

## **Anexo 3.2**

### **Comparación numérica de los resultados obtenidos al estimar la fuerza que ha de hacer el actuador, aplicando Lagrange y potencias virtuales.**

Debido a las peculiares características del programa empleado, que no permite impresiones en formato .pdf, la única solución encontrada para integrar este anexo en este documento ha sido exportarlo como fichero .rtf e imprimirlo con un programa diferente.

## PARÁMETROS GEOMÉTRICOS

$$o4 := 1 \cdot \text{m}$$

$$o2 := .25 \cdot \text{m}$$

$$b := .25 \cdot \text{m}$$

$$g2 := .3 \cdot \text{m}$$

$$g3 := .2 \cdot \text{m}$$

$$g4 := .6 \cdot \text{m}$$

$$S := 3.1416 \cdot (.016^2 - .008^2) \cdot \text{m}^2$$

$$l := .4 \cdot \text{m}$$

$$S = 6.032 \cdot 10^{-4} \cdot \text{m}^2$$

$$li := .4 \cdot \text{m}$$

$$ls := .80 \cdot \text{m}$$

$$e := .010 \cdot \text{m}$$

$$m2 := 5.0 \cdot \text{kg}$$

$$m3 := 2.0 \cdot \text{kg}$$

$$m4 := 1.0 \cdot \text{kg}$$

$$I_{g2} := \frac{1}{3} \cdot g2^2 \cdot m2$$

$$I_{g3} := \frac{1}{12} \cdot l^2 \cdot m3$$

$$I_{g4} := \frac{1}{3} \cdot g4^2 \cdot m2$$

$$t := 0, 0.01.. 1.00$$

$$a(t) := (.6 + .02 \cdot \sin(t \cdot 2 \cdot \pi)) \cdot \text{m}$$

$$vcil(t) := .04 \cdot \pi \cdot \cos(t \cdot 2 \cdot \pi) \cdot \frac{\text{m}}{\text{sec}}$$

$$acil(t) := -.08 \cdot \pi^2 \cdot \sin(t \cdot 2 \cdot \pi) \cdot \frac{\text{m}}{\text{sec}^2}$$

$$hs := ls - \frac{e}{2} - a(0)$$

$$hi := a(0) - \frac{e}{2} - li$$

## GEOMETRIA

$$d := \left| \sqrt{o4^2 + o2^2} \right|$$

$$Io2 := I_{g2} + g2^2 \cdot m2$$

$$\tan(\alpha) = \frac{o2}{o4}$$

$$Io2 = 0.6 \cdot \text{kg} \cdot \text{m}^2$$

$$d = 1.031 \cdot \text{m}$$

$$I_{g3} = 0.027 \cdot \text{kg} \cdot \text{m}^2$$

$$\alpha := \text{atan}\left(\frac{o2}{o4}\right)$$

$$\beta := \text{atan}\left(\frac{o4}{o2}\right)$$

$$I_{o4} := I_{g4} + g_4^2 \cdot m_4$$

$$I_{o4} = 0.96 \text{ kg} \cdot \text{m}^2$$

$$\phi(t) := \text{acos} \left[ \frac{d^2 + b^2 - (a(t) + l)^2}{2 \cdot d \cdot b} \right]$$

$$v_{cil}(0) = 0.126 \text{ m} \cdot \text{sec}^{-1}$$

$$\theta_4(t) := \phi(t) - \beta$$

$$h_s = 0.195 \text{ m}$$

$$h_i = 0.1'$$

$$\psi(t) := \text{acos} \left[ \frac{d^2 + (a(t) + l)^2 - b^2}{2 \cdot d \cdot (a(t) + l)} \right]$$

$$\theta_2(t) := \pi - \beta - \psi(t)$$

### ANÁLISIS CINEMÁTICO

#### VELOCIDADES

$$\omega_4(t) := \frac{1}{b} \cdot \sin(\theta_2(t) - \theta_4(t)) \cdot v_{cil}(t)$$

$$\omega_2(t) := \frac{1}{a(t) + l} \cdot \sin(\theta_2(t) - \theta_4(t)) \cdot \cos(\theta_2(t) - \theta_4(t)) \cdot v_{cil}(t)$$

$$v_{g2}(t) := \omega_2(t) \cdot g_2$$

$$v_{g3}(t) := \omega_2(t) \cdot (a(t) + g_3)$$

$$v_{g4}(t) := \omega_4(t) \cdot g_4$$

$$v_{g2x}(t) := -\omega_2(t) \cdot g_2 \cdot \sin(\theta_2(t))$$

$$v_{g3x}(t) := -\omega_2(t) \cdot (a(t) + g_3) \cdot \sin(\theta_2(t)) + v_{cil}(t) \cdot \cos(\theta_2(t))$$

