Short-Term Bidding Strategies for a Generation Company in the Iberian Electricity Market

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A mis padres

A Marco

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Abstract

The start-up of the Iberian Electricity Market introduced a set of new mechanisms in the Spanish electricity sector that forced the agents participating in the market to change their management policies. This situation created a great opportunity for studying the bidding strategies of the generation companies in this new framework. This thesis focuses on the short-term bidding strategies of a price-taker generation company that bids daily in the Iberian Electricity Market. We will center our bidding strategies on the day-ahead market because 80% of the electricity that is consumed daily in Spain is negotiated there and also because it is the market where the new mechanisms are integrated.

The liberalization of the electricity markets opens the classical problems of energy management to new optimization approaches. Specifically, because of the uncertainty that the market produces in the prices, the stochastic programming techniques have become the most natural way to deal with these problems. Notice that, in deregulated electricity markets the price is hourly fixed through a market clearing procedure, so when the agent must bid its energy it is unaware of the price at which it will be paid. This uncertainty makes it essential to use some statistic techniques in order to obtain the information coming from the markets and to introduce it in the optimization models in a suitable way. In this aspect, one of the main contributions of this thesis has been the study the Spanish electricity price time series and its modeling by means of factor models.

In this thesis, the new mechanism introduced by the Iberian Market that affects the physical operation of the units is described. In particular, it considers in great detail the inclusion of the physical futures contracts and the bilateral contracts into the day-ahead market bid of the generation companies. The rules of the market operator have been explicitly taken into account within the mathematical models, along with all the classical operational constraints that affect the thermal and combined cycle units. The expression of the optimal bidding functions are derived and proved. Therefore, the models built in this thesis provide the generation company with the economic dispatch of the committed futures and bilateral contracts, the unit commitment of the units and the optimal bidding strategies for the generation company.

Once these main objectives were fulfilled, we improved the previous models with an approach to the modeling of the influence that the sequence of very short markets have on optimal day-ahead bidding. These markets are cleared just before and during the day in which the electricity will be consumed and the opportunity to obtain benefits from them changes the optimal day-ahead bidding strategies of the generation company, as it will be shown in this thesis.

The entire models presented in this work have been tested using real data from a generation company and Spanish electricity prices. Suitable results have been obtained and discussed.

Resumen

La puesta en marcha del Mercado Ibérico de la Electricidad introdujo en el sector eléctrico español una serie de nuevos mecanismos de participación que han forzado a los agentes a renovar sus políticas de gestión. De esta nueva situación surge la oportunidad de estudiar nuevas estrategias de oferta para las compañías de generación. Esta tesis se enmarca en las estrategias de oferta a corto plazo para compañías de generación *price-taker* que participen diariamente en el Mercado Ibérico de la Electricidad. Estas estrategias se centraran en el mercado diario ya que es donde se negocia un 80% de la electricidad consumida diariamente en España y es donde se integran gran parte del resto de los mecanismos de participación.

La liberalización de los mercados eléctricos permiten aplicar nuevas técnicas de optimización a los problemas clásicos de gestión de la energía. En concreto, dada la incertidumbre en el precio existente en el mercado, las técnicas de programación estocástica se convierten en la forma más natural para abordar estos problemas. En los mercados eléctricos el precio se fija horariamente como resultado de un proceso de casación, es decir, cuando el agente debe efectuar sus ofertas desconoce el precio al que la energía le será pagada. Esta incertidumbre hace imprescindible el uso de técnicas estadísticas para obtener información del mercado e introducirla en los modelos de optimización. En este aspecto, una de las contribuciones de esta tesis es el estudio del precio de la electricidad en España y su modelado mediante modelos factoriales.

Se describen los nuevos mecanismos presentes en el Mercado Ibérico de la Electricidad que afectan directamente a la producción física de las unidades. En particular, se incluye una modelización detallada de los contratos de futuros físicos y bilaterales y su inclusión en la oferta enviada al mercado diario por las compañías de generación. En los modelos presentados se tiene en cuenta explícitamente las reglas del mercado así como las clásicas restricciones de operación de las unidades, tanto térmicas como de ciclo combinado. La expresión de la función de oferta óptima se deriva y se demuestra.

Por lo tanto, los modelos construidos son una herramienta para decidir la asignación de unidades, la generación de los contratos de futuros físicos y bilaterales a través de ellas y la oferta óptima de

una compañía de generación.

Una vez alcanzados estos objetivos, se presenta una mejora del modelo con la inclusión de la secuencia de mercados de muy corto plazo. El objetivo es modelar la influencia que esta tiene en la oferta al mercado diario. Estos mercados se casan justo antes y durante el día en el que la energía va a ser consumida y se verá cómo la posibilidad de aumentar los beneficios participando en ellos afecta a las estrategias de oferta óptima del mercado diario.

Los modelos presentados en este trabajo se han probado con datos reales procedentes del Mercado Ibérico de la Electricidad y de una compañía de generación que opera en él. Los resultados obtenidos son adecuados y se discuten a lo largo del documento.

Resum

La posada en marxa del Mercat Ibèric de l'Electricitat va introduir al sector elèctric espanyol un seguit de nous mecanismes de participació que han forçat els agents a renovar les seves polítiques de gestió. D'aquesta nova situació sorgeix l'oportunitat d'estudiar noves estratègies d'oferta a curt termini per a companyies de generació *price-taker* que participin diàriament al Mercat Ibèric de l'Electricitat. Aquestes estratègies se centraran al mercat diari, ja que és aquí on es negocia un 80% de l'electricitat que es consumeix diàriament a Espanya i on s'integren gran part de la resta de mecanismes de participació.

La liberalització dels mercats elèctrics obre a noves tècniques d'optimització els problemes clàssics de gestió de l'energia. En particular, atesa la incertesa que l'existència del mercat ocasiona als preus, les tècniques de programació estocàstiques es converteixen en la forma més natural per abordar aquests problemes. Als mercats elèctrics el preu es fixa horàriament com a resultat d'un procés de casació , és a dir que quan l'agent ha d'efectuar la seva oferta desconeix el preu al qual li vindrà remunerada l'energia. Aquesta incertesa fa imprescindible l'ús de tècniques estadístiques per obtenir informació del mercat i introduir-la als models d'optimització. En aquest aspecte, una de les contribucions d'aquesta tesi és l'estudi dels preus del mercat de l'electricitat a Espanya i el seu modelat mitjançant models factorials.

D'altra banda, s'hi es descriuen els nous mecanismes presents al Mercat Ibèric de l'Electricitat que afecten directament la producció física de les unitats. En particular, s'inclou el modelat detallat dels contractes de futurs físics i bilaterals i de la seva inclusió a l'oferta del mercat diari per part de les companyies de generació. Als models presentats, es tenen en compte explícitament les regles del mercat, així com les clàssiques restriccions d'operació de les unitats, tant tèrmiques com de cicle combinat. A més, es deriva i es demostra l'expressió de la funció d'oferta.

Per tant, els models construïts són una eina per decidir l'assignació de les unitats, la generació dels contractes de futurs físics i bilaterals a través seu i l'oferta òptima d'una companyia de generació.

Un cop s'han cobert aquests objectius, es presenta una millora dels models mitjançant la inclusió de la seqüència de mercats de molt curt termini per tal de modelar la influència que tenen en

l'oferta al mercat diari. Aquests mercats es casen just abans i durant el dia en què l'energia ha de ser consumida, i això permetrà veure com la possibilitat d'augmentar els beneficis participant-hi afecta directament les estratègies d'oferta òptima del mercat diari.

Els models presentats en aquest treball han estat provats amb dades reals provinents del Mercat Ibèric de l'Electricitat i d'una companyia de generació que hi opera. Els resultats obtinguts són adequats i es discuteixen al llarg del document.

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CHAPTER 1

Introduction

1.1 Motivation

The liberalization of the electricity sector led to a new situation in the decision-making processes of the generation companies (GenCo). In this new framework, the agents' objective is to maximize their expected profits by considering all the mechanisms in which they could participate. Specifically, in June 2005 the *White Paper on Regulatory Framework Reform for Electric Generation in Spain* (Perez-Arriaga., 2005) was published and May 2006 saw the publication of the agreement between Spain and Portugal for the creation of a joint electricity market, an agreement which establishes the basis for the *Iberian Electricity Market* (MIBEL)that started up in June 2007. This new situation was a perfect opportunity to study all the new mechanisms that were created within this new electricity market and the new problems that a Spanish generation company will have to face.

In this new framework, the Spanish generation companies have to change their bidding strategies in order to take advantage of the opportunities that the Iberian Electricity Market provides. Specifically, the creation of the *derivatives market* (DM) opens a new range of products that could help to hedge the risk produced by the uncertainty in electricity prices.

Historically, electricity generation management has been considered within the context of various planning horizons. Long-term planning (more than one year) includes strategic decisions for the GenCo, such as new investments or expansion plans. Medium-term planning (from one week to one year) focuses on the management of some limited resources(such as hydro generation) and the negotiation of bilateral and derivatives contracts. Finally, short-term decisions are the unit commitment of the units, the economic dispatch and, in the new market framework, the bidding to day-ahead or intraday markets. This short-term planning from the point of view of the generation companies is the objective of this thesis. Our study of electricity generation management will be focused company's pool of thermal units; however, all the models presented here could be expanded to include other kinds of generation units such as hydro or wind power units.

It must also be emphasized that, although the thesis is contextualized in the MIBEL, all models and results presented here can be easily applied and adapted to other electricity market situations, both those that are beginning to restructure themselves anew and to classical markets that have some mechanisms that are equivalent to the ones introduced in our models.

Other important key points coming from the liberalization of the electricity market is the price risk, i.e. the uncertainty in the price at which the generated electricity will be paid. This price is fixed in the market clearing process that occurs after the GenCos decide their bid strategy, that is to say: their generation level and price bid. Aside from some other medium-term decision for hedging the risk, the GenCo must take into account this uncertainty in its short-term planning. This is a great opportunity for the stochastic programming techniques to be used on the classical optimization problems for electricity generation management.

1.2 Objectives

The main goals of this thesis are as follows:

(i) To study in depth the new market regulations and the implications for generation companies' short-term management strategies. This objective includes the study of the derivatives market regulation and its products, the day-ahead and the secondary markets rules and the regulation on the bidding process and physical generation management in the MIBEL framework.

(ii) To develop new approximations to forecast modeling for Spanish market prices and include them into the optimization models.

(iii) To propose a model for the inclusion of physical derivatives products into the day-ahead market bidding strategies of a price-taker GenCo.

(iv) To propose a model for the inclusion of bilateral contracts into the short-term generation strategies of a price-taker GenCo. These mechanisms could include classical bilateral contracts or the new organized markets of bilateral contracts.

(v) To propose a model which integrates the short-term sequence of markets, intraday markets and ancillary services, into the day-ahead market bidding strategies of a price-taker GenCo.

1.3 Contents

After defining the fundamental concepts for the bidding strategies in Chapter 2, we start off by proposing a model that introduces futures contracts into the bidding strategies, because it applies to one of the most significant changes caused by the Iberian Electricity Market in the Spanish electricity sector: the start-up of the derivatives market. Once we have modeled these physical derivatives products, we expand the model by including the bilateral contracts. But if we look at these two approaches from a global point of view, the futures contract model presented in Chapter 4 can be seen as a particular case of the model presented in Chapter 5. Therefore, the contents of this thesis should be observed as the evolution of the work during the thesis process. The only exception to this temporal order are the last two models presented in Chapter 7. Those models were the first approaches which included the bidding function in the management of some of the new market mechanisms. It can be observed within the mathematical model that the Chapter 7 models were constructed prior to some of the previous models.

The case studies of the thesis are solved with real data from the Iberian Electricity Market and from a GenCo that participates daily in the electricity market.

The chapters structure is as follows:

1 Introduction

This chapter outlines the motivations of the study, its objectives and, a summary of the contents.

2 Background

This chapter introduces all the basic concepts needed to contextualize the thesis framework. It presents the Iberian Electricity Market, its history and main characteristics. Also, the most important mechanisms included in the models defined later are described and illustrated. A brief introduction to stochastic programming techniques is also presented, along with a definition of two-stage and multistage stochastic problems. Finally, the concepts and instruments that will be expanded on in the subsequent models are explained such as, for example, the optimal bidding curve and the matched energy.

3 Day-Ahead Market Price Scenarios

This chapter includes a study of the market price. A descriptive study of the prices from 2007 to 2010 is presented, emphasizing the main characteristics of the market price time series. Aside from other more simple approaches, an ARIMA model is adjusted and its results are used in some subsequent models. Finally, the most relevant result of the statistical study of the market price scenario is presented, a new approach by means of a factor model is adjusted and its results are compared with the ARIMA model.

4 DAMB: Futures Contracts

Chapter 4 presents the first stochastic programming model of this work. The economic dispatch of physical futures contracts is included in the short-term management of the thermal units. The result is a two-stage mixed-integer stochastic programming model for the short-term thermal optimal bidding problem that maximizes the expected benefits of the day-ahead market for a price-taker GenCo. The chapter ends with a case study based on real units participating in the Spanish electricity market.

5 DAMB: Futures and Bilateral Contracts

This chapter describes the extension of the model presented in the previous Chapter 4 with the inclusion of the bilateral contracts. In order to to maximize the benefits arising from the day-ahead market while satisfying thermal operational constraints, the optimal solution of the model determines the unit commitment of the thermal units, the optimal instrumental price bidding strategy for the generation company and the economic dispatch of the committed futures and bilateral contracts for each hour. The proposed model is tested with a case study defined by real data from a Spanish GenCo.

6 DAMB: Multimarket Problem

The model developed in Chapter 5 is extended here to include the sequence of short-term markets into the day-ahead market bidding problem, i.e. to take into account all the very short-term markets existing in the Iberian Electricity Market in the day-ahead strategies. A brief study of the secondary markets prices is made and both the reserve and the first intraday market are modeled together with economic dispatch of the futures and bilateral contracts, as a multistage stochastic programming problem. Again, the proposed model is tested with a case study based on real units participating in the Spanish electricity market.

7 DAMB: Other Extensions

This chapter is based on a joint work with M.J. Rider, which to some extend represents two extensions of the core model developed in Chapters 4 to 6. The first one describes the inclusion of combined cycle units and the definition of its optimal bid strategies. The second one models the inclusion of the opportunities coming from the virtual power plants auctions into the bidding strategies of a price-taker GenCo. The case study is made with real data from a Spanish GenCo.

8 Conclusions

General conclusions and further research are discussed, along with the scientific production generated by this thesis.

Appendix

The data for the case studies is presented in the appendix and we provide a glossary where the symbols and abbreviations used in this thesis are described. Moreover, the application of some of the techniques presented in this thesis to the Italian Electricity Market are included as an appendix.

CHAPTER 2

Background

Introduction

This chapter contains the basis for the work done in this thesis, there are described the main concepts and theoretical aspects in which the work of the thesis is based on. In Section 2.1, the electricity market under study, the Iberian Electricity Market, is described in detail. Also the market mechanism in which the generation companies can participate are studied and presented. Section 2.2 gives a brief introduction to the stochastic programming techniques and the different horizon approaches presented in this work. Finally, in Section 2.3 the *day-ahead market bidding* (DAMB) problem and all the concepts that will be extended in the subsequent chapters are presented.

2.1 Iberian Energy Market

In this section the Iberian Energy Market is presented. A brief history of its start-up process and its institutions is described and the main relevant parts and products that will be included in later models are introduced.

2.1.1 History and Structure

The organization of the electricity markets is a consequence of the historical organization of the sector and the nature of the commodity. Electricity cannot be stored and this fact forces situations and rules that distinguish it from other commodities markets. The liberalization process performed over the last decade in Europe has structured the sector in a way that follows these natural and historic characteristics. There are three vertical activities: the generation, distribution and commercialization of electricity.

The electricity distribution is based on a physical network that transports the electricity from the place where it is produced to the place where it will be consumed. This network needs a huge investment in its creation, maintenance and operation and this fact causes the monopoly to be the most efficient option for this activity. Those natural monopolies have been regulated in the reorganization process in order to open the activity to other companies by means of the payment of a regulated tariff.

The generation and commercialization of energy have been liberalized, the introduction of the market and the improvement in competition lead to more efficient management and better exploitation of the natural resources. Generation is linked to a wholesaler market, where the producers sell the energy and the buyers' agents purchase it for the buyers' own consumption or to commercialize it for final consumers. Commercialization applies to the retailer market and is therefore out of the scope of this thesis.

On the mainland of Spain, in November 1997, Electric Sector Law 54/1997 was published; it defines the Spanish Electricity Market and related institutions. The Spanish Electricity Market started up in January 1998. This reform of the Spanish electric sector has the typical triple objective: to guarantee the supply of electricity, to guarantee the quality of this supply and to guarantee this process at a lower cost. It establishes a fully competitive framework for the generation of electricity while at the same time it defines a transient process for the liberalization of retail supply.

In the Spanish Electricity Market, the market mechanisms are centralized and managed by an entity known as *market operator* (MO). To play this role, a new institution was created: the *Compañia Operadora del Mercado Español de la Electricidad*. This institution is in charge of the set of short-term market mechanisms through which the great part of the physical transactions take place. On the other hand, to guarantee the that electricity is supplied with quality, security and reliability it was necessary to create an institution that is independent from any agent that participates in the electricity market and, also, from the MO. This entity is the *independent system operator* (ISO) and it was created within the Spanish Electricity Market redefining the role played by a preexisting entity, the *Red Eléctrica de España* (REE). REE is the owner of the high-voltage transmission network and has been the system operator since 1984. In this situation, the coordination between the MO and the ISO became essential in order to guarantee that the market transactions are physically feasible and fulfill the security criteria. The Spanish Electricity Market included a day-ahead market and a set of balancing and adjustment markets. Out of the market there were still the classical bilateral contracts.

As the introduction of competition and the deregulation process did not behave as expected, the Spanish Electricity Market was improved in 2007 with the start-up of the Iberian Electricity Market (MIBEL). The MIBEL joins the Spanish and Portuguese electricity system and it complements the previous mechanisms of the Spanish Electricity Market with a derivatives market and other new market mechanisms. This derivatives market has its own MO called *Operador do Mercado Iberico de Energia - Pólo Português* (OMIP, 2008) and the old Spanish market operator is renamed as *Operador del Mercado Iberico de Energia - Polo Español* (OMEL, 2010) and it is still in charge of

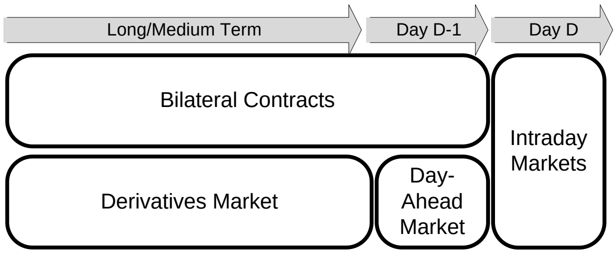

Figure 2.1: Market mechanisms.

the spot markets.

The mechanisms (Figure 2.1) in which the agents could participate in the MIBEL are:

- Bilateral contracts (BC), which could be classical bilateral trading between agents without the implication of any institution or other auctions in organized markets such as virtual capacity auction (mandatory for the dominant agents) or distribution auctions.
- Derivatives market (DM), where long term commitments for generating and buying energy are made. In this market there is either a physical or financial settlement.
- Day-ahead market (DAM), which allows the agents to execute the BCs and also allows the integration of open long-term positions with physical settlement.
- Intraday markets (IM), which are open to all agents that have participated in the DAM or who have signed BCs.
- Ancillary services auctions, which guarantee the security and reliability of the system.

The management of the day-ahead and intraday markets are entrusted to OMEL, which is also responsible for communicating payment obligations. The ancillary services are managed by REE which is also responsible for the real time management.

Figure 2.2 illustrates the sequence of spot market mechanisms and it indicates where the medium-term positions are integrated. The first node represents the GenCo agents and all the medium term products that are integrated at the GenCo DAM bids: virtual power plant options, derivatives physical products, distribution contracts and, international connection rights. The agents could participate in the spot market through seven sessions represented by the subsequent nodes. The first and main one is the day-ahead market which is followed by the technical restrictions resolution. After those first mechanisms there is the ancillary services market, also called reserve, where the ramp up and down capacity of the units is bid. Just before and during the day of study there are the six intraday sessions distributed along the 24 hours of the settlement day. Finally, real time management is the last short-term mechanism where the GenCo can participate.

Since the start up of the market in 1998, generation investments have led to an evolution in the market from a model with two generation companies with have an 80% generation market quote to a framework where the greatest quota of a participant is 22%. At the end of 2009, the number

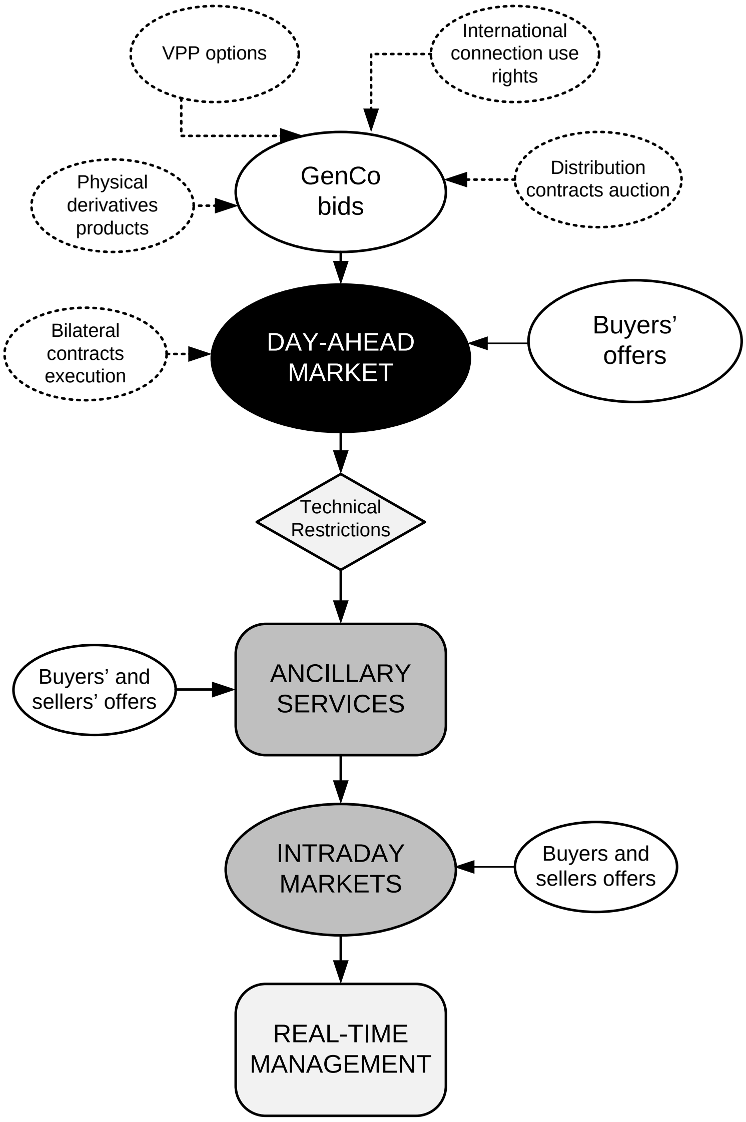

Figure 2.2: Sequence of spot market mechanisms.

of participants in the market is 1169: 918 producers (621 *special regime* producers: renewable energies, waste and cogeneration), 192 distributors or suppliers and 59 other kinds of agents. The total installed capacity of the system at December 2009 was 93729 MW.

2.1.2 Bilateral Contracts

In June 2007, not only the MIBEL was launched but also some new rules of the electrical energy production market operation for the day-ahead and intraday market were introduced. These new rules defined mechanisms to encourage competition in the production market: new kinds of bilateral contracts besides the classical ones and *virtual power plant* (VPP) capacity auctions.

From our GenCo point of view, the BCs are agreements between the company and a qualified consumer to provide a given amount of electricity at a stipulated price during a delivery period. The characteristics of the BC (energy, price and delivery period) are negotiated among the MIBEL agents, either in nonorganized or organized markets. For a GenCo, these BCs usually represent a scheduled load curve, chargeable at a fixed price that has to be optimally dispatched among the GenCo's units. In nonorganized BC markets, i.e. the classical BCs existing before the reorganization, the producers and consumers agree on the amount, price, and period of the energy delivered. This agreement is set during a private negotiation. The new mechanisms that appear with the creation of the MIBEL are two organized BC markets: the CESUR and VPP auctions (see CESUR (2010) and CNE (2010), respectively).

The CESUR are electricity auctions for the supply at regulated prices. They are new mechanisms for buying energy destined to final consumers that have recourse to a regulated tariff. The objectives of these auctions are, on the one hand, to encourage liquidity of long- and medium-term markets and, on the other hand, to stabilize the cost of energy for the final consumers, avoiding the volatility of the DAM prices. The CESUR auctions settle the BCs signed by generation and distribution companies to supply power at regulated prices. These regulated prices are the prices at which the *distribution auction generic unit* is used by the GenCo to integrate the energy matched by the CESUR auction into the energy production system. By law, a GenCo holding such a BC must use this generic unit to submit an accepting-price purchase bid to the DAM for the entire amount of the contract and, therefore, there is no room for optimization.

The virtual power plant auctions are sales of electricity capacity which, rather than physical divestments, are virtual divestments by one or more dominant firms in a market. Instead of selling the physical power plant, the firm retains management and control of the plant, but offers contracts that are intended to replicate the output of the plant. These kinds of auctions are also launched in order to stimulate liquidity in forward electricity markets and to increase the proportion of electricity that is purchased through BCs. Specifically, the MIBEL imposed on Endesa and Iberdrola (the two dominant utility companies in the Spanish electricity market in 2006) to hold a series of five auctions offering VPP capacity to any agent member of the market. In 2006, the total installed capacity of both the companies was around 47 GW, and the total installed capacity of the Spanish electricity system was 78.3 GW. Figure 2.3 shows the volumes to be auctioned by Endesa and Iberdrola; it can be observed that the greatest volumes of auctioned VPP capacity were reached from April to September 2008, with a total amount of 2000 MW. The last settlement

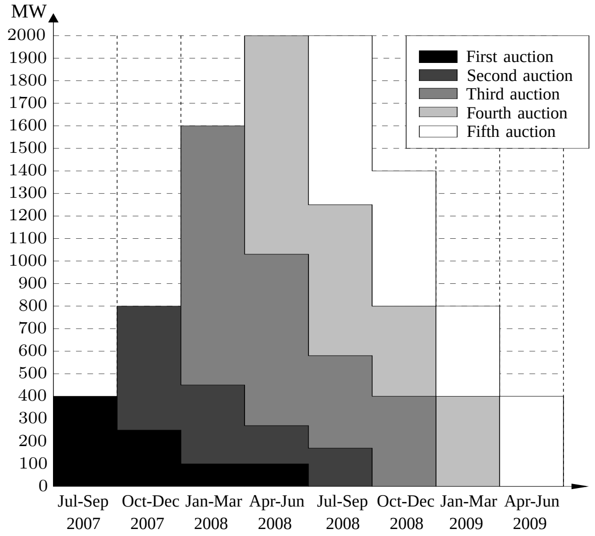

Figure 2.3: Five VPP capacity auctions in the Spanish peninsular electricity market.

dates for delivering energy bought at the last VPP auction is March 2010. Nowadays, the National Committee of Energy is making an evaluation of the real impact of the VPP auctions on market liquidity and competition. This evaluation is made through the MIBEL agents and it will be used for deciding if it is convenient to start another set of VPP auctions and if it is necessary to make some changes that improve their operation.

All these described BCs are settled *before the DAM* and the resulting dispatch must be communicated (*nominated* using the MIBEL's terminology) to the system and market operator before the closure of the DAM. There are other kind of BCs, called *after the DAM*, related to the VPP auctions that will be described below.

Thus, in the MIBEL, VPP capacity indicates that the buyer of this product will have the capacity to generate MWh at his disposal. The buyer can exercise the right to produce against an exercise price, set in advance, by paying an option premium. Hence, although Endesa and Iberdrola still own the power plants, part of their production capacity will be at the disposal of the buyers of VPP. There will be base-load and peak-load contracts with different strike prices that are defined a month before the auction. In each case, contracts with the duration of 3, 6, and 12 months will be offered.

The energy resulting from the exercise of the VPP options can be used by the buyers both as a contribution for covering the national and international BCs before the DAM as well as to sell it directly through bids to the DAM. In this latter case, the unmatched VPP energy, if any, can be sold through national BCs *after the DAM*. These new BCs after the DAM are negotiated between the agents prior to DAM gate closure, and must not be confused with other subsequent markets such as the ancillary services or balancing markets.

In 2009, in the Spanish section of the MIBEL, 34.7% of the system demand was traded through BCs. This includes the executed VPP rights and the classical BCs.

2.1.3 Derivatives Market

As we have introduced in Section 2.1.1, an important agreement between Spanish and Portuguese government lead to the creation of the MIBEL and the starting up of a regulated derivatives market managed by the Portuguese section of the MIBEL. The main objective of the participation in this long term market is to hedge risk. Financial mediators, producers, commercialization agents and other electrical agents can participate at the derivatives market sessions.

Nowadays, there are three kind of products: futures, forward and swaps. In the case of forward and swap products, they have been introduced on March 2009 and they are not totaly operative, specifically they are still out of the organized market but it is planned the negotiation through the market soon. The forward contracts will be with physical settlement and swaps contracts with financial.

Thus, the derivative product considered in this thesis is the futures contract (FC) because it was the kind of contract with physical delivery offered by the derivatives market until last year. Nowadays FCs are traded at organized derivatives markets in most electricity markets. The agents send their bids for the FCs to the market operator, OMIP, who does the clearing process.

The main characteristics of a FC are:

- *Procurement*: futures contracts could have physical or financial settlement. Physical futures contracts have cash settlement and physical delivery whereas financial ones have cash settlement only. In OMIP, the financial settlement correspond to the difference between the spot price and the futures reference price. This reference price is the last accepted transaction price.
- *Delivery period*: the delivery period defines the duration of the contract. In OMIP the delivery period could be a year, a quarter, a month or a week.
- *Load*: futures contracts could be base or peak load. In base load futures contracts the quantity to procure is constant for all the delivery period intervals. In peak load futures contracts there is procurement only in peak intervals (from 8 am to 24 pm, Monday to Friday). Nowadays, in OMIP all FCs are base load.

In the MIBEL derivatives market there are traded in average more than 3000GWh monthly.

2.1.4 Day-Ahead Market

The day-ahead market is the most important part of the electricity market with regard to physical energy exchanges. The objective of this market is to carry out the energy transactions for the next day by means of selling and buying offers presented by the market agents. The commitment of the offers to the market has the following characteristics:

- The agents owning a production unit must present a sell bid with a quantity equal to all the available energy of the unit, meaning the energy that is not committed to BCs. To avoid this rule the unit must be declared unavailable to the OM.

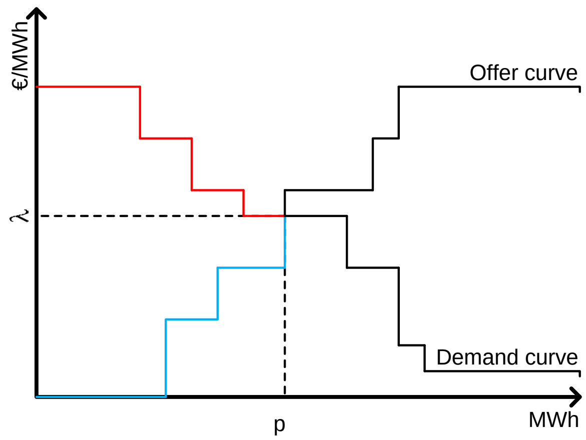

Figure 2.4: Market clearing for a certain hour.

- The external agents, commercialization agents and owners of production units in a special regime can present offers to sell.
- The distribution, commercialization or external agents and the final consumers participating in the market can present purchase offers.
- The open positions of physical derivatives products have to be integrated into the sell or purchase bids at instrumental prices.
- The energy resulting from the distribution auctions has to be integrated into the distribution companies bids at instrumental prices.
- The execution of the options obtained from the virtual power plants can be integrated into sell or purchase bids at a free price.

The clearing process is based on the construction of an offer curve and a demand curve for each hour; their intersection permits the establishment of market equilibrium and the point is determined by the result of the clearing process.

Specifically, the DAM is made up of twenty-four hourly auctions that are cleared simultaneously. Both offers from selling agents (i.e. generation companies) and bids from buying agents (i.e. distribution companies) are submitted to each auction. Each agent can submit several offers but they are unaware of the offers submitted by the rest of the agents. The offer for each interval and unit is a 25 piece-wise curve defined by a set of pairs (quantity, price) with non-decreasing price values. To derive the aggregate offer curve, offers are sorted by increasing prices and their quantities are accumulated. The clearing-price λ is determined by the intersection of the aggregate supply and demand curve (Figure 2.4). All the sale (purchase) bids with a lower (greater) bid price are matched and will be remunerated at the same clearing price λ, irrespective of the original bid price.

In 2009, in the Spanish section of the MIBEL, 76.8% of the system demand was traded on the DAM. But it must be taken into account that this percentage includes the mandatory bids as well as the products that must be integrated through the DAM bid (bilateral contracts, derivatives

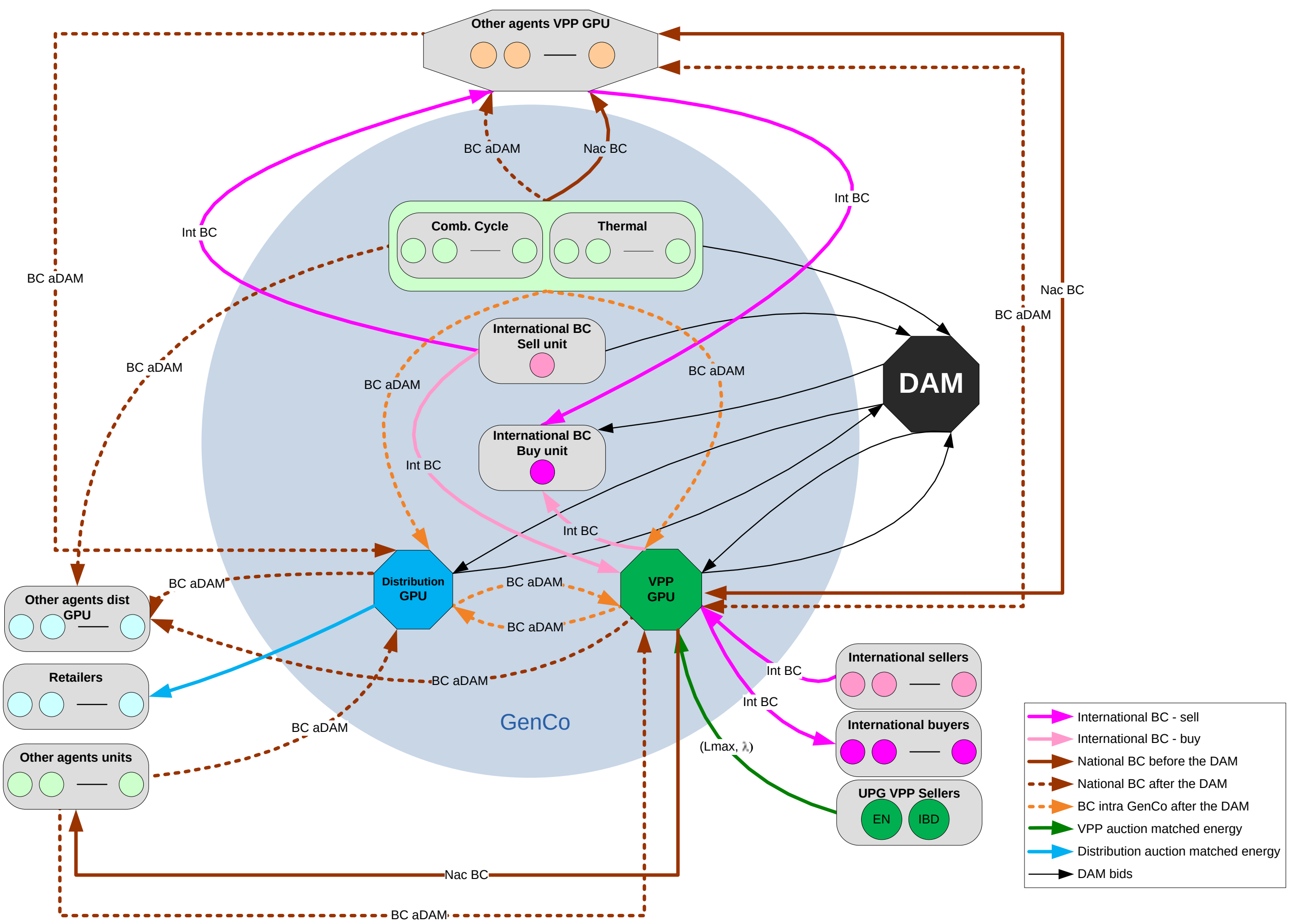

Figure 2.5: The GenCo's operation problem in the MIBEL's energy production system throughout the DAM.

products, etc.). If we extract these products, the percentage could be approximately 40% of the system demand.

2.1.4.1 Coordination between DAM and VPP Capacity

The exercised energy of the VPP is integrated into the energy production system through the *generic programming unit* (GPU). The GPUs (*VPP-GPU* node in Figure 2.5) are virtual units that bring more flexibility to the GenCo operations in the MIBEL. With the GPU, the utility can:

- Integrate the exercised VPP energy into the energy production system, both offering this energy to the pool through sales bids and allocating it among the GenCo's portfolio of national and international BCs.
- Act as a purchase agent, both sending purchase bids to the pool and acquiring energy through national and international BCs.

In summary, Figure 2.5 illustrates the set of BCs and their coordination with the DAM. As we have explained, BCs can be of two kinds: *before the DAM* (represented with continuous arcs) and *after the DAM* (dashed arcs). The elements of the MIBEL's energy production system that are relevant to the decision-making problem of a BC-owning GenCo are:

- The **programming units** owned by the GenCo. These units could be either physical (thermal, combined-cycle, etc.) or generic, i.e., virtual units (Distribution GPU, VPP GPU) through which the GenCo can operate by either selling or buying energy; in other words, by both bidding to the pool and settling BCs with the other participant in the MIBEL's energy production system.

- The **DAM** to which all the programming units of the GenCo can submit sale bids (physical production units) or sale/purchase bids (the generic units). The allowed bids are represented in Figure 2.5 by the thin black arcs.

- There are two kinds of agents that can interact with the GenCo's programming units: the **international agents** and the **MIBEL agents**. The GenCo can buy/sell energy from/to the surrounding foreign generation areas –namely, Portugal, Morocco, and France– through the *international BCs* settled with its GPU. There are also BCs signed by the GenCo and the rest of the MIBEL agents that apply to the GPU and the physical production units.

For the BCs after the DAM (dashed lines in Figure 2.5), the dispatch must be nominated following the publication of the DAM clearing results. After market clearing, the generation program of the GPU must be allocated among the GenCo's physical production units and BCs, in such a way that the net energy balance of the GPU is zero. The existence of BCs after the DAM prevents violation of the aforementioned netting energy balance condition as a consequence of possible unmatched sale or purchase bids of a GPU.

2.1.4.2 Coordination between DAM and Derivatives Market

The MIBEL regulation (OMEL, 2007) describes the coordination between the physical futures contracts portfolio and the day-ahead bidding mechanism (Figure 2.6). This coordination is structured into the following three phases:

1. For every derivatives contract in which the GenCo is interested, the corresponding *term contract unit* (UCP in the MIBEL's notation) has to be defined. A UCP is a virtual unit which is allowed to be on in the derivatives market. Each UCP is formed by a subset of the physical units of the GenCo which will generate the energy to cover the corresponding contract. For each contract, a physical unit can only participate in one virtual UCP.

2. Two days before the delivery date the GenCo receives from the derivatives market operator, OMIP, the quantity that every UCP has to produce in order to cover the matched FCs. This information is also sent to the day-ahead market operator, OMEL.

3. OMEL demands that every GenCo commit the quantity designated to FCs through the day-ahead market bidding of the physical units that form each UCP. This commitment is made by the so called *instrumental price offer*, that is, a sale offer with a bid price of 0€/MWh

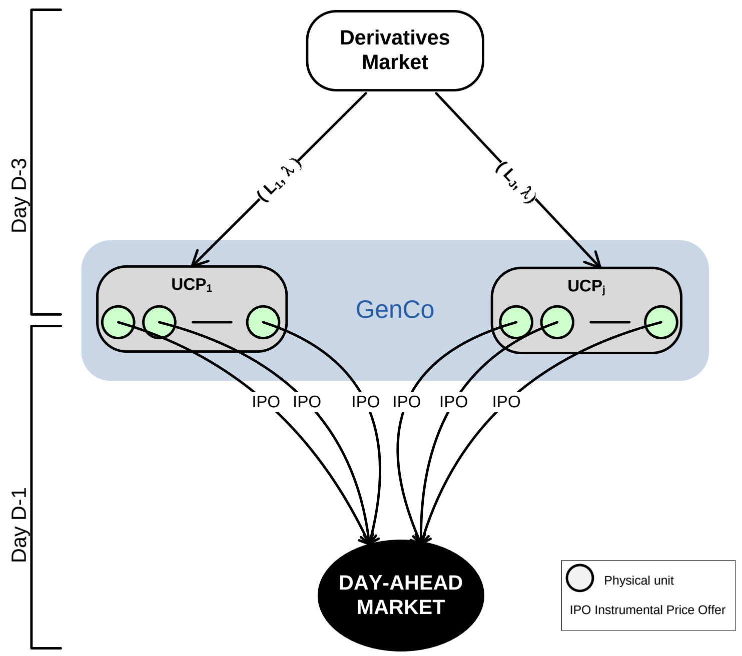

Figure 2.6: Representation of the coordination between DAM and DM.

(also called *price acceptant*). It is noteworthy that this is the main difference between FCs and BCs because, contrary to the FCs, the energy committed to the BCs must be excluded from the MIBEL's day-ahead market bid.

This regulation implies that the GenCo has to determine its unit commitment in order to be able to cover those obligations and it has to determine its optimal bid by taking into account those instrumental price offers. Due to the algorithm the market operator uses to clear the DAM, all instrumental price offers will be matched (i.e. accepted) in the clearing process; that is, this energy shall be produced and will be remunerated at the spot price.

2.1.5 Intraday Market

The *intraday market* (IM) is an adjustment market; it allows agents flexibility in optimizing its operation through a series of auctions with successive time horizons.

The IM takes place just before and during the delivery day; it is composed of 6 consecutive markets, each on comprising 24 auctions. These auctions work exactly as those of the DAM, with a matching process that is also identical (Figure 2.4).

The main difference from the DAM is that, in the IM market, all the agents can either send or buy electricity, that is, they can participate as buyers or sellers of energy. It is important to note that in a given session and hour, a unit can only submit offers to buy or sell, but not both. However, at different hours, this role can change.

The agents can participate in the IM if they have participated in the corresponding DAM session or if they have committed BCs that have been declared to the OS.

In 2009, in the Spanish section of the MIBEL, 11.50% of the system demand was traded during the six sessions of the IM.

2.1.6 Adjustment Market Services

There are a series of mechanisms that are necessary for guaranteeing the supply of electricity as well as maintaining the security and reliability of the system. The different mechanisms of the adjustment services can be mandatory or facultative and the greater part of them are managed through auction sessions. The mechanisms that are included in the adjustment services are: the technical restrictions; the primary, secondary and third regulation service (also known as *reserve*); and the real-time management of the system. In this thesis we focus on the secondary regulation service, henceforth *reserve*, because it is the only facultative regulation service between the DAM and IM.

The reserve takes place after the DAM matching process and its objective is to maintain the equilibrium between generation and demand by correcting the deviations. This service is performed during two phases: availability and use. In this thesis we focus on the first phase: the offers of availability. The agents send bids to offer its capability to increase or decrease the matched energy of the units in the DAM. If a bid is matched in the reserve, then the unit must be available to change its generation level within a given time interval on during real-time operation. For this reason, the units that participate in this market must have some specific operational characteristics that allow them to increase or decrease the generation level within a given time interval.

The adjustment market services do not represent a significant percentage of the system demand, but they are essential for proper management of the system. From a GenCo's point of view, its participation in the reserve will change its short-term operation, because their capacity for increasing or decreasing production levels will be subjected to ISO requirements.

2.2 Stochastic Programming

In this section we give a brief introduction to stochastic programming and we introduce the related terminology. A stochastic programming model is basically a mathematical programming model in which uncertain data is represented by random variables. As this thesis is devoted to application, we will present the basic class of stochastic programming problems but not the structural properties or specific solving algorithms. For an in-depth description see, for example, Kall and Wallace (1994), Prekopa (1995) or Birge and Louveaux (1997).

2.2.1 Two-stage Stochastic Program with Recourse

The basic stochastic problem is the *two-stage stochastic program with fixed recourse*. The decision are partitioned into two stages according to the information flow and we therefore refer to them as first-stage and second-stage decisions. It should be remarked that the partitioning of decisions do not actually need to reflect the separation of main decisions and recourse actions, but reflect the timing of the decisions such that first-stage decisions are to be made immediately, whereas second-stage decisions can be deferred. The decision-maker wants to minimize direct and expected future costs. Its general formulation is:

$$\begin{aligned} min \quad & z = c^T x + E_\xi[min\ q(\omega)^T y(\omega)] \\ s.t. \quad & Ax = b \\ & T(\omega)x + Wy(\omega) = h(\omega) \\ & x \geq 0 \end{aligned}$$

where x are the so-called *first-stage decision variables*, and c, b, and A are the first-stage vectors and matrices. In the second stage a number of random events $\omega \in \Omega$ may realize; $y(\omega)$ are the *second-stage decision variables* for a given realization ω; the second stage data $q(\omega)$, $h(\omega)$ and, $T(\omega)$ become known. $E[\ ]$ denotes the mathematical expectation. W is the recourse matrix, the assumption of a known fixed recourse matrix is referred to as a problem with fixed recourse. The dependency of y on ω reflects the fact that the decisions differ for different realizations of the random variables.

To illustrate the dynamics of the two-stage decision process, consider the following scheme

$$\text{decide on } x \longrightarrow \text{observe } q, h, T \longrightarrow \text{decide on } y.$$

As we have introduced, the first-stage decisions must be made with limited information regarding the future realization of the random data, and in such a way to minimize direct first-stage costs and expected second-stage costs.

Stochastic programming is based on the assumption of a known probability distribution of the random variable. Most of the time the continuos distribution is approximated through a discrete distribution with finite support. This finite number of possible realizations of the random variable define the so-called *set of scenarios*. We assume that the approximation of $\xi = (q, h, T)$ is given by a set of scenarios $\{1, ..., S\}$ that corresponds to the realizations $\xi^s = (q^s, h^s, T^s), s \in S$ and probabilities $P^s, s \in S$. The resulting two-stage program is formulated as:

$$\begin{aligned} min \quad & z = c^T x + \sum_{s=1}^{S} P^s q^s y^s \\ s.t. \quad & Ax = b \\ & T^s x + W y^s = h^s \\ & x \geq 0 \end{aligned}$$

In this thesis, the random data are the prices λ at which the energy will be paid in the different markets. These random variables will be modeled through a set of scenarios S with associated spot prices λ^s and probabilities $P^s = P(\lambda^s)$, $s \in S$.

Figure 2.7 represents a set of scenarios for a two-stage stochastic problem. The node represents the decisions points; the node to right first-stage decisions and those to the left scenario depending second-stage decisions. As it has been mentioned, it is implicitly assumed a risk neutral decision-maker, who aims to minimize an expectation based objective. In the case of other preferences or

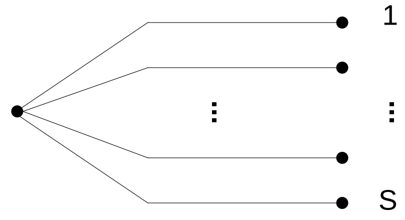

Figure 2.7: Two-stage scenario path.

other attitude toward risk, the objective function takes a different form.

2.2.2 Multistage Stochastic Program with Recourse

The previous section concerned stochastic problems with two stages, but there are many decisions that involve a sequence of decisions related with a series of outcomes over time. Those are the *multistage stochastic problems*. Multi-stage stochastic programs rely on the same idea as the two-stage version. Decisions are made without anticipating future realizations of uncertain data, which forces a partitioning of decisions into stages according to the information flow. We assume that the overall objective is to minimize expected future costs. See Römisch and Schultz (2001) for an introduction to multi-stage stochastic mixed-integer linear programming and a number of references on structural properties.

We suppose a finite horizon with K stages and fixed recourse. The stages represents the time points where the new information arrives. The general formulation for a multistage stochastic program is:

$$\begin{aligned} min\quad & z = c^1x^1 + E_\xi[min\ c^2(\omega)x^2(\omega) + ... + c^K(\omega)x^K(\omega)] \\ s.t.\quad & W^1x^1 = h^1 \\ & T^1(\omega)x^1 + W^2x^2(\omega) = h^2(\omega) \\ & \dots \\ & T^{K-1}(\omega)x^{K-1} + W^Kx^K(\omega) = h^K(\omega) \\ & x^k \geq 0,\ k = 1, \dots, K \end{aligned}$$

where x^k are the variables time depending decision variables. For $k \in K$, c^k, h^k and b^k are the random vectors and W^k is the random matrix, thus the random vector is $\xi^k = \{c^k, h^k, b^k, W^k\}$. The decision process of the decisions and observations of the random data can be summarized in the scheme:

$$\text{decide on } x \longrightarrow \dots \longrightarrow \text{observe } c^k, h^k, b^k, W^k \longrightarrow \text{decide on } x^k \longrightarrow \dots \longrightarrow \text{decide on } x^K$$

In this framework the nonanticipativity control becomes essential. Nonaticipativity implies that the *k stage* decisions are made before the realization of the *k+1 stage* and, obviously, the x decisions are taken before the realization of the random outcomes. These *nonanticipativaty constraints* prevent against the temporary violation of the random constraints at a given stage.

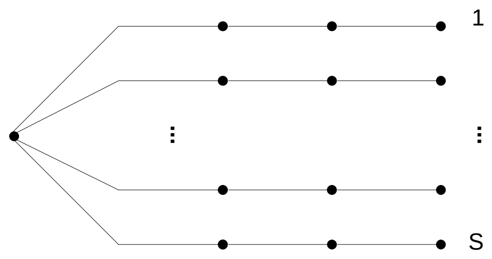

Figure 2.8: Multistage scenario path.

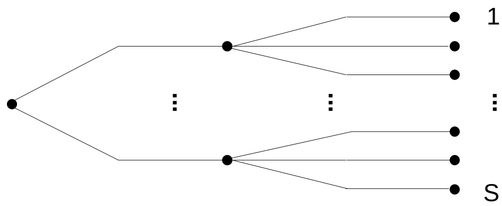

Figure 2.9: Multistage scenario tree.

There are two methods for including the nonanticipativity in the model, and they are directly related to scenario formulation. On the one hand, in a similar way as in the two-stage case, the approximate distribution of the stochastic vector $\xi^k = \{c^k, h^k, b^k, W^k\}$ is given by the scenario paths $\{\xi^{k,s}\}_{k=1}^{K} = \{c^{k,s}, h^{k,s}, b^{k,s}, W^{k,s}\}$, $s \in S$, and the scenario probabilities P^s, $s \in S$. In this case, nonanticipativity must be explicitly introduced by linear constraints that force decision variables to have the same value if they are based on the same information. This scenario paths are shown in Figure 2.8, in which the nodes again represent decision-making points. On the other hand, the scenarios can be clustered into a scenario tree and, a branching occurs when the new information arrives. Thus, decision variables are aggregated according to the available information, in other words, decision variables that are based on the same information are replaced by a single variable and automatically has the same value. In this case, nonanticipativity is introduced implicity because there is one decision variable per node. Figure 2.9 illustrates this situation, where the nodes represent decision points.

In this thesis, we use the first option, in which a set of linear constraints control the nonanticipativity of the decision variables.

2.2.3 The Value of the Stochastic Solution

The *value of the stochastic solution* (VSS) measures the effect of including stochasticity, in our case stochastic prices, explicitly into the mathematical program rather than using the expectation of the random variable and solving a deterministic program.

On the one hand, we define the *recourse problem* (RP) as:

$$RP = \min_{x} E_{\xi} z(x, \xi) \tag{2.1}$$

where $z(x, \xi)$ is the problem associated with one particular scenario ξ. On the other hand, we define, first, the *expected value problem* (EV), which is nothing more than solving the problem by replacing the random variables with their expected values:

$$EV = \min_{x} z(x, \overline{\xi}) \tag{2.2}$$

where $\overline{\xi} = E(\xi)$ denotes the expectation of ξ. We denote $\overline{x}(\overline{\xi})$ as an optimal solution to (2.2), called the *EV solution*. Secondly, we introduce the *expected result of using the EV solution* (EEV):

$$EEV = E_{\xi}(z(\overline{x}(\overline{\xi}), \xi)) \tag{2.3}$$

which measures how $\overline{x}(\overline{\xi})$ performs, allowing second stage decisions to be chosen optimally as a function of $\overline{x}(\overline{\xi})$ and ξ. The value of the stochastic solution is then defined as:

$$VSS \ = \ EEV \ - \ RP \tag{2.4}$$

These value allows us to obtain the goodness of the expected solution value when the expected values are replaced by the random values for the input variables. This indicators are classically defined for two-stage stochastic problems. There are some authors that have proposed approximations of these values for multistage problems (Escudero *et al.* (2007), Schütz and Tomasgard (2009) and, Vespucci *et al.* (2011)).

2.3 Day-Ahead Market Bidding Problem

Among the problems that concern the agents that participate in electricity markets, in this thesis we will deal with the short-term problems of a generation company and its physical trading. We have implicity assumed a risk-neutral GenCo who wants to maximize its expected benefits.

Regarding risk in the short-term horizon, there are authors who have included it in the optimization models (see for example Ni *et al.* (2004)). In this thesis, we have focused on the optimal bidding model, including the management of some medium-term products. Thus we consider that risk has been hedged when developing this medium-term products portfolio. There are some authors that also consider risk as something that must be taken into account on the mid-term horizon jointly with other mid-term strategies, such as fuel or derivatives contracts (Conejo *et al.*, 2008). For a state of the art description of risk management for electricity markets see, for example, Dahlgren *et al.* (2003) or Liu *et al.* (2006).

