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Essays on Two-Sided Markets

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This thesis is dedicated to my parents.

For their endless love, support, and encouragement.

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Introduction

Over the past 20 years, companies of a new type have emerged at the top. Tech giants like FANG (Facebook, Amazon, Netflix, and Google), which are some of the world's most valuable companies as of this writing, have profoundly transformed the global economy and are a prime testimony to how this type of companies have changed many aspects of our lives. Despite these firms' very different activities, they fall into the same general category, known as two-sided market platforms, and share many characteristics. A key characteristic is the presence of indirect network externalities: an agent from one group, by subscribing to a platform, generates value for some other agents from another group on the platform. For example, increased participation by sellers on an e-commerce platform increases value on the buyer side and vice versa. By acting as an intermediary to connect two or more distinct groups of economic agents, any successful platform incorporates these indirect network externalities in its business model. In a sense, none of the mentioned business models are really new. Newspapers have been operating for hundreds of years and Amazon is the modern equivalent of an ancient fair. However, the development of communication technologies such as the internet has enabled a huge number of potential users to connect with one another and have thus more readily facilitated the matching process between distinct sides compared to more traditional platforms.

The presence of indirect network externalities has complex implications for pricing and other business strategies by platforms. Furthermore, they are often not only attributed to being able to explain why the market tends to favor large platforms but also why rules that have been designed for traditional markets by the industrial organization literature are often not adequate to assess two-sided market outcomes. At the same time and due to their ever-larger omnipresence, platforms' conduct is under increased scrutiny by competition authorities and at the center of public debates, questioning whether observed outcomes in two-sided markets are efficient. This dissertation contains three essays that contribute to the understanding of two-sided market outcomes by studying various aspects related to how platforms operate and compete. To that extent, I focus, mostly from a theoretical perspective, on (*i*) how a change in taxes affect the demand and supply in media markets and digital newspapers in particular, (*ii*) how platforms may use long-term contracts as an anti-competitive instrument, and (*iii*) how platforms' decision to invest in their platform quality is driven by the organization mode.

In Chapter 1, titled *Promoting Pluralism and Readership in Digital News: New Lessons for Tax Policy*, I study competitive forces that determine the impact of a value-added tax (VAT) reduction for digital newspapers on readership and supply of digital news. By endogenizing entry in a model of newspaper demand in which newspapers compete for readers and advertisers, the chapter offers new insights for the debate over a tax incidence in two-sided markets and the impact of other policies in media markets. I find that a VAT reduction raises readership if the price competition, market expansion, and matching effects from additional entry outweigh the negative direct effect from a subscription price increase. I then calibrate the model parameters using Norwegian data on digital newspaper readership, prices, and revenues. The calibrated model implies that the VAT rate reduction by Norway in 2016 has increased consumer surplus and has reduced the news under-supply in the market. Finally, I explore alternative policies. By accounting for the two-sidedness of digital news, a value-added advertising subsidy is more effective than a VAT reduction in increasing readership since it leads to a direct subscription decrease as well as to additional entry.

In Chapter 2, titled *Long-Term Contracts and Entry Deterrence in Two-Sided Markets*, I study the incentives of an incumbent platform to use a long-term contract for sellers to deter entry by a more efficient platform. At the outset two-sided, market platforms appear especially susceptible to the strategic use of long-term contracts given the intense competition to get both sides on board. In the set-up, the incumbent operates for a period before the arrival of the entrant and can credibly commit to refuse to let sellers visit its platform without a contract in the first period. Whether a long-term contract is an anti-competitive instrument hinges on the homing assumptions of the two sides, that is, if a side can engage with the other side on only one or both platforms, and the size of cross-group externalities. If sellers single-home and consumers can multi-home, I formally show that the Chicago critique holds due to intense competition for sellers: sellers do not voluntarily sign a long-term contract that reduces competition unless they are fully compensated for doing so, which the incumbent cannot afford. If the homing assumptions are reversed and cross-group externalities are sufficiently large, the incumbent is strictly better off using an exclusive long-term contract. Since the platforms compete for the single-homing consumers, the sellers' surplus is extracted even in the absence of a contract and the incumbent can readily incentivize the sellers to sign a contract. The foreclosure of the more efficient platform always decreases consumer surplus and is never welfare-improving.

In Chapter 3, titled *Quality Investment by Platforms*, I study incentives to invest in platform quality by organizations that operate either in a one- or two-sided mode. Other than setting the optimal fees to the two sides, the decision to innovate is one indispensable dimension in the platform's strategic focus. In the two-sided mode, a platform acts as an intermediary between sellers and consumers. In the one-sided mode, a multi-product firm (MPF) produces and sells varieties to consumers itself. In each respective model, a platform first decides how much to invest in the quality of its platform before setting access fees for the sides. Investment in the platform's quality directly affects consumers, but not sellers, through enhancing the value of interaction with product varieties. I find that platforms in either mode underinvest relative to the social planner's outcome. Crucially, the platforms do not fully appropriate the surplus that is generated between the two sides' interdependent values by solely focusing on the marginal instead of the average user on at least one side. However, an MPF, by producing the products itself, better accounts for the cross-group benefits of products for consumers than the two-sided platform. This given the MPF an additional incentive to invest in quality to increase the customer base. Furthermore, I find that the introduction of competition leads to higher investment levels than under monopoly. Due to the business-stealing effect, competing platforms use investment as an additional instrument to attract consumers.

Chapter 1

Promoting Pluralism and Readership in Digital News: New Lessons for Tax Policy

1.1 Introduction

Newspapers have been operating in a two-sided market for hundreds of years selling information of current events to readers and eyeballs to advertisers (Foros et al., 2019). Since their beginning, newspapers have been considered to be an essential source of information for current events and important for the health of democracy.[1]

Yet markets that are decentralized may not provide the socially optimal product variety (Spence, 1976; Dixit and Stiglitz, 1977; Mankiw and Whinston, 1986; Chen and Riordan, 2007). This may be especially true in newspaper markets (Steiner, 1952; Beebe, 1977; Anderson and Coate, 2005), because the positive externality from having informed citizens through a wide offering and consumption of newspapers is not internalized by market participants. Policies to subsidize media, promote competition, and raise ideological diversity have historically been shaped by such considerations (Gentzkow et al., 2011). For instance, a preferential value-added tax (VAT) treatment where printed newspapers pay a zero or close to zero tax on sales

[1]Succinctly summarized by Gentzkow et al. (2011), theory suggests that political outcomes may be affected by news through three channels. First, media disseminates information about politics, which may increase political participation (Feddersen, 2004). Second, media may persuade voters to support particular parties or polities, or indirectly shift party support by affecting which issues are salient (Graber, 2000). Lastly, media may affect the power of incumbents, for example by informing voters about corruption and giving a platform to challengers who might be shut out of alternative communication channels (Besley and Prat, 2006).

to readers is widespread in European countries.[2,3]

Furthermore, newspapers are facing a structural consumption change: the demand for printed newspapers is shrinking rapidly, which has (mostly) been replaced by demand for digital newspapers. The case of Norway is illustrated in Figure 1.1. The young have been at the forefront of this change to the digital news medium (see Figure A.1.1 in Appendix A.1). There is little doubt that the trend away from print will continue and that digital media will play an even more presiding role in the future consumption of news.

Given the preferential VAT treatment for printed newspapers and the continued increase of digital news consumption, policy makers naturally asked whether the VAT exemption ought to be extended to digital newspapers (Foros et al., 2019). Since March 1, 2016 digital newspapers in Norway are exempt from paying any VAT.[4] On October 2, 2018 the European Commission passed legislation allowing Member States to align their VAT rates for digital and print newspapers.[5]

In the present chapter, I study economic forces that determine the impact of a VAT reduction for digital newspapers on readership and supply of digital news, and alternative policies designed to increase those. I focus on the case of Norway for two reasons. First, it was the first among the European Economic Area members to extend the VAT exemption and several years have since passed to observe the exemption's impact. Second, digital newspaper demand has reached a relatively stable level well before the policy implementation, and hence it is unlikely that interaction effects between the VAT reduction and demand transition from print to digital exist.

The Norwegian authority argues that the *direct* effect of the VAT rate reduction

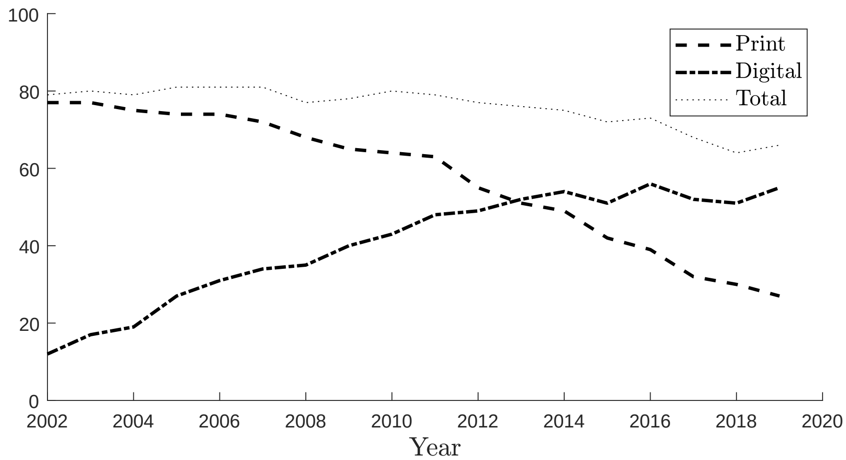
[2] Table A.1.1 in Appendix A.1 lists the standard VAT rate and the rate for printed newspapers in the ten most populous European Union countries, Norway, and the United Kingdom.

[3] Other widely applied media policies are subsidies such as production grants or distribution subsidies (for instance per the Postal Act of 1792 in the U.S.), or authorizing the formation of joint operating agreements under which competing newspapers can collude on subscription and advertisement prices while remaining separate entities (per the Newspaper Preservation Act of 1970 in the U.S., which exempts newspapers from the usual provisions of antitrust laws such as the Sherman Antitrust Act of 1890).

[4] The VAT Directive prohibited European Union member states from imposing a lower tax rate on digital newspapers at the time (European Commission, 2016). Norway was not constrained by this VAT Directive because, although a member of the European Economic Area, is not part of the European Union.

[5] The actual amendment dates by the countries listed in Table A.1.1 to apply a reduced rate to digital newspapers to their VAT systems are the following: BE on April 1, 2019; PL on November 1, 2019; DE and NL on January 1, 2020; ES on April 23, 2020; CZ and UK on May 1, 2020. EL and RO, as of July 1, 2020, have not amended their VAT systems.

Figure 1.1: Percentage of adults who read different newspaper types in Norway on an average day.



Source: Data are from Statistics Norway (2020)

for digital newspapers is that it "will lead to a reduction in the price of the good" (Kontor, 2015, p. 23), since the reduced cost of the VAT rate for newspapers is (partially) passed on to consumers. The authority furthermore argues that the VAT rate reduction has the *indirect* effect of "improving the economy of media enterprises, and thereby stimulate media pluralism and diversity, and production of news" (Kontor, 2015, p. 28). The Norwegian government concludes that said policy aids its media objective to "promote good news production and a broad and enlightened public discourse in the digital media society of the future" (Kontor, 2015, p. 4).

I model digital newspaper competition in a framework that endogenizes decisions over entry, subscription prices, and advertising prices. The model builds on the oligopoly framework by Chen and Riordan (2007), a spatial model in which differentiated firms compete in a nonlocalized fashion by first deciding whether to enter the market before competing in prices. Consumers exhibit a preference for newspapers that matches their tastes. As a first extension to Chen and Riordan (2007), I endogenize whether consumers buy, if available, only one of their two most preferred varieties (single-home) or both (multi-home), in addition to the option of buying none (Anderson et al., 2017). Due to the two-sided nature of the newspaper market, the second extension is the modeling of the advertisement side (Anderson et al., 2018). That is, after entering digital newspapers compete both in subscription and advertisement prices. The advertising model is simplified but it captures a key prediction from the two-sided market literature: platform ad price competition is influenced by the extent of newspaper readership overlap (Ambrus et al., 2016; Anderson et al., 2018, 2019; Athey et al., 2018; Anderson and Peitz, 2020).

Contrary to the Norwegian authority's intentional *direct* effect, the model predicts that the VAT reduction leads to an increase of the gross subscription price due to the two-sidedness of the digital newspaper market: while not directly affecting the advertising side of the market, a lower tax rate on the subscription revenue increases the profitability of the reader market for digital newspapers. Consequently, newspapers will focus relatively more on making profits from the reader compared to the advertising side, with the result that digital newspapers will set higher subscription prices to increase the per reader profit. The same qualitative result has been found in alternative settings from the one here (Belleflamme and Toulemonde, 2018; Kind and Koethenbuerger, 2018; Foros et al., 2019).

At the same time and pertaining to the intentional *indirect* effect, a VAT reduction increases newspapers' profits that leads to additional entry. The number of Norwegian digital newspapers has indeed increased by 16% between 2017 and 2018 (from 135 to 157).[6] It stands to reason that this increase, albeit with a one-year lag, is a consequence of the VAT policy change. Additional entry is beneficial for consumers for three reasons in the model. First, the increase in the number of newspapers causes the newspaper market to be more competitive, resulting in a reduction in the subscription price. That is, the increase in supply has the opposing effect to the initial *direct* effect on the subscription price due to the VAT reduction. This *price competition effect* sheds light on why the subscription prices, contrary to theoretical predictions (Kind and Koethenbuerger, 2018; Foros et al., 2019), have remained relatively constant in Norway.[7] Second, the increase in pluralism, i.e., the production of a diverse offering of news, enhances the likelihood that a potential reader finds a newspaper outlet she is interested in, conditional on previously not purchasing a newspaper. This is the *market expansion effect*. Lastly, some readers might switch to a newspaper they find more desirable to read according to their

[6]See Figure A.1.2 in the Appendix for the yearly number of Norwegian digital newspapers between 2010 and 2019. Furthermore, the impact of these new entrants on digital news readership has been non-negligible: while the Herfindahl index was relatively constant in a typical Norwegian region between 2012 and 2015 (with a mean of 0.20 and a standard deviation of 0.0061 across those years), the index has decreased 5% by 2019.

[7]In Appendix A.2 I report that the change in the price time trend is marginally negative but not statistically significant after the VAT exemption has been extended to digital newspapers.

tastes, i.e., the *matching effect*.

The theory makes predictions that are consistent with the pricing and entry behavior before and after the VAT exemption in Norway. In general, the policy goal to promote news consumption by reducing the VAT rate is met if the positive effects associated with additional entry outweigh the negative direct effect from the subscription price increase. This key result is governed by two parameters that are integral to the newspaper market. First, it depends on the extent newspapers derive their revenue from the advertising side (including diminishing returns to impressions, which determine the advertising competition intensity). The more important are advertisers to digital newspapers, the larger is the subscription price increase on the one hand but also the larger is additional entry on the other hand following the VAT reduction. The former dominates the latter above some threshold. Second, it depends on the value consumers assign to reading a second newspaper. While the VAT reduction does not directly interact with this value regarding the subscription price, it positively affects additional entry.

Due to the ambiguity of the theoretical result, I next calibrate the model parameters for each of the 19 Norwegian regions based on data shortly before Norway extended its VAT exemption. The calibrated model is used to simulate how the VAT reduction changes variables of interest in an average Norwegian region. In particular, the simulation results show that the number of newspapers increases (+5.6%) and the subscription price decreases (−2.7%), leading to an increase in total readership (+8%), multi-purchasing (+23.1%), and consumer surplus (+4.1%). Furthermore, the consumer surplus constitutes most of the welfare value in the calibrated model and welfare increases as a result of the VAT reduction (+3.2%).

I then compare the market outcomes under the VAT reduction to those that a social planner would choose who maximizes economic welfare while ignoring the externalities that come with having informed citizens. Relative to when the planner only chooses the optimal VAT rate or chooses both the optimal VAT rate and the number of newspapers (second best), market entry is inefficiently low and subscription prices are inefficiently high even after the VAT rate reduction. This suggests that based on the calibrated parameters there is no conflict between maximizing economic welfare and the policy goal of increasing readership and supply of digital newspapers.

Finally, I consider a range of policy experiments in comparison to the subscription VAT reduction in terms of effectiveness on welfare.[8] Allowing newspapers to collude

[8] I focus on a planner who maximizes welfare in these policy experiments since a planner who

on subscription prices reduces both welfare and consumer surplus (see footnote 3 in this chapter on collusion allowance in the U.S.). If instead newspapers are allowed to collude on advertising prices, welfare and consumer surplus increase (albeit by less than through the VAT reduction). By increasing their ad prices (by being able to extract the advertiser surplus), newspapers reduce the subscription price due to the "seesaw principle" (Rochet and Tirole, 2006). The additional entry further reduces the subscription price. This highlights the importance of taking into account the balancing of two-sided pricing by newspapers when evaluating policies.

In a similar spirit of strengthening newspaper revenue from advertising (which in turn decreases the dependence from revenues on the reader side), I find that a value-added advertisement subsidy is more effective than the VAT reduction on the reader side to increase economic welfare and consumer surplus. The reason is that a valued-added ad subsidy can replicate the number of additional entries with its associated positive effects for consumers as under a VAT reduction, but an ad subsidy also leads in contrast to a VAT reduction to a direct reduction of the subscription price. The optimal ad subsidy chosen by the social planner is such that the direct subscription price decrease and additional entry are larger than what occurs under ad price collusion. A production subsidy increases welfare through an additional supply of digital newspapers.

Related literature

My work builds on the theoretical literature on two-sided markets and relates in particular to competition and entry in advertising-financed media markets (Steiner, 1952; Beebe, 1977; Anderson and Coate, 2005. Anderson et al., 2016 provide a comprehensive overview).[9] The model extends the spokes model (Chen and Riordan, 2007) by allowing consumers to choose up to two newspapers (in the spirit of Anderson et al., 2017).[10] This recognizes the prevalence of "multi-homing" consumers

solely maximizes the consumers' well-being is the analytically uninteresting case. For instance, using a production subsidy, the planner would simply subsidize newspapers by the minimum amount that ensures that all possible varieties, as presented in the model, are supplied to maximize the consumer surplus.

[9] For a two-sided market in general, see Zhu et al. (2018) for factors that have been identified that determine entry success (or failure) of platforms.

[10] Germano (2008) and Germano and Meier (2013) model a media market also using the spokes model; however, their focus is, respectively, on the effect of media ownership on media quality and bias, and media content provided under various ownership structures.

in digital news[11] and their impact on newspaper competition for advertisers (Ambrus et al., 2016; Anderson et al., 2018; Athey et al., 2018; Anderson et al., 2019; Foros et al., 2019; Anderson and Peitz, 2020). I follow Anderson et al. (2018) to include competition for advertisers to account for the two-sidedness of the newspaper market.

Finally, I introduce a value-added tax on the reader side. The papers that examine a tax incidence in two-sided markets assume that the number of platforms is fixed and therefore cannot address the *indirect* policy goal of the VAT reduction by the Norwegian authority and the European Union (Kind et al., 2008, 2009, 2010, 2013; Belleflamme and Toulemonde, 2018; Kind and Koethenbuerger, 2018; Foros et al., 2019). To the best of my knowledge, the present paper is the first to analyze a tax incidence in a two-sided market where entry by platforms is endogeneous and consumers can choose to multi-home.

The present modeling approach follows the Norwegian authority's interpretation of *pluralism* and *diversity* as the "production of a diverse offering of [...] news and current affairs journalism" (Kontor, 2015, p. 29) by assuming that each newspaper is a variety providing a different news view (of a topic) catering to the taste of consumers. A parallel strand in the media market literature interprets *diversity* as the variety of views provided by newspapers by examining the choice of political orientation (Gabszewicz et al., 2001, 2002; Anderson and Gabszewicz, 2006; Kind et al., 2013) or content variety (Anderson et al., 2018) as a choice of spatial location. It finds that advertising financing *may* lead to minimal differentiation. This strand of literature (including the present paper) builds on the approach that media platforms provide different opinions of the same story.[12]

The rest of the chapter is organized as follows. I set up the model in Section 1.2. In Section 1.3 I analyze in a first step the *direct* policy impact of the VAT reduction by leaving the number of newspapers fixed. In a second step I allow for

[11] See Peitz and Reisinger (2015) and Athey et al. (2018). In Norway in 2019, 35% of adults read articles on at least two online newspaper websites on average per day (Statistics Norway, 2020).

[12] Another strand of related literature looks at *media bias* rather than at pluralism and diversity. See Gentzkow and Shapiro (2008) for examples of media bias. The media bias approach assumes that an objective state of the world exists and then explores why this objective state may not be reported truthfully by media platforms. Explanations offered in the literature for the existence of media bias range from demand side factors by readers (Mullainathan and Shleifer, 2005; Gentzkow and Shapiro, 2006) to supply side factors (Baron, 2006) and preferences of the advertising side (Ellman and Germano, 2009). The relation of the present paper to this literature is indirect: Gentzkow and Shapiro (2008) argue that an increase in competition, that is, pluralism and diversity, tends to reduce media bias because different views and information pieces appear in media platforms.

free-entry to examine both the *direct* and *indirect* effects of the policy. In Section 1.4 I calibrate the model parameters and present counterfactual simulations. Section 1.5 concludes. Appendix A complements the analysis in the main text.

1.2 Model

The goal of my model is to parsimoniously capture the effects of consumer preferences, subscription competition, advertising competition, and a value-added tax (VAT) rate on digital newspaper demand and supply. I approximate a set of economic forces that I believe are the most important while abstracting from many others. I defer justifications and a discussion of the model to Appendix A.3. In Appendix A.4 I examine the robustness of the theoretical main results.

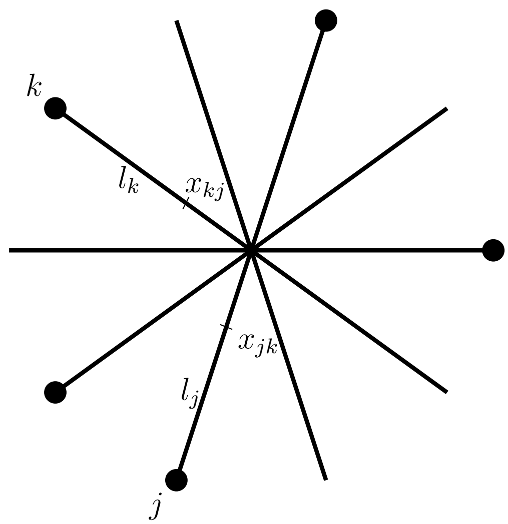
I build on the spokes model by Chen and Riordan (2007). There are $i = 1, \ldots, N$ possible differentiated newspaper varieties. Each potential variety is at the origin of a line of length $\frac{1}{2}$. Variety i's associated line is called l_i. The other ends of all the lines meet at one point, the center, forming a network that represents the preference space. A network is illustrated in Figure 1.2. There are $j = 1, \ldots, n$ platforms present in the market, with $2 \leq n \leq N$. In the analysis below I allow for free-entry and thus endogenize n. Each platform j is located at the origin of line l_j and produces a digital newspaper at marginal cost $c \geq 0$ and fixed cost $F > 0$. Newspapers sell subscriptions to consumers and eyeballs to advertisers. Denote by τ the VAT rate on consumer subscriptions for each platform.

Consumers: Consumers are uniformly distributed on the spokes network.[13] I normalize the consumers' total mass to one. A consumer derives a gross value v from reading a set of articles in a newspaper (politics, business, opinion, science, health, books & arts, food, travel, the weather, sports, etc.) A consumer's ideal point is characterized by (l_i, x_i): the consumer on l_i is at distance x_i to variety i (the origin of l_i). Variety i is (l_i, x_i)'s most preferred newspaper. Consumer (l_i, x_i) also has a second preferred variety, which is any $i' \neq i$ chosen randomly with probability $1/(N-1)$ and which she also values with v.[14]A consumer derives zero value from any variety that is not one of her two most preferred varieties. She also derives zero

[13]The qualitative results do not hinge on the uniform distribution assumption. See footnote 26.

[14]The assumption that *any* randomly chosen $i' \neq i$ is her second preferred variety implies the non-localized competition nature (Chen and Riordan, 2007).

Figure 1.2: The spokes model with $n = 5, N = 10$



value from any variety beyond her two desired brands.[15]

By symmetry of all other varieties, the distance for consumer (l_i, x_i) to any other variety $i', i' \neq i$, is $\frac{1}{2} - x_i + \frac{1}{2} = 1 - x_i$. For any consumer, let the distance be interpreted as a disalignment between the consumer's reading preferences and the newspaper she wishes to purchase. This disalignment comes at a unit cost $t > 0$.[16] I assume for simplicity that consumers are ad neutral.[17] Notice the following:

Remark. *The spokes model collapses to the Hotelling model if* $n = N = 2$.

Three consumer categories are relevant in deriving digital newspaper j's readership for any given subscription price profile $(p_1, p_2, ...p_n)$.

Category 1: consumers for whom both their preferred varieties are available, one of which is j. In particular, for any consumer on l_j or l_k, for $j, k \in \{1, \dots, n\}$,

[15]This simplification assumption is restrictive but ensures that a symmetric pure strategy equilibrium exists. See Chen and Riordan (2007, p. 903) for a justification in the original spokes model.

[16]One interpretation of the horizontal disalignment cost is to regard distance as traditional "geographical preferences", according to which each newspaper on a spoke serves a local newspaper market. . An alternative interpretation is the distance of the newspaper to the consumer's ideal political orientation, style of arguments, or types of articles. My most preferred interpretation is the former - see the "market definition" discussion in Appendix A.3.

[17]The empirical evidence for this assumption is mixed for media markets in general, but there is good support for ad-neutrality or even a positive valuation towards ads in newspaper markets (see Chapter 9 in Anderson et al., 2016 for a survey of consumers' attitude towards ads in newspapers); in contrast, consumers disvalue ads in broadcast media (TV and radio). In Appendix A.4.2 I allow for consumers' disutility from ads.

both newspapers j and k are her preferred varieties with conditional probability $1/(N-1)$. Purchasing solely newspaper j gives utility

$$u_j = \begin{cases} v - tx_j - p_j & \text{if } (l_j, x_j) \\ v - t(1 - x_k) - p_j & \text{if } (l_k, x_k), \end{cases} \tag{1.1}$$

and similar when purchasing from k. The consumer at distance x_j from j is indifferent between solely purchasing j and k if $v - tx_j - p_j = v - t(1 - x_j) - p_k$.

Since 35% of adults in Norway read articles on at least two online newspaper websites on average per day (Statistics Norway, 2020), I allow for the possibility of consumers to multi-home. Consumers endogenously choose whether to single-home (SH), i.e., purchase only from either j or k, or to multi-home (MH) on both j and k. I follow Anderson et al. (2017) and assume that the incremental value of buying newspaper k in addition to j equals $u_{jk} = u_k - d$, where $d \in (0, v)$.[18] The discount d follows an incremental value approach, supposing that a consumer only reads a fraction of articles in this second newspaper.[19]

A consumer will multi-home as long as $u_{jk} \geq 0$. Solving $u_{jk} = 0$, the marginal consumer between j and k who is indifferent between single-homing on j and multi-homing on both j and k is at distance

$$x_{jk} = 1 - \frac{v - d - p_k}{t}$$

from j. Similarly solving $u_{kj} = 0$, the marginal consumer between j and k who is indifferent between multi-homing on j and k and single-homing on k is at distance

$$x_{kj} = \frac{v - d - p_j}{t}$$

from j. These cut-offs are illustrated in Figure 1.2. For any two available newspapers j and k, the number of *category 1* consumer that multi-home is strictly positive if $x_{jk} < x_{kj}$, but zero otherwise. Thus, the number of *category 1* consumers that read

[18]Regarding the subscript terminology of u_{jk}, consumers perceive newspaper j as their primary good and newspaper k as their secondary good (Anderson et al., 2017).

[19]This might be due to time constraints or overlap of content in the articles - reading similar articles a second time is of less value to a consumer. See Appendix A.4.4 for a microfoundation of v and d and alternative multi-homing formulations. Fan (2013) and Gentzkow et al. (2014) estimate in their structural models that subscribing to a second print newspaper has a diminishing value, i.e., $d > 0$.

newspaper j is

$$\frac{2}{N}\frac{1}{N-1}\sum_{\substack{k\neq j,\\ k\in\{1,\ldots,n\}}}\left(\max\left[\min\left(x_{kj},1\right),0\right]\mathbb{1}_{x_{jk}<x_{kj}}+\max\left[\min\left(\frac{1}{2}+\frac{p_k-p_j}{2t},1\right),0\right]\mathbb{1}_{x_{jk}>x_{kj}}\right),$$

where $2/N$ is the density of consumers on l_j and l_k.

Category 2: consumers for whom variety j is their first preferred variety but their second preferred variety i is unavailable, i.e., $i \notin \{1,\ldots,n\}$. Such a consumer prefers purchasing newspaper j from not purchasing at all if $v - tx_j - p_j \geq 0$. Newspaper j's demand from this category of consumers is

$$\frac{2}{N}\frac{N-n}{N-1}\min\left[\max\left(0,\frac{v-p_j}{t}\right),\frac{1}{2}\right],$$

where $2/N$ is the density of consumers on l_j and $(N-n)/(N-1)$ is the probability that her second preferred variety is unavailable.

Category 3: consumers for whom their first preferred variety is unavailable and j is their second preferred variety. In particular, for any consumer on l_i such that $i \neq j$ and $i \notin \{1,\ldots,n\}$, variety j is her second preferred variety with probability $1/(N-1)$. She prefers purchasing j than not purchasing at all if $v-t(1-x_i)-p_j \geq 0$. Newspaper j's demand from this category of consumers is

$$\frac{2}{N}\frac{N-n}{N-1}\min\left[\max\left(0,\frac{v-p_j}{t}-\frac{1}{2}\right),\frac{1}{2}\right]$$

Newspaper j's total demand D_j is the sum of the three categories demands. *Category 4* consumers, who have neither their first nor their second preferred variety available, play no role in D_j. Note that no *category 2* or *3* consumer can multi-home because her, respectively, second and first preferred variety is not available.

Advertisers: I follow Anderson et al. (2018) to model the advertising side in the newspaper market. Platforms can costlessly place ads in their newspapers. Each platform sets a price per ad. There is a mass of homogeneous advertisers A, and each advertiser can place one ad per newspaper. The demand for ads is perfectly elastic and let $A = 1$.

Assume that each advertiser is willing to pay α per ad for a consumer that sees the ad on one and only one platform. This captures the advertiser's profit margin, or, the probability that a consumer, when seeing the ad for the first time, buys the advertised product. Second impressions are (weakly) less worth than the first impression: an advertiser's expected value of a consumer seeing its advert twice is $\alpha(1+\sigma)$, where $\sigma \in [0,1]$.[20] By the incremental pricing principle (Anderson et al., 2018), the price per ad at a given newspaper is determined by the incremental value an advertiser derives from advertising on that platform: the price per ad on newspaper j is

$$\alpha X_j^{SH} + \sigma\alpha X_j^{MH}, \tag{1.2}$$

where X_j^{SH} is the number of single-homing consumers and X_j^{MH} is the number of multi-homing consumers on newspaper j, which will be derived in Section 1.3. That is, a newspaper can extract all the advertisers' surplus from single-homing consumers but cannot charge more than $\sigma\alpha$ for consumers that it shares. Since $A = 1$, the price per ad and total ad revenue for a newspaper is an identity.

Timing: The game proceeds in four stages. First, the potential entrants choose simultaneously whether or not to enter. Second, the digital newspapers which have entered simultaneously choose their subscription prices. Third, digital newspapers simultaneously choose their advertising prices, the solution of which is given by (1.2), followed by each advertiser deciding whether or not to advertise in each newspaper. Finally, consumers make their purchasing decisions. Each newspaper's decisions are observable to all other newspapers at the end of each stage. Beginning at the end of the game and working backward, I derive the subgame perfect Nash equilibrium of the model.

1.3 Analysis

I first derive the equilibrium fees and profits for a given number of digital newspapers n and general VAT rate τ. I then investigate how the equilibrium outcomes change when the VAT rate is reduced to zero, first for a fixed n and second when allowing for free entry. Any proof not stated in the main text is relegated to Appendix A.8. I present several robustness checks in Appendix A.4. Given the rich set-up, I assume that $d > \frac{t}{2}$ to reduce the number of cases to be investigated.[21]

[20] See Anderson et al. (2018) for a formal motivation of σ. Gentzkow et al. (2014) estimate $\sigma = 0.55$ for U.S. print newspapers and Shi (2016) estimates $\sigma = 0.5$ for US magazines.

[21] That is, the multi-homing discount is sufficiently large such that the *category 1* consumer in the center derives less utility buying j as her second newspaper than a *category 3* consumer at maximum distance from j buying solely j: $v - t > v - d - t/2$.

1.3.1 Fixed number of digital newspapers

I present here only the equilibrium where each newspaper has a positive number of single- and multi-homing consumers. All other cases are relegated to Appendix A.8. Suppose that a symmetric equilibrium subscription price satisfies $p^* \in \left(v-d-t, v-d-\frac{t}{2}\right)$.[22] Then the single-homing demand for newspaper j is composed of the *category 1* consumers that single-home and the *category 2* and *3* consumers, and is given by (denoted by superscript SH)

$$X_j^{SH} = \frac{2}{N}\frac{1}{N-1}\sum_{k\neq j, k\in\{1,\ldots,n\}} x_{jk}\left(p^*\right) + \frac{2}{N}\frac{N-n}{N-1} \tag{1.3}$$

for prices in the neighborhood of p^*. The multi-homing demand for newspaper j is composed of the *category 1* consumers that multi-home and is given by (denoted by superscript MH)

$$X_j^{MH} = \frac{2}{N}\frac{1}{N-1}\sum_{k\neq j, k\in\{1,\ldots,n\}} \left[x_{kj}\left(p_j\right) - x_{jk}\left(p^*\right)\right] \tag{1.4}$$

for prices p_j in the neighborhood of p^*. Newspaper j's total consumer demand $D_j = X_j^{SH} + X_j^{MH} = \frac{2(n-1)}{tN(N-1)}\left(v-d-p_j\right) + \frac{2}{N}\frac{N-n}{N-1}$ is a downward-sloping demand curve.[23] A change in j's own price does not affect the number of single-homing consumers on j but affects its number of multi-homing consumers. This can be seen in Figure 1.2: if newspaper j increases p_j, $x_{kj}\left(p_j\right)$ decreases, whereas the cutoff that determines the single-homing consumers, $x_{jk}\left(p^*\right)$, is unaffected.

The equilibrium advertising price chosen in the third stage is given by (1.2). All advertisers buy an ad space in each digital newspaper. Then newspaper j's profit at the beginning of stage 2 is given by

$$\Pi_j = \left[(1-\tau)p_j - c + \alpha\right] X_j^{SH} + \left[(1-\tau)p_j - c + \sigma\alpha\right] X_j^{MH} - F \tag{1.5}$$

[22] Then, the conditions to have a positive number of single- and multi-homing consumers on each newspaper, $0 < x_{jk}(p^*) < x_{kj}(p^*) < 1 \ \forall j,k$, are satisfied. Notice that all consumers from the second and third category go to their, respectively, first and second preferred variety, because $p^* < v - d - \frac{t}{2} < v - t$, where the last inequality follows by the $d > \frac{t}{2}$ assumption.

[23] The incremental value of buying j in addition to k, i.e., u_{kj}, solely depends on p_j but not on the subscription prices charged by the rival newspapers; $u_{kj} = 0$ determines $x_{kj}(p_j)$. Since $x_{kj}(p_j)$ in turn determines D_j, newspaper j's total demand is independent of the subscription prices charged by the rival newspapers p_k, $\forall k \in \{1,\ldots,n\}, k \neq j$.

The first-order condition with respect to p_j can be written as

$$\frac{\partial \Pi_j}{\partial p_j} = \left[(1-\tau)D_j + [(1-\tau)p_j - c] \frac{\partial X_j^{MH}}{\partial p_j} \right] + \sigma\alpha \frac{\partial X_j^{MH}}{\partial p_j} = 0, \tag{1.6}$$

where $\frac{\partial X_j^{MH}}{\partial p_j} = -\frac{2(n-1)}{tN(N-1)} < 0$. An increase in p_j raises the profit per newspaper unit sold but reduces the multi-homing demand. Both of these effects pertain to the reader side of the news market (the terms inside the squared brackets). In addition and due to the two-sided nature of the market where second impressions have a positive value to advertisers ($\sigma\alpha > 0$), an increase in p_j reduces X_j^{MH}, meaning that j forgoes an amount of multi-homing eyeballs sold to advertisers (the term outside the squared brackets). This leads to a reduction of the consumer subscription fee compared to a one-sided market in which a digital newspaper could solely finance itself through subscriptions.

The expression in (1.6) is independent of any newspaper's price other than p_j - prices are strategically independent. Lemma 1.1 follows immediately.

Lemma 1.1. *Let the number of newspaper n be fixed. If* $v-d-\frac{c-\sigma\alpha}{1-\tau}-\frac{t(N-1)}{n-1} \in (0,t)$, *the unique symmetric subscription price in equilibrium is*

$$p_n^* = \frac{1}{2}\left(v - d + \frac{c-\sigma\alpha}{1-\tau} + \frac{t(N-n)}{n-1} \right) \tag{1.7}$$

Total equilibrium sales, number of single- and multi-homing consumers for each platform are, respectively,

$$\begin{aligned} D_n^* &= \frac{n-1}{tN(N-1)} \left[v - d - \left(\frac{c-\sigma\alpha}{1-\tau} \right) \right] + \frac{N-n}{N(N-1)} \\ X_n^{MH^*} &= 2\left(D_n^* - \frac{2N-n-1}{N(N-1)} \right), \quad X_n^{SH^*} = 2\left(\frac{2N-n-1}{N(N-1)} \right) - D_n^* \end{aligned} \tag{1.8}$$

I assume that $v-d-\frac{c-\sigma\alpha}{1-\tau}-\frac{t(N-1)}{n-1} \in (0,t)$ implicitly holds for the remainder of the chapter unless stated otherwise. This ensures that indeed a positive number of single-homing and multi-homing consumers exist on each newspaper at the equilibrium. If appropriate, I also refer to this assumption as the *relevant parameter space.*

A reduced VAT rate τ increases the newspapers' profitability from the consumer side but does not directly affect the advertiser side. Thus, a newspaper puts more emphasis on the consumer terms that are inside the squared brackets in expression (1.6), leading to a lower subscription fee.[24] A reduction in τ is equivalent to a reduction in c on the consumer side.

At the same time and due to the two-sided pricing strategy of a newspaper (Armstrong, 2006; Rochet and Tirole, 2006), the subscription fee decreases in the importance of the other side, the advertisement side: $\frac{\partial p_n^*}{\partial \sigma\alpha} < 0$. That is, the extra profit per multi-homing consumer from selling advertisement, $\sigma\alpha$, is akin to a negative marginal cost in p_n^*. Consequently, a decrease in τ increases the part in p_n^* associated with the advertisement side.[25] Therefore as τ decreases, the equilibrium subscription fee overall decreases if and only if the marginal value from selling a multi-homing consumer's eyeballs to advertisers, $\sigma\alpha$, is smaller than the marginal cost c supplying an extra newspaper copy.[26]

Since total demand D_n^* decreases in p_n^*, a VAT reduction leads to a readership decrease if and only if $\sigma\alpha > c$. The change in demand stems from the first category consumers, who are now less interested in purchasing a second variety in addition to their first preferred variety. That is, the increase in the subscription fee converts some *category 1* multi-homing consumers into single-homing consumers. In particular, the two cutoffs x_{jk} and x_{kj} between any two newspapers $j, k \in \{1, \ldots, n\}, j \neq k$ move towards each other simultaneously. Consequently, $X_n^{MH^*}$ decreases.[27] The demand by *category 2* and *3* consumers is unaffected. Lemma 1.2 formally states the discussion of how equilibrium outcomes are affected by a decrease in τ.[28]

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[24]Formally, $-\frac{\partial}{\partial \tau}\left(p_n^* + \frac{\sigma\alpha}{2(1-\tau)}\right) = -\frac{c}{2(1-\tau)^2} \leq 0$.

[25]That is, $-\frac{\partial}{\partial \tau}\left(\frac{-\sigma\alpha}{2(1-\tau)}\right) = \frac{\sigma\alpha}{2(1-\tau)^2} > 0$.

[26] The qualitative impact of a VAT reduction on the subscription price does not hinge on the consumers' uniform distribution assumption. Suppose that consumers on each spoke are distributed on $[0, 1/2]$ according to a density function $g(\cdot)$, with cumulative distribution $G(\cdot)$ such that $G(0) = 0$ and $G(1/2) = 1$. In (1.6), now $D_j = \frac{(n-1)}{N(N-1)}\left[2 - G\left(1 - (v - d - p_j)/t\right)\right] + \frac{2}{N}\frac{N-n}{N-1}$ and $\frac{\partial X_j^{MH}}{\partial p_j} = -\frac{(n-1)}{tN(N-1)} g\left(1 - (v - d - p_j)/t\right) < 0$. Assume that the second order condition for a local maximum is satisfied. By the implicit function theorem, $\frac{dp_n^*}{d\tau} \propto (c - \sigma\alpha) g\left(1 - (v - d - p_n^*)/t\right)$, which is strictly negative if and only if $\sigma\alpha > c$.

[27]Directly from (1.8): $\frac{\partial}{\partial \tau} X_n^{MH^*} = 2\frac{\partial D_n^*}{\partial \tau} = -2\frac{n-1}{tN(N-1)}\left(\frac{c-\sigma\alpha}{(1-\tau)^2}\right)$, which is positive if and only if $\sigma\alpha > c$.

[28]Other comparative statistics about $X_n^{MH^*}$ follow directly from (1.8): fewer consumers multi-home as the cost of being ideologically from one's most preferred newspaper increases (t) and fewer consumers multi-home as the discount of reading a second newspaper increases (d). If the value of second impressions ($\sigma\alpha$) increases, the more profitable it is for platforms to sell ads, which in turn makes it more lucrative for newspapers to attract multi-homing consumers through lower subscription fees (see two-sidedness discussion above), and consequently more consumers multi-home.

Lemma 1.2. *Let n be fixed. If τ decreases, p_n^* increases and the consumer surplus $CS(n)$, the fraction of consumers that multi-home $\frac{n}{2}X_n^{MH^*}$, and total readership nD_n^* decrease if and only if $\sigma\alpha > c$.*

Proof. See the main text for the impact of τ on p_n^*, $X_n^{MH^*}$ and D_n^*. Since n is fixed, the impact of τ on the fraction of consumers that multi-home, $\frac{n}{2}X_n^{MH^*}$, is the same as for $X_n^{MH^*}$. The same holds with regards to total readership. The consumer surplus is given by

$$\begin{aligned} CS(n) =& n\Bigg[\overbrace{\frac{2}{N}\frac{n-1}{N-1}\left(\int_0^{1/2}(v-tx-p_n^*)dx + \int_{x_{jk}(p_n^*)}^{1/2}(v-t(1-x)-d-p_n^*)dx\right)}^{\textit{category 1} \text{ consumers}} \\ &+ \underbrace{\frac{2}{N}\frac{N-n}{N-1}\int_0^{1/2}(v-tx-p_n^*)dx}_{\textit{category 2} \text{ consumers}} + \underbrace{\frac{2}{N}\frac{N-n}{N-1}\int_{1/2}^{1}(v-tx-p_n^*)dx}_{\textit{category 3} \text{ consumers}}\Bigg], \end{aligned} \tag{1.9}$$

from which it is immediate that the consumer surplus decreases if and only if p_n^* increases - recall that $x_{jk}(p_n^*)$ increases as p_n^* increases. □

Total readership and the fraction of consumers that multi-home can be interpreted as different measures of the degree of an "open and enlightened public discourse" by consumers (Kontor, 2015, p.3). The qualitative result of Lemma 1.2 has been already found in the two-sided media literature (see, e.g., Kind et al., 2013; Foros et al., 2019) and in a more general two-sided platform set-up (Belleflamme and Toulemonde, 2018). Hence, the spokes oligopoly extension gives the same result regarding a change in τ for a fixed n.

As a result, Foros et al. (2019, p.129) argue that "the logic of two-sided markets [...] indicates that subsidizing newspapers through reduced value-added taxes might be an ineffective or even counter-productive means to increase [multi-homing]," and Kind et al. (2013, p.1) state that "a tax increase will be welfare-enhancing." In the next subsection, I show that these qualitative results on the consumer measures may be overturned by allowing for free entry.

Remark - $\sigma\alpha$ versus c: That $\sigma\alpha > c$ holds is particularly likely for digital newspapers, where the marginal cost producing a copy c is close to or even equal to zero. If on the other hand, one were to re-interpret the model for a market of printed newspaper, the reverse relationship is likely to be true. Then according to Lemma 1.2, the historical preferential tax treatment of printed newspaper is indeed advisable from the consumers' perspective (for a fixed n). For the remainder of the chapter I assume $\sigma\alpha > c$.

1.3.2 Free entry

Suppose that up to N potential digital newspapers can enter. If n newspapers enter, the profit of each newspaper that entered, evaluated at the equilibrium prices, is

$$\Pi^*(n) = \left[\frac{1}{2}(1-\tau)\frac{tN(N-1)}{n-1}D_n^* - \alpha(1-\sigma)\right] D_n^* + 2\alpha(1-\sigma)\frac{2N-n-1}{N(N-1)} - F, \tag{1.10}$$

where D_n^* is given by (1.8).

The VAT-rate reduction affects a platform's profit on the subscription side and the advertisement side. On the reader side, per consumer profit on the one hand increases due to the subscription increase, but D_n^* on the other hand decreases.[29] On the advertisement side, by Lemma 1.2 the loss of multi-homing consumers is at twice the rate as the gain of single-homing consumers. Thus the advertisement price increases (and in turn the profit on the ad side increases) if and only if $1 > 2\sigma$. Proposition 1.1 states that the profit per newspaper overall strictly increases following the VAT reduction if subscription prices are non-negative (a sufficient condition), leading to a weak increase in the number of newspapers n^* when allowing for free entry and restricting n^* to take integer values.

Proposition 1.1. *In the relevant parameter space, the free-entry equilibrium number of digital newspapers $n^*(\tau)$ is unique. If furthermore the subscription prices are non-negative and $\sigma\alpha > c$, then $n^*(\tau)$ weakly increases as τ is reduced to zero, i.e., $\Delta n \triangleq n^*(\tau = 0) - n^*(\tau > 0) \geq 0$.*

The Proposition 1.1 finding supports the increase in the number of digital newspapers in Norway following the VAT-rate reduction, as empirically documented in Figure A.1.2.

Comparative statics: The number of additional newspapers that enter following the VAT reduction increases in the advertisers' willingness to pay per unique impres-

[29] The profit on the subscription price overall increases if and only if $\left(v - d + \frac{t(N-n)}{n-1}\right)^2 > \frac{(c+\sigma\alpha)(c-\sigma\alpha)}{1-\tau}$.

sion, α, if σ is not too large.[30] Due to the two-sidedness of the newspaper market, for one a larger α reduces the subscription price and hence profit per reader on the consumer side. However, the VAT reduction is a counter-acting force to this per consumer profit decrease due to the decrease of readership D_n^*. At the same time an increase in α increases D_n^*; although the VAT reduction diminishes this readership expansion effect, it more than compensates through a per consumer profit increase. Therefore, the profit on the consumer side overall increases in α following the VAT reduction.

On the advertisement side, an increase in α makes consumers more valuable to advertisers and allows newspapers to increase their ad prices. At the same time, the increase in α reduces the number of single-homing but increases the number of multi-homing consumers (again due to the subscription price decrease), which increases the ad price if and only if $2\sigma > 1$. Then following the VAT reduction, the described impacts from an α increase on the ad price are beneficial for the platform only if the gain from having more single-homing consumers outweighs the loss of multi-homing consumers, i.e. if $1 > 2\sigma$.

An increase in the discount from reading an additional newspaper d causes both the subscription price and demand to decrease. The VAT reduction amplifies the consumer-profit decrease. The advertisement-profit side is unaffected by the interaction between d and τ. Hence, Δn decreases in d.[31]

Impact of VAT reduction on consumers

An increase in the number of newspapers has several effects on consumers in the model.

Price effect: Observe from equation (1.7) that an increase in n^* pushes the subscription price downwards due to the familiar effect of increased competition. Thus, an increase in the number of newspapers has the opposing effect on the subscription price to the initial *direct* effect on the price due to the VAT reduction (Lemma 1.2). If the former *indirect* effect from additional entry dominates the latter *direct* effect, the equilibrium subscription price decreases. Lemma 1.3 formally states the condition.

[30] The specific condition is $\sigma < \frac{2}{3}$ when $c = 0$; this follows from $\left.\frac{\partial^2 \Pi^*(n)}{\partial \alpha \partial \tau}\right|_{c=0} \propto -\alpha\sigma^2 + 2\sigma\alpha(2\sigma - 1)$, where the first term corresponds to the profit change on the consumer side and the second term to the advertisement side. If $c > 0$, Δn increases in α if $\sigma \in \left(\frac{1}{3\alpha}\left(\alpha + c - \sqrt{\alpha^2 - \alpha c + c^2}\right), \frac{1}{3\alpha}\left(\alpha + c + \sqrt{\alpha^2 - \alpha c + c^2}\right)\right)$.

[31] $\frac{\partial^2 \Pi^*(n)}{\partial d \partial \tau} \propto v - d + \frac{t(N-n)}{n-1} > 0$

Lemma 1.3. *If the VAT rate τ is reduced to zero, the subscription price under free-entry decreases, i.e., $\Delta p \triangleq p^*(\tau = 0) - p^*(\tau > 0) < 0$, if and only if $\Delta n > \Delta \tilde{n}^p$, where $\Delta \tilde{n}^p > 0$ is implicitly defined by*

$$\frac{-\tau(c - \sigma\alpha)}{1 - \tau} = \frac{t(N-1)\Delta \tilde{n}^p}{[n^*(\tau > 0) - 1]\,[n^*(\tau > 0) + \Delta \tilde{n}^p - 1]} \tag{1.11}$$

Proof. Expression (1.11) follows from equating the subscription price given by (1.7) evaluated at $n^*(\tau > 0) + \Delta \tilde{n}^p$ and $\tau = 0$ to the price evaluated at $n^*(\tau > 0)$ and $\tau > 0$. Because $\sigma\alpha > c$ by assumption, the reduction of τ to zero leads to an initial increase in the subscription price; this is captured by the left-hand side in (1.11), which is strictly positive. Since the subscription price decreases as Δn additional newspaper enter, the right-hand side in (1.11), a continuous function in Δn that is zero if $\Delta n = 0$, increases in Δn. By Bolzano's Theorem there exists a $\Delta \tilde{n}^p > 0$ such that (1.11) is satisfied. □

The threshold $\Delta \tilde{n}^p$ is smaller the less is the direct impact on the subscription price following a reduction in the VAT rate (small initial τ and low σ and α values) compared to the indirect impact on the subscription price through the increase in the number of newspapers (large t and N and small initial number of newspapers). In the case of Norway, Lemma 1.3 suggests that Δn roughly equals $\Delta \tilde{n}^p$ to reconcile the fact that the subscription price trend for digital newspapers has not significantly changed after the VAT-rate reduction in 2016 (Figure A.2.2).

Market expansion effect: As the number of available newspapers increases, some consumers whose both preferred newspapers were previously unavailable are now able to purchase one or both of their preferred varieties. Those consumers are better off having more preferred varieties available. The market expansion effect also relates to multi-homing consumers. In particular, some *category 2* and *3* consumers are turned into *category 1* consumers; the ones close to the center in the network now consider multi-homing an attractive option. They previously only had one of their preferred varieties available so could only single-home.

Matching effect: As more newspapers become available, some consumers will be better matched regarding their preferred varieties. In particular, some *category 3* consumers are turned into *category 1* consumers, for whom their first in addition to their second preferred variety is now available. Of those consumers, the ones far away from the center will switch from their second to their first preferred variety.

The next two propositions state conditions on how many additional newspapers have to enter following the VAT reduction such that the consumer measures improve due to the three explained positive effects associated with entry for consumers.

Proposition 1.2. *If $\Delta n \geq \Delta \tilde{n}^p$, the consumer surplus, total readership and fraction of consumers that multi-home strictly increase.*

The result of Proposition 1.2 is intuitive. If the number of newspapers increases, consumers have more of their preferred varieties available (the market expansion effect) and some consumers find a better match regarding their preferred varieties (the matching effect). If also the subscription price decreases, any consumer independent of her category is better off by having to pay a lower price.

If on the other hand the VAT reduction leads to an overall higher subscription price, consumers are not necessarily worse off depending on how many additional newspapers enter. Proposition 1.3 formalizes.

Proposition 1.3. *If $\Delta n < \Delta \tilde{n}^p$,*

- *the consumer surplus increases if and only if $\Delta n > \Delta \tilde{n}^{CS}$,*
- *the fraction of consumers that multi-home increases if and only if $\Delta n > \Delta \tilde{n}^{MH}$, and*
- *total readership increases if and only if $\Delta n > \Delta \tilde{n}^{TRS}$,*

where $0 < \Delta \tilde{n}^{TRS} \leq \Delta \tilde{n}^{MH} < \Delta \tilde{n}^p$ and $\Delta \tilde{n}^{CS} \in (0, \Delta \tilde{n}^p)$.

The $\Delta \tilde{n}^{CS}, \Delta \tilde{n}^{TRS}$ and $\Delta \tilde{n}^{MH}$ cut-offs are in the proof in Appendix A.8. Proposition 1.3 formally states that even if the additional entry by newspapers following the VAT reduction does not result in an overall decrease in the subscription price, i.e, $\Delta n < \Delta \tilde{n}^p$, the consumer surplus may nevertheless increase. This requires that $\Delta n > \Delta \tilde{n}^{CS}$: the increase in the number of newspapers following the VAT reduction has to be sufficiently large such that the positive market expansion effect and matching effects outweigh the increase of the subscription price on the consumer surplus.

The change in the fraction of consumers that multi-home follows a similar logic. On the one hand existing *category 1* consumers become less likely to multi-home because of the increase in the subscription price, but on the other hand, due to the market expansion effect more consumers have the option to multi-home in the first place following an increase in the number of newspapers.[32] For the fraction of

[32]In particular, some *category 2, 3* and *4* consumers close to the center are turned into *category 1* consumers.

multi-homing consumers to increase requires that $\Delta n > \Delta\tilde{n}^{MH}$. In addition to the market expansion effect, the matching effect also means that more single-homing consumers have their preferred varieties available. This is why the threshold above which total readership increases, i.e., $\Delta\tilde{n}^{TRS}$, is lower than $\Delta\tilde{n}^{MH}$.

The Norwegian government's media objective is to "promote good news production and a broad and enlightened public discourse in the digital media society of the future" (Kontor, 2015, p. 4). By allowing for free entry and in contrast to recent theoretical predictions (see, e.g., Kind et al., 2013; Belleflamme and Toulemonde, 2018; Foros et al., 2019), Propositions 1.2 and 1.3 highlight that a reduction of the VAT rate for digital newspapers can promote this media objective. I quantify the impact of the VAT reduction in Norway in Section 1.4.

It is noteworthy that the advertiser surplus and fraction of multi-homing consumers are perfectly aligned. To see this, an advertiser's profit solely stems from multi-homing consumers, since newspapers extract the full surplus advertisers gain from single-homing consumers. A multi-homing consumer is worth $\alpha(1+\sigma)$ to an advertiser, but an advertiser only pays $2\sigma\alpha$ for her: $\sigma\alpha$ to each platform where she multi-homes. Hence, an advertiser's profit per multi-homing consumer is $\alpha(1-\sigma)$. Since each newspaper has $X_n^{MH^*}$ multi-homing consumers, an advertiser's profit is $\alpha(1-\sigma)\frac{n}{2}X_n^{MH^*}$ from placing an ad in each newspaper in equilibrium. Since furthermore the demand for ads is perfectly elastic and the mass of advertisers $A = 1$, an advertiser's surplus also equals the total advertiser surplus (AS). Hence,

$$AS = \alpha(1-\sigma)\frac{n}{2}X_n^{MH^*} \tag{1.12}$$

Thus AS and the fraction of multi-homing consumers $\frac{n}{2}X_n^{MH^*}$ are fully aligned.[33] If the VAT rate is reduced to zero, AS increases if and only if $\Delta n > \Delta\tilde{n}^{MH}$.

[33] Anderson and Peitz (2020, Sections 5 and 7) show that the consumer and advertiser interests are opposed when platforms are solely ad financed. In their model, an exogenous increase in n through entry (which here is endogenous following the VAT reduction) has two opposing effects. On the one hand, entry increases consumer participation, which is beneficial for advertisers because their ads reach a larger audience. The same effect is present here. On the other hand, the optimal response by a platform is to reduce its amount of advertising (by increasing the ad price), which hurts advertisers. The authors show that the latter effect dominates the former (and thus AS decreases), while the CS increases (more varieties are available). If instead the platforms finance themselves both through advertising and subscriptions as in the present model (Anderson and Peitz, 2020, Sections 6), the advertising decision is "uncoupled" from consumer demand. That is, the ad level per platform choice is independent of n; here the equilibrium ad price implies that the ad level is one per platform and thus is also independent of n. Thus, the above latter effect is absent under two-sided pricing and the consumer and advertiser interests are aligned.

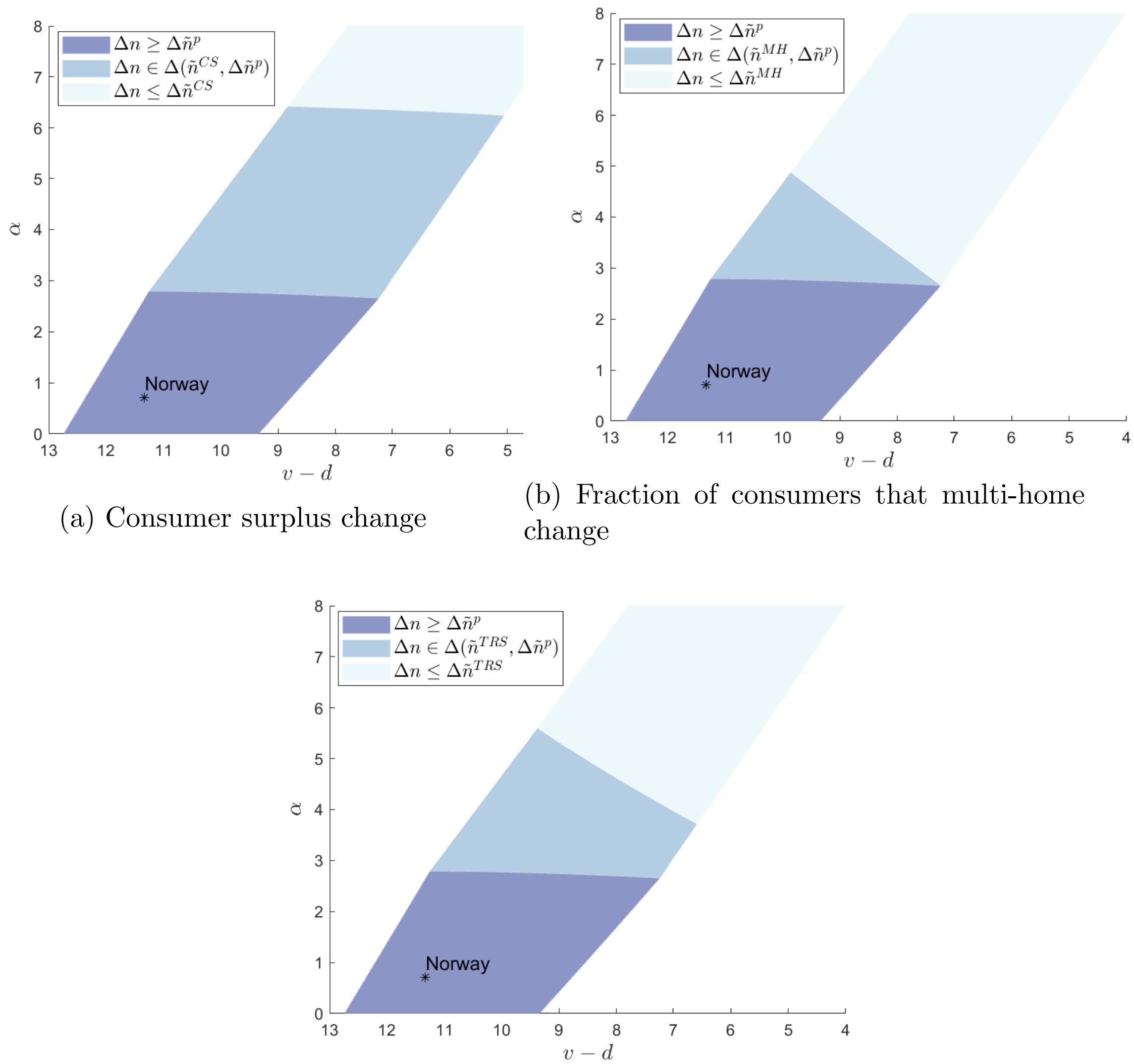
Illustration. The model can make predictions that are consistent with data from entry (Figure A.1.2) and prices (Figure A.2.2) before and after the VAT exemption in Norway. However, Propositions 1.2 and 1.3 state that the change in the subscription price, consumer surplus and digital news consumption following a VAT reduction can be either positive or negative. Figure 1.3 illustrates. Figure 1.3a highlights in the $(v - d, \alpha)$ parameter space that any of the three possible consumer surplus cases according to Propositions 1.2 and 1.3 can arise: $\Delta n \geq \Delta\tilde{n}^p$ (Proposition 1.2), $\Delta n \in \left(\Delta\tilde{n}^{CS}, \Delta\tilde{n}^p\right)$ and $\Delta n \leq \Delta\tilde{n}^{CS}$ (Proposition 1.3). I also highlight the Norwegian case based on the calibrated parameters in Section 1.4. Figure 1.3b and 1.3c repeat, respectively, the exercise for the change in the fraction of consumers that multi-home and total readership.

Figure 1.3 also qualitatively highlights the role played by the ad side on the consumer measures following the VAT reduction. Suppose $\alpha = 0$ - the digital newspapers operate in a **one-sided** market where subscriptions are the sole revenue stream. If in addition $c = 0$ (as I assume in the present illustration), the *direct* effect on the subscription price following the VAT-rate reduction is zero (see Lemma 1.2). The per-platform profit and n^* nonetheless increase, and thus the positive *indirect* effect still occurs. Then, consumers cannot be worse off in a one-sided market if the VAT rate is reduced.[34] Now suppose instead that α is positive and increasing. Recall that on the one hand the larger is α, the larger is the increase of additional newspapers and the with it associated positive effects following the VAT reduction. On the other hand the larger is α, the larger is the initial *direct* increase of the subscription price. Figure 1.3 suggests that the magnitude of the negative direct impact outweighs the positive indirect effect for the consumers above some α threshold (for a fixed $v - d$ value).

