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Essays in Business Cycles with Household Heterogeneity

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ABSTRACT

This thesis contributes to understanding how inequality affects the transmission of aggregate fluctuations through aggregate demand. First, we document how labor earnings inequality (between skilled and unskilled workers) reacts to both government spending and monetary policy shocks. We show that a contractionary monetary policy shock and an expansionary government spending shock raise labor income inequality. Government spending raises inequality because it is concentrated towards sectors that are skilled intensive and contractionary monetary policy shocks generate inequality because unskilled workers face more rigid wages. Then, we analyze the effect of these phenomena on aggregate consumption. We show that unskilled workers in the U.S. are significantly more financially constrained than skilled workers. Therefore, they are less able to smooth out income fluctuations. This fact, in addition to having different labor income fluctuations between the two skill groups, may cause dampening or amplifying effects of shocks. We show analytically and quantitatively that through aggregate demand, different wage rigidities significantly amplify monetary policy shocks and that the current distribution of government spending across the different productive sectors dampens the impact of fiscal policy. Second, we study the gains from nominal flexibility (of both prices and wages) when there is limited access to financial markets. We show that limited access to financial markets and price and wage rigidities give rise to a *distributional channel of nominal rigidities* which works through aggregate demand. In particular, we show that aggregate demand depends on the relative price and wage rigidity. Through this distributional channel, aggregate demand amplifies demand shocks if wages are more flexible than prices. This happens because in response to the shock, workers who have high marginal propensity to consume (MPC) suffer more from the shock than firm owners who have low MPCs.

RESUMEN

Esta tesis contribuye al entendimiento de cómo la desigualdad afecta la transmisión de las fluctuaciones agregadas a través de la demanda agregada. Primero, documentamos cómo la desigualdad de los ingresos laborales (entre trabajadores calificados y no calificados) aumenta en respuesta a *shocks* expansivos de gasto del gobierno y a contracciones de la política monetaria en EEUU. Aumentos del gasto del gobierno generan desigualdad porque el gasto se concentra en sectores que son más intensivos en trabajo calificado y contracciones monetarias generan desigualdad debido a que los trabajadores no calificados tienen salarios más rígidos. Luego, analizamos el efecto de estos fenómenos sobre el consumo agregado. Mostramos que los trabajadores no calificados en EEUU están más restringidos financieramente que los calificados. Entonces, los primeros son menos capaces de suavizar consumo en respuesta a fluctuaciones en su ingreso. Esto, en adición a las diferencias de respuestas del ingreso, generan amplificación o disminución del efecto de los shocks. Mostramos que a través de la demanda agregada, estas diferencias en rigideces de salarios amplifican el efecto de la política monetaria; y la actual distribución sectorial del gasto del gobierno disminuye el multiplicador fiscal si los trabajadores no calificados están más restringidos financieramente. Segundo, estudiamos las ganancias de flexibilizar precios y salarios con hogares restringidos financieramente. Mostramos que con restricciones financieras y precios y salarios rígidos, aparece un *canal distribucional de rigideces nominales*, que opera a través de la demanda agregada. Mostramos que la demanda agregada depende de la rigidez relativa de salarios y precios. En particular, que hay amplificación de shocks de demanda si los salarios son más flexibles relativo a los precios. Esto ocurre debido a que en respuesta al shock, el trabajador, que típicamente tiene una mayor propensión marginal a consumir (PMC) sufre más del shock que el dueño de la firma, que tiene una baja PMC.

PREFACE

This thesis studies the role of income inequality on the transmission of aggregate fluctuations. The thesis comprises three chapters that are self contained while related in one key aspect: the role of income inequality in determining aggregate demand. We consider three dimensions of household heterogeneity that affect the transmission of aggregate shocks through aggregate demand. These are: heterogeneity in access to financial markets; differences in the sources of income; and differences in the exposure of income to the business cycle.

Usually, the literature on household heterogeneity focuses on the effects of uninsurable income risk which gives rise to a distribution of income and wealth. By contrast, in two of the three papers we focus on a different—and ex-ante—heterogeneity: differences in workers' skill level. We find that workers of different skills face different access to financial markets and different fluctuations in their earnings. In particular, while about 30% of households in the U.S. have no access to financial markets, and hence behave as hand-to-mouth agents, this share is about 47% for workers without a college degree (the unskilled workers) and 18% with a college degree or more (the skilled workers). Therefore, unskilled workers have a higher marginal propensity to consume (MPC) than skilled workers. We explore the effects of having higher cyclical income combined with high MPCs in the transmission of monetary and fiscal policy and study the mechanisms that generate these differences. Then, in chapter three we study the effects of nominal rigidities (of prices and wages) on the transmission of aggregate shocks with limited access to financial markets. We uncover a channel that we dub *the distributional channel of nominal rigidities* in which we show that nominal rigidities enter the aggregate demand in this context.

Studying these phenomena is important for several reasons. First, it helps us understanding the transmission mechanisms of macroeconomic policies—and shocks—best. Second,

we can study the conditions under which there is amplification or dampening of shocks due to inequality. Third, to study the distributional consequences of macroeconomic policies on earnings and consumption by skill level. These are all important questions that have gained prominence in the discussions about macroeconomic policies and on central banking in particular.

Chapter one is based on recent literature that has emphasized the importance of indirect effects on the transmission of monetary policy (see e.g. [Kaplan et al. \(2018\)](#) and [Auclert \(2019\)](#)). We contribute to this literature by studying how the distribution of these effects among different households and their impact on the aggregate economy. To study that question, we show that, for the U.S., the ratio of labor income of skilled to unskilled workers (the *earnings gap*) is countercyclical and increases in response to a contractionary monetary policy shock. Those facts suggest that fluctuations impact different skill groups unevenly. By building a New Keynesian model with incomplete markets and heterogeneity in wage rigidities we find that these facts can be rationalized with gross substitutability between skilled and unskilled in production and with unskilled workers' wages being more sticky. We confirm those requirements in the data by estimating wage Phillips curves for the different groups of workers and document that unskilled workers have a flatter wage Phillips curve. We show, in the calibrated model, that if the earnings gap is countercyclical and unskilled workers are more financially constrained, the impact of monetary policy shocks is twice as strong as when there are homogeneous wage rigidities. Finally, we argue that eliminating wage rigidities only benefits the unskilled workers who have no access to financial markets, while the aggregate is negatively affected.

In chapter two, we revisit the effects of government purchases on consumption considering its effects on inequality by extending the work by [Blanchard and Perotti \(2002\)](#) and [Primeri \(2005\)](#). We show three empirical facts in this regard: (i) government spending raises labor income inequality between skilled and unskilled workers; (ii) the responses of labor income

inequality and consumption to a government spending shock are negatively related; (iii) government purchases concentrate towards sectors with a larger share of skilled workers than the overall economy. We argue that a model with two sectors, two groups of workers, and limited access to financial markets in which the government buys more proportionally the skilled intensive sector explains these empirical facts well. As a consequence of the inequality government purchases generates, the government spending multiplier can be 30 percent lower than when the government spends exclusively in the unskilled intensive sector. That occurs because due to market incompleteness, higher inequality depresses consumption because government spending delivers disproportionately more income to the skilled worker, who has a low marginal propensity to consume. Therefore, we conclude that the way government spends matters both for inequality and for the strength of government spending in stimulating the economy.

Chapter three, which is coauthored with *Damián Romero* and *Sebastián Diz*, studies wage and price flexibility as a means of absorbing adverse shocks. We focus on economies with unequal access to financial markets and where the monetary authority is constrained by the zero lower bound. We show that the economy becomes more volatile in this setting when wages are more flexible. As our model assumes financial frictions, wage flexibility translates into output volatility via a redistribution channel, which operates through the aggregate demand. We find that this volatility depends on the relative wage and price rigidity. Additionally, we show that the redistribution channel gains prominence when the central bank is at the zero lower bound. We conclude that in these kinds of economies, the usual recommendation of making labor markets more flexible to restore high output levels, is mistaken.

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THE LABOR EARNINGS GAP, HETEROGENEOUS WAGE PHILLIPS CURVES, AND MONETARY POLICY

1.1 INTRODUCTION

Recent literature has emphasized the importance of taking into account household heterogeneity to understand aggregate economic fluctuations, and the transmission of monetary policies (see e.g. [Kaplan et al. \(2018\)](#) and [Auclert \(2019\)](#)). Most studies in that literature have considered economic environments with ex-ante identical households, and where heterogeneity is the consequence of uninsurable idiosyncratic income shocks.

In this paper, we show that another dimension of heterogeneity, and namely ex-ante differences in skills/education across the population, might have important implications for the transmission of monetary policies and their welfare implications. As we argue below, over the business cycle, “unskilled” workers –i.e. people without a college degree– face more rigid wages, and experience larger fluctu-

ations in their labor income, relatively to “skilled” workers—people with college or higher degrees. Also, unskilled workers typically hold low levels of wealth, so that their consumption essentially mirrors the fluctuations in their labor income—i.e. they have a high marginal propensity to consume (MPC). Within a two-agent New-Keynesian model, we show that the combination of these features—higher wage rigidity and higher MPC of unskilled workers—leads to an amplification of the effects of monetary policy shocks. Under the baseline calibration, the (cumulative) effects of monetary shocks on GDP are twice as large as an economy with no heterogeneity.

We begin our analysis by looking at US data on wages, employment and wealth over the 1980-2018 sample, and document three main facts. First, the share of unskilled workers with zero liquid assets is 47% while this share for skilled workers is 18%. This observation suggests that a large share of unskilled workers, if affected by income shocks, cannot rely on buffer savings to avoid consumption fluctuations, and behave effectively as “hand-to-mouth” agents. Second, the *earnings gap* (which is the ratio of skilled to unskilled labor income), is countercyclical. This result holds both when looking at unconditional correlations, or when focusing exclusively on the effects on monetary shocks, through an IV-SVAR analysis (following [Miranda-Agrippino and Ricco \(2021\)](#)).¹ Third, the wage Phillips curve is much steeper for unskilled workers than for skilled workers. This suggests that skilled workers have more flexible wages than unskilled workers.² The consequence of this fact are that in an environment in which there is gross substitution between skilled and unskilled labor, in a downturn unskilled workers become relatively more expensive,

¹[Dolado et al. \(2021\)](#) also find that monetary policy affects unemployment and wages of the two skill groups in a different way, while they do not focus on the role of inequality between them.

²This analysis is similar to [Doniger \(2019\)](#) who also finds that wages of the skilled workers are more flexible than those of the unskilled workers using a panel-data analysis.

generating a further drop in their labor income.

To rationalize these facts, we build a New Keynesian model with incomplete financial markets and wage stickiness heterogeneity. We assume there are two different groups of workers, skilled and unskilled. Labor markets differ in several ways, but most importantly, they differ in their degree of wage stickiness. Firms employ these different groups of workers and aggregate them with a constant elasticity of substitution (CES) production function. Workers share bond and equity markets, and hence every agent faces the same rates of return on their assets. We assume that unskilled workers are more financially constrained. Finally, we include all the standard features of New Keynesian literature: price rigidities, monopolistic competition in intermediate goods, and a Taylor rule.

We study analytically how the interaction of labor income inequality and incomplete markets affects aggregate demand. We follow [Bilbiie \(2008\)](#) and [Debortoli and Galí \(2018\)](#) by deriving an Euler equation with incomplete markets and heterogeneity in labor income. As in the latter study, we show that the aggregate Euler equation depends on heterogeneity wedges; in particular, it depends on a *consumption gap* that summarizes how financially constrained and financially unconstrained consumption differ. We show that the consumption gap depends on the earnings gap. In turn, the earnings gap may vary in response to aggregate fluctuations, given that workers belong to different labor markets. Therefore, we show that in a model with different labor markets and financial constraints, if the earnings gap is countercyclical, there is an amplification of monetary policy shocks. This amplification effect appears because those whose income fluctuates most in response to a monetary policy shock are the workers with higher marginal propensities to consume—the unskilled workers—who are more financially constrained.

Then, we study the specifications of technology and labor markets that give rise to a countercyclical earnings gap in our model.

For the labor market arrangement, we follow [Gali \(2013\)](#), which is a model where a union representing each class of workers sets wages. This union has market power regarding the demand for workers and hence charges a markup over the marginal rate of substitution. We also assume nominal wages are sticky, which delivers fluctuating and countercyclical wage markups. Additionally, as we show that labor income shares vary over time, we consider a production technology with imperfect substitutability between workers' groups. We show that the reason why the earnings gap fluctuates in our model is the heterogeneity in labor markets. However, a necessary condition for the earnings gap to vary is gross substitution between groups of workers (which rules out the Cobb-Douglas technology, for example). And given that condition, the wages of the skilled workers must be more flexible than those of the unskilled to generate the countercyclicality of the earnings gap.

The intuition of this result is the following. In response to a contractionary monetary policy shock, wages of all groups must fall. When the wages of the unskilled are more sticky, skilled workers become relatively cheaper than unskilled workers. In those conditions, if it is easy to substitute types of workers in the production process, there is going to be a shift in demand from unskilled to skilled workers. These effects generate a more than proportional fall in unskilled workers' labor income that produces the earnings gap to increase.

We then consider a calibrated version of our model to assess quantitatively the effects of monetary shocks. We find that in the presence of labor market heterogeneity, the effects are 20% larger than in a representative agent counterpart (with no heterogeneity). This amplification is obtained because unskilled workers have more procyclical labor income and are more financially constrained than the skilled workers, making their consumption respond more strongly. We also show that the previous mechanism is the most relevant amplification channel in our model.

Finally, we conduct a welfare analysis to explore the gains from making wages more flexible in this context.³ To compute the welfare losses from wage flexibility, we follow the method proposed by [Schmitt-Grohe and Uribe \(2007\)](#). We find that in our model, there are no gains from making wages (marginally) more flexible. From our baseline calibration in which we can only make the wages of the unskilled more flexible, there are welfare losses if we let these wages adjust more freely. The losses distribute very unequally: while unskilled workers do not lose much the skilled workers lose out significantly. This effect arises from the excessive volatility of inflation generated by the higher volatility of wages of the unskilled. When the unskilled workers' wages become more flexible, all inflation rates get more volatile, while the quantities (for instance, skilled labor) do not necessarily become more stable, generating important losses from the higher flexibility of unskilled wages. Thus, in the aggregate, as the losses derive from excess volatility of inflation, we find that making wages more flexible in this context relies on the ability of the monetary authority to control it, as first stressed by [Galí \(2013\)](#).

All these findings suggest that to understand the effects of inequality on the business cycle and for the transmission of monetary policy, we must study the heterogeneity in the responses of income for different groups of workers; i.e., we must also consider the cyclicalities of the indirect effects that impact consumers and workers.

The contributions of this paper can be summarized as follows. We first contribute to the literature on labor income inequality by showing a simple measure of inequality that has aggregate effects, is countercyclical and responds to monetary policy shocks. Second, we contribute to the literature on wage rigidities by showing empirically that a particular group of agents (the skilled workers) have substantially more flexible wages. Third, we contribute to the theoretical

³See [Galí \(2013\)](#), [Galí and Monacelli \(2016\)](#), [Billi and Galí \(2020\)](#), and [Diz et al. \(2019\)](#) for other examples of this approach.

literature on the transmission of monetary policy by highlighting the importance of heterogeneity in wage rigidities for the transmission of shocks. We show that heterogeneous wage rigidities have a crucial role in the transmission of monetary policy through aggregate demand. These results, to the best of our knowledge, have not been reported in previous literature. Finally, we contribute to the literature on the gains from wage flexibility by stressing the distributional effects of making wages more flexible in this context.

Related Literature. This paper is related to three strands of the literature: on macroeconomics with heterogeneous agents, on wage rigidities with heterogeneity in labor markets, and the gains from wage flexibility.

As we mentioned above, we follow closely and extend [Bilbiie \(2008\)](#) and [Debortoli and Galí \(2018\)](#). Their work is to establish the consequences of having hand-to-mouth workers for the transmission of monetary policy. A paper that is similar to ours is the work by [Broer et al. \(2019\)](#) in which they include wage rigidities in a tractable Heterogeneous-Agent New Keynesian model (HANK). Previous papers similar to ours are [Ascari et al. \(2017\)](#) and [Colciago \(2011\)](#) that also show how incomplete markets and wage rigidities interact. They show that wage rigidities may help offset the amplifying effects of incomplete markets in Two-Agent (TANK) models. Additionally, [Furlanetto and Seneca \(2012\)](#) use a TANK model with wage rigidities to explain the negative short-run response of the economy to technology shocks. All these papers follow the earlier analysis in TANK models by [Galí et al. \(2007\)](#), who study the amplification of government spending shocks in the presence of a share of HtM agents.

This paper also relates to the HANK literature. This literature analyzes the effect of inequality on the business cycle but by assuming there is a full distribution of wealth, that they derive from idiosyncratic uncertainty. There are several works that include [Kaplan](#)

et al. (2018), Auclert et al. (2018), Luetticke (2019), and Cui and Sterk (2018), where they emphasize the role of illiquidity to explain the response of consumption to aggregate shocks. Due to the illiquid part of the portfolio, agents can not smooth consumption completely which makes them react to indirect effects. We abstract from the distribution of assets and idiosyncratic uncertainty by assuming fixed shares of HtM agents following the analysis made by Kaplan et al. (2014). Other papers that relate incomplete markets with New Keynesian features include Auclert (2019), Farhi and Werning (2017), Gornemann et al. (2016), Guerrieri and Lorenzoni (2017), McKay and Reis (2021), McKay and Reis (2016), Ravn and Sterk (2020), and many others. Our contribution with respect of the previously mentioned studies is the recognition that there are different classes of workers with their own labor markets which have different dynamics to study how labor markets interact with incomplete access to financial markets.

Patterson (2019) studies the relation between marginal propensities to consume and the cyclicalities of labor income. She estimates the exposure of income to the cycle and the marginal propensities to consume. She finds that there is a positive correlation between MPC's and labor income cyclicalities in the U.S with microeconomic data, which is similar to the facts presented above. Unlike her, we provide an explanation (different wage rigidities) and a theoretical mechanism that explains why different workers have different labor income cyclicalities and study the implications of eliminating these differences while showing that their micro evidence holds at a macroeconomic level. Also, our work is related to Dolado et al. (2021). These authors stress the differences on labor income between skilled and unskilled workers and the effect of capital-skill complementarities. Our contribution to this work is the analysis of the differences in access to financial markets that affects consumption dynamics, channels that they do not analyze. We see our work as complementary to those studies.

We are also related to the literature on wage and labor income cyclicity. The main references are [Taylor \(2016\)](#) and [Basu and House \(2016\)](#) that argue the existence of important rigidities in the way wages adjust. They also stress the possibility of heterogeneity on wage and labor market adjustments. There is also a broad literature that studies the cyclicity of wages concerning the type–and employment state–of worker. The main contributions are the ones by [Gertler et al. \(2020\)](#) and [Moscarini and Postel-Vinay \(2016\)](#). However, we concentrate on the total wage bill of workers at a different skill/educational level. In that respect, our work extends [Cairó and Cajner \(2018\)](#) and [Doniger \(2019\)](#). The former shows that unskilled workers’ unemployment is more volatile than skilled and concludes that this is due to more volatile job finding rates of unskilled workers. The latter shows that skilled workers face more flexible wages than unskilled. All this might be consistent with heterogeneity in wage rigidities. We extend these results by studying the cyclicity of labor income inequality, because what matters for consumption is the wage bill and not employment or wages separately. An additional advantage of taking labor income inequality into account is that it would serve as a sufficient statistic to evaluate the effect of inequality on the business cycle since it is what enters into the consumption equation in models with incomplete markets. That is why we study the response of the Earnings Gap to a monetary policy shock and the reason why it fluctuates.

Finally, this paper is related to the literature on the gains from wage flexibility. [Galí \(2013\)](#) starts a sequence of studies that analyze the gains from wage flexibility in New Keynesian models. The main conclusion of these studies is that there are not always gains from greater wage flexibility, as claimed by the neoclassical literature. In models with representative agents, this is due to welfare losses from the volatility of price and wage inflation. Therefore, the gains arise only if the monetary authority responds sufficiently strongly to inflation. [Galí and Monacelli \(2016\)](#) and [Billi and Galí \(2020\)](#) extend the

previous analysis to models in which the central bank is constrained by either the zero lower bound or by a currency union.

Organization of the Paper. The remaining of the paper is organized as follows: section 1.2 shows empirical evidence on the heterogeneity of labor income in the cycle and on how assets are distributed across skill groups. Section 1.3 show the effects of monetary policy on labor markets dynamics and the heterogeneity in wage Phillips curves by skill levels. Section 1.4 presents the model. Section 1.5 studies analytically how labor income inequality affects the aggregate demand and why labor income inequality fluctuates in our model. Section 1.6 studies the quantitative implications of the heterogeneity in wage rigidities. Section 1.7 analyzes the gains from making wages more flexible in this context. And finally, section 1.8 concludes.

1.2 MOTIVATING FACTS

In the present section, we provide evidence on the heterogeneity of labor markets and the distribution of assets of the different groups of workers. We first show that labor income inequality (the ratio of skilled to unskilled labor income) is countercyclical. And then, we show that skilled workers are richer and have broader access to financial markets than unskilled workers.

1.2.1 *The Earnings Gap is Countercyclical*

We first show that the labor earnings gap is countercyclical, which means that inequality falls in a boom and rises in a recession. We denote the earnings gap by η_t , which formally is given by the ratio of skilled to unskilled labor income

$$\eta_t = \frac{\text{Skilled labor income}}{\text{Unskilled labor income}}.$$

For this paper, we divide the population into these two groups, skilled and unskilled. We consider in the former group workers with a completed bachelor degree or higher while an uncompleted bachelor degree or less in the latter.⁴ We are interested in the inequality of total labor income because it is total income and not just wages or labor what determines consumption and it corresponds to the greatest part of total income in the US economy.⁵

To build η_t , we take the *Current Population Survey* (CPS) that has individual earnings and demographic data. We consider the full sample, which is the period from 1979M1 to 2018M12. We use a uniformed version of the CPS built by the *Center of Economic and Policy Research* (CEPR).⁶ The CEPR computes uniformed hourly wage and labor earnings for each period, which are comparable between panels. They also complete the sample by imputing weekly earnings from hourly wages and vice-versa if the respondent lacks one of the variables. We use the CEPR measure of total weekly labor earnings in what follows. Hence, we calculate labor income by group as its cross-sectional weighted average, representing a per-capita measure of income by group.

Figure (2.1) depicts the series of η_t . On the left hand side, we plot the twelve-month moving average of the earnings gap together with unemployment. On the right hand side, we plot the cyclical component of the earnings gap against the cyclical component of unemployment.⁷

Left panel of Figure (2.1) shows some interesting facts. First, since the 2000's, the earnings gap is high, and around 1.8. Second, our earnings gap reflects the increase on labor income inequality between

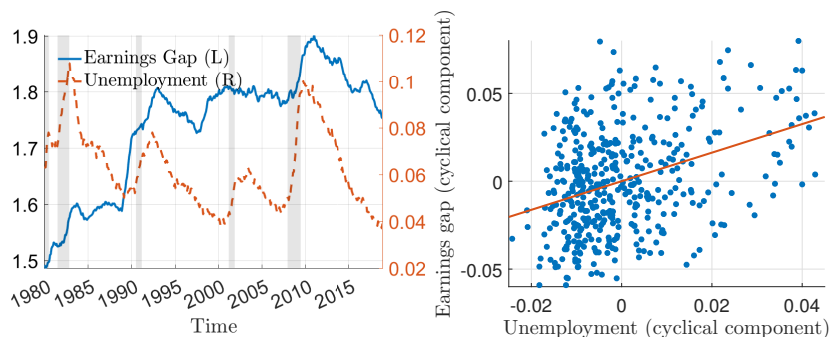
⁴According to the CPS, the average share of skilled workers was 35% for the 2000-2018 period and has been steadily increasing.

⁵Although the labor income share has been falling in the past several decades, the labor income share is higher than 60 percent.

⁶See <http://ceprdata.org/> for more information.

⁷We use the [Hamilton \(2018\)](#) filter extract the trend component of both series.

FIGURE 1.1: Labor Earnings Gap and unemployment.



Notes: This figure shows the earnings gap over the business cycle. The left-hand panel depicts the level of the earnings gap compared with the unemployment rate. The gray vertical lines correspond to the NBER recessions. The right-hand panel shows the scatter of the relationship between the cyclical component of the earnings gap and of the unemployment rate.

skilled and unskilled workers documented in previous studies. At the beginning of the 1980's the earnings gap was about 1.5; i.e, skilled workers earned 50% more than the unskilled workers. During the 1980's the gap increased substantially and rose to about 1.8, to stay around that level until the Great Recession.⁸ Third, the earnings gap increases in recessions and falls in expansions. In all the recessions except for 2001 (which seems to be a very particular one), the earnings gap has increased significantly. There are also several periods in which the gap increases even in expansions like the period prior to 1990. However, in long periods of expansion, like 1992-1997 or from 2011 to 2018, the earnings gap fell, but the fall was less pronounced than that of the unemployment rate, suggesting that the earnings gap has even more persistence than the unemployment rate.⁹ Fourth,

⁸This is consistent with the evidence on the increase of the skill premium. In general the skill premium literature only looks at the widening of the wage gap. But as we are interested in what determines consumption, we study total labor income.

⁹We do not take into account this fact in the analysis but it is certainly an interesting one.

and related to the previous point, the earnings gap seems to behave asymmetrically; i.e, the earnings gap increases sharply in recessions but seems to stay at high levels for a long period, often until the next recession takes place and pushes inequality further up.

Right panel on Figure (2.1) shows the relation between the cyclical component of unemployment and the earnings gap. Hence, not only is the medium- to long-run relation between unemployment and the earnings gap positive but their cyclical components correlate positively as well. Therefore, labor income inequality increases in recessions and falls in booms. To confirm that this is the case, we run the following regression

$$\log(\eta_t) = c + \chi u_t + \sum_{m=1}^{12} \gamma_m \mathbb{I}_m + e_t, \quad (1.1)$$

where η_t is the earnings gap, u_t is unemployment that we use as an indicator of aggregate economic conditions, and we control for monthly dummies \mathbb{I}_m .

Table (1.1) shows the results from regressing unemployment on the earnings gap for three different specifications: (i) we regress the cyclical component of unemployment on the cyclical component of the earnings gap computed using the [Hamilton \(2018\)](#) filter; (ii) the previous exercise but using the Hodrick-Prescott filter; and (iii) detrending the series with a linear and a quadratic trend as controls.

Table (1.1) confirms the observations on Figure (2.1); i.e, there is a positive relation between the earnings gap and the unemployment rate. In all the specifications, a rise in unemployment implies a rise in the earnings gap, that is always statistically significant. The specification shown in the right panel of Figure (2.1) is the one represented in the first column of Table (1.1), which shows that the earnings gap increases by 0.8% after a 1 percentage point increase in the unemployment rate.

These figures are economically significant. For example, during the Great Recession unemployment rose by about five percentage points, which meant that the Earnings Gap increased by about 4%.

Table 1.1: The relation between the Earnings Gap and unemployment.

	Dep. var: $\log(\eta_t)$		
	Cycle, Hamilton	Cycle, HP	Cycle, Qtrend
u_t	0.808*** (0.0985)	1.028*** (0.344)	0.780*** (0.121)
Adj. R -sq	0.137	0.225	0.279
N	445	480	480

Standard errors in parentheses

* $p < 0.10$, ** $p < 0.05$, *** $p < 0.01$

Notes: This table shows the regressions of Equation (1.1). We run three specifications, depending on the treatment of the data. We show the data filtered with Hamilton (2018) filter, the HP-filter, and subtracting a linear and a quadratic trend.

The aim of this exercise is to understand what kind of model fits the data best. On the one hand, these results suggest that the labor income shares of different groups shift over time. This shift means that we must consider the economy as one in which the elasticity of substitution is different from one. For instance, we must discard Cobb-Douglas technologies, because under the latter, η_t would be constant. Also, we must take into account that different groups of workers have their own labor market dynamics which account for a different jointly movement of hourly wages and hours. Next, we decompose the earnings gap.

Decomposing the Earnings Gap. We can further analyze the (log) earnings gap by decomposing it into a wage and an hours gap. Denote with w_{ht} and N_{ht} the wage and hours of group h at time t . The

earnings gap is given by $\eta_t = \frac{w_{st}N_{st}}{w_{ut}N_{ut}}$ (with s for skilled and u for unskilled). Then, the log of the earnings gap can be decomposed as

$$\begin{aligned}\log(\eta_t) &= \log\left(\frac{\text{Skilled labor income}}{\text{Unskilled labor income}}\right) = \log\left(\frac{w_{st}}{w_{ut}}\right) + \log\left(\frac{N_{st}}{N_{ut}}\right) \\ &= \log(\text{Wage Gap}) + \log(\text{Hours Gap}).\end{aligned}$$

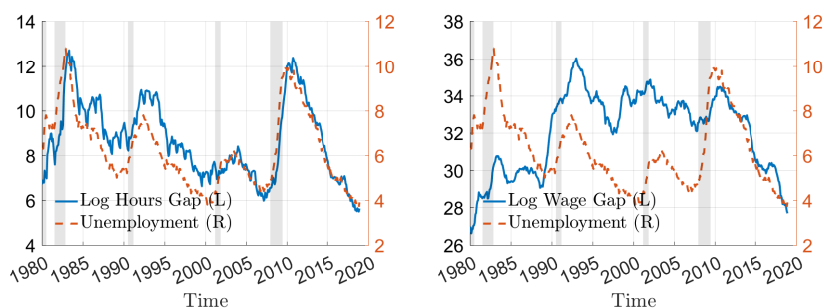
Figure (1.2) shows the previous decomposition for our data. The left-hand panel depicts the hours gap (in logs) while the right-hand panel shows the wage gap (in logs). Both the hours gap and the wage gap are countercyclical. In recessions, both wage and employment inequality increase. While hours inequality is highly correlated with unemployment, the wage gap also correlates but at a lesser extent. The wage gap suggests that the increase in wage inequality explains the upward trend the earnings gap had between early eighties and mid nineties, and also explains the greater part of the earnings gap, by accounting for about three fourths of earnings inequality. Finally, it is worth to highlight the recent recovery of the US labor market. Inequality went down both for hours and wages, following about the same path of the unemployment rate. This latter fact has not happened in past recoveries.

1.2.2 *Skilled Workers are Richer and Have More Access to Financial Markets*

Finally, we show that skilled workers own most of the assets in the U.S. economy and that unskilled workers are out of the financial markets in a higher proportion.

Table (1.2) shows the shares of zero assets individuals by skill level. We build these indicators based on Kaplan et al. (2014), using the *Survey of Consumer Finances* 2016. Kaplan et al. (2014) separate between liquid and illiquid assets. The former is composed of checking accounts, cash in hand and similar accounts, private and government bond holdings, minus revolving, and consumer credit. The latter is

FIGURE 1.2: Wage and hours gap and unemployment.



Notes: This figure shows the decomposition of Earnings Gap into an hours gap (the left-hand panel) and a wage gap (the right-hand panel). We show the log of each variable along with the unemployment rate. The gray vertical lines correspond to the NBER recessions.

composed of net housing (housing value minus mortgage-backed debt), net private businesses, direct and indirect equity holdings, and durables. Like them, we consider as zero-asset holders, individuals that hold \leq US\$1000 of net worth in absolute value (in 2004 US\$). We adjust this threshold by inflating US\$1000 to 2016 prices. Additionally, we report the share of debtors each group has.

Table 1.2: Shares of Hand-to-Mouth and debtors by educational level.

	Shares of zero assets		Debtors
	Illiq	Liq	Liq
Full sample	.17	.33	.15
\leq Some College	.24	.47	.14
\geq College grad	.09	.18	.16

Source: SCF 2016.

Notes: This table shows the shares of zero assets by educational level. We show the decomposition presented by [Kaplan et al. \(2014\)](#) by using the Survey of Consumer Finances 2016. We separate liquid from illiquid assets by educational attainment. Additionally, we show the share of debtors (in liquid assets) of each group of workers.

If we compare these figures with the ones reported by [Kaplan et al. \(2018\)](#), we can observe that they are very similar in the full sample. They show that on the SCF 2004, the share of hand-to-mouth (this is, individuals with zero liquid assets) is .28, While we find a slightly higher percentage of those individuals, .33. However, our estimates may be biased upwards since we do not conduct the imputation of cash they do.

However, our focus is on the difference between skill levels. As Table (1.2) shows, the difference between skilled and unskilled individuals is considerable. 24% of unskilled individuals hold zero illiquid assets, with this figure rising to 47% for liquid assets. Those figures imply that almost half of the uneducated-high volatility of labor income people have no means of consumption smoothing in the short run. For skilled workers, these numbers fall considerably. Only 9% of educated individuals hold zero illiquid assets, while 18% hold zero liquid. The previous facts imply that unskilled people not only earn a lower labor income, but they have more limited access to financial markets than skilled workers.

1.3 MONETARY POLICY AND THE EARNINGS GAP: AN EMPIRICAL ASSESMENT

In this section, we study the cyclicality of the earnings gap *conditional* on a shock. This allows us to answer two questions regarding the earnings gap. First, if the earnings gap continues to be countercyclical conditional on an identified shock. And second, explain why the earnings gap is countercyclical. For the former question we study the response of the earnings gap to an identified monetary policy shock, while for the latter we study the dynamic multiplier of wage inflation with respect to unemployment, which allows us to approximate the slope of the wage Phillips curve of each of the two skill

levels. We do that exercise to get an approximation of the differences in wage rigidities between groups of workers. For both exercises we estimate Bayesian Local Projection with instrumental variables following [Miranda-Agrippino and Ricco \(2021\)](#).

1.3.1 *Econometric Strategy*

To study the response of the labor market variables to a monetary policy shock, we follow [Stock and Watson \(2018\)](#) and [Gertler and Karadi \(2015\)](#) by estimating instrumental variable Local Projections and instrumental variable VARs with exogenous instruments for monetary policy, based on high-frequency identification.

Let us consider X_t , which is a $k \times 1$ vector of observable variables. We Assume X_t follows an invertible VAR(p), which has an MA representation given by

$$X_t = u_t + \psi_1 u_{t-1} + \psi_2 u_{t-2} + \dots \quad u_t \sim WN(0, \Sigma_u). \quad (1.2)$$

The process for X_t also admits a structural representation given by

$$X_t = B_0 \varepsilon_t + B_1 \varepsilon_{t-1} + B_2 \varepsilon_{t-2} + \dots \quad \varepsilon_t \sim WN(0, \mathbb{I}_k). \quad (1.3)$$

In line with the VAR literature we recover the structural shocks ε_t by assuming a structural relation given by $u_t = B_0 \varepsilon_t$. There are two assumptions required if we are to claim that the ε_t 's are proper structural shocks: (i) the econometrician when estimating the equation (1.2) includes all the information required by the structural relation (the observed equation is well specified); and (ii) there is no uncertainty about the assumed matrix B_0 .

Usually, these two requirements are not met. That is why we claim that the shocks that are obtained with the relation $\varepsilon_t = B_0^{-1} u_t$ are not always well identified. Therefore, as we can not always elude these problems, the best procedure is to instrument the shocks as proposed

by [Stock and Watson \(2012\)](#) and implemented by [Gertler and Karadi \(2015\)](#) (GK) and [Miranda-Agrippino and Ricco \(2021\)](#) (MAR).¹⁰

We implement an external instruments procedure for the estimation of the effect of a monetary policy shock as follows. Let us write the equation that relates structural (ε_t^x) and reduced-form (u_t^x) shocks as

$$\begin{pmatrix} u_t^{mp} \\ u_t^{nomp} \end{pmatrix} = \begin{pmatrix} b_1 & | & b_2 \\ [k \times 1] & & [k \times (k-1)] \end{pmatrix} \begin{pmatrix} \varepsilon_t^{mp} \\ \varepsilon_t^{nomp} \end{pmatrix}$$

where b_1 is a $k \times 1$ vector that relates the reduced-form innovation in the interest rate u_t^{mp} with all the structural shocks. The aim is to instrument the structural shock to the interest rate with an external instrument. Before explaining our procedure, let us recap the conditions for a valid instrument z_t :

Find $z_t \notin y_t$ such that:

1. Relevance: $\mathbb{E}[z_t \varepsilon_t^{mp'}] = \alpha$.
2. Exogeneity: $\mathbb{E}[z_t \varepsilon_t^{nomp'}] = 0$.
3. Lead-lag exogeneity $\mathbb{E}[z_t \varepsilon_{t+j}^i] = 0 \quad \forall j \neq 0 \text{ and } \forall i$.

Furthermore, the system to be identified can be written as:

$$\begin{pmatrix} u_t^{mp} \\ u_t^{nomp} \end{pmatrix} = \begin{pmatrix} b_{11} & | & b_{21} \\ [1 \times 1] & & [k \times (k-1)] \\ b_{12} & | & b_{22} \\ [(k-1) \times 1] & & [k \times (k-1)] \end{pmatrix} \begin{pmatrix} \varepsilon_t^{mp} \\ \varepsilon_t^{nomp} \end{pmatrix},$$

then, we multiply that system by the instrument z_t and take expectations,

$$\begin{pmatrix} \mathbb{E}(u_t^{mp} z_t') \\ \mathbb{E}(u_t^{nomp} z_t') \end{pmatrix} = B_0 \begin{pmatrix} \mathbb{E}(\varepsilon_t^{mp} z_t') \\ \mathbb{E}(\varepsilon_t^{nomp} z_t') \end{pmatrix} = \begin{pmatrix} b_1 & b_2 \end{pmatrix} \begin{pmatrix} \alpha' \\ 0 \end{pmatrix} = \begin{pmatrix} b_{11} \alpha' \\ b_{12}, \alpha' \end{pmatrix}$$

¹⁰[Stock and Watson \(2018\)](#) analyzes the properties of Local Projection- and SVAR-IV.

which implies

$$\mathbb{E}(u_t^{mp} z_t')^{-1} \mathbb{E}(u_t^{nomp} z_t') = b_{11}^{-1} b_{12} \quad (1.4)$$

Equation (1.4) represents the relations used in making an identification through an external instrument. If the instrument is valid, b_1 is consistently estimated. In practice, this method is equivalent to regressing u_t^{nomp} against u_t^{mp} using z_t as an external instrument. The procedure we follow involves four steps. First, get an estimate of u_t from a VAR(p) or a Local Projection. Second, regress u_t against z_t . Third, calculate $b_{11}^{-1} b_{12}$ as a ratio of regression coefficients. Finally, choose a normalization, for instance $b_{11} = 1$.

As in MAR, we use their informationally robust instrument and compare the responses of different estimation methods. We compare three methods: (i) a Bayesian VAR (BVAR); (ii) a Local Projection (LP); and (iii) a Bayesian Local Projection (BLP). We follow their procedure because it accounts for the bias and estimation variance trade-off that VARs and LPs have. The Bayesian VAR produces more efficient parameters than the simple VAR and LP, but it is more prone to bias if the model is misspecified. That is why VAR and LP, if they are misspecified, produce highly inaccurate estimates. According to MAR, these issues can be the reason for “puzzling” responses and lack of robustness.

This aforementioned trade-off can be accounted for by Bayesian estimation. We follow these authors and take a Bayesian approach to Local Projection, which optimally spans the model space between VAR and LP impulse-response functions. This procedure helps to unravel the puzzles that may arise from model specification. The BLP procedure requires us to specify a (Normal-Inverse Wishart) prior for the local projection coefficients at each horizon. These priors are centered around the iterated coefficients of a similarly specified VAR estimated over a pre-sample. The posterior mean of BLP responses

takes the form

$$B_{BLP}^{(h)} = \left(X'X + \left(\Omega_0^{(h)} \lambda^{(h)} \right)^{-1} \right)^{-1} \left((X'X)B_{LP}^{(h)} + \left(\Omega_0^{(h)} \lambda^{(h)} \right)^{-1} B_{VAR}^h \right)$$

where $X \equiv (x_{h+2}, \dots, x_T)'$, and $x_t = (1, y'_{t-1}, \dots, y'_{t-h})'$. Intuitively, BLP regularises LP responses by using priors centered around an iterated VAR, while allowing the data structure to select the optimal degree of departure from the priors at each horizon ($\lambda^{(h)}$'s). The procedure treats these parameters as endogenous and estimates them as the maximizers of the posterior likelihood. In this way they balance the bias and the estimation variance at all horizons, and solve the trade-off. We follow their procedure closely. A detailed analysis and description of this approach can be found in [Miranda-Agrippino and Ricco \(2021\)](#).

