# Essays on Monetary Economics

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*To my family.*

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*Quocunque Jeceris Stabit*

Abstract

This thesis consists of three chapters on monetary economics. In the first chapter, I explore if a benchmark Heterogeneous Agent New Keynesian (HANK) model fits the heterogeneous response of monetary policy shocks observed in data. The HANK model of Kaplan et al. (2018) predicts a greater increase in income for wealthier households than poorer households from an expansionary monetary policy shock, which is at odds with empirical data. I innovate on the profit distribution scheme in the model to bring the distributional response from a monetary policy shock closer to the empirical evidence. In the second chapter, I analyse how adding downward nominal wage rigidities to a standard New Keynesian model changes the response of the economy to shocks. In the third chapter, which is joint work with Luca Onorante, we explore the effectiveness of bond and corporate security purchases by a central bank within a calibrated two-country New Keynesian model featuring a banking sector.

Resum

Aquesta tesi consta de tres capítols sobre economia monetària. Al primer capítol, exploro si la resposta després d'un xoc expansiu de política monetària dins d'un model de referencia d'agents heterogenis nou keynesià (HANK) s'ajusta a la resposta heterogènia observada empíricament. Contràriament al que s'observa a les dades, després d'un xoc expansiu de política monetària, el model HANK de Kaplan et al. (2018) preveu un augment més gran dels ingressos per a les llars més riques que per a les famílies més pobres. Finalment, innovo en l'esquema de distribució de beneficis del model per tal d'apropar-lo a aquesta evidència empírica. Al segon capítol, analitzo com la introducció de rigideses salarials nominals a la baixa en un model nou keynesià afecta la resposta de l'economia davant de xocs de demanda i tecnològics. Al tercer capítol, un treball conjunt amb Luca Onorante, explorem l'eficàcia de la compra de bons i valors corporatius per part d'un banc central dins d'un model nou keynesià calibrat per a dos països amb sector bancari.

# Preface

This thesis consists of three chapters on topics in monetary economics. The first chapter, "Household Heterogeneity and the Transmission of Monetary Policy," seeks to understand if a benchmark Heterogeneous Agent New Keynesian (HANK) model fits the heterogeneous response of monetary policy shocks observed in the data? The benchmark HANK model from Kaplan et al. (2018) implies that wealthier households benefit from a greater increase in their income than poorer households from an expansionary monetary policy shock. However, this prediction is at odds with the empirical evidence. Using data on U.S. households from the Consumer Expenditure Survey I find that households across the wealth distribution have comparable income responses to an expansionary monetary policy shock, while consumption increases the most for low wealth households. Motivated by these discrepancies I innovate on the profit distribution scheme, from the bonus-based scheme (profits are distributed in proportion to labour productivity as assumed in Kaplan et al. (2018)) to a dividend-based scheme (profits are distributed in proportion to illiquid asset holdings). This innovation brings the distributional response from a monetary policy shock closer to the empirical evidence, however, a mixed scheme is required to ensure the response of aggregate investment is reasonable as it is highly dependent on the income of the wealthy hand-to-mouth households.

The second chapter, "Optimal Monetary Rules with Downward Nominal Wage Rigidity," analyses how adding in an occasionally binding constraint, downward nominal wage rigidities, to a standard model used in monetary economics can impact the response of the economy and optimal monetary policy. At the individual and country level nominal wages have been found to be downwardly rigid, such that they are more likely to increase than decrease. This has strong implications for optimal monetary policy in the standard New Keynesian model, which typically assumes flexible wages or symmetric nominal wage rigidities. This constraint causes the optimal monetary policy to react asymmetrically to symmetric shocks. Furthermore, motivated by the welfare loss generated by using a standard Taylor rule, this paper searches for a new optimal simple rule that can replicate the optimal monetary policy in this framework. As an extension I solve a non-linear model that internalises this constraint at all periods in time, which dampens wage increases in a model where agents can flexibly increase their wage, thus creating an endogenous rigidity. This work adds to the literature by introducing the downward nominal wage rigidity (DNWR) constraint of Schmitt-Grohé and Uribe (2016) into a standard New Keynesian model and finds an optimal simple rule that places a high weight on the unemployment gap.

Moreover, as with other work on DNWR, this paper finds support for 'greasing the wheels' - positive trend inflation that helps to deflate real wage increases.

The third chapter of the thesis, co-authored work with Luca Onorante titled "Conventional and Non-Conventional Monetary Policy: Between Core and Periphery," explores the effectiveness of government bond and corporate security purchases by a central bank within a calibrated two-country New Keynesian model featuring a banking sector (an extension of Gertler and Karadi (2011) and Andrade et al. (2016)) and a two-country monetary union. Focusing on the Eurozone and motivated by the extended asset purchase programme conducted by the ECB we calibrate key parameters to match Core (Germany, France, Netherlands) and Periphery (Portugal, Italy, Ireland, Greece, Spain) data. We find that corporate security purchases have a stronger impact on inflation and on lift-off time from the Effective Lower Bound than equivalent government bond purchases. This finding is in line with the ones of Gertler and Karadi (2013) for the U.S. economy.

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# Chapter 1

# HOUSEHOLD HETEROGENEITY AND THE TRANSMISSION OF MONETARY POLICY

## 1.1 Introduction

Following the aftermath of the financial crisis monetary policy tools used by central banks have come under increased scrutiny for their potential impact on inequality. The added attention from policymakers, academia and the public coupled with the increase in the availability of household-level data and improvements in computational techniques have lead to a boom in research in heterogeneous agent models. However, the evidence is mixed on how households, heterogeneous across income and wealth portfolios, are affected by interest rate changes.[1] Moreover, the efficacy of monetary policy to influence aggregate variables such as output, consumption and investment may also depend on the distribution of income and wealth in the economy. This paper seeks to add clarity to this debate by refining a benchmark heterogeneous agent New Keynesian (HANK) model and disciplining through additional empirical findings the effect of monetary policy on heterogeneous households.

[1] Coibion et al. (2017), Mumtaz and Theophilopoulou (2017), Samarina and Nguyen (2019) find that an expansionary monetary policy shock reduces income inequality whereas Inui et al. (2017) find the opposite. For a comprehensive summary of the empirical findings see Colciago et al. (2019).

The effect of monetary policy on income and wealth inequality is *a priori* ambiguous. An expansionary monetary policy shock, i.e. an exogenous, unexpected fall in the interest rate, can help boost employment and wages which would typically benefit poorer households the most. Additionally, a monetary policy stimulus generally leads to house price increases and a surge in financial markets, benefiting the richer in society. The standard theoretical model used within central banks, a representative agent New Keynesian (RANK) model, is unable to address this ambiguous response.

Moreover, the transmission of monetary policy is different in the representative agent New Keynesian model compared to the HANK model. In the RANK model direct effects of monetary policy are dominant, such that lowering the real interest rate boosts consumption as households spend rather than save, this means monetary policy works almost entirely through the substitution effect. Yet, in the HANK model the indirect effect of monetary policy is dominant, such that the consumption response from lowering the real interest rate is primarily through increased labour demand and higher wages afforded to the household, the income effect. This difference arises as HANK models feature agents who have limited liquid assets and therefore act hand-to-mouth by reacting strongly to income changes and being unable to alter their saving decisions thus reacting more to indirect than direct effects. As well as differences in the transmission mechanism, if the distribution of income and wealth of households affects the strength of monetary policy it is crucial to take this into account.

Motivated by these issues I use a benchmark HANK model of Kaplan et al. (2018) to analyse the response of consumption and income over the wealth distribution of households to an expansionary monetary policy shock. The HANK model from Kaplan et al. (2018) provides an ideal starting point for my analysis as it closely matches the distribution of asset holdings for U.S. households. In the model, income increases markedly for the wealthiest households following an expansionary monetary policy shock, driven by the return on their illiquid assets,[2] whereas this increase is more moderate for the poorer households. In contrast to their income response, the consumption response is muted for the wealthiest households, who choose to save rather than consume, compared to less wealthy, more likely to be hand-to-mouth, households who notably increase their consumption.

[2]The return on the illiquid assets is equal to the return on capital. Capital gains, the benefit from a change in the price of capital, is countercyclical in the HANK model due to countercyclical markups causing profits to be countercyclical. Therefore, from an expansionary monetary policy shock the return on illiquid assets rise whilst the price of capital falls.

Following this theoretical result I use micro data on U.S. households from the Consumer Expenditure Survey by the Bureau of Labor Statistics to test the predictions of the model. The main empirical strategy uses monetary policy shocks cleaned of information effects by Jarociński and Karadi (2020) in an Autoregressive Distributed Lag model (ARDL) with further robustness checks using a Bayesian Proxy structural vector autoregression (BP-SVAR).[3] This paper focuses on household heterogeneity across wealth levels, moreover, additional analysis corroborates the findings when households are separated by income.

Using tax rebate data from The Economic Growth and Tax Relief Reconciliation Act of 2001, following Johnson et al. (2006), I highlight that low wealth households have higher marginal propensities to consume and therefore act more hand-to-mouth than wealthier households. These findings suggest that monetary policy has distributional effects, such that an expansionary monetary policy shock decreases income and consumption inequality. This is in accordance to findings by Coibion et al. (2017).

Comparing my empirical evidence with the HANK model I find a qualitatively similar response of consumption to monetary policy shocks over the wealth distribution of households. However, the income response is at odds with my empirical exercise. One reason for this difference is that in this model, as well as other heterogeneous agent New Keynesian models, the majority share of profits are distributed as bonuses (proportional to households' labour productivity), which is defined as Case 1 within this paper. Since profits are countercyclical in Kaplan et al. (2018), and a sizable proportion of these bonuses go to low wealth households, low wealth households' income response is dampened compared to wealthier households from an expansionary monetary policy shock, which is in contrast to the empirical evidence where the income response across the wealth distribution in similar. I innovate on this scheme and assign profits in proportion to illiquid asset holdings, which is akin to equity shares, such that this scheme is comparable to dividend payouts (defined as Case 2). As in the representative agent New Keynesian model, the markups of monopolistically competitive firms are countercyclical, which induces countercyclical profits. However, if profits are distributed as dividends the countercyclical profits dampens the income response of the high wealth households. Dampening the income of the highest net worth agents causes investment to become counterfactually countercyclical as investment is wholly undertaken by the wealthiest households within the model. This is at odds with the data as investment has been found to be procyclical, see Christiano et al. (2005). A mixed profit distribution scheme that also alters the

[3] In the literature this is also known as a Bayesian SVAR-IV. Additional robustness checks use BP-Local Projection and Local Projections.

share of profits automatically invested in the firm is required to achieve the correct aggregate and household response to a monetary policy shock (defined as Case 3).

Moreover, Case 3 increases the relevance of the direct effect of monetary policy from 19% of the total effect (Case 1) to 24%, which is still notably below the RANK model, as the income of wealthier households becomes more important. The transmission channel of monetary policy is also affected by changing the profit distribution scheme. For example, the rise in wages from an expansionary monetary policy shock increases labour income but higher wages also translate into higher costs for the firm, which cause profits to fall. When profits are distributed as bonuses[4] the increase in labour income outstrips the negative effect of countercyclical profits for the wealthiest in the economy, leading to an increase in consumption from the wage increase. However, when profits are distributed as dividends the increase in wages dampens the income of the highest net worth households, the firm owners, as the fall in profits is greater than the increase in labour income and therefore the increase in wages negatively impact their consumption.

The paper is organised as follows: Section 1.2 reviews the literature. Section 1.3 derives the HANK model used with Section 1.4 providing the baseline results. Section 1.5 outlines the empirical methodology and data used. Section 1.6 highlights the identification strategy and Section 1.7 shows the empirical results. Section 1.8 innovates on the profit distribution scheme and outlines the transmission of monetary policy. Section 1.9 concludes and the Appendix includes additional empirical exercises, model derivations and further model results.

## 1.2 Literature Review

This paper is motivated by the current empirical and theoretical literature on how heterogeneity across households affect the impact of monetary policy on economic aggregates as well as the heterogeneous impact of monetary policy shocks. A short literature review follows.[5]

[4]Income from profits, which are classified as either bonuses (profits distributed by labour productivity) or dividends (profits distributed by illiquid asset holdings) are kept separate from labour income, which is the wage or salary that the employee receives.

[5]For an extensive survey on the latest empirical and theoretical work see Colciago et al. (2019).

Findings from empirical studies are mixed on the affect of monetary policy on income and wealth inequality. My work is closely related to Coibion et al. (2017), as we both analyse the household response from monetary policy shocks for the U.S. using the Consumer Expenditure Survey. The focus in Coibion et al. (2017) is on the response of consumption inequality to a contractionary monetary policy shock. They find that households at the upper end of the income distribution benefit from contractionary monetary policy shocks, and therefore consumption inequality falls from expansionary monetary policy shocks, a finding corroborated in this paper for the upper end of the wealth distribution. Moreover, using characteristics defined by Doepke and Schneider (2006) to divide households into low net-worth and high net-worth households, Coibion et al. (2017) find similar income responses across the wealth distribution from monetary policy shocks. Furthermore, Coibion et al. (2017) find that high net-worth households increased their consumption relative to low net-worth households from a contractionary monetary policy shock. Additional studies for the UK by Mumtaz and Theophilopoulou (2017) and euro area Samarina and Nguyen (2019) also find that contractionary monetary policy increases income inequality. However, this evidence is not conclusive as Cloyne et al. (2020) show for the UK and USA that mortgagors (households that own a mortgage) benefit from an increase in income over other agents within the economy from an expansionary monetary policy shock, which would increase income inequality.[6]

From Bewley (1976), Huggett (1993) and Aiyagari (1994) heterogeneous agent models have evolved with added complexity to the household and firm dimension, in part due to advances in computational techniques. This has brought with it the ability to analyse the monetary policy transmission for households that are heterogeneous. In representative agent models the monetary policy transmission is typically defined by three channels i) an income effect ii) a wealth effect and iii) a substitution effect. In a heterogeneous agent framework these effects can interact with different dimensions of the household to create distributional channels of monetary policy. These channels can be decomposed into their direct and indirect effect, or partial equilibrium and general equilibrium channels respectively. The direct channel of monetary policy is the effect a change in the interest rate has on the households' incentive to save holding prices and income fixed. This channel is the most important in RANK models as households act Ricardian such that they can save and borrow freely, meaning that monetary policy works almost entirely through intertemporal substitution. The indirect channel impacts households through general equilibrium effects such as

[6] Mortgagors in Cloyne et al. (2020) can be thought of as wealthy hand-to-mouth households of Kaplan et al. (2014) as they tend to have little liquidity, despite owning sizable illiquid assets.

wage and price changes, greatly impacting the hand-to-mouth households found in Kaplan et al. (2018) and subsequently causing the indirect channel to dominate within this setup. The presence of incomplete markets ensures that households with low levels of liquid wealth rely solely on wage changes to influence their consumption response.

Auclert (2019) measures the distributive effect of monetary policy using a cross-section of the Consumer Expenditure Survey and calculating the correlation of households' marginal propensity to consume to their net nominal position, income and unhedged interest rate exposure. Motivated by a theoretical model he finds that these are the three channels that are important to explain the winners and losers from monetary policy shocks. I construct my wealth measure based on the unhedged interest rate exposure (URE) found in Auclert (2019). The URE, measures the value of all maturing assets and liabilities at a point in time. I take the URE measure and remove the maturity transformation, thus providing a measure of end of period wealth from the Consumer Expenditure Survey. This allows for analysis of monetary policy shocks along the wealth distribution. There exists a growing literature on the distributional effects of monetary policy, analysing the impact of conventional and unconventional monetary policy along different household dimensions. With prominent examples including McKay and Reis (2016), McKay et al. (2016), Ravn and Sterk (2016), Farhi and Werning (2019), Guerrieri and Lorenzoni (2017), Debortoli and Galí (2017), Wong (2019), Cui and Sterk (2019) and Bilbiie et al. (2019).

The theoretical HANK model used is based on the baseline from Kaplan et al. (2018). This model introduces financial market incompleteness to a two-asset New Keynesian model. The model of Kaplan et al. (2018), like the standard New Keynesian model with price rigidities suffers from countercyclical markups that can cause profits to become countercyclical.[7] In a heterogeneous agent model the distribution of monopoly profits play a crucial role in the income and consumption by household net worth. The importance of which has been highlighted by Werning (2015) in determining the amplification or dampening of monetary policy shocks relative to a representative agent model. Broer et al. (2020) contrast a heterogeneous agent model of price rigidities versus one of wage rigidities when labour is the only input of production. The inclusion of wage rigidities means that the model has a more plausible response in output and hours worked from a monetary policy shock. However, as Kaplan et al. (2018)

[7] Christiano et al. (2005) show that profits are procyclical following an expansionary monetary policy shock, whereas Nekarda and Ramey (2013) show that the price-cost markup is procyclical following monetary policy shocks.

also has capital as a input of production the inclusion of wage rigidities in the same form as Broer et al. (2020) may not generate procyclical markups.

## 1.3 Model

This section outlines the HANK model used in this paper that will be empirically tested in Section 1.5. The baseline model is taken from Kaplan et al. (2018) as it provides a realistic benchmark of household heterogeneity, which includes two types of assets (liquid and illiquid) and uninsurable earning shocks, allowing the authors to closely match the distribution of wealth and marginal propensity to consume of agents seen in the data. Using this model as my baseline I analyse the response of consumption and income to a monetary policy shock for agents along the wealth and income distribution. Moreover, I innovate on the model by altering the profit distribution scheme, from a bonus based system to a more realistic scheme where profits are distributed as dividends to shareholders.[8] This change, motivated by differences from the model impulse response functions and empirical results for income changes, brings the micro results closer to my empirical exercise, however, produces unrealistic macro results.

The main innovation in the HANK model of Kaplan et al. (2018) compared to a standard RANK model is the heterogeneity added on the household side, whilst keeping the rest of the model standard. Households are able to self-insure from uninsurable idiosyncratic income risk through the use of a liquid asset, which resembles short-term bonds or money in checking accounts, and an illiquid asset, which has properties similar to housing or retirement accounts that cannot be used for instant consumption without incurring a transaction cost of liquidation. On the firm side, price changes require a payment of an adjustment cost Ã¡ la Rotemberg (1982), thus inducing the typical price stickiness in New Keynesian models that is required for non-neutrality of interest rate changes from the monetary authority. The response from the only shock analysed within this setup, a monetary policy shock, is modeled as an innovation within the Taylor rule followed by the central bank. This a one-time zero-probability shock that induces a deterministic and temporary transition away from the steady-state of the model. For ease of comparison the notation used is borrowed from Kaplan et al. (2018). The explanation of the supply side, intermediate and final good firms, are a continuous time counterpart to that found in Galí (2015) and are explained in

[8] This is analogous to moving from a "labour-based" transfer rule to a "wealth-based" as seen in Debortoli and Galí (2017).

detail in Appendix 1.A.1. The calibration follows Kaplan et al. (2018) and can be found in Appendix 1.A.2.

### 1.3.1 Households

The economy is populated with a continuum of households that receive idiosyncratic labour productivity shocks $z_t$ and hold liquid assets $b_t$ and illiquid assets $a_t$. Labour productivity evolves following an exogenous Markov process that is described in detail in Section 1.3.6. To generate a realistic number of households with zero illiquid wealth, which is seen in the data, households die with an exogenous Poisson intensity $\zeta$. New households are then born into the economy with zero illiquid wealth and receive a random draw from the labour productivity ergodic distribution. Perfect annuity markets are assumed such that the wealth of the deceased is distributed lump-sum to other individuals in proportion to their asset holdings. This distribution to surviving households is already in the return on assets. Time is continuous in this model and at each instant in time $t$ the state of the economy is defined by the joint distribution of $\mu_t(da, db, dz)$.

Each household seeks to maximise utility $u(c_t, l_t)$, through consuming a non-negative amount of consumption goods $c_t$ and supplying labour $l_t$, which provides a disutility flow but in return for working the household gains a wage $w_t$. Labour $l_t \in [0, 1]$ is modelled as a fraction of the time endowment within the economy and is normalised to 1. Preferences are separable and utility function is of the standard Constant Relative Risk Aversion (CRRA) form, with intertemporal elasticity of substitution denoted as $\gamma$ and the inverse of the Frisch elasticity $\nu$. Disutility of labour is scaled by $\varphi$.

$$u(c_t, l_t) = \frac{c_t^{1-\gamma}}{1-\gamma} - \varphi \frac{l_t^{1+\nu}}{1+\nu} \tag{1.1}$$

Due to the law of large numbers and the lack of aggregate shocks within the model there is no economy-wide uncertainty. Instead, households face uncertainty due to the idiosyncratic labour productivity shocks and thus they maximise utility condition on the expected realisation of this shock. Conditional on surviving, the households also discounts the future at rate $\rho \geq 0$:

$$E_0 \int_0^\infty e^{-(\rho+\zeta)t} u(c_t, l_t) dt \tag{1.2}$$

Households can save using liquid and illiquid assets as well as borrow up to an exogenous borrowing limit $\underline{b}$. There is an exogenous wedge $\kappa > 0$ such that the borrowing rate is strictly above the lending rate $r_t^b$, which is the real interest rate. Therefore, the interest rate at which a household can borrow is given by $r_t^{b-} = r_t^b + \kappa$.

Illiquid assets are denoted by $a$ and require a transaction cost $\chi(d_t, a_t)$, for depositing $d_t$ or withdrawing ( when $d_t < 0$) from, which depends on the amount deposited or withdrawn and the households illiquid asset holdings. Specifically, the transaction cost has a linear component that generates an inaction region as the gain from depositing or withdrawing the first dollar is smaller than the marginal cost of transacting $\chi_0 > 0$. $\chi_1$ is added to ensure that the marginal cost of transacting depends on the share of illiquid assets being transacted rather than the size of the transaction. To ensure that deposit rates are finite a convex component is added ($\chi_1 > 0, \chi_2 > 1$) and therefore $|d_t| < \infty$. The parameters within the transaction cost function, $\chi_0, \chi_1, \chi_2$, form part of the parameters used to calibrate the steady state of the model to match the distribution of liquid and illiquid wealth in the economy. The transaction cost function is given by:

$$\chi(d, a) = \chi_0 |d| + \chi_1 \left| \frac{d}{a} \right|^{\chi_2} a \tag{1.3}$$

Due to this transaction cost the illiquid asset return is strictly above the liquid asset return in equilibrium $r_t^a > r_t^b$ with short positions ( $a_t < 0$) not allowed. The illiquid asset is part capital and part equity share, which at the individual level is indeterminate. Since the household can switch between capital and equity without a transaction cost the no-arbitrage condition means that the return on illiquid assets must equate the return on capital and the return on equity. The share of profits that are distributed to the household's liquid account is given by $\pi_t^b$, which is described in greater detail in Section 1.3.2.

As in Kaplan et al. (2018) the household's positions evolve according to:

$$\dot{b}_t = (1-\tau_t)w_t z_t l_t + r_t^b(b_t)b_t + \pi_t^b + T_t - d_t - \chi(d_t, a_t) - c_t \quad (1.4)$$

$$\dot{a}_t = r_t^a a_t + d_t \quad (1.5)$$

$$b_t \geq -\underline{b}, a_t \geq 0 \quad (1.6)$$

Households maximise equation (1.2) subject to equations (1.3) to (1.6). The households takes as given the paths of the real wage $\{w_t\}_{t\geq 0}$, the real return to liquid assets $\{r_t^b\}_{t\geq 0}$, which is given by the Fisher equation and the Taylor rule, the return on illiquid assets $\{r_t^a\}_{t\geq 0}$ and taxes and transfers $\{\tau_t, T_t\}$.

The time-varying Hamilton-Jacobi-Bellman equation that summarises the household's problem is given and solved in Appendix 1.A alongside the Kolmogorov Forward Equation that shows how the distribution of households move over time.

### 1.3.2 Composition of Illiquid Wealth and Profit Distribution

As stated earlier the illiquid asset is comprised of two components, capital $k_t$ and equity shares $s_t$ in intermediate firms. This can be expressed as $a_t = k_t + q_t s_t$, where $q_t$ denotes the share price. Equity shares provide the holder a claim on the discounted future stream of the monopoly profits net of price adjustment costs produced by the intermediate firms. Therefore the dynamics of $\dot{a}_t$ is given by

$$\dot{k}_t + q_t \dot{s}_t = \left(r_t^k - \delta\right) k_t + \Pi_t s_t + d_t \quad (1.7)$$

Within the illiquid account it is assumed that the household can freely shift between capital and equity share holdings and therefore at the individual level the exact proportion of capital holdings are indeterminate. This assumption implies that the return on equity must equal the return on capital

$$\frac{\Pi_t + \dot{q}_t}{q_t} = r_t^k - \delta \equiv r_t^a. \quad (1.8)$$

A more realistic assumption would induce a transaction cost to switch between capital and equity, however, this addition would unnecessarily complicate the model with an additional state variable. Although the individual's illiquid asset portfolio is indeterminate due to the no-arbitrage condition the aggregate composition is determined. The benefit of equity is the claim on monopoly profits, however, as is typical in models or monopolistic competition with price rigidities these profits can be countercyclical as marginal cost, $m_t$, is procyclical since the price of inputs increase by more than the price that the intermediates goods are sold at, making price markups countercyclical. This feature is also present in the baseline representative agent New Keynesian model. Since prices are sticky but nominal marginal costs are not, expansionary monetary policy induces an increase in the real marginal cost (as the price of the factors of production rise) but this occurrence shrinks markups, causing in realistic calibrations countercyclical profits. In a heterogeneous agent model to whom profits are distributed is crucial in determining the strength of any policy changes as marginal propensity to consume can differ vastly over the distribution of income and wealth. Moreover, since this model features two assets further assumptions are required to determine whether profits are distributed back into the household's liquid or illiquid account. In the baseline HANK model of Kaplan et al. (2018) it is assumed that a fraction of profits $\omega \in [0, 1]$ are invested directly into the illiquid account. This fraction is set such that the effect of countercyclical profits do not weigh down directly onto the level of investment in the economy. The parameterisation that achieves this is where the share of profits distributed back into the illiquid account is equal to the capital share of output $\omega = \alpha$.

Aggregating total illiquid income flows (1.7) to the economy-wide level where aggregate equity share $S_t = 1$ and aggregate capital at time t is denoted by $K_t$. The benefit of holding capital is the return it provides subtracting the depreciation $\delta$ that it incurs, whereas the benefit of holding equity shares is in the stream of profits. This is outlined below

$$(r_t^k - \delta)K_t + \omega\Pi_t = \alpha m_t Y_t + \omega(1 - m_t)Y_t \tag{1.9}$$

$$\text{With } \omega = \alpha : \Rightarrow \alpha m_t Y_t + \omega(1 - m_t)Y_t = \alpha Y_t$$

**Case 1: Profit distribution as bonuses**

The remaining share of profits $1 - \omega$ that are not reinvested in the illiquid account are deposited lump-sum into the household's liquid account. Following Kaplan et al. (2018), these profits are distributed proportionally to household productivity

$$\pi_{it}^b = \frac{z_{it}}{\bar{z}}(1-\omega)\Pi_t, \tag{1.10}$$

where $\bar{z}$ is average productivity. This distribution scheme is best aligned with bonuses, the profit-sharing component of worker compensation due to output produced within the firm. This is the baseline profit distribution scheme used in this paper and is known as Case 1. Following the empirical exercise, in Section 1.8 I will experiment with distributing profits based on illiquid asset holdings, which closely resembles dividends.

### 1.3.3 Monetary Authority

The monetary authority sets the nominal interest rate following a Taylor rule that reacts to movements in inflation only,

$$i_t = \bar{r}^b + \phi\pi_t + \epsilon_t. \tag{1.11}$$

The nominal interest rate $i_t$ forms the nominal part of the real return on liquid bonds, given by the Fisher equation $r_t^b = i_t - \pi_t$. The central bank dislikes movements in inflation from the steady-state and $\phi$ is set such that the central bank reacts accordingly. The innovation $\epsilon_t$ will form the basis of the future analysis as a fall in $\epsilon_t$ represents an expansionary monetary policy shock. In the steady state no shocks to the Taylor rule are present and as such $\epsilon = 0$. Further extensions can be made to allow the central bank to react to the output gap or suffer from the zero-lower-bound.

### 1.3.4 Government

The government is purposely kept simple as the focus is on monetary policy instead of fiscal policy. The government serves as the sole issuer of liquid assets in the economy, which are real bonds of infinitesimal maturity $B_t^g$. Government expenditure $G_t$ is exogenous and held fixed, taxes exist on labour income and are also fixed at $\tau_t$. Therefore to balance its intertemporal budget constraint the government adjusts transfers $T_t$. This budget constraint is given by:

$$\dot{B}_t^g + G_t + T_t = \tau_t \int w_t z_t l_t(a,b,z) d\mu_t + r_t^b B_t^g \tag{1.12}$$

### 1.3.5 Equilibrium

An equilibrium in this economy, following the definition outlined in Kaplan et al. (2018), is characterised by the decisions of individual households and firms $\{a_t, b_t, c_t, d_t, l_t, n_t, k_t\}_{t\geq 0}$, input prices $\{w_t, r_t^k\}_{t\geq 0}$, return on assets, $\{r_t^b, r_t^a\}_{t\geq 0}$, share price $\{q_t\}_{t\geq 0}$ , price inflation $\{\pi_t\}_{t\geq 0}$, taxes, transfers, government expenditure and the amount of real bonds in the economy $\{\tau_t, T_t, G_t, B_t\}_{t\geq 0}$ as well as the evolution of the distribution of households $\{\mu_t\}_{t\geq 0}$, and aggregate quantities. Such that at every point in time $t$: (i) households and firms maximise their objective functions subject to their budget constraints and taking as given equilibrium prices, taxes and transfers; (ii) the sequence of distributions satisfies aggregate consistency conditions; (iii) the government budget constraint holds; and (iv) all markets clear. There are five markets in this economy: the liquid asset market, the illiquid asset market (which is the combination of the markets for capital and shares of the intermediate firm), the labour market and the goods market.

The liquid asset market clears as the total bonds in the economy are set in zero net supply. The total household holdings of liquid assets are denoted $B_t^h = \int b d\mu_t$, with the government forming the other side of the market $B_t^g$.

$$B_t^h + B_t^g = 0 \tag{1.13}$$

Illiquid asset market clears as total illiquid assets in the economy $A_t = \int a d\mu_t$ are shared between aggregate capital $K_t$ and equity shares $q_t S_t$ where the total number of shares are normalised to 1, $S_t = 1$ .

$$K_t + q_t = A_t \tag{1.14}$$

The labour market clears when the aggregate output from workers in firms $N_t$ is equal to sum of labour production by household. As flexible wages are assumed in this economy they adjust such that no unemployment exists.

$$N_t = \int z l_t(a,b,z) d\mu_t \tag{1.15}$$

The goods market closes the model as total output in the economy $Y_t$ must equate to the aggregate consumption $C_t$, investment $I_t$, government spending $G_t$, total price adjustment costs $\Theta_t$ and borrowing costs $\kappa \int \max\{-b, 0\} d\mu_t$.

$$Y_t = C_t + I_t + G_t + \Theta_t + \chi_t + \kappa \int \max\{-b, 0\} d\mu_t \tag{1.16}$$

### 1.3.6 Labour productivity dynamics

Households' labour earnings are dependent on their labour supply, the wage rate and productivity of the household. The log-earnings process below in Equation (1.17) highlights that productivity is the sum of two independent processes. Each one of these $z_{1,it}$ and $z_{2,it}$ are defined by a jump-drift process outlined in Equation (1.18). These jumps arrive at Poisson rate $\lambda_j$, where the arrival rate and size of these shocks have been estimated using the kurtosis of annual earnings changes. Two independent processes are selected, one that has small but frequent shocks and the other with large but infrequent shocks. The processes with small but frequent shocks can be thought of as advancements within the agents career, whereas large but infrequent shocks more closely match career or life changes.

$$\log z_{it} = z_{1,it} + z_{2,it} \tag{1.17}$$

Condition on a jump the new log-productivity state $z'_{j,it}$ is drawn from a normal distribution $z'_{j,it} \sim N(0, \sigma_j^2)$. $J_{j,it}$ captures jumps in the process. These processes are analogous to a discrete-time AR(1) process with a stochastic arrival of each innovation, given by $\lambda_j$

$$dz_{j,it} = -\beta_j z_{j,it} dt + dJ_{j,it} \tag{1.18}$$

## 1.4 Model Results

This section provides the results for the baseline HANK model of Kaplan et al. (2018), grouping households by their wealth holdings and decomposing their income change from an expansionary monetary policy shock.

### 1.4.1 Case 1: Profit distributed as bonuses

Case 1, following the baseline HANK model of Kaplan et al. (2018), distributes the portion of profits that flow back into the households' liquid account by relative labour productivity. As seen in Table 1.1 these profit flows form a larger share of net income for the households with the lowest net worth, compared to those in the highest quartile of net worth. In the data, using the Survey of Consumer Finances, profit flows typically form a smaller share of income as net wealth of the household decreases. The household in the bottom quartile of the net worth distribution is a borrower and therefore has to pay borrowing costs, this can be seen as the return on the liquid asset dampens the income of the poorest households. Other than the top quartile labour income forms the dominant share of income flows for these households. The return on the illiquid asset, which by the no-arbitrage condition is equal to the return on capital, includes as well the profits that are distributed directly into the illiquid account of the households ($\omega = 0.33$). The return on the illiquid asset, which due to adjustment costs between asset types is always above the liquid asset return forms the largest share of income flows for the top net worth households. The differences between quartiles in Table 1.1 helps to motivate splitting the distribution into low wealth households (bottom 75% of net worth) and high wealth households (top 25% of net worth). The empirical counterpart to Table 1.1, created using data from the Survey of Consumer Finances, can be found in Appendix 1.B.[9]

Table 1.1: Case 1: Net income sources by net worth

| | 0-25% | 25-50% | 50-75% | 75-100% |
|---|---|---|---|---|
| Labour Income | 77% | 79% | 77% | 33% |
| Transfer Income | 15% | 12% | 10% | 4% |
| Profit flows | 8% | 8% | 8% | 4% |
| Liquid Asset | -1% | 0% | 2% | 2% |
| Illiquid Asset | 0% | 0% | 3% | 58% |

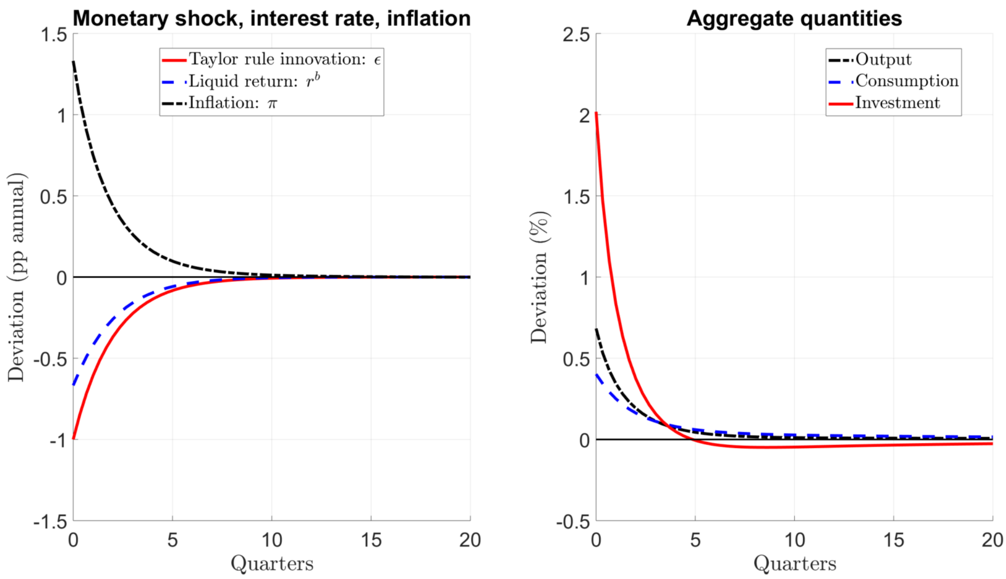
Steady state decomposition of net income by household net worth quartiles.

Figure 1.1 outlines the aggregate response to the economy of a 0.25 percentage, or 1 percentage annually, fall in the Taylor rule innovation on the aggregate variables within the economy. Figure 1.1 replicates Figure 3 of Kaplan et al. (2018) and is

[9]A further motivation for focusing on the profit distribution scheme in this paper is the discrepancies present between the theoretical income flows seen in Table 1.1 compared to their empirical counterpart in Table 1.8 and Table 1.9

empirically plausible, such that investment is the most volatile part of output with consumption being one of the least.

Figure 1.1: Case 1: Aggregate response to an expansionary MP shock

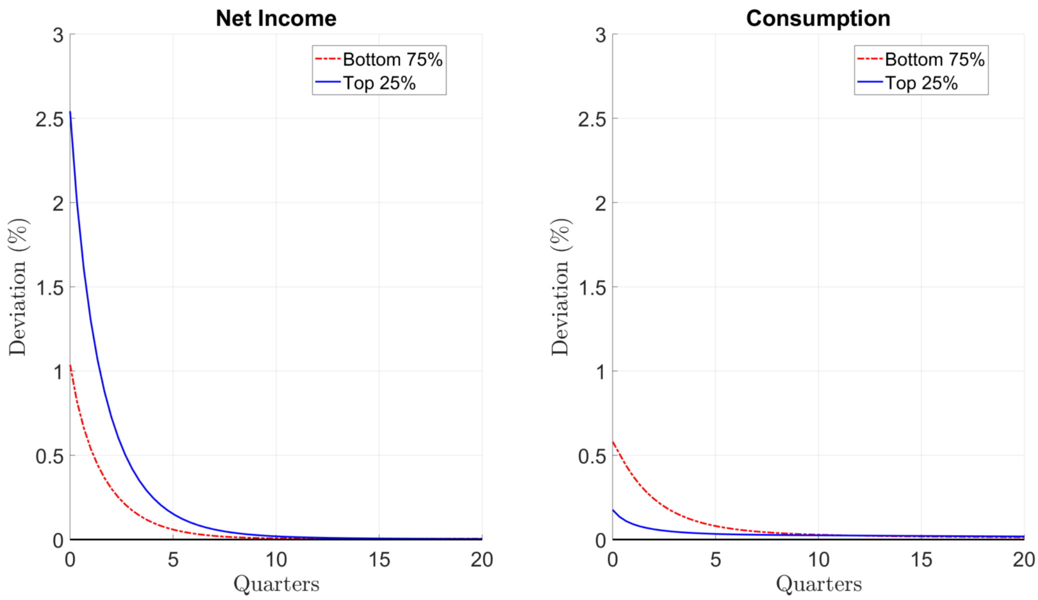



*Note:* Response of aggregate variables to a 0.25 (1% annual) fall in the innovation in the Taylor rule $\epsilon$.

Figure 1.2 shows the response of an expansionary monetary policy shock by household net worth. Net income is defined as the sum of net labour income, $(1-\tau_t)w_t z_t l_t$, transfers from the government $T_t$, income from profits, $\pi_t^b$, the return on liquid asset holdings, $r^b(b)b_t$, and the return on illiquid asset holdings, $r_t^a a_t$. Income increases by more for the households in the top 25% of the net worth distribution compared to those in the bottom 75%. Although income increases the most for the wealthiest households in the economy their consumption response is muted in comparison to the poorest households, who have a higher marginal propensity to consume.

Analysing Figure 1.3 provides the detailed breakdown of the sources of income to households by their net worth. The black dashed line is net labour income, $(1-\tau_t)w_t z_t l_t$, the dashed blue line is government transfers $T_t$, the red line is the income from profits, $\pi_t^b$, the return from liquid assets, $r^b(b)b_t$, is represented by the red dashed line and finally the pink dashed line with diamonds is the return from illiquid asset holdings, $r_t^a a_t$, which is equal to the marginal product of capital minus its depreciation rate. From this Figure it is clear that the large

Figure 1.2: Case 1: Consumption and income response by net worth




*Note:* Response of consumption and income by net worth to a 0.25 (1% annual) fall in the innovation in the Taylor rule $\epsilon$.

income increase that the highest net worth households benefit from is due to the increase in the return on illiquid assets, which they hold the majority of. The increase in labour income, which all households benefit from,[10] is offset by countercyclical markups that lead to countercyclical profit income during a period when output is increasing. This balancing of income is present for all households across the net wealth distribution, however, the poorest households hold an immaterial measure of illiquid assets and thus do not benefit from the increase in the illiquid return to the extent of the richest households. Futhermore, due to the adjustment cost between liquid and illiquid assets it is costly for households to transfer their liquid assets to benefit from the increased returns of the illiquid asset. Moreover, there is a desire of households within the economy to insure against idiosyncratic productivity shocks through holding liquid wealth, ensuring that they are unlikely to hit their budget constraint. The counteracting force of the profit income and labour income in Figure 1.3 is unsurprising as the rise in wages, above prices, is a major source of the countercyclical markups.

## 1.5 Empirical Methodology

Following from the theoretical exercise above it is important to verify if the aggregate and household response to an expansionary monetary policy shock are in line with the empirical evidence. Below I outline the empirical strategy used in this paper.

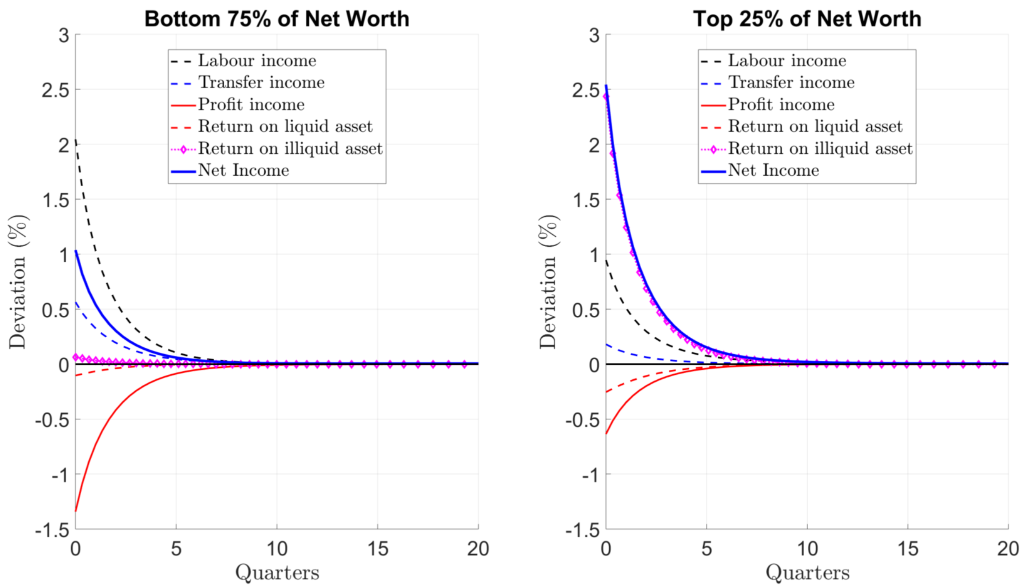
### 1.5.1 Econometric tools

The baseline empirical exercise is conducted using an autoregressive distributed lag (ARDL) model following Cloyne et al. (2020). This approach is related to those by Romer and Romer (2004) and Coibion (2012) and is shown to perform well in small samples by Choi and Chudik (2019) and is a similar approach to Jordà (2005).[11] Specifically the baseline results are obtained estimating the following relationship:

[10]The wage rises in the economy following an expansionary monetary policy shock and labour supply for the bottom 75% and top 25% of households by net worth increase comparably, as seen in Appendix 1.H.

[11]The Local Projections approach of Jordà (2005) estimates a separate regression for each horizon, whereas this is not the case with the ARDL.

Figure 1.3: Case 1: Income response decomposed by net worth




*Note:* Decomposing income response by net worth to a 0.25 (1% annual) fall in the innovation in the Taylor rule $\epsilon$. The decomposition represented is scaled by the share of net income, so that the lines (excluding net income) will sum to net income. The black dashed line is net labour income, $(1 - \tau_t)w_t z_t l_t$, the dashed blue line is government transfers $T_t$, the red line is the income from profits, $\pi_t^b$, the return from liquid assets, $r^b(b)b_t$, is represented by the red dashed line and finally the pink dashed line with diamonds is the return from illiquid asset holdings, $r_t^a a_t$.

$$Y_{i,t} = \alpha_0^i + \alpha_1^i trend + \sum_{\ell=1}^{P} b_\ell^i Y_{i,t-\ell} + \sum_{\ell=1}^{Q} c_\ell^i S_{t-\ell} + \sum_{q=2}^{4} D_q^i Z_q + u_{i,t}, \quad (1.19)$$

where the dependent variable $Y_{i,t}$ is the variable of interest - real income or non-durable consumption by wealth levels ($i = \{\text{Low Wealth}, \text{High Wealth}\}$) at quarter $t$. $S_t$ is the exogenous monetary policy shocks cleaned of any information effects taken from Jarociński and Karadi (2020) and described further in Section 1.6. $Z_t$ represents quarterly dummy variables to control for seasonal effects on consumption and income. The inclusion of $\alpha$ allows for a break in the series for the zero lower bound, which the Federal Funds rate reached in December 2008. If we let $B^i(L)$ and $C^i(L)$ represent the lag operator combined with the regression coefficients, such that $B^i(L) = \sum_{\ell=1}^{P} b_\ell^i L^\ell$ and $C^i(L) = \sum_{\ell=1}^{Q} c_\ell^i L^\ell$ with $P$ pertaining to the order of autoregressive lags and $Q$ being the lags of the monetary policy shock. Therefore, in a simplified model such as $[1 - B^i(L)]Y_{i,t} = C^i(L)S_t + u_{i,t}$, the impulse response from a monetary policy shock can be estimated as $\hat{c}^i(L) = [1 - \hat{B}^i(L)]^{-1}\hat{C}^i(L)$. Standard errors are bootstrapped using a recursive wild bootstrap following Mertens and Ravn (2013), which is robust to heteroskedasticity.

Additional empirical exercises are conducted using a Bayesian Proxy Structural VAR (BP-SVAR) following closely the approach outlined in Miranda-Agrippino and Ricco (2018a), Miranda-Agrippino and Ricco (2018b) and Caldara and Herbst (2019). This approach is described further in Appendix 1.D with the results available in Appendix 1.E.

### 1.5.2 Household level data and MPC calculation

The household data[12] used is from the Consumer Expenditure Survey (CEX), which is the most comprehensive source of consumption data for the United States currently available. Furthermore, as shown in Auclert (2019) the CEX is able to match household liabilities from the Survey of Consumer Finances (SCF) closely.[13] The CEX data is collected by the Census Bureau for the Bureau of Labor Statistics and is comprised of two surveys, the Interview Survey and the

[12]Description of the aggregate data can be found in Appendix 1.E.1.

