Universitat Autònoma de Barcelona

Essays on Political Economy

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To my mother, my father and my brother.

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Contents

Chapter 1

An Introduction to the Evolution of Political Issues

1.1 Introduction

Political issues are controversies that arise within democracies, and where a decision has to be taken. For example, whether women should be allowed to vote, or whether gay people should be allowed to get married.

Within the set of political issues, social issues are centered on "the cultural and moral concerns of a society" (Layman et al., 2006). They include, among others, the issues related to women's and minority rights, as well as racial issues. Moreover, they share the following characteristics. First, they are more easily understood than economic issues (Hunter, 1992). And second, they usually produce strong emotions in the people (Layman et al., 2006).[1]

There is not doubt that social issues were important in the past century.[2] However, it is also argued that their political salience has progressively increased in the last three decades (Hetherington, 2009). This means, on the one hand, that politicians are centering their discourses in social issues. And on the other hand, that citizens are giving more importance to social issues when choosing their vote (Ansolabehere et al., 2006).

In this paper, I show that social issues tend to follow behavioural patterns, both in terms of parties' behaviour and the public opinion. I analyse the dynamics of a broad set of social issues, making both inter-group comparisons (different issues) and intra-group comparisons (same issue in different countries). I address the issues of women's suffrage, racial segregation, same-sex marriage and abortion, among others. Then, I illustrate how these patterns can help predicting future scenarios, specially in the issues in which parties and citizens are nowadays divided.

Based on these patterns, I propose a new way of modelling parties' and citizens' behaviour. First, I model how parties strategically choose whether to be in favor or against some policy (*i.e*, women's suffrage), taking into account citizens' preferences. Then, I survey my three theoretical papers, where starting from the base model, I deepen the dynamic relationship between parties and citizens around a specific issue.

My results shed light on the following question. What makes parties be confronted about an issue? By way of summary, I identify four variables that may drive confrontation. First, parties' preferences over the issue itself. That is, the fact that parties prefer opposite policies incentivizes that they support opposite policies. Second, parties' motivation for becoming popular. For example, I show that a party may prefer confrontation because the party would steal supporters to the opponent after that. Third, parties'

[1] Social issues are often referred as "hot-button issues" because of their impact on people's emotions (Hetherington, 2009).

[2] See for example Sitkoff (1971) and Brown (1993) for a historical review of the issues of women's suffrage and racial desegregation in U.S..

influence over the future public opinion. This is a feedback effect: the fact that parties affect future opinions actually affects parties' behaviour, and, in some cases, incentivizes confrontation. And fourth, parties' uncertainty about the public opinion, which is assumed to disappear if parties support opposite policies. In this case, a party may prefer confrontation because then citizens' preferences would be observed, which increases in expectation its future payoff.

In order to better observe the dynamics, I first propose a classification of political issues. On the one hand, I define an issue as "current" when it dominates the political debate, both at a public and party levels.[3] I show three ways of detecting whether an issue is current or not. These are the frequency with which the media speak about the issue, the behaviour of political parties, and the behaviour of political activists. On the other hand, I define an issue as "latent" when neither citizens nor parties are actively discussing about the issue. Within this category, I distinguish between diverging issues (if citizens are becoming more divided), converging issues (if they are becoming less divided) or stable issues (if opinions are not changing over time).

Based on this categorization, I describe the patterns that I have observed in some social issues. I show that a broad set of issues have evolved as follows. First, the issue is latent but diverging. Then, the issue becomes current, and during that time the policy implemented changes. Eventually, the issue turns to be latent, but now the issue is converging, and in some cases citizens have reached a consensus. I show that the issues of women's suffrage, school desegregation, interracial marriage, same-sex marriage and, in some cases, abortion have shown this pattern. Then, I illustrate how these patterns can help predicting scenarios. In particular, I focus in cases where an issue is nowadays current (*e.g*, abortion in Argentina), and in cases where an issue is nowadays latent but diverging (*e.g*, death penalty in U.S.).

In the second part of the paper, I present a new way of modelling parties' and citizens' behaviour. First, I describe the base model, which is reproduced in all my papers. In the model, there are two political parties, a set of citizens, and one single issue with two possible policies. Parties strategically decide whether to support one policy or another, and one policy is implemented after parties make their decision (there may be uncertainty about the policy implemented). Parties' payoffs are a function of citizens' preferences, which are parametrized (elections are not explicitely modelled). Parties are concerned with the policy implemented (each party prefers a different policy) and with their popularity, and both variables depend on citizens' preferences. In equilibrium, parties support opposite policies only if there is not an excessively large amount of citizens in favor of

[3]The notion of "current issue" is already present in the traditional issue evolution model (Carmines and Stimson, 1986). In the model, first political parties take positions with respect to an issue, and then the issue reaches the public debate.

one policy. Otherwise, both parties will support the policy preferred by the majority of citizens.

Finally, I summarize the main features and contributions of my three theoretical papers. In Sánchez (2020c), I explore the properties of citizens' preferences that lead to party confrontation, and how parties may eventually distort these preferences. I assume that citizens not only have preferences over policies, but also over parties. Moreover, citizens may switch either of these preferences after observing the policy supported by each party. The model predicts an increase in the correlation between citizens' party and policy preferences, but only if parties have supported opposite policies. This result suggests that the sorting phenomenon is a consequence of parties becoming polarized, which goes in line with the following observation: "While U.S. parties are becoming increasingly polarized, U.S. are only better sorted" (Fiorina, 2018).

In Sánchez (2020a), the base model is repeated during a finitely number of periods, and citizens' preferences are changing over time. Moreover, I assume that parties are able to influence citizens' preferences through their policy choices. In particular, I assume that the more parties support one policy at some period, the more citizens will be in favor of that policy in the next period. I identify a feedback effect derived from parties' influence. That is, I find situations where parties would have been in consensus, but because they are able to influence future citizens' preferences, then they end up confronted. This result suggests that there may be an "excess" of party polarization due to parties' influence, which helps understanding the question of why U.S. parties seem to be more polarized than U.S. citizens (Fiorina et al. (2005), Ansolabehere et al. (2006)).

In Sánchez (2020d), I assume that when an issue moves from latent to current, then the knowledge about the state of the public opinion is improved. Then, I make use of this assumption to study parties' incentives to learn. In particular, I start from the base model, but assuming that parties do not know citizens' preferences (although they have a prior). However, parties make another policy decision before playing the base game (this decision can be interpreted as a policy proposal), and if they support opposite policies, then they learn citizens' preferences. Yet, learning will be costly if the party proposes an unpopular policy. My results suggest that learning can incentivize a party to choose confrontation, because then its future payoff may increase. I show that a party may even take the risk of suppoting an unpopular policy in order to learn citizens' preferences. However, a party is not necessarily better-off without uncertainty. In fact, in the model, a party benefits from learning if and only if the opponent does not. As a consequence of this symmetry, party confrontation will only arise as an equilibrium in mixed strategies.

The rest of the paper is as follows. In Section 1.2, I classify social issues. In Section 1.3, I describe the dynamics of several social issues. In Section 1.4, I describe the base model and I survey my theoretical papers. And in Section 1.5, I conclude with the final remarks. All historical references are described in Section 1.6.

1.2 Classifying social issues

> "Most issues most of the time lie dormant, stirring interest only in those specially informed and in those specially affected. [...] But occasionally issues rise from partisan obscurity and become so contentious, so partisan, and so long lasting that they come to define the party system in which they arise."
>
> *Carmines and Stimson (1986)*

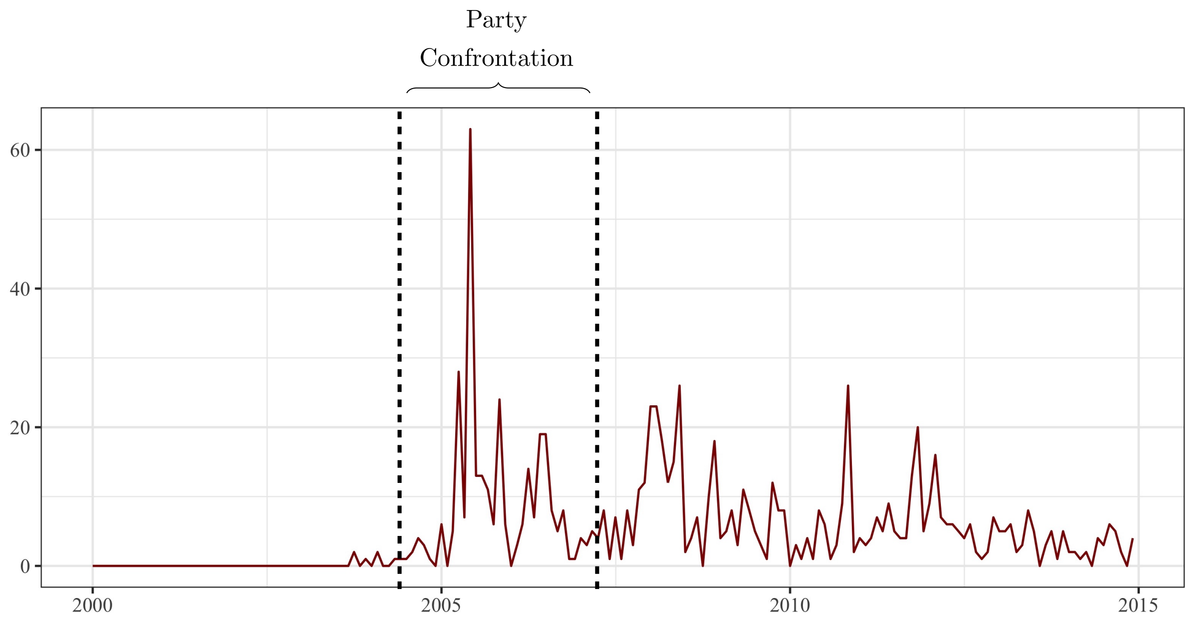
1.2.1 Current issues

I define a political issue as a "current issue" when it dominates the political debate, both at a public and party levels. In general, an issue becomes current when the policy implemented is at risk of being changed, either by law or by court decision. For example, in U.S., the issue of women's suffrage was a current issue around 1920, the time when the Nineteenth Amendment was adopted and voting discrimination in the basis of sex was prohibited.[4] Other times, an issue becomes current right after an unexpected event. For example, also in U.S., some racial issues became current in May 2020, after the death of the black citizen George Floyd.

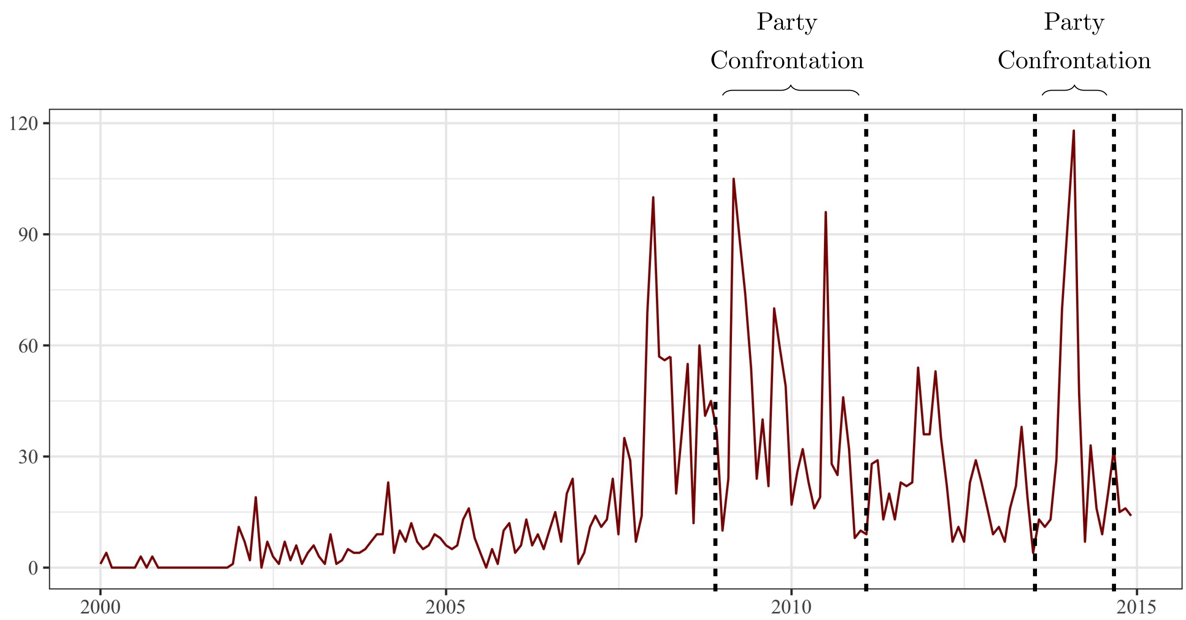
One way of detecting whether an issue has been current is through the frequency with which the issue was mentioned in the media. Apart from informing, the media promote discussions about an issue (Druckman et al., 2018) and, in some cases, stress the division between citizens' opinions (Martin and Yurukoglu, 2017). To illustrate this relation, I display in Figure 1.1 the number of articles of one Spanish newspaper mentioning the words "same-sex marriage" and "abortion" during the first fifteen years of this century.[5] The largest peaks of both series concide with the moments where the corresponding issue was a current issue. In the first case, the largest peak is around 2005, when same-sex marriage was legalized. In the second case, there are three large peaks between 2007 to 2011, and another one around 2014. The first peaks coincide with the period during which abortion was legalized under all circunstances. The last peak is when the right-wing party attempted to illegalize it (except under certain circunstances), without success.

[4] The source and a detailed explanation of all historical references made throughout the paper can be found in Section 1.6.

[5] In 2019 and 2020, *El País*, which is the newspaper analysed, has been the second most read newspaper in the country, and the first non-sport newspaper. The number of daily readers is about one millon. Source: Spanish Assotiation for Media Research, *(Asociación para la Investigación de Medios de Comunicación)*.



(a) Number of articles mentioning the words "same-sex marriage".



(b) Number of articles mentioning the word "abortion".

Figure 1.1: The frequency with which a political issue has appeared in the news help identifying the moments when the issue has been a current issue. Those moments generally coincide with the years when party confrontation was explicit.

Another way of identifying the status of an issue is by observing the behaviour of political parties. In general, when party confrontation is visible, either through voting or through political declarations, then the issue has reached the public debate. This relationship is already argued in Carmines and Stimson (1986) and Zaller (1992), and more recently shown in Druckman et al. (2013), among others. In Figure 1.1, it can be noted that the years where parties were confronted coincide with peaks in the number of news mentioning the corresponding issue. In the first case, party confrontation was visible around 2005, when the right-wing party firmly opposed same-sex marriage and voted against it. In the second case, party confrontation was visible around 2010 and later in 2014. In 2010, abortion was legalized with the opposition of the right-wing party. In 2014, the governing right-wing party promoted a law to illegalize abortion, but the party eventually backed down due to the political rejection generated.

A last way of identifying current issues is through the behaviour of political activists, whose influence over parties' positions has been widely recognized (see Fiorina et al. (2005) and Layman et al. (2006), among others). When an issue is under public discussion, then the collective actions of activists from both sides are usually intensified. One way of measuring this is with the number of demonstrations in favor and against of the policy implemented. For example, one massive protest in favor and another against same-sex marriage took place in Spain between 2004 and 2005, something that has never happened again. Similarly, four massive protests in favor and seven against abortion took place between 2008 and 2015. This coincidence had only happened in the middle of the eighties, when the issue of abortion was also under public scrutiny. At that time, abortion went from being completely illegal to legal under certain circunstances.

1.2.2 Latent issues

If the issue is not current, then I define it as "latent". And within this category, I distinguish between "diverging" ,"converging" and "stable" issues, depending on how the public opinion is evolving. An issue is latent but diverging when the share of citizens against of the policy implemented in increasing over time. For example, the issue of women's suffrage was probably latent but diverging by late 1890s, the time when the suffrage movement was expanding. An issue is latent and converging when the share of citizens against of the policy implemented is decreasing over time, and an issue is stable when the share is not changing. One particular case of a stable issue is when opinions have reached a consensus, in which case the issue is said to be "consensual". Coming back to the previous example, women's suffrage was probably latent and converging some years after the Nineteenth Amendment was adopted. Nowadays, women's suffrage has undoubtedly become latent and consensual.

Identifying whether an issue is latent or not at some period can be made through the behaviour of political parties. In general, when an issue is latent, then parties are not confronted, or at least confrontation is not visible (Carmines and Stimson, 1986). However, sensing whether an issue is diverging, converging or stable is usually more involved. One way is thourgh public opinion surveys, which can be revealing in certain cases.[6] Another way is thourgh the behaviour of political activists. If an issue is consensual, then protest actions against of the policy implemented should be very minor, often inexistent. If an issue is converging, then protest actions can be expected to happen, although their frequency has to be decreasing. On the contrary, if the issue is diverging or stable (but not consensual), then protest actions are expected either to happen with increasing frequency, or at least to happen periodically.

1.3 Dynamics of Social Issues

The previous definitions of current and latent issues, as well as the mechanisms to detect them, can be used to better understand the time evolution of social issues. I will now describe the patterns that I have observed in some of them.

There is a broad set of issues that have evolved like women's suffrage. First, they were latent but diverging issues. Then, they were current issues during several years, in which the policy implemented was switched. Then, they became latent and converging issues, and in some cases citizens have reached a consensus.

In U.S., racial issues followed this pattern during the twentieth century. For illustration, consider the issue of school segregation (Figure 1.1b). During the thirties and the fourties, school segregation was not a current issue, although the political activism against of segregation was expanding. The issue was current around 1954, when school segregation was declared ilegal *de jure*. During that time, political confrontation was highly visible, and citizens were almost evenly divided on the issue. Similarly, protest actions in favor and against of segregation happened all over the country. The issue became latent again around the seventies, with the difference that now school integration was on the way to be become consensual. In 1994, which was the last time that a survey asked about school integration, 87% of respondents declared to be in favor of it.

The issues related to the sexual minorities also tend to evolve in this way, and the pattern is also observable in other countries. For illustration, consider the issue of same-sex marriage, which has become legal in 28 countries. In many of them, its legalization derived from a law developed and implemented by one of the parties, rather than from a judicial decision. When this happens, then party confrontation usually becomes more visible, and the behavioural patterns are even more clear. This can be observed in the first three pictures of Figure 1.3. In all the three countries, parties have behaved similarly. First, none of them actively supported same-sex marriage, although its legalization had

[6]Survey data might not fully represent the status of the public opinion. Respondents' answers do not always reflect their real views (Hetherington, 2009), and they might be influenced by the question itself (Schuman and Presser, 1996). Moreover, time-series data about a single issue are usually not available.

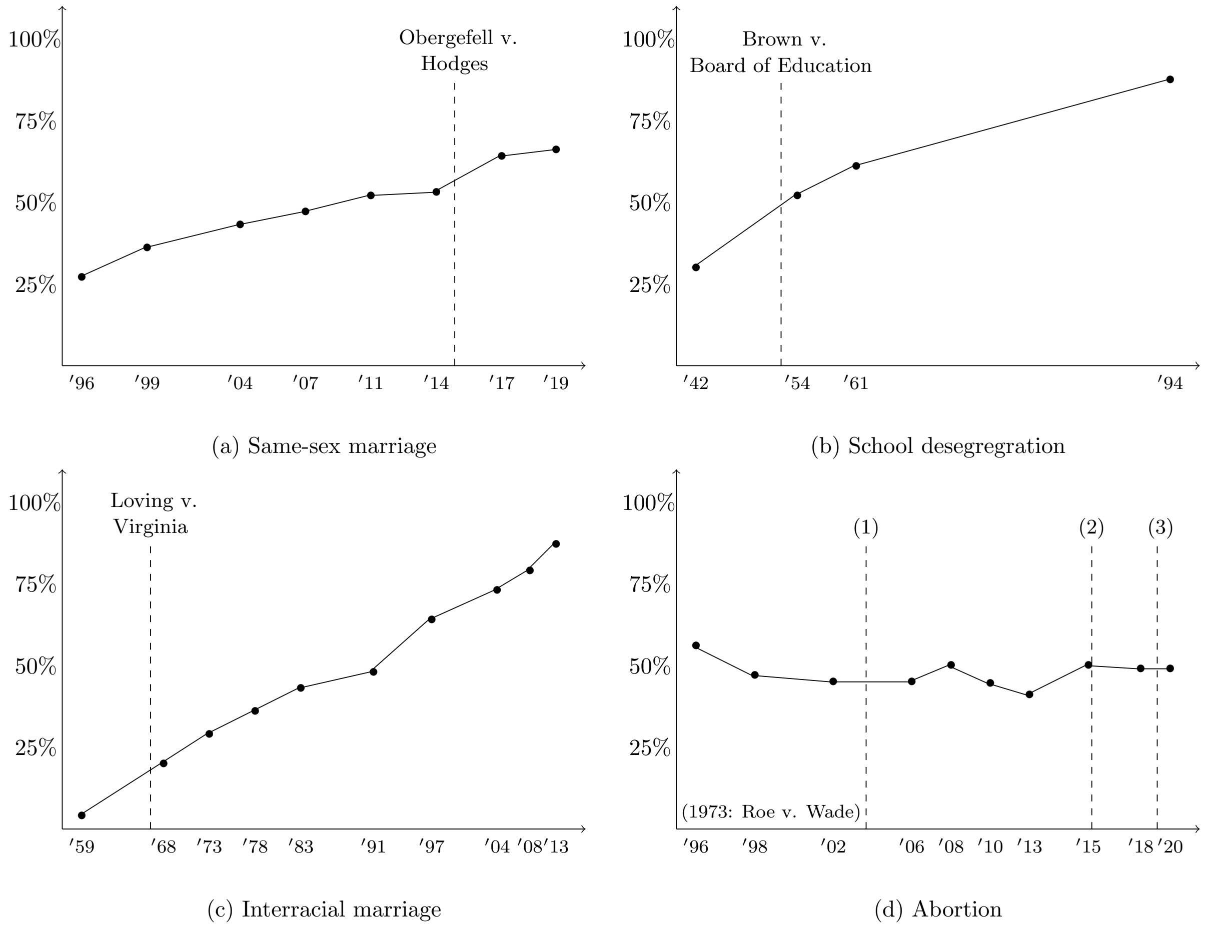
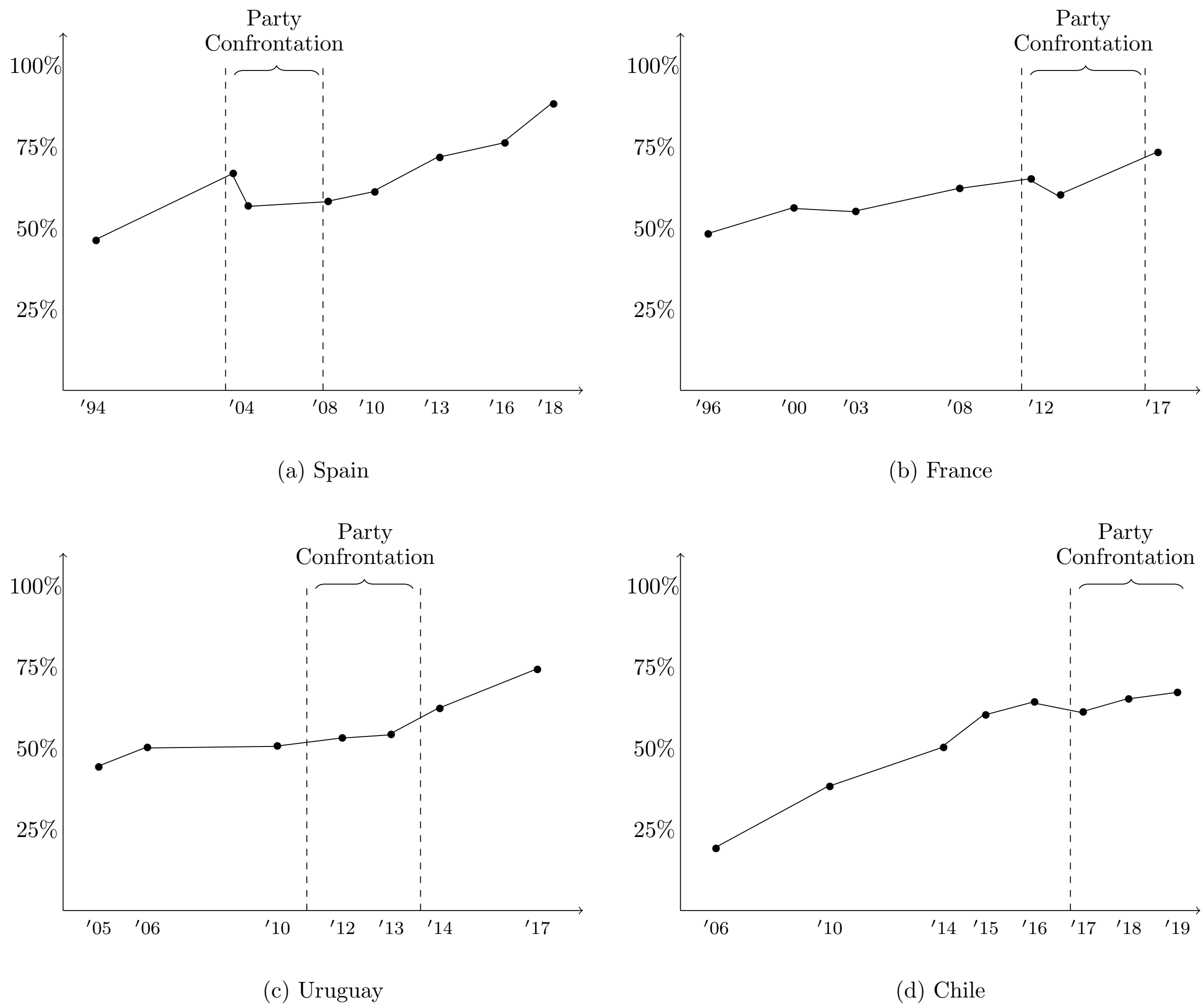


Figure 1.2: How the share of citizens in favor of several issues has evolved in U.S.. I also indicate the moment when the issue was made legal by the Supreme Court. In the case of abortion, I indicate the moments where the following political or judicial decisions were made: (1) Partial Birth Abortion Ban Act, (2) Whole Women's Health v. Hellerstedt, (3) Human Life Protection Act.

already been demanded by political activists. At some point, one party deviated and defended its legalization, while another party actively opposed it. During that period, same-sex marriage was a current issue, and it was eventually legalized. Then, there is a third period where same-sex marriage is latent and converging. None of the main parties is actively opposing it, and the mass of citizens against it is decreasing over time.

The pattern is not that clear in the abortion issue (Figure 1.4), whose peculiarity has been already acknowledged (Fiorina and Levendusky, 2006). In some countries, like in the U.K., abortion has rarely become politicized in the last decades (Moon et al., 2019). However, in others, politicians and citizens have been divided for years. This is the case of U.S.: although abortion became legal in 1973, the share of "pro-choice" citizens is around fifty percent, and it has remained stable over time (Figure 1.2d). Moreover, the issue has returned to the political arena several times since 1973, and protest actions against of abortion still happens periodically. This pattern is also observed in other countries, which



(a) Spain

(b) France

(c) Uruguay

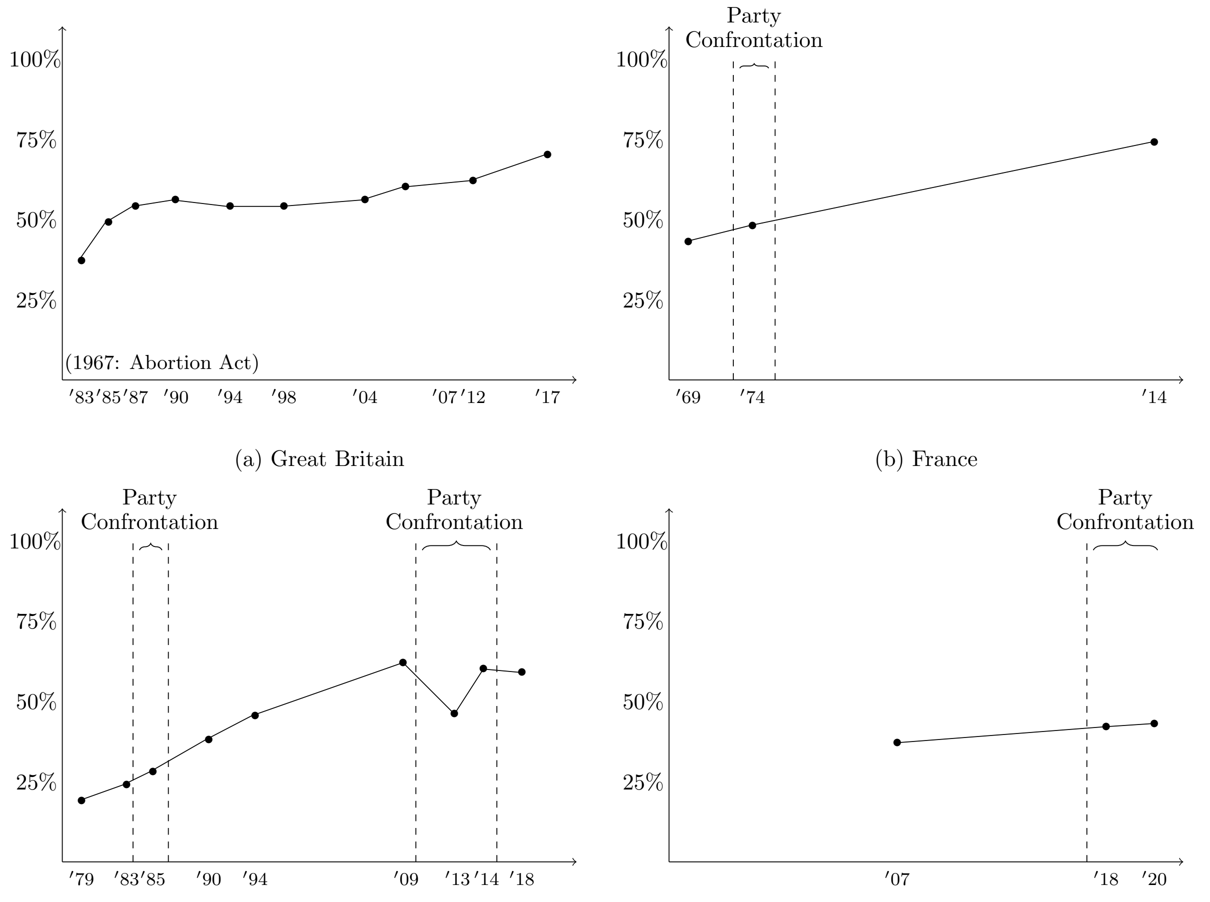
(d) Chile

Figure 1.3: How the share of citizens in favor of same-sex marriage has evolved in four countries. I also indicate the interval of years where party confrontation was visible. In Chile, same-sex marriage is not legal, but parties are currently confronted.

suggests that, unlike others, the abortion issue rarely becomes consensual.[7]

To conclude, I illustrate how those patterns can help predicting future scenarios. For example, in Chile, the issue of same-sex marriage has moved from latent to current issue (Figure 1.3d). Although same-sex marriage is not legal, parties and citizens are divided. Thus, we may expect that same-sex marriage will be legalized, and the issue will turn to be latent. In Argentina, where abortion is illegal, the issue has recently become current (Figure 1.4d). Parties and citizens are divided, and massive protests from both sides have been happening during the last years. In this case, we may expect that abortion will be legalized, but that citizens will keep divided, and maybe the issue will turn to be current in the future. Lastly, we also observe issues that are today latent and diverging. For example, the issues of death penalty in the U.S. and bullfights in Spain, where they

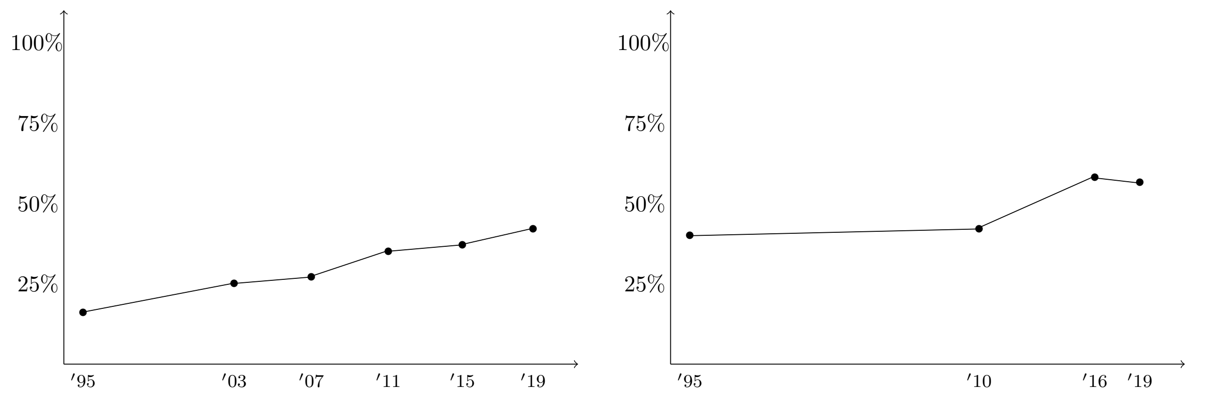
[7]For example, in all the countries analysed, a massive protest against abortion, known as “The March for Life”, takes place every year.



(a) Great Britain (b) France

(c) Spain (d) Argentina

Figure 1.4: How the share of citizens in favor of abortion "under all circumstances" has evolved in four countries. In Spain, abortion was legalized in two steps (see Section 1.6). In Argentina, abortion is not legal, but parties are currently confronted.



(a) Death penalty (U.S.) (b) Bullfights (Spain)

Figure 1.5: The share of U.S citizens against death penalty and the share of Spanish citizens against bullfights are increasing over time.

are legal. Those issues are not current, but citizens are becoming increasingly divided (Figure 1.5). Thus, we may expect that they will become current issues in some future.

1.4 Modelling parties' and citizens' behaviour

In this section, I summarize the main points of my three theoretical papers. In all of them, there are two strategic parties (or politicians) that have to decide which of the two possible policies they support (in some cases, during multiple periods). There is also a set of non-strategic citizens that are endowed with strict preferences over the policies (and, in some cases, also over the parties), and those preferences can be changing over time. First, I will describe the base model, which is reproduced in the three papers.

1.4.1 The base model

Suppose that are two parties, A and B, two policies, L and R, and a population of citizens. Each citizen is endowed with strict preferences over the policies. Parties have to decide, simultaneously, which of the two policies they support. Let $\alpha \in (0,1)$ be the share of citizens in favor of policy R, which is known by the parties. Also, let $(x_A, x_B) \in \{L, R\}^2$ be the vector indicating the policy supported by each party.

Assume that one policy is implemented after (x_A, x_B) is chosen. If $x_A = x_B$, then this common policy is implemented with probability one. If $x_A \neq x_B$, then there is uncertainty about the policy implemented. In particular, assume that policy R is implemented with probability α, and policy L with probability $1 - \alpha$.[8]

Parties' payoffs are the sum of two components. First, parties receive a payoff of value 1 only if their preferred policy is implemented. Without loss of generality, assume that party A prefers policy L and party B prefers policy R. In addition, a payoff of value k is allocated between parties depending on how popular they become. If $x_A = x_B$, then parties become equally popular, and they both receive $k/2$. If $x_A \neq x_B$, then the party supporting policy R receives $k \cdot \alpha$, and the party supporting policy L receives $k \cdot (1 - \alpha)$. In this case, the party supporting the policy preferred by the majority of citizens becomes more popular than the other, and its popularity increases with the amount of citizens preferring that policy. The parameter k reflects the degree of parties' concern with their popularity. For simplicity, assume that $k = 1$.[9]

For illustration, suppose that both parties choose policy L, and this policy is implemented with probability one. Then, party A receives a payoff of value 1, while party B does not receive any payoff from the policy implemented. In addition, both parties receive 1/2 from the popularity payoff. Hence, party A receives 3/2 in total, and party B receives 1/2. Now suppose that party A supports policy L and party B supports policy

[8] The justification of this probability is that the larger the public support that a policy has, the more likely is that this policy ends up implemented in this society. Any probability increasing with respect to α would be admissible.

[9] In Sánchez (2020c), I generalize the value of k.

