this formulation is not to rederive the differential equations governing the circuit at each switch position. These may be trivially found from (5) itself. Our purpose is to gain some insight on the physical effects of the switching action in terms of the EL parameters of the two circuit topologies. In order to use standard notation, we rewrite the input current x_1 in terms of the derivative of the circulating electric charge q_L, as $\dot q_L$. Also the capacitor voltage x_2 will be written as q_C/C where q_C is the electrical charge stored in the output capacitor.

Consider then $u = 1$. The resulting circuit is as shown in Fig. 2. In this case, two separate, or decoupled, circuits are clearly obtained, and the corresponding Lagrangian dynamics formulation can be carried out as follows.

Define $\mathcal{T}_1(\dot q_L)$ and $\mathcal{V}_1(q_C)$ as the magnetic co-energy and electric field energy of the circuit, respectively. We denote by $\mathcal{D}_1(\dot q_C)$ the Rayleigh

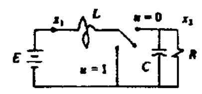

Fig. 1. Boost converter circuit.

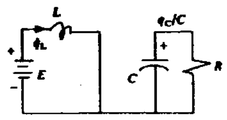

Fig. 2. Boost converter circuit ($u = 1$).

dissipation cofunction of the circuit. These quantities are readily found to be

$$\mathcal{T}_1(\dot q_L) = \tfrac{1}{2}L\dot q_L^2;\quad \mathcal{V}_1(q_C) = \frac{1}{2C}q_C^2,$$
$$\mathcal{D}_1(\dot q_C) = \tfrac{1}{2}R\dot q_C^2;\quad \mathcal{F}_{q_L} = E,\quad \mathcal{F}_{q_C} = 0, \quad (7)$$

where $\mathcal{F}_{q_L}$ and $\mathcal{F}_{q_C}$ are the *generalized forcing* functions associated with the coordinates q_L and q_C, respectively.

Evidently, the EL equations associated with these definitions immediately rederive (5), with $u = 1$, as can be verified by direct use of the general equation (2), or (3), on the set of EL parameters given in (7).

Consider now the case $u = 0$. The resulting circuit is as shown in Fig. 3. The corresponding Lagrangian dynamics formulation is carried out similarly to the case $u = 1$. That is, define $\mathcal{T}_0(\dot q_L)$ and $\mathcal{V}_0(q_L)$ as the magnetic co-energy and the electric field energy of the circuit, respectively. We denote by $\mathcal{D}_0(\dot q_L, \dot q_C)$ the Rayleigh dissipation cofunction of the circuit. These quantities are readily found to be

$$\mathcal{T}_0(\dot q_L) = \tfrac{1}{2}L\dot q_L^2;\quad \mathcal{V}_0(q_C) = \frac{2}{2C}q_C^2,$$
$$\mathcal{D}_0(\dot q_L, \dot q_C) = \tfrac{1}{2}R(\dot q_C - \dot q_L)^2, \quad (8)$$
$$\mathcal{F}_{q_L} = E,\quad \mathcal{F}_{q_C} = 0,$$

where, as before, $\mathcal{F}_{q_L}$ and $\mathcal{F}_{q_C}$ are the *generalized forcing* functions associated with the coordinates q_L and q_C, respectively.

Evidently, application of the general EL equations (3)–(8) immediately rederives (5), with $u = 0$, as can be easily verified.

The EL parameters of the two circuits, generated by the different switch-position values, result in identical magnetic co-energies, electric field energies, and forcing functions. The switching action merely changes the Rayleigh dissipation cofunction between the values $\mathcal{D}_0(\dot q_L)$ and $\mathcal{D}_1(\dot q_L, \dot q_C)$. Therefore, the *dissipation structure* of the system is the only one directly affected by the switch position function u.

Note that, according to the PWM switching policy (6), on every sampling interval of period T, the Rayleigh dissipation cofunction $\mathcal{D}_1(\dot q_L)$ is valid over only a fraction of the sampling period given by $\mu(t_k)$, while the Rayleigh dissipation cofunction $\mathcal{D}_1(\dot q_L, \dot q_L)$ is valid a fraction of the sampling period equal to $(1 - \mu(t_k))$.

There, are of course, a variety of ways in which one could reasonably propose an average value of the Rayleigh dissipation cofunction for a circuit of the form (5), undergoing a switching policy of the form (6). One possible way is to propose the following set of EL parameters:

$$\mathcal{T}_\mu(\dot q_L) = \tfrac{1}{2}L\dot q_L^2;\quad \mathcal{V}_\mu(q_C) = \frac{1}{2C}q_C^2$$
$$\mathcal{D}_\mu(\dot q_L, \dot q_C) = \tfrac{1}{2}R[\dot q_C - (1-\mu)\dot q_L]^2; \quad (9)$$
$$\mathcal{F}^\mu_{q_L} = E,\quad \mathcal{F}^\mu_{q_C} = 0.$$

Note that in the cases where μ takes the extreme saturation values $\mu = 1$ or 0, one recovers, respectively, the dissipation cofunctions $\mathcal{D}_1(\dot q_C)$ in (7) and $\mathcal{D}_0(\dot q_L, \dot q_C)$ in (2.8) from the proposed average dissipation cofunction, $\mathcal{D}_\mu(\dot q_L, \dot q_L C)$, of equation (9). Indeed, such a 'consistency' condition is verified by noting that

$$\mathcal{D}_\mu(\dot q_L, \dot q_C)|_{\mu=0} = \mathcal{D}_0(\dot q_L, \dot q_C),$$
$$\mathcal{D}_\mu(\dot q_L, \dot q_C)|_{\mu=1} = \mathcal{D}_1(\dot q_C).$$

Also, it is easy to see that the proposed average Rayleigh dissipation cofunction satisfies an important 'intermediacy' condition of the form

$$\min\{\mathcal{D}_0(\dot q_L, \dot q_C), \mathcal{D}_1(\dot q_C)\} < \mathcal{D}_\mu(\dot q_L, \dot q_C)$$
$$< \max\{\mathcal{D}_0(\dot q_L, \dot q_C), \mathcal{D}_1(\dot q_C)\}$$

for any μ lying in the open interval $(0, 1)$.

We note that the Lagrangian function associated with the above-defined average EL parameters is actually *invariant* with respect to the switch position function. Nevertheless, to keep the notation consistent, we denote it by

$$\mathcal{L}_\mu = \mathcal{T}_\mu(\dot q_L) - \mathcal{V}_\mu(q_C) = \tfrac{1}{2}L\dot q_L^2 - \frac{1}{2C}q_C^2 \quad (10)$$

One then proceeds, using the EL equations (3), to obtain the differential equations defining the average PWM model which corresponds to the proposed average EL parameters (9). Such equations are

$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}_\mu}{\partial \dot q_L}\right) - \frac{\partial \mathcal{L}_\mu}{\partial q_L} = -\frac{\partial \mathcal{D}_\mu}{\partial \dot q_L} + \mathcal{F}^\mu_{q_L},$$
$$\frac{d}{dt}\left(\frac{\partial \mathcal{L}_\mu}{\partial \dot q_C}\right) - \frac{\partial \mathcal{L}_\mu}{\partial q_L} = -\frac{\partial \mathcal{D}_\mu}{\partial \dot q_C} + \mathcal{F}^\mu_{q_C}. \quad (11)$$

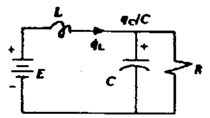

Fig. 3. Boost converter circuit ($u = 0$).

Evaluation of (11) results in the following system of differential equations:

$$L\ddot{q}_L = (1-\mu)R[\dot{q}_C - (1-\mu)\dot{q}_L] + E,$$
$$\frac{q_C}{C} = -R[\dot{q}_C - (1-\mu)\dot{q}_L], \tag{12}$$

which can be rewritten, after substitution of the second equation of (12) into the first, as

$$\ddot{q}_L = -(1-\mu)\frac{q_C}{LC} + \frac{E}{L},$$
$$\dot{q}_C = -\frac{1}{RC}q_C + (1-\mu)\dot{q}_L. \tag{13}$$

Using $z_1 = \dot{q}_L$ and $z_2 = q_C/C$ one obtains

$$\dot{z}_1 = -(1-\mu)\frac{1}{L}z_2 + \frac{E}{L},$$
$$\dot{z}_2 = (1-\mu)\frac{1}{C}z_1 - \frac{1}{RC}z_2, \tag{14}$$

where we denote by z_1 and z_2 the *average input current* and the *average output capacitor voltage*, respectively, of the PWM regulated 'boost' converter. We establish this distinction with the nonaveraged variables x_1 and x_2 so that the state variables associated with the average PWM model are not mistakenly confused with the *actual* PWM regulated circuit variables.

Note that the proposed average dynamics (14) coincides with the state average model developed by Middlebrook and Ćuk (1976), and with the infinite switching frequency model, or Filippov average model, found in Sira-Ramirez (1989) and Amran *et al.* (1991). To obtain the average model (14), one simply replaces the switch position function u in (5) by the duty ratio function μ, and the actual state variables x_1, x_2 by their averaged values z_1, z_2.

We have thus proven the following proposition.

Proposition 2.1. The state average model of the 'boost' converter (see Middlebrook and Ćuk, 1976), given by (14) is an EL system corresponding to the set of average EL parameters given by (9). These parameters are, in turn, obtained by suitable modulation, through the duty ratio function μ of the EL parameters, given by (7) and (8), which are associated to each one of the intervening circuit topologies arising from a particular value of the switch position function.

For ease of reference we will be using the following, more compact, matrix representation of (14):

$$\mathcal{D}_\mu\dot{z} - (1-\mu)\mathcal{J}_0 z + \mathcal{R}_\mu z = \mathcal{G}_\mu, \tag{15}$$

where

$$\mathcal{D}_\mu = \begin{bmatrix} L & 0 \\ 0 & C \end{bmatrix}; \quad \mathcal{J}_0 = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix};$$
$$\mathcal{R}_\mu = \begin{bmatrix} 0 & 0 \\ 0 & 1/R \end{bmatrix}; \quad \mathcal{E}_\mu = \begin{bmatrix} E \\ 0 \end{bmatrix}. \tag{16}$$

2.2.3. *An ideal circuit realization for the average PWM model.* It is easy to realize that the average model (14) has a circuit-theoretic interpretation by letting the quantity $(1-\mu)z_2$ in the first equation represent a *controlled voltage source*, while also letting the quantity $(1-\mu)z_1$ in the second equation represent a *controlled input current source*. Figure 4 depicts the ideal equivalent circuit describing the average PWM model. In such a circuit, a quadripole connects the 'input' and 'output' circuits, which effectively replaces, in an average sense, the actual switching device.

Consider the isolated quadripole constituted by the ideal controlled sources, as shown in Fig. 5. Note that the (average) input power to the quadripole, expressed as the product of the average input current z_1 times the (reflected) average input voltage $(1-\mu)z_2$ is given by

$$\mathcal{P}_{in} = \underbrace{z_1}_{\text{Input current}} \; \underbrace{(1-\mu)z_2}_{\text{Input voltage}}. \tag{17}$$

On the other hand, the (average) output power delivered by the quadripole, expressed as the product of the average output current $(1-\mu)z_1$ times the output voltage z_2 is given by

$$\mathcal{P}_{out} = \underbrace{(1-\mu)z_1}_{\text{Output current}} \; \underbrace{z_2}_{\text{Output voltage}}. \tag{18}$$

In other words, the quadripole is a lossless, ideal

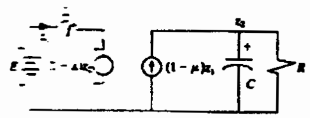

Fig. 4. Equivalent circuit of the average PWM model of the 'boost' converter circuit.

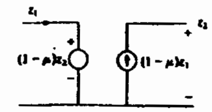

Fig. 5. Ideal transformer representing the average PWM switch position function.

