UNIVERSITAT ROVIRA I VIRGILI

PRE-SERVICE TEACHERS' MATHEMATICS TEACHING BELIEFS AND MATHEMATICAL CONTENT KNOWLEDGE

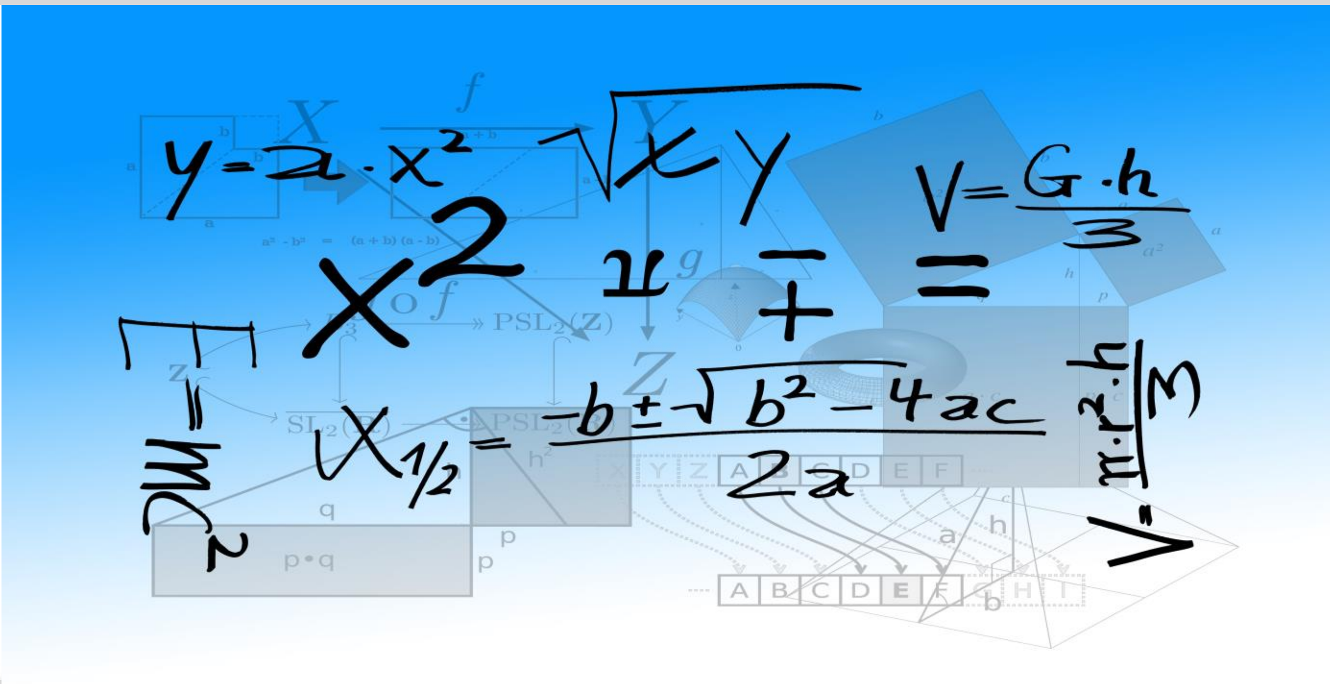
Jaime Rodrigo Segarra Escandón

UNIVERSITAT ROVIRA i VIRGILI

Pre-service teachers' mathematics teaching beliefs and mathematical content knowledge

---

JAIME RODRIGO SEGARRA ESCANDÓN



DOCTORAL THESIS
2021

UNIVERSITAT
ROVIRA i VIRGILI

ROVIRA AND VIRGILI UNIVERSITY

DEPARTMENT OF COMPUTER SCIENCE
AND MATHEMATICS OF SECURITY

Pre-service teachers' mathematics teaching beliefs and mathematical content knowledge

PHD THESIS

JAIME RODRIGO SEGARRA ESCANDÓN

Advisor: Dra. Carme Julià Ferré.

Tarragona, 2021

UNIVERSITAT
ROVIRA i VIRGILI

Rovira and Virgili University

UNIVERSITAT ROVIRA i VIRGILI

Rovira and Virgili University

Department of Computer Science
and Mathematics of Security

I STATE that the present study, entitled "Pre-service teachers' mathematics teaching beliefs and mathematical content knowledge" , presented by Jaime Rodrigo Segarra Escandón for the award of the degree of Doctor, has been carried out under my supervision at the Department Computer Science and Mathematics of Security of this university.
Approved by

February 11th, 2021.

(Signature)

MARIA CARMEN JULIÀ FERRÉ - DNI 78582424a

Firmado digitalmente por MARIA CARMEN JULIÀ FERRÉ - DNI 78582424a
Fecha: 2021.02.10 09:43:38 +01'00'

....

Dra. Carme Julià Ferré.

Tarragona, 2021

Abstract

The study of mathematical knowledge in the initial training of pre-teachers is essential, since the knowledge of the teachers influences that of the students. Besides, it is also important to measure the pre-service and in-service teachers' efficacy beliefs and attitude towards mathematics. It is thought that the self-efficacy a teacher feels whithin the mathematics area will influence the self-efficacy of her/his students. The goal of this Thesis is to study the initial mathematical knowledge of pre-service teachers and also their mathematics efficacy beliefs and attitude towards mathematics. To carry out the study of the initial knowledge of numbers and geometry, 20 items of TIMSS 2011 were selected and a test was applied to 97 first-year students from the Rovira and Virgili University.

Furthermore, in other to study the pre-service teachers' belief about the efficacy of their mathematics teaching throughout the Bachelor Degree, pre-service teachers of each year of the Primary Education Degree answered the Mathematics Teaching Efficacy Belief Instrument (MTEBI) at the end of the 2016-2017 academic year. The MTEBI is comprised of two subscales, namely, Personal Mathematics Teaching Efficacy (PMTE) and Mathematics Teaching Outcome Expectancy (MTOE).

It is also important to measure the mathematics teaching self-efficacy and outcome expectancy of pre-service teachers, novice in-service teachers (up to 10 years of experience) and experienced in-service teachers (more than 10 years of experience) are compared. The MTEBI is used to carry out the study.

Additionally, the relation between mathematics teaching efficacy beliefs of pre-service teachers, their attitude towards mathematics and their mathematics academic achievement is studied. Specifically, this part of the Thesis considers both factors together and studies their correlation with mathematics academic achievement. To evaluate mathematics academic achievement, the grades obtained in the teaching and learning mathematics course that students took in the second year of the Degree are collected. The MTEBI is used to measure the teachers' efficacy beliefs of pre-service teachers. Besides, the Attitude towards Mathematics Scale (AMS) is used to rate students' attitude towards mathematics.

Moreover, the influence of the teaching experience, the level of education and the level of teaching on mathematics teaching efficacy beliefs is studied. Concretely, the MTBEI is answered by 354 primary and secondary in-service teachers in Tarragona city.

Finally, this Thesis compares the self-efficacy for the teaching of mathematics of students in the fourth year of the Basic General Education Degree at the Azuay University, in Cuenca, Ecaudor, and the fourth year of the Primary Education Degree at the Rovira and Virgili University, in Tarragona, Spain. The subscale Personal Mathematics Teaching Efficacy is used to measure the self-efficacy of teaching mathematics.

The results of this Thesis offer potentially important information on mathematical knowledge, teachers' efficacy beliefs, self-efficacy of mathematics teaching and the attitude towards mathematics of pre-service teachers. It also provides information about in-service teachers' efficacy beliefs. These results can help policy-makers, curriculum developers, and teaching professors in higher education institutes.

Keywords

Pre-service teachers, primary education, knowledge of numbers and geometry, mathematics teaching efficacy beliefs, attitude, mathematics academic achievement.

To my wife Margarita and my children
Angel David and Jaime Matías.
Also, my parents Angel and Normelia.
Especially for all your support.

Acknowledgements

First, I would like to express my sincere gratitude to my Thesis advisor, Dra. Carme Julia, for allowing me to be part of the working team of the didactics of mathematics research group. In addition, I would like to thank my Thesis advisor for the time and effort she has put into the research projects we have carried out over the last three years.

Second, I would like to thanks to my family. I want to thank my wife Margarita and my children David and Matías for all the support they have given me in this process. Also, I want to thank my parents Normelia and Angel for all their support during this very important stage of my life.

Finally, I would like to thanks to Martí i Franqués Predoctoral Research Grant 2017PMF-PIPF (URV).

Tarragona, 25/02/2021

Jaime Rodrigo Segarra Escandón

Contents

List of Figures

List of Tables

Chapter 1

Introduction

1.1 Motivation

Nowadays, there is a widespread opinion of the importance of teacher training to achieve success in primary education. Clear evidence of this is the existence of different research, at international level, dealing with the subject (e.g., [2], [3], [4], [5], [6]). Some of the existing research is concerned about the low mathematical elementary content knowledge of pre-service teachers (e.g., [7], [4], [8], [9], [10], [11]). According to [12] and [1], the quality of mathematics teaching depends on teachers' knowledge of the content. They focused their research on determining what teachers need to know about mathematics to be successful with students in classrooms. In addition, the following question it should be considered: why Spanish primary education students do not stand out for their good results in mathematics on the international PISA (Programme for International Student Assessment) [13] and TIMSS (Trends in International Mathematics and Science Study) [14] tests?

Existing research is concerned about the low mathematical elementary content knowledge of pre-service teachers (e.g., [9], [7]). This low mathematical content knowledge might provokes poor results in their mathematical academic achievement. Indeed, in addition to elementary content knowledge of pre-service teachers, there are different variables that were found to impact on academic achievement, such as self-concept [15], values [16], or more cognitive variables, such as learning strategies [17] or previous knowledge [18].

It should be noted that it is vitally important that teachers dominate the subject that they have to explain to their students, not only at the content level, but also at the didactic level. That is, teachers need to know which difficulties their students will encounter when they learned certain content or procedures, and how they can try to help them overcome these difficulties; and that is what the Primary Education Bachelor Degree should offer them at University. The problem is that the level of knowledge of certain content or procedure is often so low that one can hardly tackle the didactic part of it. It is clear that first it is necessary to dominate the content.

Besides, knowing the level of mathematical knowledge of the students at the beginning of the Primary Education Degree is to reinforce the contents in which they present difficulties. Teachers have a very important role in education since it is considered that they directly influence the learning of students. Therefore, it is necessary to carry out research from their initial training in order to improve the learning of mathematical knowledge of future teachers. Ma [19] argued that a large part of success in the classroom depends on the teacher's own understanding of mathematics, together with an appropriate disciplinary appropriation, and security and self-confidence in the mastery of such knowledge. In this sense, Haciomeroglu [20] stated that teachers with a strong mathematical knowledge are better prepared to help their students to understand the subject significantly. Specifically, the professional knowledge that teachers must have is a determining factor for the effectiveness of teaching [21]. Without doubt, future teachers must have excellent initial training ensuring solid knowledge of content and didactics, so that they can be quality professionals in the future.

Moreover, one aspect that is considered important in the teaching-learning process is the analysis of the errors and difficulties that students have in solving certain mathematics tests. As noted by [22], research in recent years has shown the importance of focusing attention not only on students' correct answers, but also on the mistakes they make. The errors may have different origins, but they are always considered as an inadequate cognitive scheme and not only as a consequence of lack of knowledge or a mistake. These errors and difficulties are sometimes so profound that they call into question the entire teaching-learning process of mathematics [23]. Some authors recognize that the development of problems with decimals is a major source of learning difficulties with students and pre-service teachers (e.g., [24], [25]). It is therefore common for students to make repeated errors in problems with decimals.

Other researches are worried about the lack of motivation and confidence of pre-service teachers when they teach and learn mathematics (e.g., [26], [27], [28], [29], [30]). The problem is that if they do not change their beliefs, they will show them when teaching mathematics at primary education school. Bursal [31] pointed out that teachers are the most important single influence on students' attitudes toward mathematics. The author suggested that, since most of tomorrow's teachers are today's pre-service teachers, the beliefs they hold should be of concern to teacher educators. There exists a large number of researchers presenting exhaustive studies about teacher's efficacy belief of pre-service teachers and in-service teachers (e.g., [32], [31], [33]). These works highlight the importance of the teacher's efficacy belief towards mathematics. This importance is evidenced in the existing research that shows how teachers with low levels of self-efficacy are less likely to obtain positive results with their students ([34], [35]).

In the context of pre-service teachers, it is noticed that that some students of the Bachelor Degree in Primary Education present low levels of mathematics teaching efficacy beliefs and a negative attitude towards mathematics ([36], [37]). Literature research supports the importance of enhancing the pre-service teachers' mathematics teaching efficacy beliefs (e.g., [31], [38], [39]) and their attitude towards mathematics (e.g., [37], [40]), and how these factors influence to the mathematics academic achievement (e.g., [41], [42]), [43]). As pointed out in [44], it is still important and interesting to investigate the impact of variables concerning teachers' mathematics related effect on other affective or cognitive variables such as academic achievement.

1.2 Context

In Spain, students who finish high school (baccalaureate) can directly enroll to the Primary Education Degree. There is no requirement about the type of baccalaureate students need to choose during high school. In this degree, they will learn the didactic and content knowledge that will allow them to teach students between the ages of 6 and 12. From 2017, in Catalunya it is necessary to pass a Personal Aptitude Test (PAP) to access the Early Childhood and the Primary Education Bachelor Degree. The PAP test evaluates two competencies: the communicative and critical reasoning competence; and the logical-mathematical competence. Specifically, the logical-mathematical competence is the ability to use knowledge and skills relating to various areas of mathematics to solve exercises, problems and applicable situations, and the ability to analyse the results obtained from the point of view of their reasonableness. The final qualification of the PAP, which is obtained from the arithmetic mean of both exams, is either Pass or Fail.

According to the Royal Decree (476/2013) [45], primary education will be taught by Teachers in Primary Education Bachelor Degree. Teachers may teach all areas of knowledge in primary education, with the exception of music, physical education and foreign language. In addition, according to the Royal Decree (476/2013) [45], the program will have between 180 and 240 credits, which will contain all the theoretical and practical training that the student must acquire: basic aspects of the branch of knowledge, obligatory and optional courses, seminars, external practices and final degree project. Specifically, in Catalunya the Primary Education Degree has 240 ECTS.

Most of the research in this Thesis is focused on pre-service teachers of the Primary Education Degree. This Degree consists of a four-year program, composed by eight semesters (240 ECTS). The first year of the program contains only introductory pedagogical courses. The Bachelor includes three mandatory courses of Teaching and Learning Mathematics (TLM). Table 1.1 shows the distribution of the three TLM courses along the Bachelor Degree.

Table 1.1: *Distribution of the Teaching and Learning Mathematics courses (TLM) and the student teaching periods at the Primary Education Bachelor Degree (semester is denoted by S)*

	1st		2nd		3rd		4th	
	S1	S2	S3	S4	S5	S6	S7	S8
TLM				TLM1	TLM2	ST1	ST2	TLM3
Duration in weeks				17	17	13	13	17
Duration in hours				60	60	360	360	60

The TLM1 course (2nd year) presents the mathematical content pre-service teachers need to know to teach in primary school (see Appendix D for details). The students have to review mathematical content and processes, and to solve problems. In the TLM2 course (3rd year), the pre-service teachers learn how to use manipulatives and interactive applications to teach Numeracy at primary school (see Appendix D). Finally, in the TLM3 course (4th year), the students are presented materials to teach Geometry at primary school. The sessions of TLM3 consist of Geometry workshops, in which the students have to work in teams. The idea of these workshops is to promote a student-centered learning (see Appendix D).

In addition to the TLM courses, the students have two 3-month teaching periods along the Bachelor (see Table 1.1). During these student teaching periods (TS1,TS2), the pre-service teachers stay at a school under a supervised teaching practice. Particularly, they have to observe and analyse everything they see at school. Furthermore, they have to teach some lessons, in a supervised way. In the first period of student teaching, they have to analyse the mathematics teaching at the particular school and use manipulatives they studied in TLM2. After the student teaching period, they must prepare a report to conclude the TLM2 course. Therefore, the TLM2 course is linked to the first student teaching period.

1.3 Objectives

The main goal of this Thesis was to study both the mathematical content knowledge of the students of the Primary Education Bachelor Degree at the Rovira and Virgili University and also their teachers' efficacy beliefs for teaching mathematics. Furthermore, in-service teachers' efficacy beliefs for teaching mathematics is also studied. Specifically, in order to achieve the purposes of this Thesis, the following tasks were identified:

1. To study the mathematical knowledge of first-year students of the Primary Education Bachelor Degree. Besides, this work also pretends to study their common errors in detail. The idea is to detect the weaknesses that the pre-service teachers present.

2. To study the pre-service teachers' efficacy beliefs for teaching mathematics and its evolution throughout the Bachelor Degree in Primary Education.

3. To compare the mathematics teaching self-efficacy and outcome expectancy of pre-service and in-service primary education teachers.

4. To analyse the relation between mathematics teaching efficacy beliefs of pre-service teachers, their attitude towards mathematics and their mathematics academic achievement.

5. To study the influence of the level of teaching factor (primary education and secondary school) and the level of teacher training (degree and master) on mathematics teaching efficacy beliefs in-service teachers.

6. To compare the self-efficacy of pre-service teachers for the teaching of mathematics of students in the fourth year of the Basic General Education Degree at the Azuay University and the fourth year of the Primary Education Degree at the Rovira and Virgili University.

1.4 Thesis Outline

This Thesis is organised into nine chapters. Chapter (chapter 1) presents the motivation and importance of the mathematical elementary content knowledge of pre-service teachers, specifically, knowledge of geometry and numbers. Further, this chapter presents motivation and the importance of the teachers' efficacy beliefs and the attitude towards mathematics. Furthermore, this chapter provides information relating to the specific context of the study and defines the research objectives.

Chapter 2 provides a review of literature relevant to the research reported in this Thesis. Specifically, the classification of the categories of knowledge and the literature review of the pre-service teachers' knowledge in numbers and geometry are presented. Besides, self-efficacy theory and literature review the mathematics teaching efficacy belief for pre-service teachers and in-service teachers are presented. Finally, attitude towards mathematics theory and literature review of the attitude towards mathematics for pre-service teachers are presented.

In chapter 3, the results of the study the mathematical knowledge of first-year students of the Primary Education Bachelor Degree are presented. In particular, chapter3 studies the scoring and resolution processes of 20 TIMSS (Trends in International Mathematics and Science Study)-type questions of numbers and geometry. Moreover, chapter 3 studies in detail the errors made by the students in some of the problem type questions.

Chapter 4 exposes the results of the study the pre-service teachers' self-efficacy for teaching mathematics and its evolution throughout the Bachelor Degree in Primary Education at the Rovira and Virgili University. Specifically, the chapter studies the Personal Mathematics Teaching Efficacy (PMTE) and Mathematics Teaching Outcome Expectancy (MTOE).

Chapter 5 compares the mathematics teaching self-efficacy and outcome expectancy of pre-service and in-service primary education teachers. Concretely, chapter studies the Personal Mathematics Teaching Efficacy (PMTE), comparing the three groups (pre-service teachers, novice teachers and experienced teachers). In addition, chapter studies the Mathematics Teaching Outcome Expectancy (MTOE), comparing the three groups (pre-service teachers, novice teachers and experienced teachers).

In chapter 6, the correlation between mathematics teaching efficacy beliefs, the attitude towards mathematics and the mathematics academic achievement of pre-service teachers is studied.

Chapter 7 shows how the level of teaching factor (primary education and secondary school) and the level of teacher training (degree and master) influence the mathematics teaching efficacy beliefs of in-service teachers.

In chapter 8, the self-efficacy of pre-service teachers of the Primary Education Degree at the Rovira and Virgili Unversity and the one of pre-service teachers of the Basic General Education Degree at Azuay University is compared. In particular, students of the fourth year of both Degrees are considered for the study.

Finally, chapter 9 summarizes the findings obtained in the six studies carried out in this Thesis and provides future lines of research.

Chapter 2

Literature review

This chapter reviews the theoretical framework and the state of the art related to the variables of this Thesis, these being: domains of mathematical knowledge, mathematics teaching efficacy beliefs and attitude towards mathematics.

2.1 Domains of Mathematical Knowledge

The work presented in [46] suggested to consider the following categories of content knowledge; (a) content of the subject, (b) pedagogical knowledge, and (c) curriculum knowledge. It is important to note that [47] considered that mathematical knowledge for teaching has two components: knowledge of mathematical content and didactic knowledge of mathematical content. Moreover , Ball et al. [1] divided into two categories the mathematical knowledge for teaching that is composed of Specific Mathematical Knowledge and Didactic Knowledge of Content as can be seen in Figure 2.1. Maintaining the two main categories, Knowledge of Content and Knowledge of The Didactics of Content, the authors had subdivided each of them into three others.

In the first domain, a division of the Content Knowledge domain proposed by [46] is proposed by segmenting it into: Common Content Knowledge, Horizon Content Knowledge and Specialized Content Knowledge. The Common Content knowledge, which is the one that any well-educated adult must have. The Specialized Content Knowledge, which is the knowledge that only teachers need to know. Finally, the Horizon Content Knowledge, which makes reference to the relationships between mathematical subjects at different school levels. More specifically, the Common Content Knowledge corresponds to knowledge acquired in school or throughout life. It is the one that is used to solve mathematical problems, operate correctly and apply definitions and properties, so it is the knowledge of the mathematicians or of an educated adult subject [1].

In the second domain, Pedagogical Content Knowledge is divided into three subcategories: Knowledge of Content and Students, Knowledge of Content and Teaching, and Knowledge of content and Curriculum . In the Pedagogical Content Knowledge domain, Ball et al. [1] defined Knowledge Content of and students as the knowledge of content that is intertwined with knowledge of how students think, know, or learn a particular content. It is that used in teaching assignments that involve attending to specific content and particular aspects of students. It includes knowledge of common errors of students and the most common difficulties, misconceptions, strategies that can be used. All this makes the teacher able to assess the student's understanding and know how their mathematical reasoning evolves. Besides, Knowledge of Content and Teaching results from the integration of mathematical content with knowledge of the teaching of such content. Knowledge of Content and Curriculum alludes to knowledge of objectives, content, curriculum orientations, materials and resources available for teaching, which allow the teacher to guide his practice and select the appropriate tasks for learning [48].

This Thesis studies only the category of content knowledge. Specifically, this Thesis studies the mathematical content knowledge of numbers and geometry. These two content domains are select since previous studies have pointed out the importance of learning arithmetic (numbers) in the mathematical training of pre-service teachers [49] and the importance of geometry [50].

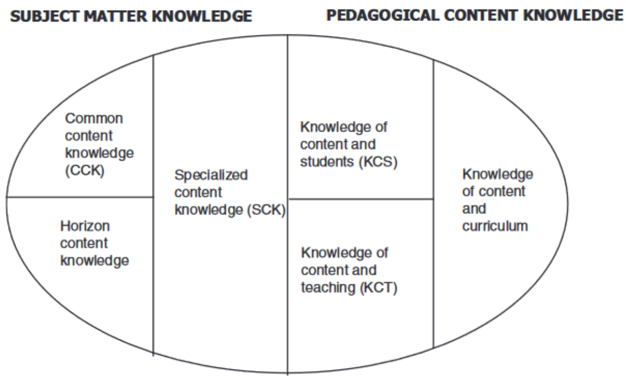


Figure 2.1: *Domains of Mathematical Knowledge for Teaching [1].*

2.1.1 Related work

This section presents the literature review of the pre-service teachers' knowledge of mathematical content. Today, it exists a clear concern about the pre-service teachers' training, since it influence the quality of the Primary Education teaching at school.

Martí [51] stated that political pressure to evaluate the performance of school students has led to the existence of educational evaluation systems that regularly provide information on the international situation of learning processes. These studies allow for comparisons between countries and are used as a relative measure of existing overall quality. The most important international studies for primary education and secondary school students are PISA (Program for International Student Assessment) and TIMSS (Third International Mathematics and Science Study). In most countries where these tests are applied, deficiencies in mathematical knowledge of primary and secondary students have been revealed.

The TIMSS project evaluates the performance of students in mathematics and science, quantifying the scope of learning in these two subjects, as well as the context in which occurs [51]. TIMSS evaluates knowledge of mathematical content across four domains: numbers, algebra, geometry, and data and probability. Each of these domains presents goals and skills or abilities. Moreover, the questions are classified according to the cognitive domains known, applyed and reasoned.

At TIMSS 2011, 2015 and 2019, 4th grade Spanish Primary Education students scored 482, 505 and 502 points, respectively. These scores are below the OECD (Organisation for Economic Cooperation and Development) scores: 522, 525 and 527, respectively. According to [4], these low results are related to poor teacher training, both in mathematical and teaching knowledge. One of the most important international study in pre-service teachers is the TEDM-S tests (Teacher Education Study in Mathematics). The comparative TEDM-S study was carried out during the years 2006-2010 for primary education and secondary school teachers in training with the participation of 17 countries. Spain participated in the study in pre-service teachers of primary education, obtaining 481 points in mathematical knowledge, below the mean (500 points), evidencing deficiencies in knowledge of mathematical content and knowledge of didactics, which puts the preparation of initial teacher training on alert.

It is important to mention the research presented in [52]. That research is focused on TEDM-S tests and it indicates that the results are not positive in the mathematical and didactic knowledge tests of mathematics compared to the countries in our region. According to [53], the studies of pre-service teachers are installed in compliance with minimums, and do not seem to find sufficient incentives to attract better students or to differentiate themselves from the rest through an improvement in the quality, both of its contents, and of the practice.

Salinas [54] studied pre-service teachers the contents referring to school mathematics, in the sense of remembering knowledge acquired in the first stages of teaching. The author discussed the importance of understanding position value to know and understand our decimal numbering system and operations. It also notes that students have gaps in knowledge and conceptual errors in mathematical content that should have been acquired in the early years of primary school.

In the study presented in [55], the authors investigated the knowledge of mathematics and the knowledge of mathematics teaching of teachers in training. The authors found a connection between knowledge of mathematics and knowledge of mathematics teaching. It is suggested that candidates for primary mathematics teachers must be educated in both the aspects of mathematical knowledge and knowledge of pedagogical content.

In [56], the authors studied the mathematical concepts of area and perimeter of teachers in training. The authors found that many teachers in training in all cohorts had a procedural understanding of area and perimeter, displayed similar misconceptions to their student counterparts, and had limited ability to demonstrate examples of the mathematical knowledge required to teach these subjects.

Nortes and Nortes [23] addressed errors and difficulties that future teachers have in solving elementary mathematics problems when developing the Mathematics test for entry into the Primary School Teachers' Training in 2013 in the Community of Madrid, for contents corresponding to 6th grade of Primary School. Most of the errors are due to poor learning of facts, skills and previous concepts produced by misused data, lack of verification of the solution and calculation errors. The percentage of error exceeds 50% in all courses, with half of the students failing the test.

In [57], the authors analyzed the common knowledge of mathematical content on fractions and decimals of students for elementary school teachers. This was done through responses to a reagent test that involved the use of fractions and decimals. Among the main results, it is worth noting that, in most cases, future teachers present greater difficulty in solving problems involving the use of fractions than those involving decimals.

In [58], the authors conducted research on mathematical knowledge of future teachers in problem solving, applying the Final Assessment of Mathematical Competence of 6th Of Primary Education test of the community of Madrid. The results indicated that one in three students does not exceed the two applied assessments and that in the second (first course with mathematics) one of the assessments does not exceed 50%. Of the total participants, three out of twenty students answer all the questions well. Students had the greatest difficulty in the equivalence between fraction, decimal and percentage, where only one in four students answers correctly.

More recently, Lo [59] studied pre-service primary teachers' (PSPTs) mathematics performance on a set of question items measuring at the sixth grade level. These items covered the five learning strands of the local primary mathematics curriculum number, algebra, measures, shape and space, and data handling. The participants were 152 PSPTs who had chosen mathematics as a major in an undergraduate teacher education program. Their strengths and weaknesses in particular mathematics topics were identified. Across all four years of the teacher education program, only 6% of the participants were able to provide fully correct mathematical responses to all five items. Further, it is surprising that merely 14% of graduating cohorts achieved a full mark for the mathematical word problem that required them to show their working steps. The authors suggested that issues relating to the preparation of mathematics specialists should be further considered in primary mathematics education.

In a different approach, Barham [60] investigated the development of problem-solving strategies demonstrated by 42 elementary pre-service mathematics teachers in problem-solving mathematics classes. The author used a mixed methods approach of quantitative and qualitative research by analyzing the collected data. The author revealed participants' weakness in applying the variety of skills required for success in problem solving, such as interpreting information, mathematical working, and logical thinking. Results also demonstrated a limited and incorrect use of mathematical terminology, as well as a lack of problem comprehension.

Considering previous studies and taking into account that initial teacher training is a key element for improving educational quality [61], one of the purposes of this Thesis is to study the knowledge of the mathematical content that students possess at the beginning of the Primary Education Degree. The idea is to detect deficiencies and difficulties that future teachers present and try to overcome them. The research consists of applying a TIMSS-type test and analyzing the knowledge of the content domains of numbering and geometry, and the cognitive domains of knowledge, application and reasoning.

2.2 Mathematics teaching efficacy beliefs

Bandura developed the self-efficacy theory ([62], [63], [64]). That theory is based on the principal assumption that psychological procedures serve as means of creating and strengthening expectations of personal efficacy. Within this analysis, efficacy expectations are distinguished from response-outcome expectancies. An outcome expectancy is defined as a person's estimate that a given behaviour will lead to certain outcomes. An efficacy expectation is the conviction that one can successfully execute the behaviour required to produce the outcomes. According to Bandura, the strength of people's convictions in their own effectiveness is likely to affect whether they will even try to cope with given situations. People fear and tend to avoid threatening situations they believe exceed their coping skills, whereas they get involved in activities and behave assuredly when they judge themselves capable of handling situations that would otherwise be intimidating. Efficacy expectations determine how much effort people will expend and how long they will persist in facing obstacles and aversive experiences. The stronger the perceived self- efficacy, the more active the efforts.