1.3.2 Monetary Policy Shocks Raise the Earnings Gap

Next, we show that a contractionary monetary policy shock increases the earnings gap. In this exercise, we are interested in the cyclicity of the earnings gap conditional on a monetary policy shock. We take a vector of monthly observed variables for the U.S., given by

$$X_t = \{IP_t, UNEM_t, \eta_t, C_t, P_t, PCOM_t; R_t\}, \quad (1.5)$$

where IP_t is the log of industrial production index, $UNEM_t$ is unemployment, η_t is the log of the earnings gap, C_t is the log of consumption of nondurables, P_t is the log of the price index, $PCOM_t$ is the log of a commodity price index, and R_t is the one-year Treasury Bond.¹¹

¹¹We build the Earnings Gap as we exposed before. We obtain the data from the database presented by [McCracken and Ng \(2016\)](#). All variables are drawn from FRED. We consider INDPRO for Industrial Production, UNRATE for unemployment, PCND for consumption, CPIAUCSL for the price index, PPIACO for the index of commodity prices, and GS1 for the interest rate.

Through our procedure, we study the effect of a shock that produces a one-percent rise in the interest rate. The sample period is 1979M1-2018M12. As we explained before, we conduct three exercises, two of them using a Bayesian approach. We set as the training sample the first eight years of data and on all the estimations we consider twelve-month lagged specifications.

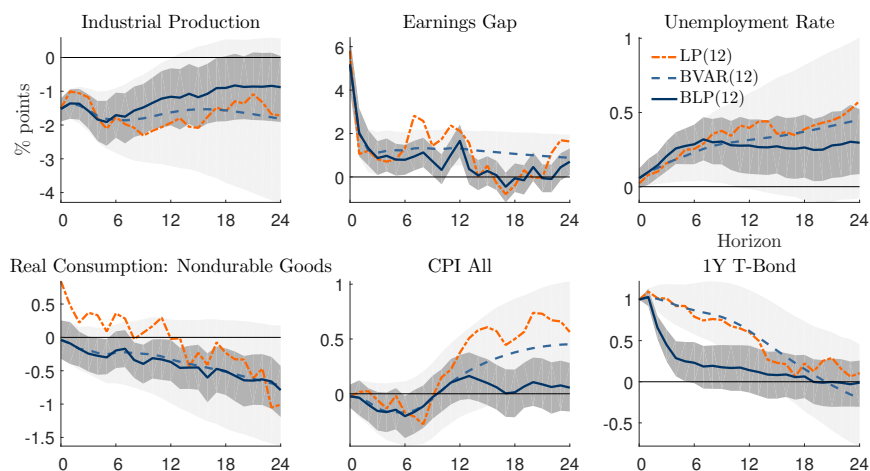
We use as an instrument the variable built by MAR, which is an “informationally robust monetary policy instrument”. They argue that the high-frequency shocks identified by GK are biased, and show that these shocks are autocorrelated and depend on the central bank’s private information. MAR point out that these biases can lead to “puzzling” responses, at least with Local Projections, as [Stock and Watson \(2018\)](#) and [Ramey \(2016\)](#) also stress. Then, the authors remove informational bias from the Fourth Federal Funds Futures (FF4) high-frequency surprise to obtain a valid instrument.¹² In the same way as the GK shocks, MAR shocks are computed for the periods from 1990M1 to 2012M12. We make the identification using this subsample while we use the whole sample to estimate the LPs and VARs.¹³

Figure (1.3) shows the responses of industrial production, the earnings gap, unemployment, consumption, CPI, and the interest rate to a contractionary monetary policy shock for the three methods: (i) Local Projection (short orange dashes); (ii) Bayesian VAR (long light blue dashes); and (iii) Bayesian Local Projection (solid blue). We report the 90% confidence bands for the BVAR (light gray) and for the BLP (gray). The monetary policy identification is normalized such that the impulse is equivalent to a one-percent increase in the one-year

¹²They regress Greenbook forecasts and revision of the forecasts for several variables (GDP growth, inflation, and the unemployment rate) on the GK FF4 shocks. They obtain the informationally robust shock as the residual of that regression.

¹³[Jarociński and Karadi \(2020\)](#) also tackle this problem by decomposing the GK shock into expansionary and contractionary monetary policy shocks, to show that effectively, the high-frequency identification also includes the information that the central bank is releasing when the monetary policy is decided.

FIGURE 1.3: Impulse-responses to an identified monetary policy shock.



Notes: This figure presents the responses of macroeconomic variables to an identified monetary policy shock for the variables in 1.5. We depict three alternative estimates: a Local Projection (short orange dashes), a Bayesian VAR (long light blue dashes), and a Bayesian Local Projection (solid blue) as proposed by [Miranda-Agrippino and Ricco \(2021\)](#). We report confidence bands at the 90% significance for both BVAR (light gray) and BLP (dark gray).

Treasury Bond.

The responses of the variables are similar for all the methods, except for non-durable consumption. Notice that the BLP helps to smooth out the responses of the Local Projection. Even though, in levels BLP is not an average of BVAR and LP, the volatility of the IRF's are in between these two methods. Finally, and more importantly, the BLP estimation is successful in improving the efficiency of the estimators, by obtaining more precise estimates with respect to both LP and BVAR. (We do not show LP confidence bands as they go off the charts.)

From the responses of the variables we see neither product nor price puzzles. After a contractionary monetary policy shock, the

industrial production index falls by about 1.5 percentage points and the CPI falls with a trough in the second quarter. Unemployment is also affected, increasing by about 0.3 percentage points at the peak response.

We find that the earnings gap increases after the contractionary monetary policy shock. The Earnings Gap increases by about 5% on impact. This effect is not very persistent but it converges to 1% after one quarter and fades away twelve months after the shock. Additionally, we want to study whether the monetary policy shock is contractionary on the demand side, as is it going to be the main channel of transmission of the earnings gap to the economy. As is common in this literature, we find that consumption drops but with a lag. Consumption falls significantly six months after the shock and this coincides with the peak unemployment rate. This implies that the shock works as a contractionary demand shock.

These results suggest that monetary policy shocks generate labor income inequality between skilled and unskilled workers, and that the earnings gap is countercyclical after an identified monetary policy shock. In the next section, we study a reason why the earnings gap is countercyclical, and then we show, with the help of a model, why these inequalities matter for the business cycle.

1.3.3 *The Wage Phillips Curve: Steeper for Skilled Workers*

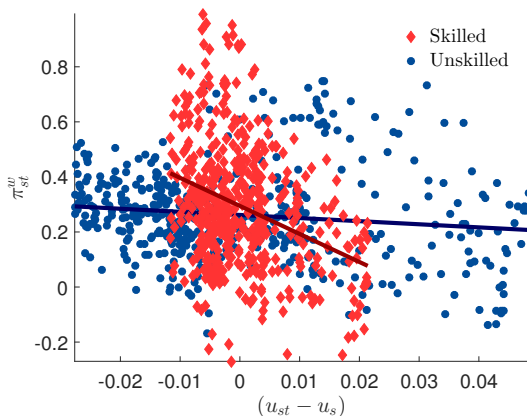
Now, we are interested in explaining why the earnings gap is countercyclical. One possible reason is that the slopes of the wage Phillips curves of the different groups of workers are different in the data. Let us define the wage Phillips curve of workers' group h as

$$\pi_{wt}^h = -\kappa_h(u_t^h - u^s) + \beta\mathbb{E}_t\pi_{wt+1}^h \quad (1.6)$$

where $\kappa_h \geq 0$, π_{wt}^h is wage inflation of group h , u_t^h is group h unemployment (with u^s being the natural unemployment rate), and β is a

time discount factor. Equation (1.6) is the usual negative relation between wage inflation and unemployment as first proposed by Phillips (1958). This version, which is forward looking, is the one introduced by Galí (2011) who extends the approach by Erceg et al. (2000). This equation can be derived from microfoundations (monopolistic competition in the labor market and nominal wage rigidities) as we explain in the next section. Then, the focus of the following exercise is on finding differences in the κ_h 's for the different groups of workers.

FIGURE 1.4: Wage inflation and unemployment by skill level.



Notes: This figure shows the relation between wage inflation π_{st}^w and unemployment $u_{st} - u_s$ for skilled and unskilled workers. We also show the linear fit for both groups of workers. The slopes of the fitted lines are -0.1 and -0.01 for skilled and unskilled, respectively. Both estimates are significant at the 95% level.

Figure 1.4 shows the scatter for unemployment and wage inflation for both groups. The first to note is the significant difference on the intervals spanned by both groups. While the skilled workers have a more volatile wage inflation, the unskilled workers have a more volatile unemployment rate. Additionally, the relationship between wage inflation and unemployment differs significantly. The slope for skilled workers is about -0.1 while it is -0.01 for unskilled workers.

However, the evidence presented in Figure 1.4 is not conclusive as the OLS estimates of these wage Phillips curves have several shortcomings. First of all, the OLS estimation of Equation (1.6) does not constitute a structural relation, as it is an ad-hoc relationship. Second, for that reason, the OLS estimate has endogeneity bias as we are omitting variables that will likely correlate with the residual, like the natural unemployment rate. For these reasons, we must switch towards a more structural relationship, which we will obtain by estimating the slopes by taking advantage of the exogenous aggregate demand shock we presented above.

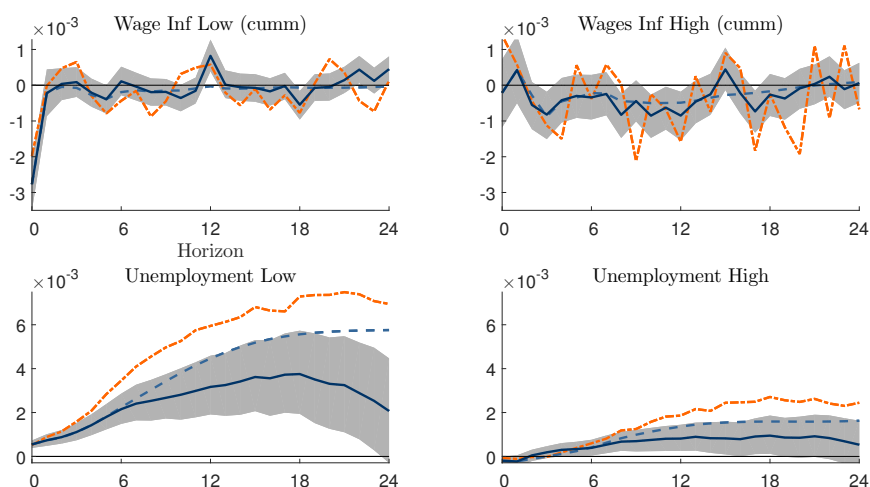
To estimate a proxy for κ_h in a semi-structural way, we augment the IV-BLP estimation of the previous section with the series of unemployment and wage inflation rates for unskilled and skilled workers, but we exclude the earnings gap.¹⁴ Then, to infer the slope of the wage Phillips curves, we compare the responses of wage inflation and unemployment with the identified demand shock in the spirit of Barnichon and Mesters (2020) and Barnichon and Mesters (2021). To do so, we follow Galí and Gambetti (2018) and Del Negro et al. (2020) who study the slopes of the Phillips curve by analyzing the relative response of wage inflation and unemployment to an identified demand shock.¹⁵ Intuitively, the procedure takes advantage of the exogeneity of the shock, which in our case, as it is a monetary policy shock, represents a demand shifter. Therefore, the resulting relative response of wage inflation and unemployment is how the demand shifts along the wage Phillips curve, which gives us a proxy of its slope (the κ_h 's).

Figure (1.5) shows the responses of wage inflation and unemployment for the two skill groups in response to an identified contrac-

¹⁴We exclude the Earnings Gap because the unemployment rate and wages are collinear with the earnings gap.

¹⁵See additionally, Gilchrist and Zakrajsek (2018) who study the slope of the Phillips curve at a sectoral level.

FIGURE 1.5: Response of wage inflation and unemployment to a monetary policy shock at the skill level.



Notes: This figure presents the responses of labor market variables at the skill level to an identified monetary policy shock where we augmented the VAR on 1.5 with wage inflation and unemployment instead of including the Earnings Gap. We depict three alternative estimates, a Local Projection (short orange dashes), a Bayesian VAR (long light blue dashes), and a Bayesian Local Projection (solid blue). We report confidence bands at the 90% significance for both BVAR (light gray) and BLP (dark gray).

tionary monetary policy shock. The response of wage inflation differs for both groups. For unskilled workers wage inflation responds negatively on impact, with the effects disappearing almost immediately. Wage inflation for skilled workers takes about three months to respond, with the effect remaining negative up to 12 periods. On the other hand, unemployment for unskilled workers goes up immediately and is much higher than for skilled workers. The peak of unemployment for unskilled workers is about four times that of the skilled workers. With these impulse-responses, we may conclude that the relative response of wage inflation to unemployment is higher for skilled workers; and hence, they have a steeper wage Phillips curve.

To study to what extent the underlying wage Phillips curves are different, we compute the Dynamic Multipliers, as defined in Galí and Gambetti (2018)

$$\Phi_w(h) = \frac{\sum_{k=0}^h \frac{\partial \pi_{w,t+k}}{\partial \varepsilon_t}}{\sum_{k=0}^h \partial \frac{u_{t+k}}{\partial \varepsilon_t}},$$

which is the relative response of wage inflation to the response of unemployment. The interpretation of this statistic is that if unemployment responds minimally and wage inflation responds substantially, the wage Phillips Curve is steep. Table (1.3) shows the dynamic multipliers at different horizons for the BLP and LP estimates.

Table 1.3: Dynamic Multipliers

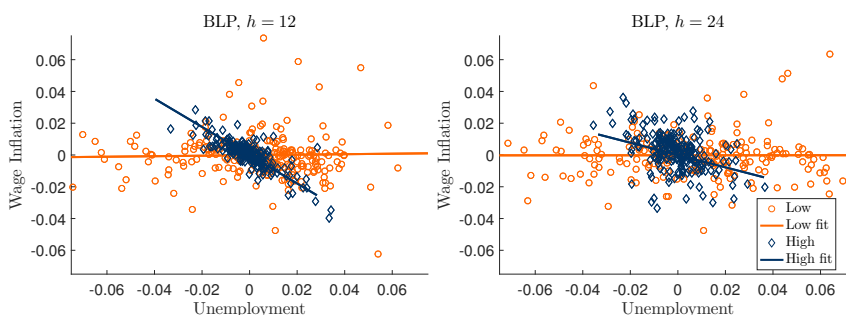
Horizon	BLP			
	Full	Unskilled	Skilled	S/U
6	-0.261	-0.534	-4.21	7.9
12	-0.156	-0.195	-1.21	6.23
18	-0.0972	-0.0884	-0.779	8.82
24	-0.0673	-0.0604	-0.543	8.99
LP				
6	-0.108	-0.217	-1.6	7.38
12	-0.0703	-0.0827	-0.442	5.34
18	-0.0706	-0.05	-0.278	5.57
24	-0.0296	-0.0314	-0.219	6.99

Notes: This table presents the empirical Dynamic multipliers estimated from augmenting 1.5 with unemployment and wage inflation at the skill level. The table reports the Dynamic Multipliers from the BLP and LP estimations at six, twelve, 18, and 24 month horizon. We also report the dynamic multiplier from aggregate wage inflation and unemployment and the ratio between the dynamic multipliers of skilled and unskilled.

As Table (1.3) shows, the dynamic multiplier for skilled workers is much larger than for unskilled. That result means that for the same response of unemployment, the reaction of wages is larger for

skilled workers than for unskilled. The differences are large, with the dynamic multiplier being about nine times larger for skilled than for unskilled.¹⁶ The differences in the dynamic multipliers are significant at the 68% level for the 12- and 18-month horizon (see Appendix (1.B) for details on the test).

FIGURE 1.6: Projected wage inflation and unemployment.



Notes: This figure presents the projections of wage inflation and unemployment rate conditional only on the identified monetary policy shock. The left-hand side depicts the projections up to 12-month and the right-hand panel up to 24-month. In both plots, the diamond blue are the projections for skilled workers and the circle orange for unskilled workers.

Finally, in Figure (1.6) we show a scatter of the projections implied by the BLP. We build the projections by using the estimated parameters to calculate projected series of unemployment and wage inflation rates for skilled and unskilled workers, using the realized shocks in the data. Hence, only we show the series that are conditional on the monetary policy shock. The rounded orange points are the projections for unskilled while the diamond blue are for skilled.¹⁷ The figure deserves some comments. First, conditional on the monetary policy

¹⁶These results are in line with the results stressed by [Doniger \(2019\)](#) in which the wages of unskilled are more rigid than those of skilled.

¹⁷These series are different from the sum of the impulse responses since the projection scales down the responses of the variables in accordance with the series of realized shocks.

shock, unskilled wages are substantially rigid; both for twelve and 24 month horizon, the implied wage Phillips curve by these estimations are flat. Second, and in line with the previous findings, the projected wage Phillips curve for skilled workers is noticeably steeper than for unskilled workers.

Discussion. The results above point towards significant differences between the slopes of the skilled and unskilled wage Phillips curves implied by our BLP analysis. Before continuing the study of the impact of these features of the labor markets on the overall economy, we must emphasize that, unlike [Barnichon and Mesters \(2021\)](#), we abstract from the endogeneity and bias that arises from estimating Phillips curves without taking into account expected inflation. As we are interested in studying the differences in the slope of the Phillips curves, we can abstract from the expectational term in Equation (1.6). Thus, if we assume that the discount factors are the same for both groups of workers, we can correctly estimate the differences in κ_h .¹⁸

To conclude this section. We find that the dynamics of the labor markets differ substantially between skill levels. These different dynamics have implications for the distribution of income in the cycle, where the burden of recessions is on unskilled workers, and a likely source of this result is the relatively stickier wages these workers face.

1.4 MODEL

Our model is a Two-Agent New Keynesian (TANK) model with wage rigidities as [Furlanetto and Seneca \(2012\)](#), following [Bilbiie \(2008\)](#). We

¹⁸We would like to estimate these equations by including expected wage inflation for each group but, unfortunately, we do not have access to expected wages at the individual level. A promising source is the *Survey of Consumer Expectations* released by the New York Fed, which contains these kinds of data; however, the series is still short.

extend these works by assuming there are two labor markets (for unskilled u and skilled s), in which wages are set by a union that is also group-specific and is subject to nominal wage adjustment costs. A measure one households populates each skill group (which we index with h). In each group, there is a share λ_h of financially constrained agents, that can not save, borrow, or hold equity. There are two types of firms, a continuum of monopolistically competitive intermediate goods producers and a final goods producer that aggregates these intermediate goods through a CES production function. These intermediate firms demand all types of labor. We embed these features into a New Keynesian model with price rigidities and monetary policy.

1.4.1 Households

We assume there are two groups of workers, skilled and unskilled. Each household belongs to a given group $h \in \{u, s\}$ with μ denoting the mass of the group of unskilled workers. We assume that a share λ_h of households in skill group h have no access to financial markets (cannot borrow or lend and cannot own shares), while the remaining $(1 - \lambda_h)$ are unconstrained. We call the former group *constrained* and the latter *unconstrained*. We index with i the access to financial markets; i.e., $i \in \{k, r\}$, with r denoting unconstrained (r for “Ricardian”) and k denoting constrained (k for “Keynesian”). Hence, household features are given by a pair of indices (i, h) .

Households derive utility from consumption and disutility from labor. We assume there is a continuum of $j \in (0, 1)$ tasks each household in (i, h) can execute. Hence, household (i, h) maximizes its lifetime utility, time-discounted at a factor $0 < \beta < 1$, given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mathcal{U} \left(c_{ht}^i, \{n_{ht}^{ij}\}_{j=0}^1 \right), \quad (1.7)$$

where c_{ht}^i is consumption and n_{ht}^{ij} is hours supplied by workers from household (i, h) to the task j . In particular, following Galí (2011), we

assume a separable utility function of the form

$$U(c_{ht}^i \{n_{ht}^{ij}\}_{j=0}^1) = \frac{(c_{ht}^i)^{1-\gamma}}{1-\gamma} - \chi \frac{\int_0^1 (n_{ht}^{ij})^{1+\varphi} dj}{1+\varphi},$$

where γ is the inverse of the intertemporal elasticity of substitution, χ is the parameter of disutility of labor, and φ is the inverse of the Frisch elasticity of the labor supply.

The Problem of Unconstrained Households. Unconstrained consumers can accumulate risk-free bonds and their borrowing constraint is given by

$$q_t b_{ht+1}^r = b_{ht}^r + \int_0^1 w_{ht}^{rj} n_{ht}^{rj} dj + D_{ht}^r - c_{ht}^r, \quad (1.8)$$

where $w_{ht}^{rj} = W_{ht}^{rj}/P_t$ is the real wage per unit of labor, n_{ht}^{rj} , where due to labor market frictions n_{ht}^{rj} is taken as given by the household as we explain below; $q_t = Q_t/P_t$ is the price of real bonds (which in equilibrium is $q_t = 1/(1+r_t)$ with r_t the real return on bonds); and D_{ht}^r are dividends delivered by firms. Hence, these workers maximize function (1.7) subject to constraint (1.8). The maximization problem of these households gives as a result the following Euler equation

$$1 = \beta(1+r_t)E_t \left(\frac{c_{ht}^r}{c_{ht+1}^r} \right)^{-\gamma}. \quad (1.9)$$

The Problem of Constrained Households. Constrained households consume their flow of income every period. Hence, constrained consumption is given by

$$c_{ht}^k = \int_0^1 w_{ht}^{kj} n_{ht}^{kj} dj, \quad (1.10)$$

where, as they are outside of the financial system, they receive only labor income.

The difference between constrained and unconstrained consumers is critical in our model because it implies different MPCs out of total income among households. From the permanent income hypothesis, we know that the MPC of unconstrained consumers is approximately $r/(1+r)$, while that of the constrained worker is equal to one, as Equation (2.10) shows. Those differences generate departing consumption dynamics between groups of workers as long as the shares of hand-to-mouth λ_h 's are distinct and labor income fluctuates differently. The group with higher λ_h has a higher average MPC; hence, their consumption responds much more to income shocks than the other groups. We exploit a similar argument as Bilbiie (2020) in which is the income cyclicality of the high-MPC consumer is what matters for the effects of inequality over the business cycle.

1.4.2 *Distribution of Monopoly Profits*

In New Keynesian models, monopoly profits are an essential source of fluctuations. As we assume monopolistic competition in intermediate markets, firms charge a markup over marginal costs. With sticky prices, this markup fluctuates. As there are differences in access to financial markets, fluctuations in markups have distributional consequences we must take into account. A widely known result is that markups (both wage and price) are countercyclical in this class of models in response to demand shocks. The implication of this is that in a boom, markups fall, so labor income gets a higher proportion of total income. This effect typically generates amplification effects from limited asset participation. That is why the distribution of monopoly profits matters. However, we can design a profit distribution rule which delivers the same amount of profits to every agent to eliminate this amplifying effect. This assumption is not realistic since poor (or unskilled) workers own a small proportion of total aggregate shares.

Therefore, to avoid “spurious” redistribution from aggregate vari-

ables from wealthier to poorer agents, we assume the distribution of profits is according to the data. In particular, we set the distribution of profits to unconstrained consumers at each workers' group to be equal to a share of total profits in the economy. This share is denoted by ϑ_h . We calibrate ϑ_h according to the *Survey of Consumer Finances 2016*. That survey shows that skilled workers own about 83% of the equity in the U.S. economy. Accordingly, we assume the dividends that delivered skilled and unskilled unconstrained are given by

$$D_t^u = \frac{\vartheta_u}{\mu(1 - \lambda_u)} D_t, \text{ and } D_t^s = \frac{\vartheta_s}{(1 - \mu)(1 - \lambda_s)} D_t. \quad (1.11)$$

1.4.3 Workers' Unions

Following [Erceg et al. \(2000\)](#), we assume that for each task-group (j, h) , there is a union that decides wages w_{ht}^j . In this setting, unions have market power as workers' tasks are in monopolistic competition. The union aggregates individual labor such that $n_{ht}^j = \lambda_h n_{ht}^{kj} + (1 - \lambda_h) n_{ht}^{rj}$. Then, we assume there is a Dixit-Stiglitz aggregator that determines aggregate labor for each labor group h , given by

$$N_{ht} = \left(\int_0^1 \left(n_{ht}^j \right)^{\frac{\varepsilon_h - 1}{\varepsilon_h}} dj \right)^{\frac{\varepsilon_h}{\varepsilon_h - 1}},$$

where ε_h is the elasticity of the demand for labor tasks in workers' group h , which is also a measure of the market power of the union. The Dixit-Stiglitz aggregator gives rise to the following demand for each task (j, h) :

$$n_{ht}^j = \left(\frac{w_{ht}^j}{w_{ht}} \right)^{-\varepsilon_h} N_{ht}. \quad (1.12)$$

We assume nominal wages are sticky and their changes are subject to the following Rotemberg adjustment costs that are measured in utility units:

$$\Gamma^h \left(\frac{W_{ht}^j}{W_{ht-1}^j} - 1 \right) = \frac{\theta_h}{2} \left(\frac{W_{ht}^j}{W_{ht-1}^j} - 1 \right)^2, \quad (1.13)$$

where θ_h is the nominal wage adjustment cost parameter, assumed to be skill-group specific. Then, the problem of the union is to choose the optimal labor, the nominal wage and the wage inflation rate by solving

$$\max_{n_{ht}^{ij}, W_{ht}^j, \pi_{wt}^{jh}} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\lambda_h U(c_{ht}^k) + (1 - \lambda_h) U(c_{ht}^r) - v(n_{ht}^{ij}) - \Gamma^s \left(\frac{W_{ht}^j}{W_{ht-1}^j} - 1 \right) \right], \quad (1.14)$$

subject to Equation (1.12), and given that wage inflation is defined as $\pi_{wt}^{jh} = \frac{W_{ht}^j - W_{ht-1}^j}{W_{ht-1}^j}$. We denote with $U(c_{ht}^i) = \frac{(c_{ht}^i)^{1-\gamma}}{1-\gamma}$ and $v(n_{ht}^j) = \chi \frac{(n_{ht}^j)^{1+\varphi}}{1+\varphi}$. This maximization problem leads to¹⁹:

$$\begin{aligned} (\pi_{wt}^h + 1) \pi_{wt}^h &= \frac{\varepsilon_h}{\theta_h} N_{ht} \left\{ v'(N_{ht}) - \frac{\varepsilon_h - 1}{\varepsilon_h} \overline{mgu}_{ht} w_{ht} \right\} \\ &+ \beta E_t \left(\pi_{wt+1}^h + 1 \right) \pi_{wt+1}^h \end{aligned} \quad (1.15)$$

where $\overline{mgu}_{ht} = \lambda_h U'(c_{ht}^k) + (1 - \lambda_h) U'(c_{ht}^r)$ is the average marginal utility of consumption of group h . Equation (1.15) is the New Keynesian Wage Phillips Curve (NKWPC) for group h .

Equation (1.15) relates the nominal wage inflation with hours worked and the aggregate group h worker's preferences; it is a version of the wage Phillips curve described by Erceg et al. (2000) adapted to heterogeneity and Rotemberg adjustment costs. Due to these labor market frictions, all workers in skill group h supply N_{ht} hours at a real wage w_{ht} . This equation allows us to calibrate the model to generate different dynamics for the two labor markets.

¹⁹See Appendix 1.C for a detailed derivation.

We can rewrite the NKWPC in a way that will be useful in the analysis below as

$$\left(\pi_{wt}^h + 1\right) \pi_{wt}^h = \kappa_{ht} \left(\frac{1}{\mathcal{M}_{wt}^h} - \frac{1}{\mathcal{M}_w^h} \right) + \beta E_t \left(\pi_{wt+1}^h + 1 \right) \pi_{wt+1}^h. \quad (1.16)$$

where $\kappa_{ht} = \frac{\varepsilon_h}{\theta_h} (w_{ht} N_{ht} \overline{mgu}_{ht}) = \frac{\varepsilon_h}{\theta_h} \aleph_{ht}$ which depends on three terms: i) the elasticity of substitution between tasks ε_h within the group h ; ii) the wage adjustment cost parameter θ_h ; and iii) \aleph_{ht} , which we call the *dynamic income effect*. The latter term, for our calibration, has little impact on the aggregate outcomes.

The wage markup \mathcal{M}_{wt}^h is the ratio of the marginal rate of substitution to the real wage

$$\frac{1}{\mathcal{M}_{wt}^h} = \frac{v'(N_{ht})}{w_{ht} \overline{mgu}_{ht}}, \quad (1.17)$$

where the wage markup is equal to $\frac{\varepsilon_h}{\varepsilon_h - 1}$ in the steady state (also called the *desired* markup). Notice that the more rigid wages are, the stronger the fluctuations on the wage markups, as can be seen in Equation (1.16). That implies that for two groups h and h' , if $\kappa_{h't} > \kappa_{ht}$ wage markups of group h are more volatile than those of group h' .

Finally, as Galí (2011) shows, Equation (1.16) can be written as the relation between wage inflation and unemployment. Let us define unemployment as the deviation of hours worked N_{ht} and labor market participation L_{ht} . We define labor market participation as the hours that the worker is willing to provide at the current labor market conditions (at the prevailing wage), in the absence of labor market frictions. Labor market participation is, then, determined by²⁰

$$w_{ht} = \frac{v'(L_{ht})}{\overline{mgu}_{ht}}. \quad (1.18)$$

²⁰See Galí (2015) Ch. 7 for a detailed explanation.

By combining Equation (1.18) with Equation (1.17) and using $v'(N) = \chi N^\varphi$, we find a mapping between markups and unemployment, with the latter defined as $U_{ht} = \frac{L_{ht}}{N_{ht}}$

$$\mathcal{M}_{wt}^h = \left(\frac{L_{ht}}{N_{ht}} \right)^\varphi = U_{ht}^\varphi.$$

Then, Equation (1.16) can be written as

$$\left(\pi_{wt}^h + 1 \right) \pi_{wt}^h = \kappa_{ht} \left(\frac{1}{U_t^{h\varphi}} - \frac{1}{U^\varphi} \right) + \beta E_t \left(\pi_{wt+1}^h + 1 \right) \pi_{wt+1}^h, \quad (1.19)$$

which writes the NKWPC as the relation between wage inflation and unemployment like Equation (1.6).

1.4.4 Firms

Final Goods Producers. A competitive representative firm produces a final good by aggregating a continuum of intermediate inputs with the following production function

$$Y_t = \left(\int_0^1 y_{ft}^{\frac{\varepsilon-1}{\varepsilon}} df \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

This composite is an aggregate of a continuum of intermediate goods with measure one. In this setting, the final firm decides how to allocate its demand among the different intermediate goods. After cost minimization, the demand for each intermediate good f , and the aggregate price index writes

$$y_{ft} = \left(\frac{p_{ft}}{P_t} \right)^{-\varepsilon} Y_t, \quad \text{and} \quad P_t = \left(\int_0^1 p_{ft}^{1-\varepsilon} df \right)^{\frac{1}{1-\varepsilon}}. \quad (1.20)$$

Intermediate Goods Producers: Labor Demand. Each intermediate good f is produced by a monopolistically competitive producer

using labor n_{fst} of both skill groups according to the production function

$$y_{ft} = \left[\omega n_{fut}^{\frac{\sigma-1}{\sigma}} + (1-\omega)n_{fst}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}},$$

which is a CES aggregator of the skill groups. Skill groups are imperfect substitutes of each other. The elasticity of substitution between skill groups is given by σ . If $\sigma > 1$ skill groups are gross substitutes while if $\sigma < 1$ they are gross complements. As explained before, the value of ω represents the labor income share of unskilled workers. Each intermediate producer hires workers from each skill group h at a real wage w_{ht} . Therefore, the demand for each class h is

$$w_{ht} = \frac{1}{\mathcal{M}_t^p} \frac{\omega_h}{\mu_h} \left(\frac{Y_t}{N_{ht}} \right)^{\frac{1}{\sigma}},$$

which corresponds to the real wage in per-capita terms. Then, N_{ht} is the class h aggregate hours worked.²¹ This way of expressing the problem of the firm, and then obtaining a per-capita wage is useful for two reasons. First, it allows us to close the model properly; and second, it allows us to split the income share received by each type of worker (given by ω_h) with the size of the group (given by μ_h) which may be different. These two parameters allow us to calibrate the Earnings Gap in steady state as well.

The index of aggregate wages is

$$w_t = \left[\omega_u w_{ut}^{1-\sigma} + \omega_s w_{st}^{1-\sigma} \right]^{\frac{1}{1-\sigma}}.$$

Due to intermediate firms' market power, there are profits in this economy. These profits are determined by a wedge $\frac{1}{\mathcal{M}_t^p}$ which is the

²¹These optimality conditions arise from minimizing:

$$\max_{n_{fut}, n_{fst}} \mu_u w_{ut} n_{fut} + \mu_h w_{ht} n_{fst} - \frac{1}{\mathcal{M}_t^p} \left(\left[\omega n_{fut}^{\frac{\sigma-1}{\sigma}} + (1-\omega)n_{fst}^{\frac{\sigma-1}{\sigma}} \right]^{\frac{\sigma}{\sigma-1}} - y_{ft} \right),$$

where $\frac{1}{\mathcal{M}_t^p}$ corresponds to the real marginal cost in equilibrium, which is equivalent to the Lagrange multiplier of the cost minimization problem.

total marginal cost consistent with equilibrium, or analogously, the inverse of the firms' price mark-up \mathcal{M}_t^p .

Intermediate Goods Producers: Price Setting. The intermediate producer chooses its price to maximize profits subject to [Rotemberg \(1982\)](#) price adjustment costs, denoted by Θ_t . These costs are quadratic on inflation and expressed as a function of produced output Y_t . This is $\Theta_t \left(\frac{p_t}{p_{t-1}} - 1 \right) = \frac{\theta}{2} \left(\frac{p_t}{p_{t-1}} - 1 \right)^2 Y_t$, where θ is the parameter that drives the degree of price rigidity.

Therefore, each intermediate producer chooses $\{p_t\}_{t \geq 0}$ to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{c_{t+1}^r}{c_t^r} \right)^\sigma \left\{ \Pi_t(p_t) - \Theta_t \left(\frac{p_t}{p_{t-1}} - 1 \right) \right\}$$

$$\text{with } \Pi_t(p_t) = \left(\frac{p_t}{P_t} - \frac{1}{\mathcal{M}_t^p} \right) \left(\frac{p_t}{P_t} \right)^{-\varepsilon} Y_t,$$

where $\beta \left(\frac{c_{t+1}^r}{c_t^r} \right)^\sigma$ is the stochastic discount factor that corresponds to the pool of unconstrained agents. Given the assumptions above, the inflation rate (after the intermediate firms optimization) is determined by the following New Keynesian Phillips curve:

$$(\pi_t + 1)\pi_t = \frac{\varepsilon}{\theta} \left(\frac{1}{\mathcal{M}_t^p} - \frac{1}{\mathcal{M}^p} \right) + \beta \left(\frac{c_{t+1}^r}{c_t^r} \right)^\sigma (\pi_{t+1} + 1)\pi_{t+1}.$$

Finally, intermediate firms generate an aggregate amount of profits in each period given by

$$D_t = \left(1 - \frac{1}{\mathcal{M}_t^p} \right) Y_t - \frac{\theta}{2} \pi_t^2 Y_t.$$

1.4.5 Monetary Authority

In the presence of nominal rigidities, the real interest rate r_t is determined by monetary policy, which sets the nominal interest rate i_t

according to a Taylor rule

$$i_t = i^* + \phi_\pi \pi_t + \varepsilon_t^{mp}.$$

where ϕ_π is the preference parameter for inflation. ε_t^{mp} is a monetary policy shock that follows an AR(1) process given by:

$$\log(\varepsilon_t^{mp}) = \rho_{mp} \varepsilon_{t-1}^{mp} + u_t^{mp}$$

Monetary authorities seek a nominal interest rate target in steady state given by i^* (where $i^* = r + \bar{\pi}$). Given the inflation level and the nominal interest rate, the real rate is determined by the Fisher equation $r_t = i_t - E_t \pi_{t+1}$.

1.4.6 Equilibrium

An equilibrium in this economy is given by paths of individual variables for households' decisions $\{b_{ht}^i, c_{ht}^i\}_{t \geq 0} \forall (i, h)$; labor market prices and quantities $\{\{N_{ht}, w_{ht}, \pi_{wt}^h\}_{h=1}^S\}_{t \geq 0}$; prices and returns $\{\pi_t, r_t, i_t\}_{t \geq 0}$, and aggregate quantities such that: (i) households maximize their objective functions taking as given both prices and aggregate quantities; and (ii) all markets clear. In our economy, we have four markets: the goods market, the market for bonds, and two labor markets.

Consumption of a given group h is given by:

$$c_{ht} = \lambda_h c_{ht}^k + (1 - \lambda_h) c_{ht}^r,$$

Hence, aggregate consumption writes

$$C_t = \mu c_{ut} + (1 - \mu) c_{st}.$$

Finally, goods market clearing holds

$$Y_t = C_t + \Theta_t.$$

where Θ_t are the price adjustment costs.

1.5 ANALYTICAL RESULTS

In this section, we obtain two analytical results that guide us in understanding the role of labor income inequality in the business cycle. First, we show how the earnings gap affects the business cycle through an aggregate demand effect. In particular, we study how, due to market incompleteness, the earnings gap influences consumption behavior represented by the aggregate Euler equation. We illustrate that if the earnings gap is countercyclical, monetary policy shocks are amplified through this aggregate demand channel. Second, we show that in the model presented above, the only reason the earnings gap fluctuates is the difference in labor markets between the skill groups. The earnings gap is countercyclical if worker groups are gross substitutes, and wages of the unskilled workers are relatively more sticky than those of the skilled workers.

To study the effect of the earnings gap in a simple way, we make the following assumptions (which we relax in the full model later): (i) the share of hand-to-mouth workers in the unskilled group is equal to one and the share of hand-to-mouth in the skilled group is zero; and (ii), there are no price rigidities nor market power on intermediate goods. This latter assumption allows us to isolate labor income as the only source of inequality since there are no profits to distribute unequally in that setup, while we maintain the aggregate demand activated with the wage rigidities.

1.5.1 *Aggregate Demand and the Earnings Gap*

We first solve for the IS equation in this economy. As [Debortoli and Galí \(2018\)](#) show, when there is limited access to financial markets, the IS equation (or the aggregate demand) depends on the inequality wedges. Recall $C_t = \mu c_{ut} + (1 - \mu) c_{st}$, where each group's consumption is given by $c_{ht} = \lambda_h c_{ht}^k + (1 - \lambda_h) c_{ht}^r$. Then, as under assumption i), $\lambda_u = 1$ and $\lambda_s = 0$, consumption of unskilled workers is $c_{ut} = c_{ut}^k$ and

that of skilled workers is $c_{st} = c_{st}^r$. Hence, aggregate consumption writes $C_t = \mu c_{ut}^k + (1 - \mu) C_{st}^r$. Notice that in this example the aggregate share of hand-to-mouth is given by μ .

Next we introduce the *consumption gap*, which is the percentage difference between the skilled and unskilled workers' consumption, as $\nu_t = 1 - \frac{c_{ut}}{c_{st}}$. According to our simplifying assumptions, unskilled and skilled workers' consumption is given by their labor income (as there is no other source of income); this is, $c_{ut} = w_{ut} N_{ut}$ for unskilled workers and $c_{st} = w_{st} N_{st}$ for skilled workers. Therefore, the consumption gap is given by

$$\nu_t = 1 - \frac{w_{ut} N_{ut}}{w_{st} N_{st}} = 1 - \frac{1}{\eta_t}. \quad (1.21)$$

Equation (1.21) shows that in this setup, the consumption gap depends only on the Earnings Gap, η_t . Then, we obtain an expression for the aggregate demand in this economy. Recall that the only agent who can save or borrow is the skilled worker. Hence, there is only one Euler equation, given by

$$\widehat{c}_{st} = E_t\{\widehat{c}_{st+1}\} - \frac{1}{\gamma} (r_t - \rho),$$

which is the loglinear approximation of Equation (1.9). Rewriting aggregate consumption as $C_t = c_{st}(1 - \mu_l \nu_t)$, it can be written, in log differences with respect to the steady state, as $\widehat{c}_t = \widehat{c}_{st} + \widehat{h}_t$, with $\widehat{h}_t = -\frac{\mu_u}{1 - \nu \mu_u} \widehat{\nu}_t$ being an inequality index. Thus, the aggregate Euler equation is given by

$$\widehat{c}_t - \widehat{h}_t = E_t\{\widehat{c}_{t+1} - \widehat{h}_{t+1}\} - \frac{1}{\gamma} (r_t - \rho). \quad (1.22)$$

Replacing the consumption gap in the inequality index \widehat{h}_t ($\widehat{\nu}_t = \widehat{\eta}_t$) we have

$$\widehat{h}_t = -\frac{\mu_u}{1 - \nu \mu_u} \widehat{\eta}_t.$$

Finally, we substitute the inequality index \widehat{h}_t in equation (1.22), and assuming goods market clearing ($\widehat{y}_t = \widehat{c}_t$), the IS equation becomes

$$\widehat{y}_t = E_t\{\widehat{y}_{t+1}\} - \frac{1}{\gamma}(r_t - \rho) + \frac{\mu_l}{1 - \nu\mu_l} E_t\{\Delta\widehat{\eta}_{t+1}\}. \quad (1.23)$$

Equation (1.23) is the dynamic IS equation of an economy with incomplete markets and inequality in labor markets. As we mentioned before, as a consequence of incomplete markets, the Euler equation depends on any form of inequality between the constrained and the unconstrained consumers. In this case, output depends on $\widehat{\eta}_t$ and on how it fluctuates over the business cycle. This relation appears because, as inequality switches, the economy distributes resources between agents. If inequality falls ($\widehat{\eta}_t$ goes down), the economy relatively distributes resources from skilled (and unconstrained) to unskilled (and constrained) agents or from low- to high-MPC agents.