In general, the power systems optimization problems are categorized according to their time horizon. Long-term problems have a horizon of up to several years; medium-term problems have an horizon of a few months to a few years and short-term of a day to a week. In a short-term framework there are also many aspects to deal with. Specifically in this thesis we will present models for solving the classical problems of thermal unit commitment and economic dispatch related with DAM optimal bid strategies.

There is also another categorization, this time for GenCos. They could be *price-maker* or *price-*

taker, according to their capacity to change the market price or not through its bid (see Conejo and Prieto, 2001). We will focus our bidding strategies on a price-taker GenCo.

In this section we present the basis for the problems that are tackled throughout this thesis. First, a review of the published works that have considered this problem. Second, we present the characteristic of the GenCo. And finally, the basis of the day-ahead market bidding problem and the common parts of the models that will be presented in the thesis are described. For the main characteristics of a thermal unit see Appendix A.

So, in summary, the models presented in this thesis are based on the following assumptions:

Assumption 2.1. *The GenCo is a price-taker i.e., the day-ahead clearing price λ_t^D does not depend on the GenCo's bidding.*

Assumption 2.2. *To guarantee the commitment of unit i in the operational programming that results from the clearing of the DAM, unit i would bid its minimum generation level $\underline{P}_i$ at zero price (instrumental bid).*

2.3.1 Literature Review

Historically, there has been approximations to power system problems under a deterministic point of view, but they are out of the scope of this thesis. Furthermore, there has been also many approximations with a stochastic programming approach where the random variable were the demand, unit failures or fuel costs. For instance, Birge *et al.* (1994) present the first formulation of the unit commitment problem in terms of stochastic mixed-integer programming. Also Takriti *et al.* (2000), Nowak and Römish (2000) or Nürnberg and Römish (2002) present stochastic approximations to short-term power system problems with the demand as uncertainty source. For a survey on stochastic programming problems in energy see Wallace and Fleten in Ruszczynski and Shapiro (2003). And for a survey on electricity market modeling see Baillo *et al.* (2006) or Ventosa *et al.* (2005).

The development of stochastic programming electricity models has grown with the deregulation of the markets, where previous obligations to satisfy the demand were replaced by the opportunity of power producers to sell their production at the market. Hence, the restructuring of the sector and the liberalization of the markets has introduced a new important source of stochasticity: market prices. Nowadays, short-term planning problems deal with DAM physical trading and market price uncertainty, Conejo and Prieto (2001) present the main changes and the new point of view that this new framework introduces to the electricity management problems.

As state in the introduction, we deal with two different aspects of the short-term electricity problems. On the one hand, there is the physical trading represented by the unit commitment and the economic dispatch problem. And, on the other hand, there is the day-ahead market bidding.

There are many works, both before and after the deregulation process, that present models for the short-term planning of a GenCo. A review of the literature that deals with the unit commitment problem can be found in Padhy (2004). Recently, Simoglou *et al.* (2010) presents a novel modeling with three different start-up types for a thermal unit, see also this work for an extended review in this field. Within the works that deals with competitive electricity markets, Sen *et al.* (2006) presents a model which integrates the unit commitment model with financial decision making taking into account and modeling the electricity demand, the forward and the market prices. An earlier model by Shrestha *et al.* (2004) presents a stochastic unit commitment problem with BC which is solved by maximizing the DAM benefit; in this case the stochasticity in the spot prices is introduced through a set of scenarios, giving rise to a two-stage stochastic programming problem. In Triki *et al.* (2005), the authors presented a mixed-integer stochastic optimization model for scheduling thermal units, where the production plans were optimized in the presence of stochastic market clearing prices. Nevertheless, the models in Shrestha *et al.* (2004), Triki *et al.* (2005) and Sen *et al.* (2006) did not propose any explicit modeling of the optimal bidding.

Several authors have proposed optimal bidding models in the DAM for thermal units, see David and Wen (2000) for a literature survey of the first works published in this framework. If we focus on those that also consider the electricity market bidding process under the price-taker assumption, the authors in Conejo *et al.* (2002a) presented a mixed integer programming model to optimize the production scheduling of a single unit with a simple bidding strategy. The approximation of the step-wise bidding curves by linear functions based on the marginal costs was already considered in Gountis and Bakirtzis (2004a). In Ni *et al.* (2004), the concept of *price-power function*, which is similar to the *matched energy function* defined in this thesis, was used to derive the optimal offer curves of a hydrothermal system under the assumption that the market prices for the day-ahead and reserve markets behave as a Markov Chain. The mixed-integer stochastic programming model presented in Nowak *et al.* (2005) distinguishes between the variables corresponding to the *bid energy* and those representing the *matched energy*, although in a price-maker framework. Fleten and Kristoffersen (2007) also distinguish between the variables representing the *bid energy* and those corresponding to the *matched energy* in the case of a price-taker GenCo. In particular, it presents a stochastic programming model to optimize the unit commitment and the day-ahead bidding of a hydropower producer in the Nord Pool. See also Fleten and Kristoffersen (2007) or Fleten and Kristoffersen (2008) for an extended review in the literature about bidding strategies for hydrothermal systems. In another framework, Nabona and Pages (2007) provide a three stage procedure for building the optimal bid based on the optimal generation quantity and the zero-price bid. Furthermore, general considerations about the bidding process in electricity markets can also be found in Anderson and Philpott (2002); Anderson and Xu (2002); Neame *et al.* (2003).

2.3.2 Day-ahead Incomes Function for a Price-Taker

This thesis focuses on the thermal units of a *price-taker* generation company, i.e. a GenCo with no capability of altering market-clearing prices (Conejo and Prieto, 2001). Therefore, the framework of this kind of GenCo could be equated to a market with perfect competition. Perfect competition is defined as a market structure in which there are large numbers of both buyers and sellers, all of them small, so that all of them act as price-takers. And it is known that in a perfectly competitive market a GenCo would maximize its profits by bidding its true marginal cost function (Gountis and Bakirtzis, 2004b).

The optimal offer curve for thermal unit i is the offer to the day-ahead market that ensures a

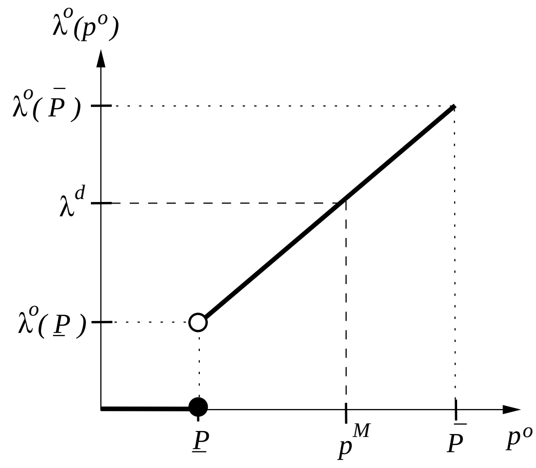

Figure 2.10: Optimal bid curve.

matched generation p_{ti}^M with a maximal benefit that is independent of the value of the clearing price λ_t^D. As stated above, in the case of a price-taker GenCo, the function that meets this condition is the *marginal cost curve.* If the cost function of thermal unit i is represented by a quadratic function, $C(p) = c_b + c_l p + c_q p^2$ where c_b is the constant operation cost, c_l is the linear cost and c_q is the quadratic cost (see Appendix A); then the optimal bid curve for this unit is:

$$\lambda_{ti}^o(p_{ti}^o) = \begin{cases} 0 & \text{if} \quad p_{ti}^o \le \underline{P}_i \\ 2c_i^q p_{ti}^o + c_i^l & \text{if} \quad \underline{P}_i < p_{ti}^o \le \overline{P}_i \end{cases} \tag{2.5}$$

Any offer of the GenCo must consist of pairs $(p_{ti}^o, \lambda_{ti}^o(p_{ti}^o))$ belonging to the optimal offer curve (Figure 2.10). By sending this offer to the DAM, the matched generation function p_{ti}^M corresponding to any clearing price λ_t^D will be:

$$p_{ti}^M(\lambda_t^D) = \begin{cases} \underline{P}_i & \text{if } \theta_{ti}(\lambda_t^D) \le \underline{P}_i \\ \theta_{ti}(\lambda_t^D) & \text{if } \underline{P}_i \le \theta_{ti}(\lambda_t^D) \le \overline{P}_i \\ \overline{P}_i & \text{if } \theta_{ti}(\lambda_t^D) \ge \overline{P}_i \end{cases} \tag{2.6}$$

where

$$\theta_{ti}(\lambda_t^D) = \left(\lambda_t^D - c_i^l\right)/2c_i^q. \tag{2.7}$$

It is easy to see that, for any clearing price λ_t^D, expression (2.6) gives the value that maximizes the benefit function taking into account the operational limits of the thermal unit:

$$B(p_{ti}) = \lambda_t^D p_{ti}^M - \left(c_i^b + c_i^l p_{ti}^M + c_i^q (p_{ti}^M)^2\right)$$

where p_{ti}^M is the matched energy.

The optimal bid curve problem for a price-taker GenCo is reduced to as many independent stochastic unit commitment problems as there are thermal units offered by the utility. If the optimal unit commitment shows that a given thermal unit must be on at interval t, then (2.5) represents the optimal offer curve to be sent to the Day-Ahead Market. The total incomes for all the committed units, I_{on_t}, will be:

$$In_t^D = \sum_{\forall i \in I_{on_t}} \lambda_t^D p_{ti}^M$$

where λ_t^D is the clearing-price and p_{ti}^M (2.6) is the matched energy that has to be produced by unit i. Thus, the incomes that a GenCo takes in from the DAM depends on the results of the clearing process. The offers are called *matched* if their price is lower or equal to the clearing-price. Only the matched offers produce benefits.

2.3.3 Matched Energy

The formulation of the models presented in this thesis will include variables representing the value of the *matched energy* for the committed thermal unit i on the t^{th} DAM. For the moment, the matched energy will be loosely defined as the accepted energy in the clearing process; that is, the energy that the thermal unit i should generate at period t and that will be rewarded at the clearing price.

This matched energy, which plays a central role in our models, is uniquely determined by the *sale bid* and the clearing price. As we have stated, a bid in the MIBEL's DAM consists of a stepwise non-decreasing curve defined by up to 25 energy (MWh)-price(€/MWh) blocks. As usual is in this kind of work (see Gountis and Bakirtzis, 2004a),we will consider a simplified model of the true sale bid through the so called *bid function* λ_{ti}^b, not necessarily stepwise:

Definition 2.1 (Bid function)**.** *A bid function for the thermal unit i is a non-decreasing function defined over the interval* $[0, \overline{P}_i]$ *that gives, for any feasible value of the bid energy* p_{ti}^b*, the asked price per MWh from the day-ahead market:*

$$\begin{array}{rrcl} \lambda_{ti}^b : & [0, \overline{P}_i] & \longrightarrow & \Re^+ \cup 0 \\ & p_{ti}^b & \longmapsto & \lambda_{ti}^b(p_{ti}^b) \end{array}$$

For a given bid function λ_{ti}^b the matched energy associated with the clearing price λ_t^D, p_{ti}^M is defined through the matched energy function:

Definition 2.2 (Matched energy function)**.** *The matched energy associated with the bid function* $\lambda_{ti}^b(p_{ti}^b)$ *is defined as the maximum bid energy with a price not greater than the clearing price* λ_t^D*, and is represented by the function:*

$$p_{ti}^M(\lambda_t^D) \stackrel{\text{def}}{=} \max\{p_{ti}^b \in [0, \overline{P}_i] \mid \lambda_{ti}^b(p_{ti}^b) \le \lambda_t^D\} \qquad (2.8)$$

As we will see, the clearing price λ_t^D is a random variable that will be modeled through a set of scenarios S with associated spot prices $\lambda^{D,s} = \{\lambda_1^{D,s}, \dots, \lambda_T^{D,s}\}$ and probabilities $P^s = P(\lambda^{D,s})$, $s \in S$. Each one of these scenarios has, for each period t, a corresponding matched energy that will be represented in the models by the second stage variable p_{ti}^s. Although our model will be developed without any assumption on the specific expression of the bid function λ_{ti}^b it is necessary, for the sake of the model's consistency, to assume the existence of a bid function with a matched energy function (2.8) that agrees with the optimal value of variables p_{ti}^s, i.e.:

	Number of variables		Number of constraints		Time (s)		Change
S	*Nabona*	*Carrion*	*Nabona*	*Carrion*	*Nabona*	*Carrion*	%
10	3200	3216	7248	7056	0.89	0.73	17.54%
50	10880	10896	32208	32016	8.29	6.46	22.03%
100	20480	20496	63408	63216	23.46	18.46	21.30%
150	30080	30096	94608	94416	41.11	37.76	8.13%
200	39680	39696	121808	121616	61.70	54.54	11.60%
250	49280	49296	157008	156816	106.67	74.79	20.88%

Table 2.1: Comparison between unit commitment formulations.

Assumption 2.3. *For any thermal unit i committed at period t there exists a bid function λ_{ti}^b such that:*

$$p_{ti}^M(\lambda_t^{D,s}) \stackrel{\text{def}}{=} p_{ti}^{M,s} = p_{ti}^{s*} \quad \forall s \in S \tag{2.9}$$

with p_{ti}^{s*} the optimal value of variable p_{ti}^s

Notice that the existence of such a bid function is not evident, as all scenarios must prove simultaneous equality (2.9).

2.3.4 Unit Commitment

The thermal *unit commitment* problem consists of the scheduling of start-up and shut-downs of the thermal units. There are many approximations to the modeling of this classical problem in the literature. During the elaboration of this thesis, the models presented have matured and, as a result, the same idea may have been modeled in many different ways. The unit commitment of the thermal units is one of the aspects that has evolved. Our first approximation was presented by Nabona and Rossell (1999) and has been used in many works. Specifically, two of the models that will be presented in this thesis included this formulation. When presenting these models, some reviewers suggested using the formulation presented by Carrión and Arroyo (2006) which reduces significantly the number of binary variables. We have supervised a work where these two formulations were compared (Nieto and Ruz, 2009). The results prove that the second approach (Carrión and Arroyo, 2006) improves the results in terms of computational time with an average of 17% (Table 2.1).

2.3.4.1 First Formulation

This formulation of the start-up and shut-down processes follows Nabona and Rossell (1999); Nabona and Pages (2007). Let $u_{ti} \in \{0,1\}$ be a first-stage binary variable expressing the off-on operating status of the i^{th} unit over the t^{th} interval ($u_{ti} = 1$ if committed, $u_{ti} = 0$ if uncommitted). The values of u_{ti} and $u_{(t+1)i}$ must obey certain operating rules in order to take into account the constraints of the minimum in service and idle time.

It is necessary to introduce two extra binary variables e_{ti} and a_{ti} for each u_{ti}. Let $e_{ti} \in \{0,1\}$ be a start-up indicator for the i^{th} unit. It has a value of one in all intervals t where the i^{th} unit has changed from $u_{(t-1)i} = 0$ to $u_{ti} = 1$, and zero elsewhere. Similarly, $a_{ti} \in \{0,1\}$ is a shut-down indicator for the i^{th} unit. It should have a value of one in all intervals t where $u_{(t-1)i} = 1$ to $u_{ti} = 0$, and zero otherwise.

The following three sets of constraints unambiguously model the commitment variable u_{ti} and the start-up and shut-down variables e_{ti} and a_{ti}:

$$u_{ti} - u_{(t-1)i} - e_{ti} + a_{ti} = 0 \qquad \forall i \in I,\ \forall t \in T \tag{2.10}$$

$$e_{ti} + \sum_{k=t}^{\min\{t+t_i^{on},|T|\}} a_{ki} \leq 1 \qquad \forall i \in I,\ \forall t \in T \tag{2.11}$$

$$a_{ti} + \sum_{k=t+1}^{\min\{t+t_i^{off},|T|\}} e_{ki} \leq 1 \qquad \forall i \in I,\ \forall t \in T \tag{2.12}$$

2.3.4.2 Second Formulation

This formulation follows the one proposed in Carrión and Arroyo (2006) for the unit commitment of the thermal units.

Let u_i^t be, as in the first formulation, the first-stage binary variable expressing the off-on operating status of the i^{th} unit. It is again necessary to introduce two extra variables, c_{ti}^u, c_{ti}^d, which represent the startup and shutdown cost, respectively, of unit i in interval t. In contrast to the previous approximation, this auxiliary variables are continuous not binary, so, the number of integer variables of the model is reduced as well as the computational cost.

It is also necessary to define two constants: G_i, which will be the number of periods that unit i must be initially online, due to its minimum up-time constraint; and H_i, which is the number of periods that unit i must be initially offline, due to its minimum down-time constraint.

The following first two sets of constraints model the start-up and shut-down costs and the next ones control minimum operation and idle time for each unit:

$$c_{ti}^u \geq c_i^{on}[u_{ti} - u_{(t-1),i}] \qquad \forall t \in T \setminus \{1\},\ \forall i \in I \tag{2.13}$$

$$c_{ti}^d \geq c_i^{off}[u_{(t-1),i} - u_{ti}] \qquad \forall t \in T \setminus \{1\},\ \forall i \in I \tag{2.14}$$

$$\sum_{j=n}^{G_i}(1 - u_{ji}) = 0 \qquad \forall i \in I \tag{2.15}$$

$$\sum_{j=1}^{H_i} u_{ji} = 0 \qquad \forall i \in I \tag{2.16}$$

$$\sum_{n=t}^{t+t_i^{on}-1} u_{ni} \geq t_i^{on}[u_{ti} - u_{(t-1),i}] \qquad \forall t = G_i + 1, \ldots, |T| - t_i^{on} + 1\ \forall i \in I \tag{2.17}$$

$$\sum_{n=t}^{t+t_i^{off}-1} (1 - u_{ni}) \geq t_i^{off}[u_{(t-1),i} - u_{ti}] \qquad \forall t = H_i + 1, \ldots, |T| - t_i^{off} + 1 \forall i \in I \tag{2.18}$$

$$\sum_{n=t}^{|T|}(u_{ni} - [u_{ti} - u_{(t-1),i}]) \geq 0 \qquad \forall t = |T| - t_i^{on} + 2, \ldots, |T|\ \forall i \in I \tag{2.19}$$

$$\sum_{n=t}^{|T|}(1 - u_{ni} - [u_{(t-1),i} - u_{ti}]) \geq 0 \qquad \forall t = |T| - t_i^{off} + 2, \ldots, |T| \;\; \forall i \in I \tag{2.20}$$

CHAPTER 3

Uncertainty Modeling

As has been introduced, in liberalized electricity markets, a generation company must build an hourly bid that is sent to the market operator, who selects the lowest price among the bidding companies in order to match the pool load. For this reason, GenCos that participate in liberalized electricity markets around the world need to estimate the prices at which the energy will be paid in order to decide how to bid and how to schedule their resources for maximizing their profit. The problem is that the market price is only known once the market has been cleared, so they need to forecast it.

The correct treatment and introduction of uncertainty into the stochastic models is one of the most important components of this kind of technique. It will determine whether or not stochastic techniques are useful, specifically whether or not it is necessary to introduce this randomness into the model. If not, then a deterministic approach is sufficient.

It is one of the arguments that will determinate if the stochastic techniques are useful or it is unnecessary to introduce this randomness into the model and a deterministic one is sufficient.

The objective of this chapter is to build a short-term forecast method for the electricity market day-ahead price in order to include it into an optimization model. Once those forecasts are obtained, it is essential to create realistic day-ahead market price scenarios in which the generation company will decide how to optimally operate and to easily update these scenarios over time.

Considering that we will introduce the random data into the model by means of a set of scenarios, this chapter can be divided into two steps: the uncertainty characterization, i.e, the correct identification of the random variable that becomes the input data of the model; and the scenario set construction. In Section 3.1, 3.2 and 3.3 we will focus on the random variable and its possible characterization through different techniques. In Section 3.4 some methods described in the literature for generating and reducing the set of scenarios are used to build the data that will be included in the models presented in the next chapters.

This chapter focuses on DAM prices, RM and IM prices are described and studied in Chapter 6.

3.1 DAM Price Characterization

As we have introduced, the start-up of the electric markets changes the uncertainties that the GenCo has to face. The demand becomes unimportant because the GenCo sends its production to the market operator through its sell bids and the generation level will be fixed in the clearing process. Meanwhile, the price at which this production will be paid becomes unknown and must be forecasted. The more accurate the forecast is, the better the GenCo can control its expected benefits and its bidding strategies.

In the field of price characterization we have used many of the published methodologies. We started out using historical data or clustering methods. These methods were very useful for the first approximations but became poor when a more accurate forecast was needed. At this moment we adjusted an ARIMA model for the day-ahead price forecast that performed correctly. Later, we realized that the ARIMA model needed continuous corrections of the estimations over time, as well as some experience in adjusting and using it correctly, which limits its practical use in an optimization framework. Thus, focusing on finding an easy method, we built a methodology based on factor models, widely used in other areas but not in the day-ahead electricity price forecast.

3.1.1 Literature Review

Electricity spot prices exhibit non-constant mean and variance, daily and weekly seasonality, calendar effects on weekends and holidays, high volatility and the presence of outliers. Those characteristics do not necessarily make it easy for electricity price short-term forecasting. Several approaches have been proposed in the power system literature which basically can be classified into parametric/nonparametric, conditional homoscedastic/heteroscedastic and others, ranging from the most popular ARIMA models belonging to the class of parametric-conditional homoscedastic models to the most sophisticated ones, as for example wavelet or neural network models. Weron (2006) classified these methodologies into six classes: production-cost models, equilibrium or game theory approaches, fundamental methods, quantitative models, statistical approaches and artificial intelligence-based techniques. In this section we point out some of the published works, we focus on the ones modeling day-ahead electricity prices and, if possible, refer to the Spanish electricity market prices. See Weron (2006), Misiorek *et al.* (2006) or Serati *et al.* (2007) for extended reviews in this field.

Nonparametric statistic methods, such as clustering or bootstrapping, applied to historical data were the first and simplest approaches. The advantage of these methods is that they are easy and computationally cheap to use but, on the other hand, they do not characterize the price distribution properly. See, for instance, Martinez-Alvarez *et al.* (2007) for an application of these techniques to the Spanish market prices.

The ARIMA models have been widely used in this field. Contreras *et al.* (2003) use this modeling technique for forecasting Spanish and Californian day-ahead electricity prices. Later, Conejo *et al.* (2005a) compared the ARIMA approach with wavelet transform and ARIMA and concluded this second approach performs better. In another work, Conejo *et al.* (2005b) compared ARIMA models, dynamic regression and transfer function. They concluded that the predictions extrapolated from dynamic regression and transfer function procedures are better than those obtained from ARIMA models whereas wavelet models which have results close to ARIMA models and neural network algorithms do not offer good forecasts. Nogales *et al.* (2002) and Nogales and Conejo (2006) presented price forecasting through transfer function and dynamic regression models for different data sets. However the residuals in some of the analyzed models exhibited non-stationary conditional variance. To solve this problem, the classical GARCH models and their variants were used for estimating the conditional heteroscedasticity of the electricity spot prices. Garcia *et al.* (2005) estimated an ARMA model with GARCH errors for the Spanish and California Electricity market, showing that this combined model overcomes the predictions obtained by the classical ARIMA model. Koopman *et al.* (2007) gave a more complex version of this model, extending it to periodic dynamic long memory regression models with GARCH errors also. Last but not least, Weron and Misiorek (2008) compared some different time series methods for day-ahead forecasting and concluded that models with system load or air temperature included as an exogenous variable give better forecasts than pure price models and that semiparametric models perform better in terms of point and interval confidence forecasts.

From another perspective, Lucia and Schwartz (2002) proposed a one and two-factor mean reverting model with deterministic seasonality, showing that the seasonal pattern could explain part of the shape of the observed term structure of futures prices. Kian and Keyhani (2001) presented another different approach with models that combine time series with stochastic methods and include information about customer energy consumption, market participant strategies or behaviors, as well as other information. We can find many other approaches. For instance, Mateo-González *et al.* (2005) adjusted non-stationary models based on input-output hidden markov models. There is also Lora *et al.* (2007), who presented weighted nearest neighbors techniques and compared them to ARIMA with wavelet transform and GARCH, concluding that they perform satisfactorily. Both approaches were analyzed using Spanish electricity market prices.

All these methods consider either one time series or random variables. Another point of view consists of considering each hour of the day as a time series and treating them separately. For instance, Garcia-Martos *et al.* (2007) decompose the hourly time series into 24 time series and model them separately with time series techniques, obtaining one day-ahead forecast for each time series. Our proposal is based on this interpretation of the electricity prices not as a single time series but a set of 24 time series, one for each hour, and aims to exploit the possibilities of factor models in the day-ahead forecasts. This is similar to Alonso *et al.* (2008), who apply the factor models for long term forecasts.

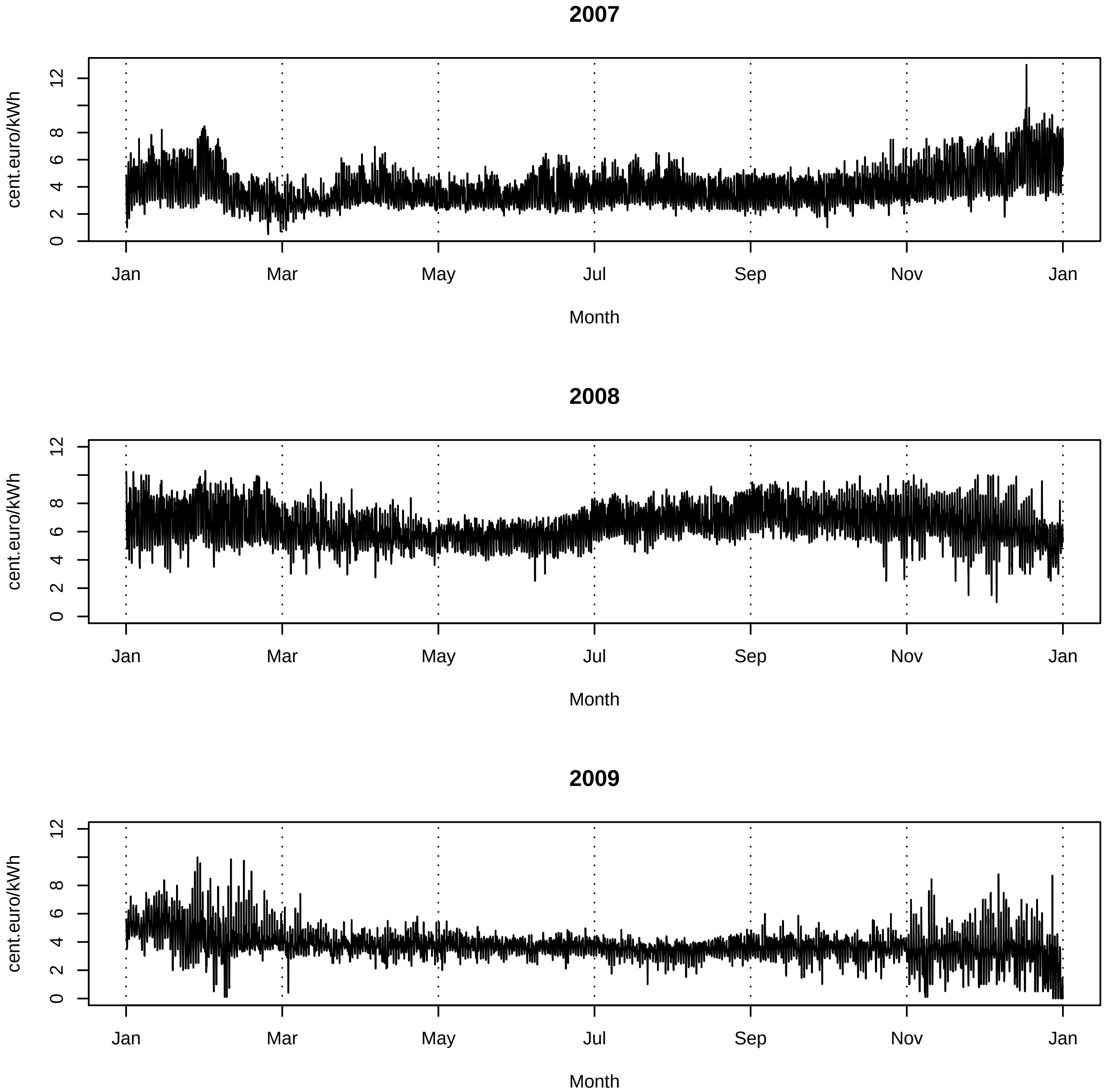

Figure 3.1: DAM hourly prices.

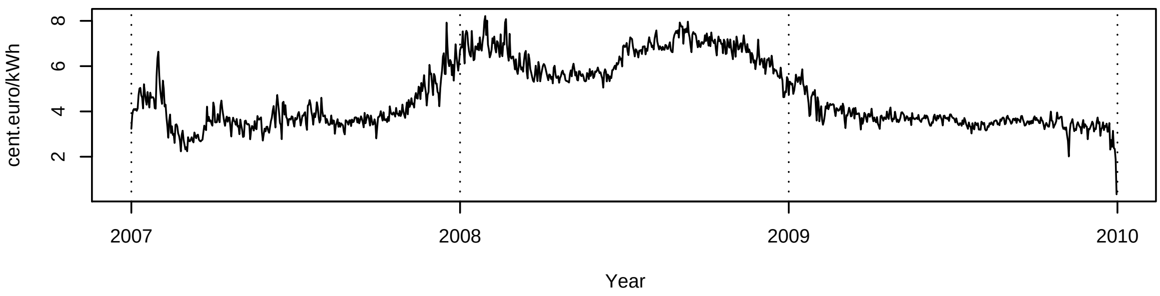

Figure 3.2: Daily mean MIBEL's DAM price.

3.2 Descriptive Statistics

Electricity prices are a sequence of data depending on the time interval where they are determined; so, a specific hour price depends on the previous hour prices. It is for this reason that the market price is usually defined and treated as a time series. We will focus on the MIBEL DAM price over one week. As with most competitive electric market prices, they present the following characteristics: high frequency, non-constant mean and variance, multiple seasonality, calendar effects, high volatility and high presence of picks.

Figure 3.1 represents the hourly Spanish DAM prices from the start-up of the MIBEL in 2007 up to December 2009. Historically, Spanish electricity prices showed a high seasonality throughout the year but this effect is not as evident nowadays. Figure 3.2 represents the daily mean of the hourly prices, also from 2007 up to 2009. This figure shows that the variation of the mean price is not directly related with the season of the year. Electricity prices are directly related to the load and other exogenous factors, as, for instance, oil or gas prices. Thus, this change in the year pattern could be explained by the changes in daily habits of small consumers (more electricity dependent), on the changes on industry holidays habits in Spain or on the combustible prices. Nowadays, almost all the weeks of the year have a similar level of demand.

If we zoom in on a weekly point of view, we can see in Figure 3.3 that the typical behavior of the day-ahead electricity prices over a week. The working days are all similar in their shape and mean and the weekend is totaly different.

3.3 Forecasting Models

MIBEL DAM prices are the variable to be forecasted. This is hourly data and the data set used to prove the following methods corresponds to the days from January 1^{st}, 2007 to December 31^{st}, 2008. This data is available at the OMEL's site (OMEL, 2010).

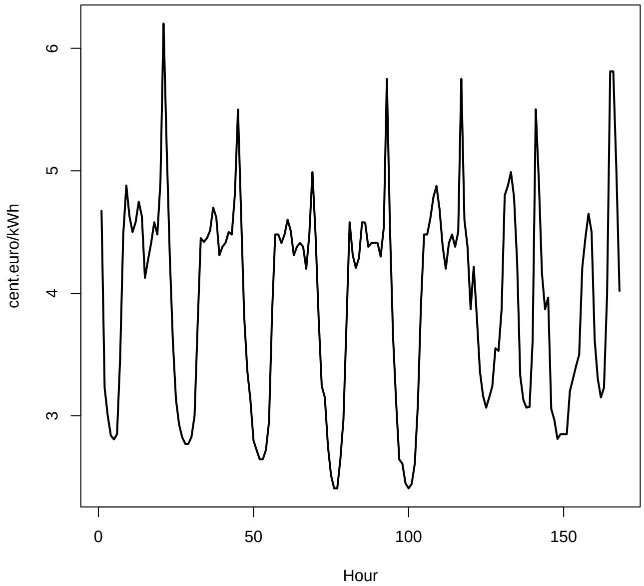

Figure 3.3: One week MIBEL's DAM price.

3.3.1 ARIMA Model

As we have introduced, our first approximation was through ARIMA models (see, for instance, Peña *et al.* (2001)). If we analyze the time series we can observe that there is non-constant mean and variance and two seasonal components: one order 24, which corresponds to the day effect, and one order 168, which corresponds to the week effect (see Figure 3.4). In order to adjust the ARIMA model, we must transform the original time series:

$$Y_t = (1 - B^{168})(1 - B)\log(\lambda_t) \tag{3.1}$$

where λ_t is the DAM hourly price. If we differentiate also the order 24, the results in terms of variance indicate that it is over-differentiated.

Thus, the resulting model for the transformed Spanish electricity price is:

$$\log(\lambda_t) \backsim ARIMA(23, 1, 13)(14, 0, 21)_{24}(0, 1, 1)_{168} \tag{3.2}$$

Table C.1, Appendix C, exhibits statistically significant parameters of the ARIMA model. Figure 3.5 represents the forecast and the confidence interval for the week of March 24^{th} to 28^{th}, 2008. In Table 3.1 there is the *mean square error* of the forecast for each hour. In this model it is not necessary to distinguish between the hours, but it is indicated in order to facilitate the comparison with the factor model forecasts to be developed in the next section.

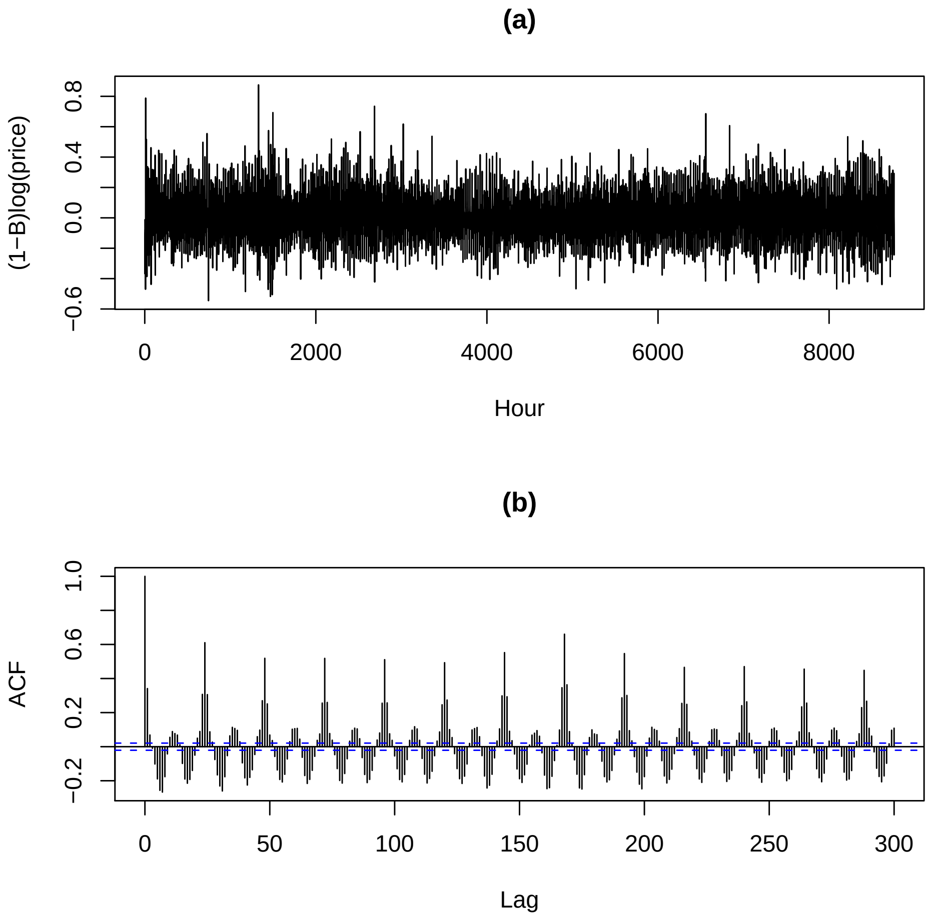

Figure 3.4: (a) Transformed time series $(1-B)\log(\lambda_t)$. (b) Autocorrelation function of $(1-B)\log(\lambda_t)$.

Hour	1	2	3	4	5	6	7	8	9	10	11	12
MSE	0.201	0.070	0.111	0.033	0.058	0.032	0.081	0.152	0.122	0.065	0.086	0.043
Hour	13	14	15	16	17	18	19	20	21	22	23	24
MSE	0.032	0.027	0.076	0.044	0.036	0.040	0.031	0.096	0.112	0.265	0.111	0.090

Table 3.1: Mean square error of the forecast for each hour with ARIMA model.

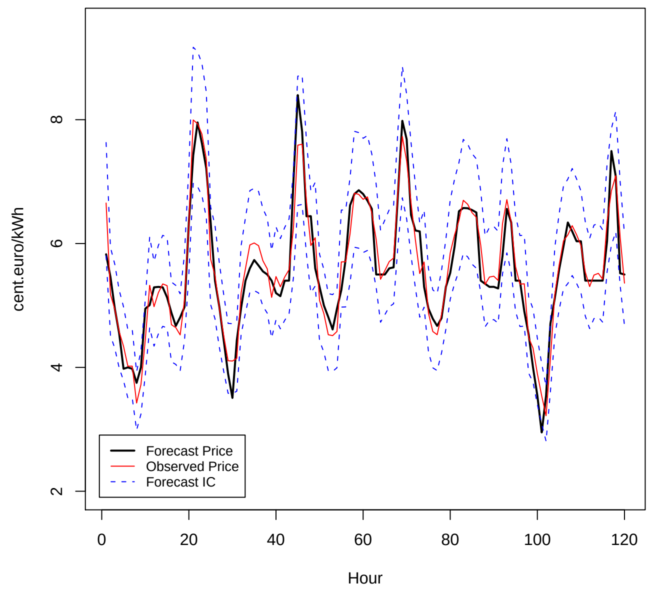

Figure 3.5: One-step-ahead forecast prices with ARIMA model.

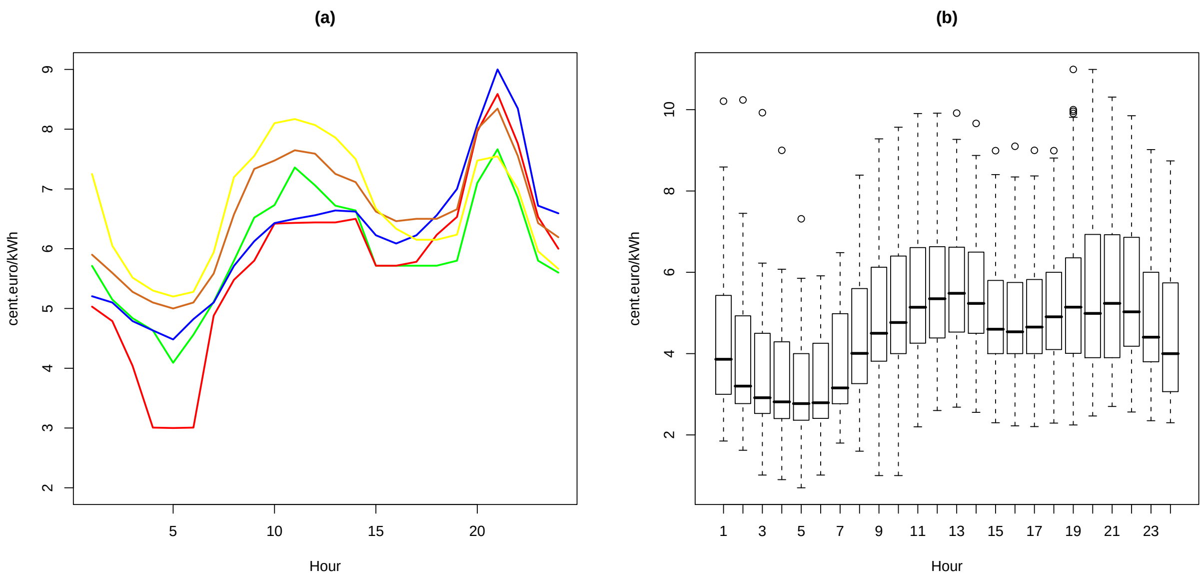

Figure 3.6: (a) Hourly DAM price for the work days of a week. (b)Boxplot of hourly DAM prices.

3.3.2 Times Series Factor Model

Despite the fact that dynamic and static factor models have been extensively used in many different frameworks (Stock and Watson (2002), Peña and Poncela (2004) and Peña and Poncela (2006)), their application to short-term electricity market prices forecasting has not been exploited. Our approach in this work is to apply the well-known methodology of factor models in order to forecast electricity market prices over a short-term horizon (24 hours). In this case, the spot prices have been interpreted not as a single time series but as a set of 24 time series, one for each hour, in a similar way to Alonso *et al.* (2008). So we will have a multivariate time series and the factor model procedure allows us to identify common unobserved factors, which represent the relationship between the hours of a day. Figure 3.6(a) shows the pattern of a typical set of hourly electricity prices in Spain. This pattern is directly related to the load of each hour and, as we will see the factor models permit us to exploit this pattern and reduce the dimensionality of the multivariate time series. Figure 3.6(b) shows the boxplot of the hourly prices and the hourly differences in level and price variability can be observed.

The estimation and forecast of price variables using factor analysis can be classified into two overarching groups: static and dynamic. The first uses *principal component analysis* whereas the second basically formulates the model into *state space* and uses the Kalman filter or the *expectation-maximization* (EM) algorithm for estimating the parameters and for forecasting the future values of the variable in question, in our case, the price.

In this work we use the alternative procedure *time series factor analysis* (TSFA), described in Gilbert and Meijer (2005). TSFA estimates the measurement model for time series data, with as few assumptions as possible about the dynamic process which governs the factors. It is an alternative to static and dynamic factor analysis. On the one hand, the static factor analysis should not be used with economic time series because the characteristics of the data do not agree with the assumptions of the method. That is, if we compare TSFA with factor analysis, we can conclude that with TSFA:

(a) The factor model has a non-trivial mean structure.

(b) The observations are allowed to be dependent over time.

(c) The data does not need to be covariance stationary as long as data with differentiation satisfies a weak boundedness condition.

TSFA differs from dynamic factor analysis in the sense that TSFA estimates parameters and predicts factor scores with few assumptions about factor dynamics; in particular, TSFA does not assume stationary covariance. Dynamic factor analysis assumes a predetermined relationship between factors in the sense that there is an assumed *a priori* relationship between the factors at time t and the factors at time $t-1$. If this relationship is misspecified, the factors estimated by dynamic factor analysis can be biased.

3.3.2.1 Factor Model Estimation

Let y_t be an M-vector of observed time series of length T and k unobserved factors ($k << M$) collected in the k-vector ξ. The relationship between the observed time series y_t and the ξ factors is assumed to be linear and described by equation:

$$y_t = \alpha_t + B\xi_t + \epsilon_t \tag{3.3}$$

where α_t is an M-vector of intercepted parameters, possibly time-varying that can be omitted without losing generality. B is an $M \times k$ matrix parameter of loadings, assumed to be time-invariant, and ϵ is a random M-vector of measurement errors. Notice that this is the standard factor analysis model with the indicators indexed by time and the intercepts explicitly included.

It is possible that y_t has a stationary first difference, defining D as the difference operator, (3.3) becomes:

$$Dy_t \equiv y_t - y_{t-1} = (\alpha_t - \alpha_{t-1}) + B(\xi_t - \xi_{t-1}) + (\epsilon_t - \epsilon_{t-1}) \tag{3.4}$$

or:

$$Dy_t = \tau_t + BD\xi_t + D\epsilon_t \tag{3.5}$$

which is also an equation with factor structure and the same loadings, so the model can be estimated with the differenced data. It is assumed that τ_t is a constant vector.

The following are the sufficient conditions that are assumed in such as that this model leads to consistent estimators (Gilbert and Meijer, 2005):

$$\sum_{t=1}^{T} \frac{D\xi_t}{T} \xrightarrow{p} \kappa, \text{ factor mean, exists and is finite}$$

$$\sum_{t=1}^{T} \frac{(D\xi_t - \kappa)(D\xi_t - \kappa)'}{T} \xrightarrow{p} \Phi, \text{ factors covariance, exists, is finite and positive definite}$$

$$\sum_{t=1}^{T} \frac{D\epsilon_t D\epsilon_t'}{T} \xrightarrow{p} \Omega, \text{ error covariance, exists, is finite and positive definite}$$

$$E(D\epsilon_t | D\xi_t) = 0$$

There are no explicit assumptions about the autocorrelation of the differenced data and they allow serial dependence in the variables. Unit roots violate the assumptions, but then the series can be differenced and the assumptions then applied to the twice-differenced variable. This process can be performed until there are no unit roots. Mean and variances are only bounded in order to obtain a probability limit; they do not have to be constant over time.

The sample mean and covariance of the differenced series Dy_t is denoted by $\overline{Dy}$ and S_{Dy}:

$$\overline{Dy} \equiv \frac{1}{T} \sum_{t=1}^{T} Dy_t$$

$$S_{Dy} \equiv \frac{1}{T} \sum_{t=1}^{T} (Dy_t - \overline{Dy})(Dy_t - \overline{Dy})'$$

and from the previous assumptions, it follows that:

$$\overline{Dy} \xrightarrow{p} \mu \equiv \tau + B\kappa$$
$$S_{Dy} \xrightarrow{p} \Sigma \equiv B\Phi B' + \Omega$$

It can be shown that consistent estimators result from estimating the parameters by maximum likelihood and minimizing the function:

$$L \equiv \lg \det \Sigma + \mathrm{tr}(\Sigma^{-1} S_{D_y})$$

See Gilbert and Meijer (2005) for a more detailed description.

3.3.2.2 Forecasting Model

The factors obtained following the previous procedure have to be implemented into a forecasting model in order to obtain the price forecasts. Stock and Watson (2002) describe forecasting models suitable to either dynamic or static factors and to any factor estimation methods. The one-step-ahead forecasting model is specified and estimated as a linear multiple regression model with the factors as predictors. It has the following form:

$$y_{t+1} = \beta \hat{\xi}_t + \alpha(L) y_t + \varepsilon_{t+1} \tag{3.6}$$

where $\hat{\xi}_t$ is the estimation of the factors, β is the regression coefficients matrix and ε_{t+1} is the resulting forecast error. Autoregressive terms are included through the polynomial of non-negative power of the lag operator L with coefficients $\alpha(L)$. The out-of-the-sample forecasts for y_{T+1}, conditional on information up to period T, are given by the conditional expectation:

$$y_{T+1|T} = \hat{\beta} \hat{\xi}_T + \hat{\alpha}(L) y_T$$

3.3.3 Factor Model Results

In this approach, we will focus on modeling the working days. As in the case of the ARIMA model, the original data is transformed and the analysis is perform with the logarithm of the prices. It is not necessary to difference the data. Following Gilbert and Meijer (2005), the number of factors is fixed, based on the eigenvalues of the sample correlation matrix of indicators, on the comparative fix index (CFI) and on the root mean square error of approximation (RMSEA). The CFI is a pseudo R^2 and RMSEA is a non-negative value that measures the lack of fit per degree of freedom, both are based on the χ^2 distribution. Table C.2, Appendix C, exhibits the factor loadings and the main goodness-of-fit measures. In our case the number of significant factors is three, the results of increasing this number up to four factors does not improve the goodness-of-fit measurements enough for justifying this new factor. In this table, it is also included the communality, which is the squared multiple correlation for the variables as dependent, using the factors as predictors. The loading matrix obtained is represented in Figure 3.7 and its relationship with the prices can be derived from the pattern observed in Figure 3.6. The behavior of the prices throughout one

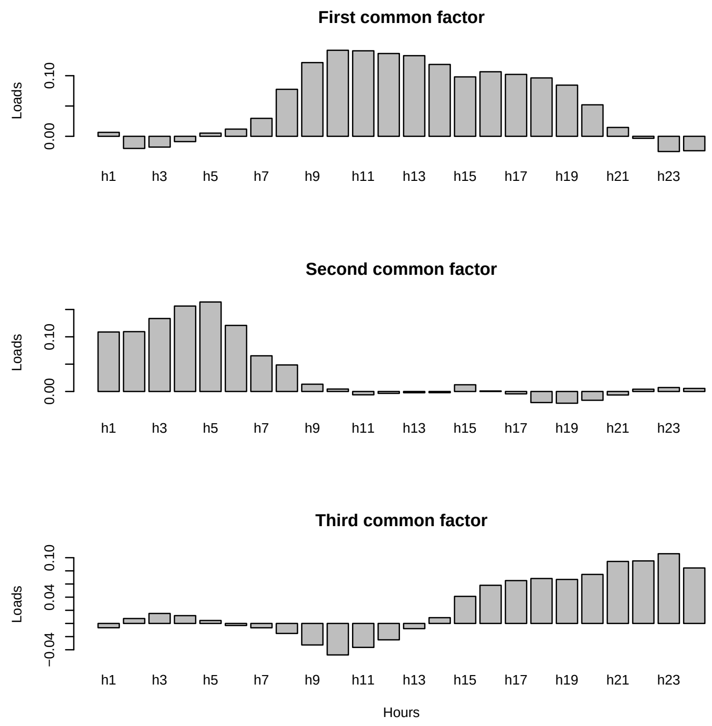

Figure 3.7: Common factors loads.

Hour	1	2	3	4	5	6	7	8	9	10	11	12
R^2	99.1	95.3	97.1	99.8	99.8	97.6	96.0	99.6	99.7	99.8	96.3	98.3
MSE	0.184	0.085	0.089	0.039	0.055	0.047	0.053	0.203	0.259	0.065	0.061	0.029
Hour	13	14	15	16	17	18	19	20	21	22	23	24
R^2	99.9	97.7	99.8	99.9	99.9	97.1	99.7	96.6	94.2	99.7	99.7	95.1
MSE	0.037	0.028	0.096	0.064	0.020	0.080	0.069	0.077	0.165	0.227	0.175	0.048

Table 3.2: Summary of the forecast models for each hour with TSFA model.

day has a particular profile, with hours called *base hours*, in which the price is low and there is lower variance. There are also hours called *peak hours*, in which there are higher prices and high variance. The first factor clearly distinguishes between night and day and it can be observed that the profile of the daily hour loads (between 8 a.m. and 8 p.m.) is similar to the profile of the prices during these hours. The second factor gives positive loads to the base hours and the third to the peak hours.

The forecasting model is based on these factors. The estimation of the 24 regression models is made with a subset of the available data (see Appendix C Table C.3). The coefficient of determination, R^2, and the MSE of the forecast regression model for each hour are shown in Table 3.2. It can be observed that these MSE are equivalent to the ones obtained with the ARIMA model. Depending on the hour, one method or the other performs better; but in general, they are both equivalents. The estimated model is used to forecast the next 5 days, as was done with the ARIMA model. In Figure 3.8, the following quantities are plotted: the real price (red line), the forecast price (black line) and the forecast confidence interval (dashed blue line) for the week May 26^{th} to 30^{th},

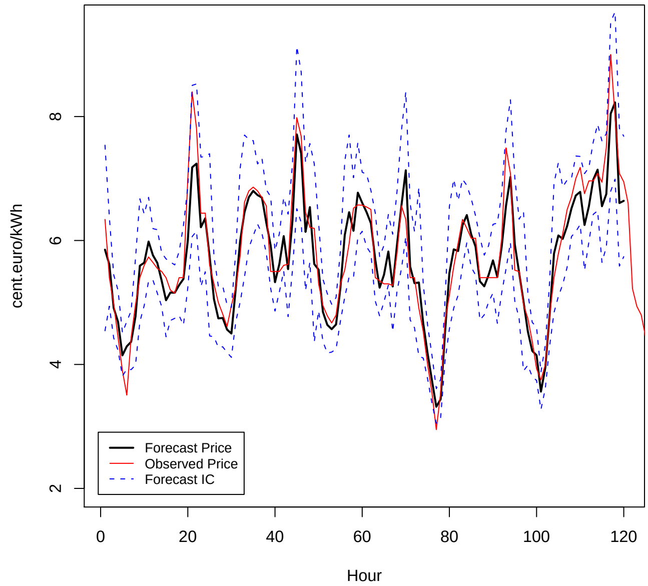

Figure 3.8: One-step-ahead forecast prices with TSFA model.

2008. It can bee observed that, once again, the results are rather similar to the ones obtained with the ARIMA model. In summary, the forecast procedure based on time series factor models gives suitable results. These results are equivalent to the ones obtained through an ARIMA model but the advantage of the procedure presented in this section lies in its simplicity. The forecast model is easiest to implement and to interpret than an ARIMA one. To build an ARIMA model for the electricity prices, a profound knowledge of times series identification is necessary, whereas such profound knowledge is not necessary for using this presented procedure. This advantage facilitates the implementation of the models automatically, so that companies can use it regularly.

This methodology depends on the particular behavior of the prices, which on its part depends on the market rules and on the country where they belong to. In the Appendix D it is developed this analysis with a set of Italian day-ahead market electricity prices with provides satisfactory forecast results.

3.4 Scenario Generation

The stochastic model is based on a representation through scenarios of the random variable involved in the problem (Birge *et al.*, 1994). In our case, the stochastic variable is the day-ahead market clearing price. So, a set of scenarios for the day-ahead market clearing price will be built from the forecasting results. Once this set of scenarios is obtained, it is introduced into the optimization model, where the convergence of the objective function in terms of the number of scenarios is analyzed. It is out of the scope of this thesis to analyze the scenario generation and reduction methods existing in the literature. In this section we will present a brief introduction to these techniques and the algorithms used to obtain the set of scenarios.

The problem of building the scenario tree has been tackled by many authors (see Kaut and Wallace (2003) and Dupacová *et al.* (2000) for a survey) and recently new proposals have been analyzed, most of them focusing on multistage scenario tree generation. These approaches can be classified into different principles (Heitsch and Römisch, 2009): *(i)* bound-based constructions, *(ii)* Monte-Carlo based schemes, *(iii)* Quasi Monte-Carlo based discretization methods, *(iv)* EVPI-based sampling and reduction within decomposition schemes, *(v)* the moment-matching principle and *(vi)* probability metric based approximations (see Heitsch and Römisch (2009) and references within for a list of references dealing with each one).