(average) power transfering device satisfying

$$P_{in} = P_{out}. \tag{19}$$

The average input voltage to the quadripole $(1-\mu)z_2$ is amplified to the value z_2 at the output, while the input current to the quadripole z_1 is attenuated to the value $(1-\mu)z_1$ at the output. The switching element has thus been effectively replaced by an *ideal transformer* with *turn ratio* parameter given by $(1-\mu)$.

2.2.4. *Input–output and internal stability properties.* Given the results of Ortega *et al.* (1995) concerning EL systems, it is expected that the averaged circuit dynamics (15) satisfies the following energy balance equation:

$$\underbrace{H(t) - H(0)}_{\text{Stored energy}} + \underbrace{\frac{1}{RC^2}\int_0^t q_C^2(\tau)\,d\tau}_{\text{Dissipated energy}} = \underbrace{\int_0^t E\dot{q}_L(\tau)\,d\tau}_{\text{Supplied energy}},$$

where $H(t) = \frac{1}{2}z^T\mathcal{D}_\mu z = \mathcal{T}_\mu + \mathcal{V}_\mu$ is the *total energy* of the average circuit model. This follows trivially by taking the time derivative of $H(t)$ along the trajectories of (15) and noting the skew symmetry of $\mathcal{J}_0$. The energy balance equation above also reveals that the forces $(1-\mu)\mathcal{J}_0 z$ appearing in (15) are *workless*, and proves the passivity of the operator $E \to \dot{q}_L$.

We proceed to establish the relationship between the equilibria of the average output voltage and the average input current. To this end assume a constant duty ratio function $\mu = U$. It easily follows from the average PWM model equations (14) that the corresponding stable equilibrium values for the average input current, denoted by I_a, and the average output voltage, denoted by V_a, are given by

$$I_a = \frac{E}{(1-U)^2R}; \quad V_a = \frac{E}{1-U}. \tag{20}$$

Henceforth, given a desired equilibrium value V_d for the output voltage, which corresponds to a constant value of the duty ratio function $\mu = U = 1 - E/V_d$, the *unique* corresponding equilibrium value for the average input current is given by

$$I_d = \frac{V_d}{R(1-U)} = \frac{1}{RE}V_d^2 \tag{21}$$

This means that if we desire to regulate z_2 towards an equilibrium value V_d which is known to correspond to a steady state value U of the duty ratio function μ, then such a regulation can be *indirectly* accomplished by stabilizing the average input current z_1 towards the corresponding equilibrium value I_d computed from (21).

Now, consider the case where the average output capacitor voltage z_2 is regarded as the output of the average PWM model (14). A straightforward elimination of z_1 from the set of differential equations (14) leads to the following nonlinear input–output differential representation:

$$\ddot{z}_2 + \left(\frac{1}{RC} + \frac{\dot{\mu}}{1-\mu}\right)\dot{z}_2 + \frac{1}{LC}\left[(1-\mu^2) + \frac{L}{R}\frac{\dot{\mu}}{1-\mu}\right]z_2 = (1-\mu)\frac{E}{LC}. \tag{22}$$

The 'zero dynamics' at an equilibrium point $z_2 = V_d$ associated with this input–output representation is obtained by letting $\dot{z}_2 = 0$ and $\ddot{z}_2 = 0$ (see Fliess, 1990). The resulting differential equation describing the 'remaining dynamics' of the duty ratio function μ is simply obtained as

$$\dot{\mu} = \frac{R(1-\mu)^2}{LV_d}[E - (1-\mu)V_d]. \tag{23}$$

The equilibrium points of (23) are given by

$$\mu = 1; \quad \mu = 1 - \frac{E}{V_d}. \tag{24}$$

The equilibrium value $\mu = U = 1 - (E/V_d)$ has physical significance, provided $V_d > E$. This fact confirms the 'amplifying' features of the 'boost' converter. However, the phase-plane diagram of equation (23), shown in Fig. 6, readily reveals that this equilibrium point is unstable. We conclude that the average PWM model of the 'boost' converter, with output represented by the average capacitor voltage z_2, is actually a *nonminimum phase* system.

Consider now the output of the circuit to be represented by the average input current z_1. One

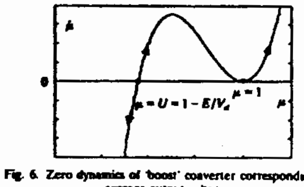

Fig. 6. Zero dynamics of 'boost' converter corresponding to average output voltage.

obtains the following differential input–output representation for the average system:

$$\ddot{z}_1+\left(\frac{1}{RC}+\frac{\dot{\mu}}{1-\mu}\right)\dot{z}_1+\left[(1-\mu)^2\frac{1}{LC}\right]z_1 = \frac{E}{L}\left(\frac{1}{RC}+\frac{\dot{\mu}}{1-\mu}\right). \quad (25)$$

The 'zero dynamics' at an equilibrium point $z_1 = I_d$, associated with the input–output representation (25), is obtained as

$$\dot{\mu}=\frac{1-\mu}{RCE}[(1-\mu)^2 RI_d - E]. \quad (26)$$

The equilibrium points of (23) are given by

$$\mu=1;\quad \mu=1-\sqrt{\frac{E}{RI_d}};\qquad \mu=1+\sqrt{\frac{E}{RI_d}}. \quad (27)$$

The equilibrium value, $\mu = U = 1-\sqrt{E/RI_d}$, has physical significance provided that RI_d, the average steady state voltage across the load resistor, satisfies $RI_d > E$. This fact confirms, once more, the 'amplifying' character of the 'boost' converter. The phase-plane diagram of equation (26), shown in Fig. 7, reveals that this equilibrium point is now locally stable. We conclude that the average PWM model of the 'boost' converter, with output represented by the average input inductor current $y = z_1$, is a *minimum phase* system.

2.3. *The 'buck–boost' converter circuit*

Consider then the switch-regulated 'buck–boost' converter circuit shown in Fig. 8. The differential equations describing the circuit are given by

$$\dot{x}_1=(1-u)\frac{1}{L}x_2+u\frac{E}{L},$$
$$\dot{x}_2=-(1-u)\frac{1}{C}x_1-\frac{1}{RC}x_2, \quad (28)$$

where x_1 and x_2 represent the input inductor current and the output capacitor voltage variables, respectively. The positive quantity E is

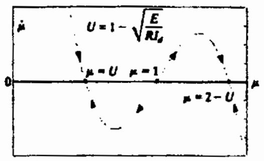

Fig. 7. Zero dynamics of 'boost' converter corresponding to average output voltage.

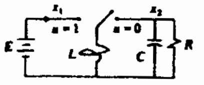

Fig. 8. The 'buck–boost' converter circuit.

the constant value of the external voltage source. The variable u is the switch position function, acting as a control input, taking values in the discrete set {0, 1}. It is assumed that a PWM regulation policy of the form (6) is available for the determination of the switch position function, as a function of time.

We summarize in the following proposition the developments demonstrating that the average PWM model of a 'buck–boost' converter is a EL system for a suitable set of average EL parameters.

Proposition 2.2. The state average model of the 'buck–boost' converter (see Middlebrook and Ćuk, 1976) given by

$$\dot{z}_1=(1-\mu)\frac{1}{L}z_2+\mu\frac{E}{L}$$
$$\dot{z}_2=-(1-\mu)\frac{1}{C}z_1-\frac{1}{RC}z_2 \quad (29)$$

is a EL system corresponding to the following set of average EL parameters:

$$\mathcal{T}_\mu(\dot{q}_L)=\tfrac{1}{2}L\dot{q}_L^2;\quad \mathcal{V}_\mu(q_C)=\frac{1}{2C}q_C^2,$$
$$\mathcal{D}_\mu(\dot{q}_L,\dot{q}_C)=\tfrac{1}{2}R[\dot{q}_C+(1-\mu)\dot{q}_L]^2, \quad (30)$$
$$\mathcal{F}^\mu_{q_L}=\mu E;\quad \mathcal{F}^\mu_{q_C}=0,$$

obtained by suitable modulation, through the duty ratio function μ, of the EL parameters associated with each one of the circuits arising from a particular value of the switch position function $u \in \{0, 1\}$.

$u = 1$:

$$\mathcal{T}_1(\dot{q}_L)=\tfrac{1}{2}L\dot{q}_L^2;\quad \mathcal{V}_1(q_C)=\frac{1}{2C}q_C^2,$$
$$\mathcal{D}_1(\dot{q}_L)=\tfrac{1}{2}R\dot{q}_C^2;\quad \mathcal{F}^1_{q_L}=E;\quad \mathcal{F}^1_{q_C}=0. \quad (31)$$

$u = 0$:

$$\mathcal{T}_0(\dot{q}_L)=\tfrac{1}{2}L\dot{q}_L^2;\quad \mathcal{V}_0(q_C)=\frac{1}{2C}q_C^2$$
$$\mathcal{D}_0(\dot{q}_L,\dot{q}_C)=\tfrac{1}{2}R(\dot{q}_L+\dot{q}_C)^2; \quad (32)$$
$$\mathcal{F}^0_{q_L}=0;\quad \mathcal{F}^0_{q_C}=0.$$

Figure 9 depicts the equivalent circuit of the average PWM regulated dynamics for the 'buck–boost' converter circuit.

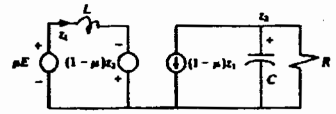

Fig. 9. Equivalent circuit of the average PWM model of the 'buck–boost' converter circuit.

We will be using the following matrix representation of (29):

$$\mathcal{D}_{BB}\dot{z}+(1-\mu)\mathcal{J}_{BB}z+\mathcal{R}_{BB}z=\mu\mathcal{E}_{BB}, \quad (33)$$

where

$$\mathcal{D}_{BB}=\begin{bmatrix}L&0\\0&C\end{bmatrix};\quad \mathcal{J}_{BB}=\begin{bmatrix}0&-1\\1&0\end{bmatrix};$$
$$\mathcal{R}_{BB}=\begin{bmatrix}0&0\\0&1/R\end{bmatrix};\quad \mathcal{E}_{BB}=\begin{bmatrix}E\\0\end{bmatrix}. \quad (34)$$

2.3.1. *Some additional facts.* Similarly to the 'boost' converter case, one can easily establish the nonminimum phase character of the average model of the PWM regulated 'buck–boost' converter system when the output of the system is taken as the average capacitor voltage z_2. When the output of the system is taken to be the average input inductor current z_1, the resulting input–output system is seen to be locally minimum phase (see Sira-Ramirez and Lischinsky-Arenas, 1991).

Given a constant duty ratio function $\mu = U$, it easily follows from the average PWM equations (29) that the corresponding stable equilibrium values for the average input current, denoted by I_d, and the average output voltage, denoted by V_d, are given by

$$I_d=\left[\frac{U}{(1-U)^2}\right]\frac{E}{R};\quad V_d=-\left(\frac{U}{1-U}\right)E. \quad (35)$$

This means that, depending on the particular value of the steady state duty ratio function U, the 'buck–boost' converter can accomplish, in steady state, either source voltage 'amplification' or 'attenuation', modulo a polarity inversion, at the load.

It follows from (35) that, given a desired equilibrium value V_d for the output voltage, which corresponds to a constant value U of the duty ratio function μ, then the *unique* corresponding equilibrium value for the average input current I_d is given by

$$I_d=-\frac{V_d}{R(1-U)}=\left(\frac{V_d}{RE}-\frac{1}{E}\right)V_d. \quad (36)$$

Hence, if we want to regulate z_2 towards an equilibrium value V_d, which corresponds to a steady state value $U = V_d/(V_d - E)$ of the duty ratio function μ, then, such a regulation can be *indirectly* accomplished by stabilizing the average input current z_1 towards the corresponding equilibrium value I_d computed from (36).