(x_A, x_B)	Popularity payoff	Policy payoff	Total payoff
(R, R)	1/2, 1/2	0, 1	1/2, 3/2
(L, R)	$1-\alpha$, α	$1-\alpha$, α	$2 \cdot (1-\alpha), 2 \cdot \alpha$
(R, L)	α, $1-\alpha$	$1-\alpha$, α	1, 1
(L, L)	1/2, 1/2	1, 0	3/2, 1/2

Table 1.1: How parties' payoffs are constructed, given $k = 1$.

$A \setminus B$	L	R
L	$3/2, 1/2$	$2 \cdot (1-\alpha), 2 \cdot \alpha$
R	$1, 1$	1/2, 3/2

Table 1.2: The game played by the parties in normal form.

R, and policy L is implemented with probability $1 - \alpha$. Then, party A receives 1 with probability $1 - \alpha$ and 0 with probability α. In addition, party A receives $1 - \alpha$ from the popularity payoff, so the party expects a total payoff of $2 \cdot (1 - \alpha)$. By symmetry, party B will expect a total payoff of $2 \cdot \alpha$. The remaning payoffs are found in Table 1.1, and the game in normal form played by the parties for a given α is displayed in Table 1.2.

In the following proposition, I describe the set of equilibria in pure strategies for every possible value of α.

Proposition 1.1. *The following statements hold for the game in normal form displayed in Table 1.2.*

1. *If $\alpha < 1/4$, then (L, L) is the unique equilibrium in pure strategies, where L is a dominant strategy for party A.*

2. *If $\alpha \in (1/4, 3/4)$, then (L, R) is the unique equilibrium in pure strategies, where both strategies are dominant.*

3. *If $\alpha > 3/4$, then (R, R) is the unique equilibrium in pure strategies, where R is a dominant strategy for party B.*

4. *If $\alpha = 1/4$ or $\alpha = 3/4$, then (L, R) is the unique equilibrium in undominated pure strategies.*

The proof is by inspection. First, observe that the best reply of party A to $x_B = L$ is always policy L, and the best reply of party B to $x_A = R$ is always policy R. Then, (R, L) is never a pure strategies equilibrium.

The cutoff 1/4 is the threshold value of α where party B is indifferent between (L, R) and (L, L). If α is below that cutoff, then party B does not support policy R (its preferred policy), because the increase of the policy payoff (due to the increase of the probability that policy R is implemented) does not compensate its loss of popularity. In this case, (L, L) becomes the unique equilibrium.

Symmetrically, the cutoff 3/4 is the threshold value of α where party A is indifferent between (L, R) and (R, R). In this case, if α is larger than this cutoff, then party A does not support policy L, and (R, R) becomes the unique equilibrium. If α is between the two cutoffs, then the dominant strategy of each party is to support its corresponding preferred policy. In this case, (L, R) is the unique equilibrium.

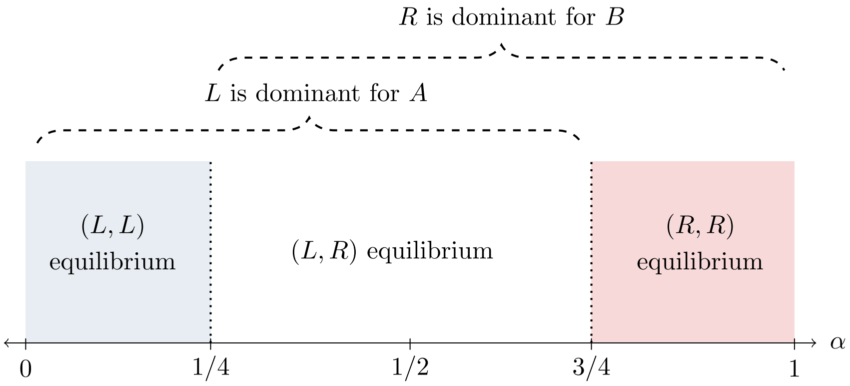
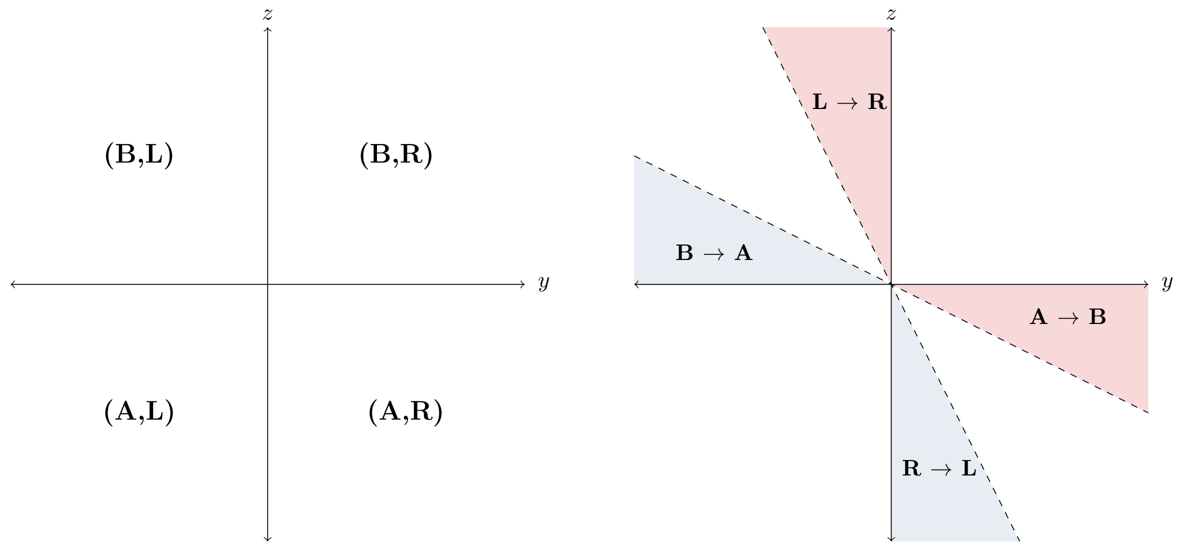


Figure 1.6: The equilibrium of the game.

1.4.2 How citizens react to parties' behaviour: the sorting phenomenon

To the best of my knowledge, traditional models have not explained two relevant questions related to the relationship between citizens' preferences and parties' behaviour. First, the claim that citizens are less polarized than political parties (see Section 1.4.3). And second, the phenomenon of party sorting, which refers to the increase in the correlation between citizens' ideology and their party affiliation. The sorting phenomenon has been already documented in U.S. (Fiorina and Levendusky (2006), Hetherington (2009)) and it can be summarized with the following statement: nowadays, someone who supports the Democratic Party is more likely to be liberal than in the past, and someone who supports the Republican Party is more likely to be conservative. Fiorina (2018) goes beyond that and formulates the following hyphothesis. "While U.S. parties are more



(a) Citizens' preferences depending on their position. (b) How citizens' preferences change after $(x_A, x_B) = (L, R)$.

Figure 1.7: Citizens' party and policy preferences are determined by the position of their vector. For example, citizens whose vector is located in the first quadrant prefer party B and policy R. If $x_A \neq x_B$, then citizens might switch one of their preferences.

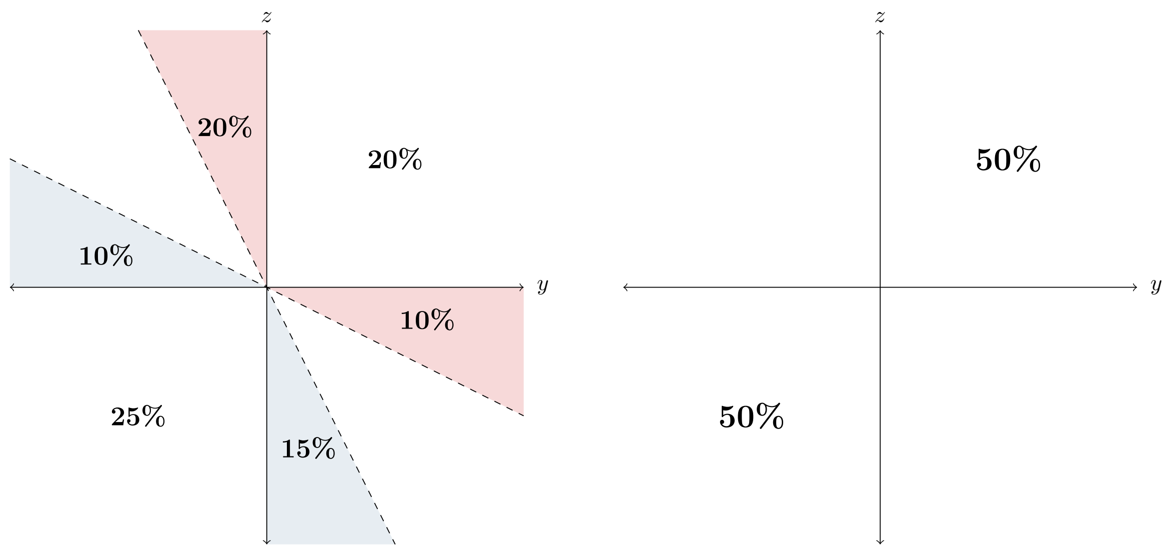
polarized than in the past, U.S. citizens are only better sorted".

In Sánchez (2020c), I deepen the relationship between citizens' preferences and parties' behaviour. I start from the base model, but I assume that citizens not only have preferences over policies, but also over parties. More formally, I assume that every citizen i has a vector $(y_i, z_i) \in \mathbb{R}^2$, such that citizen i prefers policy L to policy R if and only if $y_i < 0$, and she prefers party A to party B if and only if $z_i < 0$. In this framework, citizens' party and policy preferences are identified with the position of their vectors in the Euclidean space (see Figure 1.7a).[10] Moreover, the values of $|y_i|$ and $|z_i|$ can be interpreted as the levels of attachment toward policies and parties of citizen i. For example, citizens whose $|y_i|$ is very low and whose $|z_i|$ is very large are not very attached to their preferred policy, but very attached to their preferred party.

The key aspect of the model is that citizens' preferences might change after observing (x_A, x_B). More specfically, if $x_A \neq x_B$, then citizens whose preferred policy is different than the policy supported by their preferred party are likely to switch one of their preferences. For example, suppose that $(x_A, x_B) = (L, R)$. Then, only citizens preferring party B but policy L ($(-,+)$ quadrant), or citizens preferring party A but policy R ($(+,-)$ quadrant) might switch preferences. The rest of citizens will keep preferring the same party and policy than initially.

Moreover, whether a citizen switches or not her party or policy preferences will depend

[10]Citizens' preferences are assumed to be strict, so I withdraw $y_i = 0$ and $z_i = 0$.



(a) In this distribution, the equilibrium is (L, R).

(b) How the distribution looks after (L, R)

Figure 1.8: In (a), parties' supporters are almost equally spread among policies. The equilibrium in this distribution is (L, R), so citizens' preferences are affected. In (b), citizens belong either to the first of the third quadrant.

on her levels of attachment toward each of them. On the one hand, citizens who are sufficiently more attached to policies than to parties will switch party preferences. For example, in Figure 1.7b, where $(x_A, x_B) = (L, R)$, citizens prefering party A ($z_i < 0$) and policy R ($y_i > 0$) for whom $|z_i| < \frac{1}{2} \cdot |y_i|$ will start preferring party B. Those are called the policy-based party changes, and they are based on the traditional argument that citizens adjust their party preferences based on their ideology (Abramowitz and Saunders (1998), Putz (2002)). On the other hand, citizens who are sufficiently more attached to parties than to policies will switch policy preferences. For example, in Figure 1.7b, citizens preferring party A and policy R for whom $|z_i| > 2 \cdot |y_i|$ will start preferring policy L. Those are called the party-based policy changes, and they have been recently shown in the literature (see Section 1.4.3).

By making use of a slightly different version of the base model, I obtain parties' equilibrium behaviour for a given distribution of preferences, and then I observe how the distribution is affected. For illustration, consider the distribution of Figure 1.8a. There is at least twenty percent of citizens in each quadrant, so the correlation between citizens' party and policy preferences is not excessively large. However, I will show that the equilibrium in this distribution is (L, R), so citizens' preferences are affected. For example, ten percent of citizens switch from party A to party B, and fifteen percent of citizens switch from policy R to policy L. Now suppose that citizens' vectors are relocated according to their new preferences (for example, if a citizen initially preferring party A switches to party B, then her z_i becomes strictly positive). Then, the final distribution of preferences will be the one in Figure 1.8b, which is perfectly sorted. Now all citizens preferring policy R also prefer party B, and all citizens preferring policy L also prefer party A. This suggests that the sorting phenomenon is a consequence of party confrontation, which goes in line with the hyphothesis of Fiorina (2018).

1.4.3 How parties' influence affects parties' behaviour

In general, traditional electoral competition models do not exploit parties' influence over citizens' preferences (Sánchez, 2020a). And yet, it has been shown that parties have a strong influence over the public opinion. For example, parties bring issues into public discussion (see Section 1.2.1). But they also influence whether a new policy is implemented or not, which also affects public opinion (Flores and Barclay, 2016). Moreover, parties also influence the process by which citizens form their own political views. The reason is that citizens tend to give more weight to the arguments that reinforce the position of their most-preferred party, a phenomenon that has been called "partisan motivated reasoning" (Druckman et al. (2013), Bolser et al. (2014)).

In Sánchez (2020a), the game presented in Section 1.4.1 is repeated during a finitely number of periods, and parties are able to influence future citizens' preferences through their policy decisions. I make the following two assumptions. First, the share of citizens in favor of policy L is increasing over time (this goes in line with the patterns observed in Section 1.3). And second, parties are able to affect the rate at which this share increase. More specifically, I assume that the more parties support policy L at some t, the lower will be the value of α_{t+1}.

I identify a feedback effect derived from parties' influence. That is, I show that the fact that parties are able to influence citizens' preferences actually affects parties' behaviour. In particular, I show that party A, which is the party preferring policy L, has an additional incentive to support policy L, because then the future share of citizens in favor of that policy will be larger, and its future payoffs will also increase. Because of this effect, I find situations where parties would have been in consensus, but due to their influence over citizens' preferences, they end up confronted.

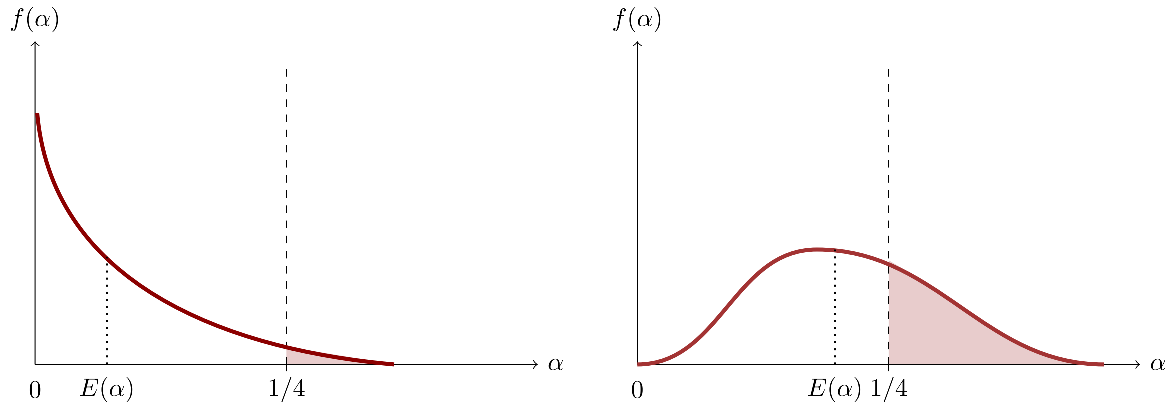
Interestingly, this result also sheds light on the question of whether U.S. parties are more polarized or not than citizens. Indeed, while party polarization is commonly accepted in the literature, popular polarization is usually very questioned. Some authors claim that U.S. citizens are also polarized (Abramowitz and Saunders, 2008), but many others argue that polarization is exaggerated (see Fiorina et al. (2005), Ansolabehere et al. (2006) or Fiorina (2018), among others). In Sánchez (2020a), I suggest that there may exist an "excess" of party polarization due to parties' influence over citizens' preferences, which may help understanding this paradox.

1.4.4 Party confrontation as a learning mechanism

In general, when party confrontation is visible, then the issue reaches the public debate (see Section 1.2.1). Based on this relationship, I make the following assumption. When parties are confronted and the issue becomes current, then the knowledge about the state of the public opinion is improved.

In Sánchez (2020d), I make use of this assumption to study parties' incentives to learn about citizens' preferences. More specifically, I start from the base model but assuming that parties only know the distribution of α. However, parties make another policy decision before playing the base game, and if they support different policies, then they learn the value of α. Then, each party faces the following dilemma. Either the party supports a different policy than the opponent, learns α, and plays the base game with perfect information, or it supports the same policy than the opponent, does not learn α, and plays the base game with imperfect information. I also assume that learning is costly if the party chooses a policy that is not preferred by the majority of citizens.

I find that a party might find confrontation profitable because α would be learned, which increases in expectation the party's future payoffs. To illustrate this, consider the two distributions displayed in Figure 1.9. In both of them, the expected value of α is below 1/4. This implies that, if α is not learned, then both parties choose policy L in the base game. However, in the right distribution, $1 - F(1/4)$ is sufficiently large. This means that, if α is learned, then it is quite likely that $\alpha > 1/4$ and parties do not play (L, L) in the base game (see Section 1.4.1). And if this happens, then party B, which is the party preferring policy R, receives a larger payoff. Thus, in this distribution, party B will choose confrontation in order to learn α. In contrast, in the left distribution, $F(1/4)$ is very close to one. As a consequence, party B's benefits from learning will be lower than the cost of learning, and thus party B will not choose confrontation.



(a) Parties do not learn α.

(b) Party B chooses confrontation in order to learn α.

Figure 1.9: Two distributions with $E(\alpha) < 1/4$.

Interestingly, parties are not necessarily better-off without uncertainty. In fact, because the model is symmetric, then a party benefits from learning if and only if the opponent does not. For example, in the distributions of Figure 1.9, the expected payoff of party A, which is the party preferring policy L, will always be lower when α is learned. As a consequence of this symmetry, party confrontation in the learning period will only arise as an equilibrium with mixed strategies.

1.5 Final remarks

In this paper, I present my research on the dynamics of political issues. First, I classify political issues depending on their salience. Then, based on this classification, I describe the dynamics of a broad set of social issues, and I highlight their behavioural patterns. These pattens reveal a dynamic interplay between political parties and the public opinion. By means of three theoretical papers, which are summarized here, I attempt to shed light on this interplay. How parties take position with respect to an issue? How the public opinion may be influenced by parties' choices?

My point of view differs from the traditional downsian approach in three main points. First, the policy space is discrete and binary, so I abandon the spatial competition framework. Second, elections are not explicitely modelled. The reason is that my objective is not to understand the electoral process itself, but the behaviour of parties and citizens around a specific political issue, at any moment in time. And third, citizens are not strategic. This is because, instead of the voting process, I model how citizens form (and update) their opinions, a much less strategic process (Kuhne and Schemer, 2015).

I show that party convergence is not necessarily the expected outcome, which is already quite innovative (De Donder and Gallego, 2017). My results shed light on the variables that drive confrontation between parties, and some of these variables actually result from their interplay with the public opinion. My results also help understanding some observed phenomena related with this interplay, like the sorting phenomenon or the excessive polarization of political parties compared to citizens.

My overall objective has been to encourage the research on the dynamics of political issues. By combining game theory with historical data, I have argued that it is possible to better understand the connection between mass behaviour and political parties. Knowing this interplay can shed light on issues in which citizens and parties are currently confronted, and where there is uncertainty about the policy outcome. This is specially relevant nowadays, as many democracies seem to be becoming increasingly divided.

1.6 Historical References

Women's suffrage in U.S.

The women's suffrage movement began to expand in the mid-19th century (Brown, 1993). The Nineteenth Amendment was adopted in 1920, within a very divided political landscape: the Amendment failed twice before it was approved in the Senate, and in the third attempt, 26% of senators voted against. Moreover, anti-suffrage movements had also expanded among the population (Maddux, 2004).

School segregation in U.S.

Racial integration became politicized around 1948 (Sitkoff, 1971), although political activism started way earlier.[11] School segregation became illegal *de jure* in 1954 after Brown v. Board of Education. Politicians and citizens were deeply divided on the issue (Patterson, 2001), and there was strong resistance to desegregation, specially in the South (Golub, 2013). The issue turned to be latent in the late sixties, along with the decline of the Civil Rights Movement (Chong, 2014).

Abortion in U.S.

The Pro-Choice movement emerged in the early sixties, and abortion was legalized in 1973 (after Roe v. Wade), within a very divided political landscape (Staggenborg, 1991). Since then, the issue has regularly returned to the political arena. For example, in the 1992 presidential elections (Abramowitz, 1995), or in 2019, coinciding with the enactment of the Human Life Protection Act.

Same-sex marriage in Spain

Political activism for same-sex marriage began its expansion in the mid nineteenth, and the left-wing party included it in its political agenda in 2004.[12] Same-sex marriage became legal in June 2005, although 43% of deputies voted against. Protests actions against of its legalization also took place at that time.[13] The issue progressively became latent and converging. Already in 2012, the right-wing party, that had voted against of same-sex marriage, did not include its illegalization in its political agenda.

[11] The National Association for the Advancement of Colored People was created in 1909.

[12] Emilio de Benito (June, 2015). "Cómo se consiguió el matrimonio gay" . *El País*. Retrieved from www.elpais.com

[13] Marta Arroyo (June, 2005). "Una multitud pide que se retire la ley del matrimonio homosexual" *El Mundo*. Retrieved from www.elmundo.es

Abortion in Spain

Abortion was forbidden in Spain until 1985, when the left-wing party legalized it under three particular circunstances: if the woman has been raped, if the woman is in danger, or if the fetus shows malformations. The law was very controversial: the right-wing party lodged an appeal to the constitutional court, and several protests against of abortion took place in the country.[14] Then, the issue became latent until 1995, when the left-wing party attempted to legalize abortion under all circunstances, without succeed.

In the late 2007, the abortion issue became current after it came to light that several private clinics in Madrid and Barcelona were doing clandestine abortions (this incident is reflected in the first large peak of news in Figure 1.1b). Then, in the late 2009, the left-wing party legalized abortion under all circunstances (before week 14 of pregnancy). Again, the new law was very controversial: the right-wing party lodged another appeal to the constitutional court, and several demonstrations against of the law took place in the country.[15]

In 2014, the issue turned to be current because the governing right-wing party attempted to implement a much more restrictive law, without succeed. During that time, massive protests in favor and against of abortion took place all over Spain.[16]

Data sources

U.S

The data for the issues of interratial marriage, same-sex marriage, abortion, dealth penalty, and school desegregation (1954, 1961, 1994) come from Gallup. The datum for school desegregation (1942) comes from NORC (National Opinion Research Center).

Spain

The data for the issue of same-sex marriage come from CIS (Centro de Investigaciones Sociológicas-Center for Sociological Research), except the datum for 2010, which comes from Metroscopia. The data for abortion come from CIS (1979, 1983, 1985, 1990, 1994, 2008), Metroscopia (2013, 2014), and Statista (2018). The data for bullfights come from CIS (1995, 2010), Ipsos (2016) and SocioMétrica (2019).

[14] "20.000 personas se manifiestan contra la ley del aborto en la madrileña calle de Serrano" . (June, 1985) *El País.* Retrieved from www.elpais.com

[15] "Más de un millón y medio de personas protestan en Madrid contra la reforma de la Ley del Aborto". (October, 2009). *La Vanguardia.* Retrieved from: www.lavanguardia.com

[16] Rubén Martínez (February, 2014) "Marcha masiva en Madrid contra la Ley del Aborto de Gallardón". *Infolibre.* Retrieved from: www.infolibre.es

France

The data for same-sex marriage come from IFOP (Institut Français d'opinion publique-French Institut of public opinion), except the datum for 2017 (Pew Research Center). The beginning of party confrontation was around 2012 presidential elections. Then, in 2013, same-sex marriage became legal, although 40% of deputies voted against. The end of party confrontation is approximated to 2017 legislative elections, when most parties gave support to same-sex marriage.

All the data for the abortion issue come from IFOP. In this case, party confrontation took place around 1974, when abortion became legal (by 227 votes against 192).

Uruguay

The data come from Research Uruguay (2005, 2006), LAPOP (Latin American Public Opinion Project) (2010, 2017), CIFRA (2012, 2013) and Pew Research Center (2014). Party confrontation began in 2012, when the issue was first discussed in the Congress. Then, in 2013, same-sex marriage became legal (by 71 votes against 21). The end of party confrontation is approximated to 2014, when no party included its illegalization in its political agenda.

Chile

The data come from Cadem, except for 2006 and 2010, which come from Universidad Pontificia Católica de Chile. The beginning of party confrontation was around 2017, when the bill was first submitted to the Congress by the governing left-wing party. In September 2019, the governing right-wing party rejected the bill.

Argentina

The data come from CEDES (Centro de Estudios de Estado y Sociedad-Centre for the Study of State and Society) (2007), Ipsos (2018) and Universidad de San Andrés (2020). Party confrontation started in 2018, when the abortion bill was first submitted to the Congress. The bill was eventually rejected by the Senate (by 38 votes against 31). In 2020, the president of Argentina has declared his intention of submitting a new abortion bill to the Congress.

Great Britain

All the data for the abortion issue comes from British Social Attitudes.

Chapter 2

On the Relationship between Parties' Behaviour and Citizens' Political Preferences

2.1 Introduction

Understanding parties' behaviour and its relationship with the public opinion is a major objective for both the Political Economy and the Political Science literatures. From the economic side, the well-known electoral competition models have focused on the strategic behaviour of parties and voters. Most of the contributions concentrate on some form of "median voter theorem", where the most common prediction is party convergence (De Donder and Gallego, 2017). However, this prediction clashes with the fact that parties do not always support the same policy in reality (Fiorina, 2018). Moreover, the fact that the models are built around elections makes them difficult to directly relate public opinion to parties' behaviour. In fact, they fail to address two relevant issues regarding this relationship. First, the argument that citizens are less polarized than political parties, which is defended by Fiorina et al. (2005) and Ansolabehere et al. (2006), among many others. Second, the phenomenon of party sorting, which refers to the increase in the correlation between citizens' ideology and their party affiliation.[1] By relating the two issues, Fiorina (2018) has formulated the following hypothesis: "While U.S. parties are more polarized than in the past, U.S. citizens are only better sorted".

In this paper, I present a new way of modelling the relationship between political parties and the public opinion. I focus on a single political issue, for which there are two policies. For instance, whether to legalize or not same-sex marriage. There are two parties that strategically decide which policy they support, although each party prefers a different policy. There is also a set of naïve citizens, who have preferences over parties and policies, and who may change them after knowing the policy supported by each party. Parties' payoffs depend on citizens' final preferences, so their relationship is endogenous. In a nutshell, there is an initial distribution of citizens' preferences, and parties make a decision taking into account how the corresponding distribution will be affected.

I analyse the conditions under which parties support different policies (party confrontation) or the same policy (party consensus). The key variable is the amount of supporters (*i.e*, the amount of citizens preferring a party) that a party might lose after confrontation. If this amount is not excessively large for either party, then confrontation is the unique equilibrium. Otherwise, the party losing supporters prefers to support the same policy than the opponent, and the equilibrium, if it exists, is a consensus. Whether a party loses or not supporters depends on how citizens update their preferences. For example, a citizen might switch her party preferences after observing parties' behaviour,

[1] Sorting here means that an individual who supports the Democrat Party is more likely to be liberal than in the past, in the same way that an individual who supports the Republican Party is more likely to be conservative. This phenomenon has been documented by Fiorina and Levendusky (2006) and Hetherington (2009), among others.

in which case one party "catches" one supporter from the other party. However, a citizen might instead switch her policy preferences, in which case parties' amount of supporters remains equal. This differentiation allows me to deepen the relationship between citizens' preferences and parties' behaviour. Interestingly, the model predicts an increase in the correlation between citizens' party and policy preferences, but only if parties have supported different policies. This result suggests that party sorting is a consequence of parties becoming polarized, which goes in line with the hyphotesis of Fiorina (2018).

In the first part of the paper, I solve the game played by the parties, without characterizing citizens' behaviour. In the game, parties receive a payoff that is increasing with their final amount of supporters. Moreover, one policy is implemented at the end of the game, and parties receive an additional payoff only if their preferred policy is implemented. I introduce uncertaintly in the model through the policy implemented. More specifically, I assume that the policy implemented depends probabilistically on parties' final amount of supporters. For example, if a party has 60% of supporters, then the policy chosen by that party will be implemented with 60% of probability.

A party is able to "catch" supporters from the opponent only when they support different policies. A party's final amount of supporters will be equal to its initial amount plus the amount of supporters that the party gains, which is a proportion of the opponent's supporters, minus the amount of supporters that the party loses, which is a proportion of its own supporters. The set of parties' payoffs will be identified with their initial amount of supporters together with four proportions. A last parameter will reflect the degree of parties' concern with the amount of supporters compared to the policy implemented.

For every combination of parameters, I characterize the set of the pure strategy equilibria of the corresponding two-by-two game. First, I show that there is never an equilibrium where each party supports its non-preferred policy. The reason is that supporting the non-preferred policy is already costly for the parties, because it decreases the probability that their preferred policy is implemented. And since parties are confronted, then one of the parties additionally loses supporters. This party will always prefer to deviate.

In the second result, I identify the conditions under which each party supports its preferred policy. I show that party confrontation is the equilibrium if no party loses an excessively large amount of supporters after that. Intuitively, supporting the preferred policy increases the probability that the corresponding policy is implemented, which is always positive for a party. If, in addition, its amount of supporters increases, then the party supports its preferred policy for sure. However, if its amount decreases, then the party faces a trade-off. I will show that if the loss of supporters is below certain threshold, then the party still supports its preferred policy. Otherwise, the party prefers to support its non-preferred policy instead of losing supporters.

In the last two results, I identify the conditions under which parties support the same policy. In general, when one party loses an excessively large amount of supporters, then the party prefers to support its non-preferred policy, and the corresponding consensus becomes the equilibrium. However, if the other party is not sufficiently policy motivated, then the party might deviate from that consensus by supporting its non-preferred policy. If this happens, then a pure strategy equilibrium will not exist.

In the second part of the paper, I characterize citizens' behaviour. At this point, I assume that citizens not only have preferences over parties, but also over policies. Moreover, citizens might have different degrees of attachment towards parties and policies. And if parties support different policies, then citizens whose preferred party does not support their preferred policy may switch one of their preferences. On the one hand, citizens who are sufficiently more attached to policies than to parties will switch party preferences. On the other hand, citizens who are sufficienly more attached to parties than to policies will switch policy preferences. Both types of preferences changes have been noted in the literature, as pointed in Layman et al. (2006).[2]

Then, I combine this characterization with the results about parties' behaviour. That is, from an initial distribution of citizens' preferences, I obtain parties' equilibrium, and then I observe how the distribution is affected. I provide two illustrative examples. In the first example, citizens' party and policy preferences are uncorrelated. That is, parties' supporters are equally spread among policies. The equilibrium is that each party supports its preferred policy, so citizens' preferences are affected. In the final distribution, the correlation has increased, as almost all supporters of a party prefer the same policy (party sorting). In the second example, there is a large majority of citizens preferring the same policy. However, the equilibrium is also party confrontation, and, as a consequence, most of the supporters of one party switch policy preferences. In the final distribution, half of the citizens prefer each policy (the society ends up more divided).

Last, I do comparative statics via simulations of the model. I show that parties are more likely to be confronted as they become more policy motivated. This is because they are willing to lose more supporters to have a chance of implementing their preferred policy. Parties are also more likely to be confronted when they initially have similar amount of supporters. Surprinsingly, if a party has lower amount of supporters initially, then it is more likely that both parties support the party's preferred policy. The reason is that, if a party's initial amount of supporters decreases, then it becomes more likely that the party gains supporters (if a party starts with zero supporters, then the party can only gain them), and that the party supports its preferred policy. At the same time,

[2]Traditionally, it was thought that citizens adjust their party preferences based on their policy preferences (Abramowitz and Saunders (1998), Putz (2002)). However, it has been shown that parties are also able to influence citizens' policy preferences (see Sánchez (2020a)).

it also becomes more likely that the other party loses supporters (if a party starts with all the supporters, then the party can only lose them), and that the party supports the same policy than the opponent.

The rest of the paper proceeds as follows. In the next section, I describe the game played by the partes. In Section 2.3, I characterize the equilibrium. In Section 2.4, I model citizens' behaviour, and I combine this characterization with the equilibrium results. In Section 2.5, I present the simulations of the model and I do the comparative statics. And in Section 4.4, I conclude. All proofs are gathered in Appendix A.

2.2 The Model

Consider a society that is made of two parties, A and B, and a finite set of citizens. There is a single issue and two policies, L (left) and R (right). Parties decide whether they support policy L or policy R. Let $(x_A, x_B) \in \{L, R\}^2$ denote the policies supported by party A and B respectively.

Citizens have strict preferences over parties. Let n_A and n_B denote the initial mass of citizens preferring party A and B respectively, where $n_A + n_B = 1$ holds. I assume that citizens might change their preferences after observing (x_A, x_B). Let $\tilde{n}_A(x_A, x_B)$ and $\tilde{n}_B(x_A, x_B)$ denote the mass of citizens preferring party A and B after observing (x_A, x_B). If a citizen becomes indifferent, then she remains in the initial preference. Hence, $\tilde{n}_A(x_A, x_B) + \tilde{n}_B(x_A, x_B) = 1$ holds for all (x_A, x_B).

One policy is implemented after (x_A, x_B) is revealed. More specifically, I assume that the policy supported by party $J \in \{A, B\}$ is implemented with probability $\tilde{n}_J(x_A, x_B)$.

2.2.1 Parties' payoffs

Parties have preferences over policies. In particular, party A prefers policy L and party B prefers policy R. Parties receive a payoff of $\pi > 0$ only if their preferred policy is implemented. This payoff is normalized to $\pi = 1$.

Parties are also concerned with the final amount of supporters. I assume that party J receives an additional payoff of $k \cdot \tilde{n}_J(x_A, x_B)$, where $k > 0$ holds. The parameter k measures the degree of parties' concern with their amount of supporters.

For illustration, consider party B, whose preferred policy is R. If both parties choose policy R, then this policy is implemented with probability 1, and party B receives $1 + k \cdot \tilde{n}_B(R, R)$. But if party A chooses policy L and party B chooses policy R, then policy R is implemented with probability $\tilde{n}_B(L, R)$, and party B receives 1 with probability $\tilde{n}_B(L, R)$ and 0 with probability $1 - \tilde{n}_B(L, R)$. Then, party B expects $(1 + k) \cdot \tilde{n}_B(L, R)$ in total.

The rest of payoffs are constructed in the same way. I will denote with $\pi_J(x_A, x_B)$ the total payoff received by party J at (x_A, x_B) (see Table 2.1).

(x_A, x_B)	$\pi_A(x_A, x_B)$	$\pi_B(x_A, x_B)$
(R, R)	$k \cdot (1 - \tilde{n}_B(R, R))$	$1 + k \cdot \tilde{n}_B(R, R)$
(L, R)	$1 - \tilde{n}_B(L, R) + k \cdot (1 - \tilde{n}_B(L, R))$	$\tilde{n}_B(L, R) + k \cdot \tilde{n}_B(L, R)$
(R, L)	$\tilde{n}_B(R, L) + k \cdot (1 - \tilde{n}_B(R, L))$	$1 - \tilde{n}_B(R, L) + k \cdot \tilde{n}_B(R, L)$
(L, L)	$1 + k \cdot (1 - \tilde{n}_B(L, L))$	$k \cdot \tilde{n}_B(L, L)$

Table 2.1: Parties' payoffs at every (x_A, x_B).

2.2.2 Final amount of supporters

I assume that citizens do not change party preferences if parties support the same policy. Hence,

$$\tilde{n}_B(L, L) = \tilde{n}_B(R, R) = n_B. \tag{2.1}$$

Suppose that party A chooses policy L and party B chooses policy R. Then, a proportion $\delta_1 \in [0, 1]$ of party A's initial mass of supporters start supporting party B, so $\delta_1 \cdot (1 - n_B)$ is the mass of supporters switching from party A to party B after (L, R). Similarly, a proportion $\delta_2 \in [0, 1]$ of party B's initial mass of supporters start supporting party A, so $\delta_2 \cdot n_B$ is the mass of supporters switching from party B to party A after (L, R). Hence,

$$\tilde{n}_B(L, R) = n_B + t, \tag{2.2}$$

where $t = \delta_1 \cdot (1 - n_B) - \delta_2 \cdot n_B$ is the net gain of supporters of party B after (L, R), or the net loss of supporters of party A. The maximum value that t can take is $1 - n_B$, when $\delta_1 = 1$ and $\delta_2 = 0$. In this situation, party B catches all party A's supporters and does not lose any of its supporters. The minimum value that t can take is $-n_B$, when $\delta_1 = 0$ and $\delta_2 = 1$. In this situation, party B loses all supporters and does not catch any of party A's supporters.