Second-Best for Consumers. Suppose next that the social planner regulates entry and chooses the VAT rate but not newspaper prices (i.e., the second-best solution). Continue to suppose that the authority is interested in the well-being of consumers. Since the consumer measures increase in the number of newspapers, clearly $n^{SB} = N$. Furthermore, because the subscription price decreases in τ (for $\sigma\alpha > c$) and a lower subscription price is beneficial for consumers, the planner chooses the VAT rate such that all consumers from the first category multi-home. That is, the planner chooses τ such that the subscription price (evaluated at $n = N$)

[34]Formally, the left-hand side in (A.24) and (A.25) are zero, and thus $\Delta n > 0$ is a necessary and sufficient condition to make consumers strictly better off.

Figure 1.3: Consumer measure changes from VAT reduction in the $(v-d, \alpha)$ space



(a) Consumer surplus change

(b) Fraction of consumers that multi-home change

(c) Total readership change

Notes: Figure 1.3 shows in separate plots the difference of the consumer surplus, fraction of consumers that multi-home, and total readership, respectively, after and before the VAT reduction in the $(v-d, \alpha)$ parameter space. Other than $v-d$ and α, I use the calibrated parameters of an average Norwegian region from Section 1.4. The changes in the three respective consumer measures are plotted for all permissible $(v-d, \alpha)$ combinations such that we are in the region with a strictly positive number of multi-homing consumers, both before and after the VAT reduction, when allowing for free-entry. The Norwegian calibrated $(v-d, \alpha)$ is highlighted in each plot. I do not impose that the free-entry equilibrium number of newspapers has to be an integer value; Figure A.1.3 in Appendix A.1 reports the same plots where I impose the number of news to take an integer value.

equals $v - d - t$, from which follows that

$$\tau^{SB} = 1 - \frac{\sigma\alpha - c}{2t - (v - d)} \tag{1.13}$$

Since $\sigma\alpha > c$, the denominator in (1.13) is positive and hence $\tau^{SB} < 1$. Note that τ^{SB} is negative if and only if $\sigma\alpha - c > 2t - (v - d)$. It is also immediate using (1.9) that the consumer surplus $CS(\tau^{SB}, n^{SB}) = \frac{d}{N}$, $\frac{n}{2} X_n^{MH^*} = 1$ and $TRS = 2$ (since all spokes are occupied every consumer is a *category 1* consumer and every consumer from this category multi-homes).

1.4 The Norwegian case

I calibrate the set of parameters for the case of Norway in a first step. In a second step, I use the calibrated parameters to simulate the model's predictions following the VAT reduction, which are then compared to the actual data. In addition, I compare the equilibrium outcomes to those that a planner who maximizes economic welfare chooses. Lastly, I present several policy experiments based on the calibrated model. I lay the theoretical foundations for the social planner and the policy experiments in Appendix A.5 and Appendix A.6, respectively. I show in Appendix A.7 that the key results are robust to alternative specifications.

1.4.1 Parameter calibration

In what follows it is assumed that the Norwegian digital newspapers played the equilibrium pricing and entry game between 2012 and 2015. This assumption seems reasonable given that most digital newspapers had entered the market by 2012,[35] and the number of newspapers was fairly stable during that period before the VAT reduction in 2016 (the coefficient of variation is 5.6%). Furthermore, the pricing data suggests that following an initial "price adjustment phase" up to and including 2011, the digital subscription prices have since remained relatively constant: the coefficient of variation is 8.5% on average for the newspapers between the second quarter of 2012 and the first quarter of 2016.[36] Based on this assumption, I calibrate

[35] In fact, the number of digital newspapers in 2010 was already 90% of the average number of newspapers between 2012 and 2015. The same figure for 2011 is 97%, but only 16% for 2008 and 61% for 2009.

[36] When I adjust the prices by the upwards trend estimated for Figure A.2.2, this figure decreases to 8.2%.

the model parameters using data from 2012 to 2015 (except for α, where I need to use data for 2016 and 2017). The calibrated parameters correspond to daily values.

As in Gentzkow (2007), I assume that the marginal cost of an additional online newspaper reader is zero, i.e., $c = 0$. I furthermore assume that $\sigma = 0.5$ (Gentzkow et al., 2014 estimate that $\sigma = 0.55$ for U.S. print newspapers; Shi, 2016 estimates that $\sigma = 0.5$ for U.S. magazines).[37]

I calibrate α by using the price per ad expression given by (1.2) and from data of daily online readership, the fraction of readers that multi-home and total online ad revenue (since I do not have said data for individual newspapers, I cannot calibrate α for each newspaper). In particular, the ad price is $\alpha_t X_{jt}^{SH} + \sigma\alpha_t X_{jt}^{MH}$ in year t from (1.2). Let x_t^{SH} and x_t^{MH} be the fraction of daily readers that, respectively, single-home and multi-home in year t. In Norway, these fractions were, respectively, 51% and 49% in both the years 2016 and 2017 (Statistics Norway, 2020). Assuming that the fraction of consumers that single-home and multi-home is the same for each newspaper, the total ad revenue expression then becomes $\sum_j^{n_t}\left(\alpha_t X_{jt}^{SH} + \sigma\alpha_t X_{jt}^{MH}\right)$ $= \alpha_t\left(x_t^{SH} + \sigma x_t^{MH}\right)\sum_j^{n_t} D_{jt}$. Note that $\sum_j^{n_t} D_{jt} = TRS_t$, the total daily demand for online newspapers. Daily readership numbers come from the Kantar TNS Norway (2019) "Consumer & Media" survey reports: $TRS_{2016} = 7.67$ million and $TRS_{2017} = 7.37$ million. Thus

$$\alpha_t = \frac{TRA_t/365}{\left(x_t^{SH} + \sigma x_t^{MH}\right)TRS_t},$$

where TRA_t is total online ad revenue in a given year. The Norwegian Media Authority (2020, Figure 21) report provides annual ad revenue estimates for online newspapers: $TRA_{2016} = 1,456$ and $TRA_{2017} = 1,447$ million NOK. It follows that $\alpha_{2016} = 0.69$ and $\alpha_{2017} = 0.73$, the average of which is $\alpha = 0.71$ NOK/day.[38] Note that α should be interpreted as the total advertisers' willingness to pay for a single contact with a consumer (per day).

For the calibration of the remaining parameters (v, d, t, F, N), I assume that local and regional newspapers only compete with other newspapers headquartered in the same region (below I explain how I account for national newspapers). Norway is divided into 19 administrative regions, called counties (*fylker*), the names of which

[37] In Appendix A.7 I show that the results are robust for varying σ values.

[38] While total advertising revenue estimates for digital newspapers are available from 2014 to 2019, estimates of the fraction of consumers that single- and multi-home are not available before 2016 and the readership figures are available up to and including 2017. I can therefore only calibrate α_t for 2016 and 2017. In Appendix A.7 I report the results when I let α vary.

are listed in column 1 of Table 1.1.[39] By ignoring the newspaper demand in "hinterland" regions, the newspapers make their pricing and entry choices in each of the 19 separate newspaper markets (one per region).[40] In fact, the average local or regional digital newspaper had 71.5 percent of its demand in its home market in Norway between 2012 and 2015 (Gentzkow et al., 2014, find in their data the figure to be 65% for U.S. print newspapers in 1924).[41] For the purpose of calibration, the market definition is thus an approximation to the observation that consumers indeed demonstrate strong but not exclusive preferences for regional news.

The average number of digital newspapers between 2012 and 2015 and across regions is 17 (the average standard deviation is 1.04). I set N, the number of possible newspaper varieties, 50% above the maximum number of newspapers observed in a region between 2012 and 2019 to leave room for entry when I investigate the policy experiments. This gives the entries in the second column of Table 1.1. The average N is 27.

I obtain $v-d$ and t using the equilibrium price and demand equations. Note that the gross value v from reading a newspaper and the discount d when reading a second newspaper are not separately identifiable according to the model expressions.[42] In equation (1.7) I use the average subscription price of the digital newspapers that are headquartered in a given region or are national newspapers for which I have data from the second quarter in 2012 to the first quarter in 2016 (adjusted for inflation). The average subscription price in the regions is 7.02 NOK per day, with a minimum of 6.58 and a maximum of 7.18.

For equation (1.8), I calculate the average daily demand of a digital newspaper in a region between 2012 and 2015. Because I only have the aggregate but not regional daily demand for a digital newspaper in a given year, in the first step I account for and weigh a newspaper's demand in the following manner: if a local or regional newspaper is headquartered in that region, I weigh this newspaper's demand by the

[39] On January 1, 2020 the Norwegian government implemented a reform to abolish some of the 19 counties and to merge them with others to form larger administrative regions. This has reduced the number of counties from 19 to 11. A (graphical) overview of the reform is provided here: `https://www.vg.no/nyheter/innenriks/i/zoEB1/dette-er-norges-nye-regioner`. The calibration results are robust to the assumption that the newspapers had already competed from 2012 to 2015 in the new 11 instead of 19 regions (see Appendix A.7).

[40] National newspapers are assumed to play the pricing and entry game in each region.

[41] I obtain this estimate using Google Trends (`https://trends.google.com/trends`), which reports the relative frequency of searches for each active digital newspaper by Norwegian regions.

[42] In Appendix A.7 I report simulation results both for various v values while holding $v - d$ constant and for various $v - d$ values.

Table 1.1: Calibration results for each Norwegian region

Region	N	$v-d$	t	F
Akershus	32	10.79	5.90	0.31
Aust-Agder	23	11.37	5.77	0.46
Buskerud	29	11.42	5.93	0.36
Finnmark	24	11.52	2.94	0.63
Hedmark	27	11.26	4.56	0.44
Hordaland	36	11.36	4.24	0.34
Møre og Romsdal	29	11.59	4.92	0.41
Nordland	35	11.72	4.52	0.36
Nord-Trøndelag	23	11.79	5.47	0.50
Oppland	24	11.15	4.60	0.48
Oslo	20	10.39	8.02	0.41
Østfold	29	11.92	5.48	0.40
Rogaland	32	11.09	4.88	0.35
Sogn og Fjordane	26	11.66	4.09	0.51
Sør-Trøndelag	24	11.39	6.11	0.42
Telemark	23	10.74	4.37	0.50
Troms	23	11.44	4.65	0.53
Vest-Agder	24	11.61	5.67	0.45
Vestfold	27	11.07	5.44	0.38
Average	27	11.33	5.13	0.43

Notes: Table reports for each of the 19 Norwegian regions the calibrated number of spokes N, the consumer utility from reading a second newspaper $v-d$, the disalignment cost t, and the fixed cost F. The last row reports the averages. See Section 1.4.1 for details.

percentage of demand that stems from that region for this newspaper according to Google Trends (otherwise, the newspaper is not included in that region). A national newspaper's demand is weighted by the population share in that region.[43] In a second step I calculate the average daily demand for a newspaper in a region. Lastly, I divide this figure by the average population in that region between 2012 and 2015. The reason for the last step is that the model assumes a unit mass of consumers. On average, a newspaper has a daily demand of 7.21% of the population in a region.

With the calculated pricing and demand values for a given region and the other parameters calibrated thus far, everything in (1.7) and (1.8) is known other than $v-d$ and t. The solution to the system of the two linear equations given by (1.7)

[43]Nine out of the on average 132 active digital newspapers between 2012 and 2015 are national newspapers, i.e., newspapers that target a nationwide audience.

and (1.8) is unique.[44] The calibrated $v-d$ and t values for each region are reported, respectively, in column three and four in Table 1.1. On average $v-d=11.33$ and $t=5.13$ (the standard deviations, respectively, are 0.39 and 1.06). The constraint to have a strictly positive number of single-homing and multi-homing consumers, i.e., $v-d-\frac{c-\sigma\alpha}{1-\tau}-\frac{t(N-1)}{n-1}\in(0,t)$, is satisfied in all but the *Finnmark* region.

I assume that the potential newspaper entrant $j\in[1,...,N]$ has a specific fixed cost F_j; I assume that F_j is gamma distributed with shape parameter μ_1 and scale parameter μ_2.[45] When ordering the N draws in ascending order in each region, let the last newspaper to enter be indexed n, which corresponds to the number of active newspapers. Using the already calibrated parameters in (1.10), column 5 of Table 1.1 reports the fixed cost values of the last newspaper to join in each region, under the assumption that this last newspaper makes zero profit. The average F equals 0.43 (with a standard deviation of 0.08) and is interpreted as the daily and per unit mass of consumers' fixed cost. Using the average of one thousand sorted N draws, I compute that $\mu_1=0.53$ and $\mu_2=0.11$ best fit the actual number of newspapers before and after the VAT reduction in an average region.

1.4.2 Equilibrium, VAT reduction, and welfare outcomes

Using the calibrated parameters, column (1) in Table 1.2 presents the model's implied number of digital newspapers, subscription price, consumer measures, and welfare including its constituent parts for an average Norwegian region before the VAT reduction. Compared to the data figures in column (2), the calibrated model fits the key features of the data reasonably well. Welfare is 28.81 NOK per day and per unit mass of consumers, which is the sum of 5.17 NOK from the newspapers' profit, 0.09 NOK from advertiser profit, 1.59 NOK from VAT revenue, and 21.96 NOK from consumer surplus.

Columns (3) and (4) report the outcomes of the model and data, respectively,

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[44] Namely,

$$v-d=\frac{c-\sigma\alpha}{1-\tau}\left[-1+\frac{2(N-n)}{\bar{D}N(N-1)}\right]+2\bar{p}\left[1-\frac{(N-n)}{\bar{D}N(N-1)}\right] \text{ and}$$
$$t=\frac{2(n-1)}{\bar{D}N(N-1)(1-\tau)}\left[-c+\sigma\alpha+\bar{p}(1-\tau)\right],$$

where $\bar{p}$ and $\bar{D}$ are, respectively, the average price and demand values calculated using the data.

[45] The heterogeneous fixed cost assumption does not affect the qualitative results in Section 1.3. Gentzkow et al. (2014) assume that their affiliation-specific cost is distributed mean-zero type-I extreme value.

Table 1.2: Equilibrium and welfare-maximizing outcomes for an average region

	Pre-VAT reduction		Post-VAT reduction		Social planner		
	Model	Data	Model	Data	VAT rate; fixed n	VAT rate; free entry	Entry and VAT rate
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Number newspapers	16.69	16.68	17.63	17.63	16.69	18.50	21.02
Subscription price	7.05	7.02	6.86	6.77	6.20	6.70	6.20
Total readership	1.13	1.17	1.22	1.20	1.26	1.30	1.58
Fraction of multi-homing consumers	0.26		0.32	0.34	0.39	0.38	0.62
Welfare	28.81		29.72		29.71	29.86	31.86
Newspaper profit	5.17		6.73		0.08	8.56	5.91
Advertiser profit	0.09		0.11		0.14	0.14	0.22
VAT revenue	1.59		0.00		6.51	−2.51	−0.17
Consumer surplus	21.96		22.87		22.98	23.67	25.89
Optimal VAT rate					0.84	−0.22	−0.02

Notes: The first column reports the simulation of the calibrated model for an average Norwegian region before the VAT reduction in 2016. Column two reports data moments before the VAT reduction. The third column reports the simulation of the calibrated model when the VAT rate is reduced to zero; the fourth column contains the data moments. In the fifth column, the number of digital newspapers is fixed at the pre-VAT reduction value and a planner chooses the welfare-maximizing VAT rate. In the sixth column, a planner continues to only choose the VAT rate but I allow for free-entry. In the last column, the social planner chooses both the VAT rate and the number of digital newspapers. In all of the three social planner columns, the newspapers compete in ad and subscription prices.

following the VAT reduction. The model continues to fit the key features of the data. One important exception is that the model over-predicts the change in readership following the VAT reduction. Following the VAT reduction, the model's prediction for the fraction of consumers that multi-home is also too large compared to the data. Unfortunately, the change in the fraction of multi-homing consumers predicted by the model cannot be compared to the data, since no data before 2016 is available on multi-homing consumers.

In the final three columns of Table 1.2, I compare the equilibrium outcomes to those that a social planner would choose to maximize total surplus (I ignore the possible practical difficulties of implementation). The social planner outcomes allow to evaluate whether the objective of maximizing total welfare is aligned with the informativeness of the public, i.e., total readership or fraction of consumers that multi-home.

Column (5) holds the number of digital newspapers from column (1) fixed, but allows the social planner to choose the optimal VAT rate. The newspapers continue to choose ad and subscription prices. Since welfare increases in the VAT rate for a fixed number of newspapers (see Appendix A.5), the planner chooses the maximal VAT rate subject to the parameter constraints. The binding constraint is that all *category 1* consumers multi-home. Thus, $\tau = 1 - \frac{\sigma\alpha - c}{\frac{t(N+n-2)}{n-1} - (v-d)} = 0.84$. The high VAT rate causes newspapers to substantially reduce the subscription prices (about 12%), which leads to an increase in total readership and fraction of multi-homing consumers (12% and 46%, respectively). Column (5) therefore quantitatively demonstrates that welfare and the consumer measures are positively aligned and increase in the VAT rate for a fixed number of newspapers (Belleflamme and Toulemonde, 2018; Foros et al., 2019; Kind and Koethenbuerger, 2018). The overall increase in welfare comes at the expense of newspapers. Note that this result is qualitatively independent of the specific calibrated Norwegian parameters.

In column (6) of Table 1.2, the social planner chooses the VAT rate that maximizes total welfare under free entry. While there can be in principal over- or under-entry in equilibrium as derived in Appendix A.5, the column (6) result suggests that there is insufficient entry in equilibrium both before and after the VAT reduction in Norway: the planner chooses a VAT rate below zero, i.e., a subscription subsidy, as an instrument to induce additional entry beyond what was observed post-VAT reduction. That is, the business-stealing and ad-price-reduction effects from additional entry are relatively small for newspapers compared to the benefits consumers and advertisers gain.

In the final column of Table 1.2, the social planner chooses the number of digital newspapers in addition to the VAT rate. This quantifies the second-best analysis since newspapers continue to choose subscription and ad prices (see Appendix A.5). The results show that the number of newspapers in market equilibrium falls well short of the social optimum. The social planner increases the number of newspapers by 26% compared to the pre-VAT reduction. Compared to the previous column, the planner chooses an even larger number of newspapers but now also a close to zero subscription subsidy: this highlights that while a smaller VAT rate leads to additional entry with its associated positive indirect effects for consumers, a smaller rate also hurts consumers through a direct increase in the subscription price.

The social planner results in Table 1.2 show that the goal of maximizing economic welfare is not in conflict with increasing the consumer measures. Policies that promote entry and/or reduce the subscription price conditional on entry are likely to enhance welfare.

Table 1.3: Policy experiments for an average region

	Baseline					Alternative instruments	
	Pre-VAT reduction	Post-VAT reduction	Allow price collusion	Allow ad collusion	Allow price and ad collusion	Ad tax	Lump-sum transfer
	(1)	(2)	(3)	(4)	(5)	(6)	(7)
Number newspapers	16.69	17.63	16.69	16.75	16.75	17.63	18.56
Subscription price	7.05	6.86	7.27	6.93	7.04	6.20	6.61
Total readership	1.13	1.22	1.10	1.15	1.14	1.33	1.32
Fraction of multi-homing consumers	0.26	0.32	0.22	0.28	0.26	0.43	0.40
Welfare	28.81	29.72	28.57	29.00	28.88	29.90	29.43
Newspaper profit	5.17	6.73	5.18	5.26	5.26	6.74	8.71
Advertiser profit	0.09	0.11	0.08	0.00	0.00	0.15	0.14
VAT revenue	1.59	0.00	1.59	1.60	1.60	1.65	1.75
Consumer surplus	21.96	22.87	21.72	22.15	22.02	23.66	23.84
Ad tax revenue						−2.30	
Lump-sum							−5.01

Notes: The two baseline columns report the simulation of the calibrated model for an average Norwegian region before and after the VAT reduction in 2016, respectively. Columns three through five are counterfactuals in which digital newspapers collude on subscription prices, advertising prices, or both subscription and ad prices, respectively, to maximize profits. Columns six and seven are counterfactuals in which the social planner implements alternative instruments and in which the VAT rate is set at the pre-reduction value: in column six the planner chooses an optimal value-added advertising subsidy per advertiser and in column seven the planner chooses an optimal lump-sum transfer per active newspaper.

1.4.3 Policy experiments

Table 1.3 presents a series of policy experiments for Norway based on the calibrated model. For reference, the first and second columns restate, respectively, the model's pre- and post-VAT reduction results of Table 1.2. The third through fifth columns relax competition policy: newspapers are allowed to collude on subscription prices only, ad prices only, and both subscription and ad prices.[46] Although the newspapers collude along one or both pricing dimensions, they remain separate entities. The last two columns evaluate the alternative instruments addressed in Appendix A.6. In all the columns the newspapers continue to make their entry decisions non-cooperatively. In all policy experiments, I set the VAT rate to the pre-reduction level for comparison to the baseline results.

[46]Online newspaper j's collusive subscription price is the jth element in the price vector $\mathbf{p^{col}}$ that solves

$$\mathbf{p^{col}} \in \underset{\mathbf{p}}{\operatorname{argmax}} \sum_{j=1}^{n} \left\{ \left[(1-\tau)p_j - c + \alpha\right] X_j^{SH}(\mathbf{p}) + \left[(1-\tau)p_j - c + \sigma\alpha\right] X_j^{MH}(\mathbf{p}) - F \right\}$$

Each active newspaper sets the collusive price $p^{col^*} = \frac{1}{2}\left(v - d + \frac{c-\sigma\alpha}{1-\tau} + \frac{\alpha(1-\sigma)}{(1-\tau)} + \frac{t(N-n)}{n-1}\right)$ in equilibrium.

Allowing subscription collusion leads to an overall welfare loss and reduction in consumer measures. For a fixed n, the price under collusion is larger compared to the baseline. Although this increase in the subscription price leads to a slight increase in newspapers, the additional competitiveness does not offset the larger collusion price when n is fixed; the subscription price overall increases from 7.05 to 7.27 NOK.

Allowing ad price collusion, newspapers now can extract the full advertiser surplus. The price per ad on newspaper j is $\alpha X_j^{SH} + \frac{\alpha(1+\sigma)}{2} X_j^{MH}$, where $\frac{\alpha(1+\sigma)}{2}$ is the average value of a multi-homing consumer to an advertiser per newspaper. Due to the ad price increase, newspapers decrease the subscription price by the "seesaw" principle in two-sided market pricing (Rochet and Tirole, 2006).[47] This reason and additional entry lead to an overall subscription price decrease (-2%) and consumer measure and welfare increase (at the expense of advertisers), which are smaller though than following the VAT reduction.

When newspapers are allowed to collude both on subscription and ad prices, the reduction in the subscription price from ad collusion and the increase from price collusion cancel each other out. Thus for a fixed number of newspapers, the consumer measures and total welfare are identical to the baseline. The sole difference is that the newspapers extract the advertiser surplus. However, precisely the larger newspaper profit leads to additional entry, which in turn leads to a small decrease of the subscription price and increase of the consumer measures and total welfare relative to the pre-VAT reduction baseline. However, the welfare gain is lower than when newspapers are only allowed to collude on ad prices.

In column (6) of Table 1.3 I evaluate a value added advertisement tax/ subsidy τ_A for digital newspapers. I compute the τ_A that maximizes welfare. If $\tau_A < 0$, i.e., a subsidy, I assume that the authority has a cost of $(1+\lambda)$ NOK per NOK transferred to a newspaper, where $\lambda = 0.3$ (Gentzkow et al., 2014). Since a necessary condition for welfare to increase in τ_A is initial over-entry (see the analysis in Appendix A.6) and the calibrated model's implication of insufficient entry in the baseline equilibrium (by the Table 1.2 results), I indeed find that the authority chooses an ad subsidy. The optimal ad subsidy is close to 280% of the ad price paid. This result is driven by the large consumer surplus gain from additional readership; an ad subsidy leads both to a direct subscription reduction by the seesaw principle and to additional entry. The welfare gains and consumer measure increases are larger

[47] The equilibrium subscription price under ad price collusion is $\frac{1}{2}\left(v - d + \frac{c-\alpha(1+\sigma)/2}{1-\tau} + \frac{t(N-n)}{n-1}\right)$.

compared to the VAT reduction.

In the final column of Table 1.3, I compute the lump-sum transfer S per online newspaper that maximizes total surplus. As for the value added ad subsidy, I assume that the authority's cost per NOK transferred is $(1+\lambda) = 1.3$. I find that the optimal lump-sum transfer amounts to 0.21 NOK, or about 34% of the calibrated fixed cost of the last newspaper to enter. The welfare gains and consumer measure increases are larger using a lump-sum transfer compared to the VAT reduction. The consumer measures increase more than even under the optimally chosen VAT rate (column (6) in Table 1.2), because a lump-sum transfer does not lead to a direct increase of the subscription price.

The results of Table 1.3 highlight that allowing for ad collusion and using an ad subsidy or lump-sum transfer are appealing policies to consider for a regulator if readership and entry are inefficiently low in a digital newspaper market, as is the case in Norway.[48] While they all lead to additional entry and in turn increase readership (by the price, market expansion, and matching effects), the former two policies also make use of the seesaw principle.[49] That is, improving the newspapers' ad revenue side causes newspapers to decrease the subscription price, which has a positive direct effect on the consumer measures. Even though more newspapers enter using a lump-sum transfer compared to an ad subsidy, the seesaw principle is absent under a lump-sum transfer and the overall subscription price decrease is smaller. The results show that the welfare and consumer measure gains using a lump-sum transfer are smaller than using an ad subsidy.

1.5 Conclusions

I study the impact of a VAT reduction for digital newspapers on readership and supply of digital news. By endogenizing entry in a model of newspaper demand in which newspapers compete for readers and advertisers, the chapter offers new insights for the debate over the impact of taxes and other policies in media markets. Regarding a VAT reduction on the reader side, the model finds that consumer surplus increases

[48] While not reported in Table 1.3, I find consistent with the results in Appendix A.6 that an ad cap κ reduces the consumer measures and total welfare. Both consumer measures and welfare strictly decrease in the fraction of advertisers that a newspaper is not allowed to carry, $1 - \kappa$.

[49] Both the consumer measures and total welfare are larger using an ad subsidy than allowing for ad collusion, even accounting for the fact that 30% of the ad subsidy is wasteful spending. The reason is that the social planner finds it optimal to choose an ad subsidy that further increases entry and decreases the subscription price compared to the ad collusion outcome.

if the positive effects from additional entry (namely, increased price competition and a market expansion and matching effect) outweigh the negative direct effect from a subscription price increase. Previous literature has solely focused on the latter (Kind et al., 2013; Belleflamme and Toulemonde, 2018; Foros et al., 2019). The theory makes predictions that are consistent regarding entry and subscription prices before and after the VAT exemption in Norway in 2016.

The calibrated model based on Norwegian data implies that the VAT reduction has increased the consumer measures and welfare; however, market entry is still inefficiently low from a social planner perspective. This suggests that policies aimed at increasing readership and supply of digital news are not in conflict with increasing economic welfare.

I also explore alternative policies to improve economic welfare and in turn readership and supply of news. By accounting for the two-sidedness of the newspaper market, the emerging theme of the most efficient policies is to strengthen newspapers' revenue from advertisements. This is notably achieved through an advertising subsidy or allowing newspapers to collude on ad prices.

Further insights regarding the impact of a VAT reduction could be developed by extending the present framework. Endogenizing content quality via costly investment in for example newsgathering is particularly important for newspapers' ability to differentiate themselves vertically (Gentzkow and Shapiro, 2008; Germano, 2008; Germano and Meier, 2013). Allowing newspapers to differentiate horizontally (Anderson and Gabszewicz, 2006; Kind et al., 2013; Gentzkow et al., 2014) may uncover additional effects from the VAT reduction due to the two-sidedness of the market and the role played by multi-homing consumers (Anderson et al., 2018). Another extension worth pursuing is to enrich the stylized advertising model by explicitly allowing for targeted advertising.

Chapter 2

Long-Term Contracts and Entry Deterrence in Two-Sided Markets

2.1 Introduction

"The body of academic knowledge [on two-sided markets] falls short of providing practical advice to antitrust enforcement agencies and courts." "Economists [...] should continue to engage in the debate about how to ascribe meaning to harm competition."

— Katz (2019)

The prevalence of prominent platforms, such as FANG mentioned in the Introduction to this dissertation, has in recent years heightened the antitrust attention that they receive.[1] While important insights for antitrust policy have been developed (Evans and Schmalensee, 2015), scholars have recently argued that modern antitrust is ill-equipped to address ways in which platforms can harm competition (Hovenkamp, 2018; Katz, 2018, 2019). For instance, these authors argue that the US Supreme Court's recent decision in *American Express* in essence illustrates a misapplication of sound antitrust policy; hence the prompt for the development of providing advice to enforcement agencies and courts.[2]

[1]Industry examples of platforms which have been subject to antitrust enforcement include newspapers (*Lorain Journal Co. v. United States*, 342 US 143 (1951); *Times Herald Printing Co. v. A.H. Belo Corp*, 820 S.W.2d 206 (1991)), a floral delivery platform (*United States v. Florist's Telegraph Delivery Ass'n*, 1956 Trade Cas. (CCH) *P* 68,367 (E.D. Mich. 1956)) , a computer operating system (*United States v. Microsoft Corp.*, 253F.3d 34, D.C. (Cir. 2001)), and credit card networks (*United States v. Visa U.S. A., Inc.*, 344F.3d 229 (2d Cir. 2003); *United States v. American Express Co.*, 88F. Supp. 3d 143 (E.D.N.Y. 2015), rev'd, 838F.3d 179 (2d Cir. 2016), *rev'd sub nom. Ohio v. American Express Co.*, 138 S. Ct. 2274 (2018)).

[2]*Ohio v. American Express Co.*, 138 S. Ct. 2274 (2018). In a comment on the court's decision,

The difficulty of developing antitrust recommendations in two-sided markets can be attributed to the role played by externalities: every agent, by subscribing to a platform, generates value through network effects for some other agents on the platform. The use of "predatory pricing" illustrates. In the absence of externalities, the standard price-below-marginal-cost test, the Areeda-Turner rule, is a straightforward approach to investigate predation in traditional one-sided markets.[3] In a two-sided market, however, a platform has to take into account the externalities between the two sides in setting its optimal fees. Setting the fee for one group below cost may be the only way for a platform to get "both sides on board."[4] Thus, a fee below marginal cost on one side cannot be used as a benchmark for two-sided platforms when evaluating predatory pricing to drive out competition (Behringer and Filistrucchi, 2015a; Evans and Schmalensee, 2015).[5]

In this chapter, I study the incentives of a platform to employ an instrument that thus far has not received any theoretical attention as a potentially anti-competitive device in two-sided markets: the requirement by a platform for sellers to sign a *long-term contract*. Long-term contracts are ubiquitous in two-sided markets. Among others, stores sign long-term rent contracts in shopping malls, credit card companies require merchants to sign up for a long-term contract for the processing and settlement of payment transactions, newspapers offer long-term contract rates

Wu (2018) writes that "Justice Stephen Breyer's dissent mocks the majority's economic reasoning, as will most economists, including the creators of the "two-sided markets" theory on which the court relied. The court used academic citations in the worst way possible - to take a pass on reality."

[3]U.S. courts apply a two-part test for predation: "First, a plaintiff seeking to establish competitive injury resulting from a rival's low prices must prove that the price complained of are below an appropriate measure of its rival's costs. [...] The second prerequisite [...] is a demonstration that the competitor had a reasonable prospect, or [...] dangerous probability, of recouping its investment in below-cost prices." (Brooke Group, Ltd. v. Brown & Williamson Tobacco Corporation, 509 U.S. 209, 222 (1993)). In the European Union, the price-cost approach states that "(a) if the price is below average variable costs, then there is a presumption of predatory pricing that the defendant can then attempt to rebut, and (b) if the price is above average variable cost but below average total cost, then the plaintiff must establish that the pricing is intended to eliminate competitors." (Case C-62/86 AKZO Chemie v Commission, [1991] ECR I-3359; Case C-333/94 P Tetra Pak International v. Commission [1996] ECR I-5951)

[4]Caillaud and Jullien (2003) coined the "divide-and-conquer" term, according to which a platform's optimal strategy in setting prices is to subsidize the participation of one group (divide), but then recover the loss from charging more the other side (conquer).

[5]For example, the U.K. Office of Fair Trading investigated in 1994 the alleged predatory pricing behavior by *The Times* of London after observing a sharp decrease in subscription prices. It is now suggested that, rather than being an episode of predation, *The Times* was the first newspaper to recognize that it was profitable to reduce subscription prices to increase readership, which in turn allowed the newspaper to extract more from advertisers (Behringer and Filistrucchi, 2015a).

for advertisers, video streaming services such as Netflix have licensing deals with studios to stream their content for a set period (typically one, three, or five years) or into perpetuity, Spotify uses licensing deals with record labels and distributors for several years, video game console producers have long-term publishing deals with popular game developers like Electronic Arts, and Amazon negotiates long-term contracts with book publishing companies.

Putting the costs of renegotiating contracts aside, it appears at the outset that two-sided market environments are especially susceptible to the strategic use of long-term contracts, given the intense competition between platforms to get both sides on board due to externalities. By preventing that the seller side goes to a rival platform in the next period, a platform ensures that it is unattractive for the consumer side to go to the other platform in the next period.

I consider a two-period model in which an incumbent platform I operates as a monopolist in the first period. Consumers, sellers, and I know that a more cost-efficient entrant platform E appears at the start of the second period. In the baseline model, E and I are homogeneous in the eyes of consumers and sellers. Cross-group externalities are present, i.e., consumers value the presence of sellers when making their platform adaption decision and vice versa. Each side pays a fee to each platform it visits in each period. I and E compete in fees in the second period. At the beginning of the first period, I decides whether it wants to incentivize sellers to sign a two-period contract to get the sellers on board for both periods.[6]

The set-up allows to study the question under which agents' *homing* settings I can profitably use a long-term seller contract as an anti-competitive instrument to enhance its position when competing with E. The agents from a given side are assumed to be single-homing agents if they can join only one platform, but are multi-homing agents if they can join both platforms.

Absent a long-term contract, E holds the upper hand in the fee competition for the sides due to its cost advantage: E can make more aggressive fee offers under any homing assumption and hence is more successful than I in attracting the agent sides in the second period. If for instance one side of the market single-homes and the other multi-homes, I and E compete intensely in fees for the single-homing side (Armstrong, 2006). Having the single-homing side on board allows a platform to

[6] In all of the above examples, sellers sign long-term contracts. The assumption that sellers sign long-term contracts in the model is without loss of generality. A platform example where consumers sign a long-term contract is an internet service provider (ISP), which connects content providers to internet users.

extract more surplus from the other side due to cross-group externalities.

The incentive for I to make use of a long-term seller contract is driven by the following trade-off: on the one hand, I has to compensate the sellers to voluntarily sign the long-term contract since sellers pay a lower fee on E than on I in the second period absent a contract. On the other hand, I becomes more attractive for consumers in the second period, having ensured that sellers are present in that period. This allows I to extract more from the consumer surplus. The presence and magnitude of the cross-group externalities and the homing assumption for each side crucially determine the net effect of this trade-off and in turn I's decision to make use of a long-term contract.

In particular, I find that the Chicago critique holds in the absence of cross-group externalities, an environment resembling two distinct one-sided markets. An agent would not sign a long-term contract that reduces competition unless she is fully compensated for doing so, but precisely I cannot afford this compensation because E is more efficient.

If cross-group externalities are present and consumers multi-home and sellers single-home, sellers are subsidized in the absence of a long-term contract but the consumers' surplus is fully extracted by E in the second period. A long-term contract is still not beneficial for I. On the one hand, I benefits by being able to extract the full consumer surplus in the second period, which I would not have been able to do absent a contract. However, this benefit is exactly offset by I having to compensate the sellers' forgone subsidy from E in the second period to incentivize sellers to accept the contract in the first place.

If both sides single-home, I might find it beneficial to use a long-term contract. The reason is that in the absence of a contract, now the sellers are less of a bottleneck in the competition between the two platforms, resulting in a lower subsidy for sellers by E. In turn, I has to compensate the sellers by a smaller amount to convince them to sign the contract, which in turn makes it more lucrative for I to create a long-term contract in the first period. If a long-term contract is in place, the market outcome from a welfare perspective is inefficient since the cost-efficient platform E is (partially) excluded in the second period. Thus, the Chicago critique may fail to hold in my set-up when cross-group externalities are present and when both sides single-home.

If consumers single-home and sellers multi-home, I is strictly better off using a long-term contract that is *exclusive*, i.e., the contract includes a clause that forbids the sellers from also going to E in the second period, and if cross-group externalities are sufficiently large and I's cost disadvantage small. Conversely to the case where sellers single-home and consumers multi-home, now the sellers' surplus is fully extracted in the second period absent a long-term contract. This plays in favor of I for the incentives to create a long-term contract by not having to compensate sellers for any surplus visiting E; I can extract the entire two-period seller surplus under a long-term contract. If instead the long-term contract is *non-exclusive*, i.e., sellers are still allowed to go to E in the second period even if they signed the contract, I is not strictly better off using such a contract. The reason is that E still attracts the consumers in the second period even under the contract, which is why in turn I does not gain a positive net benefit from either side.

Background and Related Literature

In traditional one-sided markets, the "Chicago School View" (CSV) would argue that efficiency gains are all that matter when evaluating a long-term contract: if a long-term contract is observed to be in place, such a contract must be efficient because an agent would not have voluntarily signed the contract if she is not better off (Posner, 1976; Bork, 1978). That is, if the agent is not compensated for the loss of not having access to other firms in the second period, she will not accept such a contract in the first place.

Subsequent post-Chicago models show that the CSV fails to hold in a variety of traditional one-sided market settings pertaining to exclusivity; when the entrant's cost is unknown to all agents other than the entrant (Aghion and Bolton, 1987), when buyers fail to coordinate (Rasmusen et al., 1991; Segal and Whinston, 2000; Fumagalli and Motta, 2008), when agents are downstream retail competitors (Asker and Bar-Isaac, 2014), and when an upstream firm has a strong competitive advantage and marginal prices are distorted (Calzolari et al., 2020).[7] I abstract from such considerations. Instead, I focus on the role played by externalities and homing assumptions, two integral characteristics in two-sided markets, for the incentive to use a long-term contract.

Crucially, the CSV implicitly assumes that all parties affected by an (exclusive) agreement bargain in a frictionless and efficient manner (Katz, 2018). This assumption is unlikely to be satisfied in general in two-sided markets and the reason why the CSV may fail to hold in the present set-up: if one side bargains with the platform

[7] Ide et al. (2016) study how loyalty discounts and Calzolari and Denicolò (2013) study how market-share discounts may induce exclusivity.

about a long-term contract, the outcome of that bargaining affects the welfare of the other side that did not participate in the bargaining due to cross-group externalities.

Long-term contracts have recently received attention in one-sided markets.[8] Gavin and Ross (2018) study long-term contracts as barriers to entry in a model of differentiated products. As in the present model, an incumbent operates for a period in the market before an entrant arrives. In a related paper, Bedre-Defolie and Biglaiser (2017) study contracts with breakup fees used by an incumbent facing a more efficient entrant in the future; while the products are homogeneous, consumers differ in their switching costs. In contrast to the present chapter, these papers correspond to one-sided markets where externalities play no role.

To the best of my knowledge, the present chapter is the first to study whether the use of a long-term contract by an incumbent platform in a two-sided market is an anti-competitive instrument to (partially) foreclose entry by a more efficient platform. To be clear, long-term contracts are not necessarily exclusive. If sellers can multi-home, sellers may still go in addition to E in the second period even after signing a long-term contract with I in the first period. I therefore distinguish between *exclusive* and *non-exclusive* long-term contracts if sellers can multi-home. The former contains an explicit clause forbidding sellers to go to E in the second period. If on the other hand sellers can only single-home, signing a long-term contract with I implies exclusivity for the second period.

The incentives to create a long-term contract depend crucially on the homing possibility of each side in my set-up. Armstrong (2006, pp. 669-670) succinctly highlights the impact of multi-homing agents in two-sided markets, i.e., agents that can join several platforms: "If it wishes to interact with an agent on the single-homing side, the multi-homing side has no choice but to deal with the single-homing agent's chosen platform. Thus, platforms have monopoly power over providing access to their single-homing customers for the multi-homing side, [which] leads to high prices being charged to the multi-homing side. By contrast, platforms do have to compete for the single-homing agents, and high profits generated from the multi-homing side are to a large extent passed on to the single-homing side in the form of low prices (or even zero prices)." Several papers study the competitive effects of multi-homing in two-sided markets (see, for instance, Belleflamme and Peitz, 2019b; Anderson and Peitz, 2020, and references therein), but they do not consider long-term contracts.

[8] In an early principal-agent model of multiple periods, Lambert (1983) demonstrates that when principals and agents engage in a long-term relationship, the principal can diversify away some uncertainty regarding the agent's actions.

The chapter is indirectly related to other potentially abusive practices to deter entry in two-sided markets: exclusionary predatory pricing (Vasconcelos, 2015; Amelio et al., 2020), exclusive dealing (Armstrong and Wright, 2007; Hagiu and Lee, 2011; Lee, 2013; Shekhar, 2017; Carroni et al., 2020),[9] and mergers (see Correia-da Silva et al., 2019, for a progress report). The present chapter emphasizes the long-term "nature" of contracts and not simply the exclusivity they might entail. In particular, I may exploit its first-period-presence advantage to foreclose entry in a way not considered previously in the two-sided market literature.

The rest of the chapter is organized as follows. I set up the model in Section 2.2. In Section 2.3 I first analyze the one-sided version of the model, i.e., cross-group externalities are absent, before investigating the two-sided model version under various homing assumptions. In Section 2.4 I look at various extensions of the baseline model. Section 2.5 concludes.

2.2 Model

I consider an abstract model of trade on platforms related to the early literature on two-sided markets (e.g., Armstrong, 2006; Armstrong and Wright, 2007; Caillaud and Jullien, 2003; Hagiu, 2006; Rochet and Tirole, 2003, 2006; Weyl, 2010). In particular, my model's basic ingredients are that of Armstrong (2006) without differentiation costs. While the early literature examines a static one-period set-up, I consider two periods to study the idea of "long-term" contracts.

Suppose there are two groups of agents, labeled $j = c$ and s, that interact with each other via platforms in each period. I will refer to the group c agents as consumers and s agents as sellers. Let N^c and N^s be the measure of consumers and sellers, respectively. Each agent values the number of agents on the other side that visits the same platform, but not the number of agents on the same side.[10] An agent from group j obtains benefit $\alpha^j g\left(N^{-j}\right)$ by visiting a platform that allows her to interact with N^{-j} agents from the other group, where $\alpha^j \geq 0$ measures the intensity of the cross-group externality and $g'(\cdot) \geq 0$. For simplicity of exposition, I assume that $g\left(N^{-j}\right) = N^{-j}$. In addition, each agent receives a stand-alone value from visiting each platform and in each period; v^c in the case of consumers and v^s

[9]For exclusive dealing in one-sided markets with network effects, see Doganoglu and Wright (2010), Karlinger and Motta (2012), and Hermalin and Katz (2013).

[10]See Belleflamme and Toulemonde (2016) and references within for studies modeling intra-group externalities in two-sided markets.

in the case of sellers.

In the first period, only one platform, the incumbent platform I, exists. In the second period, an entrant platform E arrives. I do not impose a fixed cost on E for entering. In the absence of a contract and depending on the specification, each agent side can join only either I or E if they single-home, or both if they multi-home. Platform $k = I, E$ sets a subscription fee $f^j_{k,t}$ to group j agents in each period $t = 1, 2$ it is present.[11]

I also decides in the first period whether or not to create a long-term contract, without loss of generality, for sellers. If I does not create a long-term contract, it operates as a monopolist in the first period but competes in fees against E in the second period. If I creates a long-term contract and sellers voluntarily sign, the sellers are bound to go to I in both periods. Furthermore, if sellers can only single-home a long-term contract is inherently exclusive, because if sellers sign the contract they cannot consider going to E in the second period. If instead sellers can multi-home, I investigate both *exclusive* and *non-exclusive* long-term contract cases (see Section 2.3.4). A seller cannot breach the contract after signing it.

Each platform incurs a fixed cost by hosting an agent on its platform in each period. I assume that $c_I > c_E \geq 0$. That is, E is the more efficient platform and the cost of serving each side is identical for platform k. I assume that each group agent can switch from I in the first period to E in the second period without incurring any costs, so that N^c and N^s can be thought of as the same set of agents that are present across the two periods. The utility of a side j agent from joining platform k in period t is given by

$$u^j_{k,t} = v^j + \alpha^j N^{-j}_{k,t} - f^j_{k,t}, \tag{2.1}$$

where $N^{-j}_{k,t} \leq N^{-j}$ is the measure of agents from group $-j$. Platforms are perceived as homogeneous by sellers and consumers (Caillaud and Jullien, 2003; Armstrong and Wright, 2007). The choice not to introduce some form of heterogeneity is deliberate to emphasize in a clear analytic way the role played by cross-group externalities. In Section, 2.4.3 I introduce heterogeneity on the consumer side in the Hotelling manner (Armstrong, 2006). I assume that both sellers and consumers have responsive expectations, i.e., each side takes into account the change of the demand configuration of the other side following a change in that side's fee.

I use the following tie-breaking rules: if an agent can only single-home and is indifferent between going to I and E, she goes to the efficient platform E. If a seller

[11] Per-transaction instead of fixed fees do not qualitatively change the results.

is indifferent between signing and not signing a long-term contract, she signs the contract.

Platform k's profit in period t is

$$\Pi_{k,t} = \left(f_{k,t}^{c} - c_k\right) N_{k,t}^{c} + \left(f_{k,t}^{s} - c_k\right) N_{k,t}^{s} \tag{2.2}$$

In the analysis sections, I add nlt and lt to the superscripts in the fee and profit expressions to distinguish, respectively, between the non-long-term and long-term contract outcomes. A bar above the seller fee, i.e., $\bar{f}_I^s$, denotes the one-time seller fee under a long-term contract.[12] I make the following viability assumption:

Assumption 2.1. $\min\{v^s, v^c\} > c_I$.

Assumption 2.1 ensures that I wants to operate even if cross-group externalities are absent, i.e., $\alpha^s = \alpha^c = 0$, or if I can solely attract agents from one side.

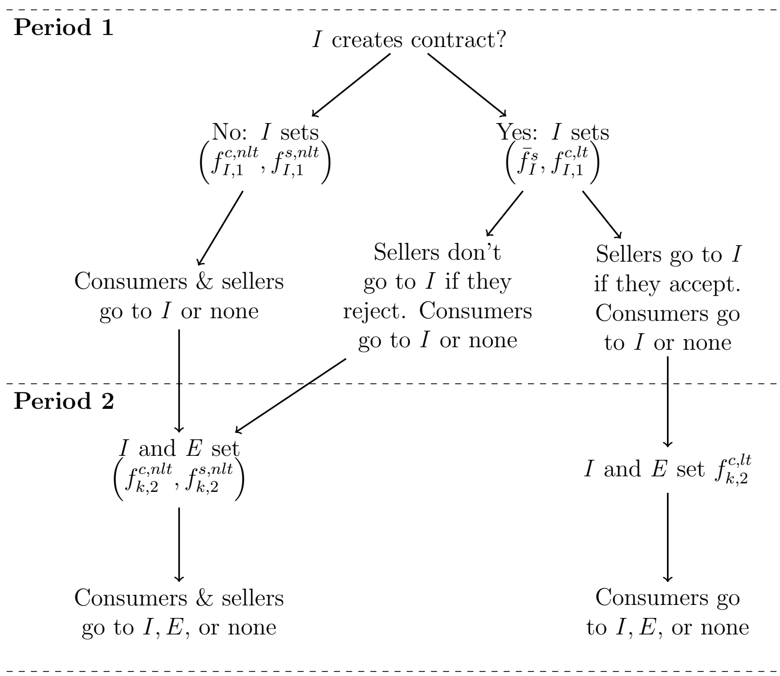
An assumption has to be made about I's ability to commit to refuse to let sellers, who declined the offered contract, visit I. In the main analysis section, I make an intermediate commitment assumption (as in Gavin and Ross, 2018). If sellers do not sign the long-term contract in the first period, I does not allow the sellers to visit its platform in the first period but does allow them in the second period. That is, I forgoes its monopoly position in the first period concerning the sellers that do not sign the contract but competes against E to attract these sellers (and consumers) in the second period.

Alternatively in Section 2.4.1, I consider an extreme commitment assumption, according to which I commits not to let sellers visit its platform outside the long-term contract for both periods. This type of commitment resembles the tying literature where a platform can refuse to sell to one side if that side does not purchase the tied goods offered by the other side. The other extreme is to assume that I does not have any commitment power; if sellers refuse to sign the long-term contract, I allows the sellers to visit its platform in both periods.

Timing: In the first period, I first decides whether or not to offer a long-term contract to sellers. If I does not offer a contract, I sets the monopoly consumer and seller fees $\left(f_{I,1}^{c,nlt}, f_{I,1}^{s,nlt}\right)$. Consumers and sellers simultaneously decide whether or not to visit I and payoffs are realized. If I offers a long-term contract to sellers it

[12]Equivalently, the total fee $\bar{f}_I^s$ could be distributed throughout the two periods.

Figure 2.1: Timing under intermediate commitment
and two-sided single-homing assumption



sets $\bar{f}_I^s$ and the first period consumer fee $f_{I,1}^{c,lt}$. Note that since sellers view platforms as homogeneous, either all or none of the N^s sellers sign the contract.

In the second period, each platform $k = I, E$ sets a consumer fee. If I did not offer a long-term contract or I offered a contract but sellers did not sign it in the first period, each platform sets in addition and simultaneously the seller fee $f_{k,2}^{s,nlt}$. If instead sellers signed a long-term contract in the first period, E sets a seller fee $f_{E,2}^{s,lt}$ only if sellers can multi-home and the contract is non-exclusive. Consumers (and sellers) simultaneously make their adaptation decisions and payoffs are realized.

Figure 2.1 depicts the timing under the intermediate commitment and two-sided single-homing assumption.

2.3 Analysis

2.3.1 One-sided market

In this subsection, I illustrate that the CSV holds in the absence of cross-group externalities. Let $\alpha^c = \alpha^s = 0$, i.e., each platform caters to two distinct and independent sides. One can think of this scenario that an active platform serves two separate one-sided markets. The agent's payoff given in (2.1) reduces to $u^j_{k,t} = v^j - f^j_{k,t}\ \forall t = 1, 2;\ j = c, s;\ k = I, E$.

Due to the independence across the sides, the participation decision by sellers bears no impact on the consumer decision and vice versa. Hence, a long-term contract would only affect the seller but not the consumer side. Therefore, I's incentive to create a long-term contract is solely based on how to make the maximum profit from sellers, and it is irrelevant whether consumers single-home or multi-home.

Furthermore, the analytically uninteresting case in absence of cross-group externalities is when sellers can multi-home: I can already extract the sellers' stand-alone benefit v^s in each period absent a long-term contract, and hence a contract cannot make I better off. Thus, assume that sellers single-home.

Suppose I does not offer a long-term contract in the first period. Then, I extracts the full seller surplus v^s in the first period. In the second period, the sellers go to the platform that offers the largest payoff. That is, each seller goes to platform k if and only if $v^s - f^{s,nlt}_{k,2} > v^s - f^{s,nlt}_{-k,2}$. By Bertrand competition and $c_I > c_E$ by assumption, the largest $f^{s,nlt}_{E,2}$ that E can set to successfully enter, and in turn keep out I, is such that $f^{s,nlt^*}_{E,2} = c_I$. The two-period profits for I and E are, respectively, $\Pi^{s,nlt^*}_{I,1} = (v^s - c_I)N^s$ and $\Pi^{s,nlt^*}_{E,2} = (c_I - c_E)N^s$.

Proposition 2.1. *If $\alpha^c = \alpha^s = 0$, I has no strict incentive to create a long-term contract, independent of the agents' homing assumption.*

Proof. From the text we know that the consumers' homing assumption is irrelevant and that I cannot be strictly better off in terms of profit if sellers multi-home. It thus remains to show that I cannot be strictly better off if sellers single-home. If sellers single-home, I has to set the two-period long-term contract fee $\bar{f}^s_I$ such that sellers are willing to accept the long-term contract (the sellers' incentive compatibility constraint):

$$2v^s - \bar{f}^s_I \geq \max\left\{v^s - f^{s,nlt}_{I,2}, v^s - f^{s,nlt}_{E,2}\right\}, \tag{2.3}$$

where the left-hand side is the seller's two-period payoff, twice v^s minus the fee $\bar{f}^s_I$, of signing the contract and hence committing to visit I in both periods. If I offers but the sellers do not accept the contract, I is committed not to sell to the sellers in the first period, and hence the sellers only can visit either I or E in the second period, with the payoffs depicted in the right-hand side of (2.3). Sellers know that if they do not sign the contract, E prevails in the competition between I and E in the second period. Thus, (2.3) reduces to $2v^s - \bar{f}^s_I \geq v^s - f^{s,nlt^*}_{E,2}$, from which it immediately follows that the largest $\bar{f}_I^s$ that I can charge sellers to convince them to sign the long-term contract is $\bar{f}_I^{s^*} = v^s + c_I$. Then, I's long-term contract profit from sellers is $\Pi_I^{s,lt^*} = \left(\bar{f}_I^{s^*} - 2c_I\right) N^s = (v^s - c_I)\, N^s$. But $\Pi_I^{s,lt^*} = \Pi_I^{s,nlt^*}$, and therefore I is not strictly better off creating a long-term contract. □

Intuitively, by using a long-term contract I has to offer the sellers a fee below $f_{E,2}^{s,nlt^*}$, but in addition can charge them v^s for having access to I in the first period. Since $f_{E,2}^{s,nlt^*} = c_I$ is the fee by E to keep out I in the second period in absence of a long-term contract, the benefit for I that sellers also visit I under a long-term contract in the second period cancels out with its cost of hosting the sellers in the second period, c_I. Thus in effect I still only reaps the benefit of sellers in the first period. Hence, I is not strictly better off creating a long-term contract.

That is, I cannot improve its profits under a long-term contract, which reduces competition, because it has to compensate the sellers for the loss of not having access to E in the second period. Precisely this compensation is not profit-enhancing for I because E is the more efficient platform. The CSV holds for $\alpha^c = \alpha^s = 0$ under any homing assumption.

2.3.2 Sellers single-home and consumers multi-home

I now re-institute cross-group externalities, i.e., $\alpha^c, \alpha^s \neq 0$, and assume that sellers single-home and consumers multi-home. Fitting two-sided market examples are internet service providers (see footnote 6) and arguably ticket marketplaces (Ticketmaster, Live Nation) and ride-hailing services (Uber, Lyft).

The following Lemma 2.1 states that E successfully attracts the consumers and the sellers in the second period in the absence of a long-term contract; nonetheless, the consumers also go to I for the stand-alone benefit.

Lemma 2.1. *If $\alpha^c, \alpha^s \neq 0$ and sellers single-home and consumers multi-home, then in the absence of a long-term contract all sellers go to E and consumers go to I and E in the second period in equilibrium. Equilibrium fees are*

$$f_{I,2}^{c,nlt^*} = v^c, f_{E,2}^{c,nlt^*} = v^c + \alpha^c N^s, f_{I,2}^{s,nlt^*} = f_{E,2}^{s,nlt^*} = c_I - \alpha^c N^c, \tag{2.4}$$

and platform profits are

$$\Pi_{I,2}^{nlt^*} = (v^c - c_I)\, N^c, \Pi_{E,2}^{nlt^*} = (v^c - c_E)\, N^c + (c_I - c_E)\, N^s \tag{2.5}$$

Proof. Suppose I did not offer or the sellers did not sign the long-term contract in the first period. Since consumers are allowed to multi-home, I always has the option to extract the stand-alone benefit consumers derive from visiting a platform without being able to interact with sellers. That is, I can always only attract consumers by setting $f_{I,2}^{c,nlt} = v^c$ and any seller fee such that $f_{I,2}^{s,nlt} > f_{E,2}^{s,nlt}$. Then, $\Pi_{I,2}^{nlt} = (v^c - c_I)\, N^c$.

The alternative for I is to compete against E for the "bottleneck" sellers. If successful, I in turn can also extract the cross-group benefits from the consumers. That is, I faces the following maximization problem in the second period if it wants to compete against E not only to attract the consumers but both sides:

$$\max_{f_{I,2}^{s,nlt}, f_{I,2}^{c,nlt}} \Pi_{I,2}^{nlt} = \max_{f_{I,2}^{s,nlt}, f_{I,2}^{c,nlt}} \left\{ \left(f_{I,2}^{c,nlt} - c_I \right) N^c + \left(f_{I,2}^{s,nlt} - c_I \right) N^s \right\} \geq (v^c - c_I)\, N^c$$

$$\text{s.t.} \quad v^c + \alpha^c N^s - f_{I,2}^{c,nlt} \geq 0, \tag{2.6}$$

$$v^s + \alpha^s N^c - f_{I,2}^{s,nlt} > v^s + \alpha^s N^c - f_{E,2}^{s,nlt}, \tag{2.7}$$

where (2.6) is the consumer participation constraint and (2.7) the sellers' incentive compatibility constraint. Clearly, (2.6) is binding. Then, $\Pi_{I,2}^{nlt}\left(f_{I,2}^{c,nlt}, f_{I,2}^{s,nlt}\right) \geq (v^c - c_I)\, N^c \iff f_{I,2}^{s,nlt} \geq c_I - \alpha^c N^c$ conditional on (2.7) being satisfied. But by Bertrand competition and the *efficient* tie-breaking rule, E sets $f_{E,2}^{s,nlt^*}$ equal to $f_{I,2}^{s,nlt^*} = c_I - \alpha^c N^c$ and consequently attracts all sellers. E can then charge $f_{E,2}^{c,nlt^*} = v^c + \alpha^c N^s$ on the consumer side and extract their full surplus.

E indeed is not only able to, but also has the incentive to undercut $f_{I,2}^{s,nlt^*}$ because

$$\begin{aligned}
\Pi_{E,2}^{nlt^*}\left(f_{E,2}^{c,nlt^*}, f_{E,2}^{s,nlt^*}\right) &= \left(f_{E,2}^{c,nlt^*} - c_E\right) N^c + \left(f_{E,2}^{s,nlt^*} - c_E\right) N^s \\
&= (v^c - c_E)\, N^c + (c_I - c_E)\, N^s \\
&> (v^c - c_E)\, N^c > 0,
\end{aligned}$$

where the first inequality states that E is strictly better off attracting the sellers and extracting the consumers' stand-alone and cross-group externality benefit rather than solely extracting the consumers' stand-alone benefit, as I does. The first inequality statement holds by the assumption that $c_I > c_E$ and the second inequality follows from Assumption 2.1. □

The equilibrium outcome is that of a "competitive bottleneck" (Armstrong, 2006): if one side (here consumers) multi-homes and the other side (the sellers) single-homes, the platforms I and E compete fiercely for the side that single-homes. Conditional on the demand from sellers, platforms do not compete to attract consumers. Consequently, E charges sellers a relatively small fee ($f_{E,2}^{s,nlt^*} - c_E = c_I - c_E - \alpha^c N^c$, which can be negative if the cross-group externalities are sufficiently strong), but makes its profit from consumers; the consumer's rent is fully extracted ($f_{E,2}^{c,nlt^*} = v^c + \alpha^c N^s$). E prevails in the competition for sellers. Because of its cost advantage, it can subsidize the sellers to a larger extent than I can.

Given that I can only extract the consumers' stand-alone benefit in the second period if I did not create a long-term contract, the question is whether I can be strictly better off over the two periods by creating a long-term contract in the first period. Proposition 2.2 answers this question with a definite "no".

Proposition 2.2. *If $\alpha^c, \alpha^s \neq 0$ and sellers single-home and consumers can multi-home, I has no strict incentive to create a long-term contract.*

Proof. I has to set the two-period long-term contract fee $\bar{f}_I^s$ such that sellers are willing to accept the long-term contract. The sellers' incentive compatibility constraint is

$$2v^s + 2\alpha^s N^c - \bar{f}_I^s \geq v^s + \alpha^s N^c - f_{E,2}^{s,nlt^*}, \tag{2.8}$$

where $f_{E,2}^{s,nlt^*}$ is given by (2.4). The left-hand side is the sellers' two-period payoff, twice v^s and $\alpha^s N^c$ minus the fee $\bar{f}_I^s$, by signing the contract and hence committing to visit I in both periods (the sellers correctly anticipate that I attracts the consumers in the second period under a long-term contract). If I offers a contract that sellers do not accept, I is committed not to sell to the sellers in the first period. Then, the sellers forgo any first-period payoff but get the second-period payoff from visiting E. Thus, by (2.8) the largest $\bar{f}_I^s$ that I can charge sellers in order to convince them to sign is $\bar{f}_I^{s^*} = v^s + \alpha^s N^c + f_{E,2}^{s,nlt^*}$.

Therefore, I's long-term contract profit

$$\begin{aligned}\Pi_I^{lt^*} &= \left(f_{I,1}^{c,lt^*} - c_I\right) N^c + \left(f_{I,2}^{c,lt^*} - c_I\right) N^c + \left(\bar{f}_I^{s^*} - 2c_I\right) N^s \\ &= 2\left(v^c + \alpha^c N^s - c_I\right) N^c + \left(v^s + \alpha^s N^c - \alpha^c N^c - c_I\right) N^s\end{aligned}$$

since $f_{I,1}^{c,lt^*} = f_{I,2}^{c,lt^*} = v^c + \alpha^c N^s$. But notice that $\Pi_I^{lt^*}$ is the same as I's two-period profit in the absence of a long-term contract: $\Pi_{I,1}^{nlt^*} + \Pi_{I,2}^{nlt^*} = \left(v^c + \alpha^c N^s - c_I\right) N^c + \left(v^s + \alpha^s N^c - c_I\right) N^s + \left(v^c - c_I\right) N^c$, since I extracts the full surplus from both sides as a monopolist in the first period and $\Pi_{I,2}^{nlt^*} = \left(v^c - c_I\right) N^c$ by Lemma 2.1. Thus,

$\Pi_I^{lt^*} - \Pi_{I,1}^{nlt^*} - \Pi_{I,2}^{nlt^*} = 0$ and, therefore, I is not strictly better off creating a long-term contract. Note that an alternative strategy for I to get the sellers to sign the contract, with the intention of not attracting the consumers in the second period, can never be profitable for I, because consumers can multi-home. □

On the one hand, it is costly on the seller side for I to implement a long-term contract. In particular, the largest fee that I can charge sellers to voluntarily sign the contract is $\bar{f}_I^{s^*} = v^s + \alpha^s N^c + f_{E,2}^{s,nlt^*}$ from the proof of Proposition 2.2. Notice that $f_{E,2}^{s,nlt^*} - c_I = -\alpha^c N^c < 0$, which is the amount each seller is charged below I's cost by E in the second period in the absence of a contract. I has to take on this "burden" to create a long-term contract.[13] Thus, I makes a total net loss of $-\alpha^c N^c N^s$ on the seller side by creating a long-term contract.