2.4. *The 'buck' converter circuit*

The 'buck' converter model is described by the following set of differential equations, with variables defined as before:

$$\dot{x}_1=-\frac{1}{L}x_2+u\frac{E}{L},$$
$$\dot{x}_2=\frac{1}{C}x_1-\frac{1}{RC}x_2. \quad (37)$$

The following proposition summarizes the EL formulation of the average 'buck' converter model.

Proposition 2.3. The state average model of the 'buck' converter (see Middlebrook and Ćuk, 1976), given by

$$\dot{z}_1=-\frac{1}{L}z_2+\mu\frac{E}{L},$$
$$\dot{z}_2=\frac{1}{C}z_1-\frac{1}{RC}z_2, \quad (38)$$

is an EL system corresponding to the following set of average EL parameters:

$$\mathcal{T}_\mu(\dot{q}_L)=\tfrac{1}{2}L\dot{q}_L^2;\quad \mathcal{V}_\mu(q_C)=\frac{1}{2C}q_C^2,$$
$$\mathcal{D}_\mu(\dot{q}_L,\dot{q}_C)=\tfrac{1}{2}R[\dot{q}_C-\dot{q}_L]^2; \quad (39)$$
$$\mathcal{F}^\mu_{q_L}=\mu E;\quad \mathcal{F}^\mu_{q_C}=0,$$

obtained by suitable modulation, through the duty ratio function μ of the EL parameters associated to each one of the circuits arising from a particular value of the switch position function $u \in \{0, 1\}$.

$u = 1$:

$$\mathcal{T}_1(\dot{q}_L)=\tfrac{1}{2}L\dot{q}_L^2;\quad \mathcal{V}_1(q_C)=\frac{1}{2C}q_C^2,$$
$$\mathcal{D}_1(\dot{q}_C)=\tfrac{1}{2}R(\dot{q}_C-\dot{q}_L)^2; \quad (40)$$
$$\mathcal{F}^1_{q_L}=E;\quad \mathcal{F}^1_{q_C}=0.$$

$u = 0$:

$$\mathcal{T}_0(\dot{q}_L)=\tfrac{1}{2}L\dot{q}_L^2;\quad \mathcal{V}_0(q_C)=\frac{1}{2C}q_C^2,$$
$$\mathcal{D}_0(\dot{q}_L,\dot{q}_C)=\tfrac{1}{2}R(\dot{q}_C-\dot{q}_L)^2; \quad (41)$$
$$\mathcal{F}^0_{q_L}=0;\quad \mathcal{F}^0_{q_C}=0.$$

A matrix form for the average model of the 'buck' converter is given by

$$\mathcal{D}_b \dot{z} + (\mathcal{J}_b + \mathcal{R}_b) z = \mu \mathcal{E}_b, \tag{42}$$

where

$$\mathcal{D}_b = \begin{bmatrix} L & 0 \\ 0 & C \end{bmatrix}; \quad \mathcal{J}_b = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix};$$
$$\mathcal{R}_b = \begin{bmatrix} 0 & 0 \\ 0 & 1/R \end{bmatrix}; \quad \mathcal{E}_b = \begin{bmatrix} E \\ 0 \end{bmatrix}. \tag{43}$$

If the output of the converter is taken as the average capacitor voltage, the input-output representation exhibits no zero dynamics and it is given by

$$\ddot{z}_2 + \frac{1}{RC}\dot{z}_2 + \frac{1}{LC} z_2 = \mu \frac{E}{LC}. \tag{44}$$

The constant equilibrium points for a constant duty ratio $\mu = U$ are found to be

$$\mu = U; \quad z_2 = V_d = UE. \tag{45}$$

If, on the other hand, the average inductor current is taken as the output of the system, the input-output representation results in

$$\ddot{z}_1 + \frac{1}{RC}\dot{z}_1 + \frac{1}{LC} z_1 = \frac{E}{L}\left(\dot{\mu} + \frac{1}{RC}\mu\right), \tag{46}$$

which exhibits the following equilibrium point, for a constant duty ratio of value U:

$$\mu = U; \quad z_1 = I_d = U\frac{E}{R}. \tag{47}$$

In this case, the zero dynamics turns out to be asymptotically stable towards the unique equilibrium point:

$$\dot{\mu} = -\frac{1}{RC}\left(\mu - \frac{RI_d}{E}\right). \tag{48}$$

3. PASSIVITY BASED PWM CONTROLLERS FOR DC-to-DC POWER CONVERTERS

3.1. *Controller design: the 'boost' converter*

3.1.1. *Direct output voltage regulation.* Suppose it is desired to regulate directly the output capacitor voltage to a constant value $z_{2d} = V_d$. Corresponding to this objective for the output voltage z_2, the required input current may be represented by a function $z_{1d}(t)$, to be determined later.

Consider then the error variables $\tilde{z}_1(t) = z_1(t) - z_{1d}(t)$ and $\tilde{z}_2(t) = z_2(t) - V_d$. We denote the average state error vector by $\tilde{z}$. Following the passivity-based methodology, we want to shape the closed-loop energy function to a desired energy function:

$$H_d = \tfrac{1}{2}\tilde{z}^T \mathcal{D}_B \tilde{z} \tag{49}$$

This choice is, as usual, motivated by the form of the total energy function of the average system model, which, as shown before, is given by $H = \frac{1}{2} z^T \mathcal{D}_B z$.

The average error vector dynamics is given by

$$\mathcal{D}_B(\dot{\tilde{z}} + (1-\mu)\mathcal{J}_B \tilde{z} + \mathcal{R}_B \tilde{z}$$
$$= \mathcal{E}_B - (\mathcal{D}_B \dot{z}_d + (1-\mu)\mathcal{J}_B z_d + \mathcal{R}_B z_d). \tag{50}$$

To ensure asymptotic stability, we also perform a *damping injection* on (50) by defining the following desired Rayleigh error dissipation term

$$\mathcal{R}_d = \tfrac{1}{2}\tilde{z}^T \mathcal{R}_{Bd} \tilde{z} = \tfrac{1}{2}\tilde{z}^T(\mathcal{R}_B + \mathcal{R}_{1B})\tilde{z}, \tag{51}$$

where

$$\mathcal{R}_{1B} = \begin{bmatrix} R_1 & 0 \\ 0 & 0 \end{bmatrix}; \quad R_1 > 0. \tag{52}$$

Adding the necessary expressions to both sides of (50) we obtain

$$\mathcal{D}_B \dot{\tilde{z}} + (1-\mu)\mathcal{J}_B \tilde{z} + \mathcal{R}_{Bd} \tilde{z} = \Psi, \tag{53}$$

where

$$\Psi := \mathcal{E}_B - (\mathcal{D}_B \dot{z}_d + (1-\mu)\mathcal{J}_B z_d - \mathcal{R}_B z_d - \mathcal{R}_{1B}\tilde{z}). \tag{54}$$

The energy shaping plus damping injection will be achieved if we can set $\Psi = 0$. In this case, the stabilization error dynamics would satisfy

$$\mathcal{D}_B \dot{\tilde{z}} + (1-\mu)\mathcal{J}_B \tilde{z} + \mathcal{R}_{Bd} \tilde{z} = 0. \tag{55}$$

To explain the rationale of the approach, consider the behavior of the desired total energy H_d, the time derivative of which along the solution of (55) results, for some strictly positive constant α, in

$$\dot{H}_d = -\tilde{z}^T \mathcal{R}_{Bd} \tilde{z} \le -\frac{\alpha}{\beta} H_d < 0 \quad \forall \tilde{z} \ne 0, \tag{56}$$

where α may be taken to be $\alpha = \min\{R_1, 1/R\}$ and $\beta = \max\{L, C\}$.

We conclude that, if the error dynamics coincides with (55), the stabilization error behavior is asymptotically stable to zero, i.e. $\tilde{z} \to 0$ independently of μ.

Thus, in order to satisfy (55), one must demand from (54) that

$$\mathcal{D}_B \dot{z}_d + (1-\mu)\mathcal{J}_B z_d + \mathcal{R}_B z_d - \mathcal{R}_{1B}\tilde{z} = \mathcal{E}_B. \tag{57}$$

These conditions are explicitly written as

$$L\dot{z}_{1d} + (1-\mu) z_{2d} - (z_1 - z_{1d}) R_1 = E,$$
$$C\dot{z}_{2d} - (1-\mu) z_{1d} - \frac{1}{R} z_{2d} = 0. \tag{58}$$

The problem thus consists of, given a desired constant output voltage value $z_{2d} = V_d$, finding a bounded function $z_{1d}(t)$ and a suitable duty ratio function μ such that (58) is satisfied. We proceed to eliminate the variable $z_1(t)$ from (58) as follows. From the second equation in (58), one obtains

$$z_{1d}(t) = \frac{V_d}{R(1-\mu(t))}. \tag{59}$$

Substituting this expression into the first equation in (58), one obtains, after some algebraic manipulations, an expression for the dynamical feedback duty ratio synthesizer of the form

$$\dot{\mu} = \frac{R(1-\mu)^2}{LV_d} \times \left[E - (1-\mu)V_d + R_1\left(z_1 - \frac{V_d}{R(1-\mu)}\right)\right] \tag{60}$$

This controller stabilizes z_1 and z_2 towards their desired values z_{1d} and z_{2d}, respectively. However, controller (60) is, unfortunately, not feasible due to its lack of stability. Indeed the 'remaining', or zero, dynamics associated with the above controller results in

$$\dot{\mu} = \frac{R(1-\mu)^2}{LV_d}[E - (1-\mu)V_d], \tag{61}$$

which coincides with the zero dynamics already found in (23) and shown to be unstable around its only physically meaningful equilibrium point.

3.1.2 *Indirect output voltage regulation.* The previous section has shown that a *direct* output voltage control scheme is unfeasible. In this section we provide a feasible regulation alternative based on an *indirect* output capacitor voltage control, achievable through the regulation of the input current. Note that some other possible alternatives include proposing a different error energy function for the system. In this instance, we have just chosen to explore the implications of using the *most natural* energy function for the system.

Suppose it is desired to regulate z_1 towards a

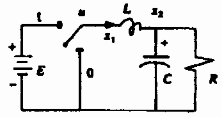

Fig. 10. 'Buck' converter circuit.

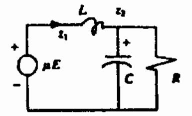

Fig. 11. Equivalent circuit of the average PWM model of the 'buck' converter circuit.

constant value $z_{1d} = I_d$. In order to find a suitable feedback controller for this task, one proceeds now to eliminate the variable z_{2d} from the set of equations (58). Using the first equation in (58), $z_{2d}(t)$ is given by

$$z_{2d}(t) = \frac{E + (z_1 - I_d)R_1}{(1-\mu(t))}. \tag{62}$$

Substituting (62) in the second equation of (58), one obtains, after some algebraic manipulations,

$$\dot{\mu} = \frac{(1-\mu)}{C[E + (z_1 - I_d)R_1]}\left\{(1-\mu)^2 I_d - \frac{E + (z_1 - I_d)R_1}{R} - \frac{R_1 C}{L}[E - (1-\mu)z_2]\right\}. \tag{63}$$

The 'remaining' dynamics associated with controller (63) is obtained by letting z_1 and z_2 coincide with their corresponding desired values. Such dynamics is given by

$$\dot{\mu} = \frac{1-\mu}{RCE}[(1-\mu)^2 R I_d - E]. \tag{64}$$

The zero dynamics (64) coincides with the zero dynamics derived in (26), which was shown to be locally stable around the only physically meaningful equilibrium point. The indirect controller (63) is, therefore, feasible.

