Jordi Cadafalch Rabasa

DOCTORAL THESIS

UPC – September 2002

UPC **CTTC**

Numerical Simulation of Turbulent Flows. Multiblock Techniques. Verification and Experimental Validation

Centre Tecnològic de Transferència de Calor
Departament de Màquines i Motors Tèrmics
Universitat Politècnica de Catalunya

Jordi Cadafalch Rabasa
Doctoral Thesis

Numerical Simulation of Turbulent Flows. Multiblock Techniques. Verification and Experimental Validation

Jordi Cadafalch Rabasa

TESI DOCTORAL

presentada al

Departament de Màquines i Motors Tèrmics
E.T.S.E.I.T.
Universitat Politècnica de Catalunya

per a l'obtenció del grau de

Doctor Enginyer Industrial

Terrassa, September 2002

Numerical Simulation of Turbulent Flows. Multiblock Techniques. Verification and Experimental Validation

Jordi Cadafalch Rabasa

Directors de la Tesi

Dr. Assensi Oliva Llena
Dr. Carlos-David Pérez-Segarra

Tribunal Qualificador

Dr. Antonio Lecuona Neumann
Universidad Carlos III de Madrid

Dr. Miquel Costa Pérez
Universitat Politècnica de Catalunya

Dr. Antonio Pascau Benito
Universidad de Zaragoza

Dr. Enric Velo García
Universitat Politècnica de Catalunya

Dr. Esteve Codina Macià
Universitat Politècnica de Catalunya

Preface

Work here presented is the result of basic research in key aspects of the currently available engineering tools and methodologies for the design, optimisation and development of thermal systems and equipment: turbulence modeling, high perfomance computing and quality tests and procedures so as to assess credibility to the numerical solutions (verification and validation).

The thesis comprises six main chapters written in a paper format. Two of them have already been published in international journals, one in the proceedings of a Spanish conference and two in proceedings of international conferences on Computational Fluid Dynamics and heat transfer. The last chapter has recently been submitted for publication to an international journal. Therefore, all the chapters are written so as to be self-contained, complete and concise. As a consecuence, some contents of the chapters such those describing the governing equations, or the verification procedure used to assess the credibility of the numerical solutions, are repeated in several of them. Furthermore, as only minor changes have been introduced in the chapters respect to the original papers, each of them reflect the know-how of the CTTC (Heat and Mass Transfer Technological Center were the research has been carried out) when they were published.

Papers presented in chapter 1 and 2 deal with turbulence modeling. A general overview is given on the formulation and numerical techniques of the different levels of turbulence modelling: Direct Numerical Simulation (DNS), Large Eddy Simulation (LES) and Reynolds Averaged Navier-Stokes Simultion (RANS). Main attention is foccused on the eddy viscosity two-equation RANS models. Their formulation is presented in more detail, and numerical solutions of the most extended Benchmark problems on turbulence modeling are given compared to the available experimental data.

Chapter 3 and 4 focuss on the use of the multiblock method (domain decomposition method), as a numerical technique that combined with the parallel computing may allow to reduce the demanding computational time and memory (high performance computing). The multiblock approach used is based on the conservation of all the physical quantites (fully conservative method) and on an explicit information exchange between the different blocks of the domain. The goal of the work presented in these two chapters is to verify that such a multiblock approach does not introduce additional uncertainty in the numerical solutions.

Chapter 5 presents a tool that has been developed at the CTTC for the verification of finite volume computations. In fact, this tool is also partilly used and described in the results presented in the previous chapters. Here, it is described and discussed in detail and it is applied to a set of different CFD and heat transfer problems in two and three dimensions, with free and forced convection, with reactive and non-reactive flows and with laminar and turbulent flows.

The last chapter shows a complete study for the development of a credible heat transfer relation for the heat evacuated from a ventilation channel. Such study comprises all the different steps that have to be accomplished so as to develope credible and applicable results in mechanical engineering. It comprises a description of the mathematical model to represent the physical phenomena in the channel, the numerical model to solve the set of coupled differential equations of the mathematical model, the construction and testing of an ad-hoc experimental set-up, and a verification and validation (V&V) test that guarantees that the numerical solution is an accurate enough approximation of the mathematical model (verification), and that it properly predicts the reality (validation).