On the other hand, precisely by ensuring through a contract that sellers are on I in the second period, who otherwise would go to E (by Lemma 2.1), I can extract the full consumers' surplus in the second period, $(v^c + \alpha^c N^s) N^c$. In contrast and in absence of a contract, I can only extract the stand-alone surplus, $v^c N^c$. Note that I can fully extract the consumers' first-period surplus in either situation. Thus, I makes a total net gain of $\alpha^c N^s N^c$ on the consumer side by creating a long-term contract.

The sum of these two effects is zero. In short, the benefit for I being able to extract the full consumer surplus in the second period is exactly offset by I having to subsidize the sellers in the second period under a long-term contract.[14]

It is worth noting that because of the absence of a long-term contract, the equilibrium outcome is socially efficient as all sellers go to E, the cost-efficient plat-

[13]At the same time and because of the commitment by I not to let sellers visit its platform if they do not sign the contract, sellers have the additional first period $v^s + \alpha^s N^c$ surplus if they sign, which I can consequently extract. This is the same amount that I extracts from sellers in the absence of a contract.

[14]The result of Proposition 2.2 holds if one assumes instead that a platform has to attract both the consumer and seller side to operate successfully. Loosely speaking, this can be the case when both sides derive a low stand-alone benefit from visiting a platform. Due to the cost advantage, E attracts both sellers and consumers, whereas I does not attract either side and consequently makes zero profit. Then, Lemma 2.1 equilibrium fees and profits are adjusted to $f_{I,2}^{c^*} = f_{E,2}^{c^*} = v^c + \alpha^c N^s, f_{I,2}^{s^*} = f_{E,2}^{s^*} = c_I - \frac{N^c}{N^s}(v^c + \alpha^c N^s - c_I)$, and $\Pi_{I,2}^{nlt^*} = 0, \Pi_{E,2}^{nlt^*} = (c_I - c_E)(N^s + N^c)$. The crucial difference is that I now does not have the "outside" option of extracting solely the consumers' stand-alone benefit. Instead, I and E compete more fiercely for the bottleneck sellers, and hence $f_{E,2}^{s^*} = c_I - \frac{N^c}{N^s}(v^c + \alpha^c N^s - c_I) < c_I - \alpha^c N^c$. Thus, although I's benefit from having access to consumers in the second period now is larger (I can extract the full consumer surplus in the second period under a long-term contract, whereas I could not extract *any* consumer surplus in this modified version absent a contract), I has to subsidize sellers by a larger amount to incentivize them to sign the contract in the first place. As before the benefits cancel each other out.

form, in the second period. That is, the two-period total welfare differential is $W^{nlt^*} - W^{lt^*} = (c_I - c_E) N^s > 0$.[15] Since I would set a long-term contract fee that makes the sellers indifferent between their long-term contract net benefit and their benefit when there is no contract, sellers are clearly indifferent between the two outcomes: $u_{I,1}^{s,nlt} + u_{E,2}^{s,nlt} = u_I^{s,lt} = v^s + (\alpha^s + \alpha^c) N^c - c_I$. The multi-homing consumers' surplus is fully extracted in both periods, in the absence of a contract or not; $u_{I,1}^{c,nlt} + u_{I,2}^{c,nlt} + u_{E,2}^{c,nlt} = u_{I,1}^{c,lt} + u_{I,2}^{c,lt} + u_{E,2}^{c,lt} = 0$.

2.3.3 Two-sided single-homing

I now assume that each side single-homes. Two-sided market examples that roughly fit this homing context include the streaming of media (Netflix, Disney+), health maintenance organizations, and online dating services.

Lemma 2.2. *If $\alpha^c, \alpha^s \neq 0$ and sellers and consumers single-home, then in the absence of a long-term contract, in equilibrium all sellers and consumers go to E in the second period. E's equilibrium fees are characterized by*

$$\left(f_{E,2}^{c,nlt^*} - c_I\right) N^c + \left(f_{E,2}^{s,nlt^*} - c_I\right) N^s + |\alpha^c - \alpha^s| N^c N^s = 0 \tag{2.9}$$

$$s.t. \quad f_{E,2}^{c,nlt^*} \leq c_I + \alpha^c N^s \quad and \quad f_{E,2}^{s,nlt^*} \leq c_I + \alpha^s N^c \tag{2.10}$$

E's profits are $\Pi_{E,2}^{nlt^*} = (c_I - c_E)(N^c + N^s) - |\alpha^c - \alpha^s| N^s N^c$. I attracts no agents and obtains zero profit.

Proof. See Appendix B.1. □

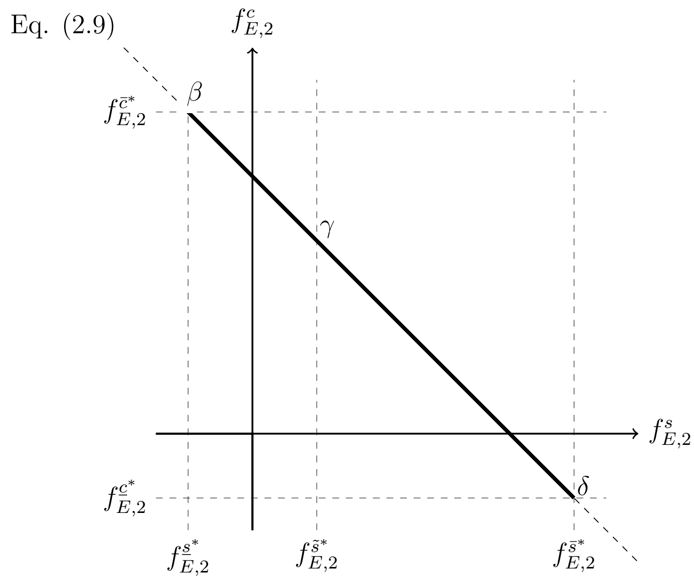
Lemma 2.2 states that E prevails in the competition with I for the consumer and seller side in the absence of a long-term contract due to its cost advantage. In contrast to Lemma 2.1, E's equilibrium fees are not unique. The $\beta\delta$ line in Figure 2.2 illustrates E's non-uniqueness of fees that satisfy (2.9) and (2.10).

Rather than exploiting one side and subsidizing the other side, E sets the fees such that the losing platform I cannot profitably undercut E's fee on one side and then extract more in fees from the other side to attract both sides. This is expressed

[15] In particular,

$$W^{nlt^*} = (3v^c + 2\alpha^c N^s - 2c_I - c_E) N^c + (2v^s + 2\alpha^s N^c - c_I - c_E) N^s \text{ and}$$
$$W^{lt^*} = (3v^c + 2\alpha^c N^s - 2c_I - c_E) N^c + 2(v^s + \alpha^s N^c - c_I) N^s$$

Figure 2.2: Illustration of Lemma 2.2 and Proposition 2.3



Note: The maximum fee $f_{E,2}^{\bar{j}^*}$ $\forall j = c, s$ follows from (2.10). The minimum fee $f_{E,2}^{\underline{j}^*}$ is obtained using $f_{E,2}^{\overline{-j}^*}$ in (2.9).

in (2.9). In addition, E has to set the fees such that I cannot attract only one side. This constraint is given by (2.10). Finally, E does not find it profitable to attract only one side: because of the threat by I to attract both sides, E would have to set a consumer fee so low to attract that side that it rather ensures to get both sides on board.

For the demand configuration in proving Lemma 2.2, I used a specific selection criterion that applied when a platform deviated. Armstrong and Wright (2007) label this refinement the *iterated best response condition*, according to which the seller and consumer side iteratively best respond to each other following a deviation in at least one fee by a platform from the equilibrium. The process is as follows. Following any fee deviation from the equilibrium, first, the sellers decide which platform to join, taking the consumers' decision as given. Second, the consumers decide which platform to visit, taking the sellers' decision as given. The process is repeated until a fixed point is reached. Agents on either side at each stage only change which platform to visit if they are strictly better off. The refinement provides a simple way to determine the demand configurations that support the equilibrium.[16]

[16]It can be readily verified that the equilibrium fees in Lemma 2.1 are robust to the iterated best response condition. First, observe that I would never want to challenge E on the seller side, i.e.,

Interestingly, both platforms are indifferent about which fee combination from (2.9) is chosen by E in the second stage when a long-term contract is absent: E's profit is independent of the fee combination and I stays out of the market and makes zero profit. However, I cares in the first period about which fee combination would be chosen by E in the absence of a contract in the second period. Proposition 2.3 formalizes.

Proposition 2.3. *If $\alpha^c, \alpha^s \neq 0$ and sellers and consumers single-home, I creates a long-term contract if I and the sellers expect that $f_{E,2}^{s,nlt^*} \in \left(f_{E,2}^{\tilde{s}^*}, f_{E,2}^{\bar{s}^*}\right]$, where $f_{E,2}^{\tilde{s}^*} = c_I - (c_E + \alpha^c N^s - c_I)\frac{N^c}{N^s}$ and $f_{E,2}^{\bar{s}^*} = c_I + \alpha^s N^c$, but does not create a long-term contract if $f_{E,2}^{s,nlt^*} \in \left[f_{E,2}^{\underline{s}^*}, f_{E,2}^{\tilde{s}^*}\right]$, where $f_{E,2}^{\underline{s}^*} = c_I - \alpha^c N^c - |\alpha^c - \alpha^s| N^c$.*

Proof. The initial steps are similar to the proof of Proposition 2.2. To implement a long-term contract, I has to set a two-period fee $\bar{f}_I^s$ that satisfies the sellers' incentive compatibility constraint (2.8), where now $f_{E,2}^{s,nlt^*}$ is given by (2.9). Thus, the largest $\bar{f}_I^s$ that I can charge sellers in order to convince the sellers to sign is $\bar{f}_I^{s^*} = v^s + \alpha^s N^c + f_{E,2}^{s,nlt^*}$.

From (2.10), the maximum fee E can charge sellers in the second period in the absence of a long-term contract is $f_{E,2}^{\bar{s}^*} = c_I + \alpha^s N^c$. The minimum seller fee, which is obtained using the maximum consumer fee $f_{E,2}^{\bar{c}^*} = c_I + \alpha^c N^s$ in (2.9), is $f_{E,2}^{\underline{s}^*} = c_I - \alpha^c N^c - |\alpha^c - \alpha^s| N^c$. The multiplicity of E's fees causes a coordination problem. If I creates a long-term contract, E's second period fees are never actually observed, and hence the sellers have to form some kind of expectation in the first period (when they have to decide whether to sign the long-term contract or not) about $f_{E,2}^{s,nlt^*}$.

If the sellers signed the long-term contract and are therefore bound to go to I in the second period, E can still compete for consumers' for their stand-alone value. For that reason E sets $f_{E,2}^{c,lt^*} = c_E$, and I sets $f_{I,2}^{c,lt^*} = c_E + \alpha^c N^s$. I creates a long-term contract that sellers voluntarily sign if and only if

$$
\begin{aligned}
\Pi_I^{lt^*} - \Pi_{I,1}^{nlt^*} &= \left(f_{I,2}^{c,lt^*} - c_I\right) N^c + \left(\bar{f}_I^{s^*} - 2c_I\right) N^s - \left(f_{I,1}^{s,nlt^*} - c_I\right) N^s \\
&= (c_E + \alpha^c N^s - c_I) N^c + \left(f_{E,2}^{s,nlt^*} - c_I\right) N^s > 0 \\
\iff f_{E,2}^{s,nlt^*} &> f_{E,2}^{\tilde{s}^*} \triangleq c_I - (c_E + \alpha^c N^s - c_I)\frac{N^c}{N^s}
\end{aligned}
$$

---

charge sellers less than E does, because it cannot recoup that loss from the consumer side, as the consumers' surplus is already being fully extracted. Second, because of the consumers' possibility to multi-home, it is never an optimal strategy to undercut E on the consumer side.

Altogether, I creates long-term contract if I and the sellers expect that $f_{E,2}^{s,nlt^*} \in (f_{E,2}^{\tilde{s}^*}, f_{E,2}^{\bar{s}^*}]$, but does not create a long-term contract if $f_{E,2}^{s^*} \in [f_{E,2}^{\underline{s}^*}, f_{E,2}^{\tilde{s}^*}]$. $\square$

The reason that I is not indifferent in the first period about which fee combination E chooses in the second period absent a contract is that I has to anticipate how much it has to compensate the sellers to forgo the opportunity to visit E in the second period. The higher I and the sellers expect E to set $f_{E,2}^{s,nlt^*}$, the larger can be the long-term contract seller fee $\bar{f}_I^s$ chosen by I. Since the attractiveness of a long-term contract for I stems from the trade-off between extracting the consumers' second-period cross-group externality surplus and having to compensate the sellers for forgoing the opportunity to go to E in the second period, a lower expected $f_{E,2}^{s,nlt^*}$ unambiguously raises I's profitability using a long-term contract. In particular, if $f_{E,2}^{s,nlt^*} \in \left(f_{E,2}^{\tilde{s}^*}, f_{E,2}^{\bar{s}^*}\right]$ I creates a long-term contract. The $\gamma\delta$ line in Figure 2.2 illustrates.

If in contrast I and the sellers expect that E would charge a sufficiently low $f_{E,2}^{s,nlt^*}$ absent a contract, I also has to set a sufficiently low long-term contract fee $\bar{f}_I^s$ to incentivize sellers to sign it, which in turn makes it less lucrative for I to offer the contract in the first place. This case corresponds to when $f_{E,2}^{s,nlt^*} \in \left[f_{E,2}^{\underline{s}^*}, f_{E,2}^{\tilde{s}^*}\right]$, as demonstrated by the $\beta\gamma$ line in Figure 2.2.

Note that the range of expected $f_{E,2}^{s,nlt^*}$ following which I creates a long-term contract is non-empty if and only if $f_{E,2}^{\bar{s}^*} > f_{E,2}^{\tilde{s}^*} \iff (\alpha^c + \alpha^s) N^s > c_I - c_E$; the cross-group externalities have to be sufficiently large compared to I's cost disadvantage. The range of expected $f_{E,2}^{s,nlt^*}$ following which I does not create a long-term contract is non-empty, because $f_{E,2}^{\underline{s}^*} < f_{E,2}^{\tilde{s}^*} \iff -|\alpha^c - \alpha^s| < \frac{(c_I - c_E)}{N^s}$, which is true by the assumption that $c_I > c_E$.

In contrast to Proposition 2.2, the equilibrium outcome now is socially inefficient whenever I uses a contract because I instead of the efficient platform E operates in the second period. That is, $W^{lt^*} - W^{nlt^*} = (c_E - c_I)(N^c + N^s) < 0$. Sellers are clearly indifferent between the long-term contract and no long-term contract, i.e., $u_I^{s,lt} = u_{I,1}^{s,nlt} + u_{E,2}^{s,nlt} = v^s + \alpha^s N^c - f_{E,2}^{s,nlt^*}$. Interestingly, consumers are worse off under the seller long-term contract.[17] The reason is that only sellers "bargain" with I about the long-term contract, but the outcome affects the consumers due to cross-group externalities. Since consumers do not participate in the bargaining process, I

[17] Formally, $u_{I,1}^{c,nlt} + u_{E,2}^{c,nlt} = v^c + \alpha^c N^s - f_{E,2}^{c,nlt^*} > u_{I,1}^{c,lt} + u_{I,2}^{c,lt} = v^c - c_E \iff f_{E,2}^{c,nlt^*} < \alpha^c N^s + c_E$, and from $f_{E,2}^{\tilde{s}^*}$ and (2.9) I makes use of a long-term contract whenever $f_{E,2}^{c,nlt^*} < f_{E,2}^{\tilde{c}^*} \triangleq \alpha^c N^s + c_E - |\alpha^c - \alpha^s| N^s < \alpha^c N^s + c_E$.

only has to include favorable terms for the sellers (such that they voluntarily sign), but not for the consumers. What therefore makes the long-term contract lucrative for I is that I can increase its two-period profits by getting the sellers on board for two periods to extract more surplus from consumers in the second period.

2.3.4 Sellers multi-home and consumers single-home

Now assume that sellers can multi-home. Sellers typically multi-home in the following two-sided market examples: credit cards, (mobile) operating systems (Windows, Android), newspapers (advertisers typically place ads in multiple newspapers), e-commerce platforms (Amazon, Alibaba), shopping malls, video game consoles (Microsoft's Xbox, Sony's PlayStation), and search engines (Google, Yahoo).

The uninteresting case is when consumers can also multi-home because then I and E in effect do not compete against each other for the two sides in the second period and each platform can exercise monopoly power over each agent side (each side is willing to go to both platforms). I can already extract the full surplus (stand-alone and cross-group externality) on each side in both periods absent a long-term contract, which cannot be improved through a contract.

Thus, I assume that consumers single-home. That is, the sides' homing opportunities from Section 2.3.2 are reversed. Absent a long-term contract, in the second period I chooses $\left(f_{I,2}^{s,nlt}, f_{I,2}^{c,nlt}\right)$ to maximize $\Pi_{I,2}^{nlt}$ subject to the sellers' participation constraint, the consumers' incentive compatibility constraint $v^c + \alpha^c N^s - f_{I,2}^{c,nlt} > v^c + \alpha^c N^s - f_{E,2}^{c,nlt}$, and the "outside option" to extract the stand-alone benefit of sellers, $\Pi_{I,2}^{nlt} = (v^s - c_I) N^s$. By similar steps as in the proof of Lemma 2.1, E successfully attracts both the consumer and seller side, whereas I only attracts the sellers. Lemma 2.3, the proof of which I skip here for brevity, states the equilibrium fees and profits.

Lemma 2.3. *If $\alpha^c, \alpha^s \neq 0$ and sellers multi-home and consumers single-home, then in the absence of a long-term contract all consumers go to E and sellers go to I and E in the second period in equilibrium. Equilibrium fees are*

$$f_{I,2}^{c,nlt^*} = f_{E,2}^{c,nlt^*} = c_I - \alpha^s N^s, f_{I,2}^{s,nlt^*} = v^s, f_{E,2}^{s,nlt^*} = v^s + \alpha^s N^c, \tag{2.11}$$

and platform profits are

$$\Pi_{I,2}^{nlt^*} = (v^s - c_I) N^s, \Pi_{E,2}^{nlt^*} = (c_I - c_E) N^c + (v^s - c_E) N^s \tag{2.12}$$

The same arguments that resulted in Lemma 2.1 apply to Lemma 2.3, but with the agent sides' roles reversed. Moving to I's incentive whether or not to employ a long-term contract in the first period, I differentiate between *exclusive* and *non-exclusive* long-term contracts. In the former case, the sellers are exclusively bound to I if they sign the contract. In the latter case, sellers commit to going to I in both periods but can consider going to E in the second period as well.

Exclusive long-term contract. First, assume that a long-term contract entails an exclusivity clause for sellers. The sellers' incentive compatibility constraint to sign the contract, inequality (2.8), is modified to $2v^s + 2\alpha^s N^c - \bar{f}_I^s \geq v^s + \alpha^s N^c - f_{E,2}^{s,nlt^*} + v^s - f_{I,2}^{s,nlt^*} = 0$. The left-hand side does not contain the additional term $v^s - f_{E,2}^{s,lt^*}$ because the sellers cannot go to E in the second period due to the exclusivity clause. Consequently, I sets $\bar{f}_I^{s^*} = 2v^s + 2\alpha^s N^c$ and the sellers accept the long-term contract. Under a long-term contract, I can charge the consumers $c_E + \alpha^c N^s$ in the second period (although the sellers only go to I in the second period under exclusivity, E still competes with I for the consumers). I's net benefit from creating a long-term contract is $\Pi_I^{lt^*} - \Pi_{I,1}^{nlt^*} - \Pi_{I,2}^{nlt^*} = (c_E + \alpha^c N^s - c_I)\, N^c + \alpha^s N^c N^s$, from which Proposition 2.4 follows.

Proposition 2.4. *If $\alpha^c, \alpha^s \neq 0$ and sellers multi-home and consumers single-home, I employs a long-term contract that is exclusive if and only if $(\alpha^c + \alpha^s)\, N^s > c_I - c_E$.*

Note that a potential alternative strategy for I is to get the sellers to sign the contract, with the intention not to attract the consumers in the second period. E attracts the consumers in the second period, so sellers only get the cross-group externality $\alpha^s N^c$ in the first but not in the second period under a contract. Hence, $\bar{f}_I^{s^*} = 2v^s + \alpha^s N^c$. Consequently, $\Pi_I^{lt^*} - \Pi_{I,1}^{nlt^*} - \Pi_{I,2}^{nlt^*} = (2v^s + \alpha^s N^c - 2c_I)\, N^s - (v^s + \alpha^s N^c - c_I)\, N^s - (v^s - c_I)\, N^s = 0$, and I is not strictly better off under a long-term contract.

Absent a long-term contract, the consumers' surplus is fully extracted in the competitive bottleneck equilibrium in Section 2.3.2. In the present section where the homing possibilities are reversed, the sellers' surplus is fully extracted. This plays in the favor of I in its incentives to create a long-term contract. By not having to compensate the sellers' low fee on E, I can extract the full seller surplus under a long-term contract in the second period. Absent a contract I could only extract the stand-alone benefit of sellers. Hence, I's net benefit of a contract on the seller side is $\alpha^s N^c$ per seller.

At the same time, I's net benefit per consumer is $c_E + \alpha^c N^s - c_I$: while I did not attract the consumers in the second period absent a contract, I has to set $c_E + \alpha^c N^s$ to ensure that E does not attract the consumers. Proposition 2.4 states that the overall benefit of a long-term contract for I is positive if the cross-group externalities are sufficiently important and I's cost disadvantage small enough. Whenever I makes use of a long-term contract, the outcome, similar to Proposition 2.3, is socially inefficient because platform I instead of E hosts the consumers in the second period; $W^{lt^*} - W^{nlt^*} = (c_E - c_I)N^c < 0$.

Non-exclusive long-term contract. Second, assume instead that a long-term contract does not entail exclusivity on sellers. The contract only states that sellers are bound to go to I in both periods, but are not forbidden to consider going to E as well in the second period.

Suppose sellers signed the long-term contract in the first period. In the second period, E attracts the sellers as well as the consumers. To see this, note that I and E compete for the consumer side in the second period because the sellers can multi-home. E can easily attract the seller side; the consumer side is the "bottleneck". The single-homing consumers go to whichever platform charges the lowest fee, knowing that sellers will be on both platforms. I does not want to charge less than $f_{I,2}^{c,lt^*} = c_I$ for consumers, because otherwise I would make a loss on the consumer side.

The difference to the proof of Lemma 2.1 with reversed homing possibilities is that I has already attracted the seller side and received $\bar{f}_I^{s^*}$ given the contract, so does not contemplate subsidizing the consumers in order to attract both sides in order to extract more from the seller side (the sellers' cross-group externality benefit). Therefore, $f_{I,2}^{c,lt^*} = c_I$ rather than $c_I - \alpha^s N^s$. E indeed wants to set $\left(f_{E,2}^{c,lt^*}, f_{E,2}^{s,lt^*}\right) = (c_I, v^s + \alpha^s N^c)$ to attract both sides, because $\Pi_{E,2}^{lt^*} = (c_I - c_E)\, N^c + (v^s + \alpha^s N^c - c_E)\, N^s > (v^s - c_E)\, N^s$. The right hand side in the inequality is E's profit if it were to only extract the sellers' stand-alone benefit and not attract the consumers. Given the second-period equilibrium fees and outcome under a long-term contract, Proposition 2.5 states that I is not strictly better off using a long-term contract.

Proposition 2.5. *If $\alpha^c, \alpha^s \neq 0$ and sellers multi-home and consumers single-home, I has no strict inventive to employ a long-term contract that is non-exclusive.*

Proof. Following up on the outcome in the second period under a long-term contract from the text, the sellers' incentive compatibility constraint (2.8) in the first period is modified to

$$3v^s + 2\alpha^s N^c - \bar{f}_I^s - f_{E,2}^{s,lt^*} \geq 0 \tag{2.13}$$

On the right-hand side, the sellers' surplus is fully extracted in absence of a long-term contract. If the sellers sign the contract that is now non-exclusive, not only do they pay $\bar{f}_I^s$ to I in the first period, but also $f_{E,2}^{s,lt^*}$ to E in the second period. The sellers receive the surplus $v^s + \alpha^s N^c$ from I in the first period, $v^s + \alpha^s N^c$ from E and v^s from I in the second period. Hence, $\bar{f}_I^{s^*} = 2v^s + \alpha^s N^c$. Importantly, I cannot charge a larger $\bar{f}_I^{s^*}$ because sellers know that if they sign the long-term contract, E will charge $f_{E,2}^{s,lt^*} = v^s + \alpha^s N^c$.

Lastly, I's net profit from a long-term contract

$$\begin{aligned} \Pi_I^{lt^*} - \Pi_{I,1}^{nlt^*} - \Pi_{I,2}^{nlt^*} &= \left(f_{I,1}^{c,lt^*} - c_I\right) N^c + \left(\bar{f}_I^{s^*} - 2c_I\right) N^s \\ &\quad - \left(f_{I,1}^{c,nlt^*} - c_I\right) N^c - \left(f_{I,1}^{s,nlt^*} - c_I\right) N^s - \left(f_{I,2}^{s,nlt^*} - c_I\right) N^s = 0 \end{aligned}$$

Therefore, I is not strictly better off creating a long-term contract. □

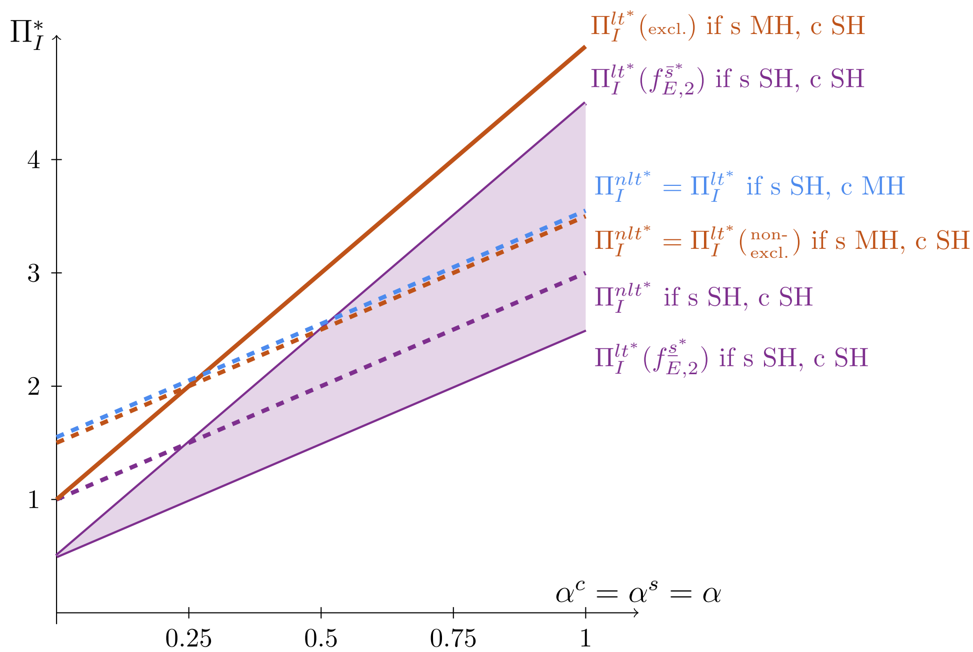
Formalized in the proof of Proposition 2.5, when I decides whether or not to use a long-term contract, I knows that it will not be able to attract the consumers (and hence their surplus) in the second period under a contract, just as in the absence of a contract. Therefore, a long-term contract entails no benefit for I on the consumer side. Since in addition $f_{E,2}^{s,lt^*} = v^s + \alpha^s N^c$, I can only charge the sellers $\bar{f}_I^{s^*} = 2v^s + \alpha^s N^c$, which is the same amount I charges the sellers over the two periods absent a contract.

We know from Proposition 2.4 that I uses an *exclusive* long-term contract if for example cross-group externalities are sufficiently large, but not otherwise. We also know from Proposition 2.5 that I never use a long-term contract that contains a *non-exclusive* seller clause. Therefore, if I is allowed to choose whether a long-term contract entails an *exclusivity* or a *non-exclusivity* clause for sellers, I would profitably choose to include the *exclusivity* clause whenever the condition in Proposition 2.4 is satisfied, but is indifferent otherwise (as I does not create a long-term contract anyway). The finding is formally stated in Corollary 2.1.

Corollary 2.1. *If $\alpha^c, \alpha^s \neq 0$ and sellers multi-home and consumers single-home, I strictly prefers an exclusive over a non-exclusive clause for sellers in a long-term contract if and only if $(\alpha^c + \alpha^s) N^s > c_I - c_E$.*

Proof. The statement directly follows from Proposition 2.4 and Proposition 2.5. □

Figure 2.3: Incumbent's profit summary



Parameters: $v^c = v^s = N^c = N^s = 1, c_I = 0.5, c_E = 0$

***I*'s profit summary.** Figure 2.3 summarizes I's profits under the three homing settings. When sellers multi-home and consumers single-home, I creates a long-term contract that is exclusive if and only if $\alpha > \frac{1}{4}$ given the parameter assumptions. When both sides single-home, $f_{E,2}^{\bar{s}^*} > f_{E,2}^{\tilde{s}^*} \iff \alpha \geq \frac{1}{4}$. In the absence of a contract, I's profit is larger if sellers multi-home and consumers single-home compared to when both sides single-home, but are equal if sellers single-home and consumers multi-home. The former holds in general because I makes a strictly positive profit in the second period if one of the sides multi-homes (I can extract that side's stand-alone value), but does not make any profit in the second period under two-sided single-homing (since E attracts both sides). The latter is true in the given example since $v^c = v^s$.

2.4 Extensions

2.4.1 Alternative commitment

I now assume that I can credibly commit not to let sellers visit its platform outside the long-term contract for both periods. It is immediate that if the sellers do not sign the long-term contract if offered, the only option for them is to visit E in the second period. This is independent of the sellers' homing assumption. Then, E faces no competition from I on the seller side in the second period and E extracts the full surplus from the seller side, i.e., $f_{E,2}^{s,nlt^*} = v^s + \alpha^s N^c$; E extracts not just the sellers' stand-alone benefit but in addition the cross-group externality benefit because E also attracts the consumers. In particular, E sets $f_{E,2}^{c^*} = v^c + \alpha^c N^s$ if consumers can multi-home (I sets $f_{I,2}^{c^*} = v^c$ and also attracts the consumers based on their stand-alone benefit), and sets $f_{E,2}^{c^*} = c_I + \alpha^c N^s$ if consumers can only single-home (the largest consumer fee E can set when competing with I for the consumers; I does not attract the consumers and makes zero profit). Note that in contrast to Lemma 2.2, the equilibrium fee combination chosen by E is unique.

When sellers can multi-home, we know from Lemma 2.3 that the sellers' surplus was already fully extracted by I and E in the second period absent a contract under the intermediate commitment assumption. Thus, the right-hand side of the sellers' incentive compatibility constraint, (2.8) adjusted for the extreme commitment assumption, remains zero, the same as in the proofs for Proposition 2.4 and 2.5. Therefore, I's incentive to create a long-term contract remains the same if sellers multi-home.

The present extreme commitment assumption, however, changes I's incentives to create a long-term contract if sellers single-home. If sellers single-home, their incentive compatibility constraint (2.8) reduces to $2v^s + 2\alpha^s N^c - \bar{f}_I^s \geq v^s + \alpha^s N^c - f_{E,2}^{s^*} = 0$. Hence, $\bar{f}_I^{s^*} = 2v^s + 2\alpha^s N^c$. Intuitively and in contrast to the intermediate commitment assumption, I does not have to compensate the sellers by any amount to forgo the benefit from having access to E in the second period, since their surplus now is zero from going to E.

If in addition consumers multi-home, I's net benefit from creating a long-term contract is

$$
\begin{aligned}
&\Pi_I^{lt^*} - \Pi_{I,1}^{nlt^*} - \Pi_{I,2}^{nlt^*} \\
=&\left(f_{I,2}^{c,lt^*} - c_I\right) N^c + \left(\bar{f}_I^{s^*} - 2c_I\right) N^s - \left(f_{I,1}^{s,nlt^*} - c_I\right) N^s - \left(f_{I,2}^{c,nlt^*} - c_I\right) N^c \\
=&\alpha^c N^s N^c + \left(v^s + \alpha^s N^c - c_I\right) N^s > 0,
\end{aligned}
$$

where $f_{I,2}^{c,lt^*} = v^c + \alpha^c N^s$ and $f_{I,2}^{c,nlt^*} = v^c$ are used. The Proposition 2.2 result is overturned. It is now always beneficial for I to use a long-term seller contract. If consumers instead single-home (sellers continue to single-home), I's net benefit is

$$
\begin{aligned}
&\Pi_I^{lt^*} - \Pi_{I,1}^{nlt^*} \\
=&\left(f_{I,2}^{c,lt^*} - c_I\right) N^c + \left(\bar{f}_I^{s^*} - 2c_I\right) N^s - \left(f_{I,1}^{s,nlt^*} - c_I\right) N^s \\
=&\left(c_E + \alpha^c N^s - c_I\right) N^c + \left(v^s + \alpha^s N^c - c_I\right) N^s,
\end{aligned}
$$

where $f_{I,2}^{c,lt^*} = c_E + \alpha^c N^s$ is used. I's net benefit is positive if the cross-group externalities α^c and α^s are relatively large compared to I's cost disadvantage, $c_I - c_E$. Proposition 2.6 summarizes.

Proposition 2.6. *Assume platform I commits not to let sellers visit its platform outside the long-term contract for both periods. If sellers single-home and consumers multi-home, I always creates a long-term contract. If both sides single-home, I creates a long-term contract if and only if* $\left(c_E + \alpha^c N^s - c_I\right) N^c + \left(v^s + \alpha^s N^c - c_I\right) N^s > 0$. *If sellers can multi-home, Propositions 2.4 and 2.5 are unchanged.*

If sellers single-home and for a given consumer homing assumption, the difference in the seller fee equilibria in the second stage between the two commitment assumptions is the crucial driver in the decision by I whether or not to use a long-term contract. In the initial set-up, I and E competed for sellers in the second period, which resulted in a low fee for sellers. This in turn meant that I had to compensate the sellers to forgo visiting E for a low fee in order to sign the long-term contract, which made it expensive and hence not lucrative for I to create a long-term contract in the first place.

Under the alternative commitment assumption, I forfeits the second period to E if sellers do not sign the long-term contract. In a sense, I disciplines the sellers if they do not sign the contract by not competing for them in the second period and in turn by letting E extract their surplus. Consequently, I now does not have to compensate the sellers, which makes it more attractive for I to create and use a long-term contract compared to the baseline commitment assumption. Here, $\bar{f}_I^{s^*}$ is the same whether consumers single- or multi-home.

Therefore, if sellers single-home the difference in I's decision to use a long-term contract is solely driven by the consumers' homing assumption. I's second period consumer net benefit from a long-term contract if consumers multi-home, $(v^c + \alpha^c N^s - c_I)\, N^c - (v^c - c_I)\, N^c = \alpha^c N^s N^c$, is larger than if consumers single-home, $(c_E + \alpha^c N^s - c_I)\, N^c$. The simple reason is that I has to set a lower consumer fee when consumers single-home, such that E cannot attract the consumers, compared to when consumers multi-home.

2.4.2 Non-negative fees

In Section 2.3, we have seen that one side of the market may be charged with a fee below marginal cost or even with a negative fee (see, for instance, the seller fee expression in Lemma 2.1). A reasonable restriction for many two-sided markets is to assume that fees must be non-negative (e.g., Armstrong and Wright, 2007). Adverse selection and moral hazard problems would arise if for example consumers are paid to own a game console, to go to a shopping mall, or if advertisers are paid to advertise on a search engine platform or in a newspaper.

When **sellers multi-home**, the non-negativity constraint has no impact on I's incentive to create a long-term contract for two reasons. First, the sellers' surplus was fully extracted by the two platforms through strictly positive fees. Therefore, the amount I has to compensate the sellers for forgoing the benefit from going to E is unaffected by the non-negativity constraint. In other words, the sellers' incentive compatibility constraint stays the same. Second, E's consumer fee absent a long-term contract, now adjusted to $\max\{0, c_I - \alpha^s N^s\}$, has no impact on I's profits as E continues to successfully attract the consumers (by the tie-breaking rule in E's favor). Therefore, the constraint has no impact on I's second-period profits absent a long-term contract, which in turn does not affect I's net benefit from applying a contract.

When **sellers single-home and consumers multi-home**, the non-negativity constraint can overturn the Proposition 2.2 result. That is, I may be strictly better off creating a long-term contract. The simple intuition is that if the seller fee charged by E were to be negative without the non-negativity restriction, the restriction means that I has to compensate the sellers less for forgoing the benefit from going to E in the second period absent a long-term contract, which in turn makes it more lucrative for I to create a contract in the first place. Proposition 2.7 formally states the result.

Proposition 2.7. *Assume that $\alpha^c, \alpha^s \neq 0$, sellers single-home and consumers can multi-home, and that the fees cannot be strictly negative. Then, I creates a long-term contract if and only if $c_I < \alpha^c N^c$.*

Proof. If $c_I \geq \alpha^c N^c$, the non-negativity constraint is not binding in the equilibrium fees in the second period absent a contract (see equation (2.4) in Lemma 2.1) and the Proposition 2.2 result continues to hold. If instead $c_I < \alpha^c N^c$, E would want to set a strictly negative seller fee to avoid that I competes for the sellers (recall that the sellers are the bottleneck in this scenario), but is not allowed to do so. Hence, $f_{I,2}^{s,nlt^*} = f_{E,2}^{s,nlt^*} = 0$. Since consumers multi-home and the platforms charge an identical seller fee, I assume a tie-breaking rule in favor of the efficient platform E.[18] Consequently, $f_{I,2}^{c,nlt^*} = v^c$, $f_{E,2}^{c,nlt^*} = v^c + \alpha^c N^s$. E indeed wants to attract the sellers since $\Pi_{E,2}^{nlt^*} = (v^c + \alpha^c N^s - c_E) N^c + (-c_E) N^s \geq (v^c - c_E) N^c$, as $\alpha^c N^c > c_I > c_E$.

From (2.8) in the $c_I < \alpha^c N^c$ case, $\bar{f}_I^{s^*} = v^s + \alpha^s N^c + f_{E,2}^{s,nlt^*} = v^s + \alpha^s N^c$. Then, following the remaining steps in the proof of Proposition 2.2 it is straightforward to show that $\Pi_I^{lt^*} - \Pi_{I,1}^{nlt^*} - \Pi_{I,2}^{nlt^*} = (\alpha^c N^c - c_I) N^s > 0$, because $\alpha^c N^c > c_I$. □

If $\alpha^c N^c > c_I$, the non-negativity constraint is binding for the seller fee, which channels some of the sellers' benefit to I. While previously the seller benefit was $v^s + (\alpha^s + \alpha^c) N^c - c_I$, it is now reduced to $u_I^{s,lt^*} = v^s + \alpha^s N^c$. The consumers' surplus is still fully extracted.

In the **two-sided single-homing** configuration, the non-negativity constraint imposes additional restrictions on the fees $\left(f_{E,2}^{c,nlt^*}, f_{E,2}^{s,nlt^*}\right)$ that E can choose such that (2.9) given in Lemma 2.2 is still satisfied. The implication on I's incentive to use a long-term contract are formalized in Proposition 2.8.

Proposition 2.8. *Assume that $\alpha^c, \alpha^s \neq 0$, sellers and consumers single-home, and that the fees cannot be strictly negative. Then, I creates a long-term contract if I and the sellers expect that $f_{E,2}^{s,nlt^*} \in \left(f_{E,2}^{\tilde{s}^*}, f_{E,2}^{\bar{s}^*}\right]$ where $f_{E,2}^{\tilde{s}^*} = c_I - (c_E + \alpha^c N^s - c_I) \frac{N^c}{N^s}$ and $f_{E,2}^{\bar{s}^*} = \min\left\{c_I + \alpha^s N^c, c_I + c_I \frac{N^c}{N^s} - |\alpha^c - \alpha^s| N^c\right\}$, but does not create a long-term contract if $f_{E,2}^{s,nlt^*} \in \left[f_{E,2}^{\underline{s}^*}, f_{E,2}^{\tilde{s}^*}\right]$ where $f_{E,2}^{\underline{s}^*} = \max\left\{0, c_I - \alpha^c N^c - |\alpha^c - \alpha^s| N^c\right\}$.*

[18] The result does not qualitatively change if I assume that sellers randomly choose with an equal probability between I and E.

Proof. The non-negativity constraint does not affect E when setting its fees in order to be immune against strategies (i) and (ii) described in the proof of Lemma 2.2. Hence, equation (2.9) remains unchanged. However, the non-negativity constraint does affect E's upper and lower bound fees, which ensure that E is immune against strategy (iii).

In particular, the upper bound of the seller fee given by (2.10), $c_I + \alpha^s N^c$, is feasible only if the corresponding consumer fee using (2.9), $c_I - \alpha^s N^s - |\alpha^c - \alpha^s| N^s$, is non-negative. If not, the largest seller fee, which corresponds to the lowest consumer fee (which is zero if the above is not satisfied), is $c_I + c_I \frac{N^c}{N^s} - |\alpha^c - \alpha^s| N^c$, by again using (2.9). The lower bound of the seller fee is the maximum of zero and the seller fee corresponding to the consumer fee $c_I + \alpha^c N^s$ from (2.10), $c_I - \alpha^c N^c - |\alpha^c - \alpha^s| N^c$. Repeating the analogous steps for the bounds of the consumer fee, the constraints given by (2.10) are adjusted and summarized as follows:

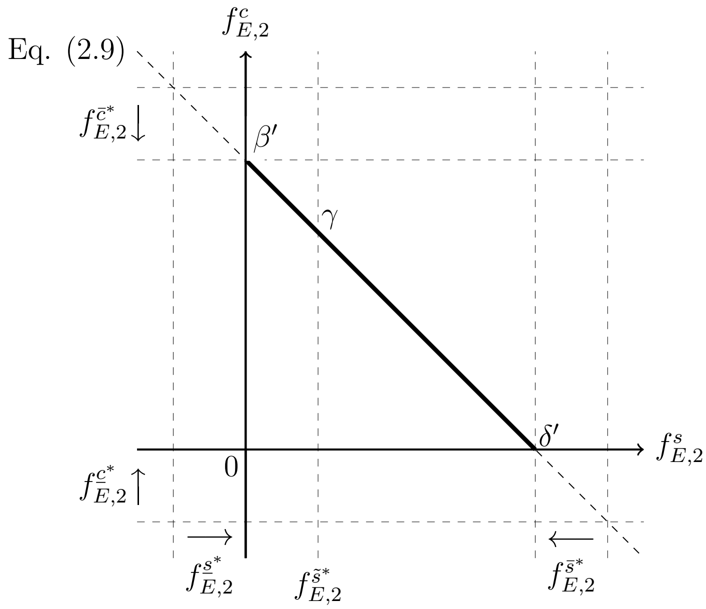
$$f_{E,2}^{s,nlt^*} \in \left(f_{E,2}^{\underline{s}^*}, f_{E,2}^{\bar{s}^*}\right) = \left(\max\left\{0, c_I - \alpha^c N^c - |\alpha^c - \alpha^s| N^c\right\}, \min\left\{c_I + \alpha^s N^c, c_I + c_I \frac{N^c}{N^s} - |\alpha^c - \alpha^s| N^c\right\}\right)$$
$$f_{E,2}^{c,nlt^*} \in \left(f_{E,2}^{\underline{c}^*}, f_{E,2}^{\bar{c}^*}\right) = \left(\max\left\{0, c_I - \alpha^s N^s - |\alpha^c - \alpha^s| N^s\right\}, \min\left\{c_I + \alpha^c N^s, c_I + c_I \frac{N^s}{N^c} - |\alpha^c - \alpha^s| N^s\right\}\right)$$

The cut-off fee $f_{E,2}^{\tilde{s}^*}$, whose derivation I omit for brevity, is identical to the one in Proposition 2.3. □

Observe that $f_{E,2}^{\underline{s}^*}$ in Proposition 2.8 is strictly larger under the non-negativity assumption if and only if $\alpha^c N^c + |\alpha^c - \alpha^s| N^c > c_I$, which holds for sufficiently large cross-group externality parameters. Thus, while E in the absence of a non-negativity constraint could substantially subsidize sellers by charging negative seller fees (and earn its profits from the consumer side), a non-negativity constraint may decrease the range of expected seller fees at the lower end. This in turn means that the range of expected seller fees for which I does not find it profitable to compensate sellers for signing a long-term contract decreases. Figure 2.4 shows the updated $\beta'\gamma$ line. Similarly, $f_{E,2}^{\bar{s}^*}$ now is smaller under the non-negativity constraint if and only if $\alpha^s N^s + |\alpha^c - \alpha^s| N^s > c_I$. That also means that the range of expected seller fees for which I does find it profitable to compensate sellers for signing a long-term contract decreases as well. Figure 2.4 shows the updated $\gamma\delta'$ line.

Re-call that the interval $\left[f_{E,2}^{\underline{s}^*}, f_{E,2}^{\tilde{s}^*}\right]$ was non-empty absent the non-negativity constraint. The interval now is empty if $\alpha^c N^c > c_I + (c_I - c_E) \frac{N^c}{N^s}$. In that case, I always creates a long-term contract.

Figure 2.4: Comparison of Propositions 2.3 and 2.8



Note: $\alpha^j N^j + |\alpha^c - \alpha^s| N^j > c_I \ \forall j = c, s$

2.4.3 Differentiation - Hotelling

I now consider a market where I and E are viewed as homogeneous by one agent side and as differentiated by the other side. This captures the idea that in many two-sided markets, consumers have a preference for using one or the other platform, whereas sellers typically do not have a preference for one platform in and of itself (controlling for the number of consumers and fees); see Armstrong and Wright (2007, Chapter 4) for justifications.

In particular, assume that I and E differ from the consumers' perspective in a standard Hotelling manner. Consumers are located uniformly along an interval of unit length; without loss of generality, I is located at the left extreme and E is located at the right extreme of that unit interval. A consumer incurs the cost $\tau^c x$ traveling distance x to the platform she visits. It suffices that $\tau^c > 0$ to ensure that the second-order conditions are satisfied. The utility of a consumer located at $x \in [0, 1]$ is

$$u^c_{k,t}(x) = v^c + \alpha^c N^s_{k,t} - \tau^c x \mathbb{1}_{(k=I)} - \tau^c (1-x) \mathbb{1}_{(k=E)} - f^c_{k,t}, \tag{2.14}$$

which is equation (2.1) adjusted for consumer heterogeneity.[19] If a consumer multi-homes, her total transportation cost is the sum of her individual costs going to each platform, i.e., τ^c. Sellers continue to regard the platforms as homogeneous. Thus, the sellers' utility expression remains unchanged.

In the first period and in each homing configuration, the condition for full versus non-full consumer market coverage is the same both under a long-term contract and absent a contract, and does not impact the net gain of a long-term contract. For brevity and for comparability to the baseline model, I only present the cases where the consumer market is fully covered in each period and in each homing configuration. The all-encompassing condition for full market coverage is given by Assumption 2.2; I state the specific parameter requirements in the corresponding homing configuration sections.

Assumption 2.2. $2\tau^c \leq v^c - c_I$

In the first period, independent of the homing configuration and if I decides not to create a long-term contract, I sets $\left(f_{I,2}^c, f_{I,2}^s\right)$ to maximize its monopoly profit.

$$\max_{f_{I,1}^c, f_{I,1}^s} \left\{\left(f_{I,1}^c - c_I\right) N_{I,1}^c + \left(f_{I,1}^s - c_I\right) N^s\right\} \quad \text{s.t. } v^s + \alpha^s N_{I,1}^c - f_{I,1}^s \geq 0,$$

where $N_{I,1}^c = \min\left\{\frac{1}{\tau^c}\left(v^c + \alpha^c N^s - f_{I,1}^c\right) N^c, N^c\right\}$. Taking first order conditions and solving gives[20]

$$\Pi_{I,1}^{nlt^*} = \left(v^c + \alpha^c N^s - \tau^c - c_I\right) N^c + \left(v^s + \alpha^s N^c - c_I\right) N^s.$$

[19]The Random Utility Model (RUM) is an alternative formulation to introduce consumer heterogeneity, according to which (2.1) is modified to $u_{k,t}^c = v^c + \alpha^c N_{k,t}^s - f_{k,t}^c + \mu\epsilon_k^c$, where ϵ_k^c is a random variable drawn from the extreme value type I distribution and $\mu > 0$ reflects the degree of platform differentiation. The baseline model is replicated by letting $\mu \to 0$. Though the RUM specification avoids having to analyze cases, I opt for the Hotelling specification for analytic tractability (the equilibrium $N_{k,t}^c$ are implicitly defined in the RUM specification) and for better comparability to the baseline model. The qualitative results remain unchanged compared to the Hotelling set-up presented here.

[20]The requirement for full market coverage, $N_{I,1}^{c,nlt^*} = N^c$, is $2\tau^c \leq v^c + \left(\alpha^c + \alpha^s\right) N^s - c_I$, which is satisfied by Assumption 2.2.

Sellers single-home and consumers multi-home[21]

In the second period absent a long-term contract, the two platforms simultaneously set the agent fees. E sets $\left(f^c_{E,2}, f^s_{E,2}\right)$ to maximize its profit and to attract the homogeneous sellers, subject to I not wanting to (deviate and) attract the sellers. Conditional on E attracting the sellers, I and E do not compete directly for consumers because consumers multi-home. Formally, E's maximization problem can be written as

$$\max_{f^c_{E,2}, f^s_{E,2}} \left\{\left(f^c_{E,2} - c_E\right) N^c_{E,2} + \left(f^s_{E,2} - c_E\right) N^s\right\}$$
$$\text{s.t.} \max_{f^{c,dev}_{I,2}} \left\{\left(f^{c,dev}_{I,2} - c_I\right) N^{c,dev}_{I,2} + \left[f^s_{E,2} + \alpha^s \left(N^{c,dev}_{I,2} - \min\left\{\frac{1}{\tau^c}\left(v^c - f^c_{E,2}\right) N^c, N^c\right\}\right) - c_I\right] N^s\right\}$$
$$\leq \max_{f^c_{I,2}} \left\{\left(f^c_{I,2} - c_I\right) \min\left\{\frac{1}{\tau^c}\left(v^c - f^c_{I,2}\right) N^c, N^c\right\}\right\} \text{ and}$$
$$v^s + \alpha^s N^c_{E,2} - f^c_{E,2} \geq v^s + \alpha^s \min\left\{\frac{1}{\tau^c}\left(v^c - f^c_{I,2}\right) N^c, N^c\right\} - f^c_{I,2},$$

where $N^c_{E,2} = \min\left\{\frac{1}{\tau^c}\left(v^c + \alpha^c N^s - f^c_{E,2}\right) N^c, N^c\right\}$ and $N^{c,dev}_{I,2} = \min\{\frac{1}{\tau^c}(v^c + \alpha^c N^s - f^{c,dev}_{I,2})N^c, N^c\}$. Taking first order conditions and solving gives $\Pi^{nlt^*}_{I,2} = \left(v^c - \tau^c - c_I\right) N^c$.

Suppose that sellers signed the long-term contract in the first period. Since sellers are assumed to single-home, they are bound to go to I in the second period, and since consumers multi-home, I and E do not compete directly with each other for the consumers in the second period. Formally,

$$I: \max_{f^c_{I,2}} \left\{\left(f^c_{I,2} - c_I\right) N^c_{I,2}\right\} \text{ and } E: \max_{f^c_{E,2}} \left\{\left(f^c_{E,2} - c_E\right) N^c_{E,2}\right\},$$

where $N^c_{I,2} = \min\left\{\frac{1}{\tau^c}\left(v^c + \alpha^c N^s - f^c_{I,2}\right) N^c, N^c\right\}$ and $N^c_{E,2} = \min\{\frac{1}{\tau^c}(v^c - f^c_{E,2})N^c, N^c\}$. It follows that $\Pi^{lt^*}_{I,2} = \left(v^c + \alpha^c N^s - \tau^c - c_I\right) N^c$ and $\Pi^{lt^*}_{E,2} = \left(v^c - \tau^c - c_E\right) N^c$.

If I wants to create a long-term contract in the first period, I faces the following

[21] The following parameter conditions ensure full market coverage: $N^{c,nlt^*}_{I,2} = N^c \iff 2\tau^c \leq v^c - c_I$, $N^{c,nlt^*}_{E,2} = N^c \iff 2\tau^c \leq v^c + \left(\alpha^c + \alpha^s\right) N^s - c_E$, $N^{c,lt^*}_{I,2} = N^c \iff 2\tau^c \leq v^c + \alpha^c N^s - c_I$, $N^{c,lt^*}_{E,2} = N^c \iff 2\tau^c \leq v^c - c_E$, and $N^{c,lt^*}_{I,1} = N^c \iff 2\tau^c \leq v^c + \left(\alpha^c + \alpha^s\right) N^s - c_I$. All are satisfied by Assumption 2.2.

maximization problem:

$$\max_{f^c_{I,1}, \bar{f}^s_I} \left\{ \left(f^c_{I,1} - c_I \right) N^c_{I,1} + \left(\bar{f}^s_I - 2c_I \right) N^s \right\}$$
$$\text{s.t. } v^s + \alpha^s N^c_{I,1} + v^s + \alpha^s N^{c,lt^*}_{I,2} - \bar{f}^s_I \geq v^s + \alpha^s N^{c,nlt^*}_{E,2} - f^{s,nlt^*}_{E,2},$$

where $N^c_{I,1} = \min\left\{\frac{1}{\tau^c}\left(v^c + \alpha^c N^s - f^c_{I,1}\right) N^c, N^c\right\}$. Taking first order conditions and solving gives $\Pi^{lt^*}_{I,1} = (v^c - \tau^c - c_I)\, N^c + \left(v^s + \alpha^s N^c - c_I - \frac{\alpha^c N^s \alpha^s N^c}{\tau^c}\right) N^s$.

The net benefit of using a long-term contract is

$$\Pi^{lt^*}_{I,1} + \Pi^{lt^*}_{I,2} - \Pi^{nlt^*}_{I,1} - \Pi^{nlt^*}_{I,2} = -\frac{\alpha^c N^s \alpha^s N^c N^s}{\tau^c} \leq 0,$$

where the strict inequality holds if $\alpha^j > 0\ \forall j = c, s$. Thus, Proposition 2.2 continues to hold when introducing consumer heterogeneity. While I is indifferent between creating a long-term contract and not for any α^j according to Proposition 2.2, I now is strictly worse off if both α^c and α^s are strictly positive.

Two-sided single-homing

I let $\alpha^c = \alpha^s = \alpha$ for the remainder of this subsection in order to reduce the cases to be investigated. The equilibrium fees chosen by I and E now are unique in the second period absent a long-term contract, which obscures the comparison between Proposition 2.3 and Proposition 2.9. The formal proof of Proposition 2.9 and the others for the remainder of this subsection are provided in Appendix B.1.

Proposition 2.9. *If sellers and consumers single-home and consumers are differentiated à la Hotelling, I creates a long-term contract that is exclusive if and only if* $\alpha\left(3\tau^c + c_E - c_I - 7\alpha N^s\right) > 0$.

The statement in Proposition 2.9 is based on the Hotelling case where $N^{c,nlt^*}_{E,2}, N^{c,lt^*}_{I,2} \in (0, N^c)$. That is, neither does E capture the entire consumer market absent a long-term contract and nor does I capture the entire consumer market in the presence of a contract in the second period. To understand Proposition 2.9, it helps to look at the total derivative of I's net benefit using a long-term contract with respect to, say, α (the individual terms' inequality signs follow from this Hotelling case's parameter restrictions):[22]

$$\underbrace{\frac{\left(c_I-c_E+3\tau^c-2\alpha N^s\right)N^cN^s}{3\tau^c}}_{\frac{\partial\Pi^{lt*}_{I,1}}{\partial\alpha}>0}+\underbrace{\frac{\left(c_E-c_I+3\tau^c+\alpha N^s\right)N^cN^s}{9\tau^c}}_{\frac{\partial\Pi^{lt*}_{I,2}}{\partial\alpha}>0}$$
$$-\underbrace{2N^cN^s}_{\frac{\partial\Pi^{nlt*}_{I,1}}{\partial\alpha}>0}-\underbrace{\frac{\left(c_I-c_E-3\tau^c+3\alpha N^s\right)N^cN^s}{3\tau^c}}_{\frac{\partial\Pi^{nlt*}_{I,2}}{\partial\alpha}<0}$$
$$=\frac{\left(c_E-c_I+3\tau^c-14\alpha N^s\right)N^cN^s}{9\tau^c}$$

If $\alpha=0$, I is indifferent between creating a long-term contract or not. If instead α is positive, an increase in α leads to an increase in I's long-term net benefit up to $\tilde{\alpha}\triangleq\frac{c_E-c_I+3\tau^c}{14N^s}>0$. The reason for small α values is that I is able to extract more from the consumer surplus in the second period by having the sellers on board, which outweighs forgoing the first-period monopoly profits and the second-period profits, which are decreasing in α (an increase in α more readily enables the efficient platform E to capture a larger consumer share). Observe that $\frac{\partial^2\left(\Pi^{lt*}_{I,1}+\Pi^{lt*}_{I,2}\right)}{\partial\alpha^2}=<0$, $-\frac{\partial^2\Pi^{nlt*}_{I,1}}{\partial\alpha^2}=0$, and $-\frac{\partial^2\Pi^{nlt*}_{I,2}}{\partial\alpha^2}<0$. Thus, I's long-term contract net benefit increases in α, conditional on $\alpha<\tilde{\alpha}$, but with decreasing returns.

If α is sufficiently large, i.e., $\alpha>\tilde{\alpha}$, I's net benefit decreases and is negative for $\alpha>2\tilde{\alpha}$. The reason is that even though I has the sellers on board, E's competitiveness through an increase in α reduces I's long-term profits to such an extent that it is not worth anymore to forgo the first-period monopoly profits.

Sellers multi-home and consumers single-home

As in Section 2.3.4, I differentiate between exclusive and non-exclusive long-term contracts when the sellers multi-home.

Exclusive long-term contract. Proposition 2.10, which builds on Proposition 2.4 introducing consumer heterogeneity, states under which conditions I uses a long-term contract that is exclusive.

Proposition 2.10. *If sellers multi-home and consumers single-home and consumers are differentiated à la Hotelling, I creates a long-term contract that is exclusive if and only if* $3\tau^c>\max\{0,c_I-c_E-\alpha N^s\}$.

[22] $\Pi^{lt*}_{I,1}+\Pi^{lt*}_{I,2}-\Pi^{nlt*}_{I,1}-\Pi^{nlt*}_{I,2}=\frac{\alpha\left(c_E-c_I+3\tau^c-7\alpha N^s\right)N^cN^s}{9\tau^c}$

Suppose $3\tau^c < c_I - c_E$. If the cross-group externalities α are sufficiently small, then even if I has the sellers exclusively signed for the second period, I does not attract any consumers. Hence, $\Pi_{I,2}^{lt^*} = 0$. Furthermore, this also unsurprisingly corresponds to the Hotelling case where I does not attract any consumers in the second period absent a contract, i.e., $\Pi_{I,2}^{nlt^*} = (v^s - c_I) N^s$. It follows that $\Pi_{I,1}^{lt^*} = \Pi_{I,1}^{nlt^*} + \Pi_{I,2}^{nlt^*}$, i.e., I's net benefit over the two periods using a contract is zero. If on the other hand α is sufficiently large, then as in Proposition 2.4 I creates a long-term contract. A decrease in competition (a larger τ^c value) raises the likelihood that I creates a contract.

Non-exclusive long-term contract. Lastly, Proposition 2.11, which builds on Proposition 2.5 introducing consumer heterogeneity, states under which conditions I uses a long-term contract that is non-exclusive.

Proposition 2.11. *If sellers multi-home and consumers single-home and consumers are differentiated à la Hotelling, I creates a long-term contract that is non-exclusive if* $v^s \leq 2$ *and* $3\tau^c > c_I - c_E + \alpha\left(\frac{3-v^s(2-v^s)}{2v^s-1}\right)N^s$ *or if* $v^s > 2$ *and* $3\tau^c > c_I - c_E + \alpha N^s$.

If $3\tau^c < c_I - c_E$, I never creates a long-term contract, independent of the magnitude of α (note that $\frac{3-v^s(2-v^s)}{2v^s-1} > 1 \; \forall v^s$). Thus, Proposition 2.5 continues to hold for sufficiently small τ^c. This corresponds to the situation where I does not attract any consumers in the second period, both in the presence and absence of a contract. Therefore, similar to the *exclusive* context, I has no additional gain on the consumer side and in turn on the seller side using a contract. However, if τ^c is sufficiently large I finds it profitable to use a long-term contract for a given α. If $3\tau^c > c_I - c_E$, an increase in α reduces the likelihood that I uses a contract.

2.5 Conclusions and discussion

In this chapter, I studied the use of a long-term contract by an incumbent platform I in a two-sided market as an anti-competitive instrument to (partially) foreclose entry by a more efficient platform E. I operates for a period prior to the arrival of E and can credibly commit to refuse to sell in the first period without a contract. Whether a long-term contract is an anti-competitive instrument crucially depends on the homing assumptions of the two sides and cross-group externalities. I show that the Chicago critique holds if sellers single-home and consumers can multi-home or if cross-group externalities are absent, but mostly fails to hold otherwise.

Since the consumers' interests are not represented in the bargaining between I and sellers but are affected by the outcome due to cross-group externalities, the Chicago critique may fail to hold in the present setting. Whenever I finds it profitable to create a long-term contract, it compensates the sellers for forgoing E's attractive fees but recoups this incentive cost from the consumer side. The foreclosure of a more efficient platform due to a long-term seller contract always decreases consumer surplus and is never welfare-improving. These results justify an intolerant competition policy stance towards long-term contracts in two-sided markets.

I consider several extensions. (*i*) I demonstrate that a long-term contract becomes more lucrative for I if I can credibly commit not to sell without a contract in both periods instead of only in the first period. By forfeiting the second period to E, the incumbent has to compensate the sellers by a smaller amount to sign a contract. (*ii*) I introduce heterogeneity on the consumer side, and (*iii*) I explore the consequence of non-negative fees. The latter increases the incumbent's likelihood to profitably use a long-term contract when sellers single-home, since it constrains the entrant's ability to subsidize the sellers absent a contract, and consequently decreases the amount with which the incumbent has to compensate the sellers.