Now suppose that party A chooses policy R and party B chooses policy L. Then, a proportion $\delta_3 \in [0, 1]$ of party A's initial mass of supporters start supporting party B, and a proportion $\delta_4 \in [0, 1]$ of party B's initial mass of supporters start supporting party A. Hence,

$$\tilde{n}_B(R, L) = n_B + r, \tag{2.3}$$

where $r = \delta_3 \cdot (1 - n_B) - \delta_4 \cdot n_B$ is the net gain of supporters of party B after (R, L), or the net loss of supporters of party A. It also lies between $-n_B$ and $1 - n_B$.

2.3 Equilibrium Analysis

First, I use (2.1), (C.1) and (2.3) to write parties' payoffs. Then, for every $k > 0$, $n_B \in (0, 1)$ and $\delta = (\delta_1, \delta_2, \delta_3, \delta_4) \in [0, 1]^4$, I look for the set of pure strategy equilibra in the two-by two game displayed in Table 2.2.[3]

$A \setminus B$	L	R
L	$1 - k \cdot n_B, k \cdot n_B$	$1 - (k+1) \cdot (n_B + t)$, $(k+1) \cdot (n_B + t)$
R	$-(k-1) \cdot (n_B + r)$, $1 + (k-1) \cdot (n_B + r)$	$-k \cdot n_B, 1 + k \cdot n_B$

Table 2.2: The game played by parties

2.3.1 Parties' best replies

By comparing $\pi_B(L, R, t)$ and $\pi_B(L, L)$ I obtain party B's best reply if party A chooses policy L. It can be shown that, for every k and n_B, there exists a unique t, denoted by t_B, where $\pi_B(L, R, t)$ is equal to $\pi_B(L, L)$.[4] Moreover, party B's best reply is policy R whenever t is larger than $t_B = -n_B/(k + 1)$ (see Figure 2.1a).

The reasoning is similar for the comparison between $\pi_A(L, R, t)$ and $\pi_A(R, R)$, which gives party A's best reply if party B chooses policy R. In this case, party A's best reply is policy L whenever t is lower than $t_A = (1 - n_B)/(k + 1)$ (see Figure 2.2a).

I now compare $\pi_B(R, L, r)$ and $\pi_B(R, R)$ to obtain party B's best reply if party A chooses policy R. It can be shown that, if k is not sufficiently large, then $\pi_B(R, L, r)$ is strictly lower than $\pi_B(R, R)$ for all r, and party B's best reply is always policy R. If k is large enough, then, given n_B, there exists a unique r, denoted by r_B, where $\pi_B(R, L, r)$ is equal to $\pi_B(R, R)$. In those cases, party B's best reply is policy R whenever r is lower than $r_B = n_B/(k - 1)$ (see Figure 2.1b).

The reasoning is similar for the comparison between $\pi_A(R, L, r)$ and $\pi_A(L, L)$, which gives party A's best reply if party B chooses policy L. If k is not sufficiently large, then

[3]For convenience, and to emphazise that they depend on t and r, I will respectively write $\pi_J(L, R)$ and $\pi_J(R, L)$ as $\pi_J(L, R, t)$ and $\pi_J(R, L, r)$, for $J \in \{A, B\}$.

[4]All the best replies are further described in Appendix A.

$\pi_A(R, L, r)$ is strictly lower than $\pi_B(L, L)$ for all r, and party A's best reply is always policy L (see Figure 2.2b) . Otherwise, party A's best reply is policy L whenever r is larger than $r_A = -(1 - n_B)/(k - 1)$.

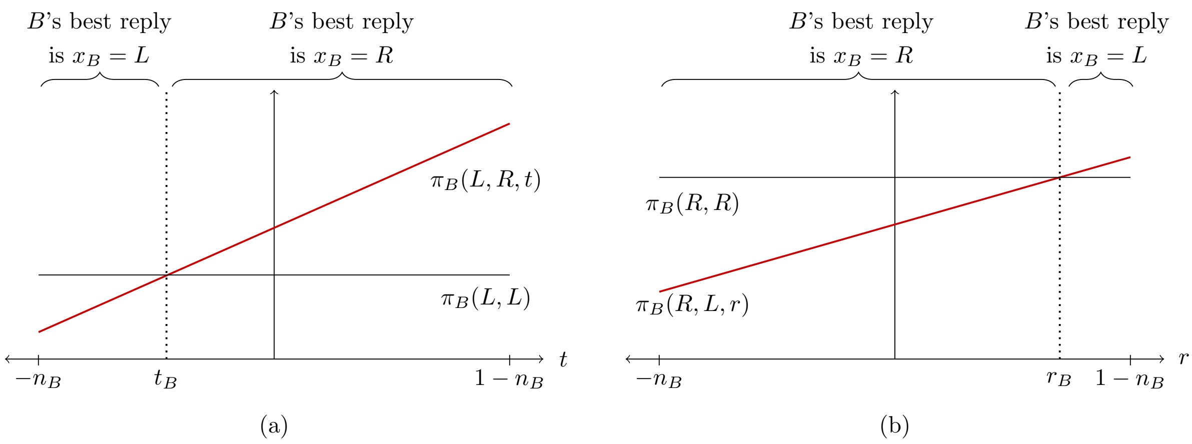


Figure 2.1: The left figure is $\pi_B(L, R, r)$ and $\pi_B(L, L)$ as a function of t. The right figure is $\pi_B(R, L, r)$ and $\pi_B(R, R)$ as a function of r.

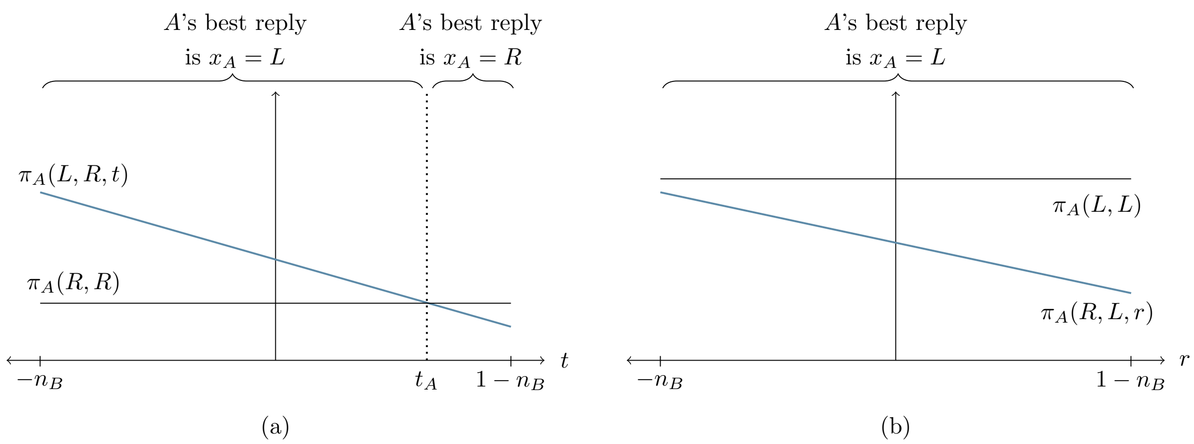


Figure 2.2: The left figure is $\pi_A(L, R, r)$ and $\pi_A(R, R)$ as a function of t. The right figure is $\pi_A(R, L, r)$ and $\pi_A(L, L)$ as a function of r when k is low.

2.3.2 Equilibria

The first result states that there is no pure strategy equilibrium in which each party supports its non-preferred policy.

Proposition 2.1. (R, L) *is never a pure strategy equilibrium.*

When k is sufficiently low and either $\pi_B(R, L, r)$ is below $\pi_B(R, R)$ for all r, or $\pi_A(R, L, r)$ is below $\pi_A(L, L)$ for all r (or both), then it is inmediate that (R, L) is not an equilibrium, because at least one of the parties has incentives to deviate.

Consider the case where k is sufficietly large, which means that $\pi_B(R, L, r)$ intersects $\pi_B(R, R)$ and $\pi_A(R, L, r)$ intersects $\pi_A(L, L)$. Then, it can be shown that r_B, which is the intersection above which party B does not deviate from (R, L), is strictly larger than r_A, which is the intersection below which party A does not deviate from (R, L). Hence, (R, L) is not an equilibrium, because one party always deviates. This is illustrated in Figure 2.3.

To understand the intuition, consider party B's best reply to $x_A = R$. If party B chooses policy R, then policy R is implemented with probability 1, and party B's mass of supporters remains equal. If party B chooses policy L, then the probability that policy R is implemented becomes lower. This is already costly for party B, because policy R is its preferred policy. Party B's best reply is policy L only if r is sufficiently large, and the gain of supporters compensates the drop of the probability that policy R is implemented. However, if party B gains supporters after (R, L), then party A loses the same amount. And party A is considering a similar dilemma, whether to choose policy L or policy R if $x_B = L$. Then, party A's best reply will never be policy R, which is party A's non-preferred policy, if this implies losing supporters.

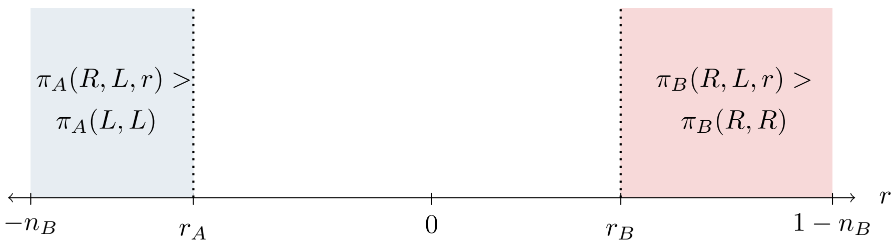


Figure 2.3: The cutoff r_B is strictly larger than r_A. Thus, If the best reply of party A to $x_B = L$ is $x_A = R$, then the best reply of party B to $x_A = R$ is not $x_B = L$.

Proposition 2.2. *If $t \in (t_B, t_A)$, then (L, R) is the unique pure strategy equilibrium.*

If t is larger than t_B, then the best reply of party B to $x_A = L$ is policy R, and if t is lower than t_A, then the best reply of party A to $x_A = R$ is policy L. Thus, if both conditions are satisfied, then (L, R) is an equilibrium (see Figure 2.4). Moreover, I will show in the Appendix that this equilibrium is unique.

To understand the intuition, consider party B's best reply to $x_A = L$. If party B chooses policy L, then policy L is implemented with probability 1, and party B's mass of supporters remains equal. But if party B chooses policy R, then policy R is implemented with positive probability. This has a positive effect over party B's payoff, because policy R is party B's preferred policy. If t is positive, then party B gains supporters after (L, R), which also has a positive effect. In this case, party B chooses policy R for sure.

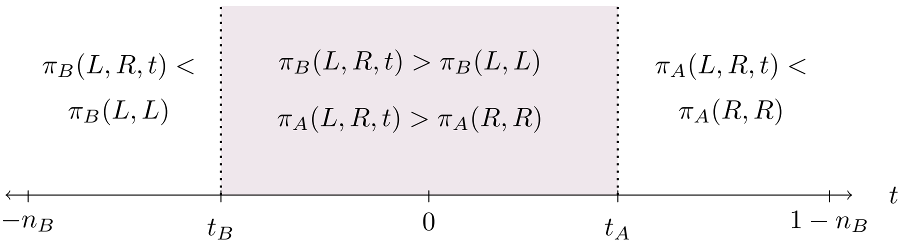


Figure 2.4: If t lies between t_A and t_B, then (L,R) is the equilibrium.

If t is negative, then party B loses supporters after (L,R), so the party faces a trade-off. Whenever t is larger than t_B, then the positive effect from the policy implemented dominates, and party B still chooses policy R, even if the amount of supporters decreases. Otherwise, party B deviates to policy L in order not lo lose supporters.

The reasoning is similar for party A's best reply. When t is positive, then party A loses supporters after (L,R). However, if t is lower than t_A, then the increase in the probability that policy L is implemented is sufficiently large, so party A does not deviate to policy R.

Proposition 2.3. *Suppose that $k \leq 1$. Then, the following statements hold.*

1. *A pure strategy equilibrium exists.*

2. *If $t < t_B$, then (L,L) is the unique equilibrium. If $t > t_A$, then (R,R) is the unique equilibrium.*

In the Appendix, I will show that if $k \leq 1$, then $\pi_B(R,L,r)$ is below $\pi_B(R,R)$ and $\pi_A(R,L,r)$ is below $\pi_A(L,L)$ for all r. The intuition is that parties are more concerned with the policy implemented than with the number of supporters. Thus, they never deviate from a consensus where their preferred policy is implemented, independently of the number of supporters that they may gain after that.

In those cases, the equilibrium only depends on the value of t. For example, suppose that t is very negative, such that party B deviates from (L,R). Since party A does not deviate from (L,L) to (R,L), then (L,L) becomes the equilibrium.

Proposition 2.4. *Suppose that $k > 1$. Then, the following statements hold.*

1. *If $t < t_B$ and $r > r_A$, then (L,L) is the unique equilibrium.*

2. *If $t > t_A$ and $r < r_B$, then (R,R) is the unique equilibrium.*

3. *If $t < t_B$ and $r < r_A$, or if $t > t_A$ and $r > r_B$, then there is no pure strategy equilibrium.*

In the Appendix, I will show that if k is large enough, then a party might deviate to (R, L) if the party gains a sufficiently large amount of supporters after that. If this happens, then a pure strategy equilibrium might not exist (see Figure 2.5).

For example, suppose that k is sufficiently large and that t is very negative. This means that party B deviates from (L, R) because it loses a very large amount of supporters. But this also means that party A deviates from (R, R) because it gains a very large amount of supporters. Now suppose that r is also very negative. By similar reasons, party B deviates from (R, L), and party A deviates from (L, L). Then, party B's best reply is always to support the opposite policy than party A, and party A's best reply is always to support the same policy than party B. Then, a pure strategy equilibrium does not exist.

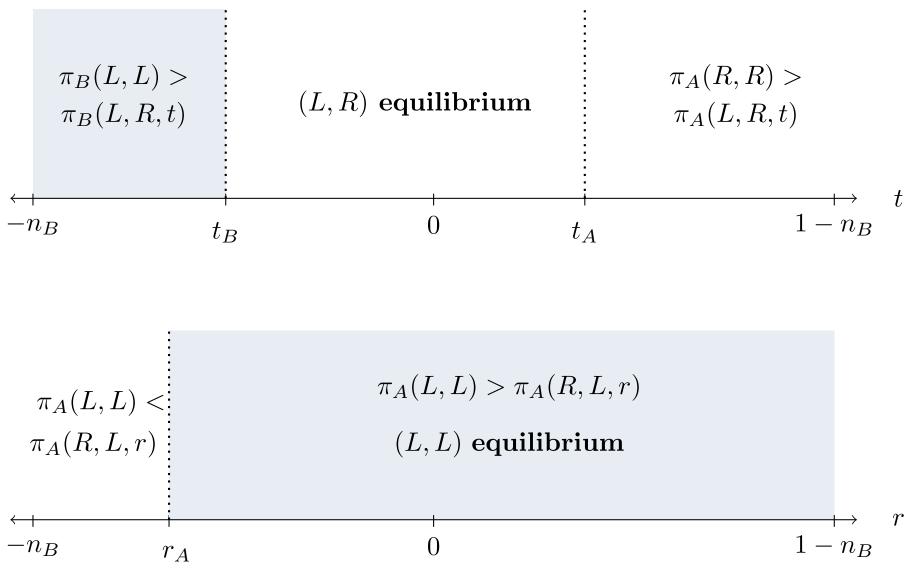


Figure 2.5: Suppose that $t < t_B$, such that party B deviates from (L, R). If $r > r_A$, then party A does not deviate from (L, L) and (L, L) becomes the equilibrium. Otherwise, a pure strategy equilibrium does not exist.

2.4 Characterization of citizens' behaviour

I now characterize how citizens naïvely update their political preferences after observing (x_A, x_B). Then, for a given initial distribution of preferences, I obtain n_B and $(\delta_1, \delta_2, \delta_4, \delta_4)$, and thus parties' decision in equilibrium. And then, I look how the updated distribution looks like.

I assume that citizens have separable and strict initial preferences over parties and policies. More specifically, I assume that every citizen i has a vector, say (y_i, z_i), such that the citizen prefers policy R to policy L if and only if $y_i > 0$, and she prefers party

B to party A if and only if $z_i > 0$. Citizens' vectors are randomly distributed along the Euclidean space (see Figure 2.6). Thus, n_A and n_B are the mass of vectors $z_i < 0$ and $z_i > 0$ respectively.

The exact location of a vector is interpreted as the degree of attachment to parties and to policies of the corresponding citizen. For example, consider citizens for whom $y_i > 0$ and $z_i < 0$ hold. Then, citizens whose y_i is sufficiently close to zero will correspond to citizens who support policy R, because $y_i > 0$, but are not excessively attached to that policy. Similarly, citizens whose z_i is sufficiently close to zero will correspond to citizens who support party A, because $z_i > 0$, but are not excessively attached to that party. The further y_i and z_i are from zero, the more attached to her preferred policy and party the citizen will respectively be. Thus, I measure a citizen's attachment to policies and to parties with $| y_i |$ and $| z_i |$ respectively.

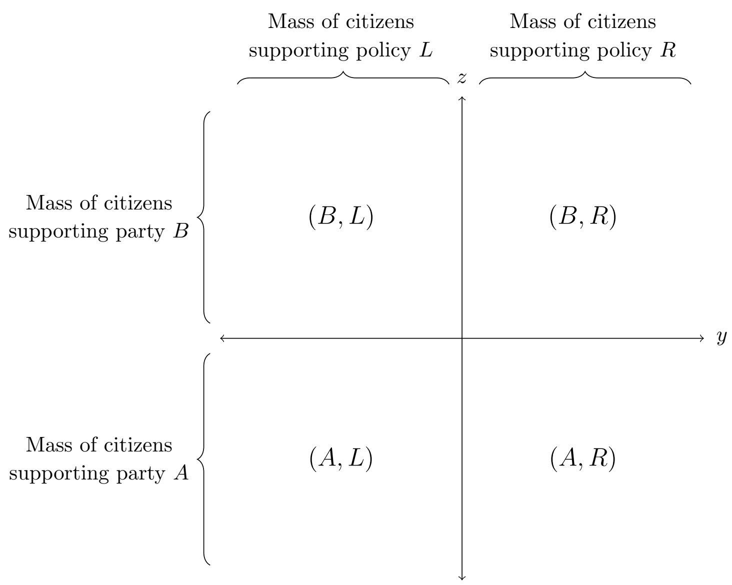
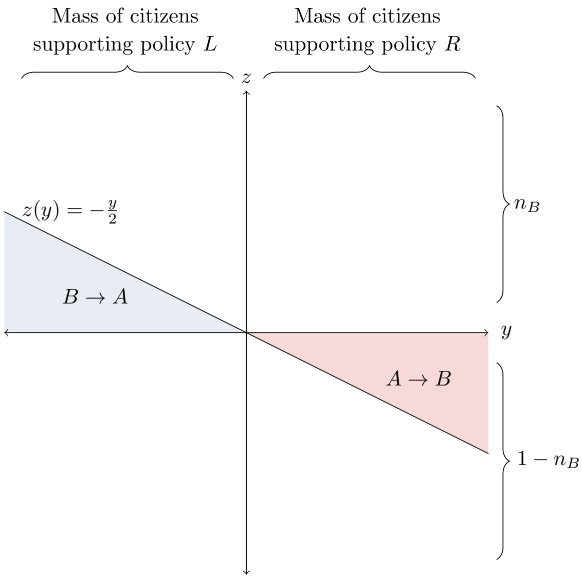


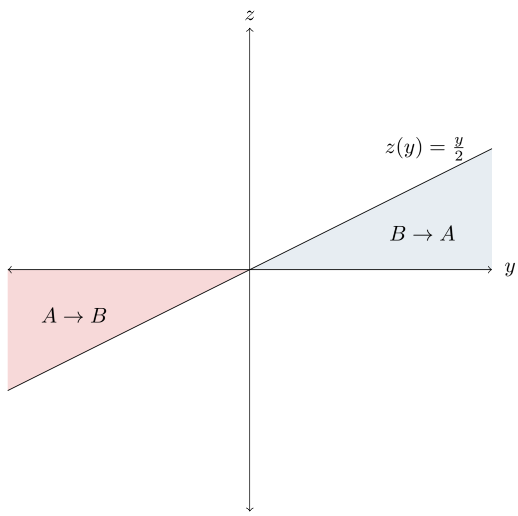
Figure 2.6: Citizens' vectors are distributed in the Euclidean space. For example, citizens whose vector is located in the first quadrant will prefer party B to party A and policy R to policy L.

2.4.1 Changes in party preferences

Following the model's assumption, if $x_A = x_B$, then citizens do not switch party preferences, so n_A and n_B remain equal. Suppose that $x_A \neq x_B$. Then, only citizens whose preferred policy is different than the policy supported by their preferred party might switch party preferences. For example, suppose that $(x_A, x_B) = (L, R)$. Then, only citizens preferring party B but policy L, or citizens preferring party A but policy R, are likely to start preferring the opposite party. The rest of citizens will keep preferring



(a) $(x_A, x_B) = (L, R)$



(b) $(x_A, x_B) = (R, L)$

Figure 2.7: (a) The line $z(y) = -\frac{y}{2}$ determines which citizens switch party preferences after (L, R). A mass $\delta_1 \cdot (1 - n_B)$ of citizens' vectors is located in $B \to A$, and a mass $\delta_2 \cdot n_B$ is located in $A \to B$. The reasoning is similar for (b), but with $\delta_3 \cdot (1 - n_B)$ and $\delta_4 \cdot n_B$ instead.

the same party than initially. This situation is captured by assuming that $\delta_1 + \delta_3 \leq 1$ and $\delta_2 + \delta_4 \leq 1$ hold.

I assume that only citizens who are sufficiently less attached to parties than to policies switch party preferences. In particular, if a citizen's relative attachment to the party with respect to the policy is below certain threshold, then the citizen switches party preferences. For example, suppose that $(x_A, x_B) = (L, R)$ and that the threshold is $\frac{1}{2}$. Then, citizens preferring party A ($z_i < 0$) and policy R ($y_i > 0$) for whom $\frac{|z_i|}{|y_i|} < \frac{1}{2}$ holds will start preferring party B. Thus, $\delta_1 \cdot (1 - n_B)$, which is the mass of citizens switching from party A to party B after (L, R), will be equal to the mass of vector satisfying the three conditions. Similarly, citizens preferring party B ($z_i > 0$) and policy L ($y_i < 0$) for whom $\frac{|z_i|}{|y_i|} < \frac{1}{2}$ holds will start preferring party A. Again, $\delta_2 \cdot n_B$, which is the mass of citizens switching from party B to party A after (L, R), will be the mass of vectors satisfying the three conditions. The reasoning is similar for when $(x_A, x_B) = (R, L)$. Both cases are illustrated in Figure 2.7.

Citizens' party preferences are sufficient for determining parties' choices in equilibrium. In the distribution shown in Figure 2.8a, both parties have the same initial amount of supporters (*i.e*, $n_B = 0.5$), and parties do not lose or gain supporters after (L, R) (*i.e*, $\delta_1 \cdot (1 - n_B) = \delta_2 \cdot n_B = 0.2$). In this distribution, the equilibrium is that each party supports its preferred policy, because they maximize the probability that the respective

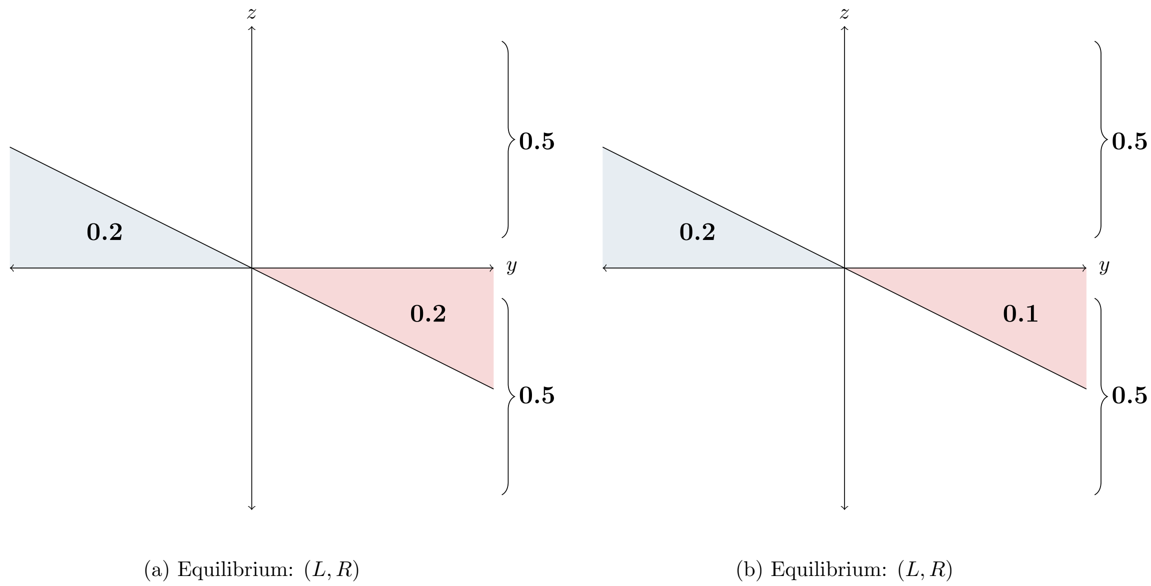


Figure 2.8: Two distributions where (L, R) is the pure strategy equilibrium, with $k = 3$.

policy is implemented. In fact, whenever $\delta_1 \cdot (1 - n_B) - \delta_2 \cdot n_B = 0$ holds, then (L, R) is the equilibrium. This can be seen in Figure 2.4, where zero always belongs to the interval delimited by the two cutoffs, independently of the value of the parameters.

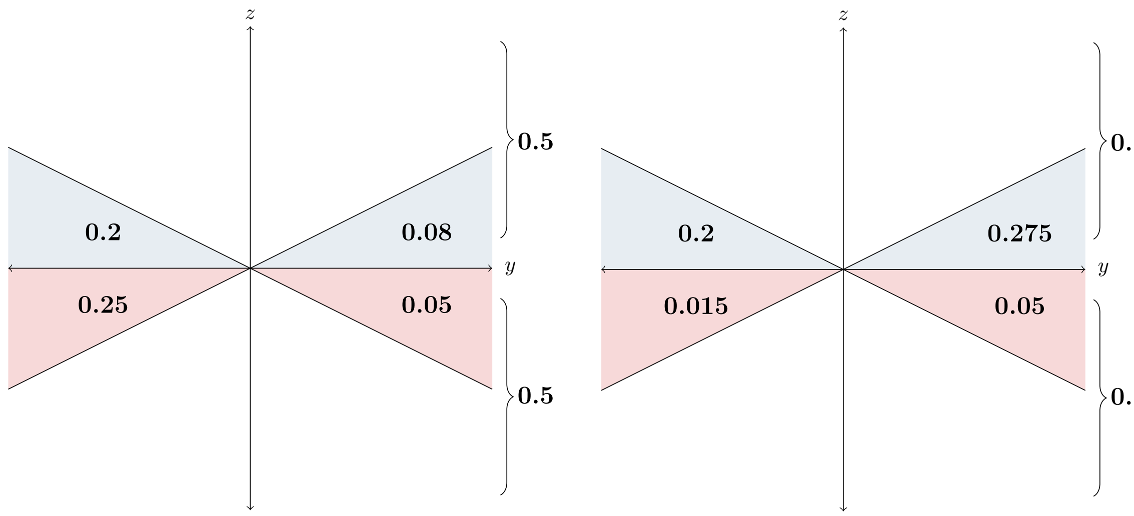
Consider a decrease of $\delta_1 \cdot (1 - n_B)$. That is, either party A starts with a lower mass of supporters, or a lower share of supporters switches to party B after (L, R). The latter case is illustrated in Figure 2.8b, where $1 - n_B$ remains equal to 0.5 but $\delta_1 \cdot (1 - n_B)$ decreases to 0.1. Because $\delta_1 \cdot (1 - n_B)$ is lower than $\delta_2 \cdot n_B$, then party A's mass of supporters increases after (L, R), so party A does not deviate. By implication, party B's mass of supporters decreases after (L, R). However, in this distribution, the expected benefits from implementing policy R compensates party B's loss of supporters, so B neither deviates from (L, R).[5]

When $\delta_1 \cdot (1 - n_B)$ is excessively low compared to $\delta_2 \cdot n_B$, then party B deviates from (L, R) by supporting policy L. This is the case of Figure 2.9a, where $\delta_1 \cdot (1 - n_B)$ has decreased to 0.05. Because $\delta_4 \cdot n_B$ is lower than $\delta_3 \cdot (1 - n_B)$, then party A's mass of supporters decreases after (R, L). As explained after Proposition 4.1, this guarantees that party A does not deviate to (R, L). Thus, in this distribution, both parties choose policy L in equilibrium.[6]

In the distribution shown in Figure 2.9b, there is no pure strategy equilibrium. Observe that both $\delta_2 \cdot n_B$ and $\delta_4 \cdot n_B$ are large enough. In constrast, $\delta_1 \cdot (1 - n_B)$ and $\delta_3 \cdot (1 - n_B)$ are very low. Consequently, both t and r are sufficiently large. Party B devi-

[5]Observe that $\delta_1 \cdot (1 - n_B) - \delta_1 \cdot n_B = -0.1$, and that $-\frac{n_B}{k+1} = -0.125$, given that $k = 3$ and $n_B = 0.5$. Since $-0.1 > -0.125$, then, by Proposition 4.2, (L, R) is the equilibrium.

[6]Observe that $\delta_1 \cdot (1 - n_B) - \delta_2 \cdot n_B = -0.15$ and $\delta_3 \cdot (1 - n_B) - \delta_4 \cdot n_B = 0.17$. Since $-\frac{n_B}{k+1} = -0.125 > -0.15$ and $-\frac{1-n_B}{k-1} = -0.25 < 0.17$, then, by Proposition 4.4, (L, L) is the equilibrium.



(a) B deviates from (L, R)
Equilibrium: (L, L)

(b) A deviates from (L, L)
No pure strategy equilibrium

Figure 2.9: In (a), the equilibrium is (L, L). In (b), a pure strategies equilibrium does not exist.

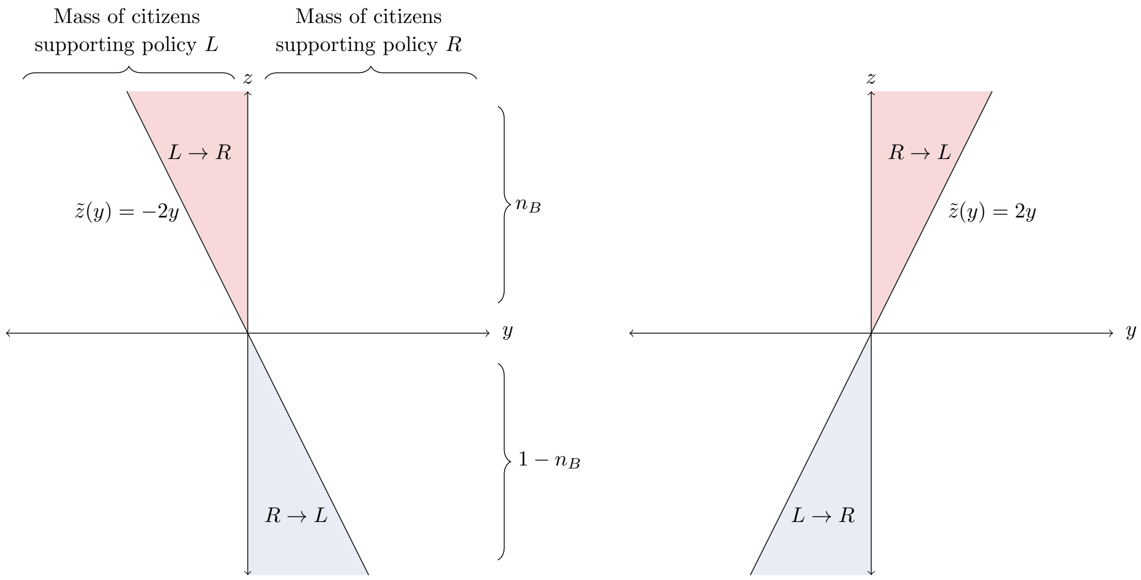
ates from (L, R) and (R, L), because it loses too many supporters in both cases. At the same time, party A deviates from (L, L) and (R, R), because it gains enough supporters in both cases. Parties play a "cat-and-mouse" game, and a pure strategies equilibrium does not exist.[7]

2.4.2 Changes in policy preferences

Suppose now that citizens might also switch policy preferences if $x_A \neq x_B$ and their preferred party does not support their preferred policy. In particular, suppose that if a citizen's relative attachment to the party with respect to the policy is above certain threshold, then the citizen switches policy preferences. I set this threshold strictly larger than the threshold below which citizens switch party preferences. This is to avoid the situation in which a citizen switches both party and policy preferences.[8] For example, suppose that $(x_A, x_B) = (L, R)$ and that the threshold is 2. Then, citizens preferring party A ($z_i < 0$) and policy R ($y_i > 0$) for whom $\frac{|z_i|}{|y_i|} > 2$ holds will start preferring policy L. Similarly, citizens preferring party B ($z_i > 0$) and policy L ($y_i < 0$) for whom $\frac{|z_i|}{|y_i|} > 2$ holds will start preferring policy R. The reasoning is similar for when $(x_A, x_B) = (R, L)$. Both cases are illustrated in Figure 2.10.

[7] Observe that $\delta_1 \cdot (1 - n_B) - \delta_2 \cdot n_B = -0.15$ and $\delta_3 \cdot (1 - n_B) - \delta_4 \cdot n_B = -0.26$. Since $-\frac{n_B}{k+1} = -0.123 > -0.15$ and $-\frac{1-n_B}{k-1} = -0.25 > -0.26$, then, by Proposition 4.4, a pure strategies equilibrium does not exist.

[8] For example, it would be irrational that a citizen stops supporting her initial preferred policy because her initial preferred party does not supports that policy, and, at the same, she also stops supporting her initial preferred party because that party does not support her initial preferred policy.



(a) $(x_A, x_B) = (L, R)$

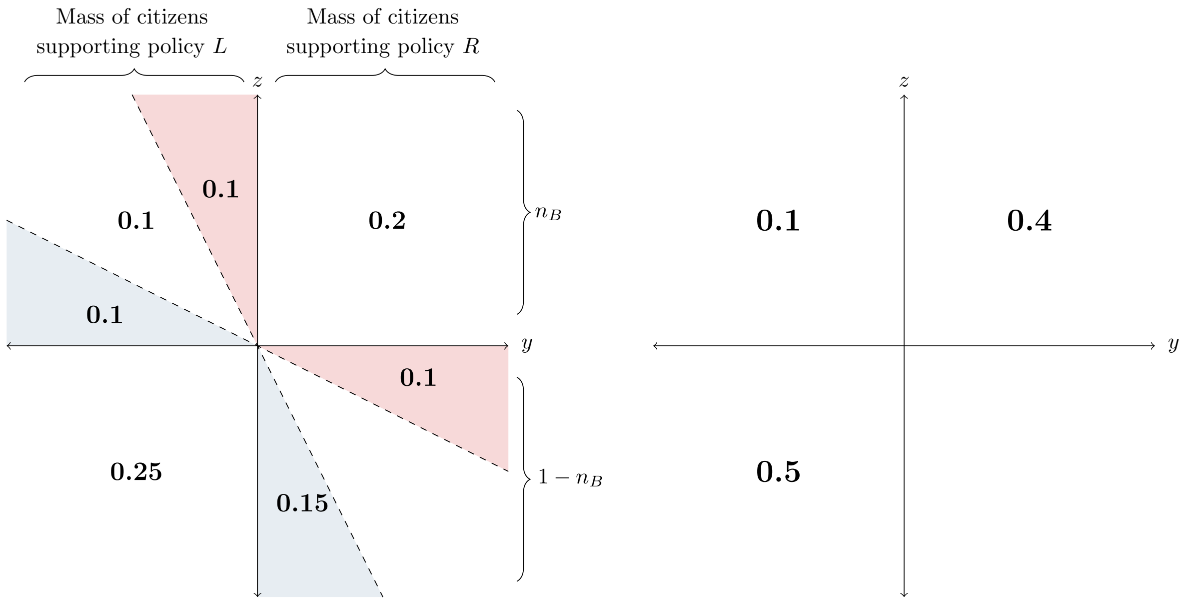
(b) $(x_A, x_B) = (R, L)$

Figure 2.10: The functions $\tilde{z}(y) = -2y$ and $\tilde{z}(y) = 2y$ determine which citizens switch policy preferences after (L, R) and (R, L) respectively.