$$v_{g4x}(t) := -\omega_4(t) \cdot g_4 \cdot \sin(\theta_4(t))$$

$$v_{g2y}(t) := \omega_2(t) \cdot g_2 \cdot \cos(\theta_2(t))$$

$$v_{g3y}(t) := \omega_2(t) \cdot (a(t) + g_3) \cdot \cos(\theta_2(t)) + v_{cil}(t) \cdot \sin(\theta_2(t))$$

$$v_{g4y}(t) := \omega_4(t) \cdot g_4 \cdot \cos(\theta_4(t))$$

#### ACELERACIONES

$$\varepsilon_4(t) := \frac{1}{b} \cdot \sin(\theta_2(t) - \theta_4(t)) \cdot a_{cil}(t) + \frac{1}{b} \cdot (\omega_2(t) - \omega_4(t)) \cdot \cos(\theta_2(t) - \theta_4(t)) \cdot v_{cil}(t)$$

$$\varepsilon_2(t) := \frac{-v_{cil}(t)^2}{(a(t) + l)^2} \cdot \frac{1}{2} \cdot \sin(2 \cdot (\theta_2(t) - \theta_4(t))) + \frac{v_{cil}(t)}{a(t) + l} \cdot (\omega_2(t) - \omega_4(t)) \cdot \cos(\theta_2(t) - \theta_4(t)) \dots$$

$$+ \frac{a_{cil}(t)}{a(t) + l} \cdot \frac{1}{2} \cdot \sin(2 \cdot (\theta_2(t) - \theta_4(t)))$$

$$a_{g2x}(t) := -\omega_2(t)^2 \cdot g_2 \cdot \cos(\theta_2(t)) - \varepsilon_2(t) \cdot g_2 \cdot \sin(\theta_2(t))$$

$$a_{g2y}(t) := -\omega_2(t)^2 \cdot g_2 \cdot \sin(\theta_2(t)) + \varepsilon_2(t) \cdot g_2 \cdot \cos(\theta_2(t))$$

$$a_{g3x}(t) := -\omega_2(t)^2 \cdot (a(t) + g_3) \cdot \cos(\theta_2(t)) - \varepsilon_2(t) \cdot (a(t) + g_3) \cdot \sin(\theta_2(t)) \dots$$

$$+ a_{cil}(t) \cdot \cos(\theta_2(t)) - 2 \cdot \omega_2(t) \cdot v_{cil}(t) \cdot \sin(\theta_2(t))$$

$$a_{g3y}(t) := -\omega_2(t)^2 \cdot (a(t) + g_3) \cdot \sin(\theta_2(t)) + \varepsilon_2(t) \cdot (a(t) + g_3) \cdot \cos(\theta_2(t)) \dots$$

$$+ a_{cil}(t) \cdot \sin(\theta_2(t)) + 2 \cdot \omega_2(t) \cdot v_{cil}(t) \cdot \cos(\theta_2(t))$$

$$a_{g4x}(t) := -\omega_4(t)^2 \cdot g_4 \cdot \cos(\theta_4(t)) - \varepsilon_4(t) \cdot g_4 \cdot \sin(\theta_4(t))$$

$$a_{g4y}(t) := -\omega_4(t)^2 \cdot g_4 \cdot \sin(\theta_4(t)) + \varepsilon_4(t) \cdot g_4 \cdot \cos(\theta_4(t))$$

$$\theta_2(0) = 1.571$$

$$\theta_4(0) = 0$$

$$\omega_2(0) = 0 \cdot \text{sec}^{-1}$$

$$\omega_4(0) = 0.503 \cdot \text{sec}^{-1}$$

### DINAMICA INVERSA

$$Fg_{2x}(t) := -m_2 \cdot ag_{2x}(t)$$

$$Fg_{2y}(t) := -m_2 \cdot ag_{2y}(t)$$

$$Mg_2(t) := -I_2 \cdot \varepsilon_2(t)$$

$$Fg_{3x}(t) := -m_3 \cdot ag_{3x}(t)$$

$$Fg_{3y}(t) := -m_3 \cdot ag_{3y}(t)$$

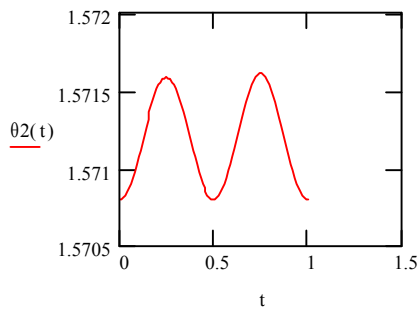
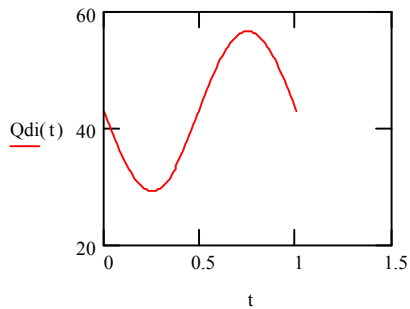
$$Mg_3(t) := -I_3 \cdot \varepsilon_3(t)$$