We will now complete the proof that the equilibrium point $(z_1, z_2, \mu) = (I_d, V_d, U)$ of the overall system, (14) and (63), is locally asymptotically stable. To this end, we introduce the following auxiliary variable:

$$\xi = \frac{1}{2}\left(\frac{E + (z_1 - I_d)R_1}{1-\mu}\right)^2 - \frac{V_d^2}{2}, \tag{65}$$

which is well defined for μ in a neighborhood of the equilibrium point $\mu = U \ne 1$. It is easy to show that ξ satisfies the following *linear* differential equation:

$$\dot{\xi} = -\frac{2}{RC}\xi + \frac{V_d^2 R_1}{RCE}(z_1 - I_d). \tag{66}$$

Recalling that $\tilde{z}_1 = z_1 - I_d \to 0$, and is exponentially fast, we conclude that $\xi \to 0$ as well. This implies that $z_{2d} \to V_d$ locally, and in turn implies that $\mu \to U$.

We have thus proven the following proposition.

Proposition 3.1. Given a desired constant value $V_d > E$ for the output capacitor voltage of a 'boost' converter, the dynamically generated duty ratio function (63), with I_d given by (21), locally asymptotically stabilizes the state trajectories of the average PWM model (14) towards the desired equilibrium point (I_d, V_d, U), with μ converging to a constant value $\bar{\mu} = U = 1 - E/V_d$.

3.1.3. *Further remarks.* The passivity-based dynamical duty ratio synthesizer design is carried out under the assumption that the average PWM model (14) of the converter captures the essential behavior of the actual switch-regulated circuit described by (5). This assumption has been shown to be only approximately valid due to the fact that, in practice, infinite sampling frequency and corresponding infinitely fast switchings are impossible to achieve. However, for *sufficiently high* sampling frequencies, feedback controllers designed on the basis of average models can indeed be used to regulate the actual switched converter, with rather satisfactory results (see Kassakian *et al.*, 1991). The scheme, shown in Fig. 12 is based in this philosophy. The underlying approach has been extensively used for similar nonlinear dynamical feedback controllers, and its validity has been justified both from a theoretical viewpoint and through extensive computer simulation results (see e.g. Sira-Ramirez and Lischinsky-Arenas, 1991, and references cited therein).

Two additional remarks are in order, regarding the use of a feedback PWM scheme such as the one shown in Fig. 12:

(i) The average-based duty ratio synthesizer produces a *computed* duty ratio function. As such, it is entirely possible that these computed values exceed the physical bounds of the required *actual* duty ratio function, which is necessarily limited, to the closed interval [0, 1]. For this reason, a *hard limiter* must be used in conjunction with the derived dynamical feedback regulator, as shown in Fig. 12. As a consequence of this limitation, only *local asymptotic stability* of the closed-loop system may actually be guaranteed. Large initial-state deviations may induce destabilizing saturation effects, which have not been accounted for in the previous developments.

(ii) The duty ratio synthesizer (63) requires the on-line values of the average PWM circuit states z_1 and z_2. These average states can be approximately obtained by *low-pass filtering* of the actual circuit states x_1 and x_2. Note, however, that in the scheme presented in Fig. 12 the *actual* circuit states x_1 and x_2 are used for feedback, rather than their averaged, or filtered, versions z_1 and z_2. It should be pointed out then that, again, for large sampling frequencies the difference between using one or the other set of states is entirely negligible, due to the underlying low-pass filtering effects of the system itself.

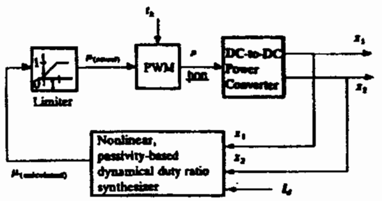

Fig. 12. PWM feedback control scheme for indirect, passivity-based, output voltage regulation for DC-to-DC power converters.

3.2. *Controller design: the 'buck–boost' converter*

Following exactly the same procedure as in the previous case, one concludes that for the 'buck–boost' converter a direct regulation policy of the output voltage is unfeasible due to nonminimum phase phenomena. We thus summarize in a proposition the dynamical feedback regulation scheme for achieving indirect output capacitor voltage regulation, towards a given desired equilibrium value V_d, through input current stabilization towards a desired constant value I_d, computable in terms of V_d, as given by (36).

Proposition 3.2. Given a desired constant value V_d for the output capacitor voltage of a 'buck–boost' converter, the dynamically generated duty ratio function given by

$$\dot{\mu} = \frac{1-\mu}{C[E + (z_1 - I_d)R_1]}\left\{(1-\mu)^2 I_d - \frac{\mu E + (z_1 - I_d)R_1}{R} - \frac{R_1 C}{L}[\mu E + z_2(1-\mu)]\right\} \quad (67)$$

locally asymptotically stabilizes the state trajectories of the average PWM model (29) towards the desired equilibrium point (I_d, V_d), with μ converging to a constant value given by $\mu = U = 1 - \sqrt{E/RI_d}$, with I_d obtained from V_d from (36).

Note that the zero dynamics associated with the controller (67) is given by

$$\dot{\mu} = \frac{(1-\mu)}{RCE}[(1-\mu)^2 RI_d - \mu E], \quad (68)$$

which has three equilibrium points given by

$$\mu = 1; \quad \mu = 1 + \frac{E}{2RI_d} \pm \sqrt{\left(\frac{E}{2RI_d}\right)^2 + \frac{E}{RI_d}}. \quad (69)$$

Two of the equilibrium points ($\mu = 1$, and the one corresponding to the plus sign of the square root) are unstable, while the remaining one, which is the only physically significant one is locally asymptotically stable.

3.3. *Controller design: the 'buck' converter*

The developments leading to a duty ratio synthesizer for the 'buck' converter are similar to those presented for the other converters. We only point out that, unlike in the previous cases, a direct output voltage regulation scheme is feasible due to the fact that the 'buck' converter exhibits no zero dynamics for such an output. In this case, the duty ratio synthesizer turns out to be *static* rather than dynamic. If, as before, we denote by V_d the desired output capacitor voltage, the passivity-based controller may be described as in the following proposition.

Proposition 3.3. Given a desired constant value $V_d < E$ for the output capacitor voltage of a 'buck' converter, the statically generated duty ratio function given by

$$\mu = \frac{V_d}{E} - \frac{1}{ER_1}\left(z_1 - \frac{V_d}{R}\right) \quad (70)$$

locally asymptotically stabilizes the state trajectories of the average PWM model (38) towards the desired equilibrium point (I_d, V_d), with I_d given by $I_d = V_d/R$ while μ converges to a constant value given by $\mu = U = V_d/E$.

It is easy to see, by substituting z_1 in terms of z_2 and $\dot{z}_2$, that this controller is just a classical 'proportional derivative' controller.

The indirect output voltage regulation scheme results in a dynamic controller with asymptotically stable zero dynamics. The details are not presented.

4. SIMULATION RESULTS

Simulations were performed for the closed-loop behavior of a 'boost' circuit regulated by means of the passivity-based indirect PWM controller (63). In order to test the effectiveness and robustness of the proposed feedback controller with respect to unmodeled parasitic resistances and unmodeled realistic switching devices, the following stochastically perturbed version of a 'boost' converter circuit, taken from the work of Czarkowski and Kazimierczuk (1993), was used for the simulations:

$$\dot{x}_1 = -\frac{1}{L}r(u) - (1-u)\frac{R}{L(R+r_C)}x_2 + \frac{E+\eta}{L} - (1-u)\frac{V_F}{L},$$
$$\dot{x}_2 = (1-u)\frac{R}{(R+r_C)C}x_1 - \frac{1}{(R+r_C)C}x_2, \quad (71)$$

where $r(u) = r_L + ur_{DS} + (1-u)(R_F + r_C \| R)$ r_L is the resistance associated with the inductor; r_{DS} is the resistance associated with the ON state of the transistor used in the realization of the switching element constituted by a transistor–diode arrangement; R_F is the forward resistance of the diode; r_C is the resistance associated with the output capacitor; and $r_C \| R$ denotes the resistance of a parallel arrangement of r_C and R. The voltage V_F represents a small constant voltage drop associated with the conducting phase of the diode. The signal η, added to the external source voltage, represents an external stochastic perturbation input affecting the system behavior.

Note that the perturbation input η is of the 'unmatched' type, i.e. it enters the system equations through an input channel vector field given by $[1/L \;\; 0]^T$ which is *not* in the range space of the control input channel, given by the vector field

$$\begin{bmatrix} -\frac{x_{DS} + R_F + r_C \| R}{L}x_1 + \frac{R}{L(R+r_C)}x_2 - \frac{V_F}{L} \\ \frac{R}{(R+r_C)C}x_1 \end{bmatrix}$$

The peak-to-peak magnitude of the noise was chosen to be approximately 20% of the value of E. The circuit parameter values were taken to be the following 'typical' values: $C = 20\ \mu F$, $R = 30\ \Omega$, $L = 20$ mH, $E = 15$ V, $r_L = 0.05\ \Omega$, $r_C = 0.2\ \Omega$, $r_{DS} = 0.1\ \Omega$, $R_s = 0.05\ \Omega$, $V_s = 0.7$ V. The sampling frequency for the PWM policy was set at 5 kHz. The duty ratio function is obtained from a sampling process carried out on the output $\mu(t)$ of the smooth dynamical duty ratio synthesizer (63). To avoid the use of low-pass filters, instead of using the averaged state variables z_1 and z_2 for feedback on the duty ratio synthesizer, we used, as it is customarily done, the actual PWM controlled states x_1 and x_2 on the controller expressions. The desired ideal average input inductor current was set to be $I_d = 3.125$ A, with a steady-state duty ratio of $U = 0.6$. This corresponds to an ideal average output voltage of $z_2 = V_d = 37.5$ V. Figure 13 shows the closed-loop state trajectories as well as the duty ratio function and a realization of the computer-generated stochastic perturbation signal η.

As can be seen from the simulations, the proposed dynamical feedback controller (63) achieves the desired indirect stabilization of the output voltage for the nonideal stochastically perturbed model around the desired equilibrium value. The average steady-state errors, with respect to the desired equilibrium values, range from approximately 2.5% in the average inductor current variable to 2.6% in the average capacitor voltage variable. The ideal duty ratio is achieved within less than 0.5% error. The controller performance also exhibits a high degree of robustness with respect to the external stochastic perturbation inputs.

Unknown load resistance variations generally affect the behavior of the closed-loop performance of the controlled converter. Simulations, shown in Fig. 14, were performed to depict the sensitivity of the regulated input current, the output capacitor voltage, and the duty ratio with respect to abrupt, but temporary, unmodeled changes in the load resistance R. An unmodeled sudden change in the load resistance was set to 80% of its nominal value. As can be seen from the figures, the controller manages rapidly to restore the desired steady-state conditions immediately after the load perturbation disappears. As expected, the state variable most affected by such a perturbation is the output voltage. Conversely, the duty ratio function is barely affected by such sudden load changes.

An extension of the above presented controller design method, which is also capable of handling unknown but constant loads, has been undertaken by the authors within the context of nonlinear *adaptive regulation*. The reader can find details in Sira-Ramirez *et al.* (1995).

5. CONCLUSIONS

Traditional state average models, or infinite switching frequency models, of DC-to-DC power converters were shown to be EL systems for a suitable set of average EL parameters. The derived average PWM models were also shown to be interpretable in terms of ideal circuit realizations, including internal controlled sources and modulated external inputs.

Physically motivated dynamic feedback duty ratio synthesizers were derived for the indirect average output voltage stabilization of DC-to-

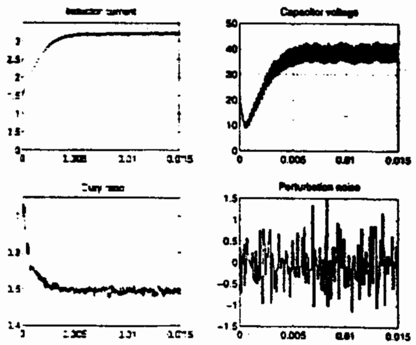

Fig. 13. Simulation results for performance evaluation of the indirect PWM controller in a realistic perturbed 'boost' converter.