[13]The Survey of Consumer Finances by the Federal Reserve Board is a triennial cross-sectional survey of U.S. families' wealth and income. The SCF is typically regarded as the most accurate and exhaustive measure of U.S. families' balance sheets.

Diary Survey. The Interview Survey details the major and recurring items purchased by households, whereas the Diary Survey focuses on minor but frequently purchased items. The research conducted focuses on the Interview Survey. The Interview Survey "provides information up to 95% of the typical household's consumption expenditures" Coibion et al. (2017). The CEX is a monthly rotating panel, such that households are interviewed for four consecutive quarters (detailing their monthly consumption and expenses), with an additional preliminary interview, before they are dropped from the sample. The survey is conducted and weighted such that it is representative of the U.S. population and about 1500-2000 households are surveyed in a given month, up to 6000 each quarter. The households report their consumption for the three months prior to the interview month, which does not need to coincide with calendar quarters. Since households tend to report smoothed consumption values within-interview (such that consumption is smoothed over the reported three months), one may be concerned that aggregating at the calendar quarterly frequency may introduce a serial correlation structure which would be problematic for VAR analysis. As shown in the Appendix of Coibion et al. (2017), this worry has no impact on their results. For the empirical exercise the data will be aggregated to the quarterly frequency and will range from 1996-Q1 to 2017-Q4, which is the span of the publicly available data. Furthermore, as explained further the data series that I will use to construct measures of household wealth are only available from 1994 in the CEX.

Consumption at the household level is the sum of non-durable expenses and services. Non-durable expenses include food and beverages, clothing, gasoline, personal care, magazines, newspapers and tobacco. Services included in this measure of consumption are household utilities, recreational services, financial services, telecommunication services and transportation services.

Net Income in the CEX is expressed as income gained over the past year. The income measure used follows Coibion et al. (2017), where it is the sum of labour income, financial income, business income and transfers (defined as other income) minus state and federal taxes. Labour income is the salary or wage earned over the last 12 months. Financial income is defined as the sum of income received from interest, dividends, royalties, estates or trusts, net rental income and income from pensions. Business income is the amount of income received from self-employment. Transfers is the total amount received from Social Security benefits, public assistance or welfare and unemployment benefits.

Following Coibion et al. (2017) tax is computed using the NBER TAXSIM calculator[14] as this helps to improve the missing tax data in the earlier sample of the CEX. To remove large anomalies in the data that could bias results I winsorise the variables used at the bottom and top 1%.

### Wealth calculation

The main measure used to separate households within this paper is wealth. Wealth here is calculated in an end-of-period fashion, such that income and consumption over the year is also taken into account. This is the natural measurement of wealth since assets and liabilities are only available in the last interview undertaken by the households in the Consumer Expenditure Survey. Equation (1.20) outlines how this wealth measure is calculated, where $Y_i$ is gross income, $T_i$ is taxes, $C_i$ is consumption, $A_i$ is assets and $L_i$ is liabilities. Net income (gross income minus taxes) as well as consumption are explained in the previous subsection. Assets are calculated as the total value of savings and checking accounts of the household as well as total bonds and securities held by the household.[15] Liabilities are calculated as the total mortgage principal and home equity loan outstanding, principal outlays on vehicles and credit card debt. Due to the lack of house values on the asset side of the household balance sheet the asset measure provides the largest data limitation to the measure of wealth from the CEX. This shortcoming is highlighted by Auclert (2019), who compares asset values in the CEX with the Survey of Consumer Finances[16]. To ensure the lack of housing data does not impact the empirical results, a robustness exercise is conducting taking housing values by age and income groups and assigning them to the equivalent household in the CEX data, results of this can be found in Appendix 1.E.6. The wealth measure used in this paper is given by

$$Wealth_i = Y_i - T_i - C_i + A_i - L_i \tag{1.20}$$

As well as analysing households by their position along the wealth distribution I also consider their position along the income distribution. Moreover, motivated

[14] See Feenberg and Coutts (1993) for further explanation of the NBER TAXSIM.

[15] House values, which is a large part of household's assets, are not available in the CEX. Therefore in Appendix 1.E.6 further robustness is undertaken using house values by income and age groups from the Survey of Consumer Finances. The underlying results are qualitatively unchanged with the inclusion of housing.

[16] The Survey of Consumer Finances, conducted tri-annually provides an in-depth measure of household wealth and liabilities but the survey occurs too infrequently to be easily used in time-series analysis.

by Auclert (2019), who highlights three channels that affect spending following a monetary policy shock, I focus on the measure of unhedged interest rate exposure (URE) as an additional robustness exercise. The URE is the differences between all maturing assets and liabilities, which also includes income minus consumption. This provides a measure of the value of currently available income and liquid assets. $A_i^m$ is assets maturing this period and $L_i^m$ is liabilities maturing this period. The asset and liability measures use the same variables as the wealth calculation but now require additional assumptions to the expected maturity. A household with a low level of URE typically acts hand-to-mouth as they have a high marginal propensity to consume out of additional cash since they have a small amount of liquid assets (or assets close to maturity). This is explored further in Auclert (2019).

$$URE_i = Y_i - T_i - C_i + A_i^m - L_i^m \tag{1.21}$$

The other channels featured in Auclert (2019), an earnings heterogeneity channel and Fisher channel from unexpected inflation are related to my two aforementioned measures. The empirical results using the URE measurement and heterogeneous income channel can be found in Appendix 1.E.5.

#### Marginal Propensity to Consume calculation

As highlighted by Auclert (2019) and going back to Tobin (1982), if households within the economy have similar marginal propensity to consume then heterogeneity across households would not matter for the aggregate response of the economy. If the marginal propensity to consume across the wealth distribution was similar then it would be more appropriate to use a RANK model rather than a HANK model. This Section shows that households across the wealth distribution have different marginal propensity to consume, providing empirical justification for using the HANK model and motivating further analysis conducted in the paper. The marginal propensity to consume calculation for each wealth group follows the exercise of Johnson et al. (2006). Their work benefits from the randomly assigned timing of tax rebates during a ten-week period from late July to the end of September during 2001. Tax rebates, due to The Economic Growth and Tax Relief Reconciliation Act of 2001 that enacted substantial reductions in federal, personal and estate tax rates, were retroactively enforced for income earned from the start of 2001. These tax rebates represented an advance payment of this tax cut and were typically \$300 to \$600 in value.

These were randomly assigned as the timing of the mailing of the rebates was due to the second-to-last digit of the Social Security number (SSN) of the tax filer that received it. This random assignment provides an ideal natural experiment to assess the marginal propensity to consume out of additional income to test whether poorer households are more hand-to-mouth, such that they consume a large majority of their additional income. Random assignment is crucial so that the timing of receiving the rebate is independent of any household characteristics. These rebates were preannounced as the Tax Act was passed in May 2001, which should dampen the consumption response if households follow the rational-expectations permanent income hypothesis.

Equation (1.22) outlines the estimation procedure conducted at the monthly level to measure the MPC of high and low wealth agents. The dependent variable is the change in monthly consumption, a seasonal dummy called $month$ is used to absorb the seasonal variation in consumption expenditures. Additional control variables, $X$, including age and family composition to absorb any preference driven factors that could influence the growth rate of consumption across households. The rebate variable $R_{i,t+1}$ is the distributed lag of the value of the rebate that uses a dummy variable, $I(Rebate > 0)$, indicating whether the rebate was received in $t+1$ along with other regressors as an instrument in a two-stage least squares (2SLS) regression. This measures the longer-run effect of the rebate on consumption up to six months after the rebate has been received[17].

$$\Delta C_{i,t+1} = \sum_{m} \beta_{0s} \times month_{s,i} + \beta_1' X_{i,t} + \beta_2' R_{i,t+1} + u_{i,t+1} \tag{1.22}$$

I cut the wealth distribution of households into the top 25%, the high wealth, and the bottom 75%, low wealth. As motivated further in Appendix 1.C, this selection provides a good measure of households with some positive wealth and should be seen as a lower bound from an alternative top 10% vs bottom 90% measure. As shown in Table 1.2 the marginal propensity to consume of the highest wealth agents is the lowest with the cumulative MPC not statistically different from zero with a large standard error of 0.53.[18] A negative MPC means that the households use the rebate to induce further savings in order to purchase goods later, as outlined further by Misra and Surico (2014) who use quantile regressions to analyse the impact of the 2001 and 2008 tax rebate. Whereas the households

[17] Due to the rolling panel data nature of the CEX using two quarters after the rebate provides a longer-run measure of consumption whilst retaining a large fraction of households.

[18] Part of this large standard error could be explained by to the existence of wealthy hand-to-mouth households within my wealth measure, as outlined by Kaplan et al. (2014).

Table 1.2: Cumulative MPC for households by wealth in 2001

| | Cumulative MPC for 2001 |
|---|---|
| Bottom 75% Wealth | 0.59 |
| | (0.26) |
| Top 25% Wealth | -0.02 |
| | (0.53) |
| Number of Observations | 11,856 |

Cumulative MPC for households in the top 25% and bottom 75% of the wealth distribution. Exercise is conducted following Johnson et al. (2006).

at the bottom 75% of the wealth distribution consume on average a significant amount of 0.59 of this rebate over two quarters with a standard error of 0.26.

This result is supported by Johnson et al. (2006), who find that older, higher income households with more liquidity all have lower MPC than younger, lower income households with less liquidity following the 2001 tax rebate. This result is also in line with additional findings by Misra and Surico (2014) who conduct a similar exercise for the tax rebates for 2001 and 2008 using quantile regressions in. Additional results analysing the marginal propensity to consume for households across the wealth distribution using tax rebate data from 2008 can be found in Appendix 1.F.

## 1.6 Identification Strategy

The preferred monetary policy surprises used directly in our baseline empirical strategy and as an instrument for the Bayesian Proxy SVAR, outlined in Appendix 1.D, are provided by Jarociński and Karadi (2020).[19] These surprises are cleaned of the superior information on the economic outlook that is released through the monetary policy announcement by the Federal Reserve's Federal Open Market Committee (FOMC). Using monetary policy surprises that are not cleaned of this informational effect "can lead to biased measurements of

[19]Further robustness checks are conducted using monetary policy surprises from Miranda-Agrippino and Ricco (2018b) as these are also cleaned of additional information on the state of the economy which have been shown to create empirical puzzles. Moreover Bayesian local projection following Miranda-Agrippino and Ricco (2018b) is also used as a further robustness to the BP-SVAR.

monetary non-neutrality," Jarociński and Karadi (2020).

Initially high-frequency movements in interest rates and asset prices surrounding 240 FOMC announcement dates, from 1990 to 2017, are used from an updated version of Gürkaynak et al. (2005). The preferred measure of interest rate surprise in Jarociński and Karadi (2020) is the 3-month fed fund futures, typically denoted as *FF4*. This duration of the futures contract coincides nicely with the date of the next FOMC meeting and therefore reflects the expected future monetary policy decision, moreover, the three month time-span is able to capture a broad measure of monetary policy - including both short-term fluctuations and near-term forward guidance. In order to clean for the additional information released during the FOMC meeting a measure of the state of the economy is required. The measure chosen is the change in the *S&P 500*, an index based on the 500 largest U.S. companies, 10 minutes before and 20 minutes after the FOMC announcement. The authors take these two surprises and construct a Bayesian Structural VAR with sign restrictions to disentangle a pure monetary policy shock from an informational shock. A monetary policy shock is identified by a negative co-movement between the interest rate surprise and the stock price surprise, whereas an information shock is found from a positive co-movement. This co-movement is informative as standard theoretical models are clear on how stock prices should react following a monetary policy shock.

## 1.7 Empirical Results

This section outlines the response of heterogeneous response of a monetary policy shock dependent on the distribution of household wealth. The response of aggregate variables to an expansionary monetary policy shock can be seen in Appendix 1.E.2.

### 1.7.1 Monetary Policy shock by wealth groups

Figure 1.4 highlights the differences in the individual responses to an expansionary monetary policy shock when households are separated due to their wealth. The expansionary monetary policy causes the increase in income for both the high wealth (top 25% wealth) and low wealth (bottom 75% wealth ) households. The wealthier household's income response is closely aligned to the lower wealth household, with very little discernible difference between the two responses. However, the wealthier households' consumption response following an expansionary monetary policy shock contrasts the low wealth households. The wealthier households do not significantly adjust consumption, whereas households in the bottom three quartiles of the wealth distribution increase their consumption markedly. The consumption response for the bottom 75% of the wealth distribution is more sensitive to monetary policy shocks. After reaching the peak response after two years from the monetary policy shock, consumption falls for the bottom 75% of the wealth distribution. The households with more wealth, likely save their income increases and choose to smooth their consumption instead - as we see consumption rising after one year of the shock.

Comparing Figure 1.4, from the empirical exercise, with Figure 1.2, from the equivalent theoretical exercise, it is apparent that the income response of households along the wealth distribution in the theoretical model is at odds with the empirical findings. In the theoretical model the income response of the lowest net worth households was muted, dampened partly by the fall in profits that were being distributed by labour productivity. The response of consumption from an expansionary monetary policy shock found in the HANK model is qualitatively similar to that in the empirical exercise, such that the highest net worth households that typically have a low marginal propensity to consume do not increase their consumption to the same extent as the other households in the economy. The magnitude of the income response is in line with the model however the consumption response of the poorer household outstrips the equivalent response in the model. This could be due to over-reporting consumption changes in the CEX or a limitation of the theoretical model.[20]

Motivated by the differences found in Figure 1.4, Figure 1.5 analyses these differences to see if they are statistically significant. The difference in income responses for the low wealth group versus the high wealth group is insignificant and signifies the lack of discrepancies that are present. The response of income is not significantly different throughout the 16 quarters observed between these two groups. The differences in consumption found previously are statistically different, which is in line with the theoretical HANK model.

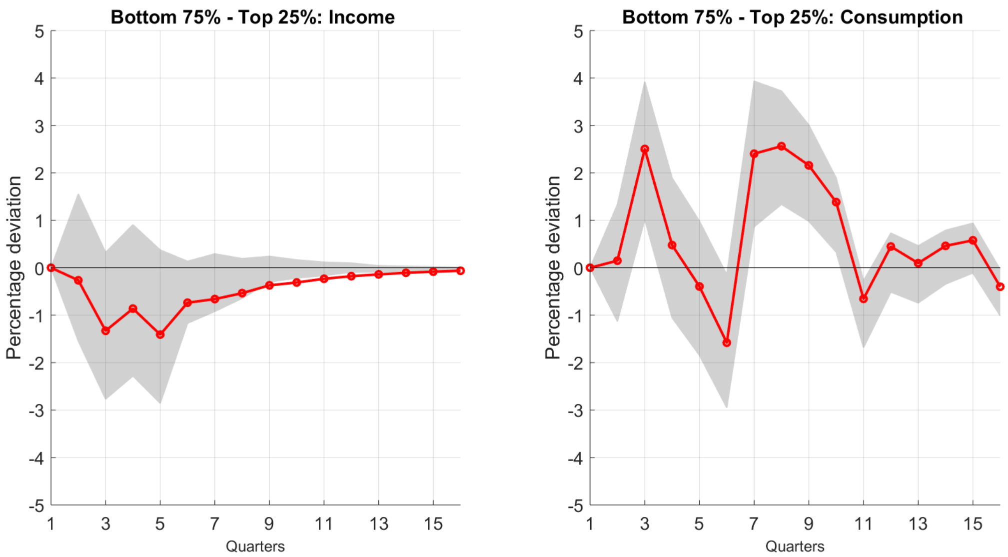
This result highlights the finding that there is an increase in disparity between consumption by high and low wealth groups from a contractionary monetary

[20]The magnitude of the response to a negative monetary policy surprise is larger than the typical response seen in similar empirical studies. This is due to the sample covering the zero-lower-bound period, where monetary policy surprises were limited but real variables were volatile. Moreover, decomposing the monetary policy surprises, following Jarociński and Karadi (2020), further diminishes the size of the clean monetary policy surprises.

Figure 1.4: Individual Response by Wealth groups


*Note:* Response to a 25bp decrease in the monetary policy surprise of Jarociński and Karadi (2020) by households grouped by wealth. Using Autoregressive Distributed Lag model from Cloyne et al. (2020). Quarterly data from 1996-2017. Wealth groups are rotated following Anderson et al. (2016). Shaded areas are 68% coverage bands obtained using 10,000 draws of the recursive wild bootstrap following Mertens and Ravn (2013).

Figure 1.5: Difference between wealth groups: Mostly in Consumption




*Note:* Response to a 25bp decrease in the monetary policy surprise of Jarociński and Karadi (2020) by households grouped by wealth. Using Autoregressive Distributed Lag model from Cloyne et al. (2020). Quarterly data from 1996-2017. Wealth groups are rotated following Anderson et al. (2016). Shaded areas are 68% coverage bands obtained using 10,000 draws of the recursive wild bootstrap following Mertens and Ravn (2013).

policy shock but less of a disparity is found for income, this finding is supported by work of Coibion et al. (2017). Using household characteristics that are associated with being close to high net worth by Doepke and Schneider (2006), namely "rich, old households" versus "young, middle-class households with fixed-rate mortgage debt," Coibion et al. (2017) analyse the response of consumption and income from a monetary policy shock. They too find that income differences between high and low wealth groups are limited whereas larger differences occur in consumption.

### 1.7.2 Decomposing the income response

Decomposing these income results further into changes in salary and changes in financial income it is apparent that the heterogeneous income sources found in the aggregate results of Figure 1.14 translate into the individual effects for households that are grouped by wealth. The top row of Figure 1.6 shows that the labour income of the households in the bottom 75% of households increase by more than the salary of the top 25 % of the wealth distribution. One explanation for this is the increased fragility of unemployment status for the low wage jobs that are held by the workers with the lowest wealth. Once a shock occurs that boosts the economy, such as an expansionary monetary policy shock, the low paid workers increase their hours of employment by more than the higher paid and likely high wealth workers. The second row of Figure 1.6 highlights the financial income response from a monetary policy shock. This response is extremely volatile, oscillating around zero and is statistically insignificant. The high wealth households are more likely to be savers, whose interest on savings and checking accounts would fall due to the fall in the interest rate. Whereas, the low wealth households are more likely to be borrowers such that as the interest rate falls the interest payment on their loans decrease and therefore financial income should rise or remain muted. Not shown here is the income from business ownership and transfers such as food stamps and unemployment benefits that make up the remaining income response.

The main conclusion from the empirical exercise is that there exists a heterogeneous response of consumption from a monetary policy shock, where the low wealth households' consumption rise by more than the wealthier households. The difference in consumption response cannot be fully explained by the income response and therefore it is likely that these households differ in their wealth holdings and their marginal propensity to consume. Moreover, in the baseline HANK model used in this paper the difference in income response between the households in the bottom 75% of the wealth distribution compared

Figure 1.6: Further decomposing the income response


*Note:* Response to a 25bp decrease in the monetary policy surprise of Jarociński and Karadi (2020) by households grouped by wealth. Using Autoregressive Distributed Lag model from Cloyne et al. (2020). Quarterly data from 1996-2017. Wealth groups are rotated following Anderson et al. (2016). Shaded areas are 68% coverage bands obtained using 10,000 draws of the recursive wild bootstrap following Mertens and Ravn (2013).

to the top 25% is at odds with the data.[21] Part of this difference can be explained by countercyclical profits dampening the income response of the lowest net worth households in the economy. However, using the CEX data it is not possible to see the share of income of low wealth households that come from bonuses.[22] This finding motivates innovating on the profit distribution scheme of the HANK model and is outlined in detail below.

## 1.8 Extending the Model: Changing the Profit Distribution Scheme

In this section I deviate away from the profit distribution scheme outlined in Kaplan et al. (2018), which distributes the majority of profits lump-sum to households dependent on their productivity share in the economy to one that distributes the profit based on their illiquid wealth share.

### 1.8.1 Case 2: Profits as Dividends

An alternative scheme, which is more realistic, has the remaining profit share deposited lump-sum into the household's liquid account proportionally to households' holdings of illiquid assets.[23] This closely resembles dividends, the share of profits paid out to shareholders.[24] As equity shares are indeterminate at the individual level I take the share of illiquid asset holdings within the economy as the measure to use when proportionally distributed the remaining profits.

[21]Moreover in Appendix 1.I the response of the Gini coefficient in the HANK model for income is at odds with the empirical finding of Coibion et al. (2017) for the U.S. economy. Comparing movements in Gini coefficients further strengthens my results and crucially does not depend on wealth data.

[22]This problem also persists in the Survey of Consumer Finances, which does not seperate labour income from bonuses.

[23]This case is similar to the ”wealth-based” rule outlined in Debortoli and Galí (2017) where profit is distributed to shareholders in proportion to their holdings of shares in an equity fund. One difference in regards to the model used in this paper, as in the HANK model of Kaplan et al. (2018), is that it includes adjustment costs between the liquid and illiquid asset.

[24]In Kaplan et al. (2018) dividends refer to altering $\omega$, the share of profits distributed into the illiquid account, whereas in this paper dividends are seen as profits distributed into the liquid account of households dependent on their illiquid asset holdings.

$$\pi_{it}^{b} = \frac{a_{it}}{\bar{a}}(1-\omega)\Pi_t \tag{1.23}$$

Case 2, completes the same exercise as seen in Section 1.3 for Case 1, except now profit is distributed as dividends, such that households with a larger proportion of illiquid assets will receive a greater share of profit flows. This change in profit distribution scheme is apparent in Table 1.3 since the highest net worth households now receive the largest share of profit as dividends. This means that the net income in steady state of the bottom three quartiles of the net wealth distribution now receive a larger share of their income as labour income compared to Case 1.

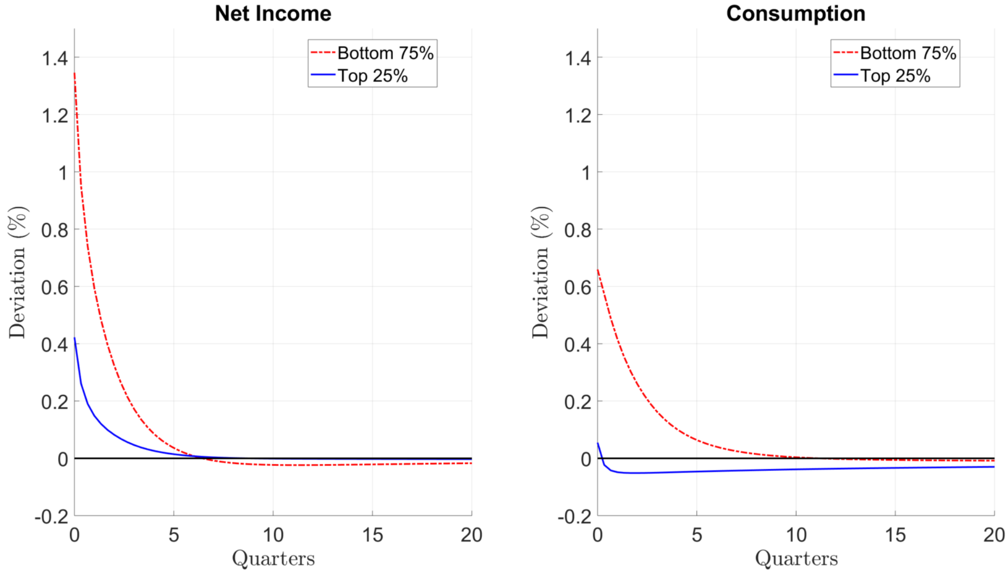
Table 1.3: Case 2: Net income sources by net worth

| | 0-25% | 25-50% | 50-75% | 75-100% |
|---|---|---|---|---|
| Labour Income | 86% | 87% | 85% | 30% |
| Transfer Income | 15% | 12% | 9% | 3% |
| Profit | 0% | 0% | 1% | 11% |
| Liquid Asset | -1% | 0% | 2% | 1% |
| Illiquid Asset | 0% | 0% | 3% | 53% |

Steady state decomposition of net income by household net worth quartiles.

Under Case 2 the highest net worth households receive income from profits into their liquid and illiquid accounts, dampening their income and causing a fall in consumption from an expansionary monetary policy shock. As seen in the left-hand-side panel of Figure 1.7 the income of the wealthiest households now increase the least. Moreover, as outlined in Figure 1.8 due to the countercyclical profits income flows into the liquid account are negative, causing the highest net worth households to reduce their consumption very slightly. This is partly because the top quartile of net worth households also include wealthy hand-to-mouth households, who do not have a buffer of liquid assets and therefore their consumption follows closely the change in their income. Furthermore, the return on illiquid assets, which is supporting the income of the wealthy households enters into their illiquid account and requires an adjustment cost to for it to be transferred to their liquid account where they can use it to purchase consumption goods. Moreover, the wealthy households that are unconstrained choose to save rather than consume, as the return on the illiquid asset increases. The lack of consumption response and more subdued income response for the wealthiest households is qualitatively in-line with the previous empirical exercise.[25] The difference in income (bottom 75% of the wealth distribution - top 25%) increased due to an expansionary monetary policy shock in the empirical exercise. The same movement is replicated here, although the income of the wealthiest households does not follow as closely the other households.

Figure 1.7: Case 2: Consumption and income response by net worth



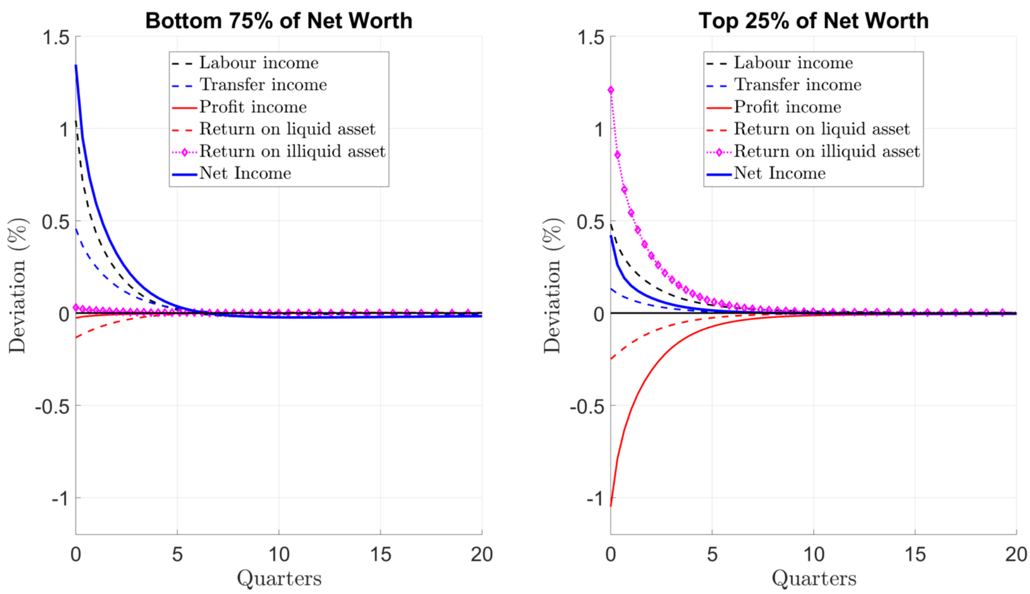

*Note:* Response of consumption and income by net worth to a 0.25 (1% annual) fall in the innovation in the Taylor rule $\epsilon$.

Decomposing the income changes by net worth it is clear to see that the negative effect of profits is primarily attributed to the income of the highest net worth households. Figure 1.8 shows that for the bottom 75% of the wealth distribution labour income continues to dominate their source of income, as seen in Case 1, but now this increase in labour income is not dampened by the profit distribution scheme allowing the income of the lowest net worth households to increase by the most. The income decomposition for the wealthy households in 1.8 sheds light on the fall in consumption seen for these households. The illiquid return is deposited back into the illiquid account and the households face a cost to transfer it over to the liquid account where it can be used for consumption. Additionally, the return on labour income, the liquid asset and profits flow into the liquid account, which due to the countercyclical profits causes a fall in the liquid

[25] The income response represents an extreme due to the profit distribution scheme chosen.

account.

Figure 1.8: Case 2: Income response decomposed by net worth

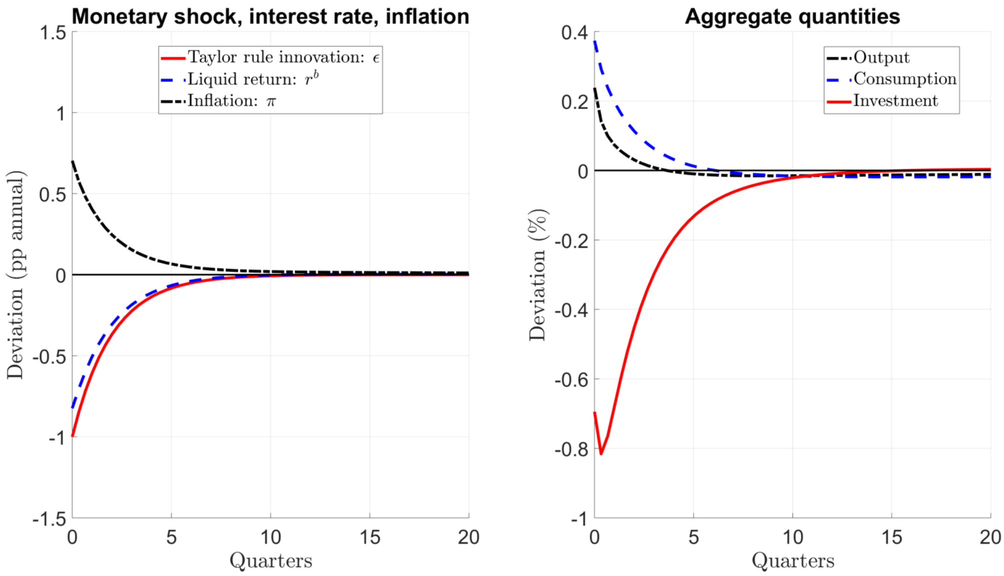



*Note:* Decomposing income response by net worth to a 0.25 (1% annual) fall in the innovation in the Taylor rule $\epsilon$. The decomposition represented is scaled by the share of net income, so that the lines (excluding net income) will sum to net income. The black dashed line is net labour income, $(1-\tau_t)w_t z_t l_t$, the dashed blue line is government transfers $T_t$, the red line is the income from profits, $\pi_t^b$, the return from liquid assets, $r^b(b)b_t$, is represented by the red dashed line and finally the pink dashed line with diamonds is the return from illiquid asset holdings, $r_t^a a_t$.

Figure 1.9 highlights the consequences of fixing the distributional impact of the monetary policy shock. Since investment in this model is determined by the change in illiquid assets and the illiquid assets are held by the highest net wealth households we see a fall in investment following an expansionary monetary policy shock. This aggregate result is counterfactual[26] and stems from the countercycical profits that is placed solely on the illiquid and liquid account of the highest net worth households, the same households that would be typically conducting investment following an expansionary monetary policy shock. Moreover, since liquid income falls for the highest net worth agents they must sell illiquid assets to try and sustain consumption, thus causing investment to fall and a negative comovement between consumption and investment, which is counterfactual.

[26]Appendix 1.E.2 shows that investment is procyclical, such that it increases following an expansionary monetary policy shock. This is a standard result in the literature.

Figure 1.9: Case 2: Aggregate response to an expansionary MP shock




*Note:* Response of aggregate variables to a 0.25 (1% annual) fall in the innovation in the Taylor rule $\epsilon$.

The share of profits that go directly into the illiquid account $\omega$ that would typically play an important role in the cyclicality of investment in Case 1, when profits are distributed proportionally to labour productivity, have little effect in Case 2. This is because lowering the share of profits that go directly into the illiquid account, by changing $\omega = 0.1$ for example, would only serve to change the account (liquid or illiquid) that the highest net worth households receive this negative profit income into.

### 1.8.2 Case 3: 50% bonus and 50% dividend profit distribution with $\omega = 0.1$

The final scheme to be analysed takes a proportion of Case 1 and Case 2 as well as lowering the share of profits that go directly into the illiquid account (lowering $\omega$). The value $\nu^{\pi}$ determines the size of the share of each Case to be used, with the value endogenously set to $50\%$. The value of $\omega$, the amount of profits automatically distributed to the illiquid account, is set sufficiently low that a large majority of profits are distributed into the liquid account.

$$\pi^b_{it} = \nu^\pi \cdot \frac{z_{it}}{\bar{z}}(1-\omega)\Pi_t + (1-\nu^\pi) \cdot \frac{a_{it}}{\bar{a}}(1-\omega)\Pi_t, \tag{1.24}$$

An agnostic approach is taken when deciding the value of the shares from the previous two cases as well as the value of $\omega$. The combined profit distribution scheme is analysed in isolation from altering $\omega$ in Appendix 1.H, as well as the effect of altering $\omega$ for the baseline Case 1.

Case 3 encompasses an equal part of the bonus distribution scheme of Case 1 and combines it with an equal share of the dividend distribution scheme of Case 2. Importantly, this case also deviates from the baseline share of profits that are distributed directly into the illiquid account $\omega$. Lowering this share so that $\omega = 0.1$, means that a larger share of profits are now distributed into the liquid account than if $\omega = 0.33$. Table 1.4 retains the appealing properties of Table 1.3 where the profit shares were highest for the higher net worth households.

Table 1.4: Case 3: Net income sources by net worth

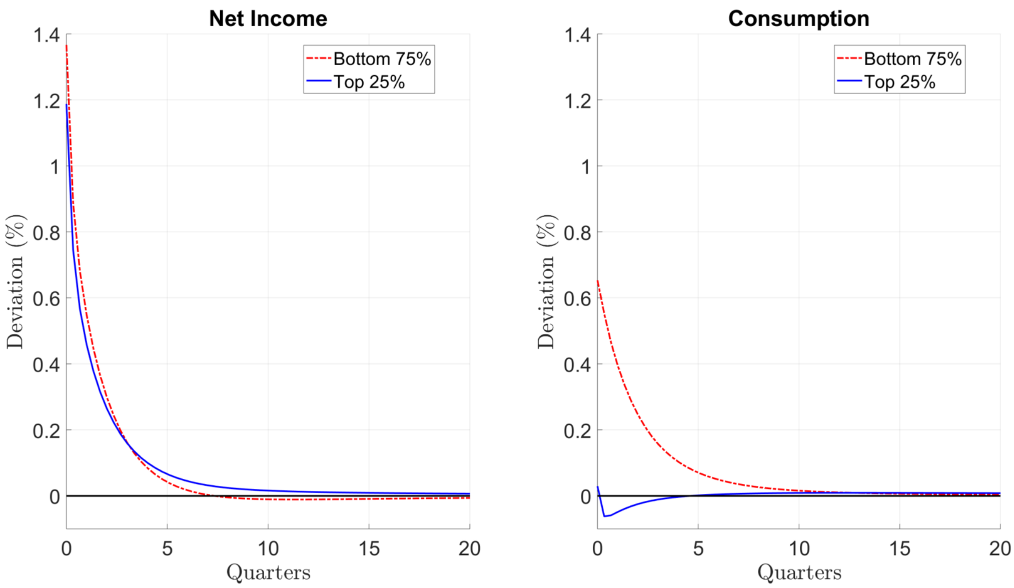
| | 0-25% | 25-50% | 50-75% | 75-100% |
|---|---|---|---|---|
| Labour Income | 79% | 82% | 81% | 34% |
| Transfer Income | 15% | 12% | 10% | 4% |
| Profit | 6% | 6% | 6% | 11% |
| Liquid Asset | -1% | 0% | 2% | 2% |
| Illiquid Asset | 0% | 0% | 2% | 49% |

Steady state decomposition of net income by household net worth quartiles.

Figure 1.10 outlines the response of consumption and income by net worth to the same expansionary monetary policy shock experienced in Case 1 and Case 2. However, the income response in the right panel of Figure 1.10 mimics the insignificant difference found in the income response of the top 25% wealth household versus the bottom 75% seen in the empirical exercise. The consumption response by net worth is also in-line with the micro evidence as the highest net worth household have a muted response in consumption as wealthy hand-to-mouth households need to transfer assets from their illiquid account to consume whereas wealthy households with a sizable liquid account wish to obtain the higher earnings from the illiquid assets and therefore save.

Figure 1.11 shows the result of selecting a mixed profit distribution scheme and lowering the share of profits distributed to the illiquid account. The negative

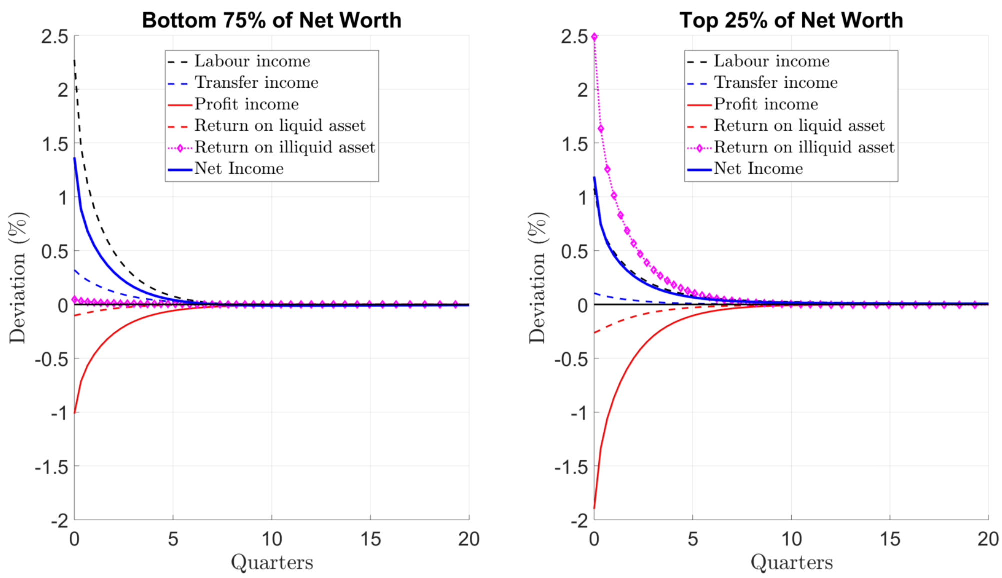
Figure 1.10: Case 3: Consumption and income response by net worth




*Note:* Response of consumption and income by net worth to a 0.25 (1% annual) fall in the innovation in the Taylor rule $\epsilon$.

income due to the countercyclical profits now appears on the liquid account of all of the households within the economy, spreading out the effect to the lower wealth households in compared to Case 2. The increase in income from the return on illiquid assets is balanced out by the negative profits that the high net worth households receive, such that on aggregate their income response is muted. For the remaining households, as with Case 1 and Case 2 labour income remains the dominant source of their income flows. However, unlike Case 1, labour income is now double the negative profit income that these households receive, helping to support their income response.

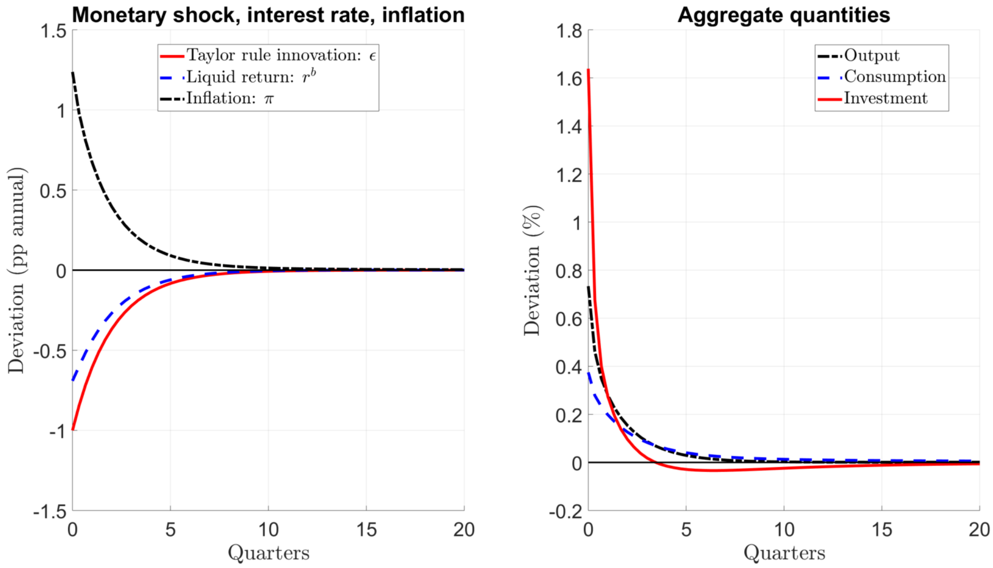
Figure 1.11: Case 3: Income response decomposed by net worth




*Note:* Decomposing income response by net worth to a 0.25 (1% annual) fall in the innovation in the Taylor rule $\epsilon$. The decomposition represented is scaled by the share of net income, so that the lines (excluding net income) will sum to net income. The black dashed line is net labour income, $(1-\tau_t)w_t z_t l_t$, the dashed blue line is government transfers $T_t$, the red line is the income from profits, $\pi_t^b$, the return from liquid assets, $r^b(b)b_t$, is represented by the red dashed line and finally the pink dashed line with diamonds is the return from illiquid asset holdings, $r_t^a a_t$.

Since the negative income flows due to the countercyclical markups causing profits to fall in response to an expansionary monetary policy is distributed amongst households, and importantly to households that do not hold illiquid assets, the investment response is upheld. Investment increases on impact with output and consumption also rising, restoring the expected aggregate response to an expansionary monetary policy shock.

Figure 1.12: Case 3: Aggregate response to an expansionary MP shock




*Note:* Response of aggregate variables to a 0.25 (1% annual) fall in the innovation in the Taylor rule $\epsilon$.

Case 3 served as an example of a fix to the counterfactual aggregate results that Case 2 suffered from, whilst still being supported by the empirical exercise conducted earlier in this paper. However, research by Christiano et al. (2005) and Nekarda and Ramey (2013) show that profits are not countercyclical following a monetary policy shock and therefore although the profit distribution scheme of Case 2 is more realistic than Case 1, the dampening effect of countercyclical profits on the income of the net worth is not observed in reality. This calls for further research into a suitable mechanism to ensure procyclical profits whilst retaining the distributional and aggregate results.

### 1.8.3 The transmission of monetary policy

Aggregate consumption $C_t$ can be written explicitly as a function of the sequence of equilibrium prices, taxes, and transfers. $\Gamma_t = \{r_t^b, r_t^a, w_t, \tau_t, T_t\}$.[27]

[27] Taxes are currently kept constant in the model and only feature in $\Gamma_t$ as a generalisation that would be useful for future work.

$$C_t\left(\{\Gamma_t\}_{t\geq 0}\right) = \int c_t\left(a, b, z; \{\Gamma_t\}_{t\geq 0}\right) d\mu_t \tag{1.25}$$

$c_t(\cdot)$ is the household consumption policy function. $\mu_t(\cdot)$ is the joint distribution of liquid and illiquid assets and idiosyncratic income. This allows through totally differentiating to decompose the total effect of monetary policy on aggregate consumption into its direct and indirect effects:

$$dC_0 = \underbrace{\int_0^\infty \frac{\partial C_0}{\partial r_t^b} dr_t^b dt}_{\text{direct effect}} + \underbrace{\int_0^\infty \left(\frac{\partial C_0}{\partial w_t} dw_t + \frac{\partial C_0}{\partial r_t^a} dr_t^a + \frac{\partial C_0}{\partial \tau_t} d\tau_t + \frac{\partial C_0}{\partial T_t} dT_t\right) dt}_{\text{indirect effects}} \tag{1.26}$$

Equation 1.26 decomposes the total change in consumption by the partial differentiation of each price change. The first term, the direct effect, reflects the impact on aggregate consumption caused by the change in the real return on liquid assets, holding wages, the return on illiquid assets, taxes and transfers constant. The direct effect is the dominant channel in the RANK model as households react to interest rate changes through intertemporal substitution.

The indirect effects, the general equilibrium effects, are dominant in HANK models. Following an expansionary monetary policy shock Ricardian agents, non-hand-to-mouth, increase their consumption as would be typical in a RANK models. This increase in consumption leads to greater demand for production of goods, which in turn causes wages to rise and hence the consumption of low wealth, hand-to-mouth- households increase as well. The increase in the return on illiquid assets following an monetary policy shock causes agents to rebalance their asset portfolios. This expansionary monetary policy shock, which boosts wages, causes an increase in labour income tax revenue received by the government. As in the baseline model government expenditure $G_t$ by is assumed to be fixed, transfers $T_t$ must adjust to balance the government's budget constraint. As tax revenue has increased following an expansionary monetary policy shock the transfers that households receive also rise, leading to an increase in consumption.

Table 1.5 outlines the elasticity of output, investment and consumption across the three different cases studied, Case 1, Case 2 and Case 3 as well as additional intermediate cases.

Table 1.5 highlights the different elasticity of investment from the profit

Table 1.5: Decomposition of the effect of monetary shock on aggregate consumption

| | Case 1 | Case 2 | Case 2 w/ $\omega = 0.1$ | 50% Case 1 & 50% Case 2 | Case 3 |
|---|---|---|---|---|---|
| Change in $r^b(pp)$ | -0.27 | -0.33 | -0.32 | -0.31 | -0.29 |
| | | | | | |
| Elasticity of Y | -3.96 | -0.73 | -1.13 | -1.70 | -3.31 |
| Elasticity of I | -9.43 | 6.10 | 4.49 | 1.98 | -4.11 |
| | | | | | |
| Elasticity of C | -2.93 | -1.86 | -1.76 | -2.21 | -2.27 |
| Partial eq. elasticity of C | -0.55 | -0.49 | -0.52 | -0.50 | -0.55 |
| | | | | | |
| *Component of percent change in C due to* | | | | | |
| | | | | | |
| Direct effect: $r^b$ | 0.19 | 0.26 | 0.29 | 0.23 | 0.24 |
| Indirect effect: $w$ | 0.51 | 0.49 | 0.45 | 0.50 | 0.47 |
| Indirect effect: $T$ | 0.32 | 0.29 | 0.25 | 0.30 | 0.23 |
| Indirect effect: $r^a$ and $q$ | -0.02 | -0.04 | 0.03 | -0.03 | 0.06 |

*Note:* Average responses over the first year to consumption following an expansionary monetary policy shock. Case 1 is the baseline specification from Kaplan et al. (2018). Case 2 implements a profit distribution scheme based on illiquid asset holdings. Case 2 with $\omega = 0.1$, reduces the share of profits that go directly into the illiquid account. 50% Case 1 & 50% Case 2 is an equal mix of Case 1 and Case 2. Case 3 uses 50% of Case 1 & 50% of Case 2 as well as $\omega = 0.1$.

distribution schemes analysed. Only Case 1 and Case 3 display procycical investment following an expansionary monetary policy shock. Moreover, the elasticity of output and elasticity of consumption are similar for Case 1 and Case 3. The lower part of Table 1.5 reports the contribution of each component to the change in consumption over the first year following the shock. Changing the profit distribution scheme from a bonus based scheme to a dividend scheme has caused the direct effect to become more relevant. However, the overall indirect effect is still dominant. The figures that decompose the change in aggregate consumption into its direct and indirect effects can be seen in Appendix 1.H.3.