The greater part of the models presented in this thesis are two-stage stochastic programs. Thus the set of scenarios consists of a set of possible values for the forecast variable $\lambda_t^s = \{\lambda_1^s, \ldots, \lambda_T^s\}$ and its corresponding probability $P^s = P(\lambda^s) \forall s \in S$. Notice that the day-ahead market is cleared once a day; therefore, we have the same information about the first or the last hour of the next day and we have to forecast them all as a vector, which means that, to obtain the forecast, we do not include in the model the hour before but the day before. Thus, for these models we will need a fan to represent the set of possible scenarios for the 24 hours of the day. We will follow the next steps:

1. Sampling: we use sampling methods in order to obtain a set of scenarios. It is known that increasing the number of scenarios, the empirical distribution function will approximate the theoretical one.

2. Reduction: given that the size and computational cost of the stochastic programming models depends on the number of scenarios, some scenario reduction techniques have to be applied in order to reduce the generated set of scenarios into one that is smaller but representative one. We have applied the scenario reduction algorithm explained in Gröwe-Kuska *et al.* (2003), which determines a subset of the initial scenario set and assigns new probabilities to the preserved scenarios.

The final number of scenarios will be fixed by increasing the number of scenarios until a stable optimal value of the objective function is obtained. We will consider that at this point the set of scenarios suitably represents the theoretic random variable.

3.4.1 Set of Scenarios for the ARIMA Model

The sampling step from the ARIMA model described in Section 3.3.1 was done by simulating new observations of a time series with the parameters obtained from the fitted model. There were 300 scenarios simulated, each one corresponding to the set of 24 hours of one day. All the scenarios were considered equiprobable. Thereupon, the reduction algorithm is applied in order to obtain subsets of the simulated set of scenarios and their corresponding probabilities. Figure 3.9 represents some of the sets of scenarios obtained for Monday March $24^t h$, 2008. These scenarios will be applied in Chapter 4.

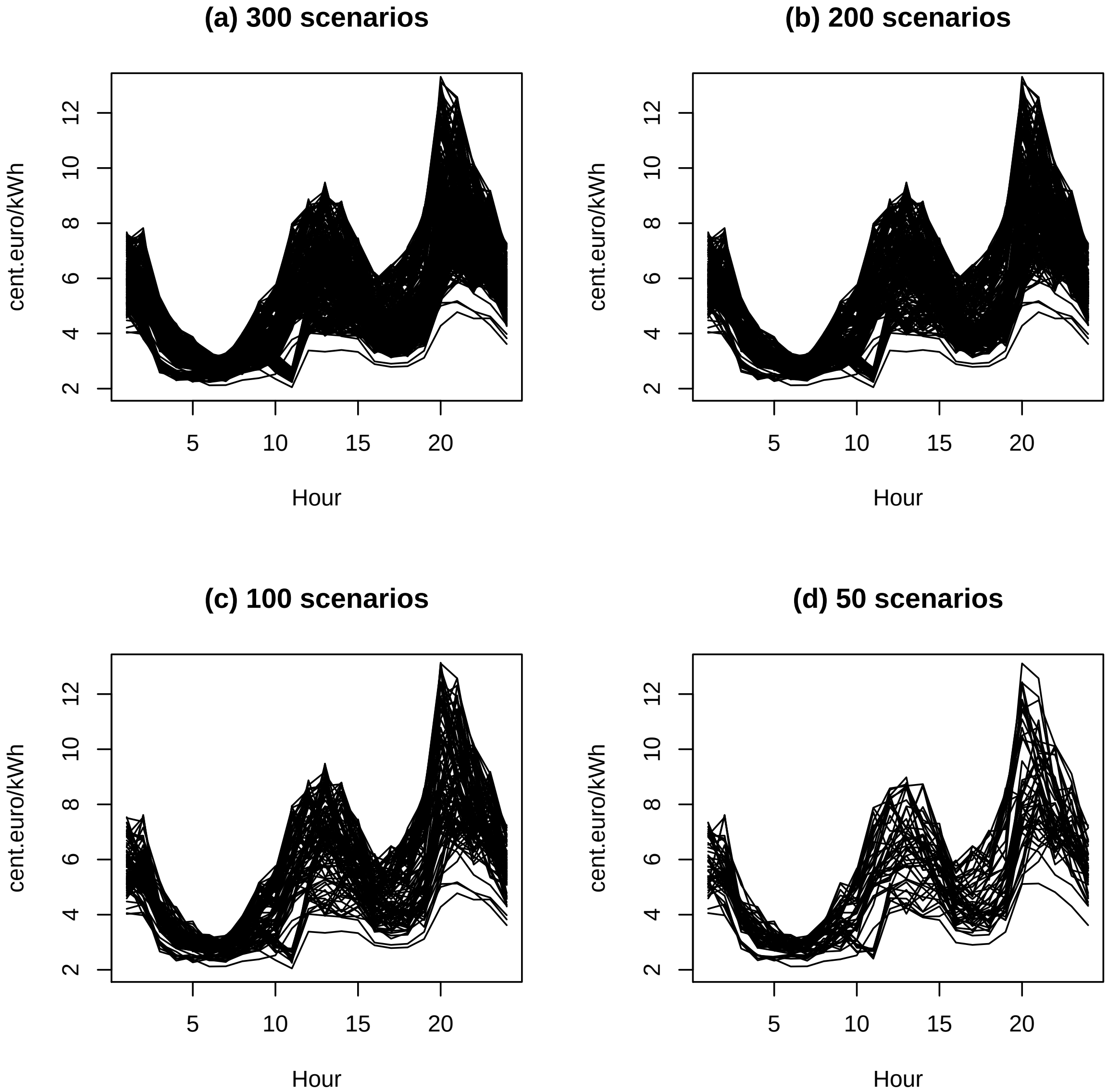

Figure 3.9: Simulated set of scenarios for the ARIMA model.

3.4.2 Set of Scenarios for the TSFA Model

In the case of the TSFA model, creating a set of scenarios is not as immediate as in the ARIMA model. Bootstrap techniques have been applied for estimating uncertainty in order to forecast and build confidence intervals (Alonso *et al.*, 2008). In this case we use this procedure to obtain a set of scenarios for our TSFA model. The bootstrap procedure consists of the following steps:

1. The model (3.3) is estimated by means of maximum likelihood and the estimators of the parameters are obtained.

2. Estimated residuals $\hat{\epsilon}_t = y_t - (\hat{\alpha}_t + \hat{B}\hat{\xi}_t)$ are obtained.

3. A *iid* resample $\tilde{\epsilon}_{e,t}$ from $F_{\hat{\epsilon}_t}$, for e=1, ..., M is obtained, where $F_{\hat{\epsilon}_t}$ is the empirical distribution function.

4. Bootstrap replica of the data $\tilde{y}_t = \hat{\alpha}_t + \hat{B}\hat{\xi}_t + \tilde{\epsilon}_t$ is built.

5. A sample of the future $\tilde{\varepsilon}_{t+h}$ are generated by resampling from $(F_{\hat{\varepsilon}_1}, ..., F_{\hat{\varepsilon}_m})$, respectively, where h =1,..., H is the forecasting horizon.

6. Future bootstrap observations are calculated for vector y_t using (3.6) $\tilde{y}_{t+h} = \hat{\beta}\hat{\xi}_t + \hat{\alpha}(L)\tilde{y}_{t+h-1} + \tilde{\varepsilon}_{t+h}$ where $\tilde{y}_T = \hat{y}_T$.

7. These steps are repeated as many times as the bootstrap replicas are needed.

This bootstrap procedure was tested using the same week that was used for testing the forecasting method. Figure 3.10 shows the real price (red line), the forecasted price (blue line) and the bootstrap replicas. Those replicas are used as scenarios for the optimization models. As in the case of the ARIMA model, all the scenarios are considered equiprobable. Finally, the scenario reduction algorithm (Gröwe-Kuska *et al.*, 2003), is used to build subsets of the initial scenario set and to assign new probabilities to the preserved scenarios. Some of the subsets built are represented in Figure 3.11.

3.5 Conclusions

Regarding the aspects of modeling uncertainty, a factor model procedure has been designed for the Iberian day-ahead market prices. This procedure has been tested in different subsets of prices giving suitable results equivalent to those obtained through a more classical approach using ARIMA models. The main advantage of this proposed procedure is its simplicity. The TSFA based forecast model is easiest to implement and to interpret, but doesn't lose the capacity for obtaining a suitable forecast with goodness-of-fit results that are similar to other more complex methods. Muñoz *et al.* (2010) is based on the work presented in this chapter.

Using this forecasting models, a set of scenarios has been built and reduced using standard techniques in order to include them in the stochastic programming models presented in this thesis.

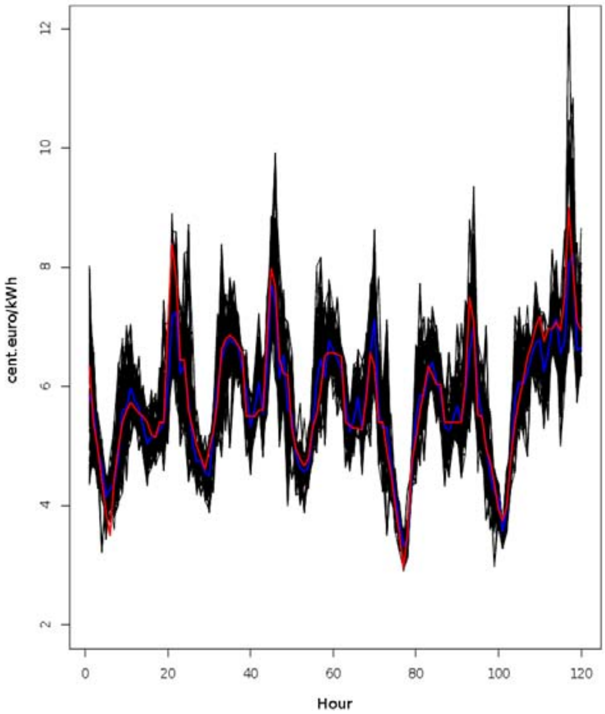

Figure 3.10: Simulated data by means of the bootstrap procedure.

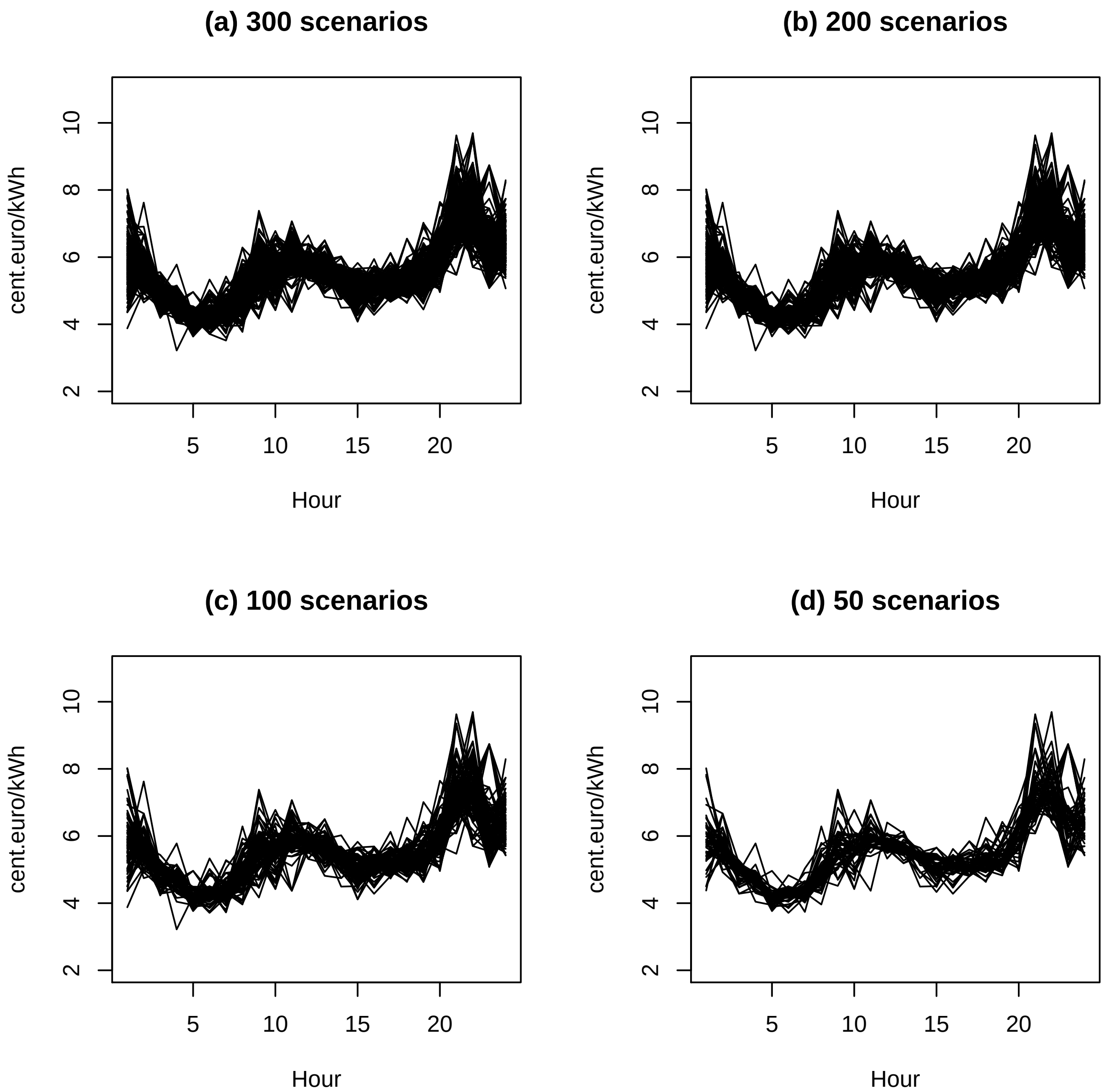

Figure 3.11: Simulated set of scenarios for the TSFA model.

CHAPTER 4

DAMB: Futures Contracts

Introduction

As we have introduced in previous chapters, this thesis focuses on the DAM bidding strategies for a GenCo. The MIBEL regulation describes the coordination between the physical futures contracts portfolio and the day-ahead bidding mechanism of the GenCo. That regulation binds the GenCo to determine its generation scheduling in order to be able to cover those obligations and to determine its optimal offer, taking into account those futures contracts. These market rules have precipitated the need for DAM and DM participation to be studied jointly, because the economic dispatch of these futures markets, along with the units, must change the DAM bid that the GenCo sends to the market operator. Thus, the main objective of this chapter is to build a stochastic programming model which includes coordination between physical futures contracts and DAM bidding, following the MIBEL rules. In other words, we want to see how the inclusion of futures contracts in the model affects the short-term strategies of the GenCo in the day-ahead market. The work in Corchero and Heredia (2009) is based on this chapter.

This chapter is organized as follows. Section 4.1 contains a brief literature review about the management of the futures contracts. In Section 4.2 it is presented the day-ahead and futures income function. Section 4.3 contains the model for the DAMB problem with futures contracts. Finally, Section 4.5 contains the computational tests and Section 4.6 discusses the main contributions of this chapter.

4.1 Literature Review

Some different approaches to the inclusion of futures contracts in the management of a GenCo can be found in the electricity market literature. Most of the works described *forward contracts* as the contracts with physical settlement and *futures contracts* as the contracts with financial settlement. The main theoretical differences between these two kinds of derivatives products is the level of standardization and the kind of market where they are traded (Hull, 2008). We focus on the inclusion of physical derivatives products in the short-term management of a GenCo. Other general considerations about futures contracts can be found in many works, for instance, Hull (2008), Collins (2002), Neuberger (1999) or Carlton (1984).

Prior to deregulation, Kaye *et al.* (1990) illustrate how physical and financial contracts can be used to hedge against the risk of profit volatility allowing for flexible responses to spot price. Once day-ahead and derivatives markets started-up, Bjorgan *et al.* (1999) described in a theoretical framework the integration of futures contracts into the risk management of a GenCo. Also, Chen *et al.* (2004) present a bidding decision-making system for a GenCo, taking into account the impacts of several types of physical and financial contracts; this system is based on a market-oriented unit commitment model, a probabilistic local marginal price simulator, and a multi-criteria decision system. Furthermore, Conejo *et al.* (2008) optimize the forward physical contracts portfolio up to one year, taking into account the day-ahead bidding; the objective of the study is to protect against the pool price volatility trough futures contracts. Moreover, on a medium-term horizon, Guan *et al.* (2008) optimize the generation asset allocation between different derivatives products and the spot market, taking into account short-term operating constraints; it considers that the price of the contracts and forecasts the spot-price is known. From another point of view, Tanlapco *et al.* (2002) make a statistical study of the reduction in risk due to forward contracts; it shows that, for a GenCo, the electricity futures contract is better for hedging price risk than other related futures such as crude oil or gas futures contracts.

As stated above, we are dealing with a new electricity futures contract situation, due to the MIBEL definition of physical futures contracts. Hence, to our knowledge, there are no previous works dealing with the short-term management of the GenCo that include the coordination between day-ahead bidding strategies and physical futures settlement.

4.2 Basic Concepts

Suppose the GenCo has a futures contracts portfolio F for day d as a result of the derivatives market clearing process. Those contracts are defined by price and quantity, $(\lambda_j^F, L_j^F),\ j \in F$. The futures contracts are settled by differences, i.e., each futures contract has daily cash settlements of the difference between the spot price and the futures settlement price. The incomes function of the derivatives markets at interval t is:

$$In_t^F = \sum_{\forall j \in F} (\lambda_j^F - \lambda_t^D) L_j^F$$

where λ_t^D is the clearing day-ahead market price.

The futures contracts included in this study are physical and base load, meaning an agreement to sell some constant quantity of electricity at some price with physical delivery and cash settlement within a specific delivery period.

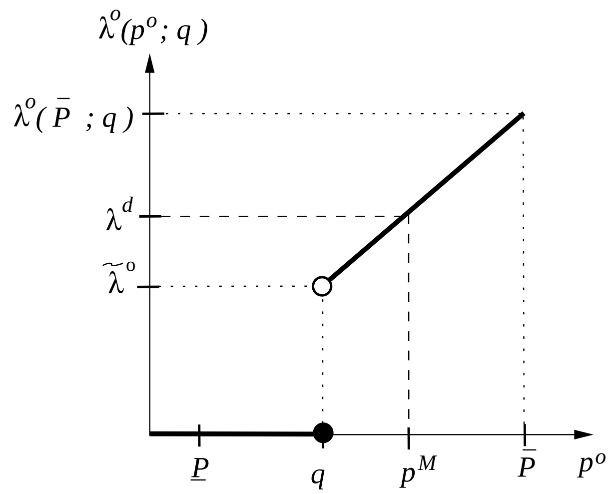

Figure 4.1: Optimal bid curve with physical futures contracts.

As the MIBEL's regulation describes, the energy L_j^F of the physical futures contract j must be allocated through the units of the GenCo that participate in this contract, $i \in I_j$, and delivered to the system through the instrumental price bid of each unit. This situation changes the structure of the optimal bid curve. Let f_{tij} be the generation of thermal i at interval t allocated to the futures physical contracts j, that is:

$$\sum_{i|i\in I_j \cap I_{on_t}} f_{tij} = L_j^F \quad \forall t \in T,\ \forall j \in F$$

where I_{on_t} is the set of thermal units available at interval t.

And let q_{ti} be the instrumental price offer which must not be less than the generation of thermal i at interval t allocated to the set of physical contracts in which it participates, $j \in F_i$:

$$q_{ti} \geq \sum_{j\in F_i} f_{tij}$$

Following the market rules, each generator sends the amount q_{ti} to the day-ahead market through an instrumental price offer. As we will show in subsequent sections, this leads to the following redefinition of the optimal offer curve (2.5) (Figure 4.1):

$$\lambda_{ti}^o(p_{ti}^o; q_{ti}) = \begin{cases} 0 & \text{if } p_{ti}^o \leq q_{ti} \\ 2c_i^q p_{ti}^o + c_i^l & \text{if } q_{ti} < p_{ti}^o \leq \overline{P}_i \end{cases} \tag{4.1}$$

The value of the matched energy depends now on the value of the market clearing price with respect to the threshold price $\tilde{\lambda}_{ti}^o$:

$$\tilde{\lambda}_{ti}^o = 2c_i^q q_{ti} + c_i^l \tag{4.2}$$

For any value $\lambda_t^D \leq \tilde{\lambda}_{ti}^o$, the matched energy is $p_{ti}^M(\lambda_t^D) = q_{ti}$. When $\lambda_t^D > \tilde{\lambda}_{ti}^o$ the matched energy coincides with expression (2.6), that is:

$$p_{ti}^{M}(\lambda_t^D) = \begin{cases} q_{ti} & \text{if } \lambda_t^D \leq \tilde{\lambda}_{ti}^o \\ \rho_{ti}(\lambda_t^D) & \text{if } \lambda_t^D > \tilde{\lambda}_{ti}^o \end{cases} \tag{4.3}$$

where

$$\rho_{ti}(\lambda_t^D) = \begin{cases} \underline{P}_i & \text{if } \theta_{ti}(\lambda_t^D) \leq \underline{P}_i \\ \theta_{ti}(\lambda_t^D) & \text{if } \underline{P}_i \leq \theta_{ti}(\lambda_t^D) \leq \overline{P}_i \\ \overline{P}_i & \text{if } \theta_{ti}(\lambda_t^D) \geq \overline{P}_i \end{cases} \tag{4.4}$$

and $\theta_{ti}(\lambda_t^D) = \left(\lambda_t^D - c_i^l\right)/2c_i^q$, as it is defined in expression (2.7).

Notice that λ_t^D and q_{ti} completely determines the amount of matched energy $p_{ti}^M(\lambda_t^D)$ through expressions (4.2) and (4.3). Using definitions (4.4) and (4.2), this matched generation with futures can be re-expressed as:

$$p_{ti}^{M}(\lambda_t^D) = \begin{cases} q_{ti} & \text{if } q_{ti} \geq \rho_{ti}(\lambda_t^D) \\ \rho_{ti}(\lambda_t^D) & \text{otherwise} \end{cases} \tag{4.5}$$

that sets the value of the matched energy as a non-differentiable function of the instrumental price offer q_{ti}, which will be part of the decision variables of the optimization model.

Therefore, in scenario s, with clearing price $\lambda_t^{D,s}$, the matched energy is given by the expression:

$$p_{ti}^{M,s} = \begin{cases} q_{ti} & \text{if } q_{ti} \geq \rho_{ti}^s \\ \rho_{ti}^s & \text{if otherwise} \end{cases} \tag{4.6}$$

where

$$\rho_{ti}^s = \begin{cases} \underline{P}_i & \text{if } \theta_{ti}^s \leq \underline{P}_i \\ \theta_{ti}^s & \text{if } \underline{P}_i \leq \theta_{ti}^s \leq \overline{P}_i \\ \overline{P}_i & \text{if } \theta_{ti}^s \geq \overline{P}_i \end{cases}$$

and $\theta_{ti}^s = \left(\lambda_t^{D,s} - c_i^l\right)/2c_i^q$.

The incomes function for the day-ahead market with futures contracts for all the committed units at interval t, In_t^{DF}, must take into account both the new expression of the matched energy (4.3) and the revenues coming from the futures portfolio:

$$In_t^{DF} = \sum_{\forall j \in F} \left(\lambda_j^F - \lambda_t^D\right) L_j^F + \sum_{\forall i \in I_{on_t}} \lambda_t^D p_{ti}^M \tag{4.7}$$

4.3 Model Description

4.3.1 Variables

For every time period $t \in T$ and thermal unit $i \in I$, the first stage variables of the stochastic programming problem are:

- The unit commitment binary variables: u_{ti}, a_{ti}, e_{ti}
- The instrumental price offer bid variables: q_{ti}.

- The scheduled energy for futures contract j variables: f_{tij}.

and the second stage variables associated with each scenario $s \in S$ are:

- Matched energy in the day-ahead market: p^s_{ti}

This model was developed with the first formulation of the unit commitment; thus equations (2.10)-(2.12) formulate the inclusion of the variables u_{ti}, a_{ti} and e_{ti} into the model.

4.3.2 FCs Covering Constraints

Let q_{ti} be the first-stage variable standing for the energy of the instrumental price offer, that is, the energy bid by unit i to the t^{th} day-ahead market at 0€/MWh. If variable f_{tij} represents the energy of the j^{th} FC allocated to thermal unit i at period t, then the following constraints must be satisfied:

$$\sum_{i \in I_j} f_{tij} = L^F_j \qquad \forall t \in T,\ \forall j \in F \tag{4.8}$$

$$q_{ti} \geq \sum_{\forall j \in F_i} f_{tij} \qquad \forall i \in I,\ \forall t \in T \tag{4.9}$$

$$f_{tij} \geq 0 \qquad \forall i \in I,\ \forall t \in T,\ \forall j \in F \tag{4.10}$$

$$\underline{P}_i u_{ti} \leq q_{ti} \leq \overline{P}_i u_{ti} \qquad \forall i \in I,\ \forall t \in T \tag{4.11}$$

where the known parameters F_i, I_j and L^F_j are, respectively, the subset of contracts in which unit i participates, the set of thermal units that participate in contract j (the units in all the UCPs that participate in contract j) and the energy that has to be settled for contract j. Constraint (4.8) ensures that the energy of the j^{th} futures contracts L^F_j will be completely dispatched among all the committed units of its associated UCPs. Constraints (4.9) formulate the MIBEL's rule that forces the energy of the futures contracts to be bid through the instrumental price offer; that is to say, the variable q_{ti} represents the quantity of the instrumental price bid and it must be not less than the sum of the energy allocated to FCs. The lower bound $q_{ti} \geq \underline{P}_i u_{ti}$ prevents committed thermal units from being matched below their minimum generation limit while the upper bound $q_{ti} \leq \overline{P}_i u_{ti}$ prevents production levels above the operational limit (4.10).

4.3.3 Matched Energy Constraints

The discussion in Sections 2.3.2 and 4.2 established that the offer curve (4.1) is completely determined by the amount of energy at instrumental price q_{ti}. In scenario s, with clearing price $\lambda^{D,s}_t$, expression (4.6) sets the value of the matched energy of thermal i at time interval t under scenario s, but it does not have to be explicitly introduced in the model because, as we will see in Section 4.4.1, the optimal value of the decision variable p^s_{ti} corresponds to $p^{M,s}_{ti}(\lambda^D_{ti})$.

The following two sets of constraints are the operational ones. They control the production of the unit, i.e., the unit will not produce above or below operational limits, and they define the relationship between both sets of variables.

$$p^s_{ti} \le \overline{P}_i u_{ti} \qquad \forall i \in I,\ \forall t \in T,\ \forall s \in S \tag{4.12}$$

$$p^s_{ti} \ge q_{ti} \qquad \forall i \in I,\ \forall t \in T,\ \forall s \in S \tag{4.13}$$

4.3.4 Objective Function

Let's consider now that the day-ahead market has been cleared, with a market price λ^D_t. For all thermal units i at time interval t, the quadratic generation costs associated with the matched energy (4.5), denoted as p^M_{ti} for the sake of simplicity, is:

$$C_t = \sum_{\forall i \in I} \left(c^b_i u_{ti} + c^{off}_i a_{ti} + c^{on}_i e_{ti} + c^l_i p^M_{ti} + c^q_i (p^M_{ti})^2 \right)$$

and the overall benefit function is:

$$B = \sum_{\forall t \in T} \left(In^{DF}_t - C_t \right) =$$

$$= \sum_{\forall t \in T} \left(\sum_{\forall j \in F} \left(\lambda^F_j - \lambda^D_t \right) L^F_j + \sum_{\forall i \in I} \lambda^D_t p^M_{ti} - \left(c^b_i u_{ti} + c^{off}_i a_{ti} + c^{on}_i e_{ti} + c^l_i p^M_{ti} + c^q_i (p^M_{ti})^2 \right) \right) =$$

$$= \sum_{\forall t \in T} \left(\sum_{\forall j \in F} \left(\lambda^F_j - \lambda^D_t \right) L^F_j - \right.$$

$$\left. - \sum_{\forall i \in I} \left(c^b_i u_{ti} + c^{off}_i a_{ti} + c^{on}_i e_{ti} + (c^l_i - \lambda^D_t) p^M_{ti} + c^q_t (p^M_{ti})^2 \right) \right) \tag{4.14}$$

As it has been shown, λ^D_t and q_{ti} completely determine the amount of matched energy p^M_{ti}. Therefore, expression (4.14) shows the dependency of the benefit function, both on the market clearing price λ^D_t and the instrumental price offer q_{ti} of the committed units. As we have indicated, the market price is modeled through a set of scenarios $\lambda^{D,s} = \{\lambda^{D,s}_1, \ldots, \lambda^{D,s}_T\}$ with probabilities $P^s = P(\lambda^{D,s})\ \forall s \in S$, where S is the number of scenarios. Thus, the expression of the day-ahead and futures benefit function for scenario s, B^s, is:

$$B^s = \sum_{\forall t \in T} \left(\sum_{\forall j \in F} \left(\lambda^F_j - \lambda^{D,s}_t \right) L^F_j - \right.$$

$$\left. - \sum_{\forall i \in I} \left(c^b_i u_{ti} + c^{off}_i a_{ti} + c^{on}_i e_{ti} + (c^l_i - \lambda^D_t) p^{M,s}_{ti} + c^q_t (p^{M,s}_{ti})^2 \right) \right) \tag{4.15}$$

The expected value of the benefit function B can be expressed as:

$$E_{\lambda^D}\left[B(u,a,e,p) \right] =$$

$$= \sum_{\forall t \in T} \sum_{\forall j \in F} \left(\lambda^F_j - \overline{\lambda}^D_t \right) L^F_j - \tag{4.16}$$

$$- \sum_{\forall t \in T} \sum_{\forall i \in I} \left[c_i^b u_{ti} + c_i^{off} a_{ti} + c_i^{on} e_{ti} \right] + \quad (4.17)$$

$$+ \sum_{\forall t \in T} \sum_{\forall i \in I} \sum_{\forall s \in S} P^s \left[\lambda_t^{D,s} p_{ti}^{M,s} - \left(c_i^l p_{ti}^{M,s} + c_i^q (p_{ti}^{M,s})^2 \right) \right] \quad (4.18)$$

where:

(4.16) is a constant term which would be excluded from the optimization, and corresponds to the incomes of the FCs, which are settled by differences. Parameter λ_j^F represents the futures settlement price and $\overline{\lambda}_t^D = \sum_{s \in S} P^s \lambda_t^{D,s}$ is the mean of the day-ahead market price scenarios.

(4.17) is the on/off fixed cost of the unit commitment of the thermal units. This term is deterministic and does not depend on the realization of the random variable λ_t^D.

(4.18) represents the expected value of the benefit from the day-ahead market, where P^s is the probability of scenario s. The first term, $\lambda_t^{D,s} p_{ti}^{M,s}$, computes the incomes from the day-ahead market based on a value p_{ti}^s of the matched energy, while the term between parentheses corresponds to the expression of the quadratic generation costs.

All the functions appearing in equations (4.17) and (4.18) are linear, except for the term (4.18), which is concave quadratic ($c_i^q \geq 0$, see Table A.1).

Terms in (4.16) are constants with respect to the decision variables and therefore the objective function $f(x)$ to be minimized in our model is:

$$f(p, q, u, a, e) = \sum_{\forall i \in I} \sum_{\forall t \in T} \left(c_t^{on} e_{it} + c_t^{off} a_{it} + c_t^b u_{it} + \sum_{s \in S} P^s \left[(c_t^l - \lambda_i^{D,s}) p_{it}^s + c_t^q (p_{ti}^s)^2 \right] \right) \quad (4.19)$$

4.3.5 Day-Ahead Market Bidding with Futures Contracts Problem

The full model developed in the preceding sections, the so-called DAMB with futures contracts problem, can be formulated as:

$$(\text{DAMB-FC}) \begin{cases} \min & f(p, q, f, u, a, e) \\ \text{s.t.} & \\ & \text{Eq. } (4.8) - (4.11) \text{ FC covering} \\ & \text{Eq. } (4.12) - (4.13) \text{ Operational constraints} \\ & \text{Eq. } (2.10) - (2.12) \text{ Unit commitment} \end{cases} \quad (4.20)$$

This formulation corresponds to an optimization problem with mixed continuous and binary decision variables, a convex quadratic objective function and a set of linear constraints. In the next section the properties of the optimal solutions of the (DAMB-FC) problem will be studied.

4.4 Optimal Bid

The preceding model (4.20) is built on Assumption 2.3, which presumes the existence of a bid function λ_{ti}^b with a matched energy function consistent with the optimal solution of the (DAMB-

FC) problem, i.e.:

$$p_{ti}^{M}(\lambda_t^{D,s}) \stackrel{\text{def}}{=} p_{ti}^{M,s} = p_{ti}^{s*} \quad \forall s \in S$$

where $p_{ti}^{M,s} = \max\{q_{ti}, \rho_{ti}^{s}\}$(4.6).

The objective of this section is the development of such a bid function, called the *optimal bid function* $\lambda_{ti}^{b*}(p_{ti}^{b})$. In order to derive this optimal bid function, the properties of the optimal solutions of Problem (4.20) will be studied in the next section and used to derive the expression of the optimal matched energy p_{ti}^{s*} in terms of the instrumental energy bid q_{ti}^{*}.

4.4.1 Optimal Matched Energy

Let $x^{*\prime} = [u^*, a^*, e^*, p^*, q^*, f^*]'$ represent the optimal solution of the (DAMB-FC) problem. Fixing the binary variables to its optimal value u^*, a^* and e^* in the formulation of the (DAMB-FC) problem, we obtain the following convex quadratic continuous problem:

$$\begin{aligned}
&(\text{DAMB-FC}^*):\\
&\underset{p,q,f}{\text{minimize}} \sum_{\forall t \in T} \sum_{\forall i \in I_{on_t}^*} \sum_{s \in S} P^s \left[(c_i^l - \lambda_t^{D,s}) p_{ti}^s + c_i^q (p_{ti}^s)^2 \right]\\
&\text{subject to}\\
&\sum_{i | i \in I_j \cap I_{on_t}^*} f_{tij} = L_j^F && \forall t \in T,\ \forall j \in F\\
&q_{ti} \geq \sum_{j \in F_i} f_{tij} && \forall t \in T,\ \forall i \in I_{on_t}^*\\
&p_{ti}^s \leq \overline{P}_i && \forall t \in T,\ \forall i \in I_{on_t}^*,\ \forall s \in S\\
&p_{ti}^s \geq q_{ti} && \forall t \in T,\ \forall i \in I_{on_t}^*,\ \forall s \in S\\
&q_{ti} \geq \underline{P}_i && \forall t \in T,\ \forall i \in I_{on_t}^*\\
&f_{tij} \geq 0 && \forall t \in T,\ \forall i \in I_{on_t}^*,\ \forall j \in F
\end{aligned}$$

with $I_{on_t}^* := \{i \in I \mid u_{ti}^* = 1\}$, the set of thermal units committed at time t. Obviously, the optimal solution of this continuous problem should coincide with the optimal value of the continuous variables of the original (DAMB-FC) problem, p^*, q^* and f^*. The (DAMB-FC*) problem is separable by intervals, being the problem associated with the t^{th} time interval in standard form:

$$\begin{aligned}
&(\text{DAMB-FC}_t^*):\\
&\underset{p_t,q_t,f_t}{\text{minimize}} \sum_{\forall i \in I_{on_t}^*} \sum_{s \in S} P^s \left[(c_i^l - \lambda_t^{d,s}) p_{ti}^s + c_i^q (p_{ti}^s)^2 \right]\\
&\text{subject to}\\
&\sum_{i | i \in I_j \cap I_{on_t}^*} f_{tij} - L_j = 0 && \forall j \in F && (\pi_{tj}) && (4.21)\\
&\sum_{j \in F_i} f_{tij} - q_{ti} \leq 0 && \forall i \in I_{on_t}^* && (\mu_{ti}^1) && (4.22)\\
&p_{ti}^s - \overline{P}_i \leq 0 && \forall i \in I_{on_t}^*,\ \forall s \in S && (\mu_{ti}^{2,s}) && (4.23)
\end{aligned}$$

$$q_{ti} - p^s_{ti} \leq 0 \qquad \forall i \in I^*_{on_t},\ \forall s \in S \qquad (\mu^{3,s}_{ti}) \tag{4.24}$$
$$\underline{P}_i - q_{ti} \leq 0 \qquad \forall i \in I^*_{on_t} \qquad (\mu^4_{ti}) \tag{4.25}$$
$$- f_{tij} \leq 0 \qquad \forall i \in I^*_{on_t},\ \forall j \in F \qquad (\mu^5_{tij}) \tag{4.26}$$

where π, μ^1, μ^2, μ^3, μ^4 and μ^5 represent the Lagrange multiplier associated with each constraint.

Following Luenberger (2004), the Karush-Kuhn-Tucker conditions of the (DAMB-FC$^*_t$) problem can be expressed as:

$$P^s \left[\left(c^l_i - \lambda^{d,s}_t \right) + 2c^q_i p^{s*}_{ti} \right] + \mu^{2,s}_{ti} - \mu^{3,s}_{ti} = 0 \qquad \forall i \in I^*_{on_t},\ \forall s \in S \tag{4.27}$$
$$- \mu^1_{ti} - \mu^4_{ti} + \sum_{\forall s \in S} \mu^{3,s}_{ti} = 0 \qquad \forall i \in I^*_{on_t} \tag{4.28}$$
$$\mu^1_{ti} + \pi_{tj} - \mu^5_{tij} = 0 \qquad \forall i \in I^*_{on_t},\ \forall j \in F_i \tag{4.29}$$
$$\mu^1_{ti} \left(\sum_{j \in F_i} f^*_{tij} - q^*_{ti} \right) = 0 \qquad \forall i \in I^*_{on_t} \tag{4.30}$$
$$\mu^{2,s}_{ti} \left(p^{s*}_{ti} - \overline{P}_i \right) = 0 \qquad \forall i \in I^*_{on_t},\ \forall s \in S \tag{4.31}$$
$$\mu^4_{ti} \left(\underline{P}_i - q^*_{ti} \right) = 0 \qquad \forall i \in I^*_{on_t} \tag{4.32}$$
$$\mu^{3,s}_{ti} \left(q^*_{ti} - p^{s*}_{ti} \right) = 0 \qquad \forall i \in I^*_{on_t},\ \forall s \in S \tag{4.33}$$
$$\mu^5_{tij} f^*_{tij} = 0 \qquad \forall i \in I^*_{on_t},\ \forall j \in F \tag{4.34}$$
$$\mu^1_{ti},\ \mu^{2,s}_{ti},\ \mu^4_{ti},\ \mu^{3,s}_{ti}, \mu^5_{tij} \geq 0 \qquad \forall i \in I^*_{on_t},\ \forall j \in F,\ \forall s \in S \tag{4.35}$$

The (DAMB-FC$^*_t$) problem is convex ($c^q_i \geq 0$) and thus the system (4.27)-(4.35) represents the necessary and sufficient optimality conditions of the (DAMB-FC$^*_t$) problem and, consequently, of the (DAMB-FC*) problem. Therefore the solution set of the preceding KKT system defines the value of variables p^s_{ti}, q_{ti} and f_{tij} over any optimal solution of the (DAMB-FC) problem associated with $I^*_{on_t}$. The following proposition states this result:

Proposition 4.1. *Let $x^{*\prime} = [u^*, a^*, e^*, p^*, q^*, f^*]'$ be an optimal solution of the (DAMB-FC) problem. Then, for any x^* there exists Lagrange multipliers, μ^1, μ^2, μ^3, μ^4 and μ^5 such that the value of variables p^*, q^* and f^* satisfy the KKT system (4.27)-(4.35). Conversely, for any solution p^*, q^* and f^* of the KKT system (4.27)-(4.35) associated with $I^*_{on_t}$ the correspondent solution x^* is optimal for the (DAMB-FC) problem.*

The fact that any solution of the (DAMB-FC) problem must satisfy the system (4.27)-(4.35) will be exploited in the next two lemmas in order to derive the expressions of the *optimal matched energy* associated with scenario s:

Lemma 4.1 (Optimal matched energy, quadratic costs)**.** *Let x^* be an optimal solution of the (DAMB-FC) problem. Then, for any unit i with quadratic a convex generation cost (i.e. $c^q_i > 0$) committed at period t (i.e. $i \in I^*_{on_t}$), the optimal value of the matched energy p^{s*}_{ti} can be expressed as:*

$$p^{s*}_{ti} = \max\{q^*_{ti}, \rho^s_{ti}\} \tag{4.36}$$

where ρ_{ti}^s *is the constant parameter*

$$\rho_{ti}^s = \begin{cases} \underline{P}_i & \textit{if } \theta_{ti}^s \leq \underline{P}_i \\ \theta_{ti}^s & \textit{if } \underline{P}_i < \theta_{ti}^s < \overline{P}_i \\ \overline{P}_i & \textit{if } \theta_{ti}^s \geq \overline{P}_i \end{cases} \tag{4.37}$$

with

$$\theta_{ti}^s = \left(\lambda_t^{D,s} - c_i^l\right)/2c_i^q \tag{4.38}$$

Proof. As Proposition 4.1 establishes, any optimal solution of the (DAMB-FC) problem must satisfy the KKT system (4.27)-(4.35). As $c_i^q > 0$, equation (4.27) allows variable p_{ti}^{s*} to be expressed as:

$$p_{ti}^{s*} = \frac{\lambda_t^{D,s} - c_i^l}{2c_i^q} + \frac{\mu_{ti}^{3,s} - \mu_{ti}^{2,s}}{2c_i^q P^s} = \theta_{ti}^s + \frac{\mu_{ti}^{3,s} - \mu_{ti}^{2,s}}{2c_i^q P^s} \tag{4.39}$$

Equations (4.23)-(4.26) establish that any optimal solution x^* of the (DAMB-FC) problem must satisfy that

$$\underline{P}_i \leq q_{ti}^* \leq p_{ti}^{s*} \leq \overline{P}_i \tag{4.40}$$

To derive the relationships (4.36), the solution of the KKT system will be analyzed in the five cases among which any optimal solution of the (DAMB-FC) problem could be classified according to (4.40). The rationale of the demonstration is to prove that, in all cases, the expression of the variable p_{ti}^{s*} derived from the KKT system (4.27)-(4.35) coincides with (4.36):

(a) $\underline{P}_i < q_{ti}^* = p_{ti}^{s*} = \overline{P}_i$: This is a trivial case, because, by definition (4.37), $\rho_{ti}^s \leq \overline{P}_i$, and then $p_{ti}^{s*} = \max\{q_{ti}^* = \overline{P}_i, \rho_{ti}^s \leq \overline{P}_i\} = \overline{P}_i$.

(b) $\underline{P}_i \leq q_{ti}^* < p_{ti}^{s*} = \overline{P}_i$: Condition (4.33) gives $\mu_{ti}^{3,s} = 0$. That, together with the non-negativity of the lagrange multipliers $\overline{\mu}_{ti}^s$ and equation (4.39) sets $\overline{P}_i \leq \theta_{ti}^s$ and, by definition (4.37), $\rho_{ti}^s = \overline{P}_i$. Then $p_{ti}^{s*} = \max\{q_{ti}^* < \overline{P}_i, \rho_{ti}^s = \overline{P}_i\} = \overline{P}_i$.

(c) $\underline{P}_i \leq q_{ti}^* < p_{ti}^{s*} < \overline{P}_i$: In this case, conditions (4.31) and (4.33) give $\overline{\mu}_{ti}^s = \mu_{ti}^{3,s} = 0$. That, together with equation (4.39) gives $p_{ti}^{s*} = \theta_{ti}^s$. Then, as it is assumed that $\underline{P}_i < p_{ti}^{s*} < \overline{P}_i$, so is θ_{ti}^s and, by definition (4.37), $\rho_{ti}^s = \theta_{ti}^s = p_{ti}^{s*} > q_{ti}^*$. Therefore $p_{ti}^{s*} = \max\{q_{ti}^*, \rho_{ti}^s = \theta_{ti}^s > q_{ti}^*\} = \rho_{ti}^s$.

(d) $\underline{P}_i < q_{ti}^* = p_{ti}^{s*} < \overline{P}_i$: In this case, condition (4.31) forces $\mu_{ti}^{2,s} = 0$ which, in combination with equation (4.39) and condition $\mu_{ti}^{3,s} \geq 0$ gives $p_{ti}^{s*} \geq \theta_{ti}^s$. As we are assuming that $q_{ti}^* = p_{ti}^{s*}$, then $q_{ti}^* \geq \theta_{ti}^s$ also holds. As $\theta_{ti}^s \leq p_{ti}^{s*} < \overline{P}_i$, definition (4.37) sets a value of ρ_{ti}^s that will be either θ_{ti}^s or $\underline{P}_i$, depending on whether $\theta_{ti}^s > \underline{P}_i$ or $\theta_{ti}^s \leq \underline{P}_i$ respectively. Nevertheless, in both cases $\rho_{ti}^s \leq q_{ti}^*$, and then $p_{ti}^{s*} = \max\{q_{ti}^*, \rho_{ti}^s \leq q_{ti}^*\} = q_{ti}^*$.

(e) $\underline{P}_i = q_{ti}^* = p_{ti}^{s*} < \overline{P}_i$: Condition (4.31) sets $\mu_{ti}^{2,s} = 0$ which, by taking into account equation (4.39) and $\mu_{ti}^{3,s} \geq 0$, provides $p_{ti}^{s*} = \underline{P}_i \geq \theta_{ti}^s$. Then, by definition (4.37), $\rho_{ti}^s = \underline{P}_i$, and $p_{ti}^{s*} = \max\{q_{ti}^* = \underline{P}_i, \rho_{ti}^s = \underline{P}_i\} = \underline{P}_i$. □

Lemma 4.2 (Optimal matched energy, linear costs). *Let x^* be an optimal solution of the (DAMB-FC) problem. Then for any unit i with a linear generation cost (i.e. $c_i^q = 0$) committed at period t (i.e. $i \in I^*_{on_t}$), the optimal value of the matched energy p^{s*}_{ti} can be expressed as:*

$$p^{s*}_{ti} = \begin{cases} q^*_{ti} & \text{if } \lambda_t^{D,s} \le c_i^l \\ \overline{P}_i & \text{if } \lambda_t^{D,s} > c_i^l \end{cases} \tag{4.41}$$

Proof. As Proposition 4.1 sets forth, any optimal solution of the (DAMB-FC) problem must satisfy the KKT system (4.27)-(4.35). When $c_i^q = 0$, equation (4.27) can be expressed as:

$$\mu_{ti}^{3,s} - \mu_{ti}^{2,s} = P^s \left(c_i^l - \lambda_t^{D,s} \right) \tag{4.42}$$

with P^s as the probability of scenario s. There are three possible cases:

(a) $\lambda_t^{D,s} < c_i^l$: In this case, equation (4.42) implies that $\mu_{ti}^{3,s} > \mu_{ti}^{2,s}$, which gives rise to two different situations. In the first one $\mu_{ti}^{3,s} > \mu_{ti}^{2,s} > 0$. That, together with equations (4.31) and (4.33) gives $p^{s*}_{ti} = q^*_{ti} = \overline{P}_i$. In the second one $\mu_{ti}^{3,s} > \mu_{ti}^{2,s} = 0$ and the same KKT conditions force $p^{s*}_{ti} = q^*_{ti} \le \overline{P}_i$.

(b) $\lambda_t^{D,s} > c_i^l$: Now equation (4.42) sets $\mu_{ti}^{2,s} > \mu_{ti}^{3,s}$, which again defines only two possibilities. In the first one the strict inequalities of $\mu_{ti}^{2,s} > \mu_{ti}^{3,s} > 0$ hold and, considering equations (4.31) and (4.33), set $p^{s*}_{ti} = q^*_{ti} = \overline{P}_i$. In the second one, $\mu_{ti}^{2,s} > \mu_{ti}^{3,s} = 0$ which, after equations (4.31) and (4.33), allows the matched energy to be expressed as $p^{s*}_{ti} = \overline{P}_i \ge q^*_{ti}$.

(c) $\lambda_t^{D,s} = c_i^l$: In this case equation (4.42) gives $\mu_{ti}^{3,s} = \mu_{ti}^{2,s}$. Two cases must be analyzed here. In the first one, where $\mu_{ti}^{3,s} = \mu_{ti}^{2,s} = 0$, the KKT system (4.27)-(4.35) doesn't impose any condition on the relation between p^{s*}_{ti}, q^*_{ti} and the bound $\overline{P}_i$. Therefore, for a given optimal solution x^* of the (DAMB-FC) problem, any feasible value of the variables p^{s*}_{ti} and q^*_{ti} are equally optimal, in particular, the value $p^{s*}_{ti} = q^*_{ti}$. The second case to be analyzed is the case where $\mu_{ti}^{3,s}$ and $\mu_{ti}^{2,s}$ are both strictly positives. Then equation (4.33) gives $p^{s*}_{ti} = q^*_{ti}$. □

Lemmas 4.1 and 4.2 establish the expressions of the optimal matched energy variable for any spot price $\lambda_t^{D,s}$ at any optimal solution of the (DAMB-FC) problem. The bid strategies consistent with such a matched energy will be developed in the next section.

4.4.2 Optimal Bid Function

In Section 2.3.3 the concepts of bid and matched energy functions were introduced. The matched energy function associated with a given bid function λ^b_{ti} was defined as

$$p^M_{ti}(\lambda^D_t) \stackrel{\text{def}}{=} \max\{p^b_{ti} \in [0, \overline{P}_i] \mid \lambda^b_{ti}(p^b_{ti}) \le \lambda^D_t\}$$

Assumption 2.3 supposes the existence of a bid function, coherent with the (DAMB-FC) problem, in the sense expressed in the following definition:

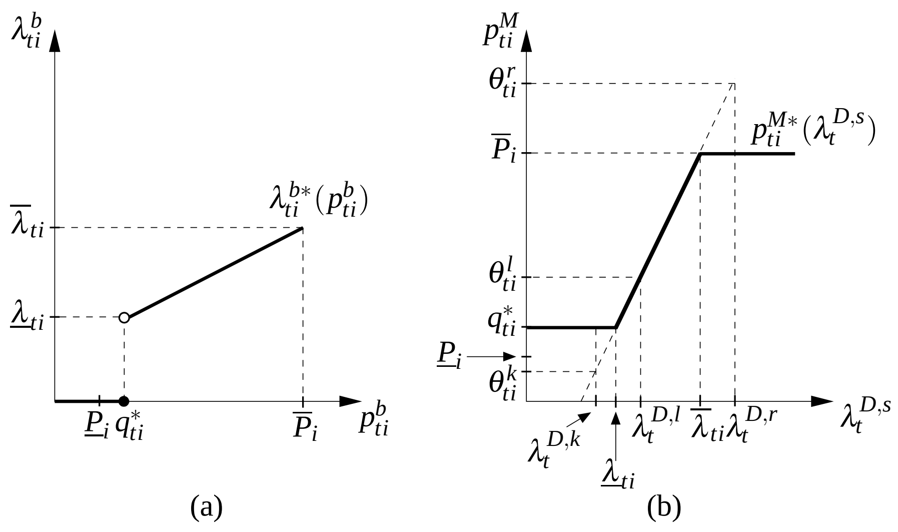

Figure 4.2: (a) Optimal bid function $\lambda_{ti}^{b*}(p_{ti}^b)$ and (b) associated matched energy function $p_{ti}^{M*}(\lambda_t^{D,s})$ for units with quadratic generation costs.

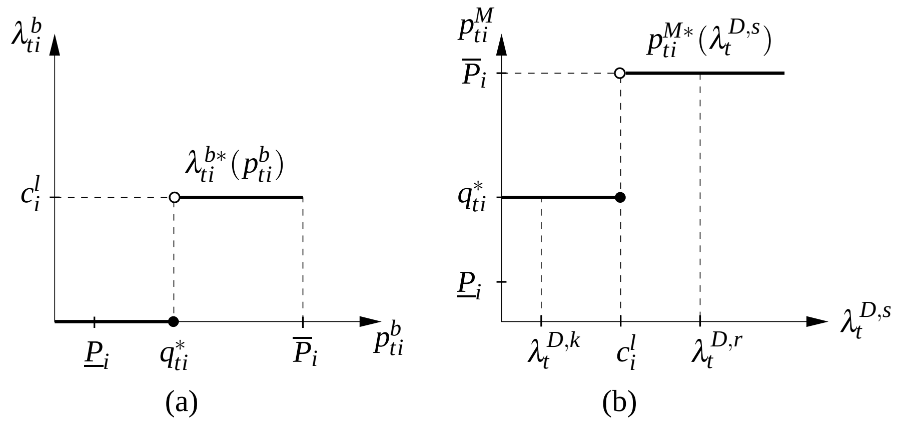

Figure 4.3: (a) Optimal bid function $\lambda_{ti}^{b*}(p_{ti}^b)$ and (b) associated matched energy function $p_{ti}^{M*}(\lambda_t^{D,s})$ for units with linear generation costs.

Definition 4.1 (Bid functions's optimality conditions). *Let $x^{*\prime} = [u^*, a^*, e^*, p^*, q^*, f^*]'$ be an optimal solution of the (DAMB-FC) problem. The bid function λ_{ti}^{b*} of a thermal unit i committed at period t (i.e. $i \in I_{on_t}^*$) is said to be optimal w.r.t. the (DAMB-FC) problem and solution x^* if the value of the matched energy function associated with any scenario's clearing price $\lambda_t^{D,s}$, $p_{ti}^{M,s}$, coincide with the optimal matched energy p_{ti}^{s*} given by expressions (4.36) and (4.41).*

The equivalence $p_{ti}^{M,s} \equiv p_{ti}^{s*}$ assures us that, if a GenCo systematically submits optimal bid functions to the day-ahead market, the expected value of the benefits will be maximized, as long as the actual behavior of the clearing price λ_t^D has been captured by the set of scenarios S. The next theorem develops the expression of the optimal bid function associated with the (DAMB-FC) problem:

Theorem 4.1 (Optimal bid function). *Let $x^{*\prime} = [u^*, a^*, e^*, p^*, q^*, f^*]'$ be an optimal solution of the (DAMB-FC) problem and i any thermal unit committed in period t of the optimal solution (i.e. $i \in I_{on_t}^*$). Then:*

(i) If the generation cost is quadratic convex, the bid function:

$$\lambda_{ti}^{b*}(p_{ti}^b, q_{ti}^*) = \begin{cases} 0 & \text{if } p_{ti}^b \leq q_{ti}^* \\ 2c_i^q p_{ti}^b + c_i^l & \text{if } q_{ti}^* < p_{ti}^b \leq \overline{P}_i \end{cases} \tag{4.43}$$

is optimal w.r.t. the (DAMB-FC) problem and the optimum x^*.

