Universitat Autònoma de Barcelona

On Education, Open Innovation and Economic Growth

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Thanks to my parents and Fernando.

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Chapter 1

General Introduction

Human capital accumulation and technology advance are among the main engines of economic growth. Human capital accumulation can directly generate growth as it is a productive factor. It can also contribute to raising technical progress and technological progress, in turn, improves the total factor of productivity and hence allows for more efficient production and brings out economic growth. Apart from education, learning by doing also contributes to the accumulation of human capital. In fact, the recent emergence of open innovation has facilitated the increased flows of knowledge and, therefore, promoted the accumulation of human capital. Together with analyzing how public policies for education affect the accumulation of human capital, this thesis aims at studying how the emergence of open goods and open innovation affect R&D competition and, then, economic growth.

In chapter 2, we analyze how public policies for self-financing education, public fund for loans and deferred deductibility of education expenses, affect growth in an overlapping generations economy where individuals can be borrowing-constrained on human capital investment. We show that public loans positively affect growth in the unconstrained economy, while how tax deductibility affects growth depends on the magnitude of both public loans and tax deductibility. In the borrowing-constrained economy, public loans positively affect growth, while tax deductibility does not affect growth. Both government policies affect the borrowing-constraint tightness and, therefore, can shift the economy from being borrowing-constrained to unconstrained or vice versa.

In chapter 3, we study how open goods affect the economy in the long run. We model an economy with open and private goods where individuals have to allocate their time for human capital acquisition, working in the private goods sector and developing open goods. We incorporate the characteristics of open goods in the maximization problems and examine how the amount of time that individuals devote for developing open goods instead of working in the private goods sector or accumulating human capital affects economic growth. We also examine the social planner problem and its difference with the market allocation.

Chapter 4 aims at studying how different types of R&D activities—open source, imitation and conventional R&D—affect innovation competition and, then, the economy in the long run. We model an economy with standardized goods and quality goods where individuals with non-homothetic preference have to allocate their budget for standardized goods and quality goods. There is a continuum of industries with duopoly production in each industry. Both industry leaders and followers invest in R&D. Technological leaders invest in R&D for higher profit of higher quality products and to reduce the risk of being copied or surpassed by followers or new entrants. Followers invests in R&D to catch up with the leaders or to gain the technological leadership. We incorporate the characteristics of conventional R&D, copying and open innovation in the maximization problems of multi-quality firms and aim at examining how open innovation affects R&D investments of firms with different technology levels and then its effects on economic growth.

Chapter 2

Self-Financing Education, Borrowing Constraints, Government Policies, and Economic Growth

2.1 Introduction

As widely accepted in the literature, human capital accumulation is one of the main engines of growth (see Lucas, 1988). In any society, young individuals are characterized as not having accumulated assets in order to pay for education, an education that provides them with a human capital level and, then, will allow them to develop better careers and earn higher salaries. Financing human capital should therefore be attached a great importance to. Apart from altruistic parents and/or public education and/or public subsidies to education, young individuals can self-finance their education by getting loans from government and/or private financial markets and pay off their loans while working later on. This paper analyzes how government policies for self-financing education affect economic growth. Specifically, we stress the connection between these policies and the borrowing-constraint tightness of young individuals.

We consider an overlapping generations economy with endogenous human capital formation depending on investment in education and the level of human capital of the previous generation. When young, individuals borrow to invest in education, which endows them with a level of human capital. Individual loans come either from private credit markets or from public funds. However, due to the supply side of the financial market, individuals could be borrowing-constrained and, then, unable to finance their desired education. When adult, individuals work and use their incomes to consume, pay back the education loans, pay lump-sum taxes and save. When old, they consume their savings returns.

We analyze the importance of two public policies on the formation of human capital and, then, growth when human capital investment has no risk. In this way, having no risk and no altruistic parents, we highlight the pure effects of these policies on the financing of human capital and, then, growth without having any indirect financing effect. These policies are a public fund for education loans and deferred deductibility of education expenses.[1] Thus, and by not considering either public education or public subsidies, these two policies imply that the education of a generation will be ultimately paid by the same generation.[2] In this way, we concentrate on to what extent the government education policies affect economic growth when education is completely self-financing. We assume that both public policies are financed through lump-sum taxes. Therefore, they imply the same negative income effect for individuals and, then, aggregate savings will decrease. The difference between public loans for education and deferred deductibility of education expenses is two-fold. Firstly, while deductibility directly distorts the price of education, public loans indirectly distort the price via a higher supply of aggregate savings. Secondly, and perhaps most importantly, public loans can alleviate or break individuals' borrowing constraints because of this increase in aggregate savings, but deductibility of education expenses. Thus, the two policies have opposite effects on the borrowing-constraint tightness: while public loans lessen the pressure in the private credit market, tax deduction tightens the borrowing constraint.

Our results are categorized into three points: the effects of government policies on economic growth when young individuals are and are not borrowing-constrained, and the effects of government policies on the borrowing constraint tightness of young individuals. First, in the unconstrained economy, public loans always positively affect economic growth since the increase in public savings more than compensate the decrease in private savings as a consequence of the negative income effect for individuals due to the lump-sum tax. However, an increase in tax deductibility has two opposite effects on the net price of education loans: a direct effect, since a higher tax deductibility implies a lower net price for education loans and, then, the loan demand increases; and an indirect effect, since this increase in the demand for loans leads to an increase in the equilibrium interest rate which, in turn, increases the net price and, then, the loans demand decreases. How this increase in tax deductibility affects economic growth depends on which effect is dominant. Thus, when the direct effect is dominant, an increase in tax deductibility positively affects economic growth since education investment increases, whereas when the indirect effect is dominant, it is the other way around. Overall, which effect is dominant depends on the magnitude of public loans, tax deductibility itself and the individual discount rate, since a higher discount rate means higher savings and, then, a lower

[1] Although we will show that in U.S. this deduction is on the 100% of the interest rate, we consider the possibility to deduct also the principal, as the case of mortgage loans in some countries.

[2] Note that, although the public fund is built up by all the previous generations, individuals have to repay their loans.

net price for education loans. Specifically, when tax deductibility is sufficiently low, the direct effect is always dominant since an increase in tax deductibility implies a considerable reduction in the net price of loans. But when tax deductibility is sufficiently high, the effect of an increase in tax deductibility depends on the magnitude of public loans. Thus, when public loans are scarce, the indirect effect is dominant since an increase in tax deductibility will lead to a considerable increase in private loan demand. As a result, the interest rate will increase considerably. When public loans are sufficiently high, the increase in private loan demand will not be high enough and, then, the direct effect will dominate.

Second, in the borrowing-constrained economy, a numerical exercise suggests that public loans for education positively affects economic growth. An increase in public loans lessens the borrowing constraint since it allows more individuals to be able to access education loans and, hence, has a positive effect on education investment that, in turn, fosters economic growth. In contrast, an increase in tax deductibility does not affect economic growth. Individuals would increase the demand of loans as its net price becomes cheaper, but since the economy is borrowing-constrained, they cannot increase their loans.

Third, we show that both government policies determine if the economy is borrowing-constrained or not. Since private lenders worry about default, individuals can borrow at most a fraction of their life-cycle income. We define this fraction as the collateral rate. Then, there exists a particular value of this collateral rate, says the critical value, such that if the collateral rate is above it then individuals are not borrowing-constrained. We show that both government policies affect this critical collateral rate and, therefore, can shift the economy from being borrowing-constrained to unconstrained or vice versa. In particular, an increase in public loans has two effects on the critical value of the collateral rate. Firstly, there is a direct effect since the demand for private loans will decrease and, as a result, the economy will be more likely to be unconstrained. This, in turn, will positively affect economic growth. And, secondly, there is an indirect effect since a higher growth rate will consequently lead to a higher demand for loans and, as a result, the economy will be more likely to be constrained. Similarly, an increase in tax deductibility has also two effects on the critical collateral rate. Firstly, there is a direct effect since the demand for private loans will increase and, as a result, the economy will be more likely to be constrained. And secondly, there is an indirect effect via the growth rate which depends on the government policy values. A numerical exercise suggests that the critical collateral rate is decreasing in public loans whereas it is increasing in tax deductibility. In conclusion, alternative government policies affect in different ways the severity of the borrowing constraint and, then, growth.

The paper is organized as follows. Following a literature review, in the next section, we present the model and define the fundamental concepts. In Section 3 and Section 4, we study the effects of the public fund and tax deduction on economic growth when the borrowing constraint is not binding and binding, respectively. In Section 5, we derive the critical value of the collateral rate which determines if the economy is constrained or unconstrained and analyze the interactions of both government policies and the borrowing constraint tightness via this critical value. Section 6 concludes the paper. All proofs are in the Appendix.

Literature review. In contrast to our paper, considerable attention of economists has focused on studying the formation of human capital, education policies and their effects on the economy in the presence of altruism. For example, Glomm and Ravikumar (1992) and Eckstein and Zilcha (1994) discuss the distinction between economies with public education and those with private education. Milesi-Ferretti and Roubini (1998) and Brauninger and Vidal (2000) study the effect of a public subsidy on private education. And Zhang (1996) and Blankenau (2005) analyze the effects of both, public education and public subsidies. But little attention has been devoted if parents are not altruistic. In this case, why then to publicly finance education if parents are not altruistic? While Soares (2003) shows that agents that get a large fraction of their income from the return on their physical capital are interested in a higher level of human capital of future workers and, therefore, support for public funding of education, Boldrin and Montes (2005) propose public education as a borrowing-lending scheme: working individuals want to pay public education to young because they will pay back a public pension when old.

In the recent years, a large body of literature document the connection between individual abilities, borrowing constraints, public policies and schooling decisions. Thus, while Abbott et al. (2016) find that the educational financial aid system in the U.S. improves welfare, and removing it would reduce GDP by 4-5 percentage points in the long run, Garriga and Keightley (2016) find that the impact of borrowing constraints on schooling enrollment are significant when the constraints are severely tightened and the option to work while in school is removed. Closely related to our work, Lochner and Monge-Naranjo (2011, 2012) examine the effects of borrowing constraints, government public loans and subsidies to education on schooling attainment in the presence of innate abilities. They suggest that endogenous borrowing constraints make human capital investment more sensitive to government education subsidies and that private lending markets play an important role in how human capital accumulation responds to changes in policies. Nevertheless, our focus is rather on the interaction between borrowing constraints, self-financing education and growth. A complementary analysis is Findeisen and Sachs (2016), who show that an education public loan system coupled with income-contingent repayment can always be designed in a Pareto optimal way. To our knowledge, only Stancheva (2016) introduces deferred tax deductibility of human capital expenses. However, different from us, she uses tax deductibility as one of the fiscal instruments in the design of a second-best optimal tax scheme for human capital accumulation over the life-cycle.

As apposed to our work where we consider no risks of human capital investment, a series of other papers study the role of government policies in education, such as taxes and subsidies, in the presence of idiosyncratic labor income risk (see Krebs, 2003, Kass and Zink, 2011, or Krueger and Ludwig, 2016) or risk during the human capital accumulation process (see Tsiddon, 1992, Kalemli-

Ozcan et al., 2000, Gottardi et al., 2015, or Lochner and Monge-Naranjo, 2016). Specifically, Krebs (2003) studies the connection between human capital risk and growth and conclude that a reduction in uninsurable idiosyncratic labor income risk decreases physical capital investment, but increases human capital investment, growth and welfare. Krueger and Ludwig (2016) find that progressive taxes provide social insurance against idiosyncratic wage risk but distort the education decision of households such that optimally chosen tertiary education subsidies mitigate these distortions. And Gottardi et al. (2015), in an environment with uninsurable risk to human capital accumulation, conclude that it is beneficial to tax both labor and capital income.

2.2 The Economy

2.2.1 Households

Consider an overlapping generations economy in which individuals live for three periods: in the first period they study, in the second period they work, and in the third period they retire. Working population at time t is N_t and grows at the rate n. An individual born at time $t-1$ has to borrow l_{t-1} to invest in education, which endows her with a number of efficiency units of labor, measured by the human capital level h_t. She is endowed with one unit of labor time that will be supplied inelastically in the second period. Human capital depends on the investment in education and the level of human capital in the previous period. In particular, we assume

$$h_t = \delta l_{t-1}^{\gamma} h_{t-1}^{1-\gamma}, \tag{2.1}$$

where $\gamma \in (0,1)$. The educational loan can be public or private. Thus, $l_{t-1} = l_{t-1}^{pr} + l_{t-1}^{pu}$, where l_{t-1}^{pr} is the private loan and l_{t-1}^{pu} is the public loan. In the second period, the individual works and gets an income $w_t h_t$, where w_t is the wage per efficiency unit of labor. She consumes c_{1t}, saves s_t, pays two lump-sum taxes v_t and m_t,[3] and repays the loan of the previous period $R_t\,(1-g_t)\,l_{t-1}$, where $R_t = 1 + r_t$ is the interest factor, r_t is the interest rate, and g_t is a proportional tax-deductible amount on the education expenses. Note that we consider the possibility to deduct both the interest rate and the principal of the loan. The budget constraint in the second period of an individual born at time $t-1$ is

$$w_t h_t - v_t - m_t = R_t\,(1-g_t)\,l_{t-1} + c_{1t} + s_t. \tag{2.2}$$

In the third period, the individual uses the return from savings $R_{t+1}s_t$ to consume c_{2t+1}. Thus,

$$c_{2t+1} = R_{t+1}s_t. \tag{2.3}$$

Moreover, since private lenders worry about default, individuals face the following borrowing constraint when asking for private loans in the first period:

[3] Although we could have only one lump-sum tax, for ease of exposition we consider two different ones.

$$l_{t-1}^{pr} \leq \phi w_t h_t, \tag{2.4}$$

where $\phi \in (0,1)$ states the maximum quantity individuals can borrow from the private capital market given their expected future income. We define this fraction as the collateral rate. Note that individuals want l_{t-1}^{pu} as big as possible, since the lower $l_{t-1}^{pr} = l_{t-1} - l_{t-1}^{pu}$, the more likely the restriction is not binding. Thus, public loans can alleviate or break individuals' borrowing constraints, but deductibility of education expenses. Combining (2.1) and (2.4), the restriction can be written as

$$\phi w_t \delta l_{t-1}^{\gamma} h_{t-1}^{1-\gamma} - l_{t-1}^{pr} \geq 0. \tag{2.5}$$

The individual maximizes $\ln c_{1t} + \beta \ln c_{2t+1}$ subject to (2.1), (2.2), (2.3) and (2.4). The optimal condition regardless of the borrowing constraint is

$$c_{2t+1} = c_{1t} \beta R_{t+1}, \tag{2.6}$$

which equates the marginal rate of substitution to the relative price. When the borrowing constraint is not binding, the optimal condition with respect to the loan is

$$R_t (1 - g_t) - \gamma w_t \delta l_{t-1}^{\gamma-1} h_{t-1}^{1-\gamma} = 0, \tag{2.7}$$

which equates the marginal income to the marginal cost of the loan. When the borrowing constraint is binding, then (2.5) holds with strict equality.[4]

2.2.2 Firms

Firms maximize profits, $(K_t)^{\alpha}(N_t h_t)^{1-\alpha} - w_t N_t h_t - R_t K_t$, where K_t is capital and $\alpha \in (0,1)$. The optimal conditions are

$$R_t = \alpha \left(\frac{K_t}{N_t h_t} \right)^{\alpha-1} = \alpha \left(\frac{k_t}{h_t} \right)^{\alpha-1} \tag{2.8}$$

and

$$w_t = (1-\alpha) \left(\frac{K_t}{N_t h_t} \right)^{\alpha} = (1-\alpha) \left(\frac{k_t}{h_t} \right)^{\alpha}, \tag{2.9}$$

where $k_t \equiv K_t / N_t$ is capital per capita. Dividing (2.8) by (2.9), we have

$$\frac{R_t}{w_t} = \left(\frac{\alpha}{1-\alpha} \right) \left(\frac{h_t}{k_t} \right). \tag{2.10}$$

[4] In this case, $R_t (1 - g_t) - \gamma w_t \delta l_{t-1}^{\gamma-1} h_{t-1}^{1-\gamma} < 0$, which means that the individual wants to increase the loan, but she cannot, since it is given by (2.5).

2.2.3 Government

The government levies workers two types of lump-sum taxes: a tax v_t to finance the tax deduction of the education loans,

$$v_t = g_t R_t l_{t-1}, \tag{2.11}$$

and a tax m_t to build a public fund for education loans. Defining F_t as the public fund, and noting that (2.11) implies that the interest rate paid for the public loan becomes a net income for the government, the fund's accumulation law is

$$F_t - F_{t-1} = N_t m_t + N_t \left(R_t - 1\right) l_{t-1}^{pu}, \tag{2.12}$$

which means that the increase in the public fund consists of the lump-sum tax and the interest rate of the public loan. Rewriting this equation in per capita terms, we have

$$f_t - \frac{f_{t-1}}{1+n} = m_t + \left(R_t - 1\right) l_{t-1}^{pu}. \tag{2.13}$$

Government loans are

$$N_{t+1} l_t^{pu} = F_t \leqslant N_{t+1} l_t$$

and, then,

$$\left(1+n\right) l_t^{pu} = f_t. \tag{2.14}$$

Combining (2.13) and (2.14), we have

$$\left(1+n\right) l_t^{pu} = m_t + R_t l_{t-1}^{pu}. \tag{2.15}$$

We assume the government fixes both g_t and l_t^{pu}. Then v_t and m_t will be endogenous.

2.2.4 Capital Market Clearing Condition

Savings $N_t s_t$ are lent to firms or to young individuals. Therefore,

$$s_t = \left(1+n\right)\left(k_{t+1} + l_t^{pr}\right). \tag{2.16}$$

Next, we derive the balanced growth path depending on the existence of financial frictions, that is, if the borrowing constraint is binding or not.

2.3 Non-Financial Frictions

2.3.1 Balanced Growth Path

Since the economy grows, we define $l_{t-1}^{pu} = \rho_{t-1} l_{t-1}$ and, as $l_{t-1} = l_{t-1}^{pr} + l_{t-1}^{pu}$, then $l_t^{pr} = \left(1 - \rho_t\right) l_t$, where $\rho_t \in (0,1)$ is the proportion of the public loan over the total loan at time t. Since the borrowing constraint is not binding, combining (2.1) and (2.7) we obtain

$$w_t h_t = \frac{R_t\left(1-g_t\right)}{\gamma} l_{t-1}. \tag{2.17}$$

Combining (2.2), (2.3), (2.6), (2.11) and (2.15) yields

$$\left(\frac{1+\beta}{\beta}\right) s_t = w_t h_t - \left(1-\rho_{t-1}\right) R_t l_{t-1} - (1+n)\, \rho_t l_t. \tag{2.18}$$

From (2.10) and (2.17), we have

$$k_t = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{1-g_t}{\gamma}\right) l_{t-1}. \tag{2.19}$$

Substituting (2.16) and (2.17) into (2.18), and after using (2.19), we obtain

$$\begin{aligned} &(1+n)\left(\left[\frac{1+\beta}{\beta}\right]\left[\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{1-g_{t+1}}{\gamma}\right)+(1-\rho_t)\right]+\rho_t\right)\varphi_{t-1} \\ &\qquad = \left[\left(\frac{1-g_t}{\gamma}\right)-\left(1-\rho_{t-1}\right)\right] R_t, \end{aligned} \tag{2.20}$$

where $\varphi_{t-1} = l_t/l_{t-1}$ is the loan's growth factor. From (2.7), (2.8) and (2.9), we have

$$R_t = \alpha^{\alpha}\left(1-\alpha\right)^{(1-\alpha)}\left(\frac{\gamma\delta}{1-g_t}\right)^{(1-\alpha)}\left(\frac{h_{t-1}}{l_{t-1}}\right)^{(1-\gamma)(1-\alpha)}. \tag{2.21}$$

And combining this equation with (2.1) yields

$$R_t = \alpha^{\alpha}\left(1-\alpha\right)^{(1-\alpha)}\left(\frac{\gamma\delta}{1-g_t}\right)^{(1-\alpha)}\left(\frac{\delta}{\varphi_{t-1}}\right)^{\frac{(1-\gamma)(1-\alpha)}{\gamma}}. \tag{2.22}$$

Finally, substituting (2.22) into (2.20) and evaluating at the balanced growth path, we obtain

$$\varphi_u = \left\{\frac{\left[\left(\frac{1-g}{\gamma}\right)-(1-\rho)\right]\left[\frac{\alpha^{\left(\frac{\alpha}{1-\alpha}\right)}(1-\alpha)\gamma\delta^{\frac{1}{\gamma}}}{(1-g)}\right]^{1-\alpha}}{(1+n)\left(\left[\frac{1+\beta}{\beta}\right]\left[\left(\frac{1-g}{\gamma}\right)\left(\frac{\alpha}{1-\alpha}\right)+(1-\rho)\right]+\rho\right)}\right\}^{\frac{1}{1+\frac{(1-\gamma)(1-\alpha)}{\gamma}}}, \tag{2.23}$$

where the subscript u denotes the unconstrained economy. Next propositions summarize the consequences on economic growth of a change in the public policy.