$$Fg_{4x}(t) := -m_4 \cdot ag_{4x}(t)$$

$$Fg_{4y}(t) := -m_4 \cdot ag_{4y}(t)$$

$$Mg_4(t) := -I_4 \cdot \varepsilon_4(t)$$

$$Q_{di}(t) := \frac{-1}{v_{cil}(t)} \cdot \left[ \begin{array}{l} Fg_{2x}(t) \cdot v_{g2x}(t) + (Fg_{2y}(t) - m_2 \cdot g) \cdot v_{g2y}(t) + Mg_2(t) \cdot \omega_2(t) \dots \\ + Fg_{3x}(t) \cdot v_{g3x}(t) + (Fg_{3y}(t) - m_3 \cdot g) \cdot v_{g3y}(t) + Mg_3(t) \cdot \omega_3(t) \dots \\ + Fg_{4x}(t) \cdot v_{g4x}(t) + (Fg_{4y}(t) - m_4 \cdot g) \cdot v_{g4y}(t) + Mg_4(t) \cdot \omega_4(t) \end{array} \right]$$



### DINAMICA DIRETTA

$$\begin{aligned}
Q_{dd}(t) := & \left( m_2 \cdot g^2 + I g_2 \right) \cdot \left[ \frac{1}{a(t)+1} \cdot \frac{1}{2} \cdot \sin(2 \cdot (\theta_2(t) - \theta_4(t))) \right]^2 \cdot acil(t) - 2 \cdot \frac{m_2 \cdot g^2 + I g_2}{a(t)+1} \cdot \omega_2(t)^2 \dots \\
& + \left( m_2 \cdot g^2 + I g_2 \right) \cdot \frac{\omega_2(t) - \omega_4(t)}{(a(t)+1)} \cdot (\cos(2 \cdot (\theta_2(t) - \theta_4(t)))) \cdot 2 \cdot \omega_2(t) \dots \\
& + \left( \frac{m_2 \cdot g^2 + I g_2}{a(t)+1} \right) \cdot \omega_2(t) \cdot (\omega_2(t) - (\omega_2(t) - \omega_4(t)) \cdot \cos(2 \cdot (\theta_2(t) - \theta_4(t)))) \dots \\
& + m_2 \cdot g \cdot g_2 \cdot \cos(\theta_2(t)) \cdot \left[ \frac{1}{a(t)+1} \cdot \frac{1}{2} \cdot \sin(2 \cdot (\theta_2(t) - \theta_4(t))) \right] \dots \\
& + \left[ m_3 \cdot (a(t) + g_3)^2 + I g_3 \right] \cdot \left[ \frac{1}{a(t)+1} \cdot \frac{1}{2} \cdot \sin(2 \cdot (\theta_2(t) - \theta_4(t))) \right]^2 \cdot acil(t) + m_3 \cdot acil(t) \dots \\
& + \left[ m_3 \cdot (a(t) + g_3)^2 + I g_3 \right] \cdot \frac{\omega_2(t) - \omega_4(t)}{(a(t)+1)} \cdot \left[ - \left( \sin(\theta_2(t) - \theta_4(t))^2 + \cos(\theta_2(t) - \theta_4(t))^2 \right) \right] \cdot 2 \cdot \omega_2(t) \dots \\
& + (2 \cdot m_3 \cdot (a(t) + g_3)) \cdot \left[ \frac{1}{a(t)+1} \cdot \frac{1}{2} \cdot \sin(2 \cdot (\theta_2(t) - \theta_4(t))) \right]^2 \cdot vcil(t)^2 + m_3 \cdot g \cdot \sin(\theta_2(t)) \dots \\
& + m_3 \cdot g \cdot (a(t) + g_3) \cdot \cos(\theta_2(t)) \cdot \frac{1}{a(t)+1} \cdot \left[ - \left( \sin(\theta_2(t) - \theta_4(t))^2 + \cos(\theta_2(t) - \theta_4(t))^2 \right) \right] \dots \\
& + - m_3 \cdot \omega_2(t)^2 \cdot (a(t) + g_3) - 2 \cdot \frac{m_3 \cdot (a(t) + g_3)^2 + I g_3}{a(t)+1} \cdot \omega_2(t)^2 \dots \\
& + \left[ \frac{m_3 \cdot (a(t) + g_3)^2 + I g_3}{a(t)+1} \right] \cdot \omega_2(t) \cdot (\omega_2(t) - (\omega_2(t) - \omega_4(t)) \cdot \cos(2 \cdot (\theta_2(t) - \theta_4(t)))) \dots \\
& + (m_4 \cdot g^4 + I g_4) \cdot \left( \frac{1}{b} \cdot \sin(\theta_2(t) - \theta_4(t)) \right) \cdot vcil(t) \cdot \frac{(\omega_2(t) - \omega_4(t)) \cdot \cos(\theta_2(t) - \theta_4(t))}{b} \dots \\
& + (m_4 \cdot g^4 + I g_4) \cdot \left( \frac{1}{b} \cdot \sin(\theta_2(t) - \theta_4(t)) \right)^2 \cdot acil(t) + m_4 \cdot g \cdot g_4 \cdot \cos(\theta_4(t)) \cdot \frac{1}{b} \cdot \sin(\theta_2(t) - \theta_4(t))
\end{aligned}$$