By combining both types of preferences' changes we can illustrate how a distribution is distorted by parties' choices. Suppose that, once (x_A, x_B) is chosen, citizens' vectors are relocated according to their final preferences. For example, if a citizen initially preferring party A switches to party B, then her z_i becomes strictly positive. Or, if a citizen initially preferring policy R switches to policy L, then her y_i becomes strictly negative. Thus, we can now compare the initial distribution with the final distribution of preferences.

For example, in the distribution shown in Figure 2.11a, there is at least twenty percent of citizens in each quadrant. Thus, the correlation between citizens' party and policy preferences is not excessively large. The equilibrium is (L, R), because $\delta_1 \cdot (1 - n_B) - \delta_2 \cdot n_B = 0$ holds, and thus citizens' preferences are affected. For example, ten percent of citizens switch from party A to party B, and fifteen percent of citizens switch from policy R to policy L. The final distribution is represented in Figure 2.11b. Interestingly, the correlation between party and policy preferences has increased. Now all citizens preferring policy R also prefer party B, and around 83 percent of citizens preferring policy L also prefer party A. The distribution is better sorted due to party confrontation.

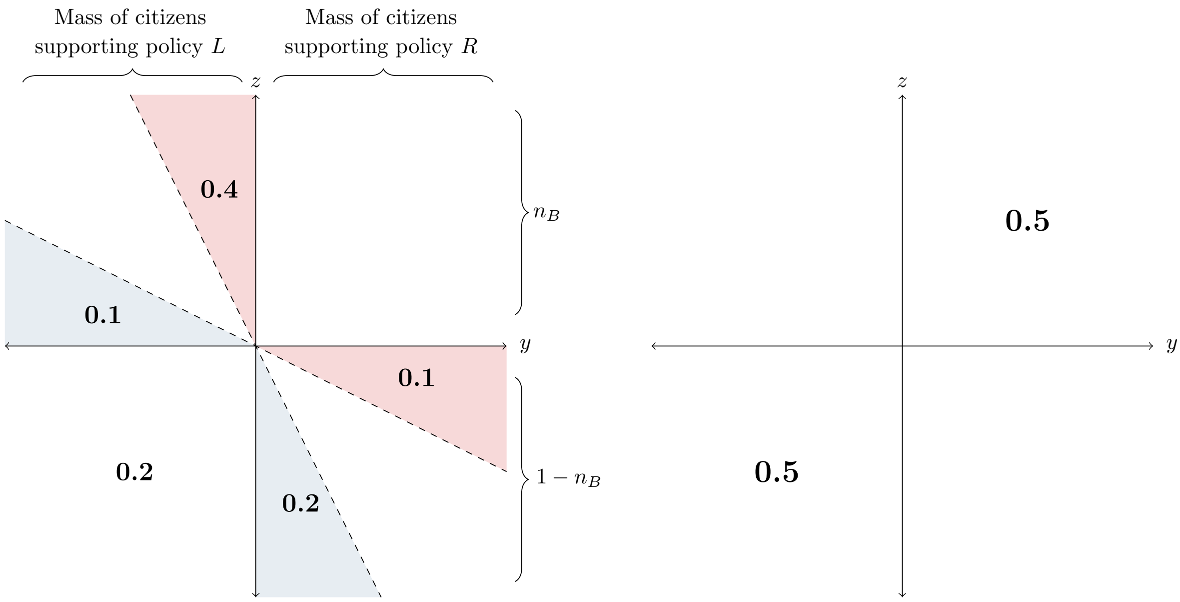
Finally, consider the distribution in Figure 2.12a. There is 70 percent of citizens who prefer policy L, but the equilibrium is (L, R). This is because a sufficiently large amount of citizen switch to policy R after party B supports that policy. The final distribution, which is represented in Figure 2.12b, is not only better sorted but also more divided, because half of the citizens prefer each policy. Again, this is due to party confrontation.



(a) Equilibrium: (L, R)

(b) The distribution after (L, R)

Figure 2.11: In (a), citizens' party and policy preferences are uncorrelated. In (b), most citizens belong either to the first or the third quadrant. The distribution in (b) is said to be better sorted than the distribution in (a).



(a) Equilibrium: (L, R)

(b) The distribution after (L, R)

Figure 2.12: In (a), there is a mass of 0.7 citizens prefererring policy L. In (b), citizens are perfectly divided.

2.5 Comparative Statics

To describe the effects of the parameters in the equilibria, I simulate the model. I fix n_B and k and I construct grids of $\delta_1, \delta_2, \delta_3$ and δ_4, setting $\delta_1 + \delta_3 \leq 1$ and $\delta_2 + \delta_3 \leq 1$. Then, I analyse how an increase of n_B or k affects the distribution of equilibria.[9]

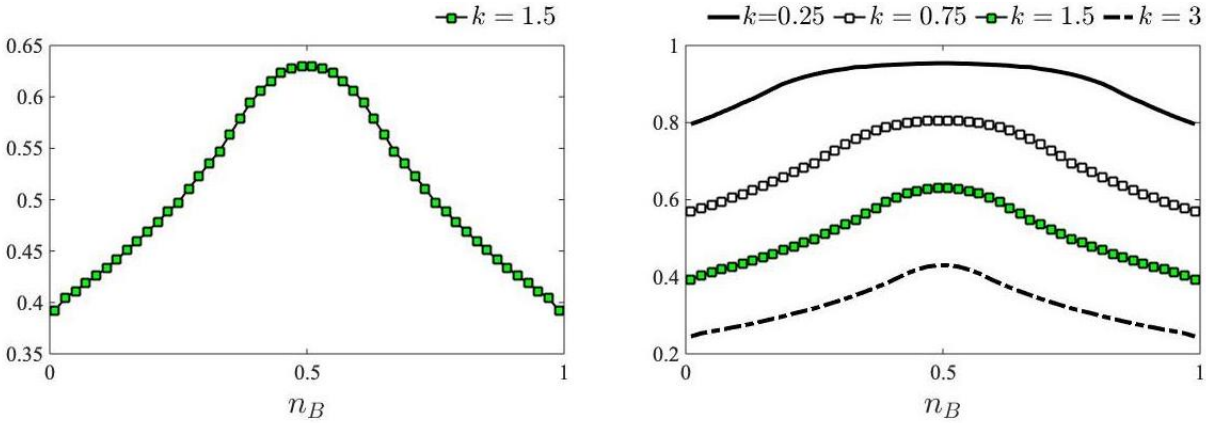


Figure 2.13: How n_B affects the proportion of cases where (L, R) is the equilibrium. The proportion is maximized at $n_B = 1/2$. As observed in the right picture, the function moves downwards if k increases.

In Figure 2.13, I show how the proportion of cases where (L, R) is the equilibrium depends on n_B. This proportion is always maximized at $n_B = 1/2$. To understand this, consider an increase of n_B. Because $\delta_2 \cdot n_B$ increases and $\delta_1 \cdot (1 - n_B)$ decreases, then $t < 0$ holds for a larger set of (δ_1, δ_2). That is, there are more values of (δ_1, δ_2) where party A gains supporters and party B loses supporters after (L, R). This has two effects over the proportion of cases where (L, R) is the equilibrium. On the one hand, a positive effect because there are less cases where party A deviates from (L, R). On the other hand, a negative effect because there are more cases where party B deviates from (L, R). If $n_B < 1/2$, then the first effect dominates. That is, there are more new values of (δ_1, δ_2) where party A does not deviate from (L, R) than new values of (δ_1, δ_2) where party B deviates from (L, R). Consequently, the proportion increases. If $n_B > 1/2$, then the second effect dominates, and the proportion decreases. The maximum is reached at

[9]For the first two simulations, which are presented in Figures 2.13 and 2.14, I fix k and I build grids of δ_1, δ_2 and n_B of 50 elements each, with increments of 0.02. In total, there are 2500 combinations of (δ_1, δ_2). For every n_B, I look for the pairs of (δ_1, δ_2) that satisfy the sufficient conditions of Proposition 4.2 and 4.3 respectively. Then, I vary k and I do the same exercise. For the third simulation, presented in Figure 2.15, I work with the same grid of n_B and I also fix k. In this case, I build grids of δ_1, δ_2, δ_3 and δ_4 of 20 elements each, with increments of 0.05. After applying $\delta_1 + \delta_3 \leq 1$ and $\delta_2 + \delta_4 \leq 1$, there are 44100 combinations of $(\delta_1, \delta_2, \delta_3, \delta_4)$. For every n_B, I look for the values of $(\delta_1, \delta_2, \delta_3, \delta_4)$ that satisfy the sufficient conditions of Proposition 4.4. Then, I vary k and I do the same exercise. For the last simulation, presented in Figure 2.16, I fix n_B and I build a grid of k (going from 1.1 to 8) made of 50 elements, with increments of approximately 0.2. Then, for every k, I look for the values of $(\delta_1, \delta_2, \delta_3, \delta_4)$ that satisfy the sufficient conditions of Proposition 4.4. Finally, I vary n_B and I do the same exercise.

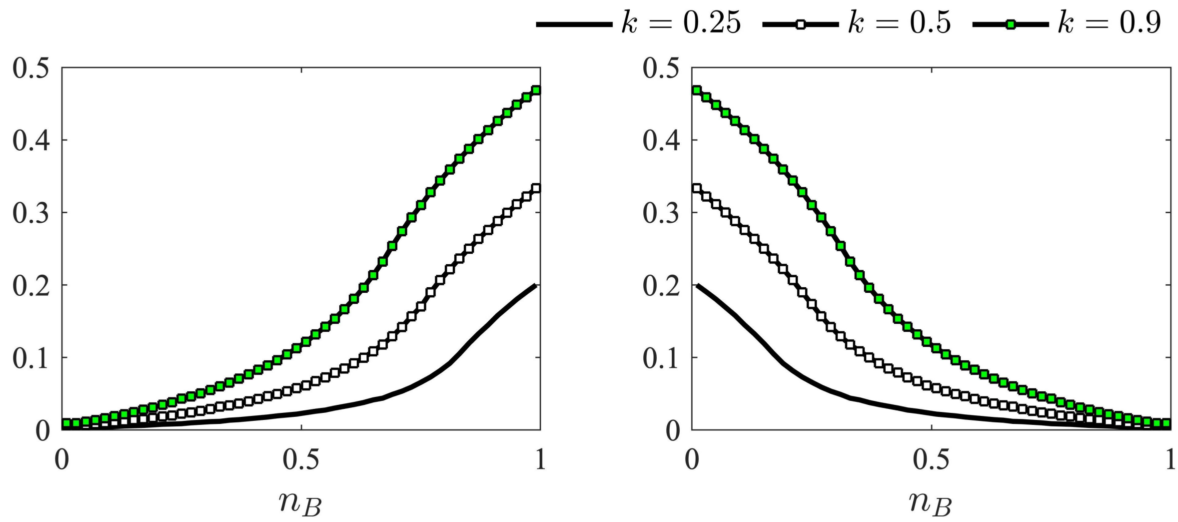


Figure 2.14: How n_B affects the proportion of cases where (L, L) (left picture) and (R, R) (right figure) is the equilibrium, when $k < 1$ holds. Functions are symmetric, and they move upwards if k marginally increases.

$n_B = 1/2$, where both effects are equal.

Similarly, the proportion increases as k becomes lower. This effect can be observed from the derivatives of t_B and t_A, which are positive and negative respectively. That is, if k decreases, then there are more values of (δ_1, δ_2) for which t lies between the two cutoffs, and thus, for which (L, R) is the equilibrium. Intuitively, a drop of k can be interpreted as parties becoming more concerned for the policy implemented. Then, parties will be more willing to lose supporters in order to have a chance of implementing their preferred policy.

The proportion of cases where (L, L) is the equilibrium increases with n_B if $k < 1$ holds. This is shown in the left picture of Figure 2.14. When n_B increases, then there are more values of (δ_1, δ_2) where party B deviates from (L, R), and when this happens, then the equilibrium becomes (L, L), because $k < 1$ holds. The proportion also increases with k. This is because party B is less willing to lose supporters, so the party deviates to (L, L) more frequently. Symmetrically, the proportion of cases where (R, R) is the equilibrium decreases with n_B and increases with k (right picture).

When $k > 1$ holds, then the effects of k and n_B can be positive or negative, as shown in Figures 2.15 and 2.16. The reason is that both k and n_B affect the proportions in two ways. For example, consider the proportion of cases where (L, L) is the equilibrium (the case of (R, R) is symmetric). If either n_B or k increases, then there are new values of (δ_1, δ_2) where party B deviates from (L, R) to (L, L), which increases the proportion. However, since $k > 1$, then there are also new values of (δ_3, δ_4) where party A deviates from (L, L) to (R, L), in which case an equilibrium in pure strategies does not exist. The

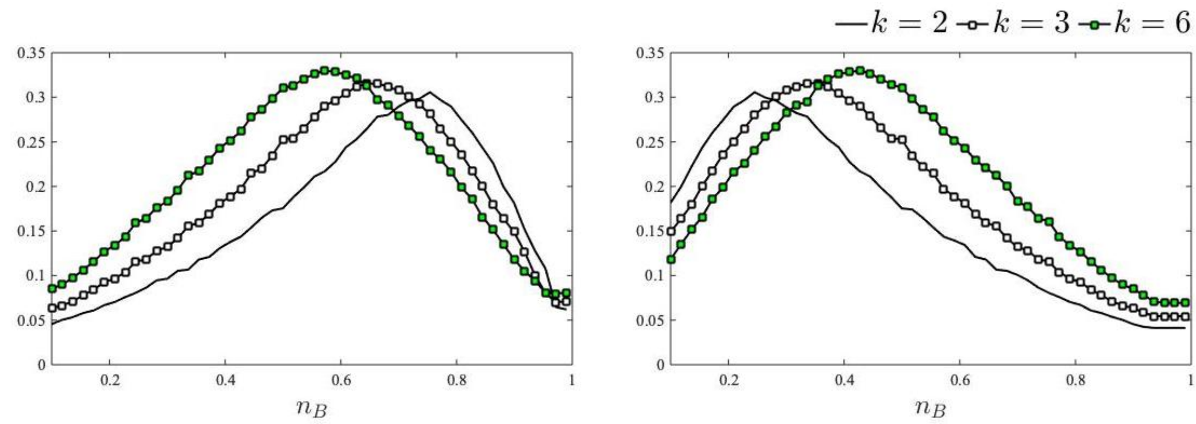


Figure 2.15: How n_B affects the proportion of cases where (L, L) (left picture) and (R, R) (right picture) is the equilibrium, when $k > 1$ holds. Whether the proportions are increasing or decreasing is determined by the value of k.

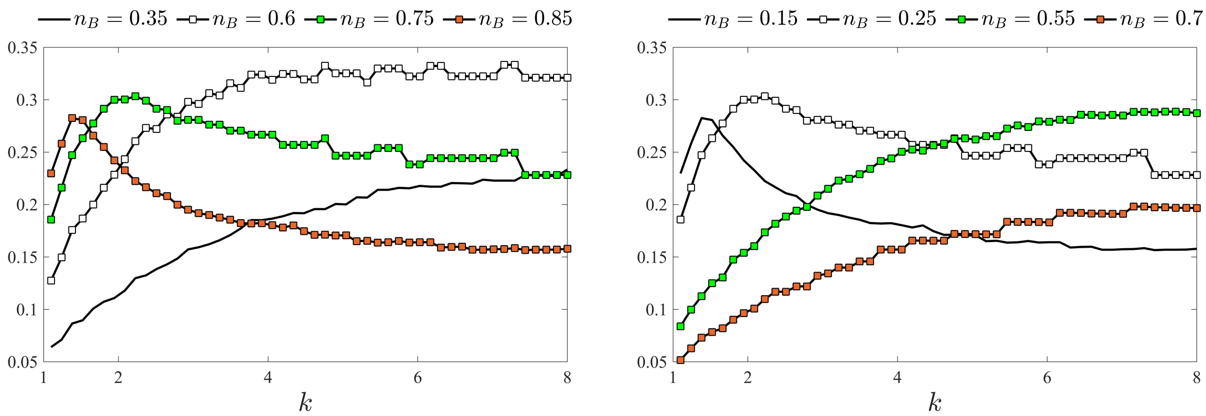


Figure 2.16: How k affects the proportion of cases where (L, L) (in the left) and (R, R) (in the right) is the equilibrium, when $k > 1$ holds. The proportion of (L, L) is increasing if n_B is low enough. Otherwise, it is increasing if k is sufficiently low. The effect is symmetric for (R, R).

final effects of n_B and k will depend on both parameters. For example, the proportion is increasing with n_B until some threshold, from which it becomes decreasing. If k increases, then the threshold decreases, so the effect of n_B turns negative faster. This is illustrated in the left picture of Figure 2.15.

The effect of k is better observed in the left figure of Figure 2.16. If k increases, then party A is less willing to lose supporters, so it deviates to (L, L) more frequently. However, party A will also prefer (R, L) to (L, L) more frequently, particularly when it gains a sufficienty big amount of supporters. The second effect dominates only if both n_B and k are sufficiently large, in which case the proportion will be decreasing with k.

2.6 Final remarks

I build a model of party competition setting the traditional approach apart. Parties directly compete for the policy implemented and for the amount of citizens supporting them, and citizens are naïve. The policy space is binary, so the question of interest is whether parties support different policies or not, rather than what is the ideological distance between the policies chosen.

On the one hand, I show that party confrontation is driven by parties' ideology. Because parties might have the chance to implement the policy chosen, then parties always have an incentive, albeit very small, to support their own preferred policy. This incentive promotes party confrontation because parties prefer opposite policies.

On the other hand, I also show that a party might find confrontation profitable because the party would "catch" supporters from the opponent. My result suggests that there is always one party, which is the one that catches supporters, that pushes confrontation. The other party, that loses supporters, remains confronted because of the chance of implementing its preferred policy. When its loss of supporters is excessively large, then the party prefers to support its non-preferred policy, and the equilibrium, if its exists, is a consensus.

I also characterize citizens' preferences and how parties influence them, and I combine this characterization with parties' equilibrium. My results suggest that party sorting, which is a very observed phenomenon, is a consequence of parties becoming polarized. They also suggest that parties do not necessarily need a large amount of citizens in favor of a policy to support that policy, because they are able to manipulate citizens' preferences. This goes in line with the feedback effect observed in (Sánchez, 2020a).

I now comment some extensions. The most inmediate one is related with the probability that each policy is implemented, that here only depends on the amount of supporters. This may be restrictive because citizens have also policy preferences. For instance, suppose that a citizen's preferred party does not support her preferred policy, but the other party does. But suppose that the citizen does not switch either of her preferences.[10] Then, the effect of this citizen on the policy implemented is ambiguous. In the current model, this citizen will "benefit" the policy supported by her preferred party (her non-preferred policy), but it may also be the other way. However, I conjecture that a small variation of the functional form of the probability will keep track of this issue. Three other extensions are the number of periods, the number of parties and the number of issues. About the last extension, there is a literature that investigates how parties behave with respect to

[10]In Figure 2.11b, I refer to the mass of 0.1 citizens that prefer policy L and party B even if $(x_A, x_B) = (L, R)$ $((-,+)$ quadrant).

multiple issues at the same time.[11] There are two opposite phenomena: the "conflict displacement" and the "conflict extension". According to the first approach, parties are only confronted on one issue at a time, while in the second approach they might be confronted on several issues. Extending the model to more than one issue might shed light on what drives the two phenomena. I will let all those extensions for future work.

[11]See for instance Layman et al. (2006).

Chapter 3

The Influence of Parties' Influence: A Feedback Effect

3.1 Introduction

A broad set of social issues have evolved as follows. First, the issue is latent, in the sense that neither parties nor citizens are discussing about it. Then, the issue becomes current, and during some years it dominates the political scene. Eventually, the issue turns to be latent, but now the policy implemented has changed. For example, the issues of women's suffrage, racial desegregation, same-sex marriage and, in some cases, abortion, have shown this pattern (Sánchez, 2020b). In all those cases, two other regularities are observed. First, the amount of citizens in favor of one of the policies has been increasing over time. And second, parties have moved from a political consensus to confrontation, and later from confrontation to another consensus.

In this dynamic setting, the relationship between parties and citizens is endogenous. That is, parties' choices are affected by citizens' preferences, and at the same time, citizens' preferences are influenced by parties' choices (Layman et al., 2006). Although traditional models have largely addressed the first effect, they generally say little about the second one.[1] And yet, it has been widely shown that parties have an strong effect over the public opinion. For example, parties bring issues into public discussion (Carmines and Stimson (1986), Zaller (1992)), and they influence whether a new policy is implemented or not, which also affects citizens' opinions (Flores and Barclay, 2016). Moreover, parties may also distort how citizens form their own political views. In a nutshell, citizens tend to give more weight to the arguments that reinforce the position of their most-preferred party, a phenomenon that has been called "partisan motivated reasoning" (Druckman et al. (2013), Bolser et al. (2014)).

In this paper, I develop a dynamic model where parties are able to influence citizens' preferences through their policy decisions. I start from the base model presented in Sánchez (2020b), where parties have to decide which policy they support, given citizens' preferences. Then, this game is repeated during a finitely number of periods, and I assume that citizens' preferences change over time. The key point of the model is that parties' choices affect future citizens' preferences. In particular, I assume that the more parties support one policy at some period, the more citizens will be in favor of that policy in the following one.

I identify a feedback effect derived from parties' influence. That is, I show that the fact that parties are able to influence citizens' preferences actually affects parties' behaviour. In particular, I find situations where parties would have been in consensus,

[1] Repeated election models generally assume that voters' policy preferences are fixed over time, so they do not exploit parties' influence over them (see Boylan and McKelvey (1995), Duggan and Fey (2006) or Banks and Duggan (2008), among others). To the best of my knowledge, only Piketty (2000) allows for voters' opinions to change. However, he does not study parties' influence over voters' opinions, but the influence of the voters themselves over the rest of opinions.

but because they are able to influence future citizens' preferences, they end up confronted. Interestingly, this effect can shed light on the question of whether U.S. parties are more polarized or not than citizens. In fact, while party polarization is commonly accepted in the literature, popular polarization is usually very questioned. Some authors claim that U.S. citizens are also polarized (Abramowitz and Saunders, 2008), but many others argue that polarization is exaggerated (see Fiorina et al. (2005), Ansolabehere et al. (2006) or Fiorina (2018), among others). In this paper, I suggest that there may exist an "excess" of party polarization due to parties' influence over citizens' preferences, which may help understanding this paradox.[2]

The benchmark model is already studied in Sánchez (2020b). In the model, there are two parties, a set of citizens, and one single issue with two possible policies (the left-wing and right-wing policy). Parties have to decide which policy they support taking into account citizens' preferences, and one policy implemented after parties' decisions are made. Parties are concerned with the policy implemented and with their popularity, and their payoffs depend on the share of citizens in favor of each policy. In equilibrium, parties will support different policies only if there is not an excessively large amount of citizens in favor of one of the policies. Otherwise, both parties will support the policy preferred by the majority of citizens.

In this paper, parties play repeatedly the previous game during a finitely number of periods. Following Sánchez (2020b), I assume that the share of citizens in favor of the left-wing policy is increasing over time. Moreover, I assume that parties are able to affect the rate of increase. In particular, the more parties support the left-wing policy at some period, the larger will be the share of citizens in favor of that policy in the next period. For the sake of comparison, I also consider the case where parties do not affect citizens' preferences, and I look for the subgame perfect equilibrium in both settings.

First, I solve the two-periods game using backwards induction. In the second period, parties only consider their inmediate payoffs, so they behave like in the base model. However, in the first period, parties also consider their future payoffs, which may depend on the first period's choices through parties' influence. When parties do not influence citizens' preferences, then they behave like in the second period. But when they do, then the party preferring the left-wing policy has an additional incentive to support it, because then the share of citizens in favor of the policy will be larger in the second period, and its future payoff will also increase. If in the first period there are very few citizens in favor of the left-wing policy, then the party faces an intertemporal trade-off, because by choosing that policy its inmediate payoff would decrease. I will show that whenever the

[2]This apparent disconnection between U.S. parties and citizens has been called "the central puzzle of modern American politics" (Fiorina and Levendusky, 2006).

influence of the party is large enough, then the party chooses the left-wing policy. And if this happens, then parties will be confronted in the first period, even though in the non-influence setting both parties would have chosen the right-wing policy.

Last, I extend the model for an arbitrarily finite number of periods. My preliminary results suggest that parties' equilibrium behaviour coincide with the patterns observed in Sánchez (2020b). That is, first parties support the right-wing policy. At some point, one of the parties start supporting the left-wing policy, and parties are confronted for some periods. Eventually, the other party also supports the left-wing policy, and parties remain in consensus for the rest of periods. In this case, the larger is the influence over citizens' preferences, the earlier the first party will start supporting the left-wing policy. As a consequence of this, the share of citizens in favor of the left-wing policy will increase faster, and the second consensus, the one where both parties choose that policy, will also be reached earlier.

The rest of the paper proceeds as follows. In Section 3.2, I describe the model. In Section 3.3, I solve the two-periods model. In Section 3.4, I extend the model to more than two periods. And in Section 3.5, I conclude. All proofs are in Appendix B.

3.2 The Model

The society is made of a set of citizens and two political parties, A and B. There is a single issue with two possible policies, L and R. During two periods, parties have to decide which of the policies they support. I will denote with $(x_A^t, x_B^t) \in \{L, R\}^2$ the policies supported by parties at time $t \in \{1, 2\}$.

Citizens have preferences over the policies. I define the public opinion at time t as a parameter $\alpha_t \in (0, 1)$ denoting the share of citizens in favor of policy R at t. Citizens' preferences are strict, so $1 - \alpha_t$ is the share of citizens in favor of policy L.

Every period, one of the policies is implemented after (x_A^t, x_B^t) is chosen. If $x_A^t = x_B^t$, then this common policy is implemented with probability one. If $x_A^t \neq x_B^t$, then I assume that policy R is implemented with probability α_t.

3.2.1 Parties' payoffs

Following Sánchez (2020b), every period parties receive a payoff that depends on (x_A^t, x_B^t) and α_t. On the one hand, parties have policy preferences. In particular, I assume that party A prefers policy L and party B prefers policy R. Parties receive a payoff of value 1 only if their preferred policy is implemented.

On the other hand, parties are also concerned with their popularity. In particular, a payoff of value 1 is allocated between parties depending on how popular they are. If $x_A^t = x_B^t$, then both parties receive 1/2. If $x_A^t \neq x_B^t$, then the party supporting policy R receives α_t, and the party supporting policy L receives $1 - \alpha_t$. In Table 3.1, I show parties' payoffs at t for every (x_A^t, x_B^t).

(x_A, x_B)	Popularity payoff	Policy payoff	Total payoff at t
(R, R)	$1/2, 1/2$	$0, 1$	$1/2, 3/2$
(L, R)	$1 - \alpha_t, \alpha_t$	$1 - \alpha_t, \alpha_t$	$2 \cdot (1 - \alpha_t), 2 \cdot \alpha_t$
(R, L)	$\alpha_t, 1 - \alpha_t$	$1 - \alpha_t, \alpha_t$	1, 1
(L, L)	$1/2, 1/2$	$1, 0$	3/2, 1/2

Table 3.1: How parties' payoffs are constructed.

3.2.2 Opinion dynamics

The value of α_2 depends on both α_1 and (x_A^1, x_B^1). I will assume three conditions for the form of $\alpha_2(x_A^1, x_B^1, \alpha_1)$. First, the share of citizens in favor of policy L is always increasing over time. Namely,

$$\alpha_2(x_A^1, x_B^1, \alpha_1) < \alpha_1 \text{ for all } (x_A^1, x_B^1). \tag{3.1}$$

Second, the size of the increase is determined by (x_A^1, x_B^1). In particular, the more parties support policy L at $t = 1$, the more citizens will be in favor policy L at $t = 2$. This assumption is identified with the following two conditions.[3]

$$\alpha_2(L, L) < \alpha_2(L, R) < \alpha_2(R, R) \tag{3.2}$$

$$\alpha_2(R, L) = \alpha_2(L, R). \tag{3.3}$$

3.3 Equilibrium Analysis

Given α_1, the strategy of party $J \in \{A, B\}$ is the policy chosen in the first period and the policy chosen in every subgame of the second period. The game in extensive form is displayed in Figure 3.1.

[3] Through the rest of the paper, the function $\alpha_2(x_A^1, x_B^1, \alpha_1)$ will be abbreviated as $\alpha_2(x_A^1, x_B^1)$.

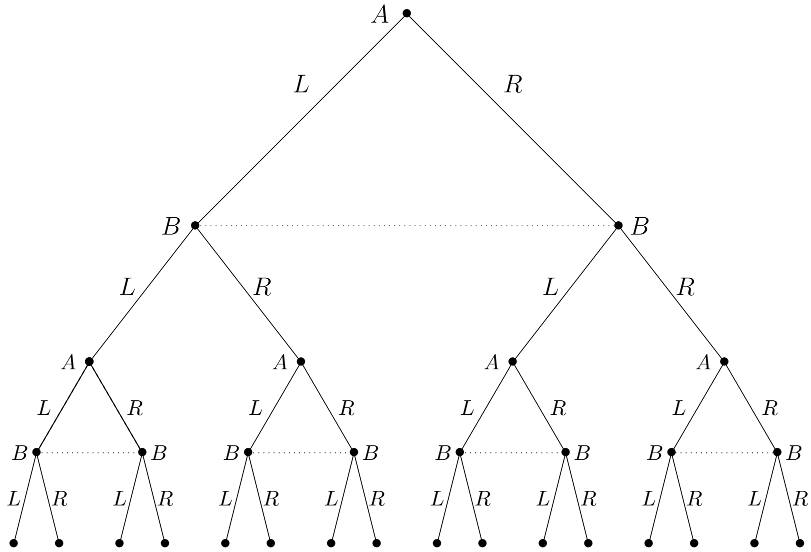


Figure 3.1: The game in extensive form.

3.3.1 Equilibrium of the subgames

In the subgames, parties are only concerned with the payoffs of the second period, because the payoffs of the first period are given. Then, the game in normal form that parties play at any (x_A^1, x_B^1) is represented in Table 3.2.

$A \setminus B$	L	R
L	$3/2, 1/2$	$2 \cdot (1 - \alpha_2(x_A^1, x_B^1))$, $2 \cdot \alpha_2(x_A^1, x_B^1)$
R	$1, 1$	$1/2$, $3/2$

Table 3.2: The game played by parties in the subgames, after substracting the payoffs of the first period (which are positive affine transformations).

In the next Proposition, I describe the possible equilibria of this game. This Proposition is proved and discussed in Sánchez (2020b). This set is illustrated in Figure 3.2.

Proposition 3.1. *Consider the game represented in Table 3.2. Then, the following statements hold.*

- *If* $\alpha_2(x_A^1, x_B^1) < 1/4$, *then* (L, L) *is the unique equilibrium, and* L *is a dominant strategy for party* A.
- *If* $\alpha_2(x_A^1, x_B^1) \in (1/4, 3/4)$, *then* (L, R) *is the unique equilibrium, and both strategies are dominant.*

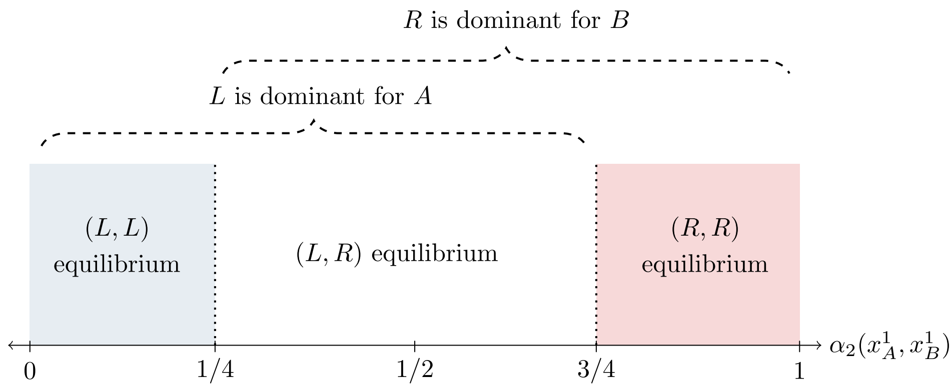


Figure 3.2: The equilibrium of the subgame.

- *If $\alpha_2(x_A^1, x_B^1) > 3/4$, then (R, R) is the unique equilibrium, and R is a dominant strategy for party B*
- *If $\alpha_2(x_A^1, x_B^1) = 1/4$ or $\alpha_2(x_A^1, x_B^1) = 3/4$, then (L, R) is the unique undominated equilibrium.*

3.3.2 Subgame perfect equilibrium

Consider the game played in the first period, and suppose that parties play in the second period following the equilibrium identified in Proposition 4.1. That is, given α_1, party A chooses policy L in the subgame starting at (x_A^1, x_B^1) if only if $\alpha_2(x_A^1, x_B^1) \leq 3/4$, and party B chooses policy R if only if $\alpha_2(x_A^1, x_B^1) \geq 1/4$.

Define $\pi_J(x_A^1, x_B^1)$ as the secon period payoff received by party J in the subgame starting at (x_A^1, x_B^1), assuming that parties play according to Proposition 4.1. For example:

$$\pi_A(x_A^1, x_B^1) = \begin{cases} 3/2 \quad \text{if } \alpha_2(x_A^1, x_B^1) < \frac{1}{4} \\ 2 \cdot (1 - \alpha_2(x_A^1, x_B^1)) \ \text{ if } \ \alpha_2(x_A^1, x_B^1) \in [\frac{1}{4}, \frac{3}{4}] \\ 1/2 \quad \text{if } \alpha_2(x_A^1, x_B^1) > \frac{3}{4}. \end{cases} \tag{3.4}$$

For any (x_A^1, x_B^1), the functions $\pi_A(x_A^1, x_B^1)$ and $\pi_B(x_A^1, x_B^1)$ are continious, symmetric, and respectively non-increasing and non-decreasing with respect to $\alpha_2(x_A^1, x_B^1)$. They are displayed in Figure 3.3.

In the first period, parties are concerned with their inmediate payoffs, which are the payoffs received in the first period, plus their future equilibrium payoffs, which are $\pi_A(x_A^1, x_B^1)$ or $\pi_B(x_A^1, x_B^1)$.

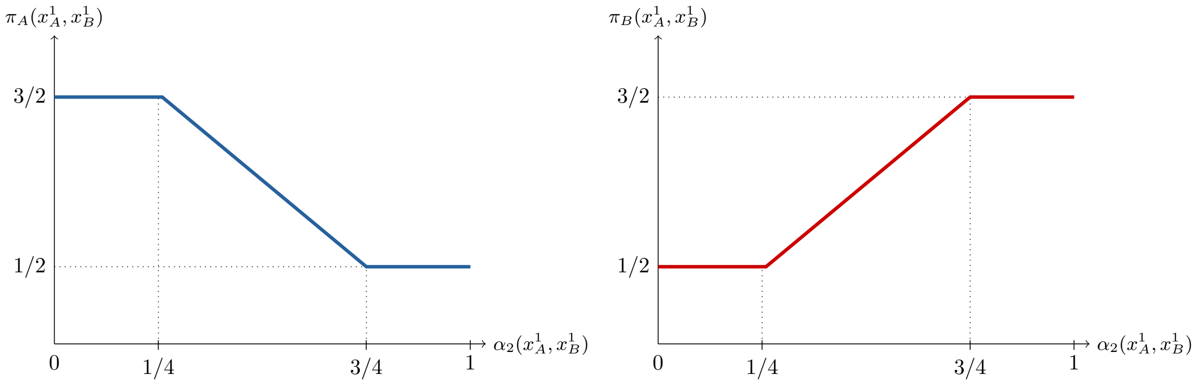


Figure 3.3: How $\pi_A(x_A^1, x_B^1)$ and $\pi_B(x_A^1, x_B^1)$ depend on $\alpha_2(x_A^1, x_B^1)$.