I believe that the model provides a simple and intuitive rationale for the use of long-term contracts as a barrier to entry in two-sided markets. While the pre-entry period of operation stresses the nature of long-term contracts, the model does not rely on aspects pointed out in earlier work (Aghion and Bolton, 1987; Rasmusen et al., 1991; Segal and Whinston, 2000; Fumagalli and Motta, 2008; Asker and Bar-Isaac, 2014; Calzolari et al., 2020) to show that the Chicago critique may fail. However, it would be of interest to incorporate several of these aspects into the model here, such as the well-known coordination challenge between consumers in two-sided markets (Caillaud and Jullien, 2003; Hagiu, 2006; Hagiu and Spulber, 2013).

For exposition purposes, in Sections 2.3.2 through 2.3.4 I assume that agents on at least one side *can* only single-home. While this is in line with some of the early two-sided market literature that assumes for exogenous reasons an agent's homing decision (Armstrong, 2006), in practice agents do get to decide whether to visit one or multiple platforms. Similar to Armstrong and Wright (2007), the "decision" to single-home can be reconciled in the model by assuming that an agent derives a low or zero intrinsic benefit from visiting a second platform due to overlap (for example functionality overlap in the case of game consoles or content overlap by newspapers) or time constraints (a shopper might only have time to go to one shopping mall).

That is, agents on one side could in principle multi-home but would never do so because of the large discount going to a second platform. Furthermore, in practice often some agents on one side single-home whereas others multi-home, as for example in the case of newspapers. It would be thus of interest to examine the impact of partial multi-homing on I's decision to use a long-term contract. To do so, a simple extension is to assume that a fraction of agents on a given side single-homes but the others multi-home. For instance, some drivers have larger costs than others adapting to multiple ride-hailing platforms to fulfill the varying requirements. Alternatively, one could endogenize an agent's homing decision as in the preceding chapter.

Further insights could be developed by adding additional periods to study optimal contract lengths. Suppose that there are $t = 1, ..., T$ periods such that $T \geq 3$ and E's entry threat exists from the second period onward. A long-term contract of any length is never socially optimal. However, I might find it the most profitable to incentivize sellers to sign a contract for T periods. In the case where sellers multi-home and consumers single-home, I can then extract the sellers' cross-group externalities for $T - 1$ instead of one additional periods through an exclusive long-term contract. Similarly, I can extract $c_E + \alpha^c N^s$ from consumers, which it would not have been able to do without a contract, at cost c_I in each of the $T-1$ additional periods. This suggests that whenever I finds it profitable to use a long-term contract in a two-period context (Proposition 2.4), I would create an exclusive contract of T periods for sellers. I leave the full analysis as a future extension.

Another extension worth pursuing is to assume that a new set of agents arrives in each period, such that the agents from the first period are an installed base at the incumbent platform in the second period (Vasconcelos, 2015). For instance, consumers that visit I in the first period can be thought of as having bought a durable product (for example, a game console) and do not consider buying again in the second period. They are captive to I and can still interact with sellers in the second period (in the example, these consumers still buy new games from sellers in the second period), but I cannot charge them a second-period fee. In that scenario, (partial) exclusion of E through the use of a long-term contract may be socially efficient, which was never the case in the baseline model. On the one hand, production inefficiencies arise as before if all sides are served by I under a long-term contract. On the other hand, society is better-off when both "old" and "new" consumers are served by I due to cross-group externalities (if sellers single-home, they can interact with all consumers). If the latter dominates the former, a long-term contract is socially optimal if I cannot attract the new set of consumers otherwise.

Finally, an assumption in the baseline model is that E's entry threat in the second period is certain and known by all players in the first period. Yet in reality, I and the consumers and sellers cannot be sure that indeed a more efficient platform E such as a new gaming console, shopping mall, video streaming service, or ride-hailing service will appear in the second period (Gavin and Ross, 2018). This might be especially true for relatively saturated markets (e.g., operating systems, credit card networks). Furthermore, I conceivably has superior knowledge than the sellers about the possible arrival of E. On the one hand, the more likely it is that E appears, the better are sellers off in the second period by having access to E. Then, I has to compensate the sellers by a larger amount for signing the contract, which reduces I's incentive to use a contract. On the other hand, an increase in the likelihood of E's arrival reduces the probability that I makes monopoly profits in the second period, which makes it less attractive for I *not* to use a contract. A thorough analysis of the impact of the assumption that E only appears with some probability is left for future research.

Chapter 3

Quality Investment by Platforms

3.1 Introduction

Even though many industries have been transformed in recent decades with the proliferation of two-sided platforms, little is known about the impact of the market structure on investment incentives (Belleflamme and Peitz, 2010), and in particular on investment in a platform's quality. Yet any platform develops or improves upon its level of technology such that it is increasingly attractive, in terms of functionality, for consumers and sellers to join and to interact with one another. That is, other than setting the optimal fees to the two sides, the decision to innovate is one indispensable dimension of the platform's strategic focus and critically impacts the likelihood of success of a platform in a given industry (Lin et al., 2011).

For example, innovation activities by credit card payment systems (MasterCard, Visa) that directly benefit consumers include the development and employment of comprehensive data security tools and protocols to protect the sensitive data of their customers and hence their trust, mobile payment capabilities, and data analysis tools to gain insights into customers' behaviors to identify their needs (Rysman and Schuh, 2017). Ride-hailing services (Uber, Lyft) undertake continuous research and innovation efforts to develop and improve upon vehicle routing models for the pickup and drop-off of passengers, matching of passengers with drivers, and payment methods (Furuhata et al., 2013; Ordónez and Dessouky, 2017).

Consumers on an e-commerce platform (such as Amazon) derive higher utility from shopping for products the easier and more intuitive the interface is to find products, the better tailored are the recommendations of similar products for them, and the better the platform can convince its customers that the online payment system is fast and secure. Video-game console platforms (Xbox, PlayStation) invest resources to attract players to their platforms not only by creating graphical performance, and hardware in general, of superior quality (Zhu and Iansiti, 2012; Tan et al., 2020), but also by providing features such as being able to play online.

In the mentioned industry examples, developing such platform characteristics and features all require costly investments by platforms. Studying investment by a two-sided proprietary platform in its own platform quality that directly benefits consumers and comparing it to the first-best solution is the first purpose of this chapter. Since network effects are a defining characteristic in two-sided markets, complex effects on the innovation incentives for platforms may arise (Rysman, 2009; Lin et al., 2011). That is, by investing in technology that enhances the functionality of the platform for the consumer side, participation on that side affects participation by the other side.

I model a monopolistic two-sided platform that enables direct interactions between consumers and sellers. Consumers are heterogeneous in their adaptation costs and sellers are heterogeneous in their marginal costs producing their product or service. The platform decides how much to invest in the quality of its platform before setting access fees for each user side. Investment in the platform's quality directly affects consumers but not sellers. After consumers and sellers have decided whether or not to visit the platform, the sellers' product prices are determined in a competitive fashion. Consumers value variety but consider products as interchangeable (Casadesus-Masanell and Llanes, 2015). I study the case in which platform quality and the number of products are complements. In addition, I assume that consumers form *passive rational* expectations about the adaptation decision by sellers to join a given platform (see Appendix C.1 for details).

I find that while the monopolistic two-sided platform does internalize cross-group externalities, it does so imperfectly when setting its fees. First, it does not fully appropriate the surplus that is generated between the two sides' interactions by focusing on the marginal users, both on the consumer and seller side, rather than the average participating agent as in the case of the social planner (Spence, 1975; Weyl, 2010). By the two-sided market logic (e.g., Armstrong, 2006), the consequent fees are too high, and too few agents on each side visit the platform relative to the first-best outcome. Second, the underprovision of product varieties and consumers is amplified by the passive expectations assumptions of consumers. Since a change in the number of sellers through a change in the fees is not accounted for by consumers, the marginal cost of marginally increasing the fees is lower and hence equilibrium fees too high.

These distortions carry over to the investment-in-quality decision by the platform. In particular, the benefit of investing is composed of directly increasing the marginal but also the inframarginal value of the platform to consumers and indirectly for sellers. Since the number of consumers and sellers that visit the two-sided platform is lower compared to the first-best levels, the platform's benefit from investing is lower.

As one main distortion in investment is caused by the market power the monopolistic platform has (e.g. Mussa and Rosen, 1978; Besanko et al., 1987), I also present the duopoly case in which two proprietary two-sided platforms compete against each other to attract the two sides. In order to make the monopoly and duopoly environment comparable, I adopt the Hotelling model with hinterlands on the consumer side (Armstrong and Wright, 2009). In addition, I assume that sellers can multi-home, i.e., they can visit both platforms, and consumers single-home, i.e., they can visit at most one platform. I find that the introduction of competition leads to larger investments by platforms in their respective platform quality, since investing, other than competing in fees, serves as an additional instrument to attract the two sides.

The second purpose of this chapter is to model investment incentives of a multi-product firm (henceforth, MPF), both in a monopoly and a duopoly market environment. In the MPF mode, a firm rather than the sellers produces a variety of products. It is conceivable that an organization positions itself as an MPF instead of a two-sided platform in several of the industries mentioned earlier: firms producing operating systems could develop software themselves and video-game platforms could produce games for their respective consoles. In fact, video-streaming services (Netflix, Disney+) create original content in addition to licensing content from studios. In the e-commerce business, Amazon, for instance, offers close to $23,000$ products from its own label brands (May, 2020).[1]

The natural question then is how an organization that operates in the MPF mode differs, if at all, along the investment dimension compared to when the organization operates in the two-sided mode. I assume that an MPF still operates with customers through the use of its platform. Hence, the conceptual idea of investing

[1] https://www.digitalcommerce360.com/2020/05/20/amazon-triples-its-private%E2%80%91label-product-offerings-in-2-years/

in a platform's quality to enhance the functionality remains the same. The MPF decides which varieties to produce and sells them to the consumers itself. As before, the MPF decides how much to invest in the quality of its platform before setting the access fee for consumers.

I find that an MPF invests relatively more than its two-sided platform counterpart, both in the monopoly and duopoly market environments. The simple reason is that an MPF, by producing the product varieties itself, takes into account the average cost and benefit of the products. That is, the MPF better internalizes the externalities product varieties have for consumers. As a consequence, the MPF produces more varieties and attracts more consumers through a lower fee. The implication for the investment stage is that the MPF has a larger marginal benefit from investing to attract additional consumers. However, the MPF still underinvests relative to the first-best outcome as it still only focuses on the preferences of the marginal consumer.

Related literature

The chapter contributes to the theoretical literature on two-sided markets. Compatible with this literature is the view that a platform enables direct interactions between consumers and sellers and, thus, can charge access fees on both sides of the market, while at the same time sellers remain in control over the price of their goods, a key term of the transactions (Hagiu and Wright, 2015b). The platform intermediary aggregates demand and balances the two sides by accounting for cross-group externalities when setting its fees. Seminal contributions are Caillaud and Jullien (2003), Rochet and Tirole (2003, 2006), Armstrong (2006), Armstrong and Wright (2007), Hagiu (2006), Rysman (2009), Choi (2010), and Weyl (2010).

Incentives to invest in platform quality by two-sided platforms versus multi-product firms versus a social planner have not been analyzed before; neither in a monopoly nor in a duopoly market environment setting. My set-up borrows from Casadesus-Masanell and Llanes (2015) regarding how agents on the two sides are modeled and affected by platform quality. The present chapter differs in three key aspects. First, instead of comparing an open-source to a proprietary two-sided platform, I compare investment incentives by a two-sided platform to an MPF.[2]

[2]In an open-source operating system, investments that enhance the platform's quality are made by the application sellers in Casadesus-Masanell and Llanes (2015). One result is that the investment level in an open platform may be higher than in a proprietary platform. A reason for

While modeling an open-source platform is appealing for the operating systems industry, in most other industries it is conceivable that the alternative organization mode to the two-sided platform mode is the MPF set-up as pointed out earlier. Second, I present the analysis for the duopoly in addition to the monopoly market environment (which is, therefore, conceptually similar to, e.g., Arrow, 1962). Indeed, one observes the presence of competing platforms in many, if not most, two-sided markets (e.g., ride-hailing services, video game consoles, e-commerce). Third, I assume that consumers form *passive* instead of responsive expectations.

While I study how investment decisions directly affect consumers, another strand in the literature examines how *sellers* are affected by investments in two-sided markets. Investment is undertaken either by sellers themselves and can for instance take the form of a cost reduction, quality improvement, or marketing measures that enhance the demand side (Hagiu, 2007; Belleflamme and Peitz, 2010; Lin et al., 2011), or by platforms that invest to reduce the costs for sellers to produce for the platform (Tan et al., 2020).

More generally, my paper contributes to the literature on the role played by the platform's quality. Several papers assume that a platform's quality is exogenously given and study competition in either a static (Mantena and Saha, 2012) or a dynamic (Zhu and Iansiti, 2012; Halaburda et al., 2020) model set-up. While several papers explicitly model investment in the platform's quality as a decision variable (Bakos and Katsamakas, 2008; Grossmann et al., 2016; Jeon and Nasr, 2016; Reisinger and Zenger, 2019), they do not consider investment outcomes across organization modes or across market environments.

My paper also contributes to an emerging literature in two-sided markets that compares the two-sided platform business mode with various alternative modes, and in particular with a reseller (Hagiu, 2007; Hagiu and Wright, 2015a), a vertically integrated firm (Hagiu and Wright, 2015b, 2019), and an agency or wholesale pricing mode (Johnson, 2017). In these papers, the key distinction between the business models is the delegation of control rights over factors that determine overall demand, such as the pricing of the goods, bundling, marketing activities, and the terms and conditions. Although I solely focus on the product price as the key term of interaction, I make the distinction of who produces the goods or services. I further contribute work in this area by endogenizing investment incentives across different

that is that an open platform may attract a larger number of users, which increases investment incentives. Another possible source is that if a decrease in the number of sellers lowers competitive pressure among them, the incentives to invest for sellers increases.

types of business models.

Finally, the present work is broadly related to the extensive literature on vertical integration (see Gibbons, 2005, for an overview). The reason is that the MPF platform in my set-up can be interpreted as a firm that is vertically integrated with all of the potential upstream sellers such that the platform controls over all of the production steps. However, I study "make-or-enable" comparisons rather than "make-or-buy" outcomes. The key difference in the economic analysis is the following: in the "make-or-buy" (vertical integration) decision, the platform contracts with and controls the sale to consumers (Hagiu and Wright, 2015b). By contrast, in the latter case of the "make-or-enable" comparison, the platform merely enables contractual relationships between sellers and buyers to which the platform is not a party.

The rest of this chapter is organized as follows. I set up the model in Section 3.2. In Section 3.3, I first analyze and compare the social planner outcome to the two organization modes (two-sided platform and MPF) in the monopoly market environment. I then repeat the analysis for the duopoly case before deriving several welfare implications. I conclude and highlight several possible extensions in Section 3.4. Appendix C complements the analysis in the main text.

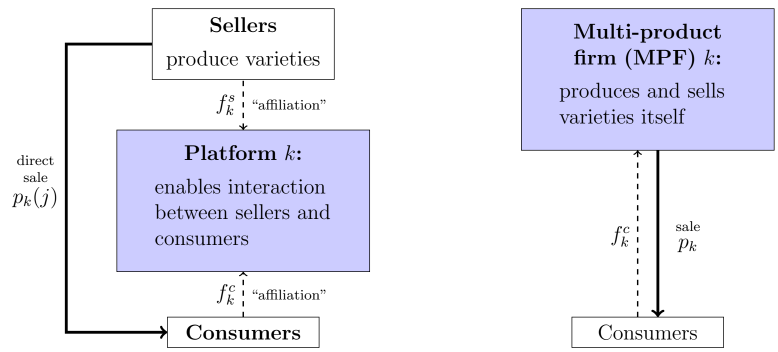
3.2 The Model

I study monopoly and duopoly competition of organizations under different modes of operation. The justification and discussion of several model assumptions are relegated to Appendix C.1. In the two-sided operation mode, platforms enable sellers to provide their service or product directly to consumers.[3] A platform is a necessary condition for consumers and sellers to be able to interact; sellers cannot sell their products themselves to consumers. Sellers and consumers are merely affiliated with a platform.[4] In the one-sided operation mode, a multi-product firm (MPF) produces the services or products itself and sells them directly to the consumers. Thus and in contrast to the two-sided mode, sellers do not exist in the one-sided operation mode. Figure 3.1 illustrates.

[3]That is, the sellers remain in control over the key terms of the transaction, i.e., the price of the good or service, as opposed to the intermediary platform taking control. I omit other terms such as bundling, marketing, or quality of the goods or services provided (Hagiu and Wright, 2015b).

[4]In the sense that they make deliberate platform-specific investments in order to interact with one another directly. Consumers for example need to buy a videogame console or spend time

Figure 3.1: Platform organization modes



(a) Two-sided mode (b) One-sided mode

I focus on the organizations' incentives to invest in the quality of the platform itself, i.e., the software or hardware that constitutes the platform. In particular, I assume that the investment in quality solely *directly* benefits the consumer's utility function from interacting with sellers. I provide examples in the introduction. Sellers *indirectly* benefit from investment in the quality, as the platform becomes more attractive to sellers the more attractive it is to consumers. I do not study incentives to invest *directly* in quality on the seller side, which has been studied elsewhere (Belleflamme and Peitz, 2010; Hagiu, 2007; Lin et al., 2011).

Each mode of operation consists of up to three types of agents: consumers, sellers, and platforms, each of which are presented next for the duopoly settings. At the beginning of Section 3.3.1 I present the monopoly adjustments.

Consumers. There exists a continuum of consumers. Each consumer is interested in joining at most one platform, i.e., consumers single-home. Independent of the operation mode, the gross indirect utility of a consumer visiting platform $k = 1, 2$ is

$$u_k = v(n_k^e, A_k) - \int_0^{n_k^e} p_k(j)dj - f_k^c,$$

where n_k^e is the expected measure of available products, $v(n_k^e, A_k)$ is the utility of consuming n_k^e products at quality A_k that the organization k invested in its platform. n_k^e is the consumers' *expected* measure of seller participation on platform k, $p_k(j)$

learning how to use an operating system (OS).

is the product price charged by seller j when the organization is in the two-sided platform operation mode or the price charged by the MPF when the organization is in the MPF mode. Lastly, f_k^c is the fixed access fee for consumers.[5]

I assume that consumers have *passive* expectations related to seller participation (see Appendix C.1 for the motivation). In particular, consumers are unaware of seller fees and thus are unable to adjust their expectations about seller participation n_k^e in response to *any* changes in platform fees. In addition, it is assumed that consumers cannot adjust the seller participation due to a change in the platform quality. Expectations are fulfilled in equilibrium. This in turn means that the platform has no choice but to treat consumers' passive expectations n_k^e as fixed when it sets its fees.

I follow the usual convention of representing partial derivatives through subscripts (e.g., $v_{nA} = \frac{\partial^2 v(n,A)}{\partial n \partial A}$). Assume that $v_A > 0$ and $v_n > 0$, that is, consumers prefer higher quality platforms and product variety, respectively. The investment in platform quality and the measure of applications are complements, i.e., $v_{nA} \geq 0$, and products are substitutes, $v_{nn} < 0$.[6] In other words, consuming more of any product decreases the marginal utility of consuming another product.

Lastly, I assume that consumers are horizontally differentiated.[7] In order to make the monopoly and duopoly market environments comparable, I adopt a "Hotelling model with hinterlands" as in Armstrong and Wright (2009). In particular, the consumer demand for platform k is given by

$$m_k = \underbrace{\frac{1}{2} + \frac{1}{2\tau}(u_k - u_{-k})}_{\text{Standard Hotelling}} + \underbrace{\lambda u_k}_{\text{Hinterland}}, \tag{3.1}$$

where τ is the differentiation cost parameter,[8] and $\lambda \geq 0$ is a parameter that represents the magnitude of market expansion possibilities (the hinterland). If $\lambda = 0$, each platform solely derives consumer demand from the consumers of mass one that are uniformly located on a line of unit length. As in the standard Hotelling model, each of the two platforms is located at one end of the unit line. In addition, if $\lambda > 0$ each platform faces a downward-sloping demand from loyal customers on its side of

[5] A two-part tariff does not affect the results. See Appendix C.2.2 for details.

[6] The cases $v_{nn} = 0$ and $v_{nn} > 0$, which I do not treat here, correspond to, respectively, products being independent and complements.

[7] The random utility model as an alternative formulation to introduce consumer heterogeneity gives the same qualitative results. The analysis is available upon request.

[8] The taste-differentiation cost τ may also be interpreted as an individual specific adaptation cost for using a platform or transportation cost.

the unit interval, referred to as *hinterland*.[9,10]

Assumption 3.1. $v(0,0) \geq \frac{\tau}{2}$

Assumption 3.1 states that the stand-alone utility for any consumer on the unit line is sufficiently large to visit one of the platforms.

Sellers and varieties. In an industry where the organizations operate as a **two-sided platform**, there exists a continuum of potential sellers, $j \in [0, \infty)$.[11] Each seller j can produce one and only one type of product. The platforms are homogeneous in the eyes of sellers, and sellers are allowed to multi-home, i.e., they can join both platforms.[12] The sellers are differentiated by their marginal costs. In particular, seller j has a specific, but constant marginal cost $c(j)$. Sellers are ordered according to the per-transaction cost such that $c_j > 0$ and $c(0) \geq 0$.

Seller j makes profit $p_k(j) - c(j)$ per customer interaction, but also has to pay the fixed seller fee f_k^s for having access to platform k. Thus, the profit of an individual seller j joining platform k is

$$\pi_k(j) = (p_k(j) - c(j))\, m_k^e - f_k^s,$$

where m_k^e is the sellers' *expectations* about consumer participation on platform k.

In an industry where the organizations operate as **MPFs**, the MPFs instead of the sellers produce and sell the products. I remain the assumptions that the potential varieties produced by MPFs are a continuum, $j \in [0, \infty)$, and that the potential varieties produced by an MPF are differentiated by the marginal cost as defined above. Then, MPF k makes profit

$$m_k^e \left(\int_0^{n_k} [p_k(j) - c(j)]\, dj \right)$$

from producing and selling n_k product varieties.

[9]Consumers in each hinterland are loyal in the sense that they can only visit the platform on the side of the unit interval they are on. The loyalty assumption precludes discontinuities in the best reply functions.

[10]The demand from consumers in the hinterland, λu_k, is derived in the following manner. Consumers are also horizontally differentiated (and incur a transportation cost parameter) and are uniformly distributed with density h in each hinterland. The marginal consumer who is indifferent between visiting and not visiting platform k is at distance m_{k_h} from platform k, such that $u_k - \tau m_{k_h} = 0$. Hence, the number of hinterland consumers is $h m_{k_h} = \lambda u_k$ with $\lambda \triangleq \frac{h}{\tau}$.

[11]The assumption of a continuum of sellers is made for the sake of exposition.

[12]I leave the reversal of the homing assumptions for future research. See Section 3.4.

I assume throughout the chapter that all sellers hold *responsive* expectations about consumer participation. In other words, they are fully informed about consumer fees f_k^c, and their expectations for consumer participation for *any* given fee always match actual consumer participation, i.e., $m_k^e = m_k$.

Let $S(n, A) \triangleq v(n, A) - \int_0^n c(j)dj$ be the social surplus of n products and platform quality A, and $s(n, A) \triangleq S_n(n, A) = v_n(n, A) - c(n)$ the marginal social surplus.

Assumption 3.2. *For all A, $s(0, A) > 0$ and $\exists n' > 0 : s(n', A) = 0$.*

The first part of Assumption 3.2 implies that having a strictly positive number of products is always desirable from a social point of view. The second part states that there exists a measure of products n' at which the marginal benefit of an additional product for a consumer equals the marginal cost of that variety.

Platforms. In each mode of operation, two platforms indexed $k = 1, 2$ are located at the ends of the Hotelling line of unit length. Each platform chooses its level of technology A_k at cost $c(A_k)$, where $c'(A_k) > 0$,[13] and its access fee(s). Other than the investment-in-quality cost and the production cost of varieties, where the latter only applies to the MPF mode, I assume that platforms' marginal and fixed costs are zero for enabling transactions between consumers and sellers.

In the **two-sided platform** set-up, each platform's profit is given by the revenue from consumer and seller fees, minus the development cost of its technology level.[14] Thus, platform k's profit is

$$m_k f_k^c + n_k f_k^s - c(A_k),$$

where m_k is given by (3.1). In the **MPF** operation mode, each MPF's profit is given by the revenue from consumer fees plus the profit from producing and selling the product varieties to its consumers, minus the technology investment cost. Hence, MPF k's profit is

$$m_k f_k^c + m_k \left(\int_0^{n_k} [p_k(j) - c(j)] \, dj \right) - c(A_k).$$

[13]Instead of being able to "choose" the level of technology, in Appendix C.2.1 each platform can only incur a costly effort to influence the probability that the level of technology increases from a default level to a higher level. I show that the results carry over.

[14]See Appendix C.1 for the discussion of an alternative interpretation of the two-sided platform set-up.

Timing. The timing of the game is as follows (and similar to Casadesus-Masanell and Llanes, 2015). First, each platform k chooses simultaneously A_k $\forall k = 1, 2$. Once the organizations have chosen their technology level, they become public knowledge. Second, each platform k chooses $(f_k^c, f_k^s), \forall k = 1, 2$ in the two-sided operation mode. In the MPF operation mode, MPF k sets f_k^c. Third, consumers and sellers simultaneously decide which, if any, platform(s) to join in the two-sided platform mode. In the MPF operation mode, each MPF chooses how many varieties n_k to produce and consumers decide which, if any, platform to join. Fourth, in the two-sided mode each seller $j \in [0, n_k]$ sets $p_k(j)$. In the MPF operation mode, MPF k sets $p_k(j)$ for each j it sells. Finally, consumers make their purchasing decisions.

The timing, according to which platforms decide on their investment in the technology level before choosing access fees, reflects the observation that the quality decision is rather a long term-decision compared to access fee decisions. The solution concept is subgame-perfect equilibria. Any proof not stated in the main text is relegated to Appendix C.4.

3.3 Analysis

3.3.1 Monopoly

In the monopoly market environment, the platforms do not compete for the consumers in the middle, i.e., the consumers on the Hotelling line between 0 and 1. Instead, half of them are captured by each of the two monopoly platforms and a monopolist can only take action to attract consumers in its own hinterland. The gross utility for a consumer visiting her "assigned" platform takes the same expression as above but with subscript k suppressed, i.e., $u = v(n^e, A) - \int_0^{n^e} p(j)dj - f^c$. Dropping subscript k, (3.1) becomes

$$m = \frac{1}{2} + \lambda \left(v(n, A) - \int_0^n p(j)dj - f^c \right) \tag{3.2}$$

The sellers' profit and a platform's profit and investment cost expressions are as in Section 3.2, but again with subscript k suppressed. Since the set-up for the two isolated monopolists is identical, I focus on deriving the results for one of the monopolists within each mode of operation. Equilibrium outcomes are denoted in the superscript by two asterisks, **, in the two-sided operation mode, by one asterisk, *, in the one-sided operation mode, and by ^{FB} for the first best.

First Best

The per-platform sum of indirect utility and profits is given by

$$\begin{aligned} W^{FB} &= \int_0^{\frac{1}{2}} [u - \tau x]\, dx + h \int_0^{m_h} [u - \tau x]\, dx + \int_0^n \pi(j)dj + mf^c + nf^s - c(A) \\ &= mv(n, A) - \tau \int_0^{\frac{1}{2}} xdx - \tau h \int_0^{m_h} xdx - m \int_0^n c(j)dj - c(A). \end{aligned}$$

Differentiating W^{FB} with respect to (m_h, n, A) and using $m = \frac{1}{2} + hm_h$, the equations characterizing the first-best solution are

$$\begin{aligned} & m^{FB} = \frac{1}{2} + \lambda S(n^{FB}, A^{FB}), \\ & s(n^{FB}, A^{FB}) = 0, \quad \text{and} \\ & m^{FB} v_A(n^{FB}, A^{FB}) - c'(A^{FB}) = 0. \end{aligned} \tag{3.3}$$

By Assumption 3.2, $n^{FB}(A^{FB})$ is strictly positive and unique. According to the social planner, the equilibrium number of consumers on the platform, m^{FB}, equals half the number of consumers on the Hotelling line between 0 and 1 plus the social surplus generated by n^{FB} products and quality A^{FB}, $S(n^{FB}, A^{FB})$, weighted by the market expansion possibility parameter λ. The last product added by the social planner has a marginal social value of zero, i.e., $s(n^{FB}, A^{FB}) = 0$. This gives the measure of products n^{FB}.

Investing in the quality of the platform has the *direct* effect of increasing the value of the platform for each of the m^{FB} consumers, $v_A(n, A) > 0$. The optimality condition of investing in quality states that the marginal benefit equals the marginal cost $c'(A^{FB})$. The equilibrium triple (m^{FB}, n^{FB}, A^{FB}) is determined by the system of equations given by (3.3) and serves as a benchmark for the analyses of the subsequent organization modes.

Observe that investing in the quality of the platform also has an *indirect* effect on the product variety side: $\frac{dn^{FB}}{dA} = \frac{v_{nA}(n,A)}{c_n(n) - v_{nn}(n,A)} > 0 \ \forall A$ because $v_{nA}(n, A) > 0$ and $v_{nn}(n, A) < 0$ by assumption; the higher the quality of the platform A^{FB}, the larger the number of varieties n^{FB}. The intuition is that the marginal social surplus $s(n, A)$ increases in A and thus n needs to be larger as well for $s(n, A) = 0$ to be satisfied.

Two-Sided Platform.

Consider the two-sided operation mode in which a monopolistic platform allows individual sellers to interact directly with consumers. Let $p^{**}(j)$ denote the **stage four** equilibrium price of the product by seller j. Since products are substitutes by assumption, the highest price a seller may charge is its marginal value for consumers, $v_n(n, A)$, on the platform (Casadesus-Masanell and Llanes, 2015). The argument is as follows: if the price of a product would be strictly greater than the marginal value of the product, consumers would be better off not consuming that product. If the price would be strictly lower than the marginal value, the seller could increase the price by a small amount $\epsilon > 0$, and all consumers on the platform would still purchase that product, giving the seller a strictly greater profit. Hence, $p^{**}(j) = v_n(n, A)$ for all j. The implication is that each consumer purchases all available products on the platform (see Appendix C.1 for further details).

Using the equilibrium product prices from the previous stage in (3.2), the measure of consumers at **stage three** is given by

$$m = \frac{1}{2} + \lambda \left[v(n^e, A) - n^e v_n(n^e, A) - f^c \right] \tag{3.4}$$

Due to the assumption that consumers have passive expectations regarding fees, at this stage consumers *ex-ante* only have an expectation about the number of sellers on each platform (n^e). Seller j joins the platform if and only if $\pi(j) \geq 0$. Ordering the sellers according to their marginal costs, the last seller to enter, indexed by n, also dictates the number of sellers that enter and is given by the expression

$$s(n, A)m - f^s = 0. \tag{3.5}$$

The marginal seller entrant n obtains zero profit in equilibrium, whereas any seller indexed $j \in [0, n)$ makes a strictly positive profit.

From equations (3.4) and (3.5), I obtain the inverse demand functions

$$f^c = \frac{1}{2\lambda} + v(n^e, A) - n^e v_n(n^e, A) - \frac{m}{\lambda} \text{ and } f^s = s(n, A)m$$

Since there is a unique pair of fees (f^c, f^s) for each pair (m, n), choosing the optimal m and n is equivalent to choosing the optimal f^c and f^s by the platform to maximize its profits, $f^c m + f^s n$, at the **second stage**. Thus, the platform's maximization problem can be written as

$$\max_{\{m,n\}} \left\{ m \left(\frac{1}{2\lambda} + v(n^e, A) - n^e v_n(n^e, A) - \frac{m}{\lambda} + ns(n, A) \right) \right\} \tag{3.6}$$

The first-order conditions with respect to (m, n) become[15]

$$m = \frac{1}{4} + \frac{\lambda}{2} \left[v(n^e, A) - n^e v_n(n^e, A) + ns(n, A) \right] \text{ and } s(n, A) + ns_n(n, A) = 0.$$

Observe that n, determined by the second first-order condition, is independent of m.[16] By imposing the rationality condition $n^e = n^{**}$ and solving, Lemma 3.1 follows.

Lemma 3.1. *The set of implicit fees charged by the monopolistic platform for a given level of technology A at the second stage are*

$$f^{c^{**}} = \frac{m^{**}}{\lambda} - n^{**} s(n^{**}, A), \tag{3.7}$$

$$f^{s^{**}} = m^{**} s(n^{**}, A), \tag{3.8}$$

where

$$m^{**} = \frac{1}{4} + \frac{\lambda}{2} \left[v(n^{**}, A) - n^{**} c(n^{**}) \right] \tag{3.9}$$

and the unique indifferent seller n^{**} is implicitly determined by

$$s(n^{**}, A) + n^{**} v_{nn}(n^{**}, A) - n^{**} c_n(n^{**}) = 0. \tag{3.10}$$

A platform will target consumers more aggressively the larger are the market expansion possibilities for the platform, λ, and the more important consumers are to the other side, the sellers (as, for instance, in Armstrong, 2006).

While the platform does internalize cross-group externalities when setting its fees, as seen in equations (3.7) and (3.8), it does so imperfectly. On the seller side, two

[15] To ensure that the second order conditions are satisfied, I impose the condition that for a given level of technology A,

$$2v_{nn}(n, A) - 2c_n(n) + nv_{nnn}(n, A) - nc_{nn}(n) < 0,$$

from which it follows that $ns(n, A)m$ is strictly concave in n.

[16] In addition, the equilibrium n is strictly positive and unique. By the first part of Assumption 3.2, $\left.\frac{\partial ns(n,A)}{\partial n}\right|_{n=0} > 0$. By the second part of Assumption 3.2 and noticing that $s_n(n, A) < 0$, because $v_{nn}(n, A) < 0$ and $c_n(n) \geq 0$ by assumption, for $n'_k > 0 : s(n'_k, A) = 0$ it follows that $\left.\frac{\partial ns(n,A)}{\partial n}\right|_{n=n'} < 0$. Since $\frac{\partial ns(n,A)}{\partial n}$ is a continuous function in n, at least one n candidate exists where $\frac{\partial ns(n,A)}{\partial n} = 0$. By the strict concavity assumption, $\frac{\partial ns(n,A)}{\partial n}$ is strictly decreasing in n. Hence there exists a unique value $n^{**} \in (0, n') : g_n(n, A) = 0$.

distinct mechanisms create distortions relative to the first best. First, the platform only takes into account the marginal cost of the marginal seller, $c(n^{**})$, rather than the average marginal cost of all product varieties to be offered as the social planner does. This is the reason for the additional term $-n^{**}c_n(n^{**})$ in equation (3.10). Second and due to the passive expectations assumption by consumers, the impact of a change in the number of sellers through a change in the fees is not accounted for by consumers, i.e., $\frac{\partial}{\partial n}(v(n^e, A) - n^e v_n(n^e, A)) = 0$. This is the reason for the additional term $n^{**}v_{nn}(n^{**}, A)$ in equation (3.10).

On the consumer side, the first distortion stems from insufficiently accounting for the importance of sellers and its resulting implication for the consumer fee. In particular, the platform, instead of taking into account the average marginal cost of the sellers as the social planner does, only focuses on the marginal seller's cost. As a result, the consumer's fee is inefficiently high, causing too few consumers to join relatively to the first-best outcome.[17] The second distortion is caused by the classic, familiar market power of any monopolist (e.g. Mussa and Rosen, 1978; Besanko et al., 1987): the two-sided proprietary platform takes into account only the preferences of the marginal consumer rather than the preferences of the average participating consumer.

Lastly, it follows from (3.10) that the sign of $\frac{dn^{**}}{dA} = \frac{v_{nA} + nv_{nnA}}{2(c_n(n) - v_{nn}) + n(c_{nn}(n) - v_{nnn})}$ is indeterminate; even though the denominator is strictly positive by the concavity assumption for this stage, $v_{nA} + nv_{nnA}$ can be either positive or negative without further (functional form) assumptions. In Appendix C.3 I show that when one assumes that $v(n, A)$ takes the CES functional form, $\frac{dn^{**}}{dA} > 0$.

Using the fee expressions from Lemma 3.1, the platform's **first stage** maximization problem to choose the optimal level of technology can be written as

$$A^{**} = \underset{A}{\operatorname{argmax}} \left\{ \frac{1}{\lambda} (m^{**})^2 - c(A) \right\} \text{ s.t. (3.9) and (3.10).}$$

Lemma 3.2 states the technology level's optimality condition.

Lemma 3.2. *The monopolistic platform's optimal investment condition at the first*

[17]To see this, observe from (3.7) that the consumer fee is adjusted downwards by $n^{**}s(n^{**}, A)$. If the two-sided platform instead were to take into account the average marginal cost of the sellers as the social planner does, the consumer fee would be adjusted downwards by $n^{**}v_n(n^{**}, A) - \int_0^{n^{**}} c(j)dj$. Hence for a given (n^{**}, A), $f^{c^{**}} = \frac{m^{**}}{\lambda} - n^{**}s(n^{**}, A) > \frac{m^{**}}{\lambda} - \left[n^{**}v_n(n^{**}, A) - \int_0^{n^{**}} c(j)dj\right]$.

stage is

$$\left[\frac{1}{4}+\frac{\lambda}{2}\left(v(n^{**},A^{**})-n^{**}c(n^{**})\right)\right]v_A(n^{**},A^{**})-c'(A^{**})=0, \tag{3.11}$$

where n^{**} *is determined by* (3.10).

Proof. The first order condition of the first stage maximization set-up with respect to A is $\frac{2}{\lambda}m^{**}\left(\frac{dm^{**}}{dA}\right)-c'(A)=0$, where m^{**} is determined by (3.9) and n^{**} by (3.10). Taking total differentials of the stage two equation (3.9) with respect to A, and noticing that $\frac{dn^e(A)}{dA}=0$ because of the passive expectations assumption, it immediately follows that $\frac{dm^{**}}{dA}=\frac{\partial m^{**}}{\partial A}+\frac{\partial m^{**}}{\partial n^e(A)}\frac{dn^e(A)}{dA}=\frac{\partial m^{**}}{\partial A}=\frac{\lambda}{2}v_A(n^{**},A)$. In equilibrium, expectations are fulfilled imposing the rationality condition: $n^e(A)=n(A)$. This gives equation (3.11).

For the second order condition to be satisfied, I impose the condition $\frac{\lambda}{2}\left[v_A(n^{**},A)\right]^2+m^{**}v_{AA}(n^{**},A)-c''(A)<0\ \forall A\geq 0$, from which it follows that $\frac{1}{\lambda}\left(m^{**}\right)^2-c(A)$ is strictly concave in A. Equivalently, the condition states that $m^{**}v_A(n^{**},A)-c'(A)$ is strictly decreasing in A. The second order condition is satisfied if λ is not too large, $c(A)$ is convex enough, or $v_A(n^{**},A)$ is sufficiently small. □

As in the first-best outcome, the marginal benefit of investing in technology is composed of increasing the value of the platform for a consumer, $v_A(n,A)$, multiplied by the number of consumers m. The equilibrium condition states that this marginal benefit equals the marginal investment cost. At the same time, m^{**} and n^{**} are implicitly determined by equation (3.9) and equation (3.10), respectively. Since both m^{**} and n^{**} are distorted, as explained above, compared to their first-best counter parts, the equilibrium level of technology A^{**} is distorted as well (compared to A^{FB}). I compare A^{**} to A^{FB} at the end of this subsubsection.

Multi-Product Firm.

Consider now the one-sided market in which the MPF chooses the varieties it produces and sells to the consumers. By the same arguments as in the two-sided market model, the MPF at **stage four** sets the price $p^*=v_n(n,A)$ for each of the varieties it produces. At **stage three**, the measure of consumers visiting the MPF is also given by equation (3.4). The MPF chooses how many *types of products* to produce to maximize $m\left(v_n(n,A)n-\int_0^n c(j)dj\right)$. Assuming for now that the number of consumers visiting the platform is strictly positive, the first order condition is given by

$$s(n,A)+nv_{nn}(n,A)=0.$$

There exists a unique equilibrium $n^* > 0$ that satisfies the first order condition, the proof of which is analogous to the steps in footnote 16.[18] Starting from its "core competence" variety (Eckel et al., 2015), i.e., the variety with the lowest cost, $c(0)$, the MPF adds new varieties up to the point where the marginal benefit (additional revenue per variety and per consumer, v_n) equals the marginal cost of producing that last variety, $c(n)$, and the marginal loss in revenue per consumer from all varieties produced because of the price reduction due to substitutability, $nv_{nn} < 0$.

At the **second stage**, the MPF chooses the consumer fee f^c to maximize profits $f^c m + m\left(v_n(n, A)n - \int_0^n c(j)dj\right)$.[19] By replacing the consumer fee with the inverse demand function using (3.4), the platform's maximization set-up becomes

$$\max_{\{m\}} \left\{ m\left[\frac{1}{2\lambda} + v(n^e, A) - n^e v_n(n^e, A) - \frac{m}{\lambda}\right] + m\left[v_n(n, A)n - \int_0^n c(j)dj\right]\right\}$$

The first order condition with respect to m can be written as

$$m = \frac{1}{4} + \frac{\lambda}{2}\left[v(n^e, A) - n^e v_n(n^e, A) + nv_n(n, A) - \int_0^n c(j)dj\right]$$

The rationality condition, $n^e = n^*$, is then applied to obtain the final equilibrium; Lemma 3.3 follows.

Lemma 3.3. *The implicit consumer fee charged by the monopolistic MPF for a given level of technology A at stage two is*

$$f^{c^*} = \frac{m^*}{\lambda} - \left(n^* v_n(n^*, A) - \int_0^{n^*} c(j)dj\right), \tag{3.12}$$

where

$$m^* = \frac{1}{4} + \frac{\lambda}{2} S(n^*, A) \tag{3.13}$$

and the number of varieties n^* is implicitly determined by

$$s(n^*, A) + n^* v_{nn}(n^*, A) = 0. \tag{3.14}$$

[18]To ensure that the second order condition is satisfied, I impose the condition that for a given level of technology A, $2v_{nn}(n, A) + nv_{nnn}(n, A) - c_n(n) < 0$, from which it follows that $m\left(v_n(n, A)n - \int_0^n c(j)dj\right)$ is strictly concave in n. Note that if $2v_{nn}(n, A) + nv_{nnn}(n, A) - c_n(n) < 0$ holds, then $2v_{nnn}(n, A) + nv_{nnn}(n, A) - 2c_n(n) - c_{nn}(n) < 0$ also satisfied, which is the second order condition for the two-sided operation mode.

[19]Alternatively, one can collapse stages two and three and write the optimization problem as

$$\max_{\{m,n\}} \left\{ m\left[\frac{1}{2\lambda} + v(n^e, A) - n^e v_n(n^e, A) - \frac{m}{\lambda}\right] + m\left[v_n(n, A)n - \int_0^n c(j)dj\right]\right\}.$$

The crucial difference between the two-sided organization mode and the MPF mode is that the MPF, by producing the product varieties itself, internalizes, like the social planner, the average marginal cost of producing these varieties, rather than focusing on the marginal cost of the marginal seller as the two-sided platform does. As a direct consequence, the term $-nc_n(n)$ is not present in (3.14), which is in contrast to (3.10). Thus, and as Lemma 3.4 shows, the MPF sells more product varieties than the two-sided platform. A further consequence is that the MPF properly adjusts the consumer fee downwards, namely by the direct value added by varieties to the MPF, i.e., by $v_n(n^*, A)n^* - \int_0^{n^*} c(j)dj$. Thus the first distortion related to m^* as in the two-sided platform case is not present here.

However, the number of varieties the MPF produces is still inefficiently low relative to the first-best outcome because of the passive consumer expectations - as documented by the additional term $n^* v_{nn}(n^*, A)$ in equation (3.14). In addition and due to its market power, the MPF still only considers the transportation cost of the marginal consumer rather than the transportation cost of the average participating consumer. Lemma 3.4 formally summarizes these comparison considerations.

Lemma 3.4. *In the monopoly environment and for a given level of technology A, $n^{FB}(A) > n^*(A) > n^{**}(A)$ and $m^{FB}(A) > m^*(A) > m^{**}(A)$.*

Hence, my set-up offers a simple rationale as to why the number of sellers on a two-sided platform in equilibrium is fewer compared to both the number of varieties on an MPF and the number of products chosen by a social planner for a given level of technology, without a commons and equilibrium selection problem (Casadesus-Masanell and Halaburda, 2014).

Using the results from Lemma 3.3, the MPF's **first stage** maximization problem becomes

$$A^* = \underset{A}{\text{argmax}} \left\{ \frac{1}{\lambda} (m^*)^2 - c(A) \right\} \text{ s.t. (3.13) and (3.14).}$$

Following the same steps as in the monopoly two-sided platform case (Lemma 3.2), the first order condition with respect to A is

$$\left[\frac{1}{4} + \frac{\lambda}{2} S(n^*, A^*) \right] v_A(n^*, A^*) - c'(A^*) = 0, \tag{3.15}$$

where n^* is determined by (3.14).[20] As in the previous operation modes, equation

[20]For the second order condition to be satisfied, I impose the condition $\frac{\lambda}{2} [v_A(n^*, A)]^2 +$

(3.15) states that the marginal benefit of investing in technology, composed of increasing the value of the platform for a consumer, $v_A(n, A)$, multiplied by the number of consumers m, equals the marginal cost of investing. Proposition 3.1 summarizes the equilibrium investment level comparisons across the operation modes.

Proposition 3.1. *In the monopoly market environment, $A^{FB} > A^* > A^{**}$ in equilibrium at stage one.*

Proposition 3.1 shows that the two-sided platform and the MPF underinvest into technology relatively to the first best. In particular, the marginal benefit of investing in technology for all operation modes is composed of increasing the value of the platform for a consumer, $v_A(n, A)$, multiplied by the number of consumers m. Since both the MPF and two-sided platform's levels of m and n are lower compared to the first-best levels, the former two modes' marginal benefit from investing is lower. That is, the imperfect internalization of externalities by the two-sided platform and MPF and the distortion due to the consumers' passive expectation assumption from stage two are carried over and amplified by the investment decision at stage one.

The MPF underinvests relatively less than the two-sided platform. By Lemma 3.4, n and consequently m are larger at the MPF platform compared to the two-sided platform at stage two. Thus, the MPF's marginal benefit from investing is larger compared to the two-sided platform's incentive.

3.3.2 Duopoly

In this section, I repeat the three operation modes' analysis for the duopoly market environment and compare outcomes to the monopoly environment. In addition to the superscripts used in Section 3.3.1, I add subscript k to differentiate the equilibrium outcomes between the duopoly and the monopoly market environments. Plugging u_k into (3.1), the measure of consumers visiting duopoly organization mode k is

$$m_k = \frac{1}{2} + \frac{1}{2\tau}\left[v(n_k^e, A_k) - v(n_{-k}^e, A_{-k}) - \int_0^{n_k^e} p_k(j)dj + \int_0^{n_{-k}^e} p_{-k}(j)dj - f_k^c + f_{-k}^c\right]$$
$$+ \lambda\left[v(n_k^e, A_k) - \int_0^{n_k^e} p_k(j)dj - f_k^c\right] \tag{3.16}$$

$m^* v_{AA}(n^*, A) - c''(A) < 0$, from which it follows that $\frac{1}{\lambda}(m^*)^2 - c(A)$ is strictly concave in A. Equivalently, $m^* v_A(n^*, A) - c'(A)$ is strictly decreasing in A.

First Best

The equations characterizing the first best under duopoly are[21]

$$
\begin{aligned}
&m_k^{FB} = \frac{1}{2} + \lambda S(n_k^{FB}, A_k^{FB}),\\
&s(n_k^{FB}, A_k^{FB}) = 0, \quad \text{and}\\
&m_k^{FB} v_A(n_k^{FB}, A_k^{FB}) - c'(A_k^{FB}) = 0.
\end{aligned} \tag{3.17}
$$

Comparing the two first-best systems of equations (3.3) and (3.17), it immediately follows that

$$m_k^{FB} = m^{FB}, n_k^{FB} = n^{FB}, \text{ and } A_k^{FB} = A^{FB} \quad \forall k = 1, 2.$$

That is, the first-best equilibrium outcomes under monopoly and duopoly are identical.[22]

Two-Sided Platform.

Consider now the two-sided operation mode under duopoly. Since stages four and three are similar to the ones in Section 3.3.1, I immediately present the stage two equilibrium fees and number of consumers and sellers in Lemma 3.5.

Lemma 3.5. *The set of implicit fees charged by platform $k = 1, 2$ for given levels of technology A_k at stage two are*

$$
\begin{aligned}
f_k^{c^{**}} &= \frac{m_k^{**}}{\frac{1}{2\tau} + \lambda} - n_k^{**} s(n_k^{**}, A_k),\\
f_k^{s^{**}} &= m_k^{**} s(n_k^{**}, A_k),
\end{aligned}
$$

[21] Let the measure of consumers on platform k on the Hotelling line between 0 and 1 be $m_{k_m} = m_k - hm_{k_h}$. Assumption 3.1 implies that $\sum_{k=1}^{2} m_{k_m} = 1$. The social planner chooses the triple (m_k, n_k, A_k) $\forall k = 1, 2$ to maximize the sum of indirect utility and profits across the two platforms:

$$W^{FB} = \sum_{k=1}^{2} \left\{ m_k v(n_k, A_k) - \tau \int_0^{m_{k_m}} x dx - \tau h \int_0^{m_{k_h}} x dx - m_k \int_0^{n_k} c(j) dj - c(A_k) \right\}.$$

Differentiating W^{FB} with respect to $(m_{k_m}, m_{k_h}, n_k, A_k)$ $\forall k = 1, 2$, imposing symmetry, i.e., $m_{k_m} = m_{-k_m} = \frac{1}{2}$, and using $m_k = m_{k_m} + hm_{k_h}$) gives the system of equations (3.17).

[22] Here I restrict attention to the case where the social planner wants to operate two active platforms. In general, there is a trade-off between saving on consumers' transportation costs (the argument that speaks for two active platforms) and the multiplicity of investment costs (the argument that speaks for one active platform). If the former dominates the latter, a social planner wants two active platforms.

where

$$m_k^{**} = \frac{1}{1 - \frac{1}{4(2\tau\lambda+1)^2}} \Bigg[\frac{1}{4} \left(1 + \frac{1}{2(2\tau\lambda+1)} \right) + \frac{1}{4\tau} \left(1 - \frac{1}{2(2\tau\lambda+1)} \right) \left[v(n_k^{**}, A_k) - v(n_{-k}^{**}, A_{-k}) - n_k^{**} c(n_k^{**}) + n_{-k}^{**} c(n_{-k}^{**}) \right] + \frac{\lambda}{4(2\tau\lambda+1)} \left[v(n_{-k}^{**}, A_{-k}) - n_{-k}^{**} c(n_{-k}^{**}) \right] + \frac{\lambda}{2} \left[v(n_k^{**}, A_k) - n_k^{**} c(n_k^{**}) \right] \Bigg], \tag{3.18}$$

and the indifferent seller $n_k^{**}(A_k)$ is implicitly determined by

$$s(n_k^{**}, A_k) + n_k^{**} \left[v_{nn}(n_k^{**}, A_k) - c_n(n_k^{**}) \right] = 0. \tag{3.19}$$

The distortions under duopoly are similar to the monopoly two-sided operation mode. I thus remark on additional observations due to two competing platforms. First, since sellers can multi-home and consumers do not internalize a seller fee change in their adaptation decision, i.e., $\frac{dm_k}{df_k^s} = 0$, platforms do not have to compete against each other to attract sellers - neither indirectly through the technology level nor directly through the fees. For these reasons, the equilibrium number of sellers on k, as implicitly determined by equation (3.19), is independent of the number of consumers and the other platform's quality level. If furthermore the quality level are the same under monopoly and duopoly, the number of sellers would be the same; this immediately follows from comparing (3.19) to (3.10).

Second and assuming that $A_k = A_{-k} = A$, platforms set a lower consumer fee in the duopoly than in the monopoly environment due to competition for the consumer in the middle of the Hotelling line.[23] Consequently, $m_k^{**}\left(n_k^{**}(A), A\right) > m^{**}\left(n^{**}(A), A\right)$ because $\frac{2(2\tau\lambda+1)}{2(2\tau\lambda+1)-1} > 1$. In other words, if the technology level is the same on the monopoly and on each of the duopoly platforms, then more consumers join a platform in the duopoly than in the monopoly environment.

---

[23] If $A_k = A_{-k} = A$, then $m_k^{**}\left(n_k^{**}(A_k), A_k\right) = \left(\frac{2(2\tau\lambda+1)}{2(2\tau\lambda+1)-1}\right)\left[\frac{1}{4} + \frac{\lambda}{2}\left(v(n_k^{**}, A_k) - n_k^{**} c(n_k^{**})\right)\right]$. Then,

$$f_k^{c^{**}}\left(n_k^{**}(A), A\right) = \left(\frac{4\tau}{2(2\tau\lambda+1)-1}\right)\left[\frac{1}{4} + \frac{\lambda}{2}\left(v(n_k^{**}(A), A) - n_k^{**}(A) c(n_k^{**}(A))\right)\right] - n_k^{**}(A) s(n_k^{**}(A), A)$$
$$< \frac{1}{\lambda}\left[\frac{1}{4} + \frac{\lambda}{2}\left(v(n^{**}(A), A) - n^{**}(A) c(n^{**}(A))\right)\right] - n^{**}(A) s(n^{**}(A), A) = f^{c^{**}}\left(n^{**}(A), A\right),$$

because $n_k^{**}(A_k) = n_{-k}^{**}(A_{-k}) = n^{**}(A)$ and $\frac{4\tau}{2(2\tau\lambda+1)-1} < \frac{1}{\lambda}$.

Lemma 3.6 states the optimal investment condition at stage one for competing two-sided platforms.

Lemma 3.6. *The symmetric equilibrium investment condition for competing two-sided platforms at the first stage is*

$$\Omega\left[\frac{1}{4}+\frac{\lambda}{2}\left(v(n_k^{**},A_k^{**})-n_k^{**}c(n_k^{**})\right)\right]v_A(n_k^{**},A_k^{**})-c'(A_k^{**})=0, \tag{3.20}$$

where $\Omega \triangleq \frac{4(2\tau\lambda+1)\left[2(2\tau\lambda+1)^2-1\right]}{(2(2\tau\lambda+1)-1)[4(2\tau\lambda+1)^2-1]} > 1$ *and* n_k^{**} *is determined by* (3.19) *for all* $k = 1, 2$.

Since $\Omega > 1$, the two-sided platforms under duopoly invest more in platform quality than their monopolistic counterpart. The intuition is straightforward: due to competition, not only does the platform benefit from investing to attract consumers from its hinterland, but investing also serves as an instrument to attract consumers from the mid-section of the Hotelling line, i.e., a business-stealing effect.

Multi-Product Firm.

Finally, consider the one-sided operation mode under duopoly. As before under monopoly, each MPF at stage four sets the price $p_k^* = v_n(n_k, A_k)$, $\forall k = 1, 2$, for each of the varieties it produces. The measure of consumers visiting MPF k at stage three is given by equation (C.4). The following expression implicitly determines the number of varieties each MPF k chooses to produce:[24]

$$s(n_k, A_k) + n_k v_{nn}(n_k, A_k) = 0. \tag{3.21}$$

At the second stage, each MPF k chooses f_k^c in order to maximize $f_k^c m_k + m_k(v_n^k n_k - \int_0^{n_k} c(j)dj)$, subject to (3.21) and (C.4). It follows that the set of implicit fees charged by MPF platform $k = 1, 2$ for given levels of technology A_k are

$$f_k^{c^*} = \frac{1}{\frac{1}{2\tau}+\lambda}m_k^* - \left(v_n(n_k^*, A_k)n_k^* - \int_0^{n_k^*} c(j)dj\right), \tag{3.22}$$

[24]Each MPF k chooses n_k to maximize $m_k\left(v_n^k n_k - \int_0^{n_k} c(j)dj\right)$. There exists a unique equilibrium $n_k^* > 0$ such that (3.21) is satisfied, the proof of which is analogous in steps to Lemma 3.3.

where

$$m_k^* = \frac{1}{1-\frac{1}{4(2\tau\lambda+1)^2}}\left[\frac{1}{4}\left(1+\frac{1}{2(2\tau\lambda+1)}\right)\right.$$
$$+\frac{1}{4\tau}\left(1-\frac{1}{2(2\tau\lambda+1)}\right)\left[S(n_k^*,A_k)-S(n_{-k}^*,A_{-k})\right]$$
$$\left.+\frac{\lambda}{4(2\tau\lambda+1)}S(n_{-k}^*,A_{-k})+\frac{\lambda}{2}S(n_k^*,A_k)\right] \quad (3.23)$$

and n_k^* is determined by (3.21) for all $k = 1, 2$.[25]

As in Lemma 3.4, it also holds in the duopoly market environment that $n_k^{FB}(A_k) > n_k^*(A_k) > n_k^{**}(A_k)$ for a given level of technology A_k.[26] Then, the next Lemma follows.

Lemma 3.7. *For a given level of technology $A_k = A$ at stage two, $n^{FB}(A) = n_k^{FB}(A_k) > n^*(A) = n_k^*(A_k) > n^{**}(A) = n_k^{**}(A_k)$ $\forall k = 1, 2$.*

Proof. By comparing the equation in (3.3) that implicitly determines n^{FB} to the equation in (3.17) that determines n_k^{FB}, it immediately follows that $n^{FB}(A) = n_k^{FB}(A_k)$ if $A_k = A$. Similarly, the other two equality are obtained by comparing (3.10) to (3.19) for the two-sided operation mode, and (3.14) to (3.21) for the MPF mode. The inequality signs stem from Lemma 3.4. □

Lemma 3.7 states that for a given level of technology, the number of products in the first-best outcome is the same in the monopoly as in the duopoly market environment. The same comparison holds across market environments for the two-sided platforms and MPFs. While the introduction of competition affects consumers at the second stage, it does not feed back into the optimal choice of varieties due to, and as indicated earlier, the consumers' passive expectations assumption. That

[25] In particular, the first order condition with respect to f_k^c of the stage two maximization problem is

$$m_k + \left[f_k^c + v_n(n_k, A_k) - \int_0^{n_k} c(j)dj\right]\frac{dm_k}{df_k^c} + m_k\left[v_n^k - c(n_k) + n_k v_{nn}^k\right]\frac{dn_k}{df_k^c} = 0.$$

Taking total differentials of the stage the equations with respect to f_k^c, one obtains that $\frac{dn_k}{df_k^c} = \frac{dn_{-k}}{df_k^c} = 0$, $\frac{dm_k}{df_k^c} = -\frac{1}{2\tau} - \lambda$, and $\frac{dm_{-k}}{df_k^c} = \frac{1}{2\tau}$. These expressions in the first order condition and imposing the rationality condition $n_k^*(n_k^e) = n_k^e$ give (3.22). Plugging $(f_k^{c^*}, f_{-k}^{c^*})$ back into (C.4) gives (3.23).

[26] The arguments to show that this is true for the duopoly case are identical to the monopoly case, because the equations (3.17), (3.19), and (3.21) solely contain in addition the subscript k in comparison to (3.3), (3.10), and (3.14).

is, an impact of competition on n through m is not present. Therefore, a platform's maximization problem with respect to n remains the same after the introduction of competition.

The optimal technology level chosen by MPF k at stage one solves the following set-up:

$$A_k^* = \underset{A_k}{\operatorname{argmax}} \left\{ \frac{1}{\frac{1}{2\tau} + \lambda} \left(m_k^*\right)^2 - c(A_k) \right\} \text{ s.t. (3.21) and (3.23).}$$

By similar steps as in the proof of Lemma 3.6, which I skip here for brevity, the symmetric equilibrium investment condition for competing MPFs is

$$\Omega \left[\frac{1}{4} + \frac{\lambda}{2} S(n_k^*, A_k^*) \right] v_A(n_k^*, A_k^*) - c'(A_k^*) = 0 \ \forall k = 1, 2, \tag{3.24}$$

where n_k^* is implicitly determined by (3.21).

The ordering of the equilibrium investment levels under duopoly is the same as under monopoly (see Proposition 3.1), i.e., $A_k^{FB} > A_k^* > A_k^{**}$.[27] The following Proposition 3.2 summarizes the equilibrium investment level comparisons between the monopoly and duopoly market environments at stage one.

Proposition 3.2. *In the symmetric equilibrium at stage one,* $A_k^{FB} = A^{FB}, A_k^* > A^*$, *and* $A_k^{**} > A^{**} \ \forall k = 1, 2$.

Proof. That $A^{FB} = A_k^{FB}$ follows directly from comparing equations (3.3) to equations (3.17). By comparing equations (3.19) and (3.20), which simultaneously determine A_k^{**} and n_k^{**}, to equations (3.10) and (3.11), which simultaneously determine A^{**} and n^{**}, the marginal benefit of investing for a two-sided platform in a symmetric duopoly is relatively higher than in a monopoly setting since $\Omega > 1$. Hence, $A_k^{**} > A^{**} \ \forall k = 1, 2$. The proof for the two-sided operation mode is analogous by comparing equations (3.21) and (3.24) to (3.14) and (3.15). □

Assuming that the social planner wants to operate two platforms, the level of investment in technology according to the first-best solution is identical in the monopoly and the duopoly environment. Proposition 3.2 also states that the two-sided market platforms and MPFs invest relatively more in technology if in the duopoly setting compared to the monopoly setting. This is due to the "business stealing" effect caused by competition: under duopoly, two-sided platforms and MPFs not only have an incentive to invest to attract consumers in their respective hinterlands but also compete for the consumers in the middle of the Hotelling line.

[27] See the second part of the Proposition 3.1 proof.

Investment in the platform's quality level is an additional instrument to attract consumers. The additional marginal benefit multiplier due to the business stealing effect is $\Omega > 1$.

Corollary 3.1 emphasizes the role played by the importance of hinterlands, i.e., the magnitude of λ, on the difference in the equilibrium technology levels across market environments for a given organization mode.

Corollary 3.1. *If* $\tau\lambda > 0.067$ *then* $\frac{d}{d\lambda}\left(A_k^{**} - A^{**}\right), \frac{d}{d\lambda}\left(A_k^{*} - A^{*}\right) < 0$ *and conversely.*

Proof. Since Ω, the marginal benefit multiplier, distinguishes the investment levels across the duopoly and monopoly environments for the two-sided and MPF platforms, it is sufficient to investigate how λ affects Ω. One can show that $\frac{\partial\Omega}{\partial\lambda} \propto 2(2\tau\lambda+1)^2\left[4(2\tau\lambda+1) - 4(2\tau\lambda+1)^2 + 1\right] - 1 < 0$ if and only if $\tau\lambda > 0.067$, from which the statement follows. □

Corollary 3.1 formally states that if $\tau\lambda$ is not too small, the difference in the equilibrium technology levels between the duopoly and monopoly market environment decreases for a given organization mode. The intuition is that as λ increases, the marginal benefit from investment to stealing consumers in the middle of the unit interval shifts towards attracting consumers in the hinterland. Since competition between platforms for the consumers in the middle distinguishes the duopoly from the monopoly market environment, an increase in λ (for $\tau\lambda$ not too small) moves the duopoly set-up closer to the monopoly set-up. For the sake of illustration, observe that $\lim_{\lambda\to\infty}\Omega = 1$; the monopoly and duopoly market environment set-ups collapse for a given organization mode.