Proposition 1 *When individuals are not borrowing-constrained, public loans for education have always a positive impact on growth. That is, the higher the value of ρ, the higher the value of φ_u.*

Public loans always positively affect economic growth since the increase in public savings more than compensate the decrease in private savings as a consequence of the negative income effect for individuals due to the tax.

Proposition 2 *When individuals are not borrowing-constrained, there exist* $\overline{\beta}(\rho)$ *and* $\overline{g}(\rho)$ *such that when* $\beta > \overline{\beta}(\rho)$, *if* $g < \overline{g}(\rho)$ *then* $\partial\varphi_u/\partial g > 0$, *and if* $g \geq \overline{g}(\rho)$ *then* $\partial\varphi_u/\partial g \leq 0$; *and when* $\beta \leq \overline{\beta}(\rho)$ *then* $\partial\varphi_u/\partial g \leq 0$.

An increase in tax deductibility has two opposite effects on the net price of education loans: a direct effect, since a higher tax deductibility implies a lower net price for education loans and, then, the loan demand increases; and an indirect effect, since this increase in the demand for loans leads to an increase in the equilibrium interest rate which, in turn, increases the net price and, then, the loans demand decreases. How this increase in tax deductibility affects economic growth depends on which effect is dominant. Thus, when the direct effect is dominant, an increase in tax deductibility positively affects economic growth since education investment increases, whereas when the indirect effect is dominant, it is the other way around. Overall, which effect is dominant depends on the magnitude of public loans, tax deductibility itself and the individual discount rate, since a higher discount rate means higher savings and, then, a lower net price for education loans. Specifically, when tax deductibility is sufficiently low, the direct effect is always dominant since an increase in tax deductibility implies a considerable reduction in the net price of loans. But when tax deductibility is sufficiently high, the effect of an increase in tax deductibility depends on the magnitude of public loans. Thus, when public loans are scarce, the indirect effect is dominant since an increase in tax deductibility will lead to a considerable increase in private loan demand. As a result, the interest rate will increase considerably. When public loans are sufficiently high, the increase in private loan demand will not be high enough and, then, the direct effect will dominate.

2.3.2 Numerical exercise

Next, we illustrate the previous proposition through a numerical exercise.[5] The strategy is as follows: firstly, we calibrate for the values of γ and δ using U.S. economy statistics; and secondly, using these calibrated parameters, we show how the combination of the values of g and ρ decides their effects on the growth rate φ_u. The below table resumes the parameter values that we use in the calibration exercise (a detailed explanation is in the Appendix). With these parameter values, from equations (2.20) and (2.22) we obtain $\gamma = 0.1040049078$ and $\delta = 2.293560488$.

α	β	g	ρ	n	φ	R
0.33	0.739	0.1897	0.3	0.24458	1.48595	2.86294

[5] According to Cameron and Taber (2004), there is no evidence of borrowing constraints in education in the U.S. Therefore, we calibrate the parameters for the unconstrained economy.

In order to show how tax deduction affects the economy, we check the sign of the derivative of the growth rate φ_u with respect to g. Since the sign of the derivative depends on the value of ρ (see the Appendix), for each value of g there exists a threshold value of $\overline{\rho}$ such that if $\rho < \overline{\rho}$ then $\partial\varphi_u/\partial g < 0$, if $\rho > \overline{\rho}$ then $\partial\varphi_u/\partial g > 0$, and if $\rho = \overline{\rho}$ then $\partial\varphi_u/\partial g = 0$. Figure 3.1. shows the associated values of $\overline{\rho}$ for each value of g. For a sufficiently low value of g, $\partial\varphi_u/\partial g > 0$ no matter the value of ρ. But for a sufficiently high value of g, the magnitude of ρ decides the sign of $\partial\varphi_u/\partial g$. In particular, $\partial\varphi_u/\partial g < 0$ and $\partial\varphi_u/\partial g > 0$ when the combinations of values of ρ and g lie on the left side and the right side of the continuous line, respectively.

There are two opposite effects of a change in tax deductibility on the net price of education loans $R_t\,(1-g_t)$: a direct effect via $(1-g_t)$ and an indirect effect via R_t. An increase in g directly implies a lower net price for education loans but, as a consequence, the demand for loans will increase and, then, leads to an increase in the interest rate R_t. Therefore, how a change in tax deductibility g affects the demand for education loans depends on which effect dominates. Figure 3.1. shows that when $g < 0.584$, the direct effect always dominates and an increase in g leads to an increase in education loans which, in turn, has a positive impact on economic growth. For $0.584 < g < 0.9558$, the dominating effect depends on the magnitude of public loans for education. When the proportion of public loans over total loans is sufficiently high, the increase in private loans due to an increase in tax deductibility will not be high enough to make the indirect effect via R_t be the dominating effect. In the U.S. economy, where there is only a 100% of tax-deduction on the interest rate and, then, $g = 0.1897$, we should deduct a considerable part of the principal of the loan in order that g decreases φ_u.

Figure 3.1. also illustrates how the results change if we use a proportional tax on income τ_t instead of a lump-sum tax to finance for tax deduction. In this case, we can define a price wedge $R_t(1-g_t)/(1-\tau_t)$ instead of the net price of loans. Now, it is more likely that tax deductibility has a positive impact on growth.

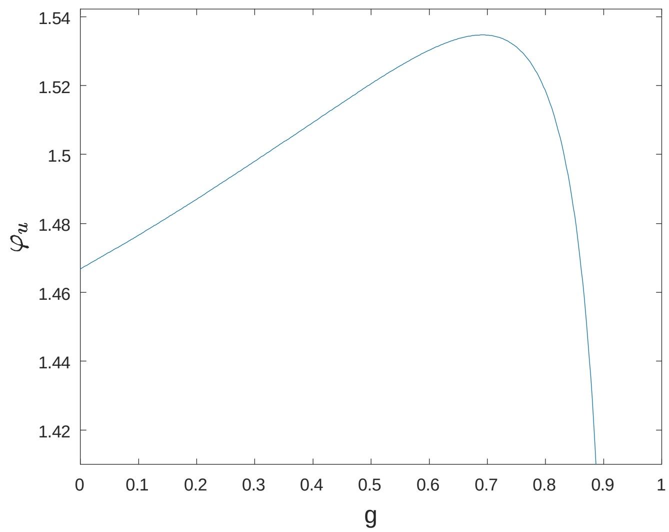

Figure 3.1. The sign of $\partial\varphi_u/\partial g$ as a function of ρ and g in the cases of a lump-sum tax and a proportional tax.

Figure 3.2 illustrates how tax deductibility affects φ_u when we set $\rho = 0.3$, as in the U.S. economy statistics. It shows that an increase in g decreases the growth rate only when a considerable part of the principal of the loan is deducted. Moreover, according to Proposition 3.2, it is the case that $\beta = 0.739 > \overline{\beta}\,(0.3)$.

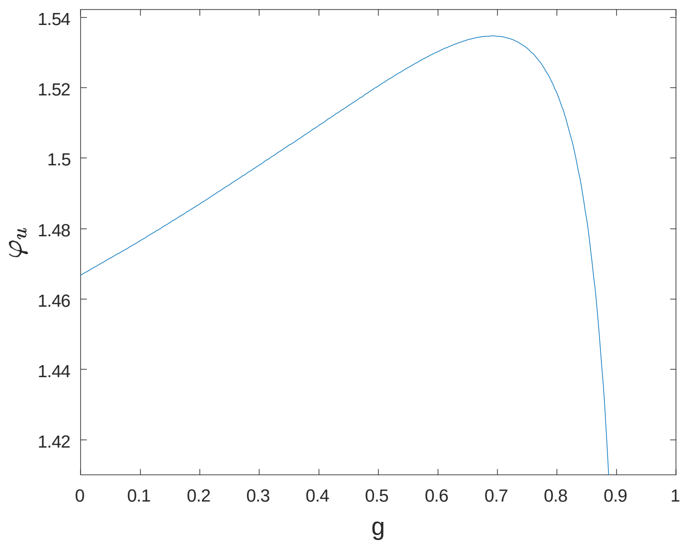

Figure 3.2. The effects of g on φ_u when $\rho = 0.3$.

2.4 Financial Frictions

2.4.1 Balanced Growth Path

When individuals are borrowing-constrained, then (2.4) is binding. Thus,

$$w_t h_t = \left(\frac{1-\rho_{t-1}}{\phi}\right) l_{t-1}. \tag{2.24}$$

From (2.16), (2.18) and (2.24), we obtain

$$\left(\frac{1+\beta}{\beta}\right)(1+n)\left[k_{t+1} + (1-\rho_t)\, l_t\right] = \left(\frac{1-\rho_{t-1}}{\phi}\right) l_{t-1} - \left(1-\rho_{t-1}\right) R_t l_{t-1} - (1+n)\,\rho_t l_t. \tag{2.25}$$

From (2.10) and (2.24) we have

$$k_t = \left(\frac{\alpha}{1-\alpha}\right)\left(\frac{1-\rho_{t-1}}{\phi}\right)\left(\frac{l_{t-1}}{R_t}\right). \tag{2.26}$$

Substituting this equation into (2.25) yields

$$(1+n)\left(\left[\frac{1+\beta}{\beta}\right]\left[\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{1-\rho_t}{\phi}\right)\frac{1}{R_{t+1}} + (1-\rho_t)\right] + \rho_t\right)\varphi_{t-1}$$

$$= \left(\frac{1-\rho_{t-1}}{\phi}\right) - \left(1-\rho_{t-1}\right) R_t. \tag{2.27}$$

Combining (2.1) and (2.24) gives

$$w_t = \left(\frac{1-\rho_{t-1}}{\phi}\right)\left(\frac{1}{\delta}\right)^{\frac{1}{\gamma}} \varphi_{t-1}^{\left(\frac{1-\gamma}{\gamma}\right)}. \tag{2.28}$$

Substituting this equation into (2.9) and using (2.8) yields

$$R_t = \alpha\left[\phi\left(\frac{1-\alpha}{1-\rho_{t-1}}\right)\right]^{\left(\frac{1-\alpha}{\alpha}\right)} \delta^{\left(\frac{1-\alpha}{\alpha\gamma}\right)} \varphi_{t-1}^{-\left(\frac{1-\gamma}{\gamma}\right)\left(\frac{1-\alpha}{\alpha}\right)}. \tag{2.29}$$

And finally, substituting (2.29) into (2.27) and evaluating at the balanced growth path, we obtain

$$(1+n)\left(\left[\frac{1+\beta}{\beta}\right]\left[\left(\frac{1-\rho}{(1-\alpha)\,\phi}\right)^{\left(2-\frac{1}{\alpha}\right)} \delta^{\left(\frac{\alpha-1}{\alpha\gamma}\right)} \varphi_c^{\left(\frac{1-\gamma}{\gamma}\right)\left(\frac{1-\alpha}{\alpha}\right)} + (1-\rho)\right] + \rho\right)\varphi_c$$

$$= \left(\frac{1-\rho}{\phi}\right) - (1-\rho)\,\alpha\left[\phi\left(\frac{1-\alpha}{1-\rho}\right)\right]^{\left(\frac{1-\alpha}{\alpha}\right)} \delta^{\left(\frac{1-\alpha}{\alpha\gamma}\right)} \varphi_c^{-\left(\frac{1-\gamma}{\gamma}\right)\left(\frac{1-\alpha}{\alpha}\right)}, \tag{2.30}$$

where the subscript c denotes the constrained economy.

Although we cannot generalize, our numerical exercise suggests that public loans for education positively affects economic growth. An increase in public loans lessens the borrowing constraint since it allows more individuals to be able to access education loans and, hence, has a positive effect on education investment that, in turn, fosters economic growth. In contrast, and as we can see from (2.30), φ_c does not depend on g. An increase of tax deductibility increases the demand of loans as its net price becomes cheaper, but since the economy is borrowing-constrained, individuals cannot increase their loans. However, it could be the case that a decrease in tax deductibility leads to a decrease in the demand of loans which, in turn, might shift the economy from being borrowing-constrained to unconstrained. Moreover, a change in public loans could also have similar effects on the economy, since it might break the borrowing constraint via affecting the demand of private loans. We analyze these effects in details in the next section.

2.4.2 Numerical Exercise

Using the same parameter values as in the previous section, Figure 4.1. shows that when the economy is constrained, the growth rate is strictly increasing and concave in ϕ. The higher the value of ϕ, the less the economy is constrained, the higher the investment in human capital and, hence, the higher the economic growth.

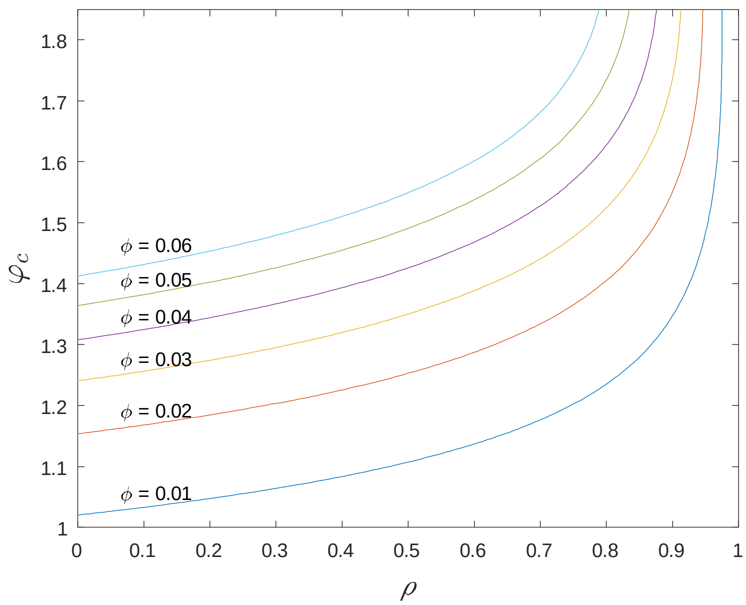

Figure 4.1. The effects of ϕ on φ_c.

We cannot plot φ_c against ρ since we have no value of ϕ in the real economy. However, Figure 4.2. shows the effects of public loans on the growth rate for different values of ϕ. For the same value of ρ, an increase in ϕ lessens the borrowing constraint and allows to increase education loans via private loans which, in turn, increases growth.

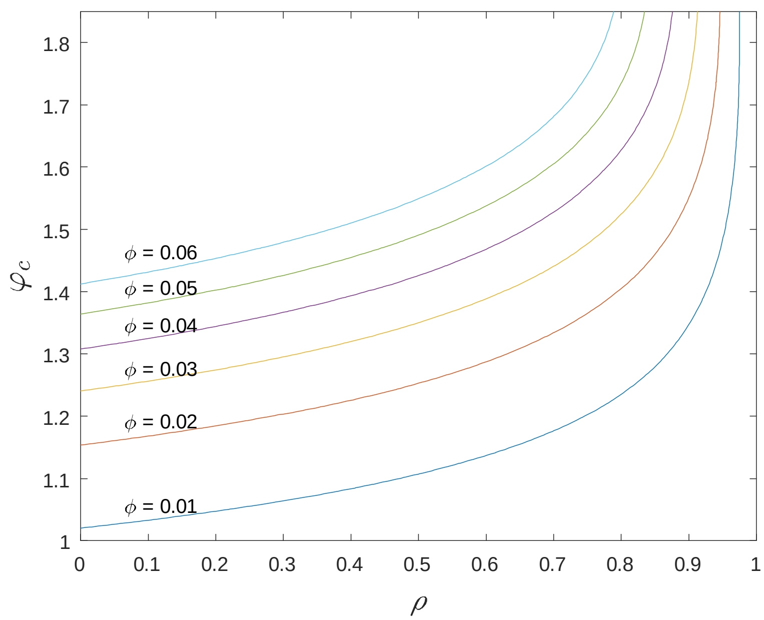

Figure 4.2. The effects of ρ on φ_c for different values of ϕ.

Note that figures 4.1. and 4.2. assume that individuals are borrowing-constrained for all values of ϕ, although this is not the case if ϕ is sufficiently high.[6] In the next section, we analyze how public policies determine if the economy is borrowing-constrained or not.

2.5 Critical Value of Collateral Rate

Since we have defined the collateral rate as the fraction of the life-cycle income that individuals can borrow at most, there exists a particular value of this collateral rate, says the critical value, such that if the collateral rate is above it then individuals are not borrowing-constrained. Define $\overline{\phi}$ as the level of the collateral rate that makes the borrowing constraint just binding. In other words, (2.7) is satisfied at the same time that (2.4) is binding.[7] Then, using (2.5) and (2.7), we have

$$\overline{\phi} = \frac{\gamma\,(1-\rho)}{R}. \tag{2.31}$$

[6] Given that we have assumed a 2% yearly growth rate, this value is 0.06, so that when $\phi \leq 0.06$ individuals are borrowing-constrained.

[7] We follow Caballé (1998), where he finds a critical value for the individual altruistic level.

Note that $\overline{\phi} = 0$ when $\rho = 1$, that is, when all the loans come from public funds, individuals have no need to ask for private loan and, therefore, they are financially unconstrained. Combining this equation with (2.22) gives

$$\overline{\phi} = \left[\frac{\gamma (1-\rho)}{\alpha^{\alpha} (1-\alpha)^{1-\alpha}} \right] \left(\frac{1-g}{\gamma\delta} \right)^{(1-\alpha)} \left(\frac{\varphi_u}{\delta} \right)^{\frac{(1-\gamma)(1-\alpha)}{\gamma}}. \tag{2.32}$$

Next proposition states when individuals are borrowing-constrained or are not.

Proposition 3 *(a) If $\phi \leq \overline{\phi}$ then the economy is financially constrained, i.e. the borrowing constraint holds with equality. (b) If $\phi > \overline{\phi}$ then the economy is financially unconstrained, i.e. the borrowing constraint does not hold.*

An increase in public loans has two effects on the critical value of the collateral rate. Firstly, there is a direct effect since the demand for private loans will decrease and, as a result, the economy will be more likely to be unconstrained. This, in turn, will positively affect economic growth. And, secondly, there is an indirect effect since a higher growth rate will consequently lead to a higher demand for loans and, as a result, the economy will be more likely to be constrained. Similarly, an increase in tax deductibility has also two effects on the critical collateral rate. Firstly, there is a direct effect since the demand for private loans will increase and, as a result, the economy will be more likely to be constrained. And secondly, there is an indirect effect via the growth rate which, as stated in Proposition 3.2., depends on the government policy values.

2.5.1 Numerical Exercise

Using the same parameter values as in the previous sections, Figure 5.1. shows that the critical value $\overline{\phi}$ is decreasing in ρ. Therefore, the direct effect dominates the indirect effect and, thus, a public fund for education loans could shift the economy from being financially constrained to financially unconstrained.