Equilibrium if parties did not influence α_2

For the sake of comparison, consider first the alternative scenario where α_2 does not depend on (x_A^1, x_B^1). Namely,

$$\alpha_2(x_A^1, x_B^1) = \overline{\alpha}_2 \text{ for all } (x_A^1, x_B^1).$$

In this case, the full game played by the parties can be represented as in Table 3.3, where $\pi_A(x_A^1, x_B^1) = \overline{\pi}_A$ and $\pi_B(x_A^1, x_B^1) = \overline{\pi}_B$ for all (x_A^1, x_B^1).

$A \setminus B$	L	R
L	$3/2 + \overline{\pi}_A$, $1/2 + \overline{\pi}_B$	$2 \cdot (1 - \alpha_1) + \overline{\pi}_A$, $2 \cdot \alpha_1 + \overline{\pi}_B$
R	$1 + \overline{\pi}_A$, $1 + \overline{\pi}_B$	$1/2 + \overline{\pi}_A$, $3/2 + \overline{\pi}_B$

Table 3.3: The game played by parties in the first period if α_2 does not depend on (x_A^1, x_B^1).

The payoffs of this game are affine transformations of the payoffs displayed in Table 3.2, except the payoffs corresponding to $(x_A^1, x_B^1) = (L, R)$, that depend on α_1 instead of $\alpha_2(x_A^1, x_B^1)$. It is inmediate that the equilibrium of this game is determined by the value of α_1 with respect to the same cutoffs of Proposition 4.1.

Proposition 3.2. *Consider the game displayed in Table 3.3. Then, the following statement hold.*

- *If $\alpha_1 < 1/4$, then (L, L) is the unique equilibrium, where L is a dominant strategy for party A.*

- *If* $\alpha_1 \in (1/4, 3/4)$*, then* (L, R) *is the unique equilibrium, where both strategies are dominant.*
- *If* $\alpha_1 > 3/4$*, then* (R, R) *is the unique equilibrium, where* R *is a dominant strategy for party* B*.*
- *If* $\alpha_1 = 1/4$ *or* $\alpha_1 = 3/4$*, then* (L, R) *is the unique undominated equilibrium.*

Intuitively, since parties are not able to influence their future payoffs, then they only consider their inmediate payoffs. This is exactly what parties do in the second period, so the cutoffs will be the same.

Equilibrium when parties influence α_2

I now return to the previous scenario, where conditions (3.1), (3.2) and (3.3) hold. In this case, parties' future payoffs also depend on (x_A^1, x_B^1), and the game played by the parties is now displayed in Table 3.4.

$A \setminus B$	L	R
L	$3/2 + \pi_A(L, L)$, $1/2 + \pi_B(L, L)$	$2 \cdot (1 - \alpha_1) + \pi_A(L, R)$, $2 \cdot \alpha_1 + \pi_B(L, R)$
R	$1 + \pi_A(R, L)$, $1 + \pi_B(R, L)$	$1/2 + \pi_A(R, R)$, $3/2 + \pi_B(R, R)$

Table 3.4: The game played by parties when α_2 depends on (x_A^1, x_B^1).

First, I focus on the cases where $\alpha_1 < 3/4$ holds.

Proposition 3.3. *Consider the game displayed in Table 3.4. Then, the following statements hold.*

- *If* $\alpha_1 < 1/4$*, then* (L, L) *is the unique equilibrium, where* L *is a dominant strategy for party* A*.*
- *If* $\alpha_1 = 1/4$ *then* (L, R) *is the unique undominated equilibrium.*
- *If* $\alpha_1 \in (1/4, 3/4)$*, then* (L, R) *is the unique equilibrium, where both strategies are dominant.*

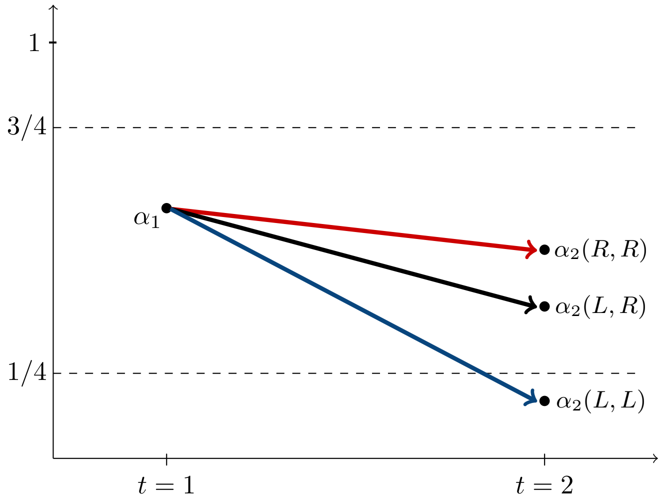


Figure 3.4: How $\alpha_2(x_A^1, x_B^1)$ may evolve if $\alpha_1 \in (1/4, 3/4)$.

If α_1 is not larger than 3/4, then parties' equilibrium choices in the first period are similar than in Proposition 4.2 . For example, suppose that $\alpha_1 < 1/4$ holds. Then, we can ensure by Condition (3.1) that $\alpha_2(x_A^1, x_B^1) < \alpha_1 < 1/4$ holds for all (x_A^1, x_B^1), and thus, $\pi_A(x_A^1, x_B^1) = 3/2$ and $\pi_B(x_A^1, x_B^1) = 1/2$ hold (see Figure 3.3) . Then, even if parties can influence α_2, they cannot affect their future payoffs, so they both choose policy L.

Consider now Figure 3.4, where $\alpha_1 \in (1/4, 3/4)$. Unlike when $\alpha_1 < 1/4$, now parties may influence their future payoffs. For example, in the case of the figure, $\pi_A(L, L) = 3/2$ because $\alpha_2(L, L) < 1/4$, while $\pi_A(L, R) = 2 \cdot (1 - \alpha_2(L, R))$ because $\alpha_2(L, R) \in (1/4, 3/4)$. However, since $\pi_A(x_A^1, x_B^1)$ and $\pi_B(x_A^1, x_B^1)$ are respectively non-increasing and non-decreasing with respect to $\alpha_2(x_A^1, x_B^1)$, then, by (3.3), we can ensure that $\pi_A(L, R) \geq \pi_A(R, R)$ and $\pi_B(L, R) \geq \pi_B(L, L)$. And thus, none of the parties deviates from (L, R).

I now focus on the scenario where $\alpha_1 > 3/4$. By Proposition 4.2, we know that both parties choose policy R if they cannot influence α_2. In this Section, I only describe the case where $\alpha_2(R, R) > 3/4$. The remaining cases, whose intuition is similar, are analysed in the Appendix.

Proposition 3.4. *Consider the game displayed in Table 3.4, and suppose that* $\alpha_1 > 3/4$ *and* $\alpha_2(R, R) > 3/4$*. Then, the following statements hold.*

- *If* $\alpha_2(L, R) > 3/2 - \alpha_1$*, then* (R, R) *is the unique equilibrium, where* R *is a dominant strategy for party* B*.*
- *If* $\alpha_2(L, R) < 3/2 - \alpha_1$*, then* (L, R) *is the unique equilibrium, where both strategies are dominant.*
- *If* $\alpha_2(L, R) = 3/2 - \alpha_1$*, then* (L, R) *is the unique undominated equilibrium.*

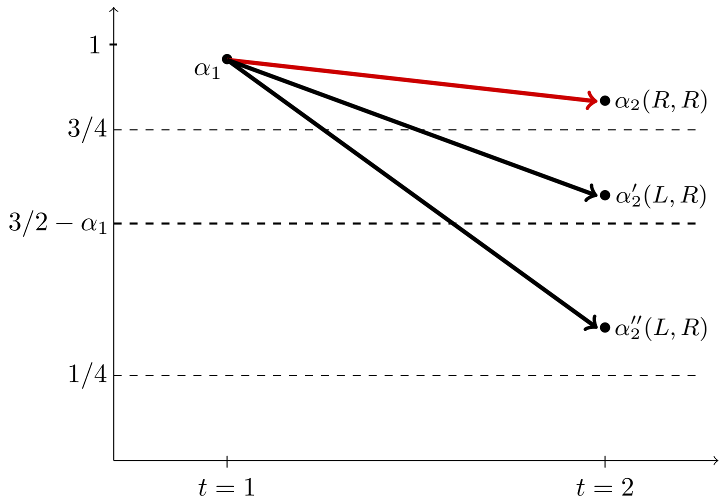


Figure 3.5: If $\alpha_1 > 3/4$ and $\alpha_2(R,R) > 3/4$, then party A chooses policy L at $t=1$ only if $\alpha_2(L,R)$ is not larger than $3/2-\alpha_1$.

The equilibrium can be either (L,R) or (R,R), because party A is facing a intertemporal trade-off. If the party chooses policy L instead of policy R, then its inmediate payoff decreases. More specifically, party A would receive $2\cdot(1-\alpha_1)$ instead of $1/2$, which is strictly larger. However, party A's future payoff would increase, because $\pi_A(L,R) \geq \pi_A(R,R)$. If this increase compensates the decrease in its inmediate payoff, then party A chooses policy L, and (L,R) becomes the equilibrium.

The case of Proposition 4.4 is illustrated in Figure 3.5. Since $\alpha_2(R,R) > 3/4$, then party A receives $\pi_A(R,R) = 1/2$ in the second period if the party does not deviate to policy L. If $\alpha_2(L,R) > 3/4$ also holds, then party A also receives $\pi_A(L,R) = 1/2$ after deviating, so, in this case, the party still chooses policy R. If $\alpha_2(L,R)$ decreases, meaning that party A's influence over α_2 is larger, then $\pi_A(L,R)$ increases, and thus party A has more incentives to choose policy L. The threshold $3/2-\alpha_1$ is where party A is indifferent between deviating to policy L or not. Whenever $\alpha_2(L,R)$ is below that threshold, then the increase in party A's future payoff will be large enough, and party A chooses policy L even if its inmediate payoff are lower.

3.4 Equilibrium with $T > 2$ periods

I now present a preliminary analysis for the case with $T > 2$ periods. I do not fully characterize the subgame perfect equilibrium, but I give an idea of parties' expected behaviour. For tractability, I will specify the law of motion of α_t, taking $\alpha_0 = 1$. First, consider the hyphotetical scenario where parties do not influence citizens' preferences. Then, given α_0, the value of α_t for $t \in \{1, ..., T\}$ is

$$\alpha_t = \frac{\alpha_{t-1}}{a \cdot \alpha_{t-1} + 1}, \tag{3.5}$$

where $a > 0$ holds. Then, the share of citizens in favor of policy R is already decreasing over time, and the parameter a measures the speed at which it decreases.

Now consider the scenario where parties are able to influence citizens' preferences. In this case, given α_0, the value of α_1 is as (3.5), and for each $t \in \{2, ..., T\}$, the value of α_t is

$$\alpha_t = \frac{\alpha_{t-1}}{\gamma(x_A^{t-1}, x_B^{t-1}) \cdot \alpha_{t-1} + 1}, \tag{3.6}$$

where

$$\gamma(x_A^{t-1}, x_B^{t-1}) = \begin{cases} a & \text{if } x_A^{t-1} = x_B^{t-1} = R \\ b & \text{if } x_A^{t-1} \neq x_B^{t-1} \\ c & \text{if } x_A^{t-1} = x_B^{t-1} = L \end{cases}$$

and $c > b > a$ holds. Thus, the share of citizens in favor of policy R is still decreasing over time, and parties are able to increase the speed at which it decreases. In particular, the more parties support policy L at $t - 1$, the lower α_t will be. If none of the parties support policy L, then citizens' preferences evolve as in (3.5).[4]

3.4.1 Equilibrium if parties do not influence citizens' preferences

Define h_t as the history of parties' choices at time $t \in \{1, ..., T\}$, with $h_1 = \emptyset$ (there is no history at $t = 1$). Then, the strategy of party J, denoted as σ_J, is the policy chosen at each possible history of choices. I denote with $\sigma_J(h_t) \in \{L, R\}$ the policy chosen by party J at history h_t.

Observe that if we replace $\alpha_2(x_A^1, x_B^1)$ by α_t in Table 3.2, then the table would represent the "stage game" that parties play at any period, with the particularity that α_t is changing over time, and all stage games are different. Then, the next result follows inmediately from Proposition 4.1 (the proof is omitted).

Proposition 3.5. *Suppose that $\alpha_0 = 1$ and for every $t \in \{1, ..., T\}$, α_t evolves as in*

[4] Observe that conditions (3.1), (3.2) and (3.3) would hold if $T = 2$.

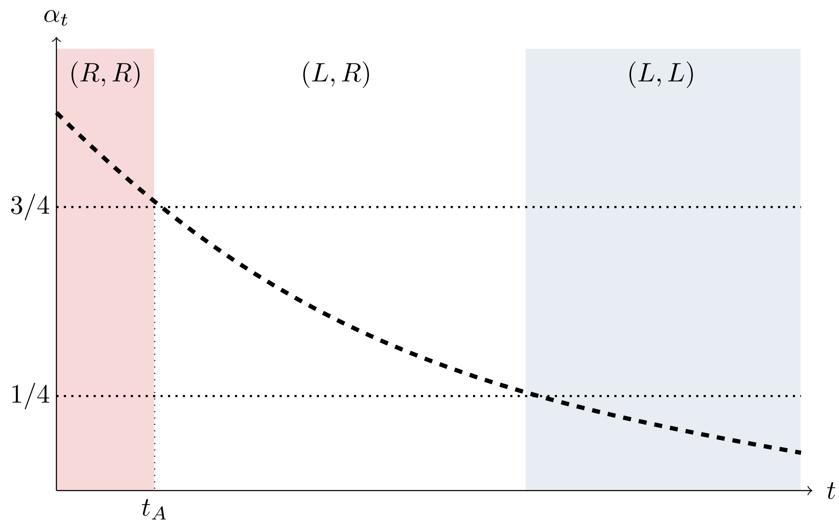


Figure 3.6: If parties do not influence α_t, then the equilibrium of the repeated game is that parties play the equilibrium of the stage game at every t.

(3.5). Then, the following profile of strategies $(\sigma_A^\star, \sigma_B^\star)$, *where*

$$\sigma_A^\star(h_t) = \begin{cases} L \textit{ for all } h_t \textit{ if } \alpha_t \leq 3/4 \\ R \textit{ for all } h_t \textit{ if } \alpha_t > 3/4 \end{cases} \quad \textit{and} \quad \sigma_B^\star(h_t) = \begin{cases} L \textit{ for all } h_t \textit{ if } \alpha_t < 1/4 \\ R \textit{ for all } h_t \textit{ if } \alpha_t \geq 1/4 \end{cases}$$

is a subgame perfect equilibrium.

If α_t evolves as in (3.5), then playing the equilibrium of the stage game at every period is a subgame perfect equilibrium.[5] Thus, we can anticipate that parties play (R, R) during $t_A - 1$ periods, where t_A is the first period where $\alpha_t < 3/4$ holds. Then, parties play (L, R) until $\alpha_t < 1/4$ holds, and from then on, they play (L, L) the rest of periods. This is illustrated in Figure 3.6.

3.4.2 Equilibrium if parties influence citizens' preferences

Applying backwards induction when α_t depends on previous parties' choices is technically very demanding, because the number of combinations of $\{\alpha_1, \alpha_2, ..., \alpha_T\}$ is excessively large. However, we can already have an intuition about parties' equilibrium behaviour. In particular, I will show that the profile (σ_A^*, σ_B^*) is not necessarily a subgame perfect equilibrium, because party A may have incentives to deviate.

First, suppose that parties still play (σ_A^*, σ_B^*). Then, the evolution of α_t is the marked line from Figure 3.7. During t_A periods, the value of α_t decreases at the same rate as when parties do not have any influence, which is the dashed line. From then on, the speed

[5] This result is analogous to the equilibrium of the finitely repeated prisoner's dilemma, where defection in every period is known to be a subgame perfect equilibrium (Benoît and Krishna, 1985).

at which α_t decreases is larger, because party A is supporting policy L. Interestingly, the total payoff of party A is also larger than in the previous setting. This can be observed in Figure 3.8, where I display the payoff that party A receives every period with (σ_A^*, σ_B^*). From t_A onwards, the payoff received is larger than when parties do not have any influence, because α_t is taking lower values.

The key point is that party A may prefer to support policy L earlier than t_A in order to increase the speed at which α_t decreases. For illustration, suppose that party A starts supporting policy L at $\tilde{\tau} < t_A$. In this case, the payoff received by party A is represented with a smooth line in Figure 3.8. Initially, the payoff is lower than with (σ_A^*, σ_B^*), because party A is choosing policy L when very few citizens are in favor of that policy. However, as soon as α_t becomes lower than 3/4 (which is earlier than t_A), the payoff will be larger than with (σ_A^*, σ_B^*). Party A will prefer to support policy L before t_A only if the increase in its future payoffs compesates its short-term losses. This intuition is similar to the two-periods case.

To get an idea of party A's optimal behaviour, let σ_A^τ denote the following strategy.

$$\sigma_A^\tau(h_t) = \begin{cases} R \text{ for all } h_t \text{ if } t < \tau \\ L \text{ for all } h_t \text{ if } t \geq \tau, \end{cases}$$

where $\tau \in \{1, ..., t_A\}$. That is, party A chooses policy R until $t = \tau$, and from then on, the party chooses policy L (it is inmediate that $\sigma_A^{t_A}(h_t) = \sigma_A^\star(h_t)$ for every h_t). For every $\tau \in \{1, ..., t_A\}$, I first compute the total payoff that party A receives in the corresponding profile $(\sigma_A^\tau, \sigma_B^*)$, and then I look for the profile that maximizes party A's payoff. The results are summarized in Table 3.5, for different values of b. If $b = a$ holds, which means that parties do not influence citizens' preferences, then party A's maximum payoff is at (σ_A^*, σ_B^*), which goes in line with Proposition C.1. If b increases, which means that parties have more influence over citizens' preferences, then the maximum is reached at some $(\sigma_A^{\tau_{max}}, \sigma_B^*)$, where $\tau_{max} < t_A$. And if b is sufficiently large, then $\tau_{max} = 1$, meaning that the maximum payoff is reached when party A chooses policy L from the beginning.

b	0.05	0.055	0.065	0.075	> 0.075
τ_{max}	66 (t_A)	51	23	1	1

Table 3.5: As b increases, A's payoff is maximized when the party starts choosing policy L earlier, given that B is playing σ_B^*. I have set $T = 500$, $a = 0.05$ and $c = b + 0.1$.

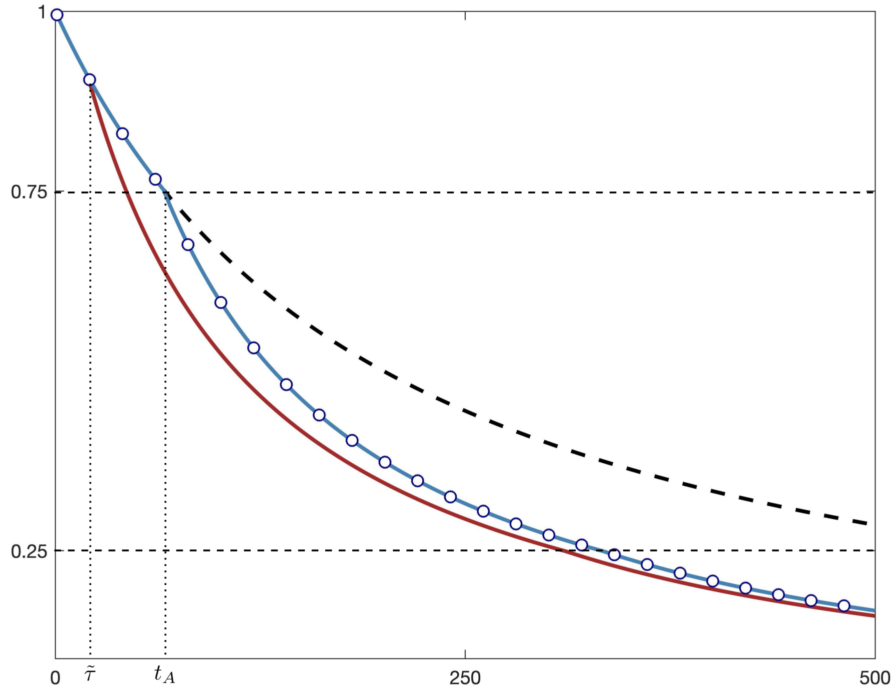


Figure 3.7: How α_t evolves over time when parties do not affect its value (dashed line), when parties affect it and they play (σ_A^*, σ_B^*) (marked line), and when parties affect it and party A starts supporting policy L at $\tilde{\tau}$ instead of t_A (smooth line). I have set $T = 500$, $a = 0.005$, $b = 0.01$ and $c = 0.012$.

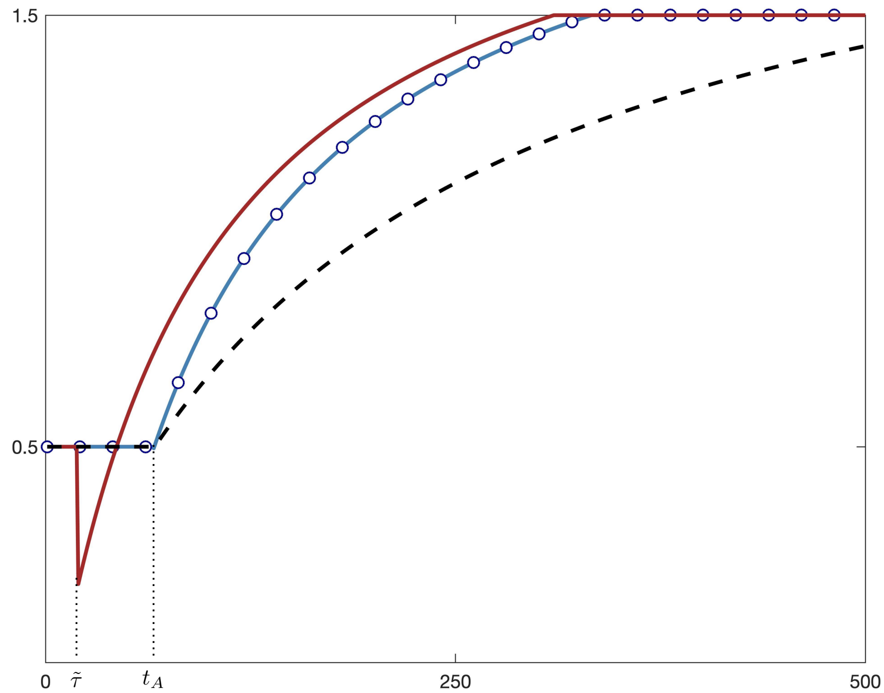


Figure 3.8: The payoff received by party A every period when parties do no influence α_t and they play (σ_A^*, σ_B^*) (dashed line), when parties influence α_t and they play (σ_A^*, σ_B^*) (marked line), and when parties influence α_t and party A starts supporting policy L at $\tilde{\tau}$ instead of t_A (smooth line).

3.5 Final remarks

This paper aims to shed light on the dynamic relationship between political parties and public opinion by putting the focus on parties' influence. I propose a dynamic model where parties' choices affect future citizens' preferences. The relationship is endogeneous, because citizens' preferences also affect parties' choices. In the equilibrium, I identify a feedback effect derived from parties' influence. That is, I show that parties' influence actually affects parties' choices, and, it may incentivize party confrontation. Interestingly, this effect may help understanding why political parties seems to be more polarized than citizens, a question that has surrounded U.S. politics for years.

The model allows for two main extensions. First, I have assumed that the amount of citizens in favor of the left-wing policy is increasing over time, which is not always the case. For example, the share of pro-choice citizens in U.S has remained around fifty percent over the last decades (Sánchez, 2020b). Or the share of U.S. citizens in favor of a stricter guns control, which has remained around sixty percent over the years.[6] Those examples encourage a more diverse analysis of the dynamics of opinions. Second, I have assumed that parties can only increase the share of citizens in favor of the left-wing policy. The idea behind this is that citizens who are very attached to a party may support the policy chosen by that party even if in other conditions the citizen would not have done it (Sánchez, 2020c). However, other types of effects could also be considered. For example, a party may decrease the speed at which citizens become in favor of a policy. Or the extreme case, where a party switches the policy preferred by the majority of citizens. By combining the previous two extensions, we could better understand the dynamics of a broader set of political issues. However, I will let this for future research.

[6] Source: Gallup.

Chapter 4

Party Confrontation as a Learning Mechanism

4.1 Introduction

When political parties are confronted about a political issue, then the issue generally becomes current, in the sense that it reaches the public debate. This argument is already present in the traditional issue evolution models, like Carmines and Stimson (1986) or Zaller (1992), and in more recent papers on public opinion formation, like Druckman et al. (2013). In contrast, when parties are not confronted, then the issue tends to be latent, without attracting much public attention (Sánchez, 2020b).

When a political issue is current, then the media generally speak more about the issue, and political activists intensify their actions (Sánchez, 2020b). Then, the following assumption can be made. When parties are confronted and the issue becomes current, the knowledge about the state of the public opinion is improved.[1]

In this paper, I make use of this assumption to study parties' incentives to learn about the public opinion.[2] More specifically, I present a dynamic model where parties do not know the state of the public opinion, but they learn it when they support different policies. The key point is that parties' payoffs depend on the public opinion, so whether to learn it or not *ex ante* becomes relevant for them.

I show that a party might find confrontation preferable because the public opinion would be observed, which increases in expectation the party's future payoff. A party might even take the risk of supporting an unpopular policy in order to learn the state of the public opinion. However, parties are not necessarily better-off without uncertainty. In fact, because parties' payoffs are symmetric, then a party benefits from learning if and only if the opponent does not. As a consequence of this symmetry, party confrontation will only arise as a mixed strategy equilibrium.

The model is made of two periods, two parties, a set of citizens, and one single issue with two possible policies (the left-wing and right-wing policy). In the second (and last) period, parties play the game presented in Sánchez (2020b). In this game, the equilibrium is determined by the amount of citizens in favor of each policy. In particular, parties are confronted in equilibrium only if there is not an excessively large amount of citizens in favor of one of the policies. Otherwise, both parties support the policy preferred by the majority of citizens.

[1]This assumption can be also deduced from parties' behaviour. For example, one can find situations where a party supports one policy, but after the party observes the public rejection, then it changes its position (Sánchez, 2020b).

[2]Parties' uncertainty about citizens' political preferences has been largely addressed in the traditional electoral competition models. Many of them randomize voters' preferences over the policies (Wittman (1983), Roemer (2001)). Others, in contrast, randomize voters' attachment towards the parties (Londregan and Romer (1993), Banks and Duggan (2005)). However, the question of whether parties have incentives to learn is always left behind. In fact, in the existing literature, citizens are generally those who learn about the state of the world, but not the other way around (Feddersen and Pesendorfer (1997), Dewan and Hortala-Vallve (2019)).

In the first period, parties make another policy decision (which can be interpreted as a proposal) without knowing citizens' preferences, but taking into account that those preferences are learned if they support different policies. Then, a party will face the following dilemma. Either the party chooses a different policy than the opponent, learns citizens' preferences and plays the second period's game with perfect information, or the party chooses the same policy than the opponent, does not learn citizens' preferences, and plays the second period's game with imperfect information. Moreover, I assume that learning is costly for a party if the party chooses a policy that is not preferred by the majority of citizens.

Learning about citizens' preferences can either increase or decrease parties' expected payoffs. For illustration, consider the following scenario. The amount of citizens in favor of the left-wing policy is expected to be low. Thus, if parties do not learn citizens' preferences, then they both choose the right-wing policy. However, the variance is quite large, so even though the expection is low, it is likely that the amount of citizens in favor of left-wing policy is larger than expected. In this case, the party preferring the left-wing policy will prefer to learn citizens' preferences, because if the amount of citizens in favor of that policy turns out to be larger, then the party will not support the right-wing policy. In contrast, the party preferring the right-wing policy will prefer not to learn citizens' preferences, because if the opponent deviates to the left-wing policy, then the payoff received by the party will be lower.

A party will prefer confrontation in the first period whenever the expected benefits from learning about citizens' preferences compesates the possible cost from supporting an unpopular policy. However, party confrontation is never a pure strategy equilibrium in the first period. The reason is that the model is perfectly symmetric, which implies that if a party prefers confrontation, then the opponent, for whom learning is not beneficial in expectation, will not.[3] In those cases, I also look for mixed strategy equilibra, bearing in mind Harsanyi's purification theorem (Harsanyi, 1973). I show that the equilibrium probabilities are always symmetric. That is, a party is as likely to support the left-wing policy as the opponent is to support the right-wing policy. Moreover, I also show that party confrontation is the most likely equilibrium.

The rest of the paper is as follows. In Section 4.2, I describe the model. In Section 4.3, I characterize the subgame perfect equilibria, both in pure and in mixed strategies. In Section 4.4, I comment the final remarks. All the proofs are gathered Appendix C.

[3]In Section 4.4, I comment some extensions under which party confrontation would likely arise as a pure strategy equilibrium. For example, I may assume that parties have different priors about citizens' preferences, or that one party is more concerned than the other with the popularity of the policy. The idea is to break the symmetry of parties' payoffs, which is the reason why confrontation is never a pure strategy equilibrium in the first period.

4.2 The Model

The society is made of two parties, A and B, two policies, L (left) and R (right), and a set of citizens. During two periods, parties have to decide which policies they support. I denote by $(x_A^0, x_B^0) \in \{L, R\}^2$ the policies supported in the first period, and by $(x_A, x_B) \in \{L, R\}^2$ the policies supported in the second period.

Public opinion is a parameter $\alpha \in (0, 1)$ that denotes the proportion of citizens in favor of policy R, and $1 - \alpha$ is the proportion of citizens in favor of policy L. I assume that α is drawn from a distribution f, and that parties do not know the value of α. I denote with $E(\alpha)$ the expected value of α.[4]

Parties learn the value of α only if $x_A^0 \neq x_B^0$ holds, and they decide (x_A, x_B) after updating the information about α. At the end of the last period, one policy is implemented. If $x_A = x_B$, then this common policy is implemented with probability 1. If $x_A \neq x_B$, then policy R is implemented with probability α, and policy L with probability $1 - \alpha$. Observe that the policy implemented does not depend on (x_A^0, x_B^0).

4.2.1 Parties' payoffs

Parties are concerned with the policy implemented at the end of the last period. In particular, party A prefers policy L and party B prefers policy R. I assume that parties receive a payoff of $\pi > 0$ only if their preferred policy is implemented. This payoff is normalized to $\pi = 1$.

Parties are also concerned with the public opinion. I assume that a payoff of value 1 is allocated between parties after (x_A, x_B). If $x_A = x_B$, then both parties receive 1/2. If $x_A \neq x_B$, then the party supporting policy R receives α and the party supporting policy L receives $(1 - \alpha)$. Another payoff of value 1 is allocated after (x_A^0, x_B^0) in the same way. Those are called the popularity payoffs (see Table 4.1).

(x_A, x_B)	Popularity payoff	Policy payoff	Second period payoff
(R, R)	$1/2, 1/2$	$0, 1$	$1/2, 3/2$
(L, R)	$1 - \alpha, \alpha$	$1 - \alpha, \alpha$	$2 \cdot (1 - \alpha), 2 \cdot \alpha$
(R, L)	$\alpha, 1 - \alpha$	$1 - \alpha, \alpha$	$1, 1$
(L, L)	$1/2, 1/2$	$1, 0$	$3/2, 1/2$

Table 4.1: How parties' payoffs of the second period are constructed. The total payoffs are obtained by adding another popularity payoff, which depends on (x_A^0, x_B^0) instead.

[4]For simplicity, I will assume that $E(\alpha) \neq 1/2$, $E(\alpha) \neq 1/4$ and $E(\alpha) \neq 3/4$.

4.3 Equilibrium Analysis

Consider the game in extensive form represented in Figure 4.1. In the first period, parties simultaneously choose (x_A^0, x_B^0) without knowning α. In the second period, parties learn α only if $x_A^0 \neq x_B^0$, and they simultaneously choose (x_A, x_B). If α is learned, then parties play the subgame starting at the corresponding (x_A^0, x_B^0, α). Otherwise, there are two information sets: (L, L), which contains all the nodes of the form (L, L, α), and (R, R), which contains all the nodes of the form (R, R, α).

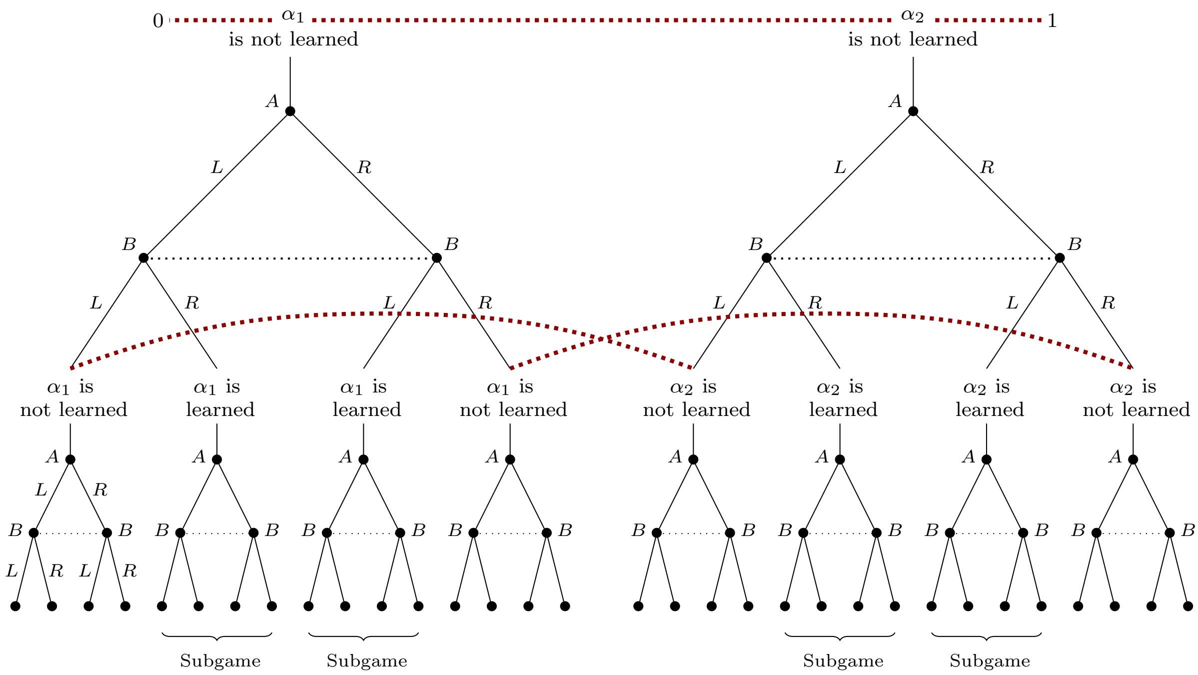


Figure 4.1: The game in extensive form played by the parties.

First, I will show that the equilibrium always exist in all the subgames, and that the equilibrium is determined by the value of α. Then, I will look for the subgame perfect equilibrium with undominated pure strategies. I will show that whether this equilibrium exists depends on the distribution of α.

4.3.1 Equilibrium of the subgames

Suppose that $x_A^0 \neq x_B^0$ and parties learn α. In the subgame (x_A^0, x_B^0, α), the popularity payoff of the first period is given, so parties maximize the payoffs that depend on (x_A, x_B). That is, the payoff from the policy implemented plus the popularity payoff of the second period. Thus, using Table 4.1, the subgame played after (R, L, α) or (L, R, α) is displayed in Table 4.2.

In the next proposition, I characterize the equilibrium of the subgame for every $\alpha \in (0, 1)$. This proposition is already proved and discussed in Sánchez (2020b).

$A \setminus B$	L	R
L	$3/2, 1/2$	$2 \cdot (1-\alpha), 2 \cdot \alpha$
R	$1, 1$	$1/2$, $3/2$

Table 4.2: The matrix payoff of the subgame played by the parties if they learn α.