## 1.9 Conclusion

In this paper I used a benchmark HANK model of Kaplan et al. (2018) to analyse the consumption and income response of an expansionary monetary policy shock over the wealth distribution. In the model studied, the income response by household wealth was at odds with the empirical findings. The income increase for the wealthier households in the model outstripped the poorer households from an expansionary monetary policy shock, whereas in the data their response was dampened. However, the consumption response across the wealth distribution is in line with my empirical findings, such that low wealth households, likely to be hand-to-mouth, respond the most.

I innovated on the profit distribution scheme, distributing profits in proportion to illiquid asset holdings, which brought the household response from a monetary policy shock in line with the data, but caused the investment response to become counterfactual. To restore plausible aggregate results whilst maintaining a household response consistent with my empirical evidence a mixed profit distribution scheme is required. Combining a profit distribution scheme based on labour productivity and illiquid asset holdings whilst lowering the share of profits that automatically are reinvested in the firm aligns the model response to an expansionary monetary policy shock with the response found in the data. The mixed distribution scheme balances dampening the income of the high net worth agent through countercyclical profits from countercyclical markups that are present in the standard New Keynesian model, spreading the negative income effect of these profits across the wealth distribution and ensuring that investment is procyclical by reducing the direct flow of countercyclical profits into investment.[28]

This paper highlights the importance of analysing the heterogeneous effects from a monetary policy shock in order to understand the aggregate response. Due to the different monetary policy transmission mechanisms in RANK and HANK models it is crucial to discipline the heterogeneous and aggregate response of the model by the empirical findings to determine the preferred model. A key policy takeaway is that the strength of the direct effect of monetary policy is determined by the response across the whole distribution of households, which in part, is sensitive to assumptions made such as the profit distribution scheme analysed in this paper.

[28]As in the HANK model of Kaplan et al. (2018), where monetary policy was amplified compared to an equivalent RANK model, under the mixed profit scheme amplification is also expected. This is because in the RANK model the profit distribution scheme is irrelevant.

## 1.A Further Derivations

This section includes further derivations and additional explanation of the model.

Equation 1.27 outlines the Hamilton-Jacobi-Bellman equation, which represents the optimisation problem for an individual household separated by their illiquid assets $a$, illiquid assets $b$ and labour productivity $z$. Households discount time at rate $\rho$ and die with probability $\zeta$. Due to perfect annuity markets the return on liquid assets is $r_t^b + \zeta$ and return on illiquid assets $r_t^a + \zeta$. There is a borrowing limit of $\bar{b}$. Households receive productivity shocks following a Markov Process and receive a share of profits $\pi_t(z)$. The productivity shocks drift towards zero at rate $\beta$.

$$
\begin{aligned}
(\rho+\zeta)V_t(a,b,z) = \max_{c,l,d} u(c,l) &+ V_{b,t}(a,b,z)[(1-\rho_t)w_t z l + (r_t^b(b)+\zeta)b + T_t \qquad (1.27)\\
&- d - \chi(d,a) + \pi_t(z) - c]\\
&+ V_{a,t}(a,b,z)((r_t^a+\zeta)a + d) + V_{y,t}(a,b,z)(-\beta z)\\
&+ \lambda \int_{-\infty}^{\infty} (V_t(a,b,x) - V_t(a,b,z))\phi(x)dx + \dot{V}_t(a,b,y)\\
&\text{s.t.}\\
&b \geq -\underline{b}; a \geq 0; 0 \leq l \leq 1; r_t^b(b) = r_t^b + I\{b \leq 0\}\kappa
\end{aligned}
$$

The boundary conditions, such that the liquid asset cannot be below an exogenous lower bound and the illiquid asset cannot fall below zero is given by:

$$V_b(a,\bar{b},z) \geq u_c(c,l) \qquad (1.28)$$

$$V_a(0,b,z) \geq u_c(c,l) \qquad (1.29)$$

Assuming standard CRRA utility form $u(c,l) = \frac{c^{1-\gamma}}{1-\gamma} - \varphi\frac{l^{1+\nu}}{1+\nu}$. The first order conditions of the household's problem can be written as:

$$u_c = V_b \qquad (1.30)$$

$$V_b\left(1+\chi_d(d,a)\right)=V_a \tag{1.31}$$

$$l=\left(\frac{V_b(1-\tau)wy}{\varphi}\right)^{1/\nu} \tag{1.32}$$

Equation 1.33 outlines the Kolmogorov Forward Equation that shows how the distribution of households move over time. Let $g_t(a,b,z)$ correspond the density function of households, dependent on their asset holdings and productivity. For starting assets and income, defined as $(a_0,b_0)$ and $g(z)$ respectively, a Dirac delta function $\delta$ is required to map the households with a point mass at zero assets.

$$\begin{aligned}\partial_t g_t(a,b,z)=&-\partial_a(s_t^a(a,b,z)g_t(a,b,z))\\&-\partial_b(s_t^b(a,b,z)g_t(a,b,z))-\partial_y(-\beta y g_t(a,b,z))\\&-\lambda g_t(a,b,z)+\lambda\phi(z)\int_{-\infty}^{\infty}g_t(a,b,x)dx-\zeta g_t(a,b,z)\\&+\zeta\delta(a-a_0)\delta(b-b_0)g_t^*(y)\end{aligned} \tag{1.33}$$

### 1.A.1 Firms

#### Final Good producers

The supply side of this model follows a standard setup and is therefore kept brief. The economy consists of a competitive representative final good firm that bundles intermediate inputs $j\in[0,1]$ into the aggregate economic output $Y_t$. This bundling takes the form:

$$Y_t=\left(\int_0^1 y_{j,t}^{\frac{\varepsilon-1}{\varepsilon}}dj\right)^{\frac{\varepsilon}{\varepsilon-1}}, \tag{1.34}$$

where the elasticity of substitution across goods is given by $\varepsilon$. The demand for intermediate good $j$ is given in the Dixit-Stiglitz fashion from cost minimization:

$$y_{j,t}(p_{j,t}) = \left(\frac{p_{j,t}}{P_t}\right)^{-\varepsilon} Y_t, \quad \text{where } P_t = \left(\int_0^1 p_{j,t}^{-1\varepsilon} dj\right)^{\frac{1}{1-\varepsilon}} \tag{1.35}$$

### Intermediate Good Producers

The intermediate inputs used in Section 1.A.1 are produced by the intermediate good producers. Each intermediate good $j$ is produced by a monopolistically competitive producer using the production function in Equation (1.36) using capital $k_{j,t}$ and labour $n_{j,t}$, with intensities $\alpha$ and $1-\alpha$ respectively:

$$y_{j,t} = k_{j,t}^{\alpha} n_{j,t}^{1-\alpha} \tag{1.36}$$

Intermediate good producers aim to maximise their output in Equation (1.36) whilst minimising their cost of production. In this case, the price at which the firm hires labour is the wage rate $w_t$ and the price for renting capital in a competitive capital market is given by $r_t^k$. The marginal cost, denoted $m_t$, is derived from the first order condition of the intermediate producers problem. Since the intermediate good producers follow a Cobb-Douglas production function the form of the marginal cost is typical :

$$m_t = \left(\frac{r_t^k}{\alpha}\right)^{\alpha} \left(\frac{w_t}{1-\alpha}\right)^{1-\alpha} \tag{1.37}$$

Each intermediate producer, differentiated by production good $j$, has a monopoly in its production and therefore chooses the price to maximise profits. Changing prices induce an adjustment cost as in Rotemberg (1982), which is the cause of the nominal rigidity within this model. This quadratic adjustment cost is given by $\Theta_t\left(\frac{\dot{p}_t}{p_t}\right) = \frac{\theta}{2}\left(\frac{\dot{p}_t}{p_t}\right)^2 Y_t$ and is expressed as a fraction of aggregate output $Y_t$.[29] For convenience, and as this problem is common across intermediate good producers $j$, $j$ is dropped from the price setting problem given in Equation (1.38) and (1.39). As briefly explained above and will shortly be explained in greater detail below, the correct interest rate to discount the flow of future profits at is $r^a$. Therefore,

[29] An alternative formulation is to use Calvo (1983) price rigidities, whereby a firm can only change prices under a certain probability, but once selected can do so costlessly has be shown to be comparable under the correct parameterisation of the model.

by choosing the price $\{p_t\}_{t\geq 0}$ to sell their intermediate good at the firm wishes to maximise

$$\int_0^\infty e^{-\int_0^t r_s^a ds} \left\{ \tilde{\Pi}_t \left(p_t\right) - \Theta_t \left(\frac{\dot{p}_t}{p_t}\right) \right\} dt \tag{1.38}$$

taking into account the flow profits before price adjustment costs are added.

$$\tilde{\Pi}_t \left(p_t\right) = \left(\frac{p_t}{P_t} - m_t\right) \left(\frac{p_t}{P_t}\right)^{-\varepsilon} Y_t \tag{1.39}$$

The maximisation problem can be written in its recursive form with $J(p,t)$ representing the real value of the firm. Equation (1.40) features the aggregate price inflation rate $\pi_t = \dot{P}_t / P_t$:

$$\begin{aligned} r^a(t) J(p,t) = \max_{\pi} \left(\frac{p}{P(t)} - m(t)\right) \left(\frac{p}{P(t)}\right)^{-\varepsilon} Y(t) - \frac{\theta}{2}\pi^2 Y(t) \\ + J_p(p,t) p\pi + J_t(p,t) \end{aligned} \tag{1.40}$$

Using the envelope condition and that the equilibrium will be symmetric as all firms face the same problem and hence $p = P$, the solution to this maximisation problem is the continuous time New Keynesian Phillips curve

$$\left(r_t^a - \frac{\dot{Y}_t}{Y_t}\right) \pi_t = \frac{\varepsilon}{\theta} \left(m_t - m^*\right) + \dot{\pi}_t, \quad m^* = \frac{\varepsilon - 1}{\varepsilon} \tag{1.41}$$

where $1/m^*$ is the markup when prices are flexible. In this model intermediate firms raise their price when their markup is above that of the flexible price case.

### 1.A.2 Calibration

The calibration strategy follows Kaplan et al. (2018) and therefore only the key elements are outlined below.

Table 1.6 outlines the parameters used in the theoretical model. The parameters follow closely Kaplan et al. (2018), with the major difference of the profit ristribution scheme shares $\nu^{\pi}$ .

On the household side of the model the intertemporal elasticity of substitution and the Frisch elasticity of labour supply are both set to 1, a standard value found in the literature. As the intertemporal elasticity of substitution is set to 1 this means that we have log preferences over consumption as the utility function is CRRA. The disutility of labour is set such that on average the household hours worked is equal to $1/2$ (with 1 representing a full day). The production side of the economy follows standard calibration values as seen in Galí (2015). The elasticity of substitution for final goods is $\varepsilon = 10$, which means that the steady-state markup $1/(\varepsilon - 1) = 11\%$. The government policy is calibrated such that labour income tax is set $\tau = 0.3$ and the lump-sum transfer is $6\%$ of steady state output. Government expenditures is then the residual of the government budget constraint in the steady-state 1.12. During the transition process following a monetary policy shock labour income tax and government expenditure is held fixed whilst transfers $T$ are allowed to fluctuate. Monetary policy follows the Taylor rule that reacts to deviations from steady state inflation with the Taylor rule coefficient $\phi = 1.25$. The parameters of the adjustment cost function and the household discount rate $\rho$ are calibrated using the model to match the steady state distribution of the mean of the illiquid and liquid wealth distribution and the fraction of poor and wealthy hand-to-mouth households from Kaplan et al. (2014). The parameters of the profit distribution form a crucial basis for the experiments conducted within this paper. The fraction of profit distributed to the illiquid account $\omega$ controls the amount of profit that flow directly back into investment. Following Kaplan et al. (2018) the baseline value of $\omega = \alpha = 0.33$, which neutralises the direct effect of countercyclical markups on investment. However, when this share is reduced less profits flow directly into the illiquid account and therefore investment is not dampened as much and increases, causing a rise in consumption and output. The baseline profit distribution scheme (Case 1), where profits are distributed related to labour productivity is when $\nu^{\pi} = 0$. Case 2, where the profits that are distributed to the liquid account in proportion to illiquid asset holdings sets $\nu^{\pi} = 1$. For case 3, half of the profits that flow into the liquid account are distributed in proportion to productivity, such as bonuses, and the other half are distributed in proportion to illiquid asset holdings, therefore $\nu^{\pi} = 0.5$. Case 2 and Case 3 are introduced in greater depth in Section 1.8.

Table 1.6: List of Parameter Values

| | Description | Value | Target/Source |
|---|---|---|---|
| *Preferences* | | | |
| $\zeta$ | Death rate | $1/180$ | Average lifespan 45 years |
| $1/\gamma$ | Intertemporal elasticity of subst. | 1 | |
| $1/\nu$ | Frisch elasticity of labour supply | 1 | |
| $\varphi$ | Disutility of labour | 2.2 | Avg. hours worked equal to $1/2$ |
| $\rho$ | Discount rate (p.a.) | $5.1\%$ | Internally calibrated |
| *Production* | | | |
| $\varepsilon$ | Demand elasticity | 10 | Profit share of 10 percent |
| $\theta$ | Price adjustment cost | 100 | Slope of Phillips curve $\varepsilon/\theta = 1$ |
| $\alpha$ | Capital share | 0.33 | |
| $\bar{\delta}$ | Steady-state depreciation rate (p.a.) | $7\%$ | |
| *Government* | | | |
| $\tau$ | Proportional labour tax | 0.3 | |
| $T$ | Lump-sum transfer (rel GDP.) | 0.06 | $40\%$ hh with net govt. transfer |
| *Monetary Policy* | | | |
| $\phi$ | Taylor rule coefficient | 1.25 | |
| $\bar{r}^b$ | Steady-state real liquid return (p.a.) | $2\%$ | |
| *Unsecured borrowing* | | | |
| $r^{borr}$ | Borrowing rate (p.a.) | $8\%$ | |
| $\underline{b}$ | Borrowing limit | $\$16,500$ | |
| *Adjustment cost function* | | | |
| $\xi_0$ | Linear Component | 0.0438 | |
| $\xi_1$ | Convex component | 0.956 | |
| $\xi_2$ | Convex component | 1.402 | |
| $\underline{a}$ | Min a in denominator | $\$1,000$ | |
| *Profit distribution* | | | |
| $\omega$ | Profit to illiquid acc. | [0,1] | Kaplan et al. (2018) |
| $\nu^{\pi}$ | Profits to liq acc. as bonus or dividend | [0,1] | Experimented (Case 1,2,3) |

## 1.B Income decomposition from the Survey of Consumer Finances

To understand the breakdown of income into their respective sources the highest quality data available for households in the United States is from the Survey of Consumer Finances (SCF) maintained by the Federal Reserve Board. The income decomposition using the SCF includes capital gains, which is not included in the CEX. Table 1.7 outlines the income sources for households grouped by their net worth levels. Labour income is income from wage and salary. Business income is defined as income from business, sole proprietorship, and farm ownership. Interest and dividend income is interest earned on savings and bonds as well as other sources of interest income and dividend income. Capital gains is the capital gains or losses from asset holdings. The variable social security includes social security and pension income. Finally, welfare is defined as income from unemployment benefits, alimony/child support, TANF/food stamps/SSI, and other income of this nature. From Table 1.7 it can be seen that the households in the bottom three quartiles of net worth receive a large majority of their income as labour income. The poorest in the economy, the households at the bottom of the net worth distribution, receive part of their income through welfare benefits. Whereas the richest in the economy have multiple income sources, such as income from business ownership, interest and dividiend income as well as capital gains (income from the changing value of their asset holdings).

Table 1.7: Income decomposition from the Survey of Consumer Finances by net worth

| | 0-25% | 25-50% | 50-75% | 75-100% |
|---|---|---|---|---|
| Labour Income | 76% | 80% | 70% | 53% |
| Business Income | 3% | 4% | 7% | 19% |
| Interest and Dividend Income | 0% | 0% | 0% | 5% |
| Capital Gains | 0% | 0% | 3% | 8% |
| Social Security | 12% | 13% | 19% | 13% |
| Welfare | 9% | 3% | 4% | 2% |

Income decomposition from the Survey of Consumer Finances using data for 2016. Variable names follow closely the summary variables that are taken from the summary extracted public data.

To compare Table 1.7 with the corresponding table of the baseline HANK model, found in Section 1.4 Table 1.1 some assumptions must be made. Firstly labour income is defined as wages and transfer income is welfare income. Profit income tries to follow the definition in HANK, whereby $1-\omega$ of business income flows back to the household, plus the dividends.[30]. The remaining half of the interest and dividend variable is attributed to liquid asset holdings. The illiquid asset return is the sum of capital gains, the remaining $\omega$ of business profits and social security benefits (as the social security accounts like a 401K account are somewhat illiquid). From these assumptions Table 1.8 is created. Comparing this table to Table 1.1 we see that labour income is still the most important part of income for the poorest households. However, the share of income from profits is now lower for the bottom three quartiles of the net wealth distribution than the top quartile. The income from liquid assets are small in the data and theoretical model. The part of illiquid income that the model is unable to capital, social security payments, as it does not feature in the model appears in the bottom row of Table 1.8.

Table 1.8: Income decomposition from the SCF put into the HANK categories

| | 0-25% | 25-50% | 50-75% | 75-100% |
|---|---|---|---|---|
| Labour Income | 76% | 80% | 70% | 53% |
| Transfer Income | 9% | 3% | 4% | 2% |
| Profit | 2% | 3% | 5% | 15% |
| Liquid Asset | 0% | 0% | 0% | 3% |
| Illiquid Asset | 13% | 14% | 21% | 27% |

Income decomposition from the Survey of Consumer Finances using data for 2016. Data is taken from the summary extracted public data. Variable transformations from the raw catagories are as follows: Labour Income = wage. Transfer Income = welfare income. Profit = $(1-\omega)$ business + 0.5 (interest and dividends). Liquid asset = 0.5(interest and dividends). Illiquid asset = Capital gains + $\omega$ business + social security.

A valid concern from Table 1.8 is the lack of negative income flows for liquid assets and the large social security payments that go to retired households that are not featured in the theoretical HANK model. To address these concerns I take the data from the Survey of Consumer Finance and restrict the sample to the working age population. Moreover, by default the calculation for liquid asset income does not include consumer debt payments. Including this category provides a fairer comparison with the theoretical HANK model, where we see in Figure 1.9 that the income from illiquid assets are still at odds with the model.

[30] An alternative assumption here for profit would be that the dividends flow black to the illiquid account. This would only serve to increase the illiquid asset income of the highest net worth household whilst lowering the profit income of these households.

Table 1.9: Income decomposition from the SCF put into the HANK categories for working age population allowing for consumer debt

| | 0-25% | 25-50% | 50-75% | 75-100% |
|---|---|---|---|---|
| Labour Income | 82% | 87% | 83% | 65% |
| Transfer Income | 9% | 4% | 3% | 2% |
| Profit | 2% | 3% | 5% | 14% |
| Liquid Asset | -1% | 0% | 0% | 2% |
| Illiquid Asset | 8% | 7% | 9% | 17% |

Income decomposition from the Survey of Consumer Finances using data for 2016. Data is taken from the summary extracted public data. Variable transformations from the raw catagories are as follows: Labour Income = wage. Transfer Income = welfare income. Profit = $(1-\omega)$ business + 0.5 (interest and dividends). Liquid asset = 0.5(interest and dividends). Illiquid asset = Capital gains + $\omega$ business + social security.

## 1.C Additional CEX info

The main empirical analysis focuses on partitioning households by end of period wealth, as further outlined above in Section 1.5.2. The wealth calculation used is replicated below. Due to the nature of the Consumer Expenditure Survey it is more natural to calculate end of period wealth as I have done below:

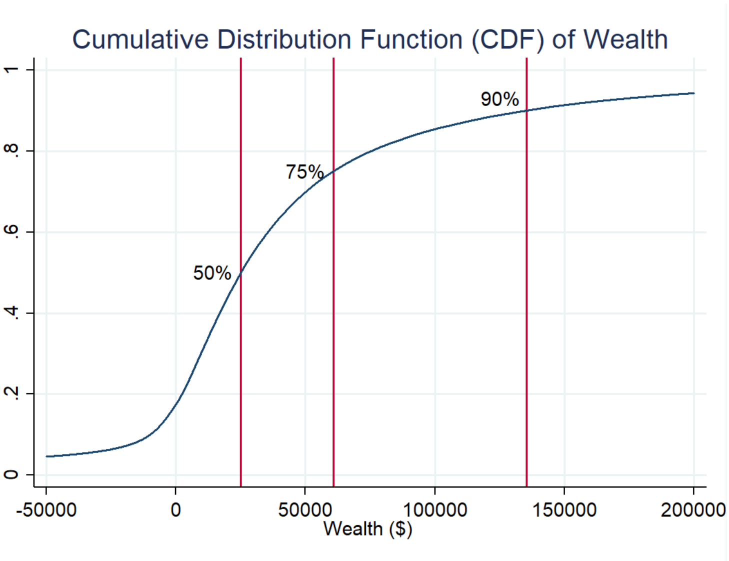
$$Wealth_i = Y_i - T_i - C_i + A_i - L_i$$

The distribution of wealth, in 2018 Q1 dollars, is provided in Figure 1.13. The red vertical lines in this figure highlight the 50%, 75% and 90% wealth values at each of this percentiles.

The dollar amount that corresponds to the percentiles at each of the red lines shown in Figure 1.13 are provided in Table 1.10. The lower end of the distribution matches up well to the Survey of Consumer Finances, however, since the CEX is unable to capture the wealth of high net worth households the top percentiles of the wealth distribution are misspecified. This point is explained in Auclert (2019), who compares the assets and liabilities of the CEX with the SCF and finds that the liabilities of households from the CEX match with those of the SCF however assets in the CEX are underrepresented.

Table 1.11 provides descriptive statistics between the two groups focused on in

Figure 1.13: Distribution of Wealth




*Note:* Real wealth in Q1 2018 dollars

Table 1.10: Dollar amount of wealth at each percentile

| | 50% | 75% | 90% |
|---|---|---|---|
| Wealth values | \$25,100 | \$60,800 | \$135,200 |

*Note:* Real wealth in Q1 2018 dollars

this paper, the bottom 75% of the wealth distribution and the top 25%. Higher wealth households are older, have a larger share of home owners and mortgage holders and are less likely to be renters. This is in line with the definition of high wealth households in Doepke and Schneider (2006). These descriptive statistics of the household characteristics provides confidence in the measure of Wealth.

Table 1.11: Descriptive statistics by Wealth

| | Average Age | Share of Home Owners | Share of Mortgagors | Share of Renters |
|---|---|---|---|---|
| Bottom 75% | 48.6 | 24% | 36% | 38% |
| Top 25% | 50.5 | 29% | 50% | 19% |

*Note:* Share of housing tenure need not sum to 100% as two categories are ignored: i) Occupied without payment of cash rent and ii) student housing.

## 1.D Additional Empirical Strategy - BVAR

The additional empirical exercise is conducted using a Bayesian Proxy Structural VAR (BP-SVAR) following closely the approach outlined in Miranda-Agrippino and Ricco (2018a), Miranda-Agrippino and Ricco (2018b) and Caldara and Herbst (2019) and combines the seminal work for the Bayesian VAR of Sims (1980) and Litterman (1979),Doan et al. (1984) with the incorporation of instrumental variables into VARs by Stock and Watson (2012) and Mertens and Ravn (2013).[31] Given the limited length of the data available, quarterly data from 1996 to 2017, overparamerisation is likely in a standard Structural Vector Autoregression making the estimation of the VAR diffifcult with standard (frequentist) techniques even if a small set of variables are used. This is known as the *curse of dimensionality* and can be efficiently dealt with by incorporating prior information about the model coefficients.

Equation 1.42 outlines the structural vector autoregression (SVAR), which cannot be estimated directly and therefore an identification scheme is required. The vector of observables is $y_t$, which is a [K x 1] vector, $B_i$ is a [K x K] matrix of coefficients with $i = 1 \ldots p$ denoting the autoregressive order, the unobserved zero-mean structural shocks are $u_t$, which is a [K x 1] vector with $E[u_t u_s'] = D$ if $t = s$ and 0 otherwise.

[31] Robustness checks are conducted using Bayesian local projections as in Miranda-Agrippino and Ricco (2018b). See Appendix 1.G for a description of this technique and corresponding results.

$$B_0 y_t = B_1 y_{1,t-1} + \cdots + B_{p,t-p} + u_t \tag{1.42}$$

The reduced form counterpart to equation 1.42 is equation 1.43, which can be obtained by pre-multiplying both sides of equation 1.42 by $B_0^{-1}$. The reduced form VAR represents the data generated from the SVAR model. The reduced form error term, also known as innovations, is $\varepsilon_t$ is also a [K x 1] vector with the standard assumptions that $E[\varepsilon_t] = 0$, $E[\varepsilon_t \varepsilon_t'] = \Sigma_\varepsilon$ and $E[\varepsilon_t \varepsilon_s'] = 0$ for $s \neq t$.

$$y_t = \underbrace{B_0^{-1} B_1}_{A_1} y_{t-1} + \cdots + \underbrace{B_0^{-1} B_p}_{A_p} y_{t-p} + \underbrace{B_0^{-1} u_t}_{\varepsilon_t} \tag{1.43}$$

Assuming invertibility of the $B$ matrix, such that $u_t = B_0 \varepsilon_t$ and that $B_0$ identifies the mapping between the structural shocks and the reduced form innovations. However, since $\Sigma_\varepsilon$ is symmetric, it only has $(K+1)/2$ independent parameters. This means that only $(K+1)/2$ can be uniquely identified out of the $K^2$ parameters in $B_0$. As the coefficients are not uniquely identified we cannot observe the causal effect of a monetary policy shock on our dependent variables without further assumptions. One such identifying assumption is a short-run restriction, recursively ordering the model variables using lower triangular Cholesky decomposition of $\Sigma_\varepsilon$ by defining a new matrix $P$ such that $PP' = \Sigma_\varepsilon$. Since $P$ is lower triangular it has $K(K-1)/2$ zero parameters and allows for exact identification of a monetary policy shock. However, recursive ordering does not always correspond well to perceptions of when a shock occurred, outlined further by Rudebusch (1998). One solution, used within this paper is to use an instrumental variables approach, instrumenting the change in the interest rate by an exogenous monetary policy surprise. The proposed instrument is the high frequency identified monetary policy surprise cleaned of any information effects derived by Jarociński and Karadi (2020) and explained in further detail in Section 1.6. This external instrument is used in the first stage regression to explain the movements in the rate of the one-year Treasury bond.[32] The advantage of this approach is that the resulting explained component, the monetary policy shock, is not contaminated by other news since the initial external instrument is a noisy measure of the true shock. Furthermore, it is important to include variables that proxy for financial conditions when using high frequency instruments, as outlined by Caldara and Herbst (2019). In this regard the excess bond premium constructed by Gilchrist and Zakrajšek (2012),

[32] Further robustness using the shadow rate from Wu and Xia (2016) is also used and the results are available upon request.

which “represents variation in the average price of bearing exposure to US corporate credit risk, above and beyond the compensation for expected defaults,” provides a good measure of the financial state of the economy.

Given an instrument $z_t$ it is possible to identify the shock of interest - monetary policy shock denoted as $u_t^{mp}$ - if :

**Instrumental Relevance**

$$(i) \quad \mathbb{E}[u_t^{mp} z_t'] = \phi \tag{1.44}$$

**Instrumental Exogeneity**

$$(ii) \quad \mathbb{E}[u_t^{\not{mp}} z_t'] = 0 \tag{1.45}$$

**Lead-lag Exogeneity**

$$(iii) \quad \mathbb{E}[u_{t+j}^{i} z_t'] = 0 \quad \forall j \neq 0 \text{ and } \forall i \tag{1.46}$$

condition (i), instrumental relevance and condition (ii), instrumental exogeneity hold. The addition of condition (iii) allows the shock to be estimated in a single regression without controls. $u_t^{mp}$ represents a monetary policy shock and $u_t^{\not{mp}}$ denotes any other shock. An advantage of using the Bayesian setting here is that weak identification does not pose a problem, as long as the prior distribution is proper, inference is possible, as highlighted by Caldara and Herbst (2019) and originally found by Poirier (1998).

To address the *curse of dimensionality* induced through the limited sample size I use a Bayesian Proxy SVAR. Following the general framework outlined in Sims and Zha (1998) the Structural VAR can be rewritten compactly as:

$$yB_0 = xB + u, \tag{1.47}$$

where y and e are vector of [TxK], with T representing the time dimension and x is the independent variables within dimension [TxN]. One can write this problem as a likelihood of form

$$p\left(y|B_0, B\right) \propto |B_0|^T \exp\left\{-\frac{1}{2}\operatorname{tr}\left[\left(yB_0 - xB\right)'\left(yB_0 - xB\right)\right]\right\}, \tag{1.48}$$

where $|B_0|$ is the determinant of $B_0$. Defining $\beta = vec(B)$ and $\beta_0 = vec(B_0)$ the SVAR coefficients can be factorised as

$$p(\beta_0, \beta) = p(\beta|\beta_0)\, p(\beta_0)\,. \tag{1.49}$$

$p(\beta_0)$ represents the marginal distribution for $\beta_0$. An assumption typically made in the literature is that the prior of $\beta$ conditional on $\beta_0$ is normal probability distribution function

$$\beta|\beta_0 \sim \mathcal{N}\left(\underline{\beta}_0, \lambda^{-1}\mathbb{I}_n \otimes \underline{\Gamma}_{\beta_0}\right). \tag{1.50}$$

The posterior distribution of $\beta$ is therefore given by

$$\beta|\beta_0, \mathbf{y} \sim \mathcal{N}\left(\bar{\beta}_0, \mathbb{I}_n \otimes \bar{\Gamma}_{\beta_0}\right) \tag{1.51}$$

where the posterior moments are updated as in the standard VAR with Normal-Inverse Wishart priors of Rao Kadiyala and Karlsson (1993). The benefit on using a Normal-Inverse Wishart prior is that the posterior distribution can be easily obtained without additional simulations and facilitates the use of the Gibbs sampler.

Ultimately the object of interest is to understand the impact of a monetary policy shock on aggregate and household specific variables. This is done through the structural impulse response function

$$IRF_h = \Theta_h B_0^{-1} \quad h = 0, \ldots, H, \tag{1.52}$$

where $\Theta_h = \sum_{\tau=1}^{h} \Theta_{h-\tau} A_\tau$. $\Theta_h$ is the matrix of dynamic multipliers, how the variable of interest changes with respect to a structural shock and $A_\tau$ are the reduced form autoregressive coefficients from Equation 1.43. The impulse response function is given as the response of a variable $i$ from shock $j$ at the horizon $h$. The impulse responses reported from the empirical exercise are point-wise estimates providing the median impulse response and the appropriate quantiles of the IRF posterior distribution to display the confidence intervals at the 68% and 90% level.

## 1.E Additional Empirical Results

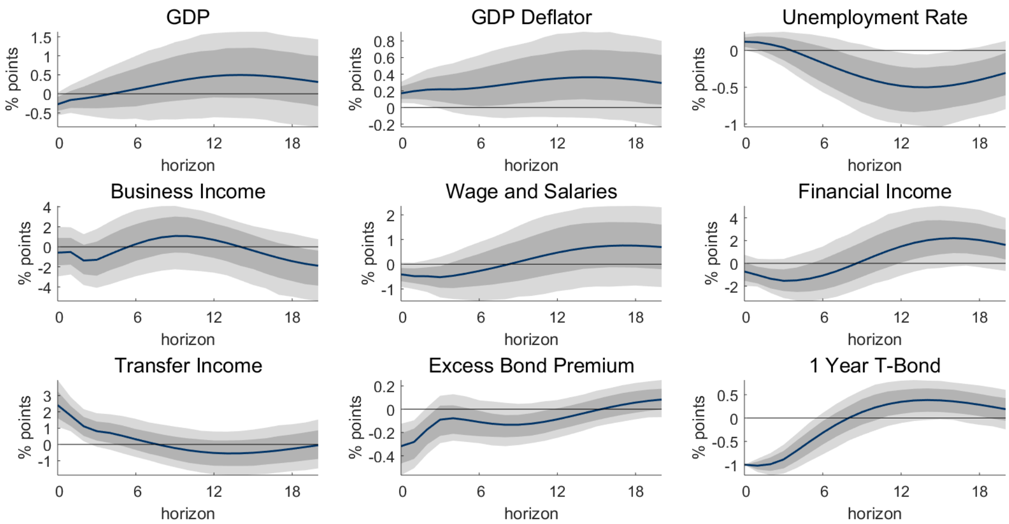
### 1.E.1 Aggregate data

The data for the United States economy comes from the Federal Reserve Economic Data (FRED) maintained by the Federal Reserve Bank of St.Louis. Analysis is conducted at the quarterly frequency and the data range from 1984-Q2 to 2017-Q4. The starting date is after the Volcker Disinflation, which is identified as ending in 1984-Q1 in Bernanke and Mihov (1998). Standard macroeconomic indicators such as real GDP, the GDP deflator and the unemployment rate are assessed in the empirical analysis with the addition of corporate profits after tax and income from inventory valuation adjustments which represents the income from business ownership. Compensation of employees wages and salary accruals used for the wage and salary variable, personal income receipts on assets is used for financial income and personal current transfer receipts is taken for transfer income. For consumption I use the real personal consumption expenditures and for aggregate investment I use real gross private domestic investment, both of these series are from FRED.

### 1.E.2 Aggregate Results

Analysing the aggregate response, although not the main purpose of this paper, provides a sanity check to ensure the shock is correctly specified and the aggregate impulse response functions behave in the manner expected. As discussed previously the aggregate Bayesian Proxy SVAR sample period is from 1984-Q2 to 2017-Q4, after the Volcker disinflationary period. From Figure 1.14 we see that an expansionary monetary policy shock causes real Gross Domestic Product to rise as well as the GDP Deflator, providing confidence in the empirical strategy. The increase on impact of the GDP Deflator shows that there is no price puzzle in this BP-SVAR, which may stem from the identification strategy of Jarociński and Karadi (2020) cleaning the monetary policy of any information effects. The median response in the SVAR is given by the blue line with the darkest grey line signifying 68% confidence bands and the lighter grey bands for 90% confidence. Unemployment and the expected bond premium fall as expected and as shown throughout the literature. The response of real wages is not significantly different from zero, whereas financial income falls, business income increases and income from transfers significantly increases and reverts back to its original level. This shows that if households have heterogeneous sources of income from a monetary policy shock this may lead to distributional changes in the consumption, income and wealth of the households.

Figure 1.14: Aggregate response of the economy to an expansionary monetary policy shock



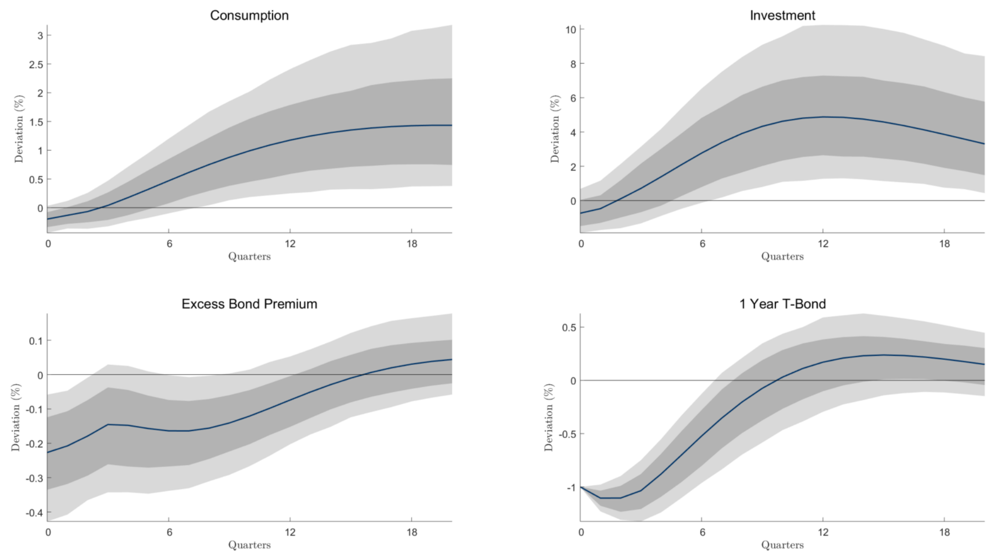

*Note:* Aggregate response to a 1% decrease in the 1 year Treasury bond . Using a Bayesian Proxy SVAR instrumented by the monetary policy surprise from Jarociński and Karadi (2020). Quarterly data from 1984-2017. Shaded areas are 68% and 90% posterior coverage bands obtained with 10,000 draws.

Figure 1.15 outlines the response of consumption and investment from an expansionary monetary policy shock. The increase in investment seen in Figure 1.15 is used to test the aggregate response from different profit distribution schemes seen in the main text of the paper.

### 1.E.3 Average CEX results

Figure 1.16 highlight the average response of income and consumption from the Consumer Expenditure Survey to a monetary policy expansion. As is expected consumption and income both increase. The increase in consumption is now statistically significant but the increase in income is. Figure 1.16 provides a further sanity check to ensure that data from the CEX respond in accordance to theory and that the monetary policy shock used and empirical specification are

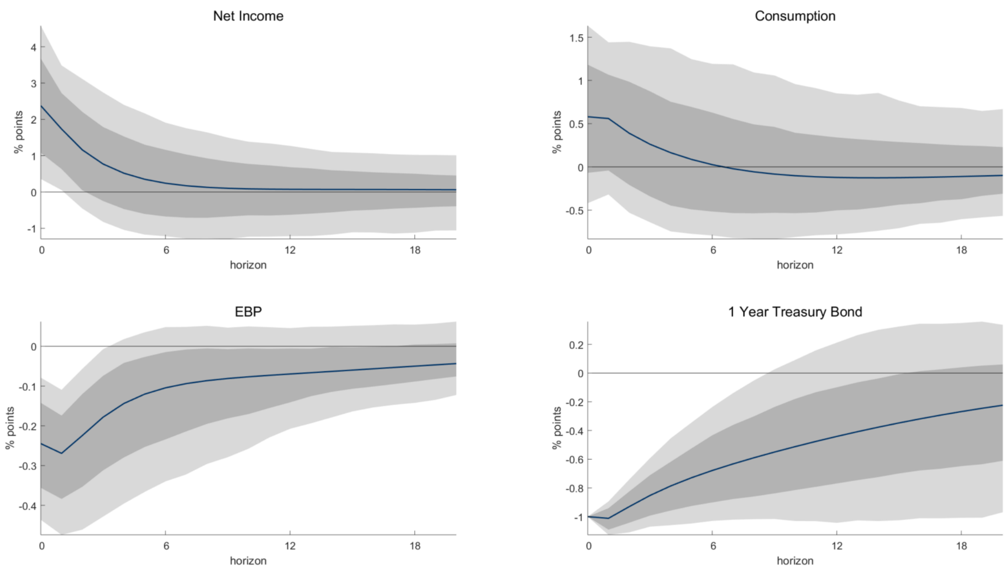
Figure 1.15: More Aggregate responses to an expansionary monetary policy shock




*Note:* Aggregate response to a 1% decrease in the 1 year Treasury bond . Using a Bayesian Proxy SVAR instrumented by the monetary policy surprise from Jarociński and Karadi (2020). Quarterly data from 1984-2017. Shaded areas are 68% and 90% posterior coverage bands obtained with 10,000 draws. GDP and GDP Deflator as added as controls but not displayed.

correct.

Figure 1.16: Mean responses from the CEX




## 1.E.4 Response by Wealth groups

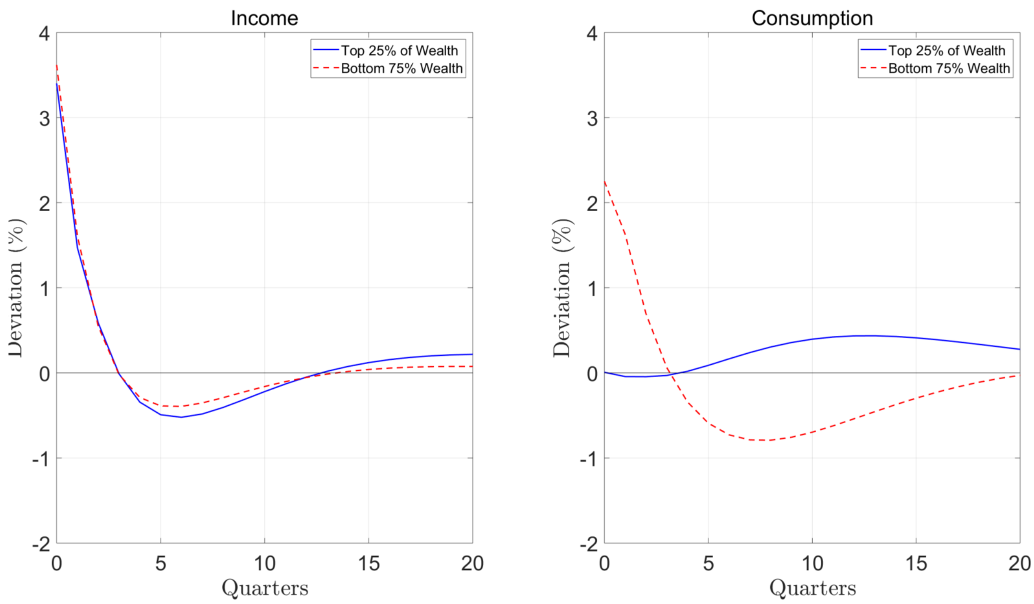
Figure 1.17 highlights the differences in the individual responses to an expansionary monetary policy shock when households are separated due to their wealth. As seen in the main text in Figure 1.4 the expansionary monetary policy causes the increase in income for both the high wealth (top 25% wealth) and low wealth (bottom 75% wealth ) households. In contrast to Figure 1.4, contemporaneous response of income and consumption is allowed. The wealthier household's income response is closely aligned to the lower wealth household, with very little discernible difference between the two responses. However, the wealthier households' consumption response following an expansionary monetary policy shock contrasts the low wealth households. Low wealth households, which typically have higher marginally propensity to consume, increase their consumption following an expansionary monetary policy shock where the consumption response for the wealthier household is subdued.[33] The households with more wealth, likely save their income increases and choose to

[33] Error bands are not shown in Figure 1.17 to highlight the differences in median response.

smooth their consumption instead - as we see consumption rising after one year of the shock.

Comparing Figure 1.17, from the empirical exercise, with Figure 1.2, from the equivalent theoretical exercise, it is apparent that the income response of households along the wealth distribution in the theoretical model is at odds with the empirical findings.

Figure 1.17: Individual Response by Wealth groups

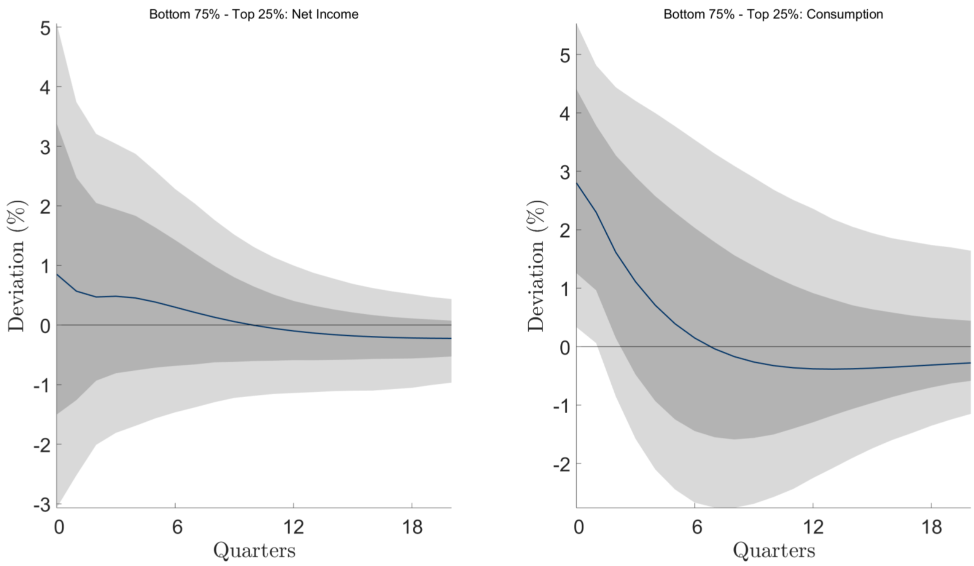



*Note:* Response to a 1% decrease in the 1 year Treasury bond by households grouped by wealth. Using a Bayesian Proxy SVAR instrumented by the monetary policy surprise from Jarociński and Karadi (2020). Quarterly data from 1996-2017. Excess bond premium and GDP Deflator are added as controls. Wealth groups are rotated following Anderson et al. (2016).

Motivated by the differences found in Figure 1.17, Figure 1.18 analyses these differences to see if they are statistically significant. The increase on impact of the difference in income responses for the low wealth group versus the high wealth group signifies the lack of discrepancies that are present. The response of income is not significantly different throughout the 20 quarters observed between these two groups. The differences in consumption found previously are statistically different at the 10% level on impact.