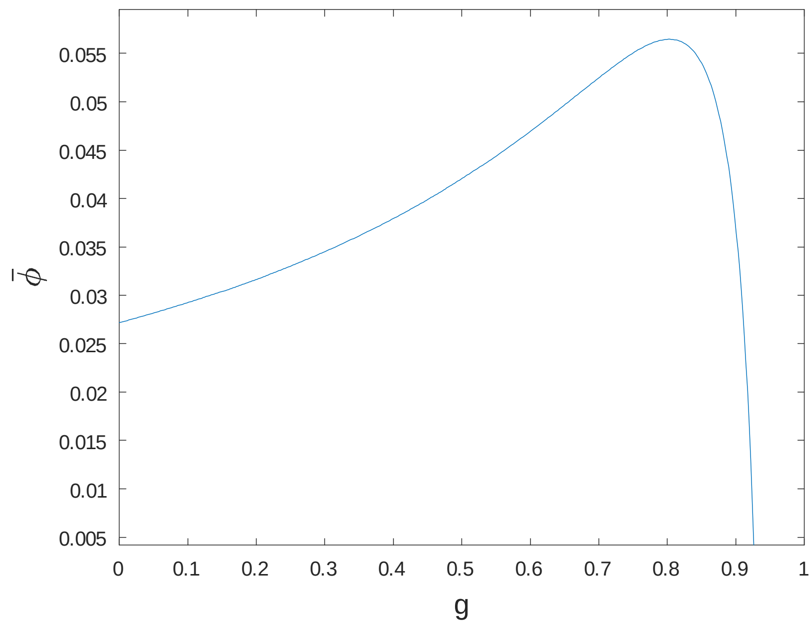

Figure 5.1. The effects of ρ on $\overline{\phi}$ when $g = 0.1897$.

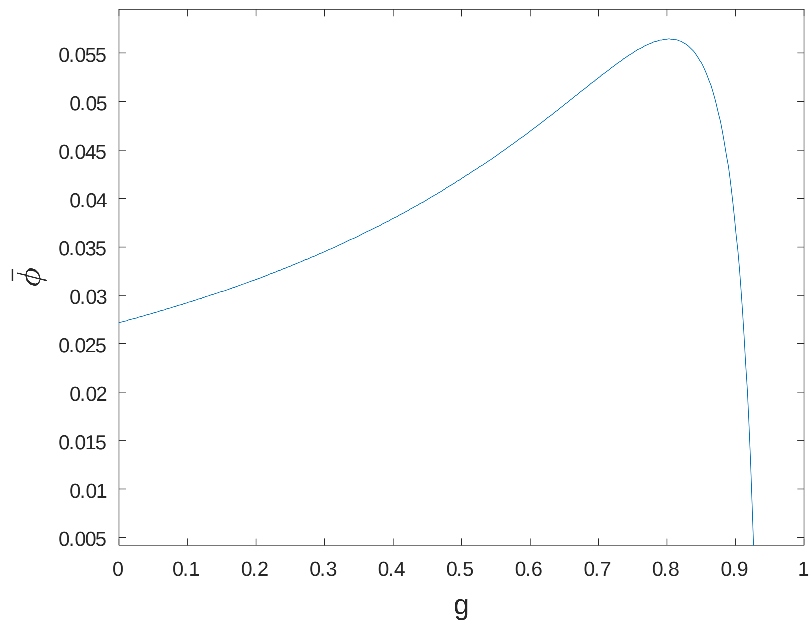

Figure 5.2. The effects of g on $\overline{\phi}$ when $\rho = 0.3$.

Figure 5.2. shows that there is a value of tax deductibility g, says $\widehat{g}$, such that if $g < \widehat{g}$ then an increase in tax deductibility will increase the critical value of collateral rate, whereas if $g > \widehat{g}$ then an increase in tax deductibility will lead to a decrease in the critical collateral value. Recall that for the U.S. case there is only a 100% of tax-deduction on the interest rate, so that $g = 0.1897 < \widehat{g}$. When tax deductibility increases, both the public and private demand for education loans increase, and this increase in private loan demand worsens the borrowing constraint.

2.6 Conclusions

In this paper, we develop a three period overlapping generations economy to analyze to what extent a public fund for education loans and deferred deductibility of education expenses affect economic growth. These two policies imply that the education of a generation is completely self-financed by the same generation. Since private lenders worry about default, individuals can borrow at most a fraction of their life-cycle income. We define this fraction as the collateral rate. Thus, individuals could be borrowing-constrained and, then, unable to finance their desired education. We show that there exists a particular value of the collateral rate, says the critical value, such that if the collateral rate is above it then individuals are not borrowing-constrained. Moreover, government policies could affect this critical collateral rate and, then, determine if the economy is borrowing-constrained or not.

We show that when young individuals are not borrowing-constrained, public loans always positively affect economic growth since the increase in public savings more than compensate the decrease in private savings as a consequence of the negative income effect for individuals due to lump-sum taxes. A numerical exercise suggests the same positive effect when young individuals are borrowing-constrained. This numerical exercise also suggests that the critical collateral rate is decreasing in public loans.

When young individuals are not borrowing-constrained, an increase in tax deductibility has two opposite effects on the net price of education loans: a direct effect, since a higher tax deductibility implies a lower net price for education loans and, then, the loan demand increases; and an indirect effect, since this increase in the demand for loans leads to an increase in the equilibrium interest rate which, in turn, increases the net price and, then, the loans demand decreases. How an increase in tax deductibility affects economic growth depends on which effect is dominant. In contrast, an increase in tax deductibility does not affect economic growth when young individuals are borrowing-constrained. A numerical exercise suggests that the critical collateral rate is increasing in tax deductibility.

In conclusion, alternative government policies affect in different ways both economic growth and the severity of the borrowing constraint. Future work should study how the endogenization of labor when young, as in Garriga and Keightley (2016) and Abbott et al. (2016), affects the relationship between both education policies and growth. While working when young reduces the demand of education loans and, hence, lessens the borrowing constraint, individuals have less time to attend classes. Thus, the final effects of both education policies on the acquisition of human capital could change.

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Appendix

Proof of Proposition 3.1.

It is straightforward to show that $\partial\varphi_u/\partial g > 0$.

Proof of Proposition 3.2.

Calculating $\partial\varphi_u/\partial g$ we obtain that

$$sign\ \ (d\varphi_u/dg) = sign\,(f(g))\,,$$

where

$$\begin{aligned} f(g) &= \left\{-\left(\frac{\alpha}{\gamma}\right)(1-g)-(1-\rho)(1-\alpha)\right\}\left\{(1+\beta)\left[\left(\frac{1-g}{\gamma}\right)\left(\frac{\alpha}{1-\alpha}\right)+1\right]-\rho\right\} \\ &\quad +\left[\left(\frac{1-g}{\gamma}\right)-(1-\rho)\right]\left[\frac{(1-g)(1+\beta)}{\gamma}\right]\left(\frac{\alpha}{1-\alpha}\right). \end{aligned}$$

Since $f(g)$ is a quadratic function of g with a positive coefficient of g^2 and $f(g)$ < 0 when $g = 1$, we can conclude that $f(g)$ has two roots, $\overline{g}$ and $\widehat{g}$, such that $\overline{g} < 1 < \widehat{g}$. If $f(0) > 0$ then $\overline{g} > 0$, $f(g) > 0$ for $0 \leq g < \overline{g} < 1$ and $f(g) < 0$ for $\overline{g} < g < 1$. Then,

$$\begin{aligned} f(0) &= \left\{-\left(\frac{\alpha}{\gamma}\right)-(1-\rho)(1-\alpha)\right\}\left\{(1+\beta)\left[\left(\frac{1}{\gamma}\right)\left(\frac{\alpha}{1-\alpha}\right)+1\right]-\rho\right\} \\ &\quad +\left[\left(\frac{1}{\gamma}\right)-(1-\rho)\right]\left(\frac{1+\beta}{\gamma}\right)\left(\frac{\alpha}{1-\alpha}\right) \end{aligned}$$

is positive whenever

$$\beta > \overline{\beta} \equiv 1 - \frac{\rho\left\{\left(\frac{\alpha}{\gamma}\right)+(1-\rho)(1-\alpha)\right\}}{\left[(2-\alpha)(1-\rho)-\left(\frac{1-\alpha}{\gamma}\right)\right]\left(\frac{\alpha}{1-\alpha}\right)\left(\frac{1}{\gamma}\right)+\left(\frac{\alpha}{\gamma}\right)+(1-\rho)(1-\alpha)}.$$

Numerical Exercise Values

We consider the U.S. economy is financially unconstrained, and that one period in our economy is equivalent to 20 years in the real economy. Therefore, we fix $\alpha = 0.33$, $\beta = 0.739$ so that the individual time discount value for one year is 0.985, $n = 0.24458$ so that population growth per year is 1.1%, $\varphi = 1.48595$ so that the economic growth rate is 2% per year, and $R = 2.86294$ so that the interest rate per year is 5.4%. Moreover, following Li (2013), we set $\rho = 0.3$. According to *https://studentaid.ed.gov/sa/types/loans/subsidized-unsubsidized*, the maximum undergraduate public student loan amount is 57,500 USD, and according to
https://www.irs.gov/publications/p970/ch04.html, for individuals with income

less than 60, 000 USD, tax deduction on student loans is only on the interest rate and with a maximum of 2, 500 USD. Taking into account that the public interest rate for student loans is 4.29%, we could assume that tax deduction covers all the student loan interest rate. Therefore, we set $g = 0.1897$ to comply with the definition of tax-deductible amount in our model.[8]

Proof of Proposition 5.1.

(a) We proceed by contradiction. Assume that the loan l is freely chosen with $\phi \leq \overline{\phi}$ and the borrowing restriction does not hold. Then, defining $\bar{l}$ as the loan associated to $\overline{\phi}$, $l \leq \bar{l}$ cannot be, since then the borrowing restriction would hold. Therefore, it must be that $l \geq \bar{l}$. Then, defining $\overline{\varphi}$ as the growth rate associated to $\overline{\phi}$, in a balanced growth path it must be true that $\varphi \geq \overline{\varphi}$,[9] otherwise it would exist a T such that $l_{t-1+T} < \bar{l}_{t-1+T}$. From (2.31), the $\overline{R}$ associated to $\overline{\phi}$ is

$$\overline{R} = \frac{1}{\overline{\phi}} \frac{\gamma (1-\rho)}{(1-g)}.$$

From (2.5) and (2.7), we have

$$R > \frac{1}{\phi} \frac{\gamma (1-\rho)}{(1-g)},$$

since the borrowing restriction does not hold. Then, $\phi \leq \overline{\phi}$ implies that $R > \overline{R}$, where the strict inequality comes from the fact that the borrowing restriction does not hold. From (2.22) and $R > \overline{R}$ we have $\varphi < \overline{\varphi}$, which cannot be.

(b) We proceed by contradiction. Assume that the borrowing restriction holds with equality with $\phi > \overline{\phi}$ so that $l \leq \bar{l}$. Then, in a balanced growth path it must be true that $\overline{\varphi} \geq \varphi$. Since the collateral restriction holds, from (2.7) it must be true that

$$R(1-g) - \gamma w \delta \left(\frac{l}{h}\right)^{\gamma-1} < 0,$$

which combined with (2.5) gives

$$R < \frac{1}{\phi} \frac{\gamma (1-\rho)}{(1-g)}.$$

Then, $\phi > \overline{\phi}$ implies that $R < \overline{R}$. From (2.29), $\phi > \overline{\phi}$ and $R < \overline{R}$ we have that $\overline{\varphi} < \varphi$, which cannot be.

[8] Instead, we use $R = 1.234$ as the interest factor to calculate this value of g since we consider an accumulative interest rate in a period of 4 years as the typical duration of undergraduate studies.

[9] In fact, $\varphi = \overline{\varphi}$ because $\overline{\varphi}$ is the growth rate associated to the unconstrained economy.

Chapter 3

The Economics of Open Goods

3.1 Types of Goods in Economics

The innovation in transportation technologies as the steam power engine at the beginning of nineteen century reduced the shipping costs considerably and, hence, allowed goods to be shipped to distant markets and marked the onset of the globalization process. Following that, the innovation in information and communication technologies has marked a new era in the process of globalization as it allows technological know-hows and ideas travel across geographical distance instantly. Apart from the free movement of ideas within internal channels where information is transmitted from one to another individually or within firms or organizations, the open platforms—such as wikipedia, the massive online open course (MOOC), edx.org or the online open source code software—grant everybody the access to knowledge freely and instantly. These channels foster the diffusion of knowledge and, hence, contribute substantially to the modern globalization process. We categorize such kind of platforms as a new type of good in the current era of advanced media technology and call it as open good. This open goods' significance has been increasing in our modern economy where the digital economy is a crucial part. In order to distinguish open goods with the conventionally characterized goods, we briefly go through how goods are categorized and then compare the characteristics of open goods with the existing categories.

Typically, goods are classified upon these two characteristics: excludability and rivalry. Excludability means that individuals can be effectively excluded from using the good whereas rivalry means that the use by one individual reduces availability to others. By considering the denial of these two characteristics, goods have been typically categorized into public goods, private goods, common-pool resources, and club goods. Table 1 summarizes the four types of goods and their characteristics.

Characteristics	*Excludable*	*Non-excludable*
Rival	**Private Goods**	**Common-Pool Resources**
Non-rival	**Club Goods**	**Public Goods**

Table 1. Classification of goods

A public good is a good that is both non-excludable and non-rival, provided by government or collective action or private agents (e.g. free-to-air television). The main problem arising from public goods is free-riding as nobody wants to pay for the creation of a public good because once it has been created everybody can enjoy it. A private good is a good that is excludable and rival, provided by government or private agents. A common-pool resource is a good that is rival but non-excludable, and provided by nature; the non-excludability of this good sometimes result in the well-known tragedy of the commons. A club good is a good that is non-rival but excludable, provided by private agents or small scale government.

As the rise of a new type of good such as the open source code software, we can see that the denial of a characteristic is not the only possible variation of it. We can also consider some degree of these characteristics and, then, make new categories. We can define an *anti-rival good* as a good that its using value to any particular individual increases as more individuals use the same good (see Weber, 2000, and von Hippel and von Krogh, 2003). By considering this anti-rival property, we could classify the goods into two more categories – network goods and open goods – as Table 2 shows.

Characteristics	*Excludable*	*Non-excludable*
Rival	**Private Goods**	**Common-Pool Resources**
Non-rival	**Club Goods**	**Public Goods**
Anti-rival	**Network Goods**	**Open Goods**

Table 2. Extended classification of goods

A network good is a good that is excludable and anti-rival, usually provided by private agents. An open good is a good that is non-excludable and anti-rival, and it is (usually) provided by private agents. Typically, open goods are associated with the Internet (e.g. some software) and, hence, are non-excludable as internet enables it to be copied an infinite number of times at no cost. Moreover, an open good is anti-rival since the society positively benefits from free riders. Free riders become an asset because although they could consume an open good without contributing to it, they still help to increase the "market share" and the importance of the consumed open good, and may help setting the quality or standards. For instance, some free riders may report any information about their consumption experience to the open good producers. Thus, the more free riders, the better (see Weber, 2000, and von Hippel and von Krogh, 2003). But then, why not all the agents are free riders? And why and how is an open good produced if its market price is zero because of its non-excludability?

One of the incentives of developers[1] when devoting time to develop the open good rather than having someone else doing it, is that they are trying to signal their abilities (see Lerner and Tirole, 2002). If the open good they are producing become a well-known open good, this could give their future potential employers information about their experience and skills and, therefore, developers would get higher chances for a better job offer later. Another reason is that a developer (or private business) can develop an open good for his own use. Some developers are paid by their employers to spend part of their time working on an open good, since these same employers are often using the open good as an input. Furthermore, spending time developing open goods could be a learning-by-doing process for developers, since they also learn from the feedbacks, critiques and corrections from the other users if the developer releases the good freely for everybody to obtain. Overall, the incentives of contributors to open goods and the properties of the open goods are summarized in the following (see Johnson, 2002):

[1] Since the most famous open goods are software programs as Linux, Apache or OpenOffice, among others, hereinafter we will call developers to the open-good producers. Thus, we will call producers to the private goods producers.

Incentives:

- i1) Preference to develop the open good rather than having someone else doing it. They could be trying to signal their abilities.
- i2) A personal or business' use of the open good they develop. This includes intrinsic rewards such as personal learning from developing. Furthermore, they also learn from the feedbacks, critiques and corrections from the other users if the developer releases the good freely for everybody to obtain.
- i3) Possible payments by their employers to spend part of their time working on an open good, since these same employers are often using the open good as an input.

Production:

- p1) In contrast with the standard collective action theory, a large group of people is more likely to provide the good than a small group. Although there is no apparent coordination among developers, a large group means that success in developing implies a higher recognition and, then, the signal about the personal abilities becomes greater.
- p2) The important thing is to promote distribution. After, development will take care of itself. Thus, open goods have different forms of property rights waiting that new goods based on open goods will become open goods, too.
- p3) Open goods are possible only when self-manufacture and/or distribution of products directly by developers can compete with commercial production and distribution. Thus, Internet has been crucial in the appearance of this type of good.
- p4) A static approach cannot capture adequately the incentives of a developer to release her work as an open good, since possible rewards will be in the future.

This paper studies how open goods affect the economy in the long run. We model an economy with open and private goods where individuals have to allocate their time for human capital acquisition, working in the private goods sector and developing open goods. We incorporate the characteristics of open goods in the maximization problems and examine how the amount of time that individuals devote for developing open goods instead of working in the private goods sector or accumulating human capital affects economic growth. We also examine the social planner problem and its difference with the market allocation. The rest of the paper is organized as follows. Section 2 models the market economy. Since the price of the open good in the market is zero, the calibration of this economy constitutes a new exercise at the same time that a challenge. Section 3 models the social planner economy. Now, the open good has a positive (shadow) price for the planner. Section 4 concludes.

3.2 Decentralized Economy with Open Goods

In this economy, there are two goods: private goods and open goods. Private goods are produced in the market by firms through capital and labor. Open goods are produced outside the market through (free) individual time, but the number of consumers affect positively its production. Individuals devote their time to work producing private goods, accumulate human capital or develop open goods. The total human capital of an individual depends on the time directly devoted for its accumulation and the time devoted to develop open goods. The human capital will be supplied in the labor market as efficiency units of labor. Individuals derive utility from private goods, but they can also derive utility either from open goods, either from the time devoted to develop open goods, or both things at the same time.