(ii) If the generation cost is linear the bid function:

$$\lambda_{ti}^{b*}(p_{ti}^b, q_{ti}^*) = \begin{cases} 0 & \text{if } p_{ti}^b \leq q_{ti}^* \\ c_i^l & \text{if } q_{ti}^* < p_{ti}^b \leq \overline{P}_i \end{cases} \tag{4.44}$$

is optimal w.r.t. the (DAMB-FC) problem and the optimum x^*.

Proof. We will consider first part (i) of the theorem. To illustrate the proof, the expression (4.43) has been represented graphically in Figure 4.2(a). It can be easily verified that the matched energy function associated with the bid function λ_{ti}^{b*} is (Figure 4.2(b)):

$$p_{ti}^{M*}(\lambda_t^D) = \begin{cases} q_{ti}^* & \text{if } \lambda_t^D \leq \underline{\lambda}_{ti} \\ \theta_{ti}(\lambda_t^D) & \text{if } \underline{\lambda}_{ti} < \lambda_t^D \leq \overline{\lambda}_{ti} \\ \overline{P}_i & \text{if } \lambda_t^D > \overline{\lambda}_{ti} \end{cases} \tag{4.45}$$

where the threshold prices $\underline{\lambda}_{ti}$ and $\overline{\lambda}_{ti}$ are defined as:

$$\underline{\lambda}_{ti} = 2c_i^q q_{ti}^* + c_i^l; \quad \overline{\lambda}_{ti} = 2c_i^q \overline{P}_i + c_i^l$$

and where $\theta_{ti}(\lambda_t^D) = (\lambda_t^D - {}_i^l)/2c_i^q$.

To prove part (i) of the theorem, it is only necessary to demonstrate that $p_{ti}^{M*}(\lambda_t^{D,s}) \stackrel{\text{def}}{=} p_{ti}^{M,s*} \equiv p_{ti}^{s*}$, where p_{ti}^{s*} is the value of the optimal matched energy at scenario s, given by (4.36). First notice that, if $\lambda_t^D = \lambda_t^{D,s}$, the spot price at scenario s, then the matched energy function (4.45) can be rewritten as:

$$p_{ti}^{M,s*} = \begin{cases} q_{ti}^* & \text{if } \lambda_t^{D,s} \leq \underline{\lambda}_{ti} \\ \theta_{ti}^s & \text{if } \underline{\lambda}_{ti} < \lambda_t^{D,s} \leq \overline{\lambda}_{ti} \\ \overline{P}_i & \text{if } \lambda_t^{D,s} > \overline{\lambda}_{ti} \end{cases} \tag{4.46}$$

where θ_{ti}^s is the parameter defined in equation (4.38). Now, the equivalence $p_{ti}^{M,s*} \equiv p_{ti}^{s*} = \max\{q_{ti}^*, \rho_{ti}^s\}$ can be easily verified for the three cases of expression (4.46) (please, refer to Figure 4.2(b) for a graphical interpretation of these three cases):

(a) If, for some $k \in S$, $\lambda_t^{D,k} \leq \underline{\lambda}_{ti}$ then $\theta_{ti}^s \leq q_{ti}^*$ and, by definition (4.37), $\rho_{ti}^k = \max\{\theta_{ti}^k, \underline{P}_i\}$, which will always be less than or equal to q_{ti}^*. Then, we can write that $p_{ti}^{M,s*} = q_{ti}^* = \max\{q_{ti}^*, \rho_{ti}^k \leq q_{ti}^*\} = p_{ti}^{k*}$.

(b) If, for some $l \in S$, $\underline{\lambda}_{ti} < \lambda_t^{D,l} \leq \overline{\lambda}_{ti}$ then $q_{ti}^* < \theta_{ti}^l \leq \overline{P}_i$ which, by definition (4.37), gives $\rho_{ti}^l = \theta_{ti}^l$ and $p_{ti}^{M,s*} = \theta_{ti}^l = \max\{q_{ti}^*, \rho_{ti}^l = \theta_{ti}^l > q_{ti}^*\} = p_{ti}^{l*}$.

(c) If, for some $r \in S$, $\lambda_t^{D,r} > \overline{\lambda}_{ti}$ then $\theta_{ti}^r > \overline{P}_i$ which, together with definition (4.37), sets $\rho_{ti}^r = \overline{P}_i$ and: $p_{ti}^{M,s*} = \overline{P}_i = \max\{q_{ti}^*, \rho_{ti}^r = \overline{P}_i > q_{ti}^*\} = p_{ti}^{r*}$.

To demonstrate the equivalence $p_{ti}^{M,s*} \equiv p_{ti}^{s*}$ when $c_i^q = 0$ (part (ii) of the theorem), observe that the optimal matched energy function associated with the optimal bid function λ_{ti}^{bl*} is:

$$p_{ti}^{M*}(\lambda_t^D) = \begin{cases} q_{ti}^* & \text{if} \quad \lambda_t^D \leq c_i^l \\ \overline{P}_i & \text{if} \quad \lambda_t^D > c_i^l \end{cases} \tag{4.47}$$

which is represented in Figure 4.3(b). Expression (4.47) is equivalent to expression (4.41), and then, $p_{ti}^{M,s*} \equiv p_{ti}^{s*}$ $\forall s \in S$. □

Observe that a direct result of Theorem 4.1 is that Assumption 2.3 always holds true for any (DAMB-FC) problem with an optimal solution.

As mentioned before, the (DAMB-FC) problem assures us that, if the optimal bids (4.43)-(4.44) are submitted to the day-ahead market, the expected value of the benefit function B (4.16)-(4.18) will be maximized. There are two important considerations about these optimal bid functions. The first one is that the optimal bid functions (4.43)-(4.44) represent to some extent a generalization of the classical self-commitment problem treated by several authors (Conejo *et al.*, 2002b; Gountis and Bakirtzis, 2004a). Effectively, if the thermal unit i does not contribute to covering futures contracts at period t (i.e., $q_{ti}^* = 0$), then the optimal bid function offers the complete production of the thermal plant p_{ti}^b at its true marginal cost, $2c_i^q p_{ti}^b + c_i^l$ or c_i^l depending on the generation costs functions. Second, the true bid function required by the market's operator in the MIBEL is a stepwise non-decreasing function. The optimal bid function (4.44) satisfies this condition, but (4.43) is not stepwise. This is an approximation commonly adopted in the literature (see Gountis and Bakirtzis, 2004a) and does not represent a serious limitation on the practical interest of the model, because it is always possible to build *a posteriori* a stepwise approximation of the resulting optimal bid (4.43).

4.5 Computational Results

In this section the set of computational tests that have been performed in order to validate the described model and its results are presented. The instances used in the test have 3 futures contracts, 9 thermal units and 24 hourly intervals.

The model has been implemented in AMPL Fourer *et al.* (2003) and solved with CPLEX 12.0 (2008) (with default options) using a SunFire V20Z with 8 Gb of RAM memory and two processor AMD Opteron 252 at 2.46GHz.

4.5.1 Scenario Set

For this model we use the set of scenarios created from the ARIMA model (Section 3.4.1). As we have introduced, in stochastic programming models, the number of scenarios is a critical decision and must be fixed for each model. We will deal with this problem by increasing the number of

$\lvert S\rvert$	c.v.	b.v.	Constraints	CPU(s)	E(benefits)(€)	$\frac{\lVert x^s - x^{150}\rVert}{\lVert x^{150}\rVert}$
10	3360	720	10872	13	13508300	0.3350
20	5760	720	20472	55	10852400	0.2997
30	8160	720	30072	112	10939000	0.2913
40	10560	720	39672	216	10810100	0.1821
50	12960	720	49272	444	11071100	0.1764
75	18960	720	73272	2100	10878600	0.0712
100	24960	720	97272	3319	10892800	0.0712
150	36960	720	145272	4244	10848800	

$|T| = 24;\ |I| = 9$

Table 4.1: Optimization characteristics of the study cases and results for different number of scenarios.

scenarios until the stabilization of the objective function optimal value. The original tree has 300 scenarios that have been reduced to sets of 150, 100, 75, 50, 40, 30, 20 and 10 scenarios. In Table 4.1 the main parameters of each test are summarized: number of scenarios (S), number of continuous variables (c.v.), number of binary variables (b.v.), number of constraints (Constraints), CPU time in seconds (CPU(s)), the value of the expected benefits (E(benefits)(€)), and the difference in the first stage variables value between the reduced set and the one with 150 scenarios, given as a fraction of unit ($\frac{\lVert x^s - x^{150}\rVert}{\lVert x^{150}\rVert}$ where $x^s = [q^*,\ u^*]'\ \forall s \in S$). The value of E(benefits) only considers the benefit from the day-ahead market (terms (4.18) and (4.17)), ignoring the constant FC income (4.16), and corresponds to minus the objective function of the (DABFC) problem. It can be observed how the CPU time increases with the number of scenarios because of the proportional relationship between them and the number of continuous variables (the number of binary variables is independent of the number of scenarios) and constraints. It can also be seen the value of the objective function stabilizes when the number of scenarios grows (Figure 4.4(a)). There is also a convergence to zero of the difference in the optimal value of the first stage decision variables between each reduced set and the largest one (Figure 4.4(b)). Both values converge from approximately 75 scenarios and the computational time is acceptable. Any increase in the number of scenarios from 75 to 100 does not improve the optimal solution accuracy enough to justify the 50% increase in CPU time. Therefore 75 will be the selected number of scenarios for the computational tests.

4.5.2 Case Study

The first computational tests are performed by changing the quantity of energy allocated to physical FCs in order to study its influence on the results. The status of the units before the first interval is fixed as *all open*, allowing them to be closed or remain opened at hour 1; this is done in order to give more freedom to the unit commitment.

The quantity allocated to FCs is confidential and therefore there is no real public data for the units in the study. The set of computational tests presented is based on the percentage of the total energy generation capacity that the GenCo has allocated to FCs, $\%\overline{P} = \sum_{\forall j \in F} L_j^F / \sum_{\forall i \in I} \overline{P}_i$. For this case study, we include the 9 available units distributed in one or more of the 3 UCPs created, each of them corresponding to one FC. In Table 4.2 the main parameters of the computational test are summarized for three different values of $\%\overline{P}$: 5%, 40% and 70%. The computational time for

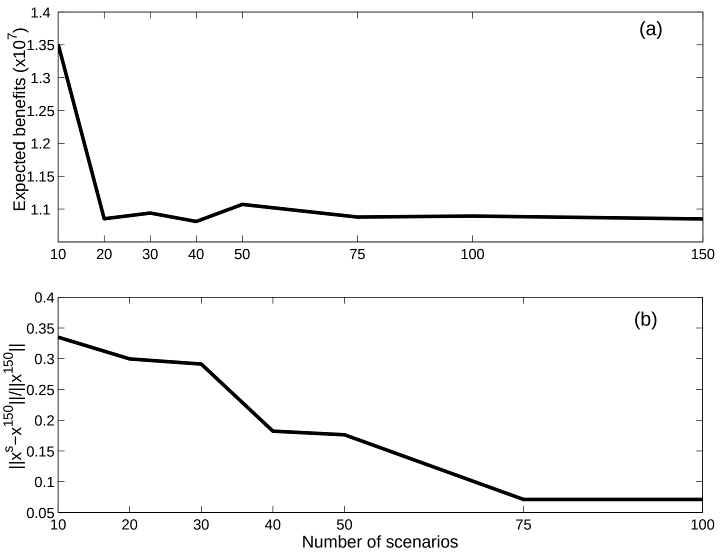

Figure 4.4: (a) Expected benefits for each reduced set of scenarios (b) First stage variables convergence evaluated as $\frac{\|x^s - x^{150}\|}{\|x^{150}\|}$, $x^s = [q^*,\ u^*]'\ \forall s \in S$.

$\%\overline{P}$	E(benefits)
5	23319100 €
40	10878600 €
70	-34164100 €

$|T| = 24$; $|I| = 9$; $|S| = 75$; b.v. $= 720$; c.v. $= 18960$

Table 4.2: Dependency of the DAM benefits with the fraction of the total generation capacity allocated to futures contracts.

the 3 cases is approximately the same but the value of the expected benefits (minus the optimal value of the objective function) differs. Observe that when $\%\overline{P} = 70\%$, the GenCo experiences a loss in the DAM, which should be compensated with the FC incomes (4.16). In Table 4.3 there are the stochastic programming indicators (see Section 7.2) and it is shown the benefits obtained by using stochastic programming instead of a deterministic approach. Figure 4.5 shows the *optimal bid function* for unit 1 at interval 12, $\lambda^{b*}_{12,1}(p^b_{12,1})$ (Section 4.4, equation (4.43)), where different values of $\%\overline{P}$ are considered. In the plot we can see the following represented: an adaptation of the optimal bid function provided by the model to the real bid function that the GenCo has to submit to the MIBEL day-ahead market operator. This bid function is composed of ten pairs *(energy, price)* with increasing prices that can be represented as a stepwise increasing curve, starting at the point defined by the instrumental price offer ($q^*_{12,1}$,0). The following steps are constructed by following

	Objective function
RP	-10878600 €
EEV	-10279300 €
VSS	599300 €

Table 4.3: Stochastic programming indicators-comentar.

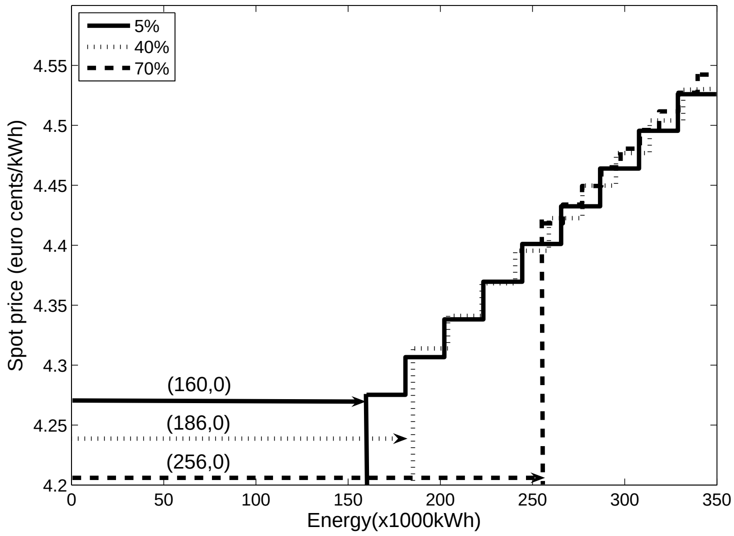

Figure 4.5: Optimal offer for unit 1 at hour 12.

the optimal bid function, in a way that the coordinates of the points are $(p^b_{12,1}, \lambda^{b*}_{12,1}(p^b_{12,1}))$, with the values of the bid energy $p^b_{12,1}$ evenly distributed between $q^*_{12,1}$ and $\overline{P}_{12}$. Notice that for the first case (solid line) the unit has no energy allocated to FCs, so the instrumental offer's energy is the minimum operational limit (160MW) because, as the unit is committed, the matched energy has to be at least this quantity. For the other two cases the energy allocated to FCs is 186MW (dotted line) and 256MW (dashed line). In the following analysis, the percentage of available energy used for physical FCs will be fixed at 40%.

Figure 4.6(a) represents variable f^*_{tij}, the optimal economic dispatch of each FC. This representation shows how the contract is settled among the different units of each UCP. Three kinds of physical FCs have been considered, 200MWh in a weekly contract, 500MWh in a monthly contract and 500MWh in a yearly contract. It can be observed that every unit of a given UCP contributes to the corresponding FC in at least one interval. Notice how in the off-peak hours (lower clearing prices), if possible, each contract is settled by the cheapest unit in the UCP, for example unit 7 in the yearly contract or unit 6 in the monthly contract. Specifically, as unit 7 cannot generate all the energy needed for the yearly contract, unit 3 has to contribute whatever is necessary in order to cover the rest of the contract. For this reason, the weekly contract is not fully covered by unit 3, which is the cheapest one, but by unit 1, since unit 3 is generating for the yearly contract. In the case of the monthly contract, since the maximum power capacity of unit 6 is insufficient, the contract must be covered with the help of the next cheapest unit, unit 4. The results of the peak hours are not as easily interpretable because day-ahead market incomes are greater and its relation with production costs allows all the units to participate both in FCs and day-ahead bidding.

Figure 4.6(b) shows variable q^*_{ti}, the *instrumental price bid*, energy for each unit and interval. The values shown in the ordinate axis are the minimum and maximum power capacity of each unit. This instrumental price bid can be either the quantity allocated to FCs or the minimum operational limit of the unit. Figure 4.6(a) also represents the unit commitment because if the unit is not producing

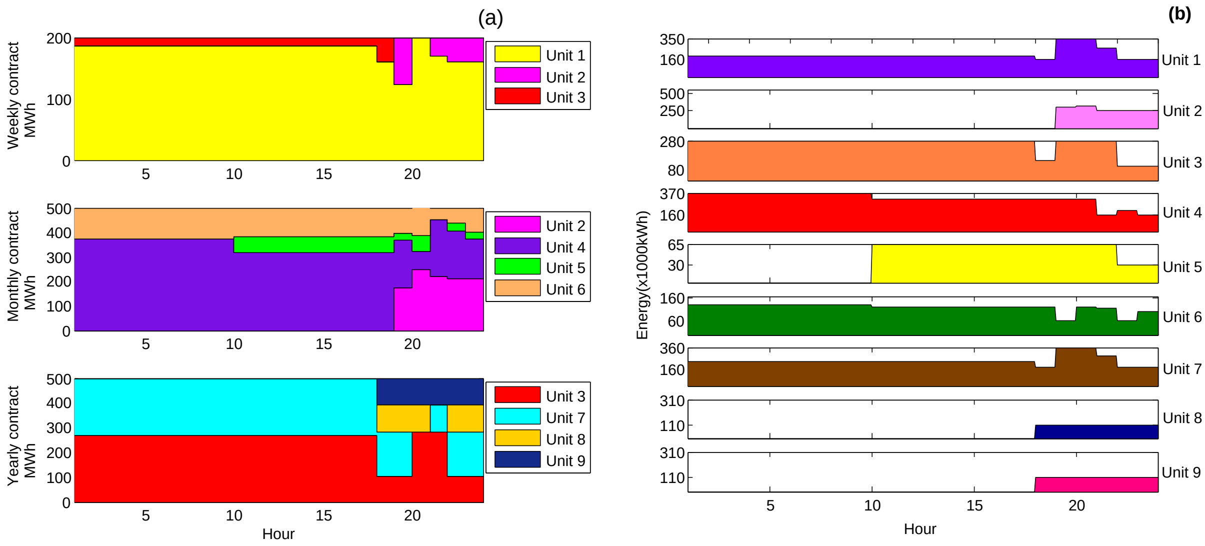

Figure 4.6: (a) Economic dispatch of each futures contracts f^*_{itj}. (b) Optimal instrumental price bid energy q^*_{it} for each unit and interval.

the minimum operational limit, it means the unit is off. We can see that unit 5 starts-up at 10 a.m. and units 2, 8 and 9 start-up after 6 p.m. This behavior is related to the price structure because, in the MIBEL, the highest prices are at noon and in the evening, the peak hours being after 6 p.m.

Figure 4.7 shows the optimal bid curves for each committed thermal unit at hour 12. The numerical values shown in the abscissa axis indicate the minimum and maximum power capacity. The first interval is always the instrumental price bid, which is indicated in parenthesis as *(price, quantity)*. Units 3, 5 and 9 have linear generation costs and its real bid coincides with the optimal bid function λ^{b*}_{ti} expressed in equation (4.44). The rest of the units have quadratic generation costs and the represented function corresponds to the optimal bid functions λ^{b*}_{ti} expressed in equation (4.43) and which are adapted to the real stepwise bid function built as in Figure 4.5. Notice that there are some thermal units that have q^*_{ti} greater than the minimum power capacity, specifically units 3, 4, 6 and 7; this is a direct consequence of the units' participation in the FC being covered.

In summary, these results give the GenCo an optimal bidding strategy which follows the market operator rules. The main difference from other bidding strategies is that the optimal value of q^*_{ti} corresponds directly to the optimal zero price bid, that is to say, the first step of the step-wise bidding curve.

4.6 Conclusions

In this chapter we have developed a mixed-integer stochastic programming model for the short-term thermal optimal bidding problem in the day-ahead market of a price-taker GenCo which also operates in the derivatives physical electricity markets. In order to maximize the benefits arising from the DAM while satisfying thermal operational constraints, the optimal solution of our model determines the unit commitment of the thermal units, the optimal instrumental price bidding

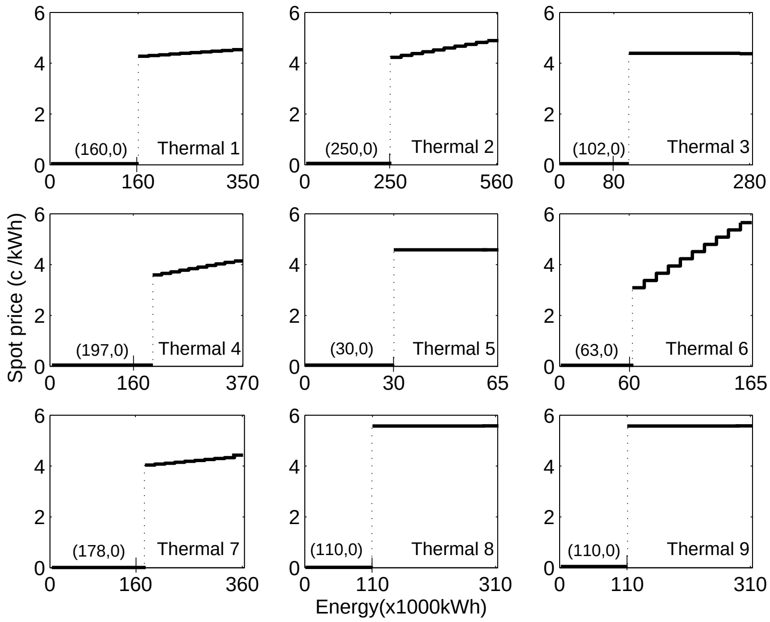

Figure 4.7: Bidding curve for each unit at hour 23.

strategy for the generation company and the economic dispatch of the committed futures contracts for each hour. The model complies with the new regulation of the MIBEL.

It has been shown through Karush-Kuhn-Tucker conditions that the optimal value of the decision variables corresponds to the theoretical optimal bidding curve for a price-taker producer.

The computational tests were performed with real data on the thermal units of a price-taker producer operating in the MIBEL. They have validated the model and they provide suitable results.

CHAPTER 5

DAMB: Futures and Bilateral Contracts

Introduction

Following the idea of building a model that includes the different mechanisms that could affect DAM bidding strategies, we expand the model presented in the previous chapter by including bilateral contracts. Thus, in this chapter we will describe de devolpment of a stochastic mixed-integer quadratic programming model for a price-taker GenCo with bilateral contract obligations as well as a physical futures contract portfolio coming from the derivatives market. The purpose of the model is to find the optimal bidding strategy for the DAM. Although BCs and FCs share some similarities, it will be shown that their mathematical formulation is different.

This chapter considers the classical BCs, i.e. an agreement between the GenCo and a qualified client to settle a given quantity of energy at a given price. The new kind of BCs that the MIBEL offers will be considered in subsequent chapters.

This chapter is organized in a similar way to Chapter 4. Section 5.1 contains a brief literature review on the bilateral contracts management. In Section 5.2 it is presented the corresponding income function and in Section 5.3 it is built the model for the DAMB problem with futures and bilateral contracts. Finally, Section 5.5 contains the computational tests and Section 5.6 discusses the main contributions of this chapter.

5.1 Literature Review

In previous sections we have provided a review of the published works that deal with DAM bidding strategies and a review of the main published works involved with futures contracts (see Sections

2.3.1 and 4.1). Thus, in this section we focus on the works related with bilateral contracts and DAM bidding strategies.

Bilateral contracts is a classic topic that has been tackled from very different points of view and there are numerous of works that analyze their characteristics, their definition and the behavior that a GenCo must have in front of them. For example, Dahlgren *et al.* (2003) provides a state of the art on the analysis of different risk-hedging mechanisms, among them BCs.

Usually, the works that study the implication of BCs into the bidding strategies focus on the medium-term management of the BC pool and not on the economic dispatch of these BCs once they are committed. For instance, Chen *et al.* (2004), analyze specifically the impact of physical and financial contracts on the bidding strategies of a GenCo. They demonstrate that the GenCo optimal bidding strategy will be affected differently, depending on which medium-term product is considered. With another point of view, Guan *et al.* (2008) present a model that joins the medium-term product allocation to the DAM bidding process. It also considers classical operation constraints as well as price risk. The main difference from our approach is that they consider the possibility of bidding into the medium-term products. One important conclusion is that there is a capacity for hedging risk through futures and bilateral contracts portfolios. Finally, Mo *et al.* (2001) present a dynamic stochastic optimization model that permits a dynamic change of the medium-term portfolio while optimizing physical generation. With a focus on the inclusion of the physical management of BCs, Shrestha *et al.* (2004) present a stochastic unit commitment problem with BC and solve it by maximizing the day-ahead market benefit. The uncertainty of the prices is introduced also through a set of scenarios, giving rise to a two-stage stochastic programming problem.

As we have pointed out, the greater part of the published works deal, not with the short-term problem of dispatching these committed BCs, but with the medium-term problem of optimizing the BC portfolio by taking into account the physical capacity of the GenCo.

5.2 Basic Concepts

Suppose that now the GenCo has, along with the futures contracts portfolio F (see Section 4.2), a bilateral contracts portfolio B, each one defined by a pair: energy, L_j^B, and price λ_j^B. Those are classical BCs, so the GenCo has agreed to physically provide an energy amount equal to the sum of all the BCs signed for the day under study. We are optimizing on a short-term horizon and with base load BCs, therefore the quantity L_j^B is constant for the T intervals under study. This assumption can be easily avoided by adding subindex t to the parameter L_j^B, so long as the electricity market has peak and base load contracts. The incomes function of the BCs portfolio is:

$$In_t^B = \sum_{j \in B} \lambda_j^B L_j^B$$

The BCs are not sent to the DAM and they do not provide incomes from this market; their benefits have been charged in advance. Therefore, the matched energy incomes function for the DAM is identical, with or without BCs (4.7).

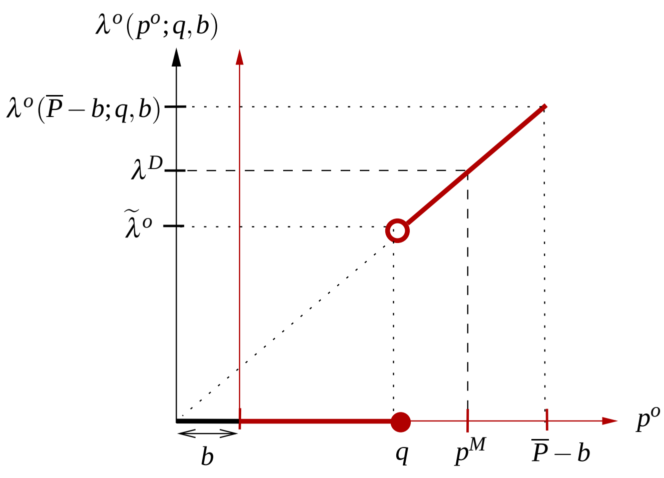

Figure 5.1: Optimal bid curve with physical futures and bilateral contracts.

As we have stated, the MIBEL's regulation indicates that the maximum energy amount that a generation unit can bid to the DAM is the difference between the maximal operational capacity of the unit and the quantity of the total BC energy committed to this unit. Let b_{ti} be the generation of thermal i at interval t allocated to settle the bilateral contracts portfolio:

$$\sum_{i \in I_{on_t}} b_{ti} = \sum_{j \in B} L_j^B \qquad \forall t \in T$$

Summarizing, the energies L_j^F and L_j^B should be integrated into the DAM bid while observing the following rules:

- If unit i contributes with a quantity f_{tij} at period t to the coverage of the FC j, then the energy f_{tij} must be offered to the DAM as an instrumental price bid.
- If unit i contributes with a quantity b_{ti} at period t to the coverage of the BCs, then the energy b_{ti} must be excluded from the DAM bid. Unit i must bid its remaining capacity $(\overline{P_i} - b_{ti})$.

When an amount b_{ti} of the total output of a generation unit is engaged with the settlement of the BC, the MIBEL rules exclude this quantity from the unit's bid. Then, the bid function is shifted by an amount b_{ti} with respect to the same bid function without a BC. This situation is represented in Figure 5.1, where the red x and y axes corresponds to the bid function, which is shifted horizontally by an amount b w.r.t. the black y axis that corresponds to the total output generation of the unit. The energy quantities on the x axis are the ones associated to the bid energy p^o. In this chapter we will show that the analytical expression of the optimal bid function for those generation units that contribute to the BC is:

$$\lambda_{ti}^o(p_{ti}^o; q_{ti}, b_{ti}) = \begin{cases} 0 & \text{if } p_{ti}^o \leq q_{ti} \\ 2c_i^q(p_{ti}^o + b_{ti}) + c_i^l & \text{if } q_{ti} < p_{ti}^o \leq (\overline{P}_i - b_{ti}) \end{cases}$$

Once again, the value of the matched energy depends on the value of the market clearing price with respect to the threshold price $\tilde{\lambda}_{ti}^o$ (4.2), but in this situation we need to define the matched energy as a function of the quantity allocated to BCs, b_{it}. First, we define the quantity $\rho^B_{ti}(\lambda^D_t)$ as:

$$\rho^B_{ti}(\lambda^D_t) = \begin{cases} [\underline{P}_i - b_{ti}]^+ & \text{if } \theta^B_{ti}(\lambda^D_t) \leq [\underline{P}_i - b_{ti}]^+ \\ \theta^B_{ti}(\lambda^D_t) & \text{if } [\underline{P}_i - b_{ti}]^+ \leq \theta^B_{ti}(\lambda^D_t) \leq \overline{P}_i - b_{ti} \\ \overline{P}_i - b_{ti} & \text{if } \theta^B_{ti}(\lambda^D_t) \geq \overline{P}_i - b_{ti} \end{cases} \tag{5.1}$$

where $\theta^B_{ti}(\lambda^D_t) = [(\lambda^D_t - c^l_i)/2c^q_i] - b_{ti}$ and $[\underline{P}_i - b_{ti}]^+ = \max\{0, (\underline{P}_i - b_{ti})\}$. Then, analogously to (4.5), we define the matched generation with futures and bilateral contracts as:

$$p^M_{ti}(\lambda^D_t) = \begin{cases} q_{ti} & \text{if } q_{ti} \geq \rho^B_{ti}(\lambda^D_t) \\ \rho^B_{ti}(\lambda^D_t) & \text{otherwise} \end{cases}$$

In order to be coherent with the stochastic programming model we must define the matched generation with respect to scenario s with clearing price $\lambda^{D,s}_t$:

$$p^{M,s}_{ti} = \begin{cases} q_{ti} & \text{if } q_{ti} \geq \rho^{B,s}_{ti} \\ \rho^{B,s}_{ti} & \text{otherwise} \end{cases} \tag{5.2}$$

where,

$$\rho^{B,s}_{ti} = \begin{cases} [\underline{P}_i - b_{ti}]^+ & \text{if } \theta^{B,s}_{ti} \leq [\underline{P}_i - b_{ti}]^+ \\ \theta^{B,s}_{ti} & \text{if } [\underline{P}_i - b_{ti}]^+ \leq \theta^{B,s}_{ti} \leq \overline{P}_i - b_{ti} \\ \overline{P}_i - b_{ti} & \text{if } \theta^{B,s}_{ti} \geq \overline{P}_i - b_{ti} \end{cases}$$

and $\theta^{B,s}_{ti} = [(\lambda^{D,s}_t - c^l_i)/2c^q_i] - b_{ti}$. Again, expression (5.2) does not have to be introduced explicitly in the model because it will be demonstrated that the optimal value of the decision variable p^s_{ti} corresponds to $p^{M,s}_{ti}$ (see Sec. 5.4).

5.3 Model Description

5.3.1 Variables

For every time period $t \in T$ and thermal unit $i \in I$, the first stage variables of the stochastic programming problem are:

- The unit commitment variables: $u_{ti} \in \{0, 1\}$, c^u_{ti}, c^d_{ti}
- The instrumental price offer bid variables: q_{ti}.
- The scheduled energy for futures contract j variables: f_{tij}.
- The scheduled energy for bilateral contract variables: b_{ti}.

and the second stage variables associated to each scenario $s \in S$ are:

- Total generation: g^s_{ti}
- Matched energy in the day-ahead market: p^s_{ti}

Note that including BCs in the model implies a new variable that represents the total generation of the unit, g^s_{ti} in order to correctly evaluate the production costs. It is important not to confuse this variable with the one representing the matched energy, p^s_{ti}, i.e., the energy that will be remunerated at market clearing price.

This model was developed with the second formulation of the unit commitment expressed by equations (2.13)-(2.20), which formulate the relation of the variables u_{ti}, c^u_{ti} and c^d_{ti} to the minimum start-up and shut-down times and initial state of the thermal generation units.

5.3.2 FCs and BCs Covering Constraints

Let b_{ti} be the energy allocated to the BC portfolio. The following constraints assure us that the total committed energy is settled among the available units:

$$\sum_{i\in I} b_{ti} = \sum_{j\in B} L^B_j \qquad \forall t \in T \tag{5.3}$$

$$0 \le b_{ti} \le \overline{P}_i u_{ti} \qquad \forall i \in I,\ \forall t \in T \tag{5.4}$$

where L^B_j is the energy that has to be settled for contract $j \in B$.

Obviously, the sets of constraints (4.8), (4.9) and (4.11), which assure us of the settlement of the futures contracts, must also be included in the model:

$$\sum_{i\in I_j} f_{tij} = L^F_j \qquad \forall t \in T,\ \forall j \in F$$

$$q_{ti} \ge \sum_{\forall j\in F_i} f_{tij} \qquad \forall i \in I,\ \forall t \in T$$

$$\underline{P}_i u_{ti} \le q_{ti} \le \overline{P}_i u_{ti} \qquad \forall i \in I,\ \forall t \in T$$

It must be noted that the formulation for futures and bilateral contracts has some similitude but they are also different in two basic aspects. First, the load coverage constraint (4.8) can be satisfied by any unit of the GenCo in the case of BCs, while each future contract j can only be covered by a pre-defined subset I_j of units. Second, and more important, the energy involved in the BC contracts must be excluded from the day-ahead bid. This mean that constraints (4.9)-(4.10), which state the relation between the energy committed to the FCs and the instrumental price offer, would not appear in the case of BCs. In this case the operational limits and the relation between the first stage variables are defined by the following set of constraints (see Figure 5.1):

$$q_{ti} \ge \underline{P}_i u_{ti} - b_{ti} \qquad \forall i \in I,\ \forall t \in T \tag{5.5}$$

which guarantee that the instrumental price bid plus the energy committed to the BC will be no less than the minimum generation output of the unit. Notice that when $u_{ti} = 0$, these constraints ensure that $q_{ti} \ge 0$.

5.3.3 Matched Energy and Total Generation Constraints

The operational limits and the relationship between the variables representing the unit's generation, following MIBEL's rules, are modeled through the following sets of constraints:

$$p_{ti}^s \leq \overline{P}_i u_{ti} - b_{ti} \qquad \forall i \in I,\ \forall t \in T,\ \forall s \in S \tag{5.6}$$

$$p_{ti}^s \geq q_{ti} \qquad \forall i \in I,\ \forall t \in T,\ \forall s \in S \tag{5.7}$$

$$g_{ti}^s = b_{ti} + p_{ti}^s \qquad \forall i \in I,\ \forall t \in T,\ \forall s \in S \tag{5.8}$$

where (5.6) and (5.7) ensure that the matched energy p_{ti}^s will be between the instrumental price bid q_{ti} and the total available energy not allocated to the BC. Equation (5.8) defines the total generation of the unit.

5.3.4 Objective Function

For all thermal units i at time interval t, the quadratic costs are associated to total generation (5.8). Those costs taking into account the start-up and shut-down costs are:

$$C_{ti} = c_{ti}^u + c_{ti}^d + c_t^b u_{ti} + c_t^l g_{ti} + c_t^q (g_{ti}^2)$$

As we have pointed out, the BCs are not sent to the DAM, so their inclusion in the short-term strategies do not change the DAM incomes (4.7). Therefore, the expression of the overall day-ahead, futures and bilateral benefit function for scenario s (4.15) only changes with respect to the benefit functio with only futures contracts in the terms related with the costs. Thus, the quadratic function that represents the expected benefits of the GenCo after participation in the DAM is:

$$E_{\lambda^D}\left[B(p^M, g, u, c^u, c^d)\right] =$$

$$= \sum_{t \in T} \left[\sum_{j \in F} (\lambda_j^F - \overline{\lambda}_i^D) L_j^F + \sum_{j \in B} \lambda_j^B L_j^B \right] - \tag{5.9}$$

$$- \sum_{t \in T} \sum_{i \in I} [c_{ti}^u + c_{ti}^d + c_i^b u_{ti}] + \tag{5.10}$$

$$+ \sum_{t \in T} \sum_{i \in I} \sum_{s \in S} P^s \left[\lambda_t^{D,s} p_{ti}^{M,s} - (c_i^l g_{ti}^s + c_i^q (g_{ti}^s)^2) \right] \tag{5.11}$$

where, as in the previous model:

(5.9) corresponds to the incomes of the FCs and the BCs and it is a constant term, equivalent to (4.16).

(5.10) is the on/off fixed cost of the unit commitment of the thermal units, deterministic and independent of the realization of the random variable λ_t^D, equivalent to (4.17).

(5.11) represents the expected value of the benefits from the DAM. The first term, $\lambda_t^{D,s} p_{ti}^{M,s}$, computes the incomes from the DAM due to a value $p_{ti}^{M,s}$ of the matched energy, while the term between brackets corresponds to the expression of the quadratic generation costs with respect to the total generation of the unit g^s_{ti}.

Then, the objective function $f(x)$ to be minimized in our model is:

$$f(p,u,c^u,c^d) = \sum_{i\in I}\sum_{t\in T}\left(c^u_{ti} + c^d_{ti} + c^b_i u_{ti} + \sum_{s\in S} P^s\left[c^l_i g^s_{ti} + c^q_i (g^s_{ti})^2 - (\lambda^{D,s}_t p^s_{ti})\right]\right) \tag{5.12}$$

5.3.5 Day-Ahead Bidding with Futures and Bilateral Contracts Problem

The final *day-ahead market bidding with bilateral and futures contracts model* (DAMB-FBC) developed in the previous sections is:

$$\text{(DAMB-FBC)}\begin{cases} \min & f(p,g,u,c^u,c^d) & \\ \text{s.t.} & & \\ & \text{Eq. (4.8) – (4.10)} & \text{FC covering} \\ & \text{Eq. (5.3) – (5.5)} & \text{BC covering} \\ & \text{Eq. (5.6) – (5.8)} & \text{Total generation} \\ & \text{Eq. (2.13) – (2.20)} & \text{Unit commitment} \end{cases} \tag{5.13}$$

This program corresponds to a mixed linearly constrained minimization problem with convex quadratic objective function and with a well defined global optimal solution.

5.4 Optimal Bid

The preceding model (5.13) is built on Assumption 2.3, which presumes the existence of a bid function λ^b_{ti} with a matched energy function that is consistent with the optimal solution of the (DAMB-FBC) problem, i.e.:

$$p^M_{ti}(\lambda^{D,s}_t) \stackrel{\text{def}}{=} p^{M,s}_{ti} = p^{s*}_{ti} \quad \forall s \in S$$

where $p^{M,s}_{ti} = \max\{q_{ti}, \rho^{B,s}_{ti}\}$ (5.2) The objective of this section is to develop such a bid function, called the *optimal bid function* $\lambda^{b*}_{ti}(p^b_{ti})$. In order to derive this optimal bid function, the properties of the optimal solutions of the problem (5.13) will be studied in the next section and used to derive the expression of the optimal matched energy p^{s*}_{ti} in terms of the instrumental energy bid q^*_{ti} and the committed energy b^*_{ti} of the bilateral contracts.

5.4.1 Optimal Matched Energy

Let $x^{*\prime} = [u^*, c^{u*}, c^{d*}, g^*, p^*, q^*, f^*, b^*]'$ represent the optimal solution of the (DAMB-FBC) problem. Fixing the unit commitment variables to its optimal value u^*, c^{u*} and c^{d*} in the formulation of the

(DAMB-FBC) problem, we obtain the following continuous convex quadratic problem:

$$
\begin{aligned}
&(\text{DAMB-FBC}^*):\\
&\min \sum_{\forall t\in T}\sum_{\forall i\in I^*_{on_t}}\sum_{s\in S} P^s\left[(c_i^l-\lambda_t^{D,s})g_{ti}^s + c_i^q(g_{ti}^s)^2\right] \\
&\text{s.t.} \sum_{i|i\in I_j\cap I^*_{on_t}} f_{tij} = L_j^F && \forall t\in T,\ \forall j\in F\\
&q_{ti} \geq \sum_{j\in F_i} f_{tij} && \forall t\in T,\ \forall i\in I^*_{on_t}\\
&\sum_{i|i\in I^*_{on_t}} b_{ti} = \sum_{j\in B} L_j^B && \forall t\in T\\
&g_{ti}^s = b_{ti} + p_{ti}^s && \forall t\in T,\ \forall i\in I^*_{on_t},\ \forall s\in S\\
&p_{ti}^s \leq \overline{P}_i - b_{ti} && \forall t\in T,\ \forall i\in I^*_{on_t},\ \forall s\in S\\
&p_{ti}^s \geq q_{ti} && \forall t\in T,\ \forall i\in I^*_{on_t},\ \forall s\in S\\
&q_{ti} \geq \underline{P}_i - b_{ti} && \forall t\in T,\ \forall i\in I^*_{on_t}\\
&b_{ti} \leq \overline{P}_i && \forall t\in T,\ \forall i\in I^*_{on_t}\\
&b_{ti} \geq 0 && \forall t\in T,\ \forall i\in I^*_{on_t}\\
&f_{tij} \geq 0 && \forall t\in T,\ \forall i\in I^*_{on_t},\ \forall j\in F
\end{aligned}
$$

with $I^*_{on_t} := \{i \in I \mid u^*_{ti} = 1\}$, the set of thermal units committed at time t. As in the case without bilateral contracts, the optimal solution of this continuous problem should coincide with the optimal value of the continuous variables of the original (DAMB-FBC) problem, g^*, p^*, q^*, b^* and f^*. The (DAMB-FBC*) problem is separable by intervals, being the problem associated with the t^{th} time interval in standard form (Luenberger, 2004):

$$
\begin{aligned}
&(\text{DAMB-FBC}_t^*):\\
&\min \sum_{\forall i\in I^*_{on_t}}\sum_{s\in S} P^s\left[(c_i^l-\lambda_t^{D,s})g_{ti}^s + c_i^q(g_{ti}^s)^2\right] \\
&s.t. \sum_{i|i\in I_j\cap I^*_{on_t}} f_{tij} - L_j^F = 0 && \forall j\in F && (\pi_{tj}^1) && (5.14)\\
&\sum_{j\in F_i} f_{tij} - q_{ti} \leq 0 && \forall i\in I^*_{on_t} && (\mu_{ti}^1) && (5.15)\\
&\sum_{i|i\in I^*_{on_t}} b_{ti} - \sum_{j\in B} L_j^B = 0 && && (\pi_t^2) && (5.16)\\
&g_{ti}^s - b_{ti} - p_{ti}^s = 0 && \forall i\in I^*_{on_t},\ \forall s\in S && (\pi_{tj}^{3,s}) && (5.17)\\
&p_{ti}^s - \overline{P}_i + b_{ti} \leq 0 && \forall i\in I^*_{on_t},\ \forall s\in S && (\mu_{ti}^{2,s}) && (5.18)\\
&q_{ti} - p_{ti}^s \leq 0 && \forall i\in I^*_{on_t},\ \forall s\in S && (\mu_{ti}^{3,s}) && (5.19)\\
&\underline{P}_i - b_{ti} - q_{ti} \leq 0 && \forall i\in I^*_{on_t} && (\mu_{ti}^4) && (5.20)\\
&-q_{ti} \leq 0 && \forall i\in I^*_{on_t} && (\mu_{ti}^5) && (5.21)\\
&b_{ti} - \overline{P}_i \leq 0 && \forall i\in I^*_{on_t} && (\mu_{ti}^6) && (5.22)
\end{aligned}
$$

$$-b_{ti} \leq 0 \qquad \forall i \in I^*_{on_t} \qquad (\mu^7_{ti}) \qquad (5.23)$$

$$-f_{tij} \leq 0 \qquad \forall i \in I^*_{on_t},\ \forall j \in F \qquad (\mu^8_{tij}) \qquad (5.24)$$

where π^1, π^2, μ^1, π^3, μ^2, μ^3, μ^4, μ^5, μ^6, μ^7 and μ^8 represent the Lagrange multipliers associated with each constraint. The Karush-Kuhn-Tucker conditions of the (DAMB-FBC$^*_t$) problem can be expressed as:

$$g^{s*}_{ti} = -\left(\frac{\pi^{2,s}_{ti}}{2c^q_i P^s}\right) - \left(\frac{c^l_i}{2c^q_i}\right) \qquad \forall i \in I^*_{on_t},\ \forall s \in S \qquad (5.25)$$

$$\pi^{2,s}_{ti} = \mu^{2,s}_{ti} - \mu^4_{ti} - P^s \lambda^{D,s}_t \qquad \forall i \in I^*_{on_t},\ \forall s \in S \qquad (5.26)$$

$$\mu^{3,s}_{ti} = \mu^1_{ti} + \mu^4_{ti} + \mu^5_{ti} \qquad \forall i \in I^*_{on_t},\ \forall s \in S \qquad (5.27)$$

$$\mu^8_{tij} = \sum_{j \in F | i \in I^*_{on_t}} \pi^1_j + \sum_{i \in I^*_{on_t} | j \in F_i} \mu^1_{ti} \qquad \forall i \in I^*_{on_t},\ \forall j \in F \qquad (5.28)$$

$$\sum_{s \in S} (\mu^{2,s}_{ti} - \pi^{3,s}_{ti}) + |I^*_{on_t}| \pi^2_{ti} + \mu^4_{ti} + \mu^6_{ti} - \mu^7_{ti} = 0 \qquad \forall i \in I^*_{on_t} \qquad (5.29)$$

$$\mu^1_{ti} (\sum_{j \in F_i} f^*_{tij} - q^*_{ti}) = 0 \qquad \forall i \in I^*_{on_t} \qquad (5.30)$$

$$\mu^{2,s}_{ti} (p^{s*}_{ti} + b^*_{ti} - \overline{P}_i) = 0 \qquad \forall i \in I^*_{on_t},\ \forall s \in S \qquad (5.31)$$

$$\mu^{3,s}_{ti} (q^*_{ti} - p^s_{ti}) = 0 \qquad \forall i \in I^*_{on_t},\ \forall s \in S \qquad (5.32)$$

$$\mu^4_{ti} (\underline{P}_i - b^*_{ti} - q^*_{ti}) = 0 \qquad \forall i \in I^*_{on_t} \qquad (5.33)$$

$$\mu^5_{ti} q^*_{ti} = 0 \qquad \forall i \in I^*_{on_t} \qquad (5.34)$$

$$\mu^6_{ti} (b^*_{ti} - \overline{P}_i) = 0 \qquad \forall i \in I^*_{on_t} \qquad (5.35)$$

$$\mu^7_{ti} b^*_{ti} = 0 \qquad \forall i \in I^*_{on_t} \qquad (5.36)$$

$$\mu^8_{tij} f^*_{tij} = 0 \qquad \forall i \in I^*_{on_t},\ \forall j \in F \qquad (5.37)$$

$$\mu^1_{ti}, \mu^4_{ti}, \mu^5_{ti}, \mu^6_{ti}, \mu^7_{ti} \geq 0 \qquad \forall i \in I^*_{on_t} \qquad (5.38)$$

$$\mu^{2,s}_{ti}, \mu^{3,s}_{ti} \geq 0 \qquad \forall i \in I^*_{on_t},\ \forall s \in S \qquad (5.39)$$

$$\mu^8_{tij} \geq 0 \qquad \forall i \in I^*_{on_t},\ \forall j \in F \qquad (5.40)$$

Analogously as in 4.4.1, one proposition an tow lemmas are introduced and demonstrated in order to state the correct definition of the *optimal matched energy.*

Proposition 5.1. *Let $x^{*\prime} = [u^*, c^{u*}, c^{d*}, g^*, p^*, q^*, f^*, b^*]'$ be an optimal solution of the (DAMB-FBC) problem. Then, for any x^* there exist Lagrange multipliers π^1, π^2, π^3, μ^1, μ^2, μ^3, μ^4, μ^5, μ^6, μ^7 and μ^8, such that the value of variables g^*, p^*, q^*, f^* and b^* satisfy the KKT system (5.25)-(5.40). Conversely, for any solution g^*, p^*, q^*, f^* and b^* of the KKT system (5.25)-(5.40) associated with $I^*_{on_t}$, the correspondent solution x^* is optimal for the (DAMB-FC) problem.*

As we have done in the case of the problem with futures contracts, we will exploit the fact that any solution of the (DAMB-FCB) problem must satisfy the system (5.25)-(5.40) to derive the expressions of the *optimal matched energy.*

Lemma 5.1 (Optimal matched energy, quadratic costs)**.** *Let x^* be an optimal solution of the (DAMB-FBC) problem. Then, for any unit i with quadratic convex generation costs (i.e. $c^q_i > 0$)*

committed at period t (i.e. $i \in I^*_{on_t}$), the optimal value of the matched energy p^{s*}_{ti} can be expressed as:

$$p^{s*}_{ti} = \max\{q^*_{ti}, \rho^{B,s}_{ti}\} \tag{5.41}$$

where $\rho^{B,s}_{ti}$ is the constant parameter

$$\rho^{B,s}_{ti} = \begin{cases} [\underline{P}_i - b_{ti}]^+ & \textit{if } \theta^{B,s}_{ti} \leq [\underline{P}_i - b_{ti}]^+ \\ \theta^{B,s}_{ti} & \textit{if } [\underline{P}_i - b_{ti}]^+ < \theta^{B,s}_{ti} < (\overline{P}_i - b_{ti}) \\ (\overline{P}_i - b_{ti}) & \textit{if } \theta^{B,s}_{ti} \geq (\overline{P}_i - b_{ti}) \end{cases} \tag{5.42}$$

with

$$\theta^{B,s}_{ti} = \frac{\left(\lambda^{d,s}_t - c^l_i\right)}{2c^q_i} - b_{ti} \tag{5.43}$$

and

$$[\underline{P}_i - b_{ti}]^+ = \max\{0, \underline{P}_i - b_{ti}\}$$

Proof. As Proposition 5.1 establishes, any optimal solution of the (DAMB-FBC) problem must satisfy the KKT system (5.25)-(5.40). Additionally, equations (5.17)-(5.19) establish that any optimal solution x^* of the (DAMB-FBC) problem must satisfy that:

$$\underline{P}_i - b_{ti} \leq q^*_{ti} \leq p^{s*}_{ti} \leq \overline{P}_i - b_{ti} \tag{5.44}$$

As we want to see that the optimal value of the matched energy p^{s*}_{ti} is equivalent to expression (5.41), we need to distinguish whether $[\underline{P}_i - b_{ti}]^+$ is equal to $(\underline{P}_i - b_{ti})$ or 0, i.e., whether $b_{ti} < \underline{P}_i$ or not. Thus, to derive the relationships (5.41), the solution of the KKT system will be analyzed in the these two situations. For each one, we will analyze five cases among which any optimal solution of the (DAMB-FBC) problem could be classified according to (5.44):

(a) $b_{ti} < \underline{P}_i \Rightarrow [\underline{P}_i - b_{ti}]^+ = (\underline{P}_i - b_{ti})$

(a.1) $(\underline{P}_i - b_{ti}) < q^*_{ti} = p^{s*}_{ti} = (\overline{P}_i - b_{ti})$: This is a trivial case, because, by definition (5.42) $\rho^{B,s}_{ti} \leq (\overline{P}_i - b_{it})$ and, $p^{s*}_{ti} = \max\{q^*_{ti} = (\overline{P}_i - b_{ti}), \rho^{B,s}_{ti} \leq (\overline{P}_i - b_{ti})\} = (\overline{P}_i - b_{ti})$.

(a.2) $(\underline{P}_i - b_{ti}) \leq q^*_{ti} < p^{s*}_{ti} = (\overline{P}_i - b_{ti})$: Condition (5.32) gives $\mu^{3,s}_{ti} = 0$ that, together with (5.27) and the non-negativity of the lagrange multipliers μ gives $\mu^1_{ti} = \mu^4_{ti} = \mu^5_{ti} = 0$ and then (5.26) gives $\pi^{2,s}_{ti} = \mu^{2,s}_{ti} - P^s\lambda^{D,s}_t$. This result, combined with the definition $g^{s*}_{ti} = p^{s*}_{ti} + b^*_{ti}$ and together with (5.25), gives that:

$$p^{s*}_{ti} = \left[\frac{\lambda^{D,s}_t - c^l_i}{2c^q_i} - b^*_{ti}\right] - \frac{\mu^{2,s}_{ti}}{2c^q_i P^s} \leq \theta^{B,s}_{ti}.$$

Then, as it is assumed that $p^{s*}_{ti} = (\overline{P}_i - b_{ti})$ and we have concluded that $\theta^{B,s}_{ti} \geq p^{s*}_{ti}$, by definition (5.42) $\rho^{B,s}_{ti} = (\overline{P}_i - b_{ti})$. So, $p^{s*}_{ti} = \max\{q^*_{ti} < (\overline{P}_i - b_{ti}), \rho B, s_{ti} = (\overline{P}_i - b_{ti})\} = (\overline{P}_i - b_{ti})$.