Decomposing these income results further into changes in salary and changes in financial income it is apparent that the heterogeneous income sources found in

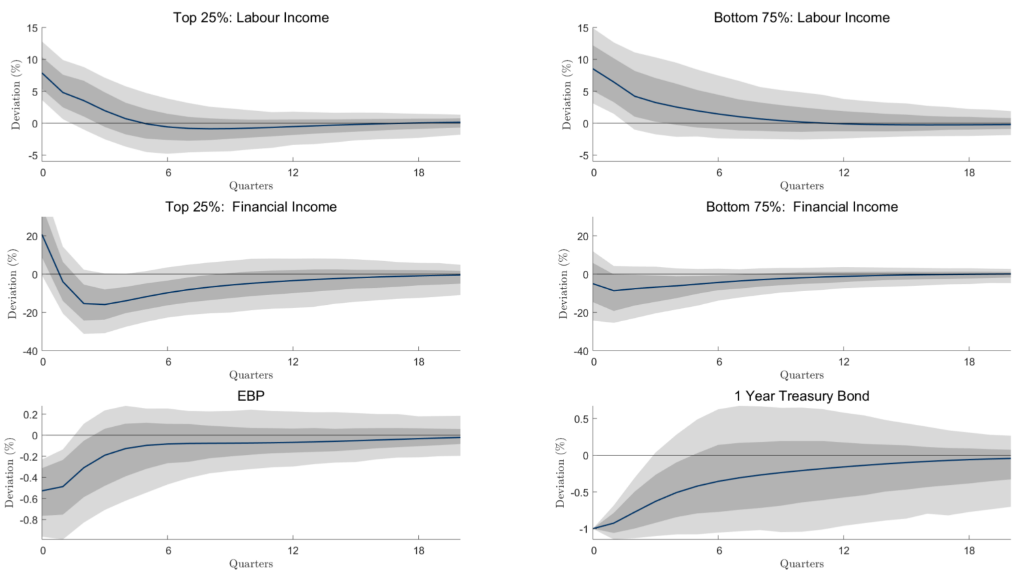
Figure 1.18: Difference between wealth groups: Mostly in Consumption




*Note:* Response to a 1% decrease in the 1 year Treasury bond by households grouped by wealth. Using a Bayesian Proxy SVAR instrumented by the monetary policy surprise from Jarociński and Karadi (2020). Quarterly data from 1996-2017. Excess bond premium and GDP Deflator are added as controls. Shaded areas are 68% and 90% posterior coverage bands obtained with 10,000 draws.

the aggregate results of Figure 1.14 translate into the individual effects for households that are grouped by wealth. The top row of Figure 1.19 shows that the labour income of the households in the bottom 75% of households increase by more than the salary of the top 25 % of the wealth distribution. The second row of Figure 1.19 highlights the financial income response from a monetary policy shock. This response is extremely volatile but shows an increase in income from the lower interest rate for the high wealth household, whereas financial income change is statistically insignificant for the low wealth households. The high wealth households are more likely to be savers, whose interest on savings and checking accounts would fall due to the lowering in the interest rate, however returns on financial assets could compensate this reduction. Not shown here is the income from business ownership and transfers such as food stamps and unemployment benefits that make up the remaining income response.

Figure 1.19: Further decomposing the income response




*Note:* Response to a 1% decrease in the 1 year Treasury bond by households grouped by wealth. Using a Bayesian Proxy SVAR instrumented by the monetary policy surprise from Jarociński and Karadi (2020). Quarterly data from 1996-2017. Excess bond premium and GDP Deflator are added as controls. Shaded areas are 68% and 90% posterior coverage bands obtained with 10,000 draws.

### 1.E.5 Response by URE and Income groups

This subsection replicates the empirical exercise seen in Section 1.7 for households distributed by their unhedged interest rate exposure and income levels.

The unhedged interest rate expose, from Auclert (2019), provides a measure of the balance sheet available to the household within this period. The URE is the difference between all maturing assets and liabilities at a point in time, provides a measure of balance sheet exposure to real interest rate changes. The calculation is replicated in Equation 1.53.

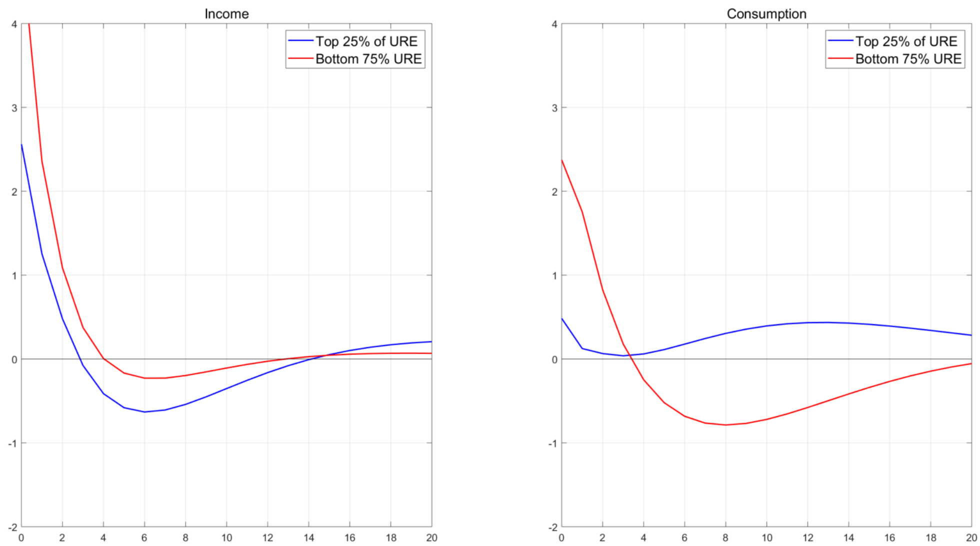
$$URE_i = Y_i - T_i - C_i + A_i^m - L_i^m \tag{1.53}$$

Similarly to household wealth I take the unhedged interest rate exposure and split it into the top 25% of households and the bottom 75%. The households at the top 25% of the URE measure typically have a large sum of liquid assets and do not react hand-to-mouth. Whereas, households with a lower value of URE may own sizeable assets but these assets are illiquid and therefore these households react more hand-to-mouth when they receive a shock. Figure 1.20 highlights the response of income and consumption for each of these groups to an expansionary monetary policy shock. The income of both types of agent, low and high URE, increase however their consumption response is markedly different. The households in the top 25% of the URE measure have a dampened response in consumption compared to those households in the bottom 75% of URE.

The difference between these households is highlighted in Figure 1.21. Although the median impulse response function is sizeable and positive between these two groups it is only statistically significant at the 68% level on impact. The response of consumption follows a similar path but is statistically significant at the 10% level on impact.

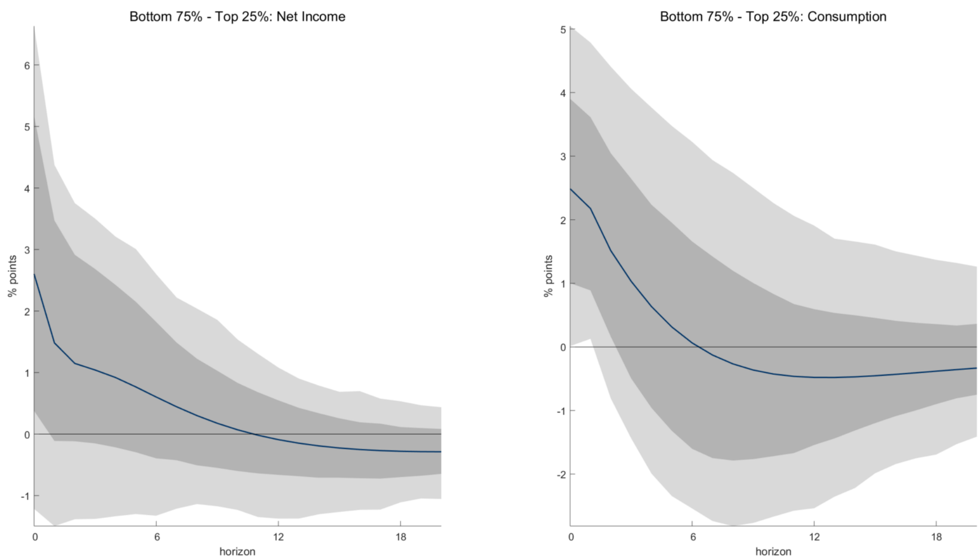
The following Figure 1.22 show the response of households when they are split by their income. The difference response in income and consumption between the groups is not statistically significant.

Figure 1.20: Individual Response by URE groups



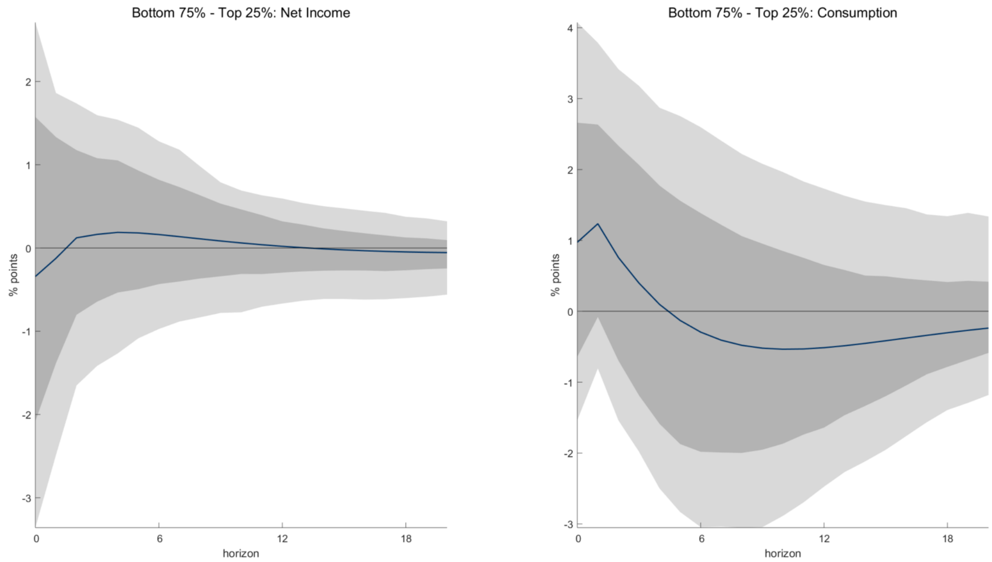

*Note:* Response to a 1% decrease in the 1 year Treasury bond by households grouped by URE. Using a Bayesian Proxy SVAR instrumented by the monetary policy surprise from Jarociński and Karadi (2020). Quarterly data from 1996-2017. Excess bond premium and GDP Deflator are added as controls. URE groups are rotated following Anderson et al. (2016).

Figure 1.21: Differences in Income and Consumption response by URE groups




*Note:* Response to a 1% decrease in the 1 year Treasury bond by households grouped by URE. Using a Bayesian Proxy SVAR instrumented by the monetary policy surprise from Jarociński and Karadi (2020). Quarterly data from 1996-2017. Excess bond premium and GDP Deflator are added as controls. Shaded areas are 68% and 90% posterior coverage bands obtained with 10,000 draws.

Figure 1.22: Differences in Income and Consumption response by Income groups




*Note:* Response to a 1% decrease in the 1 year Treasury bond by households grouped by Income. Using a Bayesian Proxy SVAR instrumented by the monetary policy surprise from Jarociński and Karadi (2020). Quarterly data from 1996-2017. Excess bond premium and GDP Deflator are added as controls. Shaded areas are 68% and 90% posterior coverage bands obtained with 10,000 draws.

### 1.E.6 Wealth calculation and Results with Housing

The initial calculation of household wealth using the Consumer Expenditure Survey lacks house values as an asset on the household's balance sheet. Housing is an important part of many households portfolio and should be taken into account for the household's wealth level. Different years of the SCF are used and households are split into 5 age groups and 10 income groups. From this split the average house price value for age-income pair is calculated and then attributed to households that own a home within the original Consumer of Expenditure Survey wealth caluclation.

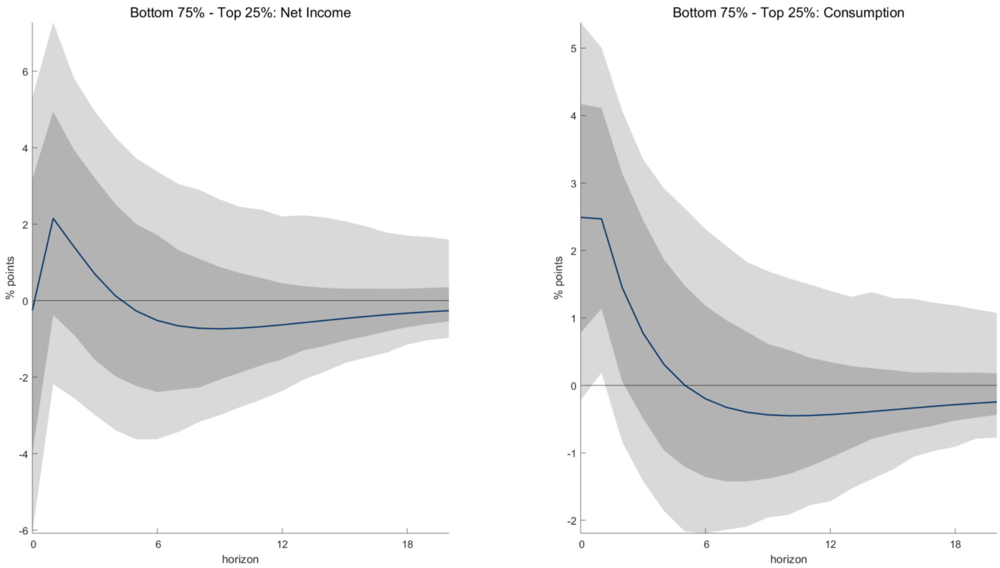
As seen in Figure 1.23 the underlying conclusions drawn from the baseline empirical exercise hold true. Such that the difference in income response between high and low wealth households is muted from an expansionary monetary policy shock. In contrast to this I still find a difference in consumption responses at the 68% level.

## 1.F Further MPC Calculation

This analysis of the 2008 U.S. tax rebate follows Parker et al. (2013), which is a follow-up paper to their analysis of the 2001 tax rebate Johnson et al. (2006) that is explained earlier in Section 1.5.2. The 2008 tax rebate is from the Economic Stimulus Act of 2008, which consisted primarily of a 100 billion dollar program that sent tax rebates, called economic stimulus payments (ESPs), to approximately 130 million US tax filers. The methodology used for analyse the 2008 tax rebate follows Johnson et al. (2006) such that we can compare the marginal propensity to consume across the to studies. The 2008 study also makes use of the randomised distribution of the tax rebate due to the recipients social security number. There are important differences between the 2001 and 2008 tax rebates. The dollar amount received in 2008 was greater than the rebate of 2001. In 2008 single individuals received between \$300 to \$600, couples received \$600 to \$1,200 and an additional \$300 per child who qualified for the child tax credit. In comparison the tax rebate in 2001 ranged from \$300 to \$600. Furthermore, the U.S. was in the midst of a recession caused by the financial crisis in 2008.

Equation (1.54) represents the regression run to analyse the impact of the tax rebate on consumption. As in the analysis in Section 1.5.2 the variable $R_i, t+1$ uses a distributed lag such that the longer duration of the tax rebate can be taken

Figure 1.23: Differences in Income and Consumption response by Wealth groups including house values




*Note:* Response to a 1% decrease in the 1 year Treasury bond by households grouped by Wealth with additional house value data from the Survey of Consumer Finances. Using a Bayesian Proxy SVAR instrumented by the monetary policy surprise from Jarociński and Karadi (2020). Quarterly data from 1996-2017. Excess bond premium and GDP Deflator are added as controls. Shaded areas are 68% and 90% posterior coverage bands obtained with 10,000 draws.

into account. Following Parker et al. (2013) standard errors are corrected to allow for heteroskedasticity and within-household serial correlation.

$$C_{i,t+1} - C_{i,t} = \sum_{s} \beta_{0s} \times \text{month}_{s,i} + \boldsymbol{\beta}_1' \mathbf{X}_{i,t} + \beta_2 R_{i,t+1} + u_{i,t+1} \tag{1.54}$$

Table 1.12 outlines the result of the 2001 and 2008 tax rebate with standard errors in parenthesis. In 2001 the households with the lowest levels of wealth consumed a large part of their tax rebate over the 6 months analysed, however, this is not the case for 2008. For 2008 the highest wealth households have a larger marginal propensity to consume out of the tax rebate compared to the less wealthy households. Due to the size of the standard errors we cannot discern a statistical difference between the high and low wealth household's MPC for 2008. This result is corroborated with that of Parker et al. (2013), where households with the highest levels of liquid wealth reacted the strongest to the 2008 tax rebate. The finding for 2008 in Table 1.12 as well as Parker et al. (2013) is at odds with the original findings for the 2001 tax rebate, where the poorer households had a higher MPC. The difference between 2001 and 2008 may be due to differences in credit constraints across households between the 2001 and 2008 recessions and differences in expectations about the length and severity of the recessions. Parker et al. (2013) also state that "Another key characteristic of the recent recession was the large decline in housing wealth and the reduced ability to borrow against home equity." Therefore with the lack of liquidity available to the higher wealth households due to the financial crisis it may rationalise the increased marginal propensity to consume of these households.

Table 1.12: Cumulative MPC calculation for 2001 and 2008

| | Cumulative MPC for 2001 | Cumulative MPC for 2008 |
|---|---|---|
| Bottom 75% of Wealth | 0.59 | 0.41 |
| | (0.26) | (0.30) |
| Top 25% of Wealth | -0.02 | 0.51 |
| | (0.53) | (0.36) |
| Number of observations | 11,856 | 9,921 |

## 1.G Bayesian Local Projection

### 1.G.1 Description of Bayesian Local Projection

Following Miranda-Agrippino and Ricco (2018b) I complete the empirical exercise found in the main body of the paper, Section 1.7, for my wealth measure using Bayesian local projection and local projection techniques.

Impulse response functions estimated using a VAR may be susceptible to model misspecification. This could occur when using a small size VAR that fails to control for any dynamic interactions that are relevant for the propagation of the shock. Moreover, the lag order may be underestimated and non-linearity are not taken into account that could potentially be important. One way to mitigate against these worries is to use a direct method such as local projections from Jordà (2005), which supposedly are more robust to misspecification. However, due to the small sample used local projections methods deliver highly imprecise estimates and noisy impulse responses. This bias-variance trade-off is addressed by Miranda-Agrippino and Ricco (2018b) through using a Bayesian Location Projection that optimally spans the model space between a VAR and LP. Bayesian Local Projection works by "specifying a (Normal-Inverse Whishart) prior for the local projection coefficients at each horizon, centred around the iterated coefficients of a similarly specified VAR estimated over a pre-sample"Miranda-Agrippino and Ricco (2018b). The posterior mean of BLP responses takes the form

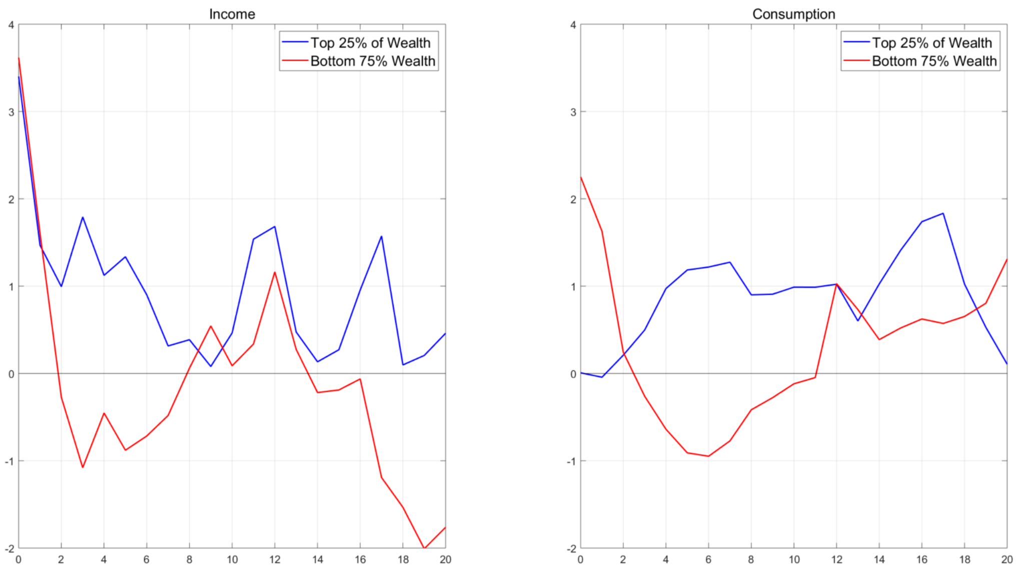
$$B_{\text{BLP}}^{(h)} \propto \left( X'X + \left( \Omega_0^{(h)} \left( \lambda^{(h)} \right) \right)^{-1} \right)^{-1} \left( (X'X) \, B_{\text{LP}}^{(h)} + \left( \Omega_0^{(h)} \left( \lambda^{(h)} \right) \right)^{-1} B_{\text{VAR}}^{h} \right), \tag{1.55}$$

where $X \equiv (x_{h+2}, \ldots, x_T)'$ with $x_t \equiv \left(1, y'_{t-h}, \ldots, y'_{t-(h+1)}\right)'$. This technique regulates the local projection impulse response function by imposing structure using priors centered around the iterated VAR. Another recent approach to discipline the local projection approach is through smoothed LP of Barnichon and Brownlees (2019).

### 1.G.2 Results

These results below, which are more robust to misspecification, outline the same conclusion found in Section 1.4 where a Bayesian Proxy SVAR was used. Although the BVAR approach produces impulse responses that are eratic the underlying message can still be gleaned from Figure 1.24. Such that the income of the wealthiest households and those in the bottom three quartiles of the wealth distribution respond similarly following an expansionary monetary policy shock. The consumption response is different, with the highest net worth households not reacting on impact and slowly increasing their consumption, whereas the poorest households immediately consume a large part of their increase in income and over the space of a year reduce their consumption back to their steady state level.

Figure 1.24: Bayesian Local Projection: Individual Response by Wealth groups




*Note:* Response to a 1% decrease in the 1 year Treasury bond by households grouped by wealth. Using a Bayesian Local Projection with the monetary policy shock from Jarociński and Karadi (2020). Quarterly data from 1996-2017. Excess bond premium and GDP Deflator are added as controls. Wealth groups are rotated following Anderson et al. (2016).

Figure 1.25 highlights the differences found in Figure 1.24. As in the main text the blue line is the median of the posterior distribution with the error bands at 68% and 90%. The difference between income responses in these two wealth groups is negligible and not statistically significant. Whereas, on impact the difference in consumption between the households in the top 25% of wealth compared to the households in the bottom 75% is statistically significant at the 10% level. Thus, finding support for differences in consumption responses across the distribution of wealth following a monetary policy shock.

Figure 1.25: Bayesian Local Projections: Difference between wealth groups


*Note:* Response to a 1% decrease in the 1 year Treasury bond by households grouped by wealth. Using a Bayesian Local Projection with the monetary policy shock from Jarociński and Karadi (2020). Quarterly data from 1996-2017. Excess bond premium and GDP Deflator are added as controls. Shaded areas are 68% and 90% posterior coverage bands obtained with 10,000 draws.

For further robustness and transparency the response of the three models used in this paper are displayed in Figure 1.26. The error bands now only represent 90% confidence with the darker band for the BLP, lighter band for BVAR and no error bands shown for LP. The three different specifications co-move, although the response from a standard local projection is very volatile.

Figure 1.26: BLP, LP, BVAR: Difference between wealth groups


*Note:* Response to a 1% decrease in the 1 year Treasury bond by households grouped by wealth. Using a Bayesian VAR (dashed teal), BLP (solid dark blue) and LP (orange) specification with the monetary policy shock from Jarociński and Karadi (2020). Quarterly data from 1996-2017. Excess bond premium and GDP Deflator are added as controls. Shaded areas are the90% posterior coverage bands obtained with 10,000 draws for the BVAR (light grey) and BLP (dark grey).

## 1.H Additional Model Results

### 1.H.1 Change in hours worked by net worth

The labour market in the HANK model of Kaplan et al. (2018) features flexible wages and different hours worked (which is the same as labour supply due to flexible wages and no unemployment) across the wealth distribution. The wage increases from an expansionary monetary policy as firms demand more labour to boost production. This increase in wage incentives households along the wealth distribution to supply more labour with the result that aggregate hours worked in the economy increases. Figure 1.27 highlights the response of hours worked averaging over the bottom 75% of the net wealth distribution and the top 25%. As seen in this Figure, the response of hours worked is similar, with the wealthiest in the economy increasing their hours by more than the poorest.[34] Although not seen in the data, one reason for this occurrence in the model is the willingness to work to sustain income in their liquid account, so they do not hit their budget constraint, whilst they are investing in the illiquid asset that has an increase in return.

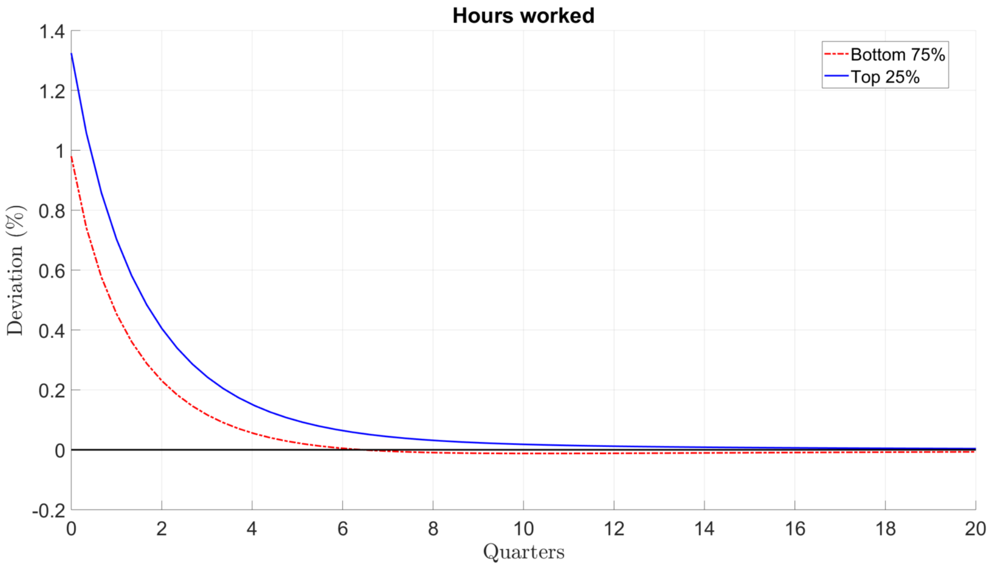
### 1.H.2 Mix of 50% Case 1 and 50% Case 2

Case 1 (profits are distributed as bonuses), Case 2 (profits are distributed as dividends) are outlined in the main body of text in Section 1.4. Case 3 combines an equal share of Case 1 and Case 2 whilst lowering the amount of profits that automatically flow back into the illiquid account ($\omega$). An obvious candidate to explore further is the mixing of Case 1 and Case 2 without altering the share of profits that are automatically put back into the firm (the illiquid account). This section highlights this result.

Table 1.13 outlines the income sources across the distribution of households by their net worth. The households in the bottom three quartiles of net worth receive the majority of their income from labour whereas the highest net worth households receive most of their income from the illiquid asset. The share of profits distributed into the liquid account of households is distributed equally as profits and as dividends. Compared to Case 1, we see a more realistic profit distribution as the highest net worth households receive a larger share, compared to the low wealth households, of their income from profits.

[34] In the steady state the households at the bottom 75% of the wealth distribution hours worked is 30% higher than the households in the top 25%.

Figure 1.27: Case 1 (KMV): Hours worked from expansionary monetary policy shock




*Note:* Response of hours worked (labour supply) by net worth to a 0.25 (1% annual) fall in the innovation in the Taylor rule $\epsilon$.

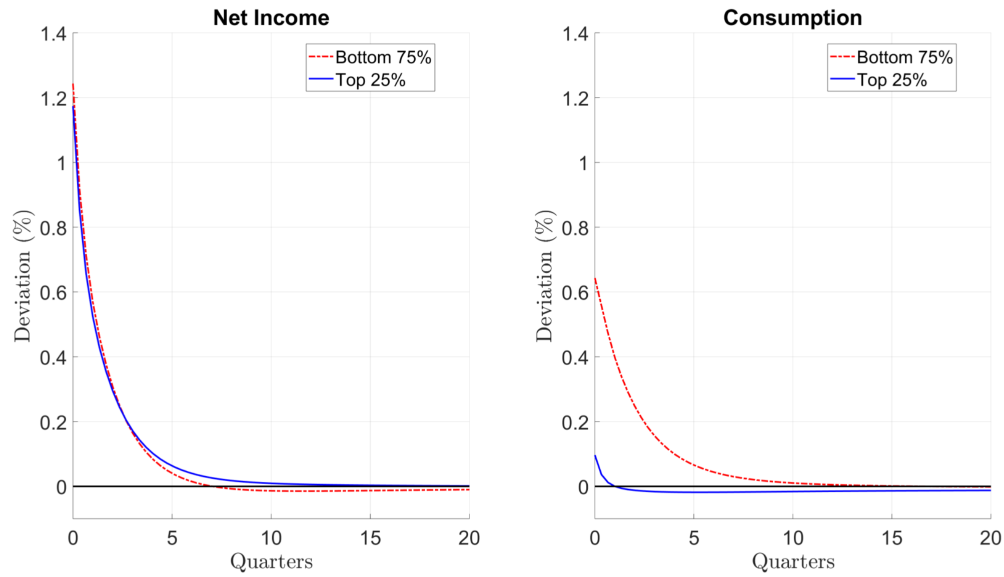
Table 1.13: Mixed Case 1 and Case 2: Net income sources by net worth

| | 0-25% | 25-50% | 50-75% | 75-100% |
|---|---|---|---|---|
| Labour Income | 81% | 83% | 81% | 32% |
| Transfer Income | 15% | 12% | 10% | 4% |
| Profit | 4% | 4% | 5% | 7% |
| Liquid Asset | -1% | 0% | 2% | 2% |
| Illiquid Asset | 0% | 0% | 3% | 56% |

Steady state decomposition of net income by household net worth quartiles.

Figure shows the resulting consumption and income responses from an expansionary monetary policy shock. The income response (right-hand-side) of the bottom 75% and top 25% of the net wealth distribution is now very similar. However, their consumption response (left-hand-side panel) differs as these households are not hand-to-mouth and instead choose to save.

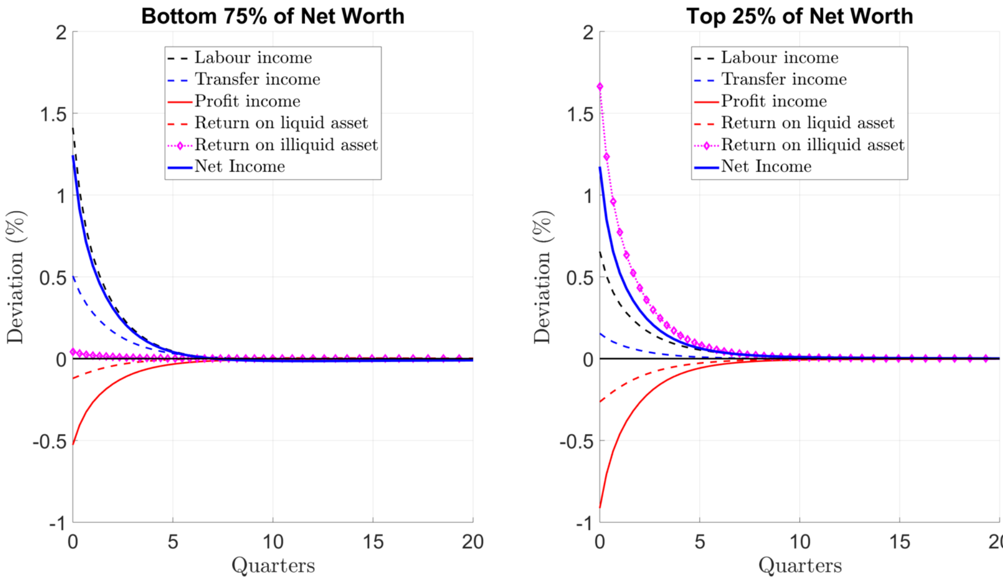
Figure 1.28: Mixed Case 1 and Case 2: Consumption and income response by net worth




*Note:* Response of consumption and income by net worth to a 0.25 (1% annual) fall in the innovation in the Taylor rule $\epsilon$.

The increase in income following an expansionary monetary policy shock comes from different sources dependent on the households net wealth, as outlined in Figure 1.29. Households in the bottom 75% of the net wealth distribution receive the majority of their income from the rise in wages following an expansionary monetary policy shock. As with Case 1 profits dampen their overall net income increase as they are now distributed, in part, due to labour productivity. The return on the liquid or illiquid asset does not directly play a significant role for their income source as they hold little of either. In contrast, the households in the top 25% of the net wealth distribution benefit from the increase in the return on the illiquid asset, pushing income up on its own by 1.5% for these households. However, since profits are distributed, in part, due to illiquid asset holdings their income is dampened due to this. Moreover, as these households hold liquid assets they also lose out when the interest rate falls. In the end, although the income sources across the distribution of net worth is different, their net income response is similar.

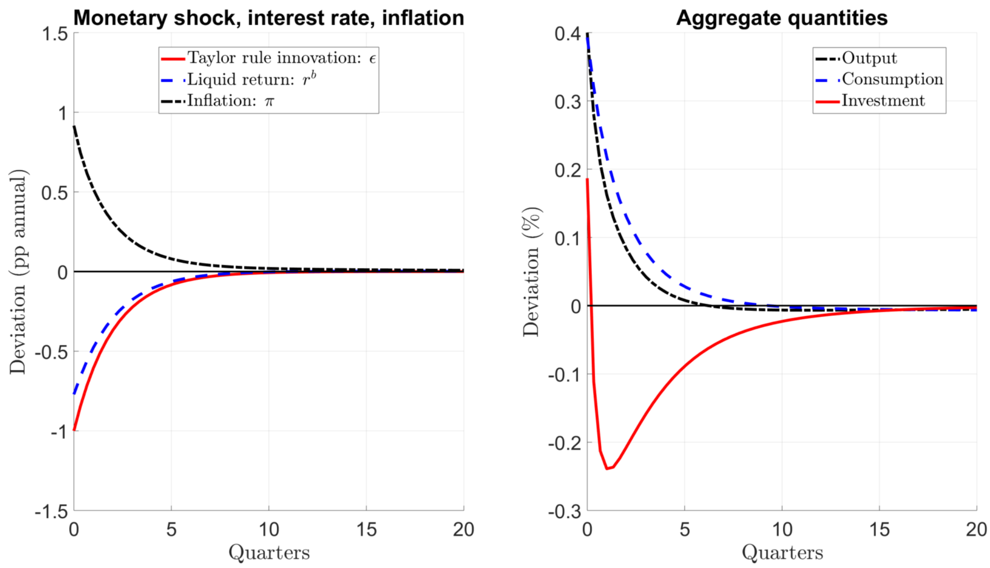
Figure 1.29: Mixed Case 1 and Case 2: Income response decomposed by net worth




*Note:* Decomposing income response by net worth to a 0.25 (1% annual) fall in the innovation in the Taylor rule $\epsilon$. The decomposition represented is scaled by the share of net income, so that the lines (excluding net income) will sum to net income. The black dashed line is net labour income, $(1-\tau_t)w_t z_t l_t$, the dashed blue line is government transfers $T_t$, the red line is the income from profits, $\pi_t^b$, the return from liquid assets, $r^b(b)b_t$, is represented by the red dashed line and finally the pink dashed line with diamonds is the return from illiquid asset holdings, $r_t^a a_t$.

Figure 1.30 outlines the aggregate response from an expansionary monetary policy shock when profits are distributed using an equal share of Case 1 and Case 2. As was the issue in Case 2 investment is now countercycical, which is in contrast to the empirical evidence. Although investment does increase on impact of the shock, it is below output and consumption, where in the data typically investment is the most volatile component. Although the highest net worth households' income increases the countercyclical profits that flow into their liquid account cause these households to transfer their assets from illiquid to liquid. Liquid assets are required to purchase consumable goods and provide a buffer to the borrowing constraint. This transfer causes investment to fall for the quarters following the monetary policy shock.

Figure 1.30: Mixed Case 1 and Case 2: Aggregate response to an expansionary MP shock




*Note:* Response of aggregate variables to a 0.25 (1% annual) fall in the innovation in the Taylor rule $\epsilon$.

### 1.H.3 Consumption decomposed

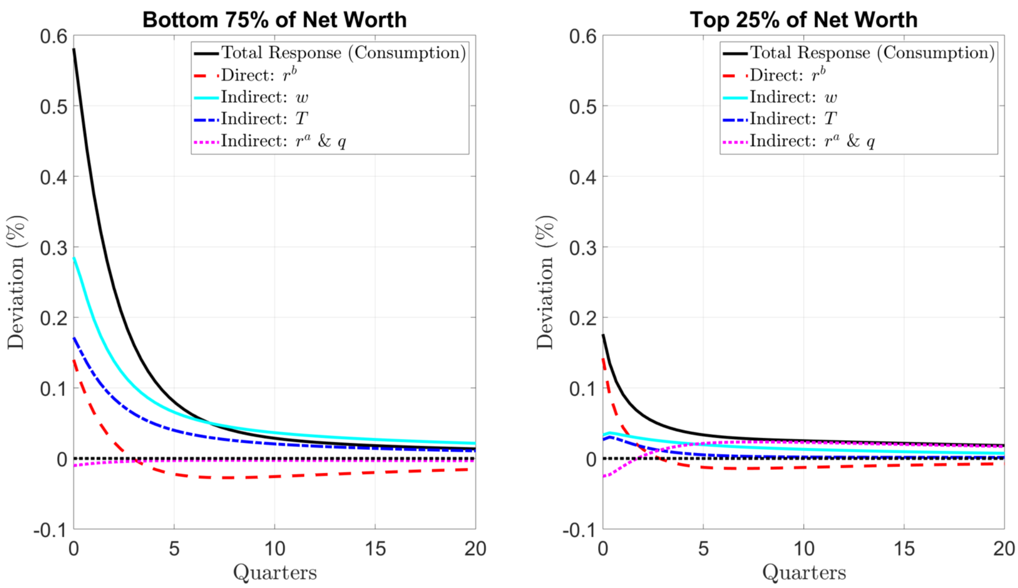
As explained in Section 1.8.3 it is possible to decompose the change in aggregate consumption due to the direct and indirect effect of monetary policy, outlined for convenience below:

$$dC_0 = \underbrace{\int_0^\infty \frac{\partial C_0}{\partial r_t^b} dr_t^b dt}_{\text{direct effect}} + \underbrace{\int_0^\infty \left( \frac{\partial C_0}{\partial w_t} dw_t + \frac{\partial C_0}{\partial r_t^a} dr_t^a + \frac{\partial C_0}{\partial \tau_t} d\tau_t + \frac{\partial C_0}{\partial T_t} dT_t \right) dt}_{\text{indirect effects}} \tag{1.56}$$

Figure 1.31 outlines the channels monetary policy impacts the bottom 75% of household net worth and top 25% of household net worth. Case 1, which is the baseline in Kaplan et al. (2018), shows that for the low wealth households transfers and wages, both indirect effects of monetary policy, play the majority role in their consumption response. Whereas the direct effect, the classical intertemporal substitution channel found in RANK models, is the largest contributor to households at the top of the net worth distribution. The wealthy households, who have enough assets to freely save, increase their consumption as the benefit of saving in the liquid account (the real interest rate on liquid bonds) falls. However, their consumption is dampened slightly as the return on illiquid assets increases in the model, which causes these households to save and benefit from the higher interest rate on illiquid assets.

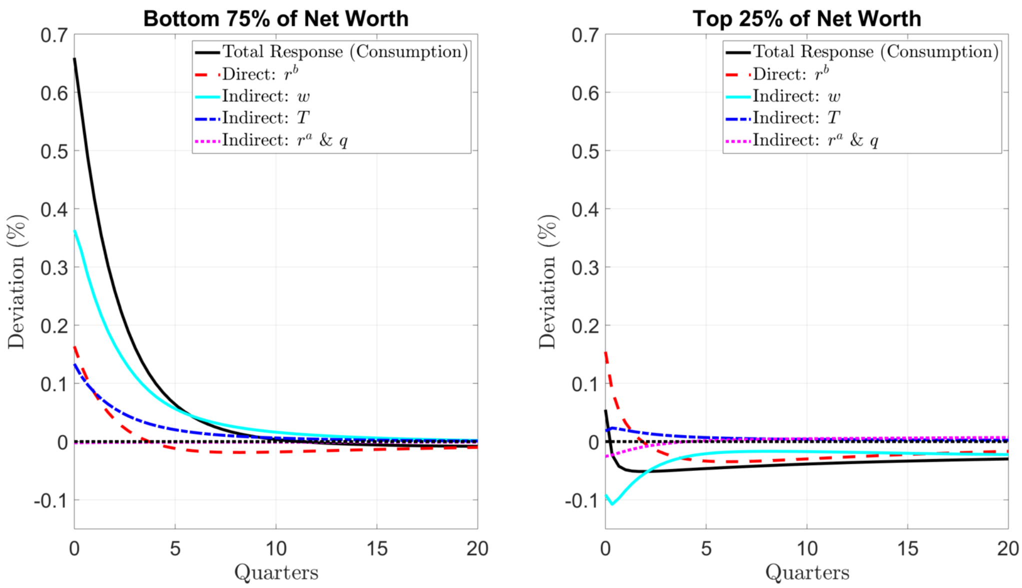
Contrasting Figure 1.31 with Figure 1.32 where profits are distributed dependent on illiquid asset holdings there are some obvious differences. For the low wealth households the picture is similar, their largest contributor is the increase in wages, which is an indirect effect of monetary policy. Moreover, for the wealthier households the fall in the return on liquid assets, the direct effect, still positively impacts their consumption response on impact. However, the increase in wages, which previously lead to an increase in consumption now negatively impacts the consumption response of the high net worth households. As seen in Section 1.4 Figure 1.8 labour income increases for the highest net worth households but wages have a negative effect on the consumption of the wealthier households. This is due to the increase in wages, the cost of labour in production, causing an increase in production costs and hence lowering the markup charges by the firm and reducing profits. Since in Case 2 profits are almost entirely distributed to households in the top 25% of net worth this rise in wages has a negative impact on their consumption.

Figure 1.31: Case 1: Consumption Decomposed




*Note:* Response of consumption by net worth decomposed into the direct and indirect effect of a 0.25 (1% annual) fall in the innovation in the Taylor rule $\epsilon$.

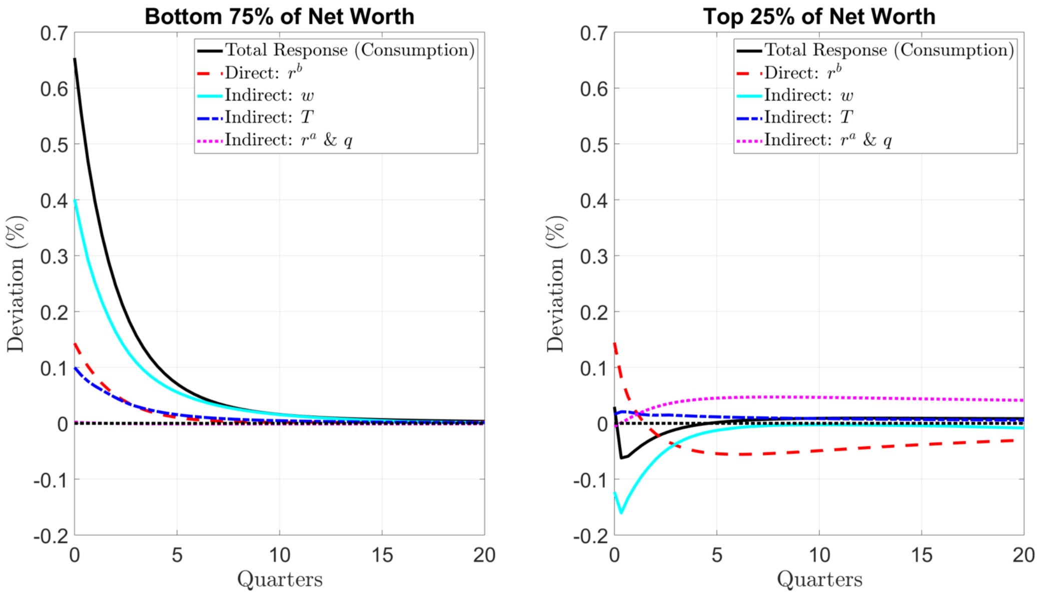
Figure 1.32: Case 2: Consumption Decomposed




*Note:* Response of consumption by net worth decomposed into the direct and indirect effect of a 0.25 (1% annual) fall in the innovation in the Taylor rule $\epsilon$.

The consumption response of Case 3, where the share of profits distributed into the liquid account increases as well as mixing the bonus based profit distribution scheme of Case 1 and the dividend distribution scheme of Case 2, is highlighted in Figure 1.33. As in Figure 1.32 wages have a negative impact on the consumption response of the highest net worth households. Moreover, wages remain to be the most important component of the consumption response of lower wealth households. Unlike in the previous cases the combined response of the illiquid asset return and the stock price now put upward pressure on the consumption response of the highest net worth.

Figure 1.33: Case 3: Consumption Decomposed




*Note:* Response of consumption by net worth decomposed into the direct and indirect effect of a 0.25 (1% annual) fall in the innovation in the Taylor rule $\epsilon$.

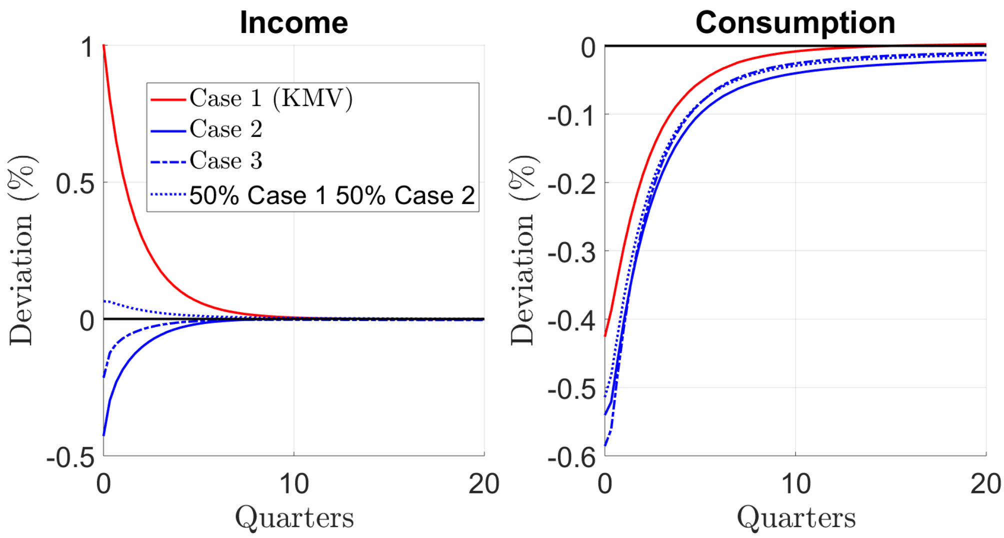
## 1.I Response of Gini Coefficients

The response of the Gini coefficient for income and consumption within the model and the extensions conducted in the main text of the paper provide an ideal measure to test against empirical findings. The response of the Gini coefficient, a measure of statistical dispersion used to quantify inequality within an economy, to monetary policy shocks has been studied previously by Coibion et al. (2017)

and Mumtaz and Theophilopoulou (2017). One strength of this approach is that it does not require the use of data on wealth, which my main empirical exercise relies upon. Coibion et al. (2017) finds that a contractionary monetary policy shock increases consumption inequality by more than income inequality. In the context of my paper, which solely focuses on expansionary monetary policy, this means that we would expect to see a fall in income and consumption inequality, with the Gini coefficient on consumption falling by more than income.