3.2.1 Individuals

In our economy, there are two types of goods: private goods y and open goods Θ. Population N grows at an exogenous rate n. Individuals live infinitely. Each individual is endowed with one unit of time and use a fraction μ of her time to work in the private goods sector, θ_1 to invest in human capital accumulation h and θ_2 to develop open goods. Individuals derive utility U from private goods consumption c and open-goods related activities. By open-goods related activities we mean the consumption of open goods as well as the joy of contributing to the development of open goods. King et al. (1988) show that in economies where consumption is not the only variable in the utility function, a balanced growth path exists if the utility function takes the following form:

$$U = \begin{cases} \frac{c^{1-\iota}}{1-\iota}.U_\Theta(\Theta,h) \text{ for } \iota > 0,\ \iota \neq 1 , \\ \ln c + U_\Theta(\Theta,h) \text{ for } \iota = 1 , \end{cases}$$

where $c^{1-\iota}/\left(1-\iota\right)$ is the CRRA utility derived from private goods consumption with intertemporal elasticity of substitution $1/\iota$, and $U_\Theta(\Theta,h)$ is the utility arising from open-goods related activities. This utility does not only depend on the open goods consumption but also on the level of efficiency units of labor $\theta_2 h$ devoted to develop open goods. The existence of a balanced growth path requires a constant intertemporal elasticity of substitution for $U_\Theta(\Theta,h)$ as well. We then assume that

$$U_\Theta(\Theta,h) = \begin{cases} \frac{J_{(\Theta,h)}^{1-\tau}}{1-\tau} \text{ for } \tau > 0,\ \tau \neq 1 , \\ \ln J \text{ for } \tau = 1 . \end{cases}$$

In order to capture the anti-rival property of open goods and the fact that developing open goods gives utility to the developer (property i2), we define $J_{(\Theta,h)}$ as:

$$J_{(\Theta,h)} = \left[\pi\left(\frac{N_\Theta}{N}\Theta\right)^\beta + (1-\pi)\left(\theta_2 h\right)^\beta\right]^{\frac{1}{\beta}} ,$$

where the anti-rival property is captured through $\Theta N_\Theta/N$, as the higher the proportion of population N_Θ/N consuming the open good the higher the use value for the individual. The second term in the brackets stands for the utility from spending time developing open goods[2]. The parameter $\beta \in (\infty, 1]$ determines the degree of substitutability of the two utilities. When $\pi = 0$, the individual does not consume open goods. When $0 < \pi < 1$, we have consumers of open goods that enjoy developing them (this captures property i2). Note that the case of $\pi = 1$ does not necessarily mean that the individual is free rider, since individuals can develop open goods without any direct increase in utility. Therefore, individuals utility at a given moment in time is defined as

$$U = \frac{c^{1-\iota}}{1-\iota}.\frac{\left[\pi\left(\frac{N_\Theta}{N}\Theta\right)^\beta + (1-\pi)(\theta_2 h)^\beta\right]^{\frac{1-\tau}{\beta}}}{1-\tau} \text{ for } \iota, \tau \neq 1.$$

The budget constraint of the individual is

$$\dot{a} = w\mu h + (r-n)a - c \ , \tag{3.1}$$

where w is the wage per efficiency unit of labor and a is the quantity of assets per capita. Note that the open good is non-excludable since its price is zero.

Human capital accumulation depends on the current level of human capital, time θ_1 devoted to increase human capital and time θ_2 devoted to develop open goods:

$$\dot{h} = h\left[\epsilon\theta_1^\upsilon + (1-\epsilon)\theta_2^\upsilon\right]^{\frac{1}{\upsilon}} - (\delta_h + n) h \ , \tag{3.2}$$

where $\epsilon \in [0, 1]$ and $\upsilon \in (\infty, 1]$ respectively captures the relative weight and complementarity between time for schooling and time for open goods, δ_h is the human capital depreciation rate. In this setting, individuals accumulate human capital not only by spending time for education but also by taking part in the production of open goods. Note that when $\epsilon = 1$ we have free riders, whereas when $\epsilon < 1$ the individual learning in producing open goods increases her human capital (this captures properties i1 and p4).

For simplicity, we consider a representative agent model where the individual consumes and produces open goods and, thus, $N_\Theta = N$. Then, the problem simplifies to

$$Max \quad \int_0^\infty e^{-(\rho-n)t}\frac{c^{1-\iota}}{1-\iota}.\frac{\left[\pi\Theta^\beta + (1-\pi)(\theta_2 h)^\beta\right]^{\frac{1-\tau}{\beta}}}{1-\tau} dt \tag{3.3}$$

subject to (3.1), (3.2) and

$$\mu + \theta_1 + \theta_2 = 1 \ . \tag{3.4}$$

The first-order conditions with respect to the control variables c, θ_1 and θ_2 of the corresponding current Hamiltonian of the maximization problem are,

[2] We could also assume that the utility is increasing in the level of human capital of the individual involved in the open goods developing.

respectively,

$$\lambda_1 = c^{-\iota} \frac{\left[\pi\Theta^{\beta} + (1-\pi)(\theta_2 h)^{\beta}\right]^{\frac{1-\tau}{\beta}}}{1-\tau} \ , \tag{3.5}$$

$$\lambda_1 wh = \lambda_2 h \left[\epsilon\theta_1^{\upsilon} + (1-\epsilon)\theta_2^{\upsilon}\right]^{\frac{1-\upsilon}{\upsilon}} \epsilon\theta_1^{\upsilon-1} \tag{3.6}$$

and

$$\lambda_1 wh =$$

$$\frac{c^{1-\iota}}{1-\iota}\left[\pi\Theta^{\beta} + (1-\pi)(\theta_2 h)^{\beta}\right]^{\frac{1-\tau-\beta}{\beta}} (1-\pi)\theta_2^{\beta-1}h^{\beta} + \lambda_2 h\left[\epsilon\theta_1^{\upsilon} + (1-\epsilon)\theta_2^{\upsilon}\right]^{\frac{1-\upsilon}{\upsilon}} (1-\epsilon)\theta_2^{\upsilon-1} \ , \tag{3.7}$$

where $\lambda_1$and λ_2 are the multipliers associated to (3.1) and (3.2), respectively.

Equation (3.5) states that the marginal utility of consuming private goods must be equal to the opportunity cost of accumulating them. Equation (3.6) and (3.7) state that, on the margin, time must be equally valuable in the three uses: working, human capital accumulation and open goods accumulation.

Substituting (3.5) and (3.6) into (3.7), we have

$$\frac{c}{1-\iota}(1-\pi)\theta_2^{\beta-1}h^{\beta-1} = \frac{\left[\pi\Theta^{\beta} + (1-\pi)(\theta_2 h)^{\beta}\right]}{1-\tau} w \left[1 - \frac{(1-\epsilon)\theta_2^{\upsilon-1}}{\epsilon\theta_1^{\upsilon-1}}\right]. \tag{3.8}$$

The rates of change of the shadow prices of the two types of capital are given by

$$\frac{\dot{\lambda}_1}{\lambda_1} = \rho - r \tag{3.9}$$

and

$$\frac{\dot{\lambda}_2}{\lambda_2} = \rho - \left[\epsilon\theta_1^{\upsilon} + (1-\epsilon)\theta_2^{\upsilon}\right]^{\frac{1-\upsilon}{\upsilon}} \epsilon\theta_1^{\upsilon-1} + \delta_h \ . \tag{3.10}$$

3.2.2 Private Goods Firms

Firms producing private goods use physical capital k and labor to produce and pay a wage w per efficiency unit of labor and an interest rate r per unit of capital. The production function is assumed to follow a CES form:

$$y = \left[\alpha k^{\sigma} + (1-\alpha)(\mu h)^{\sigma}\right]^{\frac{1}{\sigma}} \ , \tag{3.11}$$

where $\sigma \in (-\infty, 1)$. The first-order conditions of the optimal problem are

$$w = \left[\alpha k^{\sigma} + (1-\alpha)(\mu h)^{\sigma}\right]^{\frac{1}{\sigma}-1}(1-\alpha)(\mu h)^{\sigma-1} \tag{3.12}$$

and

$$r+\delta_k=\left[\alpha k^{\sigma}+(1-\alpha)(\mu h)^{\sigma}\right]^{\frac{1}{\sigma}-1}\alpha k^{\sigma-1} \ , \tag{3.13}$$

where $\delta_k \in (0,1)$ is the depreciation rate of physical capital. From these two equations we obtain

$$\frac{\mu h}{k}=\left[\frac{\left(\frac{r+\delta_k}{\alpha}\right)^{\frac{\sigma}{1-\sigma}}-\alpha}{1-\alpha}\right]^{\frac{1}{\sigma}} \tag{3.14}$$

and

$$w=\frac{1-\alpha}{\alpha}(r+\delta_k)\left[\frac{\left(\frac{r+\delta_k}{\alpha}\right)^{\frac{\sigma}{1-\sigma}}-\alpha}{1-\alpha}\right]^{\frac{\sigma-1}{\sigma}} \ . \tag{3.15}$$

3.2.3 Open Goods Accumulation Law

The open goods accumulation law is

$$\dot{\Theta}=\eta\left(\Theta\frac{N_\Theta}{N}\right)^{\zeta}\left(\int_0^{N_{\theta_2}}\theta_2 h dh\right)^{1-\zeta} \ , \tag{3.16}$$

implying that the higher the proportion of population consuming open goods N_Θ/N, the higher its production. Thus, free riders become an asset. Also note that the current stock of open good Θ affects its accumulation $\dot{\Theta}$ (both things capture property p2). The term in the second bracket captures the contribution of all the people N_{θ_2} developing open goods (this captures properties p1 and i2), where we have assumed no duplication. $\zeta \in (0,1)$ is the output elasticity of $\Theta\frac{N_\Theta}{N}$. Since we consider a representative agent model in which we have $N_\Theta = N$, $\int_0^{N_{\theta_2}}\theta_2 h dh=\theta_2 hN$. Otherwise, we would have individual heterogeneity and, then, we should require a minimum human capital level to be able to develop open goods, so that we could have at the same time free riders and developers of open goods in the economy. Note that an increase in time for schooling θ_1 might lead to a decrease in time for open good θ_2 but this could result in an increase in open good accumulation. This is because both time for schooling and for open goods matter in the human capital production (equation 3.2) and human capital might increase overall.

At this point, in the representative agent model the open good accumulation law simplifies to

$$\dot{\Theta}=\eta\Theta^{\zeta}(\theta_2 hN)^{1-\zeta} \ , \tag{3.17}$$

or

$$\frac{\dot{\Theta}}{\Theta}=\eta\theta_2^{1-\zeta}\left(\frac{hN}{\Theta}\right)^{1-\zeta} \ . \tag{3.18}$$

Then, in a balanced growth path we would observe

$$\gamma_\Theta=\frac{\dot{\Theta}}{\Theta}=\frac{\dot{h}}{h}+n \ , \text{ where } \gamma_\Theta \text{ is the growth rate of open goods.} \tag{3.19}$$

Note that an increase in the population growth rate implies an increase in the open goods growth rate, in line with property p1 as a higher population growth rate means more individuals in the production of open goods and this, in turn, means that more open goods will be developed.

3.2.4 Capital Market Clearing

In equilibrium, $a = k$, so that $\dot{a} = \dot{k}$. The budget constraint of the individual (3.1) becomes

$$\dot{k} = w\mu h + (r - n)k - c \; . \tag{3.20}$$

3.2.5 Balanced Growth Path

From (3.6), in a balanced growth path, the shadow prices of the two capitals grow at a same rate, $\dot{\lambda}_1/\lambda_1 = \dot{\lambda}_2/\lambda_2$. Combining this with (3.9) and (3.10), we have

$$r = \left[\epsilon\theta_1^{\upsilon} + (1-\epsilon)\,\theta_2^{\upsilon}\right]^{\frac{1-\upsilon}{\upsilon}} \epsilon\theta_1^{\upsilon-1} - \delta_h. \tag{3.21}$$

Using (3.8), taking ln and differentiating with respect to t, we obtain

$$\frac{\dot{c}}{c} + (\beta - 1)\,\frac{\dot{h}}{h} = \frac{\partial \ln\left[\pi\Theta^{\beta} + (1-\pi)\,(\theta_2 h)^{\beta}\right]}{\partial t}. \tag{3.22}$$

We consider the case $\beta < 0$, when devoting time to open goods is complementary with the anti-rival property of open goods (the definition of $J_{(\Theta,h)}$). In this case, since open goods grow at a higher rate than human capital, as we have seen in (3.19), asymptotically $\partial \ln\left[\pi\Theta^{\beta} + (1-\pi)\,(\theta_2 h)^{\beta}\right]/\partial t = \beta\dot{h}/h$. Then, (3.22) becomes

$$\frac{\dot{c}}{c} = \frac{\dot{h}}{h}. \tag{3.23}$$

Taking ln and differentiating with respect to t in (3.5), we obtain

$$\frac{\dot{\lambda}_1}{\lambda_1} = -\iota\frac{\dot{c}}{c} + \frac{1-\tau}{\beta}\frac{\partial \ln\left[\pi\Theta^{\beta} + (1-\pi)\,(\theta_2 h)^{\beta}\right]}{\partial t}. \tag{3.24}$$

Combining (3.24) with (3.9) and (3.22), and then using (3.23), we obtain the balanced growth path growth rate[3] γ,

$$\gamma = \frac{\dot{c}}{c} = \frac{\dot{h}}{h} = \frac{\rho - r}{1 - \tau - \iota} \; . \tag{3.25}$$

Combining (3.2) and (3.25), we obtain

$$\frac{\rho - r}{1 - \tau - \iota} = \left[\epsilon\theta_1^{\upsilon} + (1-\epsilon)\,\theta_2^{\upsilon}\right]^{\frac{1}{\upsilon}} - \delta_h - n \; . \tag{3.26}$$

[3] Recall that this is only true for the case $\beta < 0$.

Evaluating (3.8) asymptotically, we have

$$c = \left(\frac{1-\iota}{1-\tau}\right) w\theta_2 \left[1 - \frac{(1-\epsilon)\,\theta_2^{\upsilon-1}}{\epsilon\theta_1^{\upsilon-1}}\right] h \ . \tag{3.27}$$

Substituting (3.27) into (3.20) gives

$$\frac{\dot{k}}{k} = (r-n) + \left(\frac{w\mu h}{k}\right)\left\{1 - \left(\frac{1-\iota}{1-\tau}\right)\left(\frac{\theta_2}{1-\theta_1-\theta_2}\right)\left[1 - \frac{(1-\epsilon)\,\theta_2^{\upsilon-1}}{\epsilon\theta_1^{\upsilon-1}}\right]\right\} . \tag{3.28}$$

And substituting the growth rate from (3.25) into (3.28), and then using (3.14) and (3.15), we obtain

$$\frac{\rho - r}{1-\tau-\iota} =$$

$$r - n + \left[\left(\frac{r+\delta_k}{\alpha}\right)^{\frac{1}{1-\sigma}} - (r+\delta_k)\right]\left\{1 - \left(\frac{1-\iota}{1-\tau}\right)\left(\frac{\theta_2}{1-\theta_1-\theta_2}\right)\left[1 - \frac{(1-\epsilon)\,\theta_2^{\upsilon-1}}{\epsilon\theta_1^{\upsilon-1}}\right]\right\} . \tag{3.29}$$

Equations (3.21), (3.26) and (3.29) implicitly give the values of r, θ_1 and θ_2, and (3.25) gives the value of γ.

Since the open good price is zero[4], ζ has no effect on (national accounting or private goods) growth. However, while π and β have also no effect on growth, this is not the case for τ. What matters for growth are time preference, the relative preference of agents for private good consumption ι and open good related activities τ. This is consistent with the results from Rebelo (1991) and Milesi-Ferretti and Roubini (1998), in which they also have monetary utility functions of consumption and leisure.

3.2.6 Numerical Exercise

Hereinafter, we analyze the impact of open goods on growth by means of a numerical exercise. We calibrate certain parameters of the decentralized economy assuming that it is in the balanced growth path. Since open goods have arisen with the arrival of internet, we use the World Bank Data for the period 2000-2016. Note that the open good price is zero and, then, θ_2 is not observable in the market. We fix some values as follows. The population growth rate n is 0.75%[5]. The interest rate r is 2.848292%[6], and ρ equals 0.021.[7] The (average)

[4] Open goods clearly have value to consumers, but are excluded from GDP since there is no market where they are sold and bought. As a result, our measurements may not be capturing a growing share of economic activity.

[5] This is the US labor force growth rate during 1999-2015.

[6] This is the average US real interest rate during 2000-2016 according to http://data.worldbank.org.

[7] The value of ρ varies in economic literature. For exapple, in Nordhaus and Sztorc(2013) $\rho = 0.015$, in Nordhaus (2007) ρ takes 0.015 or 0.01, in Garcia-Penalosa and Turnowsky (2006) $\rho = 0.04$, in Moore & Viscusi (1990), and Weitzman (2007) $\rho = 0.02$. We set $\rho = 0.021$.

GDP growth rate is 0.94%. With these values, from the Euler equation (3.25) $\iota \in (0,\ 1.7979)$ must hold so that $\tau > 0$ and vice versa $\tau \in (0,\ 1.7979)$ must hold so that $\iota > 0$. According to King, et al. (1988), the value of ι can vary in the interval $(0.1, 10)$, in our calibration we set $\iota = 0.4$ so that we derive from (3.25) $\tau = 1.5043$. We set $\delta_k = 0.1$ as in King and Rebelo (1999), $\delta_h = 0.015$ as in Arrazola and Heiva (2004), and $\alpha = 0.33$. The time for human capital θ_1 is 0.1036[8]. Dividing both sides of equation (3.11) by k, substituting equation (3.14) into this new equation and using $y/k = 1/3$, we obtain $\sigma = 0.1415$. And using (3.21), (3.26) and (3.29), we obtain the values of υ, ϵ and θ_2. The summary of the numerical exercise is in Table 1A and Table 1B.

	Value	Description
θ_1	0.1036	*Time for human capital*
r	0.0285	*Interest rate*
δ_h	0.015	*Human capital depreciation rate*
ρ	0.021	*Time discounted rate*
α	0.33	*Capital share*
ι	0.4	*Inverse of the IES of consumption*
n	0.0075	*Population growth rate*
δ_k	0.1	*Capital depreciation rate*
γ	0.0094	*GDP growth rate*
y/k	0.33	capital-output ratio

Table 1A. Assumed values

	Value	Description
σ	0.1415	$1/(1-\sigma)$ is the *CES of physical capital production function*
τ	1.5043	*Inverse of the IES of open goods - related activities*
υ	-1.6579	$1/(1-\upsilon)$ is the *CES of human capital production function*
ϵ	0.9989	*Weight of time for school in human capital production*
θ_2	0.0005	*Time for open goods*

Table 1B. Calibrated values

Note that the value of υ equals -1.6579 which means that time for schooling and time for open goods are complementary in the formation of human capital. This suggests that time spending for open goods is more valuable for individual's human capital when more time is devoted for schooling, as schooling make them more capable of producing as well as making use of open goods. And vice versa, the contribution of time spending for schooling to human capital would

[8] We calculate the value of θ_1 as follows. From Barro-Lee statistics, the US average years of total schooling in 2010 is 13.18 years; substract this by the first 8 years that individuals cannot work during schooling time and then divide by 50 years of working time (the US average retire age in 2010 is 64). Obviously, the younger generations are attending tertiary education more. Hence, the average years of schooling would be higher for the years after 2010 but, because of the availablity of the data, we choose the statistics of the year 2010 as it it the most recent year with available data.

be manifest by the time individuals taking part in open-good related activities. Note also that schooling plays a major role in human capital formation as the weight of time for schooling in the formation of human capital is almost unity. This is due to the minority of the population involved in open-good activities.

Table 2 shows the effect of population growth on time for education and open good, and growth rate in the market economy. Not as expected, higher population growth results in lower growth rates of open goods and the market economy. This is because population growing at a faster rate has a direct negative effect (see equation (3.2)), and an indirect negative effect on human capital growth via the individual's reallocation of time. As in equation (3.17), more open goods Θ will be developed with higher population. Since individual's utility is partly derived from open goods Θ and time for developing open goods $\boldsymbol{\theta}_2$ (equation (3.3)), to obtain the same utility individuals will spend less time $\boldsymbol{\theta}_2$ for developing open goods Θ when there is more open goods Θ. And therefore time for schooling $\boldsymbol{\theta}_1$will decrease since time for schooling and time for open goods are complimentary. In turn, this negative effect of higher population growth n on $\boldsymbol{\theta}_1$and $\boldsymbol{\theta}_2$ results in lower human capital growth.

	n = 0	**n = 0.005**	**n = 0.0075**	**n = 0.0085**	**n = 0.01**
$\boldsymbol{\theta}_1$	0.1275	0.1117	0.1036	0.1003	0.0952
$\boldsymbol{\theta}_2$	0.0078	0.0063	0.0055	0.0052	0.0048
γ	0.0292	0.0159	0.0094	0.0068	0.0030
γ_Θ	0.0292	0.0209	0.0169	0.0153	0.0130

Table 2. Changes in population growth

Table 3 shows the effect of the weight ϵ of time for education in human capital production on time for education and open good, interest rate and growth rate in the market economy. As time for human capital becomes more important (higher ϵ), individual devotes more time for education and less time for open good. Overall, these changes can either have a positive or negative effect on human capital because both $\boldsymbol{\theta}_1$and $\boldsymbol{\theta}_2$ affect $\boldsymbol{\gamma}_h$ positively as in (3.2). The results in Table 3 show that growth rate increases as ϵ increases until a certain value but then decreases as ϵ gets higher. The decreases in growth rate when ϵ is high enough is partly due to the complementary property of $\boldsymbol{\theta}_1$and $\boldsymbol{\theta}_2$ in human capital production, as an increase in ϵ expands the gap of $\boldsymbol{\theta}_1$and $\boldsymbol{\theta}_2$ and this has a negative effect on human capital. When ϵ is small, the positive effect of the increase in $\boldsymbol{\theta}_1$ offsets the negative effect of the decrease in $\boldsymbol{\theta}_2$ and of the gap of $\boldsymbol{\theta}_1$and $\boldsymbol{\theta}_2$ (as the gap is sufficiently small), but as the gap gets bigger the combine negative effects of the decrease in $\boldsymbol{\theta}_2$ and of the gap of $\boldsymbol{\theta}_1$and $\boldsymbol{\theta}_2$ will offset the positive effect of the increase in $\boldsymbol{\theta}_1$.