(a.3) $(\underline{P}_i - b_{ti}) \leq q^*_{ti} < p^{s*}_{ti} < (\overline{P}_i - b_{ti})$: On the one hand, conditions (5.27), (5.32) and the non-negativity of the lagrange multipliers give $\mu^{3,s}_{ti} = \mu^1_{ti} = \mu^4_{ti} = \mu^5_{ti} = 0$. On the other hand, it is assumed that $p_{ti}^{s*} < (\underline{P}_i - b_{ti})$ and thus condition (5.31) gives $\mu_{ti}^{2,s} = 0$. These two results, combined with condition (5.26), give $\pi_{ti}^{2,s} = -P^s\lambda_t^{D,s}$, which together with (5.25) and (5.17) give:

$$p_{ti}^{s*} = \left[\frac{\lambda_t^{D,s} - c_i^l}{2c_i^q} - b_{ti}^*\right] = \theta_{ti}^{B,s}.$$

Then, as it is assumed that $(\underline{P}_i - b_{ti}) < p_{ti}^{s*} < (\overline{P}_i - b_{ti})$, so is $\theta_{ti}^{B,s}$ and, by definition (5.42) $\rho_{ti}^{B,s} = \theta_{ti}^{B,s}$. Therefore $p_{ti}^{s*} = \max\{q_{ti}^* < \theta_{ti}^{B,s} s, \rho_{ti}^{B,s} = \theta_{ti}^{B,s}\} = \theta_{ti}^{B,s}$.

(a.4) $(\underline{P}_i - b_{ti}) < q_{ti}^* = p_{ti}^{s*} < (\overline{P}_i - b_{ti})$: In this case the assumptions, together with (5.26), (5.31) and (5.33), force $\mu_{ti}^{2,s} = \mu_{ti}^4 = 0$ and $\pi_{ti}^{2,s} = -P^s\lambda_t^{D,s}$. Analogous to case (a.3), $p_{ti}^{s*} = \theta_{ti}^{B,s} = \rho_{ti}^{B,s}$ and, as it is assumed that $q_{ti}^* = p_{ti}^{s*}$, then $p_{ti}^{s*} = \max\{q_{ti}^* = \theta_{ti}^{B,s}, \rho_{ti}^{B,s} = \theta_{ti}^{B,s}\} = \theta_{ti}^{B,s}$.

(a.5) $(\underline{P}_i - b_{ti}) = q_{ti}^* = p_{ti}^{s*} < (\overline{P}_i - b_{ti})$: Condition (5.31) sets $\mu_{ti}^{2,s} = 0$ which, by taking into account condition (5.26), provides $\pi_{ti}^{2,s} = -\mu_{ti}^{4,s} - P^s\lambda_t^{D,s}$. This result, combined with the definition $g_{ti}^{s*} = p_{ti}^{s*} + b_{ti}^*$, and together with (5.25), gives that:

$$p_{ti}^{s*} = \left[\left(\frac{\lambda_t^{D,s} - c_i^l}{2c_i^q} - b_{ti}^*\right) + \frac{\mu_{ti}^{4,s}}{2c_i^q P^s}\right] \geq \theta_{ti}^{B,s}.$$

Then, as it is assumed that $p_{ti}^{s*} = (\underline{P}_i - b_{ti})$ and then $\theta_{ti}^{B,s} \leq (\underline{P}_i - b_{ti})$, by definition (5.42) $\rho_{ti}^{B,s} = (\underline{P}_i - b_{ti})$. So, $p_{ti}^{s*} = \max\{q_{ti}^* = (\underline{P}_i - b_{ti}), \rho_{ti}^{B,s} = (\underline{P}_i - b_{ti})\} = (\underline{P}_i - b_{ti})$. □

(b) $b_{ti} \geq \underline{P}_i \Rightarrow [\underline{P}_i - b_{ti}]^+ = 0$

(b.1) $0 < q_{ti}^* = p_{ti}^{s*} = (\overline{P}_i - b_{ti})$: In this case, assumptions $q_{ti} > 0$ and $q_{ti} > (\overline{P}_i - b_{ti})$ together with conditions (5.33) and (5.34) force $\mu_{ti}^4 = \mu_{ti}^5 = 0$. Then, (5.26) gives $\pi_{ti}^{2,s} = \mu_{ti}^{2,s} - P^s\lambda_t^{D,s}$ that, analogously to case (a.2) gives $\theta_{ti}^{B,s} \geq p^{s*}$, and then $\theta_{ti}^{B,s} \geq (\overline{P}_i - b_{ti})$. Therefore, by definition (5.42) $\rho_{ti}^{B,s} = (\overline{P}_i - b_{ti})$ and $p^{s*} = \max\{q_{ti}^* = (\overline{P}_i - b_{ti}), \rho_{ti}^{B,s} = (\overline{P}_i - b_{ti})\} = (\overline{P}_i - b_{ti})$.

(b.2) $0 \leq q_{ti}^* < p_{ti}^{s*} = (\overline{P}_i - b_{ti})$: This case is equivalent to (a.2) because the key is the assumption $q_{ti}^* < p_{ti}^{s*} = (\overline{P}_i - b_{ti})$. Consequently, $p_{ti}^{s*} = \max\{q_{ti}^* < (\overline{P}_i - b_{ti}), \rho_{ti}^{B,s} = (\overline{P}_i - b_{ti})\} = (\overline{P}_i - b_{ti})$.

(b.3) $0 \leq q_{ti}^* < p_{ti}^{s*} < (\overline{P}_i - b_{ti})$: The reasoning for this case is equivalent to (a.3) until the result $p_{ti}^{s*} = \theta_{ti}^{B,s}$. Then as it is assumed that $0 < p_{ti}^{s*} < (\overline{P}_i - b_{ti})$, so is $0 < \theta_{ti}^{B,s} < (\overline{P}_i - b_{ti})$ and then, by definition (5.42) $\rho_{ti}^{B,s} = \theta_{ti}^{B,s}$. Therefore, $p_{ti}^{s*} = \max\{q_{ti}^* < \theta_{ti}^{B,s}, p_{ti}^{B,s} = \theta_{ti}^{B,s}\} = \theta_{ti}^{B,s}$.

(b.4) $0 < q_{ti}^* = p_{ti}^{s*} < (\overline{P}_i - b_{ti})$: Analogously to (a.4), it is concluded that $p_{ti}^{s*} = \theta_{ti}^{B,s}$ and, as it is assumed that $0 < p_{ti}^{s*} < (\overline{P}_i - b_{ti})$, the situation is analogous to case (b.3) and therefore $p_{ti}^{s*} = \max\{q_{ti}^* < \theta_{ti}^{B,s}, \rho_{ti}^{B,s} = \theta_{ti}^{B,s}\} = \theta_{ti}^{B,s}$.

(b.5) $0 = q_{ti}^* = p_{ti}^{s*} < (\overline{P}_i - b_{ti})$: In this case, the assumption $p_{ti}^{s*} < (\overline{P}_i - b_{ti})$, together with condition (5.31), gives that $\mu_{ti}^{2,s} = 0$ and then condition (5.26) gives $\pi_{ti}^{2,s} = -\mu_{ti}^{4,s} - P^s\lambda_t^{D,s}$. Following the same reasoning as in (a.5), this result, combined with the definition $g_{ti}^{s*} = p_{ti}^{s*} + b_{ti}^*$ and expression (5.25), gives that $p_{ti}^{s*} \geq \theta_{ti}^s$. Then $\theta_{ti}^{B,s} \leq 0$ and, by definition (5.42) $\rho_{ti}^{B,s} = 0$. Therefore, $p_{ti}^{s*} = \max\{q_{ti} = 0, \rho_{ti}^{B,s} = 0\} = 0$. □

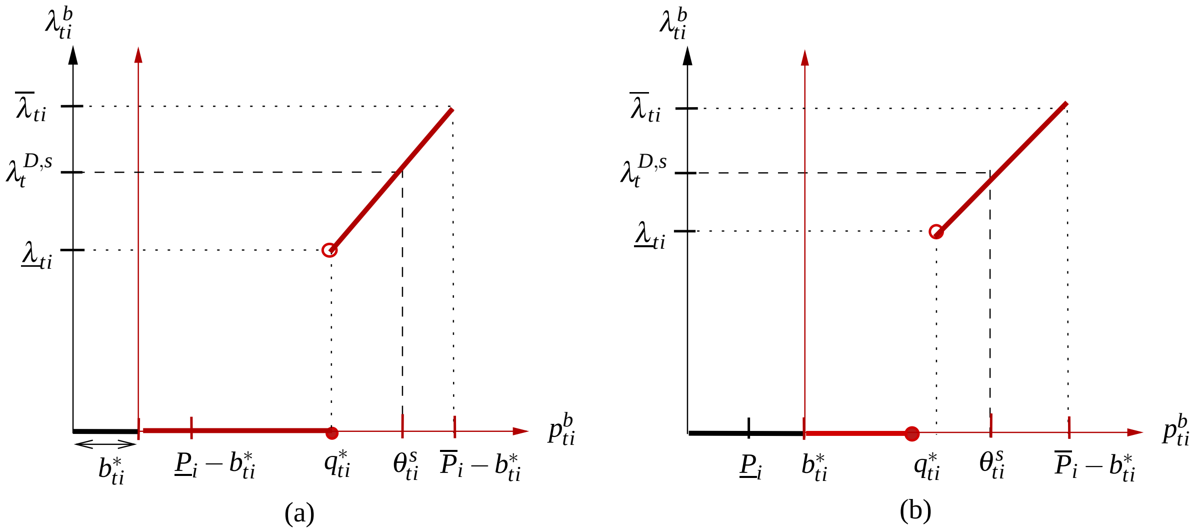

Figure 5.2: Optimal bid function $\lambda^{b*}_{ti}(p^b_{ti})$ when (a) $b^*_{ti} < P_i$ (b) $b^*_{ti} > P_i$.

Lemma 5.2 (Optimal matched energy, linear costs)**.** *Let x^* be an optimal solution of the (DAMB-FBC) problem. Then, for any unit i with linear generation costs (i.e. $c^q_i = 0$) committed at period t (i.e. $i \in I^*_{on_t}$), the optimal value of the matched energy p^{s*}_{ti} can be expressed as:*

$$p^{s*}_{ti} = \begin{cases} q^*_{ti} & \text{if } \lambda^{D,s}_t \le c^l_i \\ \overline{P}_i - b^*_{ti} & \text{if } \lambda^{D,s}_t > c^l_i \end{cases} \tag{5.45}$$

As in Lemma 5.1, any optimal solution of the (DAMB-FBC) problem must satisfy the KKT system (5.25)-(5.40). The demonstration is then analogous to the one presented for Lemma 4.2.

5.4.2 Optimal Bid Function

As it was described in Chapter 4 for the model (DAMB-FC), Assumption 2.3 supposes the existence of a bid function that in this case must be coherent with the model (DAMB-FBC) as it is expressed in the following definition, equivalent to Definition 4.1:

Definition 5.1 (Bid functions's optimality conditions)**.** *Let $x^{*\prime} = [u^*, c^{u*}, c^{d*}, g^*, p^*, q^*, f^*, b^*]'$ be an optimal solution of the (DAMB-FBC) problem. The bid function λ^{b*}_{ti} of a thermal unit i committed at period t (i.e. $i \in I^*_{on_t}$) is said to be optimal w.r.t. the (DAMB-FBC) problem and solution x^*, if the value of the matched energy function associated with any scenario's clearing price $\lambda^{D,s}_t$, $p^{M,s*}_{ti}$, coincides with the optimal matched energy p^{s*}_{ti} given by expressions (5.41) and (5.45).*

Once again, the equivalence $p^{M,s*}_{ti} \equiv p^{s*}_{ti}$ ensures that if a GenCo submits optimal bid functions, the expected value of the benefits will be maximized. The next theorem develops the expression of the optimal bid function associated with the (DAMB-FBC) problem.

Theorem 5.1 (Optimal bid function)**.** *Let $x^{*\prime} = [u^*, c^{u*}, c^{d*}, g^*, p^*, q^*, f^*, b^*]'$ be an optimal solution of the (DAMB-FBC) problem and i any thermal unit committed in period t of the optimal*

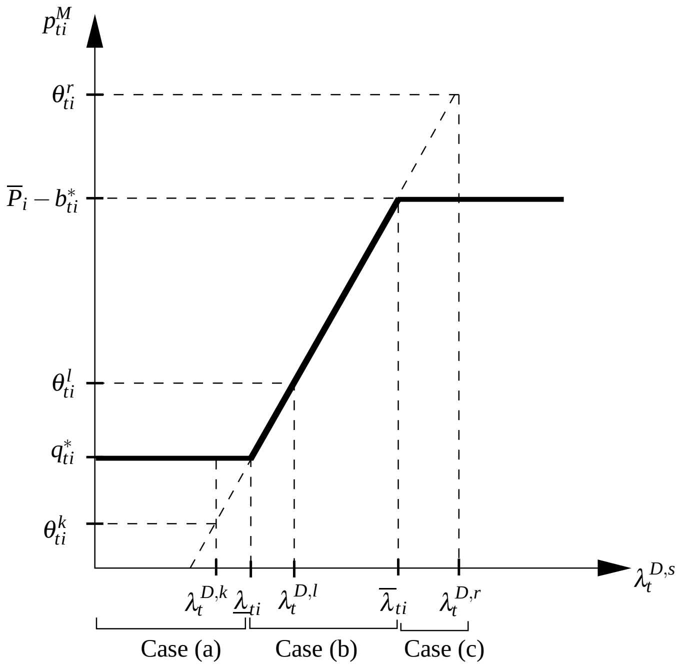

Figure 5.3: Associated matched energy function $p^{M,s*}_{ti}$.

solution (i.e. $i \in I^*_{on_t}$). Then, for a unit i with quadratic convex generation costs, the bid function:

$$\lambda^{b*}_{ti}(p^b_{ti}, q^*_{ti}, b^*_{ti}) = \begin{cases} 0 & \text{if } p^b_{ti} \leq q^*_{ti} \\ 2c^q_i(p^b_{ti} + b^*_{ti}) + c^l_i & \text{if } q^*_{ti} < p^b_{ti} \leq (\overline{P}_i - b^*_{ti}) \end{cases} \tag{5.46}$$

is optimal w.r.t. the (DAMB-FBC) problem and the optimum x^*.

Proof. First, we consider the case where $c^q_i > 0$. To illustrate this proof, the expression (5.46) has been represented graphically in Figure 5.2 for two cases: the first one, when $b^*_{ti} < \underline{P}_i$ (Figure 5.2(a)) and therefore $q^*_{ti} \geq \underline{P}_i - b^*_{ti}$ and the second one, when $b^*_{ti} > \underline{P}_i$ (Figure 5.2(b)) and therefore $q^*_{ti} \geq 0$. It is easy to see that the matched energy function associated with the bid function λ^{b*}_{ti} at scenario s (i.e $\lambda^D_t = \lambda^{D,s}_t$) is for both cases (Figure 5.3):

$$p^{M,s*}_{ti}(\lambda^{D,s}_t) = \begin{cases} q^*_{ti} & \text{if } \lambda^{D,s}_t \leq \underline{\lambda}_{ti} \\ \theta^{B,s}_{ti} & \text{if } \underline{\lambda}_{ti} < \lambda^{D,s}_t \leq \overline{\lambda}_{ti} \\ \overline{P}_i - b^*_{ti} & \text{if } \lambda^{D,s}_t > \overline{\lambda}_{ti} \end{cases} \tag{5.47}$$

where the threshold prices $\underline{\lambda}_{ti}$ and $\overline{\lambda}_{ti}$ are defined as:

$$\underline{\lambda}_{ti} = 2c^q_i(q^*_{ti} + b^*_{ti}) + c^l_i \quad ; \quad \overline{\lambda}_{ti} = 2c^q_i\overline{P}_i + c^l_i \tag{5.48}$$

and $\theta^{B,s}_{ti}$ is the parameter defined in equation (5.43). Thus, to demonstrate the optimality if bid function (5.47), it is sufficient to prove that $p^{M,s*}_{ti} \equiv p^{s*}_{ti} = \max\{q^*_{ti}, \rho^{B,s}_{ti}\}$. We verify this equivalence for the three cases of expression (5.47) (Figure (5.3):

(a) If, for some $k \in S$, $\lambda_t^{D,k} \leq \underline{\lambda}_{ti}$ then $\theta_{ti}^{B,k} \leq q_{ti}^* \leq \overline{P}_i - b_{ti}$ and, by definition (5.42), $\rho_{ti}^{B,k} = \max\{\theta_{ti}^{B,k}, [\underline{P}_i - b_{ti}^*]^+\}$, which will always be less than or equal to q_{ti}^*. Then, we can write that $p_{ti}^{k*} = \max\{q_{ti}^*, \rho_{ti}^{B,k}\} = q_{ti}^*$ and, as expression 5.47 gives $p_{ti}^{M,k*} = q_{ti}^*$, we can conclude that $p_{ti}^{M,k*} = p_{ti}^{k*}$.

(b) If, for some $l \in S$, $\underline{\lambda}_{ti} < \lambda_t^{D,l} \leq \overline{\lambda}_{ti}$ then $[\overline{P}_i - b_{ti}]^+ \leq q_{ti}^* < \theta_{ti}^{B,l} \leq (\overline{P}_i - b_{ti}^*)$ and, by definition (5.42) $\rho_{ti}^{B,l} = \theta_{ti}^{B,l}$. Then, $p_{ti}^{l*} = \max\{q_{ti}^*, \rho_{ti}^{b,l} = \theta_{ti}^{B,l} \geq q_{ti}^*\} = \theta_{ti}^{B,l}$. As expression 5.47 gives $p_{ti}^{M,l*} = \theta_{ti}^{B,l}$, we can conclude that $p_{ti}^{M,l*} = p_{ti}^{l*}$.

(c) If, for some $r \in S$, $\lambda_t^{D,r} > \overline{\lambda}_{ti}$ then $\theta_{ti}^{B,r} > (\overline{P}_i - b_{ti}^*)$ which, together with definition (5.42), sets $\rho_{ti}^{B,r} = (\overline{P}_i - b_{ti}^*)$ and thus $p_{ti}^{r*} = \max\{q_{ti}^*, \rho_{ti}^{B,r} = (\overline{P}_i - b_{ti}^*) > q_{ti}^*\} = (\overline{P}_i - b_{ti}^*)$. As expression 5.47 gives $p_{ti}^{M,r*} = (\overline{P}_i - b_{ti}^*)$, we can conclude that $p_{ti}^{M,l*} = p_{ti}^{r*}$.

Now the case with linear generation costs will be considered. If $c_i^q = 0$, the bid function 5.46 reduces to:

$$\lambda_{ti}^{b*}(p_{ti}^b, q_{ti}^*, b_{ti}^*) = \begin{cases} 0 & \text{if } p_{ti}^b \leq q_{ti}^* \\ c_i^l & \text{if } q_{ti}^* < p_{ti}^b \leq (\overline{P}_i - b_{ti}^*) \end{cases}$$

and the optimal matched energy function associated with this optimal bid function is:

$$p_{ti}^{M*}(\lambda_t^D) = \begin{cases} q_{ti}^* & \text{if } \lambda_t^D \leq c_i^l \\ \overline{P}_i - b_{ti}^* & \text{if } \lambda_t^D > \overline{c}_i^l \end{cases} \tag{5.49}$$

it is straightforward to see that this expression (5.49) is equivalent to expression (5.45) and then $p_{ti}^{M,s*} \equiv p_{ti}^{s*}$. □

5.5 Computational Results

In this section the set of computational tests that have been performed in order to validate the described model and its results are presented. The instances used in the test have a pool of bilateral contracts with 300MWh committed for each interval, a set of 3 futures contracts with 700MWh committed, 9 thermal units (see Table A.1) and 24 hourly intervals.

As it has been explained, this model is a mixed-integer quadratic program which is difficult to solve efficiently, especially for large-scale instances. It is possible to approximate the quadratic objective function by means of perspective cuts, so that this problem can be approximated by a mixed-integer linear program (MILP) and the available efficient general-purpose MILP solvers can be applied, leading to good quality solutions in a relatively short amount of time (Frangioni and Gentile, 2006).

The model has been implemented and solved in CPLEX with 12.0 (2008) using the ad-hoc implementation of the perspective cuts algorithm (Mijangos *et al.*, 2010). It has been solved using a SunFire X2200 with 32 Gb of RAM memory and two dual core processors AMD Opteron 2222 at 3 GHz.

$\lvert S\rvert$	c.v.	b.v.	CPU(s)	Objective function	$\frac{\lVert x^s - x^{200}\rVert}{\lVert x^{200}\rVert}$
25	13680	240	168	103371000	1.008
50	25680	240	730	102203000	0.940
75	37680	240	1575	102053000	0.365
100	49680	240	3664	102014000	0.365
125	61680	240	5960	101992000	0.365
150	73680	240	10205	101912000	0.365
175	85680	240	11357	101887000	0.365
200	97680	240	15141	101803000	

$\lvert T\rvert = 24$; $\lvert I\rvert = 9$

Table 5.1: Optimization characteristics of the study cases and results for different number of scenarios.

	Objective function
RP	10205300 €
EEV	10318400 €
VSS	113100 €

Table 5.2: Stochastic Programming Indicators

5.5.1 Scenario Set

For this model we use the set of scenarios created from the TSFA based model (Section 3.4.2). As we have done in Chapter 4, we will increase the number of scenarios until the stabilization of the objective function optimal value. The original tree has 300 scenarios that have been reduced to sets of 200, 175, 150, 100, 75, 50 and 25 scenarios. In Table 5.1 the main parameters of each test are summarized: number of scenarios (S), number of continuous variables (c.v.), number of binary variables (b.v.), CPU time in seconds (CPU(s)), the value of the objective function (€)), and the difference in the first stage variables value between the reduced set and the one with 200 scenarios, given as a fraction of unit ($\frac{\lVert x^s - x^{200}\rVert}{\lVert x^{200}\rVert}$ where $x^s = [q^*,\ u^*]'\ \forall s \in S$). The value of the objective function only considers the benefit from the day-ahead market (terms (5.10) and (5.11)), ignoring the constant FC and BC incomes (5.9). As in Chapter 4, it is observed how the CPU time increases with the number of scenarios and that the value of the objective function stabilizes when the number of scenarios grows (Figure 5.4(a)). There is also a convergence of the difference in the optimal value of the first stage decision variables between each reduced set and the largest one (Figure 5.4(b)). Both values converge from approximately 75 scenarios and the computational time is acceptable. Once again, an increase in the number of scenarios from 75 to 100 does not improve the optimal solution accuracy enough to justify the increase in CPU time. Therefore 75 will be the selected number of scenarios for the computational tests.

5.5.2 Case Study

In the computational tests, the status of the units before the first interval is fixed as *all open*, allowing them to be closed or remain opened at hour 1; this is done in order to give more freedom to the unit commitment.

In Table 5.2 there are the stochastic programming indicators (see Section 7.2) and it is shown the

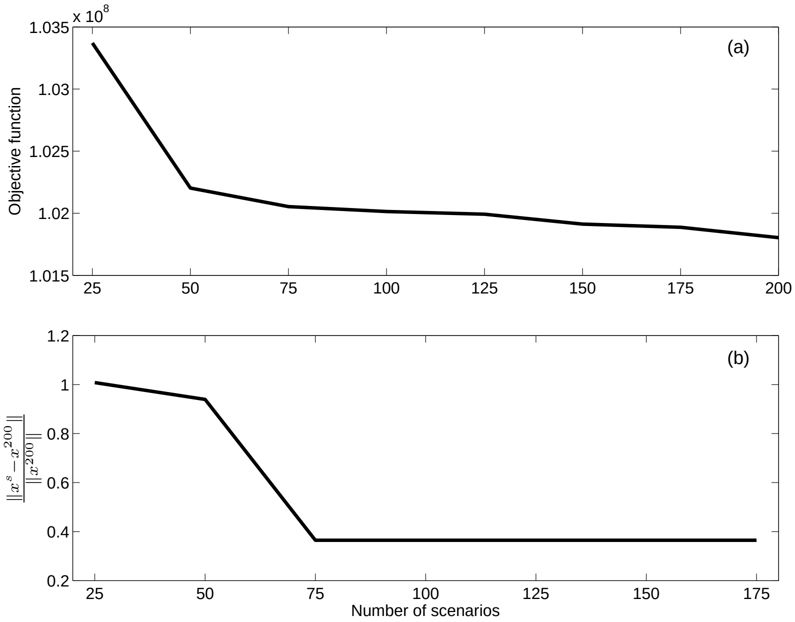

Figure 5.4: (a) Expected benefits for each reduced set of scenarios (b) First stage variables convergence evaluated as $\frac{\|x^s - x^{200}\|}{\|x^{200}\|}$, $x^s = [q^*,\ u^*]'\ \forall s \in S$.

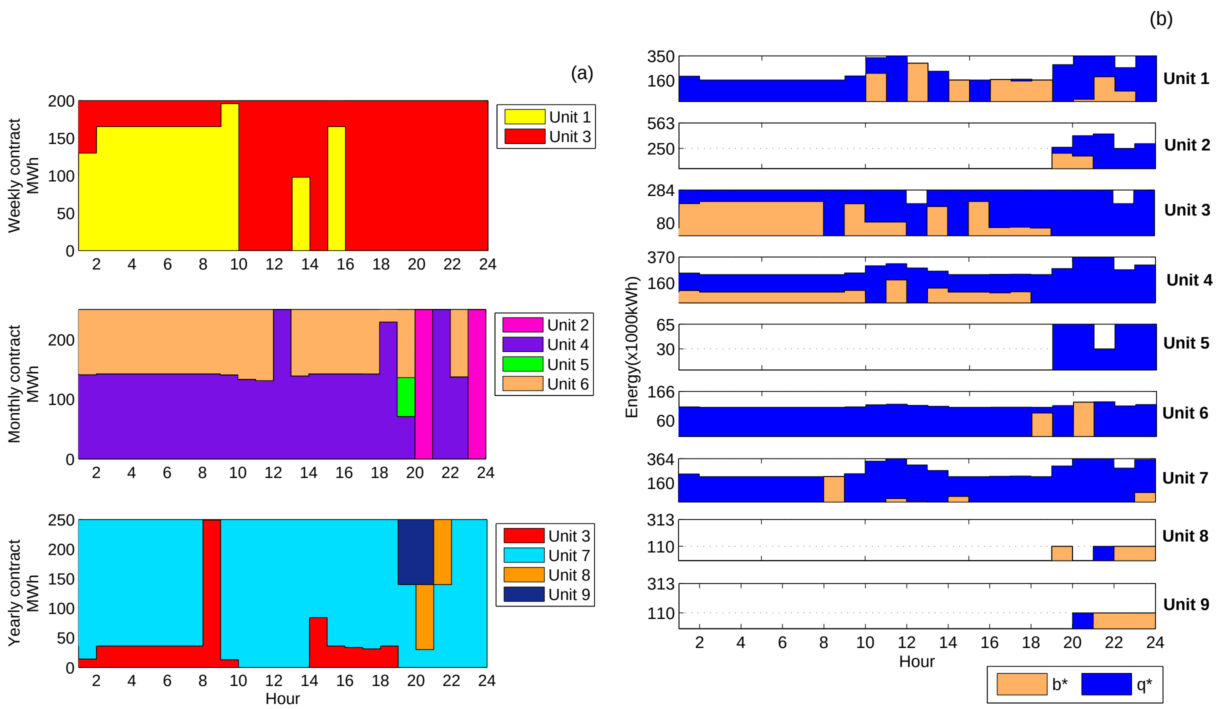

Figure 5.5: (a) Settlement of the futures contracts (q^*_{ti}) and the portfolio of bilateral contracts (b^*_{ti}) for each unit and interval (b) Economic dispatch of each futures contracts among the corresponding set of units (f^*_{itj}).

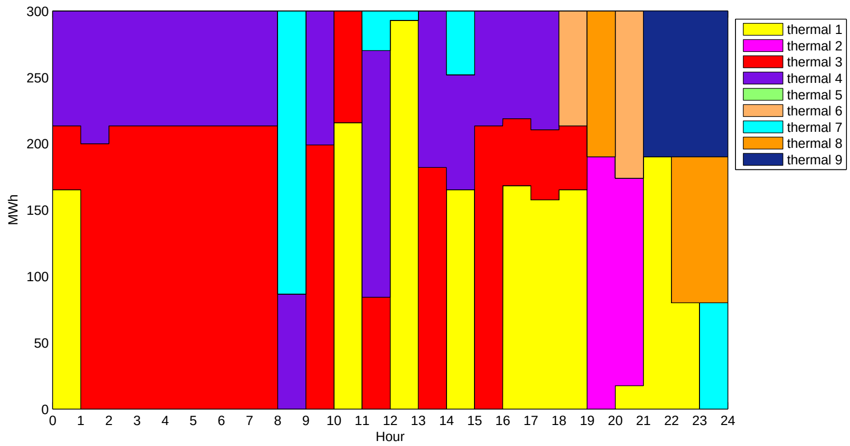

Figure 5.6: Settlement of the bilateral contracts (b^*_{ti}).

benefits obtained by using stochastic programming. In an equivalent way to Figure 4.6, Figure 5.5(a) shows the optimal value of variable f^*_{tij}, thus the optimal economic dispatch of each FC. In this case study, the three physical contracts considered are: 200MWh in a weekly contract, 250MWh in a monthly contract and 250MWh in a yearly contract. The main difference with the model from only FCs is that in this case it is essential to study both the FCs and the BCs together because their economic dispatch is closely related (Figure 5.6). Notice that the main difference between the FCs and the BCs is in their inclusion in the DAM bid but not in their economic dispatch. In this aspect, there is a slight difference in the fact that the economic dispatch requires FCs to be settled by a determined set of variables, in contrast to the BCs, which could be settled by any committed unit. Therefore, in the valley hours, each contract is covered by the cheapest unit in the UCP but, at the same time, those units will also try to cover the portfolio of BCs. See, for instance unit 3, where nearly half of the yearly FC contracts are settled by the unit in the exclusively FC model, in this model, however, the unit also participates in covering BCs, which makes it necessary for unit 7 to participate covering the FCs. As in the previous case study, the peak hours cause more variability of units behaviors. This is caused because of GenCo's interest in sending its maximum capacity to the DAM in order to maximize its benefits.

Figure 5.5(b) shows variables q^*_{ti} and b^*_{ti}: the *instrumental price bid* energy and the quantity committed to bilateral contracts for each unit and interval. It is interesting to see how the units change the quantity committed to BCs (orange area), depending on the hour and the unit cost. It is important to remember that the quantity committed to BCs changes the maximum capacity that the unit can bid to the market and therefore the benefits that it can obtain. In this figure the unit commitment is also represented. If the unit does not bid its minimum capacity to the market, it means that the unit is shut down in this interval.

Finally, to illustrate different situations concerning the bid strategy, two bid curves are represented in Figure 5.7. In Figure 5.7(a), the optimal bid curve is shown for thermal unit 1 at interval 23. It can be seen that, in this case, $b^*_{23,1} = 80$ is lower than the minimum capacity. Thus, the instrumental price bid must be at least the minimum capacity minus this quantity that is committed to BCs. In this case, the instrumental price bid quantity $q^*_{23,1} = 179$. In the other case, Figure 5.7(b),

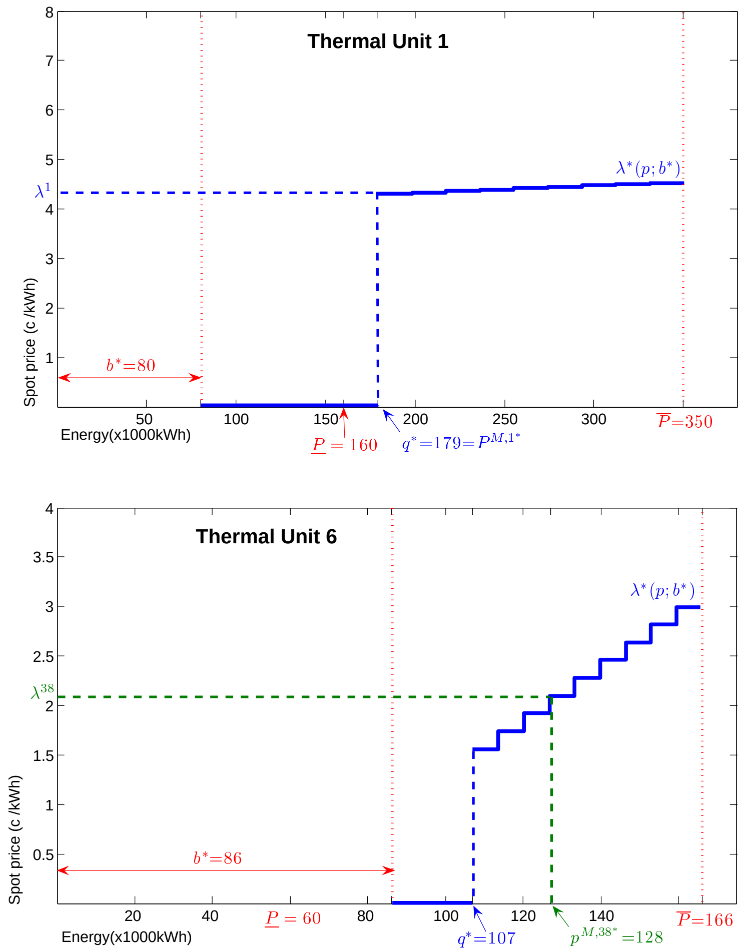

Figure 5.7: Bidding curve for (a) unit 1 at interval 23 and (b) unit 6 at interval 18.

the optimal bid curve of unit 6 at interval 18 is represented. Contrary to the previous case, the quantity committed to bilateral contracts is more than the minimum capacity ($b^*_{18,6} = 86$); so, the instrumental price bid quantity is forced, by the coverage of the FCs, to be greater than 0.

5.6 Conclusions

This chapter has presented a mixed-integer stochastic programming model for the integration of the physical futures and classical bilateral contracts into the day-ahead bidding problem of a GenCo operating in the MIBEL.

The rules for the integration of the BCs in the DAM process have been described and their relationship with FC coverage and the optimal bid curve, together with the matched energy function, are derived. The optimal solution of our model determines not only the operation of the units (unit commitment), but the quantity that must be used to cover the BCs and the quantity that must be sent to the DAM at zero price. The model explicitly follows the MIBEL regulation.

It has been shown through KKT conditions that the optimal value of the decision variables corresponds to the theoretical optimal bidding curve for a price-taker producer.

The computational tests were performed using real data on the thermal units of a price-taker producer operating in the MIBEL. They have validated the model and provide suitable results.

CHAPTER 6

DAMB: Multimarket Problem

Introduction

This chapter presents the final considered approach to the day-ahead short-term strategies of a GenCo. The models that coordinate the futures and bilateral contracts with the day-ahead market bidding process are have been presented, but as we mentioned in Section 2.1 the short-term electricity market is made up of a sequence of markets, that is, it is a multimarket enviroment. In this chapter, in order to improve the DAM bidding strategies, the possibilities that this framework provides are developed.

As was presented in Section 2.1, the case of the MIBEL sequence of short-term electricity markets includes, among others: a day-ahead market, an ancillary service market or secondary reserve market (henceforth reserve market), and a set of six intraday markets. The other smaller mechanisms, as real time operation or primary and tertiary reserve market are not taken into account because they are not related directly with the DAM bidding process; thus, its optimization can be considered as a real-time optimization matter.

It is supposed that the GenCos that participate in the electricity market could increase their benefits by jointly optimizing their participation in this sequence of electricity markets. Specifically, the DAM bidding strategies cannot be independent of the expected benefits obtained from the next markets, as the decisions taken in the DAM affect the results of the remaining short-term markets. Thus, the GenCos' objective of the generators in the short-term framework is to maximize their expected profits from participation in the day-ahead market, the reserve market and the intraday market. Moreover, the GenCo has to continue taking into account its bilateral contracts and the result of its participation in the derivatives physical markets.

The main objective of this chapter is to build a model that gives the GenCo the optimal bidding strategy for the DAM, which considers the benefits and costs of participating in the next markets and which includes both physical futures contracts and bilateral contracts.

This chapter is organized as follows: in Section 6.1, the main works published in this area are analyzed. Sections 6.2 and 6.3 present the stochastic programming model for the optimization of the day-ahead market bidding, taking into account the short-term market sequence. Finally, in Section 6.4 the first computational experiments done are presented and in Section 6.5 the first conclusions are presented.

6.1 Literature Review

The optimal multimarket bidding problem has not been studied as much as the day-ahead bidding problem and there are few research groups that have confronted this problem with stochastic techniques. The work of Plazas *et al.* (2005) is one of the first works that defines a bidding strategy for a GenCo participating in a sequence of three short-term markets. In this strategy, the unit commitment is considered known, but it is possible to engage a unit in the automatic generation control market. Notice that this is not possible in the MIBEL. They build three models that are solved successively for obtaining bidding strategies for each market, considering the expected benefits of the next markets. The work of Triki *et al.* (2005) considers a multistage stochastic model to decide the unit commitment and the capacity allocation in each market but without any bidding strategy. Furthermore, Ugedo *et al.* (2006) propose a stochastic model for obtaining the bid curve to be submitted in each market. The bidding strategies are obtained based on residual-demand curves, which represents the influence of generation offers on the clearing price. The most recent contribution, Musmanno *et al.* (2009), can be consider as an extension of Triki *et al.* (2005), where a risk aversion tool is added together with the satisfaction of the committed bilateral contracts. This work has many points in common with our approach, such as defining the economic dispatch for the BCs and considering how to include the dispatch into the DAM bid. It also includes the possibility of buying energy on the intraday markets. But there are also some differences. On the one hand, we modeled the markets according to the specific characteristics of MIBEL regulation, for example, the order in which the markets are cleared is different. On the other hand, our approach takes into account two medium-term products, BCs and FCs. A preliminary version of the model presented in the next sections has been published in Corchero and Heredia (2010).

6.2 Basic Concepts

Section 2.1 presented the sequence structure of short-term markets existing in the MIBEL. As we have introduced, in this model only the secondary reserve and the intraday markets are included. They are cleared sequentially after the DAM, as is shown in Figure 6.1. To summarize, the main characteristics of these two market processes are the following (for a detailed explanation see Sec. 2.1.6 and 2.1.5):

- Reserve market: takes place after the DAM matching process. It is an ancillary service market where the participants send bids to increase or decrease the matched energy of the

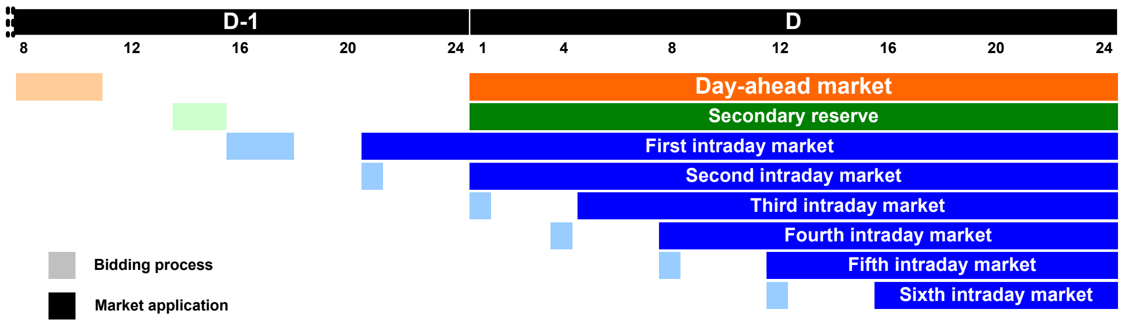

Figure 6.1: Sequence of bidding process and market application.

units in the DAM. If a bid is matched in the RM then the unit must be available to change its generation level in a given time interval during real-time operation. For this reason, the units that participate in this market must have some specific operational characteristics that allows them to increase or decrease the generation level in a given time interval.

- Intraday market: takes place just before and during the delivery day. It is composed of 6 consecutive markets. In these markets the GenCos can either send or buy electricity, that is, they can participate as buyers or sellers of energy. It works exactly as the DAM does and is used by the GenCos to change the DAM, resulting in generation scheduling. It is important to remark that at a given session and hour a unit can only submit buy or sell offers, not both; but at different hours this role can change. One unit can participate in these markets either if its bids have been matched in the DAM or if it is producing energy to settle BCs.

This set of markets that are sequentially cleared leads to a multistage stochastic programming situation (see Section 2.2.2). The multistage stochastic programming models are characterized by a set of temporary consecutive decisions each one depending on the realization of a random variable. In our case, the random variables are the price of each of the markets involved in the model. In the following sections these prices are analyzed and a set of scenarios is built.

Thus, both the decision variables and the data have to be defined depending on the stage where they take place and it is very important to carefully model the relationship between these variables and stages. Specifically, the reserve decision variables must be fixed depending on the DAM price scenario, because in the real market, the GenCo decides the reserve quantity bid once the DAM price is known. The same occurs in the case of the intraday market. When the GenCo has to decide the energy that will be bid to the intraday market, it knows the quantity and price cleared in the DAM and the RM. Thus, in the model, the IM decision variables must be fixed depending on the DAM and RM scenario. This situation is traditionally modeled through a scenario tree as is represented in Figure 6.2, where $\lambda^D = \{\lambda^{D1}..\lambda^{DSd}\}$ is the set of Sd DAM scenario prices, $\lambda^R = \{\lambda^{R1}..\lambda^{DSr}\}$ is the set of Sr RM scenario prices and $\lambda^I = \{\lambda^{I1}..\lambda^{DSi}\}$ is the set of Si IM scenario prices. There will be one set of decision variables for each branch of the tree. For this first approach we will assume some hypothesis about the RM and IM. First, we suppose that all the units in our model are capable of changing their production according to the requirements of the ISO, which means all the available units can participate in the RM. Second, we also suppose that if

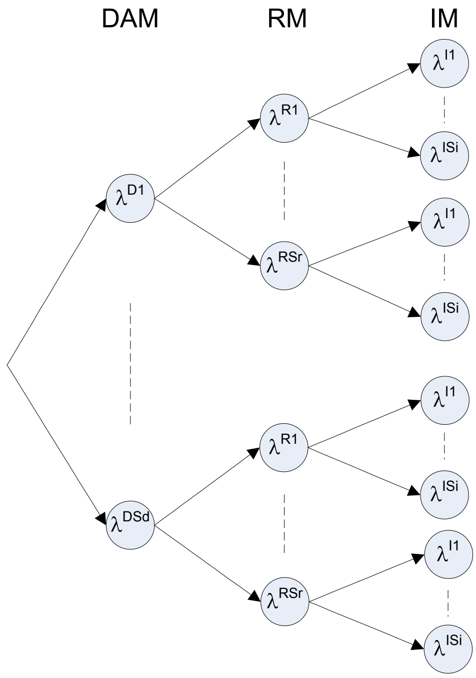

Figure 6.2: Representation of an scenario tree for the sequence of DAM, RM and IM.

the GenCo participates in the RM, then it will always bid the *automatic generation control* (AGC) capacity of the unit. The AGC capacity is an operational characteristic of each unit that indicates the quantity that the unit is able to increase or decrease in a given time. Thus, in our model the quantity submitted to the reserve market is not optimized but is always equal to the AGC capacity. We will optimize whether or not to participate in the RM. This hypothesis follows the real behavior of some GenCos observed in the MIBEL. Moreover, we work only with the first IM session. This is the session in which the greater part of the energy is negotiated and, therefore, the one that most affects the GenCo's generation scheduling. Finally, we suppose that all the energy that is bid to the RM or the IM will be matched. This can be easily forced by some bidding strategies, but this point is not dealt with in this work. These hypotheses do not limit the correct representation of the MIBEL's market sequence and they can be easily changed or adapted to different situations.

6.3 Model Description

As in previous chapters, the model is built for a price-taker GenCo owning a set of thermal generation units I (see Appendix A Table A.1).

6.3.1 Variables

For every time period $t \in T$ and thermal unit $i \in I$, the first stage variables of the stochastic programming problem are:

- The unit commitment variables: $u_{ti} \in \{0, 1\},\ c^u_{ti},\ c^d_{ti}$

- The instrumental price offer bid variables: q_{ti}.
- The scheduled energy for futures contract j variables: f_{tij}.
- The scheduled energy for bilateral contract variables: b_{ti}.

and the second and third stage variables associated with each scenario $s \in S$ are:

- Total generation: g^s_{ti}
- Matched energy in the day-ahead market: p^s_{ti}
- Reserve market related variables: r^s_{ti}
- Intraday market related variable: m^s_{ti}

Note that including the markets sequence and the BCs in the model implies, once again, that we have to introduce the total generation of the unit, g^s_{ti} in order to correctly value the production costs. It is important not to confuse this variable with the one representing the matched energy, p^s_{ti}, i.e., the energy that will be remunerated at market clearing price.

The model presented in this chapter was developed with the second formulation of the unit commitment expressed by equations (2.13)-(2.20), which formulate the relation of the variables u_{ti}, c^u_{ti} and c^d_{ti} with the minimum start-up and shut-down times and initial state of the thermal generation units.

6.3.2 FCs and BCs Covering Constraints

The bilateral and futures contracts have the same characteristics and follow the same rules as in the previous chapters (see, for instance, Sec. 2.1.2, 2.1.3 and 2.1.4).

Both the physical future and bilateral contracts coverage must be guaranteed (see Sec. 5.3.2):

$$\sum_{i\in I_j} f_{tij} = L^F_j \qquad \forall j \in F,\ \forall t \in T \tag{6.1}$$

$$\sum_{i\in I} b_{ti} = \sum_{j\in B} L^B_j \qquad \forall t \in T \tag{6.2}$$

$$f_{tij} \geq 0 \qquad \forall j \in F,\ \forall i \in I,\ \forall t \in T \tag{6.3}$$

$$0 \leq b_{ti} \leq \overline{P}_i u_{ti} \qquad \forall i \in I,\ \forall t \in T \tag{6.4}$$

6.3.3 Reserve Market Constraints

As has been explained previously, our model for the RM assumes that if the unit bids to the RM, it will bid its fixed AGC capacity, ϱ_i(MW). In this framework, the only decision to be optimized is whether the unit participates in the RM or not.

It is known that a unit can only use its AGC capacity if its generation level is constant; in other words, the unit is not increasing or decreasing its production in the corresponding interval or, equivalently, that the production level g^s_{ti} has not changed between two consecutive intervals. For all intents and purposes, the GenCo delegates its ramping capacity to the ISO.

The binary variable r^s_{ti} is introduced to trace this situation, being that $r^s_{ti} = 1$ whenever $g^s_{ti} = g^s_{(t-1),i}$ and $r^s_{ti} = 0$ otherwise.

$$g^s_{ti} - g^s_{(t-1),i} \leq (1 - r^s_{ti})R \qquad \forall i \in I,\ \forall t \in T,\ \forall s \in S \tag{6.5}$$

$$g^s_{ti} - g^s_{(t-1),i} \geq (1 - r^s_{ti})(-R) \qquad \forall i \in I,\ \forall t \in T,\ \forall s \in S \tag{6.6}$$

6.3.4 Matched Energy Constraints

The MIBEL's rules affecting the day-ahead market establishes a given relation between the variables representing the energy of the bilateral contracts b_{ti}, the energy of the future contracts f_{tij}, the instrumental price offer bid q_{ti} and the matched energy p^s_{ti}. This relation can be formulated by means of the following set of constraints:

$$p^s_{ti} \leq \overline{P}_i u_{ti} - b_{ti} \qquad \forall i \in I,\ \forall t \in T,\ \forall s \in S \tag{6.7}$$

$$p^s_{ti} \geq q_{ti} \qquad \forall i \in U_t,\ \forall t \in T,\ \forall s \in S \tag{6.8}$$

$$q_{ti} \geq \underline{P}_i u_{ti} - b_{ti} \qquad \forall i \in I,\ \forall t \in T \tag{6.9}$$

$$q_{ti} \geq \sum_{j|i \in I_j} f_{tij} \qquad \forall i \in I,\ \forall t \in T \tag{6.10}$$

where:

- (6.7) and (6.8) ensures that if a unit is on, the matched energy p^s_{ti} will be between the instrumental price bid q_{ti} and the total available energy not allocated to a BC.
- (6.9) and (6.10) guarantee respectively that the instrumental price bid will be not less than the minimum generation output of the unit if it is on, and that the contribution of the unit to the FC coverage will be included in the instrumental price bid.

6.3.5 Total Generation and Intraday Market Constraints

Finally, the total generation level of a given unit i, g^s_{ti}, is defined as the addition of the allocated energy to the BC, plus the matched energy of all the markets considered (DAM, RM and IM).

As we have introduced, our model considers the possibility of either selling or buying energy in the IM. The free variable m^s_{ti} represents, if positive, the energy of a sell offer while, if negative corresponds to the energy of a buy bid. The model ensures that m^s_{ti} is equal from 0 if the unit i is off and that it can be different to 0 if the unit i bids to the DAM or commits its generation to BCs (6.11). Notice that the variable m^s_{ti} will be positive if we produce the energy to sell it to the IM, and then will be added to the total generation level; the opposite will be the case and it will be negative if we buy the energy in the IM, where the quantity will be subtracted to the total generation level.

$$g^s_{ti} = b_{ti} + p^s_{ti} + m^s_{ti} \qquad \forall t \in T,\ \forall i \in I,\ \forall s \in S \tag{6.11}$$

The total generation must remain between the operational limits $\underline{P}_i$ and $\overline{P}_i$. But if we participate in the RM, the total generation limits change because of the energy that we must reserve in order to be able to produce it at the moment that the ISO requests:

$$\underline{P}_i u_{ti} + \varrho_i r_{it}^s \leq g_{ti}^s \leq \overline{P}_i u_{ti} - \varrho_i r_{it}^s \qquad \forall t \in T,\ \forall i \in I,\ \forall s \in S \tag{6.12}$$

And finally, if unit i is off, it cannot be offered to the reserve market:

$$r_{ti}^s \leq u_{ti} \qquad \forall t \in T,\ \forall i \in I,\ \forall s \in S \tag{6.13}$$

6.3.6 Nonanticipativity Constraints

As we have mentioned, it is necessary to control the variables through the scenario tree and this is classically done by the so called *nonanticipativity constraints* (see Section 2.2.2). The first stage variables are replicated for each of the s scenarios and those constraints ensure that if we approach it from the same previous stage scenario, the value of the decision variables will be identical. Those constraints are as follows:

$$g_{ti}^s = g_{ti}^{\hat{s}} \qquad \forall s, \hat{s} : (\lambda^{Ds} = \lambda^{D\hat{s}}),\ \forall t \in T \tag{6.14}$$

$$r_{ti}^s = r_{ti}^{\hat{s}} \qquad \forall s, \hat{s} : ((\lambda^{Ds}, \lambda^{Rs}) = (\lambda^{D\hat{s}}, \lambda^{R\hat{s}})),\ \forall t \in T \tag{6.15}$$

where (6.14) models the nonanticipativity constraints for the DAM and (6.15) models the nonanticipativity constraints for the RM.

6.3.7 Objective Function

For each thermal unit i at time interval t the quadratic costs are associated to the total generation. Taking into account the start-up and shut-down costs, the quadratic costs are:

$$C_{ti} = c_{ti}^u + c_{ti}^d + c_t^b u_{ti} + c_t^l g_{ti} + c_t^q (g_{ti}^2)$$

And the quadratic function for scenario s that represents the expected benefits of the GenCo after participation in the DAM, taking into account the overall multimarket, futures and bilateral contracts expected benefits are:

$$E_{\lambda^{D,R,I}}\left[B(g,p,m,r,u,c^u,c^d)\right] =$$

$$= \sum_{t \in T}\left[\sum_{j \in F}(\lambda_j^F - \overline{\lambda}_i^D)L_j^F + \sum_{j \in B}\lambda_j^B L_j^B\right] - \tag{6.16}$$

$$- \sum_{t \in T}\sum_{i \in I}[c_{ti}^u + c_{ti}^d + c_i^b u_{ti}] + \tag{6.17}$$

$$+ \sum_{t \in T}\sum_{i \in I}\sum_{s \in S} P^s \Big[\lambda_t^{D,s} p_{ti}^s + \lambda_t^{R,s}\varrho_i r_{ti}^s + \lambda_t^{I,s} m_{ti}^s -$$

$$-(c_i^l g_{ti}^s + c_i^q (g_{it}^s)^2)\Big] \tag{6.18}$$

where, equivalently to the previous models:

(6.16) corresponds to the incomes of the FCs and the BCs and is a constant term.

(6.17) is the on/off fixed cost of the unit commitment of the thermal units, deterministic and independent of the realization of the random variable λ_t^D.

(6.18) represents the expected value of the benefits from the DAM, the RM and the IM. The first term, $\lambda_t^{D,s} p_{ti}^s$, computes the incomes from the DAM based on a value p_{ti}^s of the matched energy. The second term, $\lambda_t^{R,s} \varrho_i r_{ti}^s$ computes the incomes from bidding the AGC capacity to the RM. The third term, $\lambda_t^{I,s} m_{ti}^s$ computes the incomes or costs from the IM, depending on the sign of m_{ti}^s. Finally, the term between brackets corresponds to the expression of the quadratic generation costs with respect to the total generation of the unit, g_{ti}^s.

Then, the objective function $f(x)$ to be minimized in our model is:

$$f(g,p,r,m,u,c^u,c^d) = \sum_{i\in I}\sum_{t\in T}\Big(c_{ti}^u + c_{ti}^d + c_i^b u_{ti} + \sum_{s\in S} P^s \left[c_i^l g_{ti}^s + c_i^q (g_{ti}^s)^2 - (\lambda_t^{D,s} p_{ti}^s) - (\lambda_t^{R,s} r_{ti}^s \varrho_i) - (\lambda_t^{I,s} m_{ti}^s)\right]\Big) \quad (6.19)$$

where $\lambda_t^{D,s}$, $\lambda_t^{R,s}$, $\lambda_t^{I,s}$ are the price scenarios for the t^{th} day-ahead, reserve or intraday market respectively.

6.3.8 Day-Ahead Market Bidding with Futures and Bilateral Contracts in a Multimarket Environment

The final *day-ahead market bidding with bilateral and futures contracts in a multimarket environment model* (DAMB-FBC-M) developed in the previous sections is:

$$(\text{DAMB-FBC-M}) \begin{cases} \min & f(g,p,r,m,u,c^u,c^d) \\ \text{s.t.} & \\ & \text{Eq. (6.1)} - \text{(6.4)} \quad \text{BC and FC covering} \\ & \text{Eq. (6.5)} - \text{(6.6)} \quad \text{Reserve market constraints} \\ & \text{Eq. (6.7)} - \text{(6.10)} \quad \text{Day-ahead market constraints} \\ & \text{Eq. (6.11)} - \text{(6.13)} \quad \text{Total generation} \\ & \text{Eq. (2.13)} - \text{(2.20)} \quad \text{Unit commitment} \end{cases} \quad (6.20)$$

This program corresponds to a mixed linearly constrained minimization problem with a convex quadratic objective function that includes a well-defined global optimal solution.