Contents

Chapter 1

Analysis of Turbulent Flows by Means of RANS Models

Main contents of this chapter are published as
C.D. Pérez-Segarra, J. Cadafalch and A. Oliva. Análisis de flujos turbulentos en base a modelos de tipo RANS. *In Proceeding of the XIII Congreso Nacional de Ingeniería Mecánica*, volume 4, pages 608-615, Terrassa, 1998.

Abstract. A general overview of the mathematical formulation and numerical techniques used for the solution of turbulence flow is presented. Among the different strategies for the analysis of these phenomena (DNS, LES, RANS, stochastic models,....), the study has been focussed on those formulations that use time-averaged variables (which are known by the acronym RANS, i.e. Reynolds Averaged Navier-Stokes Simulation). These models require the use of additional information to get a close system of equations. The closure problem introduce some kind of empirical information giving a wide range of turbulence models. In this work the formulation of different RANS turbulence models usually used in the design of thermal systems and equipment are presented.

1.1 Introduction

The turbulent flow is characterized by a transient and tri-dimensional structure, leading to a wide range of time and length scales. For high Reynolds numbers, the energy-cascade model of Kolmogorov establishes a transport of the kinetic energy from the main flow to the bigger vortexes, which are characterized by length scales with the same order of magnitude of the main flow, with a high level of anisotropy and with low fluctuation frequencies. The interaction between these vortexes leads to a stretching process (known as vortex stretching) that reduces their diameter increasing their angular velocity. In this process, the kinetic energy goes from larger vortexes to smaller vortex until it is dissipated and converted to internal energy by means of the viscous forces. The smaller scales (dissipative scales or Kolmogorov scales) are characterized by high fluctuation frequencies and a isotropic structure (high Re). In some situations, an inversion of the energy-cascade process from the smaller to the larger scales can also occur (backscatter).

As in most engineering applications the dissipative scales (Kolmogorov scales) of the turbulent flow are of several order of magnitudes bigger than the molecular scales (which in gases are characterized by the free mean path), see for example Tennekes and Lumley [1], the constitutive equations of Stokes's viscosity for Newtonian fluids and of Fourier for conductive heat transfer are suitable for the description of this kind of flows. Once introduced in the governing equations (mass, energy and momentum), the Navier-Stokes equations are obtained. Historically, the Navier-Stokes equations take account of the mass and momentum equations. However, in the CFD literature usually the Navier-Stokes equations include all the governing equations. For incompressible flows and avoiding the body forces effects, this equation system takes the form:

$$\frac{\partial u_j}{\partial x_j} = 0 \tag{1.1}$$

$$\frac{\partial(\rho u_i)}{\partial t} + \frac{\partial(\rho u_j u_i)}{\partial x_j} = -\frac{\partial p}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\mu \frac{\partial u_i}{\partial x_j}\right) \tag{1.2}$$

$$\frac{\partial(\rho T)}{\partial t} + \frac{\partial(\rho u_j T)}{\partial x_j} = \frac{1}{c_p}\frac{\partial}{\partial x_j}\left(\lambda \frac{\partial T}{\partial x_j}\right) \tag{1.3}$$

Tennekes and Lumley [1], show that the ratio between the turbulence dissipative scales and the turbulence integral scales (with the size of the larger vortexes) has an order of magnitude proportional to the turbulent Reynolds number up to the power of $-3/4$. The Direct Numerical Simulation (DNS) of these equations needs a space and time discretization fine enough so as to ensure a proper description of the smaller turbulent scales. This kind of approximation does not require any empirical information because the equation system is closed (5 equations and 5 unknown variables: p,u_i and T). In fact, results derived from the DNS are considered by the scientific community of the same nature that those derived from experimental studies. However, as direct numerical simulation is highly computational demanding, nowadays it is still restricted to flows with simple geometries and low Reynolds numbers (remember the exponential relation between the dissipative and integral turbulence scales previously described).

Therefore, as usualy it is not possible to directly solve the Navier-Stokes equations in turbulent flows, it has been necessary to transform the instantaneous equations into averaged equations. The statistical averaging process can be both in space or in time. Whatever the kind of averaging used, the averaged Navier-Stokes equations introduce additional unknowns due to the non-linearity of the convective terms. The approximation of these additional unknowns in terms of the averaged variables leads to the turbulence models. Two main analysis lines can be found: the Large Eddy Simulation models (LES), and the Reynolds Averaged Navier-Stokes Simulation (RANS).