	ϵ= **0.1**	ϵ= **0.5**	ϵ= **0.7**	ϵ= **0.8**	ϵ= **0.9**	ϵ= **0.94**	ϵ= **0.96**	ϵ= **0.98**
$\boldsymbol{\theta}_1$	0.04274	0.08072	0.09215	0.09708	0.1015	0.1032	0.1040	0.1047
$\boldsymbol{\theta}_2$	0.03004	0.02221	0.01644	0.01289	0.008431	0.006157	0.004793	0.003113
r	0.02768	0.02835	0.02847	0.02852	0.02852	0.02851	0.02849	0.02841
$\boldsymbol{\gamma}$	0.008372	0.009214	0.009358	0.009425	0.009425	0.009409	0.009388	0.009282
$\boldsymbol{\gamma}_\Theta$	0.01587	0.01671	0.01686	0.01692	0.01693	0.01691	0.01689	0.01678

Table 3. Changes in ϵ

Table 4 shows how time for education and open good, and growth rate change when υ changes. A higher complementary level of times $\boldsymbol{\theta}_1$and $\boldsymbol{\theta}_2$ in the human capital production function (lower υ) leads to a smaller gap of $\boldsymbol{\theta}_1$and $\boldsymbol{\theta}_2$.Thus, the lower υ, the higher $\boldsymbol{\theta}_2$ and lower $\boldsymbol{\theta}_1$. Because the relative weight ϵ of $\boldsymbol{\theta}_1$is very high ($\epsilon = 0.9989$) in the formulation of human capital, the negative effect (on $\boldsymbol{\gamma}_h$) of lower $\boldsymbol{\theta}_1$ offsets the positive effect of $\boldsymbol{\theta}_2$, resulting in a lower growth.

	υ= **−0.1**	υ= **−0.5**	υ= **−0.9**	υ= **−1.5**	υ= **−2**	υ= **−3**	υ= **−5**	υ= **−10**
$\boldsymbol{\theta}_1$	0.6620	0.2730	0.1679	0.1099	0.0883	0.0674	0.0516	0.0406
$\boldsymbol{\theta}_2$	0.0000	0.0002	0.0016	0.0049	0.0075	0.0118	0.0172	0.0228
$\boldsymbol{\gamma}$	0.0334	0.0146	0.0112	0.0096	0.0090	0.0086	0.0083	0.0081
$\boldsymbol{\gamma}_\Theta$	0.0409	0.0221	0.0187	0.0171	0.0165	0.0161	0.0158	0.0156

Table 4. Changes in υ

3.3 Social Planner and Open Goods

The social planner problem is

$$Max \quad \int_0^\infty e^{-(\rho-n)t}\frac{c^{1-\iota}}{1-\iota}.\frac{\left[\pi\Theta^\beta+(1-\pi)\left(\theta_2 h\right)^\beta\right]^{\frac{1-\tau}{\beta}}}{1-\tau}dt$$

subject to (3.2), (3.17) and the feasibility constraint

$$\dot{k}=\left\{\alpha k^\sigma+(1-\alpha)\left[\left(1-\theta_1-\theta_2\right)h\right]^\sigma\right\}^{\frac{1}{\sigma}}-c-\left(\delta_k+n\right)k\ . \tag{3.30}$$

The first-order conditions with respect to c, θ_1 and θ_2 of the corresponding current Hamiltonian of the maximization problem are, respectively,

$$\lambda_1=c^{-\iota}\frac{\left[\pi\Theta^\beta+(1-\pi)\left(\theta_2 h\right)^\beta\right]^{\frac{1-\tau}{\beta}}}{1-\tau}\ , \tag{3.31}$$

$$\begin{aligned} & \lambda_1\left\{\alpha k^\sigma+(1-\alpha)\left[\left(1-\theta_1-\theta_2\right)h\right]^\sigma\right\}^{\frac{1}{\sigma}-1}(1-\alpha)h^\sigma\left(1-\theta_1-\theta_2\right)^{\sigma-1} \\ = \ & \lambda_2 h\left[\epsilon\theta_1^\upsilon+(1-\epsilon)\theta_2^\upsilon\right]^{\frac{1}{\upsilon}-1}\epsilon\theta_1^{\upsilon-1} \end{aligned} \tag{3.32}$$

and

$$
\begin{aligned}
& \lambda_1 \left\{\alpha k^\sigma + (1-\alpha)\left[(1-\theta_1-\theta_2)h\right]^\sigma\right\}^{\frac{1}{\sigma}-1}(1-\alpha)h^\sigma(1-\theta_1-\theta_2)^{\sigma-1} \\
= & \frac{c^{1-\iota}}{1-\iota}\left[\pi\Theta^\beta + (1-\pi)(\theta_2 h)^\beta\right]^{\frac{1-\tau-\beta}{\beta}}(1-\pi)h^\beta\theta_2^{\beta-1} \\
& +\lambda_2 h\left[\epsilon\theta_1^\upsilon + (1-\epsilon)\theta_2^\upsilon\right]^{\frac{1}{\upsilon}-1}(1-\epsilon)\theta_2^{\upsilon-1} \\
& +\lambda_3(1-\zeta)\eta\Theta^\zeta(hN)^{1-\zeta}\theta_2^{-\zeta}, \qquad (3.33)
\end{aligned}
$$

where λ_1, λ_2 and λ_3 are the multipliers associated to (3.30), (3.2) and (3.17), respectively.

Similarly to the decentralized economy, equation (3.31) states that the marginal utility of consuming private goods must be equal to the opportunity cost of accumulating them. Equation (3.32) and (3.33) states that, on the margin, time must be equally valuable in the three uses: working, human capital accumulation and open goods accumulation. Note that the social planner takes into account the accumulation of open good in the maximization problem; therefore, on the margin, the utility gained by an extra unit of time used for open good in equation (3.33) is different from equation (3.7). Particularly, this difference is the third element on the RHS of (3.33).

The rates of change of the shadow prices of the two types of capital and the shadow price of the open goods are given by

$$
\frac{\dot{\lambda}_1}{\lambda_1} = \rho - \left[\alpha k^\sigma + (1-\alpha)\left[(1-\theta_1-\theta_2)h\right]^\sigma\right]^{\frac{1}{\sigma}-1}\alpha k^{\sigma-1} + \delta_k \ , \qquad (3.34)
$$

$$
\begin{aligned}
\dot{\lambda}_2 = & (\rho-n)\lambda_2 \\
& -\frac{c^{1-\iota}}{1-\iota}\left[\pi\Theta^\beta + (1-\pi)(\theta_2 h)^\beta\right]^{\frac{1-\tau-\beta}{\beta}}(1-\pi)h^{\beta-1}\theta_2^\beta \\
& -\lambda_1\left\{\alpha k^\sigma + (1-\alpha)\left[(1-\theta_1-\theta_2)h\right]^\sigma\right\}^{\frac{1}{\sigma}-1}(1-\alpha)h^{\sigma-1}(1-\theta_1-\theta_2)^\sigma \\
& -\lambda_2\left[\left[\epsilon\theta_1^\upsilon + (1-\epsilon)\theta_2^\upsilon\right]^{\frac{1}{\upsilon}} - \delta_h - n\right] \\
& -\lambda_3(1-\zeta)\eta\Theta^\zeta(\theta_2 N)^{1-\zeta}h^{-\zeta} \qquad (3.35)
\end{aligned}
$$

and

$$
\dot{\lambda}_3 = (\rho-n)\lambda_3 - \frac{c^{1-\iota}}{1-\iota}\left[\pi\Theta^\beta + (1-\pi)(\theta_2 h)^\beta\right]^{\frac{1-\tau-\beta}{\beta}}\pi\Theta^{\beta-1} - \lambda_3\zeta\eta\Theta^{\zeta-1}(\theta_2 hN)^{1-\zeta} \ . \qquad (3.36)
$$

Substituting (3.31) and (3.32) into (3.33) yields

$$
\begin{aligned}
& \lambda_3(1-\zeta)\eta\Theta^\zeta(hN)^{1-\zeta}\theta_2^{-\zeta} = \\
& -\lambda_2 h\left[\epsilon\theta_1^\upsilon + (1-\epsilon)\theta_2^\upsilon\right]^{\frac{1}{\upsilon}-1}\epsilon\theta_1^{\upsilon-1}\frac{1-\tau}{1-\iota}M \\
& +\lambda_2 h\left[\epsilon\theta_1^\upsilon + (1-\epsilon)\theta_2^\upsilon\right]^{\frac{1}{\upsilon}-1}\epsilon\theta_1^{\upsilon-1} \\
& -\lambda_2 h\left[\epsilon\theta_1^\upsilon + (1-\epsilon)\theta_2^\upsilon\right]^{\frac{1}{\upsilon}-1}(1-\epsilon)\theta_2^{\upsilon-1}, \qquad (3.37)
\end{aligned}
$$

where

$$M=\frac{c\left[\pi\Theta^{\beta}+(1-\pi)\left(\theta_2 h\right)^{\beta}\right]^{-1}(1-\pi)\,h^{\beta}\theta_2^{\beta-1}}{\left\{\alpha k^{\sigma}+(1-\alpha)\left[(1-\theta_1-\theta_2)\,h\right]^{\sigma}\right\}^{\frac{1}{\sigma}-1}(1-\alpha)\,h^{\sigma}\left(1-\theta_1-\theta_2\right)^{\sigma-1}}.$$

From (3.32) and (3.37), we have $\dot{\lambda}_1/\lambda_1=\dot{\lambda}_2/\lambda_2$ and $\dot{\lambda}_2/\lambda_2=\dot{\lambda}_3/\lambda_3+n$, respectively. Therefore,

$$\frac{\dot{\lambda}_1}{\lambda_1}=\frac{\dot{\lambda}_2}{\lambda_2}=\frac{\dot{\lambda}_3}{\lambda_3}+n. \tag{3.38}$$

Substituting (3.33) into (3.35) and using (3.32), we obtain

$$\frac{\dot{\lambda}_2}{\lambda_2}=\rho-\left[\epsilon\theta_1^{\upsilon}+(1-\epsilon)\,\theta_2^{\upsilon}\right]^{\frac{1}{\upsilon}-1}\epsilon\theta_1^{\upsilon-1}+\delta_h. \tag{3.39}$$

Substituting (3.33) into (3.36) and using (3.32) and (3.37), we have

$$\frac{\dot{\lambda}_3}{\lambda_3}=(\rho-n)-(1-\zeta)\,\eta\Theta^{\zeta-1}\left(\theta_2 hN\right)^{1-\zeta}\frac{\epsilon\theta_1^{\upsilon-1}\frac{1-\tau}{1-\iota}.M.\frac{\pi\Theta^{\beta}}{h^{\beta}\theta_2^{\beta}}}{\epsilon\theta_1^{\upsilon-1}-(1-\epsilon)\,\theta_2^{\upsilon-1}-\epsilon\theta_1^{\upsilon-1}\frac{1-\tau}{1-\iota}.M}-\zeta\eta\Theta^{\zeta-1}\left(\theta_2 hN\right)^{1-\zeta}\ . \tag{3.40}$$

Evaluating (3.40) asymptotically we obtain

$$\frac{\dot{\lambda}_3}{\lambda_3}=\rho-n-\zeta\frac{\dot{\Theta}}{\Theta}. \tag{3.41}$$

Since (3.31) and (3.5) are identical, we can use (3.24) in the social planner problem and combine it with (3.38) and (3.41) to obtain

$$\gamma=\frac{\dot{k}}{k}=\frac{\rho-\zeta n}{1-\tau-\iota+\zeta}. \tag{3.42}$$

And from (3.24) and (3.34), we have

$$\frac{\dot{c}}{c}=\frac{\rho-\left[\epsilon\theta_1^{\upsilon}+(1-\epsilon)\,\theta_2^{\upsilon}\right]^{\frac{1}{\upsilon}-1}\epsilon\theta_1^{\upsilon-1}+\delta_h}{1-\tau-\iota}. \tag{3.43}$$

Substituting the growth rate from (3.42) into (3.2) and (3.43) and solving, we obtain

$$\theta_1=\frac{\frac{\rho-\zeta n}{1-\tau-\iota+\zeta}+\delta_h+n}{\left[\frac{\rho+\delta_h}{\epsilon}-\frac{(\rho-\zeta n)(1-\tau-\iota)}{\epsilon(1-\tau-\iota+\zeta)}\right]^{\frac{1}{1-\upsilon}}} \tag{3.44}$$

and

$$\theta_2=\theta_1\left[\frac{\left[\frac{\rho+\delta_h}{\epsilon}-\frac{(\rho-\zeta n)(1-\tau-\iota)}{\epsilon(1-\tau-\iota+\zeta)}\right]^{\frac{\upsilon}{1-\upsilon}}-\epsilon}{(1-\epsilon)}\right]^{\frac{1}{\upsilon}}. \tag{3.45}$$

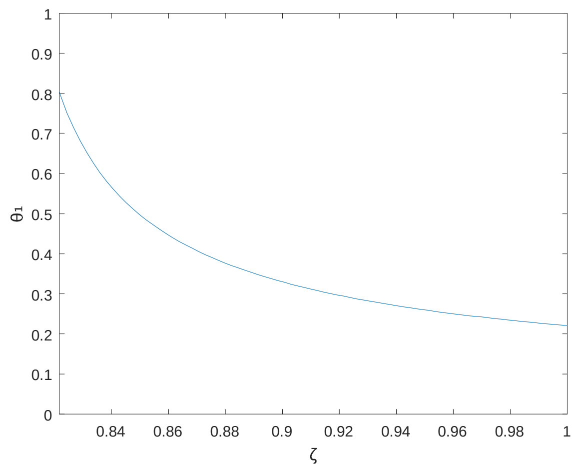

Figure 3.1: Time for education and ζ

The growth rate of the economy declines with the weight ζ of the current stock of open goods in its own accumulation. This means the more important the time for developing open goods in the accumulation of open goods, the higher the growth rate. In the numerical exercise below we will discuss in detail why this is the case. Note that similar to the market economy, π and β have also no effect on growth.

3.3.1 Numerical Exercise

Since the parameter ζ does not affect the market economy as individuals do not take into account the evolution of open goods in their optimal decisions, we could not recover the value of ζ from the market calibration. Given this lack of information, in this section we plot how the growth rate, time for education and time for open goods change as ζ changes. From (3.42), and in order to have positive growth, $\zeta > 0.7979$. Moreover, given the calibration, from (3.44) and (3.45) we have that $\zeta > 0.82181$ must be satisfied in order to have $\theta_1 + \theta_2 < 1$. Therefore, the range of ζ is (0.82181, 1).

Figures 3.1 and 3.2 show how time for education θ_1 and time for open goods θ_2 change when ζ changes along the interval (0.82181, 1). We see that θ_1 and θ_2 decrease as ζ gets bigger. This means that if time for open good and human capital play a less important role in the accumulation of open good, the planner will allocate less time for open good and schooling, and hence more time for working. Note that even if $\zeta = 1$, $\theta_2 > 0$ since time for open goods affects the accumulation of human capital.

Note that time for open goods decreases not only because time for open good is less important now, but also because so does human capital. Individuals will devote less time for open goods and schooling when human capital is less important since time for open goods constitutes the accumulation of human capital.

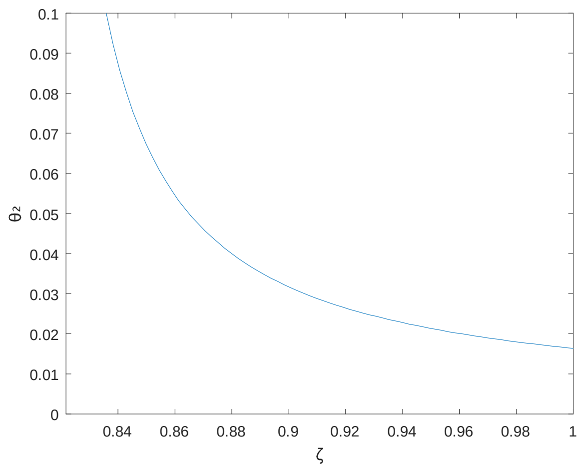

Figure 3.2: Time for open good and ζ

Figure 3.3 shows that growth is decreasing in ζ as a result of the negative relation among ζ and both time for schooling and time for open goods (and hence human capital accumulation).

As in the case of the market economy, changes in population, ϵ and ν have similar effects on time for human capital, time for open good and growth rate as illustrated in Figures 3.4-3.10.

3.4 Concluding Remarks

This paper has modeled an economy with two types of good—private good and open good—capturing the characteristics of open good to analyze how its production and use affect the economic growth. In a market economy, since the price of open good is zero and individuals do not take into account open good production in their optimal decisions, the parameters in the production of open good have no effect on growth. What matters for growth are time preference, the relative preference of agents for private good consumption and open good related activities.

