Automated Deduction with Built-in Theories

Completeness Results
and
Constraint Solving Techniques

Tesi doctoral presentada al
Departament de Llenguatges i Sistemes Informàtics
de la Universitat Politècnica de Catalunya

per a optar al grau de
Doctor en Informàtica

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Barcelona, 1 de setembre de 2001

Chapter 9

Directions for Further Research

9.1 Open problems from Chapter 4

1. Now it becomes interesting to explore completeness results for deduction modulo equational theories E for which monotonic E-compatible total orderings do not exist, but that can be handled with the techniques described in Chapter 4.
2. It has to be more carefully explored to what extent other redundancy notions, like demodulation, are applicable in our framework for ordered paramodulation.
3. In Chapter 4 we dealt with inference systems with eager selection of negative literals, that is, a negative literal is selected whenever there is any. For the moment we know of no *direct* way of proving the completeness for strategies with selection of (maximal) positive literals, although in Chapter 6 a relatively simple proof transformation technique is given for the Horn case, by which the completeness is shown for arbitrary selection strategies and arbitrary paramodulation-based inference systems that are complete with constraint inheritance and eager selection of negative literals. However, there exist examples showing the incompatibility of these techniques with tautology deletion (see Example 47 of Chapter 4). This makes it unlikely that a standard model construction-based proof, where such tautologies are redundant, can be found.

9.2 Open problems from Chapter 5

Regarding Knuth-Bendix completion, after the results of Chapter 5 the following interesting questions remain unanswered:

4. Does Theorem 52 still hold when simplification by rewriting with respect to the reduction ordering $\succ_r$ is applied?

5. Does it hold when the strict ordered paramodulation rule of Definition 28 (where inferences at topmost positions of small sides w.r.t. $\succ$ are not needed) is used?

6. Would it hold when inferences with (or on) small sides w.r.t. $\succ_r$ are not computed at all? Or, in other words, is *superposition* w.r.t. arbitrary reduction orderings complete?

9.3 Open problems from Chapter 6

The results of Chapter 6 could be extended in several directions, the most interesting ones being:

- Strategies with non-eager selection of negative equations. We think our proof transformation technique could be adapted to cover also strategies with non-eager selection of negative equations, i.e., proving that if there exists *any* (not necessarily eager negative) complete selection strategy selecting a single literal, then arbitrary selection strategies are complete. A possible way to go could be to first transform such proofs into proofs with eager selection of negative equations.

- Answer computation. In Chapter 6 we focussed on refutation completeness, but we believe that our techniques easily extend to proving the completeness for *answer computation*, thus obtaining similar results for this purpose as the ones of [Lyn97] for total reduction orderings and superposition. The key idea is that solutions σ are preserved during our transformation process.

- Non-equality constraints. Very often it is the case that an inference system works not only with equality constraints but also with other kind of constraints such as, for example, ordering constraints. But, as seen in section 6.4 of Chapter 6, the whole transformation process can be done in the same way if there exists some substitution σ that satisfies all the constraints. Note that this is a reasonable assumption for many inference systems since, in our transformation, the equations involved in the inferences of the proof by $\mathcal{A}$ are the same as in the proof by $\mathcal{N}$. It may be interesting to study with which other kind of constraints this transformation method preserves completeness.

- General clauses. Our result is for first-order Horn clauses with equality. We think that it would not be difficult to adapt our proof transformation method to the case of general clauses, provided no *factoring* inferences occur in the proofs. With factoring, incompleteness already appears in the propositional case, as shown by the following counter example from [dN96].

Example 169 Suppose we have

1. $\rightarrow p, q$
2. $\rightarrow q, r$
3. $\rightarrow r, p$
4. $p, q \rightarrow$
5. $q, r \rightarrow$
6. $r, p \rightarrow$

This set of clauses is unsatisfiable, and factoring (in this propositional case, elimination of repeated occurrences of positive literals) is needed for obtaining the empty clause. Now, suppose we choose the following arbitrary selection:

1. $\rightarrow \underline{p}, q$
2. $\rightarrow \underline{q}, r$
3. $\rightarrow \underline{r}, p$
4. $\underline{p}, q \rightarrow$
5. $\underline{q}, r \rightarrow$
6. $\underline{r}, p \rightarrow$

By applying resolution involving only the selected literals we only obtain tautologies of the form $A \rightarrow A$, and after resolving with/on them we get clauses of the initial set. Therefore this is a counter example to the completeness of arbitrary strategies when factoring is required, even if all tautologies are kept.

9.4 Open problems from Chapter 7

The first task that has to be started is the development of an experimental implementation of the inference system, combined with some simple redundancy notions. When doing this, one should not focus on efficiency of the implementation. Rather, one should carry out the development in the context of a flexible "toolkit" system like Saturate [NN93, GNN99]. Only from experimens with such an implementation it will become clearer in which directions one should continue developing the theoretical work described in this chapter.

9.5 Open problems from Chapter 8

As for Chapter 7, again the first task that has to be started is the development of an experimental implementation. In fact, the implementation of the inference system described in Chapter 7 requires at least an approximation of a constraint solving mechanism like the one described in Chapter 8. From such a constraint solver running on more or less real-life problems, one could draw conclusions about its efficiency and possible bottlenecks and enhancements.

Concerning further theoretical work, we expect that the ideas given here will provide new insights for the development of constraint solving methods for other E-compatible orderings, especially over fixed signatures.

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Index