Hence, solving Equation (1.23) forward,

$$\widehat{y}_t = -\frac{1}{\gamma} \mathbb{E}_t \sum_{s=0}^{\infty} \widehat{r}_{t+s} - \frac{\mu_l}{1 - \nu\mu_l} \widehat{\eta}_t. \quad (1.24)$$

Equation (1.24) is the expression for the output gap in our economy. As is common in New Keynesian models, the output gap depends on the path of future interest rates (or its deviations from its steady state level ρ). Additionally, in our model, the output gap depends on the contemporaneous deviation of the earnings gap. Whether fluctuations in the earnings gap are amplifying or not depends on the earnings gap's cyclicity. If inequality is countercyclical; i.e., if inequality falls in booms ($\widehat{\eta}_t < 0$ as $\widehat{y}_t > 0$), incomplete markets amplify monetary policy shocks more strongly. Whereas, if η_t is procyclical, labor income inequality stabilizes output fluctuations. Therefore, it is not only inequality in financial access that has an amplifying effect on the economy, but also the unequal fluctuations in labor earnings.

The amplification effect arises from the fact that in a recession, if the earnings gap goes up, the workers with higher MPC suffer a larger drop in their labor earnings. That implies that aggregate consumption responds more strongly.

1.5.2 *The Cyclicalilty of the Earnings Gap*

In this section, we show that the countercyclicalilty of the earnings gap holds for reasonable assumptions consistent with the empirical evidence presented above. To do so, we solve the labor market equilibrium with the assumptions we imposed at the beginning of this section.

To get a closed form expression for the earnings gap we first solve, for a generic group h , its labor income $w_{ht}N_{ht}$ by equalizing labor supply and demand

$$\mathcal{M}_{ht}N_{ht}^\varphi C_{ht}^\gamma = w_{ht} = \frac{\omega_h}{\mu_h} \left(\frac{Y_t}{N_{ht}} \right)^{\frac{1}{\sigma}},$$

which implies that labor income is given by

$$w_{ht}N_{ht} = \mathcal{M}_{ht}^{\frac{1-\sigma}{1+\varphi\sigma}} C_{ht}^{\gamma \frac{1-\sigma}{1+\varphi\sigma}} Y_t^{\frac{1}{\sigma} \frac{(\sigma-1+1+\sigma\varphi)}{1+\varphi\sigma}} \left(\frac{\omega_h}{\mu_h} \right)^{\frac{(\sigma-1+1+\sigma\varphi)}{1+\varphi\sigma}}. \quad (1.25)$$

Equation (1.25) is the labor income of workers in group h . The first point to notice is that labor income fluctuates with output. That relation depends on the elasticity of substitution between skill groups. With perfect substitutability ($\sigma = 1$), the labor income share $\left(\frac{w_{ht}N_{ht}}{Y_t} \right)$ is equal to $\frac{\omega_h}{\mu_h}$ in our specification, hence the labor income ratio is constant for every group. When that happens, all groups get a fixed labor income share, and hence, there is no effect of labor heterogeneity on the aggregate demand as labor income inequality does not fluctuate. That is the case, for instance, of Cobb-Douglas technology.

Without perfect substitutability, labor income depends on: consumption (through the income effect); the share of income earned

per-capita (ω_h/μ_h); the group's wage markup; and aggregate output. As we are interested in the impact of wage rigidities, let us study the relationship between the labor income of group h and its wage markup. This relationship depends crucially on the value of σ , the elasticity of substitution between skills in production. With gross substitutability ($\sigma > 1$), the relationship between the wage markup and labor income is negative. The intuition is the following: as markups of a group h go up, the labor supply schedule shifts upward, generating higher wages for a given level of hours supplied. This positive effect on labor income is counteracted by the impact on hours demanded by firms. When the markup of a group h goes up, its labor gets more expensive, thereby lowering the demand for labor. With gross substitution the fall in demand is amplified as firms substitute workers of the group h with workers of other groups that have become relatively cheaper. That generates a disproportionate fall in hours worked by group h .²² In this context, what dominates labor income is the effect of hours, which implies that group h labor income falls when its markup goes up. That means that markups are a crucial source of labor income fluctuations in this economy. Hence, if markups fluctuate differently for the different groups of workers there are distributional effects from aggregate fluctuations.

Next, using Equation (1.25) we compute the earnings gap by dividing the skilled labor income by the unskilled labor income. The log-deviation with respect to the steady state of the labor earnings gap can be written as

$$\widehat{\eta}_t = \underbrace{\widehat{\eta}_t^M}_{\text{labor}} + \underbrace{\widehat{\eta}_t^C}_{\text{financial}}, \quad (1.26)$$

which depends on two components, *labor market heterogeneity*, $\widehat{\eta}_t^M = \frac{1-\sigma}{1+\varphi\sigma} (\widehat{M}_{st} - \widehat{M}_{ut})$, and *financial access heterogeneity*, $\widehat{\eta}_t^C =$

²²In other words, the labor demand is more elastic the higher is σ .

$\gamma \frac{1-\sigma}{1+\varphi\sigma} (\widehat{c}_{st} - \widehat{c}_{ut})$. Equation (1.26) does not depend on heterogeneity in ω_h/μ_h as we assume they are constant.

Two points worth mentioning about Equation (1.26). First, that the labor earnings gap depends on labor markets heterogeneity $\widehat{\eta}_t^M$ as these different workers belong to different labor markets that obey their dynamics. This labor market heterogeneity component may arise from diverse labor market frictions, which, in our case, arise from heterogeneous wage rigidities. However, $\widehat{\eta}_t^M$ is not specific to our setup. Furthermore, any model that generates a labor supply where wages are not equal to the marginal rate of substitution, fits the relationship shown by Equation (1.26). For instance, a *search and matching* model would do it as well.

Second, as the labor supply in our model depends on income effects (through the effect of consumption on the labor supply), the labor earnings gap depends on the differential responses of consumption between the different groups of workers, given by $\widehat{\eta}_t^C$. This last term depends on financial frictions. Different responses of consumption appear in the presence of different financial frictions. For example, in a model with a Representative Agent, consumption of all workers moves identically, and hence $\widehat{\eta}_t^C = 0$. But since there are hand-to-mouth agents within each group and the shares of hand-to-mouth between the different workers differ in our model, the term financial access heterogeneity is different from zero as in that case, the consumption response of workers' groups are different.

According to the simplifying assumptions of this section, η_t^C depends only on the earnings gap. We can obtain an earnings gap which is simply a function of labor markets heterogeneity. As $\eta_t = \frac{c_t^s}{c_t^u}$, then the earnings gap is given by

$$\widehat{\eta}_t = \frac{1}{1-\chi} \widehat{\eta}_t^M, \quad (1.27)$$

where $\chi = \gamma \frac{1-\sigma}{1+\varphi\sigma}$.

Thus, the only reason why the earnings gap fluctuates in this setting is the difference in labor markets. Additionally, we can obtain the cyclicity of the earnings gap. Let $\frac{\partial \widehat{\mathcal{M}}_{ht}}{\partial \widehat{y}_t}$ denote the cyclicity of group h wage markup, which we consider a proxy for the degree of wage rigidity faced by the group. Recall that in our model (from Equation 1.15) when wages of a group h are more rigid, \mathcal{M}_{ht} fluctuates more strongly. Then, the cyclicity of the Earnings Gap is given by

$$\frac{\partial \widehat{\eta}_t}{\partial \widehat{y}_t} = \frac{1 - \sigma}{1 - \gamma + (\gamma + \varphi)\sigma} \left(\frac{\partial \widehat{\mathcal{M}}_{st}}{\partial \widehat{y}_t} - \frac{\partial \widehat{\mathcal{M}}_{ut}}{\partial \widehat{y}_t} \right). \quad (1.28)$$

Equation (1.28) shows that two terms drive the cyclicity of the earnings gap. On the one hand, the earnings gap depends on a term in which the key parameter is the elasticity of substitution between skills on production. The first requirement to generate a countercyclical $\widehat{\eta}_t$ is this elasticity being greater than one; i.e., $\sigma > 1$ (the skill groups are gross substitutes). If skill groups are gross substitutes, then $\frac{1-\sigma}{1-\gamma+(\gamma+\varphi)\sigma} < 0$. On the other hand, the earnings gap's cyclicity relies on endogenous variables related to differences in labor markets. This second term depends on the differential response of labor market markups in our model with heterogeneity in wage rigidities. Recall that these markups are countercyclical, this is $\frac{\partial \widehat{\mathcal{M}}_{ht}}{\partial \widehat{y}_t} < 0$ for $h = u, s$. Hence, to generate a countercyclical Earnings Gap we require that $\left| \frac{\partial \widehat{\mathcal{M}}_{st}}{\partial \widehat{y}_t} \right| < \left| \frac{\partial \widehat{\mathcal{M}}_{ut}}{\partial \widehat{y}_t} \right|$. That case occurs when unskilled worker wages are more sticky than those of the skilled workers. Under these two conditions, the earnings gap is countercyclical.

The intuition of this result is as follows. In response to a contractionary monetary policy shock, all wages should fall. If the wages of the unskilled workers are more sticky, the unskilled workers become more expensive relative to skilled workers. With gross substitution, there is a shift in demand from unskilled to skilled workers. Then, for these reasons, the labor income of the unskilled falls by more than the labor income of the skilled, thereby increasing the earnings gap.

These two requirements are supported by the evidence presented above, as well as by previous literature. On the one hand, many studies estimate the elasticity of substitution between skilled and unskilled workers. In particular, [Ciccone and Peri \(2005\)](#) show that for many specifications and instrumenting by demand and supply factors, the elasticity of substitution between skilled and unskilled workers is about 1.5. They compare their estimates with other estimates in the literature, which range between 1.3 and 2. Also, [Acemoglu \(2002\)](#) studies the elasticity of substitution between skills in the context of Directed Technical Change. He obtains an elasticity of substitution between skilled and unskilled of about 1.4.

On the other hand, [Section 1.3](#) above shows that in the data, the wage Phillips curve is steeper for skilled workers. That is consistent with skilled workers having less responsive markups in absolute values as [Equation \(1.16\)](#) describes, which implies that the second requirement also holds in the data. Next, we study to what extent these facts generate the comovement of the earnings gap and output in the full-blown calibrated model and we study the quantitative impact of these facts.

1.6 QUANTITATIVE ANALYSIS

In this section, we show quantitative results for the calibrated model. We do two experiments. First, we show the effects of the heterogeneity in wage rigidities without profits, in which the only inequality between workers is the different adjustment of wages in the cycle. Second, we show the results for the full model, with profits. We start by describing the calibration.

1.6.1 Calibration

Household Problem Parameters. We set the inverse of the intertemporal elasticity of substitution γ , the inverse of the Frisch elasticity of labor supply $1/\varphi$, and the disutility of labor χ , equal to one. The discount factor β is set such that the interest rate is one-percent quarterly. We assume that there are two groups of workers, unskilled and skilled, denoted by u and s , respectively. We set the share of the unskilled workers to be $\mu = 0.65$ while that of skilled to be $(1 - \mu) = 0.35$ according to the average 2000-2018 obtained from CPS. The shares of hand-to-mouth in each group are set according to Table (1.2); i.e., $\lambda_u = 0.47$ and $\lambda_s = 0.18$. Additionally, we observe that skilled workers hold 83% of the total equity in the economy, so we set $\vartheta_u = 0.17$ and $\vartheta_s = 0.83$.

Production and Nominal Rigidities. We set the elasticity of substitution between varieties ε at 10, implying a share of profits equal to 10% of GDP in steady state. We set the cost of adjusting prices at $\theta = 100$, which implies a slope of the Phillips curve as in a model with sticky prices a la Calvo with an average price duration of one-year. We assume the elasticity of substitution between skills to be $\sigma = 1.5$, a value consistent with the literature on skill complementarity (see [Ciccone and Peri \(2005\)](#) or [Acemoglu \(2002\)](#)). We set $\omega_u = \omega_s = 0.5$, which implies that both workers' groups receive half of the aggregate labor income (which is consistent with CPS estimates). This calibration, in addition to the μ generate an earnings gap equal to 1.85 in steady state, which is in close to what we show in Figure 2.1. Finally, we set ε_h uniformly for both workers at 10.

Government and monetary policy. Monetary policy follows a Taylor rule with $\phi_\pi = 1.15$ in the baseline calibration. We set that low parameter to avoid unintended effects from the response of monetary policy to consumption behavior. The persistence parameter of the

exogenous monetary policy shock is set to $\rho_{mp} = 0.75$, while we scale the size of the shock σ_{mp} to generate a one-percent impact increase in the real interest rate in all the exercises below.

Wage rigidities. Finally, we set the wage adjustment cost parameters to match the empirical ratio of the dynamic multipliers of the two groups of workers. We calibrate these parameters separately in our two experiments, with and without firms' profits. We target the ratio of the dynamic multipliers to be close to eight as shown in Table (1.3).

The results for the calibration in both scenarios are described in Table (1.4). The calibration implies that when we assume fully flexible prices (and no profits), the duration of unskilled workers' wages is of about a year and a half (18 months)²³, while for skilled workers' wages it is about half a year (six months). These figures imply that the average duration of wages in the economy is about a year (in line with [Le Bihan et al. \(2012\)](#)). We compare two scenarios, one in which the wage rigidities differ, which we call *baseline* and other in which the wage rigidities are the same, which we call *alternative*. In the alternative scenario we set both rigidities equal to the average of the baseline. Both cases are reported in Table (1.4).

We calibrate another version of the model in which we assume that prices are sticky and there are profits. In this case, the total stickiness in the economy is shared between wages and prices, and hence, the wage rigidities that match the ratio of dynamic multipliers fall. The duration of unskilled workers' wages is about 14 months while for skilled workers' wages it is about four months. We also study this baseline in contrast to the alternative calibration with the average wage rigidity. This calibration is exposed in Table (1.4) as well.

²³According with the equivalence between Calvo and Rotemberg proposed by [Born and Pfeifer \(2020\)](#), see Appendix 1.D.

Table 1.4: Dynamic multipliers implied by the model. With and without firms' profits.

Without profits			
	Dynamic Multipliers		
	Unskilled	Skilled	S/U
$\theta_u = \theta_s$	-0.126	-0.126	1
$\theta_u > \theta_s$	-0.088	-0.77	8.8
Calibration			
θ_h	242	9.31	
Duration	6.12	1.69	
With profits			
	Dynamic Multipliers		
	Unskilled	Skilled	S/U
$\theta_u = \theta_s$	-0.143	-0.147	1.03
$\theta_u > \theta_s$	-0.088	-0.77	8.8
Calibration			
θ_h	141	5.03	
Duration	4.78	1.43	

Notes: This table shows the calibration of the wage rigidities in our model. The left-hand table shows the case without profits and the right-hand the case with profits and price rigidities. We report in both tables the dynamic multiplier as the cummulative response of wage inflation divided by the cummulative response of unemployment at a two-year horizon. The column S/U corresponds to the ratio of dynamic multipliers. We also show the adjustment cost parameters θ_s that deliver the dynamic multipliers already described and the average duration of wages implied by the θ_s .

1.6.2 The Effects of Monetary Policy: No Profits

We first show the effect of having a countercyclical Earnings Gap in a model without profits. In this subsection, we highlight that the Earnings Gap's mechanism does not rely on the countercyclical markups, as in the baseline HANK and TANK models, to generate amplification or dampening of monetary policy shocks. Instead, the impact of inequality arises from differences in nominal rigidities faced

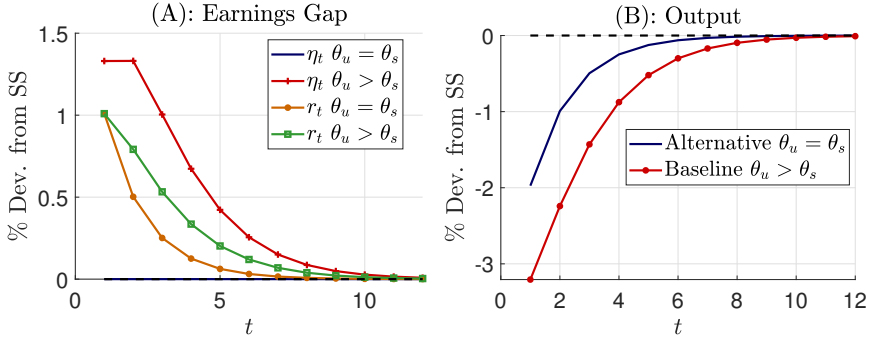
by the different workers who have different marginal propensities to consume.

Figure (1.7) shows the response of the economy to a contractionary monetary policy shock which corresponds to an increase of one percent in the real interest rate (on impact) in an economy without profits; i.e., where the differential response of labor income is the source of the redistribution in the cycle.²⁴ We show the two scenarios: the baseline ($\theta_u > \theta_s$) and the alternative ($\theta_u = \theta_s$). The left-hand panel shows the response of the interest rate and the earnings gap. The right-hand panel depicts the response of output. After a contractionary monetary policy shock, the earnings gap only rises in the case of the baseline calibration, consistent with the analytical results. The baseline calibration delivers a milder response with respect to the empirical evidence, which is close to 2.3 percent in the first quarter. For the case of output, we find significant amplification of monetary policy shocks derived from the aggregate demand effects of the inequality in wage rigidities. We find an amplification effect of about 62 percent on impact, which is more persistent than in the alternative scenario (due to the persistence in η_t). Moreover, the cumulative response of output in the baseline scenario is 2.2 times larger than in the alternative. Therefore, having a countercyclical earnings gap implies a significant amplification of monetary policy shocks in the absence of countercyclical price markups.²⁵

²⁴We scale the size of the shock to deliver a one percent increase in the real rate to get the same response as shown in the empirical evidence in both calibrations.

²⁵?? in Appendix ?? shows the responses of wage inflation and unemployment for both groups of workers in the different calibrations.

FIGURE 1.7: IRF's to a monetary policy shock. Left: earnings gap and the real interest rate. Right: output.



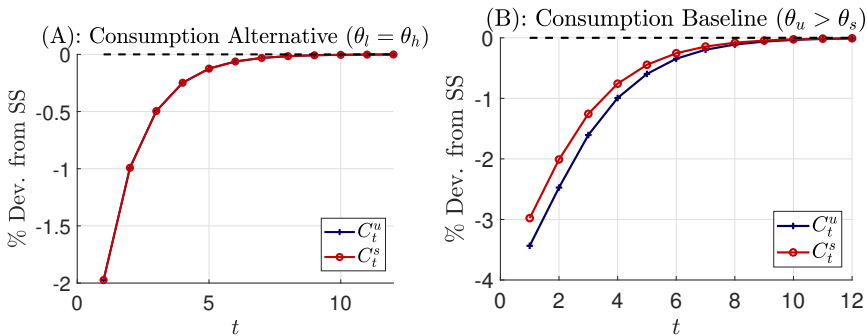
Notes: This figure presents the response of the earnings gap, the real interest rate (left-hand panel) and output (right-hand panel) to a monetary policy shock in the model. In these plots we show our two cases: the baseline ($\theta_u > \theta_s$) and the alternative economies ($\theta_u = \theta_s$). We report the response to a scaled shock that delivers a one-percent increase in the real rate on impact. All the responses correspond to deviations from the steady state. This figure shows the calibration in which we assume there are no profits nor price rigidities.

The effects of monetary policy also imply redistribution in the cycle. Figure (1.8) shows the response of groups' consumption to a monetary policy shock. The left-hand panel shows the response in the alternative scenario. In this case, consumption responses are identical, even though these groups differ in their shares of hand-to-mouth workers. As there are no profits, both workers' income fluctuate equally, and hence, their consumption reacts identically. The right-hand panel shows the baseline calibration. Two points are worth noting. First, monetary policy affects the unskilled consumers more than the skilled ones. This is a consequence of the higher volatility of labor income of unskilled workers. Second, the amplification effects from the countercyclical η_t affect both consumers, even though the skilled labor income is more stable than in the alternative scenario. This is due to a spillover effect from the excessive negative response of consumption of the unskilled workers, which pushes the aggregate

demand further down.

One can view the left-hand panel as the direct effect of monetary policy in our full model; hence, the difference between the right and left panels is approximately the indirect effect of monetary policy when we have heterogeneous wage rigidities. Given that both groups of workers have some degree of financial constraint, both groups have some departure from the representative agent, and their indirect effects are significant. Therefore, monetary policy has distributional effects, and the excess volatility from the countercyclical earnings gap affects both consumers through the indirect effect. Naturally, as we mentioned above, the most affected are unskilled workers.

FIGURE 1.8: IRF's of groups consumption to a monetary policy shock. Left: Alternative calibration. Right: Baseline calibration.



Notes: This figure presents the responses of consumption for the two skill levels in the model without firms' profits. The left-hand panel shows the alternative scenario ($\theta_u = \theta_s$) while the right-hand panel shows the baseline scenario ($\theta_u > \theta_s$).

1.6.3 The Effects of Monetary Policy: With Profits

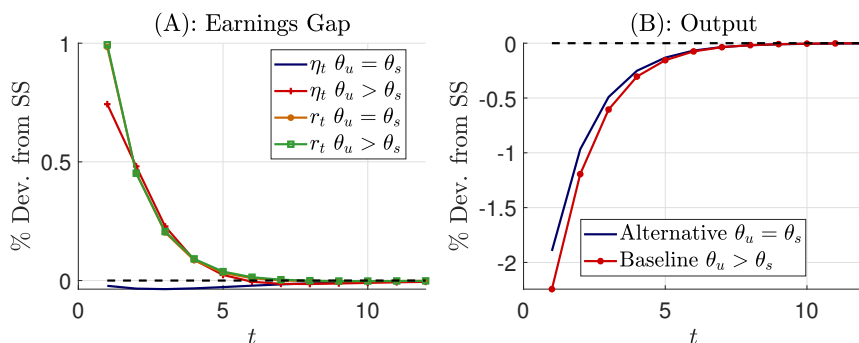
As most of the HANK and TANK literature relies on the existence of countercyclical markups to deliver aggregate effects from inequality and incomplete markets, we compare our results above with a case in which this additional redistributive channel exists. In the case with profits, as they are driven by the countercyclical markups, the effect

of wage rigidity heterogeneity weakens as wage and price markups have an inverse relationship.

Figure (1.9) shows the response of the earnings gap and output to a one-percent increase (on impact) in the interest rate. The response of output to a monetary policy shock is weaker than in the previous case. The earnings gap increases by about 0.8 percent on impact, which is far from both the data and the model without profits. In this case, we also observe amplification derived from this countercyclical earnings gap. The amplification is much lower, but still significant, about 18 percent on impact while the cumulative response is about 20 percent larger in the baseline calibration than in the alternative one.

This lower amplification effect is due to the countercyclical price markups. In models with both price and wage rigidities, the total markup (the sum of log markups) is countercyclical, causing the three markups to move in the same direction. Which markup moves more depends on the relative stickiness of the corresponding prices and wages. When prices are fully flexible, all fluctuations are absorbed by the wage markups with their differences being what generate the amplification effects previously described. If prices are sticky, price markups weaken the responsiveness of wage markups. This implies that the amplification triggered by wage stickiness is lower than with flexible prices. For instance, with fully sticky prices, the differences in wage markups have no effect. The intermediate case is what we observe in Figure (1.9).

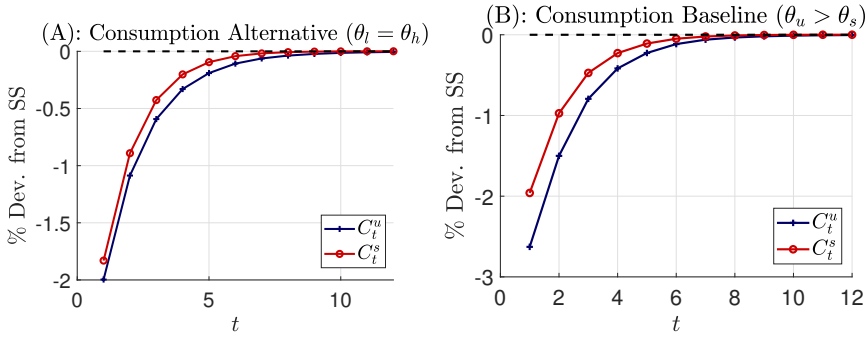
FIGURE 1.9: IRF's to a monetary policy shock. Left: earnings gap and the real interest rate. Right: output.



Notes: This figure presents the response of the earnings gap and the real interest rate (left-hand panel) and output (right-hand panel) to a monetary policy shock in the model. In these plots we show our two cases: the baseline ($\theta_u > \theta_s$) and the alternative economies ($\theta_u = \theta_s$). We report the response to a scaled shock that delivers a one-percent increase in the real rate on impact. All the responses correspond to deviations from the steady state. This figure shows the calibration in which we assume there are profits and price rigidities.

While the effect of amplification gets relatively weakened, there are still significant distributional effects of monetary policy. The response of unskilled workers' consumption is about 30 percent stronger than that of the skilled workers in the baseline calibration, as Figure (1.10) shows. This implies that unskilled workers are still worse off because of their labor market dynamics.

FIGURE 1.10: IRF's of groups consumption to a monetary policy shock. Left: Alternative calibration. Right: Baseline calibration.



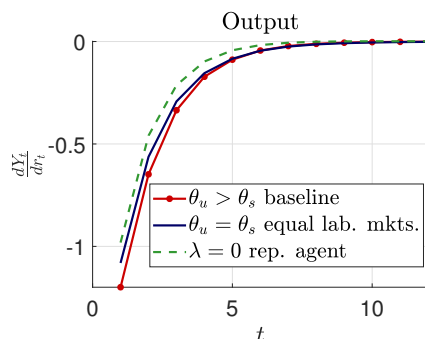
Notes: This figure presents the responses of consumption for the two skill levels in the model with firms' profits. The left-hand panel shows the alternative scenario ($\theta_u = \theta_s$) while the right-hand panel shows the baseline scenario ($\theta_u > \theta_s$).

1.6.4 The Effects of Monetary Policy: Other Benchmarks

Finally, we explore the impact of the different features of our model conditional on a monetary policy shock. Figure (1.11) shows the response of output to a monetary policy shock for different assumptions in the model with profits. We compare economies with and without incomplete markets and with equal and different labor markets. Now, we report the elasticity of output to the interest rate $\frac{\partial Y_t}{\partial r_t}$, to capture the response of output relative to the response of the real interest rate, a proxy of the slope of the IS equation. The first thing to note is that without financial frictions (with $\lambda_u = \lambda_s = 0$ hence a Representative Agent), the elasticity of output to the real interest rate does not depend on the labor market dynamics; this is because the real interest rate only drives aggregate consumption in a Representative Agent. Second, with limited financial access and equal labor markets, there is amplification, but it is small. We also interpret this as evidence of the effect of wage rigidities. When wages are rigid, profits fluctuate less (as the main component of marginal costs is wages), and the

amplification from incomplete markets gets diminished.

FIGURE 1.11: Decomposition of the response of output in the model.



Notes: This figure presents the response of output to a monetary policy shock in the model. We show the dynamic multiplier of output with respect to the interest rate; i.e., the ratio of the response of output to the interest rate. We show three alternative calibrations. The RANK case, $\lambda_h = 0$. The baseline and the alternative calibration.

However, if we consider that the earnings gap fluctuates as in our benchmark (with $\theta_u > \theta_s$), there is an amplification of shocks far beyond that implied by financial frictions. As we showed before, on impact, the effect of having different labor markets is substantial. In our model (with wage rigidities), the consequence of including incomplete markets generates an impact that is only 7% larger than the Representative Agent, while in the full model, the elasticity to the interest rate is 25% stronger than the representative agent ($\lambda = 0$). All this means that the contribution of labor market heterogeneity is about 72% of the total–cumulative– effect of jointly having incomplete markets and labor market heterogeneity. These effects imply that labor income heterogeneity may be an important source of amplification of business cycles.

1.7 INEQUALITY AND THE GAINS FROM WAGE FLEXIBILITY

Whether having more flexible wages or not is optimal depends on all the sources of welfare the different consumers have. Recall that the lifetime utility of households of type (i, h) , taking into account the wage adjustment costs, is given by

$$W_{h0}^i = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{(c_{ht}^i)^{1-\gamma}}{1-\gamma} - \chi_h \frac{(n_{ht})^{1+\varphi}}{1+\varphi} - \frac{\theta_h}{2} \pi_{ht}^2 \right\}, \quad (1.29)$$

in which households derive utility from consumption and dislike employment and wage inflation. Implicitly through consumption, all households dislike inflation volatility as aggregate consumption is given by $C_t = (1 - \frac{\theta}{2} \pi_t^2) Y_t$ in our model. Therefore, any policy that makes inflation more volatile affects welfare negatively by lowering the resources free to consume (in the Calvo model price dispersion plays this role). Then, from the perspective of welfare, when prices get more flexible, inflation gets more volatile and there is a trade-off between stabilizing quantities and prices.

Similar to price flexibility, wage flexibility in this model implies two opposite effects. On the one hand, wage inflation gets more volatile which implies welfare losses. This is present up to a maximum since if the parameter $\theta_h \rightarrow 0$ the loss from wage inflation volatility disappears. Also, a more flexible wage implies that employment is less volatile. Hence, there is an optimal level of wage rigidity for a given group of workers. On the other hand, when wages are more flexible, marginal costs, and then price inflation gets more volatile, which generates losses from inflation volatility (overall). These losses depend on the access to financial markets the different workers have since the level to which consumption is exposed to wage or price flexibility depends on how consumption is determined. To explain

this more clearly, recall constrained consumption from Equation (1.25)

$$c_{ht}^k = w_{ht}N_{ht} = \mathcal{M}_{ht}^{\frac{1-\sigma}{1+\varphi\sigma}} C_{ht}^{\gamma\frac{1-\sigma}{1+\varphi\sigma}} Y_t^{\frac{1}{\sigma}\frac{(\sigma-1+1+\sigma\varphi)}{1+\varphi\sigma}} \left(\frac{\omega_h}{\mu_h}\right)^{\frac{(\sigma-1+1+\sigma\varphi)}{1+\varphi\sigma}}. \quad (1.30)$$

As consumption of constrained consumers is essentially equal to labor income, it depends directly on wage markups, and it is affected by price inflation to a lesser extent. Thus, for these consumers, having more flexible wages makes wage markups less volatile. This implies constrained consumers gain from wage flexibility, if the elasticity of substitution between skills is different from unity, $\sigma \neq 1$. The final effect on welfare depends on the weight of employment, consumption, and wage adjustment costs on total welfare. For unconstrained consumers, Equation (1.30) does not apply, as their consumption follows the real interest rate.

To study the gains from wage flexibility, we follow [Schmitt-Grohe and Uribe \(2007\)](#) who compute welfare comparisons numerically. They compute the consumption equivalent of departing from a given benchmark. Then, the exercise we conduct is to compare the consumption equivalent of moving the degrees of wage flexibility. In our case, we move the degree of wage rigidity of the unskilled worker, while assuming that wages of the skilled are fully flexible (which means that the direct loss from their wage inflation is zero), and compare those equilibria with respect to the steady state. In this exercise, we study the welfare loss of having stochastic preference shocks; i.e., shocks to the discount factor. Hence, the relevant benchmark is the steady state as it is both the natural and the efficient allocation.

[Schmitt-Grohe and Uribe \(2007\)](#) propose the following procedure. Denote by \bar{W}_{h0}^i the welfare of the (i, h) household in the benchmark which is given by:

$$\bar{W}_{h0}^i = \mathbb{E}_0 \sum_{t=0}^{\infty} u(\bar{c}_{ht}^i, \bar{n}_{ht}^i) \quad (1.31)$$

where variables \bar{x} denote the benchmark allocations. We express the welfare of household (i, h) in the actual economy as

$$W_{h0}^i = \mathbb{E}_0 \sum_{t=0}^{\infty} u(c_{ht}^i, n_{ht}^i). \quad (1.32)$$

Then, we denote with ζ_h^i the welfare loss of departing from the benchmark as derived from

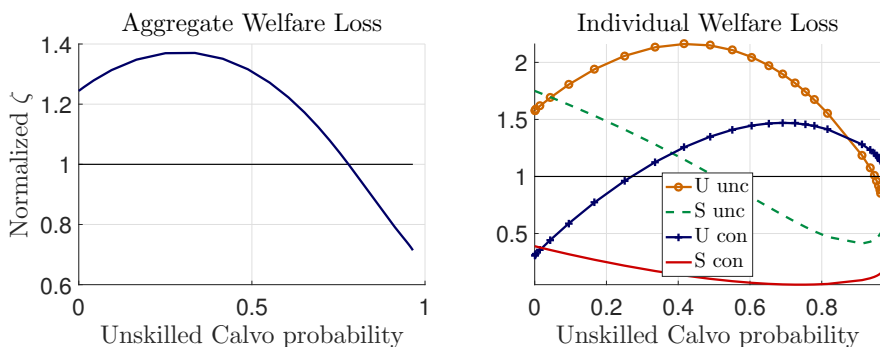
$$\bar{W}_{s0}^i = \mathbb{E}_0 \sum_{t=0}^{\infty} u((1 + \zeta_h^i)c_{ht}^i, n_{ht}^i), \quad (1.33)$$

which, as we show in Appendix 1.E, is given by

$$\zeta_h^i = \left(\frac{\bar{W}_{h0}^i - \bar{W}_{h0}^{in}}{W_{h0}^i - W_{h0}^{in} - W_{h0}^{i\pi}} \right)^{\frac{1}{1-\gamma}} - 1, \quad (1.34)$$

where \bar{W}_{h0}^{in} is the portion of welfare associated with employment in the benchmark, and W_{h0}^{in} and $W_{h0}^{i\pi}$ are the portions associated with employment and wage inflation, respectively, in the actual economies. Then, if $\zeta_h^i > 0$ the economy experiences losses with respect to the benchmark, since consumption in the actual economy is lower than consumption in the benchmark. Like [Schmitt-Grohe and Uribe \(2007\)](#), we solve the welfare loss numerically. We use a second-order approximation, because the welfare loss that arises from fluctuations depends on the second moments of the shocks. We calculate the loss ζ_h^i for all our four types of consumers and then we aggregate them proportionally to obtain the aggregate welfare loss. We are also interested in the distributional effects of making the wages of the unskilled more flexible; hence, we report the welfare losses for the four consumers in what follows. Finally, we show the role of monetary policy in this context.

FIGURE 1.12: Welfare losses as a function of wage rigidities of the unskilled.



Notes: This figure presents the welfare losses the business cycle. The left-hand panel shows the aggregate welfare loss switching the wage rigidity of the unskilled worker. We normalize the loss to be equal to one in the baseline calibration. The right-hand figure shows the welfare loss of each group of workers, skilled, unskilled, constrained, and unconstrained. We normalize the individual loss with the aggregate welfare loss in the baseline calibration.

Figure (1.12) shows the welfare effects of moving the wage rigidity parameter of the unskilled worker, assuming that the skilled workers have fully flexible wages. We report the normalized ζ , which is the ratio of the loss with respect to the baseline calibration of the previous section. Hence, when normalized ζ is equal to one, losses are those of the baseline calibration. Also, to improve the explanation, we show the Calvo probability associated with a wage stickiness parameter θ_h .²⁶ Hence, the x-axis shows the Calvo probability obtained from the adjustment cost parameter.

The left-hand panel in Figure (1.12) shows the weighted average of welfare losses. The first thing to note is a result similar to Galí (2013) and Galí and Monacelli (2016), in which there are no aggregate gains from making wages marginally more flexible. Although aggregate losses attain a maximum, the loss always stays above that of the

²⁶Appendix 1.D shows the mapping between Calvo and Rotemberg as derived by Born and Pfeifer (2020).

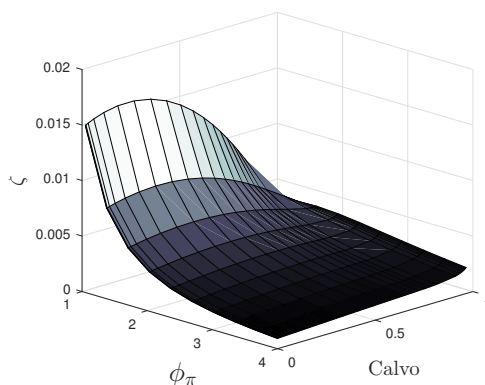
benchmark when the wages of the unskilled are more flexible. This means that the gains from making the consumption and labor of the unskilled workers less volatile do not outweigh the losses from a more volatile price inflation that arises from the higher wage inflation volatility. This result is not surprising as it resembles the results stressed by Galí (2013) in which the volatility of price inflation is first-order when evaluating the gains from wage flexibility.

However, making the wages of unskilled workers more flexible has important distributional effects. The right-hand panel in Figure (1.12) shows the disaggregated welfare losses. We report a similar exercise but now we normalize the loss with respect to the aggregate benchmark. The first thing to note is that business cycles impact very differently on different workers. Around the baseline calibration, the ones who suffer most from the business cycle are unskilled workers, regardless of their financial situation. This is due to the great volatility of hours worked that these workers face when their wages are more sticky. Unskilled workers experience a great loss from making their wages more flexible but the result differs between the constrained and the unconstrained. Constrained workers' losses have a maximum at a rigidity which is closer to the benchmark. This is for the reason we showed before: constrained workers' consumption is not greatly affected by inflation as it depends more directly on wage markups. Then, both labor and consumption become less volatile when their wages are more flexible. This implies that they actually experience gains from flexibility (starting at a high rigidity).

In contrast, skilled workers do not observe these trade-offs, since their labor income is not directly affected by moving the wage rigidity of the unskilled workers. Hence, they only suffer losses when the wages of the unskilled workers become more flexible. The reason why this happens is the relationship between inflation and wage inflation. As there is a positive relationship between these inflation rates, if inflation of unskilled workers' wages becomes more volatile, and that

generates price inflation volatility, there will be a higher volatility in wage inflation of the skilled workers. Then, the skilled workers suffer both from higher wage and price inflation volatility, while their labor does not become significantly more stable.

FIGURE 1.13: Aggregate welfare losses as a function of wage rigidities and monetary policy reaction to inflation.



Notes: This figure presents the absolute aggregate welfare losses from the business cycle as a function of the reaction of monetary policy to inflation ϕ_π and the wage rigidity of the unskilled worker.

We showed before that the main driver of welfare losses is price inflation, because as wages become more flexible, so do marginal costs and then price inflation. This implies that a way to deal with these inefficiencies is by having a monetary authority that reacts strongly to price inflation. Figure (1.13) shows how the aggregate loss varies depending on the reaction of monetary policy to inflation, ϕ_π , and the parameter of wage rigidity of the unskilled workers. As is common in this literature, if monetary policy reacts strongly to inflation we reach a point at which there are gains from wage flexibility.

1.8 CONCLUSION

In this paper, we study the role of the heterogeneity of indirect effects for the transmission of monetary policy and welfare. We first analyze, empirically, labor income inequality by building an indicator that we call the earnings gap, which corresponds to the ratio of labor income of skilled to unskilled workers. We show that the earnings gap is countercyclical and increases in response to a monetary policy shock. Additionally, to explain these patterns, we conduct a semi-structural analysis to estimate heterogeneous slopes of the wage Phillips curves for the two skill levels. We find that the slope for skilled workers is about eight times steeper than that for the unskilled workers. This suggests that the wages of skilled workers are significantly more flexible than those of the unskilled.

Then, we propose a model that rationalizes these facts. We show that if there is gross substitution between skills in production and unskilled workers have more sticky wages, the Earnings Gap is countercyclical and increases in response to a monetary policy shock. We embed these features in a New Keynesian model in which there is limited access to financial markets. We assume there are two types of workers and that within these groups there are financially constrained and unconstrained workers. We assume that the group of unskilled workers has a higher share of constrained agents, which means that they have a higher marginal propensity to consume. We find that there are significant amplification effects from the heterogeneity in wage rigidities interacting with incomplete markets. The effects of monetary policy can be twice as large with respect to the case of equal wage rigidities.

Finally, we show that eliminating wage rigidities only benefits the unskilled workers who have no access to financial markets. In fact, wage flexibility worsens aggregate welfare as it raises inflation volatility, affecting the other workers through this channel.