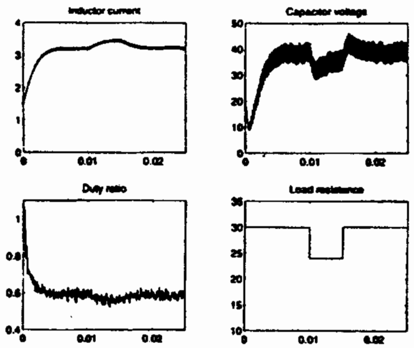

Fig. 14. Robustness evaluation of the indirect PWM controller in a realistic perturbed 'boost' converter.

DC power converters of the 'boost', 'buck–boost', and 'buck' types. The dynamic feedback controllers are based on the modification of the total energy function of the average converter circuit model. This procedure, together with the possibilities of enhancing the dissipation structure of the average models through suitable 'damping injections', were shown to yield asymptotically stable closed-loop behavior with feasible locally asymptotically stable controllers.

Other useful connections of passivity-based controllers with *differential flatness* (see Fliess *et al.*, 1992), associated with the average PWM models of DC-to-DC power converters, remain to be explored. Similarly, sliding-mode controllers, based on passivity considerations, remain to be developed for DC-to-DC power converters.

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ADAPTIVE PASSIVITY-BASED CONTROL OF AVERAGE DC-TO-DC POWER CONVERTER MODELS

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SUMMARY

An adaptive feedback regulation scheme is proposed for the stabilization of average models of dc-to-dc power converters exhibiting unknown but constant resistive loads. The scheme is based on a dynamical feedback policy which suitably modifies the total energy of the closed-loop system while inducing appropriate damping injections on the desired stabilization error dynamics. The performance of the proposed adaptative regulators is tested through computer simulations including stochastic perturbation inputs.

Key words: dc-to-dc power converters; adaptative control; passivity

1. INTRODUCTION

Feedback regulation of dc-to-dc power supplies has been extensively treated in the literature. Conference proceedings, such as the IEEE Power Electronics Specialist Conference (PESC) Records, Multi-volume Series, edited over the years,[1,2] a growing list of text books[3–7] and edited collections of research articles[8] reflect both the theoretical and practical importance of this field. We remark that dc-to-dc power supplies and, more generally, the area of *Power Electronics*, which has been traditionally credited to the discipline of *Industrial Electronics*, enjoys a growing interest in the *Automatic Control* community.

A frequent assumption in the design of feedback regulators for dc-to-dc power supplies is that the converter loads and the parameters associated with the various circuit components are

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perfectly known. In practise, however lack of precise knowledge about these parameters arises from inescapable measurement errors, unavoidable ageing effects and imperfectly modelled loads. These facts motivate the adoption of an adaptive feedback approach for the design of regulation loops in dc-to-dc power supplies. Adaptive control of dc-to-dc power supplies has been treated, from an approximate linearization viewpoint, by Sanders and Verghese in Reference 9. Their approach relies on Lyapunov stability and passivity considerations for the linear feedback controller design. A full adaptive feedback input–output linearization viewpoint for dc-to-dc power supplies was proposed by Sira-Ramirez *et al.* Reference 10. An adaptive feedback design technique that suitably combines input–output linearization, through generalized observability canonical forms as developed by Fliess in Reference 11, and the backstepping design procedure, was recently presented by Sira-Ramirez *et al.* in Reference 12, 13.

In the last few years, a feedback control design methodology for non-linear systems that exploits the physical restrictions of the system, and, in particular, its energy properties, has been developed. The approach, known as *passivity-based* controller design, consists of an energy-shaping stage where the closed-loop total energy of the system is modified, and a damping injection stage where the required dissipation is added in order to achieve asymptotic stability. The approach has been successfully used in the regulation of Euler–Lagrange (EL) systems, such as rigid and flexible robotic manipulators (see the works of Takegaki and Arimoto[14] and the subsequent developments by Ortega and Spong,[15] and Brogliato *et al.*[16] The same technique has also been used, with the same degree of success, in the regulation of electro-mechanical energy conversion devices (see the work by Ortega and Espinosa[17] and a recent article by Ortega *et al.*[18] A non-adaptive passivity-based approach has also been recently developed for dc-to-dc power converters by the authors.[19]

The main motivation of this work is to extend the developments in Reference 19, and explore the viability of applying the described passivity-based controller design methodology for the *adaptive* stabilization of a class of average models of pulse-width-modulation (PWM) regulated dc-to-dc power converters. For the sake of completeness, we treat three types of switched power supplies. Namely, 'boost' (or 'step-up' converter), 'buck-boost' (or step up-down' converter) and the 'buck' (step down) types of converters. The encouraging results corresponding to the passivity-based approach for PWM dc-to-dc power converter regulation, developed in Reference 19, have motivated an actual experimental comparison of several feedback controllers, including the classical linear controller. The several controllers that have been compared correspond to: a linear controller, a feedback linearization controller and a passivity-based controller. The results and experimental data are fully reported in a recent article by Escobar *et al.* in Reference 20 (see also Reference 21). The bottom line of the experimental results is that the passivity-based controller outperforms, both, in simplicity of implementation and robustness with respect to external noises and modelling errors, the classical and the feedback linearization schemes.

This article is organized as follows. Section 2 presents the average PWM models of the three types of dc-to-dc power converters. To make the presentation self-contained, non-adaptive passivity-based feedback regulators, such as those developed in Reference 19, are briefly revisited in Section 2 for the treated converters. Section 3 assumes that the resistive loads of the converters under study is constant but, otherwise, totally unknown. We proceed to derive adaptive feedback regulation schemes based on passivity considerations. Section 4 contains simulations of the proposed passivity-based adaptive controllers. Section 5 contains the conclusions and suggestions for further research in this area.

2. PASSIVITY-BASED REGULATION OF AVERAGE DC-TO-DC POWER CONVERTER MODELS

Average models of PWM regulated dc-to-dc switched power converters have been justified from theoretical and practical grounds in the work of many authors. Average models of dc-to-dc power converters were first introduced by Middlebrook and Ćuk in References 22, 23, under the name of *state average* models. Their approach was based on a discretization viewpoint, based on simplifying approximations of the corresponding state transition matrices of the involved linear systems. The state average models were later generalized and refined, using the analytic theory of averaging of differential equations, in the work of Krein *et al.*,[24] where a wider variety of average models with improved approximation features were shown to be entirely feasible. A completely different viewpoint based on Filippov's geometric averaging, was pursued later by Sira-Ramirez *et al.* in References 25, 26. These developments were motivated by definite mathematical connections between Sliding Mode Control and PWM regulation of non-linear systems. The approach obtained exactly the same average models initially proposed by Middlebrook and Ćuk. A physically motivated justification of the average PWM models to dc-to-dc power converters, based on an Euler–Lagrange formulation, has also been proposed by Sira-Ramirez and Delgado in Reference 27.

In this section, we use the average PWM models of dc-to-dc power converters without further justification. We proceed to find non-adaptive passivity-based feedback controllers for average models of dc-to-dc power converters without additional considerations about the nature of the approximation that such models imply when the corresponding duty ratio designs are used in actual (i.e. discontinuous) PWM feedback regulation loops. We simply point out that as the sampling frequency in the actual PWM regulation scheme is increased, the closed-loop state responses rapidly converge towards the corresponding closed-loop average state trajectories (see Reference 10).

We remark also that our controller schemes, at least for the 'boost' and 'buck-boost' converters, are of the *indirect* type, i.e. we deliberately seek to indirectly regulate the output capacitor voltage towards a feasible desired equilibrium value. For this we design a feedback controller which primarily accomplishes the asymptotic regulation of the input current towards the unique equilibrium value corresponding to the required constant output voltage. If the opposite policy is adopted, the resulting controllers are invariably *unstable* due to the well-known *non-minimum phase* character of the output voltage when taken as a converter output variable. We stress that for the average 'buck' converter case, direct, or indirect, feedback regulation policies are equally feasible and devoid of any non-minimum phase instabilities. Further details and mathematical justifications of these facts can be found in the articles by Sira-Ramirez and Lischinsky-Arenas[28] and Sira-Ramirez *et al.*[19]

2.1. A passivity-based controller for the 'boost' converter

Consider the average PWM model of a 'boost' converter circuit, shown in Figure 1 (see References [10, 19, 27]).

$$\dot{z}_1 = -(1-\mu)\frac{1}{L}z_2 + \frac{E}{L}$$

$$\dot{z}_2 = (1-\mu)\frac{1}{C}z_1 - \frac{1}{RC}z_2 \qquad (1)$$

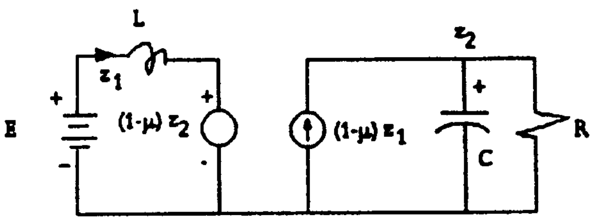

Figure 1. Average model of PWM regulated 'boost' converter

where z_1 and z_2 denote the *average input current* and the *average output capacitor voltage*, respectively. The scalar quantity μ stands for the *duty ratio* function which truly acts as the external control input to the average system model. The duty ratio is naturally constrained to take values in the interval [0, 1] of the real line.

For ease of reference we will be using the following, more compact, matrix representation of system (1):

$$\mathcal{D}_B \dot{z} + (1-\mu)\mathcal{J}_B z + \mathcal{R}_B z = \mathcal{E}_B \tag{2}$$

where

$$\mathcal{D}_B = \begin{bmatrix} L & 0 \\ 0 & C \end{bmatrix}, \quad \mathcal{J}_B = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix}, \quad \mathcal{R}_B = \begin{bmatrix} 0 & 0 \\ 0 & 1/R \end{bmatrix}, \quad \mathcal{E}_B = \begin{bmatrix} E \\ 0 \end{bmatrix} \tag{3}$$

Suppose it is desired to indirectly regulate the average output capacitor voltage to a constant equilibrium value given by, $z_2 = V_d$. Corresponding to this objective for the average output voltage z_2, the corresponding required average input current may be *uniquely* computed from (1) as

$$z_1 = \frac{V_d^2}{RE} \tag{4}$$

Consider then the error variables $\tilde{z}_1(t) = z_1(t) - z_{1d}(t)$ and $\tilde{z}_2(t) = z_2(t) - V_d$. We denote the average state error vector by $\tilde{z}^T = [\tilde{z}_1(t), \tilde{z}_2(t)]$. The average error vector dynamics is then given by

$$\mathcal{D}_B \dot{\tilde{z}} + (1-\mu)\mathcal{J}_B \tilde{z} + \mathcal{R}_B \tilde{z} = \mathcal{E}_B - (\mathcal{D}_B \dot{z}_d + (1-\mu)\mathcal{J}_B z_d + \mathcal{R}_B z_d) \tag{5}$$

One may perform a *damping injection* on (5) by considering the following desired error dissipation term:

$$\mathcal{R}_{Bd}\tilde{z} = (\mathcal{R}_B + \mathcal{R}_{1B})\tilde{z} \tag{6}$$

where,

$$\mathcal{R}_{1B} = \begin{bmatrix} R_1 & 0 \\ 0 & 0 \end{bmatrix}, \quad R_1 > 0 \tag{7}$$

Adding to both sides of equation (5) the necessary expressions, we obtain

$$\mathcal{D}_B \dot{\tilde{z}} + (1-\mu)\mathcal{J}_B \tilde{z} + \mathcal{R}_{Bd}\tilde{z} = \mathcal{E}_B - (\mathcal{D}_B \dot{z}_d + (1-\mu)\mathcal{J}_B z_d + \mathcal{R}_B z_d - \mathcal{R}_{1B}\tilde{z}) \tag{8}$$

Suppose for a moment that the right-hand side of equation (8) is identically zero. Under these circumstances the stabilization error dynamics would satisfy

$$\mathcal{D}_B \dot{\tilde{z}} + (1-\mu)\mathcal{J}_B \tilde{z} + \mathcal{R}_{Bd}\tilde{z} = 0 \tag{9}$$

Motivated by the form of the total energy function of the average system model, given by $H = \frac{1}{2} z^T \mathcal{D}_B z$, we propose a desired energy function, denoted by H_d, associated with the average state error vector as

$$H_d = \tfrac{1}{2}\tilde{z}^T \mathcal{D}_B \tilde{z} > 0 \quad \forall \tilde{z} \neq 0 \tag{10}$$

Take the expression (10) as a Lyapunov function candidate for the error dynamics (9). The time derivative of $H_d(t)$, along the solutions of (9), results, for some strictly positive constant α, in the following expression:

$$\dot{H}_d(t) = -\tilde{z}\mathcal{R}_{Bd}\tilde{z} \leq -\alpha H_d(t) < 0 \quad \forall \tilde{z} \neq 0 \tag{11}$$

We conclude that if the error dynamics coincides with (9), then the stabilization error behaviour is asymptotically stable to zero.