6.4 Numerical Examples

In this section, the descriptive statistics of the RM and IM hourly prices are presented along with the scenarios set generation and, finally, the case study for the DAMB-FBC-M problem.

The model has been implemented and solved in CPLEX with 12.0 (2008) using the ad-hoc implementation of the perspective cuts algorithm (Frangioni and Gentile, 2006) described in Mijangos *et al.* (2010). It has been solved using a SunFire X2200 with 32 Gb of RAM memory and two dual core processors AMD Opteron 2222 at 3 GHz.

6.4.1 Reserve and Intraday Electricity Market Prices

Chapter 3 presented a deep study of the Spanish DAM prices. However for the validation of the DAMB-FBC-M problem, it is necessary to also study the reserve and intraday electricity market prices. Those two prices have very different behaviors and must be analyzed independently.

6.4.1.1 Reserve Market Electricity Prices

As has been explained (see Section 2.1.6), the RM prices are not the price at which the energy production is paid but the price at which the reservation of a given level of energy is paid. In other words, the market pays the GenCo for delegating its ramping capacity to the ISO. Daily, the quantity of ramping capacity that the system needs is published. This quantity is known in advance and it can be used by the GenCos to decide how to manage their RM bid. Thus, the RM clearing price is related more to this required energy than to the expected load or the DAM prices. Figure 6.3 shows the hourly reserve market price for the first three years of the MIBEL. Some extreme outliers have been eliminated from the sample ($\lambda^R > 10$). In those figures, it is not easy to see how they differ from the DAM price; but if we zoom in to a week, we can see in Figure 6.4(a) that the RM prices over a week do not have any particular pattern. The same is observed in Figure 6.4(b): the hourly prices over a day do not follow a determined pattern. It can also be observed that the variability of the RM prices is lower than that of the DAM prices. These conclusions are corroborated by the low correlation existing between DAM and RM prices ($\rho_{2008} = 0.138$).

6.4.1.2 Intraday Electricity Prices

Contrary to the case of the RM, the intraday market has the same structure and clearing process as the DAM and the price corresponds to the quantity that the market pays to the GenCo for its production. The main difference between this market and the DAM is that the GenCos can participate as purchase or sell agents but not both simultaneously. Figure 6.5 shows the hourly clearing price for the first intraday market. If those figures are compared to Figure 3.1, the relationship is obvious; in fact, they seem to be identical. The IM prices are highly correlated with the DAM prices ($\rho_{2008} = 0.957$) and they have the same daily and weekly pattern.

6.4.2 Scenario Set

As we have introduced in Section 3.4, the generation of a multistage tree is a problem that has been widely studied (see, for instance, Heitsch and Römisch (2009)). For this first approach, all the available historical data of the sequence of market prices has been reduced in order to obtain suitable scenario sets. Initially, all the instances are equiprobable and, after applying the

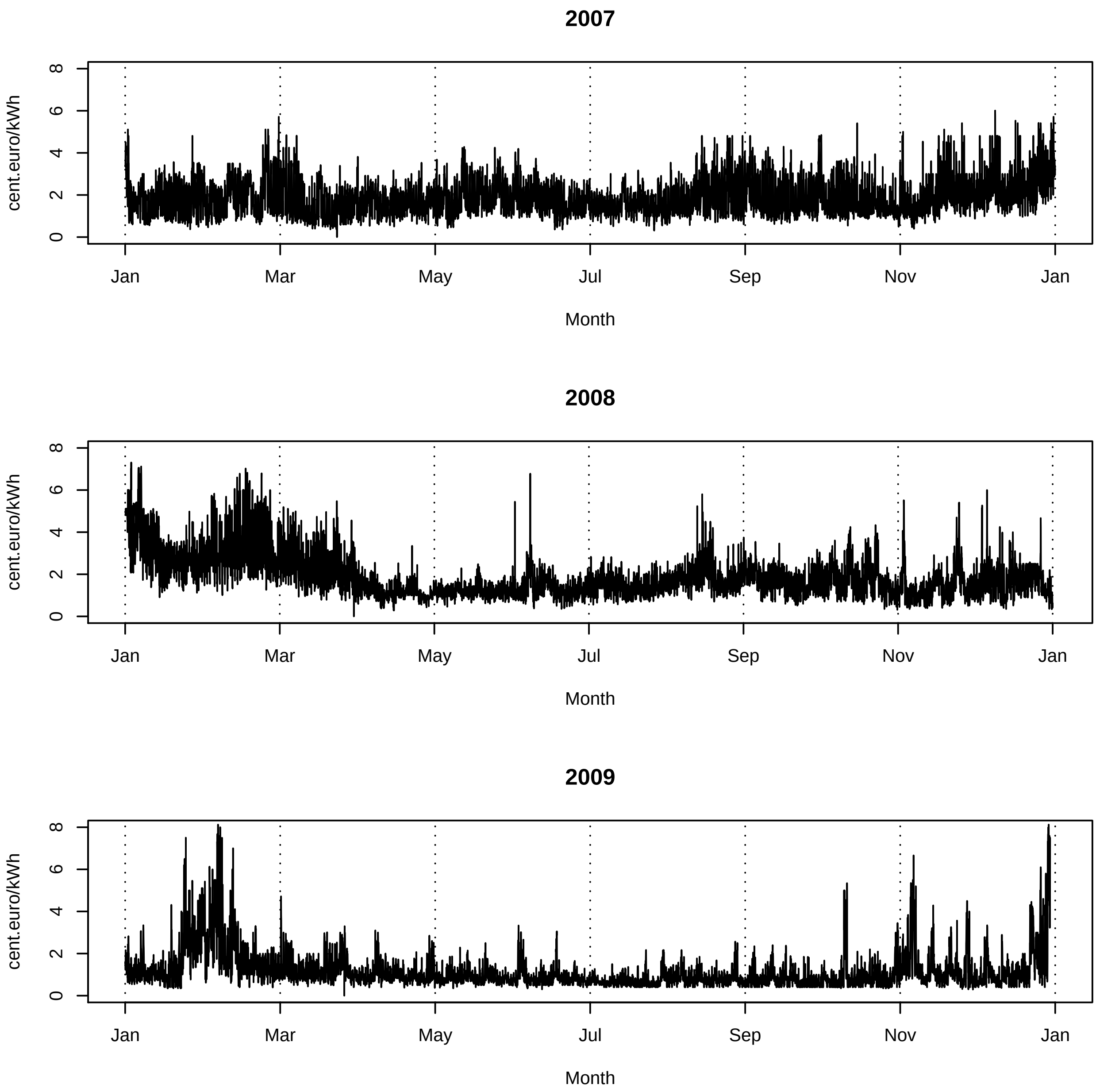

Figure 6.3: RM hourly prices.

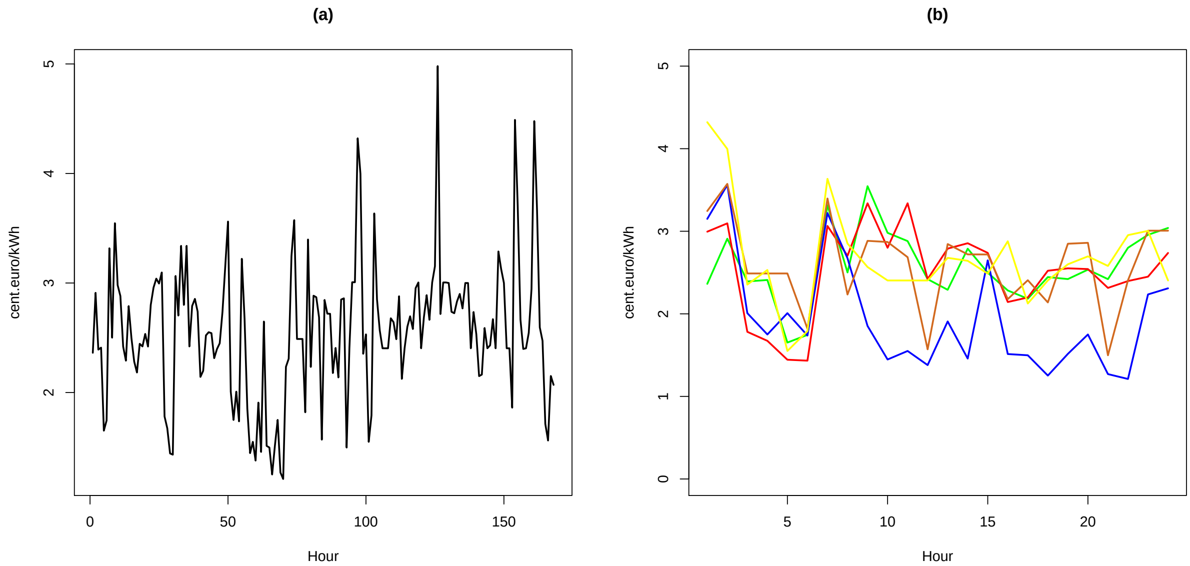

Figure 6.4: (a) MIBEL RM price over one week. (b) Hourly RM price for the working days of a week.

$\|S\|$	c.v.	b.v.	CPU(s)	Objective function	$\frac{\|x^s - x^{180}\|}{\|x^{180}\|}$
25	19680	6240	612	89230500	1,000
50	37680	12240	3093	88268300	0,001
75	55680	18240	12316	88624200	0,002
100	73680	24240	25728	88177400	0,001
120	88080	29040	32570	88209200	0,001
140	102480	33840	60030	88318100	0,002
160	116880	38640	74865	88298800	0,002
180	131280	43440	93532	88209200	

$|T| = 24$; $|I| = 9$

Table 6.1: Optimization characteristics of the cases studies and results for different number of scenarios.

reduction algorithm (Gröwe-Kuska *et al.*, 2003), the different subsets of scenarios and the respective probabilities are obtained. The main computational characteristics for each reduced set of scenarios are in Table 6.1. It can be observed (see Figure 6.6) that the objective function is nearly stable after 50 scenarios. Considering the computational burden introduced by the increase of second stage binary variables as the number of scenarios grows, we conclude that 50 scenarios retain enough information to obtain suitable results.

6.4.3 Case Study

As in previous chapters, a set of computational tests has been performed in order to validate the proposed model. The instances used in the test have 9 thermal units (see Table A.1, Appendix A). One of the objectives of the tests is to study the influence of the sequence of markets in the DAM bid. As it has been explained, the DAM bid of the GenCo will be fixed by the quantity committed to bilateral contracts, that will be excluded from the DAM bid, and the quantity committed to

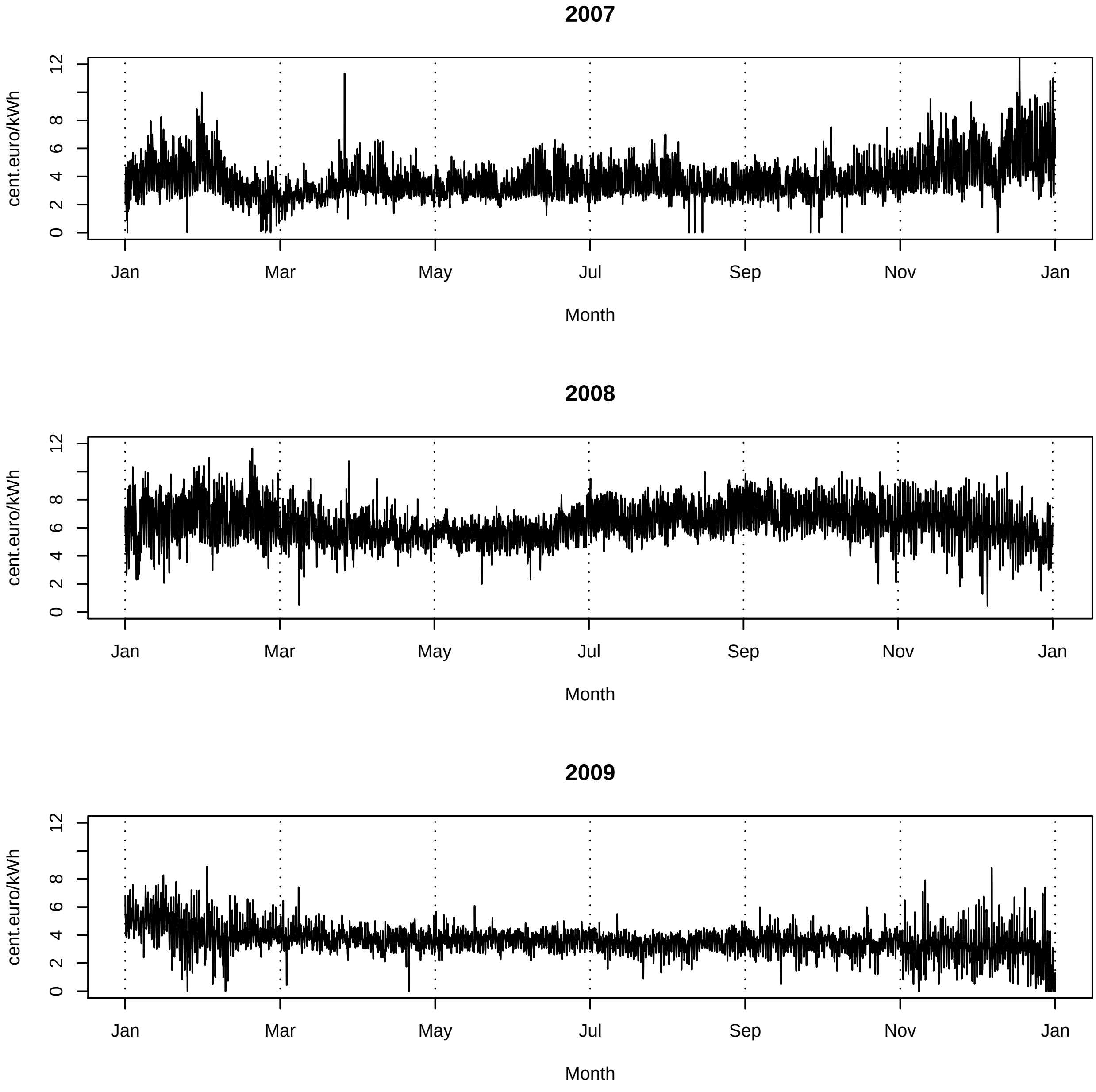

Figure 6.5: IM hourly prices.

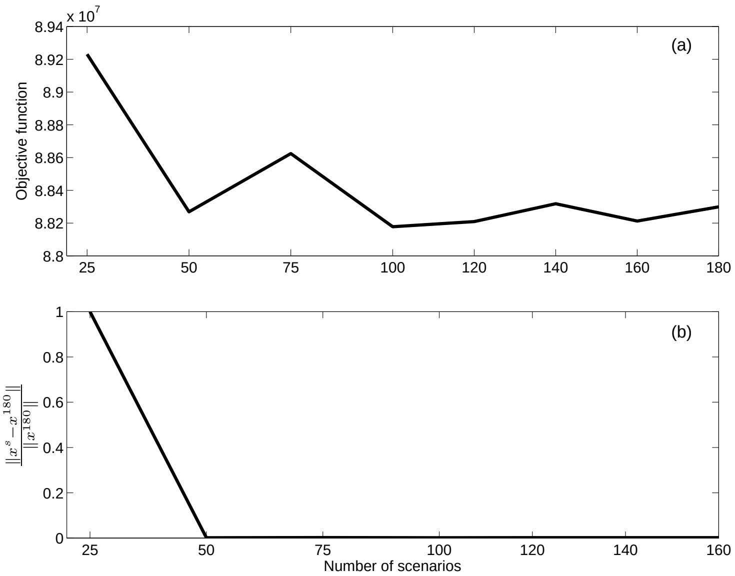

Figure 6.6: (a) Expected benefits for each reduced set of scenarios. (b) First-stage variable convergence evaluated as $\frac{\|x^s - x^{180}\|}{\|x^{180}\|}$.

futures contracts, which must be bid at the instrumental price. Thus, we focus on the two variables that represent these quantities in order to study its optimal value when taking into account, or not, the sequence of markets.

The economic dispatch of each physical futures contract among the units that participate in it is represented in Figure 6.7. Differences can be observed between the optimal values by taking into account the sequence of markets (Figure 6.7(a)) or not (Figure 6.7(b)). In the monthly contract, for example, unit 4 settles the greater part of the contract in some intervals when the sequence of markets is included. On the other hand, in the case of the optimal value without the sequence of markets included, the settlement of the monthly contract is distributed among the four units that participate in it during the same intervals.

The other important variable is one that represents the energy submitted to bilateral contracts, because this energy will be excluded from the market bidding process. Figure 6.8 represents the economic dispatch of the bilateral contracts, i.e., the quantity each unit commits to the bilateral contracts for each interval t, and the quantity to cover the futures contracts. It can be also observed the big differences among the optimal economic dispatch if we include the RM and the IM in the optimization model (Figure 6.8(a)) or not (Figure 6.8(b)). On the one hand, if a unit participates into the RM market, it must reserve a part of its participation and thus cannot use it to cover the medium-term products (see, for instance, Unit 2 at intervals 3, 7 or 8). On the other hand, they could buy or sell energy into the IM, and this can change the settlement of the medium-term products. Those differences will lead to different offer curves for each unit and interval.

At last, we will study the behavior of the GenCo in the IM. As has been introduced, the GenCo can submit either purchase or sell bids. This is controlled by the decision variable m_{ti}^s. $m_{ti}^s > 0$ means that the GenCo sells this energy and $m_{ti}^s < 0$ the GenCo buys energy in the IM. During one fixed hour the GenCo can act only as a selling or buying agent, but not both. However, throughout

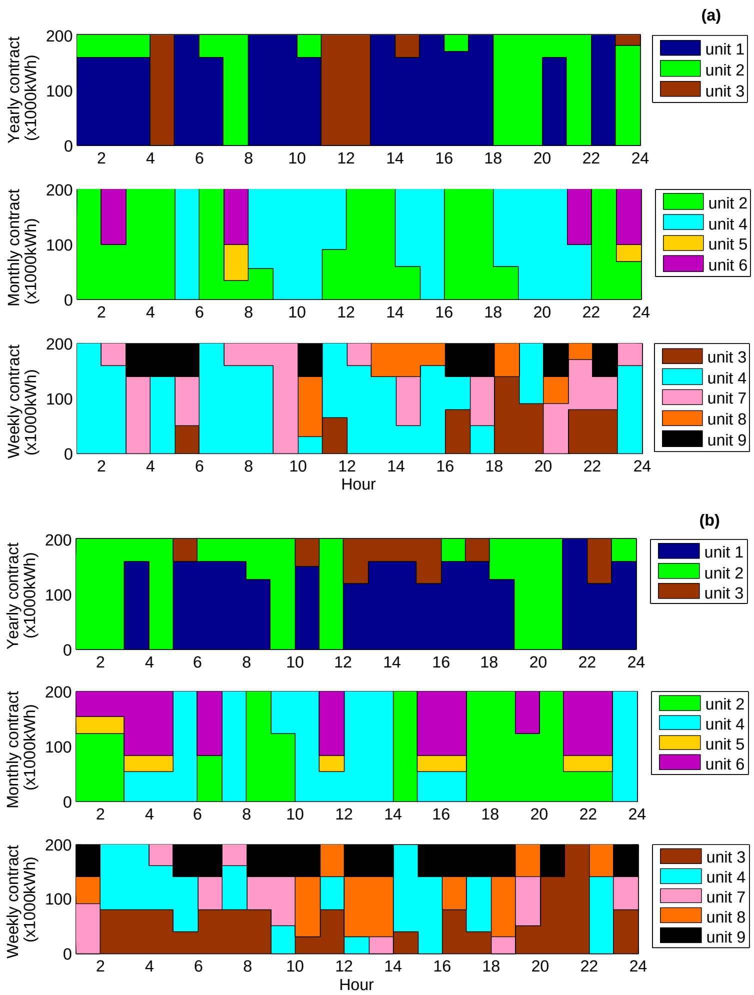

Figure 6.7: Economic dispatch of each futures contracts, f_{tij}. (a) Taking into account market sequence (b) With the DAM only.

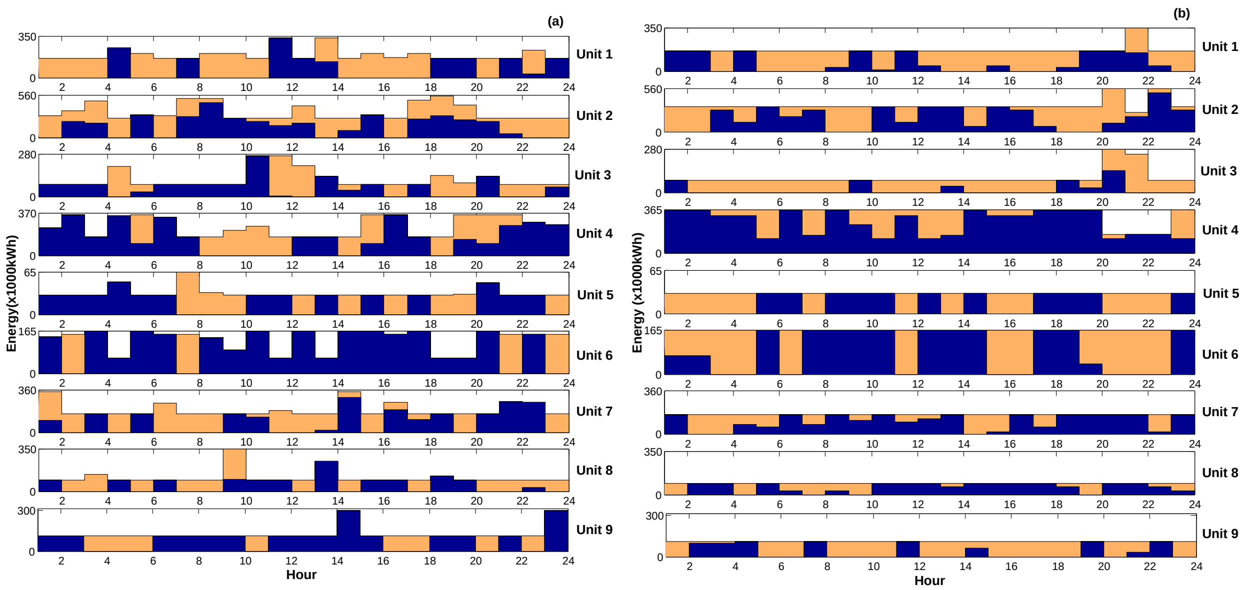

Figure 6.8: Economic dispatch of bilateral and futures contract, b_{ti} and q_{ti} . (a) Taking into account market sequence (b) With the DAM only.

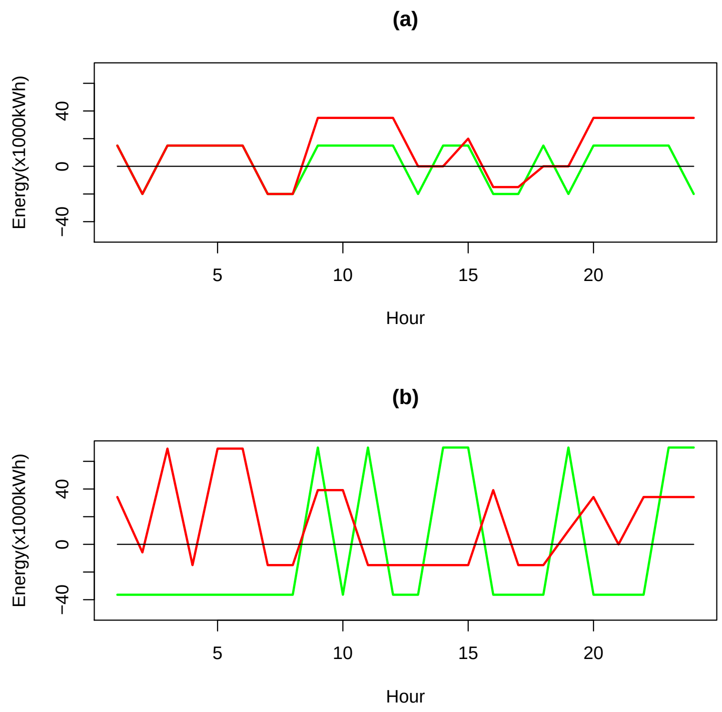

Figure 6.9: Energy send to the IM by (a) Unit 1 and (b) Unit 3 in two different scenarios.

one day it can change as many times as necessary. Figure 6.9 represents the bidding energy for two units throughout one day at two different scenarios. It can be observed that, depending on the hour, the GenCo either buys or sells energy, or it does not participate in the IM, these decisions are related with both the intraday and the day-ahead prices.

6.5 Conclusion

This work has developed a new linear mixed-integer stochastic programming model, to assist to the optimization of the day-ahead bid with futures and bilateral contracts taking into account the reserve and the intraday market.

The optimal solution of our model determines the optimal instrumental price bidding strategy and the optimal economic dispatch for the BCs and the committed FCs for each hour. The model maximizes the expected benefits of the sequence of electric markets while satisfying the thermal operational constraints and the MIBEL's rules. The results of the computational tests validate the model and show the influence of market sequence on the optimal bidding strategy of the GenCo, as well as the short-sight effect of optimizing the DAM bid without taking into account the possibilities of the next markets.

Our approach does not cover important topics which are included in our future research, such as the modeling of the sequence of market prices and and a more elaborated construction of a scenario tree that correctly reflects the three random variables involved in the three prices (DAM,RM and IM). Moreover, our aim will be to obtain a bidding curve for each unit, as the OM requires. This point has not been achieved in this first approximation of the model, but it is one of the main subjects for future research in this field.

CHAPTER 7

DAMB: Other Extensions

Introduction

In this chapter, two new approaches to the Day-Ahead Market bidding problem are presented. It is important to remark that, despite the fact they are presented last, they were built at the beginning of this work. At this time, we had not yet studied yet the optimal conditions of the models and their relationship with the theoretical definitions of the matched energy; therefore and our models contained a set of constraints that explicitly forced the optimal value of the decision variables to be equal to the theoretical definition.

We include the description of these two models because, despite its previous approaches, they exemplify two applications of our price-taker bidding strategies in other important and interesting cases, such as combined cycle units or generic programming units.

As we have indicated in the introduction, the models presented in this chapter were made in collaboration with Marcos J. Rider. The material presented in this chapter is based on the papers Heredia *et al.* (2010) and Heredia *et al.* (2009).

This chapter is organized as follows: in Section 7.1 the DAMB problem for thermal and combined cycle units is presented, taking into account a portfolio of bilateral contracts. Section 7.2 describes the DAMB problem for a price-taker GenCo that has VPP rights (see 2.1.4.1). The optimal bidding strategies for the generic programming units are presented as well.

7.1 DAMB: Thermal and Combined Cycle Units

In this section, a stochastic mixed-integer quadratic programming model for a price-taker GenCo with BC obligations is developed for for determining as optimal bidding strategy in the DAM for

	2002	2003	2004	2005	2006	2007	2008	2009
Hydro	22.598	38.874	29.777	19.170	24.761	26.381	21.428	23.862
Nuclear	63.016	61.875	63.606	57.539	60.184	55.046	58.973	52.761
Coal	78.768	72.249	76.358	77.393	66.143	71.846	46.275	33.862
Fuel	16.474	8.027	7.697	10.013	5.841	2.384	2.378	2.082
CC	5.308	14.991	28.974	48.840	63.561	68.304	91.286	78.279
Subtotal	186.164	196.016	206.412	212.955	220.490	223.962	220341	190.845
Self-consumption	-8.420	-8.162	-8.649	-9.080	-8.719	-8.655	-8.338	-7.122
Cogeneration and renewables	35.401	41.412	45.868	50.365	49.904	55.754	66.298	80.888
Total	213.145	229.266	243.631	254.240	261.675	271.061	278.301	264.612

Table 7.1: Generation of electricity in the Spanish electricity system (GWh).

thermal and *combined cycle* (CC) units while observing the MIBEL regulation.

There are many reasons to study the inclusion of the combined cycle units in the short-term bidding strategies of a GenCo. One important reason is that the CC units represent the majority of the new generating unit installations across the globe. Their main advantages are:

- High efficiency (can reach 60%, which is a 20–30% improvement over that of the traditional single-cycle thermal plants).
- Fast response (can be quite instrumental in facing rapid fluctuations in power markets), very useful nowadays for complementing renewable energies such as wind power.
- Environmental friendliness (the CO_2 production of a natural gas fueled CC plant is much lower than that of other fossil-fueled turbine technologies).
- Compact and shorter installation time.

In the Spanish Peninsular Electricity System, the first combined cycle units started generating in 2002. On December 31, 2009, the total installed capacity was 93.729 MW. This capacity has increased by 33.910 MW since 2002. The increase is chiefly attributed to the commissioning of new CCs, cogeneration, and renewable power plants, most of which comprised the wind power. Currently, the installed capacity of CC units represents 25% of the total installed capacity. Also, the total net generation in the Spanish Peninsular Electricity System is 264 TWh, with 13% from coal plants, 30% from CC plants, 31% from cogeneration and renewable plants, 20% from nuclear plants, 9% from hydro plants and 1% from fuel plants (Table 7.1). Notice that the last year both coal and CC plants presents a decrease in the total generation percentage. This is because of the increase in the renewable plants, specifically wind power, which mostly affect these two technologies. It is necessary to emphasize that, aside from this decrease, CC units have became an important key for the operating system because of their fast response; i.e. the renewable technologies introduce a large factor of uncertainty into the stability of the electricity system, so fast response units are essential for maintaining the generation levels on a very short horizon.

So, a GenCo operating in such a complex market can no longer optimize its medium- and short-term generation planning decisions without considering the relation between those markets and the increasing importance of the emission-free (wind power and hydro-generation) and low-emission technologies (combined cycle).

Thus, the model developed in this section allows the price-taker GenCo to decide the unit commitment of its thermal and CC programming units, the economic dispatch of the BC among the programming units, and the optimal bid for thermal and CC units.

7.1.1 Literature Review

In previous chapters there has been a review of the literature related to DAMB and bilateral contracts (see Sections 2.3.1 and 5.1). So, in this section is a review of literature related to the combined cycle units and their integration into short-term strategies.

One of the earlier studies (Bjelogrlic, 2000) considered CC units in short-term resource scheduling. The proposed formulation was based on the assumption that the thermal subsystem of a CC is modeled through input-output curves that are defined for all configurations and all steam load ranges. Lu and Shahidehpour (2004) presented a model for calculating the unit commitment of CC units using dynamic programming and lagrangian relaxation applied to the security-constrained short-term scheduling problem. Furthermore, Li and Shahidehpour (2005) also presented the price-based unit commitment problem based on the mixed-integer programming method for a generating company with thermal, CC, cascaded-hydro, and pumped-storage units.

To our knowledge, none of the earlier publications presented an explicit formulation of the optimal sale bid of the CC units to the DAM or any considerations about the BC.

7.1.2 Basic Concepts

Consider a price-taker GenCo possessing the set I of thermal units and a set C of CC units (a combustion turbine and a steam turbine). The set B represents the portfolio of BC duties, with known energy (L_j^B) and price (λ_j^B) for each BC contract $j \in B$, that must be dispatched at each time period $t \in T$ between the thermal and CC units.

As it was explained in Section 2.1.2 and applied in Chapter 5, the MIBEL's rules state that each GenCo must notify the scheduling of the BCs to the market operator. Subsequently, in this situation, the problem that the GenCo faces is that of maximizing its DAM's benefits for a given BC's committed energy. The objective of this section is to develop the optimal bid function, $\lambda_{ti}^o(p_{ti}^o)$, which gives the price at which the capacity generation must be bid to maximize the benefit from the pool for a given BC's committed energy.

As it has been introduced, to respect the MIBEL rules, the total contribution of unit i to the covering BC must be excluded from the bid. This implies that the bid energy must be upper bounded by $(\overline{P}_i - b_{ti})$. By assuming the quadratic thermal generation costs, the benefits obtained from the DAM as a function of the matched energy $p_{ti}^{M,s}$ for a given committed BC energy b_{ti}, under scenario s will be:

$$B^s = \lambda_t^{D,s} p_{ti}^{M,s} - \left[C(p_{ti}^{M,s} + b_{ti}) \right] \tag{7.1}$$

Then, if the GenCo is a price taker and the total contribution of unit i to the covering BC is excluded from the bid, we will see that the expression:

$$\lambda^o_{ti}(p^o_{ti}, b_{ti}) \;=\; \begin{cases} 0 & \text{if } 0 \le p^o_{ti} \le [\underline{P}_i - b_{ti}]^+ \\ 2c^q_i(p^o_{ti} + b_{ti}) + c^l_i & \text{if } [\underline{P}_i - b_{ti}]^+ < p^o_{ti} \le \overline{P}_i - b_{ti} \end{cases} \qquad \forall t \in T,\ \forall i \in U \tag{7.2}$$

with $[\underline{P}_i - b_{ti}]^+ = \max\{0, \underline{P}_i - b_{ti}\}$, is the optimal bid function of unit i for the DAM at interval t in the sense that, for any given value b_{ti}, if function (7.2) is bid, the matched energy $p^{M,s}_{ti}$ corresponding to any scenario s with market price $\lambda^{D,s}_t$, maximizes the DAM benefit function (7.1).

Moreover, the expression of $p^{M,s}_{ti}$ will prove to be:

$$p^{M,s}_{ti} = \begin{cases} [\underline{P}_i - b_{ti}]^+ & \text{if } \theta^s_{ti} \le \max\{\underline{P}_i, b_{ti}\} \\ \theta^s_{ti} - b_{ti} & \text{if } \max\{\underline{P}_i, b_{ti}\} \le \theta^s_{ti} \le \overline{P}_i \\ \overline{P}_i - b_{ti} & \text{if } \theta^s_{ti} \ge \overline{P}_i \end{cases} \qquad \forall t \in T,\ \forall i \in U \tag{7.3}$$

where $\theta^s_{ti} = \left(\lambda^{D,s}_t - c^l_i\right)/2c^q_i$.

Contrary to the previous models presented, in this case the expression of the matched energy (7.3) will be explicitly introduced through a set of constraints (see Sec. 7.1.3.4).

7.1.3 Model Description

7.1.3.1 Variables

For every time period $t \in T$ and programming unit $i \in U$ where U is the set of thermal and combined cycle units, the first stage variables of the stochastic programming problem are:

- The unit commitment binary variables: u_{ti}, a_{ti}, e_{ti}.
- The scheduled energy for bilaterals contract variables: b_{ti}.

and the second stage variables associated to each scenario $s \in S$ are:

- Matched energy in the day-ahead market: p^s_{ti}.
- Total physical production: g^s_{ti}.

Aside from these sets of variables, there are a group of auxiliary variables that are needed to model the correct inclusion of the matched energy into the model. These variables are described in Section 7.1.3.4.

This model was developed with the first formulation approach of the unit commitment for the thermal units; thus equations (2.10)-(2.12) formulate the inclusion of the variables u_{ti}, a_{ti} and $e_{ti} \forall i \in I$ into the model.

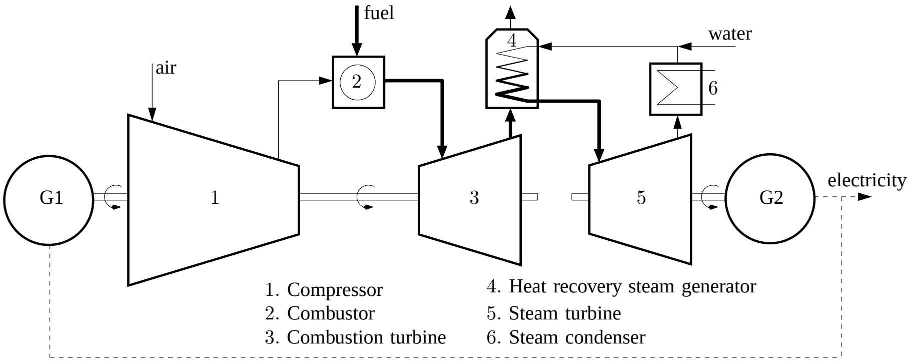

Figure 7.1: Combined cycle unit.

7.1.3.2 Thermal and Combined Cycle Units' Operation Constraints

With traditional thermal units (single-cycle thermal plants), the fuel (natural gas) and the compressed air are mixed and burnt in a combustion chamber. The energy released during combustion is used to turn a combustion turbine that drives an *electric generator* (G1), which produces electricity (Figure 7.1). Air is a relatively non-problematic and inexpensive medium, which can be used in modern gas turbines. With CC units, the heat (which would otherwise be wasted) is captured from the exhaust gas of a single-cycle *combustion turbine* (CT). The hot gas stream is used in the *heat recovery steam generator* (HRSG) that is used to turn a *steam turbine* (ST), which consequently drives an electric generator (G2) to produce additional electricity.

The CC units represent a combination of combustion and steam turbines within a power plant. Typically, a CC unit consists of several CTs and an HRSG/ST set. Based on the different combinations of CTs and HRSG/ST, a CC unit can operate in multiple states or configurations. The first two columns of Table 7.2 show the states of a CC unit with a CT and an HRSG/ST considered in this study. The operational rules of a typical CC unit were formulated by Lu and Shahidehpour (2004) with the help of the so-called *pseudo units* (PUs). As the thermal units, the PUs of each CC unit have their own unique cost characteristics, real power generation limits, minimum on-time limits, etc., and can be viewed as a special set of non-independent or coupling single thermal units. Contrary to the model proposed earlier (Lu and Shahidehpour, 2004), where each one of the three states of the CC unit had its own PU, our formulation only considered two PUs, each associated with states 1 and 2 of the CC. The on/off state of these two PUs uniquely determined the state of the CC (see columns 3 and 5 of Table 7.2), and allowed (as will be seen later) a correct model of the operation for the state 0 without the need of an additional PU.

Let us define P_c, the set of PUs of the CC unit $c \in C$, and $P = \cup_{c \in C} P_c$, the complete set of PUs. By $P_c(j)$, we denote the PU associated with the state $j \in \{1,\ 2\}$ of the CC unit c. Thus, $U = T \cup P$ represents the complete set of units (thermal and pseudo). The on/off state of each thermal and pseudo units at period t can be represented by the first-stage binary variables u_{ti}, $i \in U$. Columns 4 and 6 of Table 7.2 illustrate the relation of the commitment binary variables of the PUs, $u_{tP_c(1)}$ and $u_{tP_c(2)}$, with the state of the associated CC unit.

We can model, analogously to the operational constraints of the thermal units ((2.10)-(2.12)), the

CC unit with a CT and HRSG/ST					
State	Composition	Pseudounit 1	$u_{tP_c(1)}$	Pseudounit 2	$u_{tP_c(2)}$
0	0CT+0HRSG/ST	off	0	off	0
1	1CT+0HRSG/ST	on	1	off	0
2	1CT+1HRSG/ST	off	0	on	1

Table 7.2: States of the CC unit and its associated pseudo units.

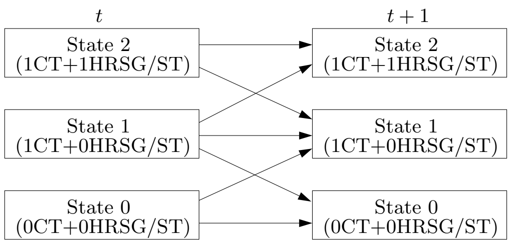

Figure 7.2: Feasible transitions of the CC unit with a CT and HRSG/ST.

start-up and shut-down process for each PU $i \in P$ that has its own minimum up time, t_i^{on}:

$$u_{ti} - u_{(t-1)i} - e_{ti} + a_{ti} = 0 \qquad \forall t \in T,\ \forall i \in P \tag{7.4}$$

$$e_{ti} + \sum_{j=t}^{\min\{t+t_i^{on},|T|\}} a_{ji} \leq 1 \qquad \forall t \in T,\ \forall i \in P \tag{7.5}$$

$$u_{ti},\ e_{ti},\ a_{ti} \in \{0,1\} \qquad \forall t \in T,\ \forall i \in P \tag{7.6}$$

Each CC unit also has a minimum down time, i.e., once shut down, the CC unit cannot be started up before t_c^C periods. Thus, we introduced the auxiliary variables u_{tc}^C, a_{tc}^C and e_{tc}^C to represent the on/off, shut-down, and start-up state, respectively, of the CC unit $c \in C$. As in the case of the thermal and pseudo units, the following constraints formulate the minimum down time condition for the CC units:

$$u_{tc}^C - u_{(t-1)c}^C - e_{tc}^C + a_{tc}^C = 0 \qquad \forall t \in T,\ \forall c \in C \tag{7.7}$$

$$a_{tc}^C + \sum_{j=t}^{\min\{t+t_c^C,|T|\}} e_{jc}^C \leq 1 \qquad \forall t \in T,\ \forall c \in C \tag{7.8}$$

$$u_{tc}^C, a_{tc}^C, e_{tc}^C \in \{0,1\} \qquad \forall t \in T,\ \forall c \in C \tag{7.9}$$

In fact, variables u_{tc}^C, a_{tc}^C and e_{tc}^C are not necessary, because they can be expressed in terms of the binary variables of the PU of P_c with the aid of the *feasible transition rules* defined in Figure 7.2:

$$u_{tc}^C = u_{tP_c(1)} + u_{tP_c(2)}\ ; e_{tc}^C = e_{tP_c(1)} - a_{tP_c(2)}\ ; a_{tc}^C = a_{tP_c(1)} - e_{tP_c(2)}$$

and, the constraints (7.7) and (7.8) can be re-expressed in terms of the PU variables as:

$$(u_{tP_c(1)} + u_{tP_c(2)}) - (u_{(t-1)P_c(1)} + u_{(t-1)P_c(2)}) +$$

$$+(a_{tP_c(1)} - e_{tP_C(1)}) - (e_{tP_c(2)} - a_{tP_C(2)}) = 0 \qquad \forall i \in I,\ \forall c \in C \qquad (7.10)$$

$$(a_{tP_c(1)} - e_{tP_c(2)}) + \sum_{j=t}^{\min\{t+t_c^C,|I|\}} e_{jP_c(1)} - a_{jP_c(2)} \leq 1 \qquad \forall i \in I,\ \forall c \in C \qquad (7.11)$$

The feasible transition rules (Figure 7.2) impose additional constraints on the operation of the PUs associated to the same CC unit, $c \in C$. First, the PUs in P_c are mutually exclusive (7.12), i.e., only one of them can be committed at a given period (a CC can only be in one state simultaneously). Second, the change of the commitment of the PUs in P_c between periods t and $t+1$ are limited to the feasible transitions depicted in Figure 7.2. These feasible transitions impose that, if the CC unit c is in state 0 at period t ($u_{tP_c(1)} + u_{tP_c(2)} = 0$), it cannot be in state 2 at period $t+1$ ($u_{(t+1)P_c(2)} = 0$) (7.13). Conversely, if $u_{tP_c(2)} = 1$, then $u_{(t+1)P_c(1)} + u_{(t+1)P_c(2)} \geq 1$ (7.14). The following set of constraints formulates the specific operation rules of the CC units:

$$\sum_{m \in P_c} u_{tm} \leq 1 \qquad \forall t \in T,\ \forall c \in C \qquad (7.12)$$

$$u_{(t+1)P_c(2)} \leq u_{tP_c(1)} + u_{tP_c(2)} \qquad \forall t \in T,\ \forall c \in C \qquad (7.13)$$

$$u_{tP_c(2)} \leq u_{(t+1)P_c(1)} + u_{(t+1)P_c(2)} \qquad \forall t \in T,\ \forall c \in C \qquad (7.14)$$

7.1.3.3 BCs Covering Constraints

As already has been stated, here we consider that the GenCo has agreed to physically provide the energy amounts L_j^B each settlement hour. This energy L_j^B can be provided by any programming unit U, both thermal and PUs:

$$\sum_{i \in U} b_{ti} = \sum_{j \in B} L_j^B \qquad \forall t \in T \qquad (7.15)$$

$$0 \leq b_{ti} \leq \overline{P}_i u_{ti} \qquad \forall i \in U,\ \forall t \in T \qquad (7.16)$$

where, as in the previous models, the total contribution of the committed unit i to the BC covering at period t is represented by the variable b_{ti}.

7.1.3.4 Matched Energy Constraints

As we have already stated, in this model the matched energy is included in the optimization model explicitly through a set of constraints. Moreover, it is necessary first to present two results: the first is the expression of the optimal bid function together with its corresponding matched energy; the second is an alternative formulation of the matched energy that will ease its modeling.

Theorem 7.1. *For a price-taker GenCo with a set of committed BCs operating in the MIBEL, the expression:*

$$\lambda_{ti}^o(p_{ti}^o, b_{ti}) = \begin{cases} 0 & \text{if } 0 \leq p_{ti}^o \leq [\underline{P}_i - b_{ti}]^+ \\ 2c_i^q(p_{ti}^o + b_{ti}) + c_i^l & \text{if } [\underline{P}_i - b_{ti}]^+ < p_{ti}^o \leq \overline{P}_i - b_{ti} \end{cases} \qquad \forall t \in T,\ \forall i \in U \qquad (7.17)$$

with $[\underline{P}_i - b_{ti}]^+ = \max\{0, \underline{P}_i - b_{ti}\}$*, is the optimal bid function of unit* i *for the DAM at interval* t*.*

Proof. The first block of the optimal bid function $\lambda^o_{ti}(p^o_{ti}, b_{ti}) = 0$ for $p^o_{it} \leq [\underline{P}_i - b_{ti}]^+$ is the instrumental bid needed to guarantee the covering of the BC contracts and the minimum operation level.

To observe the optimality of the second part of the bid function, we must maximize the DAM function (7.1) with respect to the matched energy $p^{M,s}_{ti}$ when:

$$[\underline{P}_i - b_{ti}]^+ < p^{M,s}_{ti} \leq \overline{P}_i - b_{ti}.$$

The matched energy (7.3) that maximizes the DAM benefit function for scenario s is defined as:

$$p^{M,s}_{ti} = \text{argsup}_{p^{M,s}_{ti}} \left\{ B^s \mid [\underline{P}_i - b_{ti}]^+ < p^{M,s}_{ti} \leq \overline{P}_i - b_{ti} \right\}$$

and its value can be obtained using:

$$p^{M,s}_{ti} = \begin{cases} [\underline{P}_i - b_{ti}]^+ & \text{if} \quad \theta^s_{ti} \leq \max\{\underline{P}_i, b_{ti}\} & (a) \\ \theta^s_{ti} - b_{ti} & \text{if} \quad \max\{\underline{P}_i, b_{ti}\} \leq \theta^s_{ti} \leq \overline{P}_i & (b) \\ \overline{P}_i - b_{ti} & \text{if} \quad \theta^s_{ti} \geq \overline{P}_i & (c) \end{cases} \quad i \in I,\ t \in T,\ s \in S \tag{7.18}$$

where $\theta^s_{ti} = \left(\lambda^{D,s}_t - c^l_i\right)/2c^q_i$.

Let us now analyze the expression of the matched energy associated with the bid (7.17) for all the possible values of the clearing market price $\lambda^{D,s}_t$. The following three cases can be distinguished:

$$\textbf{(a)}\ \lambda^{D,s}_t \leq \underline{\lambda}_{ti}; \quad \textbf{(b)}\ \underline{\lambda}_{it} \leq \lambda^{D,s}_t \leq \overline{\lambda}_{ti}; \quad \textbf{(c)}\ \lambda^{D,s}_t \geq \overline{\lambda}_{ti}$$

where $\underline{\lambda}_{ti}$ and $\overline{\lambda}_{ti}$ are the threshold prices:

$$\underline{\lambda}_{ti} = 2c^q_i\left([\underline{P}_i - b_{ti}]^+ + b_{ti}\right) + c^l_i \quad ; \quad \overline{\lambda}_{it} = 2c^q_i\overline{P}_i + c^l_i$$

It can be easily observed that these three cases of possible values of the clearing market price are equivalent to cases (7.18a), (b), and (c), respectively. Regarding these three cases, let us now observe the expression of the matched energy, which coincides with the optimal matched energy, $p^{M,s}_{ti}$. For the three cases of equation (7.18):

(a) $\lambda^{D,s}_t \leq \underline{\lambda}_{ti}$: If the market clearing price $\lambda^{D,s}_t$ is below the minimum non-instrumental bid price $\underline{\lambda}_{ti}$, then only the instrumental part of the bid (7.17) is accepted and the matched energy will be $[\underline{P}_i - b_{ti}]^+$,i.e, the same amount as in case (7.18(a)).

(b) $\underline{\lambda}_{it} \leq \lambda^{D,s}_t \leq \overline{\lambda}_{ti}$: When the market clearing price $\lambda^{D,s}_t$ is strictly between the threshold prices, only the bid energy with a bid price less than or equal to this clearing price will be accepted (matched). The matched energy obtained from the expression of the optimal bid function in this case is $\left[\left(\lambda^{D,s}_t - c^l_i\right)/2c^q_i\right] - b_{ti}$, which is the same expression as in case (7.18(b)).

(c) $\lambda_t^{D,s} \geq \overline{\lambda}_{ti}$: If the market clearing price $\lambda_t^{D,s}$ is above the maximum bid price $\overline{\lambda}_{it}$ then the maximum generation bid $(\overline{P}_i - b_{ti})$ is matched, which is the same amount as in case (7.18(c)).

Subsequently, it has been proved that if the proposed function (7.17) is bid to the day-ahead market, then the resulting matched energy will maximize the day-ahead benefit function (7.1). □

In order to develop a simplified expression of the optimal matched energy $p_{ti}^{M,s}$, we define the parameter ρ_{ti}^s as:

$$\rho_{ti}^s = \begin{cases} \underline{P}_i & \text{if} \quad \theta_{ti}^s \leq \underline{P}_i \\ \theta_{ti}^s & \text{if} \quad \underline{P}_i < \theta_{ti}^s < \overline{P}_i \\ \overline{P}_i & \text{if} \quad \theta_{ti}^s \geq \overline{P}_i \end{cases} \qquad \forall i \in I,\ \forall t \in T,\ \forall s \in S \tag{7.19}$$

and can be used to .

Proposition 7.1. *The optimal matched energy function (7.3) can be expressed as:*

$$p_{ti}^{M,s} = [\rho_{ti}^s - b_{ti}]^+ \qquad \forall t \in T,\ \forall i \in U \tag{7.20}$$

with the constant parameter ρ_{ti}^s defined in (7.19).

Proof To observe the equivalence of (7.3) and (7.20), the three cases (7.3) can be analyzed:

(a) $\theta_{ti}^s \leq \max\{\underline{P}_i, b_{ti}\}$: If $\max\{\underline{P}_i, b_{ti}\} = \underline{P}_i$ then, $\theta_{ti}^s \leq \underline{P}_i \Rightarrow \rho_{ti}^s = \underline{P}_i \Rightarrow [\rho_{ti}^s - b_{ti}]^+ = [\underline{P}_i - b_{ti}]^+ = p_{ti}^{M,s}$. Conversely, if $\max\{\underline{P}_i, b_{ti}\} = b_{ti}$, then either $\theta_{ti}^s \leq \underline{P}_i$, which has just been analyzed, or $\underline{P}_i < \theta_{ti}^s \leq b_{ti}$. In this last case, as $b_{ti} \leq \overline{P}_i$, (7.19) sets $\rho_{ti}^s = \theta_{ti}^s$, and both $[\rho_{ti}^s - b_{ti}]^+$ and $[\underline{P}_i - b_{ti}]^+$ take the value of zero.

(b) $\max\{\underline{P}_i, b_{ti}\} \leq \theta_{ti}^s \leq \overline{P}_i$: Through (7.19), $\rho_{ti}^s = \theta_{ti}^s$. Then, the matched energy is $[\rho_{ti}^s - b_{ti}]^+ = [\theta_{ti}^s - b_{ti}]^+ = |\theta_{ti}^s > b_{ti}| = \theta_{ti}^s - b_{ti} = p_{ti}^{M,s}$.

(c) $\theta_{ti}^s \geq \overline{P}_i$: Expression (7.19) sets $\rho_{ti}^s = \overline{P}_i$, and consequently, $[\rho_{ti}^s - b_{ti}]^+ = \left[\overline{P}_i - b_{ti}\right]^+ = |b_{ti} \leq \overline{P}_i| = \overline{P}_i - b_{ti} = p_{ti}^{M,s}$. □

Proposition 7.1 establishes that for a committed unit i that bids the function (7.2) to the DAM, the matched energy at scenario s will be $[\rho_{ti}^s - b_{ti}]^+$. However, if the unit is uncommitted, then the bid does not exist and the matched energy becomes zero. The matched energy can then be expressed through the matched energy function as a function of variables b_{ti} and u_{ti}:

$$p_{ti}^{M,s}(u_{ti}) = \begin{cases} [\rho_{ti}^s - b_{ti}]^+ & \text{if } u_{ti} = 1 \\ 0 & \text{if } u_{ti} = 0 \end{cases} \qquad \forall t \in T,\ \forall i \in U,\ \forall s \in S \tag{7.21}$$

For the sake of simplicity, as in previous models, the matched energy $p_{ti}^{M,s}$ will be represented by the variable p_{ti}^s. However, this non-differentiable expression cannot be included in the optimization model as it is. To formulate an equivalent mixed-integer linear formulation, we introduced the auxiliary binary z_{ti}^s and continuous v_{ti}^s variables. In this formulation, $z_{ti}^s = 1$ whenever $b_{ti} \geq \rho_{ti}^s$ and $z_{ti}^s = 0$ otherwise, v_{ti}^s will always be defined as $v_{ti}^s = [b_{ti} - \rho_{ti}^s]^+$.

With the help of these auxiliary variables, expression (7.21) can be transformed into the following equivalent mixed-integer linear system:

$$\left.\begin{array}{ll}
p^s_{ti} = \rho^s_{ti} u_{ti} + v^s_{ti} - b_{ti} & (a)\\
\rho^s_{ti}(z^s_{ti} + u_{ti} - 1) \leq b_{ti} & (b)\\
b_{ti} \leq \rho^s_{ti}(1 - z^s_{ti}) + \overline{P}_i(z^s_{ti} + u_{ti} - 1) & (c)\\
\rho^s_{ti}(1 - z^s_{ti}) \geq p^s_{ti} & (d)\\
\rho^s_{ti}(1 - z^s_{ti}) \leq \rho^s_{ti} u_{ti} & (e)\\
v^s_{ti} \leq (\overline{P}_i - \rho^s_{ti})(z^s_{ti} + u_{ti} - 1) & (f)\\
p^s_{ti} \in [0, \rho^s_{ti}] & (g)\\
v^s_{ti} \in [0, \overline{P}_i - \rho^s_{ti}] & (h)\\
z^s_{ti} \in \{0, 1\} & (i)
\end{array}\right\} \quad \forall t \in T,\ \forall i \in U,\ \forall s \in S \qquad (7.22)$$

The following proposition establishes the equivalence between the function (7.21) and the system (7.22) over the set of all feasible unit commitments and BC-dispatching solutions:

Proposition 7.2. *The system (7.22) and the function (7.21) are equivalent in value for the matched energy variable p^s_{ti} for every feasible solution of the system (7.22) that satisfies function (7.21).*

Proof First, let us consider the solution of system (7.22) for all the feasible solutions in Ω with $u_{ti} = 0$. As the parameter ρ^s_{ti} is always non-negative, (7.22e), $z^s_{ti} = 1$, and (7.22d), together with the bounds of (7.22g) sets $p^s_{ti} = 0$. Analogously, (7.22c) and (7.22f), together with the bounds (7.42) and (7.22h) zeroes the values of b_{ti} and v^s_{ti}, respectively. The remaining equations, (7.22a) and (7.22b), result in the redundant relations $p^s_{ti} = 0$ and $b_{ti} \geq 0$, respectively.