If, in contrast, $\tau\lambda$ is sufficiently small, an increase in λ increases the incentives to invest in the duopoly relative to the monopoly environment. The reason is that for very low $\tau\lambda$ values, the market for consumers in the middle is highly contested by competing platforms. A low, possibly negative consumer fee reflects that. Then, an increase in λ takes away some of the competitive pressure from platforms by being able to focus relatively more on the hinterland consumers. This is reflected by an increase in the consumer fee. The flip side for platform k is that platform $-k$ also increases its consumer fee, which in turn makes it more attractive to steal consumers in the middle from the other platform through a larger investment.

3.3.3 Welfare comparisons

Here I present several results pertaining to welfare using the lemmas and propositions derived in the previous two subsubsections. I start by comparing total consumer surplus across the operation modes. Lemma 3.8 serves as an intermediate step.

Lemma 3.8. *$m^{FB}(n^{FB}, A^{FB}) > m^*(n^*, A^*) > m^{**}(n^{**}, A^{**})$ and $m_k^{FB}(n_k^{FB}, A_k^{FB}) > m_k^*(n_k^*, A_k^*) > m_k^{**}(n_k^{**}, A_k^{**})$*

For the monopoly case, Lemma 3.8 is a consequence of Proposition 3.1 and Lemma 3.4. Namely, $m^{FB}(n^{FB}, A^{FB}) > m^*(n^*, A^*)$ holds because first, m^{FB} increases in A (by Proposition 3.1 $A^{FB} > A^*$), second m^{FB} increases in n (by Lemma 3.4 $n^{FB} > n^*$ for a given A), and third even for the same level of technology and number of sellers $m^{FB}(n, A) > m^*(n, A)$.[28] Intuitively, since consumers in the MPF mode enjoy a higher level of investment and a larger set of product varieties, all of which are beneficial for their utility, more consumers join the MPF than the two-sided platform. However, the consumer measure is inefficiently low under both operation modes compared to the first best.

Lemma 3.8 implies Corollary 3.2 using a geometric argument: since the MPF attracts more consumers than the two-sided platform, i.e., $m_h^* > m_h^{**}$ in the monopoly environment, a consumer that joins the MPF derives greatly stricter utility than if she joins the two-sided platform. As this holds for all consumers, the total utility for consumers under the MPF mode is also larger than under the two-sided mode.

Corollary 3.2. *$U^*(n^*, A^*) > U^{**}(n^{**}, A^{**})$ and $U_k^*(n_k^*, A_k^*) > U_k^{**}(n_k^{**}, A_k^{**})$*

Proof. Monopoly. By Lemma 3.8, $m_h^* > m_h^{**}$. This implies that $u^* - \tau x \geq u^{**} - \tau x$ x, where the strict equality holds for all $x \in [0, m_h^*)$ in the hinterland and for all consumers at the interior of the Hotelling line. Then, $U^*(n^*, A^*) = h \int_0^{m_h^*} [u^* - \tau x]\, dx + \int_0^{\frac{1}{2}} [u^* - \tau x]\, dx > h \int_0^{m_h^{**}} [u^{**} - \tau x]\, dx + \int_0^{\frac{1}{2}} [u^{**} - \tau x]\, dx = U^{**}(n^{**}, A^{**})$.

Duopoly. The steps are analogous to the steps in the monopoly case. □

Lemma 3.9 repeats the analysis of Lemma 3.8 but now across market environments and for a given operation mode. In short, competing platforms invest more heavily in technology than their monopoly counterparts, which in turn translates into attracting relatively more consumers to their platforms under duopoly.

Lemma 3.9. *$m^{FB}(n^{FB}, A^{FB}) = m_k^{FB}(n_k^{FB}, A_k^{FB}), m_k^*(n_k^*, A_k^*) > m^*(n^*, A^*)$ and $m_k^{**}(n_k^{**}, A_k^{**}) > m^{**}(n^{**}, A^{**})$.*

By the same geometric argument as before, Lemma 3.9 implies Corollary 3.3.

[28]The second and third observations also hold for the comparison between $m^*(n^*, A^*)$ and $m^{**}(n^{**}, A^{**})$. The first observation is replaced by a revealed preference argument: the MPF prefers A^* over A^{**} and thus according to Proposition 3.1 incurs a larger investment cost. But then from the MPF's profit expression, $\frac{1}{\lambda}(m^*)^2 - c(A)$, it is clear that the MPF only wants to incur a larger investment only if m^* evaluated at A^* is larger than evaluated at A^{**}.

Corollary 3.3. *$U_k^*(n_k^*, A_k^*) > U^*(n^*, A^*)$ and $U_k^{**}(n_k^{**}, A_k^{**}) > U^{**}(n^{**}, A^{**})$.*

Proof. By Lemma 3.9, $m_{k_h}^* > m_h^{**}$. This implies that $u_k^* - \tau x \geq u^* - \tau x \quad \forall x$, where the strict equality holds for all consumers that join platform k in the duopoly environment. Then, $U_k^*(n_k^*, A_k^*) = h \int_0^{m_{k_h}^*} [u_k^* - \tau x]\, dx + \int_0^{\frac{1}{2}} [u_k^* - \tau x]\, dx > h \int_0^{m_h^*} [u^* - \tau x]\, dx + \int_0^{\frac{1}{2}} [u^* - \tau x]\, dx = U^*(n^*, A^*)$. The steps to show that $U_k^{**}(n_k^{**}, A_k^{**}) > U^{**}(n^{**}, A^{**})$ are identical and omitted here. □

I now turn to the platform profit comparisons.

Corollary 3.4. *In the monopoly market environment, $\Pi^*(n^*, A^*) > \Pi^{**}(n^{**}, A^{**})$.*

In the monopoly market environment, the MPF *always* makes a strictly larger profit than the two-sided platform. Similar to what has been shown in Lemma 3.8, the cornerstone for this result is a revealed preference argument.

An interesting conjecture arises in the duopoly market environment: although an MPF better internalizes externalities, it may make a larger profit than a two-sided platform only when the market expansion possibility λ is sufficiently large. In order to see this, suppose $\lambda = 0$. Then, $m_k^* = m_k^{**} = \frac{1}{2}$; in the absence of hinterlands half of the consumers go to each platform in a given operation mode. By Proposition 3.1 $A_k^* > A_k^{**}$, including for $\lambda = 0$, and hence $\Pi_k^{**}(n_k^{**}, A_k^{**})|_{\lambda=0} = \frac{\tau}{2} - c(A_k^{**}) > \Pi_k^*(n_k^*, A_k^*)|_{\lambda=0} = \frac{\tau}{2} - c(A_k^*)$ as $c'(A_k) > 0$. Recall that the MPF always has a larger incentive to invest, even for $\lambda = 0$, because by better accounting for the importance of product varieties, which is reflected in both a lower consumer fee and a larger product variety (Lemma 3.4), the MPF more readily attracts consumers through investment than a two-sided platform.

Yet at the same time, this is the MPFs' pitfall for low λ values. If platforms can mostly only compete for consumers in the middle but not expand consumer demand through their hinterlands, a costly investment in technology is a source of wasteful expenditure from the platform's perspective. Thus, MPFs under duopoly are worse off by competing more fiercely than two-sided platforms through additional investment. In a sense, for low λ values it pays for the two-sided platforms to be *ignorant* about the importance of product varieties when it comes to investing in their own platforms' functionality.

The qualitative result ought to flip when λ is sufficiently large. The reason is that if organizations have a sufficiently large consumer market expansion possibility, it pays for MPFs to better account for the importance of product varieties on consumers than two-sided platforms. Now that platforms also have the chance to attract a sufficiently large fraction of consumers through their hinterlands, investing in technology is not just a costly source of wasteful expenditure to compete for consumers in the middle, but rather an additional tool at the platforms' disposal to attract consumers in the hinterland.

An immediate implication of this conjectured rationale is that in an industry that faces (almost) complete market coverage, as for example in the mobile OS industry (Apple, Samsung), two competing platforms prefer to be positioned as a two-sided platform rather than an MPF from the profit perspective. In an industry with sufficiently large market expansion possibilities, as for example in the ride-hailing (Lyft, Uber), e-commerce (Amazon, Alibaba), and video-game consoles (Xbox, PlayStation) industries, it pays for organizations to operate in the MPF mode.

Corollary 3.5. $\Pi^*(n^*, A^*) > \Pi_k^*(n_k^*, A_k^*)$ *and* $\Pi^{**}(n^{**}, A^{**}) > \Pi_k^{**}(n_k^{**}, A_k^{**})$.

Corollary 3.5 yet again stems from a revealed preference argument by the monopolist. That is, a monopolistic MPF chooses A^* rather than A_k^{**} (and similar in the two-sided platform mode). This and the fact that a MPF under monopoly earns a larger profit, net of investment costs, than under duopoly yields the result.[29] Intuitively, the profit of platforms under duopoly deteriorates due to competition and is not made up for through additional investment in technology (Proposition 3.2).

Corollary 3.6. $W^{FB}(n^{FB}, A^{FB}) > W^*(n^*, A^*) > W^{**}(n^{**}, A^{**})$.

By construction, the solution to the social planner's problem gives the largest total welfare. Total welfare is higher in the MPF mode than in the two-sided platform mode. The MPF, by providing a larger product variety and higher investment in its platform quality, both of which are beneficial for consumers, attracts a larger customer base than a two-sided platform.

3.4 Conclusions

I have examined incentives to invest in platform quality by organizations that operate either as an MPF or as a two-sided platform, both in monopoly and duopoly

[29] In particular, the gross profit of a monopolistic MPF $\frac{1}{\lambda}(m^*)^2 = \frac{1}{\lambda}\left(\frac{2(2\tau\lambda+1)-1}{2(2\tau\lambda+1)}\right)^2 (m_k^*)^2 > \frac{1}{\frac{1}{2\tau}+\lambda}(m_k^*)^2$ for a given level of technology and product variety.

market environments. MPFs produce and sell product varieties to consumers themselves, whereas two-sided platforms solely enable and facilitate trade between consumers and sellers. In each respective model, a platform first decides how much to invest in the quality of its platform before setting access fees for the sides. Investment in the platform's quality directly affects consumers, but not sellers, through enhancing the value of interaction with product varieties.

First, I find that platforms in either organization mode and market environment under-invest relative to the social planner's outcome. For a given level of technology, the platforms do not fully appropriate the surplus that is generated between the two sides' interactions by solely focusing on the marginal user on at least one side. This is amplified by the passive expectations assumption of consumers, who do not account for a change in the fees on the supply of product varieties. These distortions carry over to the choice of quality stage, as they reduce the benefits from investing to attract agents.

Second, I find that the introduction of competition leads to higher investment levels by platforms in their respective platform quality than under monopoly. Due to the business-stealing effect, competing platforms, in order to attract agents, have an additional incentive to invest. Other than competing in fees, investing in platform quality serves as an additional instrument in that regard.

Third, an MPF in either market environment invests more in quality than a two-sided platform. By producing the products themselves, MPFs better account for the cross-group benefits of products for consumers. That is, MPFs fully appropriate the interaction surplus from the product variety side (though still not on the consumer side). As a result, an MPF has an additional incentive to invest in quality to increase the customer base.

While the baseline model assumes that platforms can only charge a simple fixed fee, I show in Appendix C that a two-part tariff does not change the investment-in-quality results. The reason is the passive expectations assumption on the consumer side. In the two-sided operation mode, a change in the seller per-transaction fee is not internalized by consumers, removing the possibly differing responses by consumers to changes in the various seller fees. The only difference is that a continuum of equilibrium fees exists, since the platform only cares about the total fee it collects on each side, but not how it is split between the fixed and transaction fees.

Further insights regarding the investment-in-quality decision under various operation modes could be developed by extending the framework. In the present framework, I treat the organization mode as given. Endogenizing the positioning of organizations would allow studying conditions under which a platform under duopoly chooses to operate as a two-sided platform or as an MPF (Hagiu and Wright, 2015b). If platforms are allowed to make this either-or operation decision, an interesting question in the resulting two-by-two game is to examine whether asymmetric equilibria can arise. That is, whether one platform operates as an MPF whereas the other operates as a two-sided platform.

The degree of internalization of the importance of product varieties, which is carried over to the investment stage, is reflected in the (consumer) fees across the two operation modes. A reversal of the homing assumption may thus uncover additional effects in the investment comparison between the two modes, since such a reversal shifts the fee competition between platforms from the consumer to the seller side (Armstrong, 2006). Another extension worth pursuing is to enrich the model by assuming that investment not only affects consumers but also directly sellers, for instance through a reduction of the sellers' marginal cost (Tan et al., 2020).

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Appendix A

Promoting Pluralism and Readership in Digital News: New Lessons for Tax Policy

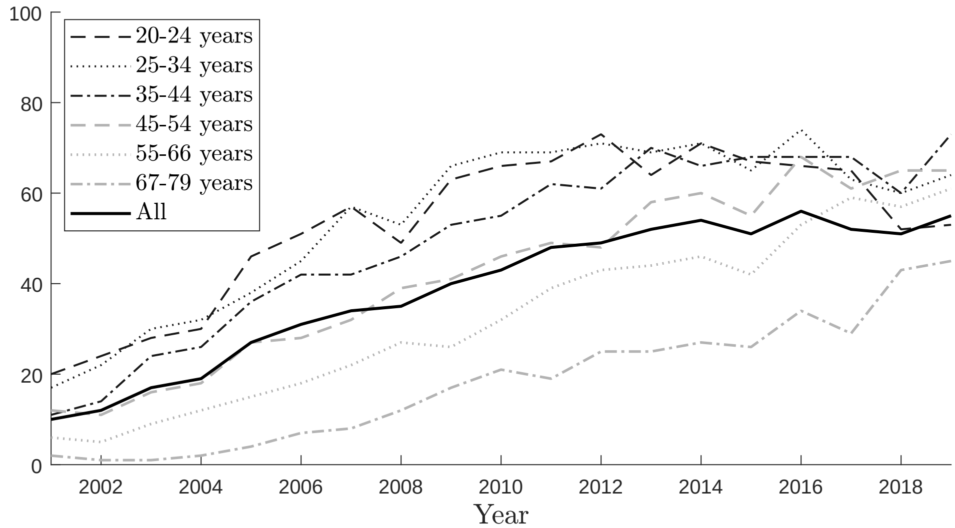
A.1 Additional figures

Table A.1.1: VAT rates in selected European countries (as of January 1, 2019)

	BE	CZ	DE	EL	ES	FR	IT	NL	NO	PL	RO	UK
Standard rate (%)	21	21	19	24	21	20	22	21	25	23	19	20
Printed newspapers (%)	0	10	7	6	4	2.1	4	9	0	8	5	0

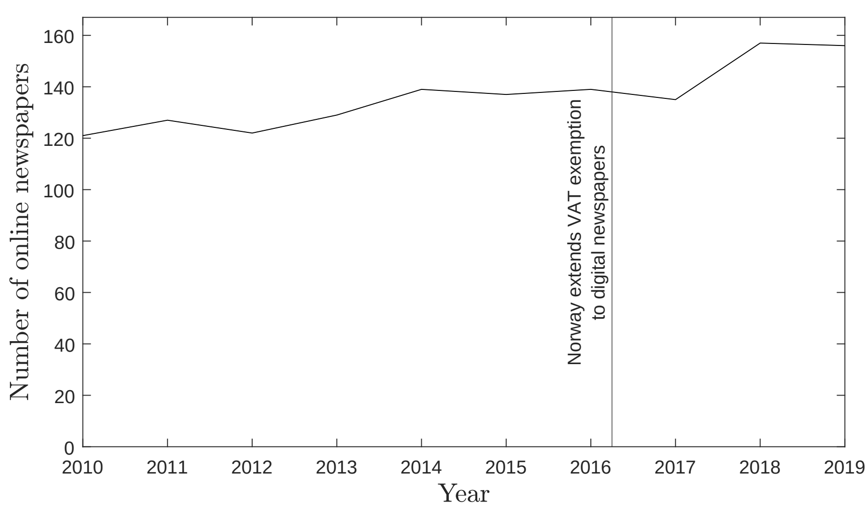
Notes: This table reports the standard value-added tax rate and the rate for printed newspapers in the ten most populous European Union countries, Norway, and the United Kingdom. For physical publications, i.e., printed books, newspapers and periodicals, all European Union member states have the option to apply a minimum reduced VAT rate of 5%. Some member states are authorized to apply super-reduced VAT rates (below 5%) or zero rates. *Data sources:* European Commission (2019) and Kontor (2015).

Figure A.1.1: Readership of online newspapers in Norway by age (in %)



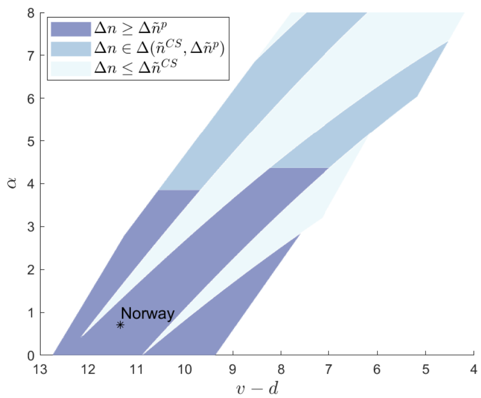
Source: Data are from Statistics Norway (2020).

Figure A.1.2: Number of Norwegian digital newspapers

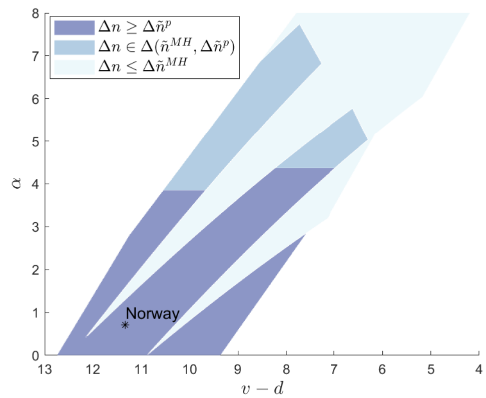


Source: Data are from Kantar TNS Norway (2019).

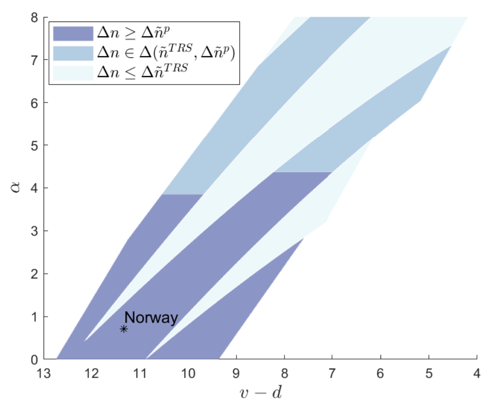
Figure A.1.3: Consumer measure changes from VAT reduction in the $(v-d, \alpha)$ space - imposing integer constraint



(a) Consumer surplus change



(b) Fraction of consumers that multi-home change



(c) Total readership change

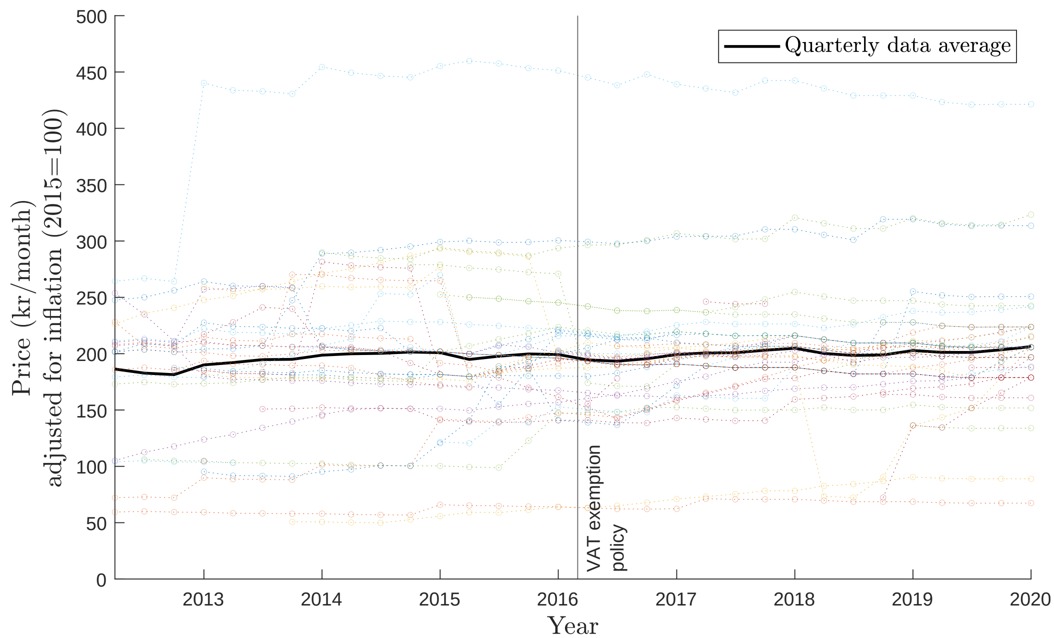
Notes: Figure A.1.3 replicates Figure 1.3 but imposes that the number of news has to take an integer value. See the notes in Figure 1.3 for parameter details.

A.2 Digital newspaper pricing data in Norway

I have collected the monthly subscription prices for the 50 largest Norwegian digital newspapers by circulation for each quarter between January 2010 and January 2020 using the *Wayback Machine* (`https://archive.org/`). These 50 newspapers encompass 94% of total digital newspaper readership in Norway in 2016 (Kantar TNS Norway, 2019). Due to the relatively few price observations up to and including the first quarter of 2012, I restrict the analysis to after Q1 2012.

Figure A.2.1 plots the monthly subscription prices adjusted for inflation for each

Figure A.2.1: Monthly subscription prices by Norwegian digital newspapers



Notes: Figure A.2.1 plots the monthly subscription prices adjusted for inflation of each of the 50 largest Norwegian digital newspapers for each quarter between April 2012 and January 2020. The solid line depicts the average monthly subscription price for these 50 newspapers.

quarter between the second quarter of 2012 and the first quarter of 2020. The graph suggests that the VAT policy change had no clear impact on the overall price trend of the digital newspapers.

The following regression analysis on the VAT-policy impact complements the purely descriptive pricing behavior reported in Figure A.2.1. I acknowledge that the estimations are limited to purely serve as a correlation test of the impact of the VAT policy change on the pricing trend. The specification is

$$p_{jt} = \alpha + \beta T_t + \gamma Policy_t + \delta Policy_t \times T_t + u_{jt}, \tag{A.1}$$

where p_{jt} is the price of newspaper j in quarter t, α is a constant, T_t is a time trend variable, and $PostPolicy_t$ is a dummy variable that takes value 0 for any quarter before Q2 2016 and the value 1 including and after Q2 2016. Finally, $u_{jt} = \alpha_j - \alpha + \epsilon_{jt}$ is the error term containing newspaper j's individual effect α_j. Then, $\frac{\partial p_{jt}}{\partial Policy_t} = \gamma + \delta T_t$ evaluated at $t =$ Q2 2016 captures the impact of the VAT-reduction policy, where the first part measures the discontinuity of and the second part measures the change in the slope of the price trend.

Table A.2.1: Regression results - prices inflation adjusted

	(1) Pooled OLS	(2) Pooled OLS	(3) Fixed Effects	(4) Fixed Effects
T	0.966	0.971**	1.141	1.141*
	(0.63)	(0.34)	(0.80)	(0.42)
Policy	87.112	20.206	68.402	68.402
	(168.71)	(116.55)	(192.86)	(97.79)
Policy $\times T$	-0.418	-0.112	-0.343	-0.343
	(0.76)	(0.52)	(0.87)	(0.44)
Constant	-14.234	-20.539	-53.211	-53.211
	(136.45)	(75.04)	(176.26)	(91.47)
N	1254	1254	1254	1254
R^2	0.004	0.458	0.041	0.041
SE type	Robust	Panel-corrected	Robust	Drisc/Kraay

Notes: Standard errors are in parentheses. * $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$. See the text in Appendix A.2 for regression details.

Column (1) in Table A.2.1 reports the pooled ordinary-least squares (OLS) regression results. Column (2) reports the pooled OLS regression results with panel corrected standard errors since it is unlikely that the u_{jt} for the newspapers are uncorrelated over time and space (Beck and Katz, 1995). In particular, the standard errors are robust to disturbances being heteroscedastic, contemporaneously cross-sectionally correlated, and autocorrelated of type AR(1).

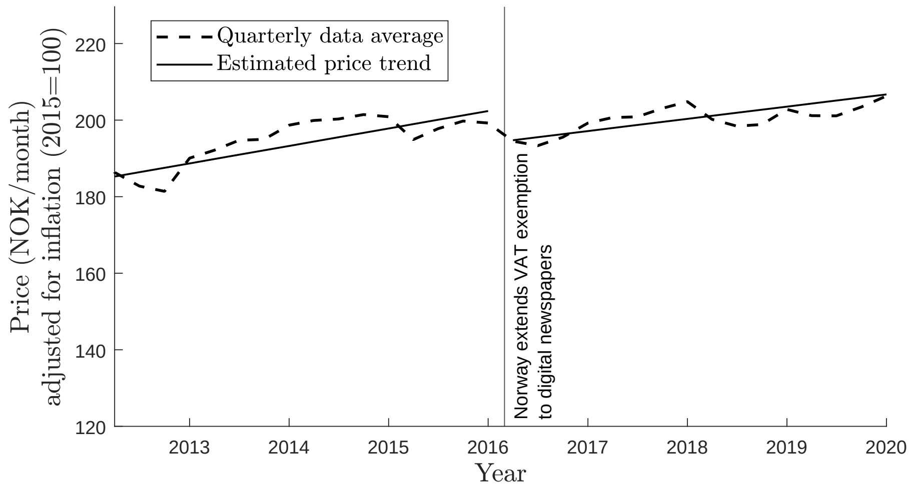
The Breusch-Pagan Lagrange multiplier test rejects the null hypothesis that variances across newspapers are zero, i.e., $H_0 : Var(\alpha_j) = 0$, and thus a panel effect is present. The test statistic is $\mathcal{X}^2(1) = 13757$. The pooled OLS estimator, therefore, is inconsistent. Column (3) in Table A.2.1 reports the fixed effects estimation results.[1,2] The fixed effects estimator is the efficient estimator of γ in (A.1) if α_j are fixed effects and the error ϵ_{jt} is iid.

The Pesaran cross-sectional independence test statistic is 7.1 and the corresponding P-value is 0.00, causing to reject the null hypothesis that residuals are not correlated across newspapers, i.e., $H_0 : cor(u_{jt}, u_{it}) = 0$ for $j \neq i$. Column (4) in Table A.2.1 presents the fixed effects estimation results of (A.1) with Driscoll and Kraay (1998) standard errors. By using Driscoll and Kraay standard errors, u_{jt} are allowed

[1] The modified Wald test for group-wise heteroskedasticity in the fixed effect regression model rejects the null hypothesis $H_0 : \sigma_j = \sigma \ \forall j$. The test statistic is $\mathcal{X}^2(49) = 2.5 \cdot 10^5$. Thus heteroskedasticity-robust standard errors are included.

[2] The Hausman test's result is $\mathcal{X}^2(3) = 2.09$ and the corresponding P-value is 0.5537.

Figure A.2.2: Price of Norwegian digital newspapers (inflation adjusted)



Note: The estimated price line plots the results from column (3) of Table A.2.1.

to be autocorrelated with MA(q), heteroscedastic, and cross-sectionally dependent (however, the assumption that the regressors are uncorrelated with u_{jt} continues to be required for a consistent estimation of the coefficients).

The estimated coefficients imply that the VAT reduction has lead to an average decrease of 8.8 NOK per monthly subscription in Q2 2016. Furthermore, the average price of Norwegian digital newspapers increases around 0.3 NOK per month *less* from quarter to quarter *after* the VAT reduction (see the estimated δ coefficients in Table A.2.1). However, the estimated (γ, δ) coefficients are never statistically significant. Repeating the analysis of the Norwegian digital newspaper price trend without adjusting the prices for inflation, the estimated (γ, δ) coefficients remain statistically insignificant. Thus, they are not driven by a change in the inflation trend that could coincide in a timely manner with the VAT policy change.

A.3 Model discussion

Market definition - the spokes model. The spokes model is a tractable tool to analyze differentiated oligopoly such as the newspaper market. I interpret the newspaper market as a regional market, in which each consumer's first preference is the newspaper from the municipality where she resides (if available). I use this regional market interpretation for calibrating the model in Section 1.4.1. A consumer's second preference is any other newspaper in that region, which implies the non-localized spatial competition in the spokes model. Nonlocalized competition seems a reasonable assumption especially regarding digital news since digital news varieties are readily identified by consumers through a simple search on the internet.[3]

In contrast to the Salop circle model, a desirable feature of the spokes model is that as the number of newspapers changes the symmetry of the model is maintained without having to change the locations of the newspapers (Chen and Riordan, 2007).[4] Furthermore, by Figure 1.1 not all consumers purchase a newspaper; in the spokes model if new newspapers enter some consumers who were previously not purchasing will now purchase at least one newspaper - a *market expansion* effect. Such a plausible effect is not present in the representative consumer and Salop model where typically full-market coverage is assumed.[5]

Market definition - online vs. offline news. For simplicity, I abstract from including print in addition to digital newspapers in the model. If indeed a digital newspaper is a close substitute to its print version (which was a common perception shared by the newspaper industry at the early stages of the internet; see Chapter 9.7 in Anderson et al., 2016), the introduction of an accompanying digital newspaper has the potential for "self-cannibalization" of the print version. This is especially true if the digital version contains "shovelware", i.e., content that has been directly copied from the print newspaper. Then in the analysis section, the marginal benefit from a digital subscription price increase is over-valued, leading to an upwards bias in the subscription price expression.

However, a digital newspaper can complement its print edition by raising awareness, e.g., through advertising the print newspaper on its online version and allowing consumers to sample content free of charge, and by providing additional services

[3]The results in Section 1.3.1 for a fixed number of newspapers do not hinge on the non-localized modeling assumption made in the spokes model. In particular, the Salop model of localized competition gives qualitatively identical results.

[4]In addition, the Salop model requires further ad-hoc assumptions on how to treat the discount d for multi-homing consumers as the number of newspapers increases and thus the distance between any two newspapers decreases. Arguably, the benefit from reading a second newspaper decreases as any two (adjacent) newspapers are more closely related to each other, i.e., d increases in n. This in turn can cause an issue of multiplicity of equilibria at free-entry. Lastly, the Salop model requires explicit modeling of how to treat the discount when a multi-homing consumer is interested in adjacent vs. non-adjacent newspapers, even if any consumer is interested in at most two newspapers.

[5]The random utility model has been used in structural estimations of print newspaper markets (Fan, 2013; Gentzkow et al., 2014) and also allows for the market expansion effect, since by definition consumers have a "draw" for the outside option of not purchasing (Anderson and de Palma, 1992). However, the random utility is less analytically tractable under the assumption of multi-homing consumers than the present spokes model.

(Kaiser and Kongsted, 2012). Regarding the latter, additional online services by the largest Norwegian newspaper *VG* include, among others, an online shop, a vehicle guide, a power supply comparison, and a coupon site. Furthermore, readers of digital news may differ in their characteristics from readers of print news (see Figure A.1.1 for the age dimension in the case of Norway), which may weaken a reader's willingness to substitute. The literature thus far finds that print versions and online companions are substitutes but of small magnitude.[6]

Digital newspapers. The model assumes that a newspaper is located at the origin of a line. Not endogenizing this form of horizontal differentiation is clearly a simplification. If one interprets the model as a model of "regional" preferences rather than a model of political orientation (as in, for instance, Gabszewicz et al., 2001, 2002; Anderson and Gabszewicz, 2006; Kind et al., 2013; Behringer and Filistrucchi, 2015b), this assumption likely is not very restrictive as newspapers cater to the consumers' preferences in that region. Ignoring other horizontal dimensions such as collusion and ownership also has implications for the parameter calibrations. While I do not treat the former formally, regarding the latter it follows from Appendix A.4.5 that the calibrated $v - d$ value is an upper bound.

Another simplification is that the model does not endogenize vertical differentiation, as v can be interpreted as a vertical quality shifter (number of articles, investigative journalism vs. "vanilla news").[7] This is clearly a dramatic simplification, as variation in quality ought to account for a large fraction of the observed differences in demand for digital newspapers. This and the uniform distribution of consumers across spokes assumption causes the model to make symmetric equilibrium demand predictions for a market in which demand clearly is not symmetric.

One complication is how to treat the fact that digital newspapers typically have

[6] Gentzkow (2007) proposes a structural framework in which products can be substitutes or complements. He finds that raising the price of a print newspaper by 33% would increase its digital readership by about 2%. Filistrucchi (2005) studies the launch of companion websites by Italian national newspapers and finds that the average short-run loss from an own website is estimated to be 3.2%. Kaiser (2006) finds that German magazines lose on average 4.2% of their potential consumers by launching an online companion, although this effect varies across age groups (smaller for young adults) and the effect ceases to exist a couple of years after the growth rate of digital readers peaks (Figure 1.1 suggests that this happened well before 2016 in Norway). Similarly, Kaiser and Kongsted (2012) find that a 1% increase in readership for a companion website is associated with a decrease in total print circulation by 0.15% on average for German magazines, but the estimate depends on reader characteristics as in Kaiser (2006).

[7] A larger v also may have positive spillover effects on society, because for example, investigative journalism leads to improved political accountability; although some consumers might not like as much investigative journalism (Hamilton, 2018).

introductory subscription offers with prices close to zero (the most common is 1 NOK for the first month). Since I do not have information on how many consumers are attracted to a newspaper due to an introductory offer, in the calibration section I assume that each consumer pays the full (monthly) subscription price.

While I endogenize entry in a simple way, a restrictive assumption is that I approximate a dynamic process of entry by a static model. This abstracts from reality that one newspaper enters and operates for some time before the next newspaper enters, and that some newspapers leave the market during that process. In the case of Norway, most digital newspapers entered between 2008 and 2011.

Consumers. The demand specification captures the disalignment between consumers and their ideal digital newspaper and allows for the possibility to purchase multiple newspapers. The former is obviously important as consumers differ in their tastes. The latter is also consistent with the observation that 35% of adults read articles on at least two online newspaper websites on average per day in Norway in 2019 (Statistics Norway, 2020).

A simplification assumption is that all consumers are homogeneous regarding the gross value v and discount d from reading a (second) newspaper. Since I do not have individual-specific demand data, I can only calibrate the v and d parameters for an average consumer. In Appendix A.4.4 I show that the analytical results are robust to alternative specifications in which v and d also depend on the location of the consumer. Another specification could assume that consumers are heterogeneous in their price-sensitivity.

As is standard in the literature where network effects are present, an implicit assumption that I make is that consumers have *responsive rational* expectations (see, e.g., Caillaud and Jullien, 2003; Rochet and Tirole, 2003, 2006; Armstrong, 2006; Armstrong and Wright, 2007; Hagiu, 2006; Rysman, 2009; Choi, 2010; Weyl, 2010). That is, consumers have full information about all prices and the ability to perfectly compute their impact on newspaper adoption. The other polar case would be to assume that consumers are uninformed about ad prices and do not adjust their expectation of newspaper adoption by advertisers in response to *any* price changes (consumers still make their purchasing decision based on the subscription prices). That is, consumers form *passive rational* expectations (Katz and Shapiro, 1985; Hurkens and Ángel L. López, 2014). In the present set-up, assuming passive expectations on the consumer side does not change the equilibria outcomes, because consumers are ad neutral and hence ad prices do not affect consumer demand.[8]

Advertisers. By following Anderson et al. (2018) in modeling the advertisement side, the subsequent equilibrium advertisement price ensures that newspapers make a strictly positive revenue on the ad side. In fact, 67% of total revenue comes from advertising for Norwegian digital newspapers during the period that I study (Norwegian Media Authority, 2020). The advertising model also captures in a simple way the desirable feature of diminishing values of duplicated impressions.

Several important simplifying assumptions are made in the advertising model. First, I assume that advertisers are homogeneous in their willingness to pay, i.e., the ad-demand is not downward-sloping as in for example Anderson and Coate (2005), and their valuation does not depend on the consumer type. The consequent equilibrium ad price causes each advertiser to demand an ad spot in each newspaper.[9] Hence, even under imperfect competition, the market allocation on the advertising side is efficient, which is a strong assumption. Though it is noteworthy that digital newspapers have the scope to negotiate ad rates individually (in contrast in *sponsored searches* - the sale of ad space on search engines such as Google - the ad space is sold to the highest bidder through an online auction), which may not lead to quantity restrictions.

Second, I assume that digital newspapers can costlessly place ads on their platforms. In practice, digital newspapers have costs of investing in the required technology and skills to handle the advertising contents for the platform and ensuring on-going development. However, most if not all of these costs can be seen as part of the fixed cost F. Marginal ad costs such as the actual placement of an ad are arguably very small, once the required technology is in place (digital newspapers typically provide HTML and resolution requirements). In fact, including a marginal ad cost in the model does not impact the equilibrium results (e.g., subscription prices set by the newspapers) as long as said marginal cost is not larger than the price per ad in the baseline model. This gives me confidence that the assumption that newspapers have no ad costs is not inappropriate.

Third, I assume that α is independent of the number of readers a newspaper attracts. Especially digital versions of newspapers can arguably offer better targeted

[8]Even if consumers are not ad neutral, ad prices do not affect consumer demand as long as the advertiser demand is perfectly inelastic (at least locally for ad prices), which is the case here.

[9]If consumers derive sufficient disutility from ads, newspapers might not place ads from all advertisers on its platform or even charge an ad price sufficiently large such that no advertiser would demand an ad space, even under the assumption of homogeneous advertisers. See Appendix A.4.2 for further details regarding consumers' disutility from ads.

advertising the larger is its audience by being able to collect and hence analyze more data to derive better insights into consumer behavior. Hence, making α dependent on the demand ought to affect the competitiveness of digital newspapers, which in turn has implications for the number of newspapers under free-entry - both before and after the VAT reduction. However, the literature finds that the competitive effects of data are ambiguous (due to specific modeling assumptions) and thus far has no single overall message (see De Cornière and Taylor, 2020, and references therein). Given the already rich set-up, I abstract from such considerations.

Finally, the timing of the game assumes that subscription prices are set before the advertising rates. The subscription pricing data for Norway suggests that, after an initial adjustment phase, newspapers infrequently and typically once at the beginning of a year adjust their prices. While I do not have information regarding the timing and frequency of ad price adjustments, it seems plausible that advertisers are responsive to consumer demand, which affects their advertising decision; the newspapers account for this when setting the ad price. The equilibrium of the game is unaffected if the simultaneous choice of both subscription and ad prices is assumed. The reason is that ad prices do not affect consumer demand.

A.4 Model robustness and extensions

A.4.1 Single-homing consumers

In this subsection I focus on the region where, both before and after the VAT reduction, *category 1* consumers buy one and only one digital newspaper.[10] The main difference is that now any pair of newspapers compete directly for *category 1* consumers whose first and second preference consist of that pair and that an increase in the subscription price results in the loss of some single-homing (instead of multi-homing) eyeballs sold to advertisers. From Appendix A.8, the subscription price

$$p_n^* = \frac{c-\alpha}{1-\tau} + \frac{t(2N-n-1)}{n-1}$$

increases as τ decreases if and only if $\alpha > c$. Since a reduction in the VAT rate results in an equal change of the subscription price by each newspaper in the symmetric equilibrium, the indifferent *category 1* consumer cut-off remains at $1/2$; the demand

[10] The pure single-homing case arises endogenously if $v - \frac{c-\alpha}{1-\tau} - \frac{2t(N-1)}{n-1} \in \left(0, d - \frac{t}{2}\right)$, which I assume holds for this subsection. That is, I focus on Region ④, which is identical to Region I in Chen and Riordan (2007) if $c = \alpha = \tau = 0$ and $t = 1$.

per platform $D_n^* = \frac{2N-n-1}{N(N-1)}$ is independent of τ. Therefore, if $\alpha > c$ the consumer surplus decreases following a VAT reduction for a fixed n - the results of Lemma 1.2 are not driven by the presence of multi-homing consumers.

Proposition A.1. *Assume that* $v - \frac{c-\alpha}{1-\tau} - \frac{2t(N-1)}{n-1} \in \left(0, d - \frac{t}{2}\right)$ *holds. Then,*

- CS *increases if and only if* $\Delta n > \Delta \tilde{n}_{SH}^{CS}$*, where* $\Delta \tilde{n}_{SH}^{CS} \in (0, \Delta \tilde{n}_{SH}^{p})$*, and*
- TRS *increases if and only if* $\Delta n \geq 1$*.*

The $\Delta \tilde{n}_{SH}^{CS}$ and $\Delta \tilde{n}_{SH}^{p}$ cut-offs are in the proof in Appendix A.8. As in the multi-homing case, the consumer surplus increases as long as the market expansion and matching effect dominate the overall price increase following the VAT reduction; this requires that $\Delta n > \Delta \tilde{n}_{SH}^{CS}$.

If a planner cares about total surplus, first note that $AS = 0$ since each advertiser pays exactly the incremental value of reaching single-homing consumers. Then,

$$W = n\Pi^*(n) + T + CS = \frac{n(2N-n-1)}{N(N-1)}(v - c + \alpha) - \frac{tn(4N-3n-1)}{4N(N-1)} - nF,$$

which is independent of the VAT rate due to the absence of multi-homing consumers, whose demand was directly dependent on τ (through the subscription price). Thus, a VAT reduction solely changes welfare through the *indirect* effect from entry: $\frac{dW}{d\tau}\Big|_{\Pi^*(n)=0} = \frac{\partial W}{\partial n}\Big|_{\Pi^*(n)=0} \frac{dn}{d\tau}$. Since $\frac{dn}{d\tau} < 0$, welfare increases following the VAT reduction if and only if $\frac{\partial W}{\partial n}\Big|_{\Pi^*(n)=0} > 0$, i.e., if there is initial under-entry. While in the multi-homing section under-entry is a necessary but not sufficient condition, here it is both. Proposition A.2 formalizes.

Proposition A.2. *Assume that* $v - \frac{c-\alpha}{1-\tau} - \frac{2t(N-1)}{n-1} \in \left(0, d - \frac{t}{2}\right)$ *holds. Then, welfare increases following the VAT reduction if and only if there is initial under-entry in equilibrium, which requires that*

$$v - c + \alpha > \frac{t(4N-6n-1)}{4(2N-2n-1)} + \frac{t(1-\tau)(2N-n-1)^2}{(n-1)(2N-2n-1)} \tag{A.2}$$

In addition, $\frac{\partial W}{\partial n}\Big|_{\Pi^*(n)=0}$ *decreases as* τ *decreases.*

For instance, under-entry is more likely as v or α increases. On the one hand, a newspaper's profit is independent of v and α, and thus plays no role in the profit externality. On the other hand, a larger v and α increase a consumer's utility (the latter through a lower subscription price due to the two-sidedness), both of which reinforce the positive consumer surplus externality from entry.

Proposition A.2 also states that the total surplus externality from entry decreases if the VAT-rate is reduced. Since the number of newspapers increases, under-entry is mitigated following the VAT reduction *if* under-entry was severe to begin with (e.g., large v and α values). If there is instead initial over-entry (e.g., small v and α values), a VAT-rate reduction exacerbates over-entry.

A.4.2 Ad nuisance

Thus far I have assumed that consumers are indifferent regarding the amount of ads they are exposed to. This was not only for simplicity but also seems to capture the newspaper market, where ad nuisance does not have a first-order effect (see Chapter 9 in Anderson et al., 2016). I now introduce consumer disutility from ads. In particular, the utility from purchasing one newspaper (1.1) is modified to

$$u_j = \begin{cases} v - tx_j - p_j - \gamma A_j^2 & \text{if } (l_j, x_j) \\ v - t(1 - x_k) - p_j - \gamma A_j^2 & \text{if } (l_k, x_k), \end{cases} \tag{A.3}$$

where $\gamma > 0$ and $A_j \in [0, 1]$ is the amount of ads chosen by the digital newspaper j. The incremental value of buying newspaper k in addition to j still equals $u_{jk} = u_k - d$. Furthermore, the subscription price and advertising amount are chosen simultaneously; the rest of the model stays the same.

With the presence of disutility from ads, a newspaper has to weigh the benefit of a larger revenue from the advertising market from an additional ad against the decrease of its attractiveness for consumers. I find the following equilibrium outcomes for a fixed n; as in the main text, I focus on the case with a strictly positive number of single-homing and multi-homing consumers.

Lemma A.1. *Let n be fixed and $\sigma = 1$. If $v - d - \frac{c}{1-\tau} + \frac{\alpha^2}{4\gamma(1-\tau)^2} - \frac{t(N-1)}{n-1} \in (0, t)$, the unique symmetric subscription price and ad volume in equilibrium are*

$$p_n^* = \frac{1}{2}\left(v - d + \frac{c}{1-\tau} - \frac{3\alpha^2}{4\gamma(1-\tau)^2} + \frac{t(N-n)}{n-1}\right) \text{ and } A^* = \frac{\alpha}{2\gamma(1-\tau)}. \tag{A.4}$$

Total equilibrium sales, number of single-homing and multi-homing consumers for each platform are, respectively,

$$\begin{aligned} D_n^* &= \frac{n-1}{tN(N-1)}\left[v - d - \frac{c}{1-\tau} + \frac{\alpha^2}{4\gamma(1-\tau)^2}\right] + \frac{N-n}{N(N-1)} \\ X_n^{MH^*} &= 2\left(D_n^* - \frac{2N-n-1}{N(N-1)}\right), \quad X_n^{SH^*} = 2\left(\frac{2N-n-1}{N(N-1)}\right) - D_n^* \end{aligned} \tag{A.5}$$

The ad level decreases in the distaste for ads ($\frac{\partial A^*}{\partial \gamma} < 0$). The reduced amount of ads increases the attractiveness of a newspaper and hence the willingness to pay for it - the subscription price increases in γ ($\frac{\partial p_n^*}{\partial \gamma} > 0$). More importantly, the subscription price still increases following the VAT-rate reduction if c is sufficiently small compared to α (if $\frac{3\alpha^2}{4\gamma}\frac{(2-\tau)}{1-\tau} > c$). Even though a consumer's utility might still increase because of the decrease of the ad volume increases when τ is reduced to zero if and only (if $\frac{\alpha^2}{4\gamma}\frac{(2-\tau)}{1-\tau} < c$), per-platform demand and number of multi-homing consumers decrease with c close to zero. Therefore, the results of Lemma 1.2 are not driven by the assumption that consumers do not derive disutility from ads.

Proposition A.3. *Let consumers derive disutility from ads as defined in* (A.3) *and let* $\sigma = 1$. *If* $\frac{\alpha^2}{4\gamma}\frac{(2-\tau)}{1-\tau} > c$, *there exist cut-off values* $\Delta\tilde{\tilde{n}}^{CS}, \Delta\tilde{\tilde{n}}^{MH}, \Delta\tilde{\tilde{n}}^{TRS} > 0$, *above which the consumer surplus, fraction of consumers that multi-home and total readership, respectively, increase following a VAT-reduction to zero. It holds that* $\Delta\tilde{\tilde{n}}^{MH} > \Delta\tilde{\tilde{n}}^{TRS}$. *If* $\frac{\alpha^2}{4\gamma}\frac{(2-\tau)}{1-\tau} \leq c$, *the consumer measures always increase.*

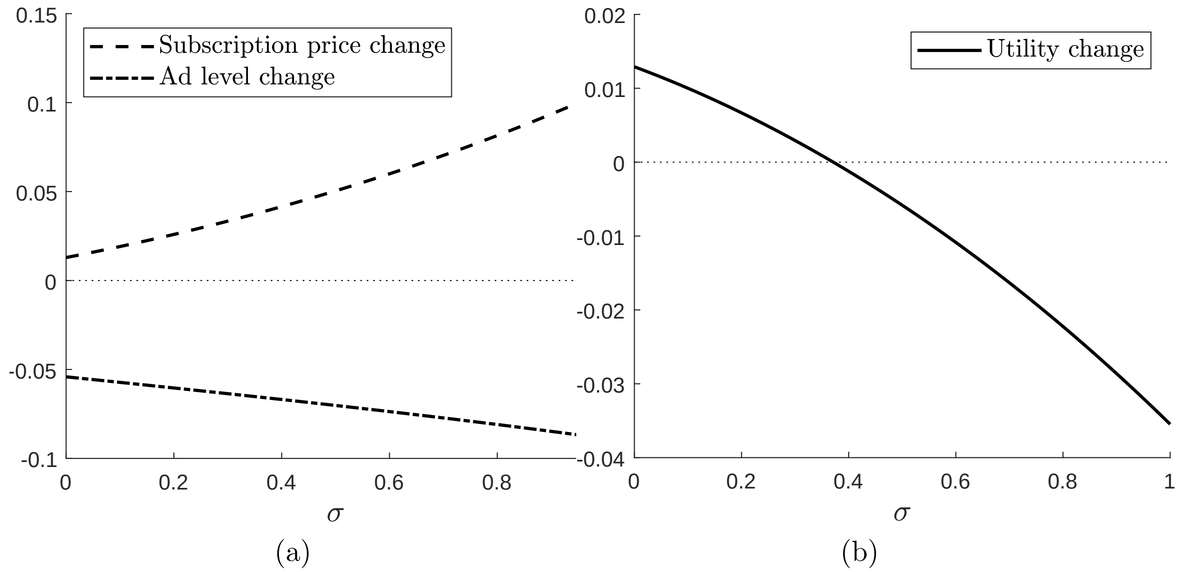
Allowing for free-entry, Proposition A.3 confirms that the main results continue to hold when consumers experience disutility from ads. That is, if the positive *indirect* effects from an increase in the number of newspapers outweigh the negative *direct* effect, which is dampened through the lower ad volume under the assumption of ad disutility, the consumers' well-being and levels of news consumption increase.

If advertisers value multi-homing consumers strictly less than single-homing consumers, i.e., $\sigma < 1$, it is not possible to derive closed-form solutions for the subscription price and ad volume for a fixed n. Using numerical analysis, I find that a digital newspaper continues to increase its subscription price following the VAT reduction and reduces its ad volume following the increased profitability from subscriptions. Figure A.4.1a reports how the equilibrium subscription prices and ad levels for various σ values change following the VAT reduction. Combining the two changes, Figure A.4.1b shows that a reduced VAT-rate may actually increase a consumer's utility: if σ is sufficiently low, the *direct* ad reduction effect can outweigh the *direct* price increase effect. In that case even for a fixed n, the consumer surplus, total readership, and fraction of consumers that multi-home all increase. The arguments are reversed if σ is sufficiently large.

A.4.3 More general advertiser demand curve

The advertiser demand curve was set deliberately simple in Section 1.2, where each advertiser had the same willingness to pay per ad. However, the results still hold

Figure A.4.1: Disutility from ads for $\sigma \in [0, 1]$



Notes: The figure illustrates the change in the subscription price, ad level chosen, and consumer utility following the VAT reduction when the number of newspapers is fixed. I use the parameters calibrated in Section 1.4.1 for an average Norwegian region. I let $\gamma = 1$; the results are qualitatively the same when I vary γ. These parameters satisfy the restriction to have a positive number of single- and multi-homing consumers.

with a more general demand curve of the advertisers. Assume instead a downward-sloping advertiser demand function, where advertisers' willingness to pay are ranked from high to low. In particular, assume that each advertiser assigns the value $\alpha r(A)$ to a unique consumer ad impression and $\alpha(1 + \sigma)r(A)$ to two impressions. Then the ad price is $\alpha X_j^{SH} r(A^m) + \sigma\alpha X_j^{MH} r(A^m)$, where A^m is the monopoly equilibrium ad level and does not depend on the composition of single- and multi-homing consumers. Thus, the only difference is that the equilibrium ad level is the monopoly ad level, which does not qualitatively change the results since $r(A^m)$ solely enters as a multiplying factor with the $\alpha\sigma$ term in the equilibrium expressions. An advertiser's profit now is $\alpha(1 - \sigma)\frac{n^*}{2} X_n^{MH^*} r(A^m)$.

A.4.4 Revisiting multi-homing discount

The gross value v from reading the first newspaper and $v - d$ from reading the second newspaper is set for simplicity and employs a modified version of Anderson et al. (2017). In particular, denote the universe of possible topics that any newspaper can deliver by Q (politics, business, opinion, tech, science, health, books & arts, food, travel, the weather, sports, etc.). Let $Q_j \subseteq Q$ be the set of topics newspaper j actually covers. Newspaper j is more attractive for consumers the larger the set Q_j. Each topic in Q is assumed to have the same intrinsic value for consumers and I normalize the measure of Q to 1. Denoting $q_j \in [0,1]$ the measure of Q_j, newspaper j has a larger "functionality" than newspaper k if $q_j > q_k$, simply because j covers a wider range of topics than k. Furthermore, suppose that each topic can be described as a point on the real line interval $[0,1]$, and that Q_j is drawn randomly from Q. That is, each point in Q is drawn randomly with probability q_j and so, by the law of large numbers, the number of topics covered by newspaper j equals q_j. Although I take the functionality as given, q_j can be interpreted as *internal diversity* of newspaper j.

The product $q_j q_k$ measures the expected overlap in topics by two available newspapers j and k ($\forall j, k \in \{1, \ldots, n\}, j \neq k$), and $(1-q_j)q_k$ is the expected measure of topics that are available in k but not j. To account for the overlap for the multi-homing decision by consumers, denote by

$$V_{jk} \equiv (1-q_j)q_k + (1-\beta)q_j q_k = q_k - \beta q_j q_k$$

the incremental value from topics when purchasing k in addition to j, where $(1-\beta)q_j q_k$ is the discounted value from reading the same topics twice and $\beta \in [0,1]$. That is, I restrict attention to the case where overlap in topics is detrimental to the perceived incremental value of a second newspaper. One extreme is where consumers place zero value on duplications ($\beta = 1$); at the other extreme the valuation of a newspaper's attributes is independent of the presence of the same attributes in the other newspaper ($\beta = 0$). Denote the consumers' reservation price by R and assume that $q_j = q \; \forall j \in \{1, \ldots, n\}$. Then,

$$v \equiv Rq \quad \text{and} \quad v - d \equiv RV = R(1-\beta q)q \tag{A.6}$$

Note that the amount of "functionality" does not interact with the distance-based utility. That is, the u_j given by (1.1) and u_{jk} formulations in Section 1.2 implicitly assume that a consumer values additional topics covered by a newspaper, whether it is her first or second purchase, irrespective of her location.

One alternative to let functionality interact with the distance-based utility, as in Anderson et al. (2017), is to re-define the single-homing and multi-homing utility expressions, respectively, as

$$u_j = (R - tx)q_j - p_j \quad \text{and} \quad u_{jk} = [R - t(1 - x)]\, V_{jk} - p_k \tag{A.7}$$

The formulation in (A.7) implies that the closer a consumer is located to the views of a newspaper, the more she values additional topics, which again holds for first and second purchases. Formally, $\frac{\partial^2 u_j}{\partial x \partial q_j} = -t < 0$ and $\frac{\partial^2 u_{jk}}{\partial x \partial q_k} = t\,(1 - \beta q_j) > 0$.[11]

The formulations in (A.6) and (A.7) both result in consumers in the middle to multi-home by construction: $\partial\,[(u_j + u_{jk}) - u_j]\,/\partial x = \partial u_{jk}/\partial x > 0$. Both formulations are versions of the following story. Consumers located close to the center insist less on being informed by one extreme view of the topics or, alternatively, have less of a regional preference regarding the news. While these consumers derive less utility from a particular newspaper than a consumer located closely to that newspaper, the flip side is that these consumers are more willing to be exposed to the view from an alternative source. This results in consumers in the middle to multi-home.

Alternatively, suppose that the location of a consumer is re-interpreted as the taste for *engagement intensity*. In particular, consumers located closely to a variety are assumed to care more about news consumption than consumers in the center of the network. It is then reasonable to think that such high-engagement news consumers are also more curious to consume news from other outlets. One way to model this multi-homing story in the present framework is to assume that the farther away a consumer is from her second choice (if available), the more she appreciates additional topics covered by that newspaper. To this end, I modify (A.7) to

$$u_j = (R - tx)q_j - p_j \quad \text{and} \quad u_{jk} = [R - tx]\, V_{jk} - p_k \tag{A.8}$$

Clearly, $\partial^2 u_{jk}/\partial x \partial q_k < 0$ and $\partial u_{jk}/\partial x < 0$. The latter results in a consumer to multi-home if she is sufficiently close to a variety (and to single-home otherwise). In this more intricate setup, an increase in p_j raises as before the profit per newspaper unit sold but reduces the multi-homing demand with the consequent loss in multi-homing eyeballs sold to advertisers. That is, fewer consumers consider buying j as a second additional purchase if j increases its price. Recall that j's own price previously did not impact its single-homing demand and that prices were strategically independent. Now two newspapers also directly compete for the single-homing consumers in the middle. Therefore, subscription prices are strategically dependent

[11] In addition, a consumer values a second newspaper less the closer she is located to that newspaper: $\frac{\partial^2 u_{jk}}{\partial x \partial \beta} = -t q_j q_k < 0$.

and an increase in p_j additionally reduces the demand from single-homing consumers with the accompanying loss in single-homing eyeballs sold to advertisers. This creates additional downward pressure for newspapers regarding consumer prices.

Repeating the analysis under the alternative specifications (A.7) and (A.8), the qualitative results remain unchanged (the analyses are available upon request). Therefore, the results are robust to modeling specifications where either the consumers at the extremes or in the center of the network multi-home.

Note that I have only addressed the intrinsic values v and $v-d$ from the consumer perspective and the heterogeneity of consumers regarding these values. A planner who cares about having informed citizens likely values R higher and β lower in (A.6)-(A.8) than consumers.

A.4.5 Ownership

Thus far I have assumed that each media "firm" owns one and only one digital newspaper. Yet in practice, many newspaper conglomerates own more than one newspaper. Let now n denote the total number of conglomerates (or owners) and suppose for simplicity that each conglomerate owns the same number $\kappa \geq 1$ of actual digital newspapers (Germano, 2008; Germano and Meier, 2013). Then, $\kappa n \leq N$ is the total number of digital newspapers in the market. Everything else stays the same; I continue to focus on the case with a strictly positive number of single-homing and multi-homing consumers. Modifying equation (1.5), a representative conglomerate's profit set-up is now given by

$$\Pi = \sum_{j \in J} \left\{ \left[(1-\tau)p_j - c + \alpha \right] X_j^{SH} + \left[(1-\tau)p_j - c + \sigma\alpha \right] X_j^{MH} - F \right\}, \qquad \text{(A.9)}$$

where J is the set of digital newspapers that belong to the conglomerate such that $|J| = \kappa$, and $X_j^{SH} = \frac{2}{N}\frac{1}{N-1} \sum_{\substack{k \neq j, \\ k \in \{1,\ldots,\kappa n\}}} x_{jk} + \frac{2}{N}\frac{N-\kappa n}{N-1}$, $X_j^{MH} = \frac{2}{N}\frac{1}{N-1} \sum_{\substack{k \neq j, \\ k \in \{1,\ldots,\kappa n\}}} (x_{kj} - x_{jk})$.

When a conglomerate increases the price of j that it owns, the effects on j are as before but now the conglomerate also takes into account the impact on the $\kappa - 1$ other newspapers that it owns. Suppose that a conglomerate owns both j and k in Figure 1.2. An increase in p_j leads to a reduction in x_{kj}. That is, some of the multi-homing consumers who buy j in addition to k are turned into single-homing consumers. Those consumers still buy from k and thus the net subscription revenue change on k is zero (the loss in multi-homing consumers that now do not buy k in addition to j anymore are already accounted for j). At the same time, single-homing consumers are more valuable than multi-homing consumers for k or any other of the other $\kappa - 1$ newspapers that the conglomerate owns, in terms of advertisement profit; the incremental ad price gain is $\alpha(1-\sigma)$. In other words, a conglomerate's subscription price increase of one newspaper has a positive externality on the ad price of all the other newspapers that it owns. Consequently, the subscription price is larger than in the baseline model.

Lemma A.2. *Let κ and n be fixed. If $v - d - \frac{c-\sigma\alpha}{1-\tau} - \frac{\alpha(1-\sigma)(\kappa-1)}{(1-\tau)(\kappa n-1)} - \frac{t(N-1)}{\kappa n-1} \in (0,t)$, the unique symmetric subscription price in equilibrium is*

$$p_n^* = \frac{1}{2}\left(v - d + \frac{c-\sigma\alpha}{1-\tau} + \frac{\alpha(1-\sigma)(\kappa-1)}{(1-\tau)(\kappa n-1)} + \frac{t(N-\kappa n)}{\kappa n-1}\right) \tag{A.10}$$

Equilibrium sales, number of single-homing and multi-homing consumers for each newspaper are, respectively,

$$\begin{aligned} D_n^* &= \frac{\kappa n-1}{tN(N-1)}\left[v - d - \frac{c-\sigma\alpha}{1-\tau} - \frac{\alpha(1-\sigma)(\kappa-1)}{(1-\tau)(\kappa n-1)}\right] + \frac{N-\kappa n}{N(N-1)} \\ X_n^{MH*} &= 2\left(D_n^* - \frac{2N-\kappa n-1}{N(N-1)}\right), \quad X_n^{SH*} = 2\left(\frac{2N-\kappa n-1}{N(N-1)}\right) - D_n^* \end{aligned} \tag{A.11}$$

A conglomerate's additional subscription price term, $\frac{\alpha(1-\sigma)(\kappa-1)}{2(1-\tau)(\kappa n-1)}$, increases in the incremental ad price gain (larger α, smaller σ) and its market power (larger κ, smaller n). Note that the results of Lemma A.2 coincide with Lemma 1.1 for $\kappa = 1$. How much a conglomerate can pass on the additional term to consumers is disciplined, like the marginal cost c, by the VAT rate. Thus, the conglomerate's ad-price-gain term is beneficial for consumers following a VAT rate reduction. Now following the VAT rate reduction, the subscription price increases, with its corresponding Lemma 1.2 results, if and only if

$$c + \frac{\alpha(1-\sigma)(\kappa-1)}{\kappa n-1} < \sigma\alpha \tag{A.12}$$

If $c = 0$ and $\alpha \neq 0$, a sufficient condition for (A.12) to hold is $\sigma(n+1) > 1$, which is satisfied for $\sigma = 1/2$ (Gentzkow et al., 2014; Shi, 2016) since $n \geq 2$.[12]

Repeating the proofs of Proposition 1.1 through Proposition 1.3 under ownership for a fixed κ, the steps of which I skip here for brevity, there exist cut-off levels for the increase in the number of newspapers following the VAT reduction, above which

[12] Since $\frac{\alpha(1-\sigma)(\kappa-1)}{\kappa n-1}$ increases in κ, $\lim_{\kappa\to\infty} \frac{\alpha(1-\sigma)(\kappa-1)}{\kappa n-1} = \frac{\alpha(1-\sigma)}{n} < \sigma\alpha \iff \sigma(n+1) > 1$.

the various consumer measures increase. Since the conglomerate's ad-price-gain term increases in the number of newspapers it owns but is disciplined by the VAT rate, the direct subscription price increase is smaller following a VAT reduction the more newspapers the conglomerate owns, i.e., $\frac{\partial^2 p_n^*}{\partial \tau \partial \kappa} = \frac{\alpha(1-\sigma)(n-1)}{2(1-\tau)^2(\kappa n-1)^2} > 0$.