Thus, in order to have (9) satisfied one must demand, from (8) that

$$\mathcal{D}_B \dot{z}_d + (1-\mu)\mathcal{J}_B z_d + \mathcal{R}_B z_d - \mathcal{R}_{1B}\tilde{z} = \mathcal{E}_B \tag{12}$$

These conditions are explicitly written as

$$\begin{aligned} L\dot{z}_{1d} + (1-\mu)z_{2d} - R_1(z_1 - z_{1d}) &= E \\ C\dot{z}_{2d} - (1-\mu)z_{1d} + \frac{1}{R}z_{2d} &= 0 \end{aligned} \tag{13}$$

The problem, thus, consists in, given a desired constant value for the input current $z_{1d} = V_d^2/RE$, finding a bounded function $z_{2d}(t)$, and a suitable duty ratio function $\mu(t)$, such that (13) is satisfied.

Suppose then that it is desired to regulate z_1 towards the constant value $z_{1d} = V_d^2/RE$. In order to find a suitable feedback duty ratio function for this task, one proceeds to eliminate the input variable μ from the set of equations (13). Using the first equation of (13), the required μ is given by

$$\mu(t) = 1 - \frac{1}{z_{2d}(t)}\left[E + R_1\left(z_1 - \frac{V_d^2}{RE}\right)\right] \tag{14}$$

Substituting (14) into the second equation of (13), one obtains, after some algebraic manipulations, the differential equation satisfied by the controller state, $z_{2d}(t)$,

$$\dot{z}_{2d} = -\frac{1}{RC}\left\{z_{2d} - \frac{V_d^2}{Ez_{2d}}\left[E + R_1\left(z_1 - \frac{V_d^2}{RE}\right)\right]\right\} \tag{15}$$

The 'remaining' dynamics associated with the controller (14) and (15), is obtained by letting z_1 coincide with its desired equilibrium value. Such a dynamics is given by

$$\dot{z}_{2d} = -\frac{1}{RC}\left(z_{2d} - \frac{V_d^2}{z_{2d}}\right) \tag{16}$$

The 'zero dynamics' (16) has two asymptotically stable equilibrium points represented by $z_{2d} = V_d$ and $z_{2d} = -V_d$. It is easy to realize that $z_{2d} = V_d$ is asymptotically stable for all initial conditions of (16) satisfying $z_{2d}(0) > 0$. Similarly, the second equilibrium point is asymptotically stable for all initial conditions satisfying, $z_{2d}(0) < 0$.

We summarize the above result in the following proposition:

Proposition 2.1.

Consider the averaged dynamics of the 'boost' converter,

$$\mathcal{D}_B\dot{z} + (1-\mu)\mathcal{J}_B z + \mathcal{R}_B z = \mathcal{E}_B \tag{17}$$

with $z^T = [z_1, z_2] \in \mathbb{R}^2$, z_1 being the average inductor current and z_2 the average capacitor voltage. The quantity $\mu \in [0, 1]$ is the duty ratio function.

Define a non-linear dynamic state feedback controller as

$$\dot{z}_{2d} = -\frac{1}{RC}\left\{z_{2d} - \frac{V_d^2}{E z_{2d}}\left[E + R_1\left(z_1 - \frac{V_d^2}{RE}\right)\right]\right\}$$

$$\mu(t) = 1 - \frac{1}{z_{2d}(t)}\left[E + R_1\left(z_1 - \frac{V_d^2}{RE}\right)\right] \tag{18}$$

where the dynamical controller initial condition is chosen so that, $z_{2d}(0) > 0$ and the constant reference value for z_2, denoted by V_d is a strictly positive quantity. The quantity R_1 is a designer-chosen constant with the only restriction of being strictly positive. Under these conditions, the closed-loop system (17) and (18) has an equilibrium point given by,

$$(z_1, z_2, z_{2d}) = \left(\frac{V_d^2}{RE}, V_d, V_d\right) \tag{19}$$

which is asymptotically stable.

2.2. A passivity-based controller for the 'buck–boost' converter

The following proposition summarizes the passivity-based controller for an average model of the buck-booster converter circuit

Proposition 2.2.

Consider the average dynamics of the 'buck–boost' converter circuit (see Figure 2),

$$\mathcal{D}_{BB}\dot{z} + (1-\mu)\mathcal{J}_{BB} z + \mathcal{R}_{BB} z = \mu\mathcal{E}_{BB} \tag{20}$$

with $z^T = [z_1, z_2] \in \mathbb{R}^2$, z_1 being the average inductor current and z_2 the average capacitor voltage, $\mu \in [0, 1]$ being the duty ratio function,

$$\mathcal{D}_{BB} = \begin{bmatrix} L & 0 \\ 0 & C \end{bmatrix}, \quad \mathcal{J}_{BB} = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}, \quad \mathcal{R}_{BB} = \begin{bmatrix} 0 & 0 \\ 0 & 1/R \end{bmatrix}, \quad \mathcal{E}_{BB} = \begin{bmatrix} E \\ 0 \end{bmatrix} \tag{21}$$

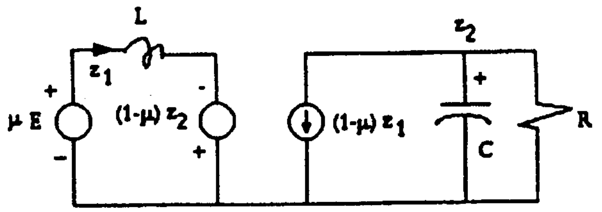

Figure 2. Average model of PWM regulated 'buck–boost' converter

Define a non-linear dynamic state feedback controller as

$$\dot{z}_{2d} = -\frac{1}{RC}\left\{z_{2d} + V_d\left(\frac{V_d}{E} + 1\right)\left[\frac{E + R_1(z_1 - (V_d/R)(V_d/E + 1))}{E - z_{2d}(t)}\right]\right\}$$

$$\mu(t) = \frac{z_{2d}(t) + R_1(z_1 - (V_d/R)(V_d/E + 1))}{z_{2d}(t) - E} \tag{22}$$

where the controller initial condition $z_{2d}(0) < E$ and where $-V_d$ is a constant reference value for z_2, with $V_d > 0$, and $R_1 > 0$ is a designer-chosen constant. Under these conditions, the closed-loop system (20–22) has an equilibrium point,

$$(z_1, z_2, z_{2d}) = \left(\frac{V_d}{R}\left(\frac{V_d}{E} + 1\right), -V_d, -V_d\right) \tag{23}$$

which is asymptotically stable.

Proof. The fact that (23) is an equilibrium point for (20)–(22) follows from direct substitution. Define,

$$z_{1d} = \frac{V_d}{R}\left(\frac{V_d}{E} + 1\right) \tag{24}$$

Note that z_{1d} and z_1 coincide at the equilibrium point. Let $\tilde{z}$ denote, as before, the error signal $z - z_d$. We can write (20) in terms of these error signals as

$$\mathcal{D}_{BB}\dot{\tilde{z}} + (1-\mu)\mathcal{J}_{BB}\tilde{z} + \mathcal{R}_{BBd}\tilde{z} = \psi \tag{25}$$

where

$$\psi = \mu\mathcal{E}_{BB} - \left[\mathcal{D}_{BB}\dot{z}_d + (1-\mu)\mathcal{J}_{BB} z_d + \mathcal{R}_{BB} z_d - \begin{bmatrix} R_1\tilde{z}_1 \\ 0 \end{bmatrix}\right] \tag{26}$$

and $\mathcal{R}_{BBd}$ is a positive-definite matrix given by

$$\mathcal{R}_{BBd} = \begin{bmatrix} R_1 & 0 \\ 0 & 1/R \end{bmatrix}, \quad R_1 > 0 \tag{27}$$

Expression (26) is explicitly written as

$$\psi_1 = -L\dot{z}_{1d} + (1-\mu)z_{2d} + R_1\tilde{z}_1 + \mu E$$

$$\psi_2 = -C\dot{z}_{2d} - (1-\mu)z_{1d} - \frac{1}{R}z_{2d} \tag{28}$$

Using (20)–(22) one has $\psi_1 = 0$ and $\psi_2 = 0$. The resulting stabilization error system is then given by the asymptotically stable dynamics,

$$\mathcal{D}_{BB}\dot{\tilde{z}} + (1-\mu)\mathcal{J}_{BB}\tilde{z} + \mathcal{R}_{BBd}\tilde{z} = 0 \tag{29}$$

Using as a Lyapunov function the total energy of the error system $H_d(t) = \frac{1}{2}\tilde{z}^T\mathcal{D}_{BB}\tilde{z} > 0$ one obtains, for some constant $\alpha > 0$, that along the trajectories of (29) the following relation is satisfied:

$$\dot{H}_d(t) = -\tilde{z}^T\mathcal{R}_{BBd}\tilde{z} \leq -\alpha\|\tilde{z}\|^2 \tag{30}$$

where α may be taken as $\min\{R_1, 1/R\}$. One concludes that $\tilde{z} \to 0$ asymptotically.

The zero dynamics associated with the proposed dynamical feedback controller is given by

$$\dot{z}_{2d} = -\frac{1}{RC}\left[z_{2d} + V_d\left(\frac{V_d}{E}+1\right)\frac{E}{E - z_{2d}}\right] \tag{31}$$

which has an asymptotic equilibrium point at $z_{2d} = -V_d$ for all initial conditions satisfying $z_{2d}(0) < E$ and it also has an equilibrium point located at $z_{2d} = E + V_d$ for all $z_{2d}(0) > E$. □

2.3. *A passivity-based controller for the 'buck' converter*

The following proposition summerizes the passivity-based controller for an average model of the 'buck' converter circuit shown in Figure 3. The proof of the result is left as an exercise for the interested reader.

Proposition 2.3.