$$Q_{dd}(0.0) = 43.14926 \text{ kg} \cdot \text{m} \cdot \text{sec}^{-2}$$

$$Q_{dd}(0.25) = 29.371299 \text{ kg} \cdot \text{m} \cdot \text{sec}^{-2}$$

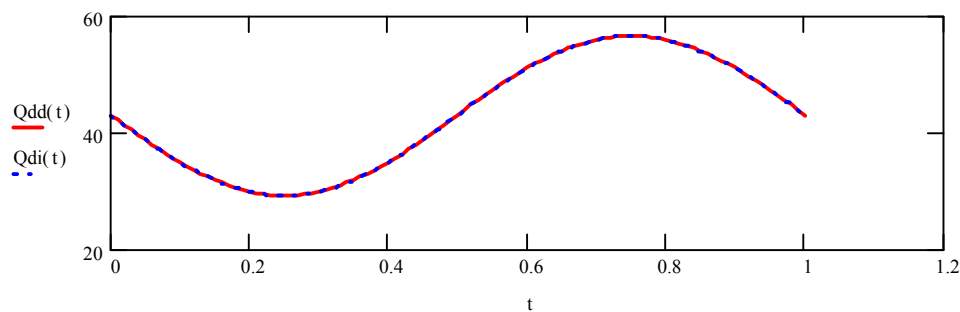
$$Q_{di}(0) = 43.14926 \text{ kg} \cdot \text{m} \cdot \text{sec}^{-2}$$

$$Q_{di}(0.25) = 29.35809 \text{ kg} \cdot \text{m} \cdot \text{sec}^{-2}$$

$$Q_{dd}(0.01) = 42.284 \text{ kg} \cdot \text{m} \cdot \text{sec}^{-2}$$

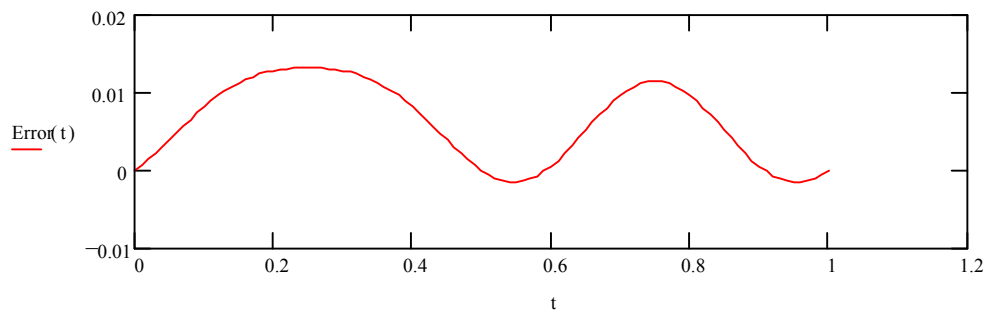
$$Q_{dd}(.75) = 56.648 \text{ kg} \cdot \text{m} \cdot \text{sec}^{-2}$$

#### COMPARACIÓN DE LOS RESULTADOS



Las 2 gráficas se superponen.

$$\text{Error}(t) := Q_{dd}(t) - Q_{di}(t)$$



El error máximo es 0.0132088 N y  $Q_{di}(t)$  siempre supera los 29 N.

$$\text{error\_relativo} := \frac{.0132088}{29}$$

$$\text{error\_relativo} = 4.555 \cdot 10^{-4}$$