A.5 Welfare and optimal number of newspapers

The theoretical analysis in Section 1.3 has solely focused on the impact of the VAT reduction on the consumer measures. Suppose instead that the authority is interested in the social surplus and not only in one of the consumer measures. This section and Appendix A.6 lay the theoretical foundations for the welfare outcomes in Section 1.4.2 and policy experiments in Section 1.4.3 for the calibrated model.

Welfare is the sum of total newspaper profit, advertiser surplus, tax revenue, and consumer surplus. That is, the social planner does not internalize any externalities associated with an "enlightened public discourse" (Kontor, 2015, p. 4). To examine the impact of the VAT reduction on total welfare, a first helpful step is to look at whether there is over- or under-entry in the free-entry equilibrium from a socially optimal perspective. During this first step the VAT rate is fixed and I ignore the integer constraint for simplicity.

Over- and under-entry in equilibrium. In the classic spokes model by Chen and Riordan (2007), the socially optimal number of firms can be greater than, equal to, or less than the equilibrium number (depending on v, N and F). Given the additional two-sidedness in the present setup, of particular interest is how the presence of multi-homing consumers and advertisers affect the socially optimal to equilibrium number of digital newspapers comparison. I continue to focus on the case with both positive numbers of single-homing and multi-homing consumers.

I decompose total welfare into its constituent parts and examine total newspaper profits, advertiser surplus, tax revenue, and consumer surplus separately. Total profit is per-platform profit, given by expression (1.10), multiplied by n. The partial derivative of total profits with respect to n evaluated at zero profit yields

$$\left.\frac{\partial\left(n\Pi^*(n)\right)}{\partial n}\right|_{\Pi^*(n)=0} = n\left[\frac{1}{2}(1-\tau)\frac{D_n^*}{n-1}\left(v-d-\left(\frac{c-\sigma\alpha}{1-\tau}\right)-\frac{t(N-1)}{n-1}-t\right) - \frac{\alpha(1-\sigma)}{tN(N-1)}\left(v-d-\left(\frac{c-\sigma\alpha}{1-\tau}\right)+t\right)\right] < 0 \tag{A.13}$$

by the relevant parameter space. The externality from entry by an additional newspaper on total newspaper profit is negative due to a business-stealing effect and an ad-price-reduction effect. Regarding the former, as a variety on spoke k enters some *category 3* consumer on spoke k far from the center, who were previously buying from j, might not buy from j anymore but only from k. Regarding the latter effect, some *category 2* and *3* buying from j and close to the center are turned into *category 1* consumers; while those consumers still visit j, the ad price is reduced because those consumers are now shared with the new newspaper k. Both of these negative effects vanish if all *category 1* consumers multi-home (because even from the additional entry of k the consumer continues to buy from j, and thus there is no business-stealing effect) and $\sigma = 1$ (second impressions have the same value to advertisers, which leaves the ad price unchanged).

The total advertisers surplus (AS) is given by (1.12). Then,

$$\frac{\partial AS}{\partial n} = \frac{\alpha(1-\sigma)(2n-1)}{tN(N-1)}\left(v - d - \left(\frac{c-\sigma\alpha}{1-\tau}\right) - \frac{t(N-1)}{2n-1}\right) > 0$$

Combining, the "total producer surplus" externality from entry is

$$\left.\frac{\partial\left(n\Pi^*(n) + AS\right)}{\partial n}\right|_{\Pi^*(n)=0} = \left(n\frac{1}{2}(1-\tau)\frac{D_n^*}{n-1} + \frac{\alpha(1-\sigma)(n-1)}{tN(N-1)}\right) \times\left(v - d - \left(\frac{c-\sigma\alpha}{1-\tau}\right) - \frac{t(N-1)}{n-1} - t\right) - \frac{\alpha(1-\sigma)}{N(N-1)} < 0$$

by the parameter restriction to have a strictly positive number of single-homing consumers from the first category, i.e., $v - d - \left(\frac{c-\sigma\alpha}{1-\tau}\right) - \frac{t(N-1)}{n-1} < t$. Thus far, even though an increase in n is beneficial for the advertisers entry is excessive from the total producer perspective. Total tax revenue T equals $\tau n p_n^* D_n^*$. Then,

$$\frac{\partial T}{\partial n} = \tau\left[p_n^* D_n^* + n p_n^* \frac{\partial D_n^*}{\partial n} + n\frac{\partial p_n^*}{\partial n} D_n^*\right],$$

which is positive if the sum of the increase of newspapers from which the authority collects taxes and the positive externality on demand per newspaper (through a reduction in the subscription price) from entry outweighs the negative externality on the subscription price.

Lastly, the consumer surplus (CS) is given by (A.26). Then,

$$\frac{\partial CS}{\partial n} = \frac{2N-2n-1}{2N(N-1)}(2d-t) + \frac{tn(2N-n-1)}{2N(n-1)^2} + \frac{t(N-2n)}{4N(N-1)} + \frac{3t}{4N} + \frac{2n-1}{16}\left(\frac{tN(N-1)}{(n-1)^2}\right)\left(X_n^{MH^*}\right)^2 + \frac{n}{4}\frac{t(N-1)}{(n-1)^2}X_n^{MH^*} + \frac{t(2N-2n-1)}{4(n-1)}X_n^{MH^*},$$

which is positive by its construction.

Denote welfare by $W = n\Pi^*(n) + AS + T + CS$. Then, there is excessive entry in equilibrium if $(\partial W/\partial n)|_{\Pi^*(n)=0} < 0$ and under-entry otherwise. Recall from the discussion following expression (A.13) that the negative newspaper profit externality from entry vanishes as $\sigma \to 1$ and all *category 1* consumers multi-home. Then, given that AS and CS increase in n, there is under-entry in equilibrium. If instead in equilibrium the full single-homing by *category 1* consumers case is approached,

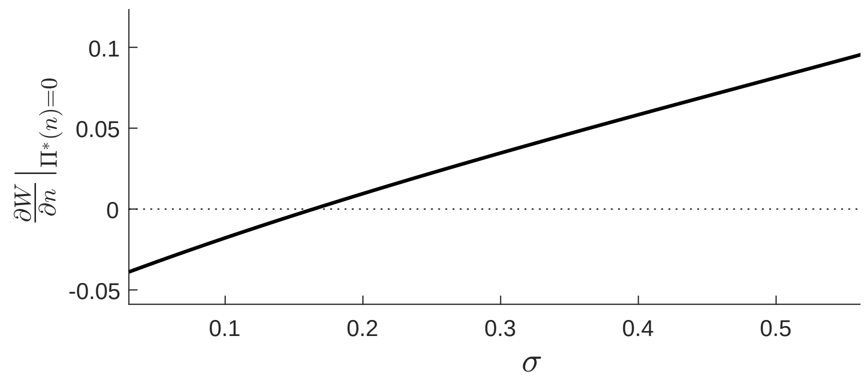
$$\begin{aligned}&\left.\frac{\partial W}{\partial n}\right|_{\substack{\Pi^*(n)=0\\ x_{jk}\to 1/2}} < 0\\ \iff &[v-c+\sigma\alpha]\left(2N-n-1+\frac{n(N-n)}{n-1}\right)\\ &< \left(d+\frac{t}{2}\right)\frac{n(N-n)}{n-1} + \frac{t(1-\tau)(2N-n-1)^2}{2(n-1)} + \frac{t(2n-1)}{4} + t(N-2n)\end{aligned} \tag{A.14}$$

It follows from (A.14) that over-entry in equilibrium is more likely when the value of second impressions for advertisers is low, i.e., small $\sigma\alpha$, and/ or when the incremental value from reading a second newspaper is low (large d). Figure A.5.1 corroborates the above derivations in a numerical example: there is excessive entry for low σ values but under-entry for high values of σ.

Impact of VAT reduction on welfare. With this preliminary work, I now address how welfare is affected by the VAT reduction. Welfare W, evaluated at the zero profit condition, is given by

$$\begin{aligned}W|_{\Pi^*(n)=0} =& n\left[v-(1-\tau)p_n^*\right]D_n^* - \frac{tn(n-1)}{4N(N-1)} - \frac{tn(N-n)}{N(N-1)}\\ &+ n\frac{\alpha}{2}(1-\sigma)X_n^{MH^*} - \frac{dn}{2}X_n^{MH^*} - \frac{tn}{4}X_n^{MH^*}\left(\frac{N(N-1)}{4(n-1)}X_n^{MH^*}+1\right)\end{aligned}$$

Figure A.5.1: Over- and under-entry in equilibrium



Parameters: $v = 2, d = t = 1, \tau = 0.2, c = 0, \alpha = 1.5, N = 20, F = 0.1$

Welfare increases following a VAT reduction if and only if

$$\frac{dW}{d\tau}\bigg|_{\Pi^*(n)=0} = \frac{\partial W}{\partial \tau}\bigg|_{\Pi^*(n)=0} + \frac{\partial W}{\partial n}\bigg|_{\Pi^*(n)=0} \frac{dn}{d\tau} \tag{A.15}$$

is negative. Lemma A.3 addresses the direct impact of a VAT reduction on W.

Lemma A.3. *Let n be fixed. If τ decreases, welfare decreases if and only if $\sigma\alpha > c$.*

Proof.

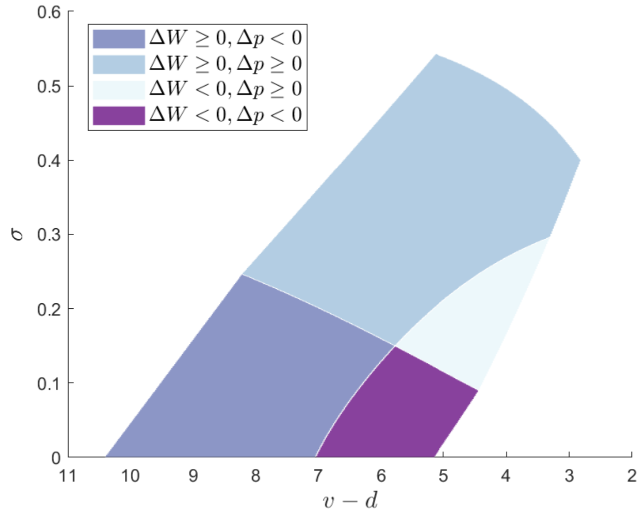
$$\frac{\partial W}{\partial \tau}\bigg|_{\Pi^*(n)=0} = n\left[v - d - (c - \sigma\alpha) - \frac{tN(N-1)}{4(n-1)}X_n^{MH^*} - \frac{t}{2}\right]\frac{\partial D_n^*}{\partial \tau},$$

where $v-d-(c-\sigma\alpha)-\frac{tN(N-1)}{4(n-1)}X_n^{MH^*}-\frac{t}{2} > v-d-(c-\sigma\alpha)-\frac{t(N-1)}{n-1} > 0\ \forall\tau$; the first inequality uses the fact that $X_n^{MH^*} < \frac{2(n-1)}{N(N-1)}$ and the last inequality follows from the parameter restriction to have a positive number of multi-homing consumers. Since $\frac{\partial D_n^*}{\partial \tau} > 0$ if and only if $\sigma\alpha > c$, the statement follows. □

Since $\frac{dn}{d\tau} < 0$ by Proposition 1.1, a necessary (but not sufficient) condition for welfare to increase following the VAT reduction is that welfare increases in n, i.e., $\frac{\partial W}{\partial n}\big|_{\Pi^*(n)=0} > 0$, which is equivalent to having under-entry. As in the case for the consumer measures, the **indirect effect** from additional entry has to outweigh the negative direct effect for W to increase following a VAT reduction. Therefore if there is (initial) under-entry, W overall increases if the increase in the number of newspapers is above some threshold $\Delta\tilde{n}^W$ and decreases if $\Delta n < \Delta\tilde{n}^W$.

However, if there is over-entry then W decreases from additional entry. Then, the indirect effect negatively impacts W. Therefore and in contrast to the consumer

Figure A.5.2: Illustration of welfare change



Notes: The figure illustrates that welfare increases or decreases following the VAT reduction in the $(v-d, \sigma)$ space when allowing for free entry. The areas also take into account whether the subscription price has increased or decreased. I use the parameters calibrated in Section 1.4.1 for an average Norwegian region except $\alpha = 9$, which in turn requires lower $v - d$ values to have a positive number of single- and multi-homing consumers. If I use the calibrated α value, welfare always increases following the VAT reduction in the then permissible $(v-d, \sigma)$ space. The number of newspapers is not restricted to take an integer value.

measures, if there is (initial) over-entry a Δn cut-off above which W overall increases does not exist. Proposition A.4 summarizes.

Proposition A.4. *If there is initial over-entry from a social perspective, a reduction in the VAT rate leads to a decrease in welfare. Otherwise, welfare increases if and only if* $\Delta n > \Delta \tilde{n}^W > 0$.

It is noteworthy that in contrast to the consumer measures, W *may* not increase even if the subscription price overall decreases. Figure A.5.2 illustrates. Similar to Figure 1.3 (though σ, not α is on the vertical axis), it shows that $(v-d, \sigma)$ regions exist where W decreases when $\Delta p \geq 0$ or W increases when the overall price change is either positive or negative.

However, there also exists a parameter region in which W decreases even though $\Delta p < 0$, which Figure A.5.2 suggests to happen for low σ values (for a sufficiently large $v-d$ value). The above discussion surrounding over- and under-entry provides an intuition why this can arise: if σ is small, the direct impact on the subscription price is small, and since n still increases following the VAT reduction the subscription price decreases overall. However, precisely for small σ values the negative externality on total profits from additional entry is large in magnitude, resulting in $\frac{\partial W}{\partial n}\Big|_{\Pi^*(n)=0} < 0$.

Welfare Second-Best. In Column (6) of Table 1.2 I compute the VAT rate that maximizes total welfare, i.e., expression (A.15) equals zero, for the calibrated model. Next, suppose that the social planner can regulate entry (but not subscription and ad prices) in addition to setting the VAT rate. By Lemma A.3 ($\frac{\partial W}{\partial \tau} > 0 \ \forall n$), the planner chooses the VAT rate such that all *category 1* consumers multi-home:

$$\tau^{SB} = 1 - \frac{\sigma\alpha - c}{\frac{t(N+n^{SB}-2)}{n^{SB}-1} - (v-d)} \tag{A.16}$$

The first-order condition of W with respect to n and by using (A.16) can be written as

$$\begin{aligned} &2[v-c](N-1) + \alpha\left[2(N-n) + (1+\sigma)(n-1)\right] - FN(N-1) \\ &+ \frac{n}{n-1}(N-1)\left[v-d-c+\sigma\alpha - t\right] - \left[\alpha(1-\sigma) + \frac{3t(2n-1)}{4} + dn\right] = 0 \end{aligned} \tag{A.17}$$

The left-hand side in (A.17) strictly decreases in n. Assuming that the left-hand side is strictly positive for $n = \max\left\{2, \frac{t(N-1)}{v-d-\frac{c-\sigma\alpha}{1-\tau}} + 1\right\}$ and strictly negative for $n = \min\left\{N, \frac{t(N-1)}{v-d-\frac{c-\sigma\alpha}{1-\tau}-t} + 1\right\}$,[13] the optimal number of newspapers chosen by the planner, n^{SB}, that solves (A.17) exists and is unique. Observe that n^{SB} increases in the willingness-to-pay by advertisers for unique consumer impressions (α), the incremental value for an advertiser when a consumer sees its ad the second time (σ) and decreases in the discount from reading a second newspaper (d).

A.6 Alternative instruments

I now explore the efficacy of instruments other than a VAT rate on subscriptions: a value-added tax or subsidy on advertisements, an ad cap, and a lump-sum transfer to digital newspapers.

Value added tax or subsidy on advertiser side. While the subject of taxation of advertising has sparked many debates and has been addressed in the context of two-sided markets for a fixed number of platforms (Kind et al., 2013; Kind and

[13] I shall require that the fixed cost F delivers such an outcome.

Koethenbuerger, 2018), in practice said tax has been rarely enacted by governments and authorities.[14]

Let $\tau \triangleq \tau_R$ be the VAT rate on the reader side as before and τ_A be the value-added tax or subsidy on the advertiser side; it is a subsidy if $\tau_A < 0$ and a tax otherwise. The rest of the model stays the same. Then, (1.5) is modified to $\Pi_j = [(1-\tau_R)p_j - c]\, D_j + (1-\tau_A)\left[\alpha X_j^{SH} + \sigma\alpha X_j^{MH}\right] - F$. The symmetric subscription price in equilibrium is[15]

$$p_n^* = \frac{1}{2}\left(v - d + \frac{c-\sigma\alpha}{1-\tau_R} + \frac{\sigma\alpha\tau_A}{1-\tau_R} + \frac{t(N-n)}{n-1}\right), \tag{A.18}$$

from which it is immediate that the subscription price is increasing in τ_A and lower than the baseline price given by (1.7) if $\tau_A < 0$ but higher otherwise. The reason is that a value-added ad subsidy increases a newspaper's profitability from the ad side: an increase in the subscription side now leads to a larger loss from selling multi-homing eyeballs to advertisers (the term outside the squared brackets in (1.6) is multiplied by $(1-\tau_A)$, which decreases in τ_A). Since an ad subsidy is equivalent to an ad price increase, the price on the other side, the subscription side, decreases by the "seesaw principle" (Rochet and Tirole, 2006). Thus, the introduction of an ad subsidy has the opposite qualitative direct effect on the **consumer measures** compared to a τ_R decrease (Lemma 1.2).

An ad subsidy clearly improves the newspapers' ad revenue source. Yet it is not obvious that n increases since the profit on the consumer side decreases (the subscription price reduction effect dominates the demand expansion effect)[16] and the price per ad increases if and only if $\sigma > 1/2$, i.e., the gain of selling eyeballs to multi-homing consumers outweighs the loss of single-homing consumers. Thus, the number of newspapers may decrease for instance for low σ values following an ad subsidy introduction.[17] However, even if n decreases following an ad subsidy,

[14]Sweden abolished its advertisement VAT of 2.5% on newspapers in 2018 and Hungary introduced an ad tax in 2014 that also applies to newspapers (the 2019 ad VAT rate is 5.3% for the taxable amount above HUF 100 million). In the U.S. in 1987, the Florida legislature enacted but soon repealed a state sales tax on all services that contain advertising (newspapers would have been subjected to this tax).

[15]At this point we require $v - d - \left(\frac{c-\sigma\alpha+\sigma\alpha\tau_A}{1-\tau}\right) - \frac{t(N-1)}{n-1} \in (0,t)$ such that a strictly positive number of *category 1* consumers single-home and multi-home.

[16]Formally, $\partial\left[(1-\tau_R)p_n^* - c\right] D_n^* / \partial\tau_A = \frac{(n-1)(1-\tau_A)(\sigma\alpha)^2}{tN(N-1)(1-\tau_R)} > 0$.

[17]This can be seen in $\frac{\partial\Pi^*(n)}{\partial\tau_A} = -\sigma\alpha D_n^* + \frac{\alpha(1-\sigma)(n-1)}{tN(N-1)}\left[v - d - \frac{c-2(1-\tau_A)\sigma\alpha}{1-\tau_R} - \frac{t(3N-n-2)}{n-1}\right]$, where $D_n^* = \frac{n-1}{tN(N-1)}\left[v - d - \left(\frac{c-\sigma\alpha(1-\tau_A)}{1-\tau}\right)\right] + \frac{N-n}{N(N-1)}$.

I do not find a numerical example where any of the consumer measure decreases overall.[18] That is, the positive direct effect of an ad subsidy dominates the negative indirect effect. In the case of total readership, $\frac{dTRS}{d\tau_A} = \frac{\partial TRS}{\partial \tau_A} + \frac{\partial TRS}{\partial n}\frac{dn}{d\tau_A} < 0$. If n instead increases, the positive indirect effect causes the consumer measures to further increase in addition to the positive direct effect.

Either way, an ad subsidy improves the consumers' well-being. From converse arguments (and numerical exercises), an ad tax always leads to a decrease in the consumer measures. The introduction of an ad subsidy is therefore likely to be more effective than a VAT reduction on the subscription side for promoting high levels of news and current affairs consumption since an ad subsidy (most likely) leads to additional entry and to a direct reduction of the subscription price.

If the authority is not just interested in the consumer measures but in **welfare**, an advertising subsidy can in principle either increase or decrease welfare. First, the direct effect of introducing $\tau_A < 0$ is positive on welfare, i.e., $\left.\frac{\partial W}{\partial \tau_A}\right|_{\Pi^*(n)=0} < 0$. Second, both $\frac{dn}{d\tau_A}$ and $\left.\frac{\partial W}{\partial n}\right|_{\Pi^*(n)=0}$ can be positive or negative from the above discussions. Then, for welfare to increase overall it has to hold that $\left.\frac{\partial W}{\partial n}\right|_{\Pi^*(n)=0}\frac{dn}{d\tau_A} < -\left.\frac{\partial W}{\partial \tau_A}\right|_{\Pi^*(n)=0}$. I find using numerical examples that W decreases only for sufficiently small σ values; in all these examples the number of newspapers has increased following the ad subsidy and there is over-entry ($\frac{dn}{d\tau_A} < 0$ and $\left.\frac{\partial W}{\partial n}\right|_{\Pi^*(n)=0} < 0$, respectively).

A cap on the advertisement level. Let $\kappa \in [0, 1]$ be the cap limit on the fraction of advertisements that a newspaper is allowed to carry (many countries limit the amount of advertising that is allowed on TV. See Anderson and Peitz, 2020, for an analysis). One possible motivation to introduce an ad cap for digital newspapers, albeit not modeled here, is that less information reaches consumers, who have a finite amount of time to spend on news consumption, if they are exposed to too much advertising.[19]

[18]In particular, for the parameters $(v, d, \alpha, \sigma, t, c, F, N)$ I create a grid of the parameter space $[0, 20]^3 \times [0, 1]^2 \times 0 \times [0, 2] \times [2, 3, ..., 40]$. This encompasses the calibrated parameters for Norway. Then for each permissible parameter combination on the grid (i.e., a positive number of consumers single- and multi-home under free-entry, $v > d$ and $d > t/2$), I numerically calculate whether a $\tau_A < 0$ rate leads to a decrease of each of the consumer measures compared to when $\tau_A = 0$. The defined parameter space is the same for the other numerical calculations in this subsection.

[19]This concern, though, is mitigated by the use of *ad-blocking* software. A second concern relates to media bias: media platforms under financial stress may be particularly vulnerable to advertisers' pressure and influence, which in turn can affect the bias (Ellman and Germano, 2009) or quality in general of newspapers.

Profit per newspaper $j \in \{1, ..., n\}$ is now modified to

$$\Pi_j = [(1-\tau)p_j - c] D_j + \kappa \left[\alpha X_j^{SH} + \sigma\alpha X_j^{MH}\right] - F$$

Except for welfare, the impact of κ is identical to the introduction of an ad VAT when $\kappa = 1 - \tau_A$, where $\tau_A \geq 0$.[20] The difference between an ad VAT and an ad cap on welfare W is that the ad VAT collected from the newspapers is added into W, which is not the case for an ad cap. In other words, an ad cap works as if taxes are collected from the advertising side but are then discarded. I do not find a numerical example where the introduction of an ad cap enhances welfare, which is in contrast to an ad VAT that may increase W.

Lump-sum transfer. Another instrument at disposal for an authority is a lump-sum transfer S per digital newspaper. In what follows, I ignore obvious adverse selection issues.[21] The Norwegian Ministry of Culture paid *production grants* to 147 newspapers (of a total of 241 newspapers) at a total value of 308 million NOK in 2015 (Kontor, 2015, p. 4). While the VAT rate applies equally to all digital newspapers, the production grants are selective in practice in Norway and are aimed at newspapers in specific and often difficult market positions (for instance Sámi newspapers, minority language publications, and niche publications). In contrast in the present symmetric set-up, I assume that the lump-sum transfers are non-competitive and that each digital newspaper receives the same amount S.

A lump-sum on the one hand does not directly affect the subscription price given by (1.7), since an incremental price change in interaction with S has no impact on the reader nor the advertiser side. On the other hand a $S > 0$ leads to additional entry (by Lemma A.4). Thus, any consumer measure and welfare is solely indirectly affected through a change of n.

The analytically straight-forward case is when the planner only cares about the **consumer** measures (and thus ignores the total amount spent on transfers). Since each consumer measure increases in n through a decrease in the subscription price and the positive market expansion and matching effects, the planner chooses a lump-

[20] For instance, the symmetric subscription price in equilibrium is $p_n^* = \frac{1}{2}\left(v - d + \frac{c - \sigma\alpha\kappa}{1-\tau} + \frac{t(N-n)}{n-1}\right)$, which is strictly larger than the baseline subscription price for any $\kappa < 1$.

[21] Adverse selection is of particular concern in a digital newspaper market if lump-sum transfers are paid, since creating a "news-site" in order to collect S is practically costless through simple copying-and-pasting.

sum transfer S per newspaper such that all spokes are occupied, i.e., $n^* = N$ under free entry. Adding S to the profit per newspaper in (1.10), the optimal lump-sum solves $\Pi^*(n = N, S^*) = 0$ and is given by

$$S^* = F - \frac{1-\tau}{2tN}\left(v - d - \frac{c - \sigma\alpha}{1-\tau}\right)^2 + \frac{\alpha(1-\sigma)}{tN}\left(v - d - \frac{c-\sigma\alpha}{1-\tau} - 2t\right) \quad \text{(A.19)}$$

If the social planner instead cares about **welfare** W, the optimal S^* is such that $\frac{\partial W}{\partial n}\big|_{\Pi^*(n)+S^*=0} = 0$ is satisfied.[22] That is, the optimal lump-sum transfer ensures that the number of newspapers under free-entry equals the number of newspapers a planner would choose to maximize welfare. Since n increases in S, the optimal lump-sum transfer is positive if there is under-entry in the absence of a transfer and negative otherwise.

A.7 Alternative specifications

In Table A.7.1, I show how the key results differ under alternative specifications. In the first five columns, I report, for each specification and counterfactual, the number of digital newspapers. In the next five columns, I show, again for each specification and counterfactual, total readership per unit mass of consumers. I report welfare in the final five columns. The first, sixth, and eleventh columns report pre-VAT reduction results for the baseline model. The second, seventh, and twelfth columns report post-VAT reductions results for the baseline model. Columns three, eight, and thirteen report the social planner results who chooses the optimal VAT rate and entry as in the final column of Table 1.2. The fourth, ninth, and fourteenth columns report results where digital newspapers are allowed to collude on ad prices. The fifth, tenth, and fifteenth columns report results where the planner chooses an optimal ad subsidy.

The first row in Table A.7.1 repeats the results from the main specification for reference. In the second through fourteenth rows, I first change a single calibrated parameter from the baseline calibration, then recalibrate the remaining parameters, and recompute counterfactuals. The second and third rows show a change in σ, the relative value of a second impression compared to the first impression to advertisers. The fourth and fifth rows show results for the advertisers' willingness to pay per

[22] This follows from $\frac{dW}{dS}\big|_{\Pi^*(n)+S^*=0} = \underbrace{\frac{\partial W}{\partial S}\bigg|_{\Pi^*(n)+S^*=0}}_{=0} + \frac{\partial W}{\partial n}\big|_{\Pi^*(n)+S^*=0}\underbrace{\frac{dn}{dS}}_{>0} = 0.$

Table A.7.1: Alternative specifications

		Number of newspapers					Total readership					Welfare				
		Basline: Pre-VAT reduction	Basline: Post-VAT reduction	Social planner	Allow ad collusion	Ad subsidy	Basline: Pre-VAT reduction	Basline: Post-VAT reduction	Social planner	Allow ad collusion	Ad subsidy	Basline: Pre-VAT reduction	Basline: Post-VAT reduction	Social planner	Allow ad collusion	Ad subsidy
(1)	Preferred calibration	16.69	17.63	21.02	16.75	17.63	1.13	1.22	1.58	1.15	1.33	28.81	29.72	31.86	29.00	29.90
Changing calibration values																
(2)	Increase σ by 25%	16.68	17.63	21.05	16.73	17.57	1.13	1.22	1.59	1.14	1.32	28.86	29.77	31.94	28.99	29.95
(3)	Decrease σ by 25%	16.69	17.63	21.01	16.77	17.66	1.13	1.22	1.58	1.16	1.32	28.74	29.64	31.75	28.99	29.76
(4)	Increase α by 25%	16.68	17.63	21.09	16.76	17.64	1.13	1.22	1.59	1.16	1.33	29.04	29.95	32.14	29.28	30.14
(5)	Decrease α by 25%	16.69	17.63	20.98	16.74	17.48	1.13	1.22	1.58	1.15	1.29	28.56	29.48	31.57	28.70	29.48
(6)	Decrease N by 25%	16.69	17.63	19.89	16.72	17.82	1.13	1.21	2.00	1.15	1.34	34.25	34.87	38.67	34.40	35.10
(7)	$F(0)$ is 1/3 of $F(n)$	16.68	17.63	20.67	16.75	17.63	1.13	1.22	1.56	1.15	1.33	27.38	28.30	30.33	27.58	28.48
(8)	$F(0)$ is 2/3 of $F(n)$	16.68	17.63	20.02	16.75	17.63	1.13	1.22	1.51	1.15	1.33	25.70	26.62	28.43	25.90	26.80
(9)	Increase $(v-d)$ by 10%	16.69	17.63	18.45	16.76	17.00	1.22	1.31	1.39	1.24	1.28	32.56	33.62	34.34	32.78	33.00
(10)	Decrease $(v-d)$ by 10%	16.68	17.63	21.28	16.73	17.86	1.04	1.12	1.60	1.06	1.27	25.18	25.95	28.54	25.34	26.25
(11)	Increase v by 25%	16.69	17.63	21.42	16.75	17.63	1.13	1.22	1.61	1.15	1.33	36.22	37.33	40.07	36.42	37.52
(12)	Decrease v by 25%	16.69	17.63	20.50	16.75	17.63	1.13	1.22	1.55	1.15	1.33	21.41	22.10	23.70	21.58	22.28
(13)	Increase t by 25%	16.69	17.63	21.51	16.71	17.84	0.95	1.02	1.62	0.97	1.13	26.98	27.78	31.35	27.10	28.02
(14)	Decrease t by 25%	16.69	17.63	18.12	16.77	16.90	1.23	1.33	1.37	1.26	1.27	29.76	30.73	31.08	29.98	30.04
Modifying calibration sample																
(15)	11 instead of 19 regions	21.55	23.18	28.72	21.64	23.25	1.09	1.21	1.65	1.12	1.33	27.37	28.57	31.06	27.57	28.80
Modifying model specifications																
(16)	Weight CS in W: 33.3%	16.69	17.63	21.56	16.75	17.63	1.13	1.22	1.63	1.15	1.33	26.53	27.44	29.93	26.72	27.82
(17)	Weight CS in W: 50%	16.69	17.63	21.56	16.75	17.63	1.13	1.22	1.63	1.15	1.33	24.24	25.15	28.05	24.43	25.74

Note: See Appendix A.7 for details.

ad and consumer. In the sixth row, I show results for a change in N. The seventh and eighth rows show the results when I increase the fixed cost of the most efficient newspaper by various degrees. These changes on the "producer" side of the model have some quantitative effects on the number of newspapers, readership, and welfare, but the qualitative insights remain unchanged.

The ninth and tenth rows show results for a change in the consumers' value from reading a second newspaper. The eleventh and twelfth rows show an increase and decrease of the value v from reading a digital newspaper. The thirteenth and fourteenth rows show results for t. These changes on the demand side of the model further highlight that consumers constitute a large fraction of total welfare and that welfare-improving policies go in hand with improving the well-being of consumers (this includes readership).

In the fifteenth row I modify the calibration sample by assuming that the digital newspapers had already competed in the new 11 instead of 19 Norwegian regions (see footnote 39).

The remaining rows of the table explore changes to the model specification. In each case, I alter a model feature, re-calibrate the model, and calculate counterfactuals. In the baseline model and calibration all constituent welfare parts have the same weight, i.e., 25%. The sixteenth and seventeenth rows present results in which I change the weight of the consumer surplus accounted for in welfare. These are 33.3% and 50%, respectively, in the two rows. Thus, a planner puts an increased emphasis on having an "enlightened public discourse" (Kontor, 2015, p. 4), which results in a larger number of newspapers and readership by an intervening planner.

A.8 Proofs

***Equilibrium subscription price for all parameter configurations & fixed* n.**

I consider in turn the seven regions of parameter values that have to be considered for the full characterization of equilibrium subscription prices when the number of digital news is fixed. In each region, I construct a unique symmetric subscription price with the property that it can only hold in the assumed parameter values of that region. Any other subscription price is then a symmetric equilibrium in a different region. The symmetric equilibrium is thus also unique. I focus on the case $d > t/2$.

Region ①: Suppose that a symmetric equilibrium price satisfies $p^* = v - d - t$. If all newspapers charge p^*, all *category 2* and *3* consumers go to their, respectively, first and second preferred variety and all *category 1* consumers multi-home: $x_{jk}(p^*) = 0, x_{kj}(p^*) = 1 \ \forall j, k \in \{1, \ldots, n\}, j \neq k$. Thus $D_j(p^*) = \frac{2}{N}$. Then,

$$D_j(p_j, p^*) = \begin{cases} \frac{2}{N} & \text{if } p_j \text{ is slightly below } v - d - t \\ \frac{2}{N(N-1)} \sum\limits_{\substack{k \neq j, \\ k \in \{1,\ldots,n\}}} \left[\frac{v - p_j - d}{t}\right] + \frac{2}{N}\frac{N-n}{N-1} & \text{if } p_j \text{ is slightly above } v - d - t \end{cases}$$

That is, there is a kink in the demand for newspaper j at $p_j = v - d - t$. In order for $p^* = v - d - t$ to be an equilibrium, a slight decrease of p_j at p^* should not increase profit: this is trivially true because D_j remains unchanged, thus a decrease in p_j decreases Π_j. In addition, a slight increase of p_j at p^* should not increase profit, i.e.,

$$\begin{aligned} \left.\frac{\partial \Pi_j}{\partial p_j}\right|_{p_j = p^*} &= (1-\tau)D_j + [(1-\tau)p_j - c + \sigma\alpha] \left.\frac{\partial X_j^{MH}}{\partial p_j}\right|_{p_j = p^*} \\ &= (1-\tau)\left[\frac{2}{N}\frac{n-1}{N-1} + \frac{2}{N}\frac{N-n}{N-1}\right] \\ &+ [(1-\tau)(v-d-t) - c + \sigma\alpha] \frac{2(n-1)}{N(N-1)}\left(-\frac{1}{t}\right) \leq 0, \end{aligned}$$

which holds if and only if $d + t + \frac{c - \sigma\alpha}{1-\tau} + \frac{t(N-1)}{n-1} < v$. Then, p^* is a local maximum. Newspaper j cannot benefit from any deviation to $p_j < v - d - t$, because the second-order condition is satisfied for $p_j < v - d - t$: $\frac{\partial^2 \Pi_j}{\partial p_j^2} = -\frac{4(n-1)(1-\tau)}{tN(N-1)} < 0$. Therefore, p^* is also globally optimal.

Region ②: Suppose that the symmetric equilibrium subscription price is such that $v - d - t < p^* < v - d - \frac{t}{2}$. If all newspapers charge p^*, the *category 2* and *3* consumers make the same adaptation decision as in Region ①. A strictly positive number of *category 1* consumers now single-home and multi-home: $0 < x_{jk}(p^*) < x_{kj}(p^*) < 1 \ \forall j, k \in \{1, \ldots, n\}, j \neq k$. Then, the demand facing newspaper j is

$$D_j = \frac{2}{N}\frac{1}{N-1} \sum_{k \neq j, k \in \{1,\ldots,n\}} [x_{jk} + (x_{kj} - x_{jk})] + \frac{2}{N}\frac{N-n}{N-1}$$

The first-order condition for newspaper j is

$$\frac{\partial \Pi_j}{\partial p_j} = (1-\tau)D_j + [(1-\tau)p_j - c + \sigma\alpha] \frac{\partial X_j^{MH}}{\partial p_j} = 0,$$

where $\frac{\partial X_j^{MH}}{\partial p_j} = \frac{2(n-1)}{N(N-1)}\left(-\frac{1}{t}\right)$. Observe that $\frac{\partial X_j^{SH}}{\partial p_j} = 0$. The symmetric equilibrium subscription price is

$$p^* = \frac{1}{2}\left(v - d + \frac{c - \sigma\alpha}{1-\tau} + \frac{t(N-n)}{n-1}\right)$$

as stated in Lemma 1.1. It is straight-forward to see that the second-order condition is satisfied and thus p^* is a local maximum: $\frac{\partial^2 \Pi_j}{\partial p_j^2} = -\frac{4(n-1)(1-\tau)}{tN(N-1)} < 0$. The requirements that $p^* < v - d - \frac{t}{2}$ and $v - d - t < p^*$ are satisfied if and only if, respectively, $d + \frac{c-\sigma\alpha}{1-\tau} + \frac{t(N-1)}{n-1} < v$ and $v < d + t + \frac{c-\sigma\alpha}{1-\tau} + \frac{t(N-1)}{n-1}$ hold. Since $v - d - t < p^* < v - d - \frac{t}{2}$, any deviation to any $p < p^*$ cannot be profitable because the second-order condition is satisfied for $p < p^*$. Thus, no global deviation is profitable. .

Region ③: Suppose that a symmetric equilibrium price satisfies $p^* = v - d - \frac{t}{2}$. If all newspapers charge p^*, all *category 2* and *3* go to their, respectively, first and second preferred variety and all *category 1* consumers single-home: $x_{jk}(p^*) = x_{kj}(p^*) = 1/2\ \forall j,k \in \{1,\ldots,n\}, j \neq k$. The *category 1* consumer at distance 1/2 from j (and k) is indifferent between single-homing and multi-homing. Thus $D_j(p^*) = \frac{2}{N(N-1)}\left[\frac{1}{2}(n-1) + N - n\right]$. Then

$$D_j(p_j, p^*) = \begin{cases} \frac{2}{N(N-1)} \sum\limits_{\substack{k\neq j,\\ k\in\{1,\ldots,n\}}} \left[\frac{1}{2} + \frac{p^* - p_j}{2t}\right] + \frac{2(N-n)}{N(N-1)} & \text{if } p_j \text{ is slightly above } v - d - \frac{t}{2} \\ \frac{2}{N(N-1)} \sum\limits_{\substack{k\neq j,\\ k\in\{1,\ldots,n\}}} \left[\frac{v - p_j - d}{t}\right] + \frac{2}{N}\frac{N-n}{N-1} & \text{if } p_j \text{ is slightly below } v - d - \frac{t}{2} \end{cases}$$

That is, there is a kink in the demand for newspaper j at $p_j = v - d - \frac{t}{2}$. A slight increase in p_j at p^* should not increase profit in order for $p^* = v - d - \frac{t}{2}$ to be an equilibrium, i.e.,

$$\begin{aligned} \left.\frac{\partial \Pi_j}{\partial p_j}\right|_{p_j = p^*} &= (1-\tau)D_j + [(1-\tau)p_j - c + \alpha]\left.\frac{\partial D_j}{\partial p_j}\right|_{p_j=p^*} \\ &= (1-\tau)\left[\frac{2}{N}\frac{n-1}{N-1}\frac{1}{2} + \frac{2}{N}\frac{N-n}{N-1}\right] \\ &+ \left[(1-\tau)\left(v - d - \frac{t}{2}\right) - c + \alpha\right]\frac{2(n-1)}{N(N-1)}\left(-\frac{1}{2t}\right) \leq 0, \end{aligned}$$

which holds if and only if $d - \frac{t}{2} + \frac{c-\alpha}{1-\tau} + \frac{2t(N-1)}{n-1} \leq v$. Also, a slight decrease of p_j at

p^* should not increase profit, i.e.,

$$\begin{aligned}\left.\frac{\partial \Pi_j}{\partial p_j}\right|_{p_j=p^*} &= (1-\tau)D_j + [(1-\tau)p_j - c + \sigma\alpha] \left.\frac{\partial X_j^{MH}}{\partial p_j}\right|_{p_j=p^*} \\ &= (1-\tau)\left[\frac{2}{N}\frac{n-1}{N-1}\frac{1}{2} + \frac{2}{N}\frac{N-n}{N-1}\right] \\ &+ \left[(1-\tau)\left(v-d-\frac{t}{2}\right) - c + \sigma\alpha\right]\frac{2(n-1)}{N(N-1)}\left(-\frac{1}{t}\right) \geq 0,\end{aligned}$$

which holds if and only if $v \leq d + \frac{c-\sigma\alpha}{1-\tau} + \frac{t(N-1)}{n-1}$. Then, p^* is a local maximum. Newspaper j cannot profitably deviate neither to $p < p^*$ nor to $p > p^*$, because the second-order condition in both cases is satisfied. Therefore, p^* is also a global optimum. For Region ③ to be non-empty, one requires that $t\left[\frac{1}{2} + \frac{N-n}{n-1}\right] \leq \frac{\alpha(1-\sigma)}{1-\tau}$.

The remaining Regions ④ - ⑦ are identical to the four regions in Chen and Riordan (2007) if $c = \alpha = \tau = 0$ and $t = 1$.

Region ④: Suppose that the symmetric equilibrium subscription price is such that $v - d - \frac{t}{2} < p^* < v - t$. Compared to Region ③ any pair of newspapers j and k ($\forall j, k \in \{1, \ldots, n\}, j \neq k$) compete in a traditional duopoly fashion for the *category 1* consumers whose first and second preferred variety is the pair j and k. Then, the demand facing newspaper j is

$$D_j = \frac{2}{N}\frac{1}{N-1}\sum_{k\neq j, k\in\{1,\ldots,n\}}\left[\frac{1}{2} + \frac{p^* - p_j}{2t}\right] + \frac{2}{N}\frac{N-n}{N-1}$$

The first-order condition for newspaper j is

$$\frac{\partial \Pi_j}{\partial p_j} = (1-\tau)D_j + [(1-\tau)p_j - c + \alpha]\frac{\partial D_j}{\partial p_j} = 0,$$

where $\frac{\partial D_j}{\partial p_j} = \frac{2(n-1)}{N(N-1)}\left(-\frac{1}{2t}\right)$. At a symmetric subscription price equilibrium,

$$p_n^* = \frac{c-\alpha}{1-\tau} + \frac{t(2N-n-1)}{n-1} \tag{A.20}$$

It is straight-forward to see that the second-order condition is satisfied and thus p^* is a local maximum: $\frac{\partial^2 \Pi_j}{\partial p_j^2} = -\frac{2(n-1)(1-\tau)}{tN(N-1)} < 0$. The requirements that $p^* < v - t$ and $v - d - \frac{t}{2} < p^*$ are satisfied if and only if, respectively, $\frac{c-\alpha}{1-\tau} + \frac{2t(N-1)}{n-1} < v$ and $v < d - \frac{t}{2} + \frac{c-\alpha}{1-\tau} + \frac{2t(N-1)}{n-1}$ hold. Observe that the parameter space for v to be in Region ④ is non-empty because $d > t/2$ by assumption. The profit of a newspaper is given by

$$\Pi^*(n) = \frac{t(1-\tau)(2N-n-1)^2}{N(N-1)(n-1)} - F \tag{A.21}$$

Region ⑤: Suppose that a symmetric equilibrium price satisfies $p^* = v - t$. If all newspapers charge p^*, all *category 2* and *3* go to their, respectively, first and second preferred variety. The *category 3* consumer located at distance 1 from j, $\forall j \in \{1, \ldots, n\}$, is indifferent between purchasing j and not purchasing at all. All *category 1* consumers single-home (competition for those *category 1* consumer is as in Region ④). Then,

$$D_j(p_j, p^*) = \begin{cases} \frac{2}{N(N-1)} \sum\limits_{\substack{k \neq j, \\ k \in \{1,\ldots,n\}}} \left[\frac{1}{2} + \frac{p^*-p_j}{2t}\right] + \frac{2(N-n)}{N(N-1)} \left(\frac{v-p_j}{t}\right) & \text{if } p_j \text{ is slightly above } p^* \\ \frac{2}{N(N-1)} \sum\limits_{\substack{k \neq j, \\ k \in \{1,\ldots,n\}}} \left[\frac{1}{2} + \frac{p^*-p_j}{2t}\right] + \frac{2(N-n)}{N(N-1)} & \text{if } p_j \text{ is slightly below } p^* \end{cases}$$

In other words, the demand for j has a kink at $p_j = v - t$. In order for $p^* = v - t$ to be an equilibrium, a slight increase of p_j at p^* should not increase profit, i.e.,

$$\begin{aligned} \left.\frac{\partial \Pi_j}{\partial p_j}\right|_{p_j = p^*} &= (1-\tau)D_j + [(1-\tau)p_j - c + \alpha] \left.\frac{\partial D_j}{\partial p_j}\right|_{p_j = p^*} \\ &= (1-\tau)\left[\frac{2}{N}\frac{n-1}{N-1}\frac{1}{2} + \frac{2}{N}\frac{N-n}{N-1}\right] \\ &+ [(1-\tau)(v-t) - c + \alpha]\left[\frac{2(n-1)}{N(N-1)}\left(-\frac{1}{2t}\right) + \frac{2(N-n)}{N(N-1)}\left(-\frac{1}{t}\right)\right] \leq 0, \end{aligned}$$

which holds if and only if $2t + \frac{c-\alpha}{1-\tau} \leq v$. In addition, a slight decrease of p_j at p^* should not increase profit, i.e.,

$$\begin{aligned} \left.\frac{\partial \Pi_j}{\partial p_j}\right|_{p_j = p^*} &= (1-\tau)D_j + [(1-\tau)p_j - c + \alpha] \left.\frac{\partial D_j}{\partial p_j}\right|_{p_j = p^*} \\ &= (1-\tau)\left[\frac{2}{N}\frac{n-1}{N-1}\frac{1}{2} + \frac{2}{N}\frac{N-n}{N-1}\right] \\ &+ [(1-\tau)(v-t) - c + \alpha]\frac{2(n-1)}{N(N-1)}\left(-\frac{1}{2t}\right) \geq 0, \end{aligned}$$

which holds if and only if $v \leq \frac{2t(N-1)}{n-1} + \frac{c-\alpha}{1-\tau}$. Then, p^* is a local maximum. Newspaper j cannot profitably deviate neither to $p < p^*$ nor to $p > p^*$, because the second-order condition in both cases is satisfied. Therefore, p^* is also a global optimum in Region ⑤. Observe that the parameter space for v to be in Region ⑤ is non-empty because $\frac{2t(N-1)}{n-1} \geq 2t$ because $n \leq N$.

Region ⑥: Suppose that the symmetric equilibrium subscription price is such that $v - t < p^* < v - \frac{t}{2}$. Now some *category 3* consumers do not purchase from their second preferred newspaper. The *category 1* and *category 2* consumers make the same adaptation decision as in Region ④ and ⑤, i.e., they all single-home. The demand facing newspaper j is

$$D_j = \frac{2}{N}\frac{1}{N-1}\sum_{k\neq j, k\in\{1,\ldots,n\}}\left[\frac{1}{2} + \frac{p^* - p_j}{2t}\right] + \frac{2(N-n)}{N(N-1)}\frac{1}{2} + \frac{2(N-n)}{N(N-1)}\left(\frac{v-p_j}{t} - \frac{1}{2}\right)$$

The first-order condition for newspaper j is

$$\frac{\partial \Pi_j}{\partial p_j} = (1-\tau)D_j + [(1-\tau)p_j - c + \alpha]\left[\frac{2(n-1)}{N(N-1)}\left(-\frac{1}{2t}\right) + \frac{2(N-n)}{N(N-1)}\left(-\frac{1}{t}\right)\right] = 0$$

At a symmetric subscription price equilibrium,

$$p^* = \frac{2v(N-n) + t(n-1) + (2N-n-1)\left(\frac{c-\alpha}{1-\tau}\right)}{4N - 3n - 1}$$

It is straight-forward to see that the second-order condition is satisfied and thus p^* is a local maximum. The requirements that $p^* < v - \frac{t}{2}$ and $v - t < p^*$ are satisfied if and only if, respectively, $t\left[\frac{1}{2} + \frac{N-1}{2N-n-1}\right] + \frac{c-\alpha}{1-\tau} < v$ and $v < 2t + \frac{c-\alpha}{1-\tau}$ hold. Observe that the parameter space for v to be in Region ⑥ is non-empty because $\frac{1}{2} + \frac{N-1}{2N-n-1} < 2$. Newspaper j cannot profitably deviate neither to $p < p^*$ nor to $p > p^*$, because the second-order condition in both cases is satisfied. Therefore, p^* is also a global optimum in Region ⑥.

Region ⑦: Suppose that a symmetric equilibrium price satisfies $p^* = v - \frac{t}{2}$. If all newspapers charge p^*, all *category 1* and *2* go to their first preferred variety. All *category 1* consumers single-home. In both categories, the consumer located at distance 1/2 from j $\forall j \in \{1, \ldots, n\}$ is indifferent between purchasing j and not purchasing at all. No *category 3* consumer purchases from j. Then

$$D_j(p_j, p^*) = \begin{cases} \frac{2}{N}\left(\frac{v-p_j}{t}\right) & \text{if } p_j \text{ is slightly above } p^* \\ \frac{2}{N(N-1)}\sum\limits_{\substack{k\neq j,\\ k\in\{1,\ldots,n\}}}\left[\frac{1}{2} + \frac{p^*-p_j}{2t}\right] + \frac{2(N-n)}{N(N-1)}\left(\frac{v-p_j}{t}\right) & \text{if } p_j \text{ is slightly below } p^* \end{cases}$$

In order for $p^* = v - \frac{t}{2}$ to be an equilibrium, a slight increase of p_j at p^* should not increase profit, i.e.,

$$\left.\frac{\partial \Pi_j}{\partial p_j}\right|_{p_j=p^*} = (1-\tau)D_j + [(1-\tau)p_j - c + \alpha] \left.\frac{\partial D_j}{\partial p_j}\right|_{p_j=p^*}$$
$$= (1-\tau)\frac{2}{N}\frac{1}{2} + \left[(1-\tau)\left(v - \frac{t}{2}\right) - c + \alpha\right] \frac{2}{N}\left(-\frac{1}{t}\right) \leq 0,$$

which holds if and only if $t + \frac{c-\alpha}{1-\tau} \leq v$. In addition, a slight decrease of p_j at p^* should not increase profit, i.e.,

$$\left.\frac{\partial \Pi_j}{\partial p_j}\right|_{p_j=p^*} = (1-\tau)D_j + [(1-\tau)p_j - c + \alpha] \left.\frac{\partial D_j}{\partial p_j}\right|_{p_j=p^*}$$
$$= (1-\tau)\frac{N-1}{N(N-1)}$$
$$+ \left[(1-\tau)\left(v - \frac{t}{2}\right) - c + \alpha\right] \left[\frac{2(n-1)}{N(N-1)}\left(-\frac{1}{2t}\right) + \frac{2(N-n)}{N(N-1)}\left(-\frac{1}{t}\right)\right] \geq 0,$$

which holds if and only if $v \leq t\left[\frac{1}{2} + \frac{N-1}{2N-n-1}\right] + \frac{c-\alpha}{1-\tau}$. Then, p^* is a local maximum. Newspaper j cannot profitably deviate neither to $p < p^*$ nor to $p > p^*$, because the second-order condition in both cases is satisfied. Therefore, p^* is also a global optimum in Region ⑦. Observe that the parameter space for v is non-empty because $1 \leq \frac{1}{2} + \frac{N-1}{2N-n-1}$. □

Proof Proposition 1.1.

Proposition 1.1 is proved using Lemmas A.4 - A.5. Assume that we are in the relevant parameter region (such that there is a positive number of multi-homing and single-homing *category 1* consumers) both before and after the VAT rate is reduced to zero.

Lemma A.4. *In the relevant parameter space,* $\frac{\partial \Pi^*(n)}{\partial n} < 0$.

Proof.

$$\frac{\partial \Pi^*(n)}{\partial n} = -\frac{1}{2}(1-\tau)\frac{t(N-1)}{(n-1)^2}D_n^* + [(1-\tau)p_n^* - c]\frac{\partial D_n^*}{\partial n} + \alpha\frac{\partial X_n^{SH^*}}{\partial n} + \sigma\alpha\frac{\partial X_n^{MH^*}}{\partial n}$$
$$\propto \left(v - d - \frac{c-\sigma\alpha}{1-\tau} - \frac{t(N-1)}{n-1} - t\right)\frac{tN(N-1)}{n-1}D_n^*$$
$$- \frac{2\alpha(1-\sigma)}{1-\tau}\left(v - d - \frac{c-\sigma\alpha}{1-\tau} + t\right) < 0,$$

since in the relevant parameter space $v - d - \frac{c-\sigma\alpha}{1-\tau} - \frac{t(N-1)}{n-1} - t < 0$ and $v - d - \frac{c-\sigma\alpha}{1-\tau} + t > 0$. □

Then, in a free-entry equilibrium and in the relevant parameter space, there are n^* active newspapers satisfying

$$\begin{cases} \Pi^*(n^*) \geq 0 > \Pi^*(n^*+1) & \text{if } n^* < N \\ \Pi^*(n^*) \geq 0 & \text{if } n^* = N \end{cases} \tag{A.22}$$

The strict inequality in the first line in (A.22) follows from Lemma A.4; thus n^* is unique (and is determined by the fixed cost F). Next for a given n, Lemma A.5 states that under mild conditions the profit per newspaper increases as τ decreases.

Lemma A.5. *Assume that either $c = 0$ and $\sigma \leq 2/3$ or $p_n^* \geq 0$ and $\sigma\alpha > c$ holds. Then in the relevant parameter space, $\frac{\partial \Pi^*(n)}{\partial \tau} < 0$.*

Proof.

$$\begin{aligned} &\frac{\partial \Pi^*(n)}{\partial \tau} < 0 \\ \iff &\left(v - d + \frac{t(N-n)}{n-1}\right)^2 + \left[-\left(\frac{c+\sigma\alpha}{1-\tau}\right) - \frac{2\alpha(1-2\sigma)}{1-\tau}\right]\left(\frac{c-\sigma\alpha}{1-\tau}\right) > 0 \end{aligned} \tag{A.23}$$

If $c = 0$ is assumed, a sufficient but not necessary condition for (A.23) to hold is that $\sigma \leq 2/3$. Alternatively, if one restricts the parameter space to non-negative subscription prices, i.e., $p_n^* \geq 0$, using (1.7) one can show that (A.23) reduces to $-\frac{2\alpha(1-\sigma)}{1-\tau}\left(\frac{c-\sigma\alpha}{1-\tau}\right) > 0$, which holds if and only if $\sigma\alpha > c$. □

It immediately follows from Lemma A.4 and Lemma A.5 that the free-entry equilibrium number of newspapers n^* weakly increases following the VAT reduction: $\Delta n \triangleq n^*(\tau = 0) - n^*(\tau > 0) \geq 0$. The inequality is always strict when the number of newspapers is not restricted to take an integer value. □

Proof Proposition 1.2.

Lemma A.6 is an intermediate step and formally confirms that the consumer surplus and total readership increase through the market expansion and matching effect as the number of available newspapers increases. For Lemma A.6 I let the subscription price be constant to isolate the market expansion and matching effect from the price effect.

Lemma A.6. *For a fixed subscription price, the consumer surplus and total readership increase if $\Delta n \geq 1$.*

Proof. Define $CS_{12} \triangleq \int_0^{1/2}(v - tx - p_n^*)dx$ as the surplus of the *category 1* and *2* consumers' first (and for *category 2* consumers only) purchase, $CS_{1MH} \triangleq \int_{x_{jk}(p_n^*)}^{1/2}(v - t(1-x) - d - p_n^*)dx$ as the surplus of *category 1*'s second purchase, and $CS_3 \triangleq$

$\int_{1/2}^{1}(v - tx - p_n^*)dx$ as the surplus of *category 3* consumers. Note that $CS_{12} > CS_3$. Then using the consumer surplus expression given by (1.9) for a fixed subscription price,

$$\begin{aligned}&\frac{N}{2}(N-1)\left[CS(n+\Delta n) - CS(n)\right]\\ =&\Delta n(N-1)(CS_{12} + CS_3) + \Delta n(2n + \Delta n - 1)(CS_{1MH} - CS_3),\end{aligned}$$

which is trivially strictly positive if $CS_{1MH} \geq CS_3$. If $CS_{1MH} < CS_3$, then using the fact that $n \leq N - \Delta n$,

$$\begin{aligned}&\frac{N}{2}(N-1)\left[CS(n+\Delta n) - CS(n)\right]\\ \geq&\Delta n(N-1)(CS_{12} - CS_3) + \Delta n(2N - \Delta n - 1)CS_{1MH} - \Delta n(1 - \Delta n)CS_3,\end{aligned}$$

which can only ever be negative if $\Delta n < 1$ because $CS_{12} > C_3$ and $(2N - \Delta n - 1) > 0$. But $\Delta n \geq 1$ by considering the integer problem.

Regarding total readership,

$$\begin{aligned}&(n+\Delta n)D_n(n+\Delta n) - nD_n(n)\\ \propto&\Delta n(2n + \Delta n - 1)(v - d - p_n^*) + \Delta n(N - 2n - \Delta n)t\\ >&\Delta n(N-1)t,\end{aligned}$$

where the strict inequality follows from the requirement that we have a strictly positive amount of *category 1* single-homing consumers, i.e., $p_n^* > v - d - t$. Clearly, $\Delta n(N-1)t > 0$ if $\Delta n > 1$. □

If $\Delta n > \Delta\tilde{n}^p$, the subscription price p^* decreases by Lemma 1.3. From (1.9) in the proof of Lemma 1.2, it is immediate that the consumer surplus increases as p^* decreases (for a fixed n^*). In addition, the consumer surplus increases in n^* by Lemma A.6. Thus the consumer surplus increases by the combination of both the decrease of the subscription price and increase of the number of digital newspapers. Similar for total readership: $n^*D_n^* = \frac{2n^*(n^*-1)}{tN(N-1)}(v - d - p^*) + \frac{2}{N}\frac{n^*(N-n^*)}{N-1}$ increases in n^* by Lemma A.6 and increases as p^* decreases. At the symmetric equilibrium, the fraction of consumers that multi-home, $\frac{n^*}{2}X^{MH^*} = \frac{n^*(n^*-1)}{tN(N-1)}[2v - 2d - 2p^* - t]$, clearly increases in n^* and decreases in p^*. □

Proof Proposition 1.3*.*

Total readership, i.e., n^*D^*, increases following the reduction in the VAT rate if and only if

$$
\begin{aligned}
-\frac{\tau(c-\sigma\alpha)}{1-\tau} <& \frac{t\Delta n[N-2n^*(\tau>0)-\Delta n]}{n^*(\tau>0)[n^*(\tau>0)-1]} \\
&+\frac{\Delta n[2n^*(\tau>0)+\Delta n-1]}{n^*(\tau>0)[n^*(\tau>0)-1]}\left[v-d-(c-\sigma\alpha)\right]
\end{aligned}
\tag{A.24}
$$

Using (1.8), the inequality follows by subtracting the total readership evaluated at $n^*(\tau > 0) + \Delta n$ and $\tau = 0$ from the total readership evaluated at $n^*(\tau > 0)$ and $\tau > 0$. The left-hand side in (A.24) is strictly positive by the $\sigma\alpha > c$ assumption and constant in Δn. In addition, the right-hand side strictly increases in Δn and is zero if $\Delta n = 0$. By Bolzano's Theorem there exists a $\Delta\tilde{n}^{TRS} > 0$ such that (A.24) holds with equality. Because the right-hand side in (A.24) is strictly larger than the right-hand side in (1.11) for any Δn (unless $\Delta n = 0$) and the left-hand side in (A.24) is identical to the left-hand side in (1.11), it follows that $\Delta\tilde{n}^{TRS} < \Delta\tilde{n}^p$.

Similarly using (1.8), the fraction of consumers that multi-home, i.e., $\frac{n^*}{2}X^{MH^*}$, increases following the reduction τ if and only if

$$
\begin{aligned}
-\frac{\tau(c-\sigma\alpha)}{1-\tau} <& -\frac{t\Delta n(N-1)}{n^*(\tau>0)[n^*(\tau>0)-1]} \\
&+\frac{\Delta n[2n^*(\tau>0)+\Delta n-1]}{n^*(\tau>0)[n^*(\tau>0)-1]}\left[v-d-(c-\sigma\alpha)\right]
\end{aligned}
\tag{A.25}
$$

The right-hand side in (A.25) strictly increases in Δn and is zero if $\Delta n = 0$. By Bolzano's Theorem there exists a $\Delta\tilde{n}^{MH} > 0$ such that (A.25) holds with equality. Because the right-hand side in (A.25) is strictly larger than the right-hand side in (1.11) for any Δn (unless $\Delta n = 0$) and the left-hand side in (A.25) is identical to the left-hand side in (1.11), it follows that $\Delta\tilde{n}^{MH} < \Delta\tilde{n}^p$.

Unless $\Delta n = 0$, the right-hand side of (A.24) is larger than the right-hand side of (A.25) if and only if $2N \geq 2n^*(\tau > 0) + \Delta n + 1$. Since $\Delta n \leq N - n^*(\tau > 0)$, $2n^*(\tau > 0) + \Delta n + 1 \leq N + n^*(\tau > 0) + 1$. In addition we required in the first place that there is room for entry, i.e., $n^*(\tau > 0) \leq N-1$. Hence, $2N \geq 2n^*(\tau > 0)+\Delta n+1$ is satisfied and therefore $\Delta\tilde{n}^{TRS} \leq \Delta\tilde{n}^{MH}$.