Figure 1.34 highlights the response from the model. In red, Case 1, which is the baseline from Kaplan et al. (2018) where profits are distributed to households depending on their labour productivity, features an increase in income inequality that is at odds with the current findings in the literature. However, the response of the Gini coefficient for consumption is in line, as it falls from an expansionary monetary policy shock. Innovating on the profit distribution scheme, as in Case 2 where profits are distributed due to illiquid asset holdings and Case 3, which features a mixture of Case 1 and Case 2 whilst increasing the amount of profits distributed feature the correct sign for the income and consumption Gini coefficients. Moreover, Case 3, where the aggregate and micro response to an expansionary monetary policy shock was in line with the empirical evidence also features the Gini coefficient on consumption falling my more than income, a finding supported by Coibion et al. (2017).

Figure 1.34: Response of Gini Coefficients




*Note:* Response of Gini coefficients for consumption and income by profit distribution scheme to a 0.25 (1% annual) fall in the innovation in the Taylor rule $\epsilon$. Case 1 is the baseline profit distribution scheme of Kaplan et al. (2018), where profits are distributed in proportion to labour productivity. Case 2 profits are distributed in proportion to illiquid assets. Case 3 combines Case 1 and 2 whilst increasing the share of profits distributed.

# Chapter 2

# OPTIMAL MONETARY RULES WITH DOWNWARD NOMINAL WAGE RIGIDITY

## 2.1 Introduction

In the standard New Keynesian model wages are assumed to be symmetrically flexible, such that wage increases or decreases are equally effortless to implement. Extensions of this simple model, such as a medium-scale dynamic stochastic general equilibrium model, add Calvo wages,[1] which symmetrically dampens wage changes. However, as shown in the data of individuals' wage changes in Daly et al. (2012) and at the country level for developed and developing countries by Schmitt-Grohé and Uribe (2016), wages are downwardly rigid - we observe increases in nominal wages more often than decreases. Adding in such a constraint into a standard model impacts how the agents in the economy react to shocks, which has further consequences for the optimal monetary policy and the optimal steady state inflation rate compared to a model with flexible wages.

This paper explores monetary policy in the New Keynesian model when the model is affected by downward nominal wage rigidities (DNWR), such that nominal wages can freely rise but are sluggish when adjusting downwards.

[1] Wages are set by labour unions that are able to adjust the wage with a certain probability, defined as Calvo wages following Calvo (1983).

Including DNWR, instead of Calvo wages or flexible wages, causes an asymmetric response of monetary policy to shocks of the same size but differing signs. Moreover, shocks that increase the nominal wage, such as a temporary positive demand shock, can create persistent effects since the wage cannot adjust down in a timely manner, which puts upward pressure on price inflation, causing the central bank to raise interest rates, which leads to households saving rather than consuming, thus lowering demand even though the wage is high, which causes unemployment in the economy, leading to a boom-bust cycle. This isn't found when Calvo wages are assumed. Furthermore, I find that the optimal monetary policy is asymmetric and allows for price inflation to deflate real wages (both of which have been found using a related framework in Kim and Ruge-Murcia (2011)). This work contributes to the literature by finding the optimal simple rule (OSR) in a New Keynesian model with a downward nominal wage rigidity constraint of the form in Schmitt-Grohé and Uribe (2016). This optimal simple rule, referred to in the paper as Simple Rule 3, places a higher weight on deviations from the natural rate of unemployment than the typical Taylor rule, referred to as Simple Rule 1 or the simple rule seen in Galí (2015), referred to as Simple Rule 2. As in, Schmitt-Grohé and Uribe (2016), when the economy suffers from unemployment the labor unions wish to lower wages, however, due to the DNWR constraint this isn't feasible, thus motivating an interest rate rule that is more sensitive to unemployment changes. In the latter part of the paper, optimal trend inflation of around 0% to 1.25%, depending on the shocks assumed, is found to be welfare improving, thus providing further support for 'greasing the wheels' of the economy. In an extension of the model, the downward nominal wage rigidity constraint is internalised, such that households optimise over it. Once the households understand the existence of this occasionally binding constraint, and that a shock may cause it to bind, it leads them to limit their wage increases even though the household can flexibly raise the wage, this is because once increased the wage is sluggish to fall, a finding made as well by Elsby (2009) and more recently in a theoretical model by Wolf (2018).

The paper is organised as follows: Section 2.2 highlights the empirical motivation. Section 2.3 discusses the papers that have implemented a downward nominal wage constraint and outlines their findings. Section 2.4 outlines the simple model used, which is a log-linearised New Keynesian model with an exogenous DNWR constraint or DNWR wage setting rule. Section 2.5 provides the impulse responses from simulating the model and presents the optimal monetary policy and optimal simple rule in this setup. Section 2.6 solves the non-linear model with a positive trend inflation and internalised DNWR constraint, allowing the households to be forward looking when setting their wages. Section 2.7 concludes and the Appendix includes details on the model equations, steady state, computational techniques used and additional tables and figures.

## 2.2 Empirical Motivation

This paper utilizes the basic New Keynesian model, which includes sticky prices in the standard form of Calvo (1983). The empirical motivation of this type of model has already been covered extensively. Therefore this section aims to empirically motivate the non-standard additions to the New Keynesian model that are used within this paper.

Previous work has been completed concerning wage rigidities, stemming from Erceg et al. (2000), which enter in symmetrically and are analogous to price rigidities. This setting has also been extended to include unemployment[2], originating from Galí (1996) and worked upon in Galí (2011) and Galí et al. (2012). The principle finding of which, "the structural wage equation derived here is shown to account reasonably well for the co-movement of wage inflation and the unemployment rate in the US economy," Galí (2011).

However, there exists empirical evidence that wages may not be symmetrically rigid and that for developed countries, as well as some developing countries, wages are downwardly rigid - such that we observe increases in nominal wages more often than decreases, Schmitt-Grohé and Uribe (2016). The parameters of the model included in this paper are based upon U.S. data and hence the evidence for DNWR presented here is U.S. focused. The three main empirical studies that use individual level data are: Gottschalk (2005), Barattieri et al. (2014) and Daly et al. (2012). Gottschalk (2005) applies new methods to data from the Survey of Income and Program Participation and finds that downward flexible wages found in individual level data is due to measurement error and once corrected the data produce findings closer to that found in firm level data where "only 2% to 3% of workers experiencing nominal-wage cuts, which implies substantial rigidity." In keeping with Gottschalk (2005), Barattieri et al. (2014) finds evidence using the same micro data "that wage changes are significantly right-skewed" therefore seeing an increase in wages is more likely than a decrease. Moreover, they also show that

[2]Staggered wage contracts and their link to unemployment can be seen even earlier in a rational expectations model of Taylor (1980).

> higher wage stickiness makes it easier for macroeconomic models to match the stylized fact that monetary shocks cause persistent changes in real output and small but relatively persistent changes in prices.(Barattieri et al., 2014)

Lastly Daly et al. (2012) analyses wage growth during the great recession of 2007 using the Current Population Survey and finds that "despite modest economic growth and persistently high unemployment, real wage growth has averaged 1.1% since 2008" and that "a significant fraction of workers are affected by downward nominal wage rigidities." One possible reason for observing DNWR during that period is that the low inflation environment meant that real wages were not being eroded by inflation. Furthermore, employers are hesitant to reduce pay as it can reduce morale and prompt resistance Kahneman et al. (1986).

Further empirical support of downward nominal wage rigidities has been found for European countries by the Wage Dynamics Network, a research network consisting of the European Central Bank (ECB) and the National Central Banks (NCBs) of the EU Member States. Using a firm-level survey spanning 15 European countries during the late 2007 and early 2008, Babecky et al. (2010) find evidence of downward nominal wage rigidity (defined in their study as wage freezes) and downward real wage rigidity (defined through wage indexation). Further evidence is provided by research conducted by the Wage Dynamics Network, which focuses on downward real wage rigidity for specific countries over a longer period of time, including Lünnemann and Wintr (2010) who focuses on Luxembourg between 2001 and 2007, and Du Caju et al. (2012) for Belgium between 1990 and 2002.

However recent work by Elsby et al. (2016), which focuses on the USA and UK labour markets, argue that downward nominal wage rigidity may be less binding than originally thought. Through the use of higher quality data - payroll data instead of self-reported surveys that may be subject to reporting error - and a comparison between male and female workers they find a higher frequency of wage reductions than previous studies. Thus motivating further empirical research into the existence and impact of downward nominal wage rigidities.[3]

[3]For example, Elsby and Solon (2019) present international evidence on the frequency of wages cuts.

## 2.3 Literature Review

Downward Nominal Wage Rigidites have been studied within economic models previously. The innovation within this paper is to include it in the New Keynesian model with unemployment using a simple constraint, exploring the optimal monetary policy and a proposed optimal simple rule. Below provides an outline and briefly discusses the most prominent papers within the literature that include DNWR.

Kim and Ruge-Murcia (2011), which builds on one of their earlier papers [4] utilize a convex cost function for changing prices and wages that can be asymmetric or reduced down to a quadratic cost a la Rotemberg (1982). Moreover, this cost function encompasses the 'L' shaped cost function of Benigno and Ricci (2011) which corresponds to the situation where cutting wages is infinitely costly and raising wages is costless. They find that 'greasing the wheels', having a low but strictly positive inflation target is welfare improving. Therefore,

> for an economy with downwardly rigid wages, the benefits of positive inflation conjectured by Tobin (1972) may overcome Friedman (1969)'s general prescription of negative inflation. (Kim and Ruge-Murcia, 2011)

This inflation target is estimated to be around 1% but will change depending on the model specifications and country estimated to. Kim and Ruge-Murcia (2011)'s paper is similar to this paper in execution and conclusion however this model utilizes fully flexible increases in nominal wages with DNWR and Calvo prices. Moreover, the model is able to analyze the response of employment, unemployment and the labor force to exogenous shocks as well as finding the coefficients for an optimal simple rule.

Benigno and Ricci (2011) introduce DNWR in a DSGE model with flexible prices and find also that 'greasing the wheels' and allowing for moderate inflation may help intratemporal and intertemporal relative wage adjustments and that "those experiencing large volatility or lower productivity growth may find it desirable to target a higher inflation rate." They also link the steepness of the Phillips Curve and wage rigidities and find that the Phillips Curve would steepen if wage rigidities declined. Furthermore, when wage rigidities are present there exists a "non-negligible long-run trade-off between inflation and the output gap," Benigno and Ricci (2011).

[4] See Kim and Ruge-Murcia (2009).

Schmitt-Grohé and Uribe (2016) motivated the manner in which the DNWR constraint is included as the constraint follows the same form utilised in their paper. Differences arise between our papers as Schmitt-Grohé and Uribe (2016) focus on developing open economies and how "the combination of a currency peg and free capital mobility creates a negative externality that causes overborrowing during booms and high unemployment during contractions." The value of the degree of DNWR, $\gamma$, is explored by Schmitt-Grohé and Uribe (2016) and is around 1 for the developed and developing countries they analyze. This paper follows Schmitt-Grohé and Uribe (2016) by choosing $\gamma$ close to one. The majority of this paper uses $\gamma = 0.9975$ such that wages decline up to 1% per year, when a different value of $\gamma$ is used it will be clearly outlined.

Fahr and Smets (2010) combines the convex cost function of Kim and Ruge-Murcia (2009) with regard to prices and wages with the labor market of Erceg et al. (2000). Using a two country model and real wage rigidity, instead of nominal rigidity, allows them to focus on transmission of monetary policy in a monetary union. Their main findings that pertain to this paper are that

> Downward nominal wage rigidities lead to a positively skewed response in nominal wage changes and a sizeable positive optimal inflation rate. This effect is stronger the lower price rigidity. The greasing effects of inflation vanish if wages are either indexed (real wage rigidity) or if adjustment costs are symmetric. (Fahr and Smets, 2010)

The range of optimal trend inflation proposed can be vast, with Gross (2018) finding optimal inflation in their model, which is an extension of Daly and Hobijn (2014), to be 5.4%. Gross (2018) uses a Calvo (1983) approach to the DNWR constraint, such that with some probability the household cannot lower their wage.

Moreover as well as adding motivation for positive trend inflation DNWR has also been shown to provide wage restraint. This has been shown in Wolf (2018), which also takes seriously the wage constraint found in Schmitt-Grohé and Uribe (2016) and uses it to assess wage inflation rates in the euro area.

More recently Dupraz et al. (2019)'s 'A Plucking Model of Business Cycles' has brought further attention to the DNWR literature utilising the constraint found in Schmitt-Grohé and Uribe (2016). They embed a strict DNWR constraint into a search and matching framework to match the dynamics of U.S. unemployment. They find large benefits of increasing the inflation target, whereby the benefits found in this paper are modest. A main difference is the lack of price rigidities as well as the inclusion of a search framework, which is the source of our differences.

Thus this paper provides further support to grease the wheels of the economy and wage restraint. Moreover, the paper advances the literature by highlighting the differences in responses when the households follow a wage setting rule versus when they are forward looking and can optimise over the DNWR constraint. Furthermore, I find an optimal simple rule, one which can be followed by a central bank, which produces impulse responses from shocks close to the optimal monetary policy.

## 2.4 Methodology

The model is a fairly simple New Keynesian model but with unemployment as in Galí et al. (2012) and the addition of downward nominal wage rigidities. The model follows closely to a standard New Keynesian model that can be found in Galí (2015) 'Chapter 6: Sticky Wages and Prices' as well as 'Chapter 7: Unemployment and Monetary Policy'. The main difference between these models is that the one used in this paper suffers from Downward Nominal Wage Rigidities instead of Calvo wages.

### 2.4.1 Firms

As in the standard New Keynesian model, a continuum of firms are assumed to exist and are indexed by $i \in [0, 1]$. Each firm produces a differentiated good with a technology represented by the production function

$$Y_t(i) = A_t N_t(i)^{1-\alpha} \tag{2.1}$$

where $A_t$ is an exogenous technology [5] parameter that is common to all firms, $Y_t(i)$ denotes the output of good $i$, and $N_t(i)$ is a labour input used by firm $i$ and can thought of as employment or hours worked. The definition of $N_t(i)$ is given

[5]Positive trend growth in technology is neglected here for simplicity, providing motivation for future extensions. Adding in positive growth in technology has important implications for optimal steady state price inflation.

by

$$N_t(i) = \left( \int_0^1 N_t(i,j)^{1-\frac{1}{\epsilon_w}} dj \right)^{\frac{\epsilon_w}{\epsilon_w - 1}}. \tag{2.2}$$

Here $N_t(i,j)$ denotes the quantity of type-j labour employed by firm $i$ in period $t$. Moreover, there is a continuum of labour types indexed by $j \in [0,1]$. The parameter $\epsilon_w$ represents the elasticity of substitution among labour types.

$W_t(j)$ denotes the nominal wage for type $j$ labour that prevails in period $t$, for all $j \in [0,1]$. Wages are set by the household and can be increased flexibly, however wages face downward rigidities such that the nominal wage cannot decrease freely - this is outlined further in Section 2.4.3.[6] Therefore the cost minimization yields a set of demand schedules for each firm $i$ and labour type $j$, given the firm's total employment $N_t(i)$, which highlights that hiring of a particular labor type is due to their relative wage and substitutability

$$N_t(i,j) = \left( \frac{W_t(j)}{W_t} \right)^{-\epsilon_w} N_t(i) \tag{2.3}$$

for all $i, j \in [0,1]$, where

$$W_t \equiv \left( \int_0^1 W_t(j)^{1-\epsilon_w} dj \right)^{\frac{1}{1-\epsilon_w}} \tag{2.4}$$

is an aggregate wage index. Through substituting equation (2.4) into equation (2.3) it can be shown that

$$\int_0^1 W_t(j) N_t(i,j) dj = W_t N_t(i),$$

which is a convenient aggregation result that will subsequently be used.

As well as hiring workers firms set the price of final goods in the economy following Calvo (1983), which is typical in a New Keynesian model. A firm in period $t$ will choose the price $P_t^*$ to maximize their current market value of profits, however, a firm may only reset their price with a probability $1-\theta$ in any given period. Hence their problem can be shown to be,

$$\max_{P_t^*} \sum_{k=0}^{\infty} \theta_p^k E_t \{ \Lambda_{t,t+k} (1/P_{t+k}) (P_t^* Y_{t+k|t} - \mathcal{C}_{t+k}(Y_{t+k|t})) \}$$

[6] The assumption of upwardly flexible wages is further explored in Appendix 2.D.3 as Calvo wage rigidity is introduced as in Galí (2015) and different probabilities for labour unions to change their wage is explored in Figure 2.10.

subject to the sequence of demand constraints

$$Y_{t+k|t} = \left( \frac{P_t^*}{P_{t+k}} \right)^{-\epsilon} C_{t+k} \tag{2.5}$$

for $k = 0, 1, 2, \ldots$ where $\Lambda_{t,t+k} \equiv \beta^k U_{c,t+k}/U_{c,t}$ is the stochastic discount factor, $\mathcal{C}(\cdot)$ is the nominal cost function. As shown in Galí (2015) solving this problem and rearranging accordingly, as well as conducting a first-order taylor-approximation in the neighborhood of the zero inflation steady state, the following equation for price inflation $\pi_t^p \equiv p_t - p_{t-1}$[7]

$$\pi_t^p = \beta E_t\{\pi_{t+1}^p - \lambda_p \hat{\mu}_t^p\} \tag{2.6}$$

where as in Galí (2015), $\hat{\mu}_t^p \equiv \mu_t^p - \mu^p$ is the deviation of the average (log) price markup from its flexible price counterpart and $\lambda_p \equiv \frac{(1-\theta_p)(1-\beta\theta_p)}{\theta_p} \frac{1-\alpha}{1-\alpha+\alpha\epsilon_p}$. Firms wish to raise their prices when the average price markup in the economy today or in the future is below the desired levels of the firms and hence prices rise when firms are able to change their prices and inflation arises from this. The average price markup is related to the output and real wage gaps, whereby using $\hat{\mu}_t^p = mpn_t - \omega_t$, one can write the New Keynesian Phillips Curve as:

$$\pi_t^p = \beta E_t\{\pi_{t+1}^p\} + \varkappa_p \tilde{y}_t + \lambda_p \tilde{\omega}_t$$

with $\varkappa_p \equiv \frac{\alpha\lambda_p}{1-\alpha}$.

### 2.4.2 Households and Unemployment

This section follows closely Galí (2015) Chapter 7 on unemployment and monetary policy.

#### Households

There exists a large number of identical households, whereby each household has a continuum of members represented by the unit square and indexed by a pair

[7]Lower case letters throughout this short paper denote the (natural) log of the corresponding variable, e.g., $\pi \equiv \log \Pi$

$(j, s) \in [0,1]$x$[0,1]$. Here $j \in [0,1]$ represents the type of labour that the household member specialises in and $s \in [0,1]$ is the disutility that each household member faces from working. Disutility from work is given by $\chi s^{\varphi}$ if he is employed and zero otherwise, where $\chi > 0$ and $\varphi > 0$ are exogenous parameters. Full risk sharing within the household is assumed and therefore given the separability of preferences this implies the same level of consumption for each household member. The household's period utility is given by the integral of its members' utilities and can therefore be written as follows

$$\begin{aligned} U(C_t\{\mathcal{N}_t(j)\}; Z_t) &\equiv \left( \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \chi \int_0^1 \int_0^{\mathcal{N}_t(j)} s^{\varphi} ds dj \right) Z_t \\ &= \left( \frac{C_t^{1-\sigma} - 1}{1-\sigma} - \chi \int_0^1 \frac{[\mathcal{N}_t(j)]^{1+\varphi}}{1+\varphi} dj \right) Z_t \end{aligned}$$

where $C_t \equiv ( \int_0^1 C_t(i)^{1-\frac{1}{\epsilon_p}} di)^{\frac{\epsilon_p}{\epsilon_p - 1}}$ is a consumption index, $C_t(i)$ is the quantity consumed of good $i$, for $i \in [0,1]$, and $\mathcal{N}_t(j) \in [0,1]$ is the fraction of members specialised in type $j$ labour who are employed in period $t$. The preference shifter, $z_t$, is assumed to follow an AR(1) process with $\rho_z = 0.5$ and $\varepsilon_t^z$ is a white noise process with zero mean and variance $\sigma_z^2 = 1$ and can be rationalised as a demand shock.

$$z_t = \rho_z z_{t-1} + \varepsilon_t^z$$

Each household seeks to maximize

$$E_0 \sum_{t=0}^{\infty} \beta^t U(C_t\{N_t(j)\}; Z_t)$$

subject to a sequence of flow budget constraints given by

$$\int_0^1 P_t(i) C_t(i) di + Q_t B_t \leq B_{t-1} + \int_0^1 W_t(j) N_t(j) dj + D_t. \tag{2.7}$$

Here $P_t(i)$ is the price of good $i$, $W_t(j)$ is the nominal wage for labour type $j$ and $B_t$ represents purchases of a nominally riskless one-period bond, $Q_t$ is the price of that bond and $D_t$ is a lump-sum component of income, which can be thought of as dividends from ownership of firms.

Optimal demand for each good resulting from utility maximization takes the form:

$$C_t(i) = \left( \frac{P_t(i)}{P_t} \right)^{-\epsilon_p} C_t \tag{2.8}$$

where $P_t \equiv \left( \int_0^1 P_t(i)^{1-\frac{1}{\epsilon_p}} di \right)^{\frac{\epsilon_p}{\epsilon_p - 1}}$ denotes the price index for final goods. This takes a familiar form and can be shown that $\int_0^1 P_t(i) C_t(i) di = P_t C_t$.

The household's intertemporal optimality condition is given by and Euler equation of the form

$$Q_t = \beta E_t \Big\{ \Big( \frac{C_{t+1}}{C_t} \Big)^{-\sigma} \Big( \frac{Z_{t+1}}{Z_t} \Big) \Big( \frac{P_t}{P_{t+1}} \Big) \Big\} \tag{2.9}$$

The wage setting is done by the workers, or a union that represents all workers specialised in it. The innovation over a standard New Keynesian model is to have wages adhere to DNWR as seen in Section 2.4.3.

### Unemployment

Unemployment in this model follows that of Galí et al. (2012), hence unemployment arises due to the discrepancy in wages set by the labour union and firm. Taking into account the household members' disutility from working that individual will be willing to work, and therefore be a part of the labour force if and only if

$$\frac{W_t(j)}{P_t} \geq \chi C_t^{\sigma} s^{\varphi}.$$

Therefore, the individual will be willing to work if the real wage achieved exceeds the disutility from working given in units of consumption, hence multiplied by the household's marginal utility of consumption.

The marginal supplier of type $j$ labour, denoted $L_t(j)$, is given by

$$\frac{W_t(j)}{P_t} = \chi C_t^{\sigma} L_t(j)^{\varphi} \tag{2.10}$$

Following this we can define the aggregate labour force by integrating over labour types $j$, such that $L_t \equiv \int_0^1 L_t(j) dj$. Taking logs and integrating over $j$ provides the following approximate relation for the real wage :

$$w_t - p_t = \sigma c_t + \varphi l_t + \xi. \tag{2.11}$$

Equation (2.11) can be thought of as a participation equation where by first-order approximation around the symmetric steady state $w_t \simeq \int_0^1 w_t(j) dj$, $l_t \simeq \int_0^1 l_t(j) dj$ and $\log(\chi) = \xi$.

Following Galí (2011); Galí et al. (2012) the unemployment rate $u_t$ is defined as the log difference between the labour force and employment:

$$u_t \equiv l_t - n_t. \tag{2.12}$$

Combining the average wage markup $\mu_t^w \equiv (w_t - p_t) - (\sigma c_t + \varphi n_t + \xi)$ with equation (2.11) and equation (2.12), provides us with a linear relation between the wage markup and the unemployment rate

$$\mu_t^w = \varphi u_t. \tag{2.13}$$

Employment is demand determined with the labour demand given by the inverse production function in logs

$$n_t = \frac{1}{1-\alpha}(y_t - a_t). \tag{2.14}$$

Following Galí (2015) the natural rate of unemployment, $u_t^n$, is defined as that which would prevail in the absence of nominal wage rigidities. The natural rate of unemployment is set to 0.05, consistent with an average unemployment rate of 5%, to take into account frictional unemployment.

$$u^n = \frac{\mu^w}{\varphi}. \tag{2.15}$$

It is important to note that due to the monopoly the households have over labour that even under flexible wages a wage markup, $\mu^w$, will still be positive and hence natural rate of unemployment will also be greater than 0.

### 2.4.3 Wage Setting and Downward Nominal Wage Rigidity

Households set wages by maximising their utility with respect to their budget constraint as well as the sequence of labour demand schedules given in equation (2.3). Since the households have market power over wage setting in a flexible wage setting environment, one without any nominal rigidities, they would set wages as in equation (2.16) as a markup over their marginal rate of substitution. The markup here is given by as $\mathcal{M}_w \equiv \frac{\epsilon_w}{\epsilon_w - 1}$.

$$\frac{W_t^*}{P_t} = \frac{W_t}{P_t} = \mathcal{M}_w \chi C_t^\sigma N_t^\varphi \tag{2.16}$$

The main innovation of this paper is the inclusion of Downward Nominal Wage Rigidities that has been inspired by Schmitt-Grohé and Uribe (2016) and empirically motivated in Section 2.2. With DNWR the occasionally binding constraint is imposed of

$$W_t \geq \gamma W_{t-1} \qquad \gamma > 0, \tag{2.17}$$

where $\gamma$ defines the degree of downward nominal wage rigidity. Such that when $\gamma = 0$ there is full wage flexibility and the higher $\gamma$, the more downwardly rigid are nominal wages. If $\gamma \geq 1$ we see absolute downward wage rigidity, found empirically in Schmitt-Grohé and Uribe (2016) for many countries[8] . The parameter $\gamma$ is chosen to emulate that of Schmitt-Grohé and Uribe (2016), such that $\gamma = 0.9975$, which lies within the estimated bounds found, and at a quarterly frequency implies that nominal wages can decline up to 1 percent per year.[9] Therefore the wage setting rule divided by the price level for convenience can now be written as:

$$\frac{W_t}{P_t} = \max\{\ \mathcal{M}_w \chi C_t^{\sigma} N_t^{\varphi}\ ,\ \gamma W_{t-1} \frac{1}{P_t}\ \} \tag{2.18}$$

For the first half of this paper the downward nominal wage rigidity constraint is added into the model exogenously. This means that the households are only able to see the constraint once they reach it. This lends itself to a first attempt at understanding the effect of adding in such an occasionally binding constraint and provides a simple modelling environment using Occbin as discussed in Guerrieri and Iacoviello (2015) and explained in Appendix 2.B for this model.

Later in section 2.6 households will be able to maximise their utility with respect to their wage while taking into account the downward nominal wage constraint. A model of this type cannot be solved using pertubation techniques and therefore I use Smolyak collocation, a projection method, to solve the model. The computational technique is outlined in Appendix 2.C.2.

[8]See Schmitt-Grohé and Uribe (2016) for an extensive list, as an example it includes countries such as Bulgaria, Ireland, Italy, Spain, Slovenia.

[9]Figure 2.9 which can be found in Appendix 2.D.2 highlights the impact of different $\gamma$ on the economy and conclusions. A lower value of $\gamma$ moves the economy closer to that of flexible wages.

### 2.4.4 Equilibrium and Calibration

Below are the equations that characterize the equilibrium conditions in the New Keynesian Framework developed above. It is important to note that these correspond exactly to that in Galí (2015) except equation (2.21), which under Calvo wages would display an equation relating wage inflation to output and the real wage gap. Due to the assumption of flexible wages and the DNWR constraint equation (2.21) shows instead how real wages are set as a markup over the marginal rate of substitution, unless the downward nominal wage rigidity is binding. Where $\tilde{y}_t = y_t - y^n$, the output gap and $\tilde{\omega}_t$ is the real wage gap.

$$\tilde{y}_t = -\frac{1}{\sigma}(i_t - E_t\{\pi^p_{t+1}\} - r^n_t) + E_t\{\tilde{y}_{t+1}\} \tag{2.19}$$

$$\pi^p_t = \beta E_t\{\pi^p_{t+1}\} + \varkappa_p \tilde{y}_t + \lambda_p \tilde{\omega}_t \tag{2.20}$$

$$\omega_t = \max\{\mu^w + \xi + \sigma c_t + \varphi n_t \ , \ \gamma + \omega_{t-1} - \pi^p_t\} \tag{2.21}$$

$$\tilde{\omega}_t \equiv \tilde{\omega}_{t-1} + \pi^w_t - \pi^p_t - \Delta\omega^n_t \tag{2.22}$$

$$\textbf{Simple Rule 1 (Taylor):} \quad i_t = \rho + \phi_\pi \pi^p_t + \phi_y \tilde{y}_t \tag{2.23}$$

The calibration used is standard except the downward nominal wage rigidity parameter $\gamma$, which is taken from Schmitt-Grohé and Uribe (2016).

Table 2.1: Table summarizing parameter values

| Symbol | Parameter | Value | Reasoning |
|---|---|---|---|
| $\alpha$ | Capital Share | 0.25 | Standard |
| $\beta$ | Discount Factor | 0.99 | Standard |
| $\varphi$ | Inverse Frisch elasticity | 5 | Galí (2015) |
| $\epsilon_p$ | Demand elasticity for goods | 9 | Galí (2015) |
| $\epsilon_w$ | Demand elasticity for labour services | 4.5 | Galí (2015) |
| $\theta_p$ | Calvo parameter for price | 0.75 | Galí (2015) |
| $\phi_\pi$ | Taylor weight on inflation | 1.5 | Galí (2015) |
| $\phi_y$ | Taylor weight on output gap | 0.125 | Galí (2015) |
| $u^n$ | Natural rate of unemployment | 5% | Galí (2015) |
| $\gamma$ | DNWR Parameter | [0.99,0.9975] | 4% to 1% wage deflation |

## 2.5 Results

This section houses the main results of the paper for the simple model, whereby the model is solved around a zero inflation steady state and exogenous downward nominal wage rigidity constraint. A more realistic model that is solved around positive trend inflation and allows the household to maximise over the occasionally binding constraint can be seen in Section 2.6.

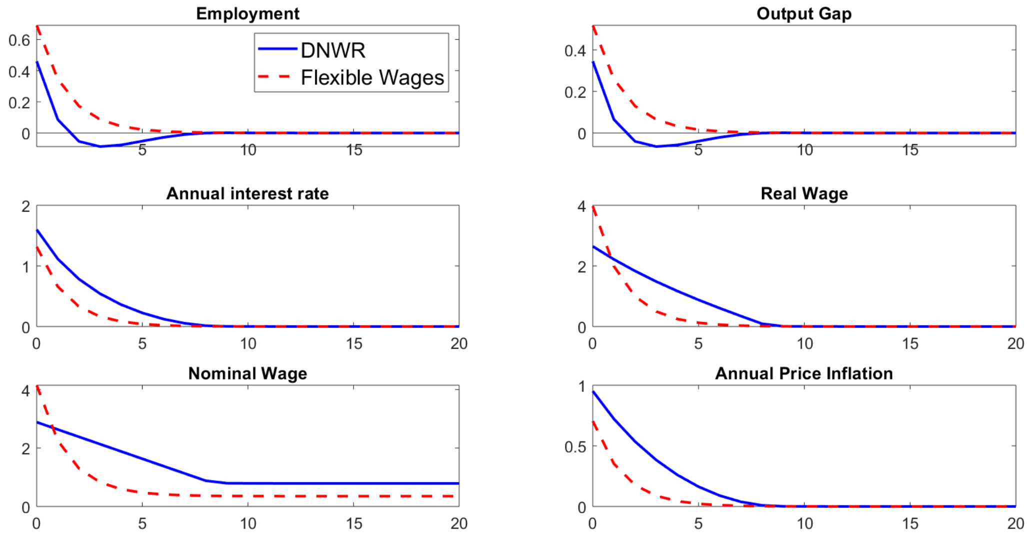
The figures below highlight how a simple New Keynesian economy is affected by adding in downward nominal wage rigidities. Of significance is the asymmetric response of the interest rate under monetary policy rules, as well as the optimal monetary policy. Furthermore, comparing the welfare loss under different monetary policy rules motivates exploration for an optimal simple rule that attempts to mimic the optimal monetary policy.

Figure 2.1 contrasts the response of a New Keynesian model with flexible wage versus downward nominal wage rigidities. As shown below, under flexible wages the wage is free to adjust such that nominal wages initially rise and then fall faster than when they are constrained by the downward nominal wage rigidity. The prolonged higher wage, when wages are constrained, pushes up prices and causes the central bank to raise the nominal interest rate, which causes households to save rather than consume. This lack of demand causes employment to fall.[10] Following this positive demand shock, due to the increased wage even when the shock subsides we see an increase in unemployment as seen in Schmitt-Grohé and Uribe (2016) as employment falls, through the change in aggregate demand resulting from the endogenous monetary policy response, but labour supply increases due to the higher wage. This results subsides when the downward nominal wage rigidity constraint is internalised in Section 2.6. The persistence in the nominal wage causes a boom-bust cycle as seen in the output gap in Figure 2.1. Since this model does not feature capital the movements in output are driven entirely by technology, $A_t$, and employment, $N_t$. The high price inflation caused by the artificially high wages helps to deflate the real wage back to its steady state value.

Figure 2.2 highlights the asymmetric response of the economy due to the inclusion of the Downward Nominal Wage Rigidity. From a positive shock, as seen in greater detail in Figure 2.1 a boom-bust cycle appears in the output gap. However, from a negative demand shock since nominal wages are unable to fall we see an amplification of the fall in output, which is caused by a greater fall in employment compared to the flexible wage counterpart. When wages are assumed to be flexible, or are afflicted by wage rigidity in the form of Calvo (1983), the outcome on the output gap is symmetric. This symmetry is broken in this model due to the inclusion of the occasionally binding constraint and thus

[10] For a clear exposition of the determinant of employment in the New Keynesian model see Galí (2013).

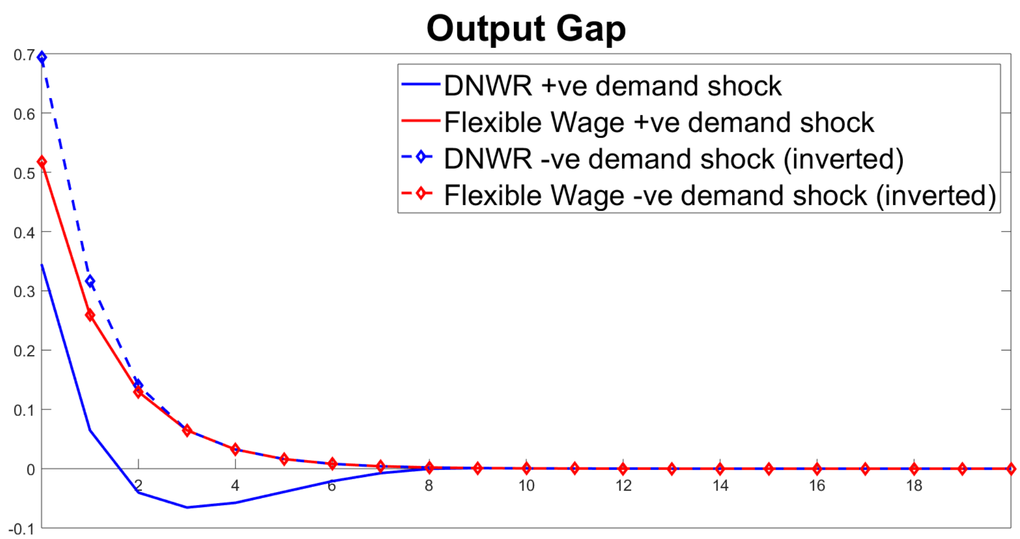
Figure 2.1: Positive Demand Shock under the Simple Rule 1 (Taylor)




Impulse response for variables facing a positive demand shock. The demand shock follows an AR(1) with $\rho_z = 0.5$ and $\sigma = 1$. The Simple Rule 1 (Taylor) is $i_t = 0.01 + 1.5\pi_t^p + 0.125\hat{y}_t + \nu_t$.

we see asymmetric response of the economy to symmetric shocks. The full response of the economy due to a negative demand shock can be seen in Figure 2.8 of Appendix 2.D.1.

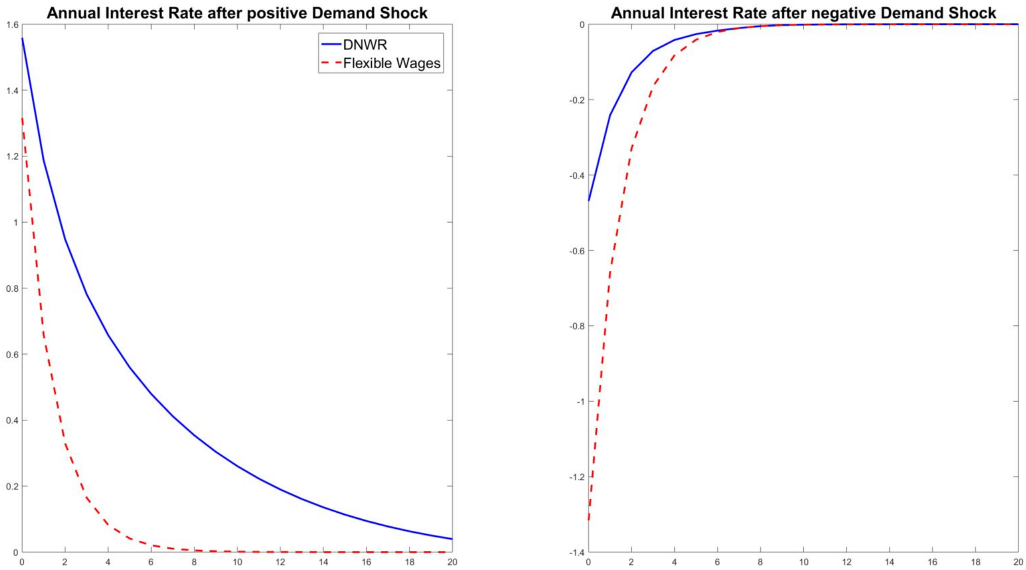
Figure 2.2: Asymmetric Output Gap Movement from symmetric Demand Shock




Output gap from a positive or negative symmetric demand shock. The demand shock follows an AR(1) with $\rho_z = 0.5$ and $\sigma = 1$. The Simple Rule 1 (Taylor) is $i_t = 0.01 + 1.5\pi_t^p + 0.125\hat{y}_t + \nu_t$.

Due to the constraint occasionally binding we see in Figure 2.3 the asymmetric response of monetary policy when the central bank follows a simple rule in the form of the standard Taylor rule. Under flexible or Calvo wages the response of the central bank is symmetric, however, when the economy suffers from DNWR the central bank reaction is stronger under a positive demand shock than a negative demand shock. This is because under a positive demand shock nominal wages rise, pushing up inflation and causing the central bank to react strongly, however, since wages cannot fall during a negative demand shock inflation will remain close to its steady state where the central bank, which cares most about inflation deviations, will not need to react as strongly as before to guide inflation back to its steady state.

Figure 2.3: Asymmetric Annual Interest Rate Response to a Symmetric Demand Shock




Impulse response of the nominal interest rate facing a positive and negative demand shock. The demand shock follows an AR(1) with $\rho_z = 0.5$ and $\sigma = 1$. The Simple Rule 1 (Taylor) is $i_t = 0.01 + 1.5\pi_t^p + 0.125\hat{y}_t + \nu_t$.

### 2.5.1 Optimal Monetary Policy

The optimal monetary policy is a perfect-foresight solution derived from minimising the discounted sum of welfare loss, shown in equation (2.24), subject to the equilibrium condition in section 2.4.4. The exogenous introduction of DNWR does not alter the steady state of the model nor the second-order approximation to the utility of the representative consumer, which remains the standard derivation as shown in Galí (2015) for a New Keynesian model with price rigidities. The introduction of the DNWR, similar to the introduction of the zero-lower-bound, restricts the set of feasible equilibrium paths such that whilst the constraint binds the optimal allocation, characterised by zero inflation and zero output gap at all times, cannot be attained. However, as is standard in the New Keynesian model the optimal monetary policy is able to fully neutralise any effect a demand shock would have on the welfare loss through changes in the interest rate. This is not the case when faced with a technology shock as it affects the natural output and interest rate of the economy. Without the DNWR constraint the optimal policy following a technology shock can replicate the flexible price/flexible wage equilibrium allocation through the necessary adjustments of the real wage (setting the real wage equal to the natural real wage at every point in time $t$). The inclusion of the DNWR breaks this relationship as wages are not able to decrease flexibly, and therefore cannot adjust adequately. Even with the inclusion of the DNWR constraint the welfare loss function used is the same as in the Calvo price but flexible wage New Keynesian model since the inclusion of this occasionally binding constraint does not affect the efficient steady state nor does it cause any wage dispersion. Therefore, as in Amano and Gnocchi (2017), which features an analogous DNWR constraint, the second-order Taylor expansion of utility is well defined because the expressions needed to derive it, the utility function and feasibility constraints, are differentiable whilst the inequality constraint is not needed in the approximation. Hence, the welfare loss function is:

$$\mathbb{W} = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \Big[ \Big( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \Big) \tilde{y}_t^2 + \frac{\epsilon_p}{\lambda_p} (\pi_t^p)^2 \Big] \tag{2.24}$$

Moreover, the average welfare loss used to compare different monetary policy rules is derived from the standard Calvo price but flexible wage New Keynesian model and is given by:

$$\mathbb{L} = \frac{1}{2} \Big[ \Big( \sigma + \frac{\varphi + \alpha}{1 - \alpha} \Big) var(\tilde{y}_t) + \frac{\epsilon_p}{\lambda_p} var(\pi_t^p) \Big] \tag{2.25}$$

The central bank's optimal monetary policy problem is outlined and solved under the more general model assumption of Calvo price and wage rigidities as well as the DNWR constraint in Appendix 2.A.

The optimal monetary policy response to a symmetric (positive or negative) demand shock is symmetric, due to the aforementioned reasons. Therefore figure 2.4 analyses if the optimal monetary policy is asymmetric following a positive and negative technology shock of one standard deviation and three standard deviations. Figure 2.4 shows that it is an optimal response of the central bank to act asymmetrically. This asymmetry arises due to the occasionally binding constraint and the central bank reacting to minimise movement in the inflation and the output gap. The kinked response of the central bank under the optimal monetary policy is amplified as the size of the shock increases, due to nominal wages stuck at the DNWR constraint for longer. A positive technology shock in the New Keynesian model with standard parameter values and flexible wages leads to a fall in inflation and employment as well as a decline in nominal wages. With DNWR and under a Taylor Rule, a similar picture emerges, albeit with nominal wages sluggishly falling to their new steady-state level. The optimal monetary policy adjusts the interest rate such that wages remain high, counteracting the relative reduction in cost of output for the firm, and price inflation stays at the steady-state. Following a negative technology shock the central bank acts to reduce fluctuations in price inflation dampening any nominal wage increases that would typically accompany this shock.

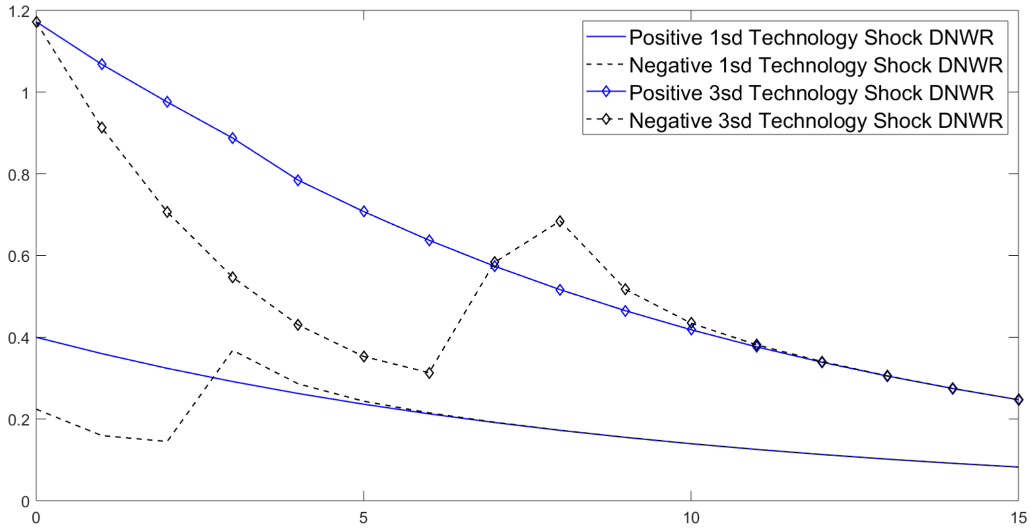
### 2.5.2 Simple Rule 2

Figure 2.5 assesses whether current monetary policy rules can match the optimal monetary policy response in this environment. To compare against Simple Rule 1 (Taylor) another simple rule is used, which is taken from Galí (2015) and performs well under Calvo price and wage rigidities. $\hat{u}_t$ here is defined as the log difference between the unemployment rate and natural level of unemployment:

$$\textbf{Simple Rule 2 (Galí):} \quad i_t = 0.01 + 1.5\pi_t^p - 0.5\hat{u}_t. \tag{2.26}$$

In Figure 2.5 it can be seen that Simple Rule 2 (Galí) proposed in Galí (2015) performs well in limiting the change in the output gap due to reacting to changes in the unemployment, however the response of price inflation is closer to the Taylor rule than the optimal monetary policy. After a positive technology shock the optimal monetary policy allows the nominal wage to rise, which adds pressure on price inflation and keeps it at its steady state level. In this scenario the positive technology shock helps to counteract the higher nominal wages to keep employment and therefore the output gap at their steady state levels.