The LES models come out from the work of Smagorinsky [2]. They are based on the volume-averaged Navier-Stokes. The averaging process (filtering) is only carried out for the smaller turbulence scales. While the tridimensional and transient structure of the larger turbulence scales are simulated in detail, the smaller scales (of the size of the discretization mesh used) are modeled. As these smaller scales are characterized by an isotropic structure (at least for high Re numbers), they are relatively easy to be modeled. Therefore, relatively coarse meshes and large time increments can be adopted. In a medium term, these models can become very relevant as a tool for the analysis of turbulent flows of engineering applications.

The RANS models are based on the time averaged Navier-Stokes equations. In principle, these equations are restricted to steady state turbulent flows (averaged values are constant in time), and to transient flows with a time scale of the medium flow significatively larger than the turbulence time scales. Details about the formulation of the RANS models will be given in the following sections.

1.2 RANS: Reynolds averaged Navier-Stokes simulations

The time averaged Navier-Stokes equations are obtained from equations (1.1), (1.2) and (1.3), substituting the instantaneous variables for its time averaged value plus a fluctuation value (i.e. $\phi = \overline{\phi} + \phi'$), and averaging in time the resulting equations:

$$\frac{\partial \overline{u}_j}{\partial x_j} = 0 \tag{1.4}$$

$$\frac{\partial(\rho \overline{u}_i)}{\partial t} + \frac{\partial(\rho \overline{u}_j \overline{u}_i)}{\partial x_j} = -\frac{\partial \overline{p}}{\partial x_i} + \frac{\partial}{\partial x_j}\left(\mu \frac{\partial \overline{u}_i}{\partial x_j} - \rho \overline{u_i' u_j'}\right) \tag{1.5}$$

$$\frac{\partial(\rho \overline{T})}{\partial t} + \frac{\partial(\rho \overline{u}_j \overline{T})}{\partial x_j} = \frac{\partial}{\partial x_j}\left(\frac{\lambda}{c_p}\frac{\partial \overline{T}}{\partial x_j} - \rho \overline{u_j' T'}\right) \tag{1.6}$$

As indicated in the previous section, the non-linearity of the convective terms introduces new unknowns: the turbulent stresses tensor (or Reynolds tensor) $\rho\overline{u_i' u_j'}$ and the turbulent heat tensor $\rho\overline{u_j' T'}$. These terms correspond to the convective transport of momentum and energy due to the fluctuating component. Therefore, the averaging process maintains the number of equations, 5, and increases the number of unknown variables, 14: $\overline{u}_i$, p, T, 6 components of the Reynolds tensor (which is symmetric), and 3 components of the heat tensor.

Representing the instantaneous momentum equation (1.2) of the velocity component u_i with the operator $M(u_i)$, a new transport equation for each one of the Reynolds tensor components can be obtained by means of the following operation

$$\overline{u_j M(u_i) + u_i M(u_j)} \tag{1.7}$$

They are known as the Reynolds stress equations and in a compact form can be written as follows:

$$C_{ij} = d_{ij} + P_{ij} + \phi_{ij} - \epsilon_{ij} \tag{1.8}$$

where:

$$C_{ij} = \frac{D\overline{u_i' u_j'}}{Dt} = \frac{\partial \overline{u_i' u_j'}}{\partial t} + \overline{u_k}\frac{\partial \overline{u_i' u_j'}}{\partial x_k} \tag{1.9}$$

$$d_{ij} = \frac{\partial}{\partial x_k}\left\{\nu \frac{\partial \overline{u_i' u_j'}}{\partial x_k} - \overline{u_i' u_j' u_k'} - \frac{\overline{p' u_i'}}{\rho}\delta_{jk} - \frac{\overline{p' u_j'}}{\rho}\delta_{ik}\right\} \tag{1.10}$$

$$P_{ij} = -\overline{u'_i u'_k}\frac{\partial \overline{u}_j}{\partial x_k} - \overline{u'_j u'_k}\frac{\partial \overline{u}_i}{\partial x_k} \tag{1.11}$$

$$\phi_{ij} = \overline{\frac{p'}{\rho}\left(\frac{\partial u'_i}{\partial x_j} + \frac{\partial u'_j}{\partial x_i}\right)} \tag{1.12}$$

$$\epsilon_{ij} = 2\nu\overline{\frac{\partial u'_i}{\partial x_k}\frac{\partial u'_j}{\partial x_k}} \tag{1.13}$$