Second, let us analyze system (7.22) for all the feasible solutions with $u_{ti} = 1$. For all these solutions, system (7.21) is reduced to:

$$\left.\begin{array}{ll}
p^s_{ti} = \rho^s_{ti} + v^s_{ti} - b_{ti} & (a)\\
\rho^s_{ti} z^s_{ti} \leq b_{ti} & (b)\\
b_{ti} \leq \rho^s_{ti}(1 - z^s_{ti}) + \overline{P}_i z^s_{ti} & (c)\\
\rho^s_{ti}(1 - z^s_{ti}) \geq p^s_{ti} & (d)\\
\rho^s_{ti}(1 - z^s_{ti}) \leq \rho^s_{ti} & (e)\\
v^s_{ti} \leq (\overline{P}_i - \rho^s_{ti}) z^s_{ti} & (f)\\
p^s_{ti} \in [0, \rho^s_{ti}] & (g)\\
v^s_{ti} \in [0, \overline{P}_i - \rho^s_{ti}] & (h)\\
z^s_{ti} \in \{0, 1\} & (i)
\end{array}\right\} \quad \forall t \in T,\ \forall i \in U,\ \forall s \in S \qquad (7.23)$$

The set of the feasible solutions with $u_{ti} = 1$ can be partitioned, depending on the value of variable b_{ti}:

(a) For those solutions with $u_{ti} = 1$ and $b_{ti} \leq \rho^s_{ti}$, (7.23b) sets $z^s_{ti} = 0$. Subsequently, (7.23f) together with the bounds (7.23h) sets $v^s_{ti} \leq 0$, and by (7.23a), $p^s_{ti} = \rho^s_{ti} - b_{ti}$, which coincides with the value given by the function (7.21) for $u_{ti} = 1$ and $b_{ti} \leq \rho^s_{ti}$. Equations (7.23c), (7.23d), and (7.23e) derive redundant expressions.

(b) Conversely, for those solutions with $u_{ti} = 1$ and $b_{ti} > \rho_{ti}^s$, (7.23c) sets $z_{ti}^s = 1$. Subsequently, (7.23d), together with the lower bound defined in (7.23g), sets $p_{ti}^s = 0$, accordingly with the value of the matched energy defined by function (7.21). The remaining equations (7.23a), (7.23b), (7.23e), and (7.23f) provide redundant constraints. □

In summary, Proposition 7.2 ensures that:

(i) Every feasible solution satisfies the equivalent matched-energy constraints (7.22)

(ii) The value of variable p_{ti}^s represents the true value of the matched energy function (7.21).

Finally, the model of the thermal units and PUs must include the following set of constraints that define the total generation output of thermal unit i at each interval t and scenario s:

$$g_{ti}^s = p_{ti}^s + b_{ti} \quad \forall t \in T,\ \forall i \in U,\ \forall s \in S \tag{7.24}$$

7.1.3.5 Objective Function

The expected value of the benefit function B (7.1) can be expressed as:

$$E_{\lambda^D}\left[B(u,a,e,p,g)\right] =$$

$$\sum_{\forall t \in T} \sum_{\forall j \in B} \lambda_j^B L_j^B - \tag{7.25}$$

$$\sum_{\forall t \in T} \sum_{\forall i \in T} \left[c_i^{on} e_{ti} + c_i^{off} a_{ti} + c_i^b u_{ti}\right] - \tag{7.26}$$

$$\sum_{\forall t \in T} \sum_{\forall c \in C} \left[c_{P_c(1)}^{on} \left(e_{tP_c(1)} - a_{tP_c(2)}\right) + c_{P_c(2)}^{on} e_{tP_c(2)} + \sum_{\forall i \in P_c} c_i^b u_{ti}\right] \tag{7.27}$$

$$+ \sum_{\forall t \in T} \sum_{\forall i \in U} \sum_{\forall s \in S} P^s \left[\lambda_t^{D,s} p_{ti}^{M,s} - c_i^l p_{ti}^s - c_i^q (p_{ti}^s)^2\right] \tag{7.28}$$

where, as in the previous models:

(7.25) is a constant term and corresponds to the BC profit.

(7.26) is the cost of the thermal units' unit commitment. It is deterministic and independent of the realization of the random variable λ_t^D.

(7.27) corresponds to the CC's start-up. Fixed generation costs are formulated. In this formulation, as in Lu and Shahidehpour (2004), only start-up costs are associated to the PU, and no cost is associated to the transition from state 2 to state 1. This is also a deterministic term (as (7.26)), so does not depend on the realization of the random variable.

(7.28) represents the expected value of the benefit from the DAM for thermal and CC units. Usually, the generation cost functions of the PUs are represented as piece-wise linear functions, but in this study, they were modeled as quadratic functions as done in a couple of earlier studies (Lu and Shahidehpour, 2004; Li and Shahidehpour, 2005).

j	$L^{BC}_{j=1...24}$ MW	$\lambda^{BC}_{j=1...24}$ €/MWh
1	200	75
2	150	73
3	250	78

Table 7.3: Characteristics of the bilateral contracts.

All the functions appearing in (7.26) and (7.28) are linear except for the generation costs in (7.28), which are concave quadratic ($c^q_i \geq 0$, see Tables A.1 and A.2).

7.1.3.6 Final Model

The final model developed in the previous sections is as follows:

$$\left\{\begin{array}{lll} \max & E_{\lambda^D}\left[B(u,a,e,p,g)\right] & \\ \text{s.t. :} & & \\ & Eq.(2.10)-(2.12) & \text{Thermal unit commitment constraints} \\ & Eq.(7.4)-(7.6),(7.10)-(7.14) & \text{Combined cycle unit commitment const.} \\ & Eq.(7.15)-(7.16) & \text{Bilateral contracts dispatching const.} \\ & Eq.(7.22) & \text{Optimal matched energy equivalent const.} \\ & Eq.(7.24) & \text{Definition of the total unit's generation } g^s_{ti} \end{array}\right. \tag{7.29}$$

The deterministic equivalent of the two-stage stochastic problem (7.29) is a mixed, continuous-binary concave quadratic maximization problem with linear constraints that can be solved with the help of standard optimization software, as illustrated in the following section.

7.1.4 Computational Results

Model (7.29) has been tested and the results are reported in this section. 3 bilateral contracts, 4 thermal units, 2 combined cycle units with a CT and a HRSG/ST and 24 hours of study were used in the tests. The characteristics of the thermal and CC units and BCs are shown in Tables A.1, A.2 Appendix A and Table 7.3, respectively. The thermal units were all set *on*, allowing them to be shut down or continue producing during the first interval. The minimum off time for both CC units was set to 3 hours Both CC units were also considered shut-down for 3 hours previous to the first optimization period. In this case study, previous to the models presented in Chapter 3, a set of 25 scenarios has been used. It has been obtained as the result of the application of the scenario reduction algorithm (Gröwe-Kuska *et al.*, 2003) to the complete set of available historic data from June 2007 to the day under study.

A summary of the characteristics of the optimization problem and its solution is shown in Table 7.4. In Table 7.5 the usual stochastic programming indicators needed to evaluate the goodness of the stochastic approximation are reported. VSS (2.4), which is the measure of the advantage of using the stochastic programming model instead of the deterministic one, shows that it is possible to increase the expected benefits by 19.363 € (2.33%) through use of the stochastic optimal solution.

Constraints	Real variables	Binary variables	E(Benefits) €	CPU s
31927	9915	5240	850.058	893

Table 7.4: Optimization characteristics of the study case.

RP	850.058 €
EEV	830.695 €
VSS	19.363 €

Table 7.5: Stochastic programming indicators.

The optimal unit commitment of thermal and CC units is shown in Figure 7.3. The three states or configurations of the CC units are represented as white (state 0), gray (state 1, $P_c(1)$) and black (state 2, $P_c(2)$), in hourly blocks. Notice how the operation of the CC units obey both the minimum up-time as well as the feasible transition rules expressed by (7.4-7.6) and (7.12-7.14) respectively. When started up, both CC units stay in state 1 longer than the minimum *on time* $t_i^{on} = 2$ before switching to the state 2, where they remain for the rest of the optimization period.

Figure 7.4 shows the aggregated economic dispatch of the three BCs (600MWh) by the thermal (white bars) and the CC (black bars) units. It can be observed that, depending on the period, the BC portfolio is covered exclusively by the thermal units (periods 1,2,10,15,19 and 24), or by a combination of thermal and CC units (the rest of the periods).

The optimal bid functions (7.2) for the thermal and CC units are represented in Figure 7.5 respectively, where $b_{t...k}$ represents the value of b_{ti} at different periods t, and b_* corresponds to the remaining periods that are not explicitly indicated. To help in understanding of these graphics, let us analyze the most simple case; thermal unit T4, which is committed throughout all the periods except for the first one. First, observe the piecewise discontinuous thick line, with a first block going from 0 to the minimum output $\underline{P}$, and a second block between $\underline{P}$ and $\overline{P}$, with a slope equal to the marginal cost of the thermal unit $2c_4^q$. Both blocks correspond, respectively, to the two blocks defining the optimal bid function (7.2). We know that this thick line represents the optimal bid function only in those periods where $b_{ti} = 0$, (periods $t \in \{2, 9, 11-14, 16, 18, 20-23\}$ for the thermal unit T4). Moreover, for those periods where $b_{ti} > 0$, the optimal bid function corresponds to the part of the thick line between the auxiliary second vertical axis shown in Figure 7.5, located

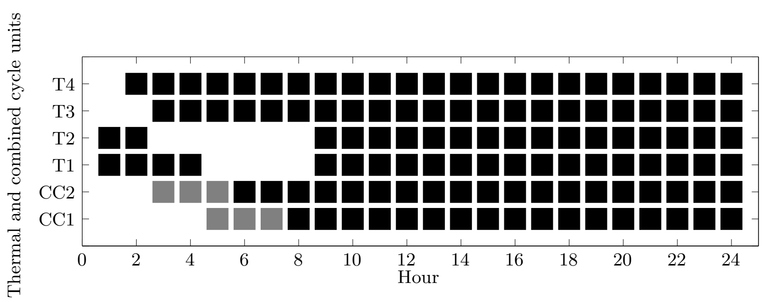

Figure 7.3: The unit commitment of thermal and CC units.

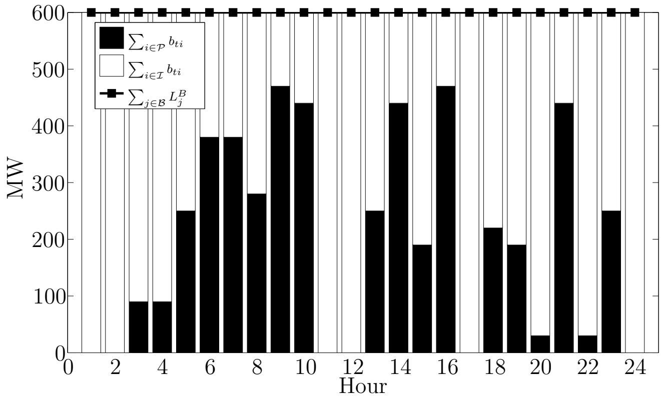

Figure 7.4: Management of the bilateral contracts ($\sum_{j\in BC} L_{tj}^{BC} = 600$MW) between thermal and combined cycle units.

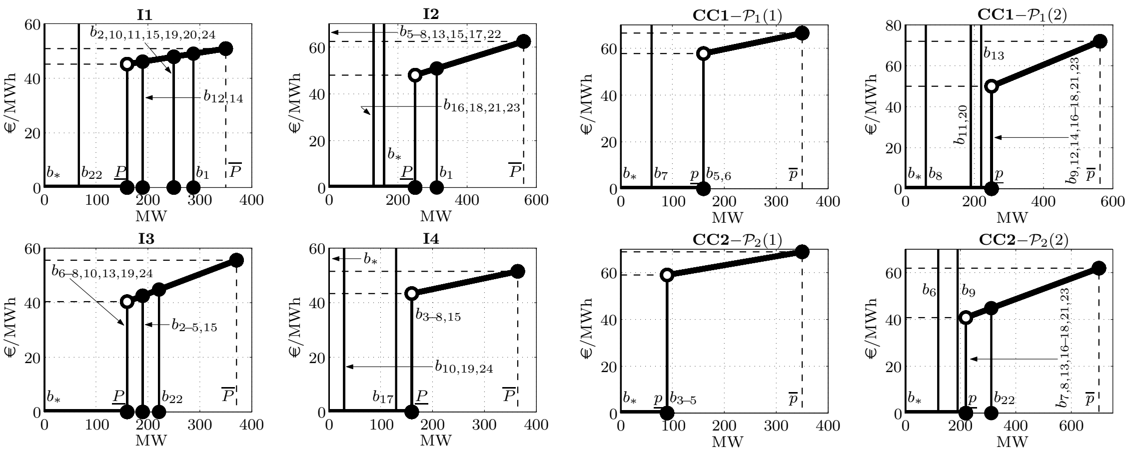

Figure 7.5: Representation of the optimal bid function $\lambda_{ti}^{b*}(p_{ti}^{b}, b_{ti})$ of the thermal and CC units.

at b_{ti}, and $\overline{P}_i$. In the case of thermal unit T4, $b_{i,4} = \underline{P}_4 = 160$MW for periods $t \in \{3-8, 15\}$, $b_{t,4} = 130$MW for period $t = 17$ and $b_{t,4} = 30$MW for periods $t \in \{10, 19, 24\}$. Although $b_{t,4} \leq \underline{P}_4$ $\forall t$ in the case of thermal unit T4, this could not be the case for other thermal units: see for instance the optimal bid function of thermal unit I3, where $b_{t,3} = 190$MW $\forall t \in \{2-5, 15\}$ and $b_{22,3} = 221.5$MW, in which case both values are above the minimum generation $\underline{P}_3 = 160$MW. The optimal bid functions of the remaining thermal units in Figure 7.5 can be interpreted in a similar way.

Let us now focus our attention on the optimal bid functions of the CC units (Figure 7.5). First observe how each CC has two different sets of optimal bid functions, depending on the state of the CC unit at each period t. The CC unit 1 would send the optimal bid functions CC1-$P_1(1)$ to periods 5, 6 and 7, where this CC unit is in state 1 (gray blocks of Figure 7.3), and the optimal bid functions CC1-$P_1(2)$ to the rest of the periods (black blocks of Figure 7.3). The same happens with the second CC unit, CC2. Please notice that the optimal bid function of each state of the same CC unit has its own slope, which corresponds to the marginal cost of each PU.

7.1.5 Conclusions

This section has a procedure for a price-taker GenCo, operating in the MIBEL, to optimally manage a pool of thermal and combined cycle units. A two-stage stochastic mixed-quadratic programming problem, which observes the MIBEL regulation has been proposed for help in deciding the optimal unit commitment and sale bid to the DAM, and the optimal economic dispatch of the bilateral contracts for all the thermal and combined cycle units. The objective of the producer is to maximize the expected profit from its involvement in the spot market.

The most relevant contribution of this study is to include the integration of a new model for the combined cycle units with the thermal units and the bilateral contracts settlement and a new model of the optimal sale bid for combined cycle units with respect to the dispatched energy of the bilateral contracts.

The model was tested with real data on market prices and programming units from a GenCo operating in the Spanish electricity market. Suitable results were provided.

7.2 DAMB: Thermal and Generic Programming Units

In this section, a stochastic programming model is presented, one that integrates the DAM optimal bidding problem with the BC rules of the MIBEL, giving special consideration to the mechanism for balancing production market: the *virtual power plants* (VPP) auctions. By observing the MIBEL regulation, the model allows the GenCo to decide on the unit commitment of the thermal units, the economic dispatch of the BCs between the thermal units and the generic programming unit, and the optimal sale/purchase bids for all units. The objective of this model has been to find the optimal bidding strategy of both the thermal production units and the *generic programming units* (GPU) in the DAM regarding the BC rules.

As was introduced in Section 2.1.4.1, the VPP capacity indicates that the buyer of this product will have the capacity to generate MWh at his disposal. The buyer, in this case our GenCo under study, can exercise the right to produce against an exercise price, set in advance, by paying an option premium. Hence, although Endesa and Iberdrola still own the power plants, part of their production capacity will be at the disposal of the VPP buyers, who are the subjects of our study. There will be base-load and peak-load contracts with different strike prices that are defined a month before the auction. In each case, contracts with a duration of 3, 6, and 12 months will be offered. Furthermore, all the products will be offered simultaneously using an electronic auction.

The energy resulting from the exercise of the VPP options can be used by the buyers both as a contribution for covering the national and international BCs before the DAM as well as to sell it to the DAM. In this latter case, the unmatched VPP energy, if any, can be sold through national BCs after the DAM. These BCs after the DAM are negotiated between the agents prior to DAM gate closure, and must not be confused with other subsequent markets such as the reserve or balancing markets.

The GPU and VPP are new elements in the MIBEL, whose utilities need to be integrated into its daily optimal bid strategy. To our knowledge, these elements have not been considered previously in the literature. The model presented is the first attempt at both using and analyzing these elements with an aim toward encouraging competition in the MIBEL, and it can be of great economic interest for any GenCo operating a GPU. Regarding the VPP, the model provides the GenCo with tool for deciding wether the energy rights of the VPP should be nominated or not. Regarding the GPU, the model's output determines its optimal bid to the market and the participation in the BCs. Finally, it must be mentioned that another novelty is the consideration, for the first time to our knowledge, of the BCs after the day-ahead market, which is another characteristic of the MIBEL.

The model presented in this study has been tuned to incorporate the specificities of the MIBEL energy production system. Nevertheless, the proposed model could also be of interest for other electricity markets with VPP, such as the Belgian and German markets.

7.2.1 Literature Review

In Sections 2.3.1 and 5.1 we have presented a review of the literature in general about the DAM bidding strategies and specifically about the inclusion of BC. As we have stated, in this chapter the model includes two mechanisms that are new into the DAM bidding process: the VPP capacity auctions with their associated GPU, and the bilateral contracts after the DAM. To our knowledge, there are no publications that consider either of them.

7.2.2 Basic Concepts

Suppose that the GenCo now has, aside from the set of I thermal units, a GPU associated with a VPP with known capacity ($\overline{p}^V$ MWh) and an exercise price (λ^V €/MWh). There is also the portfolio B of BCs before the DAM with known energy (L_j^B MWh) and price (λ_j^B €/MWh). The energy L_j^B can be delivered at time period t both by the GPU (variables b_t^G) and by any combination of the thermal units (variables b_{ti}). Finally, consider that there is an agreement for

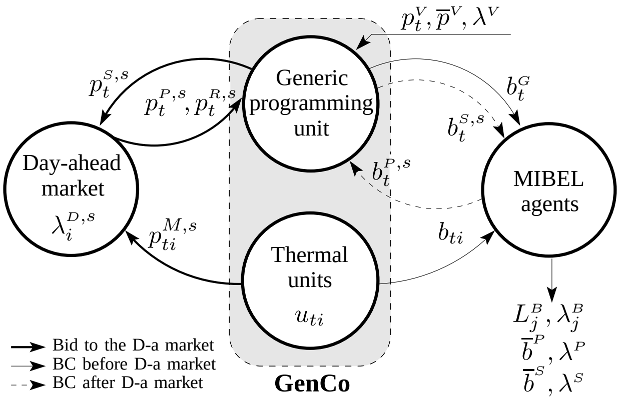

Figure 7.6: Case study.

selling (purchasing) BCs after the DAM up to a quantity $\overline{b}^S$ MWh ($\overline{b}^P$ MWh) at a price λ^S €/MWh (λ^P €/MWh). We assume that it is not possible to obtain net gain from those contracts ($\lambda^P > \lambda^S$). Figure 7.6 represents the part of the whole MIBEL energy production system considered in this study.

In the next two sections we will describe the optimal bidding function. In this case, there are two definitions, one for the thermal units and one for the generic programming unit.

7.2.2.1 Optimal Thermal Bidding Function

In the previous Section 7.1.3 the expression of the optimal bid function for a thermal unit that contributes to the BC's portfolio was proven to be:

$$\lambda_{ti}^o(p_{ti}^o, b_{ti}) = \begin{cases} 0 & \text{if } 0 \le p_{ti} \le [\underline{P}_i - b_{ti}]^+ \\ 2c_i^q(p_{ti}^o + b_{ti}) + c_i^l & \text{if } [\underline{P}_i - b_{ti}]^+ < p_{ti}^o \le \overline{P}_i - b_{ti} \end{cases} \quad \forall i \in I,\ \forall t \in T \tag{7.30}$$

where $[\underline{P}_i - b_{ti}]^+ = \max\{0, \underline{P}_i - b_{ti}\}$, and the variable b_{ti} is the total energy production of unit i assigned to the whole portfolio of BCs. This expression is analogous to equation 7.2.

Consequently, the thermal matched energy function under scenario s, $p_{ti}^{M,s}$, associated with the optimal thermal bidding function (7.30) is the analogous to expression (7.21)):

$$p_{ti}^{M,s}(u_{ti}) = \begin{cases} [\rho_{ti}^s - b_{ti}]^+ & \text{if } u_{ti} = 1 \\ 0 & \text{if } u_{ti} = 0 \end{cases} \quad \forall i \in I,\ \forall t \in T,\ \forall s \in S \tag{7.31}$$

with the constant parameter ρ_{ti}^s defined in (7.19).

As in the previous model, this discontinuous and non differentiable function can be alternatively formulated as the system of linear constraints (7.22) and introduced into the optimization model. For sake of simplicity, as in previous models, the matched energy $p_{ti}^{M,s}$ will be represented by the variable p_{ti}^s.

7.2.2.2 Optimal Generic Programming Unit Bidding Model

In this section, the optimal bidding and the matched energy functions for a GPU will be derived. First, variable b_t^G represents the total contribution of the GPU to the coverage of the BCs before the day-ahead market. Second, we assume that p_t^V, the exercised energy of the VPP, depends on the value of the binary variable x_t^V, a binary variable that indicates if the VPP rights are exercised ($x_t^V = 1$) as follows:

$$p_t^V = \overline{P}^V x_t^V \quad \forall t \in T \tag{7.32}$$

Under this assumption, the expression of the optimal GPU bid function can be developed by analyzing the two cases, $x_t^V = 0$ and $x_t^V = 1$:

(a) $x_t^V = 0$: VPP rights are not exercised and the energy b_t^G must be either acquired from the pool or provided by the BCs after the DAM at an agreed price λ^P, which is the maximum price that we were willing to pay to the pool for that amount of energy. Therefore, the optimal purchase bid (*energy*, *price*) pair is:

$$(b_t^G, \lambda^P) \quad \text{if} \quad x_t^V = 0 \tag{7.33}$$

(b) $x_t^V = 1$: the VPP rights are exercised and the exercise price is paid. Subsequently, two different situations must be considered:

   (i) $b_t^G \leq \overline{P}_t^V$: After covering the energy b_t^G with the VPP, there is an energy surplus of $[\overline{P}_t^V - b_t^G]$ that can be sold either to the pool, at unknown spot price λ_t^D, or to the BCs after the DAM, at known sale price λ^S. Subsequently, the energy surplus should be offered to the DAM at a price not less than λ^S, which is the optimal sale bid:

$$([\overline{P}_t^V - b_t^G], \lambda^S) \quad \text{if} \quad x_t^V = 1 \text{ and } b_t^G \leq \overline{P}_t^V \tag{7.34}$$

   (ii) $b_t^G > \overline{P}_t^V$: analogously to the case $x_t^V = 0$, to fulfill the uncovered part of the BC's duty, the following optimal purchase bid must be submitted:

$$([b_t^G - \overline{P}_t^V], \lambda^P) \quad \text{if} \quad x_t^V = 1 \text{ and } b_t^G > \overline{P}_t^V \tag{7.35}$$

As a result of the preceding analysis, the *optimal sale and purchase bid for the GPU* (7.33)-(7.35) can be expressed in the following compact form:

$$\text{OSB}_t = \left([p_t^V - b_t^G]^+, \lambda^S\right) \tag{7.36}$$

$$\text{OPB}_t = \left([b_t^G - \overline{P}^V]^+ + \min\{b_t^G, \overline{P}^V - p_t^V\}, \lambda^P\right) \tag{7.37}$$

It can be easily verified that for any given value of the first stage variables b_t^G and p_t^V, (7.36)-(7.37) correspond to the optimal bidding rules developed in (7.33)-(7.35). Equations (7.36)-(7.37) can be used to derive the expressions of the matched energy at each scenario $s \in S$, as functions of the first stage variables p_t^V and b_t^G. For the sake of clarify the notation, we define the two following sets of scenarios:

$$\begin{aligned} M_t^S &:= \left\{ s \in S \mid \lambda_t^{D,s} \geq \lambda^S \right\} \\ M_t^P &:= \left\{ s \in S \mid \lambda_t^{D,s} < \lambda^P \right\} \end{aligned} \qquad \forall t \in T$$

The set M_t^S includes those scenarios where, at the t^{th} DAM auction, the optimal sale bid (7.36), if any, will be accepted. Then, with respect to (7.36), the *matched sale energy function* will be:

$$p_t^{S,s} = \begin{cases} [p_t^V - b_t^G]^+ & \text{if } s \in M_t^S \quad (a) \\ 0 & \text{if } s \notin M_t^S \quad (b) \end{cases} \quad \forall t \in T,\ \forall s \in S \tag{7.38}$$

Analogously, the set M_t^P includes those scenarios where, at the t-th day-ahead auction, the optimal purchase bid (7.37), if any, will be accepted. For the clarity of the exposition, the two terms of the total matched purchase energy of (7.37) will be represented by two separate matched functions, the *matched purchase energy function*:

$$p_t^{P,s} = \begin{cases} \min\{b_t^G, \overline{P}^V - p_t^V\} & \text{if } s \in M_t^P \quad (a) \\ 0 & \text{if } s \notin M_t^P \quad (b) \end{cases} \quad \forall t \in T,\ \forall s \in S \tag{7.39}$$

and the *residual matched residual energy function*:

$$p_t^{R,s} = \begin{cases} [b_t^G - \overline{P}^V]^+ & \text{if } s \in M_t^P \quad (a) \\ 0 & \text{if } s \notin M_t^P \quad (b) \end{cases} \quad \forall t \in T,\ \forall s \in S \tag{7.40}$$

On observing (7.38),(7.39) and (7.40), it becomes evident that actually, the value of the matched sale energy will be the same for any scenario in M_t^S, and the same happens with the matched purchase energies and the scenarios in M_t^P. Nevertheless, the superscript s will be conserved for the sake of clarity and to strengthen the fact that these are actually second-stage variables, as there will be scenarios with nonzero matched energies, while in others, those energies will be zero. Another issue to mention is that, as we have assumed that $\lambda^S < \lambda^P$, the intersection set:

$$M_t^S \cap M_t^P = \{s \in S \mid \lambda_t^{D,s} \in [\lambda^S, \lambda^P]\}$$

could be nonempty. This fact does not reveal any inconsistency of the model, because (7.38), (7.39) and (7.40) are formulated in a way that, for any $s \in M_t^S \cap M_t^P$, only the matched sale energy $p_t^{S,s}$ or the total matched purchase energy $p_t^{P,s} + p_t^{R,s}$ can be greater than zero, but never simultaneously. Hence, for those scenarios in $M_t^S \cap M_t^P$, only a sale bid or a purchase bid will be submitted, depending on the value of the variables b_t^G and p_t^V. Each one of the three nondifferential functions (7.38), (7.39) and (7.40) will be conveniently incorporated into the optimization model through an associated system of equivalent generic matched energy constraints.

7.2.3 Model Description

7.2.3.1 Variables

This problem has been modeled as a mixed integer quadratic two-stage stochastic optimization problem. The main first stage variables of the model for each period $t \in T$ which corresponds to quantities that the GenCo has to decide on before the DAM clearing, are:

- The on/off state of the thermal units: u_{ti}, a_{ti}, e_{ti}.
- The BC energy allocated to each thermal unit: b_{ti}.
- The decision whether or not exercise the VPP rights: $x_t^V \in \{0, 1\}$.
- The exercised VPP energy for the GPU: p_t^V.
- The BC energy allocated to the GPU before the DAM: b_t^G.

The second stage variables for each scenario $s \in S$, are:

- The matched energy for the thermal selling bids: p_{ti}^s.
- The matched energy for the GPU sale/purchase bids: $p_t^{S,s}$ and $p_t^{P,s}$ respectively.
- The energy allocated to the sale/purchase bilateral contracts after the DAM: $b_t^{S,s}$ and $b_t^{P,s}$ respectively.
- Total physical production: g_{ti}^s.

Aside from these sets of variables, there are a group of auxiliary variables that are needed. These auxiliary variables must be included into the model throughout the four expressions of the matched energy (presented in the previous section) (7.22), (7.38)-(7.40).

This model follows the first formulation approach of the unit commitment constraints for the thermal units. Thus equations (2.10)-(2.12) formulate the inclusion of the variables u_{ti}, a_{ti} and e_{ti} into the model.

7.2.3.2 Bilateral Contracts Constraints

The GenCo has agreed to physically provide the energy amounts L_j^B at every interval of the settlement day for each one of the $j \in B$ BCs. This energy L_j^B can be provided both by the real thermal units I and the virtual GPU:

$$\sum_{i \in I} b_{ti} + b_t^G = \sum_{j \in B} L_j^B \qquad \forall t \in T \qquad (7.41)$$

$$b_{ti} \geq 0 \qquad \forall t \in T,\ \forall i \in I \qquad (7.42)$$

$$b_t^G \geq 0 \qquad \forall t \in T,\ \forall i \in I \qquad (7.43)$$

7.2.3.3 Matched Energy Constraints

Although function $p_{ti}^{M,s}$ is discontinuous and nondifferentiable, it can be formulated as a system of linear constraints. With the help of the auxiliary variables z_{ti}^s (binary) and v_{ti}^s (continuous), the nondifferentiable expression (7.31) can be shown (Proposition 7.2) to be equivalent to the following mixed-integer linear system:

$$\left.\begin{array}{l} p^s_{ti} = \rho^s_{ti} u_{ti} + v^s_{ti} - b_{ti} \\ \rho^s_{ti}(z^s_{ti} + u_{ti} - 1) \leq b_{ti} \\ b_{ti} \leq \rho^s_{ti}(1 - z^s_{ti}) + \overline{P}_i(z^s_{ti} + u_{ti} - 1) \\ 0 \leq p^s_{ti} \leq \rho^s_{ti}(1 - z^s_{ti}) \leq \rho^s_{ti} u_{ti} \\ 0 \leq v^s_{ti} \leq (\overline{P}_i - \rho^s_{ti})(z^s_{ti} + u_{ti} - 1) \\ b_{ti} \in [0, \overline{P}_i] \\ z^s_{ti} \in \{0, 1\} \end{array}\right\} \quad \forall i \in I \quad \forall t \in T \quad \forall s \in S \qquad (7.44)$$

The equivalence between (7.31) and (7.44) indicates that, for every possible value of u_{ti} and b_{ti}, there is a unique feasible value of the matched energy p^s_{ti} with respect to (7.44), and that this value satisfies (7.31).

In a similar fashion, the matched sale energy function $p^{S,s}_t$ (7.38), matched purchase energy function $p^{P,s}_t$(7.39) and residual matched purchase energy function $p^{R,s}_t$(7.40) associated with the GPU must be conveniently incorporated into the optimization model through an equivalent mixed-linear modeling. First, consider (7.38), which expresses the matched sale energy $p^{S,s}_t$ as a function of variables b^G_t and p^V_t. This non-differential expression can be included into the optimization model through the following equivalent set of linear constraints, using the auxiliary variables w^S_t (continuous) and y^S_t (binary):

$$\left.\begin{array}{lll} p^{S,s}_t = 0 & \forall s \notin M^S_t & (a) \\ p^{S,s}_t = p^V_t + w^S_t - b^G_t & \forall s \in M^S_t & (b) \\ 0 \leq p^{S,s}_t \leq \overline{P}^V(1 - y^S_t) \leq p^V_t & \forall s \in M^S_t & (c) \\ \overline{P}^V(y^S_t - 1) + p^V_t \leq b^G_t & & (d) \\ b^G_t \leq \overline{P}^V(1 - y^S_t) + \sum_{\forall j \in B} L^B_j y^S_t & & (e) \\ 0 \leq w^S_t \leq (L^B_t - \overline{P}^V) y^S_t + \overline{P}^V - p^V_t & & (f) \\ y^S_t \in \{0, 1\} & & \end{array}\right\} \quad \forall t \in T \qquad (7.45)$$

and where the constant $\sum_{\forall j \in B} L^B_j$ is used as a trivial upper bound of the variable b^G_t. The equivalence between (7.45) and (7.38) can be easily observed, in the sense that for every possible combination of the values of variables b^G_t and p^V_t, the value uniquely assigned by (7.38) to the matched sale energy variable $p^{S,s}_{ti}$ satisfies (7.45). First, it can be observed that for those $s \notin M^S_t$, both (7.38) and (7.45) zero $p^{S,s}_{ti}$. We analyzed the equivalency for the remaining scenarios $s \in M^S_t$:

(a) $p^V_t = 0$: (7.45c) sets $y^s_t = 1$ and $p^{S,s}_t = 0$, which coincides with the value of the matched sale energy function associated with (7.38). The rest of the system (7.45) is reduced to the redundant condition $0 \leq w^s_t = b^G_t \leq L^B_t$.

(b) $p^V_t = \overline{P}^V$: if $b^G_t \leq p^V_t$, the value of the matched sale energy function (7.38) will be $p^{S,s}_t(b^G_t, \overline{P}^V) = p^V_t - b^G_t$. It is easy to verify that when $b^G_t \leq p^V_t$, (7.45d) and (7.45f) set $y^S_t = 0$ and $w^S_t = 0$. Consequently, through (7.45b), $p^{S,s}_{ti} = p^V_t - b^G_t$, which is the same value given by the function (7.38). The remaining equations of system (7.45) provide redundant bounds.

(c) $p_t^V = \overline{P}^V$ and $b_t^G > p_t^V$: expression (7.38) gives $p_t^{S,s}(b_t^G, \overline{P}^V) = 0$. By assuming $b_t^G > p_t^V$, the only feasible value of y_t^S permitted by (7.45e) is $y_t^S = 1$, which, together with (7.45c), determines $p_{ti}^{S,s} = 0$. The rest of system (7.45) derives redundant expressions.

By applying a similar analysis to the expression of the matched purchase energy function (7.39), it is possible to verify the equivalence between expression (7.39) and the system of linear constraints (7.46):

$$\left.\begin{array}{ll} p_t^{P,s} = 0 & \forall s \notin M_t^P \\ p_t^{P,s} = b_t^G - w_t^P & \forall s \in M_t^P \\ \overline{P}^V y_t^P \leq p_t^{P,s} \leq \overline{P}^V - p_t^V & \forall s \in M_t^P \\ \overline{P}^V y_t^P \leq b_t^G & \\ b_t^G \leq \overline{P}^V (1 - y_t^P) + L_t^B (y_t^P + x_t^V) - p_t^V \quad) & \\ 0 \leq w_t^P \leq (L_t^B - \overline{P}^V) y_t^P + L_t^B x_t^V & \\ y_t^P \in \{0,1\} & \end{array}\right\} \forall t \in T \tag{7.46}$$

where again, the auxiliary variables w_t^P (continuous) and y_t^P (binary) were introduced. Finally, proceeding in a similar way, the residual matched-purchase energy function (7.40), is introduced in the model through the following set of equivalent linear constraints:

$$\left.\begin{array}{ll} p_t^{R,s} = 0 & \forall s \notin M_t^P \\ p_t^{R,s} = b_t^G + w_t^R - \overline{P}^V & \forall s \in M_t^P \\ 0 \leq p_t^{R,s} \leq L_t^B y_t^R & \forall s \in M_t^P \\ \overline{P}^V y_t^R \leq b_t^G & \\ b_t^G \leq \overline{P}^V (1 - y_t^R) + L_t^B y_t^R & \\ 0 \leq w_t^R \leq \overline{P}^V (1 - y_t^R) & \\ y_t^R \in \{0,1\} & \end{array}\right\} \forall t \in T \tag{7.47}$$

Again, w_t^R (continuous) and y_t^R (binary) represent auxiliary variables.

Finally, we can define the second-stage variables g_{ti}^s that represent the total generation of the thermal unit i at period t conditioned to scenario s, expressed as:

$$\left.\begin{array}{l} g_{ti}^s = p_{ti}^s + b_{ti} \\ \underline{P}_i u_{ti} \leq g_{ti}^s \leq \overline{P}_i u_{ti} \end{array}\right\} \quad \forall i \in I, \ \forall t \in T, \ \forall s \in S \tag{7.48}$$

7.2.3.4 Generic Programming Unit's Net Energy Balance

At every hour, any GPU operating in the MIBEL must ensure that the net energy balance of the GPU be zero, with the help, if necessary, of the BCs after the DAM. Following this rule, we assume that, for each scenario $s \in S$, the energies $b_t^{P,s}$ and $b_t^{S,s}$ are purchased and sold through these new BCs up to a given maximum quantity at known prices λ^P and λ^S respectively (note that $\lambda^S < \lambda^P$). Thus, the GPU's net energy balance constraints for each hour t and scenario s are:

$$\left.\begin{array}{l} p_t^V + p_t^{P,s} + p_t^{R,s} + b_t^{P,s} = p_t^{S,s} + b_t^{S,s} + b_t^G \\ 0 \leq b_t^{P,s} \leq \bar{b}^P \\ 0 \leq b_t^{S,s} \leq \bar{b}^S \end{array}\right\} \forall s \in S, \ \forall t \in T \tag{7.49}$$

7.2.3.5 Objective Function

The expected value of the benefit function B can be expressed as:

$$E_{\lambda^D}\left[B(u,a,e,g,p,p^V,p^S,p^P,p^R,b^S,b^P)\right] =$$

$$\sum_{t\in T}\sum_{j\in BC}\lambda^B_{tj}L^B_{tj} \tag{7.50}$$

$$-\sum_{t\in T}\sum_{i\in I}\left[c^{on}_i e_{ti} + c^{off}_i a_{ti} + c^b_i u_{ti}\right] - \sum_{t\in T}\lambda^V p^V_t \tag{7.51}$$

$$+\sum_{t\in T}\sum_{i\in I}\sum_{s\in S}P^s\left[\lambda^{D,s}_t p^s_{ti} - c^l_i g^s_{ti} - c^q_i (g^s_{ti})^2\right] \tag{7.52}$$

$$+\sum_{t\in T}\sum_{s\in S}P^s\left[\lambda^{D,s}_t (p^{S,s}_t - p^{P,s}_t - p^{R,s}_t)\right] \tag{7.53}$$

$$+\sum_{t\in T}\sum_{s\in S}P^s\left[\lambda^S b^{S,s}_t - \lambda^P b^{P,s}_t\right] \tag{7.54}$$

where

(7.50) represents the total income of the BCs before the DAM (constant) and can be ignored in the optimization.

(7.51) corresponds to the on/off fixed cost of the unit commitment and the exercise cost of the VPP energy and does not depend on the realization of the random variable $\lambda^{D,s}_t$.

(7.52) is the expected value of the benefit coming from the DAM of the thermal units.

(7.53) is the expected value of the benefit coming from the DAM of the GPU.

(7.54) is the expected value of the benefit coming from the DAM of the BCs after the DAM.

All the functions in (7.51)-(7.54) are linear, except for the generation costs of the thermal units (7.52), which are concave quadratic ($c^q_i \geq 0$).

7.2.3.6 Final Model

The final model developed in the previous sections is:

$$\left\{\begin{array}{lll}
\max & E_{\lambda^D} & \left[B(u,a,e,g,p,p^V,p^S,p^P,p^R,b^S,b^P)\right] \\
\text{s.t.:} & & \\
& Eq.(2.10)-(2.12) & \text{Unit commitment constraints} \\
& Eq.(7.41)-(7.43) & \text{BCs covering constraints} \\
& Eq.(7.44) & \text{Thermal's matched energy constraints} \\
& Eq.(7.48) & \text{Thermal's total generation definition} \\
& Eq.(7.32) & \text{VPP's energy nomination } p^V_t \text{ def.} \\
& Eq.(7.45)-(7.47) & \text{GPU's matched energy constraints} \\
& Eq.(7.49) & \text{GPU's net energy balance const.}
\end{array}\right. \tag{7.55}$$

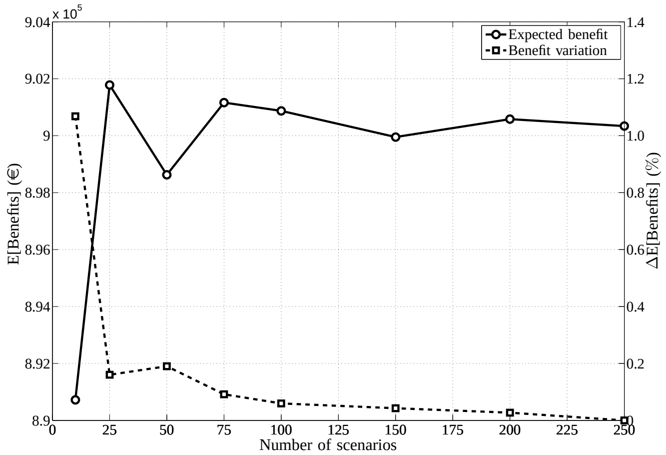

Figure 7.7: Expected benefit value and difference between the expected benefit of the complete set and each reduced set, as function of the number of scenarios.

RP	901.164 €
EEV	848.528 €
VSS	52.636 €

Table 7.6: Stochastic programming indicators.

7.2.4 Computational Results

In this section the set of computational tests that have been performed in order to validate the described model and its results are presented.

7.2.4.1 Scenario Set

The scenario set used in this chapter is previous to the study presented in Chapter 3. The creation of new BCs and the application of VPP auctions started in June 2007 and this study was done in May 2008. Thus, the complete available set of equiprobable scenarios has been obtained using all the available market prices from June 2007.

The reduction algorithm was applied, resulting in subsets of 10, 25, 50, 75, 100, 150, 200, and 250 scenarios. Figure 7.7 shows how the optimal objective function value changes as the number of scenarios increases. It also contains (right axis) the difference in the percentage between the expected benefits of the complete group of scenarios and each reduced set (ΔE[Benefits](%)). It can be observed that any increase above 75 scenarios improved the expected benefits by less than 0.09%, while the CPU time increased by almost 14 times (from 442 s with 75 scenarios to 6554 s with 100 scenarios). As a consequence, (7.55) was tested by a fan with 75 scenarios, for which the objective function value became stable and the computational time cost remained acceptable. In Table 7.6 the stochastic programming indicators needed to evaluate the advantage of the stochastic approximation are reported. VSS shows that it is possible to increase the expected benefits by 52.636 €, by using the stochastic optimal solution. Therefore, we can conclude that the solution obtained through the stochastic programming model increases our expected profits by 6.02% with over the deterministic model.

j	$L^B_{1\ldots24}$ MWh	$\lambda^B_{1\ldots24}$ €/MWh
1	1100	52
2	400	63

Table 7.7: Characteristics of the bilateral contracts.

$\overline{P}^V$ MWh	λ^V €/MWh	λ^S €/MWh	$\overline{b}^S$ MWh	λ^B €/MWh	$\overline{b}^B$ MWh
800	38	20	200	100	200

Table 7.8: Characteristic of the VPP capacity and the BC after DAM.

7.2.4.2 Case Study

The characteristics of the thermal units, BCs, and VPP capacity are shown in Tables A.1 Appendix A, 7.7, and 7.8, respectively. A set of computational tests were performed to evaluate the influence of the GPU and VPP in the GenCo's optimal bidding strategy in the MIBEL. For this reason, the proposed stochastic programming model was tested for three different cases (see Table 7.9 for a summary of the optimization problem's dimensions and solutions):

(a) a GenCo with GPU and VPP capacity;

(b) a GenCo with GPU but without VPP capacity; and

(c) a GenCo without GPU.

The worst expected profit was obtained in case (c), where the thermal units were the only ones responsible for fulfilling the BCs before the day-ahead market. Case (b) obtained a greater expected profit than case (c), due to the possibility of being able to buy cheaper energy from the pool to cover the BC and to avoid the use of expensive thermal units. The greatest expected profit was obtained in case (a), where the VPP capacity was used to sell in the day-ahead market and to cover part of the BC, using the same advantages in case (b).

The optimal management of the GPU in case (a) can be analyzed with the help of Figures 7.8 and 7.9. Figure 7.8 shows the aggregated economic dispatch of the two BCs (1.500 MWh) in thermal units (b^T_t, white bars) and GPUs (b^G_t, black bars), together with the exercised VPP energy p^V_t (small circles). Figure 7.9 shows the optimal GPU's sale bid (OSB_t, positive values) and purchase bid (OPB_t, negative values) for both cases (a) and (b) (black and white bars respectively). On

Case	Constraints	Real variables	Binary variables	E(Benefits) €	CPU s
(a)	134034	56002	18816	901.164	442
(b)	128503	52364	18792	665.530	214
(c)	119399	46895	18720	610.264	142

Table 7.9: Optimization characteristics of the study cases.

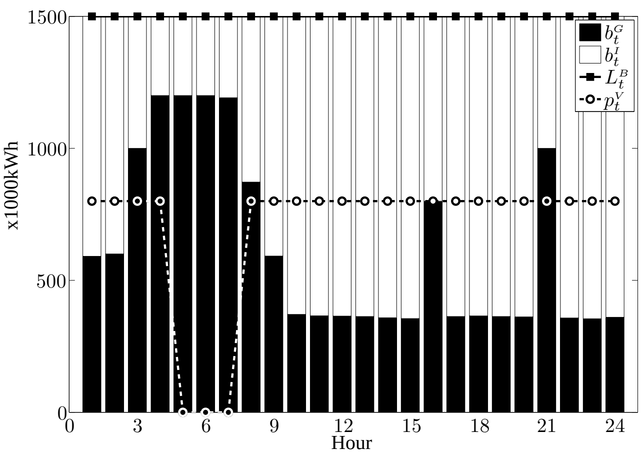

Figure 7.8: Aggregated economic dispatch of the two BCs between the thermal units and the GPU for the study case (a). Exercised VPP energy is also shown.

observing both of the graphs within the whole 24-hour optimization horizon, it is clear that the GPU exhibits a differentiated behavior, depending on the time period considered:

(i) $t \in \{5, 6, 7\}$: the GenCo does not exercise its VPP rights ($p_t^V = 0$). For those time periods, all the energy b_t^G allocated to the BC must be purchased in the day-ahead market (purchase bids, black negative bars in Figure 7.9) or from the BC after the day-ahead market. For the rest of the time periods, the GenCo does exercise its VPP rights completely ($p_t^V = \overline{P}_t^V$).

(ii) $t = 16$: where the exercised energy coincides with the energy allocated to the BC ($b_{16}^G = p_{16}^V$).

(iii) $t \in \{3, 4, 8, 21\}$ the allocated energy exceeds the exercised energy ($b_t^G > p_t^V$). The surplus energy, $b_t^G - p_t^V$, must be obtained either from the day-ahead market (see the purchase bids for those time periods, black negative bars in Figure 7.9) or from the BC after the day-ahead market.

(iv) $t \in \{1, 2, 9-15, 17-20, 22-24\}$: only part of the exercised VPP energy is used to satisfy the BC, and the rest is submitted to the DAM (sale bids for those time periods, black positive bars in Figure 7.9)

Case (b) corresponds to those GenCos operating in the MIBEL, which are not allowed to acquire any VPP capacity rights. These GenCos are not allowed in order to prevent from becoming price-makers. Under the assumptions of model (7.55), such a GenCo can use the GPU to purchase energy from the DAM in the most convenient manner, resulting in an optimal purchase bid pattern that is depicted by the white bars in Figure 7.9. The energy of the optimal purchase bid coincides in this case with the contribution of the GPU to the BC at each time period, b_t^G.

Finally, the optimal thermal unit's bidding is analyzed. The thick line in Figure 7.10 shows the optimal thermal bid function $\lambda_{ti}^*(p_{ti}, b_{ti})$ of the three thermal units (3, 4, and 6) for all the case studies at each hour. It must be noted that b_t is the energy allocated to the BC in such a way that the submitted bidding comprises energies between b_t and $\overline{P}_i$. The symbol b_* is used to point out the BC contribution for the remaining hours not shown explicitly in each sub-figure. From

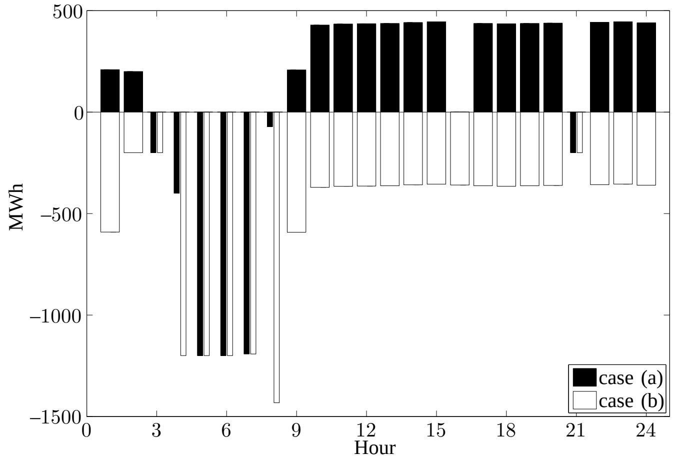

Figure 7.9: Sold and bought optimal bidding of the GPU for the study cases (a) and (b).

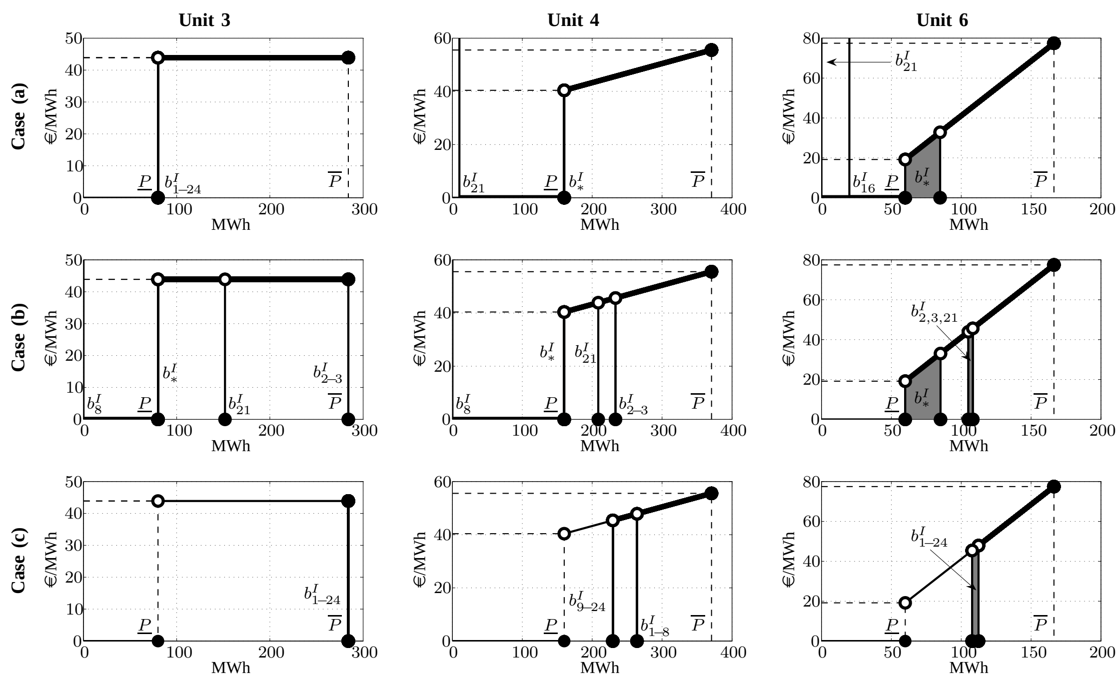

Figure 7.10: Bidding curve of thermal production units 3, 4, and 6 for all the case studies. Shadowed zones are regions with a high concentration of values of b_t.

Figure 7.10, it is clear that the presence of the GPU and VPP capacity allows the thermal units to submit more energy to the pool. For instance, consider the extreme case of thermal unit 3: without GPU (case (c)), the generation of this unit is exclusively dedicated to the BC ($b_{t,3} = \overline{P}_3\ \forall t$); while with GPU and VPP capacity (case (a)), all the production output within the operation limits are submitted to the pool ($b_{t,3} = \underline{P}_3\ \forall t$). The rest of the thermal units exhibit a similar behavior. It can also be observed how the availability of the GPU allows the bidding of thermal unit 6 to adapt itself to the different periods, which is in contrast to case (c), where the bidding is almost identical in all the time periods. In general, Figure 7.10 shows that the optimal thermal unit's bidding is affected significantly when a GPU is considered, which drastically changes the optimal bidding in a nontrivial way that increases the opportunity of the GenCo to take benefits from the pool.

Finally, based on an estimate of the additional profit expected from the VPP holding, we have presented a comment on the unsuitability of the proposed model to assess the optimal bid of the GenCo in the VPP auction. It must be noted that, although a GenCo can use the proposed model to evaluate the expected increase in profits over a 1-day period (compare case studies (a) and (b) in Table 7.9), the products auctioned in the VPP market are energy delivered over 6 months and 1 year. In other words, in order to evaluate the overall expected increase in benefits, it would be necessary to use mid-term optimization models and not a short-term optimization model such as the one presented.