Consider the averaged dynamics of the 'buck' converter,

$$\mathcal{D}_b\dot{z} + (\mathcal{J}_b + \mathcal{R}_b)z = \mu\mathcal{E}_b \tag{32}$$

with $z^T = [z_1, z_2] \in \mathcal{R}^2$, z_1 being the average inductor current and z_2 the average capacitor voltage, $\mu \in [0, 1]$ being the duty ratio function.

$$\mathcal{D}_b = \begin{bmatrix} L & 0 \\ 0 & C \end{bmatrix};\quad \mathcal{J}_b = \begin{bmatrix} 0 & 1 \\ -1 & 0 \end{bmatrix};\quad \mathcal{R}_b = \begin{bmatrix} 0 & 0 \\ 0 & 1/R \end{bmatrix};\quad \mathcal{E}_b = \begin{bmatrix} E \\ 0 \end{bmatrix} \tag{33}$$

Define a linear time-varying state feedback controller as

$$\dot{z}_{2d} = -\frac{1}{RC}\left(z_{2d} - V_d\right)$$

$$\mu(t) = \frac{z_{2d}(t) - R_1(z_1 - V_d/R)}{E} \tag{34}$$

where the controller initial condition, $z_{2d}(0)$, satisfies, $E > z_{2d}(0) > 0$, and where $V_d > 0$, is a constant reference value for z_2. The parameter, $R_1 > 0$, is a designer chosen constant. Under

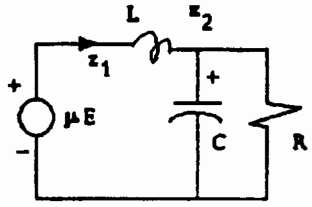

Figure 3. Average model of PWM regulated 'buck' converter

these conditions, the closed-loop system (32)–(34) has an equilibrium point,

$$(z_1, z_2, z_{2d}) = \left(\frac{V_d}{R}, V_d, V_d\right) \tag{35}$$

which is asymptotically stable.

3. PASSIVITY-BASED ADAPTIVE CONTROLLER DESIGN FOR DC-TO-DC POWER CONVERTERS

In this section we assume that the resistive loads, in the various converters, are known to be constant, but otherwise completely unspecified. This type of uncertain situation is commonly present in many practical applications of dc-to-dc power converters. To handle this type of uncertainty, adaptive versions of the previously developed feedback controllers are developed. We emphasize that the results here presented can be extended, with little difficulty, to the case of uncertainties in the rest of the circuit component parameters, L, C and E, of the dc-to-dc power converter circuit.

3.1. *An adaptive controller for the 'boost' converter*

Proposition 3.1.

Consider the averaged dynamics (17) of the 'boost' converter, where $C > 0$, $L > 0$, $E > 0$ are known constants representing the capacitance, inductance and external voltage, respectively, and $R > 0$ is the *unknown* load charge resistance. Define an *adaptive* non-linear dynamic state feedback controller as

$$\dot{z}_{2d} = -\frac{\theta}{C}\left\{z_{2d} - \frac{V_d^2}{Ez_{2d}}\left[E + R_1\left(z_1 - \theta\frac{V_d^2}{E}\right) + L\frac{V_d^2}{E}z_{2d}(z_2 - z_{2d})\right]\right\}$$

$$\mu = 1 - \frac{1}{z_{2d}}\left[E + R_1\left(z_1 - \theta\frac{V_d^2}{E}\right) + L\frac{V_d^2}{E}z_{2d}(z_2 - z_{2d})\right]$$

$$\dot{\theta} = -z_{2d}(z_2 - z_{1d}) \tag{36}$$

where the dynamical controller initial condition is chosen so that, $z_{2d}(0) > 0$ and $\hat{\theta}(0) > 0$. The constant reference value for z_2, denoted by V_d, is a strictly positive quantity. The quantity $\hat{\theta}$ denotes the estimate of $1/R$. The parameter R_1 is a designer-chosen constant with the only restriction of being strictly positive. Under these conditions, it is always possible to choose the controller's initial state $z_{2d}(0)$ and $\hat{\theta}(0)$, such that the closed-loop system (17) and (36) has an equilibrium point given by

$$(z_1, z_2, z_{2d}, \hat{\theta}) = \left(\frac{1}{R}\frac{V_d^2}{E}, V_d, V_d, \frac{1}{R}\right) \tag{37}$$

which is *asymptotically stable*.

Proof. It can be verified, by direct substitution, that (37) represents an equilibrium point for the closed-loop system.

Define

$$z_{1d} = \hat{\theta}\frac{V_d^2}{E} \tag{38}$$

Note that z_{1d} and z_1 coincide at the equilibrium point. Let, again, $z - z_d$ stand for the error vector $\tilde{z}$.

In terms of the error signals, (17) is rewritten as

$$\mathscr{D}_B\dot{\tilde{z}} + (1-\mu)\mathscr{J}_B\tilde{z} + \mathscr{R}_{Bd}\tilde{z} = \psi \tag{39}$$

where

$$\psi = \mathscr{E}_B - \left[\mathscr{D}_B\dot{z}_d + (1-\mu)\mathscr{J}_B z_d + \mathscr{R}_B z_d - \begin{bmatrix} R_1\tilde{z}_1 \\ 0 \end{bmatrix}\right] \tag{40}$$

and $\mathscr{R}_{Bd}$ is a positive-definite matrix given by

$$\mathscr{R}_{Bd} = \begin{bmatrix} R_1 & 0 \\ 0 & 1/R \end{bmatrix}, \quad R_1 > 0 \tag{41}$$

Expression (40) is explicitly written as

$$\psi_1 = -L\dot{z}_{1d} - (1-\mu)z_{2d} + R_1\tilde{z}_1 + E$$
$$\psi_2 = -C\dot{z}_{2d} + (1-\mu)z_{1d} - \frac{1}{R}z_{2d} \tag{42}$$

Using (38) and (36) one has $\psi_1 = 0$ and $\psi_2 = \tilde{\theta}z_{2d}$, where $\tilde{\theta} = \hat{\theta} - 1/R$.

The resulting stabilization error system is then given by the following perturbed dynamics:

$$\mathscr{D}_B\dot{\tilde{z}} + (1-\mu)\mathscr{J}_B\tilde{z} + \mathscr{R}_{Bd}\tilde{z} = \begin{bmatrix} 0 \\ \tilde{\theta}z_{2d} \end{bmatrix} \tag{43}$$

Using as a Lyapunov function the total energy of the stabilization error system plus the 'energy' associated with the parameter estimation error

$$H_d(t) = \frac{1}{2}\left[\tilde{z}^T\mathscr{D}_B\tilde{z} + \tilde{\theta}^2\right] \tag{44}$$

one verifies that, along the trajectories of (43), the following relation is satisfied:

$$\dot{H}_d(t) = -\tilde{z}^T\mathscr{R}_{Bd}\tilde{z} + \tilde{\theta}\left[\dot{\tilde{\theta}} + z_{2d}(z_2 - z_{2d})\right] \tag{45}$$

Using the last equation in (36) and the fact that $\dot{\tilde{\theta}} = \dot{\hat{\theta}}$, one obtains

$$\dot{H}_d(t) = -\tilde{z}^T\mathscr{R}_{Bd}\tilde{z} \leqslant -\alpha\|\tilde{z}^2\| \tag{46}$$

where α may be taken to be $\min\{R_1, 1/R\}$. One concludes that $\tilde{z}$ and $\tilde{\theta}$ *are bounded and that* $\tilde{z}$ is square integrable. To actually show that $\tilde{z} \to 0$ asymptotically, it must be verified that $\tilde{z}$ is uniformly continuous. For this, it suffices to show that $\dot{\tilde{z}}$ is bounded. From the perturbed error dynamics (43), and the established boundedness of $\tilde{\theta}$ and $\tilde{z}$, it follows that $\dot{\tilde{z}}$ is bounded if, and only if, z_{2d} is bounded. In order to prove that z_{2d} is bounded, note first that its associated zero dynamics, given by

$$\dot{z}_{2d} = -\frac{\hat{\theta}}{C}\left(z_{2d} - \frac{V_d^2}{z_{2d}}\right) \tag{47}$$

is asymptotically stable towards the equilibrium point located at $z_{2d} = V_d$, for all initial conditions satisfying $z_{2d}(0) > 0$, provided $\hat{\theta} > 0 \; \forall t$. The dynamics (47) is also asymptotically stable towards a second equilibrium point, located at $z_{2d} = -V_d$, for all initial conditions satisfying $z_{2d}(0) < 0$, provided $\hat{\theta} > 0 \; \forall t$.

Take as a Lyapunov function candidate for the controller dynamics, $V_2 = C/2(z_{2d} - V_d)^2$. The time derivative of V_2 along the trajectories of (36) results in the following expression:

$$\dot{V}_2 = -\hat{\theta}(z_{2d} - V_d)\left\{z_{2d} - \frac{V_d^2}{z_{2d}}\left[\left(1 + \frac{R_1}{E}\tilde{z}_1\right) + L\frac{V_d^2}{E}z_{2d}\dot{z}_2\right]\right\} \tag{48}$$

Then, by virtue of the boundedness of $\tilde{z}_1$, $\dot{z}_2$, and $\hat{\theta}$, and the fact that initial conditions for such variables can be entirely chosen at will and, also provided that $\hat{\theta} > 0 \; \forall t$, it follows that given positive constants β_1 and β_2, with

$$0 < \beta_1 < \frac{E}{R_1}, \quad 0 < \beta_2 < \frac{2E^2}{LV_d^2}\sqrt{\frac{R_1}{E}\beta_1} \tag{49}$$

such that initial conditions for the error vector components satisfy, $|\tilde{z}_1| < \beta_1$; $|\dot{z}_2| < \beta_2$, then, the time derivative of V_2, given by (48), is strictly negative *outside* the closed interval $[Z_m, Z_M]$ of the real line, containing in its interior the equilibrium point, V_d, for z_{2d}, where

$$Z_m = V_d\left\{\left[\sqrt{1 - \frac{R_1}{E}\beta_1 + \frac{L^2V_d^4}{4E^4}\beta_2^2}\right] - \frac{LV_d^2}{2E^2}\beta_2\right\}$$
$$Z_M = V_d\left\{\left[\sqrt{1 + \frac{R_1}{E}\beta_1 + \frac{L^2V_d^4}{4E^4}\beta_2^2}\right] - \frac{LV_d^2}{2E^2}\beta_2\right\} \tag{50}$$

We conclude that $\dot{\tilde{z}}$ is absolutely continuous and hence $\lim_{t\to\infty}\tilde{z}(t) = 0$. Moreover, given that z_2 asymptotically converges to the same equilibrium point of z_{2d}, given by V_d, it follows z_1 converges to its corresponding equilibrium value, V_d^2/RE. Since z_1 and z_{1d} asymptotically converge to the same equilibrium point, then, it follows that, necessarily, $\hat{\theta} \to 1/R$. □

3.2. *An adaptive controller for the 'buck boost' converter*

The following proposition summarizes the properties of a passivity-based non-linear adaptive dynamical controller for the 'buck–boost' converter. The proof follows similar arguments to those already used in the previous proposition.

Proposition 3.2.

Consider the averaged dynamics (20) and (21) of the 'buck–boost' converter circuit, where $C > 0$, $L > 0$, $E > 0$ are *known* constants representing the capacitance, inductance and external voltage, respectively, and $R > 0$ is the *unknown* load charge resistance.
Define an adaptive non-linear dynamic state feedback controller as

$$\dot{z}_{2d} = -\frac{\theta}{C}\left\{z_{2d} + V_d\left(\frac{V_d}{E}+1\right)\left[\frac{E + LV_d(V_d/E+1)\dot{z}_{2d}(z_2 - z_d) + R_1(z_1 - V_d(V_d/E+1)\theta)}{E - z_{2d}(t)}\right]\right\}$$

$$\mu(t) = \frac{z_{2d}(t) + LV_d(V_d/E+1)\dot{z}_{2d}(z_2 - z_d) + R_1(z_1 - V_d(V_d/E+1)\theta)}{z_{2d}(t) - E} \qquad (51)$$

where the dynamical controller initial condition is chosen so that, $z_{2d}(0) < E$ and $\hat{\theta}(0) > 0$. The constant reference value for z_2, denoted by $-V_d$, is a strictly negative quantity. The quantity θ denotes the estimate of $1/R$. The parameter R_1 is a designer-chosen constant with the only restriction of being strictly positive. Under these conditions, it is always possible to choose the controller's initial state $z_{2d}(0)$ and $\hat{\theta}(0)$, such that the closed-loop system (20) and (51) has an equilibrium point given by

$$(z_1, z_2, z_{2d}, \theta) = \left(\frac{V_d}{R}\left(\frac{V_d}{E}+1\right), -V_d, -V_d, \frac{1}{R}\right) \qquad (52)$$

which is *asymptotically stable*.