These equations describe the convective transport and storage process (C_{ij}), the diffusive transport (d_{ij}), generation (P_{ij}), redistribution (ϕ_{ij}) and destruction (ϵ_{ij}) of each one of the components of the Reynolds tensor $\rho\overline{u'_i u'_j}$. Although the mathematical processes described yields to a new equation for each one of the Reynolds tensor terms, 22 new additional unknowns are also derived ($\overline{u'_i u'_j u'_k}$, $\overline{p' u'_i}$, ϕ_{ij}, ϵ_{ij}). Similar mathematical procedures can be used to obtain new additional equations for the third and higher order momentums, however the resulting equation system would never be closed because the number of unknowns would ever be larger than the number of equations.

The averaging process obviously leads to a los of some of the information contained within the instantaneous Navier-Stokes equations. This lack of information is overcome by means of approximations of those unknown terms as a function of the average variables (closure problem). The approximations used in the closure problem must be as general and as accurate as possible so as to be able to model different turbulent phenomena. Different procedures used to handle the closure problem lead to the RANS turbulent models.

For the turbulent heat flux, see equation (1.6), and in general for all the turbulent terms corresponding to scalar variables, similar transport equations to that of the Reynolds tensor components can be obtained.

1.2.1 Turbulent kinetic energy

Before going through the different levels of RANS analysis, it is worth to keep attention on the turbulent kinetic energy transport equation ($k = \overline{u'_i u'_i}/2$). This equation can be directly obtained summing up the normal components of the Reynolds tensor and dividing the result by 2:

$$C_k = d_k + P_k - \epsilon \tag{1.14}$$

where:

$$C_k = \frac{\partial k}{\partial t} + \overline{u_j}\frac{k}{\partial x_j} \tag{1.15}$$

$$d_k = \frac{\partial}{\partial x_j}\left\{\nu\frac{\partial k}{\partial x_j} - \overline{u'_j\left(\frac{p'}{\rho} + \frac{u'_i u'_i}{2}\right)}\right\} \tag{1.16}$$

$$P_k = -\overline{u'_i u'_j}\frac{\partial \overline{u}_i}{\partial x_j} \tag{1.17}$$

$$\epsilon = \nu\overline{\frac{\partial u'_i}{\partial x_j}\frac{\partial u'_i}{\partial x_j}} \tag{1.18}$$

All the terms of this equation can be calculated except d_k and ϵ, which respectively account for the diffusion and dissipation of turbulent kinetic energy, and they have to be modeled (approximated).

1.2.2 Levels of RANS analysis

Among all the different possibilities to handle the closure problem, three levels of analysis commonly used in engineering applications can be distinguished:

- Reynolds Stress Models (RSM) or Differentially Second-Moment Closures. These models make use of the differential transport equations of the Reynolds tensor, equations (1.8) (and in general of the turbulent fluctuation terms), the turbulent kinetic energy k equation, equation (1.14), and of an additional differential equation for the dissipation of turbulent kinetic energy ϵ (instead, other dissipation variables can also be used such as $\omega = \epsilon/k$, with vorticity dimensions, or $\tau = k/\epsilon$, with time dimensions). The remaining unknown terms are modeled. For example, the triple correlations of velocities are commonly modeled as follows:

$$-\overline{u'_i u'_j u'_k} \approx C_s\frac{k}{\epsilon}\overline{u'_k u'_l}\frac{\partial \overline{u'_i u'_j}}{\partial x_l} \tag{1.19}$$

  where C_s is an empirical constant. In these models, major difficulties appear in the formulation of the equation for the dissipation of the turbulent kinetic energy and in the approximation of the terms that account for the pressure redistribution in the zones close to the solid boundaries.