The consumer surplus can be re-written by plugging in the subscription price (1.7) in (1.9) as

$$
\begin{aligned}
CS(n,\tau) =& \left(\frac{n(N-n)}{N(N-1)}+\frac{n}{N}\right)\frac{1}{2}\left[v-t+d-\left(\frac{c-\sigma\alpha}{1-\tau}\right)-\frac{t(N-1)}{n-1}\right]+\frac{2n(N-n)}{N(N-1)}\frac{t}{8} \\
&+\frac{2n}{N}\frac{3t}{8}+\frac{n(n-1)}{N(N-1)}\frac{1}{4t}\left[v-d-\left(\frac{c-\sigma\alpha}{1-\tau}\right)-\frac{t(N-1)}{n-1}\right]^2
\end{aligned}
\tag{A.26}
$$

On the one hand for a fixed number of newspapers, the reduction in the VAT rate has a negative *direct* impact on the consumer surplus through an increase of the subscription price (Lemma 1.2), i.e., $CS(n, \tau = 0) < CS(n, \tau > 0)$. On the other hand the *indirect* effect, an increase in the number of newspapers, has a positive impact on the consumer surplus because of the market expansion and matching effect (Lemma A.6) and the opposing effect on the subscription price to the initial *direct* effect on the price due to the VAT reduction (Lemma 1.2). That is, $\frac{\partial}{\partial \Delta n}\left(CS(n+\Delta n, \tau) - CS(n, \tau)\right) > 0 \; \forall \Delta n$. By Bolzano's Theorem there exists a $\Delta\tilde{n}^{CS} > 0$ such that $CS(n + \Delta\tilde{n}^{CS}, \tau = 0) = CS(n, \tau > 0)$ is satisfied. Lastly, we need to show that $\Delta\tilde{n}^{CS} < \Delta\tilde{n}^{p}$. Suppose instead that $\Delta\tilde{n}^{CS} \geq \Delta\tilde{n}^{p}$. Furthermore suppose that $\Delta\tilde{n}^{CS} > \Delta n > \Delta\tilde{n}^{p}$. Using Proposition (1.2), according to which the consumer surplus increases, we have arrived at a contradiction. □

Proof Proposition A.1.

From the derivation of the equilibrium subscription prices for all parameters configurations above, a newspaper's profit at the symmetric equilibrium is

$$\Pi^*(n) = \frac{t(1-\tau)(2N-n-1)^2}{N(N-1)(n-1)} - F$$

Clearly, $\Pi^*(n)$ decreases in n and τ. Thus in the now relevant single-homing parameter space, the free-entry equilibrium number of digital newspapers $n^*(\tau)$ is unique and weakly increases as τ is reduced to zero, i.e., $\Delta n \triangleq n^*(\tau = 0) - n^*(\tau > 0) \geq 0$.

Repeating Lemma 1.3, the subscription price under free-entry decreases if and only if $\Delta n > \Delta\tilde{n}^{p}_{SH}$, where $\Delta\tilde{n}^{p}_{SH} > 0$ is given by

$$\frac{-\tau(c-\alpha)}{1-\tau} = \frac{2t(N-1)\Delta\tilde{n}^{p}_{SH}}{\left[n^*(\tau>0)-1\right]\left[n^*(\tau>0)+\Delta\tilde{n}^{p}_{SH}-1\right]} \tag{A.27}$$

Repeating the proof of Proposition 1.3, total readership TRS increases if and only if

$$TRS(\tau = 0) - TRS(\tau > 0) \propto \Delta n(2N - 2n - \Delta n - 1) > 0, \tag{A.28}$$

which is satisfied for $\Delta n > 0$ since $\Delta n \leq N - n$ and $n \leq N - 1$ (if the latter inequality does not hold, all spokes would be already occupied in the initial $\tau > 0$ free-entry equilibrium and so there would be no room for an additional newspaper to enter).

Regarding the consumer surplus, the consumers who have their first preferred variety available, travel on average a distance of $1/4$. Since a fraction n/N of consumers have their first preferred variety available and the transport cost is linear, the consumer surplus for those consumers is

$$\frac{n}{N}\left(v - p^*_n - \frac{t}{4}\right)$$

The consumers on the remaining $(N-n)$ spokes (with a mass of $2/N$ on each spoke of length $1/2$) have their second preferred variety available with probability $n/(N-1)$ and travel on average a distance of $3/4$. Thus, the surplus serving those consumers is

$$\frac{n}{N}\frac{N-n}{N-1}\left(v-p_n^*-\frac{3t}{4}\right)$$

In sum,

$$CS(n)=\frac{n}{N}\left(\frac{2N-n-1}{N-1}\right)\left(v-\frac{c-\alpha}{1-\tau}-\frac{t(2N-n-1)}{n-1}\right)-\frac{tn(4N-3n-1)}{4N(N-1)} \tag{A.29}$$

Observe that $CS(n,\tau=0)<CS(n,\tau>0)$ if $\alpha>c$. On the other hand, $\partial CS(n)/\partial n>0$ by the market expansion and matching effects for single-homing consumers. Then, there exists a $\Delta\tilde{n}_{SH}^{CS}>0$ such that $CS(n+\Delta\tilde{n}_{SH}^{CS},\tau=0)=CS(n,\tau>0)$ is satisfied. The cut-off $\Delta\tilde{n}_{SH}^{CS}>0$ is given by

$$\begin{aligned}\frac{-\tau(c-\alpha)}{1-\tau}=&\frac{2t\Delta n(N-1)}{(n+\Delta n-1)(n-1)}+\frac{t\Delta n(4N-2n-\Delta n+3)}{4n(2N-n-1)}\\&+\frac{\Delta n(2N-2n-\Delta n-1)}{n(2N-n-1)}\left[v-(c-\alpha)-\frac{t(2N-n-\Delta n-1)}{n+\Delta n-1}-t\right]\end{aligned} \tag{A.30}$$

Since the sum of the last two rows in (A.30) is strictly positive and by comparing (A.30) to (A.27), it follows that $\Delta\tilde{n}_{SH}^{CS}<\Delta\tilde{n}_{SH}^{p}$. □

Proof Proposition A.2.

The partial derivative of total profits with respect to n evaluated at zero profit yields

$$\left.\frac{\partial\left(n\Pi^*(n)\right)}{\partial n}\right|_{\Pi^*(n)=0}=-\frac{t(1-\tau)n(2N-n-1)(2N+n-3)}{N(N-1)(n-1)^2}<0$$

Due to the absence of multi-homing consumers, any advertiser's surplus is fully extracted by the newspapers. Thus, $AS=0$ and $W=n\Pi^*(n)+T+CS$. Then,

$$\left.\frac{\partial W}{\partial n}\right|_{\Pi^*(n)=0}=(v-c+\alpha)\frac{2N-2n-1}{N(N-1)}-\frac{t(4N-6n-1)}{4N(N-1)}-\frac{t(1-\tau)(2N-n-1)^2}{N(N-1)(n-1)}$$

from which the inequality (A.2) in Proposition A.2 immediately follows. Furthermore,

$$\frac{d}{d\tau}\left(\left.\frac{\partial W}{\partial n}\right|_{\Pi^*(n)=0}\right) = \frac{t(2N-n-1)^2}{N(N-1)(n-1)}\frac{d\tau}{d\tau} - \frac{1}{N(N-1)}\left[2(v-c+\alpha)-\frac{t}{2}-\tau t-\frac{4t(1-\tau)(N-1)^2}{(n-1)^2}\right]\frac{dn}{d\tau} \tag{A.31}$$

Clearly, the first term in (A.31) is positive. While the second term inside the squared bracket in the second row may be negative (for large N and t and small n values), which causes the second row to be negative overall since $\frac{dn}{d\tau} < 0$, the direct effect dominates the indirect effect; thus, $\left.\frac{\partial W}{\partial n}\right|_{\Pi^*(n)=0}$ decreases as τ decreases. □

Proof Lemma A.1.

The marginal consumer (l_j, x_j) between j and k who is indifferent between single-homing on j and multi-homing on both j and k is at distance $x_{jk} = 1 - \frac{v-d-p_k-\gamma A_k^2}{t}$ from j, and the marginal consumer between j and k who is indifferent between single-homing on k and multi-homing on k and j is at distance $x_{kj} = \frac{v-d-p_j-\gamma A_j^2}{t}$ from j. Suppose that the symmetric subscription price p^* and advertisement amount A^* are such that $p^* + \gamma {A^*}^2 \in \left(v-d-t, v-d-\frac{t}{2}\right)$. Then, $0 < x_{jk}(p^*, A^*) < x_{kj}(p^*, A^*) < 1\ \forall j,k \in \{1,\dots,n\}, j \neq k$. Notice that all consumers from the second and third category go to their, respectively, first and second preferred variety, because $p^* + \gamma {A^*}^2 < v-d-\frac{t}{2} < v-t$, where the last inequality follows by the $d > \frac{t}{2}$ assumption. The single-homing and multi-homing demand for newspaper j are, respectively, given by

$$X_j^{SH} = \frac{2}{N}\frac{1}{N-1}\sum_{k\neq j, k\in\{1,\dots,n\}} x_{jk}\left(p^*, A^*\right) + \frac{2}{N}\frac{N-n}{N-1},$$

$$X_j^{MH} = \frac{2}{N}\frac{1}{N-1}\sum_{k\neq j, k\in\{1,\dots,n\}} \left[x_{kj}\left(p_j, A_j\right) - x_{jk}\left(p^*, A^*\right)\right]$$

for prices p_j and ad levels A_j in the neighborhood of (p^*, A^*). Newspaper j's total consumer demand is $D_j = X_j^{SH} + X_j^{MH}$. Newspaper j's profit is

$$\Pi_j = \left[(1-\tau)p_j - c + \alpha A_j\right] X_j^{SH} + \left[(1-\tau)p_j - c + \sigma\alpha A_j\right] X_j^{MH} - F$$

The first-order conditions are

$$\frac{\partial \Pi_j}{\partial p_j} = (1-\tau)D_j + [(1-\tau)p_j - c + \sigma\alpha A_j]\,\frac{\partial X_j^{MH}}{\partial p_j} = 0, \tag{A.32}$$

$$\frac{\partial \Pi_j}{\partial A_j} = \alpha X_j^{SH} + \sigma\alpha X_j^{MH} + [(1-\tau)p_j - c + \sigma\alpha A_j]\,\frac{\partial X_j^{MH}}{\partial A_j} = 0, \tag{A.33}$$

where $\frac{\partial X_j^{MH}}{\partial p_j} = \frac{2(n-1)}{N(N-1)}\left(-\frac{1}{t}\right)$ and $\frac{\partial X_j^{MH}}{\partial A_j} = \frac{2(n-1)}{N(N-1)}\left(-\frac{2\gamma A_j}{t}\right)$. I assume for now that $\sigma = 1$, which allows me to derive closed-form solutions from (A.32) and (A.33). Solving gives the symmetric equilibrium price and ad levels in (A.4). In addition,

- $\frac{\partial^2 \Pi_j}{\partial p_j^2} = -\frac{4(n-1)(1-\tau)}{tN(N-1)} < 0,$
- $\left.\frac{\partial^2 \Pi_j}{\partial A_j^2}\right|_{\sigma=1, p_j=p^*, A_j=A^*} = \frac{(1-\tau)}{4}\frac{2(n-1)}{tN(N-1)}\left[-\frac{9\alpha^2}{(1-\tau)^2} + \frac{4\gamma c}{1-\tau} - 4\gamma\left(v - d + \frac{t(N-n)}{n-1}\right)\right],$
- $\left.\frac{\partial^2 \Pi_j}{\partial p_j^2}\cdot\frac{\partial^2 \Pi_j}{\partial A_j^2}\right|_{\sigma=1, p_j=p^*, A_j=A^*} - \left(\left.\frac{\partial^2 \Pi_j}{\partial p_j \partial A_j}\right|_{\sigma=1, A_j=A^*}\right)^2$
  $= -\frac{(1-\tau)^2}{2}\left(\frac{2(n-1)}{tN(N-1)}\right)^2\left[-\frac{\alpha^2}{(1-\tau)^2} + \frac{4\gamma c}{1-\tau} - 4\gamma\left(v - d + \frac{t(N-n)}{n-1}\right)\right]$

Therefore, a necessary and sufficient condition for the second-order condition is $c < \frac{\alpha^2}{4\gamma(1-\tau)} + (1-\tau)\left(v - d + \frac{t(N-n)}{n-1}\right)$.

The requirements that $p^* + \gamma {A^*}^2 < v - d - \frac{t}{2}$ and $v - d - t < p^* + \gamma {A^*}^2$ are satisfied if and only if, respectively, $d + \frac{c}{1-\tau} - \frac{\alpha^2}{4\gamma(1-\tau)^2} + \frac{t(N-1)}{n-1} < v$ and $v < d + t + \frac{c}{1-\tau} - \frac{\alpha^2}{4\gamma(1-\tau)^2} + \frac{t(N-1)}{n-1}$ hold. Lastly, plugging the equilibrium subscription price and ad level into X_j^{SH}, X_j^{MH}, and D_j gives (A.5). □

Proof Proposition A.3.

At the symmetric equilibrium, a firm's profit is

$$\Pi^*(n) = \frac{1}{2}\left[(1-\tau)\left(v - d + \frac{t(N-n)}{n-1}\right) + \frac{\alpha^2}{4\gamma(1-\tau)^2}\right] D_n^* - F,$$

where D_n^* is given by (A.5). As in the proof of Proposition 1.1, one can show that $\frac{\partial \Pi^*(n)}{\partial n} < 0$ and $\frac{\partial \Pi^*(n)}{\partial \tau} < 0$ in the relevant parameter space. For the latter, non-negative subscription prices are a sufficient condition. The free-entry equilibrium number of digital newspapers $n^*(\tau)$ is unique and if the subscription prices are non-negative, then $n^*(\tau)$ weakly increases as τ is reduced to zero, i.e., $\Delta n \triangleq n^*(\tau = 0) - n^*(\tau > 0) \geq 0$.

The increase in the number of newspapers has the opposing effect on the subscription price as the initial direct effect on the price due to the VAT reduction. Repeating Lemma 1.3, the subscription price under free-entry decreases if and only if $\Delta n > \Delta\tilde{\tilde{n}}^p$, where $\Delta\tilde{\tilde{n}}^p > 0$ is given by

$$\frac{-\tau c}{1-\tau} + \frac{3\alpha^2}{4\gamma}\frac{\tau(2-\tau)}{(1-\tau)^2} = \frac{t\Delta\tilde{\tilde{n}}^p(N-1)}{[n^*(\tau>0)-1]\left[n^*(\tau>0)+\Delta\tilde{\tilde{n}}^p-1\right]} \tag{A.34}$$

Repeating the proof of Proposition 1.3, the $\Delta\tilde{\tilde{n}}^{TRS}$ cut-off above which total readership increases is implicitly defined by

$$\begin{aligned}&\frac{-\tau c}{1-\tau} + \frac{\alpha^2}{4\gamma}\frac{\tau(2-\tau)}{(1-\tau)^2}\\ =&\frac{t\Delta\tilde{\tilde{n}}^{TRS}(N-1) + \Delta\tilde{\tilde{n}}^{TRS}\left[2n^*(\tau>0)+\Delta\tilde{\tilde{n}}^{TRS}-1\right]\left[v-d-t-c+\frac{\alpha^2}{4\gamma}\right]}{n^*(\tau>0)[n^*(\tau>0)-1]},\end{aligned} \tag{A.35}$$

and $\Delta\tilde{\tilde{n}}^{MH}$ is implicitly defined by

$$\begin{aligned}&\frac{-\tau c}{1-\tau} + \frac{\alpha^2}{4\gamma}\frac{\tau(2-\tau)}{(1-\tau)^2}\\ =&\frac{-t\Delta\tilde{\tilde{n}}^{MH}(N-1) + \Delta\tilde{\tilde{n}}^{MH}\left[2n^*(\tau>0)+\Delta\tilde{\tilde{n}}^{MH}-1\right]\left[v-d-c+\frac{\alpha^2}{4\gamma}\right]}{n^*(\tau>0)[n^*(\tau>0)-1]}\end{aligned} \tag{A.36}$$

By the same steps as in the proof of Proposition 1.3, one can show that $\exists\Delta\tilde{\tilde{n}}^{MH} > \Delta\tilde{\tilde{n}}^{TRS} > 0$. The consumer surplus

$$\begin{aligned}CS(n) = \frac{2n}{N(N-1)}\Bigg[&(n-1)\int_0^{1/2}(v-tx-p_n^*-\gamma A^{*^2})dx\\ &+(n-1)\int_{x_{jk}(p_n^*,A^*)}^{1/2}(v-t(1-x)-d-p_n^*-\gamma A^{*^2})dx\\ &+(N-n)\int_0^{1/2}(v-tx-p_n^*-\gamma A^{*^2})dx\\ &+(N-n)\int_{1/2}^{1}(v-tx-p_n^*-\gamma A^{*^2})dx\Bigg]\end{aligned} \tag{A.37}$$

decreases in $p_n^* + \gamma A^{*^2}$ for a fixed number of newspapers. That is, the reduction in the VAT rate has a negative *direct* impact on the consumer surplus through the combination of the increase of the subscription price and the decrease in the ad volume, i.e., $CS(n,\tau=0) < CS(n,\tau>0)$. On the other hand, an increase in the number of newspapers has a positive *indirect* effect on the consumer surplus because of the market expansion and matching effect (by repeating Lemma A.6 for the disutility from ads extension) and the opposing effect on the subscription price to the initial *direct* effect on the price due to the VAT reduction . That is, $\frac{\partial}{\partial \Delta n}(CS(n+\Delta n,\tau) - CS(n,\tau)) > 0 \ \forall \Delta n$. By Bolzano's Theorem there exists a $\Delta\tilde{\tilde{n}}^{CS} > 0$ such that $CS(n+\Delta\tilde{\tilde{n}}^{CS}, \tau = 0) = CS(n, \tau > 0)$ is satisfied. □

Appendix B

Long-Term Contracts and Entry Deterrence in Two-Sided Markets

B.1 Proofs

Proof Lemma 2.2.

In what follows, I show that I cannot set fees $\left(f_{I,2}^{c}, f_{I,2}^{s}\right)$ to disrupt the (exclusionary) equilibrium described in Lemma 2.2 to either attract both consumers and sellers or only one agent side, and that E wants to attract both agent sides rather than just one. There are two possible strategies by the losing platform I to attract both sides (conditional on E setting fees to attract both the consumer and seller side): (i) steal the sellers from E to extract more surplus from the consumers, or (ii) steal the consumers from E to extract more surplus from the sellers. In other words, E has to set $\left(f_{E,2}^{c,nlt^*}, f_{E,2}^{s,nlt^*}\right)$ such that I cannot profitably undercut its fee on one side to attract agents from that side in order to extract more surplus from agents on the other side. Strategy (iii) by I is to attract only one agent side. Lastly, I show that I wants to attract both sides rather than just one side.

I first consider strategy (i). In any equilibrium where E attracts the consumers, the condition that sellers prefer E is $f_{E,2}^{s,nlt^*} \leq \alpha^s N^c + f_{I,2}^{s,nlt}$ by the sellers' incentive compatibility constraint. Given the iterated best response condition (see details in the main text), I can attract all agents by setting $f_{I,2}^{s,(i)} = f_{E,2}^{s,nlt^*} - \alpha^s N^c - \epsilon^s$, where $\epsilon^s > 0$. In particular, the sellers take as given that consumers go to E in the first iteration, and therefore I has to compensate the sellers by $\alpha^s N^c + \epsilon^s$ for forgoing the cross-group externality benefit. But then the most I can charge consumers to attract them is $f_{I,2}^{c,(i)} = f_{E,2}^{c,nlt^*} + \alpha^c N^s - \epsilon^c$. By the consumers' incentive compatibility constraint and the second iterated best response, consumers now expect that sellers go to I. Thus, E has to choose its fee combination such that strategy (i) is unprofitable for I, i.e., $\Pi_{I,2}^{nlt,(i)} = \left(f_{I,2}^{c,(i)} - c_I\right) N^c + \left(f_{I,2}^{s,(i)} - c_I\right) N^s < 0$. It follows that $\left(f_{E,2}^{c,nlt^*}, f_{E,2}^{s,nlt^*}\right)$ has to be such that $\left(f_{E,2}^{c,nlt^*} - c_I\right) N^c + \left(f_{E,2}^{s,nlt^*} - c_I\right) N^s +$

$(\alpha^c - \alpha^s)\, N^c N^s = 0$.

I now consider strategy (ii). In any equilibrium where E attracts the sellers, the condition that consumers prefer E is $f_{E,2}^{c,nlt^*} \leq \alpha^c N^s + f_{I,2}^{c,nlt}$ by the consumers' incentive compatibility constraint. Given the iterated best response condition, I can attract all agents by setting $f_{I,2}^{c,(ii)} = f_{E,2}^{c,nlt^*} - \alpha^c N^s - \epsilon^c$. Here, the first iteration is skipped. In the second iteration the consumers take that the sellers go to E as given, and therefore I has to compensate the consumers by $\alpha^c N^s + \epsilon^c$ for forgoing the cross-group externality benefit. But then the largest fee I can charge to attract sellers is $f_{I,2}^{s,(ii)} = f_{E,2}^{s,nlt^*} + \alpha^s N^c - \epsilon^s$; by the sellers' incentive compatibility constraint and the third iteration, sellers now expect that consumers go to I. Thus, E has to choose its fees such that strategy (ii) is unprofitable for I, i.e., $\Pi_{I,2}^{nlt,(ii)} = \left(f_{I,2}^{c,(ii)} - c_I\right) N^c + \left(f_{I,2}^{s,(ii)} - c_I\right) N^s < 0$. It follows that $\left(f_{E,2}^{c,nlt^*}, f_{E,2}^{s,nlt^*}\right)$ has to be such that $\left(f_{E,2}^{c,nlt^*} - c_I\right) N^c + \left(f_{E,2}^{s,nlt^*} - c_I\right) N^s + (\alpha^s - \alpha^c)\, N^c N^s = 0$.

Consequently for E to be immune against both strategies (i) and (ii), it has to hold that $\left(f_{E,2}^{c,nlt^*} - c_I\right) N^c + \left(f_{E,2}^{s,nlt^*} - c_I\right) N^s + |\alpha^c - \alpha^s|\, N^c N^s = 0$, which is equation (2.9).

In addition, for sellers to want to join E, even if I is not an option, any equilibrium requires that the sellers' participation constraint is satisfied, i.e., $f_{E,2}^{s,nlt^*} \leq v^s + \alpha^s N^c$. Similarly, the consumers' participation constraint is satisfied if and only if $f_{E,2}^{c,nlt^*} \leq v^c + \alpha^c N^s$. For E to be immune against strategy (iii) by I, E has to set $f_{E,2}^{s,nlt^*} \leq c_I + \alpha^s N^c$ and $f_{E,2}^{c,nlt^*} \leq c_I + \alpha^c N^s$ so that I cannot attract solely sellers or consumers, respectively. Since $\min\{v^s, v^c\} > c_I$ by assumption, the latter two inequalities are stated by (2.10) as equilibrium requirement. Using (2.9) in E's profit expression, $\Pi_{E,2}^{nlt^*} = \left(f_{E,2}^{c,nlt^*} - c_I\right) N^c + \left(f_{E,2}^{s,nlt^*} - c_I\right) N^s = (c_I - c_E)(N^c + N^s) - |\alpha^c - \alpha^s|\, N^s N^c$.

Lastly, it needs to be verified that E indeed wants to attract *both* agent sides rather than just one side. Suppose instead that E sets $\left(f_{E,2}^{c,nlt}, f_{E,2}^{s,nlt}\right)$ with the intention to only attract the consumer side (the argument for only attracting the seller side is analogous and omitted here). Then in response, I can set $\left(f_{I,2}^{c,nlt}, f_{I,2}^{s,nlt}\right)$ with the aim to either attract both sides, only the seller side, or only the consumer side. The latter case is not a profitable strategy for I, because E wins the Bertrand competition with I when only competing for the consumer side due to the cost asymmetry.

Thus in response to E's fees, I considers to either attract both sides or only the seller side. The maximum fees I can set to attract both sides are $f_{I,2}^{c,nlt} = f_{E,2}^{c,nlt} + \alpha^c N^s - \epsilon^c$ and $f_{I,2}^{s,nlt} = f_{E,2}^{s,nlt} + \alpha^s N^c - \epsilon^s$, and $f_{I,2}^{s,nlt} = f_{E,2}^{s} - \epsilon^s$ to only attract the seller side. In turn, E has to set $\left(f_{E,2}^{c,nlt}, f_{E,2}^{s,nlt}\right)$ such that I, from a profit perspective, only wants to attract the seller side but not both agent sides, because otherwise no agents will go to E and consequently $\Pi_{E,2}^{nlt} = 0 < \Pi_{E,2}^{nlt^*}$. Formally, E has to set $\left(f_{E,2}^{c,nlt}, f_{E,2}^{s,nlt}\right)$ such that the following holds:

$$\left(f_{E,2}^{c,nlt} + \alpha^c N^s - \epsilon^c - c_I\right) N^c + \left(f_{E,2}^{s,nlt} + \alpha^s N^c - \epsilon^s - c_I\right) N^s < \left(f_{E,2}^{s,nlt} - \epsilon^s\right) N^s$$

It follows that the maximum $f_{E,2}^{c,nlt}$ that E can set to only attract the consumer side is $c_I - (\alpha^c + \alpha^s) N^s$. But then, $\Pi_{E,2}^{c,nlt} = (c_I - (\alpha^c + \alpha^s) N^s - c_E) N^c < (c_I - c_E)(N^c + N^s) - (\alpha^c + \alpha^s) N^s N^c < (c_I - c_E)(N^c + N^s) - |\alpha^c - \alpha^s| N^s N^c = \Pi_{E,2}^{nlt^*}$, and therefore E does not want to attract only the consumer side. □

Proof Proposition 2.9.

The following parameter conditions have to be satisfied for the derivations below:
$u_{E,2}^{c,nlt^*}\left(\frac{N_{E,2}^{c,nlt^*}}{N^c}\right) \geq 0 \iff 2v^c + 3\alpha N^s - c_I - c_E - 3\tau^c \geq 0$, $u_{E,2}^{c,lt^*}\left(\frac{N_{E,2}^{c,lt^*}}{N^c}\right) \geq 0 \iff 2v^c + \alpha N^s - c_I - c_E - 3\tau^c \geq 0$, $N_{I,1}^{c,lt^*} = N^c \iff 2\tau^c \leq v^c + 2\alpha N^s - c_I$.

In the second period absent a long-term contract, E sets $\left(f_{E,2}^c, f_{E,2}^s\right)$ to maximize its profit and to attract the homogeneous sellers, subject to I not wanting to deviate to attract the sellers but instead only competes for the consumers with E without the sellers on board. Formally,

$$\begin{aligned}
&\max_{f_{E,2}^c, f_{E,2}^s} \left\{\left(f_{E,2}^c - c_E\right) N_{E,2}^c + \left(f_{E,2}^s - c_E\right) N^s\right\} \\
&\quad \text{s.t. } \max_{f_{I,2}^{c,dev}} \left\{\left(f_{I,2}^{c,dev} - c_I\right) N_{I,2}^{c,dev} + \left[f_{E,2}^s + \alpha^s \left(N_{I,2}^{c,dev} - \left(N^c - N_{I,2}^{c,dev}\right)\right) - c_I\right] N^s\right\} \\
&\quad\quad \leq \max_{f_{I,2}^c} \left\{\left(f_{I,2}^c - c_I\right)\left(N^c - N_{E,2}^c\right)\right\},
\end{aligned}$$

such that $N_{E,2}^c = \left(\frac{1}{2} + \frac{f_{I,2}^c - f_{E,2}^c + \alpha^c N^s}{2\tau^c}\right) N^c \in [0, N^c]$ and $N_{I,2}^{c,dev} = \left(\frac{1}{2} + \frac{f_{E,2}^c - f_{I,2}^{c,dev} + \alpha^c N^s}{2\tau^c}\right) N^c \in [0, N^c]$. I let $\alpha^c = \alpha^s = \alpha$ in order to reduce the cases to be investigated. Then, taking first order conditions and solving gives (Lemma 2.2 adjusted for consumer heterogeneity):

- Case $\text{NLT}_{\text{i}}^{\text{sh}}$: If $\tau^c > \frac{c_I - c_E}{3} + \alpha N^s$,

$$\begin{cases}
f_{I,2}^{c,nlt^*} = \tau^c - \alpha N^s + \dfrac{1}{3}(c_E + 2c_I), & f_{E,2}^{c,nlt^*} = \tau^c - \alpha N^s + \dfrac{1}{3}(2c_E + c_I), \\
f_{E,2}^{s,nlt^*} = c_I - \alpha N^s + \dfrac{2\alpha N^c}{3\tau^c}(c_I - c_E), & N_{E,2}^{c,nlt^*} = \left(\dfrac{1}{2} + \dfrac{c_I - c_E + 3\alpha N^s}{6\tau^c}\right) N^c, \\
\Pi_{I,2}^{nlt^*} = \frac{1}{18\tau^c}\left(c_E - c_I + 3\tau^c - 3\alpha N^s\right)^2 N^c &
\end{cases}$$

- Case $\text{NLT}^{\text{sh}}_{\text{ii}}$: If $\tau^c \le \frac{c_I - c_E}{3} + \alpha N^s$ and $\tau^c \ge \alpha N^s$,

$$\begin{cases} f^{c,nlt^*}_{I,2} = c_I, & f^{c,nlt^*}_{E,2} = c_I - \tau^c + \alpha N^s, \\ f^{s,nlt^*}_{E,2} = c_I + \dfrac{\alpha N^c}{\tau^c}\left(\tau^c - 2\alpha N^s\right), & N^{c,nlt^*}_{E,2} = N^c, \\ \Pi^{nlt^*}_{I,2} = 0,\ \Pi^{nlt^*}_{E,2} = \left[c_I - c_E + \frac{\alpha N^c}{\tau^c}\left(\tau^c - 2\alpha N^s\right)\right] N^c - \left(c_E - c_I + \tau^c - \alpha N^s\right) N^s \end{cases}$$

- Case $\text{NLT}^{\text{sh}}_{\text{iii}}$: If $\tau^c \le \frac{c_I - c_E}{3} + \alpha N^s$ and $\tau^c < \alpha N^s$,

$$\begin{cases} f^{c,nlt^*}_{I,2} = c_I, & f^{c,nlt^*}_{E,2} = c_I - \tau^c + \alpha N^s, \\ f^{s,nlt^*}_{E,2} = c_I - 3\alpha N^c + 2\tau^c \dfrac{N^c}{N^s}, & N^{c,nlt^*}_{E,2} = N^c, \\ \Pi^{nlt^*}_{I,2} = 0,\ \Pi^{nlt^*}_{E,2} = \left(c_I - c_E\right)\left(N^c + N^s\right) + \left(\tau^c + 2\alpha N^s\right) N^c \end{cases}$$

Suppose that sellers signed the long-term contract in the first period, and since sellers are assumed to single-home, they are bound to only go to I in the second period. I and E still compete for the consumers in the second period by simultaneously setting the consumer fees.

$$I : \max_{f^c_{I,2}} \left\{ \left(f^c_{I,2} - c_I\right) N^c_{I,2} \right\} \text{ and } E : \max_{f^c_{E,2}} \left\{ \left(f^c_{E,2} - c_E\right)\left(N^c - N^c_{I,2}\right) \right\},$$

where $N^c_{I,2} = \left(\frac{1}{2} + \frac{f^c_{E,2} - f^c_{I,2} + \alpha^c N^s}{2\tau^c}\right) N^c \in [0, N^c]$. I obtain the following cases:

- Case $\text{LT}^{\text{sh}}_{\text{i}}$: If $\tau^c \ge \min\left\{\frac{c_I - c_E}{3} - \frac{\alpha N^s}{3}, \frac{c_E - c_I}{3} + \frac{\alpha N^s}{3}\right\}$,

$$\begin{cases} f^{c,lt^*}_{I,2} = \tau^c + \dfrac{c_E + 2c_I + \alpha N^s}{3}, & f^{c,lt^*}_{E,2} = \tau^c + \dfrac{2c_E + c_I - \alpha N^s}{3}, \\ N^{c,lt^*}_{I,2} = \left(\dfrac{1}{2} + \dfrac{c_E - c_I + \alpha N^s}{6\tau^c}\right) N^c, \\ \Pi^{lt^*}_{I,2} = \frac{1}{18\tau^c}\left(c_E - c_I + 3\tau^c + \alpha N^s\right)^2 N^c,\ \Pi^{lt^*}_{E,2} = \frac{1}{18\tau^c}\left(-c_E + c_I + 3\tau^c - \alpha N^s\right)^2 N^c \end{cases}$$

- Case $\text{LT}^{\text{sh}}_{\text{ii}}$: If $\tau^c < \frac{c_E - c_I}{3} + \frac{\alpha N^s}{3}$,

$$\begin{cases} f^{c,lt^*}_{I,2} = c_E - \tau^c + \alpha N^s, \quad f^{c,lt^*}_{E,2} = c_E, \quad N^{c,lt^*}_{I,2} = N^c, \\ \Pi^{lt^*}_{I,2} = \left(c_E - c_I - \tau^c + \alpha N^s\right) N^c,\ \Pi^{lt^*}_{E,2} = 0 \end{cases}$$

- Case $\text{LT}^{\text{sh}}_{\text{iii}}$: If $\tau^c < \frac{c_I - c_E}{3} - \frac{\alpha N^s}{3}$,

$$\begin{cases} f^{c,lt^*}_{I,2} = c_I, \quad f^{c,lt^*}_{E,2} = c_I - \tau^c - \alpha N^s, \quad N^{c,lt^*}_{I,2} = 0, \\ \Pi^{lt^*}_{I,2} = 0,\ \Pi^{lt^*}_{E,2} = \left(c_I - c_E - \tau^c - \alpha N^s\right) N^c \end{cases}$$

In the first period, if I creates a long-term contract I has to incentivize the sellers to sign it and faces the following maximization problem:

$$\max_{f^c_{I,1},\bar{f}^s_I} \left\{ \left(f^c_{I,1} - c_I\right) N^c_{I,1} + \left(\bar{f}^s_I - 2c_I\right) N^s \right\}$$
$$\text{s.t. } v^s + \alpha^s N^c_{I,1} + v^s + \alpha^s N^{c,lt^*}_{I,2} - \bar{f}^s_I \geq v^s + \alpha^s N^{c,nlt^*}_{E,2} - f^{s,nlt^*}_{E,2}$$

where $N^c_{I,1} = \min\left\{\frac{1}{\tau^c}\left(v^c + \alpha^c N^s - f^c_{I,1}\right) N^c, N^c\right\}$. Taking first order conditions and solving gives

$$\Pi^{lt^*}_{I,1} = \left(v^c + \alpha N^s - \tau^c - c_I\right) N^c + \left(v^s + \alpha N^c + \alpha N^{c,lt^*}_{I,2} - \alpha N^{c,nlt^*}_{E,2} + f^{s,nlt^*}_{E,2} - 2c_I\right) N^s,$$

where $\left(N^{c,lt^*}_{I,2}, N^{c,nlt^*}_{E,2}, f^{s,nlt^*}_{E,2}\right)$ are given by the cases above.

The net benefit for I using a long-term contract when both sides single-home (Proposition 2.3 adjusted for consumer heterogeneity),

$$\Pi^{lt^*}_{I,1} + \Pi^{lt^*}_{I,2} - \Pi^{nlt^*}_{I,1} - \Pi^{nlt^*}_{I,2}$$
$$= \begin{cases} \frac{\alpha(c_E - c_I + 3\tau^c - 7\alpha N^s) N^c N^s}{9\tau^c} & \text{if NLT}^{\text{sh}}_{\text{i}} \text{ \& LT}^{\text{sh}}_{\text{i}} \\ \frac{(2(c_E - c_I) + 3\tau^c + 5\alpha N^s) N^c}{6} + \frac{\left(-32\alpha^2 {N^s}^2 + 5\alpha(c_E - c_I)N^s + (c_E - c_I)^2\right) N^c}{18\tau^c} & \text{if NLT}^{\text{sh}}_{\text{ii}} \text{ \& LT}^{\text{sh}}_{\text{i}} \\ \frac{(2(c_E - c_I) + 15\tau^c - 19\alpha N^s) N^c}{6} + \frac{\left(4\alpha^2 {N^s}^2 + 5\alpha(c_E - c_I)N^s + (c_E - c_I)^2\right) N^c}{18\tau^c} & \text{if NLT}^{\text{sh}}_{\text{iii}} \text{ \& LT}^{\text{sh}}_{\text{i}} \\ -\frac{2\alpha^c N^c {N^s}^2}{\tau^c} & \text{if NLT}^{\text{sh}}_{\text{ii}} \text{ \& LT}^{\text{sh}}_{\text{iii}} \\ 2\left(\tau^c - 2\alpha N^s\right) N^c & \text{if NLT}^{\text{sh}}_{\text{iii}} \text{ \& LT}^{\text{sh}}_{\text{iii}} \\ \left(c_E - c_I + \tau^c - 2\alpha N^s\right) N^c & \text{if NLT}^{\text{sh}}_{\text{iii}} \text{ \& LT}^{\text{sh}}_{\text{ii}} \end{cases}$$

It immediately follows from case $\text{NLT}^{\text{sh}}_{\text{i}}$ & $\text{LT}^{\text{sh}}_{\text{i}}$, which requires $3\tau^c > c_I - c_E + 3\alpha N^s$ to hold, that I creates a long-term contract if and only if $3\tau^c > c_I - c_E + 7\alpha N^s$. One can then show that I's net benefit from creating a long-term contract is never strictly positive in all the other cases. For example, $\Pi^{lt^*}_{I,1} + \Pi^{lt^*}_{I,2} - \Pi^{nlt^*}_{I,1} - \Pi^{nlt^*}_{I,2}$ clearly is negative in cases $\text{NLT}^{\text{sh}}_{\text{ii}}$ & $\text{LT}^{\text{sh}}_{\text{iii}}$ and $\text{NLT}^{\text{sh}}_{\text{iii}}$ & $\text{LT}^{\text{sh}}_{\text{ii}}$; the latter because $c_E - c_I + \tau^c - 2\alpha N^s < c_E - c_I - \alpha N^s < 0$, where the first inequality follows from the $\text{NLT}^{\text{sh}}_{\text{iii}}$ case condition $\tau^c < \alpha N^s$. In the $\text{NLT}^{\text{sh}}_{\text{iii}}$ & $\text{LT}^{\text{sh}}_{\text{i}}$ case,

$$\Pi^{lt^*}_{I,1} + \Pi^{lt^*}_{I,2} - \Pi^{nlt^*}_{I,1} - \Pi^{nlt^*}_{I,2} > 0$$
$$\iff \Delta_I \triangleq 3\tau^c\left[2(c_E - c_I) + 3\tau^c + 5\alpha N^s\right] - 32\alpha^2 {N^s}^2 + 5\alpha(c_E - c_I)N^s + (c_E - c_I)^2 > 0$$

If $c_E - c_I + 8\alpha N^s \geq 0$ and using $3\tau^c < c_I - c_E + 3\alpha N^s$, $\Delta_I < -8\alpha^2 {N^s}^2 \leq 0$. If instead $c_E - c_I + 8\alpha N^s < 0$, by using the case's parameter restrictions $\tau^c > \alpha N^s$, $3\tau^c < c_I - c_E + 3\alpha N^s$ and $3\tau^c > c_E - c_I + \alpha N^s$, one can show that $\Delta_I \leq 0$, where the strict inequality holds if $\alpha > 0$. It follows from comparing the case parameter restrictions that I creates a long-term contract if and only if $\alpha\left(3\tau^c + c_E - c_I - 7\alpha N^s\right) > 0$, as stated in Proposition 2.9. □

Proof Proposition 2.10.

The following parameter conditions have to be satisfied: $u_{E,2}^{c,nlt^*}\left(\frac{N_{E,2}^{c,nlt^*}}{N^c}\right) \geq 0 \iff 2v^c - c_E - c_I - 3\tau^c + 3\alpha N^s + \alpha v^s N^s \geq 0$, $u_{E,2}^{c,lt^*}\left(\frac{N_{E,2}^{c,lt^*}}{N^c}\right) \geq 0 \iff 2v^c + \alpha N^s - c_I - c_E - 3\tau^c \geq 0$, $N_{I,1}^{c,lt^*} = N^c \iff 2\tau^c \leq v^c + 2\alpha N^s - c_I$.

In the second period absent a long-term contract, now that sellers can multi-home I and E compete in the more classical Hotelling fashion for the consumers. In particular, each platform $k = I, E$ faces the following maximization problem:

$$\max_{f_{k,2}^c, f_{k,2}^s} \left\{\left(f_{k,2}^c - c_k\right) N_{k,2}^c + \left(f_{k,2}^s - c_k\right) N^s\right\} \text{ s.t. } v^s + \alpha^s N_{k,2}^c - f_{k,2}^s \geq 0,$$

where $N_{k,2}^c = \left(\frac{1}{2} + \frac{f_{-k,2}^c - f_{k,2}^c}{2\tau^c}\right) N^c$. Here again I let $\alpha^c = \alpha^s = \alpha$, this time for the sake of I's long-term contract net benefit expressions. Noticing that the seller fee constraint clearly is binding and taking first order conditions with respect to $f_{k,2}^c$ and solving gives (Lemma 2.3 adjusted for consumer heterogeneity):

- Case $\text{NLT}_{\text{i}}^{\text{mh}}$: If $v^s > 1$ and $\tau^c > \frac{c_I - c_E}{3} + \alpha\frac{(v^s-1)}{3}N^s$,

$$\begin{cases} f_{I,2}^{c,nlt^*} &= \tau^c + \dfrac{1}{3}\left(c_E + 2c_I - (v^s+2)\alpha N^s\right), \\ f_{E,2}^{c,nlt^*} &= \tau^c + \dfrac{1}{3}\left(2c_E + c_I - (2v^s+1)\alpha N^s\right), \\ f_{E,2}^{s,nlt^*} &= v^s + \dfrac{\alpha N^c}{6\tau^c}\left(c_I - c_E + 3\tau^c + (v^s-1)\alpha N^s\right), \\ N_{E,2}^{c,nlt^*} &= \left(\dfrac{1}{2} + \dfrac{c_I - c_E + (v^s-1)\alpha N^s}{6\tau^c}\right) N^c, \\ \Pi_{I,2}^{nlt^*} &= \dfrac{1}{3}(c_E - c_I + (1-v^s)\alpha N^s)N^c + (v^s - c_I)N^s \\ &\quad + \dfrac{\tau^c N^c}{2} + \dfrac{1}{18\tau^c}\left((v^s-1)\alpha N^s + c_I - c_E\right)^2 N^c \end{cases}$$

- Case $\text{NLT}_{\text{ii}}^{\text{mh}}$: If $v^s > 1$ and $\tau^c \leq \frac{c_I - c_E}{3} + \alpha\frac{(v^s-1)}{3}N^s$,

$$\begin{cases} f_{I,2}^{c,nlt^*} = c_I - \alpha N^s, \quad f_{E,2}^{c,nlt^*} = c_I - \tau^c - \alpha N^s, \\ f_{E,2}^{s,nlt^*} = v^s + \alpha N^c, \quad N_{E,2}^{c,nlt^*} = N^c, \\ \Pi_{I,2}^{nlt^*} = (v^s - c_I)\, N^s,\ \Pi_{E,2}^{nlt^*} = (c_I - c_E - \tau^c - \alpha N^s)\, N^c + (\alpha v^s N^c - c_E)\, N^s \end{cases}$$

Suppose that sellers signed the exclusive long-term contract in the first period, and therefore sellers are bound to only go to I in the second period. I and E

still compete for the consumers in the second period by simultaneously setting the consumer fees. Thus, under an exclusive long-term contract, the second-period platforms' maximization problem is identical to the two-sided single-homing case.

The first period maximization problem is given by:

$$\max_{f^c_{I,1},\bar{f}^s_I} \left\{ \left(f^c_{I,1} - c_I\right) N^c_{I,1} + \left(\bar{f}^s_I - 2c_I\right) N^s \right\} \text{ s.t. } 2v^s + \alpha^s N^c_{I,1} + \alpha^s N^{c,lt^*}_{I,2} - \bar{f}^s_I \geq 0,$$

where $N^c_{I,1} = \min\left\{\frac{1}{\tau^c}\left(v^c + \alpha^c N^s - f^c_{I,1}\right) N^c, N^c\right\}$. Re-call that the homogeneous sellers' surplus was fully extracted in the second period absent a contract if sellers multi-home. One derives that

$$\Pi^{lt^*}_{I,1} = \left(v^c + \alpha N^s - \tau^c - c_I\right) N^c + \left(2v^s + \alpha N^c + \alpha N^{c,lt^*}_{I,2} - 2c_I\right) N^s,$$

where $N^{c,lt^*}_{I,2}$ is given by the cases above.

The net benefit for I using a long-term contract when both sides single-home (Proposition 2.4 adjusted for consumer heterogeneity),

$$\Pi^{lt^*}_{I,1} + \Pi^{lt^*}_{I,2} - \Pi^{nlt^*}_{I,1} - \Pi^{nlt^*}_{I,2}$$
$$= \begin{cases} \frac{\alpha\left((2v^s+3)(c_E-c_I+3\tau^c+\alpha N^s)-\alpha {v^s}^2 N^s\right)N^c N^s}{18\tau^c} & \text{if } \mathrm{NLT}^{\mathrm{mh}}_{\mathrm{i}} \,\&\, \mathrm{LT}^{\mathrm{sh}}_{\mathrm{i}} \\ \frac{(c_E-c_I+3\tau^c+\alpha N^s)(c_E-c_I+3\tau^c+4\alpha N^s)N^c}{18\tau^c} & \text{if } \mathrm{NLT}^{\mathrm{mh}}_{\mathrm{ii}} \,\&\, \mathrm{LT}^{\mathrm{sh}}_{\mathrm{i}} \\ 0 & \text{if } \mathrm{NLT}^{\mathrm{mh}}_{\mathrm{ii}} \,\&\, \mathrm{LT}^{\mathrm{sh}}_{\mathrm{iii}} \\ \left(c_E - c_I - \tau^c + 2\alpha N^s\right) N^c & \text{if } \mathrm{NLT}^{\mathrm{mh}}_{\mathrm{ii}} \,\&\, \mathrm{LT}^{\mathrm{sh}}_{\mathrm{ii}} \\ \frac{(4(c_E-c_I)-9\tau^c+10\alpha N^s+2\alpha v^s N^s)N^c}{6} - \frac{(c_E-c_I+\alpha(1-v^s)N^s)^2 N^c}{18\tau^c} & \text{if } \mathrm{NLT}^{\mathrm{mh}}_{\mathrm{ii}} \,\&\, \mathrm{LT}^{\mathrm{sh}}_{\mathrm{ii}} \end{cases}$$

Note that the case $\mathrm{NLT}^{\mathrm{mh}}_{\mathrm{i}}$ & $\mathrm{LT}^{\mathrm{sh}}_{\mathrm{ii}}$ can only arise if $v^s < 2$. One can then show that I's net benefit from creating a long-term contract is strictly positive in all the cases other than if $\mathrm{NLT}^{\mathrm{mh}}_{\mathrm{ii}}$ & $\mathrm{LT}^{\mathrm{sh}}_{\mathrm{iii}}$. For example in the $\mathrm{NLT}^{\mathrm{mh}}_{\mathrm{i}}$ & $\mathrm{LT}^{\mathrm{sh}}_{\mathrm{i}}$ case, $\Pi^{lt^*}_{I,1} + \Pi^{lt^*}_{I,2} - \Pi^{nlt^*}_{I,1} - \Pi^{nlt^*}_{I,2} > 0 \iff 3\tau^c > c_I - c_E + \alpha\left(\frac{-3-2v^s+{v^s}^2}{3+2v^s}\right) N^s$. Using the parameter restriction to be in the $\mathrm{NLT}^{\mathrm{mh}}_{\mathrm{i}}$ case, $3\tau^c > c_I - c_E + \alpha\left(v^s - 1\right) N^s > c_I - c_E + \alpha\left(\frac{-3-2v^s+{v^s}^2}{3+2v^s}\right) N^s$ because $v^s - 1 > \frac{-3-2v^s+{v^s}^2}{3+2v^s}$ $\forall v^s > 0$. Therefore, whenever we are in the $\mathrm{NLT}^{\mathrm{mh}}_{\mathrm{i}}$ & $\mathrm{LT}^{\mathrm{sh}}_{\mathrm{i}}$ case, I creates a long-term contract. It follows from comparing the case parameter restrictions that I creates a long-term contract if and only if $3\tau^c > \max\left\{0, c_I - c_E - \alpha N^s\right\}$, as stated in Proposition 2.10. □

Proof Proposition 2.11.

The following parameter conditions have to be satisfied: $u^{c,nlt^*}_{E,2}\left(\frac{N^{c,nlt^*}_{E,2}}{N^c}\right) \geq 0 \iff 2v^c - c_E - c_I - 3\tau^c + 3\alpha N^s + \alpha v^s N^s \geq 0$, $u^{c,lt^*}_{E,2}\left(\frac{N^{c,lt^*}_{E,2}}{N^c}\right) \geq 0 \iff$

$2v^c + 3\alpha N^s - c_I - c_E - 3\tau^c \geq 0$, $N_{I,1}^{c,lt^*} = N^c \iff 2\tau^c \leq v^c + 2\alpha N^s - c_I$. The second period outcome absent a long-term contract is identical to the one analyzed for Proposition 2.10.

Suppose that sellers signed the non-exclusive long-term contract in the first period. Then E can still attract the sellers and I and E compete for the consumers in the second period. The second-period platforms' maximization problem is given by:

$$I : \max_{f_{I,2}^c} \left\{ \left(f_{I,2}^c - c_I\right) N_{I,2}^c \right\},$$
$$E : \max_{f_{E,2}^c, f_{E,2}^s} \left\{ \left(f_{E,2}^c - c_E\right)\left(N^c - N_{I,2}^c\right) + \left(f_{E,2}^s - c_E\right) N^s \right\} \text{ s.t. } v^s + \alpha^s \left(N^c - N_{I,2}^c\right) - f_{E,2}^s \geq 0,$$

where $N_{I,2}^c = \left(\frac{1}{2} + \frac{f_{E,2}^c - f_{I,2}^c}{2\tau^c}\right) N^c \in [0, N^c]$. One obtains the following cases:

- Case $\text{LT}_{\text{i}}^{\text{mh}}$: If $\tau^c \geq \frac{c_I - c_E + \alpha N^s}{3}$,

$$\begin{cases} f_{I,2}^{c,lt^*} = \tau^c + \dfrac{c_E + 2c_I - \alpha N^s}{3}, & N_{I,2}^{c,lt^*} = \left(\dfrac{1}{2} + \dfrac{c_E - c_I - \alpha N^s}{6\tau^c}\right) N^c, \\ f_{E,2}^{c,lt^*} = \tau^c + \dfrac{2c_E + c_I - 2\alpha N^s}{3}, & f_{E,2}^{s,lt^*} = v^s + \alpha \left(\dfrac{1}{2} + \dfrac{c_I - c_E + \alpha N^s}{6\tau^c}\right) N^c, \\ \Pi_{I,2}^{lt^*} = \dfrac{1}{18\tau^c} \left(c_E - c_I + 3\tau^c - \alpha N^s\right)^2 N^c & \end{cases}$$

- Case $\text{LT}_{\text{ii}}^{\text{mh}}$: If $\tau^c < \frac{c_I - c_E + \alpha N^s}{3}$,

$$\begin{cases} f_{I,2}^{c,lt^*} = c_I, \quad f_{E,2}^{c,lt^*} = c_I - \tau^c, \quad f_{E,2}^{s,lt^*} = v^s + \alpha N^c, \quad N_{I,2}^{c,lt^*} = 0, \\ \Pi_{I,2}^{lt^*} = 0, \ \Pi_{E,2}^{lt^*} = \left(v^s - c_E + \alpha N^s\right) N^s + \left(c_I - c_E - \tau^c\right) N^c \end{cases}$$

The first period maximization problem is given by:

$$\begin{aligned} &\max_{f_{I,1}^c, \bar{f}_I^s} \left\{ \left(f_{I,1}^c - c_I\right) N_{I,1}^c + \left(\bar{f}_I^s - 2c_I\right) N^s \right\} \\ &\text{s.t. } 3v^s + \alpha^s N_{I,1}^c + \alpha^s N_{I,2}^{c,lt^*} - \bar{f}_I^s + \alpha^s (N^c - N_{I,2}^{c,lt^*}) - f_{E,2}^{s,lt^*} \\ &\quad = 3v^s + \alpha^s N_{I,1}^c + \alpha^s N^c - \bar{f}_I^s - f_{E,2}^{s,lt^*} \geq 0, \end{aligned}$$

where $N_{I,1}^c = \min\left\{\frac{1}{\tau^c}\left(v^c + \alpha^c N^s - f_{I,1}^c\right) N^c, N^c\right\}$. One derives that

$$\Pi_{I,1}^{lt^*} = \left(v^c + \alpha N^s - \tau^c - c_I\right) N^c + \left(3v^s + 2\alpha N^c - f_{E,2}^{c,lt^*} - 2c_I\right) N^s,$$

where $f_{E,2}^{s,lt^*}$ is given by the cases above.

The net benefit for I using a long-term contract when both sides single-home

(Proposition 2.5 adjusted for consumer heterogeneity),

$$\Pi_{I,1}^{lt^*} + \Pi_{I,2}^{lt^*} - \Pi_{I,1}^{nlt^*} - \Pi_{I,2}^{nlt^*}$$
$$= \begin{cases} \frac{\alpha((2v^s-1)(3\tau^c - c_I + c_E) - 3\alpha N^s + \alpha v^s(2-v^s)N^s)N^c N^s}{18\tau^c} & \text{if } \mathrm{NLT}_{\mathrm{i}}^{\mathrm{mh}} \text{ \& } \mathrm{LT}_{\mathrm{i}}^{\mathrm{mh}} \\ \frac{(c_E - c_I + 3\tau^c - \alpha N^s)(c_E - c_I + 3\tau^c + 2\alpha N^s)N^c}{18\tau^c} & \text{if } \mathrm{NLT}_{\mathrm{ii}}^{\mathrm{mh}} \text{ \& } \mathrm{LT}_{\mathrm{i}}^{\mathrm{mh}} \\ -\frac{(c_E - c_I + 3\tau^c + \alpha(1-v^s)N^s)^2 N^c}{18\tau^c} & \text{if } \mathrm{NLT}_{\mathrm{i}}^{\mathrm{mh}} \text{ \& } \mathrm{LT}_{\mathrm{ii}}^{\mathrm{mh}} \\ 0 & \text{if } \mathrm{NLT}_{\mathrm{ii}}^{\mathrm{mh}} \text{ \& } \mathrm{LT}_{\mathrm{ii}}^{\mathrm{mh}} \end{cases}$$

First note that the case $\mathrm{NLT}_{\mathrm{ii}}^{\mathrm{mh}}$ & $\mathrm{LT}_{\mathrm{i}}^{\mathrm{mh}}$ only arises if $v^s > 2$, and case $\mathrm{NLT}_{\mathrm{i}}^{\mathrm{mh}}$ & $\mathrm{LT}_{\mathrm{ii}}^{\mathrm{mh}}$ only if $v^s \leq 2$. In the $\mathrm{NLT}_{\mathrm{ii}}^{\mathrm{mh}}$ & $\mathrm{LT}_{\mathrm{i}}^{\mathrm{mh}}$ case, $\Pi_{I,1}^{lt^*} + \Pi_{I,2}^{lt^*} - \Pi_{I,1}^{nlt^*} - \Pi_{I,2}^{nlt^*} > 0$ because $c_E - c_I + 3\tau^c - \alpha N^s > 0$ and $c_E - c_I + 3\tau^c + 2\alpha N^s > 0$ by the $\mathrm{LT}_{\mathrm{i}}^{\mathrm{mh}}$ parameter requirement.

In the $\mathrm{NLT}_{\mathrm{i}}^{\mathrm{mh}}$ & $\mathrm{LT}_{\mathrm{i}}^{\mathrm{mh}}$ case, $\Pi_{I,1}^{lt^*} + \Pi_{I,2}^{lt^*} - \Pi_{I,1}^{nlt^*} - \Pi_{I,2}^{nlt^*} > 0 \iff 3\tau^c > c_I - c_E + \alpha\left(\frac{3-v^s(2-v^s)}{2v^s-1}\right)N^s$. Then if $v^s > 2$, the parameter requirement is $3\tau^c > c_I - c_E + \alpha\left(v^s - 1\right)N^s$. Since $v^s - 1 > \frac{3-v^s(2-v^s)}{2v^s-1}$ if $v^s > 2$, I always creates a long-term contract. If $v^s \leq 2$, the parameter requirement is $3\tau^c > c_I - c_E + \alpha N^s$. Since $\frac{3-v^s(2-v^s)}{2v^s-1} > 1 \ \forall v^s$, I creates a long-term contract if and only if $3\tau^c > c_I - c_E + \alpha\left(\frac{3-v^s(2-v^s)}{2v^s-1}\right)N^s$. $\square$

Appendix C

Quality Investment by Platforms

C.1 Discussion

Model set-up. The justification and discussion of several explicit and implicit assumptions in the model are appropriate. First, I take a "micro" model approach in the sense that I explicitly model the valuations that users on the two sides derive from *directly* interacting with each other when joining a platform (several papers in the two-sided market literature take this route, e.g. Hagiu, 2006; Zhu and Iansiti, 2012; Hagiu and Wright, 2015b; Casadesus-Masanell and Llanes, 2015). At the same time, I impose few restrictions on $v(n, A)$. In Appendix C.3 I illustrate the main results using the CES functional form assumption, which encompasses a large and widely used class of utility functions. "Macro" models in contrast take user valuations as exogenous to any direct interactions between the two sides (e.g., Caillaud and Jullien, 2003; Rochet and Tirole, 2006; Armstrong, 2006; Tan et al., 2020).

Second, network effects for consumers are across, but not within. That is, consumers benefit from a wider range of product varieties but do not care about participation and usage of fellow consumers on the same side.[1] I abstract for simplification from within network effects on the consumer side, which has been studied elsewhere (see, e.g., Belleflamme and Peitz, 2019a). On the other hand, network effects for sellers are both across and within: sellers benefit from having more consumers on the other side (as they can sell their products to more customers), but are hurt by an increase in the presence of sellers (which pushes down the product price).

Third, price discrimination on any side is not allowed by platforms when setting their access fees. That is, each platform has to set a uniform fee for each side.

[1] For example in the video-game consoles market, players value the presence of other players when playing online.

Fourth, the model assumes that users on one side, say consumers, are of equal value to users on the other side. As pointed out by Weyl (2010), this for example rules out that video games are especially valued and hence engaged with by (a subset) of players or high advertising-value readers at a newspaper.

Fifth and as highlighted in Section 3.2, the introduction of hinterlands as in Armstrong and Wright (2009) allows for a tractable comparison between the monopoly and duopoly market environments. The second advantage of adopting a "Hotelling model with hinterlands" is that one can properly account for the various degrees of consumer adaptation observed across industries. For instance in the (mobile) operating system and credit card industries, arguably most if not every consumer chooses between one or the other platform, and thus the hinterlands play a small role. This corresponds to a low λ scenario. Yet in other industries, hinterlands are of larger importance and hence correspond to a relatively large λ value: not every (potential) consumer owns a video-game console, and not every consumer uses ride-hailing services (Uber, Lyft).

Finally, I assume that consumers form *passive rational* expectations about the adaptation decision by sellers to join a given platform.[2] This assumption merits further discussion since it arguably is the least common assumption used in the two-sided market literature. The majority of the existing literature in the presence of network effects assumes that users on *all* sides have full information about *all* the actions taken by platforms and the ability to perfectly compute actions' impact on platform adaptation.[3] In other words, users are assumed capable to perfectly adjust their expectations in response to changes in platform actions. Here, actions are setting the fees and investing in the platform's quality level.

[2]Passive expectations were first introduced in one-sided markets with network effects by Katz and Shapiro (1985). Two recent examples with one-sided network effects are Griva and Vettas (2011), who find that price competition between horizontally and vertically differentiated firms is more intense when responsive instead of fixed expectations are prevalent, and Hurkens and Ángel L. López (2014), who resolve a puzzle in the mobile termination literature by assuming passive expectations: the literature predicts that firms prefer lower (below-cost) termination charges, while in reality firms prefer above termination charges and thereby oppose regulators' attempts to push termination rates down.

[3]See, for instance, Armstrong (2006), Armstrong and Wright (2007), Caillaud and Jullien (2003), Choi (2010), Halaburda and Yehezkel (2013), Rochet and Tirole (2003), Rochet and Tirole (2006), and Weyl (2010). Only some recent papers have incorporated passive expectations in two-sided markets: Gabszewicz and Wauthy (2014) find that responsive expectations lead to wider participation on the platform than passive expectations and Hagiu and Halaburda (2014) investigate *hybrid* expectations. Alternatively, Belleflamme and Peitz (2019c) study a set-up in which some users on one or both sides of the market are unaware of the price charged to market participants on the other side.

In most real-world settings, however, users arguably are unable to calculate the effect of at least *some* platform action on the adaptation decision by other users - whether they are users on the same side or the other side of the market. One reason is that one side, typically consumers, may for instance simply not be aware of the fee charged to the other side by the platform (Hagiu and Halaburda, 2014). For example, few mobile OS users are aware of the fees charged by Apple or Samsung to third-party app sellers. Another reason is that users, even when platform fees are known, may not have enough information to formulate an aggregate demand change if fees change. Instead, they rely on external information (e.g., press releases, market reports, word of mouth) to form *passive* expectations about the total number of sellers who will participate.

For these and tractability reasons, I assume that *consumers* form *passive rational* expectations regarding the sellers' adaptation decision due to the platform's actions. Users do, however, change their individual adoption decisions based on the platform's quality level they observe and the fee they are being charged. All types of expectations are rational, i.e., fulfilled in equilibrium. In contrast to the consumers, I assume throughout the chapter that the seller side has full information about all the actions taken and forms *rational and responsive* expectations about user participation.

Alternative interpretation of the two-sided platform. The two-sided platform outcomes in the present chapter can be replicated by a **re-seller** set-up. The idea is that sellers in a first step sell their goods or services to a re-seller who then in a second step sells the goods or services to the consumers (as in Hagiu and Wright, 2015b). The distinction between a two-sided platform and a re-seller rests on which agent is in command of the residual control right, i.e., setting the product price, over the goods traded: in the two-sided platform case the sellers retain the control right and interact directly with the consumers, whereas in the re-seller case the re-seller platform holds the residual control right and sellers do not interact directly with consumers.