The calibration results in the market economy suggest that time for schooling and time for open goods are complementary in the formation of human capital. Time spending for open goods is more valuable when more time is devoted for schooling as schooling make them more capable of producing as well as making use of open goods. And vice versa, the contribution of schooling time to human

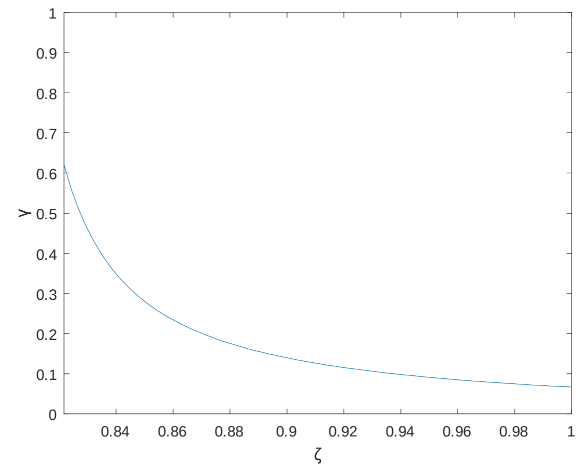

Figure 3.3: Relationship between growth and ζ

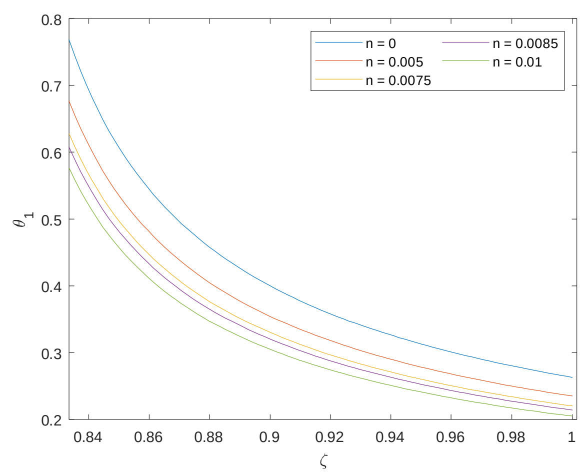

Figure 3.4: Population changes and θ_1

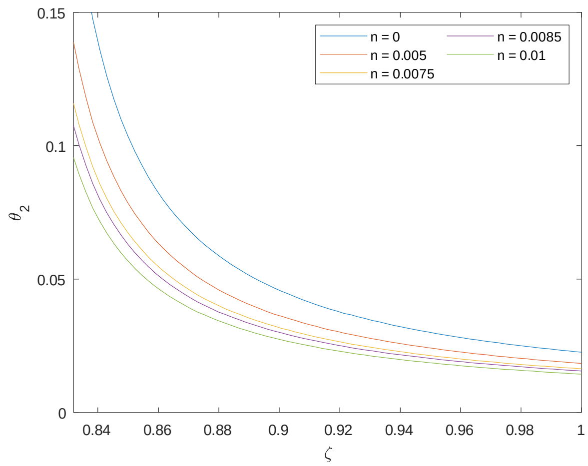

Figure 3.5: Population changes and θ_2

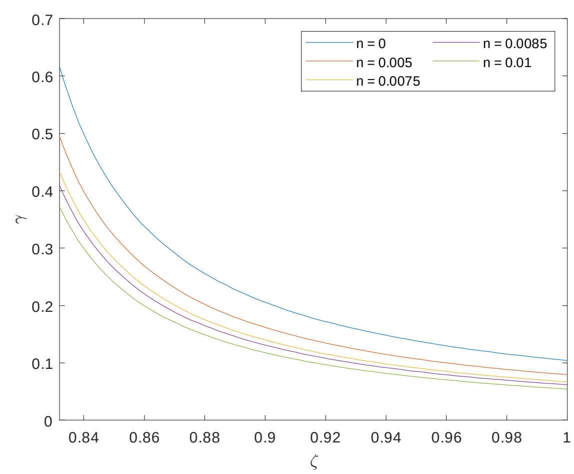

Figure 3.6: Population changes and growth

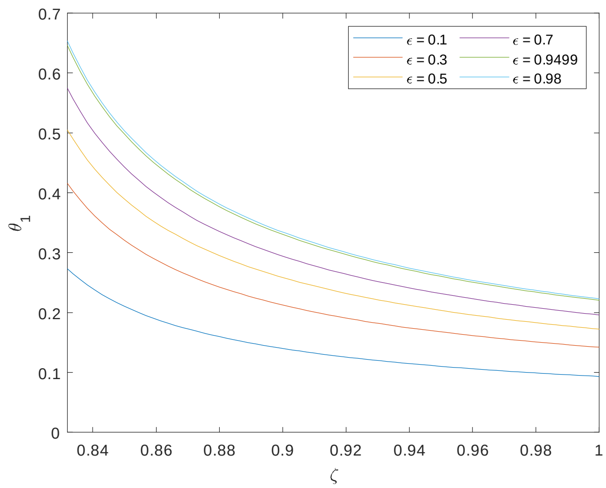

Figure 3.7: ε changes and θ_1

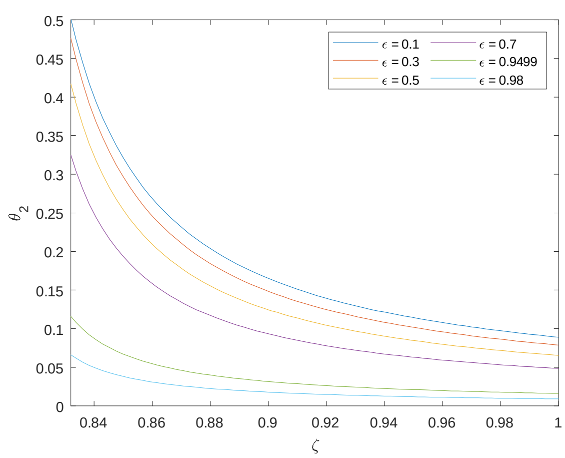

Figure 3.8: ε changes and θ_2

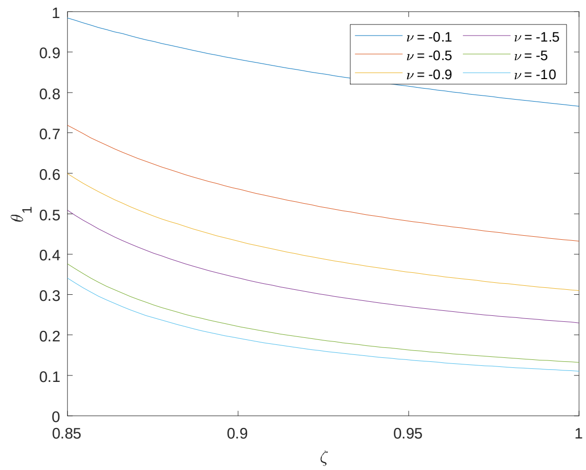

Figure 3.9: ν changes and θ_1

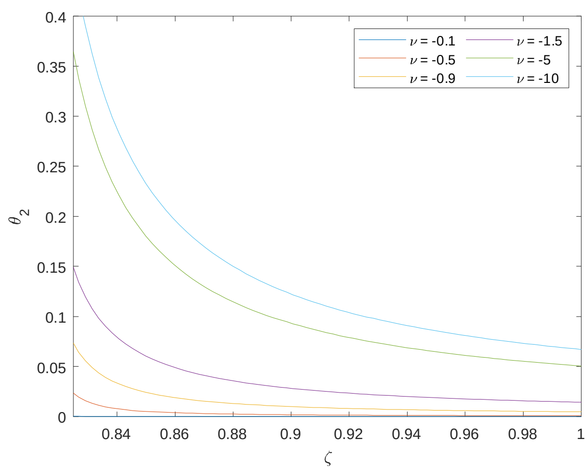

Figure 3.10: ν changes and θ_2

capital is also manifested by the time individuals taking part in open-good related activities. The calibration results also suggest that schooling plays a major role in human capital formation as the weight of time for schooling in the formation of human capital is almost unity. This is in line with the fact that a minority of the population use open good and devotes their time to develop it.

The numerical exercises show that population growth and economic growth are negatively related both in the market economy and the social planner economy. Moreover, in the market economy, as time for schooling becomes more important in the human capital accumulation, individual devotes more time for schooling and less time for open good. This changes can either have positive or negative effects on growth. Particularly, growth increases as the weight of time for schooling increases until a certain value and then the relation is negative. The decreases in growth rate when the weight of time for schooling is high enough is partly due to the complementary property of time for schooling and time for open goods in human capital production. Higher weight of time for schooling will expands the gap between time for schooling and time for open goods. And since these two types of time are complementary, this increasing gap has a negative effect on human capital. When the weight is high enough, this negative effect combining with the negative effect of the decrease in time for open goods will be strong enough to offset the positive effect of the increase in time for schooling on human capital.

Furthermore, the more complementary time for schooling and time for open goods in the human capital production function are, the smaller gap of the two purposes of time is. The aggregate effect is that growth gets lower. Because the relative weight of time for schooling is very high in the formulation of human capital, the negative effect on human capital of lower time for schooling offsets the positive effect of higher time for open goods, resulting in a lower growth. In the social planner economy the changes in the weight of time for schooling and in the complementary level of the two purposes time have similar effects on how planner allocate time.

In the social planner's economy, economic growth declines with the weight of the stock of open goods, and therefore increases in the level of human capital and time for open good, in its accumulation. This is because when time for open good and human capital is less important in the accumulation of open good, the planner allocate less time for open good as well as schooling. This results in less human capital accumulated as both time for open goods and schooling matter in the accumulation of human capital, and economic growth rate gets lower eventually. The numerical exercises enhance this results.

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Chapter 4

Open Innovation, Imitation and R&D Competition

4.1 Introduction

Traditionally, the birth of products and the improvement in quality of the existing products are the results of the investment in R&D of firms. In this conventional closed innovation paradigm, profit-seeking firms invest in R&D in order to improve their products to gain advantage over their rivals in the market or to launch a new product line that enables them to capture a huge profit as they become a monopoly in the market. In this innovation paradigm, as firms use their own internal resources in the R&D process to develop a better or new product, the proprietary right over the innovation is protected and the innovation could only be commercialized by the firm that has the proprietary right to the innovation. However, recently, we have observed the emergence of open-source software (OSS) and, following, open-source hardware (OSH). This open-source paradigm has some core differences to the conventional innovation paradigm in the way the development of the innovation is carried out as well as the proprietary right over the innovation.

OSS products such as Apache web server, Linux, Android were developed in a different fashion from the conventional innovation paradigm in two major aspects. First, OSS products are developed by a public community of software developers which is far beyond the limited resources of any private firm. The open-source code platform ensures that the public can freely access, modify and update the codes of the software being developed; this allows the OSS project to utilize the collective contribution of the elite public. Second, the propriety right mechanism of the OSS products ensures that they are free of use and distribution to the public. The success of OSS has led to the emergence of OSH.

With a similar open paradigm, several OSH projects such as RepRap (Replicating Rapid Prototyper), Arduino have proved the prospect of open source in technology innovation. Henceforth, we refer open source in technology innovation as open innovation (OI). RepRap is an OSH project that initiated in 2005 to make low-cost self-replicable 3D printers. It gathered worldwide contribution of the community of technical students, engineers, professionals and other people who shared the common interests to create the biggest 3D technology community online and successfully developed the first self-replicable 3D printer. Arduino is an open-source electronics platform developed by the Ivrea Interaction Design Institute. This platform is based on easy-to-use hardware and software and has gathered worldwide community of students, engineers, hobbyists to contribute to an accessible stock of technical knowledge to the peers. All Arduino boards are completely open-source, this allows users to build them independently to adapt to their own needs.

The onset of OI paradigm hints a new era for the R&D activities and innovation competition. We have observed in many industries that big leading firms who invest significantly in R&D are the ones who would be more likely to win the R&D race due to the advantages of better technology and bigger investment in comparison with small firms. However, different from conventional innovation paradigm, in some specific industries, OI paradigm allows small firms or left-behind firms in the R&D race who have financial and technological disadvantages in the innovation competition to be able to come up with a major innovation that enables these firms to catch up with the leaders in the industries or even become a leader or a monopoly firm in a new industry. The low-cost open R&D activities that seek for collective contribution from the public community of interested user and professionals therefore could change the R&D behavior of firms in some certain industries.

This paper studies how open innovation affects the R&D competition and then the economy in the long run. We model an economy with standardized goods and quality goods where individuals with non-homothetic preference have to allocate their budget for standardized goods and quality goods. We incorporate the characteristics of conventional R&D, copying and open innovation in the maximization problems of multi-quality firms and aim at examining how open innovation affects R&D investments of firms with different technology levels and then its effects on economic growth.

The analysis in this article relates to the strand of literature of R&D-driven endogenous growth literature (Romer (1990), Grossman and Helpman (1991), Aghion and Howitt (1992), Aghion et al. (2014)). In Grossman and Helpman (1991) and Aghion and Howitt (1992), income inequality has no impact on research activities because consumers have homothetic preferences. Zweimüller et al. (2005) and Latzer (2018) study how income inequality affects R&D investments and, consequently, economic growth. Latzer (2018) with a vertical innovation framework with multi-product firms shows that income inequality has positive effects on R&D investments of industry leaders. Glass (1995) builds a model with different tastes about quality of two different types of households to study the effects of income inequality on R&D-driven growth. Similar to Zweimüller et al. (2005) and Latzer (2018), we assume that individuals are different in their wealth endowment and they consume maximally one unit of each type of the quality goods and spend the rest of their budgets on a standardized goods.

Our study joins a handful of models where incumbent leaders have incentive to invest in R&D. In Segerstrom, et al. (1990), Grossman and Helpman (1991a) and Aghion and Howitt (1992), firms invest in R&D to improve the quality of existing products,but once firms win the R&D race and become industry leaders, they rest on their past accomplishments and do not try to improve their own products as it is not profit-maximizing for them to invest in R&D activities. In these models, incumbent leaders are monopolists and have weaker incentive to invest in R&D than the followers, so with constant return to scale in R&D and without any R&D advantage, industry leaders will prioritize investing in R&D in other industries to become leaders in these industries instead of aiming to improve their own products. Different to these papers, our paper allows innovations by incumbent leaders. Similar in spirit are Barro and Sala-i-Martin (1995, ch. 7), Segerstrom and Zolnierek (1999), Aghion et al.(2001) and Latzer (2018).

Finally, our study adds to previous works studying imitation, innovation and R&D-driven growth. Segerstrom (1991) develops a dynamic general equilibrium model where firms can invest in both innovative and imitative R&D. Aghion et al. (1997) and Aghion et al.(2001) considered the effects of imitation and product market competition on growth. We contribute to this strand of literature by incorporating open innovation in the model and examining the effects of open innovation on growth.

4.2 The Model

4.2.1 Consumers

There are a continuum of quality goods indexed by $\omega \in [0, 1]$ and firms that produce goods at several levels of quality j. Firms are different in the highest quality that they can produce. We call a firm with its finest quality level j as firm j. We assume that when there are more than one firms operating at a certain level of j then the price and quantity of the market j will be determined by Cournot - Nash equilibrium.[1]

There is a fixed number L of infinitely-lived consumers i who are identical in their preferences and their constant wage w but different in their wealth endowment A_i. Thus, consumer's income is $y_i(t) = w(t) + r(t)A_i$, where r is the interest rate. There are two types of goods which are the standardized goods and the quality goods indexed by $\omega \in [0, 1]$ .For simplicity, we normalize the price of standardized goods to 1 and we denote the price of quality goods as $p(\omega)$. Consumer i uses her income to buy c_i units of standardized goods and

[1] In a typical quality ladder model, when a follower catches up with a leader, it is assumed Bertrand competition in the market. Game theory indicates that this is not a credible threat for the leader since the leader can still have profit by sharing the market. In fact, the leader always has incentives to invest in R&D to maintain being a monopoly.

quality goods. Here we assume that the consumption of the second unit of the same quality goods generates zero utility; therefore, consumers only consume at most 1 unit of each quality good. Consumer i decides how much to consume standardized goods and whether or not to consume quality goods and what quality if she does.

The consumer i maximization problem is

$$\max u_i(t) = \max \int_{\tau}^{\infty} \ln \left(c_i(t) Q_i(t)\right) e^{t-\tau} dt, \tag{4.1}$$

with

$$Q_i(t) = \int_0^1 z_i(\omega, t)\, q_i^j(\omega, t)\, d\omega \tag{4.2}$$

being the quality index of consumer type i. Here, $z_i(\omega, t)$ takes value 1 if consumer i consumes the quality good ω and takes value 0 otherwise, $q_i^j(\omega, t)$ is the the quality of good ω that individual i consumes.

Consumer's budget constraint is

$$\int_{\tau}^{\infty} \left(c_i(t) + P_i(\omega, t)\right) e^{-R(t,\tau)} dt \leqslant A_i(\tau) + \int_{\tau}^{\infty} w(t)\, e^{-R(t,\tau)} dt, \tag{4.3}$$

where $p^j(\omega, t)$ is the price of good ω at quality j,

$$P_i(t) = \int_0^1 z_i(\omega, t)\, p^j(\omega, t)\, d\omega \tag{4.4}$$

is the total expenditures that a consumer type i spends on quality goods $\int_0^1 z_i(\omega, t)\, q_i^j(\omega, t)\, d\omega$, and $R(t, \tau) = \int_{\tau}^{t} r(s) d(s)$ is the accumulative interest rate between time τ and t.

The first-order condition of the maximization problem gives

$$\frac{1}{c_i(t)} = \lambda_i(t), \tag{4.5}$$

where $\lambda_i(t)$ is the associated multiplier of (4.3), it equals the marginal utility of standardized goods $c_i(t)$. Due to separability in utility both over time and across goods, for any given time path $P_i(t)$ of expenditures spending on quality goods $\int_0^1 z_i(\omega, t)\, q_i^j(\omega, t)\, d\omega$, the optimal path of expenditures spending on standardized goods $c_i(t)$ satisfies

$$\frac{\dot{c}_i(t)}{c_i(t)} = r(t) - \rho. \tag{4.6}$$

Similar to Latzer (2018), the first-order conditions for the discrete consumption choice of the quality goods are:

$$\begin{aligned}
& \left\{z_i(\omega,t), q_i^j(\omega,t)\right\} \qquad (4.7)\\
= & \left\{1, k^{n(\omega,t)}\right\} \text{ if } \chi_i(t)k^{n(\omega,t)} - p^n(\omega,t) \geq \max \left\{\chi_i(t)k^{n(\omega,t)-1} - p^{n-1}(\omega,t), ..., \chi_i(t) - p^0(\omega,t), 0\right\}\\
or = & \{1,1\} \text{ if } \chi_i(t) - p^0(\omega,t) \geq \max \left\{\chi_i(t)k^{n(\omega,t)-1} - p^{n-1}(\omega,t), ..., \chi_i(t)k - p^1(\omega,t), 0\right\}\\
or = & \{0,.\} \text{ otherwise}
\end{aligned}$$

where $\chi_i(t) = \frac{1}{\lambda_i(t)Q_i(t)} = \frac{c_i(t)}{Q_i(t)}$ is the willingness of consumer i to pay per unit of quality. In words, the first order conditions (4.7) states that consumer decides to buy quality good ω at a certain quality q^n when the price of this quality good is lower than the consumer's willingness to pay and this quality q^n is the quality that generates the biggest gain in utility among all the qualities of good ω.

4.2.2 Market Structures and Prices

Market Structures

Assume that in each type of quality goods, there is an R&D race between only two firms producing this good and due to limited capacity, each firm can only produce at 2 quality levels. Assume that the production cost at a certain quality level is the same for the two firms, therefore, Cournot competition implies that each firm will get 1/2 of the market. Population L is equally divided into 4 groups, namely rich, upper medium, lower medium and poor. These 4 groups are different in their initial wealth endowment A_i, $i \in (R, UM, LM, P)$.

The solely copying project can only allow follower to imitate the next higher quality (no matter the gap) available in the market while open source not only generate the possibility of copying but also brings up a chance to win the next technological race. All the firms has the incentive to move up in the quality ladder to charge a higher price for higher quality.

Since we have 2 firms and each firm produces 2 qualities, the competition of leader and follower in the market could generate either one of the following 3 cases. First, when the technological gap of the two firms is more than 1 step in the quality ladder, the leader becomes monopoly in the 2 top qualities and the follower is a monopoly in the 2 lower qualities. Second, when the technological gap is one step in the quality ladder, the leader is a monopoly in the top quality and share the market with the follower in the second highest quality, and the third quality is solely sold by the follower. Third, when leader and follower are neck-and-neck competitors, both firms share the market of the top and the second qualities.

With 4 groups of consumer, in the first case only the leader sells the top 2 qualities to groups R and UM while only the follower sells the 2 lower qualities to groups LM and P. In the second case, only the leader sells the top quality to group R and share groups UM and LM with the follower at the second best quality, and only the follower sells the third quality to group P. In the third case, both firms sell the top quality to share groups R and UM and also sell the second quality to share groups LM and P.

Prices

Since firms face four groups of consumers with different budget constraints and there are different qualities being offered in the market, when setting prices for the quality goods firms must take into account the prices of the other qualities. We denote $\widehat{p}_i^j$ as the maximum price that consumer i is willing to pay for quality q^j given that p^{j-m} is the price of quality q^{j-m} being offered in the market. Consumer is indifferent in consuming qualities q^j and q^{j-m}, given their prices, when

$$\chi_i(t)k^j - \widehat{p}_i^j = \chi_i(t)k^{j-m} - p^{j-m}. \tag{4.8}$$

Rearranging, we obtain

$$\widehat{p}_i^j = \chi_i(t)k^{j-m}\left(k^m - 1\right) + p^{j-m}. \tag{4.9}$$

Equation (4.9) states that the maximum price firms can charge consumer type i for quality q^j depends on the willingness to pay per unit of quality of consumer i and the price p^{j-m} of quality q^{j-m} being offered in the market. Moreover, since $k > 1$, the price of quality q^j is higher than the price of quality q^{j-m}.