The self-efficacy is one's personal judgement about how well they can deal with a specific situation or task (social cognitive theory) [65]. A person's self-efficacy could influence the effort they put forth in a given situation and how long they persist on a given task [66]. According to Bandura, people fear and tend to avoid threatening situations they believe exceed their coping skills, whereas they get involved in activities and behave assuredly when they judge themselves capable of handling situations that would otherwise be intimidating. Efficacy expectations determine how much effort people will expend and how long they will persist in facing obstacles and aversive experiences. The stronger the perceived self-efficacy, the more active the efforts.

Taking into account Bandura's theory ([62], [67]), studies on teachers' efficacy beliefs consider two separate dimensions [68]. The first dimension, personal teaching efficacy, represents a teacher's belief in their skills and abilities to be an effective teacher. The second dimension, teaching outcome expectancy, is a teacher's belief that effective teaching can bring about student learning regardless of external factors such as home environment, family background, and parental influences [69]).

Some researchers report that there exists relationship between efficacy belief and academic performance in the area of mathematics (e.g., [70]). In addition, [71] determined that self-efficacy is a strong predictor of performance in the pre-service teachers and that the outcome expectation is positively related to their performance. Furthermore, [72] investigated the resources of mathematics self-efficacy and perception of science self-efficacy as predictors of academic achievement. According to their obtained results, resources of mathematics self-efficacy and perception of science self-efficacy are significantly correlated with academic achievement in high levels of influence.

2.2.1 Related work

Literature research shows that teacher's efficacy belief is an important construct in teaching. Students learn more from teachers with high self-efficacy than from those whose self-efficacy is low [32]. In the first part of this section, existing researchers who studied the mathematics teaching efficacy belief for pre-service teachers is presented.

Utley et al. [73] investigated the relationship between science and mathematics teaching efficacy of pre-service teachers. Additionally, the authors investigated if the teacher efficacy beliefs about mathematics and science teaching change during participation in methods courses and student teaching. Their results showed that both the personal teaching efficacy and the outcome expectancy beliefs increase during the science and mathematics courses. However, they decrease slightly after the student teaching period. They pointed out that as the pre-service teachers progress in their college coursework the student's optimism and enthusiasm tend to increase and then become blemished when confronted with the reality of the classroom.

In [69], the authors presented a longitudinal study of elementary pre-service teachers' mathematics beliefs and content knowledge. They conclude that the teacher preparation programs can have an impact on the beliefs of pre-service teachers about mathematics teaching and learning. At the end, they pointed out that their research give them information about how to construct their program, which may provide more opportunities to enforce the pre-service beliefs about their efficacy.

Bursal [31] investigated Turkish pre-service elementary teachers' self-efficacy beliefs regarding mathematics and science teaching. Particularly, that work is focused on the teaching self-efficacy beliefs of participants in elementary mathematics and science (PMTE and PSTE). The outcome expectancy is not studied, since according to Bursal, it is usually viewed to be inconsistent among pre-service teachers. The reported results manifest that participants with mathematics/science high school majors have significantly higher PMTE and PSTE scores than those with other high school majors.

In a different approach, Newton et al. [74] examined the relationship between mathematics content knowledge and teacher efficacy during an elementary mathematics methods course. The authors founded a positive moderate relationship between content knowledge and personal teaching efficacy. However, the authors did not find any relationship between content knowledge and outcome expectancy.

Moody and DuCloux [38] studied the mathematics teaching efficacy of traditional and non-traditional elementary pre-service teachers enrolled in a three-course, three-semester mathematics sequence. The authors showed that non- traditional elementary pre-service teachers improve their self-efficacy beliefs with regard to both personal teaching efficacy and outcome expectancy. However, traditional pre-service teachers' beliefs concerning outcome expectancy do not change significantly.

A study presented in [75] investigated primary school teachers' characteristics by comparing their mathematics teaching self-efficacy beliefs. Their results showed that teachers with a higher self-efficacy belief show a higher level of effort and persistence with students. Besides, those teachers believe in students' achievements and take responsibilities for students' success.

In [27], the authors analysed the elementary pre-service teachers' self- efficacy for teaching mathematics. The authors showed that the pre-service teachers had positive levels of efficacy regarding their mathematics teaching abilities as well as positive levels of outcome expectancy for their students in mathematics.

In a more recent approach [76], the authors investigated Influence of Self-Efficacy and Self-Esteem on Attitude of Pre-Service Teachers towards Teaching Profession. The authors revealed among others that self-efficacy, and self-esteem have positive influence on attitude of Pre-Service teachers towards teaching profession . It was concluded that pre-service teachers with high self-efficacy and self-esteem has the potentials of excelling in teaching profession.

Mutlu et al. [77] the authors analysed pre-service mathematics teachers' self-efficacy beliefs about using concrete models in teaching mathematics. The authors demonstrated that the instruction had positive contributions on the pre-service teachers' self-efficacy beliefs. In addition, the authors revealed that pre-service teachers had confidence in themselves about using concrete models both as learners and as teachers. Moreover, they believed that using concrete models would have positive consequences in teaching process and students' learning. However, pre-service teachers had relatively low efficacies about classroom management.

In the second part of this section, existing research that studied the mathematics teaching efficacy belief for in-service teachers is presented.

There exist works that studied the influence of teaching experience on teacher's efficacy belief. Liu et al. [78] investigated 282 Taiwan elementary teachers' views of teaching efficacy and outcome expectations. Their study advocated the position that the years of teaching experience of elementary science teachers have a significant impact on personal science teaching efficacy and science teaching outcome expectations.

In [74], the authors examined the relationship between mathematics content knowledge and teacher efficacy during an elementary mathematics methods course. The authors founded a positive moderate relationship between content knowledge and teaching self- efficacy. However, they did not find any relationship between content knowledge and outcome expectancy.

Holzberger et al. [79] explored reciprocal effects of teachers' self-efficacy and instructional quality in a longitudinal panel study. The authors studied if the teachers' self-efficacy beliefs affect their instructional quality or if it is a re- verse effect perhaps more likely, with positive experiences in the classroom positively impacting the development of teachers' self-efficacy. Their findings showed that teachers with higher self-efficacy did not necessarily provide higher quality instruction, when measured with a 1-year time lag. On the other hand, the authors showed that the self-efficacy of teachers not only changes over the course of a school year but also increases in response to experiences of success in the classroom.

A study presented in [75] investigated primary school teachers' characteristics by comparing their mathematics teaching self-efficacy beliefs. The author stated that teachers with a greater belief in self-efficacy show a higher level of effort and persistence with students, being more open to new ideas and new methods. In addition, these teachers believe in students' academic achievements and take responsibility for student success.

Chang [33] examined the relationships between the efficacy of primary math teachers with the self-efficacy and mathematical performance of their students. Their results showed that mathematics teachers' beliefs of efficacy have a significant influence on the self-efficacy and performance of their students in mathematics.

More recently, Julaihi et al. [80] examined the confidence level and self-efficacy beliefs of Mathematics teachers, using an instrument adapted from two reliable questionnaires: Trend in International Mathematics and Sciences Study (TIMSS) and Teachers' Sense of Teaching Mathematics Efficacy Scale (TSES). The respondents were 49 teachers from both primary and secondary schools. The analysis reported that teachers have shown practically high confidence level in teaching mathematics and they closely agreed on their beliefs about their own teaching mathematics efficacy. The authors reported that there were no significant differences in mean scores between teachers' confidence level and self-efficacy beliefs across gender, highest education attained and years of mathematics teaching. The correlations analysis showed a significant positive strong relationship between teachers' confidence level and self- efficacy beliefs.

Thomson and Walkowiak [81] studied the development of mathematics efficacy trajectories for novice teachers over the course of their teacher preparation and into their first two years of teaching. Participants were graduates of an elementary primary program. Overall, quantitative results showed that participants' efficacy trajectory increased during their teacher preparation program and decreased during their first years of teaching. Additionally, efficacy trajectories were identified for particular groups that displayed patterns of either increased or decreased efficacy over time.

Finally, there is research that studies factors that may influence teacher's efficacy beliefs. Some of these researches are detailed below.

The research presented in [82] analyzed the relationships between four factors of teacher's efficacy beliefs. Among the important factors analyzed are teaching experience and level of teaching. The authors found, in a sample of 321 teachers, that experience and level of teaching influence the self-efficacy of teachers. In addition, the results indicated that teachers who believe they can motivate students to achieve better academic performance give less evidence of stress than teachers who believe that neither they nor other teachers can affect student performance.

Gavora et al. [83] studied the measurement of the self-efficacy of in-service teachers. Specifically, the authors studied the teaching experience factor and found, in a sample of 217 teachers, that serving teachers with more than 5 years of teaching experience have higher arithmetic mean scores than teachers with less than 5 years of experience in the teacher 's efficacy beliefs. The authors also indicated that highly qualified teachers in teaching score high scores in self-efficacy.

In a different study, Takunyaci and Takunyaci [84] determined preschool teacher's efficacy beliefs concerning mathematics teaching using the Mathematics Teaching Efficacy Belief Instrument (MTEBI). The authors showed that the teachers' efficacy beliefs on teaching mathematics depends on their years of experience. Particularly, their results evidenced that the preschool teachers who have 13 and more years of teaching experience presented significant higher values of teachers' efficacy beliefs than the teachers did with less years of teaching experience.

In [85], the authors studied the correlation between perception of self-efficacy and teaching performance and posed six research questions. One of the questions posed: is there any difference in teacher self-efficacy in relation to the level they teach? The researchers found, in a sample of 26 primary and pre-primary teachers, that teachers teaching in primary education reported a higher arithmetic average of self-efficacy in all its dimensions than pre-primary teachers.

Ngidi and Ngidi [86] examined the factors that influence the sense of self efficacy of pre-service teachers. The authors revealed that pre-service teachers differed in terms of the extent of their teaching self-efficacy. Furthermore, the authors also showed that the program of study emerged as a significant predictor of efficacy in the classroom management subscale of teaching self-efficacy.

Results obtained in the above presented studies evidence the importance of improving the primary pre-service and in-service teachers' self-efficacy for teaching mathematics. As mentioned earlier, this Thesis studies teachers' efficacy beliefs of pre-service and in-service teachers.

2.3 Attitude towards mathematics

Attitude is a learned disposition or tendency on the part of an individual to respond positively or negatively to some object, situation, concept or another person [87]. Neale [88] defined attitude to mathematics as an aggregated measure of a liking or disliking of mathematics, a tendency to engage or avoid mathematical activities, a belief that one is good or bad at mathematics, and a belief that mathematics is useful or useless. Attitude refers to a learned tendency of a person to respond positively or negatively towards an object, situation, a concept, or a person. It is also regarded as a belief held by individuals that reflects their opinions and feelings and can be sometimes manifested in behaviour [89]. Attitudes, behaviour, and feelings are interrelated in such a way that people's attitudes determine their behaviour towards objects, situations, and people [90].

In [91], the authors defined university academic achievement as a result of learning, aroused by the educational activity of the teacher and produced in the student, although it is clear that not all learning is the product of teaching action. The performance is expressed in a qualitative, quantitative and qualitative in many cases, a note that if it is consistent and valid will be the reflection of a certain learning or the achievement of pre-established objectives. It is necessary to establish the difference between school achievement and academic performance. Academic or school performance is based on the budget that the student is responsible for their performance. In addition, school achievement refers to the teaching-learning process, whose levels of efficiency are responsible for both the teacher and the learner [92]. Some researchers reported that it has been shown that there is a statistically significant relationship between achievement in mathematics and the attitude towards mathematics [93], [94].

Several researchers suggested that the pre-service teachers should have a positive attitude towards mathematics, because it influences their mathematics academic achievement. Moreover, they can have a positive effect on the learning of their students in the future ([95], [43]). However, results of previous studies show that teachers of all levels of education often have negative attitudes towards mathematics (e.g., [36], [37]). Besides, other researchers indicated that primary pre-service teachers, worldwide, possessed negative attitudes towards mathematics and fears about the teaching of mathematics (e.g., [96], [42]).

2.3.1 Related work

There are several researchers who highlighted the importance of studying teachers' efficacy beliefs in teaching mathematics and attitude towards mathematics as both factors significantly influence the students' academic performance (e.g., [97], [98], [75], [33] [99]). In [98], the authors studied the relationship between the mathematical anxiety of elementary school teachers, mathematics instructional practices, and students' performance in mathematics. The authors showed that low-level beliefs in self-efficacy to teach mathematics could cause mathematical anxiety, which, at the same time, can negatively influence student performance. In addition, the authors found a positive relationship between mathematics anxiety and anxiety about teaching mathematics. Moreover, they found that the increase in student performance in mathematics was related to lower levels of anxiety from teaching mathematics, but was not related to general anxiety about mathematics.

The results presented in [100] showed that the attitude towards Mathematics in primary pre-teachers is very low. In addition, the authors indicated that the students perceive the teacher's attitude. Therefore, if the attitude of the teacher is not positive, it negatively influences the attitude of their students. Flores and Auzmendi [99] stated that the attitude towards mathematics of future teachers is generally high positive. The authors indicated that the teachers have the responsibility to develop positive attitudes among their students.

In a different study, Fachrudin et al. [101] examined Pre-Service Mathematics Teachers' (PSMTs) general knowledge of PISA and their attitudes and beliefs towards using PISA-based problem in mathematics. The authors revealed the poor score results of PSMTs knowledge of PISA. Additionally, they stated that the teacher education program needs to improve the PSMTs PISA skills and knowledge. Also, the result regarding PSMTs attitude and belief indicate that almost all the participants have positive attitude and belief toward using PISA-Based problem in teaching mathematics.

More recently [39], the authors studied the relationship between teachers' knowledge, beliefs and instructional practices based on a sample of 495 pre-service mathematics teachers. Results indicated that Chinese pre-service mathematics teachers tend to hold mixed beliefs about the nature of mathematics, and a constructivist view about mathematics teaching and learning, and that they are inclined to report that their teaching is inquiry-oriented.

Taking into account the above presented works, this Thesis aims at studying the relationship between pre-service teachers' attitude towards mathematics with their mathematics academic achievement.

Chapter 3

Knowledge of Numbers and Geometry

This chapter aims at studying the mathematical knowledge of first-year students of the Primary Education Bachelor Degree. Besides, this work also pretends to analyse their common errors in detail. To carry out this study, 20 questions from the TIMSS 2011 test were selected. Specifically, the selected questions correspond to numbers and geometry content.

3.1 Introduction

The study of mathematical knowledge in the initial training of pre-teachers is essential, since the knowledge of the teachers influences that of the students. In fact, there are a series of research that assess the knowledge of teachers in training (e.g., [55], [102], [4], [103], [104]).

3.2 Purpose of the study

In a first part, scores obtained in each of the questions are studied. The goal is to detect possible weakness and misunderstandings that the pre-teachers may present. Additionally, the procedures they use to solve the problem type questions are also studied. Specifically, in the first part of the analysis, obtained scores in each question are shown considering numbers and geometry domains separately. Besides, obtained grades of students at the test are studied. Moreover, percentages of obtained correct, blank, and wrong problem questions are also exposed. Finally, the mistakes made by pre-service teachers in the problem type questions of the test are studied in detail. In a second part of the analysis, four hypotheses are constructed, considering four different variables:

1. H1.There is no significant difference between the mean score of students graduated from the Baccalaureate in Science and those in Humanities and Social Sciences.

2. H2. There is no significant difference between the mean score of students who have not taken mathematics in the last two years and those who have.

3. H3. There is no significant difference between the average score of the questions corresponding to numbers content and those corresponding to geometry content.

4. H4. There is no significant difference between the mean scores obtained in the questions corresponding to each cognitive domain (knowing, analysing and reasoning).

3.3 Methodology

In this research, descriptive and inferential statistics are used for the analysis of the information, with the purpose of of analyzing the results.

3.3.1 Participants

The sample used in this investigation is a purposive sample [105]. The population under study corresponds to the first year students of the Primary Education Degree at the Rovira and Virgili University in the 2018/2019 academic year. Specifically, the test was applied to 97 students, representing 71% of the total population enrolled in the first year. For more details of the of the Bachelor Degree in Primary Education at Rovira and Virgili University see section 1.2. The participation was anonymous and voluntary. It is important to note that students in the Primary Education Degree do not receive a specific training as mathematics teachers. All students have three mandatory mathematics teaching and learning courses.

3.3.2 Instrument

The instrument used to carry out the research gathers information on the knowledge of mathematical content in numbers and geometry. These two content domains were selected since previous studies have pointed out the importance of learning arithmetic (numbers) (e.g., [49]) and geometry (e.g., [50]) in the mathematical training of future teachers.

Measuring teacher knowledge is a complex task and there is little consensus on how it should be done [106]. From the beginning, the scheme and procedures of the TIMSS study were consolidated as valid for assessing mathematical knowledge [4]. For this reason, and also to consider international statements, the test was prepared from the questions released from TIMSS 2011. Considering the Mathematical Content Domain of Secondary Compulsory Education (ESO), 20 questions were selected: 12 questions from the Number Content Domain (Q1-Q12), representing the 60% of the test, and 8 questions from the Geometry Content Domain (Q13-Q20), representing the 40% of it. It should be highlighted that eleven questions were posed to force the students to write the process they followed to answer the question. These items will be referred as problem questions (Q5, Q6, Q7, Q8, Q9, Q10, Q11, Q12, Q17, Q18, Q19 and Q20). The other questions maintain the structure of the TIMSS test and will be referred as objective questions.

The details of the used test are presented in the Appendix A.

Reliability

To determine the construct validity of the test, the Exploratory Factor Analysis methods were used. In particular, the Kaiser-Meyer-Olkin (KMO) and Bartlett's Sphericity Test (BTS) ([107], [108]) were used $(KMO = 0.64, p < 0.001)$. Moreover, the Kaiser-Meyer- Olkin test (KMO) indicates the adequacy of the sample size used. Besides, the Bartlett Sphericity Test (BTS) indicates that the correlations between items is not an identity matrix. The extracted factors explain 65% of the total variance of the data. The item-corrected item-total correlations of the items in the scale vary 0.32 and 0.62. In addition, the reliability of the test was determined by the Cronbach alpha coefficient of internal consistency [109]. The obtained value of $\alpha = 0.79$ evidences that the reliability is acceptable, following the rules presented in [110].

3.3.3 Data analysis

The students who attended the first day of classes were given 40 minutes to answer the 20 questions of the test. Then, we proceeded to generate a database with all the information provided by the test.

In order to grade the test, the following details were set: the objective questions are scored 0 or 1, depending on whether they were correct or incorrect. In the case of the problem type questions, the procedure of solving the questions is also taken into account in addition to the answers. Specifically, the score is 0 if both the answer and procedure are incorrect, 0.5 if the answer or procedure is correct, and 1 if both are correct. The test is scored on a scale from 0 up to 10. All calculations of the descriptive and inferential statistics were performed using the R programming language. The graphs were generated through Microsoft Excel and R programming language.

3.4 Results

In this section, results obtained in the TIMSS type test are analyzed in detail. Concretely, results are studied by considering two clearly different parts, as mentioned in the introduction section. The first part consists of studying the obtained scores, considering numbers and geometry content domains separately. Additionally, the mistakes made by the pre-service teachers in the problem type questions of the test are also studied. In the second part, four hypothesis are constructed, each one studying a different variable.

3.4.1 Studying scores and solving processes of the questions

Comparing scores obtained in numbers and geometry contain domains

This subsection compares results obtained in numbers and geometry content domains. Figure 3.1 shows the mean obtained in each question of the numbers content domain. If the scores corresponding to the 12 numbers content questions are taken, a mean of 6.54 and a standard deviation of 1.65 are obtained (the mean obtained by taking all the questions of the test is 6.14).

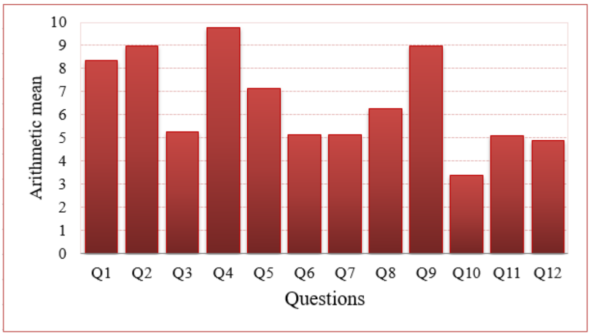


Figure 3.1: *Results of numbers content domain questions.*

Figure 3.1 shows that the question with the highest score is Q4, with a mean of 9.79. The questions with the lowest scores are Q10 and Q12, with a mean of 3.4 and 4.9, respectively.

According to TIMSS 2011 [111], question Q4 is in the content domain of numbers and the cognitive domain of applying. Within the content domain of numbers, it is associated with the topic of fractions and decimals. The objective of the question is to represent and operate with fractions and decimals, using models (number lines), and to identify and use these representations.

Question Q10 corresponds to the cognitive domain of applying and it is associated with the topics ratio, proportion, and percentage. The goal is to identify and find equivalent ratios, model a given situation using a ratio.

Lastly, question Q12 is classified within the cognitive domain of reasoning and is on the topic of organization and representation of data. The goal is to read scales and data from tables, pictograms, bar charts, pie charts, and line charts. Additionally, this question is associated with deducing the pattern that the values represent in the given graphs.

Similarly, Figure 3.2 illustrates the mean obtained in each of the questions in the geometry content domain. The mean obtained in this domain is 5.54 and the standard deviation is 2.57. Recall that the mean and standard deviation obtained by taking all the questions are 6.14 and 1.76, respectively.

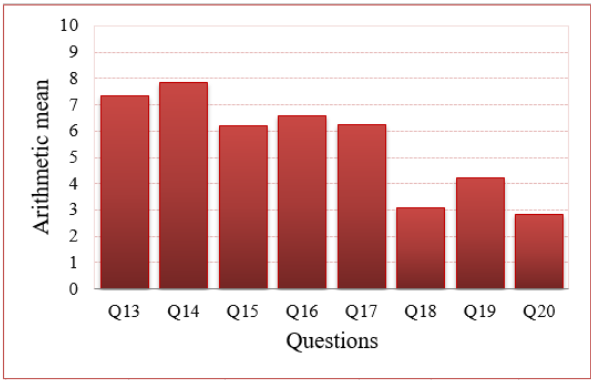


Figure 3.2: *Results of geometry content domain questions.*

In Figure 3.2, the question with the highest score is Q14, with an average of 7.84. The questions with the lowest scores are Q18, Q19 and Q20, with a mean of 3.09, 4.23 and 2.84, respectively.

Question Q14 is in the applying cognitive domain. It belongs to the theme of geometric shapes and spatial reasoning. The goal is to recognize geometric properties in two dimensions or three dimensions forms, including linear and rotational symmetry. Both Q18 and Q19 are from the cognitive domain of applying and question Q20 is from reasoning. These questions refer to the topic of geometric measurements and are associated with the objective of selecting and using appropriate measurement formulas for perimeters, circumferences, areas, surfaces, and volumes.

Score range analysis: suspended, approved, notable and excellent

In this subsection, the obtained scores in the test are classified in suspense $(0-4.9)$, approved $(5-6.9)$, notable $(7-8.9)$ and excellent $(9-10)$. Figure 3.3 shows the percentage of pre-service teachers that obtain each of these scales.

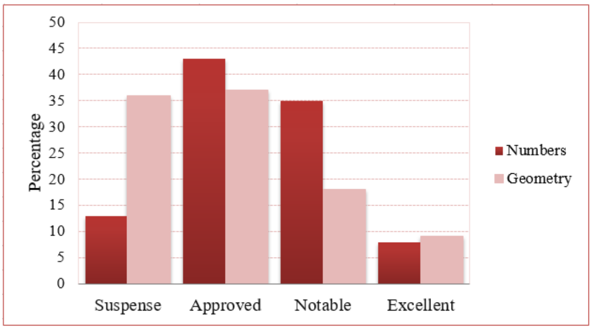


Figure 3.3: *Percentage of pre-teachers obtaining each scale.*

As shown in Figure 3.3, there is a considerable percentage of students at the exceptionally low learning level (suspended): 14% in numbers content and 36% in geometry content, a remarkably higher number.

At the approved level, we have 43% in numbers content compared to 37% in geometry content, and in notable level we get 35% in numbers content and 18% in geometry content. Notice that at this level there is a considerable difference between the two domains. Finally, the excellent level has 8% in numbers content and 9% in geometry content.

Results of correct, blank and erroneous questions

This section analyses the answers that the students have provided in the case of problem type questions. Specifically, Figure 3.4 shows, for each question, the percentage of students who answered it well, the ones who left it blank and the ones that made mistakes.

In Figure 3.4, it is observe that the question with the greatest difficulty

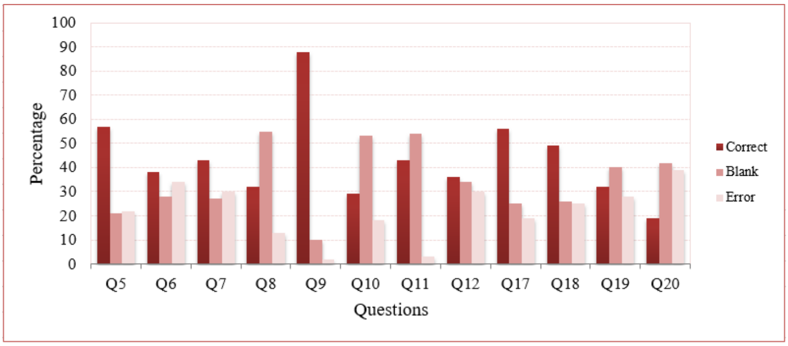


Figure 3.4: *Percentage of correct, blank and error answers in each of the questions.*

in the number domain is Q10: only 29% of the students answer it correctly, 53% make some kind of error and 18% of students let it blank. This question belongs to the cognitive domain of applying. In contrast, Q9 is the question with the highest percentage of students who answer it well, with 88% of correct answers, 10% of erroneous answers and only 2% of blank answers. This question is in the cognitive domain of knowledge.

In the case of the geometry content domain, the most difficult question for the students is Q20, in the reasoning cognitive domain: 19% answered it correctly, 42% made mistakes, and 39% of students leave it blank. Question Q17, on the other hand, is the question with the highest percentage of correct answers, with 56% of correct answers, 25% of erroneous answers and 19% of blank answers. This question is in the cognitive domain of knowledge. Moreover, it can be seen that the percentage of students who did not include the procedure of the problems in the resolution is very high, especially in the following questions: Q6 and Q12 (34%), Q7 (27%), Q18 (30%), Q19 (28%) and Q20 (39%). It is also important to highlight that some questions have a high error rate: Q6 (28%), Q7 and Q12 (30%), Q8 (55%), Q10 (53%), Q11 (54%), Q18 (46%), Q19 (40%) and Q20 (42%). Lastly, notice that the percentage of corrected answers is very low in some questions. Specifically, the ones with less than 40% of corrected answers are the following: Q6 (38%), Q8 (32%), Q10 (29%), Q12 (36%), Q18 (24%), Q19 (32%) and Q20 (19%).

3.4.2 Common errors in the problem type questions

General errors

This section studies in detail the errors made by the students in some of the problem type questions. Specifically, to carry out that study, the problem type questions with an arithmetic mean of less than 5 in an interval from 0 to 10 are considered. In the number content domain, the questions correspond to Q10 and Q12, and in the Geometry content domain they correspond to Q18, PQ9 and Q20.

Each of the questions and examples of the common mistakes made by the students are shown below. Specifically, for each studied error, the percentage of students who make it is detail.

Q10 (*Ana and Jenny divide* 560 *euros between them. If Jenny gets* $\frac{3}{8}$ *of the money, how many Euros will Ana have? See the complete test in Appendix A*). Figure 3.5 show two of the most common errors found in this question.

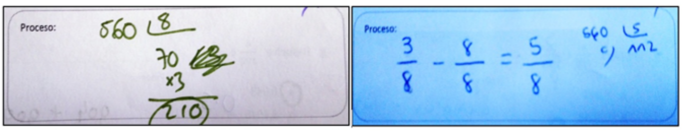


Figure 3.5: *Examples of errors in question P10.*

The error shown in Figure 3.5 (left) is due to a lack of understanding of the problem and/or incorrect reasoning. The 25% of students who make mistakes in this question present this difficulty. The students calculate the money obtained from Jenny and not from Ana. It is observed that they perform the calculation of $\frac{3}{8}$ of 560 obtaining a result of 210. Hence, the process is incomplete since the subtraction of $560 - 210 = 350$ is missing. The 75% of students make errors in which the non-understanding of fractions is identified. See Figure 3.5 (right) as example. Moreover, it is also detected that the students have not carried out a final check to validate if they have really responded what is requested.

Question Q12 (*The graph shows the sales of two types of soda for 4 years. If sales trends continue for the next 10 years, determine the year in which the sales of Guinda Cola will be equal to the sales of Limón Cola. See Appendix A for details*). In this case, 34% of the students the students leave the question blank; and in 30% of the incorrect answers, the pre-service teachers do not include a correct reasoning. The wrong answers are diverse, most are meaningless and do not provide important information. Therefore, no example was added.

Q18 (*The area of a square is* $144cm^2$*. What is the perimeter of the square? See Appendix A for details*). The 34% of students make mistakes like the one shown in Figure 3.6 (left). This error occurs because they do not know the formula for the area of the square. Notice that to calculate the length of the side of the square, the students divide by 2 instead of calculating the square root. The value of the side length of the square wrongly calculated by the students is $72cm$ and the perimeter is $288cm$. The 53% of students make errors following the logic shown in Figure 3.6 (right). It is observed that they do calculate the square root of 144, but they do not multiply the result by 4, leaving 12 as the result, which represents the length of the side of the square. The remaining 13% of students present remarkably diverse errors, evidencing the lack of knowledge of the formulas and the process to be carried out. It is manifested that they do not have a clear notion of the area nor of the perimeter.