Proposition 4.1. *Consider the subgame represented in Table 4.2. Then, the following statements hold.*

- *If $\alpha < 1/4$, then (L, L) is the unique equilibrium, and L is a dominant strategy for party A.*
- *If $\alpha \in (1/4, 3/4)$, then (L, R) is the unique equilibrium, and both strategies are dominant.*
- *If $\alpha > 3/4$, then (R, R) is the unique equilibrium, and R is a dominant strategy for party B.*
- *If $\alpha = 1/4$ or $\alpha = 3/4$, then (L, R) is the unique undominated equilibrium.*

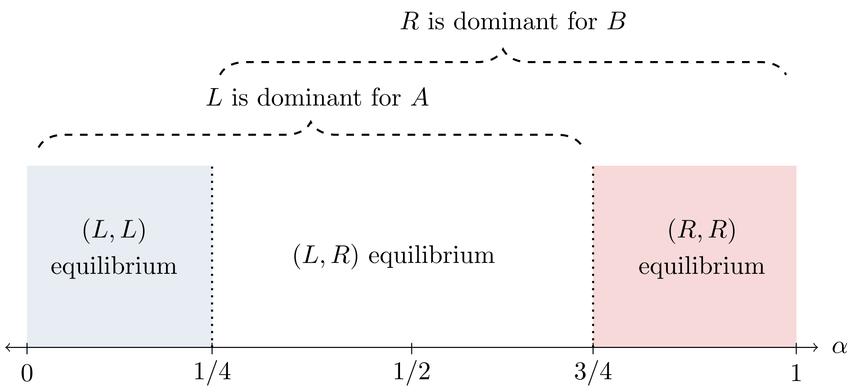


Figure 4.2: The equilibrium of the subgame.

4.3.2 Subgame perfect equilibrium

Suppose that, when $x_A^0 \neq x_B^0$, parties play at each subgame (x_A^0, x_B^0, α) the equilibrium identified in Proposition 4.1. Namely, for every (x_A^0, x_B^0) with $x_A^0 \neq x_B^0$, party A chooses policy L in the subgame starting at (x_A^0, x_B^0, α) if and only if $\alpha \leq 3/4$, and party B chooses policy R if and only if $\alpha \geq 1/4$.

I describe parties' choices at the first period and at the nodes (L, L) and (R, R) as a vector $\sigma_J \in \{L, R\}^3$, where the first component, denoted with σ_J^1, is the policy chosen by party J in the first period, and the second and third components, denoted with σ_J^2 and σ_J^3, are the policies chosen by party J after (L, L) and (R, R) respectively. For example, $\sigma_A = (L, L, L)$, which is abbreviated as $\sigma_A = LLL$, is the strategy where party A plays according to Proposition 4.1 at each subgame of the form (x_A^0, x_B^0, α) with $x_A^0 \neq x_B^0$, and chooses policy L in the other three information sets.

The strategy set of party J is identified with its eight possible strategies. The game in normal form, which is an 8×8 matrix, will be obtained by computing the expected payoff for each of the strategy profiles.

First, consider a profile (σ_A, σ_B) where $\sigma_A^1 \neq \sigma_B^1$ holds. That is, a profile where α is learned and parties play the corresponding subgame $(\sigma_A^1, \sigma_B^1, \alpha)$. Define $\pi_J(\sigma_A^1, \sigma_B^1, \alpha)$ as the payoff received by party J in that subgame, assuming that parties play according to Proposition 4.1.[5] Namely,

$$\pi_A(\alpha) = \begin{cases} 3/2 & \text{if } \alpha < 1/4 \\ 2 \cdot (1-\alpha) & \text{if } \alpha \in [1/4, 3/4] \\ 1/2 & \text{if } \alpha > 3/4 \end{cases} \qquad \pi_B(\alpha) = \begin{cases} 1/2 & \text{if } \alpha < 1/4 \\ 2 \cdot \alpha & \text{if } \alpha \in [1/4, 3/4] \\ 3/2 & \text{if } \alpha > 3/4. \end{cases}$$

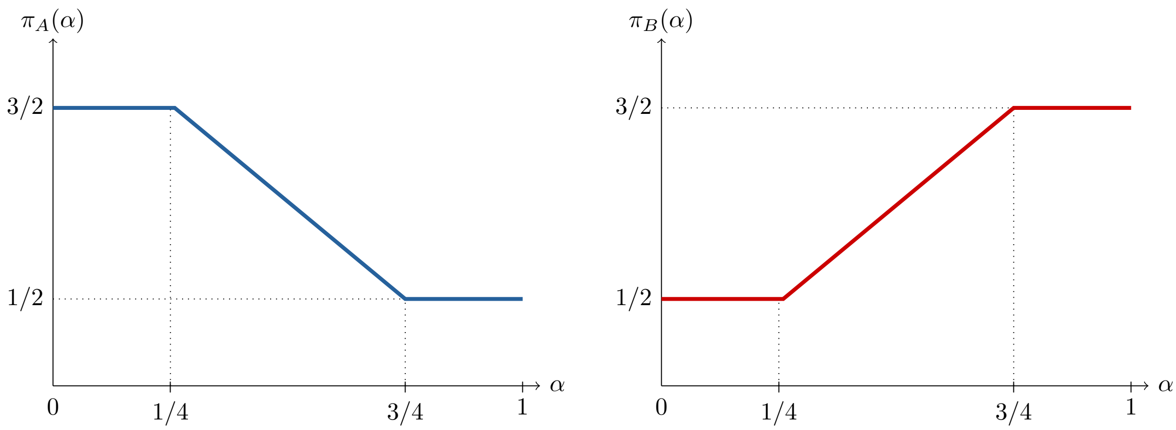


Figure 4.3: The functions $\pi_A(\alpha)$ and $\pi_B(\alpha)$.

The functions $\pi_A(\alpha)$ and $\pi_B(\alpha)$ are continuous, symmetric, and respectively non-increasing and non-decreasing with respect to α. They are displayed in Figure 4.3.

Hence, at the beginning of the game, the payoff that party J expects to receive in the subgames where α is learned is $E(\pi_J(\alpha))$.[6] Party J also expects either $E(\alpha)$ or $1 - E(\alpha)$ from the popularity payoff of the first period. Hence, in a profile where $\sigma_A^1 \neq \sigma_B^1$

[5]Since the equilibrium of each subgame only depends on α, $\pi_J(R, L, \alpha) = \pi_J(L, R, \alpha)$ always holds. Thus, I will abbreviate $\pi_J(\sigma_A^1, \sigma_B^1, \alpha)$ as $\pi_J(\alpha)$.

[6]Observe that $\pi_A(\alpha) + \pi_B(\alpha) = 2$ holds for all α, and thus, $E(\pi_A(\alpha)) + E(\pi_B(\alpha)) = 2$ also holds.

holds, the total payoff that party J expects to receive is either $E(\alpha) + E(\pi_J(\alpha))$ or $1 - E(\alpha) + E(\pi_J(\alpha))$, depending on whether $\sigma_J^1 = R$ or $\sigma_J^1 = L$.

Now consider a profile where $\sigma_A^1 = \sigma_B^1$ holds and α is not learned. In this case, the payoff received in the second period is determined by either the second or the third component of each strategy, depending on whether $\sigma_J^1 = L$ or $\sigma_J^1 = R$. For example, if both parties choose policy L in the first period, then the payoff depends on σ_A^2 and σ_B^2. If $\sigma_A^2 = \sigma_B^2 = L$, then party A receives 3/2 and party B receives 1/2. But if $\sigma_A^2 = L$ and $\sigma_B^2 = R$, then party A expects $2 \cdot (1 - E(\alpha))$, and party B expects $2 \cdot E(\alpha)$. In addition to this payoff, parties also receive 1/2 from the popularity payoff of the first period.

Since parties' payoffs never depend on all three components, then parties' strategies can be grouped in pairs. For example, parties always expect the same payoff at LLL than at LLR, independently of the strategy of the opponent. After this arrangement, the game in normal form is represented in Table 4.3, where, $\Gamma_J = E(\alpha) + E(\pi_J(\alpha))$, and $\Upsilon_J = 1 - E(\alpha) + E(\pi_J(\alpha))$.

$A \setminus B$	LLL ⋮ LLR	LRL ⋮ LRR	RLL ⋮ RRL	RLR ⋮ RRR
LLL / LLR	2, 1	$\frac{1}{2} + 2 \cdot (1 - E(\alpha))$, $\frac{1}{2} + 2 \cdot E(\alpha)$	Υ_A, Γ_B	Υ_A, Γ_B
LRL / LRR	$\frac{3}{2}$, $\frac{3}{2}$	1, 2	Υ_A, Γ_B	Υ_A, Γ_B
RLL / RRL	Γ_A, Υ_B	Γ_A, Υ_B	2, 1	$\frac{1}{2} + 2 \cdot (1 - E(\alpha))$, $\frac{1}{2} + 2 \cdot E(\alpha)$
RLR / RRR	Γ_A, Υ_B	Γ_A, Υ_B	$\frac{3}{2}$, $\frac{3}{2}$	1, 2

Table 4.3: The game in normal form, after regrouping parties' strategies.

The set of payoffs from the profiles where $\sigma_A^1 = \sigma_B^1$ are grouped in two identical submatrices, which are affine transformations of the payoffs from Table 4.2 (after adding 1/2 the payoffs of the two parties), except that now α remains unknown (see Table 4.4). In fact, if either of those submatrices constituted a game itself, then the equilibrium would be like in Proposition 4.1, but depending on $E(\alpha)$ instead. For example, suppose that party B chooses LRL. Then, party A will prefer LLL to LRL only if $E(\alpha)$ is lower than 3/4, which is one of the cutoffs identified in the proposition. Through the rest of the section, I fix the value of $E(\alpha)$ at each interval and solve the corresponding game. I

$A \setminus B$	LLL ⋮ LLR	LRL ⋮ LRR	RLL ⋮ RRL	RLR ⋮ RRR
LLL / LLR	$2,\ 1$	$\frac{1}{2}+2\cdot(1-E(\alpha)),\ \frac{1}{2}+2\cdot E(\alpha)$	Υ_A, Γ_B	Υ_A, Γ_B
LRL / LRR	$\frac{3}{2},\ \frac{3}{2}$	$1,\ 2$	Υ_A, Γ_B	Υ_A, Γ_B
RLL / RRL	Γ_A, Υ_B	Γ_A, Υ_B	$2,\ 1$	$\frac{1}{2}+2\cdot(1-E(\alpha)),\ \frac{1}{2}+2\cdot E(\alpha)$
RLR / RRR	Γ_A, Υ_B	Γ_A, Υ_B	$\frac{3}{2},\ \frac{3}{2}$	$1,\ 2$

Table 4.4: If we fix $\sigma_A^1 = \sigma_B^1$, then parties' preferences over the strategies are determined by $E(\alpha)$. More specifically, $1/2 + 2 \cdot (1 - E(\alpha)) > 1$ holds if and only if $E(\alpha) < 3/4$ (red arrows), and $1/2 + 2 \cdot E(\alpha) > 1$ holds if and only if $E(\alpha) > 1/4$ (blue arrows).

also make the following assumption on the distribution of α.[7]

$$E(\alpha) < 1/2 \iff \Gamma_B < 1/2 + 2 \cdot E(\alpha) \iff \Upsilon_B > 1/2 + 2 \cdot E(\alpha). \tag{4.1}$$

$E(\alpha) < 1/4$

First, suppose that $E(\alpha) < 1/4$.[8] Since $E(\alpha) < 3/4$, then the following strategies are weakly dominated.

- $\sigma_A = RLR$ and $\sigma_A = RRR$ are weakly dominated by $\sigma_A = RLL$ and $\sigma_A = RRL$.
- $\sigma_A = LRL$ and $\sigma_A = LRR$ are weakly dominated by $\sigma_A = LLL$ and $\sigma_A = LLR$.

After their elimination, and since $E(\alpha) > 1/4$, then the following strategies are now weakly dominated.

- $\sigma_B = RLR$ and $\sigma_B = RRR$ are weakly dominated by $\sigma_B = RLL$ and $\sigma_B = RRL$.
- $\sigma_B = LRL$ and $\sigma_B = LRR$ are weakly dominated by $\sigma_B = LLL$ and $\sigma_B = LLR$.

After their elimination, parties play the game represented in Table **??**. Parties always have to decide between learning α or not, independently of the strategy of the opponent.

[7] In the Appendix, I show that Assumption (4.1) holds with the most well-known probability distributions with support $[0, 1]$ (see Figures C.2 and C.3).

[8] This case is symmetric to $E(\alpha) > 3/4$, which is studied in the Appendix.

$A \setminus B$	LLL ⋮ LLR	RLL ⋮ RRL
LLL / LLR	$2, 1$	Υ_A, Γ_B
RLL / RRL	Γ_A, Υ_B	$2, 1$

Table 4.5: The game played by parties if $E(\alpha) < 1/4$, after iterative elimination of weakly dominated strategies.

And if α is not learned, then both parties support policy L in the second period. For example, suppose that either $\sigma_A = LLL$ or $\sigma_A = LLR$. That is, party A chooses policy L in the first period, and policy L after (L, L). In this case, party B has to decide between choosing policy R in the first period and learning α ($\sigma_B = \underline{R}LL$ or $\sigma_B = \underline{R}RL$) or not learning α and choosing policy L afterwards ($\sigma_B = \underline{LL}L$ or $\sigma_B = \underline{LL}R$).

Proposition 4.2. *Suppose that $E(\alpha) < 1/4$ holds and consider the game displayed in Table 4.5. Then, the following statements hold.*

- *If $\Gamma_B < 1$, then there are four subgame perfect equilibra that survive iterative elimination of dominated pure strategies. Namely,*

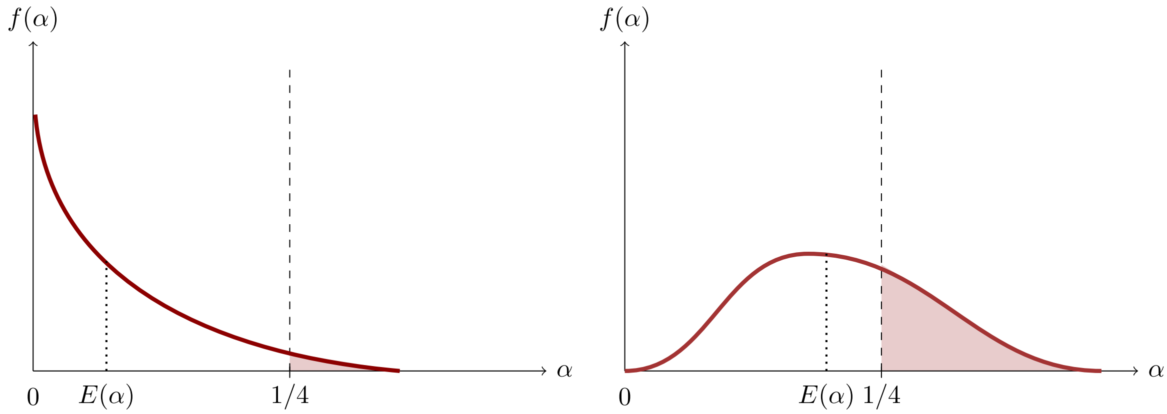
$$(LLx, LLy) \text{ for } x, y \in \{L, R\} .$$

- *If $\Gamma_B = 1$, then there are eight subgame perfect equilibra that survive iterative elimination of dominated pure strategies. Namely,*

$$(LLx, LLy),\ (LLv, RwL) \text{ for } x, y, v, w \in \{L, R\} .$$

- *If $\Gamma_B > 1$, then there is no subgame perfect equilibrium with undominated pure strategies.*

In the Appendix, I will show that $\Gamma_B = 3 - \Upsilon_A$ (see Equation (C.2)), which means that $\Gamma_B < 1$ holds if and only if $\Upsilon_A > 2$ holds. Thus, if $\Gamma_B < 1$, then in all the equilibria parties play (L, L) in the first period, they do not learn α, and then they play (L, L) in the second period. If $\Gamma_B > 1$, then party B deviates to $\sigma_B^1 = R$ in order to learn α, but then party A deviates to $\sigma_A^1 = R$ in order not to learn α, and a pure strategy equilibrium does not exist.



(a) Parties do not learn α.

(b) Party B deviates to policy R to learn α.

Figure 4.4: Two distributions of α with $E(\alpha) < 1/4$.

Whether $\Gamma_B < 1$ holds or not depends on $F(1/4)$. For example, when $F(1/4)$ is close to one, then parties do not learn α.[9] The intuition is the following. If $F(1/4)$ is very large, like in Figure 4.4a, then learning α does not excessively affect in expectation the payoffs received in the second period, because $E(\pi_A(\alpha))$ and $E(\pi_B(\alpha))$ are close to 3/2 and 1/2 respectively, which are the payoffs received if α is not learned. Then, parties only compete for the popularity payoff of the first period, and they both choose the policy preferred by the majority of citizens, which is policy L.

If $F(1/4)$ decreases, then $E(\pi_B(\alpha))$ increases, because $\pi_B(\alpha) > 1/2$ holds with more probability, but $E(\pi_A(\alpha))$ decreases, because $\pi_A(\alpha) < 3/2$ also holds with more probability (see Figure 4.3). Hence, party B has more incentives to learn α, while party A has less incentives to do so. For illustration, consider the distribution of Figure 4.4b. Since $E(\alpha) < 1/4$, then parties play (L, L) if they do not learn α, and parties A and B receive 3/2 and 1/2 respectively. However, the probability that $\alpha \in (1/4, 3/4)$ is quite large, which means that, if parties learn α, then it is likely that they play (L, R) in the subgame. Then, party B would receive $\pi_B(\alpha) > 1/2$, and party A would receive $\pi_A(\alpha) < 3/2$. Thus, the lower is $F(1/4)$, the larger will be $E(\pi_B(\alpha))$ with respect to 1/2, and the lower will be $E(\pi_A(\alpha))$ with respect to 3/2. When $F(1/4)$ is low enough (taking into account that $E(\alpha) < 1/4$ must hold), then party B deviates to policy R in order to learn α. However, party A will then deviate to policy R in order not to learn α, and a pure strategies subgame perfect equilibrium will not exist.

[9] Observe that if $F(1/4) = 1$, then $E(\pi_B(\alpha)) = 1/2$ (see Figure 4.3), and thus $\Gamma_B = E(\alpha) + 1/2$, which is lower than one because $E(\alpha) < 1/2$ holds.

$E(\alpha) \in (1/4, 3/4)$

Suppose that $E(\alpha) \in (1/4, 3/4)$. In this case, the following strategies are weakly dominated.

- $\sigma_A = RLR$ and $\sigma_A = RRR$ are weakly dominated by $\sigma_A = RLL$ and $\sigma_A = RRL$.
- $\sigma_A = LRL$ and $\sigma_A = LRR$ are weakly dominated by $\sigma_A = LLL$ and $\sigma_A = LLR$.
- $\sigma_B = LLL$ and $\sigma_B = LLR$ are weakly dominated by $\sigma_B = LRL$ and $\sigma_B = LRR$.
- $\sigma_B = RLL$ and $\sigma_B = RRL$ are weakly dominated by $\sigma_B = RLR$ and $\sigma_B = RRR$.

After their elimination, parties now play the game represented in Table 4.6. The main difference with respect to previous game is that if α is not learned, then parties play (L, R) instead of (L, L) in the second period.

$A \setminus B$	LRL | LRR	RLR | RRR
LLL / LLR	$\frac{1}{2} + 2 \cdot (1 - E(\alpha))$, $\frac{1}{2} + 2 \cdot E(\alpha)$	Υ_A, Γ_B
RLL / RRL	Γ_A, Υ_B	$\frac{1}{2} + 2 \cdot (1 - E(\alpha))$, $\frac{1}{2} + 2 \cdot E(\alpha)$

Table 4.6: The game played by parties if $E(\alpha) \in (1/4, 3/4)$, after elimination of weakly dominated strategies.

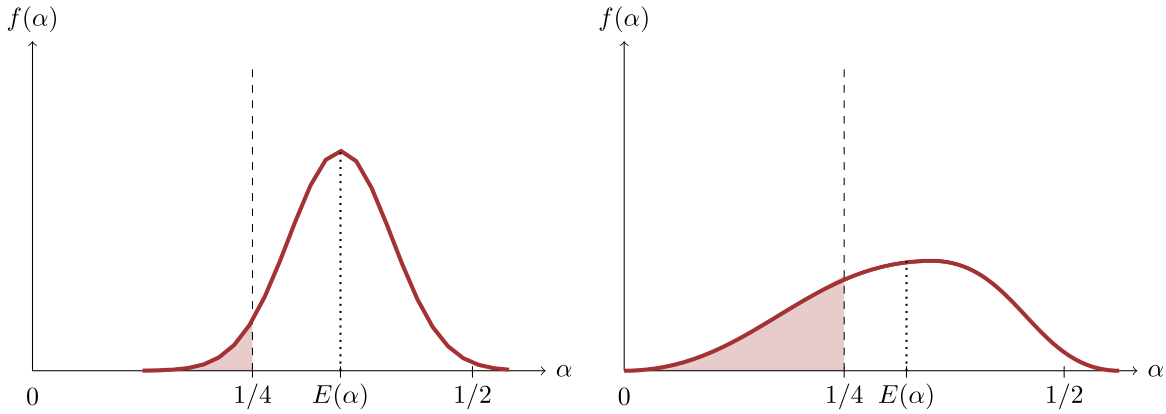
Proposition 4.3. *Suppose that $E(\alpha) \in (1/4, 3/4)$ and Assumption (4.1) holds. Then, the following statements hold.*

- *If $E(\alpha) < 1/2$, then there are four subgame perfect equilibria with undominated pure strategies. Namely,*

$$(LLx, LRy) \text{ for } x, y \in \{L, R\}.$$

- *If $E(\alpha) > 1/2$, then there are four subgame perfect equilibria with undominated pure strategies. Namely,*

$$(RxL, RyR) \text{ for } x, y \in \{L, R\}.$$



(a) Party B has low incentives to learn α.

(b) Party B has larger incentives to learn α.

Figure 4.5: Two distributions of α with $E(\alpha) \in (1/4, 1/2)$.

If Assumption (4.1) holds, then parties do not learn α. Instead, they both choose in the first period the policy preferred by the majority of citizens, and then they play (L, R) in the second period.

Although parties never learn α, they may still have incentives to do so. For illustration, consider the two distributions of Figure 4.5. In both of them, $E(\alpha) \in (1/4, 1/2)$ holds, so parties choose (L, R) if they do not learn α. In the left distribution, $F(1/4)$ is very low, and $E(\pi_B(\alpha))$ and $E(\pi_A(\alpha))$ are close to $2 \cdot E(\alpha)$ and $2 \cdot (1 - E(\alpha))$ respectively, which are the payoffs that parties expect to receive if α is not learned. Thus, learning α does not affect in expectation parties' future payoffs, so like previously, they both choose policy L. In the right distribution, $F(1/4)$ is larger, which means that if α is learned, then it is more likely that parties play (L, L) in the subgame. Then, B would receive $\pi_B(\alpha) = 1/2$, which is larger than $2 \cdot \alpha$. Hence, B has now more incentives to choose $\sigma_B^1 = R$ and learn α. However, the expected popularity payoff that party B would receive in the first period, which is $E(\alpha)$, is likely to be very low, given that $F(1/4)$ has increased. If Assumption (4.1) holds, then the loss of popularity will always be larger than the expected future gain from learning α, and the party will never deviate to policy R.

4.3.3 Mixed strategies

For the cases where there is no pure strategies subgame perfect equilibrium, I look for the mixed strategies equilibria. I still assume that parties play in the subgames following the equilibria identified in Proposition 4.1, and that parties never play the strategies that were iteratively eliminated. In this section, I will describe the case where $E(\alpha) < 1/4$ and $\Gamma_B > 1$ hold. The procedure for $E(\alpha) > 3/4$ is similar, and it will be studied in the Appendix.

Suppose that $E(\alpha) < 1/4$ and $\Gamma_B > 1$, and let $p = (p_1, p_2, p_3, p_4)$ and $q = (q_1, q_2, q_3, q_4)$

<table>
<tr><td></td><td></td><td>q_1</td><td>q_2</td><td>q_3</td><td>q_4</td></tr>
<tr><td></td><td>$A \setminus B$</td><td>LLL</td><td>LLR</td><td>RLL</td><td>RRL</td></tr>
<tr><td>p_1</td><td>LLL</td><td colspan="2" rowspan="2">$2, 1$</td><td colspan="2" rowspan="2">Υ_A, Γ_B</td></tr>
<tr><td>p_2</td><td>LLR</td></tr>
<tr><td>p_3</td><td>RLL</td><td colspan="2" rowspan="2">Γ_A, Υ_B</td><td colspan="2" rowspan="2">$2, 1$</td></tr>
<tr><td>p_4</td><td>RRL</td></tr>
</table>

Table 4.7: The mixed extension of the game played by parties if $E(\alpha) < 1/4$.

denote two vectors indicating the probabilities that parties A and B play each of the undominated strategies. For example, p_1 is the probability that party A chooses $\sigma_A = LLL$ (see Table 4.7). It is inmediate that $p_3 + p_4 = 1 - p_1 - p_2$ and $q_3 + q_4 = 1 - q_1 - q_2$ hold.

On the one hand, it can be shown that party A is indifferent between its four strategies if and only if:

$$q_1 + q_2 = \frac{2 - \Upsilon_A}{4 - \Upsilon_A - \Gamma_A}.$$

If $q_1 + q_2$ is strictly larger than this expression, then party A chooses either $\sigma_A = LLL$ or $\sigma_A = LLR$. The reason is that party A, which is the one that does not benefit from learning α, always prefers to choose the same policy than party B in the first period (see Section 4.3.2) . That is, if party B chooses a strategy with $\sigma_B^1 = L$, then party A prefers the strategies with $\sigma_A^1 = R$, and vice versa. Thus, party A chooses $\sigma_A^1 = L$ in the mixed extension only if $q_1 + q_2$, which is the probability that party B chooses $\sigma_B^1 = L$, is sufficiently large.

On the other hand, it can be shown that party B is indifferent between its fourth strategies if and only if:

$$p_1 + p_2 = \frac{\Upsilon_B - 1}{\Gamma_B + \Upsilon_B - 2}.$$

In this case, if $p_1 + p_2$ is strictly lower that this expression, then party B chooses either $\sigma_B = LLL$ or $\sigma_B = LLR$. The reasoning is similar that previously, except that now party B, which is the one that benefits from learning α, always prefers to choose the opposite policy than party A in the first period. Hence, party B now chooses $\sigma_B^1 = L$ in the mixed extension only if $p_1 + p_2$, which is the probability that party A chooses $\sigma_A^1 = L$, is sufficiently low.

Proposition 4.4. *Suppose that $E(\alpha) < 1/4$ and $\Gamma_B > 1$. Then, any pair $(p^\star, q^\star)$ where:*

$$p_1^* + p_2^* = \frac{\Upsilon_B - 1}{\Gamma_B + \Upsilon_B - 2} \quad \textit{and} \quad q_1^* + q_2^* = \frac{2 - \Upsilon_A}{4 - \Upsilon_A - \Gamma_A}$$

$A \setminus B$	LLL	LLR	RLL	RRL
LLL	$(p_1^* + p_2^*) \cdot (1 - p_1^* - p_2^*)$		$(p_1^* + p_2^*)^2$	
LLR				
RLL	$(1 - p_1^* - p_2^*)^2$		$(p_1^* + p_2^*) \cdot (1 - p_1^* - p_2^*)$	
RRL				

Table 4.8: The probabilities of every equilibrium.

is a mixed strategies equilibrium of the game displayed in Table 4.7. Moreover,

$$\frac{2 - \Upsilon_A}{4 - \Upsilon_A - \Gamma_A} = 1 - \frac{\Upsilon_B - 1}{\Gamma_B + \Upsilon_B - 2},$$

and thus, $q_1^* + q_2^* = 1 - p_1^* - p_2^*$ *holds.*

It can be also shown that $p_1^* + p_2^* > 1/2$, and thus $q_1^* + q_2^* < 1/2$, hold.[10] As a consequence, in all the mixed strategies equilibria, $(\sigma_A^1, \sigma_B^1) = (L, R)$ is the most likely policy vector of the first period (see Table 4.8).

4.4 Final remarks

The objective of this paper has been to study parties' incentives to learn about the state of the public opinion. I have assumed that parties might have some belief about the public opinion, but they only learn its true state when they support different policies. Then, I have found that a party might find confrontation profitable because citizens' preferences would be observed, which increases in expectation the party's future payoffs.

Because of the symmetry of the model, then party confrontation is never a pure strategy equilibrium in the learning period. I now comment two extensions under which confrontation might arise as an equilibrium. First, I may assume that each party has its own belief about citizens' preferences, in which case the model would be of incomplete information (as long as the beliefs are private information). Then, it can happen that both parties benefit in expectation from learning, which is not possible in the current model, and thus that both parties prefer confrontation. Second, I may also vary parties'

[10]The reason behind this is that $E(\alpha) < 1/2$, which implies that parties always prefer to be the one supporting policy L in the strategies where $\sigma_A^1 \neq \sigma_B^1$, because policy L is the policy preferred by the majority of citizens. For example, party A always receives a larger expected payoff at $(\sigma_A, \sigma_B) = (LLL, RLL)$ than at $(\sigma_A, \sigma_B) = (RLL, LLL)$, and vice versa. This can be interpreted as a positive valence towards policy L which leads to $p_1^* + p_2^* > 1/2$, so if for example $p_1 + p_2 = 1/2$ (there is 1/2 probability that party A chooses $\sigma_A^1 = L$), then party B still chooses $\sigma_B^1 = L$ with probability one.

features. For example, I may assume that one party is more concerned with its own popularity than the other. In this case, parties' payoffs would no longer be symmetric, and then confrontation might arise.

As a final remark, I have not imposed strong assumptions on the distribution that parties know about citizens' preferences. Hence, my equilibrium results are quite general, but they can also be seen as too arbitrary. By focusing on particular distributions, the results may become more precise. For example, we could assume that citizens' preferences can only be of two types, and parties decide whether to learn or not which type it is. This kind of simplification may allow to study the effect of learning in a less abstract way. Yet, I will let all those extensions for future work.

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Appendix A

Proofs of Results of Chapter 2

A.0.1 Best replies

I obtain the best replies of the game displayed in Table 2.2. Define $BR_A(x_B)$ and $BR_B(x_A)$ as the best reply correspondences of party A and party B to $x_B \in \{L, R\}$ and $x_A \in \{L, R\}$ respectively.

1. $BR_A(R)$

After replacing (2.1) and $(C.1)$, $\pi_A(L, R, t)$ and $\pi_A(R, R)$ can be written as[1]

$$\pi_A(L, R, t) = 1 - (k + 1) \cdot (n_B + t)$$
$$\pi_A(R, R) = -k \cdot n_B,$$

where, given n_B and k, $\pi_A(L, R, t)$ is continuous and strictly decreasing with respect to t and $\pi_A(R, R)$ is constant. Observe that if $t = -n_B$, which is the minimum value that t can take, then $\pi_A(L, R, t) = 1$, which is strictly above $\pi_A(R, R)$. Moreover, if $t = 1 - n_B$, which is the maximum value that t can take, then $\pi_A(L, R, t) = -k$, which is strictly below $\pi_A(R, R)$. Because $\pi_A(L, R, t)$ is continuous and strictly decreasing, then $\pi_A(L, R, t)$ is equal to $\pi_A(R, R)$ at a unique value of t, denoted by t_A. Moreover $\pi_A(L, R, t) < \pi_A(R, R)$ for all t above t_A, and $\pi_A(L, R, t) > \pi_A(R, R)$ for all t below t_A.

[1]In Table 2.1, it can be noted that the constant k appears in every payoff of party A. Then, since equilibria are invariant with respect to positive affine transformation of the payoffs, I have substracted k to every payoff of party A.

To find the intersection, I set $\pi_A(L, R, t_A) = \pi_A(R, R)$ and solve for t_A:

$$1 - (k+1) \cdot (n_B + t_A) = -k \cdot n_B$$
$$\Longrightarrow t_A = \frac{1 + k \cdot n_B}{k+1} - n_B$$
$$\Longrightarrow t_A = \frac{1 - n_B}{k+1}.$$

which is strictly positive and lower than $1 - n_B$. Then,

$$BR_A(R) = \begin{cases} \{L\} & \text{if} \quad t < t_A \\ \{L, R\} & \text{if} \quad t = t_A \\ \{R\} & \text{if} \quad t > t_A. \end{cases} \tag{A.1}$$

$BR_A(R)$ is illustrated in Figure 2.2a.

2. $BR_B(L)$

After replacing (2.1) and (C.1), $\pi_B(L, R, t)$ and $\pi_B(L, L)$ can be written as

$$\pi_B(L, R, t) = (k+1) \cdot (n_B + t)$$
$$\pi_B(L, L) = k \cdot n_B,$$

where, given n_B, $\pi_B(L, R, t)$ is continuous and strictly increasing with respect to t, and $\pi_B(L, L)$ is constant. If $t = -n_B$ (its minimum), then $\pi_B(L, R, t) = 0$, which is strictly below $\pi_B(L, L)$. And if $t = 1 - n_B$ (its maximum), then $\pi_B(L, R, t) = k$, which is strictly above $\pi_B(L, L)$. Because $\pi_B(L, R, t)$ is continuous and strictly decreasing, then $\pi_B(L, R, t)$ is equal to $\pi_B(L, L)$ at a unique value of t, denoted by t_B. Moreover $\pi_B(L, R, t) > \pi_B(L, L)$ for all t above t_B, and $\pi_B(L, R, t) < \pi_B(L, L)$ for all t below t_B To find the intersection, I set $\pi_B(L, R, t_B) = \pi_B(L, L)$ and solve for t_B:

$$(k+1) \cdot (n_B + t_B) = k \cdot n_B$$
$$\Longrightarrow t_B = \frac{k \cdot n_B}{k+1} - n_B$$
$$\Longrightarrow t_B = -\frac{n_B}{k+1},$$

which is strictly negative and greater than $-n_B$. Then,

$$BR_B(L) = \begin{cases} \{L\} & \text{if} \quad t < t_B \\ \{L, R\} & \text{if} \quad t = t_B \\ \{R\} & \text{if} \quad t > t_B. \end{cases} \tag{A.2}$$

$BR_B(L)$ is illustrated in Figure 2.1a.

3. $BR_A(L)$

After replacing (2.1) and (2.3), $\pi_A(R, L, r)$ and $\pi_A(L, L)$ can be written as

$$\pi_A(R, L, r) = -(k-1) \cdot (n_B + r)$$
$$\pi_A(L, L) = 1 - k \cdot n_B.$$

Given n_B, the function $\pi_A(R, L, r)$ is strictly decreasing with respect to r if and only if $k > 1$, and $\pi_A(L, L)$ is constant. If $r = 1 - n_B$ (its maximum), then $\pi_A(R, L, r) = 1 - k$, which is strictly below $\pi_A(R, R)$. If $k \leq 1$, then $\pi_A(R, L, r)$ is non-decreasing, so we can ensure that $\pi_A(R, L, r) < \pi_A(R, R)$ for all r between $-n_B$ and $1 - n_B$. Thus, if $k \leq 1$:

$$BR_A(L) = \{L\} \quad \text{for all } \ r. \tag{A.3}$$

This case is illustrated in Figure 2.2b. Now suppose that $k > 1$, such that $\pi_B(R, L, r)$ is strictly decreasing. If $r = -n_B$ (its minimum), then $\pi_A(R, L, r) = 0$, which is strictly below $\pi_A(L, L)$ if and only if:

$$0 < 1 - k \cdot n_B,$$

which is equivalent to $k < \frac{1}{n_B}$, where $\frac{1}{n_B} > 1$ holds. Thus, if k belongs to $\left(1, \frac{1}{n_B}\right)$, then $\pi_A(R, L, r)$ is strictly decreasing but still below $\pi_A(L, L)$ for all r, so $BR_A(L) = \{L\}$ also holds. If $k \geq \frac{1}{n_B}$, then $\pi_A(R, L, r)$, which is continuous, is equal to $\pi_A(L, L)$ at a unique value of r, denoted by r_A. Moreover, $\pi_A(R, L, r) > \pi_A(L, L)$ for all r below r_A, and $\pi_A(R, L, r) < \pi_A(L, L)$ for all r above r_A. To find the intersection, I set $\pi_A(R, L, r_A) = \pi_A(L, L)$ and solve for r_A:

$$-(k-1) \cdot (n_B + r_A) = 1 - k \cdot n_B$$
$$\Longrightarrow r_A = \frac{k \cdot n_B - 1}{k - 1} - n_B$$
$$\Longrightarrow r_A = -\frac{1 - n_B}{k - 1},$$

which is strictly negative. Thus, if $k > 1$, then I can express $BR_A(L)$ as follows:

$$BR_A(L) = \begin{cases} \{R\} & \text{if} \quad r < r_A \\ \{L, R\} & \text{if} \quad r = r_A \\ \{L\} & \text{if} \quad r > r_A \end{cases} \tag{A.4}$$

This expression holds if $k \in \left(1, \frac{1}{n_B}\right)$, in which case $r > r_A$ holds for all r between $-n_B$ and $1 - n_B$, so $BR_A(L) = \{L\}$.