3.3. *An adaptive controller for the 'buck' converter*

The following proposition summarizes the passivity-based adaptive controller for an average model of the 'buck' converter circuit. The proof of the result is left as an excercise for the interested reader.

Proposition 3.3.

Consider the averaged dynamics (32) and (33) of the 'buck' converter circuit, where $C > 0$, $L > 0$, $E > 0$ are *known* constants representing the capacitance, inductance and external voltage, respectively, and $R > 0$ is the *unknown* load charge resistance.
Define a linear adaptive time-varying state feedback controller as

$$\dot{z}_{2d} = -\frac{\theta}{C}\left(z_{2d} - V_d\right)$$

$$\mu(t) = \frac{-LV_d\dot{z}_{2d}(z_2 - z_{2d}) + z_{1d}(t) - R_1(z_1 - V_d\theta)}{E}$$

$$\dot{\theta} = -z_{2d}(z_2 - z_{2d}) \qquad (53)$$

where the controller initial condition, $z_{2d}(0)$, satisfies, $E > z_{2d}(0) > 0$, $\hat{\theta}(0) > 0$ and where $V_d > 0$, is a constant reference value for z_2. The parameter, $R_1 > 0$, is a designer-chosen constant. Under these conditions, the closed-loop system (32) and (53) has an equilibrium point,

$$(z_1, z_2, z_{2d}, \theta) = \left(\frac{V_d}{R}, V_d, V_d, \frac{1}{R}\right) \qquad (54)$$

which is *asymptotically stable*.

4. SIMULATION RESULTS

Simulations of the closed-loop behaviour of the average boost converter and the passivity-based indirect adaptative feedback controller were performed on the following perturbed version of the 'boost' converter circuit:

$$\dot{z}_1 = -(1-\mu)\frac{1}{L}z_2 + \frac{E+\eta}{L}$$

$$\dot{z}_2 = (1-\mu)\frac{1}{C}z_1 - \frac{1}{RC}z_2 \qquad (55)$$

where η represents an external stochastic perturbation input affecting the system behaviour directly through the external voltage source value. Note that this perturbation input is of the "unmatched" type, i.e., it enters the system equations through an input channel vector field, given by $[1/L\ 0]^T$ which is *not* in the range space of the control input channel, given by the vector field $[z_2/L\quad z_1/C]^T$. The magnitude of the noise was chosen to be, approximately, 5% of the value of E. The circuit parameter values were taken to be the following 'typical' values:

$$C = 20\,\mu\text{F},\quad R = 30\,\Omega,\quad L = 20\,\text{mH},\quad E = 15\,\text{V}$$

$z_{1d} = 3{\cdot}125$ A, with a steady-state duty ratio of $U = 0.6$. This corresponds to a nominal average output voltage, $z_2 = V_d = 37{\cdot}5$ V. Figure 4 shows the closed-loop state trajectories corresponding to the feasible adaptive duty ratio synthesizer derived for the 'boost' converter. This figure also presents the trajectory of the duty ratio function, the trajectory of the parameter estimation values and a realization of the computer-generated stochastic perturbation signal η/L, addressed to as the 'total perturbation noise'.

The simulation show that the proposed controller achieves the desired indirect stabilization of the output voltage around the desired equilibrium value while exhibiting a high degree of robustness with respect to the external stochastic perturbation input.

4.1. *'Bust–Boost' converter*

Simulations were also carried for evaluating the closed-loop behaviour of an indirectly adaptively regulated, perturbed 'buck–boost' converter The converter parameter values were chosen to be identical to those of the previously considered 'boost' converter simulation example. The perturbed switch-regulated model used in the simulations was taken to be

$$\dot{z}_1 = -(1-u)\frac{1}{L}z_2 + u\frac{E+\eta}{L}$$

$$\dot{z}_2 = -(1-u)\frac{1}{C}z_1 - \frac{1}{RC}z_2 \qquad (56)$$

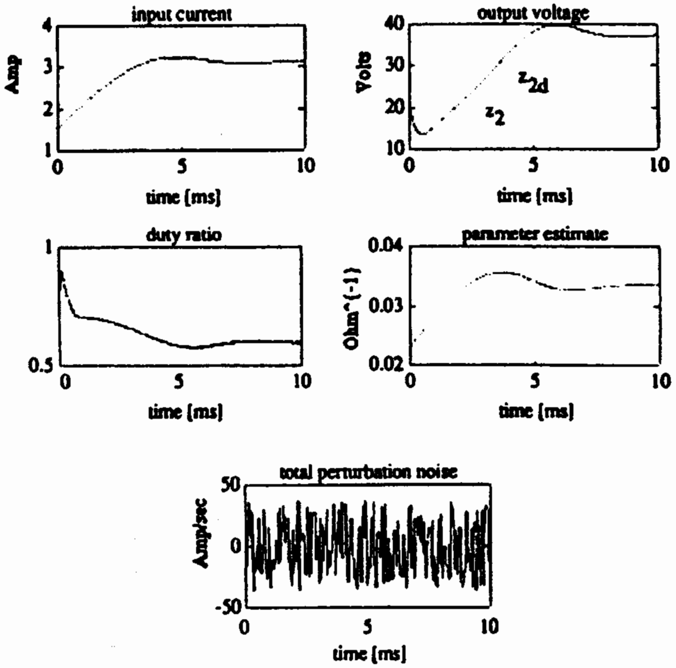

Figure 4. Simulation results for performance evaluation of the indirect adaptive controller in a perturbed average 'boost' converter

The desired average input inductor current was set to be $z_{1d} = I_d = 1{\cdot}875$ A, with a steady-state duty ratio of $U = 0{\cdot}6$. This corresponds to a nominal average output voltage, $z_1 = -V_d = -22{\cdot}5$ V. Figure 5 shows the closed-loop state trajectories corresponding to the adaptive duty ratio synthesizer derived for the 'buck-boost' converter. This figure also presents the trajectory of the duty ratio function, the trajectory of the parameter estimation values and

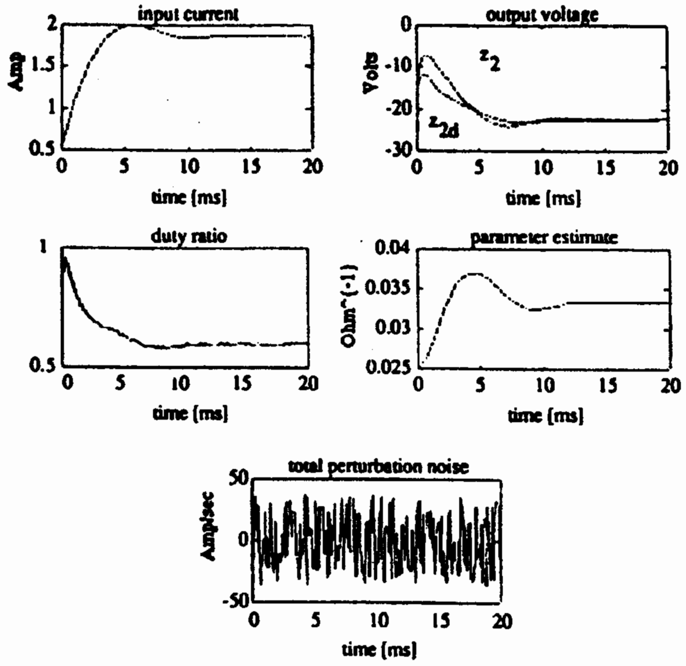

Figure 5. Simulation results for performance evaluation of the indirect adaptive PWM controller in a perturbed average 'buck-boost' converter

a realization of the total perturbation noise signal, η/L. The magnitude of the perturbation noise η was chosen to be, approximately, 5% of the value of E.

As it can be seen from the simulations, the proposed adaptive controller achieves the desired indirect stabilization of the output voltage around the desired equilibrium value while exhibiting a high degree of robustness with respect to the 'unmatched' external stochastic perturbation input.

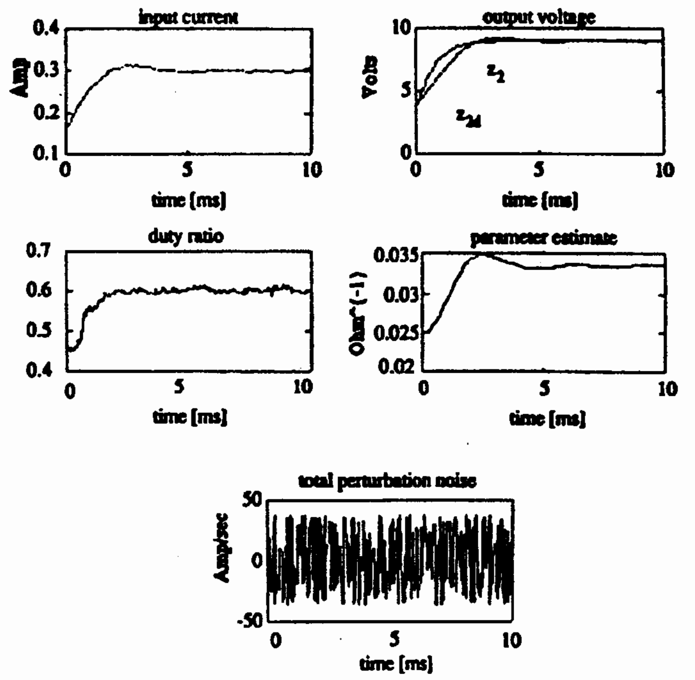

Figure 6. Simulation results for performance evaluation of the indirect adaptive controller in a perturbed 'buck' converter

4.2. 'Buck' converter

Simulations were also carried for an indirect regulation scheme acting on a perturbed, 'buck' converter of the form,

$$\dot{z}_1 = -\frac{1}{L}z_2 + u\frac{E+\eta}{L}$$

$$\dot{z}_2 = \frac{1}{C}z_1 - \frac{1}{RC}z_2 \qquad (57)$$

The desired average input inductor current was set to be $z_{1d} = 0.3$ A, with a steady-state duty ratio of $V = 0.6$. This corresponds to a nominal average output voltage, $z_2 = V_d = 9$ V. Figure 6 shows the closed-loop state trajectories corresponding to the adaptive duty ratio synthesizer derived for the "buck" converter. This figure also presents the trajectory of the duty ratio function, the trajectory of the parameter estimation values and a realization of the total perturbation noise signal η/L. The magnitude of the perturbation noise η was chosen to be, approximately, 5% of the value of E.

5. CONCLUSIONS

In this article a passivity-based regulation scheme has been developed for the on-line feedback specification of the stabilizing duty ratio function in various kinds of dc-to-dc power converters. The controller designs were first tackled under the assumptions of perfectly known loads and then they were extended to handle, in an adaptative fashion, the more realistic case of uncertain resistive loads. The proposed approach is based on a combination of closed-loop energy shaping and appropriate stabilizing damping injection through dynamical state feedback. The proposed technique, which uses the total energy of the system as a Lyapunov function was shown to easily accommodate for parameteric uncertainties at the load. The results may also be extended to those cases where other important circuit parameters are also regarded to be constant but unknown. Based on the encouraging experimental results reported in Reference 20, 21, further work is in progress to implement, in a laboratory set-up, several non-linear adaptive regulation schemes including the one desired in this article, for some of the dc-to-dc power converters here described.

Average models dc-to-dc power converters have been known to be differentially flat (see Reference 29) i.e. all system variables are differential functions of the total energy of the system, which then qualifies as a linearizing output. As such, an interesting line of research can be proposed which exploits the differential flatness property of the system in connection with the possibilities of energy shaping and damping injecion, i.e. passivity, controller design techniques.

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