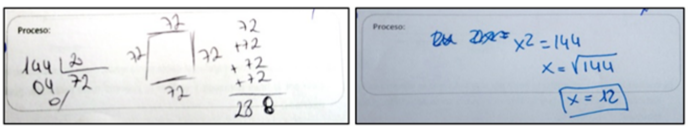


Figure 3.6: *Examples of errors in question P18.*

Q19 (*In the figure below, what is the area in cm of the shaded region? See Appendix A for details*). The 30% of the students calculate the area of the unshaded region, as shown in Figure 3.7 (left). It seems that they do not understand correctly the problem statement or that they do not know how to solve it. The 20% of student try t o calculate the shaded area, as shown in Figure 3.7 (right), but the students do not know the formulas to reach the correct solution.

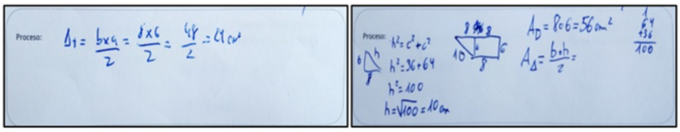


Figure 3.7: *Examples of errors in question Q19.*

Q20 (*Raúl is packing books in a rectangular box. All books are the same size. What is the largest number of books that will fit in the box? See Appendix A for details*). Some of the procedures given by the students are shown in Figure 3.8.

Figure 3.8: *Examples of errors in question Q20.*

The errors shown in Figure 3.8 evidence that students do not understand the problem or do not know how to reach the solution. It should be noted that, unlike other questions, this requires an effort of reasoning. A percentage of 39% of the students leave this question blank. Others try to solve it, but few succeed. In fact, in this question the students present a great diversity of errors. For example, in the case of Figure 3.8 (left), the procedure used by the student shows that he/she is not considering the organization of the books: he leaves spaces in the box. In the procedure shown in Figure 3.8 (right), the student makes adequate reasoning, but he/she does not correctly master the basic operations and this prevents him/her from reaching the correct solution.

Errors about the decimal number system

This subsection shows examples of errors that students present in considerably basic concepts and procedures. There exist important deficiencies when manipulating the numbers, showing little control over the properties of the decimal number system. Below are some of these errors, grouped by similarity.

Figure 3.9 shows the first group of errors which are due to difficulties in understanding the positional value of the digit.

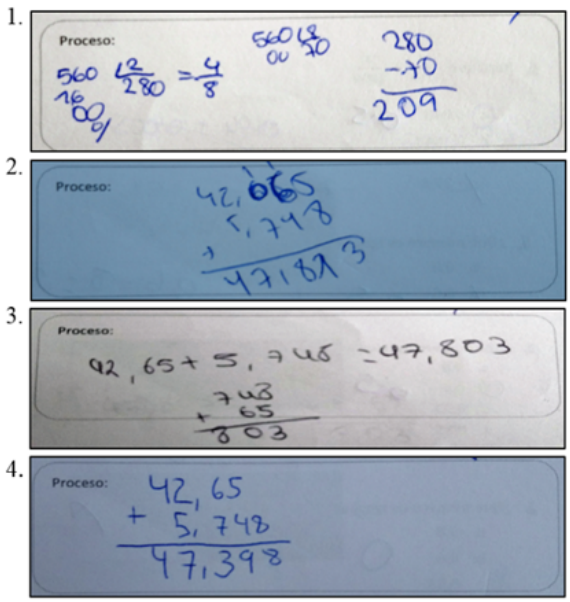


Figure 3.9: *Basic errors 1 (positional value of the digits.*

In the first example (Figure 3.9 (1)), although whole tens are subtracted, the result does not correspond to whole tens ($280 - 70$ gives 209). The other three examples correspond to sums with decimal numbers. In Example 2, the student adds a 0 to the tenths place. In Example 3, the student adds the decimal part and the integer part separately. Finally, in Example 4, the student does not add the numbers correctly. Recall that both the positional value of the digits and the decimal numbers are contents that are taught in Primary Education.

Figure 3.10 shows examples where students make an error when computing a product between two quantities. It is interesting to highlight the case in which the student responds that 6×8 is 49, an odd number. This fact shows that this student does not understand some of the basic properties of numbers (the product of two even numbers cannot result in an odd number). As in the case of the previous basic errors (Figure 3.11), these types of properties are taught in Primary Education.

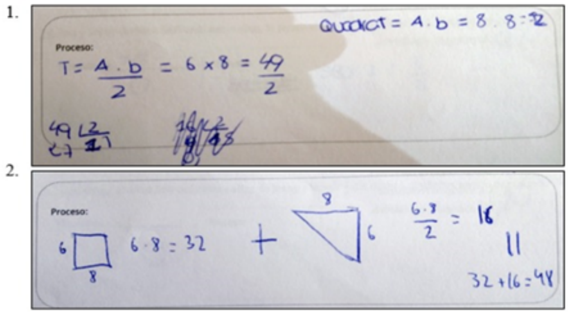


Figure 3.10: *Basic errors 2 (Multiplication tables).*

The examples of errors in the Figure 3.11 show that students present difficulties in developing the division algorithm (examples 1 and 2) and conceptual errors in the properties of decimal numbers (example 3).

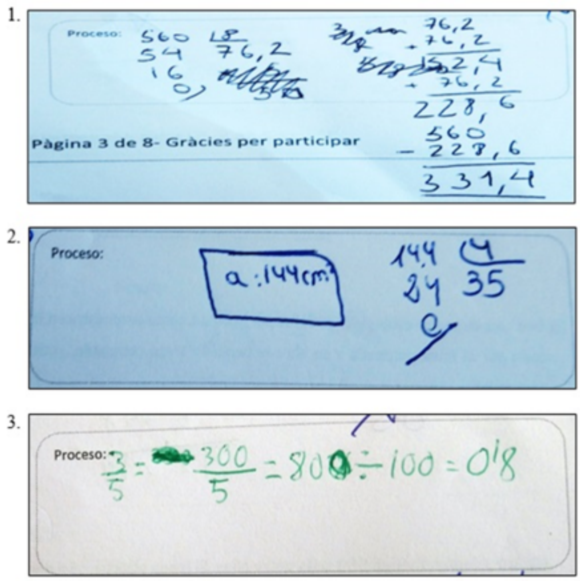


Figure 3.11: *Basic errors 3 (Division algorithm).*

3.4.3 Studying the relation between results and four different variables

Type of baccalaureate studied by students of the Primary Education Degree

Hypothesis 1. In this subsection, the scores obtained by the students who completed the Baccalaureate in Science are compared with those who completed the Baccalaureate of Humanities and Social Sciences. In the current sample, 22% of students come from the Baccalaureate in Sciences, 61% from the Baccalaureate in Humanities and Social Sciences, 4% from the Baccalaureate in Arts and 13% other studies.

Figure 3.12 shows the box plot of the total scores obtain by the students who came from the Baccalaureate in Sciences and those who attended the Baccalaureate of Humanities and Social Sciences. This type of diagram allows to identify the first and third quartiles, the median (horizontal line) and the mean (marked with a dot). The whiskers extend from the box to the maximum and minimum values. Three diagrams are shown: the one corresponding to the scores of the entire test (Total), and, separately, the scores corresponding to the numbers geometry domains, respectively.

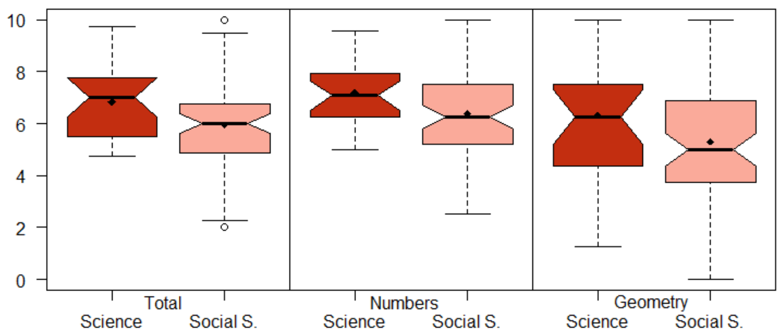


Figure 3.12: *Total scores, scores of number questions and scores of geometry questions obtained according to the type of baccalaureate.*

Table 3.1 shows that the scores obtained by the students coming from the Baccalaureate in Sciences are in general superior to the ones who come from the Baccalaureate of Humanities and Social Sciences. In the results of the test that includes the two domains of mathematical content (Total), 25% of the students from Science (Q1) have grades lower than 5.44 and another 25% (Q2) grades lower than 7. In the case of students who come from Humanities and Social Sciences, 25% of the marks are less than 4.75 (Q1) and 50% less than 6 (Q2). In addition, the two types of baccalaureate obtain a negative asymmetric distribution (that is, in both cases there is greater dispersion of data in values below the median). Notice that the difference between both groups of baccalaureate is similar in the case of numbers content and geometry content.

Table 3.1 summarizes the values of the mean and standard deviation of the scores obtained considering the questions all the questions (Total), the ones corresponding to the numbers content domain and the ones corresponding to geometry domain. A statistical test is performed to analyse whether the difference between the means obtained in each domain is statistically significant. Specifically, the t-student test is applied, taking the parameter $\alpha = 0.05$ with a significance level of 5%.

Table 3.1: *Mean and standard deviation in the case of students from the Baccalaureates of Sciences and of Humanities and Social Sciences*

	Total		Numbers		Geometry	
	Science	Social S.	Science	Social S.	Science	Social S.
Mean	6.83	5.94	7.17	6.38	6.30	5.29
S. D.	1.50	1.78	1.33	1.74	2.39	2.57
$p-valor$	0.016		0.017		0.049	

Taking into account the obtained $p - value$ ($p = 0.016$), in H1 it could be said than, there is significant difference between the mean score of the students graduated from Baccalaureate Science with those of Humanities and Social Sciences. Besides, there is a significant difference in the mean score obtained in each content domain: numbers ($p = 0.017$) and geometry ($p = 0.049$). Thus, it can be affirmed that there are significant differences between the results of the students from the two baccalaureates considering all the questions (total) and also taking the ones corresponding to each content domain.

Time without receiving lessons of mathematics

Hypothesis 2. This subsection studies the relationship of the results corresponding to students who have had the subject of mathematics in the last two years (39% of the participants) with those of students who have not studied mathematics in the last two years (61% of participants).

Figure 3.13 shows the box-plot of the results corresponding to two studied groups of students ($<= 2, > 2$). As in the previous case, the total score and the scores obtained in each mathematical content domain are shown.

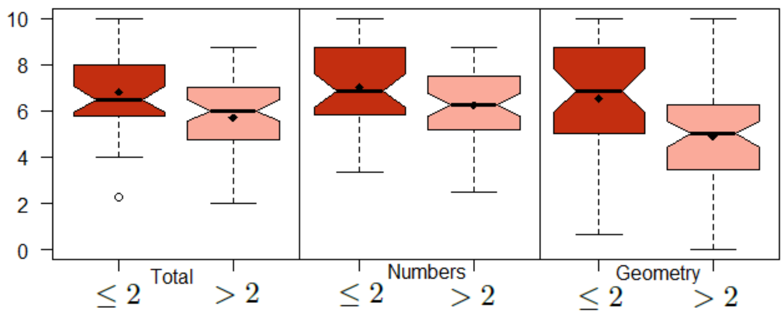


Figure 3.13: *Total scores, scores of number questions and scores of geometry questions obtained depending on the time without receiving lessons of mathematics.*

Figure 3.13 shows that the students who have studied mathematics in the last two years clearly score higher. The median obtained on the test by these students is 6.50, while the one obtained by students who have not had mathematics in the last two years is 6.00. According to the mathematical content domain, the most noticeable difference between both groups of students is given in geometry, obtaining in the first and second groups a median of 6.88 and 5.00, respectively.

Table 3.2 shows the mean and standard deviation obtained by each studied group of questions (total, numbers, and geometry) and the $p - value$ resulting from the applying of the $t - Student$ statistical test.

With a value of $p = 0.0012$, the null hypothesis is rejected . In other words, the difference between the mean of the scores obtained is statistically signif-

Table 3.2: *Mean, standard deviation and $p-value$ between means of the two studied groups*

	Total		Numbers		Geometry	
	≤ 2	> 2	≤ 2	> 2	$\leq$	> 2
Mean	6.82	5.70	7.01	6.23	6.53	4.90
Standard deviation	1.78	1.60	1.77	1.50	2.47	2.44
$p-valor$	0.012		0.014		0.0011	

icantly greater in students who have had mathematics in the last two years. This significant difference is evidenced in the two domains of mathematical content: numbers $(p = 0.014)$ and geometry $(p = 0.0011)$.

Significant differences between numbers and geometry

Hypothesis 3. This subsection aims at studying if there is a significant difference between the mean obtained in the questions corresponding to numbers content domain and the ones corresponding to the geometry domain. Figure 3.14 shows the box-plot of the distribution of the scores obtained in the questions corresponding to the numbers and geometry content domains.

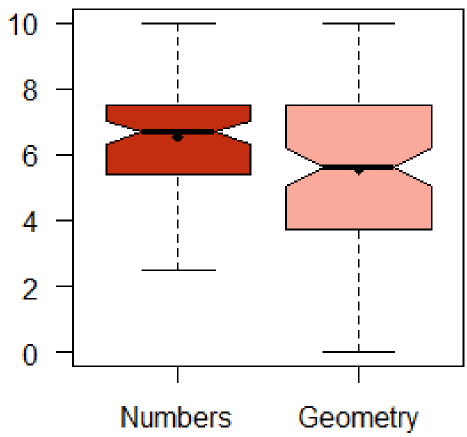


Figure 3.14: *Scores obtained in the questions corresponding to numbers and geometry content domains.)*

In the numbers content domain, 50% of students score below 6.67. In the geometry domain, on the other hand, the median is 5.63, indicating that 50% of students have grades below that value. In the number domain, the obtained distribution is a negative asymmetric one, while in the geometry domain the distribution is symmetric.

Table 3.3 shows the mean and standard deviation of the scores obtained in the number and geometry domains, respectively. In addition, a statistical test has been carried out to analyse whether the difference between the means is significant.

Table 3.3: *Mean, standard deviation and $p - value$ between means of the two studied groups*

	Numbers	Geometry
Mean	6.54	5.54
Standard deviation	1.65	2.57
p-valor	0.0015	

From the results shown in Table 3.3 ($p = 0.00153$), the null hypothesis is rejected. Thus, there is a significant difference between the results corresponding to questions of the number domain and the ones of the geometry domain.

Scores according to cognitive domain (knowing, applying and reasoning).

Hypothesis 4. In this section we analyse the results obtained according to the cognitive domain: knowing, applying, and reasoning. Figure 3.15 shows the box-plot of the scores obtained by the students in the test in the questions corresponding to each cognitive domain.

In knowledge, 50% of students have grades below 7.50, while in applying and reasoning the obtained medians are 5.83 and 4, respectively. To check the hypothesis of the homogeneity of the cognitive domain groups of the applied test, the analysis of variance (ANOVA) is used. The ANOVA test results ($F = 56.6; p < 0.001$), which implies that there are statistically significant differences in the studied groups. Then, Tukey's HSD (Honestly Significant Dif-

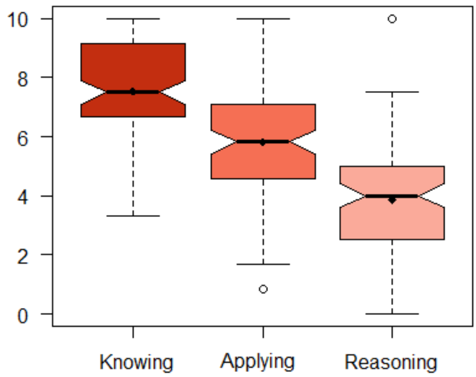


Figure 3.15: *Scores according to cognitive domain (knowing, applying and reasoning.*

ference) statistical test is applied to determine in which groups that difference exists. The HSD test shows that there are significant differences between: applying and knowing ($p < 0.01$), reasoning and knowing ($p < 0.01$), and reasoning and applying ($p < 0.01$).

3.5 Discussion

In the first part of the analysis, the results were studied by content. At this point, it is evident that the pre-service teachers have more difficulty in solving problems of geometry content than that of numbers content. Moreover, the level of student performance was analysed and classified as: suspended, approved, notable and excellent. There is a clear concern about the high percentage of students who failed the applied test. Results showed that students have more difficulty in geometry than in numbers, as in [112].

Besides, the results for each of the problem type questions were analysed separately into correct, blank and wrong. In numbers, 6 of the 8 questions did not reach 50% of correct answers, while in geometry, that happened in 3 of the 4 questions. Furthermore, in 3 number questions, more than 50% of the students made some kind of mistake. In 3 of the geometry questions, more than 40% of the students made a mistake. These results are alarming since the test is composed of content taught in Primary Education. This situation is also evident in other research on future teachers, as for example in [102], which shows that pre-service teachers do not have the mathematical skills they are supposed to have after completing their compulsory education.

The first part of analysis ends showing some of the mistakes made by students in the problem type questions. First, we study the errors that some students made in questions with an arithmetic mean of less than 5. Students have difficulty in remembering the formulas of area and perimeter of the square and rectangle. Similarly, they do not know the formula for the volume of a prism. These results are in accord with the research presented in [56], where the authors indicated that students have deficiencies in solving area and perimeter problems. Finally, the errors on the decimal number system are analysed. In [19] , authors highlighted concerns about some aspects of teacher content knowledge practice. Specifically, these concerns are mainly about the knowledge of decimals. The current research verifies that some students made mistakes in basic operations. Concretely, students made errors in addition, multiplication, and the division algorithm. It is important to note that some studies have showed that limited decimal knowledge affects the ability of teachers-in-training to identify errors in student thinking and to apply appropriate teaching approaches [113].

In the second part of the analysis, four hypotheses were constructed. In the first hypothesis the obtained results were analysed according to the type of baccalaureate (H1). It has been shown that there are significant differences between the two groups. First year students of the Primary Education Degree who come from a Science Baccalaureate obtained a higher average than those who studied the Humanities and Social Sciences Baccalaureate. This result corroborates what was said in ([114], [115]). Specifically, they pointed out that students coming from the Bachelor of Science obtained better results in the applied mathematics tests and were the most motivated. Then, hypothesis 2 was verified: students who have had the subject of mathematics in the last two years obtained a higher average than those who have not had mathematics in the last two years. Statistical analysis has shown that the difference between both averages is statistically significant. In the mathematical content domain, answering the H3, a statistically significant difference is evident between the mean of the scores obtained in the numbers domain and in the geometry domain. We agree with [116], where geometry is the block with the worst results. Moreover, in the cognitive domain, there are significant differences in the three groups posed in the test, rejecting the null hypothesis posed in H4. It is observed that the students have difficulties in developing the problems aimed at applying and reasoning (assessment by skills). These results are consistent with those presented in [117] in which it was shown that students have great difficulty in solving real-life problem situations.

We believe that in order to improve the quality of teacher training and, consequently, the quality of mathematics teaching at Primary Education level, it would be interesting to apply a diagnostic test at the beginning of the first teaching and learning mathematics course included in the Bachelor Degree. In fact, this is what it has been done in this current chapter. Then, the errors and difficulties that students present in this test would have to be analysed and tackled during that course. We agree with the idea exposed in [118] that claims that a youngster can learn from his mistakes since it allows his peers or the teacher to help him complete the additional knowledge.

Based on the errors and difficulties presented by the students in this current chapter, it would be necessary to adapt the content of the teaching and learning mathematics course with the purpose of reinforcing some contents and including others.

One of the key points to improve the quality of teacher education would be to achieve that all students entering the Primary Education Bachelor Degree understood the basic mathematical knowledge learned in primary and secondary school. In some cases, students may have forgotten them, but in most cases, the problem is that they did not understand them at the time. Therefore, this research proposes to design an initial course of mathematical knowledge for students who have not had mathematics in the last two years and students who have weaknesses in learning numbers content and geometry content. This course would have two thematic blocks, namely numbers content and geometry content. It would aim at strengthening the cognitive abilities to analyse and reason of pre-teachers.

Finally, it is important to emphasize that the research presented in this chapter can be useful for any University dedicated to initial teacher training, and also for primary and secondary schools. The purpose is to improve the quality of teaching and learning mathematics.

Chapter 4

Pre-service teachers' belief about the efficacy of their mathematics teaching

This chapter aims at analysing the pre-service teachers' self-efficacy for teaching mathematics and its evolution throughout the Bachelor Degree in Primary Education at the Rovira and Virgili University. Specifically, the current chapter aims at analysing if there exists a significant difference (in positive) between the first measured self-efficacy, which is measured before having taken any maths subject, and the last one, which is measured just before ending the Bachelor Degree in Primary Education.

4.1 Introduction

As mentioned in section 1.1, there are several works focused on improving mathematical pre-service teacher education. Those works emphasize the importance of the teacher's positive attitude towards mathematics in order to obtain the same attitude among their students. Newton suggests that, since most of tomorrow's teachers are today's pre-service teachers, the beliefs they hold should be of concern to teacher educators, teacher efficacy is also considered an important factor in high-quality mathematics instruction [74]. Indeed, it is a factor not only considered in the case of mathematics, but also in science and technology.

4.2 Methodology

This section exposes the methodology adopted in this chapter. Pre-service teachers of each year of the Primary Education Degree answer the MTEBI.

4.2.1 Participants

This study was held on the 2016-2017 academic year at the University Rovira i Virgili. Participants were enrolled at the 4-year Bachelor Degree in Primary Education in the Faculty of Science Education and Psychology, located in a small city, Tarragona. Students of all the years of the Bachelor were invited to answer the MTEBI. Therefore, the participation was optional and completely anonymous. The sample consists of the students that assisted at the session when the test was applied. Specifically, the number of participants of each year of the Bachelor was 133, 107, 95 and 84, respectively. Hence, the test was administrated to 419 pre-service teachers, which represents the 76% of the enrolled students that academic year. Notice that the four studied groups correspond to individuals at different moments of the Bachelor Degree, rather than the same individuals over time.

As explained in detail in section 1.2, the Primary Education Degree consists of a four-year program containing three mandatory courses of Teaching and Learning Mathematics (TLM).

4.2.2 Instrument

The Mathematics Teaching Efficacy Belief Instrument (MTEBI) for pre-service teachers results from the modification of the Science Teaching Efficacy Belief Instrument STEBI-B [119] and aims at reflecting future mathematics teaching beliefs. Indeed, the MTEBI for pre-service teachers has been widely used to measure mathematics teaching self-efficacy and teaching outcome expectancy (e.g., [27], [38], [74], [69]). The original MTEBI consisted of 21 items in a five-point Likert scale measuring one (strongly disagree) to five (strongly agree). As in [78], the third Likert scale item, uncertain, was deleted in the current work to encourage all participants to indicate a level of certainty.

The MTEBI is comprised of two subscales, Personal Mathematics Teaching Efficacy (PMTE) and Mathematics Teaching Outcome Expectancy (MTOE). The PMTE subscale consists of 13 items (2, 3, 5, 6, 8, 11, 15, 16, 17, 18, 19, 20, and 21). Eight of these items are reverse scored. That is, they are stated in negative manner (3, 6, 8, 15, 17, 18, 19 and 21). Notice that the responses corresponding to these items have to be inverted before being added into the total PMTE score (4 = 1, 3 = 2, 2 = 3, 1 = 4). The MTOE subscale consists of 8 items (1, 4, 7, 9, 10, 12, 13, and 14). Thus, PMTE scores range from 13 to 52 while MTOE scores range from 8 to 32.

The questions that conform the MTEBI in the case of pre-service teachers are detailed in Appendix B. The MTEBI subscales were explained in detail in subsection 4.2.2.

Reliability

In order to determine the reliability of the obtained results, the Cronbach's alpha ($\alpha - Cronbach$) is calculated. Specifically, the obtained Cronbach's alpha is 0.77 and 0.74 for the PMTE and MTOE subscale, respectively.

4.3 Results

This section reports the results obtained in the current chapter. Recall that the pre-service teachers' efficacy beliefs was measured at the end of the academic year, considering each of the four years of the Bachelor. The PMTE and MTOE subscales were studied separately.

4.3.1 Personal Mathematics Teaching Efficacy (PMTE)

This section studies the scores given by the students to the PMTE subscale items. First, Table 4.2 shows the mean and standard deviation of the scores that each group of students give to the PMTE subscale items. In order to emphasize the evolution of the scores along the years of the Primary

Education Degree, and also for the lack of space in the table, only the scores corresponding to pre-teachers of the 1*st* and 4*th* years are shown. The rank of the students responses based on the mean score is also included in the table. In the reverse questions, the inverted scores are shown.

Notice that the PMTE scores given by students of the 1st year are in general lower than the ones given by students of the 4th year. In the 1st year responses, there are 6 questions with a score above 3.20 (questions 20, 2, 8, 19 and 6). In the case of the 4th year, there are 9 questions with a score above 3.20 (questions 20, 15, 2, 19, 8, 21, 6, 3 and 16).

The highest score in the 1*st* and the 4*th* years is obtained in the question 20 (*When teaching mathematics, I will usually welcome student questions*), with a mean score of 3.77 and 3.79, respectively. The two-lowest scores, on the contrary, are obtained in both years in the questions 17 and 5. Specifically, in the 1st year, the lowest score was obtained in the question 17 (*I wonder if I will have the necessary skills to teach mathematics*), with a mean score of 2.41. In the 4*th* year case, the lowest score was obtained in the question 5 (*I know how to teach mathematics concepts effectively*), with a mean score of 2.83. However, recall that even these worst results correspond to a mean score of 60% and 70%, respectively.

Table 4.1: *Mean and standard deviation of the scores that participants give to the PMTE subscale items*

No	Question	first			fourth		
		M	SD	rank	M	SD	rank
2	I will continually find better ways to teach mathematics.	3.52	0.62	2	3.69	0.47	3
3(R)	Even if I try very hard, I will not teach mathematics as well as I will most subjects.	3.17	0.99	6	3.27	0.91	8
5	I know how to teach mathematics concepts effectively.	2.47	0.81	12	2.83	0.69	13
6(R)	I will not be very effective in monitoring mathematics activities.	3.21	0.87	5	3.35	0.80	7
8(R)	I will generally teach mathematics ineffectively.	3.46	0.83	3	3.49	0.87	5
11	I understand mathematics concepts well enough to be effective in teaching elementary mathematics.	2.95	0.85	10	3.19	0.74	10
15(R)	I will find it difficult to use manipulatives to explain to students why mathematics works.	3.16	0.80	7	3.71	0.63	2
16	I will typically be able to answer students' questions.	3.06	0.69	9	3.20	0.65	9
17(R)	I wonder if I will have the necessary skills to teach mathematics.	2.41	0.90	13	2.85	0.94	12
18(R)	Given a choice, I will not invite the principal to evaluate my mathematics teaching.	2.86	0.95	11	3.05	1.07	11
19(R)	When a student has difficulty understanding a mathematics concept, I will usually be at a loss as to how to help the student understand it better.	3.39	0.81	4	3.58	0.68	4
20	WWhen teaching mathematics, I will usually welcome student questions.	3.77	0.53	1	3.79	0.44	1
21(R)	I do not know what to do to turn students on to mathematics..	3.11	0.75	8	3.36	0.85	6

Figure 4.1 shows the mean score obtained in each of the PMTE questions through all the years of the Primary Education Degree. Again, in the reverse items, the inverted scores are provided.

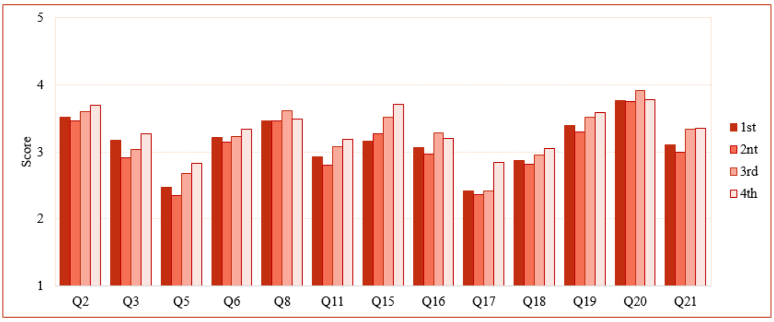


Figure 4.1: *Mean of the scores corresponding to the PMTE subscale items through all the years of the Degree.*

It can be seen that the rank of the questions is the same or very similar in all years of the Degree. It is noteworthy to see the evolution of some of the questions along the Degree. Notice, for instance, the positive evolution of question 17: in the fourth year, students are more positive with the fact of having the necessary skills to teach mathematics. In the case of question 15, the students clearly improved their confidence on using manipulative to teach mathematics. Another example of positive evolution is question 21, which corresponds to students' belief about their capacity to turn students on to mathematics. However, it should be remarked that the PMTE scores given by students in the 2nd year of the Degree are in most cases lower than the ones given by students in the 1st year of the Bachelor.

Additionally, Figure 4.2 shows the individual students average scores corresponding to the PMTE items (that is, the PMTE scores averaged by the number of PMTE items scored by the student), for every year of the Primary Education Degree. In particular, the box plot enclose data in between lower and upper quartiles (medians are represented by horizontal lines in thinner regions and means are denoted with points). This representation permits to study the distribution of the scores given by all the students.

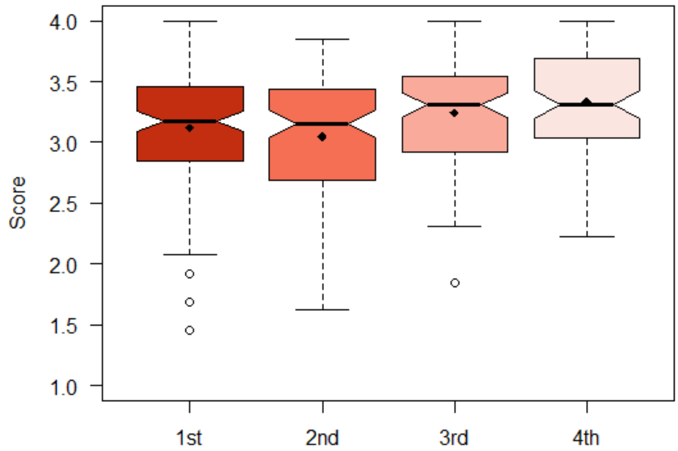


Figure 4.2: *Individual pre-teacher average scores given to the PMTE subscale items.*

It is noteworthy to remark results obtained in the 2nd year of the Bachelor Degree: the obtained mean, median and minimum values are smaller than the ones obtained in the 1st year. In the 3rd year, on the contrary, the improvement is clear. Finally, it is interesting to highlight the good results obtained in the 4th year: recall that the distribution of the scores is nearly symmetric (mean is similar to median, but a little higher). The minimum obtained score is 2.23 (55.77%), which is quite high, compared to the minimum values obtained the other years. The other three years, the mean is under the median and the range of values is larger due to some obtained small scores.