APPENDIX

APPENDIX 1.A THE EARNINGS GAP WITH THE SIPP

A useful test of the robustness of finding in Section 1.2 is to compare them with a different survey. To do so, we take the *Survey of Income and Program Participation* (SIPP) that has data of individual workers. We consider the period from 1990 to 2012. The advantage of this survey is that it provides regularly updated data on wages, income, and educational and demographic characteristics of workers, the disadvantage is that this survey has some missing periods, and hence is not suitable to conduct a VAR analysis. However, we can exploit the panel dimension and also construct the series of the earnings gap.

The survey is composed of several panels, in which about 10,000 households are followed over four years in a four-month frequency. Each four month window is called a *wave* in which households report their job status, their wage, and the social benefits being received, as well as households' individual characteristics. Each panel lasts for about four years and is constructed to be representative of the

U.S. population. We use a uniformed version of the SIPP panels built by the *Center of Economic and Policy Research (CEPR)*.²⁷ The CEPR compute uniformed hourly wage and labor earnings for each period which are comparable between panels. They also complete the sample by imputing monthly earnings from hourly wages and vice-versa if the respondent lacks one of the variables. We use the CEPR measure of total monthly labor earnings in what follows. Hence, we calculate labor income by group as the cross-sectional weighted average.

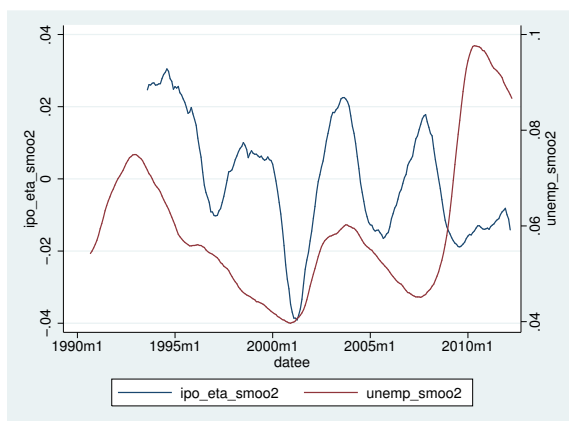


FIGURE 1.A.1: Labor Earnings Gap and unemployment.

Figure (1.A.1) depicts the detrended series of the earnings gap in blue, and the unemployment rate in red.²⁸ The figure shows some interesting patterns. First, labor income inequality is characterized by large fluctuations. Our series shows several peaks and troughs, with the sharpest around the dot-com crisis in 2001. Second, that in recessions, the earnings gap increases. In the 1991 and the 2008

²⁷See <http://ceprdata.org/> for more information.

²⁸We plot the annual moving average. The SIPP is incomplete in some periods. To get a complete series of η_t , we interpolate η_t in those periods. (this happens in eight months in 2000 and four in 2008, where they did not conduct the survey) In the regressions that follow, we take out these interpolated periods and run the regressions only with the available data.

recessions, the earnings gap went up considerably, while in the 2001 recession, the earnings gap was already large prior to the recession. Third, on average, the level of the earnings gap (not shown in this plot) is about two, which means that skilled workers get about twice as much labor income as unskilled workers.

Next we calculate the unconditional relation of the earnings gap with the cycle. To study this relation, we run the following regression:

$$\eta_t = c + \chi u_t + \beta W_t + \varepsilon_t, \quad (1.35)$$

where we regress our η_t with aggregate unemployment as a measure of the business cycle. Hence, we are interested in $\hat{\chi}$. We include some controls W_t that may be time trends and/or monthly dummies depending on the specification used.

These results are shown in Table (1.A.1). We consider both four-month and monthly data. We also consider specifications in levels and filtered. We filter the data using the Hamilton filter.²⁹ Each regression controls for monthly dummies and the robust standard deviations are shown in parenthesis. Additionally, we consider two definitions for η_t : without unemployment insurance (panel (A)) and with unemployment insurance b_t^k (panel (B)). We check the results by running the same regression with different treatments of the data. As mentioned before, we consider both monthly data (which is the frequency of the survey) and four-monthly data (which is the frequency of the waves). Additionally, we run the regressions with the data both in levels and filtered, to study the effect of the high persistence of unemployment on labor income inequality.

Several lessons can be taken from Table (1.A.1). First, that for all specifications, the correlation between unemployment and η_t is positive. Therefore, inequality *rises* with unemployment. In a recession, unskilled workers receive even lower labor income with respect to

²⁹Hamilton (2018)

Table 1.A.1: Cyclicity of labor income inequality, for different specifications.

		(A) Dep var. $\hat{\eta}_t$ (labor income)					
u_t	0.97*** (0.18)	0.99*** (0.42)	0.79*** (0.19)	0.78*** (0.32)	1.10*** (0.17)	1.25*** (0.37)	
		(B) Dep var. $\hat{\eta}_t$ (labor income + u benefits)					
u_t	0.94*** (0.18)	0.98** (0.41)	0.66*** (0.17)	0.65** (0.32)	1.02*** (0.15)	1.16*** (0.33)	
Frequency	1m	4m	1m	4m	1m	4m	
Filtered	✓	✓	✗	✗	✗	✗	
Trend	✗	✗	✓	✓	✗	✗	
Month dumm	✓	✓	✓	✓	✓	✓	

Note: Robust standard errors in parentheses.

* $p < 0.1$, ** $p < 0.05$, *** $p < 0.01$

skilled workers. In all the specifications where we consider only labor income, the estimated coefficients are significant.

Second, this correlation is economically important. Take, for instance, the second column in panel (A), that is our preferred specification. For every one percentage point increase in unemployment, inequality rises by 0.83 percentage points.

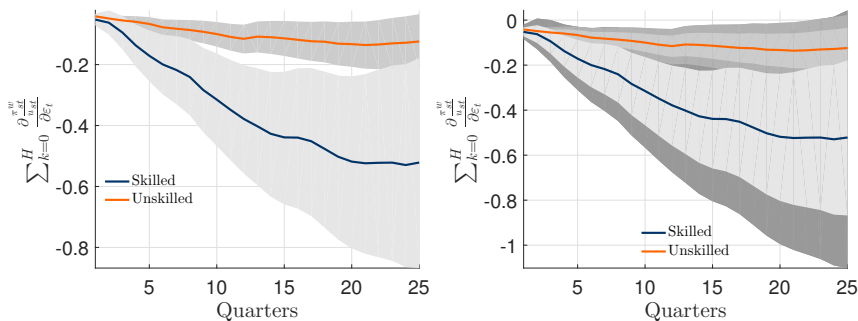
That implies that in a recession which makes unemployment increase by two percentage points, labor income for unskilled workers is almost one and a half points less than for skilled workers. Moreover, as panel (B) in Table (1.A.1) shows, unemployment benefits do not “help” to solve this cyclicity, they only help to diminish the coefficients of the relation of labor income inequality with unemployment, but these coefficients remain high.

APPENDIX 1.B TEST FOR THE DYNAMIC MULTIPLIER

To conduct the test of the significance of the difference between the slopes of the Phillips curve, we run the Bayesian BLP by including the ratio π_{ht}/u_{ht} instead of π_{ht} and u_{ht} separately. The reason why we do that is because the distribution of the actual dynamic multipliers get undetermined as the response of unemployment, $\sum_{l=0}^L \frac{\partial u_{ht+l}}{\partial \varepsilon_t}$, is zero with a high frequency, and hence the dynamic multiplier gets undefined. Instead, we show $\sum_{l=0}^L \frac{\partial \frac{\pi_{ht+l}}{u_{ht+l}}}{\partial \varepsilon_t}$ which can be seen as a more restrictive test for the actual dynamic multiplier. It is more restrictive since it would require that at *every* point of the IRF's, the two dynamic multipliers must differ.

Figure 1.B.1 shows the test for $\sum_{l=0}^L \frac{\partial \frac{\pi_{ht+l}}{u_{ht+l}}}{\partial \varepsilon_t}$ for different horizons. We report the median and the 68% confidence interval on the left-hand panel, while we add the 90% confidence interval in the right hand panel. The test delivers significance at the 68% confidence from month eight. Also, we document “weak” significance at the 90% level. If we consider the Unskilled as the benchmark, the skilled median falls outside the 90% confidence interval of the unskilled, which is still significant with this criteria. Recall that this is a more strict test since it requires the ratio π_{ht}/u_{ht} and not the variables separately to respond in different way.

FIGURE 1.B.1: Test for the Dynamic multiplier.



APPENDIX 1.C WAGE PHILLIPS CURVE DERIVATION

To solve for the wage Phillips curves, unions solve

$$\max_{w_{ht}^j, \pi_{ht}^j} \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left[\lambda_h U(c_{ht}^k) + (1 - \lambda_h) U(c_{ht}^r) - v(n_{ht}^j) - \frac{\theta_h}{2} \left(\frac{w_{ht}^j}{w_{ht-1}^j} - 1 \right)^2 \right] \quad (1.36)$$

subject to

$$n_{ht}^j = \left(\frac{w_{ht}^j}{w_{ht}} \right)^{-\varepsilon_h} n_{ht} \quad (1.37)$$

The FOCs are

$$\begin{aligned} & \lambda_h U'(c_{ht}^k) \frac{\partial c_{ht}^k}{\partial w_{ht}^j} + (1 - \lambda_h) U'(c_{ht}^r) \frac{\partial c_{ht}^r}{\partial w_{ht}^j} - v'(n_{ht}^j) \frac{\partial n_{ht}^j}{\partial w_{ht}^j} \\ & - \theta_h (\pi_{ht}^{hj} + 1) \pi_{ht}^{hj} \frac{1}{w_{ht}^j} + \beta \theta_h \mathbb{E}_t (\pi_{ht+1}^{hj} + 1) \pi_{wt+1}^{hj} \frac{1}{w_{ht}^j} = 0 \end{aligned} \quad (1.38)$$

where we used $\pi_{st} = \frac{w_{st}}{w_{st-1}}$. First, notice that

$$\frac{\partial n_{ht}^j}{\partial w_{ht}^j} = -\varepsilon_h \frac{n_{ht}^j}{w_{ht}^j}, \quad \text{and} \quad \frac{\partial c_{ht}^k}{\partial w_{ht}^j} = \frac{\partial c_{ht}^r}{\partial w_{ht}^j} = (1 - \varepsilon_h) n_{ht}^j \quad (1.39)$$

$$\begin{aligned} & \left(\lambda_h U'(c_{ht}^k) + (1 - \lambda_h) U'(c_{ht}^r) \right) (1 - \varepsilon_h) n_{ht}^j + v'(n_{ht}^j) \varepsilon_h \frac{n_{ht}^j}{w_{ht}^j} \\ & - \theta_h (\pi_{wt}^{hj} + 1) \pi_{wt}^{hj} \frac{1}{w_{ht}^j} + \beta \theta_h \mathbb{E}_t (\pi_{wt+1}^{hj} + 1) \pi_{wt+1}^{hj} \frac{1}{w_{ht}^j} = 0 \end{aligned} \quad (1.40)$$

Define $\overline{mgu}_{ht} = \lambda_h U'(c_{ht}^k) + (1 - \lambda_h) U'(c_{ht}^r)$ and after symmetry

$$(\pi_{wt}^h + 1) \pi_{wt}^h = \frac{\varepsilon_h}{\theta_h} N_{ht} \left\{ v'(N_{ht}) - \frac{\varepsilon_h - 1}{\varepsilon_h} \overline{mgu}_{ht} w_{ht} \right\} + \beta \theta_h \mathbb{E}_t (\pi_{wt+1}^h + 1) \pi_{wt+1}^h \quad (1.41)$$

APPENDIX 1.D ROTEMBERG-CALVO EQUIVALENCE

To compute the equivalence between calvo and rotemberg, we follow [Born and Pfeifer \(2020\)](#). The relation between Calvo and Rotemberg is given by:

$$\frac{(1 - \aleph_h)}{\aleph_h} (1 - \beta \aleph_h) = (\varepsilon_h - 1) (1 - \alpha) \frac{(\varepsilon - 1)}{\varepsilon} \frac{1}{\theta_h} \quad (1.42)$$

where \aleph_h is the Calvo probability of not adjusting wages, and θ_h is the parameter of the wage adjustment cost with Rotemberg.

APPENDIX 1.E WELFARE COMPUTATION

In section blah we compute the welfare loss of the different equilibria. To get that expression we proceed as follows.

Let W_{ht}^i and \overline{W}_{ht}^i be the lifetime welfare associated to an equilibrium of the economy and the benchmark for an individual household on (i, h) , which in our case is the steady state. Let ζ_h^i be the consumption *loss* from an equilibrium with respect to the benchmark. Noting

that welfare is given by

$$W_{h0}^i = E_0 \sum_{t=0}^{\infty} \beta^t \left\{ \frac{(c_{ht}^i)^{1-\gamma}}{1-\gamma} - \chi \frac{(n_{ht})^{1+\varphi}}{1+\varphi} - \frac{\theta_h}{2} \pi_{ht}^2 \right\}, \quad (1.43)$$

it can be split in three terms

$$W_{h0}^i = W_{h0}^{ic} + W_{h0}^{in} + W_{h0}^{i\pi} \quad (1.44)$$

which can also be written in recursive form as

$$W_{ht}^{ic} = \frac{(c_{ht}^i)^{1-\gamma}}{1-\gamma} + \beta \mathbb{E}_t W_{ht+1}^{ic} \quad (1.45)$$

$$W_{ht}^{in} = -\chi \frac{(n_{ht})^{1+\varphi}}{1+\varphi} + \beta \mathbb{E}_t W_{ht+1}^{in} \quad (1.46)$$

$$W_{ht}^{i\pi} = -\frac{\theta_h}{2} \pi_{ht}^2 + \beta \mathbb{E}_t W_{ht+1}^{i\pi} \quad (1.47)$$

$$\bar{W}_{ht}^{ic} = \frac{(1 + \zeta_h^i)^{1-\gamma} (c_{ht}^i)^{1-\gamma}}{1-\gamma} + \beta \mathbb{E}_t \bar{W}_{ht+1}^{ic} \quad (1.48)$$

$$(1.49)$$

Combining equations (1.48) with (1.45)

$$\frac{\bar{W}_{h0}^{ic}}{1-\beta} = (1 + \zeta_h^i)^{1-\gamma} \frac{W_{h0}^{ic}}{1-\beta} \quad (1.50)$$

Which gives rise to

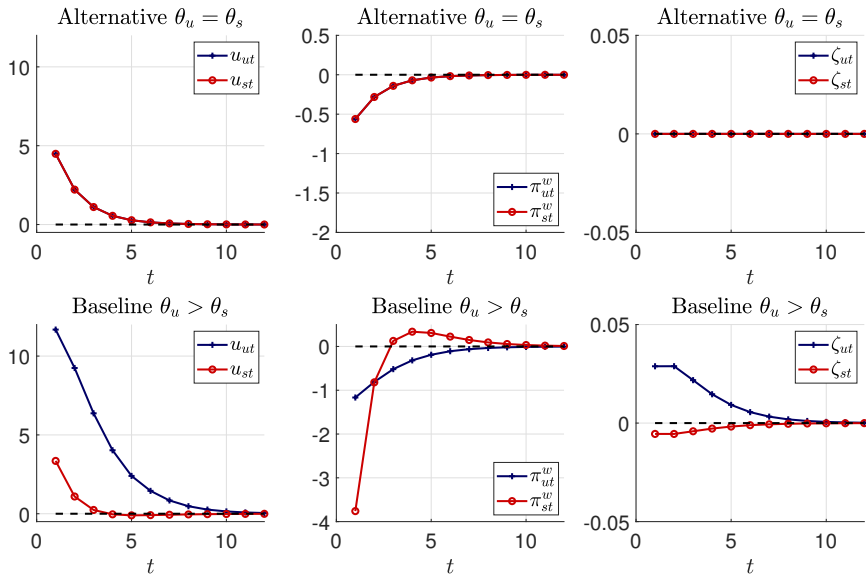
$$\zeta_h^i = \left(\frac{\bar{W}_{h0}^{ic}}{W_{h0}^{ic}} \right)^{\frac{1}{1-\gamma}} - 1 \quad (1.51)$$

Using (1.44) and $W_{h0}^{i\pi}$:

$$\zeta_h^i = \left(\frac{\bar{W}_{h0}^i - \bar{W}_{h0}^{in}}{W_{h0}^{ic} - W_{h0}^{in} - W_{h0}^{i\pi}} \right)^{\frac{1}{1-\gamma}} - 1 \quad (1.52)$$

APPENDIX 1.F EXTRA IRFS

FIGURE 1.F.1: IRFs of groups consumption to a monetary policy shock.
 Left: Alternative calibration. Right: Baseline calibration.



GOVERNMENT PURCHASES, THE LABOR EARNINGS GAP, AND CONSUMPTION DYNAMICS

2.1 INTRODUCTION

There is an extensive literature that studies the effects of government spending on aggregate outcomes. While this literature agrees that a rise in government spending increases output, the conclusion about its effects on consumption is still open (see [Ramey \(2016\)](#)). Understanding the effects of government purchases on consumption is essential for at least two reasons. First, the response of consumption is crucial to assess the welfare implications of government spending programs. Second, the relationship between consumption and government spending helps us understand the underlying economic model. Some researchers claim that finding a fall in consumption in response to a rise in government purchases favors the Real Business Cycles model (RBC) over the New Keynesian model (or Keynesian) as a consequence of a crowding-out effect of government spending

on consumption.

In this paper, we study the effects of government spending on consumption by considering its effects on labor income inequality. The previous literature shows that when consumers have imperfect access to financial markets (and they cannot smooth out consumption perfectly), all sources of income determine consumption in the cycle.¹ Hence, in economies where consumers are heterogeneous and have different MPCs, the effect of government spending on the different types of income matters. If the government affects the distribution of income, it is changing the average MPC of the economy, and thus, it is affecting aggregate consumption and the aggregate demand.²

To study empirically to what extent government purchases affect income inequality and then consumption, by using the *Current Population Survey* we build an index of inequality that we call the *earnings gap* for the U.S., which we define as the ratio of skilled to unskilled labor income.³ We embed this variable into a Bayesian Structural Vector Autoregression following the specification used by Galí et al. (2007) and show that government spending increases this gap. That means that government spending raises the labor income inequality between skilled and unskilled workers. In addition, we study whether the earnings gap is related to aggregate consumption conditional on government spending shocks. We estimate a Time-Varying Structural Vector Autoregression as in Primiceri (2005) and show a negative relationship over time between the responses of consumption and the

¹See, for instance, Kaplan et al. (2018).

²This concept goes back to Keynes (1936) Ch.19, where he mentions that wages enter the aggregate demand if wage fluctuations affect the average MPC of the economy. In this work, we exploit a similar argument in which as a consequence of government spending, there is a redistribution of resources between agents with different MPC, making the average MPC fluctuate. That, in turn, affects aggregate consumption. A more modern approach is studied by Bilbiie (2020), where he shows that is the income cyclicality of the high-MPC consumer that matters for the effects of inequality in the business cycle. We exploit these concepts in this work.

³Skilled are the workers with completed bachelor's degree or higher; the rest are classed as unskilled.

earnings gap following a government spending shock. That means that when government spending shocks generate higher inequality, consumption responds less strongly. Third, we use microdata for the U.S. government purchases to show that government spending concentrates on sectors with a higher share of skilled workers than the overall economy. For example, Aerospace and R&D services account for about 25 percent of total government spending, and these are sectors with a large skilled income share (66 and 82 percent, respectively) which is higher than the 50 percent the overall economy has). That means that an additional dollar of government spending accrues disproportionately to skilled workers. This pattern of distribution could be behind the increase in labor income inequality in response to a government spending shock. Finally, we show that unskilled workers are more financially constrained than skilled workers. We measure the share of Hand-to-Mouth (HtM) consumers as in [Kaplan et al. \(2014\)](#) in both groups of workers.⁴ We show that the share of HtM of unskilled workers is about three times higher than that of the skilled workers. This means that unskilled workers have on average a higher MPC.

In the second part, we rationalize these facts in a New Keynesian model with limited asset market participation. We assume there are two productive sectors that supply goods and two groups of workers –skilled and unskilled. Sectors are in monopolistic competition and are subject to price rigidities. Both groups of workers supply labor to both sectors, but in different proportions. To be consistent with the empirical evidence above, we give the two groups of workers different levels of access to financial markets. This generates heterogeneous MPCs between the groups of workers. Finally, consumers and the

⁴We measure the share of Hand-to-Mouth as the share of households who hold zero liquid assets, where zero is defined as having 30 percent (in absolute value) of their income (or less) in these types of assets in a given period. See [Kaplan et al. \(2014\)](#) for more details.

government buy goods from both sectors, with unequal weights.

We develop two analytical results with the model. First, we show that due to the different access to financial markets that different groups of workers have, there is a negative relationship between the earnings gap and consumption; i.e., consumption falls in response to an increase in the earnings gap. That happens because when the earnings gap goes up, skilled workers earn more relative to unskilled workers. As skilled workers have more access to financial markets, they smooth out consumption more, and hence, the average MPC falls, lowering consumption. An interesting consequence of the latter is that if the response of the earnings gap is strong enough, it may reverse the sign of the consumption response (counteracting the positive effect of government spending on consumption highlighted by Galí et al. (2007)). Therefore, we show that consumption may also fall in New Keynesian models with incomplete markets if we consider this dimension of heterogeneity (which is consistent with the evidence provided by Ramey (2011)).

We show the conditions for the earnings gap to rise in response to a government spending shock. We show that two channels drive the response of labor income inequality. A *direct channel*, which operates through direct government purchases of the two goods. If the government purchases the high skilled intensive good in a higher proportion than the economy, government purchases raise the earnings gap. The intuition is straightforward: a large proportion of the extra income generated by the government is accrued to skilled workers, increasing labor income inequality. The second channel is a *general equilibrium channel*, which operates through the responses of aggregate variables. We show that the earnings gap depends on output and prices as well as government spending.

Finally, we study these questions quantitatively in a model by analyzing different calibrations focusing on the way government spends. We compare a baseline calibration (where the government

spends more proportionally in the skilled intensive sector) with an alternative in which all the spending is in the unskilled intensive sector. We show that switching from the calibration of the actual economy to spending only on unskilled intensive sectors, the response of consumption to a government spending shock is 45 percent larger. That implies that the fiscal multiplier rises by about a third when government spending switches to unskilled intensive sectors.

The main takeaway is that the size of the government spending multiplier is affected by the impact of government spending on earnings inequality. To the extent that, as we show below, an increase in government spending raises earnings inequality, the size of the multiplier also reduces through a dampening effect on private consumption.

Related Literature. This paper is related to two strands of the literature, the literature on the effects of government spending on macroeconomic aggregates and the theoretical mechanisms through which government spending operates.

The former is comprehensively accounted by [Ramey \(2016\)](#), who summarizes the state of the art on the effects of government spending. We use [Blanchard and Perotti \(2002\)](#) (BP) identification, who find a positive effect of government spending on consumption. Additionally, they report a fiscal multiplier between 0.6-1.2. A paper which is related to ours is [Galí et al. \(2007\)](#) who extend the BP identification scheme to focus on the effects of government spending on consumption. They show that government spending raises output and consumption, with the response of the latter following disposable income. That finding motivates considering models for consumption behavior with limited access to financial markets as the one we emphasize. The other strand of the empirical literature is the one led by [Ramey \(2011\)](#) who extends [Ramey and Shapiro \(1998\)](#) by considering a the period from 1939 to 2006 and building the expected value of

military buildups. She finds negative effects of government spending on consumption, unlike BP. The reason why she find these effects is that according to her view, the BP shocks are anticipated while the military spending shocks are not. In the main exercises we use BP since as [Ramey \(2016\)](#) shows, the military spending shocks do not pass the test of weak instruments for the period after the Korean War.

On the theoretical and microdata side, our paper is related to [Cox et al. \(2020\)](#) who study the sectoral composition of government spending, and show that government purchases are concentrated towards sectors that have more sticky prices, which raises the fiscal multiplier. A similar argument is raised by [Bouakez et al. \(2021\)](#) who study the size of the fiscal multiplier in a production-network economy where sectors differ in their price rigidity, factor intensities and use of intermediate inputs. They find that the multiplier rises by 75% with respect to a one-good economy. The amplification they find is due to input-output linkages and sectoral heterogeneity in price rigidity. Another work which studies the effects of the sectoral composition of government spending is by [Boehm \(2020\)](#) who shows that a reason why the fiscal multiplier is relatively low is because it is concentrated towards investment goods.

The most related paper to ours is [Flynn et al. \(2021\)](#) who study the sectoral composition of government spending and the effect of the network structure of firms and labor markets taking into account the heterogeneity in MPCs of different households. They show that fiscal multipliers vary substantially depending on where spending and transfers are targeted. They show that to be more effective, government policies must be directed towards higher MPC households.

Our paper is complementary to these in several ways. We exploit the skill composition of the different sectors, extending [Cox et al. \(2020\)](#) and showing that government spends on sectors more skilled intensive than the economy as a whole. Second, we show theoretically, that the previous fact has an aggregate demand channel through

heterogenous MPC's; mechanism which is very similar to [Flynn et al. \(2021\)](#) but we only consider two consumers/workers with a different concept of heterogeneity, the one that comes from permanent differences (at least at the business cycle frequency) between households, the skill level. This allows us to abstract from idiosyncratic risk (since the mean income for the different groups would average out the idiosyncratic shocks) and helps us in proposing a more clear target for policies, given by an observable feature. Finally, we contribute in testing the inequality channel on aggregate consumption by showing that this mechanism is present in the data through the negative relationship over time in the size of the responses of consumption and labor income inequality to the government spending shock.

Layout. The remainder of the paper is as follows. Section [2.2](#) studies empirically, the effects of government spending on consumption and labor income inequality. Section [2.3](#) describes the model. Section [2.4](#) develops two analytical results, the solution for the aggregate demand and studies why the earnings gap fluctuates in our model. Section [2.5](#) presents quantitative results for different calibrations of our model. Finally, section [2.6](#) concludes.

2.2 EMPIRICAL EVIDENCE

In this section, we revisit empirically the effects of government purchases on consumption taking into account its effects on labor income inequality. We first show that an increase in government spending generates income inequality and raises consumption. Then, we show that the size of the responses of consumption and labor income inequality to a government spending shock, are negatively related. Finally, we show that government purchases are concentrated towards sectors with a high share of skilled workers.

2.2.1 *The Earnings Gap*

In this subsection, we introduce the variable that we will work with throughout the paper: the earnings gap. We denote the earnings gap by η_t , which formally we define it as the ratio of skilled to unskilled average labor income

$$\eta_t = \frac{\text{Skilled labor income}}{\text{Unskilled labor income}}.$$

For this paper, we divide the population into these two groups, skilled and unskilled. We consider in the former group workers with a completed bachelor degree or higher while an uncompleted bachelor degree or less in the latter.⁵ We are interested in the inequality of total labor income (total hours and wages), because it is these together which determine consumption.

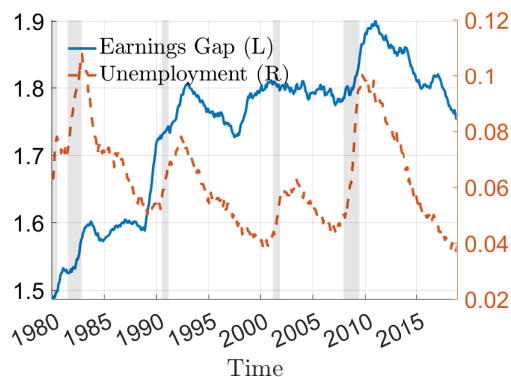
To build η_t , we take the *Current Population Survey* (CPS) that has individual earnings and demographic data. We consider the period from 1979M1 to 2018M12. We use a uniformed version of the CPS built by the *Center of Economic and Policy Research* (CEPR).⁶ The CEPR computes uniformed hourly wage and labor earnings for each period, which are comparable between surveys. They also complete the sample by imputing weekly earnings from hourly wages and vice-versa if the respondent lacks one of the variables. We use the CEPR measure of total weekly labor earnings in what follows and we calculate the cross-sectional weighted average of labor income by group. Hence, the earnings gap is a measure of per-capita labor income inequality.

Figure (2.1) displays the earnings gap. Four observations worth commenting. First, that since the 2000's, the earnings gap is high, and around 1.8. Second, our earnings gap accounts for the increase on labor income inequality between skilled and unskilled workers documented in previous studies (see [Acemoglu \(2002\)](#), for example). At

⁵According to the Current Population survey, the share of skilled workers is about 40% and the share of unskilled is about 60% by 2018.

⁶See <http://ceprdata.org/> for more information.

FIGURE 2.1: Labor earnings gap and unemployment.



Notes: This figure shows the Earnings Gap in the business cycle. The left-hand panel depicts the level of the earnings gap compared with the unemployment rate. The gray vertical lines correspond to the NBER recessions.

the beginning of the 1980s the earnings gap was about 1.5; i.e, skilled workers earned 50% more than the unskilled on average. During the 1980s the gap increased substantially and rose to about 1.8, to stay around that level until the Great Recession.⁷ Third, the earnings gap increases in recessions and falls in expansions. In all the recessions except for 2001 (which seems to be a very particular one), the earnings gap has increased significantly. There are also several periods in which the gap increases even in expansions like the period prior to 1990. However, in long periods of expansion, like 1992-1997 or from 2011 to 2018, the earnings gap fell, but the fall was less pronounced than that of the unemployment rate, suggesting that the earnings gap has even more persistence than the unemployment rate. Fourth, and related to the previous point, the earnings gap seems to behave

⁷This is consistent with the evidence on the increase of the skill premium. In general, the skill premium literature only looks at the widening of the wage gap. But as we are interested in what determines consumption, we study total labor income since fluctuations in employment also play a role in determining the earnings gap.

assymmetrically; i.e, the earnings gap increases sharply in recessions but seems to stay at high levels for a long period, often until the next recession takes place and pushes inequality further up.⁸

2.2.2 *Government Spending Raises the Earnings Gap: Evidence from a Bayesian SVAR*

The baseline VAR includes the earnings gap, government expenditures in consumption and investment, government receipts, GDP, consumption of non-durables and services, fixed non-residential investment, and unemployment. All the quantity variables are real, are divided by the working-age population, and enter the regressions in logarithms. Data is quarterly and we consider the period 1981Q1-2018Q4 which is the longest sample available for calculating the earnings gap. Finally, we include four lags in the estimation.⁹

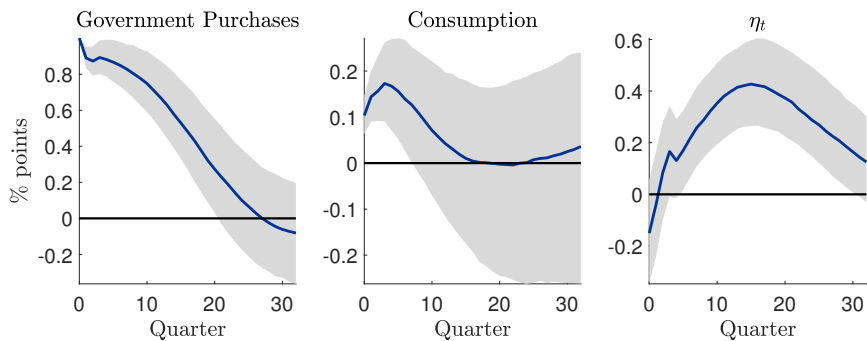
We first estimate a Bayesian SVAR with Normal-Inverse Wishart priors, where we study the response of the economy to a government purchases shock. We identify this shock through a Cholesky identification (following [Blanchard and Perotti \(2002\)](#)), by ordering government spending first; i.e., government spending does not respond contemporaneously to the state of the economy. The reason why we use a Bayesian approach is the small sample available (at least for the earnings gap) and the large number of parameters to estimate

⁸In Chapter 1 we study the reasons why the earnings gap is countercyclical also in response to a monetary policy shock. As we will show below, this countercyclicality is unconditional and does not hold for all aggregate shocks.

⁹We build the Earnings Gap as we exposed before and we average the monthly data in every quarter. We obtain the data from the database presented by [McCracken and Ng \(2016\)](#). All variables drawn from FRED. We consider GCEC1 for government purchases, FGRECPT for government receipts, the sum of the sum of PCESV and PCND for consumption, PNF1 for investment, and UNRATE for unemployment. We use this specification in this section to be able to compare the results with [Galí et al. \(2007\)](#).

in the VAR.¹⁰ In the estimation, we follow [Miranda-Agrippino and Ricco \(2021\)](#) and run the Bayesian SVAR, choosing the hyperparameters optimally as in [Giannone et al. \(2015\)](#), who estimate the prior tightness parameters by maximizing a joint maximum likelihood for the Bayesian SVAR model.¹¹

FIGURE 2.2: IRF's to a unitary shock on government spending BSVAR with Cholesky identification. Sample: 1981Q1-2018Q4



Notes: This figure depicts the responses of Government purchases, aggregate consumption, and the Earnings Gap to a unitary government spending shock. These responses are obtained from a Bayesian VAR which includes government revenues, output, investment, and unemployment too. The figure plots the median response from 4000 draws and reports the 68% confidence areas.

Figure 2.2 depicts the response of the three variables of our interest: government purchases, consumption, and the earnings gap in response to a one-percent increase in government purchases. The solid-blue line shows the median response from the draws in our Bayesian VAR, while the gray areas represent the 68% confidence bands. Two observations worth mentioning. First, that for our sample, consumption increases in response to the government spending

¹⁰With four lags, seven endogenous variables and including a constant, the number of parameters is $7 \times (1 + 4 \times 7) = 203$.

¹¹For more details, we refer the reader to these articles. And we thank Silvia Miranda-Agrippino for sharing her codes with us.

shock; although the response is small, it confirms the existence of a Keynesian effect for government spending as pointed out by Galí et al. (2007). And second, that the earnings gap increases significantly and persistently. This implies that when government spending goes up, labor income inequality between skilled and unskilled workers rise.

The reader might argue that the identification scheme used is not the best to study this question since this way of computing government spending shocks does not deliver really exogenous shocks. Unfortunately, for the sample we have the earnings gap available, the shocks as those built with news about military spending are weak instruments, as pointed out by Ramey (2016). Therefore, the best—and simpler—way to conduct this exercise is to consider the identification by Blanchard and Perotti (2002). However, we still provide an estimate (though with these weak instruments) using news about military spending in Appendix 2.A Figure 2.A.1. We find that with this alternative identification, the earnings gap still rises and responds in a very similar shape compared to the estimates with Blanchard and Perotti (2002)'s identification. Moreover, the effects are stronger, which confirm the finding that government spending generates labor income inequality between skilled and unskilled workers. In our specification we find that as in Ramey (2016), consumption falls in response to the government spending shock.

A second issue that may arise is the endogeneity of the government spending shocks with respect to inequality when estimating with a Cholesky identification scheme. One may wonder if government spending reacts contemporaneously to inequality and that is why we observe the positive relationship between government spending and the earnings gap. We can analyze if by switching the ordering between government spending and the earnings gap the result changes. As Appendix 2.B shows, the change in the ordering does not affect the response of the earnings gap and consumption to the government spending shock; the results are almost identical to

the ones shown in Figure 2.2.¹²

2.2.3 *The Size of the Responses of Consumption and the Earnings Gap are Negatively Related: Evidence from a TVC-SVAR*

We are also interested in the evolution of the response of consumption and the earnings gap to a government spending shock. One of our hypothesis is that a lower (even negative) response of consumption is related to a higher (positive) response of the earnings gap, as our theory will make clear below. In this subsection we evaluate this hypothesis. To do so, we estimate a Time-Varying Bayesian SVAR (TVC-SVAR) as in Primiceri (2005). The econometric model we assume is the following

$$y_t = c_t + B_{1,t}y_{t-1} + \dots + B_{k,t}y_{t-k} + u_t, \quad t = 1, \dots, T, \quad (2.1)$$

where y_t is an $n \times 1$ a vector of endogenous observable variables, c_t is an $n \times 1$ vector of time-varying constants, $B_{i,t}$ are $n \times n$ matrices of time-varying coefficients with k the order of the VAR, and u_t is an $n \times 1$ vector of heteroscedastic unobservable shocks with variance-covariance matrix Ω_t . Consider the triangular reduction of Ω_t , $A_t\Omega_tA_t' = \Sigma_t\Sigma_t'$, where A_t is lower-triangular and Σ_t is diagonal. That allows us to write the model in Equation (2.1) as

$$y_t = X_t' B_t + A_t^{-1} \Sigma_t \varepsilon_t, \quad (2.2)$$

with $X_t' = I_n \otimes [\mathbf{1}, y_{t-1}', \dots, y_{t-k}']$ and B_t is the matrix which contains the matrices $B_{i,t}$ stacked.

As in Primiceri (2005), the time-varying matrices A_t and Σ_t are crucial for the exercise that follows. We want to study both a time-varying relationship between the variables, contained in B_t , while we also want to exploit the time-varying variance-covariance structure of shocks. That implies having both heteroscedastic structural shocks as

¹²We also tried with ordering the earnings gap last, and found no differences.

well as a time-varying contemporaneous relation between them. More importantly, all these elements together bring time-varying impulse responses of the different variables to a government spending shock. To conduct the estimation of model in Equation (2.2) we must assume a time-varying dynamics for the parameters. Let α_t be the nonzero elements of matrix A_t and σ_t the vector of diagonal elements of Σ_t . The dynamics of the parameters are

$$\begin{aligned} B_t &= B_{t-1} + \nu_t, \\ \alpha_t &= \alpha_{t-1} + \xi_t, \\ \log \sigma_t &= \log \sigma_{t-1} + \zeta_t. \end{aligned}$$

We assume that the parameters B_t and α_t follow a random walk, while the elements of Σ_t follow a geometric random walk.

We assume the innovations in the model follow a joint normal distribution where the variance covariance matrix is given by

$$V = \text{var} \left(\begin{bmatrix} \varepsilon_t \\ \nu_t \\ \xi_t \\ \zeta_t \end{bmatrix} \right) = \begin{bmatrix} \mathbf{I}_n & 0 & 0 & 0 \\ 0 & Q & 0 & 0 \\ 0 & 0 & S & 0 \\ 0 & 0 & 0 & W \end{bmatrix}.$$

We use Bayesian methods to evaluate the posterior distributions of the parameters of interest, the sequences of vectors B^T , α^T , σ^T , given the hyperparameters in V . We use Gibbs sampling to evaluate these posterior distributions. In particular, to compute the time-varying parameters, we use the [Carter and Kohn \(1994\)](#) algorithm. As [Primiceri \(2005\)](#), we assume Normal-Inverse Wishart prior which we calibrate by estimating a VAR on the first ten years of data.¹³

We are interested in the effect of government spending on the earnings gap and consumption simultaneously. To do not saturate the

¹³We set the same hyperparameters [Primiceri \(2005\)](#) use. We also find that the results are robust to the choice of the hyperparameters (see Appendix 2.C). We thank Gary Koop for having available the codes for this procedure.

model with a large number of parameter estimates, we run a small VAR which only includes government purchases, consumption, and the earnings gap (in that order). We divide aggregate data by the population 16-64 year old, and detrend them with a second order deterministic trend. As in the previous estimation, we identify the government spending shock recursively assuming that government purchases are ordered first. The data is quarterly and the estimation is carried out for the 1981Q1-2018Q4 period as well. We set priors for the starting values $(B_0, \alpha_0, \sigma_0)$ with an estimate of a static VAR as in [Primiceri \(2005\)](#). We use the first 40 periods which we drop for the subsequent estimation; hence, we obtain time-varying parameters for the period 1991Q1-2018Q4. We consider four lags in the estimation. The number of replications in the bayesian estimation is set to 15000 where we burn 5000 draws.

We estimate the impulse responses of our variables of interest in a time-varying fashion. We compute the cumulative response of consumption, the earnings gap, and government spending, and then, we calculate the *dynamic multiplier* of the variables with respect to government spending. We define the dynamic multiplier as the ratio of the sum of the impulse response of the variable of interest to the sum of the response of government spending, which we denote by \mathcal{D}_{Xs} and is given by

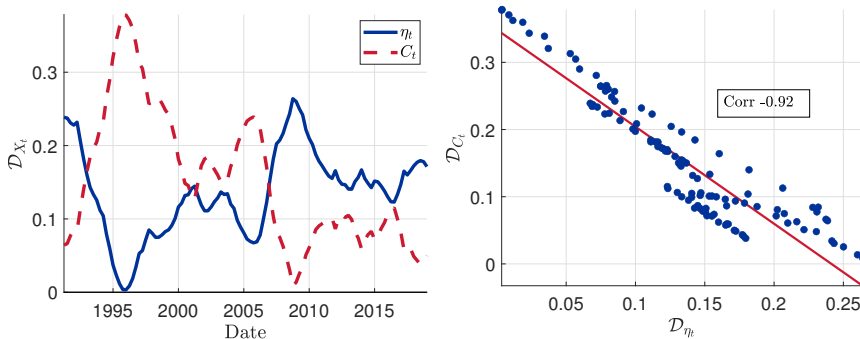
$$\mathcal{D}_{Xs} = \frac{\sum_{k=0}^K IRF_{Xg,k}^s}{\sum_{k=0}^K IRF_{gg,k}^s}.$$

which represents the dynamic multiplier of variable X in period s . We calculate these indicators for a horizon of 20 quarters ($K = 20$) at every period in our time-varying estimates. The dynamic multiplier is an appropriate statistic since it allows us to take into account the response of government spending as well, by *discounting* for the evolution of government spending, as if may vary in the different periods.¹⁴

¹⁴As [Mountford and Uhlig \(2009\)](#) propose.