7.2.5 Conclusions

This section provides a procedure for optimally managing a pool of thermal production units and a GPU. A two-stage stochastic mixed quadratic programming problem has been proposed for deciding on: the optimal unit commitment of the thermal units; the optimal economic dispatch of the BCs from thermal production units to a GPU; and the optimal bid for thermal production units and a GPU. In all of these cases, the programming problem observes the MIBEL regulation. The objective of the producers is to maximize the expected profit from their involvement in the spot market, BCs and VPP capacity.

The results of the computational experiments illustrate that the optimal bid policy furnished by the proposed model can increase the DAM benefits of a GenCo that operates a GPU of at least 10% (case study (b) of Table 7.9) or even more, and that holds a VPP capacity (case study (a) of Table 7.9).

The main contributions of this model are the mathematical modeling of the GPU and the VPP, the modeling of the optimal bid functions and matched energy of the GPU, and the inclusion of BCs after the day-ahead market into the optimization model.

CHAPTER 8

Conclusions and Further Research

8.1 Conclusions

The objectives proposed for this thesis have been fulfilled. Throughout the chapters, an exhaustive study of the Iberian Market legislation has been conducted and models were created for the optimal bid strategies of a price-taker GenCo. Further, the Iberian Electricity Market rules have been taken into account. The models presented have been formulated, computationally implemented and tested with real data from the Iberian Electricity Market. Although each chapter contains in its discussion section the main contributions of the different models presented so far, here we will summarize the general contributions of the whole thesis following the five objectives proposed in Chapter 1:

(i) Market regulations:

- The MIBEL regulation has been deeply studied (see Section 2.1). The day-ahead, derivatives and sequence of short-term markets are modeled following the MIBEL rules. The regulation of the bidding process and physical generation in the MIBEL has been taken into account when building the models presented throughout this thesis.
- A rigorous study of the optimal bid function is presented, one which corresponds to the various developed models that follow the MIBEL regulation. The analytical expression of the optimal bid functions associated to the different DAM stochastic programming models has been obtained and analyzed (see Theorems 4.1 and 5.1).

(ii) Forecast modeling for Spanish market prices:

- One of the contributions of this thesis is the design of a new factor model procedure for the Spanish day-ahead market prices (see Section 3.3). This procedure gives equivalent results to those obtained through a more classical approach using ARIMA models. The main advantage of this proposed procedure is its simplicity in being implemented and interpreted. What is more, it does not lose its capacity for obtaining a suitable forecast. This factor model procedure has also been tested satisfactorily also on the Italian day-ahead market prices (see Appendix D).

(iii) Model for the inclusion of physical derivatives products:

- A new and detailed formulation is presented for integrating futures contracts into DAM bidding strategies following the MIBEL rules(see Section 4.3). This formulation is combined with the classical operation management of the units.
- The influence of the FC's economic dispatch on the unit commitment and on the so called instrumental price bid has been illustrated (see Section 4.5).

(iv) Model for the inclusion of bilateral contracts:

- A detailed formulation for the integration in the DAM bidding strategies of the bilateral contracts, together with the physical derivatives products following MIBEL rules, is presented. This formulation is again combined with the classical operation management of the units (see Section 5.3).
- The relationship between BC and FC coverage and their influence in the operational behavior of the units is illustrated (see Section 5.5).

(v) Model which integrates the short-term sequence of markets:

- A model for the joint optimization of the medium-term products, futures and bilateral contracts, taking into account the very short-term mechanisms of the market, intraday and ancillary services markets, is developed. This model is optimized jointly with the classical operation constraints of the thermal units (see Section 6.3).
- The results obtained show that the DAM bidding strategies of the GenCo change when the market sequence is considered. It can also be observed that, in order to change its total physical production level, the GenCo takes advantage of the possibility of acting as a selling or buying agent in the intraday market (see Section 6.4).

Aside from the initial objectives, the proposed optimal bid model has been applied in some joint works, in order to include it in: (a) a formulation for the combined cycle units (see Section 7.1.3); (b) a model for the management of the generic programming units and the integration of the bilateral contracts after the DAM is proposed (see Section 7.2.3); and (c) a model for an Italian GenCo with hydro and thermal units (see Appendix E).

8.2 Topics for Further Research

There are some avenues for further research, if we follow the objectives stated in Chapter 1:

(ii) Forecast modeling for Spanish market prices:

- Exhaustive study of the intraday and ancillary services prices and their modeling by means of factor models, if possible. Given the strong relationship between day-ahead and intraday market prices, factor models are expected to perform satisfactorily with the two time series. It has to be analyzed how to introduce the reserve prices, which are apparently independent.
- To generate a multistage scenarios based on the adjusted models for the three market prices involved in the models. The literature in this area has been deeply studied and some of the published techniques can be implemented (Heitsch and Römisch, 2009).

(ii-iii) Model for the inclusion of physical derivatives products and bilateral products:

- To build a DAM bidding problem with futures and bilateral contracts for a GenCo that owns a hydrothermal and combined cycle set of units, together with a generic programming unit. This is a natural step because it will bring together the different models presented in this thesis.
- To study other decision-makers behaviors, such as risk aversion, or the introduction of some risk measurements. All the presented models are based on mean-risk approaches, but changing this point of view will change both the objective function, the bidding strategies and optimal bidding functions of the GenCo.

(v) Model which integrates the short-term sequence of markets:

- Improvements are needed in the modeling of the markets sequence, in particular, in the modeling of the reserve market.
- To include other kinds of units, especially hydro units. Their participation in bilateral contracts management is well known and it could be interesting to see how the optimal bid strategies change when introducing into the models a low cost technology, such as hydro units.
- It could also be very interesting to introduce wind power into an optimization model jointly with a combined cycle and ancillary services market. The quick response of combined cycle units is being studied as a useful tool for controlling the uncertainty inherent in wind power plant generation.

Regarding the resolution of the models, it is necessary to introduce specific optimization algorithms in order to reduce computational time. Otherwise, including new mechanisms in the models produces prohibitive results in regard to computational time, due mostly to the increasing number of binary variables. A first step towards this objective was the solution of the DAMB-FBC and DAMB-FBC-M models presented in Chapters 5 and 6 respectively, with the Perspective-Cut method (Mijangos *et al.*, 2010); it produced very satisfactory results. Our intention is to try Branch-and-Fix-Coordination (Alonso-Ayuso *et al.*, 2003) and proximal Bundle methods (Hiriart-Urruty and Lemaréchal, 1993) as an alternative to the Perspective-Cut methodology.

8.3 Publications and presentations generated by this thesis

Published:

- F.J. Heredia, M. Rider, C. Corchero (2009). *Optimal Bidding Strategies for Thermal and Combined Cycle Units in the Day-ahead Electricity Market with Bilateral Contracts.* IEEE Proceedings of the 2009 Power Engineering Society General Meeting, vol. 1, pp. 1-6. The contents of this paper are partially reproduced in Chapter 7.

- F.J. Heredia, M. Rider, C. Corchero (2010). *Optimal Bidding Strategies for Thermal and Generic Programming Units in the Day-Ahead Electricity Market.* IEEE Transactions on Power Systems, vol. 25 (3), pp. 1504 - 1518. The contents of this paper are partially reproduced in Chapter 7.

- C. Corchero, F.J. Heredia (2010). *Optimal Day-Ahead Bidding in the MIBEL's Multimarket Energy Production System.* IEEE Proceedings of the 7th Conference on European Energy Market EEM10, vol. 1, pp. 1-6. The contents of this paper are partially reproduced in Chapter 6.

Accepted:

- M.P. Muñoz, C. Corchero and F.J. Heredia (2010). *Improving Electricity Market Price Scenarios by Means of Forecasting Factor Models.* Accepted in International Statistical Review. The contents of this paper are partially reproduced in Chapter 3.

Submitted:

- F.J. Heredia, M. Rider, C. Corchero. *A stochastic programming model for the optimal electricity market bid problem with bilateral contracts for thermal and combined cycle units.* Submitted in April 2009 to Annals of Operations Research. Third revision in progress. The contents of this paper are partially reproduced in Chapter 7.

- C. Corchero and F.J. Heredia. *A Stochastic Programming Model for the Thermal Optimal Day-Ahead Bid Problem with Physical Futures Contracts.* Submitted in March 2010 to Computers & Operations Research. Second revision in progress. The contents of this paper are partially reproduced in Chapter 4.

- E. Mijangos , C. Corchero and F.J. Heredia. *Perspective Cuts for Some Class of Electricity Market Optimization Problems.* Submitted in November 2010 to Engineering Optimization.

Presentations:

- C. Corchero and F.J. Heredia. *Optimal Short-Term Strategies for a Generation Company in the MIBEL.* APMOD 2006 Conference (Madrid, June 2006).

- C. Corchero and F.J. Heredia. *A mixed-integer stochastic programming model for the day-ahead and futures energy markets coordination.* EURO 2008 Conference (Prague, July 2007).

- M.Rider, F.J. Heredia and C. Corchero. *Optimal thermal and virtual power plants operation in the day-ahead electricity market.* APMOD 2008 Conference (Bratislava, May 2008).
- C. Corchero and F.J. Heredia. *Stochastic programming model for the day-ahead bid and bilateral contracts settlement problem.* IWOR 2008 Workshop (Madrid, June 2008).
- M.Rider, F.J. Heredia and C. Corchero. *Optimal thermal and virtual power plants operation in the day-ahead electricity market.* IWOR 2008 Workshop (Madrid, June 2008).
- F.J. Heredia and C. Corchero. *Stochastic programming models for optimal bid strategies in the Iberian Electricity Market.* 20th ISMP Symposium (Chicago, June 2009).
- M.P. Muñoz, C. Corchero and F.J. Heredia. *Improving electricity market price scenarios by means of forecasting factor models.* 57th Session of the International Statistical Institute (Durban, August 2009).
- C. Corchero and F.J. Heredia. *Optimal Day-Ahead Bidding Strategy in the MIBEL's Multimarket Energy Production System.* 7th International Conference on the European Energy Market (Madrid, June 2010).
- C. Corchero, F.J. Heredia, M.P. Muñoz. *Optimal Day-Ahead Bidding Strategy with futures and bilateral contracts. Scenario generation by means of factor models.* 24th EURO Conference (Lisbon, July 2010).
- F.J. Heredia, C. Corchero, M.P. Muñoz and E. Mijangos. *Electricity Market Optimization: finding the best bid through stochastic programming.* Conference on Numerical Optimization and Applications in Engineering 2010 (Barcelona, October 2010).

During the research stay in the Department of Mathematics, Statistics, Computing and Applications of the Università degli studi di Bergamo (Italy) some of the modeling concepts presented in this thesis were applied to the Italian Market with data from an Italian GenCo. This collaboration has also generated the scientific production that follows:

- M.T. Vespucci, C. Corchero, F.J. Heredia and M. Innorta (2009). *A decision support procedure for the short-term scheduling problem of a generation company operating on day-ahead and physical derivatives electricity markets.* Published at Proceedings of the *11^{th} International Conference on the Modern Information Technology in the Innovation Processes of the Industrial Enterprisers.*
- M.T. Vespucci, C. Corchero, F.J. Heredia and M. Innorta. *A Short-term Scheduling Model for a Generation Company operating on Day-Ahead and Physical Derivatives Electricity Markets.* Presentation in the *3rd FIMA International Conference* (Gressoney Saint Jean, January 2009).
- C. Corchero, M.T. Vespucci, F.J. Heredia and M. Innorta. *A decision support procedure for the short-term scheduling problem of a Generation Company operating on Day-Ahead and Physical Derivatives Electricity Markets.* Presentation in the *43rd Euro Working Group on Financial Modelling Meeting* (London, September 2008).

- C. Corchero, M.T. Vespucci, F.J. Heredia and M. Innorta. *A stochastic approach to the decision support procedure for a Generation Company operating on Day-Ahead and Physical Derivatives Electricity Market.* Presentation in the *EURO 2009* conference (Bonn, July 2008).

- M.T. Vespucci, C. Corchero, F.J. Heredia and M. Innorta.*A decision support procedure for a Price-Taker producer operating on Day-Ahead and Physical Derivatives Electricity Markets.* Presentation in the *V International Summer School in Risk Measurement and Control* (Roma, July 2008).

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APPENDIX A

Data

A.1 Thermal Units Characteristics

The relevant parameters of a thermal unit are:

- quadratic generation costs with constant, linear and quadratic coefficients, c_i^b (€), c_i^l (€/MWh) and c_i^q (€/MWh)^2 respectively, for unit i.
- $\overline{P}_i$ and $\underline{P}_i$ the upper and lower bound, respectively, on the energy generation (MWh) of a committed unit i.
- start-up, c_i^{on}, and shut-down, c_i^{off}, costs (€) for unit i.
- minimum operation and minimum idle time, t_i^{on} and t_i^{off} respectively, for unit i, i.e., the minimum number of hours that the unit must remain in operation once it is started up and the minimum number of hours that the unit must remain idle once it has been shut down before being started up again, respectively.

See Table A.1 for characteristics of the units used in this thesis.

A.2 Combined Cycle Units Characteristics

The relevant parameters of a combined cycle units are:

- P_c set of *pseudo units* of the CC unit $c \in C$.
- quadratic generation costs with constant, linear and quadratic coefficients, c_c^b (€), c_c^l (€/MWh) and c_i^q (€/MWh)^2 respectively, for unit c.

i	c_i^b €	c_i^l €/MWh	c_i^q €/MWh^2	$\underline{P}_i$ MW	$\overline{P}_i$ MW	c_i^{on} €	c_i^{off} €	$t_i^{on/off}$ h
1	151.08	40.37	0.015	160.0	350.0	412.80	412.80	3
2	554.21	36.50	0.023	250.0	563.2	803.75	803.75	3
3	97.56	43.88	0.000	80.0	284.2	244.80	244.80	3
4	327.02	28.85	0.036	160.0	370.7	438.40	438.40	3
5	64.97	45.80	0.000	30.0	65.0	100.20	100.20	3
6	366.08	-13.72	0.274	60.0	166.4	188.40	188.40	3
7	197.93	36.91	0.020	160.0	364.1	419.20	419.20	3
8	66.46	55.74	0.000	110.0	313.6	1298.88	1298.88	3
9	372.14	105.08	0.000	90.0	350.0	1315.44	1315.44	3

Table A.1: Operational characteristics of the thermal units

c	P_c	c_c^b €	c_c^l €/MWh	c_c^q €/MWh^2	$\underline{P}_c$ MW	$\overline{P}_c$ MW	c_c^{on} €	$t_c^{on/off}$ hr
1	5	151.08	50.37	0.023	160.0	350.0	803.75	2
1	6	224.21	32.50	0.035	250.0	563.2	412.80	2
2	7	163.11	55.58	0.019	90.0	350.0	320.50	2
2	8	245.32	31.10	0.022	220.0	700.0	510.83	2

Table A.2: Operational Characteristics of the Combined Cycle Units

- $\overline{P}_c$ and $\underline{P}_c$ the upper and lower bound, respectively, on the energy generation (MWh) of a committed unit c.
- start-up, c_c^{on}, and shut-down, c_c^{off}, costs (€) for unit c.
- minimum operation and minimum idle time, t_c^{on} and t_c^{off} respectively, for unit c.

See Table A.2 for characteristics of the units used in this thesis.

A.3 Data Sources

The sources for all data used in the case studies are described below. All the data of this thesis is public and it is either directly available in the web pages indicated or it has been calculated using some other public data.

- **Market data:** the day-ahead and intraday market prices are available at OMEL's site (www.omel.es) since January 1998 until today. In this work we use the data from January 1^{st}, 2004 to December 31^{th}, 2009. The reserve market prices are available at ESIOS site (www.esios.ree.es). Generic data about the quantities and clearing prices of the FCs is available at OMIP's site (www.omip.pt), this data has been used to define some examples of FCs.
- **Generation Company data:** the information about the thermal and combine cycle units in the study belongs to a GenCo that bids daily in the day-ahead market and also participates in the derivatives market (Table A.1). Most of the information about the generation units is available at the CNE's site (www.cne.es).

APPENDIX B

Glossary of symbols

Sets

B	Bilateral contract portfolio
C	Combined cycle units
F	Futures contract portfolio
F_i	Futures contracts in which unit i participates
I	Thermal units
I_{on_t}	Committed thermal units at interval t
I_j	Thermal units that participate in FC j
M^P	Scenarios where $\lambda^{D,s} < \lambda^P$
M^S	Scenarios where $\lambda^{D,s} > \lambda^S$
P	Pseudo units
P_C	Pseudo units of combined cycle unit c
S	Scenarios
T	Intervals
U	Pseudo and thermal units

Parameters

$\bar{b}^S$	Maximum quantity for selling BCs after the DAM
$\bar{b}^P$	Maximum quantity for purchase BCs after the DAM
c_i^b	Base generation cost of unit i
c_i^l	Linear generation cost of unit i
c_i^q	Quadratic generation cost of unit i
c_i^{on}	Turn-on cost of unit i
c_i^{off}	Shut-down cost of unit i
G_i	Number of periods that unit i has been on
In_t^D	Day-ahead market incomes on period t
In_t^{DF}	Day-ahead market and futures contracts incomes on period t
In_t^F	Futures contracts incomes on period t
H_i	Number of periods that unit i has been off
λ_j^B	Bilateral contract price for contract j
λ_t^D	Day-ahead market clearing price on period t
$\overline{\lambda}_t^D$	Mean of the day-ahead market price scenarios on period t
$\lambda_t^{D,s}$	Period t day-ahead market clearing price scenario s
λ_j^F	Futures contract price for contract j
$\lambda_t^{I,s}$	Period t intraday market clearing price scenario s
λ^P	Purchase price of the BC after the DAM
$\lambda_t^{R,s}$	Period t reserve market clearing price scenario s
λ^S	Sell price of the BC after the DAM
λ^V	Exercised price for the GPU
$\tilde{\lambda}^o{}_{ti}$	Threshold price for unit i on period t
$\overline{\lambda}_{ti}$	Threshold price for unit i on period t
$\underline{\lambda}_{ti}$	Threshold price for unit i on period t
L_j^B	Bilateral contract energy for contract j
L_j^F	Futures contract energy for contract j
μ	Lagrange multiplier
P^s	Probability of scenario s
$\underline{P}_i$	Minimum generation capacity of unit i
$\overline{P}_i$	Maximum generation capacity of unit i
p_{ti}^b	Bid energy for unit i on period t
p_{ti}^M	Matched energy for unit i on period t
p_{ti}^o	Bid curve energy for unit i on period t
$\overline{p}^V$	Maximum capacity of the GPU
π	Lagrange multiplier
ϱ_i	AGC capacity of unit i
ρ_{ti}^s	Auxiliary parameter
$\rho_{ti}^{B,s}$	Auxiliary parameter

t_c^C	Minimum idle time for unit c
t_i^{on}	Minimum start-up time for unit i
t_i^{off}	Minimum idle time for unit i
θ_{ti}^s	Auxiliary parameter
$\theta_{ti}^{B,s}$	Auxiliary parameter

Functions

$B(\ )$	Benefit function
$C(\ )$	Generation cost function
$\lambda_{ti}^b(\ )$	Bid function for unit i and period t
$\lambda_{ti}^o(\ )$	Optimal bid function for unit i and period t
$p_{ti}^M(\ )$	Matched energy function for unit i and period t
$p_{ti}^{M,s}(\ )$	Matched energy function for scenario s, unit i and period t
$p_{ti}^{P,s}(\ )$	Matched purchase energy function for scenario s, unit i and period t
$p_{ti}^{R,s}(\ )$	Matched residual sale energy function for scenario s, unit i and period t
$p_{ti}^{S,s}(\ )$	Matched sale energy function for scenario s, unit i and period t
$\rho_{ti}(\ )$	Auxiliary function
$\rho_{ti}^B(\ )$	Auxiliary function
$\theta_{ti}(\ )$	Auxiliary function
$\theta_{ti}^B(\ )$	Auxiliary function

Variables

a_{ti}	Thermal unit i turned on at interval t (binary)
a_{tc}^C	Combined cycle unit c turned on at interval t (binary)
b_{ti}	Generation of thermal unit i at interval t allocated to BC
b_{ti}^G	Generation of GPU unit at interval t allocated to BC
$b_t^{P,s}$	Generation allocated to the purchase BC after the DAM at interval t
$b_t^{S,s}$	Generation allocated to the sale BC after the DAM at interval t
c_{ti}^u	Start-up cost of unit i at interval t
c_{ti}^d	Shut-down cost of unit i at interval t
e_{ti}	Thermal unit i turned off at interval t (binary)
e_{tc}^C	Combined cycle unit c turned off at interval t (binary)
f_{tij}	Generation of unit i at interval t allocated to FC j
g_{ti}^s	Total physical production of unit i at interval t and scenario s
m_{ti}^s	Matched energy at the IM of unit i at interval t and scenario s
p_{ti}^s	Matched energy variable of unit i at interval t and scenario s
p_t^V	Exercised energy of the VPP at interval t
$p_t^{P,s}$	Matched energy for the GPU purchase bids at scenario s and interval t
$p_t^{S,s}$	Matched energy for the GPU sale bids at scenario s and interval t
q_{ti}	Instrumental offer quantity for unit i at interval t
r_{ti}^s	Participation at the RM of unit i at interval t and scenario s (binary)
u_{ti}	Thermal unit i start-up at interval t (binary)
u_{tc}^C	Combined cycle unit c start-up at interval t (binary)
w_{ti}^P	Auxiliary variable
w_{ti}^R	Auxiliary variable
w_{ti}^S	Auxiliary variable
v_{ti}^s	Auxiliary variable
x_t^V	VPP rights exercised at interval t (binary)
y_{ti}^P	Auxiliary variable
y_{ti}^R	Auxiliary variable
y_{ti}^S	Auxiliary variable
z_{ti}^S	Auxiliary variable

Abbreviations

AGC	Automatic Generation Control
BC	Bilateral contract
CC	Combined cycle
CT	Combustion turbine
D	Day in study
DAM	Day-ahead market
DAMB	Day-ahead market bidding problem
DAMB-FC	Day-ahead market bidding with futures contracts problem
DAMB-FBC	Day-ahead market bidding with futures and bilateral contracts problem
DAMB-FC	Day-ahead market bidding with futures and bilateral contracts in a multimarket environment problem
DM	Derivatives market
EEV	Expected result of using EV solution
EV	Expected value problem
FC	Futures contract
GenCo	Generation company
GPU	Generic programming units
HRSG	Heat recovery steam generator
IM	Intraday market
ISO	Independent system operator
MIBEL	Iberian Electricity Market
MO	Market operator
OMEL	Iberian market operator
OMIP	Iberian derivatives market operator
PU	Pseudo unit
REE	Red Eléctrica de España
RM	Ancillary service or reserve market
RP	Recourse problem
ST	Steam turbine
TSFA	Time series factor analysis
UCP	Term contract unit
VPP	Virtual power plant
VSS	Value of the stochastic solution

APPENDIX C

Coefficients of the Forecasting Models

μ	ϕ_{12}	ϕ_{13}	ϕ_{23}	ϕ_{96}	ϕ_{120}	ϕ_{144}	ϕ_{168}	ϕ_{336}	θ_1	θ_2
0.078	-0.038	0.143	0.037	0.132	0.078	0.095	0.525	0.045	0.133	0.170
θ_3	θ_4	θ_5	θ_6	θ_{13}	θ_{24}	θ_{48}	θ_{72}	θ_{168}	θ_{504}	θ_{168}
0.147	0.110	0.106	0.085	0.195	-0.222	-0.141	-0.151	0.394	0.010	0.986

Table C.1: ARIMA model significant coefficients.

Hour	Factor 1	Factor 2	Factor 3	Communality
1	0.007	0.109	-0.006	0.419
2	-0.020	0.109	0.007	0.669
3	-0.018	0.133	0.015	0.844
4	-0.009	0.156	0.011	**0.944**
5	0.005	0.164	0.005	**0.911**
6	0.011	0.121	-0.003	**0.818**
7	0.030	0.065	-0.006	0.547
8	0.078	0.049	-0.015	0.461
9	0.122	0.013	-0.033	0.585
10	0.142	0.004	-0.048	0.737
11	0.141	-0.006	-0.036	**0.903**
12	0.137	-0.003	-0.025	**0.932**
13	0.133	-0.002	-0.008	**0.895**
14	0.119	-0.002	0.008	**0.839**
15	0.098	0.012	0.041	0.730
16	0.106	0.001	0.058	**0.795**
17	0.102	-0.004	0.065	**0.822**
18	0.096	-0.020	0.068	**0.832**
19	0.084	-0.021	0.067	0.719
20	0.052	-0.016	0.075	0.633
21	0.015	-0.006	0.094	0.557
22	-0.003	0.004	0.095	0.608
23	-0.025	0.007	0.106	0.561
24	-0.024	0.006	0.084	0.366
CFI	0.347	0.587	0.755	
RMSEA	0.289	0.241	0.190	

Table C.2: Factors loadings and measures of goodness of fit.

Hour	f_1	f_2	f_3	ϕ_1	ϕ_2	ϕ_3	ϕ_4	ϕ_5	ϕ_{24}	ϕ_{48}	ϕ_{72}	ϕ_{96}	ϕ_{120}
1	0.023			0.705				-0.167	0.237				
2		0.021		0.635				0.058			0.077		0.061
3		0.036	0.013	0.877					-0.184				
4	-0.007		0.014	1.00				-0.106			0.041		
5		-0.044	0.010	1.167		-0.170		-0.077	0.314				
6		-0.021		0.627		0.172			0.238	0.093			
7	-0.011	-0.024	0.011	0.825	-0.178	-0.248		0.319	0.427				
8		-0.031	0.022	0.982				-0.216	0.335				
9			0.021		0.701				0.214				
10	0.024		0.015	0.850	-0.093				-0.104	0.054			
11	0.013	0.007	0.953	-0.119		-0.108							0.059
12	0.008			1.022		-0.140							
13	-0.040		0.015	1.373	-0.505				0.268	0.080			
14	-0.028			1.128	-0.222				0.285	0.058			
15	-0.013		0.012	0.818					0.168		0.072	-0.084	
16		-0.006	0.940				-0.193	0.271		0.048		-0.050	
17	-0.014			1.133	-0.368	0.247		-0.071	0.121	0.055			
18			0.013	0.954		-0.404	0.397						
19	-0.051	0.016	-0.020	1.031	-0.580	0.301			0.535	0.108	0.096		
20	-0.034	0.019	-0.043	0.666		-0.207	0.219		0.587		0.136		
21	-0.019		-0.025	0.696	-0.281	0.293			0.440		0.136		
22		0.010	-0.057	0.487	-0.266	0.208	0.203		0.701				
23	0.010	0.011		0.547				0.252					
24	-0.015			0.714			0.298	-0.193	0.115		0.149		0.092

Table C.3: Regression coefficients.

APPENDIX D

Italian Day-Ahead Market Price

In this appendix, the time series factor analysis is performed for the data of the Italian day-ahead market price. The data is obtained from the Mercato Elettrico web site (`http://www.mercatoelettrico.org/`.

The same steps developed in section 3.3.2 are perform with this set of prices. It can be observed in Figure D.1(a) that the shape of the prices along one day is slightly different to the Spanish one.

In this case 4 factors are needed, they are shown in Figure D.2 and their interpretation is similar to the Spanish case. The different to zero loads represents the group of hours that behave similar, the first factor represents clearly the working hours, notice that this factor ends before than the Spanish one because of the different habits in each country. The second factor represent the first hours of the day, the opposite to the third factor which represents the last ones. Finally, the fourth factor is the most difficult to interpret, but it corresponds to the very peak hours of the day, that means, the hour with maximum load and variability.

Finally, Figure D.1(b) shows the forecasts, the real prices and the forecast IC for a week. The MSE for the forecasts in this case is 0.521.

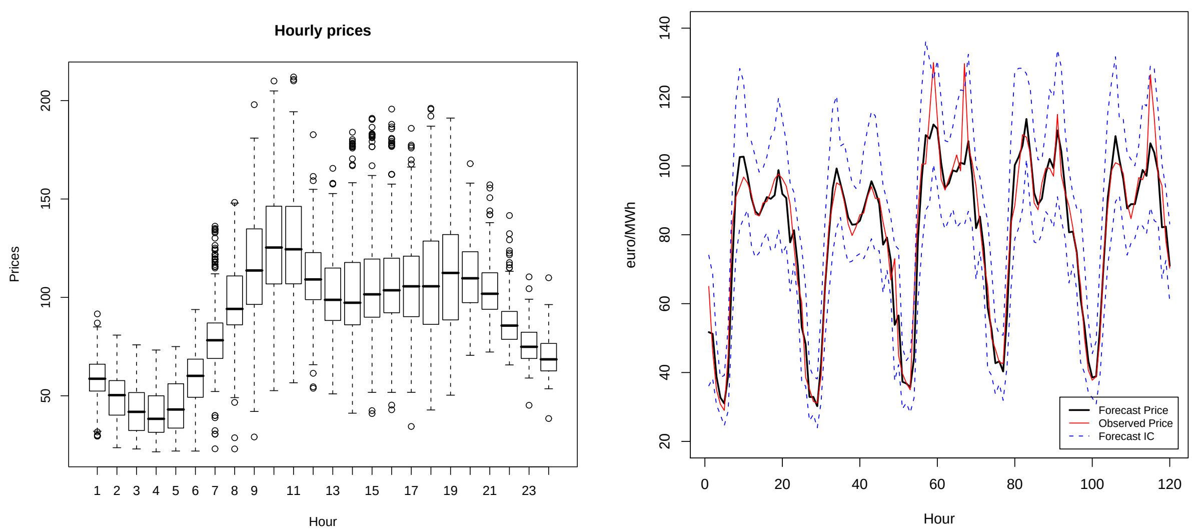

Figure D.1: (a)Boxplot of Italian hourly prices. (b)One-step-ahead forecast prices with TSFA model for the Italian prices.

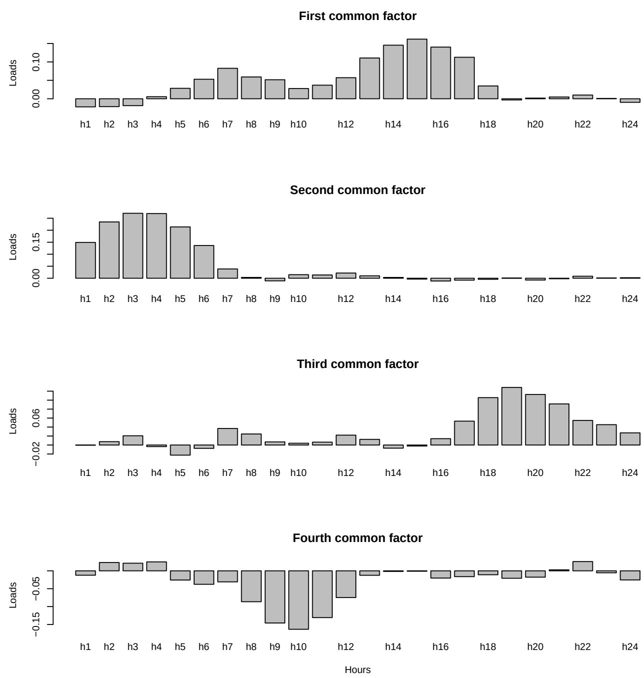

Figure D.2: Loads of the common factors for the Italian prices.

APPENDIX E

A Stochastic Approach to the Decision Support Procedure for a GenCo Operating in a Day-Ahead and Physical Derivatives Electricity Market

Introduction

This appendix presents work performed jointly with professors Maria Teresa Vespucci and Mario Innorta, from the *Università degli studi di Bergamo* (Italy). The work consists of building the stochastic version and including physical derivatives products in an existing model (Vespucci *et al.*, 2008) for the GenCo short-term operation. The motivation for this work arose because the Italian electricity market was planning the creation of an derivatives market with some similarities to that which was being created in Spain. Both the model and the data are built on the assumptions of the Italian Electricity Market (`http://www.mercatoelettrico.org/`). The only external issue is the inclusion of the Spanish rules for the physical derivatives products, because the Italian rules were expected to be very similar to the Spanish ones.

The most significant differences from Chapters 4 and 5 are the inclusion of the hydro units into the model and the hypothesis for the market participation of a price-taker GenCo. Specifically, instead of studying the optimal bidding curve, it is supposed that the quantity sent to the DAM bid is the remaining quantity after covering the bilateral contracts. The rules for the DAM participation are equal to the ones presented in Section 2.1.4, except for the obligatory nature of participation in the market. Simultaneous to participation in the DAM, we supposed that participation in the derivatives market follows the Spanish rules defined in Section 2.1.4.2.

Therefore, in this work a decision support procedure has been developed for the short-term hydro-thermal resource scheduling problem of a GenCo who operates in the Italian Electricity Market and who aims to maximize its own profit. The generation company is supposed to be a price-taker and the resources owned by the producer are hydroelectric plants and thermal units. On the short-term horizon, the generation company has to solve the unit commitment problem for the thermal units, the economic dispatch problem for the available hydro plants and the committed thermal units and settlement of the futures contracts which controls the quantity that each committed thermal unit or hydro plant has to produce to fulfill them. Its decisions must be compatible with both technical constraints and market constraints. The model is a stochastic mixed integer linear programming model, where the objective function represents the total profit determined on the basis of price scenarios and the constraints describe the hydro system, the thermal system, the futures contracts settlement and the market.

Notice that the notation in this chapter is independent to the general notation used in the rest of this thesis. The original notation of the work has been maintained.

E.1 Model

In this section the mathematical model is introduced. The planning horizon is short-term with periods of 1 hour. Let T denote the number of periods considered and let t, $0 \leq t \leq T$, denote the period index, where $t = 0$ denotes the last hour of the planning horizon immediately preceding the one in consideration.

E.2 Model of the Hydroelectric System

The hydro system consists of a number of cascades, i.e. sets of hydraulically interconnected hydro plants, pumped-storage hydro plants and basins. It is mathematically represented by a directed graph (see Figure E.1): water storages (basins) correspond to a set J of nodes; water flows correspond to a set I of arcs; and the interconnections are represented by the arc-node incidence matrix, whose (i, j)-entry is denoted by $A_{i,j}$ ($A_{i,j} = -1$, if arc i leaves node j; $A_{i,j} = 1$ if arc i enters node j and; $A_{i,j} = 0$ otherwise). The GenCo has to determine the optimal use of the hydro resources which are available in the planning period. They are given by the initial storage volumes $v_{j,0}(m^3)$ in all basins $j \in J$ and the natural inflows $B_{j,t}(m^3/h)$ in all basins $j \in J$ and hours $t \in T$. The GenCo must schedule the hourly production of each hydro plant, which is expressed as the product of the hydro plant energy coefficient times the turbine volume in hour t, as well as the hourly pumped and spilled volumes. The decision variables of the hydro scheduling problem are:

- Water flow: $q_{i,t}(m^3/h)$ in arc i along hour t
- Storage volume: $v_{j,t}(m^3)$: in basin j at the end of hour t

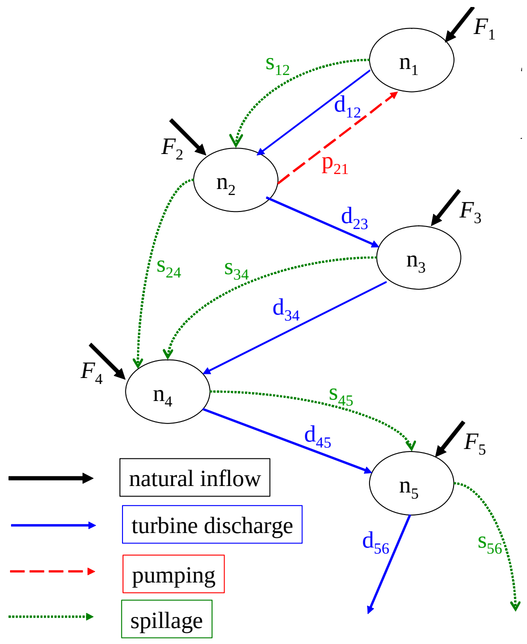

Figure E.1: Multi-graph representing the hydro system.

The mathematical relations that describe the hydro system are the following:

$$0 \leq q_{i,t} \leq \overline{q}_i \qquad i \in I,\ 1 \leq t \leq T \tag{E.1}$$

$$0 \leq v_{j,t} \leq \overline{v}_j \qquad j \in J,\ 1 \leq t \leq T \tag{E.2}$$

$$\underline{v}_{j,T} \leq v_{j,T} \qquad j \in J \tag{E.3}$$

$$v_{j,t} = v_{j,t-1} + B_{j,t} + \sum_{i \in I} A_{i,j} q_{i,t-\rho_i} \qquad j \in J,\ 1 \leq t \leq T \tag{E.4}$$

where (E.1) represents requirements that the water flow in arc i in hour t is nonnegative and bounded above by the maximum volume $\overline{q}_i$ that can be either turbined, or pumped, or spilled. (E.2) requires that the storage volume in basin j at the end of hour t is nonnegative and bounded above by the maximum storage volume $\overline{v}_j$. Equation (E.3) imposes that the storage volume in basin j at the end of hour T is bounded below by the minimum storage volume $v_{j,T}$, determined by the medium-term resource scheduling, in order to provide the required initial storage volume at the beginning of the next planning period. Finally, constraint (E.4) imposes that the storage volume in basin j at the end of hour t equals the basin storage volume at the end of hour $t-1$ plus inflows in hour t, taking into account the time $\rho_i(h)$ required by the water flow leaving node i to reach node j, minus outflows in hour t. A positive energy coefficient $k_i(MWh/m^3)$ is associated to every arc i representing generation; the product $k_i q_{i,t}$ expresses the energy produced in hour t. A negative energy coefficient k_i is associated to every arc i representing pumping; the product $k_i q_{i,t}$ represents the energy used for pumping in hour t. Zero energy coefficients k_i are associated to arcs representing spillage.

E.3 Model of the Thermal System

Let K denote the set of thermal units owned by the power producer. For every unit $k \in K$ the producer must decide on the unit commitment, taking into account minimum up-time and minimum down-time constraints; and the hourly production of each committed unit, taking into account lower and upper bounds on production levels and ramping constraints. The decision variables in the thermal system problem are:

- Production level: $p_{k,t}(MWh)$ of unit k in hour t
- Unit commitment binary variables: $\alpha_{k,t}$ representing the start-up; $\beta_{k,t}$ representing the shut-down; and $\gamma_{k,t}$ representing that unit is *on* , for unit $k \in K$ in hour $t \in T$

The set of constraints that control the unit commitment taking into account the minimum up- and minimum down-time are the following:

$$\gamma_{k,t-1} + \alpha_{k,t} = \gamma_{k,t} + \beta_{k,t} \qquad k \in K,\ 1 \leq t \leq T \qquad \text{(E.5)}$$

$$\text{if } \gamma_{k,0} = 1 \quad \text{then } \gamma_{k,t} = 1 \qquad 1 \leq t \leq ta_k - nh_k \qquad \text{(E.6)}$$

$$\sum_{\tau=t+1}^{\min(t+ta_k-1,T)} \gamma_{k,\tau} \geq \alpha_{k,t} \min\left(ta_k - 1, T - t\right) \qquad k \in K,\ 1 \leq t \leq T \qquad \text{(E.7)}$$

$$\text{if } \gamma_{k,0} = 0 \quad \text{then } \gamma_{k,t} = 0 \qquad 1 \leq t \leq ts_k - nh_k \qquad \text{(E.8)}$$

$$\sum_{\tau=t+1}^{\min(t+ts_k-1,T)} \gamma_{k,\tau} \leq (1 - \beta_{k,t}) \min\left(ts_k - 1, T - t\right) \qquad k \in K,\ 1 \leq t \leq T \qquad \text{(E.9)}$$

where (E.5) fixes the values of the binary variables representing states in hours $t-1$ and t and manoeuvres in hour t so that they are coherent, i.e. no status change can take place without the corresponding manoeuvre. Information about the status of unit k at the beginning of the scheduling period are given by the following data $\gamma_{k,0}$, the state at the last hour of the previous planning period, and $nh_k(h)$, number of hours in which unit k has been in state $\gamma_{k,0}$ since the last change in the previous scheduling period. If $\gamma_{k,0} = 1$ and $nh_k \neq 0$ indicate that unit k as *on* at the beginning of the scheduling period and was started-up in hour $T - nh_k$ of the previous scheduling period, $\gamma_{k,0} = nh_k = 0$ indicates that unit k is *off* at the beginning of the scheduling period. The unit commitment must satisfy minimum up-time constraints and minimum down-time imposed by constraints (E.6)-(E.8) where $ta_k(h)$ is the minimum number of hours unit k must be *on* after start-up and $ts_k(h)$ minimum number of hours unit k must be *off* after shut-down.

Moreover, the hourly production levels $p_{k,t}$ are subject to the following constraints:

$$\gamma_{k,t}\underline{p}_k \leq p_{k,t} \leq \gamma_{k,t}\overline{p}_k \qquad k \in K,\ 1 \leq t \leq T \qquad \text{(E.10)}$$

$$p_{k,t} - p_{k,t-1} \leq \delta u_k + \alpha_{k,t}\left(vsu_k - \delta u_k\right) \qquad k \in K,\ 1 \leq t \leq T \qquad \text{(E.11)}$$

$$p_{k,t} - p_{k,t-1} \geq -\delta d_k + \beta_{k,t}\left(-vsd_k + \delta d_k\right) \qquad k \in K,\ 1 \leq t \leq T \qquad \text{(E.12)}$$

Those constraints model the decisions that the producer has to made in order to fix the production level $p_{k,t}$:

(E.10) If unit k is *on* in hour t, the hourly production $p_{k,t}$ must be neither less than the minimum level $\underline{p}_k$ nor greater than the maximum level $\overline{p}_k$; if unit k is *off* in hour t, the hourly production must be zero.

(E.11) *Ramp-up* constraint: if unit k is started-up in hour t, the hourly production $p_{k,t}$ cannot be greater than vsu_k (maximum production of the unit at start-up); moreover, if the production levels in two subsequent hours, $t-1$ and t, are such that $p_{k,t-1} \leq p_{k,t}$, the production variation is bounded above by δu_k (maximum production increase per hour).

(E.12) *Ramp-down* constraint: if unit k is shut-down in hour t, the hourly production $p_{k,t}$ cannot be greater than vsd_k (maximum production at shut-down) and if the production levels in two subsequent hours $t-1$ and t are such that $p_{k,t-1} \geq p_{k,t}$, the production variation is bounded above by δd_k (maximum production decrease per hour).

E.4 Futures Contract Dispatch and Market Constraints

Let F denote the set of futures contracts assigned to the GenCo and let I_f and K_f denote the subset of hydro plants and the subset of thermal units, respectively, assigned to futures contract f. Moreover, let L_f denote the constant quantity of electricity to be delivered every hour and let λ_f denote the futures contract settlement price. The decision variables of the futures contracts dispatch problem are:

- Energy to be produced by hydro plant: $gh_{i,t,f}$(MWh) hydro plant i at hour t for Futures Contract f
- Energy to be produced by thermal unit: $gt_{k,t,f}$(MWh) thermal unit k at hour t for Futures Contract f

And the Futures Contract dispatch problem is subject to the following restrictions:

$$0 \leq \sum_{f \in F_i} gh_{i,t,f} \leq k_i q_{i,t} \qquad i \in I_g,\ 1 \leq t \leq T \tag{E.13}$$

$$\sum_{f \in F_k} gt_{k,t,f} \leq p_{k,t} \qquad k \in K,\ 1 \leq t \leq T \tag{E.14}$$

$$\sum_{i \in I_f} gh_{i,t,f} + \sum_{k \in K_f} gt_{k,t,f} = L_f \qquad f \in F,\ 1 \leq t \leq T \tag{E.15}$$

where, (E.13)-(E.14) assures the nonnegativity of the decision variables while imposing that the production used for FC covering does not exceed the total hourly production. Note that F_i is the subset of futures contracts in which hydro plant i participates and F_k is the subset of futures contracts in which thermal unit k participates. Constraint (E.15) imposes that the energy quantity L_f is only produced by the unit assigned to futures contract f where I_f represents the subset of hydro plants that participate in FC f; and K_f represents the subset of thermal units that participate in FC f.

Moreover, the producer must satisfy in every hour t the commitments derived from bilateral contracts. The energy car_t(MWh) to be delivered on the basis of a BC may be either produced or bought in the DAM. If the total production in hour t exceeds the load from BC, the excess quantity $sell_{t,s}$(MWh) is sold on the DAM; if its total production is less than the load from the BC, the producer has to buy in the DAM the amount of energy $buy_{t,s}$(MWh) necessary to meet the BC load demand. These two variables are the only two-stage variables of the model because they are the ones that depend on the DAM price and, therefore, on the price scenarios. The market constraints are represented by equations (E.16):

$$\sum_{i \in I} k_i q_{i,t} + \sum_{k \in K} p_{k,t} + buy_{t,s} = car_t + sell_{t,s} + \sum_{j=1}^{L} L_j \qquad 1 \leq t \leq T,\ 1 \leq s \leq S \tag{E.16}$$

E.5 Objective Function

The power producer is assumed to be a price-taker. It is assumed that the hourly sell price λ_t(€/MWh) can be represented by a set of scenarios $\lambda_{t,s},\ s \in S$, as well as the hourly purchase price $\mu_{t,s},\ \mu_{t,s} \geq \lambda_{t,s}$.

Two types of costs are associated to thermal production: manoeuvres costs and generation costs. For every unit k, costs csu_k(€) and csd_k(€) are associated to every start-up and shut-down manoeuvre, respectively. The thermal generation cost $G_{k,t}$ of unit k in hour t is assumed to be a convex quadratic function of the production level $p_{k,t}$.

$$G_{k,t}(p_{k,t}) = g_{2,k} p_{k,t}^2 + g_{1,k} p_{k,t} + g_{0,k}$$

where $g_{2,k}$ (€/MWh^2), $g_{1,k}$(€/MWh) and $g_{0,k}$(€) are the quadratic generation cost coefficients for unit k. In order to obtain a mixed integer linear programming model the generation cost functions are linearized. For every unit k the interval $\left[\underline{p}_k, \overline{p}_k\right]$ is divided in H subintervals of width $\overline{pl}_{k,h}$, $1 \leq h \leq H$. Let $p_{k,t,h-1}$ and $p_{k,t,h}$ denote the extreme points of subinterval h; let $gl_{k,h}$ denote the slope of the straight line segment passing through points $(p_{k,t,h-1}, G_{k,t}(p_{k,t,h-1}))$; and $(p_{k,t,h}, G_{k,t}(p_{k,t,h}))$ and let $pl_{k,t,h}$ denote the real variable associated to subinterval h. The linearized generation costs of thermal unit k in hour t are then given by

$$G_{k,t}^{lin}(p_{k,t}) = \left(g_{2,k} \underline{p}_k^2 + g_{1,k} \underline{p}_k + g_{0,k}\right) \gamma_{k,t} + \sum_{h=1}^{H} gl_{k,h} pl_{k,t,h}$$

where variables $pl_{k,t,h}$ are subject to the constraints

$$p_{k,t} = \underline{p}_k \gamma_{k,t} + \sum_{h=1}^{H} pl_{k,t,h} \qquad k \in K,\ 1 \leq t \leq T \tag{E.17}$$

$$0 \leq pl_{k,t,h} \leq \overline{pl}_{k,h} \qquad k \in K,\ 1 \leq t \leq T,\ 1 \leq h \leq H \tag{E.18}$$

Then, the following objective function represents the power producer's profits:

$$f(sell, buy, \alpha, \beta, p) = \sum_{t=1}^{T} \left[\sum_{s=1}^{S} P_s \left(\lambda_{t,s} sell_{t,s} - \mu_{t,s} buy_{t,s} \right) - \sum_{k \in K} \left(csu_k \alpha_{k,t} + csd_k \beta_{k,t} + G_{k,t}^{lin} \left(p_{k,t} \right) \right) \right]$$

E.6 Complete Model

The full model developed in the preceding sections can be formulated as:

$$\begin{cases} \max & f(sell, buy, \alpha, \beta, p) \\ \text{s.t.} & \\ & \text{Eq. } (E.1) - (E.4) \quad \text{Hydro system} \\ & \text{Eq. } (E.5) - (E.12) \quad \text{Thermal system} \\ & \text{Eq. } (E.13) - (E.15) \quad \text{FC covering} \\ & \text{Eq. } (E.16) \quad \text{Market constraint} \\ & \text{Eq. } (E.17) - (E.18) \quad \text{Linearization constraints} \end{cases} \tag{E.19}$$

This formulation corresponds to an optimization problem with mixed continuous and binary decision variables, linear objective function and a set of linear constraints.

E.7 Computational Results

A set of computational tests has been performed in order to validate the described model. The model has been implemented in GAMS and solved by CPLEX. Real data from an Italian GenCo generation company, with 17 thermal units and 12 hydro plants was used. The prices have been modeled through factor models, as has been explained in Appendix D, and a set of 50 scenarios has been built.

Figure E.2 shows the generation used to cover the bilateral and futures contracts for each thermal unit and interval. As in previous case studies, it can be seen that there are some units that submit nearly the maximum generation to these products while others use only a small percentage of their capacity. It must be noted that the level of production of all units is almost constant; this occurs because the Italian prices hardly vary throughout the day. Italian DAM prices do not fall at the midday so much as Spanish prices, and this causes the generation level of the units to be more constant after 7 a.m., once the prices achieve their daily mean.

Figure E.3 shows the same as Figure E.2, but for the hydro units. It has to be noted that a hydro system has not production costs, therefore the behavior of hydro units is totally different from that of thermal units. They do not depend so much on the prices as on the natural inflows and the expected level at the end of the day. It is also natural to send the hydro capacity to the market because, although the prices were not high enough to cover thermal production costs, the hydro units always generate benefits. Figure represents the generation level of two units for

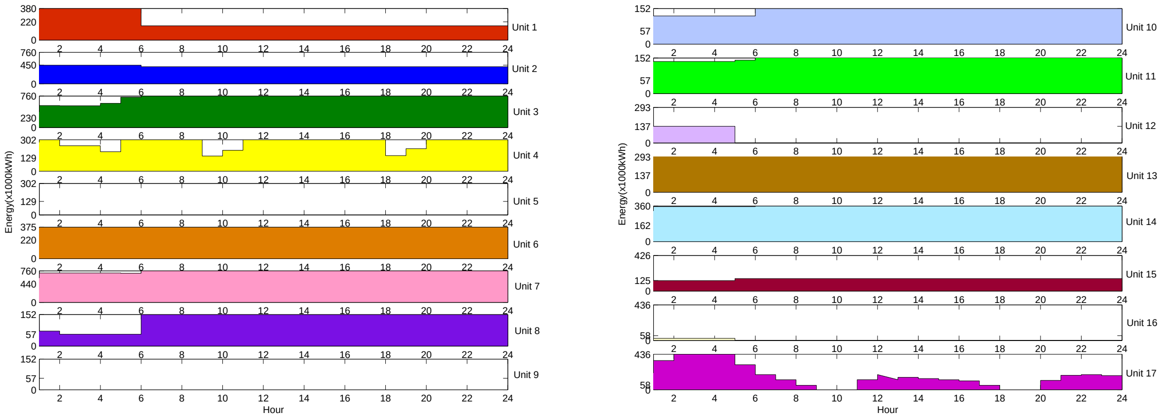

Figure E.2: Energy committed to bilateral and futures contracts by each thermal unit.

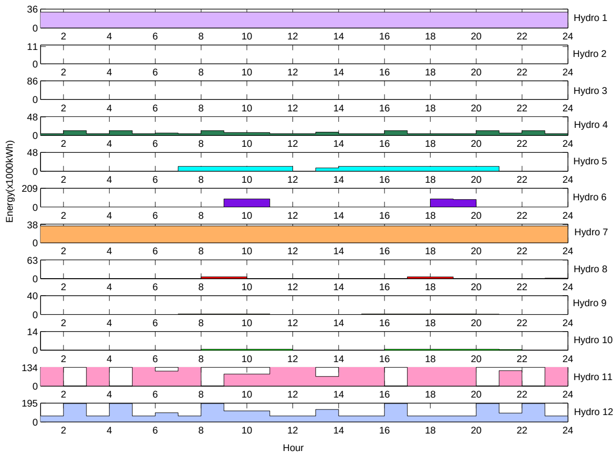

Figure E.3: Energy committed to bilateral and futures contracts by each hydro unit.

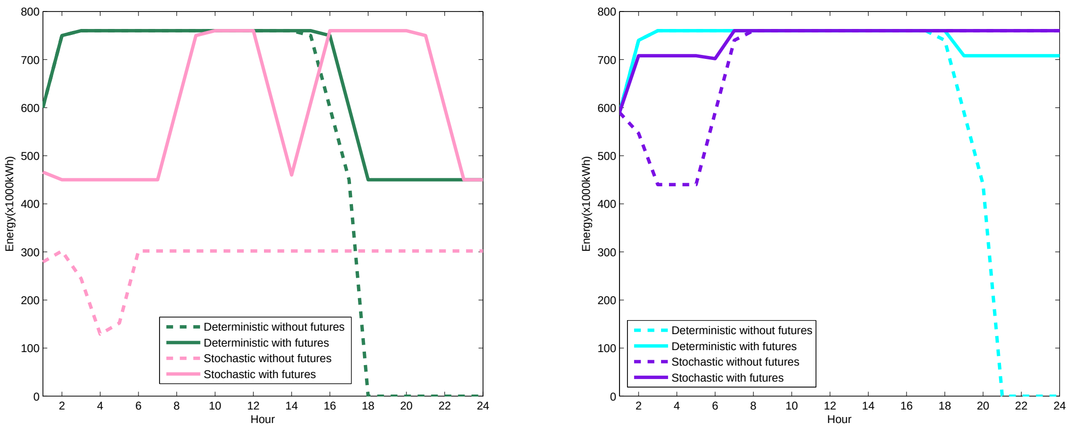

Figure E.4: Level of production of two units for each hour in the four possible models.

all the intervals. It shows the difference between the model that takes into account coordination with the futures contract covering and the one without this coordination, as well as the difference between the deterministic and the stochastic approach. It can be seen that the pattern present large differences between the four models which confirms to us the influence of stochasticity and the inclusion of futures and bilateral contracts in the optimal generation of the units.

E.8 Conclusions

A mixed integer linear programming model has been defined for the profit maximizing short-term resource scheduling problem of a GenCo operating in the Italian Electricity Market. Energy delivery commitments derived from bilateral and futures contracts are used to hedge against DAM price uncertainty. The optimization model allows the producer to determine the optimal resource schedule, taking into account the operational constraints of hydro and thermal plants. DAM price stochasticity is represented by through use of a scenario fan.