In order to compare the global performance of the students of each year of the Degree, Table 4.2 shows the mean and standard deviation of the scores given by each group of students to the PMTE items. Additionally, an ANOVA test was performed to see if the difference among means obtained in each year are statistically significant.

The ANOVA gives the following values: $F = 7.243$; $p = 9.55 \times 10^{-5} < 0.001$. Hence, the difference among means is statistically significant. Then, a Tukey's HSD (honestly significant difference) test is applied to study the relation between each pair of groups. Specifically, the difference between the obtained means is statistically significant between 2nd and 3rd years ($p =$

0.01379) and between 1st and 4th years ($p = 0.00731$).

Table 4.2: *PMTE (Mean and standard deviation)*

	1st	2nd	3rd	4th
Mean	3.12	3.046	3.24	3.33
%	77.96	76.15	81.09	83.37
SD	0.49	0.495	0.40	0.43
%	12.28	12.38	9.95	10.89

4.3.2 Mathematics Teaching Outcome Expectancy (MTOE)

This section studies the scores given by the students to the MTOE subscale items. First, Table 4.2 shows the mean and standard deviation of the scores given by the participants of the 1st and 4th years of the Bachelor to the MTOE subscale items.

In this subscale, the highest mean score obtained in the 1st and the 4th years of the Bachelor corresponds to question 9 (*The inadequacy of a student's mathematics background can be overcome by good teaching*), with a mean score of 3.36 and 3.44, respectively. The lowest mean score was obtained in question 1 (*When a student does better than usual in mathematics, it is often because the teacher exerted a little extra effort*), with a mean score of 2.44 and 2.42 in the 1st and 4th years, respectively.

Figure 4.3 shows the mean of the scores corresponding to each of the MTOE subscale items through all the years of the Degree.

As in the PMTE subscale, the rank of the questions is the same or very similar in every group of students. However, the positive difference between first and last grades is not as notable as in the PMTE subscale. Indeed, the evolution of the scores is different in each item of the MTOE subscale.

Table 4.3: *Mean and standard deviation of the scores given for the participants to the questions corresponding to the MTOE subscale*

No	Question	first			fourth		
		M	SD	rank	M	SD	rank
1	When a student does better than usual in mathematics, it is often because the teacher exerted a little extra effort.	2.44	0.83	8	2.42	0.66	8
4	When the mathematics grades of students improve, it is often due to their teacher having found a more effective teaching approach.	3.33	0.65	2	3.05	0.58	2
7	If students are underachieving in mathematics, it is most likely due to ineffective mathematics teaching.	2.84	0.76	5	2.74	0.89	6
9	The inadequacy of a student's mathematics background can be overcame by good teaching.	3.36	0.62	1	3.44	0.57	1
10	When a low-achieving child progresses in mathematics, it is usually due to extra attention given by the teacher.	2.81	0.77	7	2.93	0.65	4
12	The teacher is generally responsible for the achievement of students in mathematics.	2.94	0.70	3	2.98	0.66	3
13	Students' achievement in mathematics is directly related to their teacher's effectiveness in mathematics teaching.	2.89	0.69	4	2.86	0.64	5
14	If parents comment that their child is showing more interest in mathematics at school, it is probably due to the performance of the child's teacher	2.83	0.75	6	2.59	0.70	7

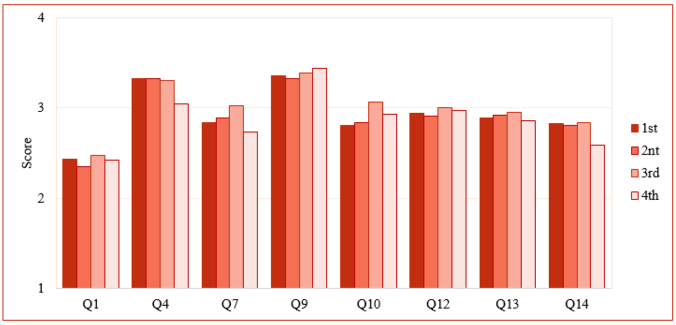


Figure 4.3: *Mean of the scores corresponding to the MTOE subscale items through all the years of the Degree.*

Additionally, Figure 4.4 shows the mean of the values that each pre-service teacher gives to the MTOE subscale items.

It can be seen that the lowest values of MTOE are obtained in the 1st year of the Degree. The minimum value is very small: 1.75 (43.75%). Besides, there is a clear improvement on the 3rd year values. Notice that the minimum value in the 3rd year is 2.25 (56.25%) and the distribution is nearly symmetric (the mean and median are nearly equal). It should be noticed that the mean and median decrease in the 4th year with respect to the 3rd year.

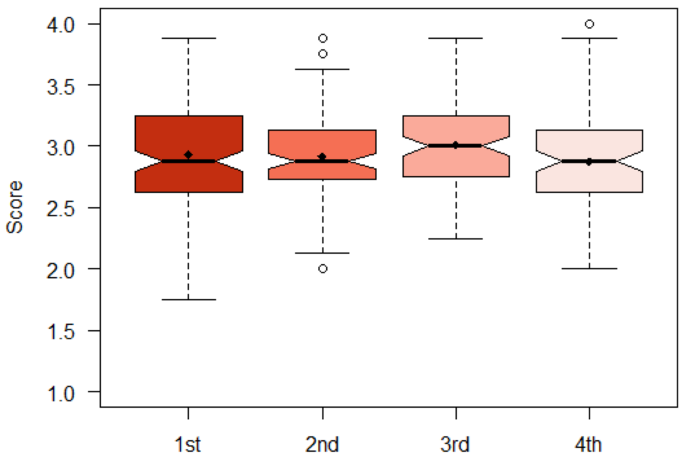


Figure 4.4: *Individual pre-teacher average scores given to the MTOE subscale items.*

Finally, Table 4.4 shows the mean and standard deviation of the scores given by each group of students to the MTOE items. In this subscale, the ANOVA gives the following values: $F = 1.828$ and $p = 0.14 > 0.05$. Therefore, there is no significant difference among groups.

Table 4.4: *MTOE (mean and standard deviation)*

	1st	2nd	3rd	4th
Mean	2.93	2.91	3.01	2.87
%	73.23	72.84	75.22	71.91
SD	0.42	0.37	0.37	0.41
%	10.53	9.22	9.32	10.19

4.4 Discussion

Obtained results evidence that the current Primary Education Degree program does have an impact on the pre-service teachers' personal mathematics teaching efficacy (PMTE). It should be remarked the statistically significant difference obtained between the PMTE values at 1st and 4th years ($p = 0.00731$). These results are in accord with the ones presented in ([69]), where the authors show that pre-teacher training programmes can have an impact on pre-teachers beliefs about mathematics teaching and learning.

The reported study also shows the positive and statistically significant evolution between 2nd and 3rd years of the current Bachelor Degree in Primary Education ($p = 0.0138$). It is noteworthy to remark that, as mentioned above, the students take the TLM2 course connected to a student teaching period during the 3rd year. Concretely, students have to analyse the relation between the content of the TLM2 course and the reality observed at school. Besides, they apply some methodologies presented during the TLM2 at school, they see how they teach at a real classroom and they observe how children learn. Therefore, the design of this 3rd year of the Bachelor Degree definitely helps to improve the PMTE of students.

Notice that the values of PMTE decrease between 1st and 2nd years. Specifically, the PMTE values obtained at the end of the 2nd year are clearly the smallest obtained during the Bachelor. This is due to the content of the TLM course they attend during that year, mainly consisting of mathematical content. There is a high percentage of pre- service teachers that have not been receiving mathematics formation for 2 or 3 years when they arrive at the Bachelor. Sometimes, they do not remember mathematical concepts and they do not perform well when solving problems. The difficulties they have and the bad results obtained in most cases produce low levels of PMTE.

Obtained results do not show a positive evolution of the pre-teachers' mathematics teaching outcome expectancy (MTOE) during the Bachelor Degree. These results are in accord with the ones presented in [120] and [32] in the case of science teaching self-efficacy. In both investigations, they show a positive effect of their approaches on personal science self-efficacy (PSTE) and a lack of positive change in the science teaching outcome expectancy beliefs (STOE).

Results evidence that the smallest mathematics teaching outcome expectancy (MTOE) values are obtained in the 1st year. In the 2nd year, the obtained values are similar. Recall that the PMTE clearly decreases the 2nd year, which surely affects the MTOE values.

Besides, it is noteworthy the MTOE improvement obtained in the 3rd year. As exposed in the PMTE case, the students attend the TLM2 course linked to a 3-month student teaching period. The clear relation between the theory taught at the University and the reality of the class reinforces the MTOE of the students.

However, the MTOE scores decrease the 4th year of the Bachelor, when students have assisted to the second student teaching period. We attribute to several points that decreasing. First, the TLM3 course they have just finished promotes a student-centered learning approach. Hence, it should be expected that the students did not have high levels of MTOE. Second, they assisted to the second student teaching period before taking the TLM3 course. Therefore, they do not have any mathematical task assigned during the second student teaching period. Finally, they are finishing the Bachelor and they know they will not receive more formation. Hence, they may feel uncertainty about being in a class and producing effect in the students learning.

Chapter 5

Comparing Mathematics Teaching Self-efficacy and Outcome Expectancy of Pre-service and In-service Teachers

This chapter aims at comparing the mathematics teaching self-efficacy and outcome expectancy of pre-service and in-service Primary Education teachers. Concretely, two different groups of in-service teachers are considered: novice teachers (up to 10 years of experience) and experienced teachers (more than 10 years of experience). We defined these two groups of in-service teachers after exploring the results considering different ranges of time. The number of years that made the difference was 10.

5.1 Purpose of the study

It should be remarked that this is not a longitudinal study. Ideally, it would be very interesting to work with the same participants, beginning when they are pre-service teachers, continuing with their first 10 years of experience and finishing when they have more than 10 years of experience. However, several factors make that kind of study very difficult to consider. First of all, it would take more than 10 years just to collect the data. People, society and school change remarkably in 10-12 years. Would they be the same participants? Would it be the same society? Would the schools share the current characteristics? Secondly, it would be very difficult to track so many subjects. Thirdly, that study would imply to answer the same test 10 or

12 times. For all those reasons, this chapter is based on cross-sectional data.

Specifically, the current chapter was designed to answer the following questions:

1. Do the three studied groups present significant differences in their mathematics teaching self-efficacy beliefs?

2. Do the three studied groups present significant differences in their mathematics teaching outcome expectancy feelings?

3. Does the teaching experience help to enforce those beliefs?

5.2 Methodology

In order to find the answers to the questions formulated above, the current work compares the obtained scores in the MTEBI items considering the three groups: pre-service, novice and experienced Primary Education teachers. Additionally, this chapter studies if there are significant differences between the mathematics teaching self-efficacy and outcome expectancy of the three studied groups.

5.2.1 Participants

The participants comprised a purposeful sample [105]. Specifically, the sample in this study consisted of three independent groups. The first group of participants consists of pre-service teachers enrolled at the Bachelor Degree in Primary Education at the University Rovira i Virgili. Students of all the years of the Bachelor Degree were invited to answer the MTEBI. Therefore, the participation was optional and completely anonymous. At the end, 419 pre-service teachers assisted at the session when the test was applied, which represents the 76% of the enrolled students that academic year. For more details of the of the Bachelor Degree in Primary Education at the Rovira and Virgili University see section 1.2.

Besides, the MTEBI survey was adapted for in-service teachers, as explained in short, and created in Google Forms. Directors of Primary Education schools in the city were sent the link of the survey during the second trimester of 2018-19 school academic year. They shared the link so that novice and experienced teachers were able to answer it confidentially. As a result, the second group of participants are 69 novice teachers in Primary Education. The last group of participants consists of 176 experienced teachers in Primary Education. Recall that the number of novice teachers is notably inferior to the number of experienced teachers. This is due to the average years of Primary Education teachers' teaching experience in the current country, which is 14 years. Indeed, the average age of Primary Education teachers is 42 years. Therefore, it was expected to obtain a higher percentage of experienced teachers (more than 10 years of teaching experience) when considering a purposeful sample.

5.2.2 Instrument

As in chapter 4, the Mathematics Teaching Efficacy Belief Instrument (MTEBI) for pre-service teachers was used in this chapter.

It should be highlighted that the MTEBI was adapted for novice and experienced teachers using the present tense instead of the future (recall that the original MTEBI was designed for pre-service teachers). In fact, the MTEBI for in service teachers was used in several works(e.g., [78], [84]).

Reliability

In order to determine the reliability of the instrument for the current context, the Cronbach's alpha test was applied to both subscales (namely, PMTE and MOTE). Table 5.1 shows the values of Cronbach's alpha for the PMTE and MTOE considering each studied group. According to the scale proposed by [110], the obtained values of Cronbach's alpha are good (>0.8) or acceptable (>0.7) in all the cases.

Table 5.1: $\alpha - Cronbach$ (PMTE, MTOE)

	Pre-service	Novice	Experienced
PMTE ($\alpha - Cronbach$)	0.86	0.86	0.81
MTOE ($\alpha - Cronbach$)	0.70	0.82	0.78

Validity

To study the validity of the instrument for the current context, the Kaiser-Meyer-Olkin (KMO) is used to relate the correlation coefficients and the Bartlett's Test of Sphericity (BTS) is applied to contrast the null hypothesis that the items are not correlated with each other, contrasting the observed matrix with the identity matrix.

Table 5.2: *Kiser-Meyer-Olkin (KMO) and Bartlett's Test of Sphericity (BTS)*

	Pre-service	Novice	Experienced
KMO	0.88	0.67	0.81
BTS ($p - valor$)	< 0.001	< 0.001	< 0.001

Results summarized in Table 5.2 allow to conclude that the Exploratory Factor Analysis [107], [108] can be applied in the three studied groups (KMO > 0.60 and p < 0.001). Table 5.3 shows the individual item-total correlations.

Table 5.3: *Item-Total Correlations values*

Items (PMTE)	Item-Total Correlations Pre-service	Item-Total Correlations Novice	Item-Total Correlations Experienced
Q2	0.45	0.37	0.33
Q3	0.71	0.72	0.60
Q5	0.61	0.60	0.68
Q6	0.67	0.78	0.58
Q8	0.52	0.64	0.44
Q11	0.72	0.57	0.58
Q15	0.54	0.57	0.41
Q16	0.72	0.61	0.55
Q17	0.55	0.76	0.53
Q18	0.56	0.40	0.33
Q19	0.67	0.60	0.50
Q20	0.51	0.43	0.44
Q21	0.64	0.61	0.57
Items(MTOE)			
Q1	0.51	0.53	0.38
Q4	0.60	0.65	0.44
Q7	0.64	0.56	0.40
Q9	0.48	0.45	0.47
Q10	0.57	0.67	0.50
Q12	0.60	0.63	0.51
Q13	0.66	0.68	0.66
Q14	0.57	0.69	0.52

From the results shown in Table 5.3, it can be said that item-total correlations are adequate, since all items have a factor loading above 0.30 [121]. Hence, the Exploratory Factor Analysis confirms that two subscales are defined (PMTE, MTOE).

5.3 Results

This section presents the results of the application of the Mathematics Teaching Efficacy Belief Instrument (MTEBI). The mathematics teaching self-efficacy (PMTE) subscale and the mathematics teaching outcome expectancy (MTOE) subscale are studied separately.

5.3.1 PMTE subscale

In this section, the average scores given for the participants to the questions corresponding to the PMTE subscale are studied ($\bar{x}$). Therefore, the average scores for individual items range from 1 to 4. Figure 5.1 shows the average scores obtained in the 13 items of the PMTE.

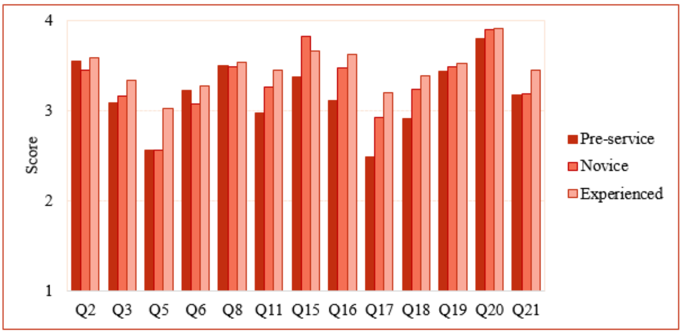


Figure 5.1: *Average score obtained in each PMTE item.*

In the case of teaching self-efficacy (PMTE), the three groups express in general a high level of mathematics teaching self-efficacy (see Figure 5.1). Notice that there are only few questions scored with less than 3 points. Table 5.4 summarizes these results.

Table 5.4 shows that only 4 items are scored, in average, below 3 in the case of pre-service teachers. This group gives the minimum score to Q17 (*I wonder if I will have the necessary skills to teach mathematics*). The novice teachers score only 2 items, in average, below 3. The experienced teachers score all the items above 3. Both groups of in-service teachers give the minimum score to Q5 (*I know how to teach mathematics concepts effectively*). The three groups of participants give the maximum score value to Q20 (*When teaching mathematics, I will usually welcome student questions, without will in in-service teachers*).

Table 5.4: *Comparison of obtained values in the PMTE subscale questions*

Mean value	Pre-service	Novice	Experienced
< 3	Q5 (I know how to teach mathematics concepts effectively)	Q5	
	Q11 (I understand mathematics concepts well enough to be effective in teaching elementary mathematics)		
	Q17 (I wonder if I will have the necessary skills to teach mathematics)	Q17	
	Q18 (I wonder if I will have the necessary skills to teach mathematics)		
> 3	Q3,Q6,Q8,Q15,Q16,Q19,Q20,Q21		
		Q11, Q18	Q11, Q18
			Q5, Q17
Minimum	Q17	Q5	Q5
Maximum	Q20 (When teaching mathematics, I will usually welcome student questions)	Q20	Q20

Additionally, a differential analysis of items is performed to see if the three groups answer specific questions differently. To determine the significant differences observed among the three groups and according to the conditions of the data, the analysis of ANOVA test and the Tukey's HSD (honestly significant difference) are applied. Table 5.5 summarizes the obtained results when comparing consecutive groups.

Table 5.5: *Comparison of obtained values in the PMTE subscale questions*

p-value	comparing pre-teachers and novice teachers	comparing novice teachers and experienced teachers
no significant difference ($p > 0.05$)	Q2, Q3, Q6, Q8, Q19	
significant difference ($p < 0.01, *p < 0.05$)		Q5 (I know how to teach mathematics concepts effectively)
	Q11 (I understand mathematics concepts well enough to be effective in teaching elementary mathematics)	
	Q15 (I will find it difficult to use manipulatives to explain to students why mathematics works)	
	Q16 (I will typically be able to answer students' questions)	
	Q17 (I wonder if I will have the necessary skills to teach mathematics)	
	Q18 (Given a choice, I will not invite the principal to evaluate my mathematics teaching)	
		Q21 (I do not know what to do to turn students on to mathematics)*

Table 5.5 shows that there are 6 questions for which there exists a significant difference between pre-teachers and novice teachers. Comparing the two groups of in-service teachers, there are only 2 questions for which there exists a significant difference between their scores (Q5 and Q21). Although it is not included in the table, the difference between pre-teachers and experi-

enced teachers was statistically significant in 9 of the 13 questions.

Comparing the three groups

The goal of this section is to study if there are statistically significant differences among the three group of participants in the mathematics teaching self-efficacy (PMTE). To carry out the study, all the items of the PMTE subscale are studied together. That is, the total score of the PMTE subscale items obtained by each participant is computed (the total score is averaged by the number of items scored by the participant). In order to study the distribution of the scores given by the participants, Figure 5.2 shows the average scores of the teaching self-efficacy corresponding to the pre-service teachers, novice teachers and experienced teachers. Specifically, the boxplot allows to identify the quartiles, the median (represented by horizontal lines in thinner regions) and the mean (denoted with points).

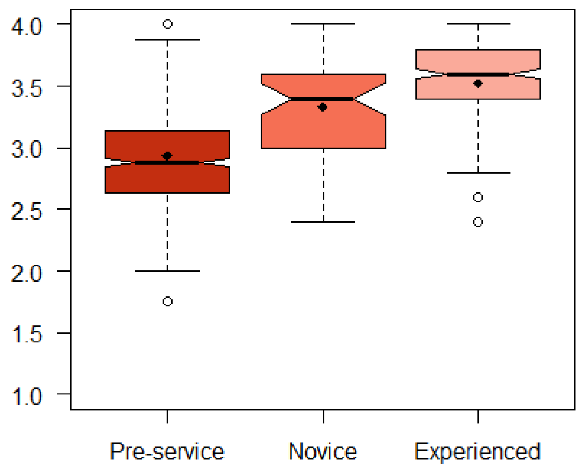


Figure 5.2: *Individual pre-service teachers, novice teachers and experienced teachers average scores given to PMTE subscale.*

Notice in Figure 5.2 that the range of values of the pre-teachers is clearly wider than the ones of the other two groups. Specifically, the minimum value of the pre-teachers is the smallest one (2). In the case of novice teachers, the number of small values (below 2.5) has been reduced (the minimum is 2.31).

In the case of experienced teachers, there are not so many small values: there are two outliers with values 2.46 and 2.69, and the minimum value is 2.77.

The specific values of the mean $(\bar{x})$, median $(\hat{x})$ and standard deviation (σ) of scores given by pre-service teachers, novice teachers and experienced teachers in the case of the PMTE subscale are shown in Table 5.6.

Table 5.6: *PMTE (mean, median and standard deviation of the average value that each participant gives to the PMTE subscale questions)*

	$\bar{X}$	$\hat{X}$	σ
Pre-service	2.93	2.88	0.40
Novice	3.33	3.40	0.39
Experienced	3.52	3.60	0.34

In order to analyze the significance of the difference in the groups, first, the hypotheses of normality (Shapiro-Wilk test) and homoscedasticity (Fligner-Killeen test) of the three groups of participants are verified. According to the conditions of the data, the Kruskal-Wallis test is used as the nonparametric alternative of ANOVA. After applying the test $(p < 0.01)$ it can be concluded that there are significant differences between medians of the groups in teaching self-efficacy.

Secondly, a post-hoc analysis is used to determine the significant differences observed among consecutive groups. Specifically, Tukey's range test is applied. Obtained p-values allow to affirm that the difference between the obtained medians is statistically significant between pre-service teachers and novice teachers $(p < 0.01)$ and between novice teachers and experienced teachers $(p < 0.01)$.

5.3.2 MTOE subscale

As in the PMTE subscale, the average scores $(\bar{x})$ that the participants of each group gives to each individual item are studied. Figure 5.3 shows the average scores obtained in the eight items of the outcome expectancy subscale (MTOE).

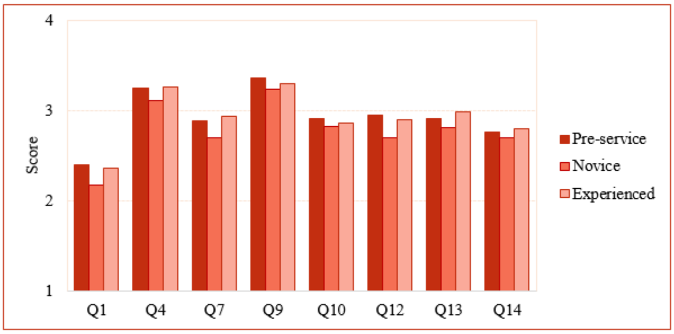


Figure 5.3: *Average score obtained in each MTOE item.*

Figure 5.3 shows that the three studied groups surpass the average score value 3 in only two items: Q4 and Q9. Notice that the average scores of experienced teachers exceed pre-service teachers in just four questions. Besides, it is noteworthy to remark that in all the items novice teachers give lower scores (in average) than pre-service teachers. Table 5.7 summarizes the obtained results.

As in the PMTE subscale case, a differential analysis of items is performed to see if the three groups answer specific questions differently. Specifically, an ANOVA test and the Tukey's HSD (honestly significant difference) are applied. Obtained $p - values$ allow to conclude that there is only significant difference between the mean obtained in Q12 (The teacher is generally responsible for the achievement of students in mathematics) by considering consecutive groups. That is, between pre-teachers and novice teachers and between novice and experienced teachers $(p < 0.05)$.

Table 5.7: *Comparison of obtained values in the MTOE subscale questions*

Mean value	Pre-service	Novice	Experienced
< 3	Q1, Q7, Q10, Q12, Q13, Q14		
> 3	Q4 (When the mathematics grades of students improve, it is often due to their teacher having found a more effective teaching approach)	Q4	Q4
	Q9 (The inadequacy of a student's mathematics background can be overcome by good teaching)	Q9	Q9
Minimum	Q1 (When a student does better than usual in mathematics, it is often because the teacher exerted a little extra effort)	Q1	Q1
Maximum	Q9	Q9	Q9

Comparing the three groups

The goal of this section is to study if the differences among the three group of participants are statistically significant in the MTOE subscale.

Figure 5.4 shows the distribution of the individual outcome expectancy values of pre-service teachers, novice teachers and experienced teachers.

Table 5.8 shows the mean, median and standard deviation of scores obtained by participants in the case of the outcome expectancy.

Table 5.8: *MTOE (mean, median and standard deviation of the average value that each participant gives to the MTOE subscale questions)*

	$\bar{X}$	$\hat{X}$	σ
Pre-service	2.91	2.89	0.39
Novice	2.76	2.75	0.48
Experienced	2.94	3.01	0.43

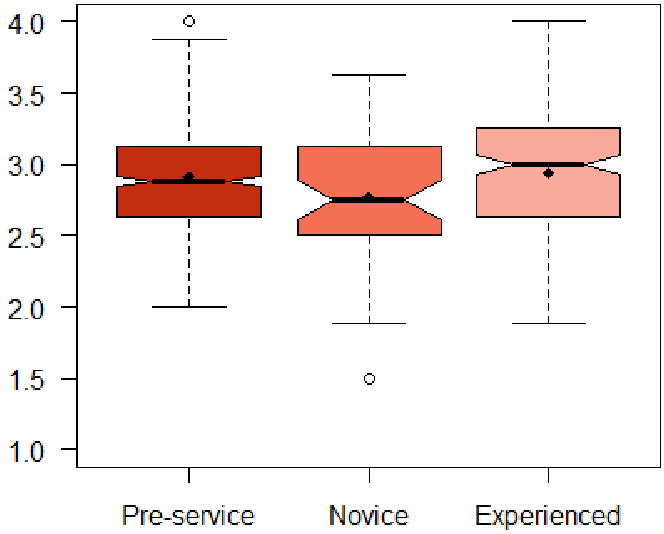


Figure 5.4: *Individual pre-service teachers, novice teachers and experienced teachers average scores given to MTOE subscale.*

Figure 5.4 shows that the distribution of individual MTOE values are similar in the three groups of participants. Recall that the range of values is slightly wider in the case of experienced teachers ($minimum = 1.88, maximum = 4$). However, a very similar distribution is obtained in the case of pre-service teachers. The novice teachers present clearly smaller values than the other two groups. Notice that the maximum value in this case is 3.63 and there is an outlier with a value of 1.50.

Additionally, a statistical analysis is carried out to study the significance of the obtained results. First, compliance with the conditions of hypothesis of normality (Shapiro-Wilk) and homoscedasticity (Bartlett) is verified. Then, according to the conditions of the data, the analysis of ANOVA test is applied ($F = 3.53, p = 0.03$). The obtained values confirm that there are significant differences among the groups.

Second, the Tukey's HSD (honestly significant difference) test is applied, in a post-hoc analysis. According to the results, there is a significant difference between pre-service teachers and novice teachers ($p = 0.04$) and between novice teachers and experienced teachers ($p = 0.02$).

5.4 Discussion

In order to answer the first chapter question, the mean scores of each question of the PMTE subscale were studied for each of the three groups of participants (pre-service teachers, novice teachers, and experienced teachers). Our findings indicate that in the PMTE questions experienced teachers give higher scores than novice teachers and the latter give higher scores than pre-service teachers. Only in question Q15 (*I will use manipulatives to explain to students why mathematical solutions work*) novice teachers give a higher average than experienced teachers. We believe that this may occurs because some experienced teachers did not learn how to use manipulatives in the mathematics teaching and learning courses they took during their training. Manipulatives are currently more studied and used in mathematics education. Besides, a statistically analysis of items is performed to see if the three groups answer specific questions differently. It is noteworthy to remark that there are 6 questions to which pre-teachers and novice teachers give statistically significant different score values. In the case of novice teachers and experienced teachers that occurs only in 2 questions. There are 5 questions whose scores do not present significant differences between consecutive groups of participants. Therefore, the difference is more significant between pre-service teachers and novice teachers than between the latter and experienced teachers.

Furthermore, a statistical analysis of the PMTE values given by each group was carried out in this study. The results show that there are statistically significant differences among the three groups of participants regarding the PMTE subscale. Specifically, experienced teachers obtain a significantly higher median than novice teachers and the latter have significantly higher median than pre-service teachers.

Analogously, a similar approach have been used to compare the mathematics teaching outcome expectancy of the three studied groups (second chapter question). First of all, the mean scores of each MTOE subscale question and for each group of participants (pre-service teachers, novice teachers and experienced teachers) were studied. Obtained results show that the group of experienced teachers gives higher scores with respect to the group of novice teachers. That is, the results show that the teaching experience helps to promote the MTOE.

However, it is important to note that novice teachers have lower scores than the pre-service teachers in all items. Thus, MTOE values decline in the first few years of school teaching. These results are consistent with those presented in [69] , where MTOE values increase during university courses, but decrease during the first years of teaching students. Swars et al. [69], claim that teachers in training have an unrealistic optimism that is modified with the teaching of students. MTOE values decrease during the first years of teaching at school because, although pre-service teachers attended student practice at Primary school, they have not already faced the reality of being a teacher. Once at school, teachers need to master certain skills such as managing the group, organizing time, proposing activities, and so on. At the beginning, they may dedicate more effort to learn to be a teacher than to try to influence their students' learning.