Figure 2.3 shows the dynamic multipliers of consumption and the earnings gap in the left-hand panel, and a scatterplot for these two estimates in the right-hand panel. The left-hand panel shows that for all our sample, the dynamic multiplier of the earnings gap and consumption are positive. This means that throughout the full sample, from 1991 to 2018, increases in government spending raised labor income inequality between skilled and unskilled workers. On the other hand, we find that the dynamic multiplier of consumption is also positive throughout the full sample. Both dynamic multipliers fluctuate strongly with peaks and troughs in periods that coincide. More interestingly, the responses of the earnings gap and consumption have a negative relationship, which is confirmed in the right-hand panel that shows the scatterplot of the two dynamic multipliers. Therefore, when the earnings gap increases by more, consumption rises by less in response to a government spending shock.¹⁵

FIGURE 2.3: Time-Varying Dynamic Multipliers of C_t and η_t



Notes: Cumulative responses of consumption and the earnings gap estimated with a time varying coefficients VAR. The cumulative response is obtained by summing up to 20 quarters. The left-hand panel shows the cumulative responses over time and the right-hand side shows the relationship between the two responses.

¹⁵Appendix 2.C shows the responses of consumption and the earnings gap to the government spending shock. We find that the IRF's are very similar to the ones obtained in the time-invariant Bayesian SVAR.

We consider these results as evidence of the relationship between labor income inequality and consumption conditional on government spending shocks. In the next subsection we explore a reason why this relationship exists.

2.2.4 *Government Spending is Concentrated Towards Skilled Intensive Sectors*

A possible reason for the positive effect of government purchases on the earnings gap is that government purchases are concentrated on skilled intensive sectors. This implies that when government increases spending, a higher proportion of this demand is directed towards skilled workers, increasing their labor income with respect to the labor income of the unskilled workers.

To study this question, we use the most comprehensible government spending database, available at usaspending.gov. This database contains all the procurement transactions between private firms and the Federal Government in the US. It has the awarded amounts given to firms at the transaction level. The database is publicly available on their website, and it runs from 2001 to present. Government purchases released by USA Spending are composed by an average of about 3 million yearly transactions, with a scope on about 160 thousand companies each year and covering nearly all the sectors in the economy. An extensive analysis of the features of this database is made by [Cox et al. \(2020\)](#) where they report that government spending is concentrated in few sectors and firms, in sectors that have more sticky prices, that government contracts are short, and fluctuations in aggregate government spending are driven mainly by granular fluctuations in the sense of [Gabaix \(2011\)](#). More importantly, they show that the data on procurement is a good representation of total government spending.

Here we study at what extent government spending is concen-

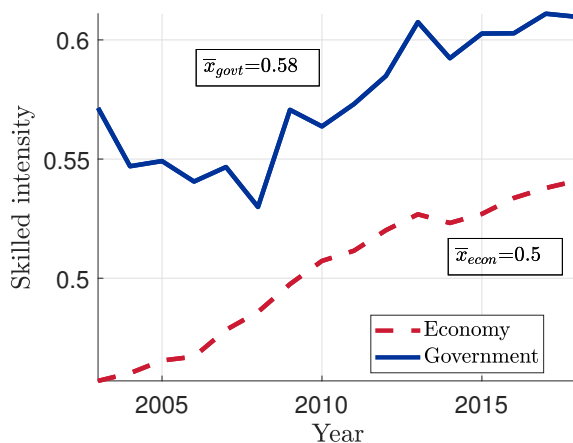
trated towards more or less skilled intensive sectors. To do so, we calculate in each year, the share of government spending on each sector. Then, with the CPS data, we calculate the share of labor income paid to skilled and unskilled within each sector. With the latter, we obtain the skill intensity by sector, and also we can obtain a measure of the average skill intensity of the economy. Having the skill intensities of every sector in the economy, we can calculate the average share of skilled and unskilled income of government spending. That is a measure of the level of skills the government is buying from the private sector in average.

Table ?? shows the five main sectors that supply goods to the government. We display the share of spending on that sector out of total government spending G_j/G , the cumulative share, and the share of skilled income of each sector. What can be seen from that table is that government spending is concentrated towards few sectors. In fact, 44% of the purchases is concentrated on these five sectors. Aerospace manufactures distinguishes as it exceeds by several percentage points the second largest sector. Aerospace manufactures accounts for 13.6% of the government spending in the period 2001-2020. This, followed by R&D services. As we may suppose, these sectors have a large share of skilled workers.

The purpose of this analysis is to study the share of skilled income embedded in government purchases as a whole. Figure 2.4 displays the average share of skilled income of the economy (red-dashed line) and the average share of skilled income of government purchases (blue-solid line). Two main conclusions arise from this picture. First, that the skilled income share embedded in government purchases is at all times larger than that of the overall economy. The skilled income share on government purchases (on average) is about 58% for the 2003-2018 sample, while that share on the economy as a whole is about 50%. Second, the skilled income share in both measures is increasing, at least from 2008. The trend is a consequence of the rise

in the share of skilled workers in the population, which increased from 30% to 40% between 2003 and 2018, according to CPS. Third, the share of skilled in government purchases fluctuates more than that of the economy. In fact this share fell from 2003 to 2008, while it begun an upward trend in the periods after.

FIGURE 2.4: Average Share of Skilled Income



Notes: This figure depicts the average share of skilled workers. The red-dashed line shows the average share of skilled labor income in the overall economy calculated from the Current Population Survey. The blue-solid line shows the average share of skilled income weighted by sectoral government spending according to data from [usaspending.gov](https://www.usaspending.gov).

Summary of the Empirical Findings. First, we find that the earnings gap increases in response to a government spending shock. We show this by estimating a Bayesian VAR augmented with the gap and find that government purchases generate inequality. Second, we show that over time, there is a negative relationship between the responses of consumption and the earnings gap to a government spending shock. And finally, we show that the government purchases goods from sectors that hire a higher share of skilled workers than the overall economy. We think these findings are important for sev-

eral reasons. First, because government spending, while generating inequality it is distributing income between different worker types. And second, because these redistribution of resources might impact the response of consumption as the estimations show.

2.3 MODEL

Our model is a Two-Agent New Keynesian (TANK) model with two sectors, two groups of workers (skilled and unskilled), and a share of financially constrained households within the groups. The model is an extension of [Debortoli and Galí \(2018\)](#) and [Bilbiie \(2008\)](#). We assume that for each group of workers, wages are determined by a union on behalf of households. A continuum of measure one households populates each skill group, where there is a fixed share of financially constrained agents. These constrained households can not save, borrow, or hold equity; while the rest have full access to financial markets. We assume there are two sectors which require different shares of skilled and unskilled workers in technology. Production of both sectors is demanded by the government and households. There are two types of firms in each sector. There is a continuum of monopolistically competitive intermediate goods producers and a final goods producer that aggregates these intermediate goods through a CES production function. The intermediate producers demands workers of both types and are subject to price adjustment costs. We close the model with a Taylor rule.

2.3.1 *Government*

The key feature of our model is how government spending is distributed among the different sectors. If government spending is distributed differently among the sectors, an increase in *total* government purchases has distributional effects. That has consequences on the dis-

tribution of income between skilled and unskilled households as long as the sectors hire the two types of workers in different proportions.

The government in this model has preferences over the sectors of the economy. The government solves a static problem in which it delivers utility from a Cobb-Douglas composite of the sectors

$$G_t = G_{1t}^{\aleph} G_{2t}^{1-\aleph},$$

where \aleph is the share of spending on sector one out of total spending. From now on, we consider the sector one as the skilled intensive sector. Hence, \aleph is the share of government spending in the skilled intensive sector. The government solves the following static cost minimization problem

$$\min_{G_{1t}, G_{2t}} P_{1t} G_{1t} + P_{2t} G_{2t} - P_t^G \left(G_{1t}^{\aleph} G_{2t}^{1-\aleph} - G_t \right)$$

where P_t^G is a Lagrange multiplier that coincides with the government price index. Cost minimization implies the following government demands for each good

$$G_{1t} = \aleph \left(\frac{P_{1t}}{P_t^g} \right)^{-1} G_t, \quad G_{2t} = (1 - \aleph) \left(\frac{P_{2t}}{P_t^g} \right)^{-1} G_t, \quad (2.3)$$

where P_t^g is the government's price index which is given by

$$P_t^G = \frac{1}{\aleph^{\aleph} (1 - \aleph)^{1-\aleph}} P_{1t}^{\aleph} P_{2t}^{1-\aleph}.$$

This price index is different to the consumer price index as long as household's preferences are different to government's preferences. To finance purchases, the government sets a flat rate on labor income τ_t . We assume the government finances spending with a budget balance, which requires

$$G_t = \tau_t W_t N_t,$$

where $W_t N_t$ denotes aggregate labor income. We assume that aggregate government spending G_t is exogenous and follows an AR(1) process with persistence ρ_g . In the exercises below we study the effects of an increase in total government spending G_t which is distributed according to the demands in Equation (2.3), instead of analyzing the impact of raising G_{1t} or G_{2t} separately.

2.3.2 Households

There are two groups of workers, skilled and unskilled, denoted by s and u , respectively. Each household belongs to a group $h \in \{s, u\}$ with μ the share of unskilled workers while $(1 - \mu)$ the share of skilled workers. We assume that a share λ_h of households in skill group h have no access to financial markets (cannot borrow, lend, and own firms' shares), while the remaining $(1 - \lambda_h)$ are unconstrained (they can borrow, lend, and own firms). We index with i the dimension of access to financial markets; i.e., $i \in \{k, r\}$, with r denoting unconstrained (with r for Ricardian) and k denoting constrained (with k for Keynesian). Hence, household features are given by a pair (i, h) of indices.

A household (i, h) derives utility from consumption and disutility from labor, maximizing its lifetime utility, time-discounted at a factor $0 < \beta < 1$, given by

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \mathcal{U}(c_{ht}^i, n_{ht}), \quad (2.4)$$

where c_{ht}^i is total real consumption and n_{ht} is hours supplied by the household. In particular, we assume a separable utility function of the form

$$\mathcal{U}(c_{ht}^i, n_{ht}^i) = \frac{(c_{ht}^i)^{1-\gamma}}{1-\gamma} - \chi_h \frac{(n_{ht})^{1+\varphi}}{1+\varphi},$$

where γ is the inverse of the intertemporal elasticity of substitution, φ is the inverse of the Frisch elasticity of the labor supply, and χ_h is the parameter of disutility of labor of each worker's group.

As the economy is two-sector, total household's consumption is given by a bundle of the two goods. We assume total household's consumption is derived from a Cobb-Douglas composite of the goods produced by these two sectors,

$$c_{ht}^i = (c_{1ht}^i)^\xi (c_{2ht}^i)^{1-\xi}. \quad (2.5)$$

c_{jht}^i is consumption of good j by household i who belongs to group h at time t . ξ is the share of spending on good one, the skilled intensive. The Cobb-Douglas aggregator implies the following demands for each good:

$$c_{1ht}^i = \xi \left(\frac{P_{1t}}{P_t^C} \right)^{-1} c_{ht}^i, \quad c_{2ht}^i = (1 - \xi) \left(\frac{P_{2t}}{P_t^C} \right)^{-1} c_{ht}^i. \quad (2.6)$$

With these demands, we can derive the consumer price index which is given by

$$P_t^C = \frac{1}{\xi(1-\xi)} P_{1t}^\xi P_{2t}^{1-\xi}. \quad (2.7)$$

In what follows, we assume all consumers have the same preferences for the different goods; i.e. ξ is equal for all consumers. This assumption simplifies aggregation, and allows us to determine final consumption as the numeraire good. Then, when we mention sectoral prices, we are referring to the price *relative* to the final consumption good. Although there are no differences in preferences between the different workers, they do have differences in their access to financial markets and income, so their total consumption fluctuates differently.

Unconstrained Households' Problem. Unconstrained households can accumulate risk-free bonds and their budget constraint is given

by

$$b_{ht+1}^r = (1 + r_t)b_{ht}^r + (1 - \tau_t^n)\tilde{w}_{ht}n_{ht} + D_{ht}^r - c_{ht}^r, \quad (2.8)$$

where $\tilde{w}_{ht} = \tilde{W}_{ht}/P_t^C$ is the real per capita wage per unit of labor n_{ht} . We assume $\tilde{w}_{st} = w_{st}/(1 - \mu)$ and $\tilde{w}_{st} = w_{ut}/\mu$, where w_{ht} are the wages at which firms demand the different workers. Due to labor market frictions n_{ht} is taken as given by the household; r_t is the real return on risk-free bonds; and D_{ht}^r are firm's dividends accrued by unconstrained households of group h , who receive a fixed fraction of total shares that are distributed among unconstrained households of both types of workers as we explain below. Hence, these workers maximize function (2.4) subject to constraint (2.8). The maximization problem of these households gives as a result the Euler equation

$$1 = \beta(1 + r_t)\mathbb{E}_t \left(\frac{c_{ht}^r}{c_{ht+1}^r} \right)^{-\gamma}. \quad (2.9)$$

Constrained Households' Problem. Constrained households consume their flow of disposable income every period. Hence, consumption is given by

$$c_{ht}^k = (1 - \tau_t^n)\tilde{w}_{ht}n_{ht}, \quad (2.10)$$

where as they are out of the financial market, consume their disposable income, which is their labor income after taxes. They are also subject to frictions in labor markets so they take n_{ht} as given, which is determined by the union as we explain below.

The difference between constrained and unconstrained consumers is crucial in our model because it implies different MPCs out of total income among households. From the permanent income hypothesis, we know that the MPC of unconstrained consumers is approximately $r/(1+r)$, while that of the constrained worker is equal to one, as Equation (2.10) shows. Those differences generate departing consumption dynamics between groups as long as the shares of hand-to-mouth λ_h 's are distinct and labor income fluctuates differently. The group with

higher λ_h has a higher average MPC; hence, their consumption responds much more to labor income fluctuations than the other group. These are the features that we will exploit in the analysis below.

Distribution of Monopoly Profits. In New Keynesian models, monopoly profits are an essential source of fluctuations. As we assume monopolistic competition in intermediate markets, firms charge a markup over marginal costs. With sticky prices, this markup fluctuates. As there are differences in access to financial markets and the sources of income of the different consumers are different, fluctuations in markups have distributional effects we must take into account. A widely known result is that markups are countercyclical in response to demand shocks in this class of models. The implication of this is that in a boom, markups fall, so labor income gets a higher proportion of total income. This effect typically generates amplification effects from limited asset participation. That is why the distribution of monopoly profits matters.

Therefore, to avoid “spurious” redistribution from aggregate variables to not wealthy agents, we assume the distribution of profits is according to the data. In particular, we set the distribution of profits to unconstrained consumers in each group of workers to be equal to a share of total profits in the economy. This share is denoted by ϑ_h , which we calibrate according to the *Survey of Consumer Finances 2016*. Therefore, per-capita dividends are given by

$$D_t^u = \frac{\vartheta_u}{\mu(1 - \lambda_u)} D_t \quad \text{and} \quad D_t^s = \frac{\vartheta_s}{(1 - \mu)(1 - \lambda_s)} D_t. \quad (2.11)$$

2.3.3 Labor Supply

We assume that due to labor market frictions both the constrained and unconstrained workers of a group h supply the same quantity of labor. In our setting, the labor supplied is determined by a union that represents each worker type h and sets a common labor supply

for all households in the same worker group. Essentially, we split the consumption-labor problem described above in two: the consumption and the labor problem. The union solves the latter by maximizing the average utility of workers in group h :

$$\begin{aligned} \max_{c_{ht}^r, c_{ht}^k, n_{ht}} \quad & \lambda_h \frac{(c_{ht}^k)^{1-\gamma}}{1-\gamma} + (1-\lambda_h) \frac{(c_{ht}^r)^{1-\gamma}}{1-\gamma} - \frac{n_{ht}^{1+\varphi}}{1+\varphi} \\ \text{s.t.} \quad & \\ & b_{ht+1}^r = (1+r_t)b_{ht} + (1-\tau_t^n)\tilde{w}_{ht}n_{ht} + D_{ht}^r - c_{ht}^r, \\ & c_{ht}^k = (1-\tau_t^n)\tilde{w}_{ht}n_{ht} \end{aligned}$$

The solution of this problem delivers the following labor supply for each workers' group h :

$$\tilde{w}_{ht} = \chi_h \frac{\overline{mg}_{ht} n_{ht}^\varphi}{(1-\tau_t)}$$

where $\overline{mg}_{ht} = (\lambda_h (c_{ht}^k)^{-\gamma} + (1-\lambda_h)(c_{ht}^r)^{-\gamma})^{-1}$, which implies that the labor supply in our model depends on the average marginal rate of substitution of constrained and unconstrained in the group h . We assume this to avoid insurance with labor. In this case, we obtain well behaved labor supplies whereas if individual consumers are let to determine their own supply, the constrained consumer could have an inelastic labor supply (if preferences for consumption are logarithmic) as shown by [Bilbiie \(2008\)](#).¹⁶

Finally, we assume each workers' group work in both sectors, and hence the supply of labor must meet the sum of the demands from all sectors:

$$n_{ht} = n_{1ht} + n_{2ht}.$$

where n_{jht} is the total hours worked by workers' group h in sector j at a given period t .

¹⁶This approach is also used by [Auclert et al. \(2018\)](#) when studying the effects of fiscal transfers in HANK.

2.3.4 Firms

The two sectors in this economy are populated by a continuum measure one of intermediate goods producers that are in monopolistic competition. These sectors demand both types of workers in a different proportion, which we consider as a technological feature. Next, we describe the setup and optimality conditions for a sector $j \in \{1, 2\}$.

Final Goods Producers. In sector j , a competitive representative firm produces a final good by aggregating a continuum of intermediate inputs with a CES production function,

$$Y_{jt} = \left(\int_0^1 y_{jt}(m)^{\frac{\varepsilon-1}{\varepsilon}} dm \right)^{\frac{\varepsilon}{\varepsilon-1}}.$$

This composite aggregates a continuum of intermediate goods with measure one, with $m \in [0, 1]$. In this setting, the final firm decides how to allocate its demand among the different intermediate goods. After cost minimization, the demand for each intermediate good m , and sector's j price index write

$$y_{jt}(m) = \left(\frac{p_{jt}(m)}{P_{jt}} \right)^{-\varepsilon} Y_t, \quad \text{and} \quad P_{jt} = \left(\int_0^1 p_{jt}(m)^{1-\varepsilon} dm \right)^{\frac{1}{1-\varepsilon}}.$$

Intermediate Goods Producers: Labor Demand. Each intermediate good m in sector j is produced by a monopolistically competitive producer using labor of both skill groups according to the production function

$$y_{jt}(m) = A_{jt} n_{jst}(m)^{\omega_j} n_{jut}(m)^{1-\omega_j},$$

where ω_j is the share of (total) skilled income in sector j and A_{jt} is the productivity of the sector that allows us to calibrate the size of the sector.

Each intermediate producer hires workers from each skill group h at a real wage w_{ht} . Therefore, the demand of the sector for the workers

of group h implies per-capita wages given by¹⁷

$$w_{ht} = mc_{jt} \omega_j \frac{Y_{jt}}{n_{jst}} \text{ and } w_{ut} = mc_{jt} (1 - \omega_j) \frac{Y_{jt}}{n_{jut}}.$$

Intermediate Goods Producers: Price Setting. In each sector, the intermediate producer chooses its price to maximize profits subject to [Rotemberg \(1982\)](#) price adjustment costs, denoted by $\Theta_{jt}(m)$. These adjustment costs are quadratic in the rate of price change $\frac{p_{jt}(m)}{p_{jt-1}(m)} - 1$ and are expressed as a fraction of output $p_{jt}(m)y_{jt}(m)$:

$$\Theta_{jt}(m) = \frac{\theta_j}{2} \left(\frac{p_{jt}(m)}{p_{jt-1}(m)} - 1 \right)^2 p_{jt}(m)y_{jt}(m).$$

Therefore, each intermediate producer chooses $\{p_{jt}(m)\}_{t \geq 0}$ to maximize

$$\mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \left(\frac{c_{t+1}^r}{c_t^r} \right)^\gamma \{ \Pi_t(p_{jt}(m)) - \Theta_{jt}(m) \},$$

with

$$\Pi_{jt}^m(p_{jt}(m)) = \left(\frac{p_{jt}(m)}{P_{jt}} - mc_{jt}(m) \right) \left(\frac{p_{jt}(m)}{P_{jt}} \right)^{-\varepsilon_j} y_{jt},$$

where $\beta \left(\frac{c_{t+1}^r}{c_t^r} \right)^\gamma$ is the stochastic discount factor of the pool of unconstrained agents, and $mc_{jt}(m)$ is the marginal cost. Given the assumptions above, the inflation rate (after the intermediate firms

¹⁷These optimality conditions arise from minimizing costs subject to technology (after symmetry):

$$\min_{n_{jst}, n_{jut}} w_{st}n_{jst} + w_{ut}n_{jut} - mc_{jt} \left(A_{jt} n_{jst}^{\omega_j} n_{jut}^{1-\omega_j} - Y_{jt} \right)$$

where mc_{jt} is the Lagrange multiplier of the cost minimization problem of firms, which in equilibrium corresponds to the real marginal cost (nominal marginal cost of sector j divided by the price level). Firms minimize a per-capita labor cost, taking into account the shares of population of the different workers. This is an assumption which allows us to close the model.

optimization) is determined by the following New Keynesian Phillips curve for sector j :

$$\begin{aligned} (\pi_{jt} - \bar{\pi}_j)\pi_{jt} = & \frac{\varepsilon_j}{\theta_j} \left(\frac{mc_{jt}}{p_{jt}} - \frac{\varepsilon_j - 1}{\varepsilon_j} \right) \\ & + \beta \mathbb{E}_t \left[\beta^t \left(\frac{c_{t+1}^r}{c_t^r} \right)^\gamma (\pi_{jt+1} - \bar{\pi}_j)\pi_{jt+1} \frac{p_{jt+1}y_{jt+1}}{p_{jt}y_{jt}} \right], \end{aligned} \quad (2.12)$$

with

$$\pi_{jt} = \frac{p_{jt}}{p_{jt-1}} \pi_t, \quad (2.13)$$

where π_t denotes CPI inflation.¹⁸

Intermediate firms generate each period an aggregate amount of profits given by

$$D_{jt} = (1 - mc_{jt})Y_{jt} - \frac{\theta_j}{2} \pi_{jt}^2 Y_{jt},$$

that are distributed among households according to the rules described above.

2.3.5 Monetary Authority

In the presence of nominal rigidities, the return on assets r_t is affected by monetary policy, which sets the nominal interest rate i_t according to a Taylor rule

$$i_t = \rho + \phi_\pi \mathbb{E}_t \pi_{t+1},$$

where ϕ_π is the preference parameter of the monetary authority with respect to expected inflation and ρ is the steady state interest rate which is equal to the discount rate. Given the inflation level and the nominal interest rate, the real return on the risk-free asset is determined by the Fisher equation $r_t = i_t - \mathbb{E}_t \pi_{t+1}$.

¹⁸This expression arises from the definition of sectoral inflation $\pi_{jt} = \frac{p_{jt}}{p_{jt-1}} = \frac{p_{jt}P_t^C}{p_{jt-1}P_{t-1}^C} = \frac{p_{jt}}{p_{jt-1}} \pi_t$.

2.3.6 Equilibrium

An equilibrium of this economy is given by paths of individual variables for households' decisions $\{c_{ht}^i\}_{t \geq 0} \forall (i, h)$; labor market prices and quantities $\{\{n_{1st}, n_{2st}, w_{st}\}, h \in \{u, s\}\}_{t \geq 0}$; prices and returns $\{p_{1t}, p_{2t}, \pi_{1t}, \pi_{2t}, r_t, i_t\}_{t \geq 0}$, and aggregate quantities such that: (i) households maximize their objective functions taking as given both prices and aggregate quantities; (ii) the government budget constraint holds; and (iii) all markets clear. In our economy, we have five markets: two goods markets, the market for bonds, and two labor markets.

As we assume that each class of workers are split between constrained and unconstrained, the aggregation of group h consumption is

$$c_{ht} = \lambda_h c_{ht}^k + (1 - \lambda_h) c_{ht}^r,$$

then, aggregate consumption writes

$$C_t = \mu_u c_{ut} + \mu_s c_{st}.$$

Goods market clearing in each sector is given by

$$y_{jt} = C_{jt} + G_{jt} + \Theta_{jt},$$

where C_{jt} and G_{jt} are given by the demand for each good. And finally, aggregate goods market clearing implies

$$GDP_t = p_{1t} y_{1t} + p_{2t} y_{2t} = C_t + p_t^g G_t + \Theta_{1t} + \Theta_{2t}$$

2.4 ANALYTICAL RESULTS

In this section, we obtain two analytical results that guide us in understanding the role of labor income inequality in the transmission of government spending shocks to consumption and output. First,

we show how the earnings gap affects aggregate consumption. In particular, we study how, due to market incompleteness, the earnings gap influences consumption behavior by entering in the aggregate Euler equation. There we show that if the earnings gap increases in response to a government spending shock, the response of consumption is dampened. Second, we show the conditions under which the earnings gap increases in response to a government spending shock in our model.

To study these questions in a simple and tractable way, throughout this section, we assume that the share of hand-to-mouth workers in the unskilled group of workers is equal to one and the share of hand-to-mouth in the skilled group is zero. Additionally, we study a symmetric equilibrium in steady state, in which wages of both workers are equal which implies that sectoral prices are the same.

2.4.1 *Aggregate Demand and the Earnings Gap*

In what follows, we show that in the presence of financial markets incompleteness, the earnings gap enters the aggregate demand equation.

Following [Debortoli and Galí \(2018\)](#), we exploit that when there is limited access to financial markets, the IS equation (or the aggregate demand) depends on inequality wedges. Recall aggregate consumption $C_t = \mu c_{ut} + (1 - \mu)c_{st}$, where each group's consumption is given by $c_{ht} = \lambda_h c_{ht}^k + (1 - \lambda_h)c_{ht}^r$. Under the assumptions we mentioned above ($\lambda_u = 1$ and $\lambda_s = 0$), consumption of the unskilled workers is $c_{ut} = c_{ut}^k$ and consumption of the skilled workers is $c_{st} = c_{st}^r$. Notice that the aggregate share of hand-to-mouth is now given by μ . Then in this case, c_t^s is determined by a Euler equation, and c_t^u is equal to labor income.

Next, we introduce the *consumption gap*, which is defined as the percent difference between workers' consumption with respect to

skilled consumption, given by $\nu_t = 1 - \frac{c_{ut}}{c_{st}}$. In equilibrium, consumption of unskilled workers is $c_{ut} = (1 - \tau_t)w_{ut}n_{ut}$ while that of skilled workers is $c_{st} = (1 - \tau_t)w_{st}n_{st} + \frac{1}{1-\mu}D_t$. Therefore, the consumption gap writes

$$\nu_t = 1 - \frac{w_{ut}n_{ut}}{w_{st}n_{st} + \frac{1}{1-\mu}D_t} = 1 - \frac{1}{\eta_t + \frac{1}{1-\mu}\delta_t}. \quad (2.14)$$

Equation (2.14) shows that in this setup, the consumption gap depends on two variables, the earnings gap $\eta_t = \frac{w_{st}n_{st}}{w_{ut}n_{ut}}$, and the ratio of dividends to labor income of the unskilled, $\delta_t = \frac{D_t}{(1-\tau_t)w_{ut}n_{ut}}$. The log-linear approximation of the consumption gap (Equation (2.14)) is given by

$$\widehat{\nu}_t = \nu_\eta \widehat{\eta}_t + \nu_\delta \widehat{\delta}_t, \quad (2.15)$$

with $\nu_\eta = \frac{\eta}{\nu} \frac{1}{(1-\nu)^2}$ and $\nu_\delta = \frac{\delta}{\nu} \frac{1}{1-\mu} \frac{1}{(1-\nu)^2}$. Recall that the agents who can save or borrow are the skilled workers; hence, there is only one Euler equation which writes

$$\widehat{c}_{st} = \mathbb{E}_t\{\widehat{c}_{st+1}\} - \frac{1}{\gamma}(r_t - \rho), \quad (2.16)$$

where γ is the inverse of the intertemporal elasticity of substitution, and ρ is the time discount rate. Rewrite aggregate consumption as $C_t = c_{st}(1 - \mu\nu_t)$, which in log deviations with respect to the steady state is

$$\widehat{c}_t = \widehat{c}_{st} - \frac{\mu}{1 - \nu\mu} \widehat{\nu}_t.$$

Using this equation and replacing it in Equation (2.16), we obtain the aggregate Euler equation:

$$\widehat{c}_t + \frac{\mu}{1 - \nu\mu} \widehat{\nu}_t = \mathbb{E}_t\left\{\widehat{c}_{t+1} + \frac{\mu}{1 - \nu\mu} \widehat{\nu}_{t+1}\right\} - \frac{1}{\gamma}(r_t - \rho). \quad (2.17)$$

Substituting Equation (2.15) in Equation (2.17) and solving forward, we obtain the expression for contemporaneous consumption

$$\widehat{c}_t = -\frac{1}{\gamma} \mathbb{E}_t \sum_{s=0}^{\infty} \widehat{r}_{t+s} - \frac{\mu\nu\eta}{1 - \nu\mu} \widehat{\eta}_t - \frac{\mu\nu\delta}{1 - \nu\mu} \widehat{\delta}_t. \quad (2.18)$$

Equation (2.18) shows how aggregate consumption is related to the earnings gap. When the earnings gap increases in response to a government spending shock, consumption goes down, all else being equal. The relationship between the earnings gap and consumption is negative because when the earnings gap rises, there is a redistribution of resources (on average) from high- to low-MPC consumers; i.e., the labor income of the skilled with low-MPCs rises by more than the labor income of the unskilled with high-MPCs. This effect is similar to the one emphasized by Bilbiie (2020) which is that the cyclical income of the high-MPC consumer is what matters for the amplification or dampening effects of inequality. We extend his argument by exploring the impact of different cyclicalities among workers in segmented labor markets.

Our model also features the effect of inequality typical of TANK models. This effect operates through profits $\hat{\delta}_t$. Recall that $\hat{\delta}_t = \hat{d}_t - (\hat{w}_{ut} + \hat{n}_{ut} - \hat{r}_t)$ and notice that dividends \hat{d}_t are countercyclical in response to demand shocks. In New Keynesian models markups are countercyclical, while unskilled labor income is procyclical. That means that δ_t falls whenever output goes up. This fall has a positive effect on consumption because the income of the unskilled worker is increasing by more than the profits delivered to skilled workers. Therefore, fluctuations in markups generate a redistribution of resources from low- to high-MPC consumers. This is the channel emphasized by Galí et al. (2007) who generate a positive response of output to a government spending expansion. We can use countercyclical markups to obtain a rise in consumption in two ways: (i) with a highly responsive markup (with very rigid prices); and (ii) with a high share of HtM agents (here μ). The latter is the one explored by Galí et al. (2007) who use the estimates for the share of HtM obtained Campbell and Mankiw (1989). Therefore, two forces depending on inequality affect consumption in our model: the cyclicality of the earnings gap and countercyclical markups.

These results are consistent with two of the empirical findings described above. First, consumption may rise in response to a government spending shock; countercyclical markups mainly drive this effect. Second, there is a negative relationship between the earnings gap and consumption. Interestingly, even if the mechanism implied by δ_t is strong, there might be a negative response of consumption. Hence, how the earnings gap responds can reverse the sign of the response of consumption to a rise in government spending. That result depends on the relative strength of the earnings gap response to that of markups.

Finally, to show how this translates to the response of output, we impose goods market clearing ($\hat{y}_t = (1 - \gamma_g)\hat{c}_t + \gamma_g\hat{g}_t$) to obtain the IS equation

$$\hat{y}_t = -\frac{1 - \gamma_g}{\gamma} \mathbb{E}_t \sum_{s=0}^{\infty} \hat{r}_{t+s} - \frac{\mu\nu\eta(1 - \gamma_g)}{1 - \nu\mu} \hat{\eta}_t - \frac{\mu\nu\delta(1 - \gamma_g)}{1 - \nu\mu} \hat{\delta}_t + \gamma_g\hat{g}_t. \quad (2.19)$$

Equation (2.19) is the expression for output. Government spending enters directly as in the baseline New Keynesian model; however, the response of output (the fiscal multiplier), now depends on the response of the earnings gap and markups. If the earnings gap increases, the multiplier falls; otherwise, the fiscal multiplier increases due to countercyclical markups $\hat{\delta}_t$.

2.4.2 Government Purchases and the Earnings Gap

Next, we derive an expression for the earnings gap that depends on government spending. Recall that government spending is exogenous in our model, and it distributes among the sectors. As the labor demand depends on firms' output, labor income is a function of government spending through the demand for production. Moreover, the earnings gap depends on government purchases since the govern-

ment demands the two sectors (which have different shares of skilled workers) in different proportions.

Take the labor demand for group h from sector j

$$\widehat{w}_{ht} = \widehat{m}c_{jt} + \widehat{y}_{jt} - \widehat{n}_{jht}, \quad \text{for } j \in [1, 2] \text{ and } h \in [s, u], \quad (2.20)$$

where \widehat{w}_{ht} and $\widehat{m}c_{jt}$ are the real wages and marginal costs with respect to the price index, respectively. Take the loglinear approximation of the labor supply and the aggregate labor by worker groups

$$\widehat{w}_{ht} = \varphi \widehat{n}_{ht} + \widehat{m}g_{ht} + \widehat{\tau}_t,$$

and

$$\widehat{n}_{ht} = \kappa_{1h} \widehat{n}_{1ht} + \kappa_{2h} \widehat{n}_{2ht},$$

with $\kappa_{jh} = \frac{n_{jh}}{n_h}$ the share of labor supplied to sector j by a given workers' group h in steady state. By definition, $\kappa_{1h} + \kappa_{2h} = 1$.¹⁹

We can obtain a total demand for the workers of group h , by taking the weighted sum of the demands from each sector, from Equation (2.20). These demands write:

$$\widehat{w}_{ht} = \kappa_{1h}(\widehat{m}c_{1t} + \widehat{y}_{1t}) + \kappa_{2h}(\widehat{m}c_{2t} + \widehat{y}_{2t}) - \widehat{n}_{ht} \quad \text{for } h \in [s, u].$$

Then, equilibrium labor income is given by

$$\widehat{w}_{ht} + \widehat{n}_{ht} = \kappa_{1h}(\widehat{m}c_{1t} + \widehat{y}_{1t}) + \kappa_{2h}(\widehat{m}c_{2t} + \widehat{y}_{2t}),$$

and the earnings gap ($\widehat{\eta}_t = [\widehat{w}_{st} + \widehat{n}_{st} - (\widehat{w}_{ut} + \widehat{n}_{ut})]$) is

$$\widehat{\eta}_t = (\kappa_{1s} - \kappa_{1u})(\widehat{y}_{1t} - \widehat{y}_{2t}) + (\kappa_{1s} - \kappa_{1u})(\widehat{m}c_{1t} - \widehat{m}c_{2t}).$$

To obtain an expression for the earnings gap depending on government spending, we need expressions for production in sectors one

¹⁹We derive the expression for κ_{jh} in Appendix 2.D, where we show that the shares of labor in the different sectors depend on the skilled and unskilled intensities and the relative sizes of the sectors.

and two. Taking $\hat{y}_{jt} = \frac{C_j}{Y_j}\hat{c}_{jt} + \frac{G_j}{Y_j}\hat{g}_{jt}$ for $j = \{1, 2\}$, it can be shown that around a symmetric steady state (where $P_1 = P_2 = P^C$):

$$\begin{aligned}\hat{y}_{1t} &= \frac{\xi(1-\gamma_g)}{n}\hat{c}_t + \frac{\aleph\gamma_g}{n}(\hat{g}_t + \hat{p}_t^G) - \hat{p}_{1t}, \\ \hat{y}_{2t} &= \frac{(1-\xi)(1-\gamma_g)}{1-n}\hat{c}_t + \frac{(1-\aleph)\gamma_g}{1-n}(\hat{g}_t + \hat{p}_t^G) - \hat{p}_{2t},\end{aligned}$$

with $n = \xi(1-\gamma_g) + \aleph\gamma_g$ the size of sector one. Plugging $\hat{y}_{1t} - \hat{y}_{2t}$ into the Earnings Gap, and using the expression for aggregate output $\hat{p}_t^Y + \hat{y}_t = (1-\gamma_g)\hat{c}_t + \gamma_g(\hat{p}_t^G + \hat{g}_t)$, we obtain the Earnings Gap depending on aggregate variables,

$$\hat{\eta}_t = \Upsilon_\eta\hat{g}_t - \Upsilon_\eta\hat{y}_t + \Upsilon_p(\hat{p}_{1t} - \hat{p}_{2t}) \quad (2.21)$$

Equation (2.21) is the earnings gap in our economy. The earnings gap depends on government purchases, output, and prices. The parameters Υ_x are the relationships between labor income inequality and the different variables. First, we have the relationship with aggregate variables, given by $\Upsilon_\eta = \frac{(\kappa_{1s} - \kappa_{1u})}{n(1-n)}(\aleph - \xi)\gamma_g$. Notice that this parameter governs both the cyclicity of the Earnings Gap and its direct relationship with government spending. This is an important result of our model. The earnings gap rises (in response to an increase in government spending) if two conditions hold. First, sector one is the more skilled intensive, i.e. $\kappa_{1s} > \kappa_{1u}$. Second, the share of government spending on sector one is larger than the share of private consumption in that sector, $\aleph > \xi$. Therefore, if government spending is concentrated on skilled sectors in a higher proportion than the overall economy, the earnings gap rises in response to an increase in government spending, which is consistent with the empirical evidence presented in Section 2.2.

Additionally, under the conditions above, the earnings gap is countercyclical. This is a consequence of the crowding out effect of government spending on consumption. When there is an increase

in \hat{g}_t , the economy distributes resources to workers, who spend their resources in the two sectors. However, in this setup, the increase in production may not be enough to satisfy the greater demand, and hence, the crowding out takes place. Therefore, the positive relationship between the earnings gap and government spending relies also on $\hat{g}_t > \hat{y}_t$, which holds for any reasonable calibration of our model.

Finally, the earnings gap depends on price dispersion. This relationship is given by $\Upsilon_p = \frac{(\kappa_{1s} - \kappa_{1u})}{n(1-n)} (\aleph - \xi)^2 \gamma_g (1 - \gamma_g)$. This dependence arises from the differences between spending by government and private consumption, and represent the differences in price index of private and public spending. The relation is positive because when prices in sector one rise by more than in sector two, the government (and the economy) must spend more resources in the skilled intensive sector, increasing the relative income earned by skilled workers. However, this effect is quantitatively unimportant as it is two orders of magnitude lower than v_η .

Next, we study the role of government spending composition in the transmission of government spending to consumption and output in the model without the simplifying assumptions we made in this section.

2.5 QUANTITATIVE RESULTS

In this section, we study quantitatively at what extent the the sectoral distribution of government spending affects inequality, consumption, and the fiscal multiplier. We first describe the calibration we set to match the empirical facts presented in section 2.2. Then, we explore the role of government spending distribution and the role of financial constraints in explaining those facts.

2.5.1 Calibration

Household Problem Parameters. We set the inverse of the intertemporal elasticity of substitution γ , the inverse of the Frisch elasticity of labor supply φ , and the disutilities from labor χ_h , equal to one. We calibrate the shares of unskilled and skilled workers as $\mu = 0.35$ and $(1 - \mu) = 0.65$, that we obtain from the CPS for period 2001-2020.²⁰ The discount factor β is set such that the interest rate is one-percent quarterly in steady state. We assume the shares of hand-to-mouth in each group of workers equal to $\lambda_u = 0.47$ and $\lambda_s = 0.18$. Additionally, we observe that skilled workers hold 83% of the total equity in the economy. Hence, we set $\vartheta_s = 0.83$ and $\vartheta_u = 0.17$.²¹

Production and Price Rigidities. We build two sectors that produce goods requiring different skill intensities. We assume sector one is the skilled intensive sector and set $\omega_1 = 0.7$; we assume sector two is the unskilled intensive sector and we set $\omega_2 = 0.3$.²² We set the price adjustment cost parameters equal to 100 in both sectors ($\theta_1 = \theta_2 = 100$). We set the elasticity of substitution ε_1 and ε_2 equal to 10 for both sectors.²³ We calibrate the remainder of parameters symmetric between the two sectors, such that both are the same size and deliver the same aggregate income for both types of workers, as in Figure 2.4; i.e., the productivities and the size of the sectors are equal in steady state. The purpose of this is to assume sectors that are symmetric on everything except from the share of income

²⁰According to the CPS, these are the average shares (in hours) of the two types of workers for the period 2000-2019.