Figure 2.4: Asymmetric Optimal Monetary Policy: Annual Interest Rate response from positive and negative Technology Shocks

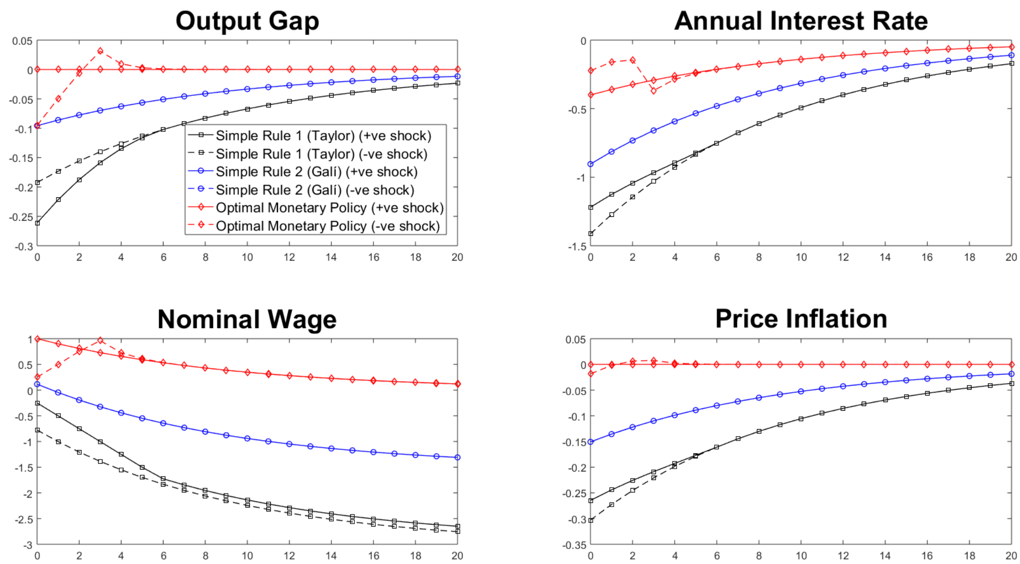



Impulse response of the nominal interest rate under optimal monetary policy facing a positive and negative technology shock. The technology shock follows an AR(1) with $\rho_a = 0.9$ and $\sigma = \{1, 3\}$.

In contrast, the optimal response, the response which minimises the welfare loss, allows the wage to fall (shown as an increase in the Figure as the impulse response has been multiplied by -1) and minimises any movement in price inflation and the output gap from a negative productivity shock. The fall in the real wage may seem counter intuitive but due to a fall in aggregate productivity, $A_t$, the marginal cost of production increases and therefore allowing wages to fall helps to offset this increased cost of production. Simple Rule 2, which is the simple rule of Galí (2015), where the central bank reacts to inflation and unemployment, outperforms the standard Taylor rule (Simple Rule 1) and does not react asymmetrically to the productivity shocks as nominal wages do not hit the occasionally binding constraint.

Figure 2.5 highlights the asymmetric response to a symmetric technology shock, as can be seen in the movements in the annual interest rate under the optimal monetary policy and the Taylor Rule.

Figure 2.5: Optimal Policy vs Simple Rule 1 (Taylor) vs Simple Rule 2 (Galí): Positive and Negative Technology Shocks




Impulse response for variables facing a positive and negative technology shocks. The technology shocks follows an AR(1) with $\rho_a = 0.9$ and $\sigma = 1$. Simple Rule 1 (Taylor) is $i_t = 0.01 + 1.5\pi_t^p + 0.125\hat{y}_t + \nu_t$ and Simple Rule 2 (Galí) is $i_t = 0.01 + 1.5\pi_t^p - 0.5\hat{u}_t$. The impulse responses for the negative technology shock have been multiplied by -1.

### 2.5.3 Simple Rule 3

The relatively good performance of the Simple Rule 2 (Galí) in Figure 2.5 and welfare losses presented in Table 2.2 and Table 2.5 in Appendix 2.D.4 compared with the Simple Rule 1 (Taylor) motivated a search to see if another simple rule could minimise the welfare loss. The coefficients on the new optimal simple rule are found by simulating the economy over demand and technology shocks and optimising these values to produce the minimise the welfare loss. The starting point of this search is a general rule of the form:

$$i_t = 0.01 + \phi_i i_{t-1} + \phi_\pi \pi_t^p + \phi_{\hat{y}} \hat{y}_t + \phi_{\pi^w} \pi_t^w + \phi_{\hat{u}} \hat{u}_t + \nu_t, \tag{2.27}$$

where $\phi$ dictates the sensitivity of the nominal interest rate to each corresponding variable. Simple Rule 3, the optimal simple rule displayed in equation (2.28) is similar to the simple rule provided in Galí (2015), however, reacts stronger to both inflation and the unemployment gap - difference in unemployment and its natural rate. Moreover, this new optimal simple rule assigns a higher weight on deviations in unemployment compared compared to inflation than Simple Rule 2 (Galí) in equation (2.26).[11] [12] Here $\rho = -\log(\beta) = 0.01$ and therefore the OSR can be given as:

$$\textbf{Simple Rule 3 (Optimal Simple Rule)}: \quad i_t = 0.01 + 4.0\pi_t^p - 2.5\hat{u}_t. \tag{2.28}$$

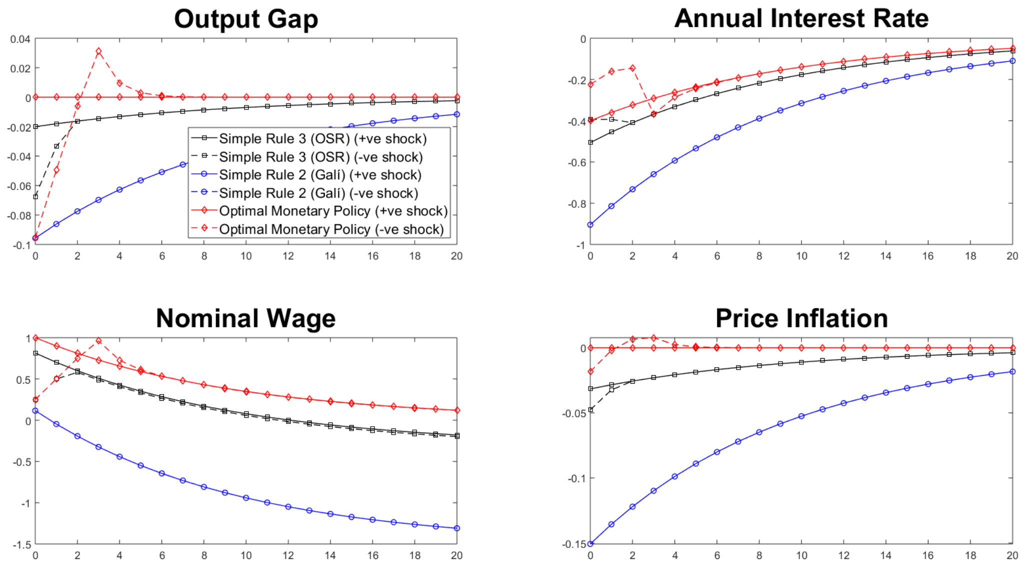
Figure 2.6 presents the impulse response from positive and negative technology shocks comparing the optimal monetary policy, optimal simple rule and the simple rule. The optimal simple rule closely follows the optimal monetary policy and is able to approximately replicate the optimal monetary policy. In comparison to Figure 2.5, which shows the outcome under a Taylor rule, output gap deviations have been significantly dampened. Paradoxically the optimal monetary policy and optimal simple rule allow for increases in nominal wage from a positive technology shock even though wage decreases are sluggish. Allowing a higher wage allows for price inflation to fall less and the labour force

[11] Debortoli et al. (2019) also find welfare improvements through imposing a higher weight on the unemployment gap or the output gap than the standard Taylor rule.

[12] Finding the optimal simple rule coefficients numerically has the drawback that additional welfare loss minimisation can still be achieved. However, the only improvement to this rule found so far requires very large coefficients on the simple rule for minimal gains.

decrease to be muted. Under the optimal simple rule the output gap also deviates less compared to the alternative simple rule, due to employment staying closer to its steady state value. Simple Rule 3, the optimal simple rule, exhibits asymmetry following symmetric technology shocks, with a similar motive to the optimal monetary policy - adjusting to the interest rate to allow for nominal wages to fall sufficiently to counteract the lower aggregate productivity from a negative technology shock. Unlike the optimal monetary policy, which also benefits from the assumption that the central bank acts under commitment, the response following a positive and negative technology shock only deviate under Simple Rule 3 (OSR) for two quarters.

Figure 2.6: Optimal Monetary Policy, Simple Rule 3 (OSR), Simple Rule 2 (Galí): Positive and Negative Technology Shocks




Impulse response for variables facing a positive and negative technology shocks. The technology shocks follows an AR(1) with $\rho_a = 0.9$ and $\sigma = 1$. Simple Rule 2 (Galí) is $i_t = 0.01 + 1.5\pi_t^p - 0.5\hat{u}_t$ and Simple Rule 3 (OSR) is $i_t = 0.01 + 4.0\pi_t^p - 2.5\hat{u}_t$. The impulse responses for the negative technology shock have been multiplied by -1.

Table 2.2 displays the welfare loss, using equation (2.25), from positive technology and demand shocks. The negative shock counterpart to this Table can be found in Appendix 2.D.4 Table 2.5. Strict targeting rules keep price inflation and wage inflation, respectively, at their steady state values and adjust the interest rate accordingly. The optimal rule provides a lower bound on the welfare loss in the Table. From Table 2.2 it is evident that Simple Rule 3 (OSR) performs well with both positive technology and demand shocks. This is in contrast to Simple Rule 1 (Taylor), which provides a relatively high welfare loss in comparison to the other rules in Table 2.2.

Table 2.2: Evaluation of MP rules following **positive** Technology and Demand Shocks

| | Optimal | Strict Targeting | | Simple Rules 1, 2, & 3 | | |
|---|---|---|---|---|---|---|
| | | Price | Wage | Taylor | Galí | OSR |
| Technology shocks | | | | | | |
| $\sigma(\pi^p)$ | 0 | 0 | 0.025 | 0.076 | 0.042 | 0.009 |
| $\sigma(\pi^w)$ | 0.1905 | 0.1905 | 0 | 0.090 | 0.054 | 0.158 |
| $\sigma(\tilde{y})$ | 0 | 0 | 0.0334 | 0.664 | 0.026 | 0.06 |
| $\mathbb{L}$ | 0 | 0 | 0.133 | 1.232 | 0.364 | 0.016 |
| Demand Shocks | | | | | | |
| $\sigma(\pi^p)$ | 0 | 0 | 0 | 0.059 | 0.111 | 0.030 |
| $\sigma(\pi^w)$ | 0 | 0 | 0 | 0.551 | 0.824 | 0.367 |
| $\sigma(\tilde{y})$ | 0 | 0 | 0 | 0.067 | 0.114 | 0.054 |
| $\mathbb{L}$ | 0 | 0 | 0 | 0.756 | 2.6501 | 0.206 |

Simple Rule 1 (Taylor) is the Taylor rule with $i_t = 0.01 + 1.5\pi_t^p + 0.125\hat{y}_t + \nu_t$ and Simple Rule 2 (Galí) is $i_t = 0.01 + 1.5\pi_t^p - 0.5\hat{u}_t$. Simple Rule 3 (OSR) is $i_t = 0.01 + 4.0\pi_t^p - 2.5\hat{u}_t$.

## 2.6 Extensions

This section houses part of the extensions of the model presented in the main text above. The main differences are i) the model is not log-linearised around a zero percent steady-state inflation rate and ii) the model internalises the occasionally binding constraint, which allows the households to maximise their utility taking into account that wages are downwardly rigid. This moves the model to a more appealing setting, enables the exploration of the optimal trend inflation rate and provides more sensible results to exogenous shocks[13]. The results in this section are presented with Simple Rule 1 (Taylor) assumed.

Internalising the downward norminal wage rigidity constraint means that the household's maximisation problem needs to be revisited. Since the only optimisation problem impacted is the wage maximisation this is the focus on the equations below. The variables in parenthesis $\lambda_t$ and $\Omega_t$ correspond to the

[13]The main illustration of this is the fall in employment from a positive demand shock seen in Figure 2.1.

lagrange multipliers, or shadow cost of the constraints. The wage setter (household or labour union for a worker of type $j$) seeks to maximise their utility flow subject to labour demand, the budget constraint and the downward nominal wage rigidity.

$$\max_{W_t(j)} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t(j)^{1+\eta}}{1+\eta} \right] \right\} Z_t$$

subject to:

$$N_t(j) = \left( \frac{W_t(j)}{W_t} \right)^{-\epsilon_w} N_t$$

$$(\lambda_t) \;\; P_t C_t + E_t[Q_t D_{t+1}] \leq D_t + W_t(j) N_t(j) - T_t$$

$$(\Omega_t) \;\; W_t(j) \geq \gamma W_{t-1}(j)$$

The solution to this problem, combining the previous first order conditions found in Section 2.4, can be seen below. It is convenient when simulating the model to represent this condition in terms of nominal wage inflation $\Pi_t^w = W_t / W_{t-1}$ and the real wage rather than solely nominal wages and nominal wage changes.

$$\Pi_t^w \Omega_t = (\epsilon_w - 1) \frac{W_t}{P_t} Z_t C_t^{-\sigma} N_t - \epsilon_w N_t^{1+\eta} Z_t + \beta E_t[\Omega_{t+1} \Pi_{t+1}^w]$$

$$\text{Complementary slackness: } \Omega_t(\Pi_t^w - \gamma) = 0$$

Non-negativity constraints can be shown to be:

$$\Omega_t \geq 0$$

$$\Pi_t^w - \gamma \geq 0$$

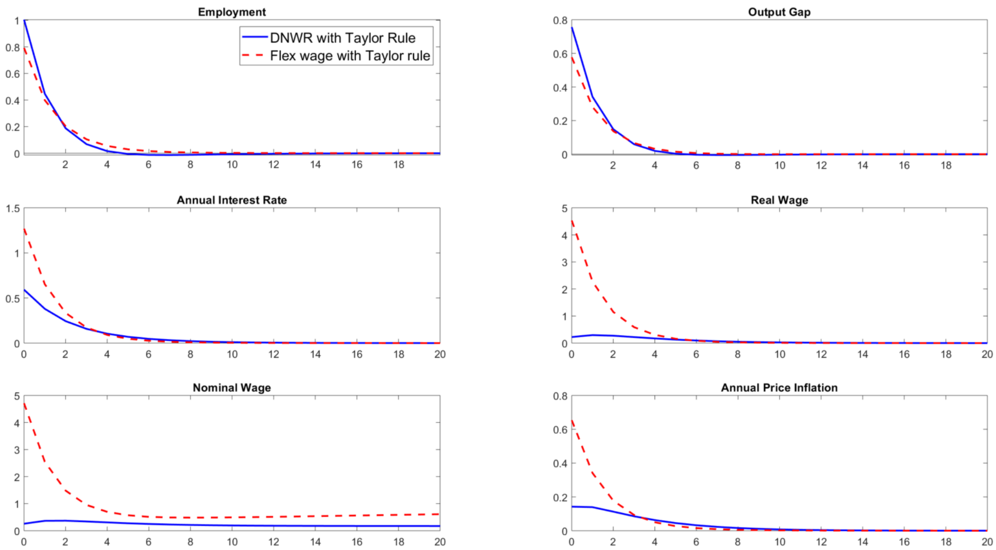
Therefore when the DNWR constraint does not bind the associated lagrange multiplier will equal zero, $\Omega_t = 0$, and we are back to the flexible wage schedule where the real wage is a markup over the marginal rate of substitution between consumption and labour. A detailed derivation of this problem can be found in Appendix 2.B.1.

### 2.6.1 Extension Results

Figure 2.7 displays the impulse response from a one standard deviation positive demand shock, comparing the response of an economy with downward nominal wage rigidities and flexible wages under a Taylor rule with $\phi_\pi = 1.5$ and $\phi_{\hat{y}} = 0.125$ . For this figure no positive trend inflation is assumed, which allows for a direct comparison to Figure 2.1. Figure 2.7 can be used as a comparison to Figure 2.1 which had exogenous DNWR and a zero inflation steady state. Internalising the occasionally binding constraint means that the households now barely increase their nominal wage, which is in contract to the increase of of 2% witnessed previously. Therefore the existence of the constraint causes wage increases to be muted, creating an endogenous rigidity when increasing wages. Figure 2.7 also shows a more sensible response of the labour market to a positive demand shock since the labour force response is muted (not shown) due to the subdued wage, whilst employment still rises, hence unemployment falls on impact of the shock and output increases similarly with the flexible wage model.

Wage restraint, the phenomenon displayed in Figure 2.7, has also been found empirically in Elsby (2009) and motivated by a stylised model of workers resistant to nominal wage cuts. Instead in this paper workers understand that wages are downwardly rigid and therefore limit their demand for higher wages as unemployment will arise when the DNWR constraint binds. This mechanism is due to the equilibrium wage being artificially high if the DNWR constraint binds, which causes labour supply to remain high, the wage to not adjust downwards and therefore the firm cannot afford to hire all available workers and unemployment arises. This form of wage restraint is similar to the benchmark case of including Calvo wage rigidity into a model with downward nominal wage rigidities as seen in Appendix 2.D.2 and specifically Figure 2.10.

Figure 2.7: Taylor Rule - Positive Demand Shock




Impulse response for variables facing a positive demand shock using projection methods. The demand shock follows an AR(1) with $\rho_z = 0.5$ and $\sigma = 1$. The Taylor rule is $i_t = 0.01 + 1.5\pi_t^p + 0.125\hat{y}_t + \nu_t$. Wages are assumed to be either flexible or suffer from downward nominal wage rigidities that are internalised by the labour union. The DNWR parameter is $\gamma = 0.9975$.

### 2.6.2 Optimal Trend Inflation and Taylor Rule

Unlike in Section 2.5, I now introduce welfare as the present discounted value of the flow utility of a representative agent, which will be used to assess the optimal steady state inflation rate and Taylor rule coefficients under DNWR. The previous measurement of optimality, which used a second order approximation around a zero inflation steady state, cannot be used to assess positive trend inflation in that form. Moreover, this measure should be able to handle the highly non-linear nature of the occasionally binding constraint and therefore provide a more accurate measure of welfare. The optimal inflation and Taylor rule coefficients will disciplined by choosing the values which maximise the present discounted value of the flow utility of a representative agent seen in equation (2.29).

$$V_t = U_t(C_t, N_t) + \beta E_t V_{t+1} \tag{2.29}$$

In contrast, the Ramsey Planner, which provides the optimal solution to this model, would maximise the households welfare taking into account the first order conditions from our non-linear model seen in Appendix 2.B. For now I focus on a standard Taylor rule that focuses on deviations in inflation and output from their steady state levels, outlined below.

$$\frac{R_t}{R} = \left(\frac{\Pi_t}{\Pi}\right)^{\phi_\pi} \left(\frac{Y_t}{Y}\right)^{\phi_y} \tag{2.30}$$

Using a grid search method[14] over $\{\Pi, \phi_\pi, \phi_y\}$ the economy is simulated for 300,000 periods of shocks[15] and the mean value of the households welfare is calculated $V_t$ and transformed into its consumption equivalent amount in comparison to a zero trend inflation steady state and Taylor rule coefficients $\{\phi_\pi = 1.5, \phi_y = 0.25\}$. The finding is suggestive of 'greasing the wheels' and is within a sensible range of what others have found. In this model the optimal trend inflation rate is dependent on shocks and is between $0\%$ to $1.25\%$ with $\phi_\pi \in [2, 5]$ and $\phi_y \in [0, 0.25]$. The higher trend inflation helps to deflate the

[14]Future work which will provide a robustness check will use a non-linear solver such as the Newton-Raphson Method to determine the optimal $\{\Pi, \phi_\pi, \phi_y\}$. Moreover, technological growth will need to be added to the model as well as this plays an important role in finding the optimal level of trend inflation.

[15]Technology and Demand shocks are simulated separately and the welfare values from the simulation are then compared.

Downward Nominal Wage Rigidity with the higher-than-typical reaction to inflation likely being needed to assure determinacy of the model. Other papers, such as Kim and Ruge-Murcia (2009) who use asymmetric wage adjustment costs, find that the optimal trend inflation is 0.35%. However, using asymmetric wage adjustment costs and heterogeneous agents Fagan and Messina (2009) find a much larger range of optimal trend inflation, 0% to 5% depending on calibration used. Tables 2.3 and 2.4 summarise the optimal calibration exercise:

Welfare Analysis: Optimal calibration of Taylor Rule

Table 2.3: Demand shocks

| $\Pi$ | $\phi_\pi$ | $\phi_y$ | Consumption Equivalence |
|---|---|---|---|
| 0.25% | 2 | 0 | 0.64% |
| 0% | 2 | 0 | 0.63% |
| 0% | 1.5 | 0 | 0.62% |

Table 2.4: Technology shocks

| $\Pi$ | $\phi_\pi$ | $\phi_y$ | Consumption Equivalence |
|---|---|---|---|
| 1.00% | 5 | 0.25 | 0.71% |
| 1.00% | 4.5 | 0.25 | 0.69% |
| 1.25% | 5 | 0.25 | 0.67% |

Consumption equivalence is calculated from differences in the mean of discounted household flow utility $(V_t = U_t(C_t, N_t) + \beta E_t V_{t+1})$ after 300,000 periods of uniformly distributed shocks. The welfare compared is from a Taylor rule with $\{\Pi = 0\%, \phi_\pi = 1.5, \phi_y = 0.25\}$. DNWR parameter $\gamma = 0.9975$ allowing for an annual decrease of nominal wages by $1\%$.

Further work will be completed to extend the model, mimicking the work done above to find an optimal simple rule in this set-up - providing a further contribution to the literature.

## 2.7 Conclusion

In conclusion adding an occasionally binding constraint into the New Keynesian model such as a downward nominal wage rigidity seen throughout this paper can distort the standard results of a New Keynesian model with flexible or Calvo wages. This work has found that a DNWR constraint can cause boom-bust cycles from a positive demand shock if the agents within the model are not affected by the constraint until they receive a shock that will cause them to reach the constraint. The optimal monetary policy in this setup is asymmetric and there are gains in welfare to be made over the Taylor rule by finding a new optimal simple rule - one that reacts stronger to changes in unemployment. Taking the constraint seriously and embedding it into the households problem and solving the non-linear model with positive trend inflation leads to support for 'greasing the wheels', allowing positive inflation in the steady state to deflate real wage changes, which leads to welfare gains. Once the constraint is fully internalised, such that the wage setters understand its existence even during periods that they are unconstrained, wage increases become dampened even though they are flexible upwards, a finding also shown in Elsby (2009) and Wolf (2018). The main contribution of this work comes from embedding the DNWR constraint from Schmitt-Grohé and Uribe (2016) into a New Keynesian model, finding wage restraint and a new optimal simple rule whilst providing more support to positive trend inflation.

## 2.A Derivation of Optimal Monetary Policy under DNWR

### 2.A.1 A model with staggered wage and price setting

This section outlines the central bank's problem under a more general New Keynesian model, where wages as well as prices are now Calvo rigid. Under the assumption of staggered wage setting, workers specialised in any given labour type can reset their nominal wage only with probability $1 - \theta_w$, independently of the time elapsed since their last adjustment. The labour union seeks to maximise the labour type's utility subject to the sequence of labor demand schedules, outlined in the main text. In this problem the labour union must take into account the future probability of setting their wage. Given the assumed wage setting strcture, the evolution of the aggregate wage index is given by

$$W_t = \left(\theta_w W_{t-1}^{1-\epsilon_w} + (1-\theta_w)(W_t^*)^{1-\epsilon_w}\right)^{\frac{1}{1-\epsilon_w}}$$

Log-linearising around the zero wage inflation steady state yields

$$w_t = \theta_w w_{t-1} + (1-\theta_w) w_t^*$$

Combining the log-linearised wage setting rule from the labour unions problem discussed above[16] and the previous equation, and letting $\pi_t^w = w_t - w_{t-1}$ the wage inflation equation is

$$\pi_t^w = \beta E_t\{\pi_{t+1}^w\} - \lambda_w \hat{\mu}_t^w$$

where $\lambda_w \equiv \frac{(1-\theta_w)(1-\beta\theta_w)}{\theta_w(1+\epsilon_w\varphi)}$. The wage markup is $\hat{\mu}_t^w = \omega_t - mrs_t - \mu^w$.[17] Therefore it can be shown that

[16]The problem is standard and therefore not derived algebraically. For the full problem outlined in detail see Galí (2015)

[17]The wage markup can also be written in terms of the unemployment gap, such that $\varphi \hat{u}t = \hat{\mu}_t^w$, where $\hat{u}_t \equiv u_t - u^n$.

$$\pi_t^w = \beta E_t\{\pi_{t+1}^w\} + \varkappa_w \tilde{y}_t - \lambda_w \tilde{\omega}_t$$

where $\varkappa_w \equiv \lambda_w(\sigma + \frac{\varphi}{1-\alpha})$ . By defining the real wage gap as the difference between the real wage and the natural real wage (real wage with no rigidities), $\tilde{\omega}_t \equiv \omega_t - \omega_t^n$, hence

$$\tilde{\omega}_t \equiv \tilde{\omega}_{t-1} + \pi_t^w - \pi_t^p - \Delta\omega_t^n.$$

In the main text it is assumed that wages are flexible, $\theta_w = 0$ which therefore means that $\lambda_w \rightarrow \infty$, however with Calvo wages the wage inflation equation takes the place of the real wage equation.

### 2.A.2 The optimal monetary policy problem

The central bank, under optimal policy with commitment, seeks to minimise equation (2.31)

$$\mathbb{W} = \frac{1}{2} E_0 \sum_{t=0}^{\infty} \beta^t \Big[ \Big( \sigma + \frac{\varphi + \alpha}{1-\alpha} \Big) \tilde{y}_t^2 + \frac{\epsilon_p}{\lambda_p} (\pi_t^p)^2 + \frac{\epsilon_w (1-\alpha)}{\lambda_w} (\pi_t^w)^2 \Big] \tag{2.31}$$

subject to equation (2.32), (2.33) and, (2.34) for $t = 0, 1, 2, \ldots$ :

$$\pi_t^p = \beta E_t\{\pi_{t+1}^p\} + \varkappa_p \tilde{y}_t + \lambda_p \tilde{\omega}_t \tag{2.32}$$

$$\pi_t^w = \max\{\beta E_t\{\pi_{t+1}^w\} + \varkappa_w \tilde{y}_t - \lambda_w \tilde{\omega}_t \ , \ \log(\gamma)\} \tag{2.33}$$

$$\tilde{\omega}_t \equiv \tilde{\omega}_{t-1} + \pi_t^w - \pi_t^p - \Delta\omega_t^n \tag{2.34}$$

With the Lagrange multipliers $\zeta_{1,t}$, $\zeta_{2,t}$, $\zeta_{3,t}$, respectively. The first order conditions below must hold when the DNWR constraint, $\pi^w \geq \log(\gamma)$, is not binding:

$$\left(\sigma + \frac{\varphi + \alpha}{1 - \alpha}\right)\tilde{y}_t + \varkappa_p \zeta_{1,t} + \varkappa_\omega \zeta_{2,t} = 0 \tag{2.35}$$

$$\frac{\epsilon_p}{\lambda_p}\pi_t^p - \Delta\zeta_{1,t} + \zeta_{3,t} = 0 \tag{2.36}$$

$$\frac{\epsilon_w(1-\alpha)}{\lambda_w}\pi_t^w - \Delta\zeta_{2,t} - \zeta_{3,t} = 0 \tag{2.37}$$

$$\lambda_p \zeta_{1,t} - \lambda_\omega \zeta_{2,t} + \zeta_{3,t} - \beta E_t\{\zeta_{3,t+1}\} = 0 \tag{2.38}$$

and slackness conditions

$$\zeta_{2,t} \geq 0; \;\; \pi_t^w \geq log(\gamma); \;\; \zeta_{2,t}(\pi_t^w - log(\gamma)) = 0$$

as well as initial conditions $\zeta_{1,-1} = \zeta_{2,-1} = 0$ and an initial condition for $\tilde{\omega}_{-1}$. Whenever the DNWR constraint is not binding the model is standard and differentiable. However, when the DNWR constraint binds equation (2.37) does not hold, which is communicated to the Levenberg-Marquardt mixed complementarity problem (LMMCP) solver through inserting $\pi_t^w \geq log(\gamma)$ under the mixed complementary problem (mcp) tag for equation (2.37) into the Dynare mod file.[18] Thereby I avoid using the complementary slackness condition that would give rise to a singular Jacobian. This procedure is equivalent to solving for the optimal monetary policy with commitment for the non-negativity constraint of the zero lower bound in Chapter 5 of Galí (2015).

## 2.B Derivation of the theoretical model

Appendix 2.B houses the equilibrium equations for the full non-linear New Keynesian model with Downward Nominal Wage Rigidities. In the latter part of this section the steady state of this model is outlined. Since the nominal wage is not constrained in the steady-state the lagrange multiplier associated with the

[18]The LMMCP solver is used instead of Occbin by Guerrieri and Iacoviello (2015) to solve the optimal monetary policy.

downward nominal wage rigidity constraint is zero, such that $\Omega_t = 0$, and the steady state equations are similar to those found in most medium-scale New Keynesian models.

### 2.B.1 Detailed derivation of Internalised DNWR

The household is the monopoly supplier of labour within this model and therefore is the wage setter. One can think that the household forms a trade union per differentiated skill $j$ and sets wages to maximise utility whilst adhering to the demand for differentiated labour, their budget constraint and the Downward Nominal Wage Rigidity.

$$\max_{W_t(j)} E_0 \left\{ \sum_{t=0}^{\infty} \beta^t \left[ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{N_t(j)^{1+\eta}}{1+\eta} \right] \right\} Z_t$$

subject to:

$$N_t(j) = \left( \frac{W_t(j)}{W_t} \right)^{-\epsilon_w} N_t$$

$$(\lambda_t) \;\; P_t C_t + E_t[Q_t D_{t+1}] \leq D_t + W_t(j) N_t(j) - T_t$$

$$(\Omega_t) \;\; W_t(j) \geq \gamma W_{t-1}(j)$$

Reformulating this problem as a Lagrange, substituting the demand for labour type $j$ given by $N_t(j)$ and $\Pi_t^W(j) = \frac{W_t(j)}{W_{t-1}(j)}$:

$$\mathcal{L} = \left[ \frac{C_t^{1-\sigma}}{1-\sigma} - \frac{\left( \frac{W_t(j)}{W_t} \right)^{-\epsilon_w(1+\eta)} N_t^{1+\eta}}{1+\eta} \right] Z_t$$
$$- \lambda_t (D_t + W_t(j) \left( \frac{W_t(j)}{W_t} \right)^{-\epsilon_w} N_t - T_t - P_t C_t - E_t[Q_t D_{t+1}]) + \Omega_t (\Pi_t^W(j) - \gamma)$$

$$\frac{\partial \mathcal{L}}{\partial W_t(j)} = \frac{\epsilon_w(1+\eta)}{W_t(1+\eta)}\left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w(1+\eta)-1} N_t^{1+\eta} Z_t + \lambda_t(1-\epsilon_w)\left(\frac{W_t(j)}{W_t}\right)^{-\epsilon_w} N_t$$
$$+\,\Omega_t \frac{1}{W_{t-1}(j)} - \beta\Omega_{t+1}\frac{W_{t+1}(j)}{W_t(j)^2} = 0$$

Using $\lambda_t = \frac{1}{P_t} C_t^{-\sigma} Z_t$ from the Household's FOC of consumption and the solution of each labour type is identical, therefore $W_t(j) = W_t \Rightarrow$

$$\frac{\epsilon_w}{W_t} N_t^{1+\eta} Z_t + \frac{1}{P_t} C_t^{-\sigma} Z_t(1-\epsilon_w) N_t + \Omega_t \frac{1}{W_{t-1}} - \beta\Omega_{t+1}\frac{W_{t+1}}{W_t^2} = 0$$

$$\epsilon_w N_t^{1+\eta} Z_t + \frac{W_t}{P_t} C_t^{-\sigma} Z_t(1-\epsilon_w) N_t + \Omega_t \frac{W_t}{W_{t-1}} - \beta\Omega_{t+1}\frac{W_{t+1}}{W_t} = 0$$

$$\epsilon_w N_t^{1+\eta} Z_t + \frac{W_t}{P_t} C_t^{-\sigma} Z_t(1-\epsilon_w) N_t + \Omega_t \Pi_t^w - \beta\Omega_{t+1}\Pi_{t+1}^w = 0$$

Hence, we arrive at the solution presented in the main body of the paper. When the DNWR constraint does not being $\Omega_t = 0$ and the wage is set flexibly as a markup over the marginal rate of substitution.

$$\Pi_t^w \Omega_t = (\epsilon_w - 1)\frac{W_t}{P_t} Z_t C_t^{-\sigma} N_t - \epsilon_w N_t^{1+\eta} Z_t + \beta\Omega_{t+1}\Pi_{t+1}^w$$

Additionally:

Complementary slackness:

$$\Omega_t(\Pi_t^w - \gamma) = 0$$

Feasibility (non-negativity constraints):

$$\Pi_t^w - \gamma \geq 0$$

$$\Omega_t \geq 0$$

must hold.

### 2.B.2 Equations of the full model

The equations of the model outlined in Section 2.6 where the labour union internalises the downward nominal wage rigidity constraint is outlined below:

$$Q_t = \frac{\beta \left(\frac{C_{t+1}}{C_t}\right)^{(-\sigma)} \frac{Z_{t+1}}{Z_t}}{\Pi_{t+1}} \tag{2.39}$$

$$R^n{}_t = \frac{1}{Q_t} \tag{2.40}$$

$$Y_t = A_t \left(\frac{N_t}{S_t}\right)^{1-\alpha} \tag{2.41}$$

$$R^n{}_t = \Pi_{t+1}\, R^r{}_t \tag{2.42}$$

$$R^n{}_t = \frac{\Pi_{SS}}{\beta} \left(\frac{\Pi_t}{\Pi_{SS}}\right)^{\phi_\pi} \left(\frac{Y_t}{(\bar{Y})}\right)^{\phi_y} e^{\mu_t} \tag{2.43}$$

$$C_t = Y_t \tag{2.44}$$

$$log\left(A_t\right) = \rho_a\, log\left(A_{t-1}\right) + \varepsilon_{at} \tag{2.45}$$

$$log\left(Z_t\right) = \rho_z\, log\left(Z_{t-1}\right) - \varepsilon_{zt} \tag{2.46}$$

$$MC_t = \frac{\frac{W}{P\,t}}{S_t\, \frac{Y_t\,(1-\alpha)}{N_t}} \tag{2.47}$$

$$1 = \theta\, \Pi_t{}^{\epsilon-1} + (1-\theta)\ \Pi^*{}_t{}^{1-\epsilon} \tag{2.48}$$

$$S_t = (1-\theta)\; {\Pi^*}_t^{\frac{(-\epsilon)}{1-\alpha}} + \theta\, {\Pi_t}^{\frac{\epsilon}{1-\alpha}}\, S_{t-1} \tag{2.49}$$

$$ {\Pi^*}_t^{1+\epsilon\, \frac{\alpha}{1-\alpha}} = \frac{\epsilon\, \frac{x_{1t}}{x_{2t}}}{\epsilon - 1} \tag{2.50}$$

$$x_{1t} = MC_t\, Y_t\, Z_t\, {C_t}^{(-\sigma)} + \beta\, \theta\, {\Pi_{t+1}}^{\epsilon + \frac{\alpha\, \epsilon}{1-\alpha}}\, x_{1t+1} \tag{2.51}$$

$$x_{2t} = Y_t\, Z_t\, {C_t}^{(-\sigma)} + \beta\, \theta\, {\Pi_{t+1}}^{\epsilon - 1}\, x_{2t+1} \tag{2.52}$$

$$\Pi_t^w \Omega_t = (\epsilon_w - 1) \frac{W_t}{P_t} C_t^{-\sigma} N_t - \epsilon_w N_t^{1+\eta} + \beta \Omega_{t+1} \Pi_{t+1}^w \tag{2.53}$$

$$\frac{W_t}{P_t} = \frac{\Pi_t^w}{\Pi_t} \frac{W_{t-1}}{P_{t-1}} \tag{2.54}$$

$$\mu_t = \rho_a\, \mu_{t-1} + \epsilon_{\mu t} \tag{2.55}$$

$$V_t = Z_t\, \left( log\,(C_t) - \frac{{N_t}^{1+\varphi}}{1+\varphi} \right) + \beta\, V_{t+1} \tag{2.56}$$

$$\Omega_t (\Pi_t^w - \gamma) = 0 \tag{2.57}$$

$$\Pi_t^w - \gamma \geq 0 \tag{2.58}$$

$$\Omega_t \geq 0 \tag{2.59}$$

### 2.B.3 Steady State

State state of the model with internalised DNWR is outlined below. The steady state is not affected by the DNWR constraint.

$$A = 1$$

$$\mu = 0$$

$$Z = 1$$

$$\Pi^* = \left[\frac{1 - \theta\Pi^{\epsilon-1}}{1-\theta}\right]^{\frac{1}{1-\epsilon}}$$

$$S = \frac{(1-\theta)\Pi^{\frac{-\epsilon}{1-\alpha}}}{1 - \theta\Pi^{\frac{\epsilon}{1-\alpha}}}$$

$$MC = \frac{\epsilon - 1}{\epsilon}\Pi^{*\frac{1+\alpha\epsilon}{1-\alpha}} 1 - \beta\theta\Pi^{\epsilon + \frac{\alpha\epsilon}{1-\alpha}} \frac{1}{1 - \beta\theta\Pi^{\epsilon-1}}$$

$$Q = \frac{\beta}{\Pi}$$

$$R = \frac{1}{Q}$$

$$r = \frac{R}{\Pi}$$

$$N = \left[MC(1-\alpha)\frac{\epsilon_w - 1}{\epsilon_w} S^{\sigma - \alpha\sigma + \alpha}\right]^{\frac{1}{(1-\sigma)\alpha + \varphi + \sigma}}$$

$$C = A\left(\frac{N}{S}\right)^{1-\alpha}$$

$$Y = C$$

$$\frac{W}{P} = w = \frac{MC \cdot S \cdot Y(1-\alpha)}{N}$$

$$x_1 = \frac{C^{-\sigma}Y \cdot MC}{1 - \beta\theta\Pi^{\frac{\epsilon+\alpha\epsilon}{1-\alpha}}}$$

$$x_2 = \frac{C^{-\sigma}Y}{1 - \beta\theta\Pi^{\epsilon-1}}$$

$$\Omega = 0$$

## 2.C Computational technique

Two computation techniques have been used in this project. Firstly, Occbin by Guerrieri and Iacoviello (2015) is used as a first attempt to analyse the effect of DNWR on a standard New Keynesian Model. Latterly, Smolyak Projection Method by Smolyak (1963) is used to provide more realistic analysis as it allows the agents to understand that the DNWR constraint exists. Below I outline both of these techniques used.

### 2.C.1 Occbin

Most of the model simulations, impulse response functions and welfare losses were calculated using Dynare[19], an extension to Matlab used for DSGE models. Dynare cannot typically be used when there is an occasionally binding constraint such as the DNWR, however, with help of the Occbin toolbox seen in Guerrieri and Iacoviello (2015) it is possible. Occbin uses first order pertubation but allows the solution to be highly non-linear. One disadvantage is that all agents within the model have no prior knowledge of the existence of the occasionally binding constraint, and therefore this technique does not capture precautionary behaviour.

At the start of the period the model is at the steady state and then when the households' wish to lower the nominal wage after the monetary policy shock, the model switches to that of the binding constraint and the wage reduction is forced to be sluggish. Appendix 2.C.2 highlights a projection method, which provides a global solution, used to interalise this occasionally binding constraint and will form the

### 2.C.2 Smolyak Approximation

Section 2.6 displays the non-linear model with positive trend inflation and an occasionally binding constraint that the households maximise over. The model is solved using the Smolyak collocation method laid out in Malin et al. (2011) and implemented for a New Keynesian model with a Zero-Lower-Bound constraint in Fernández-Villaverde et al. (2015). My solution technique closely follows the

[19]The dynare files used were adapted from those created by Dr Johannes Pfeifer to replicate Galí (2015) and provided freely for use, as of which I am extremely grateful.

exercise provided by Fernández-Villaverde et al. (2015). Smolyak collocation allows for more state variables than other common projection methods as the number of terms of the approximating polynomial and grid points do not grow exponentially and therefore do not suffer as much as other techniques from the curse of dimensionality. One prominent example is Fernández-Villaverde and Levintal (2018), which uses 12 state variables and still retains accuracy and speed of computation.

Smolyak's algorithm introduced in Smolyak (1963) is a numerical technique using a sparse grid to efficiently solve multi-dimensional hypercubes. The technique ordered and selected the solution to a tensor-product rule importance of finding the quality of approximation to the problem. Smolyak's algorithm was then adapted by Krueger and Kubler (2004) to be used in an economic setting.

Following the steps found in the technical appendix of Fernández-Villaverde et al. (2015) I start by defining a state vector:

$$\mathbb{S}_t = (S_{t-1}, A_t, Z_t, w_{t-1})$$

With the exogenous states in logs:

$$\hat{\mathbb{S}}_t = (S_{t-1}, \log(A_t), \log(Z_t), w_{t-1})$$

The equilibrium functions $f = (f^1, f^2, f^3, f^4)$ characterize the dynamics of the model:

$$\begin{aligned}
\log(C_t) &= f^1(\hat{\mathbb{S}}_t) \\
\log(\Pi_t) &= f^2(\hat{\mathbb{S}}_t) \\
\log(x_{1t}) &= f^3(\hat{\mathbb{S}}_t) \\
\log(\Pi_t^w) &= f^4(\hat{\mathbb{S}}_t) \\
\Omega_t &= f^5(\hat{\mathbb{S}}_t)
\end{aligned}$$

To define the hypercube (grid points) we then choose bounds on the state variables around their steady state levels. The bounds for the exogenous state variables are determined by their unconditional standard deviation.

Then to solve for $f$ I use a time-iteration procedure:

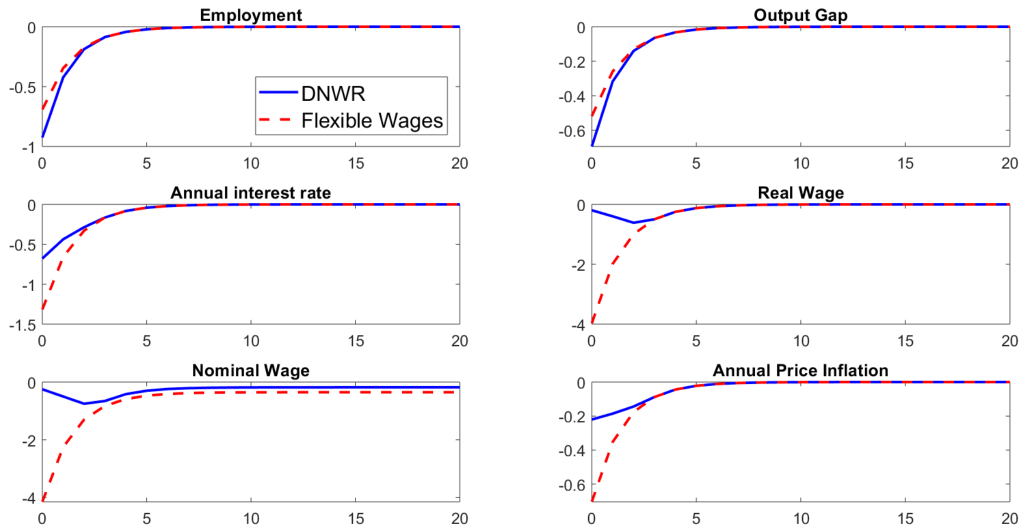
- Guess on : $\{\Pi_t,\ \Pi_t^w\}$
- Update state to obtain: $= \{S_t,\ \log(A_{t+1}),\ \log(Z_{t+1}),\ w_t\}$
- Using the state today and weights from a monomial rule calculate expectations of time $t+1$ variables in the model.
- Check whether initial guess was correct by using the euler equation, real wage equation and complementary slackness for the occasionally binding constraint - iterate over guess if not correct.
- With the time $t$ equilibrium found at each of the collocation points, check if they differ from the $t+1$ values. If they are similar up to a tolerance level then stop.

## 2.D Additional figures or tables

### 2.D.1 Negative Demand Shock- amplification

Figure 2.8 outlines the response to a negative demand shock when the economy suffers from DNWR in comparison to the economy under flexible wages. Due to the fall in demand, wages fall, however as wages hit the DNWR constraint they are artificially higher. These high wages are a cost to the firm and therefore employment falls more relative to when wages are flexible. In this labour-driven economy the fall in the output gap is amplified when wages are downwardly rigid compared to their flexible counterpart.

Figure 2.8: Negative demand shock with DNWR

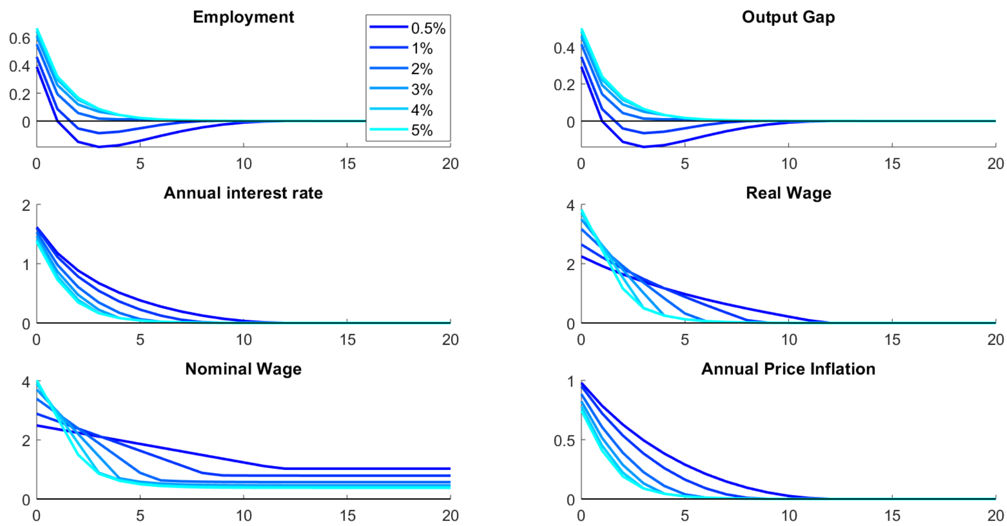



Impulse response for variables facing a negative demand shock. The demand shock follows an AR(1) with $\rho_z = 0.5$ and $\sigma = 1$. The Taylor rule is $i_t = 0.01 + 1.5\pi_t^p + 0.125\hat{y}_t + \nu_t$.