A possible idea to foster the MTOE of novice teachers would be to assist at school as apprentice teachers the first year (or even more than one year). The key point is that they would be accompanied by a teacher to gain confidence and to acquire skills that every teacher should have. Our proposal is in accord with the finding presented in [79], where they evidenced that the self-efficacy of teachers, even those with many years of teaching experience, increases in response to experiences of success in the classroom. They proposed to consider the potential of intervention programs that can help teachers to reflect on their successes and to modify their self-efficacy beliefs accordingly.

Moreover, as in the PMTE subscale, the scores obtained in each item of the MTOE subscale are compared considering pairs of consecutive groups in order to see if their answers are statistically different. Results evidence that there is only significant difference between the mean obtained in one item.

The other 7 items of the MTOE subscale do not present significant differences between consecutive groups of participants. This finding reinforces the fact that MTOE does not vary as much as the PMTE along time.

Additionally, a statistical analysis of MTOE values obtained by each group was carried out. Results show that there are statistically significant differences between the groups of teachers considered in this study. Concretely, experienced teachers get significantly higher values than novice teachers. In the case of the pre-service teachers and the novice teachers, the MTOE is also significantly different, but the MTOE values are lower for novice teachers.

The significant difference that has been shown between the different groups, both in the case of PMTE and MTOE subscales, reinforces the robustness of the initial choice of the groups of teachers in service: novice teachers (up to 10 years of experience) and experienced teachers (more than 10 years of experience).

Obtained results allow to affirm that the teaching experience enforce the mathematics teaching efficacy beliefs (third chapter question). Particularly, it has been shown that the teaching experience strengthen the personal mathematics teaching efficacy (PMTE), since as teachers gain experience, its value is higher, coinciding with the results of previous studies [84], [122]. In the case of mathematics teaching outcome expectancy (MTOE), our findings evidence that the teaching experience also enforce those beliefs, but only in the case of in-service teachers.

Chapter 6

Mathematics teaching efficacy belief and attitude toward mathematics: two key factors for pre-service teachers' mathematics academic achievement

This chapter aims at studying the relation between mathematics teaching efficacy beliefs of pre-service teachers, their attitude towards mathematics and their mathematics academic achievement. Additionally, a multiple regression is applied to assess the dependence of each variable with academic achievement. Participants are pre-service teachers of the third year of the Primary Education Degree. To evaluate mathematics academic achievement, the grades obtained in the teaching and learning mathematics course that students took in the second year of the Degree are collected.

6.1 Introduction

In the context of pre-service teachers, authors noticed that some students of the Bachelor Degree in Primary Education present low levels of mathematics teaching efficacy beliefs and a negative attitude towards mathematics. Literature research supports the importance of enhancing the pre-service teachers' mathematics teaching efficacy beliefs (e.g., [31], [38], [39]) and their attitude towards mathematics (e.g., [37], [40]), and how these factors influence to the mathematics academic achievement (e.g., [41], [42], [43]). As pointed out in [44], it is still important and interesting to investigate the impact of variables concerning teachers' mathematics related effect on other affective or cognitive variables such as academic achievement.

6.2 Purpose of the study

Taking into account the above presented works, this chapter aims at studying, on the one hand, the relationship between pre-service teachers' mathematics teaching efficacy beliefs and their attitude towards mathematics with their mathematics academic achievement. Both factors are considered important in the literature, but there is not any work studying both factors together, as far as we are concerned. Additionally, multiple linear regression is used in this chapter to determine the influence of each studied factor to academic achievement. Besides, another goal of this work is to study if there are subscales of the considered factors with which there is a greater correlation with academic achievement. Therefore, the correlation among six subscales of the two factors (two in the case of teaching efficacy beliefs and five in the case of attitude towards mathematics) and mathematics academic achievement is also studied. Specifically, the current study was designed to answer the following questions:

1. How is the correlation between pre-service teachers' mathematics teaching efficacy beliefs, their attitude towards mathematics and their mathematics academic achievement?

2. How is the correlation between the two subscales of mathematics teaching efficacy beliefs, the five subscales of the attitude towards mathematics and the mathematics academic achievement?

6.3 Methodology

This section presents the methodology adopted in this chapter.

6.3.1 Participants

The participants in this study are students of the Bachelor Degree in Primary Education at the Rovira and Virgili University (details on the Bachelor in section 1.2). During several years, it has been noticed low levels of mathematics teaching efficacy beliefs and negative attitudes toward mathematics among pre-service teachers enrolled in the Bachelor Degree in Primary Education. However, there is not any study quantifying those feelings at the current University. Concretely, the most negative feelings and the worst mathematics academic achievement occur during the TLM 1 course, which mainly includes mathematical content. The participants of this study are students of the third year of the Bachelor. They have only coursed the TLM 1 course, which is given during the second year of the Bachelor, as shown in section 1.2.

As detailed in short, the mathematics academic achievement was assessed by taken the grade they were given in the TLM 1 course during the 2018-2019 academic year. A total of 97 students were enrolled to TLM 1 course. Moreover, students of the third year that attended at the TLM 2 course class at the beginning of the 2019-2020 academic year were invited to answer the MTBI and the AMS. Therefore, participation was voluntary and anonymous. Students indicated the initials of their name in order to stablish correspondences with the marks obtained in the TLM 1 course (the marks were provided by the corresponding teacher and by using the same code). Once the correspondences were made, all codes were deleted.

Specifically, the sample of this study correspond to 56 students out of the 70 enrolled in the third year of the Degree of Primary Education (81% of the total population). Notice that only the students that had already attended to the TLM 1 course could be selected for the study.

6.3.2 Instrument

As mentioned above, two instruments were used in this chapter: 1) the Mathematics Teaching Efficacy Belief Instrument (MTEBI) for pre-service teachers [123], which was presented in detail in subsection 4.2.2; and 2) the Attitude towards Mathematics Scale (AMS) for pre-service teachers [97].

The Mathematics Teaching Efficacy Belief Instrument (MTEBI)

The Mathematics Teaching Efficacy Belief Instrument (MTEBI) (MTEBI), presented in detail in subsection 4.2.2, for pre-service teachers [123] is also used in the current chapter.

The Attitude towards Mathematics Scale (AMS)

There exist different instruments measuring the attitude towards mathematics. Some of them are studied in (e.g., [124], [125], [97], [126], [127], [97]) drew up one of the most cited attitude scale towards mathematics from those performed in the Spanish language. The author justifies the construction of a new scale in the absence of this type of instrument developed in Spanish and Latin American. The author was inspired by the characteristics that represent a scale of attitudes in [128]. The current chapter uses the attitude towards the mathematics scale (AMS) proposed in [97], since it is an instrument designed in Spanish and specifically developed for pre-service teachers. The AMS has five subscales (pleasure, anxiety, usefulness, motivation and confidence). According to Auzmendi, the pleasure towards mathematics refers to the enjoyment caused by mathematical work. The anxiety towards mathematics refers to the feeling of anxiety, fear that the person manifests in the matter of mathematics. The usefulness subscale refers to the value that a student attaches to mathematics, to the usefulness that he perceives that this subject may have for professional life. The motivation subscale represents how the student feels towards the study and the use of mathematics. Finally, the confidence subscale is the feeling caused by the ability towards mathematics.

The AMS consists of 25 items on a five-point Likert scale that measures from one (strongly disagree) to five (strongly agree) and is used in several investigations (e.g., [129], [130], [131]). The items of the AMS are detailed in Appendix C . As in the MTEBI, the third element of the Likert scale, which was in the original version of the AMS, was removed. As mentioned above, the AMS establishes five subscales: pleasure (items 4, 9, 14 and 24), anxiety (items 2, 3, 7, 8, 12, 13, 17, 18 and 22), motivation (items 5, 10, and 25), usefulness (items 1, 6, 15, 16, 19 and 21) and confidence (items 11, 20 and 23). Ten of the AMS items have a reverse score (items 2, 5, 7, 10, 12, 15, 16, 17, 22 and 25). The answers corresponding to these items must be reversed before being added to the total AMS score (4 = 1, 3 = 2, 2 = 3 and 1 = 4).

The questions that conform the AMS in the case of pre-service teachers are detailed in Appendix C.

Mathematical content exam

The mathematics academic achievement was assessed during the TLM 1 by using an exam. The items of the exam aimed at evaluating if pre-service teachers had learned the mathematical content they were taught during the TLM 1 course. In addition, they had to know how to explain the problems correctly. Specifically, the exam consists of Arithmetic and Geometry problems.

Reliability

In order to determine the reliability of the obtained results, Cronbach's alpha test was applied [109]. Table 6.1 shows Cronbach's alpha coefficients for the MTEBI and its two subscales (PMTE and MOTE) and for the AMS and its five subscales (pleasure, anxiety, motivation, usefulness and confidence). Rules proposed in [110] are considered to interpret the obtained alpha values. In the case of motivation and confidence subscales, the values of Cronbach's alpha are poor and unacceptable, respectively. Due to that, the confidence subscale was eliminated before the analysis. It is important to note that Cronbach's alpha for AMS is excellent and for MTEBI it is acceptable.

Table 6.1: *Cronbach's Alpha*

	α-Cronbach	Meaning
MTEBI	0.75	Acceptable
Teaching self-efficacy	0.82	Good
Outcome expectancy	0.66	Questionable
AMS	0.93	Excellent
Pleasure	0.82	Good
Anxiety	0.93	Excellent
Motivation	0.55	Poor
Usefulness	0.81	Good
Confidence	0.36	Unacceptable

6.3.3 Data analysis

To answer the two research questions in this chapter, we use the mean $(\bar{X})$, the standard deviation (σ), the Pearson's correlation coefficient (r) and the simple regression analysis for the variables in each case. In addition, we study the contrast of the normal population distribution (Shapiro-Wilk) and homoscedasticity (Fligner-Killeen). The tests indicate that data normality is accepted in each of the subscales $(p - value > 0.05)$ [132]. In the case of the variation uniformity test, the Fligner-Killeen test indicates that there are not significant differences between the variances of the groups $(p - value > 0.05)$.

Considering the results of the Shapiro-Wilk and Fligner-Killeen tests, the Pearson's correlation coefficient is used in this study.Table 6.2 shows the classification of correlation coefficients considered in this chapter. This classification has been used by several researchers (e.g., [133], [134]).

Table 6.2: *Correlation coefficient*

Value	Meaning
1	Large and perfect correlation
0.9 − 0.99	Very high correlation
0.7 − 0.89	High correlation
0.40 − 0.69	Moderate correlation
0.20 − 0.39	Low correlation
0.01 − 0.19	Very low correlation
0	Null correlation

6.4 Results

This section is organized into two subsections, each of which corresponds to each of the research questions. The first subsection examines the correlation between the pre-service teachers' mathematics teaching efficacy beliefs, their attitude towards mathematics and their mathematics academic achievement. The second subsection studies the correlation between subscales of MTEBI (self-efficacy of the teaching of mathematics and outcome expectancy of the teaching of mathematics), subscales of AMS (pleasure, anxiety, motivation, usefulness and confidence) and mathematics academic achievement.

6.4.1 Correlation between mathematics teaching efficacy beliefs, attitude towards mathematics and mathematics academic achievement

First, the mean $(\bar{X})$ and the standard deviation (σ) obtained in the MTEBI and AMS are computed. Besides, the marks obtained in the TLM 1 course are collected as mathematics academic achievement. Table 6.3 shows the mean and standard deviation of the MTEBI, the AMS and the academic achievement obtained by the pre-service teachers. Recall that the MTEBI and the AMS are given values up to 4, while the academic achievement is measured up to 10.

Table 6.3: *Mean, standard deviation, and correlation between MTEBI and academic achievement*

	$\bar{X}$	σ
MTEBI	3.02	0.30
AMS	2.85	0.56
Academic achievement	5.82	1.52

Table 6.3 shows that the mean of the MTEBI values is higher than the one obtained in the AMS. In addition, it is observed that the deviation of the AMS is wider than the MTEBI.

Moreover, we study the relationship between the pre-service teachers' efficacy beliefs, attitude towards mathematics and academic achievement through the Pearson's correlation coefficient. Table 6.4 summarizes the obtained results.

Results shown in Table 6.4 indicate that there is a significant moderate correlation between teachers' efficacy beliefs and academic achievement, and also between attitude towards mathematics and academic achievement ($0.69 > r > 0.40$). In addition, there is a significant moderate correlation between the teachers' efficacy beliefs and between the attitude towards mathematics ($0.69 > r > 0.40$).

Table 6.4: *Pearson's correlation coefficient between the three pairs of variables. (** The correlation is significant ($p - value < 0.001$))*

	Academic achievement	Teachers' efficacy beliefs	Attitude
Academic achievement	1		
Teachers' efficacy beliefs	0.66^{**}	1	
Attitude	0.60^{**}	0.47^{**}	1

Finally, a multiple linear regression analysis between the teachers' efficacy beliefs and attitude towards mathematics (independent variable predictors) and mathematics academic achievement (dependent variable) is carried out. Results of the multiple linear regression analysis are shown in Table 6.5.

The variables teachers' efficacy beliefs and attitude towards mathematics show a significant relationship with academic achievement. The coefficient of determination indicates that 54% of the variation in the output variable (predicted variable) is explained by the input variables (predictor variables). In other words, the two predictor variables explain 54% of the change in academic achievement score.

Table 6.5: *Multiple linear regression model summary*

R	R^2	$R^2 adjusted$	F	$Sig.$
0.73	0.54	0.52	29.56	0.000

Furthermore, the obtained Standardized Beta Coefficient indicates that the most powerful variable is teachers' efficacy beliefs ($\beta = 0.54, p < 0.001$). In the case of the attitude towards mathematics, the Standardized Beta Coefficient is $\beta = 0.29, p = 0.001$. These values evidence that the two factors predict significantly academic achievement.

Finally, for a better interpretation of the correlation of the data, Figure 6.1 shows the scatter plot between the teachers' efficacy beliefs and academic achievement. There is a direct linear relationship between the teachers' efficacy beliefs and the academic achievement of pre-service teachers with a stochastic dependence. The regression line expresses the trend between beliefs and academic achievement. Specifically, this means that students' academic achievement will be higher as the teachers' efficacy beliefs are also higher.

Analogously, Figure 6.2 shows the dispersion of data between the attitude towards mathematics and academic achievement. It is shown that there is a direct correlation between the attitude towards mathematics and the aca-

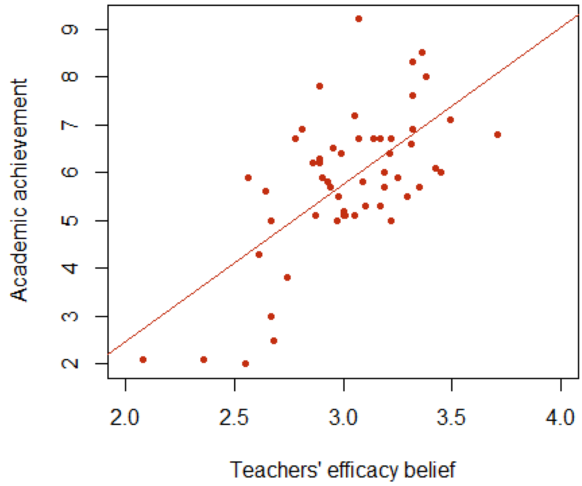


Figure 6.1: *Scatter plot between the teachers' efficacy beliefs and academic achievement.*

demic achievement of pre-service teachers with a stochastic dependence.

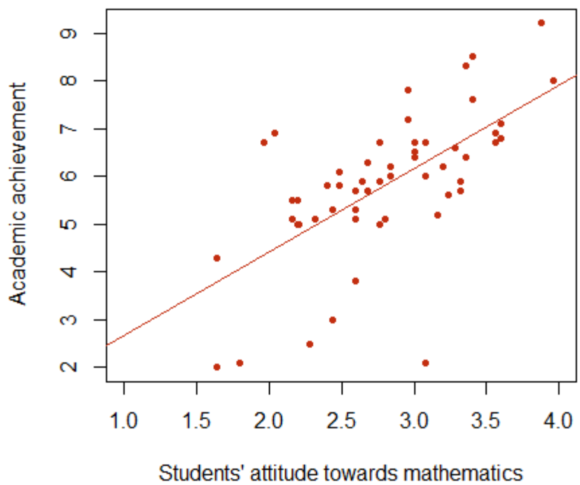


Figure 6.2: *Scatter plot between the attitude mathematics and academic achievement.*

6.4.2 Correlation among subscales of MTEBI, subscales of AMS and mathematics academic achievement

This section studies together the two subscales that conform the MTEBI (mathematics teaching self-efficacy and outcome expectancy) and the four considered from the AMS (pleasure, anxiety, motivation and usefulness). Recall that the subscale confidence was removed due to the unaccepted reliability provided by Cronbach alpha.

Table 6.6 shows the mean ($\bar{X}$), the standard deviation (σ), and the Pearson's correlation coefficient value (r) between the two subscales of MTEBI, the four subscales of the AMS, and academic achievement. It should be noted that the anxiety subscale has been reversed scored. That is, the higher the score is, the less the anxiety.

In addition, in the case of MTEBI, the PMTE subscale has the highest mean. In the AMS, the motivation subscale has the highest mean. The lowest score, on the contrary, is obtained in the pleasure subscale.

Results summarized in Table 6.6 shows that there is a significant high correlation between the teaching self-efficacy (PMTE) and academic achievement. In the case of the outcome expectation (MTOE), its correlation with the academic achievement is very low and it is not statistically significant. In the case of the AMS subscales, results show that the correlation between both anxiety and usefulness and academic achievement is moderate ($0.70 > r > 0.39$). Specifically, the highest correlation with academic achievement is found in the case of the anxiety subscale ($r = 0.57, p < 0.05$). The lowest correlation, on the contrary, is obtained between the motivation subscale and academic achievement ($r = 0.27$) and in this subscale, the obtained result is not significant ($p > 0.05$).

Table 6.6: *Mean and standard deviation obtained in each studied subscale, and correlation between each of them and academic achievement*

	$\bar{X}$	σ	r	$p - valor$
PMTE	3.17	0.30	0.72	< 0.01
MTOE	2.87	0.45	0.16	0.11
Pleasure	2.15	0.78	0.54	< 0.01
Anxiety	2.81	0.69	0.57	< 0.01
Motivation	3.28	0.57	0.27	0.024
Usefulness	2.78	0.66	0.55	< 0.01

Finally, a multiple linear regression analysis between the six subscales and mathematics academic achievement is carried out taking the independent variables predictors (self-efficacy of the teaching of mathematics, outcome expectancy of the teaching of mathematics, pleasure, anxiety, motivation and usefulness) and the dependent variable (mathematics academic achievement).Table 6.7 summarizes obtained results.

Table 6.7: *Multiple linear regression model summary*

R	R^2	$R^2 adjusted$	F	$Sig.$
0.77	0.59	0.54	11.52	0.000

Results shown in Table 6.7 indicate that there is a significant dependent relationship between the subscales and academic achievement. The coefficient of determination ($R^2 = 0.59$) indicates that 59% of the variation in the output variable (predicted variable) is explained by the input variables (predictor variables). Therefore, the six predicted variables explain 59% of the change in academic score. According to the obtained Standardized Beta Coefficient values, the order of influence of the predictor variables on academic achievement is the following: mathematics teaching self-efficacy ($\beta = 0.58$), outcome expectation ($\beta = 0.14$), pleasure ($\beta = 0.13$), anxiety ($\beta = 0.08$), usefulness ($\beta = 0.06$) and motivation ($\beta = 0.05$).

Considering the obtained correlation ($p > 0.05$) and Standardized Beta Coefficient ($\beta = 0.13$, $p > 0.05$), the MTOE subscale is removed from the initial model. Five independent variables predictors (self-efficacy of the teaching of mathematics, pleasure, anxiety, motivation and usefulness) and dependent variable (mathematics academic achievement) define the new model. The values of the new model are equal to model 1, but the Standardized Beta Coefficient values are slightly modified. Again, the most powerful variable is mathematics teaching self-efficacy ($\beta = 0.59$), followed by pleasure ($\beta = 0.19$).

6.5 Discussion

This chapter aimed at studying pre-service teachers' mathematics teaching efficacy beliefs, their attitude towards mathematics and their mathematics academic achievement. Besides, it also aimed at studying the correlation between subscales of MTEBI (mathematics teaching self-efficacy and outcome expectancy), subscales of AMS (pleasure, anxiety, motivation, usefulness and confidence) and mathematics academic achievement. Regarding the first research question, results reaffirm that there is a significant moderate correlation between the mathematics teaching efficacy beliefs and academic achievement $(r = 0.66)$. That is, teachers with greater teachers' efficacy beliefs showed a better academic achievement. This result is in accord with the work presented in [135], where the self-efficacy is related to mathematics academic achievement. Results also show a moderate correlation between the attitude towards mathematics and academic achievement $(r = 0.60)$. This result is consistent with previous research that supports the influence of the attitude towards mathematics of pre-service teachers in their academic achievements in mathematics [93]. Additionally, results manifest a significant moderate correlation between both factors $(r = 0.47)$. This result allows to conclude that both factors are not redundant (its correlation is not high), so they provide different and useful information.

From the multiple linear regression analysis it can be concluded that mathematics teaching efficacy beliefs is the most powerful factor to predict academic achievement ($\beta = 0.54$).

Therefore, this chapter emphasizes that both positive mathematics teaching efficacy beliefs and a positive attitude towards mathematics of pre-service teachers have a positive effect on their academic achievement.

Concerning the second research question, it is shown that the subscale PMTE presents a significant high correlation with academic achievement ($r = 0.72$). The MTOE subscale, on the contrary, presents a low correlation and it is not significant ($r = 0.16, p = 0.11$). These results are consistent with the theoretical predictions based on Bandura's works [64], [67], where it is affirmed that self-efficacy beliefs predict academic outcomes. Obtained results also coincide with other previous researches (e.g., [136], [137], [138], [72]), where authors indicated that there is a relationship between self-efficacy and academic achievement.

The second highest correlation coefficient is obtained between the anxiety subscale and academic achievement ($r = 0.57$). Some researchers also determined that there is a relationship between anxiety and academic achievement ([137], [28], [139]). That is, more anxiety implies less academic achievement.

Results obtained applying a multiple linear regression show that the most powerful variable to predict academic achievement is PMTE subscale ($\beta = 0.59$). The second powerful variable to predict academic achievement is pleasure ($\beta = 0.19$). In the current context, pleasure is the subscale with the lowest given mean. Therefore, a clear implication of this chapter would be to try to improve the pleasure pre-service teachers feel towards mathematics. According to the obtained results in this chapter, that improvement will benefit the mathematical academic achievements.

Indeed, considering the impact of the mathematics teaching efficacy beliefs and the attitude towards mathematics with respect to academic achievement, teachers training programs should help pre-service teachers to develop more positive beliefs and attitudes towards mathematics. A clear implication of this chapter would be to include an evaluation of the mathematics teaching efficacy beliefs and the attitude toward mathematics of the pre-service teachers. Additionally, its evolution throughout the Bachelor Degree should also be studied.

Although this chapter is focused on a reduced population and a specific context, we believe that the significant relationships found in this investigation allow to extend the same conclusions to a larger population and other contexts.

Chapter 7

Studying mathematics teaching efficacy beliefs of primary and secondary in-service teachers

This chapter compares the mathematics teaching efficacy beliefs of in-service primary and secondary education teachers. Additionally, the influence of the level of teacher training on mathematics teaching efficacy beliefs is also studied. Specifically, beliefs of teachers with a bachelor degree are compared with the ones of teachers who have a Master. The two factors (level of teaching and level of teacher training) are considered important in the literature.

7.1 Introduction

Several researchers have shown that efficacy belief of in-service teachers' mathematics is important because it affects their teaching practices ([64], [67], [136], [138], [78], [83], [84], [72]). Specifically, researchers have indicated the importance of self-efficacy in improving student learning.

Indeed, many researchers have studied factors that influence teachers' efficacy beliefs ([82], [84], [86]). In this sense, in [140], the authors groups the factors that influence teachers' efficacy beliefs into two dimensions: teacher related variables and contextual variables. The following are considered teacher-related variables: gander, causal attributions, teaching experience and teacher training level.Besides, the contextual variables are: level where teachers teach, characteristics of the group the teacher teaches and collaboration between teachers. Specifically, in this chapter we study the factors: level where teachers teach and level of training. Hoy et al. [141] propose the level of academic training of teachers as a determining factor in their sense of self-efficacy. According to them, teachers with a higher level of training tend to enjoy greater self-efficacy, perhaps because more extensive training contributes to the acquisition of more teaching skills. Moreover, the level of teaching factor is a contextual variable. The level of teaching at which the teacher teaches represents the contextual variable that influence on his expectations of self-efficacy [82].

7.2 Purpose of the study

As discussed above, the objective of this chapter is to study two factors that can influence the mathematics teaching efficacy beliefs in primary education and secondary school teachers. Specifically, two factors are studied: teaching level (primary and secondary) and level of training (degree, master). To meet the objective, two research questions are defined, one for each factor:

1. Does the teaching level factor (level where teachers teach) influence the teaching self-efficacy and the outcome expectancy?

2. Does the level of training (degree and master) influence the teaching self-efficacy and the outcome expectancy?

7.3 Methodology

In this chapter, for the analysis of information, descriptive and inferential statistics are used, once the necessary conditions for its application are met.

7.3.1 Participants

The sample in this study was comprised of a total of 354 teachers, 252 (71%) of which were primary school teachers and 102 (29%) were secondary school mathematics teachers. The MTEBI survey was adapted for in-service teachers, and created in google forms. Directors of Primary Education schools and Secondary school in the city of Tarragona were sent the link of the survey during the second trimester of 2018-2019 school academic year. They shared the link so that in-service teachers were able to answer it confidentially.

In Spain primary education teachers obtain their bachelor degree in Primary Education and can teach students between 6 and 12 years old. Besides, it is important to point out that in Spain there is not a specific bachelor degree to become a secondary education teacher. Secondary school mathematics teachers mostly have a degree in Mathematics or related areas, and subsequently obtain their postgraduate studies in education. In fact, today, all secondary school teachers have to study a master's degree in education, in the corresponding speciality. Secondary school teachers teach students between the ages of 13 and 18.

7.3.2 Instrument

The Mathematics Teaching Efficacy Belief Instrument (MTEBI) for pre-service teachers is again used in this chapter.

7.3.3 Data analysis

In order to determine the reliability of the instrument for the current context, the Cronbach's alpha test was applied to both subscales (namely, PMTE and MOTE) [109]. The values of Cronbach's alpha for the PMTE and the MTOE are 0.84 and 0.81, respectively. According to the scale proposed by [110], the obtained values of Cronbach's alpha are good (>0.8).

In addition, the contrast of the normal population distribution (Shapiro-Wilk [132]) and homoscedasticity (Fligner-Killeen) is studied. Depending on the conditions of the data, the Kruskal-Wallis (Non-Parametric ANOVA) test is used to verify significant differences between different groups [142]. A post-hoc analysis determines the significant differences observed in the data groups. Specifically, a Tukey (non-parametric) test is applied.

Further, in the case of two analysis groups, the $t - student$ test is used to verify significant differences between these groups (parametric data).

7.4 Results

This section presents the results of the implementation of the The Mathematics Teaching Efficacy Belief Instrument (MTEBI). The Personal Mathematics Teaching Efficacy (PMTE) and Mathematics Teaching Outcome Expectancy (MTOE) subscales are studied separately. In particular, the two subscales are studied in primary education and secondary school teachers. The results of this research are organized into two subsections, each of which corresponds to each research question.

7.4.1 Comparing primary and secondary teachers

In the first subsection, the factor level of studies where the teachers teach is studied. That is to say, the mathematics teaching efficacy beliefs of primary and secondary teachers are compared. In order to meet the objective of this section, the scores of the mean, median and standard deviation are studied.

The normality hypothesis is studied, the Shapiro-Wilk test is applied and normality is accepted ($p > 0.05$) for the two groups of the PMTE and MTOE subscales. In addition, the homoscedasticity of the variances ($p > 0.05$) in the PMTE and MTOE subscales is accepted.

Mathematics teaching self-efficacy (PMTE subscale)

The primary teachers' mean score is $\bar{x} = 3.47$ and the standard deviation $\sigma = 0.36$. Besides, the secondary school teachers obtained a mean score of $\bar{x} = 3.59$ and a standard deviation of $\sigma = 0.33$.

A $t - student$ test is then performed to compare whether there are significant differences between the mean scores obtained in each group. The value of p (0.022) indicates that there is a significant difference between the mean scores of the two groups (primary and secondary education teachers). It is important to note that secondary school teachers have a significantly higher mean score than primary education teachers. Figure 7.1 shows the box plot of the total mean scores obtained by primary education and secondary school teachers in the teaching self-efficacy (PMTE) subscale.

Furthermore, Figure 7.1 shows that the range of values for primary education teachers is clearly wider than that of secondary school teachers. Specifically, in the case of primary teachers, an outlier (1.25) is observed. In the case of secondary school teachers, no outliers are observed, while the minimum value is 2.8. The minimum value for secondary school teachers is higher than the minimum value for primary teachers. Figure 7.1 shows that secondary school teachers obtained higher mean scores in teaching self-efficacy than primary teachers.