²¹According to the Survey of Consumer Finances 2016.

²²We set $\omega_1 = 0.7$ because it is a midpoint between 0.58 and 1, and $\omega_2 = 0.3$ because it is in the midpoint between 0 and 0.5. These are the bounds for these parameters according to the data as shown in Figure 2.4.

²³This calibration is equivalent to having a Calvo parameter given by 0.75, which is in the upper bound of the empirical estimates. According to this calibration, prices last 4 quarters. We use the correspondence between Calvo and Rotemberg proposed by Ascari et al. (2011) $\theta = \frac{(\varepsilon-1)\zeta}{(1-\zeta)(1-\beta\theta)}$, with ζ the Calvo probability of keeping prices.

delivered to each type of worker driven by the ω_j s, and the demands for each sector. With this calibration, and the household parameters, we obtain an earnings gap in steady state equal to 1.85, which is about the average of the period 1990-2018.

Demand for Goods. In the baseline calibration, we set the share of spending of government in sector one \aleph according to Figure 2.4, to satisfy $0.58 = \aleph\omega_1 + (1 - \aleph)\omega_2$. That implies that in the baseline calibration the share of government spending in the skilled intensive sector is given by $\aleph = 0.7$. Similarly, we set the private spending share ξ according to $0.5 = (1 - \gamma_g)(\xi\omega_1 + (1 - \xi)\omega_2) + \gamma_g 0.58$, which with $\gamma_g = 0.2$, gives $\xi = 0.45$.

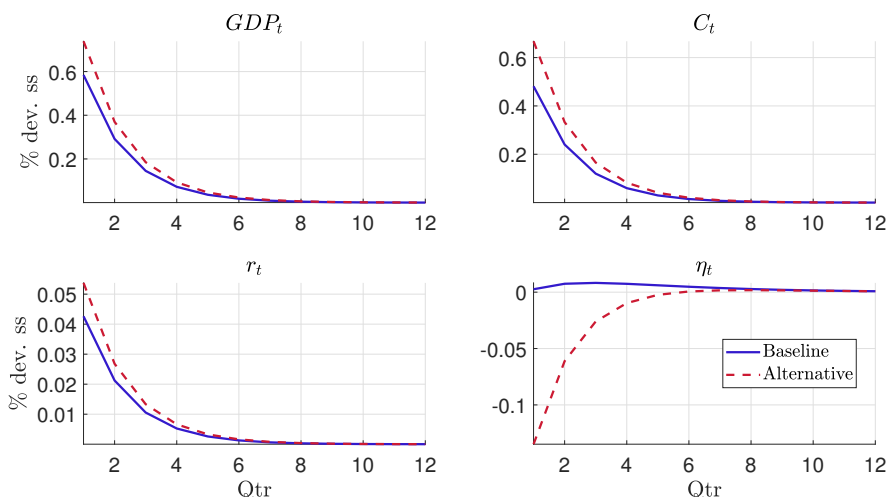
Government and Monetary Policy. Monetary policy follows a Taylor rule with $\phi_\pi = 1.5$. We assume that government spending as a share of GDP γ_g equal to 0.2. Then, we calibrate the labor tax rate in steady state to satisfy the budget constraint of the government with zero debt, and let it adjust in response to shocks to government spending. The persistence parameter of the exogenous government spending shock is set to $\rho_g = 0.5$.

2.5.2 How Government Spends Matters

The first exercise we make is to show that the way the government spends matters. To do so, we compare two different calibrations. One in which the economy is in the *baseline* calibration, with $\aleph = 0.7$, which as we mentioned above, implies that the average share of skilled workers in government spending is equal to 0.58. Another, which we call the *alternative* calibration where we assume that the government takes the extreme stance in which it spends only on the unskilled intensive sector $\aleph = 0$. We compare these two calibrations maintaining the remainder of parameters. The idea is to compare a case in which government preferences switch from the actual spending distribution

to one in which it spends all in the unskilled intensive sector, all else equal.

FIGURE 2.5: IRFs to a one-percent increase in government spending. Baseline and Alternative calibrations.



Notes: this figure shows the responses of GDP, consumption, the real interest rate, and the Earnings Gap in response to a 1% increase in government spending in the quantitative model. We show the percent deviations from the steady state at a quarterly frequency. This case compares the baseline calibration and the alternative in which all government consumption is on the unskilled intensive sector, $\aleph = 0$.

Figure 2.5 shows the responses of GDP, consumption, the real interest rate, and the earnings gap to a one-percent government spending shock in the model. We omit government spending responses as they are exogenous and equal in all the exercises we make below. In both calibrations, consumption increases in response to the government spending shock. In both cases, the interest rate and GDP also rise. The earnings gap has different responses: in the baseline, it rises in response to the government spending shock, while in the alternative, it falls. This means that when the government spends more in unskilled intensive sectors, labor income inequality falls. Therefore, the gov-

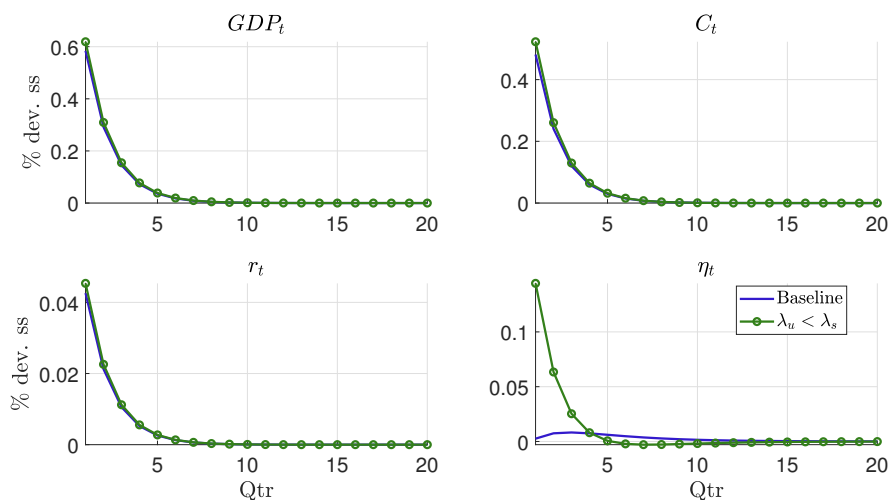
ernment in the baseline calibration generates inequality, consistently with the empirical findings.

The previous result matters for what we showed in the empirical evidence: there is a negative relationship between the earnings gap and consumption in the presence of incomplete markets. That implies that when the earnings gap rises by more, consumption increases by less, which is what we observe in Figure 2.5. When the government reverts the way it spends, the consumption response is stronger than the baseline. That occurs because when the government spends in the sector that hires unskilled workers in a higher proportion, it is transferring resources towards workers with higher MPCs. That means that government spending translates into consumption in a stronger way. In this stylized exercise, the strength of government spending in stimulating consumption rises by about 39 percent on impact. We find that generating inequality, for this reason, can reduce the government spending multiplier as well. By switching the way government spends, the effect on output can rise by about 26 percent on impact, even though we assume a responsive interest rate.²⁴

Therefore, the pattern of access to financial markets skilled and unskilled workers have may be why there is a negative relationship between the responses of consumption and the earnings gap in the data. To explore the quantitative importance of this feature, we switch the pattern of financial access. We take the baseline calibration and compare it with the situation in which the MPCs of the different classes of workers is switched: now $\lambda_u < \lambda_s$, such that the group with higher MPC is the group of skilled workers.

²⁴As Woodford (2011) shows, all these results, especially the fiscal multiplier, depend on the monetary policy response. Woodford points out that the multiplier is maximized when monetary policy follows a constant real rate rule. In our case, the differences between the baseline and the alternative calibrations are also maximized if monetary policy follows that kind of rule.

FIGURE 2.6: IRFs to a one-percent increase in government spending. Different distribution of HtM.



Notes: this figure shows the responses of GDP, consumption the real interest rate, and the Earnings Gap in response to a 1% increase in government spending in the quantitative model. We show the percent deviations from the steady state at a quarterly frequency. This case compares the baseline calibration with an scenario in which $\lambda_u < \lambda_s$.

In Figure 2.6 we show the simulations for that exercise. The responses of the relevant variables are similar to those shown in Figure 2.5; however, the responses of consumption and the earnings gap are not negatively related. The relationship is the opposite: when the earnings gap rises by more, consumption increases by more. This result contradicts the empirical findings shown above that there is a negative relationship between consumption and the earnings gap responses to a government spending shock. Therefore, having heterogeneity in asset markets participation in which the unskilled group is more financially constrained than the skilled group is essential for explaining the empirical facts we showed before.

2.6 CONCLUSION

In this paper, we revisit the effects of government purchases on consumption by considering its effects on inequality. We show three empirical results in this regard. First, we estimate a SVAR following [Blanchard and Perotti \(2002\)](#) identification augmented by labor income inequality and show that government spending increases this indicator strongly and persistently. Second, we estimate a time-varying structural VAR as in [Primiceri \(2005\)](#) and uncover that the responses of the earnings gap and consumption to the government spending shock are negatively related. And third, we show that government spending is concentrated towards sectors that hire skilled workers in a higher proportion than the economy as a whole.

To rationalize these facts, we build a two workers, two agent, two sector model in which we assume skilled and unskilled workers work in different sectors, and crucially, have different access to financial markets (where the unskilled worker is more financially constrained than the skilled worker). We show both analytically and quantitatively that the responses of labor income inequality and consumption to a government spending shock can be explained by the patterns of financial constraints (in which unskilled workers are more financially constrained) and the pattern of spending of the government, which is more concentrated towards sectors that hire skilled workers in a higher proportion. The reason is that when the government spends more on the skilled intensive sector, it is distributing resources towards the workers with lower marginal propensity to consume. This implies that the response of consumption is lower than when the government spends on the unskilled intensive sector. The previous result implies that the effects of government spending on consumption can rise as much as 45% if the sectoral spending pattern switches to spend on the unskilled intensive sector. That alone would raise the government spending multiplier by 32 percent.

While the distribution of spending across sectors is a political decision, it is important to emphasize that how government spends matters. And matters a great deal especially if policymakers are interested in inequality and its aggregate consequences.

APPENDIX

APPENDIX 2.A THE EFFECT OF GOVERNMENT SPENDING ON THE EARNINGS GAP USING RAMEY NEWS SHOCKS

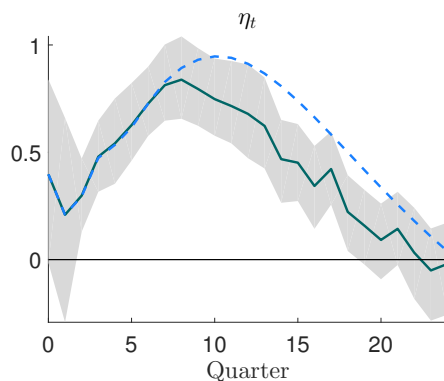
In this section, we estimate the effects of government spending shocks on the earnings gap by considering the News Shocks by [Ramey \(2011\)](#), that are claimed to be exogenous. We use the method proposed by [Miranda-Agrippino and Ricco \(2021\)](#) who propose the Bayesian Local Projection, which is a method that combines the estimation of a Bayesian VAR with Local Projection in an optimal way, to account for the problems these two method alone have. This allows us to estimate a instrumental variable VAR using the Ramey news shocks as instruments.²⁵

We estimate the model including the same variables considered in the body of the paper: The earnings gap, government expenditures in consumption and investment, government receipts, GDP, consumption of non-durables and services, fixed non-residential investment,

²⁵For more details, we refer the reader to [Miranda-Agrippino and Ricco \(2021\)](#).

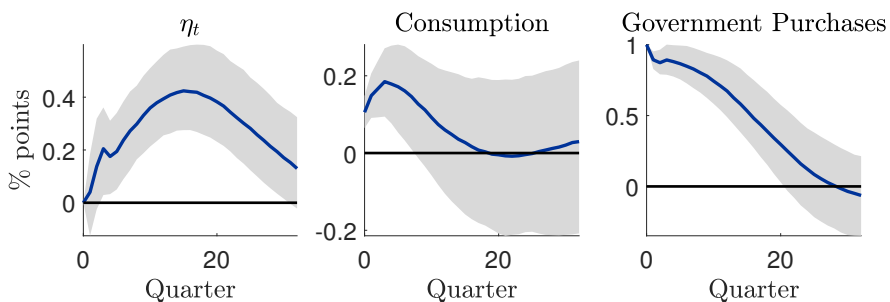
and unemployment. Figure 2.A.1 shows the responses of the earnings gap to a one-percent increase in government spending. In green-solid we plot the BLP and in blue-dashed the IV-BVAR. We observe in this picture that the earnings gap also rises in response to the government spending shock. If anything, this response is stronger than the one estimated with Blanchard and Perotti (2002) and has a very similar shape. Unfortunately, as Ramey (2016) point out, her exogenous shocks do not pass the test for weak instruments for the period we have the earnings gap available (1980-2018). Therefore, we stick with Cholesky identification in the main analysis.

FIGURE 2.A.1: Response of the earnings gap to a unitary shock to government spending in with BLP using Ramey News about military spending shocks.



APPENDIX 2.B ALTERNATIVE ORDERING IN THE BSVAR

FIGURE 2.B.1: IRF's to a unitary shock to government spending BSVAR with Cholesky identification ordering η_t first, then government spending. Sample: 1981Q1-2018Q4



APPENDIX 2.C ROBUSTNESS TO THE HYPERPARAMETERS

FIGURE 2.C.1: 1991Q3-2018Q4

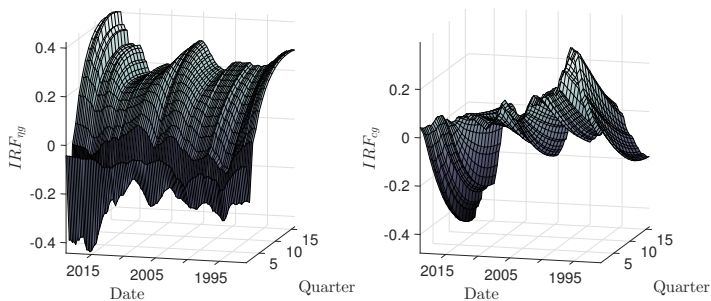
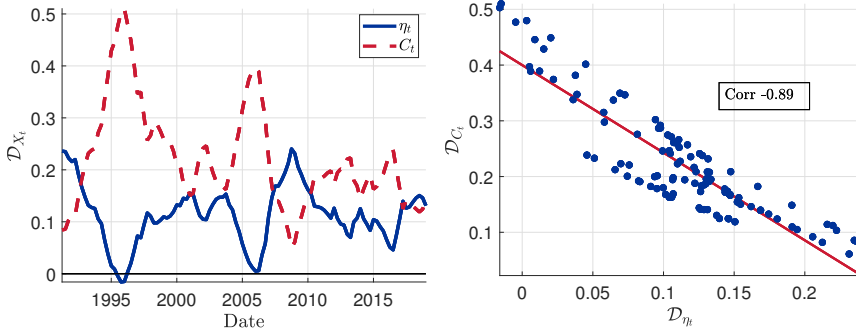


FIGURE 2.C.2: Time-Varying Dynamic Multipliers of C_t and η_t



APPENDIX 2.D DERIVATIONS

Shares of sectors in the symmetric steady state. In a symmetric equilibrium, the share of sector one in the total economy is given by

$$Y_1 = \xi \left(\frac{P_1}{PC} \right)^{-1} C + \aleph \left(\frac{P_1}{PG} \right)^{-1} G$$

$$n = \frac{Y_1}{Y} = \xi \left(\frac{P_1}{PC} \right)^{-1} \frac{C}{Y} + \aleph \left(\frac{P_1}{PG} \right)^{-1} \frac{G}{Y} \quad (2.22)$$

In a symmetric equilibrium (where $P_1 = P_2$), the share of sector 1 in total production is given by

$$n = \xi(1 - \gamma_g) + \aleph\gamma_g \quad (2.23)$$

where $\gamma_g = \frac{G}{Y}$ is the share of government spending in total output.

The symmetric equilibrium is attained when wages are equal. This can be attained by setting χ_s and χ_u , such that:

$$w_s = \chi_s N_s^\varphi C_s^\gamma = \chi_u N_u^\varphi C_u^\gamma = w_u. \quad (2.24)$$

When $w_s = w_u$, the marginal costs in both sectors are the same. And then, if $\varepsilon_1 = \varepsilon_2$, prices are equal in both sectors. That allows us to

ignore prices in the steady state and in the deviations from the steady state.

Under the assumptions of section 2, the condition for a symmetric equilibrium is:

$$\frac{\chi_s}{\chi_u} = \frac{(\omega_1 n + \omega_2(1 - n))(\varepsilon - 1)}{(\omega_1 n + \omega_2(1 - n))(\varepsilon - 1) + 1} \quad (2.25)$$

Share of labor in the different sectors. In steady state the demands for skilled labor are given by:

$$w_s = \omega_1 \frac{Y_1}{n_{1s}} mc_1, \quad w_s = \omega_2 \frac{Y_2}{n_{2s}} mc_2$$

With these demands, we can obtain the total demanded labor for skilled workers

$$n_s = \omega_1 mc_1 \frac{Y_1}{w_s} + \omega_2 mc_2 \frac{Y_2}{w_s}$$

Then,

$$\kappa_{1s} = \frac{\omega_1 mc_1 \frac{Y_1}{w_s}}{\omega_1 mc_1 \frac{Y_1}{w_s} + \omega_2 mc_2 \frac{Y_2}{w_s}} = \frac{\omega_1 mc_1 Y_1}{\omega_1 mc_1 Y_1 + \omega_2 mc_2 Y_2}$$

Assuming a symmetric equilibrium $mc_j = \frac{p_j}{M_j^p} = \frac{p}{M^p}$, and $n = \frac{Y_1}{Y}$

$$\kappa_{1s} = \frac{\omega_1 n}{\omega_1 n + \omega_2(1 - n)}, \quad \kappa_{2s} = \frac{\omega_2(1 - n)}{\omega_1 n + \omega_2(1 - n)} \quad (2.26)$$

$$\kappa_{1u} = \frac{(1 - \omega_1)n}{(1 - \omega_1)n + (1 - \omega_2)(1 - n)}, \quad \kappa_{2s} = \frac{(1 - \omega_2)(1 - n)}{(1 - \omega_1)n + (1 - \omega_2)(1 - n)}. \quad (2.27)$$

which implies that the share work in the different sectors (the κ_s) depend on the technology parameters and sizes of the different sectors.

INEQUALITY, NOMINAL RIGIDITIES, AND AGGREGATE DEMAND

Joint with Sebastián Diz and Damián Romero

3.1 INTRODUCTION

Letting wages adjust freely after an economic downturn is one of the main elements of the classical economists' toolkit. According to this argument, if wages fall, labor demand increases and output returns to its potential level. However, as [Galí \(2013\)](#) shows this is not necessarily true in the presence of price and wage rigidities. In this paper, we extend this analysis to an heterogeneous agent economy, at the zero lower bound. We show that in a model where there is heterogeneity in marginal propensities to consume (MPC), there is a distributional channel of nominal rigidities that exacerbates the losses from wage flexibility depending on the degree of price rigidities and the degree of inequality.

Making wages more flexible to restore full employment was first challenged by [Keynes \(1936\)](#). He disagreed with the classical theory

by postulating that aggregate demand (AD) matters for the determination of output. He also pointed out that the classical theory is wrong in assuming that the AD does not depend on wages. In his view, the AD depends on wages as long as they affect: (i) the return on assets or (ii) the average MPC. The first channel operates if wages affect the interest rate in the economy, switching the incentives to consume and invest. This mechanism was explored by Galí (2013), among others who show that this channel matters if wages alter the real interest rate; i.e., wages affect the AD only indirectly through the endogenous response of the central bank. The second channel operates if wage adjustments redistribute resources between agents with different MPCs, affecting their levels of consumption and aggregate demand. Recent literature on monetary policy has emphasized that in the presence of market incompleteness the indirect effects of monetary policy dominate, where one important component are fluctuations in labor income. (see Kaplan et al. (2018) and Auclert (2019) among others).

This paper builds on the second channel—the average MPC. We start from the observation that, when there are agents who are unable to save or borrow, the AD depends on the distribution of all income sources (in particular, labor income) and not only on the interest rate, as in the baseline New Keynesian model. In such a case, the shift of resources in the cycle between workers and firm owners affects the extent to which wages determine consumption dynamics. There is a shift in the average aggregate MPC when: (i) income from assets (which includes bonds and firms' profits) and labor is unequally distributed; (ii) the MPCs of workers and other agents differ; and (iii) wages fluctuate differently from other sources of income. That is, specifically, the intuition put forward by Keynes (1936) Chapter 19.

In this paper, we show both analytically and numerically that the relative rigidity of wages and prices can drive this redistribution. These relative rigidities affect the evolution of the real wage, which

has an active role determining the AD in the presence of limited access to financial markets. In short, the relative rigidities determine who gets the resources from aggregate fluctuations: workers or firm owners. If prices fall less than wages in a downturn, then there is a redistribution of resources from workers to firm owners. Hence, if workers have more restricted access to financial markets and their MPCs are higher, the average MPC of the economy shifts because of the differential nominal rigidities that the different agents face. In turn, limited access to financial markets activates the channels proposed by Keynes, and the final effect is governed by the relative nominal rigidities. In this context, wage flexibility might not be desirable if it generates countercyclical redistribution from high- to low-MPC consumers. We show that this is especially acute if the degree of price rigidity is high.

The existence of prices and wage rigidities has been broadly studied in the literature. [Dhyne et al. \(2006\)](#) provide evidence on price rigidities, finding that the average spell of prices in the Euro Area is about one year, while in the U.S. it is about two quarters. While [Bils and Klenow \(2004\)](#) and [Nakamura and Steinsson \(2013\)](#) show a shorter duration of prices (about four months), they do not rule out the existence of price rigidities. Regarding nominal wage rigidities, there is broad evidence reviewed by [Taylor \(2016\)](#). He highlights the evidence from France studied by [Le Bihan et al. \(2012\)](#) and Iceland collected by [Sigurdsson and Sigurdardottir \(2011\)](#) which shows that wages in both cases stay fixed, on average, for about one year.

We rationalize the concepts above by building a textbook New Keynesian model with limited asset participation and price and wage rigidities as in [Colciago \(2011\)](#) and [Furlanetto and Seneca \(2012\)](#). To capture market incompleteness, we assume there is a share of agents without access to financial markets as in [Galí et al. \(2007\)](#) and [Bilbiie \(2008\)](#)¹, implying different MPCs across the population. We call these

¹See also the [Debortoli and Galí \(2018\)](#) who compare the results of Two-Agent

constrained agents *Hand to Mouth* (HtM).²

The main result of this paper is an analytical characterization of the equilibrium in a simplified economy where prices and wages are set in advance. We derive an aggregate demand that depends on wages, as emphasized by Keynes (1936). We show that with wage rigidities and a share of HtM agents, the AD (and output) depends on wage and price inflation and in particular on the wage and price processes, which are given by the respective degrees of stickiness. This dependence arises from the fact that when there are wage rigidities, the price adjustment process is not isomorphic to the wage adjustment. When prices and wages fluctuate separately, the income of firm owners and of workers fluctuates differently. That implies that the redistribution between workers and firm owners can arise from nominal rigidities. If workers are financially constrained and firms' owners do not, different price and wage rigidities generate switches in the average MPC of the economy.

In our model the final effect of the different features, conditional on demand shocks, operates through two channels: an *interest rate channel* and a *distributional channel*. The former is the conventional procyclical response of monetary policy to the different shocks when the Taylor rule responds to endogenous variables. The latter corresponds to how aggregate demand is affected by the redistribution of resources in the cycle among different households. On the one hand, we show that through the distributional channel, wage flexibility amplifies the cycle by making income of the high MPC agent more volatile. On the other hand, we show that wage flexibility stabilizes output if the monetary policy response to this excessive volatility is strong enough. The final effect depends on the share of HtM agents, and more interestingly,

and Heterogeneous-Agent New Keynesian models.

²Kaplan et al. (2014) provide evidence of the existence of HtM households in the US and Europe. They find that 30% of households do not hold assets in average. Our calculations find that this share has been barely stable over the 2000-2018 period.

on the degree of price rigidities. The importance of price rigidities for the gains from wage flexibility was already emphasized by Galí and Monacelli (2016), who showed that more flexible wages translates into a more stable economy (with lower prices and higher demand for its goods) if prices are sufficiently flexible. Our argument is similar, though we highlight an alternative channel which operates through redistribution of resources between workers and firm owners.

We show that the conduct of monetary policy—and what it reacts to—matters. As prices and wages do not have the same effect on aggregate demand (as in a model without wage rigidities and financial frictions), reacting to price inflation is not enough to stabilize output. If prices are too rigid, the distributional channel gains prominence. Therefore, we show that monetary policy should react to wage inflation to dampen the distributional channel. In our model, monetary policy is more effective when it reacts to wage inflation than when it reacts to price inflation.

Finally, we study the gains from wage flexibility using the full model. We show that the results presented in the simple model still stand. In this setup, we find no gains from wage flexibility with limited access to financial markets, since it stimulates the distributional channel. This result is especially acute if monetary policy cannot react, i.e., is in the zero lower bound. We show that there are losses from wage flexibility caused by an excessive volatility of prices, wages, and output.

This paper contributes to the literature mainly because it helps to clarify the effects of the interaction between incomplete markets and nominal rigidities. We obtain a closed-form characterization of the economy subject to these three frictions: price rigidities, wage rigidities, and limited asset markets participation. Previous literature has not emphasized the role of the three frictions together, but rather detracted from the role of price rigidities in shaping redistribution. This paper is similar to Broer et al. (2019). However, we show ana-

lytically how the gains from wage flexibility may depend on price rigidities and the monetary policy stance, and not only how wage rigidities affect the aggregate outcome. Moreover, we uncover the distributional channel of nominal rigidities which arises in models with incomplete markets and nominal rigidities.³

The remainder of the paper is organized as follows. Section 3.2 describes the model. In Section 3.3 we solve analytically a simple version of the model where we assume prices and wages are set in advance. In Section 3.4 we conduct quantitative exercises in the full model. And finally, we present our conclusions in Section 3.5.

3.2 MODEL

Our setup is a New Keynesian model with limited asset market participation and wage rigidities, building on the work of Bilbiie (2008), Furlanetto and Seneca (2012), Debortoli and Galí (2018), among others. In particular, we assume there is a fraction of agents that cannot borrow or lend and cannot own firms. Workers supply labor in a monopolistically competitive environment and are subject to staggered wage setting. Firms are also subject to price rigidities and supply their goods in a monopolistically competitive environment. Additionally, we assume that monetary policy follows a Taylor rule which is bounded from below by zero.

Households. The economy is populated by a continuum of households of mass 1, where a fraction λ cannot borrow, lend or own firms while the remainder $1 - \lambda$ has full access to financial markets and owns the firms in the economy. We refer to the former as constrained agents, denoted by c , and to the latter as unconstrained, denoted by u . Each household is composed by a continuum of members that supply

³Additionally, Colciago (2011) study this question while he does not show explicitly how these mechanisms interact.

differentiated labor varieties denoted by $j \in [0, 1]$. We assume there is perfect insurance within the household, which equalizes members' consumption.

Households' lifetime utility is given by:

$$\mathbb{E}_t \sum_{k=0}^{\infty} \beta^k \chi_{t+k} \left(\frac{(C_{t+k}^K - 1)^{1-\sigma}}{1-\sigma} - \int_0^1 \frac{N_{t+k}^K(j)^{1+\varphi}}{1+\varphi} dj \right), \quad (3.1)$$

for $K \in \{u, c\}$, where χ_t represents a shock to preferences, C_t^K is final good consumption and $N_t^K(j)$ denotes hours worked supplied to variety j .

Households face the following period resource constraint:

$$P_t C_t^K + Q_t B_t^K = B_{t-1}^K + \int_0^1 W_t(j) N_t^K(j) dj + D_t^K. \quad (3.2)$$

Earnings are given by labor income $\int_0^1 W_t(j) N_t^K(j) dj$ and profits D_t^K that proceed from firm ownership. $P_t C_t^K$ is total (nominal) expenditure on the final good. And $Q_t B_t^K$ are bond purchases, where P_t is the price of the final good while Q_t is the price of bonds.

We assume the preference shock follows an exogenous AR(1) process given by:

$$\log \bar{\chi}_t = (1 - \rho_\chi) \log \bar{\chi} + \rho_\chi \log \bar{\chi}_{t-1} + \sigma_\chi \eta_t.$$

Intertemporal optimization implies the following Euler equation for unconstrained households

$$1 = R_t \mathbb{E}_t \left\{ \beta \frac{\chi_{t+1}}{\chi_t} \left(\frac{C_t^u}{C_{t+1}^u} \right)^\sigma \frac{1}{\Pi_{p,t+1}} \right\}, \quad (3.3)$$

where we define $R_t = 1/Q_t$. Constrained households have no access to financial markets. Hence, their consumption equals current income from labor:

$$C_t^c = \frac{W_t}{P_t} N_t. \quad (3.4)$$

Finally, aggregate consumption is given by:

$$C_t = (1 - \lambda)C_t^u + \lambda C_t^c. \quad (3.5)$$

Final Good Producers. Firms producing the final good operate in a perfectly competitive environment and combine a continuum of measure one of intermediate goods $Y_t(i)$ to produce a homogeneous final good Y_t according to

$$Y_t = \left(\int_0^1 Y_t(i)^{\frac{\varepsilon_p - 1}{\varepsilon_p}} di \right)^{\frac{\varepsilon_p}{\varepsilon_p - 1}}, \quad (3.6)$$

where ε_p is the elasticity of substitution among good varieties.

Solving the optimization problem of the firm, we obtain the following demand function for intermediate inputs:

$$Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon_p} Y_t, \quad (3.7)$$

where $P_t \equiv \left(\int_0^1 P_t(i)^{1 - \varepsilon_p} di \right)^{\frac{1}{1 - \varepsilon_p}}$ is the price of the final good.

Intermediate Goods Producers. There is a continuum of intermediate firms, indexed by $i \in [0, 1]$. These firms operate in a monopolistic competitive environment. Hence, each firm produces a single-differentiated good and operates as a monopoly in its own market. Intermediates production technology is given by

$$Y_t(i) = N_t(i)^{1 - \alpha}, \quad (3.8)$$

where $Y_t(i)$ is firm i output and $N_t(i)$ is labor input. We assume that each firm i demands different kinds of labor provided by the households, with an elasticity of substitution ε_w . Thus, we have $N_t(i) = \left(\int_0^1 N_t(i, j)^{\frac{\varepsilon_w - 1}{\varepsilon_w}} dj \right)^{\frac{\varepsilon_w}{\varepsilon_w - 1}}$, where $N_t(i, j)$ is the amount of labor

variety j demanded by firm i . Then, a standard cost minimization problem derives the demand for each labor variety

$$N_t(i, j) = \left(\frac{W_t(j)}{W_t} \right)^{-\varepsilon_w} N_t(i), \quad (3.9)$$

where $W_t(j)$ is the wage of labor variety j . From Equation (3.9), note that the total demand across firms for variety j is given by $N_t(j) = \left(\frac{W_t(j)}{W_t} \right)^{-\varepsilon_w} N_t$, which will be useful later on.

Firms also face price rigidities. In the next sections we consider two different types of rigidities: prices set in advance and Calvo pricing, which we will describe in detail later. Assuming prices are set in advance allows us to study the role of nominal rigidities in Aggregate Demand analytically. On the other hand, with Calvo pricing, we take into account the role of expectations and dynamics to describe the impact of nominal rigidities in the presence of incomplete markets.

Wage Setting. The wage for each labor variety is set by a union operating in a monopolistically competitive market. Unions choose the wage rate that maximizes a weighted average of unconstrained and constrained lifetime utility, which, for example, with Calvo pricing writes

$$\begin{aligned} \mathbb{E}_t \sum_{k=0}^{\infty} (\beta \theta_w)^k \chi_{t+k} & \left[(1 - \lambda) \left(\frac{(C_{t+k}^u - 1)^{1-\sigma}}{1 - \sigma} - \int_0^1 \frac{(N_{t+k|t}^u)^{1+\varphi}}{1 + \varphi} dj \right) \right. \\ & \left. + \lambda \left(\frac{(C_{t+k}^c - 1)^{1-\sigma}}{1 - \sigma} - \int_0^1 \frac{(N_{t+k|t}^c)^{1+\varphi}}{1 + \varphi} dj \right) \right], \end{aligned} \quad (3.10)$$

where θ_w is the parameter of wage rigidities.

In the same way, in the next sections we consider two different types of rigidities: wages set in advance and Calvo wage adjustment. We will describe these problems in detail below.

Monetary Policy. We assume that the monetary authority follows a Taylor rule which is subject to the zero lower bound, given by:

$$R_t = \max \left\{ \bar{R} \left(\frac{\Pi_{p,t}}{\bar{\Pi}_p} \right)^{\phi_\pi}, 1 \right\} \quad (3.11)$$

where $R_t = \frac{1}{Q_t}$, and parameters ϕ_π is the response of the central bank to deviations of inflation from its steady state level.

Equilibrium. In this economy all production is consumed

$$Y_t = C_t. \quad (3.12)$$

The relation between aggregate output and employment can be written as⁴

$$N_t = \Delta_{w,t} \Delta_{p,t} Y_t^{\frac{1}{1-\alpha}}, \quad (3.13)$$

where $\Delta_{w,t} \equiv \int_0^1 \left(\frac{W_t(j)}{W_t} \right)^{-\varepsilon_w} dj$ and $\Delta_{p,t} \equiv \int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{\frac{-\varepsilon_p}{1-\alpha}} di$.

Finally, we assume bonds are in zero net supply. Hence, equilibrium in the bonds market requires:

$$(1 - \lambda)B_t^u + \lambda B_t^c = 0. \quad (3.14)$$

Since constrained agents have no access to financial markets, the last expression implies $B_t^u = B_t^c = 0$.

3.3 AGGREGATE DEMAND WITH PRICES AND WAGES SET IN ADVANCE

In this section, we illustrate how the interaction between price and wage rigidities shapes redistribution over the business cycle and how

⁴See Appendix 3.B.1 for a derivation.

such interaction affects aggregate demand. To clarify the mechanisms of our model, we assume the wage and price setting processes are as follows. Workers supply labor in a monopolistically competitive environment. However, we include a staggered wage setting by assuming that a fraction of workers set nominal wages in advance (i.e., before the shocks are realized). The remaining workers are not constrained to set wages. We use the same simplification for firms' pricing problem.

The aim of this section is to solve the model to obtain an aggregate demand equation when there is limited access to financial markets. This explains how market incompleteness interacts with price and wage rigidities in shaping aggregate demand.

3.3.1 The Consumption Gap

Combining equations (3.3) to (3.5) we can solve for the following Euler equation of aggregate consumption (in log deviations from the steady state):⁵

$$\begin{aligned} \widehat{c}_t = & \mathbb{E}_t \{ \widehat{c}_{t+1} \} - \frac{1}{\sigma} (\widehat{r}_t - \mathbb{E}_t \{ \widehat{\pi}_{p,t+1} \}) - (1 - \rho_\chi) \widehat{\chi}_t \\ & + \frac{\lambda}{(1 - \lambda)\gamma + \lambda} \mathbb{E}_t \{ \Delta \widehat{\gamma}_{t+1} \}, \end{aligned} \quad (3.15)$$

where $\gamma_t \equiv \frac{C_t^u}{C_t^c}$ is the *consumption gap* between the unconstrained and the constrained households. Notice that Equation (3.15) is the usual Euler equation with an additional term that depends on the growth rate of consumption inequality. This equation can be solved forward to obtain

$$\widehat{c}_t = - \frac{\lambda}{(1 - \lambda)\gamma + \lambda} \widehat{\gamma}_t - \frac{1}{\sigma} \mathbb{E}_t \sum_{k=0}^{\infty} (\widehat{r}_{t+k} - \widehat{\pi}_{p,t+k+1} - (1 - \rho_\chi) \widehat{\chi}_{t+k}). \quad (3.16)$$

⁵In what follows hat (\widehat{x}) variables correspond to log-deviations with respect to the steady-state.

Equation (3.16) shows that consumption, through aggregate demand, is directly affected by inequality. The direction of this dependence is determined by the cyclicality of the consumption gap, which is an endogenous variable. To derive the consumption gap, recall that unconstrained agents work and own the firms; hence their income is given by the sum of labor and profit earnings, i.e. $C_t^u = \frac{W_t}{P_t} N_t + \frac{1}{1-\lambda} \frac{D_t}{P_t}$. Constrained households, meanwhile, only receive labor income; hence, their consumption is given by $C_t^c = \frac{W_t}{P_t} N_t$. Then, in equilibrium, it must be that the consumption gap is given by

$$\gamma_t = \frac{W_t N_t + \frac{1}{1-\lambda} D_t}{W_t N_t}.$$

As [Debortoli and Galí \(2018\)](#) show, the consumption gap can be written in terms of the economy's price markup

$$\gamma_t = \frac{1 - \alpha + \frac{1}{1-\lambda} (\mathcal{M}_t^p - (1 - \alpha))}{1 - \alpha}, \quad (3.17)$$

where \mathcal{M}_t^p is the average price markup.⁶ Equation (3.17) in log-deviations from the steady state reads

$$\widehat{\gamma}_t = \Psi \widehat{\mu}_t^p, \quad (3.18)$$

where $\Psi \equiv \frac{\mathcal{M}^p}{(1-\lambda)(1-\alpha + \frac{1}{1-\lambda}(\mathcal{M}^p - (1-\alpha)))}$. Equation (3.18) represents a relation that is at the core of the results that follow: it is only the price markup that determines the consumption gap, as the only source of inequality in the model is the ownership of firms. Importantly, notice that the coefficient Ψ , which determines the relationship between the consumption gap and markups, depends negatively on the share of unconstrained agents; i.e., of the fraction of firms owners. This occurs because having a lower share of firms' owners implies

⁶To obtain this expression, we used $\frac{W_t N_t}{P_t Y_t} = (1 - \alpha) \frac{Y_t}{\mathcal{M}_t^p N_t} \frac{N_t}{Y_t} = \frac{1-\alpha}{\mathcal{M}_t^p}$ and $\frac{D_t}{P_t Y_t} = \frac{P_t Y_t - W_t N_t}{P_t Y_t} = 1 - \frac{1-\alpha}{\mathcal{M}_t^p}$.

that any increase in the price markup (and hence in firms' profits) is distributed among a smaller share of agents. Therefore, firm owners experience a greater increase in their income, leading to a larger rise in the consumption gap between firm owners and workers. The next step is to understand how the price markup evolves over the business cycle.

Firms Average Markup. As Equation (3.18) shows, firms' markups are important in the equilibrium of the model. This is not only because they are a source of fluctuations, like in any New Keynesian model, but because they shape inequality and its effects on aggregate demand. Regarding the firms' labor demand, the average markup is given by

$$\mathcal{M}_t^p = (1 - \alpha) \frac{P_t Y_t}{W_t N_t},$$

which log-linearized around the steady state yields

$$\widehat{\mu}_t^p = -\frac{\alpha}{1 - \alpha} \widehat{y}_t - \widehat{\omega}_t. \quad (3.19)$$

From (3.18) and (3.19) we get

$$\widehat{\gamma}_t = -\Psi \left(\frac{\alpha}{1 - \alpha} \widehat{y}_t + \widehat{\omega}_t \right). \quad (3.20)$$

Equation (3.20) describes the evolution of the consumption gap and its drivers. We highlight two results from this expression. First, the consumption gap depends negatively on output. This occurs because decreasing returns on labor imply a reduction in the firms' average markup following an increase in production (and hence employment). Second, the gap depends negatively on real wages. That occurs as wages raise marginal costs and hence drive firms' markups down. As a result, income is redistributed towards workers; i.e., the consumption gap drops.

3.3.2 Equilibrium Wages

In this subsection, we derive the equilibrium real wage. To do so, we first obtain wage and price inflation schedules. We derive two Phillips-like equations for prices and wages and show how the real wage depends on relative rigidities of prices and wages. To obtain closed-form solutions for price and wage inflation, we make some simplifying assumptions summarized in Proposition 1.