4. $BR_B(R)$

After replacing (2.1) and (2.3), $\pi_A(R, L, r)$ and $\pi_A(R, R)$ can be written as

$$\pi_B(R, L, r) = 1 + (k-1) \cdot (n_B + r)$$
$$\pi_B(R, R) = 1 + k \cdot n_B.$$

Given n_B, the function $\pi_B(R, L, r)$ is strictly increasing with respect to r if and only if $k > 1$, and $\pi_B(R, R)$ is constant. If $r = -n_B$ (its minimum), then $\pi_B(R, L, r) = 1$, which is strictly below $\pi_B(R, R)$. If $k \leq 1$, then $\pi_B(R, L, r)$ is non-increasing, so we can ensure that $\pi_B(R, L, r)$ is below $\pi_B(R, R)$ for all r between $-n_B$ and $1 - n_B$. Thus, if $k \leq 1$:

$$BR_B(R) = \{R\} \quad \text{for all } r. \tag{A.5}$$

Now suppose that $k > 1$, which means that $\pi_B(R, L, r)$ is strictly increasing. If $r = 1 - n_B$ (its maximum), then $\pi_B(R, L, r) = k$, which is strictly below $\pi_B(R, R)$ if and only if:

$$k < 1 + n_B \cdot k,$$

which is equivalent to $k < \frac{1}{1-n_B}$, where $\frac{1}{1-n_B} > 1$ holds. Thus, if $k \in \left(1, \frac{1}{1-n_B}\right)$, then $\pi_B(R, L, r)$ is strictly increasing but still below $\pi_B(R, R)$ for all r, so $BR_B(R) = \{R\}$ holds. If $k \geq \frac{1}{1-n_B}$, then $\pi_B(R, L, r)$ is equal to $\pi_B(R, R)$ at a unique r, denoted by r_B. Moreover, $\pi_B(R, L, r) > \pi_B(R, R)$ for all r above r_B, and $\pi_B(R, L, r) < \pi_B(R, R)$ for all r below r_B. To find the intersection, I set $\pi_B(R, L, r_B) = \pi_B(R, R)$ and solve for r_B:

$$1 + (k-1) \cdot (n_B + r_B) = 1 + k \cdot n_B$$
$$\Longrightarrow r_B = \frac{k \cdot n_B}{k-1} - n_B$$
$$\Longrightarrow r_B = \frac{n_B}{k-1},$$

which is strictly positive. Thus, If $k > 1$, then I can express $BR_B(R)$ as follows:

$$BR_B(R) = \begin{cases} \{R\} & \text{if} \quad r < r_B \\ \{L, R\} & \text{if} \quad r = r_B \\ \{L\} & \text{if} \quad r > r_B \end{cases} \tag{A.6}$$

The expression holds if $k \in \left(1, \frac{1}{1-n_B}\right)$, in which case $r < r_B$ holds for all r between $-n_B$ and $1 - n_B$, so $BR_B(R) = \{R\}$. Figure 2.1b illustrates $BR_B(R)$.

A.0.2 Proofs of the propositions

Proof of Proposition 4.1. Suppose that $k \leq 1$ holds. Then, by (A.3), $BR_A(L) = \{L\}$, so (R, L) is not a fixed point. Suppose now than $k > 1$. Then, by (A.4), $BR_A(L) = \{R\}$ if $r < r_A$, and by (A.6), $BR_B(R) = \{L\}$ if $r > r_B$. Because $r_B > r_A$, then (R, L) is never a fixed point.

■

Proof of Proposition 4.2. By (A.1), if $t < t_A$, then $BR_A(R) = \{L\}$. By (A.2), if $t > t_B$, then $BR_B(L) = \{R\}$. Hence, if $t \in (t_B, t_A)$, then (L, R) is a fixed point.

Moreover, (R, R) and (L, L) are not equilibria, because $BR_A(R) = \{L\}$ and $BR_B(L) = \{R\}$. By Proposition 4.1, (R, L) is neither an equilibrium. Hence, (L, R) is the unique equilibrium. ■

Proof of Proposition 4.3 . **1**. Suppose that $k \leq 1$ holds. To obtain a contradiction, suppose that a pure strategy equilibrium does not exist. Then, either *i)* $BR_A(R) = L$, $BR_B(L) = L$, $BR_A(L) = R$ and $BR_B(R) = R$ hold, or *ii)* $BR_A(R) = R$, $BR_B(R) = L$, $BR_A(L) = L$ and $BR_B(L) = R$ hold.

By (A.3), $BR_A(L) = \{L\}$, so *i)* cannot be satisfied. Also, by (A.5), $BR_R(R) = \{R\}$, so *ii)* can neither be satisfied. Thus, if $k \leq 1$, then a pure strategy equilibrium exists.

2. By (A.3) and (A.5), if $k \leq 1$, then $BR_A(L) = \{L\}$ and $BR_B(R) = \{R\}$. Suppose that $t < t_B < t_A$. Then, by (A.1) and (A.2), $BR_B(L) = \{L\}$ and $BR_A(R) = \{L\}$, so (L, L) is the unique fixed point.

Suppose that $t > t_A$. Then, by (A.1) and (A.2), $BR_A(R) = \{R\}$ and $BR_B(L) = \{R\}$, so (R, R) is the unique fixed point. ■

Proof of Proposition 4.4. Suppose that $k > 1$ holds.

1. Suppose that $t < t_B$ and $r > r_A$ hold. Then, by (A.1), (A.2) and (A.4), $BR_A(R) = \{L\}$, $BR_B(L) = \{L\}$ and $BR_A(L) = \{L\}$.

Suppose that $r \in (r_A, r_B)$. Then, by (A.6), $BR_B(R) = \{R\}$, and the unique fixed point is (L, L). Suppose that $r \geq r_B$. Then, by (A.6), $L \in BR_B(R)$, so the unique fixed point is also (L, L).

2. Suppose that $t > t_A$ and that $r < r_B$ hold. Then, by (A.1), (A.2) and (A.6), $BR_A(R) = \{R\}$, $BR_B(L) = \{R\}$ and $BR_B(R) = \{R\}$.

Suppose that $r \in (r_A, r_B)$. Then, by (A.4), $BR_A(L) = \{L\}$, and the unique fixed point is (R, R). Suppose that $r \leq r_A$. Then, by (A.4), $R \in BR_A(L)$, and the unique fixed point is also (R, R).

3. Suppose that $t < t_B$ and $r < r_A$ hold. Then, by (A.1), (A.2), (A.4) and (A.6), $BR_A(R) = \{L\}$, $BR_B(L) = \{L\}$, $BR_A(L) = \{R\}$ and $BR_B(R) = \{R\}$, so a pure strategy equilibrium does not exist.

Suppose that $t > t_A$ and that $r > r_B$ holds. Then, by (A.1), (A.2), (A.4) and (A.6), $BR_A(R) = \{R\}$, $BR_B(L) = \{R\}$, $BR_A(L) = \{L\}$ and $BR_B(R) = \{L\}$, so a pure strategy equilibrium does not exist.

■

Appendix B

Proofs of Results of Chapter 3

B.0.1 Best replies

Consider the game displayed in Table 3.4. Define $BR_A(x_B^1)$ and $BR_B(x_A^1)$ as the best reply of party A and party B to x_B^1 and x_A^1 respectively. Also, recall that $\pi_A(x_A^1, x_B^1)$ and $\pi_B(x_A^1, x_B^1)$ are defined as follows.

$$\pi_A(x_A^1, x_B^1) = \begin{cases} 3/2 \quad \text{if } \alpha_2(x_A^1, x_B^1) < \frac{1}{4} \\ 2 \cdot (1 - \alpha_2(x_A^1, x_B^1)) \text{ if } \alpha_2(x_A^1, x_B^1) \in [\frac{1}{4}, \frac{3}{4}] \\ 1/2 \quad \text{if } \alpha_2(x_A^1, x_B^1) > \frac{3}{4}. \end{cases} \tag{B.1}$$

$$\pi_B(x_A^1, x_B^1) = \begin{cases} 1/2 \quad \text{if } \alpha_2(x_A^1, x_B^1) < \frac{1}{4} \\ 2 \cdot \alpha_2(x_A^1, x_B^1) \text{ if } \alpha_2(x_A^1, x_B^1) \in [\frac{1}{4}, \frac{3}{4}] \\ 3/2 \quad \text{if } \alpha_2(x_A^1, x_B^1) > \frac{3}{4}, \end{cases} \tag{B.2}$$

where $\pi_A(x_A^1, x_B^1)$ and $\pi_B(x_A^1, x_B^1)$ are respectively non-increasing and non-decreasing with respect to $\alpha_2(x_A^1, x_B^1)$.

1. B's best reply to $x_A^1 = R$

Fix $x_A^1 = R$. Then, party B chooses policy R if $3/2 + \pi_B(R, R) > 1 + \pi_B(R, L)$ holds. By Condition (3.3), $\alpha_2(R, R) > \alpha_2(R, L)$. Since $\pi_B(x_A^1, x_B^1)$ is non-decreasing with $\alpha_2(x_A^1, x_B^1)$, then $\pi_B(R, R) \geq \pi_B(R, L)$, and thus $3/2 + \pi_B(R, R) > 1 + \pi_B(R, L)$. Hence,

$$BR_B(R) = \{R\}. \tag{B.3}$$

2. A's best reply to $x_B^1 = L$

Fix $x_B^1 = L$. Then, party A chooses policy L if $3/2 + \pi_A(L, L) > 1 + \pi_A(R, L)$ holds. By Condition (3.3), $\alpha_2(L, L) < \alpha_2(R, L)$. Since $\pi_A(x_A^1, x_B^1)$ is non-increasing with $\alpha_2(x_A^1, x_B^1)$, then $\pi_A(L, L) \geq \pi_A(R, L)$, and thus $3/2 + \pi_A(L, L) > 1 + \pi_A(R, L)$. Hence,

$$BR_A(L) = \{L\}. \tag{B.4}$$

3. B's best reply to $x_A^1 = L$

Fix $x_A^1 = L$. Then, party B chooses policy R if $2 \cdot \alpha_1 + \pi_B(L, R) > 1/2 + \pi_B(L, L)$ holds. Otherwise, party B chooses policy L (if both sides are equal, then party B is indifferent between L and R).

By Condition (3.3), $\alpha_2(L, R) > \alpha_2(L, L)$. Since $\pi_B(x_A^1, x_B^1)$ is non-decreasing with $\alpha_2(x_A^1, x_B^1)$, then $\pi_B(L, R) \geq \pi_B(L, L)$. Thus, whenever $\alpha_1 > 1/4$ then $2 \cdot \alpha_1 + \pi_B(L, R) > 1/2 + \pi_B(L, L)$. In this case, $BR_B(L) = \{R\}$.

If $\alpha_1 < 1/4$, then, by Condition (3.1), $\alpha_2(L, L) < 1/4$ and $\alpha_2(L, R) < 1/4$ hold, so $\pi_B(L, L) = \pi_B(L, R) = 1/2$. Then, it is inmediate that $2 \cdot \alpha_1 + \pi_B(L, R) < 1/2 + \pi_B(L, L)$ holds. In this case, $BR_B(L) = \{L\}$.

If $\alpha_1 = 1/4$, then, by Condition (3.1), $\alpha_2(L, L) < 1/4$ and $\alpha_2(L, R) < 1/4$ hold, so $\pi_B(L, L) = \pi_B(L, R) = 1/2$. Then, it is inmediate that $2 \cdot \alpha_1 + \pi_B(L, R) = 1/2 + \pi_B(L, L) = 1$, so $BR_B(L) = \{L, R\}$. Hence,

$$BR_B(L) = \begin{cases} \{L\} & \text{if } \alpha_1 < 1/4 \\ \{L, R\} & \text{if } \alpha_1 = 1/4 \\ \{R\} & \text{if } \alpha_1 > 1/4. \end{cases} \tag{B.5}$$

3. A's best reply to $x_B^1 = R$

Fix $x_B^1 = R$. Then, party A chooses policy L if the following inequality holds.

$$2 \cdot (1 - \alpha_1) + \pi_A(L, R) > 1/2 + \pi_A(R, R). \tag{B.6}$$

Otherwise, party A chooses policy R (if both sides of (B.6) are equal, then party A is indifferent between L and R).

By Condition (3.3), $\alpha_2(L, R) < \alpha_2(R, R)$. Since $\pi_A(x_A^1, x_B^1)$ is non-increasing with $\alpha_2(x_A^1, x_B^1)$, then $\pi_A(L, R) \geq \pi_A(R, R)$. Thus, whenever $\alpha_1 < 3/4$ then (B.6) holds. Thus,

$$\text{If } \alpha_1 < 3/4, \text{ then } BR_A(R) = \{L\}. \tag{B.7}$$

If $\alpha_1 > 3/4$, then the form of the best reply depends on $\alpha_2(R,R)$ and $\alpha_2(L,R)$. There are six possible cases.

Case 1: $\alpha_1 > 3/4$, $\alpha_2(R,R) > 3/4$, and $\alpha_2(L,R) > 3/4$. In this case, $\pi_A(R,R) = \pi_A(L,R) = 1/2$. Then, it is inmediate that $2 \cdot (1-\alpha_1) + \pi_A(L,R) < 1/2 + \pi_A(R,R)$ holds, so $BR_A(R) = \{R\}$.

Case 2: $\alpha_1 > 3/4$, $\alpha_2(R,R) > 3/4$, and $\alpha_2(L,R) \in [1/4, 3/4]$. In this case, $\pi_A(R,R) = 1/2$ and $\pi_A(L,R) = 2 \cdot (1 - \alpha_2(L,R))$. By replacing them in (B.6), we find that $BR_A(R) = \{L\}$ if $\alpha_2(L,R) < 3/2 - \alpha_1$ holds.

If $\alpha_2(L,R) > 3/2 - \alpha_1$ holds, then $BR_A(R) = \{R\}$. If $\alpha_2(L,R) = 3/2 - \alpha_1$ holds, then $BR_A(R) = \{L,R\}$.

Case 3: $\alpha_1 > 3/4$, $\alpha_2(R,R) > 3/4$, and $\alpha_2(L,R) < 1/4$. In this case, $\pi_A(R,R) = 1/2$ and $\pi_A(L,R) = 3/2$. By replacing them in (B.6), we find that $BR_A(R) = \{L\}$ if $\alpha_1 < 5/4$, which always holds. Then, $BR_A(R) = \{L\}$.

Observe that, since $\alpha_1 \in (3/4, 1)$, then $3/4 > (3/2 - \alpha_1) > 1/4$, so if for example $\alpha_2(L,R) > 3/4$ holds, then $\alpha_2(L,R) > 3/2 - \alpha_1$ also holds. Thus, we can summarize the previous three cases as follows.

$$\text{If } \alpha_1 > 3/4,\ \alpha_2(R,R) > 3/4, \text{ then } BR_A(R) = \begin{cases} \{L\} & \text{if } \alpha_2(L,R) < 3/2 - \alpha_1 \\ \{L,R\} & \text{if } \alpha_2(L,R) = 3/2 - \alpha_1 \\ \{R\} & \text{if } \alpha_2(L,R) > 3/2 - \alpha_1. \end{cases} \tag{B.8}$$

Cases 1 to 3 are illustrated in Figure B.1.

Case 4: $\alpha_1 > 3/4$, $\alpha_2(R,R) \in [1/4, 3/4]$, and $\alpha_2(L,R) \in [1/4, 3/4]$. In this case, $\pi_A(R,R) = 2 \cdot (1 - \alpha_2(R,R))$ and $\pi_A(L,R) = 2 \cdot (1 - \alpha_2(L,R))$. By replacing them in (B.6), we find that $BR_A(R) = \{L\}$ if $\alpha_2(R,R) - \alpha_2(L,R) > \alpha_1 - 3/4$ holds.

If $\alpha_2(R,R) - \alpha_2(L,R) < \alpha_1 - 3/4$, then $BR_A(R) = \{R\}$. If $\alpha_2(R,R) - \alpha_2(L,R) = \alpha_1 - 3/4$, then $BR_A(R) = \{L,R\}$ (Case 4 is illustrated in Figure B.2a). Thus,

$$\text{If } \alpha_1 > 3/4,\ \alpha_2(R,R) \in [1/4,3/4],\ \alpha_2(L,R) \in [1/4,3/4], \text{ then}$$
$$BR_A(R) = \begin{cases} \{L\} & \text{if } \alpha_2(R,R) - \alpha_2(L,R) > \alpha_1 - 3/4 \\ \{L,R\} & \text{if } \alpha_2(R,R) - \alpha_2(L,R) = \alpha_1 - 3/4 \\ \{R\} & \text{if } \alpha_2(R,R) - \alpha_2(L,R) < \alpha_1 - 3/4. \end{cases} \tag{B.9}$$

Case 5: $\alpha_1 > 3/4$, $\alpha_2(R,R) \in [1/4, 3/4]$, and $\alpha_2(L,R) < 1/4$. In this case,

$\pi_A(R,R) = 2 \cdot (1 - \alpha_2(R,R))$ and $\pi_A(L,R) = 3/2$. By replacing them in (B.6), we find that $BR_A(R) = \{L\}$ if $\alpha_2(R,R) > \alpha_1 - 1/2$ holds.

If $\alpha_2(R,R) < \alpha_1 - 1/2$, then $BR_A(R) = \{R\}$. If $\alpha_2(R,R) = \alpha_1 - 1/2$, then $BR_A(R) = \{L,R\}$. (Case 5 is illustrated in Figure B.2b). Thus,

$$\text{if } \alpha_1 > 3/4,\ \alpha_2(R,R) \in [1/4, 3/4], \text{ and } \alpha_2(L,R) < 1/4,\ \text{then}$$

$$BR_A(R) = \begin{cases} \{L\} & \text{if } \alpha_2(R,R) > \alpha_1 - 1/2 \\ \{L,R\} & \text{if } \alpha_2(R,R) = \alpha_1 - 1/2 \\ \{R\} & \text{if } \alpha_2(R,R) < \alpha_1 - 1/2. \end{cases} \tag{B.10}$$

Case 6: $\alpha_1 > 3/4$, $\alpha_2(R,R) < 1/4$, and $\alpha_2(L,R) < 1/4$. In this case, $\pi_A(R,R) = \pi_A(L,R) = 3/2$. Then, it is inmediate that $2 \cdot (1 - \alpha_1) + \pi_A(L,R) < 1/2 + \pi_A(R,R)$ holds. Thus,

$$\text{if } \alpha_1 > 3/4,\ \alpha_2(R,R) < 1/4, \text{ and } \alpha_2(L,R) < 1/4,\ \text{then } BR_A(R) = \{R\}. \tag{B.11}$$

If $\alpha_1 = 3/4$, then the best reply also depend on $\alpha_2(R,R)$ and $\alpha_2(L,R)$. In this case, there are three possible cases.

Case 7: $\alpha_1 = 3/4$, $\alpha_2(R,R) \in [1/4, 3/4]$, and $\alpha_2(L,R) \in [1/4, 3/4]$. In this case, $\pi_A(R,R) = 2 \cdot (1 - \alpha_2(R,R))$ and $\pi_A(L,R) = 2 \cdot (1 - \alpha_2(L,R))$. Since by Condition (3.3), $\alpha_2(L,R) < \alpha_2(R,R)$ holds, we have that $BR_A(R) = \{L\}$.

Case 8: $\alpha_1 = 3/4$, $\alpha_2(R,R) \in [1/4, 3/4]$, and $\alpha_2(L,R) < 1/4$. In this case, $\pi_A(R,R) = 2 \cdot (1 - \alpha_2(R,R))$ and $\pi_A(L,R) = 3/2$. Since by assumption $\alpha_2(R,R) > 1/4$, then we have that $BR_A(R) = \{L\}$.

Case 9: $\alpha_1 = 3/4$, $\alpha_2(R,R) < 1/4$, and $\alpha_2(L,R) < 1/4$. In this case, $\pi_A(R,R) = \pi_A(L,R) = 3/2$. Then, $2 \cdot (1 - \alpha_1) + \pi_A(L,R) = 1/2 + \pi_A(R,R) = 2$. Thus, $BR_A(L) = \{L,R\}$.

Case 7 to 9 can be summarized as follows.

$$\text{If } \alpha_1 = 3/4,\ \text{then } BR_A(R) = \begin{cases} \{L\} & \text{if } \alpha_2(R,R) \geq 1/4 \\ \{L,R\} & \text{if } \alpha_2(R,R) < 1/4. \end{cases} \tag{B.12}$$

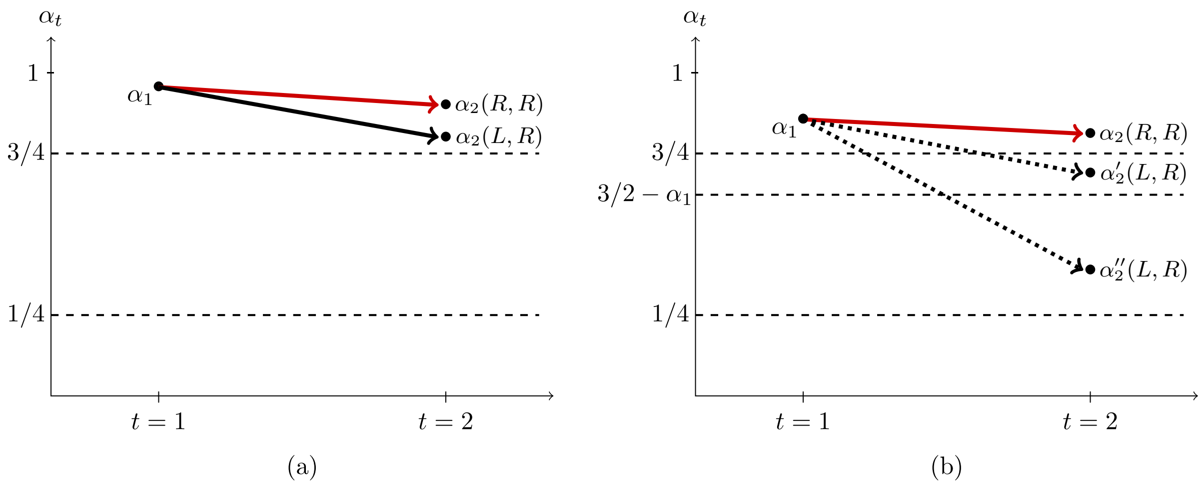


Figure B.1: In (a), A's best reply to $x_B^1 = R$ is $x_A^1 = R$. In (b), A's best reply to $x_B^1 = R$ is $x_A^1 = R$ only if $\alpha_2(L, R)$ is lower than $3/2 - \alpha_1$.

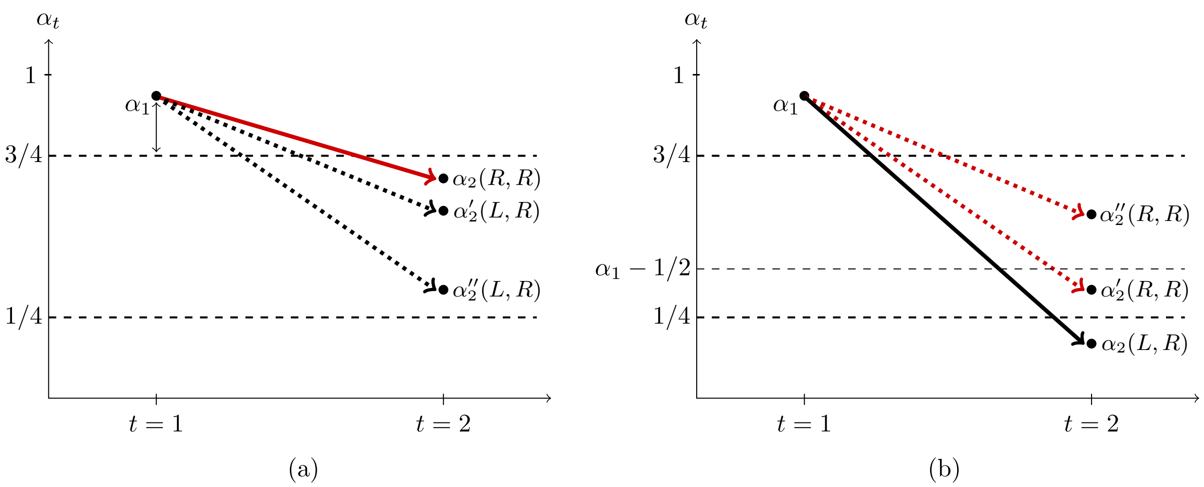


Figure B.2: In (a), A's best reply to $x_B^1 = R$ is $x_A^1 = L$ only if the difference between $\alpha_2(R, R)$ and $\alpha_2(L, R)$ is larger than $3/4-\alpha_1$. In (b), A's best reply to $x_B^1 = R$ is $x_A^1 = L$ only if $\alpha_2(R, R)$ is larger than $\alpha_1 - 1/2$.

B.0.2 Proofs of the propositions

Proof of Proposition 4.3. By (B.3) and (B.4), we know that

$$BR_B(R) = \{R\} \quad \text{and} \quad BR_A(L) = \{L\}. \tag{B.13}$$

Suppose that $\alpha_1 < 1/4$. Then, by (B.5), $BR_B(L) = \{L\}$, and by (B.7), $BR_A(R) = \{L\}$. Thus, by (B.13), (L, L) is the unique fixed point, and policy L is a dominant strategy for A (Figure B.3a). This proves the first statement.

Suppose that $\alpha_1 = 1/4$. Then, by (B.5), $BR_B(L) = \{L, R\}$, and by (B.7), $BR_A(R) = \{L\}$. Thus, by (B.13) (L, R) and (L, L) are two fixed points. However, policy R is weakly dominated for party B, because $BR_B(R) = \{R\}$. Thus, (L, R) is the unique undominated equilibrium (Figure B.3b). This proves the second statement.

Suppose that $\alpha_1 \in (1/4, 3/4)$. Then, by (B.5), $BR_B(L) = \{R\}$, and by (B.7), $BR_A(R) = \{L\}$. Thus, by (B.13) (L, R) is the unique fixed point, anb both strategies are dominant (Figure B.3c) . This proves the last statement.

■

Proof of Proposition 4.4. Fix $\alpha_1 > 3/4$ and $\alpha_2(R, R) > 3/4$. By (B.3), (B.4), and (B.5) we know that

$$BR_B(R) = \{R\}\ ,\ BR_A(L) = \{L\} \ \text{ and } \ BR_B(L) = \{R\}. \tag{B.14}$$

Suppose that $\alpha_2(L, R) > 3/2 - \alpha_1$. Then, by (B.8), $BR_A(R) = \{R\}$. Thus, by (B.14), (R, R) is the unique fixed point, and R is a dominant strategy for party B (Figure B.4a). This proves the first statement.

Suppose that $\alpha_2(L, R) = 3/2 - \alpha_1$. Then, by (B.8), $BR_A(R) = \{L, R\}$. Thus, by (B.14), (L, R) and (R, R) are two fixed points. However, R is weakly dominated for party A, because $BR_A(L) = \{L\}$. Thus, (L, R) is the unique undominated equilibrium (Figure B.4b). This proves the second statement.

Suppose that $\alpha_2(L, R) < 3/2 - \alpha_1$. Then, by (B.8), $BR_A(R) = \{L\}$. Thus, by (B.14), (L, R) is the unique fixed point, and both strategies are dominant (Figure B.4b). This proves the last statement.

■

Proposition B.1. *Consider the game displayed in Table 3.4, and suppose that* $\alpha_1 > 3/4$, $\alpha_2(R, R) \in [1/4, 3/4]$ *and* $\alpha_2(L, R) \in [1/4, 3/4]$*. Then, the following statements hold.*

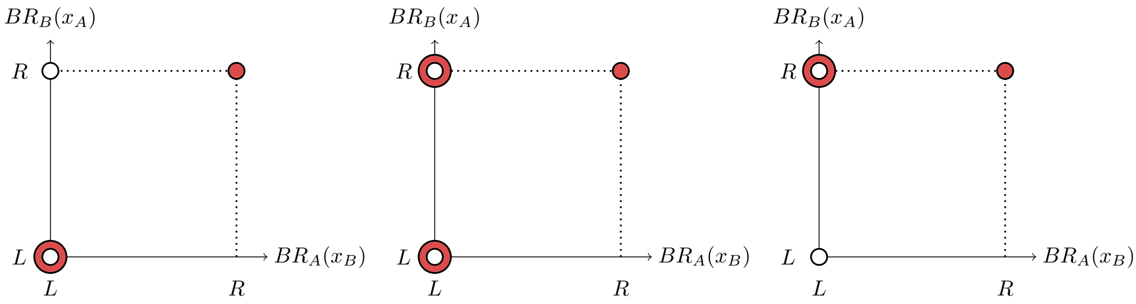
1. *If* $\alpha_2(R, R) - \alpha_2(L, R) < 3/4 - \alpha_1$*, then* (R, R) *is the unique equilibrium, where* R *is a dominant strategy for party* B*.*

2. *If* $\alpha_2(R, R) - \alpha_2(L, R) = 3/4 - \alpha_1$*, then* (L, R) *is the unique undominated equilibrium.*

3. *If* $\alpha_2(R, R) - \alpha_2(L, R) > 3/4 - \alpha_1$*, then* (L, R) *is the unique equilibrium, where both strategies are dominant.*

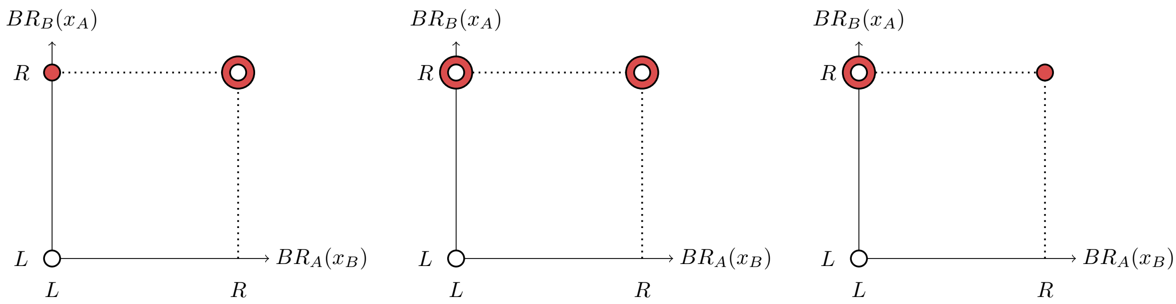
Proof of Proposition B.1. The proof is identical to the proof of Proposition 4.4, except that now I fix $\alpha_1 > 3/4$, $\alpha_2(R, R) \in [1/4, 3/4]$ and $\alpha_2(L, R) \in [1/4, 3/4]$, and I apply (B.9) instead of (B.8).

■



(a) $\alpha_1 < 1/4$ (b) $\alpha_1 = 1/4$ (c) $\alpha_1 \in (1/4, 3/4)$

Figure B.3: Parties' best replies when $\alpha < 3/4$. The coloured circle corresponds to $BR_B(x_A^1)$.



(a) $\alpha_1 > 3/4$, $\alpha_2(R,R) > 1/4$ and $\alpha_2(L,R) > 3/2 - \alpha_1$ (b) $\alpha_1 > 3/4$, $\alpha_2(R,R) > 1/4$ and $\alpha_2(L,R) = 3/2 - \alpha_1$ (c) $\alpha_1 > 3/4$, $\alpha_2(R,R) > 1/4$ and $\alpha_2(L,R) < 3/2 - \alpha_1$

Figure B.4: Parties' best replies when $\alpha_1 > 3/4$ and $\alpha_2(R,R) > 3/4$. The coloured circle corresponds to $BR_B(x_A^1)$.

Proposition B.2. *Consider the game displayed in Table 3.4, and suppose that $\alpha_1 > 3/4$, $\alpha_2(R,R) \in [1/4, 3/4]$ and $\alpha_2(L,R) < 1/4$. Then, the following statements hold.*

1. *If $\alpha_2(R,R) < \alpha_1 - 1/2$, then (R,R) is the unique equilibrium, wher R is a dominant strategy for party B.*
2. *If $\alpha_2(R,R) = \alpha_1 - 1/2$, then (L,R) is the unique undominated equilibrium.*
3. *If $\alpha_2(R,R) > \alpha_1 - 1/2$, then (L,R) is the unique equilibrium, where both strategies are dominant.*

Proof of Proposition B.2. The proof is identical to the proof of Proposition 4.4, except that now I fix $\alpha_1 > 3/4$ and $\alpha_2(R,R) \in [1/4, 3/4]$, and I apply (B.10) instead of (B.7). ■

Proposition B.3. *Consider the game displayed in Table 3.4, and suppose that $\alpha_1 = 3/4$. Then, the following statements hold.*

1. *If* $\alpha_2(R, R) \geq 1/4$*, then* (L, R) *is the unique equilibrium, and both strategies are dominant.*

2. *If* $\alpha_2(R, R) < 1/4$*, then* (L, R) *is the unique undominated equilbrium.*

Proof of Proposition B.3. The proof is identical to the proof of Proposition 4.4, except that now I fix $\alpha_1 = 3/4$ and I apply (B.11) instead of (B.7). ■

Appendix C

Proofs of Results of Chapter 4

C.0.1 Best replies

Consider the game displayed in Table 4.3, where:

$$\begin{aligned}\Gamma_A &= E(\alpha) + E(\pi_A(\alpha)) \\ \Upsilon_A &= 1 - E(\alpha) + E(\pi_A(\alpha)) \\ \Gamma_B &= E(\alpha) + E(\pi_B(\alpha)) \\ \Upsilon_B &= 1 - E(\alpha) + E(\pi_B(\alpha)).\end{aligned}$$

Recall that:

$$\pi_A(\alpha) = \begin{cases} 3/2 & \text{if } \alpha < 1/4 \\ 2 \cdot (1-\alpha) & \text{if } \alpha \in [1/4, 3/4] \\ 1/2 & \text{if } \alpha > 3/4 \end{cases} \qquad \pi_B(\alpha) = \begin{cases} 1/2 & \text{if } \alpha < 1/4 \\ 2 \cdot \alpha & \text{if } \alpha \in [1/4, 3/4] \\ 3/2 & \text{if } \alpha > 3/4. \end{cases}$$

Thus, $E(\pi_J(\alpha)) \in [1/2, 3/2]$ for $J \in \{A, B\}$. Moreover, $\pi_A(\alpha) = 2 - \pi_B(\alpha)$ for all α, and thus,

$$E(\pi_A(\alpha)) = 2 - E(\pi_B(\alpha)). \tag{C.1}$$

After replacing (C.1) in Υ_A and Γ_A, the following relationships hold.

$$\Upsilon_A = 3 - \Gamma_B. \tag{C.2}$$

$$\Gamma_A = 3 - \Upsilon_B. \tag{C.3}$$

Moreover, the following assumption is made.

Assumption 1: $E(\alpha) < 1/2 \Longleftrightarrow \Gamma_B < 1/2 + 2 \cdot E(\alpha) \Longleftrightarrow \Upsilon_B > 1/2 + 2 \cdot E(\alpha)$.

In Figures C.2 and C.3, I show that this assumption is verified for beta, truncated normal and triangular distributions with support $[0, 1]$.[1]

Now define $BR_A(\sigma_B)$ and $BR_B(\sigma_A)$ the best reply correspondences of parties A and B to σ_B and σ_A respectively. I will obtain parties' best replies by distinguishing among the cases where $E(\alpha)$ is below 1/4, between 1/4 and 3/4, and above 3/4.

1. $E(\alpha) < 1/4$

First, suppose that $E(\alpha) < 1/4$. Then, after iterative elimination of weakly dominated strategies, parties play the following game.

$A \setminus B$	LLL ⋮ LLR	RLL ⋮ RRL
LLL / LLR	$2, 1$	Υ_A, Γ_B
RLL / RRL	Γ_A, Υ_B	$2, 1$

Table C.1: The game played by the parties if $E(\alpha) < 1/4$, after iterative elimination of weakly dominated strategies.