Let R be the fixed take-it-or-leave-it amount the monopolistic re-seller offers to each potential seller j for buying the amount m, which is the amount the re-seller will sell of each product variety.[4] At stage three, the last seller indexed n (and hence

[4] The notion of "buying" and "re-selling" can be adapted to service industries (e.g., ride-hailing services) by thinking of R as a fixed wage for which the re-seller contracts the service providers (drivers in the example).

the measure of sellers) to sell its product to the re-seller is given by the expression $R - c(n)m = 0$.

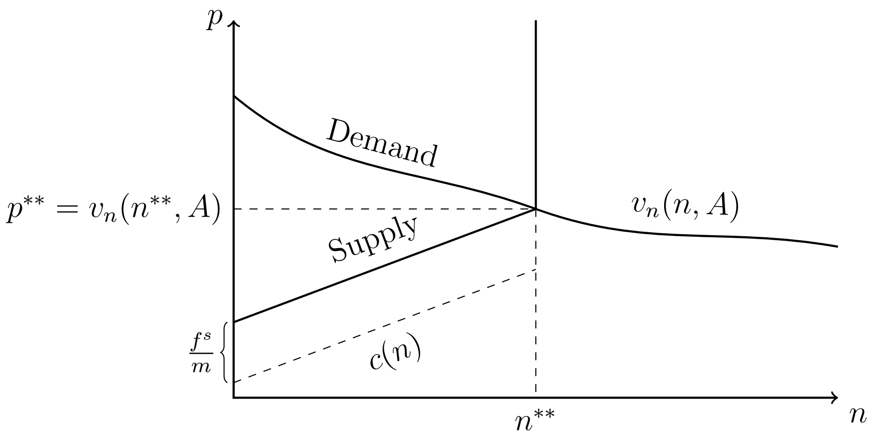
R is chosen by the re-seller at the second stage. The re-seller can sell the merchandise at a total revenue $v_n m$ per variety to the consumers, where again v_n is the product price the re-seller charges in equilibrium. Thus, the re-seller's profit is $mf^c + n(v_n m - R) - c(A) = mf^c + nm(v_n - c(n)) - c(A)$, which becomes the same maximization problem of choosing (m, n) as in the two-sided platform mode when using the inverse consumer demand function - see (3.6).

The reason why the two-sided platform and the re-seller set-ups collapse here is because, in contrast to Hagiu and Wright (2015b), I do not model non-contractible residual control rights such as marketing activities other than the product price. The crucial assumption in their paper is that sellers (in the two-sided platform mode) and intermediaries (in the re-seller mode) have different information relevant to the tailoring of the marketing activity for the products. In my model, however, the sellers and two-sided platforms have identical information along the product price dimension, the sole residual control right.

On the measure of sellers and the competitive product price. Note that consumers do not care about the index of the seller j from which they purchase products. While consumers care about product variety, each product is homogeneous in the eyes of each consumer according to the definition of u. Since consumers can observe the price of all products offered, they only buy the most inexpensive products. While that means that consumers *per se* are not forced to buy all products that are offered on the platform, in the subsequent analysis I will show that sellers enter only if they can actually sell their products, which then implies that consumers do purchase all available products. Selling its product to the consumers is a requisite for a seller to make a positive profit.

Consumers are willing to purchase an additional product as long as the price for that product is less or equal than the marginal value they obtain from purchasing that product, denoted by $v_n(n, A)$. The inverse demand curve $v_n(n, A)$ is decreasing in n because $v_{nn}(n, A) < 0$ by assumption. In the two-sided market operation mode, each seller decides whether to join the platform or not. Seller j has to earn a product price greater or equal than its marginal cost $c(j)$ plus the fixed seller fee per consumer, f^s/m (assuming for now that j sells to all consumers), for j wanting to join the platform and to sell its product. Formally, $p(j) \geq c(j) + f^s/m$. Since the sellers are ordered by their marginal cost index and $c_j > 0$, the supply curve is

Figure C.1.1: Competitive product price equilibrium



given by $c(n) + f^s/m$.

The supply and demand curves are illustrated in Figure C.1.1. The intersection of the supply and demand curves gives the number of sellers and the competitive product price in equilibrium. For a given level of technology (stage one) and seller fee (stage two), sellers enter at stage three up to the point where the measure of sellers is n^{**} such that $(v_n(n^{**}, A) - c(n^{**}))\, m - f^s = 0$ is satisfied. The consequent equilibrium price is $p^{**} = v_n(n^{**}, A)$.

Suppose instead that $n' < n^{**}$. Then, $p' = v_n(n', A)$ and $p' = c(n') + f^s/m > p^{**} = c(n^{**}) + f^s/m$ and all sellers $j \in [0, n']$ make a strictly positive profit. Additional sellers with a higher index than n' join up to the point where the seller measure equals n^{**}. No seller indexed $j > n^{**}$ will join the platform because that seller j would make a profit $(v_n(n^{**}, A) - c(j))m - f^s < 0$.

In the MPF case, although the MPF sets the product prices for all of its products offered, the MPF does not have an incentive to deviate from setting $p(j) = v_n(n^*, A)$ $\forall j \in [0, n^*]$. Changing the product price (increasing it) of one product has no impact (because the mass of one product is zero), but then the MPF should change the price of all other products as well. Yet in equilibrium $p(j) = v_n(n^*, A)$ is the price that maximizes the MPF's profit by the same argument as before: consumers are willing to purchase an additional product as long as the price for that product is less or equal than the marginal value they obtain from purchasing that product.

C.2 Extensions

C.2.1 Effort extension

Instead of choosing the level of technology, assume that platform k can only incur a non-negative effort e_k at cost $c(e_k)$ to influence the probability that the level of technology increases from a default low level $A_k = A^L$ to a higher level $A_k = A^H = \gamma A^L$, where $\gamma > 1$. Specifically, the level of technology is high ($A_k = A^H$) with probability

$$p(e_k) \triangleq \mathbb{P}(A_k = A^H | e_k)$$

and remains at the low default level ($A_k = A^L$) with complementary probability $1 - p(e_k)$. I assume that $p(e_k)$ is strictly increasing and concave in e_k, and that $c(e_k) = e_k$. For ease of presentation, I let $\lambda = 0$, i.e., hinterlands are omitted in this subsection. The rest of the model stays the same as introduced in Section 3.2.

I am interested in whether Proposition 3.1 still holds for the duopoly market environment. The analysis in stages two to four is analogous to the baseline model. Following up on the results from Section 3.3.2, if *both* platforms are either successful or unsuccessful improving the technology, $m_k^{**} = m_k^* = \frac{1}{2}$. In the two-sided operation case, if platform k succeeds but $-k$ does not succeed $m_k^{**} = \frac{1}{2} + \frac{1}{2}\Gamma^{**}$, where

$$\Gamma^{**} = \frac{1}{3\tau}\Big[v\left(n^{**}(A^H), A^H\right) - v\left(n^{**}(A^L), A^L\right) \\ - n^{**}(A^H)c\left(n^{**}(A^H)\right) + n^{**}(A^L)c\left(n^{**}(A^L)\right)\Big].$$

As in the baseline model $n^{**}(A)$ is implicitly defined by equation (3.19). Conversely, if platform $-k$ succeeds but k does not, $m_k^{**} = \frac{1}{2} - \frac{1}{2}\Gamma^{**}$.[5]

The first part in Assumption C.1 ensures that a successful innovation rewards the platform with an increased consumer share. The second part ensures that the market in either operation mode is still being served by both platforms if one platform succeeds in innovating but the other does not succeed. The last part assumes that effort is sufficiently productive at $e_k = 0$ to ensure that platform k chooses a strictly positive effort according to the first-order conditions.

Assumption C.1. $\min\{\Gamma^{**}, \Gamma^*\} > 0, \max\{\Gamma^{**}, \Gamma^*\} < 1, \frac{\tau}{2}p_e(0)\left(\Gamma^{**}(2 - \Gamma^{**})\right) > 1.$

[5]Similarly in the MPF operation mode: $m_k^* = \frac{1}{2} + \frac{1}{2}\Gamma^*$ if platform k succeeds but $-k$ does not succeed, and $m_k^* = \frac{1}{2} - \frac{1}{2}\Gamma^*$ if platform $-k$ succeeds but k does not, where $\Gamma^* = \frac{1}{3\tau}\left[v\left(n^*(A^H), A^H\right) - v\left(n^*(A^L), A^L\right) - \int_0^{n^*(A^H)} c(j)dj + \int_0^{n^*(A^L)} c(j)dj\right]$ and $n^*(A)$ is implicitly defined by equation (3.21).

At the first stage, platform k in the two-sided operation mode chooses e_k to maximize its expected profit. In particular,

$$\Pi_k^{**}(e_k, e_{-k}) = \max_{e_k} \Bigg\{ p(e_k)p(e_{-k})\frac{\tau}{2} + p(e_k)(1-p(e_{-k}))\frac{\tau}{2}\left[1+\Gamma^{**}\right]^2$$
$$+ (1-p(e_k))p(e_{-k})\frac{\tau}{2}\left[1-\Gamma^{**}\right]^2 + (1-p(e_k))(1-p(e_{-k}))\frac{\tau}{2} - e_k \Bigg\}$$
$$\text{s.t. equation (3.19).}$$

The first-order conditions with respect to e_k can be written as

$$\frac{\partial \Pi_k^{**}(e_k, e_{-k})}{\partial e_k} = \frac{\tau}{2} p_e(e_k)\Big(\Gamma^{**}(2+(1-2p(e_{-k}))\Gamma^{**})\Big) - 1 \leq 0,$$
$$e_k \geq 0, \quad e_k \frac{\partial \Pi_k^{**}(e_k, e_{-k})}{\partial e_k} = 0.$$

Notice that $\Gamma^{**}(2+(1-2p(e_{-k}))\Gamma^{**})$ is strictly positive, strictly decreasing in e_{-k}, strictly increasing in Γ^{**}, and takes the maximum value of 3 when $\Gamma^{**}=1$ and $e_{-k}=0$. Since $p(e_k)$ is strictly concave in e_k and bounded from above by 1, $p_e(e_k)$ is strictly decreasing in e_k and

$$\lim_{e_k \to \infty} p_e(e_k) = 0$$

Thus, $\lim_{e_k\to\infty} \frac{\partial \Pi_k^{**}(e_k,e_{-k})}{\partial e_k} = -1 < 0$ for all e_{-k}. Since $\frac{\partial \Pi_k^{**}(e_k,e_{-k})}{\partial e_k}$ is continuous in e_k, there exists a unique value of e_k at which $\frac{\partial \Pi_k^{**}(e_k,e_{-k})}{\partial e_k} = 0$ if and only if

$$\frac{\partial \Pi_k^{**}(0, e_{-k})}{\partial e_k} = \frac{\tau}{2} p_e(0)\Big(\Gamma^{**}(2+(1-2p(e_{-k}))\Gamma^{**})\Big) - 1 > 0,$$

which is satisfied by the third part of Assumption C.1 for all permissible values of e_{-k} and Γ^{**}.[6] In the symmetric equilibrium, $e_k^{**} = e_{-k}^{**} = e^{**}$. Hence, the unique e^{**} is implicitly defined by

$$\frac{\partial \Pi_k^{**}(e^{**}, e^{**})}{\partial e_k} = \frac{\tau}{2} p_e(e^{**})\Big(\Gamma^{**}(2+(1-2p(e^{**}))\Gamma^{**})\Big) - 1 = 0. \tag{C.1}$$

Repeating the analysis in the MPF operation mode, the unique symmetric equilib-

[6]The second-order condition for a maximum, $\frac{\tau}{2} p_{ee}(e_k)\Big(\Gamma^{**}(2+(1-2p(e_{-k}))\Gamma^{**})\Big) < 0$, trivially holds by the strict concavity assumption, $p_{ee}(e_k) < 0$.

rium effort e^* is implicitly defined by

$$\frac{\partial \Pi_k^*(e^*, e^*)}{\partial e_k} = \frac{\tau}{2} p_e(e^*)\Big(\Gamma^*(2 + (1 - 2p(e^*))\Gamma^*)\Big) - 1 = 0. \qquad \text{(C.2)}$$

Observe that in both operation modes (suppressing equilibrium notation)

$$\frac{de}{d\Gamma} = -\frac{2p_e(e)\left[1 + (1 - 2p(e))\Gamma\right]}{p_{ee}(e)\left[\Gamma(2 + (1 - 2p(e))\Gamma)\right] - 2p_e(e)\Gamma^2} > 0,$$

because the numerator is strictly positive and the denominator is strictly negative: $p_e(e) > 0, p_{ee} < 0$ by assumption, $p(e) \in [0, 1]$ by definition, and $\Gamma < 1$ by Assumption C.1. Thus, applying this comparative statistics result for the comparison of equations (C.1) and (C.2), we immediately see that $e^* > e^{**}$, the effort equivalent to Proposition 3.1, if and only if $\Gamma^* > \Gamma^{**}$.

C.2.2 Two-part tariff

In the analysis thus far consumers and, when applicable, sellers are charged a fixed fee to join a platform. Yet in practice, one observes that two-part tariffs are prevalent, at least on one agent side, in several of the industries outlined. In the video-game console industry, for example, developers of games are charged a fixed fee for having access to the platform's development kit and pay royalties for the games they sell to consumers.

Here I present a two-part tariff set-up, according to which agents pay both a fixed fee and a per-transaction fee (Armstrong, 2006). The model is modified as follows. Let t^c and t^s be the per-transaction fee charged to consumers and sellers, respectively, in the monopoly market environment. The fixed fees are as above. Thus, a two-sided platform now has four degrees of freedom in its tariff choice, (f^c, f^s, t^c, t^s). In the MPF organization mode, a platform has two degrees of freedom (f^c, t^c).

I find that the introduction of per-transaction fees has no bearing on the previously derived results, substantiating the justification to work with a simple fixed fee, a type of *one-part tariff*, in the baseline model. Furthermore, I find that the results are identical. Here I present the results for the monopoly market environment. The analysis for the duopoly case is analogous.

First, the **MPF** case. At stage four the equilibrium product price the MPF now charges is $p^* = v_n(n, A) - t^c$, because the marginal benefit for a consumer to purchase the last product has changed from $v_n(n, A)$ to $v_n(n, A) - t^c$ since the consumer also has to pay a transaction fee. Combining stages three and two, the MPF chooses

(f^c, t^c, n) to maximize $mf^c + t^c mn + m\left[n\left(v_n(n,A) - t^c\right) - \int_0^n c(j)dj\right]$, subject to $m = \frac{1}{2} + \lambda\left[v(n,A) - n\left(v_n(n,A) - t^c\right) - nt^c - f^c\right]$. It follows immediately that the per-transaction fee t^c cancels out in both expressions. Hence, the maximization problem for the MPF is identical at the second stage to the baseline set-up. The reason is that on the consumer side, a per-transaction fee t^c, on the one hand, makes the platform nt^c less attractive as consumers have to pay this fee per transaction to the platform, but on the other hand, the product price has been reduced by the same amount. On the consumer side, these two effects cancel each other out. Hence, t^c has no impact on m. On the variety side, on the one hand, the change of the product price by $-t^c$ decreases the MPF's profit by mnt^c, but on the other hand, the MPF recoups this loss from the consumer side by earning an extra mnt^c. Since the second stage is unaffected by a two-part tariff, the MPF's maximization problem at the first stage is identical to the baseline set-up. Thus, the introduction of a per-transaction fee t^c has no overall impact on the outcomes.

Second, the **two-sided platform** case. At stage four, $p(j)^{**} = v_n(n,A) - t^c$ $\forall j \in [0,n]$. At stage two the platform chooses (f^c, t^c, f^s, t^s) to maximize $mf^c + nf^s + mn(t^c + t^s)$, subject to the number of consumers $m = \frac{1}{2} + \lambda[v(n^e,A) - n^e(v_n(n^e,A) - t^c + t^c) - f^c]$ and the number of sellers, given by $\left[v_n(n,A) - t^c - t^s - c(n)\right]m - f^s = 0$, from the third stage.

As in the MPF case, introducing t^c does not affect consumer participation. Combining the first order conditions with respect to f^c and t^c, it follows that $f^{c^{**}} = \frac{m^{**}}{\lambda} - n^{**}s(n^{**},A)$ and $m^{**} = \frac{1}{4} + \frac{\lambda}{2}\left[v(n^{**},A) - n^{**}c(n^{**})\right]$, which are equivalent to (3.7) and (3.9), respectively. That is, the implicit equilibrium consumer fixed fee and consumer participation expressions are independent of t^c.

Each of the first order conditions with respect to f^s, t^s, and t^c can be written as

$$f^{s^{**}} + m^{**}\left(t^{c^{**}} + t^{s^{**}}\right) = m^{**}n^{**}\left[c_n(n^{**}) - v_{nn}(n^{**},A)\right] \tag{C.3}$$

Thus, a continuum of equilibria regarding the triplet $(f^{s^{**}}, t^{s^{**}}, t^{c^{**}})$ exists such that (C.3) is satisfied. For instance in the case of sellers, the reason for the multiplicity of fees is that the platform only cares about the total fee paid by sellers, not how it is split between the fixed fee and transaction fees. That is, different combinations of the two seller fees yield the same profit.[7] Furthermore, since the consumer per-

[7]Formally, the first order condition with respect to f^s and t^s are respectively given by $n + f^s\frac{dn}{df^s} + (t^c + t^s)m\frac{dn}{df^s} = 0$ and $f^s\frac{dn}{dt^s} + mn + (t^c + t^s)m\frac{dn}{dt^s} = 0$, where $\frac{dn}{dt^s} = m\frac{dn}{df^s} = \frac{1}{v_{nn}(n,A) - c_n(n)} < 0$ (because of the passive expectations assumption, $\frac{dm}{df^s} = \frac{dm}{dt^s} = 0$). An increase in f^s on the one

transaction fee does not affect m, both per-transaction fees have the same impact on the platform's profit. Plugging (C.3) into the seller participation equation gives $s(n^{**}, A) + n^{**}v_{nn}(n^{**}, A) - n^{**}c_n(n^{**}) = 0$, which is identical to (3.10).

The emergence of a continuum of equilibria is a typical feature of two-part tariffs in two-sided markets (see, for instance, Armstrong, 2006). While an undesirable consequence of the continuum of equilibria is that the model is stripped of its predictive power as the question arises which of the equilibria that satisfy (C.3) is selected,[8] the first stage is unaffected by this multiplicity. To see this, the stage one maximization problem using the stage two results is

$$\begin{aligned} A^{**} &= \underset{A}{\text{argmax}} \left\{ m^{**} f^{c^{**}} + n^{**} f^{s^{**}} + \left(t^{c^{**}} + t^{s^{**}}\right) m^{**} n^{**} - c(A) \right\} \\ &= \underset{A}{\text{argmax}} \left\{ \frac{1}{\lambda} \left(m^{**}\right)^2 - c(A) \right\} \\ &\text{s.t. (3.9) and (3.10).} \end{aligned}$$

Therefore, the introduction of a two-part tariff has no impact, neither qualitatively nor quantitatively, on the results compared to the baseline model. The reason, which is in contrast to Armstrong (2006), stems from the passive expectations assumption on the consumer side: a change in the seller per-transaction fee is not internalized by consumers, i.e., a change in the seller participation, and thus the cross-group externality from the consumers' perspective is not affected by a change in the seller per-transaction fee. Instead, a change in the per-transaction fee (multiplied by m) has the same effect as a change in the fixed seller fee. Hence the fixed and per-transaction seller fees are perfect substitutes from the platform's profit perspective.

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hand increases the fee revenue from sellers n, but on the other hand reduces seller participation $\frac{dn}{df^s} < 0$, thereby reducing the fee revenue through the fixed fee, $f^s \frac{dn}{df^s} < 0$, and the revenue from per-transaction fees, $(t^c + t^s)m\frac{dn}{df^s} < 0$. Similar regarding a larger t^s: the revenue from the per-transaction seller fee increases, $mn > 0$, but the reduction in seller participation reduces the revenue from the fixed fee, $f^s \frac{dn}{dt^s} < 0$, and the revenue from per-transaction fees, $(t^c + t^s)m\frac{dn}{dt^s} < 0$.

[8] Reisinger (2014) assumes heterogeneous interaction behavior among agents on both sides to solve this selection problem. In particular, some agents (labeled as "small" agents) only interact with a fraction of agents on the other side, whereas the other agents (labeled as "normal" agents) interact with all agents on the other side. This formulation yields a unique equilibrium in the fee game.

C.3 Illustration

I illustrate here the results obtained using functional form assumptions. In particular, I let the consumers' utility consuming n products at quality A take the constant elasticity of substitution (CES) functional form $v(n, A) = u_0 + [\theta_1 n^\rho + \theta_2 A^\rho]^{\frac{1}{\rho}}$, where the coefficients θ_1 and θ_2 are share parameters for n and A respectively, $\rho \in (-\infty, 1)$,[9] and u_0 is the stand-alone utility value. Since $\rho < 1$, the utility assumptions set out in Section 3.2, namely $v_n(n, A) > 0, v_A(n, A) > 0, v_{nn}(n, A) < 0, v_{nA} \geq 0$, hold for all values of $(u_0, \theta_1, \theta_2, n, A)$.

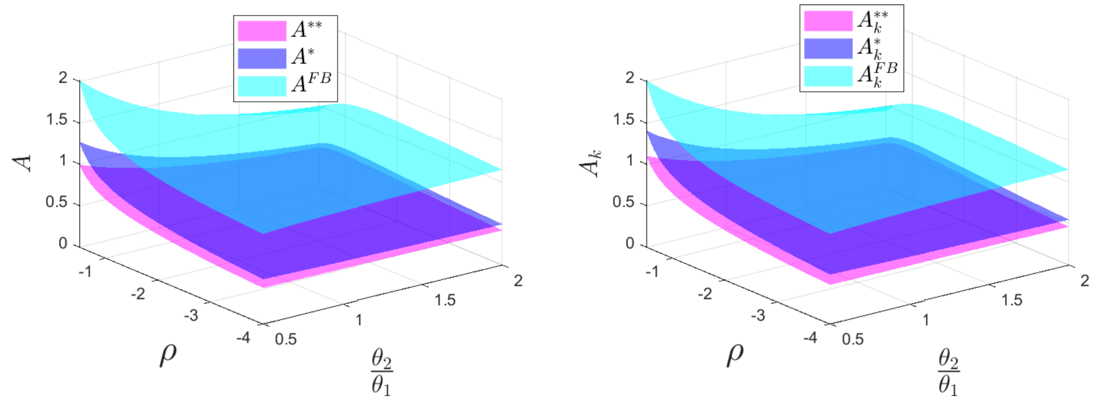
Furthermore, I assume that the heterogeneity of marginal cost for producing the varieties is linear, $c(j) = \alpha j$ where $\alpha > 0$. Lastly, I let the cost of investment $c(A) = \frac{\kappa}{x+1} A^{x+1}$, where $\kappa > 0$. The following systems of nonlinear equations determine the equilibrium levels of technology and number of product varieties in the monopoly environment.

$$\begin{aligned}
\left(n^{FB}, A^{FB}\right): \quad & \theta_1 \left(\theta_1 n^\rho + \theta_2 A^\rho\right)^{\frac{1}{\rho}-1} n^{\rho-1} - \alpha n = 0 \text{ and} \\
& \left[\frac{1}{2} + \lambda \left(u_0 + \left(\theta_1 n^\rho + \theta_2 A^\rho\right)^{\frac{1}{\rho}} - \frac{\alpha}{2} n^2\right)\right] \theta_2 \left(\theta_1 n^\rho + \theta_2 A^\rho\right)^{\frac{1}{\rho}-1} A^{\rho-1} - c'(A) = 0, \\
\left(n^{**}, A^{**}\right): \quad & \theta_1 \left(\theta_1 n^\rho + \theta_2 A^\rho\right)^{\frac{1}{\rho}-2} n^{\rho-1} \left(\theta_1 n^\rho + \rho \theta_2 A^\rho\right) - 2\alpha n = 0 \quad \text{and} \\
& \left[\frac{1}{4} + \frac{\lambda}{2} \left(u_0 + \left(\theta_1 n^\rho + \theta_2 A^\rho\right)^{\frac{1}{\rho}} - \alpha n^2\right)\right] \theta_2 \left(\theta_1 n^\rho + \theta_2 A^\rho\right)^{\frac{1}{\rho}-1} A^{\rho-1} - c'(A) = 0, \\
\left(n^{*}, A^{*}\right): \quad & \theta_1 \left(\theta_1 n^\rho + \theta_2 A^\rho\right)^{\frac{1}{\rho}-2} n^{\rho-1} \left(\theta_1 n^\rho + \rho \theta_2 A^\rho\right) - \alpha n = 0 \quad \text{and} \\
& \left[\frac{1}{4} + \frac{\lambda}{2} \left(u_0 + \left(\theta_1 n^\rho + \theta_2 A^\rho\right)^{\frac{1}{\rho}} - \frac{\alpha}{2} n^2\right)\right] \theta_2 \left(\theta_1 n^\rho + \theta_2 A^\rho\right)^{\frac{1}{\rho}-1} A^{\rho-1} - c'(A) = 0.
\end{aligned}$$

The following figures provide a graphical illustration of the results using the specified functional form assumption in the $\left(\frac{\theta_2}{\theta_1}, \rho\right)$ space, where $\alpha = \lambda = u_0 = 1, \kappa = \tau = \frac{1}{3}, x = 2, \theta_1 = \frac{1}{2}$. The panels in Figure C.3.1 taken separately illustrate Proposition 3.1. Proposition 3.2 is illustrated comparing Figure C.3.1a to Figure C.3.1b.

[9] If $\rho = 1$, the utility indifference curves become linear. As $\rho \to 1$, the CES utility function represents the Cobb-Douglas utility function $v(n, A) = n^{\theta_1} A^{\theta_2}$. As $\rho \to -\infty$, in the limit $v(n, A)$ has the indifference map of the Leontief utility function $v(n, A) = \min\{n, A\}$. $s = \frac{1}{1-\rho} \in (0, \infty)$ is the elasticity of substitution.

Figure C.3.1: Equilibrium technology level in $(\theta_2/\theta_1, \rho)$ space.



(a) Monopoly (b) Duopoly

Lemma C.1. *For the permissible parameter values $(\rho, \theta_1, \theta_2)$ under the CES functional form assumption, the number of varieties strictly increases in the level of technology, i.e. $\frac{dn}{dA} > 0$, in all organization modes and market environments.*

Proof. From (3.3) and (3.17), $\frac{dn^{FB}}{dA}, \frac{dn_k^{FB}}{dA_k} > 0$ if and only if $v_{nA}(n, A) > 0$, which holds by assumption. From (3.10), (3.14), (3.19) and (3.21), all $\frac{dn^{**}}{dA}, \frac{dn^{*}}{dA}, \frac{dn_k^{**}}{dA_k}, \frac{dn_k^{*}}{dA_k} > 0$ respectively if and only if $v_{nA}(n, A) + nv_{nnA}(n, A) > 0$, which in the in the CES functional form assumption requires that $(1-\rho)\theta_1 n^\rho + \rho\theta_2 A^\rho > 0$.

If $\rho > 0$, the condition is trivially satisfied. If $\rho < 0$, assume that $(1-\rho)\theta_1 n^\rho + \rho\theta_2 A^\rho < 0$ for some parameter values $(\rho, \theta_1, \theta_2)$. Then, observe that $0 > (1-\rho)\theta_1 n^\rho + \rho\theta_2 A^\rho > \theta_1 n^\rho + \rho\theta_2 A^\rho$. In addition, $v_n(n, A) + nv_{nn}(n, A) > 0$ if and only if $\theta_1 n^\rho + \rho\theta_2 A^\rho > 0$ for all (n, A). Thus, if $(1-\rho)\theta_1 n^\rho + \rho\theta_2 A^\rho < 0$, then $v_n(n, A) + nv_{nn}(n, A) < 0$. But then according to equations (3.10), (3.14), (3.19) and (3.21), no $n^{**}, n^{*}, n_k^{**}, n_k^{*} > 0$ respectively exist. Thus, we have arrived at a contradiction in the $\rho < 0$ case. Therefore, one requires parameter values $(\rho, \theta_1, \theta_2)$ such that $(1-\rho)\theta_1 n^\rho + \rho\theta_2 A^\rho > 0$ for all (n, A). Hence, the permissible parameter values $(\rho, \theta_1, \theta_2)$ imply that $v_{nA}(n, A) + nv_{nnA}(n, A) > 0$ for all (n, A), from which the statement follows. □

Lemma C.1 finds for the CES functional form case that the higher the investment in the platform quality, the larger the number of product varieties on a platform under any operation mode and market environment. Re-call the earlier discussion that this result does not hold in general, since $v_{nA}(n, A) + nv_{nnA}(n, A)$ may be negative.

Corollary C.1. *For the permissible parameter values $(\rho, \theta_1, \theta_2)$ under the CES functional form assumption, $n^{FB}(A^{FB}) > n^*(A^*) > n^{**}(A^{**})$ in the monopoly environment equilibrium and $n_k^{FB}(A_k^{FB}) > n_k^*(A_k^*) > n_k^{**}(A_k^{**})$ in the duopoly environment.*

Proof. The result immediately follows by combining the results of Proposition 3.1, Lemma C.1 and Lemma 3.4. □

Corollary C.1 states that within each market environment, the equilibrium number of products produced and sold is highest in the first-best outcome and followed by, in decreasing order, the MPF, and two-sided platform. The panels on the right in Figure C.3.2 and C.3.3 graphically illustrate Corollary C.1.

The left panels in Figure C.3.2 and C.3.3, taken separately, illustrate Lemma 3.8. Lemma 3.9 is illustrated comparing the left panel in Figure C.3.2 to the left panel in Figure C.3.3.

Figure C.3.2: Monopoly equilibrium consumer and seller participation in $(\theta_2/\theta_1, \rho)$ space.

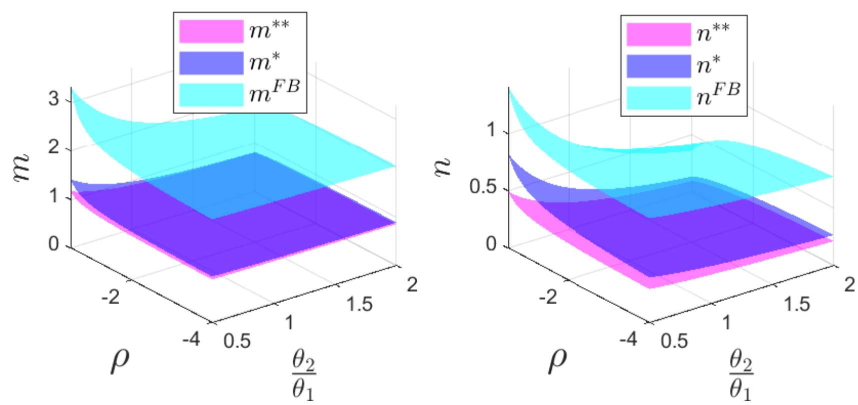
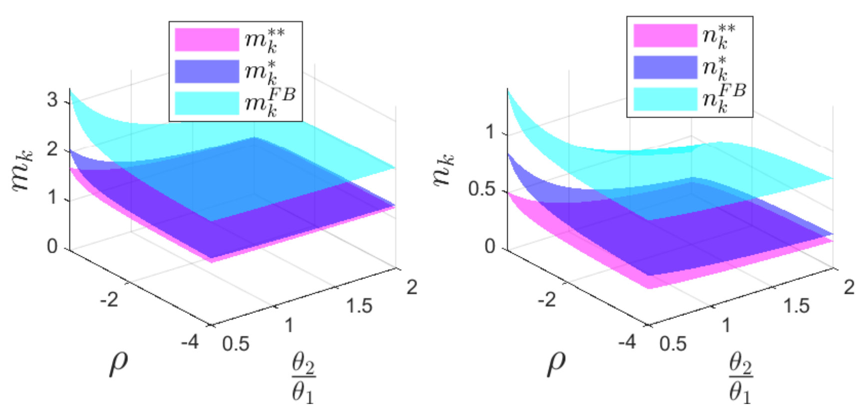


Figure C.3.3 replicates Figure C.3.2 for the duopoly environment.

Figure C.3.3: Duopoly symmetric equilibrium consumer and seller participation in $(\theta_2/\theta_1, \rho)$ space.



C.4 Proofs

Proof Lemma 3.4.

By the optimality conditions (3.3), $n^{FB}(A) : s(n^{FB}, A) = 0$. Since $nv_{nn}(n, A) < 0$, $s(n^{FB}, A) + n^{FB}v_{nn}(n^{FB}, A) < 0$. By the strict concavity assumption of $v_n(n, A)n - \int_0^n c(j)dj$, $s(n, A) + nv_{nn}(n, A)$ is a strictly decreasing function in n. Then for $n^* : s(n^*, A) + n^*v_{nn}(n^*, A) = 0$ to hold, it immediately follows that $n^*(A) < n^{FB}(A)$.

Then, $m^{FB}(n^{FB}(A), A) > m^{FB}(n^*(A), A)$ because $\frac{\partial m^{FB}}{\partial n}\Big|_{n^*} = s(n, A)|_{n^*} > 0$ and $n^{FB}(A) > n^*(A)$. By comparing (3.3) to (3.13), $m^{FB}(n^*(A), A) > m^*(n^*(A), A)$. Hence, $m^{FB}(n^{FB}, A) > m^*(n^*, A)$.

Similarly, $ns_n(n, A) + s(n, A) < s(n, A) + nv_{nn}(n, A)\ \forall n$. Hence, $s(n^*, A) + ns_n(n^*, A) < 0$. By the strict concavity assumption of $ns(n, A)$, $s(n, A) + ns_n(n, A)$ is a strictly decreasing function in n. Then for $n^{**} : s(n^{**}, A) + ns_n(n^{**}, A) = 0$ to be satisfied, it has to hold that $n^{**}(A) < n^*(A)$.

Then, $m^*(n^*(A), A) > m^*(n^{**}(A), A)$ because $\frac{\partial m^*}{\partial n}\Big|_{n^{**}} = \frac{\lambda}{2}s(n, A)\Big|_{n^{**}} > 0$ and $n^*(A) > n^{**}(A)$. By comparing (3.13) to (3.9), it is immediate that $m^*(n^{**}(A), A) > m^{**}(n^{**}(A), A)$. Hence, $m^*(n^*, A) > m^{**}(n^{**}, A)$. □

Proof Proposition 3.1.

Monopoly. First, I compare A^* to A^{FB}. Let $(A^*, n^*(A^*))$ such that (3.15) and (3.14) are satisfied. $n^{FB}(A^*) > n^*(A^*)$ by Lemma 3.4. Observe that

$$\frac{\partial}{\partial n}\left(\left[\frac{1}{2} + \lambda S(n, A)\right] v_A - c'(A)\right)\Bigg|_{n^{FB}(A^*), A^*} = \lambda s(n, A)v_A + \left[\frac{1}{2} + \lambda S(n, A)\right] v_{nA}\Bigg|_{n^{FB}(A^*), A^*}$$

$$= \left[\frac{1}{2} + \lambda S(n, A)\right] v_{nA}\Bigg|_{n^{FB}(A^*), A^*} > 0,$$

because $v_A >$ and $v_{nA} > 0$ by assumption, $s(n^{FB}(A^*), A^*) = 0$ by (3.3), and

$$\left[\frac{1}{2} + \lambda S(n, A)\right]\Bigg|_{n^{FB}(A^*), A^*} = m^{FB}\big|_{n^{FB}(A^*), A^*} > 0$$

by (3.3). Hence,

$$\left[\frac{1}{2} + \lambda S(n, A)\right] v_A - c'(A)\Bigg|_{n^{FB}(A^*), A^*} > \left[\frac{1}{2} + \lambda S(n, A)\right] v_A - c'(A)\Bigg|_{n^*(A^*), A^*}$$

$$> \left[\frac{1}{4} + \frac{\lambda}{2}S(n, A)\right] v_A - c'(A)\Bigg|_{n^*(A^*), A^*} = 0$$

By the strict concavity assumption of the stage 1 maximization program, $\left[\frac{1}{2} + \lambda S(n, A)\right] v_A - c'(A)$ is strictly decreasing in A. Then for (3.3) to hold, A needs to be larger than A^*. Hence, $A^{FB} > A^*$.

Second, A^{**} versus A^*. Let $A^{**}, n^{**}(A^{**})$ such that (3.11) and (3.10) are satisfied. $n^*(A^{**}) > n^{**}(A^{**})$ by Lemma 3.4. Observe that

$$\frac{\partial}{\partial n}\frac{\partial \Pi^*}{\partial A}\bigg|_{n^*(A^{**}),A^{**}} = \frac{\lambda}{2}s(n,A)v_A + \left[\frac{1}{4}+\frac{\lambda}{2}S(n,A)\right]v_{nA}\bigg|_{n^*(A^{**}),A^{**}} > 0,$$

because $v_A >$ and $v_{nA} > 0$ by assumption,

$$s(n^*(A^{**}),A^{**}) > s(n^*(A^{**}),A^{**}) + n^*(A^{**})v_{nn}(n^*(A^{**}),A^{**}) = 0$$

by (3.14), and $\left[\frac{1}{4}+\frac{\lambda}{2}S(n,A)\right]\big|_{n^*(A^{**}),A^{**}} > 0$ by (3.13). Hence,

$$\left[\frac{1}{4}+\frac{\lambda}{2}S(n,A)\right]v_A - c'(A)\bigg|_{n^*(A^{**}),A^{**}} > \left[\frac{1}{4}+\frac{\lambda}{2}S(n,A)\right]v_A - c'(A)\bigg|_{n^{**}(A^{**}),A^{**}}$$
$$> \left[\frac{1}{4}+\frac{\lambda}{2}\left(v - nc(n)\right)\right]v_A - c'(A)\bigg|_{n^{**}(A^{**}),A^{**}} = 0$$

By the strict concavity assumption of the stage 1 maximization program, $[\frac{1}{4} + \frac{\lambda}{2}S(n,A)]v_A - c'(A)$ is strictly decreasing in A. Then, for (3.15) to hold, A needs to be larger than A^{**}. Hence, $A^* > A^{**}$.

Duopoly. First, A_k^{**} versus A_k^*. Let $A_k^{**}, n_k^{**}(A_k^{**})$ such that the conditions (3.20) and (3.19) at a symmetric equilibrium are satisfied. $n_k^*(A_k^{**}) > n_k^{**}(A_k^{**})$ by Lemma 3.4.

Observe that at a symmetric equilibrium,

$$\frac{\partial}{\partial n_k}\frac{\partial \Pi_k^*}{\partial A_k}\bigg|_{n_k^*(A_k^{**}),A_k^{**}}$$
$$= \Omega\frac{\lambda}{2}s(n_k,A_k)v_A(n_k,A_k) + \Omega\left[\frac{1}{4}+\frac{\lambda}{2}S(n_k,A_k)\right]v_{nA}(n_k,A_k)\bigg|_{n_k^*(A_k^{**}),A_k^{**}} > 0$$

To see this, first note that $v_A >$ by assumption and $s(n_k^*(A_k^{**}), A_k^{**}) > 0$, because $s(n_k(A_k),A_k) > 0$ for all $n_k(A_k) < n_k^{FB}(A_k)$ and $\forall A_k$ and by Lemma 3.4 $n_k^*(A_k^{**}) < n_k^{FB}(A_k^{**})$. Second, $v_{nA} > 0$ by assumption and $\left[\frac{1}{4}+\frac{\lambda}{2}S(n_k,A_k)\right]\big|_{n_k^*(A_k^{**}),A_k^{**}} > 0$ by (3.23); otherwise, $m_k^*(n_k^*(A_k^{**}),A_k^{**})$ would be negative. Re-call that $\Omega > 1$ for all positive τ, λ. In short, the first order condition of the MPF, evaluated at a symmetric equilibrium, increases in the increase from n_k^{**} to n_k^*:

$$\Omega\left[\frac{1}{4}+\frac{\lambda}{2}S(n_k,A_k)\right]v_A(n_k,A_k) - c'(A_k)\bigg|_{n_k^*(A_k^{**}),A_k^{**}}$$
$$> \Omega\left[\frac{1}{4}+\frac{\lambda}{2}S(n_k,A_k)\right]v_A(n_k,A_k) - c'(A_k)\bigg|_{n_k^{**}(A_k^{**}),A_k^{**}}$$

The strict inequality also holds when $\lambda = 0$. Hence,

$$\begin{aligned}
\left.\frac{\partial \Pi_k^*}{\partial A_k}\right|_{n_k^*(A_k^{**}),A_k^{**}} &= \Omega\left[\frac{1}{4}+\frac{\lambda}{2}S(n_k,A_k)\right]v_A(n_k,A_k)-c'(A_k)\Big|_{n_k^*(A_k^{**}),A_k^{**}} \\
&> \Omega\left[\frac{1}{4}+\frac{\lambda}{2}S(n_k,A_k)\right]v_A(n_k,A_k)-c'(A_k)\Big|_{n_k^{**}(A_k^{**}),A_k^{**}} \\
&\geq \Omega\left[\frac{1}{4}+\frac{\lambda}{2}\left(v(n_k,A_k)-n_k c(n_k)\right)\right]v_A(n_k,A_k)-c'(A_k)\Big|_{n_k^{**}(A_k^{**}),A_k^{**}} \\
&= \left.\frac{\partial \Pi_k^{**}}{\partial A_k}\right|_{n_k^{**}(A_k^{**}),A_k^{**}} = 0
\end{aligned}$$

The inequality is strict when $\lambda > 0$. By the strict concavity assumption of the stage 1 maximization program, $\Omega\left[\frac{1}{4}+\frac{\lambda}{2}S(n_k,A_k)\right]v_A(n_k,A_k)-c'(A_k)$ is strictly decreasing in A_k. Then for (3.24) to hold, A_k^* needs to be larger than A_k^{**}. Hence, $A_k^* > A_k^{**}$.

Second, A_k^* versus A_k^{FB}. Let $(A_k^*, n_k^*(A_k^*))$ such that (3.24) and (3.21) are satisfied. $n_k^{FB}(A_k^*) > n_k^*(A_k^*)$ by Lemma 3.4. Observe that

$$\begin{aligned}
&\frac{\partial}{\partial n_k}\left(\left[\frac{1}{2}+\frac{1}{2\tau}\left[S(n_k,A_k)-S(n_{-k},A_{-k})\right]+\lambda S(n_k,A_k)\right]v_A^k-c'(A_k)\right)\Big|_{n_k^{FB}(A_k^*),A_k^*} \\
&= \left(\frac{1}{2\tau}+\lambda\right)s(n_k,A_k)v_A^k+\left[\frac{1}{2}+\frac{1}{2\tau}\left[S(n_k,A_k)-S(n_{-k},A_{-k})\right]+\lambda S(n_k,A_k)\right]v_{nA}^k\Big|_{n_k^{FB}(A_k^*),A_k^*} \\
&= \left[\frac{1}{2}+\frac{1}{2\tau}\left[S(n_k,A_k)-S(n_{-k},A_{-k})\right]+\lambda S(n_k,A_k)\right]v_{nA}^k\Big|_{n_k^{FB}(A_k^*),A_k^*} > 0,
\end{aligned}$$

because $v_A^k >$ and $v_{nA}^k > 0$ by assumption, $s(n_k^{FB}(A_k^*), A_k^*) = 0$ by (3.17), and

$$\left[\frac{1}{2}+\frac{1}{2\tau}\left[S(n_k,A_k)-S(n_{-k},A_{-k})\right]+\lambda S(n_k,A_k)\right]v_{nA}^k\Big|_{n_k^{FB}(A_k^*),A_k^*} = m_k^{FB} > 0 \text{ by (3.3).}$$

Hence, at a symmetric equilibrium

$$\begin{aligned}
&\left[\frac{1}{2}+\lambda S(n_k,A_k)\right]v_A^k-c'(A_k)\Big|_{n_k^{FB}(A_k^*),A_k^*} > \left[\frac{1}{2}+\lambda S(n_k,A_k)\right]v_A^k-c'(A_k)\Big|_{n_k^*(A_k^*),A_k^*} \\
&> \Omega\left[\frac{1}{4}+\frac{\lambda}{2}S(n_k,A_k)\right]v_A^k-c'(A_k)\Big|_{n_k^*(A_k^*),A_k^*} = 0.
\end{aligned}$$

The last inequality follows from the fact that $\frac{\Omega}{2} < 1$. By the strict concavity assumption of the first stage maximization program, $\left[\frac{1}{2}+\lambda S(n_k,A_k)\right]v_A^k-c'(A_k)$ is strictly decreasing in A_k. Then for (3.17) to hold, A_k needs to be larger than A_k^*. Hence, $A_k^{FB} > A_k^*$. □

Proof Lemma 3.5.

By the same arguments as in Section 3.3.1, at stage four $p_k^{**}(j) = v_n^k$ for all j and k. Then, at stage three the measure of consumers on platform k is

$$m_k = \frac{1}{2} + \frac{1}{2\tau}\left[v(n_k^e, A_k) - v(n_{-k}^e, A_{-k}) - n_k^e v_n(n_k^e, A_k) + n_{-k}^e v_n(n_{-k}^e, A_{-k}) - f_k^c + f_{-k}^c\right] \\ + \lambda\left[v(n_k^e, A_k) - n_k^e v_n(n_k^e, A_k) - f_k^c\right] \quad \text{(C.4)}$$

The indifferent seller indexed $n_k, \forall k = 1, 2$ is given by

$$s(n_k, A_k)m_k - f_k^s = 0. \quad \text{(C.5)}$$

The second stage fees (f_k^c, f_k^s) are chosen in order to maximize profits $f_k^c m_k + f_k^s n_k$ subject to equations (C.4)-(C.5). The first order conditions are

$$f_k^c : m_k + f_k^c \frac{dm_k}{df_k^c} + f_k^s \frac{dn_k}{df_k^c} = 0 \text{ and}$$
$$f_k^s : f_k^c \frac{dm_k}{df_k^s} + n_k + f_k^s \frac{dn_k}{df_k^s} = 0.$$

The optimal choices depend on the derivatives of m_k, n_k with respect to $(f_k^c, f_k^s, f_{-k}^c, f_{-k}^s)$. Taking the total differentials of the indifference equations (C.4)-(C.5) from stage 3, each with respect to f_k^c and f_k^s, and solving that system of equations, we obtain that

$$\frac{dm_k}{df_k^c} = \frac{-1}{2\tau} - \lambda, \frac{dm_{-k}}{df_k^c} = \frac{1}{2\tau}, \frac{dn_k}{df_k^c} = \frac{\left(\frac{-1}{2\tau} - \lambda\right)(v_n^k - c(n_k))}{m_k(c_n(n_k) - v_{nn}^k)}, \frac{dn_{-k}}{df_k^c} = \frac{\frac{1}{2\tau}\left(v_n^{-k} - c(n_{-k})\right)}{m_{-k}(c_n(n_{-k}) - v_{nn}^{-k})},$$
$$\frac{dm_k}{df_k^s} = \frac{dm_{-k}}{df_k^s} = \frac{dn_{-k}}{df_k^s} = 0, \text{ and } \frac{dn_k}{df_k^s} = \frac{-1}{m_k(c_n(n_k) - v_{nn}^k)}.$$

The final equilibrium is then obtained by imposing the rationality condition $n_k^e = n_k^{**}$. Plugging the total differential expressions back into the first order conditions and solving the system of linear equations, we obtain the following implicit fee expressions:

$$f_k^c = \frac{m_k^{**}}{\frac{1}{2\tau} + \lambda} - n_k^{**} s(n_k^{**}, A_k),$$
$$f_k^s = m_k^{**} s(n_k^{**}, A_k),$$

Plugging the implicit seller fee into the indifferent seller equation (C.5), and using the fact that by assumption we are at an interior solution for m_k, one obtains $s(n_k, A_k) + ns_n(n_k, A_k) = 0$. There exists a unique equilibrium $n_k^{**} > 0 : s(n_k, A_k) + ns_n(n_k, A_k) = 0$, the proof of which is identical to the uniqueness proof of n^{**} in Lemma 3.1. This is equation (3.19). Plugging the implicit consumer fee expressions from above into the indifferent consumer expression (C.4) gives the equilibrium indifferent consumer expressed by equation (3.18). □

Proof Lemma 3.6.

Using the Lemma 3.5 results, the first stage maximization problem for platform k can be written as

$$A_k^{**} = \underset{A_k}{\operatorname{argmax}} \left\{ \frac{1}{\frac{1}{2\tau} + \lambda} \left(m_k^{**}\right)^2 - c(A_k) \right\} \text{ s.t. (3.18) and (3.19).}$$

The first order condition with respect to A_k is given by $\frac{2}{\frac{1}{2\tau}+\lambda} m_k^{**} \left(\frac{dm_k^{**}}{dA_k}\right) - c'(A_k) = 0$. Taking total differentials of the stage 2 equation (3.18) with respect to A_k and observing that $\frac{dn_k^e(A_k)}{dA_k} = \frac{dn_{-k}^e(A_k)}{dA_k} = 0$, one obtains that

$$\frac{dm_k^{**}}{dA_k} = \frac{1}{2\tau} \left(\frac{(2\tau\lambda + 1)\left(2(2\tau\lambda + 1)^2 - 1\right)}{4(2\tau\lambda + 1)^2 - 1} \right) v_A(n_k^{**}, A_k).$$

By the symmetric set-up $A_k^{**} = A_{-k}^{**}$, which implies that $n_k(A_k^{**}) = n_{-k}(A_{-k}^{**})$ and results in the stage one first order condition (3.20). For the second order condition to be satisfied, I impose the condition $\left(\frac{4(2\tau\lambda+1)^2-2}{4(2\tau\lambda+1)^2-1}\right) \left[m_k^{**} v_{AA}(n_k^{**}, A_k) + \frac{dm_k^{**}}{dA_k} v_A(n_k^{**}, A_k)\right] - c''(A_k) < 0$, from which it follows that $\frac{1}{\frac{1}{2\tau}+\lambda} \left(m_k^{**}\right)^2 - c(A_k)$ is strictly concave in A_k. □

Proof Lemma 3.8.

Monopoly. First, I show that $m^{FB}(n^{FB}, A^{FB}) > m^*(n^*, A^*)$.

$\left.\frac{dm^{FB}}{dA}\right|_{n^{FB}} = \lambda v_A(n, A) + \lambda s(n, A) \left.\frac{v_{nA}(n,A)}{(-v_{nn}(n,A)+c_n(n))}\right|_{n^{FB}} = \lambda v_A(n, A)|_{n^{FB}} > 0$ for all A, since $v_A(n, A) > 0$ by assumption and $s(n, A)|_{n^{FB}} = 0$. Hence,

$$m^{FB}(n^{FB}(A^{FB}), A^{FB}) > m^{FB}(n^{FB}(A^*), A^*)$$

as $A^{FB} > A^*$ by Proposition 3.1. In addition, $m^{FB}(n^{FB}(A^*), A^*) > m^{FB}(n^*(A^*), A^*)$ because $\left.\frac{\partial m^{FB}}{\partial n}\right|_{n^*} = s(n, A)|_{n^*} > 0$ and $n^{FB}(A) > n^*(A)$ for all A by Lemma 3.4. By comparing (3.3) to (3.13), $m^{FB}(n^*(A^*), A^*) > m^*(n^*(A^*), A^*)$. Hence, $m^{FB}(n^{FB}, A^{FB}) > m^*(n^*, A^*)$.

Second, I show that $m^*(n^*, A^*) > m^{**}(n^{**}, A^{**})$.

By definition $A^* = \operatorname{argmax}_A \left\{\frac{1}{\lambda}\left(m^*\right)^2 - c(A)\right\}\Big|_{(3.13),\,(3.14)}$ and thus

$$\left.\frac{1}{\lambda}\left(m^*\right)^2 - c(A)\right|_{A^*, n^*(A^*)} > \left.\frac{1}{\lambda}\left(m^*\right)^2 - c(A)\right|_{A^{**}, n^*(A^{**})}$$

Since $A^* > A^{**}$ by Proposition 3.1 and therefore $c(A^*) > c(A^{**})$, it follows that $\frac{1}{\lambda}(m^*)^2\big|_{A^*,n^*(A^*)} > \frac{1}{\lambda}(m^*)^2\big|_{A^{**},n^*(A^{**})} + c(A^*) - c(A^{**}) > \frac{1}{\lambda}(m^*)^2\big|_{A^{**},n^*(A^{**})}$. Thus, $m^*(n^*(A^*), A^*) > m^*(n^*(A^{**}), A^{**})$.

In addition, $m^*(n^*(A^{**}), A^{**}) > m^*(n^{**}(A^{**}), A^{**})$ because $\frac{\partial m^*}{\partial n}\big|_{n^{**}} = \frac{\lambda}{2}s(n, A)\big|_{n^{**}} > 0$ and $n^*(A) > n^{**}(A)$ for all A by Lemma 3.4. By comparing (3.13) to (3.9), it is immediate that $m^*(n^{**}(A^{**}), A^{**}) > m^{**}(n^{**}(A^{**}), A^{**})$. Hence, $m^*(n^*, A^*) > m^{**}(n^{**}, A^{**})$.

Duopoly. First, to show that $m_k^{FB}(n_k^{FB}, A_k^{FB}) > m_k^*(n_k^*, A_k^*)$ is analogous to the monopoly case. The last inequality, $m_k^{FB}(n_k^*(A_k^*), A_k^*) > m_k^*(n_k^*(A_k^*), A_k^*)$, follows from the fact that $2 > \frac{2(2\tau\lambda+1)}{2(2\tau\lambda+1)-1}$. Second, the steps to show that $m_k^*(n_k^*, A_k^*) > m_k^{**}(n_k^{**}, A_k^{**})$ are analogous to the monopoly case. □

Proof Lemma 3.9.

By comparing (3.3) and (3.17), it follows immediately that $m^{FB}(n^{FB}, A^{FB}) = m_k^{FB}(n_k^{FB}, A_k^{FB})$. Here I show that $m_k^*(n_k^*, A_k^*) > m^*(n^*, A^*)$; the steps to show that $m_k^{**}(n_k^{**}, A_k^{**}) > m^{**}(n^{**}, A^{**})$ are identical and omitted here. By comparing the two first order conditions (3.15) and (3.24) from stage one, $m_k^*(n_k^*, A_k^*) > m^*(n^*, A^*)$ if and only if

$$\left.\frac{c'(A_k)}{\left(\frac{4(2\tau\lambda+1)^2-2}{4(2\tau\lambda+1)^2-1}\right)v_A(n_k, A_k)}\right|_{n_k^*(A_k^*),A_k^*} > \left.\frac{c'(A)}{v_A(n, A)}\right|_{n^*(A^*),A^*} = \left.\frac{c'(A)}{v_A(n, A)}\right|_{n_k^*(A^*),A^*}$$

The equality follows from Lemma 3.7. One can easily verify that $\left(\frac{4(2\tau\lambda+1)^2-2}{4(2\tau\lambda+1)^2-1}\right) < 1$. By Proposition 3.2, $A_k^* > A^*$ and thus $c'(A_k^*) > c'(A^*)$. Lastly, $v_A(n_k^*(A_k^*), A_k^*) < v_A(n_k^*(A^*), A^*)$ because $v_{AA} < 0$ by assumption. Therefore, the inequality statement is true and consequently $m_k^*(n_k^*, A_k^*) > m^*(n^*, A^*)$. □

Proof Corollary 3.4.

By definition, $A^* = \text{argmax}_A \left\{\frac{1}{\lambda}(m^*)^2 - c(A)\right\}\big|_{(3.13),\ (3.14)}$ and thus

$$\left.\frac{1}{\lambda}(m^*)^2 - c(A)\right|_{A^*,n^*(A^*)} > \left.\frac{1}{\lambda}(m^*)^2 - c(A)\right|_{A^{**},n^*(A^{**})}$$

Next,

$$\left.\frac{1}{\lambda}(m^*)^2 - c(A)\right|_{A^{**},n^*(A^{**})} > \left.\frac{1}{\lambda}(m^{**})^2 - c(A)\right|_{A^{**},n^{**}(A^{**})}$$
$$\iff m^*(n^*(A^{**}), A^{**}) > m^{**}(n^{**}(A^{**}), A^{**})$$

First, $m^*(n^*(A^{**}), A^{**}) > m^*(n^{**}(A^{**}), A^{**})$, because $\frac{\partial m^*}{\partial n}\Big|_{n^{**}} = \frac{\lambda}{2} s(n,A)\Big|_{n^{**}} > 0$ and $n^*(A) > n^{**}(A)$ for all A by Lemma 3.4. Second, by comparing (3.13) to (3.9) it is immediate that $m^*(n^{**}(A^{**}), A^{**}) > m^{**}(n^{**}(A^{**}), A^{**})$. Hence, $m^*(n^*(A^{**}), A^{**}) > m^{**}(n^{**}(A^{**}), A^{**})$ and, therefore,

$$\Pi^*(n^*, A^*) = \max_A \frac{1}{\lambda}(m^*)^2 - c(A)\Big|_{(3.13),\ (3.14)}$$
$$> \Pi^{**}(n^{**}, A^{**}) = \max_A \left\{ \frac{1}{\lambda}(m^{**})^2 - c(A) \right\}\Big|_{(3.9),\ (3.10)}$$

□

Proof Corollary 3.5.

By definition, $A^* = \text{argmax}_A \left\{ \frac{1}{\lambda}(m^*)^2 - c(A) \right\}\big|_{(3.13),\ (3.14)}$ and thus

$$\Pi^*(n^*, A^*) = \frac{1}{\lambda}(m^*)^2 - c(A_k)\Big|_{A^*, n^*(A^*)} > \frac{1}{\lambda}(m^*)^2 - c(A_k)\Big|_{A_k^*, n^*(A_k^*)}$$

From (3.13) and (3.23), the latter evaluated at $A_k = A_{-k}$, $m^* = \frac{2(2\tau\lambda+1)-1}{2(2\tau\lambda+1)} m_k^*$ for a given level of technology. Then,

$$\frac{1}{\lambda}(m^*)^2 - c(A_k)\Big|_{A_k^*, n_k^*(A_k^*)} = \frac{1}{\lambda}\left(\frac{2(2\tau\lambda+1)-1}{2(2\tau\lambda+1)}\right)^2 (m_k^*)^2 - c(A_k)\Big|_{A_k^*, n_k^*(A_k^*)}$$
$$> \frac{1}{\frac{1}{2\tau}+\lambda}(m_k^*)^2 - c(A_k)\Big|_{A_k^*, n_k^*(A_k^*)} = \Pi_k^*(n_k^*, A_k^*)$$

because $\frac{1}{\lambda}\left(\frac{2(2\tau\lambda+1)-1}{2(2\tau\lambda+1)}\right)^2 > \frac{1}{\frac{1}{2\tau}+\lambda}$ for all λ, τ. Thus, $\Pi^*(n^*, A^*) > \Pi_k^*(n_k^*, A_k^*)$. The steps to show that $\Pi^{**}(n^{**}, A^{**}) > \Pi_k^{**}(n_k^{**}, A_k^{**})$ are identical and omitted here. □

Proof Corollary 3.6.

Total welfare is expressed by

$$W = m\left[v(n,A) - \int_0^n c(j)dj\right] - \tau\int_0^{\frac{1}{2}} x dx - h\tau \int_0^{m_h} x dx - c(A)$$

evaluated at the equilibrium (m, n, A) for each of the three organization modes. By definition $(m^{FB}, n^{FB}, A^{FB}) = \text{argmax}_{m,n,A} W$. Since (m^{FB}, n^{FB}, A^{FB}) is unique and $m^* \neq m^{FB}, n^* \neq n^{FB}, A^* \neq A^{FB}$ from the first order conditions, it immediately follows that $W^{FB}(n^{FB}, A^{FB}) > W^*(n^*, A^*)$ by the revealed preference of the social planner, who maximizes W.

Total welfare can be written as $W^*(n^*, A^*) = U^*(n^*, A^*) + \Pi^*(n^*, A^*)$ in the MPF case and as $W^{**}(n^{**}, A^{**}) = U^{**}(n^{**}, A^{**}) + \Pi^{**}(n^{**}, A^{**}) + m^{**}\left(n^{**}c(n^{**}) - \int_0^{n^{**}} c(j)dj\right)$ in the two-sided platform case. Since $U^*(n^*, A^*) > U^{**}(n^{**}, A^{**})$ by Corollary 3.2, a sufficient but not necessary condition for $W^*(n^*, A^*) > W^{**}(n^{**}, A^{**})$ to hold is that $\Pi^*(n^*, A^*) > \Pi^{**}(n^{**}, A^{**}) + m^{**}\left(n^{**}c(n^{**}) - \int_0^{n^{**}} c(j)dj\right)$.

By definition, $A^* = \text{argmax}_A \, \Pi^*|_{(3.13),\,(3.14)} = \text{argmax}_A \left\{\frac{1}{\lambda}(m^*)^2 - c(A)\right\}\Big|_{(3.13),\,(3.14)}$ and thus $\frac{1}{\lambda}(m^*)^2 - c(A)\Big|_{A^*, n^*(A^*)} > \frac{1}{\lambda}(m^*)^2 - c(A)\Big|_{A^{**}, n^*(A^{**})}$. One can show doing some algebraic manipulation that

$$
\begin{aligned}
&\frac{1}{\lambda}(m^*)^2 - c(A)\Big|_{A^{**}, n^*(A^{**})} \\
=&\frac{1}{\lambda}(m^{**})^2 + \left[nc(n) - \int_0^n c(j)dj\right]\left[m^{**} + nc(n) - \int_0^n c(j)dj\right] - c(A)\Big|_{A^{**}, n^*(A^{**})} \\
>&\frac{1}{\lambda}(m^{**})^2 + \left[nc(n) - \int_0^n c(j)dj\right]m^{**} - c(A)\Big|_{A^{**}, n^*(A^{**})} \\
>&\frac{1}{\lambda}(m^{**})^2 + \left[nc(n) - \int_0^n c(j)dj\right]m^{**} - c(A)\Big|_{A^{**}, n^{**}(A^{**})} \\
=&\Pi^{**}(n^{**}, A^{**}) + m^{**}\left(n^{**}c(n^{**}) - \int_0^{n^{**}} c(j)dj\right).
\end{aligned}
$$

The first inequality holds because $nc(n) - \int_0^n c(j)dj > 0$ by construction and the second inequality holds because 1) $\frac{\partial m^{**}}{\partial n}\Big|_{n^{**}} = \frac{\lambda}{2}(s(n, A) - nc_n(n))\Big|_{n^{**}} > 0$, 2) $\frac{\partial}{\partial n}\left(nc(n) - \int_0^n c(j)dj\right) = nc_n(n) > 0$ by assumption and 3) $n^*(A) > n^{**}(A)$ for all A by Lemma 3.4. Hence, $\Pi^*(n^*, A^*) > \Pi^{**}(n^{**}, A^{**}) + m^{**}\left(n^{**}c(n^{**}) - \int_0^{n^{**}} c(j)dj\right)$ and therefore $W^*(n^*, A^*) > W^{**}(n^{**}, A^{**})$. □