Proposition 1 (Price and wage dynamics). *Assume there is a continuum of measure one of firms (unions) in a monopolistic competitive environment, in which a share θ_p (θ_w) of firms (unions) set prices (wages) in advance, while the remainder $1 - \theta_p$ ($1 - \theta_w$) set prices (wages) considering the value of the shocks in t . Assume also, that firms maximize profits by taking into account their production function, Equation (3.8); while unions maximize aggregate welfare of the members of the union of each task, Equation (3.40).*

Under these assumptions, the evolution of price inflation is given by

$$\widehat{\pi}_t^p = \kappa_\pi \left(\widehat{\omega}_t + \frac{\alpha}{1 - \alpha} \widehat{y}_t \right) + \mathbb{E}_{t-1} \widehat{x}_t^p, \quad (3.21)$$

where $\kappa_\pi \equiv \frac{1 - \theta_p}{\theta_p} \frac{1 - \alpha}{1 - \alpha + \alpha \varepsilon_p}$ and $\widehat{x}_t^p \equiv \widehat{\omega}_t + \alpha \widehat{n}_t^m(j) + \widehat{\pi}_t^p$.

While the evolution of real wages is given by

$$\widehat{\omega}_t = \kappa_\omega (\varpi_1 \widehat{y}_t + \varphi \widehat{n}_t) - \varsigma \widehat{\pi}_t^p + \mathbb{E}_{t-1} \widehat{x}_t^w, \quad (3.22)$$

where $\kappa_\omega \equiv \frac{1 - \theta_w}{1 + \theta_w \varphi \varepsilon_w - (1 - \theta_w) \varpi_2}$, $\varsigma \equiv \frac{\theta_w (1 + \varphi \varepsilon_w)}{1 + \theta_w \varphi \varepsilon_w - (1 - \theta_w) \varpi_2}$ and $\widehat{x}_t^w \equiv \varsigma (\varpi_1 \widehat{y}_t + \varpi_2 \widehat{\omega}_t + \varphi \widehat{n}_t^m(j) + \widehat{\pi}_t^p)$.

Proof. See Appendix 3.B.2. □

Proposition 1 describes the evolution of inflation and the real wage. Equation (3.21) shows that in our setting, price inflation depends on wages and output; while Equation (3.22) describes the relationship between the real wage with output, labor and price inflation. Both equations are a Phillips-like relationship for prices and wages as we

obtain a positive relationship between the output gap and price inflation on the one hand and labor supply and wages on the other. These relationships depend on the degrees of price and wage stickiness, θ_p and θ_w . If wages or prices are fully flexible, these relationships break. If prices are fully flexible, price inflation is free to evolve (and the aggregate supply becomes infinitely inelastic). Also, if wages are fully flexible, the real wage is given by the labor supply at all times.^{7,8} Notice that the real wage depends negatively on the price inflation rate, with this relation given by the parameter ς which is a function of the degree of wage rigidities. Therefore, the real wage depends on the price inflation rate because of the wage rigidities.

Our price and wage arrangements assume prices are set in advance. This implies that firms and workers set wages taking an expectation of the future demand for goods and labor before the shocks realize (in $t - 1$). That is why the terms $\mathbb{E}_{t-1}\hat{x}_t^p$ and $\mathbb{E}_{t-1}\hat{x}_t^w$ appear in the prices and wage setting schedules. When prices and (or) wages are fully sticky, the evolution of these variables are given by these expectations that are the best the restricted agents can do. Throughout this section we assume shocks are iid with zero mean, so these expectation terms are zero⁹.

As the real wage depends on price inflation, by combining Equations (3.21) and (3.22), we obtain the *real wage schedule*, which is presented in the following proposition.

Proposition 2 (The real wage schedule). *From the evolution of the real wage and price inflation, the real wage is given by*

$$\hat{\omega}_t = \Xi_y \hat{y}_t + \mathbb{E}_{t-1} \hat{x}_t, \quad (3.23)$$

⁷With flexible wages $\hat{\omega}_t = \bar{\omega} \hat{y}_t + \bar{\varphi} \hat{n}_t$.

⁸These two Phillips-like equations are very close to the ones derived in the basic New Keynesian model with wage rigidities. The difference is the backward looking nature of these ones, while in the New Keynesian they are forward-looking.

⁹We introduce this assumption in Proposition 4 below.

where $\Xi_y \equiv \frac{\kappa_\omega \left(\varpi_1 + \frac{\varphi}{1-\alpha} \right) - \varsigma \kappa_\pi \frac{\alpha}{1-\alpha}}{1 + \varsigma \kappa_\pi}$ and $\hat{x}_t \equiv \frac{\hat{x}_t^w - \varsigma \hat{x}_t^p}{1 + \varsigma \kappa_\pi}$.

Proof. This result follows directly from Proposition 1. \square

Equation (3.23) describes the real wage of this economy, which is a function of the output gap. The parameter Ξ_y governs the cyclicity of the real wage, which depends on the relative wage and price rigidities.

3.3.3 Aggregate Demand

Aggregate demand in our economy, as in any New Keynesian model, corresponds to the aggregate Euler equation in addition to goods market clearing. In our model, as Equation (3.15) shows, the aggregate Euler equation depends on the consumption gap. Hence, before solving for aggregate demand, we present the equilibrium consumption gap, which is characterized in Proposition 3.

Proposition 3 (The Consumption Gap). *The equilibrium consumption gap is given by:*

$$\hat{\gamma}_t = -\Theta_y \hat{y}_t - \Psi \mathbb{E}_{t-1} \hat{x}_t, \quad (3.24)$$

where

$$\Theta_y \equiv \Psi \left(\underbrace{\frac{\alpha}{1-\alpha}}_{\text{Employment}} + \underbrace{\frac{\kappa_\omega \left(\varpi_1 + \frac{\varphi}{1-\alpha} \right) - \varsigma \kappa_\pi \frac{\alpha}{1-\alpha}}{1 + \varsigma \kappa_\pi}}_{\text{Real Wage}} \right).$$

Proof. This result follows directly from replacing Equation (3.23) in the expression for the consumption gap (3.20). \square

Equation (3.24) shows that consumption inequality depends on output where the coefficient Θ_y represents the cyclicity of the consumption gap. The cyclicity depends on two channels: *Employment* and *Real Wage*. The former derives from the switch in labor quantity required by firms in the presence of decreasing returns on labor. The

latter enters due to the dependence of the consumption gap (through markups) on the real wage. More importantly, with wage rigidities, the real wage depends on both price and wage inflation. Hence, the cyclical nature of the consumption gap depends on the dynamics of nominal wages and prices, represented by the parameters κ_ω and κ_π .

Price and wage rigidities have different effects on the cyclical nature of the consumption gap. While both κ_ω and κ_π fall when wages or prices become more rigid, their impact on inequality is different. When prices are more rigid (given a degree of wage stickiness) consumption inequality becomes more procyclical, whereas when wages are more sticky inequality becomes less procyclical. The intuition is that these rigidities generate a distribution of resources between workers and firms' owners. When there is a recession and wages do not fall by much, it is firms' owners who bear the shock. This implies that the consumption gap does not react as much as in the case with flexible wages.

Next, we show how the previous result translates into aggregate demand (and output). We present our main result: aggregate demand equation with wage and price rigidities and limited access to financial markets. Given (3.15) and the definition of the consumption gap, we derive the IS equation, as presented by Proposition 4.

Proposition 4 (The IS equation). *Under iid shocks, the IS equation of this economy with financial frictions and price and wage rigidities is given by*

$$\hat{y}_t = -\frac{1}{\sigma} \frac{1}{1 - \frac{\lambda}{(1-\lambda)^{\gamma+\lambda}} \Theta_y} (\hat{r}_t - \hat{\chi}_t), \quad (3.25)$$

Proof. See Appendix 3.B.3. □

Equation (3.25) presents the Euler equation after deriving the consumption gap, replacing the gap into the original Euler (Equation (3.3)), and assuming goods market clearing. The main difference between this Euler equation and the one derived from a representative

agent without wage rigidities is that the slope (the relationship between output and the interest rate) is significantly affected by other features of the model. In particular, the slope depends on the way resources are distributed in the cycle, which in this case depends on the rigidities as we explain below.

Equilibrium in the Simplified Economy. If the economy is subject to *iid* shocks, the equilibrium in this economy is summarized by the following equations:

$$\hat{y}_t = -\frac{1}{\sigma} \frac{1}{1 - \frac{\lambda}{(1-\lambda)\gamma+\lambda} \Theta_y} (\hat{r}_t - \hat{\chi}_t), \quad (3.26)$$

$$\hat{\pi}_t = \Upsilon_y \hat{y}_t, \quad (3.27)$$

$$\hat{\omega}_t = \Xi_y \hat{y}_t, \quad (3.28)$$

$$\hat{r}_t = \phi_\pi \hat{\pi}_t + \phi_\omega \hat{\pi}_t^\omega + \varepsilon_t^{mp} \quad (3.29)$$

Equations (3.26)-(3.29) characterize: (i) the IS equation; (ii) the relation between price inflation and output (obtained by replacing the equation for real wages into the equation for price inflation); (iii) the cyclicity of real wages; and (iv) the evolution of the interest rate. Notice that due to the *iid* shocks assumptions we made, the terms with expectations disappear (both past and future). Therefore, what follows in this section can be interpreted as the *impact* responses of the variables to the different shocks.

3.3.4 The Distributional Channel of Nominal Rigidities

As we can observe in Proposition 4, output depends directly on price and wage rigidities. This dependence arises because nominal rigidities affect how income is distributed in the cycle, distorting the average marginal propensity to consume. In this way, we obtain the mechanism proposed by Keynes (1936) which is that wages enter aggregate demand if they distort the average MPC. Our approach

to obtain this result is through nominal rigidities, and we call this *the distributional channel of nominal rigidities*. Next, we study how the distributional channel affects the variance of the output gap.¹⁰

To study the role of this channel, let us for now assume that monetary policy fully controls the real interest rate, i.e., $\hat{r}_t = \varepsilon_t^{mp}$, where ε_t^{mp} is an exogenous monetary policy shock.¹¹ With these assumptions, the output gap (in the absence of preference shocks) is given by

$$\hat{y}_t = -\frac{1}{\sigma} \frac{1}{1 - \frac{\lambda}{(1-\lambda)\gamma+\lambda} \Psi \left(\frac{\alpha}{1-\alpha} + \frac{\kappa_\omega (\varpi_1 + \frac{\bar{\varphi}}{1-\alpha})}{1+\zeta\kappa_\pi} - \frac{\zeta\kappa_\pi \frac{\alpha}{1-\alpha}}{1+\zeta\kappa_\pi} \right)} \varepsilon_t^{mp}, \quad (3.30)$$

where we use the expression for Θ_y . Hence, through the coefficient Θ_y , output depends explicitly on the relative wage and price rigidities. This can be observed by the dependence of output on the parameters κ_ω and κ_π . Hence, in this model, amplification of the monetary policy transmission is not just obtained from incomplete markets but from a higher degree of price stickiness relative to wage stickiness. That is the distributional channel of nominal rigidities on aggregate demand. Notice that when prices get more sticky (given a degree of nominal wage rigidity), the parameter κ_π falls, and the response of output to the monetary policy shock is amplified¹². The intuition of this result is simple. The stickier prices are, the more price markups rise in a downturn. That implies that workers with high MPCs (as they are more financially constrained) lose more than firm owners with low

¹⁰We are interested on the second moment because is the welfare relevant indicator.

¹¹Another way of obtaining this type of rule is by having a monetary policy rule that fully targets the expected inflation $r_t = \mathbb{E}_t \pi_{t+1} + \varepsilon_t^{mp}$ as in [Bilbié \(2020\)](#). Notice that with our assumptions of iid shocks we have $\mathbb{E}_t \pi_{t+1} = 0$.

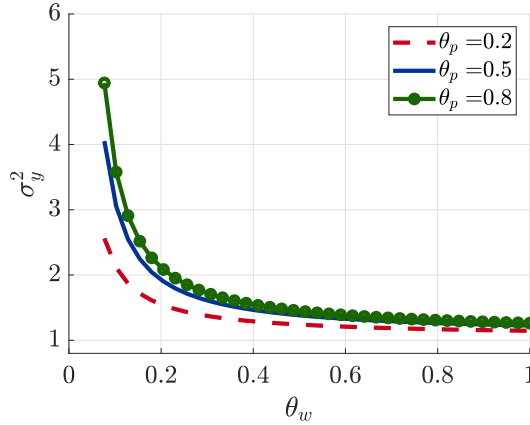
¹²In [Appendix 3.E](#) we show that the derivative of Equation (3.30) is negative, meaning that the economy becomes more sensitive to monetary policy shocks when price rigidity increases.

MPCs (who are unconstrained). Therefore, the response of consumption (and then output) is amplified by the higher countercyclicality of markups generated by high price stickiness.

However, wage rigidity dampens the effect of monetary policy through redistribution. Having more rigid wages implies that workers are more protected from aggregate shocks as their income fluctuates less (in our setup). Therefore, firm owners bear the costs of recessions when wages are more rigid. In that case the distributional channel is weaker and the economy stabilizes. This implies that as a consequence of the distributional channel, there are no gains from wage flexibility, conditional on a monetary policy shock.

Figure 3.1 describes how the distributional channel operates depending on price and wage rigidities. It shows that, conditional on monetary policy shocks (i) there are never gains from wage flexibility: the variance of output monotonically increases with wage flexibility ($\theta_\omega \rightarrow 0$); and (ii) the variance of output increases with price rigidity ($\theta_p \rightarrow 1$).

FIGURE 3.1: Variance of output for different calibrations, conditional on monetary policy shocks.



Notes: This figure shows the variance of the output gap for different levels of price rigidities, θ_π as a function of wage rigidities θ_ω . The calibration assumed in this figure is the following: $\lambda = 0.3$, $\alpha = 0.25$, $\varepsilon = \varepsilon_w = 6$, and $\sigma = \varphi = 1$.

Importantly, we obtain this dependence of output in the relative wage and price rigidity because of wage rigidities. Recall that the parameters ς , κ_ω , and κ_π depend on the degrees of price and wage rigidities (θ_π and θ_ω). Recall also that $\varsigma = \frac{\theta_\omega(1+\varphi\varepsilon_w)}{1+\theta_\omega\varphi\varepsilon_w-(1-\theta_\omega)\varpi_2}$, which is equal to zero if wages are fully flexible ($\theta_\omega = 0$). This implies that when wages are fully flexible, aggregate demand does not depend on price rigidities. The reason why this is the case is that with wage rigidities the real wage depends explicitly on price inflation. That dependence is given by the parameter ς which is the pass-through from price inflation to the real wage (see Equation 3.22). This pass-through is stronger when wages are more rigid. That happens because whenever nominal wages are rigid an increase in price inflation makes the real wage fall. Naturally, the higher the wage rigidity, the stronger the relationship between the real wage and price inflation. Then, if wages are flexible, it can be shown that aggregate demand, conditional only on monetary policy shocks, does not depend on any rigidity and

we get

$$\hat{y}_t = -\frac{1}{\sigma} \frac{1}{1 - \frac{\lambda}{(1-\lambda)\gamma+\lambda} \Psi \left(\varpi_1 + \frac{\alpha+\varphi}{1-\alpha} \right)} \varepsilon_t^{mp}, \quad (3.31)$$

which is the expression obtained by [Bilbiie \(2008\)](#) and [Debortoli and Galí \(2018\)](#). Notice that in this subsection we have only highlighted the aggregate demand effects of rigidities, abstracting from the equilibrium effects of the slopes of the wage and price Phillips curves.

3.3.5 *Wage Flexibility and the Role of Monetary Policy with Inequality*

As [Galí \(2013\)](#) uncovered, the effect of wage flexibility in New Keynesian models hinges crucially on how monetary policy is conducted. In this section, we show that in models with heterogeneity, monetary policy rules that only respond to price inflation are not sufficient to counteract the effect of highly volatile wages. In a Representative Agent economy (RANK), targeting price inflation is isomorphic to targeting wage inflation (unless we are interested in welfare). That is because prices are the only channel through which wage fluctuations affect output. Then, the role of higher–or lower– wage flexibility depends primarily on the response of price inflation to it. Hence, having a rule that reacts to wages or prices has similar qualitative effects on the economy.

Studying monetary policy design in models with heterogeneity and market incompleteness is relevant. Most of the models with household heterogeneity assume simple monetary policy rules. They do so, because the focus is on the impact of heterogeneity and not the conduct of monetary policy. However, as we explained above, with income heterogeneity, price and wage inflation affect aggregate demand directly (through the real wage). Moreover, aggregate demand depends differently on prices and wages, which means that the responses to price and wage inflation might no longer be equivalent.

Let us assume that the Taylor Rule is given by

$$\hat{r}_t = \phi_\omega \hat{\pi}_t^\omega + \phi_\pi \hat{\pi}_t^p,$$

where monetary policy reacts to deviations of the nominal wage and the price inflation rate from their steady states (assumed at zero). Substituting the Taylor rule into the Euler equation delivers

$$\hat{y}_t = -\frac{1}{\sigma} \frac{1}{1 - \frac{\lambda}{(1-\lambda)\gamma+\lambda} \Theta_y} [\phi_\omega \hat{\pi}_t^\omega + \phi_\pi \hat{\pi}_t^p - \hat{\chi}_t].$$

Recalling that $\hat{\pi}_t^\omega = \hat{\omega}_t + \hat{\pi}_t^p$, $\hat{\omega}_t = \Xi_y \hat{y}_t$, and $\hat{\pi}_t = \Upsilon_y \hat{y}_t$, the previous expression implies

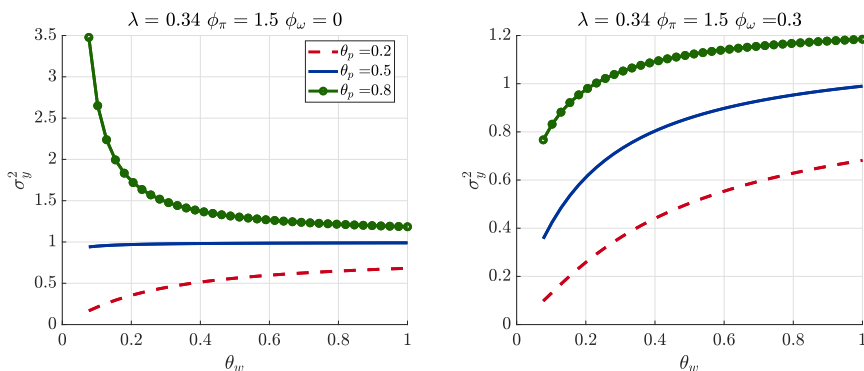
$$\hat{y}_t = -\frac{1}{\sigma} \left(1 - \frac{\lambda}{(1-\lambda)\gamma+\lambda} \Theta_y + \frac{1}{\sigma} [\phi_\omega (\Xi_y + \Upsilon_y) + \phi_\pi \Upsilon_y] \right)^{-1} \hat{\chi}_t. \quad (3.32)$$

Hence, output depends on the cyclicality of prices Υ_y and the cyclicality of wages Ξ_y through the interest rate response, in addition to the distributional channel represented by the expression $\frac{\lambda}{(1-\lambda)\gamma+\lambda} \Theta_y$.

With this type of policy rule, monetary policy has the ability to directly counteract the excessive volatility of wages if the response to them is sufficiently strong. Figure (3.2) shows the variance of the output for two alternative Taylor rules. The left-hand panel shows the the variance of output with a Taylor rule which does not respond to wages, while the right-hand panel shows the variance if monetary policy reacts to nominal wage inflation too. When monetary policy does not react to wages, it does not offset the amplifying effects of redistribution since the real wage is still too volatile (if prices are too sticky). That translates to output through aggregate demand. However, if monetary policy reacts (strongly) to wage inflation, it activates an additional countercyclical response. In this case, monetary policy reacts to the high volatility of wages and counteracts the distributional effect of high wage flexibility.

Therefore, we have two opposing effects from wage flexibility in our model. One depends on the share of HtM agents and the other on the ability of monetary policy to react to aggregate outcomes, whether prices or wages. The former controls the degree of redistribution, and the latter acts as a countercyclical device. The left-hand panel in Figure (3.2) shows that the strength of these effects depends on the degree of price rigidities. If prices are flexible, wage flexibility is stabilizing while if prices are sticky wage flexibility is expansionary. An additional result from this is that there is a threshold for θ_π which turns wage flexibility from expansionary to stabilizing. These results depend on the response of monetary policy to wages or prices, as the right-hand panel shows. For that specific calibration, monetary policy can restore the ability of wage flexibility to stabilize output.

FIGURE 3.2: Variance of Output with alternative Taylor rules.



Notes: This figure shows the variance of the output gap for different levels of price rigidities, θ_π as a function of wage rigidities θ_w . The calibration assumed in this figure is the following: $\lambda = 0.3$, $\alpha = 0.25$, $\varepsilon = \varepsilon_w = 6$, and $\sigma = \varphi = 1$.

However, the previous result does not hold for every $\phi_\omega > 0$. To show this, we compute the value of ϕ_ω which turns wage rigidities

from amplifying to stabilizing, which is given by ¹³

$$\bar{\phi}_\omega = \frac{\sigma\lambda(\varepsilon_p - 1)(1 - \alpha)(1 - \alpha + \alpha\varepsilon_p)\theta_\pi}{(\varepsilon_p - \lambda(\varepsilon_p - 1)(1 - \alpha))(1 - \alpha + \theta_\pi\alpha\varepsilon_p)} - \frac{(1 - \theta_\pi)(1 - \alpha)}{1 - \alpha + \theta_\pi\alpha\varepsilon_p}\phi_\pi, \quad (3.33)$$

which we refer to as the threshold ϕ_ω . Equation (3.33) shows that the threshold depends on the share of HtM and the degree of wage rigidities. Figure 3.3 displays $\bar{\phi}_\omega$.

As the share of HtM and price rigidities increase, so too does the threshold. These two are the main drivers of the distributional channel. Hence, to offset the distributional channel, monetary policy must react to wage inflation sufficiently strongly.

In the case of Figure (3.3), for some combinations of parameters (i.e., low λ and low θ_p) the threshold is negative. We interpret that as combinations of parameters in which wage fluctuations are not a constraint for monetary policy when stabilizing output. The negative values are a consequence of having $\phi_\pi > 0$, which helps in stabilizing output when responding to prices. However, with extremely sticky prices, the role of ϕ_π disappears. Equation (3.33) also shows that when prices are fully sticky, the required response to wages is bounded. This means that monetary policy counteracts the distributional channel more effectively when it responds to wage inflation; i.e., monetary policy does not need to set $\phi_\omega \rightarrow \infty$ to stabilizing output. This is more evident if we compare the threshold for ϕ_ω with the one for ϕ_π , which reads

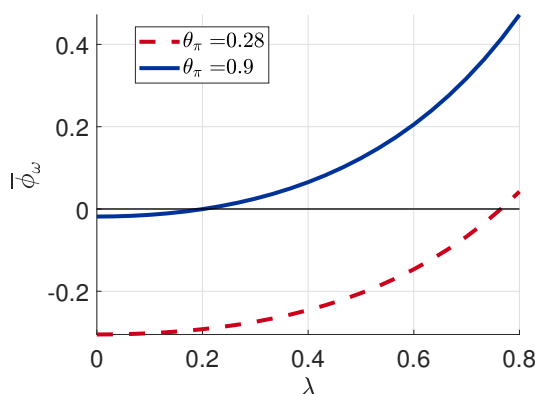
$$\bar{\phi}_\pi = \frac{\sigma\lambda(\varepsilon_p - 1)(1 - \alpha + \alpha\varepsilon_p)}{\varepsilon_p - \lambda(\varepsilon_p - 1)(1 - \alpha)} \frac{\theta_\pi}{(1 - \theta_\pi)} - \frac{1 - \alpha + \alpha\varepsilon_p\theta_\pi}{(1 - \theta_\pi)(1 - \alpha)}\phi_\omega. \quad (3.34)$$

Notice that if prices are fully sticky ($\theta_p = 1$), $\bar{\phi}_\pi \rightarrow \infty$. Whereas, in the case of ϕ_ω that is not the case. This means that monetary policy is

¹³In Appendix 3.F we get this threshold by computing the derivative of the variance of output to θ_w , equalizing equalize that to zero, and solving for the minimum parameter required to turn wage flexibility from amplifying to stabilizing.

not sufficiently effective to counteract the impact of wage flexibility if prices are sticky. Therefore, policy makers should consider responding to wages.

FIGURE 3.3: Threshold for ϕ_ω .



Notes: This figure shows the variance of the output gap for different levels of price rigidities, θ_π as a function of wage rigidities θ_ω . The calibration assumed in this figure is the following: $\lambda = 0.3$, $\alpha = 0.25$, $\varepsilon = \varepsilon_w = 6$, and $\sigma = \varphi = 1$.

The main takeaway of this exercise is that monetary policy plays a vital role in offsetting the effects of redistribution. When wages are too volatile with respect to prices, monetary policy should react to wages to offset the distributional effects of shocks and stabilize aggregate demand. Therefore, in economies with income inequality and incomplete markets, in which wage inflation is more volatile than price inflation, the monetary authority should target wages instead of prices to stabilize output effectively.

3.4 GAINS FROM WAGE FLEXIBILITY: CALVO PRICE AND WAGE ADJUSTMENT

In this section, we use our model to quantitatively investigate the gains from wage flexibility and how such gains depend on: the relative nominal rigidities; the degree of market incompleteness; and the zero lower bound on the nominal interest rate (ZLB). We consider the latter because it helps us study the effects of the distributional channel of nominal rigidities in a model without the simplifications we made in Section 3.3. Now we switch from a setup where prices are set in advance to one in which prices and wages are subject to Calvo pricing. This allows us to take into account agents' expectations and the dynamics of the economy, unlike in Section 3.3, and analyze if the main results we showed previously still hold.

3.4.1 Price and Wage Setting à la Calvo

Now firms face price stickiness à la Calvo. Hence, in every period, they reset prices with probability $(1 - \theta_p)$. A firm that is able to reset prices in period t , chooses the price P_t^* that maximizes the following sum of discounted profits:

$$\mathbb{E}_t \sum_{k=0}^{\infty} \theta_p^k \{ Q_{t,t+k} (P_t^* Y_{t+k|t} - TC_{t+k}(Y_{t+k|t})) \} \quad (3.35)$$

subject to the demand constraint given by

$$Y_{t+k|t} = \left(\frac{P_t^*}{P_{t+k}} \right)^{-\varepsilon_p} Y_{t+k}, \quad (3.36)$$

which is the demand faced in $t + k$ by a firm that sets prices optimally in t . Total costs of producing $Y_{s,t+k|t}$ units is defined as $TC_{t+k}(Y_{t+k|t}) \equiv W_{t+k} \left(\frac{Y_{t+k|t}}{A_{t+k}} \right)^{\frac{1}{1-\alpha}}$. And $Q_{t,t+k} = \beta^k \left(\frac{C_{t+k}^u}{C_{t+k}^u} \right)^{-\sigma}$ is the stochastic discount factor, which depends only on the consumption

of the unconstrained agent. The first-order condition for profits maximization reads

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\theta_p)^k \{Q_{t,t+k} Y_{t+k|t} (P_t^* - \mathcal{M}^p MC_{t+k|t})\} = 0, \quad (3.37)$$

where $\mathcal{M}^p \equiv \frac{\varepsilon_p}{\varepsilon_p - 1}$ is the desired markup and $MC_{t+k|t}$ is the nominal marginal cost.

The wage for each labor variety is set by a union operating in a monopolistically competitive market. The union faces Calvo wage stickiness where the probability of adjusting wages is given by $(1 - \theta_w)$. Unions choose the wage rate that maximizes a weighted average of unconstrained and constrained lifetime utility, given by

$$\begin{aligned} \mathbb{E}_t \sum_{k=0}^{\infty} (\beta \theta_w)^k \chi_{t+k} & \left[(1 - \lambda) \left(\frac{(C_{t+k}^u - 1)^{1-\sigma}}{1 - \sigma} - \int_0^1 \frac{(N_{t+k|t}^u)^{1+\varphi}}{1 + \varphi} dj \right) \right. \\ & \left. + \lambda \left(\frac{(C_{t+k}^c - 1)^{1-\sigma}}{1 - \sigma} - \int_0^1 \frac{(N_{t+k|t}^c)^{1+\varphi}}{1 + \varphi} dj \right) \right], \end{aligned} \quad (3.38)$$

subject to households' resource constraint and the sequence of demands for the labor variety they represent, given by

$$N_{t+k|t}^K = \left(\frac{W_t^*}{W_{t+k}} \right)^{-\varepsilon_w} N_{t+k}^K, \quad (3.39)$$

where W_t^* is the optimal wage chosen by a union that last resets its wage at t , $N_{t+k|t}^K$ is labor supply for the household's members whose wage was last reoptimized in period t and ε_w is the elasticity of substitution among labor varieties.

Assuming firms demand for constrained and unconstrained workers labor is the same, i.e. $N_t(j)^u = N_t(j)^c = N_t(j)$, the first-order

condition of the union is

$$\mathbb{E}_t \sum_{k=0}^{\infty} (\beta \theta_w)^k \chi_{t+k} N_{t+k|t}^{1+\varphi} \left[\left(\frac{(1-\lambda)}{(C_{t+k}^u)^\sigma N_{t+k|t}^\varphi} + \frac{\lambda}{(C_{t+k}^c)^\sigma N_{t+k|t}^\varphi} \right) \frac{W_t^*}{P_{t+k}} - \mathcal{M}^w \right] = 0, \quad (3.40)$$

where $\mathcal{M}^w \equiv \frac{\varepsilon_w}{\varepsilon_w - 1}$ is the desired markup and $\Pi_{w,t} \equiv \frac{W_t}{W_{t-1}}$ is the gross inflation rate of wages.

3.4.2 Quantitative Analysis

Calibration. For the baseline calibration we set the parameter α to 0.25 and the discount factor β to 0.994. We initially set the Calvo price and wage parameters to 0.75, which implies an average contract duration of four quarters. We set the parameters ε_p and ε_w to six, which implies a steady state markup of about 17%. We assume an inverse of the intertemporal elasticity of substitution, σ , and the inverse of the Frisch elasticity, φ , equal to one. Additionally, we fix the coefficient for price inflation of the Taylor rule, ϕ_π , to 1.5 and the one of wage inflation, ϕ_ω , to 0.¹⁴ The interest rate smoothing parameter ρ_r is set to 0.8. We assume, ρ_c , the autoregressive coefficient of the exogenous preference shock is 0.8. Regarding the fraction of constrained agents, we assume two scenarios: a Representative Agent (RANK) economy where all households are unconstrained (i.e. $\lambda = 0$) and an economy with a positive fraction of constrained agents, where $\lambda = 0.3$. We solve the model with the extended path method to implement the zero lower bound, and solve all the versions with this method for comparability.

The Gains from Wage Flexibility without the ZLB. We simulate the response of the economy to a contractionary preference shock on

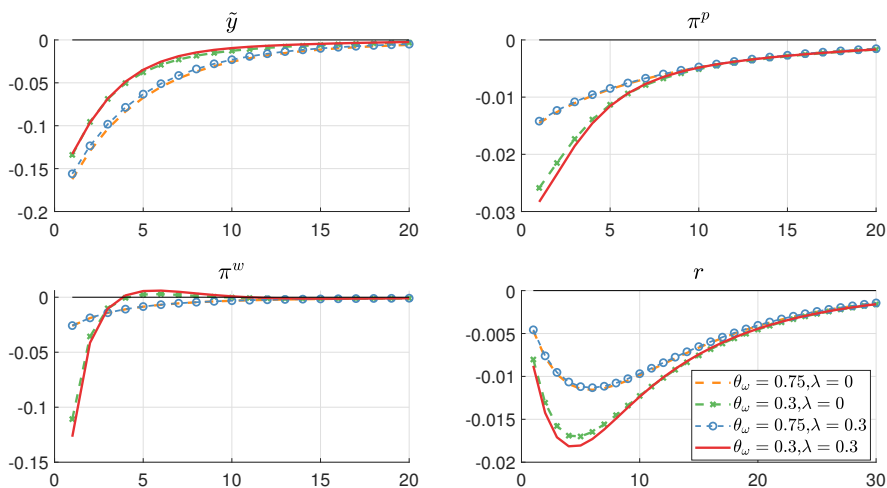
¹⁴We set these parameters to describe how the distributional channel affects the dynamics of the economy for a monetary policy that does not consider wages and prices as variables with different effects.

different scenarios depending on the degree of wage flexibility and the access to financial markets. We solve the model for combinations of $\theta_\omega = \{0.3, 0.75\}$, a flexible and a rigid wage case; and $\lambda = \{0, 0.3\}$, without and with inequality, and keep the remaining parameters as we described in the calibration. We report the results for the four combinations of these parameters in the plots that follow.

Figure 3.4 shows the responses of output, price inflation, wage inflation, and the nominal interest rate to a demand shock in the four afore-mentioned cases. The differences in the responses of output are mostly driven by differences in wage rigidity. The more rigid wages are, the stronger the response of output. Naturally, in the case where wages are flexible, wage inflation falls considerably more than in the case in which wages are rigid. More volatile wages are transmitted to price inflation. Since prices are relatively sticky in this example, the response of inflation is not as strong as the wage inflation rate. However, it is still different enough to trigger a substantial response in the interest rate compared to the case of having flexible wages.

Although the responses of the interest rate with and without inequality are different, they are not quantitatively important. Therefore, in this calibration, monetary policy is successful in counteracting the distributional channel of nominal rigidities. Then, wage flexibility reduces output volatility.

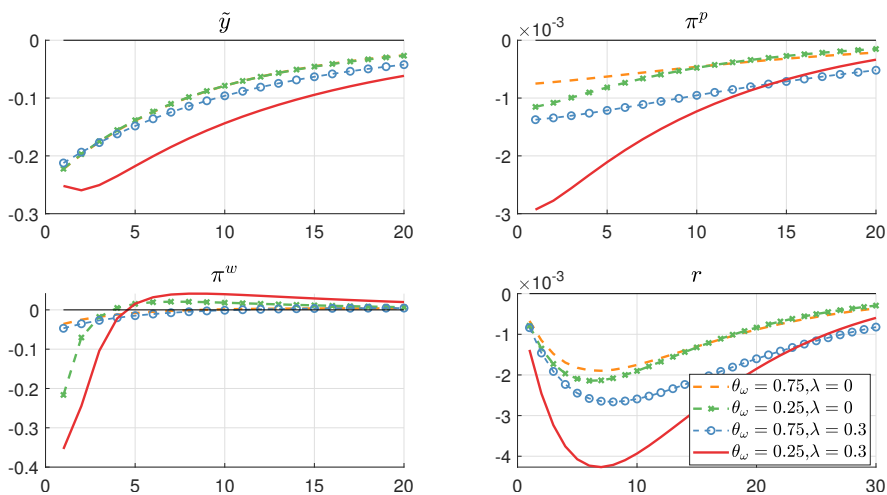
FIGURE 3.4: Response of output, prices and wage inflation, and the nominal interest rate to a contractionary preference shock. Baseline calibration.



Notes: This figure shows the responses of output, price and wage inflation, and the nominal interest rate to a contractionary preference shock. We show four calibrations for combinations of $\theta_\omega = \{0.3, 0.75\}$ and $\lambda = \{0, 0.3\}$. This plot assumes the baseline calibration.

In Figure 3.5 we set out the role of the distributional channel by assuming prices are almost fully sticky ($\theta_p=0.95$). When prices are sticky, and monetary policy only responds to price inflation, the distributional channel comes into play. In the case of Figure 3.5, output with inequality and flexible wages falls persistently more than in the other three cases (in which we observe no differences). This is because now monetary policy does not stimulate output, and the distributional effect operates very strongly. Moreover, the case in which there is inequality and wages are sticky behaves like the representative agent case. This suggests that wage rigidity acts as an insurance device for workers even if prices are highly sticky.

FIGURE 3.5: Response of output, prices and wage inflation, and the nominal interest rate to a contractionary preference shock. High price rigidity, $\theta_p = 0.95$.



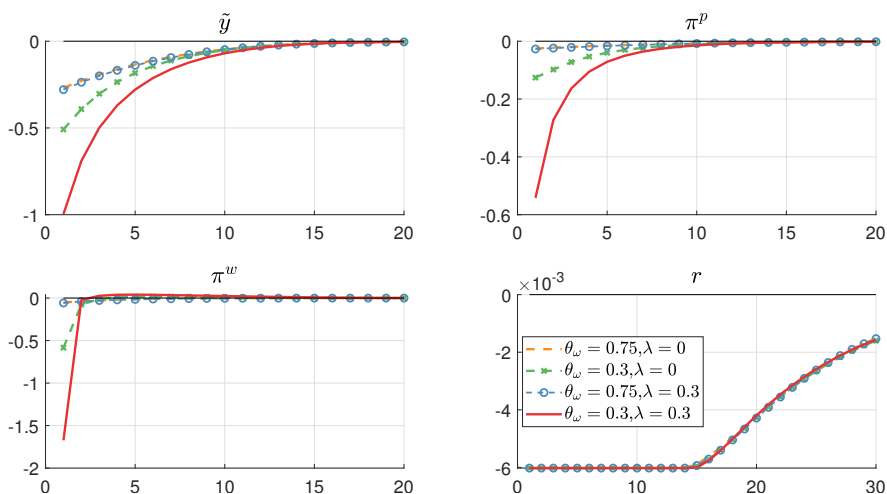
Notes: This figure shows the responses of output, price and wage inflation, and the nominal interest rate to a contractionary preference shock. We show four calibrations for combinations of $\theta_\omega = \{0.3, 0.75\}$ and $\lambda = \{0, 0.3\}$. This plot considers $\theta_p = 0.95$.

The Gains from Wage Flexibility with the ZLB. Another way of analyzing the distributional channel is by imposing the ZLB. If there is a contractionary demand shock and monetary policy cannot react, the difference between the responses (with and without inequality) would be generated only by the distributional channel (as we showed above). We show this in Figure 3.6. When the ZLB is present, wage flexibility is undesirable. This observation was first made by [Billi and Galí \(2020\)](#) who showed that in the presence of the ZLB, there are always losses from wage flexibility. In addition to that, in our model which includes inequality, the distributional channel has an impact on the economy. For our calibration, in a RANK economy, the impact of having highly flexible wages makes output fall by twice as much

as in the case of rigid wages.

In the economy with inequality, the effect is even larger. We observe that the output gap falls twice as much as in the RANK case. Therefore, in our calibration, the distributional channel generates a further amplification of shocks, beyond those observed in RANK models. All the previous analysis implies that nominal rigidities determines the distributional channel, and monetary policy is a key determinant in the transmission of shocks.

FIGURE 3.6: Response of output, prices and wage inflation, and the nominal interest rate to a contractionary preference shock in the ZLB.



Notes: This figure shows the responses of output, price and wage inflation, and the nominal interest rate to a contractionary preference shock. We show four calibrations for combinations of $\theta_\omega = \{0.3, 0.75\}$ and $\lambda = \{0, 0.3\}$. This plot assumes the baseline calibration and monetary policy subject to the ZLB.

Output Volatility and Welfare. Now, we generate artificial time series for several variables subject to demand shocks. We consider the two alternative calibrations of the Calvo wage parameter and generate these series using the extended path method to explore the effects of greater wage flexibility. We set the volatility of the innovation so

that the ZLB binds 5% of the time. Figure 3.A.1 in Appendix 3.A shows that in the RANK economy, having flexible wages is associated with lower output variability in periods when monetary policy is active. However, volatility increases in periods when the ZLB binds.¹⁵ When there are financial frictions (Figures 3.A.2 and 3.A.3 in Appendix 3.A), higher wage flexibility exacerbates the contraction of output in periods when monetary policy is constrained by the ZLB. In line with our previous discussion, the more severe contraction is explained by a larger drop in unconstrained agents' consumption. These agents are responding to increased deflationary expectations, and, to a larger extent, redistribution. Importantly, notice that higher flexibility particularly affects constrained households, whose income is severely reduced due to a large cut in wages, triggering a sizable cut in spending.

How does wage flexibility affect volatility and welfare? Table 3.1 presents the results for the volatility of output, the rates of inflation, and the consumption gap in different calibrations of θ_w . In the RANK economy, higher flexibility is associated with a more stable output (while price and wage inflation volatility greatly increases) than in the TANK economy. We observe that with inequality: (i) more flexible wages destabilize output, and (ii) price and wage inflation volatility rise. In terms of welfare losses, Table 3.2 shows that in an economy with limited asset markets participation, greater wage flexibility increases losses related to all welfare-relevant variables.¹⁶

¹⁵Note that the reduction in rigidities has only a modest effect on output dynamics. Conversely, the volatility of price and wage inflation is greatly affected by the reduction in rigidities.

¹⁶See Appendix 3.D for a derivation of the welfare loss function.