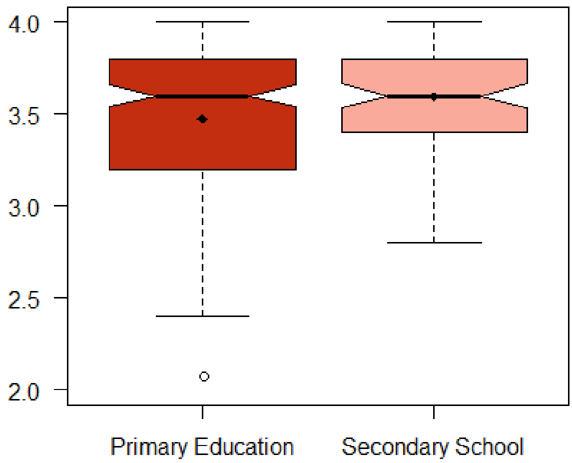


Figure 7.1: *Individual mean scores given to in-service teachers, teachers mean scores given to PMTE subscale.*

Outcome expectation (MTOE subscale)

Analogously, the mean and standard deviation scores of the questions corresponding to the outcome expectation subscale are calculated for each group of participants from the outcome expectation. Primary education teachers get a score of the mean of $\bar{x} = 2.90$ and a standard deviation of $\sigma = 0.46$. Moreover, secondary school teachers obtain a score of the mean of $\bar{x} = 2.54$ and a standard deviation of $\sigma = 0.52$. The $t - student$ test is used to verify the difference between obtained means of the data. In this case, the difference between the means obtained is statistically significant ($p = 3.99 \times 10^{-10}$). It should be noted that, unlike teaching self-efficacy, in the outcome expectation primary education teachers obtain a significant higher mean than secondary school teachers. Figure 7.2 shows the box plot of the total mean scores obtained by primary education and secondary school teachers in the subscale outcome expectation.

Figure 7.2 shows that for both primary education and secondary school teachers there are outliers of less than 1.5. However, it can be clearly seen that the mean scores of primary teachers are higher than those of secondary

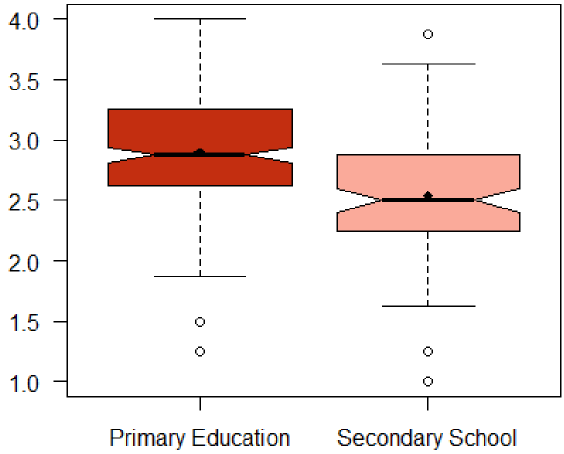


Figure 7.2: *Individual mean scores given to in-service teachers, teachers mean scores given to MTOE subscale.*

school teachers. Specifically, 50% of primary teachers' mean scores are below 2.88. Moreover, 50% of secondary teachers' mean scores are below 2.50.

7.4.2 Level of teacher training

In this section, the training level factor is studied. Concretely, the results obtained by teachers who have a bachelor degree are compared with those of teachers that have a master degree. In order to meet the purpose of this section, the scores given by primary education and secondary school teachers to the items of both subscales PMTE and MTOE are studied. In this case, the population of primary education and secondary school teachers is divided into two groups: teachers with a bachelor degree (70%) and teachers with a master degree (30%). In the case of secondary school teachers, 47% have of secondary school teachers have a bachelor degree and 53% have a master degree. Besides, 80% of primary education teacher have a bachelor degree and the other 20% have a master degree.

The hypothesis of normality is studied, the Shapiro-Wilk test is applied, and the normality ($p > 0.05$) of the two groups of the PMTE and MTOE subscales is accepted. In addition, the homoscedasticity of the variances ($p > 0.05$) in the PMTE and MTOE subscales is accepted.

Mathematics teaching self-efficacy (PMTE subscale)

This section studies the scores given by teachers to the items of teaching self-efficacy. In this subscale, the group of participants of teachers with a bachelor's degree obtains a mean of $\bar{x} = 3.46$ and a standard deviation of $\sigma = 0.37$, while the group of teachers with a master's degree obtains a mean of $\bar{x} = 3.57$ and a standard deviation of $\sigma = 0.34$. The $t - student$ test is applied to verify the significance of the means of the data obtained by the participants and it is evident that significant differences exist between the groups of participants ($p = 0.04$). These results indicate that teachers with a master's degree obtain higher mean scores than teachers with a bachelor's degree. Figure 7.3 shows the box plot of total scores obtained by primary education and secondary school teachers in the teaching self-efficacy subscale (PMTE).

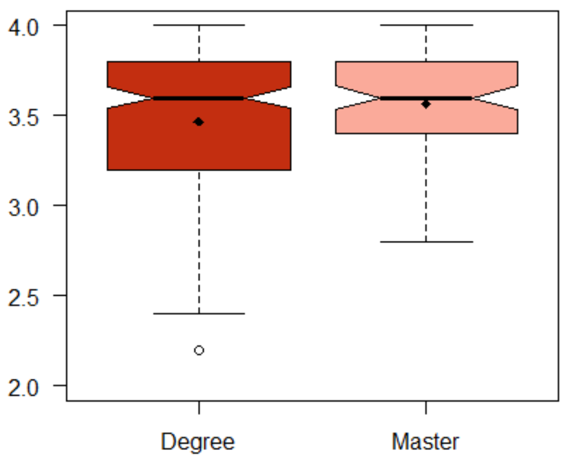


Figure 7.3: *Individual in-service teachers, teachers average scores given to PMTE subscale.*

Further, Figure 7.3 shows that the range of values for teachers with a bachelor's degree is clearly wider than for teachers with master's degree training. Specifically, an outlier (2.2) is observed for teachers with a bachelor's degree. In the case of teachers with a master's degree, there are no outliers. The minimum values and quartile 1 of the teachers with a master's degree are clearly higher than those of the teachers with a bachelor's degree. The median in both cases is 3.6.

Outcome expectation (MTOE subscale)

Similarly, in this subscale, the group of teachers with a bachelor degree training obtains a mean of $\bar{x} = 2.74$ and a standard deviation of $\sigma = 0.54$, while, the group of teachers with a master degree training obtains a mean of $\bar{x} = 2.81$ and a standard deviation of $\sigma = 0.48$. Results of the test $t - student$ verifies that there is no significant difference between the means of the two groups of teachers $(p = 0.10)$. Figure 7.4 shows the box plot of the total mean scores obtained by primary education and secondary school teachers in the subscale outcome expectation by training level of education.

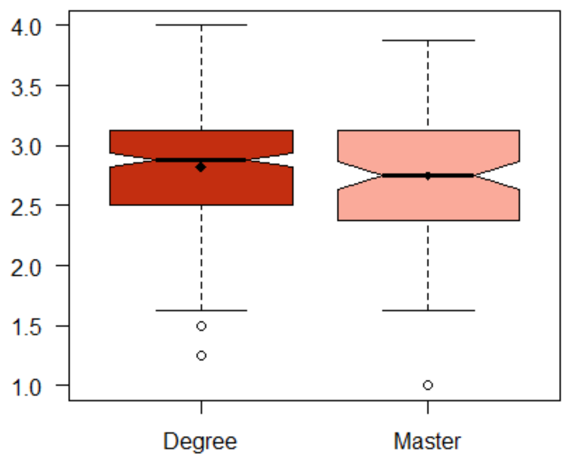


Figure 7.4: *Individual in-service teachers, teachers average scores given to MTOE subscale.*

In addition, Figure 7.4 shows that the range of values for teachers with a bachelor's degree is similar to that of teachers with a master's degree. Specifically, two outliers (1.25,1.5) are observed in the case of teachers with a bachelor's degree. In the case of teachers with a master's degree, an outlier is observed (1.00). The minimum values, quartile 1, 2 and 3 in the two cases are similar. This representation allows visualizing the mean scores given by each teacher, which vary from 1.63 to 4 and from 1.63 to 3.88 in the case of teachers with bachelor's and master's degrees, respectively. These scores show that there is homogeneity in the data.

7.5 Discussion

The current chapter studies the teachers' mathematics teaching self-efficacy (PMTE) and mathematics teaching outcome expectations (MTOE) of primary education and secondary school teachers. Specifically, this chapter studies if the level of teaching and the level of teacher training are factors that influence teaching self-efficacy and outcome expectations of primary and secondary education teachers. To meet the objective of this chapter, two research questions were posed, one for each factor.

Answering the first question, this study indicates that the teaching level factor influences the teaching self-efficacy. Particularly, this study determines that secondary school teachers get a significantly higher mean score than primary education teachers. These results are in accord with previous studies ([82], [143]) with the fact that the teaching level factor represents the contextual variable that influences the teaching self-efficacy. Besides, the results show that in the outcome expectation primary education teachers get a significantly higher mean score than secondary school teachers.

Considering the results of the teaching level factor, we believe that the teaching self-efficacy of secondary school teachers is greater than that of primary education school teachers because secondary school teachers are specialists in the mathematics subject. These results may be due to the fact that secondary school teachers are mathematics graduates or a related grade and dominate the subject. However, primary education teachers are generalists and work with several subjects at once, which implies that they may have higher teaching self-efficacy in another area of teaching. Moreover, in the outcome expectation, one reason that secondary school teachers may perform at a lower mean than primary education teachers may be because a significant percentage of secondary school teachers do not have a bachelor's/postgraduate degree in education, and thus do not have specific training in didactics. This fact implies that they do not have scientific knowledge such as: group techniques, strategies to maintain discipline in the classroom, techniques to carry out evaluation, etc. These variables can diminish the teaching self-efficacy and outcome expectation according to ([64], [67]).

Finally, in the second question, this chapter shows that the training level factor influences the teaching self-efficacy. Concretely, teachers with a master's degree obtain significantly higher mean scores than teachers with a bachelor's degree. Obtained results coincide with [141], who indicated that the level of academic training of teachers is a determining factor in their self-efficacy. Besides, no evidence was found that the level of teacher training influences the outcome expectation.

Considering these results, it can be concluded that teachers without specific training in didactics should receive didactics training courses before starting their work as teachers. Another important implication is that the mathematical content knowledge of primary school teachers should be strengthened.

Chapter 8

A comparison between pre-service teachers' mathematics teaching self-efficacy at Rovira and Virgili University and at Azuay University

This chapter aims at comparing pre-service teachers' mathematics teaching self-efficacy in two different contexts. Concretely, students in the fourth year of the Basic General Education Degree at the Azuay University and the fourth year of the Primary Education Degree at the Rovira and Virgili University are compared. Self-efficacy is measured just before completing the Basic General Educational and the Bachelor's Degree in Primary Education, respectively.

8.1 Introduction

Dilekli and Tezcib [144] indicated that teachers' self-efficacy belief for teaching thinking changes depending on teachers' country, gender, teaching field and professional seniority. Except the mutual effect of gender and professional seniority, mutual effect of country and gender, gender and teaching field were found significant. Teachers' self-efficacy belief for teaching thinking were found high in all countries. Besides, Zamora [145] and Rosario [146] highlight the importance of the teachers' mathematics teaching self-efficacy in these students as a predictor of mathematical academic performance, and point out that the variables of sex, scholarship holding, educational level of students Parents, career choice and career level, as well as other motivational, socio-educational and school context factors strongly influence mathematical academic performance, albeit indirectly.

This chapter compares the self-efficacy of teaching mathematics of two different groups: students of at Azuay University, Cuenca, in Ecuador, and students of at Rovira and Virgili University, Tarragona, in Spain. This study is carried out in different contexts (countries).

8.2 Research Question

To meet the objective, a research question is defined:

Do the two studied groups present significant differences in their teachers' self -efficacy? It is studied how to increase the self-efficacy of the teaching of mathematics, considering the strategies used in the Mathematics and didactics course and / or Teaching and Learning Mathematics course.

8.3 Methodology

In this chapter, descriptive and inferential statistics are used for the analysis of the information, once the necessary conditions for its application are met. The Personal Mathematics Teaching Efficacy subscale of the Mathematics Teaching Efficacy Belief Instrument (MTEBI) [123] is used to measure the mathematics teaching self-efficacy of pre-service teachers. The test of mathematics teaching self-efficacy was sent to students at the Rovira and Virgili University (group 1) using a Google form at the end of the 2019-2020 academic period. Further, students at the Azuay University (group 2) was administered the test in person at the beginning of the 2019-2020 academic period. They had approximately 15 minutes to answer the questions.

8.3.1 Participants

The sample used in this research is an intentional sample. Specifically, there are 26 (96%) students at the University of Azuay in Ecuador out of a total of 27 enrolled students. This group is denoted as group 1. In addition, there are 31 (88%) students from the Rovira i Virgili University of Spain out of a total of 35 enrolled students of the Terres of l'Ebre campus. This group is denoted as group 2. In the first case, they are students of the General Basic Education Degree who in the future will teach basic subjects (mathematics, natural sciences, social studies and language and literature) to boys and girls and young people between 6 and 15 years old. In the second case, they are students of the Primary Education Degree who will teach the basic subjects (mathematics, Catalan language and literature, Spanish language and literature, social and cultural environment, and natural environment) to boys and girls between 6 and 12 years.

The details of the Basic General Education Degree and the content of mathematics courses at Azuay University is detailed below. Recall that the details of the Primary Education Bachelor Degree at Rovira and Virgili University were presented in section 1.2.

Azuay University

The Basic General Education Degree consists of a four and a half year program, composed of 9 semesters. The degree includes four mandatory courses of Mathematics and Didactics, which begin in the first and end in the fourth semesters, the profile of the future professional seeks to respond to the entire the sublevel of General Basic Education, proposed by the Ministry of Education of the Ecuador, that is: elementary, medium and superior. Table 8.1 shows the distribution of the four courses of the Mathematics and Didactics throughout the Bachelor's Degree.

Mathematics I and Didactics covers the contents of the second, third and fourth years of Basic General Education, considering the details of the 2010

Table 8.1: *Distribution of Mathematics and Didactics (MD) at the General Primary Education Bachelor Degree (semester is denoted by S)*

	1st		2nd		3rd		4th		5th
	S1	S2	S3	S4	S5	S6	S7	S8	S9
MD	MD1	MD2	MD3	MD4	ST1	ST2	ST3	ST4	
Durationin weeks	16	16	16	16	6	6	6	6	
Durationin hours	80	80	80	80	120	120	120	120	

Curricular Update and Strengthening Document. This document aims at achieving the the effective development of skills with performance criteria in boys and girls, facilitating good knowledge, management and application of the curricular program of these sub-levels to the student. Besides, it guides the teacher's orientation safely and increase awareness of the importance of mathematics in the integral development of students. The course favors reasoning, methodologies and logical sequences that allow the student to obtain the bases for understanding Mathematics in general.

Mathematics II and Didactics develops the conceptual elements that serve as a basic orientation to carry out the teaching work in learning the mathematical blocks for students in the fifth and sixth year of Basic General Education. Through this course, the student could conveniently handle the programs proposed by the national educational authority for the aforementioned sub-levels, while discovering new ways to stimulate logical thinking through the topics covered and the proposed methodology. The course includes the last basic topics that must be assimilated by students to cover a good part of the demands of the current quality standards requested by the Ministry for the third level. Also, it contributed to the development of logical thinking pursued by the entire curriculum proposal.

Mathematics III and Didactics covers the contents for seventh and eighth of Basic Education, providing the student with a good knowledge, management and application of the curricular program of these levels, to guide the teaching orientation safely and increase awareness of the importance of mathematics in the integral development of students. The subject strengthens reasoning, methodologies and logical sequences, promotes creativity, abstraction, order, perseverance, integrates values and transversal knowledge allowing the student to obtain the bases for understanding Mathematics related to university education.

Mathematics IV and Didactics studies the details, topics and contents for the ninth and tenth year of Basic General Education with appropriate methodologies. It articulates the course of Mathematics I, Mathematics II, Mathematics III and Didactics, because it complements the basic concepts and methodologies addressed in the didactics of mathematics so that students can apply them in the different pre-professional practices of Basic General Education.

In addition to the MD courses, the students have 480 hours teaching periods along the Bachelor (Table 8.1). During these student teaching periods (TS1, TS2, TS3 and TS4), the pre-service teachers stay at a school under a supervised teaching practice. In the first period of student teaching, they have to carry out pre-professional practices and link with the Community of First grade of Basic General Education. In the second period of student teaching, they have to carry out pre-professional practices and link with the Community of Second grade of Basic General Education. In the third period of student teaching, they have to carry out pre-professional practices and link with the Community from Third to Seventh grade of Basic General Education. Finally, they have to carry out pre-professional practices and link with the Community from eighth to tenth grade of Basic General Education.

8.3.2 Instrument

To study the self-efficacy of mathematics teaching, one of the subscales of the MTEBI was chosen: the Personal Mathematics Teaching Efficacy subscale (PMTE).

8.3.3 Data analysis

To determine the reliability of the results obtained, the Cronbach's alpha test is applied [109]. The Cronbach's alpha coefficient in group 1 is $\alpha = 0.78$ and for group 2 $\alpha = 0.72$. According to the rules provided by [110], these alpha values are acceptable. Additionally, the normal distribution of the population in each proposed group [132] and homoscedasticity are studied using the Fligner-Killeen test. Depending on the data conditions, the Kruskal-Wallis test (non-parametric) or $t - Student$ (parametric data) will be used to verify significant differences between the groups.

8.4 Results

Let us remember that the pre-service teachers' mathematics teaching self-efficacy was measured at the end of the academic year, considering the fourth year at the Basic Education at Azuay University and Primary Education at Rovira and Virgili University in Ecuador and Spain, respectively.

8.4.1 Self-efficacy of the teaching of mathematics

The scores given by fourth-year students to the items of mathematics teaching self-efficacy are studied. First, Table 8.2 shows the mean and standard deviation of the scores corresponding to the fourth-year Primary Education students of the two groups. Also, the ranking of student responses based on the mean score is included in the table. In inverse questions, inverted scores are displayed.

Table 8.2: *Mean and standard deviation of the scores that participants give to the self-efficacy items*

No	Question	Group 1			Group 2		
		M	SD	rank	M	SD	rank
1	I will continually find better ways to teach mathematics.	3.38	0.75	5	3.68	0.47	5
2(R)	Even if I try very hard, I will not teach mathematics as well as I will most subjects.	2.88	0.59	11	3.32	0.91	
3	I know how to teach mathematics concepts effectively.	3.00	0.48	10	2.71	0.69	13
4(R)	I will not be very effective in monitoring mathematics activities.	2.85	0.67	12	3.71	0.80	4
5(R)	I will generally teach mathematics ineffectively.	3.42	0.57	4	3.81	0.87	2
6	I understand mathematics concepts well enough to be effective in teaching elementary mathematics.	3.08	0.62	8	3.26	0.74	10
7(R)	I will find it difficult to use manipulatives to explain to students why mathematics works.	3.46	0.64	3	3.77	0.62	3
8	I will typically be able to answer students' questions.	3.15	0.54	7	3.35	0.60	8
9(R)	I wonder if I will have the necessary skills to teach mathematics.	2.12	0.58	13	2.84	0.94	12
10(R)	Given a choice, I will not invite the principal to evaluate my mathematics teaching.	3.04	0.66	9	2.84	0.94	12
11(R)	When a student has difficulty understanding a mathematics concept, I will usually be at a loss as to how to help the student understand it better.	3.23	0.58	6	3.68	0.68	6
12	WWhen teaching mathematics, I will usually welcome student questions.	3.77	0.42	1	3.97	0.44	1
13(R)	I do not know what to do to turn students on to mathematics.	3.50	0.51	2	3.39	0.85	7

Note that the self-efficacy scores given by the students in group 1 are generally lower than those given by the students in group 2. In the answers in group 1, there are 7 questions with a score lower than 3.20 (questions 2, 3 , 4, 6, 8, 9 and 10). In the case of group 2, there are 3 questions with scores lower than 3.20 (questions 9 and 10).

The highest mean in group 1 and group 2 of the fourth year is in question 12, with values of the arithmetic mean of 3.77 and 3.97, respectively. On the contrary, in group 1, the lowest value of the mean was obtained in question 9 (2.12). On the other hand, in group 2, the lowest value is given in question 3 (2.71).

Figure 8.1 shows the values of the mean obtained in each of the self-efficacy questions by the students of the two groups. For inverse items, inverted scores are provided.

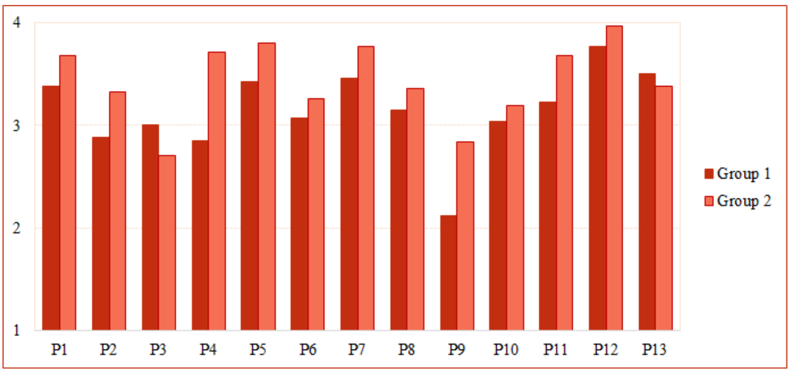


Figure 8.1: *Mean of the scores corresponding to the self-efficacy items of the two groups.*

In Figure 8.1, it can be seen that the ranges of the questions are different in most of the questions. It is noteworthy the difference in some of the questions between the two groups. Note, for example, the score in question 9: group 1 $(\bar{x} = 2.12)$, group 2 $(\bar{x} = 2.84)$. Students of group 2 are clearly more positive about having the necessary skills to teach mathematics. Also, in question 7, the students of group 2 have greater confidence in the use of manipulative to teach mathematics. Another example would be question 4, in which students in group 2 are more effective at monitoring math activities. However, it should be noted that only in question 3 and 13 does group 1 have a higher average than group 2. According the the results, group 1 think that they will teach mathematical concepts more effectively than group 2. Also, group 1 think that they will know what to do to turn students on to mathematics.

Additionally, a differential analysis is performed on each of the questions to find out if the two groups answer the specific questions differently. Compliance with the normality hypothesis conditions is verified in all pairs of questions (Shapiro-Wilk) and homoscedasticity (Bartlett) ($p - value > 0.05$). According to the data conditions, the Student t test is applied. Applying the t-Student, significant differences are found between the arithmetic means in 7 of the 13 questions. Concretely, in the following questions: 2, 3, 4, 5, 7, 9 and 11. In 6 of the 7 questions, group 2 obtained higher scores than group 1.

Figure 8.2 shows the mean scores of the self-efficacy of the participants corresponding to group 1 and group 2. To study the distribution of the scores, the total score of the self-efficacy questions obtained by each participant is averaged (the total score it is averaged by the number of questions scored by the participant).

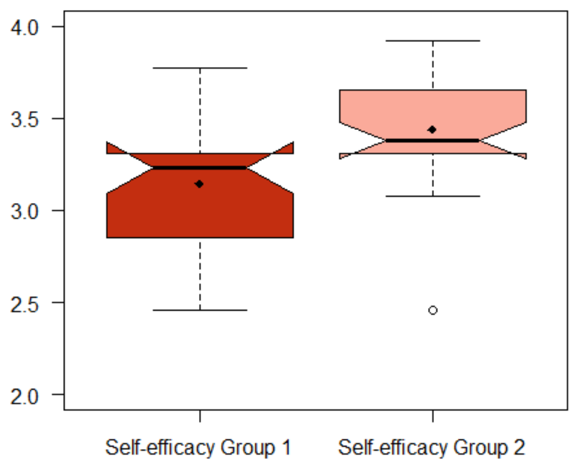


Figure 8.2: *Individual mean scores given to pre-service teachers, teachers mean scores given to self-efficacy items of the two groups.*

Figure 8.2 shows that the range of values of pre-service teachers in group 1 is clearly wider than that of group 2. In particular, the minimum value of group 1 is the smallest (2.46). In the case of group 2, the minimum value is 3.08 and an outlier of 2.46 can be observed. Note that in group 1, 50% of the participant obtained their lowest values between 2.46 and 3.23; the other 50% obtain higher values of the mean between 3.23 and 3.77. Moreover, in group 2, 50% of participants obtained their lowest scores between 3.08 and 3.38; the other 50% obtain higher values of the mean between 3.38 and 3.92.

Additionally, the overall performance of the students in each group is compared. First, the fulfillment of the normality hypothesis conditions is verified in the two groups (Shapiro-Wilk) and homoscedasticity (Bartlett) $(p - value > 0.05)$. Next, the t-Student test was performed to see if the difference between the means obtained in each group is statistically significant. The t-Student test gives the following value $(p - value = 0.00078)$. Therefore, the difference between the means is statistically significant, the participants in group 2 have a higher mean than those in group 1.

Table 8.3 shows the mean and standard deviation of the scores obtained in the mathematics teaching self-efficacy for the two groups.

Table 8.3: *Self-efficacy (mean and standard deviation)*

	Group 1	Group 2
Mean	3.15	3.44
SD	0.31	0.30

8.5 Discussion

To answer the research question, the means of each question of the mathematics teaching self-efficacy were studied for each of the two groups of participants. Our findings indicate that students in the Primary Education grade (group 2) obtain higher scores than students in the Basic Education grade (group 1). Only in question Q3 (I know how I will teach mathematical concepts effectively) and Q13 (I know what to do to turn students on to mathematics), the students in group 1 have a higher mean.

Also, a statistical analysis of the items was performed to see if the two groups answer specific questions differently. It is noteworthy that there are 7 questions to which students give statistically significant different scores (only in Q3 the relationship is the other way around, group 1 highest score). We believe that this occurs due to the difference in the mathematical content of the Mathematics and Didactics course (group 1) and the Teaching and Learning of Mathematics course (group 2). Likewise, we think that the difference in the number of hours of the teaching practice periods influences the mathematics teaching self-efficacy, in this case, group 2 has 240 hours more practice than group 1, in the same way group 2 has higher self-efficacy than group 1. Also, they are finishing their degree and the students in group 1 have not received math training for a long time (2 years). Therefore, they may feel insecure about being in a class and have a negative effect on self-efficacy. Therefore, it is recommended that in group 1 a change be made in its curriculum, the Mathematics 1 and Didactics course begin in S4, as is done in the Teaching and Learning Mathematics course, in addition to adding more hours of teaching practice.

Additionally, the results obtained show that there are statistically significant differences between the values of the means of the mathematics teaching self-efficacy of the fourth-year students of the two universities. These results are in accord with those presented in [144], where the authors show that the self-efficacy of teachers changes depending on the country. The results of the self-efficacy of mathematics teaching of students in group 1 is relatively low compared to students in group 2.

Importantly, although pre-service teachers arrive at degree with established mathematical beliefs, researchers have shown that pre-service teachers programs can influence the self-efficacy of future teachers ([69], [30]). Considering this aspect, we propose to reinforce the contents of the Mathematics and Didactics course and include more practical sessions (group 1).

Another possible solution for group 1 could be to promote the content knowledge mathematics of pre-service teachers by including an introduction to basic mathematics in the first year of the degree. Considering that, [74] found a moderate positive relationship between content knowledge and teacher self-efficacy. Also, note that the students in group 1 do not have mathematics in the last two years of the degree, it would be pertinent to analyze the curriculum and the possibility of assigning hours of study of the teaching of mathematics in the last two years of the degree.

Finally, we believe that it would be pertinent that the course of Mathematics and Didactics (group 1) include the how to use manipulatives and interactive applications to teach mathematics in Basic Education. In group 2, in the TLM2 course, the pre-service teachers learn how to use manipulatives and interactive applications to teach Numeracy at Primary school. In addition, it would be pertinent to include manipulable materials in the course of Mathematics and Didactics for the teaching of Geometry in Primary, as in TLM3, at the URV. Besides, studying question 3 (group 1 is more positive when answering: I know how I will teach mathematical concepts effectively), so it would be necessary to include more theoretical topics with reference to the teaching of mathematics in the Teaching and Learning Mathematics course (group 2).

Chapter 9

Conclusions

In this chapter, conclusions and future lines of research are summarized.

9.1 Summary

In this Thesis, the main goal was to study both the mathematical content knowledge of the students of the Primary Education Bachelor Degree at the Rovira and Virgili University and also their teachers' efficacy beliefs for teaching mathematics. Furthermore, in-service teachers' efficacy beliefs for teaching mathematics is also studied. The idea is to study which contents and procedures are more difficult for student teachers. In addition, the pre-service and in-service teachers' mathematics teaching self-efficacy is studied. Specifically, the pre-service teachers' self-efficacy for teaching mathematics and its evolution throughout the Bachelor Degree in Primary Education at a University Rovira i Virgili is analysed. Furthermore, this Thesis compares the mathematics teaching self-efficacy and outcome expectancy of pre-service and in-service Primary Education teachers. Besides, this Thesis studies the relation between mathematics teaching efficacy beliefs of pre-service teachers, their attitude towards mathematics and their mathematics academic achievement. Additionally, this Thesis studies the influence of the factors teaching experience, level of education and level of teaching on mathematics teaching efficacy beliefs. Finally, this Thesis compared the mathematics teaching self-efficacy of students in the fourth year of the General Basic Education (Azuy University) and Primary Education Degree (Rovira and Virgili University. The main contributions and conclusions of this Thesis are summarized in the following sections. Furthermore, future lines of research are also detailed.

9.2 Knowledge of Numbers and Geometry

The objective of the chapter 3 was to study the initial knowledge of the first year students of the Primary Education Degree and, above all, to detect possible deficiencies and difficulties that they presented in the field of mathematics. Errors and difficulties presented by the students were analysed in detail.

The results of the chapter 3 indicated that the questions with the highest percentage of unanswered questions, those with a mean of less than 5 and those with a high percentage of error are in the cognitive application and reasoning domains. Students better understand the questions corresponding to the cognitive domain of knowledge. In addition, a group of errors is identified that are due to working exclusively on numbers digit by digit and not considering them globally. In most cases, the errors demonstrate a lack of knowledge of basic properties of the decimal system. In others, it is evident that some algorithms, like the one of the division algorithm, are forgotten by lack of their use. Multiplication tables are also forgotten.

Another important result highlighted by this chapter is a lack of understanding in some specific areas, in the domain of number content: reading and interpreting data from bar charts and identifying the pattern that the chart follows; and in the areas of ratio, proportion and percentage. In geometry, the topic of geometric measurement, the weakness students have is in the application of formulas of area, surface, and volume.