Quality gap n $\geqslant$ 2 : The leader sells the best quality j to group R and quality $j-1$ to group UM while follower sells quality q^{j-n} to group LM and quality q^{j-n-1} to group P. The prices of different qualities are given as follows.

We first examine the prices that the follower offers for quality q^{j-n} and q^{j-n-1}. Given that quality q^{j-n-2} is offered at limit price (marginal cost wa), from (4.9) we have the price that the follower offers for quality q^{j-n-1} to capture the market P is given by:

$$p^{j-n-1} = \frac{c_P\left(t\right)}{Q_P(t)}\left(k-1\right)k^{j-n-2} + wa. \tag{4.10}$$

Similarly, given p^{j-n-1} as above, p^{j-n}, p^{j-1} and p^j respectively are

$$p^{j-n} = \left[\frac{c_{LM}\left(t\right)}{Q_{LM}(t)}k + \frac{c_P\left(t\right)}{Q_P(t)}\right]\left(k-1\right)k^{j-n-2} + wa\ , \tag{4.11}$$

$$p^{j-1} = \left[\frac{c_{UM}\left(t\right)}{Q_{UM}(t)}k^n + \frac{c_{LM}\left(t\right)}{Q_{LM}(t)}k + \frac{c_P\left(t\right)}{Q_P(t)}\right]\left(k-1\right)k^{j-n-2} + wa \tag{4.12}$$

and

$$p^j = \left[\frac{c_R\left(t\right)}{Q_R(t)}k^{n+1} + \frac{c_{UM}\left(t\right)}{Q_{UM}(t)}k^n + \frac{c_{LM}\left(t\right)}{Q_{LM}(t)}k + \frac{c_P\left(t\right)}{Q_P(t)}\right]\left(k-1\right)k^{j-n-2} + wa. \tag{4.13}$$

Therefore, the profit of the follower in an industry gap $n \geqslant 2$ is the sum of the profit in market P and market LM,

$$\overline{\Pi}_n = L^P \frac{c_P(t)}{Q_P(t)} (k-1) k^{j-n-2} + L^{UM} \left[\frac{c_{LM}(t)}{Q_{LM}(t)} k + \frac{c_P(t)}{Q_P(t)} \right] (k-1) k^{j-n-2} . \tag{4.14}$$

Since $L^P = L^{LM} = L^{UM} = L^R = \frac{1}{4} L$, (4.14) can be written as:

$$\overline{\Pi}_n = \frac{1}{4} L (k-1) k^{j-n-2} \left[2 \frac{c_P(t)}{Q_P(t)} + \frac{c_{LM}(t)}{Q_{LM}(t)} k \right] . \tag{4.15}$$

Similarly, the profit of a leader in an industry gap $n \geqslant 2$ is the sum of the profit in market UM and market R,

$$\Pi_n = \frac{1}{4} L (k-1) k^{j-n-2} \left[\frac{c_R(t)}{Q_R(t)} k^{n+1} + 2 \frac{c_{UM}(t)}{Q_{UM}(t)} k^n + 2 \frac{c_{LM}(t)}{Q_{LM}(t)} k + 2 \frac{c_P(t)}{Q_P(t)} \right] . \tag{4.16}$$

Quality gap n = 1 : In this case, there are three qualities being sold in the market. The follower sells the third best quality q^{j-2} to group P and the second best quality q^{j-1} to the group LM. The leader has 2 options: (i) selling the second best quality q^{j-1} to group UM at the price that the follower offers to group LM and the best quality to group R, or (ii) selling the first quality q^j to groups UM and R at the price low enough to attract group UM (and therefore also group R). Whether the leader chooses option (i) or (ii) depends on the inequality level of wealth endowment between the groups. Specifically, the leader chooses option (i) if wealth endowment inequality is high enough and vice versa.

We assume that the inequality of wealth endowments between groups are high enough in order to ensure that it is more profitable for the leader to chooses option (i).[2] In this case, the prices of different qualities are

$$p^{j-2} = \frac{c_P(t)}{Q_P(t)} (k-1) k^{j-3} + wa, \tag{4.17}$$

[2] In case of option (ii), leader sells the first quality to group R and UM at the price that attracts UM (therefore also R). This price is defined as

$$p^j = \left[\frac{c_{UM}(t)}{Q_{UM}(t)} k^2 + \frac{c_{LM}(t)}{Q_{LM}(t)} k + \frac{c_P(t)}{Q_P(t)} \right] (k-1) k^{j-3} + wa.$$

The profit of the leader is

$$\Pi_1' = \frac{1}{4} L (k-1) k^{j-3} \left[2 \frac{c_{UM}(t)}{Q_{UM}(t)} k^2 + 2 \frac{c_{LM}(t)}{Q_{LM}(t)} k + 2 \frac{c_P(t)}{Q_P(t)} \right].$$

The leader prefers option (i) if to option (ii) if $\Pi_1' < \Pi_1$. This happens when $2 \frac{c_{UM}(t)}{Q_{UM}(t)} < \frac{c_R(t)}{Q_R(t)}$.

$$p^{j-1} = \left[\frac{c_{LM}(t)}{Q_{LM}(t)}k + \frac{c_P(t)}{Q_P(t)}\right](k-1)\,k^{j-3} + wa \tag{4.18}$$

and

$$p^{j} = \left[\frac{c_R(t)}{Q_R(t)}k^2 + \frac{c_{LM}(t)}{Q_{LM}(t)}k + \frac{c_P(t)}{Q_P(t)}\right](k-1)\,k^{j-3} + wa. \tag{4.19}$$

The profit of the follower and leader in industry gap $n = 1$ respectively are

$$\overline{\Pi}_1 = \frac{1}{4}L\,(k-1)\,k^{j-3}\left[\frac{c_{LM}(t)}{Q_{LM}(t)}k + 2\frac{c_P(t)}{Q_P(t)}\right], \tag{4.20}$$

and

$$\Pi_1 = \frac{1}{4}L\,(k-1)\,k^{j-3}\left[\frac{c_R(t)}{Q_R(t)}k^2 + 2\frac{c_{LM}(t)}{Q_{LM}(t)}k + 2\frac{c_P(t)}{Q_P(t)}\right]. \tag{4.21}$$

Quality gap n = 0 (neck-and-neck firms): Both firms sell the best quality j to the mass of groups R and UM (we call this combined group as RM) and the second best quality $j-1$ to the mass of groups LM and P (we call this combined group as PM). The prices of the different qualities are

$$p^{j-1} = \frac{c_P(t)}{Q_P(t)}(k-1)\,k^{j-2} + wa \tag{4.22}$$

and

$$p^{j} = \left[\frac{c_{UM}(t)}{Q_{UM}(t)}k + \frac{c_P(t)}{Q_P(t)}\right](k-1)\,k^{j-2} + wa. \tag{4.23}$$

The profit of a neck-and-neck firm in industry gap $n = 0$ is

$$\Pi_0 = \frac{1}{4}L\,(k-1)\,k^{j-2}\left[\frac{c_{UM}(t)}{Q_{UM}(t)}k + 2\frac{c_P(t)}{Q_P(t)}\right]. \tag{4.24}$$

4.2.3 R&D and Innovation

The technological leader invests in R&D to come up with innovation that improves its existing quality a step $k > 1$. Hence, $q^j = k^j q^0$. For simplicity, we set $q^0 = 1$. Therefore, $q^j = k^j$. A firm who is not in the technological frontier can obtain higher technology by (i) copying the next higher product in the quality ladder available in the market , or (ii) open its technology and seek for higher quality by open innovation project. We assume that the costs of copying and for open innovation are increasing with the gap of technology. Labor is the only input of R&D production. Henceforth, we call an industry is in state (j, n) if the best quality level of this industry is j and the leader is n steps ahead of the follower and a firm (leader or follower) is in state (j, n) if this firm belongs to industry state (j, n).

Innovate new technology

A firm in the technological frontline with the highest quality j wants to obtain a higher technology $j+1$ by investing in R&D hires $x(\phi) = \beta\frac{\phi^2}{2}$ units of labor to realize an innovation with a Poisson hazard rate ϕ, $\beta > 0$, thus, the more a firm investing in R&D the more likely it would come up with an innovation.[3]

Copying

A firm which is not in the technological frontline in industry state (j, n) can coppy the next higher technology $j-1$ in the quality ladder with a Poisson hazard rate $\phi + h - \delta^{n-1}$ by hiring $x(\phi) = \beta\frac{\phi^2}{2}$ units of labor in R&D, where $h > 0$ captures the idea that it is easier to imitate than to innovate and $\delta > 0$ captures the idea that the larger the gap n the harder to imitate. We assume that δ is sufficiently small so that $h-\delta^{n-1} > 0$ to ensure that imitating is always easier than innovating.

Open to catch up with higher technology

A follower in state (j, n) can also obtain higher technology by hiring $x(\phi) = \gamma\frac{\phi^2}{2}\frac{1}{O^\epsilon}$ units of labor to work on an R&D open innovation project. This open innovation project yields either a hazard rate of $\phi - \sigma^{n+1}$ that firm will get the next technological innovation race or a probability $\phi + g - \sigma^{n-1}$ that firm will end up with imitating the next upper technology $j-1$ in the market. $\sigma > 0$ captures the idea that the larger the gap n the more difficult it is for the follower to innovate or to imitate. $g > 0$ captures the idea that it is easier to imitate than to innovate. $\epsilon > 0$ and O^ϵ capture the external effects of the crowd that participates in the open project.

It is reasonable to assume that given the same cost (the amount of labor), among the three ways of obtaining higher technology, opening is the most difficult way, innovating comes next and the easiest is copying.

Firm's R&D Problems

The leader's problem is to decide how much to invest in R&D effort in order to realize the next innovation. The follower has the problem of deciding how much to invest in solely copying and how much to invest in open innovation project.

We denote $\phi_{I_n^j}$ as the R&D effort of a leader in state (j, n) and $\phi_{C_n^j}$, $\phi_{O_n^j}$, respectively, as the imitating effort and open source R&D effort of a follower in state (j, n). We denote Π_n^j and V_n^j, respectively, as the profit and the expected present value of a leader in state (j, n). Similarly, $\overline{\Pi}_n^j$ and $\overline{V}_n^j$ respectively are the profit and the expected present value of a follower in state (j, n). Π_0^j and V_0^j respectively are the profit and the expected present value of neck-and-neck firms in state $(j, 0)$.

[3] We follow Aghion et al. (2001) for the R&D cost function.

Quality gap n ⩾ 2 : In this case, the leader is a monopoly of the best two quality markets and the follower is a monopoly in the third and forth best quality markets. Only the leader sells the best quality product to the group R and the second best quality to the group UM; only the follower sells the third best quality product to the group LM and the forth best quality to the group P.

Industry in state (j, n) could move to state $(j+1, n+1)$ with probability $\phi_{I_n^j} dt$; this happens when the leader is successful in innovating to move one step forward in the quality ladder. Industry in state (j, n) could become state $(j, 1)$ if the follower succeeds to imitate quality $j-1$ with probability $\left(\phi_{C_n^j} + h - \delta^{n-1}\right) dt + \left(\phi_{O_n^j} + g - \sigma^{n-1}\right) dt$, or state $(j+1, 1)$ if the follower wins the innovation race with probability $\left(\phi_{O_n^j} - \sigma^{n+1}\right) dt$ by the open innovation project. Note that a leader who is n steps ahead could become a follower with 1 step behind when the follower wins the innovation race. Finally, industry state (j, n) remains in state (j, n) with probability $1 - \phi_{I_n^j} dt - \left(\phi_{C_n^j} + h - \delta^{n-1}\right) dt - \left(\phi_{O_n^j} + g - \sigma^{n-1}\right) dt - \left(\phi_{O_n^j} - \sigma^{n+1}\right) dt$ if both leader and follower fail to improve their qualities.

The leader faces the following Hamilton–Jacobi–Bellman equation:

$$V_n^j = \max\{\left(\Pi_n^j - \beta\frac{\phi^2_{I_n^j}}{2}w\right)dt + e^{-rdt}\begin{bmatrix} \phi_{I_n^j} dt V_{n+1}^{j+1} \\ +\left(\phi_{C_n^j} + h - \delta^{n-1} + \phi_{O_n^j} + g - \sigma^{n-1}\right) dt V_1^j \\ +\left(\phi_{O_n^j} - \sigma^{n+1}\right) dt \overline{V}_1^{j+1} \\ \left(1 - \phi_{I_n^j} dt - \left(\phi_{C_n^j} + h - \delta^{n-1}\right) dt - \left(\phi_{O_n^j} + g - \sigma^{n-1}\right) dt - \left(\phi_{O_n^j} - \sigma^{n+1}\right) dt\right) V_n^j \end{bmatrix}\}. \tag{4.25}$$

For a small dt, it can be written as

$$rV_n = \max\begin{bmatrix} \left(\Pi_n^j - \beta\frac{\phi^2_{I_n^j}}{2}w\right) + \phi_{I_n^j}\left(V_{n+1}^{j+1} - V_n^j\right) \\ +\left(\phi_{C_n^j} + h - \delta^{n-1} + \phi_{O_n^j} + g - \sigma^{n-1}\right)\left(V_1^j - V_n^j\right) \\ +\left(\phi_{O_n^j} - \sigma^{n+1}\right)\left(\overline{V}_1^{j+1} - V_n^j\right) \end{bmatrix}. \tag{4.26}$$

Similarly, the follower's Hamilton–Jacobi–Bellman equation is given by:

$$r\overline{V}_n^j = \max\begin{bmatrix} \left(\overline{\Pi}_n^j - \beta\frac{\phi^2_{C_n^j}}{2}w - \gamma\frac{\phi^2_{O_n^j}}{2}\frac{1}{O^\epsilon}w\right) + \phi_{I_n^j}\left(\overline{V}_{n+1}^{j+1} - \overline{V}_n^j\right) \\ +\left(\phi_{C_n^j} + h - \delta^{n-1} + \phi_{O_n^j} + g - \sigma^{n-1}\right)\left(\overline{V}_1^j - \overline{V}_n^j\right) \\ +\left(\phi_{O_n^j} - \sigma^{n+1}\right)\left(V_1^{j+1} - \overline{V}_n^j\right) \end{bmatrix} \tag{4.27}$$

All the firms choose its R&D effort to maximize the right hand side of their Hamilton–Jacobi–Bellman equation. The first order conditions of (4.26) and (4.27) respectively yield:

$$\phi_{I_n^j} = \frac{V_{n+1}^{j+1} - V_n^j}{\beta w}, \tag{4.28}$$

$$\phi_{C_n^j} = \frac{\overline{V}_1^j - \overline{V}_n^j}{\beta w} \tag{4.29}$$

and

$$\phi_{O_n^j} = \frac{\overline{V}_1^j - \overline{V}_n^j + V_1^{j+1} - \overline{V}_n^j}{\gamma w} O^{\epsilon}. \tag{4.30}$$

Quality gap $\mathbf{n = 1}$: In this case, the leader is a monopoly of the best quality market and shares the second-best-quality market with the follower. The third best quality market is monopolized by the follower. Only leader sells the best-quality product to the group R, the leader and the follower sell the second-best-quality product to the groups UM and LM, and only the follower sells the third-best-quality product to the group P.

The leader in industry state $(j, 1)$ could become the leader in industry $(j+1, 2)$ with probability $\phi_{I_1^j} dt$, this happens when the leader is successful in innovating to move one step forward. The leader in state $(j, 1)$ could become a neck-to-neck firm in industry state $(j, 0)$ if follower succeeds to imitate with probability $\left(\phi_{C_1^j} + h - \delta\right) dt + \left(\phi_{O_1^j} + g - \sigma\right) dt$. The leader in industry $(j, 1)$ could become a follower of industry $(j+1, 1)$ with probability $\left(\phi_{O_1^j} - \sigma^2\right) dt$ when the follower wins the innovation race by the open innovation project. Finally, the leader in industry $(j, 1)$ remains unchanged with probability

$$1 - \phi_{I_1^j} dt - \left(\phi_{C_1^j} + h - \delta\right) dt - \left(\phi_{O_1^j} + g - \sigma\right) dt - \left(\phi_{O_1^j} - \sigma^2\right) dt$$

if both leader and follower fail to improve their qualities. Therefore, the leader faces the following Hamilton–Jacobi–Bellman equation:

$$rV_1^j = \max \left[\begin{array}{c} \left(\Pi_1^j - \beta \frac{\phi_{I_1^j}^2}{2} w\right) + \phi_{I_1^j} \left(V_2^{j+1} - V_1^j\right) \\ + \left(\phi_{C_1^j} + h - \delta + \phi_{O_1^j} + g - \sigma\right) \left(V_0^j - V_1^j\right) \\ + \left(\phi_{O_1^j} - \sigma^2\right) \left(\overline{V}_1^{j+1} - V_1^j\right) \end{array} \right]. \tag{4.31}$$

The value of the leader in state $(j, 1)$ at time t is equal the profit flow $\Pi_1^j dt$ minus the R&D cost $\beta \frac{\phi_{I_1^j}^2}{2} w dt$ plus the expected discounted value of the new states of the leader in the next period.

Similarly, the follower faces the following Hamilton–Jacobi–Bellman equation:

$$r\overline{V}_1^j = \max \left[\begin{array}{c} \left(\overline{\Pi}_1^j - \beta \frac{\phi^2_{C_1^j}}{2} w - \gamma \frac{\phi^2_{O_1^j}}{2} \frac{1}{O^\epsilon} w \right) + \phi_{I_1^j} \left(\overline{V}_2^{j+1} - \overline{V}_1^j \right) \\ + \left(\phi_{C_1^j} + h - \delta + \phi_{O_1^j} + g - \sigma \right) \left(V_0^j - \overline{V}_1^j \right) \\ + \left(\phi_{O_1^j} - \sigma^2 \right) \left(V_1^{j+1} - \overline{V}_1^j \right) \end{array} \right]. \quad (4.32)$$

In words, the value of the follower in state $(j, 1)$ at time t is equal the profit flow $\overline{\Pi}_1^j dt$ minus the R&D cost $\left(\beta \frac{\phi^2_{C_1^j}}{2} w + \gamma \frac{\phi^2_{O_1^j}}{2} \frac{1}{O^\epsilon} w \right) dt$ plus the expected discounted value of the new states of the follower in the next period.

The first order conditions of (4.31),(4.32) and (4.36) respectively yield:

$$\phi_{I_1^j} = \frac{V_2^{j+1} - V_1^j}{\beta w}, \quad (4.33)$$

$$\phi_{C_1^j} = \frac{V_0^j - \overline{V}_1^j}{\beta w} \quad (4.34)$$

and

$$\phi_{O_1^j} = \frac{V_0^j - \overline{V}_1^j + V_1^{j+1} - \overline{V}_1^j}{\gamma w} O^\epsilon. \quad (4.35)$$

Quality gap $\mathbf{n = 0}$: In this case, the leader and the follower are identical. Both neck-and-neck firms have the same technology and share the whole market. Both leader and follower sell the best quality product to groups R and UM and the second best quality to groups UM and P.

The industry is in state $(j, 0)$. Either one of the two firm has a probability of $\phi_{I_0^j} dt$ to win the R&D race and be the leader in state $(j+1, 1)$ and a probability of $1 - 2\phi_{I_0^j} dt$ to remain in state $(j, 0)$ as neither of the firms proceed to improve.