	\tilde{y}	π^P	π^w
$\theta_w = 0.75$	0.030	0.002	0.002
$\theta_w = 0.30$	0.029	0.004	0.015
Ratio	0.96	2.7	6.4

(a) $\lambda = 0$

	\tilde{y}	γ	π^P	π^w
$\theta_w = 0.75$	0.027	0.011	0.001	0.002
$\theta_w = 0.30$	0.036	0.055	0.006	0.019
Ratio	1.3	5	3.8	8.9

(b) $\lambda = 0.3$

Table 3.1: Standard deviation

	\tilde{y}	π^P	π^w	Total Loss
$\theta_w = 0.75$	0.0012	0.0003	0.0010	0.0025
$\theta_w = 0.30$	0.0011	0.0019	0.0022	0.0052
Ratio	0.9	7.1	2.1	2.1

(a) $\lambda = 0$

	\tilde{y}	γ	π^P	π^w	Total Loss
$\theta_w = 0.75$	0.0010	0.0000	0.0002	0.0008	0.0021
$\theta_w = 0.30$	0.0017	0.0003	0.0032	0.0034	0.0087
Ratio	1.7	24.9	14.7	4.1	4.2

(b) $\lambda = 0.3$

Table 3.2: Consumption equivalent welfare losses

3.5 CONCLUSION

In this paper, we analyze the consequences of nominal rigidities in the presence of incomplete markets and the zero lower bound. We show both analytically and numerically that in the presence of limited asset market participation, the relative price and wage rigidities enter

aggregate demand; i.e., both nominal rigidities play a crucial role in determining the response of the economy to shocks. That is due to a distributional effect: if the distribution of income is unequal and there are agents with limited access to financial markets when prices fluctuate less strongly than wages, the distributional channel gains prominence, and the effects of shocks amplify. That is a consequence of the larger fluctuation of wages relative to prices. When that is the case, workers (who are usually more financially constrained than the owners of firms) suffer more from fluctuations. That implies that wage flexibility may amplify the cycle if prices are sufficiently more rigid than wages.

As [Billi and Galí \(2020\)](#), we show that the conduct of monetary policy is important for all the results exposed above. We find that with inequality, responding to price inflation it is not longer isomorphic to responding to wage inflation. If monetary policy only reacts to price inflation, it misses the effects that higher wage volatility has on aggregate demand. We show that in our model, monetary policy is more effective if it responds to wage inflation rather to price inflation.

Understanding the interaction of these three features (price and wage rigidities and limited access to financial markets) is important for several reasons. First, there is a growing literature that uses these features to study diverse macroeconomic questions, like the effects of fiscal and monetary policy in the presence of incomplete markets. We show that frequently used calibrations may generate unintended effects of some shocks. Second, it is important to understand the effects of labor market policies, particularly the policies that pretend to stabilize the economy through wage deflation. We show that these kinds of policies are not desirable under some circumstances since they generate significant aggregate demand effects that could further depress the economy. Then, with high inequality, the economy may experience a sharp contraction from making wages more flexible.

APPENDIX

APPENDIX 3.A FIGURES

FIGURE 3.A.1: Fluctuations under preference shocks ($\lambda = 0$).

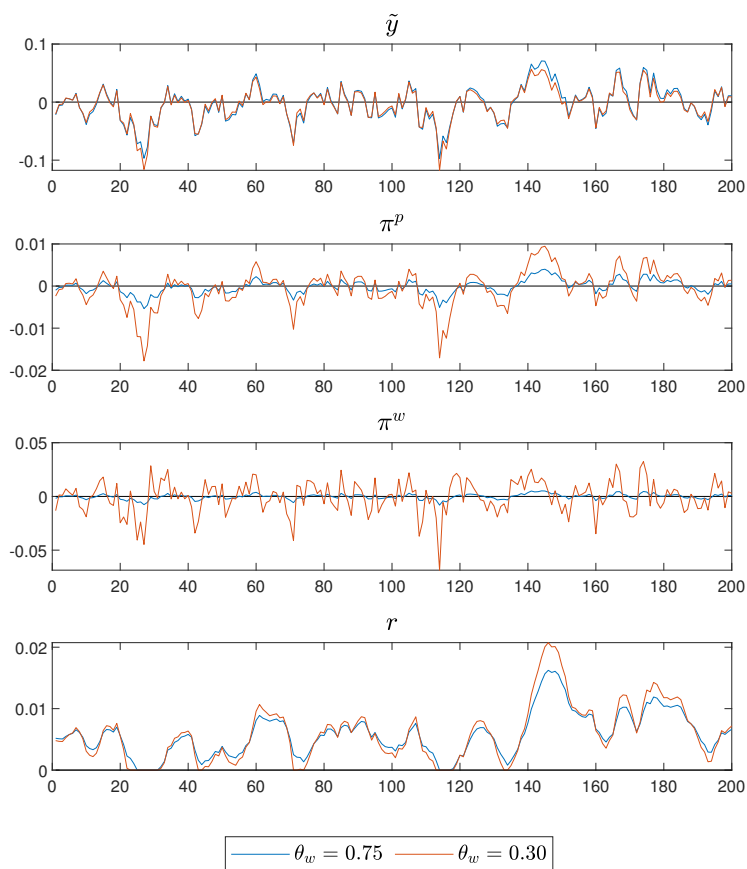


FIGURE 3.A.2: Fluctuations under preference shocks ($\lambda = 0.3$).

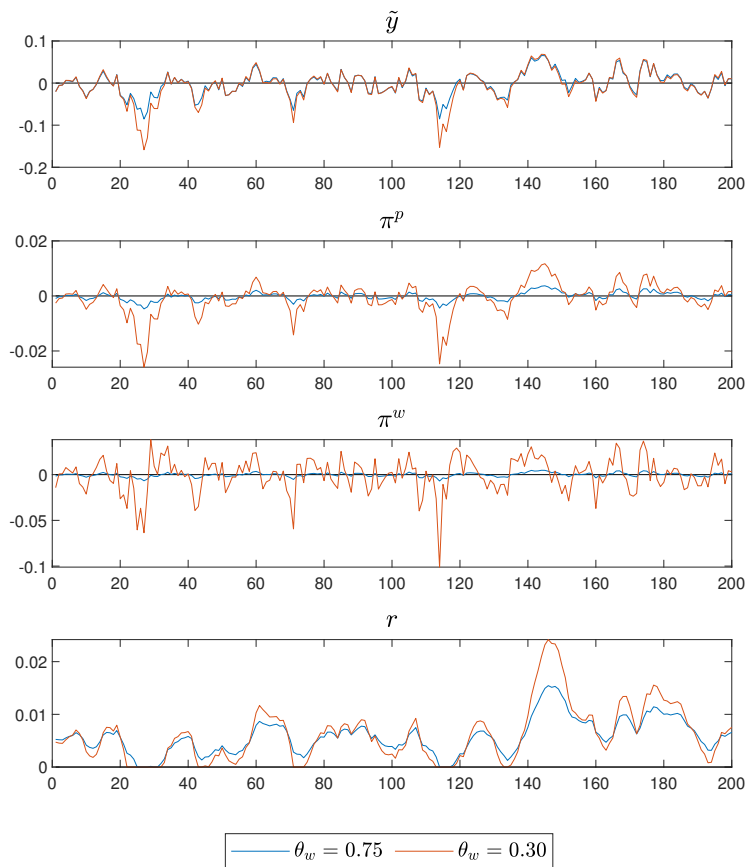
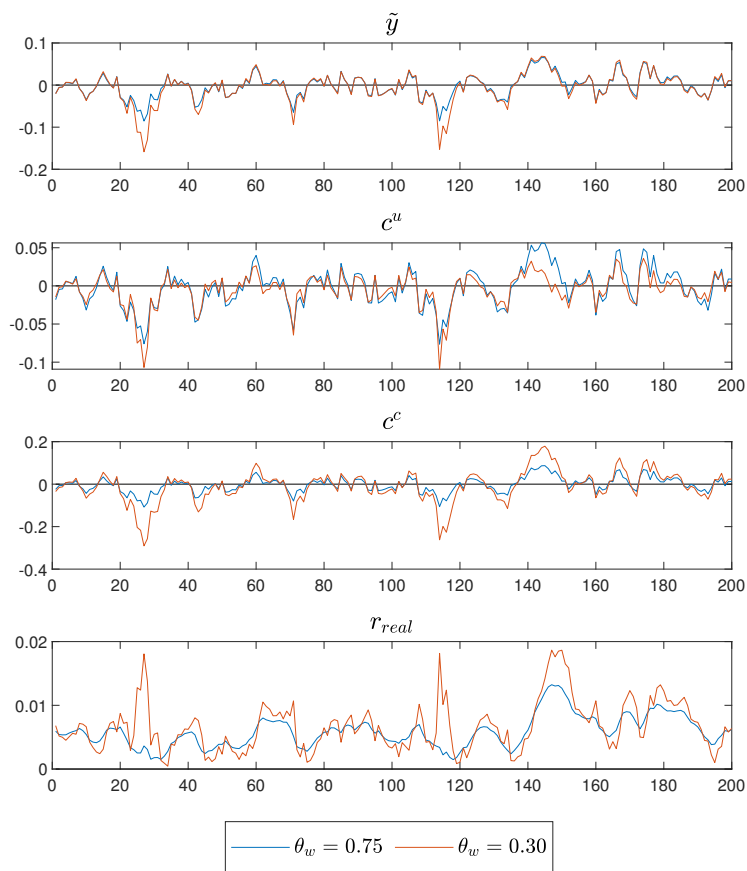


FIGURE 3.A.3: Fluctuations under preference shocks ($\lambda = 0.3$).



APPENDIX 3.B PROOFS AND DERIVATIONS

3.B.1 Aggregation

Total labor supply must be equal to total demand. This is, $N_t = \int_0^1 \int_0^1 N_t(i, j) di dj$, where i denotes firms and j labor varieties. From the demand of each firm i for variety j we have

$$N_t = \int_0^1 \left(\frac{W_t(j)}{W_t} \right)^{-\varepsilon_w} \left(\int_0^1 N_t(i) di \right) dj.$$

Recalling the demand for each firm i 's variety $Y_t(i) = \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon_p} Y_t$ and from the production function of each firm i $N_t(i) = \left(\frac{Y_t(i)}{A_t} \right)^{\frac{1}{1-\alpha}}$, we have

$$\begin{aligned} N_t &= \int_0^1 \left(\frac{W_t(j)}{W_t} \right)^{-\varepsilon_w} \left(\int_0^1 \left(\left(\frac{Y_t}{A_t} \right) \left(\frac{P_t(i)}{P_t} \right)^{-\varepsilon_p} \right)^{\frac{1}{1-\alpha}} di \right) dj \\ &= \left(\frac{Y_t}{A_t} \right)^{\frac{1}{1-\alpha}} \underbrace{\left(\int_0^1 \left(\frac{P_t(i)}{P_t} \right)^{\frac{-\varepsilon_p}{1-\alpha}} di \right)}_{\equiv \Delta_{p,t}} \underbrace{\left(\int_0^1 \left(\frac{W_t(j)}{W_t} \right)^{-\varepsilon_w} dj \right)}_{\equiv \Delta_{w,t}}, \end{aligned}$$

which is the same expression in the main text.

3.B.2 Proof of Proposition 1

We separate the proof in three parts. First, we describe the evolution of price inflation. Second, we describe the labor supply in the economy with heterogeneity and wage rigidities. Finally, we describe the process for real wages.

Part 1: Price inflation

We know there is a mass $1 - \theta_p$ of optimizing firms (denoted by superindex o) while the remainder θ_p set prices before the shock is realized (denoted by superindex m).

We start by describing the price setting problem optimizers face. Given the (log-linearized) demand function for a given variety i :

$$y_t^o(i) = y_t - \varepsilon_p(p_t^o(i) - p_t) \quad (3.41)$$

firms maximize profits setting their price as a markup μ^p over the marginal cost. Optimal pricing implies:

$$p_t^o(i) = \mu^p + w_t - \log(1 - \alpha) + \frac{\alpha}{1 - \alpha} y_t^o(i) - \frac{1}{1 - \alpha} a_t \quad (3.42)$$

where $\mu^p = \log(\mathcal{M}^p)$. Substituting the demand (3.41) into the firm optimality condition (3.42) and rearranging yields:

$$p_t^o(i) = \frac{1 - \alpha}{1 - \alpha + \alpha \varepsilon_p} \left(\mu^p + w_t - \log(1 - \alpha) + \frac{\alpha}{1 - \alpha} y_t - \frac{1}{1 - \alpha} a_t \right) + p_t, \quad (3.43)$$

where $\omega_t \equiv w_t - p_t$ is the real wage.

Consider next the price setting problem that non-optimizers face. These firms set prices at the end of period $t - 1$, and hence, they make their pricing decisions for period t based on the information set available at $t - 1$. Optimization implies:

$$p_t^m(i) = \mathbb{E}_{t-1} (\mu^p + w_t - a_t - \log(1 - \alpha) + \alpha n_t^m(i)) \quad (3.44)$$

On the other hand, the aggregate price index is given by:

$$P_t \equiv \left(\int_0^1 P_t(j)^{1-\varepsilon_p} dj \right)^{\frac{1}{1-\varepsilon_p}}$$

which implies:

$$P_t = \left((1 - \theta_p)(P_t^o)^{1-\varepsilon_p} + \theta_p(P_t^m)^{1-\varepsilon_p} \right)^{\frac{1}{1-\varepsilon_p}}$$

Accordingly, the following relation holds around steady state:

$$p_t = (1 - \theta_p)p_t^o + \theta_p p_t^m$$

Substituting the pricing rules from optimizers (3.43) and non-optimizers (3.44) into the aggregate price index yields the following price inflation equation:

$$\widehat{\pi}_t^p = \frac{1 - \theta_p}{\theta_p} \frac{1 - \alpha}{1 - \alpha + \alpha\varepsilon_p} \left(\widehat{\omega}_t + \frac{\alpha}{1 - \alpha} \widehat{y}_t - \frac{1}{1 - \alpha} \widehat{a}_t \right) + \mathbb{E}_{t-1} \widehat{x}_t^p \quad (3.45)$$

where hat variables correspond to log-deviations with respect to steady-state and $\widehat{x}_t^p \equiv \widehat{\omega}_t - a_t + \alpha \widehat{n}_t^m(j) + \widehat{\pi}_t^p$. Finally, define $\kappa_\pi \equiv \frac{1 - \theta_p}{\theta_p} \frac{1 - \alpha}{1 - \alpha + \alpha\varepsilon_p}$, which concludes the first part of the proof, for price inflation.

Part 2: Labor supply

Before solving for the wage schedule, we first solve for the *actual* labor supply. This is needed since the labor supply in our model with inequality depends on the consumption gap as it depends on the average marginal utility (thus, depends on both constrained and unconstrained consumption). In our setting, unions maximize households utility by setting the wage (in real terms) to be a markup μ^w over the marginal rate of substitution. However, in a heterogeneous

economy like ours, the average marginal utility depends on the fraction of each group of consumers, in particular, it depends on the share of constrained consumers λ . That is why it does not directly depend on the output gap as usual.

To start, we compute three intermediate results. First we show how the average marginal utility of consumption depends on inequality (λ) and the level of consumption of each group. Then we show the relation between individual constrained and unconstrained consumption with GDP and consumption gap. Finally, we show how average marginal utility of consumption depends on GDP and real wages.

Lemma 1. *The average marginal utility of consumption, U , as a function of individual consumptions can be approximated as*

$$\hat{u}_t = -\sigma(u_c \hat{c}_t^c + u_u \hat{c}_t^u), \quad (3.46)$$

with $u_c = \frac{\lambda}{\lambda + (1-\lambda)\gamma^{-\sigma}}$ and $u_u = 1 - u_c$, and $\gamma \equiv \frac{C^U}{C^C}$ being the steady-state consumption gap.

Proof. The marginal utility of consumption of agent $K \in \{c, u\}$ is $(C_t^K)^{-\sigma}$. therefore, the average marginal utility of consumption, U , can be written as $U = \lambda(C_t^c)^{-\sigma} + (1 - \lambda)(C_t^u)^{-\sigma}$. Taking a first order approximation around steady-state, we get $\hat{u}_t = -\sigma(u_c \hat{c}_t^c + u_u \hat{c}_t^u)$, with $u_c = \frac{\lambda C_c^{-\sigma}}{\lambda C_c^{-\sigma} + (1-\lambda) C_u^{-\sigma}}$. Replacing $C_u = \gamma C_c$ we get $u_c = \frac{\lambda}{\lambda + (1-\lambda)\gamma^{-\sigma}}$ and $u_u = 1 - u_c$. \square

Lemma 1 shows that the average marginal utility of consumption, which is a relevant piece of information for unions to set nominal wages and total hours, depends on the fraction of constrained agents, the income effect on labor supply (given by σ) and the steady-state consumption gap, γ . Clearly, whenever $\lambda = 0$ ($\lambda = 1$), $\hat{c}_t^c = \hat{c}_t^u = \hat{c}_t$, the consumption gap is zero and $u_u = 1$ ($u_c = 0$), so $\hat{u}_t = -\sigma \hat{c}_t$, which

is the same marginal utility of consumption of a representative agent model.

Lemma 2 shows the aggregate relation between consumption on each segment of the population, GDP and the consumption gap.

Lemma 2. *Consumption of constrained and unconstrained agents can be approximated as*

$$\begin{aligned}\widehat{c}_t^c &= \widehat{y}_t - \frac{(1-\lambda)\gamma}{\lambda + (1-\lambda)\gamma} \widehat{\gamma}_t \\ \widehat{c}_t^u &= \widehat{y}_t + \frac{\lambda}{\lambda + (1-\lambda)\gamma} \widehat{\gamma}_t\end{aligned}$$

Proof. From market clearing in the market of final good and the definition of aggregate consumption we have $Y_t = C_t = \lambda C_t^c + (1-\lambda)C_t^u$. Using the definition of consumption gap, the previous expression is $Y_t = C_t^U \left(\frac{\lambda}{\gamma_t} + 1 - \lambda \right)$. This can be approximated as $\widehat{y}_t = \widehat{c}_t^u - \frac{\lambda}{(1-\lambda)\gamma + \lambda} \widehat{\gamma}_t$. On the other hand, the consumption gap is $\widehat{\gamma}_t = \widehat{c}_t^u - \widehat{c}_t^c$. Using these two equations to solve for constrained and unconstrained consumption as a function of GDP and consumption gap gives the desired equations. \square

Finally, Lemma 3 fully characterizes the behavior of the average marginal utility of consumption as a function of GDP and real wages.

Lemma 3. *The average marginal utility of consumption can be written in terms of GDP and real wages as*

$$\widehat{u}_t = -\varpi_1 \widehat{y}_t - \varpi_2 \widehat{\omega}_t, \quad (3.47)$$

with $\varpi_1 \equiv \sigma + \bar{u}\Psi \frac{\alpha}{1-\alpha}$ and $\varpi_2 \equiv \bar{u}\Psi$, and where $\bar{u} \equiv -\sigma \frac{\lambda(1-\lambda)\gamma^{-\sigma} - \lambda(1-\lambda)\gamma}{[(1-\lambda)\gamma + \lambda][\lambda + (1-\lambda)\gamma^{-\sigma}]}$ and $\Psi \equiv \frac{M^p}{(1-\lambda)(1-\alpha + \frac{1}{1-\lambda}(M^p - (1-\alpha)))}$ comes from the relation between consumption inequality and price markup in equation (3.18).

Proof. First note that by combining the results from Lemma 1 and Lemma 2, we get $\hat{u}_t = -\sigma\hat{y}_t + \bar{u}\hat{\gamma}_t$, with \bar{u} previously defined. Then, replacing equation (3.19) into (3.18) (average price markup into consumption gap), we get $\hat{\gamma}_t = \Psi\left(-\frac{\alpha}{1-\alpha}\hat{y}_t - \hat{\omega}_t\right)$. Combining these results and re-arranging, we get equation (3.47). \square

Now we are ready to solve for the actual labor supply schedule. The optimality condition for the union is $\frac{W_t}{P_t}\left(\frac{\lambda}{(C_t^c)^{-\sigma}} + \frac{1-\lambda}{(C_t^u)^{-\sigma}}\right) = \mathcal{M}^w N_t^\varphi$, where \mathcal{M}^w is the wage markup and we impose the condition that all workers have the same wage and the same number of hours. Note that the term in parentheses is the average marginal utility of consumption across workers. Taking a log-linear approximation, we get $\hat{\omega}_t + \hat{u}_t = \varphi\hat{n}_t$. Replacing the average marginal utility of consumption (3.47), we obtain

$$\hat{\omega}_t - \varpi_1\hat{y}_t - \varpi_2\hat{\omega}_t = \varphi\hat{n}_t.$$

Re-ordering and defining $\varpi = \frac{\varpi_1}{1-\varpi_2}$ and $\bar{\varphi} = \frac{\varphi}{1-\varpi_2}$, we get the average labor supply

$$\hat{\omega}_t = \bar{\varphi}\hat{n}_t + \varpi\hat{y}_t, \tag{3.48}$$

where $\bar{\varphi}$ and ϖ are parameters that depend on inequality, λ .

Part 3: Real wage

Finally, we can describe the evolution of real wages. As in the case of price inflation, we can separate the problem between optimizers and non-optimizers. Consider first the optimizers problem. Unions optimally set the wage according to:

$$\hat{\omega}_t^o(j) - \varpi_1\hat{y}_t - \varpi_2\hat{\omega}_t = \varphi\hat{n}_t^o(j). \tag{3.49}$$

Given the demand function

$$n_t^o(j) = n_t - \varepsilon_w(\omega_t^o(j) - \omega_t) \quad (3.50)$$

we obtain:

$$\hat{\omega}_t^o(j) = \frac{1}{1 + \varphi\varepsilon_w}(\varpi_1\hat{y}_t + \varphi\hat{n}_t + (\varphi\varepsilon_w + \varpi_2)\hat{\omega}_t) \quad (3.51)$$

Consider next the wage setting problem non-optimizers face. Identical to the firms problem, non-optimizing unions decide wages for period t based on the information set available at $t - 1$, implying:

$$\mathbb{E}_{t-1}(\hat{\omega}_t^m(j) - \varpi_1\hat{y}_t - \varpi_2\hat{\omega}_t - \varphi\hat{n}_t^m(j)) = 0$$

which can be rewritten as:

$$\hat{\omega}_t^m(j) = -\hat{\pi}_t^p + \mathbb{E}_{t-1}(\varpi_1\hat{y}_t + \varpi_2\hat{\omega}_t + \varphi\hat{n}_t^m(j) + \hat{\pi}_t^p) \quad (3.52)$$

On the other hand, the aggregate wage is given by:

$$W_t \equiv \left(\int_0^1 W_t(i)^{1-\varepsilon_w} di \right)^{\frac{1}{1-\varepsilon_w}}$$

implying:

$$W_t = (\theta_w(W_t^m)^{1-\varepsilon_w} + (1 - \theta_w)(W_t^o)^{1-\varepsilon_w})^{\frac{1}{1-\varepsilon_w}}.$$

The previous expression can be written in log-deviation from the steady state as:

$$\hat{\omega}_t = \theta_w\hat{\omega}_t^m + (1 - \theta_w)\hat{\omega}_t^o$$

Finally, substituting optimizers and non-optimizers wage setting rules (3.51) and (3.52) into the aggregate wage equation and imposing market clearing condition yields:

$$\hat{\omega}_t = \kappa_\omega(\varpi_1 \hat{y}_t + \varphi \hat{n}_t) - \varsigma \hat{\pi}_t^p + \mathbb{E}_{t-1} \hat{x}_t^w. \quad (3.53)$$

Defining $\kappa_\omega \equiv \frac{1-\theta_w}{1+\theta_w \varphi \varepsilon_w - (1-\theta_w) \varpi_2}$, $\varsigma \equiv \frac{\theta_w(1+\varphi \varepsilon_w)}{1+\theta_w \varphi \varepsilon_w - (1-\theta_w) \varpi_2}$ and $\hat{x}_t^w \equiv \varsigma(\varpi_1 \hat{y}_t + \varpi_2 \hat{\omega}_t + \varphi \hat{n}_t^m(j) + \hat{\pi}_t^p)$, we get the result.

3.B.3 Derivation of the IS equation

From (3.5) we can get:

$$C_t = C_t^u \left((1-\lambda) + \lambda \frac{1}{\gamma_t} \right)$$

which can be rewritten in log-deviation from steady state as:

$$\hat{c}_t = \hat{c}_t^u - \frac{\lambda}{(1-\lambda)\gamma + \lambda} \hat{\gamma}_t. \quad (3.54)$$

On the other hand, the Euler equation of unconstrained agents is $\hat{c}_t = \mathbb{E}_t \{ \hat{c}_{t+1} \} - \frac{1}{\sigma} \mathbb{E}_t [\hat{r}_t - \hat{\pi}_{p,t+1} - (1-\rho_\chi) \chi_t]$. Replacing (3.24) and (3.54) into the previous expression and imposing market clearing we obtain:

$$\begin{aligned} & \hat{y}_t + \frac{\lambda}{(1-\lambda)\gamma + \lambda} (-\Theta_y \hat{y}_t + \Theta_a a_t - \Psi \mathbb{E}_{t-1} x_t) \\ &= \mathbb{E}_t \left[\hat{y}_{t+1} + \frac{\lambda}{(1-\lambda)\gamma + \lambda} (-\Theta_y \hat{y}_{t+1} + \Theta_a a_{t+1} - \Psi \mathbb{E}_t x_{t+1}) \right] \\ & - \frac{1}{\sigma} \mathbb{E}_t (\hat{r}_t - \hat{\pi}_{p,t+1} - (1-\rho_\chi) \hat{\chi}_t). \end{aligned}$$

Assume next that the economy starts at steady state in period $t-1$ and that shocks are iid. Since shocks are unexpected, then at $t-1$

agents forecast that the economy at t will remain at steady state, i.e. $\mathbb{E}_{t-1}\hat{x}_t = 0$. Additionally, since shocks have no persistence, all real variables return to steady state at $t + 1$.¹⁷ The nominal variables at $t + 1$ on the other hand will be determined by monetary policy. We assume the central bank implements a policy such that $\hat{\pi}_{p,t+1} = 0$. Accordingly, we have $\mathbb{E}_t\hat{y}_{t+1} = \mathbb{E}_t\hat{\pi}_{t+1}^p = \mathbb{E}_t\hat{x}_{t+1} = 0$, and hence the aggregate Euler equation can be written as:

$$\hat{y}_t = -\frac{1}{\sigma} \frac{1}{1 - \frac{\lambda}{(1-\lambda)\gamma + \lambda} \Theta_y} \mathbb{E}_t \left(\hat{r}_t - \hat{\chi}_t + \frac{\lambda}{(1-\lambda)\gamma + \lambda} \sigma \Theta_a a_t \right) \quad (3.55)$$

Expression (3.55) is the Euler equation under our simplifying assumptions. Notice that the response of output to the interest rate not only depends on the intertemporal elasticity of substitution, σ , but also on another term, which involves the market incompleteness parameter λ . If $\lambda = 0$ the economy is a RANK and the slope of the Euler equation is given by $-1/\sigma$. However, when market incompleteness is present (with $\lambda > 0$) the elasticity to the real rate depends on another parameter, Θ_y , which governs the cyclicity of the consumption gap.

APPENDIX 3.C CAN THE CONSUMPTION GAP BE PROCYCLICAL?

An interesting question arising from the discussion above is whether sufficiently large values of the wage stickiness, θ_w , reverse the cyclicity of the consumption gap. In such a situation, the real wage would remain high during economic downturns, and there is redistribution in favor of workers; i.e., the consumption gap can turn procyclical.

¹⁷At t agents correctly anticipate no shocks at $t + 1$, hence it is as if the economy was not affected by any friction at $t + 1$ and thus the allocation must coincide with that of the flexible price economy. The latter ensures all real variables return to steady state at $t + 1$.

To answer this question, recall that the coefficient $\Theta_y \equiv \Psi \left(\Xi_y + \frac{\alpha}{1-\alpha} \right)$ determines the cyclicity of the consumption gap. This coefficient is positive under sticky prices and flexible wages, which implies a countercyclical gap.

Since $\Psi \equiv \frac{M^p}{(1-\lambda)(1-\alpha + \frac{1}{1-\lambda}(M^p - (1-\alpha)))} > 0$, it follows that to overturn the cyclicity of $\hat{\gamma}_t$, we require $\Xi < -\frac{\alpha}{1-\alpha}$. As we show in the Appendix 3.C, it turns out that for any parameter calibration $\Xi \geq -\frac{1}{1-\alpha}$ holds, i.e., the consumption gap can never be procyclical.

To provide an intuition for this result, let us consider the response of our economy to a negative iid preference shock. Equation (3.18) shows that a procyclical consumption gap requires the price markup to turn negative in the face of the adverse shock, i.e., markups need to reduce below their desired level. Nevertheless, such circumstance is not possible in our setting. In fact, at t , when the contractive shock hits, employment must fall along with output while wages experience downward pressure due to reduced employment and consumption. Accordingly, nominal marginal costs reduce. The latter implies a rise in the markup, and so firms cut prices. However, such a reduction in prices can never lead to a negative markup, for this would imply markups fall below the desired level, which is sub optimal. Accordingly, that result rules out redistribution in favor of workers.

Let us now consider the role of nominal rigidities in the context of supply shocks. As stressed earlier, the degree of nominal rigidities can affect the demand effects of productivity shocks, as captured by Θ_a . Precisely, given $\frac{\partial \Xi_a}{\partial \theta_p} < 0$ and $\frac{\partial \Xi_a}{\partial \theta_w} > 0$, the contractive demand effect of a positive productivity shock is increasing in the degree of price rigidity and decreasing in the degree of wage stickiness. Against this background, an important question is whether a high degree of wage stickiness, relative to price stickiness, overturns the contractive demand effect of a positive productivity shock. Intuitively, in such a scenario, the positive supply shock would lead to a rise in the real wage (due to a cut in prices following the drop in marginal costs). If

the real wage increase is large enough relative to the magnitude of the employment drop resulting from a higher productivity level, labor income and workers' earnings rise relative to firm owners' income. Given such income redistribution, aggregate demand would increase.

However, as shown in the appendix, $\Xi_a = f(\alpha, \sigma, \varphi, \theta_p, \varepsilon_p, \theta_w, \varepsilon_w) \leq \frac{1}{1-\alpha}$, and hence $\Theta_a \geq 0$, which implies that a positive productivity shock can never expand aggregate demand. To understand this outcome, notice that given the level of nominal prices and wages, a rise in productivity leads to increased firms' markups (over the desired level) due to a drop in marginal costs. Therefore, there is a redistribution in favor of firms' owners. As a result of the surge in markups, firms cut prices.

Nevertheless, such a cut can never bring prices below the desired markup, as this would be suboptimal. Hence, we discard redistribution in favor of workers as markups do not fall below desired levels. Accordingly, income redistribution arising from the positive supply shock can never expand aggregate demand.

In our previous discussion, we have stressed the negative relationship between the degree of wage rigidities and the slope of the Euler equation. That result seems to indicate that, under limited financial market participation, more flexible wages would necessarily lead to an increase in output volatility in the face of demand shocks. Besides, we have seen that supply improvements are contractive, and the more so, the higher the degree of wage flexibility. However, note that our earlier analysis abstracted from any endogenous response of the policy rate to shocks. Nevertheless, as we will see next, the rate response becomes critical for the outcome from more flexible wages.

Case when $\theta_p = 0$:

$$\lim_{\theta_p \rightarrow 0} \Xi = \frac{N}{D} = \frac{\frac{1-\theta_w}{1+\theta_w\bar{\varphi}\varepsilon_w} \left(\varpi + \frac{\bar{\varphi}}{1-\alpha} \right) - \frac{\theta_w(1+\bar{\varphi}\varepsilon_w)}{1+\theta_w\bar{\varphi}\varepsilon_w} \frac{1-\theta_p}{\theta_p} \frac{1-\alpha}{1-\alpha+\alpha\varepsilon_p} \frac{\alpha}{1-\alpha}}{1 + \frac{\theta_w(1+\bar{\varphi}\varepsilon_w)}{1+\theta_w\bar{\varphi}\varepsilon_w} \frac{1-\theta_p}{\theta_p} \frac{1-\alpha}{1-\alpha+\alpha\varepsilon_p}} = \frac{\infty}{\infty}$$

Apply L'Hopital:

$$\lim_{\theta_p \rightarrow 0} \frac{\frac{\partial N}{\partial \theta_p}}{\frac{\partial D}{\partial \theta_p}} = \frac{-\frac{\theta_w(1+\bar{\varphi}\varepsilon_w)}{1+\theta_w\bar{\varphi}\varepsilon_w} \frac{1-\alpha}{1-\alpha+\alpha\varepsilon_p} \frac{\alpha}{1-\alpha} \frac{-\theta_p-(1-\theta_p)}{\theta_p^2}}{\frac{\theta_w(1+\bar{\varphi}\varepsilon_w)}{1+\theta_w\bar{\varphi}\varepsilon_w} \frac{1-\alpha}{1-\alpha+\alpha\varepsilon_p} \frac{-\theta_p-(1-\theta_p)}{\theta_p^2}} = -\frac{\alpha}{1-\alpha}$$

Is Ξ a monotonic function of θ_p ?

$$\frac{\partial \Xi}{\partial \theta_p} = \frac{\left(\frac{\alpha}{1-\alpha} + \frac{1-\theta_w}{1+\theta_w\bar{\varphi}\varepsilon_w} \left(\varpi + \frac{\bar{\varphi}}{1-\alpha} \right) \right) \frac{\theta_w(1+\bar{\varphi}\varepsilon_w)}{1+\theta_w\bar{\varphi}\varepsilon_w} \frac{1-\alpha}{1-\alpha+\alpha\varepsilon_p} \frac{1}{\theta_p^2}}{\left(1 + \frac{\theta_w(1+\bar{\varphi}\varepsilon_w)}{1+\theta_w\bar{\varphi}\varepsilon_w} \frac{1-\theta_p}{\theta_p} \frac{1-\alpha}{1-\alpha+\alpha\varepsilon_p} \right)^2} > 0$$

We have shown that for any calibration of parameters $\varpi, \bar{\varphi}, \varepsilon_p, \theta_w, \varepsilon_w$ the coefficient Ξ is an increasing function of θ_p with lower limit $\Xi = -\frac{\alpha}{1-\alpha}$. Accordingly, for any parameter calibration we have that $\Xi \geq -\frac{\alpha}{1-\alpha}$.

APPENDIX 3.D WELFARE LOSSES

We derive a general welfare loss function for the economy, taking into account limited asset market participation. We assume that the central bank seeks to minimize the weighted utility of constrained and unconstrained agents (with weights given by their relative sizes).¹⁸ Taking a second order approximation of utility around the efficient steady state with no inequality, average welfare losses can be expressed as:

$$L = \frac{1}{2} \left[\left(\sigma + \frac{\varphi + \alpha}{1-\alpha} \right) \text{var}(\tilde{y}_t) + \sigma \lambda (1-\lambda) \text{var}(\hat{\gamma}_t) + \frac{\varepsilon_p}{\lambda_p} \text{var}(\pi_t^p) + \frac{(1-\alpha)\varepsilon_w}{\lambda_w} \text{var}(\pi_t^w) \right]$$

¹⁸For simplicity, we further assume the existence of a labor subsidy that corrects for the inefficiencies generated by monopolistic competition, and transfers that equate the steady state consumption of constrained and unconstrained households.

where $\lambda_p \equiv \frac{(1-\beta\theta_p)(1-\theta_p)}{\theta_p} \frac{1-\alpha}{1-\alpha+\alpha\varepsilon_p}$ and $\lambda_w \equiv \frac{(1-\beta\theta_w)(1-\theta_w)}{\theta_w(1+\varphi\varepsilon_w)}$. Welfare losses are a function of the output gap, price and wage inflation volatility, and the consumption gap. The latter term captures inequality and arises from the existence of limited asset markets participation. This will be the metric we use to analyze gains from wage flexibility, as well as the degree of output volatility.

APPENDIX 3.E THE SLOPE OF THE IS CONDITIONAL ON MONETARY POLICY SHOCKS

A way to write the slope of the IS (denoted by S) is

$$S = -\frac{(1-\alpha)(1+\varsigma\kappa_\pi)}{\sigma(1-\alpha)(1+\varsigma\kappa_\pi) + (\alpha + \kappa_\omega x)\Omega^*} \quad (3.56)$$

with $\Omega^* = -\sigma\Gamma\Psi = -\sigma\frac{\lambda(\varepsilon_p-1)(1-\alpha)}{\varepsilon_p-\lambda(\varepsilon_p-1)(1-\alpha)}$ and $x = \varpi(1-\alpha) + \bar{\varphi}$. Then, we compute the derivative of this slope, $\frac{\partial S}{\partial\theta_p}$ as

$$\frac{\partial S}{\partial\theta_p} = -(1-\alpha)\varsigma(\alpha + \kappa_\omega x)\sigma\frac{\lambda(\varepsilon_p-1)(1-\alpha)}{\varepsilon_p-\lambda(\varepsilon_p-1)(1-\alpha)}\frac{(1-\alpha)}{1-\alpha+\alpha\varepsilon_w} \quad (3.57)$$

Notice that as $x > 0$, and all the remaining terms are positive, the slope of the IS increases with the price rigidity in absolute value. This is, output through the IS equation is more volatile conditional on monetary policy shocks.

APPENDIX 3.F COMPUTING THE THRESHOLD $\bar{\phi}_w$

$$S = -\frac{(1-\alpha)(1+\varsigma\kappa_\pi)}{\sigma(1-\alpha)(1+\varsigma\kappa_\pi) + (\alpha + \kappa_\omega x)\tilde{\Omega} + \phi_\omega(\kappa_\omega x - \varsigma\kappa_\pi\alpha)} \quad (3.58)$$

with $\tilde{\Omega} = \kappa_\pi(\phi_\omega + \phi_\pi) - \sigma\Gamma\Psi$ and $\Gamma\Psi = \frac{\lambda(\varepsilon_p-1)(1-\alpha)}{\varepsilon_p-\lambda(\varepsilon_p-1)(1-\alpha)}$

$$\frac{\partial S}{\partial \theta_\omega} = \{\kappa_\pi(\alpha + \kappa_\omega x) + (1 + \varsigma\kappa_\pi)x\} (\tilde{\Omega} + \phi_\omega) \frac{1-\alpha}{D^2} \frac{\partial \kappa_\omega}{\partial \theta_\omega} \quad (3.59)$$

Notice that $\varsigma = 1 - \kappa_\omega$:

$$\frac{\partial S}{\partial \theta_\omega} = \{\kappa_\pi(\alpha + x) + x\} (\tilde{\Omega} + \phi_\omega) \frac{1-\alpha}{D^2} \frac{\partial \kappa_\omega}{\partial \theta_\omega} \quad (3.60)$$

All the elements of this derivative are positive for any κ_π , except for the element $(\tilde{\Omega} + \phi_\omega)$, which sign depends on the degree of price stickiness through κ_π . Recall that $\Gamma\Psi = \frac{\lambda(\varepsilon_p-1)(1-\alpha)}{\varepsilon_p-\lambda(\varepsilon_p-1)(1-\alpha)}$, and $\kappa_\pi = \frac{1-\theta_\pi}{\theta_\pi} \frac{1-\alpha}{1-\alpha+\alpha\varepsilon_p}$. then

$$\tilde{\Omega} + \phi_\omega = \frac{1-\theta_\pi}{\theta_\pi} \frac{1-\alpha}{1-\alpha+\alpha\varepsilon_p} (\phi_\pi + \phi_\omega) - \sigma \frac{\lambda(\varepsilon_p-1)(1-\alpha)}{\varepsilon_p-\lambda(\varepsilon_p-1)(1-\alpha)} + \phi_\omega \quad (3.61)$$

Hence, the threshold is given by the following expression:

$$\frac{1-\theta_\pi}{\theta_\pi} \frac{1-\alpha}{1-\alpha+\alpha\varepsilon_p} \phi_\pi + \frac{1}{\theta_\pi} \frac{1-\alpha+\theta_\pi\alpha\varepsilon_p}{1-\alpha+\alpha\varepsilon_p} \phi_\omega = \sigma \frac{\lambda(\varepsilon_p-1)(1-\alpha)}{\varepsilon_p-\lambda(\varepsilon_p-1)(1-\alpha)} \quad (3.62)$$

$$\bar{\phi}_\omega = \frac{\sigma\lambda(\varepsilon_p-1)(1-\alpha)(1-\alpha+\alpha\varepsilon_p)\theta_\pi}{(\varepsilon_p-\lambda(\varepsilon_p-1)(1-\alpha))(1-\alpha+\theta_\pi\alpha\varepsilon_p)} - \frac{(1-\theta_\pi)(1-\alpha)}{1-\alpha+\theta_\pi\alpha\varepsilon_p} \phi_\pi \quad (3.63)$$

Now we can obtain for $\bar{\phi}_\pi$ which is given by:

$$\bar{\phi}_\pi = \frac{\sigma\lambda(\varepsilon_p-1)(1-\alpha+\alpha\varepsilon_p)}{\varepsilon_p-\lambda(\varepsilon_p-1)(1-\alpha)} \frac{\theta_\pi}{(1-\theta_\pi)} - \frac{1-\alpha+\alpha\varepsilon_p\theta_\pi}{(1-\theta_\pi)(1-\alpha)} \phi_\omega \quad (3.64)$$

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