In summary, first-year students in the primary grade who have not studied mathematics in the last two years score lower than those who have studied in the last two years. Also, it is important to note that in the analysis of the cognitive domain, knowing, applying and reasoning. Students to teachers performed better in the cognitive facet of knowledge and find it difficult to answer the questions of the cognitive aspects of applying and reasoning.

Finally, some of the results obtained are a warning indicator that shows the need to improve the reasoning and understanding of the decimal numbering system during the primary education stage. This work could be used to propose teacher training courses that insist on the importance of working on reasoning and reinforcing the understanding of the decimal numbering system during the Primary Education stage.

The content taught in the teaching-learning subjects of Primary Education Mathematics should be reinforced, taking into consideration the deficiencies that students present in geometry subjects. However, this study provides results that indicate that we should strengthen the cognitive domain of reasoning.

Future work

In future research, the test will be reapplied when students are in a higher grade, to verify whether their knowledge of mathematical content has had any variation once they have been trained in mathematics. In another future investigation, the similar test should be applied to students in all primary school grade courses, in order to verify whether the difficulties and errors that students make decrease and whether their mathematical knowledge improves over the course of the courses.

9.3 Pre-service teachers' belief about the efficacy of their mathematics teaching

Chapter 4 aimed at analysing the pre-service teachers' mathematics teaching self-efficacy and its evolution throughout the Bachelor Degree in Primary Education. To carry out the study, the pre-teachers of each year of the Bachelor Degree in Primary Education were invited to answer the MTEBI at the end of the 2016-2017 academic course. The MTEBI is comprised of two subscales, namely, Personal Mathematics Teaching Efficacy (PMTE) and Mathematics Teaching Outcome Expectancy (MTOE).

The obtained results evidence the clear influence of the Bachelor Degree Curriculum in the pre-service teachers' personal mathematics teaching efficacy (PMTE). Specifically, the difference between the mean PMTE value in the 1st and the 4th years was statistically significant. In the case of the MTOE, the obtained values did not differ significantly among the different years of the Bachelor.

Chapter 4 provide interesting insights for teacher educators. Results obtained from the particular context and the Bachelor Degree studied in this chapter can be adapted and extended to other contexts and other Bachelor Degrees in Primary Education. The teaching and learning mathematics (TLM) courses included in the current Degree can provide ideas to redesign TLM courses in other Universities in order to reinforce the primary pre-service teachers' belief about the efficacy of their mathematics teaching.

Taking into account the PMTE values obtained at the end of the 2nd year of the Bachelor Degree, a clear implication of this chapter would be to redesign the TLM1 course by providing experiences to the students that allow them to improve their self-efficacy. One idea could be to include some mathematical didactic content and to apply it to school in a short-term teaching practice period.

Regarding the obtained MTOE values, it is difficult to improve them, when the Bachelor Degree program promotes a student-centered learning approach. However, one possibility to try to change it could be to redesign the 4th-year of the Bachelor. Particularly, pre-service teachers could take the courses that conform the 4th year of the Bachelor (including TLM3) during the first semester and assist to the student teaching period during the second semester. Hence, they could use the workshops presented in TLM3 to teach Geometry at school. This kind of real intervention at school might support the pre-service teachers to improve their outcome expectancy beliefs.

Future work

Future lines of research include, first of all, to collect data considering the same students along the Bachelor Primary Degree in order to study their evolution along the different academic courses. Further, it would be desirable to analyse the relation between mathematics pre-service teachers' background and mathematics self-efficacy. Finally, we would also like to analyse the relation among the pre-service teachers' self- efficacy and their achievements in the TLM courses of the Primary Education Bachelor Degree.

9.4 Comparing mathematics teaching eelf-efficacy and outcome expectancy of pre-service and in-service teachers

Chapter 5 compares the mathematics teaching self-efficacy and outcome expectancy of three groups of participants: pre-service teachers, novice teachers and experienced teachers. For this purpose, the Mathematics Teaching Efficacy Belief Instrument (MTEBI), which measures the mathematics teaching self-efficacy (PMTE), and the mathematics teaching outcome expectancy (MTOE) was used.

In the PMTE subscale, the obtained PMTE values of novice teachers are significantly higher than those of pre-service teachers and experienced teachers have significantly higher PMTE values than novice teachers. Furthermore, this study shows that 6 of the 13 items present significant mean values when comparing pre-service teachers and novice teachers. In the case of novice and experienced teachers, that occurs with only 2 items of the PMTE subscale. The average means of the other 5 items do not present significant differences between consecutive groups of participants. Therefore, the difference is more significant between pre-service and novice teachers than between the two groups of in-service teachers.

In the case of MTOE subscale, results manifest that experienced teachers give values significantly higher than those of novice teachers and pre-service teachers give significantly higher values than novice teachers. These later results are attributed to the optimistic scores that pre-service teachers give to the items of the MTOE subscale, which in many cases may be due to lack of experience in a real school classroom. Additionally, results evidence that the average scores of 7 items of the MTOE subscale do not present significant differences between consecutive groups. Only 1 item is given significant different scores between consecutive groups of participants.

As a conclusion, chapter 5 shows that the teaching experience strengthen the mathematics teaching efficacy beliefs. Specifically, obtained values in the PMTE subscale are smallest in the case of pre-service teachers and highest in the case of experienced teachers. In the case of MTOE, the novice teachers present the smallest values, while the other groups present similar MTOE beliefs. It can be concluded that in the case of in-service teachers, the MTOE beliefs are enforced with mathematics teaching experience.

The findings of this study could be useful for pre-service and in-service teacher educators. Concretely, obtained results suggest that specific formation about managing a school classroom would help to reduce the negative MTOE beliefs teachers will possibly have during their first in-service teaching year. In addition, longer teaching study periods would help pre-teachers to know better the reality of a classroom. These two ideas could give a more realistic perception of being a teacher in school. In the case of novice teachers, the figure of apprentice, as mentioned above, could improve their MTOE beliefs.

Future work

A future line of research would be to put in practice the idea proposed in this work in order to foster the MTOE of novice teachers. Specifically, a pilot project consisting of guiding novice teachers in their first year of school teaching could be started. Their MTOE could be measured at the beginning and at the end of the academic school year. Furthermore, the relation between the PMTE and the mathematics academic achievements could be analyzed.

9.5 Mathematics teaching efficacy belief and attitude toward mathematics: two key factors for pre-service teachers' mathematics academic achievement

The purpose of the chapter 6 was to study the relationship between mathematics teachers' efficacy beliefs and attitude towards mathematics of primary pre-service teachers and their academic achievement.

Two instruments were selected to study each of the factors: the MTEBI aims at measuring the teachers' efficacy beliefs, while the AMS aims at assessing the attitude towards mathematics. It should be highlighted that each instrument is composed by subscales (two and four, respectively). This chapter studies in detail each of the six subscales, in order to determine the correlation between each of them and academic achievement. Unlike the referenced literature, two factors (beliefs and attitude) and their six subscales were studied together in this chapter.

Results reaffirm that the mathematics teaching efficacy beliefs and the attitude towards mathematics have a significant moderate correlation with academic achievement. Furthermore, the two factors are moderately correlated one to each other. Beside, the multiple linear regression evidences that mathematics teaching efficacy beliefs is more powerful than the attitude towards mathematics to predict mathematics academic achievement.

Additionally, results show that the teaching self-efficacy (PMTE) subscale has the highest correlation with academic achievement of pre-service teachers, followed by the anxiety subscale. The multiple linear regression analysis shows that the PMTE subscale is the most powerful variable to predict academic achievement, followed by pleasure.

This chapter can be very useful for institutions that design pre-teachers training programs. They can realize that it is important to promote the mathematics teaching efficacy beliefs and the attitude towards the mathematics of their students, since both factors contribute significantly to academic achievement. Furthermore, they can obtain valuable information by analyzing in detail the results obtained in each studied subscales. That analysis would allow to detect possible weakness of students. These weaknesses could be tackled by adapting the curriculum of the Degree correspondingly.

Future work

A future line of research would be to collect data on beliefs and attitudes towards mathematics considering all the students of the Primary Education Degree and to study their evolution throughout the different academic courses. The idea would be to analyze the beliefs and attitudes towards mathematics at the end of the mathematics teaching and learning courses conforming the degree, to study the impact that the methodological strategies applied in each of them have on these two factors.

9.6 Studying mathematics teaching efficacy beliefs of primary and secondary in-service teachers

This research was intended to study two factors that may influence the teaching self-efficacy and outcome expectation of teaching mathematics in primary education and secondary school teachers. Specifically, two factors were studied: level of teaching and level of training.

An important result of the chapter 7 is that statistical analysis significant differences have been found between the different groups studied. Specifically, it can be said that the two factors studied are influential variables in the mathematics teaching self-efficacy (PMTE). In the case of the level of teaching factor, secondary school teachers have better levels of teaching self-efficacy than primary education teachers. Finally, at the level of training, teachers with a master's degree gain better self-efficacy than teachers who have a degree level.

Additionally, the influence of the two factors (teaching experience, level of teaching and level of training) on the outcome expectation (MTOE) of primary education and secondary school teachers was analyzed. Specifically, after statistical analysis, significant differences have been found and it can be said that in this study it was determined that the level of teaching factors influence the outcome expectation of primary education and secondary school teachers. In the case of the level of teaching, primary education teachers obtain a better outcome expectation than secondary school teachers.

These results can be useful for authorities and teachers in primary and secondary education institutions to reflect on the impact of the factors analysed. In addition, this research work can provide ideas to reinforce the PMTE and MTOE subscales of primary education and secondary school teachers.

Future work

A future line of research would be to verify if a training in didactics improves the outcome expectation (MTOE) of secondary school teachers. Furthermore, would be to verify if a training in content mathematical improves the teachers' mathematics teaching self-efficacy (PMTE) of Primary Education teachers. In addition, other research would be to collect data on the in-service teachers' self-efficacy for teaching mathematics and the academic achievement of their students. The idea would be to analyze the relationship of the teaching experience factor and teachers' self-efficacy with the academic achievement of its students.

9.7 A comparison between pre-service teachers' mathematics teaching self-efficacy at Rovira and Virgili University and at Azuay University

The goal of this research work was to compare the teachers' mathematics teaching self-efficacy of students in the fourth year of the General Basic Education and Primary Education degree. To meet the objective, the personal mathematics teaching efficacy subscale (teachers' self-efficacy) of the Mathematics Teaching Efficacy Belief Instrument (MTEBI) was used.

The results obtained in chapter 8 show that the values obtained by the students in the Elementary Education Degree (group 2) are significantly higher than the students in Basic Education (group 1). In addition, this study shows that 7 of the 13 items present significantly different mean values when comparing the two groups. Specifically, we think that this value is given by the difference of the topics and the number of training hours of the Mathematics and Didactics course and the Teaching and Learning of Mathematics course. Additionally, we assume that the number of hours of teaching practice affects self-efficacy.

Future work

A future line of investigation, first, would be to modify the course of Didactics of Mathematics (Azuay University) of the degree with the considerations presented in chapter 8. Besides, it would be interesting to collect data on the self-efficacy of mathematics teaching considering students of all the Bachelor's degrees to study their evolution with the change of curriculum.

9.8 Students follow-up proposal

Considering the results of the research presented in this Thesis, this section presents a proposal for adapting the Teaching and Learning Mathematics courses at the URV. The working plan is summarized in Table 9.1.

Table 9.1: *Working plan*

1st		2nd		3rd		4th	
S1	S2	S3	S4	S5	S6	S7	S8
			TLM1	TLM2	ST1	ST2	TLM3
TIMSS 1		Course 0	MTEBI		MTEBI		MTEBI and TIMSS 2

*1st-4th: each of the years of the Primary Education Bachelor Degree. S1-S8: semesters of the Bachelor Degree. TLM1-TLM3: Teaching and Learning Mathematics courses. ST1-ST2: student teaching practice periods.

Students who begin the Bachelor Degree must dominate the school mathematical contents. Therefore, the proposal of this Thesis is to apply a TIMSS 1-type test (denoted as TIMSS 1 in Table 9.1) to first grade students, in order to verify the initial knowledge with which pre-service teachers arrive. The test will have two dimensions, content and cognitive; the content dimension will contain the subject of numbers and geometry. Besides, the cognitive dimension will contain knowledge, application and reasoning. Specifically, a test similar to the one applied in this Thesis to first grade students (S1) should be prepared.

Considering the results of the application of the TIMSS 1 test, the proposal is to design a virtual course in Moodle (course 0) that will have two dimensions, content and cognitive; the content dimension will contain the subject of numbers and geometry. Concretely, it will be applied for those students who obtain less than 5 over 10 in the TIMSS 1 test, in the first academic year (S3). This type of training will help pre-service teachers to strengthen their school math skills, in addition, this training will help develop mathematics teaching efficacy belief.

The proposal is that the errors and difficulties that students present in TIMSS 1 test would have to be analyzed and tackled during Teaching and Learning Mathematics course(TLM1, S4). The study of errors and difficulties that students present can contribute positively to the teaching process.

Moreover, this Thesis aims at exploring the beliefs about teaching mathematics. It is proposed to apply the MTBEI tests at the end of each year (TLM1, TLM2 and ST1). The idea of applying the MTEBI test at the end of each year is to verify the impact of the methodological strategies on the teachers' efficacy beliefs. Besides, these tests will allow to continuously adapt the methodological strategies that are applied in the Teaching and Learning Mathematics courses.

In addition, to verify that the pre-service teachers have acquired the basic knowledge to be able to teach in Primary Education, it is recommended to create another TIMSS 2-type instrument with different items to the TIMSS 1 test. In the same way it is intended to evaluate the knowledge of numbers and geometry at the end of the degree (S8). In particular, this test must have a higher degree of difficulty than the test applied in S1.

The objective of applying these tests is to monitor the students and adapt the activities and content in courses of the Primary Education Bachelor Degree.

Publications

1. **Segarra, J.** & Julià, C. Mathematics teaching self-efficacy and outcome expectancy of pre-service and in-service Primary Education teachers. *Revista Ensino de Ciencias e Matemática, 22*, (6), 89-108, 2020. SJR, Q4, 0.13.

2. **Segarra, J.** & Julià, C. Mathematical knowledge of elementary education student teachers: variable analysis. *Uniciencia, 35* (1), 1-20, 2021. SJR, 0.10.

3. **Segarra, J.** & Julià, C. Comparison of attitude towards mathematics in students of 5th of primary education, 2nd ESO and 3rd of pre-service teachers. *EDETANIA, 58* , 79-104, 2020. LATINDEX.

4. **Segarra, J.** & Julià, C. Knowledge of Numbers and Geometry of the Primary Education Bachelor Degree's students. *Revista Ensino de Ciencias e Matemática*, under review (August 2020). SJR, Q4, 0.13.

5. **Segarra, J.** & Julià, C. Factors that influence about the mathematics teaching efficacy beliefs in-service teachers. *Universitas Psychologica*, under review (March 2020). SJR, Q3, 0.23. JCR, Q4, 0.517.

6. **Segarra, J.**, Julià, C. & Valls, C. Pre-service teachers' belief about the efficacy of their mathematics teaching. *Education and Research*, under review (August 2020). SJR, Q3, 0.31.

7. **Segarra, J.** & Julià, C. Mathematics teaching efficacy belief and attitude toward mathematics: two key factors for pre-service teachers' mathe-

matics academic achievement. *Revista Ensino de Ciencias e Matemática*, under review (December 2020). SJR, Q4, 0.13.

8. **Segarra, J.**, Bueno, A., Barrazueta, J., & Julià, C. (2021). Study of the self-efficacy of the mathematics teachings of the fourth year students of University Azuy and University Rovira and Virgili *Revista de Investigación en Didáctica de la Matemática*, under review (February 2021). SJR Q4, 0.13.

9. **Segarra, J.**, Chocho, X., Cáceres, B., Morán, B., & Julià, C. (2021). Influential variables in the self-efficacy of teaching mathematics in-service teachers. *Uniciencia*, under review (February 2021). SJR, 0.13.

10. **Segarra, J.** A study about the mathematical knowledge of pre-service teachers. *In 5th URV Doctoral Workshop in Computer Science and Mathematics*. 97-103, 2019.

11. **Segarra, J.** Pre-service teachers' efficacy beliefs and attitude towards mathematics. *In 6th URV Doctoral Workshop in Computer Science and Mathematics*. 49-52, 2020.

Bibliography

[1] D. Loewenberg Ball, M. H. Thames, and G. Phelps, "Content knowledge for teaching: What makes it special?" *Journal of teacher education*, vol. 59, no. 5, pp. 389–407, 2008.

[2] B. B. Leapard, "Using think-alouds to examine pre-service elementary teachers' visualization of fractional concepts," *The Scholarship of Teaching and Learning at EMU*, vol. 2, no. 1, p. 7, 2009.

[3] A. K. Morris, J. Hiebert, and S. M. Spitzer, "Mathematical knowledge for teaching in planning and evaluating instruction: What can preservice teachers learn?" *Journal for research in mathematics education*, pp. 491–529, 2009.

[4] A. Gutiérrez Gutiérrez, P. Gómez Guzmán, and L. Rico, "Conocimiento matemático sobre números y operaciones de los estudiantes de magisterio," *Educación XXI: revista de la Facultad de Educación*, vol. 19, no. 1, pp. 135–158, 2016.

[5] L. Bowie, H. Venkat, and M. Askew, "Pre-service primary teachers' mathematical content knowledge: An exploratory study," *African Journal of Research in Mathematics, Science and Technology Education*, vol. 23, no. 3, pp. 286–297, 2019.

[6] J.-W. Son and M. Y. Lee, "Exploring the relationship between preservice teachers' conceptions of problem solving and their problem-solving performances," *International Journal of Science and Mathematics Education*, pp. 1–22, 2020.

[7] L.-C. Wu, L.-l. Chao, P.-Y. Cheng, H.-L. Tuan, and C.-J. Guo, "Elementary teachers' perceptions of their professional teaching competencies: Differences between teachers of math/science majors and non-math/science majors in taiwan," *International Journal of Science and Mathematics Education*, vol. 16, no. 5, pp. 877–890, 2018.

[8] M. Á. Montes, L. C. Contreras, M. del Mar Liñán, M. C. Muñoz-Catalán, N. Climent, and J. Carrillo, "The arithmetic knowledge of prospective teachers. strengths and weaknesses1 conocimiento de aritmética de futuros maestros. debilidades y fortalezas," *Revista de educación*, vol. 367, pp. 36–62, 2015.

[9] D. C. Mapolelo and M. K. Akinsola, "Preparation of mathematics teachers: Lessons from review of literature on teachers' knowledge, beliefs, and teacher education," *American Journal of Educational Research*, vol. 3, no. 4, pp. 505–513, 2015.

[10] H. P. Osana and D. A. Royea, "Obstacles and challenges in preservice teachers' explorations with fractions: A view from a small-scale intervention study." 2011.

[11] J. Ryan and J. Williams, "Mathsmaps for diagnostic assessment with pre-service teachers: Stories of mathematical knowledge," *Research in Mathematics Education*, vol. 9, no. 1, pp. 95–109, 2007.

[12] D. L. Ball, H. C. Hill, and H. Bass, "Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide?" 2005.

[13] S. F. De Frutos and V. R. Santaren, "La influencia del género en el aprendizaje matemático en españa. evidencias desde pisa," *Revista de Sociología de la Educación-RASE*, vol. 13, no. 1, pp. 63–80, 2020.

[14] M. A. F. Rey, "Análisis de procesos de resolución de problemas en preguntas liberadas de timss-2011," *Edma 0-6: Educación Matemática en la Infancia*, vol. 7, no. 2, pp. 38–54, 2018.

[15] B. M. Byrne, “Self-concept and academic achievement: Investigating their importance as discriminators of academic track membership in high school,” *Canadian Journal of Education/Revue canadienne de l’education*, pp. 173–182, 1990.

[16] J. S. Eccles and A. Wigfield, “Motivational beliefs, values, and goals,” *Annual review of psychology*, vol. 53, no. 1, pp. 109–132, 2002.

[17] C. Senko and K. M. Miles, “Pursuing their own learning agenda: How mastery-oriented students jeopardize their class performance,” *Contemporary Educational Psychology*, vol. 33, no. 4, pp. 561–583, 2008.

[18] J. M. Harackiewicz, K. E. Barron, J. M. Tauer, and A. J. Elliot, “Predicting success in college: A longitudinal study of achievement goals and ability measures as predictors of interest and performance from freshman year through graduation.” *Journal of educational psychology*, vol. 94, no. 3, p. 562, 2002.

[19] L. Ma, “Knowing and teaching elementary mathematics: Teachers’ understanding of fundamental mathematics in china and the united states,” 2010.

[20] G. Haciomeroglu, “Prospective secondary teachers’ subject matter knowledge and pedagogical content knowledge of the concept of function,” 2006.

[21] D. Mahler, J. Großschedl, and U. Harms, “Using doubly latent multilevel analysis to elucidate relationships between science teachers’ professional knowledge and students’ performance,” *International Journal of Science Education*, vol. 39, no. 2, pp. 213–237, 2017.

[22] M. Socas, “Dificultades y errores en el aprendizaje de las matemáticas. análisis desde el enfoque lógico semiótico,” 2007.

[23] R. Nortes Martinez-Artero and A. Nortes Checa, “Problem solving, errors and difficulties in the primary school degree,” *RIE-REVISTA DE INVESTIGACION EDUCATIVA*, vol. 34, no. 1, pp. 103–117, 2016.

[24] K. Stacey, S. Helme, V. Steinle, A. Baturo, K. Irwin, and J. Bana, "Preservice teachers' knowledge of difficulties in decimal numeration," *Journal of mathematics teacher education*, vol. 4, no. 3, pp. 205–225, 2001.

[25] B. Ubuz and B. Yayan, "Primary teachers' subject matter knowledge: Decimals," *International Journal of Mathematical Education in Science and Technology*, vol. 41, no. 6, pp. 787–804, 2010.

[26] I. Moses, A. Berry, N. Saab, and W. Admiraal, "Who wants to become a teacher? typology of student-teachers' commitment to teaching," *Journal of Education for tEaching*, vol. 43, no. 4, pp. 444–457, 2017.

[27] B. K. Giles R and B. A, "An investigation of elementary preservice teachers' self-efficacy for teaching mathematics," *Cogent Education*, vol. 3, no. 1, pp. 1–11, 2003.

[28] S. L. Swars, C. Daane, and J. Giesen, "Mathematics anxiety and mathematics teacher efficacy: What is the relationship in elementary preservice teachers?" *School Science and Mathematics*, vol. 106, no. 7, pp. 306–315, 2006.

[29] M. G. Gill, P. T. Ashton, and J. Algina, "Changing preservice teachers' epistemological beliefs about teaching and learning in mathematics: An intervention study," *Contemporary Educational Psychology*, vol. 29, no. 2, pp. 164–185, 2004.

[30] J. L. Wilkins and B. R. Brand, "Change in preservice teachers' beliefs: An evaluation of a mathematics methods course," *School Science and Mathematics*, vol. 104, no. 5, pp. 226–232, 2004.

[31] M. Bursal, "Turkish preservice elementary teachers'self-efficacy beliefs regarding mathematics and science teaching," *International Journal of Science and Mathematics Education*, vol. 8, no. 4, pp. 649–666, 2010.

[32] J. Mulholland, J. P. Dorman, and B. M. Odgers, "Assessment of science teaching efficacy of preservice teachers in an australian university," *Journal of Science Teacher Education*, vol. 15, no. 4, pp. 313–331, 2004.

[33] Y.-L. Chang, "Examining relationships among elementary mathematics teacher efficacy and their students' mathematics self-efficacy and achievement," *Eurasia Journal of Mathematics, Science and Technology Education*, vol. 11, no. 6, pp. 1307–1320, 2015.

[34] H. Sivri and E. Balcı, "Pre-service teachers' classroom management self-efficacy beliefs." *International Online Journal of Educational Sciences*, vol. 7, no. 4, 2015.

[35] R. García Ros, M. C. Fuentes, and B. Fernández, "Teachers' interpersonal self-efficacy: evaluation and predictive capacity of teacher burnout." *Electronic Journal of Research in Educational Psychology*, vol. 13, no. 3, 2015.

[36] J. Boaler, "Setting, social class and survival of the quickest," *British educational research journal*, vol. 23, no. 5, pp. 575–595, 1997.

[37] L. Michaluk, R. Stoiko, G. Stewart, and J. Stewart, "Beliefs and attitudes about science and mathematics in pre-service elementary teachers, stem, and non-stem majors in undergraduate physics courses," *Journal of Science Education and Technology*, vol. 27, no. 2, pp. 99–113, 2018.

[38] V. R. Moody and K. K. DuCloux, "Mathematics teaching efficacy among traditional and nontraditional elementary pre-service teachers." *European Journal of Science and Mathematics Education*, vol. 3, no. 2, pp. 105–114, 2015.

[39] X. Yang, G. Kaiser, J. König, and S. Blömeke, "Relationship between pre-service mathematics teachers' knowledge, beliefs and instructional practices in china." *ZDM Mathematics Education*, vol. 52, pp. 281–294, 2020.

[40] M. Hourigan and A. M. Leavy, "The influence of entry route to teaching on irish pre-service primary teachers' attitudes towards mathematics," *Journal of Further and Higher Education*, vol. 43, no. 7, pp. 869–883, 2019.

[41] X. Ma and N. Kishor, "Assessing the relationship between attitude toward mathematics and achievement in mathematics: A meta-analysis," *Journal for research in mathematics education*, pp. 26–47, 1997.

[42] C. Y. Charalambous, A. Panaoura, and G. Philippou, "Using the history of mathematics to induce changes in preservice teachers' beliefs and attitudes: Insights from evaluating a teacher education program," *Educational Studies in Mathematics*, vol. 71, no. 2, p. 161, 2009.

[43] A. Leavy, M. Hourigan, and C. Carroll, "Exploring the impact of reform mathematics on entry-level pre-service primary teachers attitudes towards mathematics," *International Journal of Science and Mathematics Education*, vol. 15, no. 3, pp. 509–526, 2017.

[44] M. S. Hannula, "Exploring new dimensions of mathematics-related affect: embodied and social theories," *Research in Mathematics Education*, vol. 14, no. 2, pp. 137–161, 2012.

[45] C. Ministry of Education and Sports, *Royal Decree 476/2013, of June 21, which regulates the qualification and training conditions that teachers of private Early Childhood Education and Primary Education centers must have.*, 2013.

[46] L. S. Shulman, "Those who understand: Knowledge growth in teaching," *Educational researcher*, vol. 15, no. 2, pp. 4–14, 1986.

[47] L. Shulman, "Knowledge and teaching: Foundations of the new reform," *Harvard Educational Review*, vol. 57, no. 1, pp. 1–23.

[48] N. Rojas González *et al.*, "University of granada faculty of sciences of the education department of didactic of mathematics knowledge para the teaching and mathematical quality of the instruction of the concept of fraction: case study of a chilean professor," 2010.

[49] A. Castro, N. Gorgorió, and M. Prat, "Conocimiento matemático fundamental en el grado de educación primaria: sistema de numeración decimal y valor posicional (fundamental mathematical knowledge in the

primary education degree: decimal number system and place value)," 2015.

[50] G. Torregrosa Gironés, H. Quesada Vilella, and M. C. Penalva Martínez, "Razonamiento configural como coordinación de procesos de visualización (configural reasoning as coordination of visualization processes)," *Enseñanza de las Ciencias*, vol. 28, no. 3, pp. 0327–340, 2010.

[51] M. L. Martí Selva and R. Puertas Medina, *Comparativa de la eficiencia educativa de Europa y Asia: TIMSS 2015: Comparison of educational efficiency in Europe and Asia: TIMSS 2015.* Ministerio de Educación.

[52] J. Lacasa and J. Rodríguez, "Diversidad de centros, conocimientos matemáticos y actitudes hacia la enseñanza de las matemáticas de los futuros maestros en españa," *TEDS–M Estudio Internacional sobre la formación inicial en Matemáticas de los maestros. IEA. Informe español. Volumen II. Análisis secundario*, pp. 65–97, 2013.

[53] I. Egido and E. López, "Análisis del practicum en los estudios de magisterio en españa a partir de los datos de teds-m," *TEDS-M. Estudio internacional sobre la formación inicial en matemáticas de los maestros. Informe español*, vol. 2, pp. 108–135, 2013.

[54] M. J. Salinas, "Errores sobre el sistema de numeración decimal de estudiantes de magisterio," 2007.

[55] E. B. Turnuklu and S. Yesildere, "The pedagogical content knowledge in mathematics: Pre-service primary mathematics teachers' perspectives in turkey." *Issues in the Undergraduate Mathematics Preparation of School Teachers*, vol. 1, 2007.

[56] S. Livy, T. Muir, and N. Maher, "How do they measure up? primary pre-service teachers' mathematical knowledge of area and perimeter." *Mathematics Teacher Education and Development*, vol. 14, no. 2, pp. 91–112, 2012.

[57] J. F. González Retana and D. Eudave Muñoz, "Conocimiento común del contenido del estudiante para profesor sobre fracciones y decimales," *Educación matemática*, vol. 30, no. 2, pp. 106–139, 2018.

[58] R. N. Martínez-Artero and A. N. Checa, "Conocimientos matemáticos de futuros maestros en resolución de problemas de 6. º de primaria," *Educatio Siglo XXI*, vol. 36, no. 3 Nov-Feb1, pp. 201–230, 2018.

[59] W. Y. Lo, "Examining the mathematical content knowledge of pre-service primary teachers at the highest primary school level in hong kong," *Teacher Development*, pp. 1–18, 2020.

[60] A. I. Barham, "Investigating the development of pre-service teachers' problem-solving strategies via problem-solving mathematics classes." *European Journal of Educational Research*, vol. 9, no. 1, pp. 129–141, 2020.

[61] G. A. Muñoz-Fernández, P. Rodríguez-Gutiérrez, and M. Luque-Vílchez, "La formación inicial del profesorado de educación secundaria en españa: perfil y motivaciones del futuro docente," *Educación XX1*, vol. 22, no. 1, 2019.