- Algebraic Stress Models (ASM) or Algebraic Second Moment Closures. The main idea in these models is based on the transformation of the differential equations of the Reynolds tensor terms (and of all the turbulent fluctuation terms) into algebraic equations. The primary step of the procedure is based on the following assumption:

$$C_{ij} - d_{ij} \approx \frac{\overline{u_i' u_j'}}{k}(C_k - d_k) = \frac{\overline{u_i' u_j'}}{k}(P_k - \epsilon) \tag{1.20}$$

  The last equality can be obtained from the equation of k, equation (1.14). These models require the use of two additional differential equations (as the RSM models), one for the turbulent kinetic energy k and another for its dissipation rate ϵ (or another dissipation quantity such as ω or τ). The standard version of the ASM models is fully implicit, and the stiffness of the resulting equations makes the convergence difficult. Speziale (1996) has proposed explicit versions that overcome some of the numerical problems of the standard version.

- Eddy Viscosity Models (EVM). They are the most popular models in engineering. Main relevant aspects of these models will be explained in the following sections.

1.3 EVM: Eddy viscosity models

1.3.1 Constitutive law of the turbulent stresses

In these models, the turbulent stresses are calculated as a function (linear or not) of the mean velocity gradients adopting the idea of turbulent viscosity ν_t.

In linear models (L-EVM), the "constitutive law" of the turbulent stresses is obtained from a linear relation between the turbulent stresses and the corresponding terms of the stress tensor:

$$\overline{u_i' u_j'} - \frac{2}{3} k \delta_{ij} = -2\nu_t S_{ij} \tag{1.21}$$

where:

$$S_{ij} = \frac{1}{2}\left(\frac{\partial \overline{u}_i}{\partial x_j} + \frac{\partial \overline{u}_j}{\partial x_i}\right) \tag{1.22}$$

This kind of models assume the Reynolds tensor to have an isotropic behaviour (i.e., the function relating the stresses and the deformations is maintained invariable for any rotation of axis of the co-ordinates system).

Due to its linearity and isotropy, the L-EVM model cannot properly evaluate flows characterized by curvature effects, body forces, etc. Recently, new improved versions based on non-linear models (NL-EVM) have been coming up in the literature. Speziale [3] developed an explicit non-linear version taking into account the second order terms:

$$\overline{u_i' u_j'} - \frac{2}{3} k \delta_{ij} = -2\nu_t S_{ij} + 4 \frac{C_D \nu_t^2}{k} \left[S_{ik} S_{kj} - \frac{1}{3} S_{kl} S_{kl} \delta_{ij} + \tilde{S}_{ij} - \frac{1}{3} \tilde{S}_{kk} \delta_{ij} \right] \tag{1.23}$$

where:

$$\tilde{S}_{ij} = \frac{\partial S_{ij}}{\partial t} + \overline{u}_k \frac{\partial S_{ij}}{\partial x_k} - S_{kj} \frac{\partial \overline{u}_i}{\partial x_k} - S_{ki} \frac{\partial \overline{u}_j}{\partial x_k} \tag{1.24}$$

New other expressions have been proposed by Speziale and other authors. As an example, Craft, Launder and Suga [4] propose a formulation considering the third order non-linear terms.

1.3.2 Turbulent viscosity

Three different models are usually adopted for the evaluation of the turbulent viscosity ν_t. They are hereafter indicated from less to more generality (range of application):

- Eddy-viscosity 0-equations models. Algebraic models based on expressions such as

$$\nu_t = l_m^2 (2 S_{ij} S_{ij})^{1/2} \tag{1.25}$$

  where l_m is the mixing length and is evaluated by means of algebraic expressions obtained from empirical analysis. The Prandtl mixing length is one of the most popular expressions for l_m. It is discussed in most of the basic books on fluid mechanics and heat transfer.

- Eddy-viscosity one-equation models. Models based on expressions obtained from dimensional analysis or from analogy to the molecular viscosity of the kinetic-molecular theory. Typical expressions are

$$\nu_t = C_\mu k^{1/2} l \tag{1.26}$$

The characteristic velocity, $k^{1/2}$, is calculated by means of the corresponding differential transport equation (1.14) with the turbulent diffusion and dissipation terms properly modeled; C_μ is an empirical constant; and the length scale, l, is calculated using semi-empirical algebraic expressions.