### 2.D.2 Varying the degree of DNWR

As outlined in Figure 2.9, the degree to which wages are allowed to fall is crucial in driving the boom-bust cycle from a positive demand shock, the amplification from a negative demand shock and also the motivation for a higher trend inflation in the extended model. In Figure 2.9, the degree of DNWR is varied such that wages are allowed to fall from 0.5% a year to 5%. Currently the baseline model allows wages to fall by 1% a year, which corresponds to $\gamma = 0.9975$, a figure within the bounds of the estimated DNWR for various countries by Schmitt-Grohé and Uribe (2016). In contrast wages are allowed to fall by 4% per year , $\gamma = 0.99$, in the baseline case of Schmitt-Grohé and Uribe (2016). As can be seen in the Figure 2.9, allowing for nominal wages to fall 2% per year removes the boom-bust result highlighted in the main text of the paper. However, in this case the outcome of the economy is still distorted as the downward nominal wage rigidity constraint is still binding. The binding of the constraint, even if the boom-bust cycle is not apparent, still causes a sharper fall in employment and the output gap compared to a case where the wage is allowed to fall by more (e.g. 5% per year).

Figure 2.9: Varying degrees of Downward Nominal Wage Rigidity

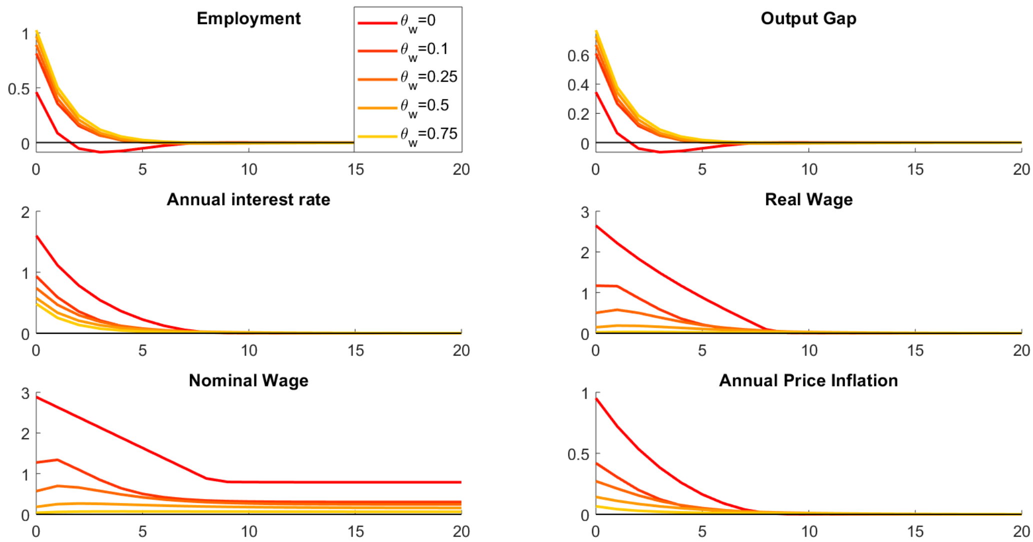



Impulse response for variables facing a positive demand shock. The demand shock follows an AR(1) with $\rho_z = 0.5$ and $\sigma = 1$. The degree to which wages are allowed o fall per year are varied from 0.5% to 5%. The Taylor rule is $i_t = 0.01 + 1.5\pi_t^p + 0.125\hat{y}_t + \nu_t$.

### 2.D.3 Varying the degree of wage stickiness

The previous models abstracted away from wage stickiness, the phenomenon that wages may be nominally rigid as wage changes are often infrequent. I follow the standard procedure from adding Calvo wage rigidity outlined in Galí (2015) to the New Keynesian model with DNWR and vary the degree to which these wages are rigid. In the benchmark case, $\theta_w = 0.75$, and as nominal wages are muted following a positive demand shock outlined in Figure 2.10 the DNWR does not impact the economy. In fact, it is only when wages are fully flexible upwards do we see the boom-bust cycle returning to the economy due to a positive demand shock. Adding in an extra rigidity, Calvo wages, to any degree that resembles the parameter values commonly used in the literature removes the boom-bust result. Furthermore, even when $\theta = 0.1$, and hence wages are allowed to adjust regularly but not fully flexibly, we see the downward nominal wage rigidity constraint binding only for a few periods and the outcome to the output gap is very similar to when $\theta = 0.75$. Therefore, the introduction if Calvo wage rigidity into the model, somewhat dominates the downward nominal wage rigidity.

Figure 2.10: Varying degrees of Calvo Wage Rigidity




Impulse response for variables facing a positive demand shock. The demand shock follows an AR(1) with $\rho_z = 0.5$ and $\sigma = 1$. The degree of wage rigidity is varied, $\theta_w$ goes from 0, full wage flexibility, to $\theta_w = 0.75$, the benchmark case in the New Keynesian model with wage rigidity. The Taylor rule is $i_t = 0.01 + 1.5\pi_t^p + 0.125\hat{y}_t + \nu_t$.

The culmination of Figure 2.9 and 2.10 highlight the important prerequisites needed to drive the results laid out in the main body of the paper. Under the current calibration and adapting the simple New Keynesian model flexible wages and a high enough degree of downward nominl wage rigidity is needed for boom-bust cycles to be present following a positive demand shock. However, even with relatively accommodate wage deflation asymmetric response of the economy to symmetric shocks would still persist. Furthermore, Figure 2.10, when wages are sufficiently sticky, closely resembles the response of the economy in the extended model where households internalise the downward nominal wage rigidity constraint and maximise their utility and wage setting decision taking it into account, as seen in Figure 2.7.

### 2.D.4 Welfare loss for negative shock

With an occasionally binding constraint the response of a central bank following an interest rate rule or an optimal monetary policy can be asymmetric. Therefore it is important to look at welfare loss for different interest rate rules under positive and negative shocks separately. Hence, table 2.5 displays the welfare loss from negative shocks to provide a comparison with table 2.2 found in the main body of the paper.

As with a positive demand shock the optimal policy is able to change the interest rate such that no welfare is lost from the shock. The optimal simple rule in this scenario also does well, for negative technology and demand shocks. Strict price targeting performs well under demand shocks however this regime performs poorly under technology shocks relative to the optimal monetary policy or optimal simple rule.

Table 2.5: Evaluation of MP rules following **negative** Technology and Demand Shocks

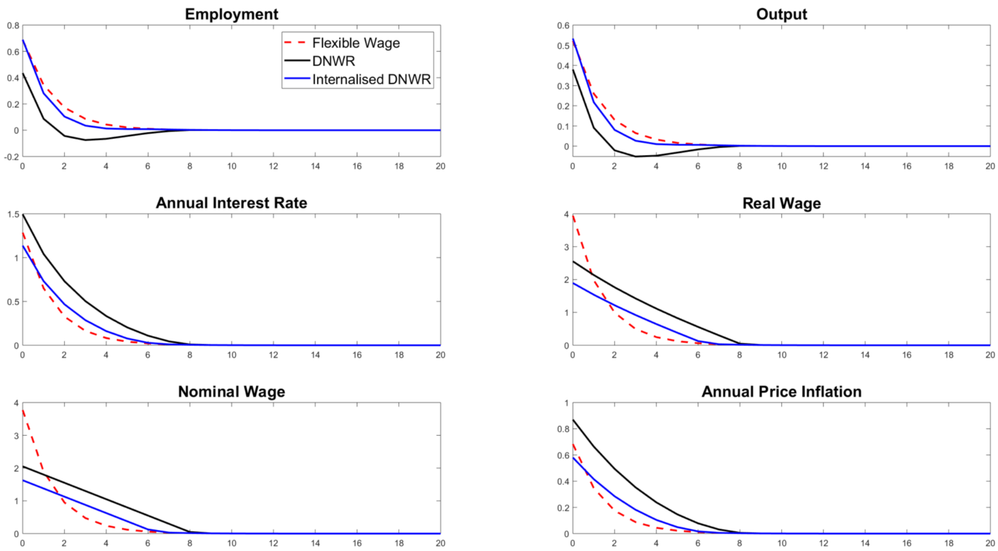
| | Optimal | Strict Targeting | | Simple Rules 1, 2, & 3 | | |
|---|---|---|---|---|---|---|
| | | Price | Wage | Taylor | Galí | OSR |
| Technology shocks | | | | | | |
| $\sigma(\pi^p)$ | 0.004 | 0 | 0.0134 | 0.063 | 0.031 | 0.008 |
| $\sigma(\pi^w)$ | 0.105 | 0.047 | 0 | 0.089 | 0.036 | 0.042 |
| $\sigma(\tilde{y})$ | 0.021 | 0.254 | 0.020 | 0.040 | 0.020 | 0.008 |
| $\mathbb{L}$ | 0.005 | 0.840 | 0.133 | 1.469 | 0.364 | 0.024 |
| Demand Shocks | | | | | | |
| $\sigma(\pi^p)$ | 0 | 0 | 0 | 0.014 | 0.015 | 0.010 |
| $\sigma(\pi^w)$ | 0 | 0 | 0 | 0.096 | 0.099 | 0.093 |
| $\sigma(\tilde{y})$ | 0 | 0 | 0 | 0.138 | 0.083 | 0.029 |
| $\mathbb{L}$ | 0 | 0 | 0 | 0.120 | 0.075 | 0.026 |

Simple Rule 1 (Taylor) is the Taylor rule with $i_t = 0.01 + 1.5\pi_t^p + 0.125\hat{y}_t + \nu_t$ and Simple Rule 2 (Galí) is $i_t = 0.01 + 1.5\pi_t^p - 0.5\hat{u}_t$. Simple Rule 3 (OSR) is $i_t = 0.01 + 4.0\pi_t^p - 2.5\hat{u}_t$.

### 2.D.5 Perfect Foresight Solution

Figure 2.11 highlights the difference between the model with flexible wages, when wages are downwardly rigid and when wages are downwardly rigid but the labour union internalises the downward nominal wage rigidity constraint as explained further in Section 2.6. Once the labour union internalises the DNWR constraint the wage increase from the positive demand shock is dampened compared to a case where the constraint is not internalised.

Figure 2.11: Positive demand shock under perfect foresight




Impulse response for variables facing a positive demand shock. The demand shock follows an AR(1) with $\rho_z = 0.5$ and $\sigma = 1$. The figure represents the response of the economy when wages are assumed to be flexible, follow downward nominal wage rigidity and when the wage setters internalise the DNWR constraint.

# Chapter 3

# CONVENTIONAL AND NON-CONVENTIONAL MONETARY POLICY: BETWEEN CORE AND PERIPHERY

with Luca Onorante*

## 3.1 Introduction

The Great recession disrupted the traditional transmission mechanism of monetary policy and led the major Central Banks into the unchartered territory of non conventional measures. After 2008, the European Central Bank used non standard measures at first in conjunction with standard monetary policy, to allow its correct operation amid market disruption. Since 2015, as the interest rate reached its Effective Lower Bound (ELB), the APP (Asset Purchase Programme) complemented the conventional monetary policy. Between 9 March 2015 and 19 December 2018 the Eurosystem conducted net purchases of public sector securities under the public sector purchase programme (PSPP). Additionally, as of 2016 the ECB added to the APP the net purchases of corporate sector bonds

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under the corporate sector purchase programme (CSPP)[1].

The use of non conventional measures and their effectiveness in times of distress and even as a substitute for interest-rate based monetary policy has been widely debated. A related question concerns the relative effect of the different parts of the programme. Finally, as the sovereign and banking debt crisis particularly hit periphery countries of the euro area, the analysis of the possible asymmetric effects of common policies became important.

In this paper, we explore the effectiveness of government bond and corporate security purchases within a calibrated two-country New Keynesian model with a banking sector and a monetary union. We combine three important dimensions of the European experience after the crisis and analyze their interactions.

i) We focus on the effect of unconventional policies in three different regimes: during normal times, in "difficult times" when the banking system lacks liquidity and the transmission of a change in the interest rates is impaired, and under the ELB, when rates simply cannot be reduced further and unconventional policies essentially act as a substitute.

ii) We deal with the possible asymmetric effects of common policies by examining them in the context of a two-country monetary union, where the two countries represent the core of the euro area and the countries that most suffered during the recession (periphery), respectively. Due to the rich structure of our banking sector, where Core and Periphery banks are able to hold Core and Periphery government bonds and corporate securities, we are able to analyze the transmission of shocks through the interbank market. The two-country setup further allows considering different economic structures and banking systems and to assess their role in the transmission of policy shocks.

iii) Finally, we distinguish between two different classes of assets targeted by APP. More specifically, we differentiate between long-term government bonds and stocks, thereby reproducing the different mechanism behind the PSPP and CSPP. The presence of short-term investment and long-term government bonds also accounts for different maturities.

Our initial findings pertain to the propagation of shocks within our Monetary Union. We find that a capital destruction shock in the Periphery causes a fall in the output of the entire union, and this propagation is amplified if financial

[1]A smaller part of the APP also includes the asset-backed securities purchase programme (ABSPP) and the covered bond purchase programme (CBPP)

markets are fluid such that banks and households can freely trade government bonds. To have a strong real effect on the monetary union we need to assume extreme fluidity of the financial markets. However, even under more realistic calibration we are still able to find propagation effects to the financial market from a 1% capital destruction shock in the Periphery.

Our second focus of the paper is to analyze the interaction between the ELB and the effectiveness of various QE programmes implemented by the ECB. We calibrate the model to 2012 and use a capital destruction shock in each region followed by a demand shock, thereby forcing the economy to the Effective Lower Bound. We are able to see that, under our calibration and shocks, the real GDP loss due to the inability of the central bank to lower the interest rate is roughly 1% at its peak during 6 quarters at the ELB.

The loss of the instrument of the policy rate at the ELB forced the ECB to turn to unconventional monetary policy. We find that corporate security purchases are more effective than government bond purchases. This finding is mechanical due to our collateral constraint a la Gertler and Karadi (2013). However, due to calibrating to the eurozone, and hence an economy with less reliance on corporate securities, we find that the effectiveness of corporate security purchases versus government bond purchases are dampened.

## 3.2 The European Experience

In their survey of 20 years of ECB experience, Hartmann and Smets (2018) identify four time periods, and implicitly three regimes. In the first regime (from 1999 to 2007) the ECB was able to use standard monetary policy to achieve its inflation objective. The policy rate during this period varied between 2 and 5 percent, far from the ELB,and the ECB operated accordingly to the so-called Separation Principle: liquidity operations and asset purchases addressed malfunctioning interbank money markets and sovereign bond markets and thereby facilitated the transmission of monetary policy, interest rates focused on maintaining price stability over the medium term.

A second regime (from 2008 to 2013) started with the collapse of Lehman Brothers, the following Great Recession and the beginning of the sovereign crisis. In these “difficult times” the ECB maintained the Separation Principle, but conventional and non conventional policies were used jointly. The ECB lowered its key policy rate to an unprecedented level of 1%. At the same time, to respond to the increased demand for liquidity and reduce the risk of financial disruptions, the ECB introduced a number of non conventional measures. Starting in October 2009, the Main Refinancing Operations (MROs) were conducted with full allotment, in practice letting demand decide the amounts allocated at the MRO interest rate. Additional measures included the expansion of the list of marketable assets accepted as collateral in Eurosystem credit operations, and a Covered Bond Purchase Programme.

In the third and final regime (from 2014 onwards) the ECB used non conventional measures to overcome the Effective Lower Bound on interest rates. When the Effective Lower Bound (ELB) of the interest rate was approached, non conventional policies acted as a substitute while the ECB counteracted the risk of deflation and attempted at bringing inflation back to close to two percent. During this phase, policies such as funding for lending, forward guidance and (most of all) quantitative easing determined an expansion of the balance sheet of the ECB, both in size and variety of assets. The ECB's assets reached 3 trillions in the course of 2019; most of these securities are held for monetary policy purposes (see Figure 3.1).

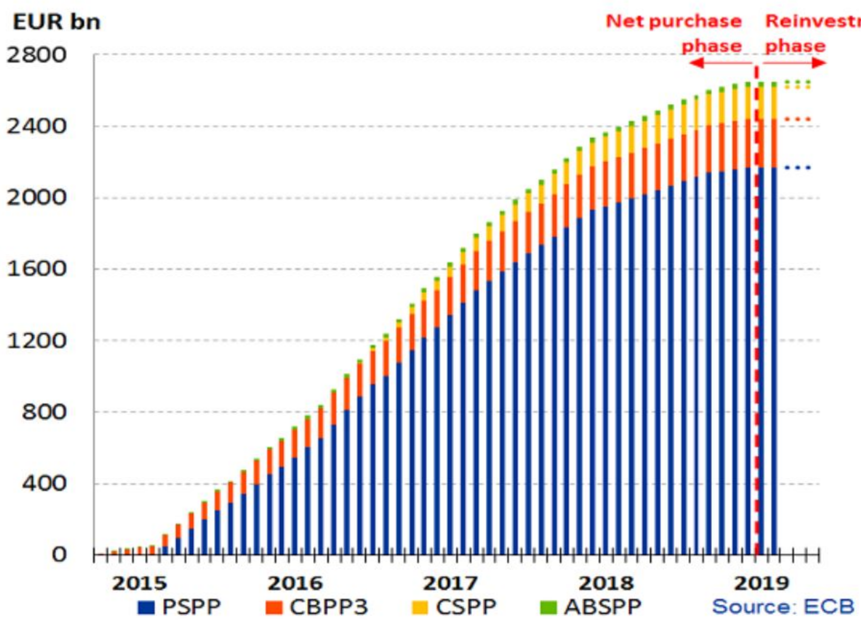



Figure 3.1: Operations conducted by the Eurosystem in the context of implementing its monetary policy. Source: ECB.

## 3.3 Literature Review

From a theoretical perspective one of the most influential works on Quantitative Easing is from Eggertsson and Woodford (2003, 2004), where the authors analyse the effects of open-market operations. Their main finding is that

> Quantitative easing that implies no change in the interest-rate policy should neither stimulate real activity nor halt deflation; and this is equally true regardless of the kind of assets purchased by the central bank. (Eggertsson and Woodford, 2004)

The previous quote was also theorized earlier in Wallace (1981).

Theoretically this view has been challenged using models that include the short-term interest rate at the zero lower bound. An example of this is from Bernanke and Reinhart (2004) who present a model with the ZLB and financial frictions, which during crises prevent arbitrage across asset classes and drive changes in term premia of assets. As a consequence,

> QE can take the risk of default out of the balance sheet of the banks and into the balance sheet of the central bank, reducing the extent of the credit crunch and increasing the effective supply of safe asset. (Reis, 2016)

If the distress in the economy is due to a fragile financial sector then credit easing, purchasing risky assets and providing safe reserves, reduces the risks and the fragility of financial intermediation.

We break the irrelevance result[2] of quantitative easing theoreticized by Wallace (1981) in the same spirit as Gertler and Karadi (2011). The main friction is a collateral constraint, or an incentive compatibility constraint, which means that bankers are only trusted to hold a certain amount of corporate securities and government bonds and they take this into account when maximising their lifetime net worth. Since there are limits to arbitrage, central bank intermediation increases overall asset demand and does not solely displace the private

[2] Another technique to break the irrelevance result is through a preferred-habit model of the term structure of interest rates, whereby agents in the model prefer to hold assets of different maturities, this has been popularised by Vayanos and Vila (2009) and more recently features heavily in Ray et al. (2019).

intermediation one-for-one, this increased demand increases the price. The Gertler and Karadi (2011) paper has formed the basis of many further research works, such as their own theoretical study calibrated to the U.S. experience of using quantitative easing, as seen in Gertler and Karadi (2013). More recently, this work has been extended to also analyse the impact on the eurozone by the quantitative easing and forward guidance conducted by the European Central Bank, Andrade et al. (2016).

A salient feature of Gertler and Karadi (2013) is that corporate security purchases (defined as claims on firm's capital) have a larger impact on the economy than government bond purchases, due to their riskier nature as they are easy to be absconded with compared to government bonds and therefore intermediation by the central bank is more beneficial. Kurtzman and Zeke (2020) show that if central bank purchases from large firms this reduces the incentive to invest from smaller firms whose debt is not purchased and therefore induces non-neglible misallocation costs. If these misallocation costs are sizable then security purchases can be less effective than government bond purchases in stimulating the economy. Since we do not embed a heterogeneous firm structure into our model this misallocation effect is not present and thus we also find that corporate securities purchases are more effective than government bond purchases.

A closely related paper that analyses quantitative easing within a two-country monetary union is Bletzinger and von Thadden (2018). They include short-term and long-term government bonds in a symmetric and asymmetric monetary union whilst taking the fiscal structure of each country seriously. Our paper differs from theirs by focusing on the effectiveness of government bond purchases versus private security purchases, which is not included in their model. On the other hand, to keep the model tractable our fiscal structure is purposely kept simple. Another paper that focuses on a two-country DSGE model of a monetary union estimated to fit Core and Periphery of the eurozone is Poutineau and Vermandel (2015). Their work focuses on the cross-border transmission of shocks and find that national variables, for example regional production and consumption, are less sensitive to financial shocks whilst investment is more sensitive. Our findings are in a similar vein as we experiment with opening and closing the financial transmission in our model to study how shocks in our region propagate to the other region. Moreover, they find little difference in the sensitivity of national variables to shocks when they move from banking autarky to a cross-border banking parameterization. Although we do not completely turn off the banking sector, as in Poutineau and Vermandel (2015), the limited difference in movement in national variables is echoed in our work when we experiment with fluid and rigid banking sectors. The closest paper to ours is

Auray et al. (2018), who evaluate PSPP with and without the ELB, but they do not focus on CSPP or maturity effect from long term government bonds and also differ by adding government default risk to their model. One way in which we try to demonstrate an added riskiness of the Periphery region is through a higher probability of Periphery banks failing, thus leading to a higher Periphery government bond premium compared to the Core region.

Although our paper does not focus on the empirical results from the European Central Bank's programmes, we utilise evidence by Andrade et al. (2016) to motivate our work. They find that "the programme produced significant effects upon announcement, on 22 January 2015" and that these effects are expected to last "approximately as long as in the case of standard monetary policy announcements." There also seems to be an effect other than the signalling channel, more specifically:

> We show that average yields (in basis points) plotted relative to the day prior to the PSPP announcement, dropped on average by about 13 basis points after the announcement and an additional 14 basis points after the implementation. Andrade et al. (2016).

Further compelling evidence of the impact of QE on the economy is shown in Haldane et al. (2016), who focus on the experience of the main economies that conduced QE. They find reasonably strong evidence the suggest that QE has had an impact on financial markets, loosing credit constraints, as well as on the real economy through temporarily boosting GDP and prices.

## 3.4 Model - Two Countries

### 3.4.1 Layout

We build a two-country New Keynesian model à la Galí (2015) with a banking sector motivated by Gertler and Karadi (2011). There are two regions denoted as Core and Periphery, one central bank and two fiscal authorities. Figure 3.2 represents the model layout, stars denote the Periphery region. Households are either workers or bankers. Workers supply labour, deposit into banks and hold Core and Periphery government bonds. Bankers wish to maximise their lifetime net worth taking into account their budget constraint, collateral constraint and the probability of survival ($\sigma$). Bankers hold Core and Periphery government bonds as well as corporate securities, which are modeled as claims on capital. The governments are kept purposely simple and solely finance the net interest on a fixed amount of government bonds through lump-sum taxes. The central bank sets the interest rate on safe deposits for both regions following a Taylor rule and conducts asset purchases dictated currently by an exogenous AR(1) shock[3].

### 3.4.2 Households

Our model derivation is focused on the Core region since the theoretical setup between Core and Periphery economies are symmetric. Households in the Core region (symmetric for Periphery) gain utility from consumption and disutility from working. The utility function includes habit formation, as this is shown to improve the empirical fit of the model, and takes the form:

$$E_t \sum_{j=0}^{\infty} \beta_{t,t+j} \left[ ln(C_{t+j} - hC_{t+j-1}) - \chi \frac{(L_{t+j})^{1+\varphi}}{1+\varphi} \right] \zeta_t \tag{3.1}$$

with $0 < \beta < 1$, $0 < h < 1$, $\chi > 0$ and $\varphi > 0$ all taking values calibrated to the euro area. $\zeta_t$ is added as a preference shifter (pure demand shock) and will be assumed to follow a persistent AR(1) process. Labour is a composite of heterogeneous labour services provided by the household and the economy is considered to be at limit where it becomes cashless as in Woodford (2011) and

[3] Although an AR(2) process more closely represents asset purchases and the expected path of these purchases by the ECB, we currently use an AR(1) for simplicity.

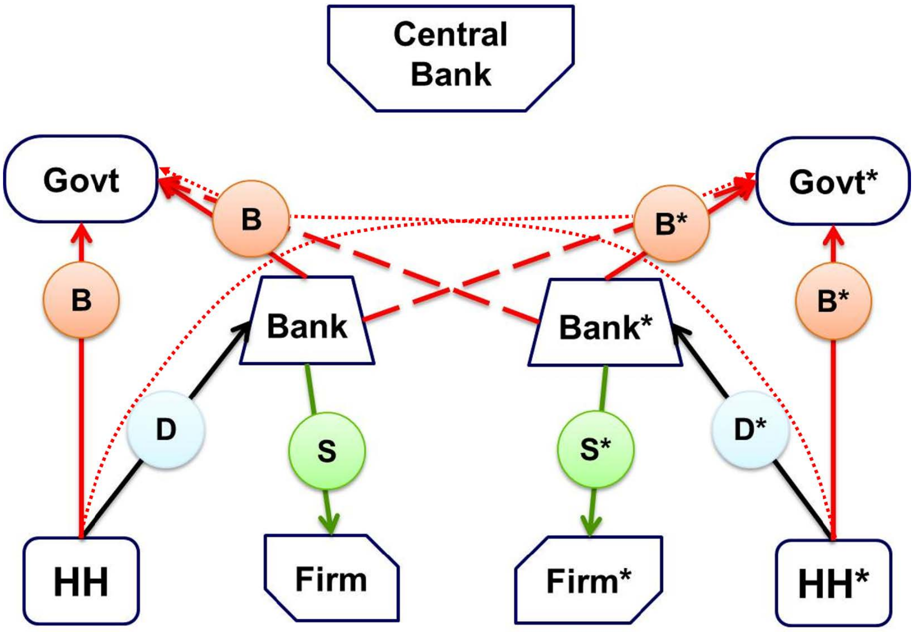



Figure 3.2: Basic structure of the model. Stars represents Periphery. B denotes government bonds, D deposits, S corporate securities.

Galí (2015), hence the convenience yield of real money balances are ignored. There is a unit continuum of households within the model, where a household belongs to the Core region if $j = [0, n)$ and the Periphery region if $j \in [n, 1]$. Households that are part of the Core are able to purchase goods from the Periphery, and vice-versa, with Periphery goods being denoted with superscript $^*$ when clarification is necessary. Consumption by the Core households of Core goods is given by $c$ and Core household consumption of Periphery goods is denoted as $c^*$. Aggregate consumption $C_t$ in the Core is the share of consumption of Core goods $c$ and Periphery goods $c^*$, which is a Dixit-Stiglitz aggregator of consumption goods from each region taking into account a home bias[4]:

$$C_t \equiv \left[ (\nu)^{\frac{1}{\theta_T}} (c_t)^{\frac{\theta_T - 1}{\theta_T}} + (1 - \nu)^{\frac{1}{\theta_T}} (c_t^*)^{\frac{\theta_T - 1}{\theta_T}} \right]^{\frac{\theta_T}{\theta_T - 1}} \tag{3.2}$$

The above equation characterizing total consumption in the Core region allows for home bias through $\nu \in [0, 1]$. This home bias, therefore, affects the price index for the region and the currency union. $\theta_T$ measures the elasticity of substitution

[4]For further details on the Dixit-Stiglitz aggregation see the Appendix Section 3.A.

between goods in the Core and Periphery.

Price index for the Core region takes the form:

$$P_t \equiv \left[\nu(p_t)^{1-\theta_T} + (1-\nu)(p_t^*)^{1-\theta_T}\right]^{\frac{1}{1-\theta_T}}$$

The household can consume either Core or Periphery final goods and deposit their savings into a bank in the Core region. The household receives a wage for working, the net worth of a Core bank $\Xi_t$ when the bank ceases business, transfers from the government $T_t$, interest payments from her previous period deposits $R_t D_t$. It needs to be noted that the rate of return for deposits here are in real terms and therefore are deflated by the price index $P_t$. Households' save using short-term bank deposits $D_{ht}$ and long-term government bonds $B_{ht}$, where subscript $h$ means that is held by the household, subscript $b$ means that is from the banking sector. To account for limited participation in asset markets by households, which provides us with limited arbitrage among assets, holdings of government bonds comes with a cost equal to the percentage of total government bonds held above a threshold $\bar{B}_h$[5]. As is typically assumed the long-term government bonds are perpetuities, however, to add realism to our government bonds we take inspiration from Woodford (2001) and allow the bonds to decay at rate $\rho$. Following Auray and Eyquem (2017) the decay rate $\rho$ is calibrated to match a 10 year or 40-quarter government bond with $\mathcal{M}$ denoting the maturity of the bond and $\beta$ the households discount rate.

$$\mathcal{M} = \frac{1}{1-\beta\rho} = 40$$

Therefore the interest rate on the government bond can be defined as:

$$R_{b,t} = \frac{1+\rho Q_{b,t}}{Q_{b,t-1}}$$

[5]Adding in the long-term bonds allows for asset purchase analysis of government bonds vs private loans (PSPP vs CSPP), it is possible to simplify this further by taking out the role of long-term government bonds

The budget constraint can be shown to be:

$$C_t + D_t + Q_{b,t}[B_{h,t} + \frac{1}{2}\kappa(B_{h,t} - \bar{B}_h)^2] + Q^*_{b,t}[B^*_{h,t} + \frac{1}{2}\kappa^*(B^*_{h,t} - \bar{B}^*_h)^2]$$
$$= \frac{W_t}{P_t}L_t + \Xi_t + T_t + R_t D_{t-1} + R_{b,t}Q_{b,t-1}B_{h,t-1} + R^*_{b,t}Q^*_{b,t-1}B^*_{h,t-1}$$

Households optimize equation (3.1) using $\{C_t, L_t, B_{ht}, B^*_{ht}, D_{ht}, W_t\}$ subject to the budget constraint. Given the assumption of flexible wages, the real wage will be a markup over the marginal rate of substitution. Collating the first order conditions of this problem:

$$E_t\left[\beta\frac{U_{C,t+1}}{U_{C,t}}R_{t+1}\right] = E_t[\Lambda_{t,t+1}R_{t+1}] = 1$$

$$\frac{W_t}{P_t} = \chi(L_t)^{\varphi}\zeta_t\frac{1}{U_{C,t}}$$

$$B_{h,t} = \bar{B}_h + \frac{E_t[\Lambda_{t,t+1}(R_{b,t+1} - R_{t+1})]}{\kappa}$$

$$B^*_{h,t} = \bar{B}^*_h + \frac{E_t[\Lambda_{t,t+1}(R^*_{b,t+1} - R_{t+1})]}{\kappa^*}$$

Where $\Lambda_{t,t+1} \equiv \beta\frac{U_{C,t+1}}{U_{C,t}}$ is utilised.

### 3.4.3 Banks

The banking system is modeled as a two-country extension of Gertler and Karadi (2013) or Andrade et al. (2016). Banks receive deposits from households and use these to make loans to firms and purchase government bonds.

Lending goes entirely to domestic non-financial firms. The return for holding a claim on a non-financial firm $R_{k,t+1}$ is equal to the marginal productivity of the capital lent to the firm ($Z_{t+1}$) plus the value of this capital leftover (after depreciation) $(1-\delta)Q_{t+1}$ divided by the cost of this asset today ($Q_t$, or the cost of capital). Region specific capital quality shocks are given as in Gertler and Karadi (2011) by $\xi_{t+1}$. This can be summarised as:

$$R_{k,t+1} = \frac{Z_{t+1} + (1-\delta)Q_{s,t+1}}{Q_{s,t}}\xi_{t+1}$$

Additionally, banks have access to a common financial market through their ability to purchase government bonds from the domestic and foreign government. Focusing on the Core region and writing the relative preference for domestic and foreign government bond holdings as a CES function, as in Auray et al. (2018), Core and Periphery government bonds are only partially substitutable, with $\iota$ being the elasticity of substitution. Additionally, a parameter $\upsilon$ is added to calibrate the well-documented home bias in government bonds.

$$\mathscr{b}_t = \left(\upsilon_b^{\frac{1}{\iota}}\, b_t^{\frac{\iota-1}{\iota}} + (1-\upsilon_b)^{\frac{1}{\iota}}\; b_t^{*\frac{\iota-1}{\iota}}\right)^{\frac{\iota}{\iota-1}}$$

The interest rate received on the bank's government bond portfolio can be written in a similar fashion as the CES government bond structure and is defined as $\mathscr{R}_{b,t}$. The value of the government bond portfolio held by the Core bank is given by:

$$\mathbb{Q}_{b,t}\mathscr{b}_t = Q_{b,t}b_t + Q^*_{b,t}b^*_t$$

The banks' activities in the balance sheet includes claims on firms $Q_{s,t}s_t$ (at a regional market price), domestic government bond holdings $Q_{b,t}b_t$ and foreign government bond holdings $Q^*_{b,t}b^*_t$. This is equal to the banks' net worth $n_t$ plus deposits received this period $d_t$. Combining the banks interim balance sheet and flow of funds gives the evolution of the bank's net worth[6]:

$$n_t = (R_{k,t} - R_t)Q_{s,t-1}s_{t-1} + (\mathscr{R}_{b,t} - R_t)\mathbb{Q}_{b,t-1}\mathscr{b}_{t-1} + R_t n_{t-1}$$

We now turn to the maximization problem of the banker. As bankers are detached from the household, their objective is to maximise their net worth and the payments to the household. Their discount factor is the same as the households' intertemporal marginal rate of substitution $\Lambda_{t,t+j}$, augmented with

[6]For a full derivation of the banks problem see Appendix 3.C

the probability $1 - \sigma$ that the banker will cease business and return to the household, transferring the remaining net worth to the household as a lump-sum payment. The maximization problem can be written as:

$$\max_{b_t, s_t} V_t = E_t \sum_{j=0}^{\infty} (1-\sigma)\sigma^{j-1}\Lambda_{t,t+j} n_{t+j} \tag{3.3}$$

Finally, we add the incentive compatibility constraint. As in Gertler and Karadi (2011), the bankers are able to divert a proportion of funds back to their own household. The incentive to default reduces the amount the depositors are willing to lend to the banks. It is assumed here that diverting funds from private loans (loans made to firms) is easier than diverting funds from government bonds. Specifically, the banker can divert $\theta$ from their private loans and $\theta\Delta$, with $0 < \Delta < 1$, from government bonds. We assume that it is equally difficult to abscond with Core government bonds as it is with Periphery government bonds. The incentive compatibility constraint is then given as:

$$V_t \geq \theta Q_{s,t} s_t + \Delta\theta(\mathbb{Q}_{b,t} b_t) \tag{3.4}$$

Adding a moral hazard or costly enforcement problem is essential to make financial markets non-frictionless and therefore to induce non-neutral asset purchases by the central bank, breaking the irrelevance result of Eggertsson and Woodford (2003).

The solution of the maximization problem under compatibility constraint results in a risk-adjusted leverage constraint, where $\phi_t$ is the leverage ratio and the below inequality will hold with equality.

$$Q_{s,t} s_t + \Delta(\mathbb{Q}_{b,t} b_t) \leq \phi_t n_t$$

The leverage ratio, $\phi_t$, is an adjusted measure of assets to net worth representing the maximal value of assets the bank is able to hold without violating the incentive compatibility constraint. Tighter scrutiny on the bank reduces the ability of the bank to divert funds and increases trust in the bank, lowering $\theta$ and increasing the amount a bank can hold and the leverage ratio $\phi_t$: [7]

[7] Bankers' problem is is laid out in greater detail in Appendix 3.C. The leverage ratio is found by guess-and-verify, where $\Omega_{t,t+1} = \Lambda_{t,t+1}[1-\sigma+\sigma\theta\phi_{H,t+1}]$ , is the banks' augmented discount factor, reflecting the shadow value of a unit of net worth.

$$\phi_t = \frac{E_t \Omega_{t,t+1} R_{t+1}}{\theta - E_t \Omega_{t,t+1}(R_{k,t+1} - R_{t+1})}$$

Additionally, a bank must be indifferent between investing in firms or purchasing government bonds. Therefore, in expectation, the following arbitrage condition must hold over Core and Periphery government bonds and loans to non-financial Home firms:

$$\Delta E_t \Omega_{t+1}(R_{k,t+1} - R_t) = E_t \Omega_{t+1}(\mathscr{R}_{b,t+1} - R_t) \tag{3.5}$$

### 3.4.4 Aggregation of banks

All banks within a given region are identical;[8] the equivalent equilibrium conditions are therefore given as the incentive compatibility constraint and the evolution of total net worth.

$$Q_{s,t} S_{b,t} + \Delta(\mathbb{Q}_{b,t} \mathscr{B}_{b,t}) \leq \phi_t N_t$$

$$N_t = \sigma \Bigg( (R_{k,t} - R_t) \frac{Q_{s,t-1} S_{b,t-1}}{N_{t-1}} + (\mathscr{R}_{b,t} - R_t) \frac{\mathbb{Q}_{b,t-1} \mathscr{B}_{b,t-1}}{N_{t-1}} + R_t \Bigg) N_{t-1} + \omega$$

### 3.4.5 Regional Governments

There are two identical regional governments. Government bonds are assumed to be in fixed supply and their quantities are calibrated to the debt-over-GDP of

[8]Uppercase variables are the aggregate versions of their lowercase counterparts. Therefore $S_{b,t}$ is defined as the aggregate claims by financial firms on non-financial firms within the economy. $B_{b,t}$ and $B^*_{b,t}$ are defined as total Core and Periphery, government bonds, respectively, held by Core banks.

each region. The governments pay net interest on bonds and balance their budget through taxes levied on the households in their region.

$$(R_{b,t} - 1)\bar{B}_t = T_t$$

The total amount of government bonds of the Core region is:

$$B_t = \bar{B}_t$$

and the total amount of government bonds in the monetary union is exogenous and defined as:

$$B_t^U = \bar{B}_t + \bar{B}_t^*$$

### 3.4.6 Central Bank

A central bank conducts monetary policy for the whole union. The Central Bank's objective is to set the nominal interest rate in order to minimize deviations of inflation from its steady state (or target) value and output from its natural level (the level of output that would prevail if no frictions were applied to the model).

Conventional monetary policy sets the common interest rate on deposits following a non-linear interest rate rule defined on a harmonized index of consumer prices, $P_t^U$, and the growth (inflation) of these prices $\Pi_{t+1}^U$ within the monetary union. We assume interest rate smoothing governed by the parameter $\phi_i$.

$$1 + i_t^U = \max\left\{\left[\frac{1}{1+i^U}\left(\frac{\Pi_t^U}{\Pi^U}\right)^{\phi_\Pi}\left(\frac{Y_t^U}{Y^U}\right)^{\phi_y}\right]^{1-\phi_i}\left[1 + i_{t-1}^U\right]^{\phi_i}, 1\right\}$$

$i_t$ is the net nominal interest rate. The nominal interest rate maps into the real interest rate on deposits through deflating by inflation:

$$1 + i_t^U = R_t^U \Pi_{t+1}^U$$

The Harmonised Index on Consumer Prices (HICP) is given by weighting the price levels of both regions by their relative size, $n$.

$$P_t^U = \left(P_t\right)^n \left(P_t^*\right)^{1-n}$$

After hitting the Zero-Lower-Bound on interest rates the central bank can use Unconventional Monetary Policy (UMP) to stimulate the economy. UMP in this model takes the form of purchasing government bonds or claims on financial firms, thus increasing their price within the economy and lowering the excess return on these assets. For comparability purposes the central bank purchases of private assets , $\psi_{S;t}$, and government bonds , $\psi_{B,t}$, are expressed as a share of GDP. Following the ECB's practice of purchasing according to capital key[9], the share of bonds/assets purchased is assumed proportional to the size of the two countries and determined by where the bond/asset originated from and not the location of the bank that holds it. [10] Subscript $g$ is used to denote assets held by the central bank.

$$B_{g,t} = \varphi_{B,t} B_t$$

$$S_{g,t} = \varphi_{S,t} S_t$$

These purchases follow an AR(1) process

$$\varphi_{B,t} = \rho_B \varphi_{B,t-1} + \varepsilon_{B,t}$$
$$\varphi_{S,t} = \rho_S \varphi_{S,t-1} + \varepsilon_{S,t}$$

The central bank finances its purchases through issuing central bank reserves, $D_{g,t}$, which pay the safe interest rate $R_{t+1}$, and from interest on previously held government bonds and corporate securities.

[9]The capital key governs the proportion of bonds the ECB can buy from each country.

[10]As in Gertler and Karadi (2011), the central bank must pay an additional efficiency cost $\tau$ to hold onto these assets. In their model, this cost ensures that the central bank does not take over the intermediation role of a financial firm permanently. Assuming the central bank is less efficient can be rationalized through additional monitoring costs that a central bank will need to complete while holding the asset. As our asset purchases are stylized and do not follow an asset purchasing rule that depends on interest rate spreads, this efficiency cost is redundant and added in an attempt to more accurately portray the costs and benefits of asset purchases.

This short term debt is issued to households. An equivalent, but more realistic way, to model central bank reserves is to have them held by banks. If private banks are unable to abscond with central bank reserves, which are held at the central bank, then this will lead to identical results. The balance sheet of the central bank is:

$$Q_{s,t}S_{g,t} + Q^*_{s,t}S^*_{g,t} + Q_{b,t}B_{g,t} + Q^*_{b,t}B^*_{g,t} = D_{g,t}$$

The total amount of the government bonds in the economy is exogenous fixed, calibrated to the debt-to-GDP ratio of each region. Central bank purchases of government bonds therefore push up the price of this asset and push down the bond yield, lowering the government bond premium.

$$B^U_t = B_t + B^*_t + B_{g,t} + B^*_{g,t}$$

Unlike government bonds, which are in a positive fixed supply, corporate securities (that are claims on capital) can increase due to rising investment within the economy. When the central bank purchases these assets they are taking over the intermediation of firms without the limit of the moral hazard problem faced by private banks. Therefore the total amount of securities in the Core region is given by those held by the private bank, $S_{b,t}$, and central bank, $S_{g,t}$.

$$S_t = S_{b,t} + S_{g,t}$$

### 3.4.7 Rest of the model

The rest of the model follows a two-country version of the standard New Keynesian setup à la Galí (2015) .

### 3.4.8 Non-Financial Firms: Intermediate good producers

The intermediate good firms produce their goods following a Cobb-Douglas production function using capital, $K_t$, and labour , $L_t$, available within the region. Intermediate goods produced in the Core region are sold at price $P_{m,t}$ .

Output in the Core region, $Y_t$, is produced using a technology common to all intermediate good producers $A_t$. The output elasticity of capital is given by $\alpha$ and labour elasticity is $1 - \alpha$.

$$Y_t = A_t(\xi_t K_t)^{\alpha}(L_t)^{1-\alpha}$$

The firms demand for labour, where $P_{m,t}$ is the price of intermediate goods, is equal to the marginal productivity of labour.

$$\frac{W_t}{P_t} = P_{m,t}(1-\alpha)\frac{Y_t}{L_t}$$

Gross profit per unit of capital in the Core region is given by $Z_t$:

$$Z_t = P_{m,t}\alpha\frac{Y_t}{\xi_t K_t}$$

The capital stock evolves according to regional investment $I_t$, a region-specific capital quality shock $\xi_{t+1}$, and depreciates at rate $\delta$:

$$K_{t+1} = I_t + (1-\delta)K_t\xi_{t+1}$$

### 3.4.9 Capital good producers

Capital goods producers are owned by households and therefore discount their expected future profits at the stochastic discount rate $\Lambda_{t,t}$. They produce capital through investment $I_t$, using final output as an input.[11] They sell capital to firms at the price $Q_t$. Therefore they choose $I_t$ to solve:

$$\max_{I_t} E_t \sum_{\tau=t}^{\infty} \Lambda_{t,\tau}\left\{Q_{s,\tau}I_\tau - \left[1 + f\left(\frac{I_\tau}{I_{\tau-1}}\right)\right]I_\tau\right\}$$

[11]Investment is done on a per-region basis and therefore only Core capital producers can invest to produce capital used in Core production, there is no trade in capital or cross-country investment in this model.

Where $f\left(\frac{I_t}{I_{t-1}}\right) = \frac{\eta}{2}\left(\frac{I_t}{I_{t-1}} - 1\right)^2$ is the adjustment cost of net investment.