1.1. B's best reply to $\sigma_A = LLL$ or $\sigma_A = LLR$

Fix $\sigma_A = LLL$ or $\sigma_A = LLR$. If $\Gamma_B < 1$, then party B chooses $\sigma_B = LLL$ or $\sigma_B = LLR$. If $\Gamma_B > 1$, then B chooses $\sigma_B = RLL$ or $\sigma_B = RRL$. And if $\Gamma_B = 1$, then B

[1] I simulate 3000 beta distributions of 1000 elements each, half of them with $E(\alpha) < 1/2$. Recall that the beta distribution is identified with two positive parameters, say γ_1 and γ_2. The expected value is $\gamma_1/(\gamma_1 + \gamma_2)$, which is lower than 1/2 if and only if $\gamma_2 > \gamma_1$ holds. Thus, I randomly select (γ_1, γ_2) satisfying either $\gamma_2 > \gamma_1$ or $\gamma_2 < \gamma_1$, and then I generate the corresponding distribution.

Also, I simulate 2500 truncated normal distributions of 1000 elements each, around half of them with $E(\alpha) < 1/2$. In this case, the distribution is identified with the truncation points (which are zero and one), together with the mean (μ) and the standard deviation (σ). I build grids of $\mu \in (0, 1)$ and $\sigma \in (0, 1)$ of 50 elements each. For each combination (μ, σ), I generate the corresponding distribution, and I check whether $E(\alpha) < 1/2$ holds or not.

Finally, I simulate 2700 triangular distributions of 1000 elements each, around half of them with $E(\alpha) < 1/2$. Now the distribution is identified with the lower bound (a), the upper bound (b) and the mode (c). I fix $c \in \{0.4, 0.5, 0.6\}$ and I build grids of $a \in [0, c - 0.05]$ and $b \in [c + 0.05, 1]$ of 30 elements each. For every combination (a, b, c), I generate the corresponding distribution, and I check whether $E(\alpha) < 1/2$ holds or not.

is indifferent between the four strategies. Hence, if $E(\alpha) < 1/4$, then,

$$BR_B(LLL) = BR_B(LLR) = \begin{cases} \{LLL, LLR\} \text{ if } \Gamma_B < 1 \\ \{LLL, LLR, RLL, RRL\} \text{ if } \Gamma_B = 1 \\ \{RLL, RRL\} \text{ if } \Gamma_B > 1. \end{cases} \tag{C.4}$$

1.2. ***A*'s best reply to $\sigma_B = RLL$ or $\sigma_B = RRL$**

Fix $\sigma_B = RLL$ or $\sigma_B = RRL$. If $\Upsilon_A < 2$, then party A chooses $\sigma_A = LLL$ or $\sigma_A = LLR$. If $\Upsilon_A < 2$, then A chooses $\sigma_A = RLL$ or $\sigma_A = RRL$. And if $\Upsilon_A = 2$, then A is indifferent between the four strategies.

Observe that, after applying (C.2) to Υ_A, the following relationships hold.

$$\Upsilon_A > 2 \Longleftrightarrow \Gamma_B < 1.$$

$$\Upsilon_A < 2 \Longleftrightarrow \Gamma_B > 1. \tag{C.5}$$

Hence, if $E(\alpha) < 1/4$, then,

$$BR_A(RLL) = BR_A(RRL) = \begin{cases} \{LLL, LLR\} \text{ if } \Gamma_B < 1 \\ \{LLL, LLR, RLL, RRL\} \text{ if } \Gamma_B = 1 \\ \{RLL, RRL\} \text{ if } \Gamma_B > 1. \end{cases} \tag{C.6}$$

1.3. ***B*'s best reply to $\sigma_A = RLL$ or $\sigma_A = RRL$**

Fix $\sigma_A = RLL$ or $\sigma_B = RRL$. If $\Upsilon_B > 1$, then party B chooses $\sigma_B = LLL$ or $\sigma_B = LLR$. Since $E(\alpha) < 1/2$ and $E(\pi_B(\alpha)) \geq 1/2$ hold, then $1 - E(\alpha) + E(\pi_B(\alpha)) > 1/2 + 1/2$, and thus $\Upsilon_B > 1$ holds. Hence, if $E(\alpha) < 1/4$, then

$$BR_B(RLL) = BR_B(RRL) = \{LLL, LLR\}. \tag{C.7}$$

1.4. ***A*'s best reply to $\sigma_B = LLL$ or $\sigma_B = LLR$**

Fix $\sigma_B = LLL$ or $\sigma_B = LLR$. If $\Gamma_A < 2$, then party A chooses $\sigma_A = LLL$ or $\sigma_a = LLR$. Since $E(\alpha) < 1/2$ and $E(\pi_A(\alpha)) \leq 3/2$, then $E(\alpha) + E(\pi_A(\alpha)) < 1/2 + 3/2$, and thus $\Gamma_A < 2$ holds. Hence, if $E(\alpha) < 1/4$, then

$$BR_A(LLL) = BR_A(LLR) = \{LLL, LLR\}. \tag{C.8}$$

2. $E(\alpha) \in (1/4, 3/4)$

Suppose that $E(\alpha) \in (1/4, 3/4)$. Then, after iterative elimination of weakly dominated strategies, parties play the following game.

<table>
<tr><th>$A \setminus B$</th><th>LRL ⋮ LRR</th><th>RLR ⋮ RRR</th></tr>
<tr><td>LLL</td><td rowspan="2">$\frac{1}{2} + 2 \cdot (1 - E(\alpha))$,
$\frac{1}{2} + 2 \cdot E(\alpha)$</td><td rowspan="2">$\Upsilon_A, \Gamma_B$</td></tr>
<tr><td>LLR</td></tr>
<tr><td>RLL</td><td rowspan="2">Γ_A, Υ_B</td><td rowspan="2">$\frac{1}{2} + 2 \cdot (1 - E(\alpha))$,
$\frac{1}{2} + 2 \cdot E(\alpha)$</td></tr>
<tr><td>RRL</td></tr>
</table>

Table C.2: The game played by the parties after elimination of weakly dominated strategies if $E(\alpha) \in (1/4, 3/4)$.

2.1. B's best reply to $\sigma_A = LLL$ or $\sigma_A = LLR$

Fix $\sigma_A = LLL$ or $\sigma_A = LLR$. If $\Gamma_B < 1/2 + 2 \cdot E(\alpha)$, then party B chooses $\sigma_B = LRL$ or $\sigma_B = LRR$. If $\Gamma_B > 1/2 + 2 \cdot E(\alpha)$, then B chooses $\sigma_B = RLR$ or $\sigma_B = RRR$. Hence, by Assumption (4.1), if $E(\alpha) \in (1/4, 3/4)$, then,

$$BR_B(LLL) = BR_B(LLR) = \begin{cases} \{LRL, LRR\} & \text{if } E(\alpha) < 1/2 \\ \{RLR, RRR\} & \text{if } E(\alpha) > 1/2. \end{cases} \tag{C.9}$$

2.2. A's best reply to $\sigma_B = RLR$ or $\sigma_B = RRR$

Fix $\sigma_B = RLR$ or $\sigma_B = RRR$. If $\Upsilon_A > 1/2 + 2 \cdot (1 - E(\alpha))$, then party A chooses $\sigma_A = LLL$ or $\sigma_A = LLR$. If $\Upsilon_A < 1/2 + 2 \cdot (1 - E(\alpha))$, then A chooses $\sigma_A = RLL$ or $\sigma_A = RRL$.

Observe that, after applying (C.2) to Υ_A, the following relationships hold.

$$\Upsilon_A > 1/2 + 2 \cdot (1 - E(\alpha)) \Longleftrightarrow \Gamma_B < 1/2 + 2 \cdot E(\alpha).$$

$$\Upsilon_A < 1/2 + 2 \cdot (1 - E(\alpha)) \Longleftrightarrow \Gamma_B > 1/2 + 2 \cdot E(\alpha).$$

Hence, by Assumption (4.1), if $E(\alpha) \in (1/4, 3/4)$, then,

$$BR_A(RLR) = BR_A(RRR) = \begin{cases} \{LLL, LLR\} & \text{if } E(\alpha) < 1/2 \\ \{RLL, RRL\} & \text{if } E(\alpha) > 1/2. \end{cases} \tag{C.10}$$

2.3. B's best reply to $\sigma_A = RLL$ or $\sigma_A = RRL$

Fix $\sigma_A = RLL$ or $\sigma_A = RRL$. If $\Upsilon_B > 1/2+2\cdot E(\alpha)$, then party B chooses $\sigma_B = LRL$ or $\sigma_B = LRR$. If $\Upsilon_B < 1/2+2\cdot E(\alpha)$, then B chooses $\sigma_B = RLR$ or $\sigma_B = RRR$. Hence, by Assumption (4.1), if $E(\alpha) \in (1/4, 3/4)$, then,

$$BR_B(RLL) = BR_B(RRL) = \begin{cases} \{LRL, LRR\} & \text{if } E(\alpha) < 1/2 \\ \{RLR, RRR\} & \text{if } E(\alpha) > 1/2. \end{cases} \tag{C.11}$$

2.4. A's best reply to $\sigma_B = LRL$ or $\sigma_B = LRR$

Fix $\sigma_B = LRL$ or $\sigma_B = LRR$. If $\Gamma_A < 1/2 + 2\cdot(1 - E(\alpha))$, then party A chooses $\sigma_A = LLL$ or $\sigma_A = LLR$. If $\Gamma_A > 1/2 + 2\cdot(1-E(\alpha))$, then A chooses $\sigma_A = RLL$ or $\sigma_A = RRL$.

Observe that, after applying (C.3) to Γ_A, the following relationships hold.

$$\Gamma_A < 1/2 + 2\cdot(1-E(\alpha)) \iff \Upsilon_B > 1/2 + 2\cdot E(\alpha).$$

$$\Gamma_A > 1/2 + 2\cdot(1-E(\alpha)) \iff \Upsilon_B < 1/2 + 2\cdot E(\alpha).$$

Hence, by Assumption (4.1), if $E(\alpha) \in (1/4, 3/4)$, then,

$$BR_A(LRL) = BR_A(LRR) = \begin{cases} \{LLL, LLR\} & \text{if } E(\alpha) < 1/2 \\ \{RLL, RRL\} & \text{if } E(\alpha) > 1/2. \end{cases} \tag{C.12}$$

3. $E(\alpha) > 3/4$

Suppose that $E(\alpha) > 3/4$. Since $E(\alpha) > 1/4$, then the following strategies are weakly dominated in Table 4.3 .

- $\sigma_B = RLL$ and $\sigma_B = RRL$ are weakly dominated by $\sigma_B = RLR$ and $\sigma_B = RRR$.
- $\sigma_B = LLL$ and $\sigma_B = LLR$ are weakly dominated by $\sigma_B = LRL$ and $\sigma_B = LRR$.

After their elimination, and since $E(\alpha) > 3/4$, the following strategies are weakly dominated.

- $\sigma_A = RLL$ and $\sigma_A = RRL$ are weakly dominated by $\sigma_A = RLR$ and $\sigma_A = RRR$.
- $\sigma_A = LLL$ and $\sigma_A = LLR$ are weakly dominated by $\sigma_A = LRL$ and $\sigma_A = LRR$.

After their elimination, parties play the game represented in Table C.3.

$A \setminus B$	LRL ⋮ LRR	RLR ⋮ RRR
LRL / LRR	$1, 2$	Υ_A, Γ_B
RLR / RRR	Γ_A, Υ_B	$1, 2$

Table C.3: The game played by the parties if $E(\alpha) > 3/4$, after iterative elimination of weakly dominated strategies.

3.1. A's best reply to $\sigma_B = RLR$ or $\sigma_B = RRR$

Fix $\sigma_B = RLR$ or $\sigma_B = RRR$. If $\Upsilon_A < 1$, then party A chooses $\sigma_A = RLR$ or $\sigma_A = RRR$. If $\Upsilon_A > 1$, then A chooses $\sigma_A = LRL$ or $\sigma_A = LRR$. And if $\Upsilon_A = 1$, then A is indifferent between the four strategies. Hence, if $E(\alpha) > 3/4$, then,

$$BR_A(RLR) = BR_A(RRR) = \begin{cases} \{LRL, LRR\} \text{ if } \Upsilon_A > 1 \\ \{LRL, LRR, RLR, RRR\} \text{ if } \Upsilon_A = 1 \\ \{RLR, RRR\} \text{ if } \Upsilon_A < 1. \end{cases} \tag{C.13}$$

3.2. B's best reply to $\sigma_A = LRL$ or $\sigma_A = LRR$

Fix $\sigma_A = LRL$ and $\sigma_A = LRR$. If $\Gamma_B > 2$, then, party B chooses $\sigma_B = RLR$ or $\sigma_B = RRR$. If $\Gamma_B < 2$, then B chooses $\sigma_B = LRL$ or $\sigma_B = LRR$. And if $\Gamma_B = 2$, then B is indifferent between the four stretegies.

Observe that, after applying (C.2) to Γ_B, the following relationships hold.

$$\Gamma_B > 2 \Longleftrightarrow \Upsilon_A < 1.$$

$$\Gamma_B < 2 \Longleftrightarrow \Upsilon_A > 1.$$

Hence, if $E(\alpha) > 3/4$, then,

$$BR_B(LRL) = BR_B(LRR) = \begin{cases} \{LRL, LRR\} \text{ if } \Upsilon_A > 1 \\ \{LRL, LRR, RLR, RRR\} \text{ if } \Upsilon_A = 1 \\ \{RLR, RRR\} \text{ if } \Upsilon_A < 1. \end{cases} \tag{C.14}$$

3.3. A**'s best reply to** $\sigma_B = LRL$ **or** $\sigma_B = LRR$

Fix $\sigma_B = LRL$ or $\sigma_B = LRR$. If $\Gamma_A > 1$ then, party A chooses $\sigma_A = RLR$ or $\sigma_A = RRR$. Since $E(\alpha) > 1/2$ and $E(\pi_A(\alpha)) \geq 1/2$, then $E(\alpha) + E(\pi_A(\alpha)) > 1/2 + 1/2$ holds. Then, if $E(\alpha) > 3/4$, then,

$$BR_A(LRL) = BR_A(LRR) = \{RLR, RRR\}. \tag{C.15}$$

3.4. B**'s best reply to** $\sigma_A = RLR$ **or** $\sigma_A = RRR$

Fix $\sigma_A = RLR$ or $\sigma_A = RRR$. If $\Upsilon_B < 2$, then party B chooses $\sigma_B = RLR$ or $\sigma_B = RRR$. Since $E(\alpha) > 1/2$ and $E(\pi_B(\alpha)) \leq 1/2$, then $1 - E(\alpha) + E(\pi_B(\alpha)) < 1/2 + 3/2$ holds. Hence, if $E(\alpha) > 3/4$, then,

$$BR_B(RLR) = BR_B(RRR) = \{RLR, RRR\}. \tag{C.16}$$

C.0.2 Proofs of the Propositions

Proof of Proposition 4.2. Suppose that $E(\alpha) > 1/4$. Thus, by (C.7) and (C.8), $BR_B(RLL) = BR_B(RRL) = \{LLL, LLR\}$ and $BR_A(LLL) = BR_A(LLR) = \{LLL, LLR\}$.

In addition, suppose that $\Gamma_B < 1$ holds. Then, by (C.4) and (C.6), $BR_B(LLL) = BR_B(LLR) = \{LLL, LLR\}$ and $BR_A(RLL) = BR_A(RRL) = \{LLL, LLR\}$. Hence, there are four equilibria. Namely,

$$(LLx, LLy) \text{ for } x, y \in \{L, R\}.$$

This proves the first statement.

Now suppose that $\Gamma_B = 1$ holds. Then, by (C.4) and (C.6), $BR_B(LLL) = BR_B(LLR) = \{LLL, LLR, RLL, RRL\}$ and $BR_A(RLL) = BR_A(RRL) = \{LLL, LLR, RLL, RRL\}$. Hence, there are eight equilibria. Namely,

$$(LLx, LLy),\ (LLv, RwL) \text{ for } x, y, v, w \in \{L, R\}.$$

This proves the second statement.

Finally, suppose that $\Gamma_B > 1$ holds. Then, by (C.4) and (C.6), $BR_B(LLL) = BR_B(LLR) = \{RLL, RRL\}$ and $BR_A(RLL) = BR_A(RRL) = \{RLL, RRL\}$. Hence, a pure strategies equilibrium does not exist. This proves the third statement.

■

Proof of Proposition 4.3. Suppose that $E(\alpha) \in (1/4, 1/2)$. Then, by (C.9), (C.10), (C.11) and (C.12), $BR_B(LLL) = BR_B(LLR) = \{LRL, LRR\}$, $BR_A(RLR) = BR_A(RRR) = \{LLL, LLR\}$, $BR_B(RLL) = BR_B(RRL) = \{LRL, LRR\}$ and $BR_A(LRL) = BR_A(LRR) = \{LLL, LLR\}$. Then, there are four equilibria. Namely,

$$(LLx, LRy) \text{ for } x, y \in \{L, R\}.$$

This proves the first statement.

Now suppose that $E(\alpha) \in (1/2, 3/4)$. Then, by (C.9), (C.10), (C.11) and (C.12), $BR_B(LLL) = BR_B(LLR) = \{RLR, RRR\}$, $BR_A(RLR) = BR_A(RRR) = \{RLL, RRL\}$, $BR_B(RLL) = BR_B(RRL) = \{RLR, RRR\}$ and $BR_A(LRL) = BR_A(LRR) = \{RLL, RRL\}$. Then, there are four equilibria. Namely,

$$(RxL, RyR) \text{ for } x, y \in \{L, R\}.$$

This proofs the second statement.

■

Proof of Proposition 4.4. Suppose that $E(\alpha) < 1/4$ and $\Gamma_B > 1$ hold, and consider the mixed extension of the game played by parties, which is represented in Table 4.7. On the one hand, party A is indifferent between its four strategies, LLL, LLR, RLL and RRL, if and only if

$$(q_1 + q_2) \cdot 2 + (1 - q_1 - q_2) \cdot \Upsilon_A = (q_1 + q_2) \cdot \Gamma_A + (1 - q_1 - q_2) \cdot 2 \Longrightarrow$$

$$q_1 + q_2 = \frac{2 - \Upsilon_A}{4 - \Upsilon_A - \Gamma_A}, \tag{C.17}$$

where $q_1 + q_2 \in (0, 1)$ holds.

On the other hand, party B is indifferent between its four strategies, LLL, LLR, RLL and RRL, if and only if

$$(p_1 + p_2) \cdot 1 + (1 - p_1 - p_2) \cdot \Upsilon_B = (p_1 + p_2) \cdot \Gamma_B + (1 - p_1 - p_2) \cdot 1 \Longrightarrow$$

$$p_1 + p_2 = \frac{\Upsilon_B - 1}{\Gamma_B + \Upsilon_B - 2}, \tag{C.18}$$

where $p_1 + p_2 \in (0, 1)$ holds.

Hence, any pair $p = (p_1, p_2, p_3, p_4)$ and $q = (q_1, q_2, q_3, q_4)$ satisfying (C.17) and (C.18) is a mixed strategies equilibrium of the game displayed in Table 4.7. Moreover, after applying (C.2) and (C.3) to Υ_A and Γ_A,

$$\begin{aligned}
\frac{2 - \Upsilon_A}{4 - \Upsilon_A - \Gamma_A} &= \frac{\Gamma_B - 1}{\Gamma_B + \Upsilon_B - 2} \\
&= \frac{\Gamma_B - 1 + \Upsilon_B - \Upsilon_B + 1 - 1}{\Gamma_B + \Upsilon_B - 2} \\
&= \frac{\Gamma_B + \Upsilon_B - 2 - \Upsilon_B + 1}{\Gamma_B + \Upsilon_B - 2} \\
&= 1 - \frac{\Upsilon_B - 1}{\Gamma_B + \Upsilon_B - 2},
\end{aligned}$$

and thus $q_1 + q_2 = 1 - p_1 - p_2$ holds.

■

The following proposition characterizes the equilibria for when $E(\alpha) > 3/4$. This case, which is symetric to $E(\alpha) < 1/4$, is illustrated in Figure C.1.

Proposition C.1. *Suppose that $E(\alpha) > 3/4$ holds and consider the game displayed in Table C.3. Then, the following statement hold.*

- *If $\Upsilon_A < 1$, then there are four subgame perfect equilibra that survive iterative elimination of dominated pure strategies. Namely,*

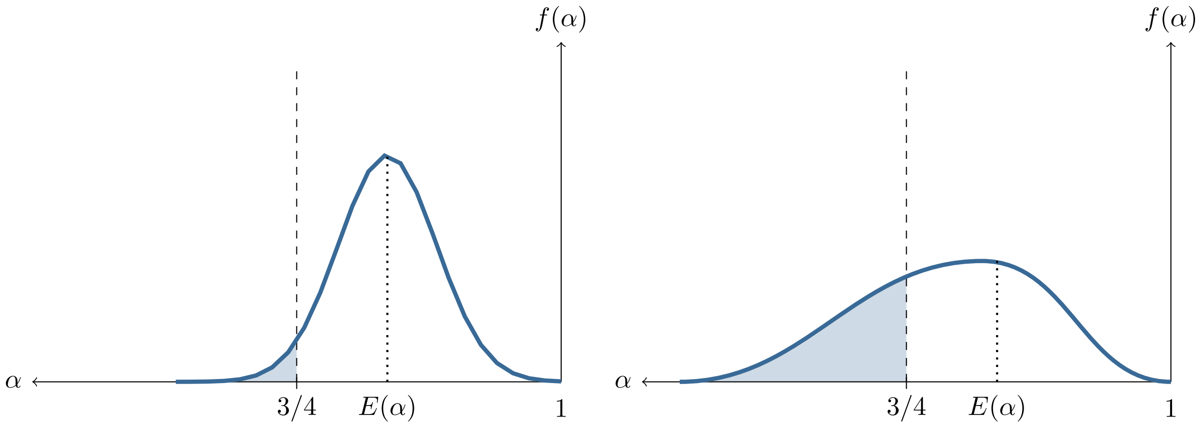
$$(RxR, RyR) \text{ for } x, y \in \{L, R\} .$$

- *If $\Upsilon_A = 1$, then there are eight subgame perfect equilibra that survive iterative elimination of dominated pure strategies. Namely,*

$$(RxR, RyR), (LRv, RwR) \text{ for } x, y, v, w \in \{L, R\} .$$

- *If $\Upsilon_A > 1$, then there is no subgame perfect equilibrium with undominated pure strategies.*

Proof of Proposition C.1. The proof is identical to the proof of Proposition 4.2, but using (C.13), (C.14), (C.15) and (C.16). ■



(a) $\Upsilon_A < 1$: parties do not learn α.

(b) $\Upsilon_A > 1$: A deviates to policy L to learn α.

Figure C.1: Two distributions with $E(\alpha) > 3/4$. Whether an equilibrium exists or not now depends on $F(3/4)$.

The following proposition characterizes the mixed strategy equilibrium of the game played by the parties if $E(\alpha) > 3/4$ and $\Upsilon_A > 1$. The game is displayed in Table C.4. This case is symmetric to $E(\alpha) < 1/4$ and $\Gamma_B > 1$.

<table>
<tr><td></td><td>$A \setminus B$</td><td>LRL (s_1)</td><td>LRR (s_2)</td><td>RLR (s_3)</td><td>RRR (s_4)</td></tr>
<tr><td>r_1</td><td>LRL</td><td colspan="2" rowspan="2">$1, 2$</td><td colspan="2" rowspan="2">Υ_A, Γ_B</td></tr>
<tr><td>r_2</td><td>LRR</td></tr>
<tr><td>r_3</td><td>RLR</td><td colspan="2" rowspan="2">Γ_A, Υ_B</td><td colspan="2" rowspan="2">$1, 2$</td></tr>
<tr><td>r_4</td><td>RRR</td></tr>
</table>

Table C.4: The mixed extension of the game played by parties if $E(\alpha) > 3/4$.

Proposition C.2. *Suppose that $E(\alpha) > 3/4$ and $\Upsilon_A > 1$. Then, any pair (r^*, s^*) where:*

$$r_1^* + r_2^* = \frac{2 - \Upsilon_B}{4 - \Upsilon_B - \Gamma_B} \quad \text{and} \quad s_1^* + s_2^* = \frac{\Upsilon_A - 1}{\Upsilon_A + \Gamma_A - 2}$$

is a mixed strategies equilibrium of the game displayed in Table C.4. Moreover,

$$\frac{\Upsilon_A - 1}{\Upsilon_A + \Gamma_A - 2} = 1 - \frac{2 - \Upsilon_B}{4 - \Upsilon_B - \Gamma_B},$$

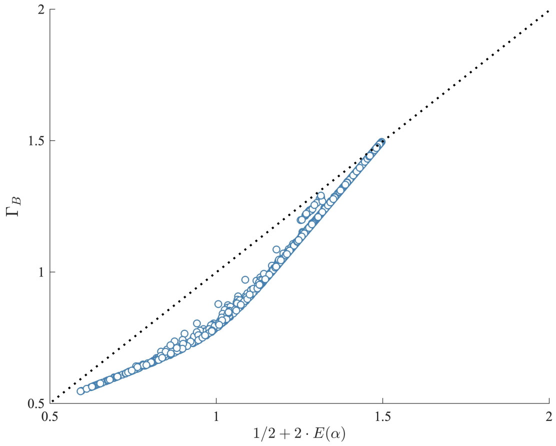
and thus, $s_1^* + s_2^* = 1 - r_1^* - r_2^*$ holds.

Proof of Proposition C.2. The proof is identical to the proof of Proposition C.1, but solving the game displayed in Table C.4. ■

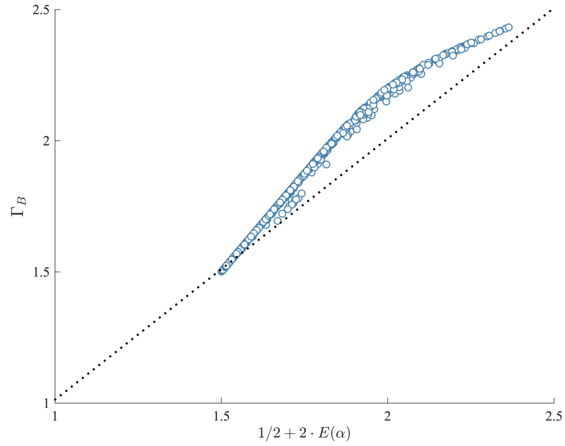
<table>
<tr><td>$A \setminus B$</td><td>LRL</td><td>LRR</td><td>RLR</td><td>RRR</td></tr>
<tr><td>LRL</td><td colspan="2" rowspan="2">$(r_1^* + r_2^*) \cdot (1 - r_1^* - r_2^*)$</td><td colspan="2" rowspan="2">$(r_1^* + r_2^*)^2$</td></tr>
<tr><td>LRR</td></tr>
<tr><td>RLR</td><td colspan="2" rowspan="2">$(1 - r_1^* - r_2^*)^2$</td><td colspan="2" rowspan="2">$(r_1^* + r_2^*) \cdot (1 - r_1^* - r_2^*)$</td></tr>
<tr><td>RRR</td></tr>
</table>

Table C.5: The probabilities of every equilibrium.

Beta distributions

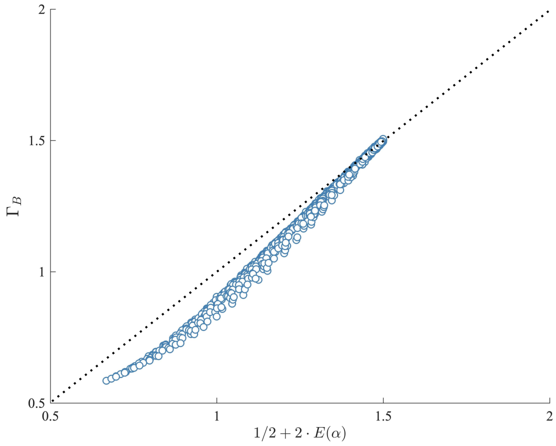


(a) $E(\alpha) < 1/2$

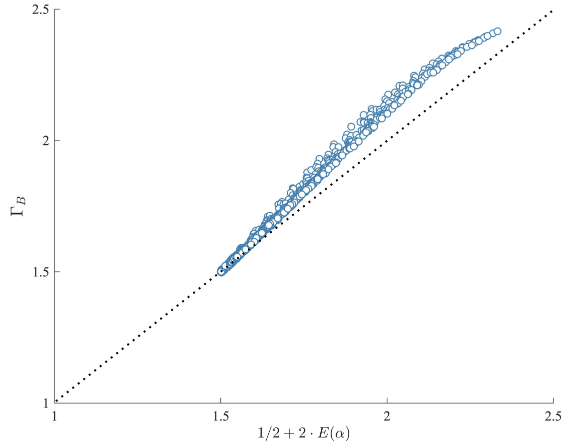


(b) $E(\alpha) > 1/2$

Truncated normal distributions

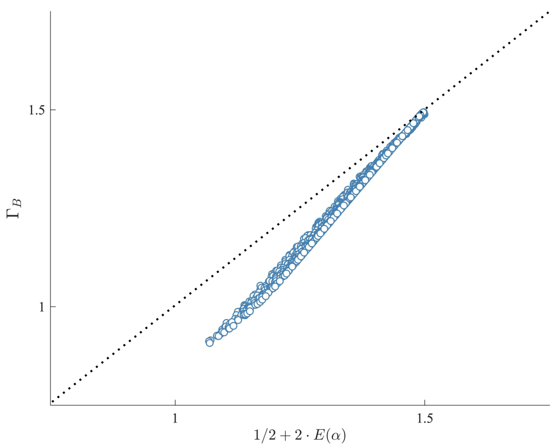


(a) $E(\alpha) < 1/2$

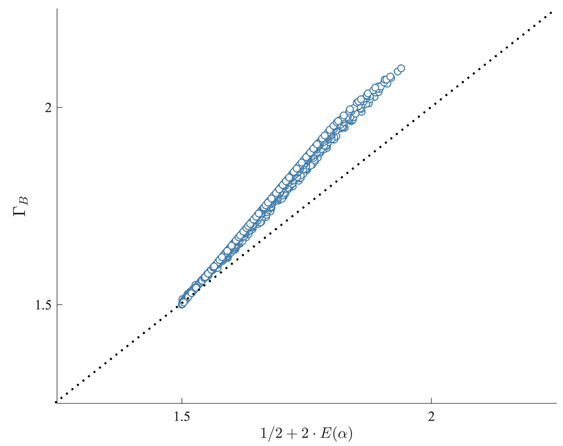


(b) $E(\alpha) > 1/2$

Triangular distributions



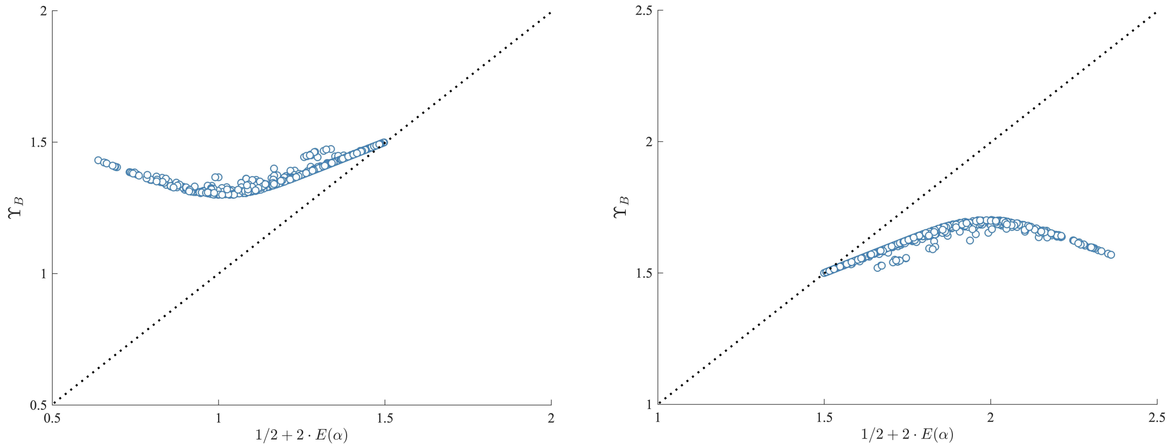
(a) $E(\alpha) < 1/2$



(b) $E(\alpha) > 1/2$

Figure C.2: In all the simulations, $E(\alpha) < 1/2$ if and only if $\Gamma_B < 1/2 + 2 \cdot E(\alpha)$.

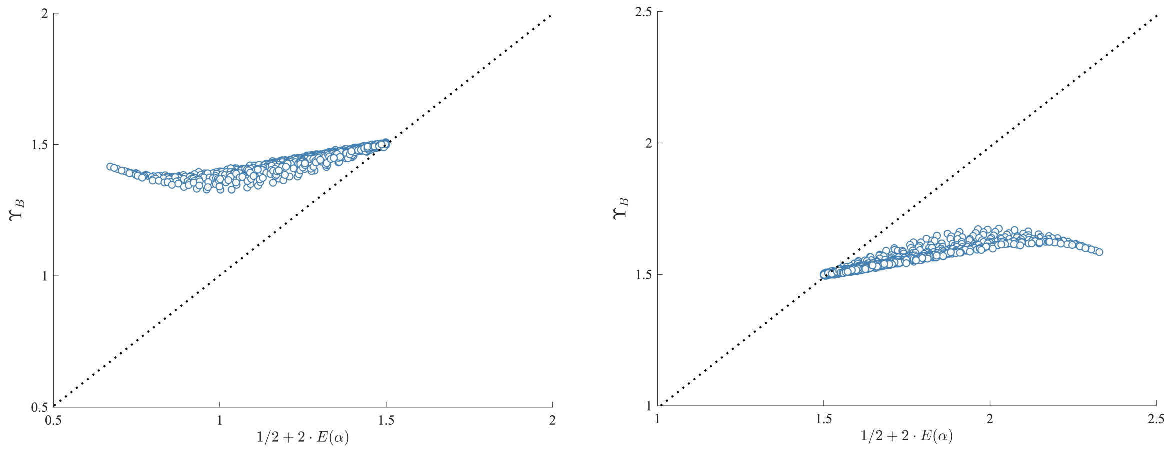
Beta distributions



(a) $E(\alpha) < 1/2$

(b) $E(\alpha) > 1/2$

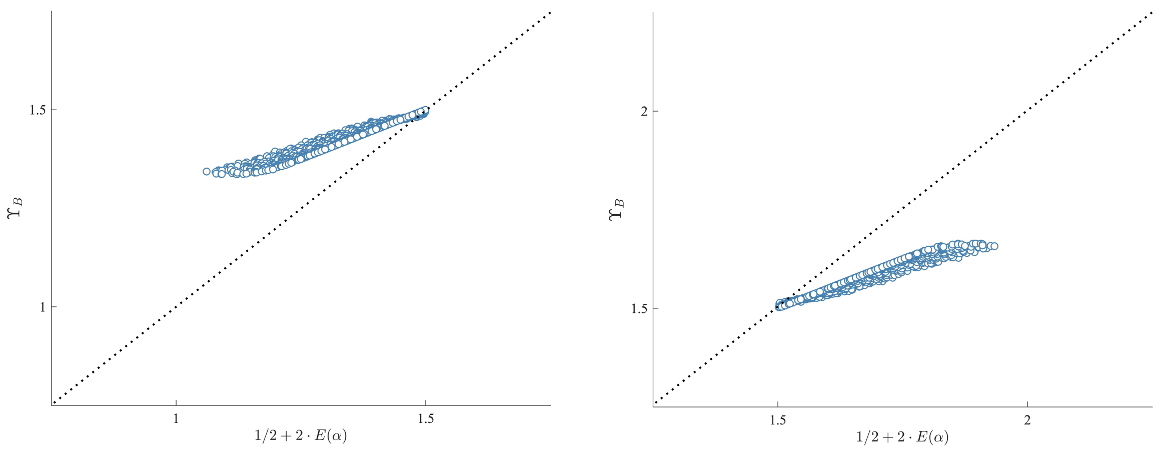
Truncated normal distributions



(a) $E(\alpha) < 1/2$

(b) $E(\alpha) > 1/2$

Triangular distributions



(a) $E(\alpha) < 1/2$

(b) $E(\alpha) > 1/2$

Figure C.3: In all the simulations, $E(\alpha) < 1/2$ if and only if $\Upsilon_B > 1/2 + 2 \cdot E(\alpha)$.