[62] A. Bandura, "Self-efficacy: toward a unifying theory of behavioral change." *Psychological review*, vol. 84, no. 2, p. 191, 1977.

[63] ——, "Self-efficacy mechanism in human agency." *American psychologist*, vol. 37, no. 2, p. 122, 1982.

[64] ——, "Social foundations of thought and action," *Englewood Cliffs, NJ*, vol. 1986, 1986.

[65] ——, "Social cognitive theory: An agentic perspective," *Annual review of psychology*, vol. 52, no. 1, pp. 1–26, 2001.

[66] R. D. Goddard, W. K. Hoy, and A. W. Hoy, "Collective efficacy beliefs: Theoretical developments, empirical evidence, and future directions," *Educational researcher*, vol. 33, no. 3, pp. 3–13, 2004.

[67] A. Bandura, *Self-efficacy: The exercise of control*. New York: W.H., 1997.

[68] A. Hassan *et al.*, "Science teaching self-efficacy and outcome expectancy beliefs of secondary school teachers in uae," *International Journal for Research in Education*, no. 32, pp. 1–22, 2012.

[69] S. Swars, L. C. Hart, S. Z. Smith, M. E. Smith, and T. Tolar, "A longitudinal study of elementary pre-service teachers' mathematics beliefs and content knowledge," *School Science and Mathematics*, vol. 107, no. 8, pp. 325–335, 2007.

[70] F. Pajares, "Self-efficacy beliefs in academic settings," *Review of educational research*, vol. 66, no. 4, pp. 543–578, 1996.

[71] S. Perkmen and S. Pamuk, "Social cognitive predictors of pre-service teachers' technology integration performance," *Asia Pacific Education Review*, vol. 12, no. 1, pp. 45–58, 2011.

[72] D. Kaya and H. C. Bozdag, "Resources of mathematics self-efficacy and perception of science self-efficacy as predictors of academic achievement." *European Journal of Contemporary Education*, vol. 18, no. 4, pp. 438–451, 2016.

[73] J. Utley, C. Moseley, and R. Bryant, "Relationship between science and mathematics teaching efficacy of preservice elementary teachers," *School science and Mathematics*, vol. 105, no. 2, pp. 82–87, 2005.

[74] K. J. Newton, J. Leonard, B. R. Evans, and J. A. Eastburn, "Preservice elementary teachers' mathematics content knowledge and teacher efficacy," *School Science and Mathematics*, vol. 112, no. 5, pp. 289–299, 2012.

[75] Ö. Nurlu, "Investigation of teachers' mathematics teaching self-efficacy," *International Electronic Journal of Elementary Education*, vol. 8, no. 1, pp. 21–40, 2015.

[76] M. Bala and J. Adamu, "Influence of self-efficacy and self-esteem on attitude of pre-service teachers towards teaching profession in bauchi state, nigeria," *ATBU Journal of Science, Technology and Education*, vol. 7, no. 4, pp. 224–230, 2019.

[77] M. P. Tunç, E. Çakıroğlu, and S. Bulut, "Exploring self-efficacy beliefs within the context of teaching mathematics with concrete models," *Elementary Education Online*, vol. 19, no. 1, pp. 100–117, 2020.

[78] C.-J. Liu, B. M. Jack, and H.-L. Chiu, "Taiwan elementary teachers' views of science teaching self-efficacy and outcome expectations," *International Journal of Science and Mathematics Education*, vol. 6, no. 1, pp. 19–35, 2008.

[79] D. Holzberger, A. Philipp, and M. Kunter, "How teachers' self-efficacy is related to instructional quality: A longitudinal analysis." *Journal of Educational Psychology*, vol. 105, no. 3, p. 774, 2013.

[80] N. Julaihi, L. Ying, V. Li, and S. A. Bakri, "Confidence level and self-efficacy beliefs of mathematics teachers: Evidence from sarawak," *International Journal of Service Management and Sustainability*, vol. 4, no. 2, pp. 1–23, 2020. [Online]. Available: http://103.8.145.246/index.php/IJSMS/article/view/8145

[81] M. M. Thomson, T. A. Walkowiak, A. N. Whitehead, and E. Huggins, "Mathematics teaching efficacy and developmental trajectories: A mixed-methods investigation of novice k-5 teachers," *Teaching and Teacher Education*, vol. 87, p. 102953, 2020.

[82] G. E. Greenwood, S. F. Olejnik, and F. W. Parkay, "Relationships between four teacher efficacy belief patterns and selected teacher characteristics." *Journal of research & Development in Education*, 1990.

[83] P. Gavora *et al.*, "Measuring the self-efficacy of in-service teachers in slovakia," *Orbis scholae*, vol. 5, no. 2, pp. 74–94, 2011.

[84] M. Takunyaci and M. Takunyaci, "Preschool teachers' mathematics teaching efficacy belief," *Procedia-Social and Behavioral Sciences*, vol. 152, pp. 673–678, 2014.

[85] M. Cocca, A. Cocca, A. Martínez, E. Alvarado, and M. G. Rodríguez Bulnes, "Correlation between self-efficacy perception and

teaching performance: The case of mexican preschool and primary school teachers," *Arab World English Journal (AWEJ) Volume*, vol. 9, 2018.

[86] D. Ngidi and S. Ngidi, "Determination of factors influencing pre-service teachers' sense of self-efficacy," *South African Journal of Higher Education*, vol. 33, no. 5, pp. 98–111, 2019.

[87] L. R. Aiken Jr, "Attitudes toward mathematics," *Review of educational research*, vol. 40, no. 4, pp. 551–596, 1970.

[88] D. C. Neale, "The role of attitudes in learning mathematics," *The Arithmetic Teacher*, vol. 16, no. 8, pp. 631–640, 1969.

[89] G. Joseph, "A study on school factors influencing students' attitude towards learning mathematics in the community secondary schools in tanzania: The case of bukoba municipal council in kagera region," 2013.

[90] M. Y. Mazana, C. S. Montero, and R. O. Casmir, "Assessing students' performance in mathematics in tanzania: The teacher's perspective," *International Electronic Journal of Mathematics Education*, vol. 15, no. 3, p. em0589, 2020.

[91] J. Touron, "The prediction of academic performance: procedures, results and implications," *Revista española de pedagogía*, pp. 473–495, 1985.

[92] Y. N. Reyes Tejada, "Relationship between academic performance, test anxiety, personality traits, self-concept and assertiveness in first-year students of psychology at unmsm," 2003.

[93] B. Hemmings and R. Kay, "Prior achievement, effort, and mathematics attitude as predictors of current achievement," *The Australian Educational Researcher*, vol. 37, no. 2, pp. 41–58, 2010.

[94] M. P. Michelli, "The relationship between attitudes and achievement in mathematics among fifth grade students," 2013.

[95] E. Cakiroglu and j. v. n. p. y. p. Isiksal, M., "Preservice elementary teachers' attitudes and self-efficacy beliefs toward mathematics."

[96] W. H. Cockcroft *et al.*, *Mathematics counts*. HM Stationery Office London, 1982.

[97] E. Auzmendi Escribano, "Las actitudes hacia la matemática-estadística en las enseñanzas media y universitaria (attitudes towards mathematics-statistics in middle and university education)," *Características y medición. Ed mensajero. España*, 1992.

[98] K. M. Hadley and J. Dorward, "The relationship among elementary teachers' mathematics anxiety, mathematics instructional practices, and student mathematics achievement." *Journal of Curriculum & Instruction*, vol. 5, no. 2, 2011.

[99] W. Flores and E. Auzmendi, "Actitudes hacia las matemáticas en la enseñanza universitaria y su relación con las variables género y etnia (attitudes towards mathematics in university education and its relationship with gender and ethnic variables)," *Profesorado, Revista de Currículo y Formación del Profesorado*, vol. 22, no. 3, pp. 231–248, 2018.

[100] R. Nortes Martínez-Artero, A. Nortes Checa *et al.*, "Actitud hacia las matemáticas en futuros docentes de primaria y de secundaria," *Edetania*, no. 44, pp. 47–76, 2013.

[101] A. Fachrudin, S. Widadah, and I. Kusumawati, "Pre-service mathematics teachers' knowledge, beliefs, and attitude toward using pisa-based problem in mathematics education," in *Journal of Physics: Conference Series*, vol. 1200, no. 1. IOP Publishing, 2019, p. 012013.

[102] T. Muir and S. Livy, "What do they know? a comparison of pre-service teachers' and in-service teachers' decimal mathematical content knowledge," *International Journal for Mathematics Teaching and Learning*, pp. 1–15, 2012.

[103] Z. Putra and C. Winsløw, "A framework for a comparative study of pre-service elementary teachers' knowledge of rational numbers," *Educação Matemática Pesquisa: Revista do Programa de Estudos Pós-Graduados em Educação Matemática*, vol. 21, 2019.

[104] A. Wilkinson, "Pre-service teachers' perceptions of theory–the case of compressed knowledge in mathematics," in *Research Proceedings of the 9th British Congress on Mathematics Education*, 2020.

[105] M. Q. Patton, *Qualitative evaluation and research methods*. SAGE Publications, inc, 1990.

[106] D. L. Ball, S. T. Lubienski, and D. S. Mewborn, "Research on teaching mathematics: The unsolved problem of teachers' mathematical knowledge," *Handbook of research on teaching*, vol. 4, pp. 433–456, 2001.

[107] H. F. Kaiser, "An index of factorial simplicity," *Psychometrika*, vol. 39, no. 1, pp. 31–36, 1974.

[108] M. S. Bartlett, "Tests of significance in factor analysis," *British Journal of statistical psychology*, vol. 3, no. 2, pp. 77–85, 1950.

[109] L. J. Cronbach, "Coefficient alpha and the internal structure of tests," *psychometrika*, vol. 16, no. 3, pp. 297–334, 1951.

[110] D. George and M. Mallery, "Using spss for windows step by step: a simple guide and reference," 2003.

[111] *TIMSS 2011. Marcos de la evaluación.* INEE, 2012.

[112] R. Nortes and A. Nortes, "Matemáticas escolares en futuros maestros: un estudio necesario," *Profesorado, Revista de Currículum y Formación del Profesorado*, vol. 20, pp. 368–381, 2016.

[113] N. Maher and T. Muir, "I think they think that zero point something is less than zero: Investigating pre-service teachers' responses to mathematical tasks," in *Australian Association for Research in Education Conference (AARE 2011 Conference)*, 2011, pp. 1–16.

[114] A. Nortes and R. Martínez, "Actitud, aptitud y rendimiento en matemáticas: un estudio en primero de magisterio," *Suma*, vol. 10, pp. 36–40, 1992.

[115] J. R. Gauna, J. García, and J. Sarasua, "Perspectiva de los alumnos de grado de educación primaria sobre las matemáticas y su enseñanza," *Números. Revista de Didáctica de las Matemáticas*, vol. 82, pp. 5–15, 2013.

[116] R. N. Martínez-Artero and A. N. Checa, "Competencia matemática, actitud y ansiedad hacia las matemáticas en futuros maestros," *Revista Electrónica Interuniversitaria de Formación del Profesorado*, vol. 20, no. 3, pp. 145–160, 2017.

[117] M. Bocco and C. Canter, "Errores en geometría: clasificación e incidencia en un curso preuniversitario," *Revista Iberoamericana de educación*, vol. 53, no. 2, p. 1, 2010.

[118] J. Kilpatrick, P. Gómez, and L. Rico, *Educación matemática. Errores y dificultades de los estudiantes. Resolución de problemas. Evaluación. Historia*. una empresa docente, 1998.

[119] L. G. Enochs and I. M. Riggs, "Further development of an elementary science teaching efficacy belief instrument: A preservice elementary scale," *School science and mathematics*, vol. 90, no. 8, pp. 694–706, 1990.

[120] L. L. Liang and G. M. Richardson, "Enhancing prospective teachers' science teaching efficacy beliefs through scaffolded, student-directed inquiry," *Journal of Elementary Science Education*, vol. 21, no. 1, pp. 51–66, 2009.

[121] S. Buyukozturk, "Sosyal bilimler için veri analizi el kitabi: Istatistik, arastirma deseni spss uygulamalari ve yorum," 2013.

[122] M. Ay, "Sınıf öğretmenlerinin matematik öğretimine ilişkin öz yeterlik algıları," Ph.D. dissertation, University of Hacettepe, 2005.

[123] L. G. Enochs, P. L. Smith, and D. Huinker, "Establishing factorial validity of the mathematics teaching efficacy beliefs instrument," *School Science and Mathematics*, vol. 100, no. 4, pp. 194–202, 2000.

[124] L. R. Alken, "Two scales of attitude toward mathematics," *Journal for research in Mathematics Education*, pp. 67–71, 1974.

[125] E. Fennema and J. A. Sherman, "Fennema-sherman mathematics attitudes scales: Instruments designed to measure attitudes toward the learning of mathematics by females and males," *Journal for research in Mathematics Education*, vol. 7, no. 5, pp. 324–326, 1976.

[126] M. Tapia, G. E. Marsh *et al.*, "An instrument to measure mathematics attitudes," *Academic Exchange Quarterly*, vol. 8, no. 2, pp. 16–22, 2004.

[127] J. L. Adelson and D. B. McCoach, "Development and psychometric properties of the math and me survey: Measuring third through sixth graders' attitudes toward mathematics," *Measurement and Evaluation in Counseling and Development*, vol. 44, no. 4, pp. 225–247, 2011.

[128] P. M. Vallejo, *Medición de actitudes en psicología y educación: construcción de escalas y problemas metodológicos*. Univ Pontifica Comillas, 2006, vol. 80.

[129] E. J. D. Morales, "Escala de actitudes hacia la estadística (attitude scale towards statistics)," *Psicothema*, vol. 12, no. Suplemento, pp. 175–178, 2000.

[130] R. F. Cézar and C. A. Pérez, "Actitudes iniciales hacia las matemáticas de los alumnos de grado de magisterio de educación primaria: Estudio de una situación en el eees ((initial attitudes towards the mathematics of elementary school teaching students: Study of a situation in the eees)," *Unión: Revista Iberoamericana de educación matemática*, vol. 23, pp. 107–116, 2010.

[131] R. N. Martínez-Artero and A. Nortes Checa, "¿ tienen ansiedad hacia las matemáticas los futuros matemáticos? (do the future mathematicians have math anxiety?)," 2014.

[132] S. S. Shapiro and M. B. Wilk, "An analysis of variance test for normality (complete samples)," *Biometrika*, vol. 52, no. 3/4, pp. 591–611, 1965.

[133] H. A. Fragoso Cano and D. Pineda Domínguez, "Modelo de gobierno de tecnología de la información para mejorar el desempeño de proyectos de negocio minorista (information technology governance model

to improve the performance of retail business projects)," *Investigación administrativa*, vol. 47, no. 122, pp. 0–0, 2018.

[134] J. J. A. Zavala, J. Isea, and C. Ianni, "Relación entre el desempeño laboral y la calidad de servicio de la industria venezolana de cemento (invecem) (relationship between job performance and quality of service in the venezuelan cement industry)," *Emprende y Transforma*, vol. 1, no. 1, pp. 63–69, 2019.

[135] D. H. Schunk and C. A. Mullen, "Self-efficacy as an engaged learner," in *Handbook of research on student engagement*. Springer, 2012, pp. 219–235.

[136] M. F. Pajares, "Teachers' beliefs and educational research: Cleaning up a messy construct," *Review of educational research*, vol. 62, no. 3, pp. 307–332, 1992.

[137] F. Pajares and M. D. Miller, "Role of self-efficacy and self-concept beliefs in mathematical problem solving: A path analysis." *Journal of educational psychology*, vol. 86, no. 2, p. 193, 1994.

[138] B. J. Zimmerman, "Self-efficacy: An essential motive to learn," *Contemporary educational psychology*, vol. 25, no. 1, pp. 82–91, 2000.

[139] A. Devine, K. Fawcett, D. Szűcs, and A. Dowker, "Gender differences in mathematics anxiety and the relation to mathematics performance while controlling for test anxiety," *Behavioral and brain functions*, vol. 8, no. 1, p. 33, 2012.

[140] J. A. Ross, J. B. Cousins, and T. Gadalla, "Within-teacher predictors of teacher efficacy," *Teaching and teacher education*, vol. 12, no. 4, pp. 385–400, 1996.

[141] W. K. Hoy and A. E. Woolfolk, "Teachers' sense of efficacy and the organizational health of schools," *The elementary school journal*, vol. 93, no. 4, pp. 355–372, 1993.

[142] M. Gómez-Gómez, C. Danglot-Banck, and L. Vega-Franco, "Non parametric statistical tests synopsis. when are they used?" *Revista Mexicana de Pediatría*, vol. 70, no. 2, pp. 91–99, 2003.

[143] L. Navarro, "El análisis de las creencias de autoeficacia: un avance hacia el desarrollo profesional del docente," *Revista de teología y ciencias humanas*, vol. 60, no. 107, pp. 591–612, 2002.

[144] Y. Dilekli and E. Tezci, "A cross-cultural study: Teachers' self-efficacy beliefs for teaching thinking skills," *Thinking Skills and Creativity*, vol. 35, p. 100624, 2020.

[145] J. A. Zamora-Araya, J. D. Cruz-Quesada, and M. S. Amador-Montes, "Autoeficacia y su relación con el rendimiento académico en estudiantes de enseñanza de la matemática," *Innovaciones Educativas*, vol. 22, no. 32, pp. 137–150, 2020.

[146] P. J. S. L. d. F. Rosário, A. Lourenço, O. Paiva, A. Rodrigues, E. Tuero Herrero, and A. Valle Arias, "Predicción del rendimiento en matemáticas: efecto de variables personales, socioeducativas y del contexto escolar," *Psicothema*, 2012.

[147] N. Buchholtz and G. Kaiser, "Improving mathematics teacher education in germany: Empirical results from a longitudinal evaluation of innovative programs," *International Journal of Science and Mathematics Education*, vol. 11, no. 4, pp. 949–977, 2013.

Appendices

Appendix A

Instrument TIMMS

1. Q1. Which of these shows how 36 can be expressed as a product of prime factor?

    (a) 6×6

    (b) 4×9

    (c) $4 \times 3 \times 3$

    (d) $2 \times 2 \times 3 \times 3$

2. Q2. Which fraction is equivalent to 0.125?

    (a) $\frac{125}{100}$

    (b) $\frac{125}{1000}$

    (c) $\frac{125}{10000}$

    (d) $\frac{125}{100000}$

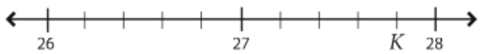
3. Q3. Which shows a correct method for findings $\frac{1}{3} - \frac{1}{4}$?

    (a) $\frac{1-1}{4-3}$

    (b) $\frac{1}{4-3}$

(c) $\frac{3-4}{3\times 4}$
(d) $\frac{4-3}{3\times 4}$

4. Q4. What number K represents on this number line?



(a) 27.4
(b) 27.8
(c) 27.9
(d) 28.2

5. Q5. Which number is equal to $\frac{3}{5}$?

(a) 0.8
(b) 0.6
(c) 0.53
(d) 0.35

6. Q6. $\frac{4}{100} + \frac{3}{1000} =$

(a) 0.043
(b) 0.1043
(c) 0.403
(d) 0.43

7. Q7. The fractions $\frac{4}{14}$ and $\frac{a}{21}$. What is the value of a?

(a) 0.043
(b) 0.1043
(c) 0.403
(d) 0.43

8. Q8. A workman cut off $\frac{1}{5}$ of a pipe. The piece he cut off was 3 meters long. How many meters long was the original pipe?

   (a) 8 m

   (b) 12 m

   (c) 15 m

   (d) 18 m

9. Q9. 42.65 + 5.748 =

   Answer:

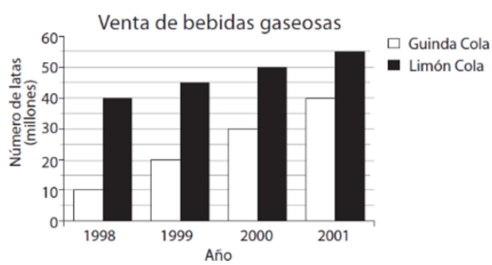
10. Q10. Ann and Jenny divide 560 euros between them. If Jenny gets $\frac{3}{8}$ of money, how many euros will Ann get?

    Answer:

11. Q11. Carla is packing eggs into boxes. Each box holds 6 eggs. She has 94 eggs. What is the smallest number of boxes she needs to pack all the eggs?

    Answer:

12. Q12. The graph shows the sales of two types of soft drink over 4 years. If the sales trends continue for the next 10 years, determine the year in which the sales of Cherry Cola will be the same as the sales of Lemon Cola.



   (a) 2003

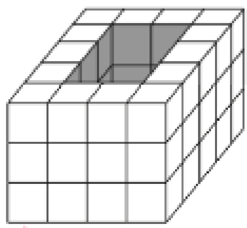
   (b) 2004

(c) 2005
(d) 2006

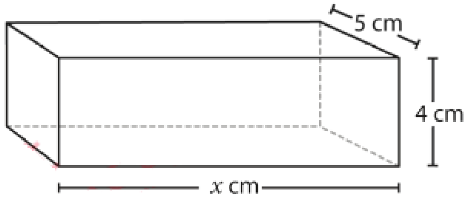
13. Q13. The length of side of each of the small square represents 1 cm. Draw an isosceles triangle with base of 4 cm and a height of 5 cm.

14. Q14. The figure down shows a shape made up cubes that are all the same size. There is a hole all way through the shape. How many cubes would be needed to fill the hole?

    (a) 6
    (b) 12
    (c) 15
    (d) 18



15. Q15. The volume of the rectangular box is 200 cm^3. What is the value of x?



Answer:

16. Q16. A piece of paper in the shape of a rectangle is folded in half as shown in the figure down. It is then cut along the dotted line, and the small piece that is cut is opened. What is the shape of the cutout figure?

(a) an isosceles triangle
(b) two isosceles triangles
(c) a right triangle
(d) an equilateral triangle

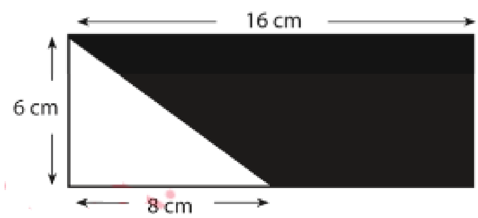
17. Q17. The perimeter of a square is 36 cm. What is the area of this square?

(a) 81 cm^2
(b) 36 cm^2
(c) 24 cm^2
(d) 18 cm^2

18. Q18. The area of a square is 144 cm^2. What is the perimeter of the square?

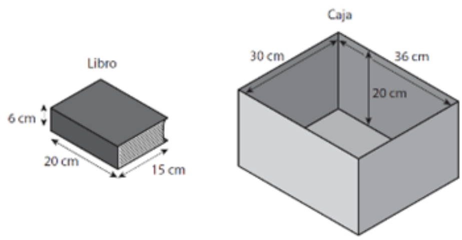
(a) 12 cm
(b) 48 cm
(c) 288 cm
(d) 576 cm

19. Q19. In the figure down, what is the area of the shaded region in cm^2 ?



(a) 24
(b) 44
(c) 48
(d) 72

20. Q20. Ryan is packing books into a rectangular box. All the box are the same size.



What is the largest number of number of books that will fit inside the box?.

Answer:

Appendix B

Instrument MTEBI

Table B.1: *MTEBI (Mathematics Teaching Efficacy Belief Instrument) for pre-service teachers*

	Question
Q1	When a student does better than usual in mathematics, it is often because the teacher exerted a little extra effort.
Q2	I will continually find better ways to teach mathematics.
Q3	Even if I try very hard, I will not teach mathematics as well as I will most subjects.
Q4	When the mathematics grades of students improve, it is often due to their teacher having found a more effective teaching approach.
Q5	I know how to teach mathematics concepts effectively.
Q6	I will not be very effective in monitoring mathematics activities.
Q7	If students are underachieving in mathematics, it is most likely due to ineffective mathematics teaching.
Q8	I will generally teach mathematics ineffectively.
Q9	The inadequacy of a student's mathematics background can be overcome by good teaching.
Q10	When a low-achieving child progresses in mathematics, it is usually due to extra attention given by the teacher.
Q11	I understand mathematics concepts well enough to be effective in teaching elementary mathematics.
Q12	The teacher is generally responsible for the achievement of students in mathematics.
Q13	Students' achievement in mathematics is directly related to their teacher's effectiveness in mathematics teaching.
Q14	If parents comment that their child is showing more interest in mathematics at school, it is probably due to the performance of the child's teacher.
Q15	I will find it difficult to use manipulatives to explain to students why mathematics works.
Q16	I will typically be able to answer students' questions.
Q17	I wonder if I will have the necessary skills to teach mathematics.
Q18	Given a choice, I will not invite the principal to evaluate my mathematics teaching.
Q19	When a student has difficulty understanding a mathematics concept, I will usually be at a loss as to how to help the student understand it better.
Q20	When teaching mathematics, I will usually welcome student questions.
Q21	I do not know what to do to turn students on to mathematics.

Appendix C

Instrument AMS

Table C.1: *AMS (Attitude towards mathematics scale (subscales and items)*

Subscales		Question
Pleasure	Q4	Using mathematics is fun.
	Q9	I enjoy talking to others about mathematics.
	Q14	Mathematics is enjoyable and stimulating to me.
	Q24	If I had the chance, I would enroll in more mathematics courses than are required.
Anxiety	Q2	I am pretty bad at mathematics.
	Q3	Studying or working with mathematics does not scare me at all.
	Q7	Mathematics is one of the subjects that I fear the most.
	Q8	I am confident when I face with a mathematics problem.
	Q12	When I face a mathematics problem, I feel unable to think clearly.
	Q13	I am calm when I face with a mathematics problem.
	Q17	Working with math makes me feel nervous.
	Q18	I do not get upset when I have to work on mathematics problems.
	Q22	Mathematics makes me feel uncomfortable and nervous.
Motivation	Q5	Mathematics is too theoretical to be of any use to me.
	Q10	Mathematics can be useful for those who decide to pursue a "science" degree, but not for the rest of the students.
	Q25	The topics taught in math classes is very uninteresting..
Usefulness	Q1	I consider mathematics to be a very necessary subject in my studies.
	Q6	I want to get a deeper understanding of mathematics.
	Q15	I hope to have little use of mathematics in my professional life.
	Q16	I consider that there are other subjects more important than mathematics for my future profession.
	Q19	I would like to have an occupation in which I had to use mathematics.
	Q21	For my professional future, mathematics is one of the most important subjects I have to study.
Confidence	Q11	Having good mathematics skills will increase my job prospects.
	Q20	It gives me great satisfaction to be able to solve mathematics problems.
	Q23	If I set my mind to it, I think I would master the mathematics well.

Appendix D

Distribution of subject of Learning Mathematics courses (TLM1, TLM2 and TLM3) at the Primary Education Bachelor Degree

Table D.1: *TLM1*

Subject	subtopic
Introduction to mathematical education and numerical language.	School mathematics. Curriculum. Mathematical competence. Numbers and numbering. Natural numbers and discrete quantity. Numbering systems. Numerical patterns. The numbers of the measure.
Learning the addition and subtraction calculation.	The beginnings of numbering and calculation with Primary Education students. Didactic resources for learning. Representations and strategies. Mental calculation and calculation algorithms.
Learning the calculation of multiplication	Notions of product and division. Calculation strategies. Representations and resources for learning. The measurement of calculation skills.
Rational numbers in Primary Education	Notions of fraction and rational number. Ability to calculate with fractions. Representations and resources. Use of fraction in probability contexts.
Elemental geometry	The classification of polygons. The circle. Geometric puzzles. Geometric representations. Geometric constructions with GeoGebra. Calculation of perimeters and surfaces. Practical problems.

Table D.2: *TLM2*

Subject	subtopic
The current curriculum. Content blocks and processes. Mathematical skills to be developed.	The curriculum. Concepts and nomenclature to use. The sequencing of learning. The initial training work in this subject: Mathematical competence and didactic competence of the teachers. Concepts and basic notions of the didactics of the numerical language in the stage. Introduction to the design of classroom activities.
Positional numbering and addition and subtraction operations	Numerical meaning and understanding of positional numbering. The acquisition of additive structures: Sense of operations. Mental calculation skills. Number-to-number calculations.
Product and division operations in Primary Education.	Senses of operations. Mental calculation and written calculation. Algorithms. Multiple and divisor relations. Problem solving in Primary Education. Application problems and strategy problems.
The numbers of the measure: Fractions and decimal number	The measurement of linear magnitudes. Uses of fractions: the part / all relationship; the partitions of the units of measure, points on the line. Concept and use of decimal numbers. Operations with decimals. The division with decimal quotient. The quotient as a mixed number. Proportional reasoning Notion and calculation of percentage.
The didactics of the Statistics	Double entry tables and Venn diagrams. Frequency graphs. Bar chart. Pie chart. Other graphics. Idea of statistical distribution. Reading values. Arithmetic median. Mode. Sure, possible, impossible success. Probability measurement.

Table D.3: *TLM3*

Subject	subtopic
Curriculum and standards of measurement and geometry.	The measurement section and the space section in the official primary education school curriculum. Documentation and collection of resources.
The didactics of the measurement of magnitudes throughout Primary Education.	Measurable attributes. Idea of magnitude. Units, systems and processes of measurement. International system of measurement and relationship with the value of position. Measuring instruments and estimates. Problems.
Analysis of the characteristics and properties of geometric figures.	Elements and properties of polygons. Elementary classifications. The circle and the circumference. Round bodies and polyhedra. Composition and decomposition of figures. Problems.
Location and description of spatial relationships.	Position and orientation in space. Grid paths. Coordinate reading. Plans, maps and scales. Problems.
Identification and application of geometric transformations.	Movements in the plane (symmetries, turns, translations). Angles. Polyhedral truncations. Flat developments of polyhedra, cylinders and cones. Problems.
Use of visualization and geometric models to solve problems.	Notions of perimeter and area. Geometric representation of the product and surface of the square and rectangle. Construction of parallelepipeds with polycubes and introduction to the notion of volume. Problems.