The resulting selling price is given by

$$Q_{s,t} = 1 + \frac{\eta}{2}\Big(\frac{I_t}{I_{t-1}} - 1\Big)^2 + \frac{I_t}{I_{t-1}}\eta\Big(\frac{I_t}{I_{t-1}} - 1\Big) - E_t\Lambda_{t,t+1}\Big(\frac{I_{t+1}}{I_t}\Big)^2\eta\Big(\frac{I_{t+1}}{I_t} - 1\Big)$$

### 3.4.10 Retail firms

Retail firms, in a similar vein to households, are a unit continuum where a firm $f$ belongs to the Core region if $f \in [0, n)$, these firms are given the marker $h$, and the foreign region if $f \in [n, 1]$. We focus on retail firms in the Core region but keep the generic identifier $f$ for firms to derive the problem of the retailer. The retailer bundles (CES aggregator) intermediate output at purchasing cost $P_{m,t}$ and sells it at price $P_t(f)$ as a final good for consumption. The CES aggregator of output in the Core region and Periphery can be written as:

$$Y_t = \left(\left(\frac{1}{n}\right)^{\frac{1}{\epsilon}} \int_0^n y_t(f)^{\frac{\epsilon-1}{\epsilon}} df\right)^{\frac{\epsilon}{\epsilon-1}} \quad Y_t^* = \left(\left(\frac{1}{1-n}\right)^{\frac{1}{\epsilon}} \int_n^1 y_t(f)^{\frac{\epsilon-1}{\epsilon}} df\right)^{\frac{\epsilon}{\epsilon-1}}$$

The demand function for goods produced by the individual firm is derived using the standard Dixit-Stiglitz problem, and the demand for the final good (as a share of total demand in that region) produced by firm $f$ in the Core region depends on the relative price.

$$y_t(f) = \left(\frac{P_t(f)}{P_t}\right)^{-\epsilon} \frac{Y_t}{n} \quad \forall \; f \in [0, n)$$

A monopolistic retailer wishes to maximise profits $\pi_t$ by choosing the price $P_t(f)$ to sell the final good, taking into account their input cost of intermediate goods and adjustment cost of prices à la Rotemberg (1982) :

$$\pi_t = P_t(f)y_t(f) - P_{m,t}^N y_t(f) - \frac{\psi}{2}\Big(\frac{P_t(f)}{P_{t-1}(f)} - 1\Big)^2 P_t Y_t$$

Dividing the profit by the price level and then taking the first order condition with respect to $P_t(f)$ gives us:

$$\frac{\partial}{\partial P_t(f)} = (1-\epsilon)\left(\frac{P_t(f)}{p_t}\right)^{-\epsilon}\frac{Y_t}{nP_t} + \epsilon\frac{P^N_{m,t}}{P_t}\frac{1}{p_t}\left(\frac{P_t(f)}{p_t}\right)^{-\epsilon-1}\frac{Y_t}{n}$$
$$-\psi\frac{1}{P_{t-1}(f)}\left(\frac{P_t(f)}{P_{t-1}(f)}-1\right)\frac{p_t}{P_t}Y_t + E_t\Lambda_{t,t+1}\psi\frac{P_{t+1}(f)}{P_t(f)^2}\left(\frac{P_{t+1}(f)}{P_t(f)}-1\right)\frac{p_{t+1}}{P_{t+1}}Y_{t+1} = 0$$

Since this is an identical problem for all firms within a region, each firm will choose the same price level, and $P_t(f) = p_t \;\; \forall \;\; f \in [0, n)$.[12] Price inflation of the final goods produced in the Core region is defined as $\Pi_{C,t} = \frac{p_t}{p_{t-1}}$. We can then rewrite the FOC as:

$$(1-\epsilon) + \epsilon P_{m,t} P_t \frac{1}{p_t} - \psi \Pi_{C,t}(\Pi_{C,t} - 1) + E_t\Lambda_{t,t+1}\psi\frac{\Pi^2_{C,t+1}}{\Pi_{t+1}}(\Pi_{C,t+1} - 1)\frac{Y_{t+1}}{Y_t} = 0$$

When $\psi = 0$ we are in a flexible price equilibrium and therefore price is set as a markup over marginal cost $\frac{P_{m,t}P_t}{p_t} = \frac{\epsilon-1}{\epsilon}$. Due to the two-country setup there is a difference between producer price inflation $\Pi_{C,t}$ of goods produced in the Core region and consumer price inflation $\Pi_t$, which is the one faced by Core consumers and also takes into account their consumption of goods produced in the Periphery [13].

### 3.4.11 Closing the model

To close the model we state the resource constraint, a Fisher equation and the link between corporate securities and capital.

The resource constraint of output in the Core region is determined by total

[12] Recall that $p_t$ represents the price of Core goods, this differs from $P_t$, which is a weighted sum of the price level faced by Core households and therefore also feature the price of Periphery produced goods consumed by Core households.

[13] In a standard one-country model where $\Pi_{C,t} = \Pi_t$ and $p_t = P_t$ the above equation reverts back to the standard Rotemberg (1982) pricing form with $P_{m,t}$ thought of as the real marginal cost.

consumption of goods produced from the Core region[14], investment cost for the capital production firm, cost of asset purchases for the home region $\Phi_{C,t} = \tau(\psi_{s,t-1}Q_{s,t-1}S_{t-1} + \psi_{B,t}Q_{b,t-1}B_{t-1})$ and the adjustment cost paid by the retail firm to change their prices. Consumption of goods produced in the Core region is comprised of consumption of Core goods by Core households, $c_t$, and consumption of Core goods by periphery households $c_{p,t}$.

$$Y_t = c_t + c_{P,t} + [1 + f\Big(\frac{I_t}{I_{t-1}}\Big)]I_t + \Phi_{C,t} + \frac{\psi}{2}(\Pi_{C,t} - 1)^2 Y_t$$

The total amount of corporate securities in the Core region is given by the investment conducted in that region by the capital producer and the remaining capital in the economy discounted at the standard rate $\delta$. Since we are assuming that foreign banks cannot hold domestic region corporate securities this equation is identical to a one-country model equivalent.

$$S_t = I_t + (1 - \delta)K_t$$

Total output of the currency union, $Y_t^U$, is defined as the weighted sum of output from each region weighted by relative region size $n$.

$$Y_t^U = nY_t + (1 - n)Y_t^*$$

[14]Where $c_t = \left(\frac{p_t}{P_t}\right)^{-\theta} \nu C_t$ and $c_t^* = \left(\frac{P_t^*}{P_t}\right)^{-\theta} (1 - \nu)C_t$ . $\nu$ represents home bias, and $\theta$ is the elasticity of substitution between goods in the Core and Periphery regions - see Appendix for further explanation.

### 3.4.12 Calibration

The model is calibrated to the largest economies within the Eurozone and split into two regions, Core and Periphery. The Core is comprised of Germany, France and the Netherlands. The Periphery is comprised of Portugal, Italy, Ireland, Greece and Spain. The calibration seen in Table 3.1 focuses on 2012 and draws on national statistics, data from the IMF and household consumption bias from Bussière et al. (2013). Specifically, country size is set proportional to the Gross Domestic Product of each region. The statistic for home bias is taken from Bussière et al. (2013), who derive import contents of consumption up to 2005[15] for major world economies. We take the total debt over GDP, gross position for 2012, from the World Economic Outlook produced by the International Monetary Fund and use a weighted sum to arrive at 65.77% for the Core and 78.34% for the Periphery. Debt held by households and banks is calibrated on data by the European Central Bank. Lastly, the fraction of time spent working, which determines the steady state level of labour $L$ and is chosen by adjusting, $\chi$, the disutility of labour, is the weighted average of the number of people employed and hours worked in the Core and Periphery taken from the OECD. The rest of the calibration is standard and is drawn from Gertler and Karadi (2013) and Galí (2015). The effective lower bound is introduced into the paper using Occbin by Guerrieri and Iacoviello (2015). Occbin is a piecewise linear perturbation method that can handle occasionally binding constraints and is applicable to models with a large number of state variables.

[15]An underlying assumption is that import contents of consumption has been stable from 2005 to 2012 such that we are able to derive home bias from this statistic.

Table 3.1: Calibration

| Description | Variable | **Core** | **Periphery** |
|---|---|---|---|
| Model Specific | | | |
| Country Size | $n$ | 0.61 | 0.39 |
| Home Bias in final goods | $\nu$ | 0.77 | 0.77 |
| Home bias in bonds (banking) | $\nu_b$ | 0.81 | 0.61 |
| Debt to GDP | $by_t$ | 65.77 | 78.34 |
| Percent of Core debt held by households | $\bar{B}_h$ | 0.35 | 0.21 |
| Percent of Periphery debt held by households | $\bar{B}_h^*$ | 0.13 | 0.49 |
| Percent of core debt held by banks | $b$ | 0.33 | 0.11 |
| Percent of periphery debt held by banks | $b^*$ | 0.12 | 0.27 |
| Fraction of time spent working | $L$ | 0.24 | 0.30 |
| Conventional Parameters | | | |
| Capital share | $\alpha$ | 0.36 | 0.36 |
| Discount factor | $\beta$ | 0.9975 | 0.9975 |
| Persistence of monetary policy decisions (Monetary union) | $\phi_i$ | 0.5 | 0.5 |
| Inflation feedback Taylor Rule (Monetary Union) | $\phi_\pi$ | 2 | 2 |
| Output feedback Taylor Rule (Monetary Union) | $\phi_y$ | 0.125 | 0.125 |
| Demand Elasticity | $\epsilon$ | 3.857 | 3.857 |
| Elasticity of labour supply | $\varphi$ | 2 | 2 |
| Adjustment cost of Households holding bonds | $\kappa$ | 1 | 1 |
| Absconding Rate | $\theta$ | 0.3 | 0.3 |
| Absconding for government bonds | $\Delta$ | 0.7 | 0.7 |
| Bankers startup fund | $\omega$ | 0.0047 | 0.0047 |
| Probability of banker survival | $\sigma$ | 0.95 | 0.917 |
| Adjustment cost of investment | $\eta$ | 5.169 | 5.169 |
| Adjustment cost for Rotemberg Pricing | $\psi$ | 34.03 | 34.03 |
| Steady state inflation | $\Pi_{ss}$ | 1 | 1 |
| Discount rate of capital | $\delta$ | 0.025 | 0.025 |
| Inefficiency of government purchases | $\tau$ | 0.001 | 0.001 |
| Elasticity of substitution between goods | $\theta_T$ | 5 | 5 |
| Elasticity of substitution between bonds (banking) | $\iota$ | 1.1 | 1.1 |
| Persistence of technology shock | $\rho_a$ | 0.9 | 0.9 |
| Persistence of monetary policy shock | $\rho_\nu$ | 0.9 | 0.9 |
| Persistence of demand shock | $\rho_\zeta$ | 0.9 | 0.9 |
| Persistence of capital quality shock | $\rho_\xi$ | 0.7 | 0.7 |
| Persistence of securities purchase shock | $\rho_{st}$ | 0.9 | 0.9 |
| Persistence of bond purchase shock | $\rho_{bt}$ | 0.9 | 0.9 |

## 3.5 Results and policy simulations

This section analyses the financial pass-through of a capital destruction shock from the Periphery region to the Core region. We explore the propagation of this shock under a fluid bond market compared to a rigid market. Moreover, the impact of the ELB on the real economy is detailed as well as the effect of government bond purchases and corporate security purchases within the monetary union.

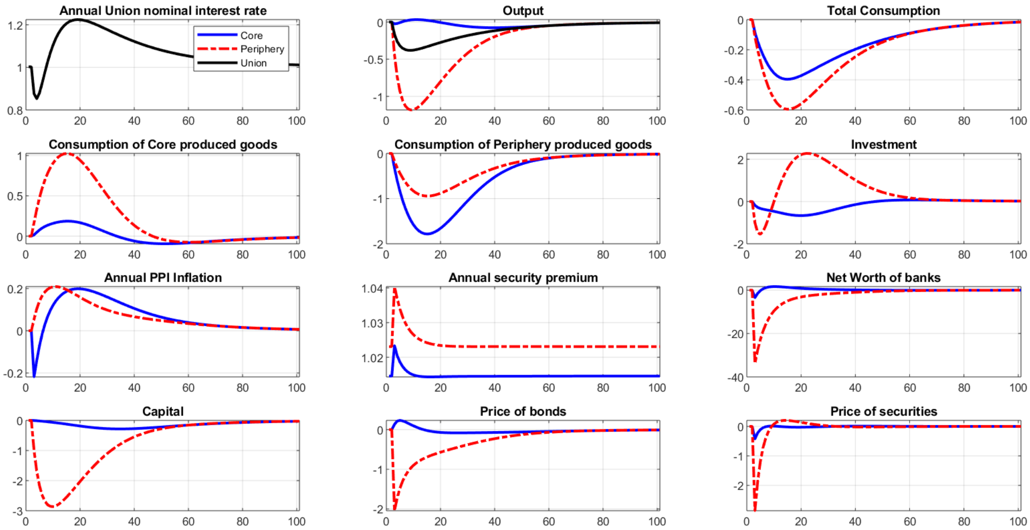
### 3.5.1 Scenario 1: The role of bond market and financial pass-through

Figure 3.3 and 3.4 highlight the transmission through the banking sector of a one-percent-annualised capital destruction shock in the Periphery region. When government bonds are not easily substitutable, banks and households are less inclined to change their positions and the bond market is less fluid.

Figure 3.3 highlights the effect of the capital destruction shock and weak financial pass-through. With low elasticity of substitution the Periphery government bond premium and corporate security premium rises more, the value of Periphery government bonds and corporate securities fall more and Periphery investment is lower. Households from both regions increase their consumption of the relatively cheaper Core produced final goods, which helps to support Core output. The destruction of capital in the Periphery region also lowers the net worth of Periphery banks, causing them to sell off government bonds in order to adhere to their collateral constraint. Periphery banks sell Core and Periphery government bonds and households from both regions as well as the Core banks purchase these bonds.

The main differences between Figure 3.3 and 3.4 can be seen in the role of the financial sector. When elasticity is high and households can easily trade government bonds, this capital destruction shock spreads across the monetary union. Both banks react more. Due to the higher fluidity of the bond market (primarily due to lowering the adjustment cost of government bonds for households $\kappa$ ), we see an unrealistic sell-off of Periphery government bonds. When the bond market was rigid the price of government bonds and corporate securities in the Core region rose, as demand for the safer Core government bonds increased. However, with higher pass-through we find that the shock spreads to the Core and the annual corporate security premium, as well as the

Figure 3.3: Capital destruction shock in Periphery with rigid bond market




One-percent capital quality shock in the Periphery region. Bond elasticity $\iota = 1.1$ and household bond adjustment cost $\kappa = 1$

annual government bond premium (not shown)[16], is much closer between Core and government bonds, signalling that the perceived risk of both regions is now similar. Moreover, we see lower investment in the Core region in Figure 3.4 compared to Figure 3.3 and a drop in consumption by the Core and Periphery households. When pass-through is high the central bank must react more to the capital destruction shock as the latter has a larger impact on the union as a whole, since it also affects the Core region.

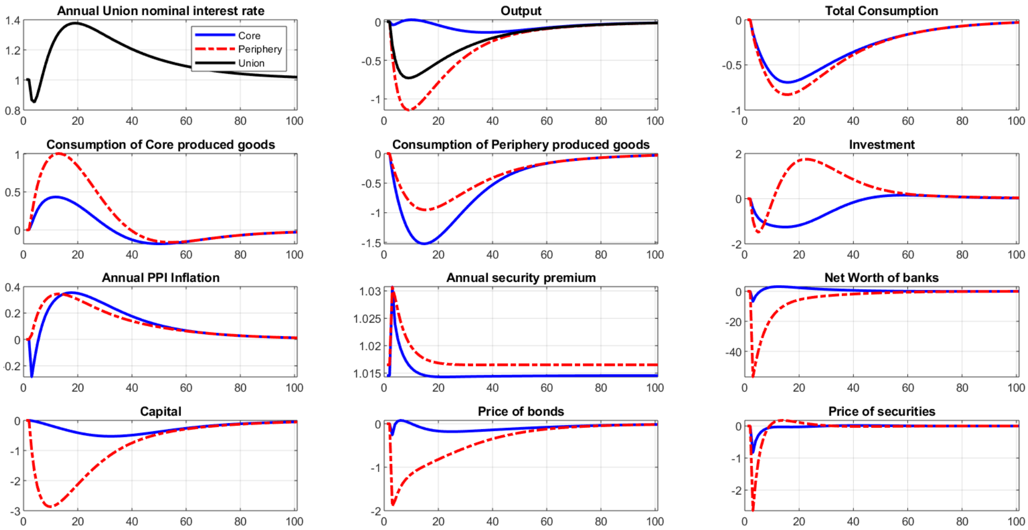
### 3.5.2 Scenario 2: Effective lower bound and Asset Purchases

Figure 3.5 displays the impact of reaching the effective lower bound from a series of capital destruction shocks and demand shocks[17] in both the Core and Periphery. The economy is at the effective lower bound for 6 quarters and restricting the central bank's ability to lower the interest rate negatively impacts

[16]The bond premium and security premium co-move as they are linked due to the collateral constraint on the banking sector.

[17]Due to calibrating the model to 2012, and therefore the interest rate is set to 1%, it only takes two quarters of 1% capital destruction coupled with a 1.5% demand shock to both regions to reach the ELB.

Figure 3.4: Capital destruction shock in Periphery with fluid bond market

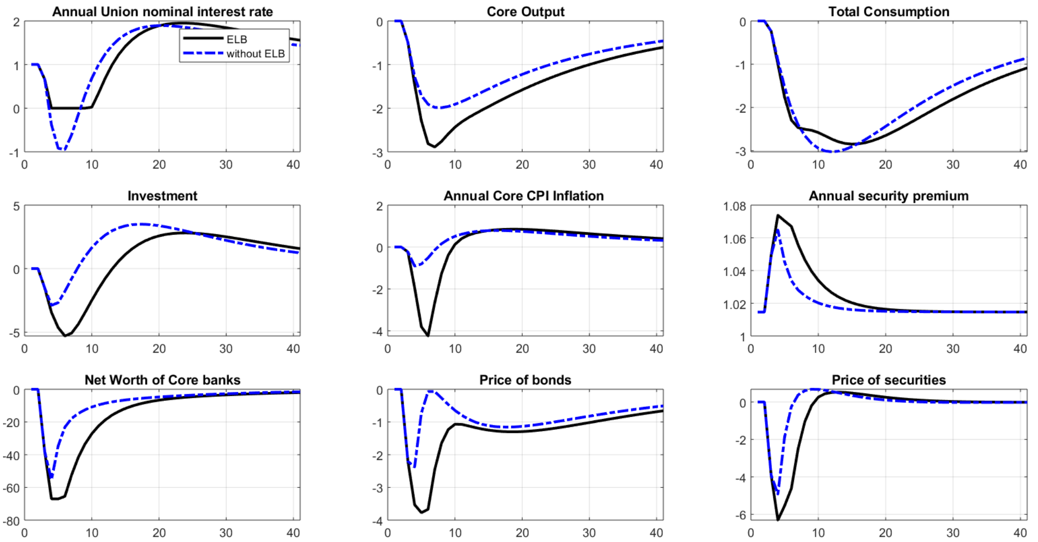



One-percent annualised capital quality shock in the Periphery region. Bond elasticity $\iota = 100$ and $\kappa = 0.25$ .

consumption, output and inflation. In this scenario Core and Union-wide output is 1% lower than it otherwise would be if the central bank could lower the interest rate below zero and the CPI deflates by more than an additional 3%. Due to capital quality and negative demand shocks investment is reduced and the premium on both government bonds and corporate securities rise. Households, who wish to postpone consumption due to a falling demand, purchase government bonds from both Core and Periphery banks who are selling their bonds. The rebound in the price of government bonds one period after their fall is the result of this demand shock, which we assume impacts the model one period after the capital destruction shock. As previously shown, the capital destruction shock forces banks to sell government bonds on their balance sheet and their net worth falls. This effect is amplified if the interest rate, which is also the deposit rate, is held artificially high (as in the ELB), increasing the cost to the banks of household deposits.

Figure 3.6 highlights the benefit of the central bank conducting government bond purchases. Bond purchases increase the price of bonds, supporting the banks and households balance sheet. These bond purchases support consumption and price inflation, allowing the central bank to escape the ELB earlier. The government bond purchases are calibrated to 10% of GDP in both the Core and Periphery regions for two quarters and then following an AR(1) process shown earlier. This

Figure 3.5: Effective lower bound in the Core region




Capital destruction and demand shocks in both regions force the monetary union to the ELB. The IRF focuses on the variables in the Core region. The effective lower bound is reached by a series of capital quality and then demand shocks.

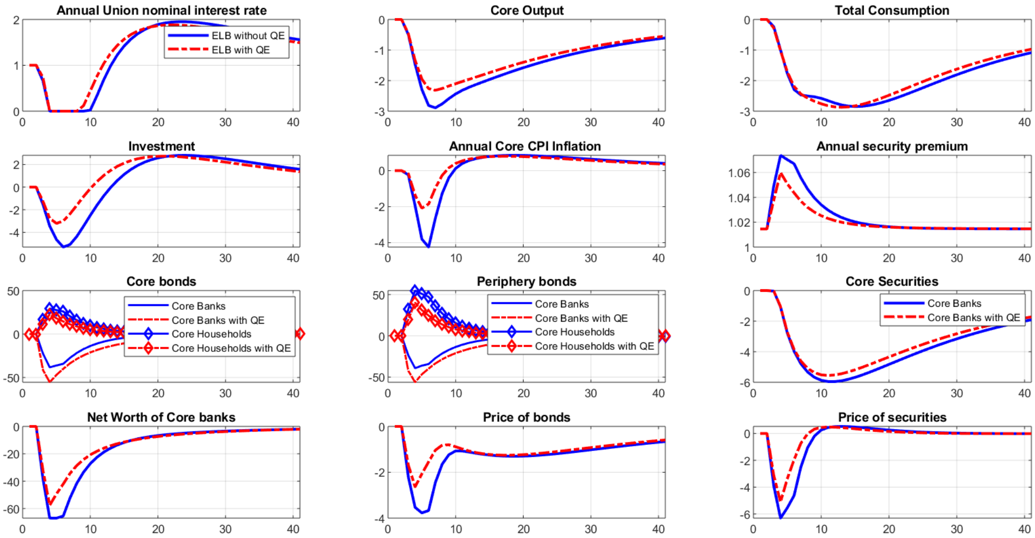
process is set to be very persistent, however, not as severe as similar AR(2) processes. Due to modeling sovereign bonds a la Woodford (2001), and as such as a perpetuity with a decaying coupon, which have a maturity of 10 years, the price-quantity nexus of public debt is altered as shown in Auray and Eyquem (2017).[18] As is found in Auray and Eyquem (2017) longer bond maturities dampen the movement in output and consumption.

Finally, government bond purchases help to support bond prices, which means that banks wish to sell more government bonds and purchase corporate securities, which offer a higher return. These purchases help to limit the rise in the annual government bond and corporate security premium for the Core region (shown in the figure) and Periphery region, representing a lowering of risk in the market. Investment and CPI inflation are also supported by this intervention.

The effect of corporate security purchases by the central bank can be seen in

[18] Auray and Eyquem (2017) find that longer maturities are associated with low steady state bond levels but higher bond prices, such that debt-to-GDP is the same but the amount of government bonds purchased is reduced. This is why in figure 3.6 the quantity of bonds purchased by the monetary union is minimal since we are assuming these bonds have a long duration and their steady-state price is high relative to a bond of one quarter duration.

Figure 3.6: Effective Lower Bound and Bond Purchases (10% of GDP)

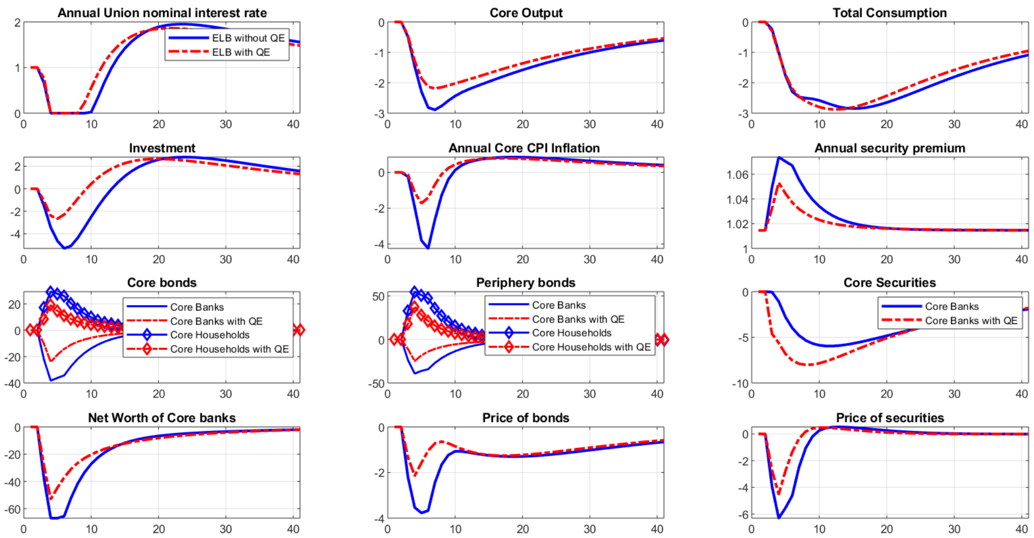



Figures show response of the Core region only. The effective lower bound is reached by a series of capital quality and then demand shocks. Bond purchase shock is calibrated to reach 10% of GDP for 2 quarters and follow an AR(1) process.

Figure 3.7. The impact on the economy from corporate security purchases is larger than that of government bond purchases, primarily due to the collateral constraint introduced on the banking sector and the higher risk of absconding with corporate securities than with government bonds. This assumption implies that, by buying corporate securities, the Central Bank obtains a larger relaxation of the incentive compatibility constraint, and a larger impact on the economy. The transmission channel from the corporate security purchase shock seen in Figure 3.7 is through the portfolio re-balance channel as banks move back into government bonds and households sell bonds to deposit into banks, thereby also benefiting from the asset purchases. The main difference between Figure 3.6 and Figure 3.7 can be seen in the response of banks as when the Central Bank conducts bond purchases the banks sell more bonds and move into corporate securities whereas when the Central Bank purchases corporate securities the banks sell more corporate securities and hold more government bonds.[19]

[19] Additional results can be found in Appendix 3.D where the impact of asset purchases in the Periphery versus Core on union-wide output and inflation are assessed.

Figure 3.7: Effective Lower Bound and Security Purchases (10% of GDP)




The figure focuses on the response of Core region variables. The effective lower bound is reached by a series of capital quality and then demand shocks. The corporate security purchase shock is calibrated to reach 10% of GDP for the first two quarters and follow an AR(1) process.

## 3.6 Closing remarks

This paper explores the effectiveness of government bond and corporate security purchases by a central bank within a calibrated two-country New Keynesian model featuring a banking sector (an extension of Gertler and Karadi (2011) and Andrade et al. (2016)) and a two-country monetary union. It further explores the propagation of economic shocks from one region to another in a monetary union and how these propagation depends on the banking sector.

We propose a rich setup, where households also hold government bonds and capital is region-restricted, and we account for the maturity effect of longer-term government bonds. We find that a negative (capital destruction) shock in the Periphery causes a fall in the output of the entire union, and this propagation is amplified if financial markets are integrated so that banks and households can freely trade government bonds. The impact on the financial economy is always sizeable, even with a small adjustment to the fluidity of the government bond market.

Our second finding concerns the effect of non-conventional monetary policy, as we focus on the effectiveness of government bond purchases versus corporate security purchases. As in Gertler and Karadi (2013) we analyse the impact of government bond purchases versus corporate security purchases at the effective lower bound. Due to the nature of the collateral constraint we find that corporate security purchases have a stronger impact on inflation and on lift-off time from the Effective Lower Bound than equivalent government bond purchases. This finding is in line with the ones of Gertler and Karadi (2013) for the U.S. economy. However, the large difference, seen in Gertler and Karadi (2013), between these two quantitative easing policies is not present in our model. This is likely due to our calibration to the eurozone, which has a smaller share of corporate securities than the U.S.

There are many experiments that can be done with our model setup: analysing the propagation of different shocks and the effect of different degrees of symmetry among them, testing a more realistic QE schedule (AR(2) rather than AR(1)), adjusting the size and stability of a region to see how this impacts the monetary union. These are left for future work.

## 3.A CES aggregator and Dixit-Stiglitz

We use the CES formulation for consumption, prices and bond holdings of private banks in our model. As is standard in the literature we also derive the consumption of domestic and imported goods as a share of total consumption using the Dixit-Stiglitz aggregator, from Dixit and Stiglitz (1977).

Focusing on the Core region, since the problem is symmetric, we define aggregate consumption $C_t$ in the Core region as the share of domestic consumption $c_t$ and foreign consumption $c_t^*$, taking into account home bias in consumption $\nu \in [0,1]$ and the elasticity of substitution, $\theta$, between Core and Periphery goods:

$$C_t \equiv \left[ (\nu)^{\frac{1}{\theta}} (c_t)^{\frac{\theta-1}{\theta}} + (1-\nu)^{\frac{1}{\theta}} (c_t^*)^{\frac{\theta-1}{\theta}} \right]^{\frac{\theta}{\theta-1}} \tag{3.6}$$

The price index in the Core region follows a similar form to the consumption CES. It combines the price of Core goods $p_t$ and the price of Periphery goods $p_t^*$, weighted by the respective amount the representative Core consumer would purchase.

$$P_t \equiv \left[ \nu(p_t)^{1-\theta} + (1-\nu)(p_t^*)^{1-\theta} \right]^{\frac{1}{1-\theta}}$$

Using the standard Dixit-Stiglitz miniminsation it is possible to derive the the demand functions $c_t$ and $c_t^*$ as a share of total consumption in the Core region weighted by the relative price of the goods and the home bias:

$$c_t = \left( \frac{p_t}{P_t} \right)^{-\theta} \nu C_t$$

$$c_t^* = \left( \frac{p_t^*}{P_t} \right)^{-\theta} (1-\nu) C_t$$

Allowing $\epsilon$ to be defined as the elasticity of substitution across the differentiated goods within a region. Therefore it is possible to derive the price aggregators for goods originating in the Core and Periphery regions by minimising the cost of each bundle, taking the prices of the differentiated goods as given. The result is:

$$p_t \equiv \left[ \frac{1}{n} \int_0^n p_t(c)^{\frac{\epsilon-1}{\epsilon}} dc \right]^{\frac{\epsilon}{\epsilon-1}}$$

$$p_t^* \equiv \left[\frac{1}{1-n}\int_n^1 p_t(p)^{\frac{\epsilon-1}{\epsilon}}dp\right]^{\frac{\epsilon}{\epsilon-1}}$$

The second stage of the Dixit-Stiglitz aggregator allows us to derive the consumption of each good consumed in the Core region, $c_t$ and $c_t^*$, as an index of consumption across the continuum of differentiated goods. Where the size of the Core region is denoted as $n$.

$$c_t \equiv \left[\left(\frac{1}{n}\right)^{\frac{1}{\epsilon}}\int_0^n c_t(c)^{\frac{\epsilon-1}{\epsilon}}dc\right]^{\frac{\epsilon}{\epsilon-1}}$$

$$c_t^* \equiv \left[\left(\frac{1}{1-n}\right)^{\frac{1}{\epsilon}}\int_n^1 c_t(p)^{\frac{\epsilon-1}{\epsilon}}dp\right]^{\frac{\epsilon}{\epsilon-1}}$$

$$c_t(c) = \left(\frac{P_t(c)}{p_t}\right)^{-\epsilon}\frac{c_t}{n}$$

$$c_t(p) = \left(\frac{P_t(p)}{p_t^*}\right)^{-\epsilon}\frac{c_t^*}{1-n}$$

Combining the demand and price equation allows us to rewrite consumption in the Core region by the differentiated goods $c$ and $p$.

$$\Rightarrow c_t(c) = \left(\frac{p_t(c)}{p_t}\right)^{-\epsilon}\frac{\nu}{n}\left(\frac{p_t}{P_t}\right)^{-\theta}C_t$$

$$\Rightarrow c_t(p) = \left(\frac{p_t(p)}{p_t^*}\right)^{-\epsilon}\frac{1-\nu}{1-n}\left(\frac{p_t^*}{P_t}\right)^{-\theta}C_t$$

## 3.B Relation of prices in the model

It is useful to have the relation of prices explicitly written out as they are used extensively in multi-country models. .

Terms of trade (in producer pricing):

$$T_t = \frac{p_t^*}{p_t}$$

Harmonised index of consumer prices:

$$\Pi_{t+1}^U = \frac{P_{t+1}^U}{P_t^U}$$

Consumer Price Inflation in Core:

$$\Pi_{t+1} = \frac{P_{t+1}}{P_t}$$

Harmonised level of consumer prices:

$$P_t^U = \Big(P_t\Big)^n \Big(P_t^*\Big)^{1-n}$$

Price of consumption by households:

$$P_t \equiv \left[\nu\Big(p_t\Big)^{1-\theta} + (1-\nu)\Big(p_t^*\Big)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

Further price relations:

$$\frac{P_t}{p_t} = \left[\nu\left(\frac{p_t}{p_t}\right)^{1-\theta} + (1-\nu)\left(\frac{p_t^*}{p_t}\right)^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

$$\Rightarrow \frac{P_t}{p_t} = \left[\nu + (1-\nu)T_t^{1-\theta}\right]^{\frac{1}{1-\theta}}$$

$$\frac{P_t^*}{p_t^*} = \left[\nu^* + (1-\nu^*)T_t^{\theta-1}\right]^{\frac{1}{1-\theta}}$$

$$\frac{P_t}{p_t^*} = \left[\nu T_t^{\theta-1} + (1-\nu)\right]^{\frac{1}{1-\theta}}$$

$$\frac{P_t^*}{p_t} = \left[\nu^* T_t^{1-\theta} + (1-\nu^*)\right]^{\frac{1}{1-\theta}}$$

$$\frac{P_t^U}{P_t} = \left[\frac{\nu^* T_t^{1-\theta} + (1-\nu^*)}{\nu + (1-\nu)T_t^{1-\theta}}\right]^{\frac{1-n}{1-\theta}}$$

$$\frac{P_t^U}{P_t^*} = \left[\frac{\nu T_t^{\theta-1} + (1-\nu)}{\nu^* + (1-\nu^*)T_t^{\theta-1}}\right]^{\frac{n}{1-\theta}}$$

$$\frac{P_t^*}{P_t} = \left[\frac{\nu^* T_t^{1-\theta} + (1-\nu^*)}{\nu + (1-\nu)T_t^{1-\theta}}\right]^{\frac{1}{1-\theta}}$$

Change in the terms of trade is the change in price inflation of the Periphery produced goods compared to Core, (PPI inflation).

$$\frac{T_t}{T_{t-1}} = \frac{\Pi_{P,t}}{\Pi_{C,t}}$$

Where $\Pi_{P,t}$ is the PPI of Periphery produced goods and $\Pi_{C,t}$ is the PPI of the Core produced goods.

## 3.C Two country bank problem derivation

This section derives the banker's problem seen in the main text Section 3.4.3. We focus on the Core banker and assume that it is equally difficult to abscond with domestic government bonds as it is to abscond with Periphery government bonds within a monetary union. For ease of reference we outline again the CES form that the bond holdings of the banker takes:

$$\mathscr{b}_t = \left(\nu_b^{\frac{1}{\iota}} b_t^{\frac{\iota-1}{\iota}} + (1-\nu_b)^{\frac{1}{\iota}} b_t^{*\frac{\iota-1}{\iota}}\right)^{\frac{\iota}{\iota-1}}$$

$$\mathbb{Q}_{b,t}\mathscr{b}_t = Q_{b,t}b_t + Q^*_{b,t}b^*_t$$

Using this CES structure allows us to solve the bankers problem in an analogous manner to the one-country bank model. Therefore we set up two value functions, the end of period value function $V_{t-1}$ and the beginning of next period value function $W_t$. As before the value of a bank at the end of the period is equal to the franchise value of the bank with assets: $s_{t-1}, \mathscr{b}_{t-1}, n_{t-1}$. We write this by equating the beginning of next periods value taking into account the survival probability $\sigma$:

$$V_{t-1}(s_{t-1}, \mathscr{b}_t, n_{t-1}) = E_{t-1}\Lambda_{t-1,t}\{(1-\sigma)n_t + \sigma W_t(n_t)\}$$

To solve this problem first conjecture that the value function is linear in state variables with the coefficients: $\mu_{s,t}, \mu_{b,t}$, to be determined.

$$V_t = \mu_{s,t}Q_{s,t}s_t + \mu_{b,t}\mathbb{Q}_{b,t}\mathscr{b}_t + \upsilon_t n_t$$

The banks problem is to select assets, $s_t, \mathscr{b}_t$, to maximise its net worth while still respecting the collateral constraint (incentive compatibility constraint) imposed by the households. The collateral constraint enforces a limit on the leverage ratio of the bank.

$$W_t(n_t) = \max_{s_t, \mathscr{b}_t} V_t(s_t, \mathscr{b}_t, n_t)$$

Subject to:

$$V_t(s_t, \mathscr{b}_t, n_t) \geq \theta Q_{s,t}s_t + \Delta\theta(\mathbb{Q}_{b,t}\mathscr{b}_t)$$

The above problem can be rewritten using the incentive compatibility constraint with a lagrange multiplier $\lambda_t$ associated with this constraint:

$$\max \quad V_t(\cdot) + \lambda_t(V_t(\cdot) - \theta Q_{s,t}s_t + \Delta\theta\mathbb{Q}_{b,t}\mathscr{b}_t)$$

$$\Rightarrow (1+\lambda_t)V_t(\cdot) - \lambda_t(\theta Q_{s,t}s_t + \Delta\theta \mathbb{Q}_{b,t}\boldsymbol{b}_t)$$

Writing out the equation to maximise can be summarized as:

$$(1+\lambda_t)[\mu_{s,t}Q_{s,t}s_t + \mu_{b,t}\mathbb{Q}_{b,t}\boldsymbol{b}_t + v_t n_t] - \lambda_t(\theta Q_{s,t}s_t + \Delta\theta \mathbb{Q}_{b,t}\boldsymbol{b}_t)$$

The first order conditions are therefore:

$$\begin{aligned}
\frac{\partial}{\partial s_t} &= (1+\lambda_t)\mu_{s,t}Q_{s,t} = \lambda_t \theta Q_{s,t} \\
\Rightarrow \mu_{s,t} &= \frac{\lambda_t}{1+\lambda_t}\theta \\
\frac{\partial}{\partial \boldsymbol{b}_t} &= \mu_{b,t}\mathbb{Q}_{b,t}(1+\lambda_t) = \lambda_t \Delta\theta \mathbb{Q}_{b,t} \\
\Rightarrow \mu_{b,t} &= \frac{\lambda_t}{1+\lambda_t}\Delta\theta = \Delta\mu_{s,t}
\end{aligned}$$

The complementary slackness condition (lagrange multiplier times the constraint) is written below. It must hold that either the constraint is binding and therefore the lagrange multiplier is non-zero (positive) or the constraint does not bind and the lagrange multiplier $\lambda_t$ is zero.

$$\lambda_t[\mu_{s,t}Q_{s,t}s_t + \mu_{b,t}\mathbb{Q}_{b,t}\boldsymbol{b}_t + v_t n_t - (\theta Q_{s,t}s_t + \Delta\theta \mathbb{Q}_{b,t}\boldsymbol{b}_t)] = 0$$

Since we are assuming that the constraint binds with equality it must be that the terms inside the bracket are zero, therefore using the complementary slackness condition we can write:

$$\Rightarrow \mu_{s,t}Q_{s,t}s_t + \mu_{b,t}\mathbb{Q}_{b,t}b_t + v_t n_t = \theta Q_{s,t}s_t + \Delta\theta \mathbb{Q}_{b,t}\boldsymbol{b}_t$$

Rewriting the constraint in terms of the net worth $n_t$ :

$$\Rightarrow v_t n_t = (\theta - \mu_{s,t})Q_{s,t}s_t + \Delta(\theta - \mu_{s,t})\mathbb{Q}_{b,t}\boldsymbol{b}_t$$

$$\Rightarrow v_t n_t = (\theta - \mu_{s,t})[Q_{s,t}s_t + \Delta\mathbb{Q}_{b,t}\boldsymbol{b}_t$$

$$\frac{v_t}{\theta - \mu_{s,t}}n_t = Q_{s,t}s_t + \Delta\mathbb{Q}_{b,t}\boldsymbol{b}_t$$

$$\phi_t n_t = Q_{s,t}s_t + \Delta\mathbb{Q}_{b,t}\boldsymbol{b}_t$$

Where

$$\phi_t = \frac{v_t}{\theta - \mu_{s,t}} \quad \text{Leverage Ratio in the Core region}$$

The leverage ratio, $\phi_t$, is the maximum value of assets over net worth that the banker can hold without violating its incentive compatibility constraint. If the incentive constraint binds (assumed it does) then this is the leverage of the bank.

The beginning of period value function $W_t$ is also linear and can be written as a function of the net worth of the banker:

$$\begin{aligned} W_t(n_t) &= \mu_{s,t}(Q_{s,t}s_t + \Delta\mathbb{Q}_{b,t}\mathscr{b}_t + v_t n_t \\ &= (\mu_{s,t}\phi_t + v_t)n_t \\ &= \theta\phi_t n_t \end{aligned}$$

Using the beginning of period value function we can derive the end of period value function by rewriting $V_{t-1}$ and inserting $W_t(n_t)$:

$$\begin{aligned} &\mu_{s,t-1}Q_{s,t-1}s_{t-1} + \mu_{b,t-1}\mathbb{Q}_{b,t-1}\mathscr{b}_{t-1} + v_t n_{t-1} \\ &\qquad = E_{t-1}\Lambda_{t-1,t}\{(1-\sigma)n_t + \sigma W_t(n_t)\} \end{aligned}$$

The flow of funds for the banker is given by the returns on holding securities and the returns on holding Core and Periphery bonds minus the interest payments that the banker owes the households for their deposits.

$$n_t = R_{k,t}Q_{s,t-1}s_{t-1} + \mathscr{R}_{b,t}\mathbb{Q}_{b,t-1}\mathscr{b}_{t-1} - R_t d_{t-1}$$

Net worth develops as the benefit of holding claims on non-financial firms $s_{t-1}$, which is $(R_{K,t} - R_t)Q_{s,t-1}$ and the benefit of holding government bonds plus the previous periods net worth.

$$n_t = (R_{K,t} - R_t)Q_{s,t-1}s_{t-1} + (\mathscr{R}_{b,t} - R_t)\mathbb{Q}_{b,t-1}\mathscr{b}_{t-1} + R_t n_{t-1}$$

Using what we have derived above it is possible to find the values of the coefficients $\mu_{s,t}, \mu_{b,t}$ of our linear value function:

$$\begin{aligned} &\mu_{s,t-1}Q_{s,t-1}s_{t-1} + \mu_{b,t-1}\mathbb{Q}_{b,t-1}\mathscr{b}_{t-1} + v_t n_{t-1} \\ &\qquad = E_{t-1}\Lambda_{t-1,t}\{[(1-\sigma) + \sigma\theta\phi_t](R_{K,t} - R_t)Q_{s,t-1}s_{t-1} \\ &\qquad\qquad + (\mathscr{R}_{b,t} - R_t)\mathbb{Q}_{b,t-1}\mathscr{b}_{t-1} + R_t n_{t-1}]\} \end{aligned}$$

Which therefore means that:

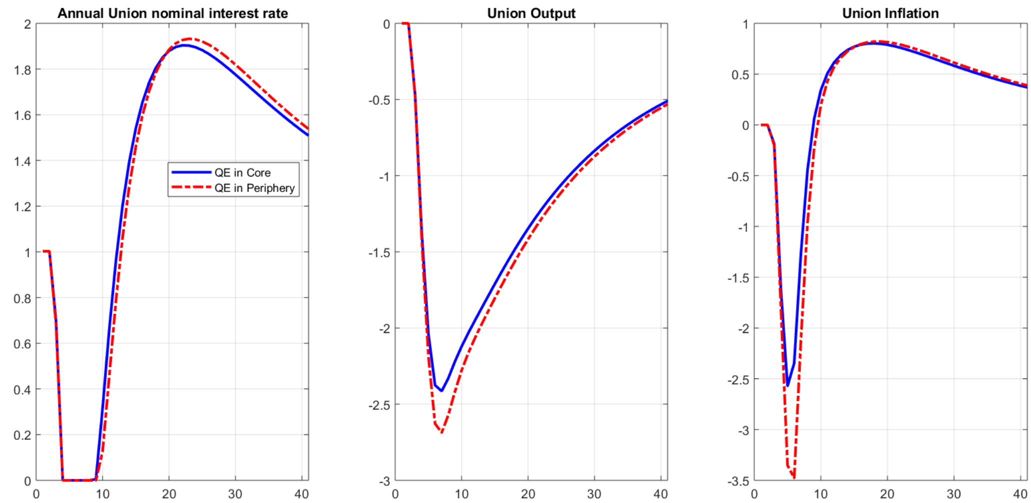
$$\begin{aligned}
\mu_{s,t} &= E_t\Omega_{t,t+1}(R_{k,t+1} - R_{t+1}) \\
\mu_{b,t} &= E_t\Omega_{t,t+1}(\mathscr{R}_{b,t+1} - R_{t+1}) \\
\upsilon_t &= E_t\Omega_{t,t+1}R_{t+1} \\
\Omega_{t-1,t} &= \Lambda_{t-1,t}[1 - \sigma + \sigma\theta\phi_t]
\end{aligned}$$

The bank's stochastic discount factor is given by $\Omega_{t-1,t}$, this is derived from the household's discount factor but additionally takes into account the probability of the banker exiting, thus it is augmented by the shadow value of unit of the net worth of the bank. This reflects the benefit of holding a larger amount of net worth, allowing the banker to retain more assets (whilst respecting the leverage restraint), and forms a crucial part of the financial accelerator mechanism. The amount of assets a bank is allowed to hold is partly determined by the absconding rate $\theta$. The above solution shows that the end of period value function for the banker is linear and the coefficients of this are independent of bank specific variables. This means that it is possible to aggregate the banking sector and solve the model as if there was only one large bank (a representative bank) or a multitude of identical banks per region. The aggregation (which is used in the computation) can be found in the main text.

## 3.D Asset Purchases in Core or Periphery

This section outlines the impact of bond purchases by the monetary union in either the Core or Periphery region. Figure 3.8 displays the result of the monetary union purchasing bonds worth 10% of GDP in the Periphery region compared to the equivalent value of bonds purchased in the Core region. Purchasing bonds in the Core region helps to support output and inflation by more than in the Periphery. It is important to stress that this finding is partly due to assuming the Core region has a higher level of technology, $A_t$, and therefore supporting production in the Core region is more beneficial than supporting production in the Periphery. Moreover, since households are able to freely borrow and save their marginal propensities to consume across the monetary union are similar, which may not be the case in reality. For instance, if a greater share of households act hand-to-mouth in the Periphery then monetary stimulus would lead the households in the Periphery to increase their consumption by more than households in the Core and therefore support output by more.

Figure 3.8: Bond Purchases in Core versus Periphery




The effective lower bound is reached by a series of capital quality and then demand shocks. The bond purchase shock is calibrated to reach 10% of GDP for the first two quarters in the Periphery region and equivalent value for the Core. The bond purchases follow an AR(1) process.

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