- Eddy-viscosity two-equations models. The third level of modeling is based on the resolution of two transport equations properly modeled: the equation of the turbulent kinetic energy k, and another equation accounting for the dissipation term $k^n \epsilon^m$ (commonly used variables are ϵ, ω or τ). The turbulent viscosity is obtained from dimensional analysis or from analogy to the molecular viscosity of the kinetic-molecular theory (as done in the previous level of modeling):

$$\nu_t = C_\mu f_\mu k^{1/2} l \tag{1.27}$$

where the length scale is obtained from $l = k^{3/2}/\epsilon$ (or $l = k^{1/2}/\omega$ or $l = k^{1/2}\tau$), C_μ is a empirical constant, and f_μ is an empirical function introduced so as to account for those zones with low turbulent Reynolds numbers (if the turbulent Reynolds is high, $f_\mu = 1$). While the turbulent kinetic energy equation only requires the modeling of two terms (turbulent diffusion and dissipation), the equation of dissipation energy is almost fully empirical.

1.4 Eddy-viscosity two-equations models

Presently, these are the most extended models in the engineering world, because they combine generality, reasonable accuracy, simplicity (they are quite easy to be implemented in a Computational Fluid Dynamics code) and they do not demand a high computational effort.

Different models can be found in the literature. The standard model is prepared for the modeling of high turbulent Reynolds flows. Hence, it cannot properly describe the flow close to solid boundaries. In these situations empirical functions, known as wall functions, are usually used. The wall functions express the shear stress (or the heat fluxes) at the walls in terms of the velocities (or temperatures) at the mesh node closer to the wall (usually situated at the logarithmic zone, i.e. $y^+ = yu_t/\nu = 40 \div 100$, where y is the wall distance and $u_t = (\tau_w/\rho)^{1/2}$ is the friction velocity).

More general models, known as low-Reynolds number two-equations models, also take into account the effects in zones with low turbulent Reynolds number. Therefore, they can directly be used for the modeling of the zones close to the walls (laminar subboundary layer and transition zone). These models require a finer mesh (and, therefore, more computational effort) so as to be able to properly describe the flow close to the solid boundaries. In spite of their higher computational demands, the use of the low-Reynolds number two-equations models is preferable, because they are prepared to handle a major variety of flow phenomena (they are more general).

The modeled transport equations for the turbulent kinetic energy k and for the dissipation term φ can be expressed in a generic form as follows:

$$\frac{D(\rho k)}{Dt} = \frac{\partial}{\partial x_j}\left[\left(\frac{\mu}{\sigma_k^*} + \frac{\mu_t}{\sigma_k}\right)\frac{\partial k}{\partial x_j}\right] + P_k - \beta^* \rho \epsilon + D \tag{1.28}$$

$$\frac{D(\rho\varphi)}{Dt} = \frac{\partial}{\partial x_j}\left[\left(\frac{\mu}{\sigma_\varphi^*} + \frac{\mu_t}{\sigma_\varphi}\right)\frac{\partial \varphi}{\partial x_j}\right] + c_{\varphi 1} f_1 \frac{P_k}{T_\varphi} - c_{\varphi 2} f_2 \frac{\rho\hat{\varphi}}{T_\varphi} + E \tag{1.29}$$

In the widely extended k-ϵ model of Launder and Sharma [5], the dissipation variable is $\varphi = \hat{\varphi} = \epsilon$, while the empirical functions and constants are: $T_\varphi = k/\epsilon$, $\beta^* = 1$, $D = 2\mu(\partial k^{\frac{1}{2}}/\partial x_j)^2$, $E = 2(\mu\mu_t/\rho)(\partial^2 U_i/\partial x_j \partial x_k)^2$, $f_1 = 1$, $f_2 = 1 - 0.3exp(-R_t^2)$, $f_\mu = exp(-3.4/(1+R_t/50)^2)$, $\sigma_k = \sigma_k^* = 1$, $\sigma_\epsilon = 1.3$, $\sigma_\epsilon^* = 1$, $c_{\epsilon 1} = 1.44$, $c_{\epsilon 2} = 1.92$, $c_\mu = 0.09$. At the walls: $k_w = \tilde{\epsilon_w} = 0$.