Firm's Hamilton–Jacobi–Bellman equation is

$$rV_0^j = \max\{\Pi_0^j - \beta \frac{\phi^2_{I_0^j}}{2} w + \phi_{I_0^j} \left(V_1^{j+1} - V_0^j \right) + \phi_{I_0^j} \left(\overline{V}_1^{j+1} - V_0^j \right)\}. \quad (4.36)$$

All the firms choose its R&D effort to maximize the right hand side of their Hamilton–Jacobi–Bellman equation. The first order condition of (4.36) yields

$$\phi_{I_0^j} = \frac{V_1^{j+1} - V_0^j}{\beta w}. \quad (4.37)$$

4.3 Steady-State Innovation Growth Rate

In a balanced growth path, the proportion of industries with technological gap n must be constant. We denote θ_n^j as the proportion of industries in state (j, n), and θ_n as the proportion of all the industries with technological gap n. By definition we have

$$\theta_n = \sum_{j \in J(n)} \theta_n^j$$

and

$$\sum_{n \geqslant 0} \sum_{j \in J(n)} \theta_n^j = 1, \tag{4.38}$$

where $J(n)$ is the set of the best qualities of all the industries with technological gap n between the leader and the follower.

Quality gap $\mathbf{n} \geqslant \mathbf{2}$: The inflow of the industries with technological gap $n \geq 2$ in interval time dt is defined as

$$\sum_{j \in J(n-1)} \theta_{n-1}^j \phi_{I_{n-1}^j} dt$$

That is the inflow of industries with technological gap $n \geq 2$ is all the industries in state $(j, n-1)$ in which the leaders succeed to innovate to increase the gaps with the followers one more step and hence these industries turn into state$(j+1, n)$.

The outflow of the industries with technological gap $n \geq 2$ in interval time dt is

$$\sum_{j \in J(n)} \theta_n^j \left(\phi_{I_n^j} + \phi_{C_n^j} + h - \delta^{n-1} + \phi_{O_n^j} + g - \sigma^{n-1} + \phi_{O_n^j} - \sigma^{n+1} \right) dt.$$

There are $\sum_{j \in J(n)} \theta_n^j \theta_n^j \phi_{I_n^j} dt$ industries in state (j, n) in which leaders succeed to innovate and increase the gap with the followers one more step and hence these industries turn into state $n+1$. There are $\sum_{j \in J(n)} \theta_n^j \theta_n^j \left(\phi_{C_n^j} + h - \delta^{n-1} \right) dt$ industries in state (j, n) in which followers succeed to imitate and there are $\sum_{j \in J(n)} \theta_n^j \theta_n^j \left(\phi_{O_n^j} + g - \sigma^{n-1} \right) dt$ industries in which followers succeed to imitate by open innovation projects and, hence, these industries turn into state $n = 1$.There are $\sum_{j \in J(n)} \theta_n^j \theta_n^j \left(\phi_{O_n^j} - \sigma^{n+1} \right) dt$ industries in state (j, n) in which the followers win the technological race by open innovation projects and, hence, these industries turn into state $n = 1$.

Since θ_n is constant, we must have

$$\sum_{j\in J(n-1)} \theta_{n-1}^j \phi_{I_{n-1}^j} = \sum_{j\in J(n)} \theta_n^j \left(\phi_{I_n^j} + \phi_{C_n^j} + h - \delta^{n-1} + \phi_{O_n^j} + g - \sigma^{n-1} + \phi_{O_n^j} - \sigma^{n+1}\right). \tag{4.39}$$

Quality gap $\mathbf{n = 1}$: Similarly, we have

$$\sum_{j\in J(0)} 2\theta_0^j \phi_{I_0^j} + \sum_{n\geqslant 2}\sum_{j\in J(n)} \theta_n^j \left(\phi_{C_n^j} + h - \delta^{n-1} + \phi_{O_n^j} + g - \sigma^{n-1} + \phi_{O_n^j} - \sigma^{n+1}\right)$$
$$= \sum_{j\in J(1)} \theta_1^j \left(\phi_{I_1^j} + \phi_{C_1^j} - \delta + h + \phi_{O_1^j} + g - \sigma\right). \tag{4.40}$$

The inflow of industries in state $n = 1$ consists of three elements. First, $\sum_{j\in J(0)} 2\theta_0^j \phi_{I_0^j} dt$ industries in state $n = 0$ in which either one of the two neck-to-neck competitors wins the race. Second, the aggregate of all the industries in state $n \geqslant 2$ that have the followers succeed to imitate the upper technology level in the quality ladder either by copying effort or by open innovation projects. And third, the aggregate of all the industries in state $n \geqslant 2$ where the followers have won the technological race by open innovation projects.

The outflow of industries in state $n = 1$ consists of three elements. First, $\sum_{j\in J(1)} \theta_1^j \phi_{I_1^j} dt$ industries in state $n = 1$ in which the leaders succeed to innovate to increase the gap with the followers one more step and, hence, these industries turn into state $n = 2$. Second, $\sum_{j\in J(1)} \theta_1^j \left(\phi_{C_1^j} + h - \delta\right) dt$ industries in state $n = 1$ in which the followers succeed to imitate and, hence, these industries turn into state $n = 0$. And, third, $\sum_{j\in J(1)} \theta_1^j \left(\phi_{O_1^j} + g - \sigma\right) dt$ industries in which the followers succeed to imitate by open innovation projects and, hence, these industries also turn into state $n = 0$.

Quality gap $\mathbf{n = 0}$: Similarly, we have

$$\sum_{j\in J(1)} \theta_1^j \left(\phi_{C_1^j} + h - \delta + \phi_{O_1^j} + g - \sigma\right) = \sum_{j\in J(0)} 2\theta_0^j \phi_{I_0^j}. \tag{4.41}$$

The inflow of industries in state $n = 0$ consists of industries in state $n = 1$ that have the followers succeeded to imitate the upper technology level in the quality ladder either by copying effort or by open innovation projects. The outflow of industries in state $n = 0$ consists of $\sum_{j\in J(0)} 2\theta_0^j \phi_{I_0^j} dt$ industries in state $n = 0$ in which either one of the two neck-to-neck competitors wins the race and, hence, these industries turn into state $n = 1$.

Group Quality Indices

Given θ_n^j as the proportion of industries in state (j,n), the quality index of the four groups in equation (4.2) are defined as

$$Q_P = \sum_{n\geqslant 2}\sum_{j\in J(n)}\int_{\theta_n^j} k^{j-n-1}(\omega)d\omega + \sum_{j\in J(1)}\int_{\theta_1^j} k^{j-2}(\omega)d\omega + \sum_{j\in J(0)}\int_{\theta_0^j} k^{j-1}(\omega)d\omega, \tag{4.42}$$

$$Q_{LM} = \sum_{n\geqslant 2}\sum_{j\in J(n)}\int_{\theta_n^j} k^{j-n}(\omega)d\omega + \sum_{j\in J(1)}\int_{\theta_1^j} k^{j-1}(\omega)d\omega + \sum_{j\in J(0)}\int_{\theta_0^j} k^{j-1}(\omega)d\omega, \tag{4.43}$$

$$Q_{UM} = \sum_{n\geqslant 2}\sum_{j\in J(n)}\int_{\theta_n^j} k^{j-1}(\omega)d\omega + \sum_{j\in J(1)}\int_{\theta_1^j} k^{j-1}(\omega)d\omega + \sum_{j\in J(0)}\int_{\theta_0^j} k^{j}(\omega)d\omega \tag{4.44}$$

and

$$Q_R = \sum_{n\geqslant 2}\sum_{j\in J(n)}\int_{\theta_n^j} k^{j}(\omega)d\omega + \sum_{j\in J(1)}\int_{\theta_1^j} k^{j}(\omega)d\omega + \sum_{j\in J(0)}\int_{\theta_0^j} k^{j}(\omega)d\omega. \tag{4.45}$$

Standardized Goods Consumption

Given θ_n^j as the proportion of industries in state (j,n), the total expenditures that a consumer in group P spends on quality goods in equation (4.4) is defined as

$$P_P = \sum_{n\geqslant 2}\sum_{j\in J(n)}\int_{\theta_n^j} p^{j-n-1}(\omega)d\omega + \sum_{j\in J(1)}\int_{\theta_1^j} p^{j-2}(\omega)d\omega + \sum_{j\in J(0)}\int_{\theta_0^j} p^{j-1}(\omega)d\omega. \tag{4.46}$$

Substituting (4.10), (4.17) and (4.22) to (4.46) and using $y_i = c_i + P_i$ we obtain

$$\begin{aligned} P_P &= \sum_{n\geqslant 2}\sum_{j\in J(n)}\int_{\theta_n^j} \frac{y_P - P_P}{Q_P}(k-1)\,k^{j-n-2}d\omega + \sum_{j\in J(1)}\int_{\theta_1^j} \frac{y_P - P_P}{Q_P}(k-1)\,k^{j-3}d\omega \\ &\quad + \sum_{j\in J(0)}\int_{\theta_0^j} \frac{y_P - P_P}{Q_P}(k-1)\,k^{j-2}d\omega + 3wa. \end{aligned} \tag{4.47}$$

Combining (4.47) with (4.42) and isolating P_P we obtain

$$P_P = \frac{(k-1)\,y_P + 3kwa}{2k-1}. \tag{4.48}$$

Similarly, the total expenditures that a consumer in groups LM, UM and R spend on quality goods respectively are

$$P_{LM} = \frac{kP_P + (k-1)\, y_{LM} \sum_{n \geqslant 1} \sum_{j \in J(n)} \theta_n^j}{k + (k-1) \sum_{n \geqslant 1} \sum_{j \in J(n)} \theta_n^j}, \tag{4.49}$$

$$P_{UM} = \frac{kP_{LM} + (k-1)\, y_{UM} \left(\sum_{n \geqslant 2} \sum_{j \in J(n)} \theta_n^j + \sum_{j \in J(0)} \theta_0^j \right)}{k + (k-1) \left(\sum_{n \geqslant 2} \sum_{j \in J(n)} \theta_n^j + \sum_{j \in J(0)} \theta_0^j \right)} \tag{4.50}$$

and

$$P_R = \frac{kP_{UM} + (k-1)\, y_R \sum_{n \geqslant 1} \sum_{j \in J(n)} \theta_n^j}{k + (k-1) \sum_{n \geqslant 1} \sum_{j \in J(n)} \theta_n^j}. \tag{4.51}$$

Rewriting (4.48), (4.49), (4.50) and (4.51) we obtain the standardized consumption of consumers P, LM, UM and R respective are

$$c_P = \frac{k}{2k-1} \left(y_P - 3wa \right), \tag{4.52}$$

$$c_{LM} = \frac{k \left(y_{LM} - y_P + c_P \right)}{k + (k-1) \sum_{n \geqslant 1} \sum_{j \in J(n)} \theta_n^j}, \tag{4.53}$$

$$c_{UM} = \frac{k \left(y_{UM} - y_{LM} + c_{LM} \right)}{k + (k-1) \left(\sum_{n \geqslant 2} \sum_{j \in J(n)} \theta_n^j + \sum_{j \in J(0)} \theta_0^j \right)} \tag{4.54}$$

and

$$c_R = \frac{k \left(y_R - y_{UM} + c_{UM} \right)}{k + (k-1) \sum_{n \geqslant 1} \sum_{j \in J(n)} \theta_n^j}. \tag{4.55}$$

4.3.1 Innovation growth rate

The quality index of quality goods given that q^j is the highest quality of industry ω is defined as

$$\ln Q(t) = \int_0^1 \ln q^j \left(\omega, t \right) d\omega. \tag{4.56}$$

The growth rate of $Q(t)$ is

$$g=\frac{d}{dt}\ln Q(t)=\frac{d}{dt}\int_0^1 \ln q^j\left(\omega,t\right)d\omega=\frac{d}{dt}\ln q^j\left(\omega,t\right). \tag{4.57}$$

When industry ω reach to quality j, the value of $\ln q^j$ rises $j\ln k$.Thus,

$$g=\lim_{\Delta t\to\infty}\frac{\Delta\ln q^j\left(\omega,t\right)}{\Delta t}=\lim_{\Delta t\to\infty}\frac{j}{\Delta t}\ln k. \tag{4.58}$$

Industry ω reaches quality q^j when innovation has occurred j times. Recall that innovations could be made by either a neck-and-neck firm, leader, or follower with open innovation project. Therefore, $\lim_{\Delta t\to\infty}\frac{j}{\Delta t}$ is the sum of (i) the number of times that neck-to-neck firms innovate, and (ii) the number of times that leader innovates and (iii) the number of times that followers innovate by open innovation project.

When $\Delta t\to\infty$, the number of times that neck-to-neck firms innovate is equal to the flow of industry from state $n=0$ to state $n=1$, which equals $\sum\limits_{j\in J(0)} 2\theta_0^j\phi_{I_0^j}$.

The number of times that the leader innovates equals the product of $n-1$ and the frequency that industries with gap $n\geqslant 2$ turn into gap 1 because followers imitate leaders' second-best quality or followers win the technological race by open innovation, aggregating this product for all $n\geqslant 2$ we get[4]

$$\sum_{n\geqslant 2}(n-1)\sum_{j\in J(n)}\theta_n^j\left(\phi_{C_n^j}+h-\delta^{n-1}+\phi_{O_n^j}+g-\sigma^{n-1}+\phi_{O_n^j}-\sigma^{n+1}\right).$$

The number of times that followers innovate by an open innovation project is $\sum\limits_{n\geqslant 1}\sum\limits_{j\in J(n)}\theta_n^j\left(\phi_{O_n^j}-\sigma^{n+1}\right)$. Therefore, we have

$$\begin{aligned}\lim_{\Delta t\to\infty}\frac{j}{\Delta t} &= \sum_{j\in J(0)}2\theta_0^j\phi_{I_0^j}+\sum_{n\geqslant 1}\sum_{j\in J(n)}\theta_n^j\left(\phi_{O_n^j}-\sigma^{n+1}\right)\\ &\quad+\sum_{n\geqslant 2}(n-1)\sum_{j\in J(n)}\theta_n^j\left(\phi_{C_n^j}+h-\delta^{n-1}+\phi_{O_n^j}+g-\sigma^{n-1}+\phi_{O_n^j}-\sigma^{n+1}\right).\end{aligned} \tag{4.59}$$

[4] In a certain cycle, an industry is in state n because the incumbent leader innovates $n-1$ times after a neck-to-neck firm innovates or a follower innovates by open innovation project.

The third term on the right hand side can be written as

$$\begin{aligned}
& \sum_{n\geqslant 2}(n-1)\sum_{j\in J(n)}\theta_n^j\left(\phi_{C_n^j}+h-\delta^{n-1}+\phi_{O_n^j}+g-\sigma^{n-1}+\phi_{O_n^j}-\sigma^{n+1}\right)\\
= & \sum_{n\geqslant 2}\sum_{j\in J(n)}\theta_n^j\left(\phi_{C_n^j}+h-\delta^{n-1}+\phi_{O_n^j}+g-\sigma^{n-1}+\phi_{O_n^j}-\sigma^{n+1}\right)\\
& +\sum_{n\geqslant 3}\sum_{j\in J(n)}\theta_n^j\left(\phi_{C_n^j}+h-\delta^{n-1}+\phi_{O_n^j}+g-\sigma^{n-1}+\phi_{O_n^j}-\sigma^{n+1}\right)\\
& +... \qquad (4.60)
\end{aligned}$$

Moreover, we have

$$\begin{aligned}
& \sum_{n\geqslant k}\sum_{j\in J(n)}\theta_n^j\left(\phi_{C_n^j}+h-\delta^{n-1}+\phi_{O_n^j}+g-\sigma^{n-1}+\phi_{O_n^j}-\sigma^{n+1}\right)\\
= & \sum_{n\geqslant 2}\sum_{j\in J(n)}\theta_n^j\left(\phi_{C_n^j}+h-\delta^{n-1}+\phi_{O_n^j}+g-\sigma^{n-1}+\phi_{O_n^j}-\sigma^{n+1}\right)\\
& -\sum_{n=2}^{k-1}\sum_{j\in J(n)}\theta_n^j\left(\phi_{C_n^j}+h-\delta^{n-1}+\phi_{O_n^j}+g-\sigma^{n-1}+\phi_{O_n^j}-\sigma^{n+1}\right) \qquad (4.61)
\end{aligned}$$

From (4.39) we have

$$\begin{aligned}
& \sum_{n=2}^{k-1}\sum_{j\in J(n)}\theta_n^j\left(\phi_{I_n^j}+\phi_{C_n^j}+h-\delta^{n-1}+\phi_{O_n^j}+g-\sigma^{n-1}+\phi_{O_n^j}-\sigma^{n+1}\right)\\
= & \sum_{j\in J(1)}\theta_1^j\phi_{I_1^j}-\sum_{j\in J(k-1)}\theta_{k-1}^j\phi_{I_{k-1}^j}. \qquad (4.62)
\end{aligned}$$

Substituting (4.62) into (4.61) we obtain

$$\begin{aligned}
& \sum_{n\geqslant k}\sum_{j\in J(n)}\theta_n^j\left(\phi_{C_n^j}+h-\delta^{n-1}+\phi_{O_n^j}+g-\sigma^{n-1}+\phi_{O_n^j}-\sigma^{n+1}\right)\\
= & \sum_{n\geqslant 2}\sum_{j\in J(n)}\theta_n^j\left(\phi_{C_n^j}+h-\delta^{n-1}+\phi_{O_n^j}+g-\sigma^{n-1}+\phi_{O_n^j}-\sigma^{n+1}\right)\\
& -\sum_{j\in J(1)}\theta_1^j\phi_{I_1^j}+\sum_{j\in J(k-1)}\theta_{k-1}^j\phi_{I_{k-1}^j}. \qquad (4.63)
\end{aligned}$$

Combining with (4.40) and (4.41), we obtain

$$\sum_{n\geqslant k}\sum_{j\in J(n)}\theta_n^j\left(\phi_{C_n^j}+h-\delta^{n-1}+\phi_{O_n^j}+g-\sigma^{n-1}+\phi_{O_n^j}-\sigma^{n+1}\right)=\sum_{j\in J(k-1)}\theta_{k-1}^j\phi_{I_{k-1}^j}. \qquad (4.64)$$

Substituting (4.64) into (4.60) and using the result to substitute into (4.59) we obtain

$$\lim_{\Delta t \to \infty} \frac{j}{\Delta t} = \sum_{j \in J(0)} 2\theta_0^j \phi_{I_0^j} + \sum_{n \geqslant 1} \sum_{j \in J(n)} \theta_n^j \left(\phi_{O_n^j} - \sigma^{n+1} \right) + \sum_{n \geqslant 1} \sum_{j \in J(n)} \theta_n^j \phi_{I_n^j}.$$

Therefore, (4.58) can be written as

$$g = \left[\sum_{j \in J(0)} 2\theta_0^j \phi_{I_0^j} + \sum_{n \geqslant 1} \sum_{j \in J(n)} \theta_n^j \left(\phi_{O_n^j} - \sigma^{n+1} \right) + \sum_{n \geqslant 1} \sum_{j \in J(n)} \theta_n^j \phi_{I_n^j} \right] \ln k, \tag{4.65}$$

where θ_n^j $(n \geqslant 0)$ are determined by equations (4.38)-(4.41) and $\phi_{I_0^j}, \phi_{I_n^j}$, and $\phi_{O_n^j}$ $(n \geqslant 1)$ are determined by equations (4.26)-(4.37). Equation (4.65) states that the growth rate of innovation is the product of logarithm of the magnitude of innovation and the aggregate frequency of all the innovations carried out by neck-and-neck firms, industry leaders and followers with open source projects.

4.4 Conclusions

We model an economy with standardized goods and quality goods where individuals with non-homothetic preference have to allocate their budget for standardized goods and quality goods. We incorporate the characteristics of conventional R&D, copying and open innovation in the maximization problems of multi-quality firms and examine how open innovation affects R&D investments of firms with different technology levels and then its effects on economic growth. Calibration and more analysis on the impacts of open innovation on growth as well as how inequality among different population groups affects the market price structures and, thus, incentives of firms to engage in different types of R&D investment.

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