In the Wilcox k-ω model [6], the dissipation variable is $\varphi = \omega$ and $\hat{\varphi} = k\omega$, and the empirical functions and constants are: $T_\varphi = k/\omega$, $\beta^* = (9/100)[5/18 + (R_t/R_\beta)^4]/[1 + (R_t/R_\beta)^2]$, $D = E = 0$, $f_1 = [\alpha_o + R_t/R_\omega]/[c_\mu f_\mu (1 + R_t/R_\omega)]$, $f_2 = 1$, $f_\mu = [\alpha_o^* + R_t/R_k]/[1 + R_t/R_k]$, $\sigma_k = 2$, $\sigma_k^* = 1$, $\sigma_\omega = 2$, $\sigma_\omega^* = 1$, $c_{\omega 1} = 5/9$, $c_{\omega 2} = 3/40$, $c_\mu = 1$, $R_k = 6$, $R_\omega = 2.7$, $R_\beta = 8$, $\alpha_o = 0.1$, $\alpha_o^* = c_{\omega 2}/3$. At the walls: $k_w = 0$ and for the node closer to the wall $\omega_1 = [2u_\tau^2]/[\nu\beta^* y_1^{+2}]$.

Results of free and forced convection flows obtained with different two-equation models including the two here referenced can be found in [7] and [8].

1.5 Conclusions

A general outlook of the problems involved in the mathematical formulation of turbulent flows has been presented. Kicking of from a general point of view, the different strategies for the resolution of the turbulent phenomena (levels of simulation) have been discussed: Direct Numerical Simulation (DNS) vs. statistical models such as Large Eddy Simulation (LES) and Reynolds Averaged Navier-Stokes Simulations (RANS). Main attention has been kept on RANS, the most commonly used strategy in the engineering field: Reynolds Stress Models (RSM), Algebraic Stress Models (ASM) and Eddy Viscosity Models (EVM). Within the group of Eddy Viscosity Models, the models called Two-Equation Eddy-Viscosity Models, have been discussed in more detail. In these models, two additional transport differential equations must be solved accounting for the turbulent kinetic energy and its dissipation energy.

Generally, the “best” level of turbulence simulation is that with a larger field of applicability (generality), with the best accuracy and with the lowest computational requirements. Unfortunately, this model does not exists. Furthermore, the most general model it is not always the most accurate. Therefore, a complete understanding of all the simulation levels is a key aspect, because the most suitable model depends on the design requirements and the application being studied.

1.6 Acknowledgments

This work has been funded by the Comisión Interministerial de Ciencia y Tecnología of Spain, (project TIC724-96), and by the Comissionat per Universitats i Recerca of the Generalitat of Catalonia.

References

[1] H. Tennekes and J.L. Lumley. *A first course in turbulence.* The MIT Press, 1972.

[2] J. Smagorinsky. General circulation experiments with the primitive equations, part. I: the basic experiment. *Monthly Weather Rev.*, 91:99–164, 1963.

[3] C.G. Speziale. Comparison of explicit and traditional algebraic stress models of turbulence. *AIAA Journal*, 35(9):1506–1509, 1997.

[4] T.J. Craft, B.E. Launder, and K. Suga. Extending the applicability of eddy viscosity model through the use of deformation invariant and non-linear elements. In *Proceedings 5th Int. IAHR Symp. on Refined Flow Modeling and Turbulence Measurements*, pages 0–0, 1993.

[5] B.E. Launder and B.I. Sharma. Application of the energy dissipation model of turbulence to the calculation of flow near a spinning disc. *Letters in Heat Transfer*, 1:131–138, 1974.

[6] D.C. Wilcox. *Turbulence modeling for CFD.* DCW Industries, Inc. CA, 1993.

[7] C.D. Pérez-Segarra, A. Oliva, M. Costa, and F. Escanes. Numerical experiments in turbulent natural and mixed convection in internal flows. *International Journal for Numerical Methods for Heat and Fluid Flow*, 5(1):13–33, 1995.

[8] C.D. Pérez-Segarra, A. Oliva, and R. Cònsul. Analysis of some numerical aspects in the solution of the navier-stokes equations using non-orthogonal collocated finite-volume methods. In *Proceedings of the Second European Congress on Computational Methods in Applied Sciences and Engineering (ECCOMAS)*, pages 505–511, 1996.